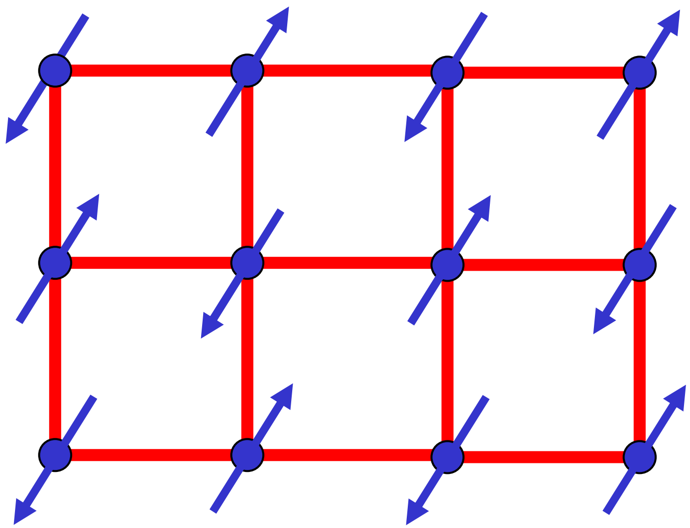
Breakdown of the Landau-Ginzburg-Wilson paradigm at quantum phase transitions

Science **303**, 1490 (2004); cond-mat/0312617
cond-mat/0401041

Leon Balents (UCSB)
Matthew Fisher (UCSB)
T. Senthil (MIT)
Ashvin Vishwanath (MIT)

Parent compound of the high temperature superconductors: La_2CuO_4

A Mott insulator



$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with angular momentum $S=1/2$

Ground state has long-range spin density wave (Néel) order at wavevector $\boldsymbol{K}=(\pi,\pi)$

spin density wave order parameter:

$$\vec{\varphi} = \eta_i \frac{\vec{S}_i}{S} \quad ; \quad \eta_i = \pm 1 \text{ on two sublattices}$$

$$\left\langle \vec{\varphi} \right\rangle \neq 0$$

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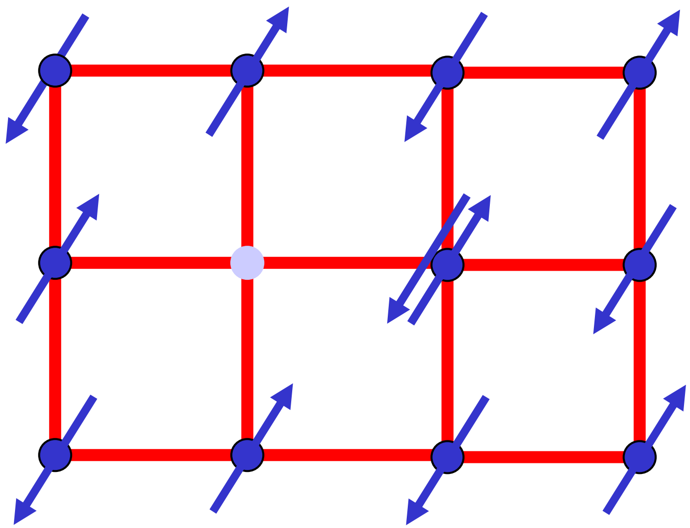

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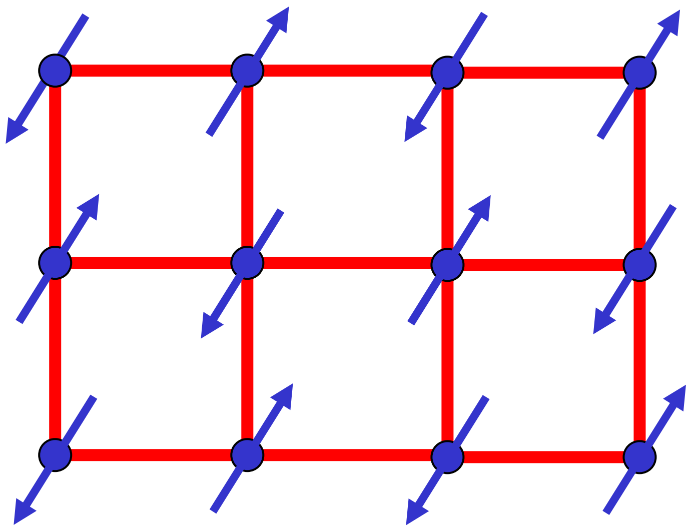

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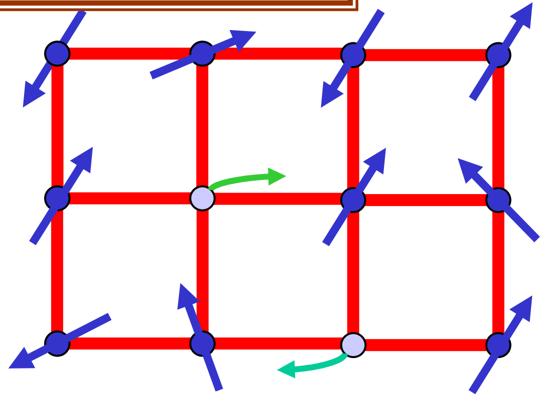
Superconductivity in a doped Mott insulator

Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$



Doped state is a paramagnet with $\langle \vec{\varphi} \rangle = 0$

and also a high temperature superconductor with

the BCS pairing order parameter $\langle \Psi_{BCS} \rangle \neq 0$.

$\Rightarrow$ With increasing δ, there must be one or more

quantum phase transitions involving

(i) onset of a non-zero $\langle \Psi_{BCS} \rangle$

(ii) restoration of spin rotation invariance by a transition

from $\langle \vec{\varphi} \rangle \neq 0$ to $\langle \vec{\varphi} \rangle = 0$

First study magnetic transition in Mott insulators…………

Outline

A. Magnetic quantum phase transitions in “dimerized” Mott insulators

Landau-Ginzburg-Wilson (LGW) theory

B. Mott insulators with spin S=1/2 per unit cell

Berry phases, bond order, and the breakdown of the LGW paradigm

C. Technical details

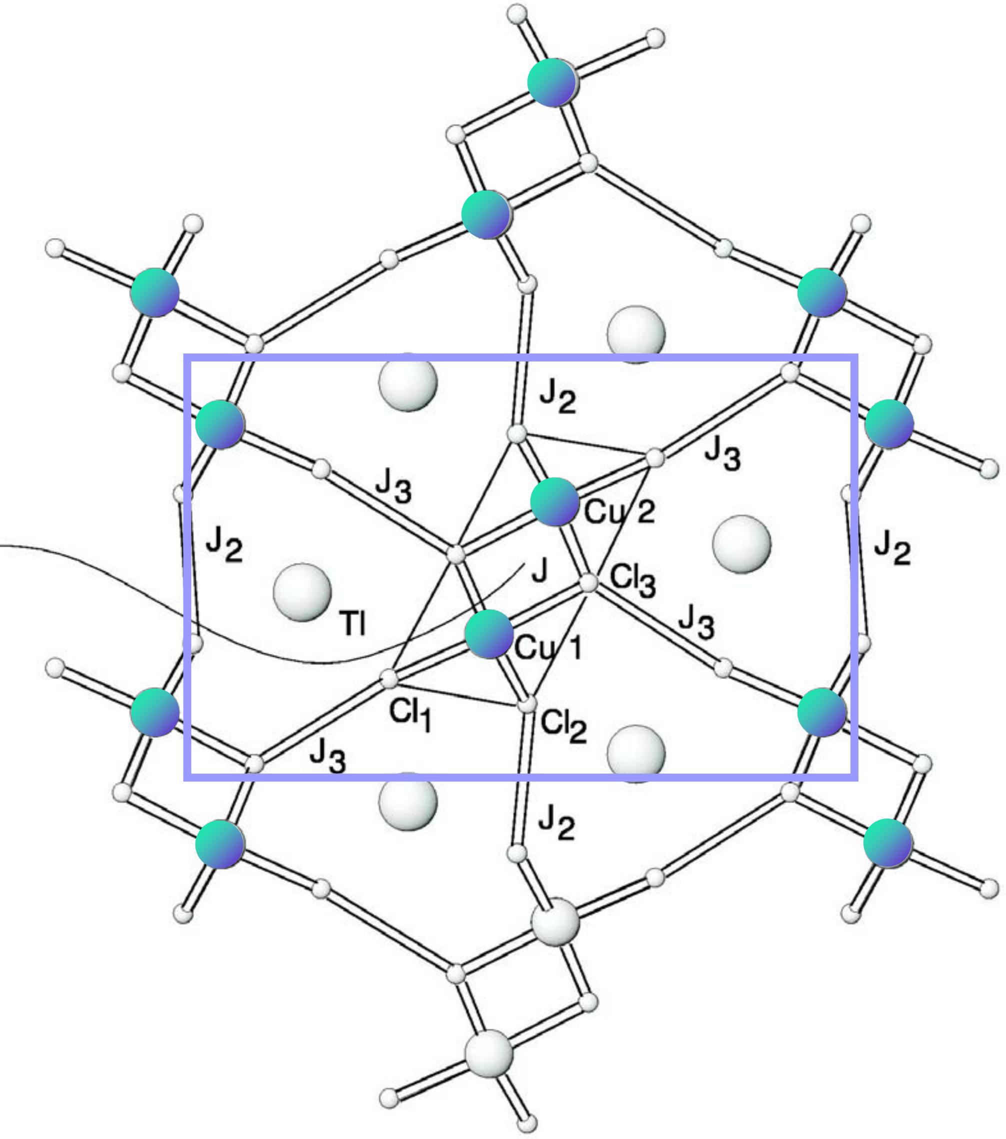
Duality and dangerously irrelevant operators

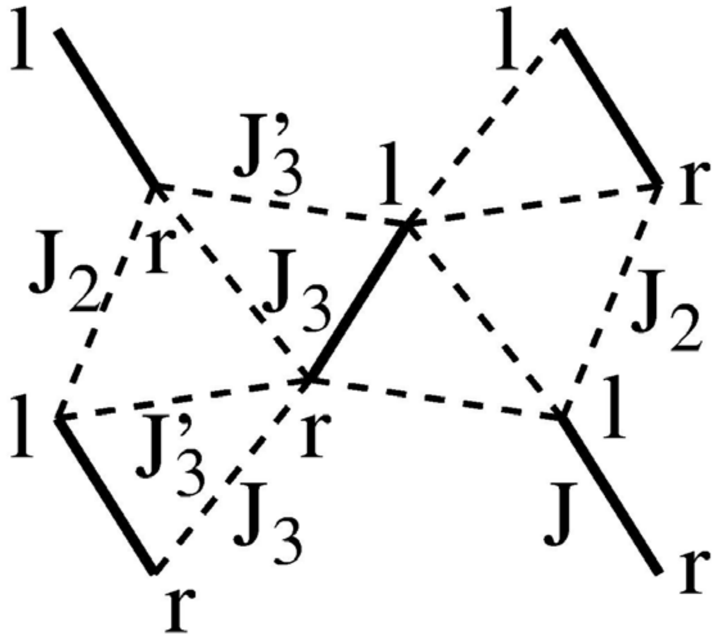
A. Magnetic quantum phase transitions in "dimerized" Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:

*Second-order phase transitions described by fluctuations of an **order parameter** associated with a **broken symmetry***

$TlCuCl_3$



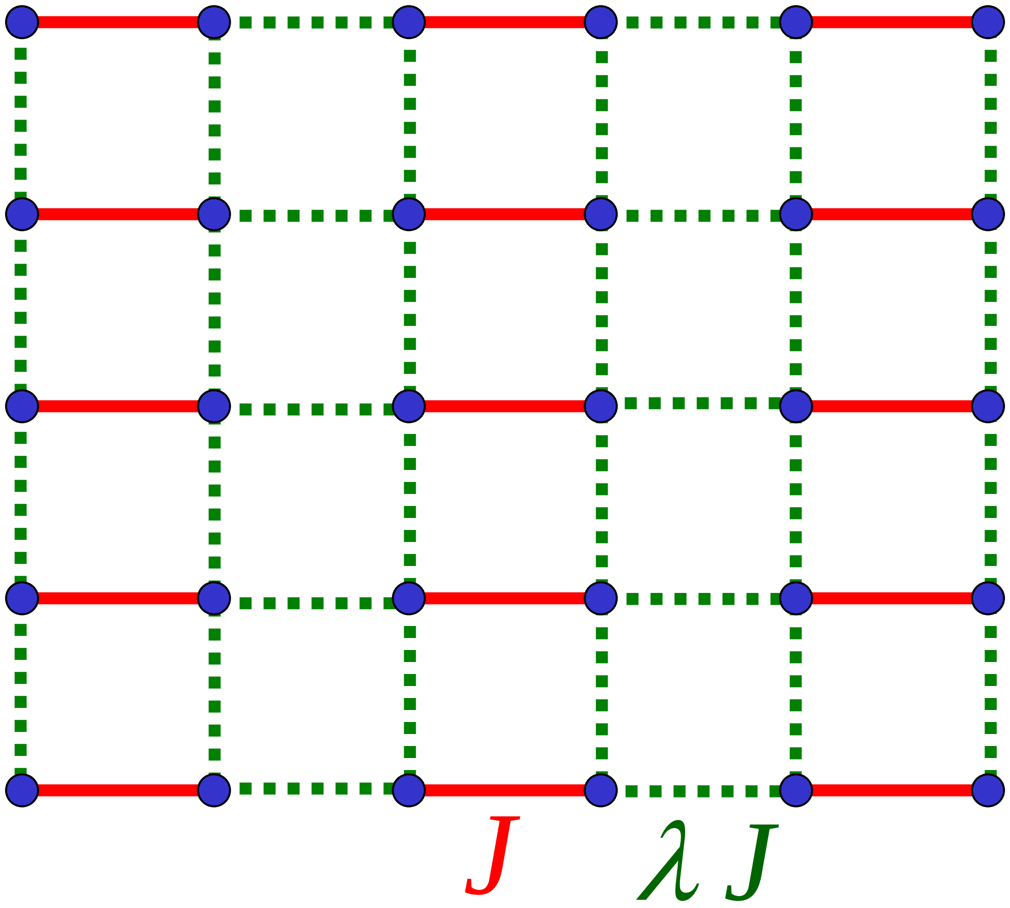


M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

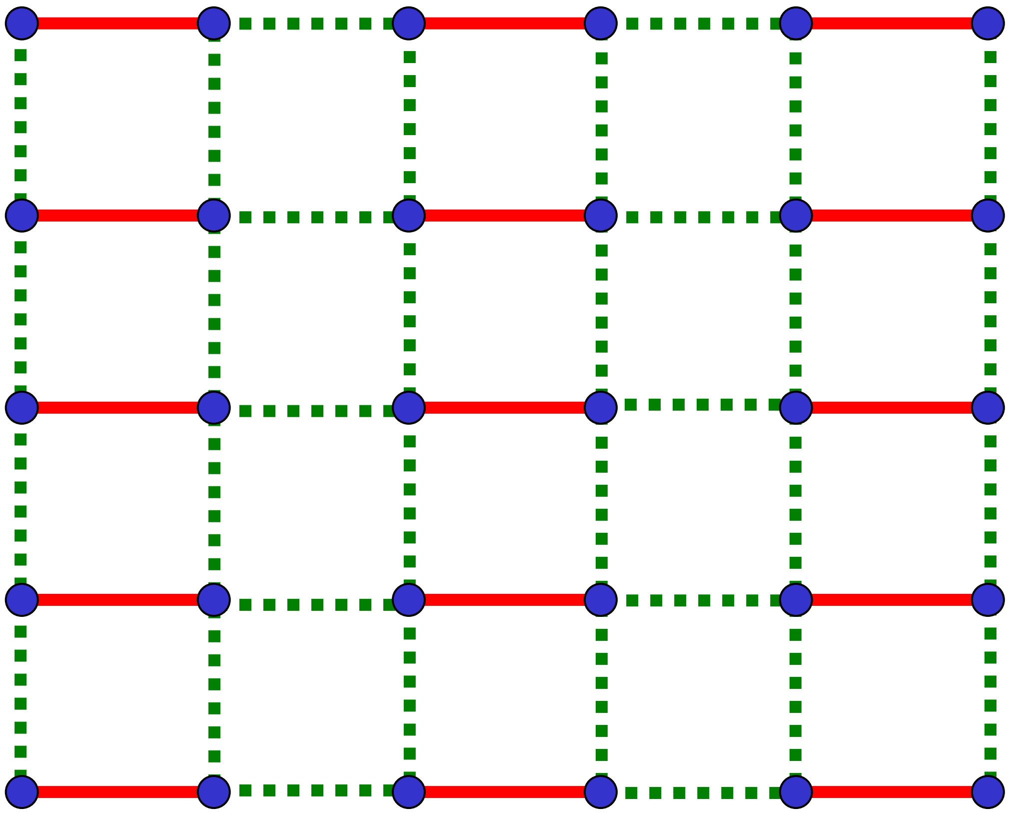
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



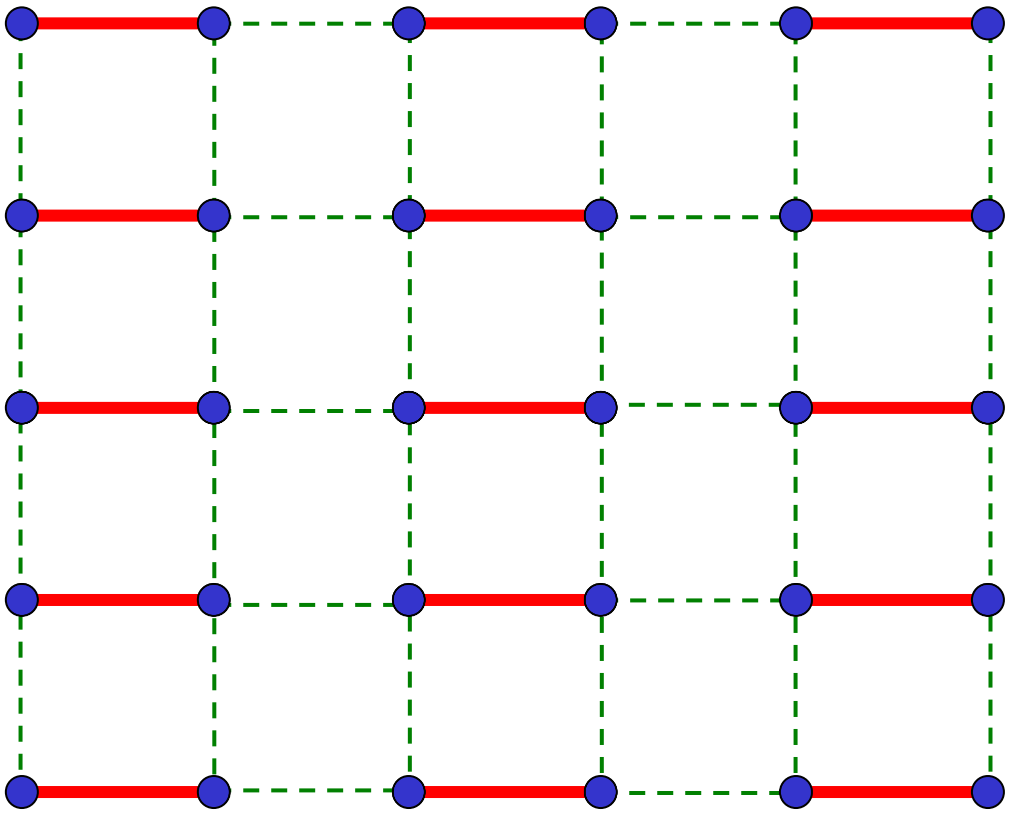
$$H = \sum_{<ij>} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$



λ close to 0

Weakly coupled dimers



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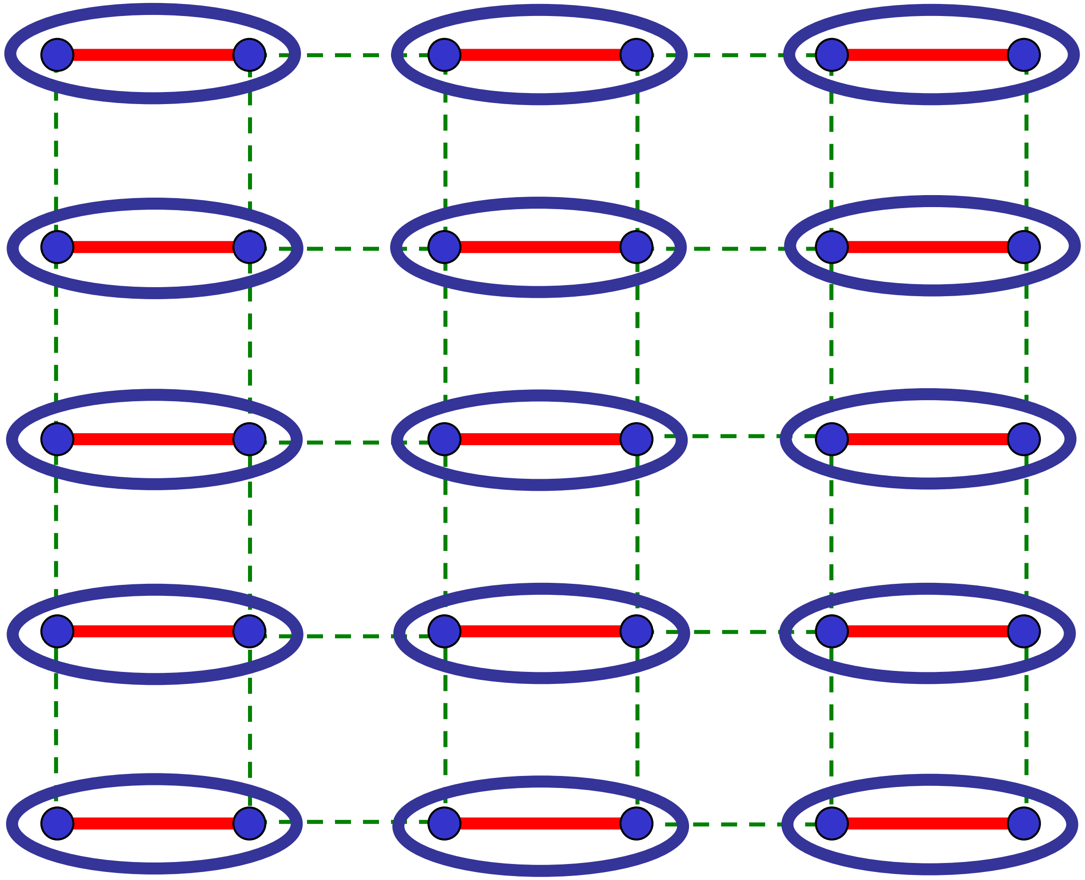
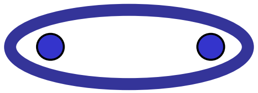




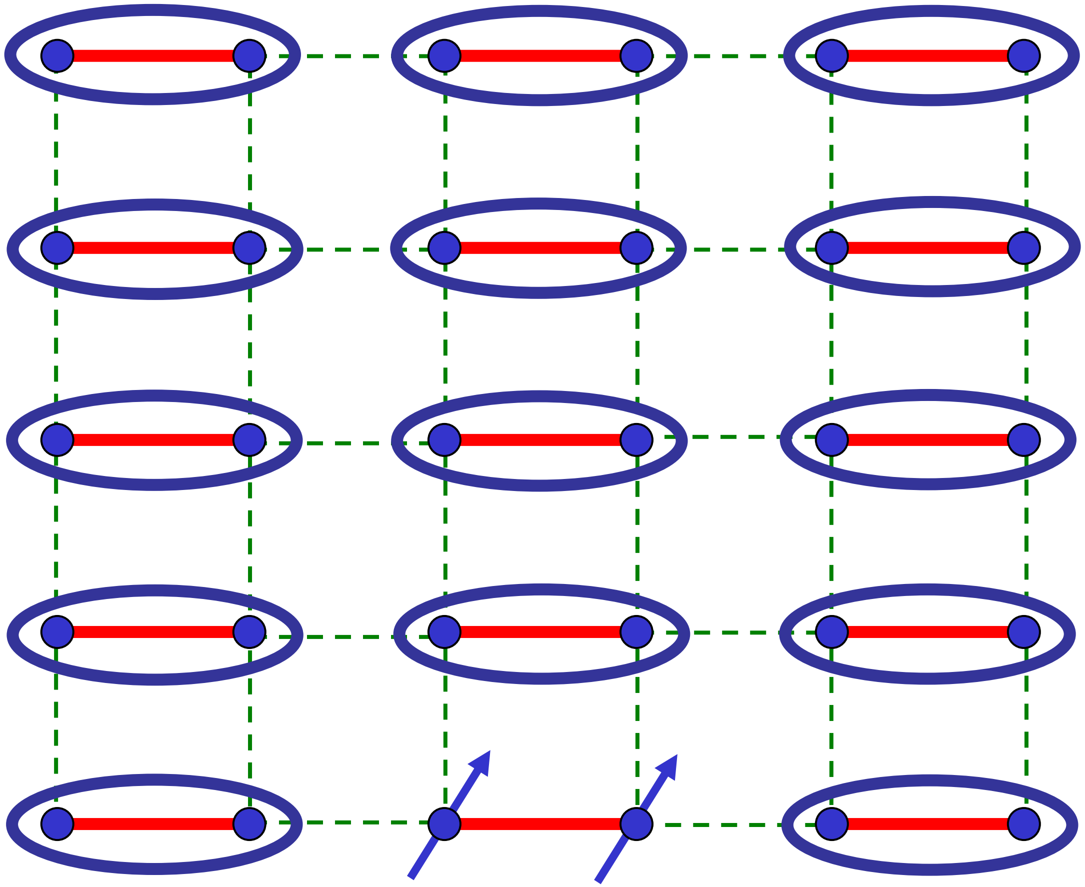
$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

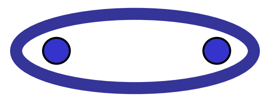
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0, \; \left\langle \vec{\varphi} \right\rangle = 0$$

λ close to 0

Weakly coupled dimers

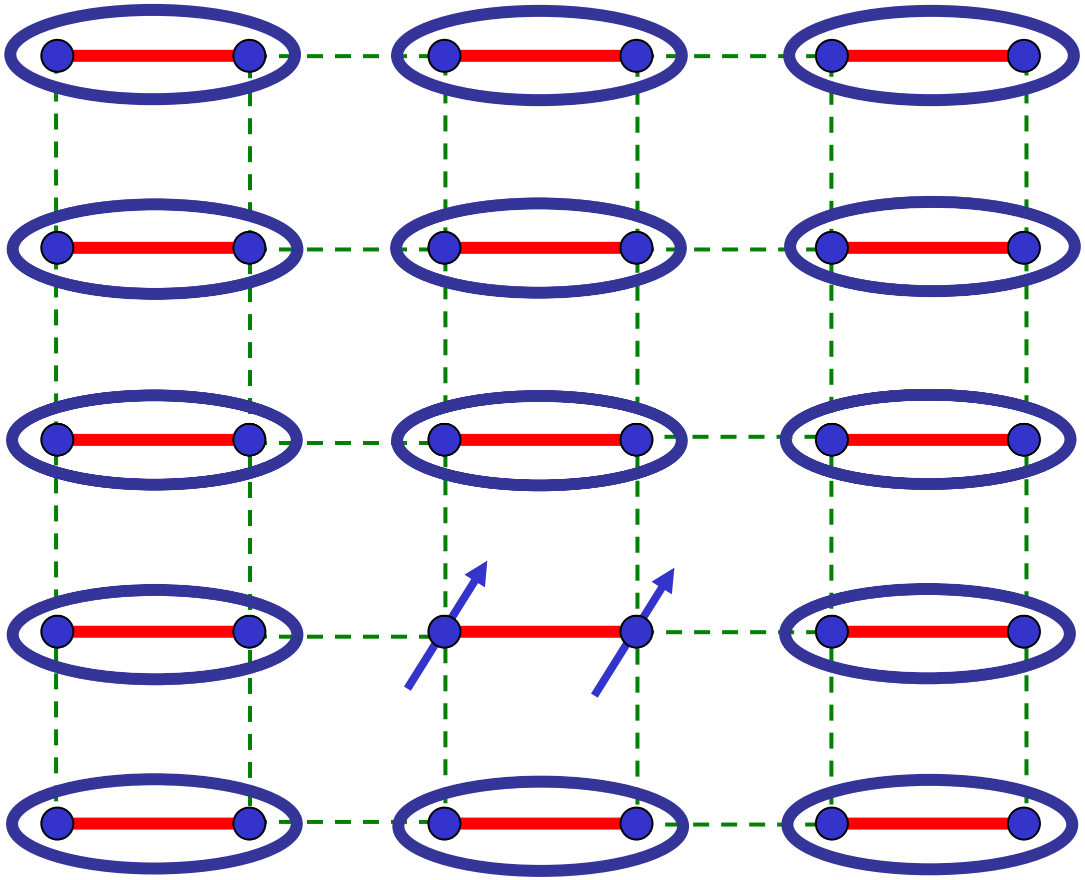


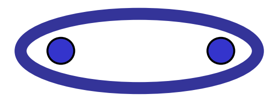
 $= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers





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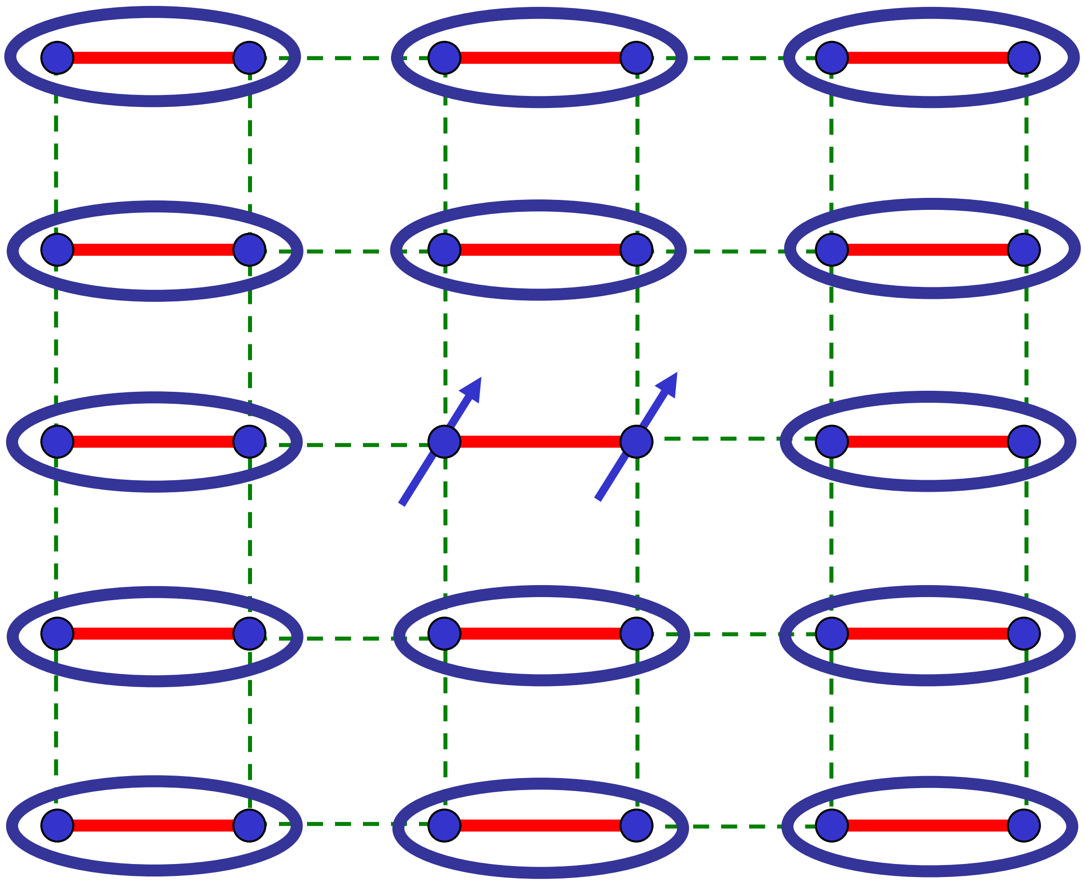

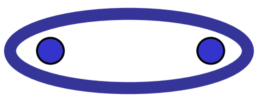

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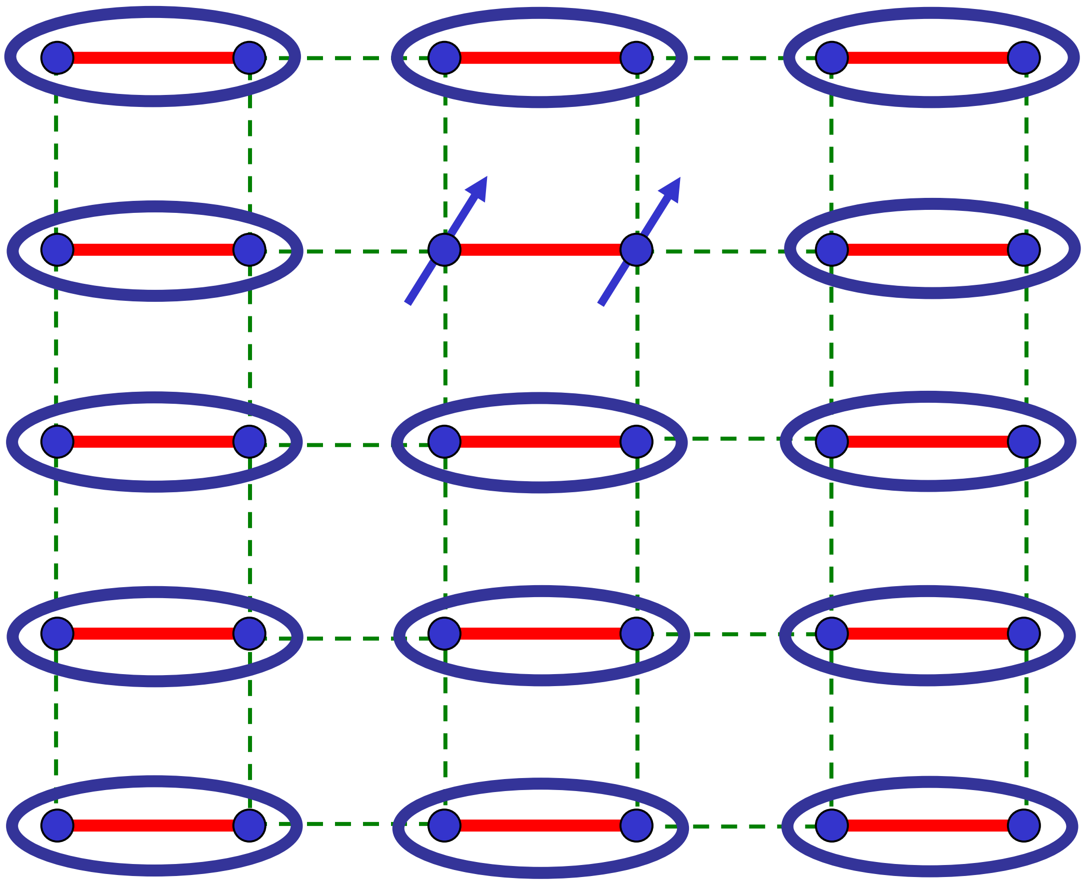

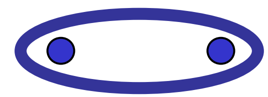

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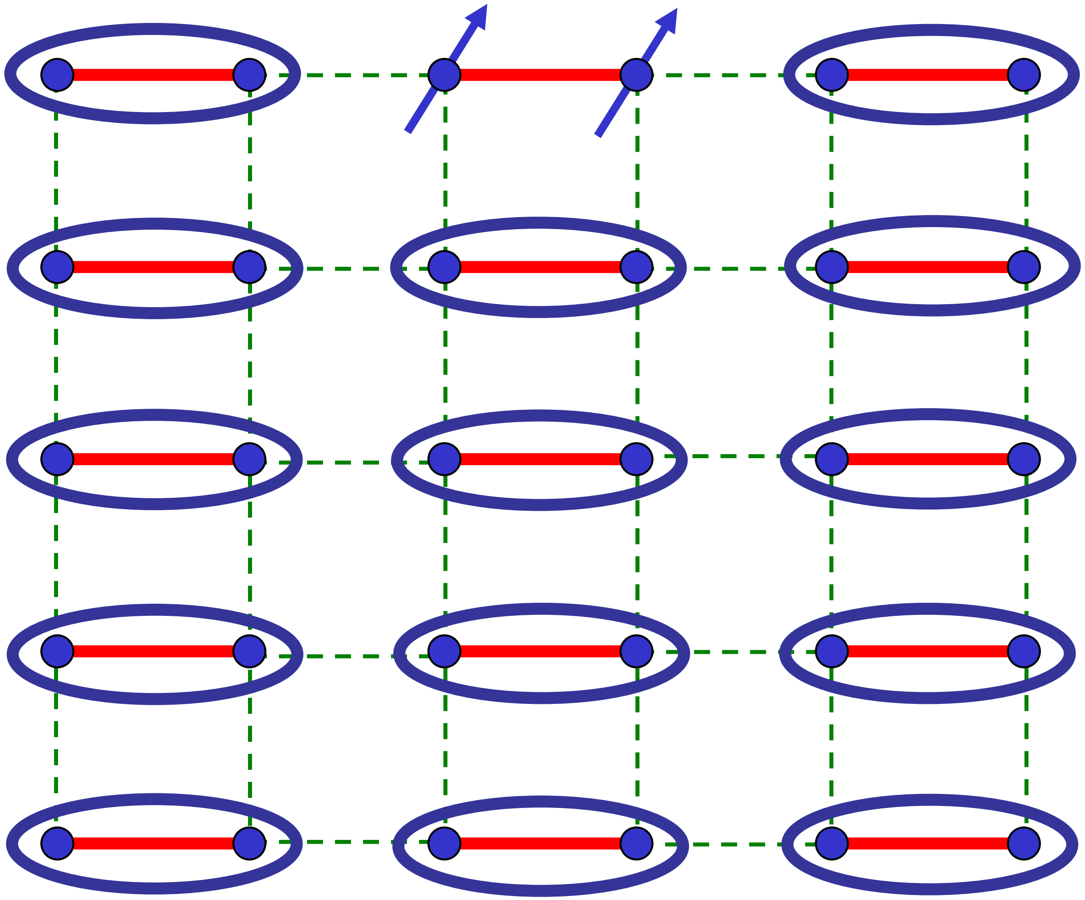

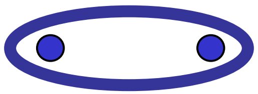 $= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$

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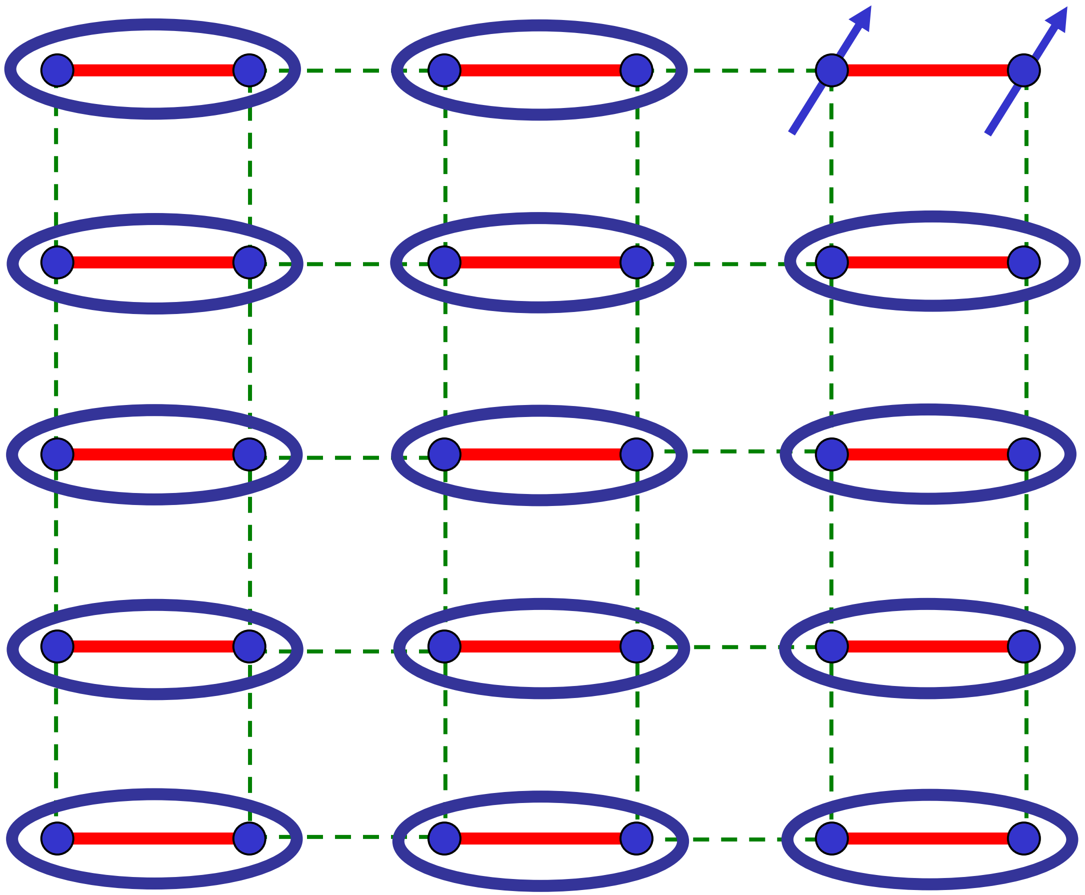
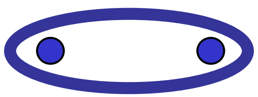
Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

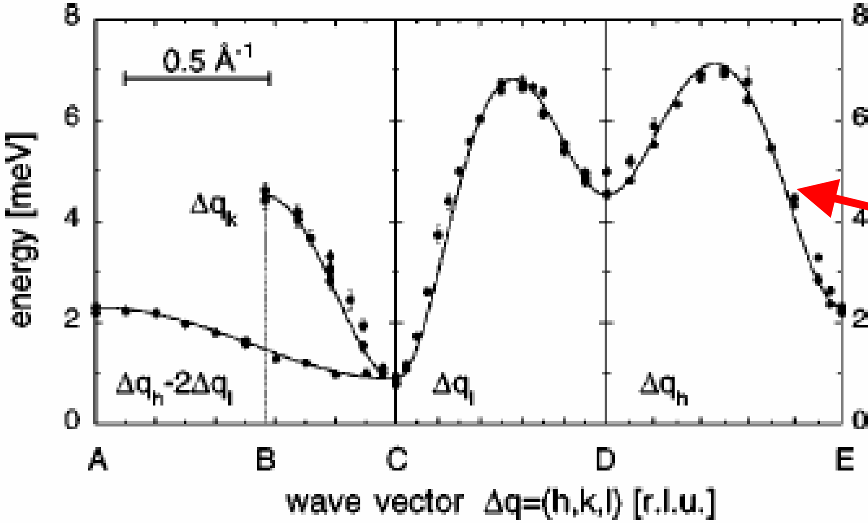
Excitation: S=1 ***triplon***
(***exciton***, spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

$TlCuCl_3$



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

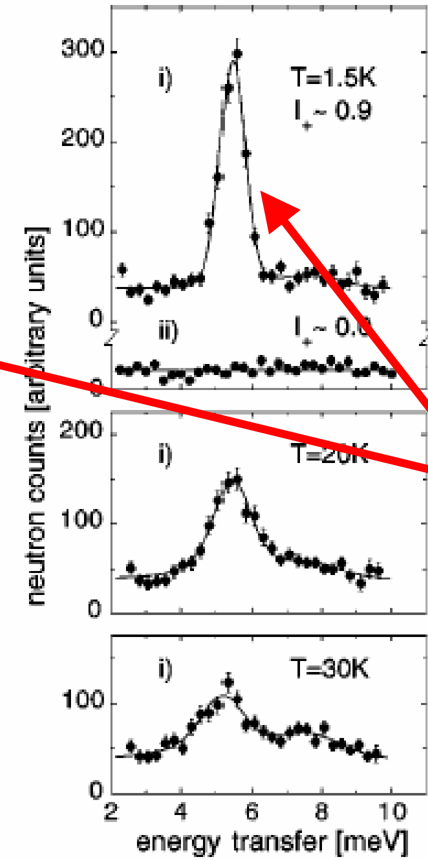


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

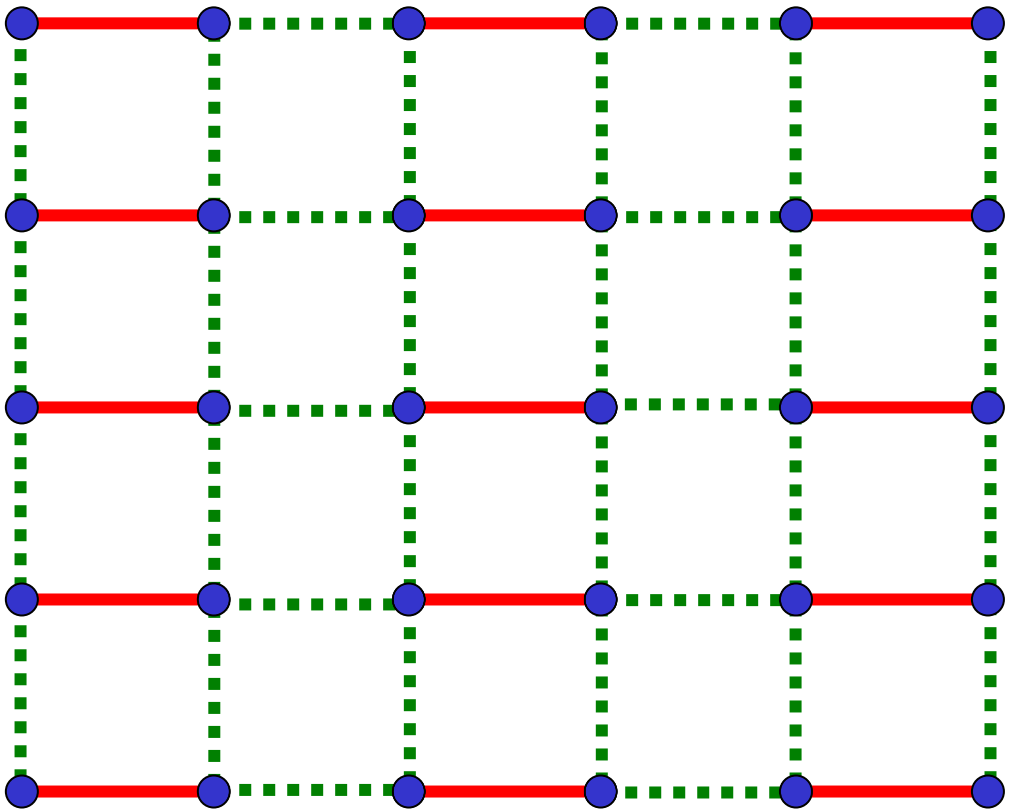
"triplon"

For quasi-one-dimensional systems, the triplon linewidth takes the exact universal value $=1.20 k_B T e^{-\Delta/k_B T}$ at low T

K. Damle and S. Sachdev, *Phys. Rev.* B **57**, 8307 (1998)

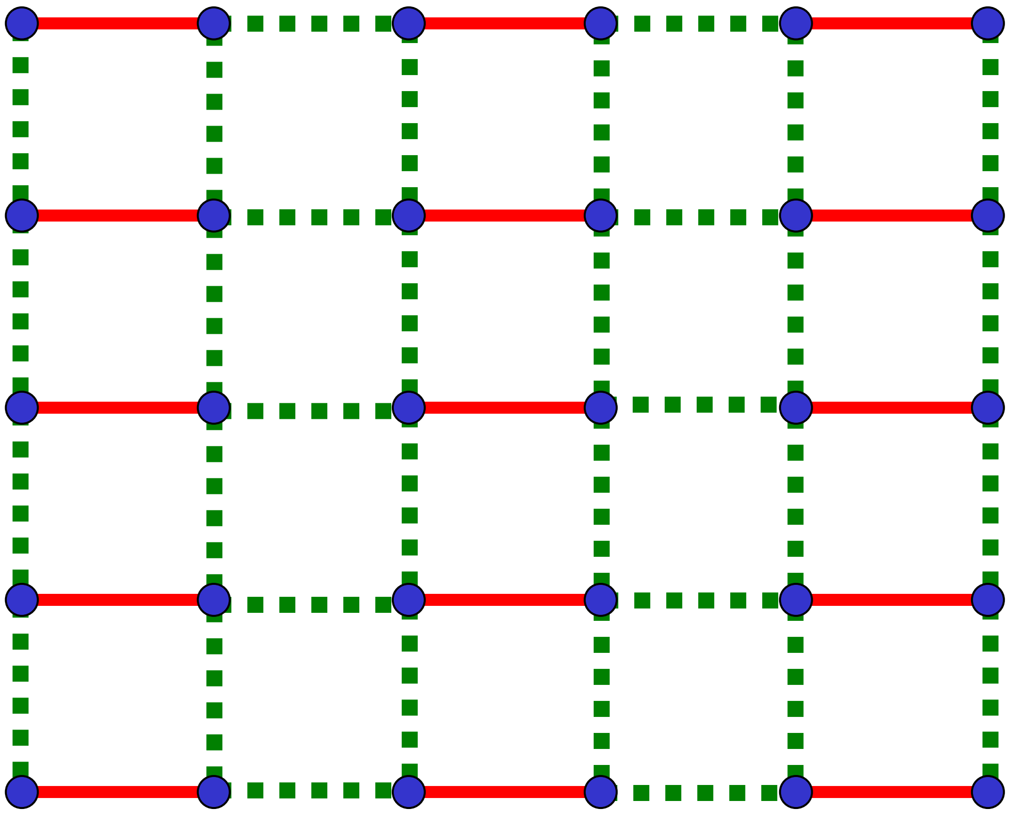
This result is in good agreement with observations in $CsNiCl_3$ (M. Kenzelmann, R. A. Cowley, W. J. L. Buyers, R. Coldea, M. Enderle, and D. F. McMorrow *Phys. Rev.* B **66**, 174412 (2002)) and Y_2NiBaO_5 (G. Xu, C. Broholm, G. Aeppli, J. F. DiTusa, T.Ito, K. Oka, and H. Takagi, preprint).

Coupled Dimer Antiferromagnet



λ close to 1

Weakly dimerized square lattice



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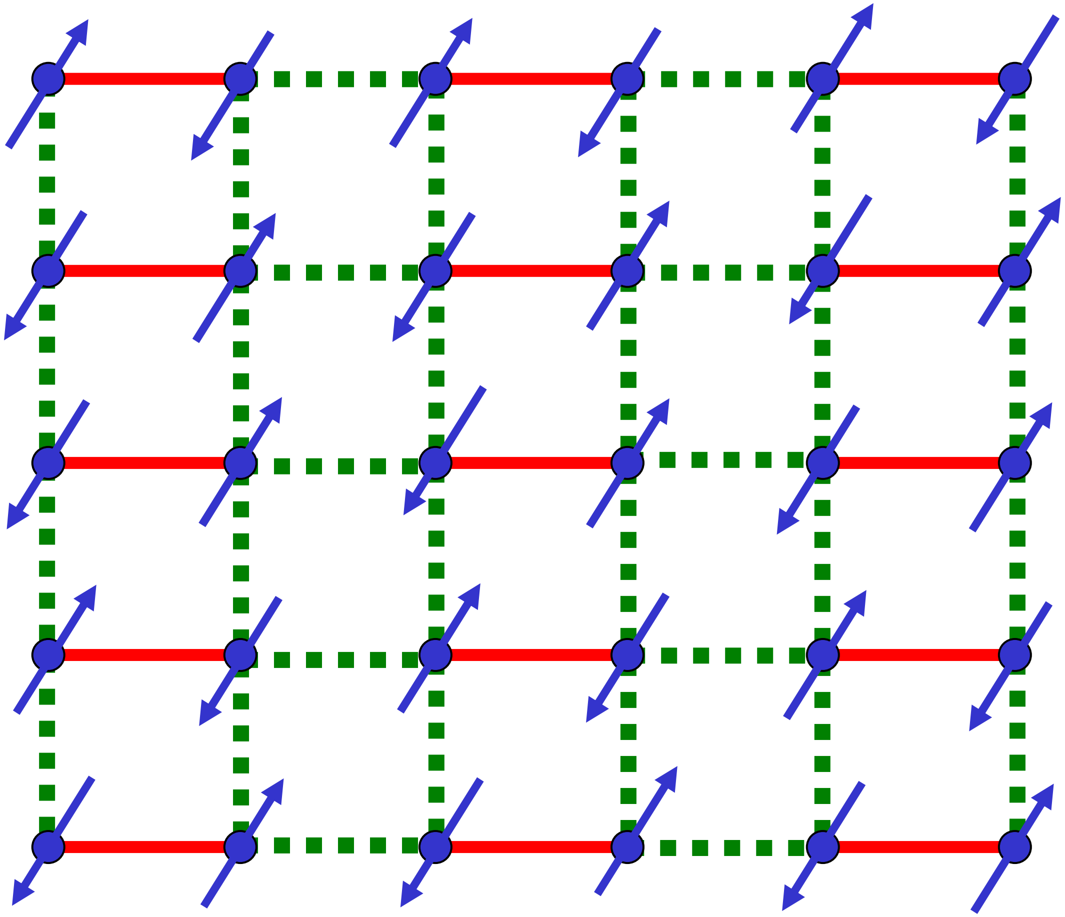

Excitations:
2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

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spin density wave order parameter: $\vec{\varphi} = \eta_i \frac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

$TlCuCl_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

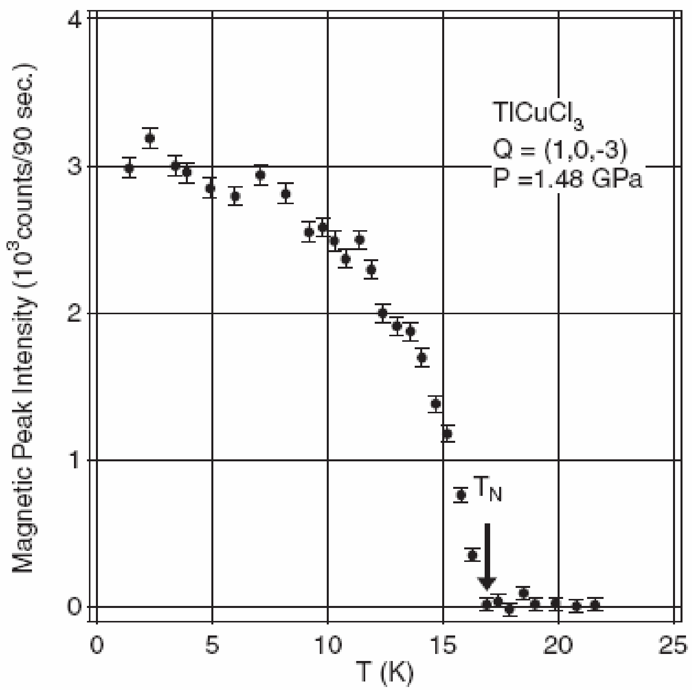


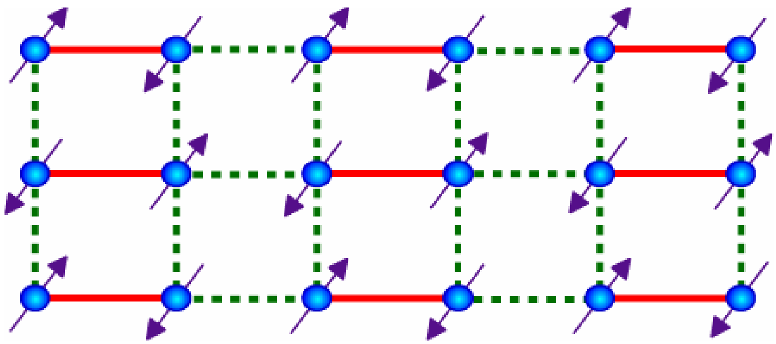
Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1,0,-3)$ reflection measured at $P = 1.48$ GPa in $TlCuCl_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

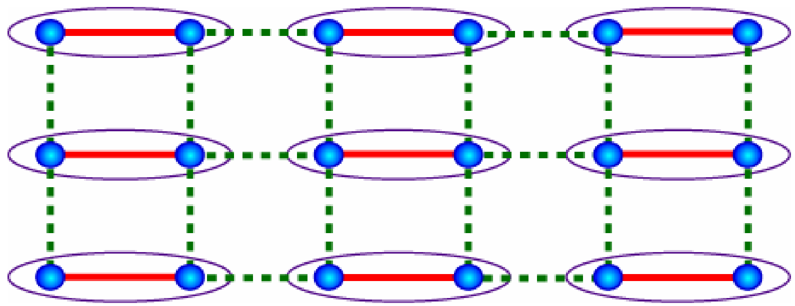
$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



Néel state

$\langle \vec{\varphi} \rangle \neq 0$



Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$

λ 1 λ_c

Pressure in $TlCuCl_3$

The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

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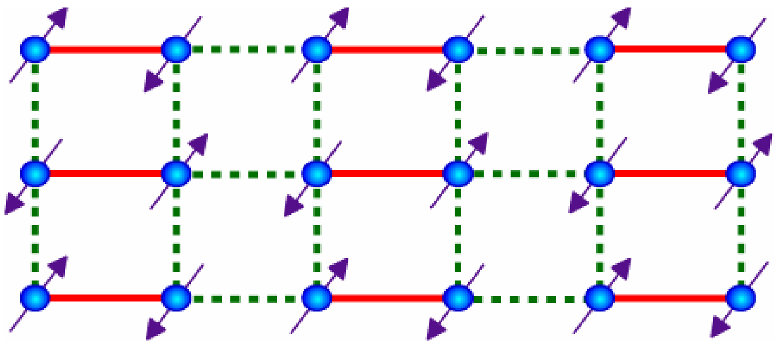

Néel state

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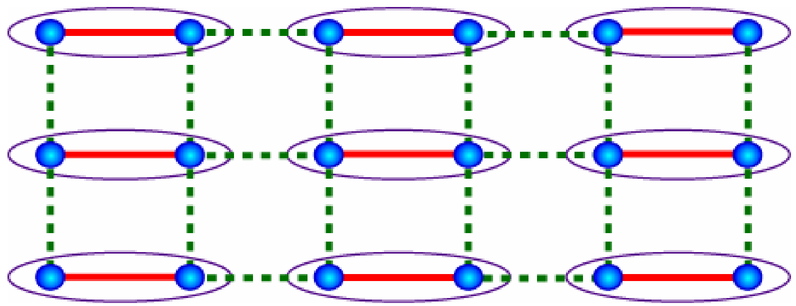


Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$

Magnetic order as in La_2CuO_4

Electrons in charge-localized Cooper pairs

λ 1 λ_c

δ in cuprates ?

The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

LGW theory for quantum criticality

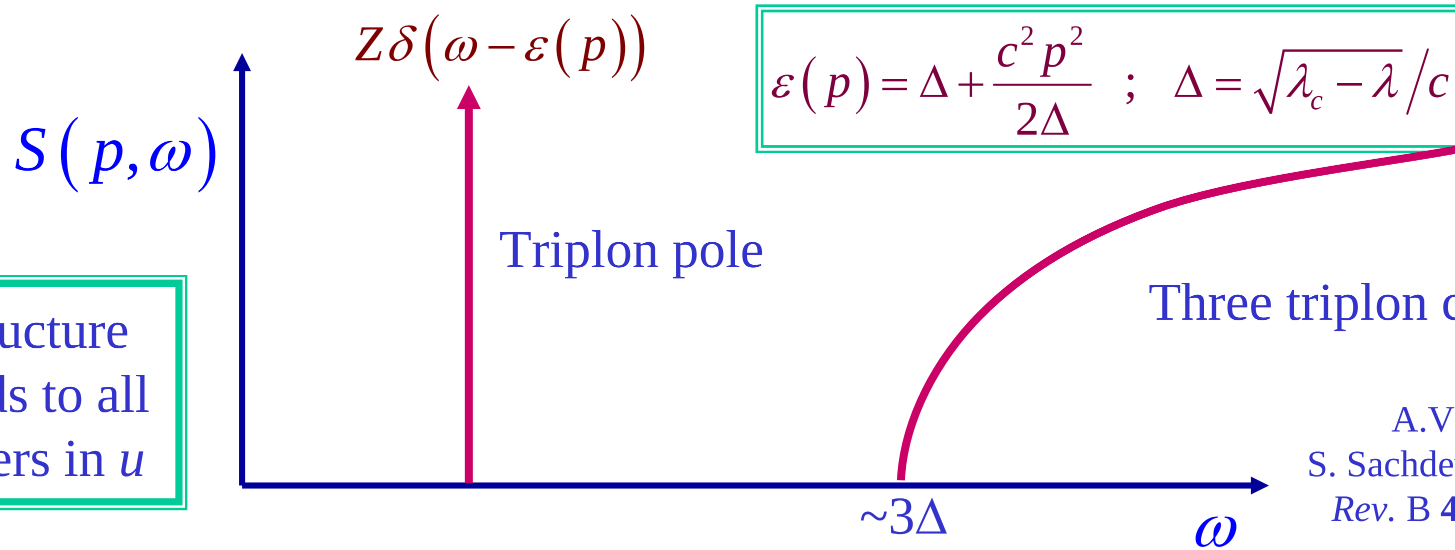
Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + \frac{1}{c^2}\left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right)\vec{\varphi}^{\,2} \right) + \frac{u}{4!}\left(\vec{\varphi}^{\,2}\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$ oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ lead to the following structure in the dynamic structure factor $S(p,\omega)$

$$\varepsilon(p) = \Delta + \frac{c^2 p^2}{2\Delta} \quad ; \quad \Delta = \sqrt{\lambda_c - \lambda}\,/c$$



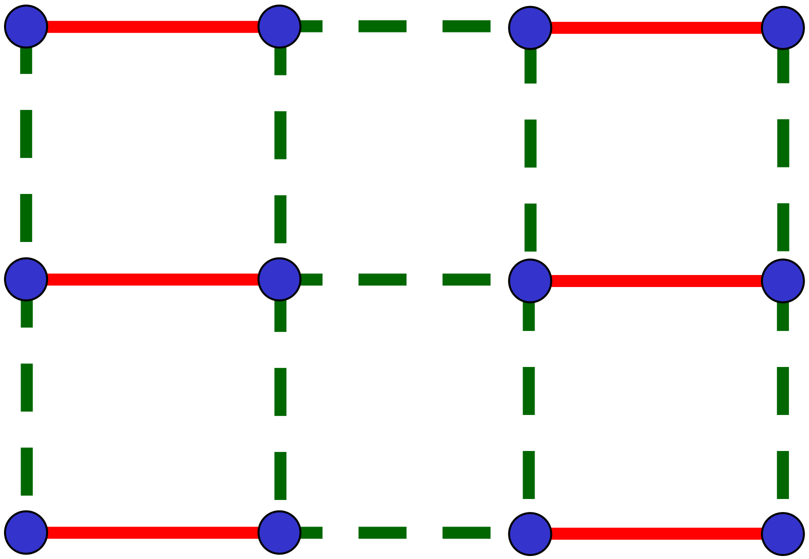
Structure holds to all orders in u

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

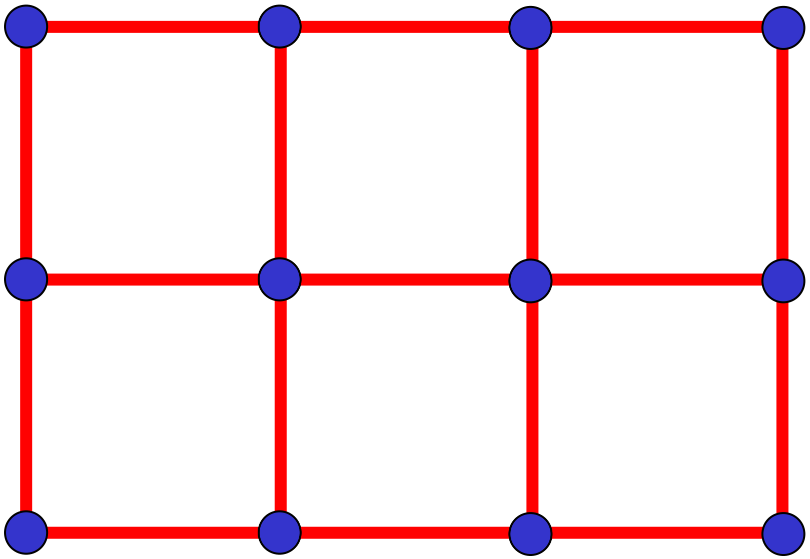
B. Mott insulators with spin S=1/2 per unit cell:

Berry phases, bond order, and the breakdown of the LGW paradigm

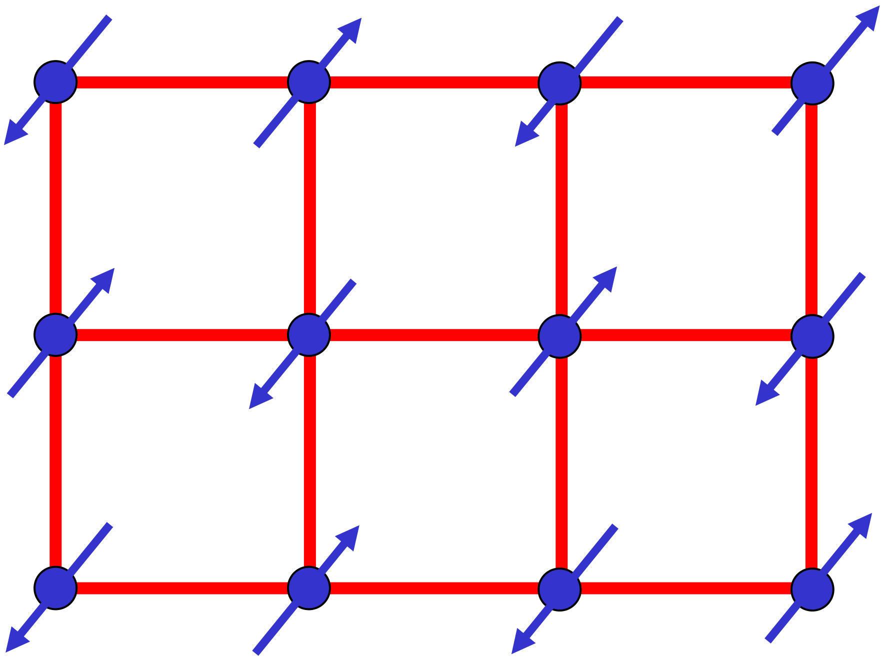
Mott insulator with two S=1/2 spins per unit cell



Mott insulator with one S=1/2 spin per unit cell

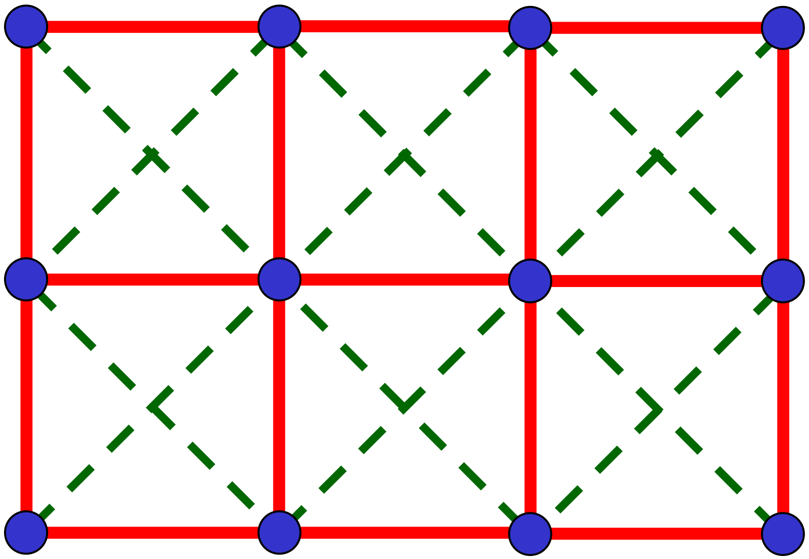


Mott insulator with one S=1/2 spin per unit cell



Ground state has Neel order with $\vec{\varphi} \neq 0$

Mott insulator with one S=1/2 spin per unit cell

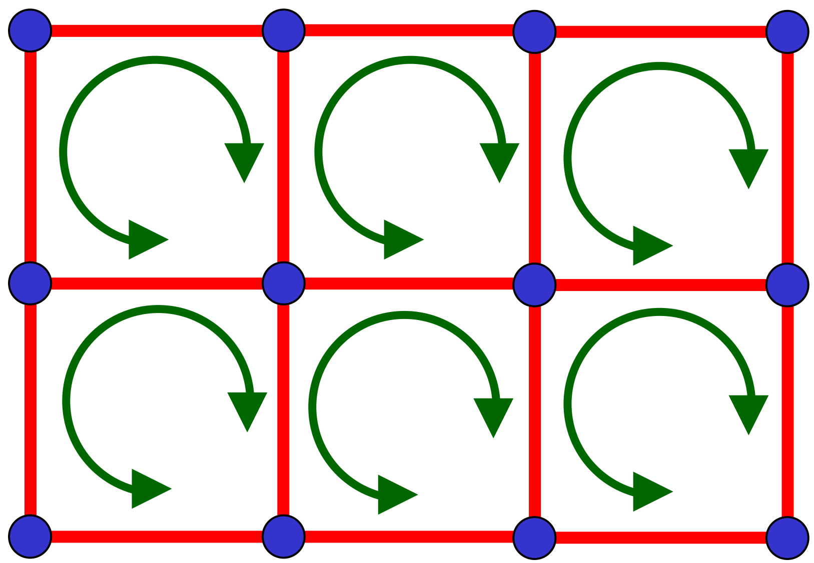


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange. The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell



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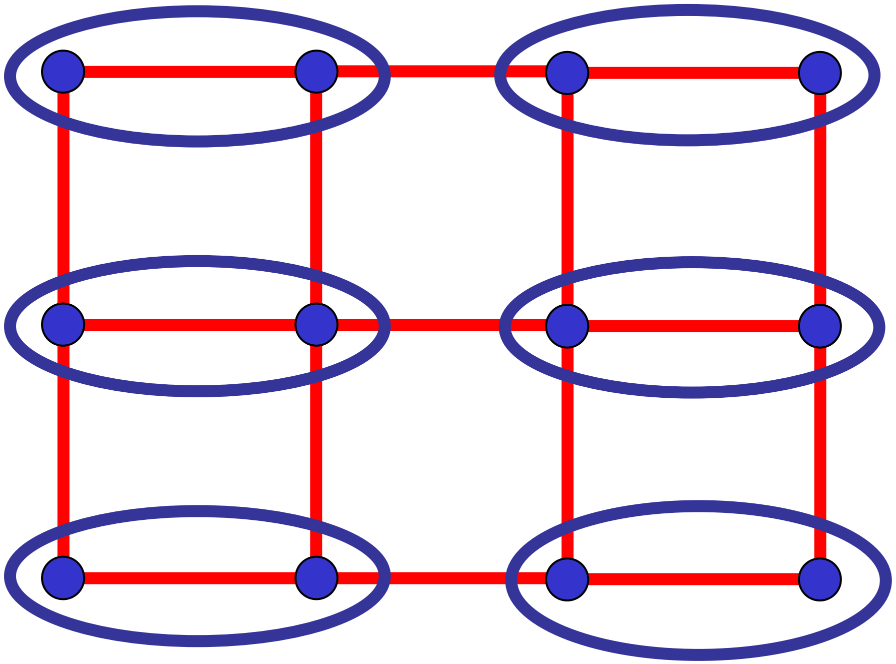
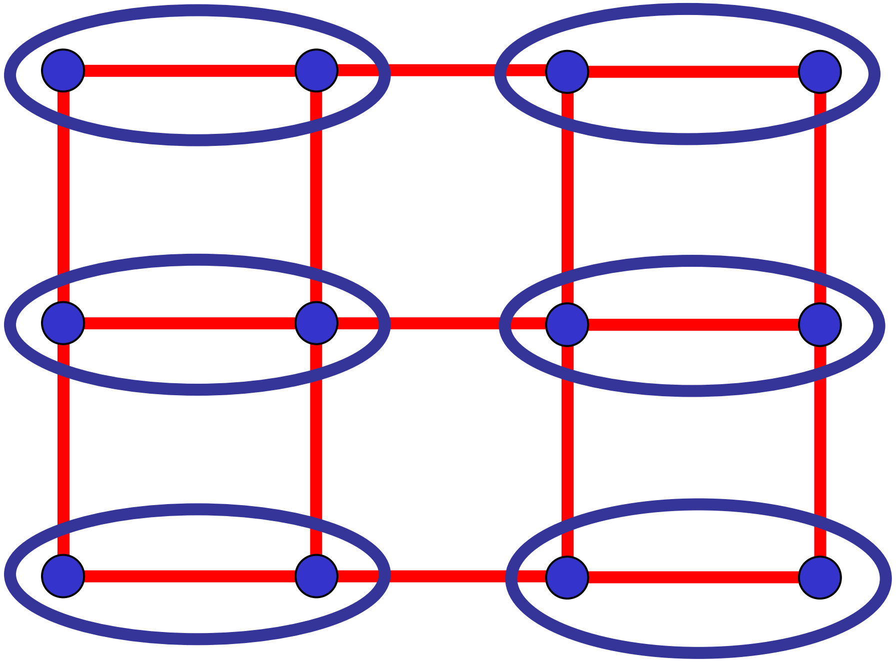
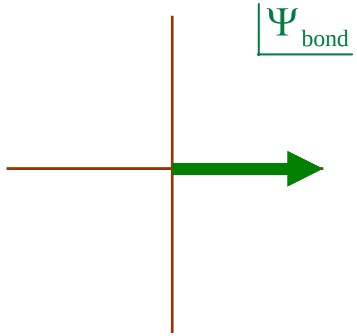


Possible large g paramagnetic ground state (Class A) with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell





Possible large g paramagnetic ground state (Class A) with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

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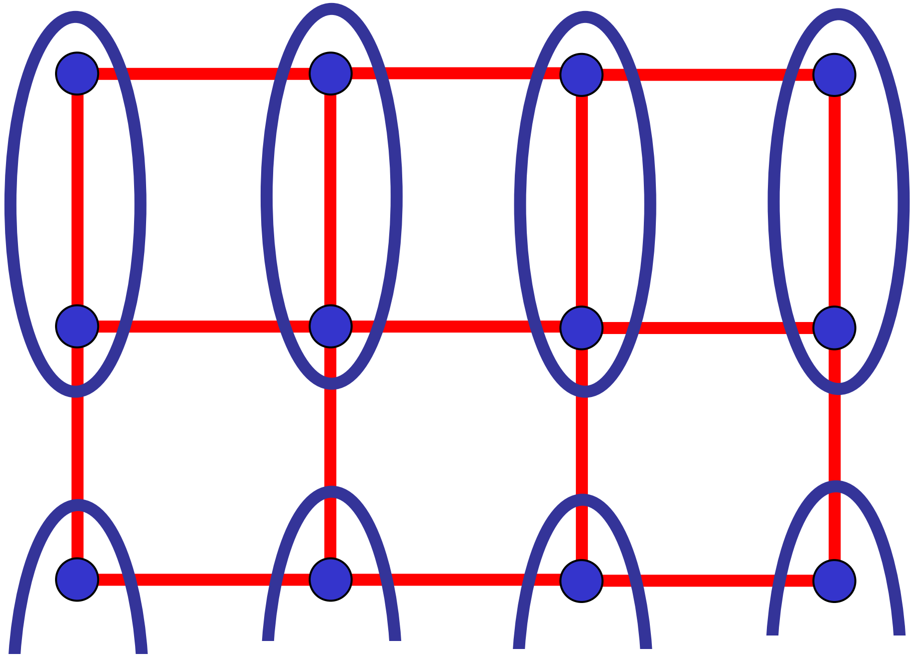
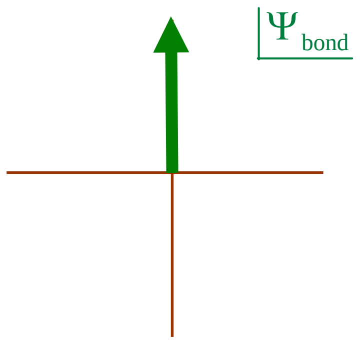
Mott insulator with one S=1/2 spin per unit cell





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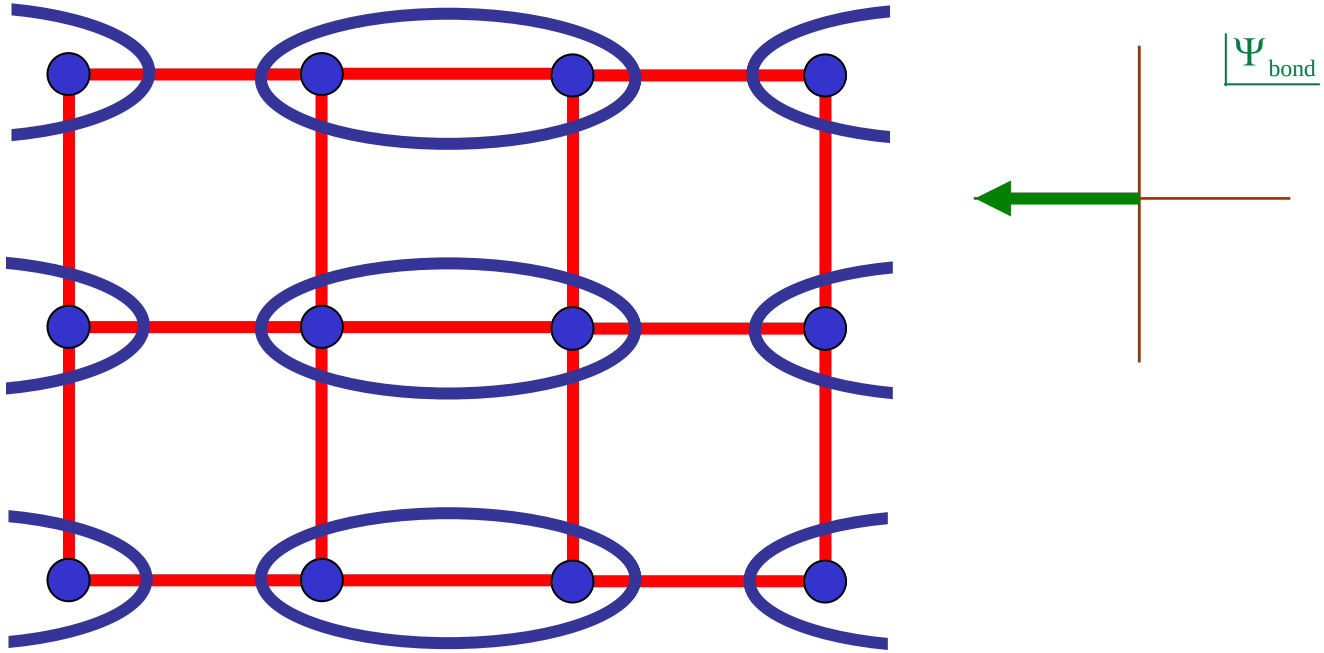

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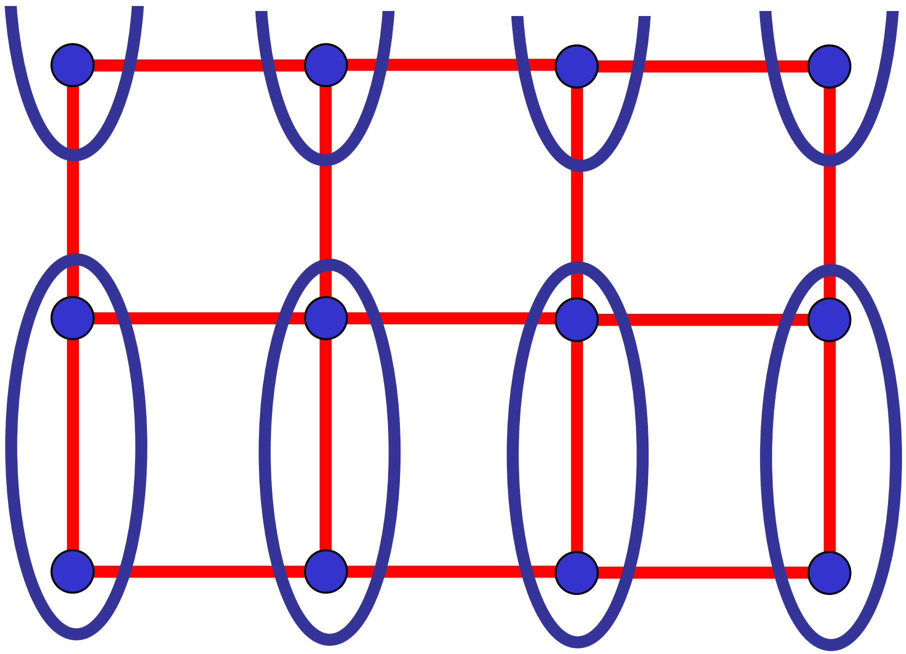

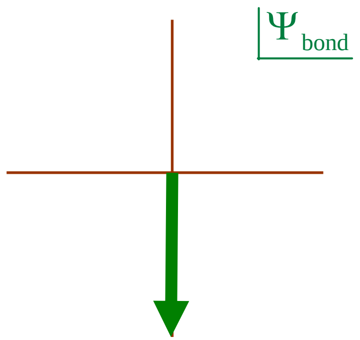

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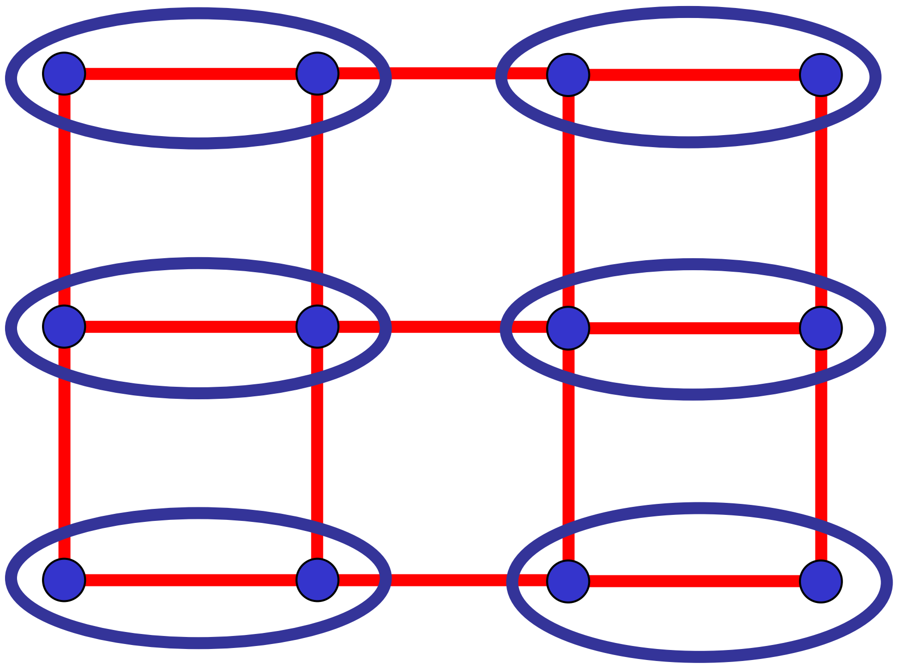

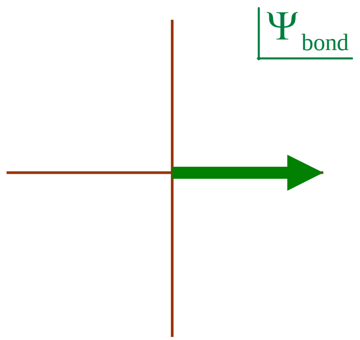

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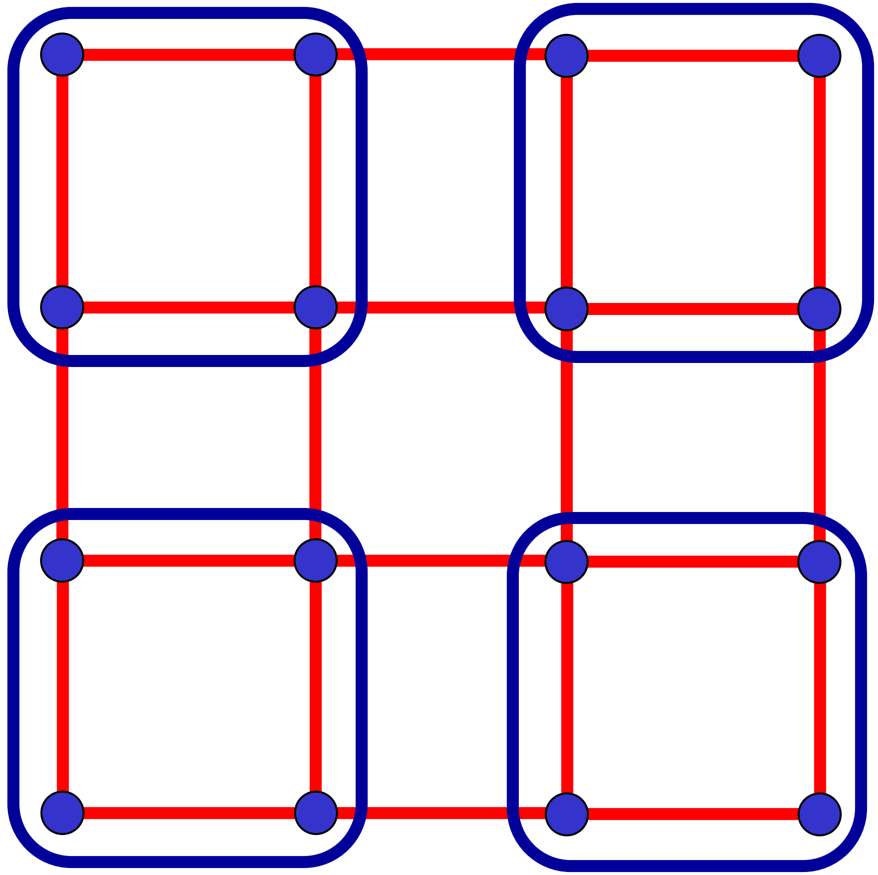

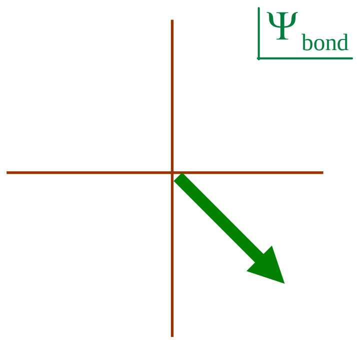

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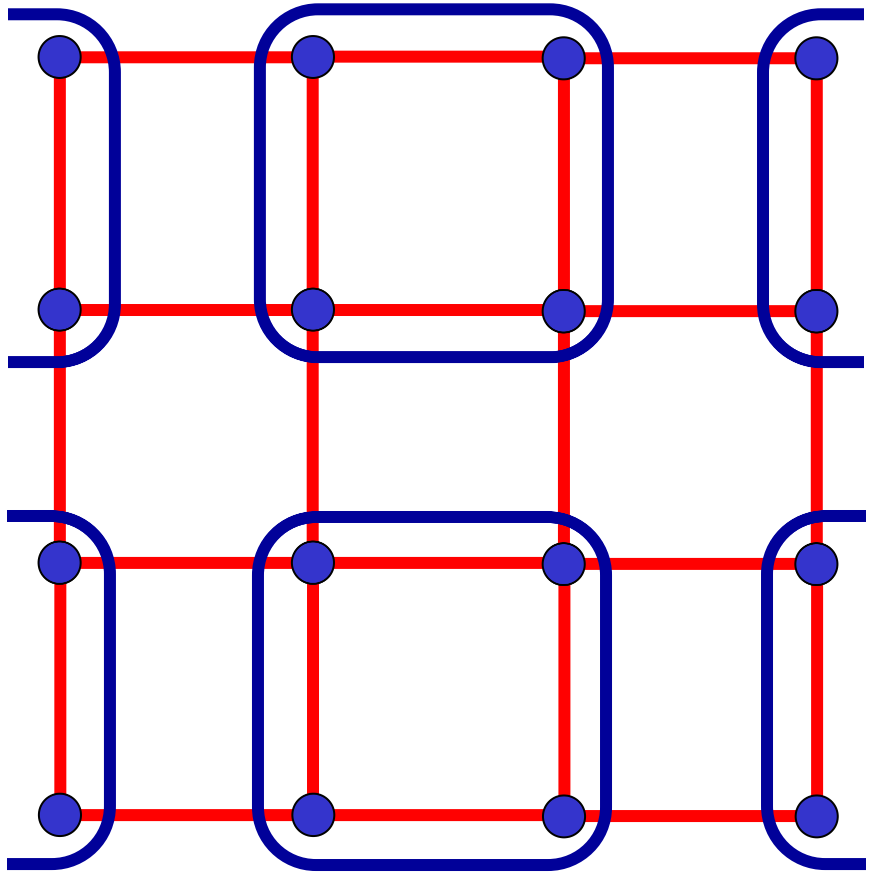

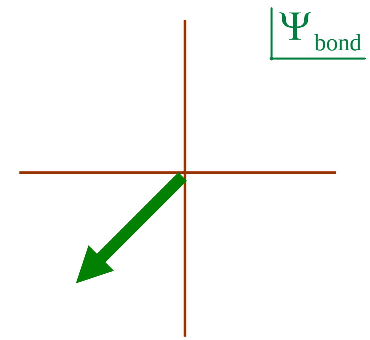

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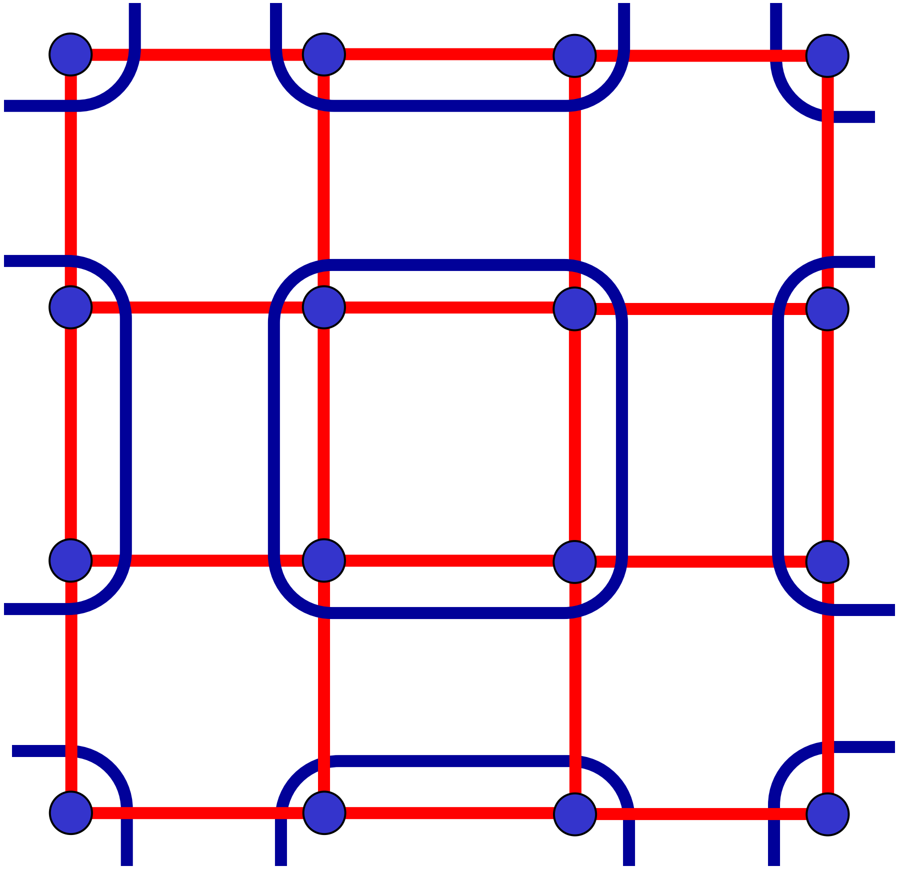

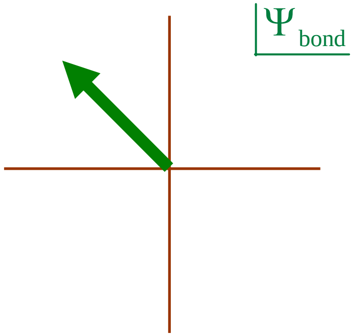

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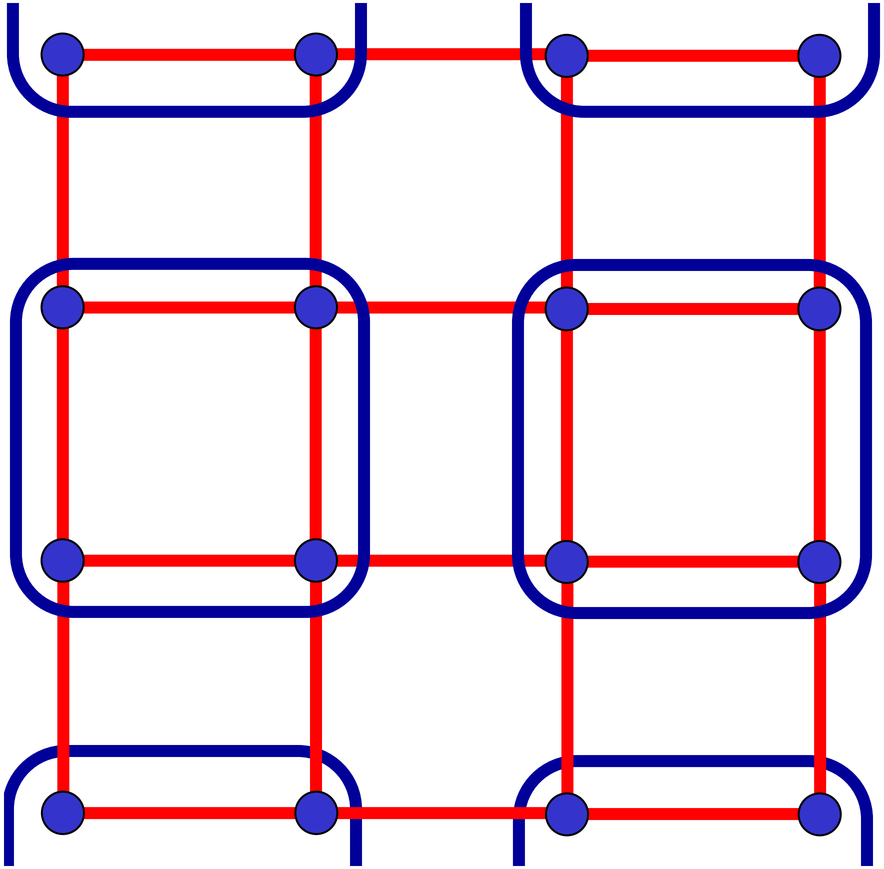

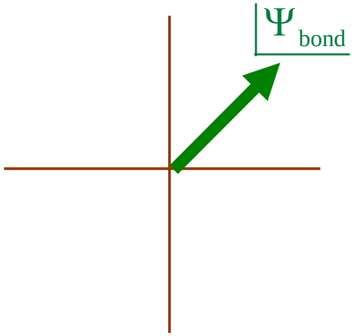

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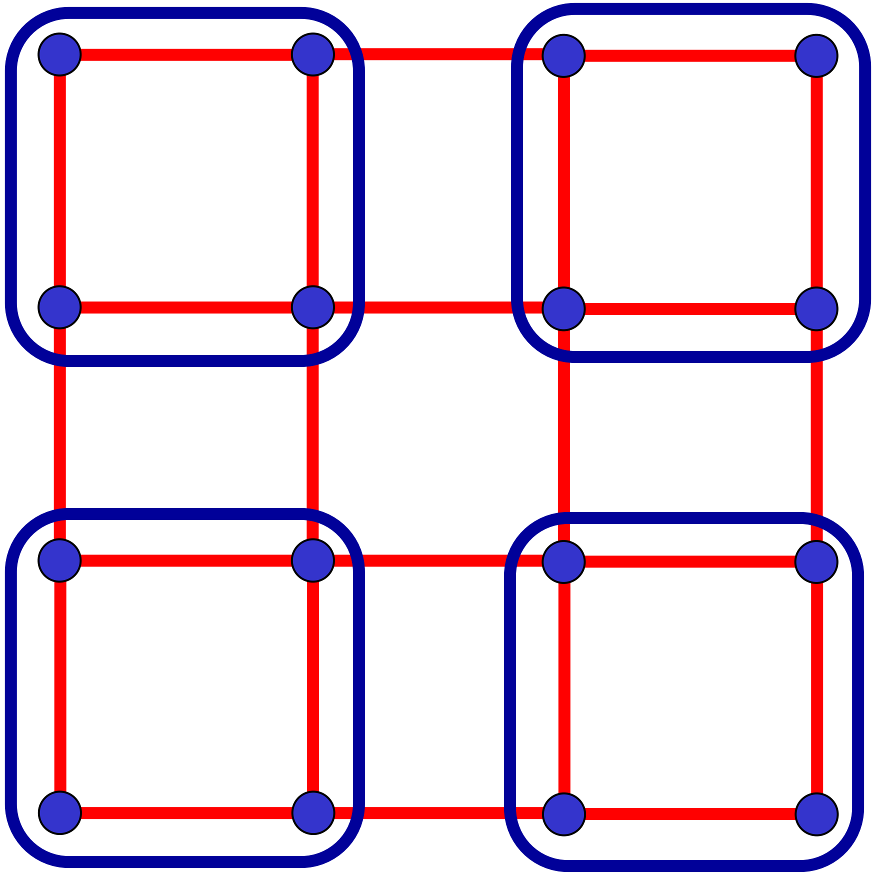

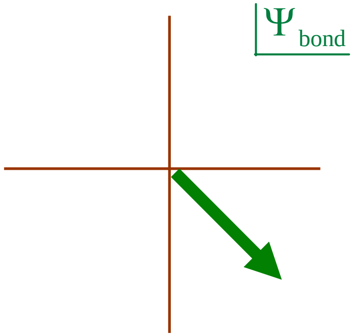

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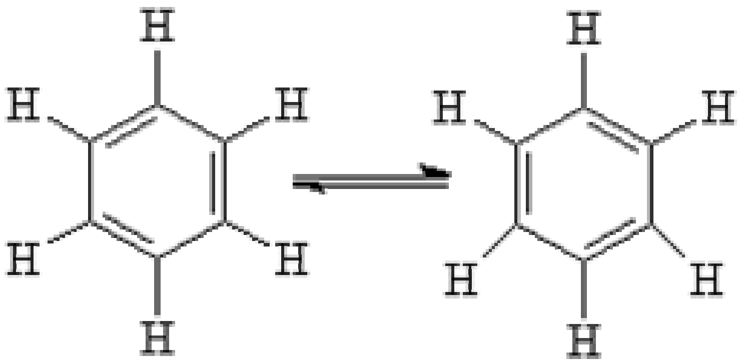
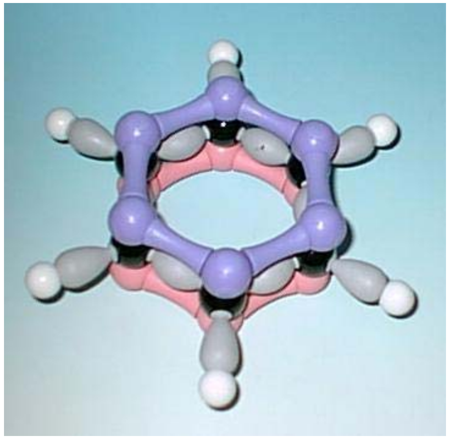
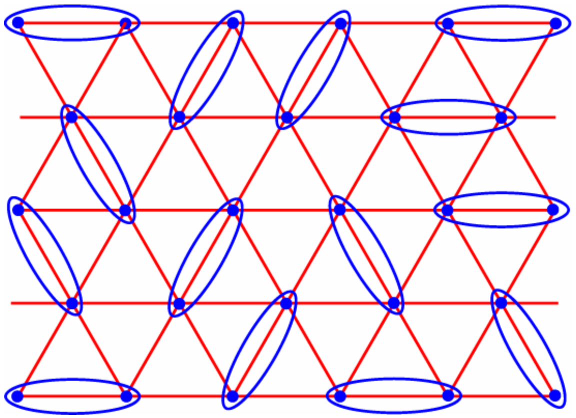
Resonating valence bonds





Resonance in benzene leads to a symmetric configuration of valence bonds
(*F. Kekulé, L. Pauling*)



Different valence bond pairings resonate with each other, leading to a resonating valence bond *liquid*, (Class B paramagnet) with $\langle \Psi_{\text{bond}} \rangle = 0$

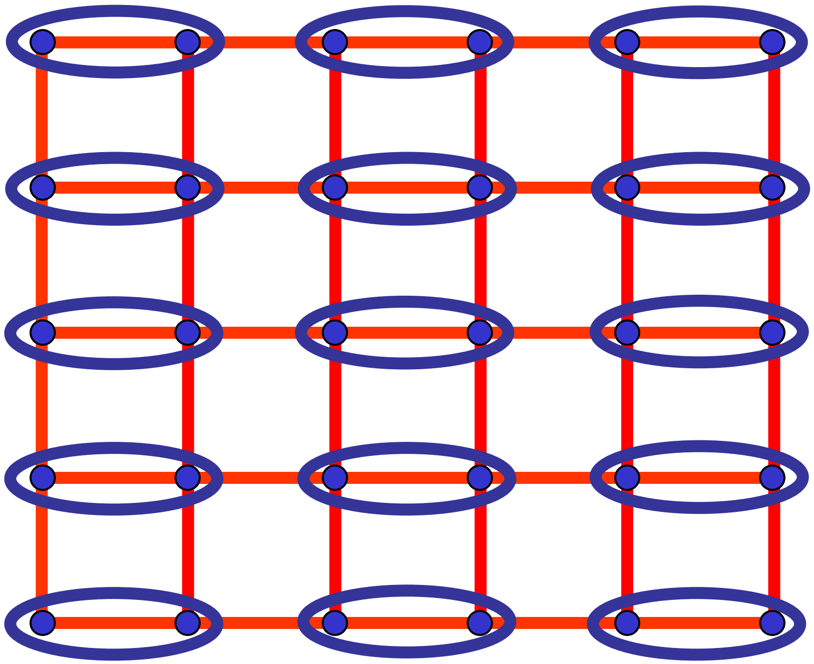
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974); P.W. Anderson 1987

Such states are associated with non-collinear spin correlations, Z_2 gauge theory, and topological order.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991); X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).

Excitations of the paramagnet with non-zero spin

$\langle \Psi_{\text{bond}} \rangle \neq 0$; Class A



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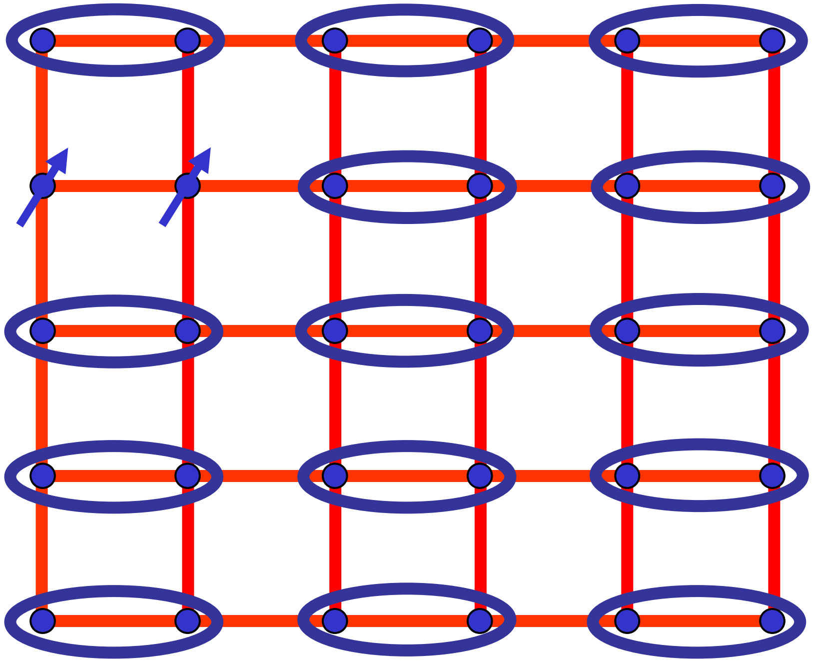

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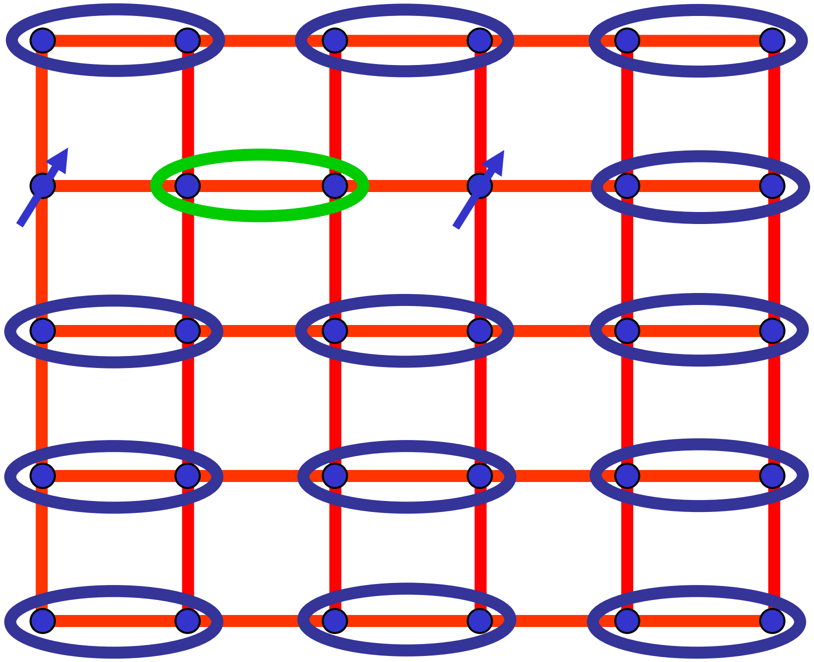

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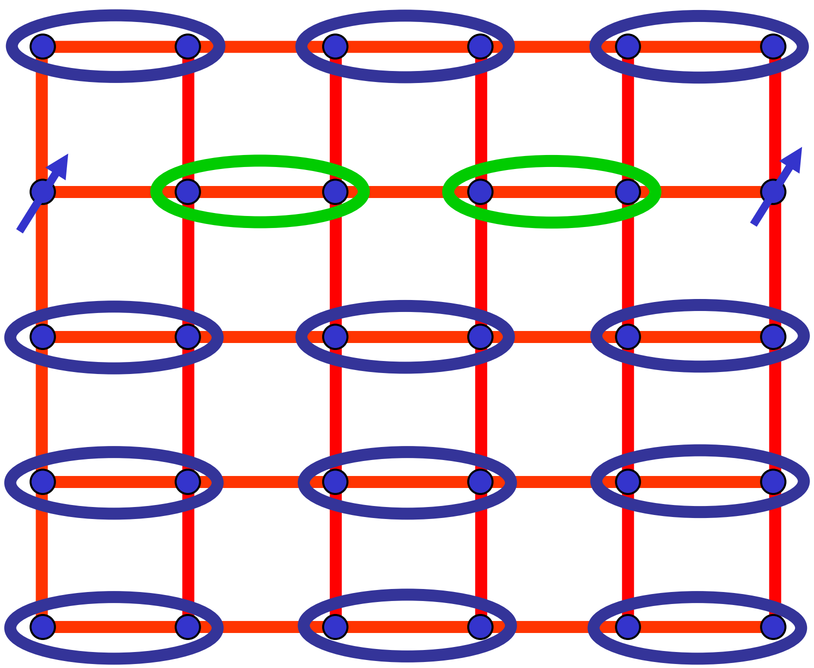

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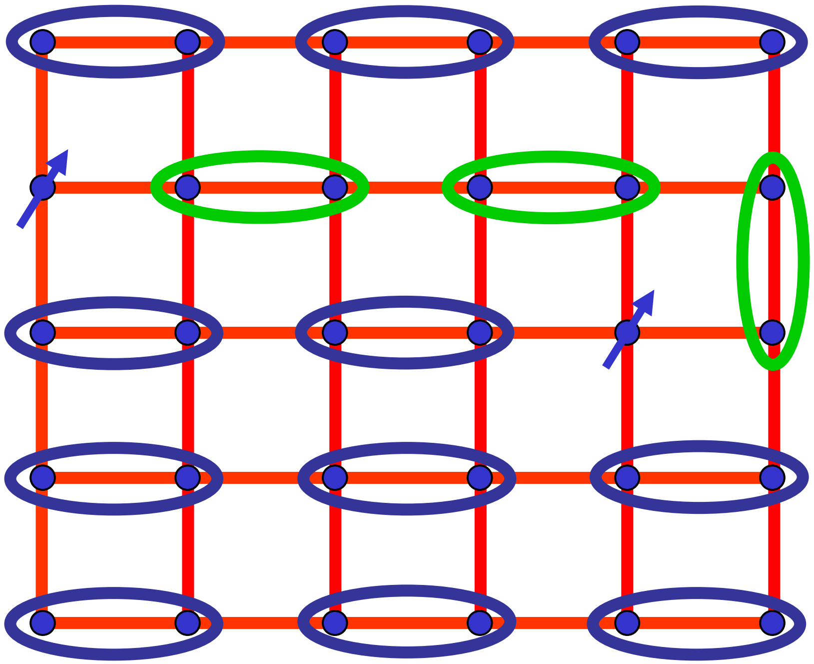

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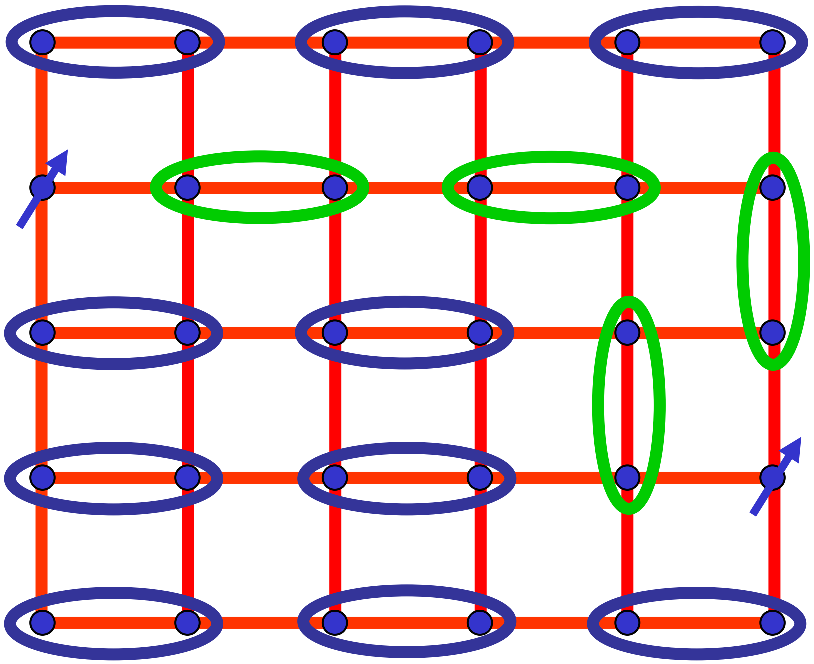

S=1/2 ***spinons***, z_α, are *confined* into a S=1 ***triplon***, $\vec{\varphi}$

$$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

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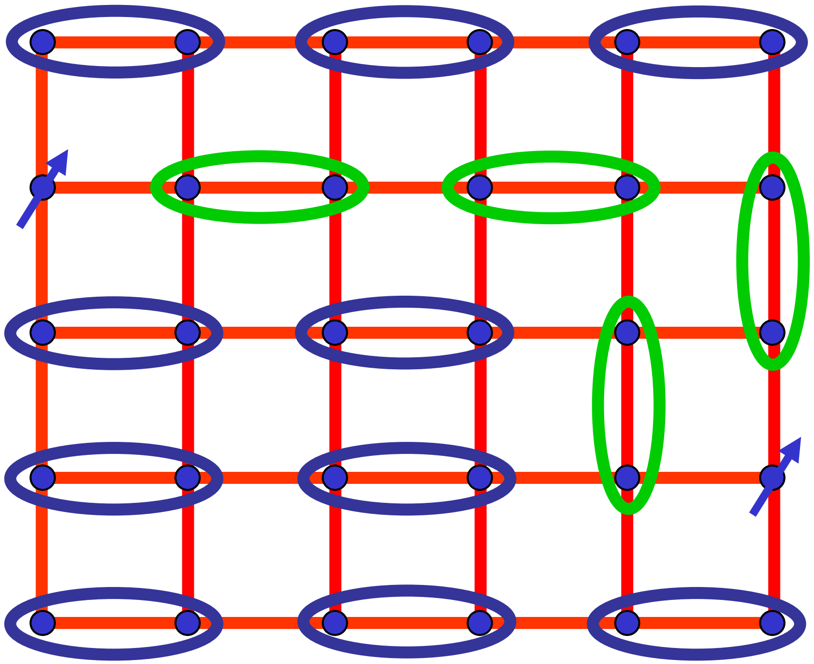

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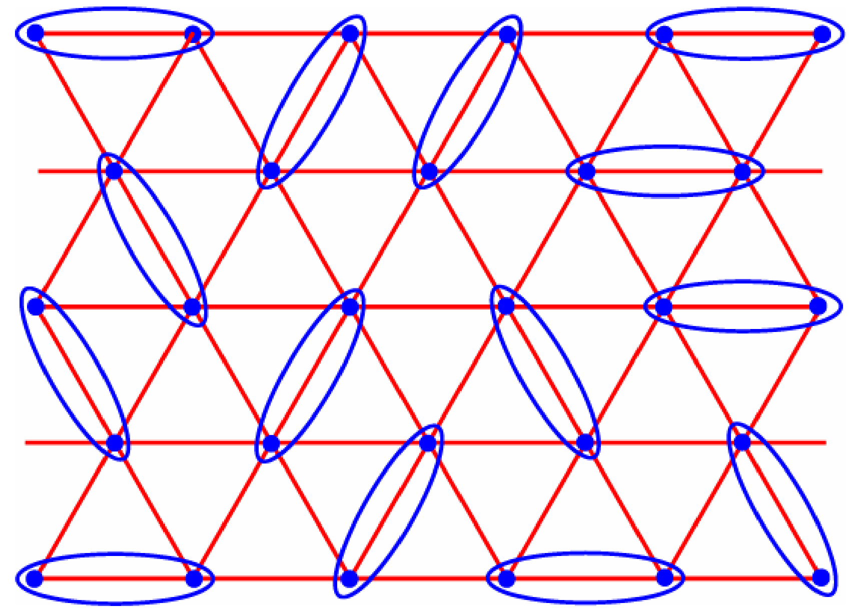

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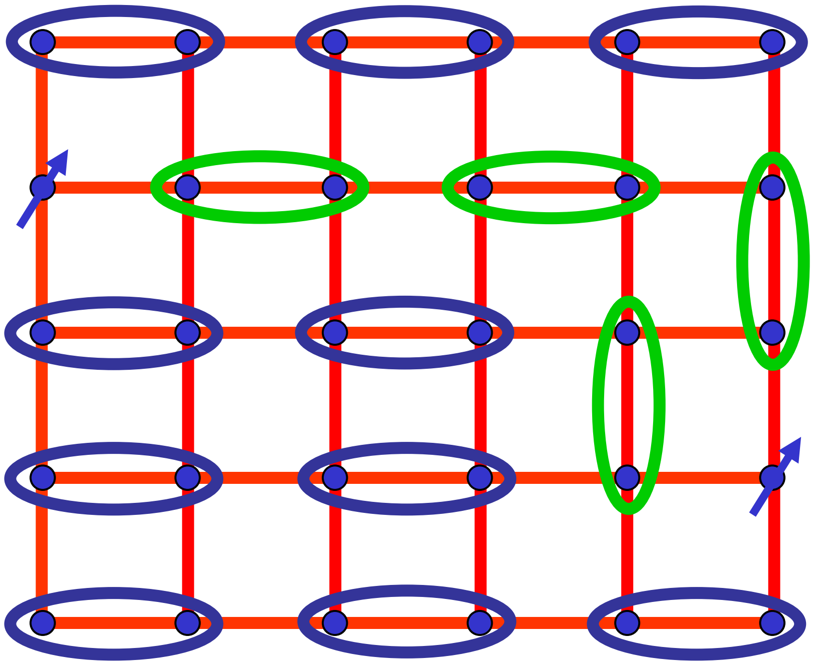

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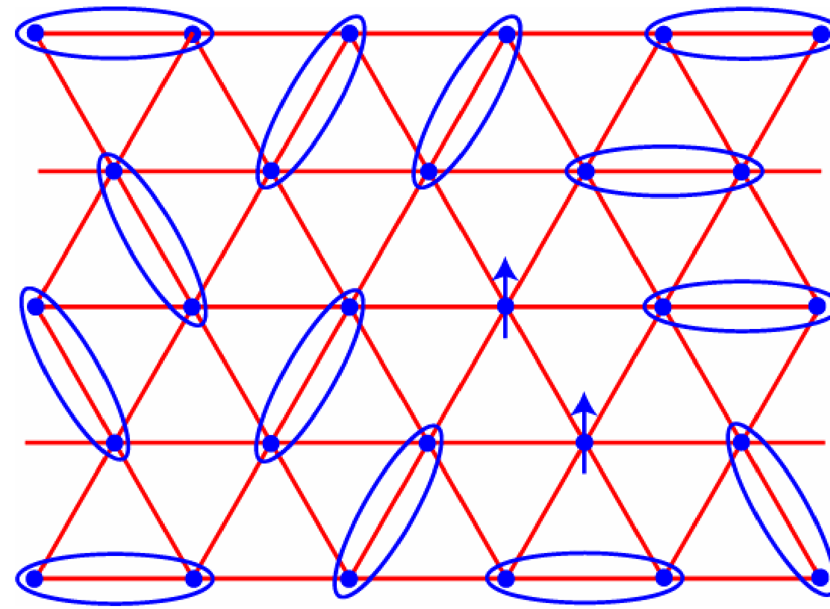

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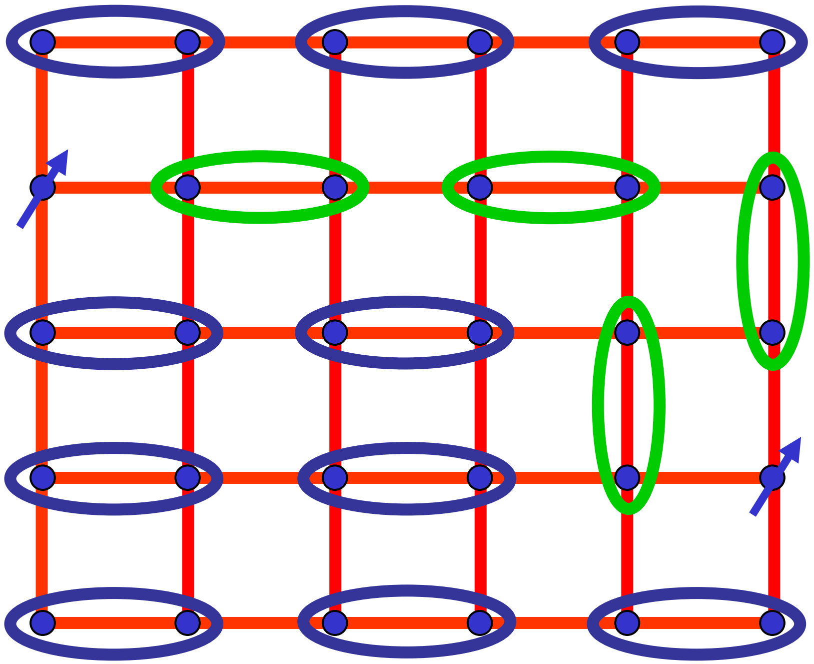

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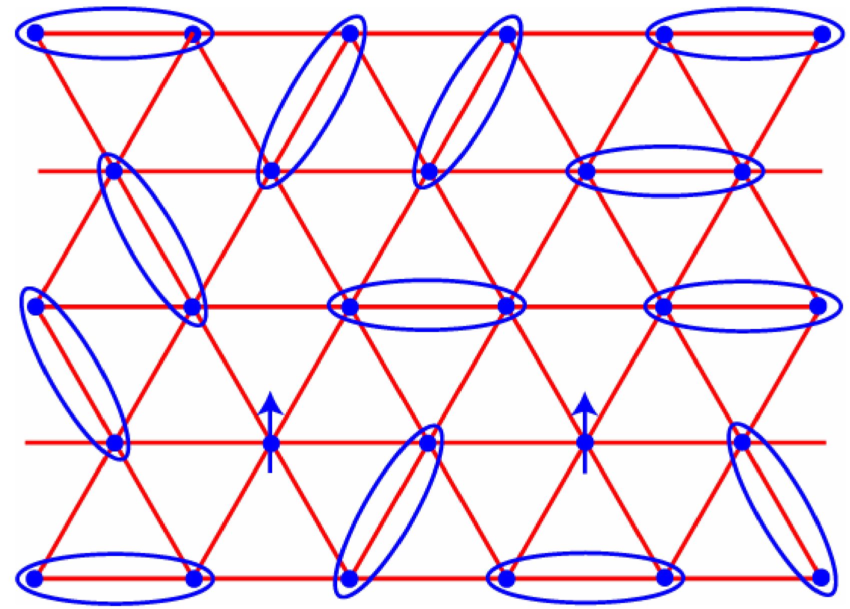

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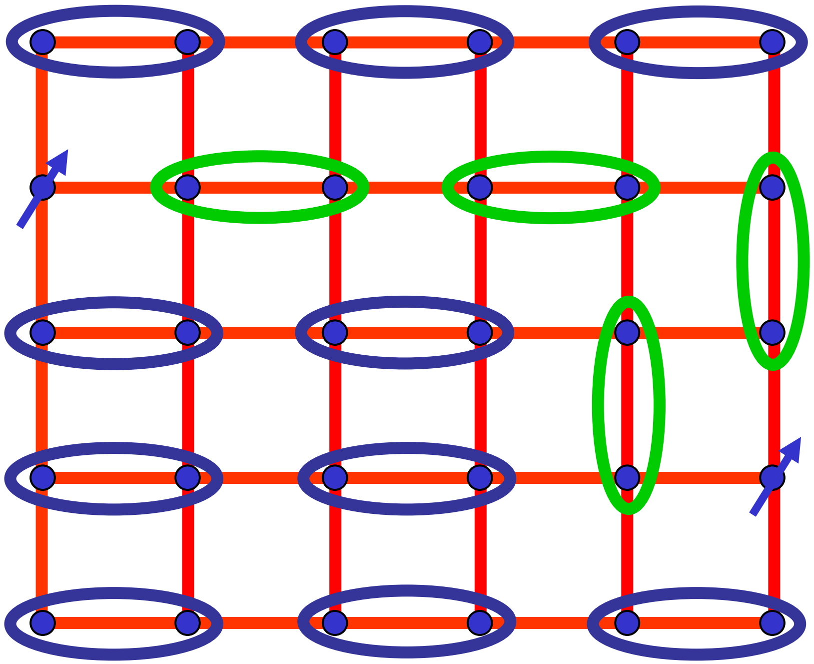

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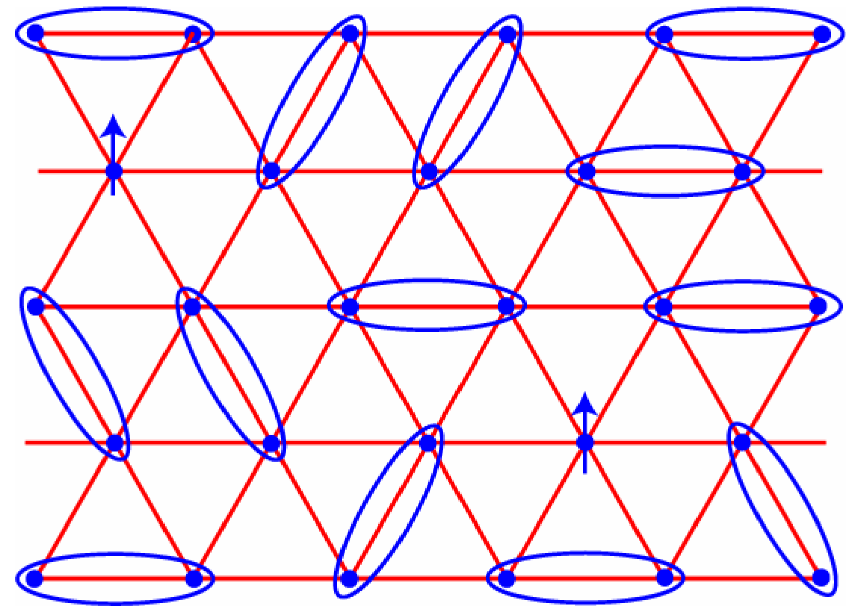
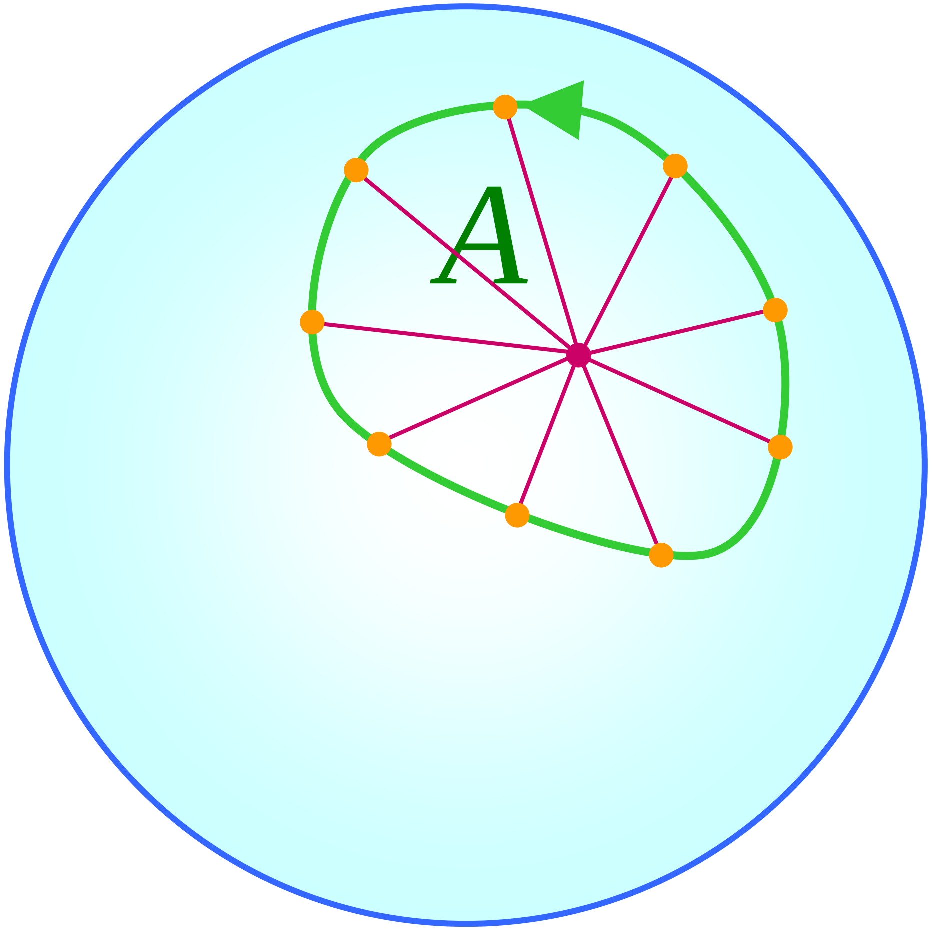


S=1/2 ***spinons*** can propagate independently across the lattice

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

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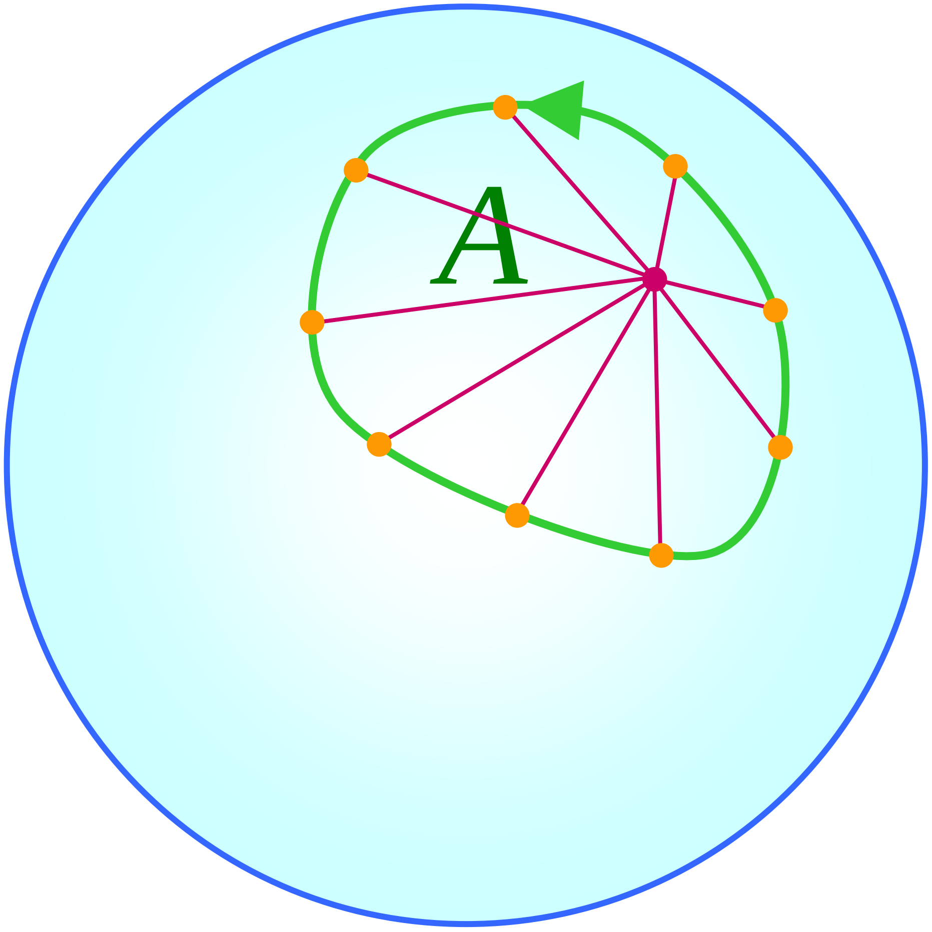

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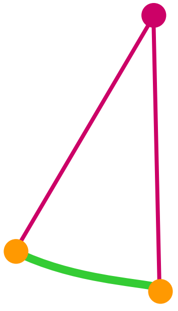
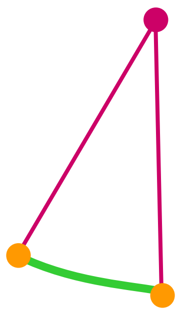


Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*



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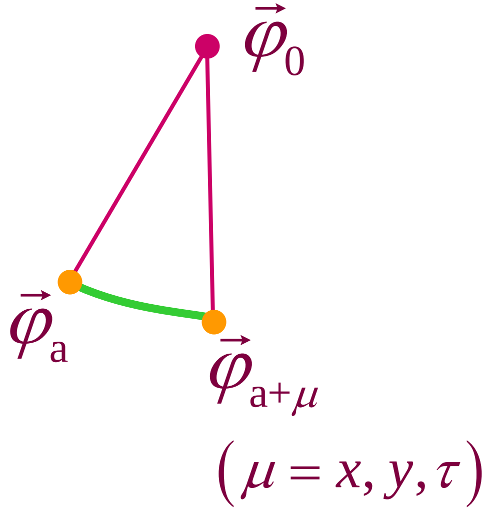

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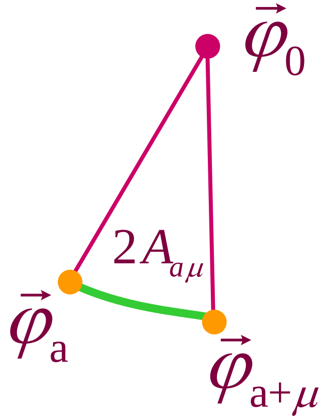

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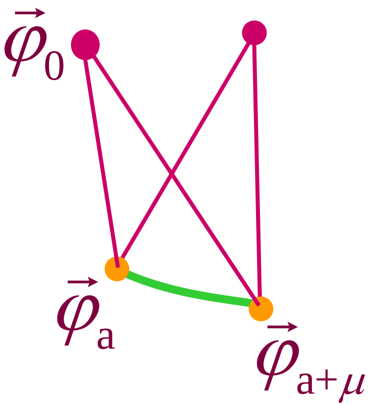

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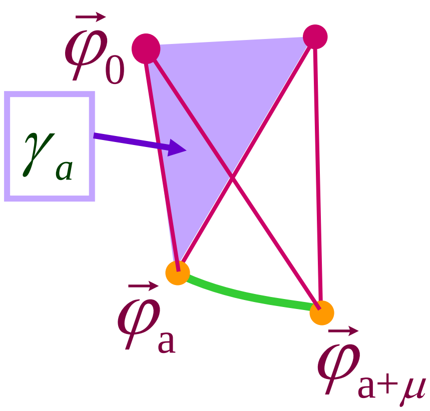

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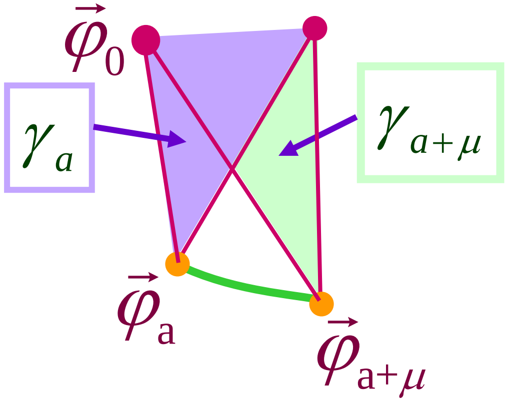

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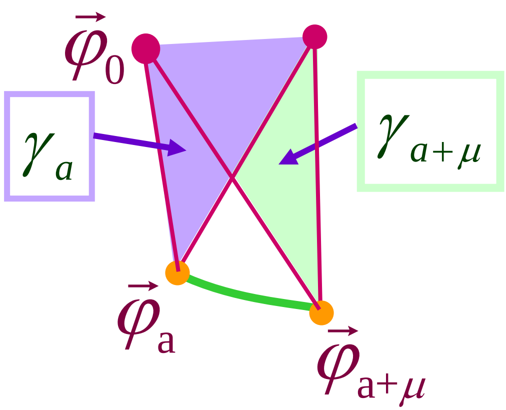
$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

Change in choice of $\vec{\varphi}_0$ is like a "gauge transformation"



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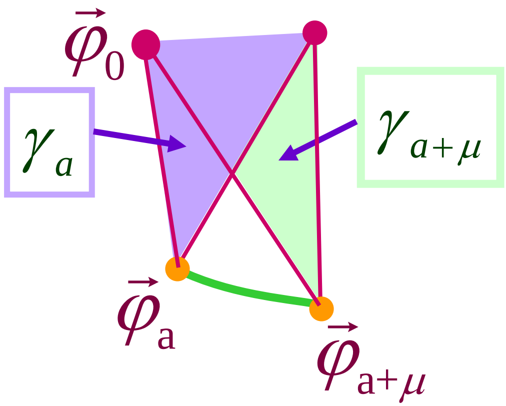
$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$

$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

Change in choice of $\vec{\varphi}_0$ is like a "gauge transformation"

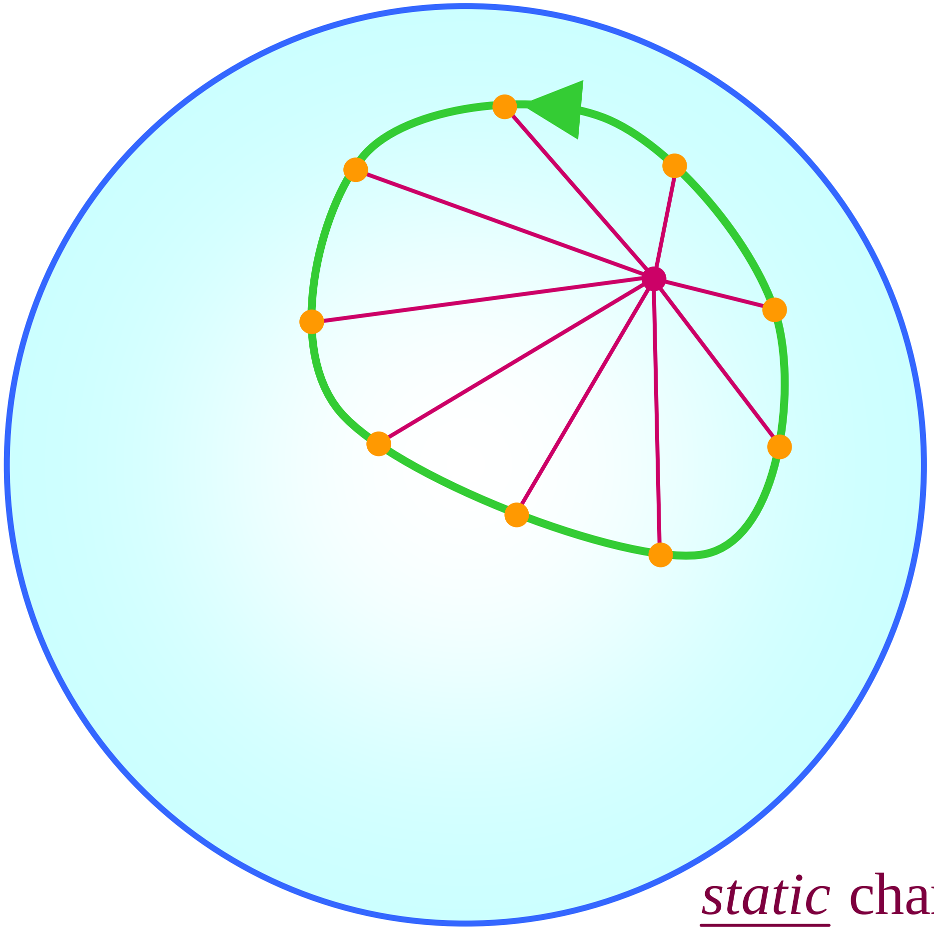


The area of the triangle is uncertain modulo 4π, and the action has to be invariant under $A_{a\mu} \rightarrow A_{a\mu} + 2\pi$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$\exp\left(i \sum_a \eta_a A_{a\tau} \right)$$

Sum of Berry phases of all spins on the square lattice.

$$= \exp\left(i \sum_{a,\mu} J_{a\mu} A_{a\mu} \right)$$

with "current" $J_{a\mu}$ of

static charges ± 1 on sublattices

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\left\langle \vec{\varphi} \right\rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\left\langle \vec{\varphi} \right\rangle = 0$

Berry phases lead to large cancellations between different time histories $\rightarrow$ need an effective action for $A_{a\mu}$ at large g

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) + i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

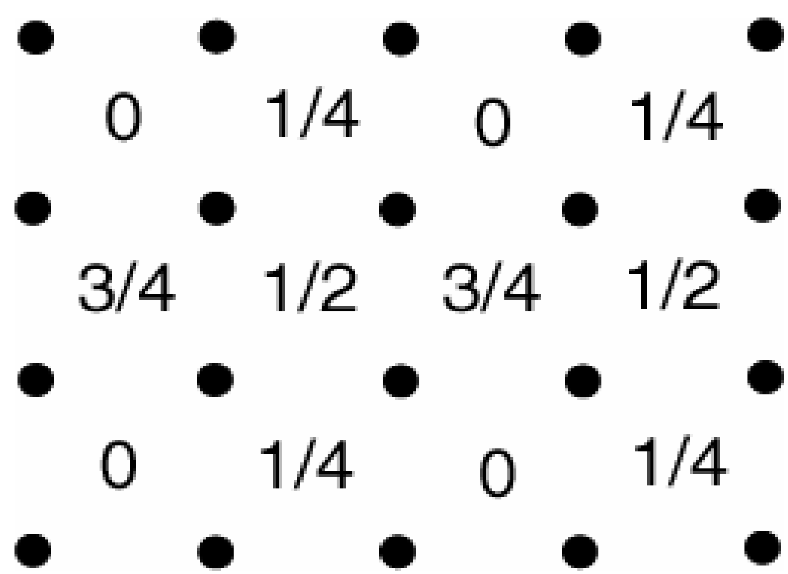
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields a representation in terms of a Coulomb gas of monopoles

$$Z_{\text{dual}} = \sum_{\{m_{\bar{\jmath}}\}} \exp\left(-\frac{\pi}{2e^2} \sum_{\bar{\jmath},\bar{\jmath}'} \frac{m_{\bar{\jmath}} m_{\bar{\jmath}'}}{|r_{\bar{\jmath}} - r_{\bar{\jmath}'}|} + 2\pi i \sum_{\bar{\jmath}} m_{\bar{\jmath}} \mathcal{X}_{\bar{\jmath}} \right)$$

with the $m_{\bar{\jmath}}$ integer monopole charges. Each monopole carries a Berry phase (F.D.M. Haldane, *Phys. Rev. Lett.* **61**, 1029 (1988)) determined by the fixed $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



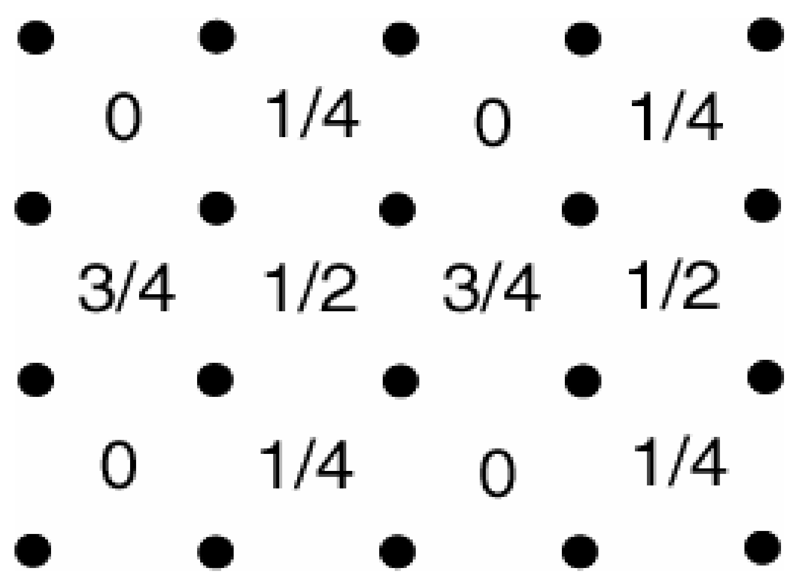
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

Alternative representation is in terms of a "height" model

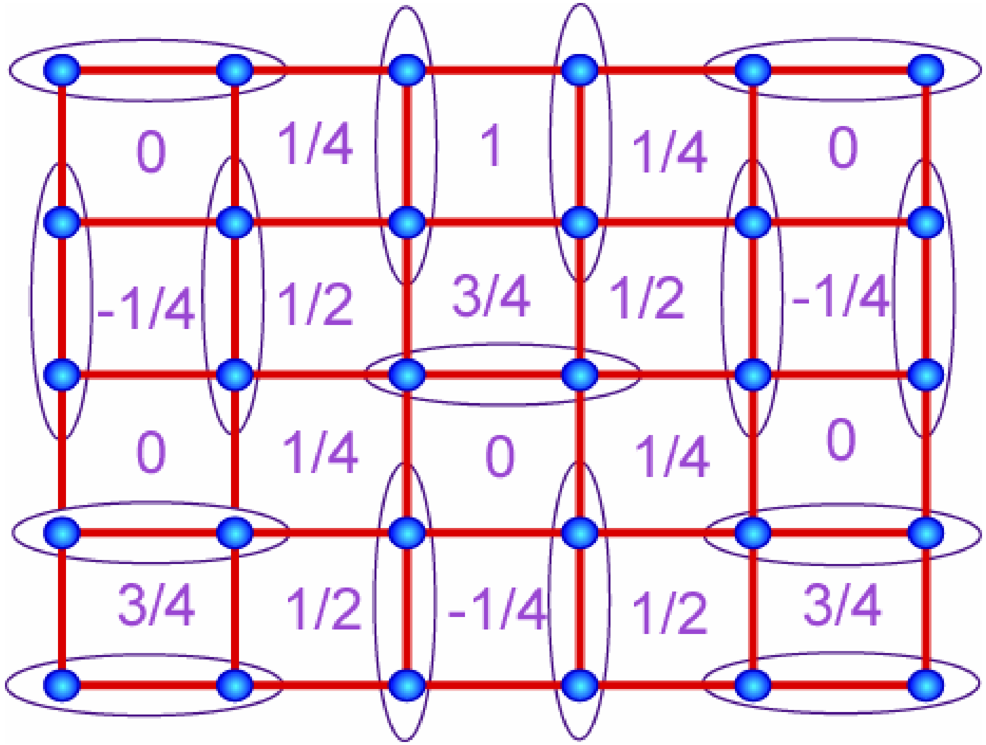
$$Z_{\rm dual} = \sum_{\{h_{\bar{\jmath}}\}} \exp\left(-\frac{e^2}{2} \sum_{\bar{\jmath}} \left(\Delta_\mu h_{\bar{\jmath}} - \Delta_\mu \mathcal{X}_{\bar{\jmath}}\right)^2 \right)$$

with the $h_{\bar{\jmath}}$ integer heights.

The Berry phases now lead to height 'offsets' $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with nearest-neighbor valence bond pairings of the sites square lattice



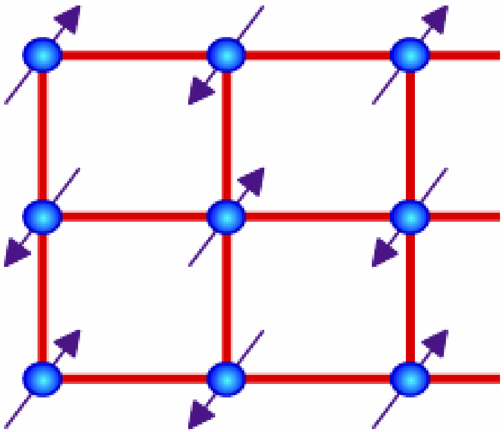
There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

⟹ There is a definite average height of the interface

⟹ **Ground state has bond order.**

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

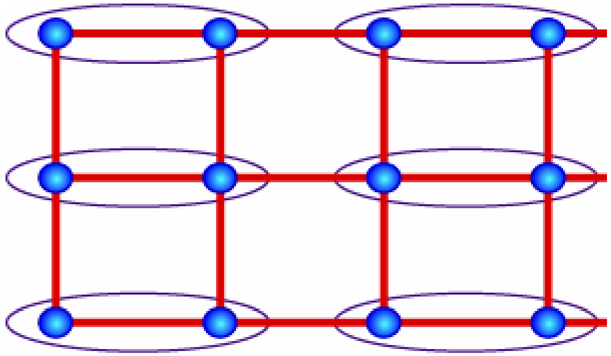
$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$



Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



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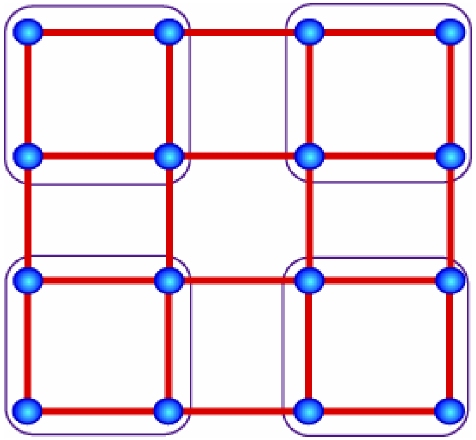


Bond order

$\langle \Psi_{\mathrm{bond}} \rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0 g

Naïve approach: add bond order parameter to LGW theory "by hand"

Neel order

$$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{bond}} \rangle = 0$$

First order transition

Bond order

$$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{bond}} \rangle \neq 0$$

g

Neel order

$$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{bond}} \rangle = 0$$

Coexistence

$$\langle \vec{\varphi} \rangle \neq 0,$$
$$\langle \Psi_{\text{bond}} \rangle \neq 0$$

Bond order

$$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{bond}} \rangle \neq 0$$

g

Neel order

$$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{bond}} \rangle = 0$$

"disordered"

$$\langle \vec{\varphi} \rangle = 0,$$
$$\langle \Psi_{\text{bond}} \rangle = 0$$

Bond order

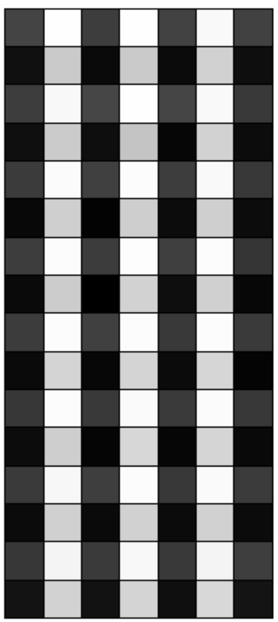
$$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{bond}} \rangle \neq 0$$

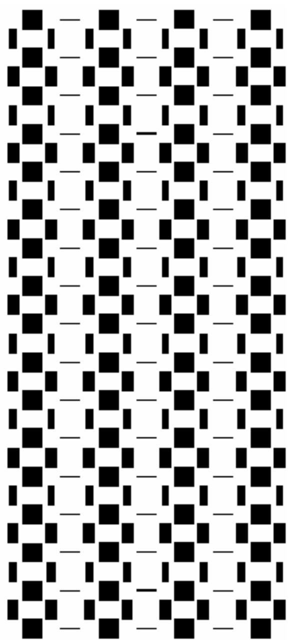
g

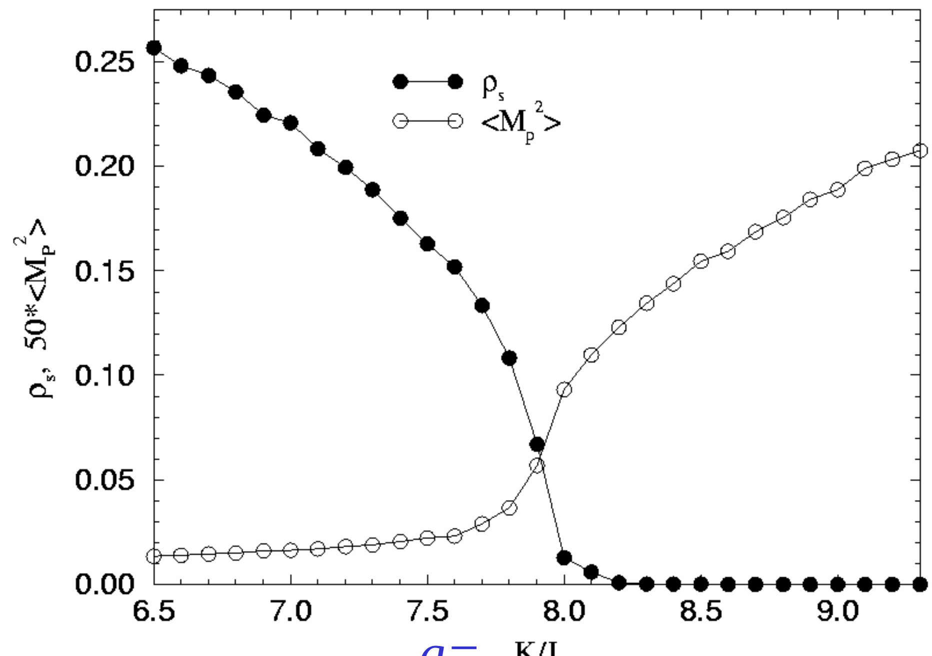
Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First <u>*large scale*</u> (> 8000 spins) numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry



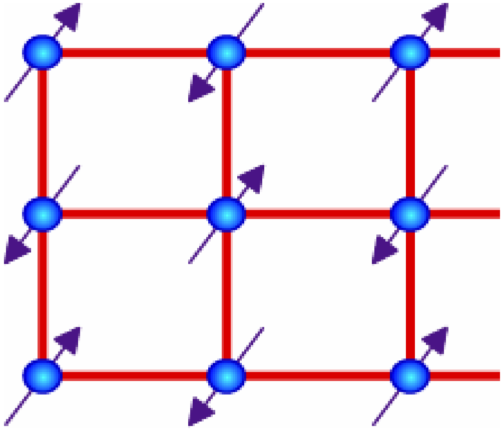




$g=$ K/J

$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\Box}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

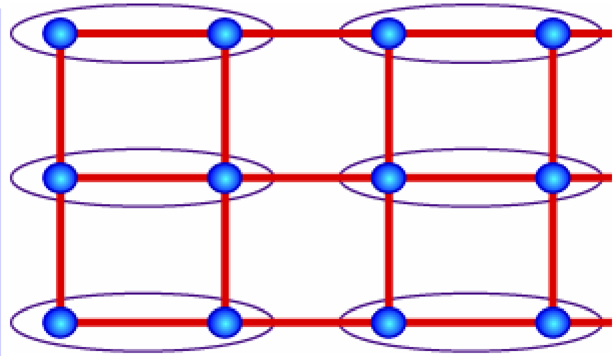
$$Z = \prod_a \int d\vec{\varphi}_a \, \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$



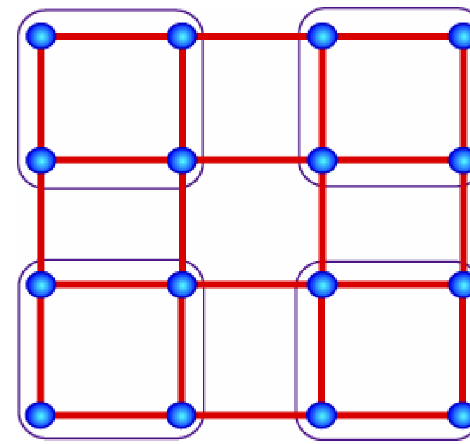
Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



or



Bond order

$\langle \Psi_{\text{bond}} \rangle \neq 0$

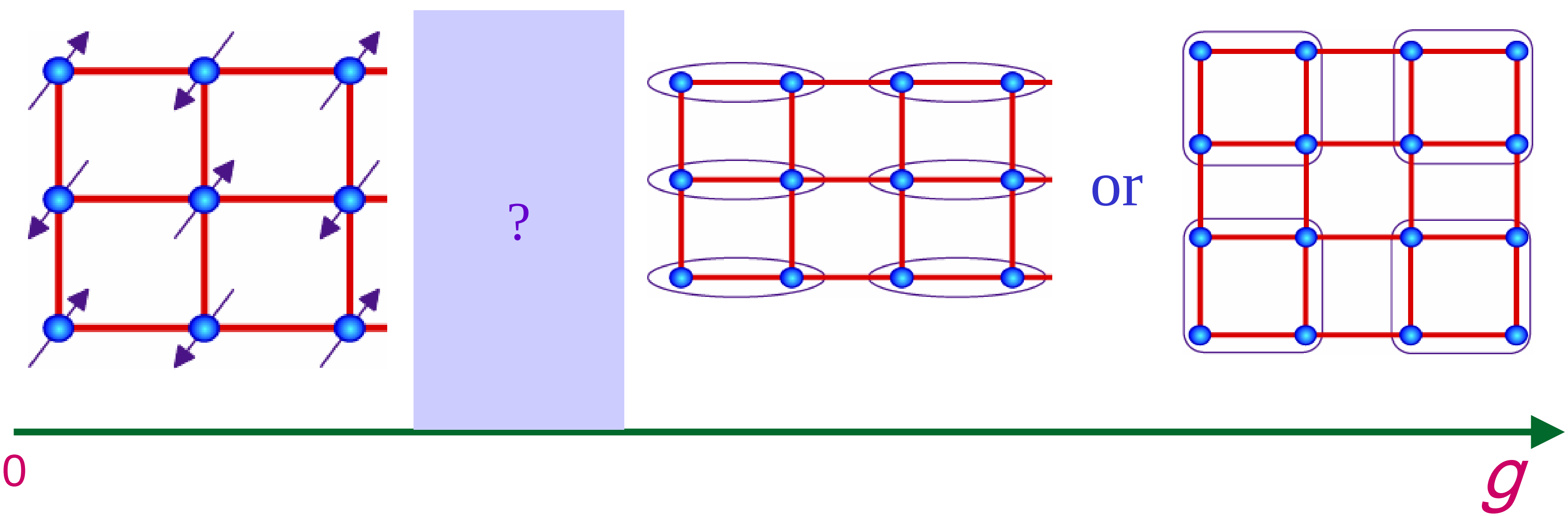
Not present in LGW theory of $\vec{\varphi}$ order

0 $\longrightarrow$ g

Alternative formulation to describe transition:

Express theory in terms of a complex spinor $z_{a\alpha}$, $\alpha = \uparrow, \downarrow$, with

$$
\begin{aligned}
\mathbf{n}_a &= z^*_{a\alpha} \sigma_{\alpha\beta} z_{a\beta} \\
Z &= \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right) \\
& \qquad \exp\left(\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i \sum_a \eta_a A_{a\tau} \right)
\end{aligned}
$$



S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
S. Sachdev and K. Park, *Annals of Physics* **298**, 58 (2002).

Theory of a second-order quantum phase transition between Neel and bond-ordered phases

At the quantum critical point:

- $A_\mu \rightarrow A_\mu + 2\pi$ periodicity can be ignored

(Monopoles interfere destructively and are dangerously irrelevant).

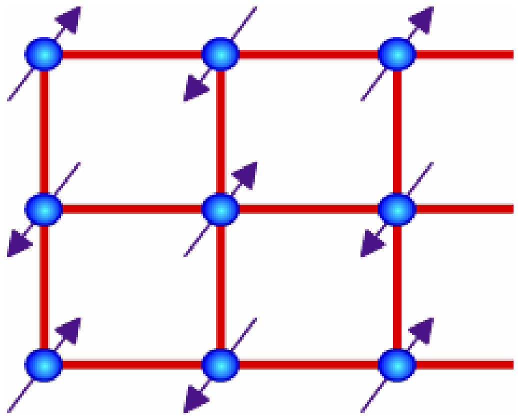
- $S=1/2$ *spinons* z_α, with $\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, are globally propagating degrees of freedom.

Second-order critical point described by emergent fractionalized degrees of freedom (A_μ and z_α); Order parameters ($\vec{\varphi}$ and Ψ_{bond}) are "composites" and of secondary importance

S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990); G. Murthy and S. Sachdev, *Nuclear Physics* B **344**, 557 (1990); C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001); S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002); O. Motrunich and A. Vishwanath, cond-mat/0311222.

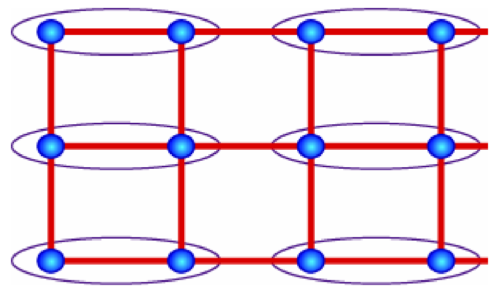
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Phase diagram of S=1/2 square lattice antiferromagnet

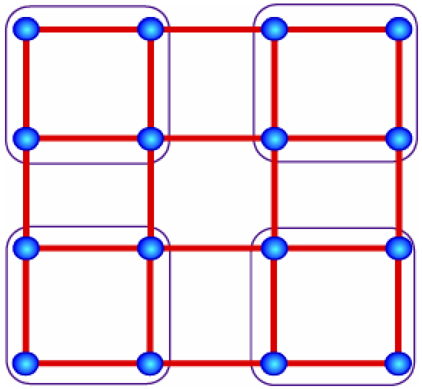


Neel order

$$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$$



or



Bond order $\langle \Psi_{\text{bond}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

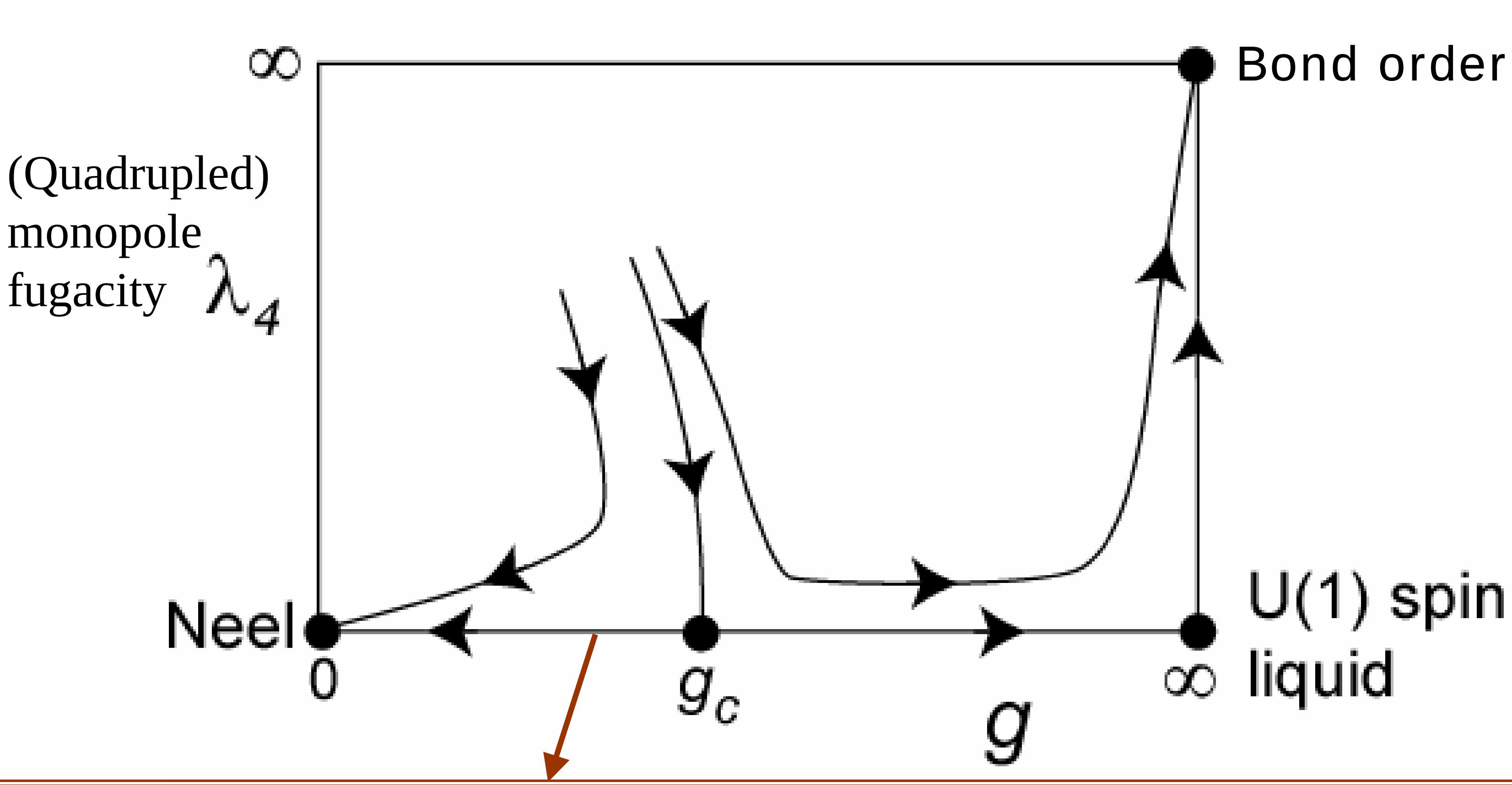
$S = 1$ triplon excitations

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + r |z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

at its critical point $r = r_c$, where A_μ is *non-compact*

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).
S. Sachdev cond-mat/0401041.



The line $\lambda_4 = 0$ is described by

$$\mathcal{S} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + r\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

where A_μ is *non-compact*

B. Mott insulators with spin S=1/2 per unit cell:

Berry phases, bond order, and the breakdown of the LGW paradigm

Order parameters/broken symmetry

+

Emergent gauge excitations, fractionalization.

C. Technical details

Duality and dangerously irrelevant operators

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i\sum_a \eta_a A_{a\tau} + \frac{1}{e^2}\sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right)\right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

A. Non-compact QED with scalar matter
B. Compact QED with scalar matter
C. *N=1*: Compact QED with scalar matter and Berry phases
D. $N \to \infty$ theory
E. Easy plane case for *N=2*

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g}\sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i\sum_a \eta_a A_{a\tau} + \frac{1}{e^2}\sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right)\right)$$

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A. Non-compact QED with scalar matter

B. Compact QED with scalar matter

C. $N=1$: Compact QED with scalar matter and Berry phases

D. $N \to \infty$ theory

E. Easy plane case for $N=2$

A. *N=1*, non-compact U(1), no Berry phases

Use $z_a = e^{i\theta_a}$ and then

$$Z = \prod_a \int d\theta_a dA_{a\mu} \exp\left(-\frac{1}{2e^2}\sum_{\square}(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu})^2 + \frac{1}{g}\sum_{a,\mu}\cos(\Delta_\mu \theta_a - A_{a\mu})\right)$$

Standard duality maps, similar to those discussed earlier, show that this theory is equivalent to an **inverted XY model**, described by the field theory

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp\left(-\int d^2x d\tau \left(|\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4\right)\right)$$

Here ψ is a *dual* field which orders in the paramagnetic phase *i.e.* $\langle\psi\rangle \neq 0$ where $\langle e^{i\theta}\rangle = 0$, and vice versa. The field ψ is a creation operator for *vortices* in the original theory of a "Ginzburg-Landau superconductor" coupled to "electromagnetism".

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981).

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta \left(|z_{a\alpha}|^2 - 1 \right)$$

$$\exp \left(\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos \left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) \right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

A. Non-compact QED with scalar matter
B. Compact QED with scalar matter
C. *N=1*: Compact QED with scalar matter and Berry phases
D. $N \to \infty$ theory
E. Easy plane case for *N=2*

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i\sum_a \eta_a A_{a\tau} + \frac{1}{e^2}\sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right)\right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

A. Non-compact QED with scalar matter
B. Compact QED with scalar matter
C. $N=1$: Compact QED with scalar matter and Berry phases
D. $N \to \infty$ theory
E. Easy plane case for $N=2$

B. *N=1*, compact U(1), no Berry phases

Use $z_a = e^{i\theta_a}$ and then

$$Z = \prod_a \int d\theta_a dA_{a\mu} \exp\left(\frac{1}{e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) + \frac{1}{g} \sum_{a,\mu} \cos\left(\Delta_\mu \theta_a - A_{a\mu}\right) \right)$$

The Dasgupta-Halperin mapping now yields the dual theory

$$Z_{\rm dual} = \int \mathcal{D}\psi \exp\left(- \int d^2x d\tau \left(|\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 - y_m(\psi + \psi^*) \right) \right)$$

Here y_m is a *monopole fugacity*, and the last term in $Z_{\rm dual}$ accounts for the fact that vortex lines can end in monopoles.

This dual theory is an **inverted XY model in a "magnetic" field** and it has no phase transition. In the direct theory, the monopoles are a relevant perturbation, and they destroy the "superconducting" phase.

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) \right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

A. Non-compact QED with scalar matter
B. Compact QED with scalar matter
C. $N=1$: Compact QED with scalar matter and Berry phases
D. $N \to \infty$ theory
E. Easy plane case for $N=2$

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i\sum_a \eta_a A_{a\tau} + \frac{1}{e^2}\sum_{\square}\cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right)\right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

A. Non-compact QED with scalar matter
B. Compact QED with scalar matter
C. *N=1*: Compact QED with scalar matter and Berry phases
D. $N \to \infty$ theory
E. Easy plane case for *N=2*

C. *N=1*, compact U(1), Berry phases

Upon including Berry phases, the previous theory becomes

$$Z = \prod_a \int d\theta_a dA_{a\mu} \exp\left(\frac{1}{e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) + \frac{1}{g} \sum_{a,\mu} \cos\left(\Delta_\mu \theta_a - A_{a\mu}\right) + i \sum_a \eta_a A_{a\tau} \right)$$

The Dasgupta-Halperin duality can also be extended to this theory, and we obtain

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp\left(- \int d^2x d\tau \left(|\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 - \tilde{y}_m(\psi^4 + \psi^{*4}) \right) \right)$$

S. Sachdev and R. Jalabert, Mod. Phys. Lett. **4**, 1043 (1990).

C. *N=1*, compact U(1), Berry phases

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp\left(- \int d^2x d\tau \left(|\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 - \tilde{y}_m(\psi^4 + \psi^{*4}) \right) \right)$$

This is an **inverted XY model with a four-fold anisotropy**, *i.e.* a Z_4 clock model. The four-fold anisotropy is irrelevant at the critical point (J.M. Carmona, A. Pelissetto, E. Vicari, Phys. Rev. B **61**, 15136 (2000)), and hence there is a XY transition to a four-fold degenerate state with $\langle \psi \rangle \neq 0$. In the direct theory, this is the *bond-ordered* paramagnet.

S. Sachdev and R. Jalabert, Mod. Phys. Lett. **4**, 1043 (1990).

C. *N=1*, compact U(1), Berry phases

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp\left(- \int d^2x d\tau \left(|\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 - \tilde{y}_m(\psi^4 + \psi^{*4}) \right) \right)$$

Reinterpretation by T. Senthil: In the direct theory, the irrelevance of $\tilde{y}_m$ implies that the Berry phases have cancelled out the monopole contributions. So monopoles are 'dangerously irrelevant' at the critical point, and the critical theory is *the same Dasgupta-Halperin inverted XY model describing the non-compact theory without monopoles or Berry phases!*

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i\sum_a \eta_a A_{a\tau} + \frac{1}{e^2}\sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right)\right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

Identical critical theories !

A. Non-compact QED with scalar matter

B. Compact QED with scalar matter

C. *N=1*: Compact QED with scalar matter and Berry phases

D. $N \to \infty$ theory

E. Easy plane case for *N=2*

Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right)$$

$$\exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i\sum_a \eta_a A_{a\tau} + \frac{1}{e^2}\sum_{\square}\cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right)\right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

Identical critical theories !

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E. Easy plane case for *N*=2

D. $N \to \infty$, compact U(1), Berry phases

Near the critical point of the $N = \infty$ non-compact theory, integrate out z_α quanta (with gap Δ) in the presence of a Dirac monopole with $A_\mu = A_\mu^D$ with magnetic charge q. The functional determinant yields the action of such a monopole, and the scaling dimension of the monopole insertion

$$\mathcal{S}_{\text{monopole}} = N \operatorname{Tr} \ln \left[\frac{-(\partial_\mu - i A_\mu^D)^2 + \Delta^2 + V(r)}{-\partial_\mu^2 + \Delta^2} \right] - \frac{N}{g} \int d^3 r V(r)$$

$$\text{where } \frac{\delta \mathcal{S}_{\text{monopole}}}{\delta V(r)} = 0 \text{ and } V(r \to \infty) = 0.$$

Evaluation of functional determinant for $q = 4$ shows

$$\mathcal{S}_{\text{monopole}} = 0.815787 N \ln \left(\frac{\Lambda}{\Delta} \right)$$

This computation shows that the scaling dimension of $q = 4$ monopoles is $3 - 0.815787N$

Monopoles are irrelevant both with and without Berry phases for large N.

G. Murthy and S. Sachdev, *Nucl. Phys.* B **344**, 557 (1990).

E. Easy plane case for *N*=2

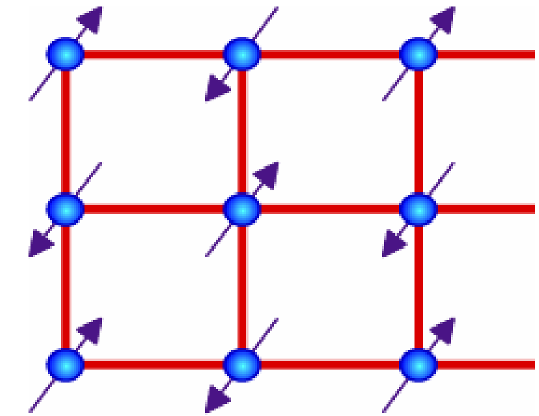
Explicit duality mappings show that the physical situation is as for $N = 1$:

- monopoles are relevant without Berry phases,
- monopoles are irrelevant at the critical point in the presence of Berry phases, and
- monopoles drive the appearance of bond order in the paramagnetic phase.

$$Z_{\text{dual}} = \int \mathcal{D}\psi_1 \mathcal{D}\psi_2 \mathcal{D}a_\mu \exp\Bigg(- \int d^2x d\tau \Big(|(\partial_\mu - i a_\mu)\psi_1|^2 + |(\partial_\mu - i a_\mu)\psi_2|^2 + r\left(|\psi_1|^2 + |\psi_2|^2\right) + \frac{u}{2}\left(|\psi_1|^4 + |\psi_2|^4\right) - \tilde{y}_m \left((\psi_1^* \psi_2)^4 + (\psi_1 \psi_2^*)^4\right) \Big) \Bigg)$$

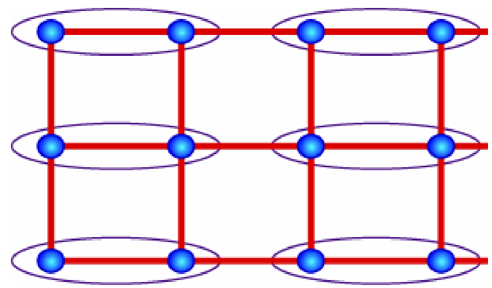
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001).
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002).
O. Motrunich and A. Vishwanath, cond-mat/0311222
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, *Science* **303**, 1490 (2004).

Phase diagram of S=1/2 square lattice antiferromagnet

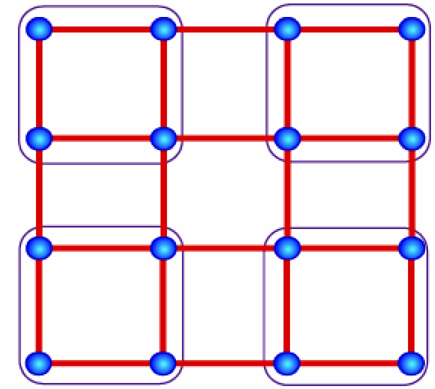


Neel order

$$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$$



or



Bond order $\langle \Psi_{\text{bond}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + r|z_\alpha|^2 + \frac{u}{2}\left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

at its critical point $r = r_c$, where A_μ is *non-compact*

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).
S. Sachdev cond-mat/0401041.