# Strange metals and black holes

The Society of Physics
St. Xavier's College, Mumbai
August 26, 2021

Subir Sachdev



Talk online: sachdev.physics.harvard.edu


INSTITUTE FOR ADVANCED STUDY

PHYSICS
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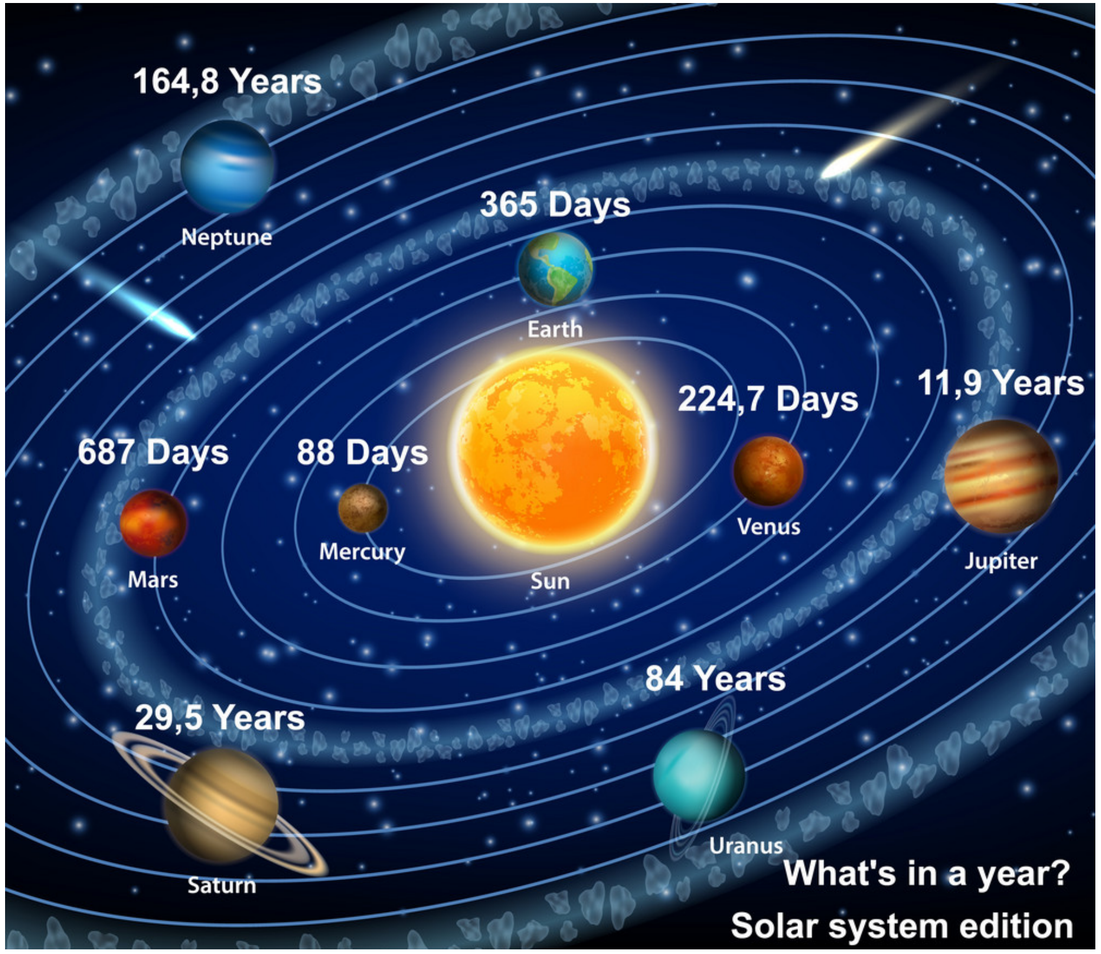
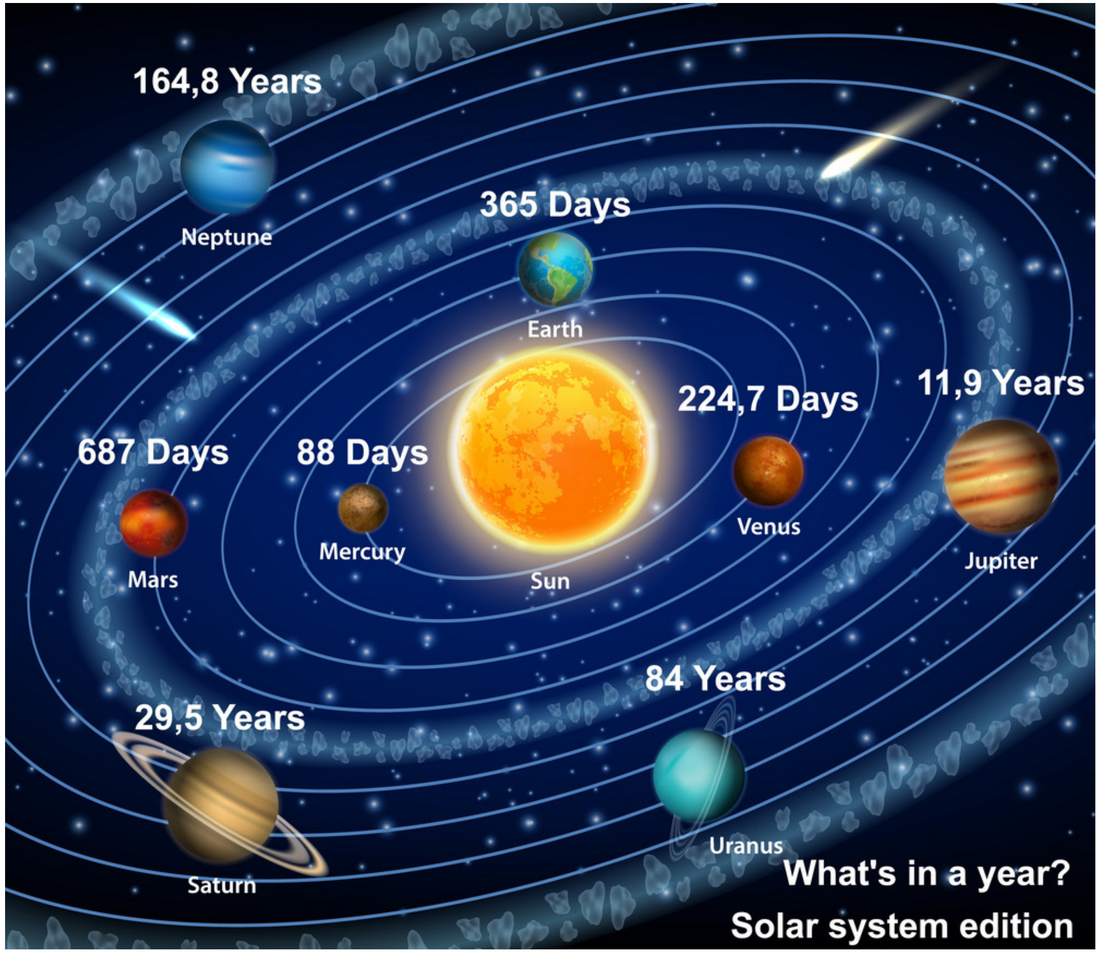
HARVARD




Newton showed (1687) that the same laws of motion applied on planetary length scales ($\sim$ 1 trillion meters) and the length scale of an apple tree (1 meter).

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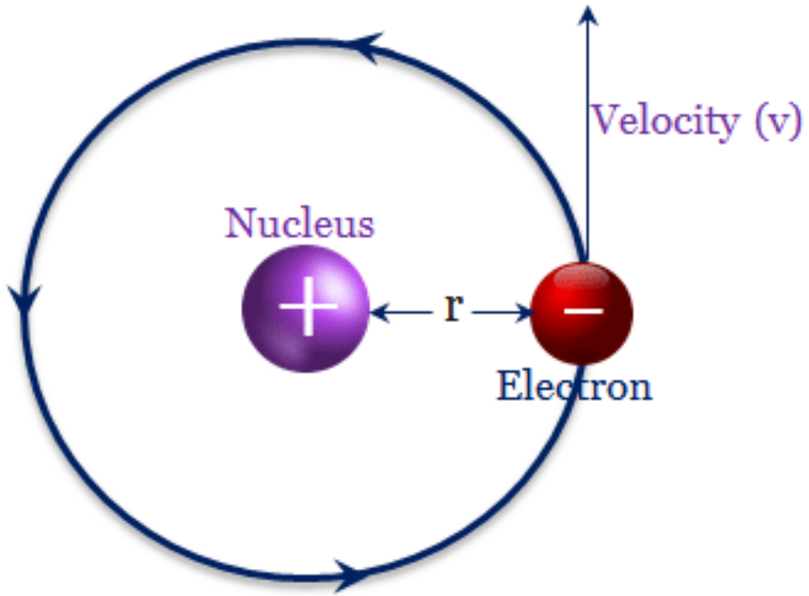
What happens on smaller distances ?

# Quantum theory of electrons, one at a time: metals and insulators

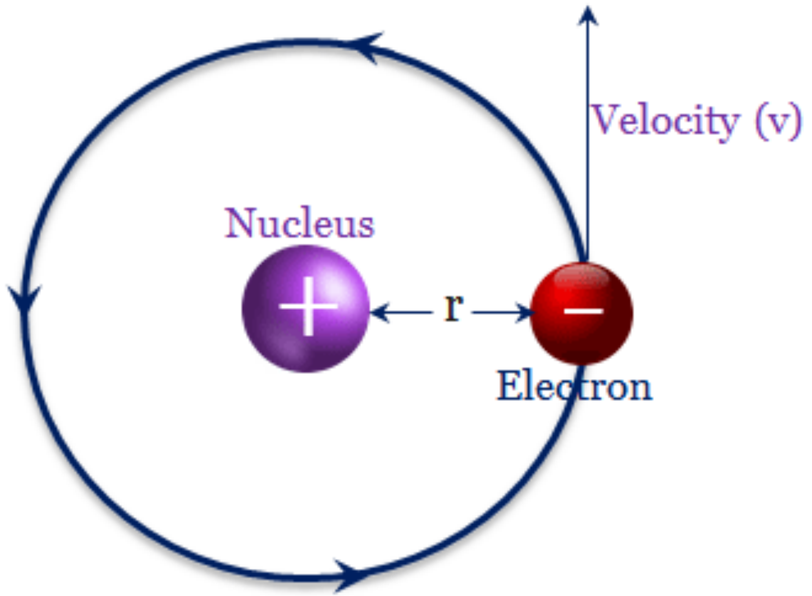
# Hydrogen atom




$\Rightarrow 10^{-10}$ meters $\Leftarrow$

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

# Hydrogen atom

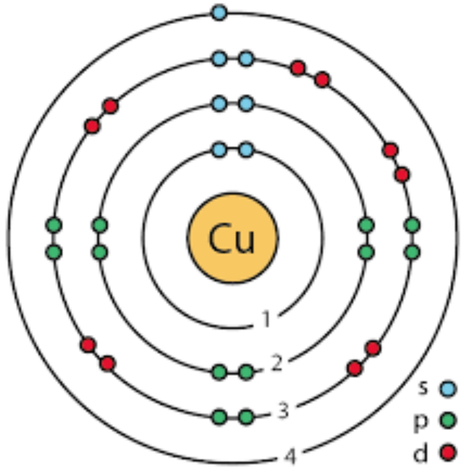


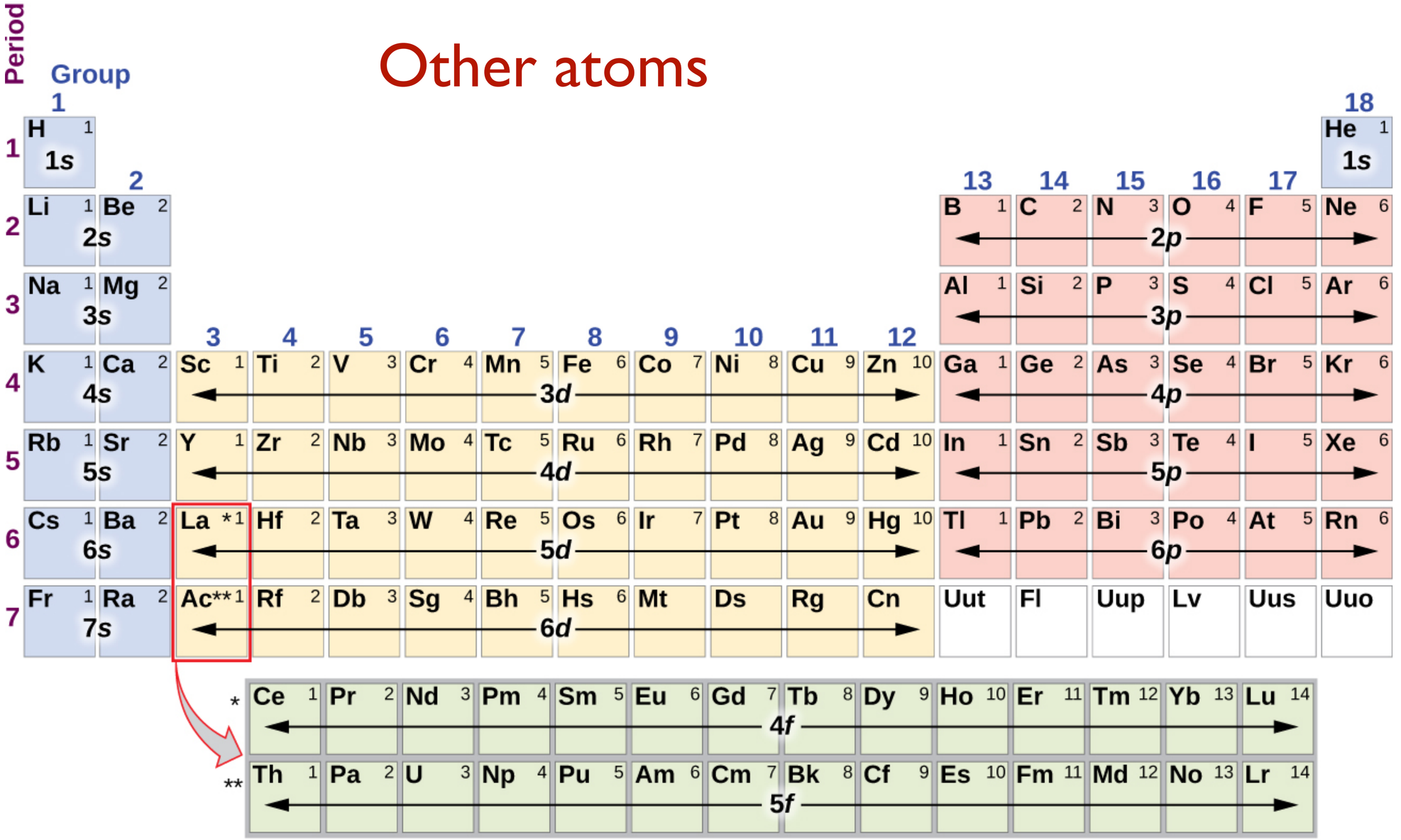

$\Rightarrow 10^{-10}$ meters $\Leftarrow$

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

It is described by the quantum theory of Schrödinger and Heisenberg (1925).

# Other atoms



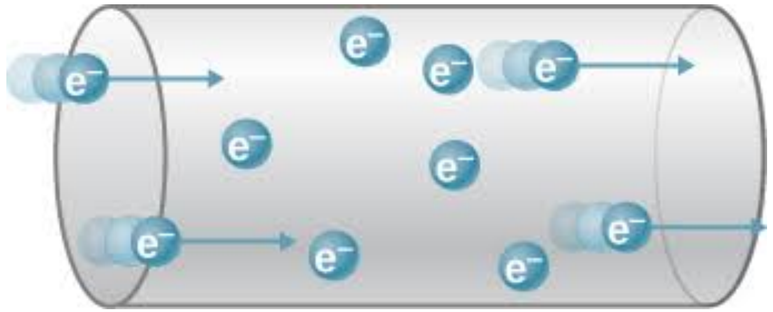



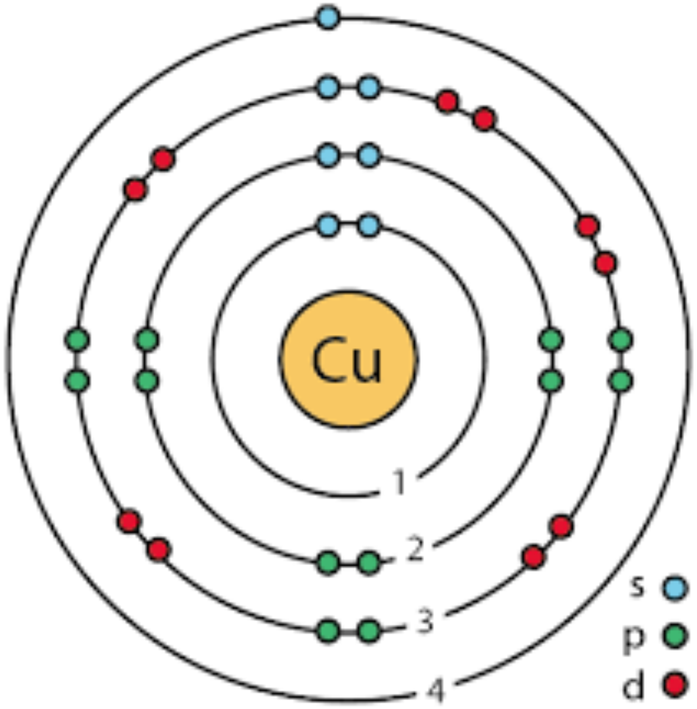

The periodic table follows from $(i)$ the exclusion principle, and $(ii)$ each electron has 2 spin states $|\uparrow\rangle$, and $|\downarrow\rangle$.

# *Ordinary metals*


Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

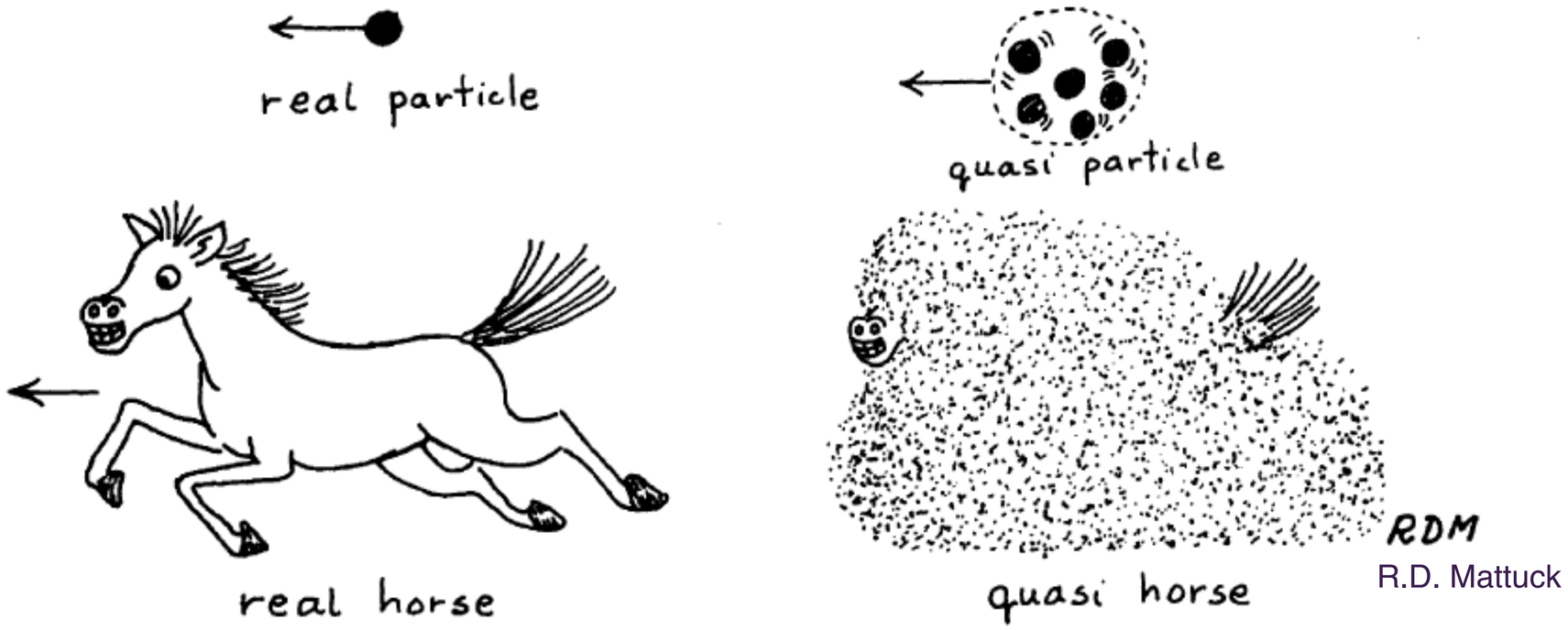
# *Copper*



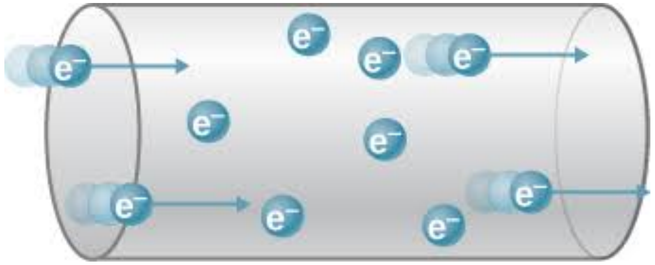

Each copper atom donates its outermost electron
These electrons move freely throughout the crystal and carry current

*Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle. The existence of quasiparticles implies limited many-particle entanglement*




R.D. Mattuck

## *Current flow with quasiparticles in Copper*

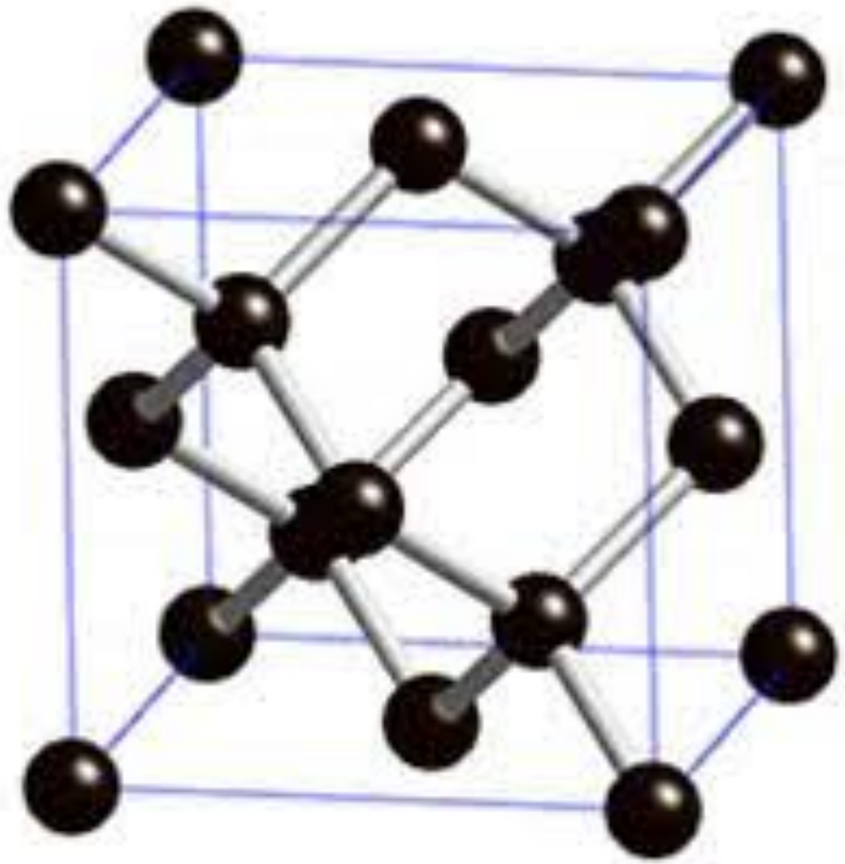


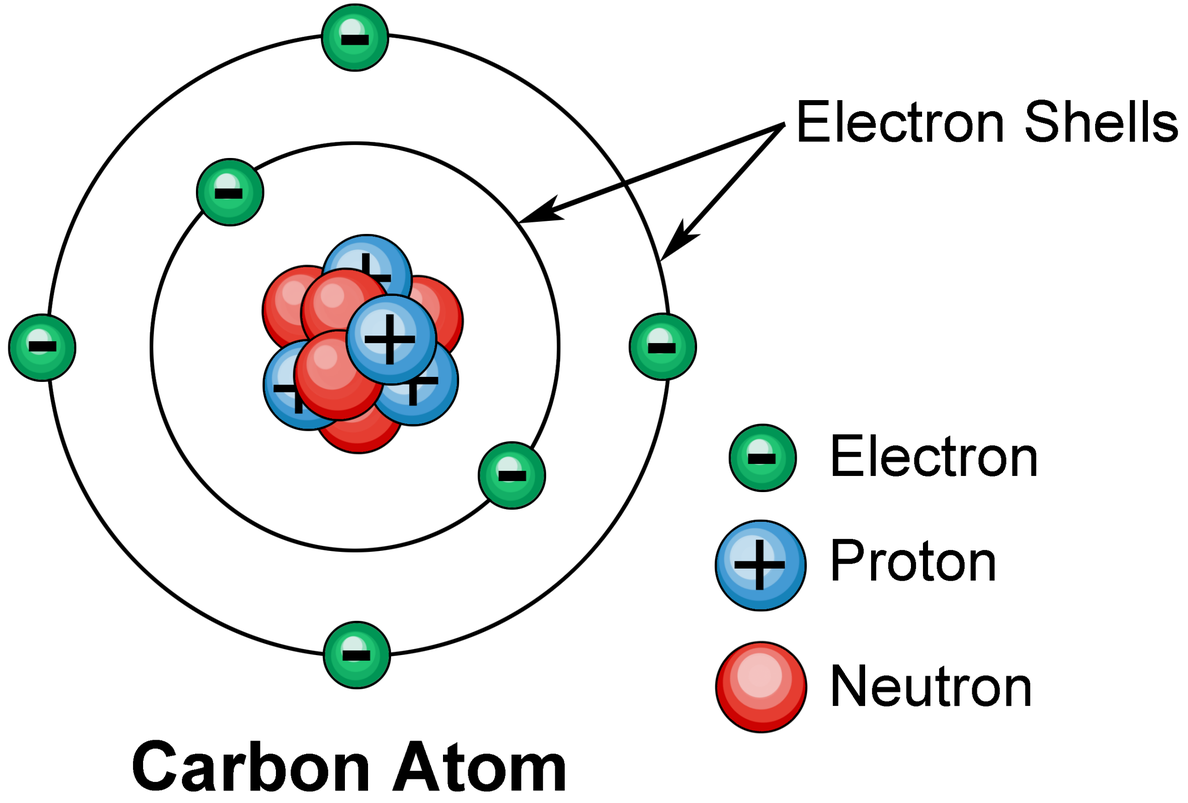

Flowing quasiparticles scatter off each other in a typical scattering time $\tau$

This time is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

# *Diamond - a very good insulator*






Each carbon atom donates 4 electrons

These electrons occupy filled "bands" and are not able to carry current

# Quantum entanglement of electron pairs: superconductivity

The most remarkable new idea in the quantum theory is the
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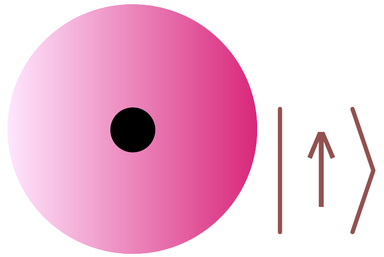
a physical system can be in a
superposition of two (or more) distinct states.

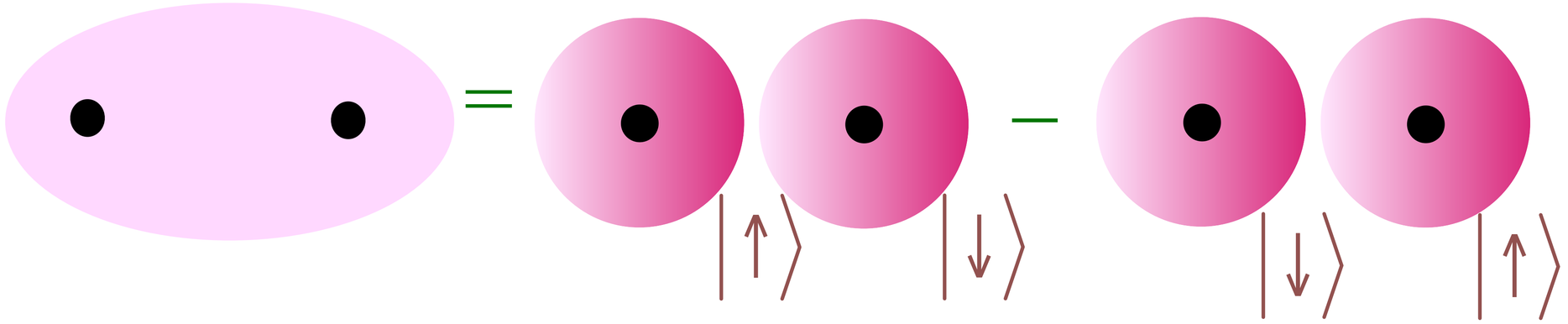
# Molecules

Hydrogen atom:



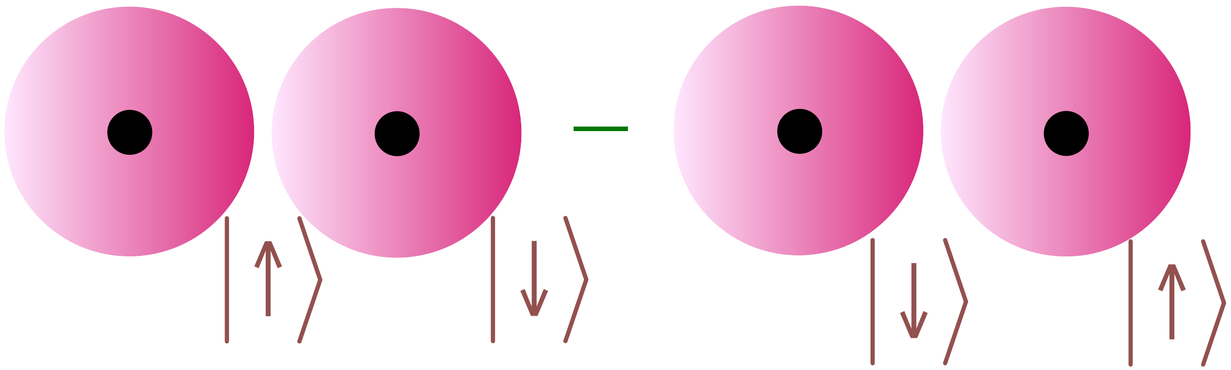

# Covalent bond

Hydrogen molecule:



# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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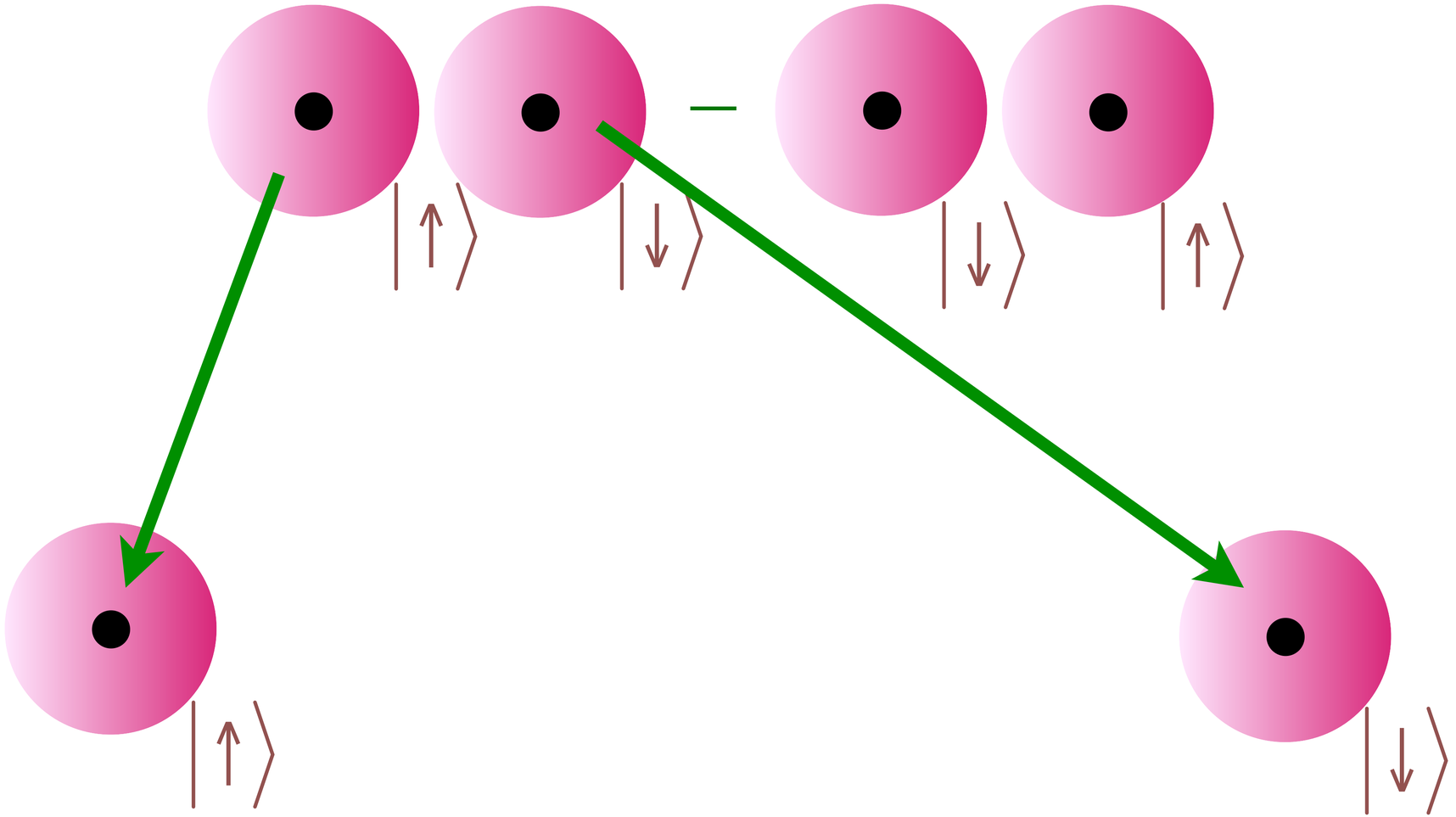

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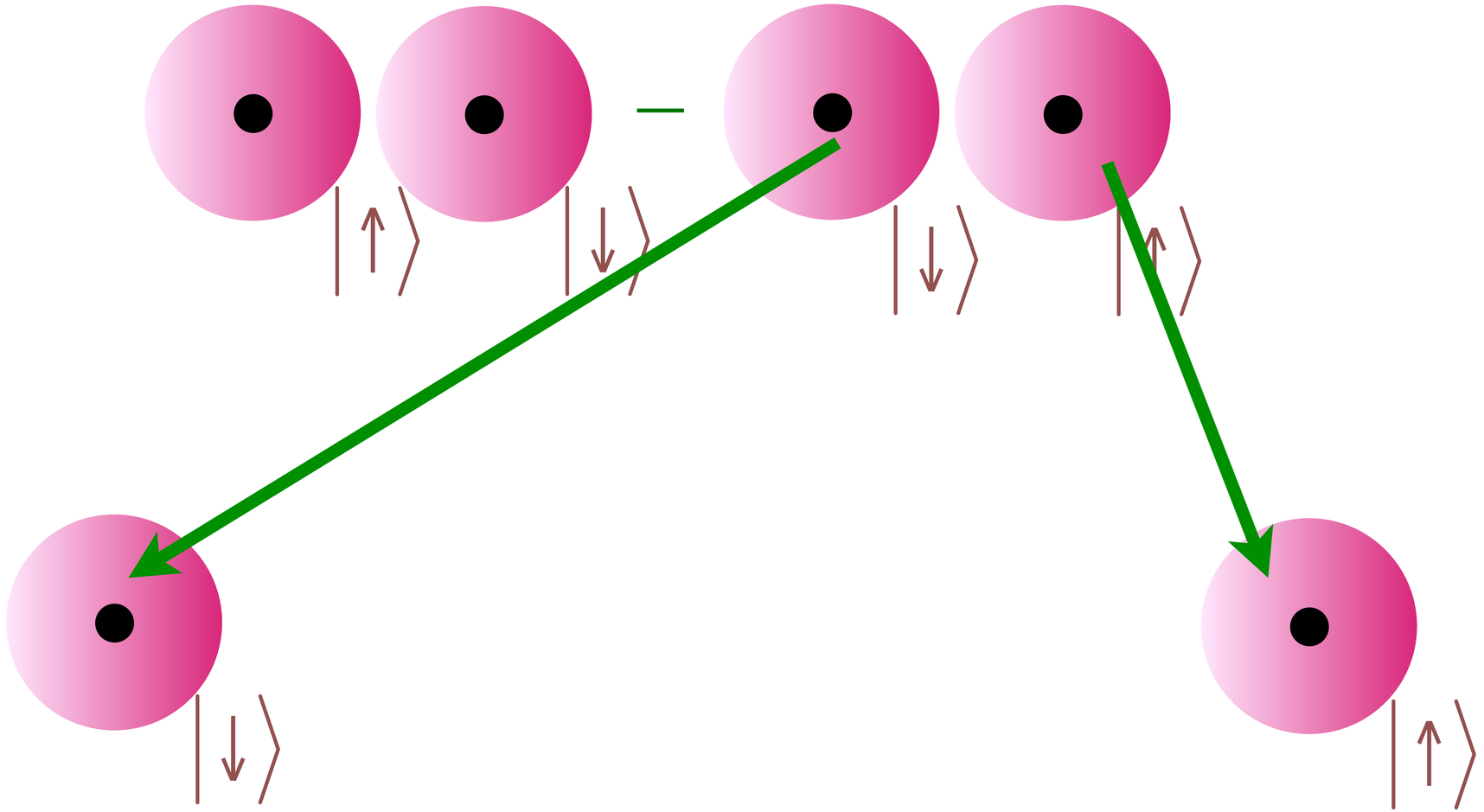

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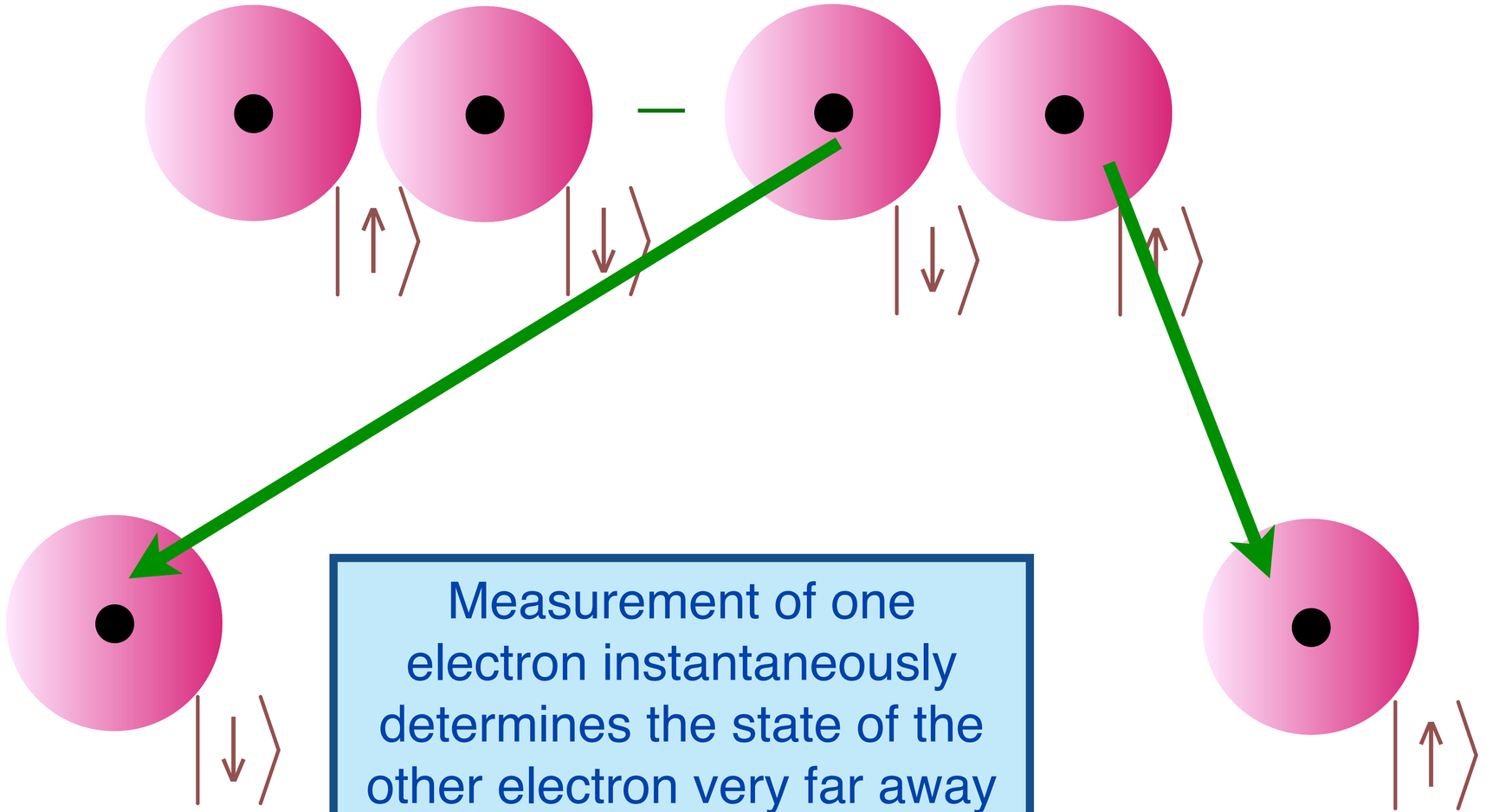

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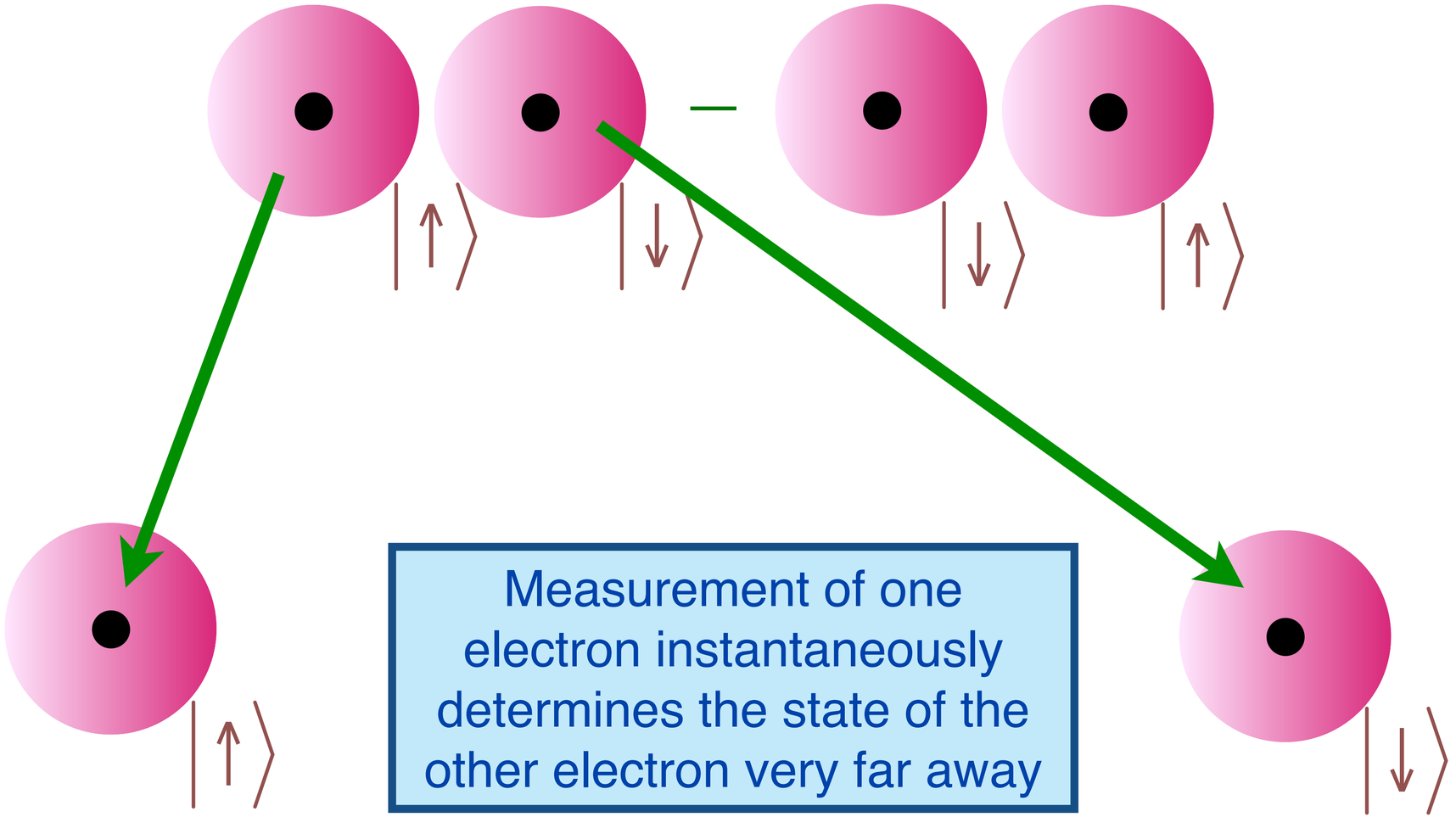
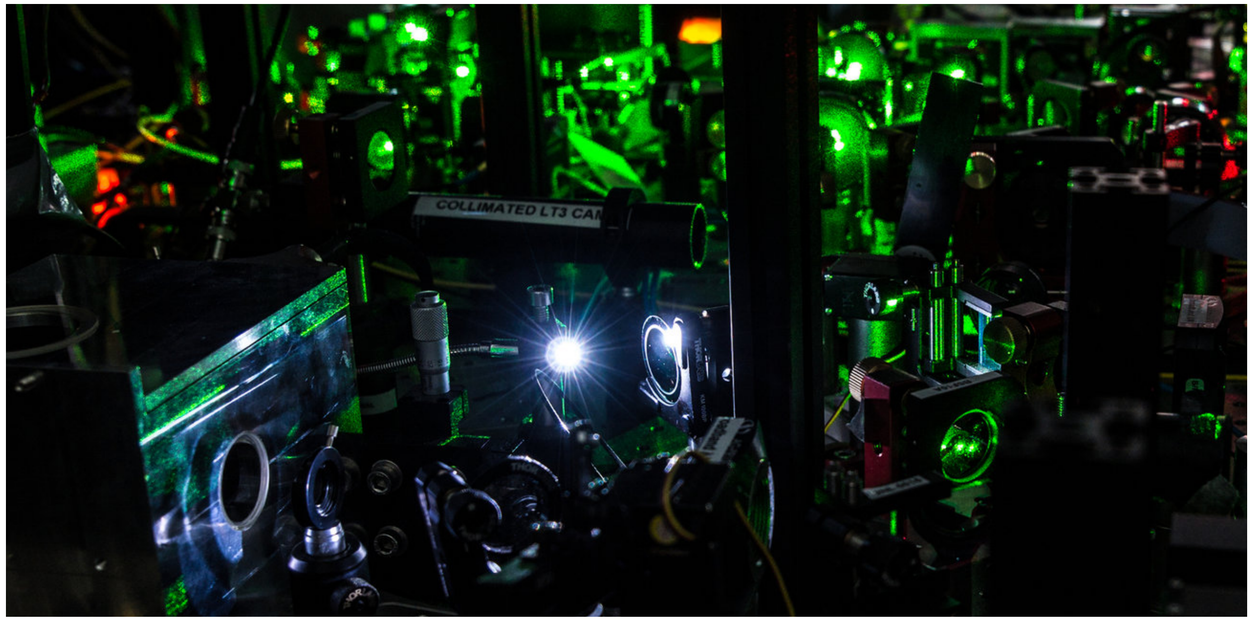


The New York Times

# Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

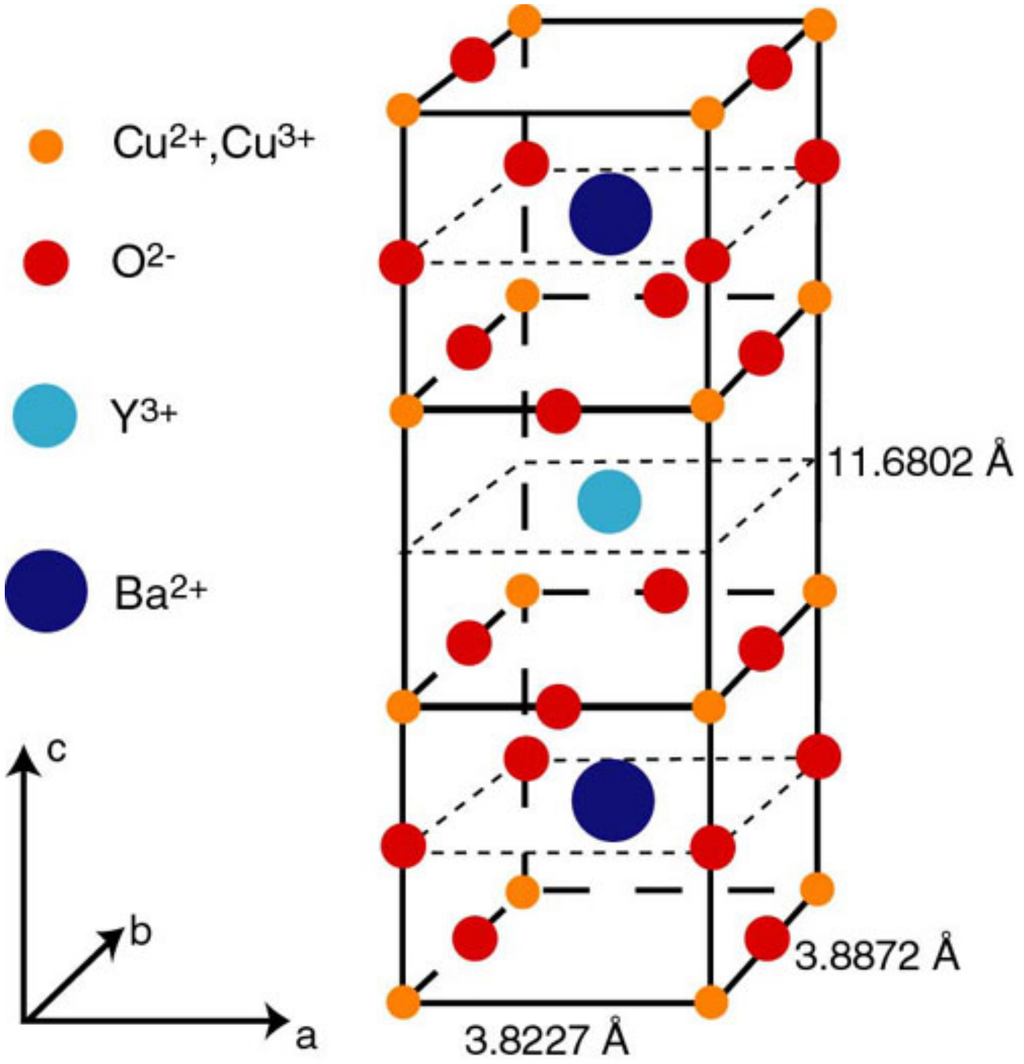
By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

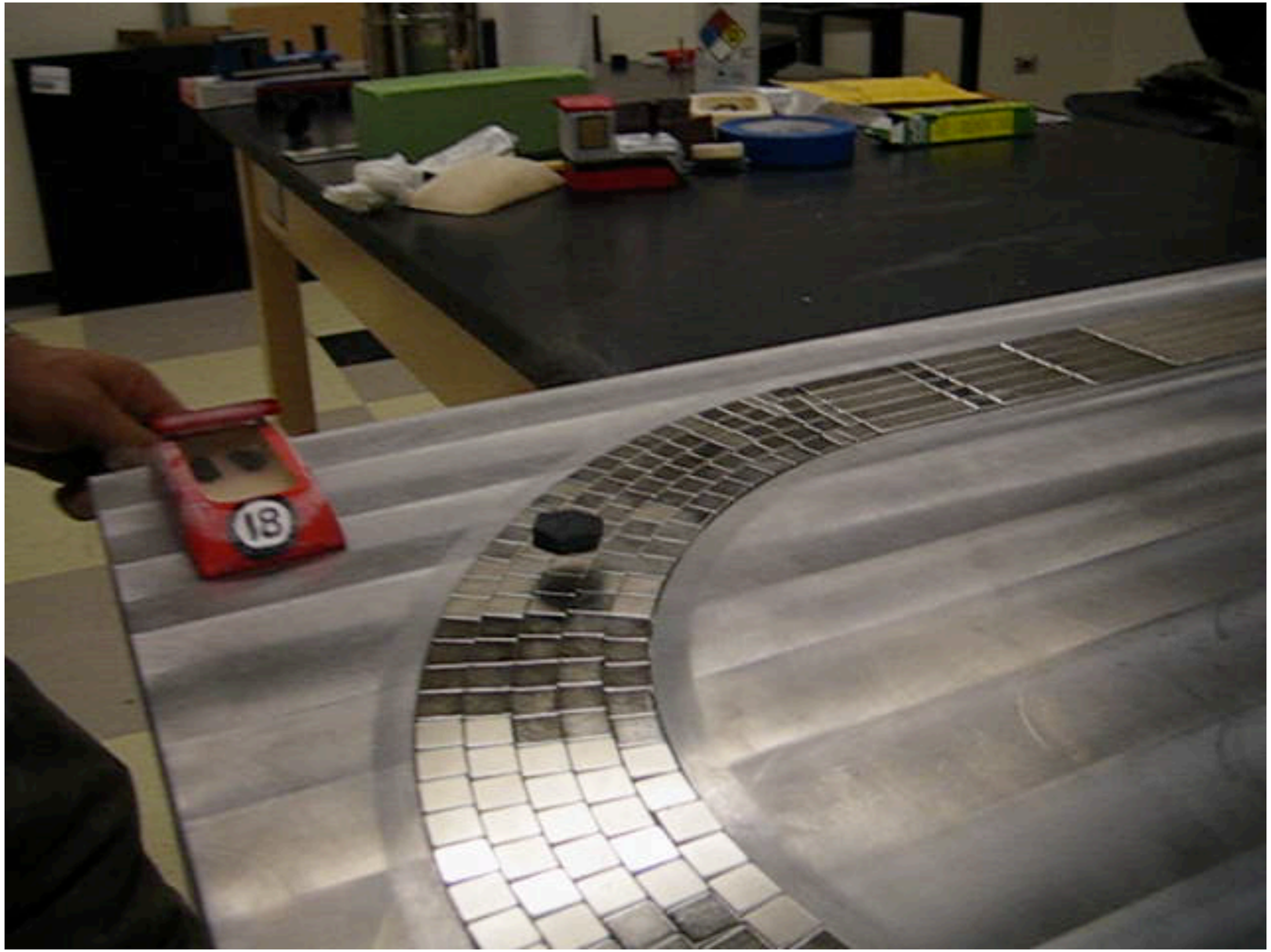



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

# High temperature superconductors

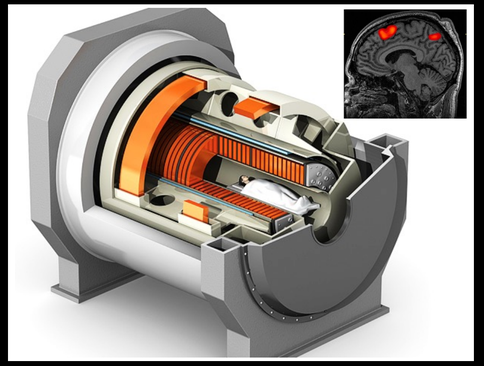

$$YBa_2Cu_3O_{6+x}$$



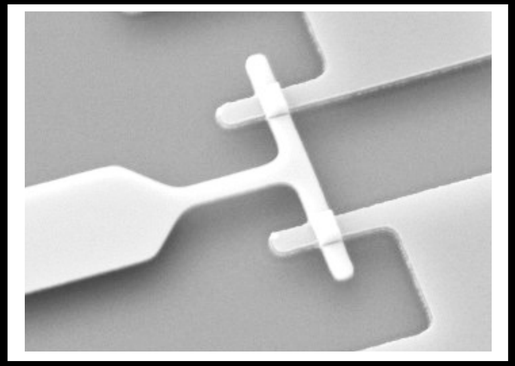

Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

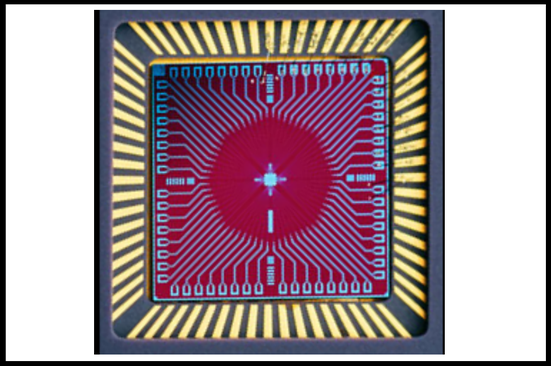
# SUPERCONDUCTIVITY: SCIENTIFIC APPLICATIONS



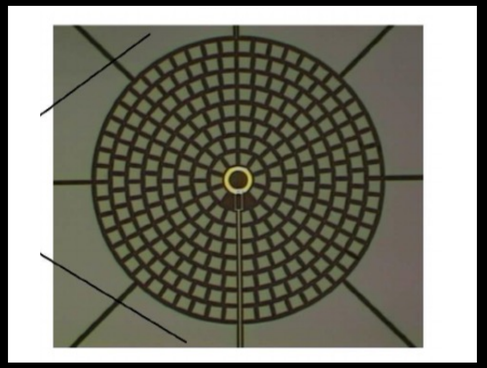
FUNCTIONAL MRI



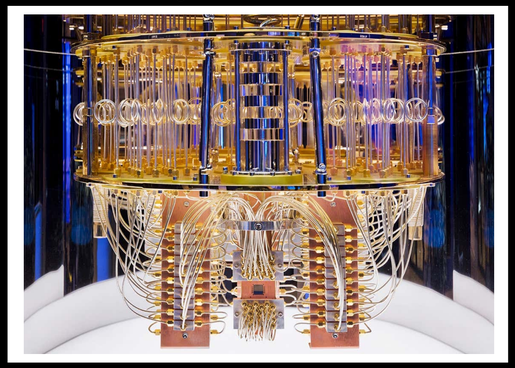
SQUID SENSORS



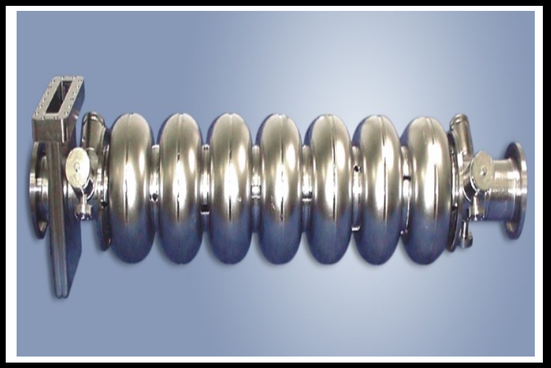
SINGLE PHOTON IMAGING



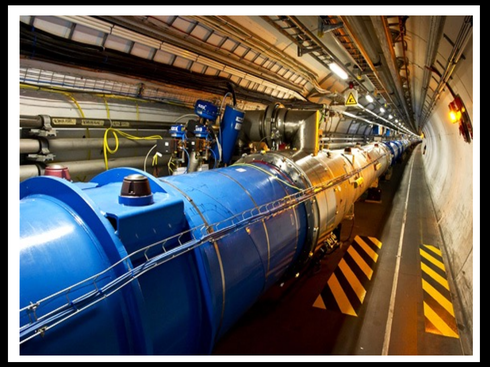
TRANS. EDGE BOLOMETER



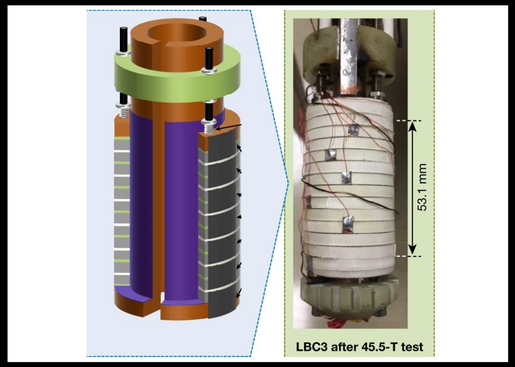
QUANTUM INFO. TECH.



HIGH ENERGY PHYSICS



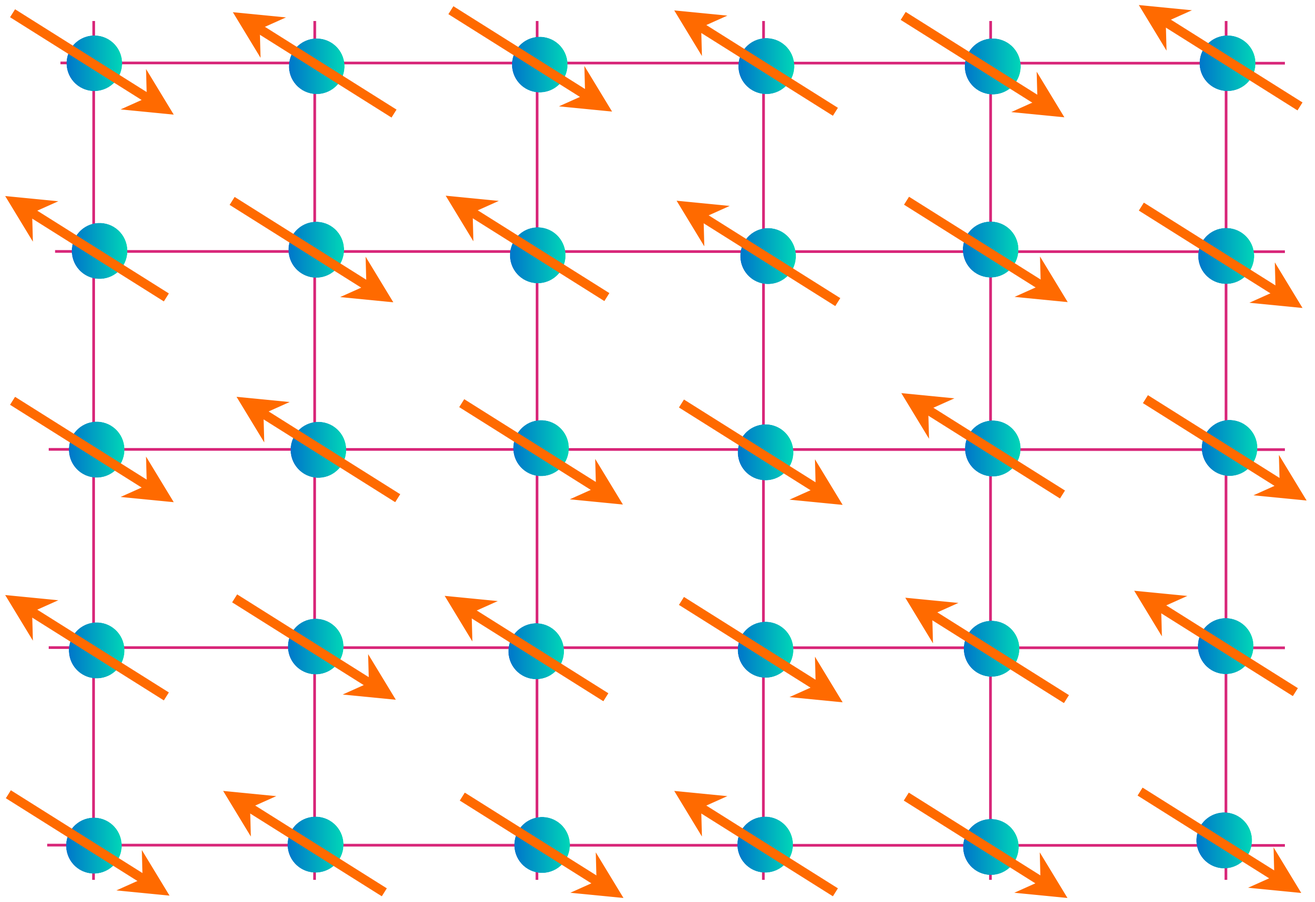
ACCELERATORS

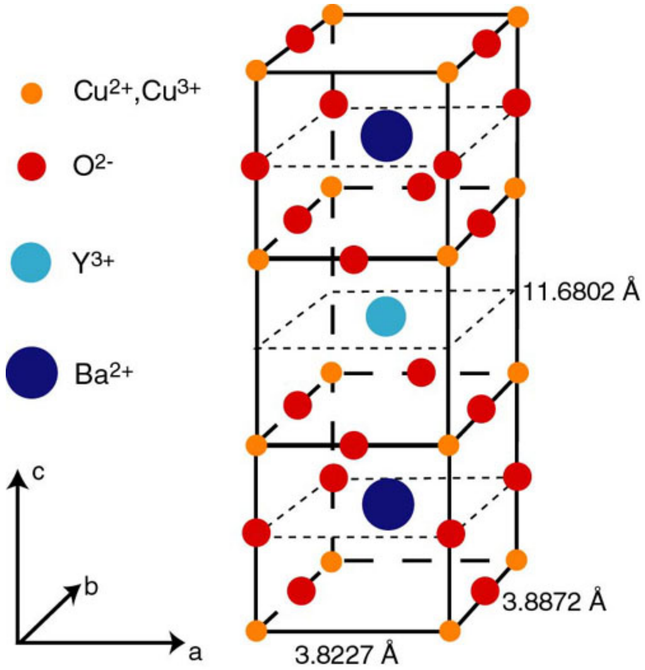



50+ TESLA MAGNETS

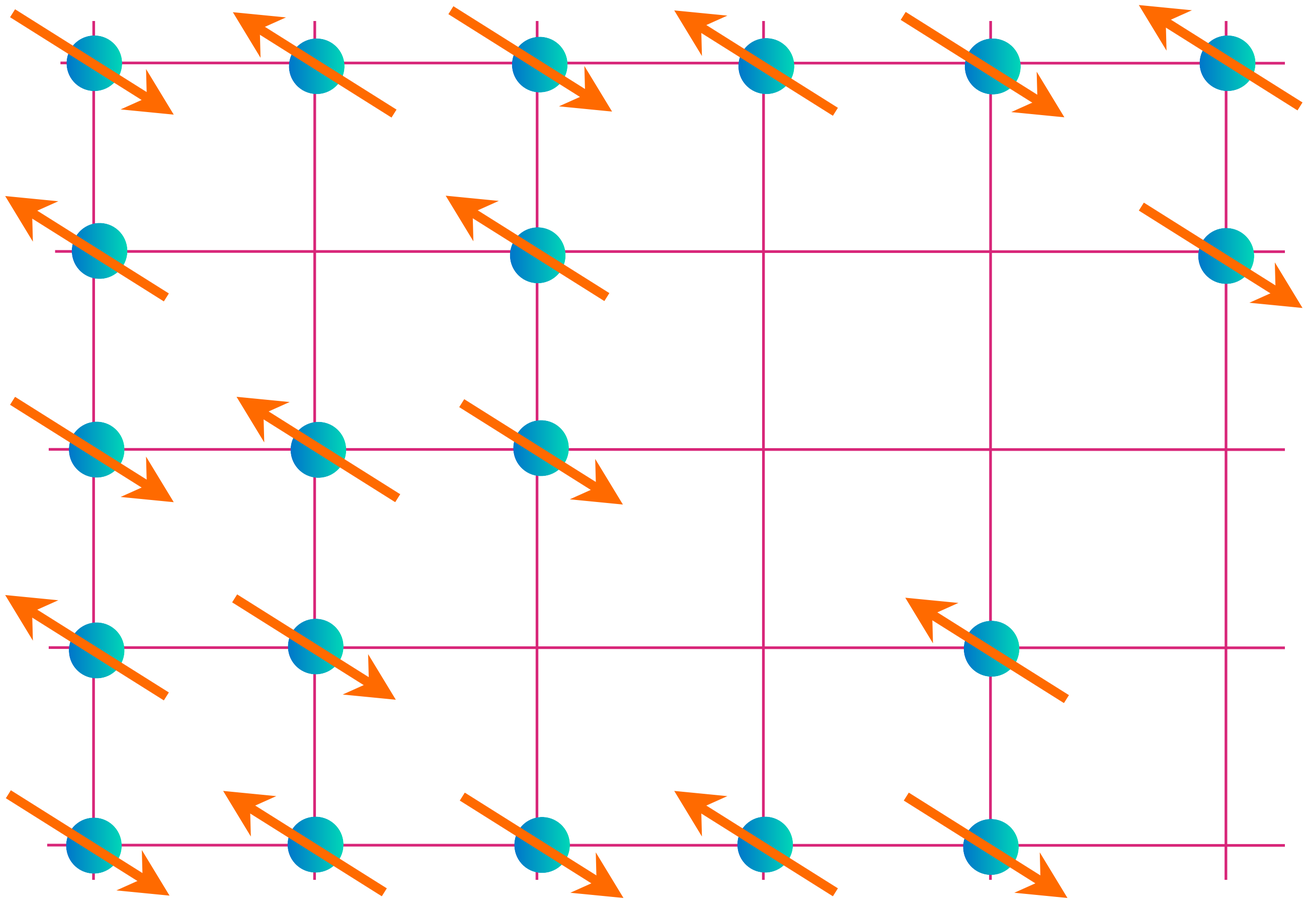
TOKOMAK FUSION

# Insulating antiferromagnet





# Antiferromagnet doped with hole density $p$



Remove fraction $p$ electrons

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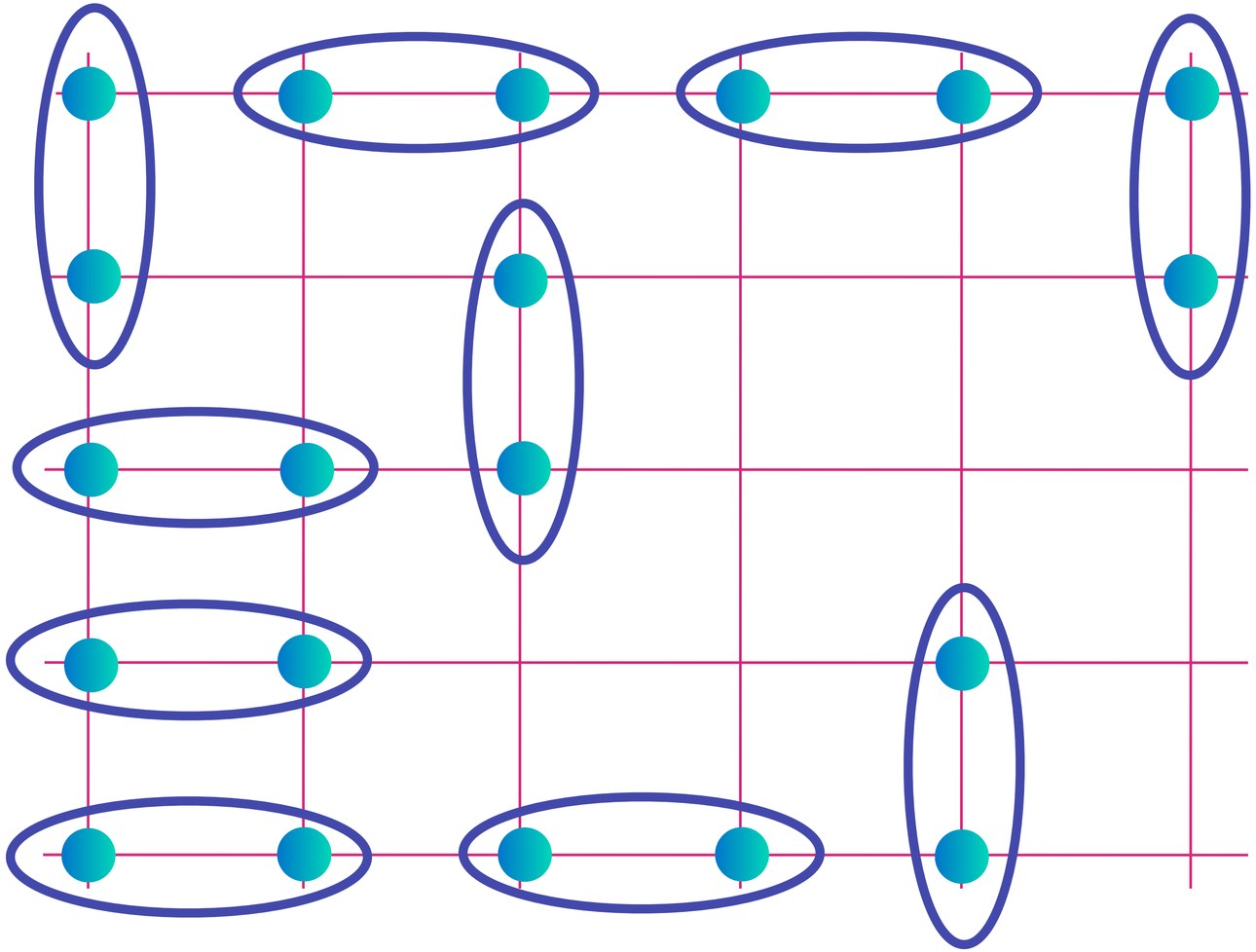

$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Motion of electron pairs leads to Bose-Einstein condensation and superconductivity

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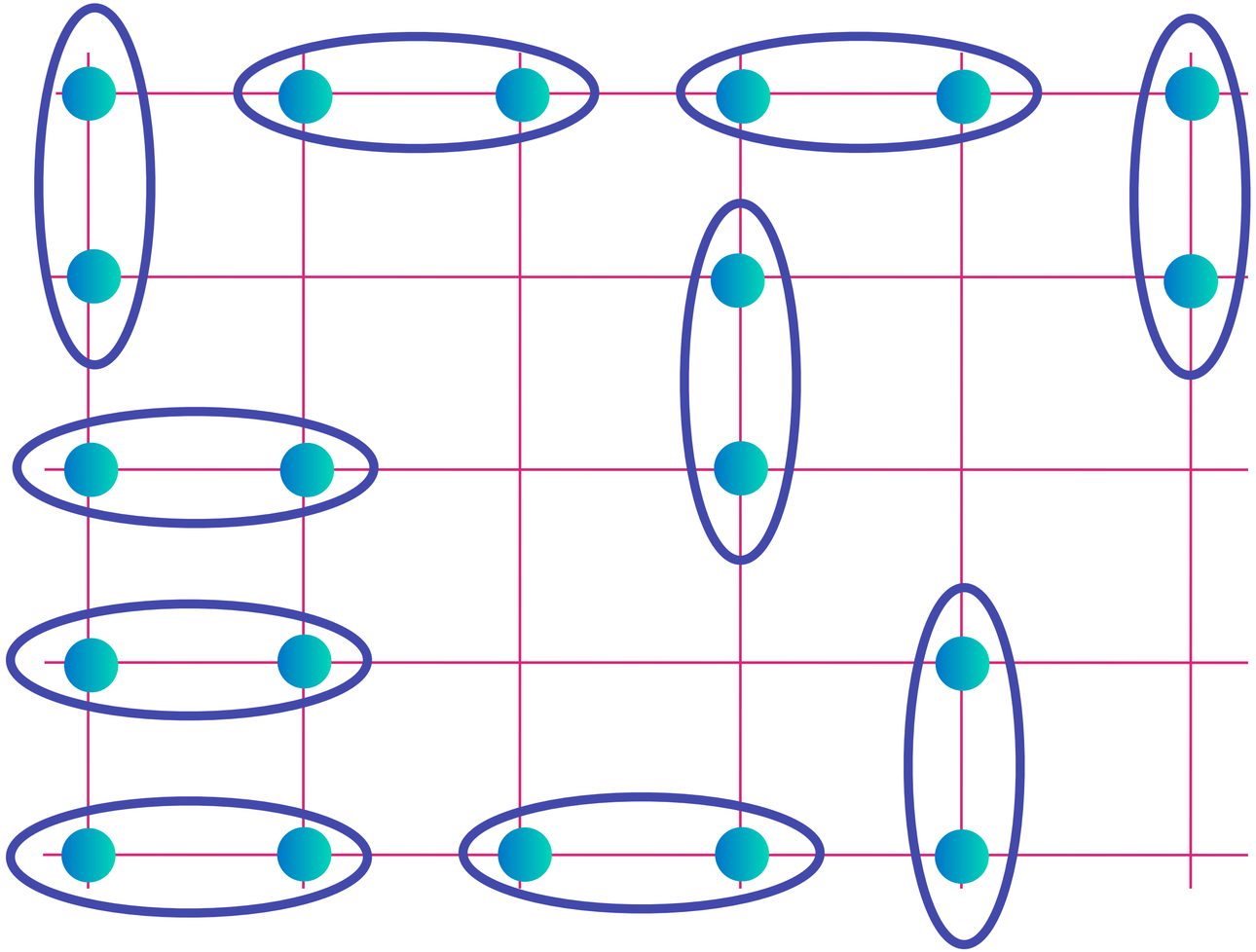

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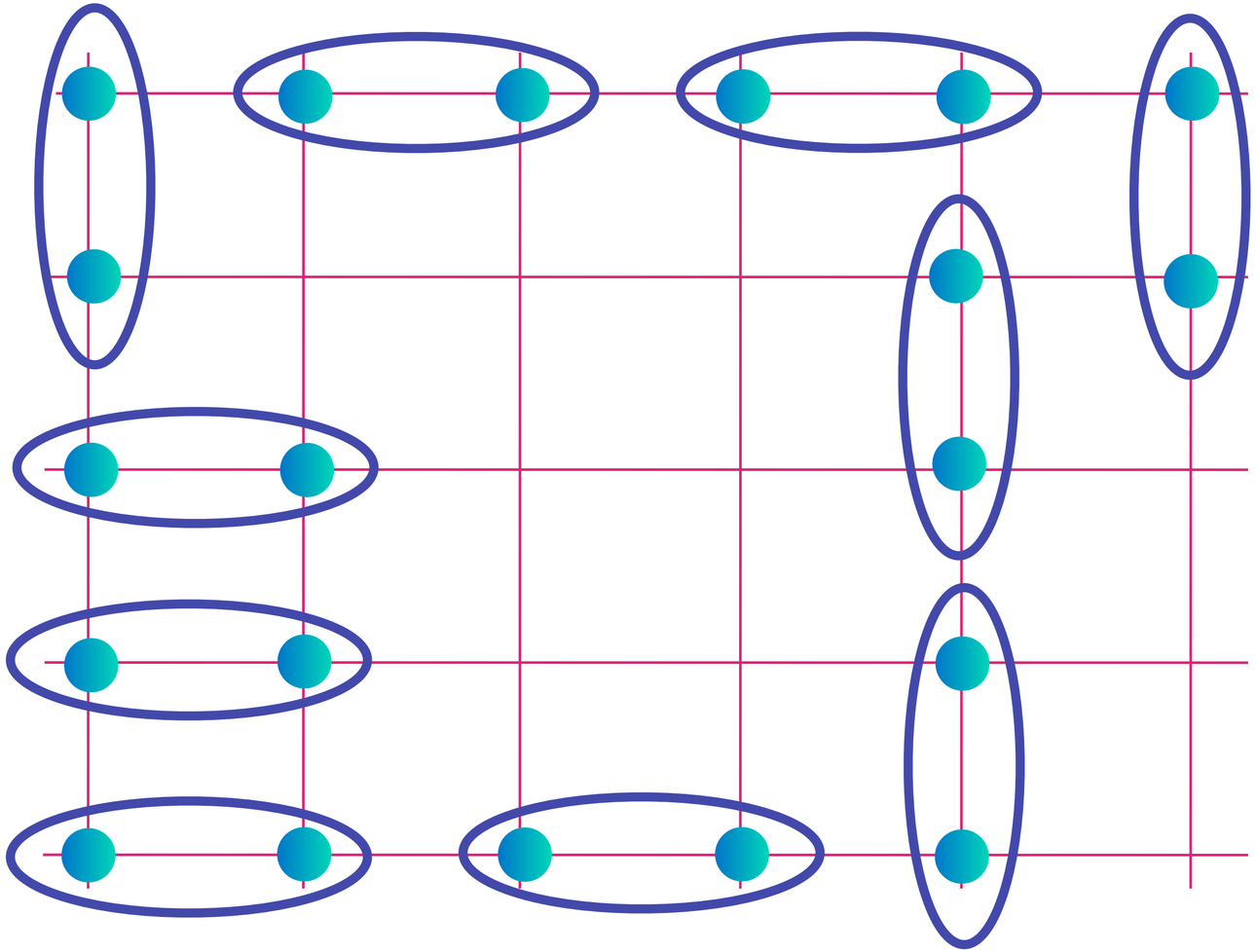

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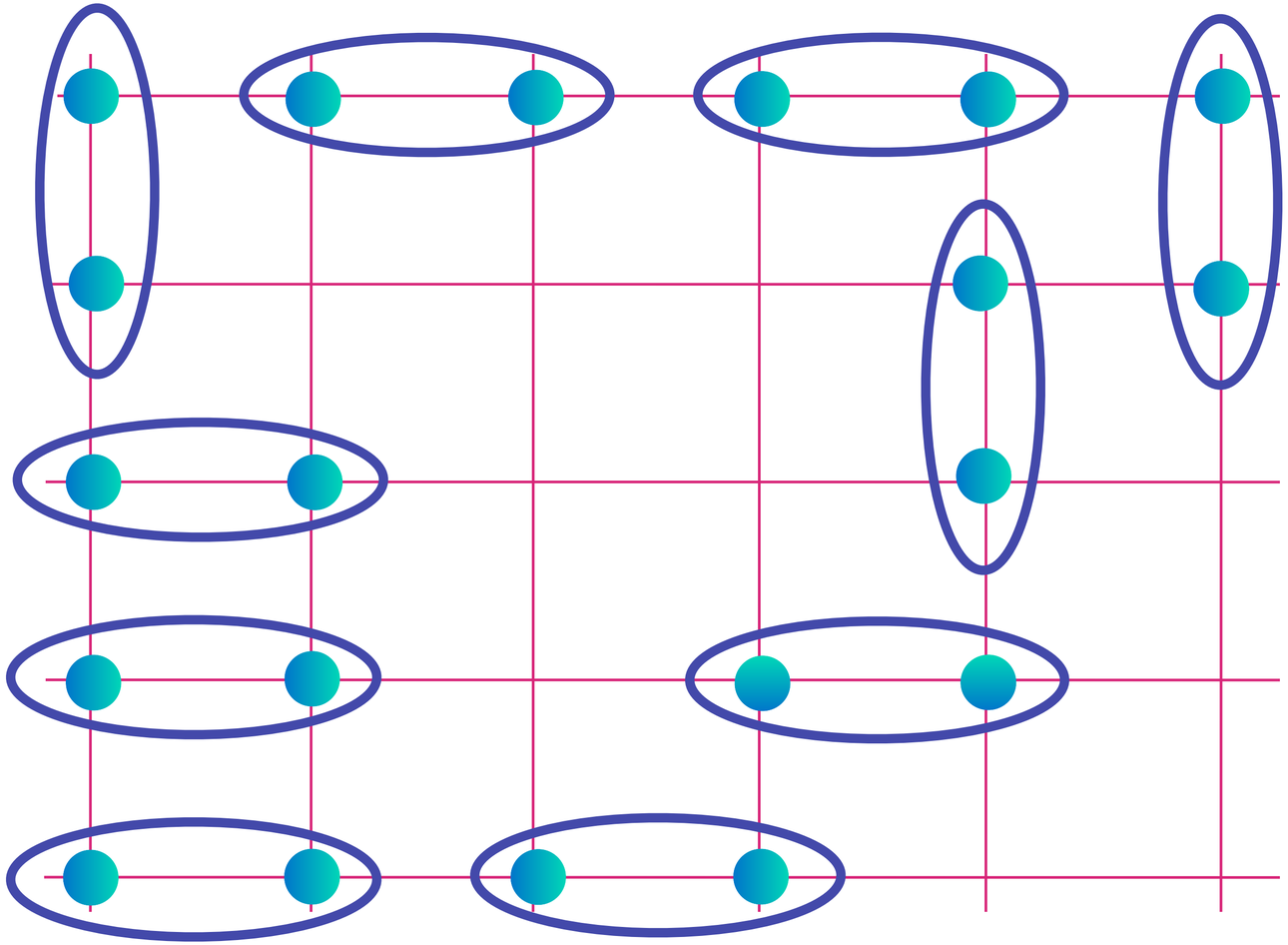

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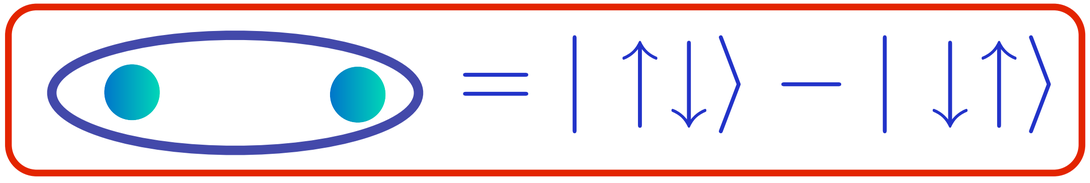

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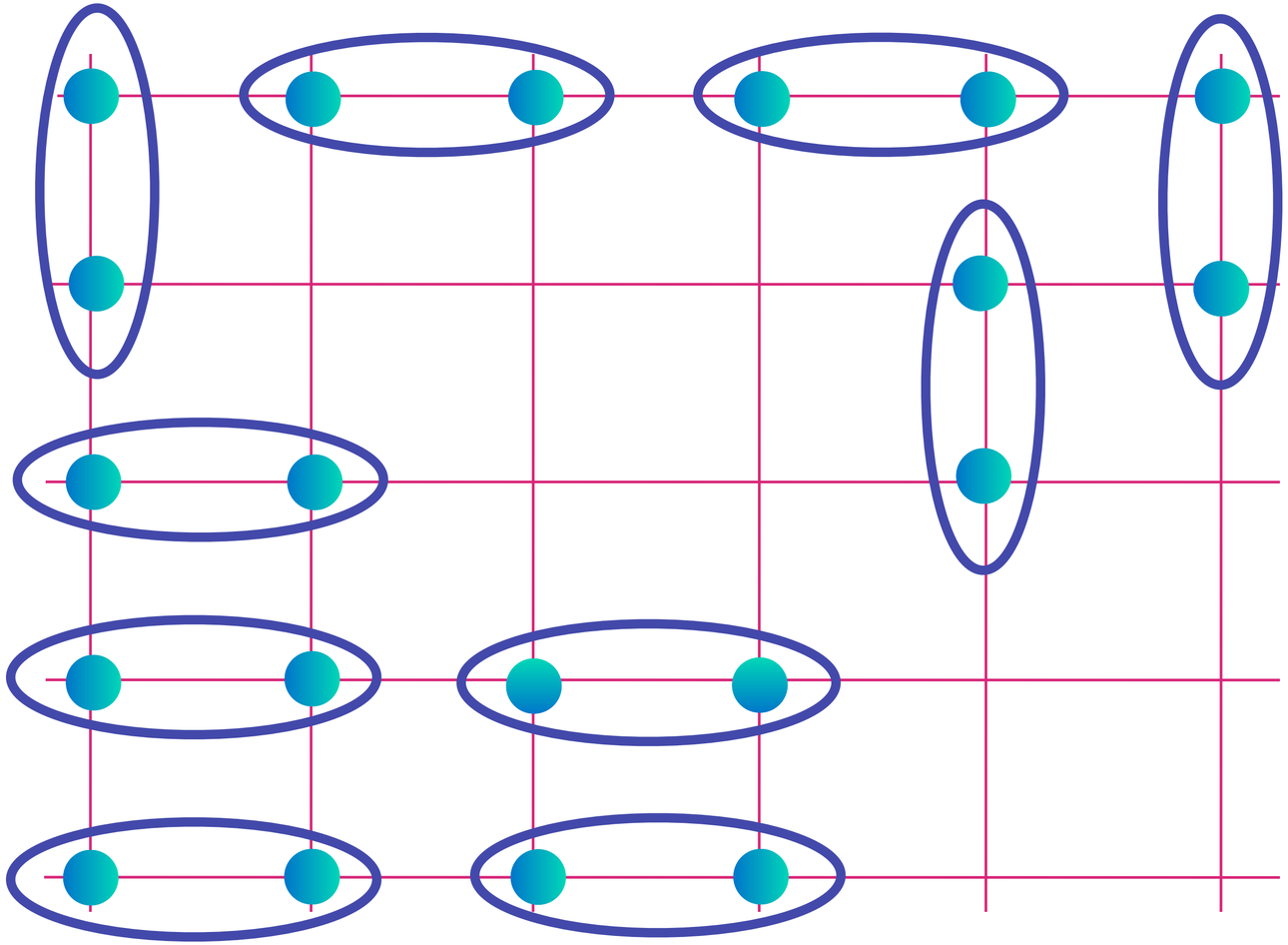

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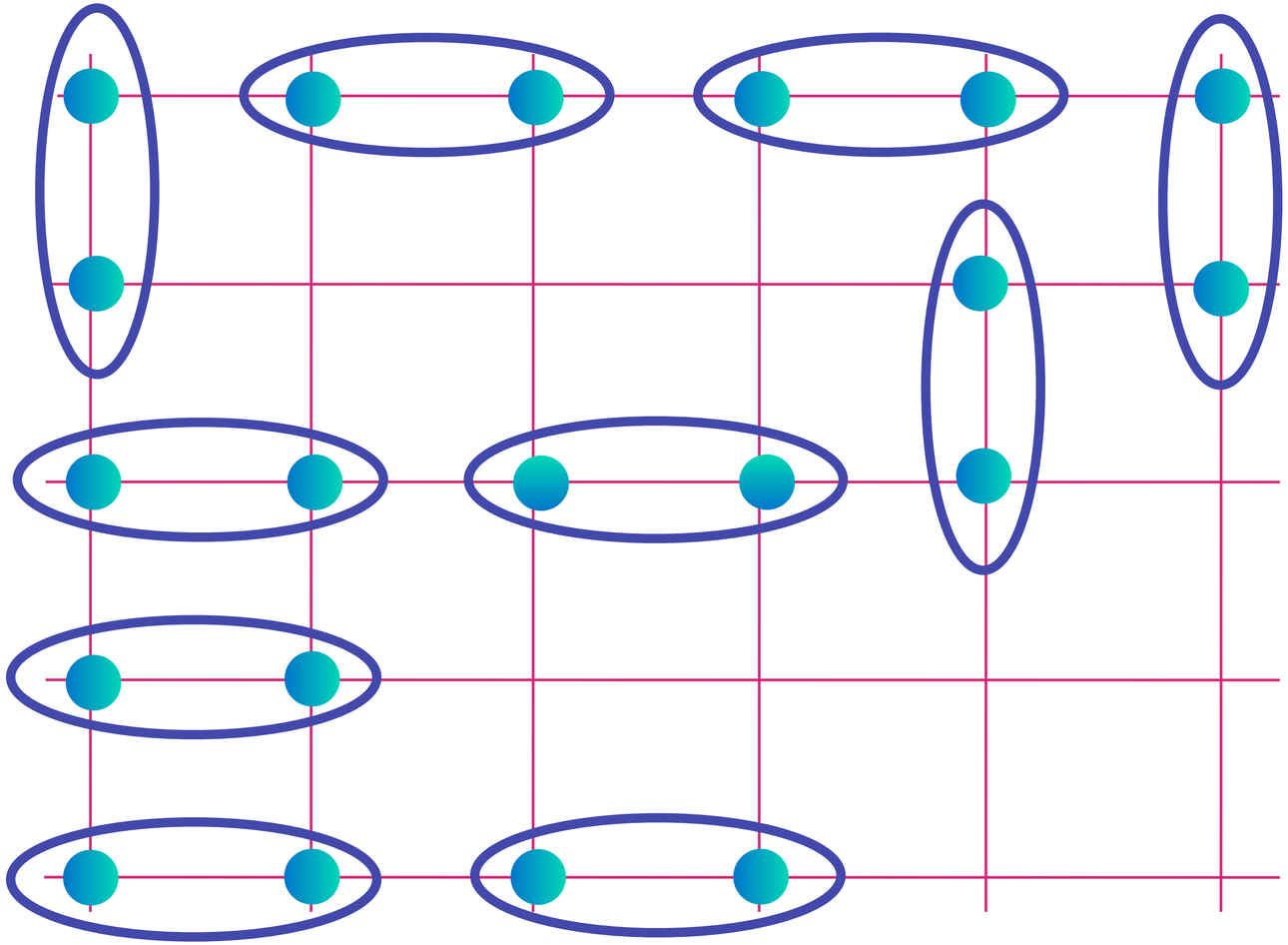

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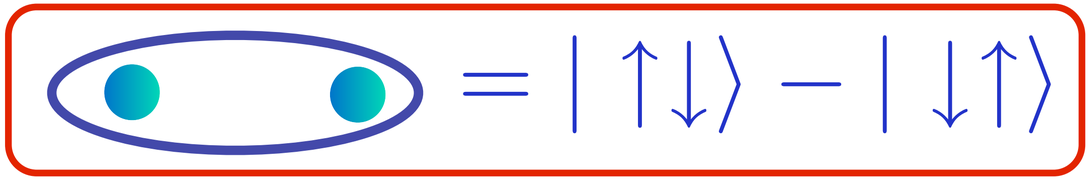

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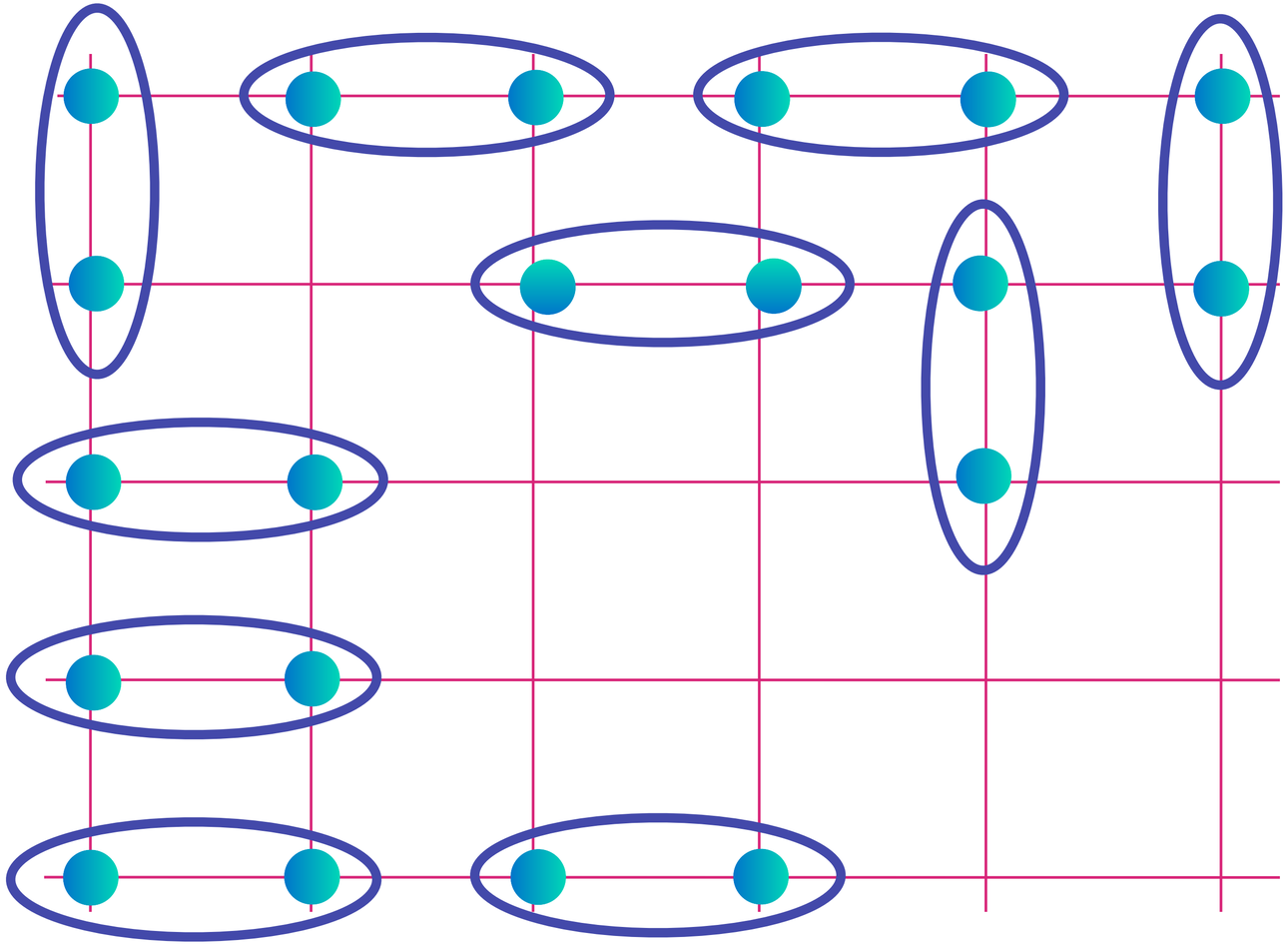

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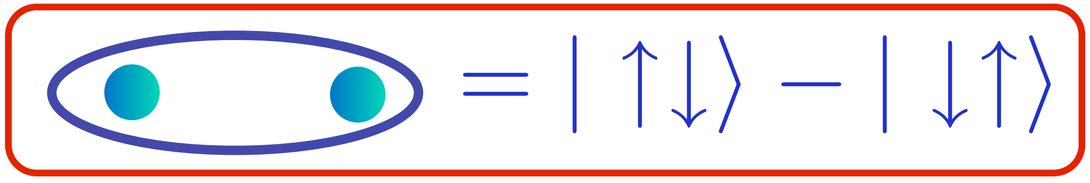

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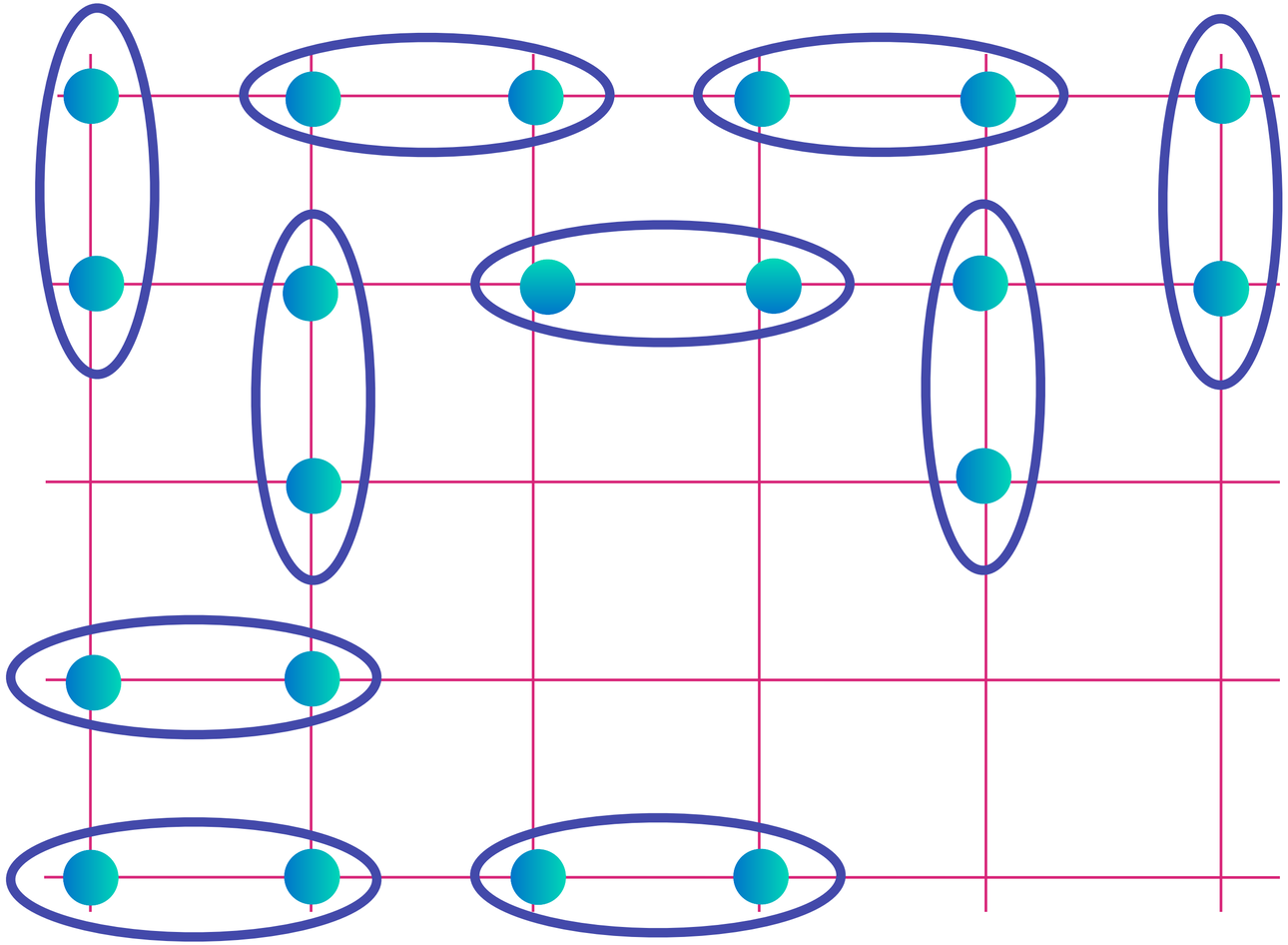

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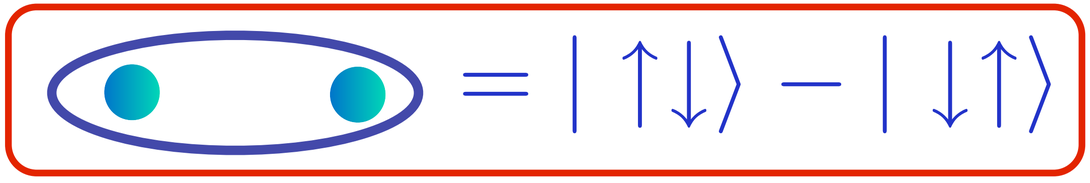

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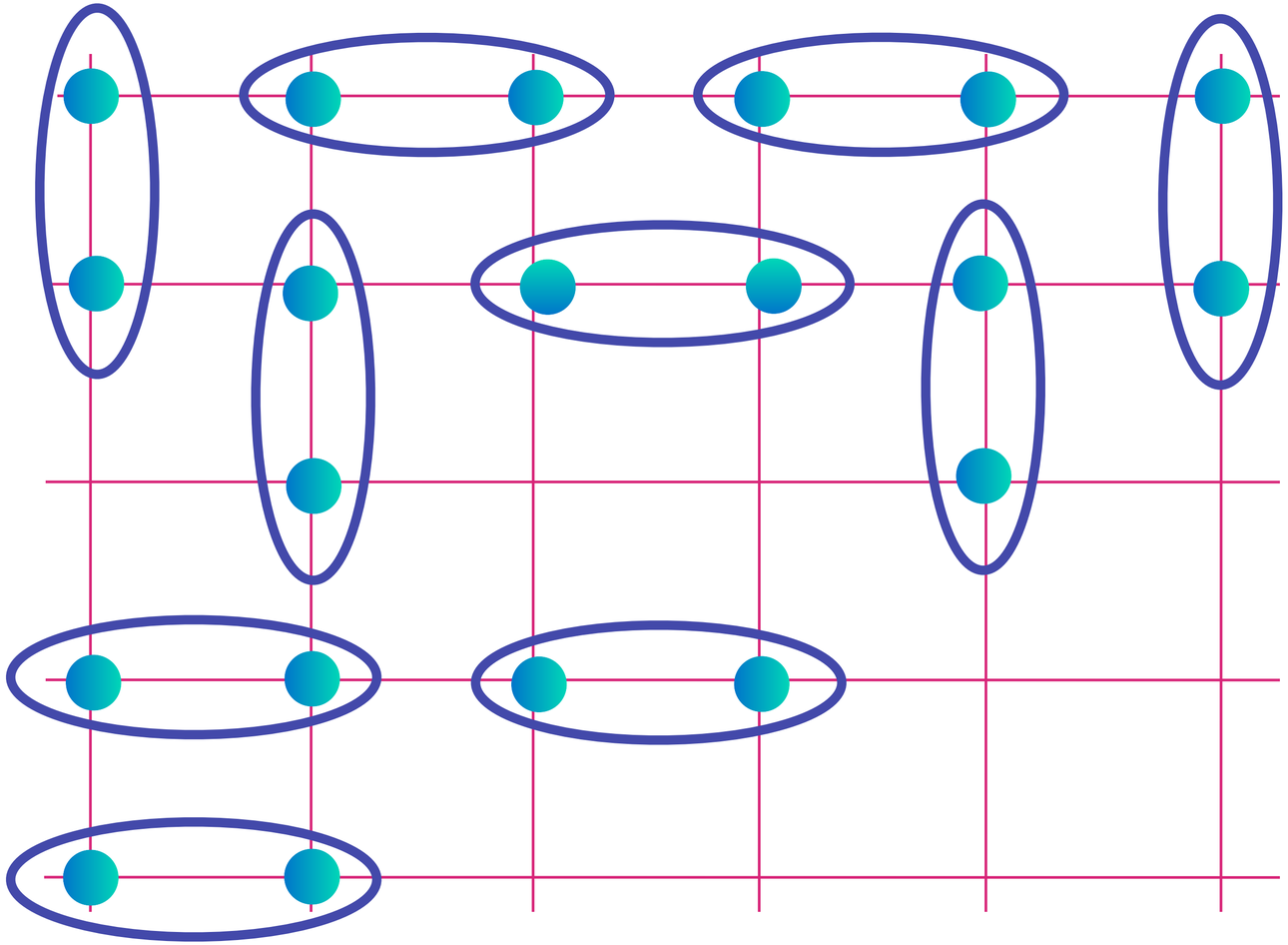

Motion of electron pairs leads to Bose-Einstein condensation and superconductivity

$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# Quantum entanglement of 2,3,4,……$\infty$ electrons: strange metals

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Can we entangle more than two electrons ?

# Molecules

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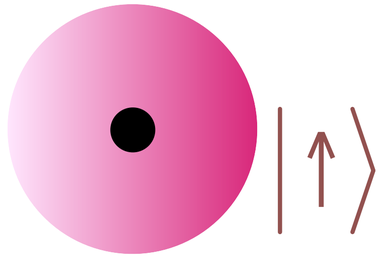


# Covalent bond

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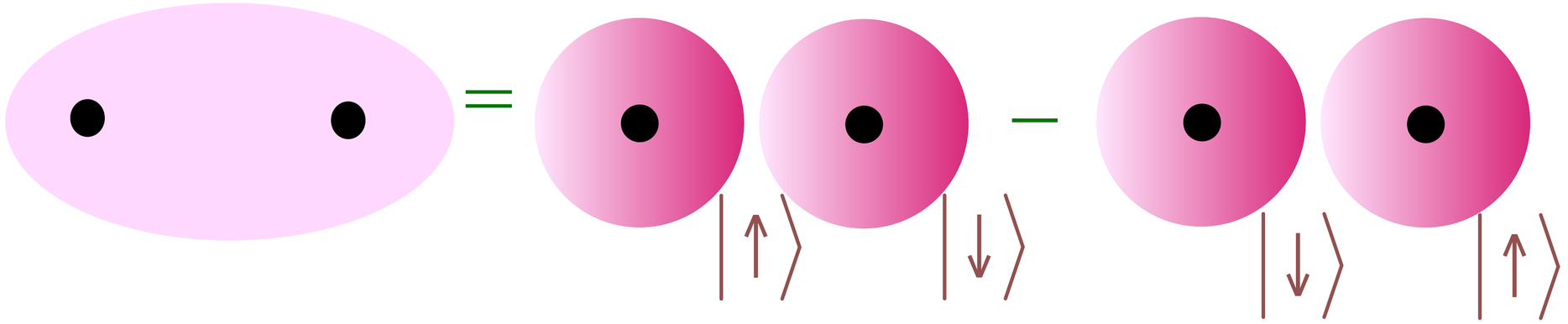

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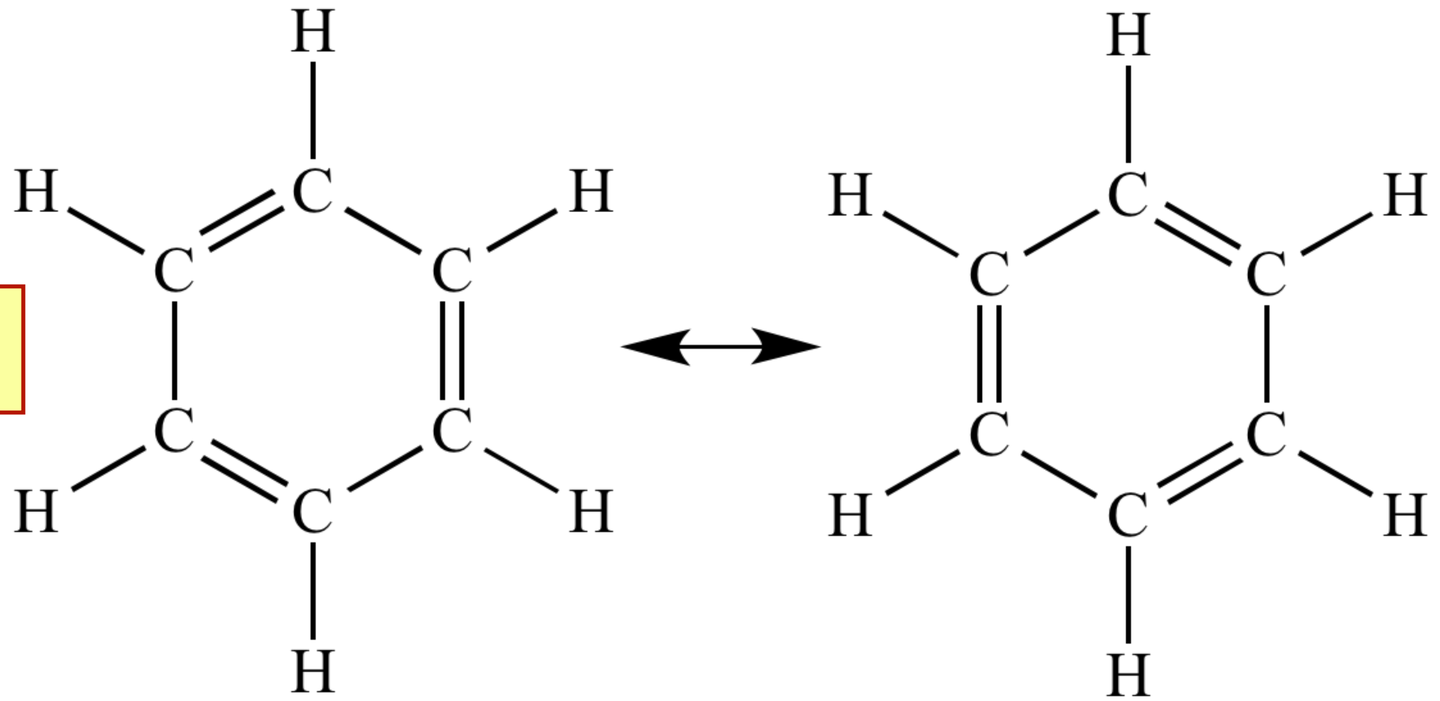
Benzene



"Resonating" covalent bonds

Benzene has a superposition of *covalent bonds*,
each of which is a superposition of a pair of electrons!

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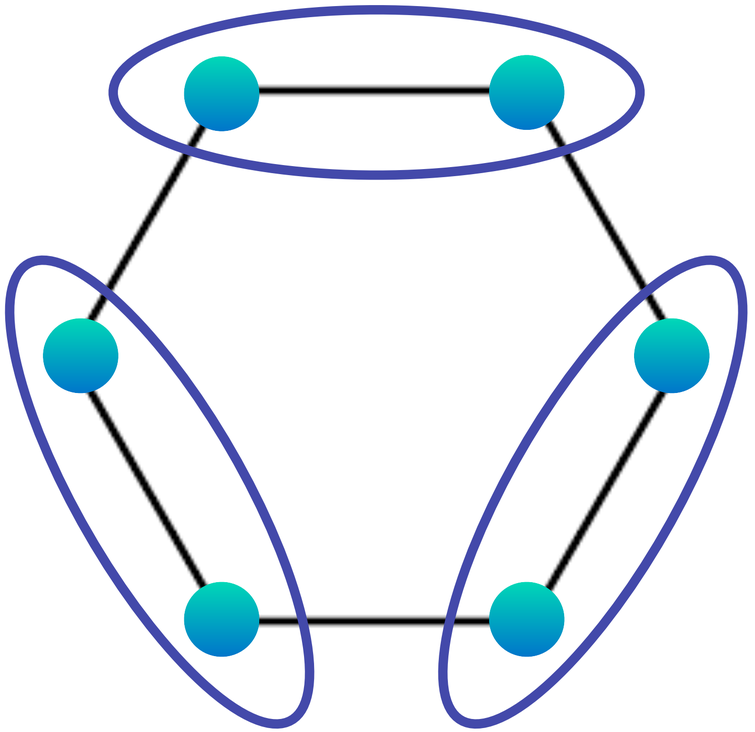

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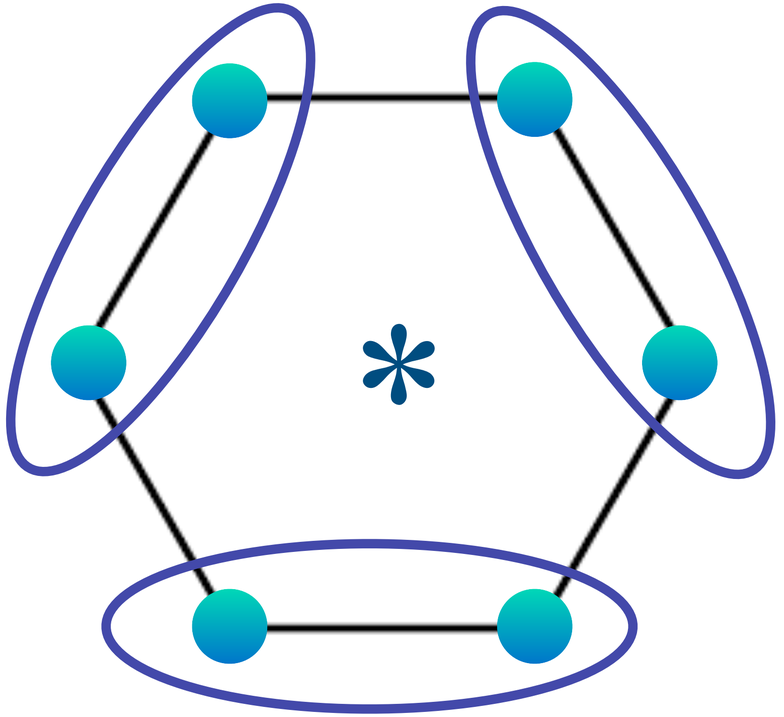
a physical system can be in a
superposition of two (or more) distinct states.

Benzene



"Resonating" covalent bonds

Benzene has a superposition of *covalent bonds*,
each of which is a superposition of a pair of electrons!

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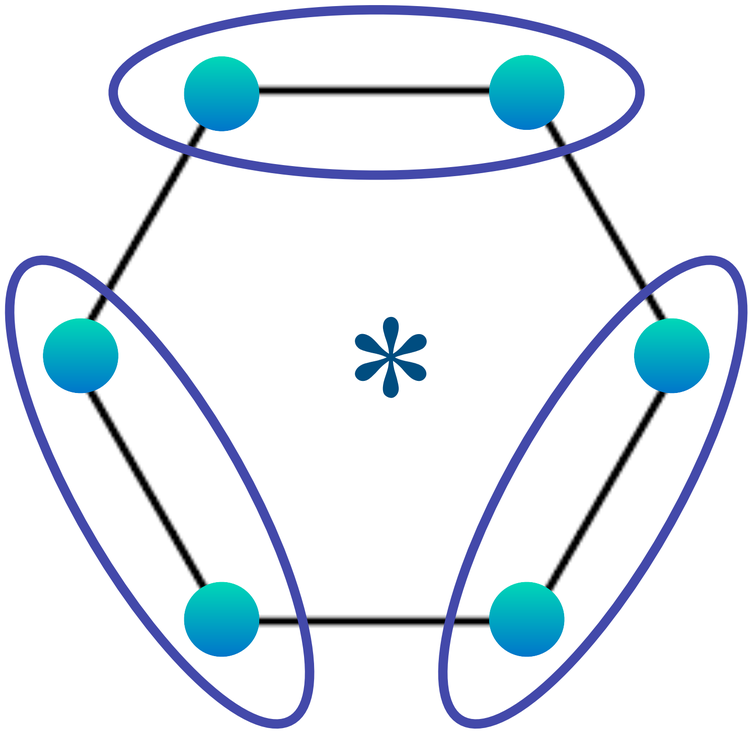

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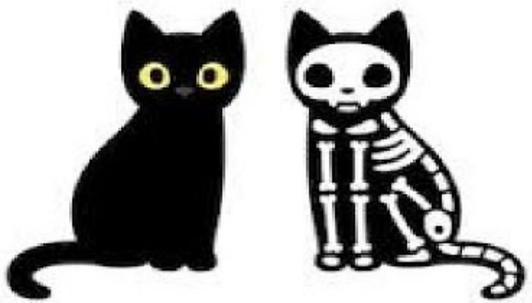
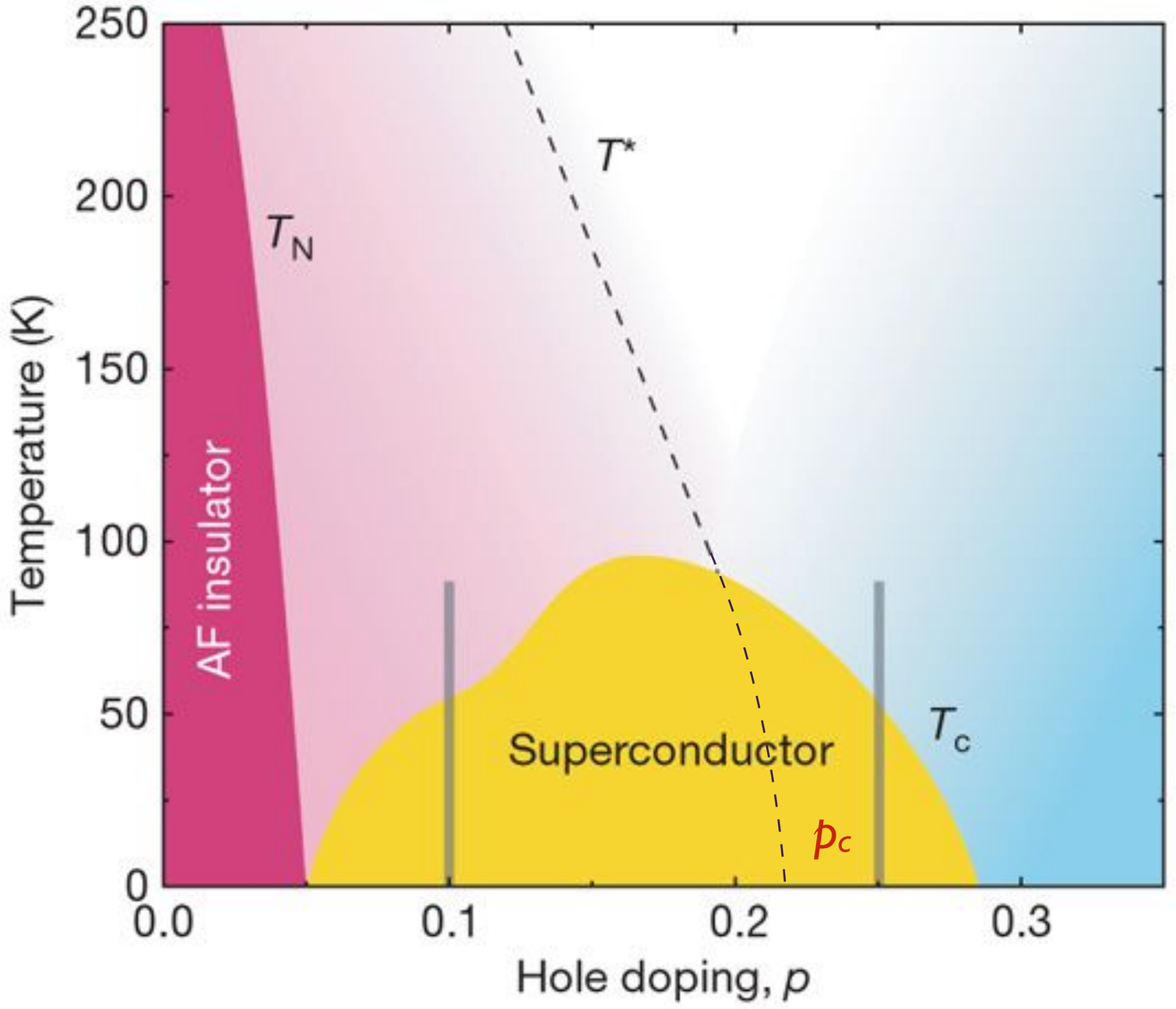
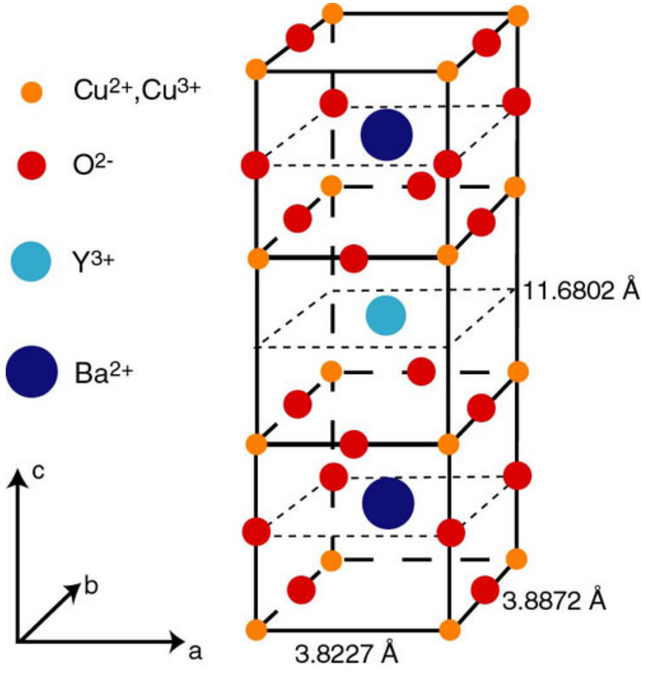
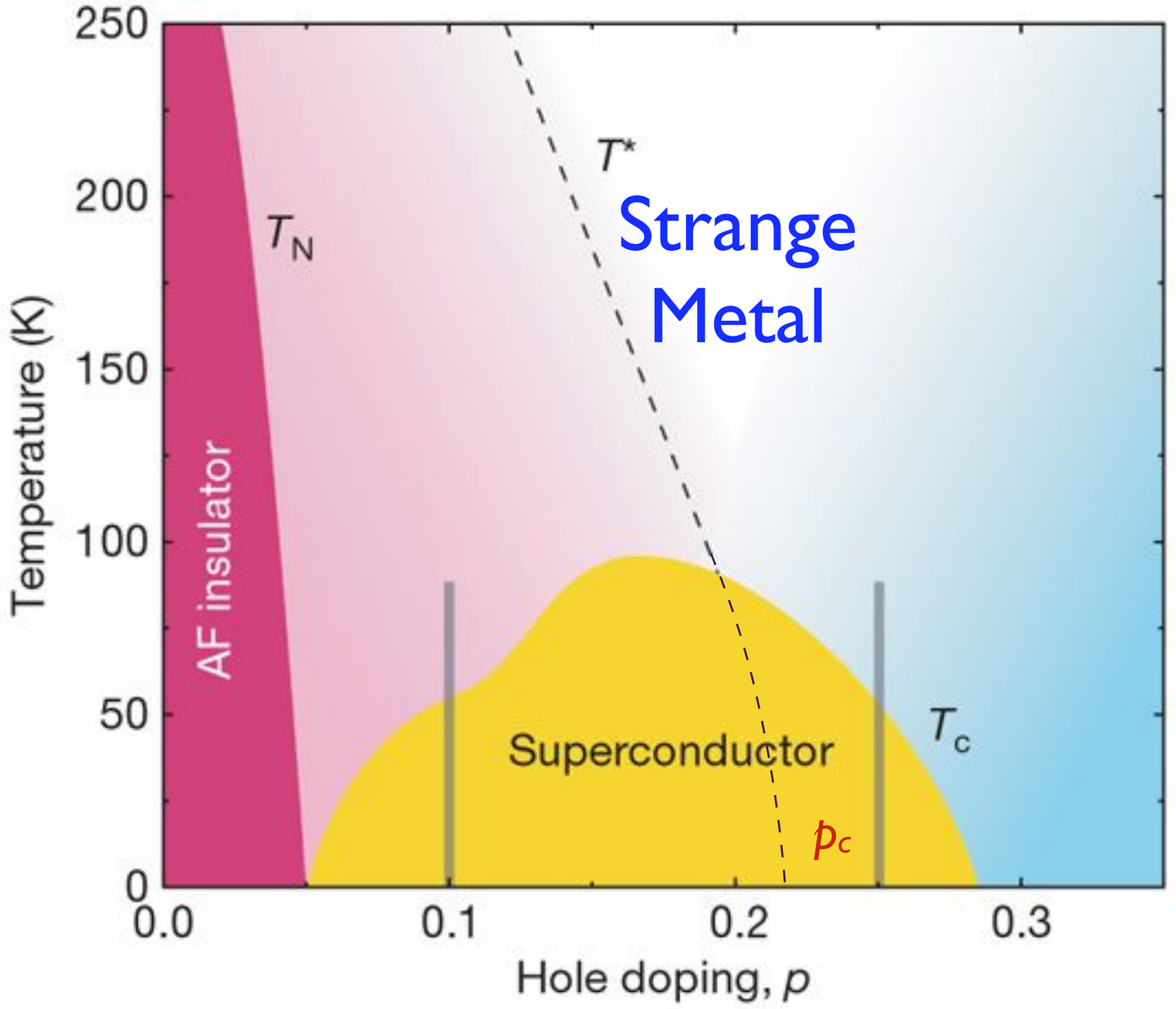
## Schrodinger's Cat

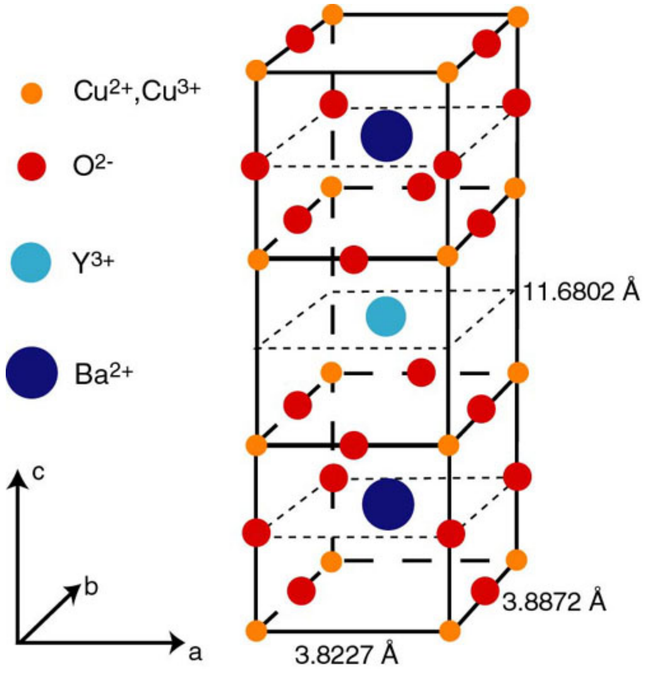


$$\frac{1}{\sqrt{2}}|\text{live cat}\rangle + \frac{1}{\sqrt{2}}|\text{dead cat}\rangle$$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
$T_N$
$T^*$
$T_c$
AF insulator
Superconductor
$p_c$


$Cu^{2+}$,$Cu^{3+}$
$O^{2-}$
$Y^{3+}$
$Ba^{2+}$
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
$T_N$
$T^*$
$T_c$
AF insulator
Superconductor
$p_c$
Strange
Metal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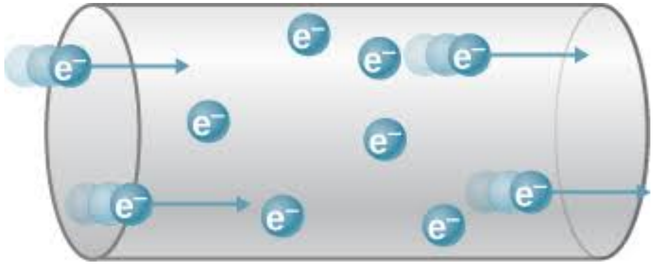
## *Current flow with quasiparticles in Copper*




Flowing quasiparticles scatter off each other in a typical scattering time $\tau$

This time is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

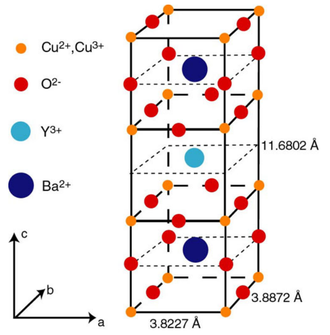
The long scattering time implies that quasiparticles are well-defined.

A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

| Material | | $n$ ($10^{27}$ m$^{-3}$) | $m^*$ ($m_0$) | $A_1 / d$ (Ω / K) | $h / (2e^2 T_F)$ (Ω / K) | α |
|---|---|---|---|---|---|---|
| Bi2212 | $p = 0.23$ | 6.8 | 8.4 ± 1.6 | 8.0 ± 0.9 | 7.4 ± 1.4 | 1.1 ± 0.3 |
| Bi2201 | $p \sim 0.4$ | 3.5 | 7 ± 1.5 | 8 ± 2 | 8 ± 2 | 1.0 ± 0.4 |
| LSCO | $p = 0.26$ | 7.8 | 9.8 ± 1.7 | 8.2 ± 1.0 | 8.9 ± 1.8 | 0.9 ± 0.3 |
| Nd-LSCO | $p = 0.24$ | 7.9 | 12 ± 4 | 7.4 ± 0.8 | 10.6 ± 3.7 | 0.7 ± 0.4 |
| PCCO | $x = 0.17$ | 8.8 | 2.4 ± 0.1 | 1.7 ± 0.3 | 2.1 ± 0.1 | 0.8 ± 0.2 |
| LCCO | $x = 0.15$ | 9.0 | 3.0 ± 0.3 | 3.0 ± 0.45 | 2.6 ± 0.3 | 1.2 ± 0.3 |
| TMTSF | $P = 11$ kbar | 1.4 | 1.15 ± 0.2 | 2.8 ± 0.3 | 2.8 ± 0.4 | 1.0 ± 0.3 |

Electron scattering time $\tau$ in 7 different strange metals

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



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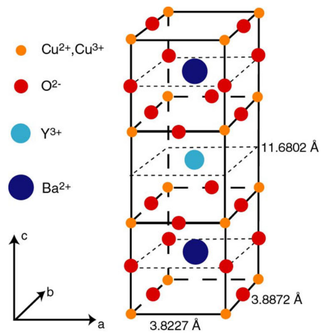
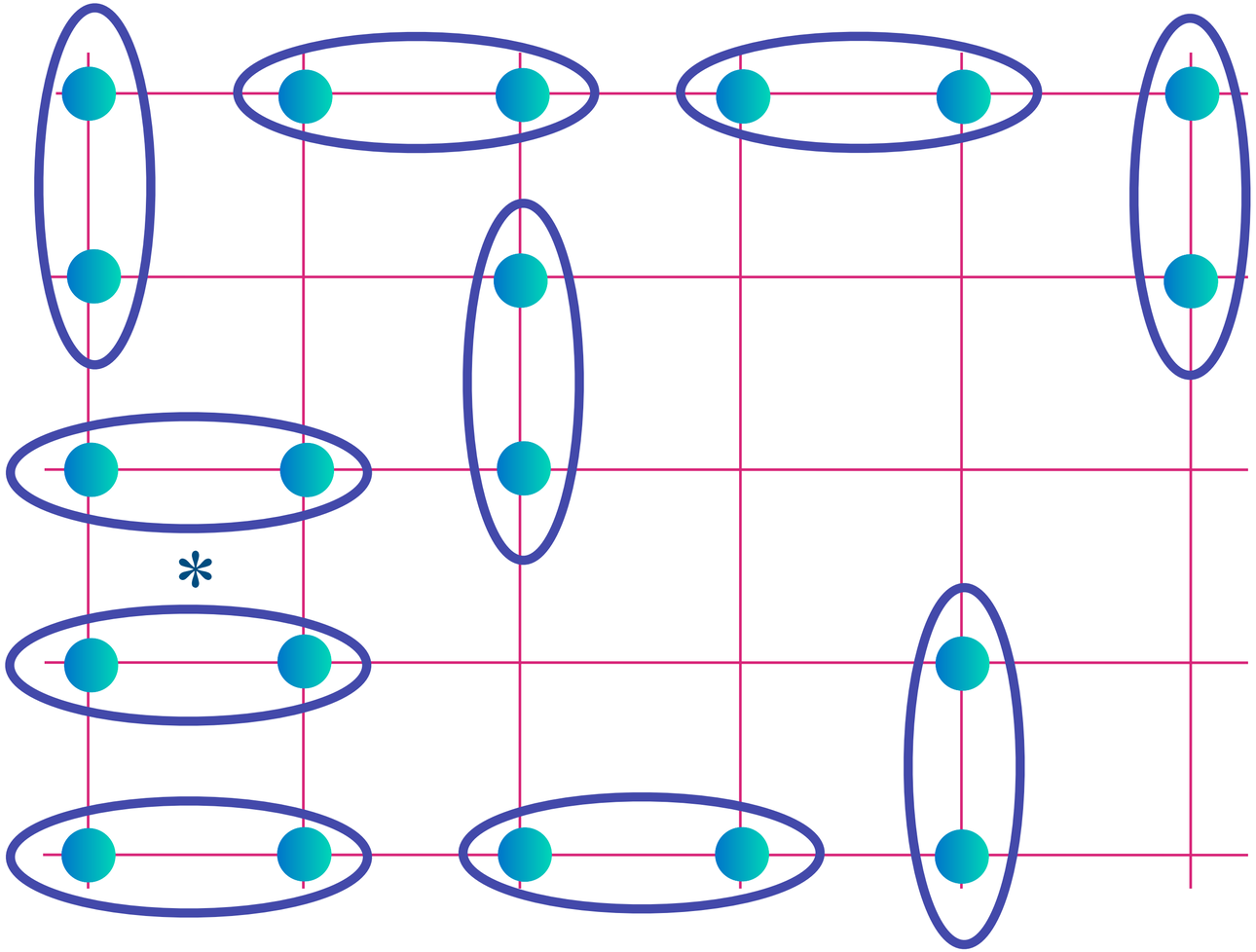



Current flow without quasiparticles

# Antiferromagnet doped with hole density $p$



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Electrons entangle "en masse" by exchanging partners, and there is long-range quantum entanglement

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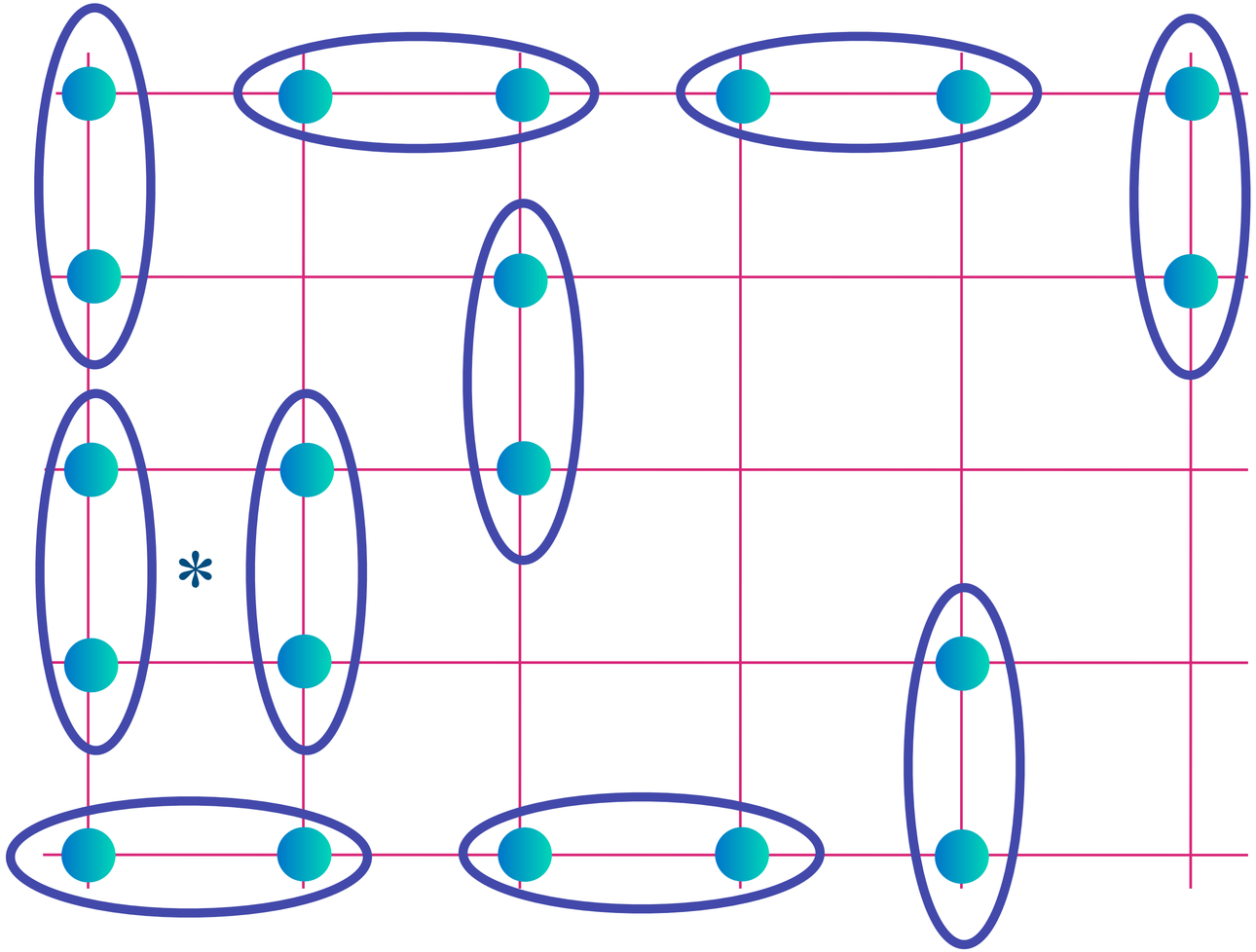

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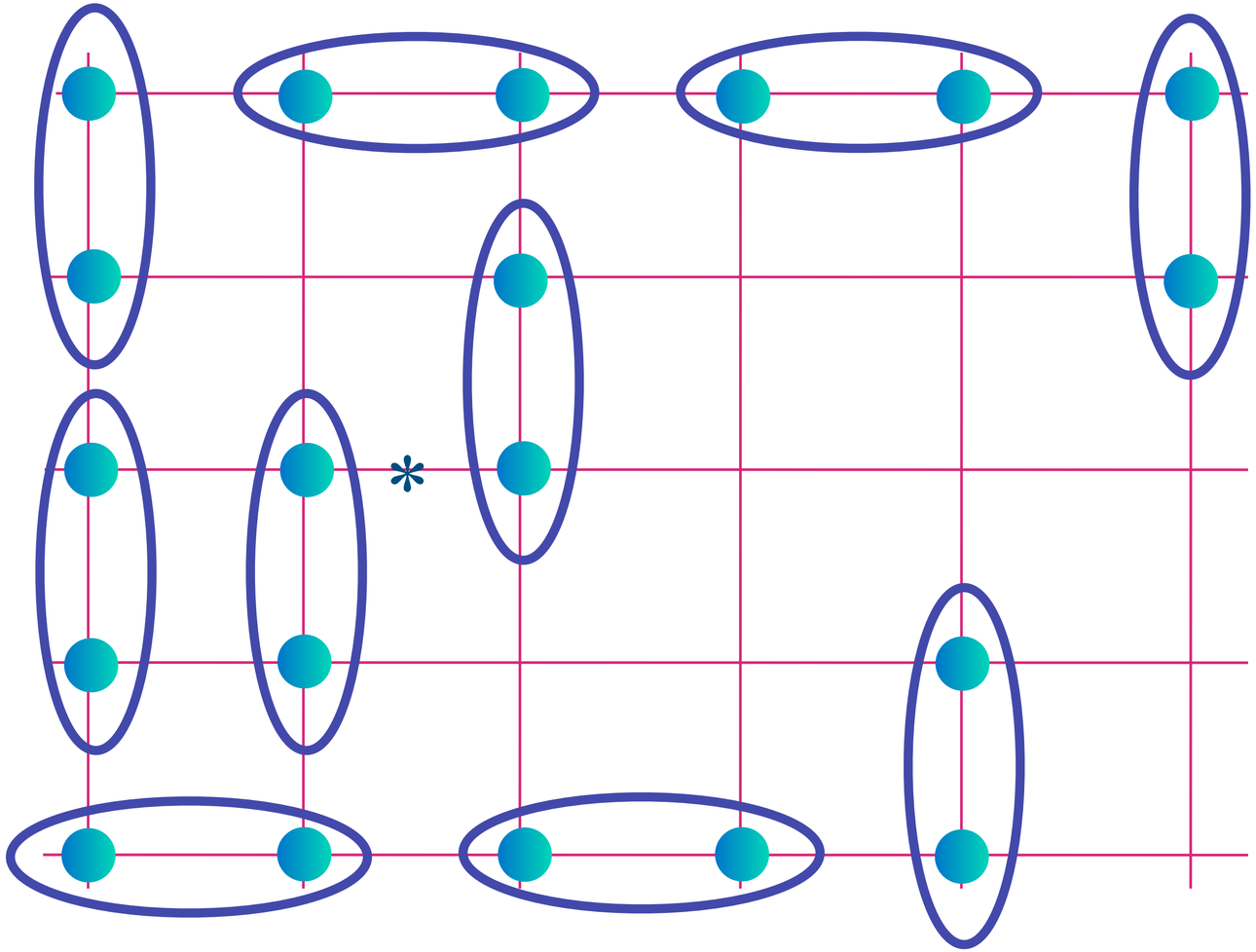

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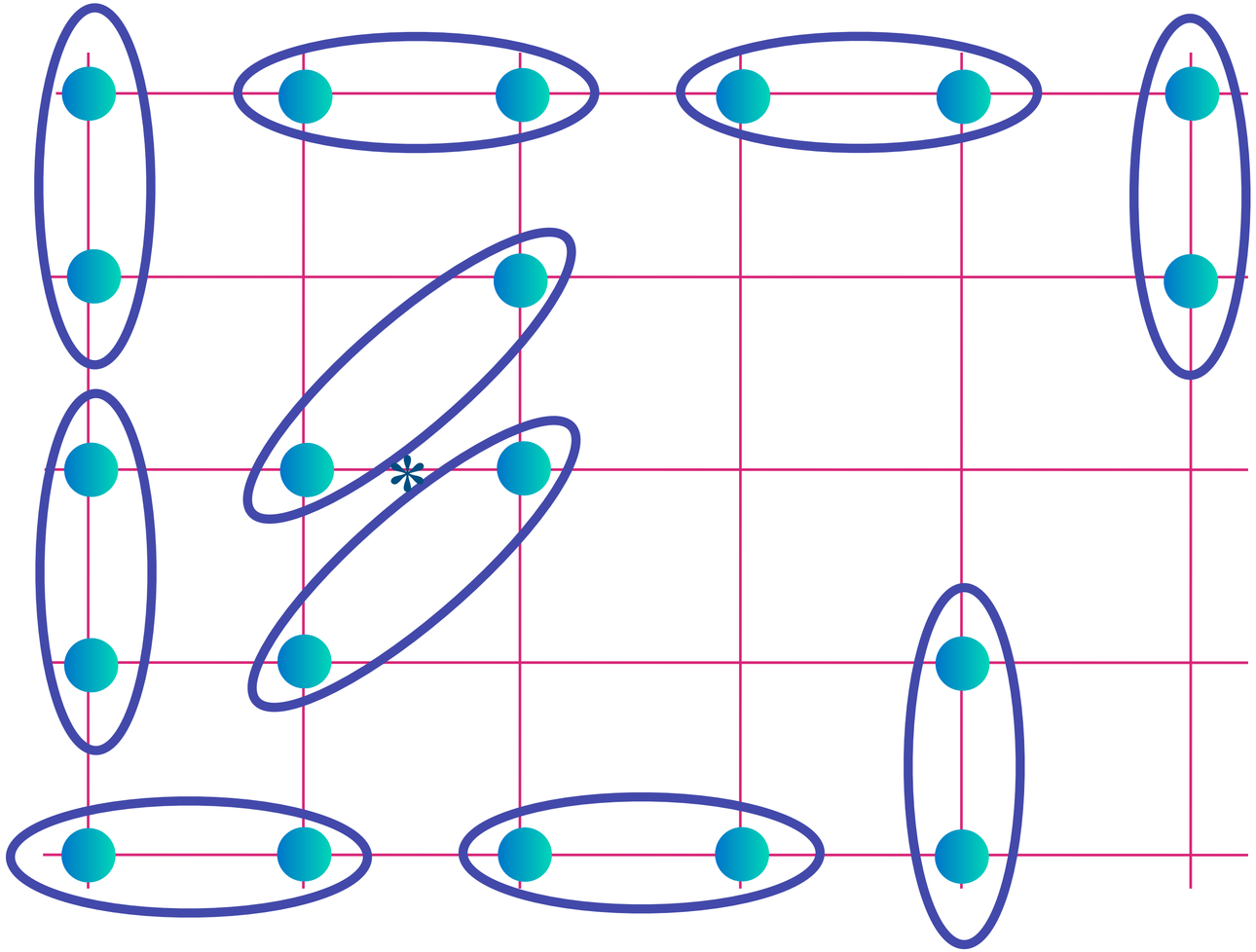

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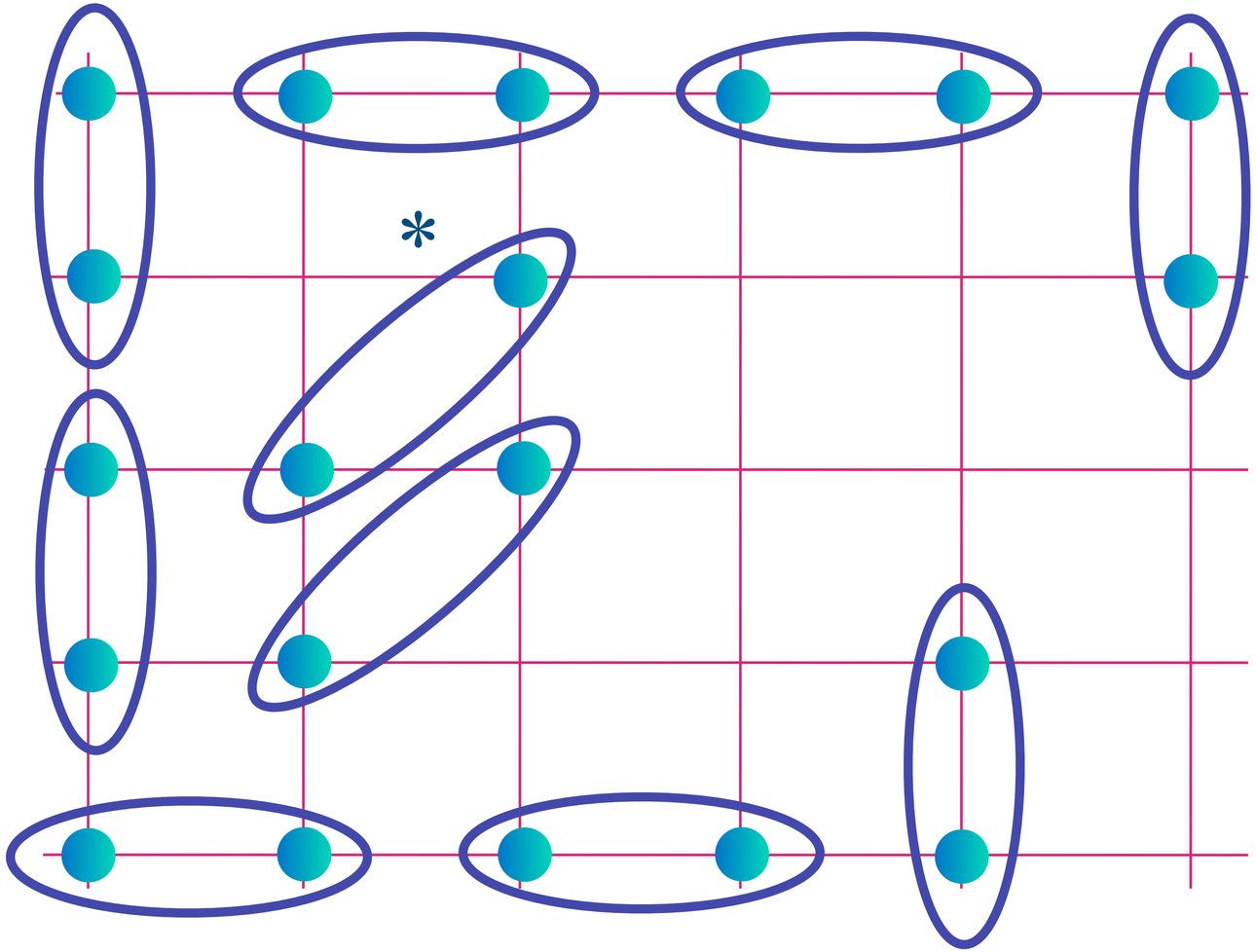

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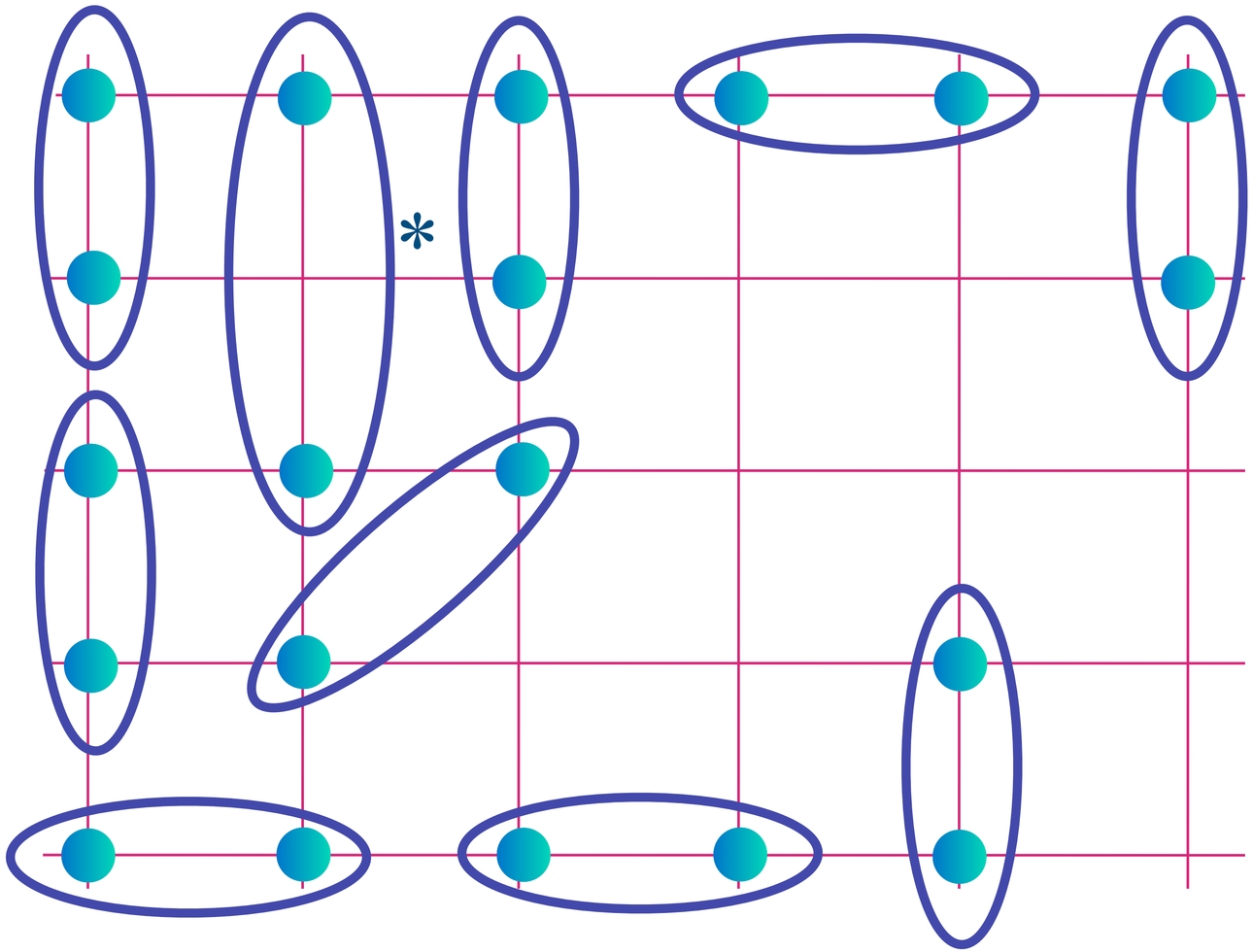

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# The Sachdev-Ye-Kitaev (SYK) model

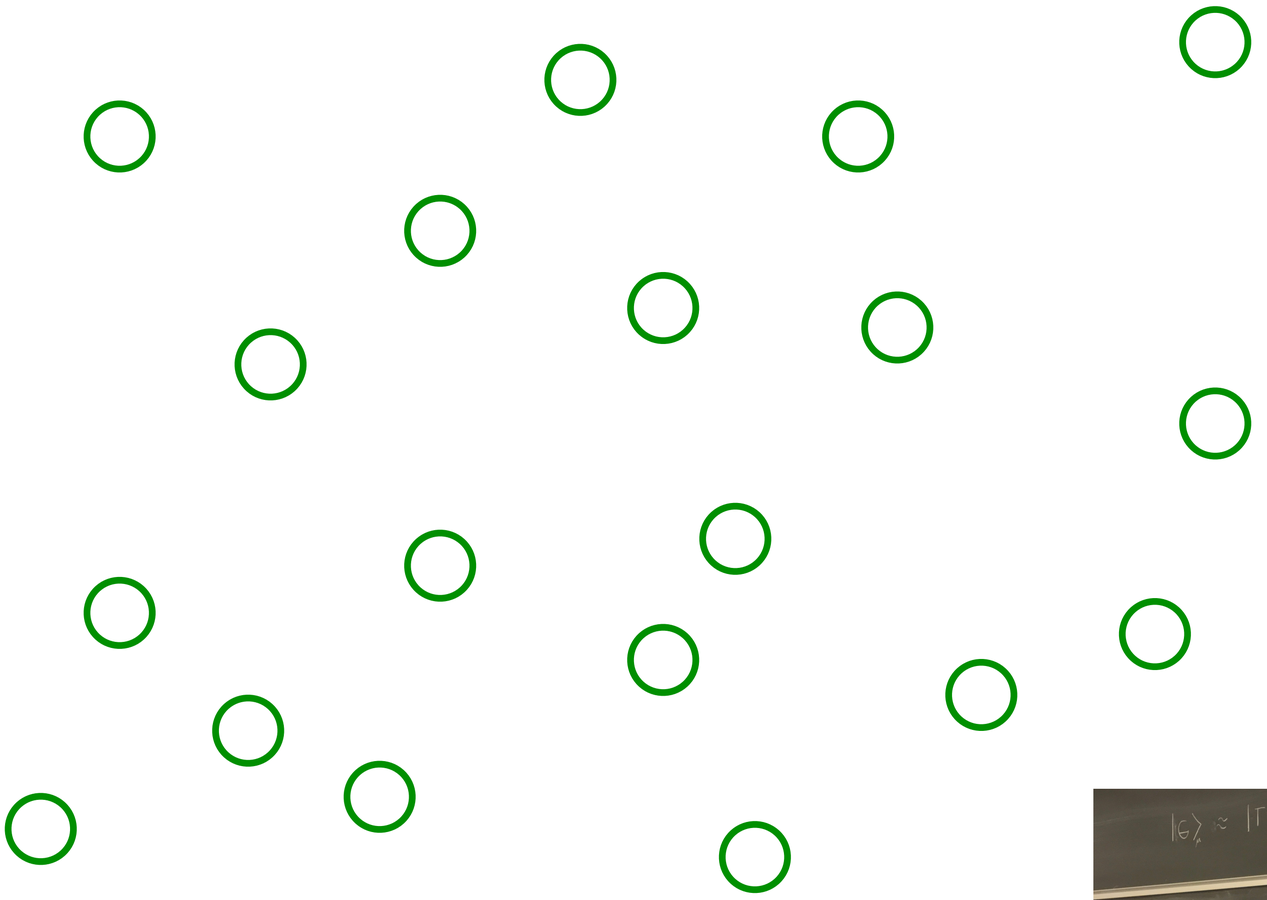
The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

It describes
certain ***strange metals***

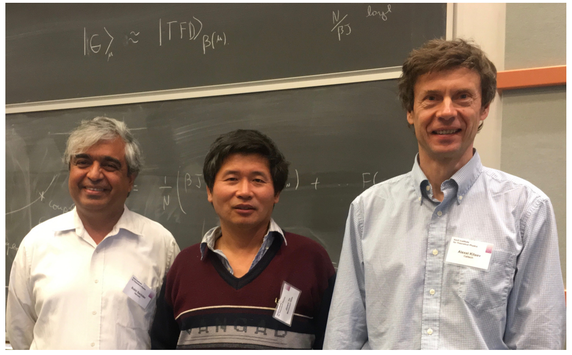
Sachdev, Ye (1993)

# The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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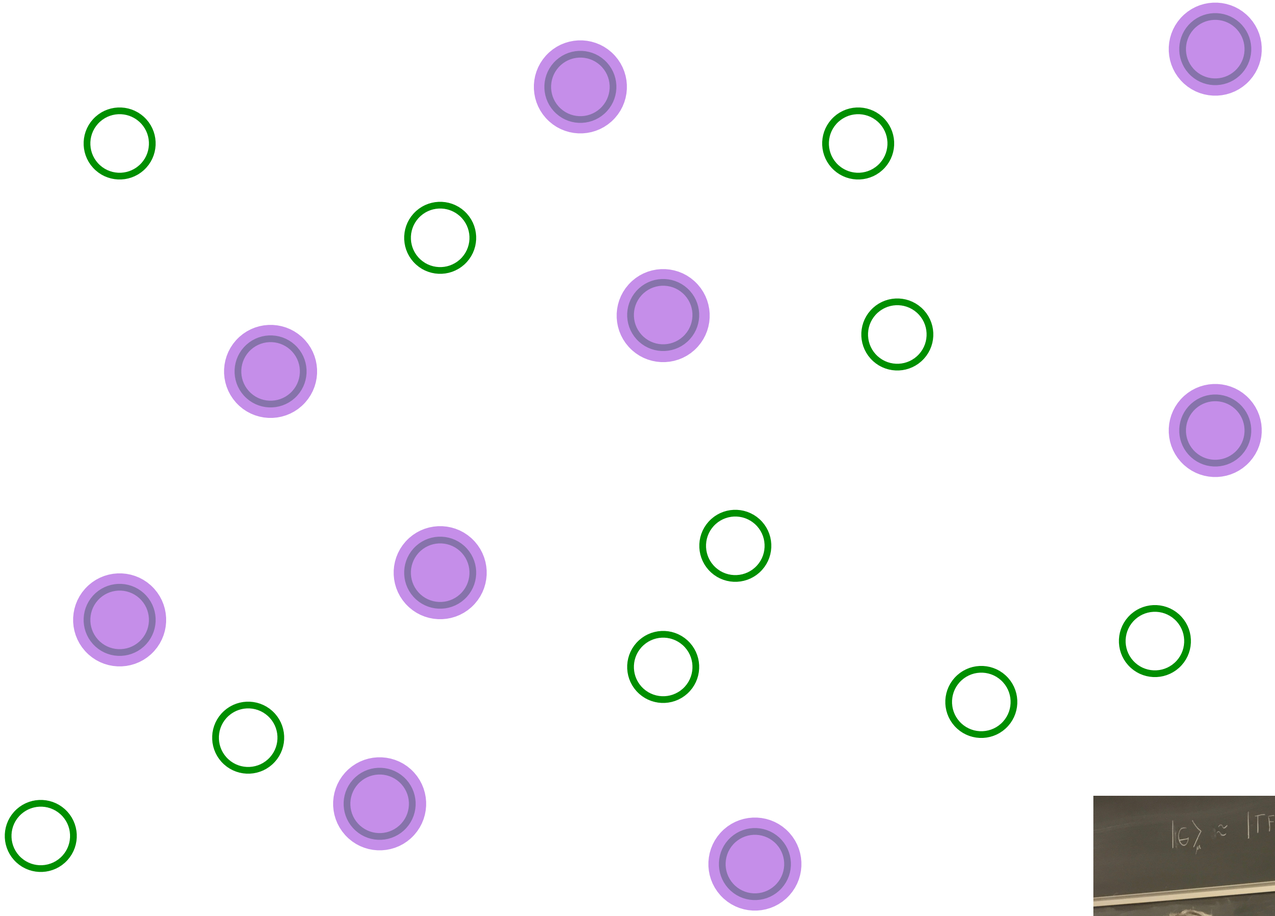
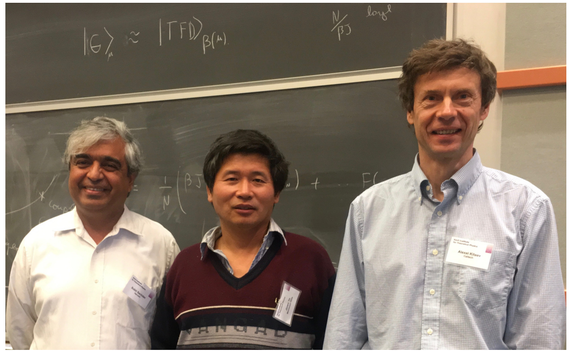
Sachdev, Ye (1993); Kitaev (2015)



Place electrons randomly on some sites



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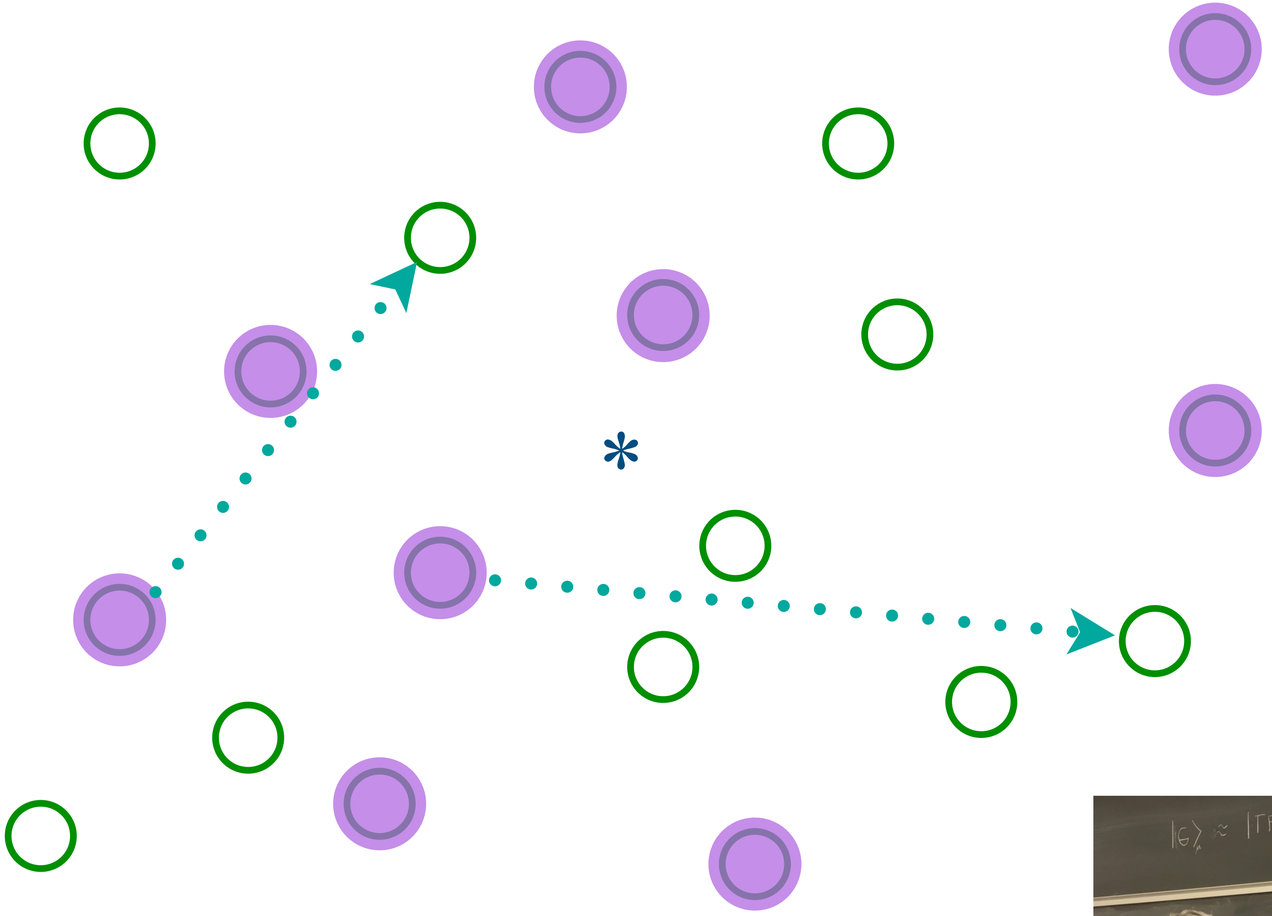

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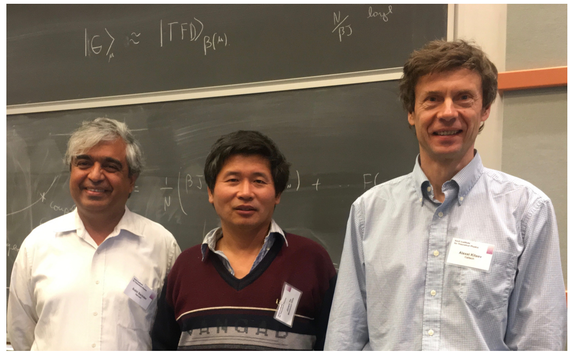

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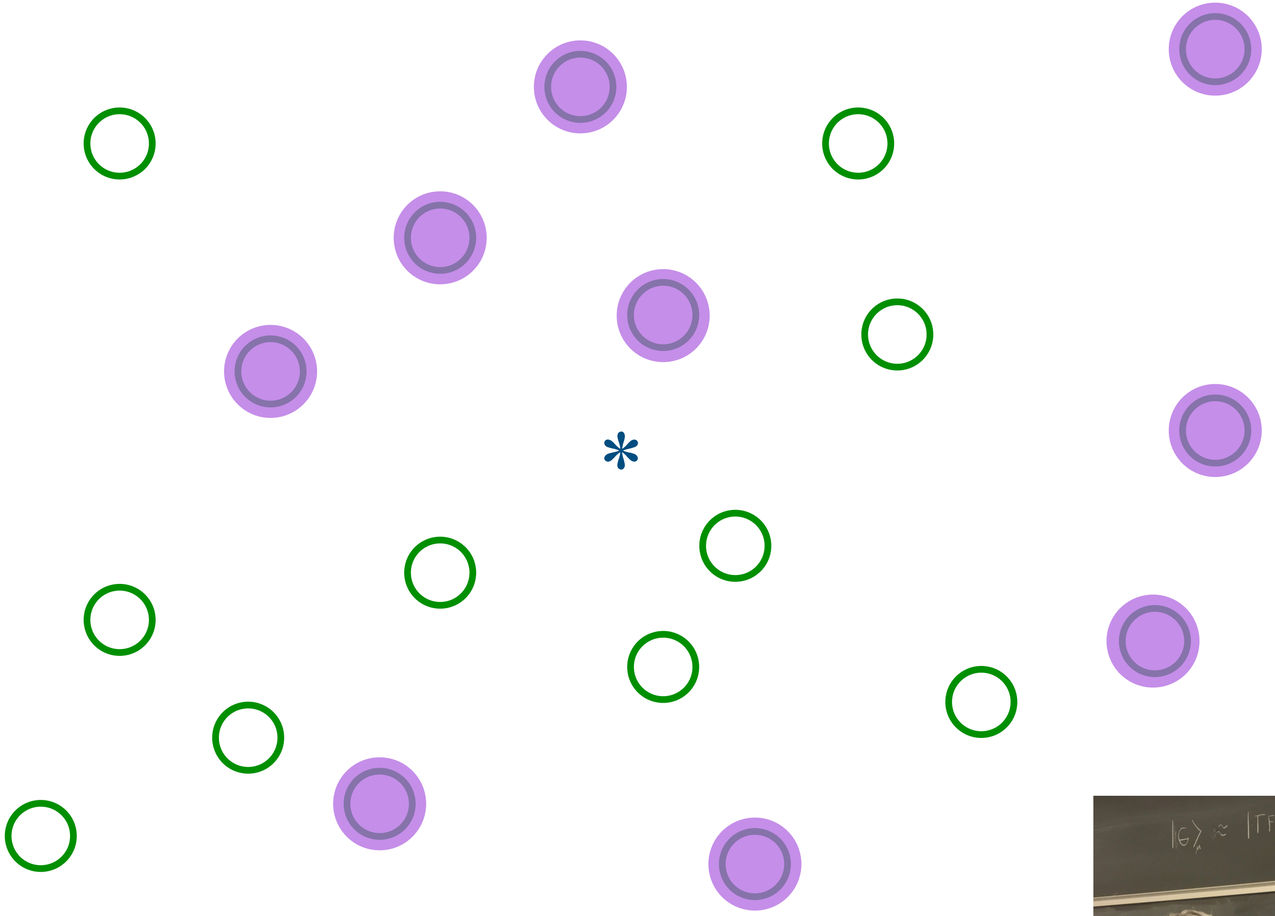
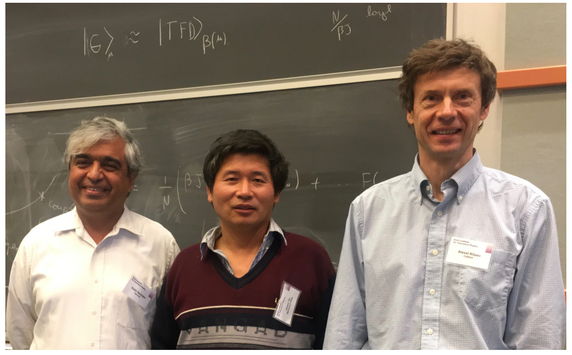


Entangle electrons pairwise randomly



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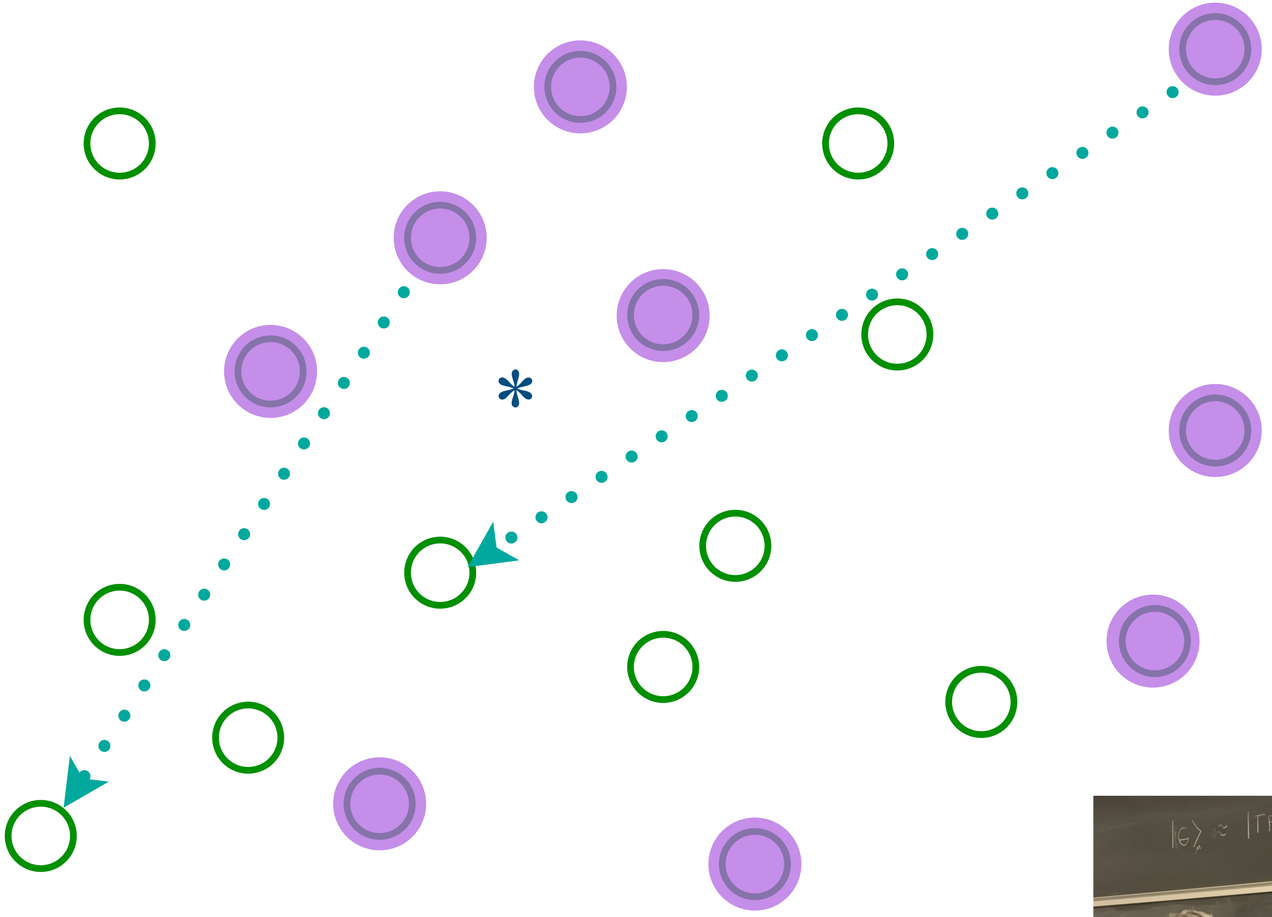

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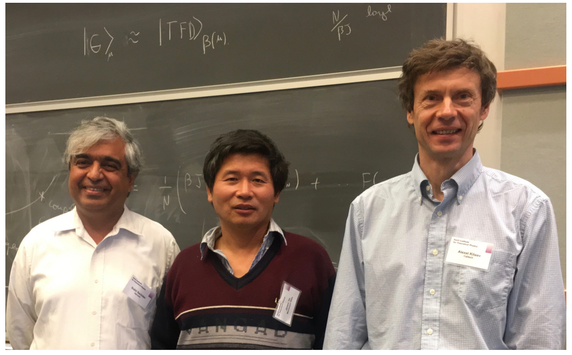

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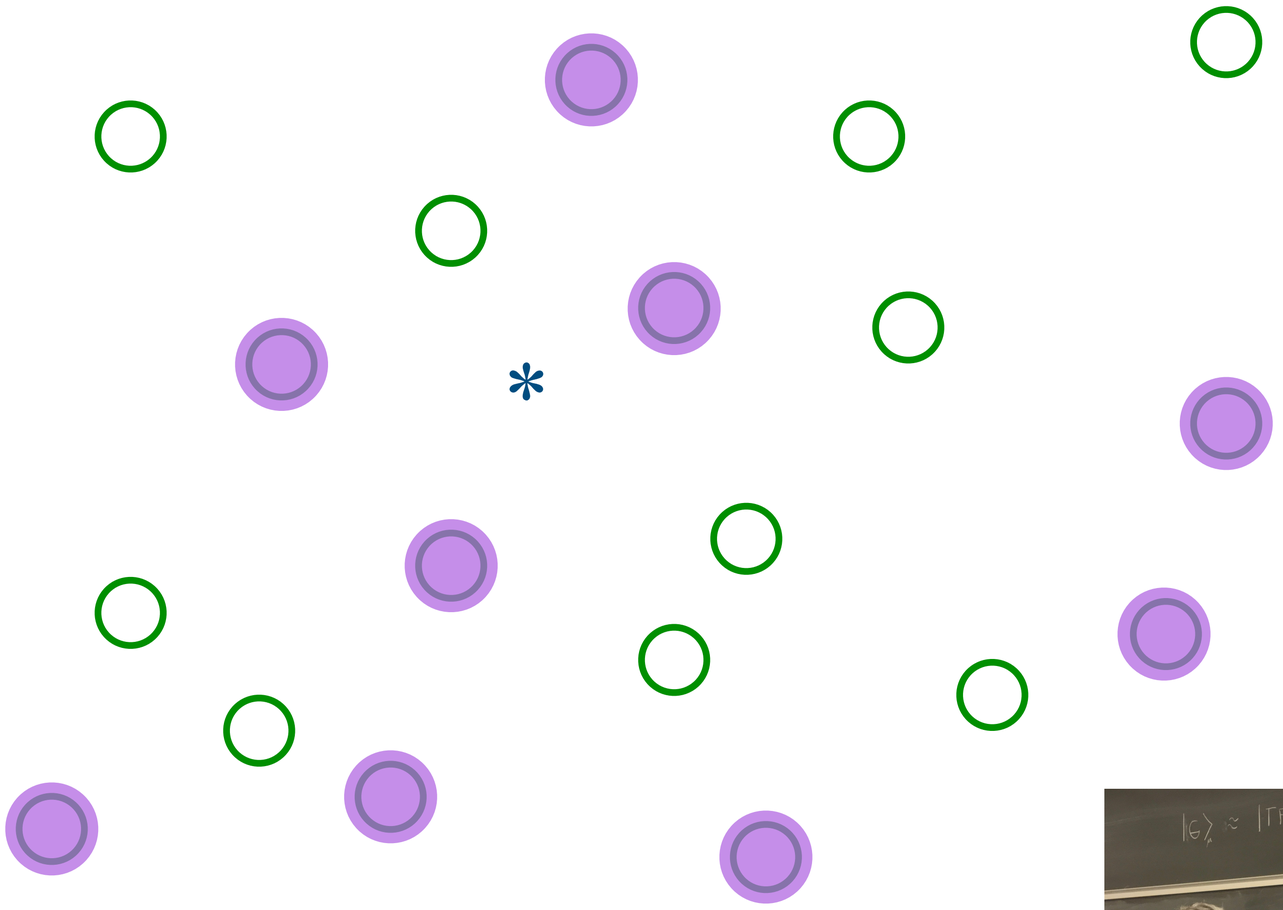

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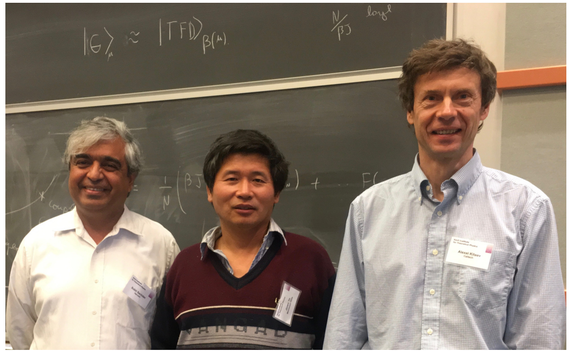

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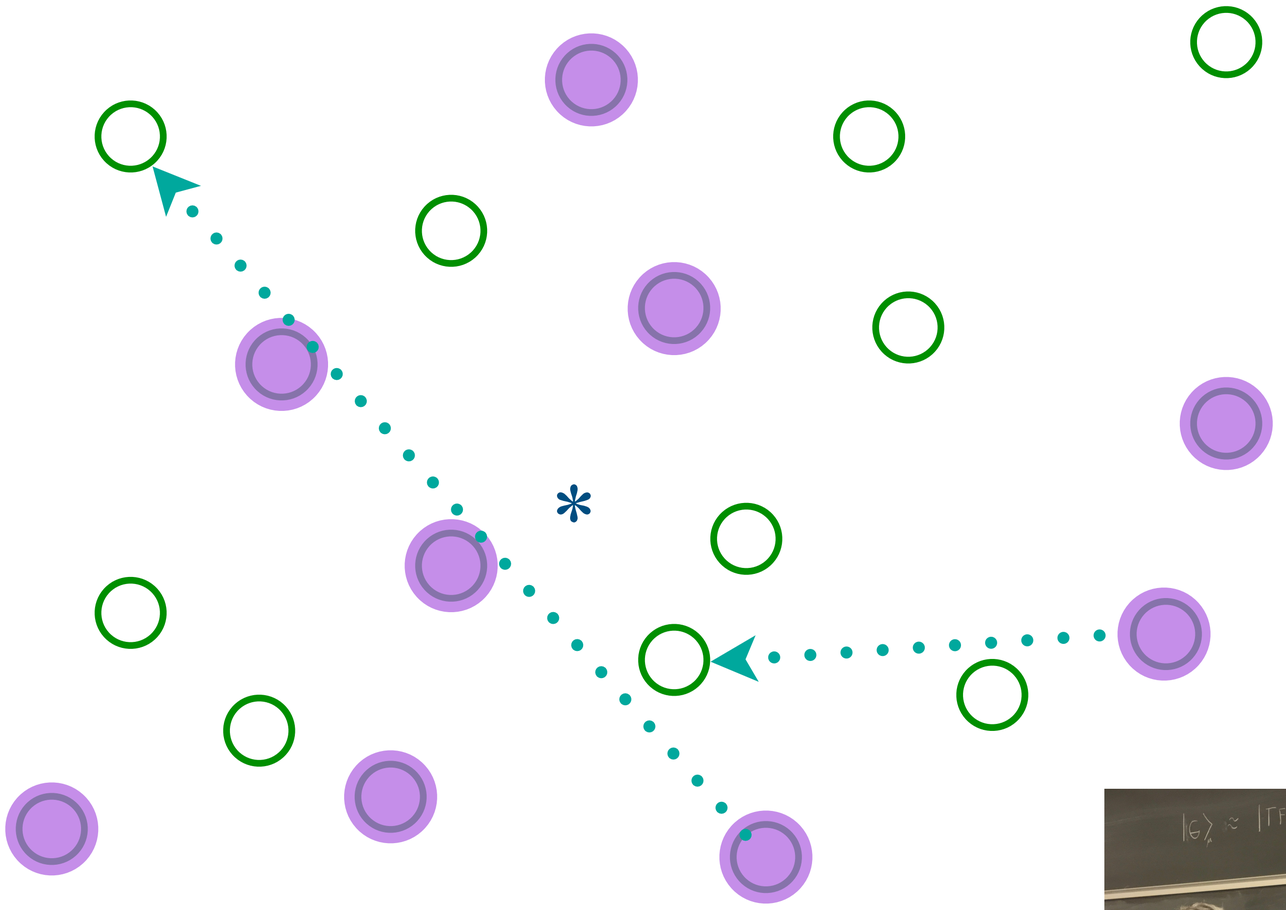

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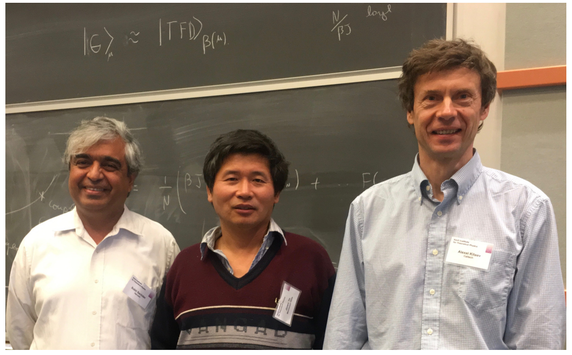

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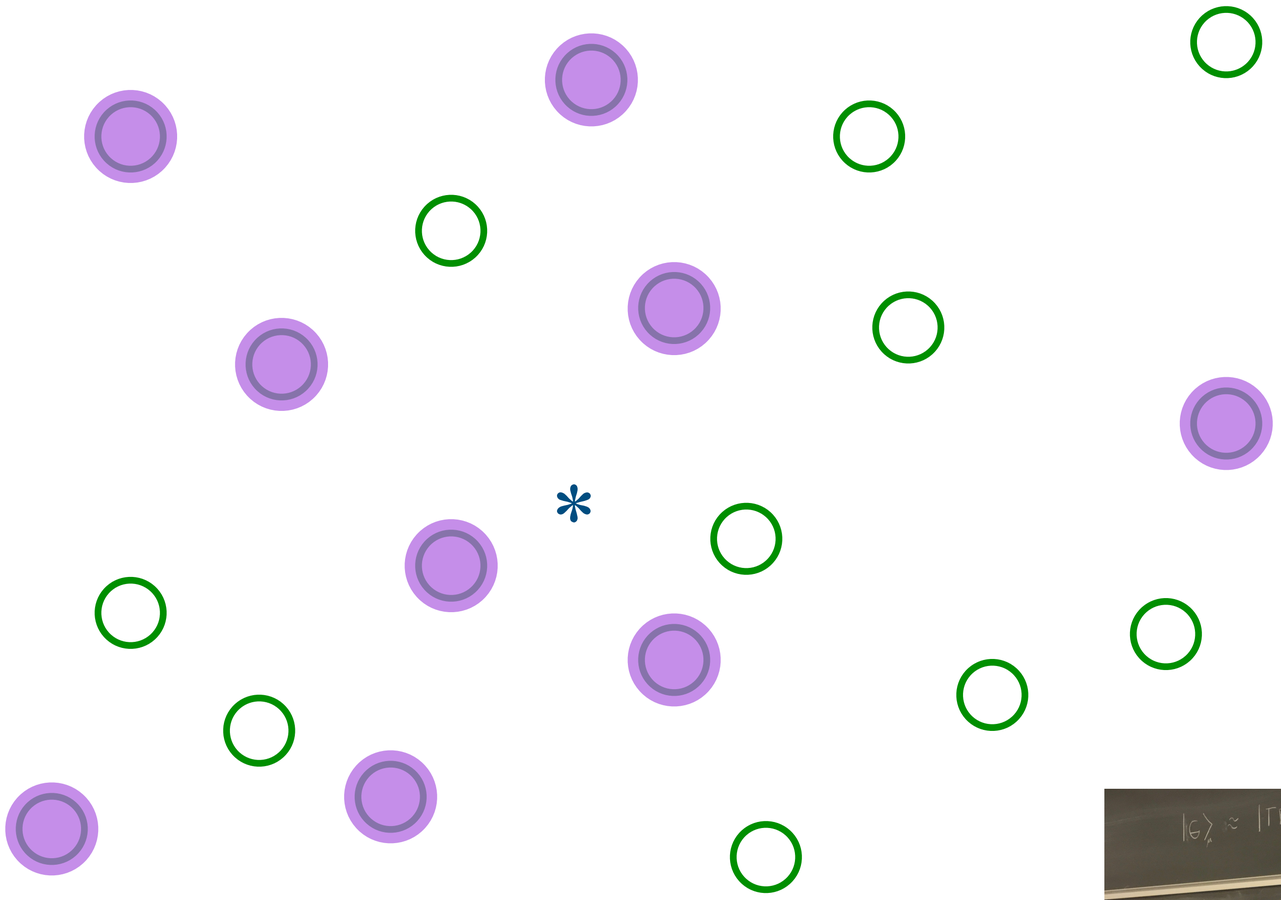

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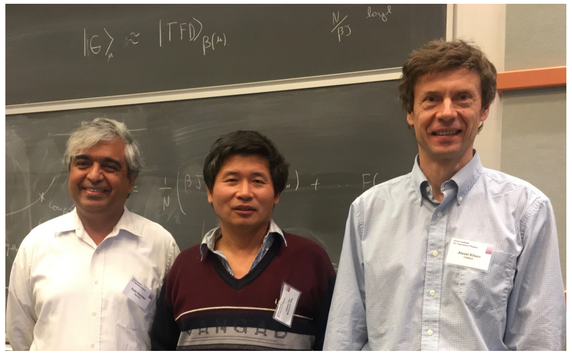

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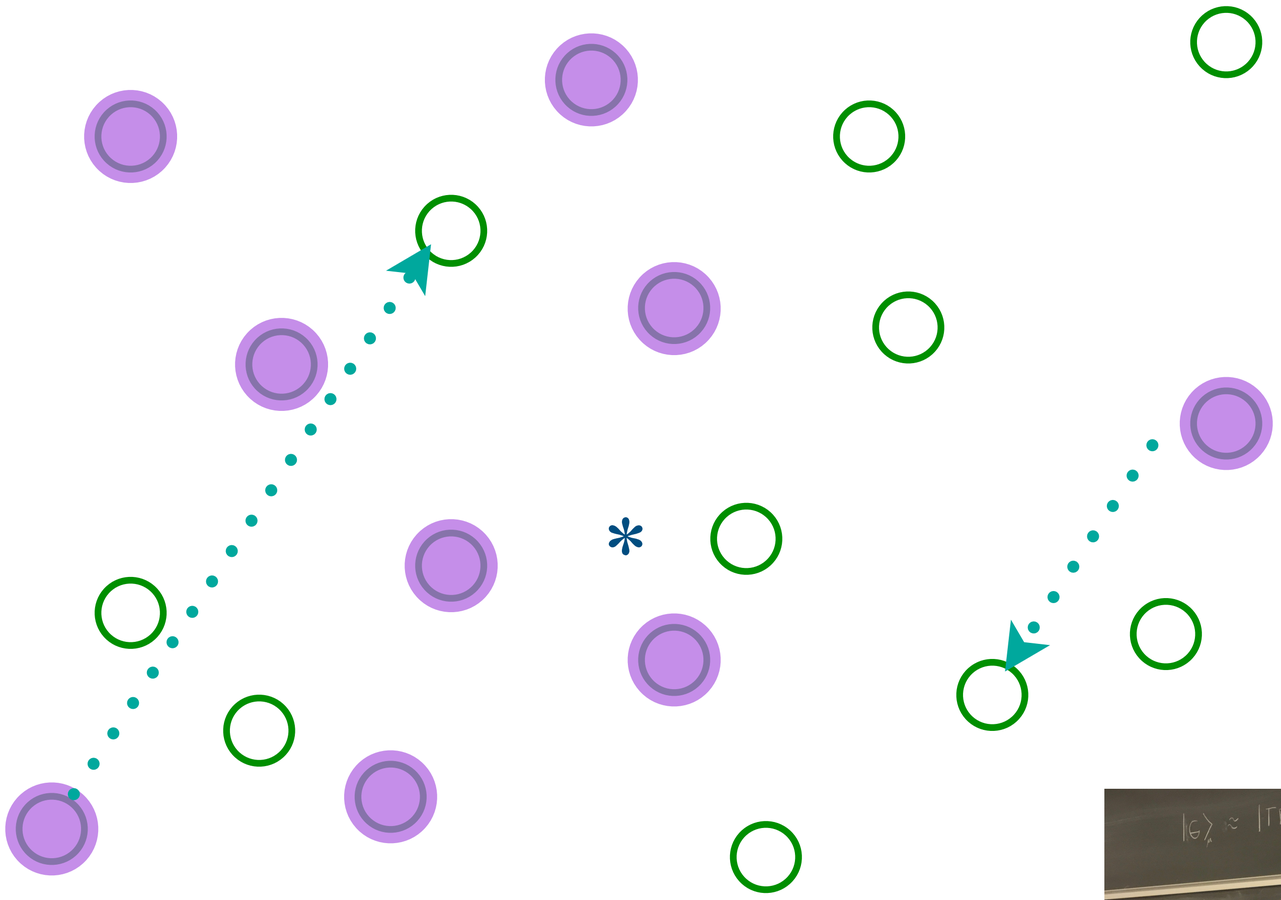

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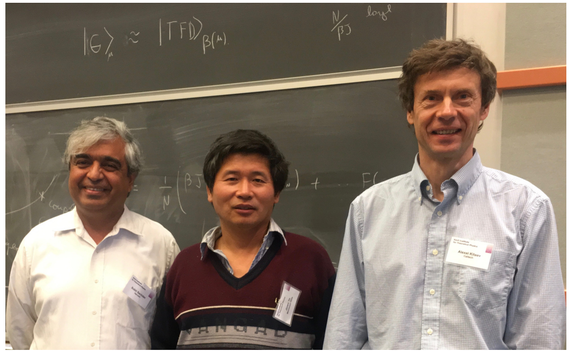

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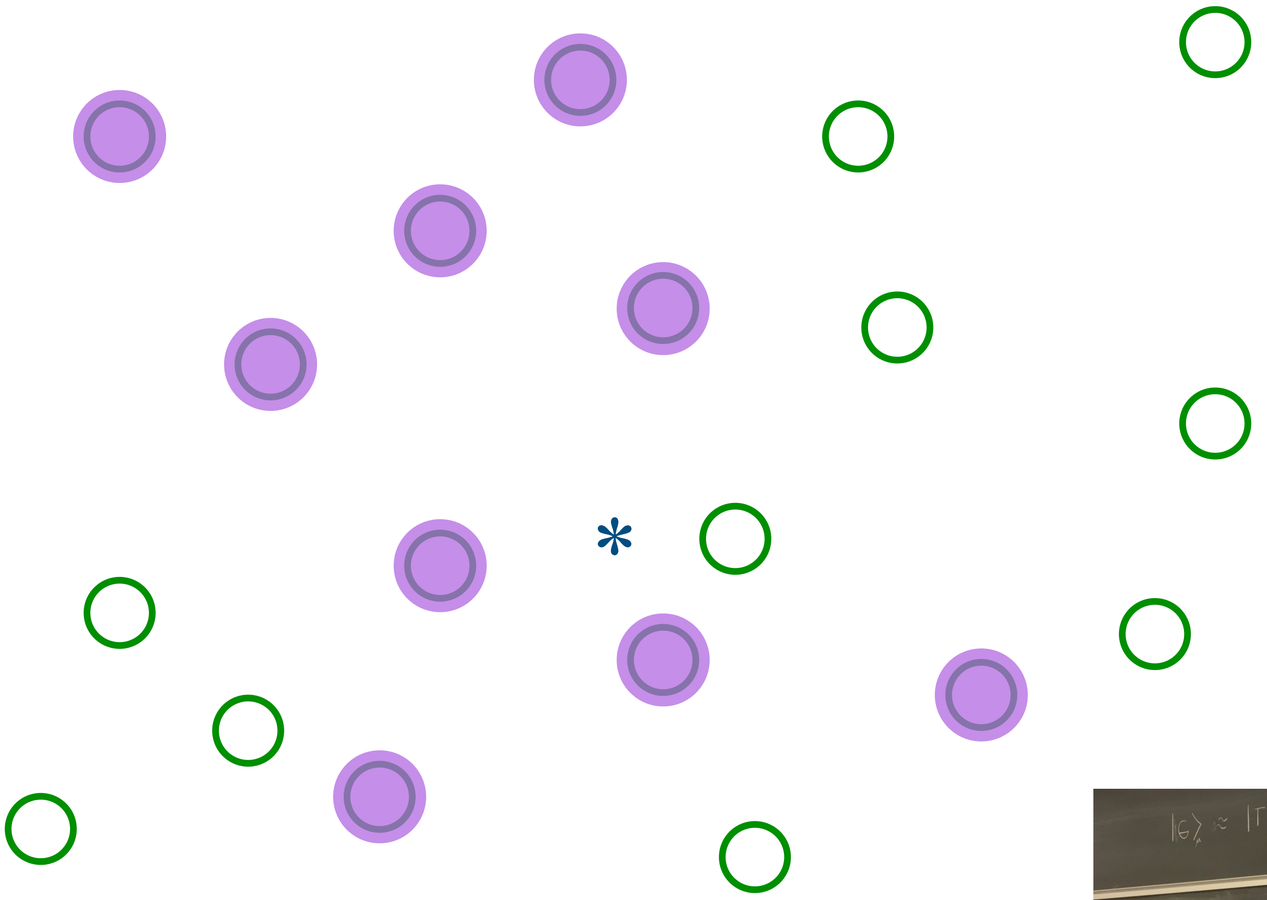

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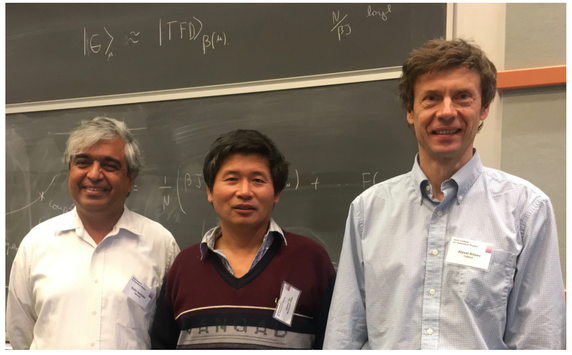

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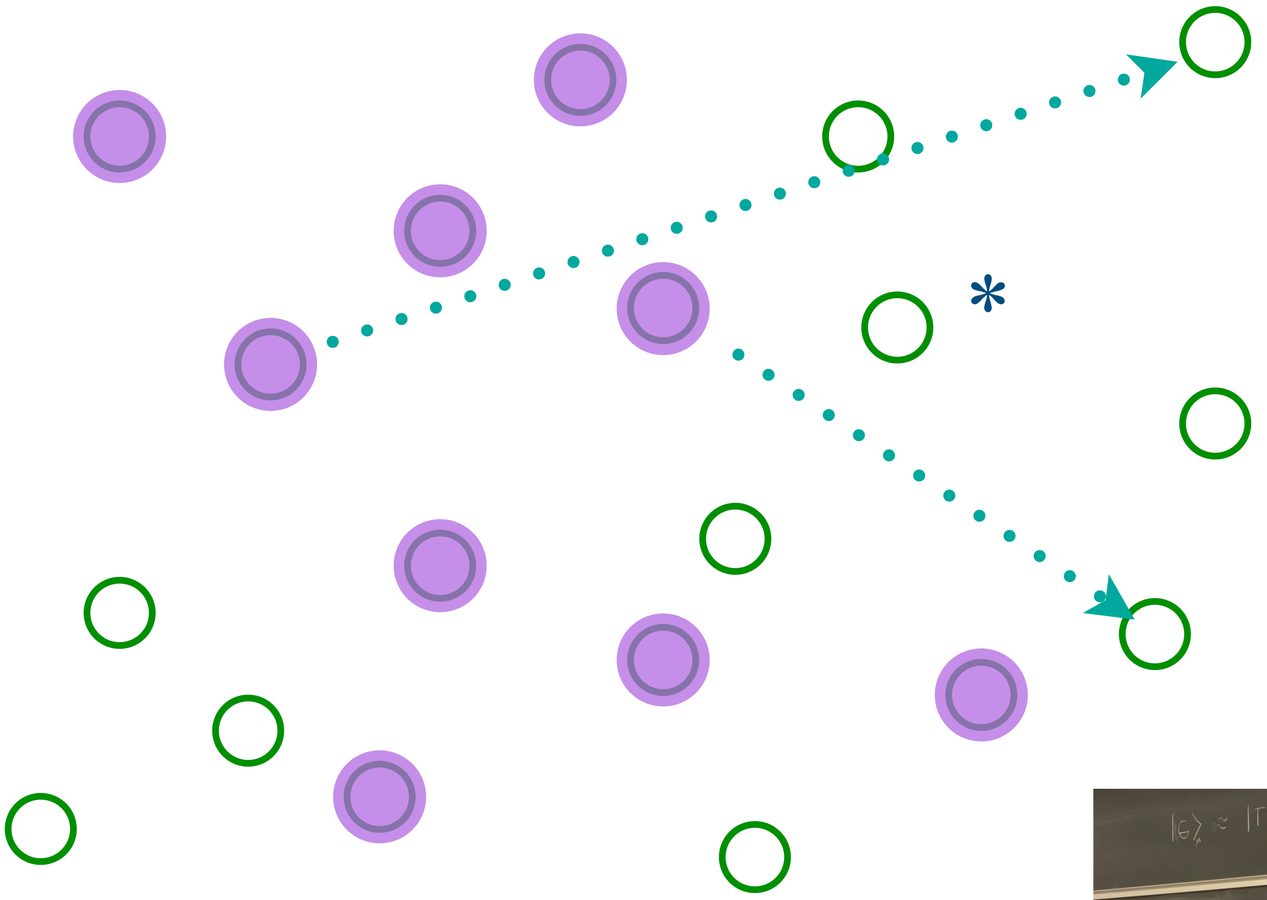

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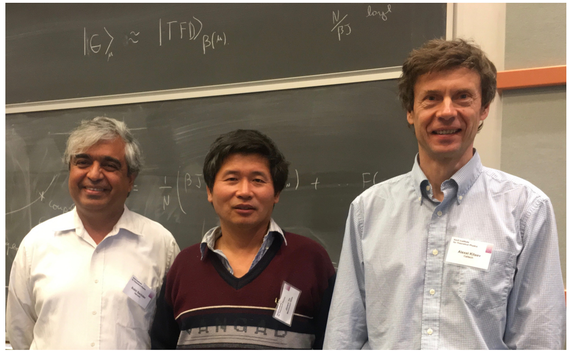

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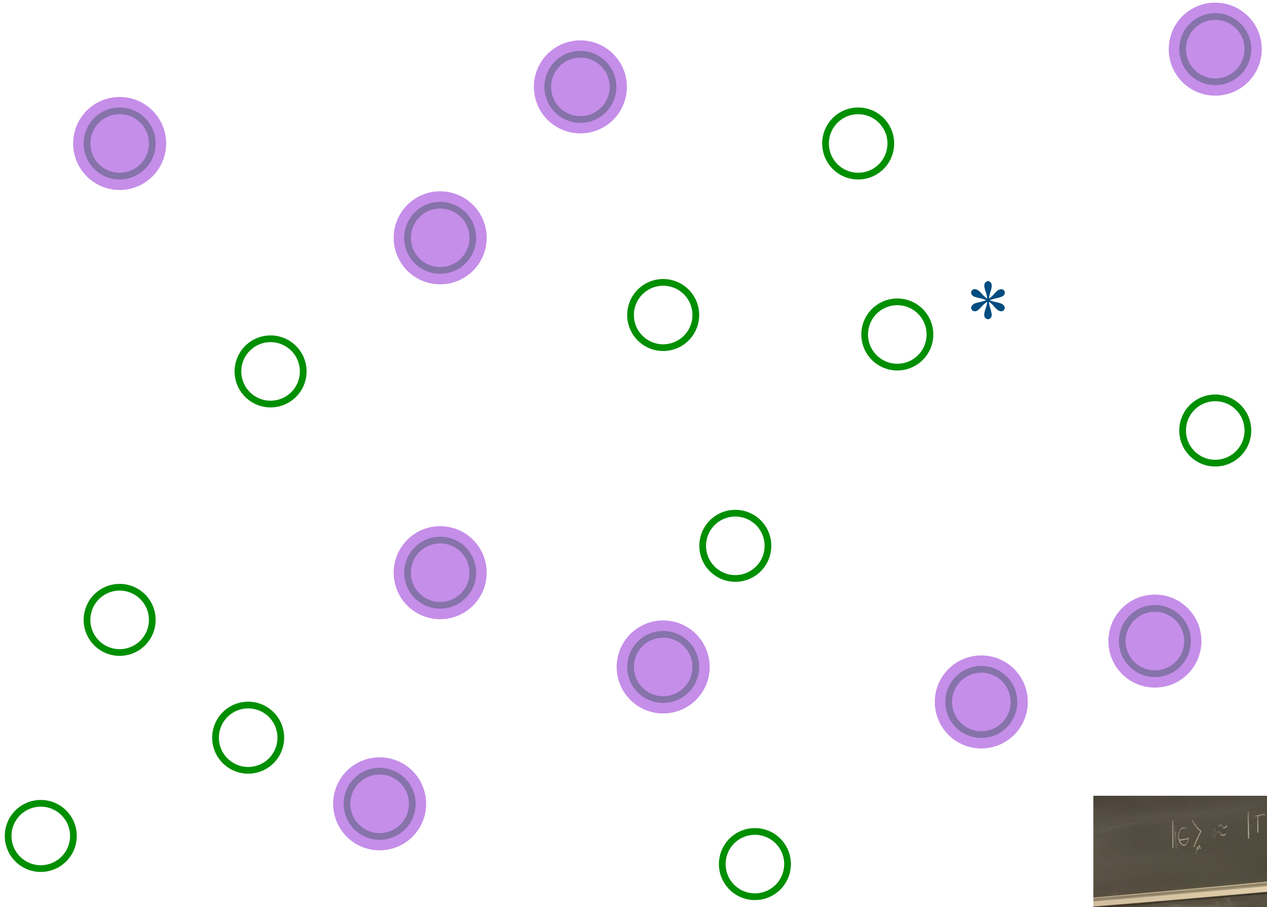

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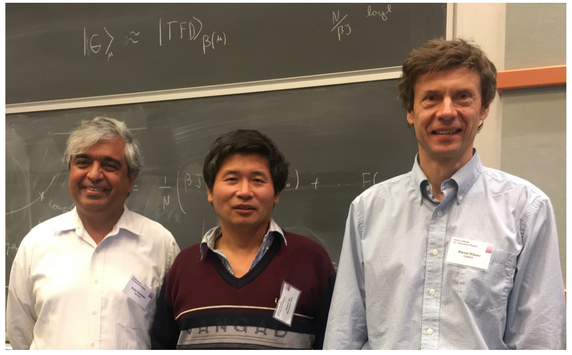

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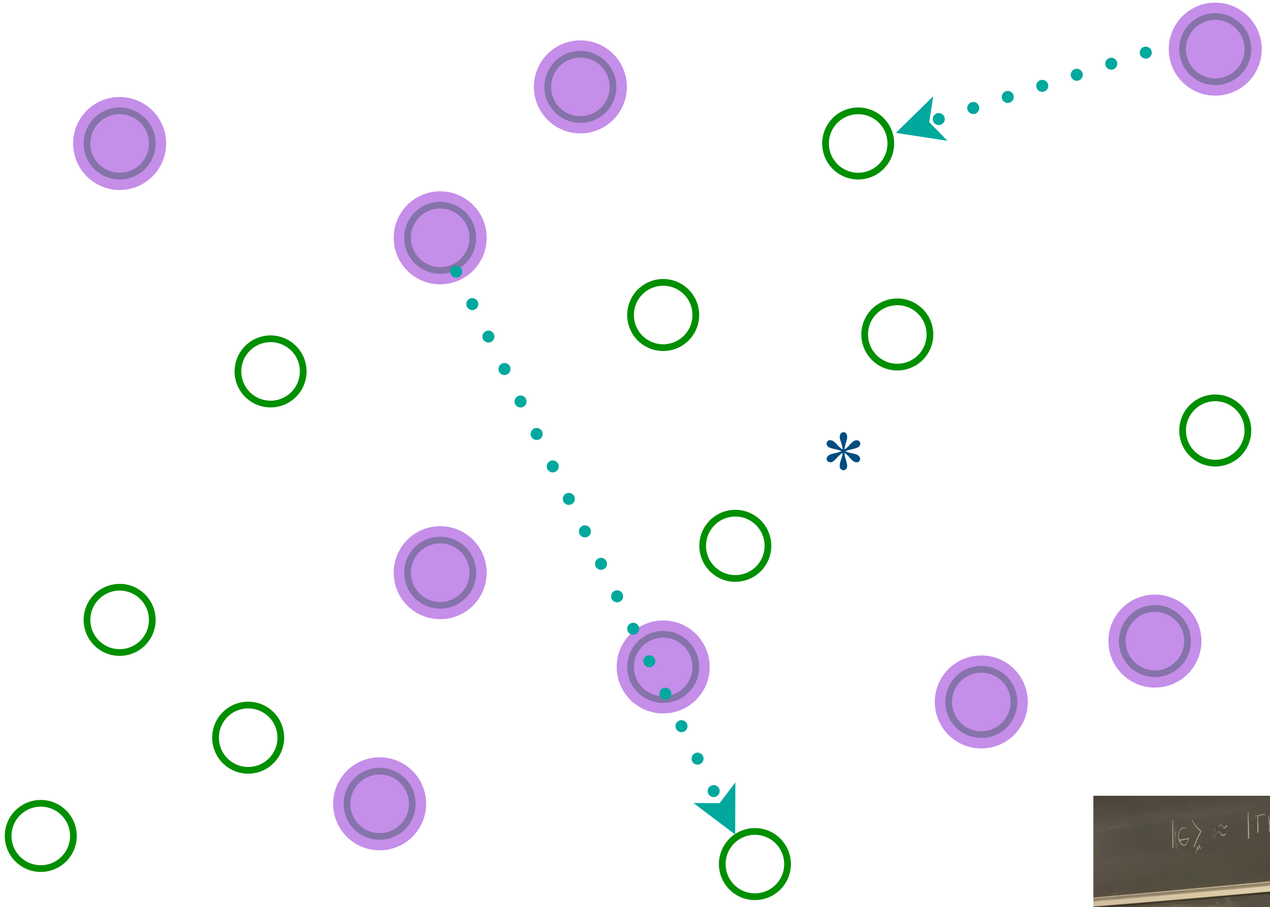

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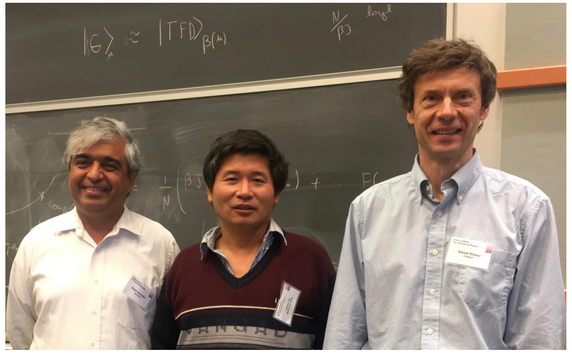

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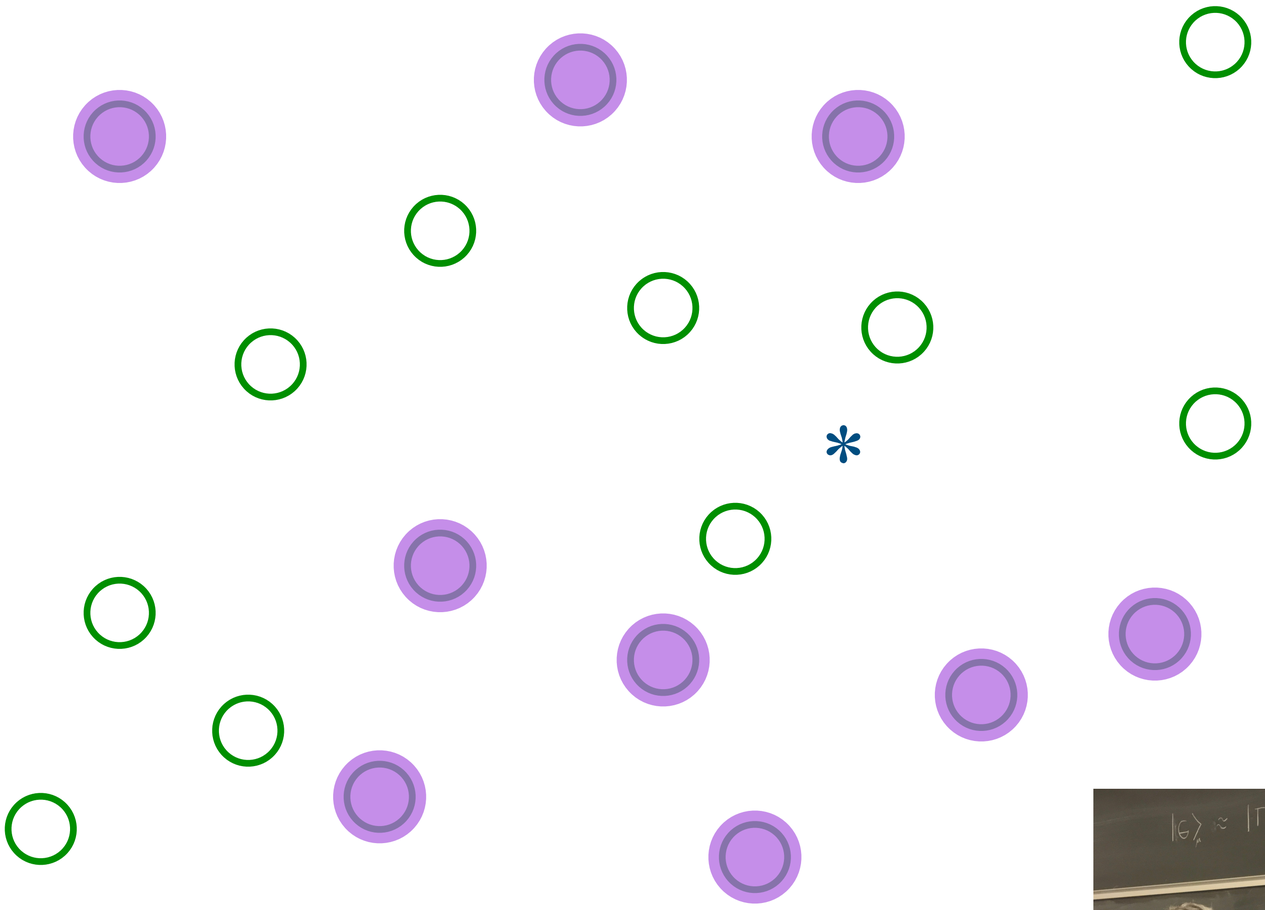
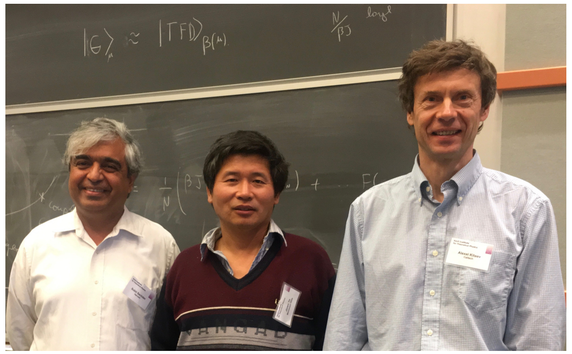
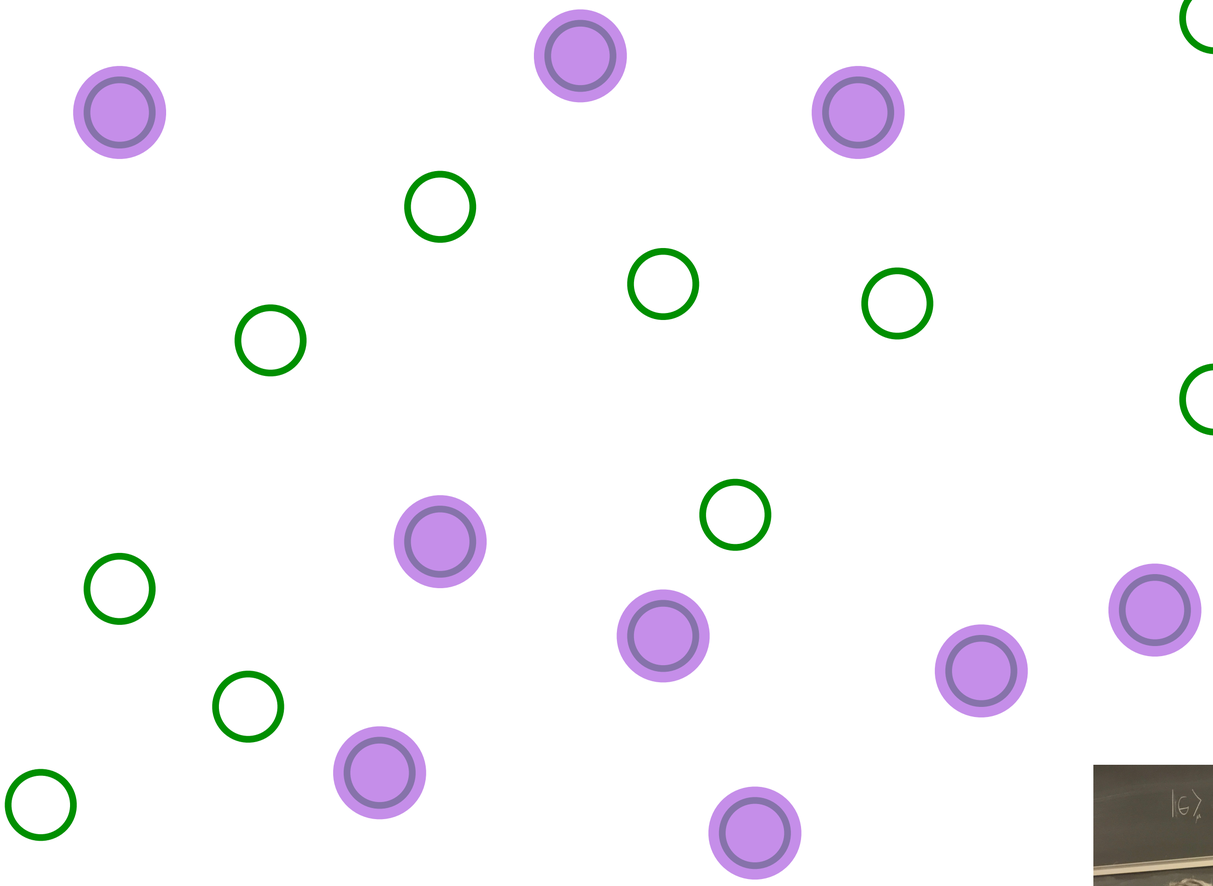


Entangle electrons pairwise randomly



# The SYK model

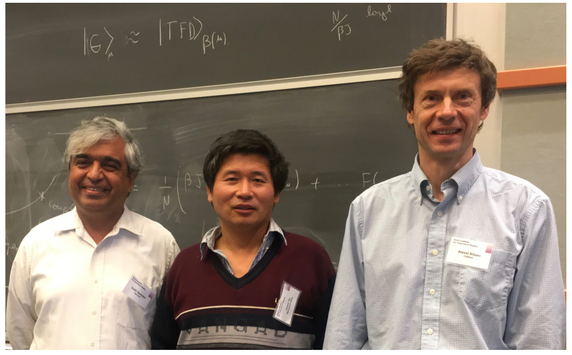
Sachdev, Ye (1993); Kitaev (2015)



Electron scattering time $\tau$ in the SYK model

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

Entangle electrons pairwise randomly

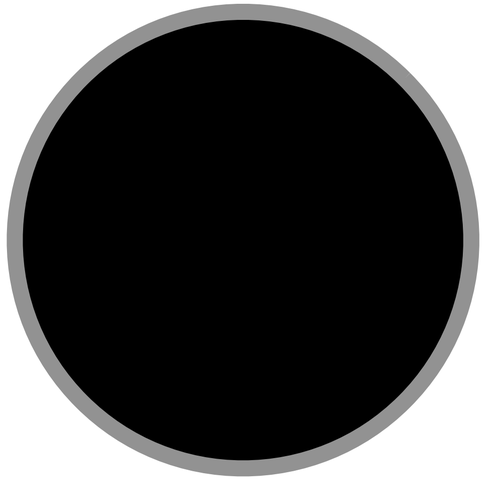


# Complex quantum entanglement in black holes

# Black Holes

Objects so dense that light is gravitationally bound to them.

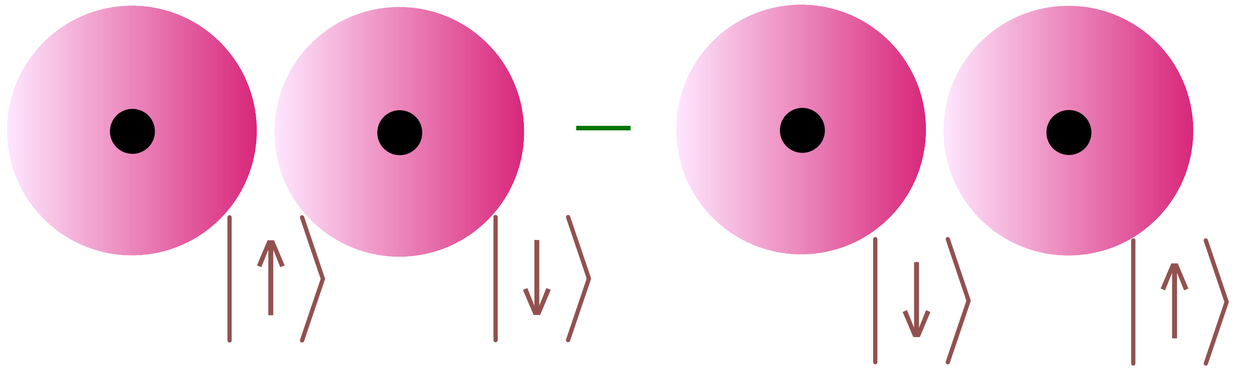
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



$G$ Newton's constant, $c$ velocity of light, $M$ mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

# Quantum Entanglement across a black hole horizon



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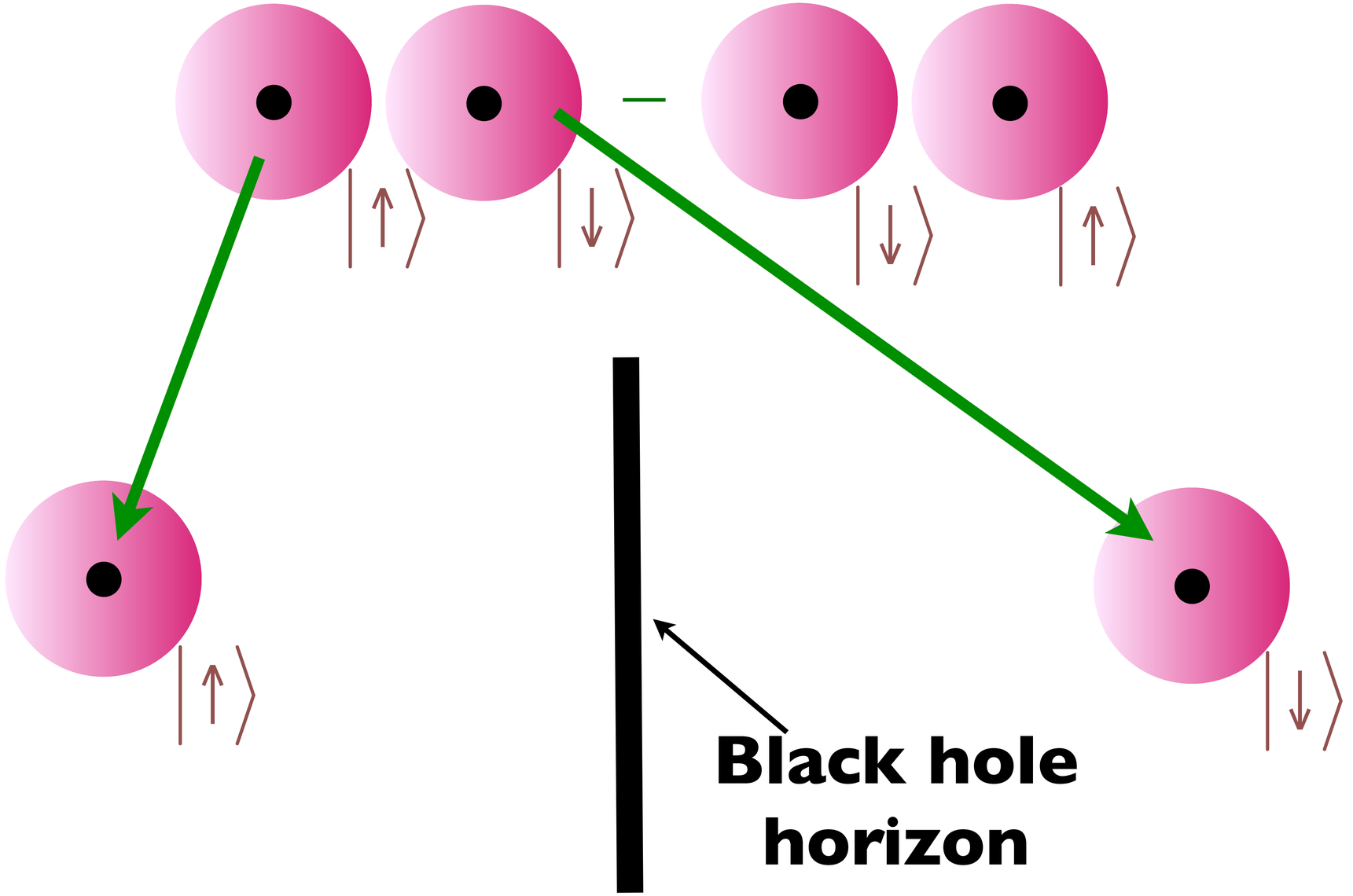

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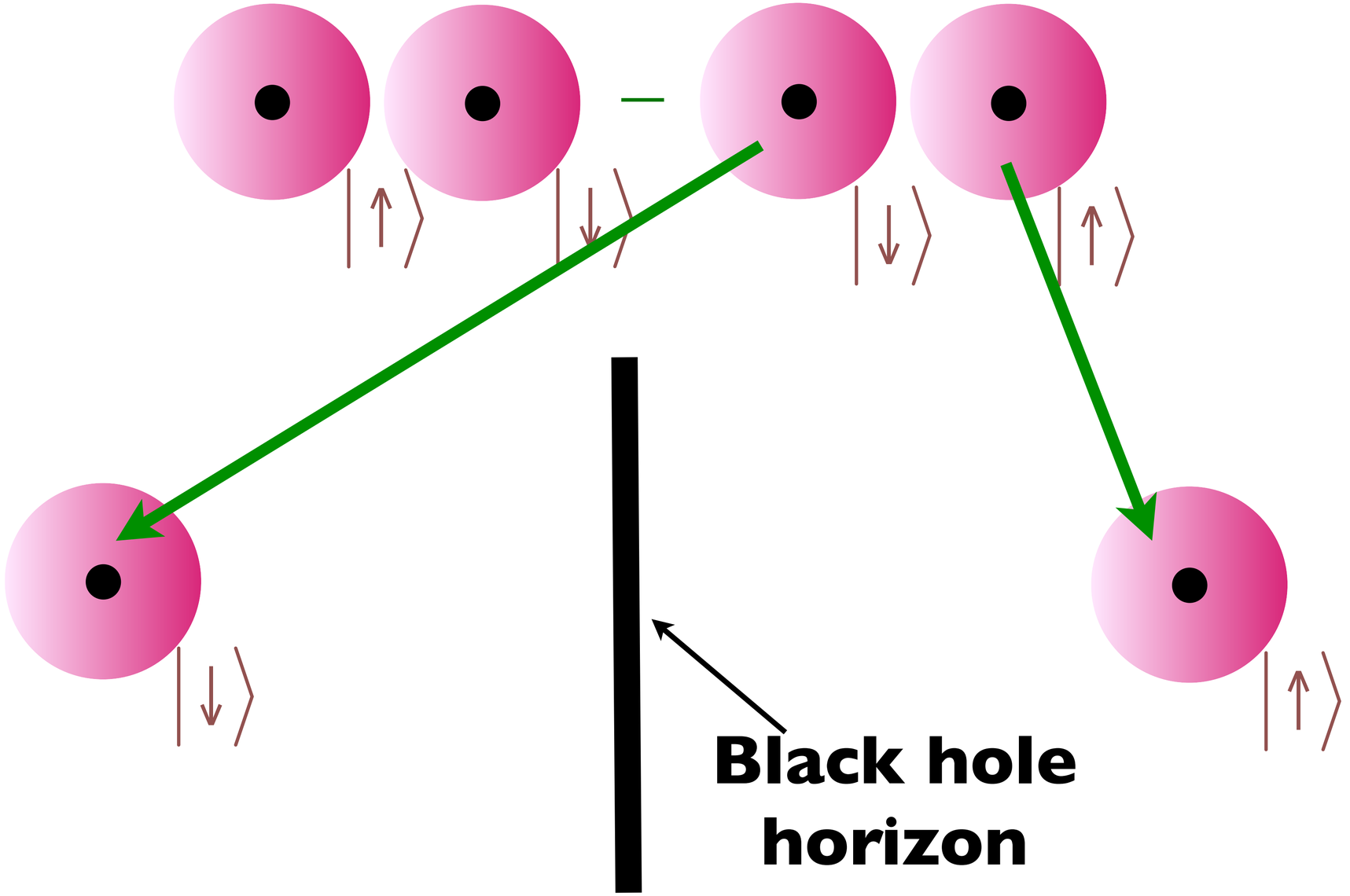
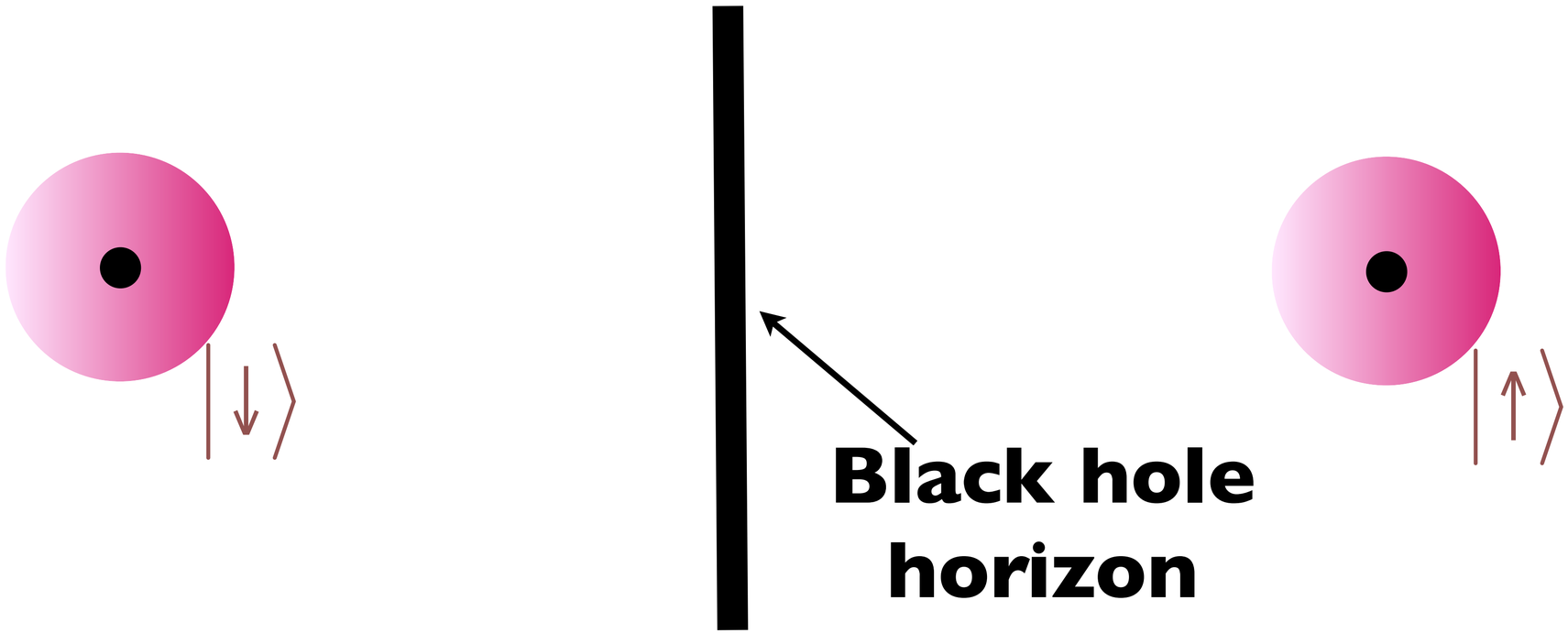


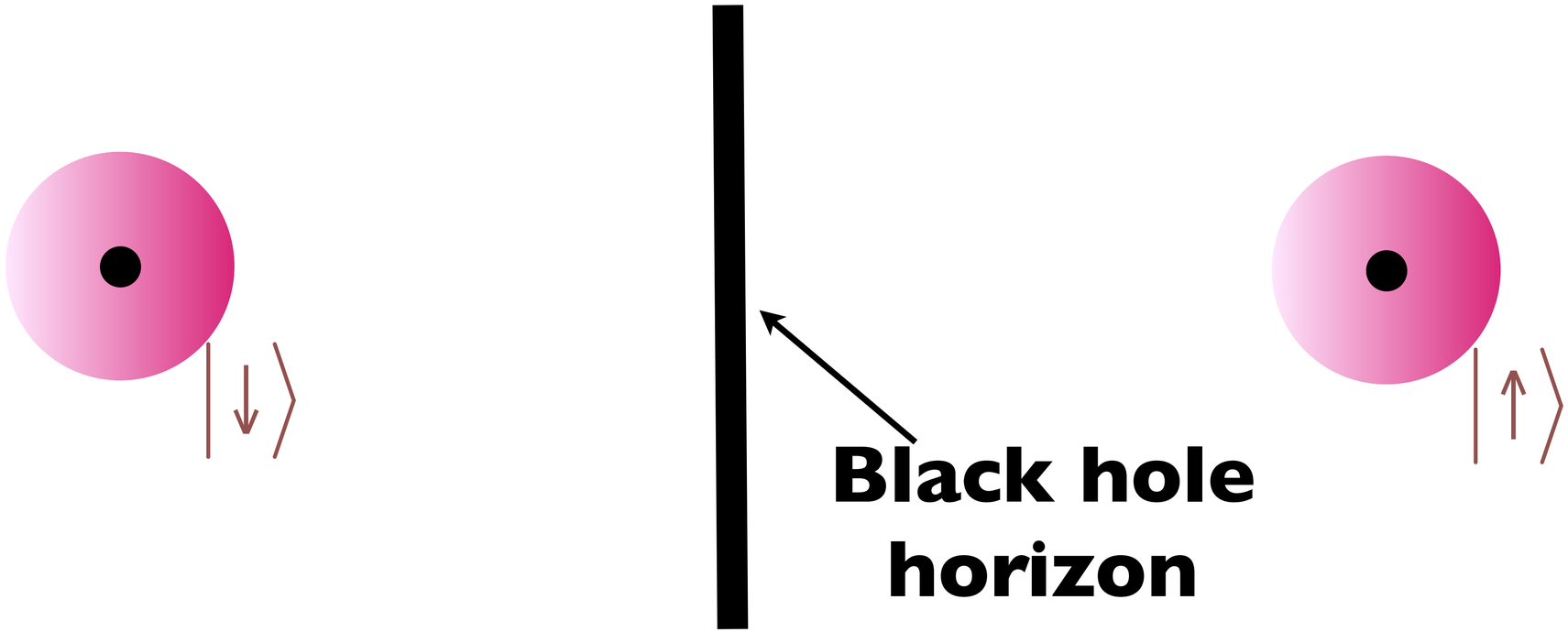
# Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



# Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state. )



# Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)

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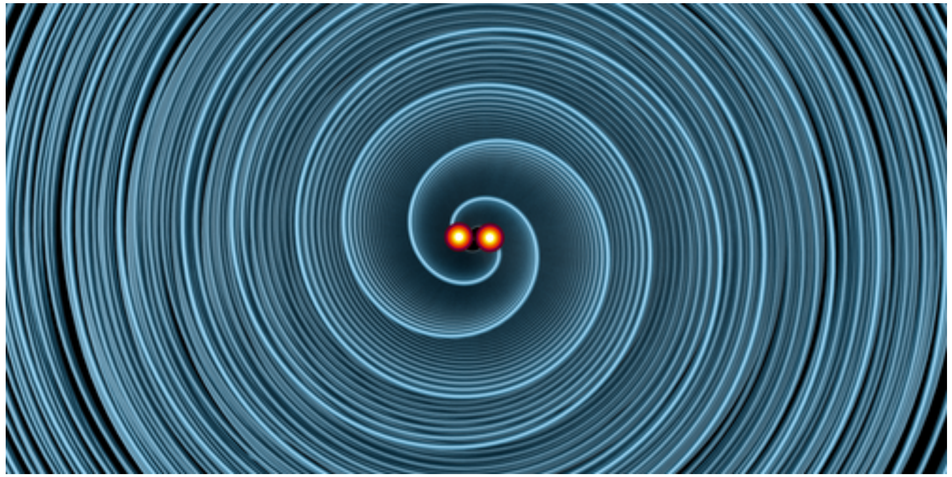
All many-body quantum systems
(without quantum gravity)
have an entropy
proportional to their volume !?!?

# Black Holes Obey Information-Emission Limits

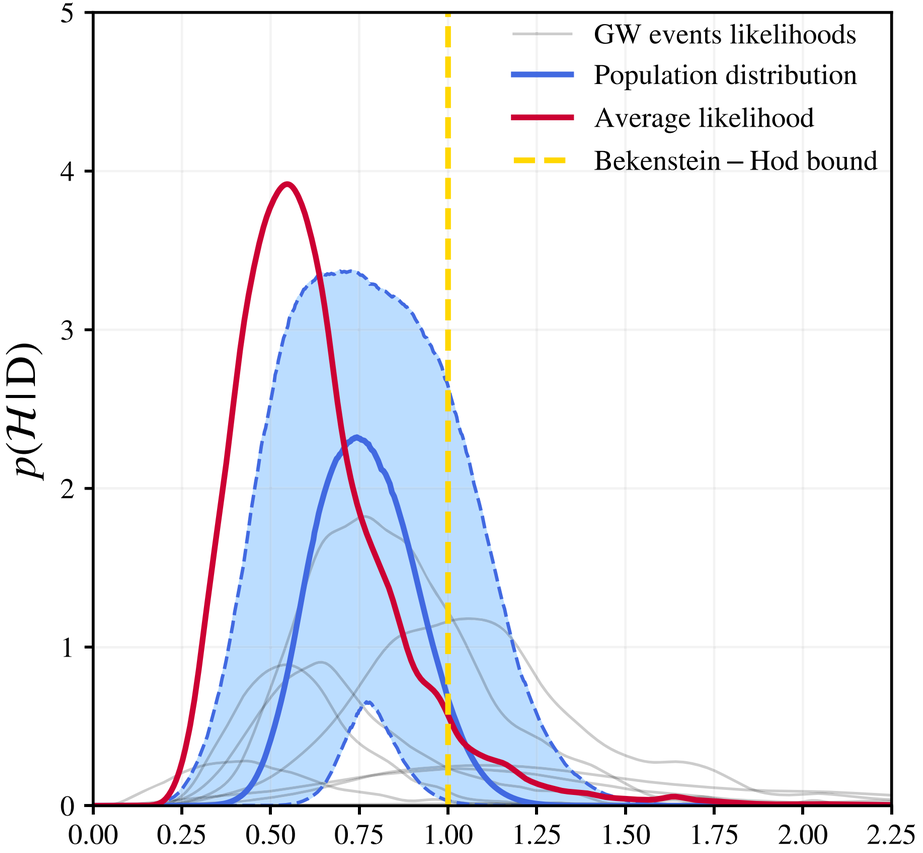
April 22, 2021 • *Physics* 14, s47 –Christopher Crockett
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

**An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.**



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$




$$\mathcal{H} = \frac{1}{\pi} \frac{\hbar/\tau}{k_B T}$$

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- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

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Black holes are represented as a `*hologram*' by a quantum many-body system in one lower dimension.

*Duality*: a `*change of variables*' between the many-particle configurations and the metric of spacetime

Susskind, Maldacena…..

# Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
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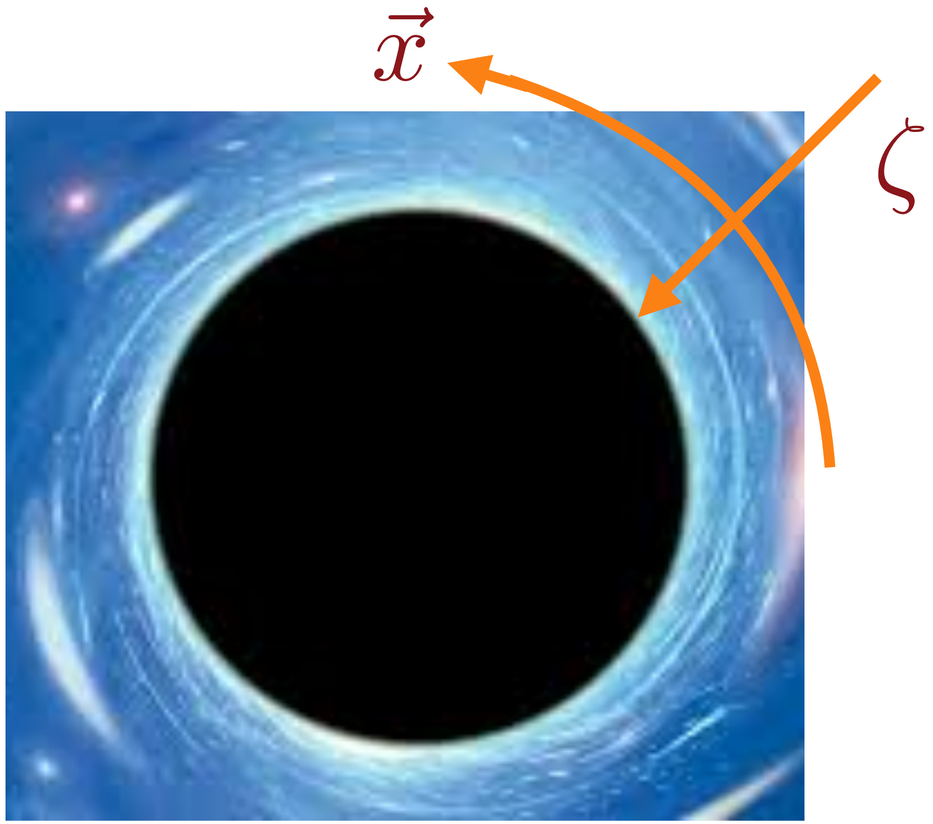
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

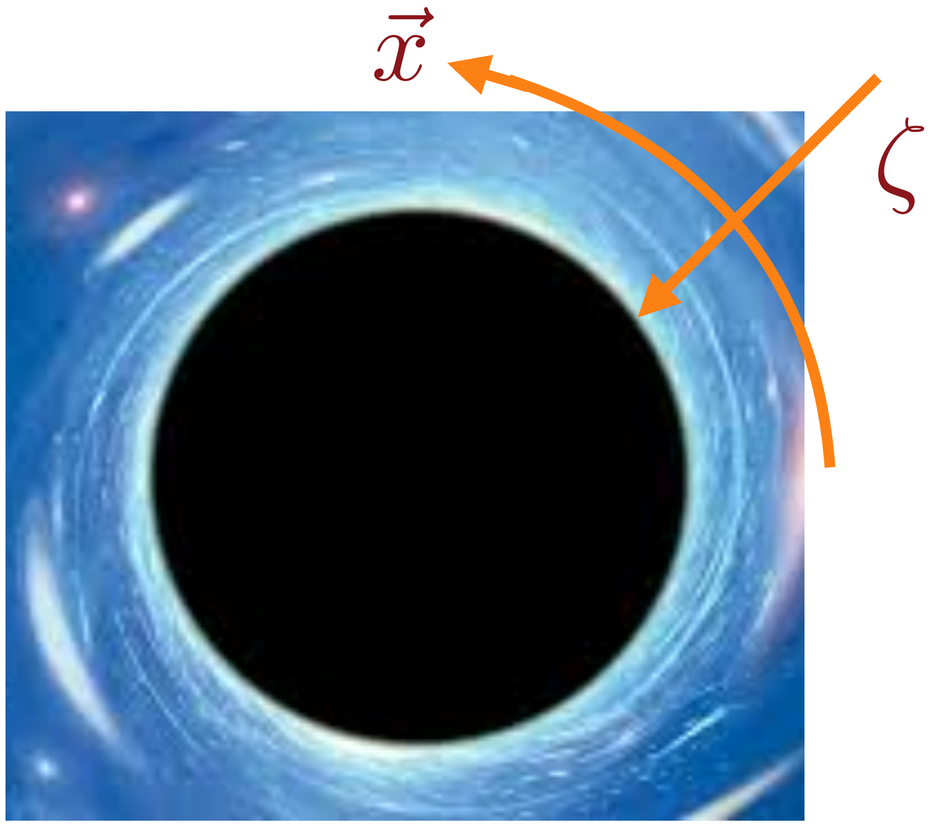
The hologram of a black hole
in $d$ dimensions
is a quantum many-particle system
in $(d-1)$ dimensions
which relaxes to thermal equilibrium
in a Planckian time $\sim \hbar/(k_B T)$


# Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge




The near-horizon geometry of a charged black hole is one-dimensional ($\zeta$)


# Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge




The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016) ; Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

# The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

It describes
certain ***strange metals***

Sachdev, Ye (1993)

# The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

In one set of variables, it describes certain ***strange metals***

Sachdev, Ye (1993)

In a *dual* set of variables it describes certain

***black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

Quantum theory of electrons,
one at a time:
metals and insulators

Quantum entanglement of
electron pairs:
superconductivity

Quantum entanglement of 2,3,4,……$\infty$ electrons: strange metals

Complex quantum entanglement in black holes