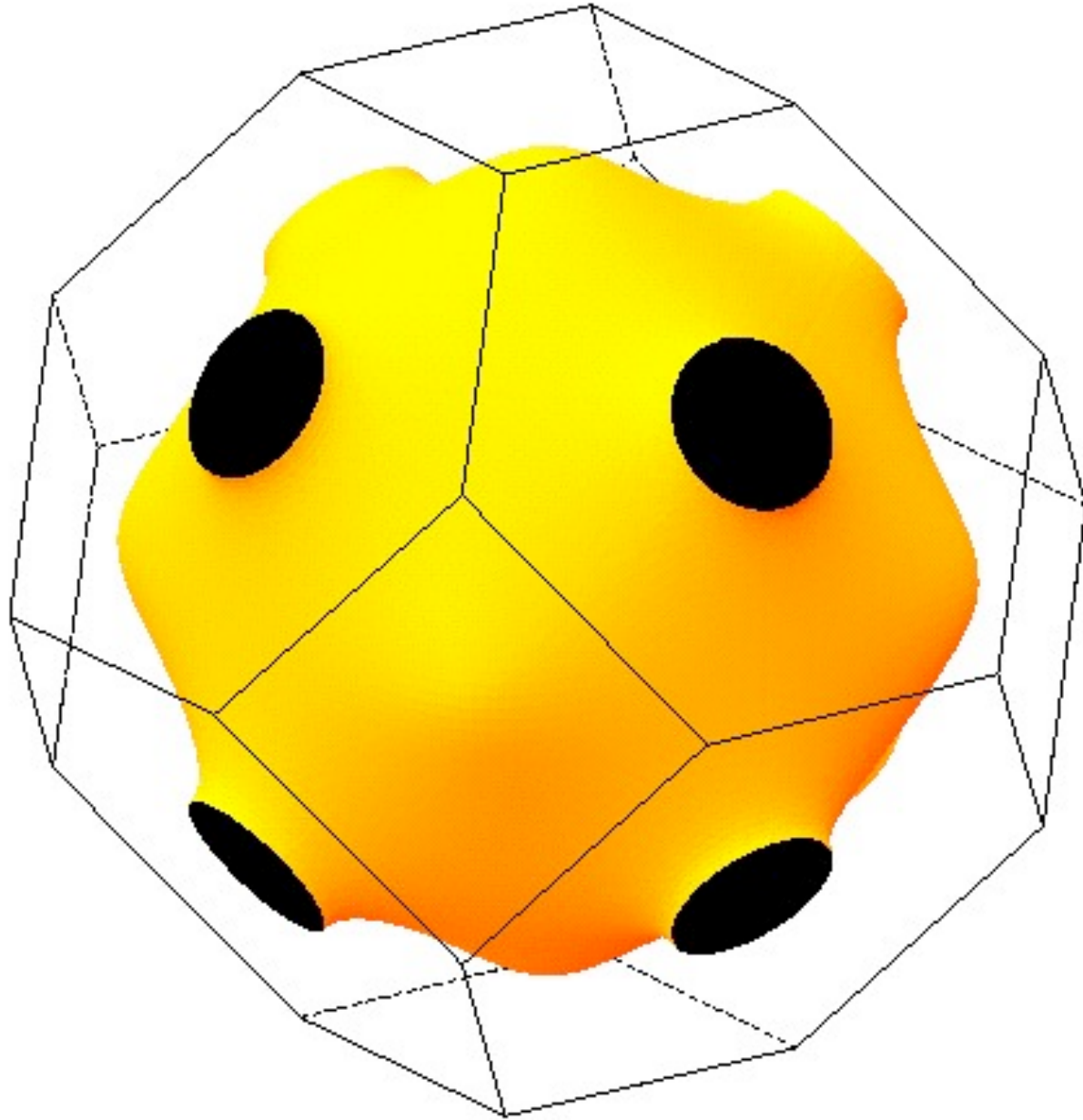


Metals, Insulators, Superconductors,.....

S. Sachdev

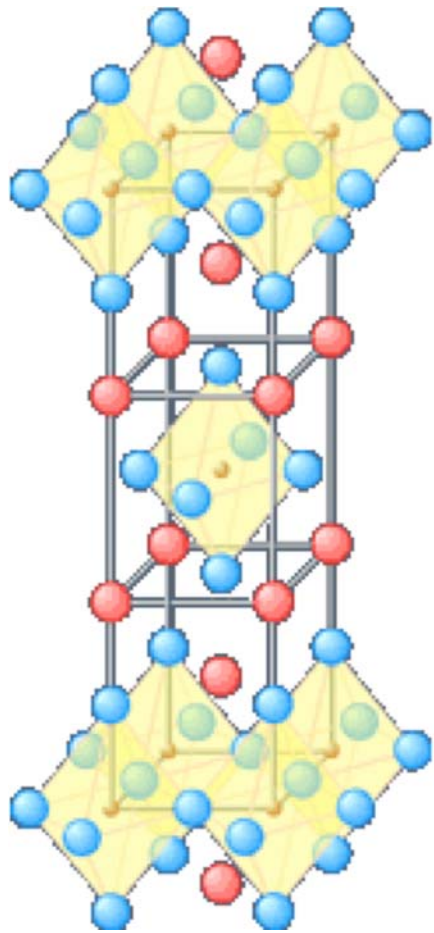
<http://qpt.physics.harvard.edu/p168.pdf>

Metals



Fermi surface of Cu

Sr_2RuO_4



RuO_2

SrO

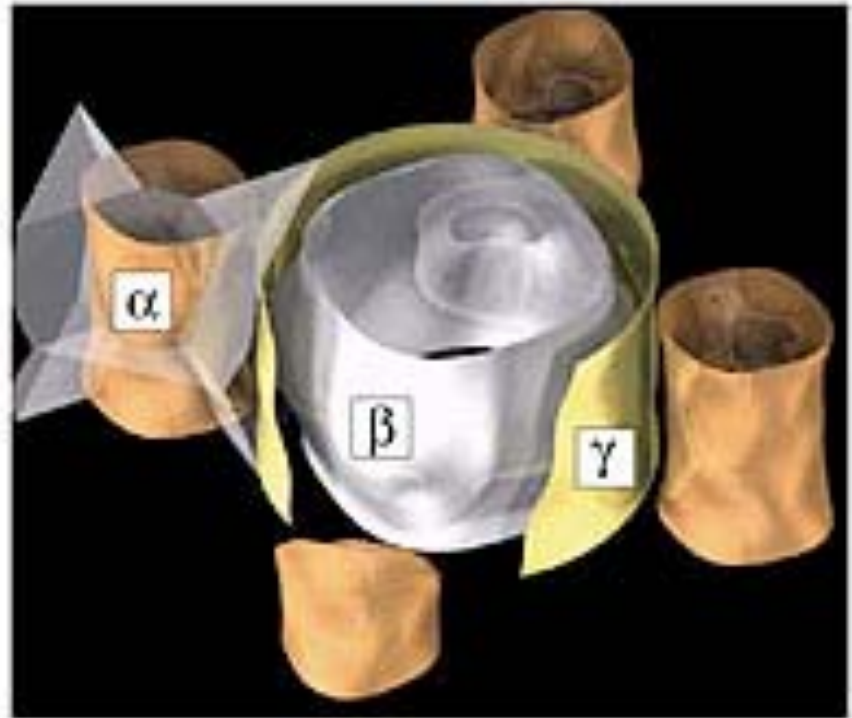
SrO

RuO_2

SrO

SrO

RuO_2



de Hass-van Alphen oscillations

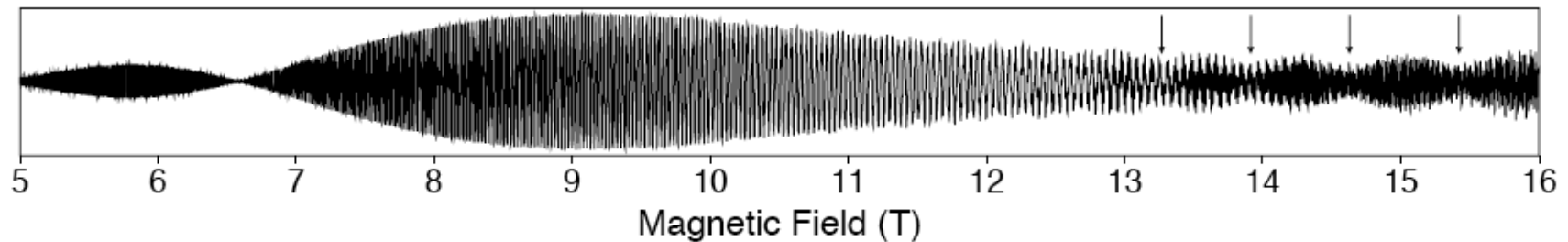


FIG. 1. Example of a dHvA field sweep on Sr_2RuO_4 , for $\theta = 9.8^\circ$ off the c axis in the $[001] \rightarrow [110]$ rotation study. The vertical axis is the pickup signal (in arbitrary units) at the second harmonic of the excitation frequency. At high fields, beats in the β oscillations are visible, as indicated by arrows.

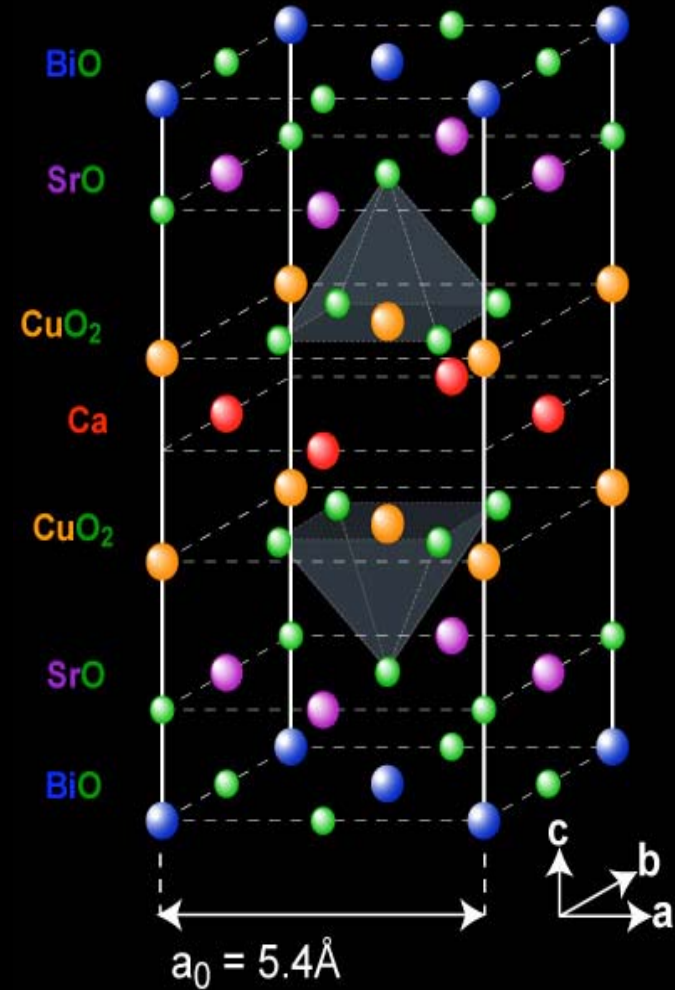
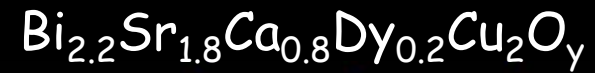
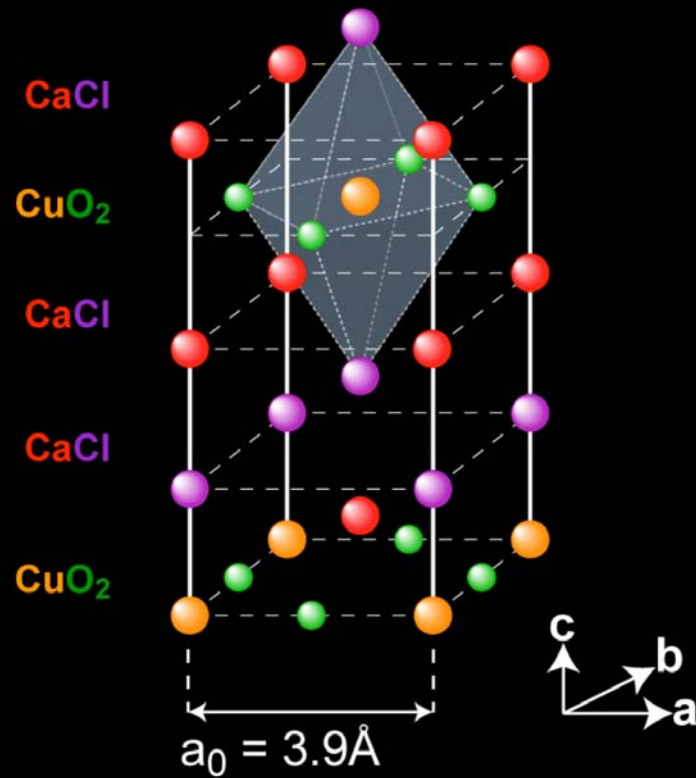
$$\text{Period of oscillation in } 1/H = \frac{2\pi e}{\hbar c} \frac{1}{\text{extremal area}}$$

Luttinger relation:

$$2 \times \frac{\text{Volume enclosed by Fermi surface}}{8\pi^3} \\ = \text{density of electrons modulo}(2 \times \text{volume of Brillouin zone})$$

Insulators

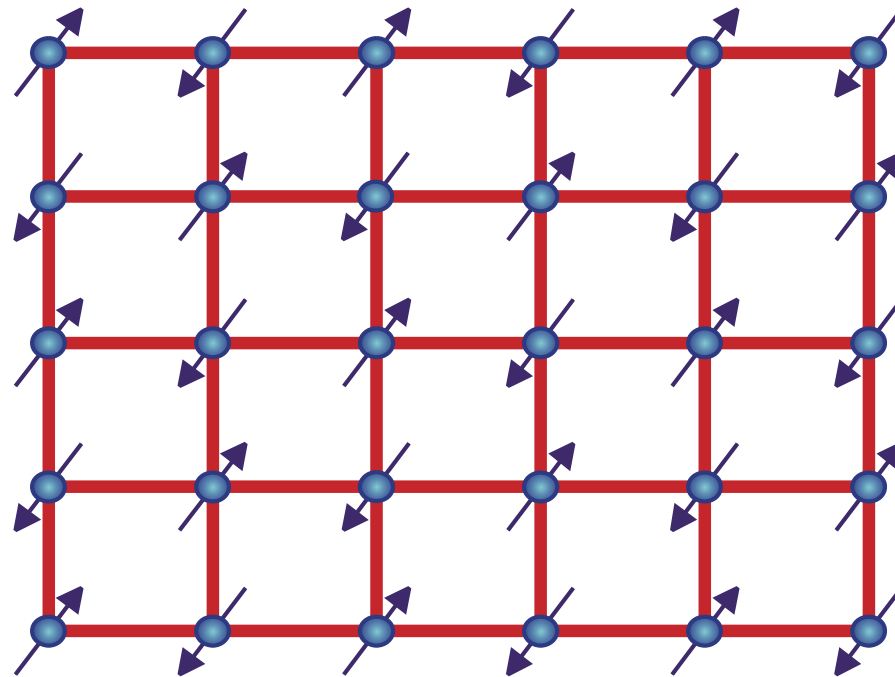
Antiferromagnetism in the cuprate superconductors



S=1/2 Heisenberg antiferromagnets on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

For small Q

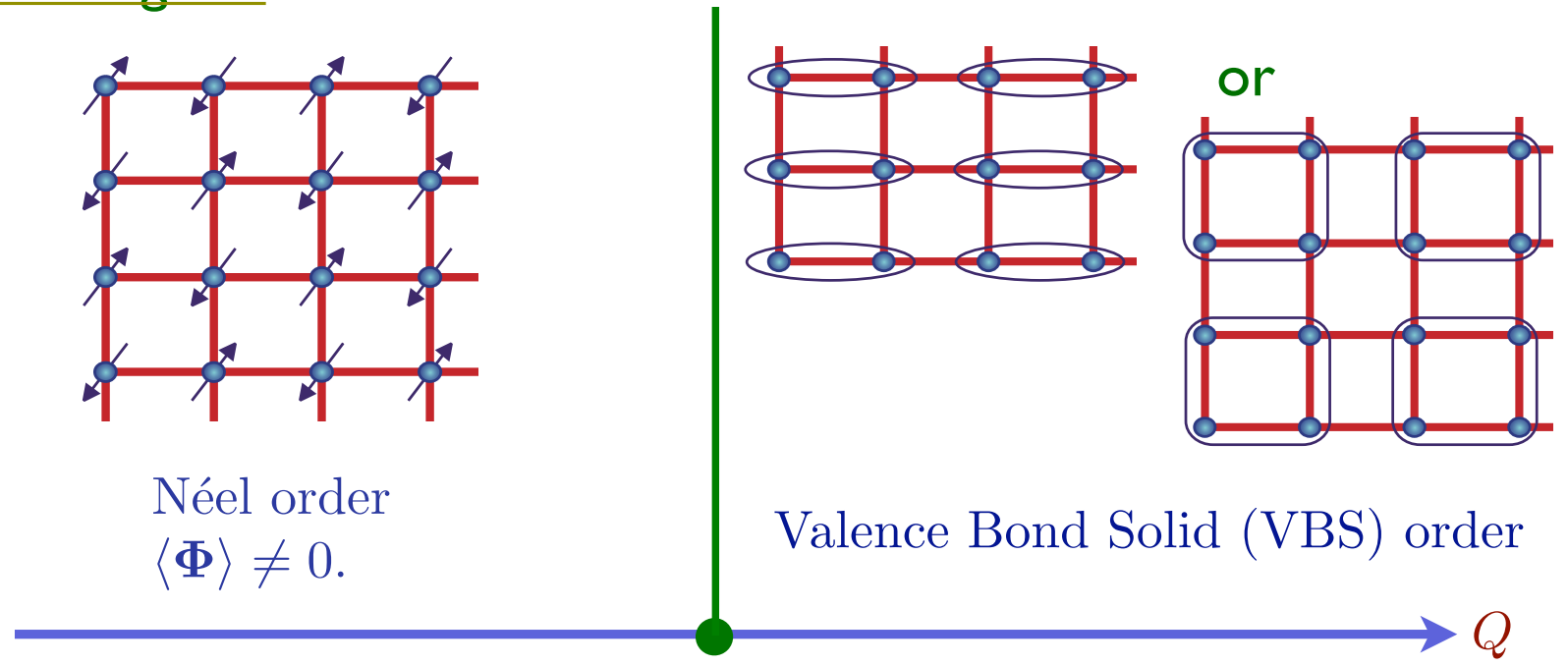


$$\text{Order parameter } \Phi = (-1)^i \mathbf{S}_i$$

S=1/2 Heisenberg antiferromagnets on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

Phase diagram

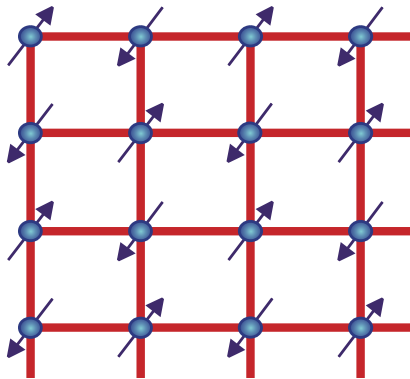


$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

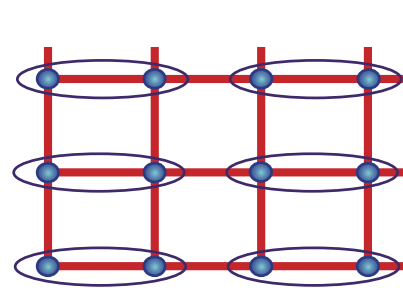
S=1/2 Heisenberg antiferromagnets on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

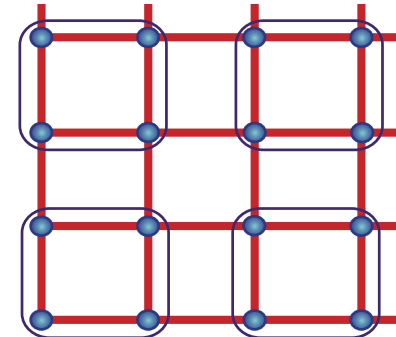
Phase diagram



Higgs phase: Néel order
 $\langle z_\alpha \rangle \neq 0, \langle \Phi \rangle \neq 0.$



or



“Coulomb” phase: VBS order,
 Monopole $\langle V \rangle \neq 0$

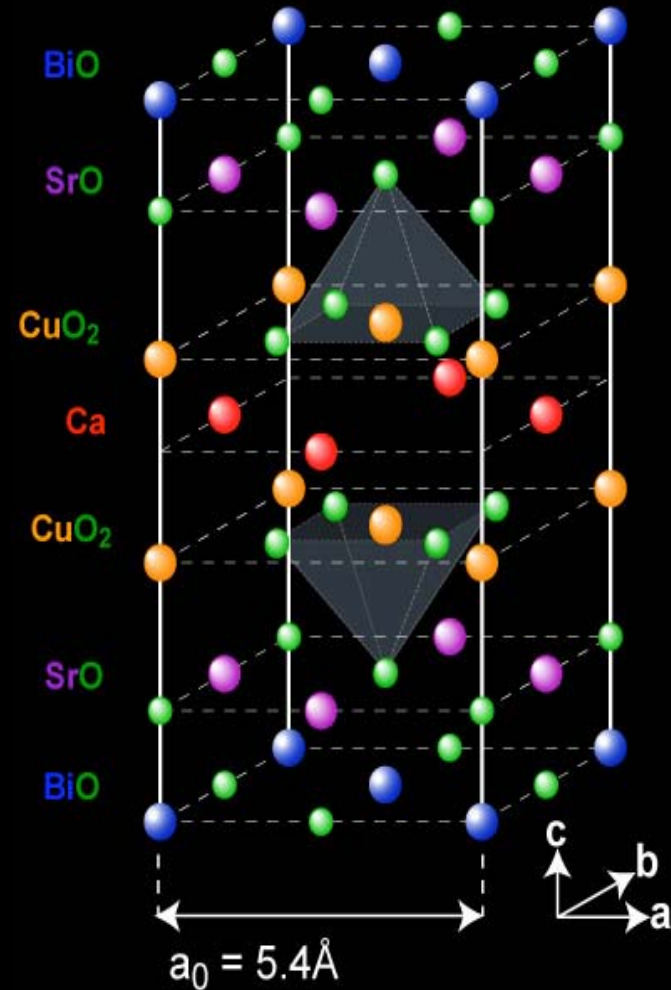
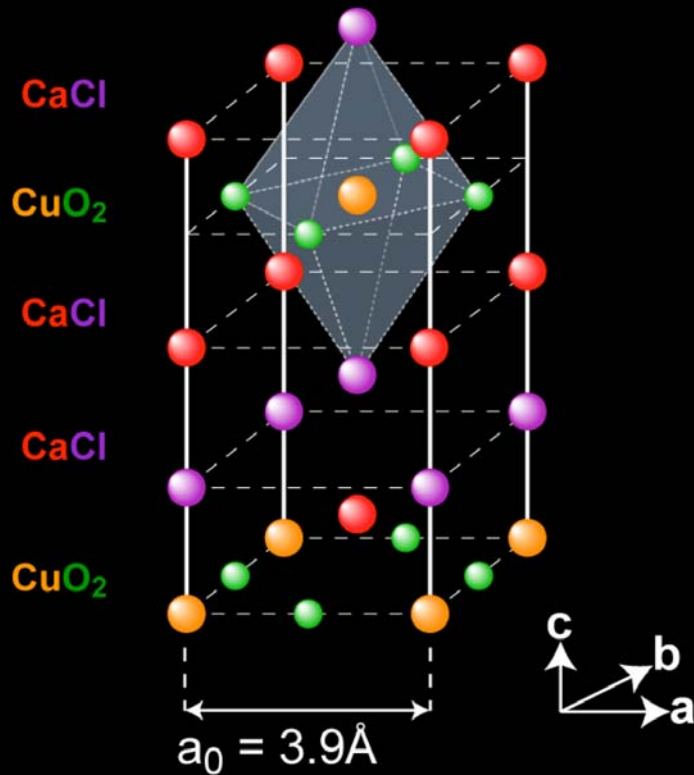


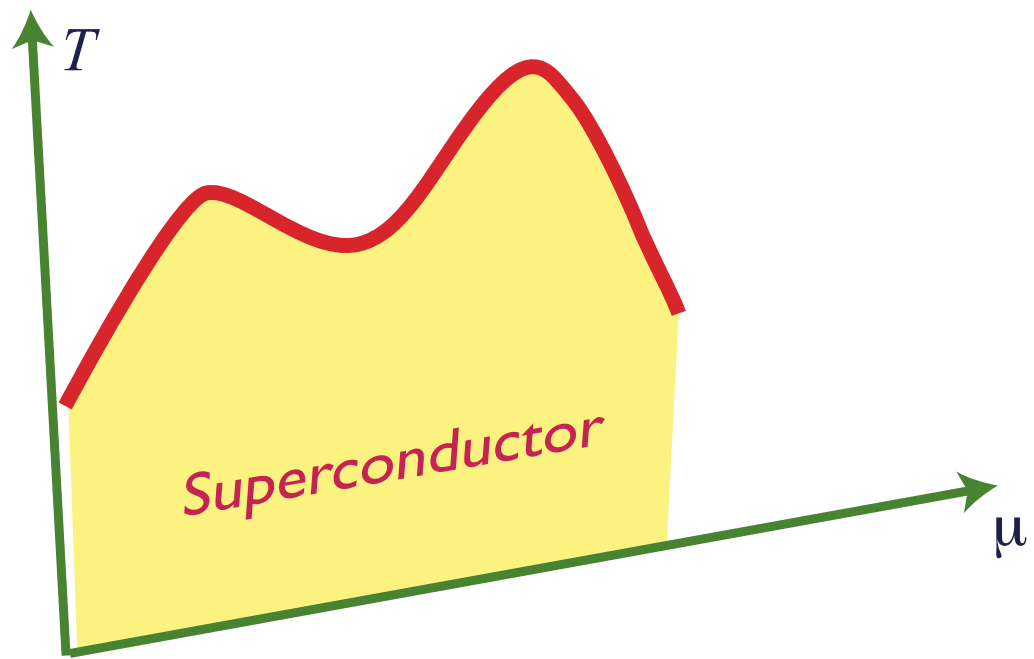
$$\mathcal{S}_z = \int d^2r d\tau \left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

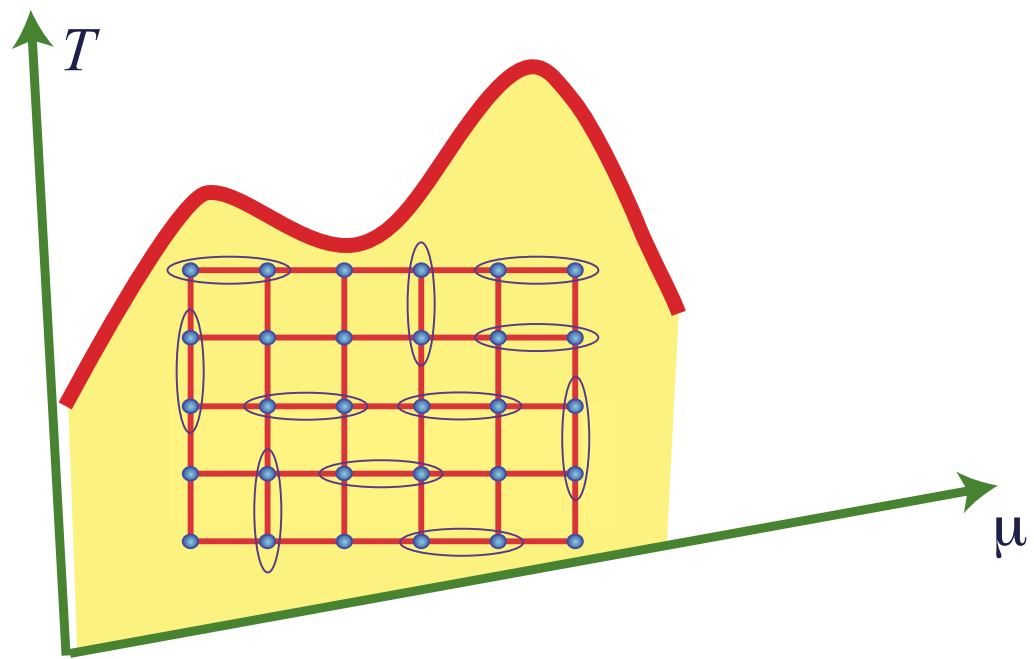
Similar phase diagram for $\mathcal{N} = 4$ supersymmetric quantum electrodynamics with 2 matter hypermultiplets

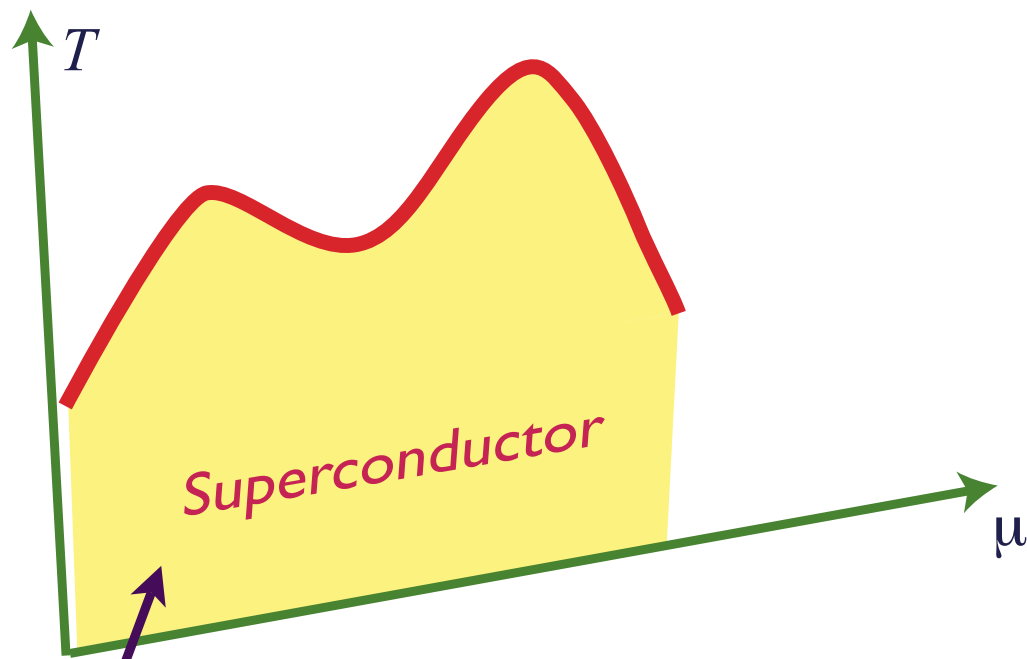
Superconductors

Dope the antiferromagnets with charge carriers of density x by applying a chemical potential μ



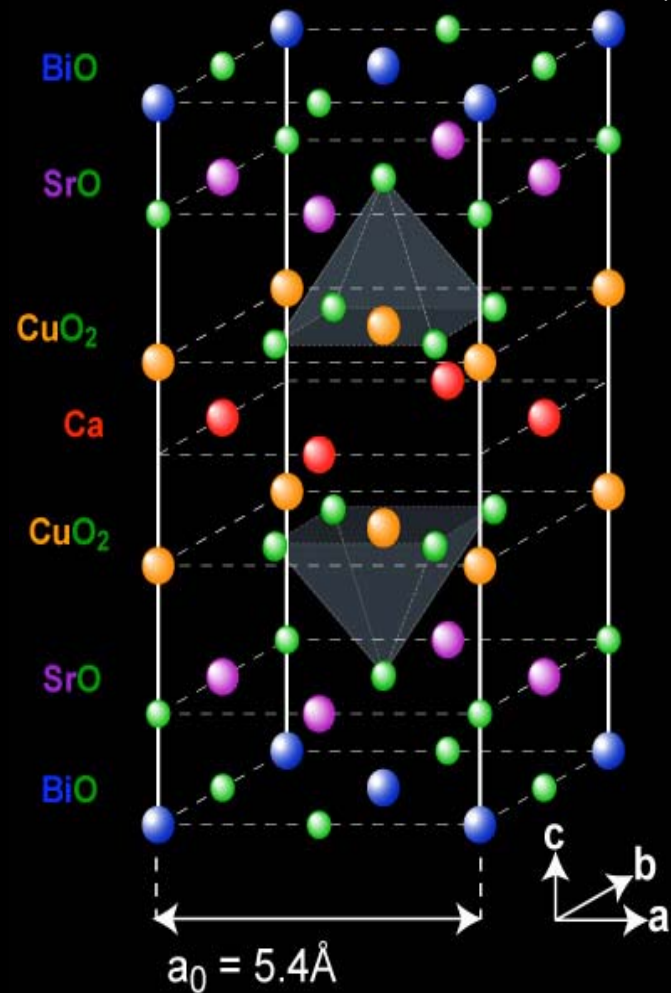
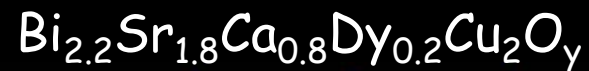
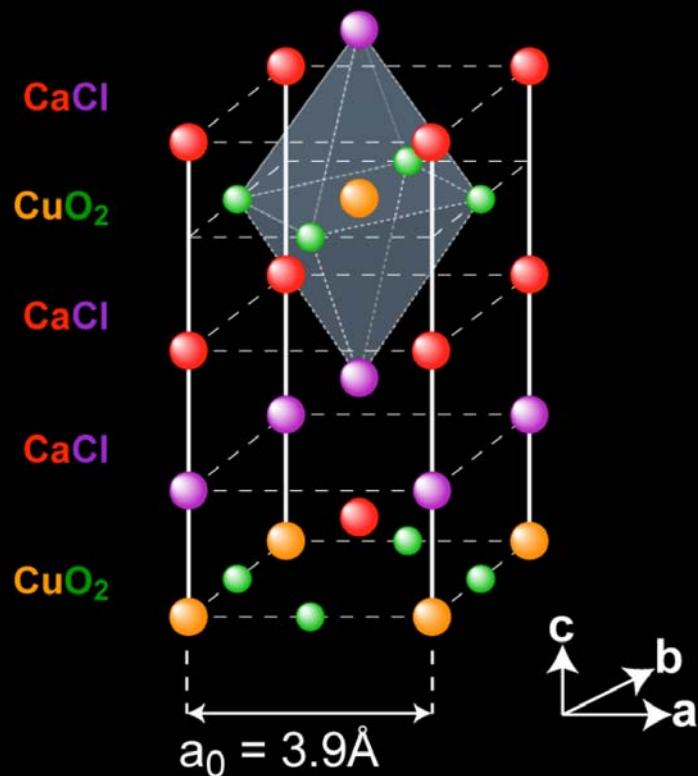




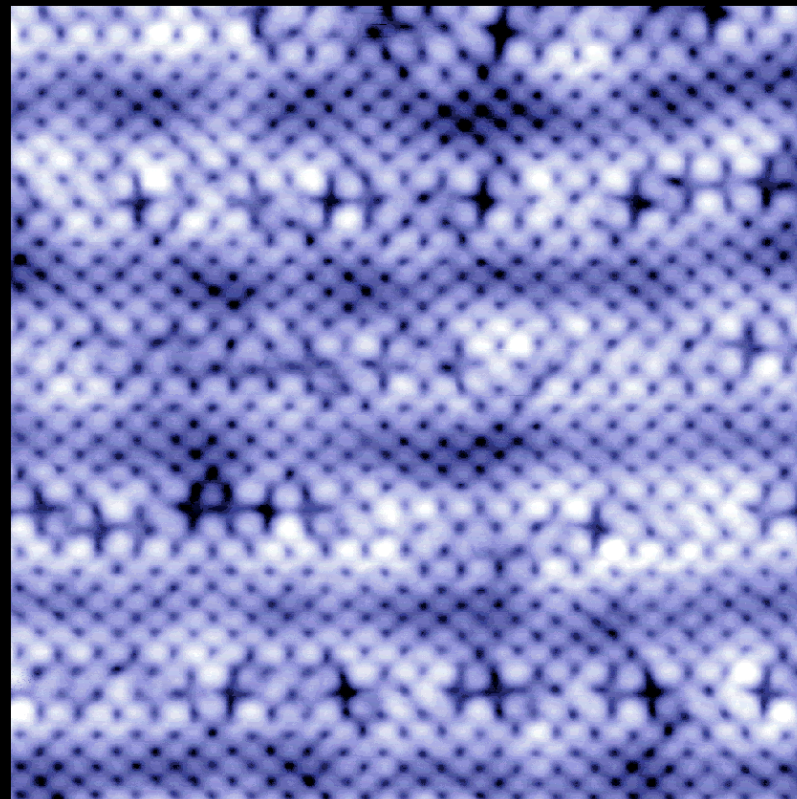
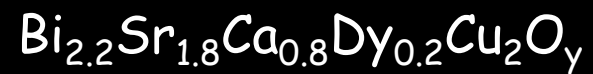
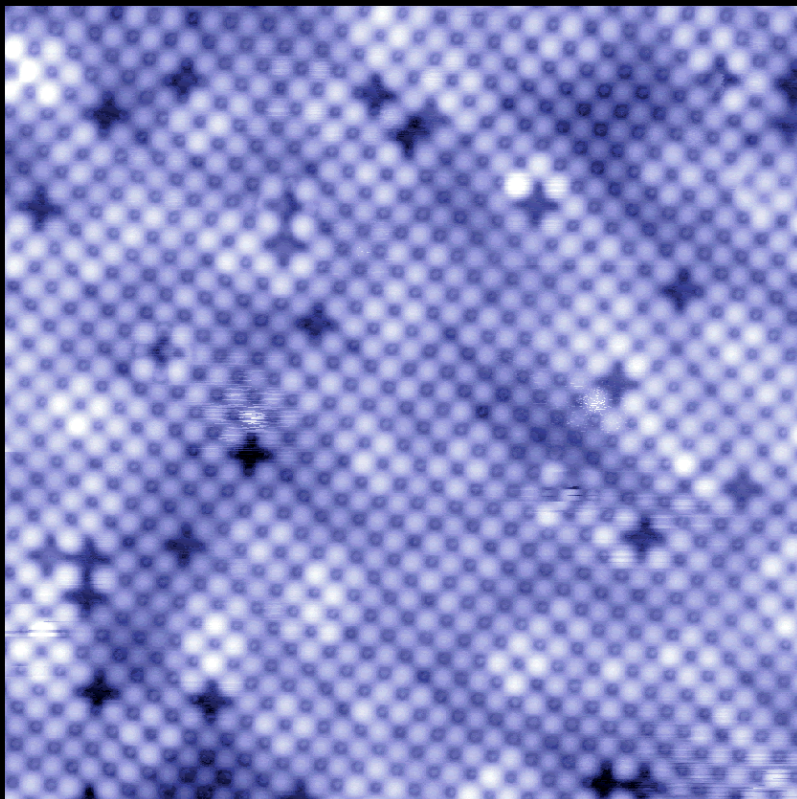
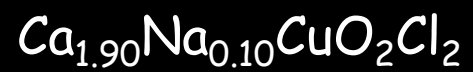


Scanning tunnelling microscopy

STM studies of the underdoped superconductor

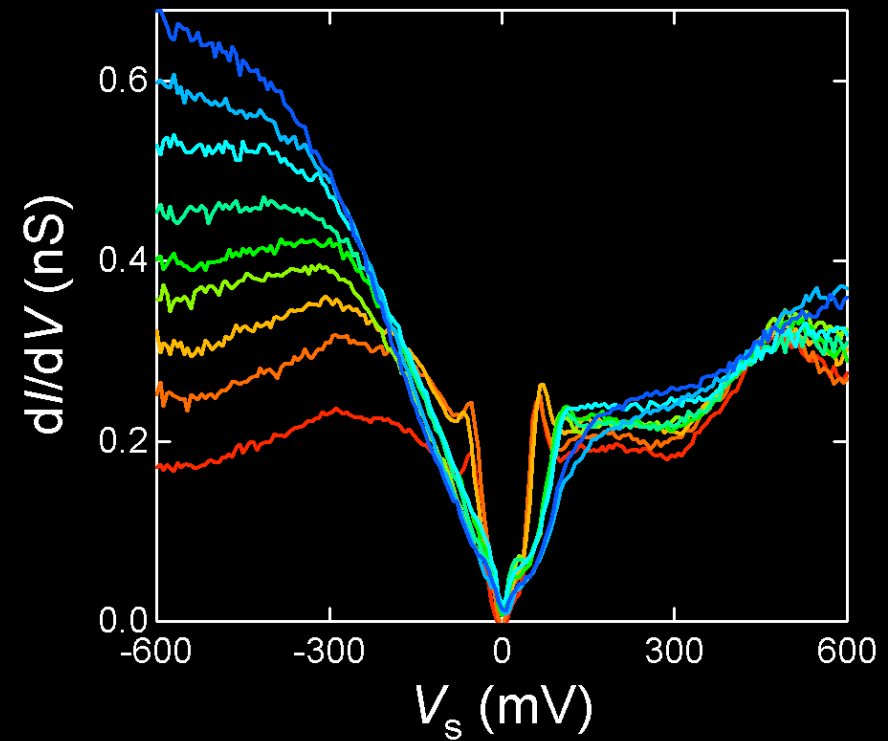
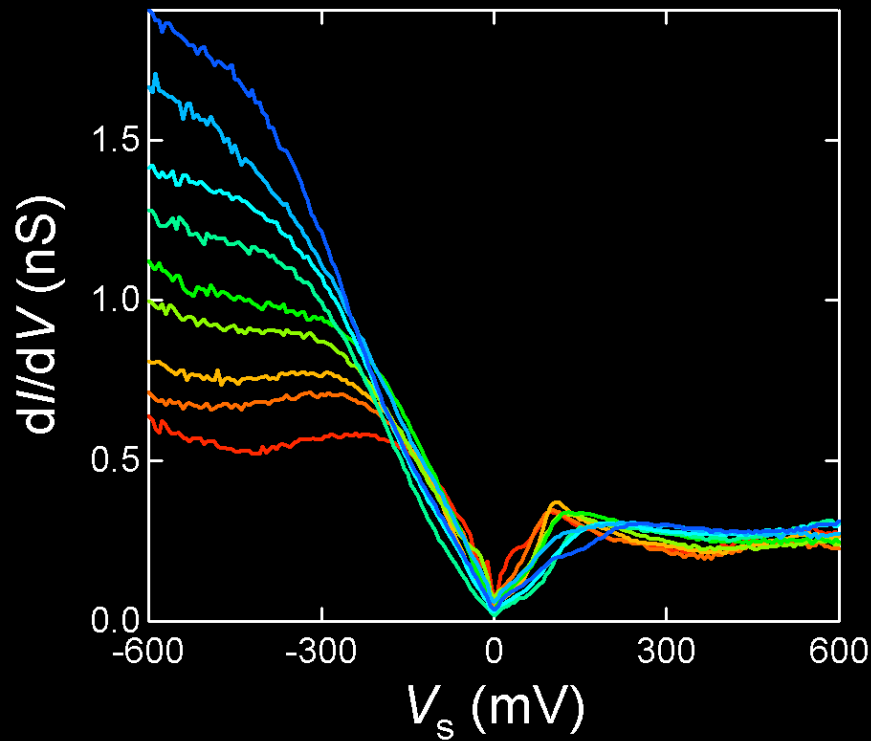


Topograph



12 nm

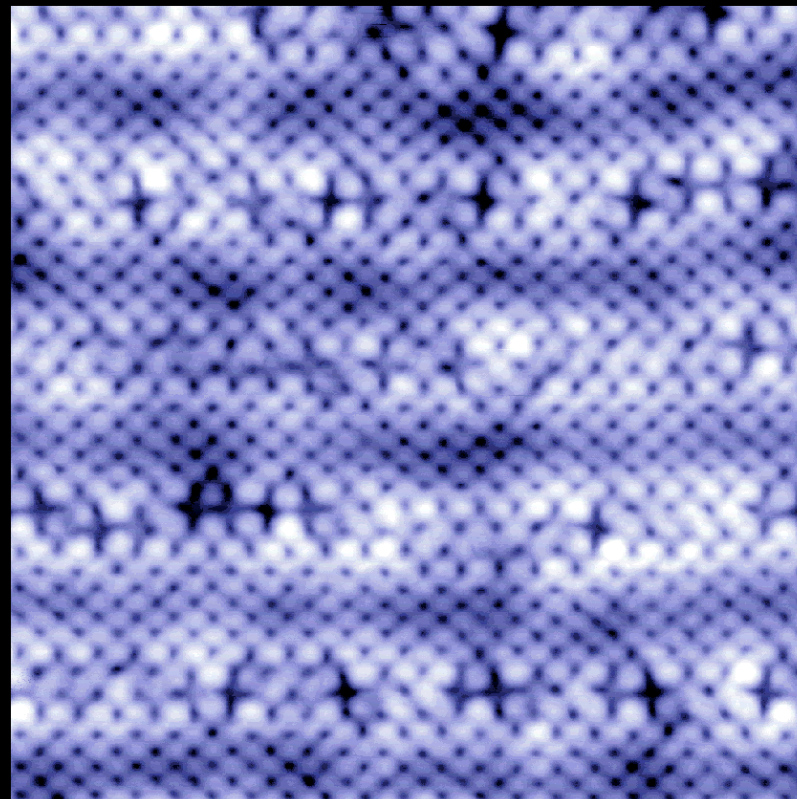
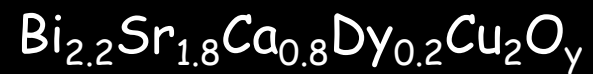
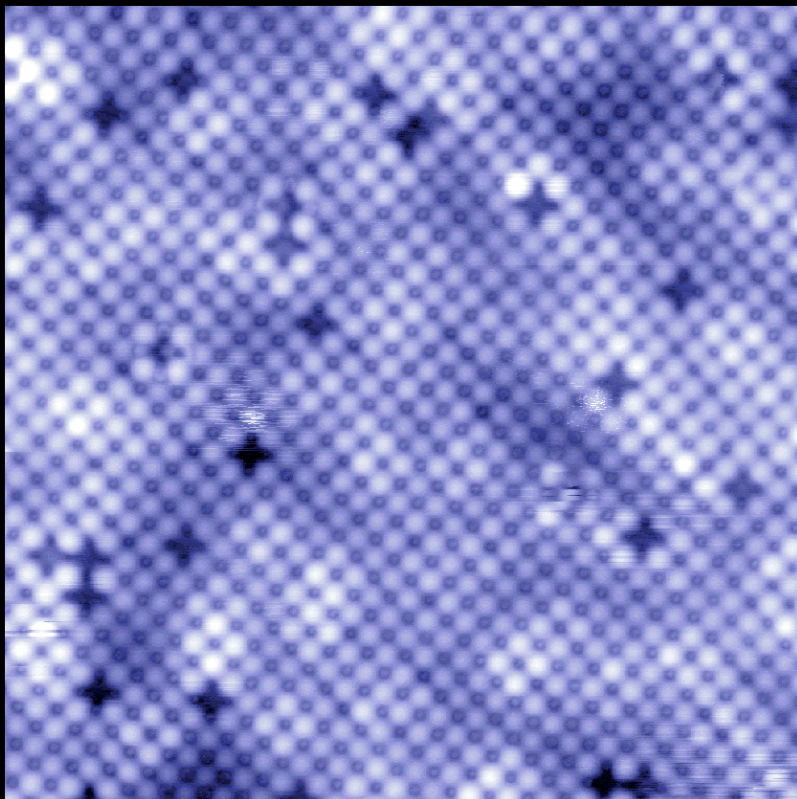
dI/dV Spectra



Intense Tunneling-Asymmetry (TA)
variation are highly similar

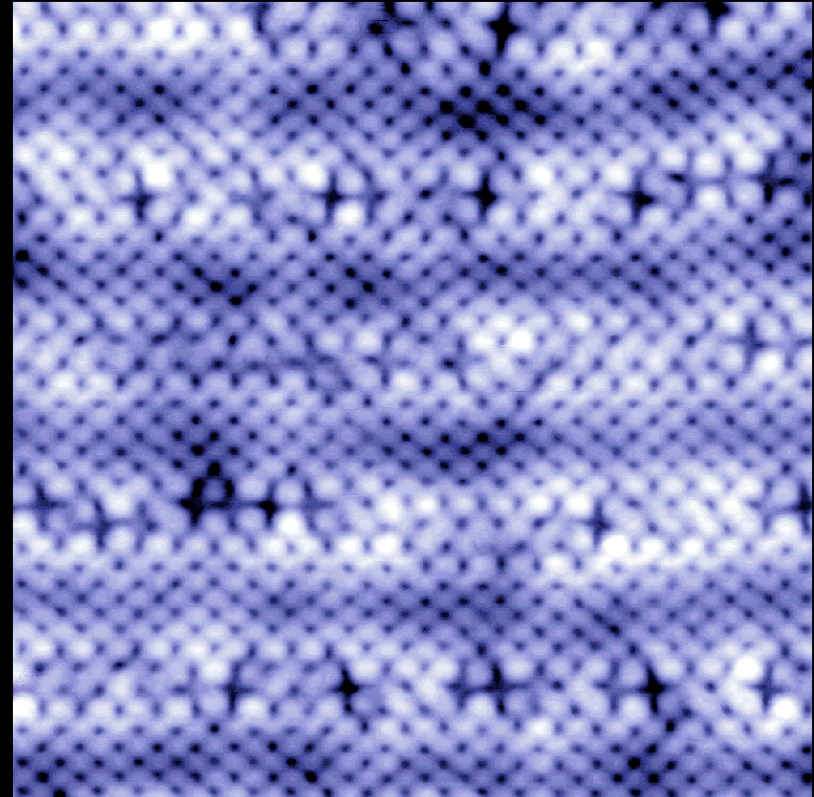
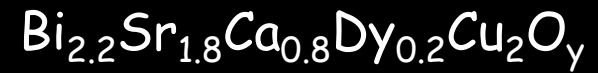
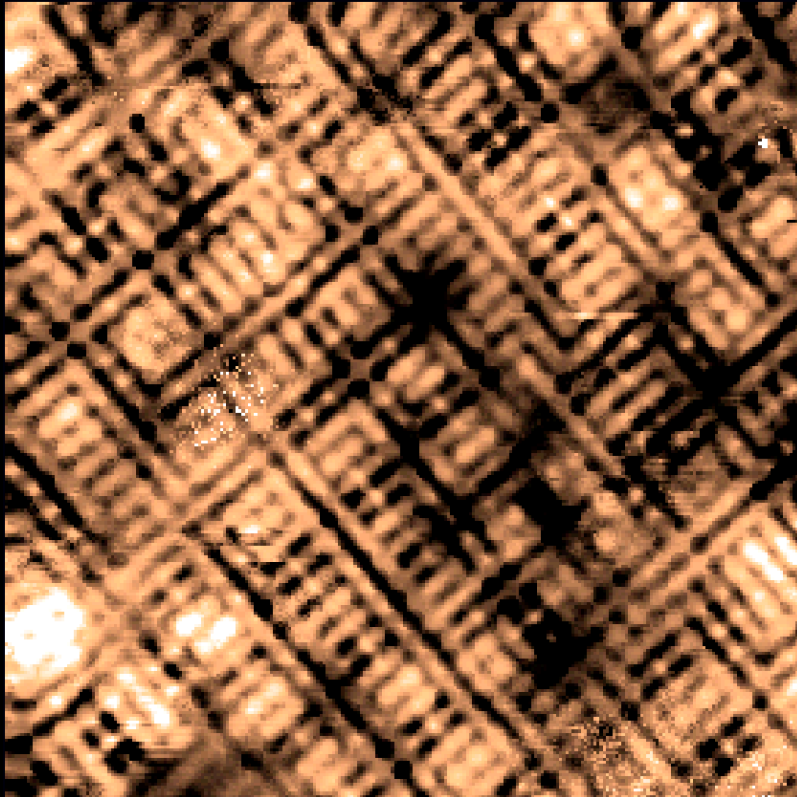
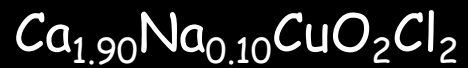
Y. Kohsaka et al. Science 315, 1380 (2007)

Topograph



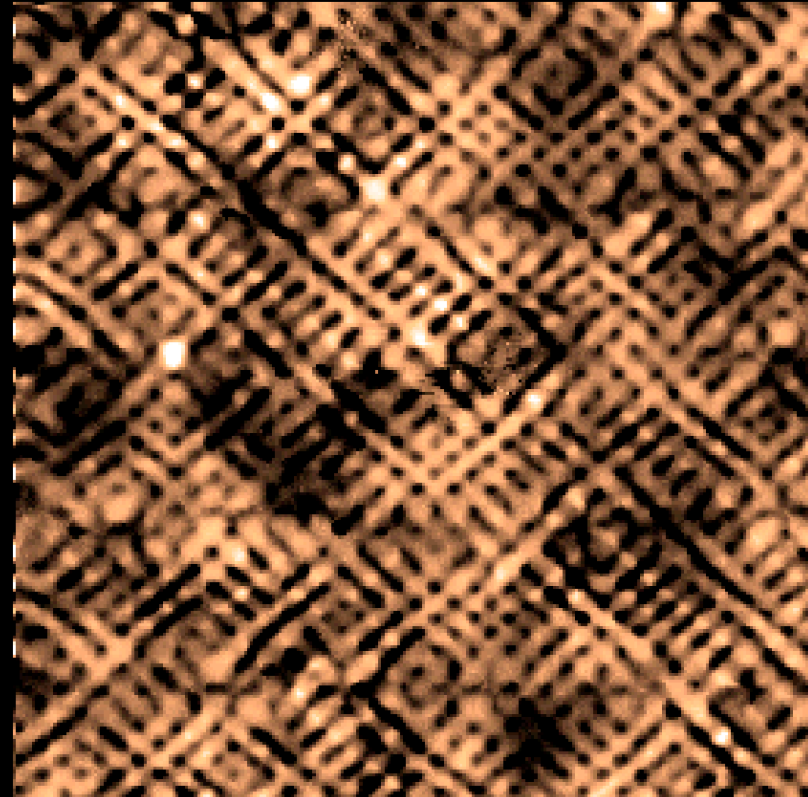
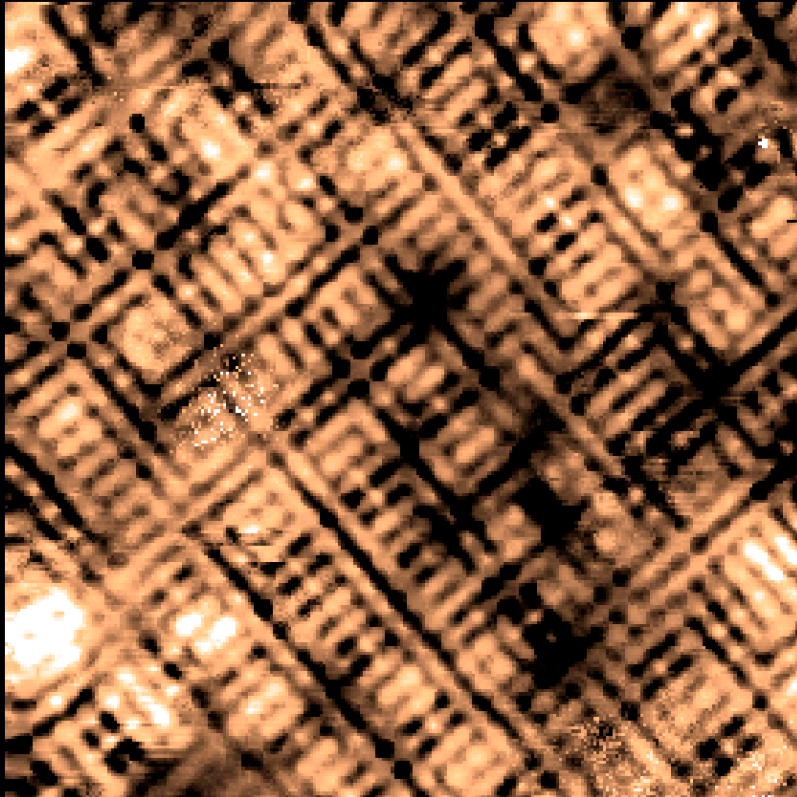
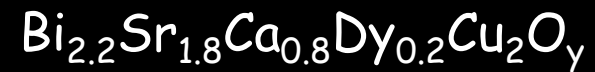
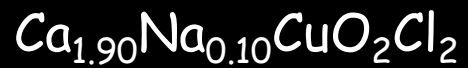
12 nm

Tunneling Asymmetry (TA)-map at $E=150\text{meV}$



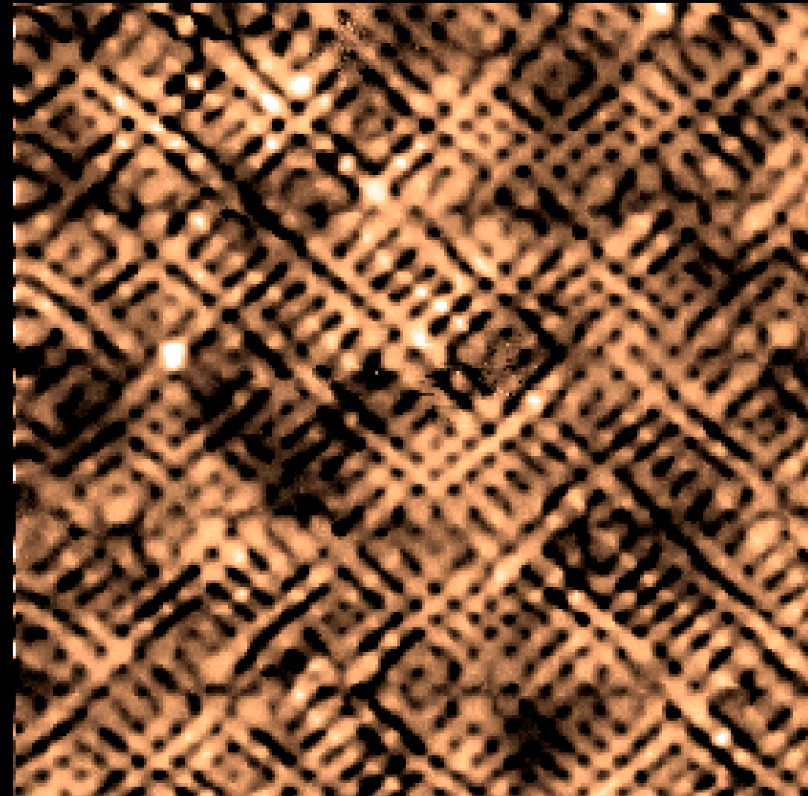
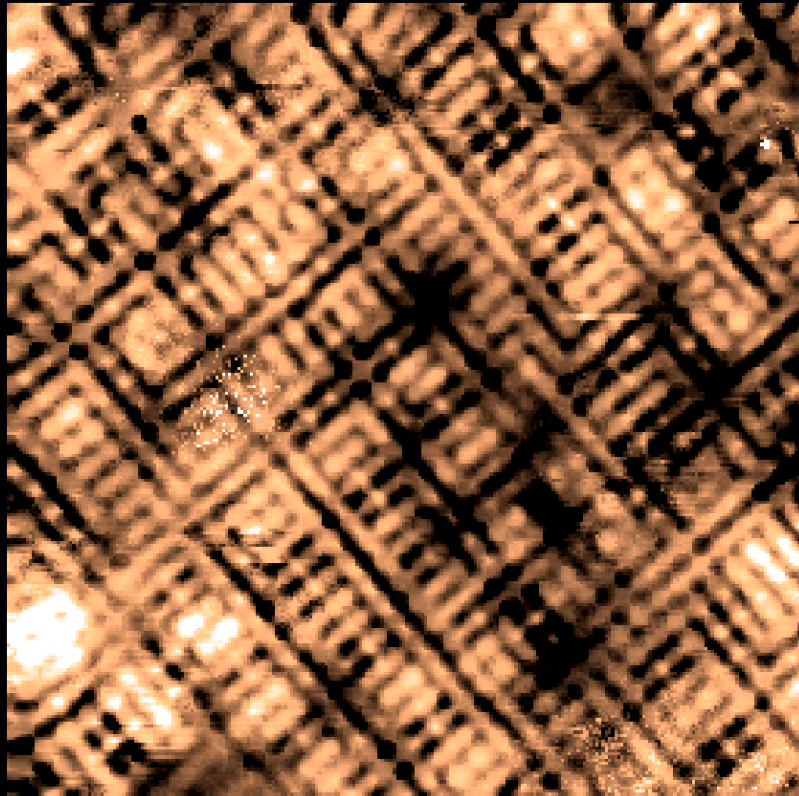
12 nm

Tunneling Asymmetry (TA)-map at $E=150\text{meV}$



12 nm

Tunneling Asymmetry (TA)-map at $E=150\text{meV}$



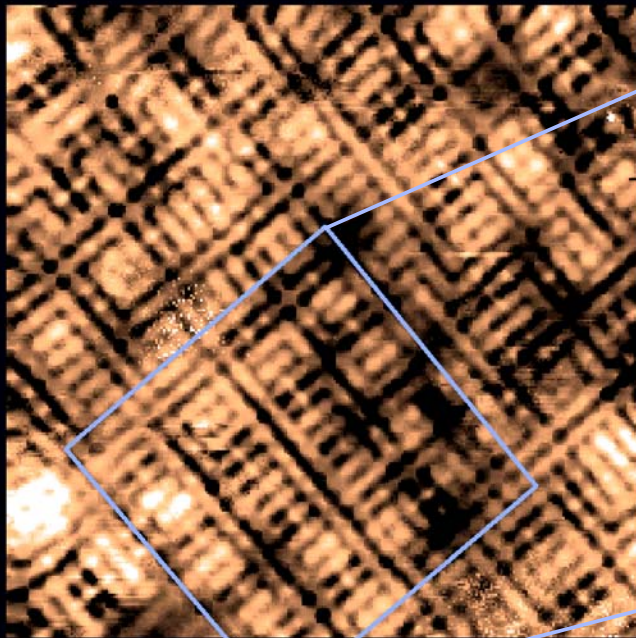
12 nm

Indistinguishable bond-centered TA contrast
with disperse $4a_0$ -wide nanodomains

Y. Kohsaka et al. Science 315, 1380 (2007)

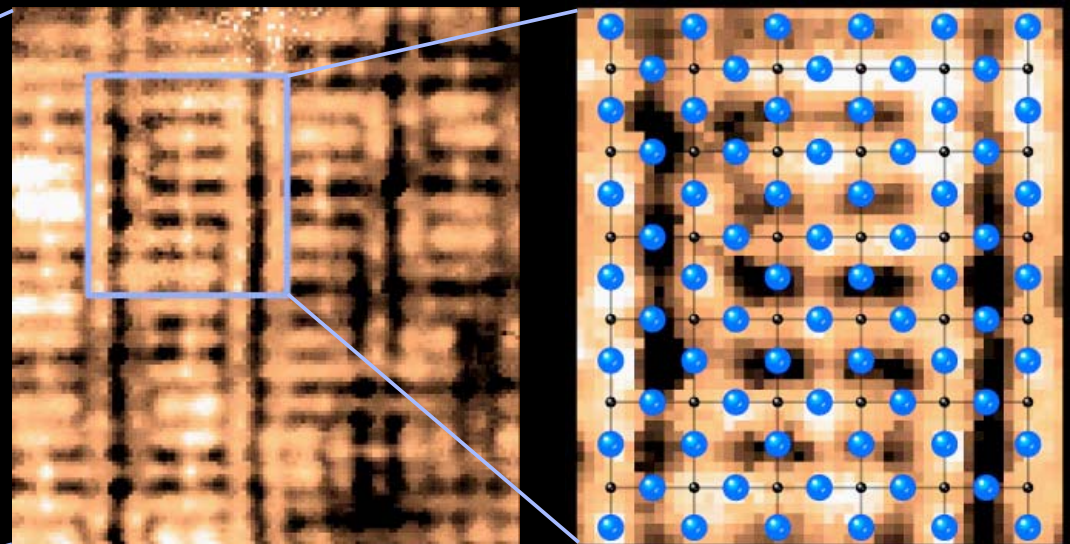
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)



12 nm

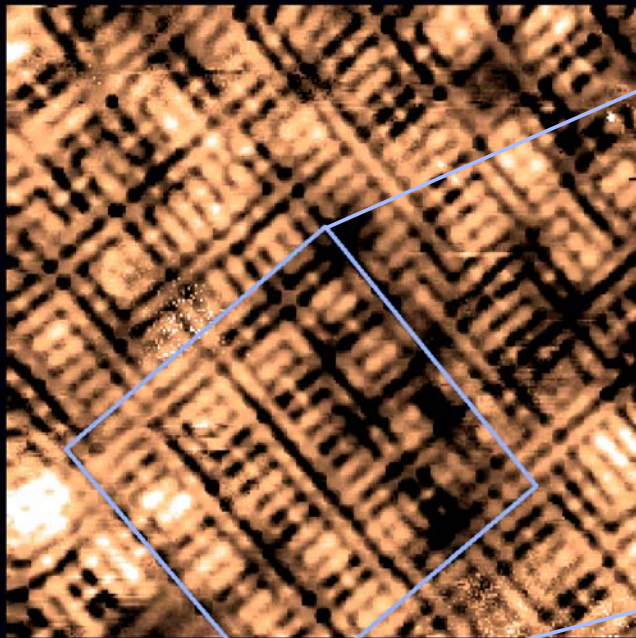
$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$, 4 K



$4a_0$

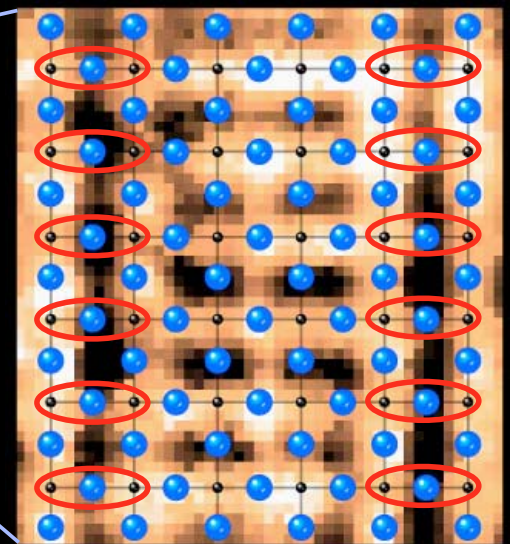
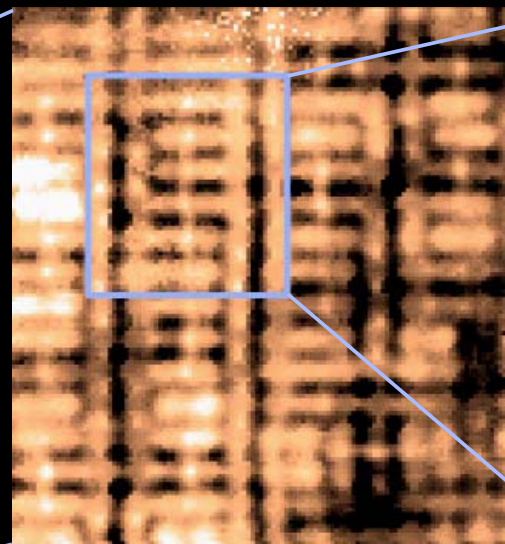
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)



12 nm

$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$, 4 K



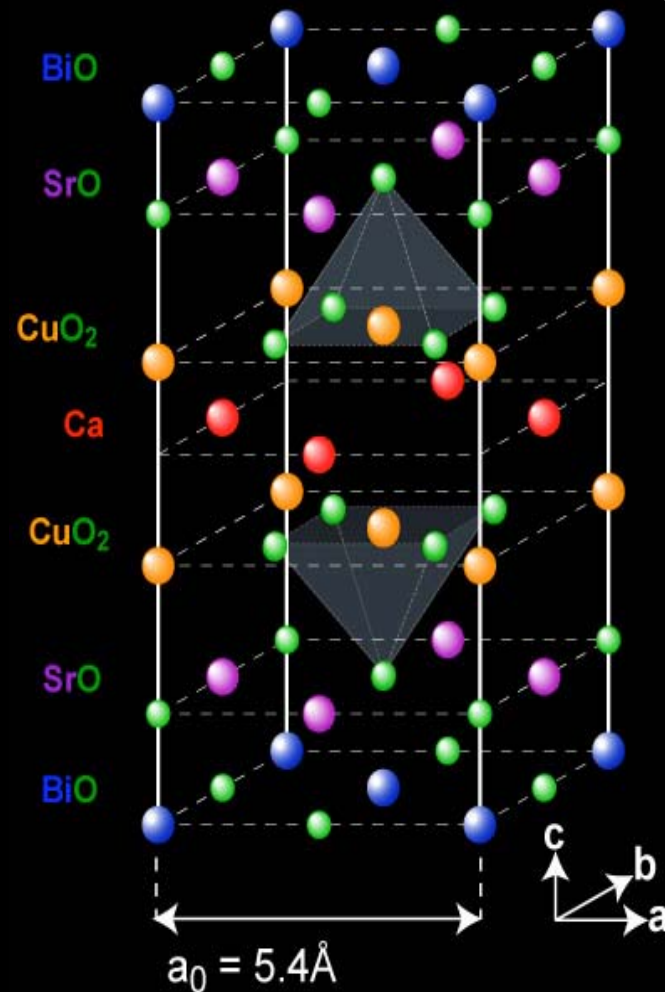
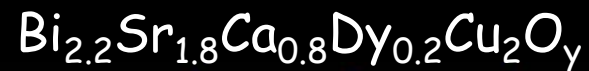
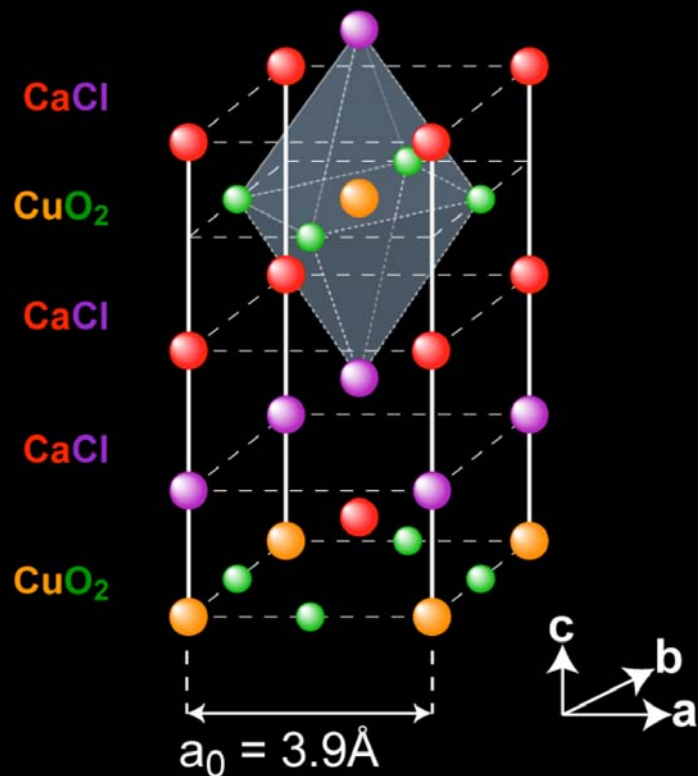
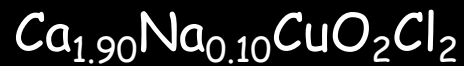
$4a_0$

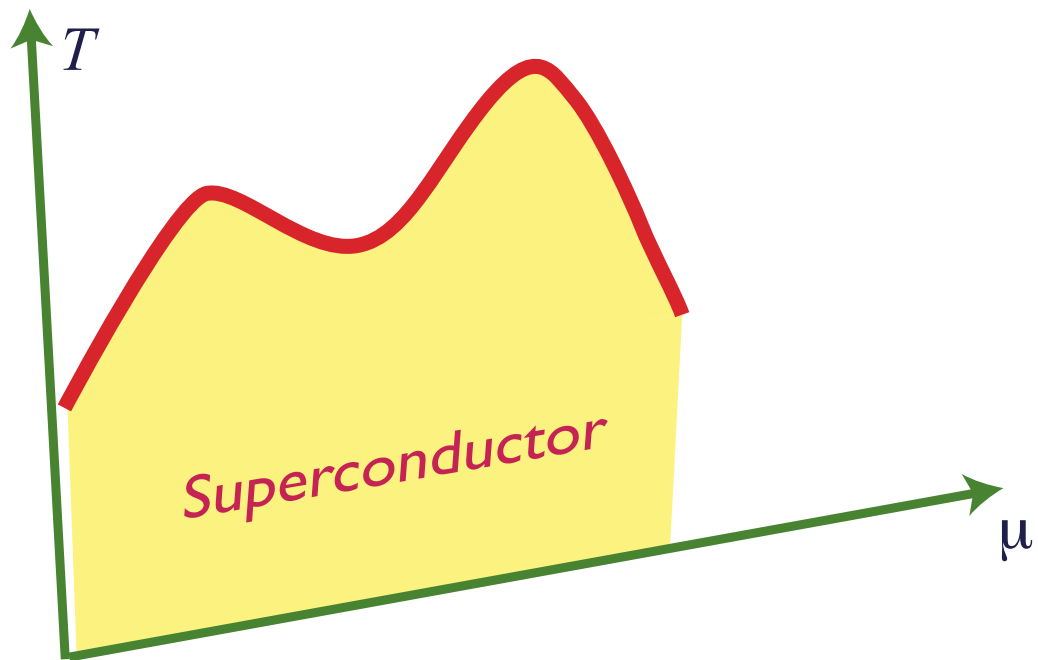
Evidence for VBS order - a valence bond supersolid

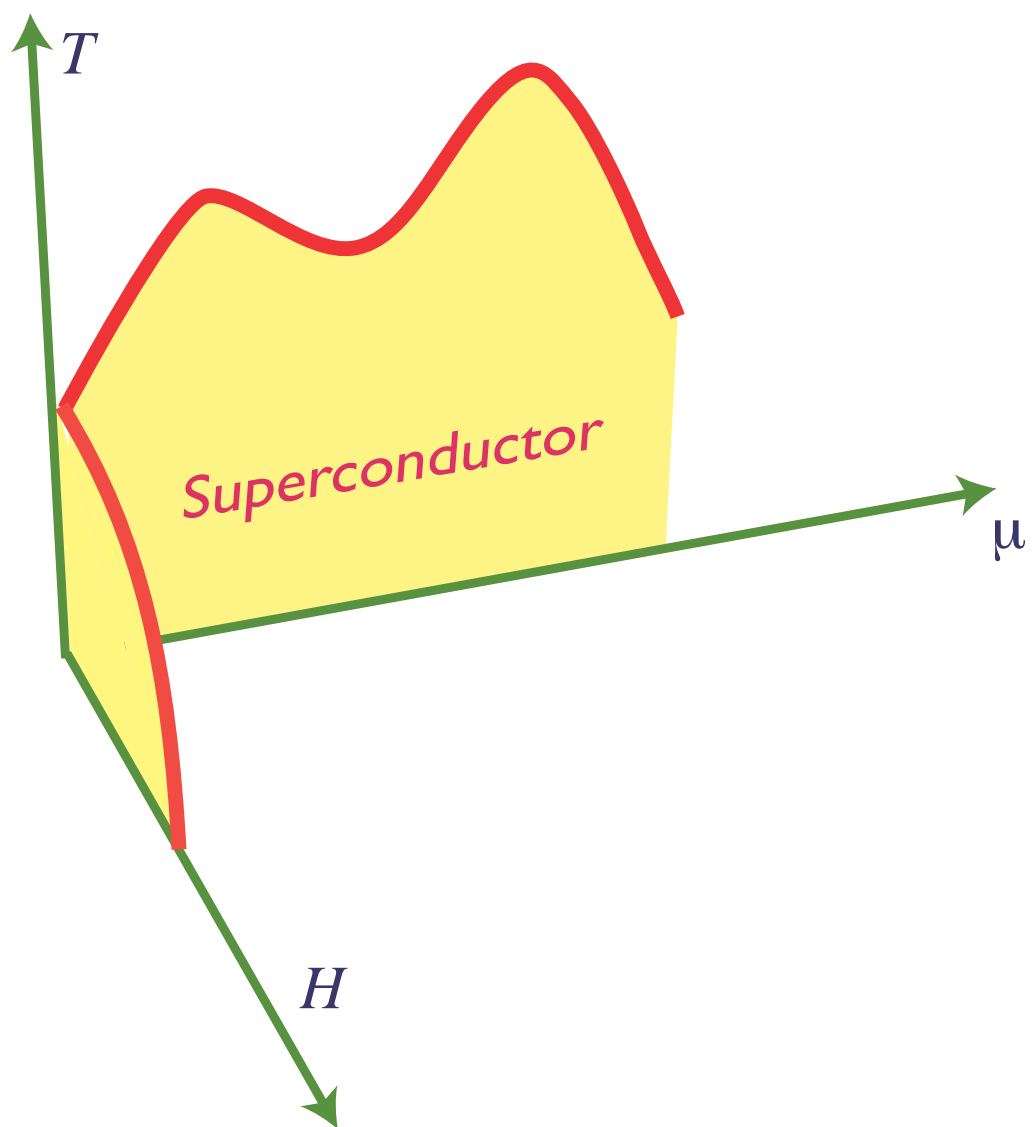
S. Sachdev and N. Read, *Int. J. Mod. Phys. B* 5, 219 (1991).

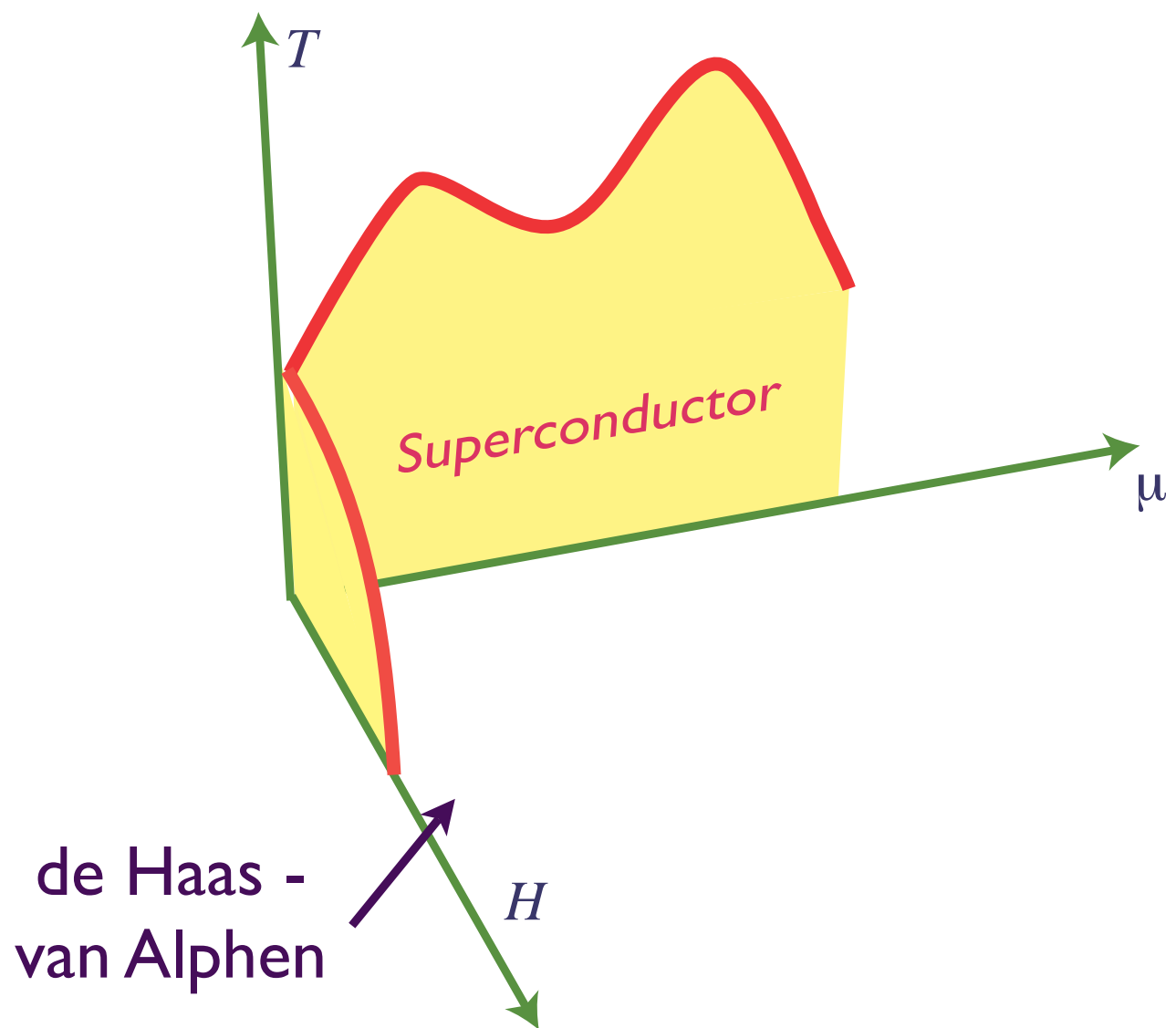
■ ■ ■ ■

High field “normal state” of the cuprate superconductors

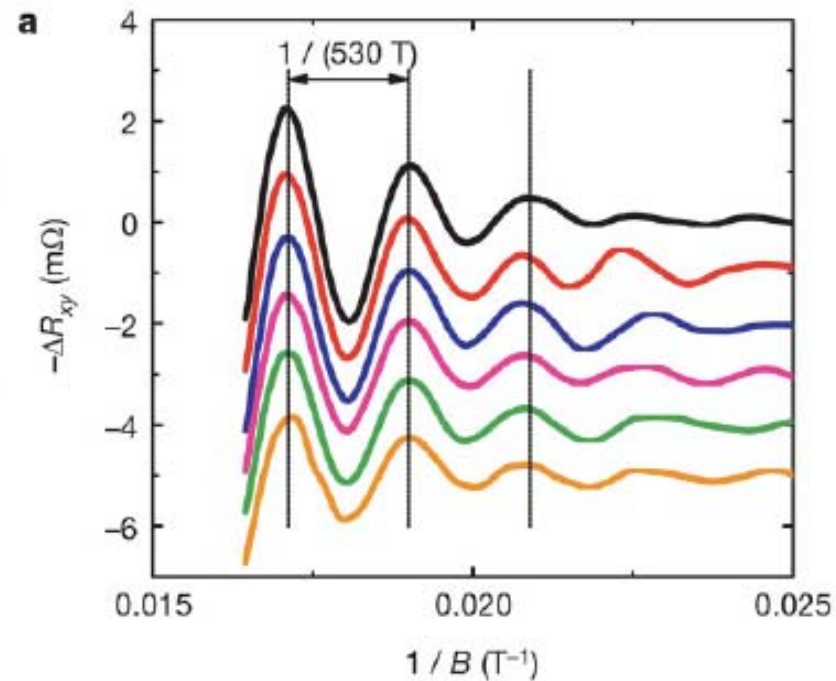




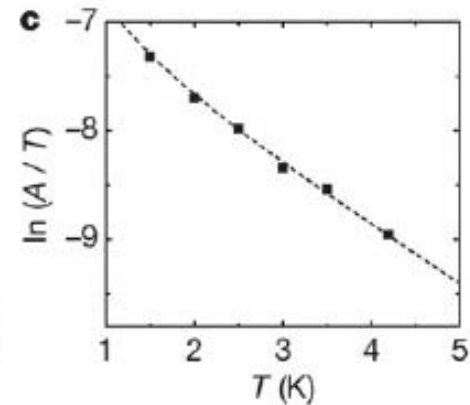
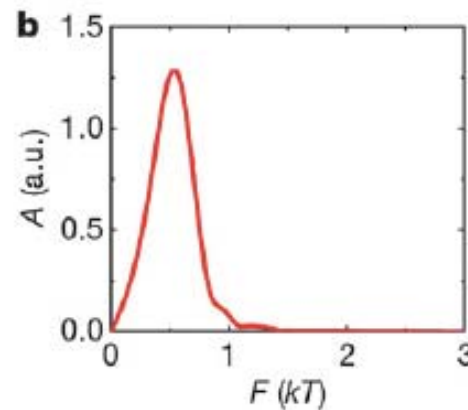




Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $\text{YBa}_2\text{Cu}_3\text{O}_{6.5}$).

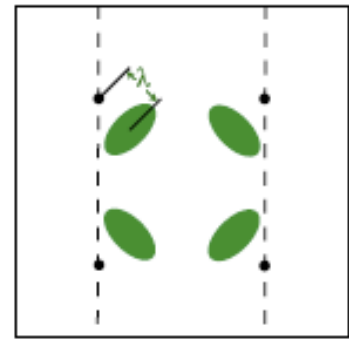
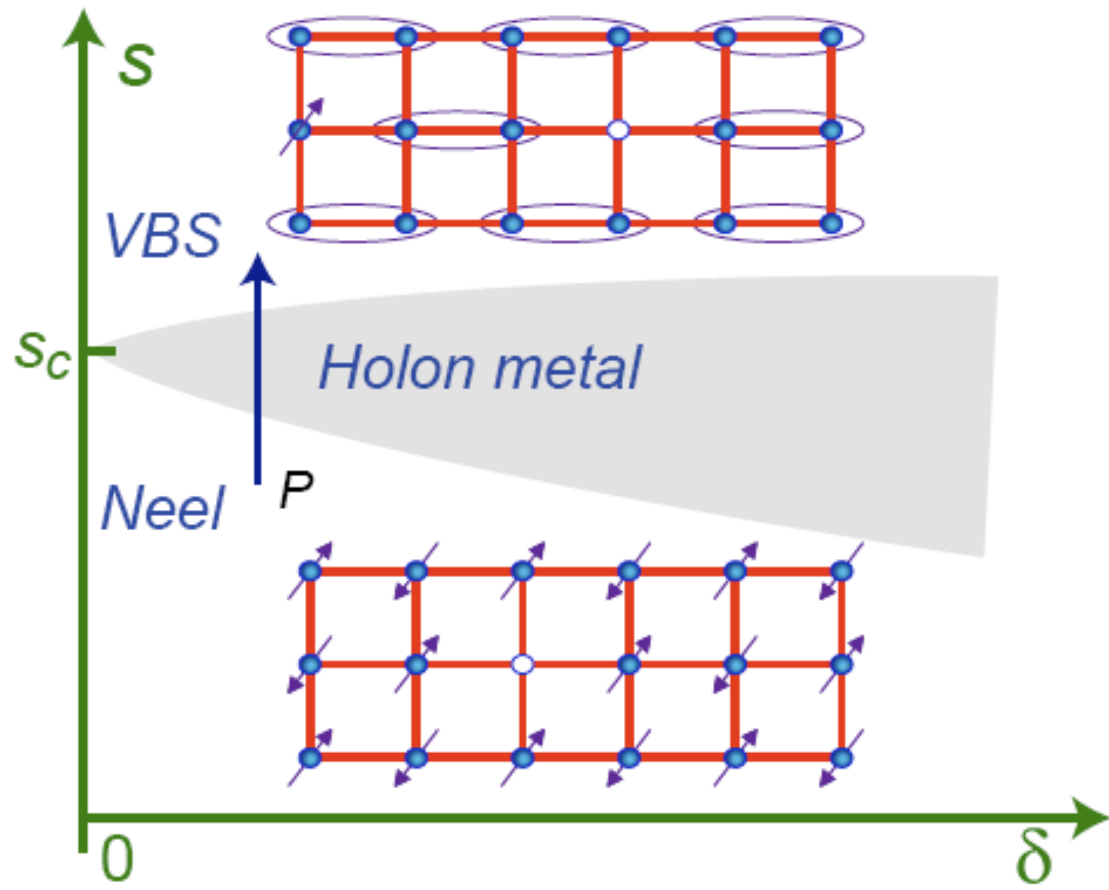


N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaïson, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)



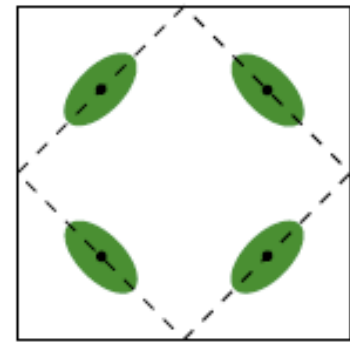
Period of oscillations indicates a carrier density which is the density of holes, and violates the Luttinger count

Phase diagram of lightly doped antiferromagnet



VBS
 $\mathcal{A} = (2\pi)^2 x/8$

$\mathcal{A} = (2\pi)^2 x/4$



Neel