Entanglement, holography, and the quantum phases of matter

Utrecht University, June 13, 2012

Subir Sachdev

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World
arXiv:1203.4565

sachdev.physics.harvard.edu

PHYSICS
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Liza Huijse

Max Metlitski

Brian Swingle

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle

quantum entanglement

"Complex entangled" states of quantum matter in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals in higher temperature superconductors, spin liquids

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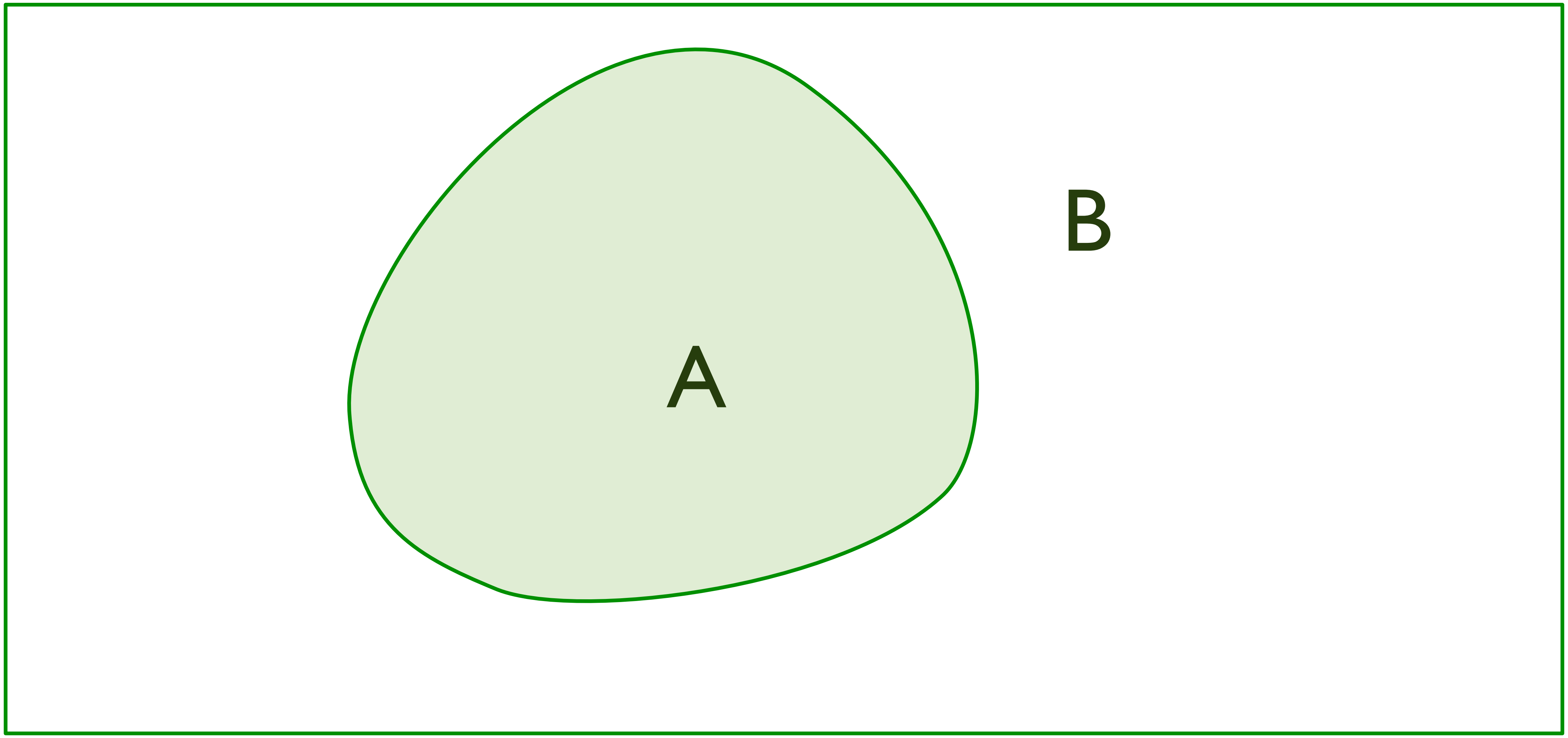
Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

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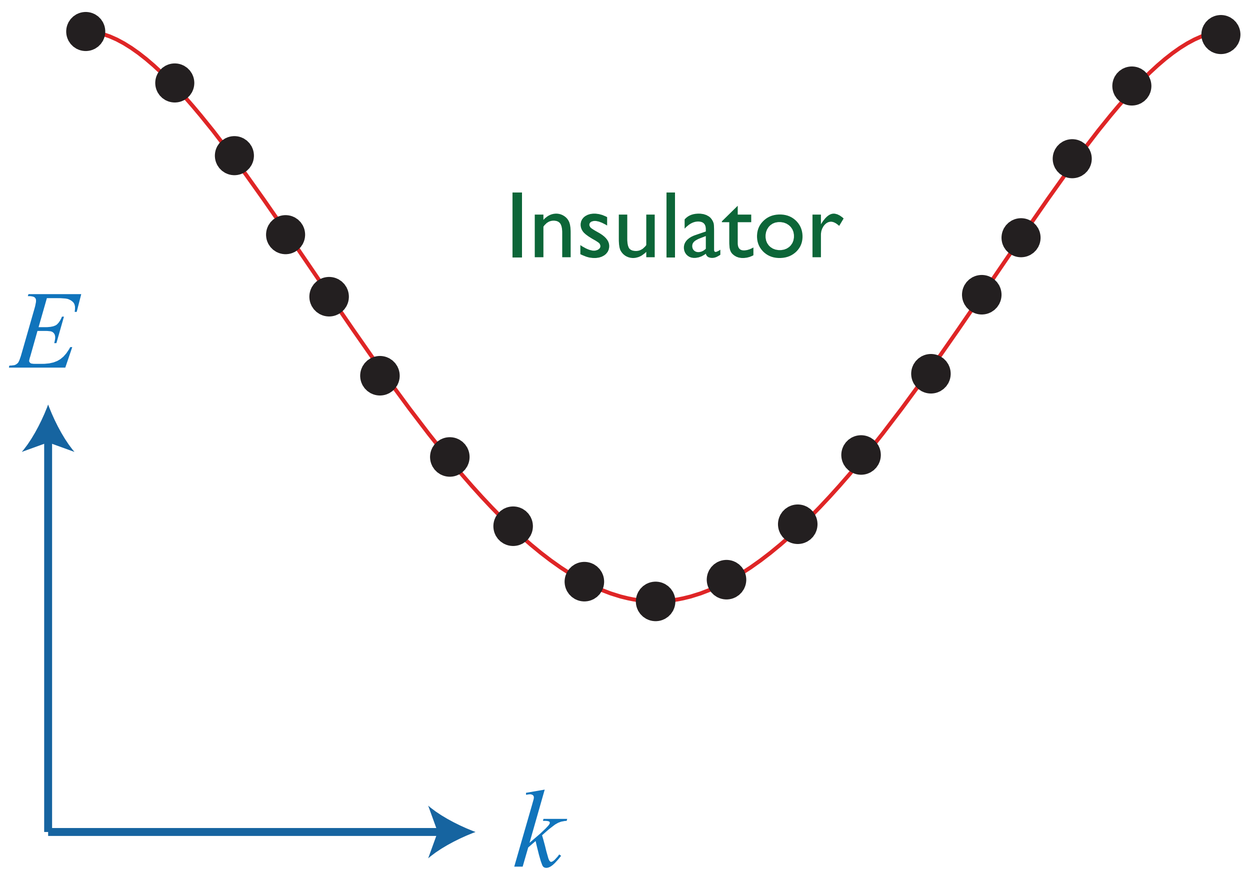
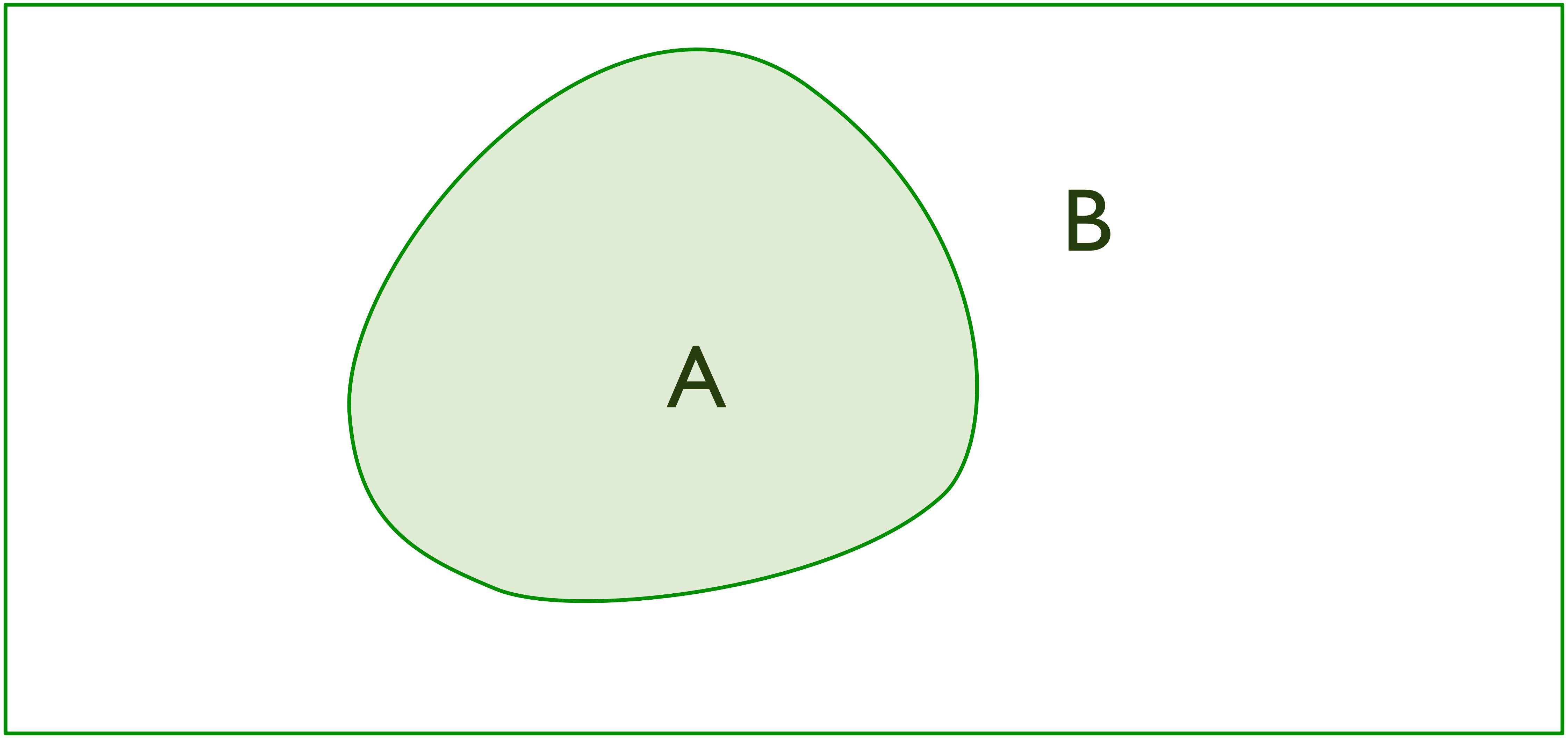
Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator



$$S_E = aP - b\exp(-cP)$$

where P is the surface area (perimeter) of the boundary between A and B.

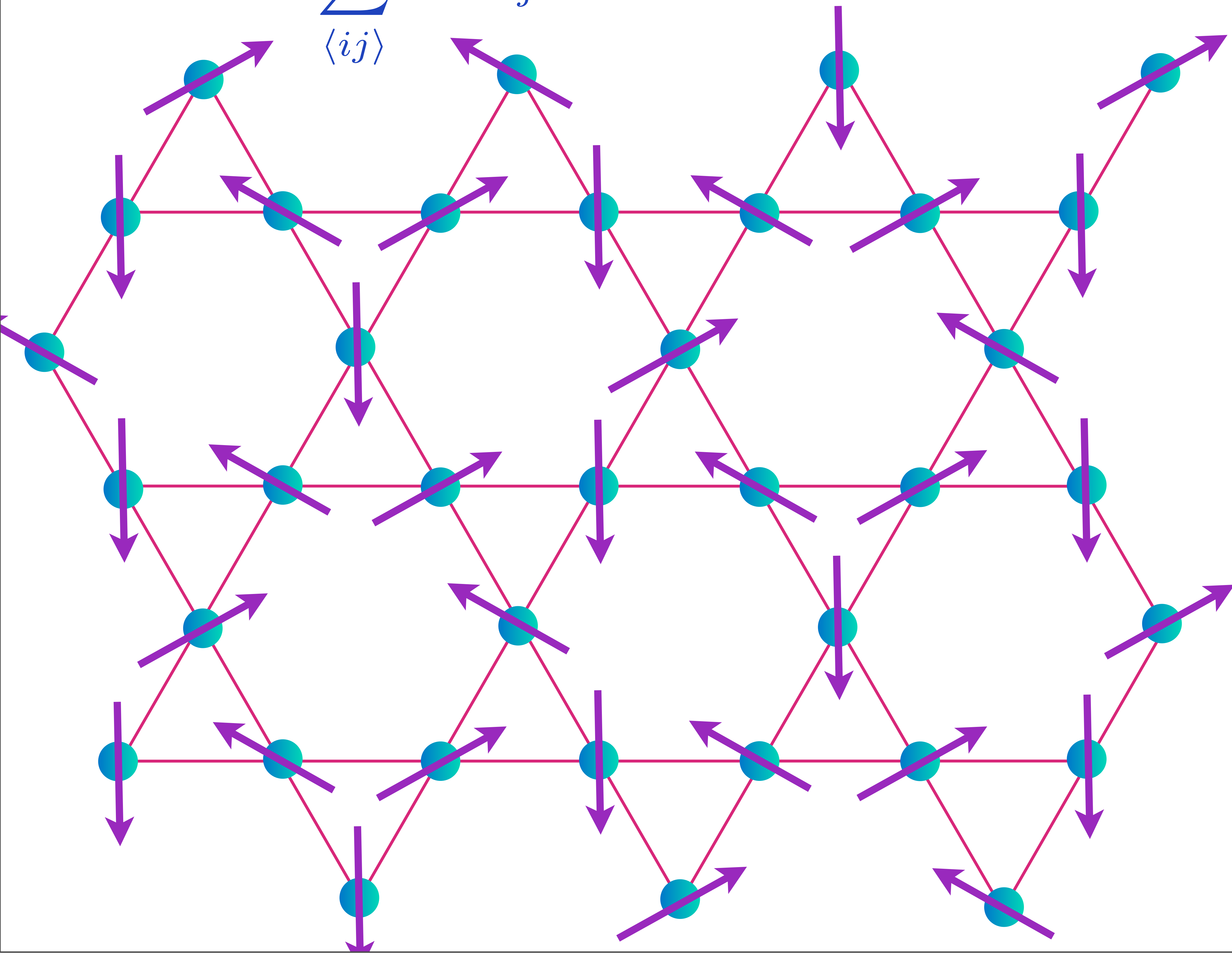
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



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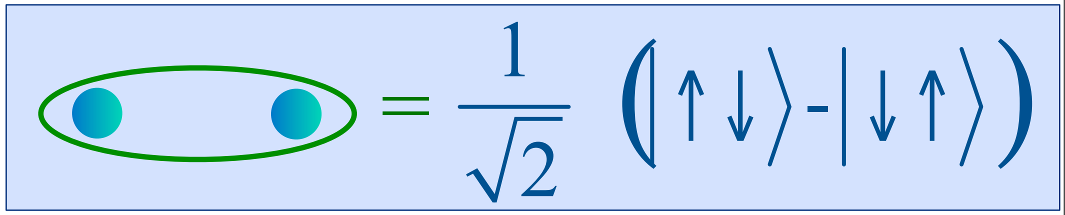

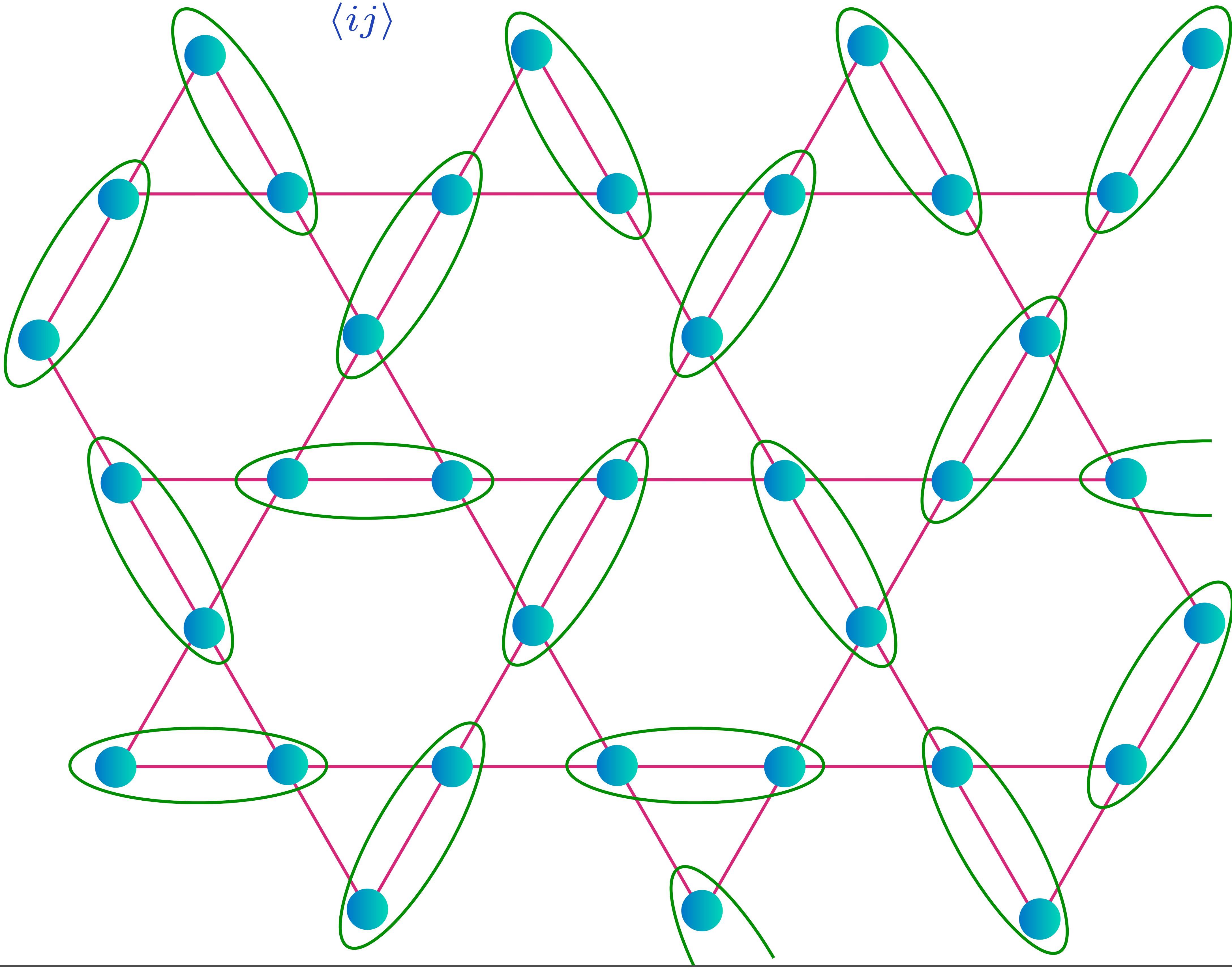

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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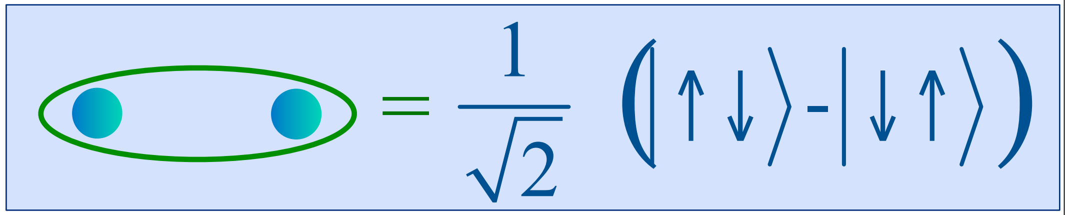

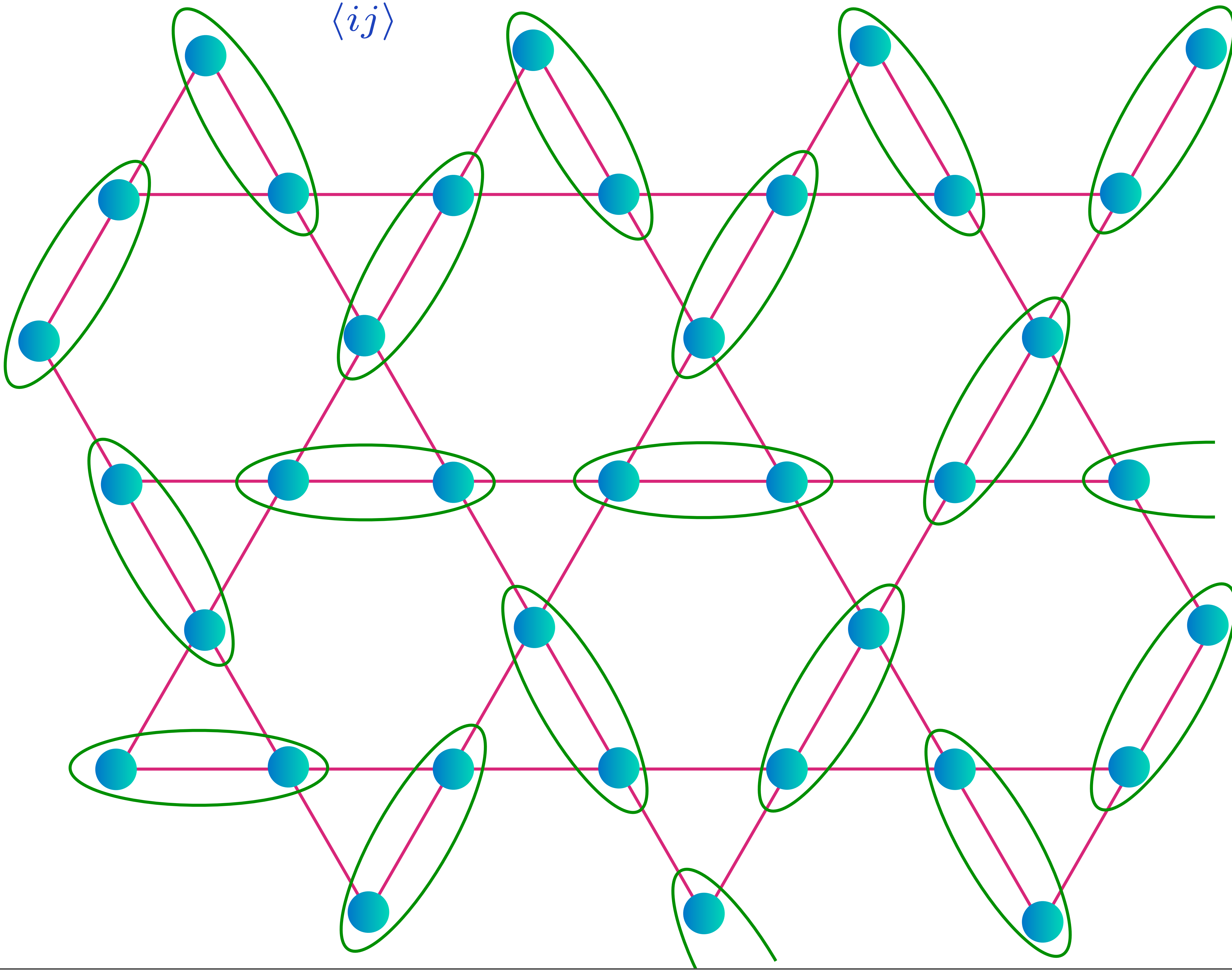

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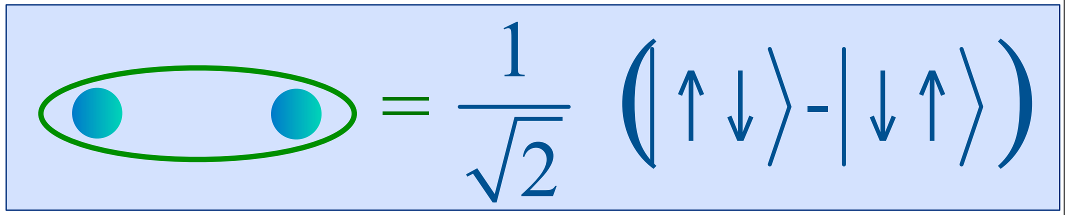

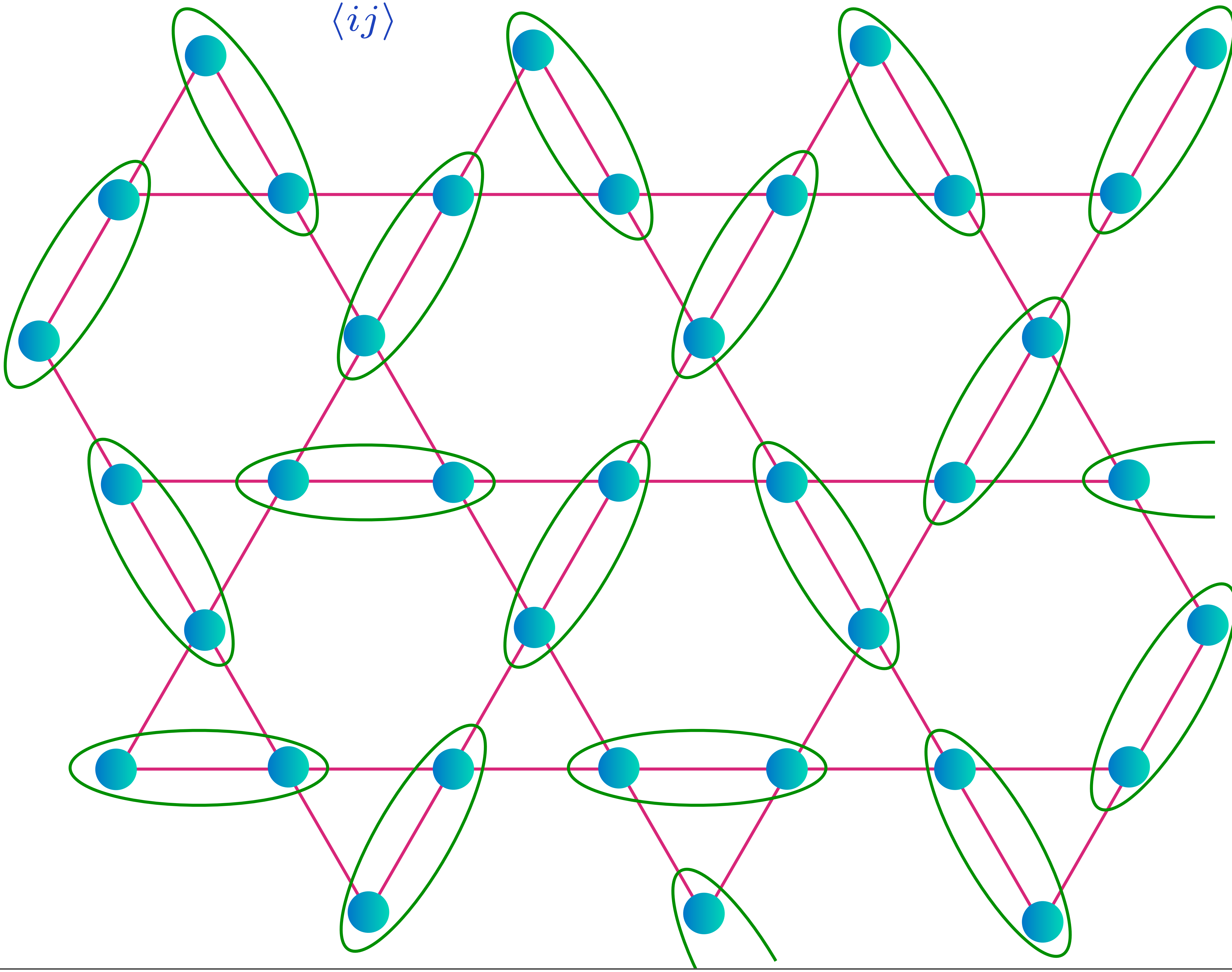

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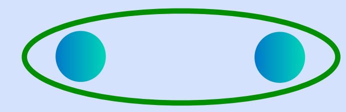
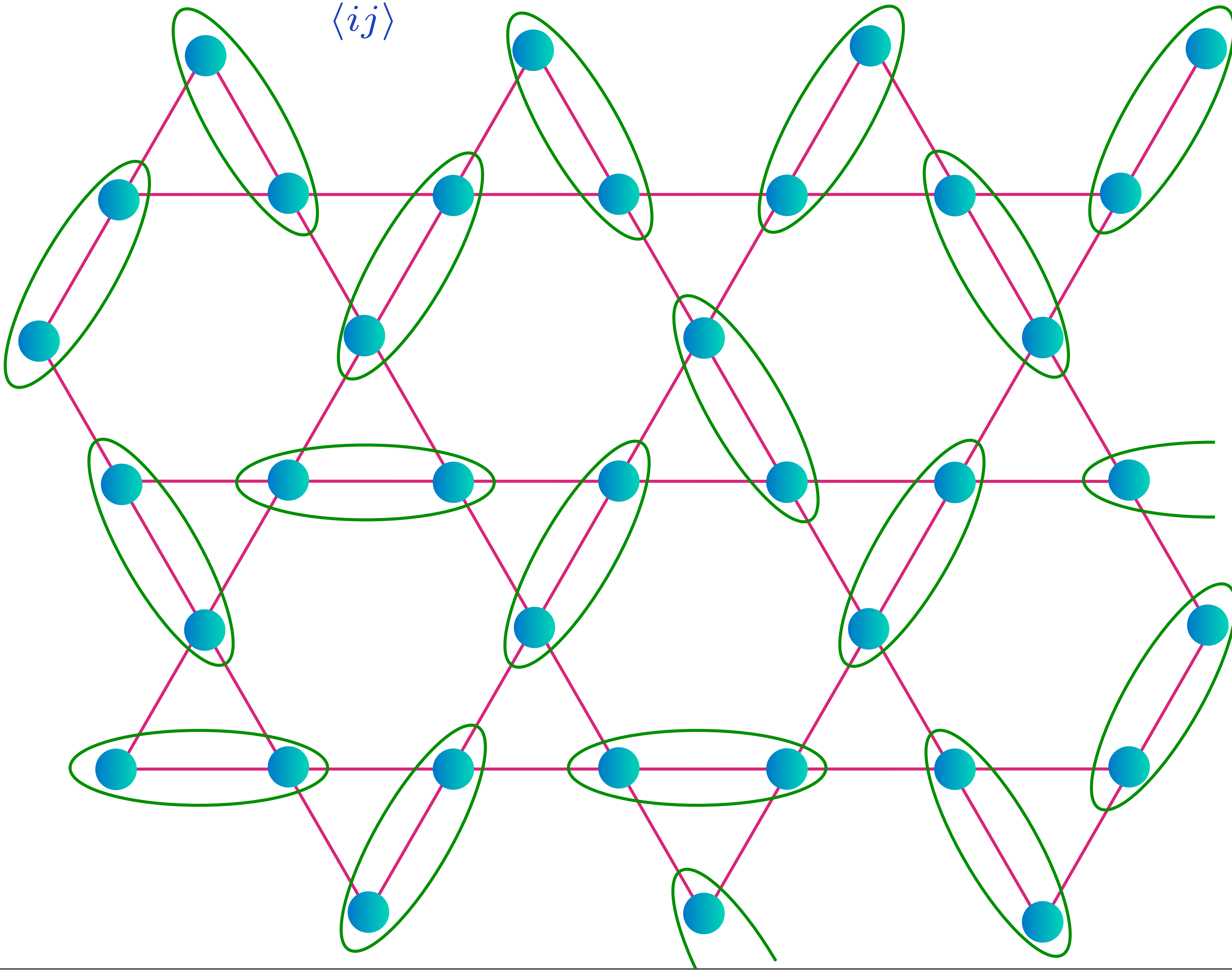

$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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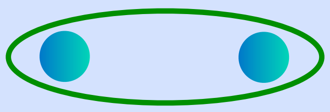

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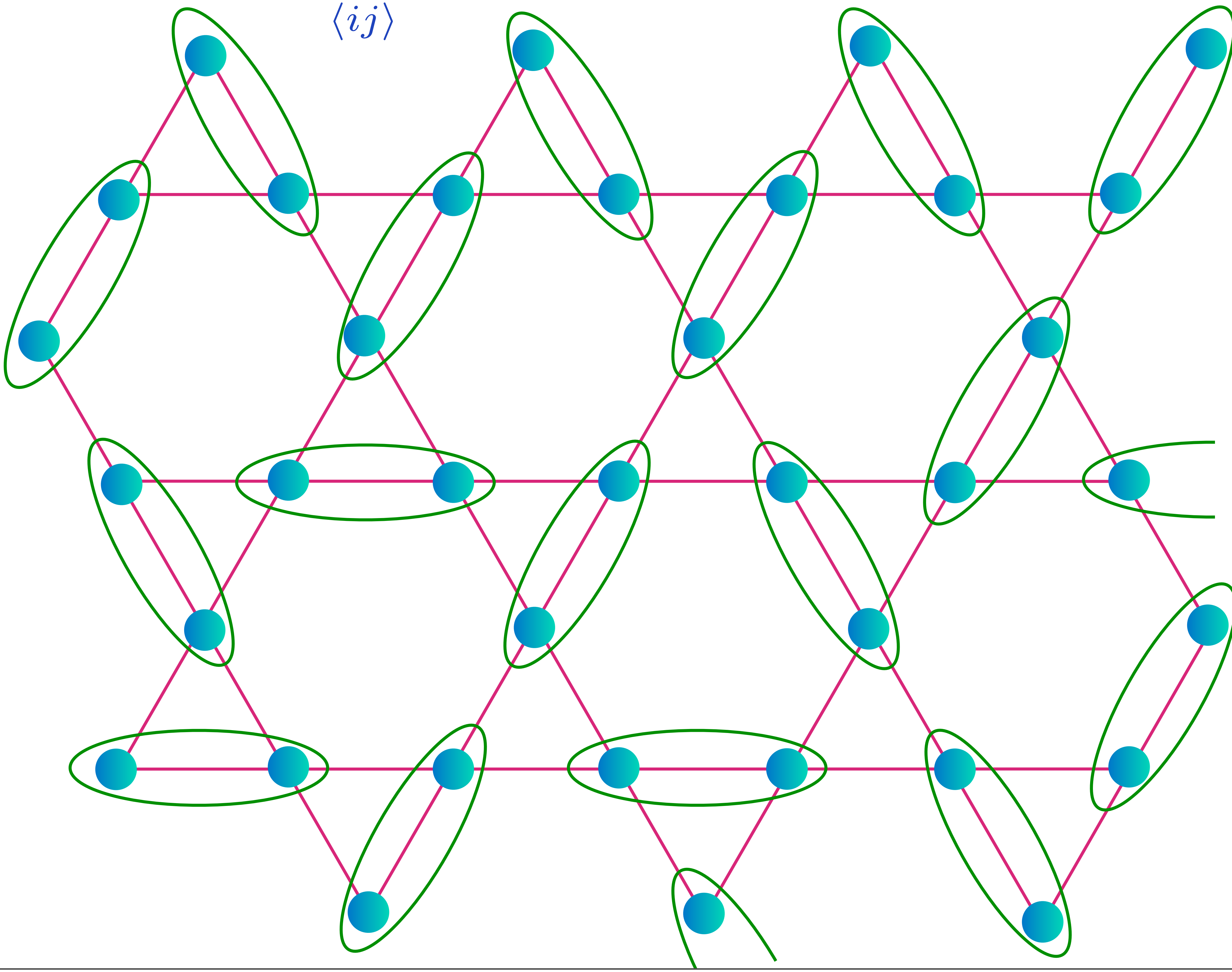

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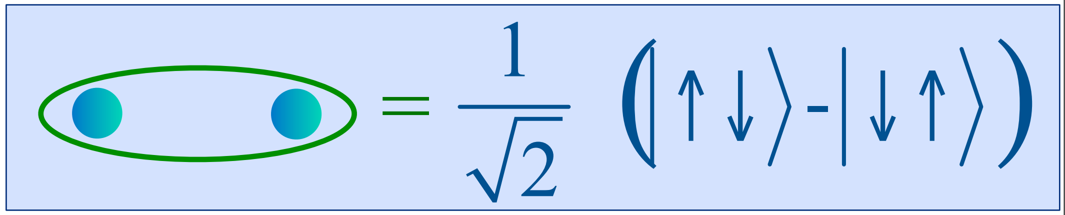

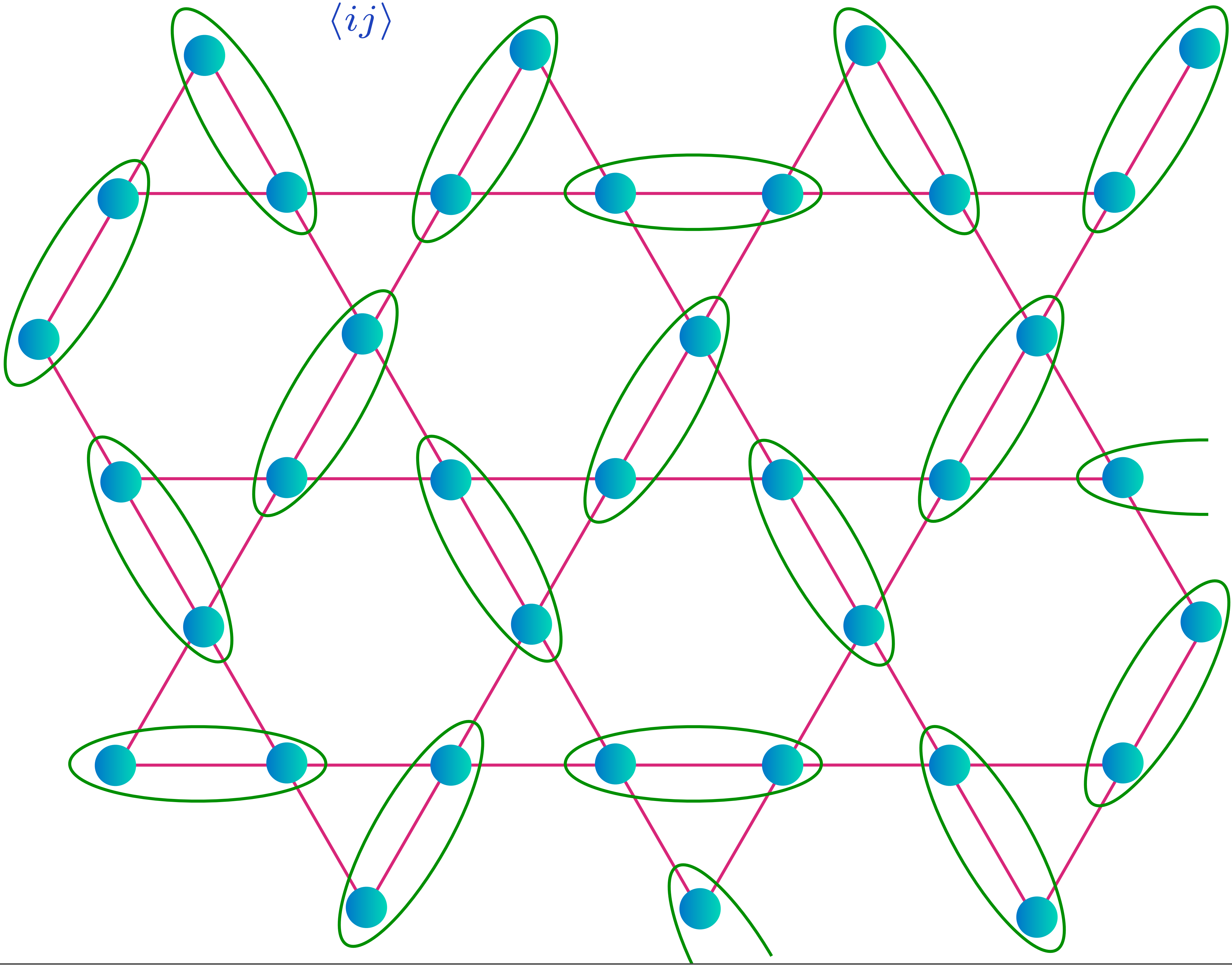

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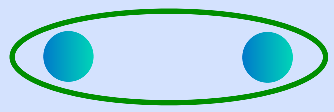
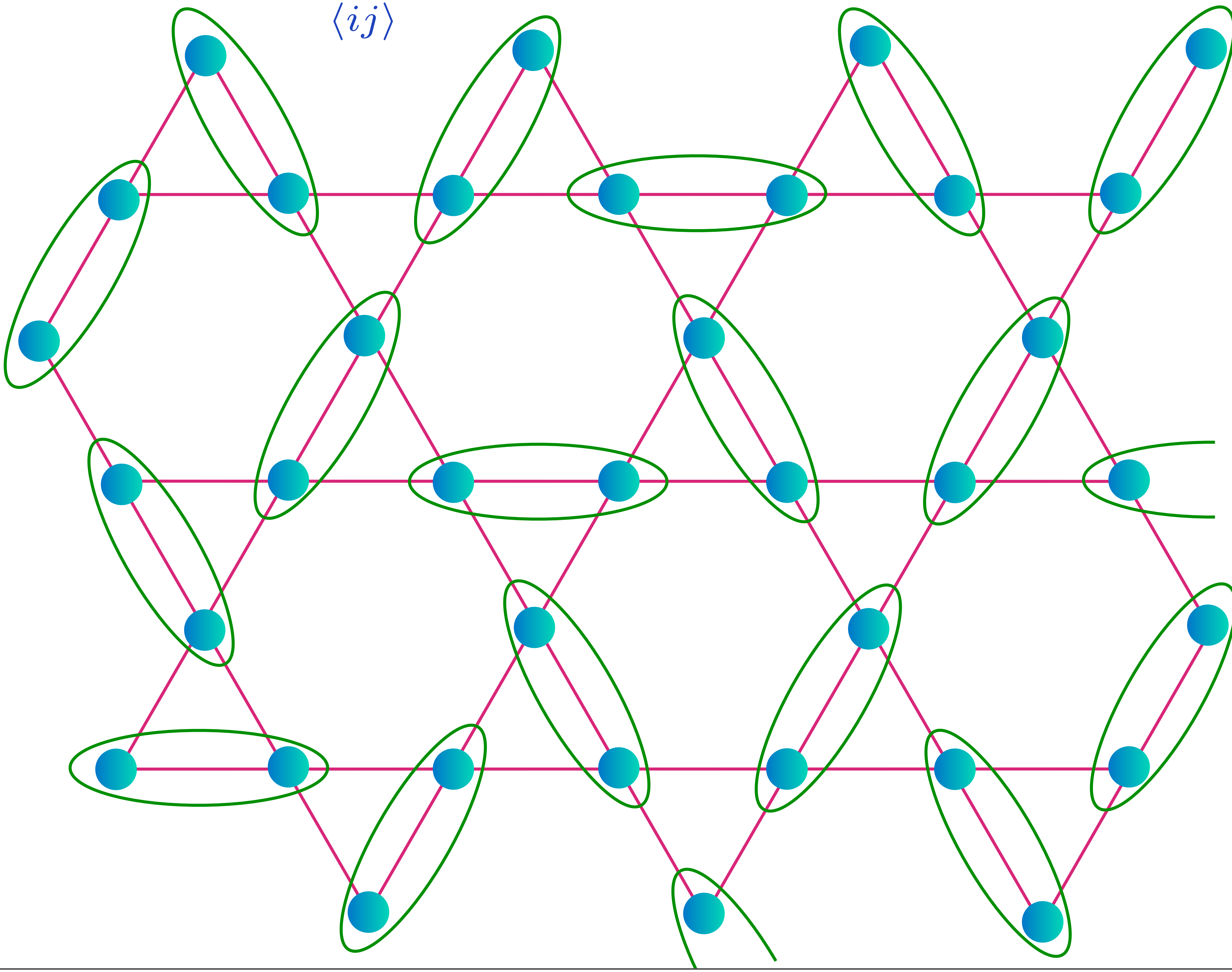
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

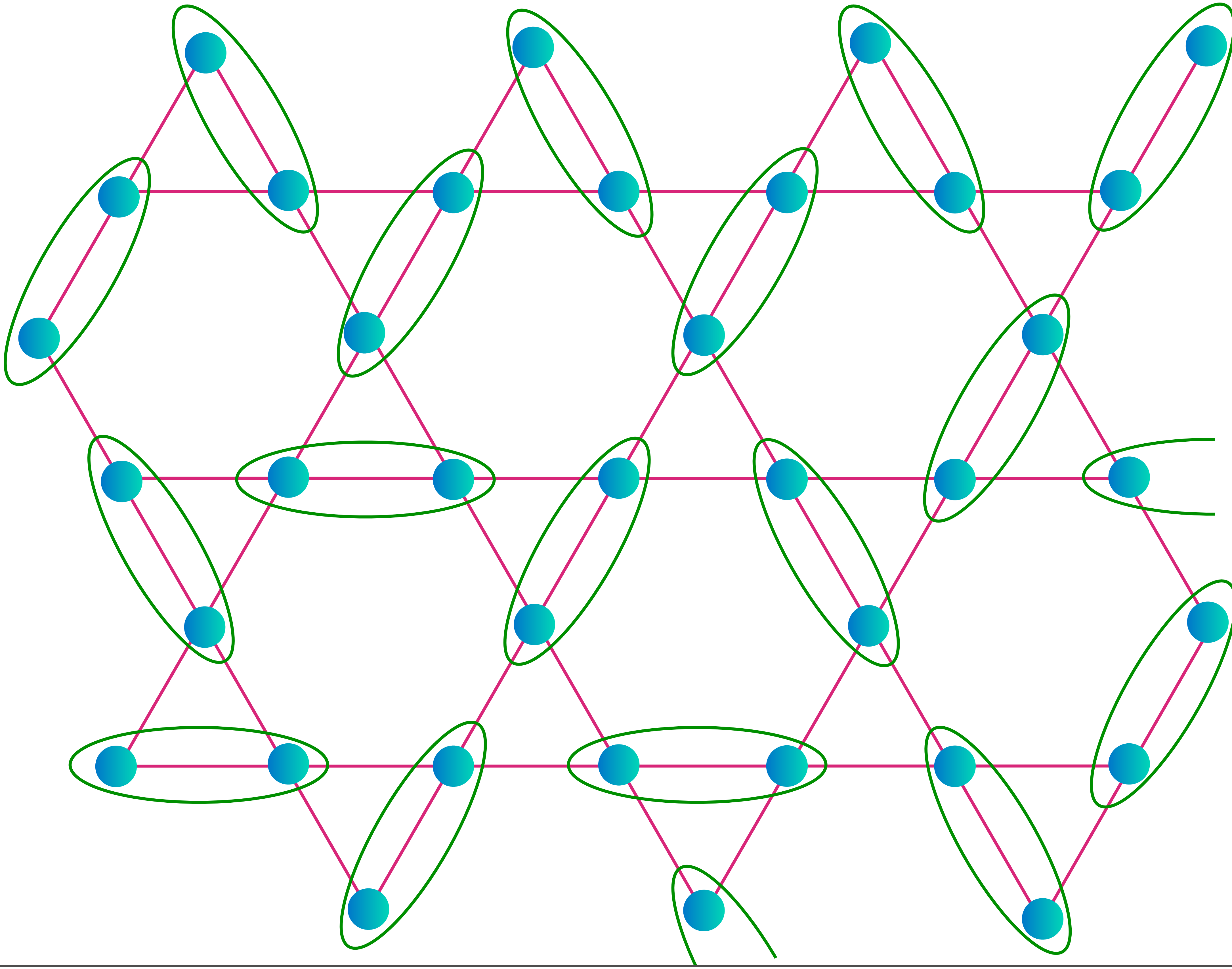


P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration

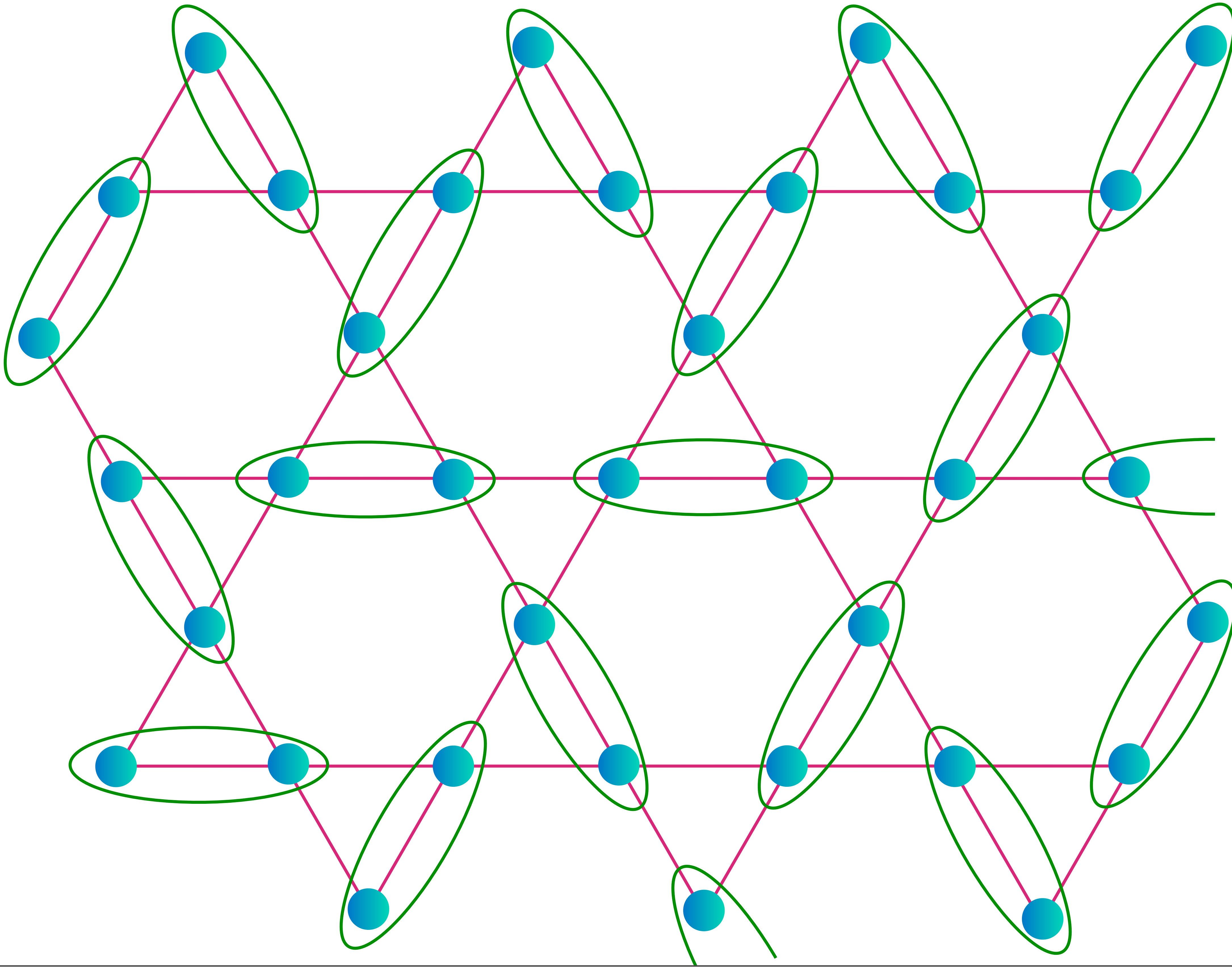


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

A nearby configuration

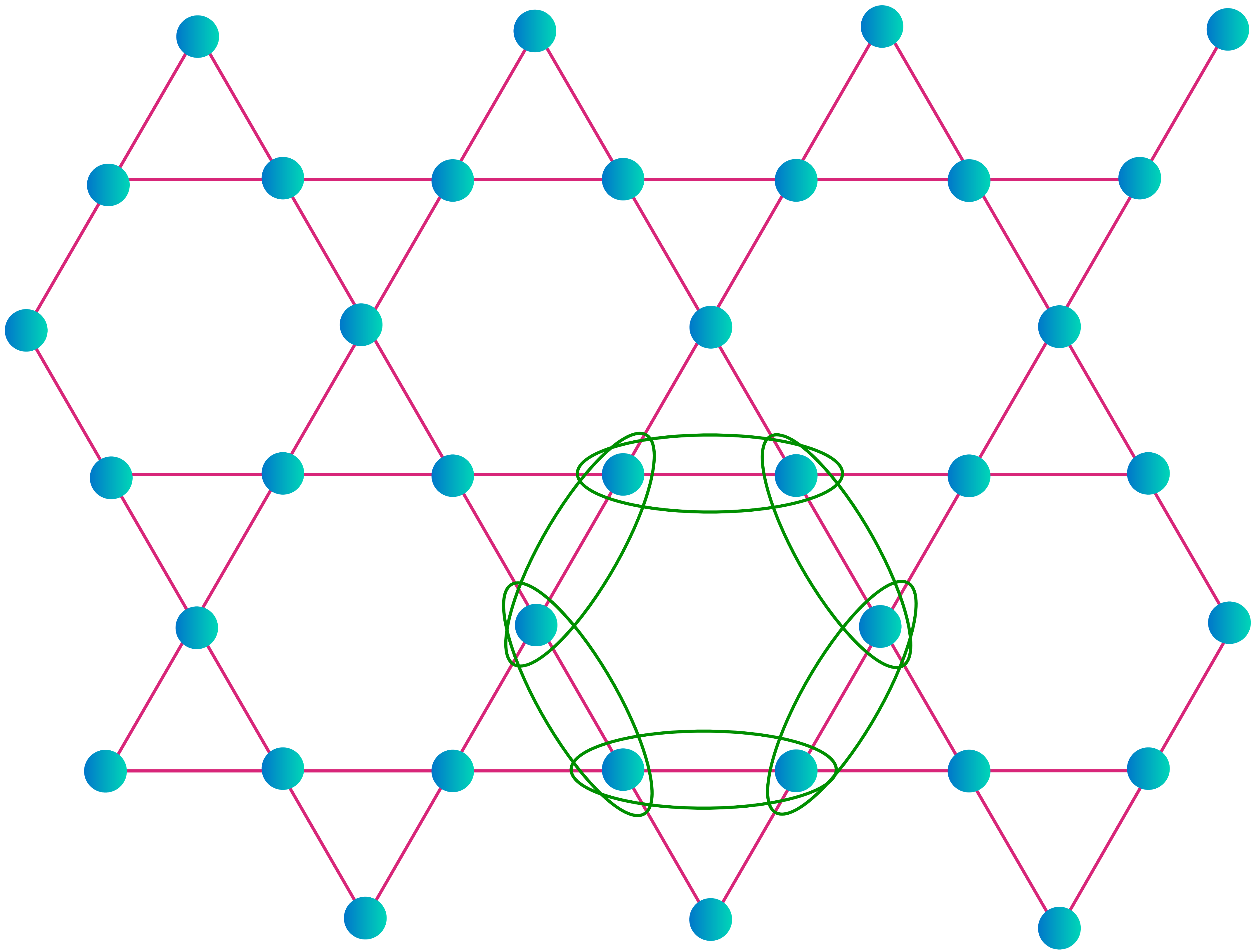


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Difference: a closed loop



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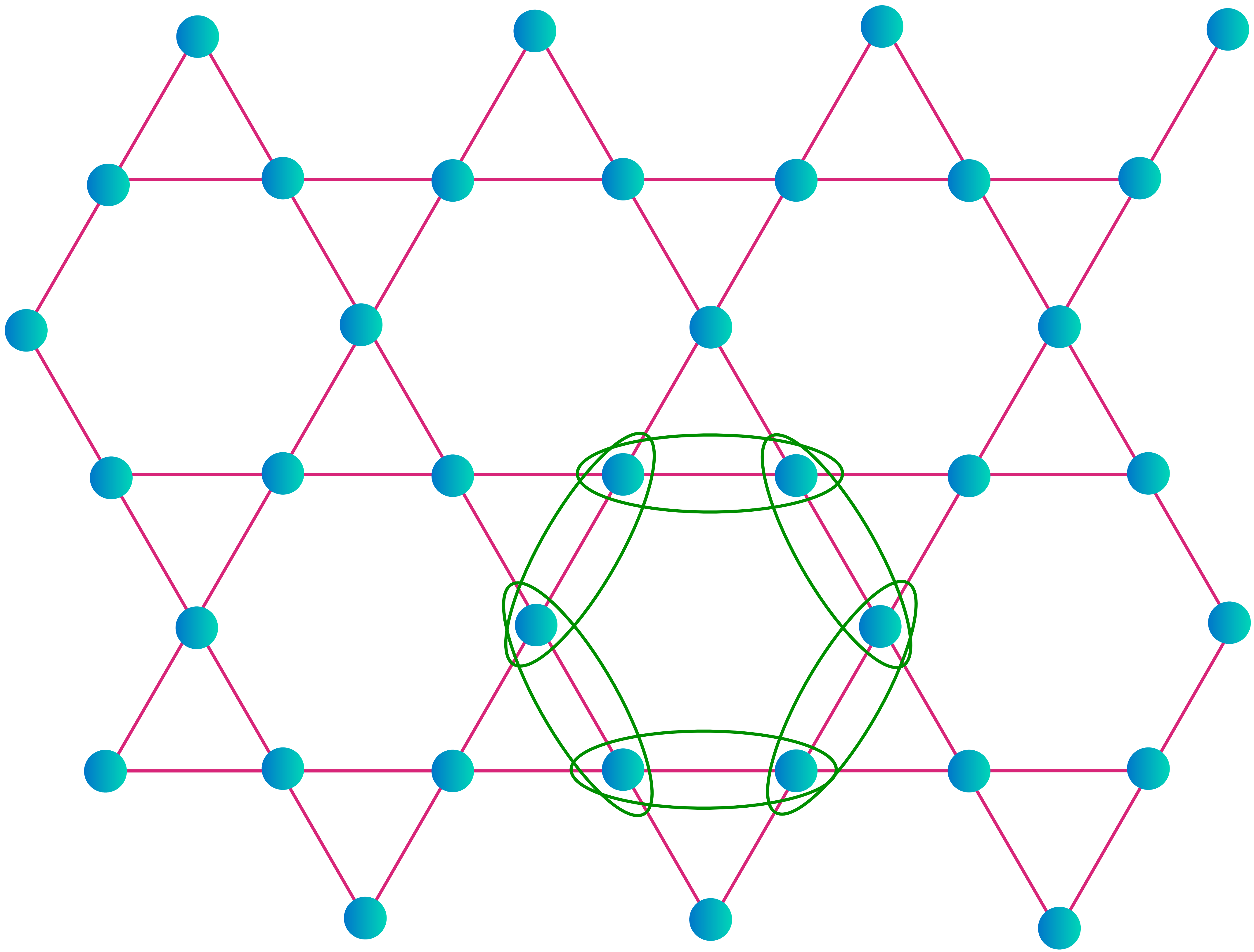
Ground state: sum over closed loops



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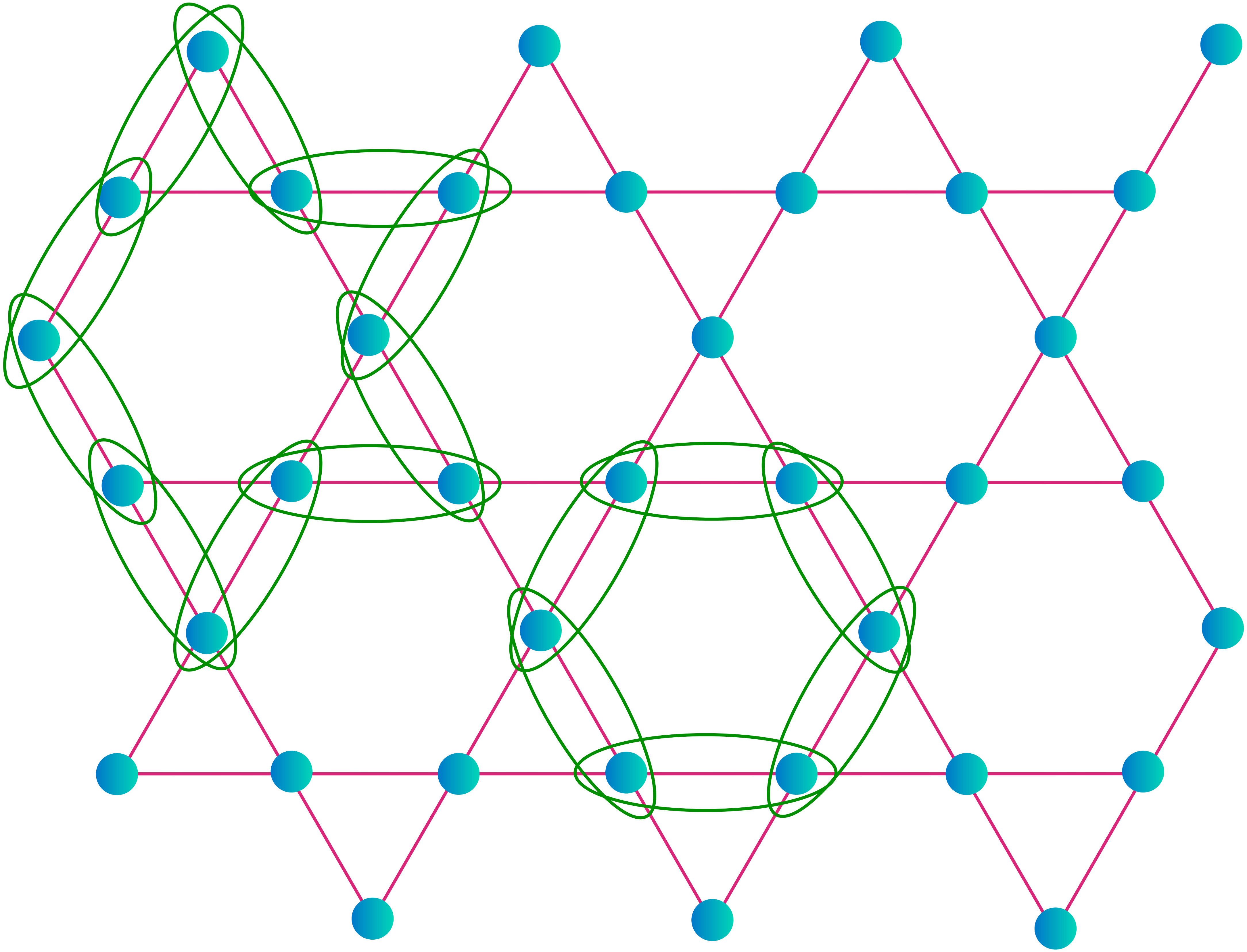

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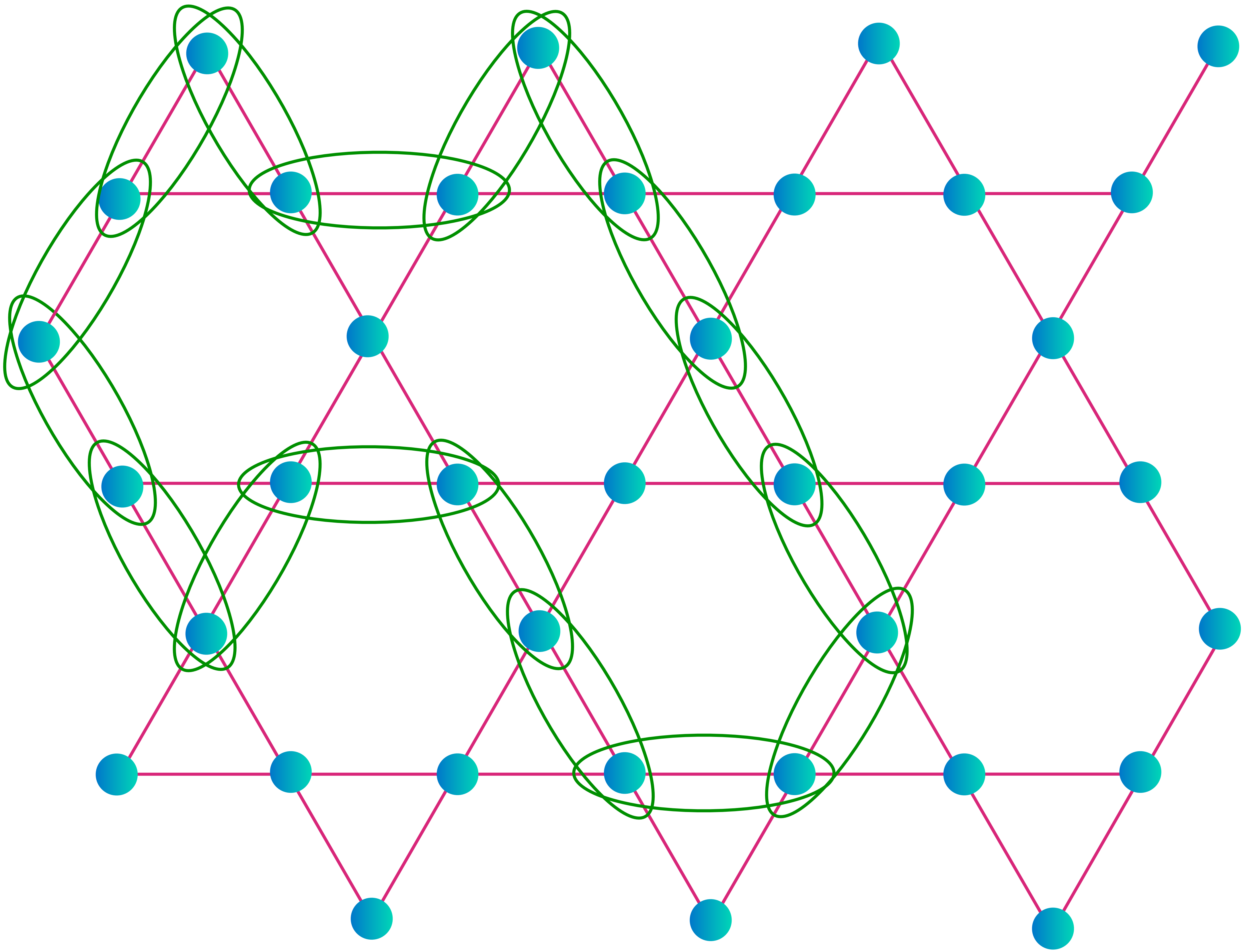

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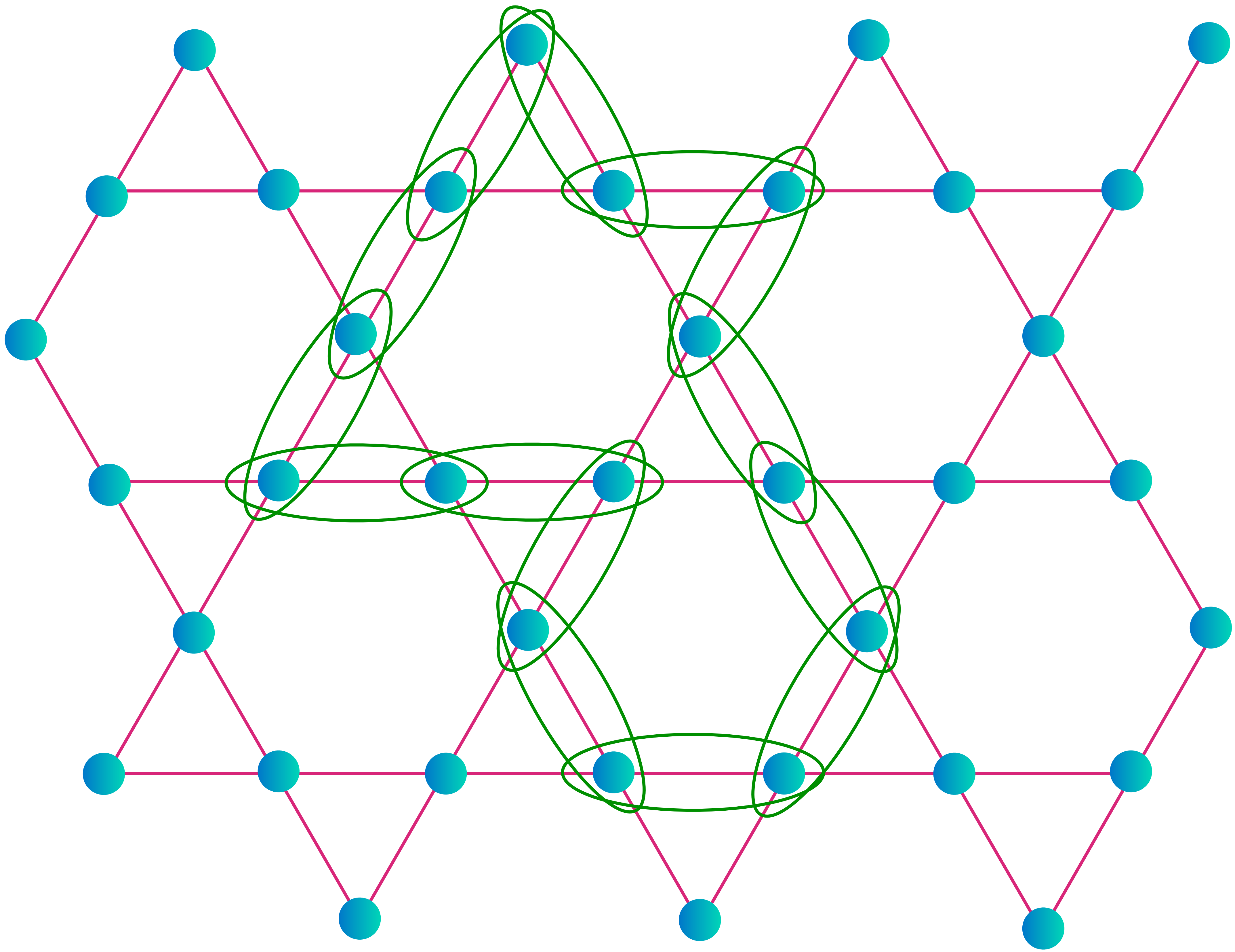

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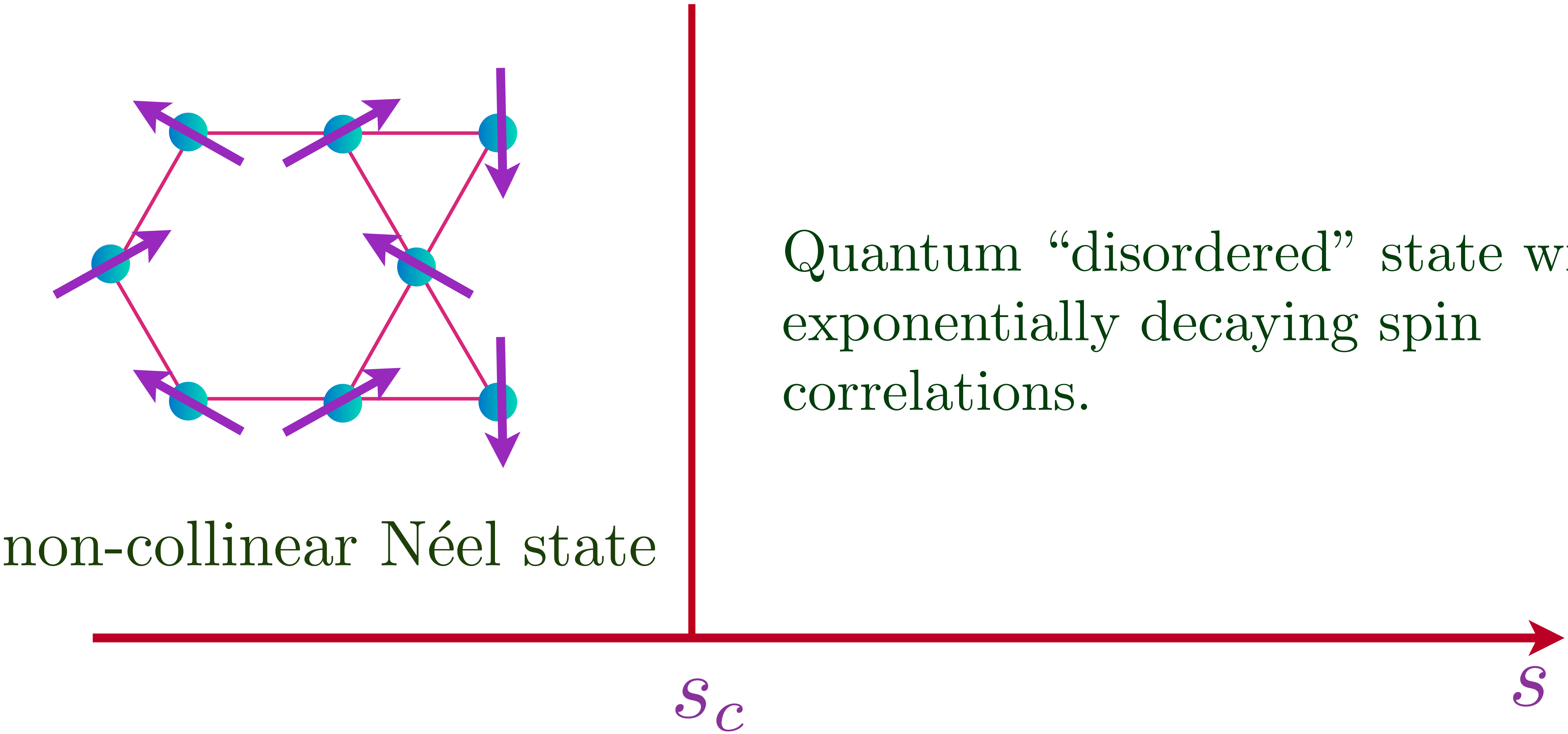

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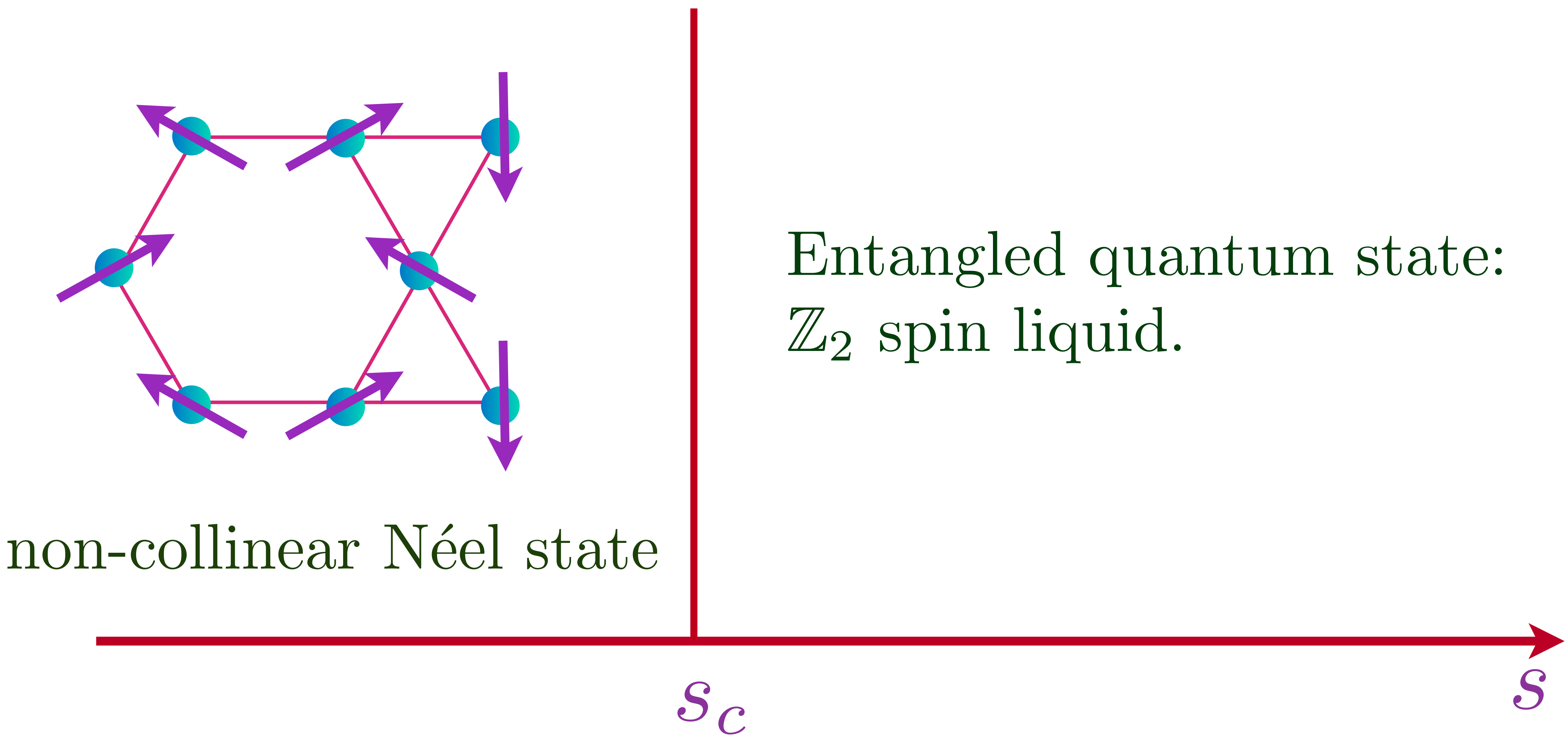


N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)
S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)

Mott insulator: Kagome antiferromagnet

$\mathbb{Z}_2$ spin liquid: parton construction

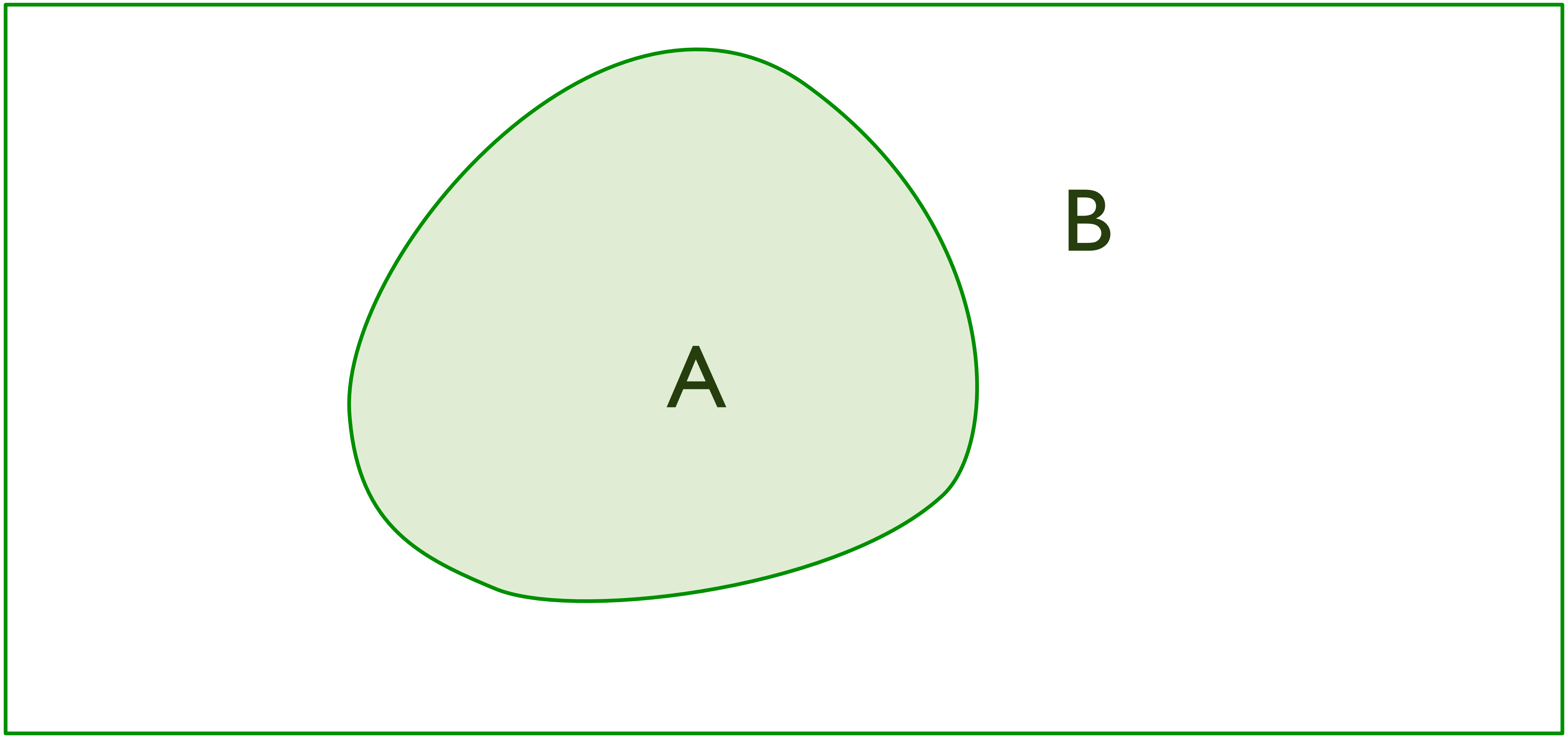
Write spin operators in terms of $S = 1/2$ 'partons' $\vec{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} b_{i\beta}$. The ground state is

$$|\Psi\rangle = \mathcal{P}_{n_b=1} \exp\left(f(i-j) \varepsilon^{\alpha\beta} b^\dagger_{i\alpha} b^\dagger_{j\alpha} \right) |0\rangle$$

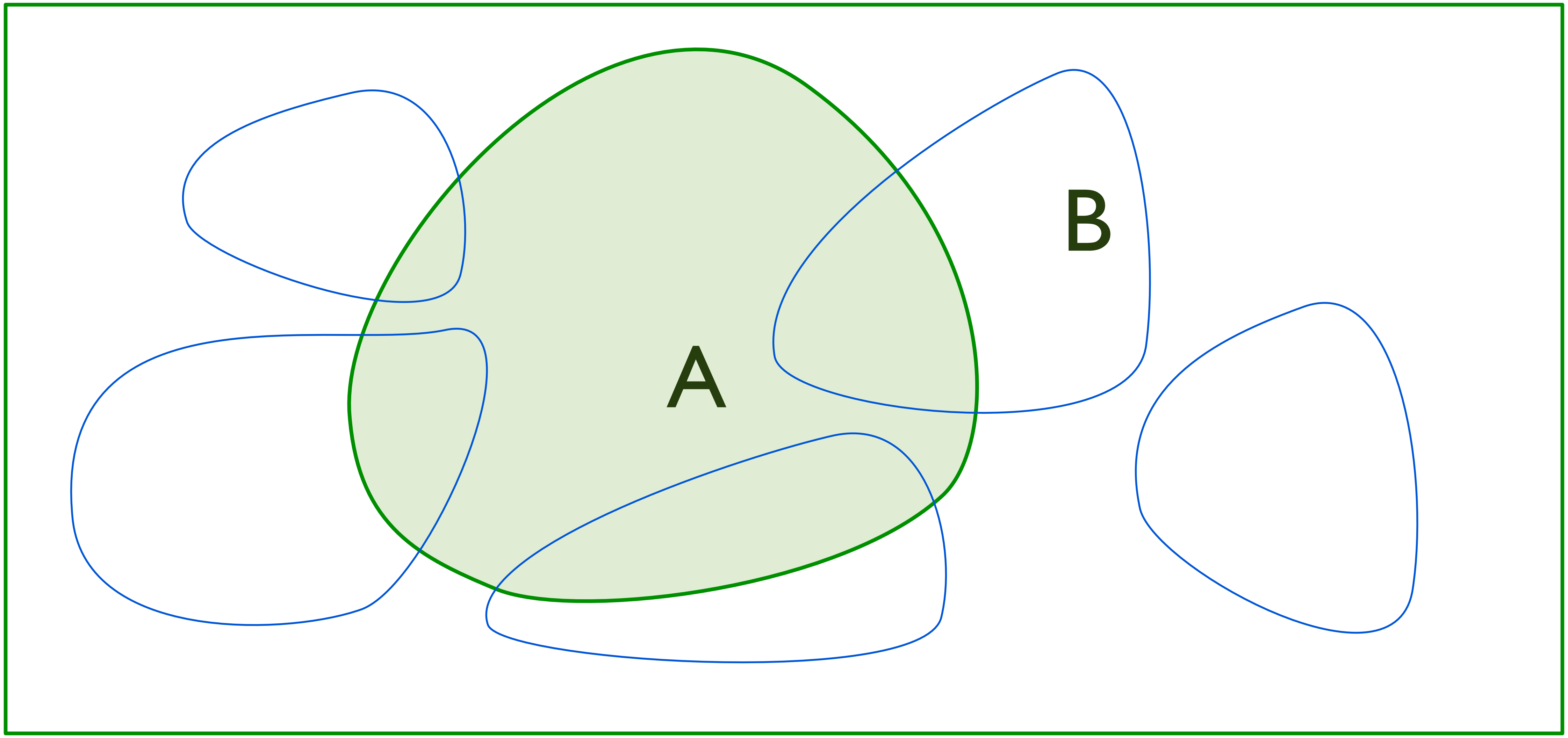
Leads to a description of fractionalized 'spinon' and 'vison' excitations coupled to an emergent $\mathbb{Z}_2$ gauge field.

S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)
Y. Huh, M. Punk, and S. Sachdev, *Phys. Rev.* B **84**, 094419 (2011)

Entanglement in the Z_2 spin liquid

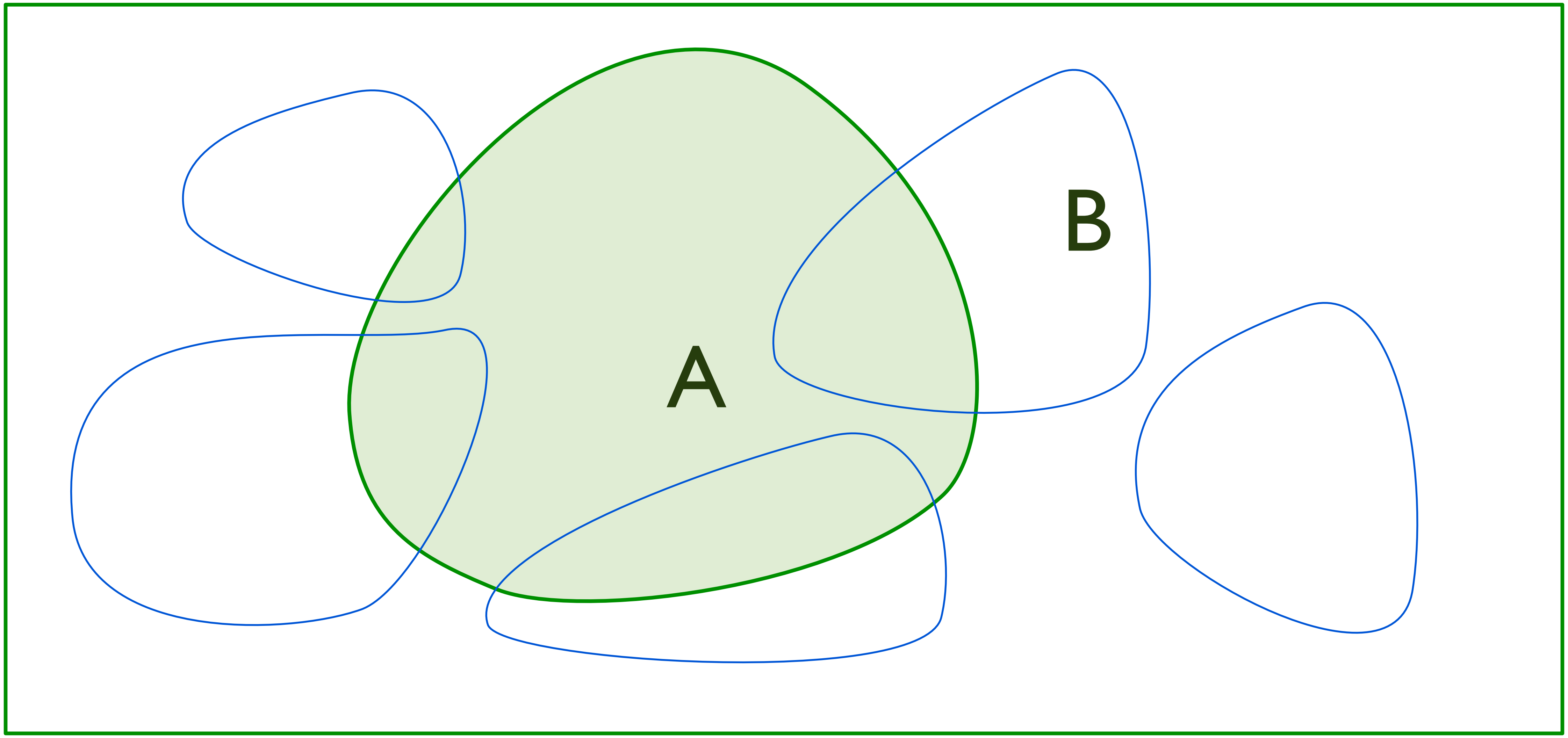


Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

Entanglement in the Z_2 spin liquid

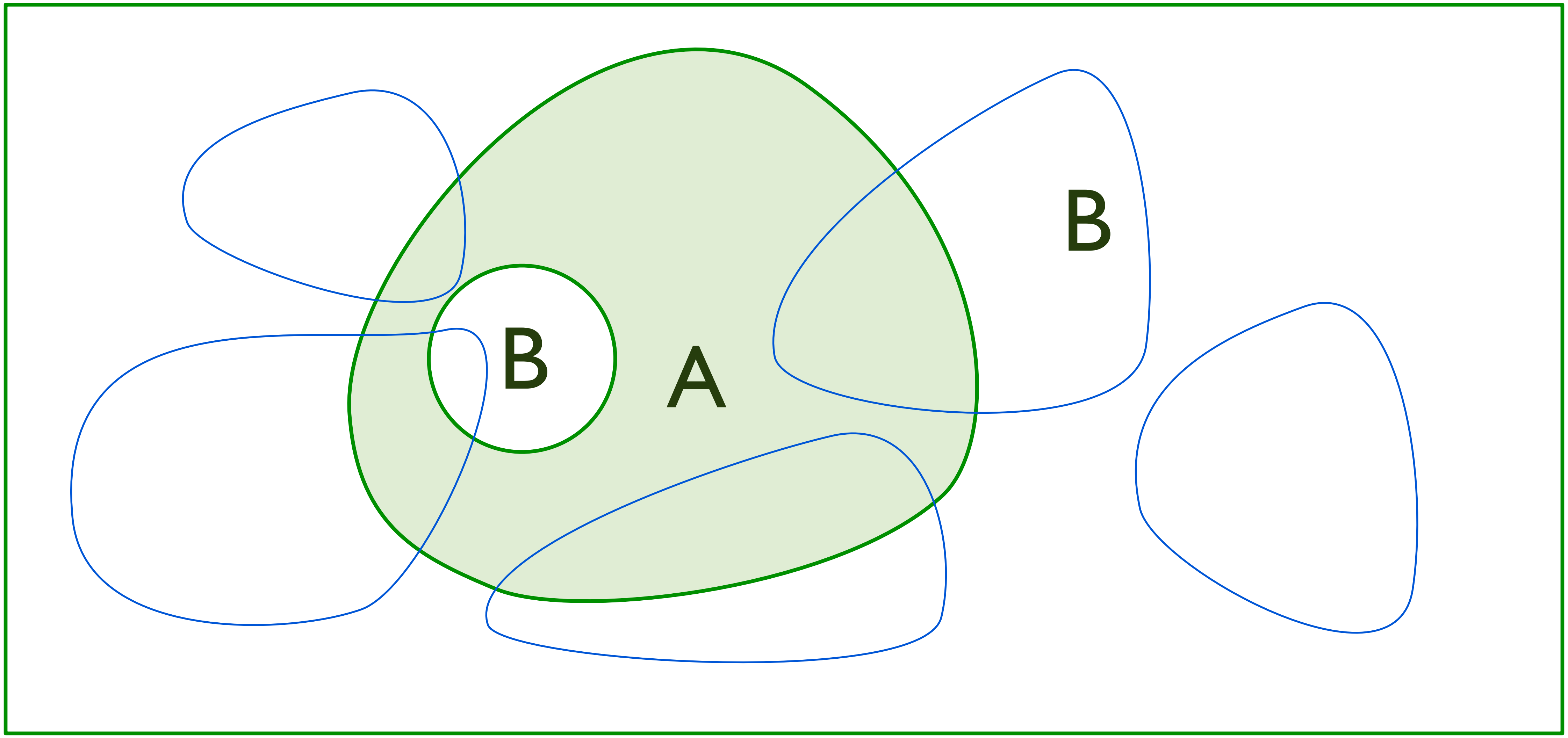


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006); Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Entanglement in the Z_2 spin liquid



$$S_E = aP - \ln(4)$$

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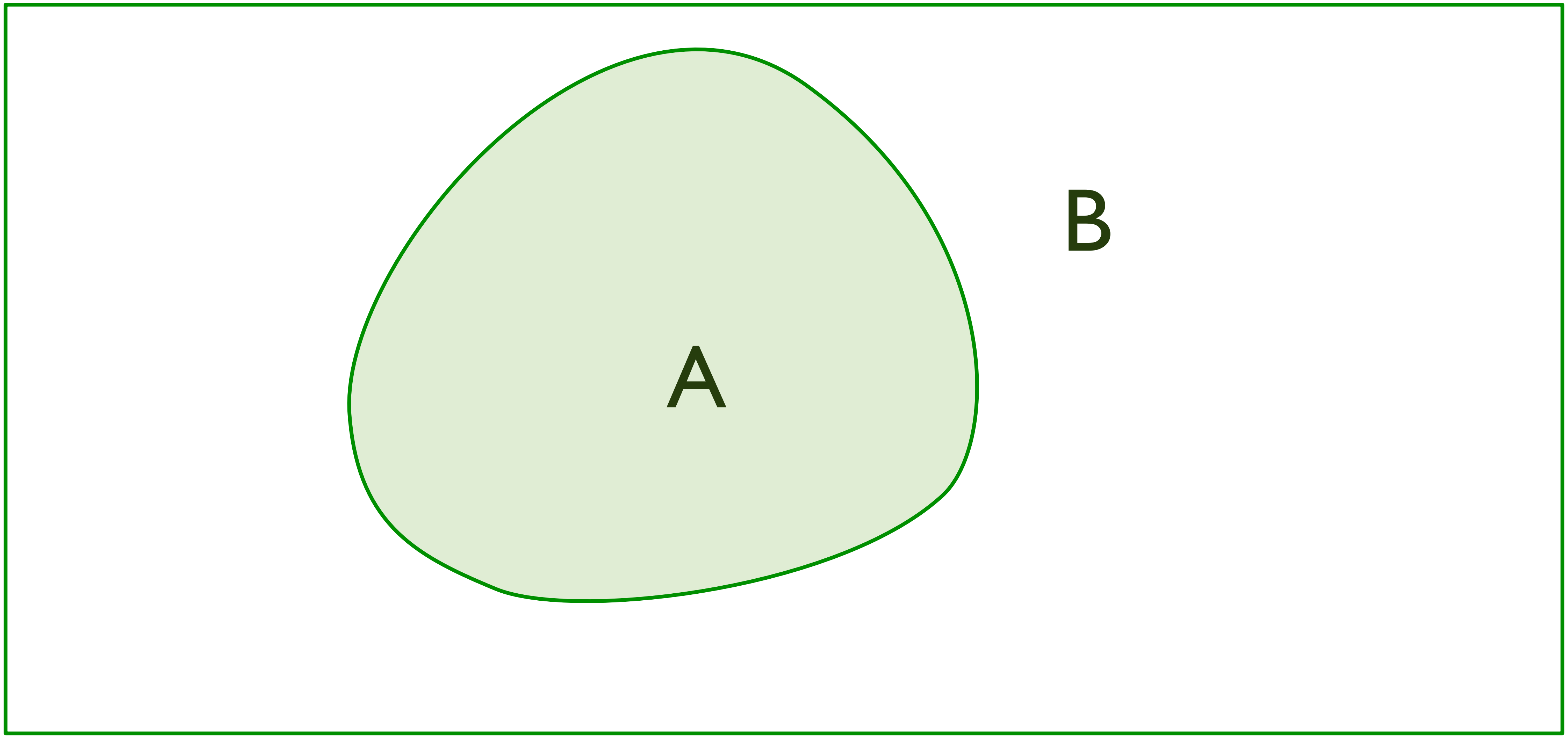
M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Entanglement in the Z_2 spin liquid



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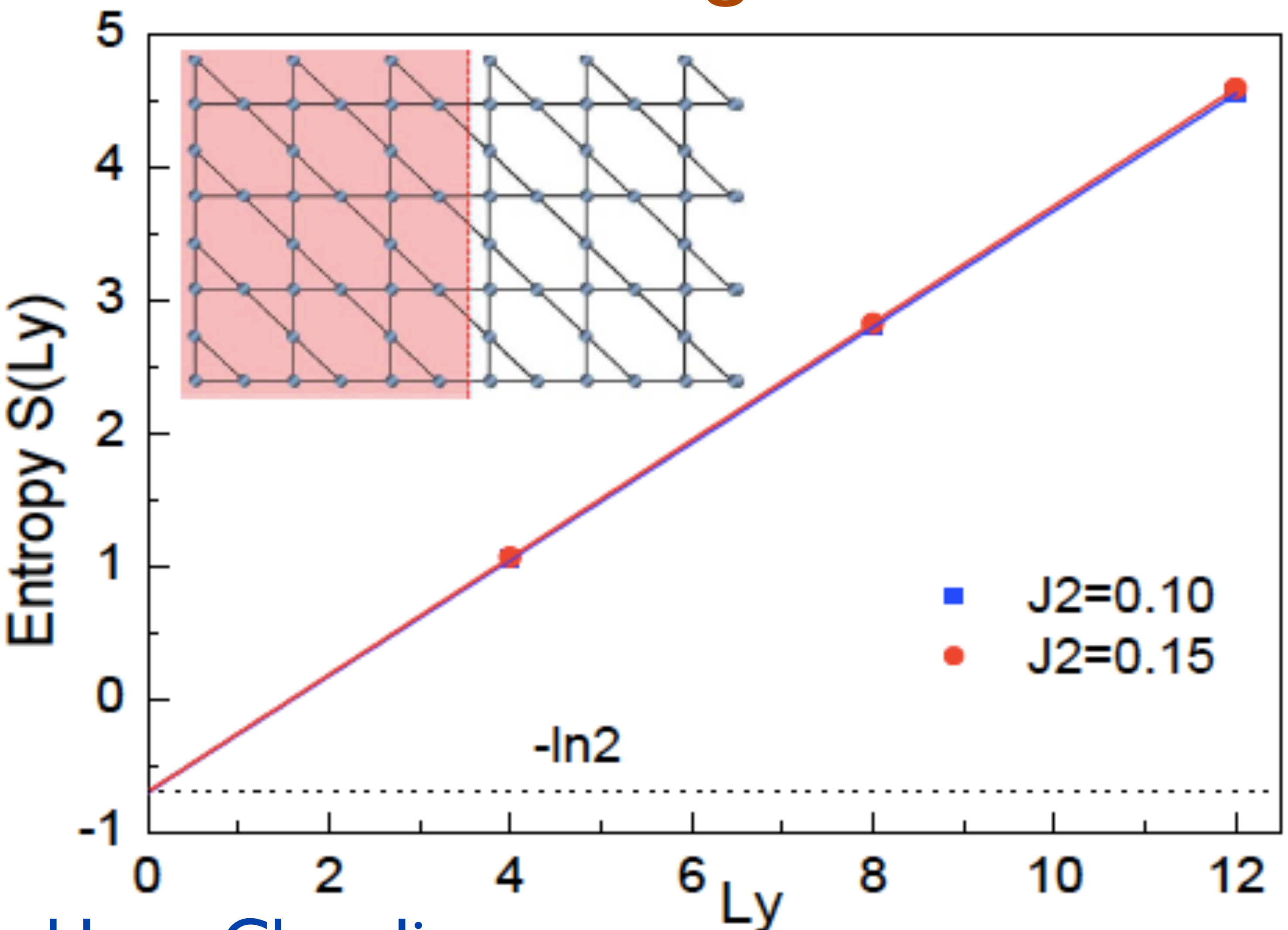
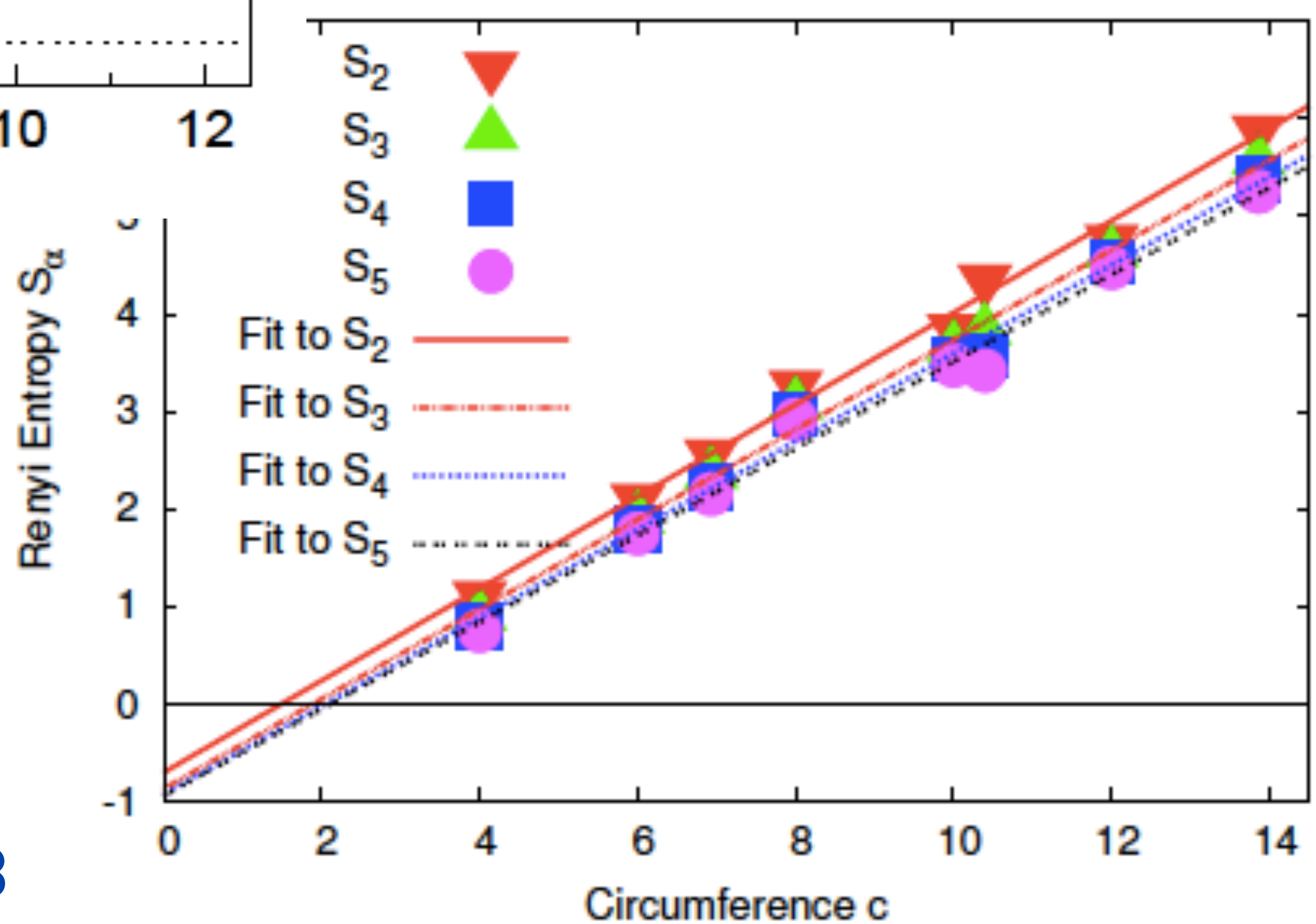


Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

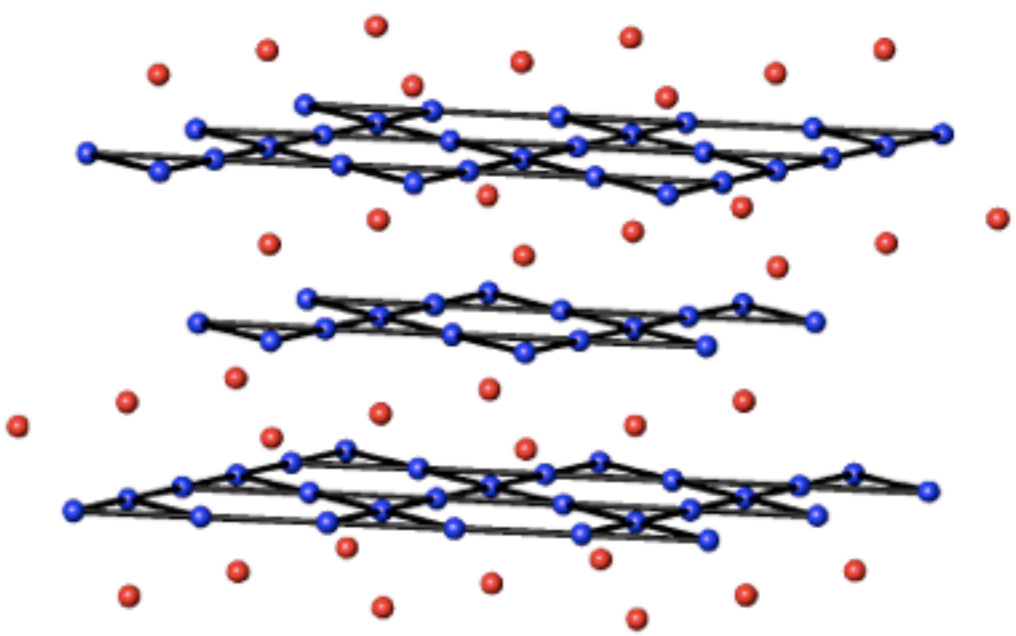
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

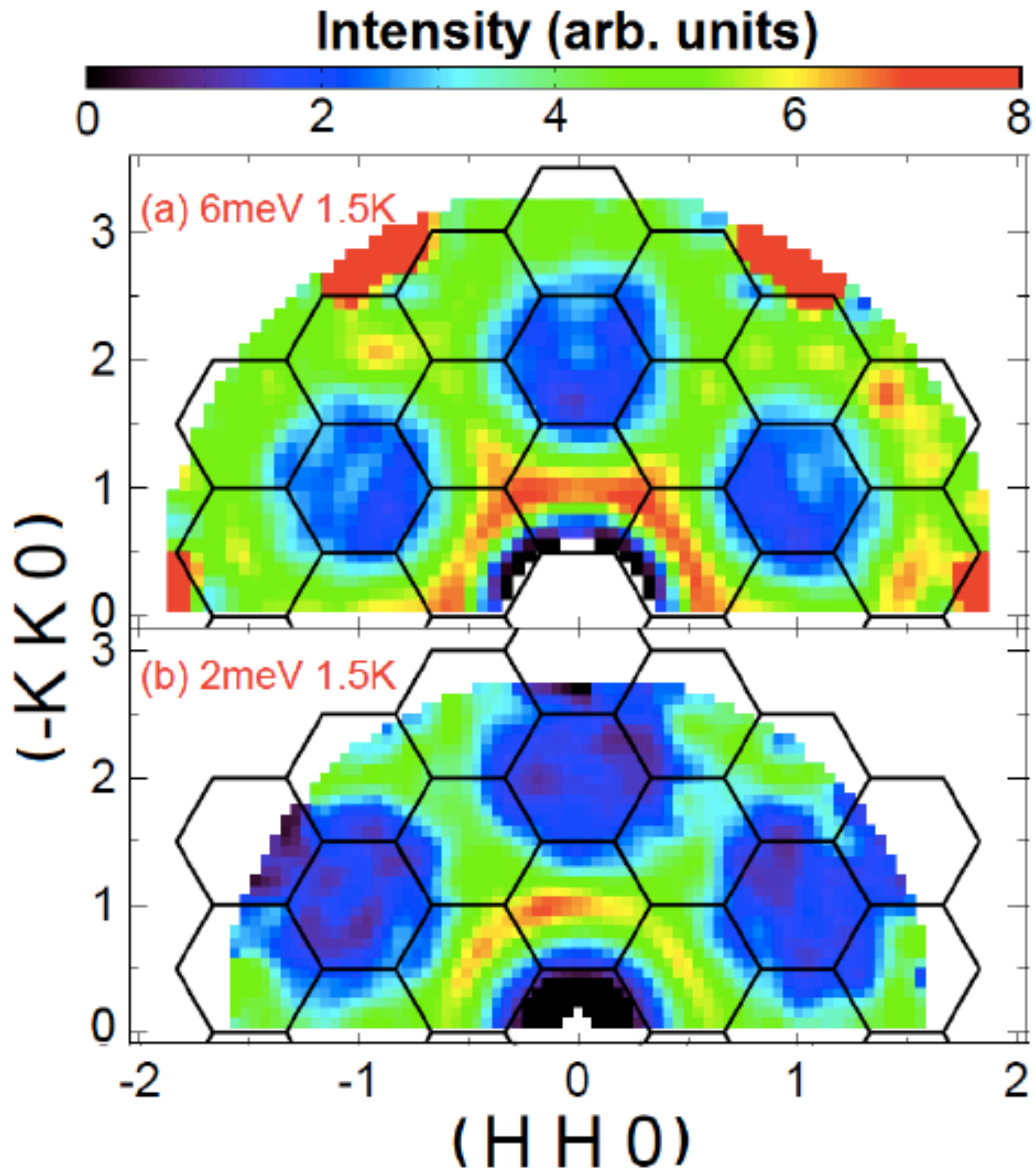


Mott insulator: Kagome antiferromagnet

Evidence for spinons
Young Lee,
APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





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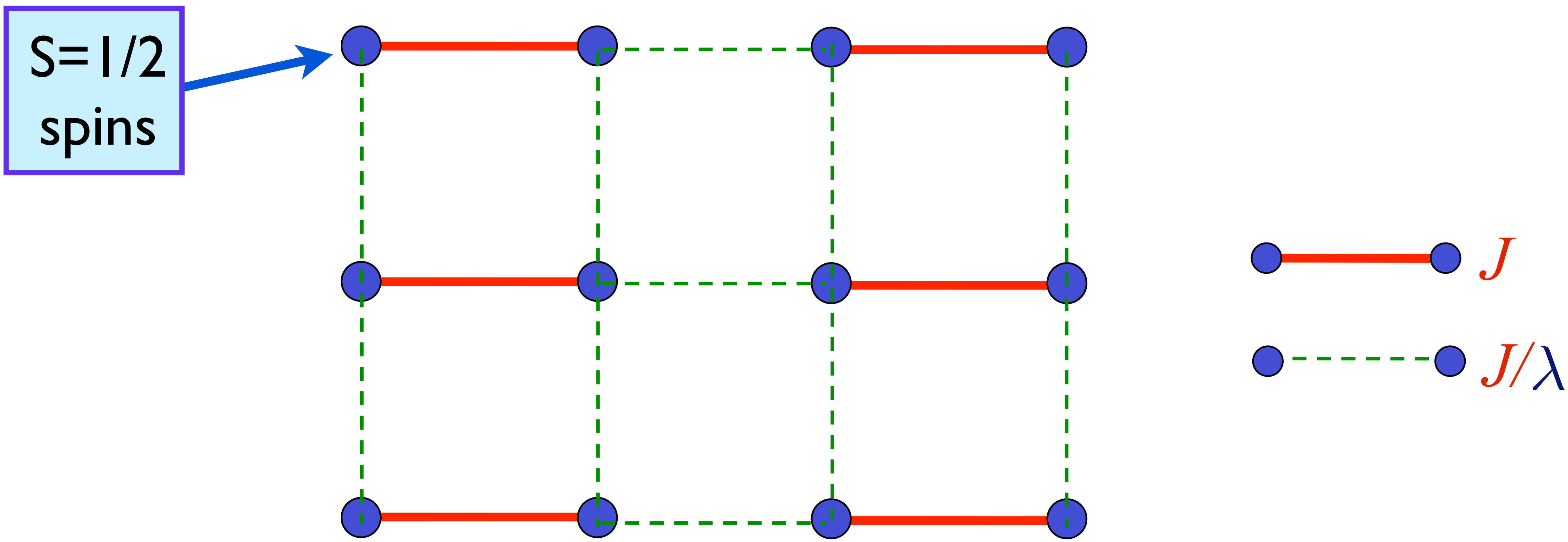
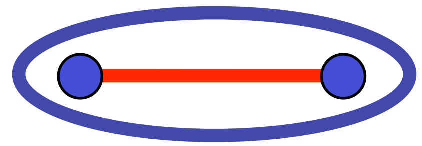
Spinning electrons localized on a square lattice

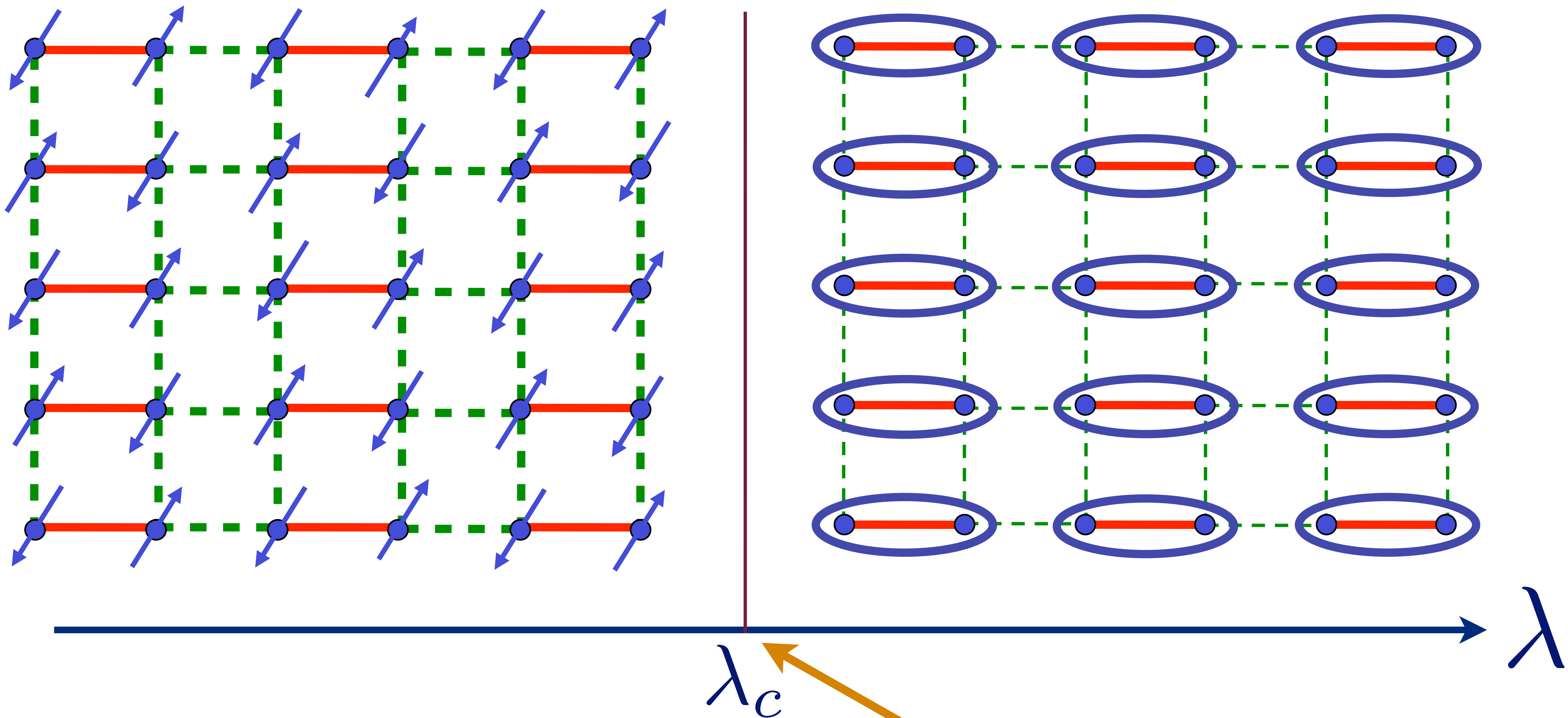
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ



$$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$$

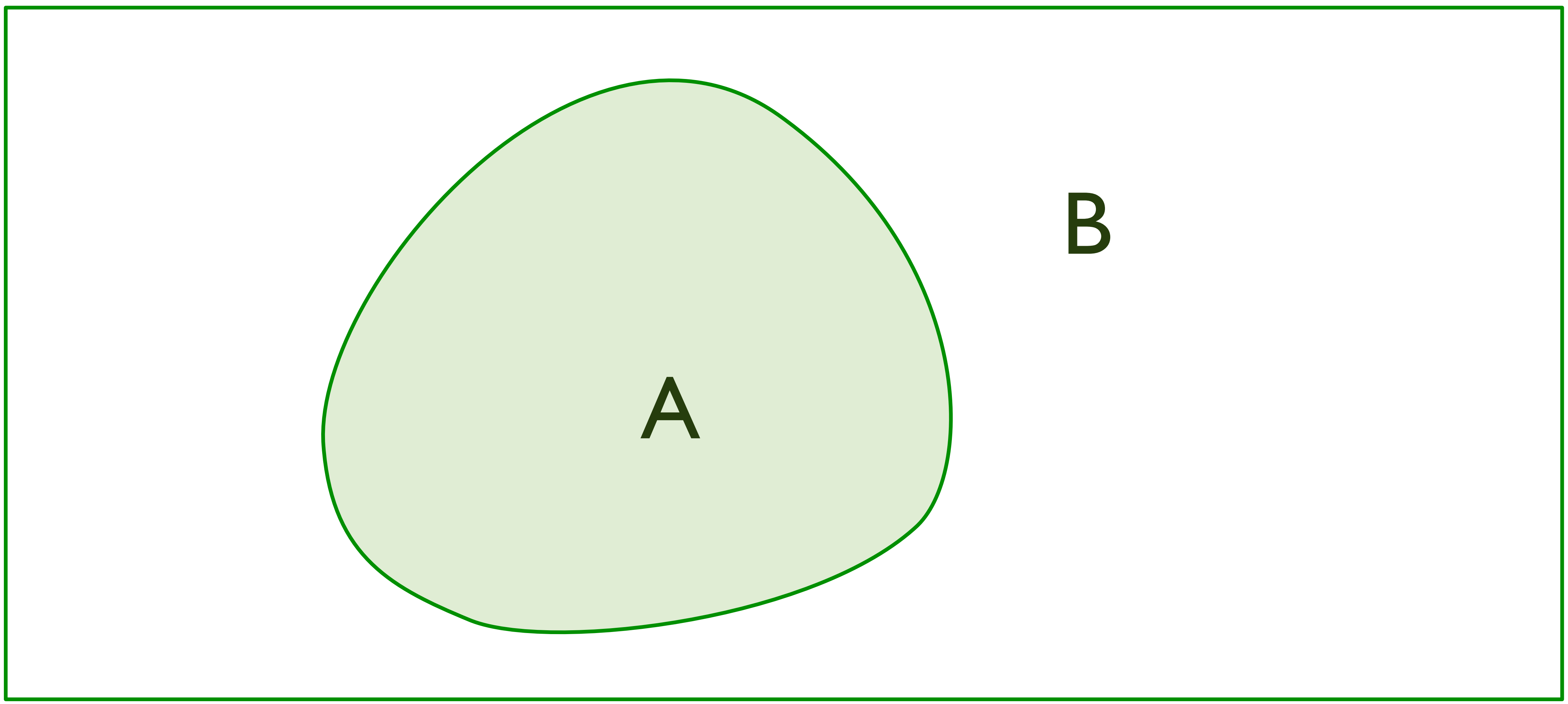


Quantum critical point described by a CFT3 (O(3) Wilson-Fisher)

A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

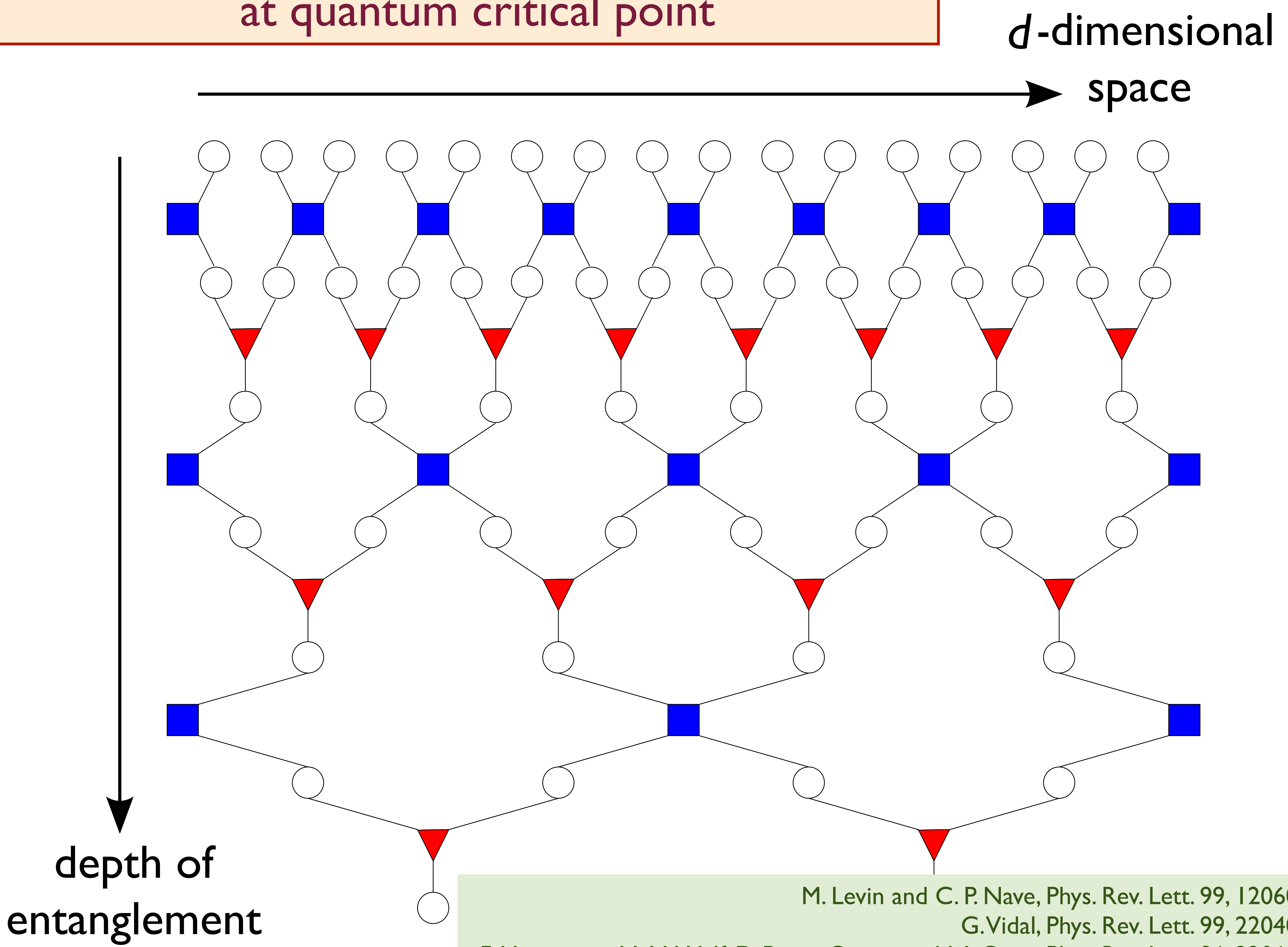
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

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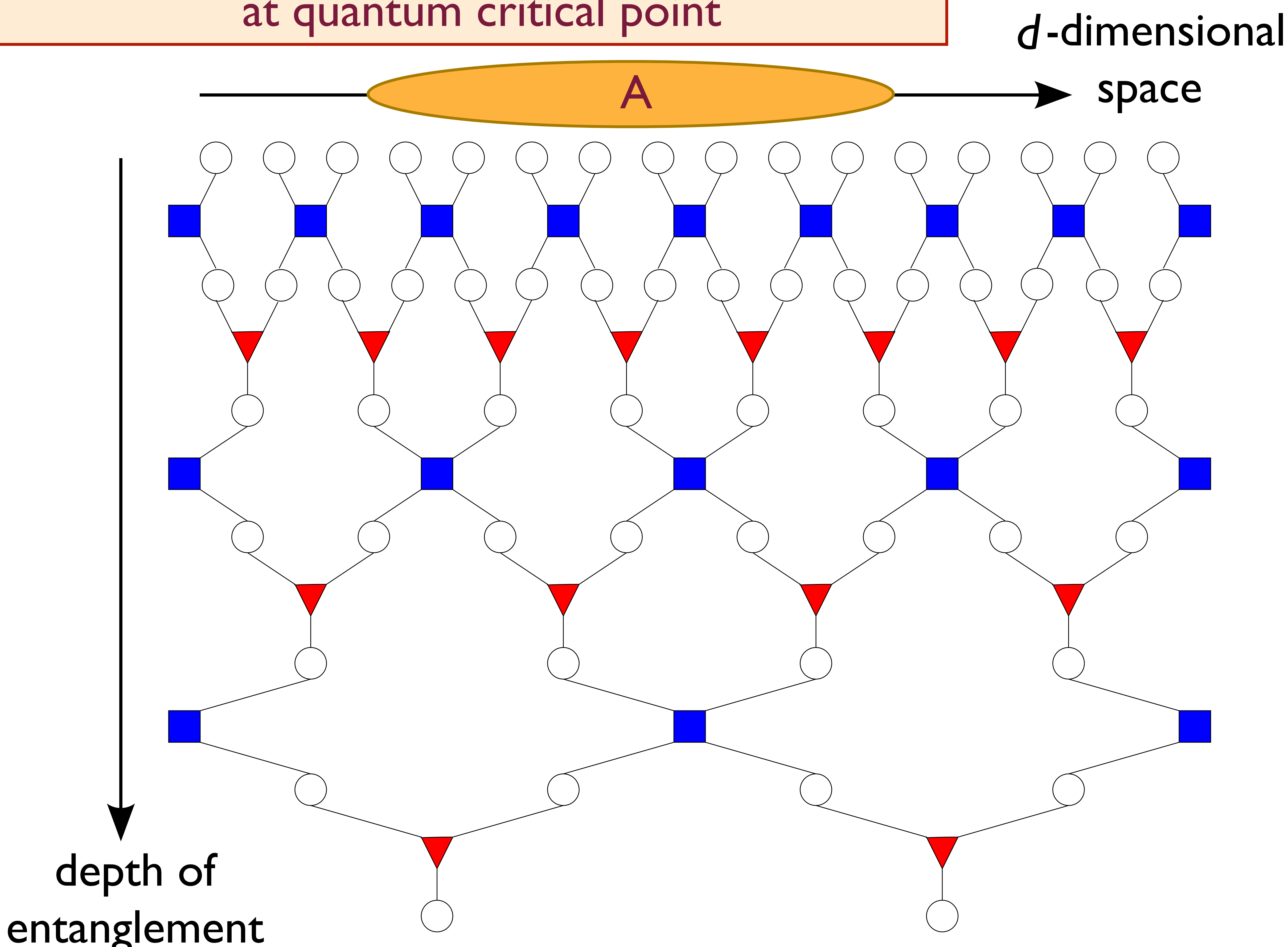

Brian Swingle, arXiv:0905.1317

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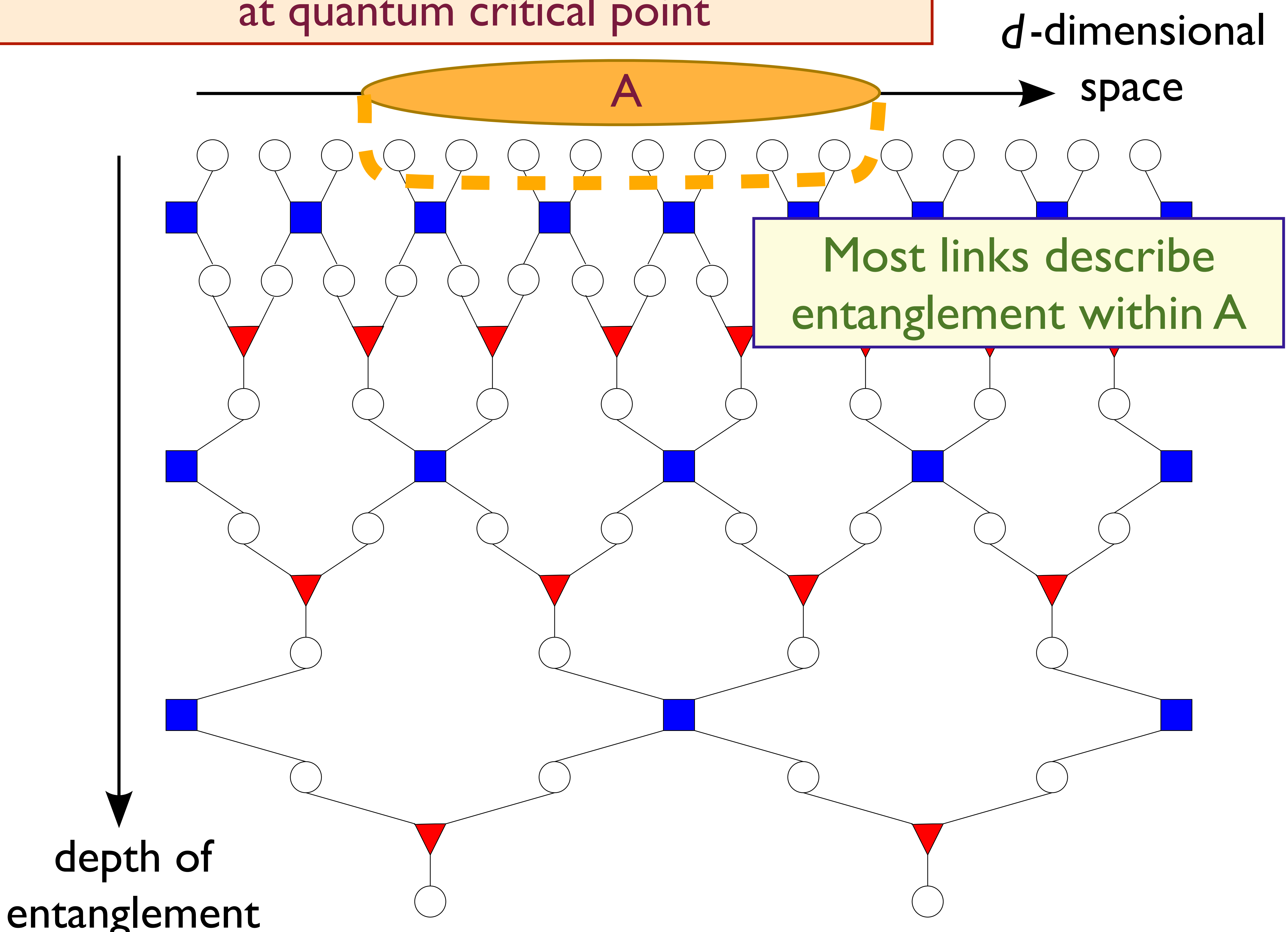

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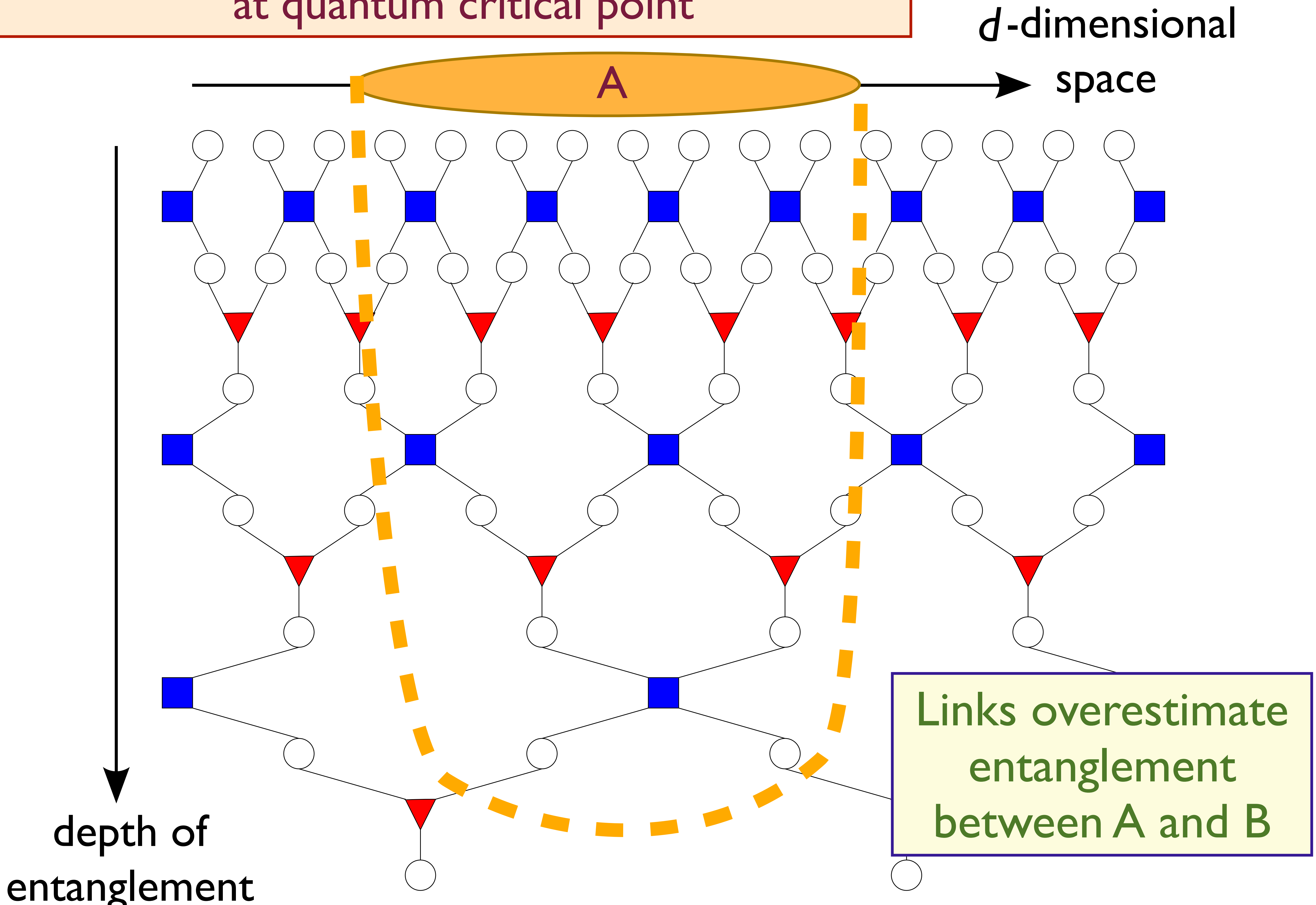

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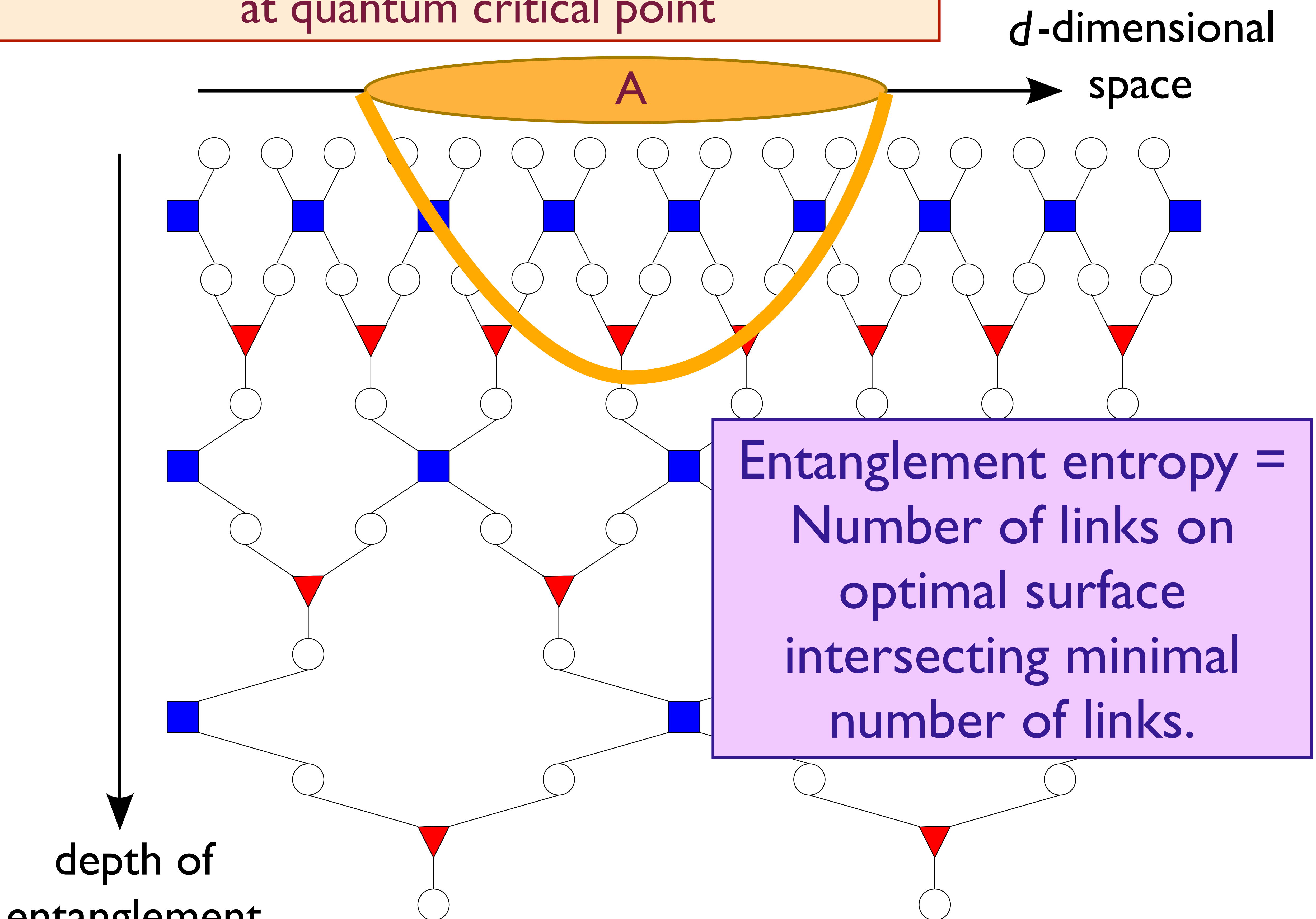
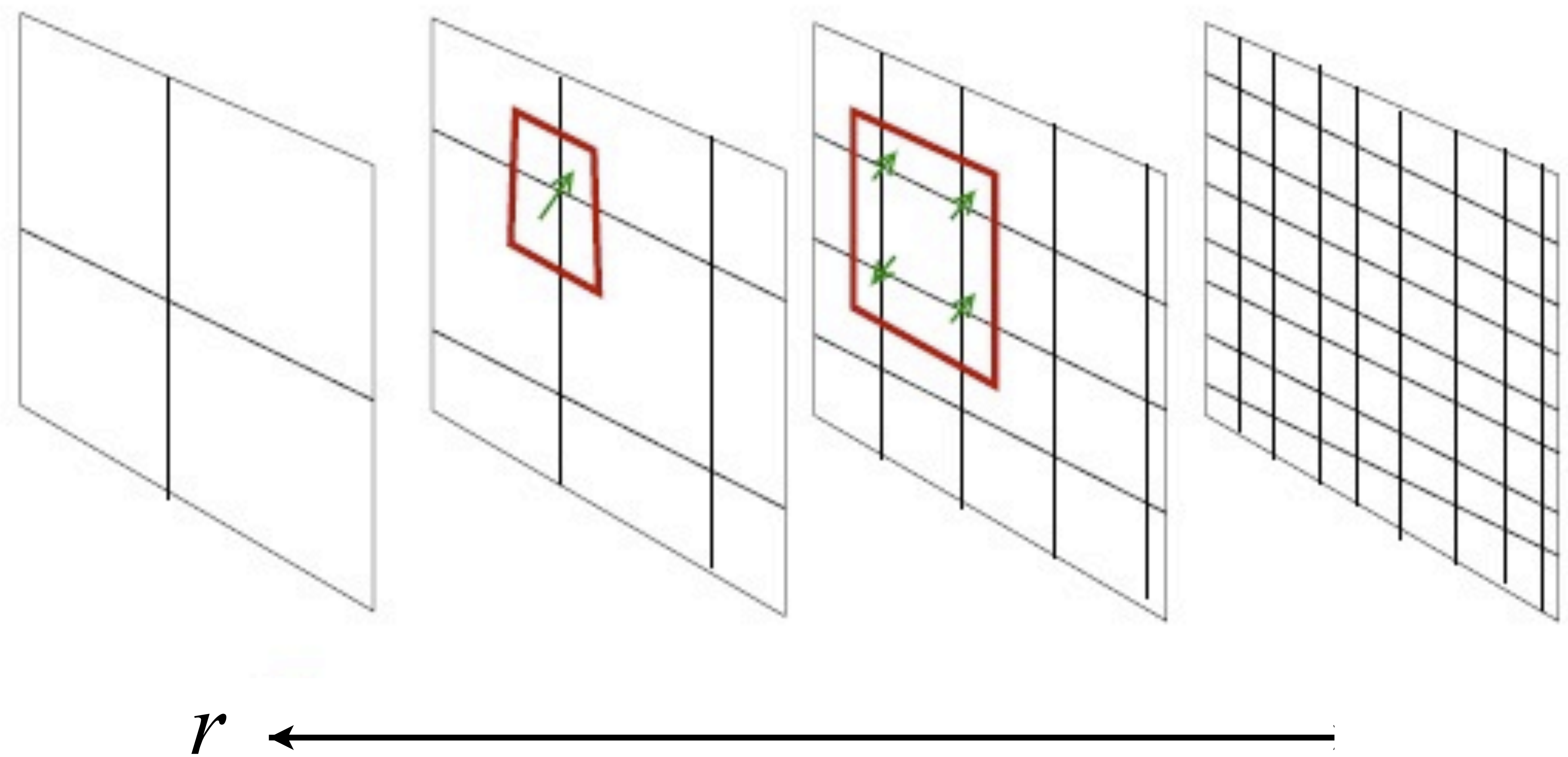


Brian Swingle, arXiv:0905.1317

Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \dots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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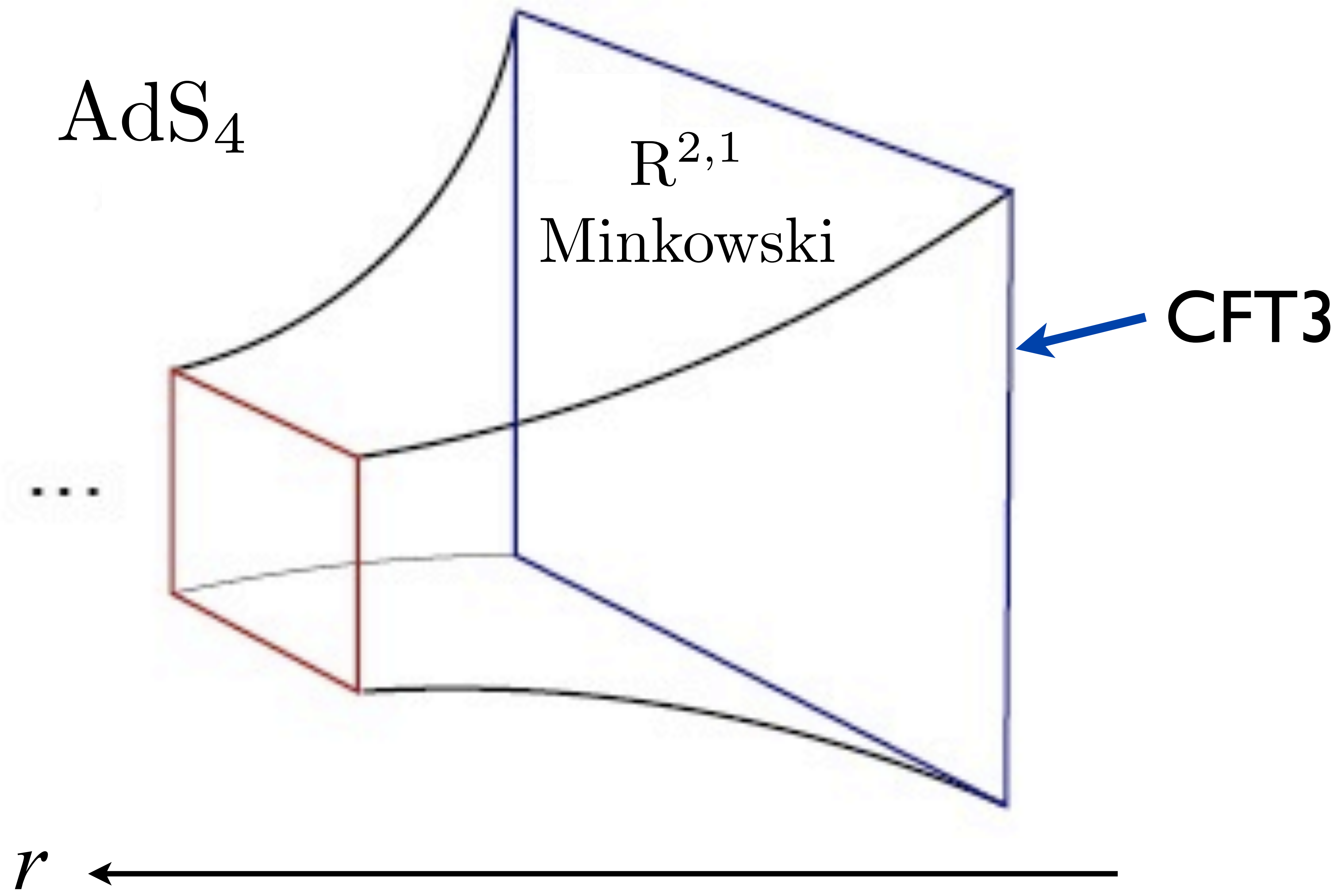
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS/CFT correspondence



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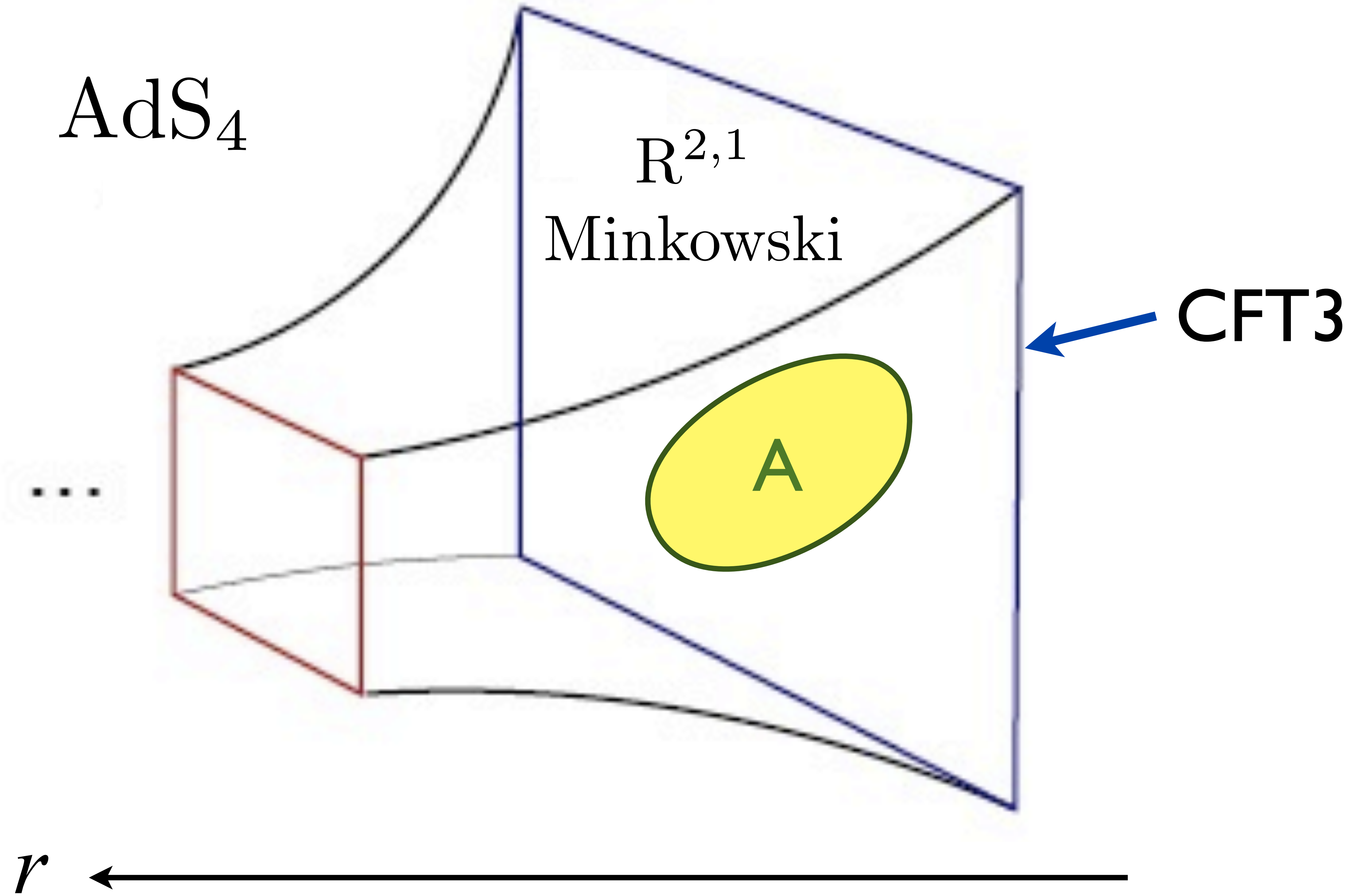

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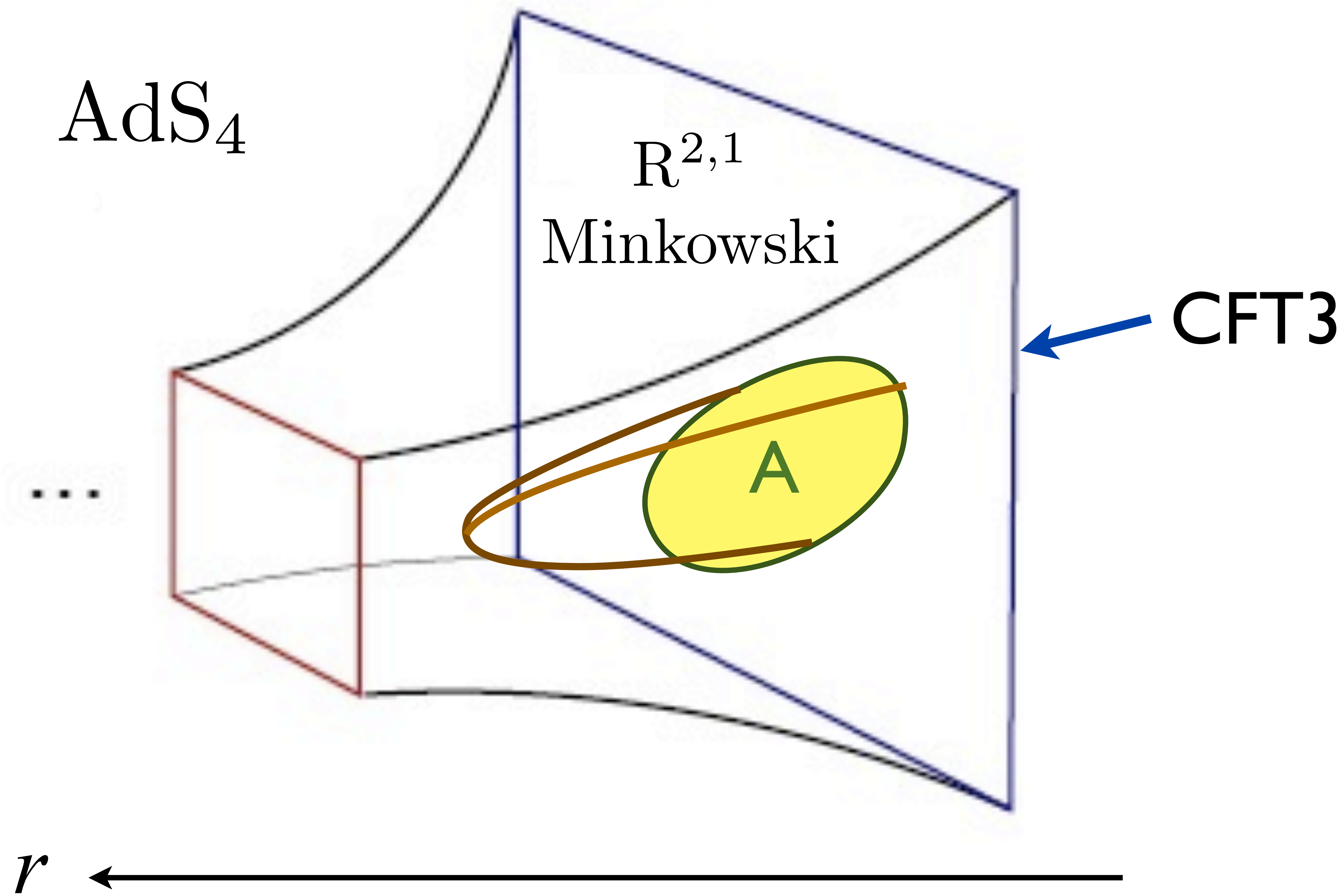
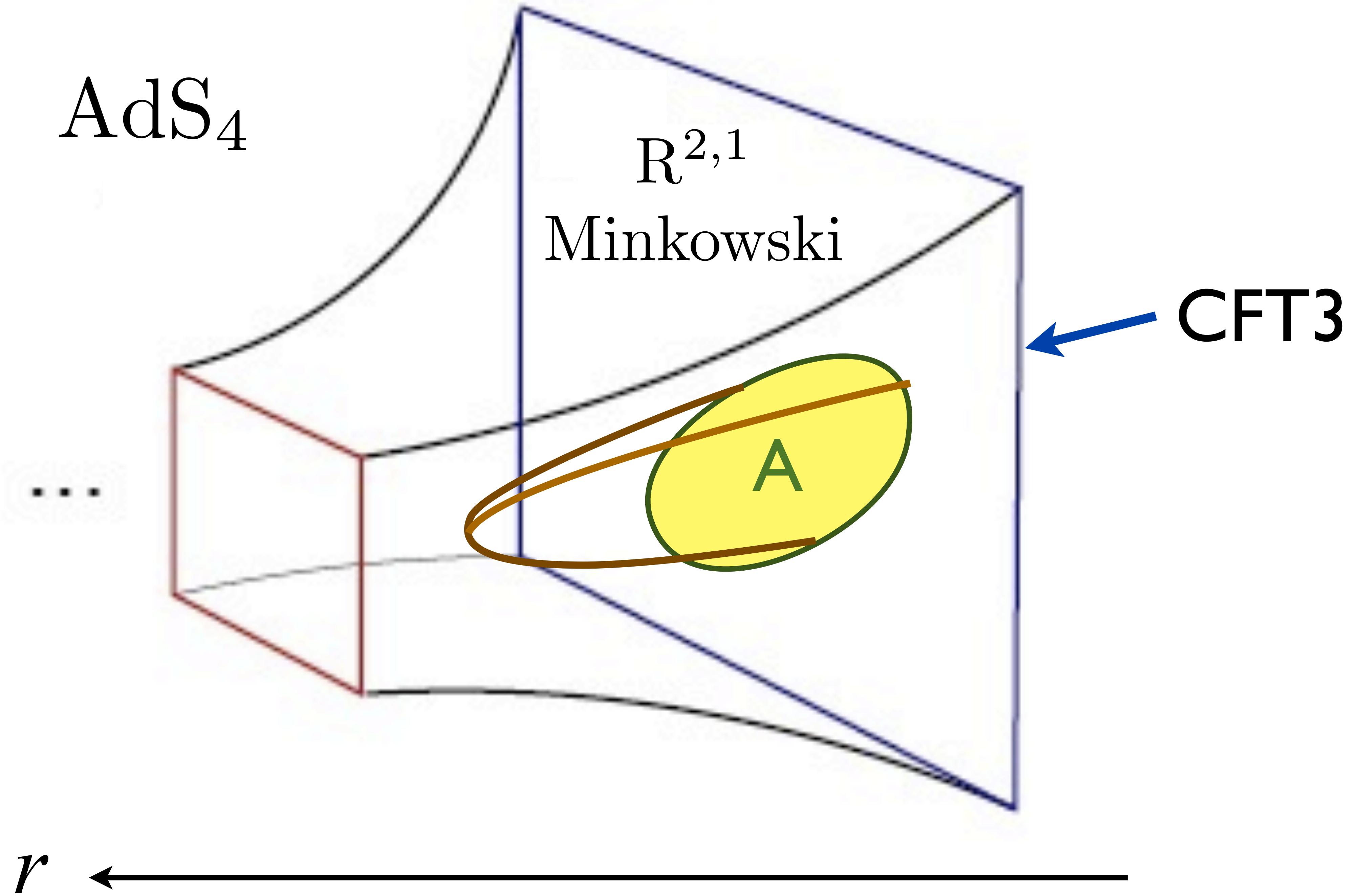


Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

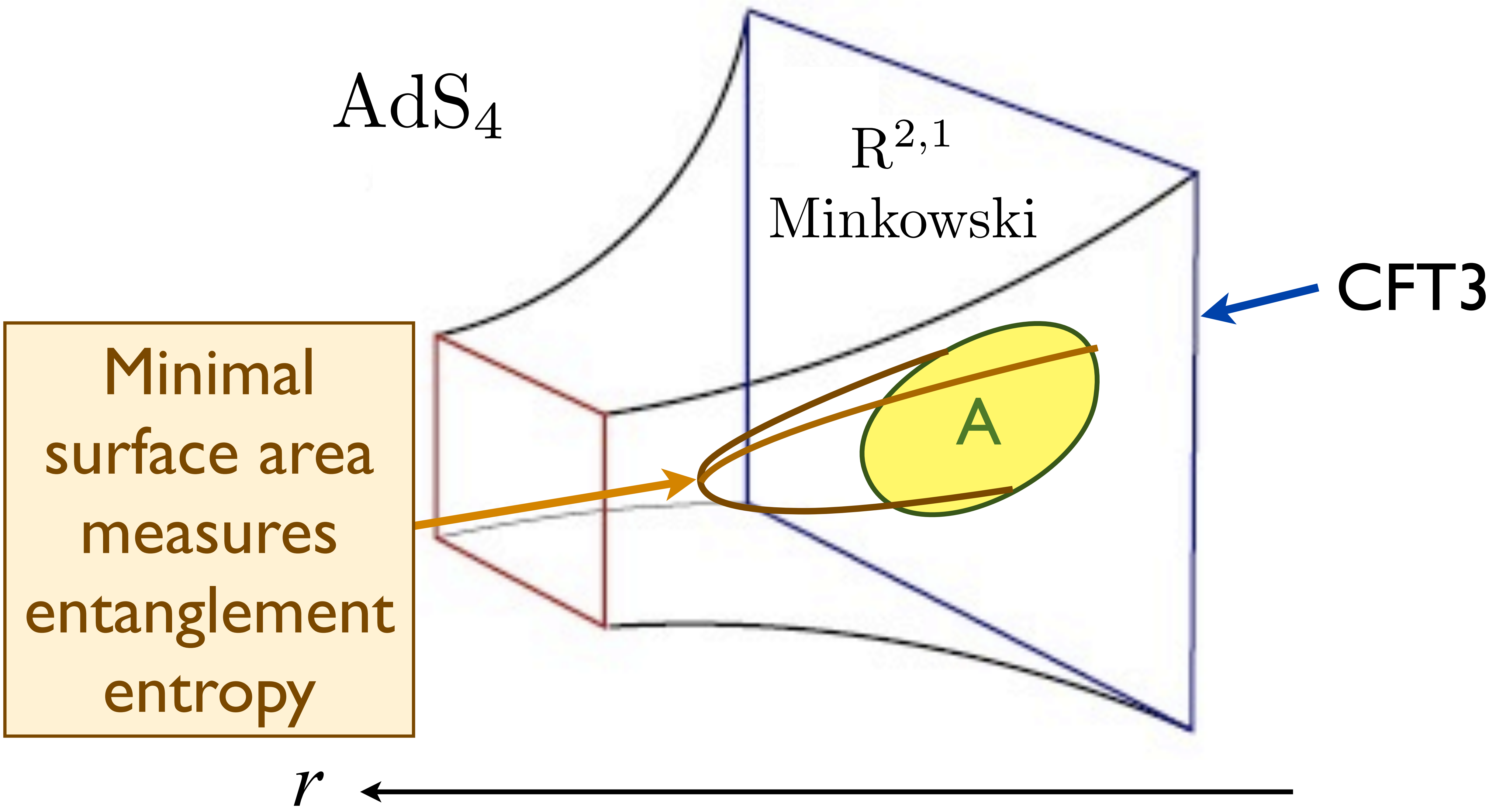
AdS/CFT correspondence



The entropy of this region is bounded by its surface area (Bekenstein-Hawking-'t Hooft-Susskind)

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



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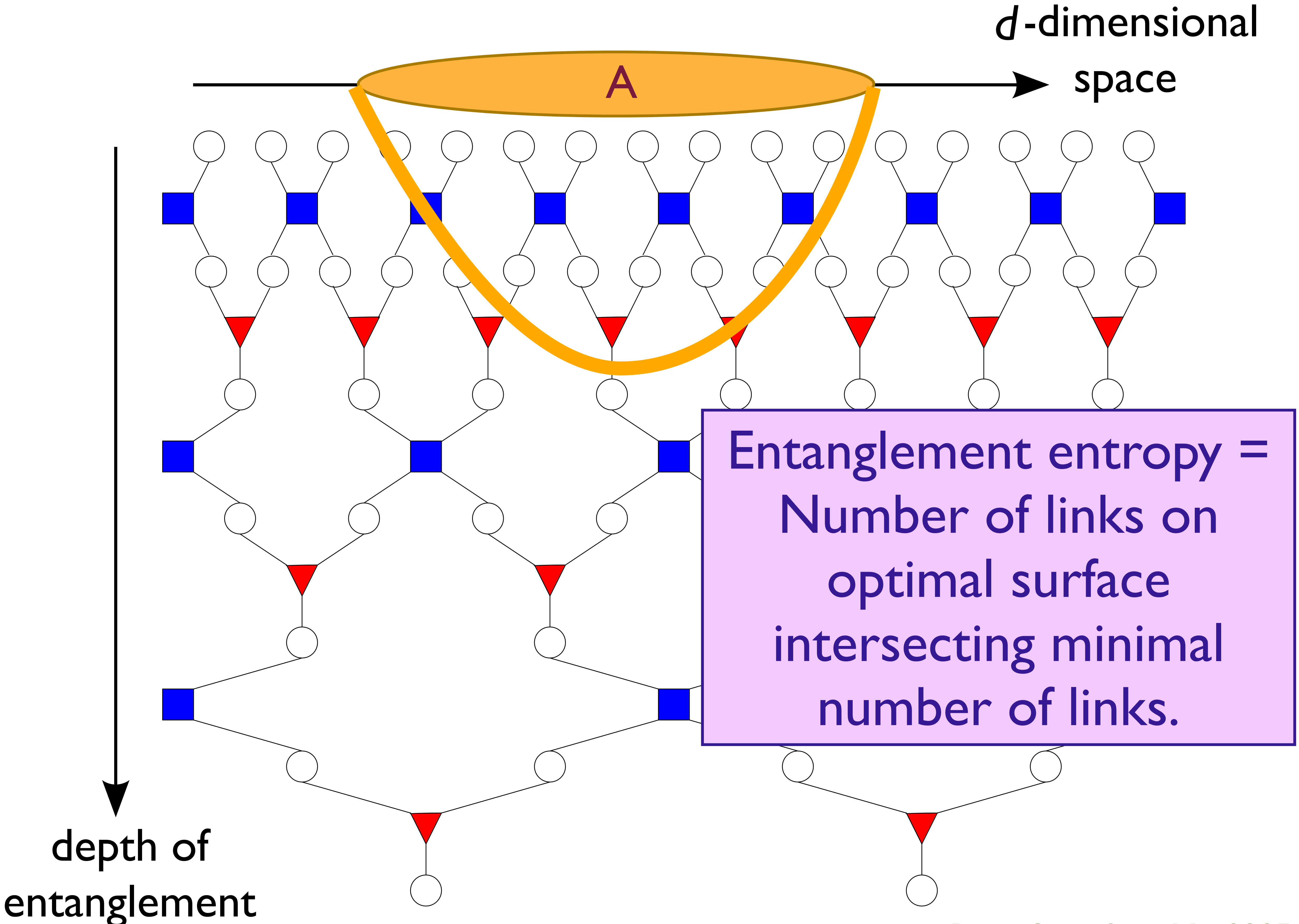
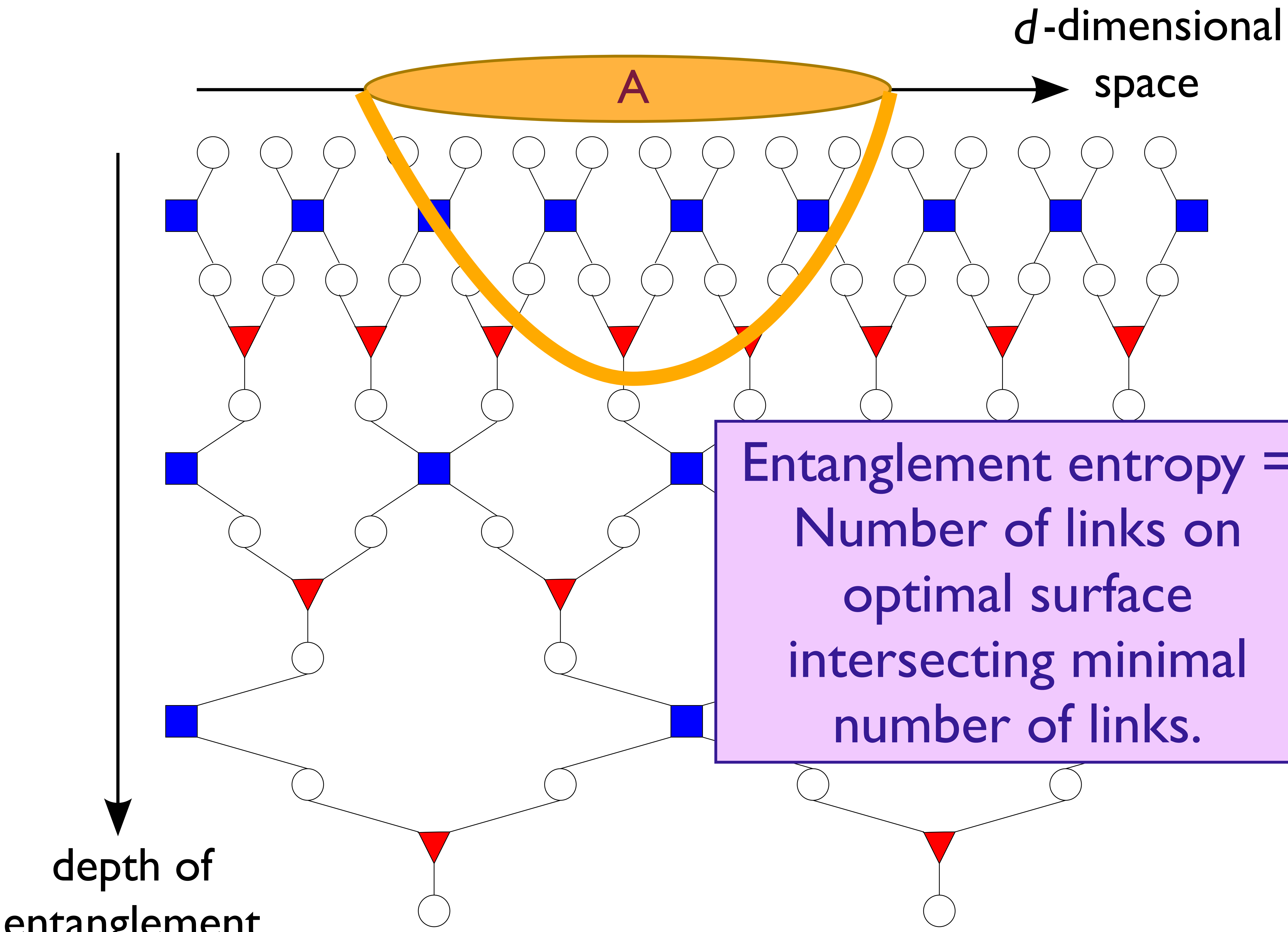
Entanglement entropy



Brian Swingle, arXiv:0905.1317

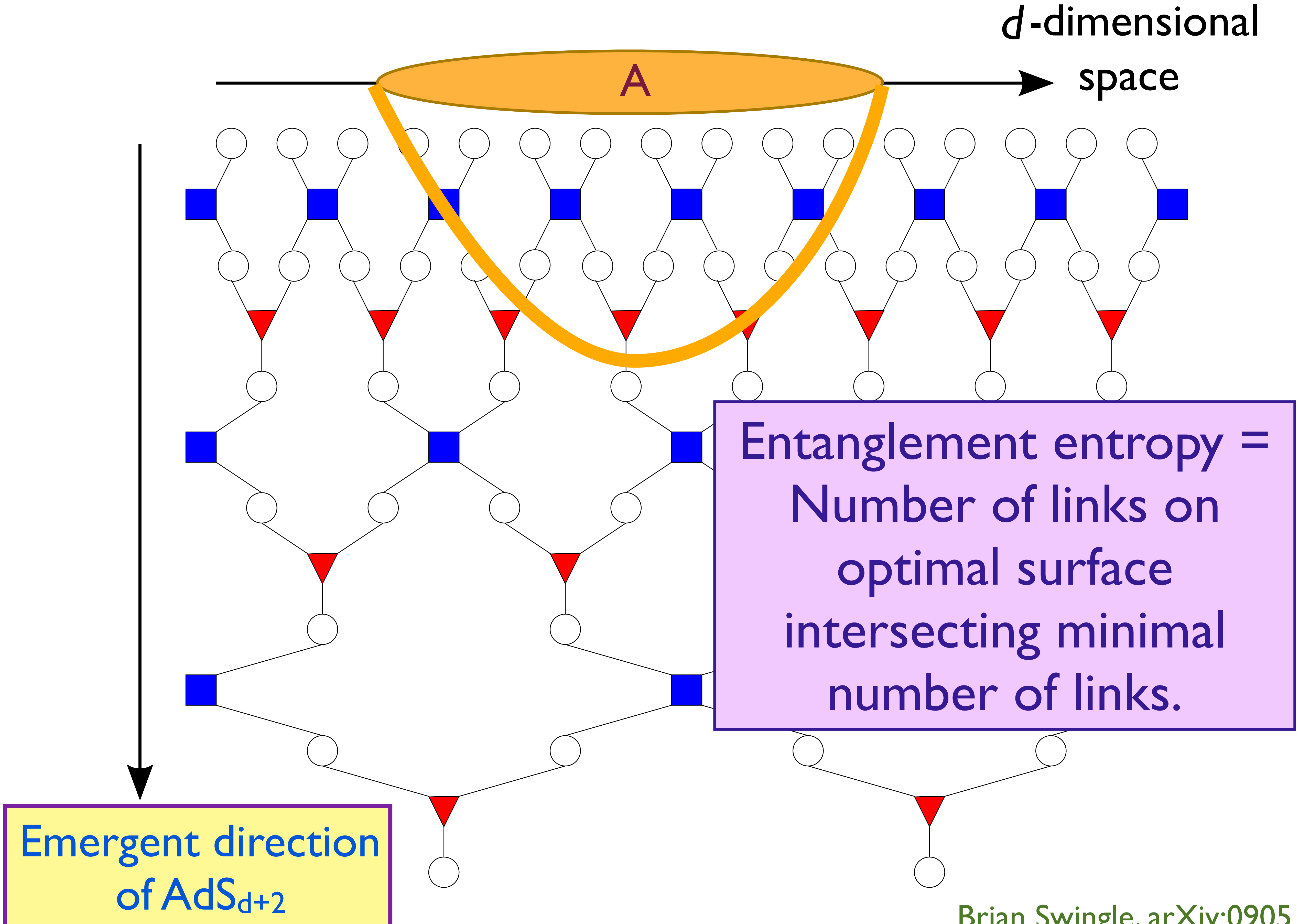
Entanglement entropy



Entanglement entropy =
Number of links on
optimal surface
intersecting minimal
number of links.

Brian Swingle, arXiv:0905.1317

Entanglement entropy



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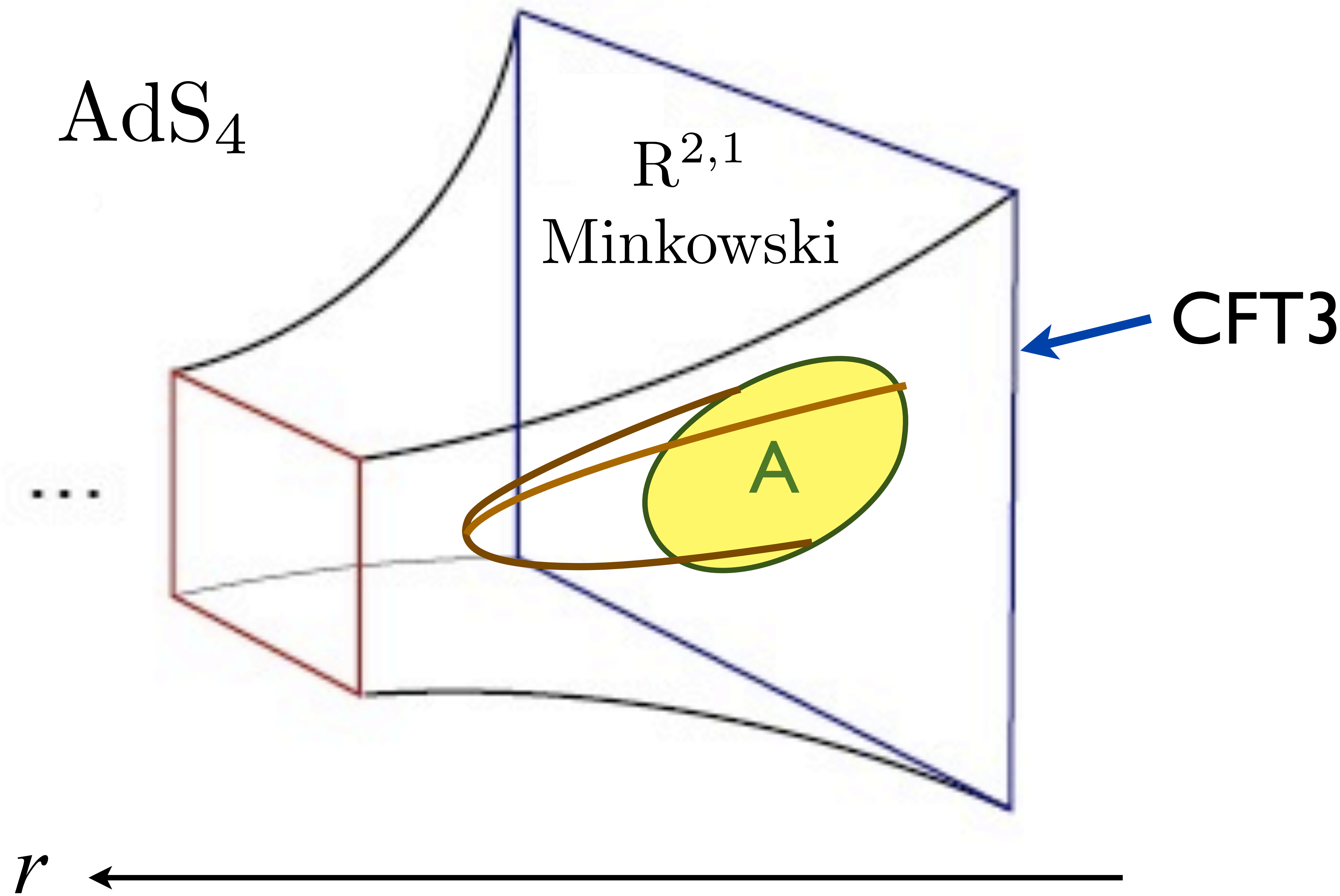


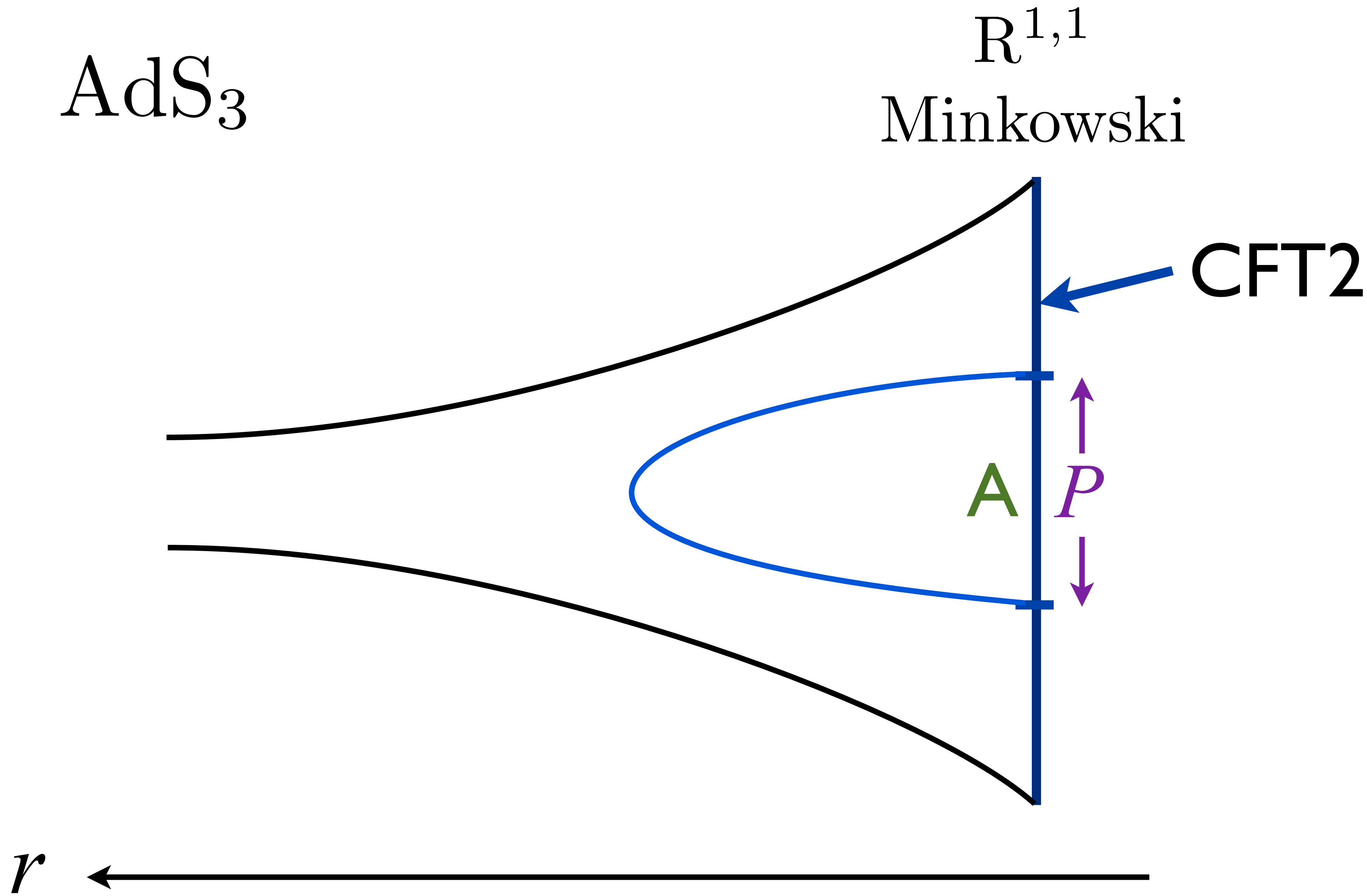
- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



- Computation of minimal surface area, or direct computation in CFT2, yield $S_E = (c/6)\ln P$, where c is the central charge.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

"Complex entangled" states of quantum matter in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

“Complex entangled” states of quantum matter in *d* spatial dimensions

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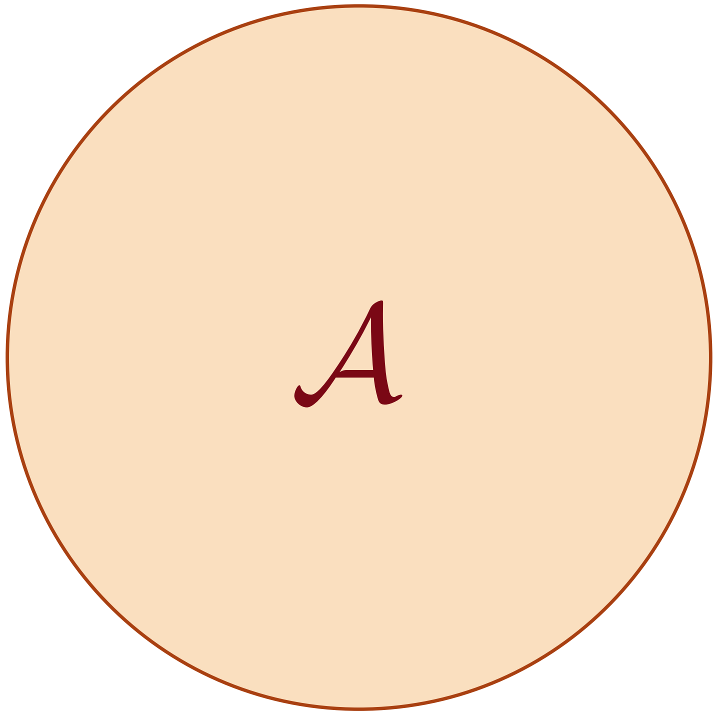
Graphene, strange metals in high temperature superconductors, spin liquids

The Fermi liquid



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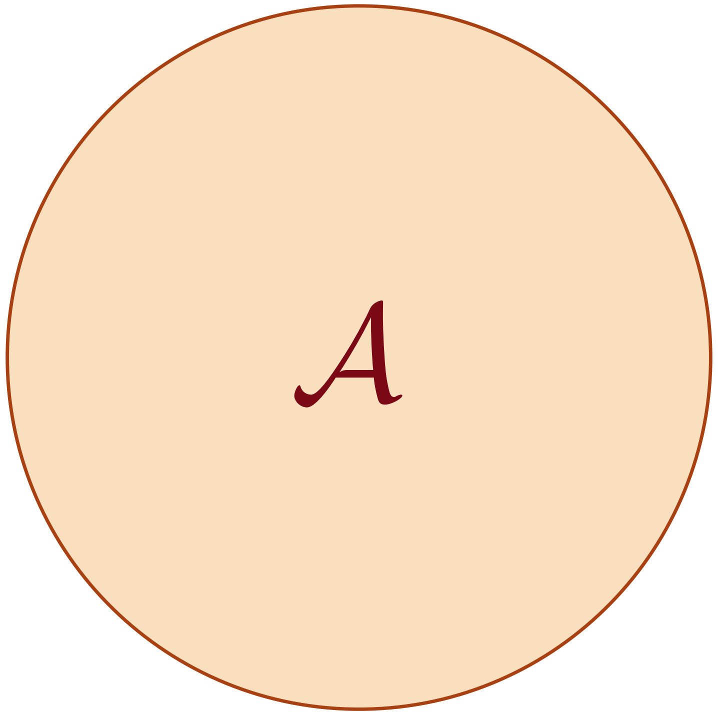

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

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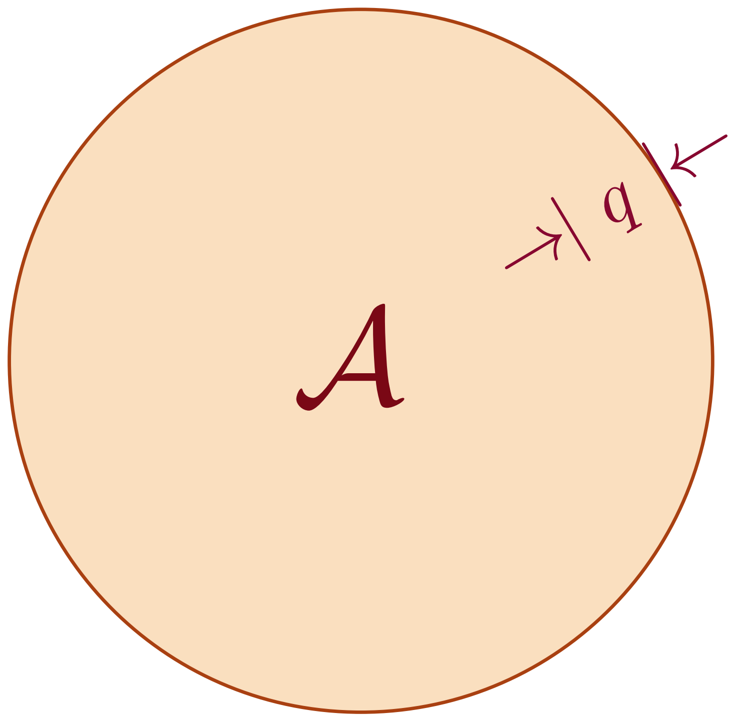

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- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|$.

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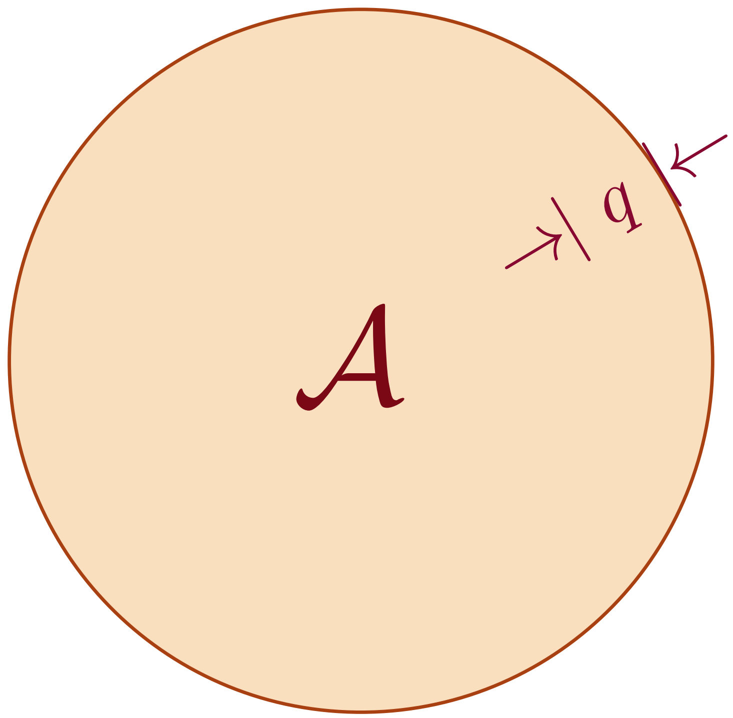

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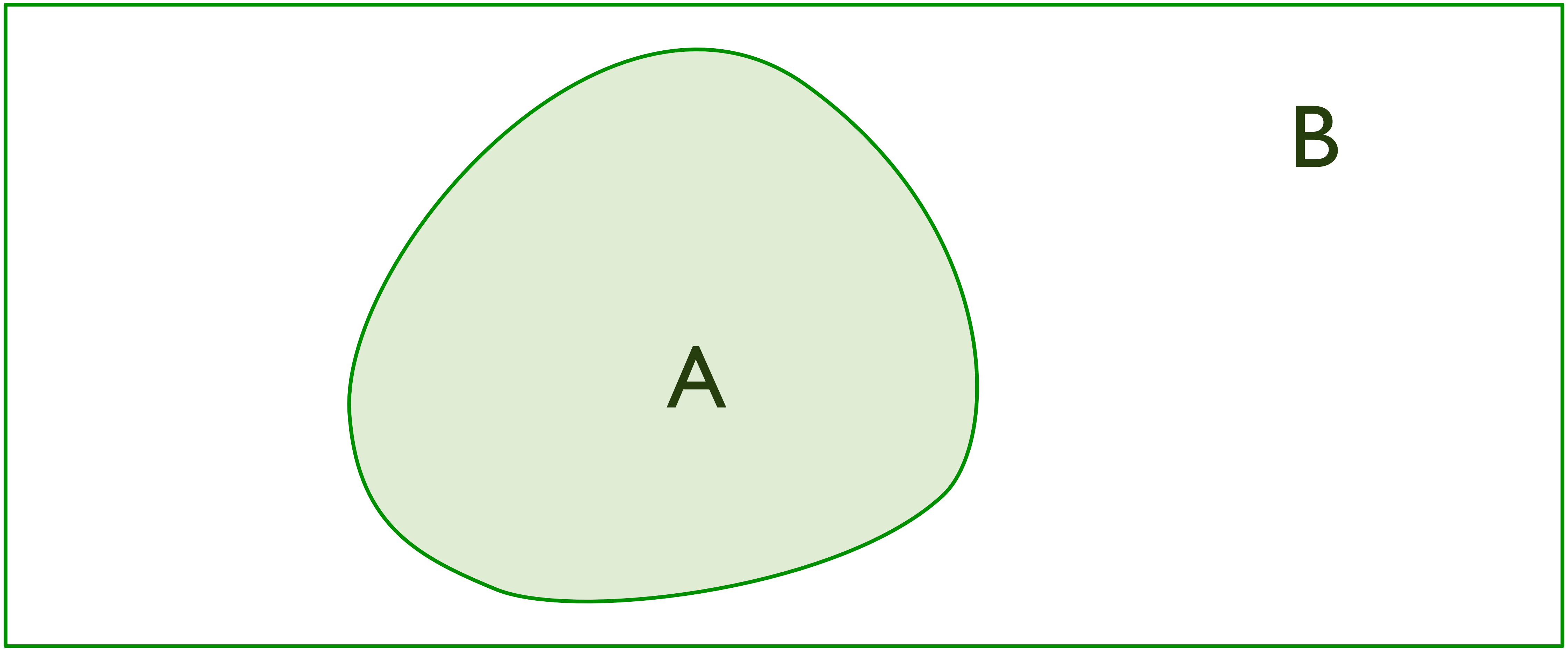
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}}$ with $d_{\text{eff}} = 1$.

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Strange metals

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless scalar field, ϕ, which has low energy excitations near $\mathbf{q} = 0$.

Strange metals

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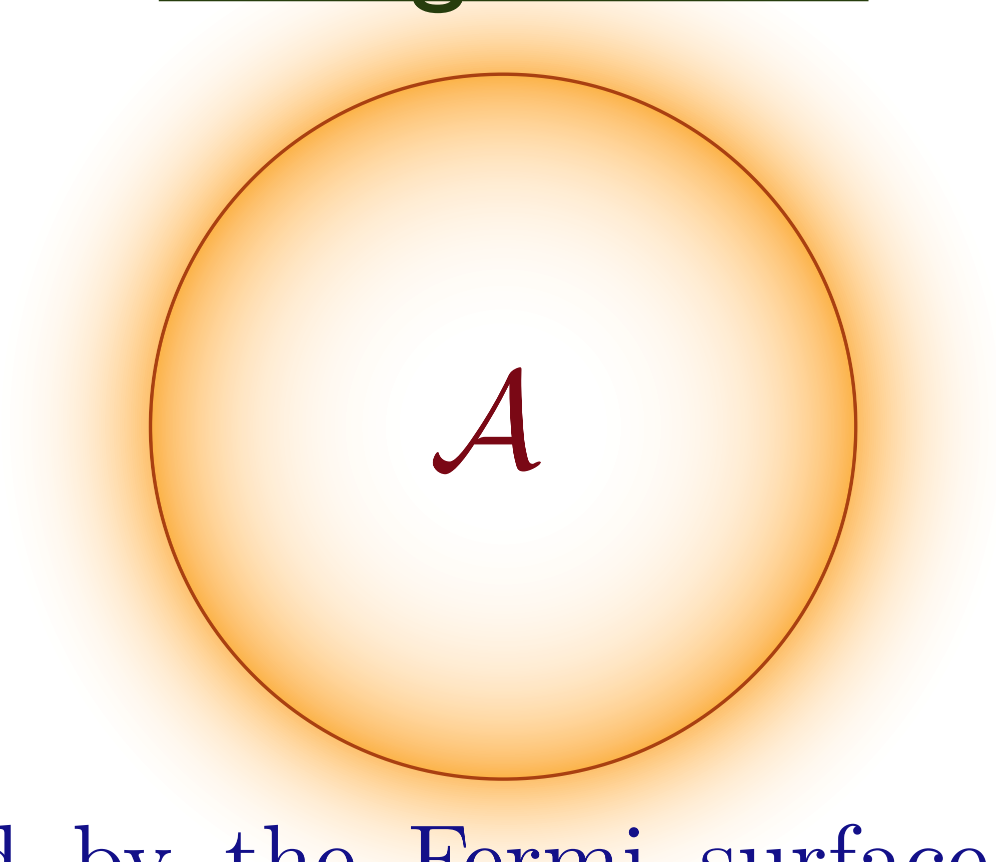
- ferromagnetic order
- breaking of point-group symmetry (Ising-nematic order)
- breaking of time-reversal symmetry
- circulating currents
- transverse component of an Abelian or non-Abelian gauge field.
- ...

Strange metals



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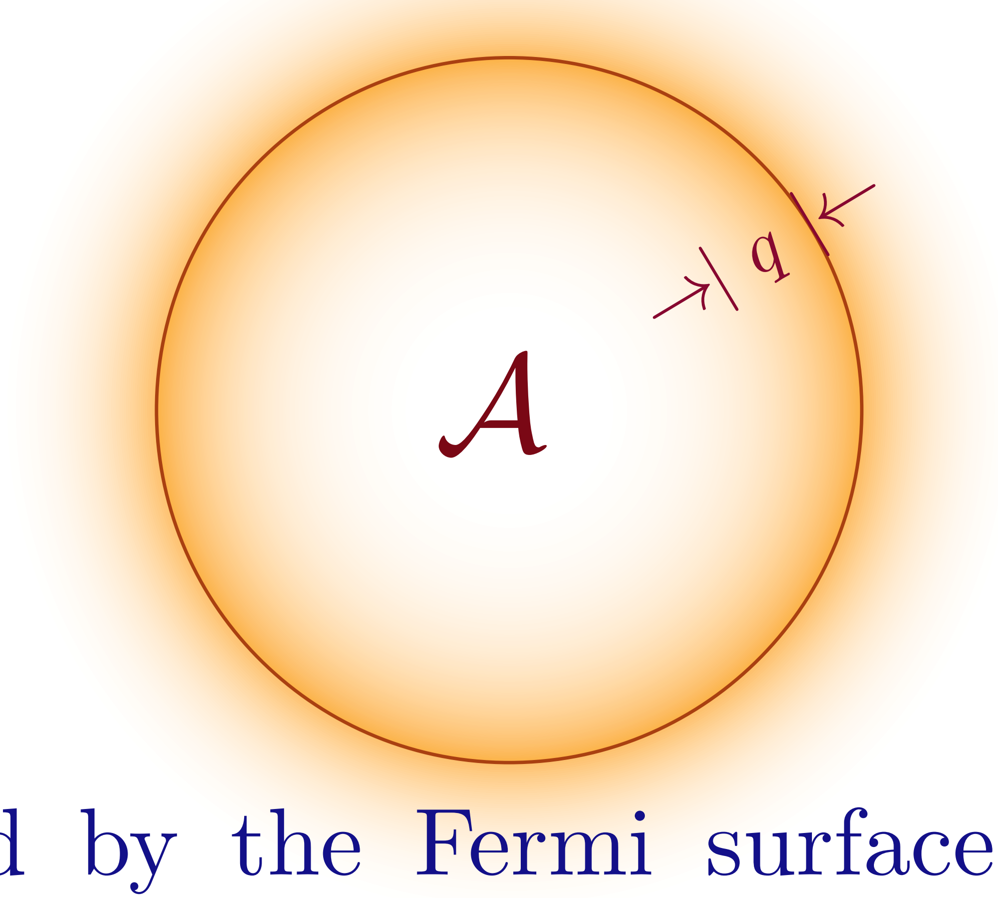
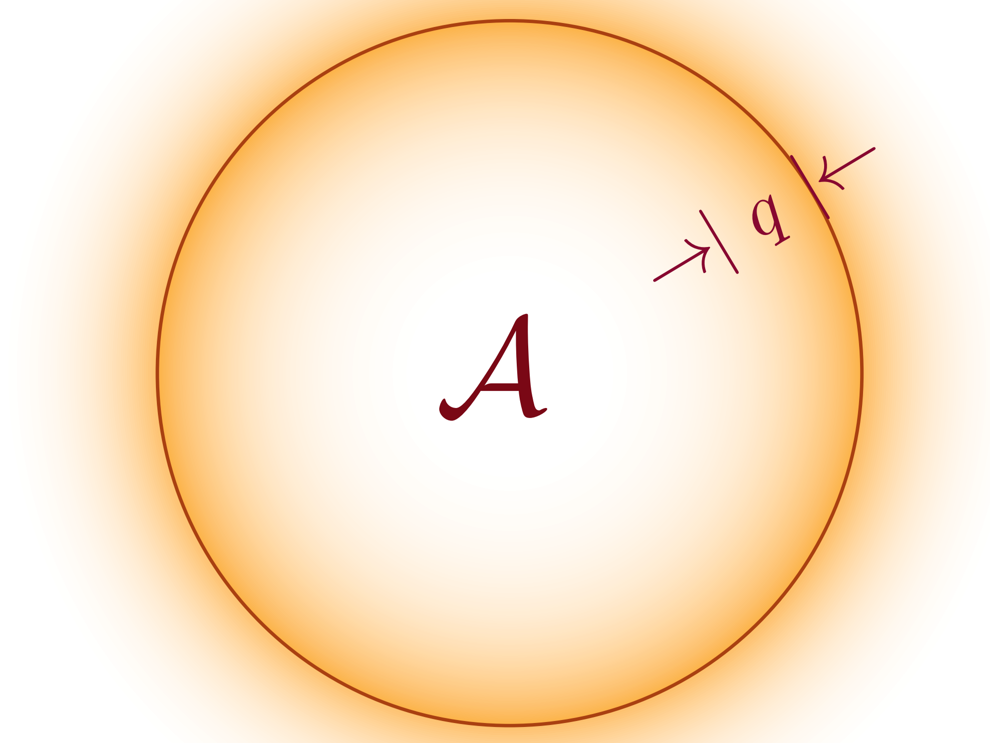


- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$; three-loop computation shows $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Strange metals



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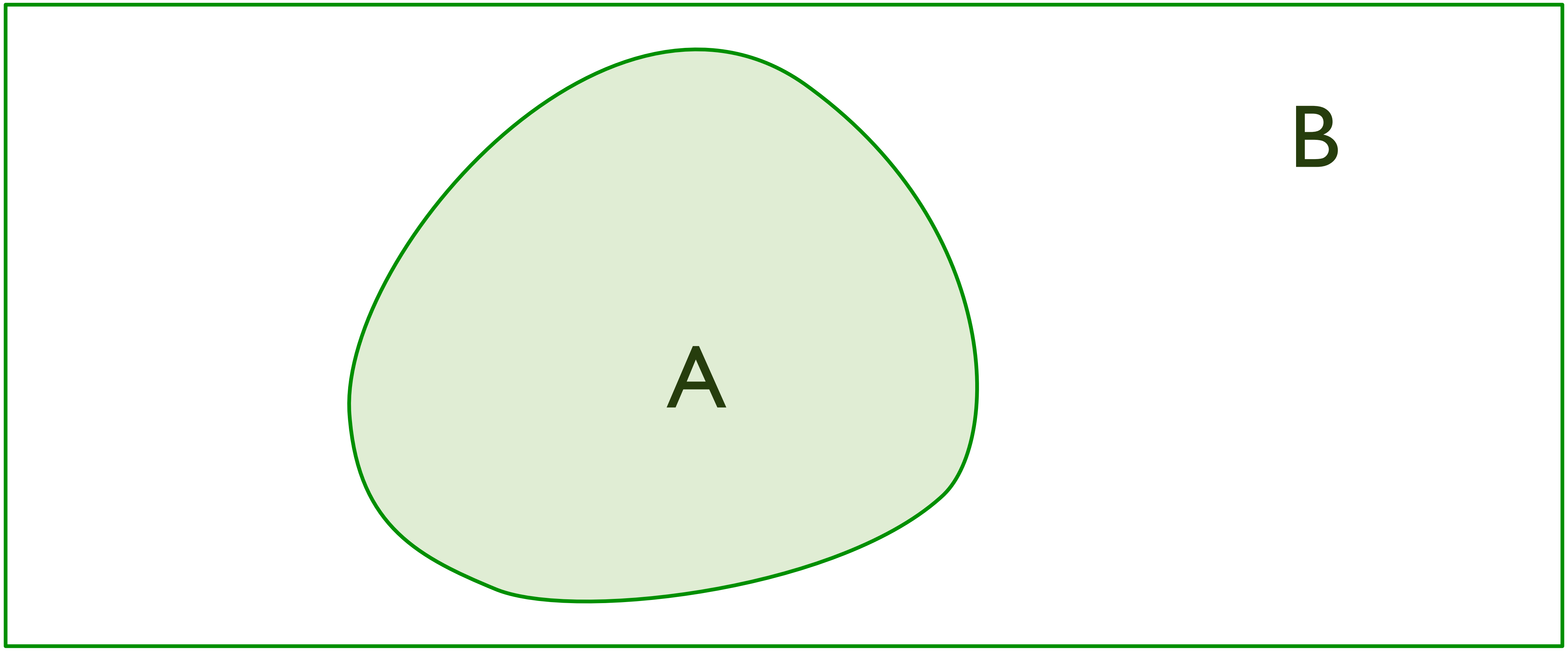

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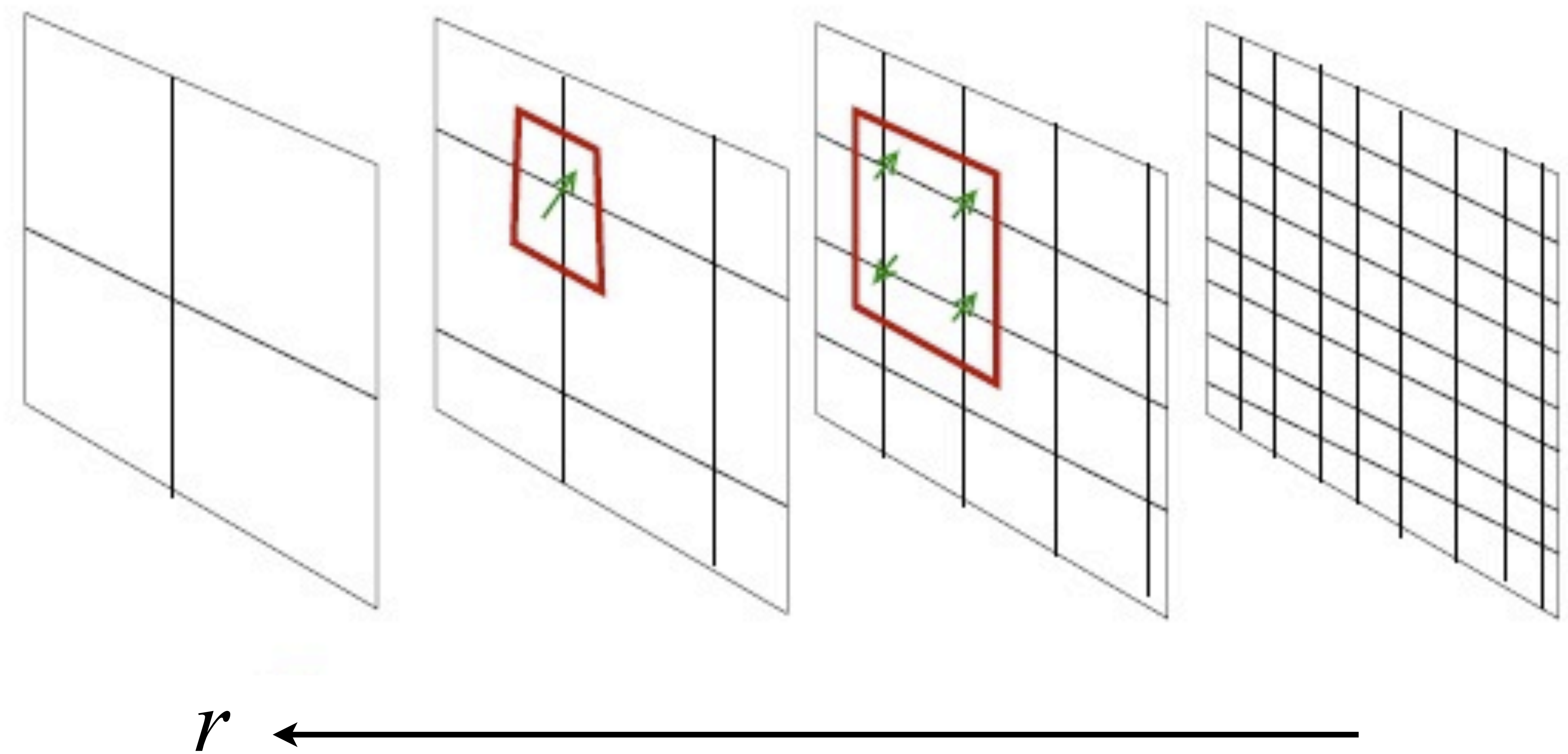
where P is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$
ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)
$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

- The entanglement entropy, S_E, of an entangling region with boundary surface 'area' P scales as

$$S_E \sim \begin{cases} P & , \quad \text{for } \theta < d-1 \\ P \ln P & , \quad \text{for } \theta = d-1 \\ P^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases}$$

All local quantum field theories obey the "area law" (upto log violations) and so $\theta \leq d-1$.

- The null energy condition implies $z \geq 1 + \dfrac{\theta}{d}$.

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d.
  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

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$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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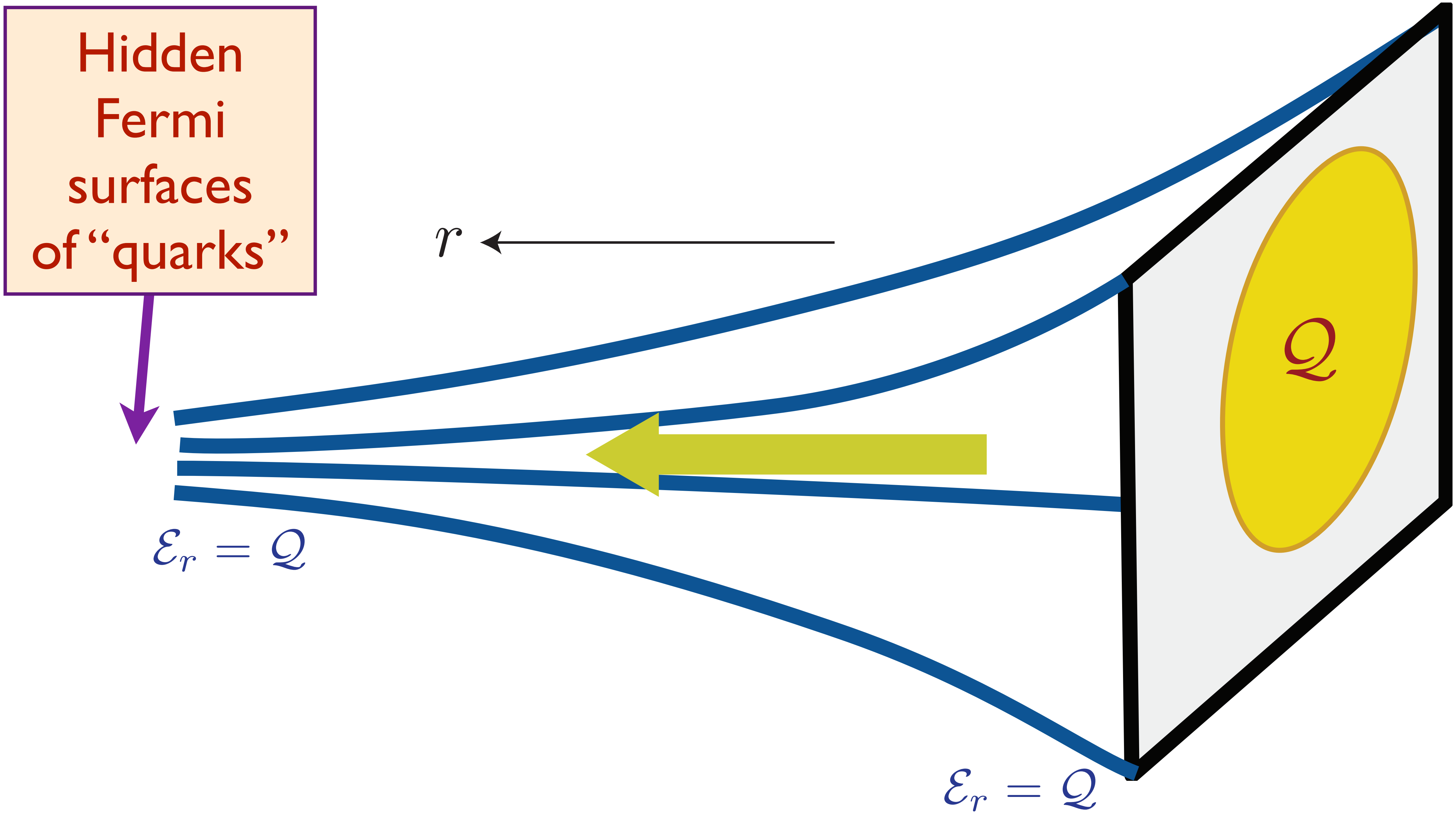
- This metric can be realized in a Maxwell-Einstein-dilaton theory, which may be viewed as a "bosonization" of the non-Fermi liquid state. The entanglement entropy of this theory has log-violation of the area law with

$$S_E = \Xi\, \mathcal{Q}^{(d-1)/d} P \ln P.$$

where P is surface area of the entangling region, and Ξ is a dimensionless constant which is independent of all UV details, of $\mathcal{Q}$, and of any property of the entangling region.
Note $\mathcal{Q}^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals



Gauss Law and the "attractor" mechanism
$\Leftrightarrow$ Luttinger theorem on the boundary

Conclusions

Gapped quantum matter

- Numerical and experimental observation of a spin liquid on the kagome lattice. Likely a Z_2 spin liquid.

Conclusions

Conformal quantum matter

- Numerical and experimental observation in coupled-dimer antiferromagnets, and at the superfluid-insulator transition of bosons in optical lattices.

Conclusions

Compressible quantum matter

- Field theory of a non-Fermi liquid obtained by coupling a Fermi surface to a gapless scalar field with low energy excitations near zero wavevector. Obtained promising holographic dual of this theory.

Conclusions

Compressible quantum matter

Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

After fixing $\theta = d-1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E.
- Leading-log S_E independent of shape of entangling region.
- The $d = 2$ bound $z \geq 3/2$, compared to $z = 3/2$ in three-loop field theory.
- Evidence for Luttinger theorem in prefactor of S_E.


Thank you !