The quantum mechanics of two dimensional superfluids

Physical Review B **71**, 144508 and 144509 (2005),
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Talk online: Google Sachdev

Outline

I. Bose-Einstein condensation and superfluidity

II. The superfluid-Mott insulator quantum phase transition

III. The cuprate superconductors

Superfluids proximate to finite doping Mott insulators with VBS order ?

IV. Vortices in the superfluid

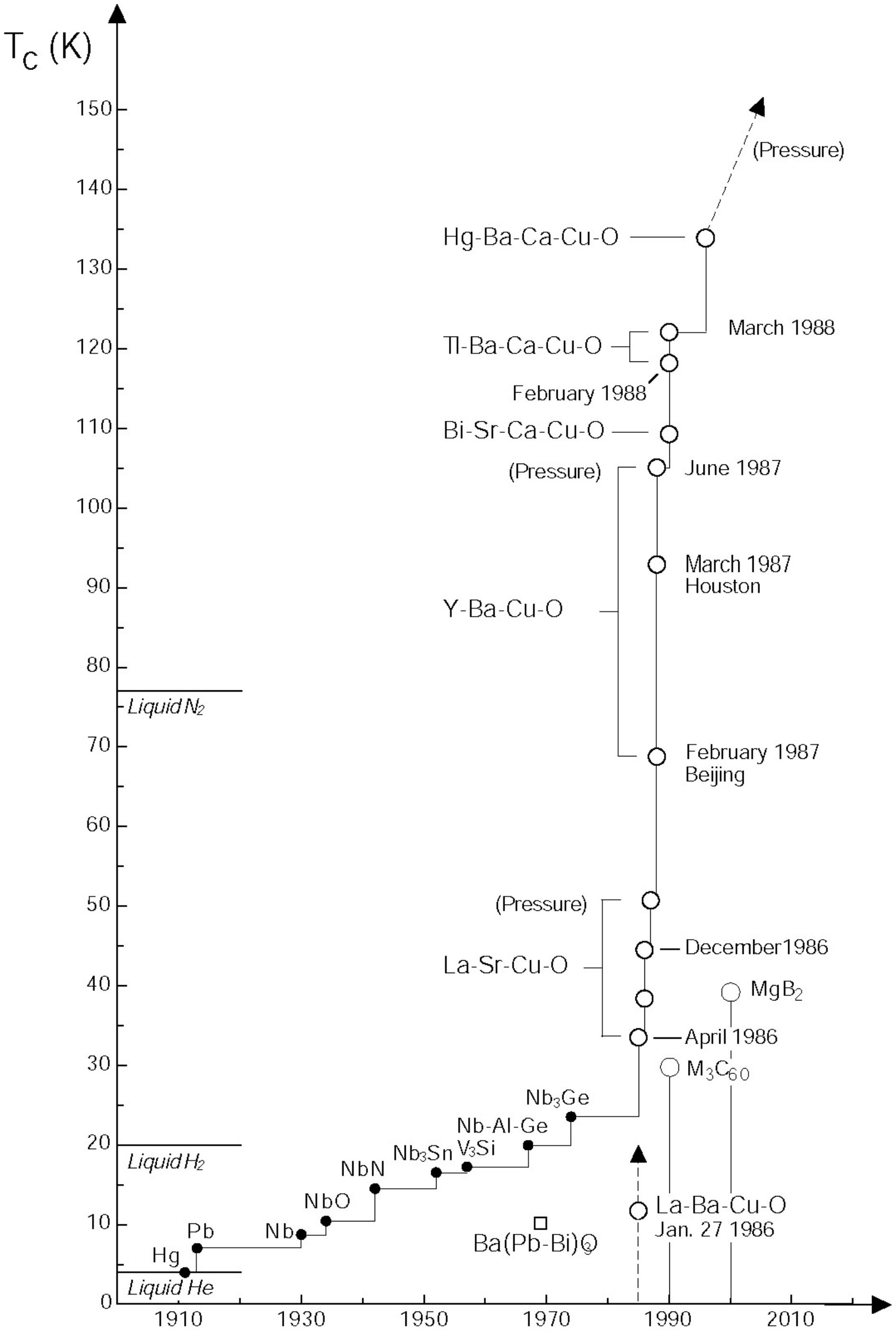
V. Vortices in superfluids near the superfluid-insulator quantum phase transition

The "quantum order" of the superconducting state: evidence for vortex flavors

I. Bose-Einstein condensation and superfluidity

Superfluidity/superconductivity occur in:

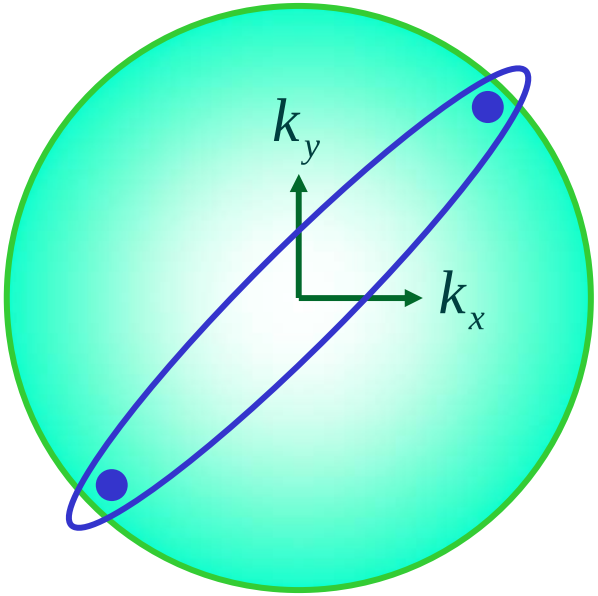
- liquid ^{4}He
- metals Hg, Al, Pb, Nb, Nb_3Sn…..
- liquid ^{3}He
- neutron stars
- cuprates $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{6+y}$….
- M_3C_{60}
- ultracold trapped atoms
- MgB_2



The Bose-Einstein condensate:

A macroscopic number of bosons occupy the lowest energy quantum state

Such a condensate also forms in systems of fermions, where the bosons are Cooper pairs of fermions:

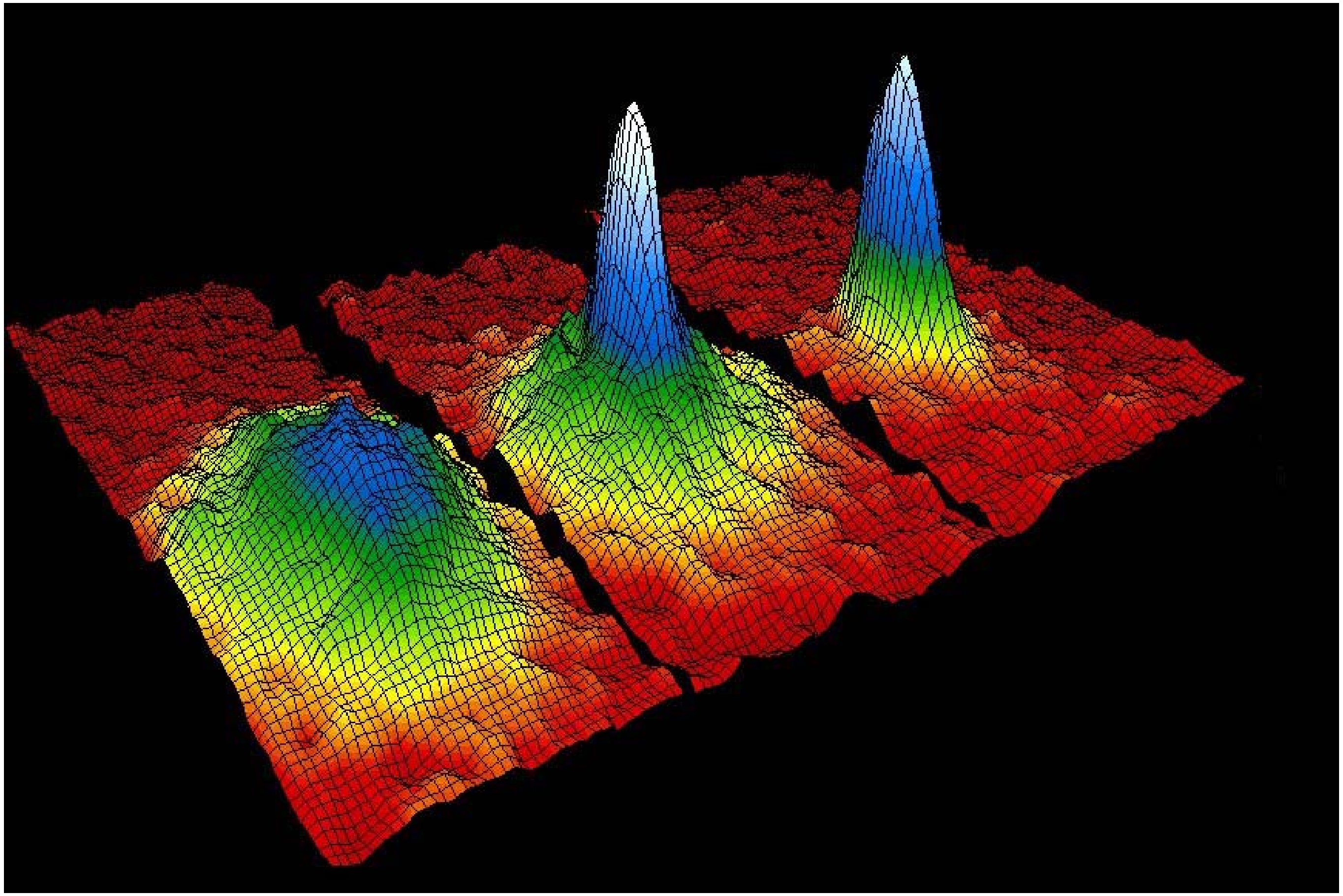


Pair wavefunction in cuprates:

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

Velocity distribution function of ultracold ^{87}Rb atoms



M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman
and E. A. Cornell, *Science* **269**, 198 (1995)

Superflow:

The wavefunction of the condensate

$$\Psi \rightarrow \Psi e^{i\theta(\boldsymbol{r})}$$

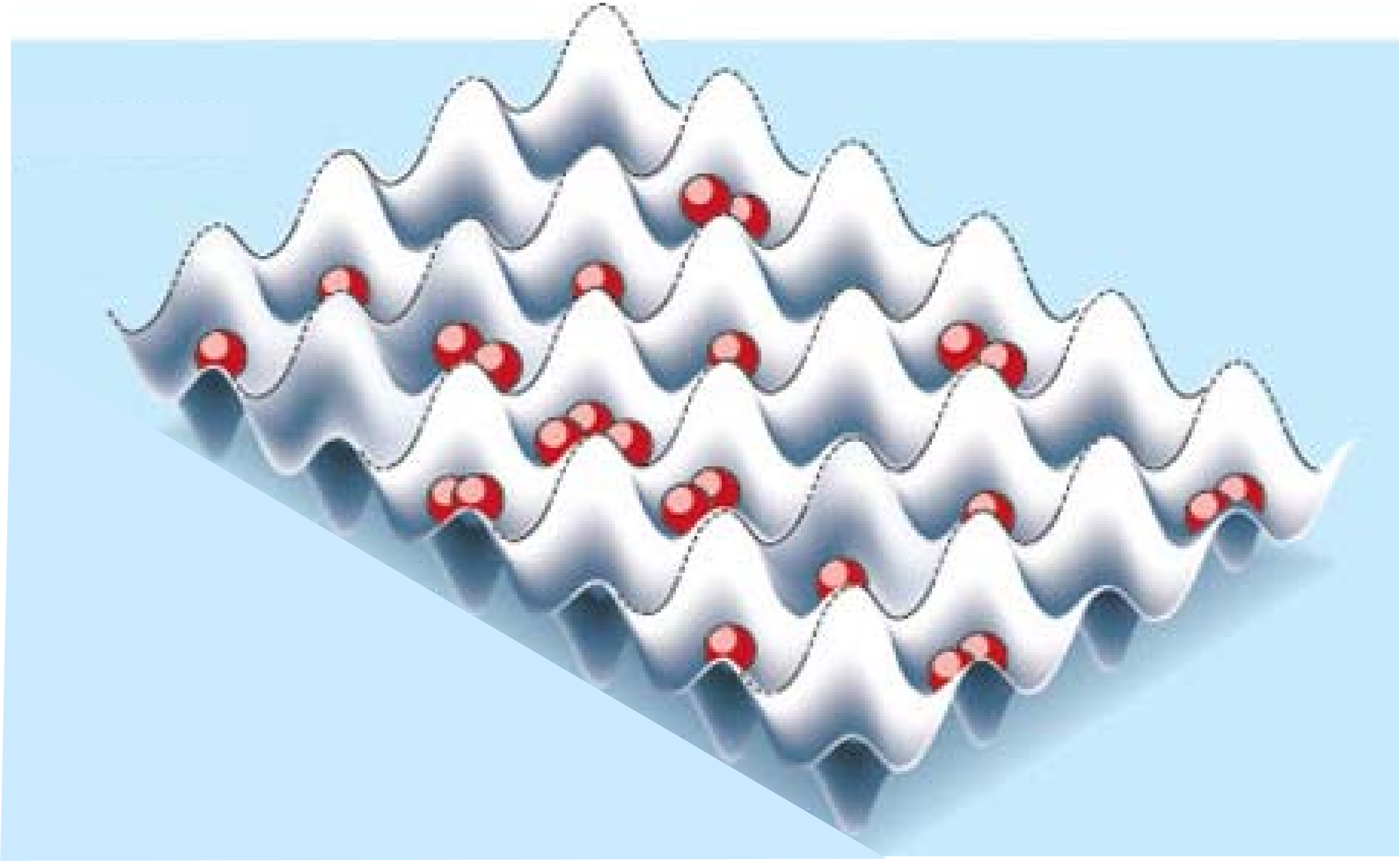
Superfluid velocity

$$\boldsymbol{v}_s = \frac{\hbar}{m} \nabla \theta$$

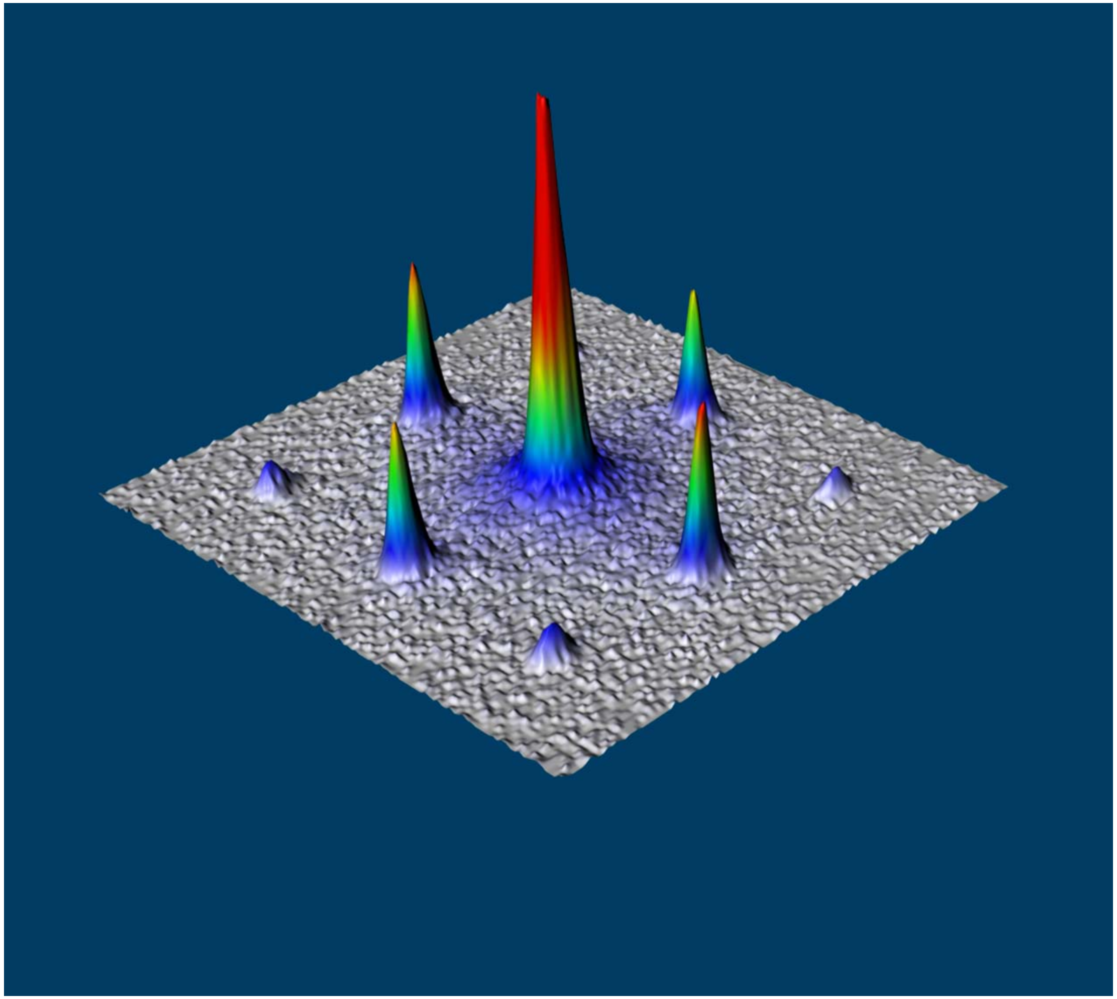
(for non-Galilean invariant superfluids,
the co-efficient of $\nabla \theta$ is modified)

II. The superfluid-Mott insulator quantum phase transition

Apply a periodic potential (standing laser beams) to trapped ultracold bosons (^{87}Rb)



Momentum distribution function of bosons



Bragg reflections of condensate at reciprocal lattice vectors

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

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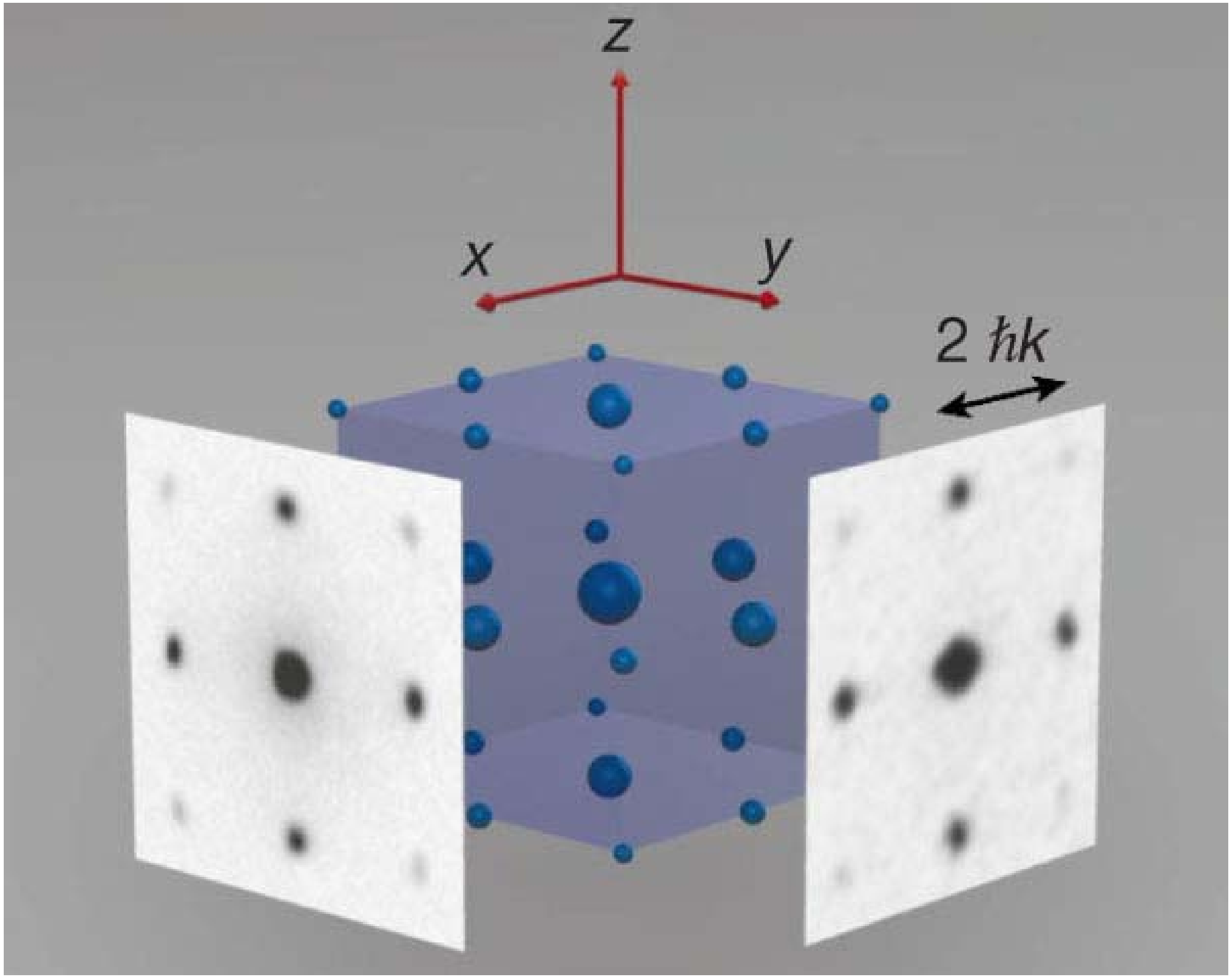

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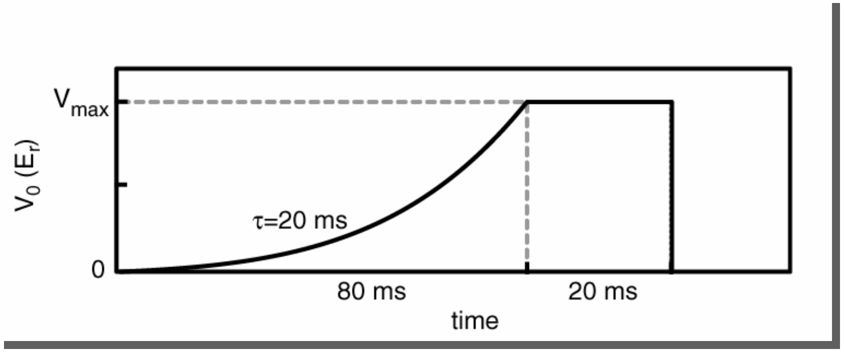
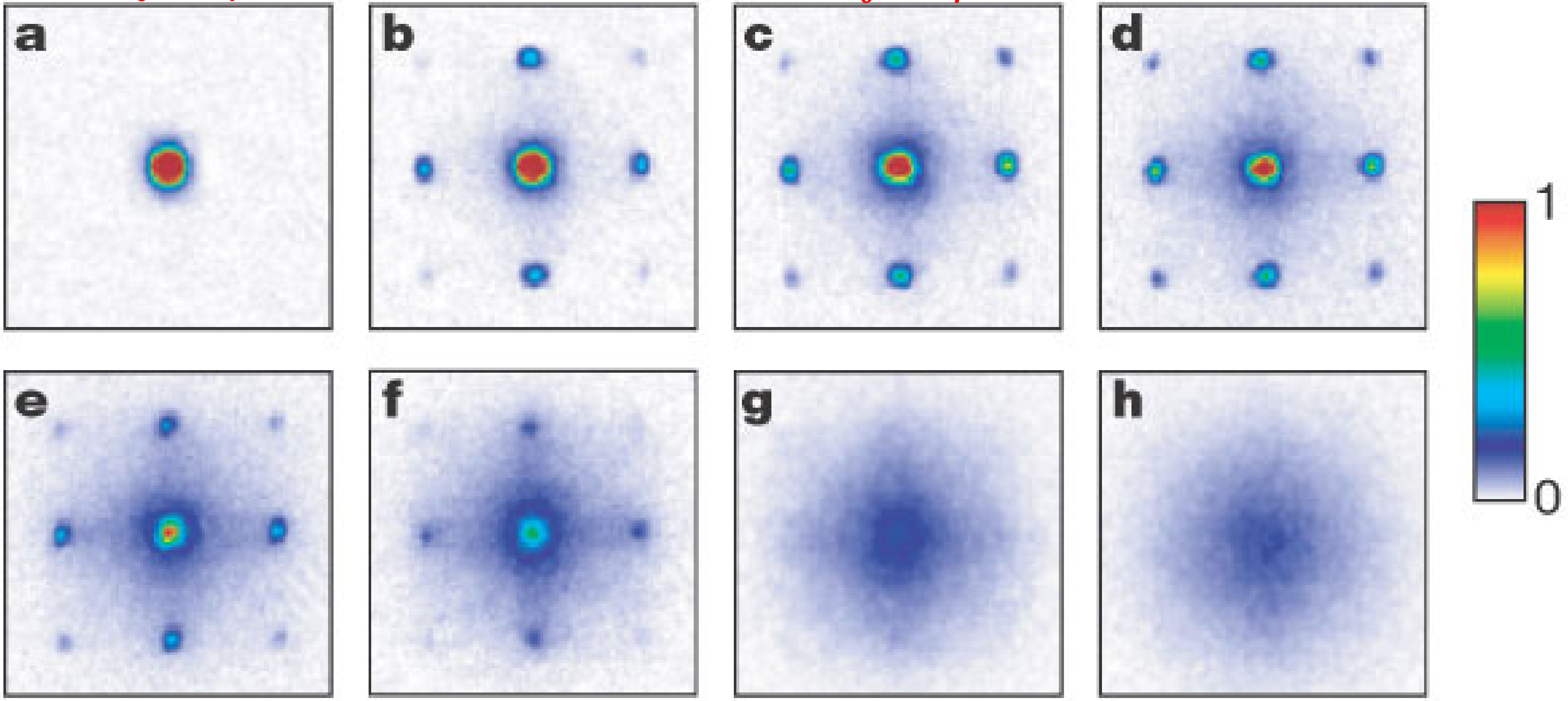
Superfluid-insulator quantum phase transition at T=0



$V_0=0E_r$ $V_0=3E_r$ $V_0=7E_r$ $V_0=10E_r$



$V_0=13E_r$ $V_0=14E_r$ $V_0=16E_r$ $V_0=20E_r$

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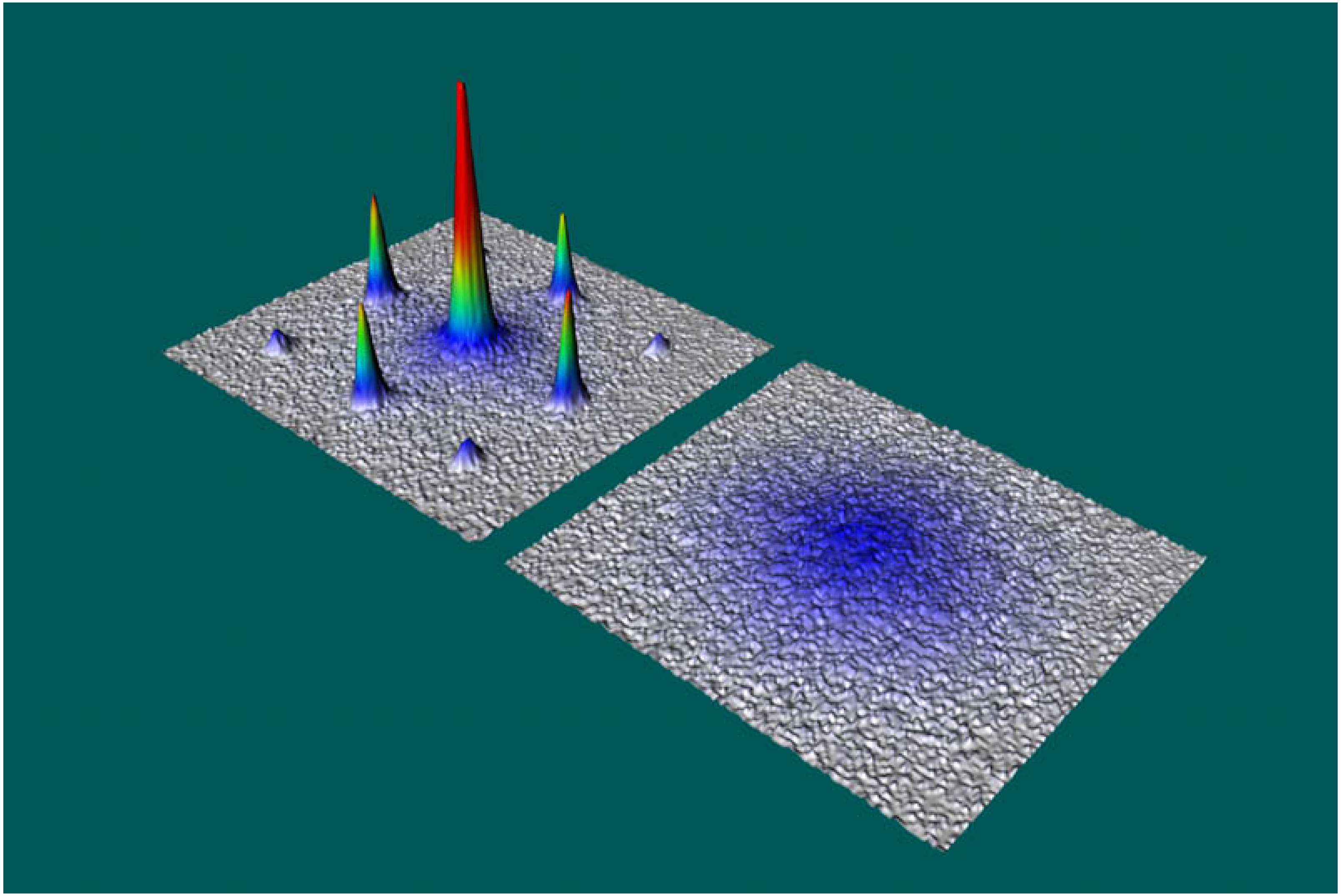

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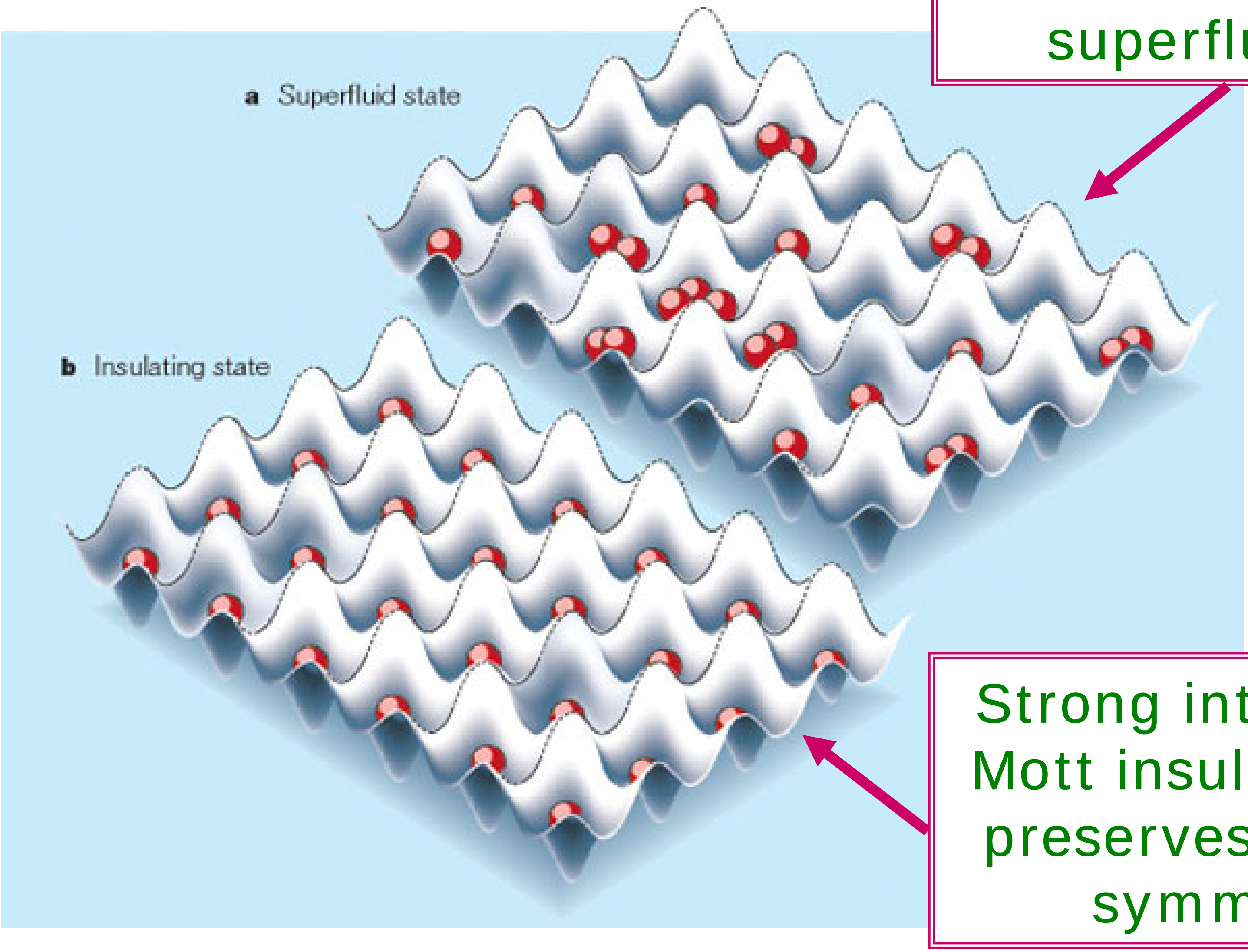
Bosons at filling fraction $f = 1$



Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

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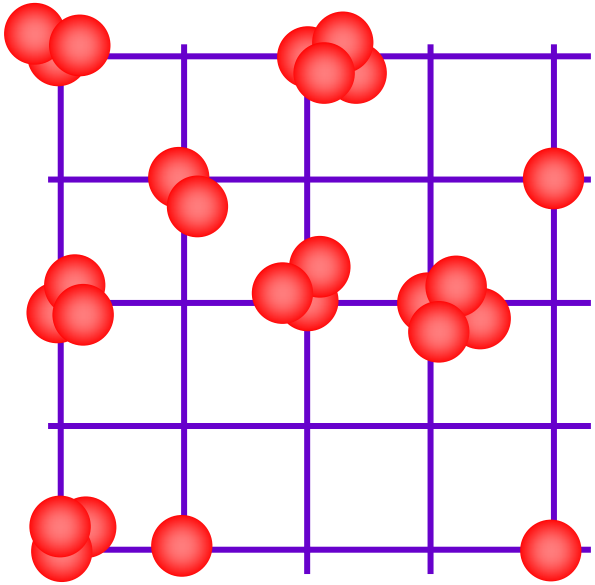

$$\langle \Psi \rangle \neq 0$$

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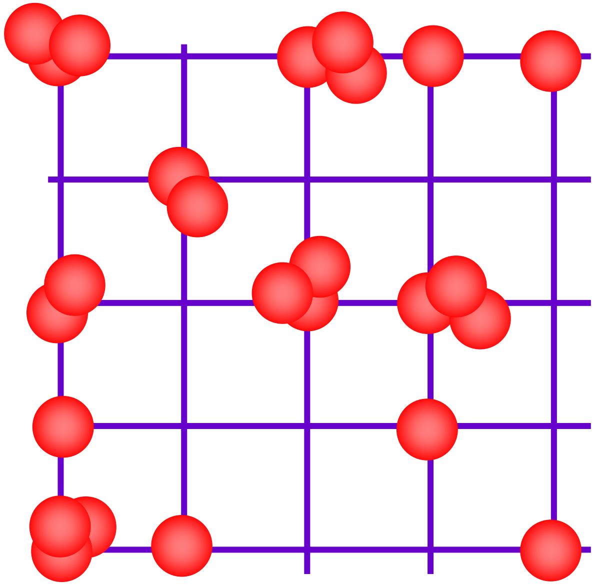

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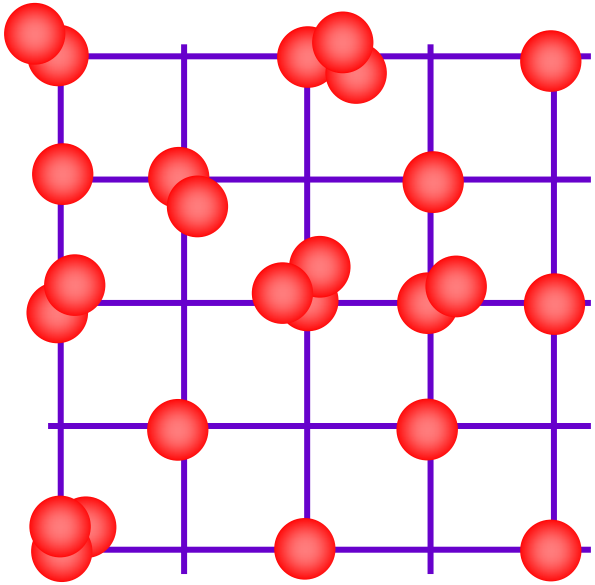

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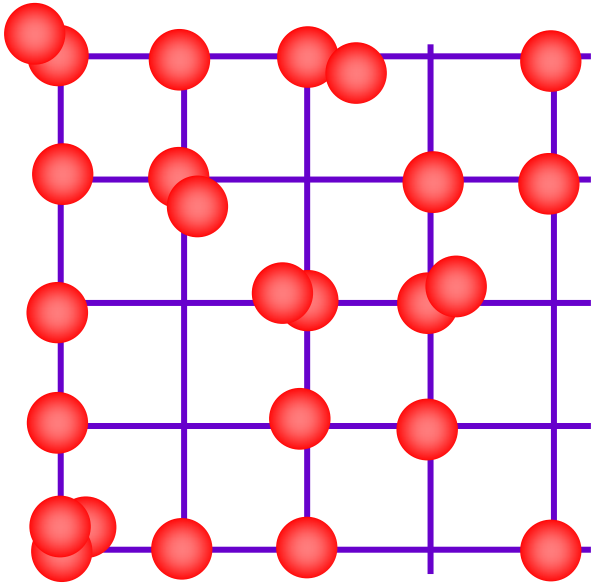

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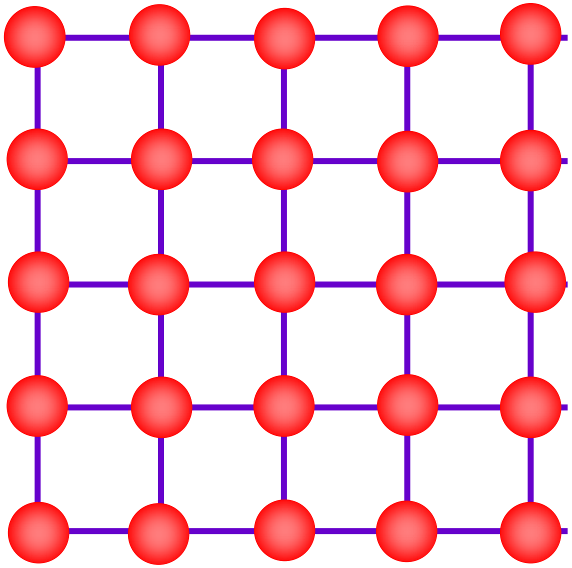

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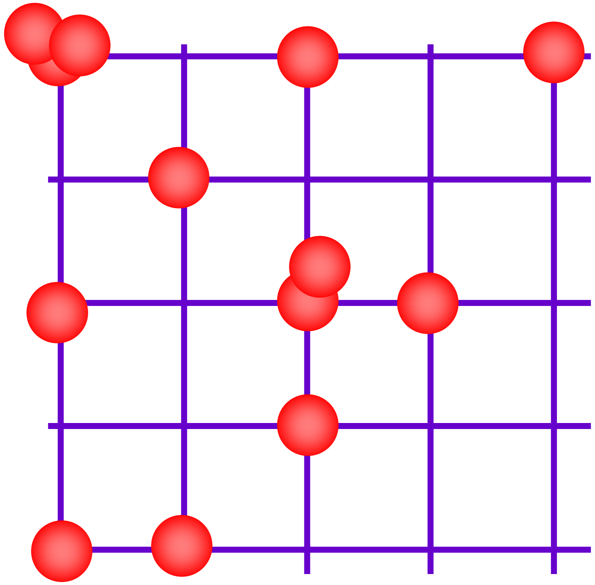
Strong interactions: insulator

Bosons at filling fraction f = 1/2



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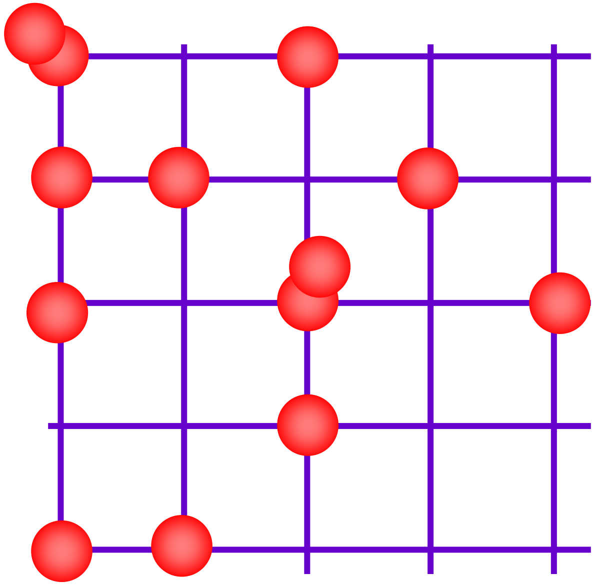

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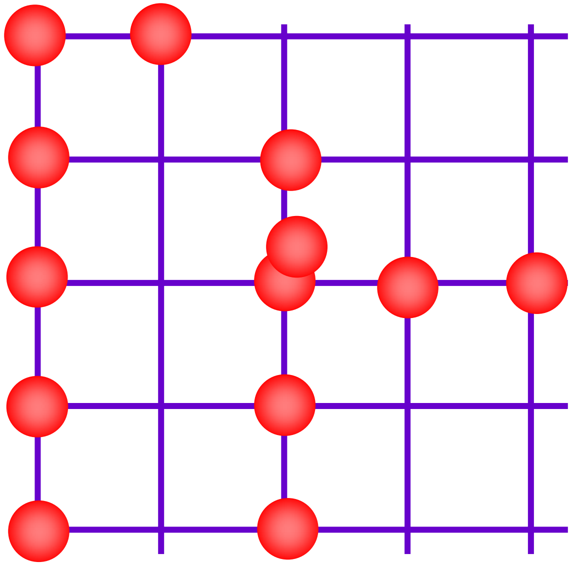

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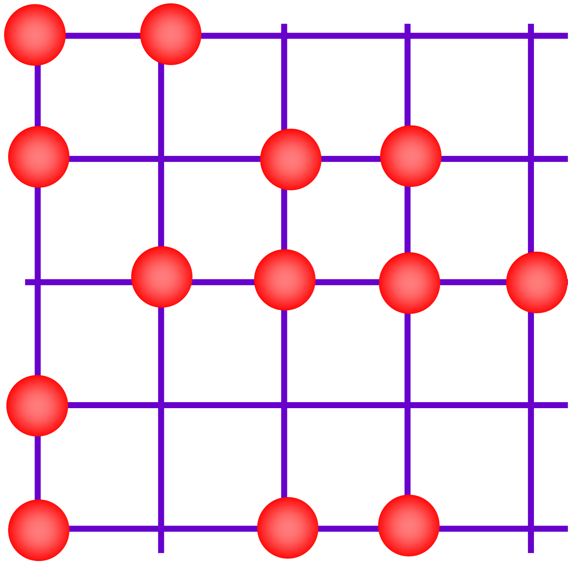

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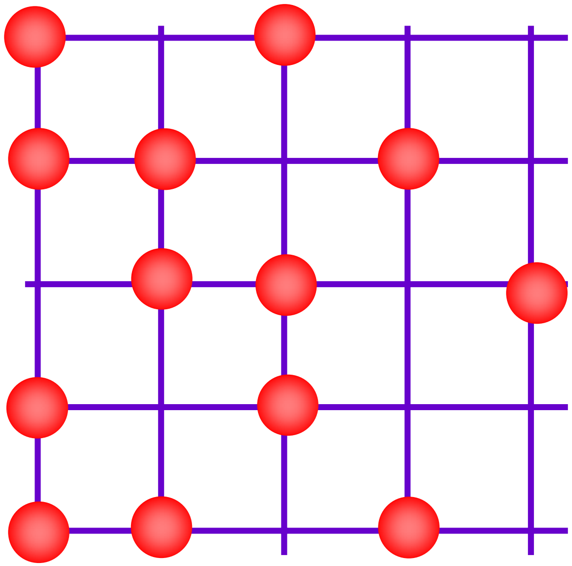

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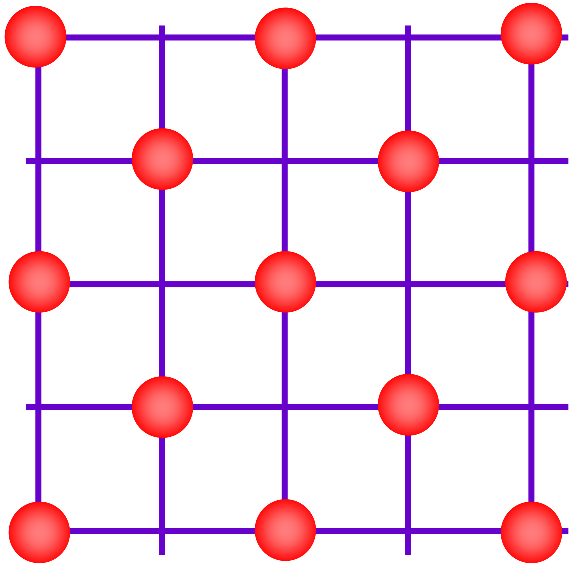

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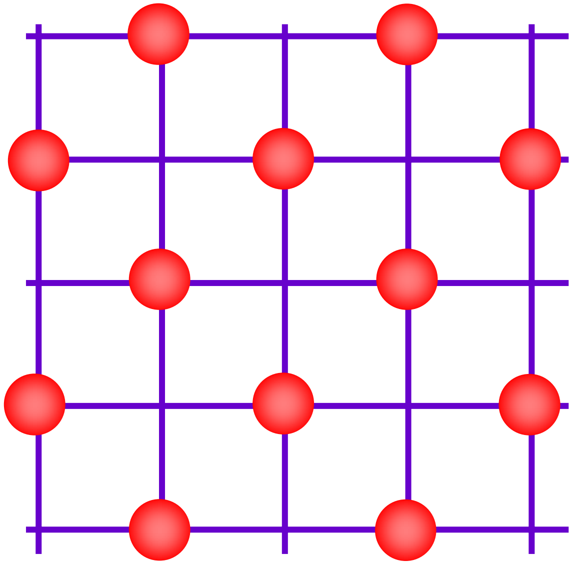

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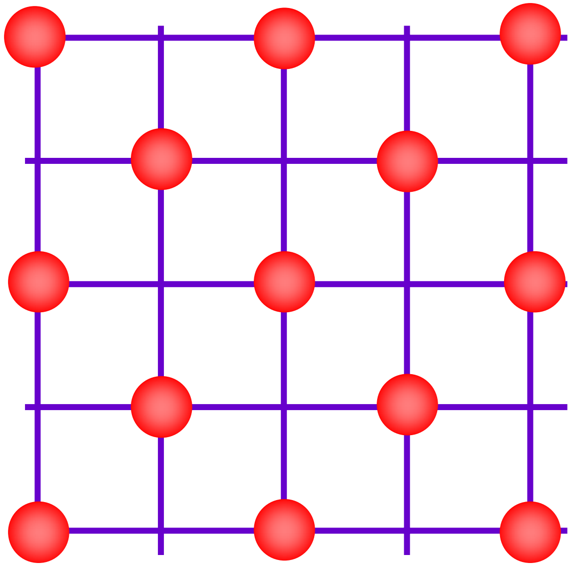
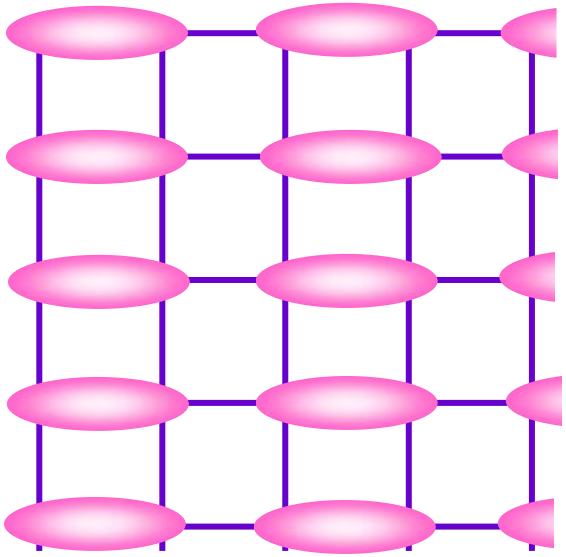
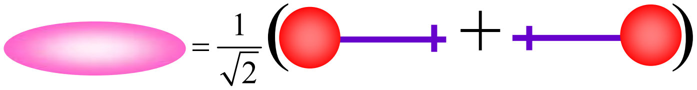
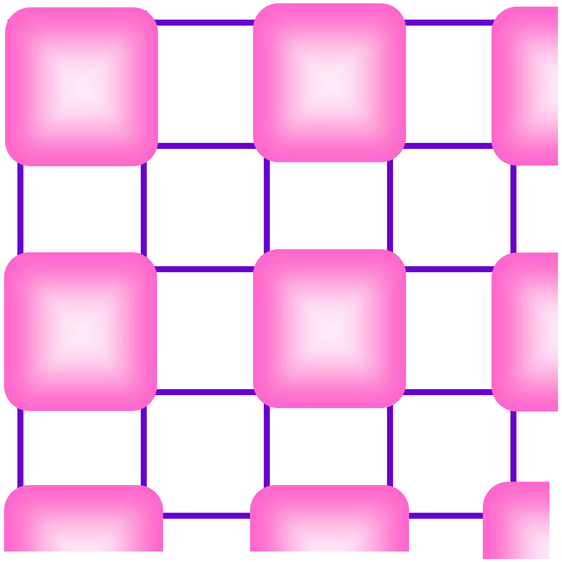


Charge density wave (CDW) order





Valence bond solid (VBS) order



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Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

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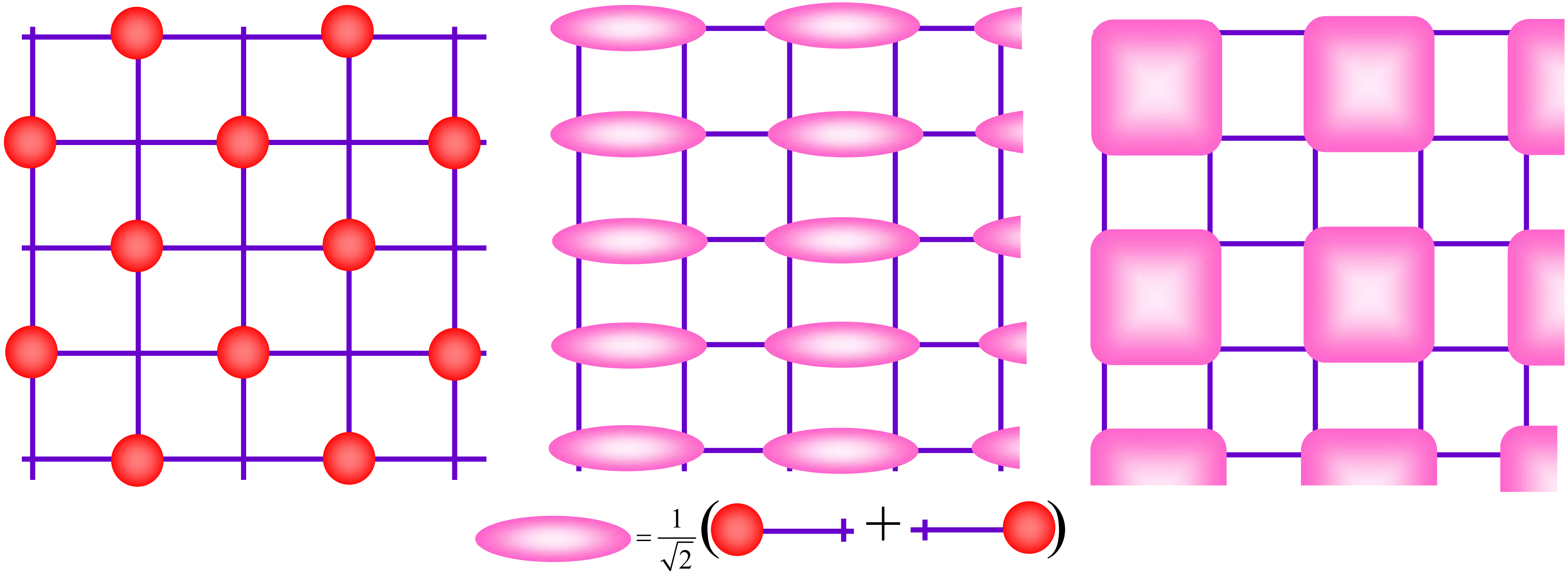

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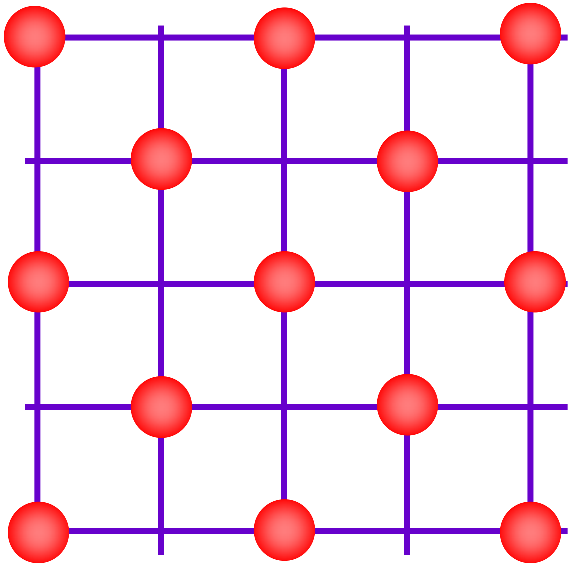

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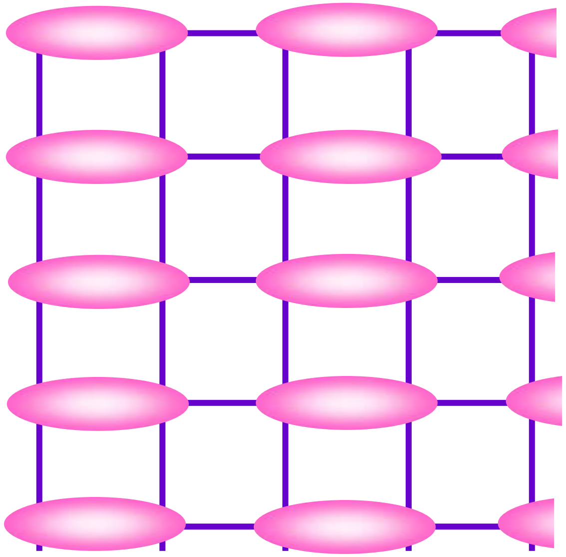

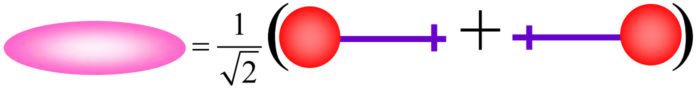

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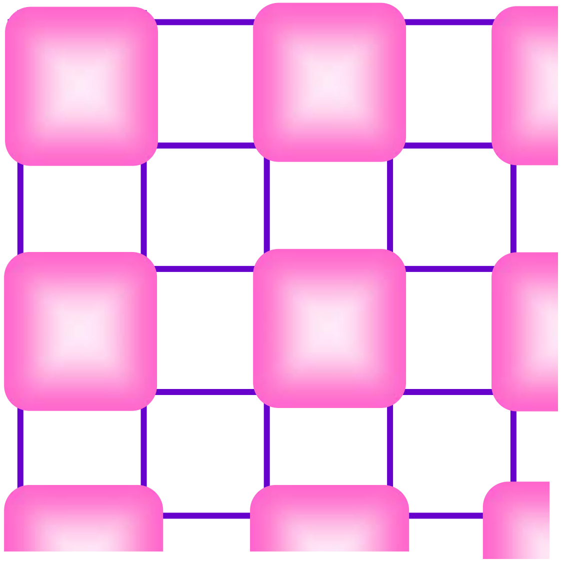

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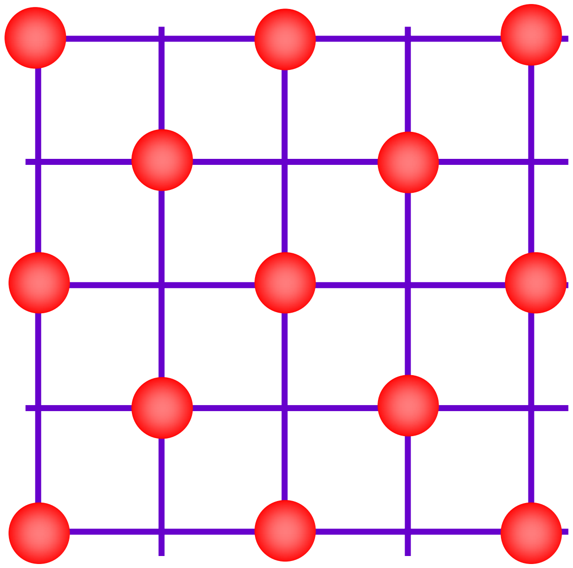

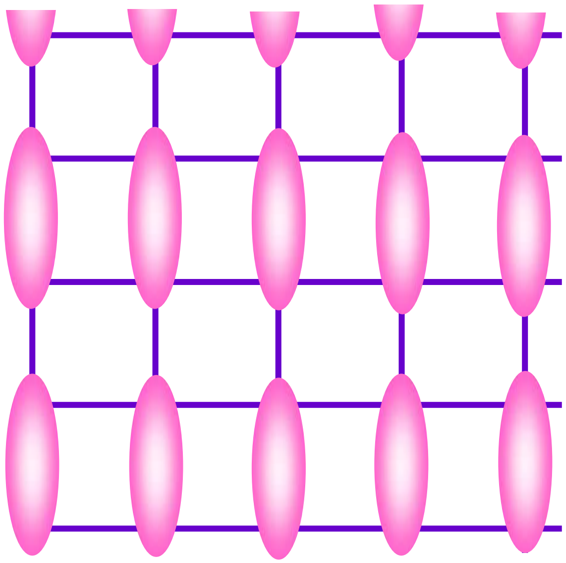

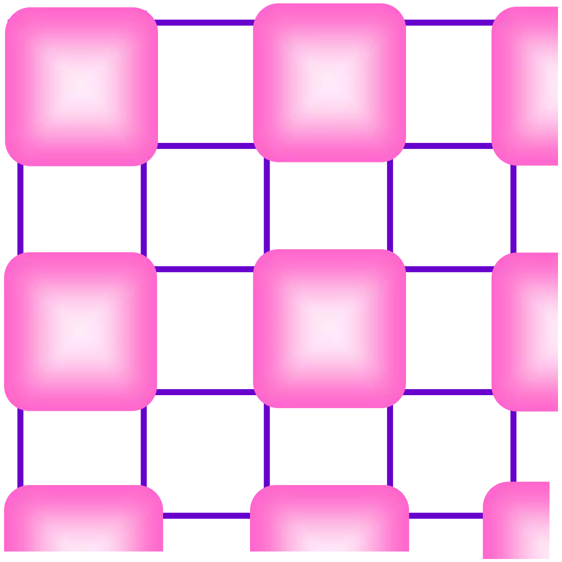

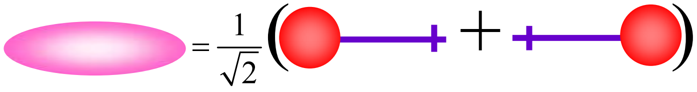

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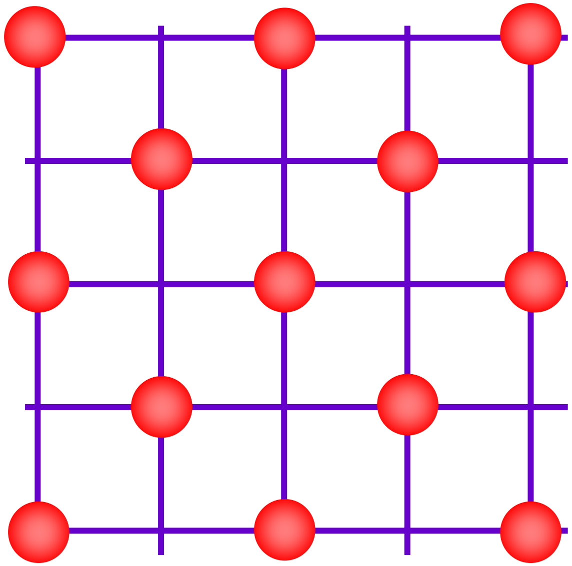

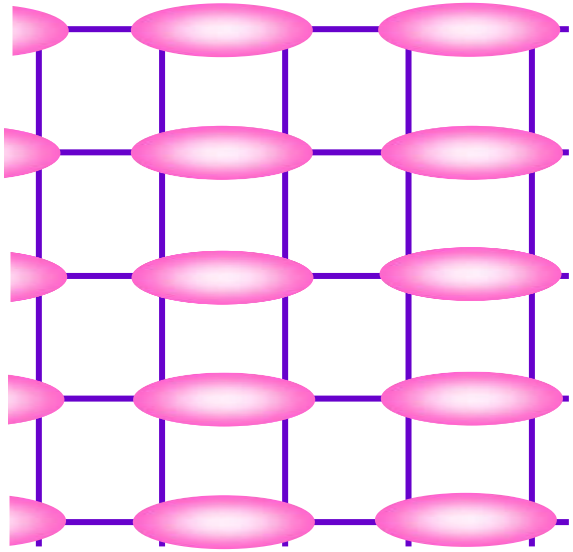

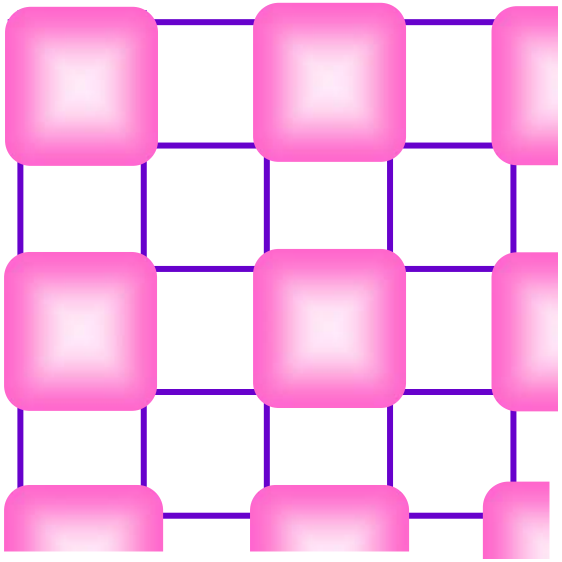

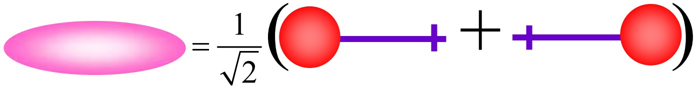

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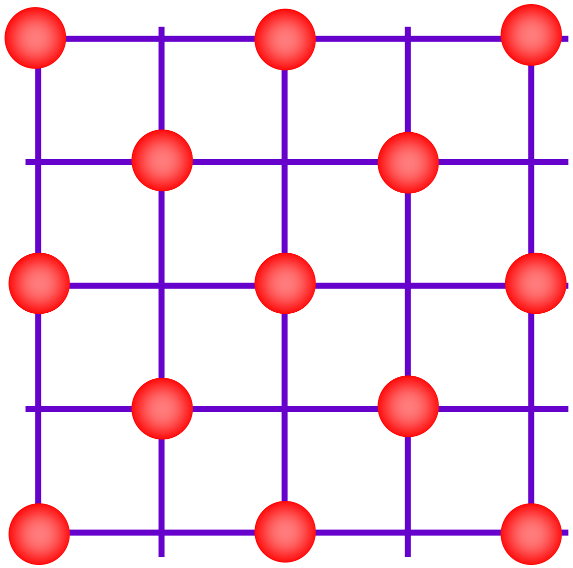

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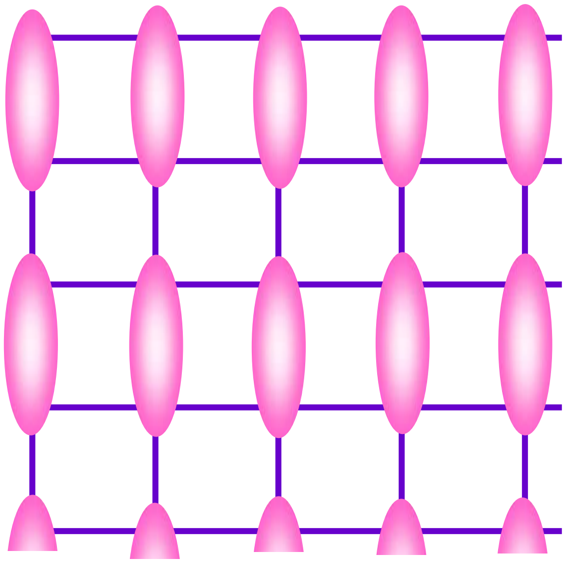

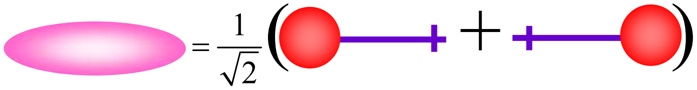

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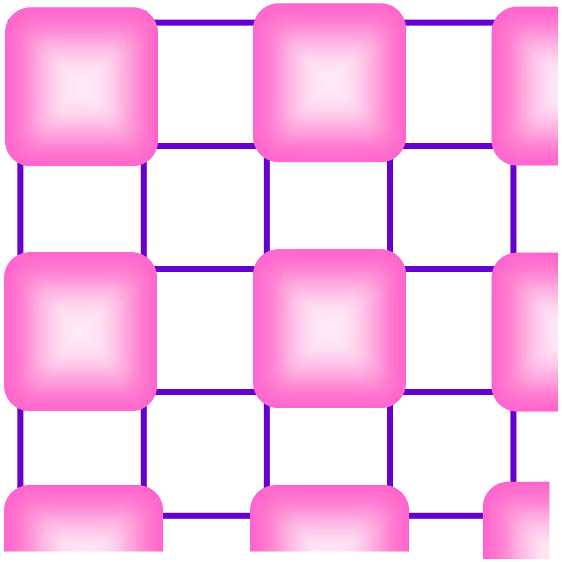

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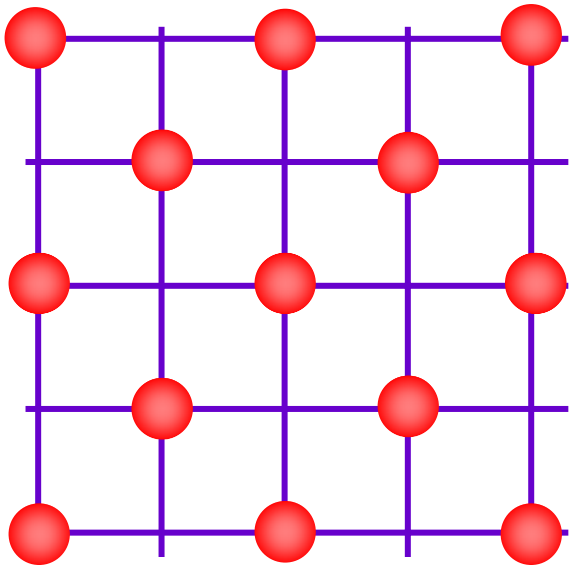

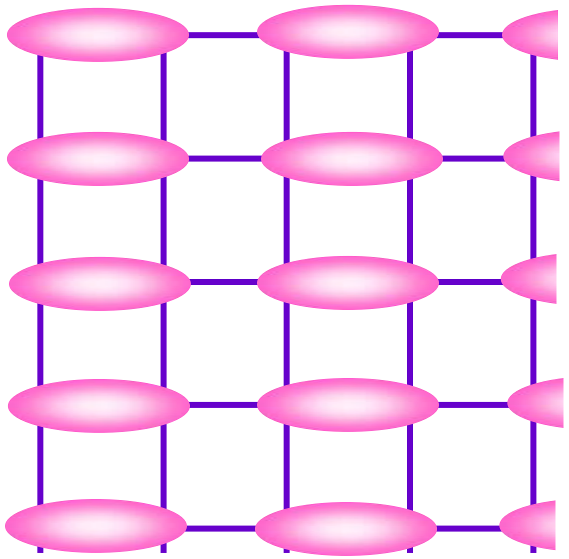

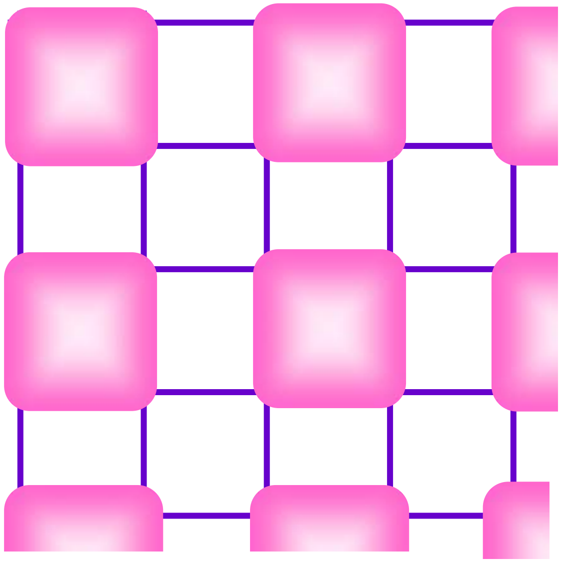

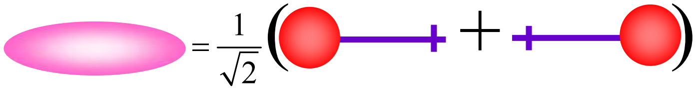

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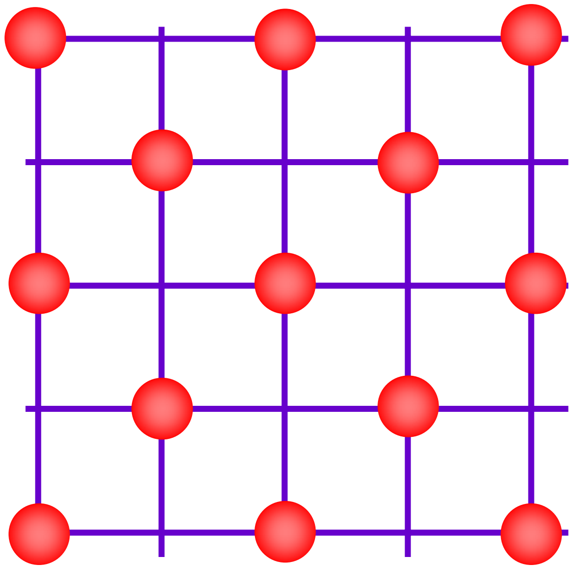
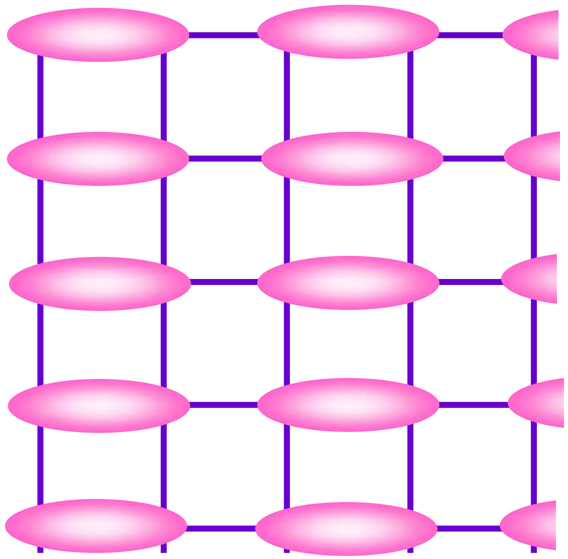
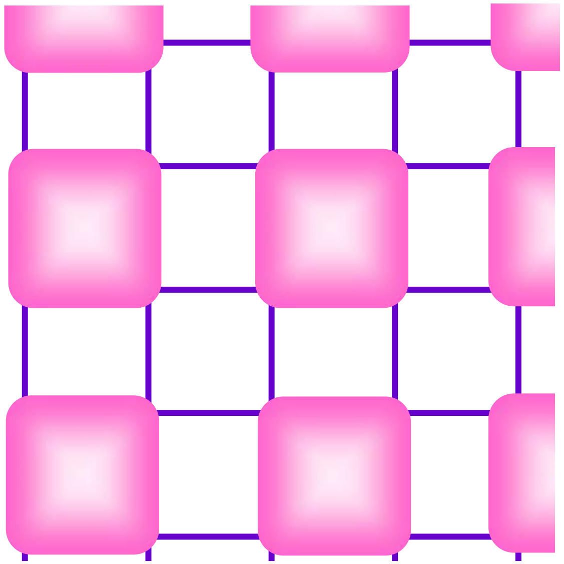
Insulating phases of bosons at filling fraction $f = 1/2$



Charge density wave (CDW) order



Valence bond solid (VBS) order



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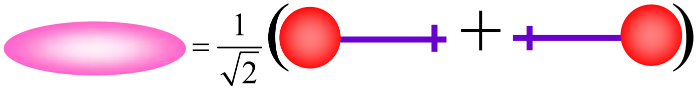

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

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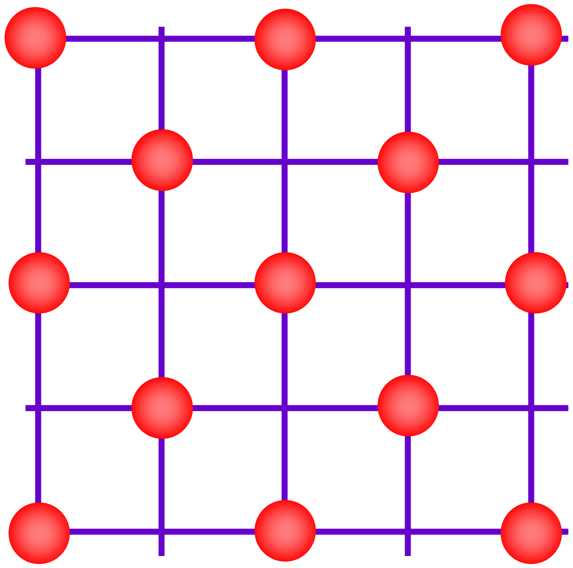

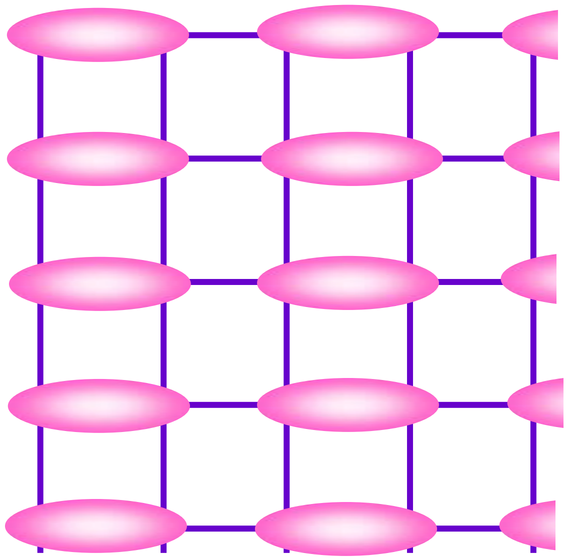

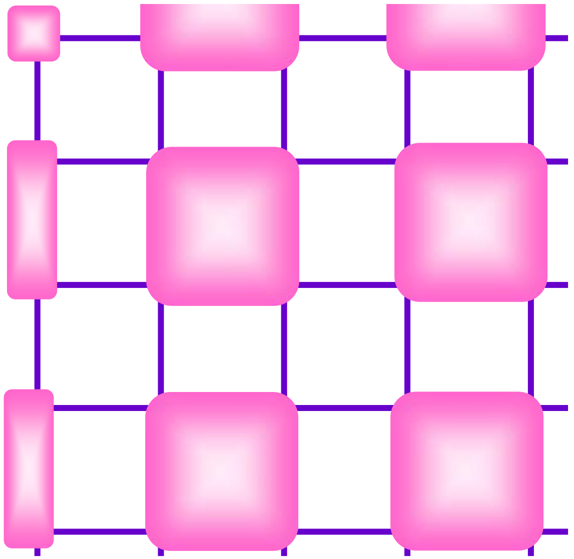

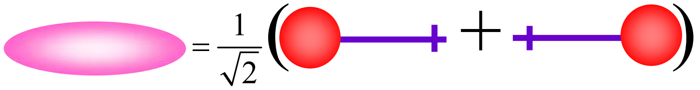

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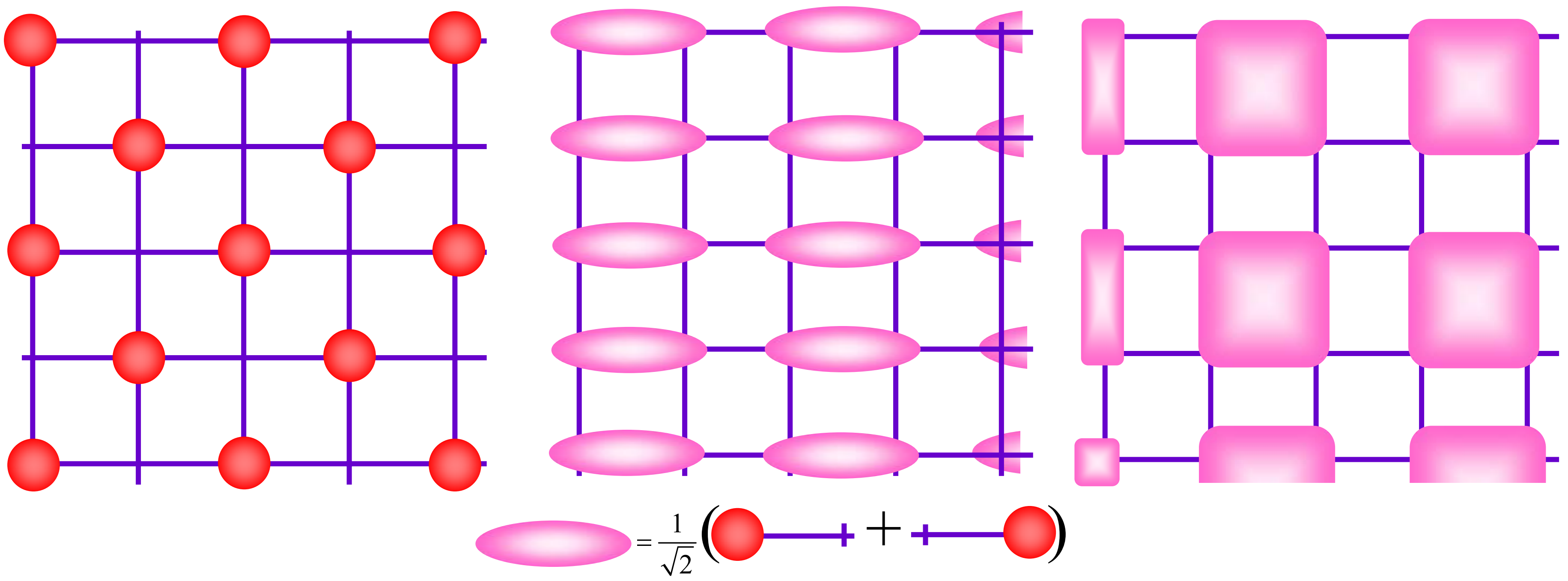

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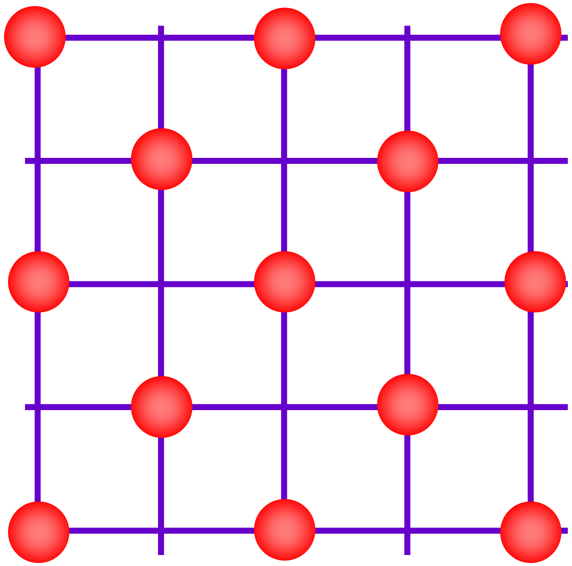

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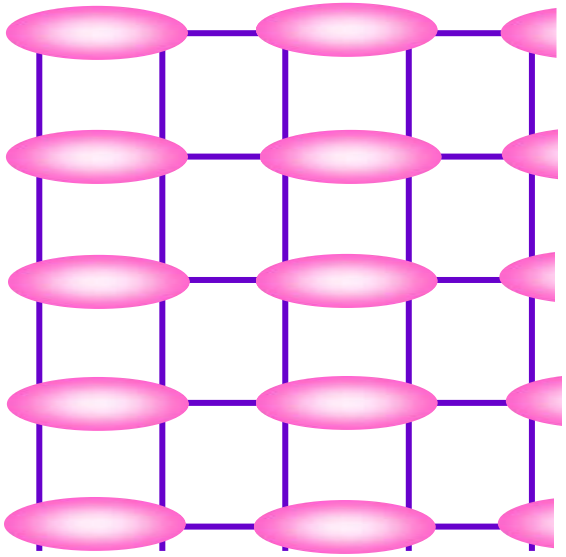

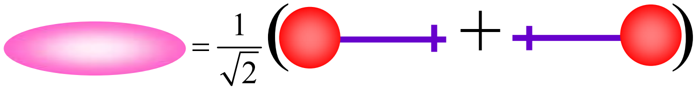

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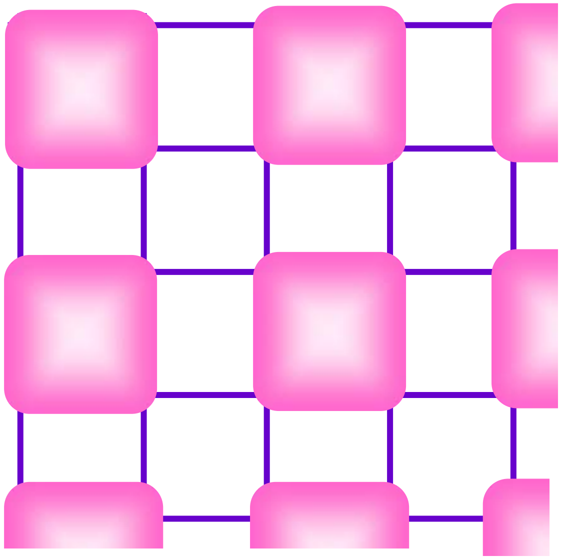

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Superfluid-insulator transition of bosons at generic filling fraction f

The transition is characterized by multiple distinct order parameters (boson condensate, VBS/CDW order)

Traditional (Landau-Ginzburg-Wilson) view:

Such a transition is first order, and there are no precursor fluctuations of the order of the insulator in the superfluid.

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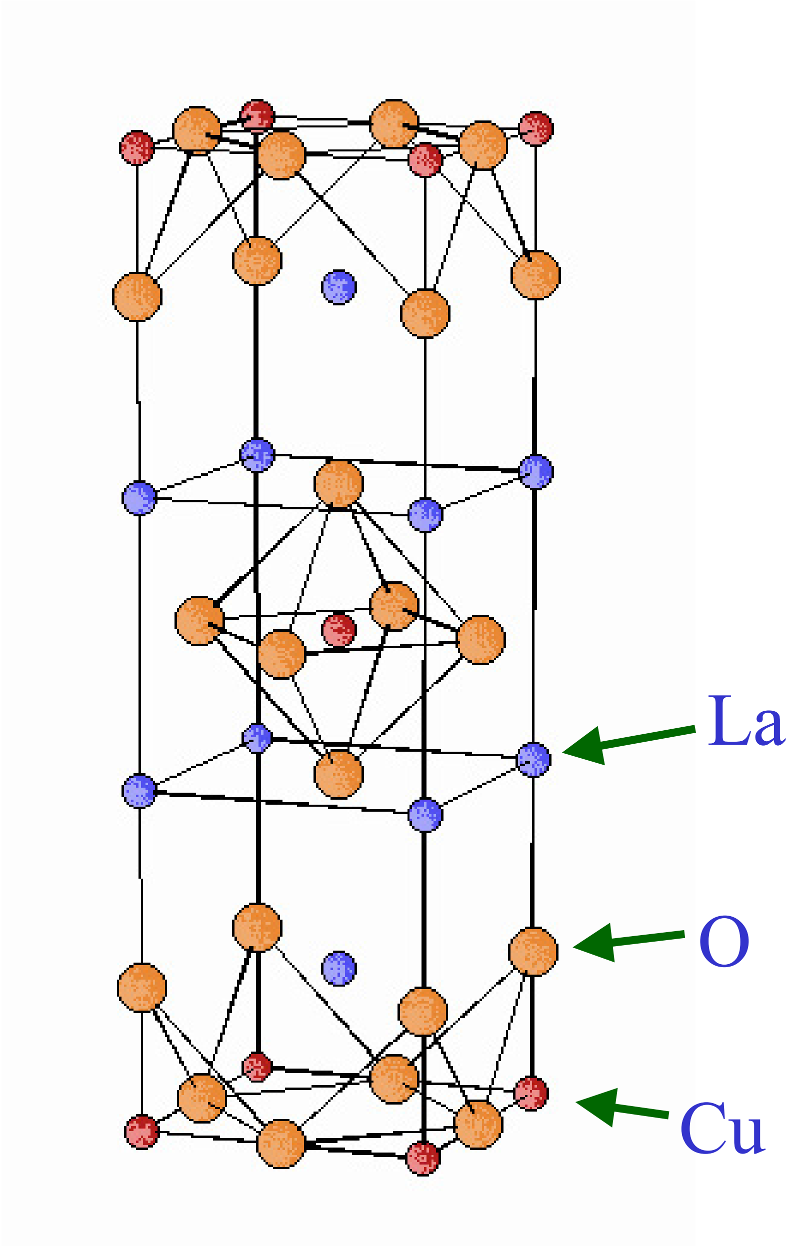
Recent theories:

Quantum interference effects can render such transitions second order, and the superfluid does contain VBS/CDW fluctuations.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

III. The cuprate superconductors

Superfluids proximate to finite doping Mott insulators with VBS order ?



La

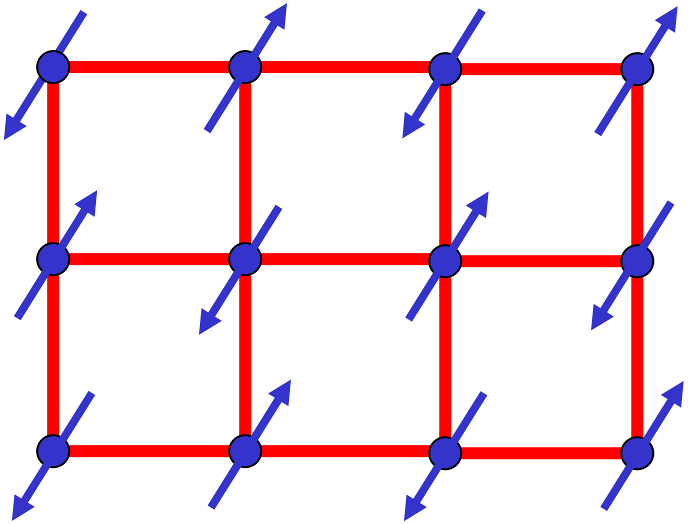
O

Cu

La_2CuO_4

La_2CuO_4

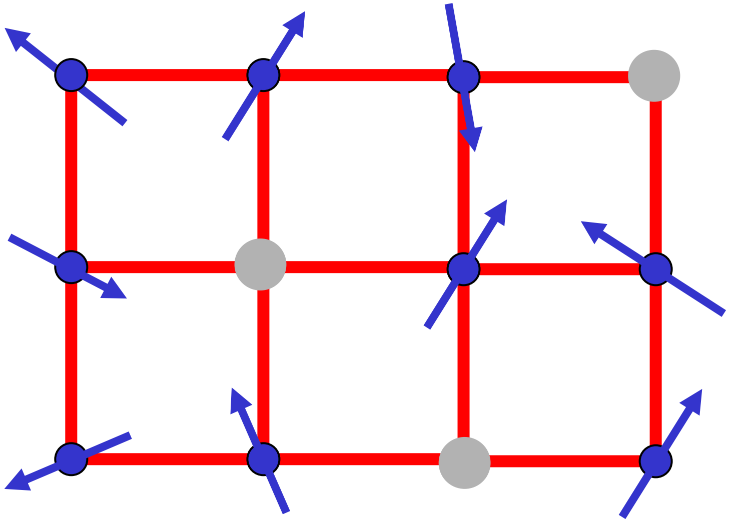
Mott insulator: square lattice antiferromagnet



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$La_{2-\delta}Sr_{\delta}CuO_4$

Superfluid: condensate of paired holes



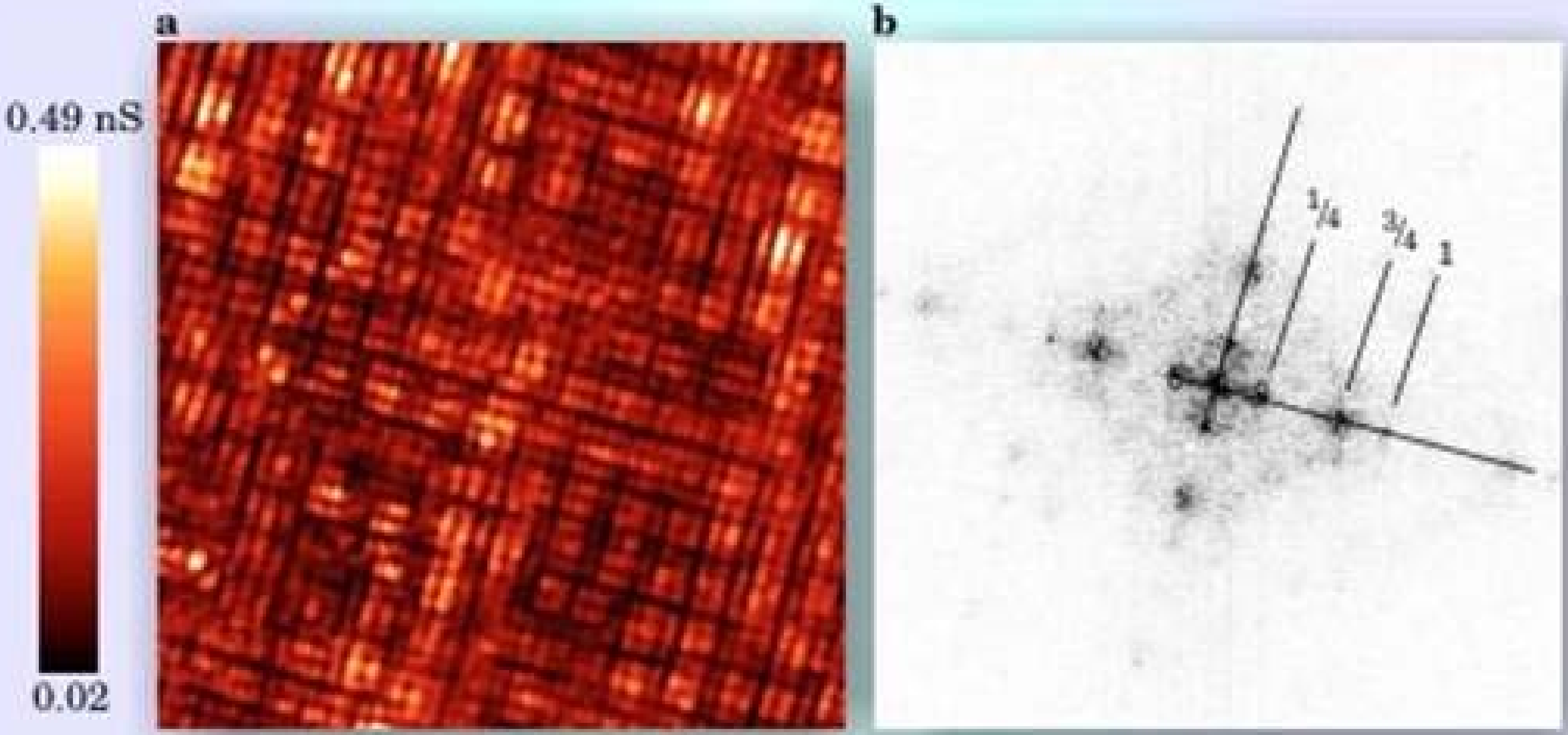
$$\left\langle \vec{S} \right\rangle = 0$$

Many experiments on the cuprate superconductors show:

- Tendency to produce modulations in spin singlet observables at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

- Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range charge modulations at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$



T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano,
H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

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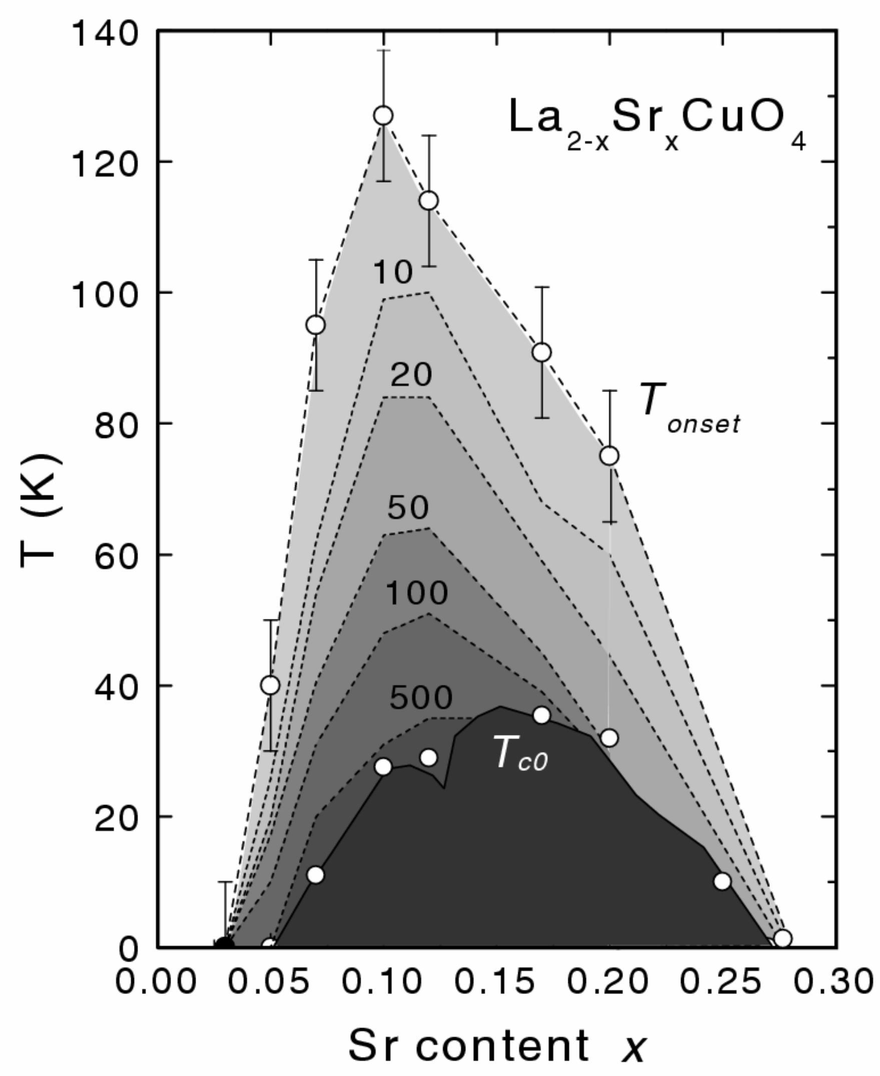
Superfluids proximate to finite doping Mott insulators with VBS order ?

Experiments on the cuprate superconductors also show strong vortex fluctuations above T_c



Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

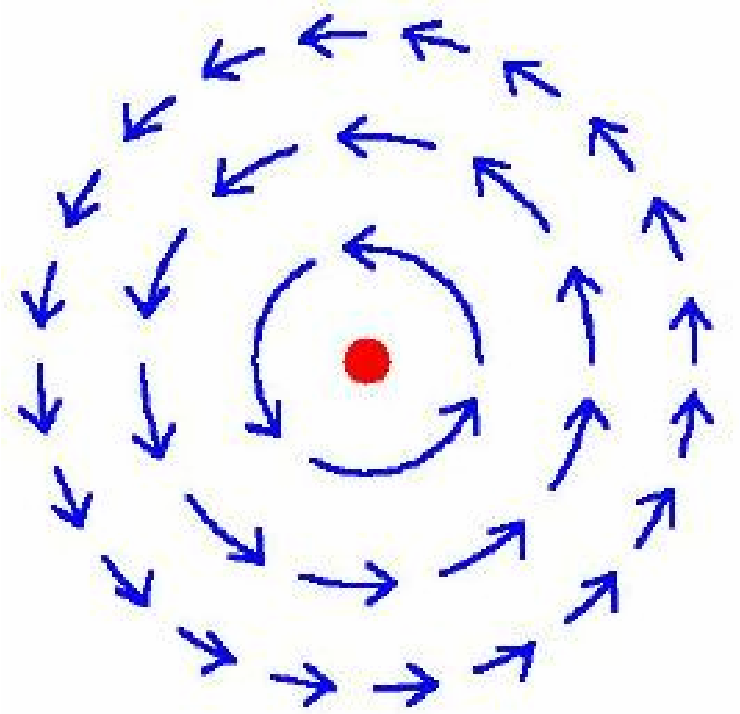
Main claims:

- *There are precursor fluctuations of VBS order in the superfluid.*
- *There fluctuations are intimately tied to the quantum theory of vortices in the superfluid*

IV. Vortices in the superfluid

Magnus forces, duality, and point vortices as dual “electric” charges

Excitations of the superfluid: **Vortices**

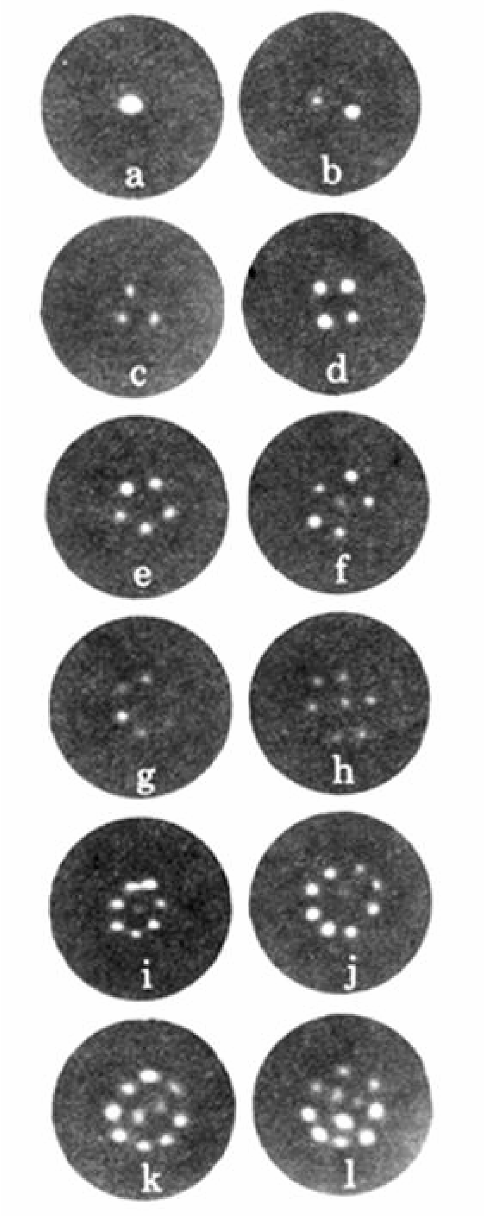


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

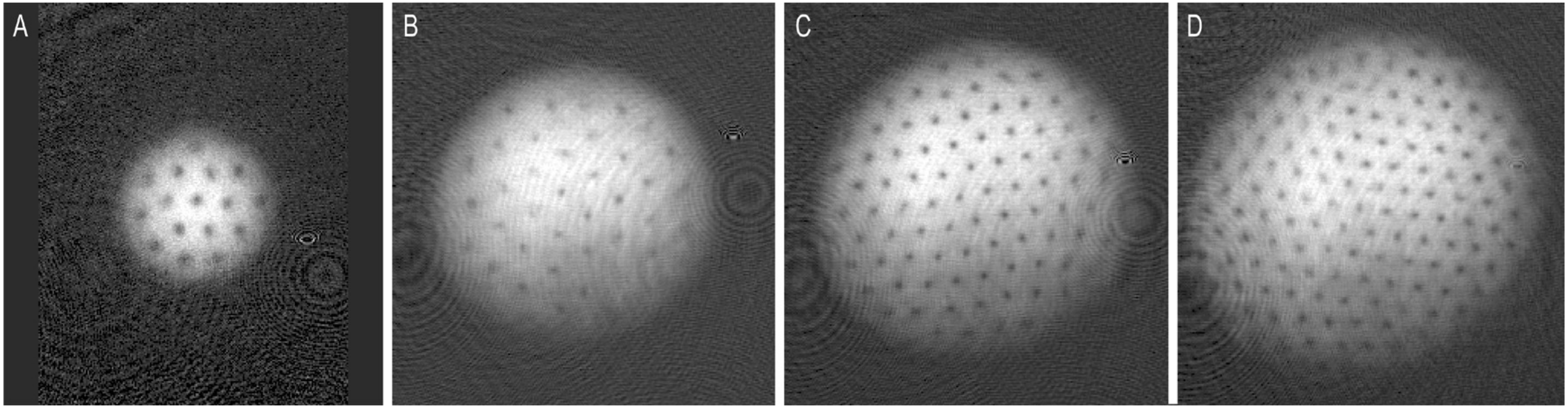
where n is an integer.

Observation of quantized vortices in rotating ^{4}He



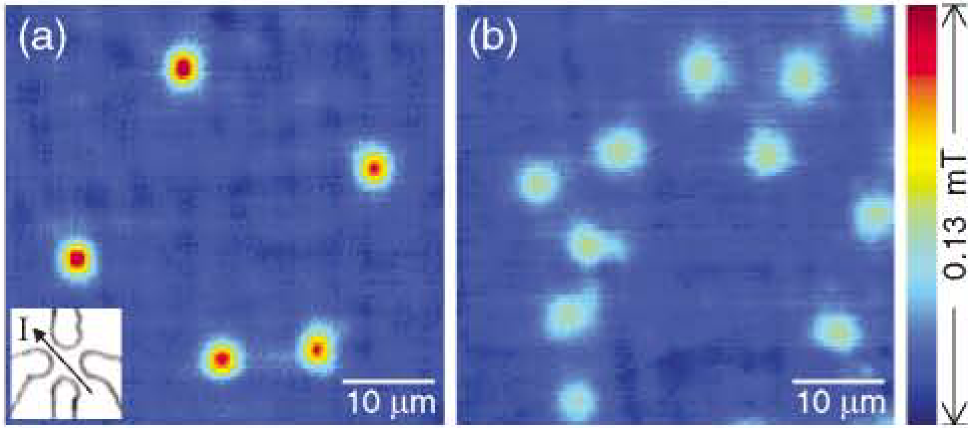
E.J. Yarmchuk, M.J.V. Gordon, and
R.E. Packard,
Observation of Stationary Vortex Arrays in Rotating Superfluid Helium,
Phys. Rev. Lett. **43**, 214 (1979).

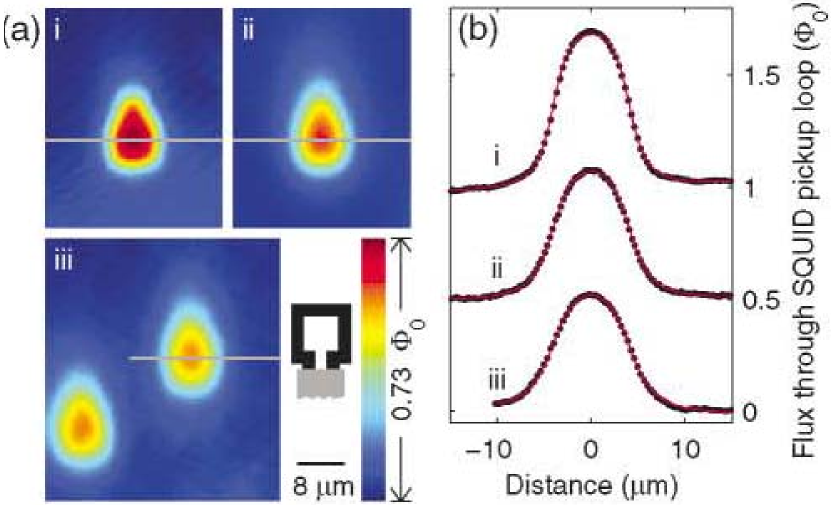
Observation of quantized vortices in rotating ultracold Na



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle,
Observation of Vortex Lattices in Bose-Einstein Condensates,
Science **292**, 476 (2001).

Quantized fluxoids in $YBa_2Cu_3O_{6+y}$



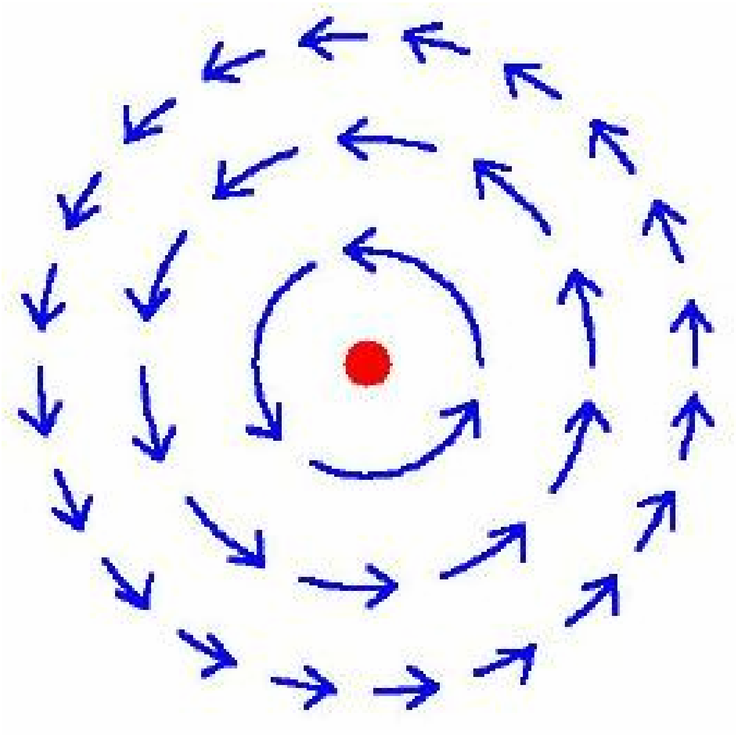


J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett*. **87**, 197002 (2001).

In superconductors, vortices carry quantized magnetic flux:

$$\int \mathbf{B} \cdot d\mathbf{S} = n\frac{hc}{2e}$$

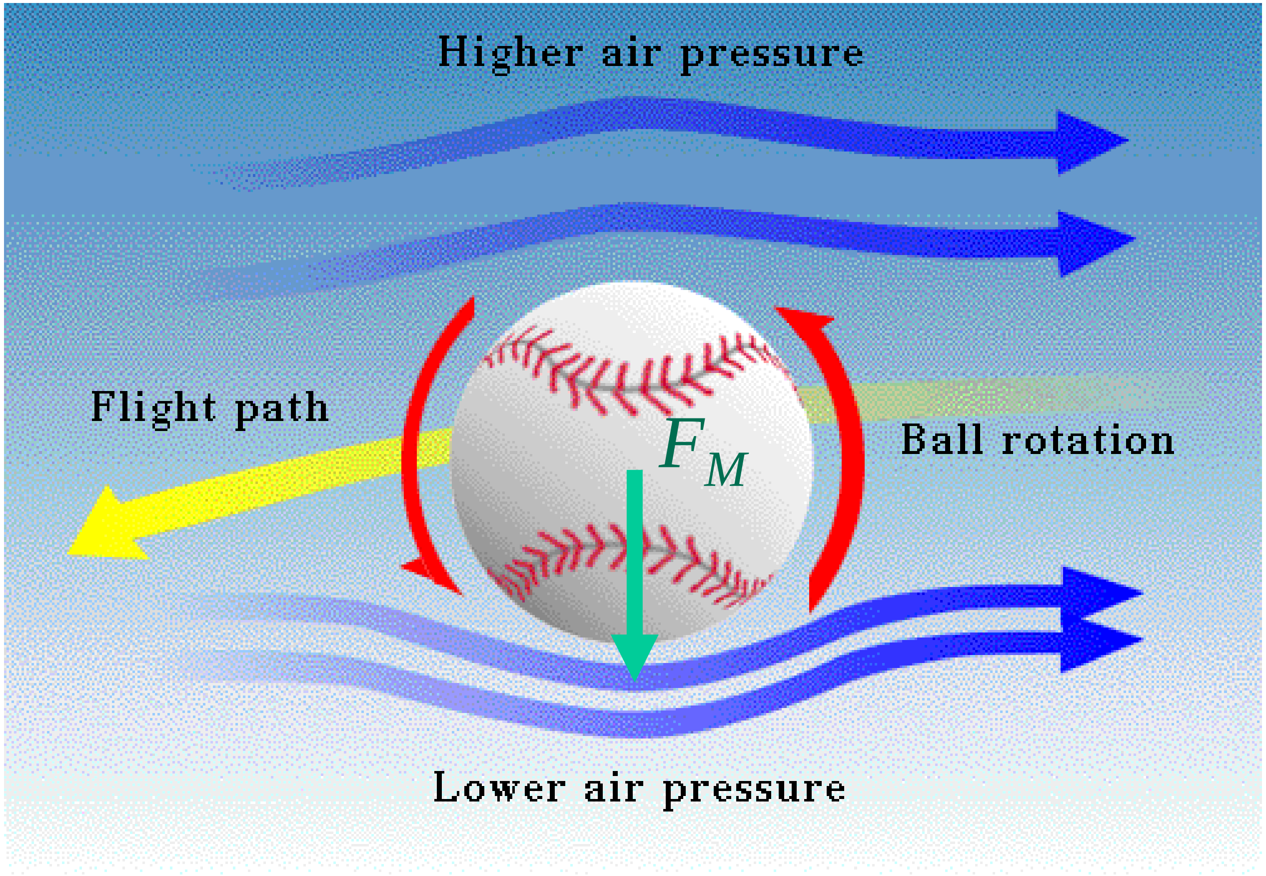
Excitations of the superfluid: **Vortices**



<u>Central question</u>:

In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these "particles" ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \bullet (\text{velocity of ball}) \bullet (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right) \\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\end{aligned}$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vortex in a superfluid, this is

$$\begin{aligned} \mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right) \\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}} \\ &= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt} \times \mathbf{B}\right) \end{aligned}$$

where $\mathbf{E} = \rho \mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

Dual picture:

The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = *h*×(number density of Bose particles)

V. Vortices in superfluids near the superfluid-insulator quantum phase transition

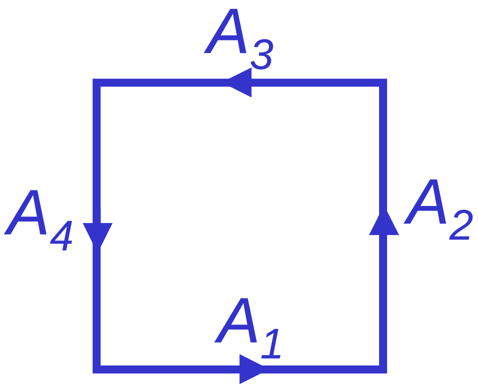
The "quantum order" of the superconducting state: evidence for vortex flavors

- The vortices are quantum particles moving in a periodic potential with the symmetry of the square lattice, and in the presence of a dual "magnetic" field of strength $= h\rho$, where ρ is the number density of bosons per unit cell.

- The vortex motion can be described by the effective Hofstadter Hamiltonian:

$$\mathcal{H}_v = -t \sum_{\langle ij \rangle} \left(e^{iA_{ij}} \varphi_i^* \varphi_j + \text{c.c.} \right)$$

where φ_i is an operator which annihilates a vortex particle at site i of a square lattice.



$$A_1+A_2+A_3+A_4= 2\pi f$$

where *f* is the boson filling fraction.

Bosons at filling fraction $f = 1$

- At f=1, the “magnetic” flux per unit cell is 2π, and the vortex does not pick up any phase from the boson density.

- The effective dual “magnetic” field acting on the vortex is zero, and the corresponding component of the Magnus force vanishes.

Bosons at rational filling fraction f=p/q

Quantum mechanics of the vortex "particle" in a periodic potential with *f* flux quanta per unit cell

Space group symmetries of Hofstadter Hamiltonian:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

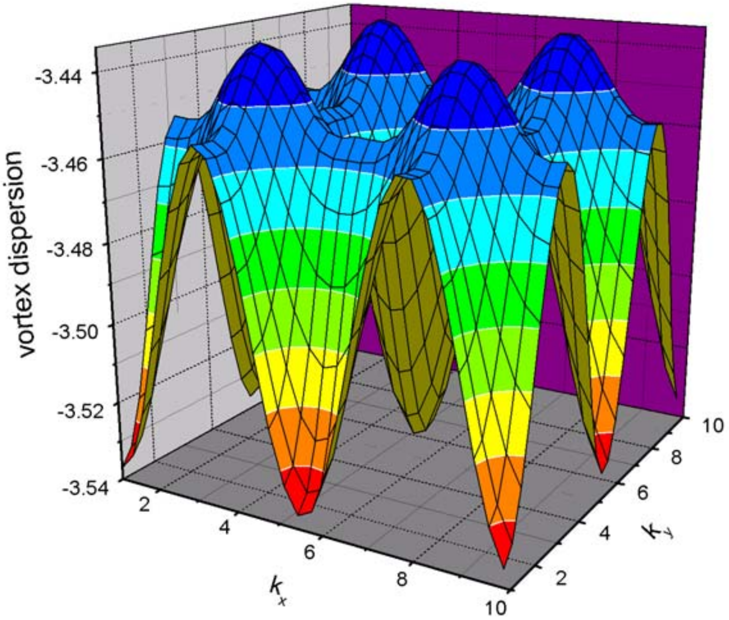
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

The low energy vortex states must form a representation of this algebra

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Hofstadter spectrum of the quantum vortex "particle" with field operator φ



At filling $f=p/q$, there are q species of vortices, φ_ℓ (with $\ell=1\ldots q$), associated with q degenerate minima in the vortex spectrum. These vortices realize the smallest, q-dimensional, representation of the magnetic algebra.

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

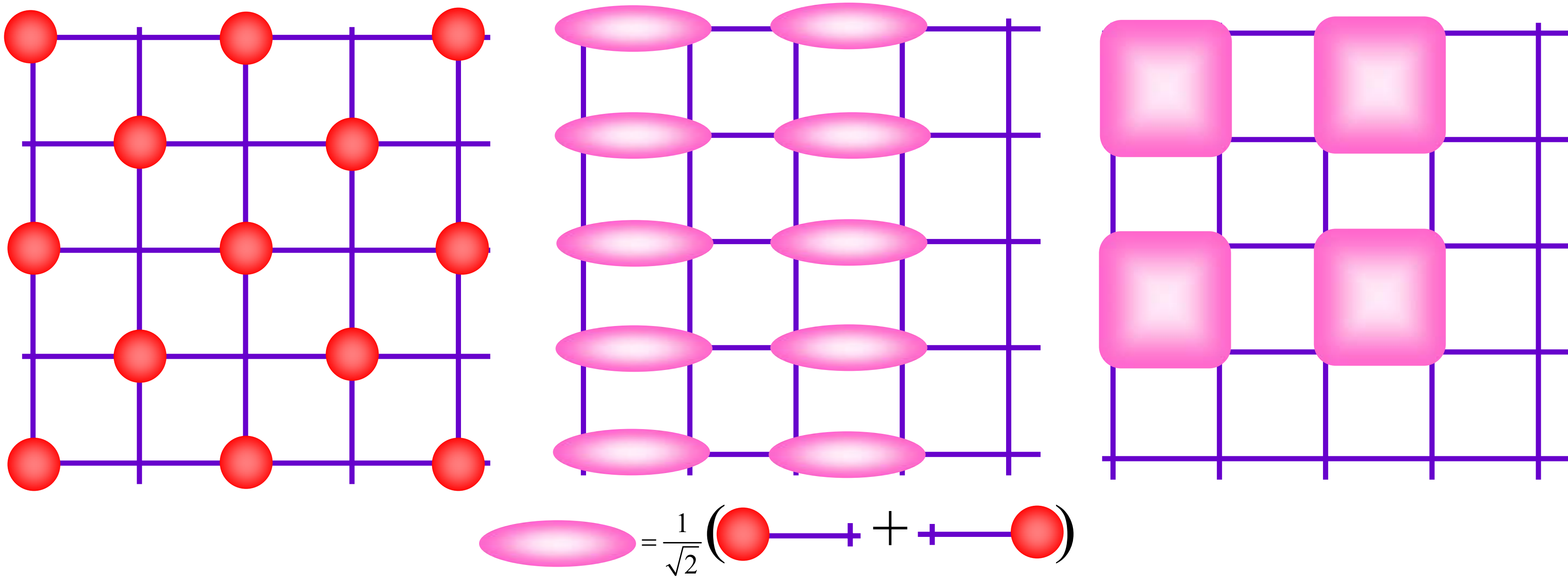
Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

- The orientation of the vortex in flavor space implies a particular configuration of VBS order in its vicinity.

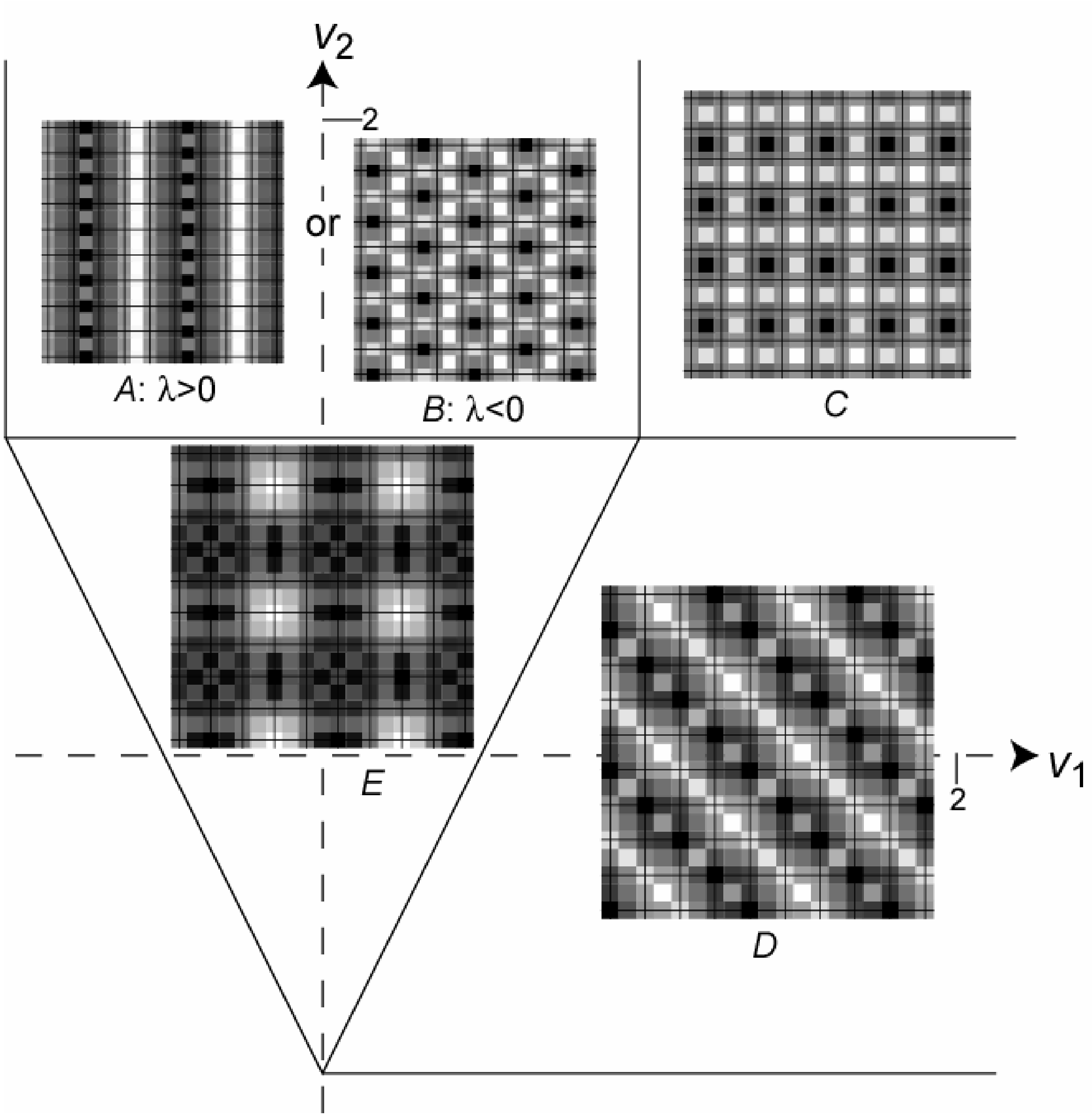
Mott insulators obtained by “condensing” vortices

Spatial structure of insulators for q=2 (f=1/2)



Field theory with projective symmetry

Spatial structure of insulators for $q=4$ ($f=1/4$ or $3/4$)



$a \times b$ unit cells;

q/a, q/b, ab/q

all integers

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

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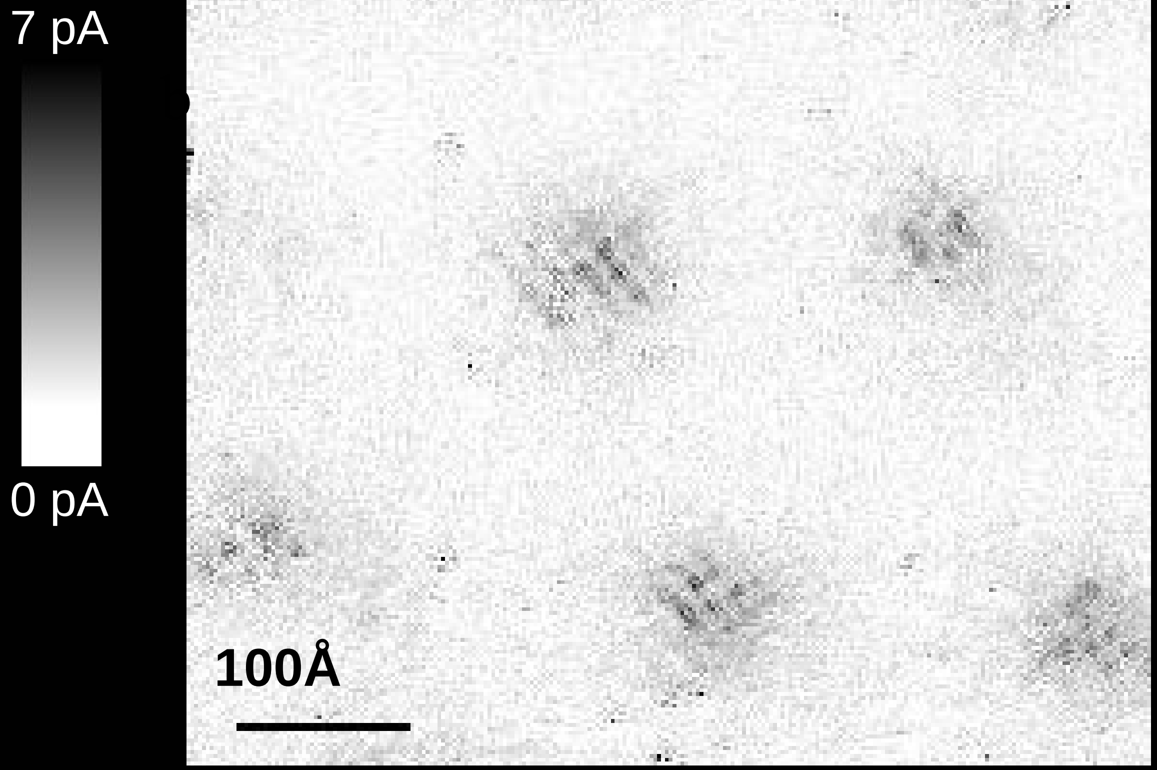
Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

- The orientation of the vortex in flavor space implies a particular configuration of VBS order in its vicinity.

- Any pinned vortex must pick an orientation in flavor space: this induces a halo of VBS order in its vicinity

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

J. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Measuring the inertial mass of a vortex

The spatial extent of the LDOS modulations measures the region over which the vortex executes its zero-point motion. The size of this region can be determined by solving the equations of motion

$$m_v \frac{d^2\mathbf{r}}{dt^2} = F_M$$

and so is determined by the inertial vortex mass m_v.

Measuring the inertial mass of a vortex

Preliminary estimates for the BSCCO experiment:

Inertial vortex mass $m_v \approx 10 m_e$

Vortex magnetoplasmon frequency $\nu_{\mathrm{p}} \approx 1\ \mathrm{THz} = 4\ \mathrm{meV}$

Future experiments can directly detect vortex zero point motion by looking for resonant absorption at this frequency.

Vortex oscillations can also modify the electronic density of states.

Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, $f = p/q$ per site, while the density of the superfluid is close (but need not be identical) to this value

- Vortices with flux *h/(2e)* come in multiple (usually q) “flavors”
- The lattice space group acts in a projective representation on the vortex flavor space.
- These flavor quantum numbers provide a distinction between superfluids: they constitute a “quantum order”
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.