# The quantum phase transition between a superfluid and an insulator: *applications to trapped ultracold atoms and the cuprate superconductors.*

# The quantum phase transition between a superfluid and an insulator:
## *applications to trapped ultracold atoms and the cuprate superconductors*.

Leon Balents (UCSB)
Lorenz Bartosch (Harvard)
Anton Burkov (Harvard)
Predrag Nikolic (Harvard)
Subir Sachdev (Harvard)
Krishnendu Sengupta (HRI, India)

Talk online at http://sachdev.physics.harvard.edu

# Outline

I. Bose-Einstein condensation and superfluidity.

II. The superfluid-insulator quantum phase transition.

III. The cuprate superconductors, and their proximity to a superfluid-insulator transition.

IV. Landau-Ginzburg-Wilson theory of the superfluid-insulator transition.

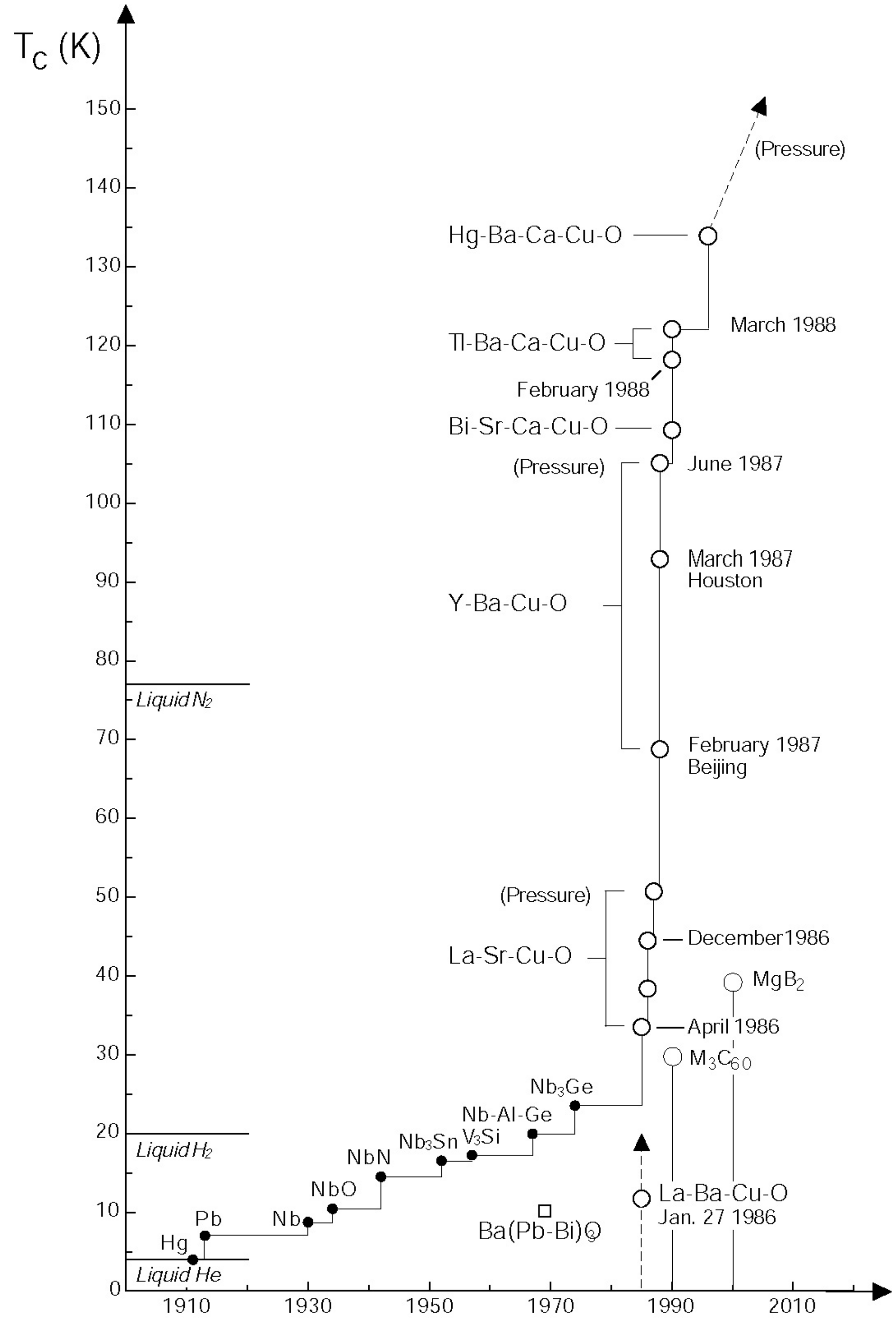
V. Beyond the LGW paradigm: continuous quantum transitions with multiple order parameters.

VI. Experimental tests in the cuprates.

# I. Bose-Einstein condensation and superfluidity

Superfluidity/superconductivity occur in:

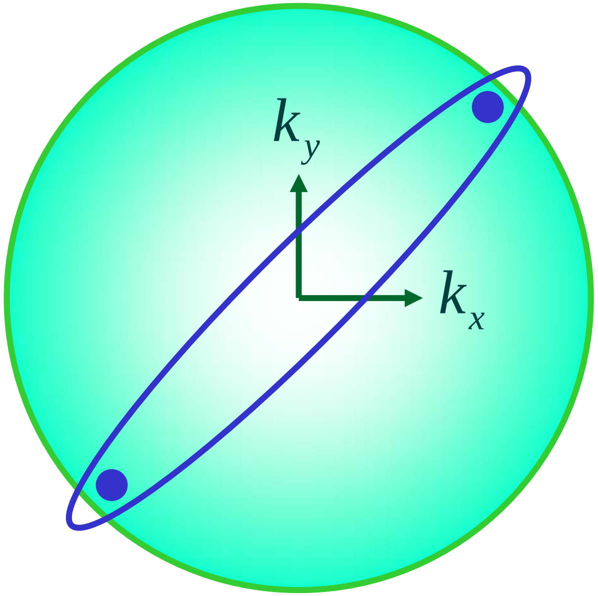
- liquid $^4He$
- metals Hg, Al, Pb, Nb, $Nb_3Sn$…..
- liquid $^3He$
- neutron stars
- cuprates $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{6+y}$….
- $M_3C_{60}$
- ultracold trapped atoms
- $MgB_2$



# The Bose-Einstein condensate:

A macroscopic number of bosons occupy the lowest energy quantum state

Such a condensate also forms in systems of fermions, where the bosons are Cooper pairs of fermions:

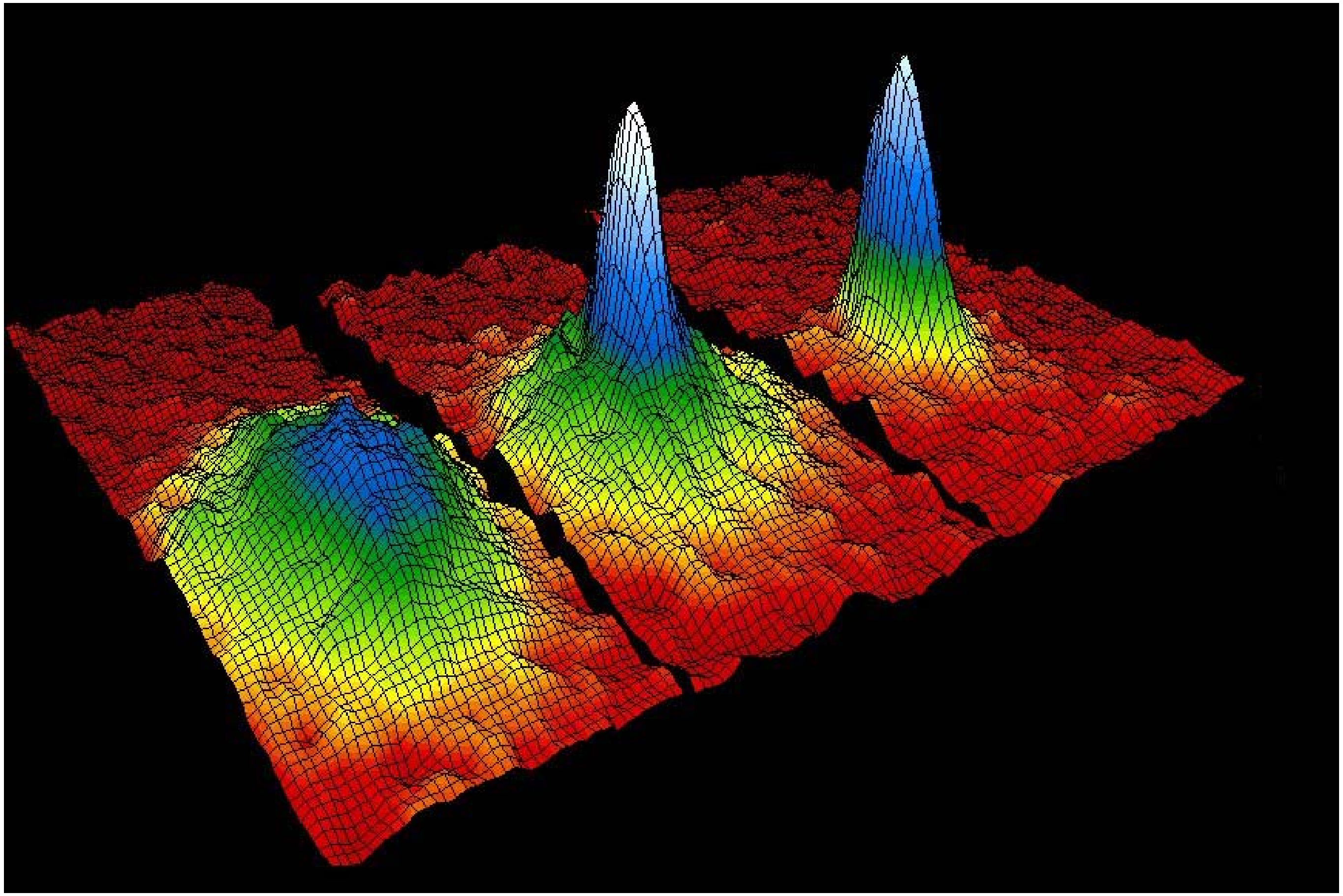



Pair wavefunction in cuprates:

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

# Velocity distribution function of ultracold $^{87}$Rb atoms



M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

# Superflow:

The wavefunction of the condensate

$$\Psi \rightarrow \Psi e^{i\theta(\boldsymbol{r})}$$

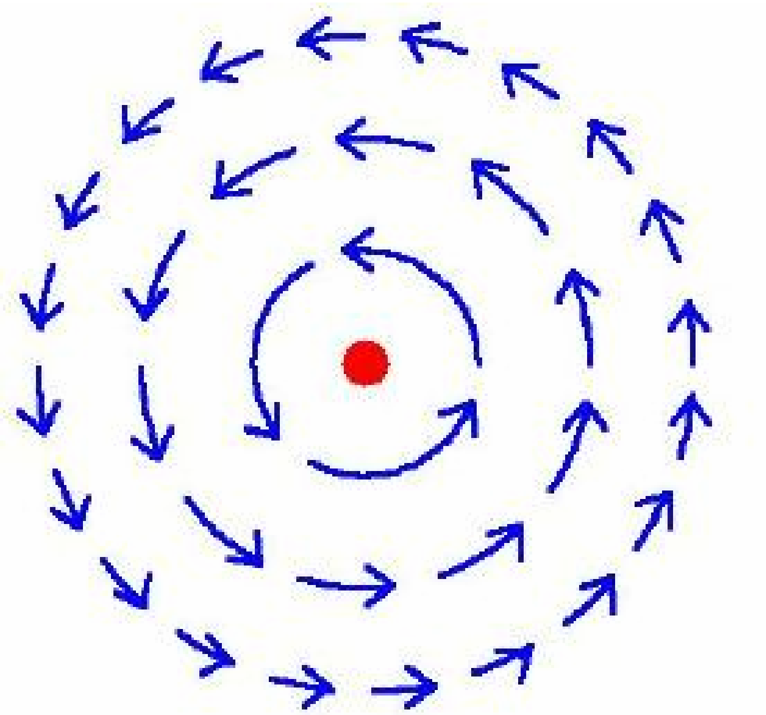
Superfluid velocity

$$\boldsymbol{v}_s = \frac{\hbar}{m} \nabla \theta$$

(for non-Galilean invariant superfluids,

the co-efficient of $\nabla \theta$ is modified)

# Excitations of the superfluid: **Vortices**

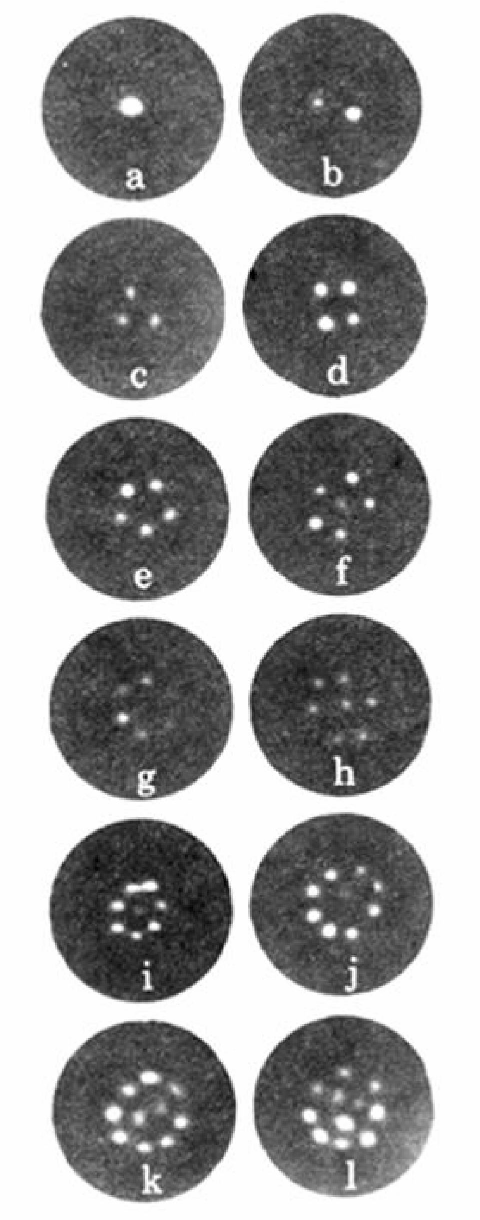


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

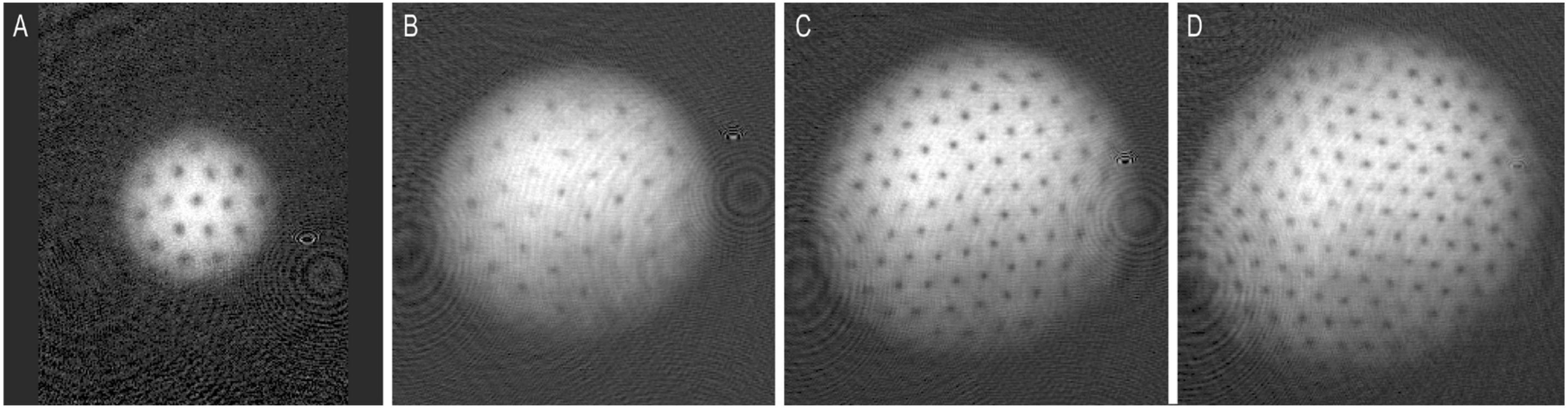
where $n$ is an integer.

# Observation of quantized vortices in rotating $^4$He



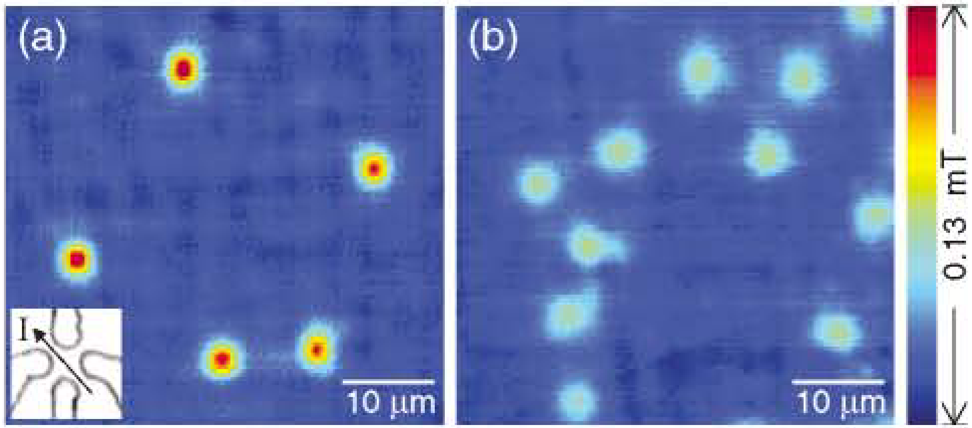

E.J. Yarmchuk, M.J.V. Gordon, and R.E. Packard, *Observation of Stationary Vortex Arrays in Rotating Superfluid Helium*, Phys. Rev. Lett. **43**, 214 (1979).

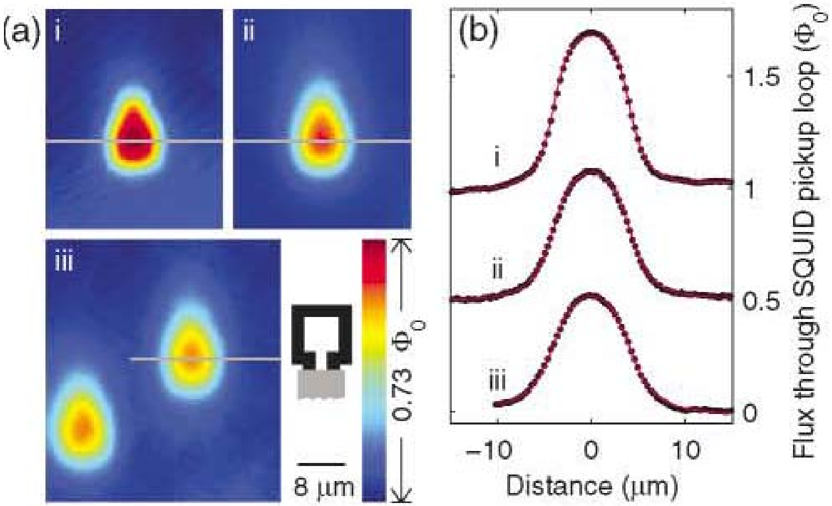
# Observation of quantized vortices in rotating ultracold Na




J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle,
*Observation of Vortex Lattices in Bose-Einstein Condensates*,
Science **292**, 476 (2001).

# Quantized fluxoids in $YBa_2Cu_3O_{6+y}$




J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

In superconductors, vortices carry quantized magnetic flux:

$$\int \mathbf{B} \cdot d\mathbf{S} = n \frac{hc}{2e}$$

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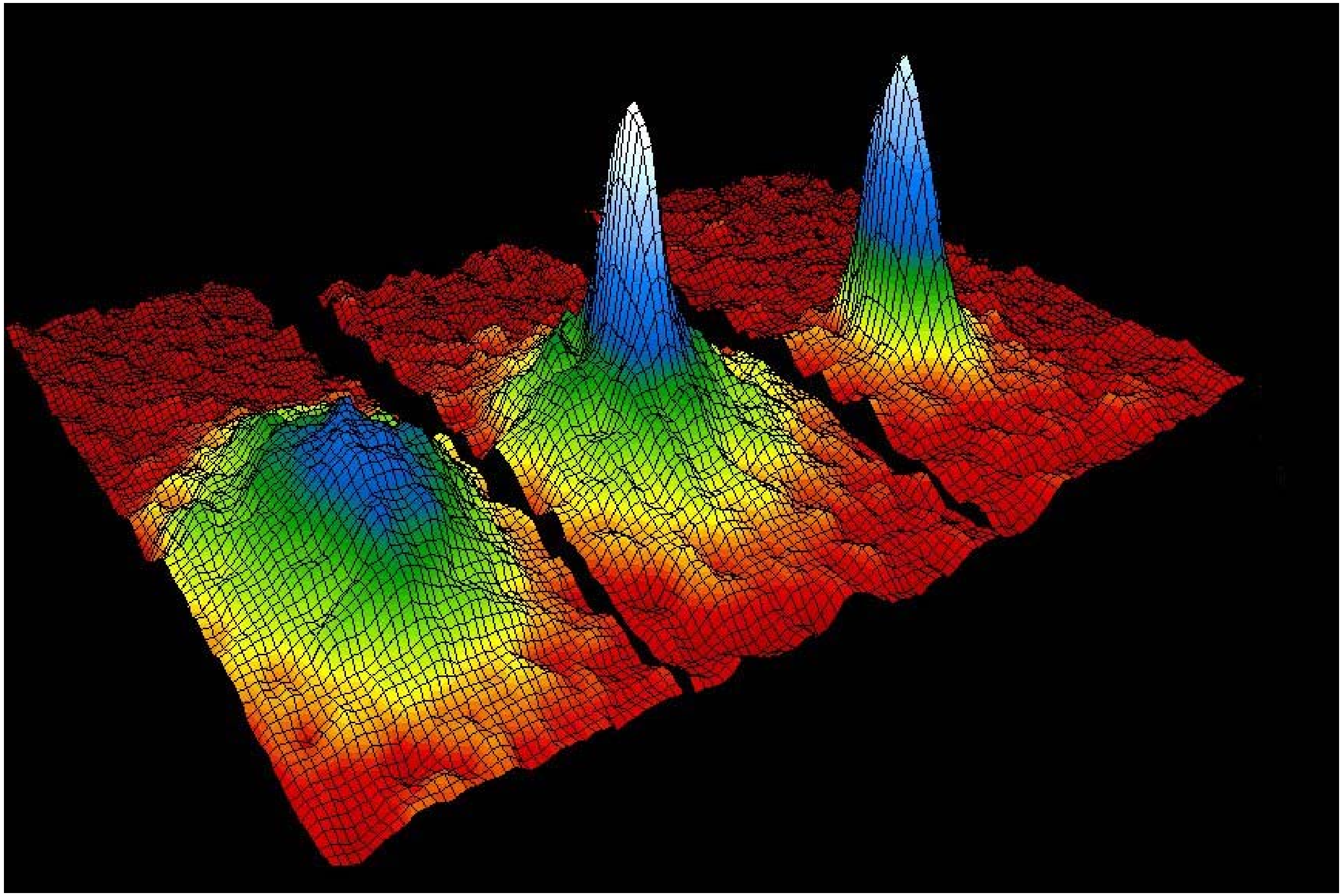
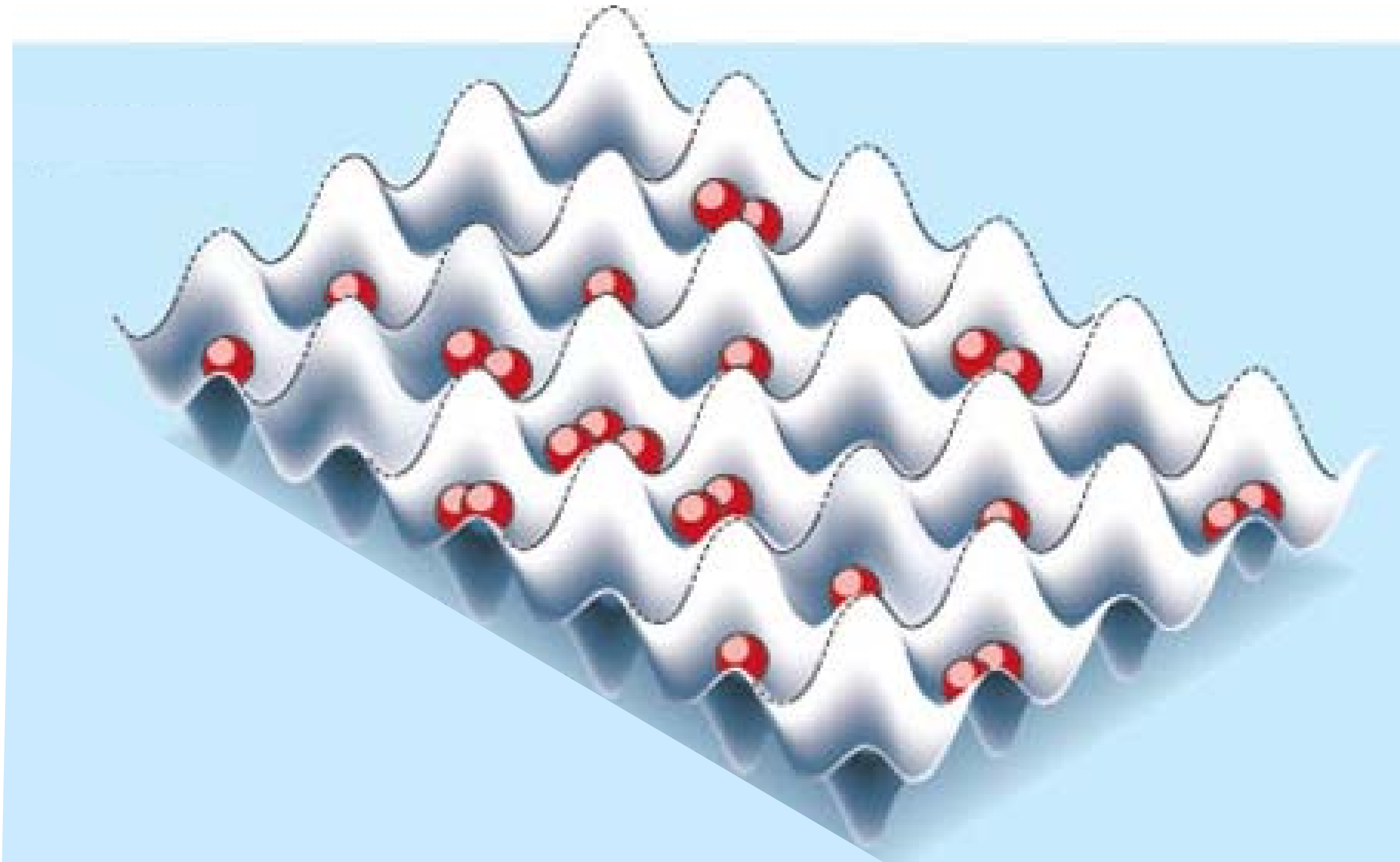
# II. The superfluid-insulator quantum phase transition

# Velocity distribution function of ultracold $^{87}$Rb atoms



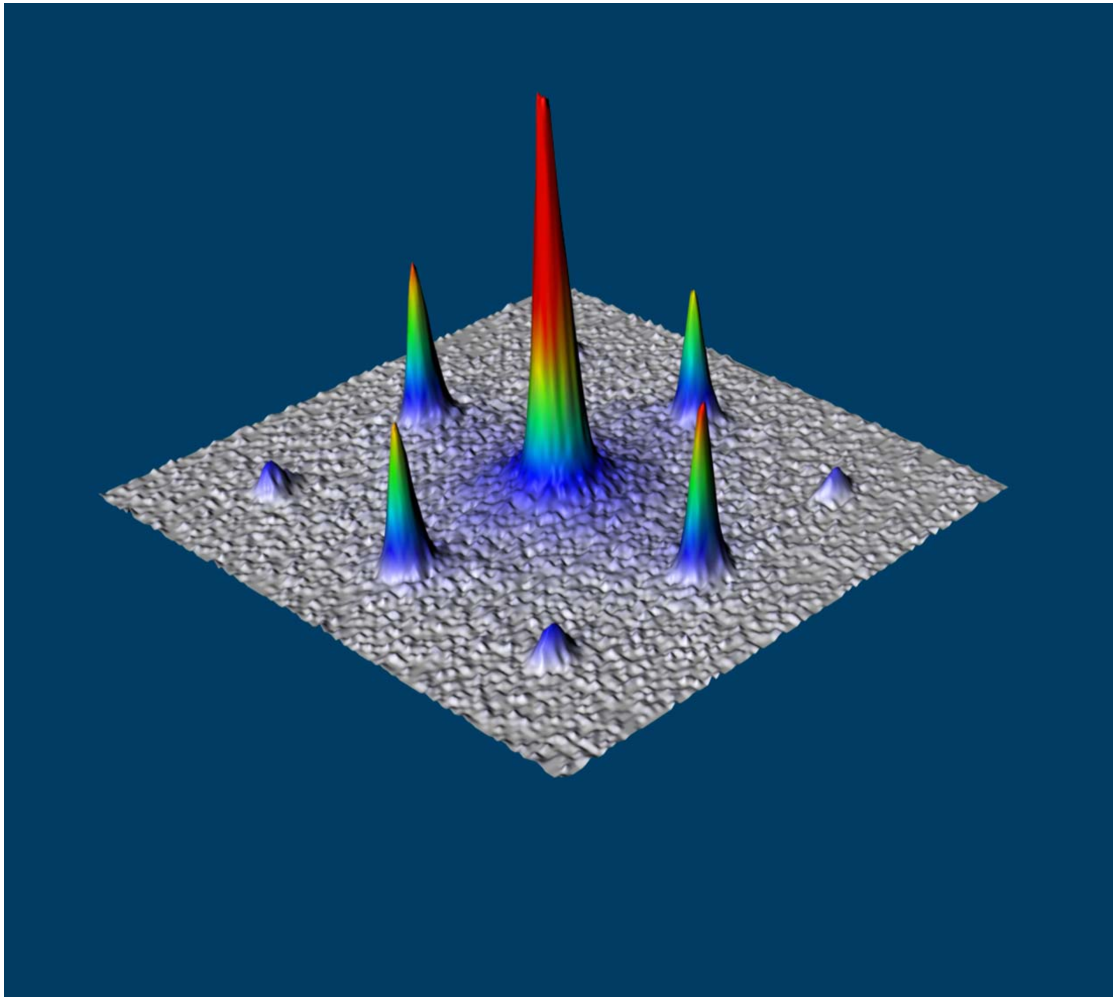
M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

# Apply a periodic potential (standing laser beams) to trapped ultracold bosons ($^{87}$Rb)



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

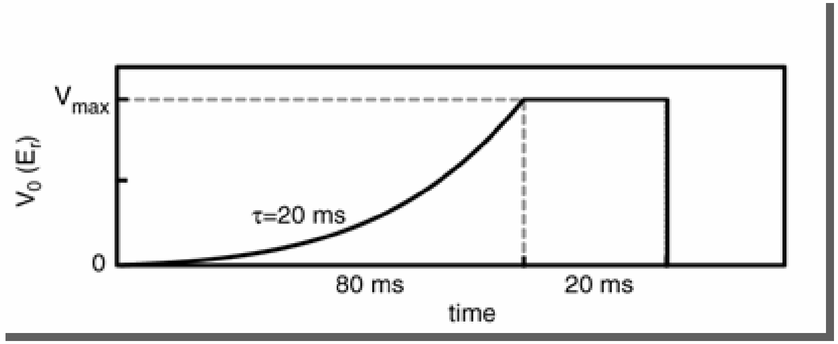
# Momentum distribution function of bosons



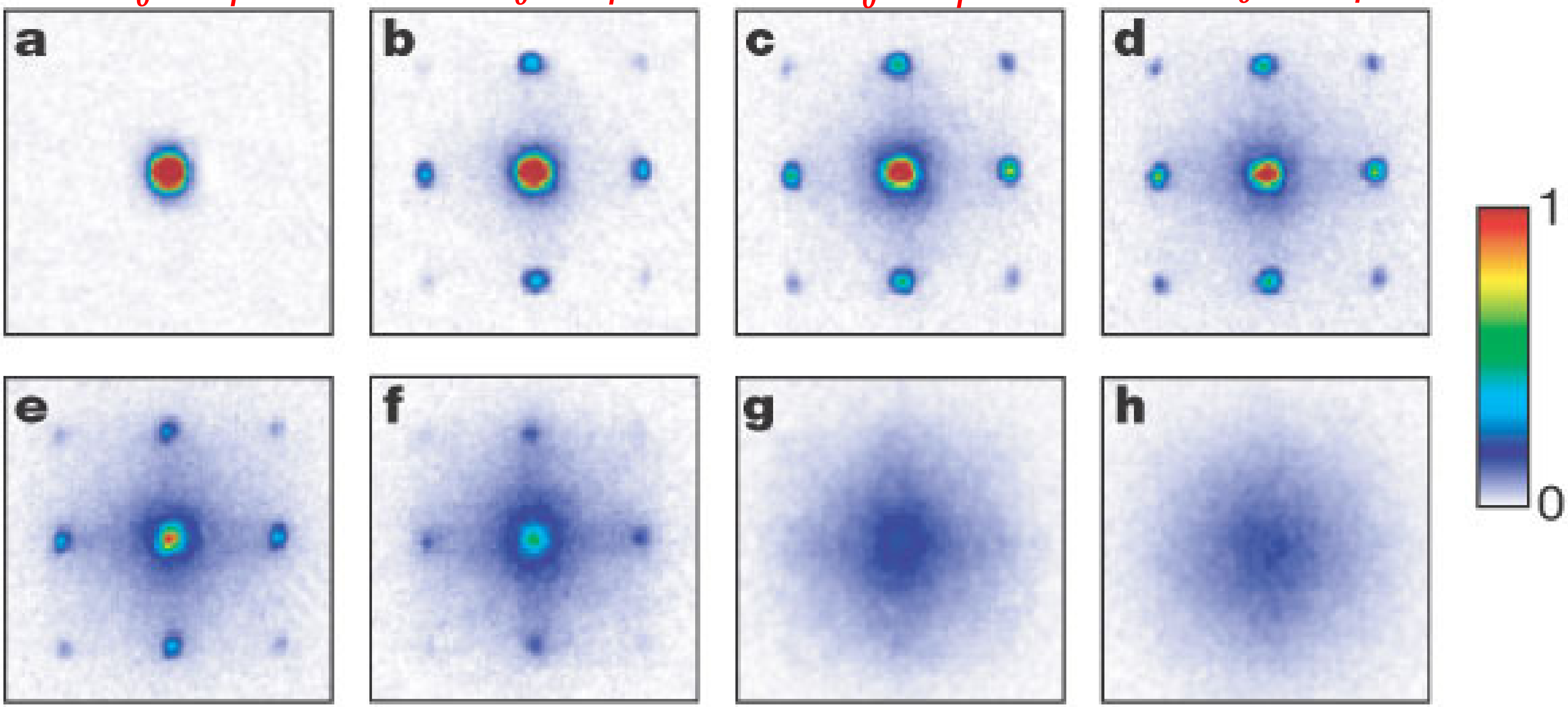
Bragg reflections of condensate at reciprocal lattice vectors

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

# Superfluid-insulator quantum phase transition at $T$=0

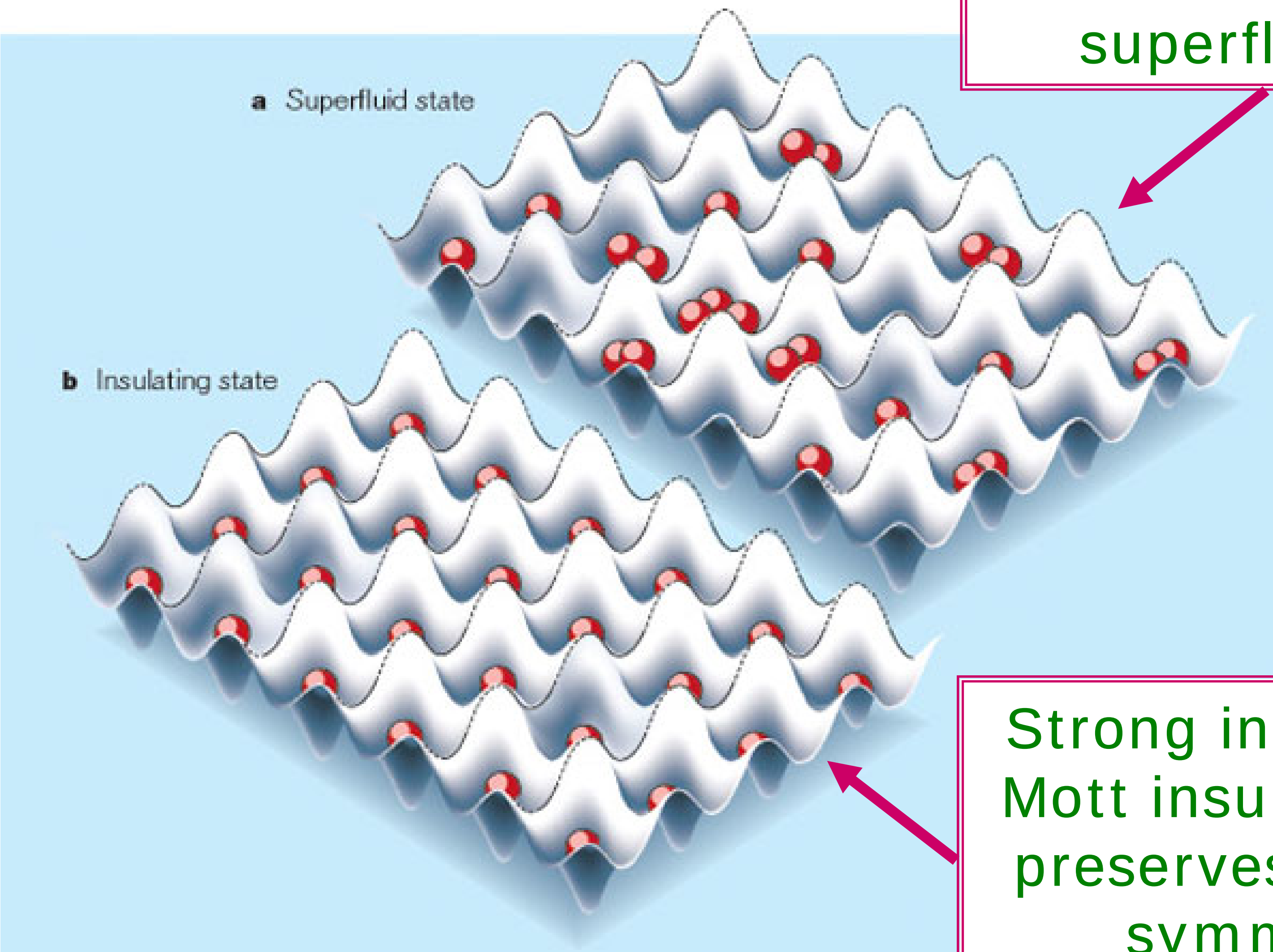



$V_0=0E_r$ $V_0=3E_r$ $V_0=7E_r$ $V_0=10E_r$




$V_0=13E_r$ $V_0=14E_r$ $V_0=16E_r$ $V_0=20E_r$

# *Bosons at filling fraction $f = 1$*

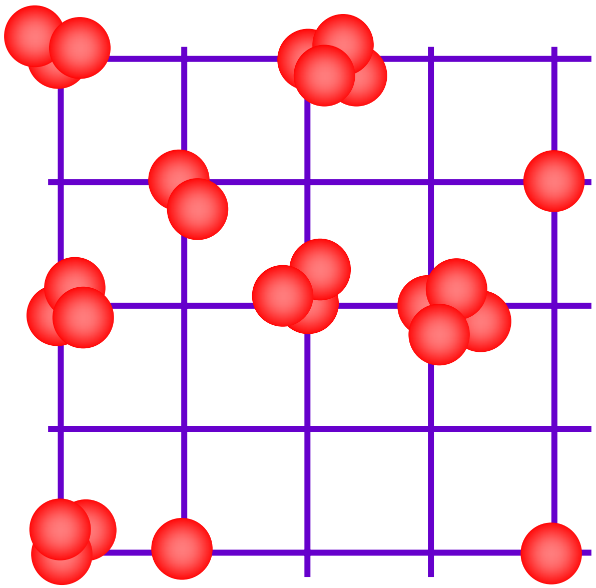



Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

## *Bosons at filling fraction* $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

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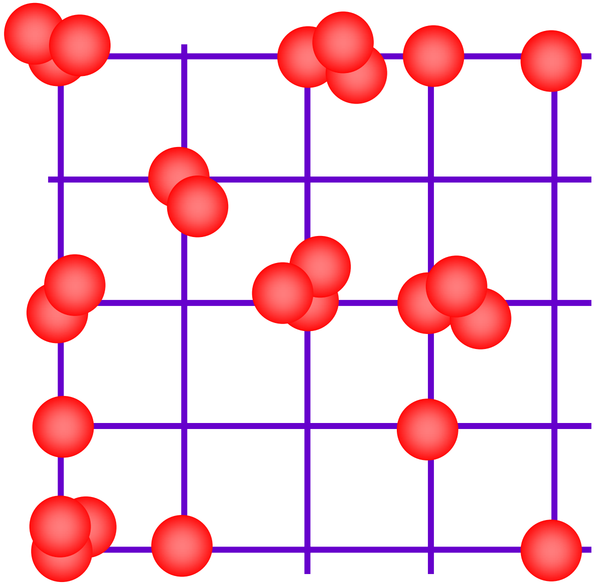

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Weak interactions: superfluidity

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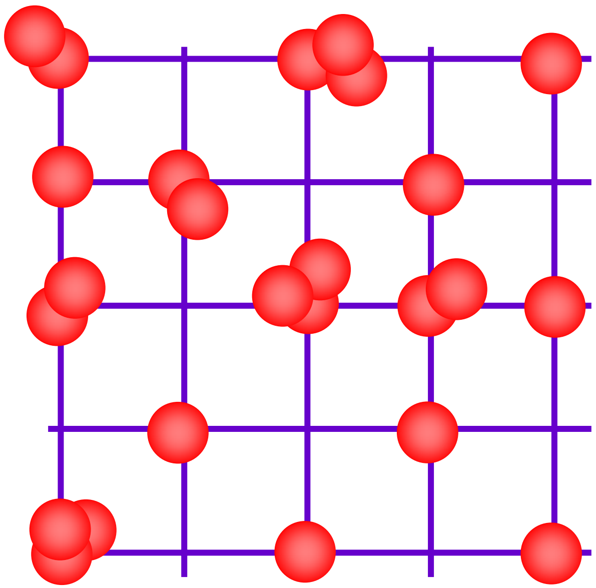

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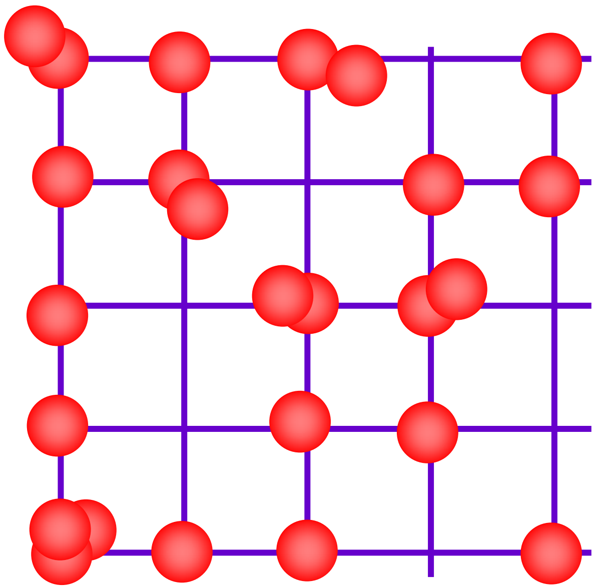

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**Weak interactions: superfluidity**

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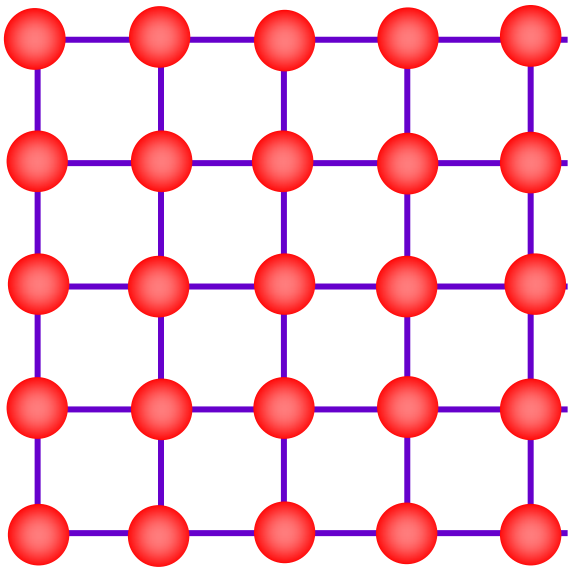
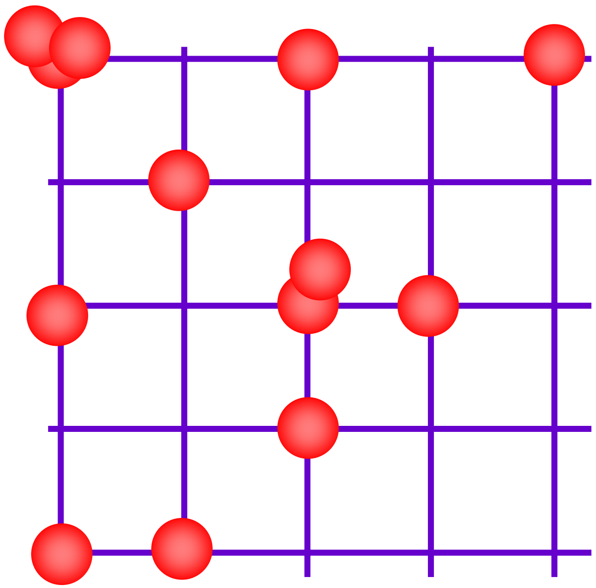


$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

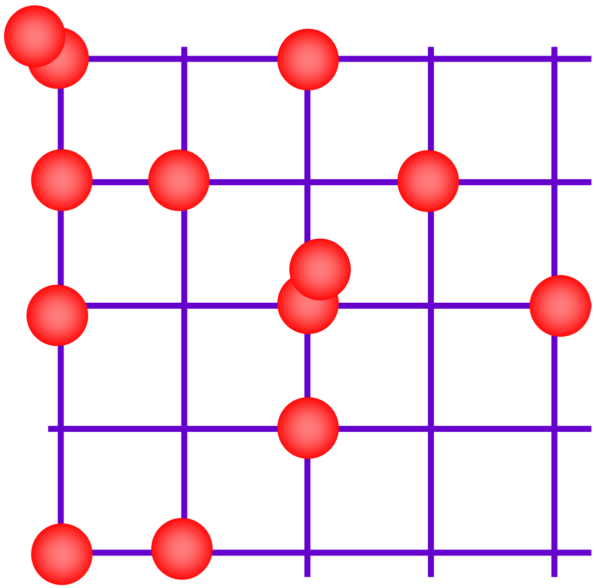
## *Bosons at filling fraction* $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

# *Bosons at filling fraction $f = 1/2$*



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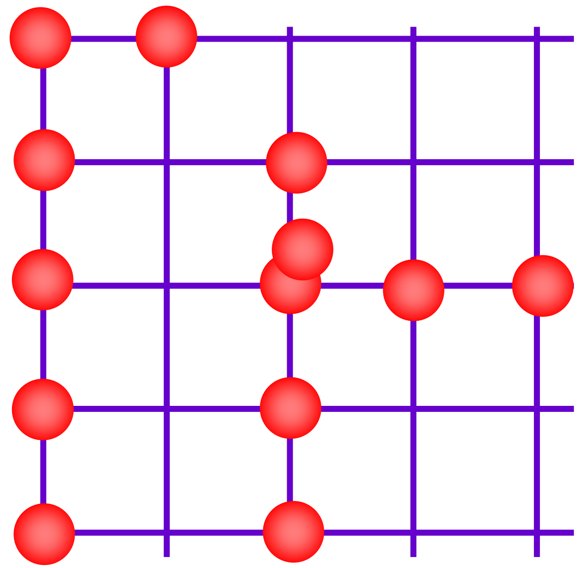

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Weak interactions: superfluidity

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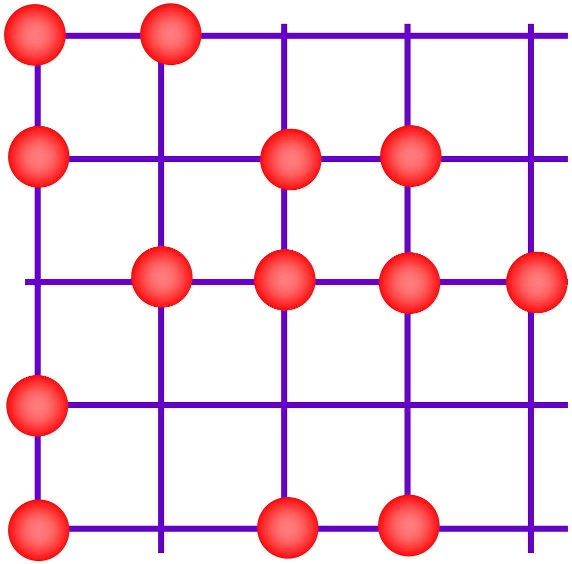

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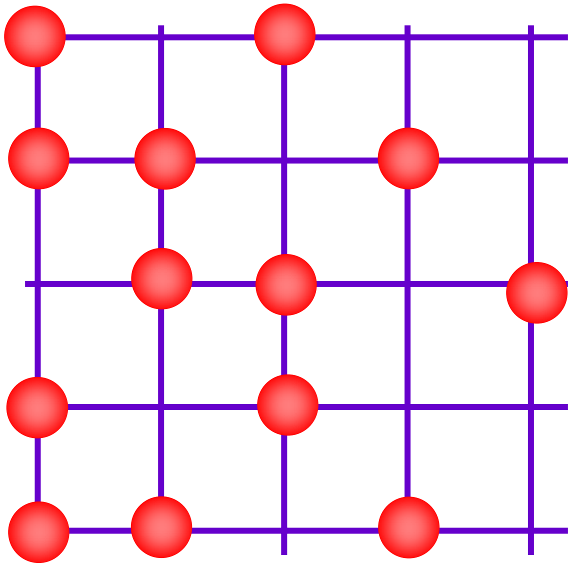

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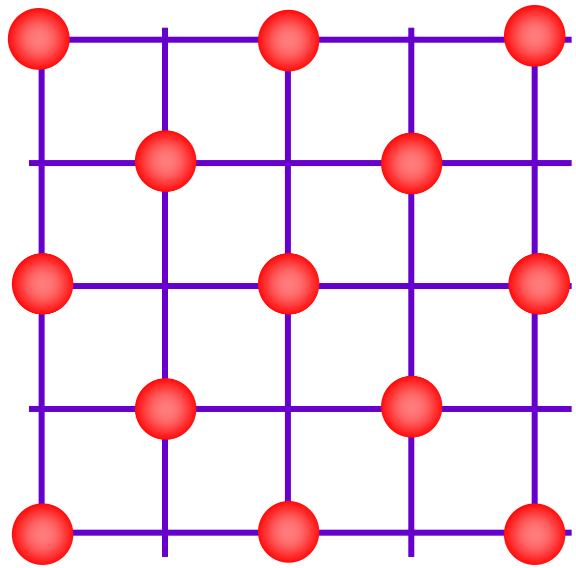

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Strong interactions: insulator

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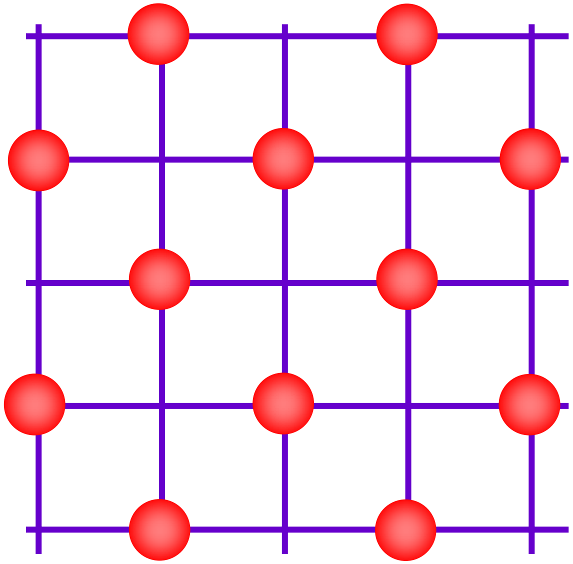

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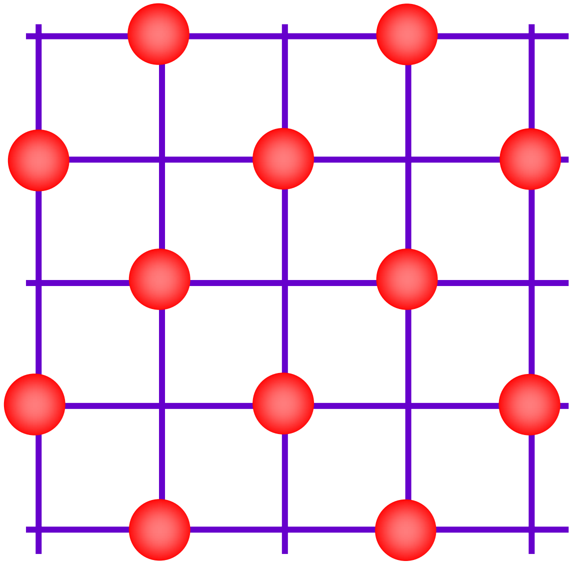
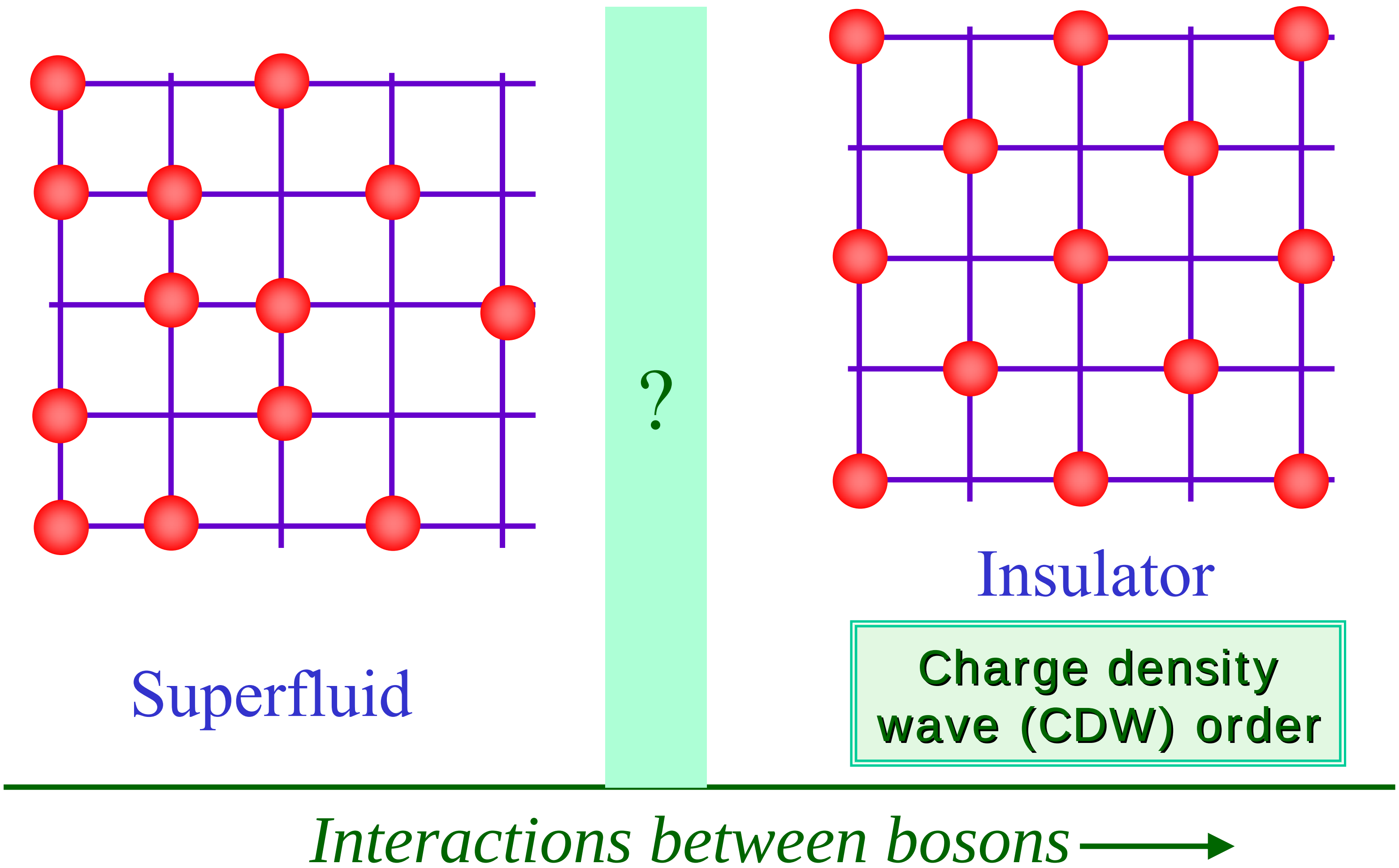


$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Insulator has "density wave" order

# *Bosons on the square lattice at filling fraction f=1/2*



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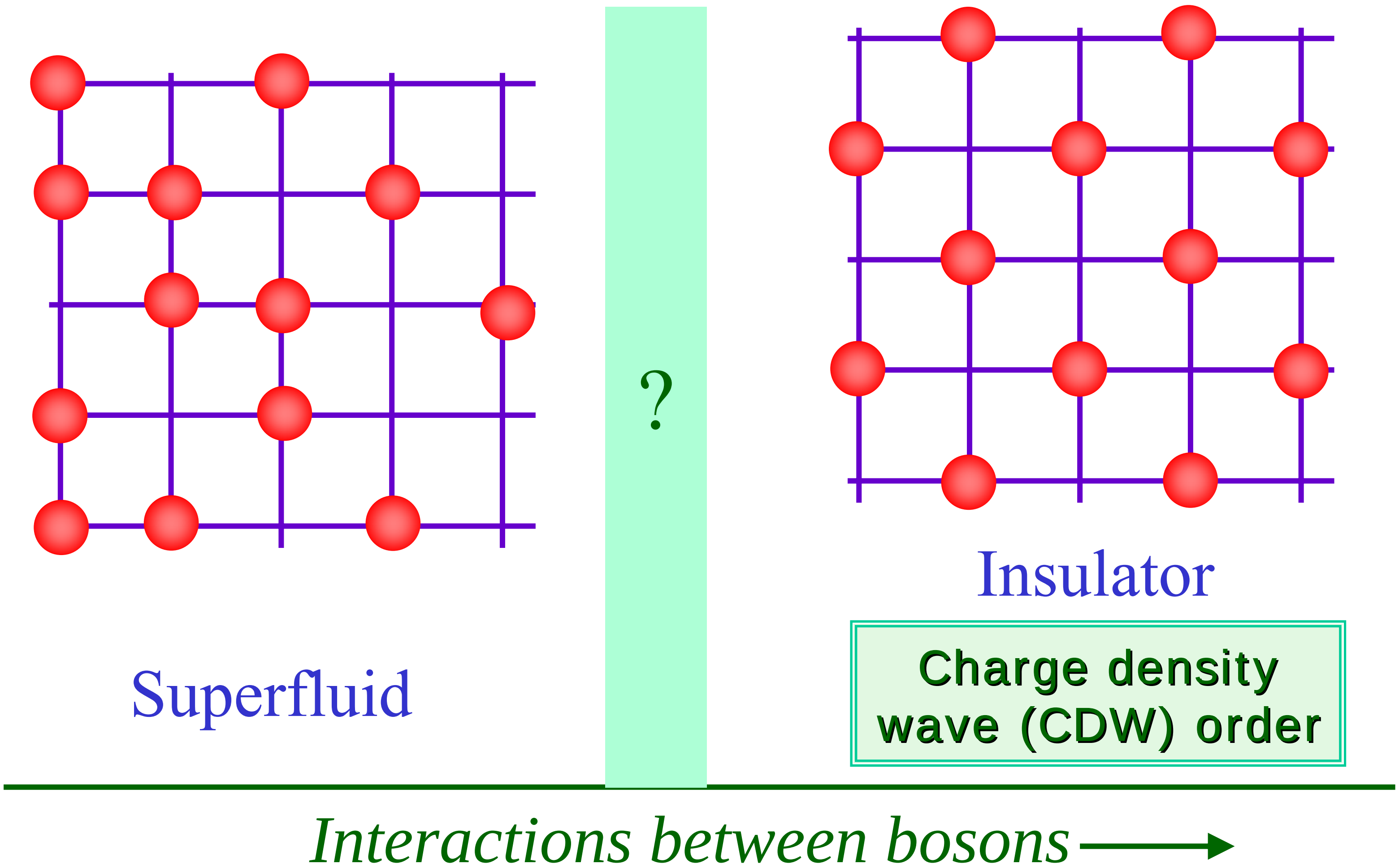

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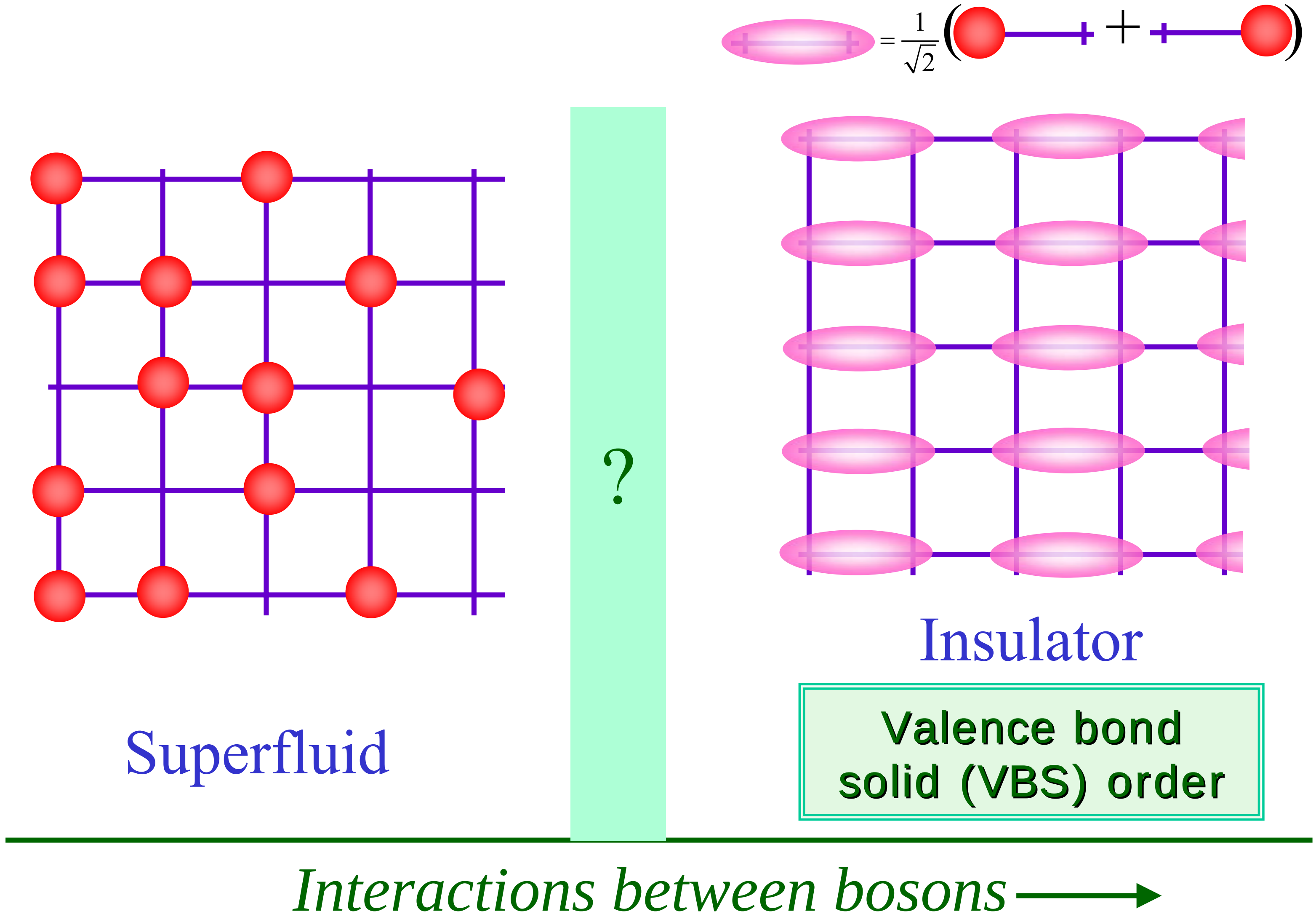


N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

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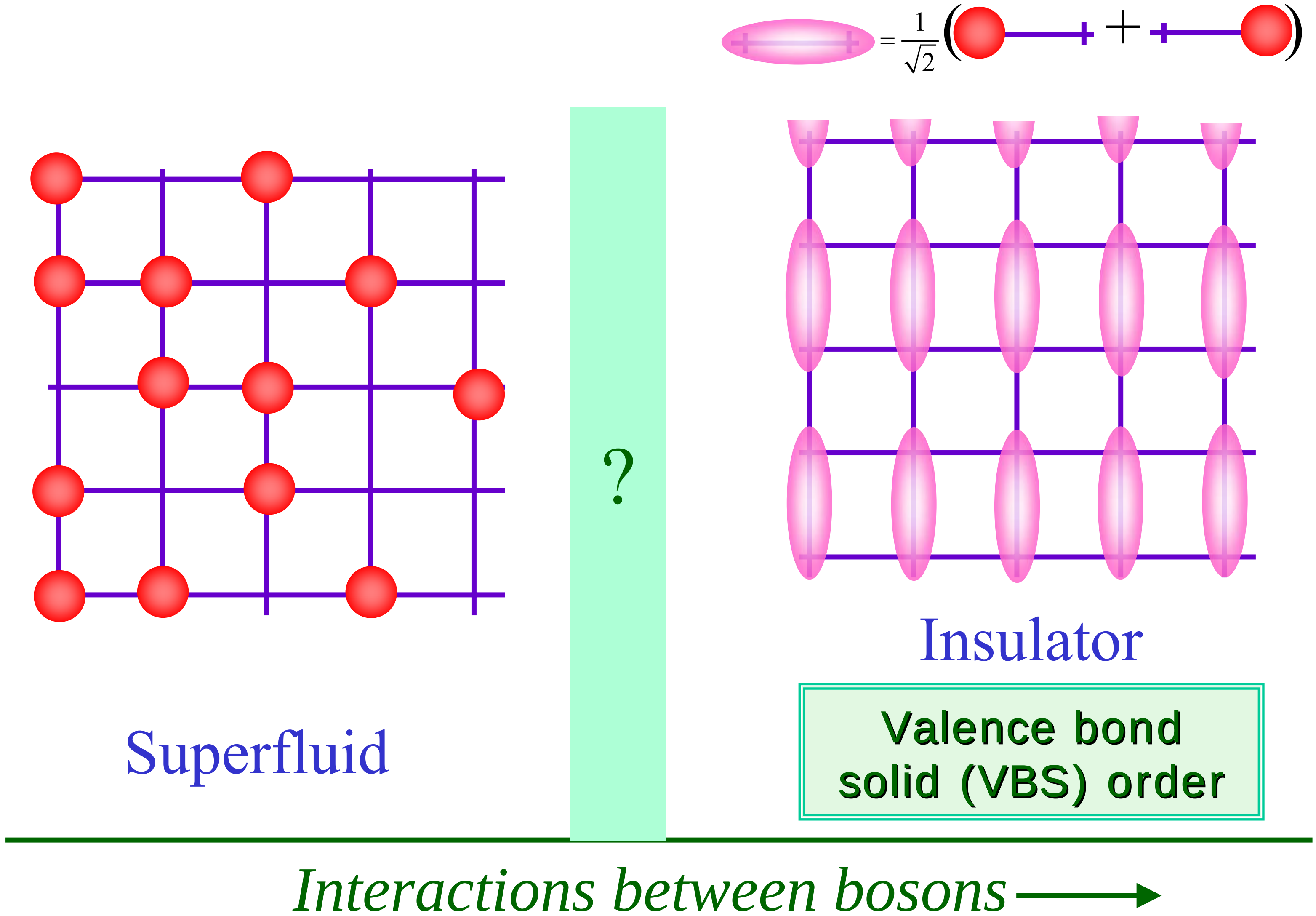


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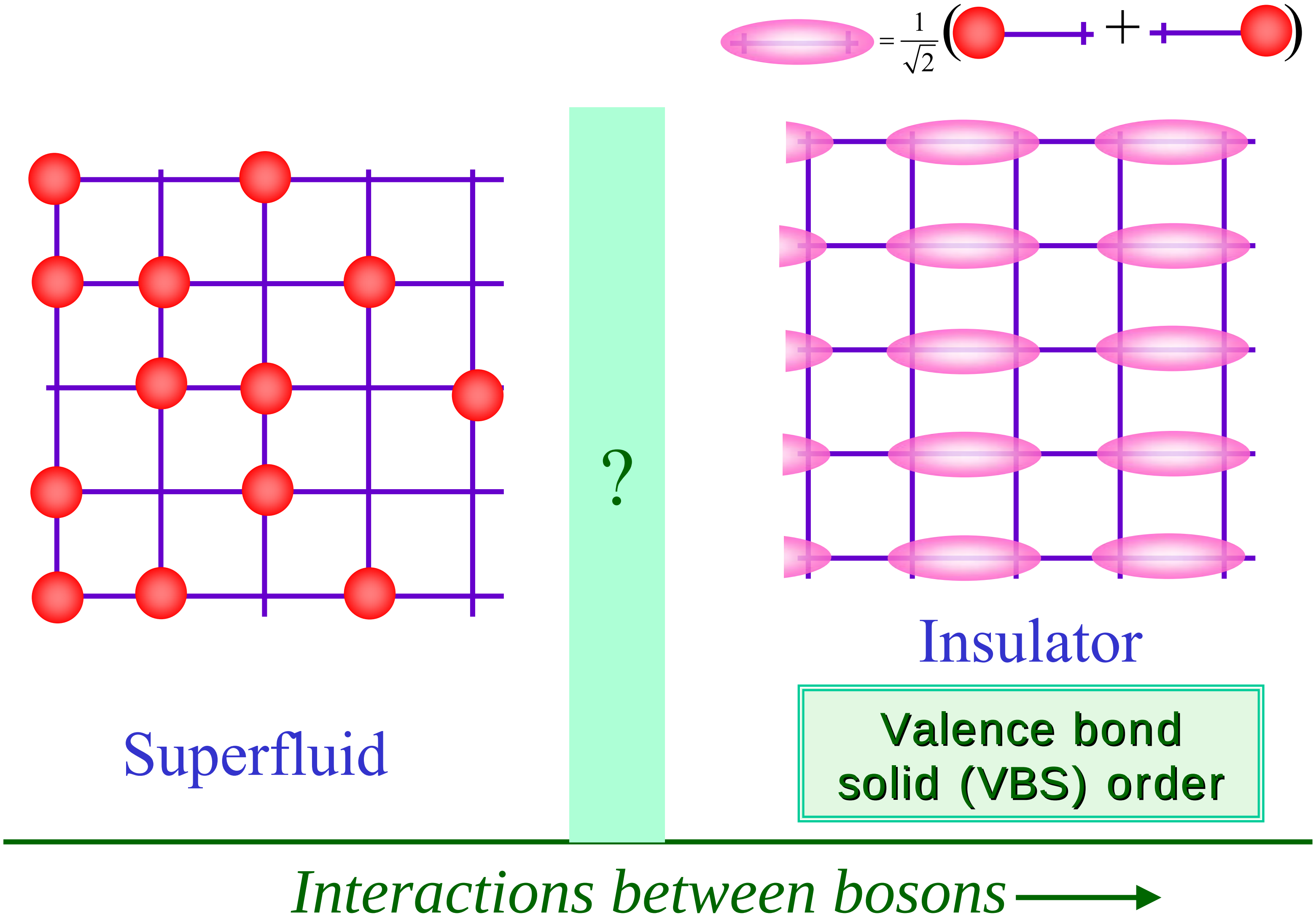


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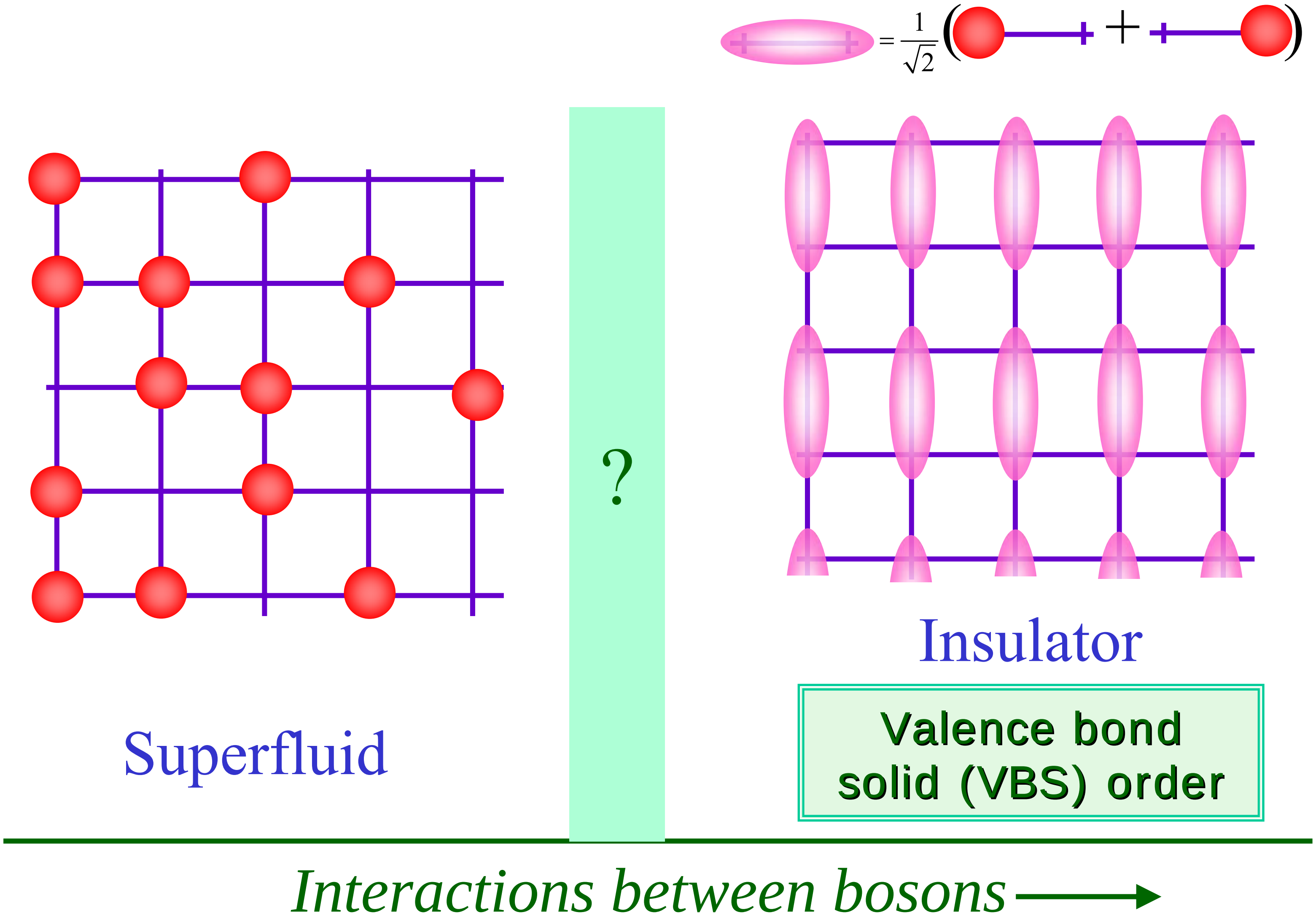


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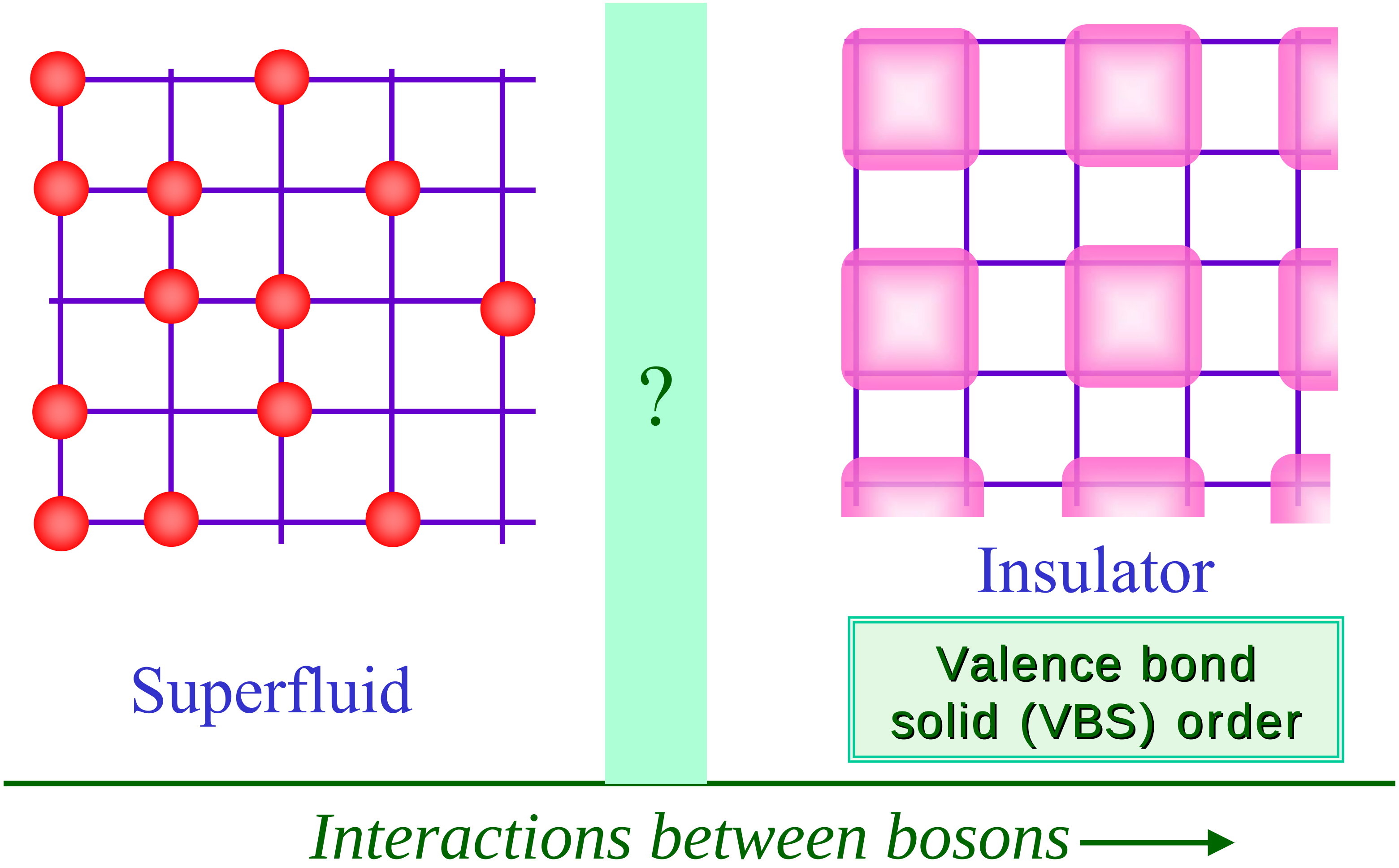


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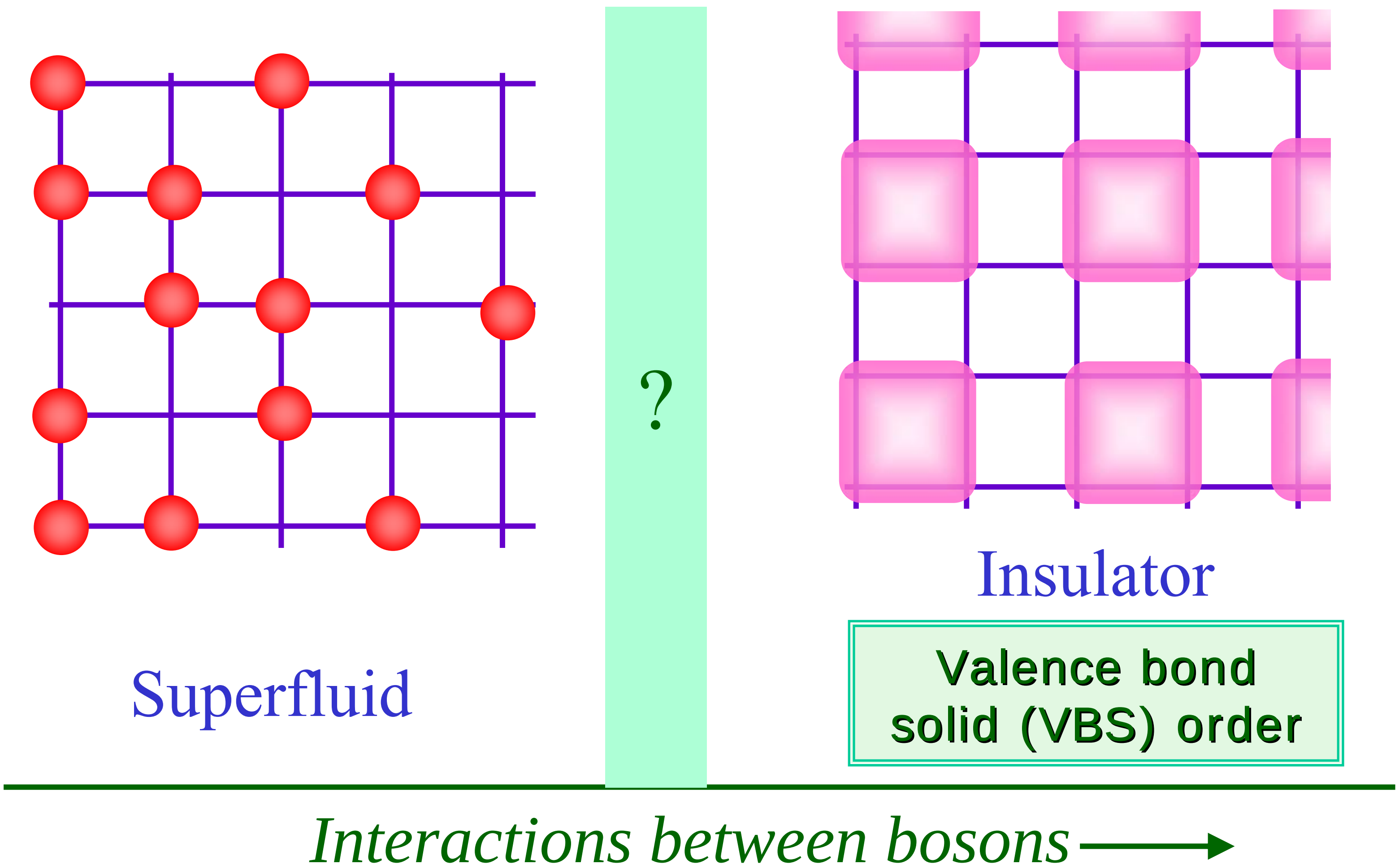


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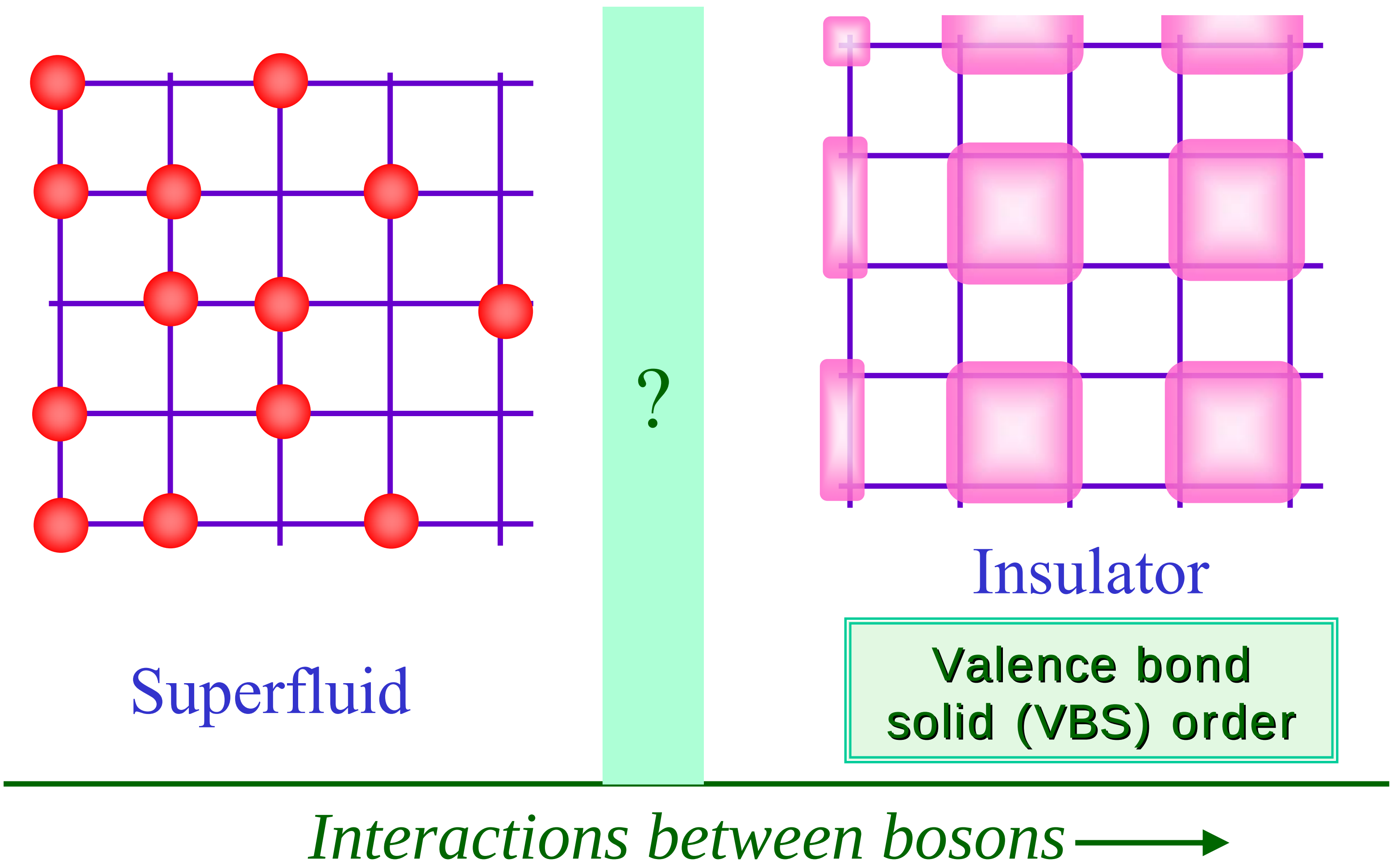


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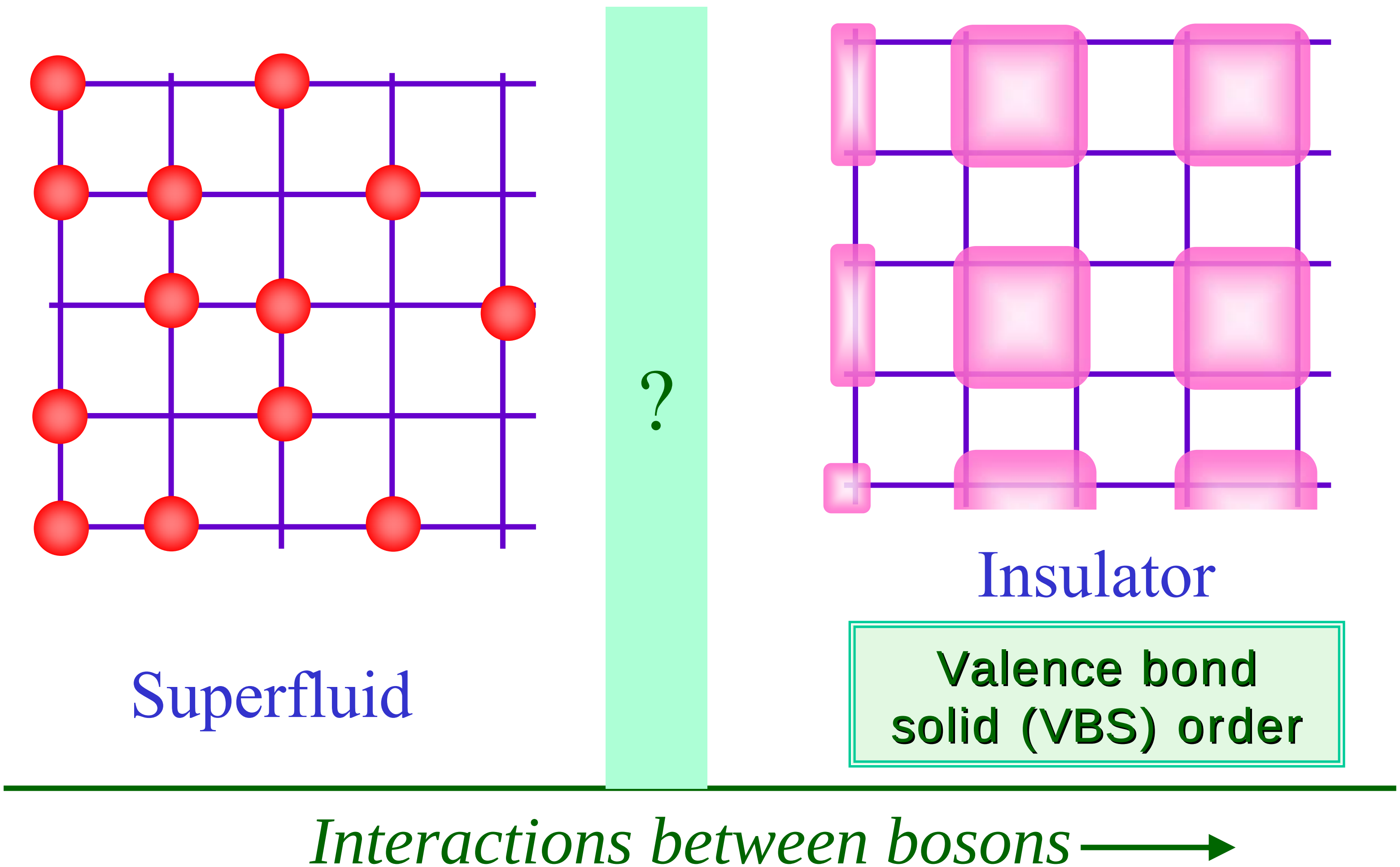


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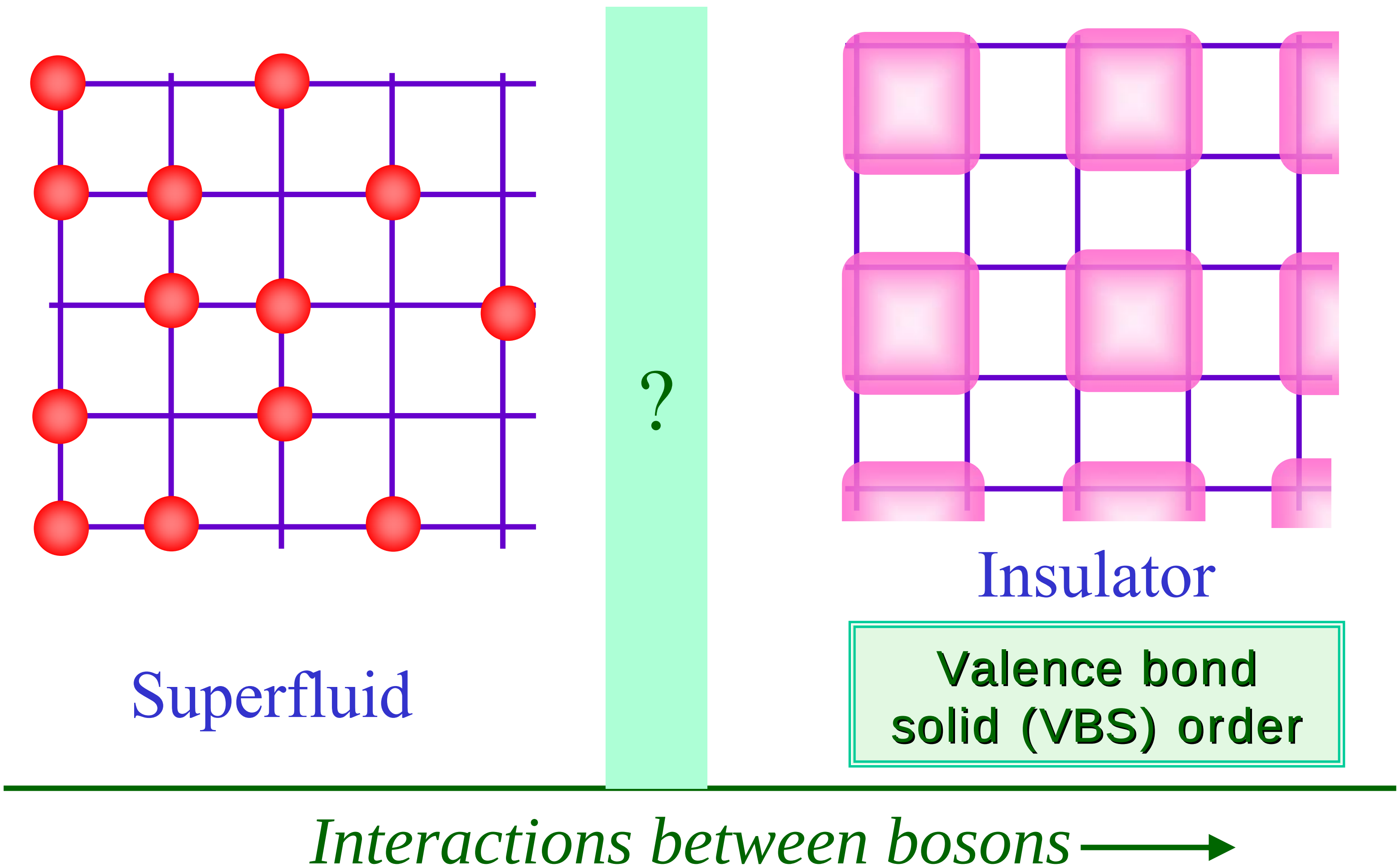


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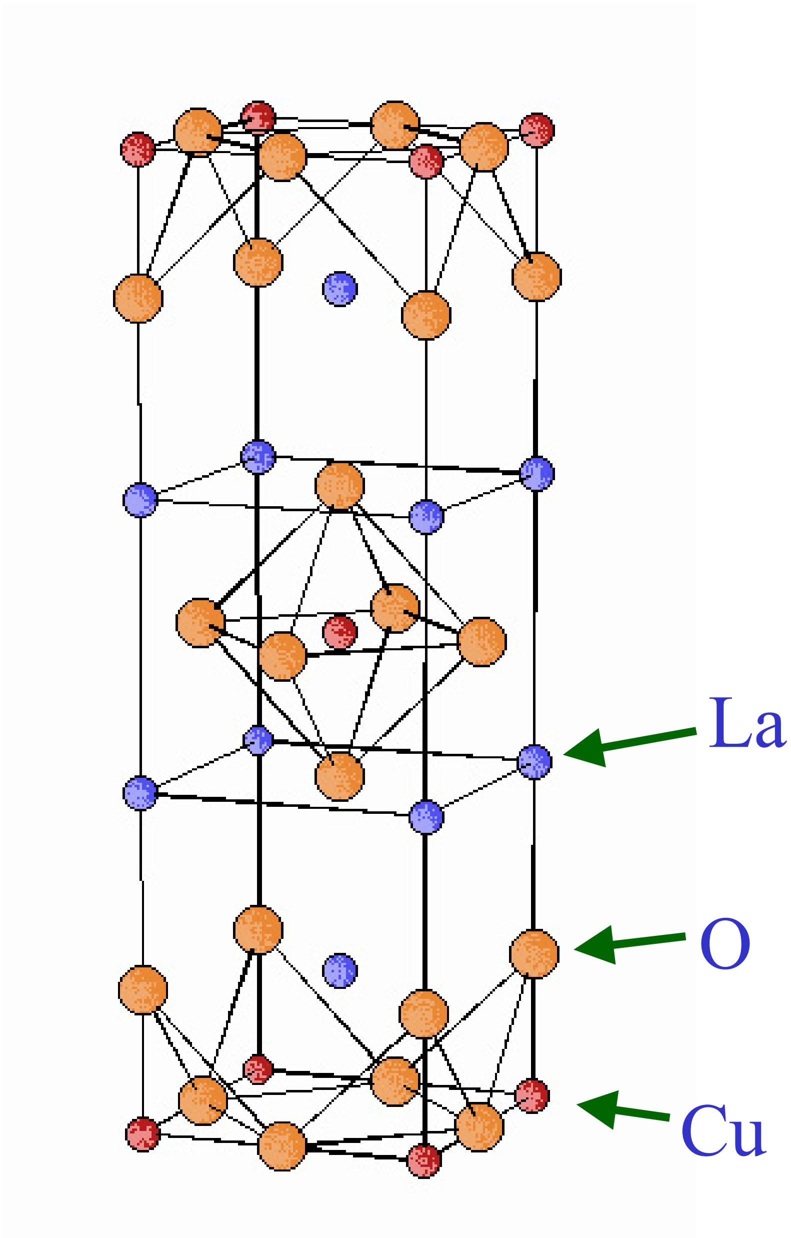
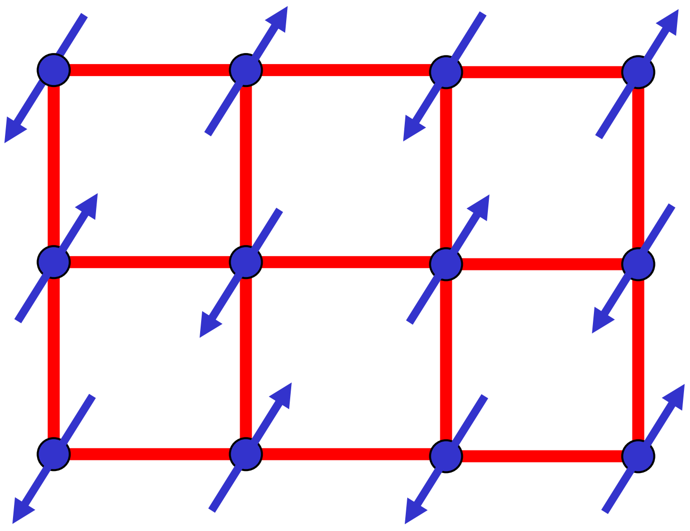
# III. The cuprate superconductors and their proximity to a superfluid-insulator transition

$La_2CuO_4$



# $La_2CuO_4$

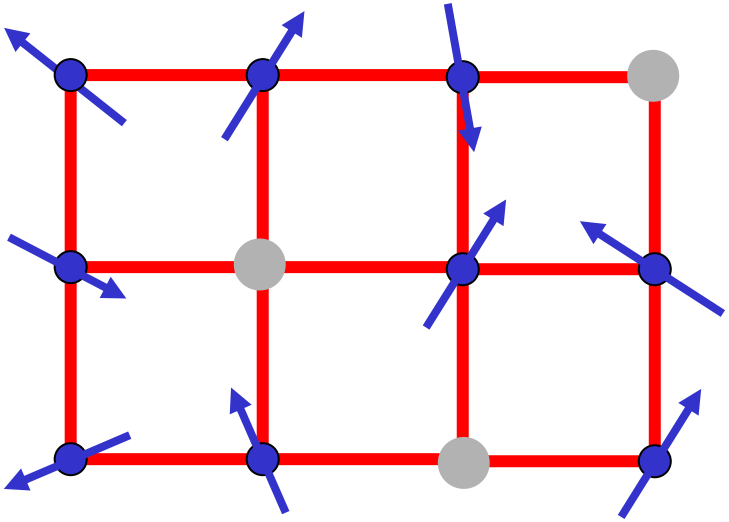
## Mott insulator: square lattice antiferromagnet



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

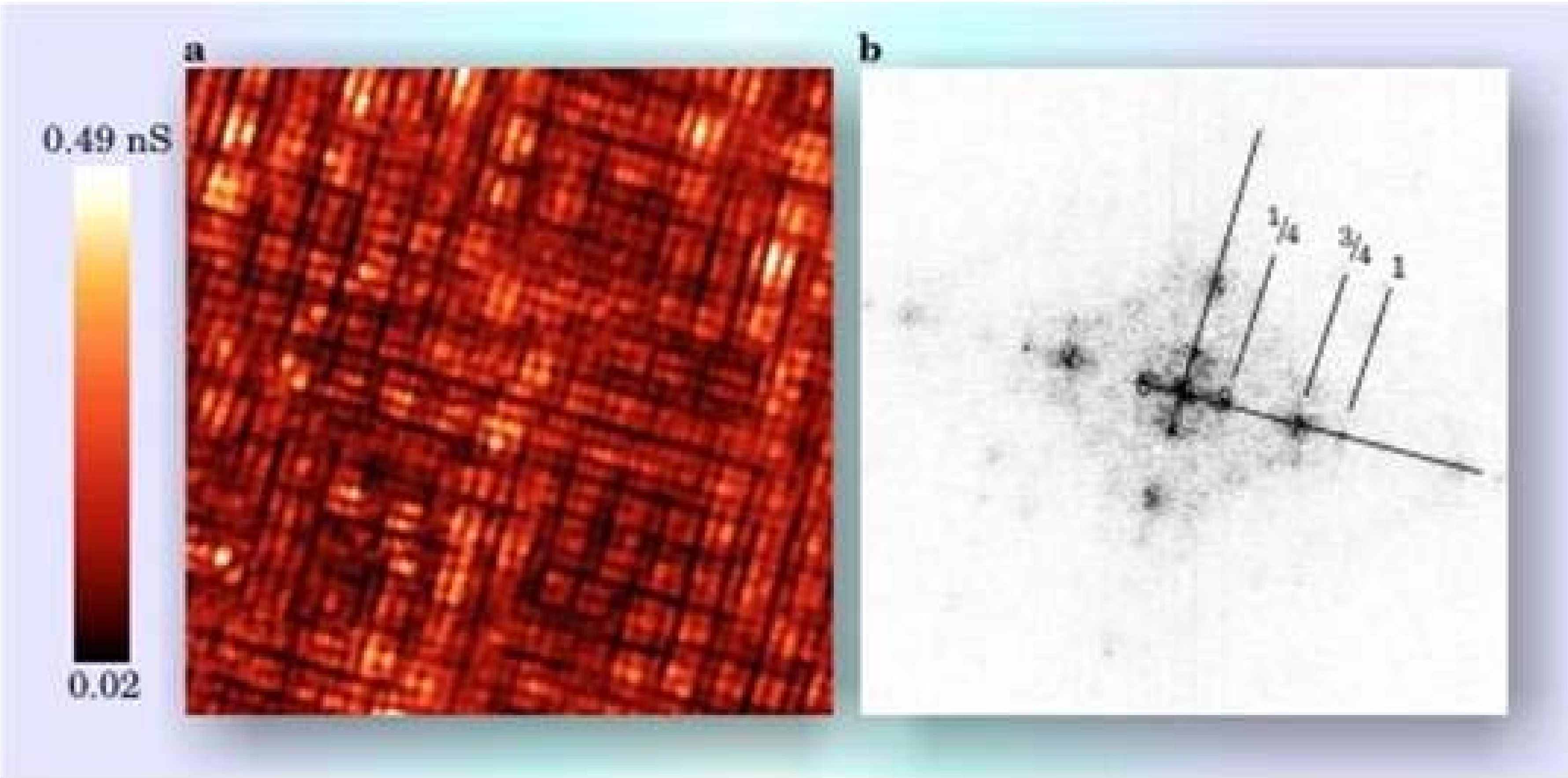
## $La_{2-\delta}Sr_{\delta}CuO_4$

Superfluid: condensate of paired holes



$$\langle \vec{S} \rangle = 0$$

# The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$




T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

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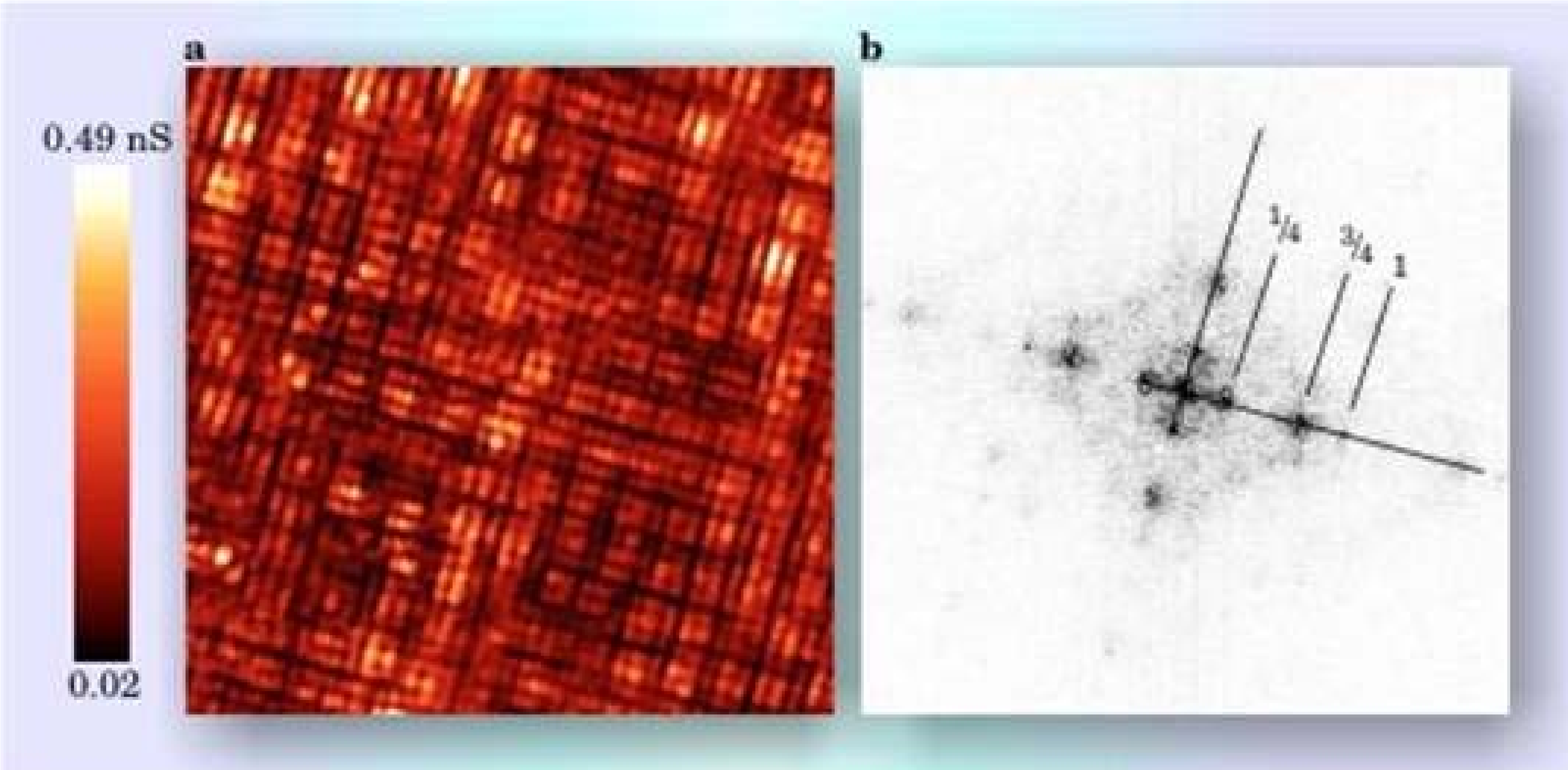



Evidence that holes can form an insulating state with period ≈ 4 modulation in the density


T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

Experimental evidence of paired hole states in model high-$T_c$ compounds


A. Rusydi,[1,2,3] P. Abbamonte,[1,4] M. Berciu,[5] S. Smadici,[1,4] H. Eisaki,[6]
Y. Fujimaki,[7] S. Uchida,[7] M. Rübhausen,[3] and G. A. Sawatzky[5]

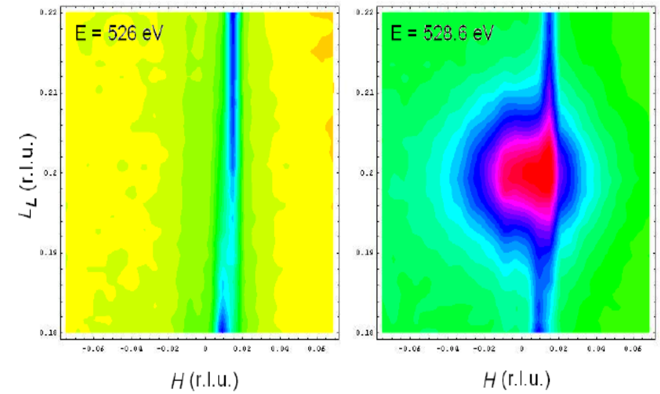
[1]*National Synchrotron Light Source, Brookhaven National Laboratory, Upton, NY, 11973-5000, USA*
[2]*Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[3]*Institut für Angewandte Physik, Univeristät Hamburg, Jungiusstraße 11, D-20355 Hamburg, Germany*
[4]*Physics Department and Frederick Seitz Materials Research Laboratory, University of Illinois, Urbana, IL, 61801*
[5]*Department of Physics and Astronomy, University of British Columbia, Vancouver, B.C., V6T-1Z1, Canada*
[6]*Nanoelectronics Research Institute, AIST, 1-1-1 Central 2, Umezono, Tsukuba, Ibaraki, 305-8568, Japan*
[7]*Department of Superconductivity, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan*



(Dated: May 17, 2006)

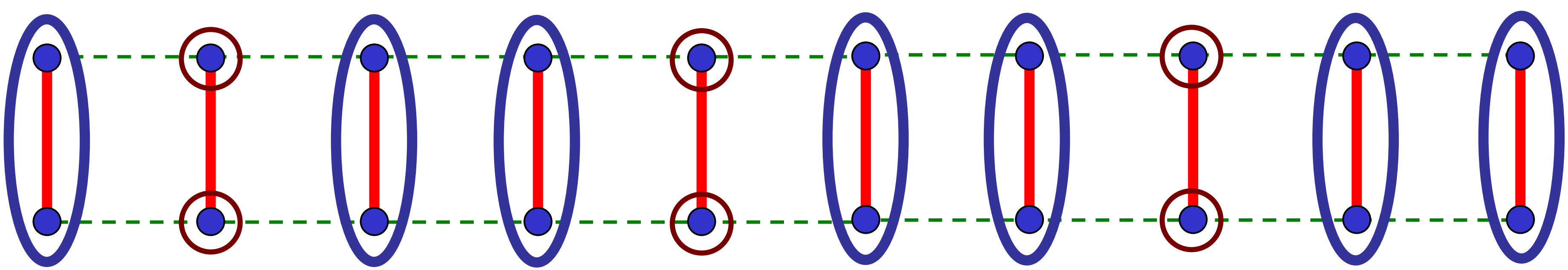


The distribution of holes in $Sr_{14-x}Ca_xCu_{24}O_{41}$ (SCCO) is revisited with semi-emperical reanalysis of the x-ray absorption (XAS) data and exact-diagonalized cluster calculations. A new interpretation of the XAS data leads to much larger ladder hole densities than previously suggested. These new hole densities lead to a simple interpretation of the hole crystal (HC) recently reported with 1/3 and 1/5 wave vectors along the ladder. Our interpretation is consistent with paired holes in the rung of the ladders. Exact diagonalization results for a minimal model of the doped ladders suggest that the stabilization of spin structures consisting of 4 spins in a square plaquette as a result of resonance valence bond (RVB) physics suppresses the hole crystal with a 1/4 wave vector.


*$Sr_{24-x}Ca_xCu_{24}O_{41}$*

*Nature* **431**, 1078 (2004); cond-mat/0604101



Resonant X-ray scattering evidence that the modulated state has one hole pair per unit cell.

Experimental evidence of paired hole states in model high-$T_c$ compounds

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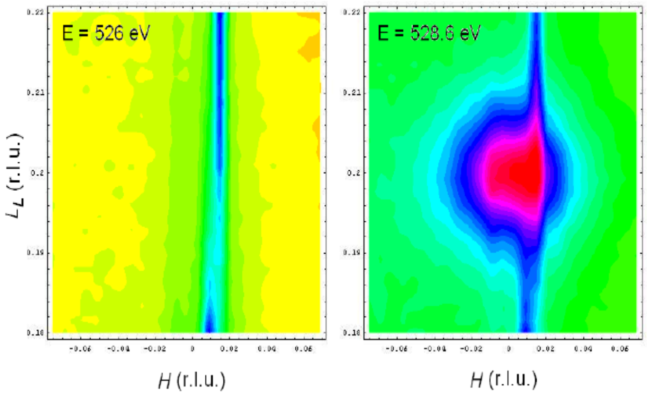
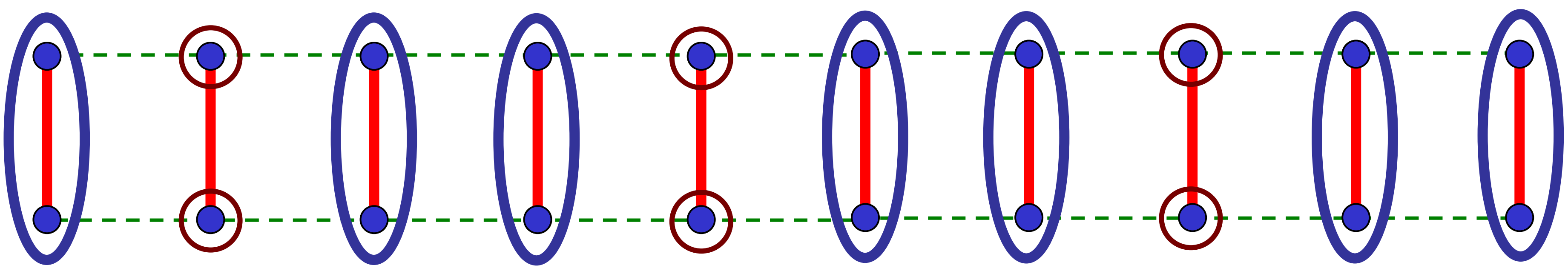
(Dated: May 17, 2006)

The distribution of holes in $Sr_{14-x}Ca_xCu_{24}O_{41}$ (SCCO) is revisited with semi-emperical reanalysis of the x-ray absorption (XAS) data and exact-diagonalized cluster calculations. A new interpretation of the XAS data leads to much larger ladder hole densities than previously suggested. These new hole densities lead to a simple interpretation of the hole crystal (HC) recently reported with 1/3 and 1/5 wave vectors along the ladder. Our interpretation is consistent with paired holes in the rung of the ladders. Exact diagonalization results for a minimal model of the doped ladders suggest that the stabilization of spin structures consisting of 4 spins in a square plaquette as a result of resonance valence bond (RVB) physics suppresses the hole crystal with a 1/4 wave vector.




$Sr_{24-x}Ca_xCu_{24}O_{41}$

*Nature* **431**, 1078 (2004); cond-mat/0604101



*Similar to the superfluid-insulator transition of bosons at fractional filling*

# Outline

I. Bose-Einstein condensation and superfluidity.

II. The superfluid-insulator quantum phase transition.

III. The cuprate superconductors, and their proximity to a superfluid-insulator transition.

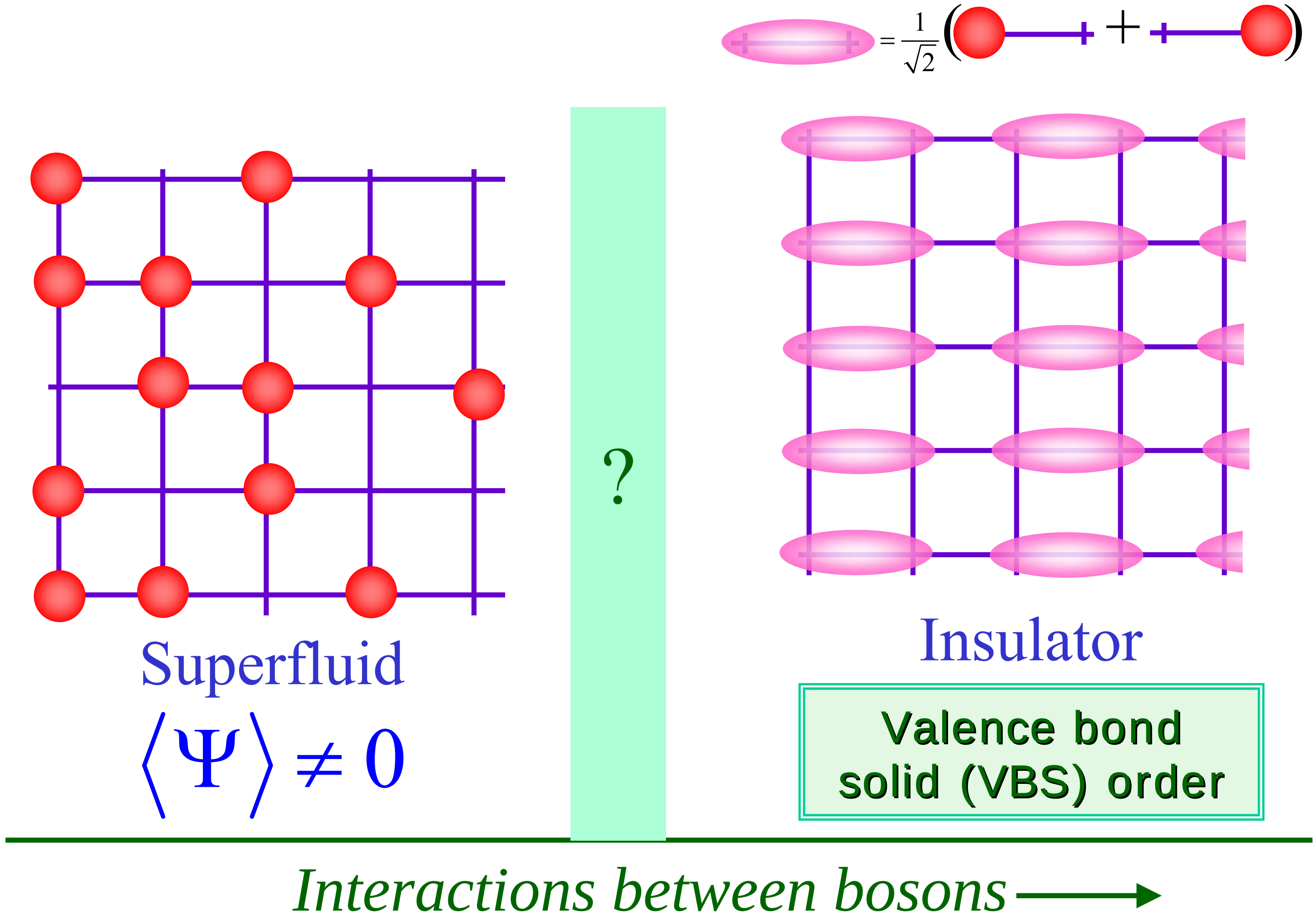
IV. Landau-Ginzburg-Wilson theory of the superfluid-insulator transition.

V. Beyond the LGW paradigm: continuous quantum transitions with multiple order parameters.

VI. Experimental tests in the cuprates.

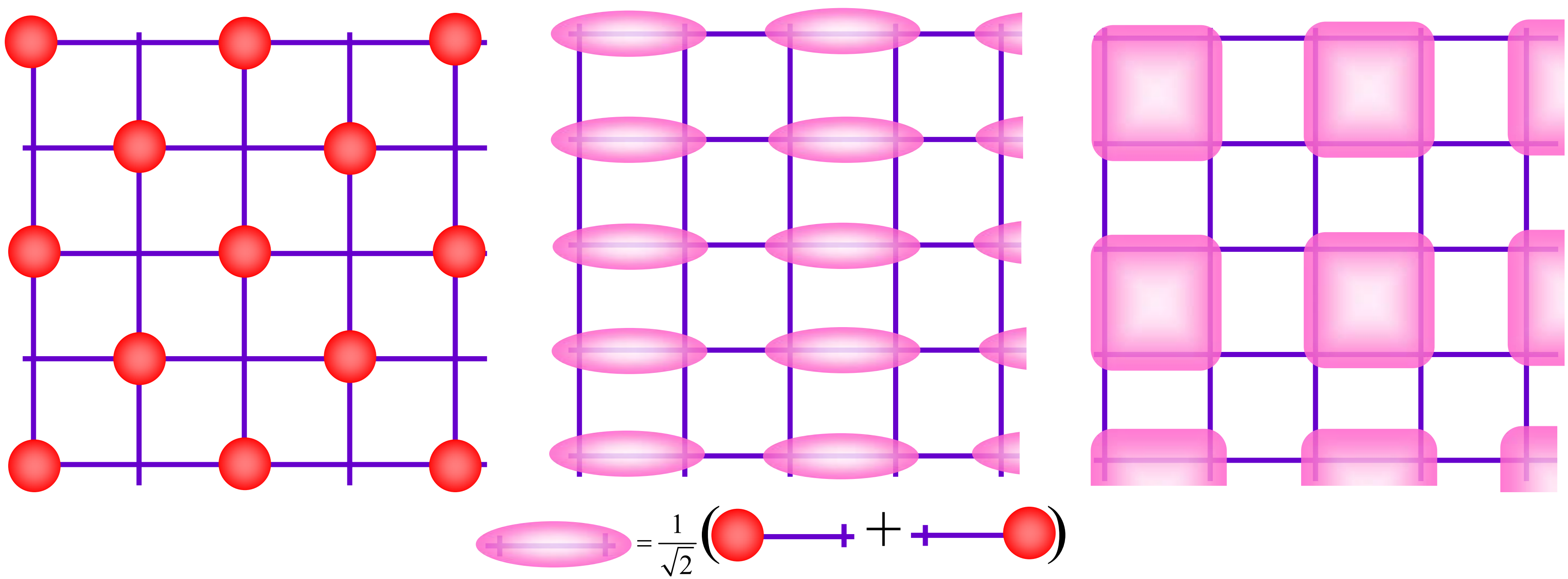
# IV. Landau-Ginzburg-Wilson theory of the superfluid-insulator transition

# *Bosons on the square lattice at filling fraction f=1/2*




N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

# Insulating phases of bosons at filling fraction $f = 1/2$



**Charge density wave (CDW) order**

**Valence bond solid (VBS) order**

**Valence bond solid (VBS) order**

Can define a common CDW/VBS order using a generalized "density" $\rho(\boldsymbol{r}) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\boldsymbol{Q}} \rangle \neq 0$ for certain $\boldsymbol{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

## Landau-Ginzburg-Wilson approach to multiple order parameters:

$$F = F_{sc}\left[\Psi\right] + F_{\text{charge}}\left[\rho_{\mathbf{Q}}\right] + F_{\text{int}}$$

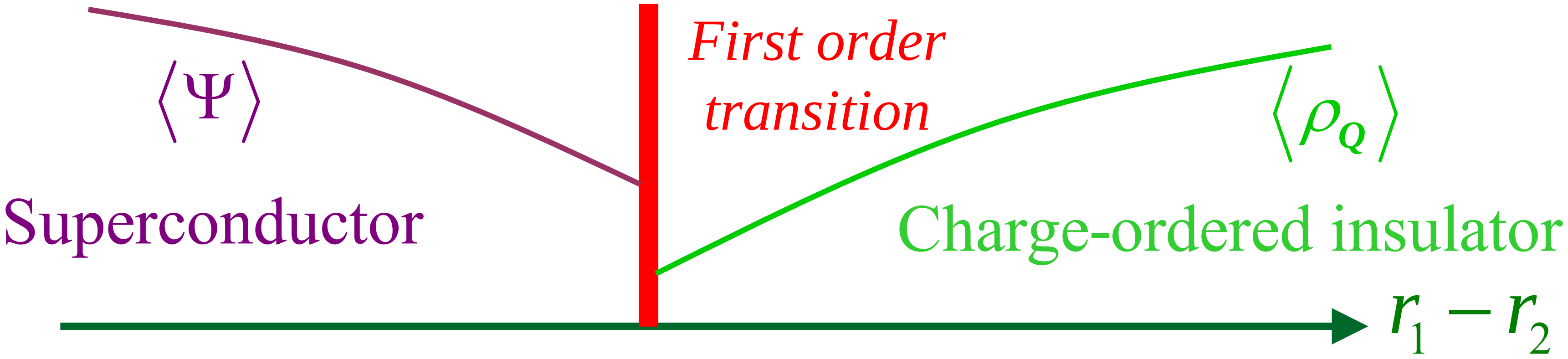
$$F_{sc}\left[\Psi\right] = r_1\left|\Psi\right|^2 + u_1\left|\Psi\right|^4 + \cdots$$

$$F_{\text{charge}}\left[\rho_{\mathbf{Q}}\right] = r_2\left|\rho_{\mathbf{Q}}\right|^2 + u_2\left|\rho_{\mathbf{Q}}\right|^4 + \cdots$$

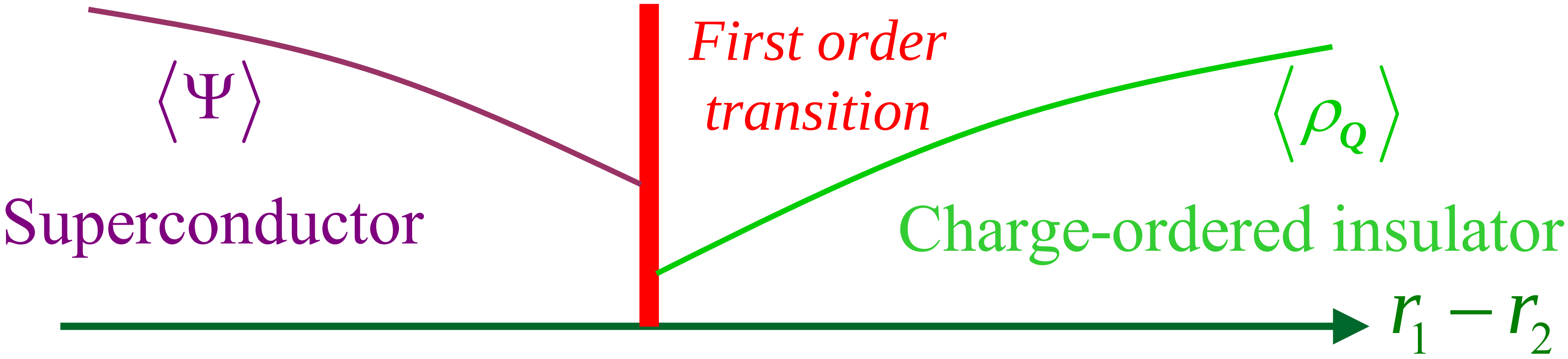
$$F_{\text{int}} = v\left|\Psi\right|^2\left|\rho_{\mathbf{Q}}\right|^2 + \cdots$$

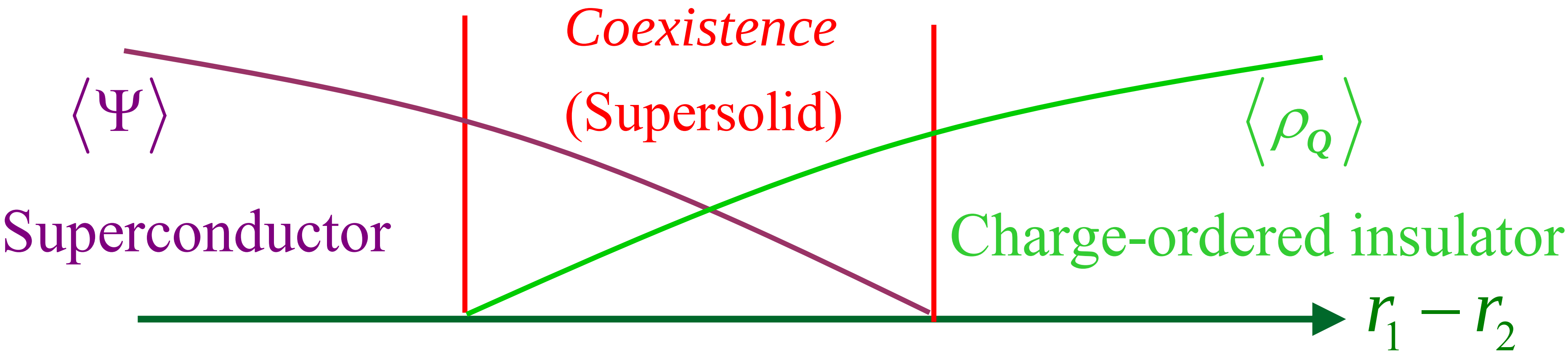
Distinct symmetries of order parameters permit couplings only between their energy densities

# Predictions of LGW theory

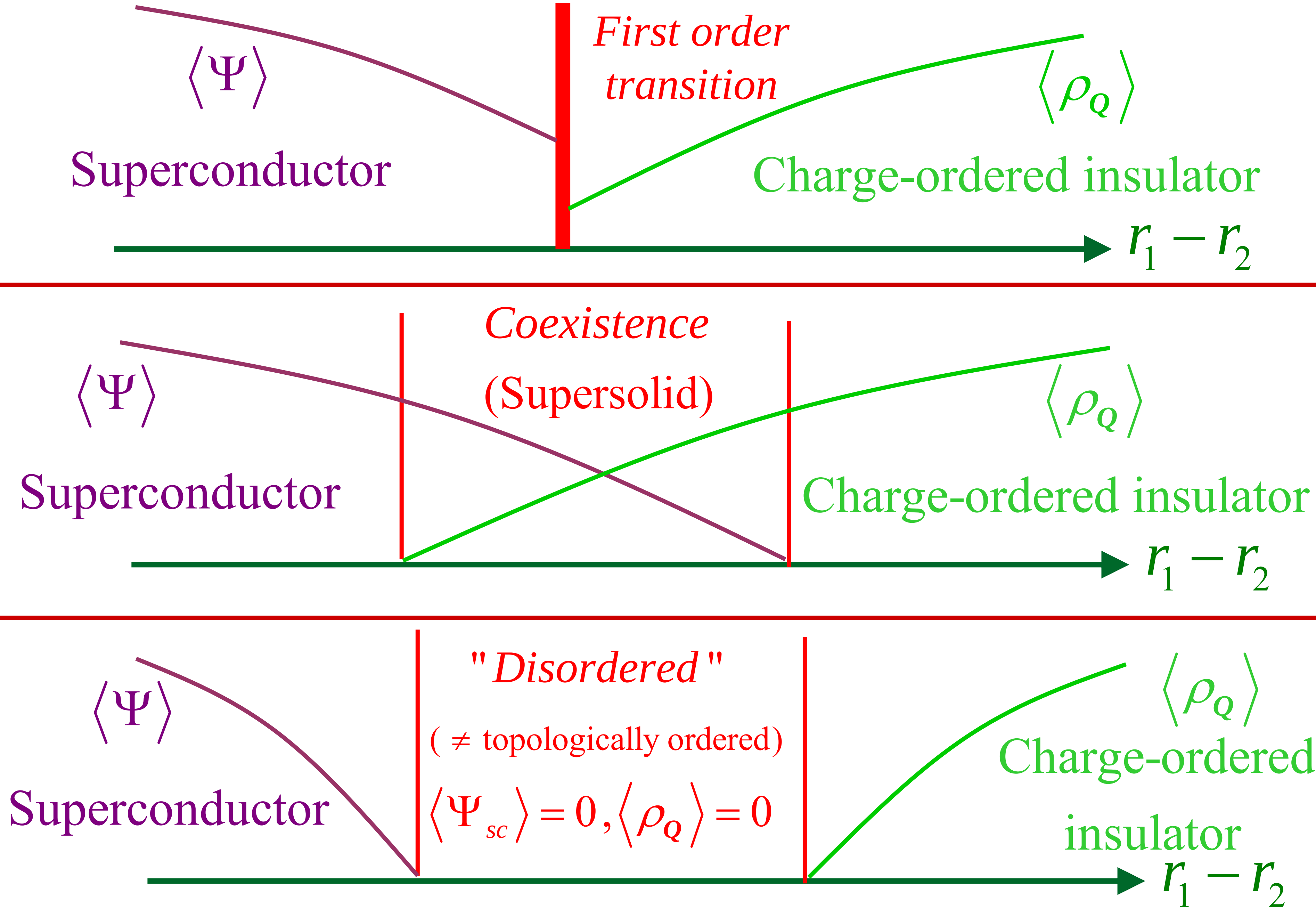


# Predictions of LGW theory

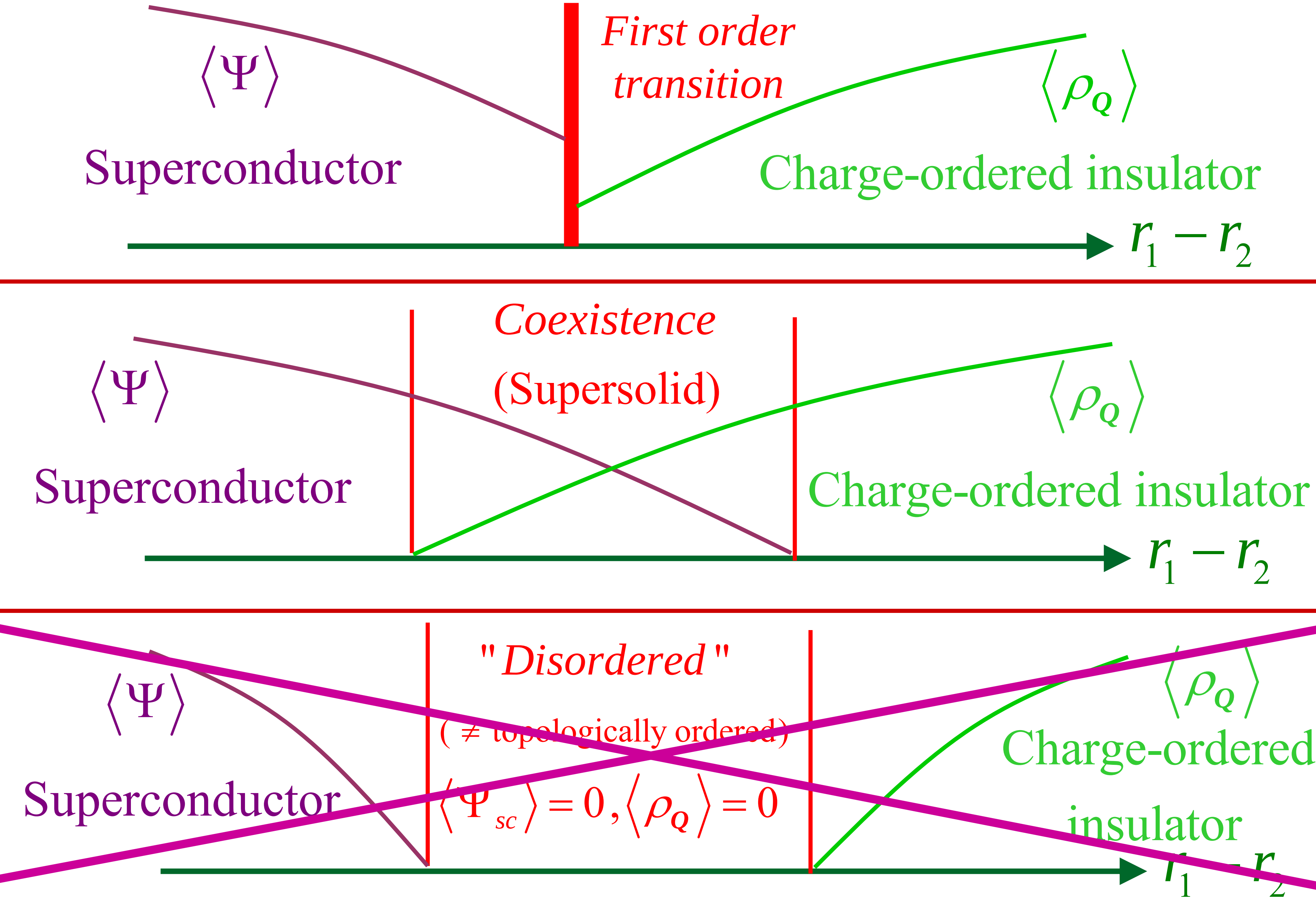


# Predictions of LGW theory



# Predictions of LGW theory



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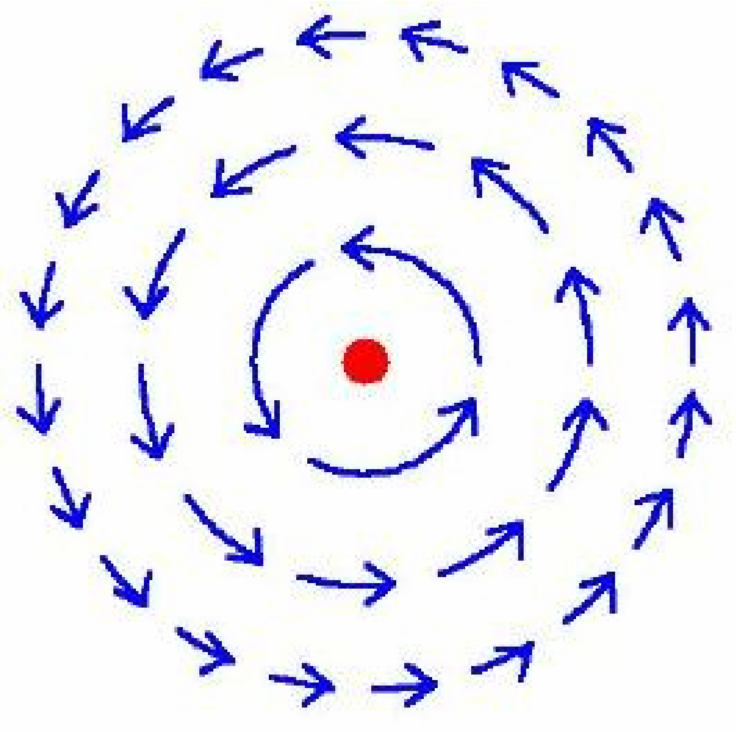
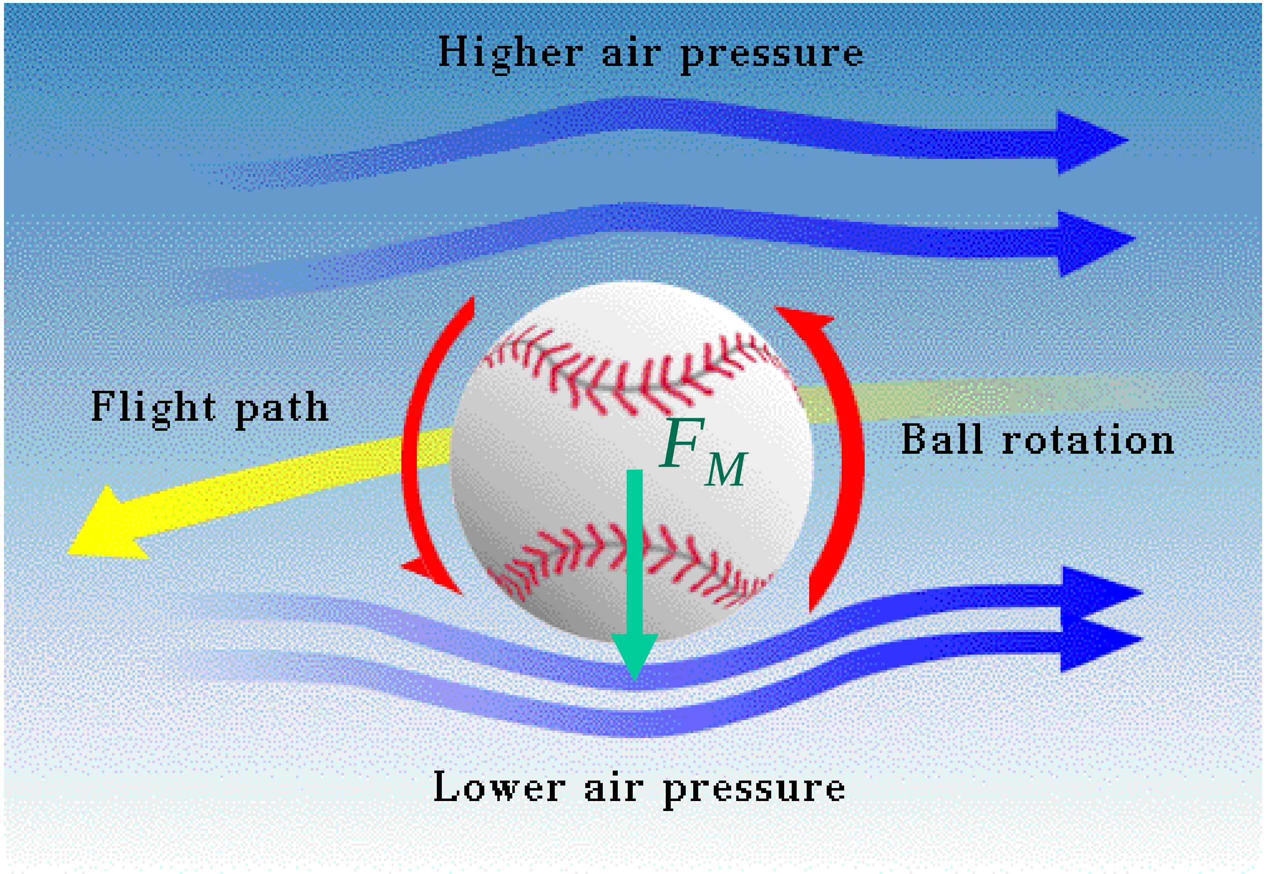
# V. Beyond the LGW paradigm: continuous transitions with multiple order parameters

# Excitations of the superfluid: **Vortices and anti-vortices**



**Central question**:
In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these "particles" ?

# In ordinary fluids, vortices experience the Magnus Force




$$F_M = (\text{mass density of air}) \bullet (\text{velocity of ball}) \bullet (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned} \mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right) \\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}} \end{aligned}$$

where $\rho$ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vertex in a superfluid, this is

$$\mathbf{F}_M = (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s\cdot d\mathbf{r}\right)$$
$$= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}$$
$$= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt}\times\mathbf{B}\right)$$

where $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

**Dual picture**:
The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = *h*×(number density of Bose particles)

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989)

Let the Hamiltonian of a single vortex be $\mathcal{H}_v$.

In general, this is a very complicated object, but we can obtain all needed information by symmetry considerations.

The Hamiltonian $\mathcal{H}_v$ should commute with $T_x$, the operator which translates the square lattice by one site in the $x$ direction (and similarly for $T_y$):

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

However, $T_x$ and $T_y$ do not commute with each other.

Under translation along a distance $\mathbf{s}$, a vortex picks up a a Aharanov-Bohm phase factor $\exp\left(i\int_0^{\mathbf{s}} d\mathbf{r}\cdot\mathbf{A}\right)$.

Consequently

$$\boxed{T_x T_y = \exp\left(i\phi\right) T_y T_x}$$

where $\phi$ is the dual "flux" through a unit cell, This "flux" has the value

$$\boxed{\phi = 2\pi f}$$

where $f$ is the filling fraction of bosons (Cooper pairs). We will consider the case of rational filling fraction $f = p/q$, where $p$, $q$ are relatively prime integers.

# *Bosons on the square lattice at filling fraction f=p/q*

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

# *Bosons on the square lattice at filling fraction f=p/q*

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

- **Any pinned vortex exhibits modulations in "density"-like observables at the wavevectors $\mathbf{Q} = (2\pi p/q)(m, n)$ over the region in which the vortex executes its quantum zero-point motion.**

# *Bosons on the square lattice at filling fraction f=p/q*

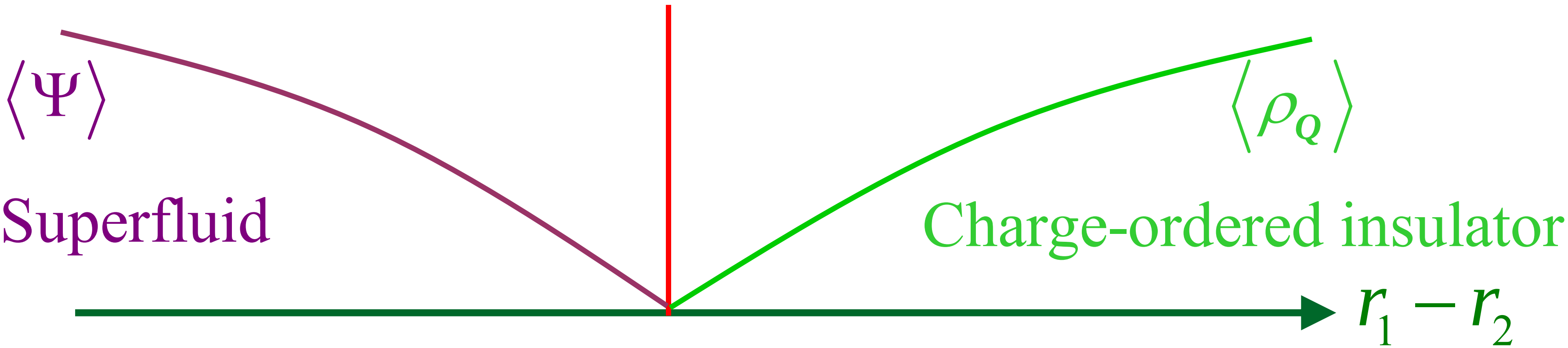
$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

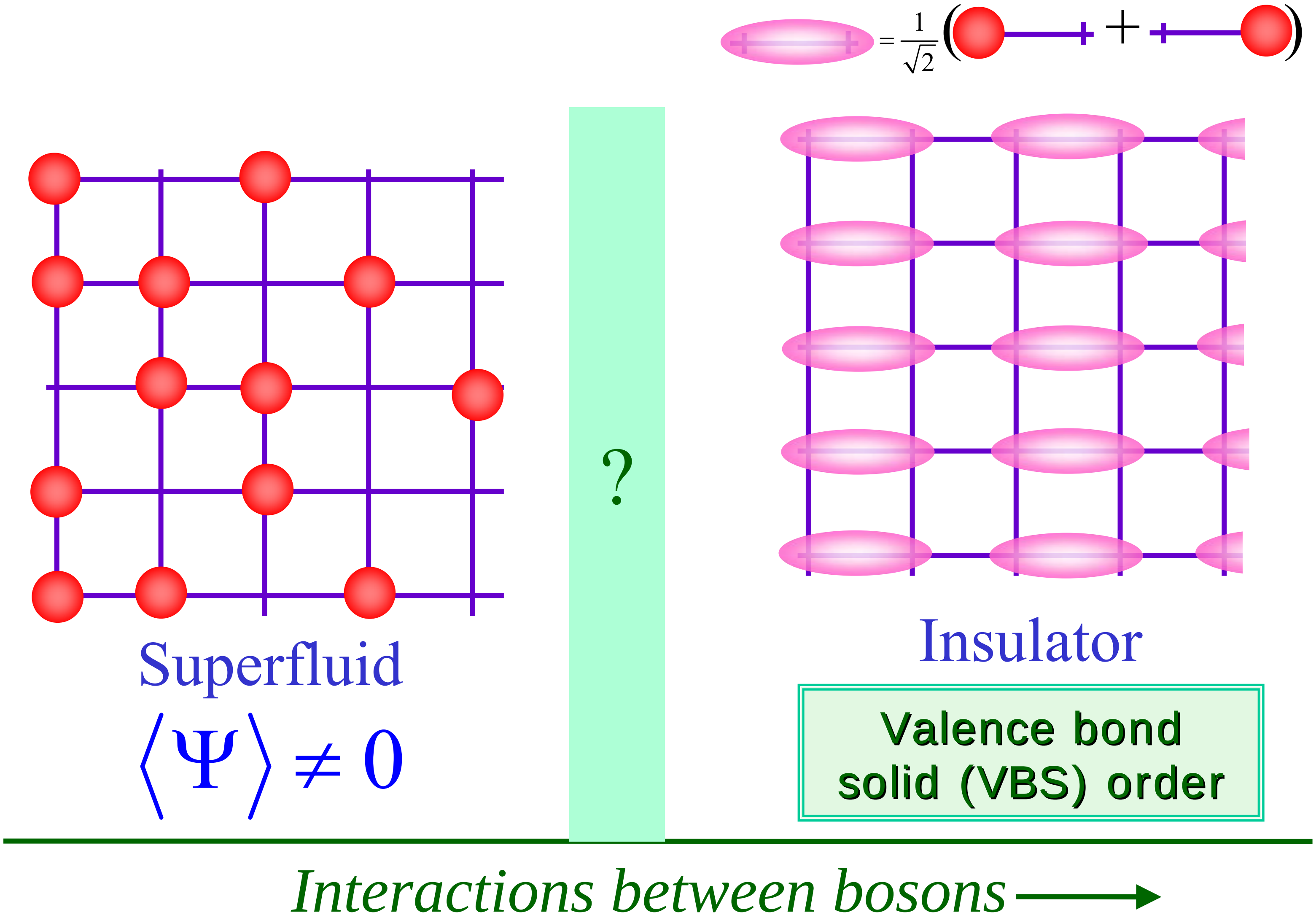
- **The "condensation" of vortices/anti-vortices transforms the superfluid to an insulator with density modulations, and an integer number of bosons per unit cell.**

# A Landau-forbidden continuous transitions



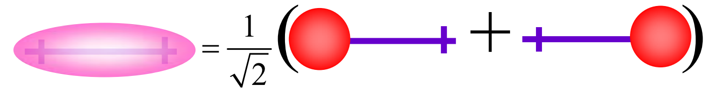

Vortices in the superfluid have associated quantum numbers which determine the local “charge order”, and their proliferation in the superfluid can lead to a continuous transition to a charge-ordered insulator

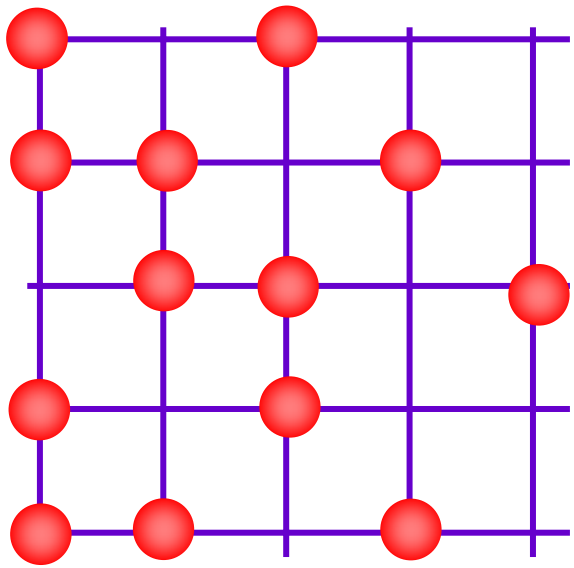
# Bosons on the square lattice at filling fraction f=1/2




N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

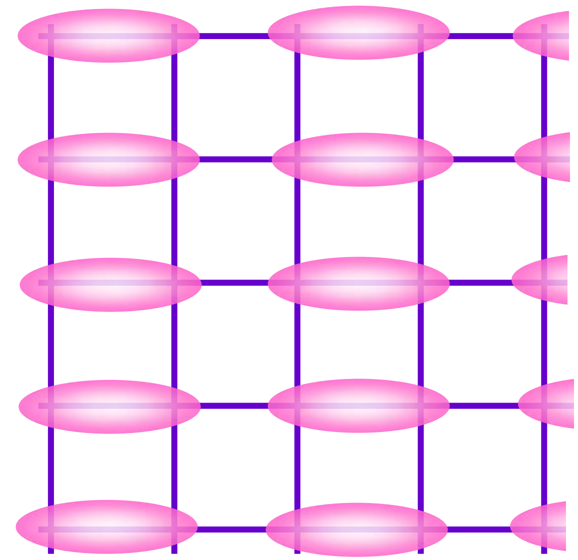
# *Bosons on the square lattice at filling fraction f=1/2*





Superfluid

$$\langle \Psi \rangle \neq 0$$



Insulator

**Valence bond solid (VBS) order**

*"Aharanov-Bohm" or "Berry" phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.*

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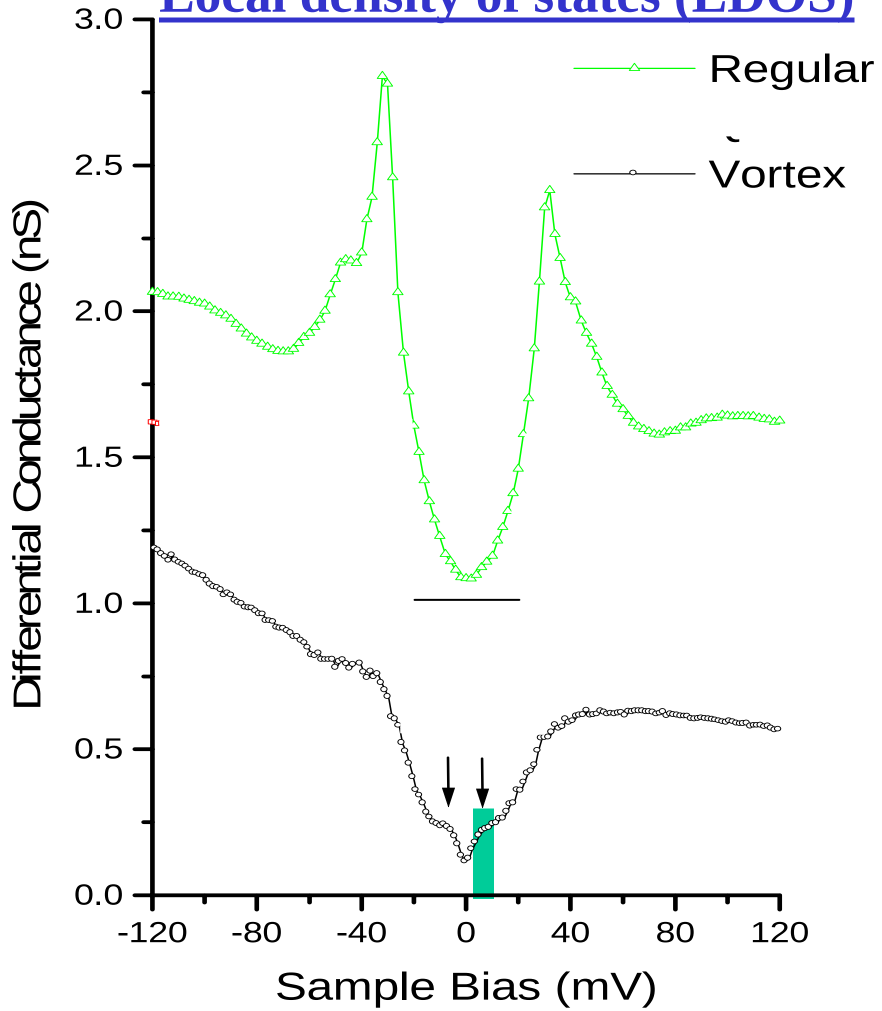
# STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

## Local density of states (LDOS)

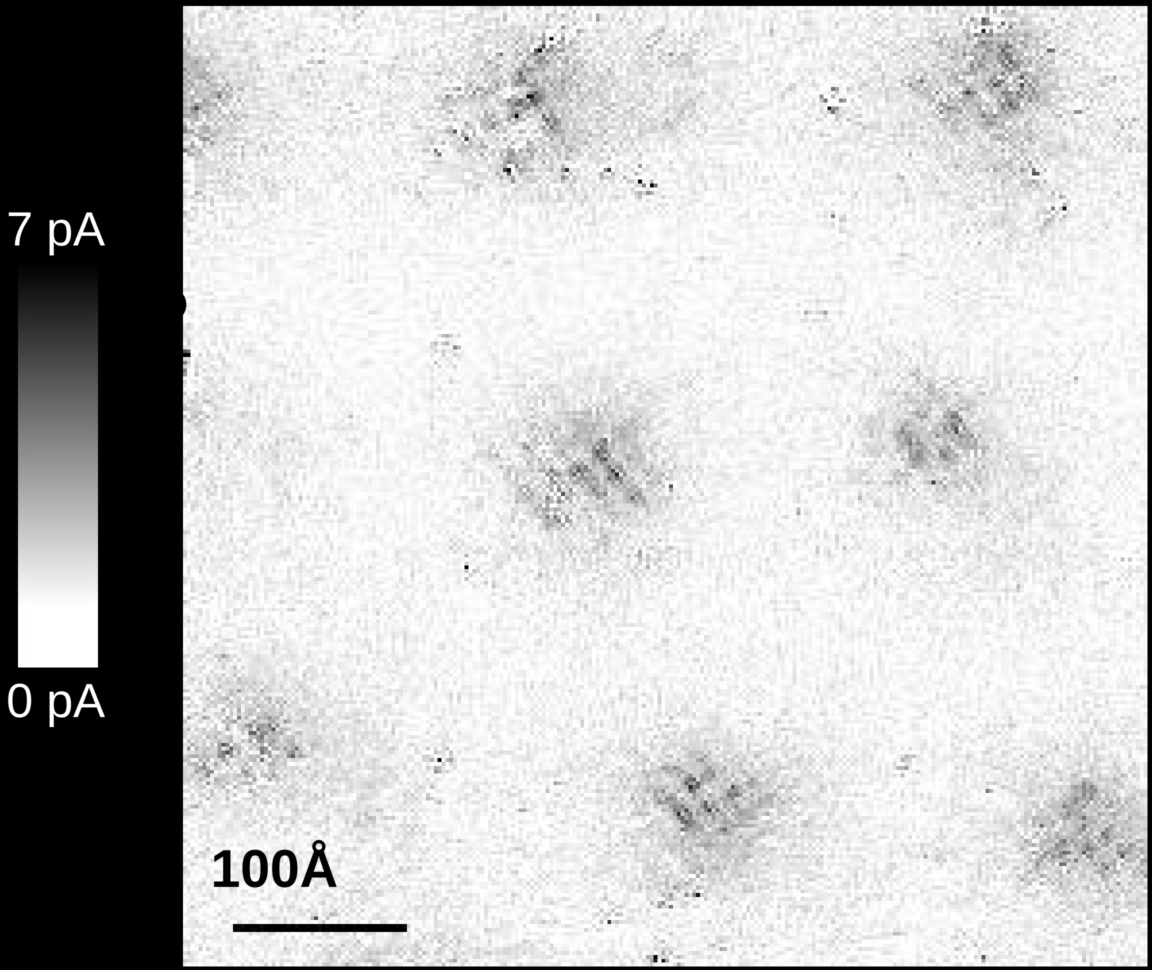



S.H. Pan *et al. Phys. Rev. Lett.* **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

( 1meV to 12 meV) at B=5 Tesla.

# Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K




Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

**J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).**

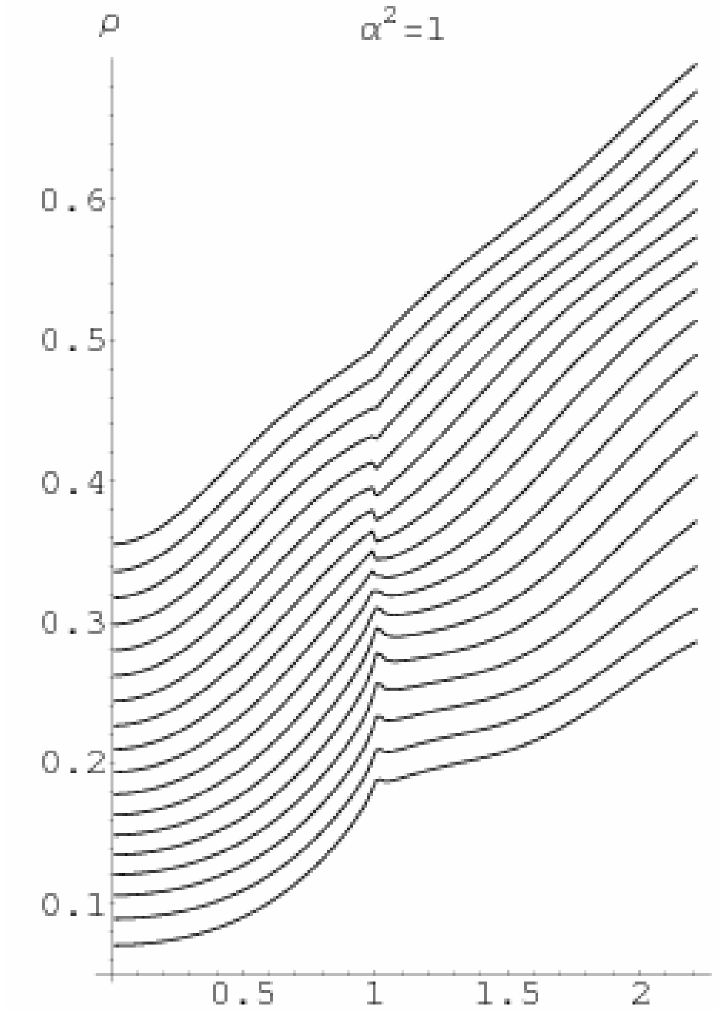
Prediction of periodic LDOS modulations near vortices: K. Park and S. Sachdev, Phys. Rev. B 64, 184510 (2001).

- **Vortex quantum zero-point motion leads to a natural explanation of STM experiments.**

- **Vortex quantum zero-point motion leads to a natural explanation of STM experiments.**

Using as <u>input</u> ($i$) the size of the "checkerboard halo" in STM as a measure of the zero-point motion radius of the vortex, and ($ii$) the forces between the vortices as determined from an estimate of the superfluid stiffness, we obtain as <u>output</u> an estimate of $m_v \approx 2 - 9m_e$ and the vortex oscillation frequency $\omega_v \approx 2 - 7$ meV.

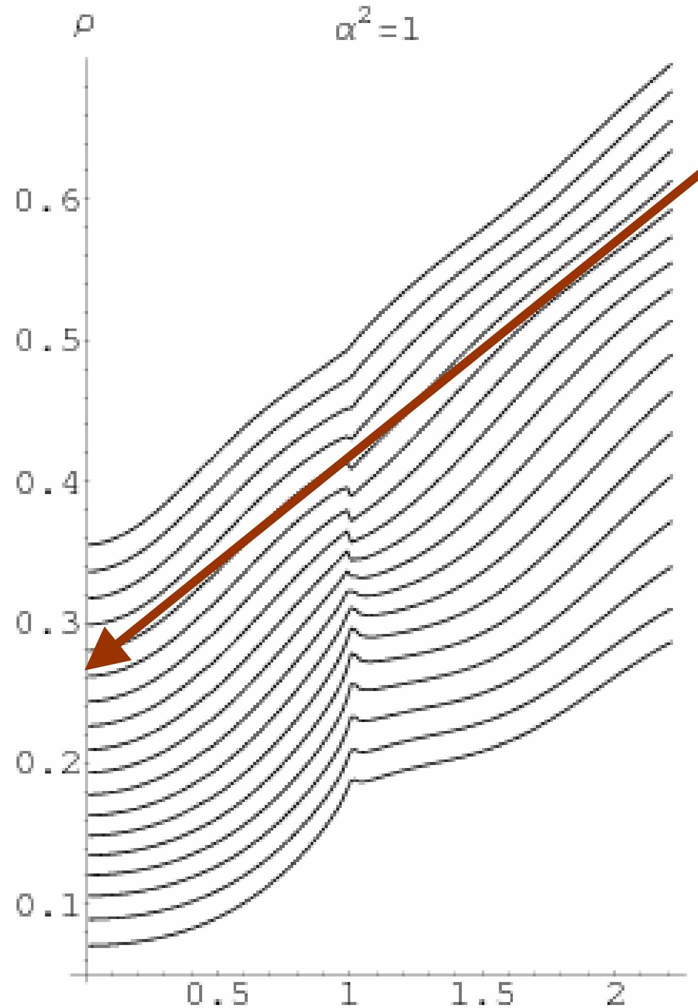
# *Influence of the quantum oscillating vortex on the LDOS*




$$\omega / \omega_v$$

$$\alpha^2 = \frac{mv_F^2}{\omega_v} = 1$$

# Influence of the quantum oscillating vortex on the LDOS

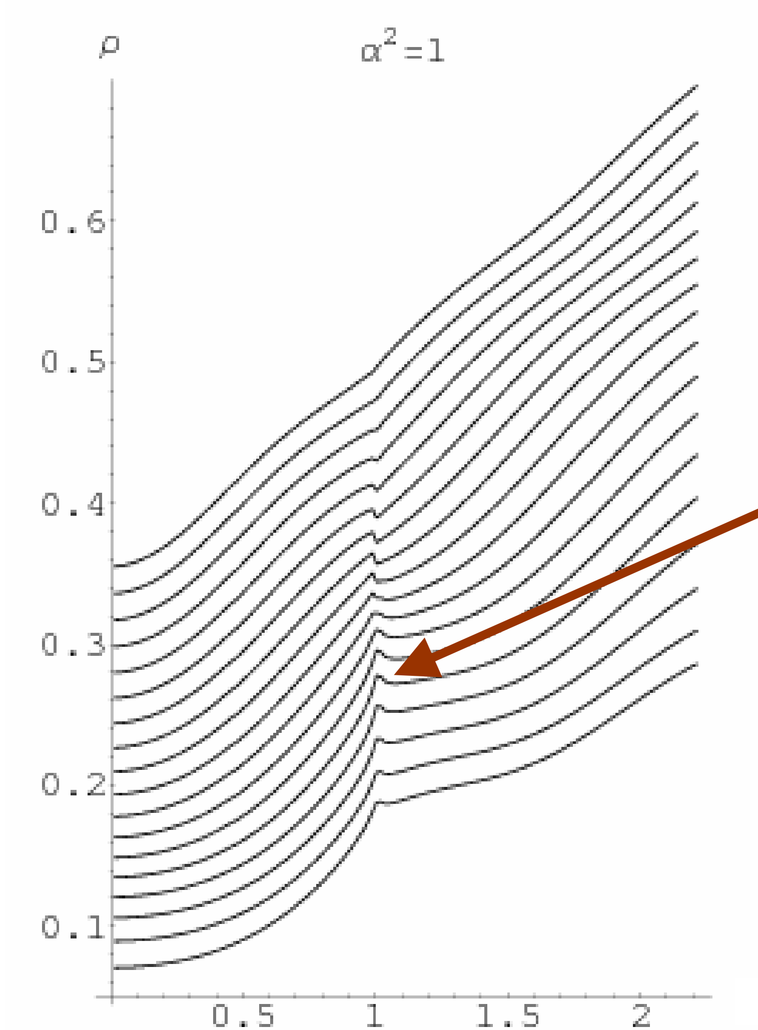



$\omega / \omega_v$

$$\alpha^2 = {mv_F^2}/{\omega_v} = 1$$

No zero bias peak.

# *Influence of the quantum oscillating vortex on the LDOS*

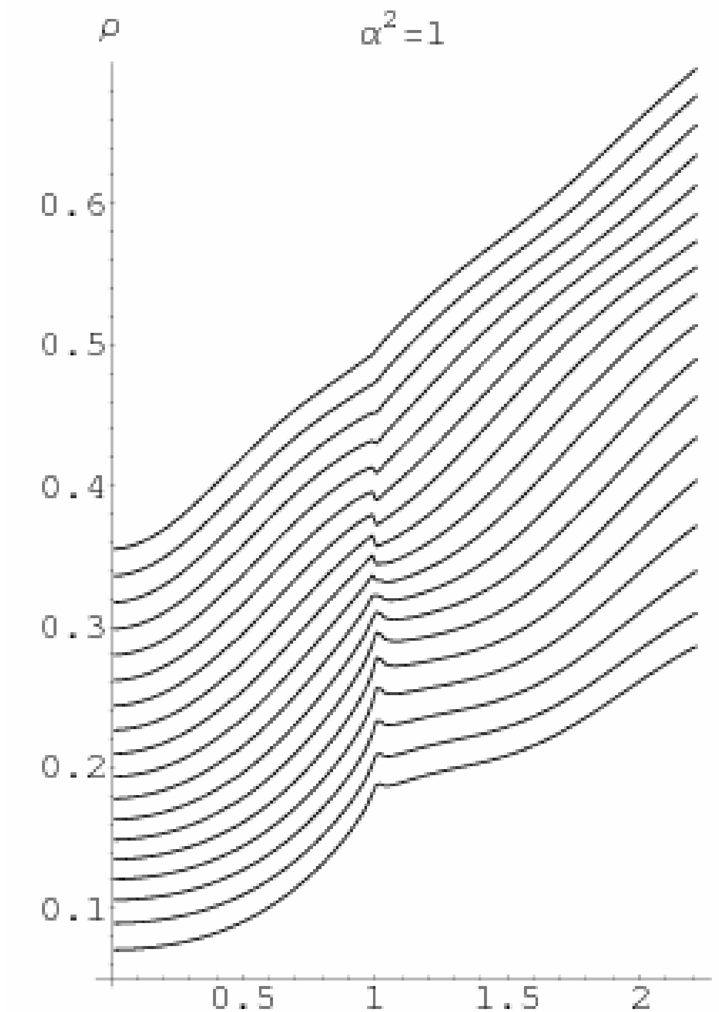



$$\omega / \omega_v$$

$$\alpha^2 = \frac{mv_F^2}{\omega_v} = 1$$

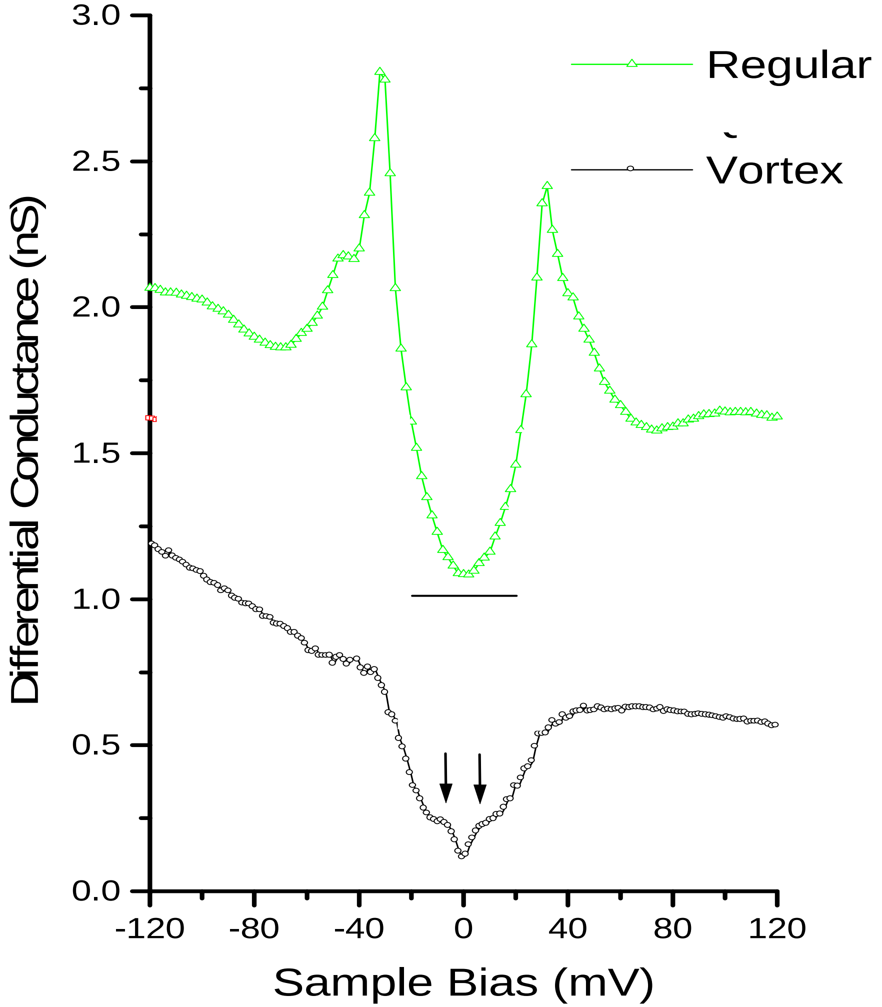
Resonant feature near the vortex oscillation frequency

# *Influence of the quantum oscillating vortex on the LDOS*




$\omega / \omega_v$

$$\alpha^2 = {mv_F^2}/{\omega_v} = 1$$




I. Maggio-Aprile *et al. Phys. Rev. Lett.* **75**, 2754 (1995)
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

# Conclusions

- Quantum zero point motion of the vortex provides a natural explanation for LDOS modulations observed in STM experiments.
- Size of modulation halo allows estimate of the inertial mass of a vortex
- Direct detection of vortex zero-point motion may be possible in inelastic neutron or light-scattering experiments
- The quantum zero-point motion of the vortices influences the spectrum of the electronic quasiparticles, in a manner consistent with LDOS spectrum
- "Aharanov-Bohm" or "Berry" phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.