Classifying two-dimensional superfluids: why there is more to cuprate superconductivity than the condensation of charge -2*e* Cooper pairs

cond-mat/0408329, cond-mat/0409470, and to appear

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Talk online: Google Sachdev

Outline

I. Bose-Einstein condensation and superfluidity

II. Vortices in the superfluid

Magnus forces, duality, and point vortices as dual "electric" charges

III. The superfluid-Mott insulator quantum phase transition

IV. Vortices in superfluids near the superfluid-insulator quantum phase transition

The Hofstadter Hamiltonian and vortex flavors

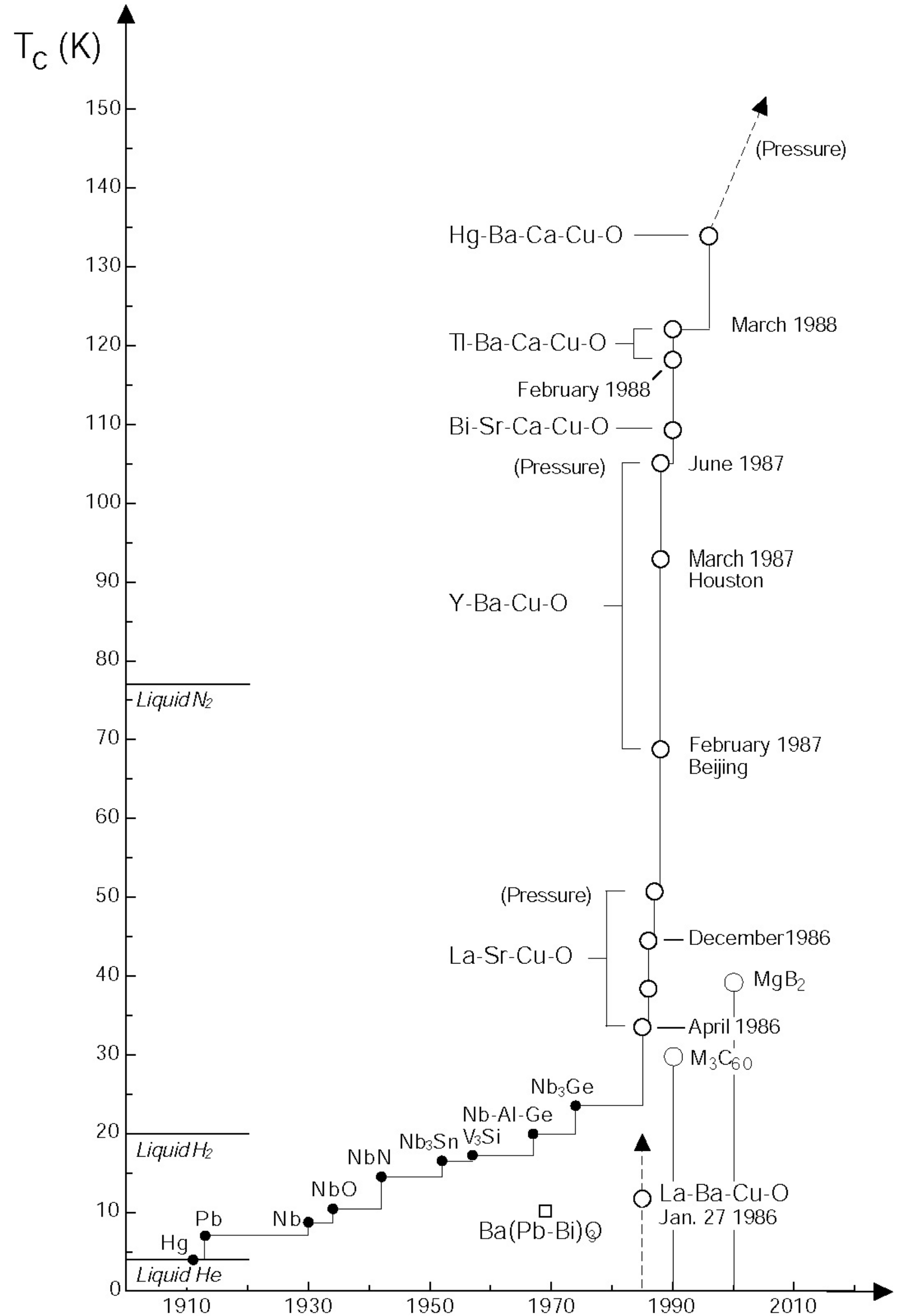
V. The cuprate superconductors

The "quantum order" of the superconducting state: evidence for vortex flavors

I. Bose-Einstein condensation and superfluidity

Superfluidity/superconductivity occur in:

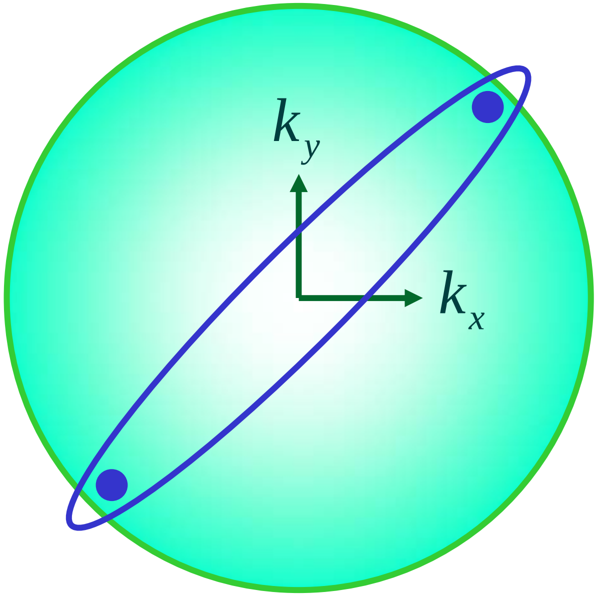
- liquid ^4He
- metals Hg, Al, Pb, Nb, Nb_3Sn…..
- liquid ^3He
- neutron stars
- cuprates $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{6+y}$….
- M_3C_{60}
- ultracold trapped atoms
- MgB_2



The Bose-Einstein condensate:

A macroscopic number of bosons occupy the lowest energy quantum state

Such a condensate also forms in systems of fermions, where the bosons are Cooper pairs of fermions:

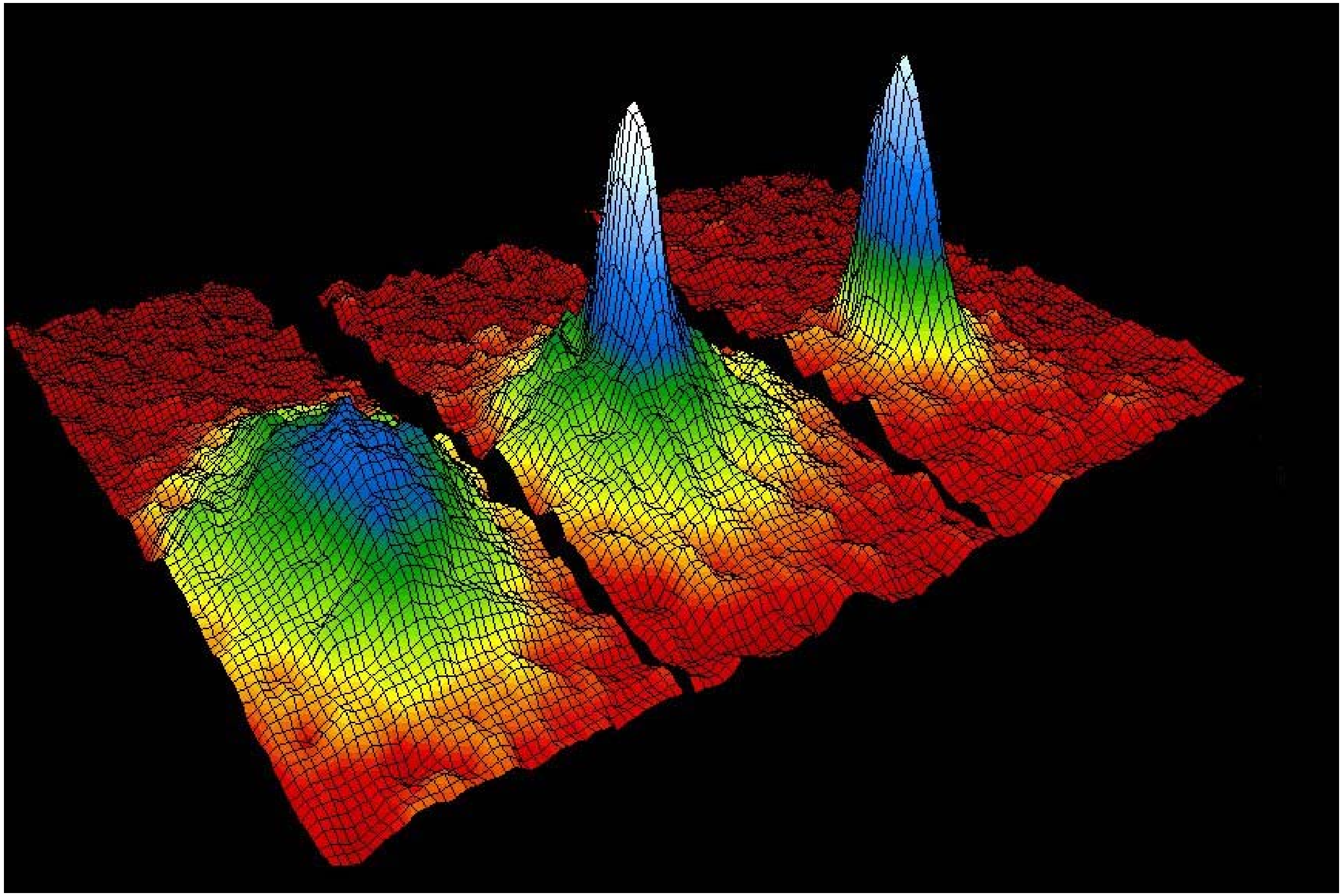


Pair wavefunction in cuprates:

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

Velocity distribution function of ultracold ^{87}Rb atoms



M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

Superflow:

The wavefunction of the condensate

$$\Psi \rightarrow \Psi e^{i\theta(\boldsymbol{r})}$$

Superfluid velocity

$$\boldsymbol{v}_s = \frac{\hbar}{m} \nabla \theta$$

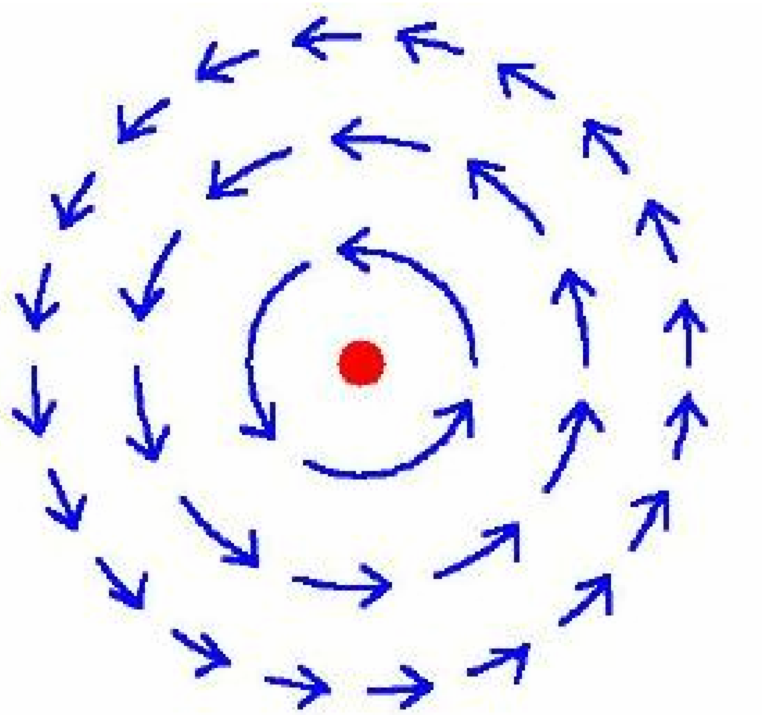
(for non-Galilean invariant superfluids,

the co-efficient of $\nabla \theta$ is modified)

II. Vortices in the superfluid

Magnus forces, duality, and point vortices as dual "electric" charges

Excitations of the superfluid: **Vortices**

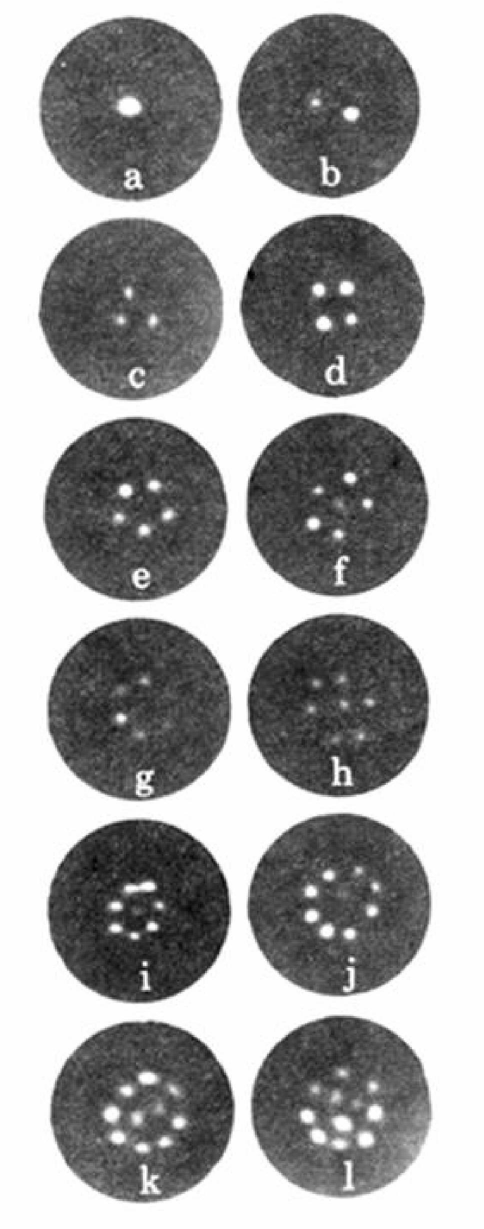


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

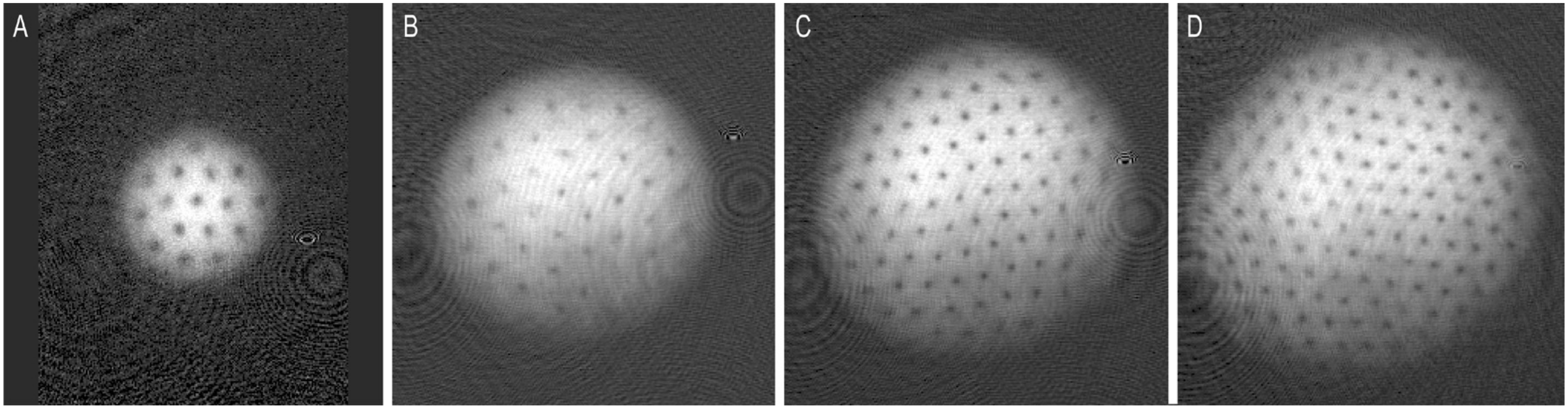
where n is an integer.

Observation of quantized vortices in rotating ^{4}He



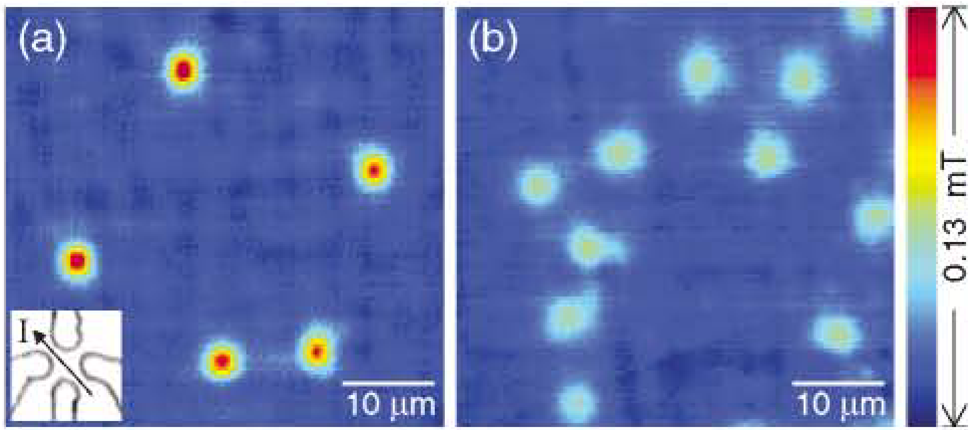
E.J. Yarmchuk, M.J.V. Gordon, and R.E. Packard, *Observation of Stationary Vortex Arrays in Rotating Superfluid Helium*, Phys. Rev. Lett. **43**, 214 (1979).

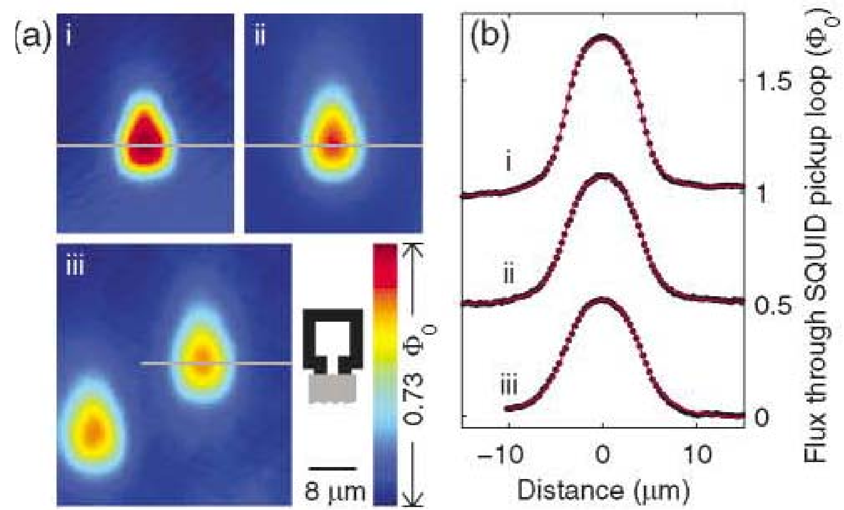
Observation of quantized vortices in rotating ultracold Na



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle,
Observation of Vortex Lattices in Bose-Einstein Condensates,
Science **292**, 476 (2001).

Quantized fluxoids in $YBa_2Cu_3O_{6+y}$



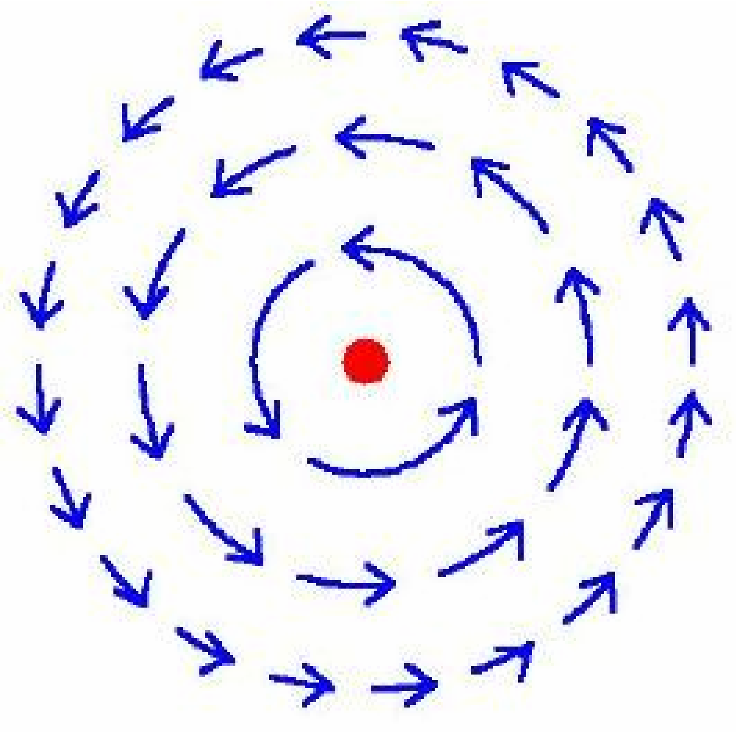


J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

In superconductors, vortices carry quantized magnetic flux:

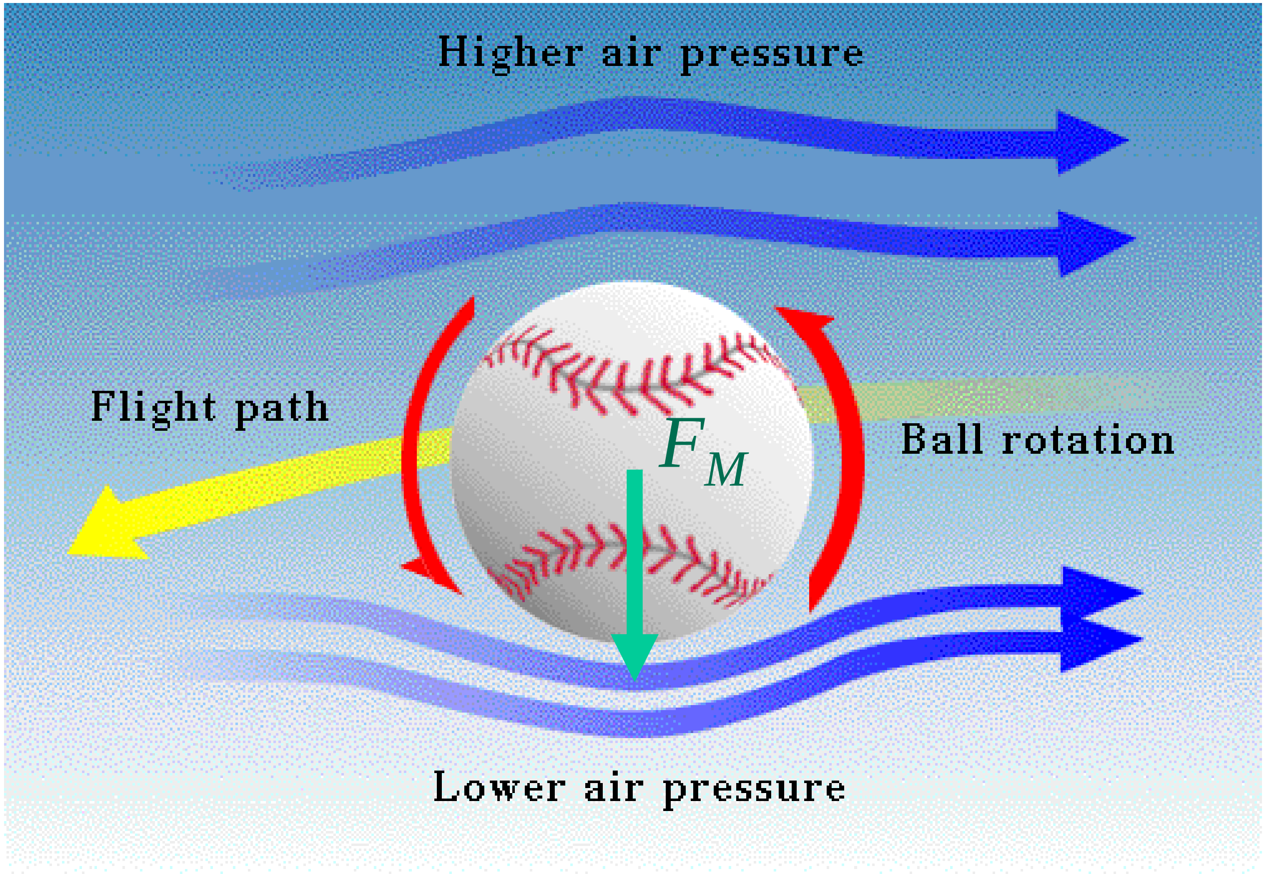
$$\int \mathbf{B} \cdot d\mathbf{S} = n\frac{hc}{2e}$$

Excitations of the superfluid: **Vortices**



<u>Central question</u>:
In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these "particles" ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \bullet (\text{velocity of ball}) \bullet (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s\cdot d\mathbf{r}\right)\\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\end{aligned}$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vortex in a superfluid, this is

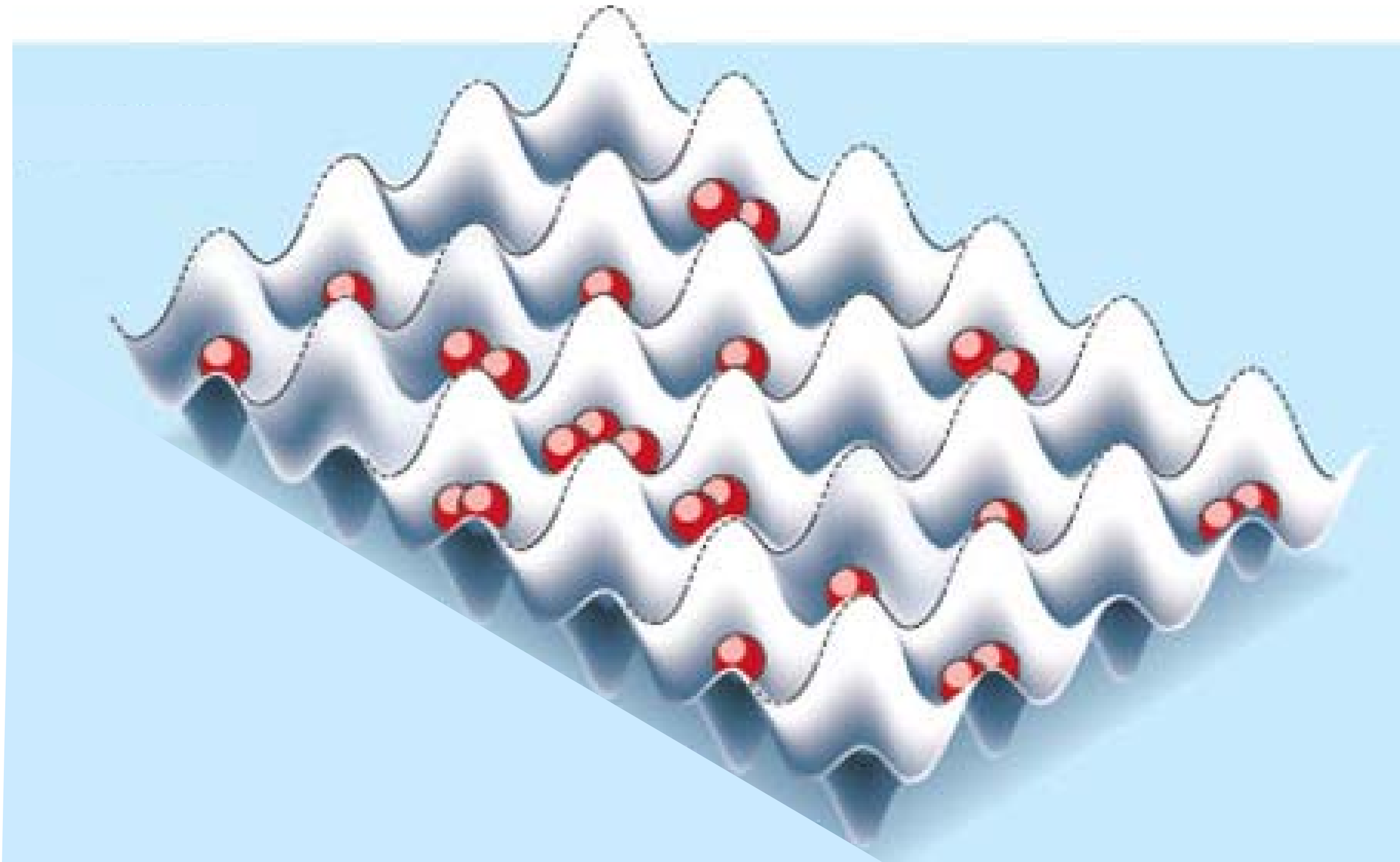
$$\begin{aligned}
\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right) \\
&= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}} \\
&= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt} \times \mathbf{B}\right)
\end{aligned}$$

where $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

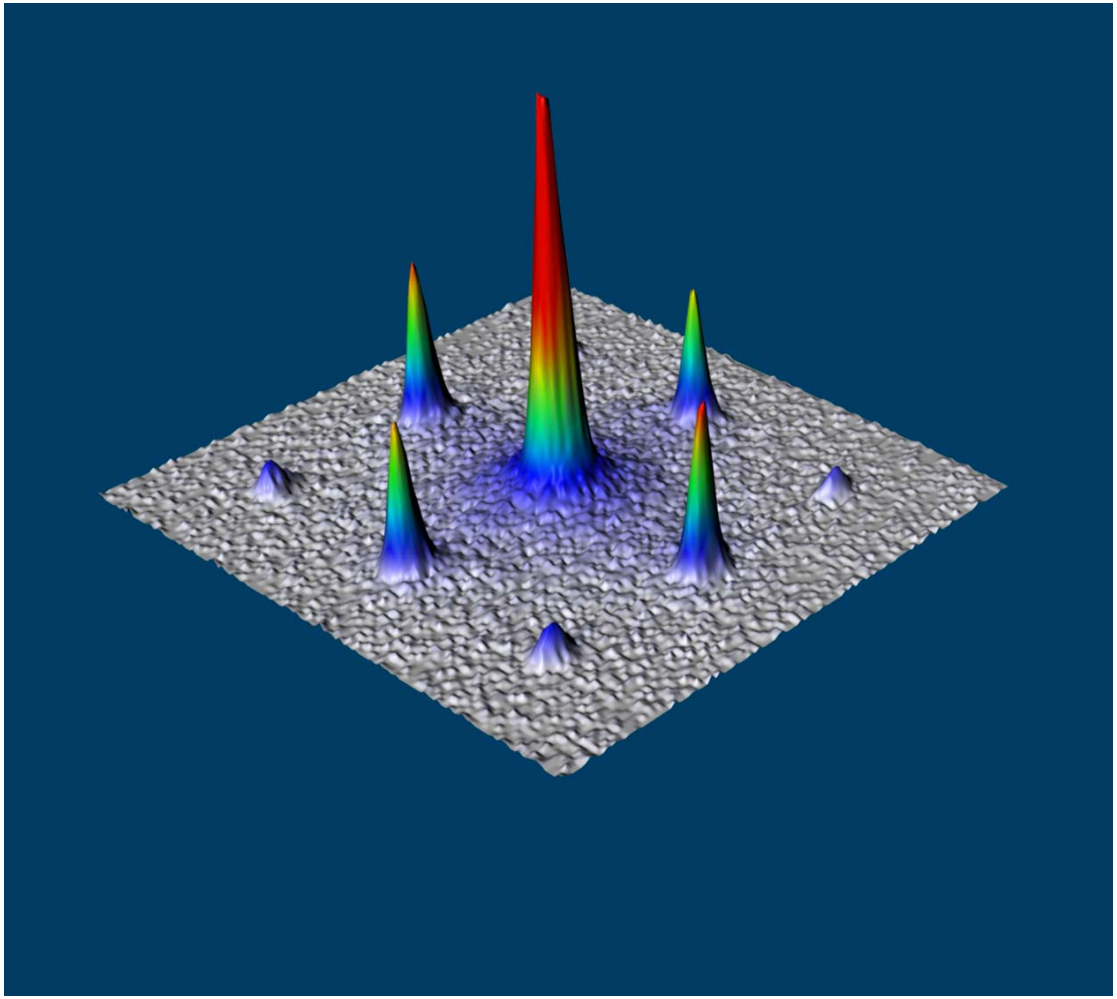
Dual picture:
The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = *h*×(number density of Bose particles)

III. The superfluid-Mott insulator quantum phase transition

Apply a periodic potential (standing laser beams) to trapped ultracold bosons (^{87}Rb)



Momentum distribution function of bosons



Bragg reflections of condensate at reciprocal lattice vectors

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

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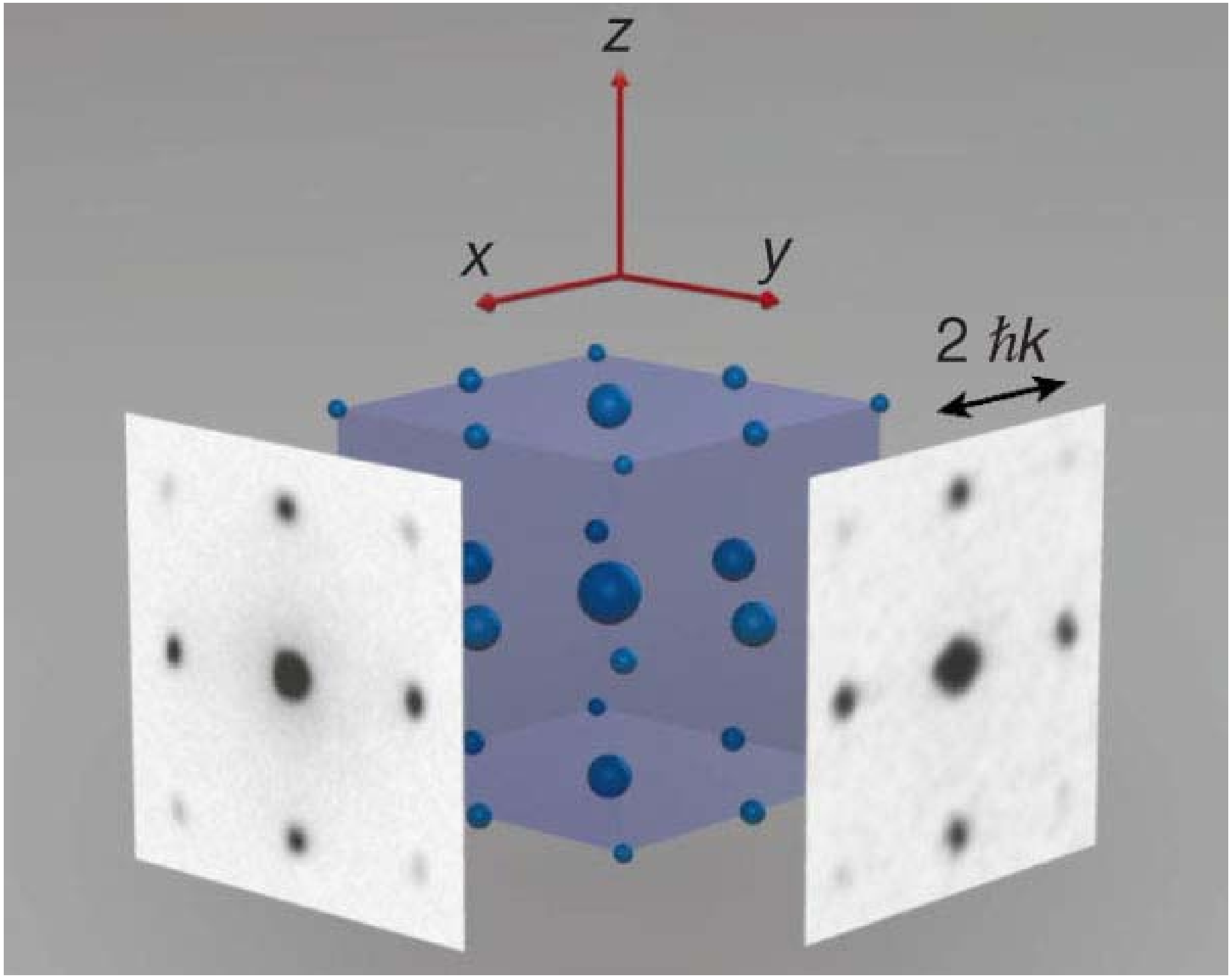

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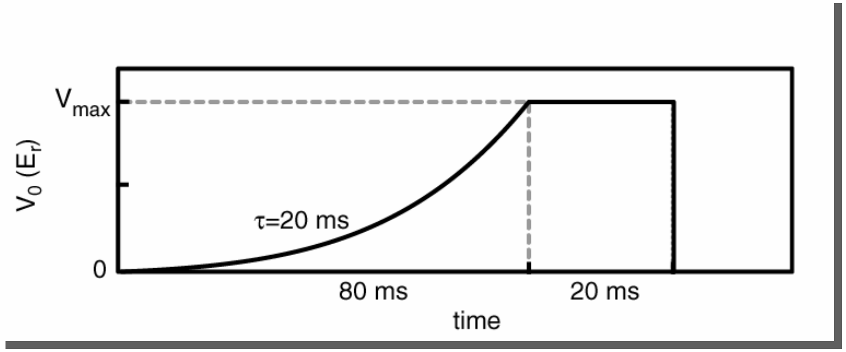
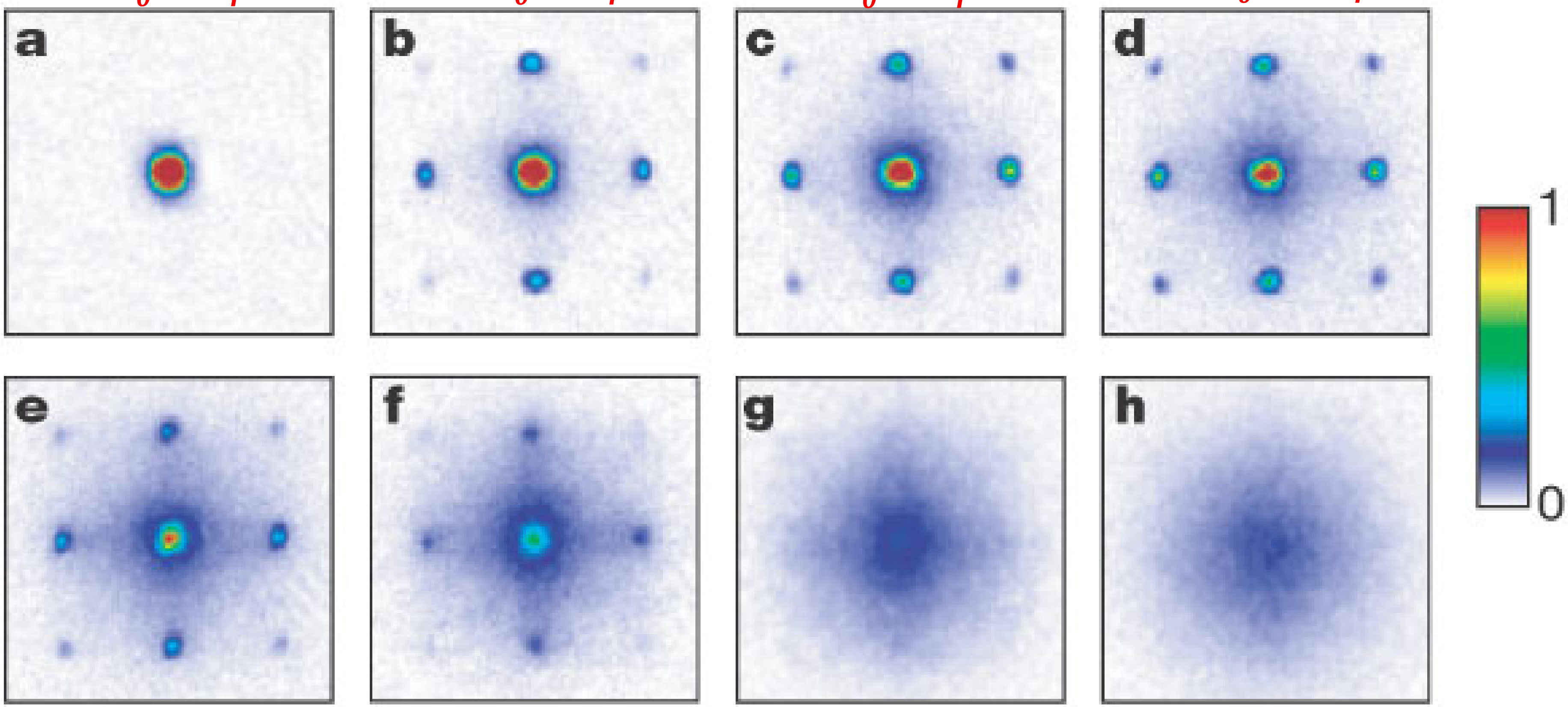
Superfluid-insulator quantum phase transition at T=0



$V_0=0E_r$ $V_0=3E_r$ $V_0=7E_r$ $V_0=10E_r$



$V_0=13E_r$ $V_0=14E_r$ $V_0=16E_r$ $V_0=20E_r$

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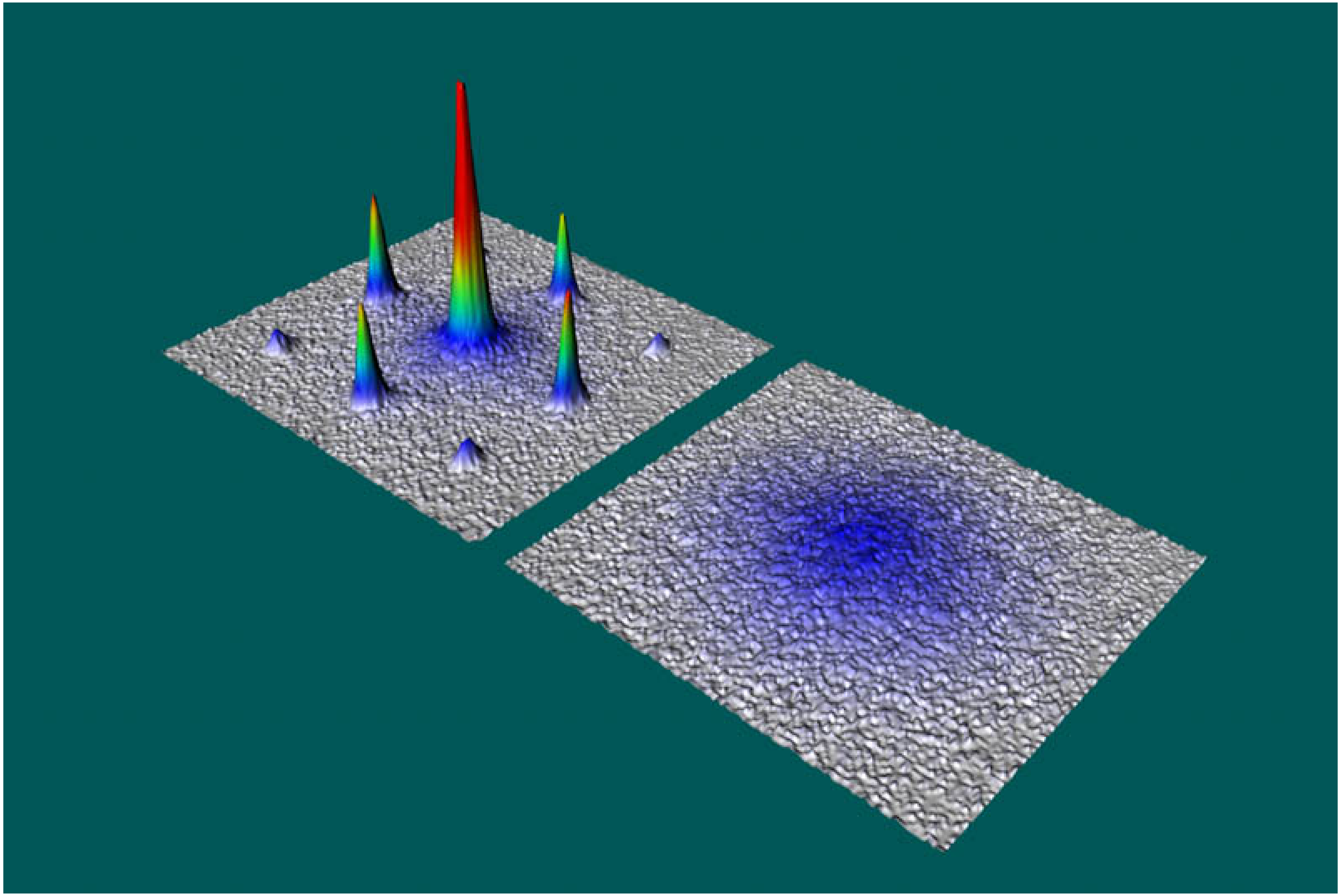

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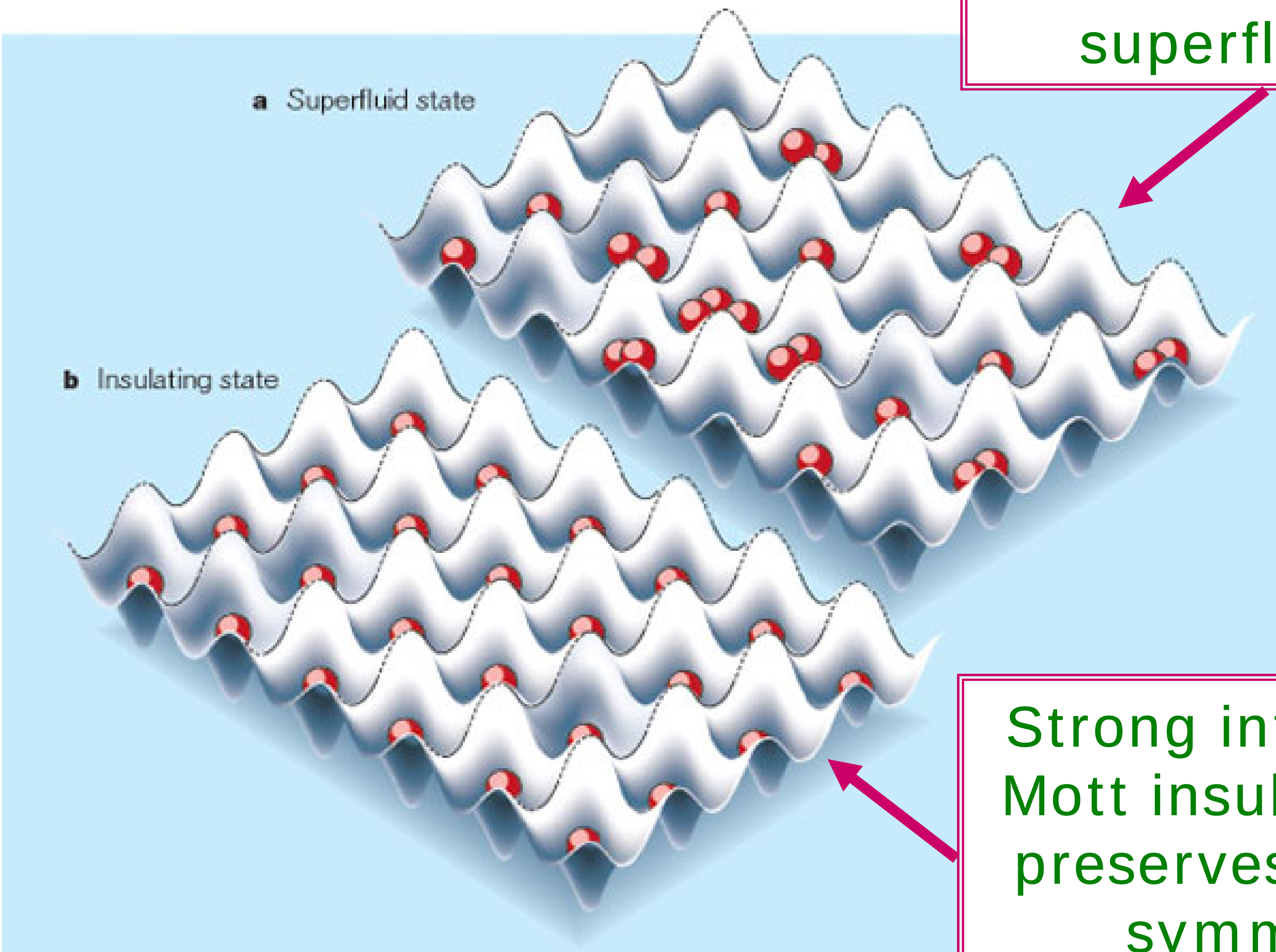
Bosons at filling fraction f = 1

Weak interactions: superfluidity



Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

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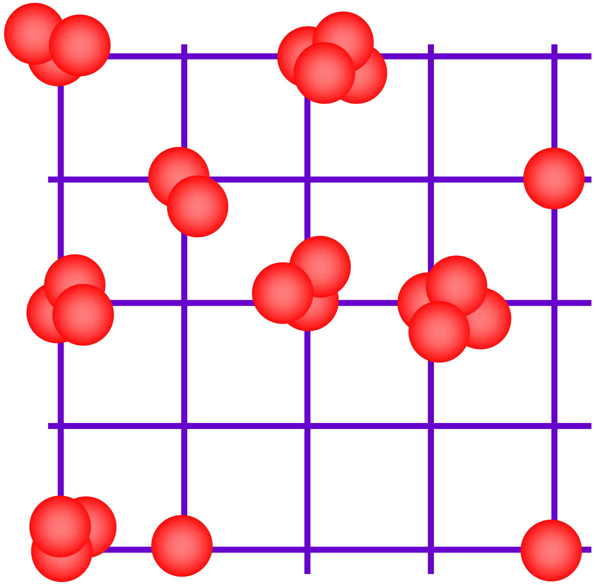

$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

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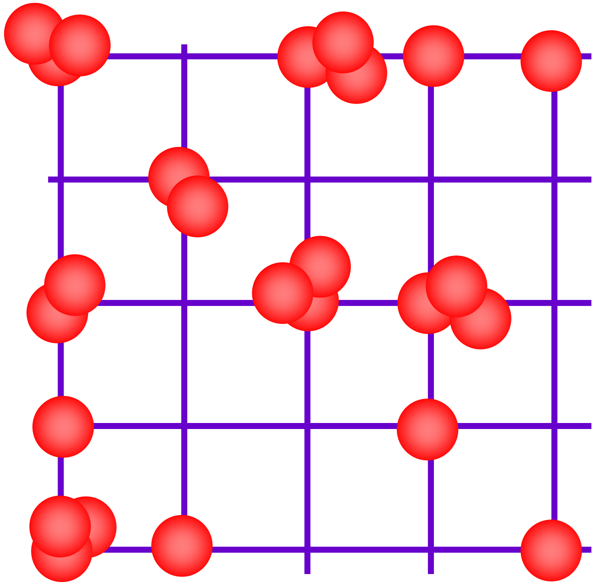

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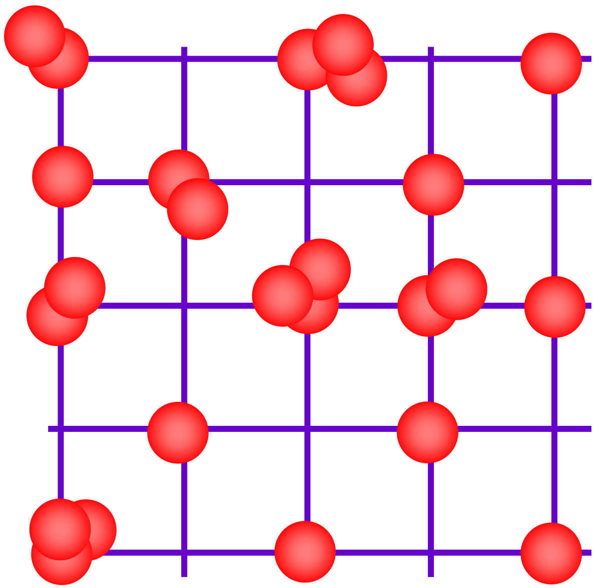

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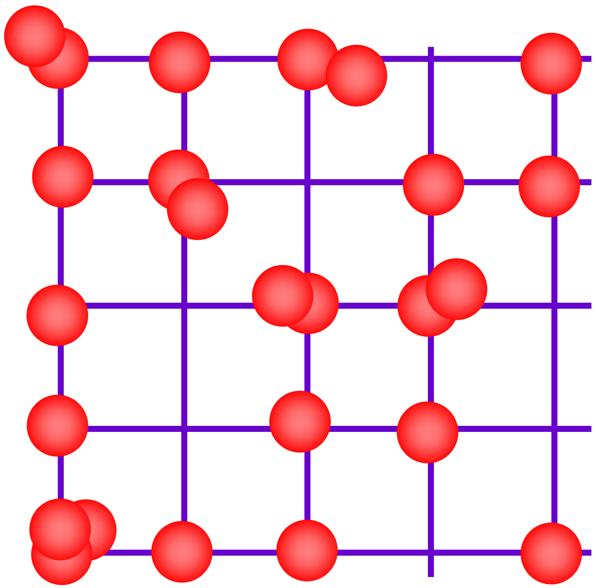

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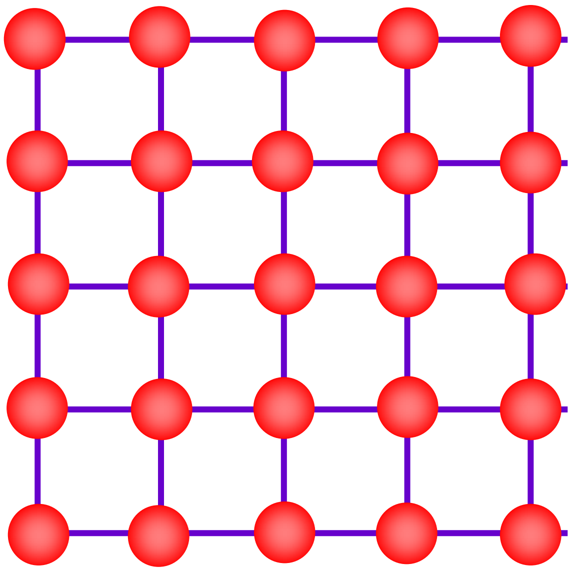

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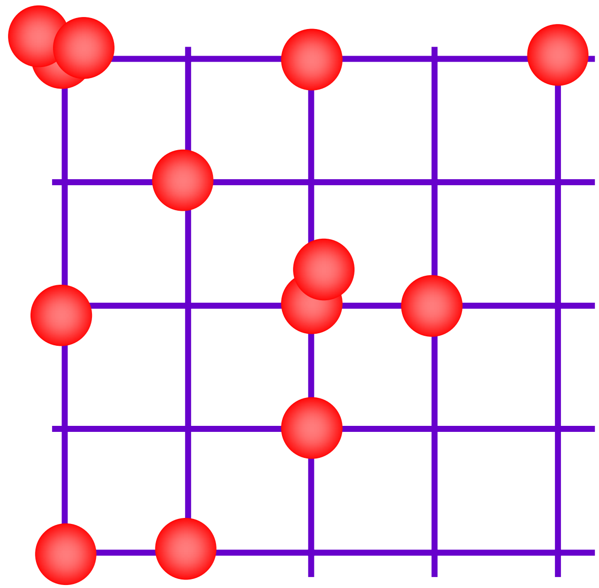
Strong interactions: insulator

Bosons at filling fraction $f = 1/2$



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S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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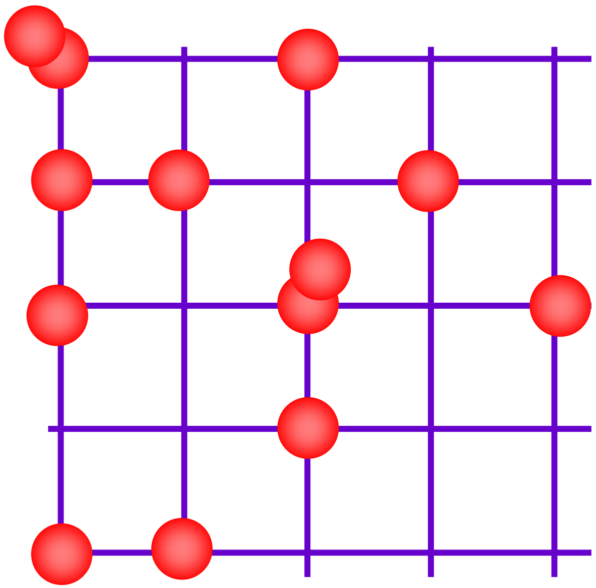

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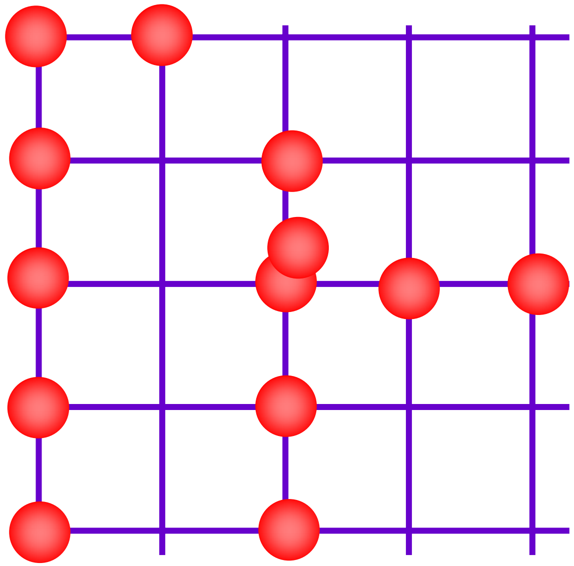

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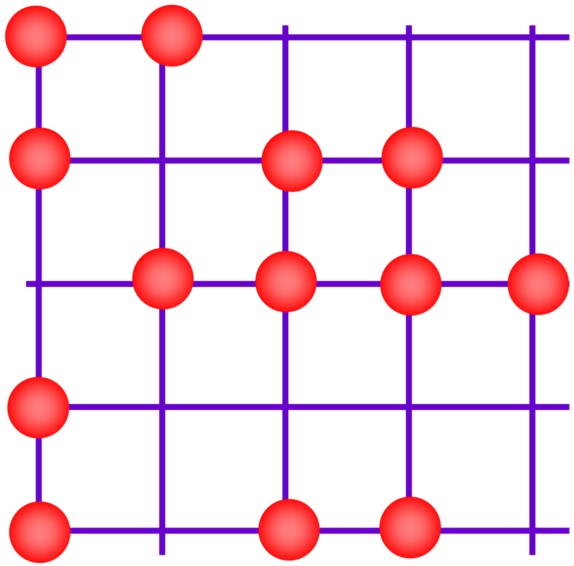

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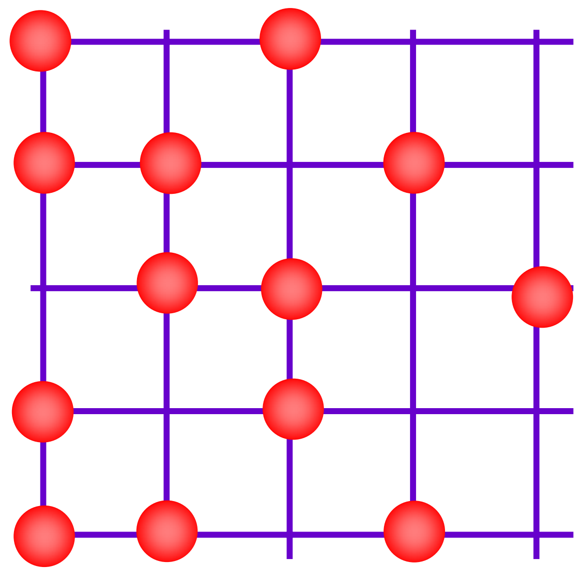

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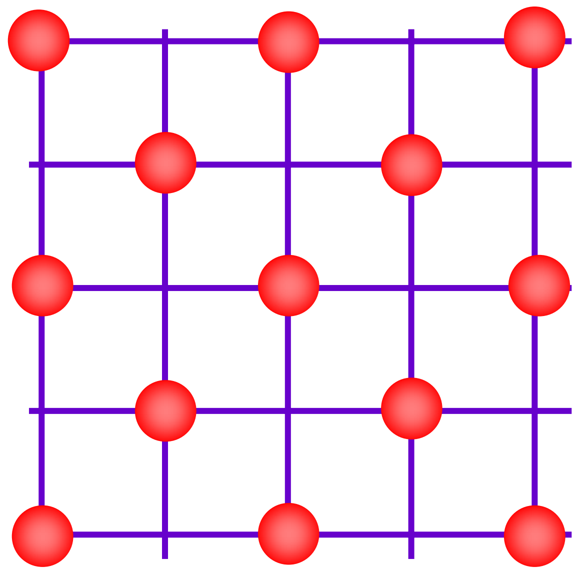

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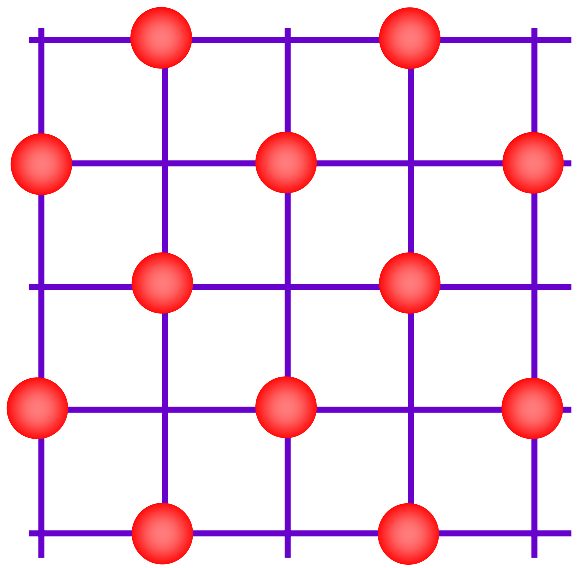

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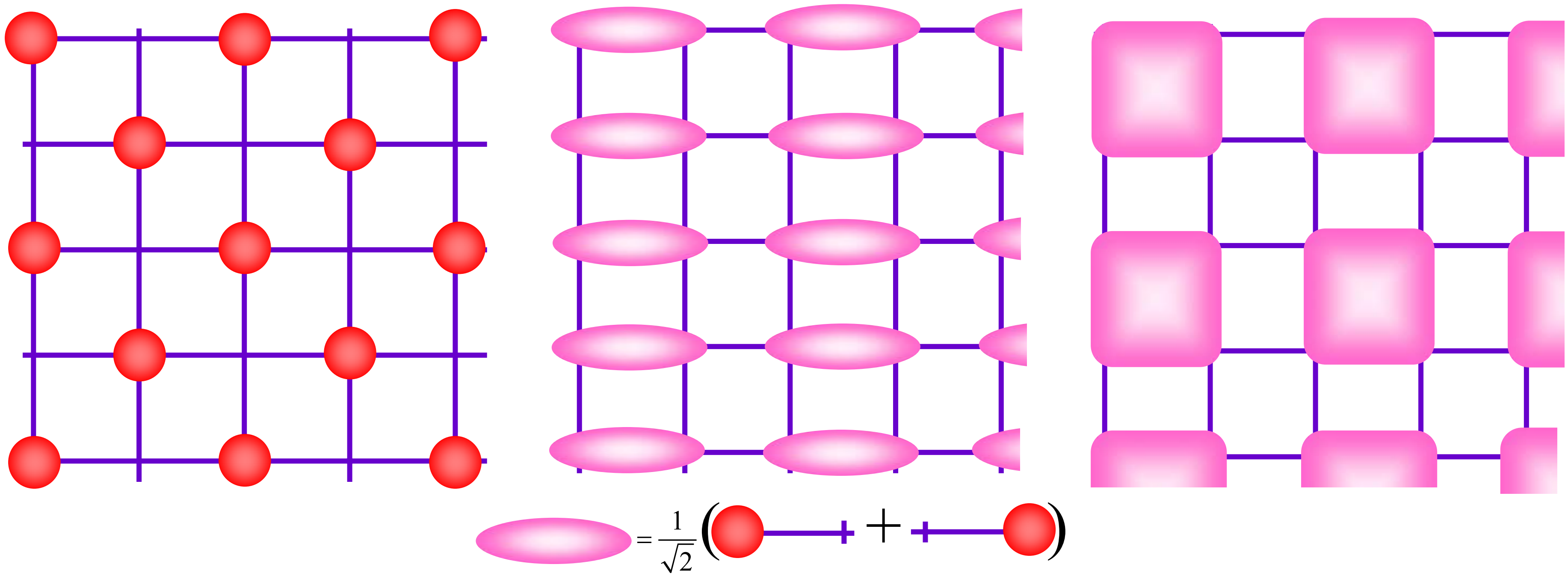
Insulating phases of bosons at filling fraction $f = 1/2$



Charge density wave (CDW) order	Valence bond solid (VBS) order	Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

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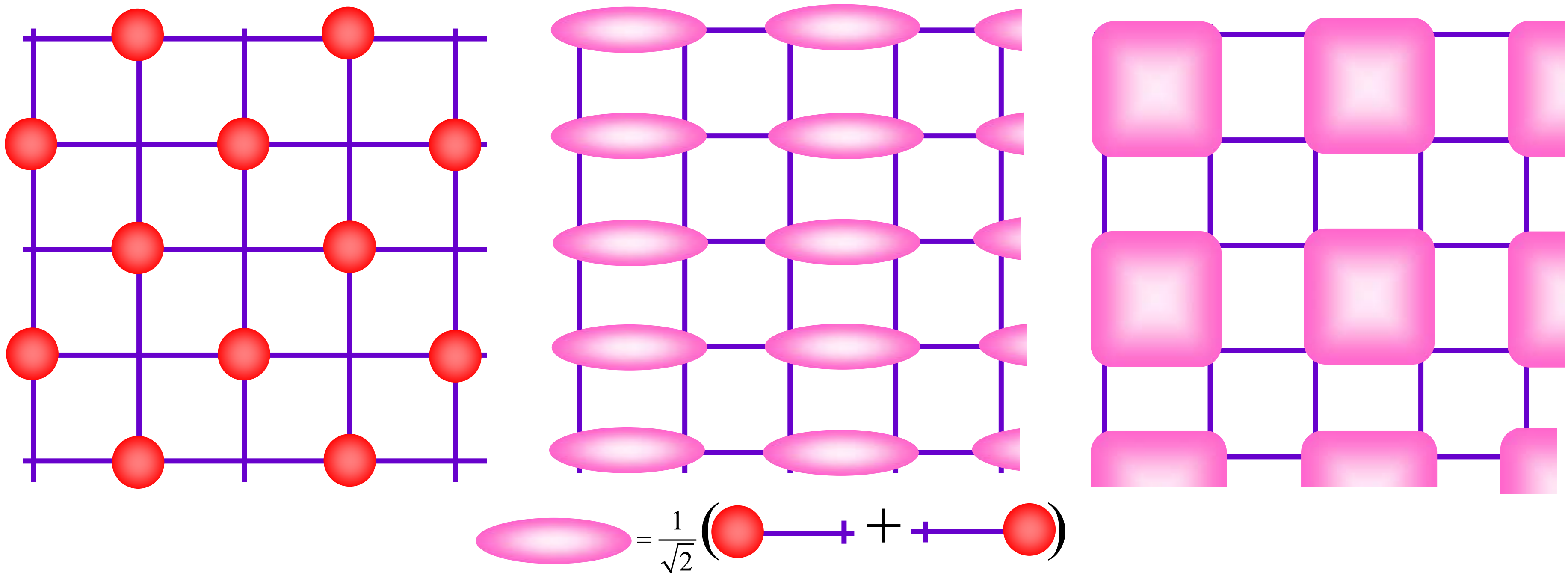

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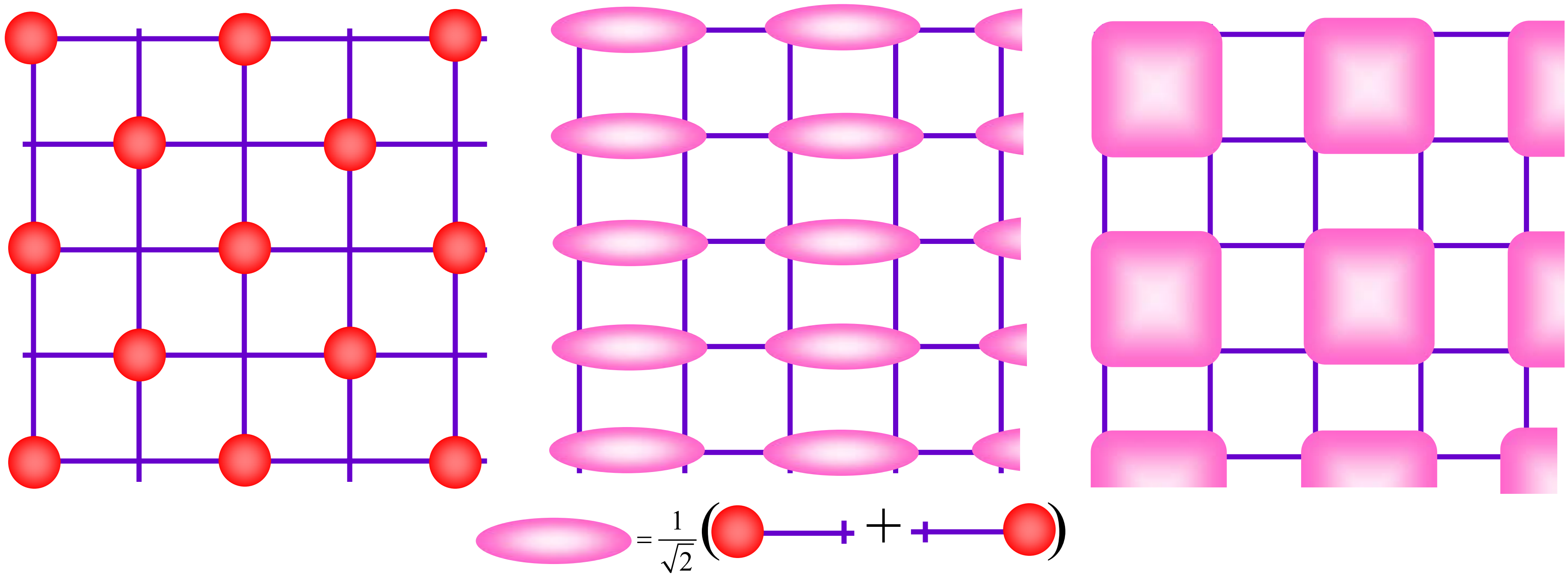

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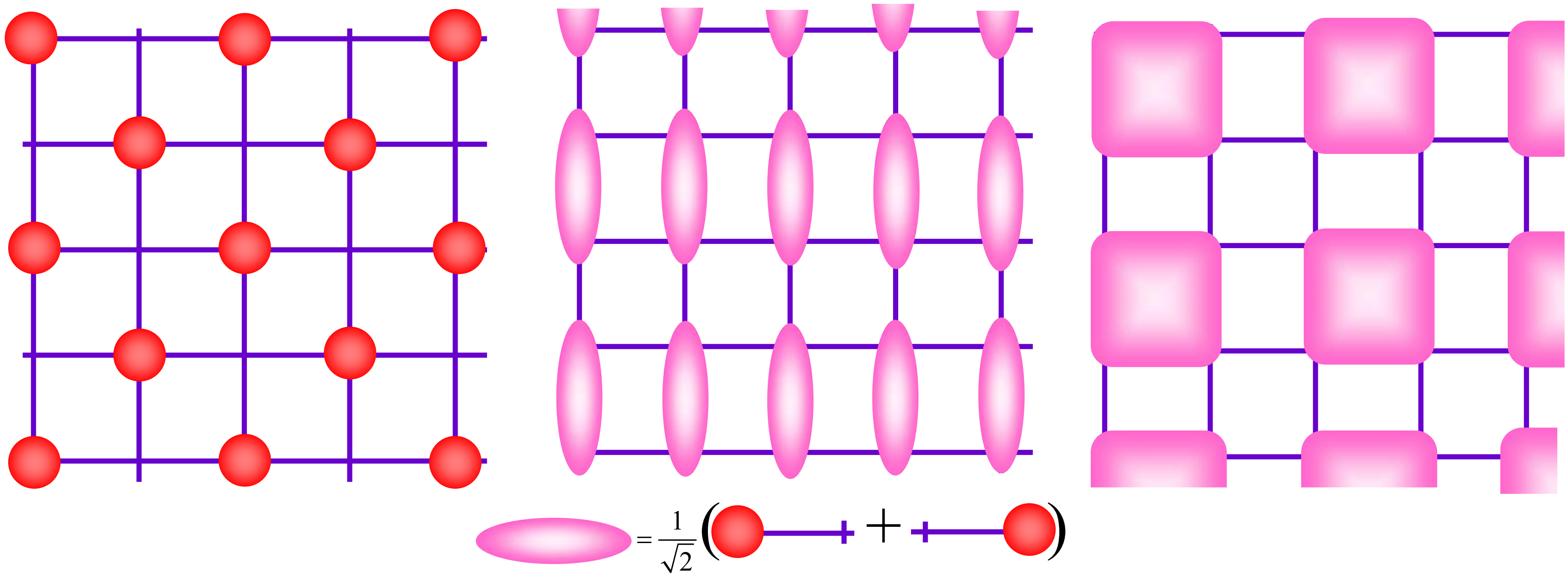

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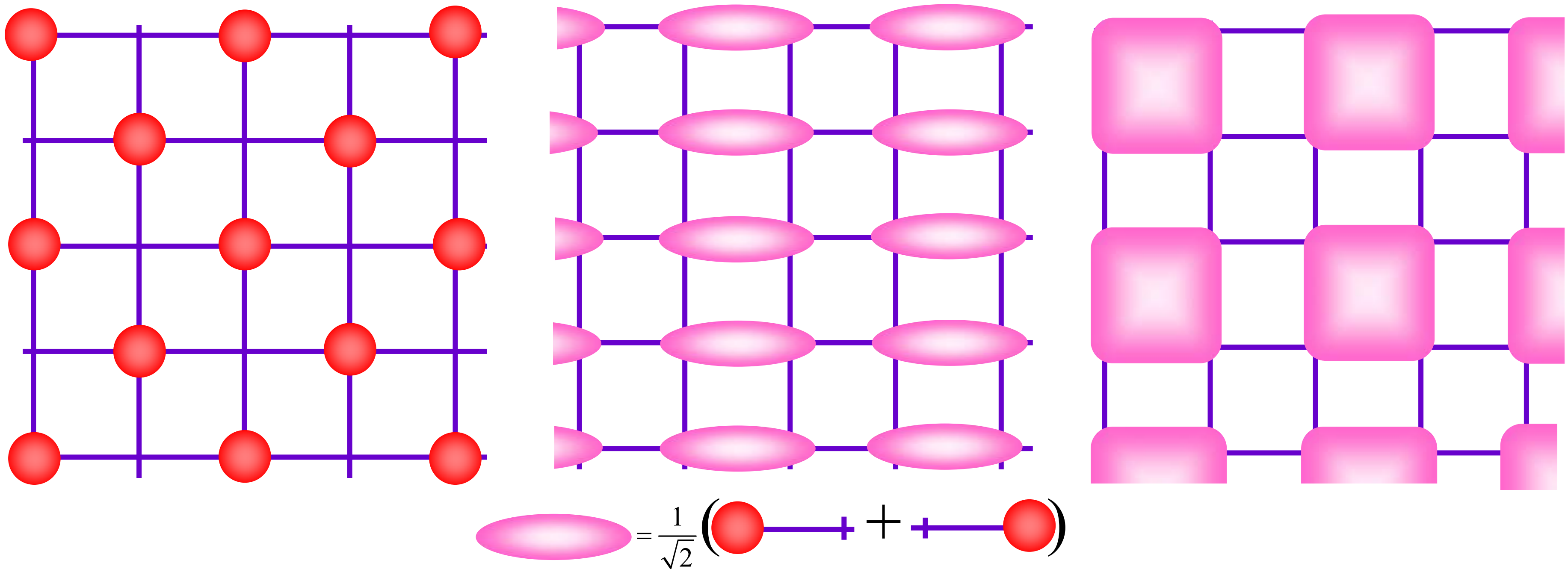

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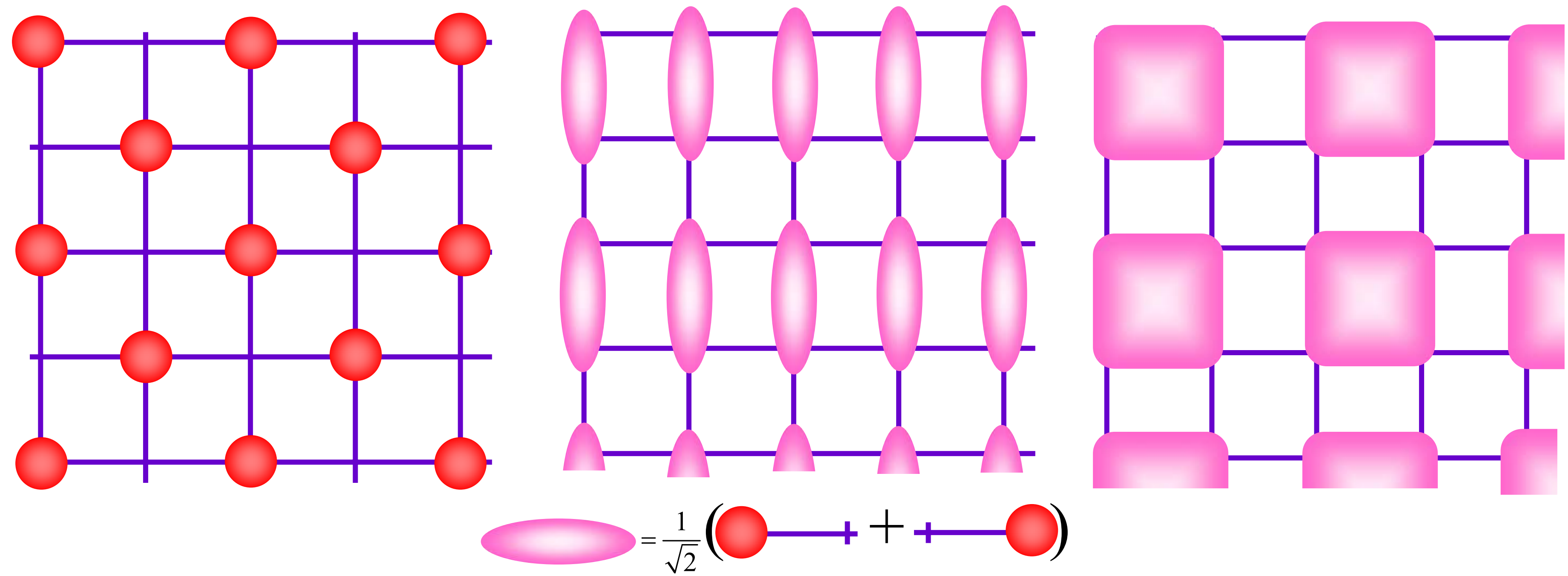

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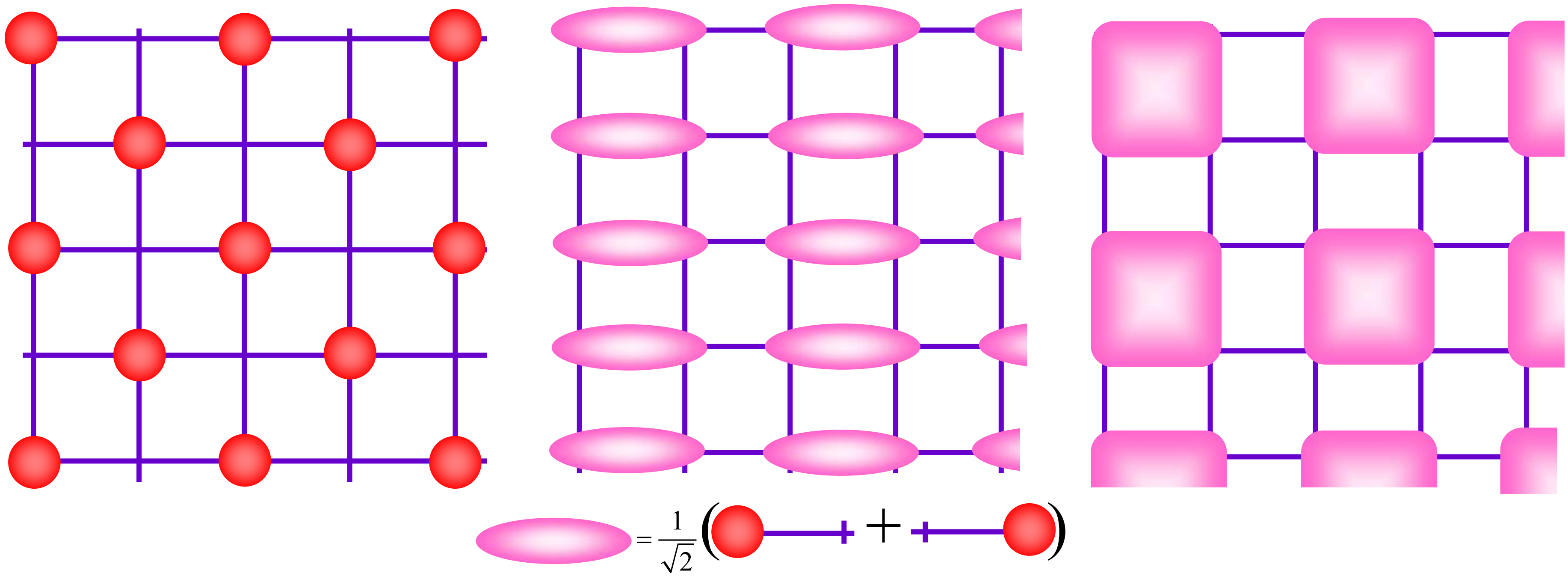

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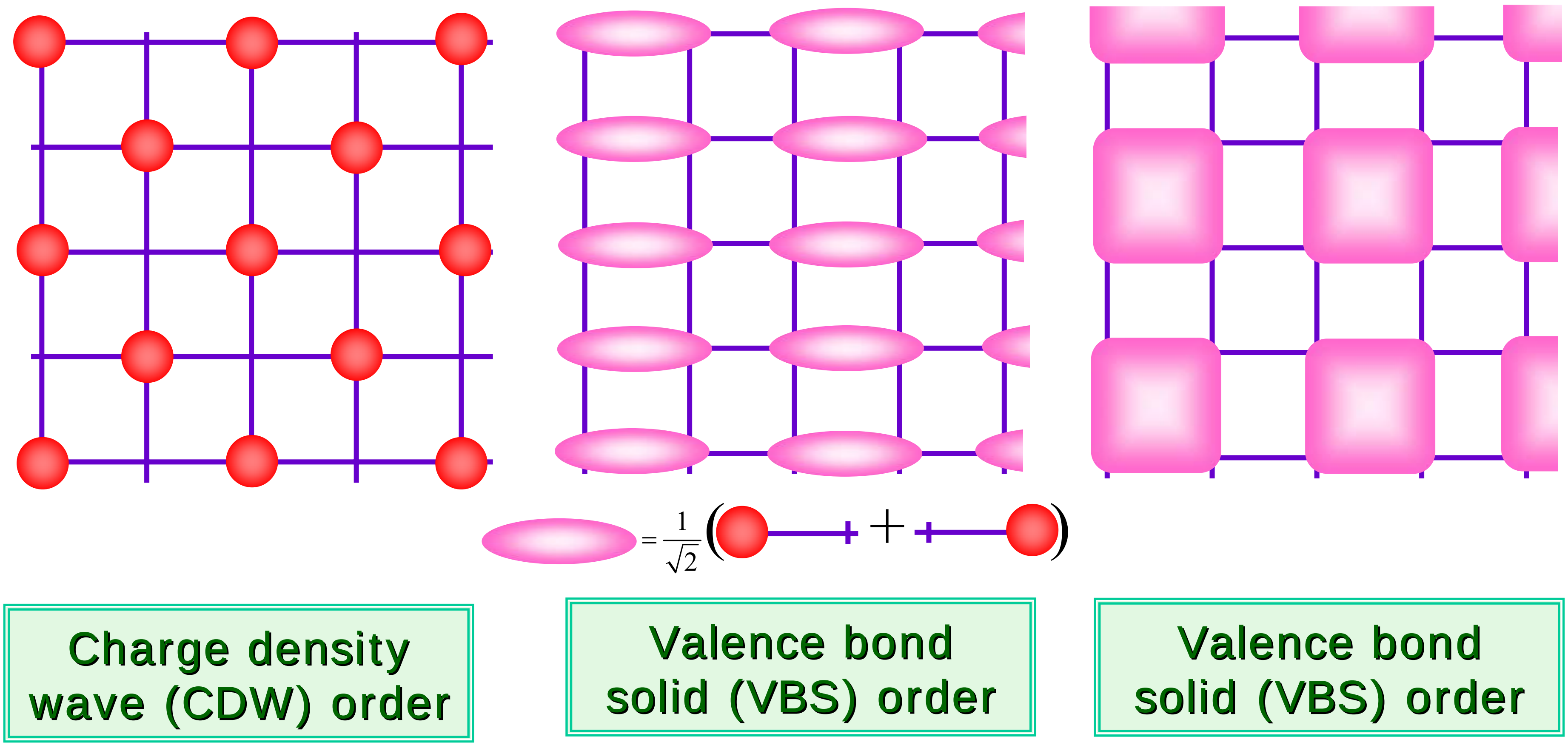

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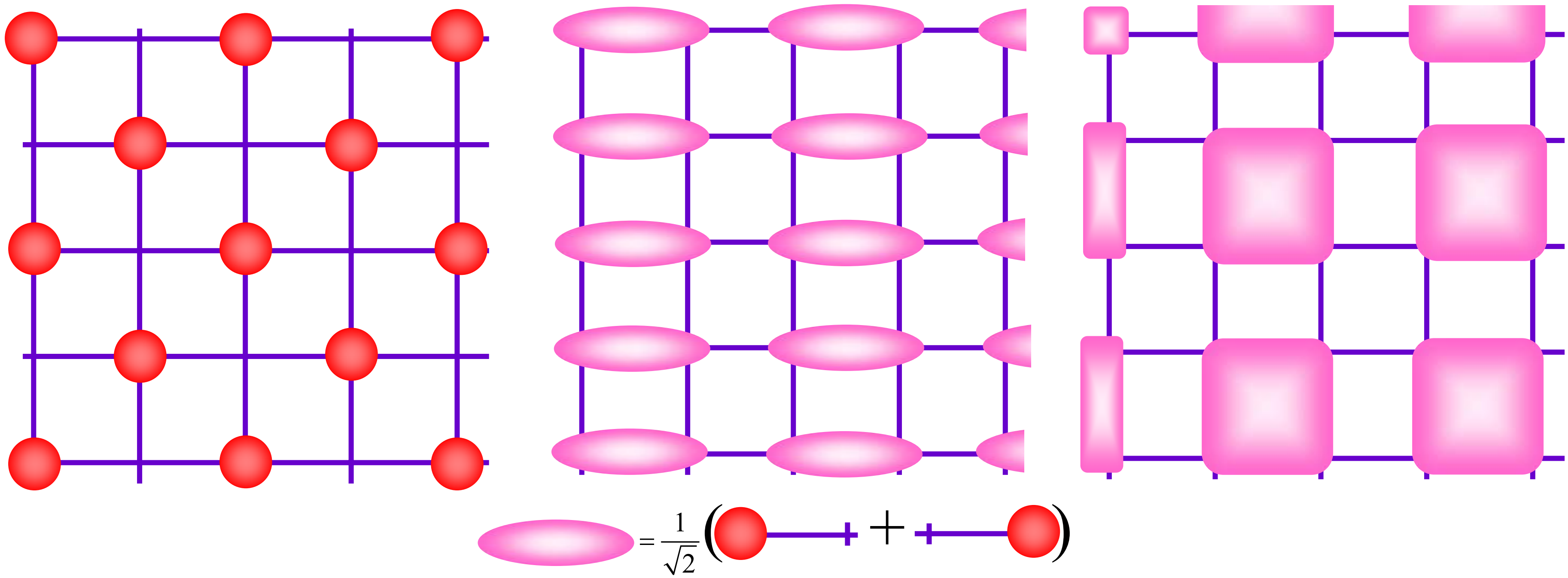

Charge density wave (CDW) order	Valence bond solid (VBS) order	Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

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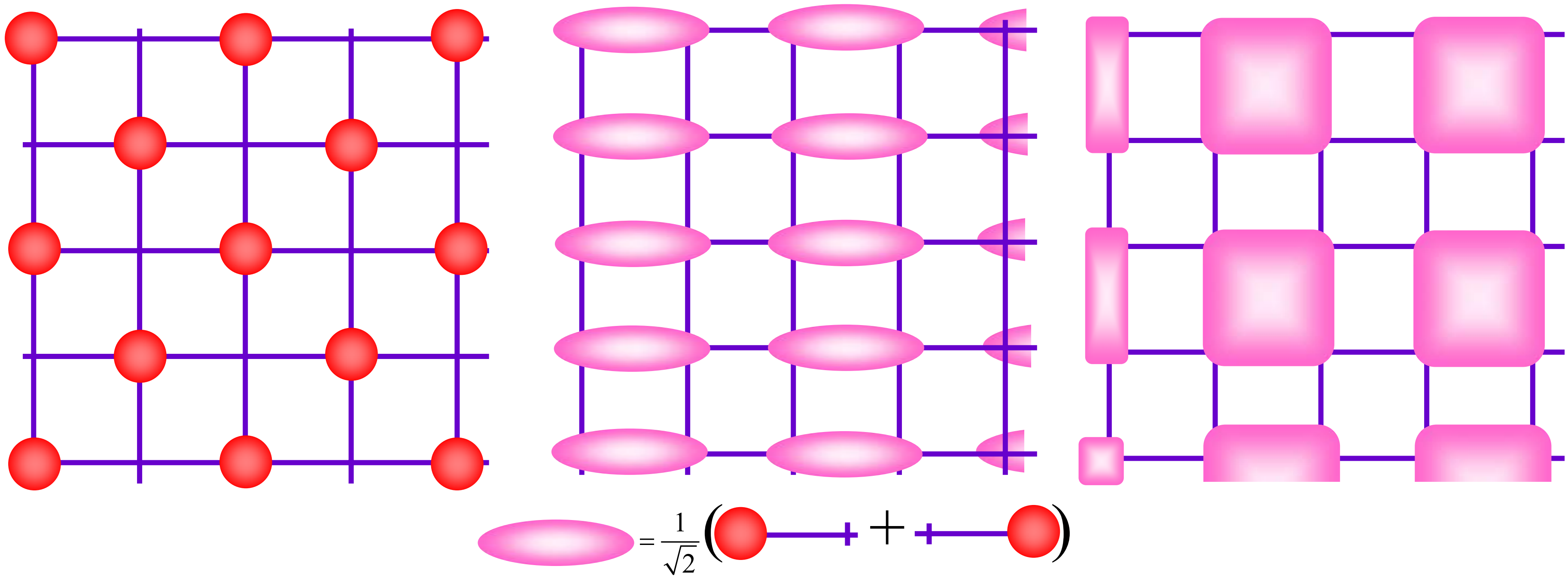

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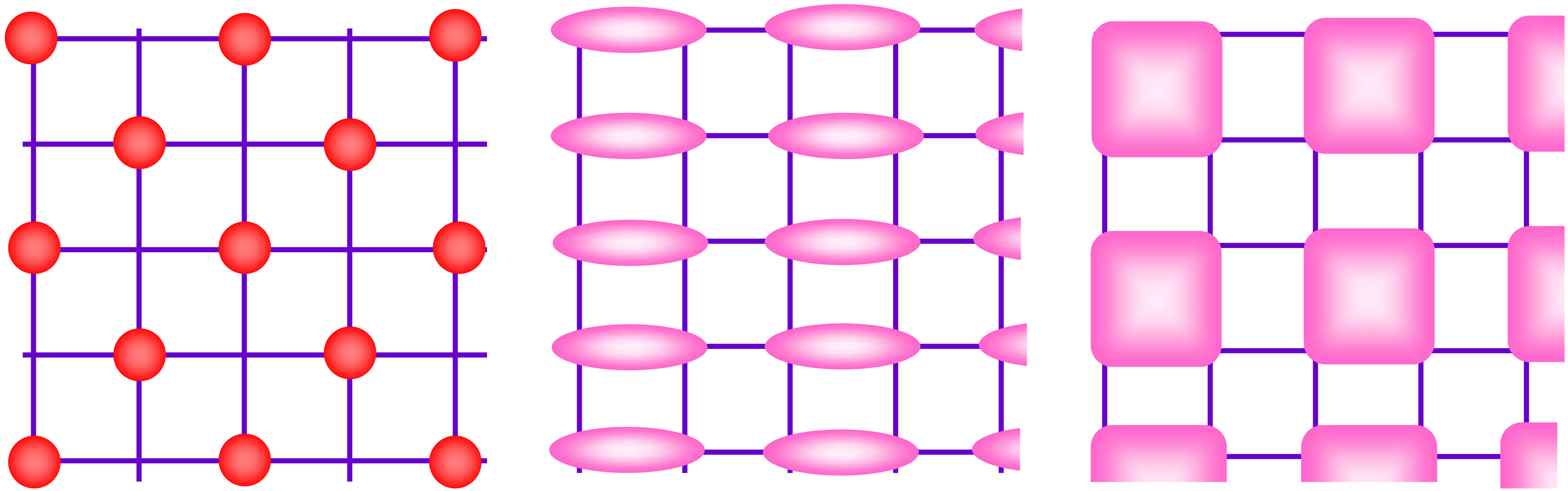

$$\text{(ellipse)} = \frac{1}{\sqrt{2}}\left(\bullet\!\!-\!\!- \; + \; -\!\!-\!\!\bullet \right)$$

Charge density wave (CDW) order

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S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

IV. Vortices in superfluids near the superfluid-insulator quantum phase transition

The Hofstadter Hamiltonian and vortex flavors

Upon approaching the insulator, the phase of the condensate becomes “uncertain”.

Vortices cost less energy and vortex-anti-vortex pairs proliferate.

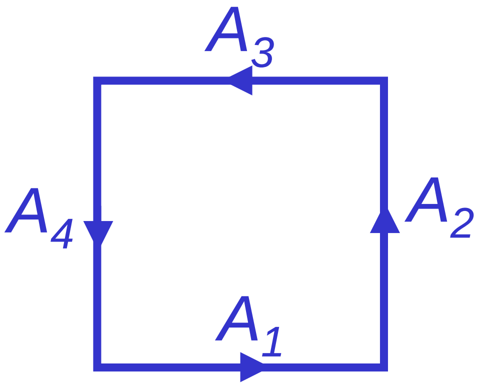
The quantum mechanics of vortices plays a central role in the superfluid-insulator quantum phase transition.

- The vortices are quantum particles moving in a periodic potential with the symmetry of the square lattice, and in the presence of a dual "magnetic" field of strength $= h\rho$, where ρ is the number density of bosons per unit cell.

- The vortex motion can be described by the effective Hofstadter Hamiltonian:

$$\mathcal{H}_v = -t \sum_{\langle ij \rangle} \left(e^{iA_{ij}} \varphi_i^* \varphi_j + \text{c.c.} \right)$$

where φ_i is an operator which annihilates a vortex particle at site i of a square lattice.



$$A_1 + A_2 + A_3 + A_4 = 2\pi f$$

where f is the boson filling fraction.

Bosons at filling fraction $f = 1$

• At f=1, the "magnetic" flux per unit cell is 2π, and the vortex does not pick up any phase from the boson density.

• The effective dual "magnetic" field acting on the vortex is zero, and the corresponding component of the Magnus force vanishes.

Bosons at rational filling fraction f=p/q

Quantum mechanics of the vortex "particle" in a periodic potential with *f* flux quanta per unit cell

Space group symmetries of Hofstadter Hamiltonian:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

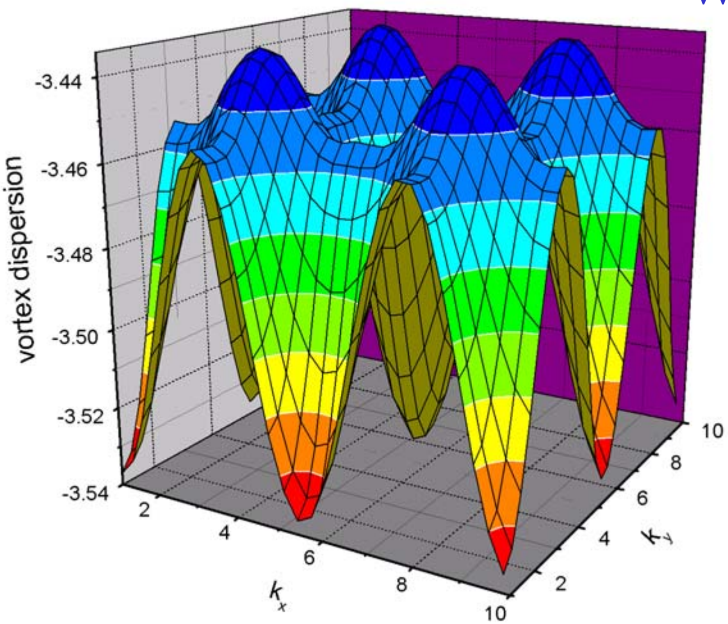
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

The low energy vortex states must form a representation of this algebra

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Hofstadter spectrum of the quantum vortex "particle" with field operator φ



At filling $f=p/q$, there are q species of vortices, φ_ℓ (with $\ell=1\ldots q$), associated with q degenerate minima in the vortex spectrum. These vortices realize the smallest, q-dimensional, representation of the magnetic algebra.

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

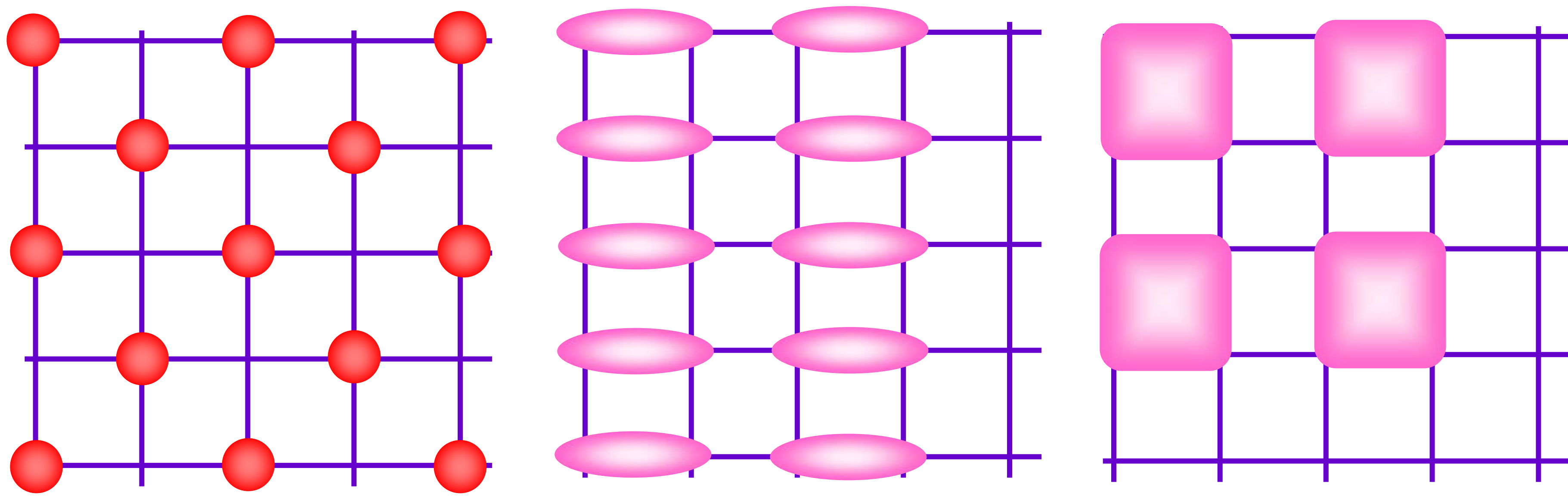
- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

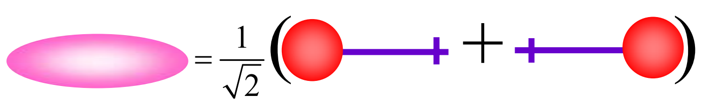
- The Mott insulator is a Bose-Einstein condensate of vortices, with $\left\langle \varphi_\ell \right\rangle \neq 0$

- Any set of values of $\left\langle \varphi_\ell \right\rangle$ breaks the space group symmetry, and the orientation of the vortex condensate, $\left\langle \varphi_\ell \right\rangle$, in flavor space determines the CDW/VBS order.

Mott insulators obtained by condensing vortices

Spatial structure of insulators for $q=2$ ($f=1/2$)

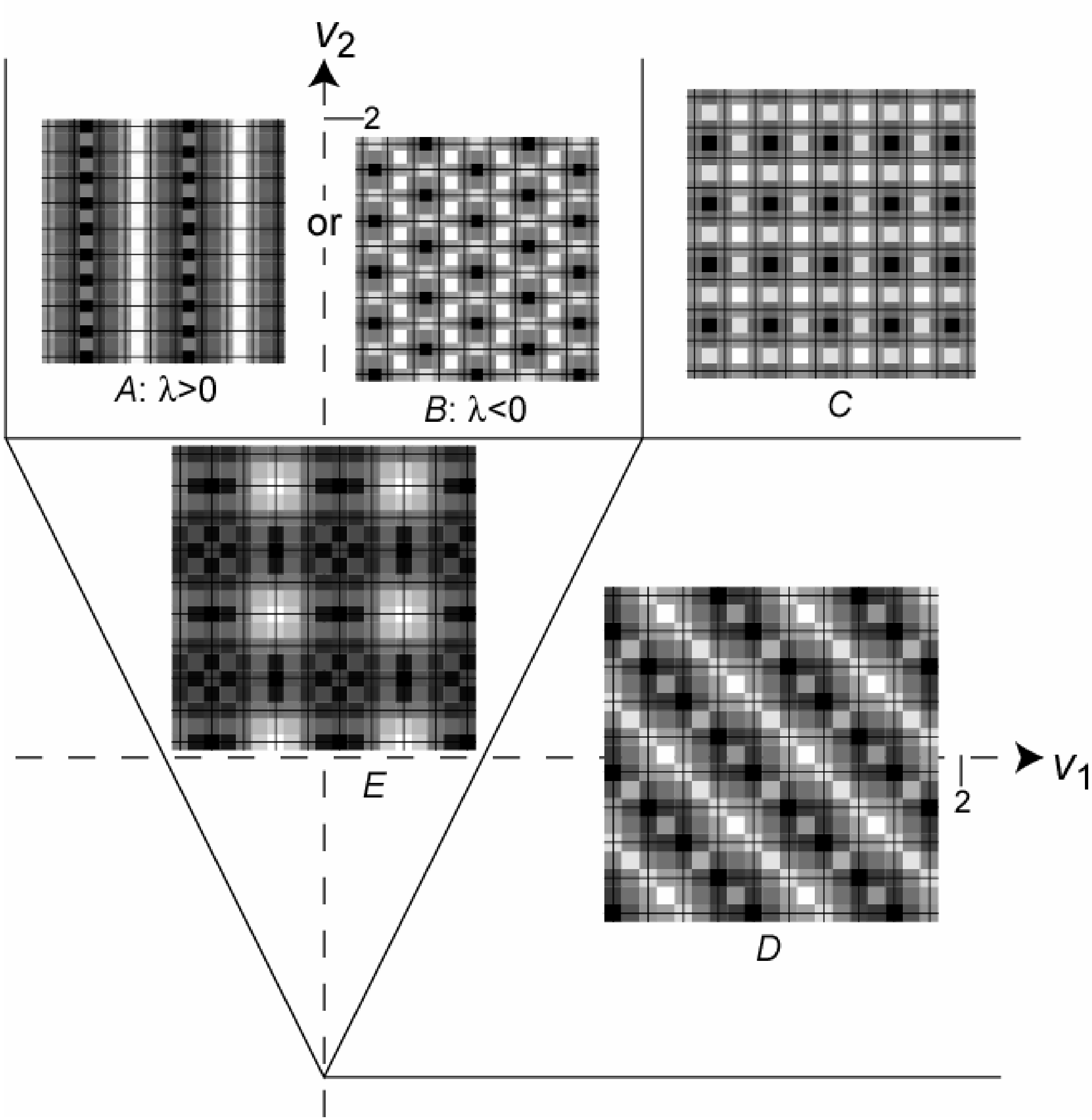




$$= \frac{1}{\sqrt{2}}\left(\quad + \quad \right)$$

Field theory with projective symmetry

Spatial structure of insulators for q=4 (f=1/4 or 3/4)



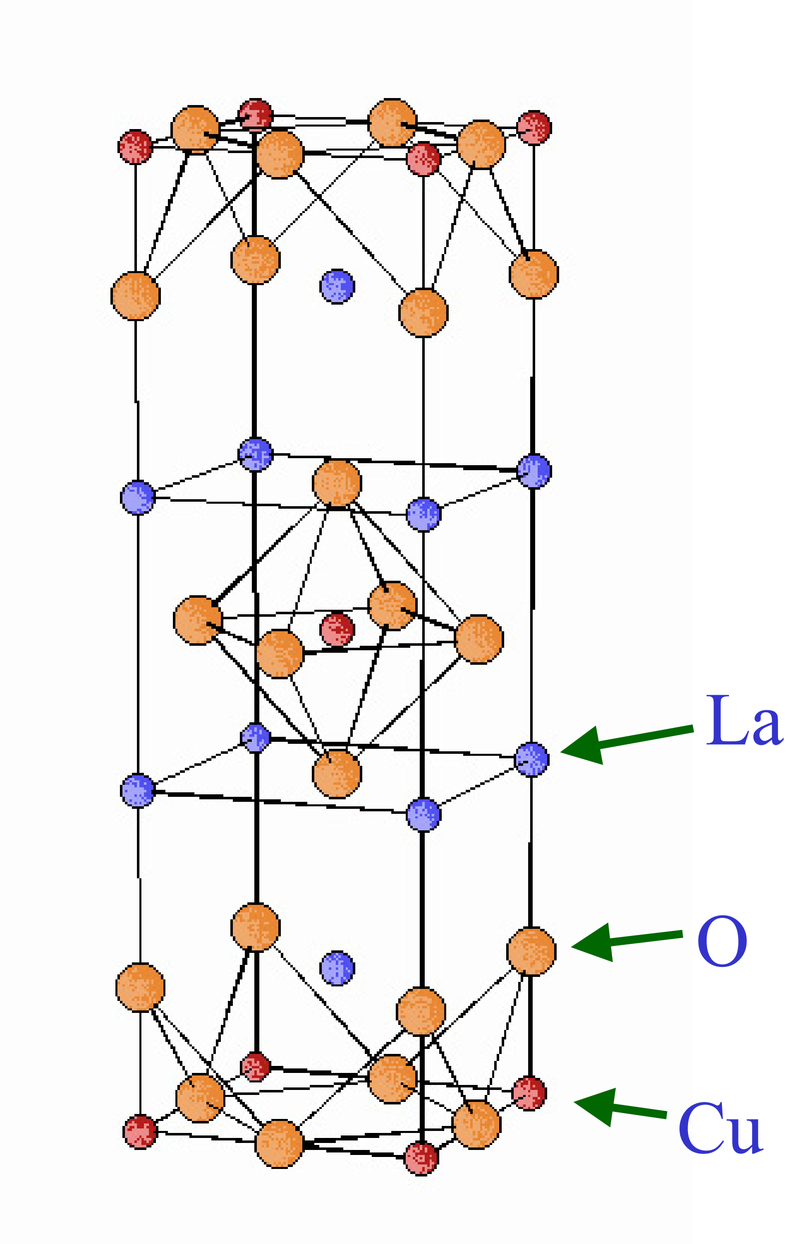
$a \times b$ unit cells;

q/a, q/b, ab/q

all integers

V. The cuprate superconductors

The "quantum order" of the superconducting state: evidence for vortex flavors

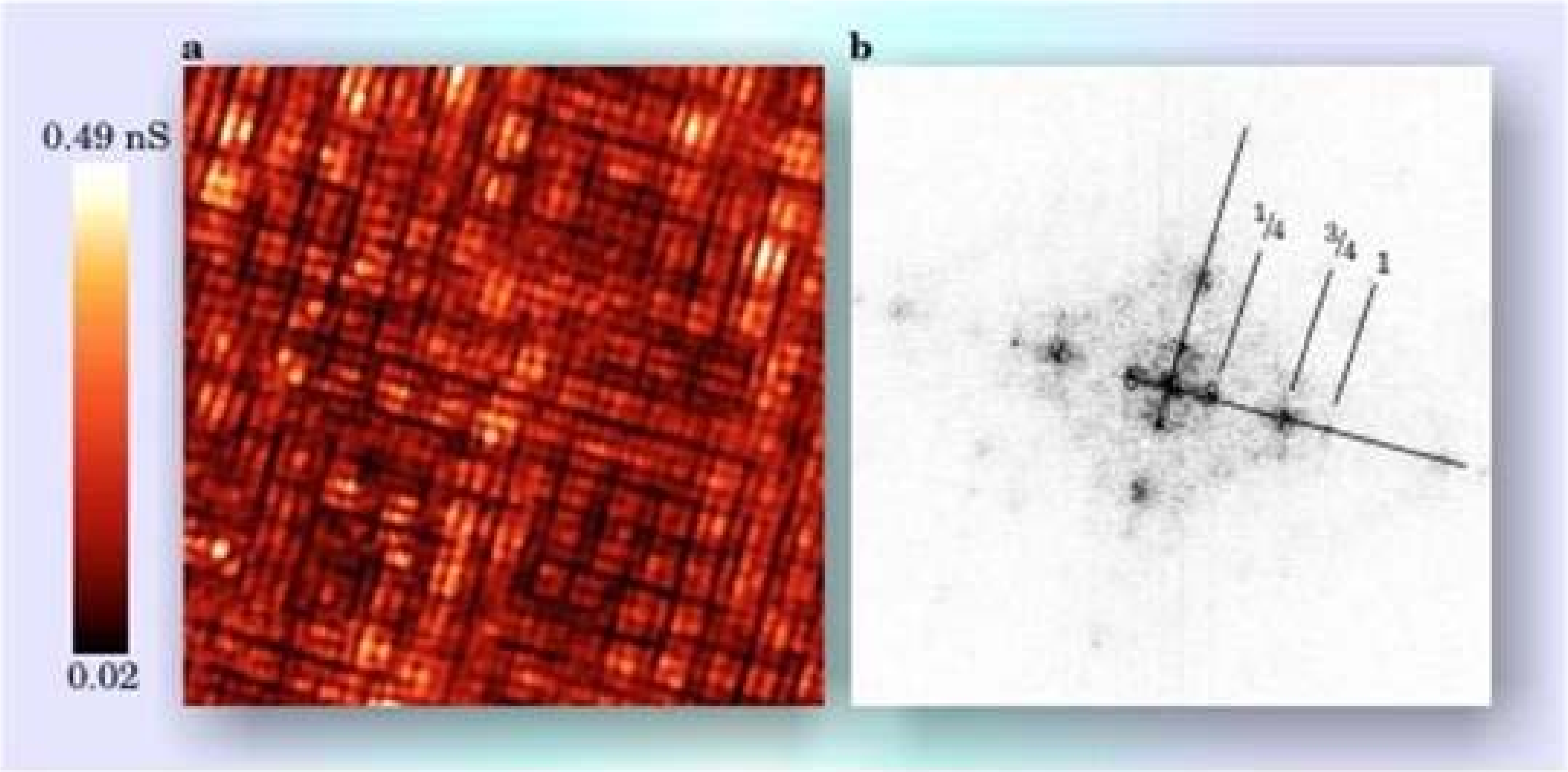


Superconductivity of holes, of density δ, moving on the square lattice of Cu sites.

Experiments on the cuprate superconductors show:

- Tendency to produce "density" wave order near wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.
- Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range "density" wave order at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.
- Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$

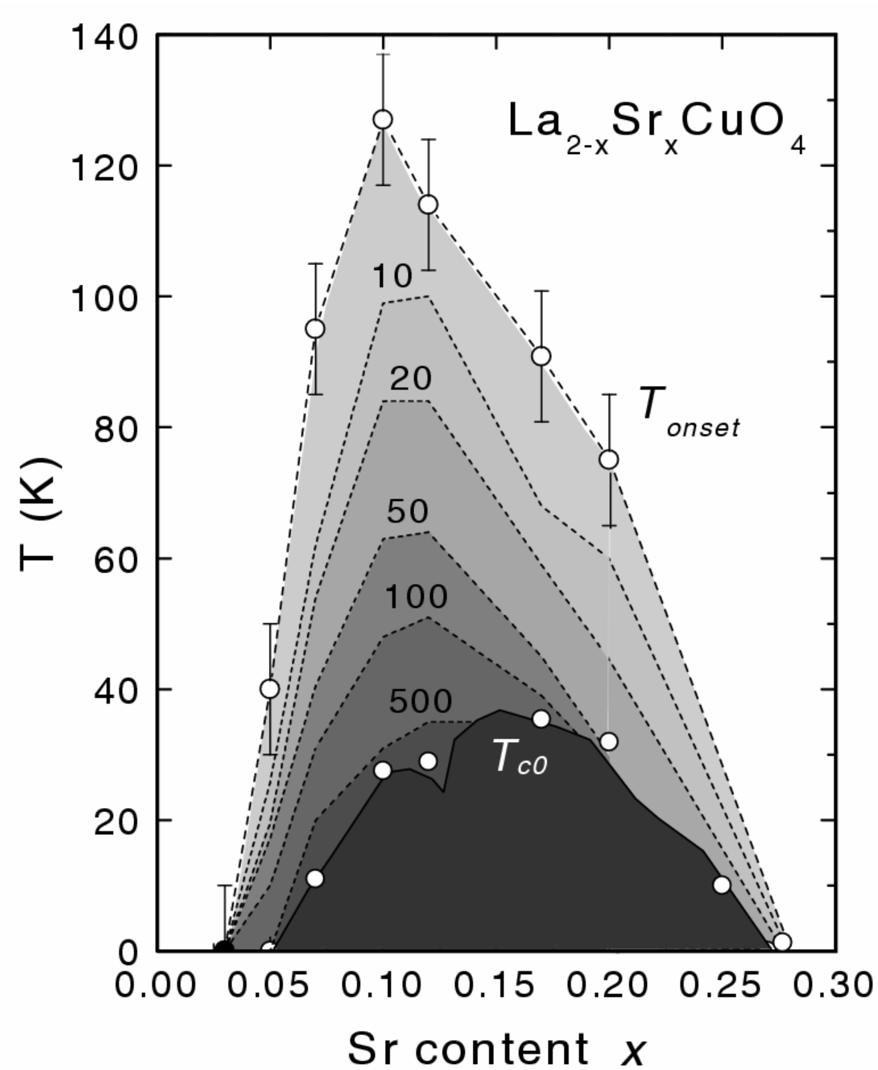


Multiple order parameters: superfluidity and density wave.
Phases: Superconductors, Mott insulators, and/or supersolids

T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

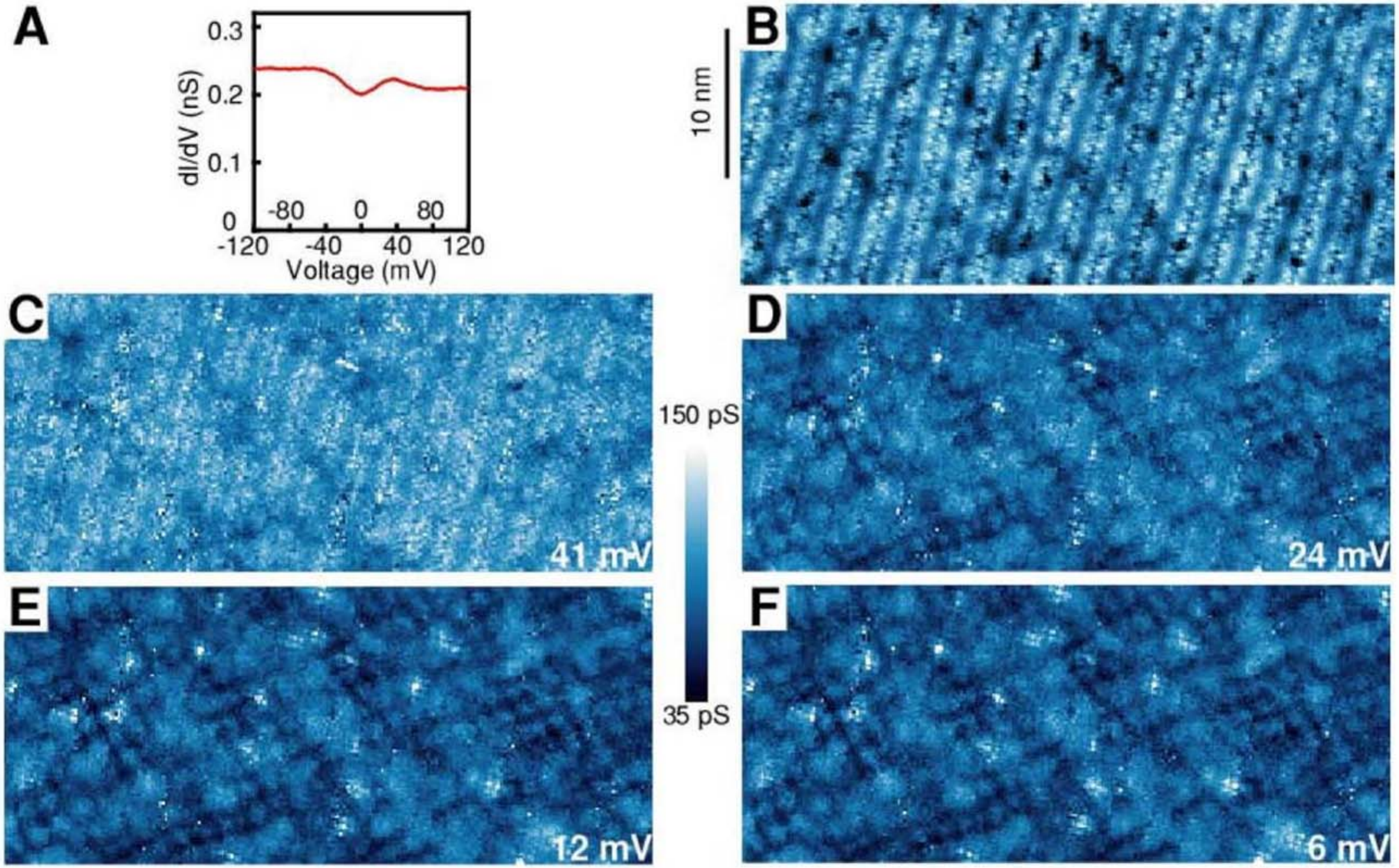


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

STM measurements observe "density" modulations with a period of ≈ 4 lattice spacings



LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

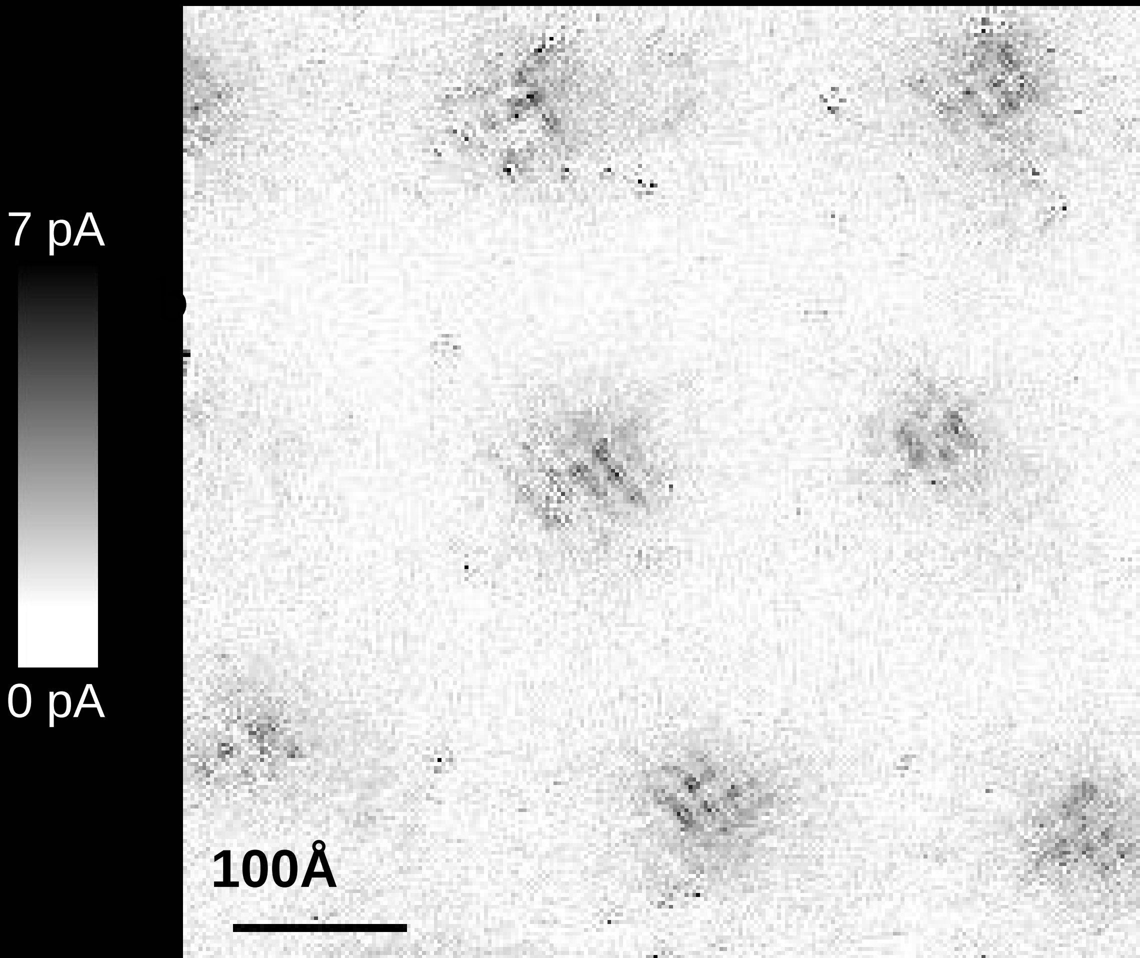
M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Pinned vortices in the superfluid

Any pinned vortex breaks the space group symmetry, and so has a preferred orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.

In the cuprates, assuming boson density=density of Cooper pairs we have $\rho_{\mathrm{MI}} = 7/16,$ and $q = 16$ (both models in part B yield this value of q). So modulation must have period $a \times b$ with $16/a,$ $16/b,$ and $ab/16$ all integers.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

J. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Measuring the inertial mass of a vortex

The spatial extent of the LDOS modulations measures the region over which the vortex executes its zero-point motion. The size of this region can be determined by solving the equations of motion

$$m_v \frac{d^2\mathbf{r}}{dt^2} = F_M$$

for a triangular lattice of vortices. Defining

$$u_{\rm rms} = \text{rms displacement of vortex from its equilibrium position},$$

we obtain from the vortex 'magnetophonon' spectrum

$$\begin{aligned} m_v &= 0.0419 \frac{\hbar^2 A_0}{\rho_s u_{\rm rms}^4} F\left(\frac{u_{\rm rms}^2 B}{\hbar}\right) \\ F(x) &\approx 0.5039 + \sqrt{0.2461 + 0.4147 x^2} \end{aligned}$$

where A_0 is the area of a vortex lattice unit cell, and $B = -h(\rho - \rho_{MI})$.

Measuring the inertial mass of a vortex

Preliminary estimates for the BSCCO experiment:

Inertial vortex mass $m_v \approx 10 m_e$

Vortex magnetoplasmon frequency $\nu_{\mathrm{p}} \approx 1\ \mathrm{THz} = 4\ \mathrm{meV}$

Future experiments can directly detect vortex zero point motion by looking for resonant absorption at this frequency.

Vortex oscillations can also modify the electronic density of states.

Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, $f = p/q$ per site, while the density of the superfluid is close (but need not be identical) to this value

- Vortices with flux *h/(2e)* come in multiple (usually q) "flavors"
- The lattice space group acts in a projective representation on the vortex flavor space.
- These flavor quantum numbers provide a distinction between superfluids: they constitute a "quantum order"
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.