Competing order and quantum criticality

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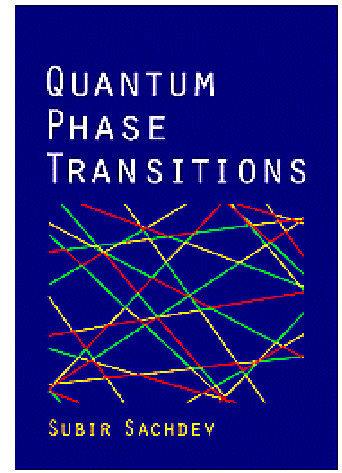
Lecture based on the review article cond-mat/0109419



Quantum Phase Transitions
Cambridge University Press

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

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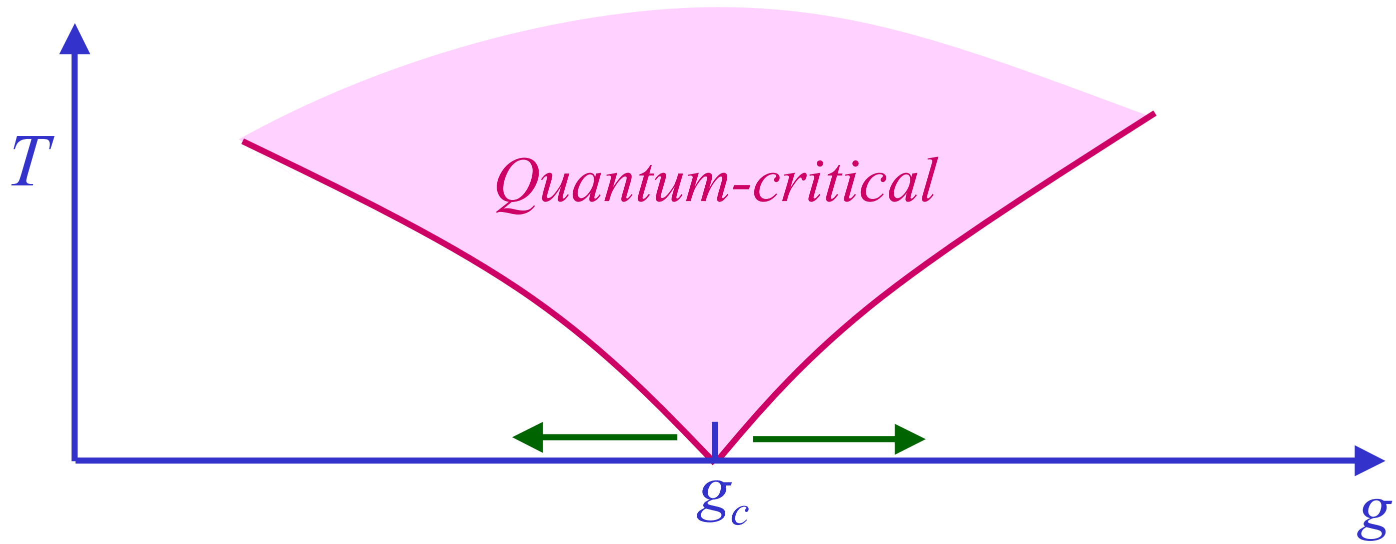
Quantum phase transition: ground states on either side of g_c have distinct "order"



- Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

- Critical point is a novel state of matter without quasiparticle excitations

- Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Important property of ground state at $g=g_c$: temporal and spatial scale invariance; characteristic energy scale at other values of g: $\Delta \sim \left| g - g_c \right|^{z\nu}$

Outline

I. **Quantum Ising Chain**

II. Coupled Ladder Antiferromagnet
  A. Coherent state path integral
  B. Quantum field theory for critical point

Single order parameter.

III. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell.
  A. Collinear spins, Berry phases, and bond-order.
  B. Non-collinear spins and deconfined spinons.

Multiple order parameters.

IV. Quantum transition in a BCS superconductor

Transition between $d_{x^2-y^2}$ and $d_{x^2-y^2}+id_{xy}$ pairing

V. Conclusions

I. Quantum Ising Chain

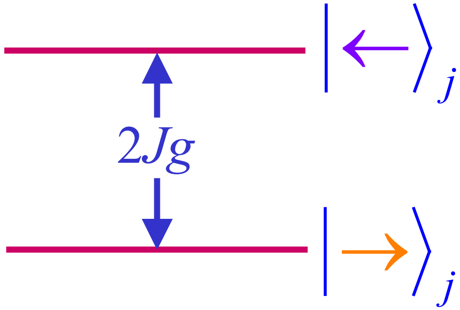
Degrees of freedom: $j = 1 \ldots N$ qubits, N "large"

$$\left|\uparrow\right\rangle_j , \left|\downarrow\right\rangle_j$$

or $$\left|\rightarrow\right\rangle_j = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_j + \left|\downarrow\right\rangle_j\right) , \quad \left|\leftarrow\right\rangle_j = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_j - \left|\downarrow\right\rangle_j\right)$$

Hamiltonian of decoupled qubits:

$$H_0 = -Jg \sum_j \sigma_j^x$$



Coupling between qubits:

$$H_1 = -J \sum_j \sigma_j^z \sigma_{j+1}^z$$

$$\Big(\left|\rightarrow\right\rangle_j \left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_j \left\langle\rightarrow\right| \Big) \Big(\left|\rightarrow\right\rangle_{j+1} \left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_{j+1} \left\langle\rightarrow\right| \Big)$$

Prefers neighboring qubits

are *either* $\left|\uparrow\right\rangle_j \left|\uparrow\right\rangle_{j+1}$ *or* $\left|\downarrow\right\rangle_j \left|\downarrow\right\rangle_{j+1}$

(not entangled)

Full Hamiltonian

$$H = H_0 + H_1 = -J \sum_j \left(g \sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$

leads to entangled states at g of order unity

Weakly-coupled qubits $(g \gg 1)$

Ground state:

$$|G\rangle = |\cdots \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle$$

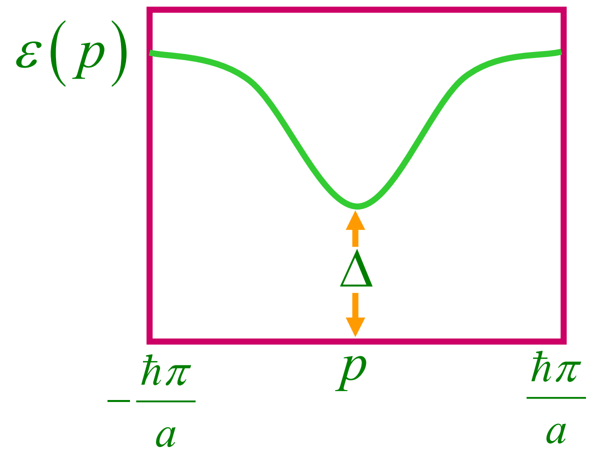
$$-\frac{1}{2g}|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\leftarrow\leftarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle - \cdots$$

Lowest excited states:

$$|\ell_j\rangle = |\cdots \rightarrow\rightarrow\rightarrow\rightarrow\underset{j}{\leftarrow}\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle + \cdots$$

Coupling between qubits creates "flipped-spin" *quasiparticle* states at momentum p

$$|p\rangle = \sum_j e^{ipx_j/\hbar}|\ell_j\rangle$$



Excitation energy $\varepsilon(p) = \Delta + 4J\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^{-1}\right)$

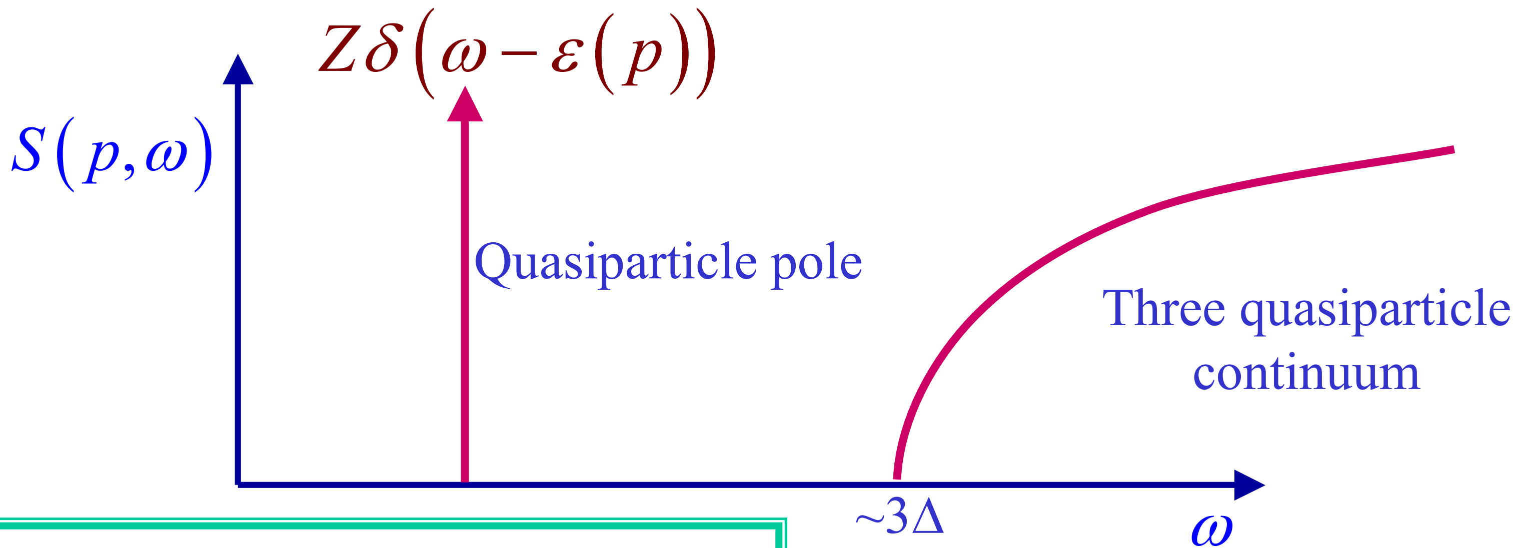
Excitation gap $\Delta = 2gJ - 2J + O\left(g^{-1}\right)$

Entire spectrum can be constructed out of multi-quasiparticle states

Dynamic Structure Factor $S(p,\omega)$:

Weakly-coupled qubits $(g \gg 1)$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



Structure holds to all orders in $1/g$

At $T > 0$, collisions between quasiparticles broaden pole to a Lorentzian of width $1/\tau_\varphi$ where the ***phase coherence time*** τ_φ is given by

$$\frac{1}{\tau_\varphi} = \frac{2k_B T}{\pi\hbar} e^{-\Delta/k_B T}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Strongly-coupled qubits $\left(g \ll 1\right)$

Ground states:

$$\left|G\uparrow\right\rangle = \left|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\right\rangle$$

$$-\frac{g}{2}\left|\cdots\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\right\rangle - \cdots$$

Ferromagnetic moment

$N_0 = \left\langle G\right|\sigma^z\left|G\right\rangle \neq 0$

Second state $\left|G\downarrow\right\rangle$ obtained by $\uparrow \Leftrightarrow \downarrow$

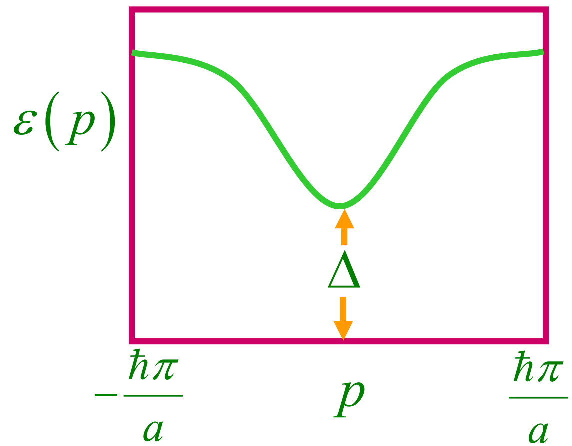
$\left|G\downarrow\right\rangle$ and $\left|G\uparrow\right\rangle$ mix only at order g^N

Lowest excited states: domain walls

$$\left|d_j\right\rangle = \left|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow_j\downarrow\downarrow\downarrow\downarrow\downarrow\cdots\right\rangle + \cdots$$

Coupling between qubits creates new "domain-wall" *quasiparticle* states at momentum p

$$\left|p\right\rangle = \sum_j e^{ipx_j/\hbar}\left|d_j\right\rangle$$



Excitation energy $\varepsilon(p) = \Delta + 4Jg\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^2\right)$

Excitation gap $\Delta = 2J - 2gJ + O\left(g^2\right)$

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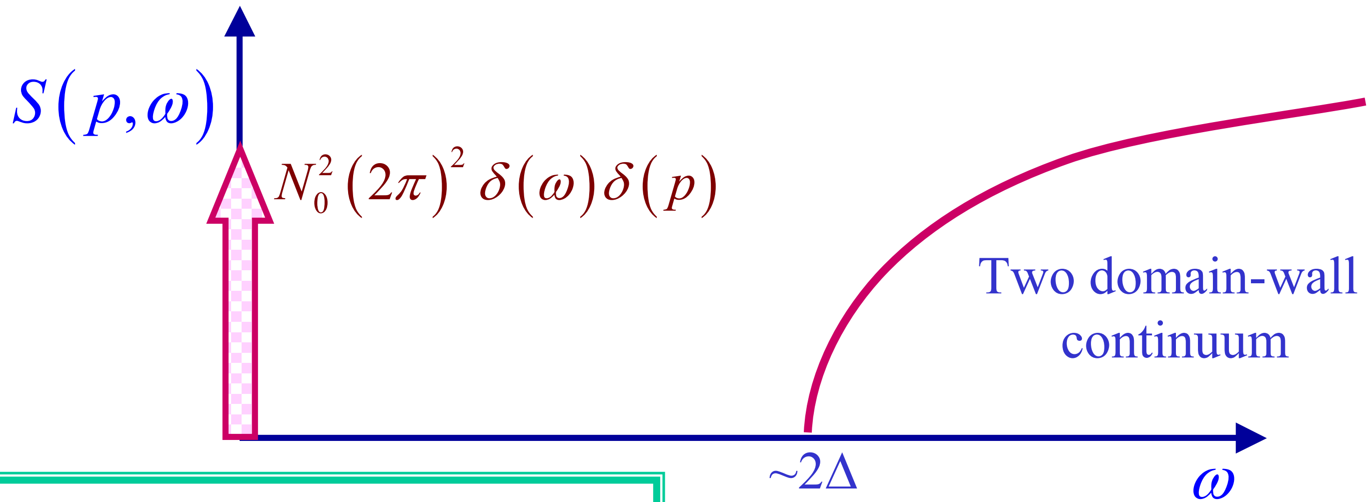
Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



Structure holds to all orders in g

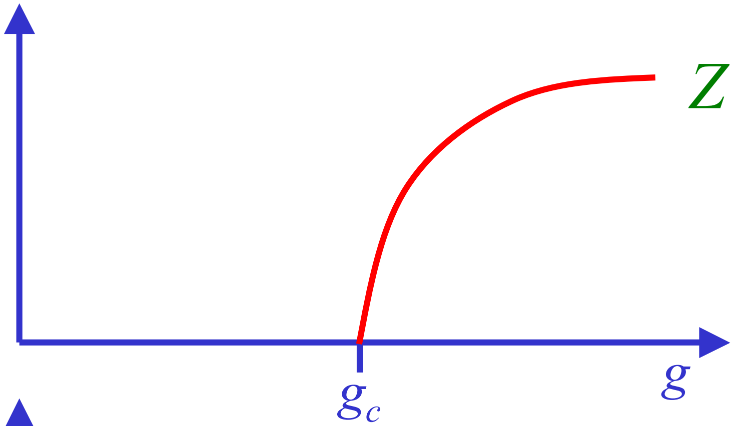
At $T>0$, motion of domain walls leads to a finite ***phase coherence time*** τ_φ, and broadens coherent peak to a width $1/\tau_\varphi$ where

$$\frac{1}{\tau_\varphi} = \frac{2k_BT}{\pi\hbar}e^{-\Delta/k_BT}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Entangled states at g of order unity

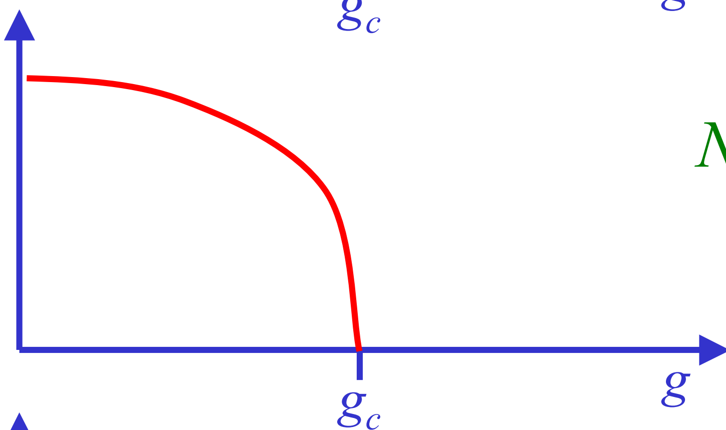
"Flipped-spin" Quasiparticle weight Z



$$Z \sim (g - g_c)^{1/4}$$

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

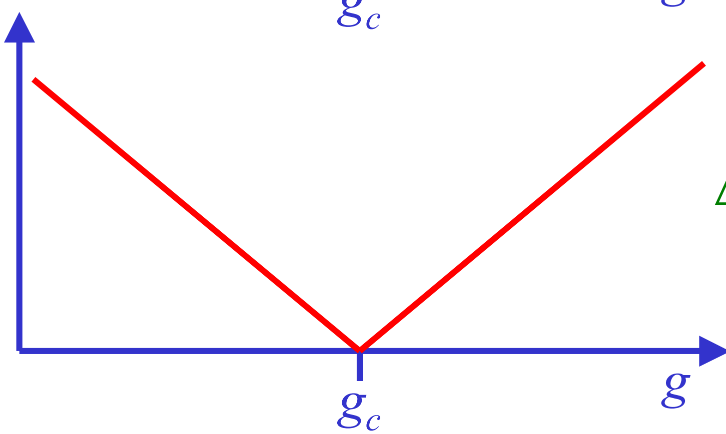
Ferromagnetic moment N_0



$$N_0 \sim (g_c - g)^{1/8}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

Excitation energy gap Δ



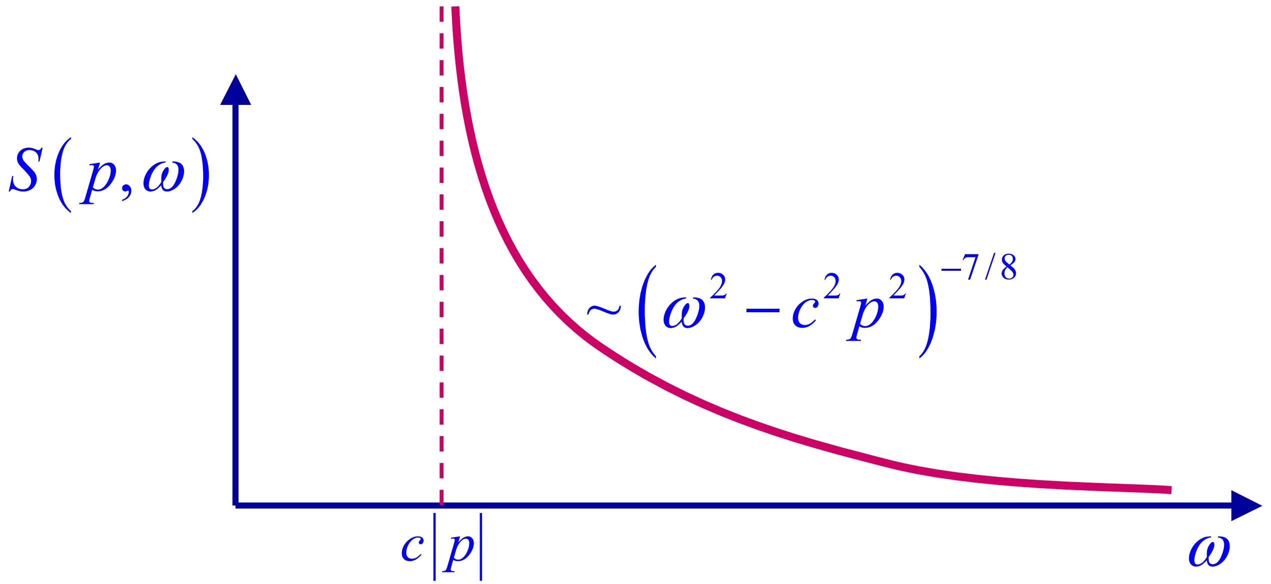
$$\Delta \sim |g - g_c|$$

Dynamic Structure Factor $S(p,\omega)$:

Critical coupling $(g = g_c)$

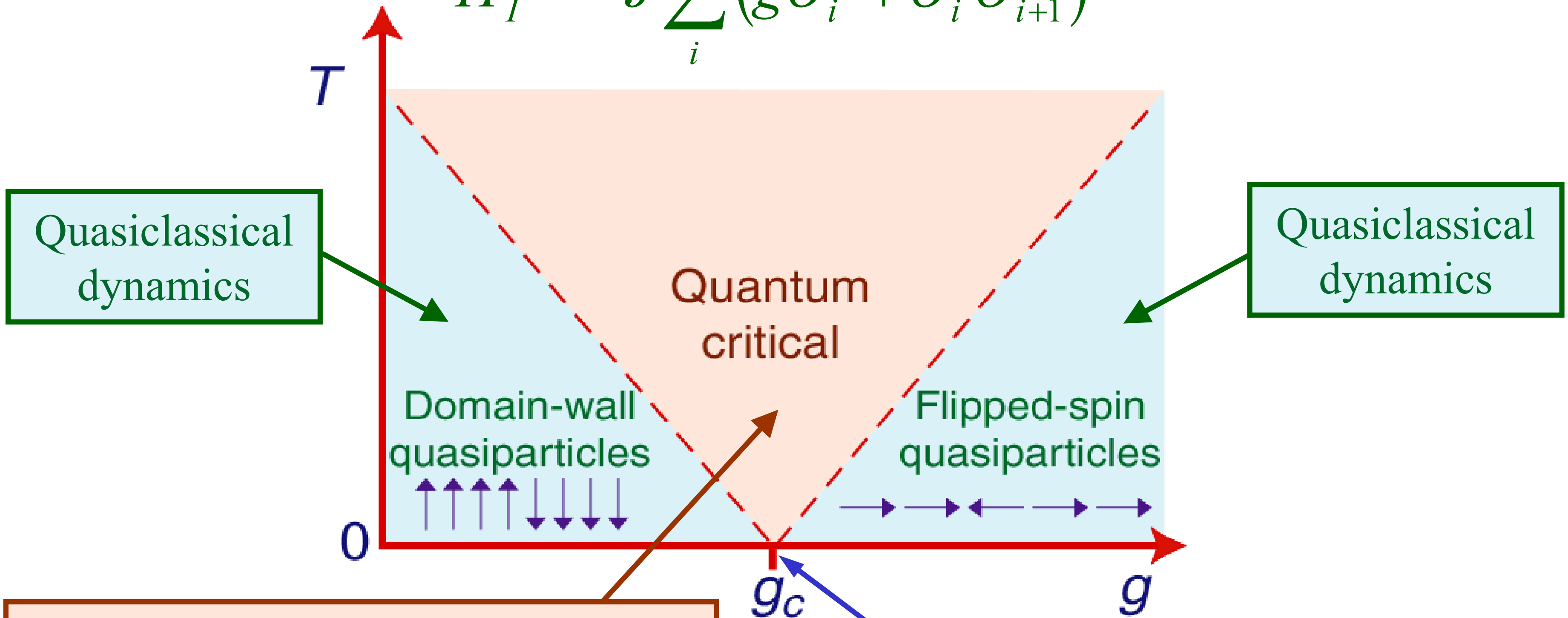
Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa)

while transferring energy $\hbar\omega$ and momentum p



No quasiparticles --- dissipative critical continuum

$$H_I = -J \sum_i \left(g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$



$$\chi(\omega) = \frac{i}{\hbar} \sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4} \left(1 - i\omega / \Gamma_R + \ldots \right)}$$

$$\Gamma_R = \left(2 \tan \frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{\left| j - k \right|^{1/4}}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

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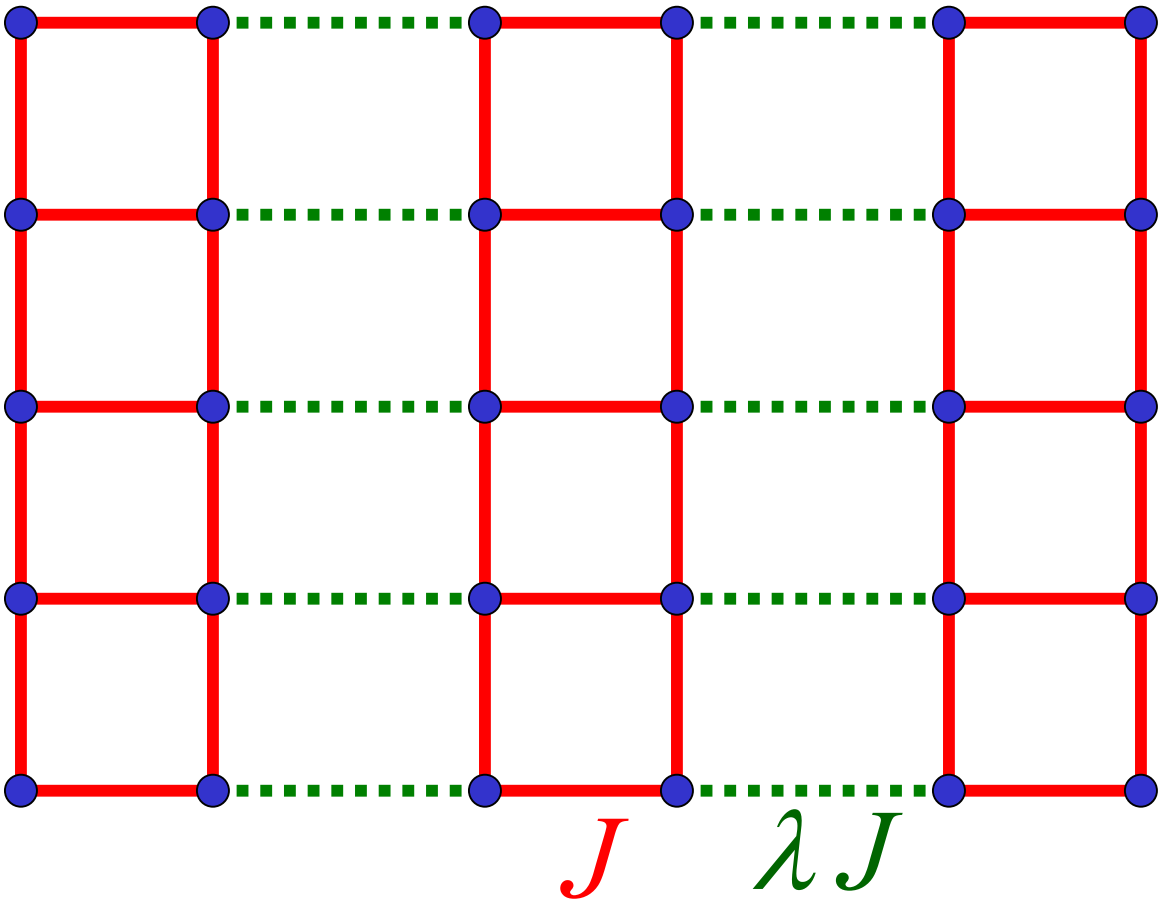
V. Conclusions

II. Coupled Ladder Antiferromagnet

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled 2-leg ladders



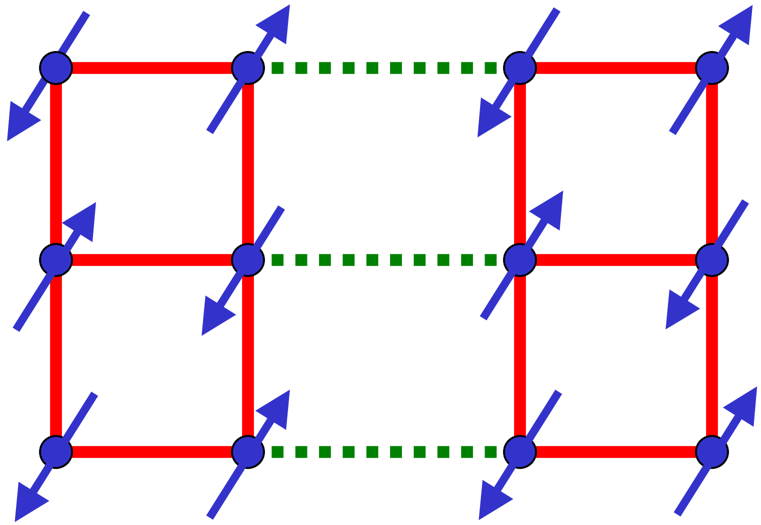
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



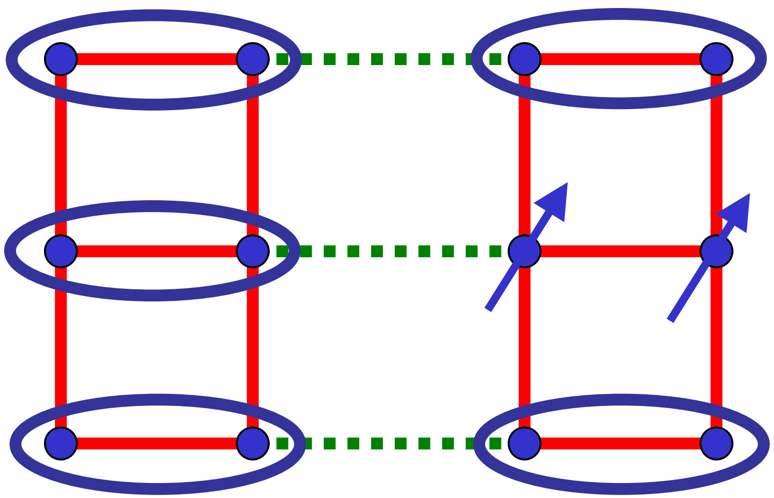
Ground state has long-range magnetic (Neel) order

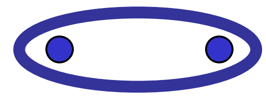
$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

Paramagnetic ground state

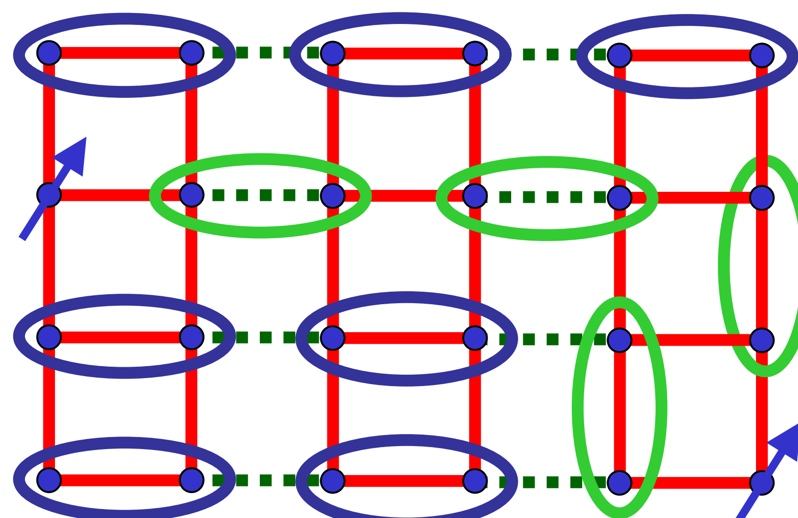
$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: S=1 ***exciton***

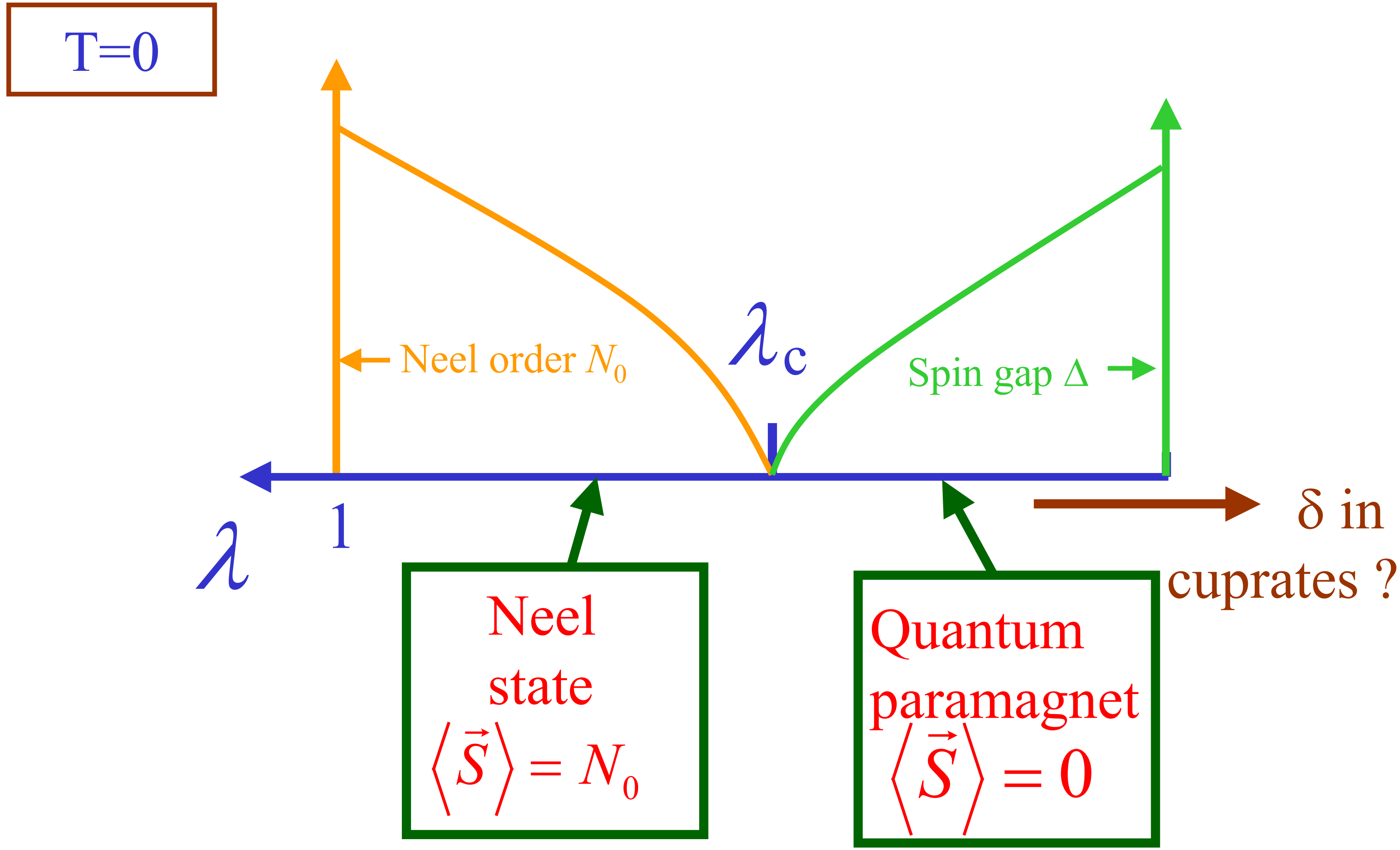
(spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$



S=1/2 spinons are ***confined*** by a linear potential.



T=0
Neel order N_0
λ_c
Spin gap Δ
λ
1
δ in
cuprates ?
Neel
state
$\langle \vec{S} \rangle = N_0$
Quantum
paramagnet
$\langle \vec{S} \rangle = 0$

II.A Coherent state path integral

See Chapter 13 of *Quantum Phase Transitions*, S. Sachdev, Cambridge University Press (1999).

Path integral for a single spin

$$Z = \mathrm{Tr}\left(e^{-H[\boldsymbol{S}]/T}\right)$$

$$= \int \mathcal{D}\boldsymbol{N}(\tau)\,\delta\left(\boldsymbol{N}^2 - 1\right)\exp\left(-iS\int A_\tau(\tau)\,d\tau - \int d\tau H\left[S\boldsymbol{N}(\tau)\right]\right)$$

$A_\tau(\tau)\,d\tau =$ Oriented area of triangle on surface of unit sphere bounded by $\boldsymbol{N}(\tau), \boldsymbol{N}(\tau + d\tau)$, and a fixed reference $\boldsymbol{N}_0$

Action for lattice antiferromagnet

$$\boldsymbol{N}_j(\tau) = \eta_j \boldsymbol{n}(x_j, \tau) + \boldsymbol{L}(x_j, \tau)$$

$\eta_j = \pm 1$ identifies sublattices

$\boldsymbol{n}$ and $\boldsymbol{L}$ vary slowly in space and time

Integrate out $\boldsymbol{L}$ and take the continuum limit

$$Z = \int \mathcal{D}\boldsymbol{n}(x,\tau)\,\delta\left(\boldsymbol{n}^2 - 1\right)\exp\left(-iS\sum_j \int \eta_j A_\tau(x_j,\tau)\,d\tau - \frac{1}{2g}\int d^2x\,d\tau\left(\left(\partial_\tau \boldsymbol{n}\right)^2 + c^2\left(\nabla_x \boldsymbol{n}\right)^2\right)\right)$$

$\eta_j = \pm 1$ identifies sublattices

$g < g_c \Leftrightarrow \lambda > \lambda_c$

$g > g_c \Leftrightarrow \lambda < \lambda_c$

Berry phases can be neglected for coupled ladder antiferromagent

(justified later)

Discretize spacetime into a cubic lattice

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right)\exp\left(\frac{1}{g}\sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu}\right)$$

$a \to$ cubic lattice sites; $\mu \to x, y, \tau$;

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989).

Quantum path integral for two-dimensional quantum antiferromagnet
$\Leftrightarrow$ Partition function of a classical three-dimensional ferromagnet at a "temperature" g

Quantum transition at $\lambda=\lambda_c$ is related to classical Curie transition at $g=g_c$

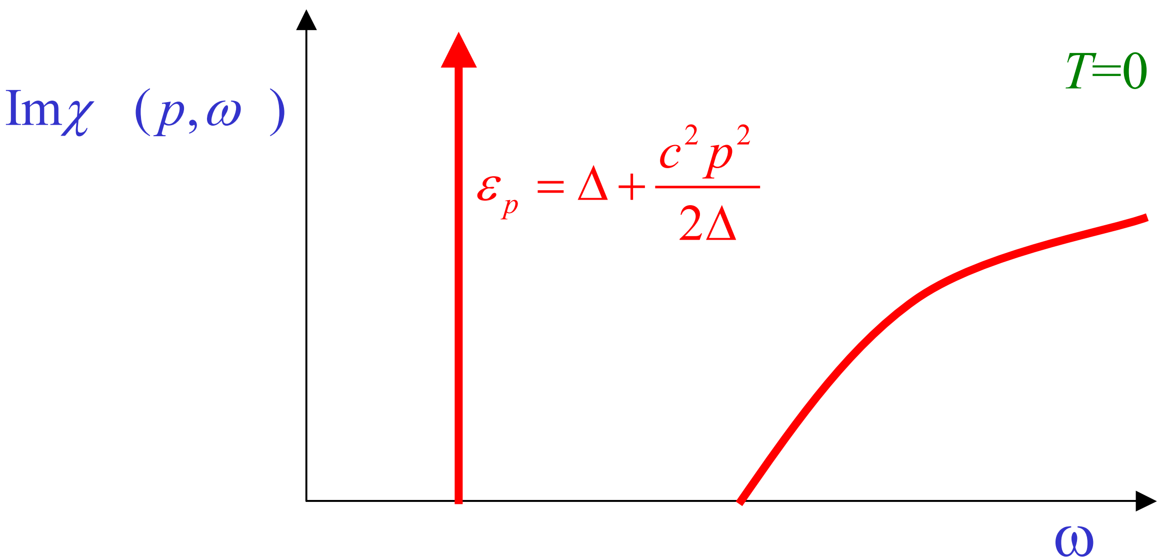
II.B Quantum field theory for critical point

λ close to λ_c : use "soft spin" field

$$\mathcal{S}_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 + \left(\lambda_c - \lambda \right) \phi_\alpha^2 \right) + \frac{u}{4!} \left(\phi_\alpha^2 \right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

Oscillations of ϕ_α about zero (for $\lambda < \lambda_c$)
→ spin-1 collective mode

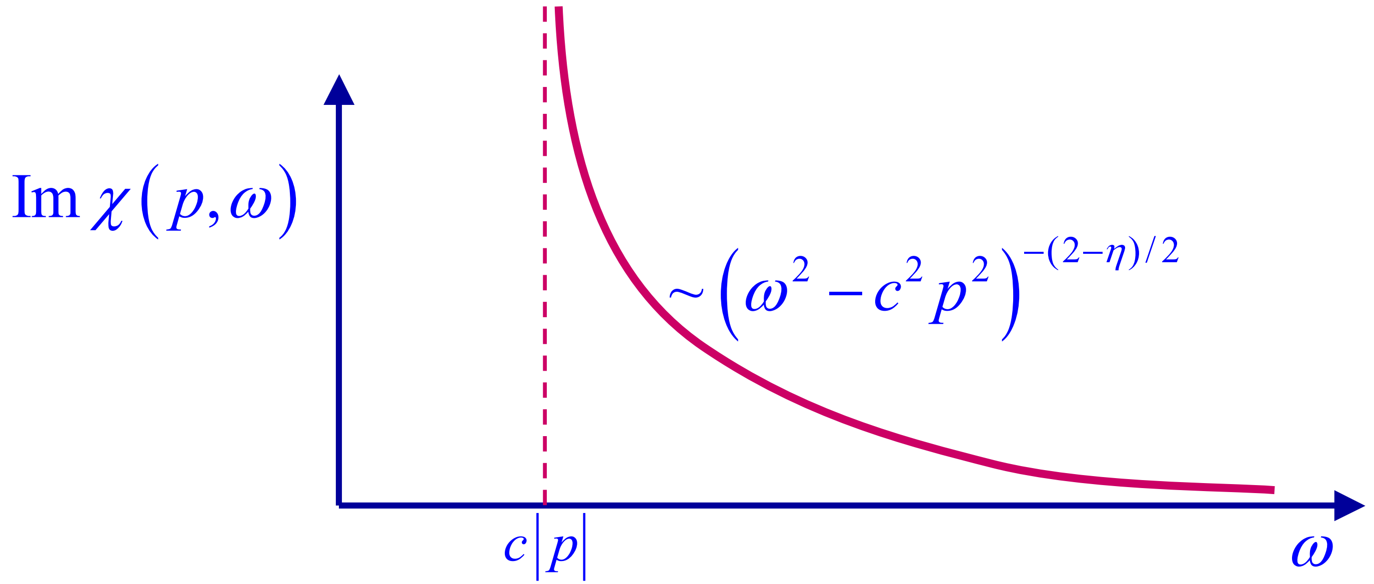


T=0 spectrum

$$\Delta = \sqrt{\lambda_c - \lambda} / c$$

Critical coupling $\left(\lambda = \lambda_c\right)$

Dynamic spectrum at the critical point



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III.A Collinear spins, Berry phases, and bond-order

S=1/2 square lattice antiferromagnet with non-nearest neighbor exchange

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\rm a} \rightarrow \pm 1$ on two square sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

Small g $\rightarrow$ Spin-wave theory about Neel state receives minor modifications from Berry phases.

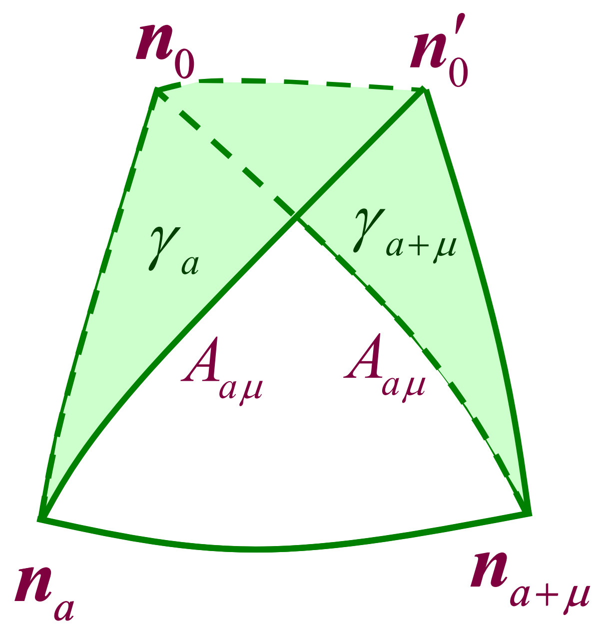
Large g $\rightarrow$ Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \mathbf{n}_a \rangle = 0$

Integrate out $\mathbf{n}_a$ to obtain effective action for $A_{a\mu}$

Change in choice of $\mathbf{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\mathbf{n}_a$ and the two choices for $\mathbf{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda} \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 2+1 dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is ***always*** in a *confining* phase:

There is an energy gap and the ground state has a **bond order wave**.

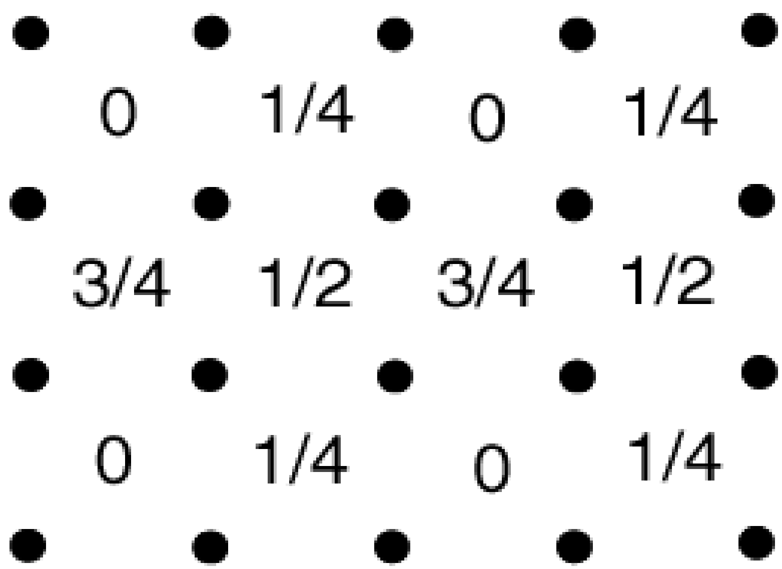
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev*. B **65**, 220405 (2002).

Exact duality transform on periodic Gaussian ("Villain") action for compact QED yields

$$Z = \sum_{\{h_{\bar{\jmath}}\}} \exp\left(-\frac{e^2}{2}\sum_{\bar{\jmath}}\left(\Delta_\mu h_{\bar{\jmath}} - \Delta_\mu \mathcal{X}_{\bar{\jmath}}\right)^2\right)$$

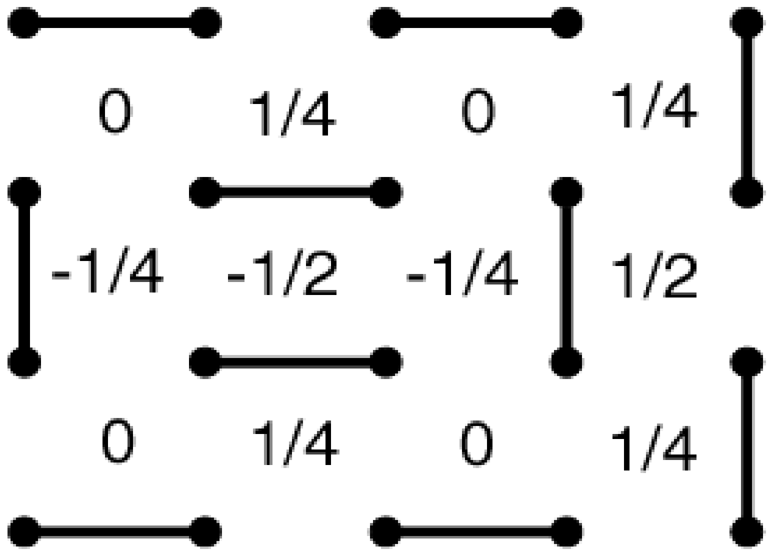
with $h_{\bar{\jmath}}$ integer.

Height model in 2+1 dimensions with 'offsets' $\mathcal{X}_{\bar{\jmath}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice

$\Longrightarrow$ 2+1 dimensional height model is the path integral of the **Quantum Dimer Model**



There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

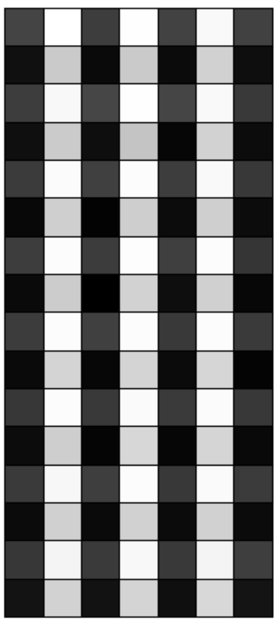
$\Longrightarrow$ There is a definite average height of the interface

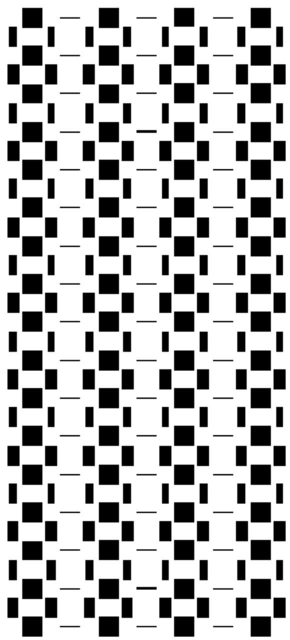
$\Longrightarrow$ **Ground state has a bond order wave.**

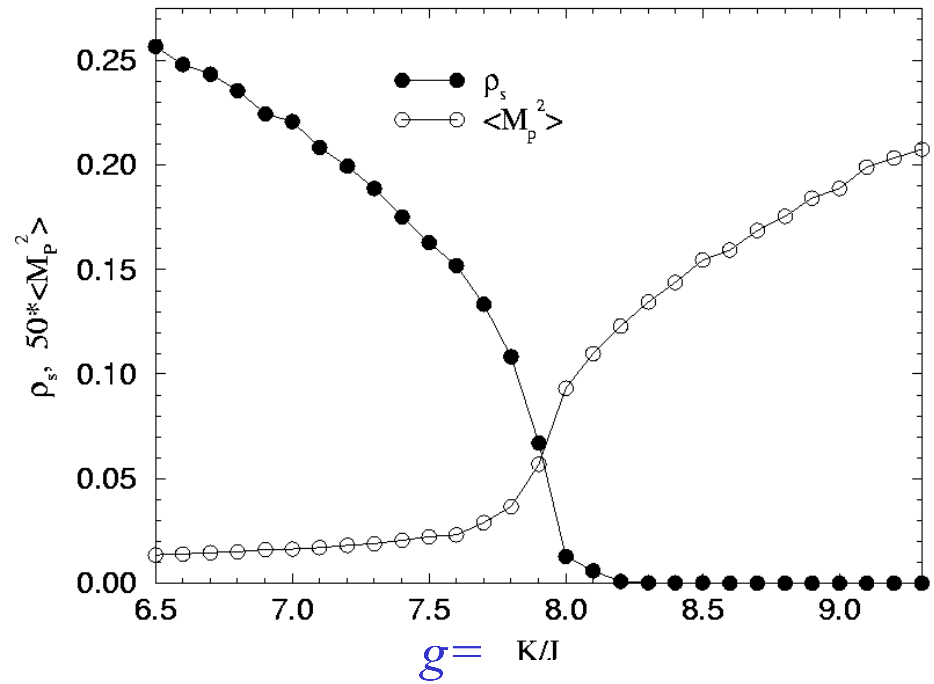
Bond order wave in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in *S=1/2* antiferromagnet with full square lattice symmetry







$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

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Single order parameter.

III. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell.
  A. Collinear spins, Berry phases, and bond-order.
  B. Non-collinear spins and deconfined spinons.

Multiple order parameters.

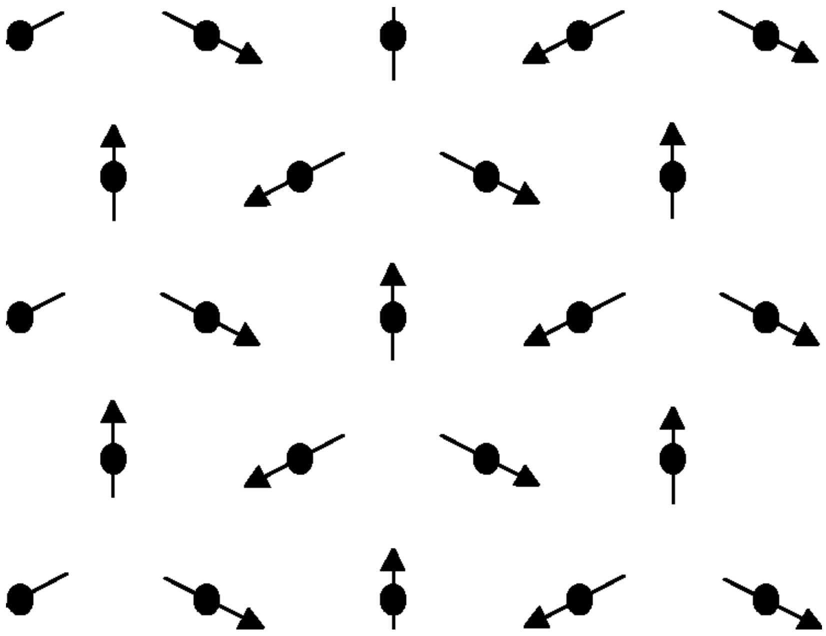
IV. Quantum transition in a BCS superconductor

Transition between $d_{x^2-y^2}$ and $d_{x^2-y^2}+id_{xy}$ pairing

V. Conclusions

III.B Non-collinear spins and deconfined spinons.

Magnetically ordered state:



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q}\bullet\boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q}\bullet\boldsymbol{r})$$

$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \; \vec{N}_1^{\,2} = \vec{N}_2^{\,2} = 1 \,; \; \vec{N}_1 \bullet \vec{N}_2 = 0$$

Solve constraints by writing:

$$\vec{N}_1 + i\vec{N}_2 = \varepsilon_{ac} z_c \vec{\sigma}_{ab} z_b$$

where $z_{1,2}$ are two complex numbers with

$$|z_1|^2 + |z_2|^2 = 1$$

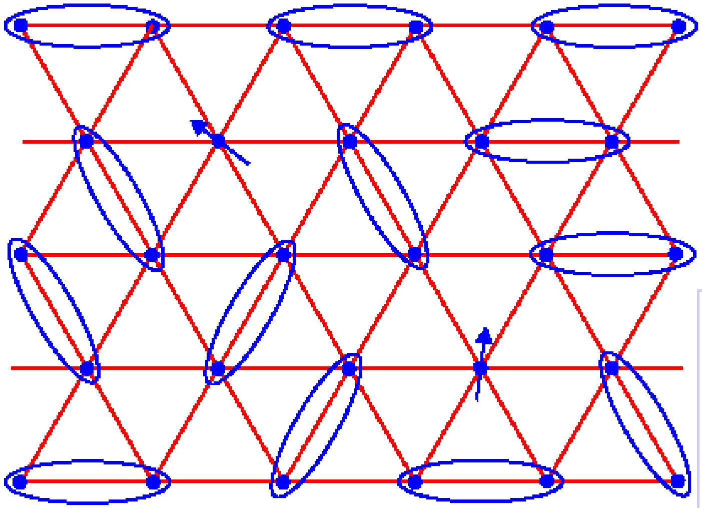
Order parameter space: S_3/Z_2

Physical observables are invariant under the Z_2 gauge transformation $z_a \to \pm z_a$

Non-magnetic state

Fluctuations can lead to a "quantum disordered" state in which z_a are globally well defined. This requires a topologically ordered state in which vortices associated with $\pi_1(S_3/Z_2)=Z_2$ ["visons"] are gapped out. This is an RVB state with deconfined $S=1/2$ spinons z_a

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991).
X. G. Wen, *Phys. Rev.* B **44**, 2664 (1991).
A.V. Chubukov, T. Senthil and S. Sachdev, *Phys. Rev. Lett.* **72**, 2089 (1994).
T. Senthil and M.P.A. Fisher, *Phys. Rev.* B **62**, 7850 (2000).



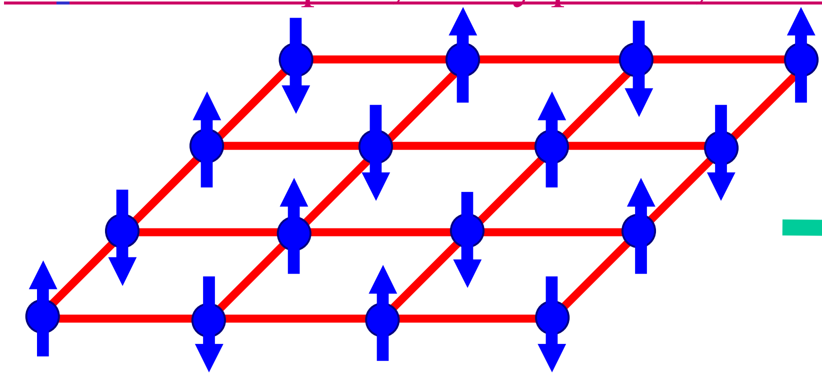
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).
S. Sachdev, *Phys. Rev.* B **45**, 12377 (1992).
G. Misguich and C. Lhuillier, *Eur. Phys. J.* B **26**, 167 (2002).
R. Moessner and S.L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001).

Recent experimental realization: Cs_2CuCl_4
R. Coldea, D.A. Tennant, A.M. Tsvelik, and Z. Tylczynski, *Phys. Rev. Lett.* **86**, 1335 (2001).

2D Antiferromagnets with an odd number of *S=1/2* spins per unit cell

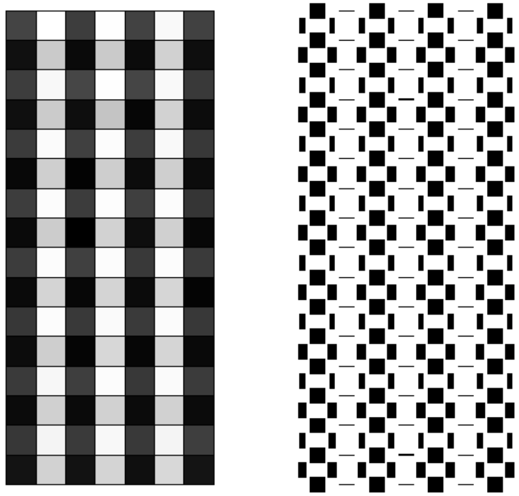
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991; *Int. J. Mod. Phys.* B **5**, 219 (1991).

A. Collinear spins, Berry phases, and bond-order



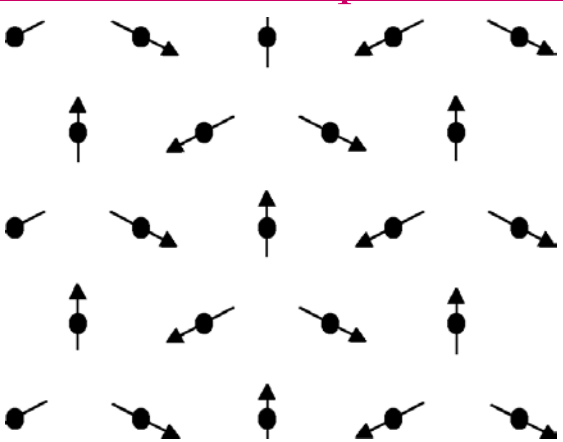
Néel ordered state

Quantum transition restoring spin rotation invariance



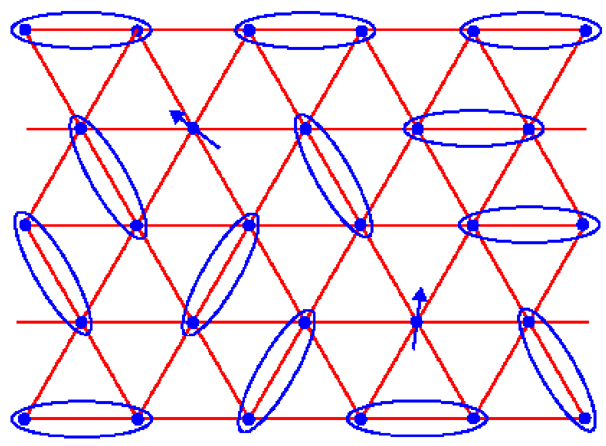
Bond-ordered state with confinement of spinons and *S=1* spin exciton

B. Non-collinear spins and deconfined spinons.



Non-collinear ordered antiferromagnet

Quantum transition restoring spin rotation invariance



RVB state with visons, deconfined *S=1/2* spinons, Z_2 gauge theory

Outline

I. Quantum Ising Chain

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} *Single order parameter.*

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} *Multiple order parameters.*

IV. Quantum transition in a BCS superconductor

Transition between $d_{x^2-y^2}$ and $d_{x^2-y^2} + i d_{xy}$ pairing

V. Conclusions

IV. Quantum transitions between BCS superconductors

From numerical/analytical/RG studies of the square lattice Hubbard model we know that ground state has

$\Rightarrow$ $d_{x^2-y^2}$ superconductivity near half filling

C.J. Halboth and W. Metzner Phys. Rev. Lett. **85**, 5162 (2000).

$\Rightarrow$ d_{xy} superconductivity for small electron density

M.A. Baranov and M. Yu Kagan, Z. Phys. B **86**, 237 (1992).
M.A. Baranov, A.V. Chubukov, and M. Yu Kagan, Int. J. Mod. Phys. B **6**, 2471 (1992).

We model this phenomenologically:

$$H = \sum_k \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J_1 \sum_{j,\mu} \mathbf{S}_j \cdot \mathbf{S}_{j+\hat{\mu}} + J_2 \sum_{j,\nu} \mathbf{S}_j \cdot \mathbf{S}_{j+\hat{\nu}}$$

where the sum on μ is over x, y, that on ν is over $x+y$, $x-y$,

$$\mathbf{S}_j \equiv \frac{1}{2} c_{j\sigma}^\dagger \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'}$$

with $\vec{\sigma}$ the Pauli matrices, and the dispersion

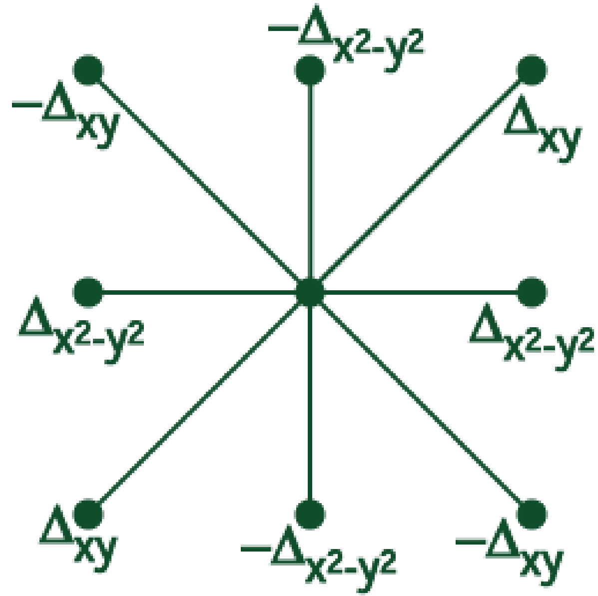
$$\varepsilon_k = -2t_1(\cos(k_x) + \cos(k_y)) - 2t_2(\cos(k_x+k_y) + \cos(k_x-k_y)) - \mu$$

BCS mean-field theory

$$H_{BCS} = \sum_k \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} \quad - \quad \frac{J_1}{2} \sum_{j,\mu} \Delta_\mu (c^\dagger_{j\uparrow} c^\dagger_{j+\hat{\mu},\downarrow} - c^\dagger_{j\downarrow} c^\dagger_{j+\hat{\mu},\uparrow}) + \text{h.c.}$$
$$\quad - \quad \frac{J_2}{2} \sum_{j,\nu} \Delta_\nu (c^\dagger_{j\uparrow} c^\dagger_{j+\hat{\nu},\downarrow} - c^\dagger_{j\downarrow} c^\dagger_{j+\hat{\nu},\uparrow}) + \text{h.c.}$$

We choose

$$\begin{aligned} \Delta_x &= -\Delta_y \equiv \Delta_{x^2-y^2} \\ \Delta_{x+y} &= -\Delta_{x-y} \equiv \Delta_{xy} \end{aligned}$$



The complex numbers $\Delta_{x^2-y^2}$ and Δ_{xy} are to be determined by minimizing the ground state energy per site

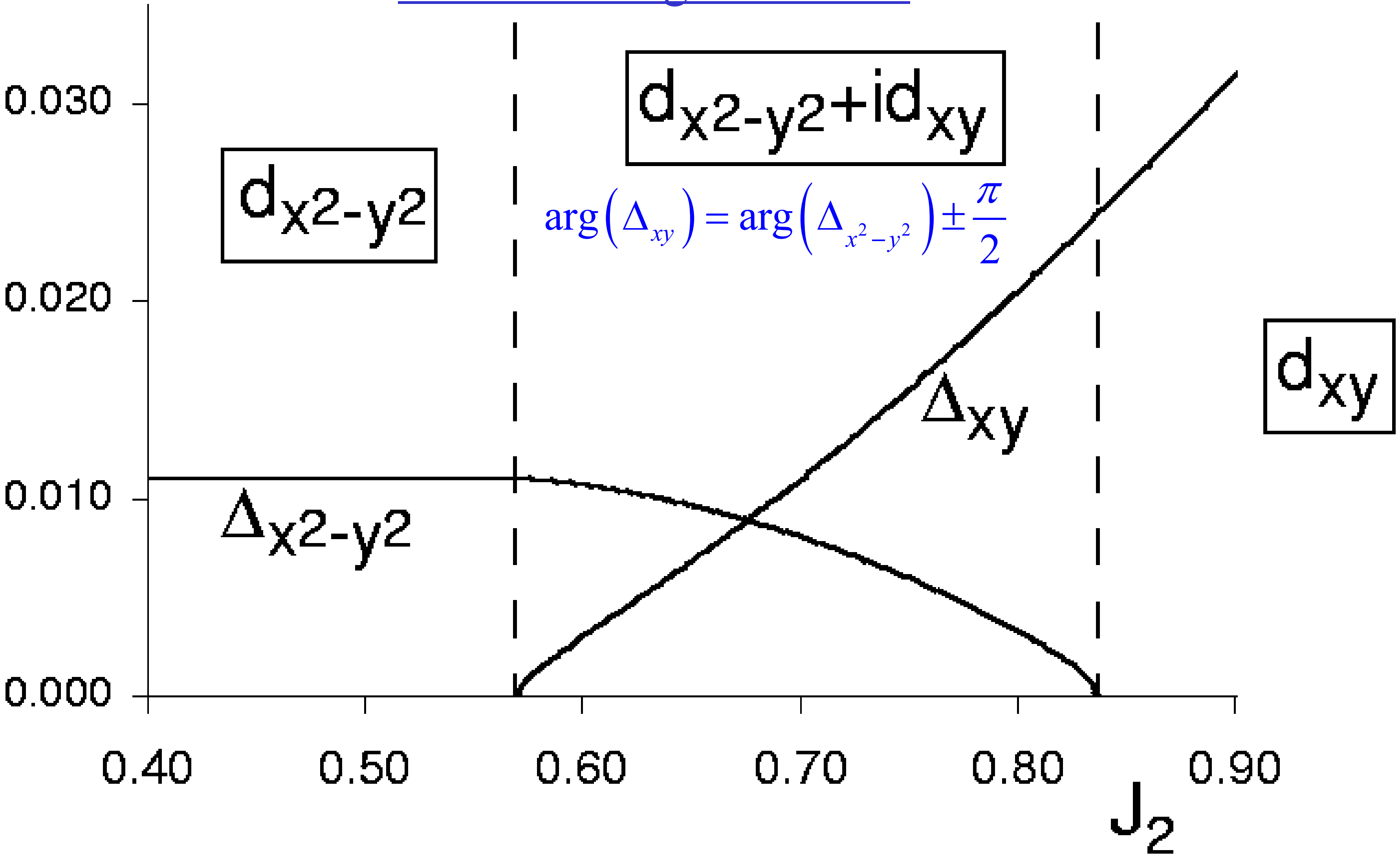
$$E_{BCS} = J_1 |\Delta_{x^2-y^2}|^2 + J_2 |\Delta_{xy}|^2 - \int \frac{d^2k}{4\pi^2} \left[E_k - \varepsilon_k \right]$$

where the fermionic quasiparticle dispersion is

$$E_k = \left[\varepsilon_k^2 + |J_1 \Delta_{x^2-y^2} (\cos k_x - \cos k_y) + 2 J_2 \Delta_{xy} \sin k_x \sin k_y|^2 \right]^{1/2}$$

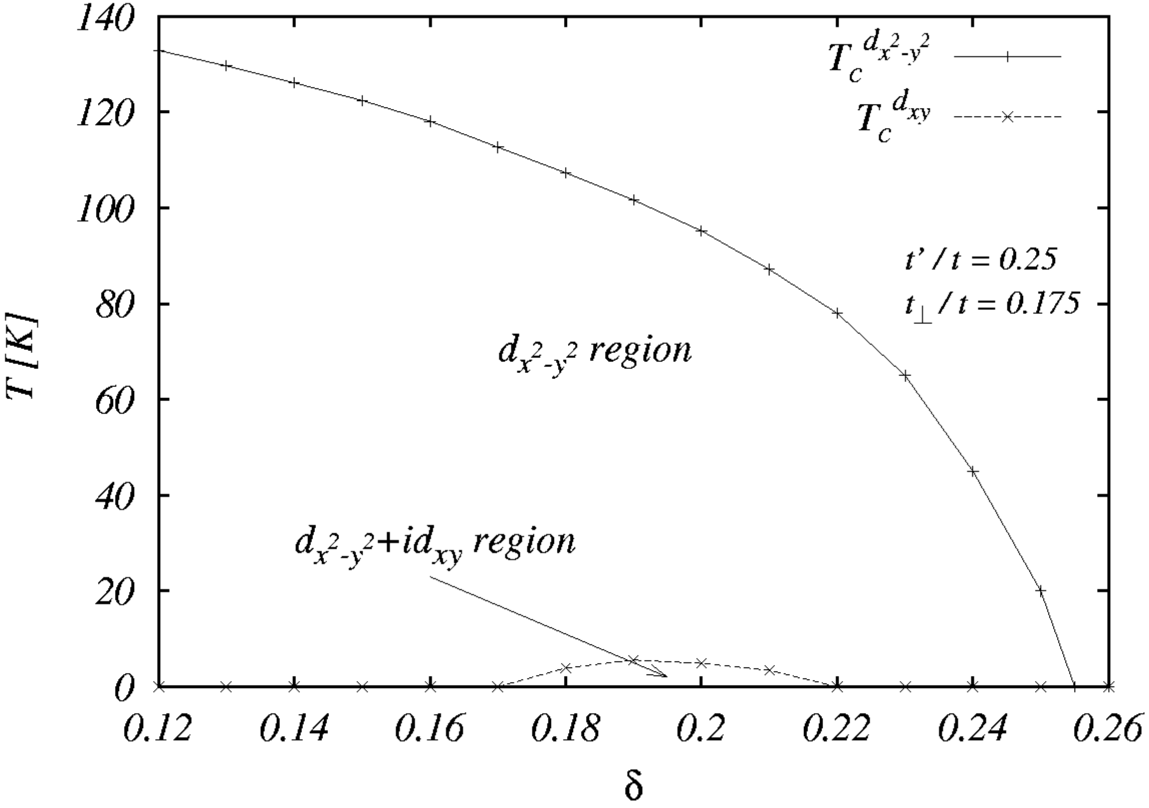
The energy only depends upon the relative phase between $\Delta_{x^2-y^2}$ and Δ_{xy}.

Evolution of ground state



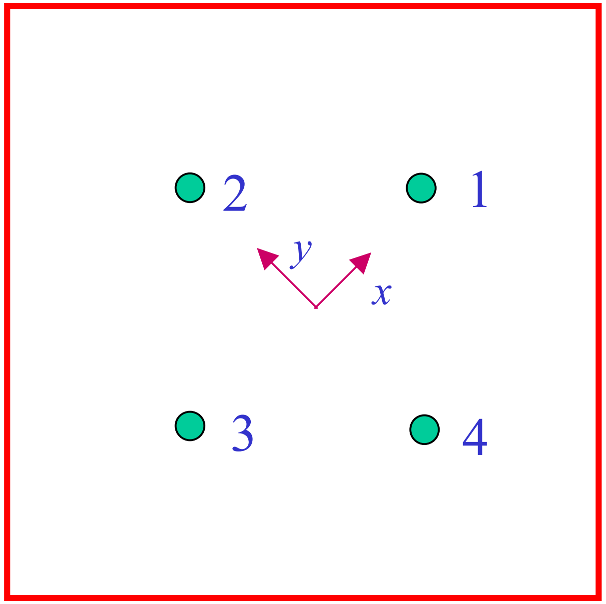
BCS theory fails near quantum critical points

Microscopic study of square lattice model



G. Sangiovanni, M. Capone, S. Caprara, C. Castellani, C. Di Castro, M. Grilli, cond-mat/0111107

Field theory for transition from $d_{x^2-y^2}$ to $d_{x^2-y^2}+id_{xy}$ superconductivity



Gapless Fermi Points in a d-wave superconductor at wavevectors $(\pm K, \pm K)$

K=0.391π

$$\Psi_1 = \begin{pmatrix} f_{1\uparrow} \\ f_{3\downarrow}^\dagger \\ f_{1\downarrow} \\ -f_{3\uparrow}^\dagger \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_{2\uparrow} \\ f_{4\downarrow}^\dagger \\ f_{2\downarrow} \\ -f_{4\uparrow}^\dagger \end{pmatrix}$$

$$S_\Psi = \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_1^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x \right) \Psi_1$$

$$+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_2^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x \right) \Psi_2$$

Ising order parameter for transition $\phi \sim i\Delta_{xy}$

Coupling to low energy fermions

$$i\lambda \int d^2x d\tau \left[\phi \left(f_{1\uparrow}^\dagger f_{3\downarrow}^\dagger - f_{1\downarrow}^\dagger f_{3\uparrow}^\dagger - f_{2\uparrow}^\dagger f_{4\downarrow}^\dagger - f_{2\downarrow}^\dagger f_{4\uparrow}^\dagger \right) + \text{h.c.} \right]$$

$$= \int d^2x d\tau \left[\lambda \phi \left(\Psi_1^\dagger \tau^y \Psi_1 - \Psi_2^\dagger \tau^y \Psi_2 \right) \right]$$

Action for low energy fluctuations near critical point

$$S = \int d^2x d\tau \left[\frac{1}{2} \left(\partial_\tau \phi \right)^2 + \frac{c^2}{2} \left(\nabla \phi \right)^2 - \frac{s}{2} \phi^2 + \frac{u}{24} \phi^4 \right.$$
$$+ \Psi_1^\dagger \left(\partial_\tau + i v_F \partial_x \tau^z + i v_\Delta \partial_y \tau^x \right) \Psi_1$$
$$+ \Psi_2^\dagger \left(\partial_\tau + i v_F \partial_y \tau^z + i v_\Delta \partial_x \tau^x \right) \Psi_2$$
$$\left. + \lambda \phi \left(\Psi_1^\dagger \tau^y \Psi_1 - \Psi_2^\dagger \tau^y \Psi_2 \right) \right]$$

{For $v_F = v_\Delta$ terms with fermions

$$= \overline{\Psi}_1 \gamma_\mu \partial_\mu \Psi_1 + \overline{\Psi}_2 \gamma_\mu \partial_\mu \Psi_2 + \lambda \phi \left(\overline{\Psi}_1 \Psi_1 - \overline{\Psi}_2 \Psi_2 \right)$$

Chiral symmetry breaking in the Higgs-Yukawa model}

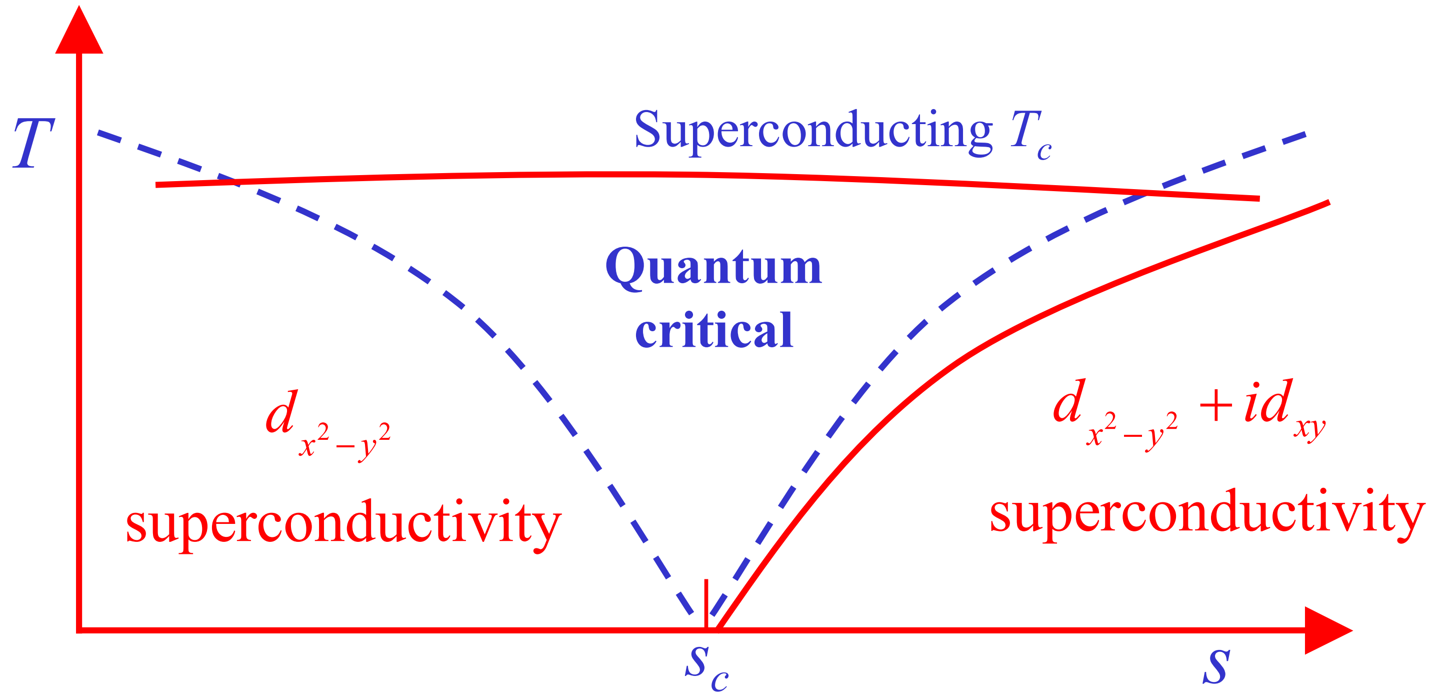
Momentum shell renormalization group equations

$$\frac{d\lambda}{d\ell} = \frac{(3-d)}{2}\lambda - C_1 \lambda^3$$

$$\frac{du}{d\ell} = (3-d)u - C_2 u^2 + C_3 \lambda^4 - C_4 \lambda^2 u$$

where C_{1-4} are functions of v_F / v_Δ; similar flow equation for v_F / v_Δ. Critical point is controlled by fixed point $u = u^*,\ \lambda = \lambda^*, v_F^* = v_\Delta^*$, which obeys hyperscaling, and is Lorentz invariant.

Crossovers near transition in d-wave superconductor



M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev. Lett*. **85**, 4940 (2000).

In a Fermi liquid, quasiparticle relaxation rate $\sim T^2$

In a BCS $d_{x^2-y^2}$ superconductor, quasiparticle relaxation rate $\sim T^3$

In a gapped BCS $d_{x^2-y^2}+id_{xy}$ superconductor, (as in the gapped Ising chain) quasiparticle relaxation rate $\sim e^{\Delta/T}$

In quantum critical region:

$$G_F(k,\omega)=\frac{\Lambda^{-\eta_F}}{(k_BT)^{1-\eta_F}}\Phi\left(\frac{ck}{k_BT},\frac{\hbar\omega}{k_BT}\right)$$

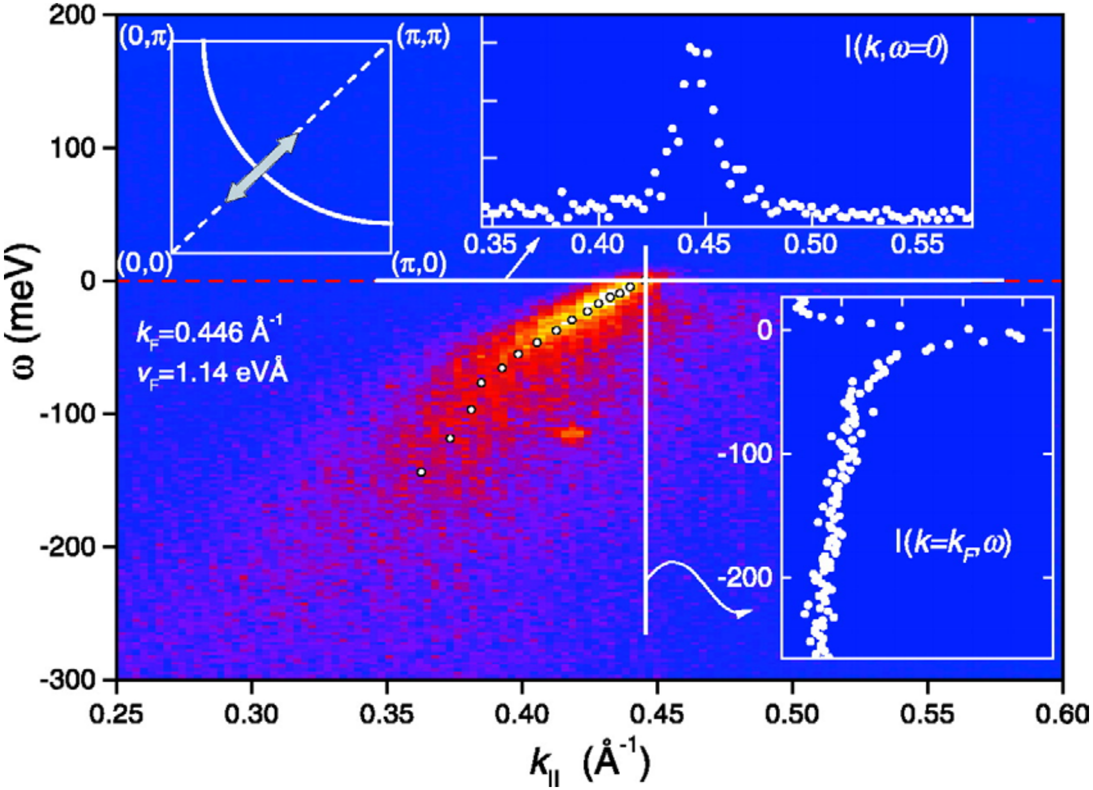
Nodal quasiparticle Green's function

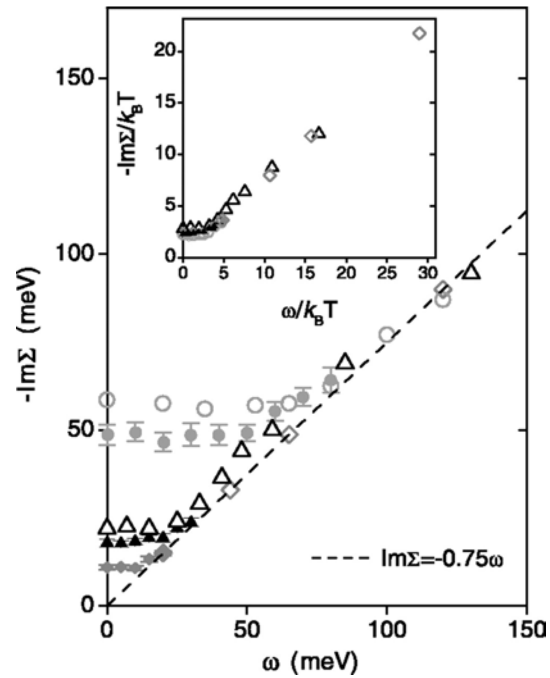
k → wavevector separation from node

Damping of Nodal Quasiparticles

Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))





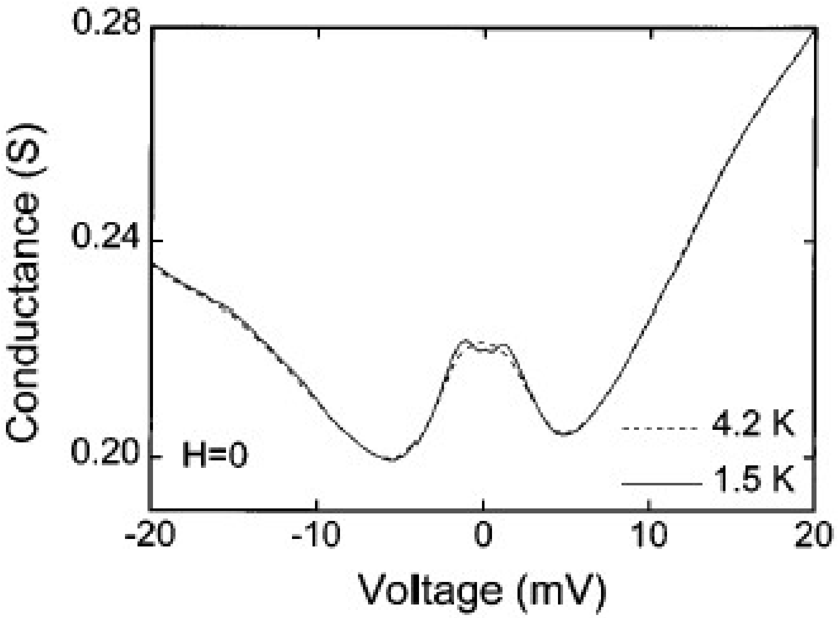
Width of quasiparticle energy distribution curve (EDC) ~ $k_B T$

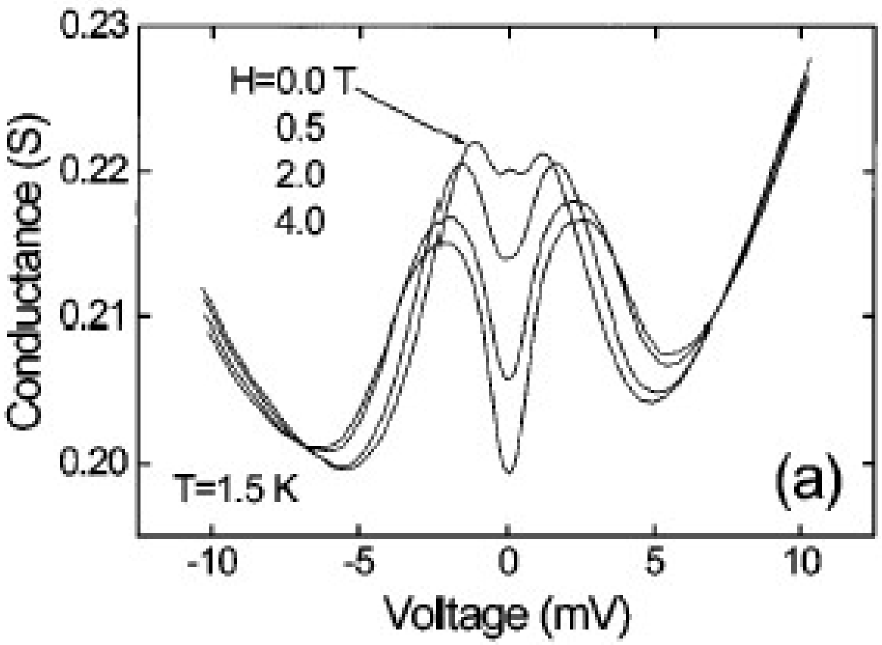
Yoram Dagan and Guy Deutscher, *Phys. Rev. Lett.* **87**, 177004 (2001).

Observations of splitting of the ZBCP

Spontaneous splitting (zero field)

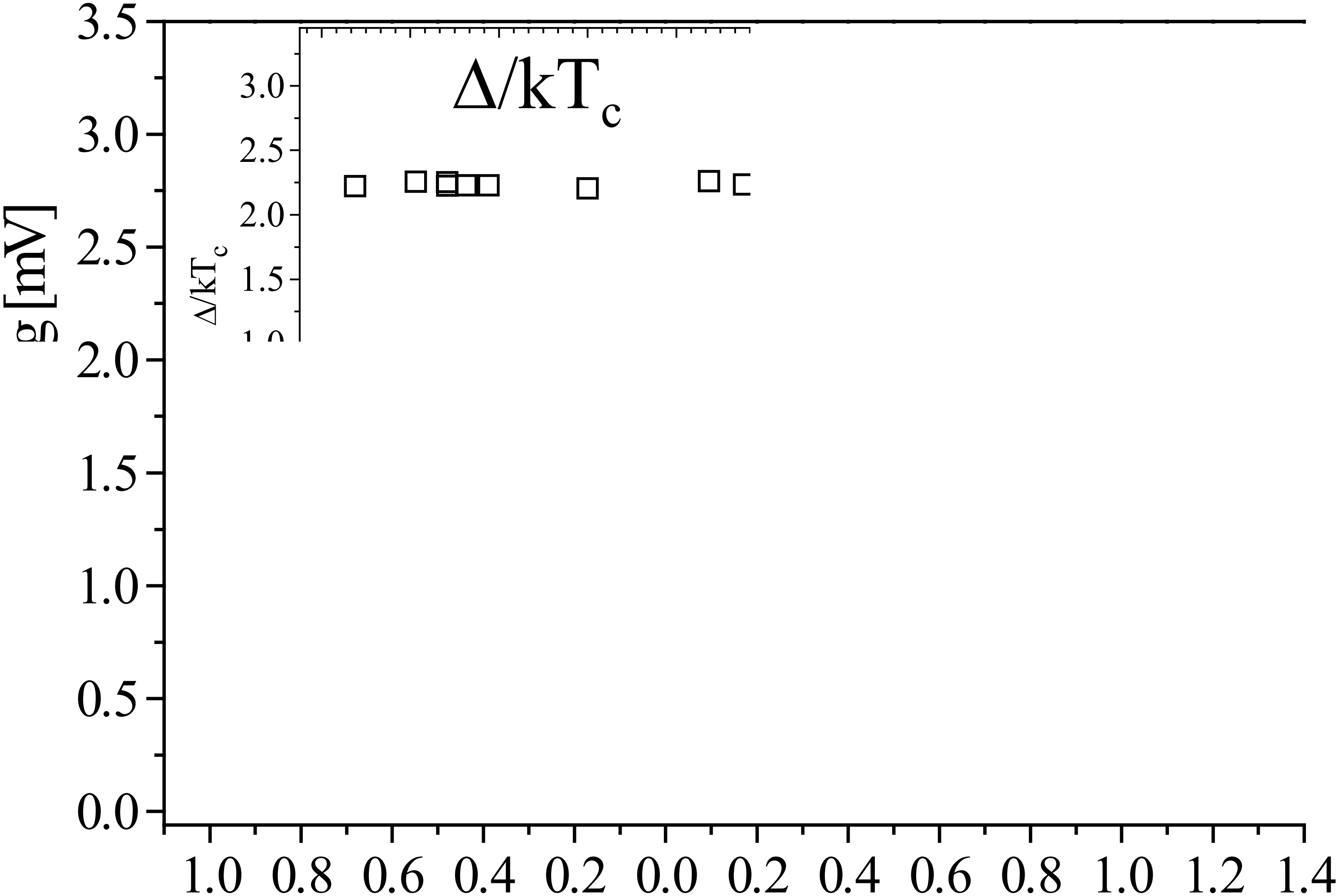
Magnetic field splitting





Covington, M. *et al.* Observation of Surface-Induced Broken Time-Reversal Symmetry in $YBa_2Cu_3O_{7-\delta}$ Tunnel Junctions, *Phys. Rev. Lett*. **79**, 277-281 (1997)

Zero Field splitting and χ^{-1} versus $[\Delta_{max}-\Delta]^{1/2}$ All YBCO samples



Conclusions: Phase transitions of BCS superconductors

Examined general theory of all possible candidates for zero momentum, spin-singlet order parameters which can induce a second-order quantum phase transitions in a *d*-wave superconductor

Only cases

(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ pairing and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$ pairing

have renormalization group fixed points with a non-zero interaction strength between the bosonic order parameter mode and the nodal fermions, and so are candidates for producing damping $\sim k_B T$ of nodal fermions.

Independent evidence for (B) from tunneling experiments.

M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev. Lett*. **85**, 4940 (2000).

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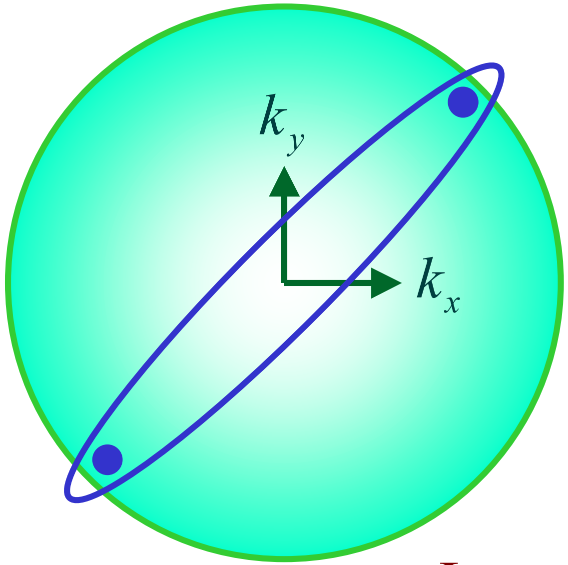
V. Conclusions

Competing order in the cuprate superconductors

Eugene Demler (Harvard)
Kwon Park
Anatoli Polkovnikov
Subir Sachdev
Matthias Vojta (Augsburg)
Ying Zhang

Lecture based on the article
cond-mat/0112343 and the reviews
cond-mat/0108238 and cond-mat/0203363

Talk online at
http://pantheon.yale.edu/~subir



Hole-doped cuprates are BCS superconductors with

$$\left\langle c^{\dagger}_{\boldsymbol{k}\uparrow} c^{\dagger}_{\boldsymbol{-k}\downarrow} \right\rangle \equiv \Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y \right)$$ d-wave pairing

$$\left\langle \vec{S} \right\rangle = 0$$ spin-singlet

Low energy excitations:

Superflow: $\Delta_0 \to \Delta_0 e^{i\theta}$

$S = 1/2$ fermionic quasiparticles: $E_{\boldsymbol{k}} = \sqrt{\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2}$

BCS theory also predicts that the Fermi surface, with gapless quasiparticles, will reveal itself when $\Delta_0 \to 0$, either locally or globally at low temperatures. Δ_0 can be suppressed by a strong magnetic field, and near vortices, impurities and interfaces.

Superconductivity in a doped Mott insulator

Hypothesis: cuprate superconductors have low energy excitations associated with additional order parameters

Theory and experiments indicate that the most likely candidates are spin density waves and associated "charge" order

Superconductivity can be suppressed globally by a strong magnetic field or large current flow.

Competing orders are also revealed when superconductivity is suppressed locally, near impurities or around vortices.

S. Sachdev, *Phys. Rev.* B **45**, 389 (1992);
N. Nagaosa and P.A. Lee, *Phys. Rev.* B **45**, 966 (1992);
D.P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang *Phys. Rev. Lett.* **79**, 2871 (1997);
K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).

Outline

I. **Experimental introduction**

II. Spin density waves (SDW) in LSCO
    Tuning order and transitions by a magnetic field.

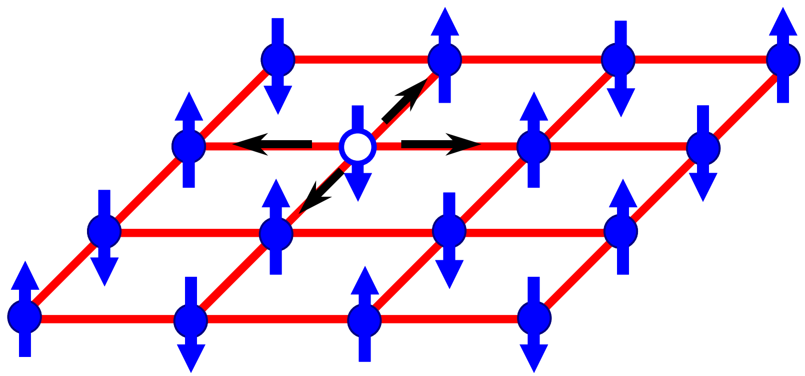
III. Connection with "charge" order – phenomenological theory
    STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

IV. Connection with "charge" order – microscopic theory
    Theories of magnetic transitions predict bond-centered modulation of exchange and pairing energies with even periods---a bond order wave

V. Conclusions

I. Experimental introduction

The doped cuprates



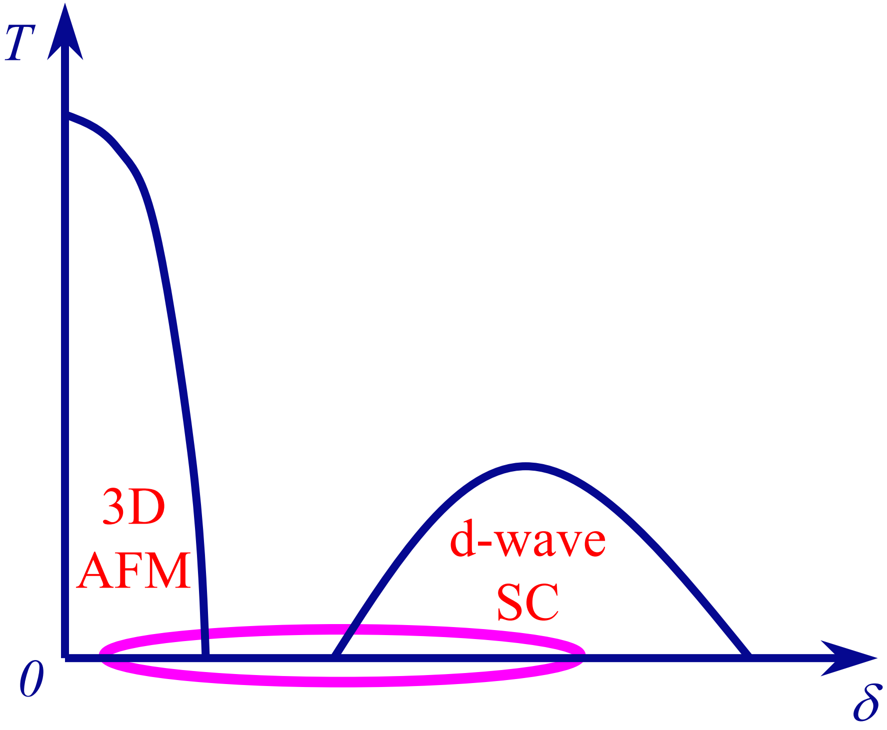
2-D CuO_2 plane

2-D CuO_2 plane

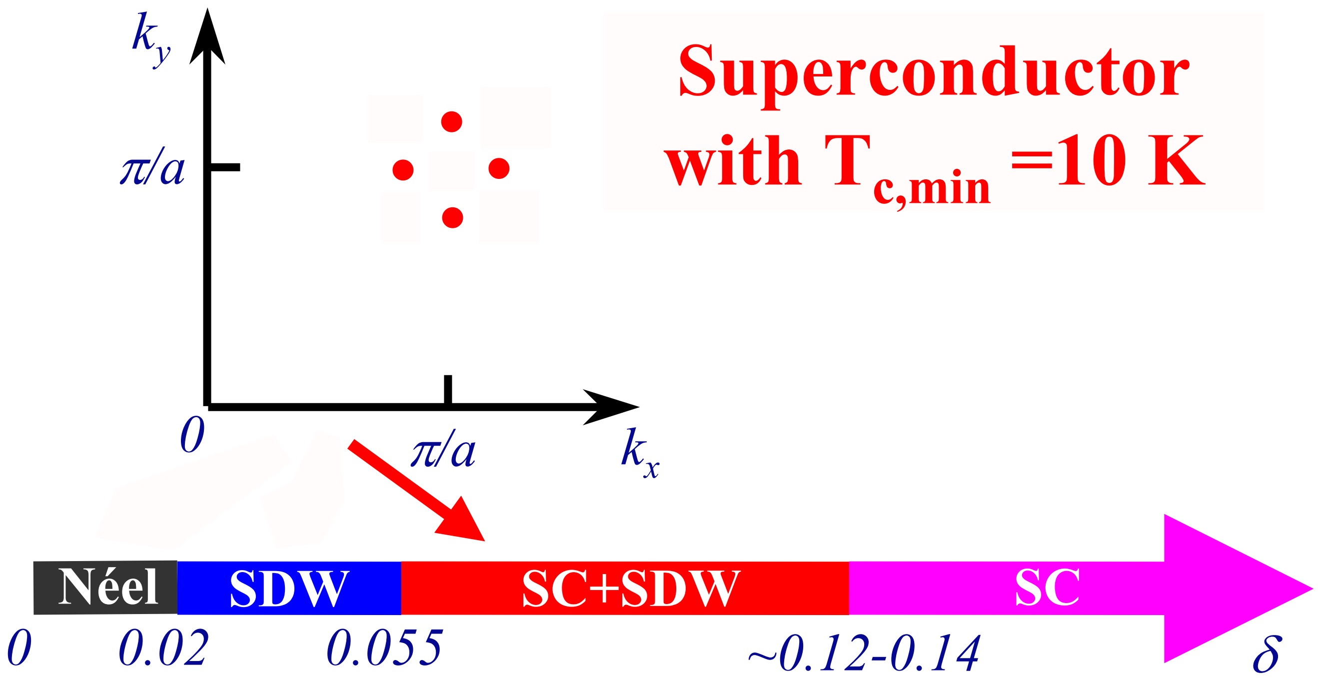
with finite hole doping

Néel ordered ground state at zero doping

Phase diagram of the doped cuprates



$T = 0$ phases of LSCO



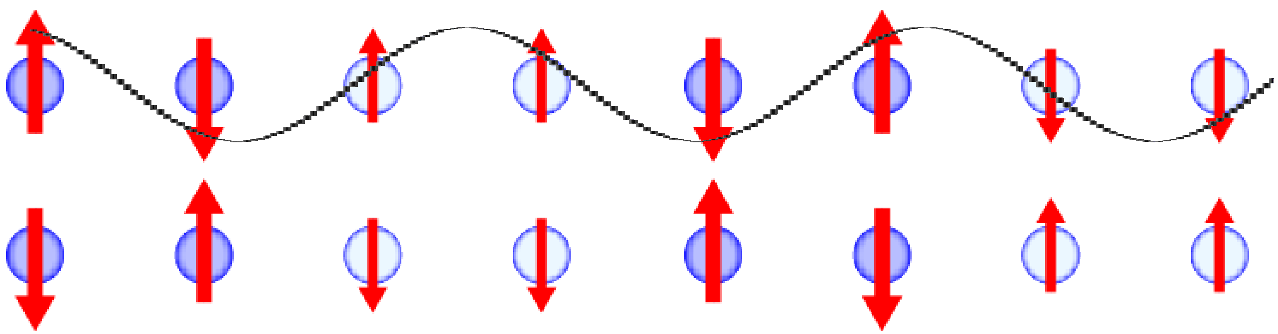
J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996)
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999).

SDW order parameter for general ordering wavevector

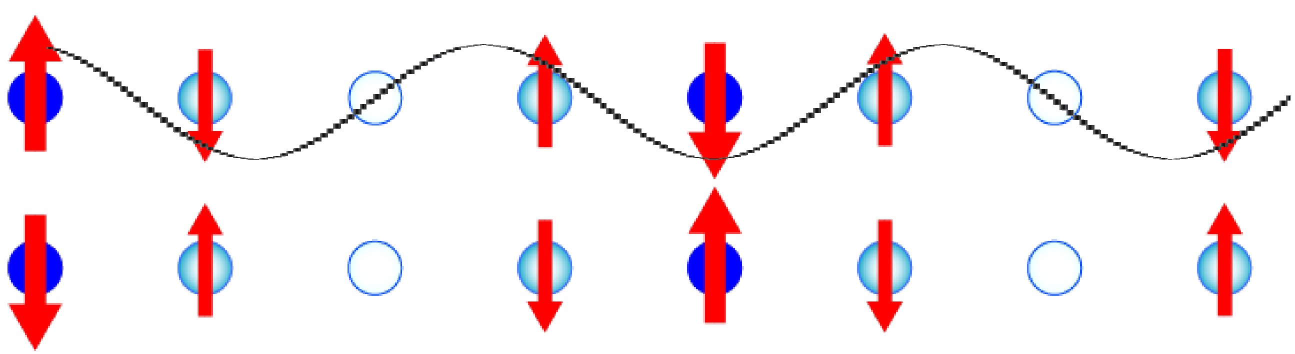
$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K}=(3\pi/4,\pi)$

Spin density wave is ***longitudinal*** (and not spiral): $\Phi_\alpha = e^{i\theta} n_\alpha$



Bond-centered



Site-centered

Outline

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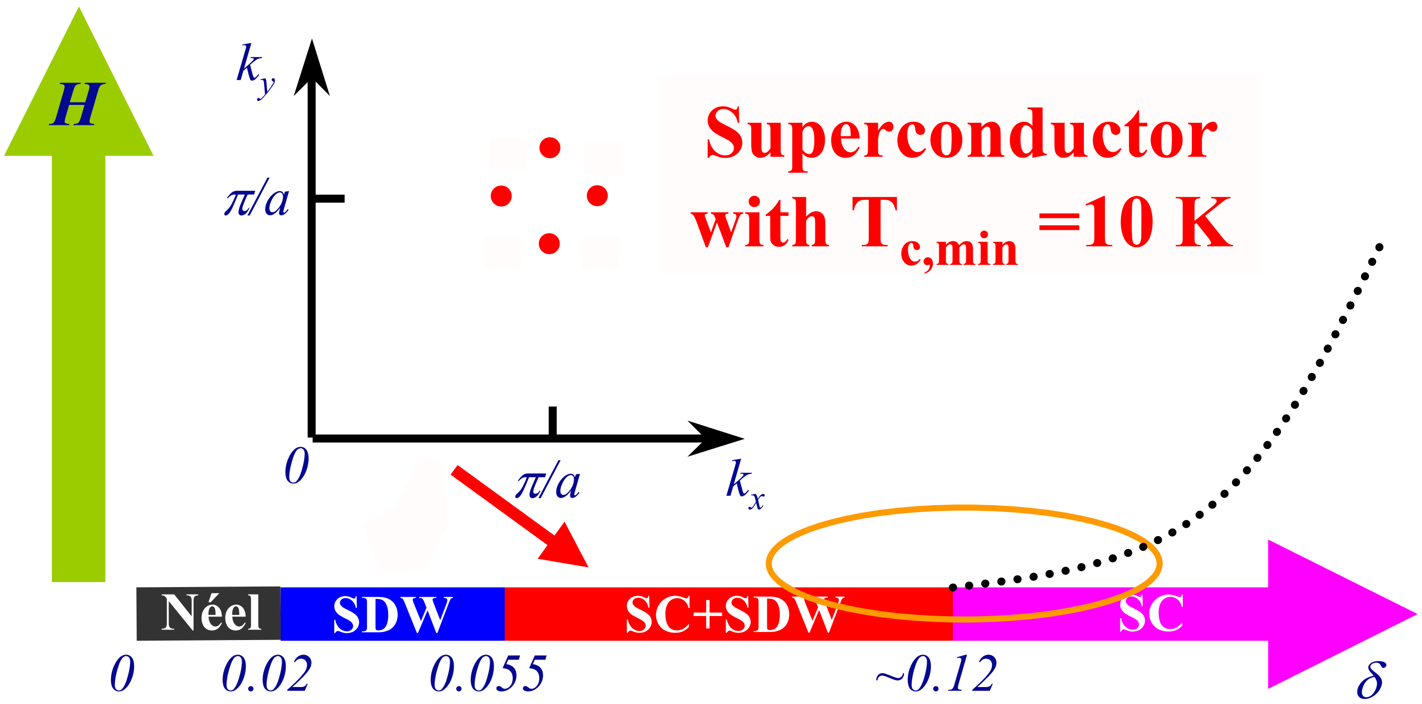
IV. Connection with "charge" order – microscopic theory

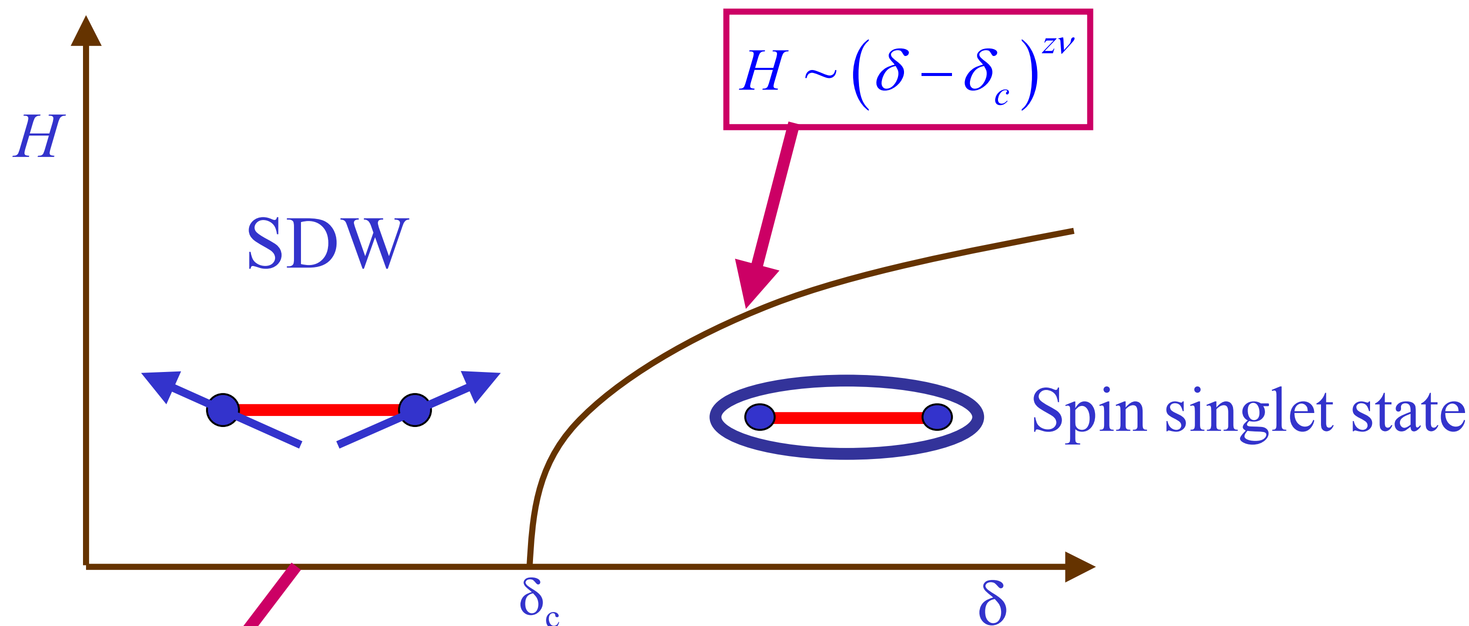
Theories of magnetic transitions predict bond-centered modulation of exchange and pairing energies with even periods---a bond order wave

V. Conclusions

II. Effect of a magnetic field on SDW order with co-existing superconductivity



Effect of the Zeeman term: precession of SDW order about the magnetic field



Elastic scattering intensity

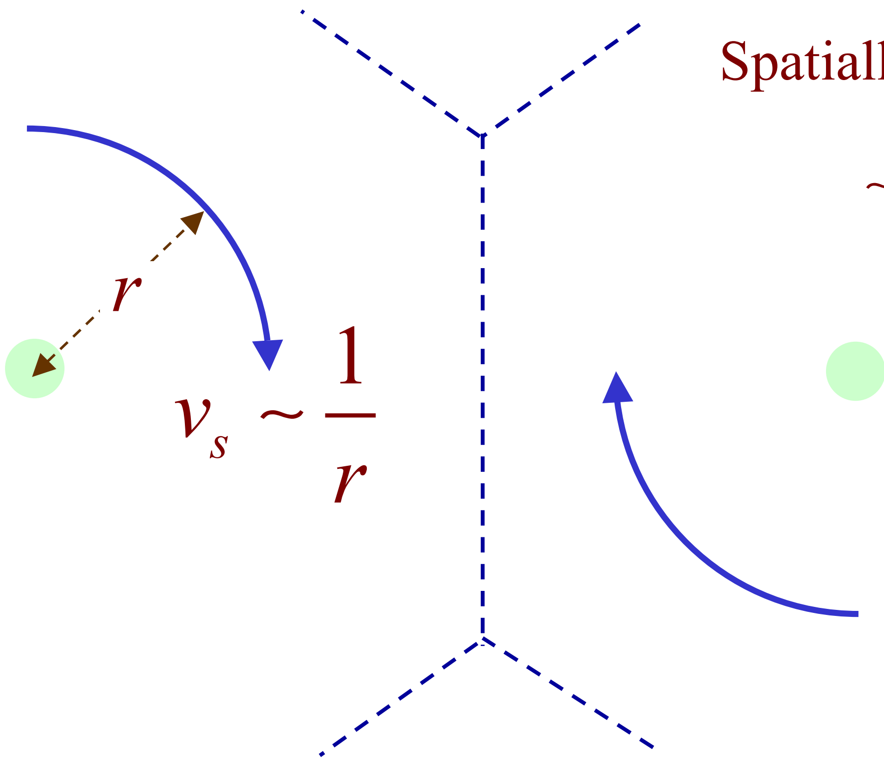
$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Effect is negligible over experimental field scales

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

The presence of the field replaces δ by

$$\delta_{eff}\left(H\right) = \delta - C\frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

Infinite diamagnetic susceptibility of *non-critical* superconductivity leads to a strong effect.

- Theory should account for *dynamic* quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a *static* Ginzburg-Landau theory

$$\mathcal{S}_b = \int d^2 r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]$$

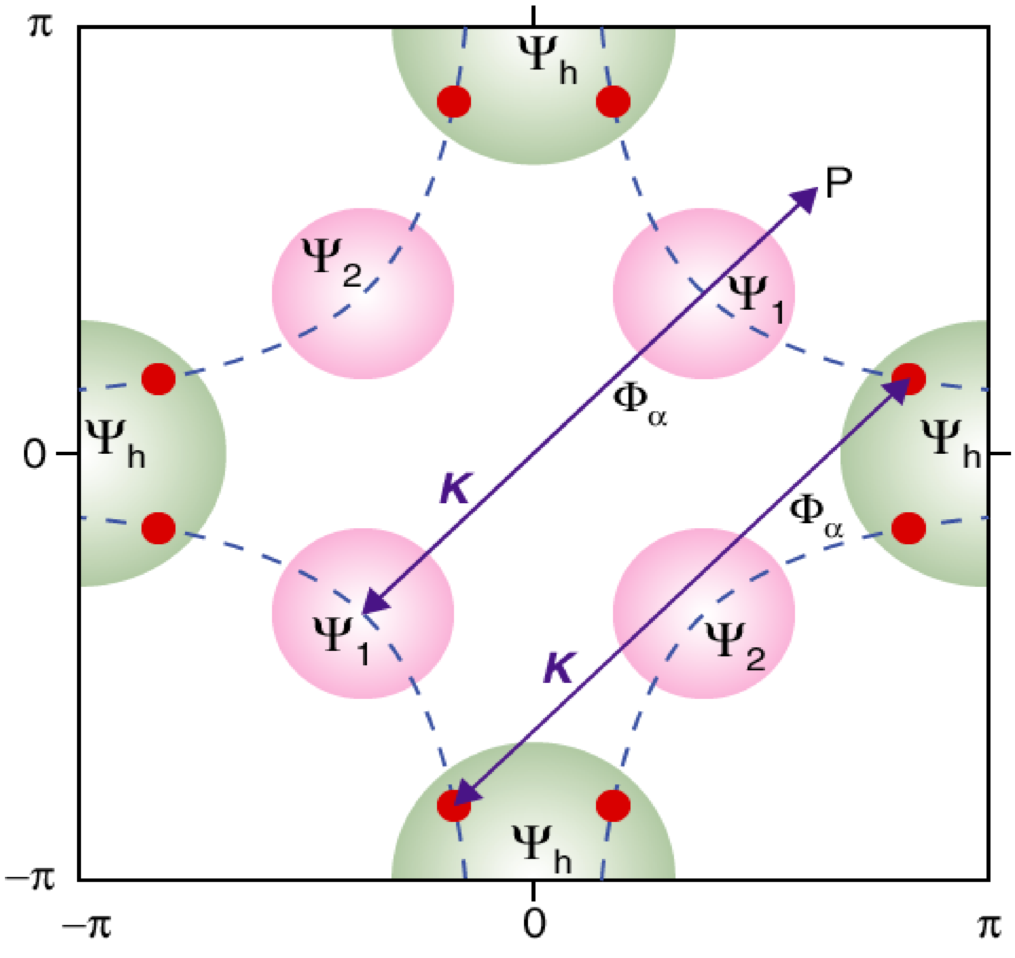
$$\mathcal{S}_c = \int d^2 r d\tau \left[\frac{v}{2} \left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]$$

$$Z\left[\psi(r)\right] = \int D\Phi(r,\tau)\, e^{-F_{GL} - \mathcal{S}_b - \mathcal{S}_c}$$

$$\frac{\delta \ln Z\left[\psi(r)\right]}{\delta \psi(r)} = 0$$

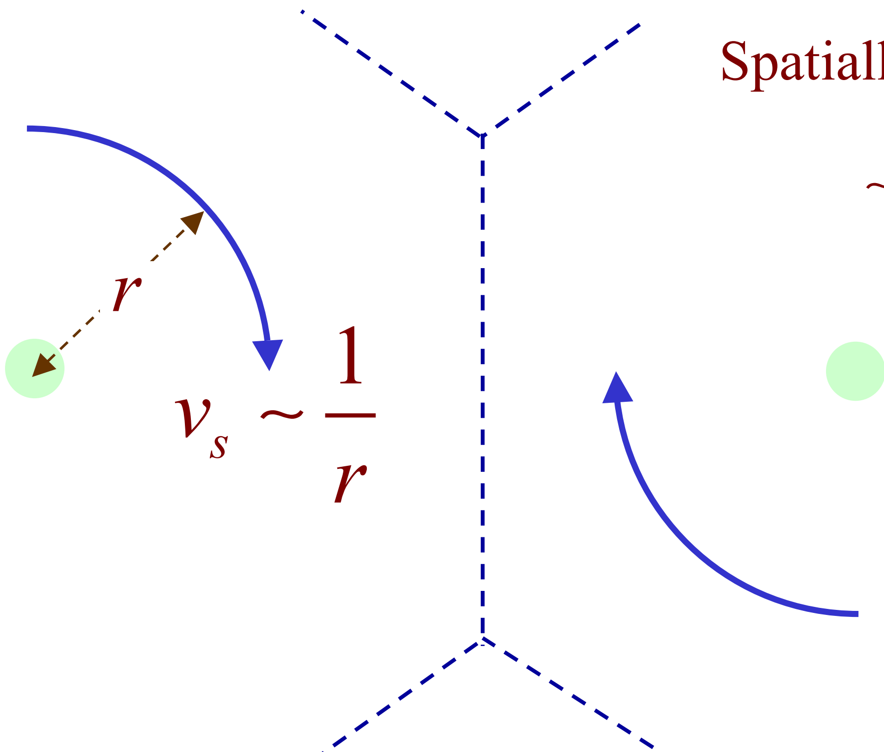
$$F_{GL} = \int d^2 r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]$$

Coupling between S=1/2 fermionic quasiparticles and collective mode of spin density wave order



Strong constraints on the mixing of quasiparticles and the Φ_α collective mode by momentum and energy conservation: nodal quasiparticles can be essentially decoupled from nonzero ***K*** collective modes

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

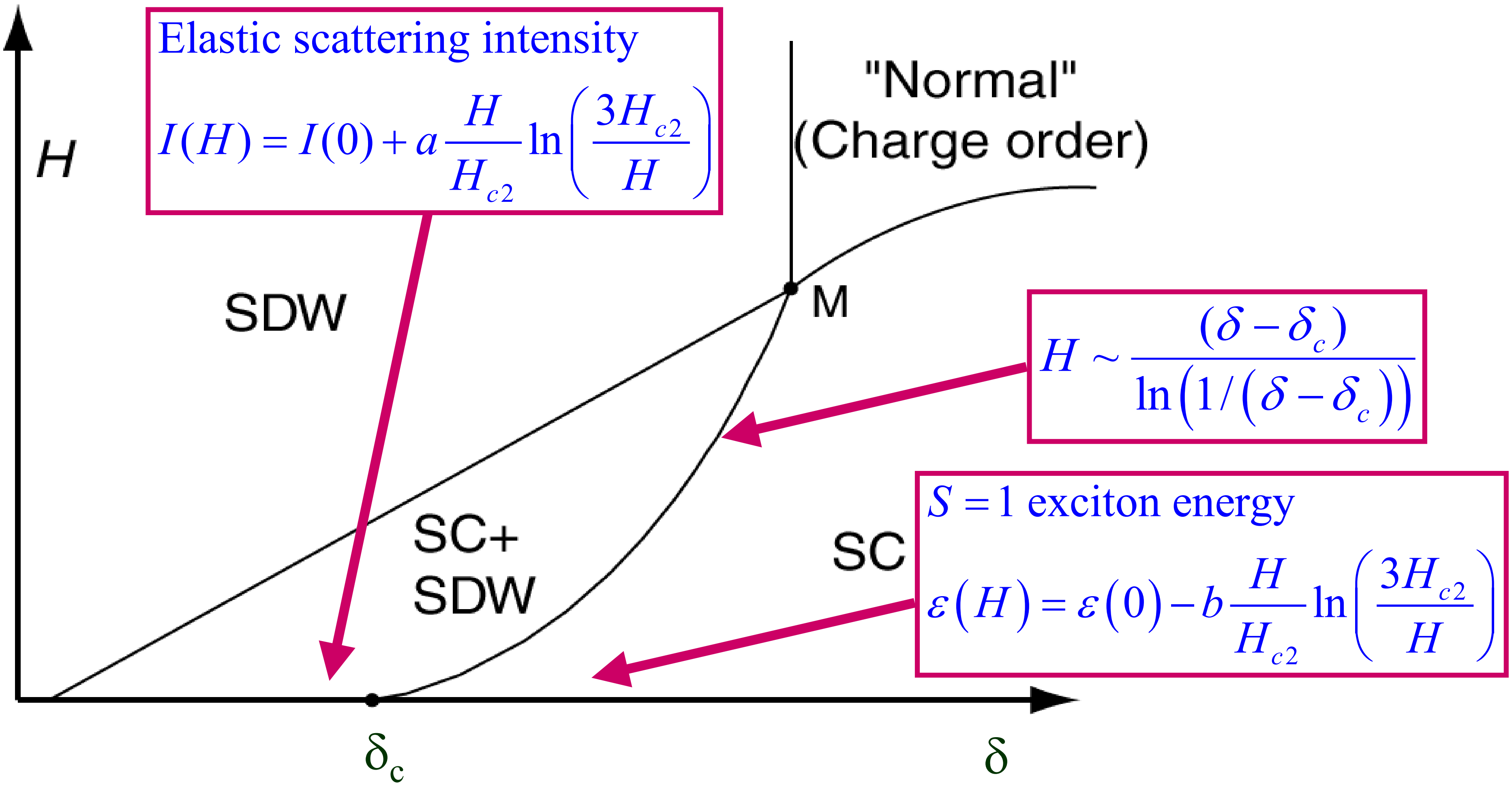
The presence of the field replaces δ by

$$\delta_{eff}\left(H\right) = \delta - C\frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

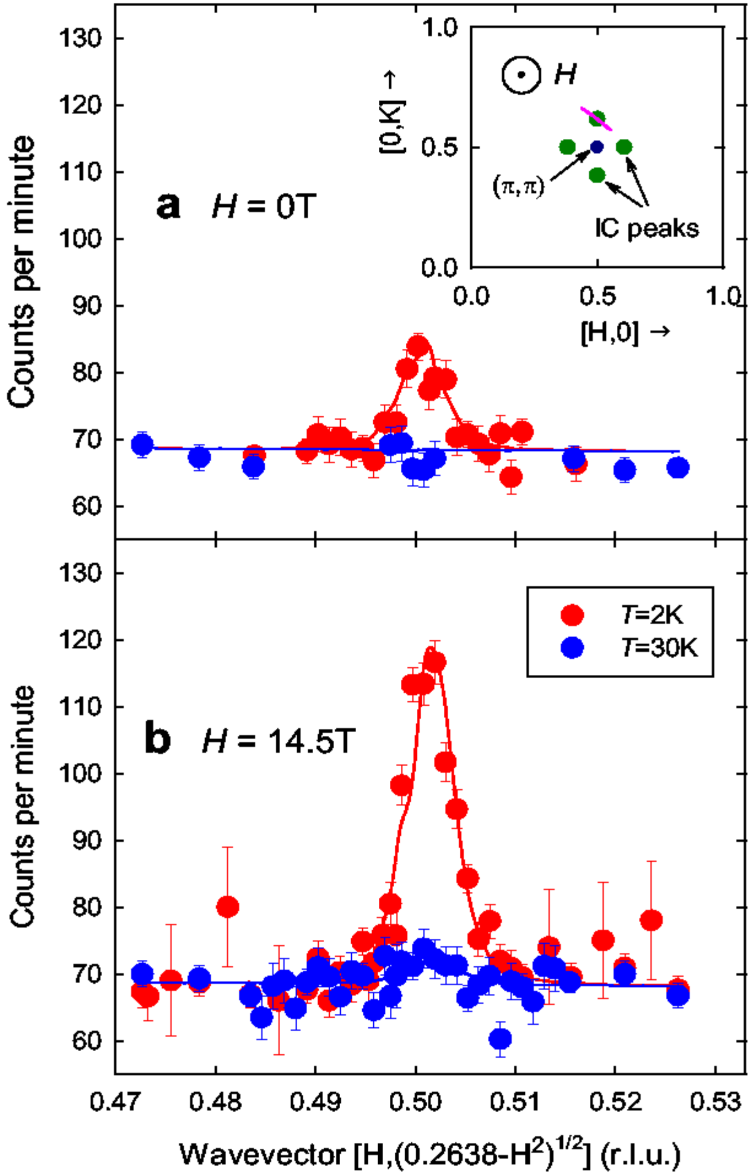
Main results

T=0

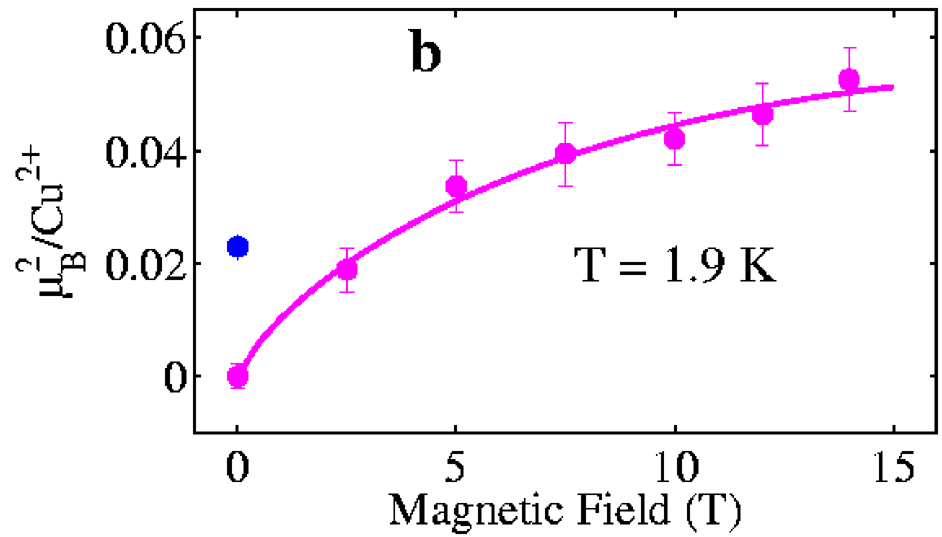


E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).

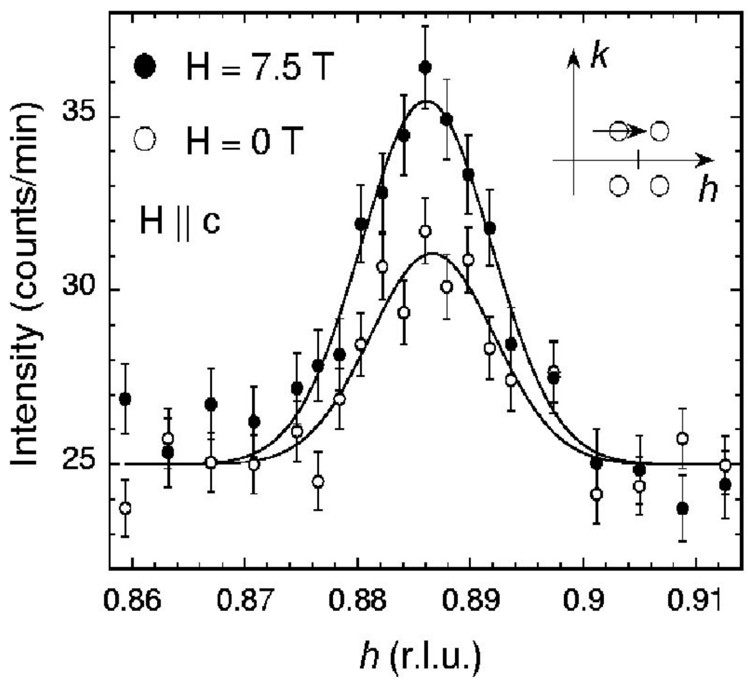


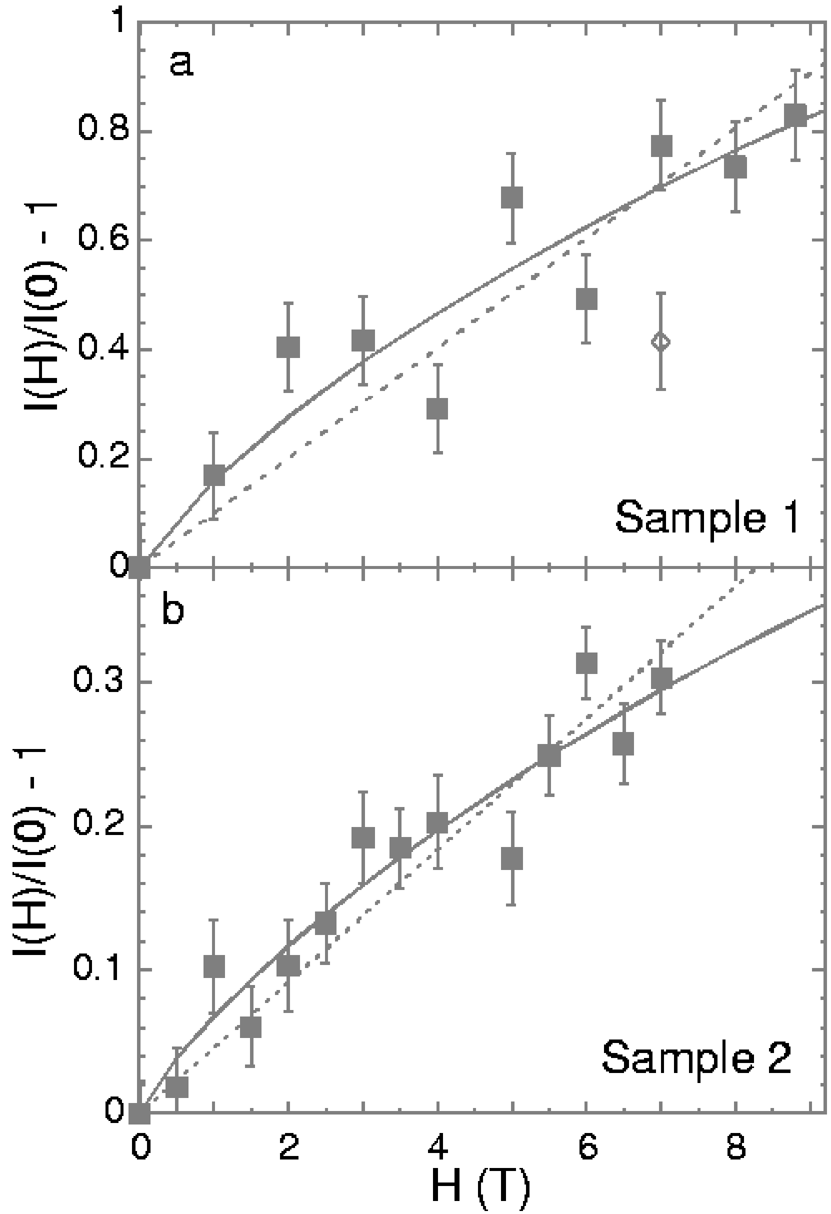
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, cond-mat/0112505.





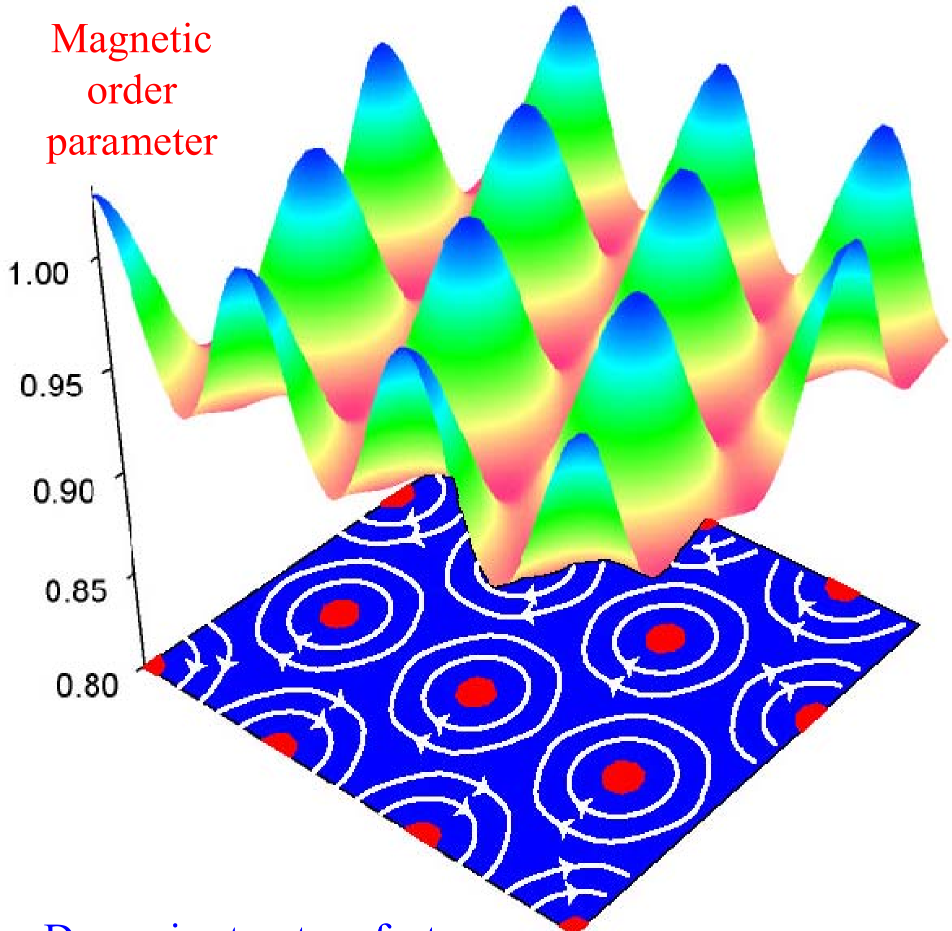
Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

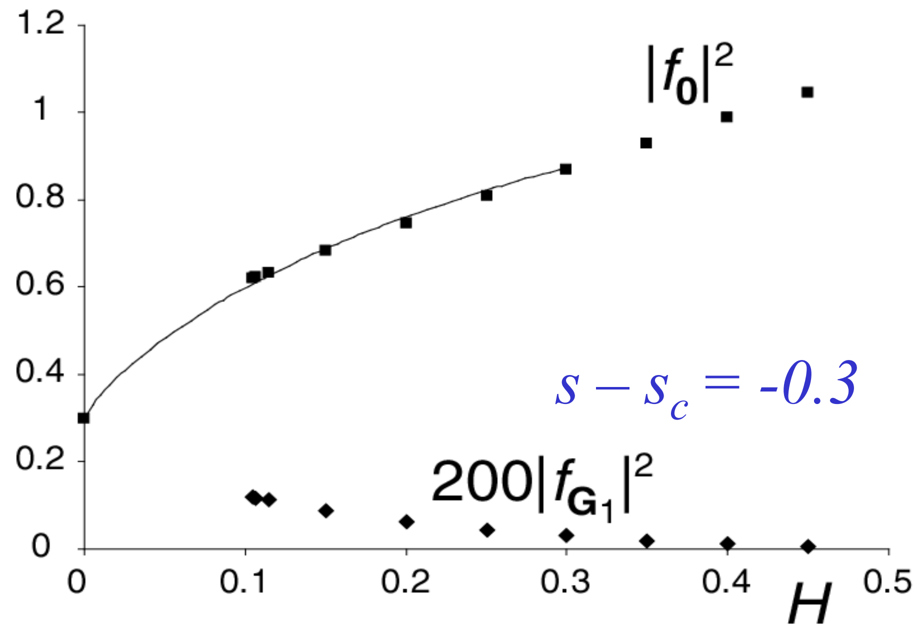
Best fit value - $a = 2.4$ with $H_{c2} = 60$ T

Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



$$\delta \left| f_0 \right|^2 \propto H \ln(1/H)$$



Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\boldsymbol{G}} \left| f_{\boldsymbol{G}} \right|^2 \delta(\boldsymbol{k}-\boldsymbol{G}) + \cdots$$

$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) discussed static magnetism within the vortex cores in the SC phase.Their model implies a ~*H* dependence of the intensity

Outline

I. Experimental introduction

II. Spin density waves (SDW) in LSCO
    Tuning order and transitions by a magnetic field.

III. Connection with "charge" order – phenomenological theory
    STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

IV. Connection with "charge" order – microscopic theory
    Theories of magnetic transitions predict bond-centered modulation of exchange and pairing energies with even periods---a bond order wave

V. Conclusions

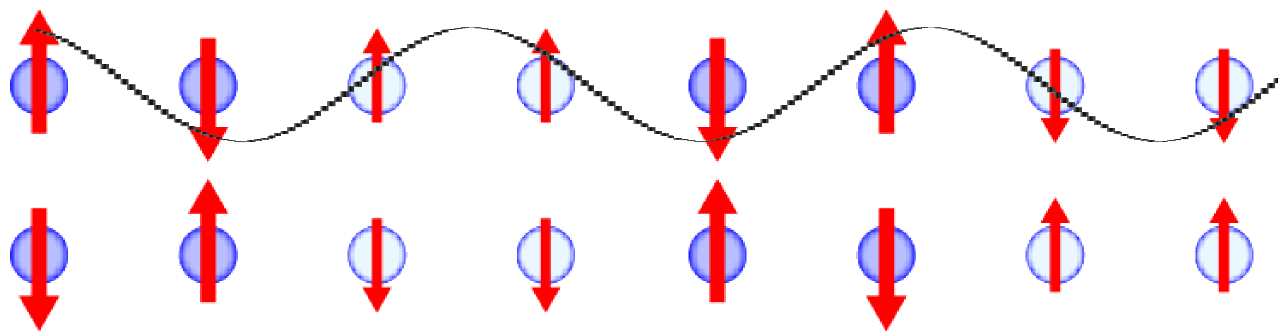
III. Connections with "charge" order – phenomenological theory

Spin density wave order parameter for general ordering wavevector

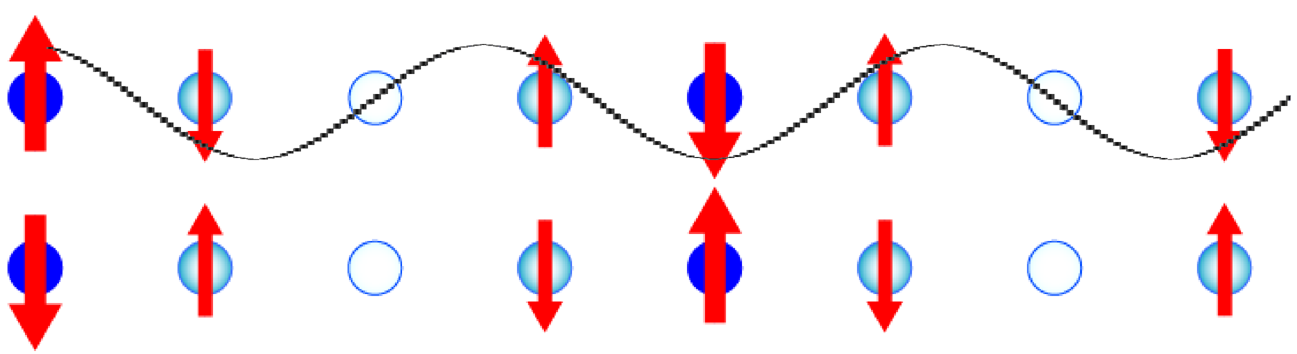
$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K}=(3\pi/4,\pi)$

Spin density wave is ***longitudinal*** (and not spiral): $\Phi_\alpha = e^{i\theta} n_\alpha$



Bond-centered



Site-centered

A longitudinal spin density wave necessarily has an accompanying modulation in the site charge densities, exchange and pairing energy per link etc. at half the wavelength of the SDW

"Charge" order: periodic modulation in local observables invariant under spin rotations and time-reversal.

$$\text{Order parmeter} \sim \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r})$$

$$\delta\rho(\boldsymbol{r}) \propto S_{\alpha}^{2}(\boldsymbol{r}) = \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
K. Machida, Physica **158C**, 192 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

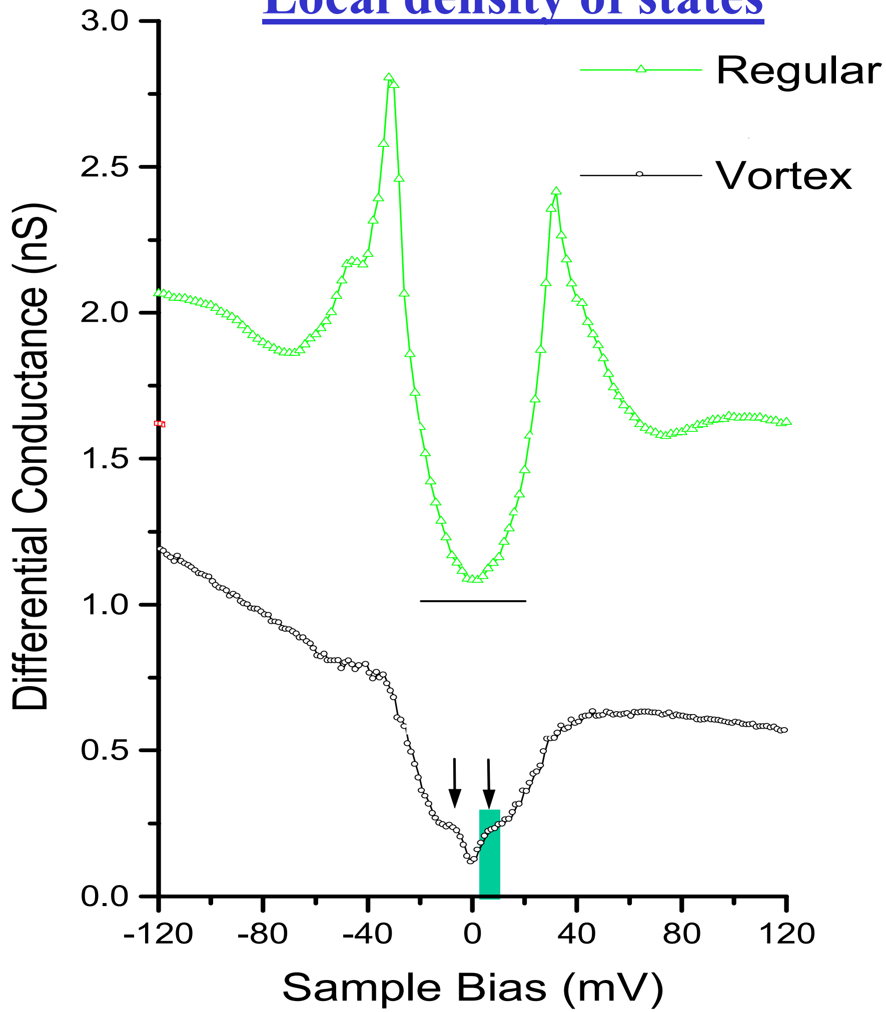
Prediction: Charge order should be pinned in halo around vortex core

K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

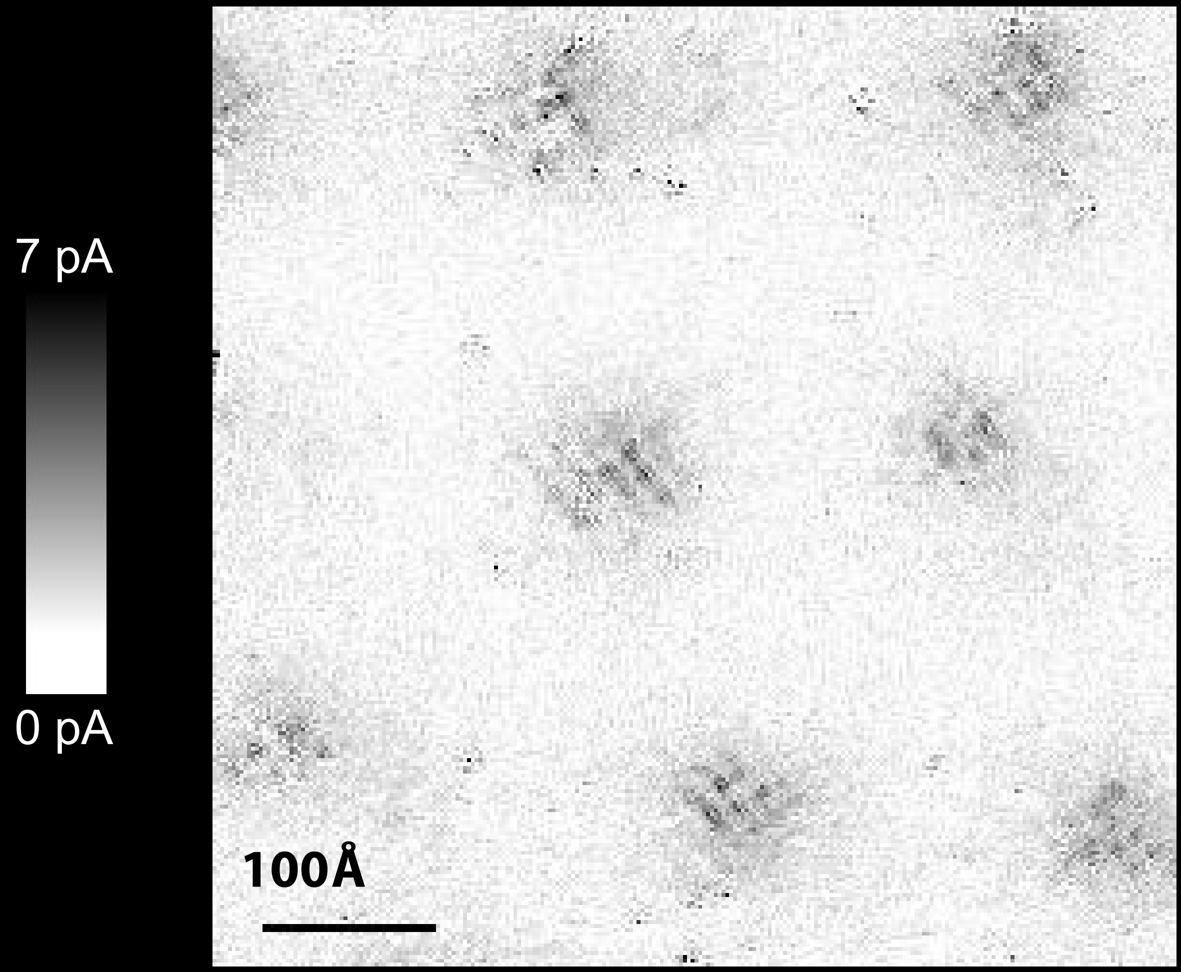


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

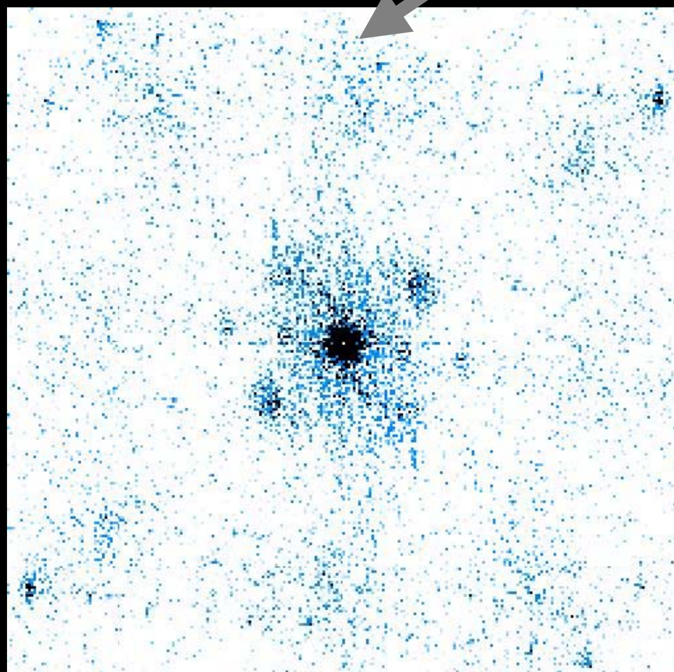
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

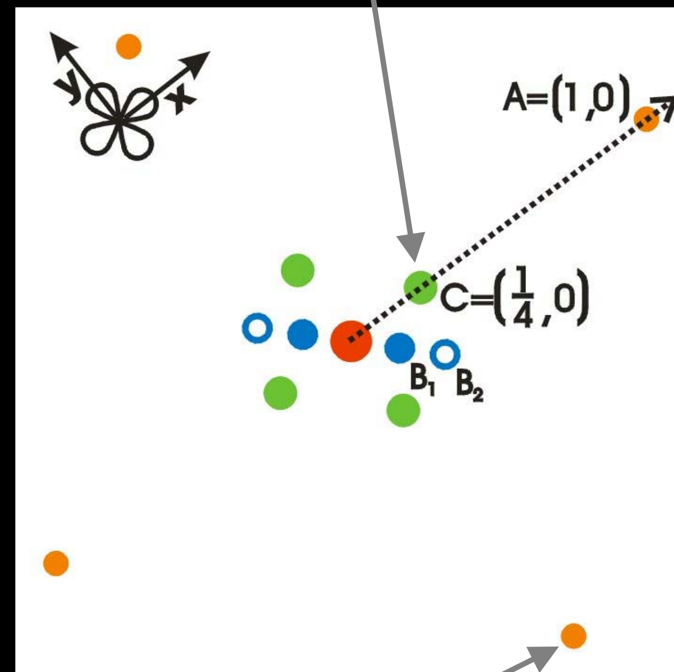


J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Fourier Transform of Vortex-Induced LDOS map

K-space locations of vortex induced LDOS





K-space locations of Bi and Cu atoms

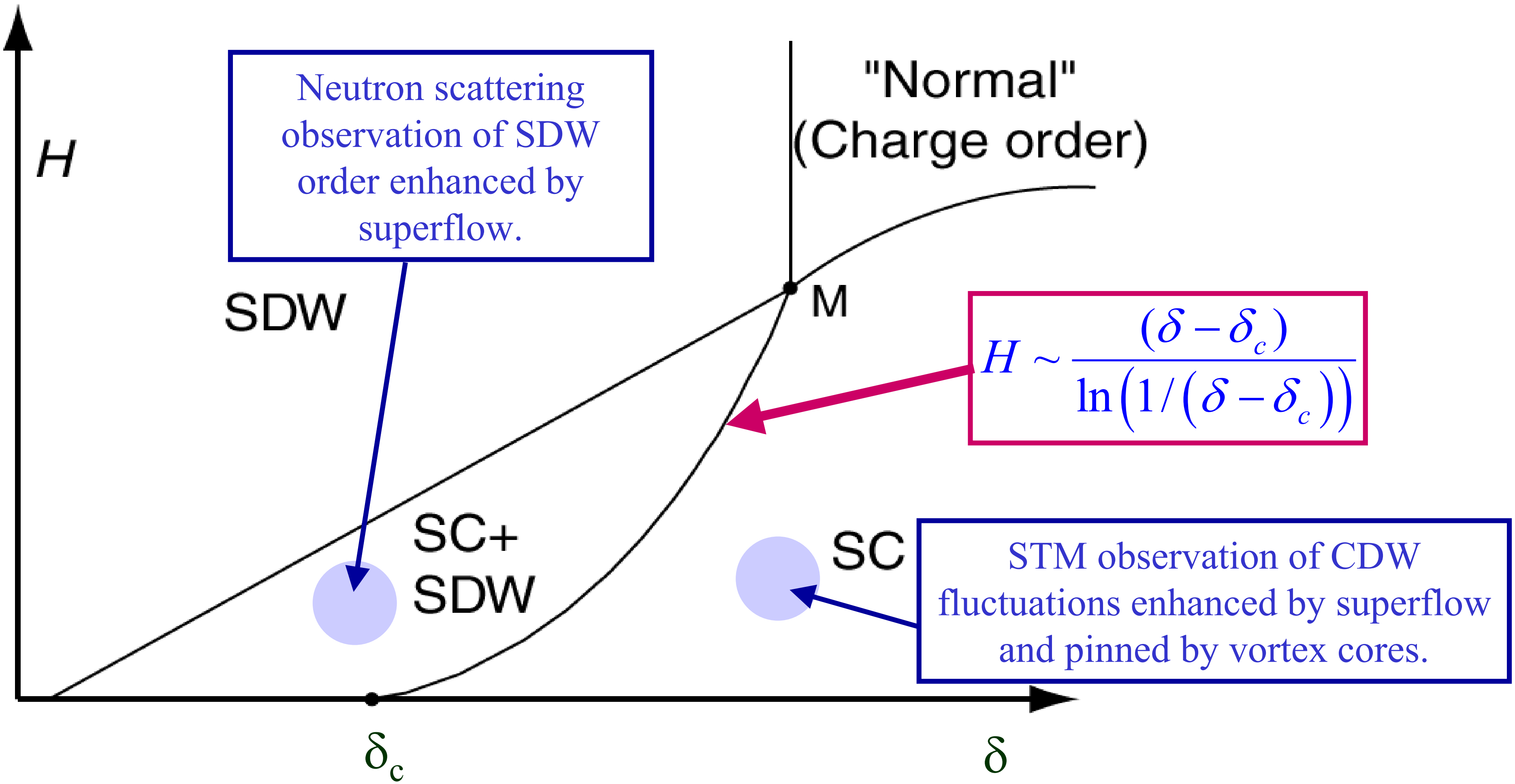
Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

J. Hoffman *et al. Science*, **295**, 466 (2002).

Summary of theory and experiments

(extreme Type II superconductivity)

T=0



E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

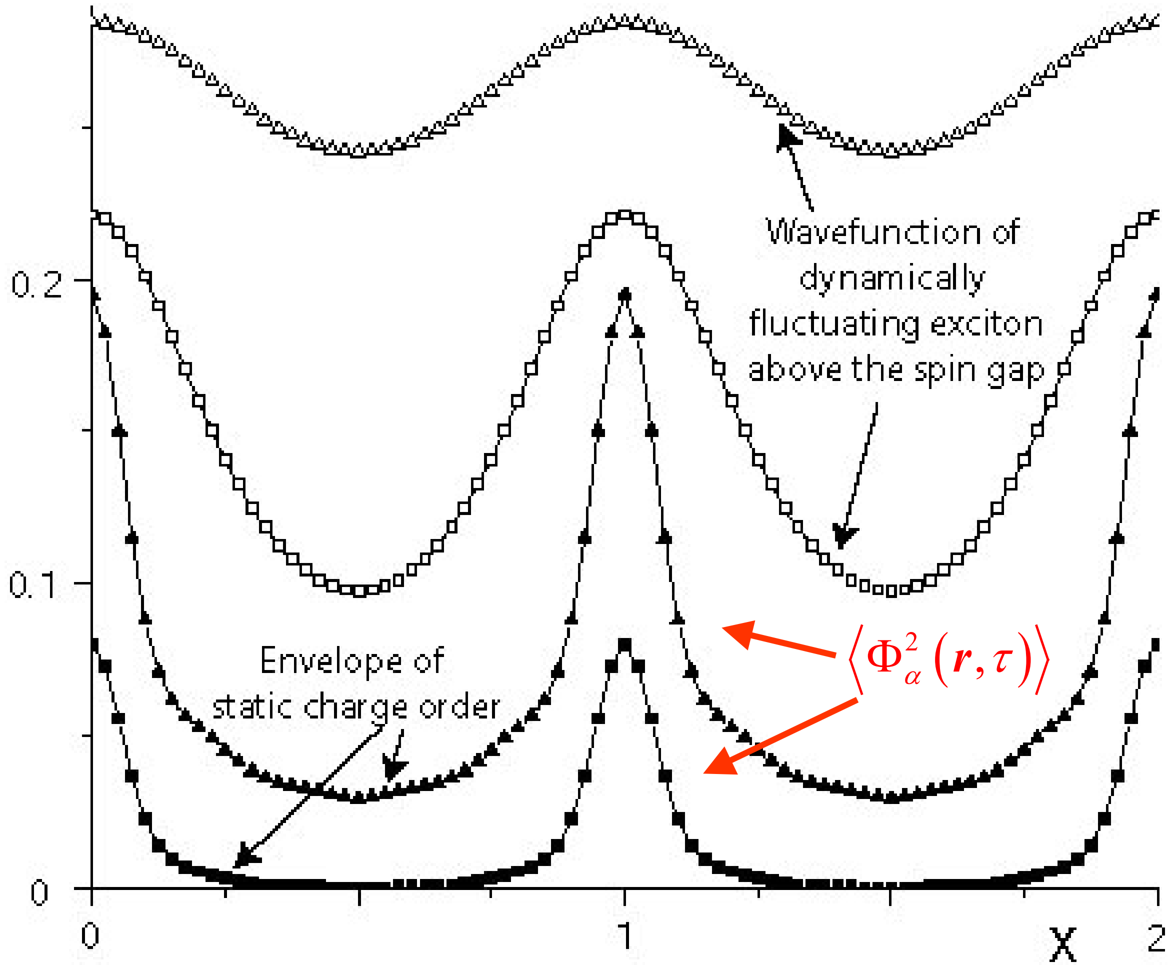
Quantitative connection between the two experiments ?

Pinning of CDW order by vortex cores in SC phase

$$\mathcal{S}_{\text{pin}} = \zeta \sum_{\text{All } \boldsymbol{r}_v \text{ where } \psi(\boldsymbol{r}_v)=0} \int_0^{1/T} d\tau \left[\Phi_\alpha^2(\boldsymbol{r}_v) e^{i\theta} + \text{c.c.} \right]$$

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

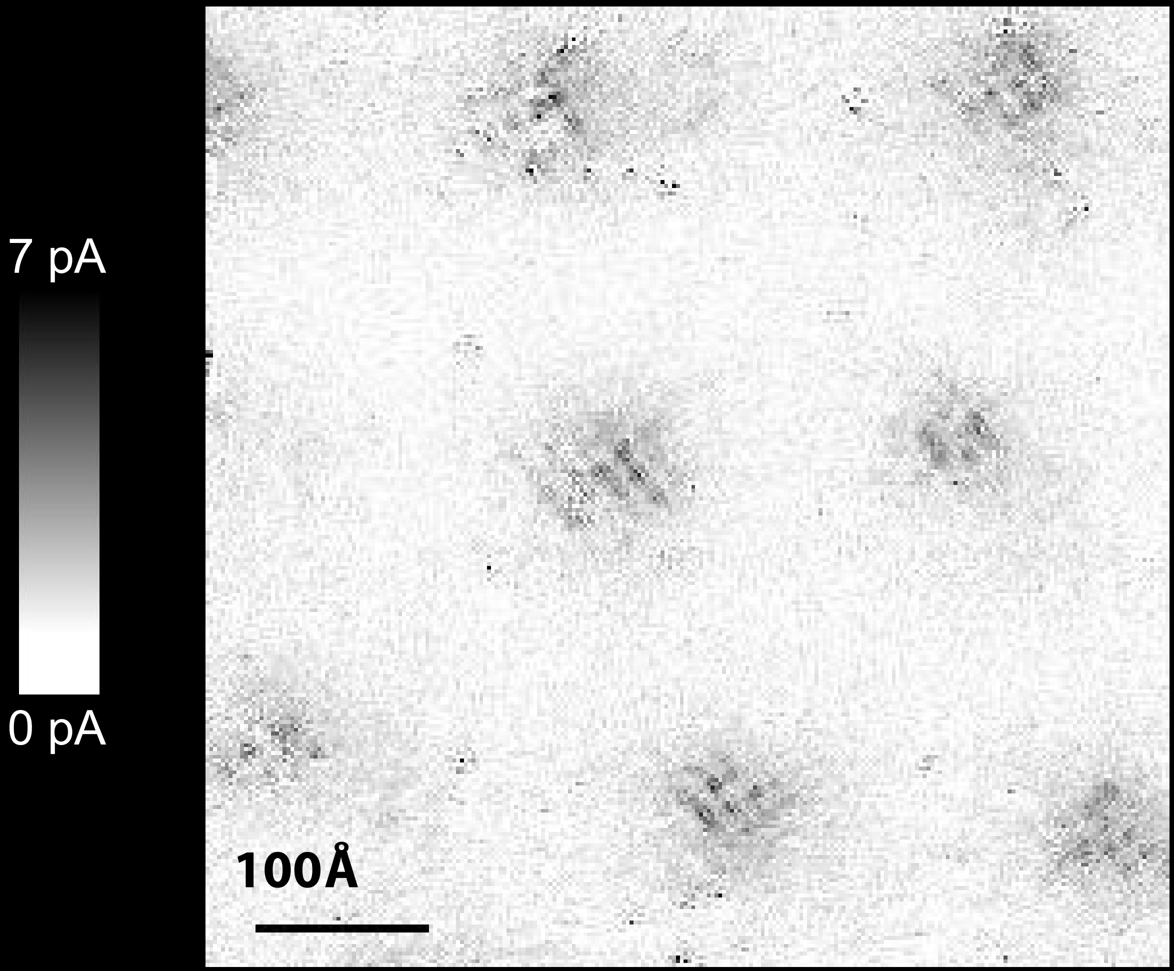
$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau) \right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau) \Phi_\alpha^*(\boldsymbol{r}_v,\tau_1) \right\rangle^2$$



□→ low magnetic field

△→ high magnetic field near the boundary to the SC+SDW phase

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

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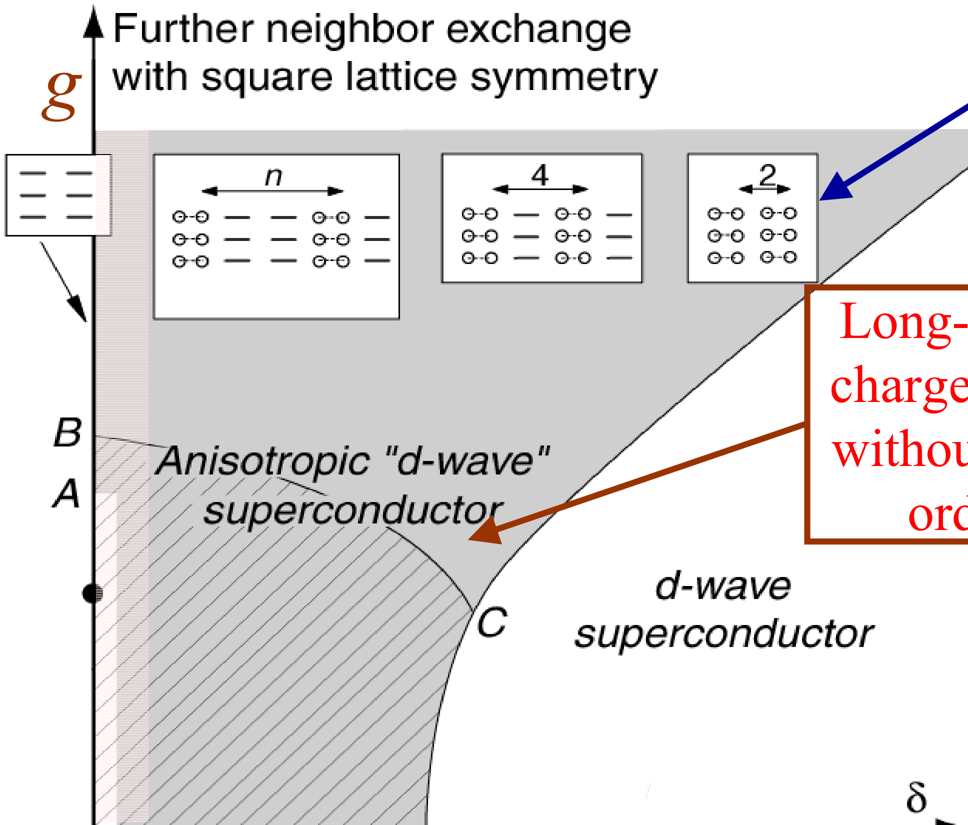
Theories of magnetic transitions predict bond-centered modulation of exchange and pairing energies with even periods---a bond order wave

V. Conclusions

IV. Microscopic theory of the charge order: Mott insulators and superconductors

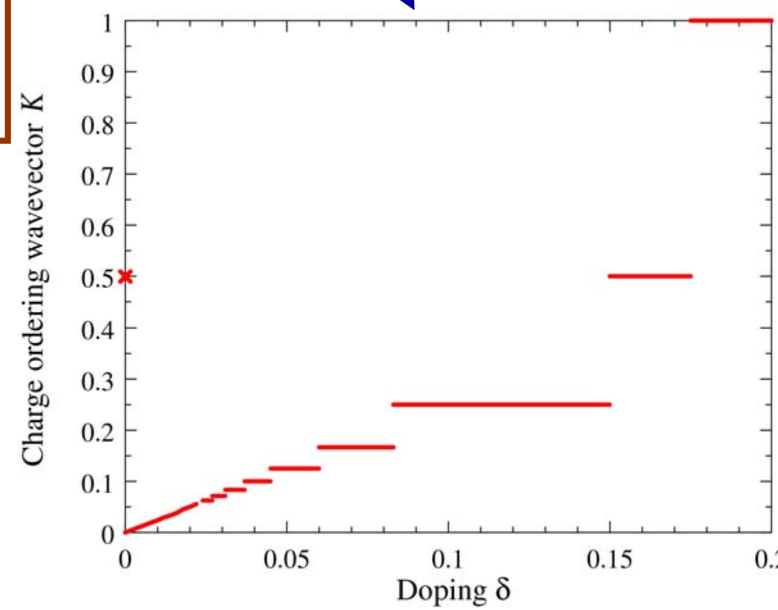


Hatched region --- spin order
Shaded region ---- charge order

Long-range charge order without spin order

"Large *N*" theory in region with preserved spin rotation symmetry

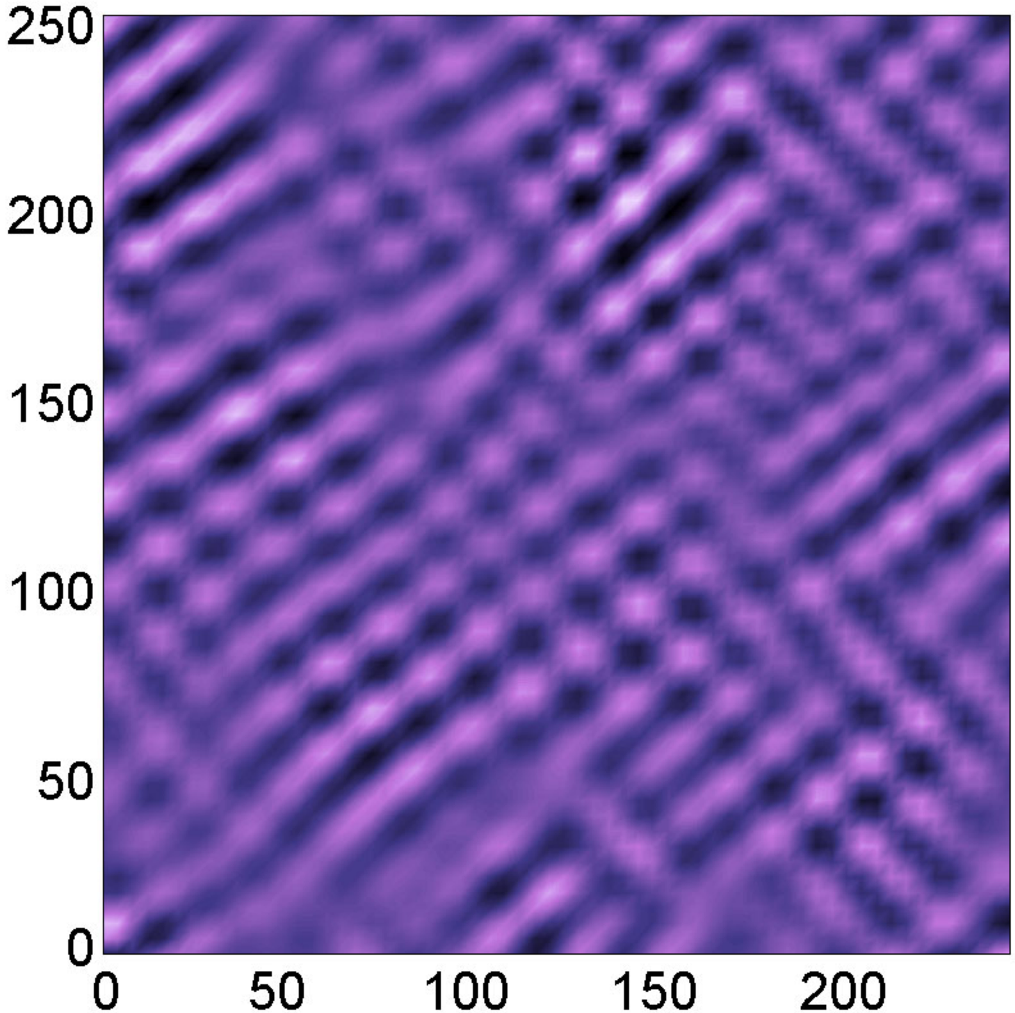
S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).

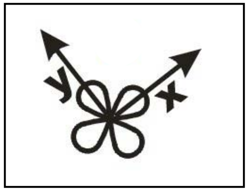


Charge order is bond-centered and has an even period.

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

IV. STM image of pinned charge order in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

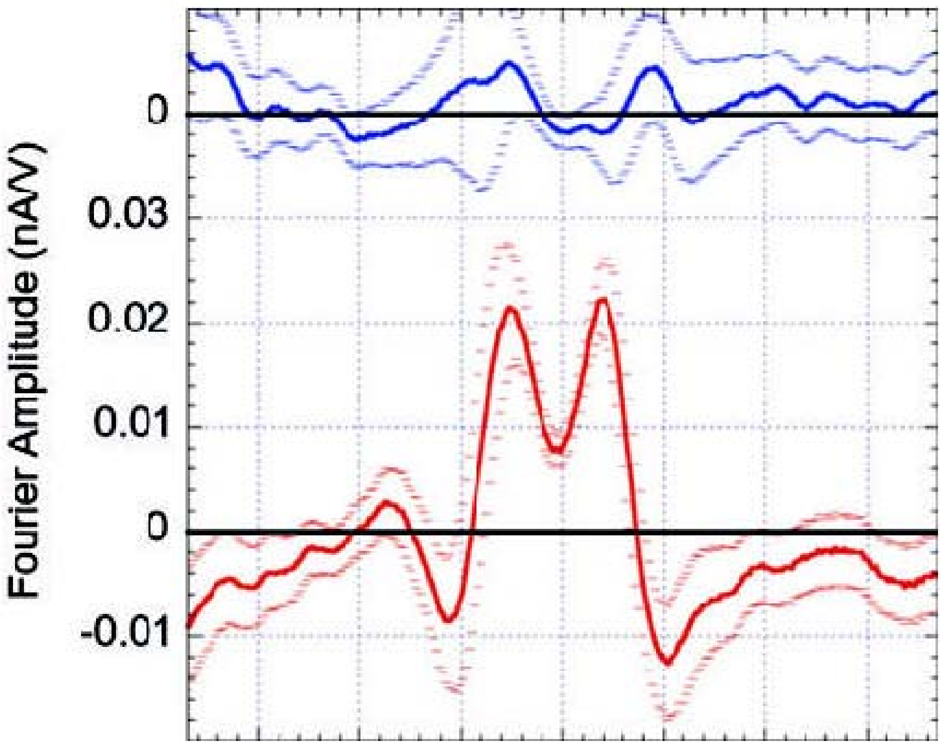




Charge order period = 4 lattice spacings

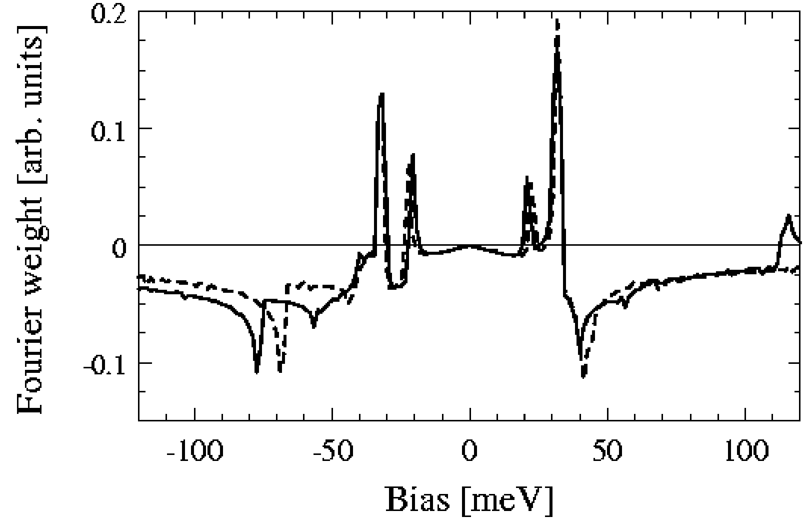
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, cond-mat/0204284.
D. Podolsky, E. Demler,
K. Damle, and B.I. Halperin,
cond-mat/0204011

IV. Neutron scattering observation of static charge order in $YBa_2Cu_3O_{6.35}$

(spin correlations are dynamic)

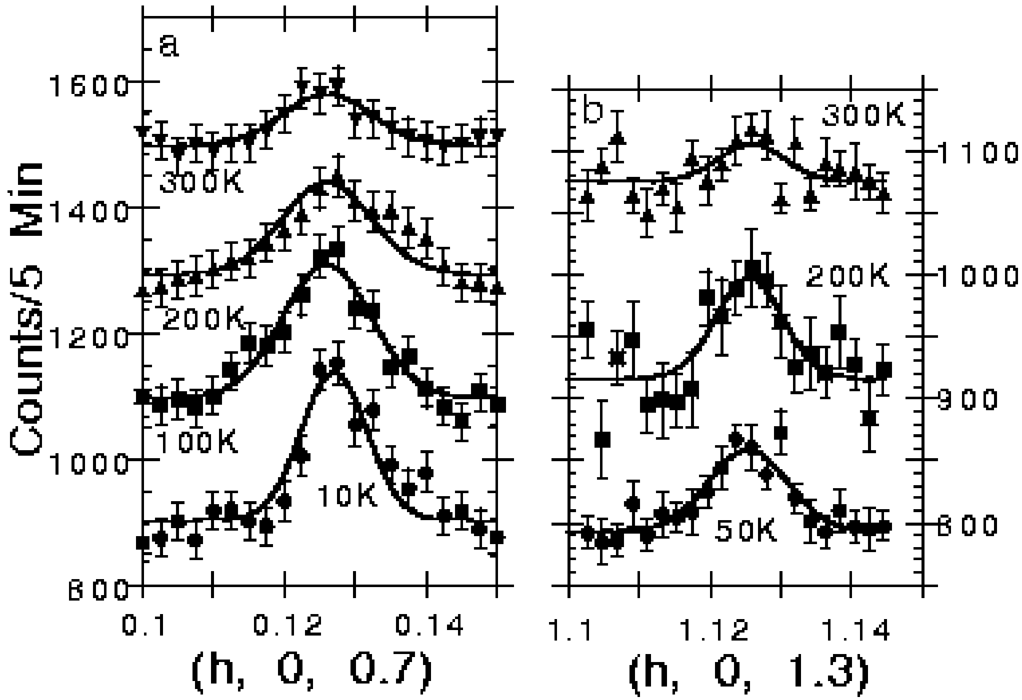


FIG. 1. Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) r.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

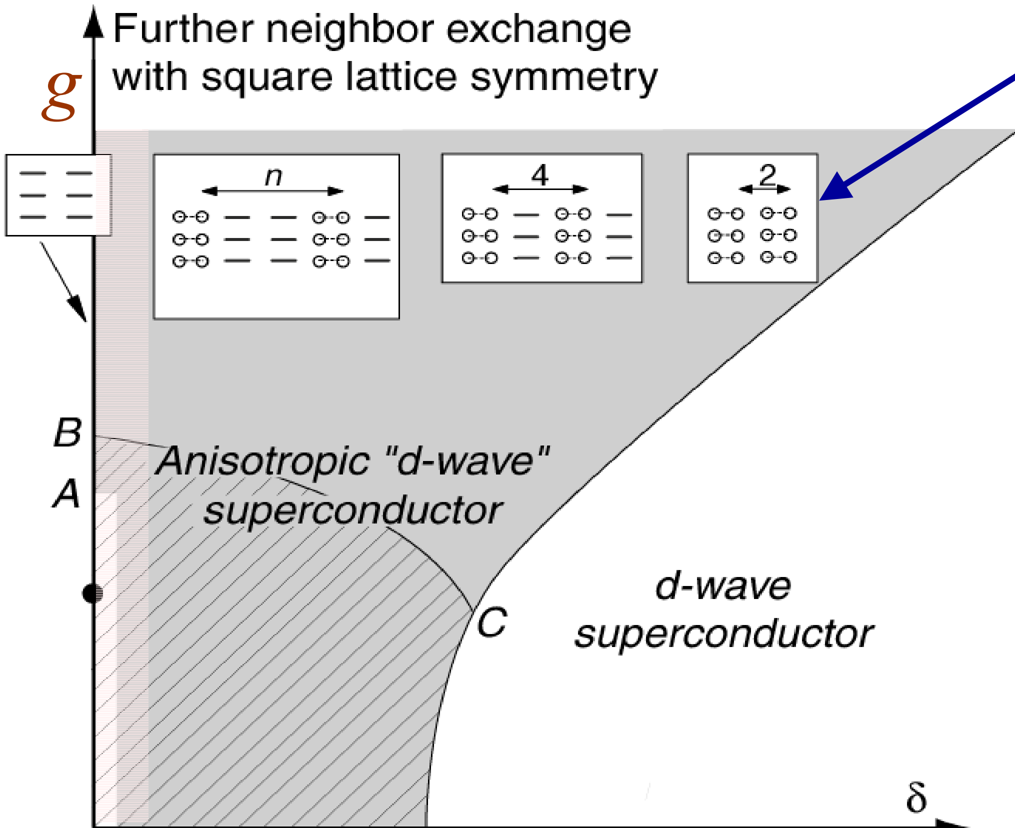
Charge order period = 8 lattice spacings

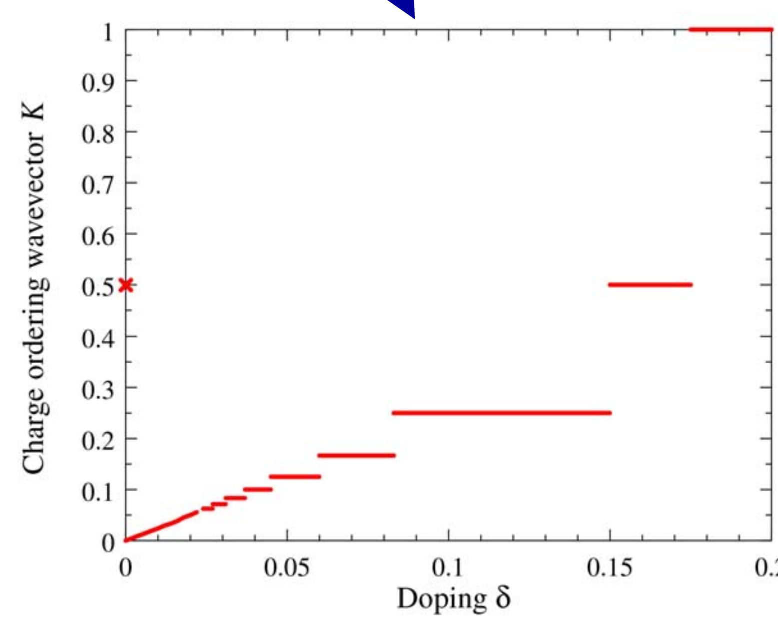
H. A. Mook, Pengcheng Dai, and F. Dogan
Phys. Rev. Lett. **88**, 097004 (2002).

IV. Bond order waves in the superconductor.

“Large *N*” theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).





Hatched region --- spin order
Shaded region ---- charge order

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has bond-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?