Quantum matter without quasiparticles

50 years of science for the future ICTP 50th Anniversary, Trieste, Italy, October 9, 2014

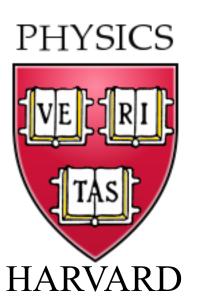
Subir Sachdev

Talk online: sachdev.physics.harvard.edu





JOHN TEMPLETON FOUNDATION



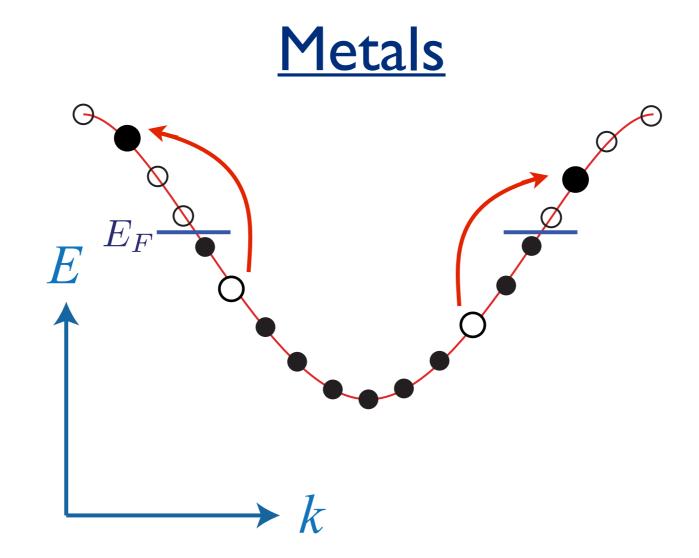
Foundations of quantum many body theory:

I. Ground states <u>connected</u> adiabatically to independent electron states

Metals E k

Foundations of quantum many body theory:

- I. Ground states <u>connected</u> adiabatically to independent electron states
- 2. Boltzmann-Landau theory of quasiparticles



Modern phases of quantum matter:

- I. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement
 - 2. Boltzmann-Landau theory of quasiparticles

Famous example:

The <u>fractional quantum Hall</u> effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

- I. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement
 - 2. No quasiparticles

Modern phases of quantum matter:

- I. Ground states disconnected from independent electron states: many-particle entanglement
 - 2. No quasiparticles

Only 2 examples:

- I. Conformal field theories in spatial dimension d > 1
- 2. Quantum critical metals in dimension d=2

Outline

- 1. Conformal field theories in 2+1 dimensions
 - Superfluid-insulator transition
 - A. Boltzmann dynamics
 - B. Conformal / holographic dynamics

- 2. Non-Fermi liquid in 2+1 dimensions
 - Strange metal in the high temperature superconductors
 - A. Lessons from holography
 - B. Field theories and memory functions

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Emanuel Katz Boston University

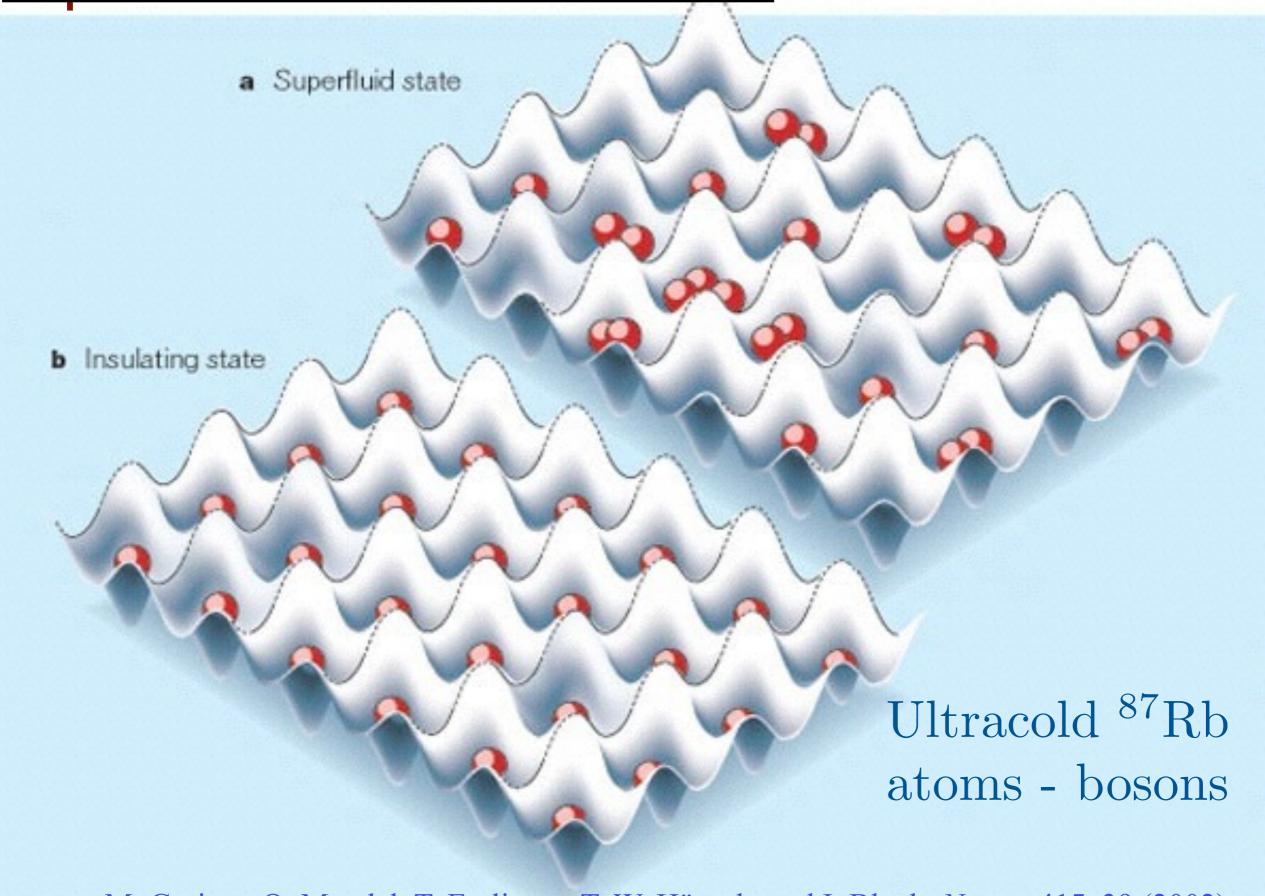


William Witczak-Krempa Perimeter



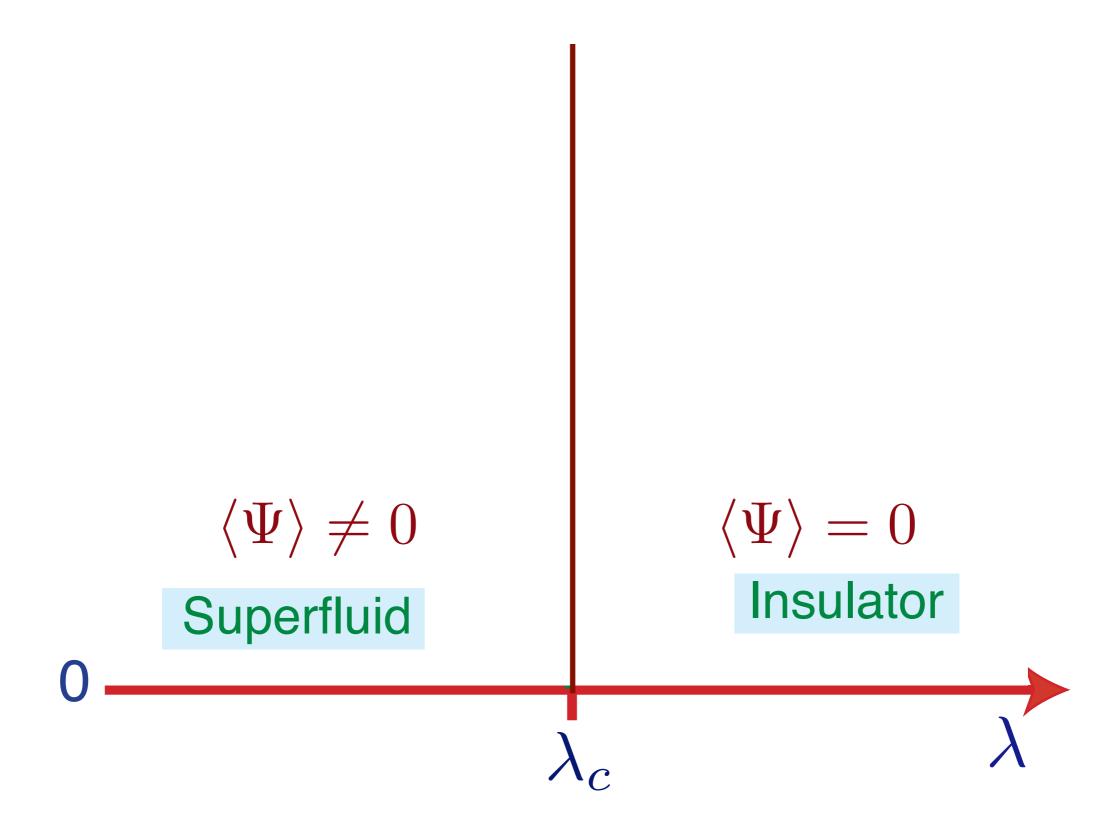
Erik Sorensen McMaster

Superfluid-insulator transition



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).

 $\Psi \to \text{a complex field representing the}$ Bose-Einstein condensate of the superfluid



$$\mathcal{Z} = \int \mathcal{D}\Psi(r,\tau) \exp\left(-\int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2|\nabla_r \Psi|^2 + V(\Psi)\right]\right)$$

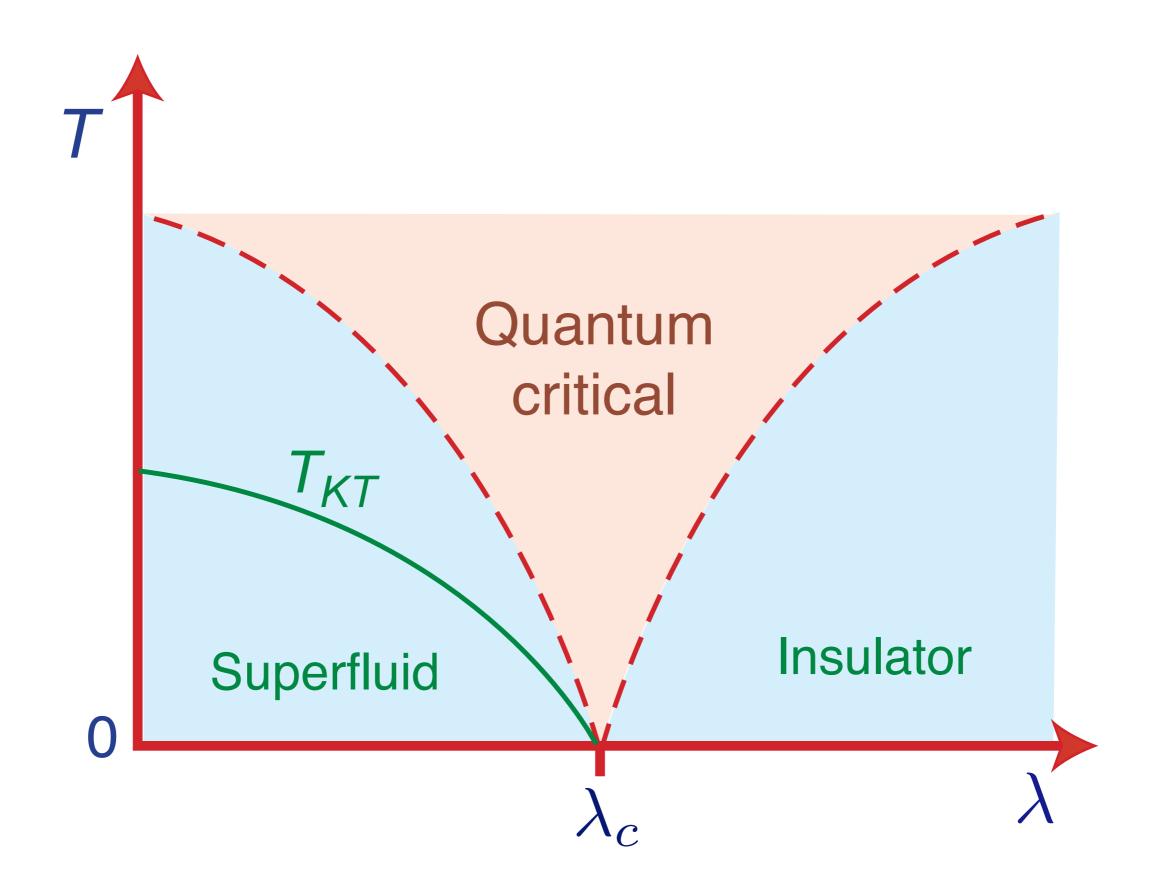
$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$

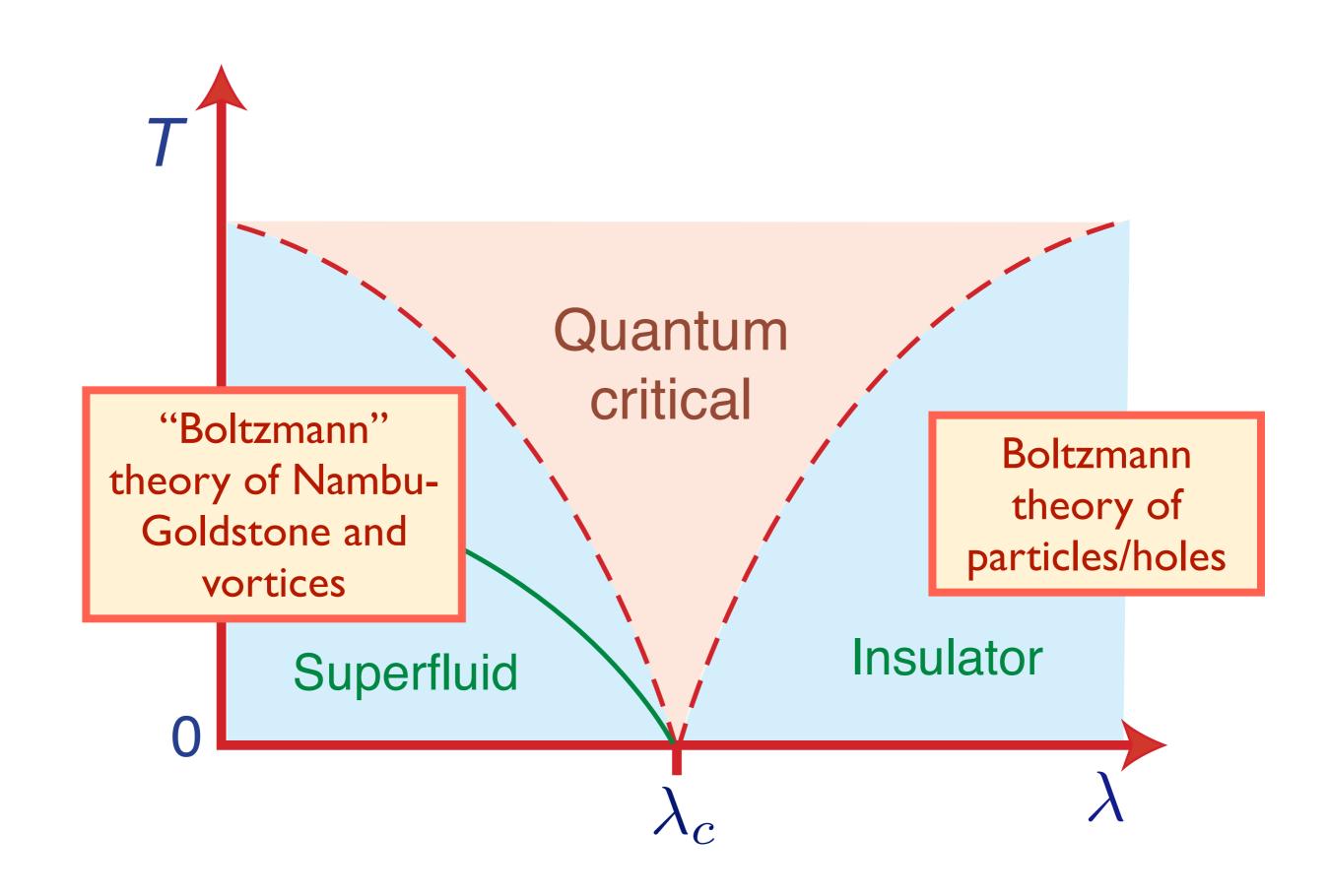
$$\langle \Psi \rangle \neq 0$$

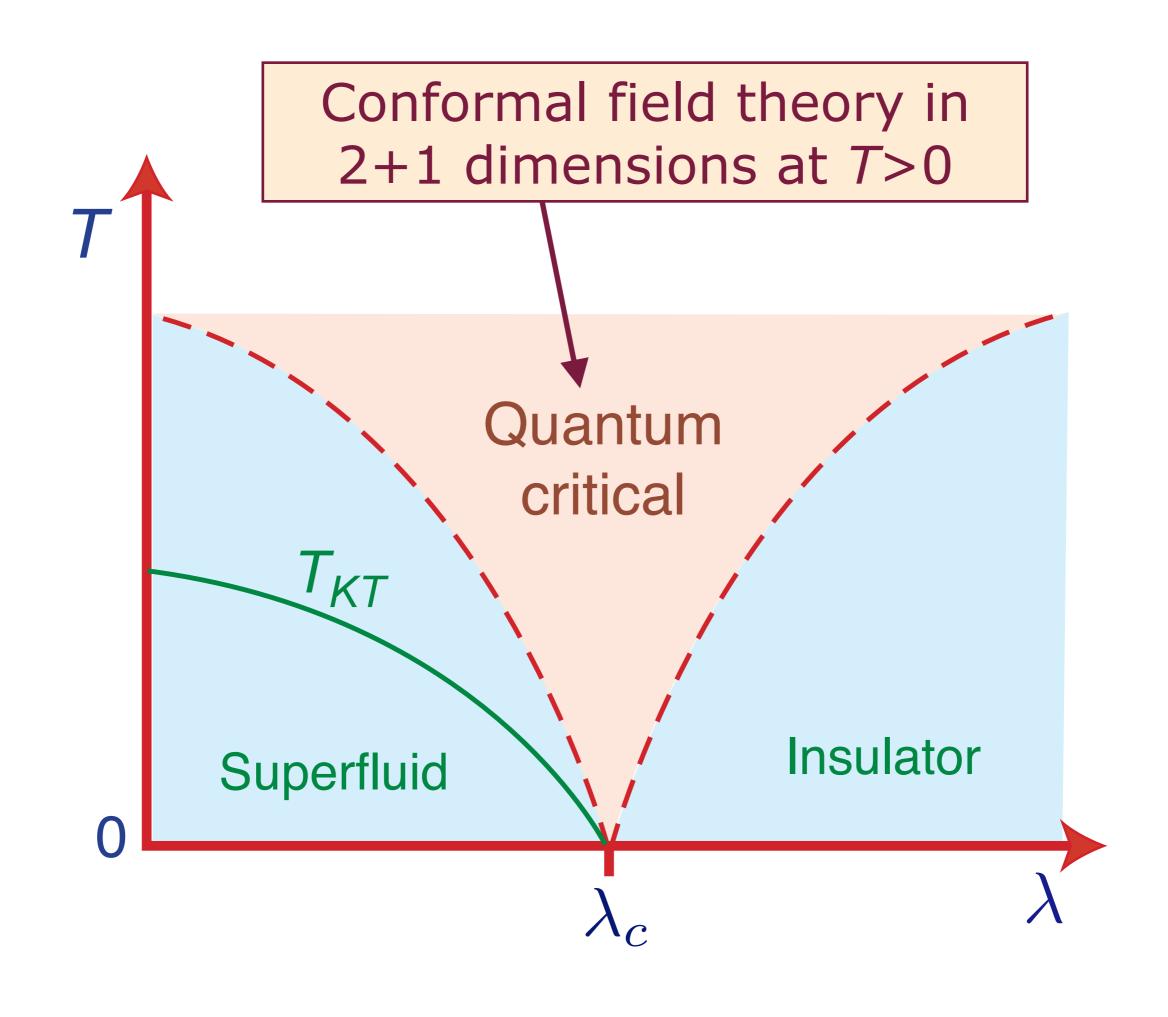
$$\langle \Psi \rangle = 0$$
Superfluid
$$0$$
Insulator

$$\mathcal{Z} = \int \mathcal{D}\Psi(r,\tau) \exp\left(-\int d^2r d\tau \left[|\partial_{\tau}\Psi|^2 + c^2|\nabla_{r}\Psi|^2 + V(\Psi)\right]\right)$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$
 A conformal field theory in 2+1 spacetime dimensions (CFT3): the O(2) Wilson-Fisher CFT3
$$\langle \Psi \rangle \neq 0$$
 Superfluid
$$0$$
 Insulator







Outline

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Strange metal in the high temperature superconductors

A. Lessons from holography

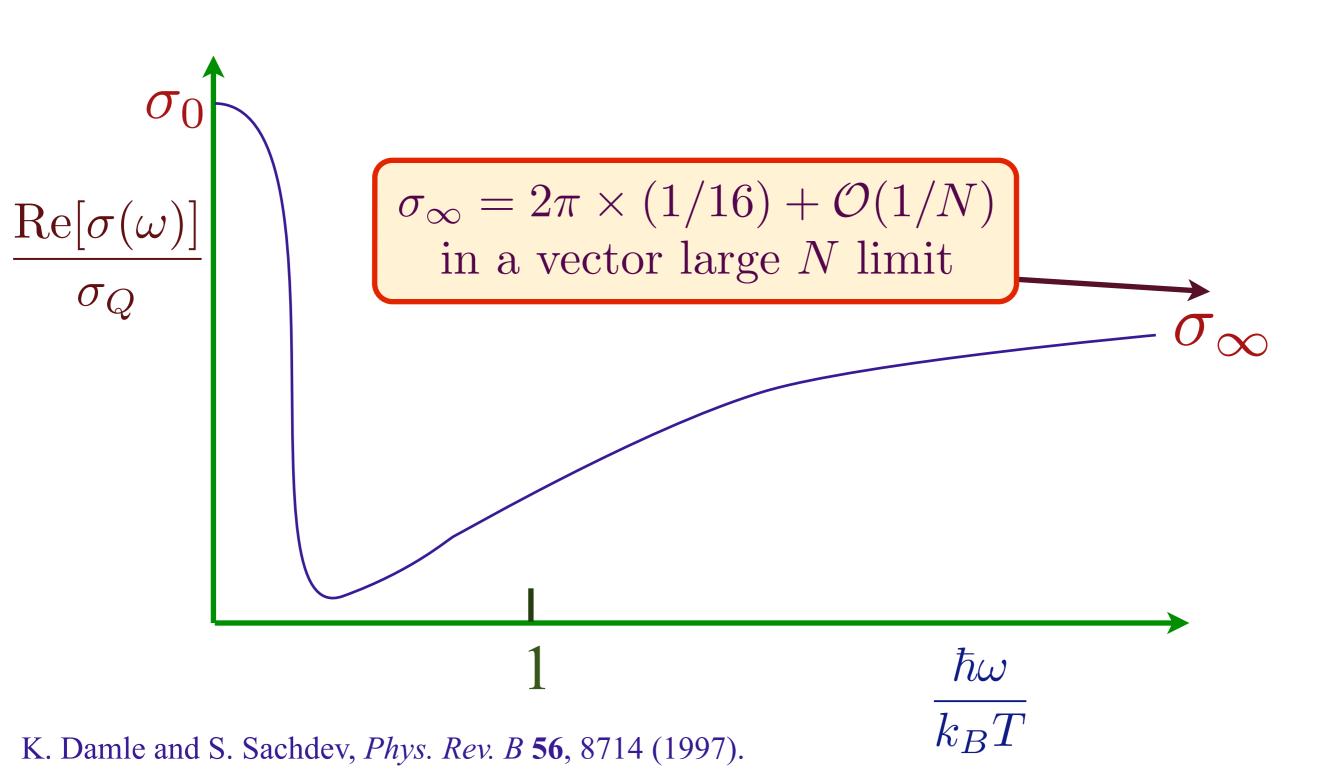
B. Field theories and memory functions

- ♦ Identify quasiparticles and their dispersions
- ♦ Compute scattering matrix elements of quasiparticles

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- \Diamond Compute dissipative properties at $\omega \ll$ quasiparticle-collision-rate

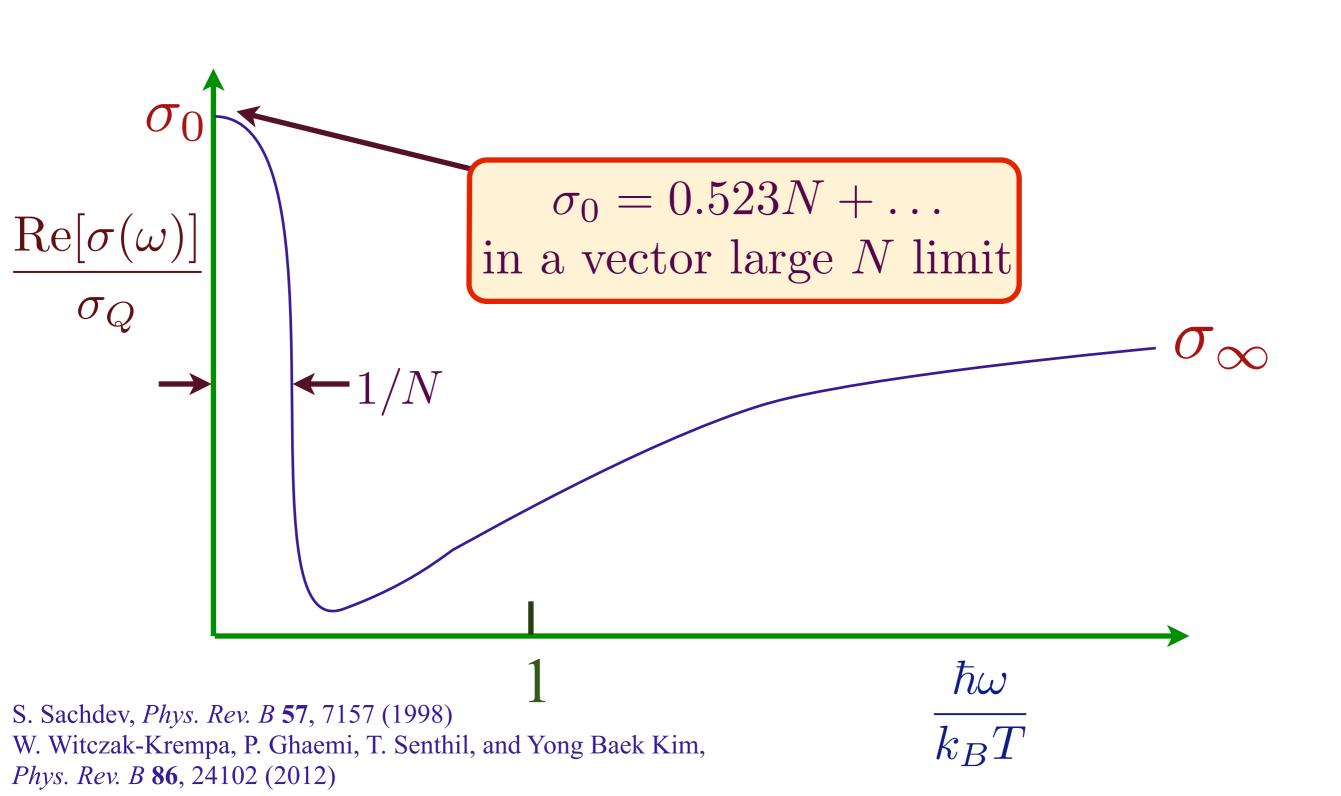
Quasiparticle view of quantum criticality (Boltzmann equation): Transport of O(N) current for a (weakly) interacting CFT3

 $\sigma_Q = e^2/h$, the quantum unit of conductance



Quasiparticle view of quantum criticality (Boltzmann equation): Transport of O(N) current for a (weakly) interacting CFT3

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 - A. Holographic model
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Dynamics without quasiparticles



 \bigstar Using scaling dimensions and operator product expansions (OPE) of the CFT, compute conductivity at $\hbar\omega \gg k_BT$

Basic characteristics of CFT3s

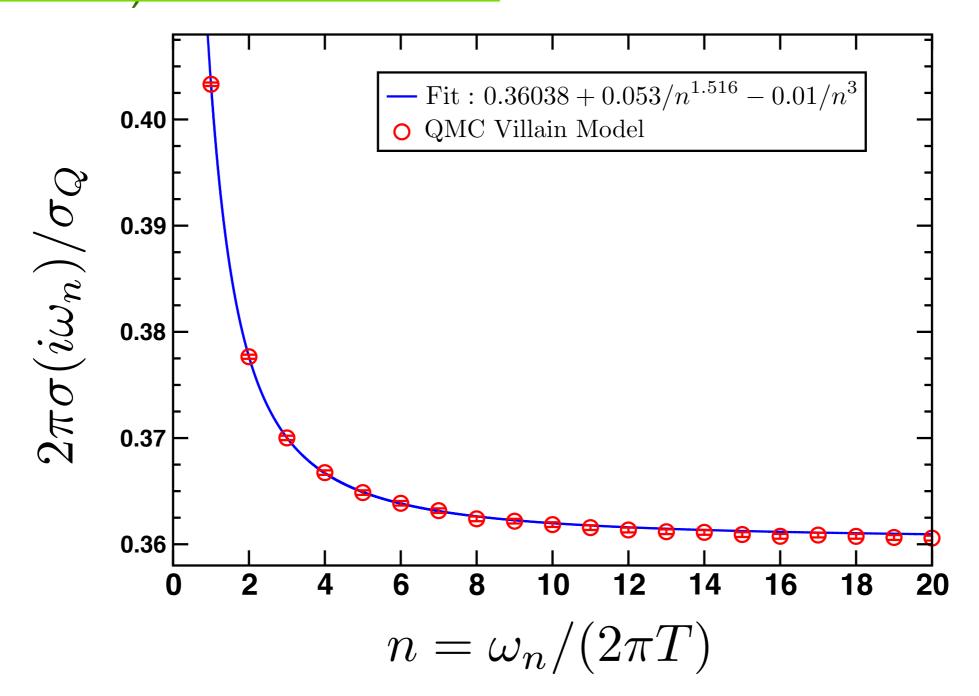
The thermal average of the OPE of two O(2) current operators yields for $\omega \gg T$

$$\frac{\sigma(\omega)}{\sigma_Q} = \sigma_{\infty} + b_1 \left(\frac{T}{\omega}\right)^{3-1/\nu} + b_2 \left(\frac{T}{\omega}\right)^3 + \dots$$

where $b_{1,2}$ are universal numbers dependent upon OPE coefficients.

- b_1 depends on a relevant scalar operator with dimension $3 1/\nu$; for the O(2) Wilson-Fisher CFT3, $\nu \approx 0.6717(1)$.
- b_2 depends on OPE with the energy-momentum tensor.

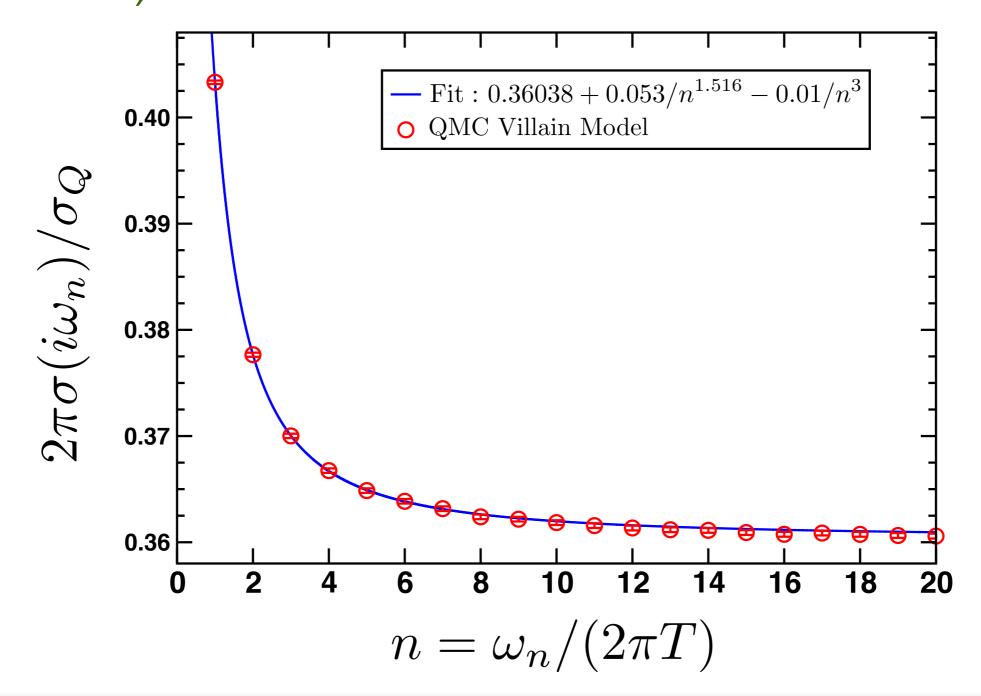
Quantum Monte Carlo for lattice model of integer currents (Villain model) in Euclidean time



Excellent agreement with OPE

E. Katz, S. Sachdev, E. Sorensen, and W. Witczak-Krempa, arXiv: I 409.384 I

Quantum Monte Carlo for lattice model of integer currents (Villain model) in Euclidean time



QMC fails for Minkowski frequencies $\hbar\omega \ll k_BT$

E. Katz, S. Sachdev, E. Sorensen, and W. Witczak-Krempa, arXiv: I 409.384 I

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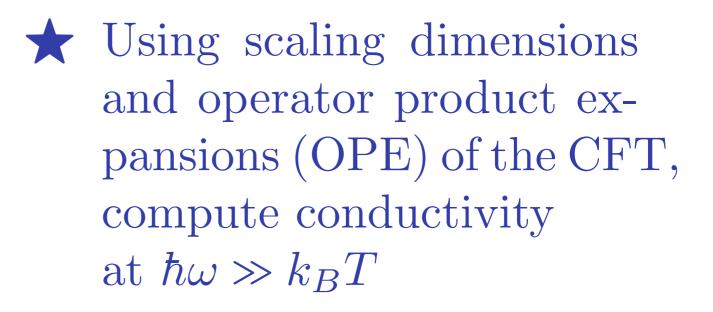


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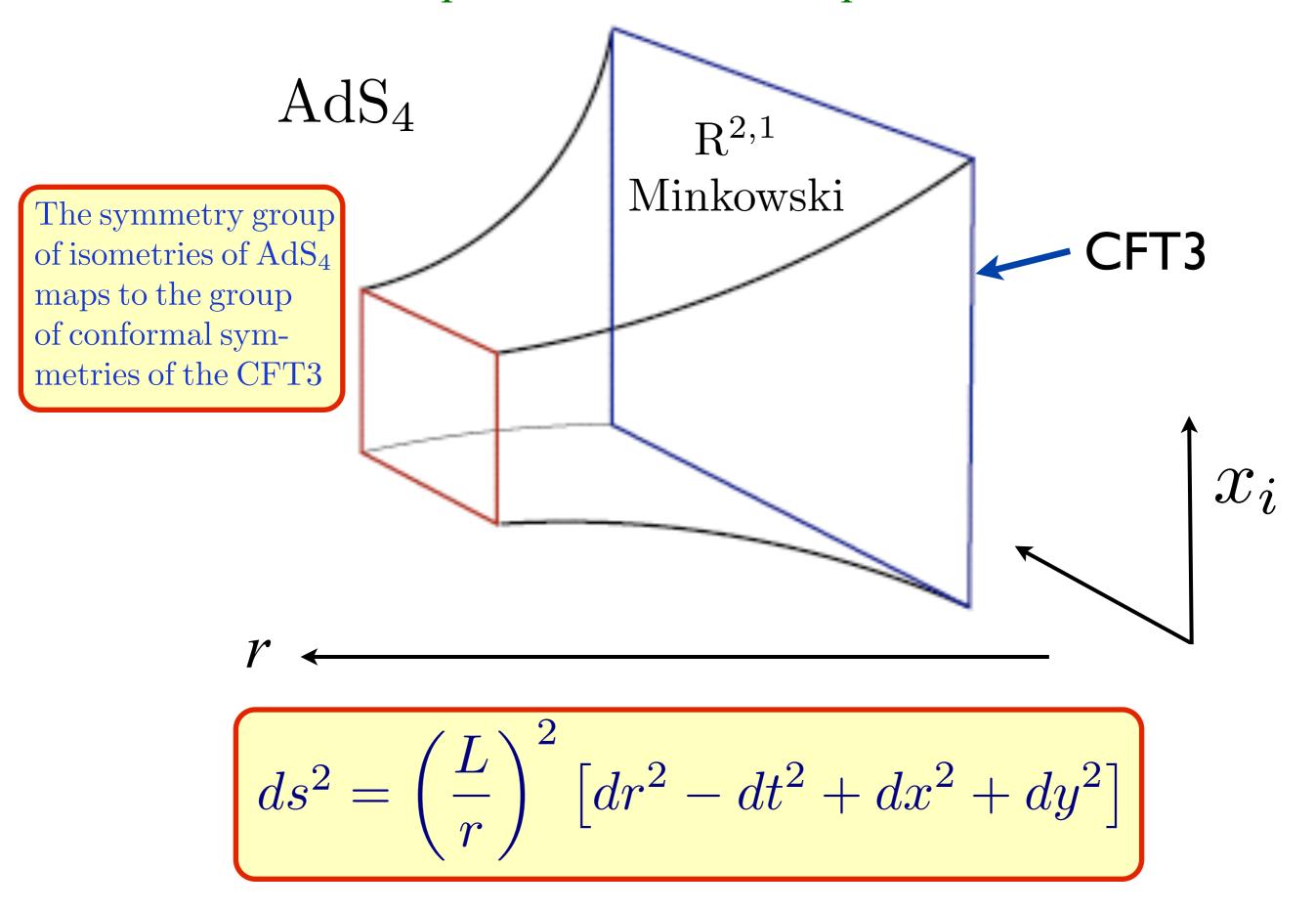
Dynamics without quasiparticles





Relate OPE coefficients to couplings of an effective gravitational theory on AdS

AdS/CFT correspondence at zero temperature



AdS/CFT correspondence at zero temperature

To fully match the OPE of the current operators, we need an *Einstein-Maxwell-Weyl-scalar* theory

$$S_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} \left[1 + \alpha \varphi(x) \right] F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right]$$

$$+ \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) + g^{ab} \partial_a \varphi \partial_b \varphi + m^2 \varphi^2 \right],$$

where C_{abcd} is the Weyl tensor. Stability constraints on this action restrict $|\gamma| < 1/12$, in agreement with results from the CFT3. The scalar field φ is conjugate to the CFT operator \mathcal{O} with scaling dimension $3 - 1/\nu$, which fixes its mass m. The coupling α is determined by the OPE of the currents with \mathcal{O} .

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* 83, 066017 (2011)

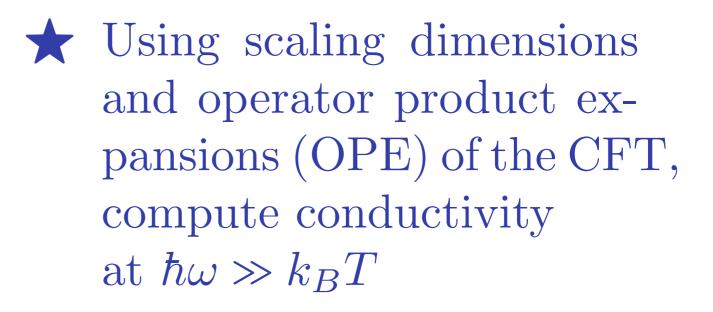
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* 87, 085138 (2013).

E. Katz, S. Sachdev, E. Sorensen, and W. Witczak-Krempa, arXiv:1409.3841

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Dynamics without quasiparticles



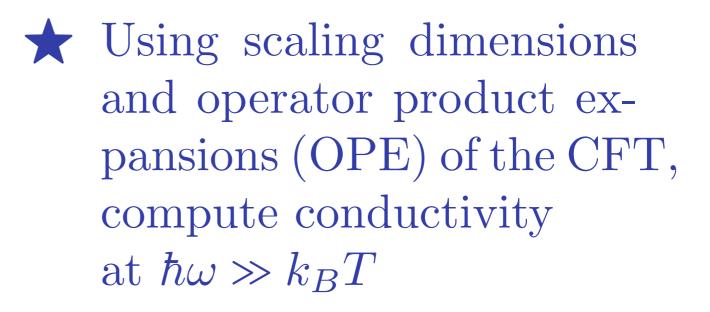


Relate OPE coefficients to couplings of an effective gravitational theory on AdS

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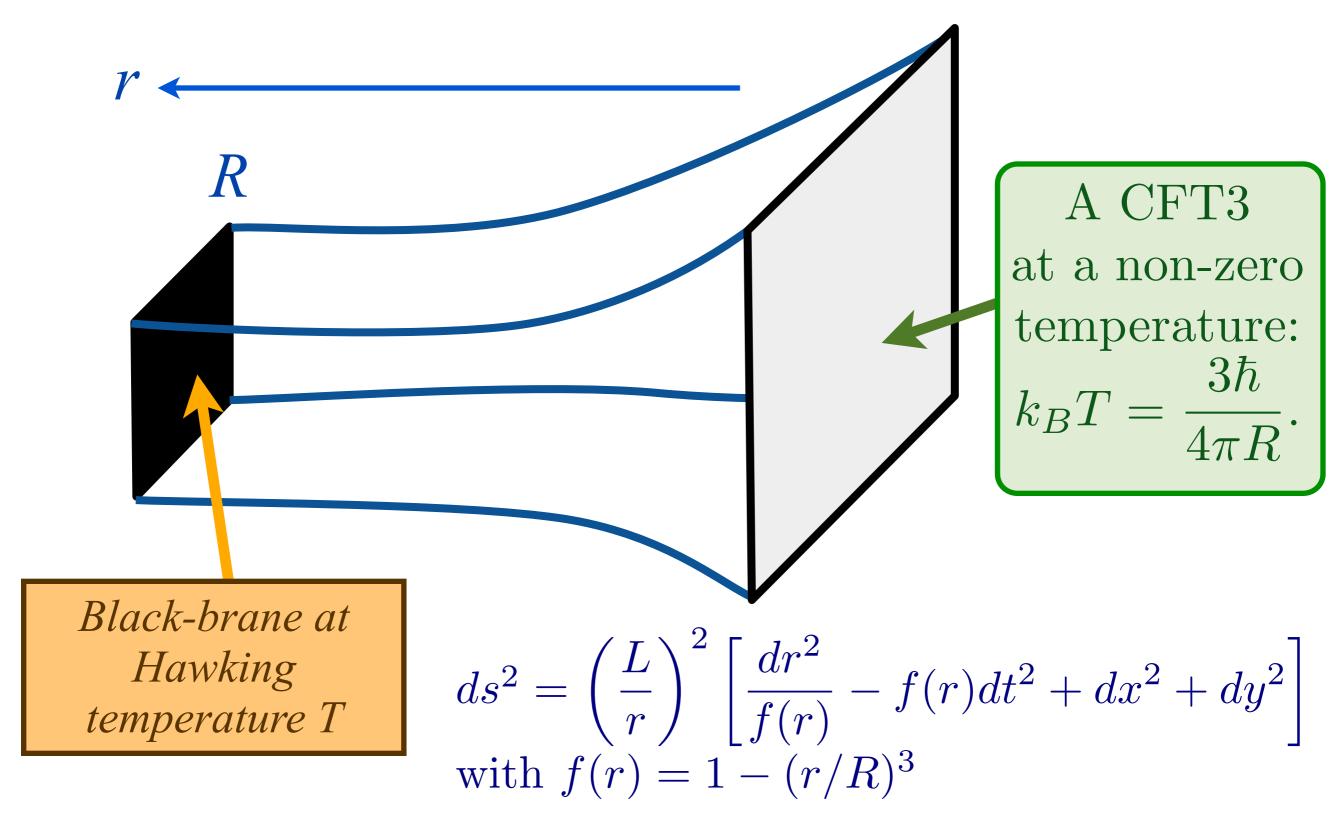


Relate OPE coefficients to couplings of an effective gravitational theory on AdS

 \bigstar Dynamics of a "horizon" in gravitational theory yields info at $\hbar\omega \ll k_BT$.

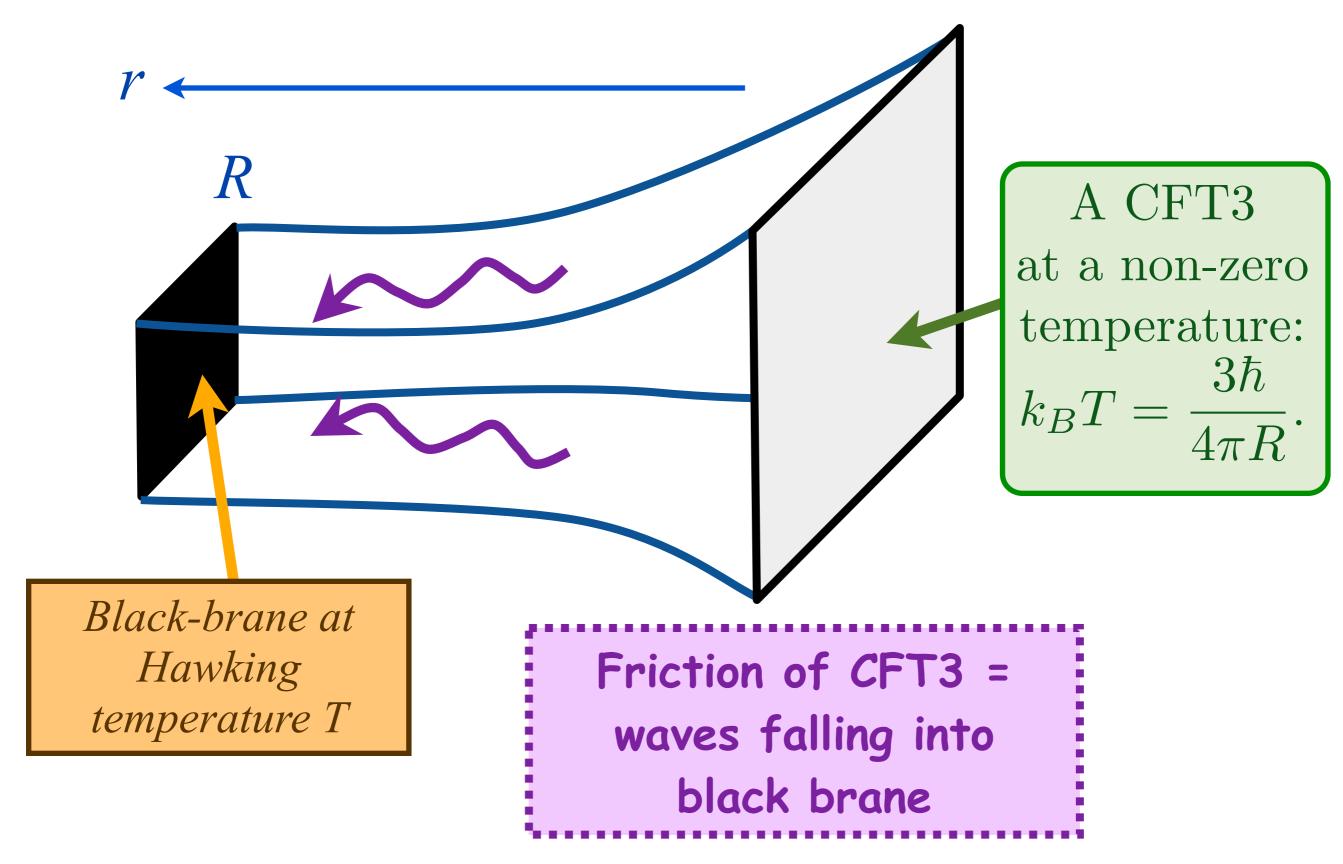
AdS/CFT correspondence at non-zero temperatures

AdS₄-Schwarzschild black-brane



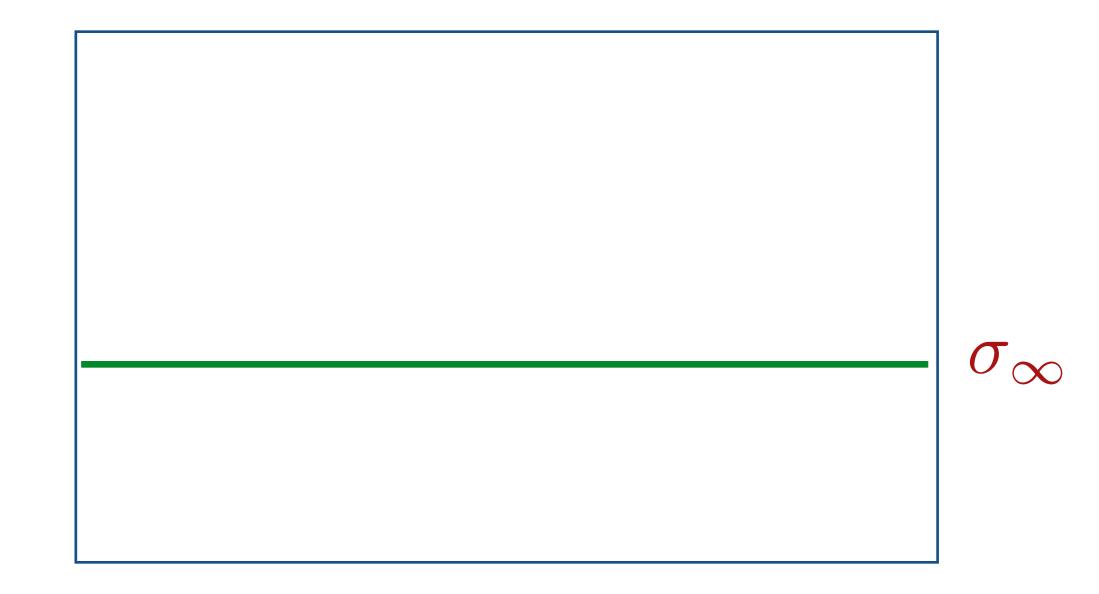
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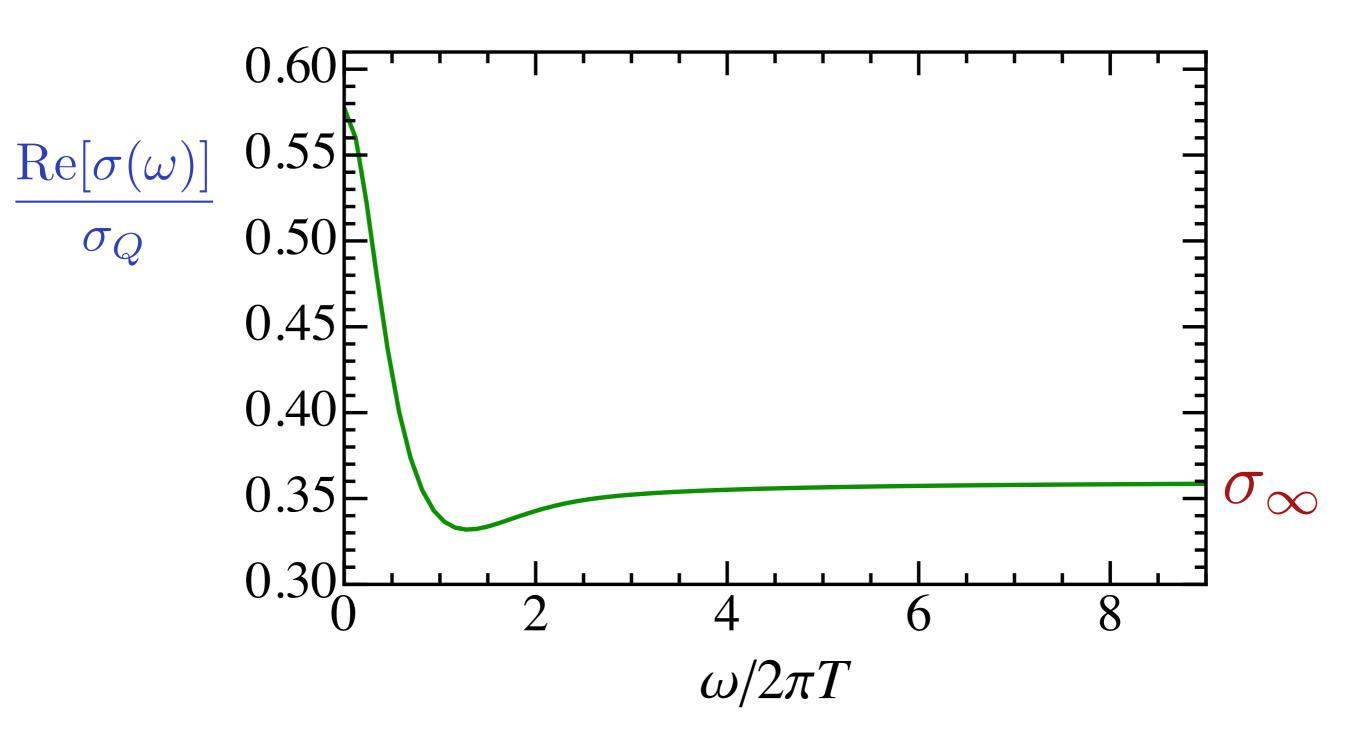
Conductivity of Einstein-Maxwell theory

$$\frac{\mathrm{Re}[\sigma(\omega)]}{\sigma_Q}$$



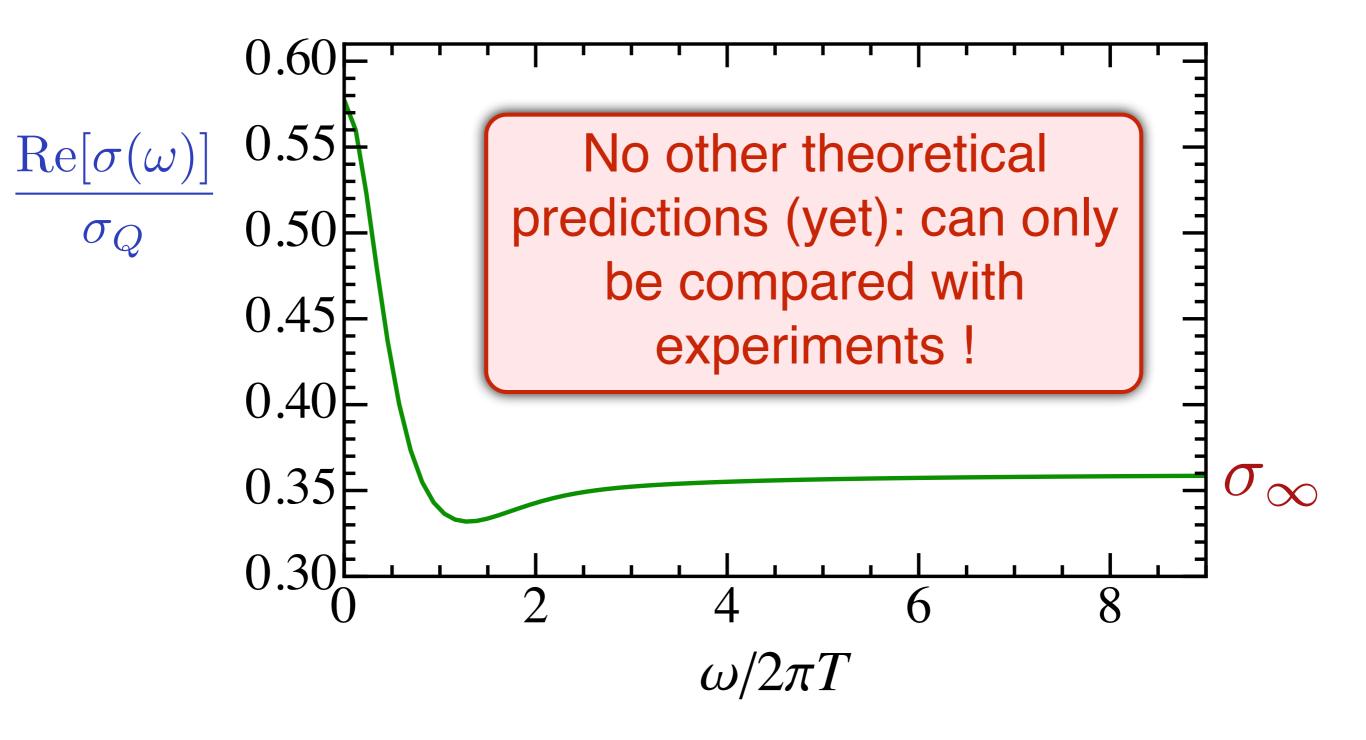
 $\omega/2\pi T$

Numerical solution of Einstein-Maxwell-Weyl-scalar theory + OPE info from QMC



E. Katz, S. Sachdev, E. Sorensen, and W. Witczak-Krempa, arXiv: I 409.3841

Numerical solution of Einstein-Maxwell-Weyl-scalar theory + OPE info from QMC



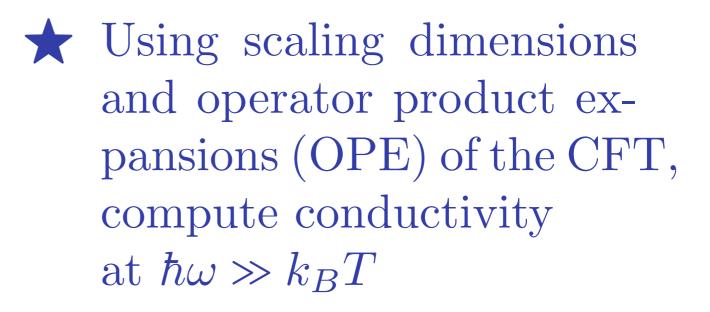
E. Katz, S. Sachdev, E. Sorensen, and W. Witczak-Krempa, arXiv: I 409.3841

Traditional CMT

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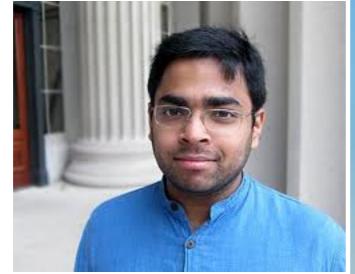
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Sean Hartnoll Stanford



Raghu Mahajan Stanford



Innsbruck

Matthias Punk



Andrea Allais



Koenraad Schalm Leiden



Andrew Lucas



Aavishkar Patel

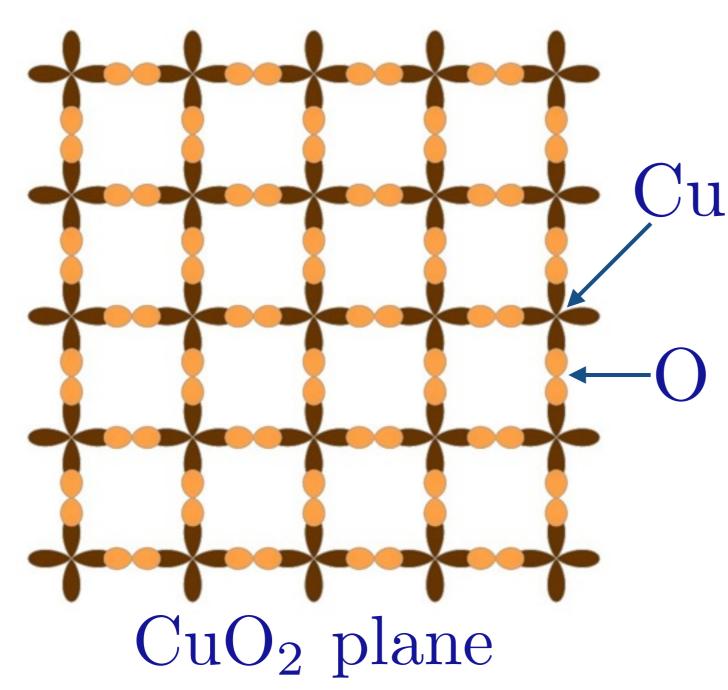


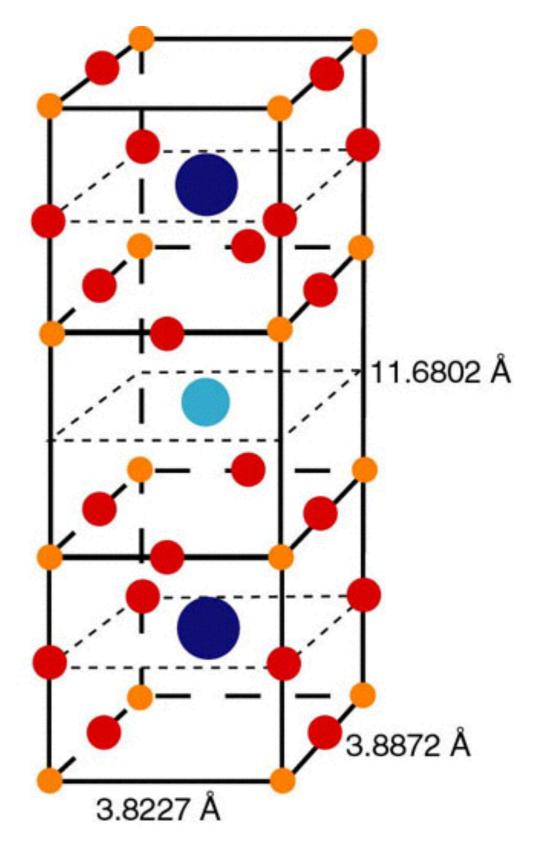
Debanjan Chowdhury



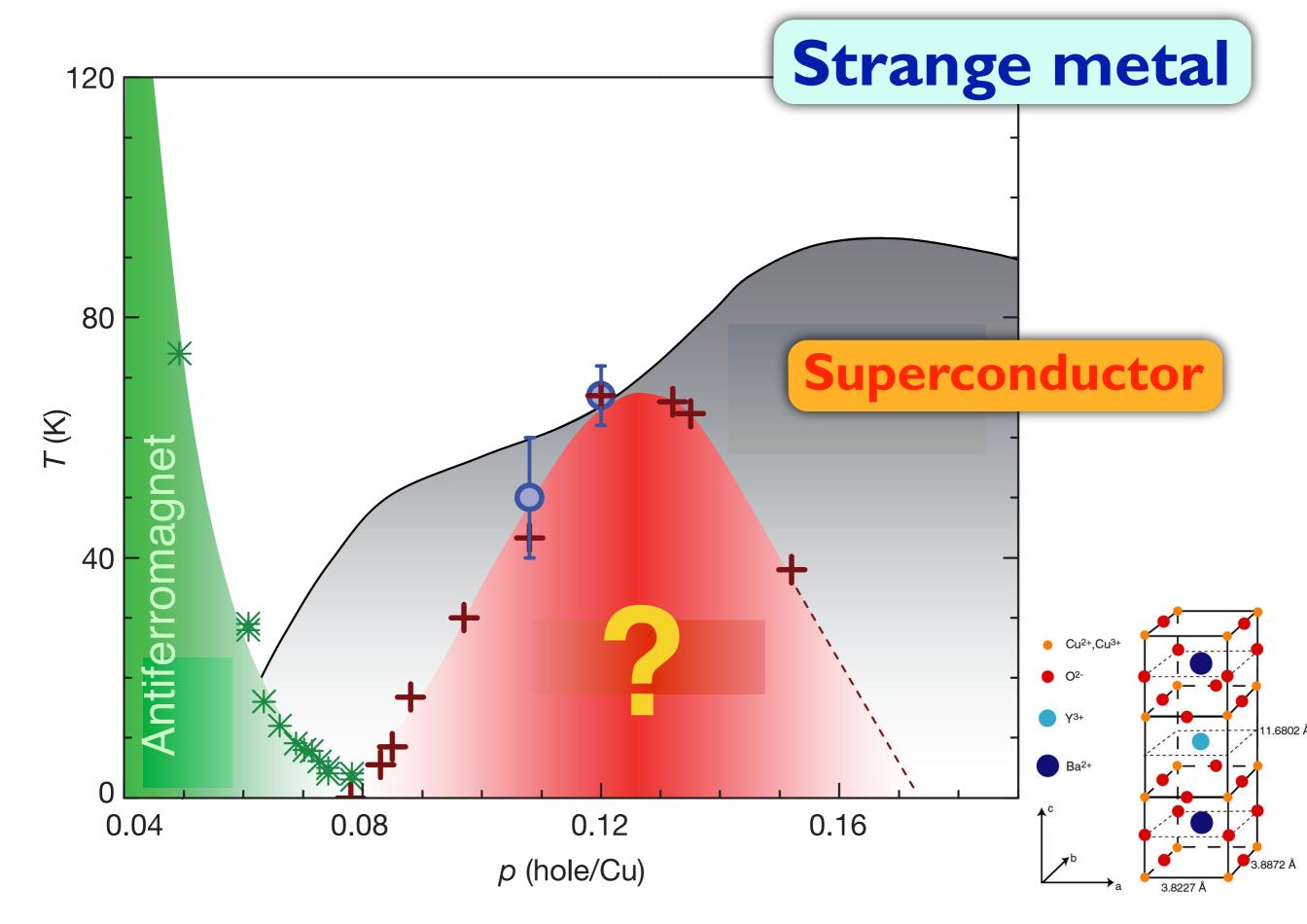
Alexandra Thomson

High temperature superconductors

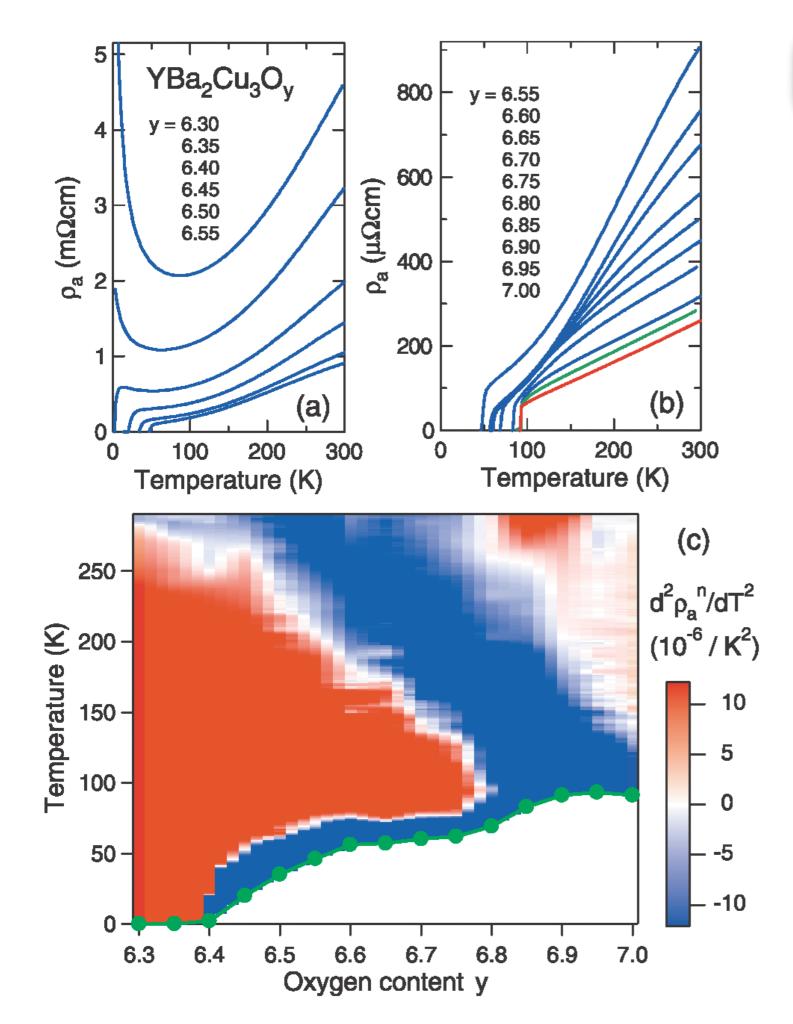




 $YBa_2Cu_3O_{6+x}$



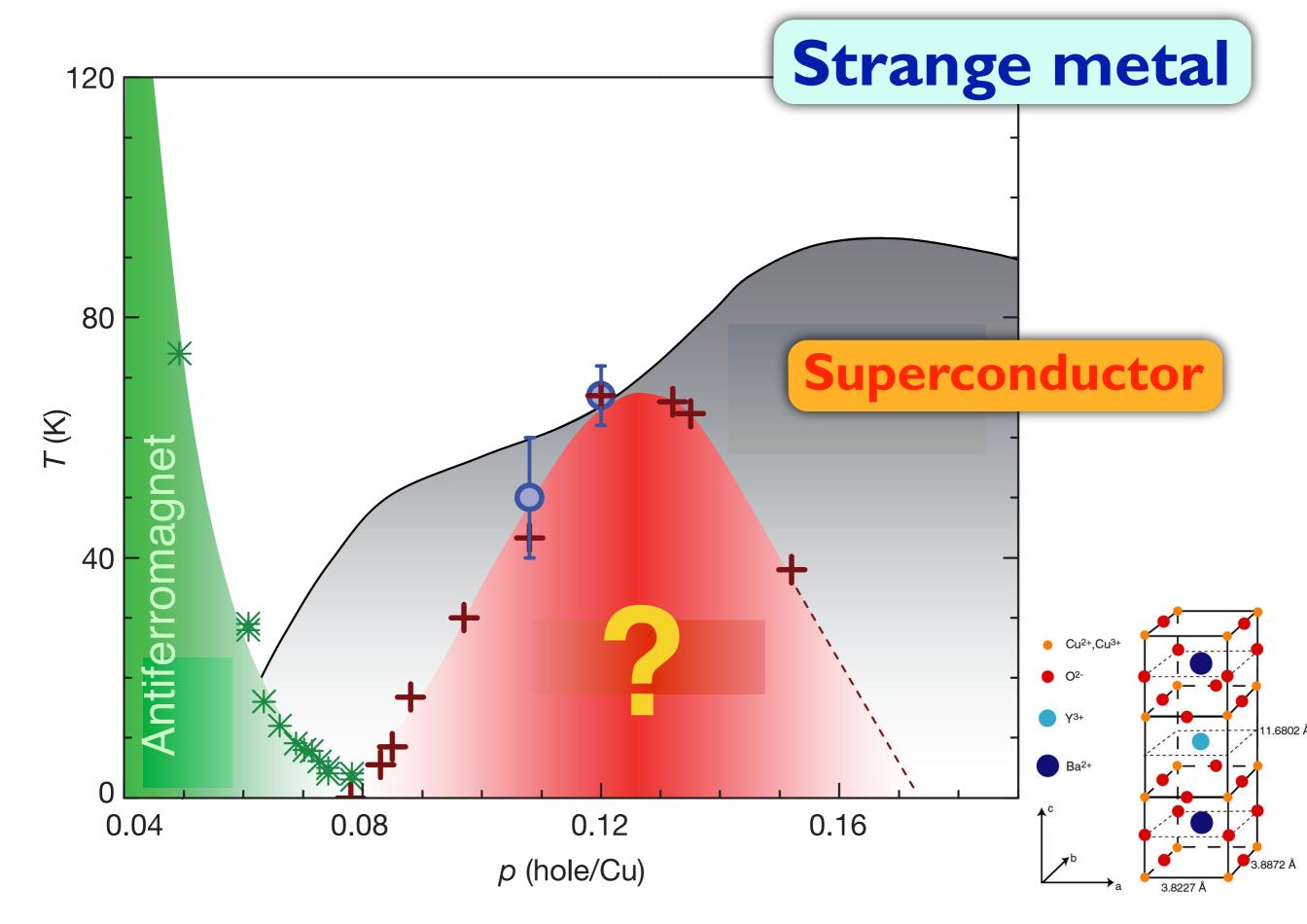
T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, Nature 477, 191 (2011).



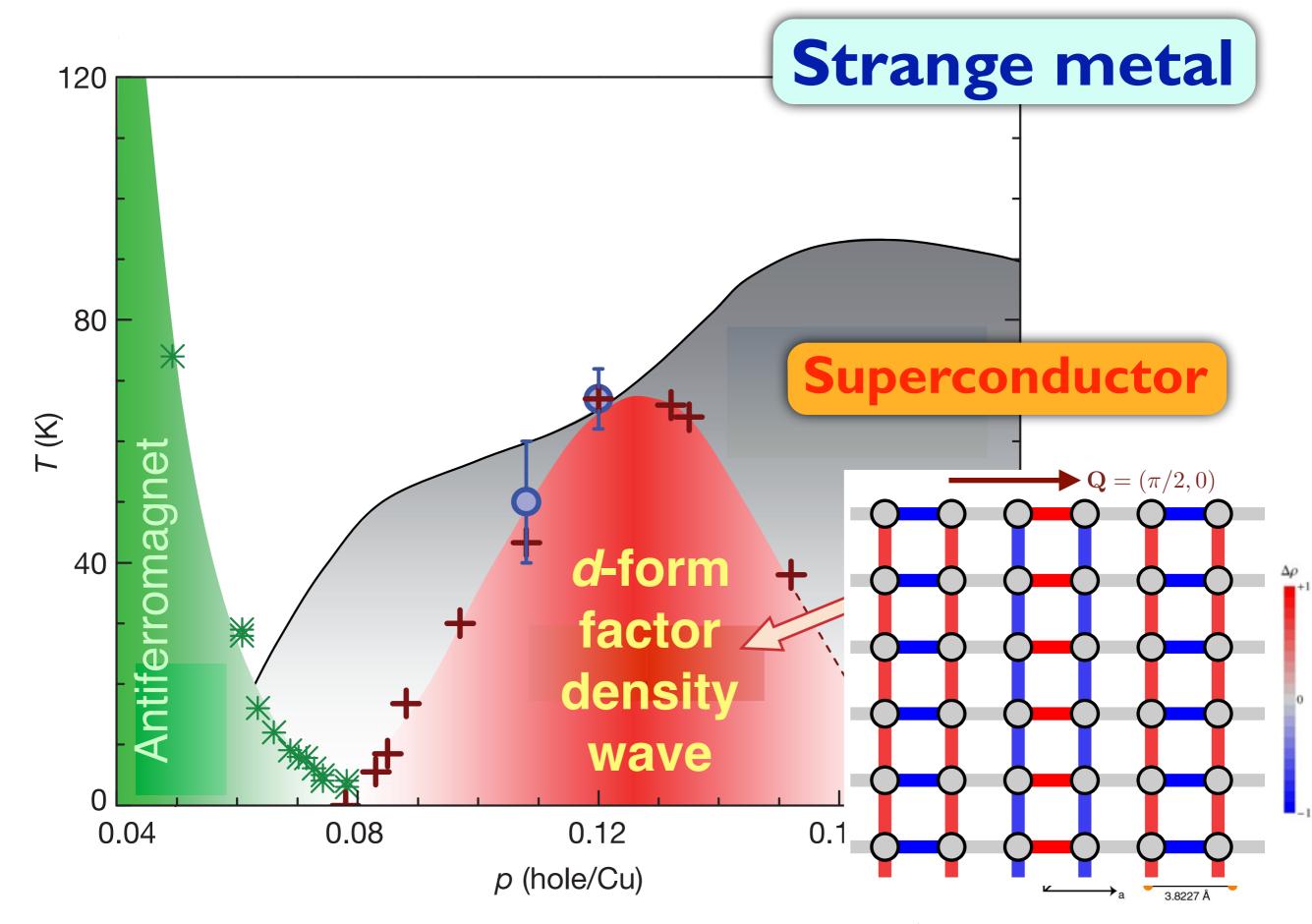
Strange metal

YBCO at optimal doping has resistivity $\rho(T) \sim T$.

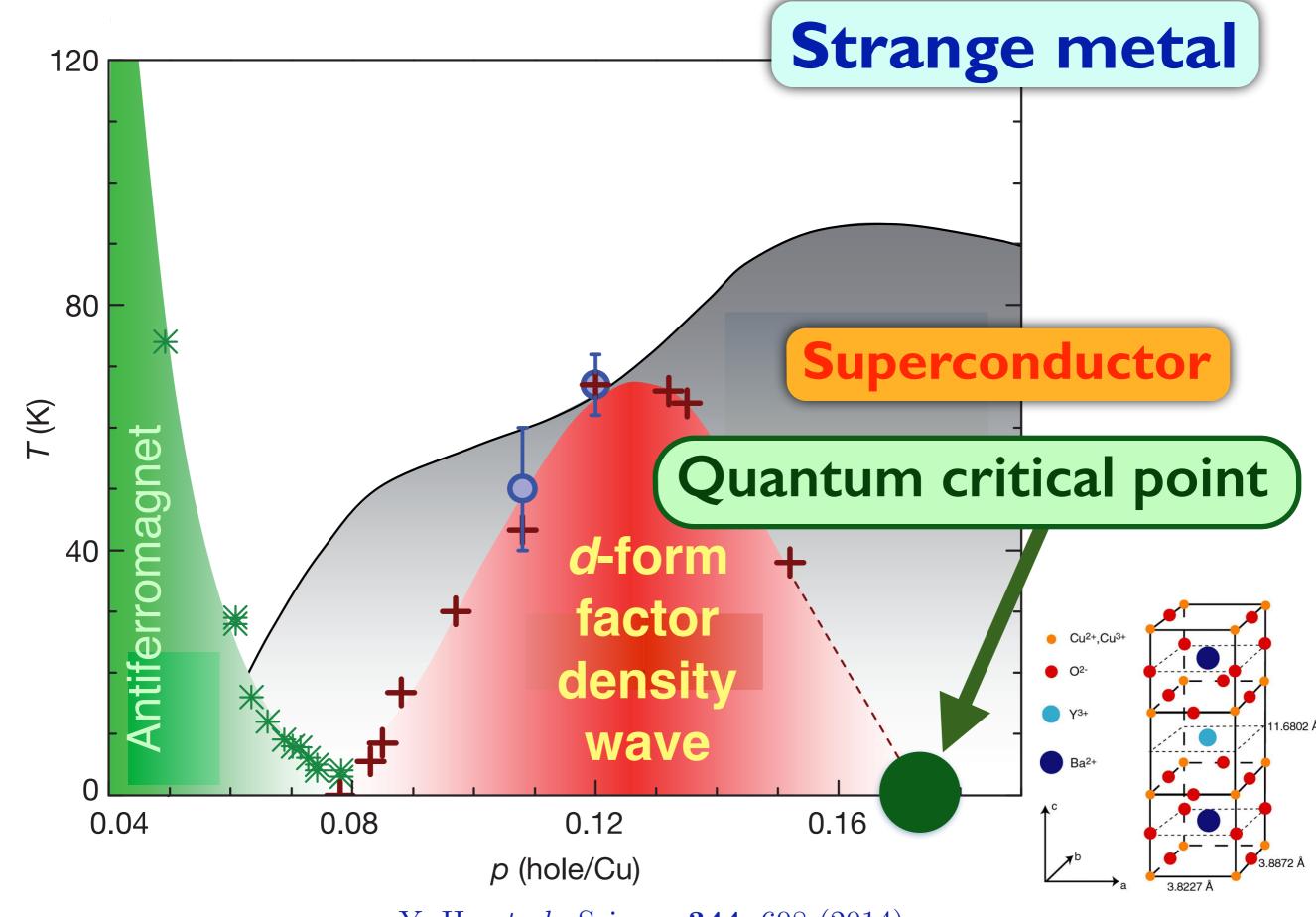
Yoichi Ando, Seiki Komiya, Kouji Segawa, S. Ono, and Y. Kurita, Phys. Rev. Lett. **93**, 267001 (2004)



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, Nature 477, 191 (2011).



K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



Y. He *et al.*, Science **344**, 608 (2014) K. Fujita *et al.*, Science **344**, 612 (2014)

Write the electron operator c_{α} ($\alpha = \uparrow, \downarrow$ are spin indices) as

$$\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = R \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix}$$

where R is a SU(2) matrix which determines the orientation of the local antiferromagnetic order, and ψ_{\pm} are spinless fermions which carry the global electron U(1) charge.

This parameterization is invariant under a SU(2) <u>gauge</u> transformation

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \to U \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} ; \quad R \to RU^{\dagger}$$

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B 80, 155129 (2009)

Assume field R is non-critical.

- Fermion ψ , transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface.
- A SU(2) gauge boson.
- A real Higgs field, H, transforming as a gauge SU(2) adjoint, carrying lattice momentum (π, π) . Condensation of the Higgs breaks $SU(2) \rightarrow U(1)$, and transforms the <u>large</u> Fermi surface to a <u>small</u> Fermi surface.

• The quantum critical theory is the Higgs transition where the gauge "symmetry" breaks from SU(2) down to U(1), in the presence of a Fermi surface of fermions carrying fundamental SU(2) charges.

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- The Higgs condensation does not give the fermions a "mass"; instead it reconstructs the Fermi surface from *large* to *small*.

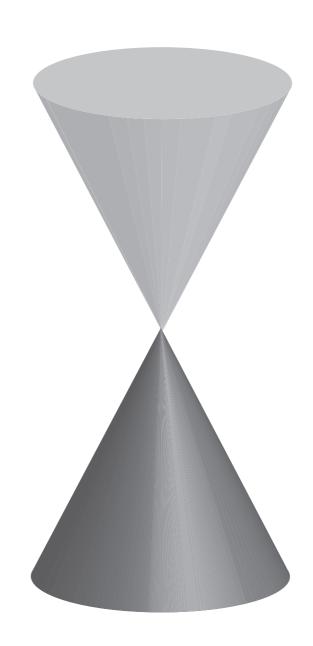
- The quantum critical theory is the Higgs transition where the gauge "symmetry" breaks from SU(2) down to U(1), in the presence of a Fermi surface of fermions carrying fundamental SU(2) charges.
- The Higgs condensation does not give the fermions a "mass"; instead it reconstructs the Fermi surface from *large* to *small*.
- The quantum phase transition has no gaugeinvariant "order parameter", and it does not break any global symmetries.

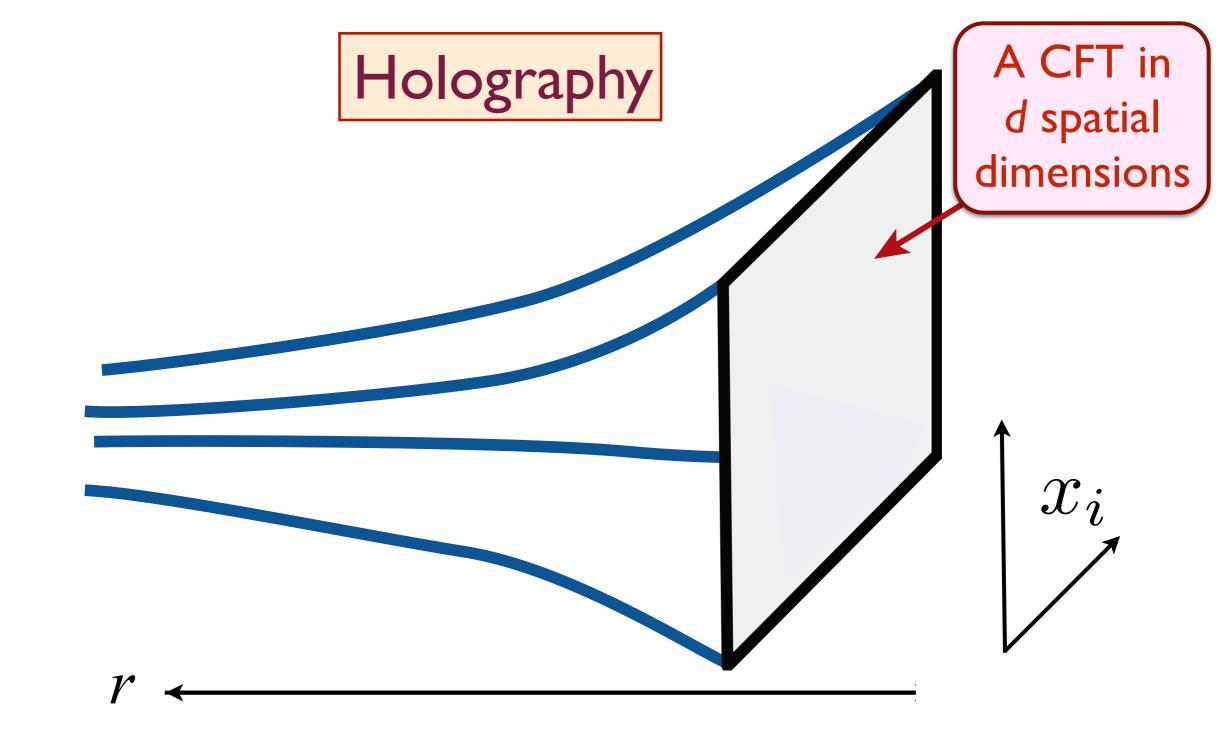
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A CFT

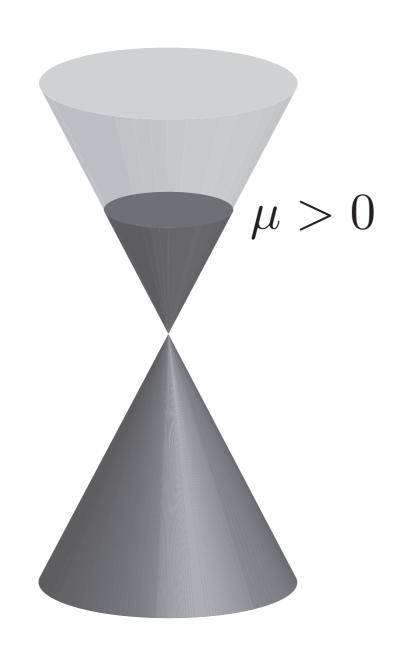


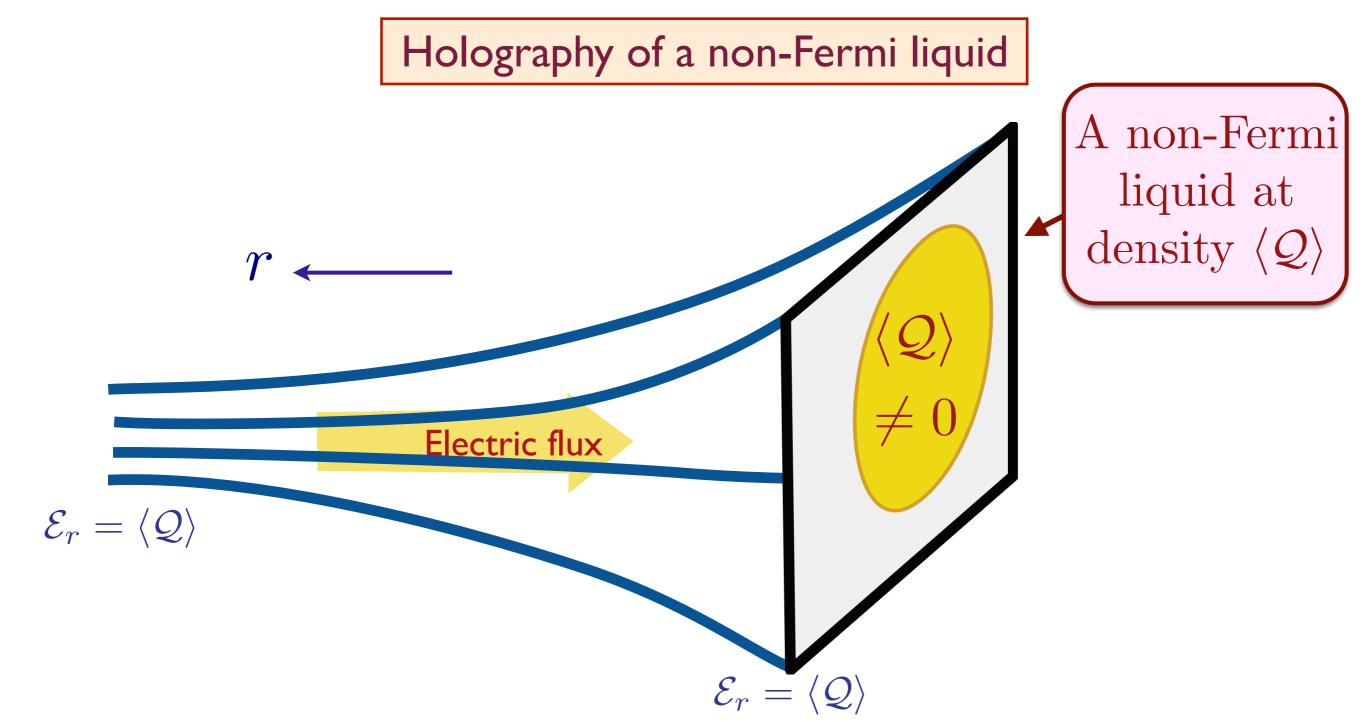


$$ds^{2} = \frac{1}{r^{2}} \left(-dt^{2} + dr^{2} + dx_{i}^{2} \right)$$

This is the metric of anti-de Sitter space AdS_{d+2} .

Apply a chemical potential





The most general metric with scale-invariance at long distances/times

$$ds^{2} = \frac{1}{r^{2}} \left(-\frac{dt^{2}}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^{2} + dx_{i}^{2} \right)$$

L. Huijse, S. Sachdev, B. Swingle, Physical Review B 85, 035121 (2012)

★ Computation of resistivity in gravitational theory yields zero resistance at all temperatures, $\rho(T) = 0!$

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This can be understood by

- Conservation of total momentum, P,
- Non-zero value of $\chi_{JP} = \langle \vec{P}; \vec{J} \rangle$ when $\langle \mathcal{Q} \rangle \neq 0$ $(\vec{J} \text{ is the O(2) current}).$
- i.e. Momentum drags current.

To relax momentum, add a random perturbation coupling to the operator \mathcal{O} :

$$\mathcal{S} \to \mathcal{S} + \int d^d r d\tau h(r) \mathcal{O}(r, \tau) \quad \text{with } \overline{h(r)} = 0 \text{ and } \overline{h(r)h(r')} = h_0^2 \delta^d(r - r')$$

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Solution of gravitational equations for small h_0 yields the resistivity

$$\rho(T) \sim h_0^2 T^{2(1+\Delta-z)/z}$$
,

where Δ is the dimension of \mathcal{O} . This agrees precisely with the $\underline{memory\ function}$ computation on a field theory with the operator $\overline{\mathcal{O}}$, and with $\chi_{JP} \neq 0$!

The memory-function result for the resistivity for current along angle ϑ

$$\rho(T) = \frac{1}{\chi_{\mathbf{J},\mathbf{P}}^2} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \frac{\operatorname{Im} G_{\mathcal{O},\mathcal{O}}^R(\omega, \mathbf{k})}{\omega}.$$

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Gravity + Holography "know" about

• classical hydrodynamics

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Gravity + Holography "know" about

- classical hydrodynamics
- quantum corrections beyond hydrodynamics

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Main assumption:

All points on the Fermi surface have a rapid relaxation to local thermal equilibrium by processes which conserve a suitably defined momentum.

Relaxation of the momentum occurs at a slower rate.

The resistivity of this strange metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic.

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There is a dominant contribution $\rho(T) \sim T$ by the coupling of long-wavelength disorder to the gauge-invariant operator $\mathcal{O} \sim H^2$. Quantitative predictions for transport in 2+1 dimensional CFTs obtained by combining the operator product expansion, quantum Monte Carlo, and the dynamics of black branes.

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- Proposed theory of linear-T resistivity in strange metals involving a Higgs transition in a SU(2) gauge theory of Fermi surface reconstruction.