

Quantum entanglement and the phases of matter

TIFR, Mumbai
January 17, 2012

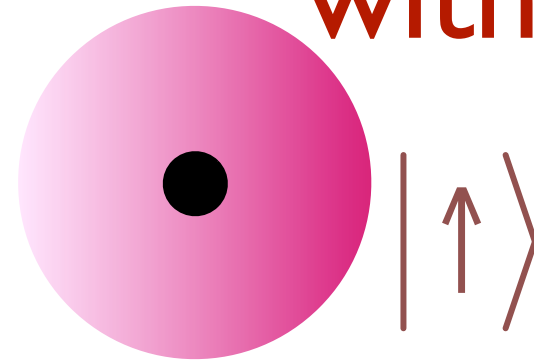
sachdev.physics.harvard.edu



Quantum Entanglement: quantum superposition with more than one particle

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Hydrogen atom:

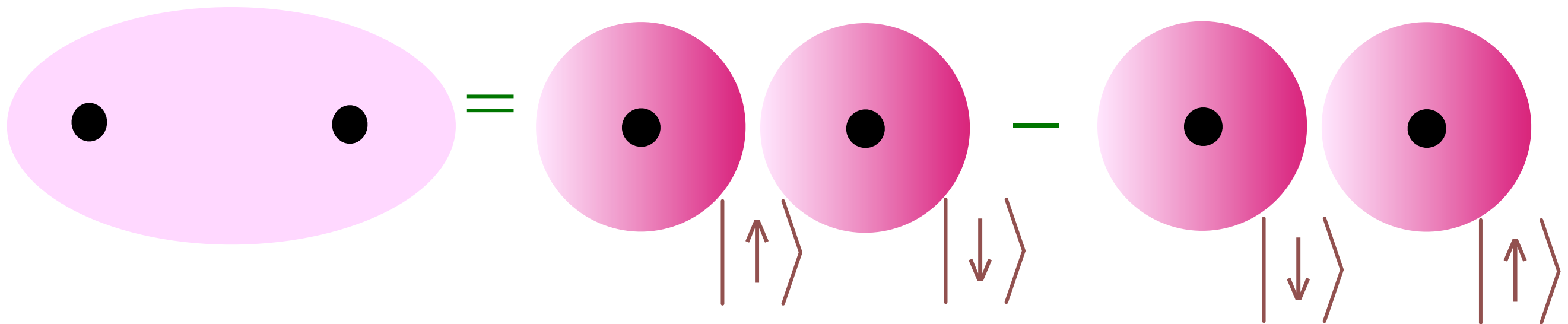


Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



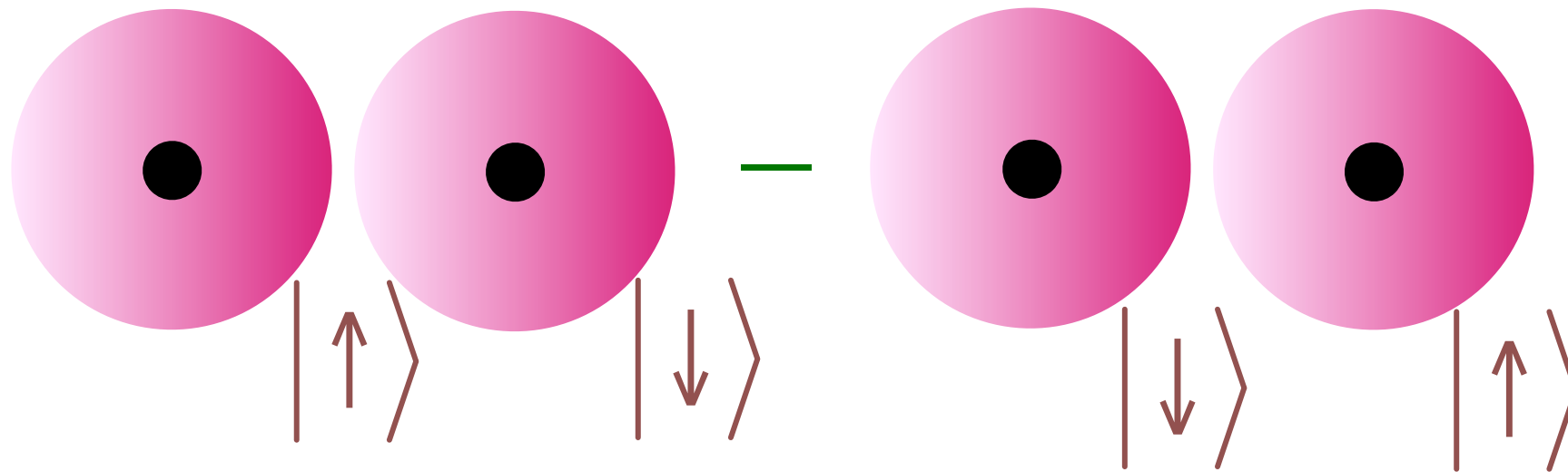
Hydrogen molecule:



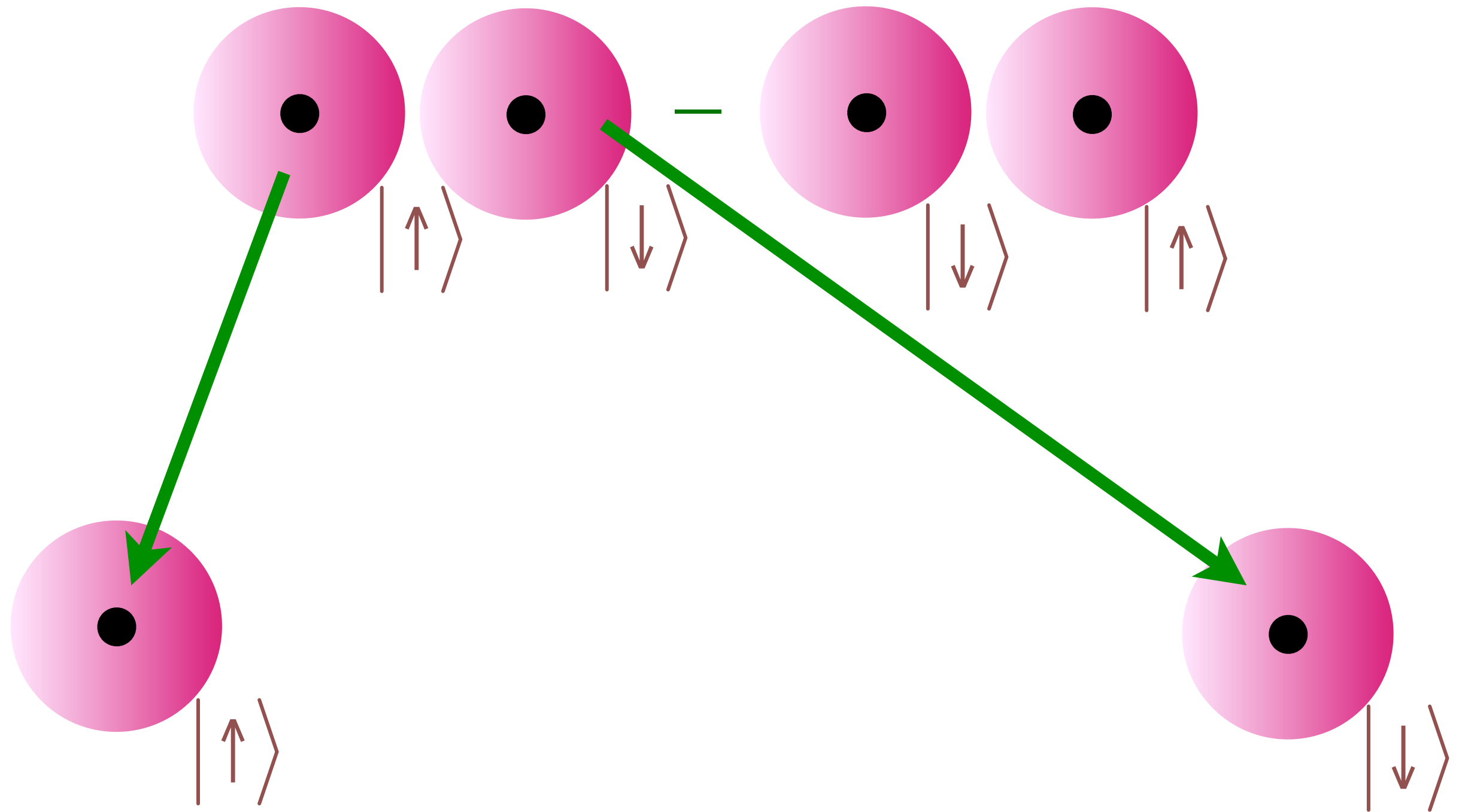
$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Superposition of two electron states leads to non-local
correlations between spins

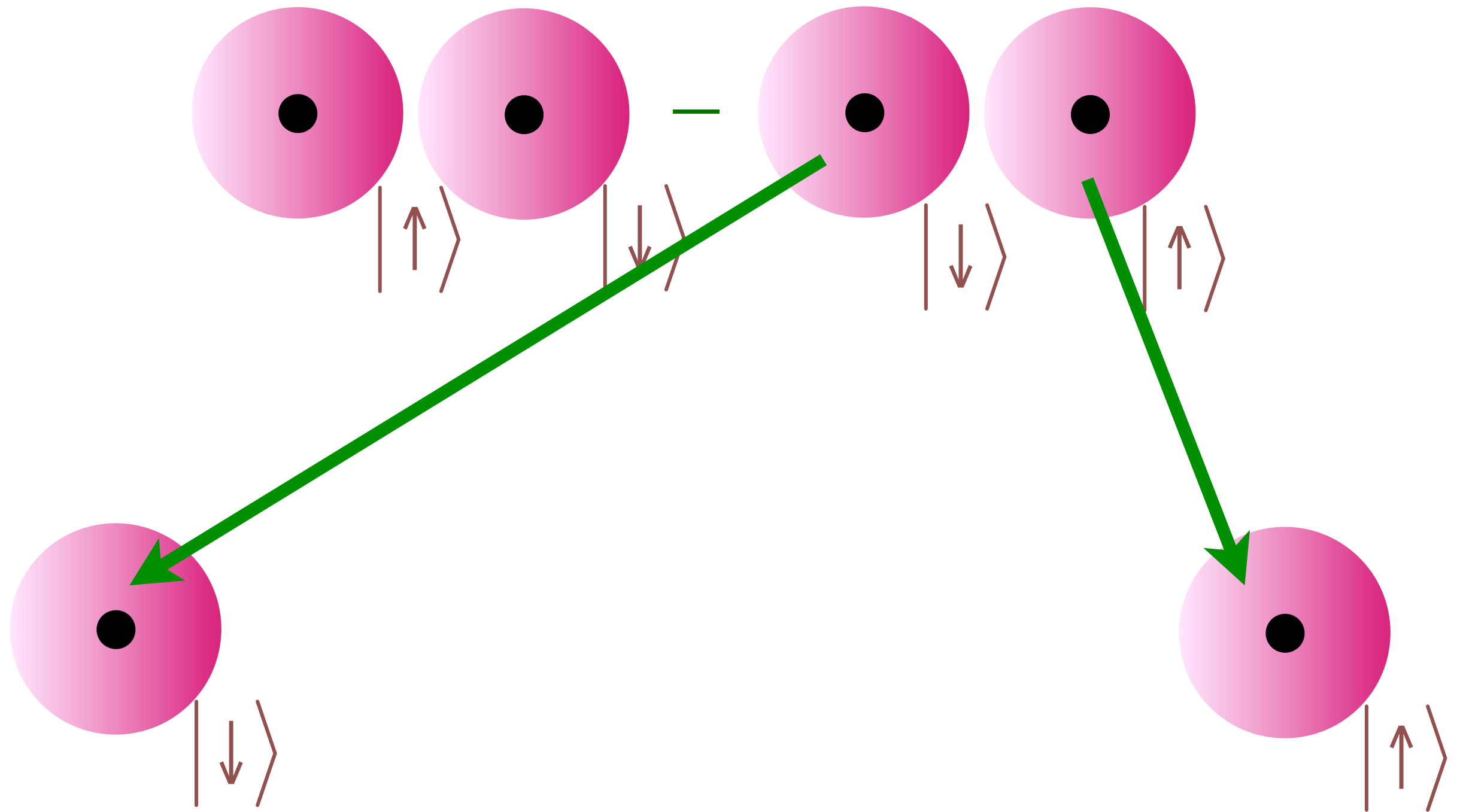
Quantum Entanglement: quantum superposition with more than one particle



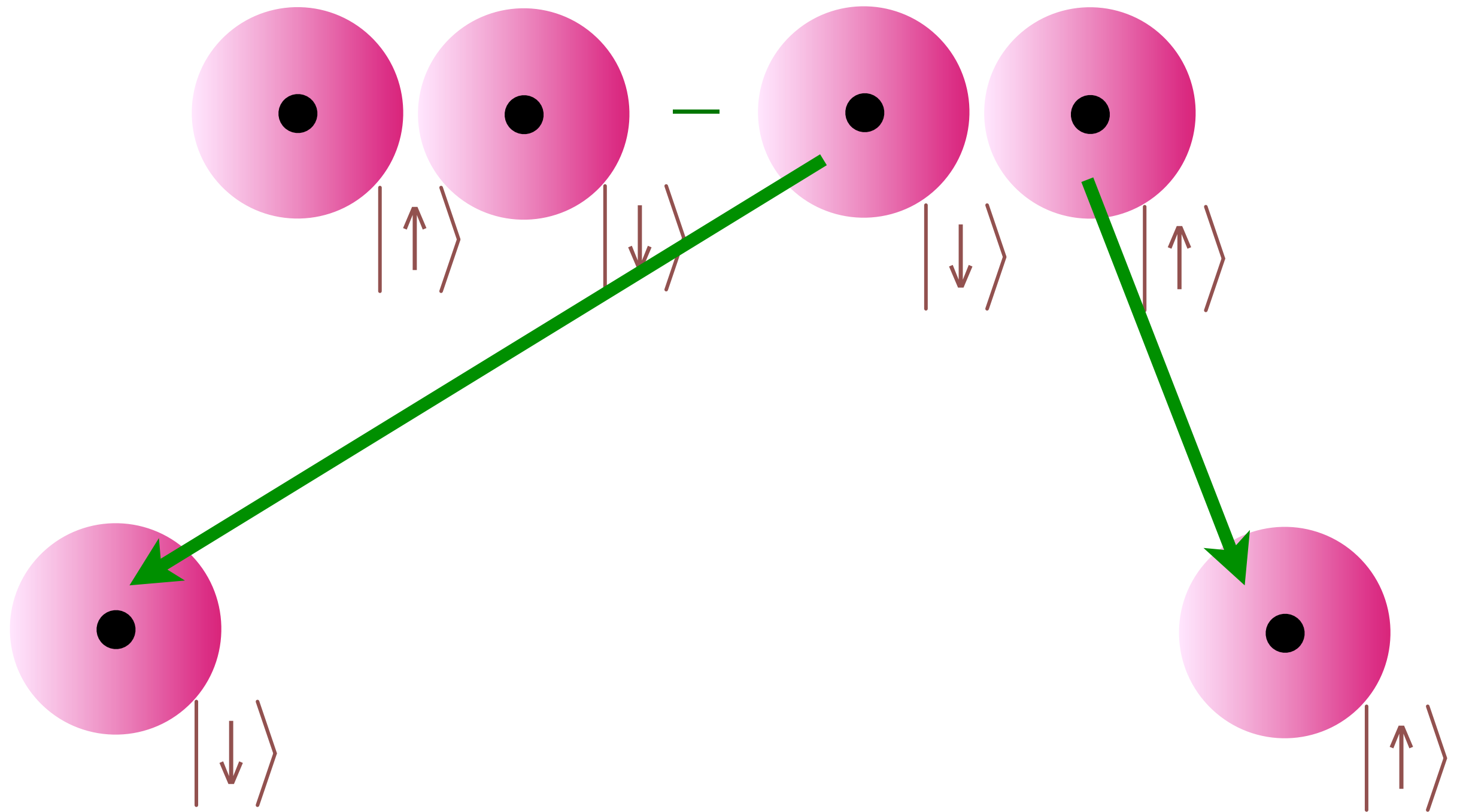
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart

Outline

1. Quantum critical points and string theory
Entanglement and emergent dimensions

2. High temperature superconductors
and strange metals
Holography of compressible quantum phases

Outline

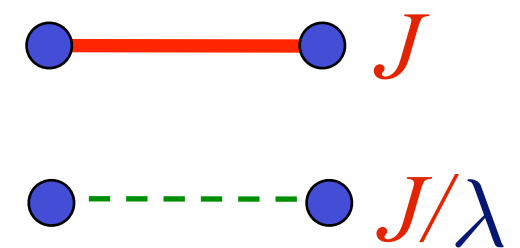
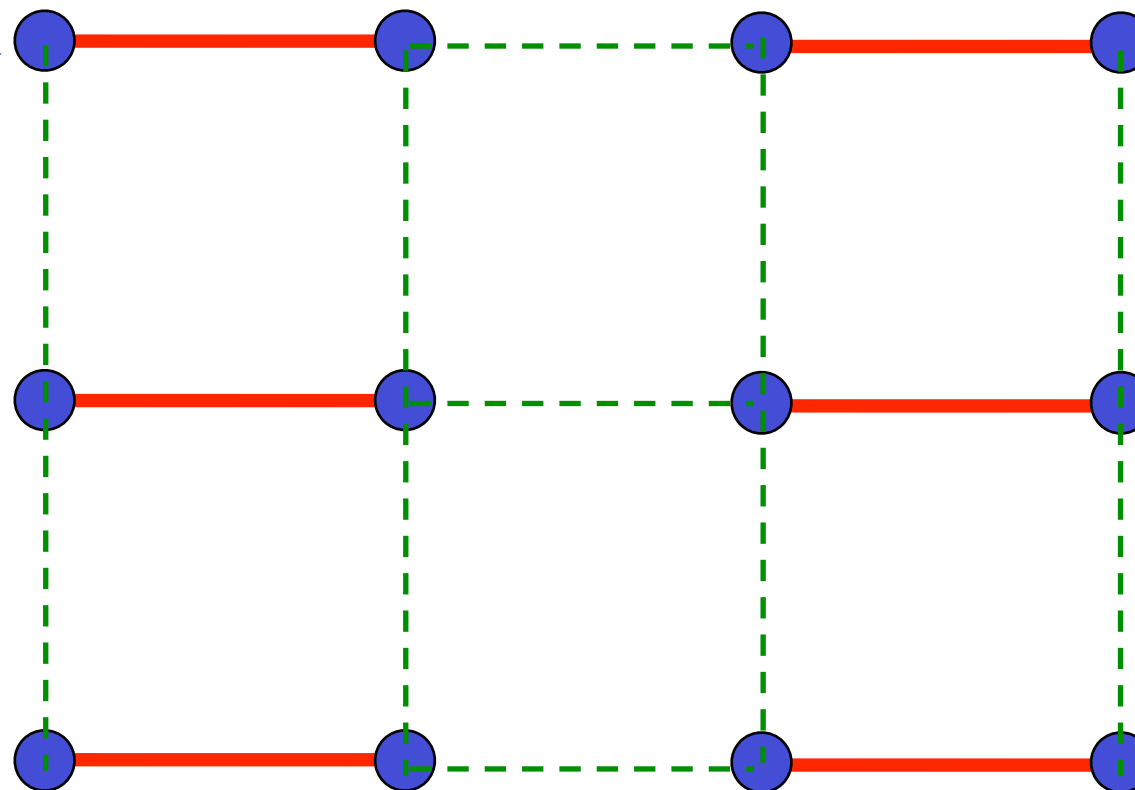
1. Quantum critical points and string theory
Entanglement and emergent dimensions

2. High temperature superconductors
and strange metals
Holography of compressible quantum phases

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

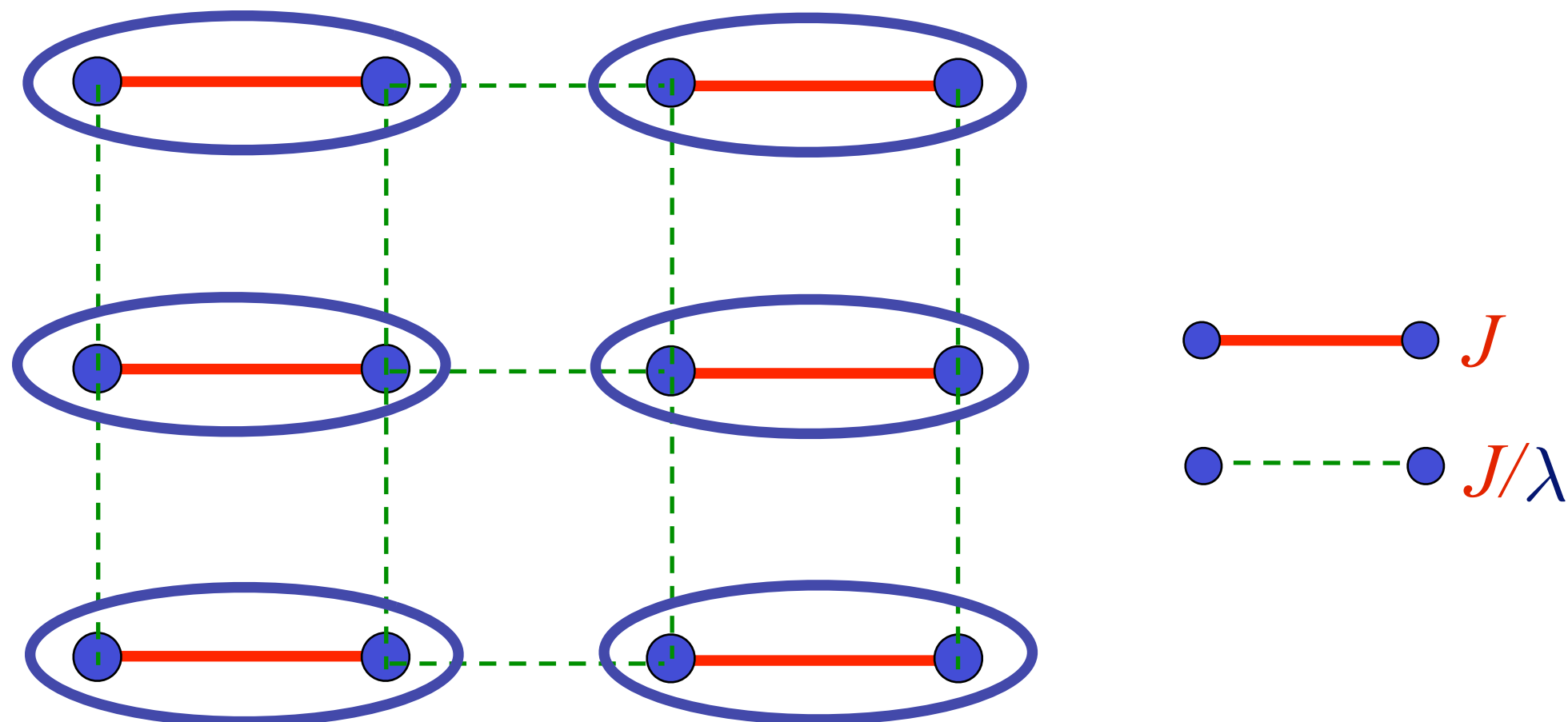
$S=1/2$
spins



Examine ground state as a function of λ

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

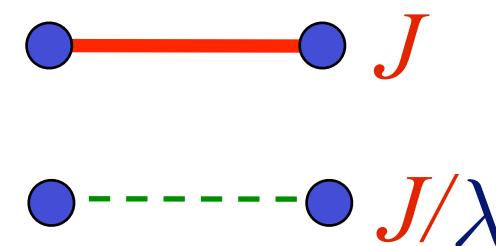
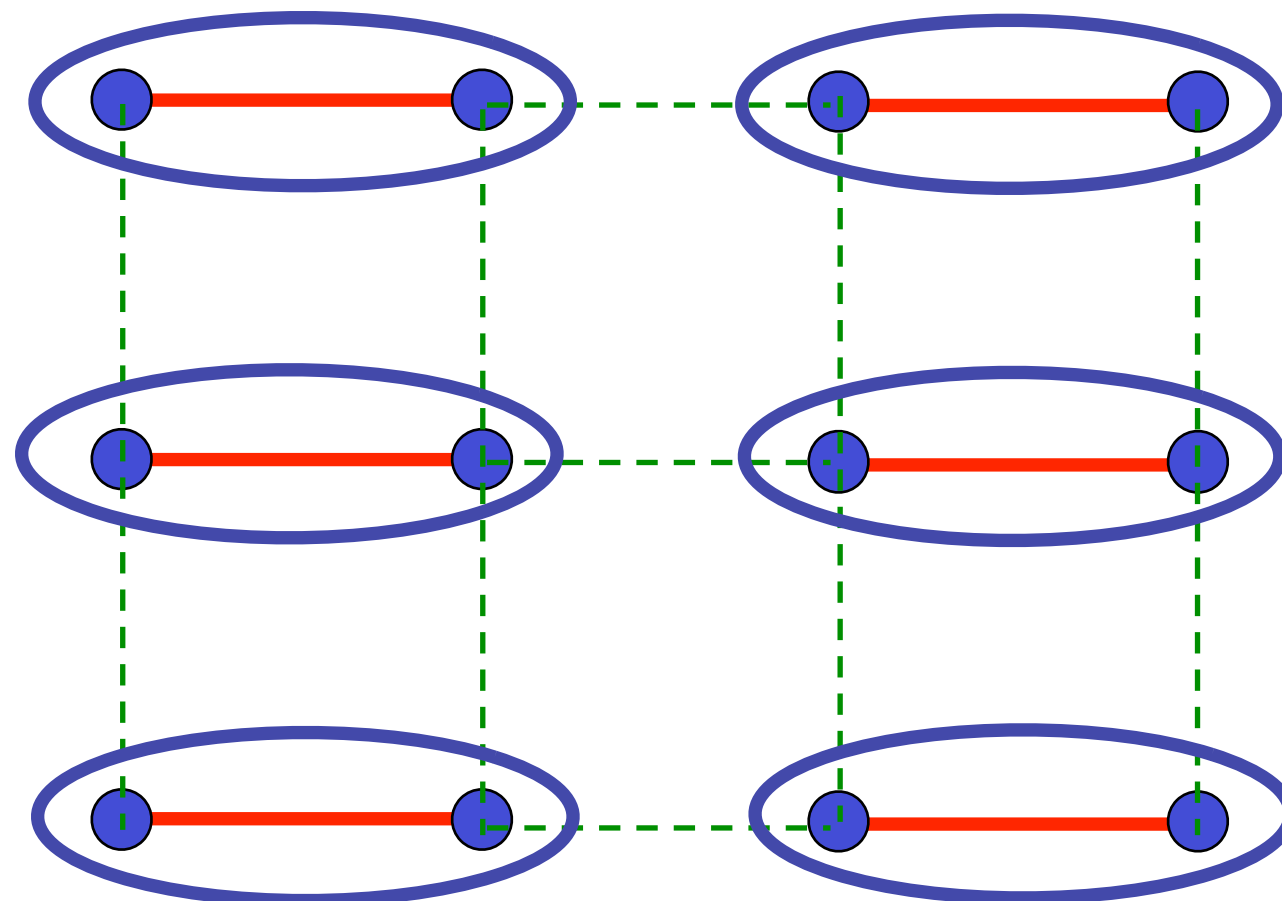


$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

At large λ ground state is a “quantum paramagnet” with spins locked in valence bond singlets

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

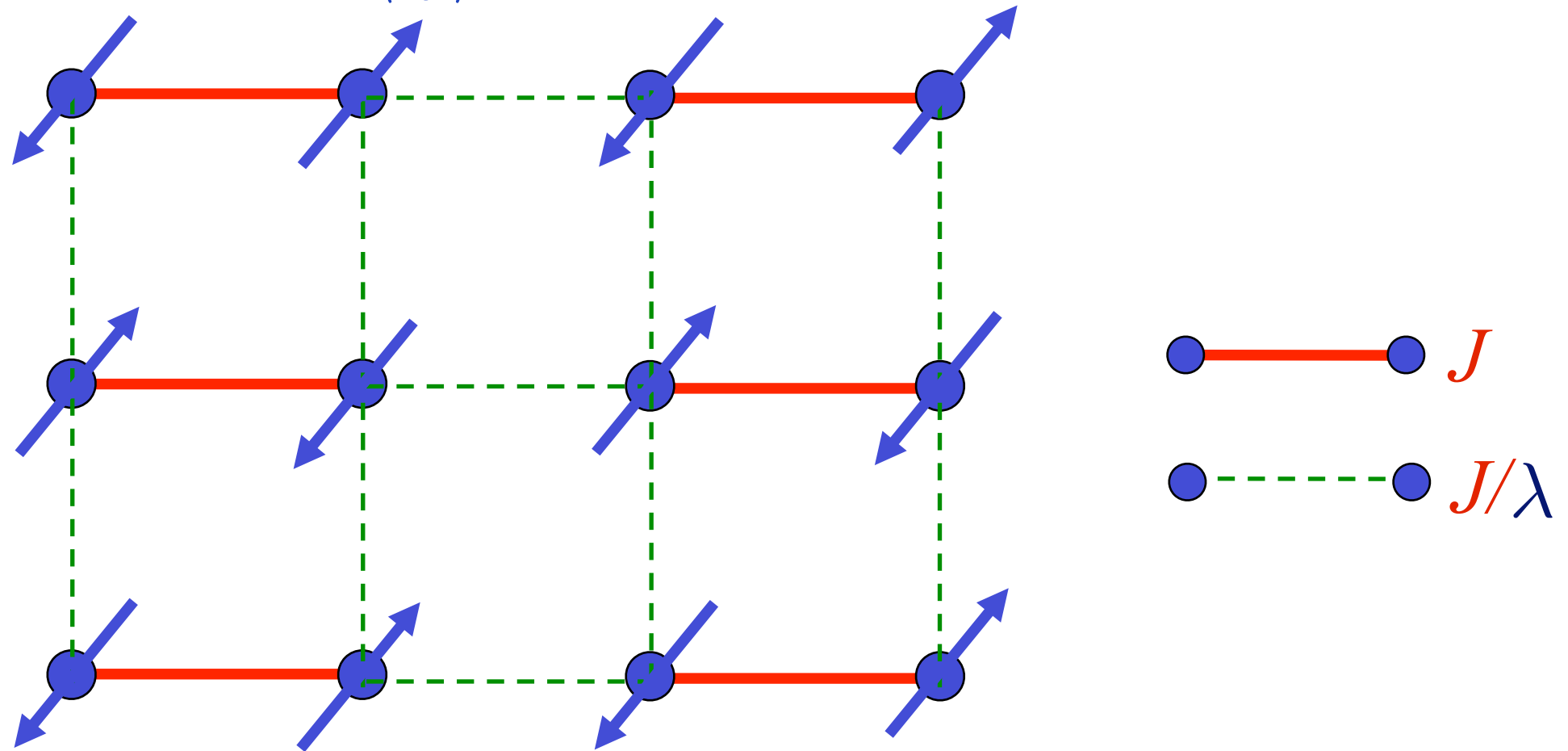


$$\text{[Diagram of two spins in a blue oval]} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Nearest-neighbor spins are “entangled” with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Square lattice antiferromagnet

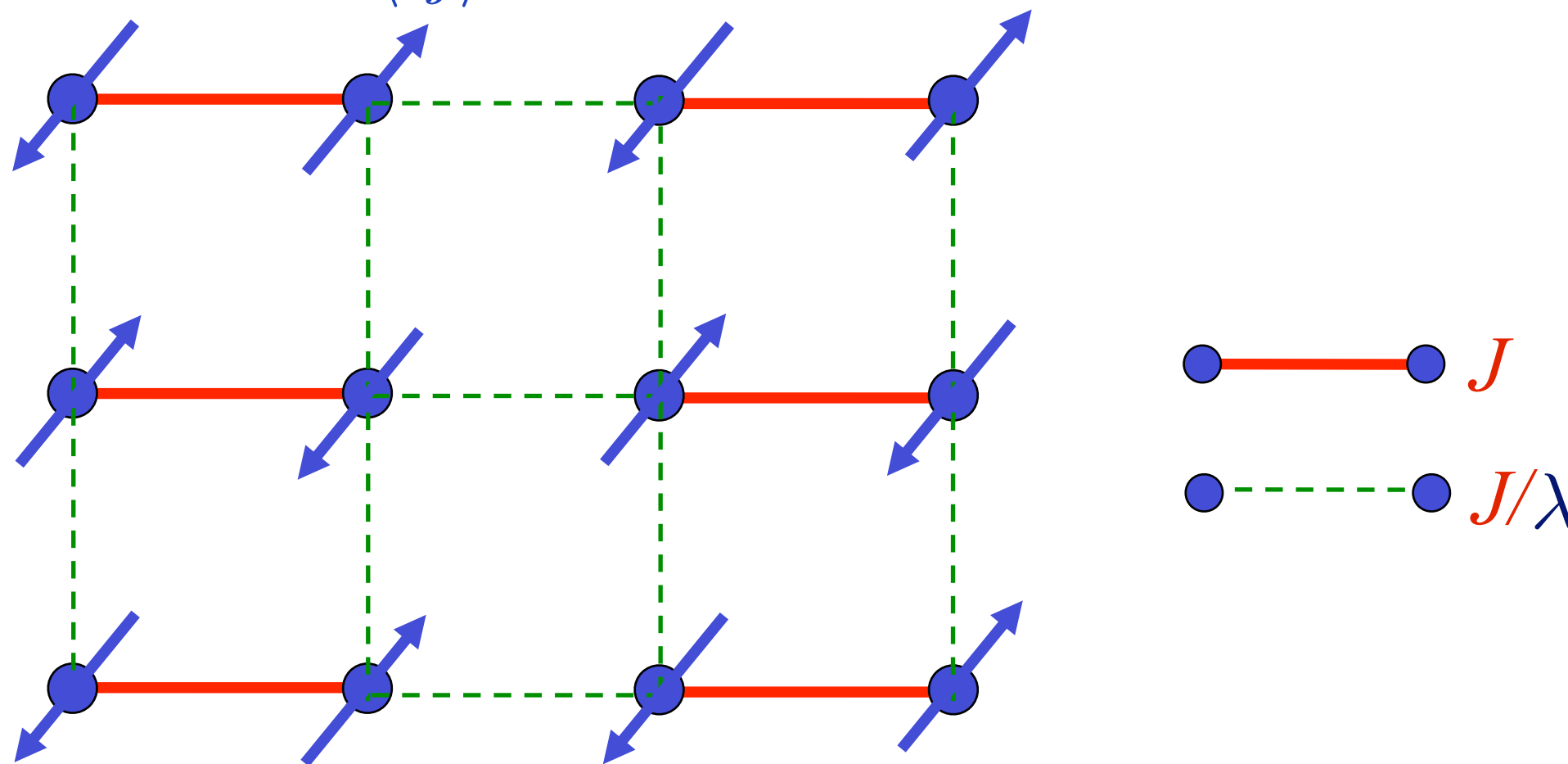
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic (“Néel”) order, and the spins align in a checkerboard pattern

Square lattice antiferromagnet

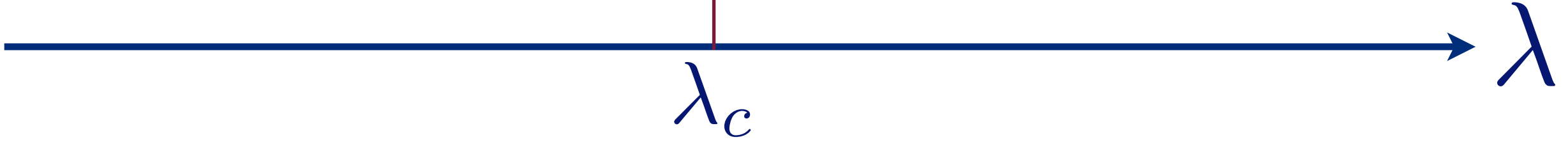
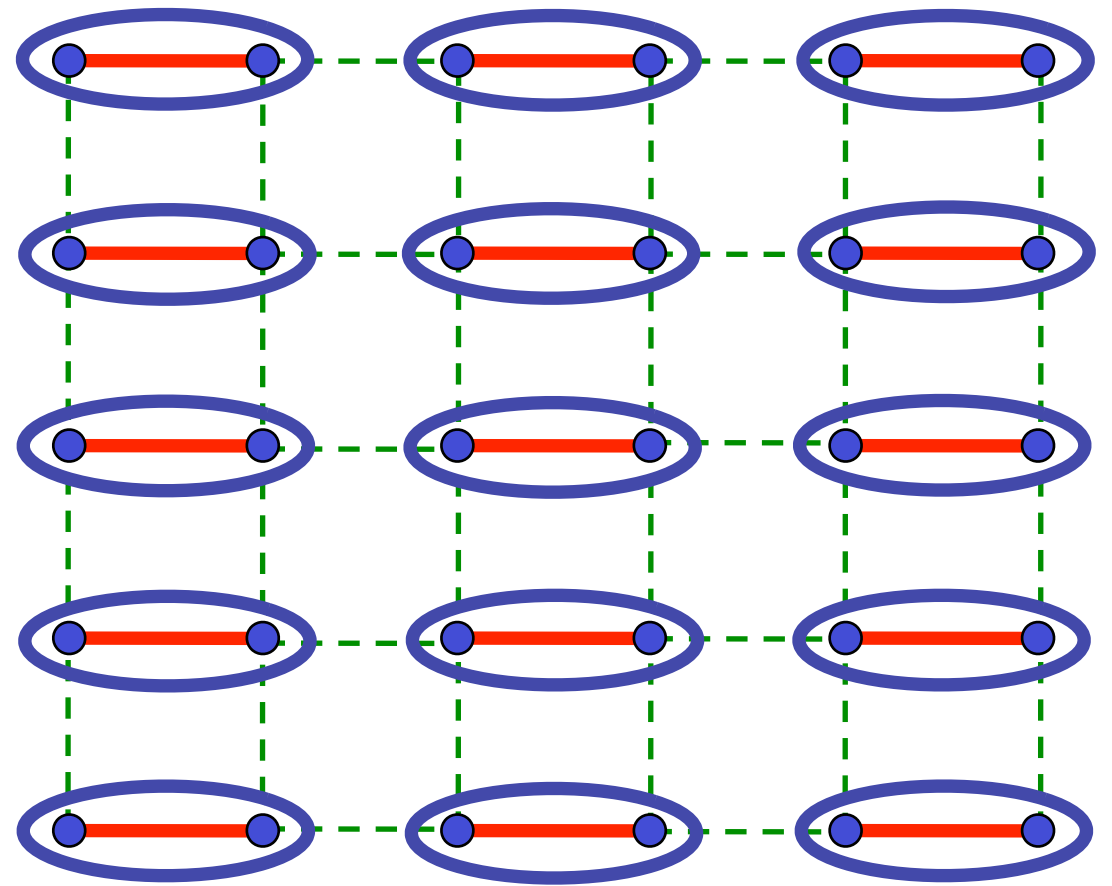
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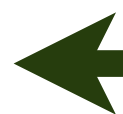
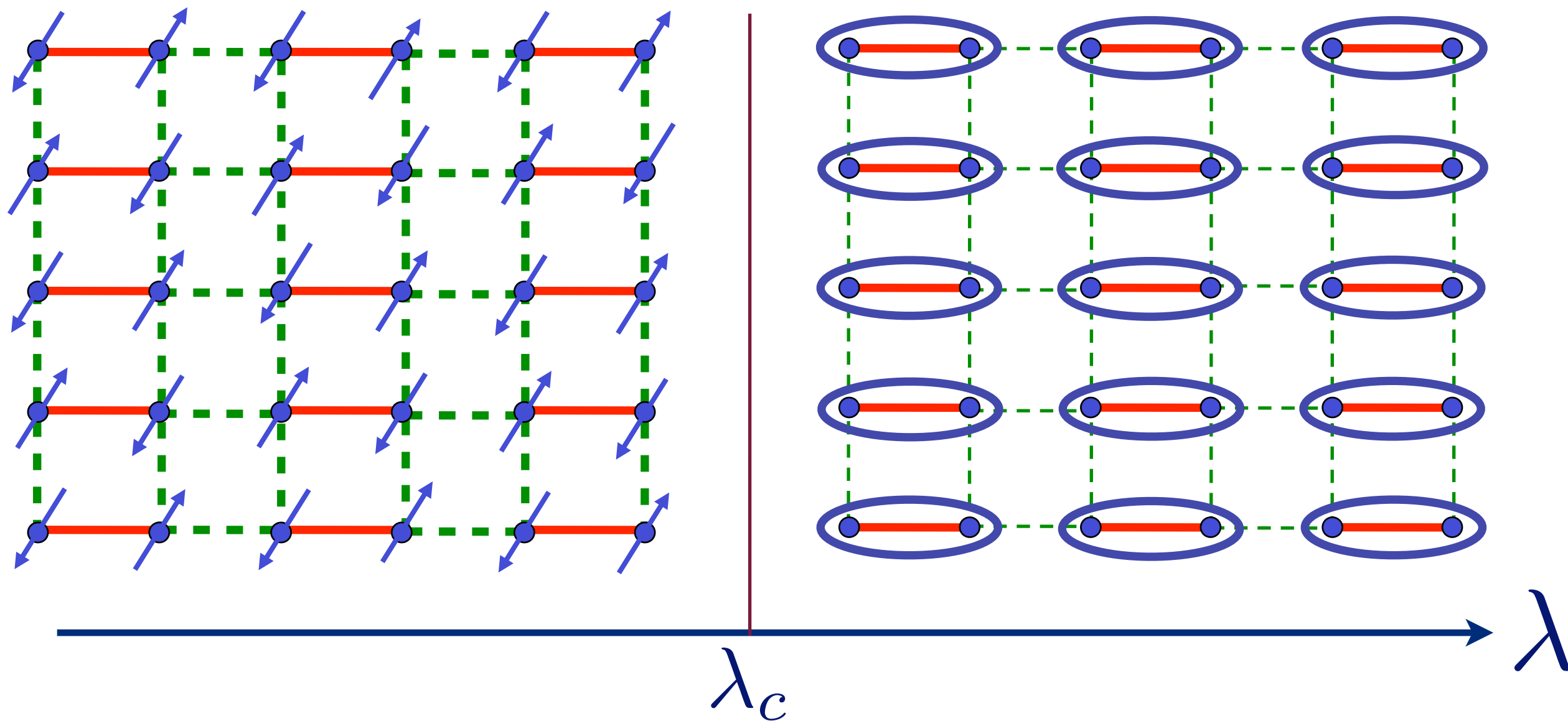
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No EPR pairs

$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



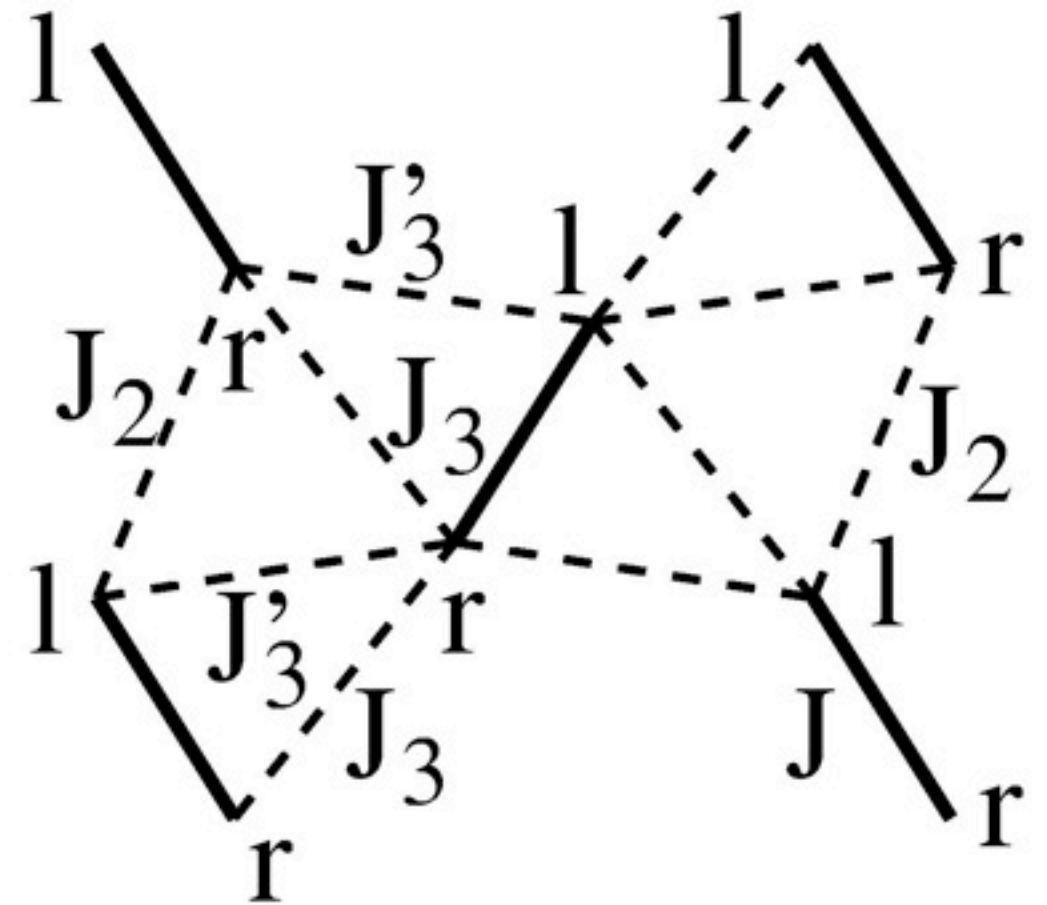
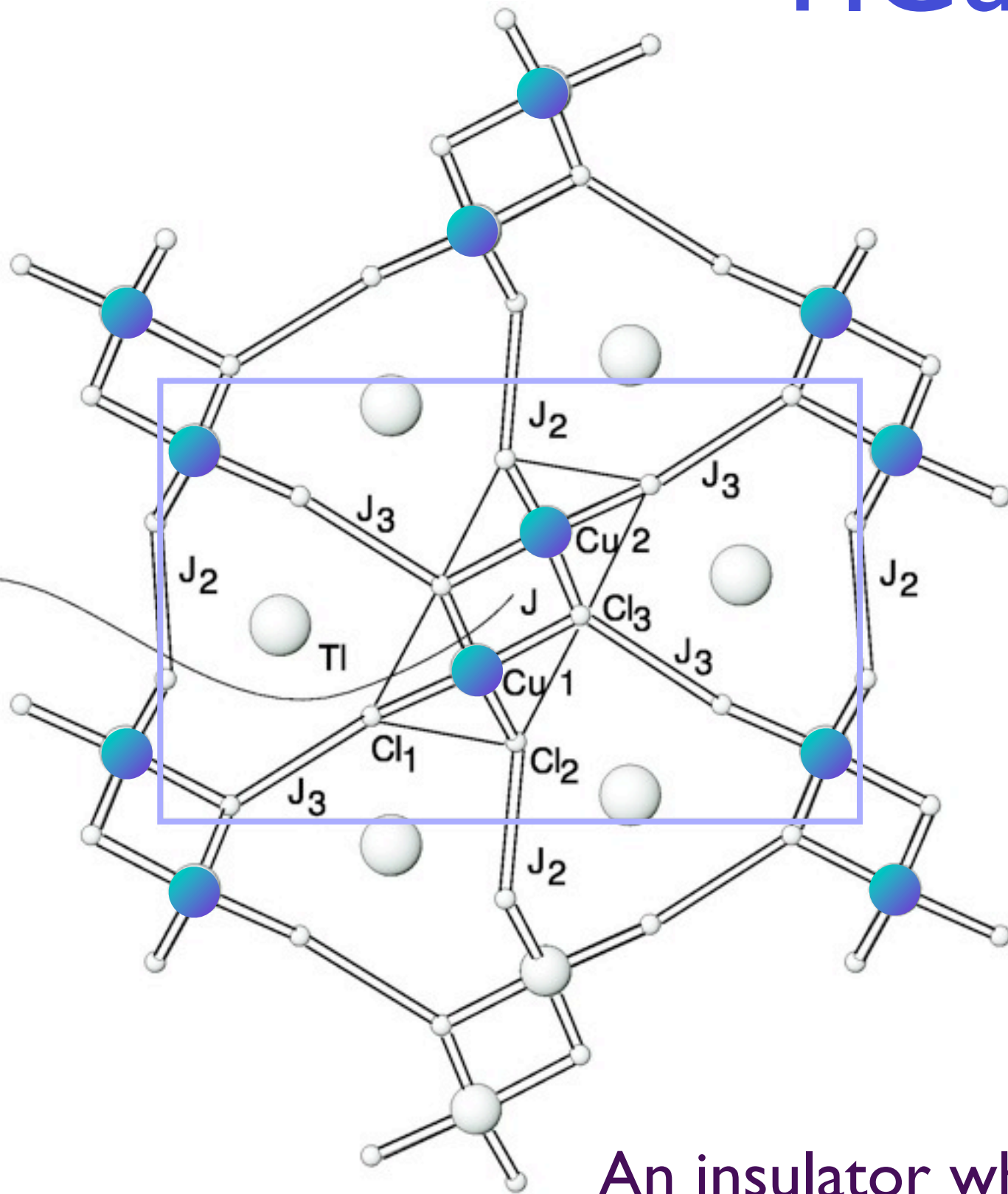
$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Pressure in TlCuCl_3

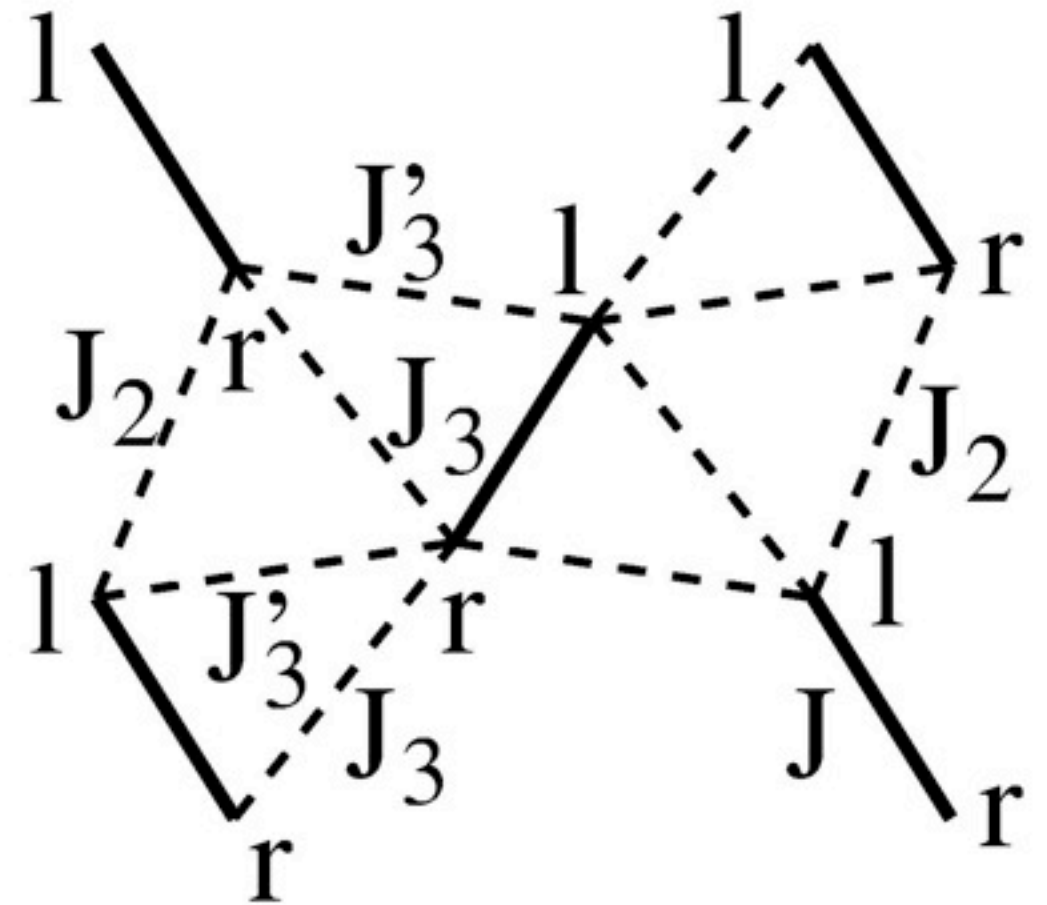
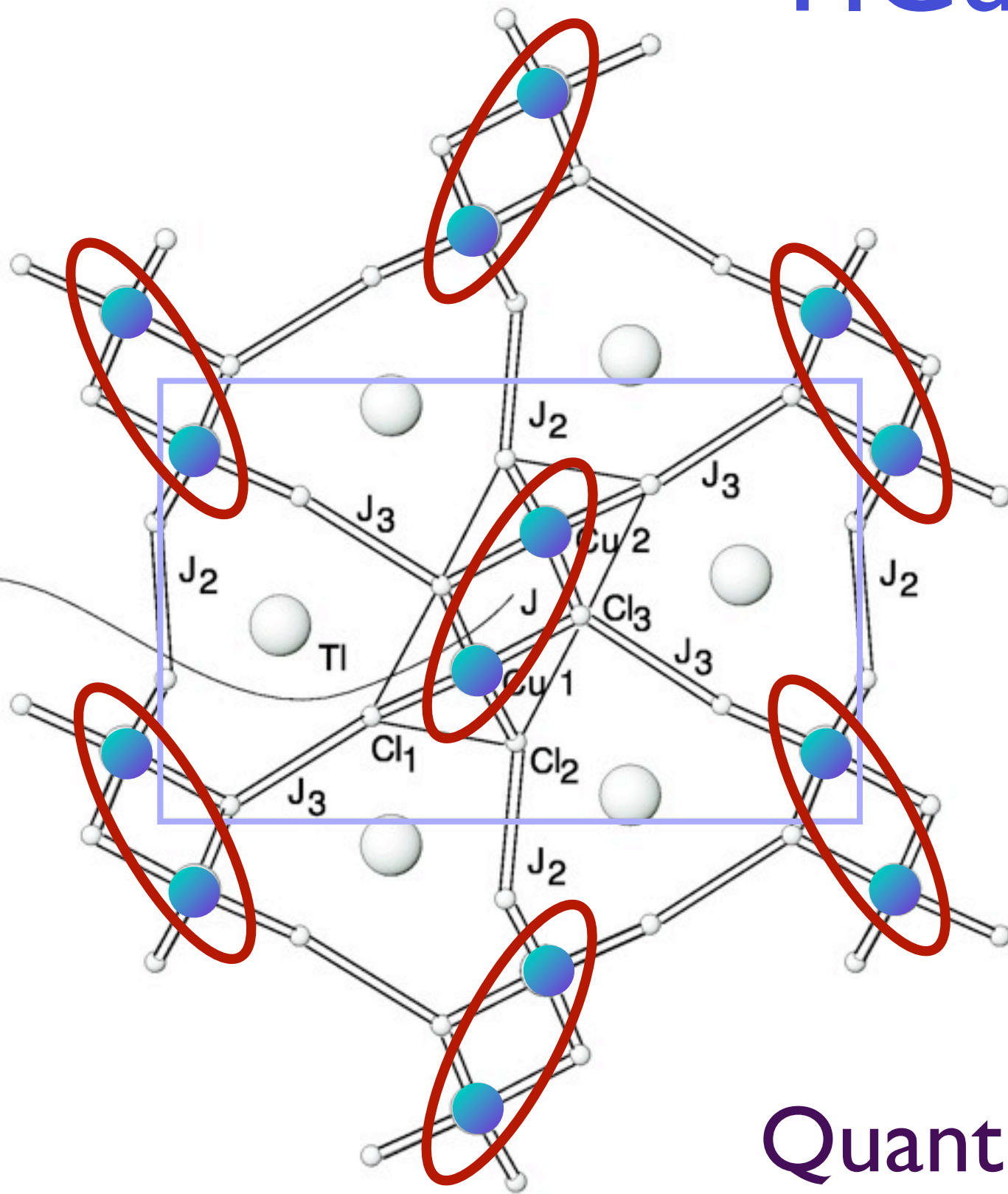
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
Journal of the Physical Society of Japan, **73**, 1446 (2004).

TlCuCl₃



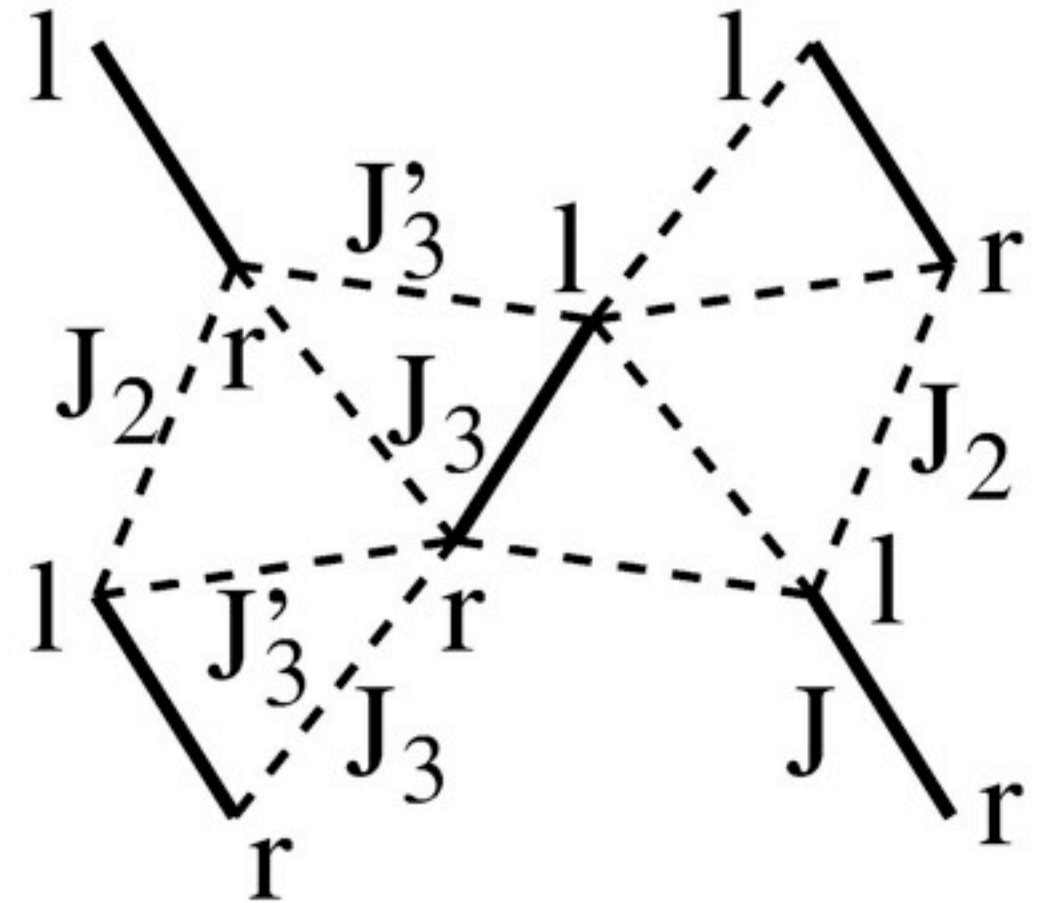
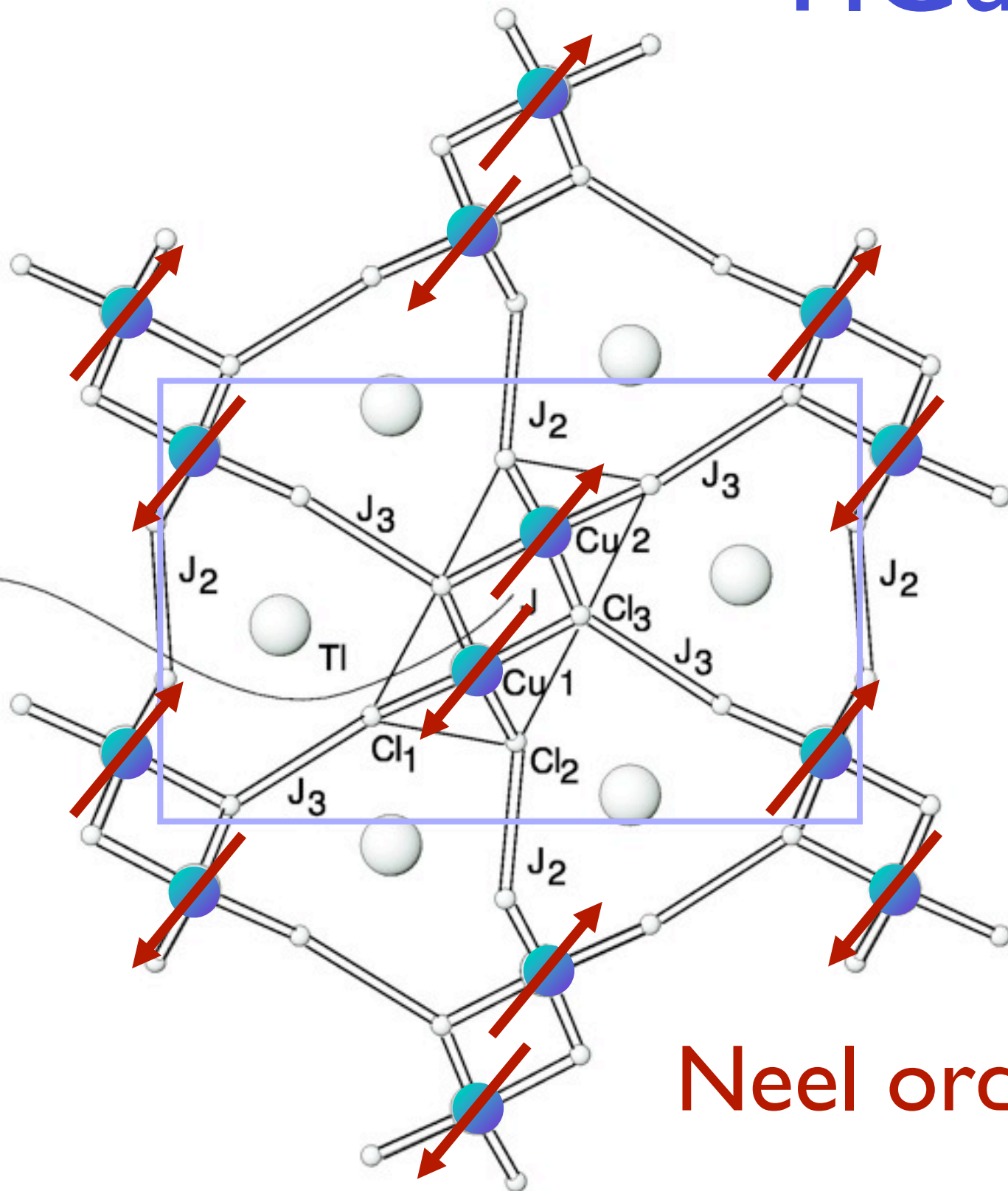
An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

TlCuCl₃



Quantum paramagnet at ambient pressure

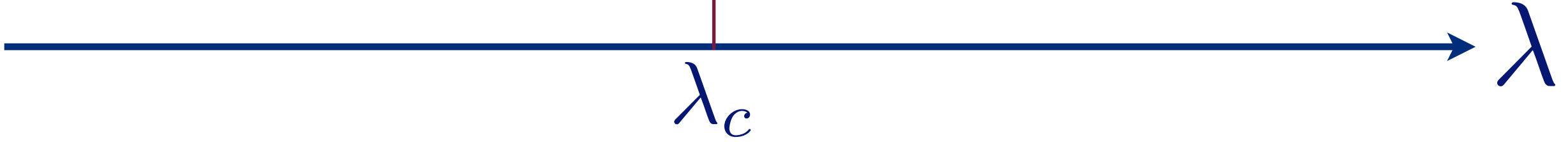
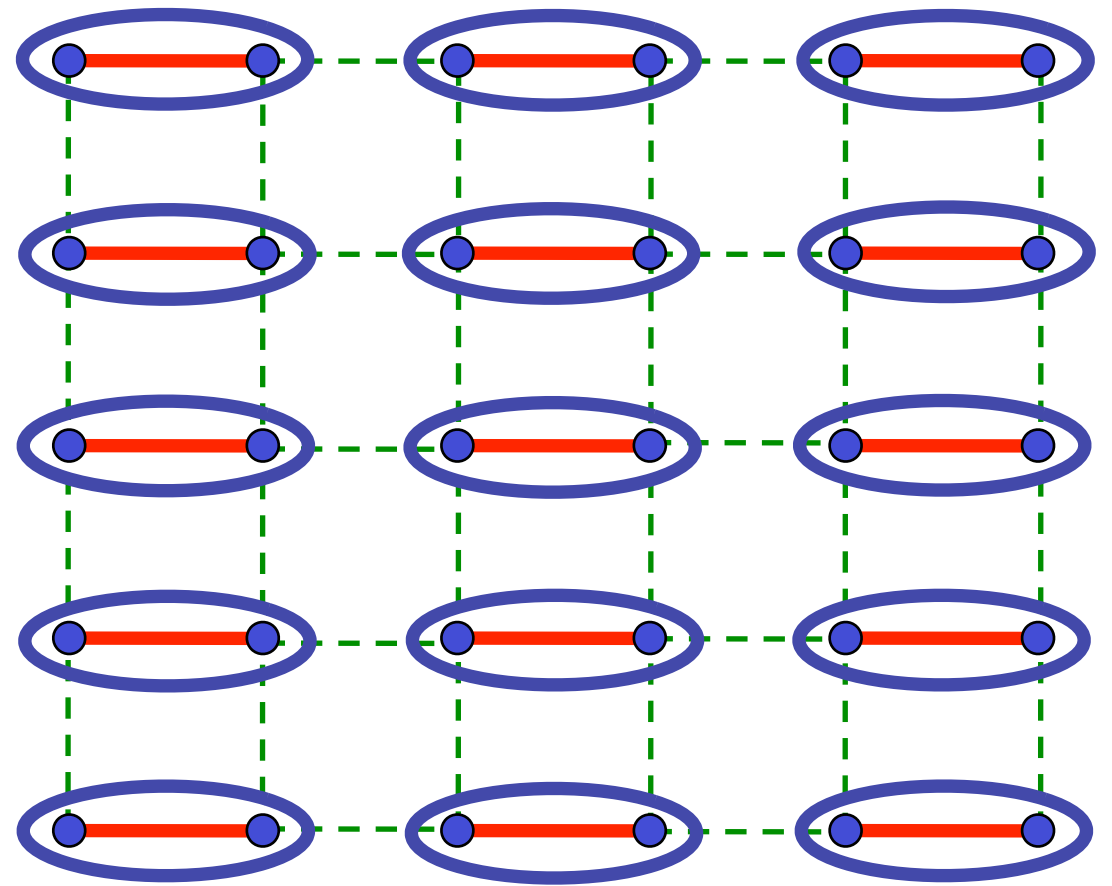
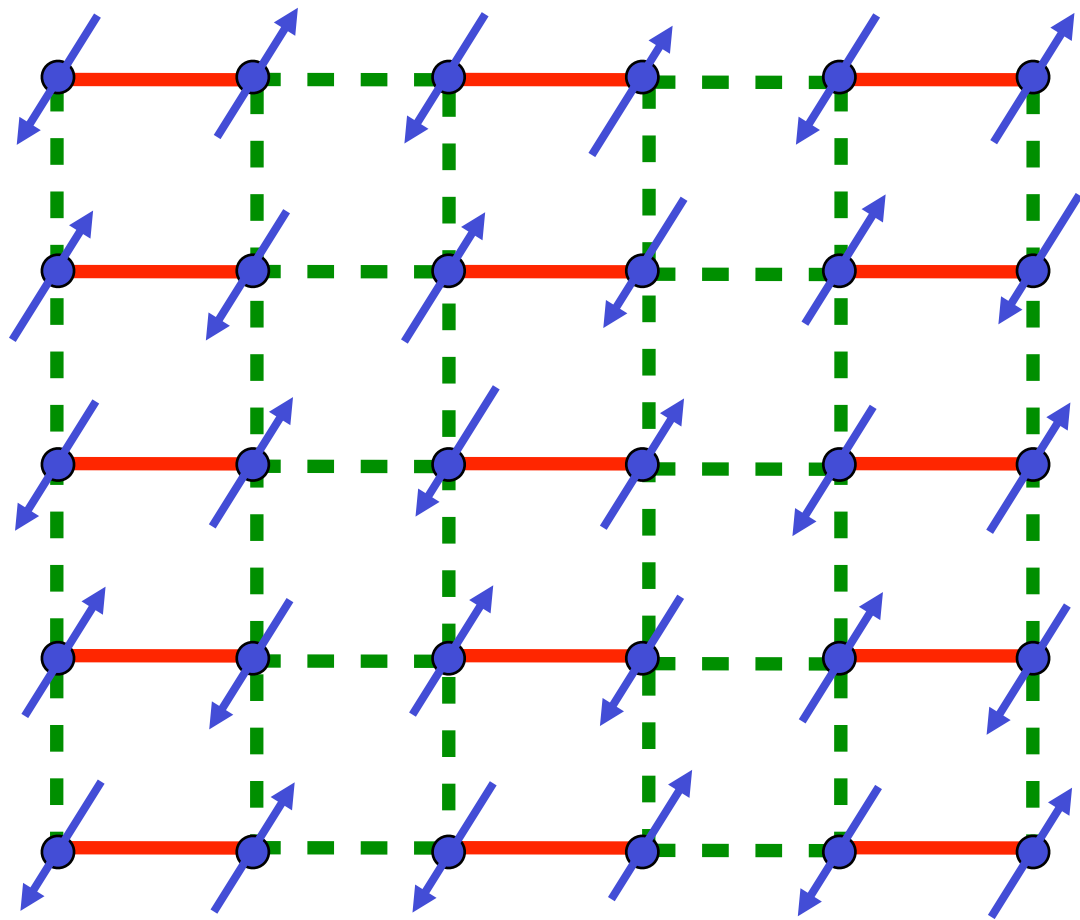
TlCuCl₃



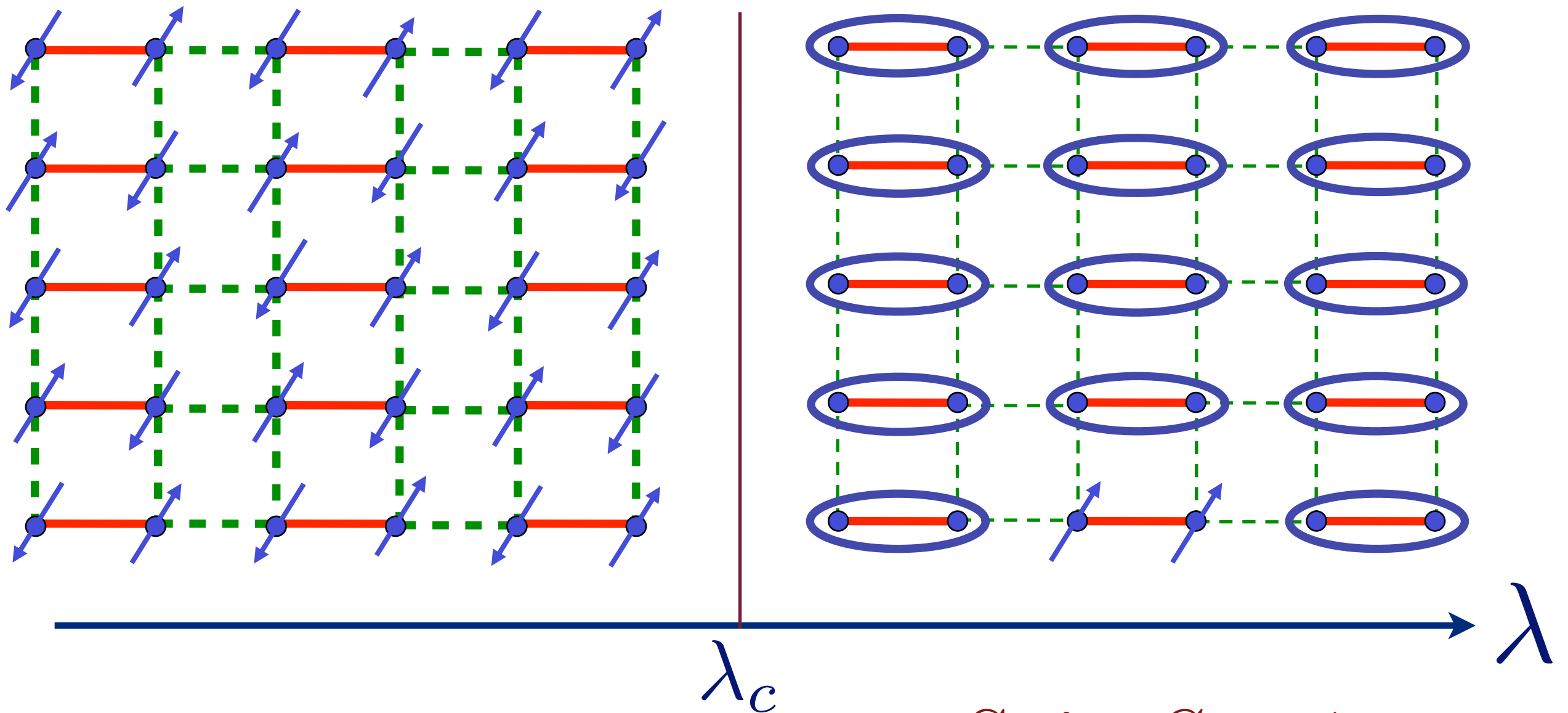
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
Journal of the Physical Society of Japan, **73**, 1446 (2004).

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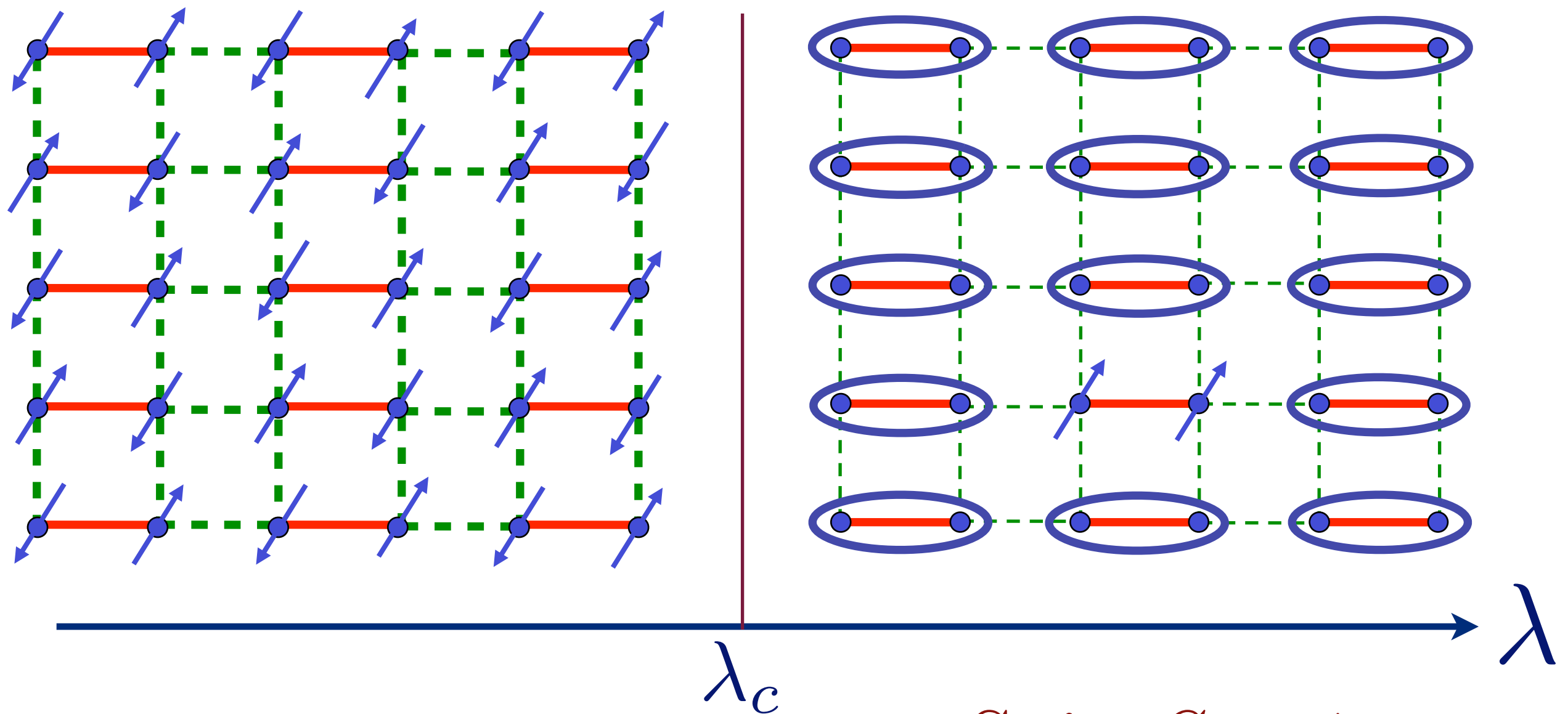


Excitation spectrum in the paramagnetic phase



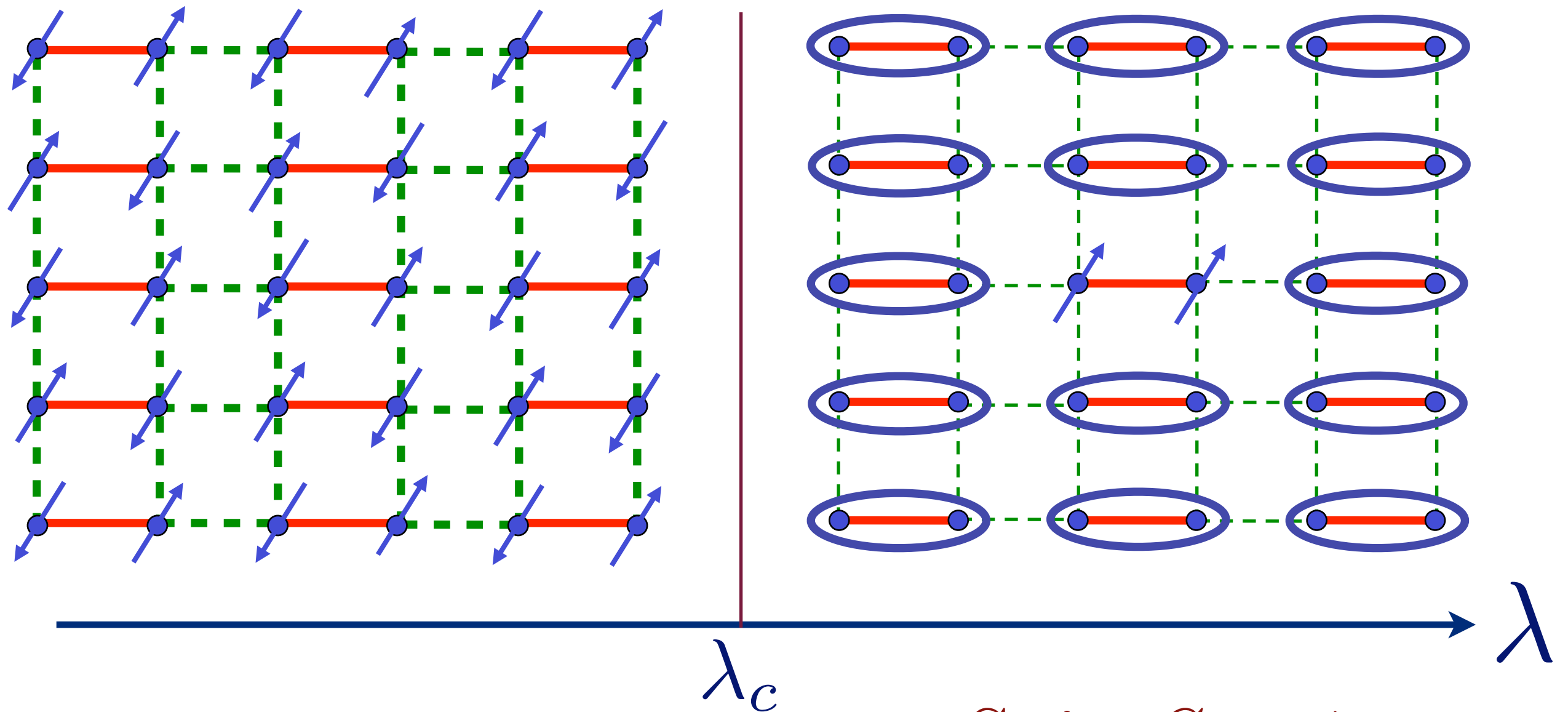
Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



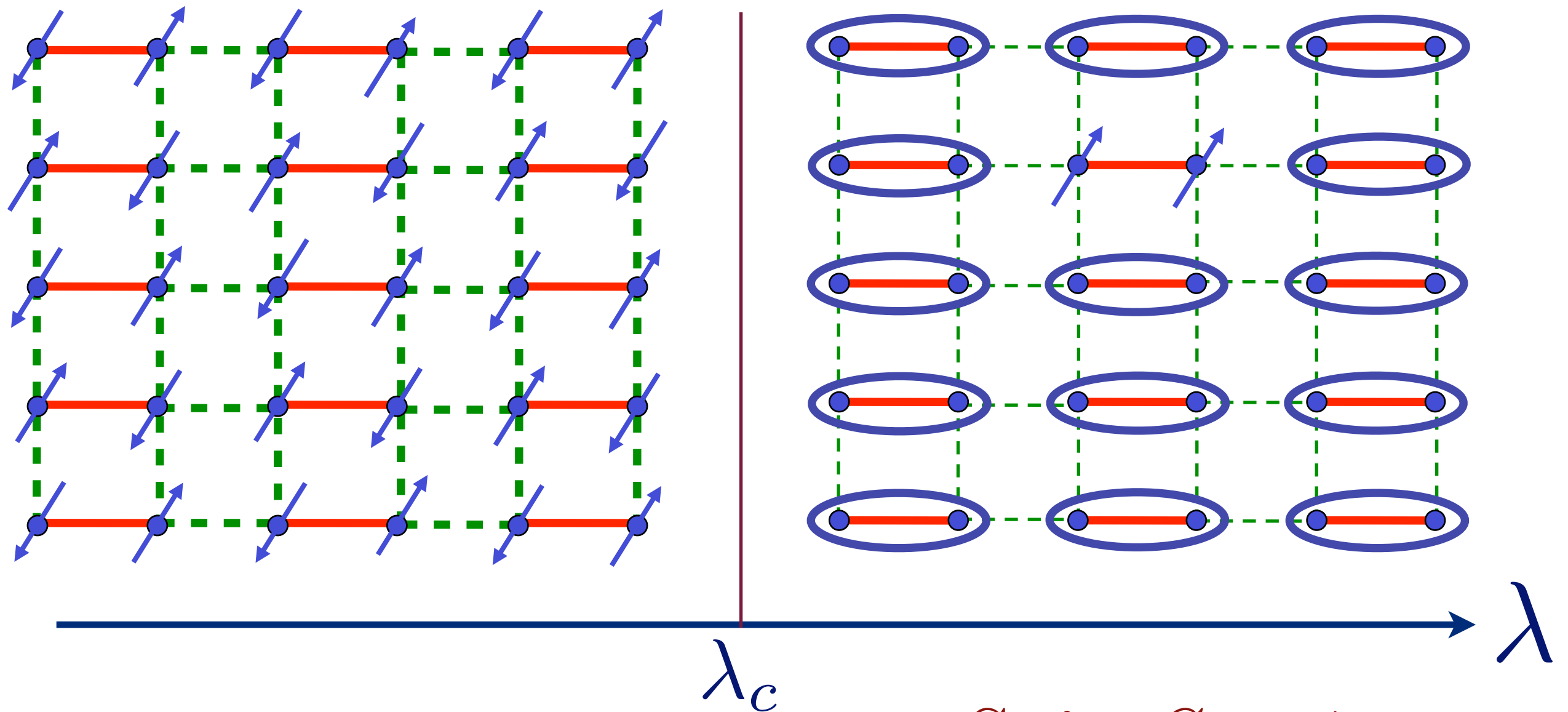
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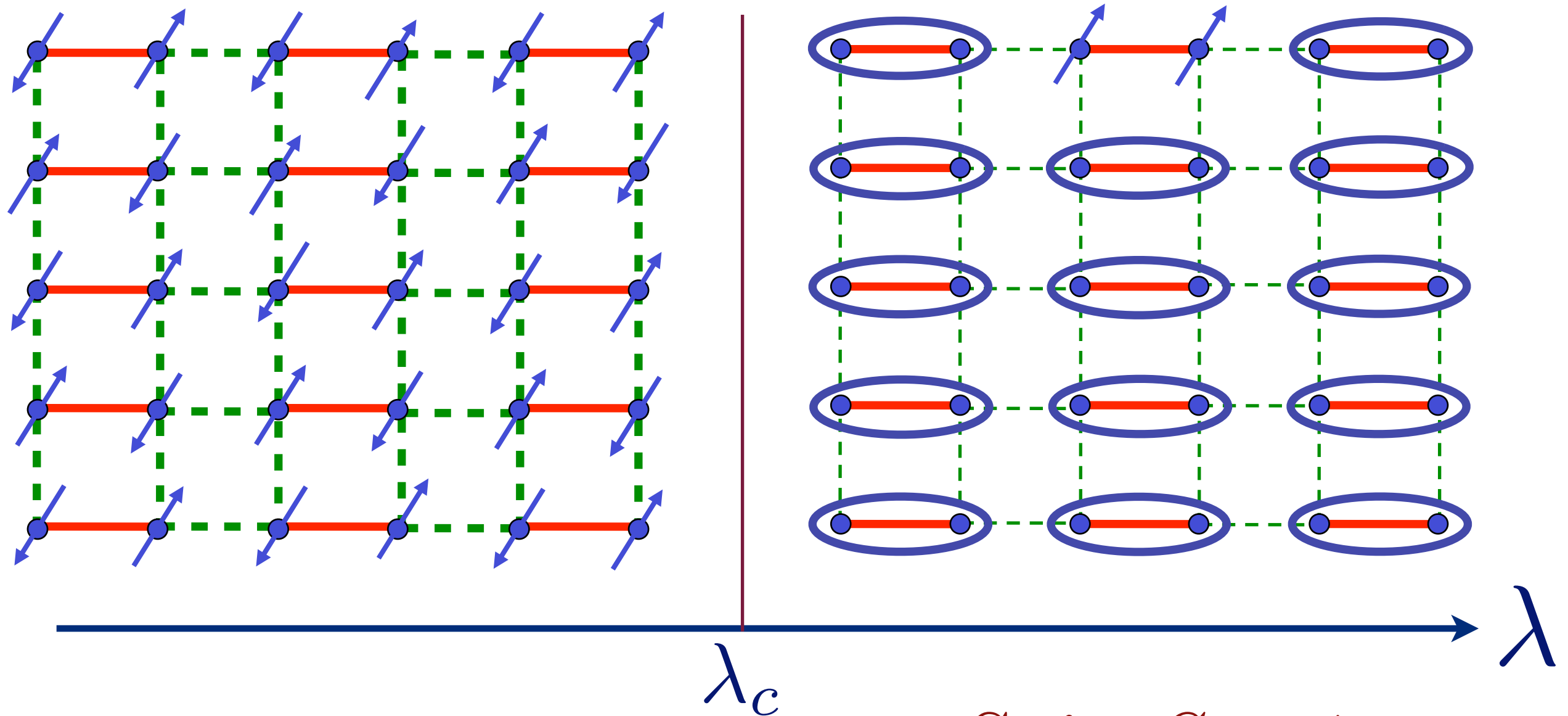
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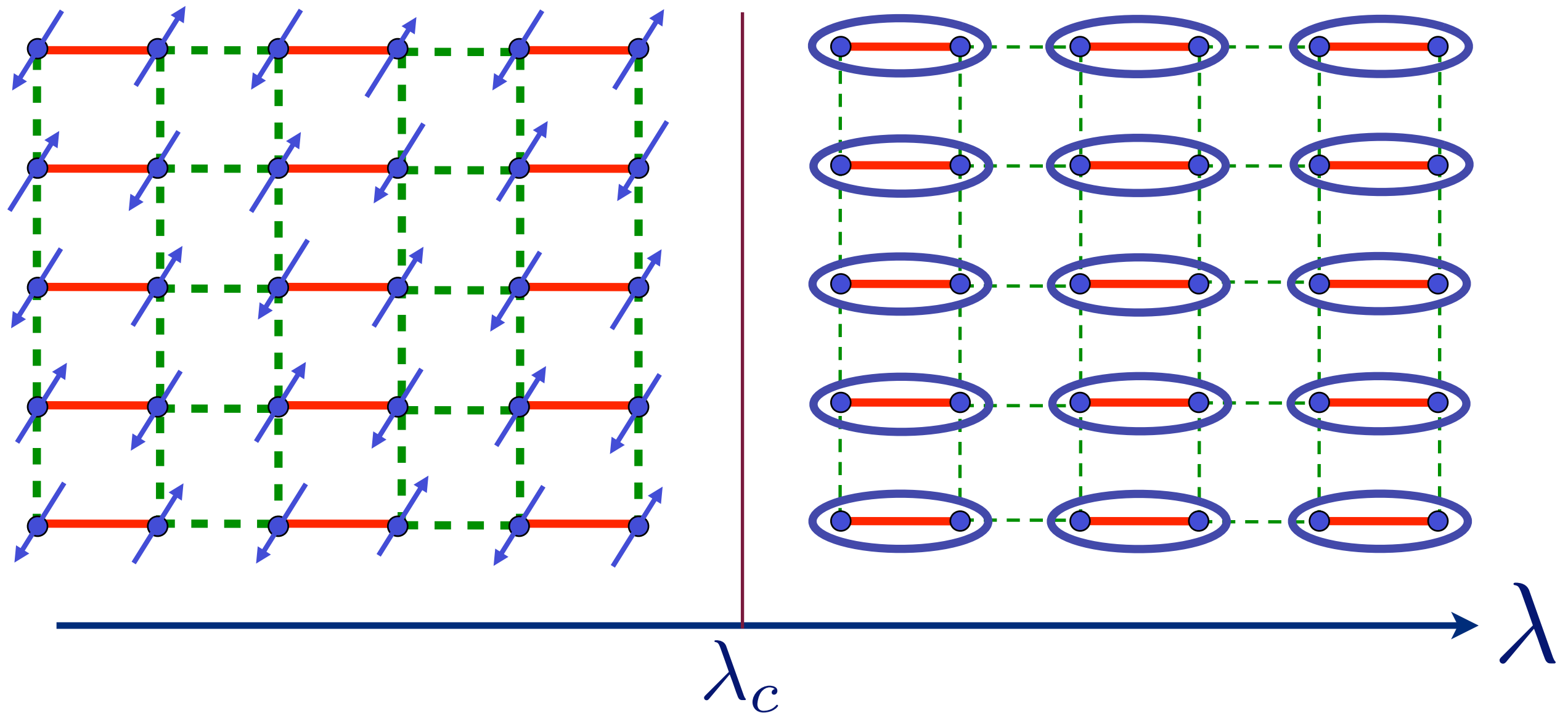
Spin $S = 1$
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Excitation spectrum in the paramagnetic phase



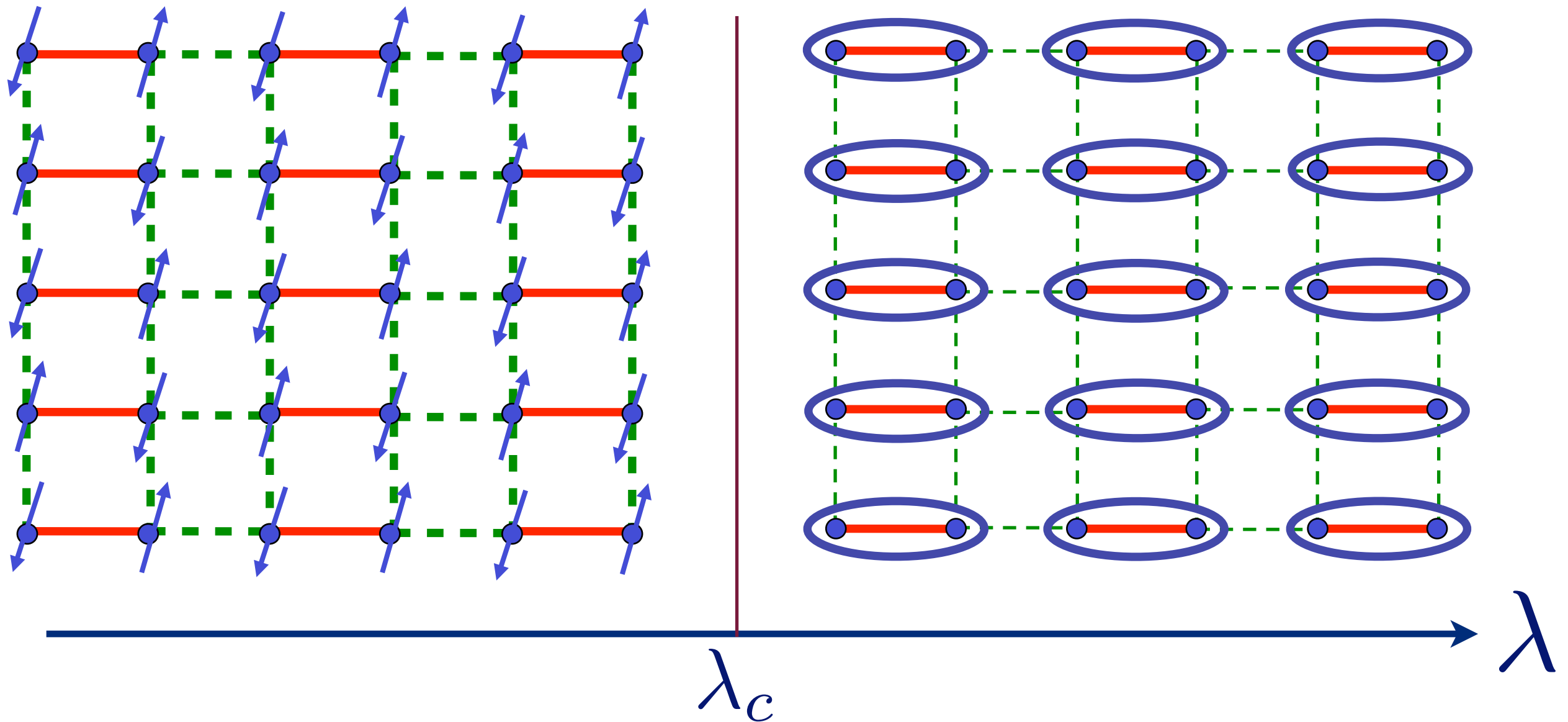
Spin $S = 1$
“triplon”

Excitation spectrum in the Néel phase



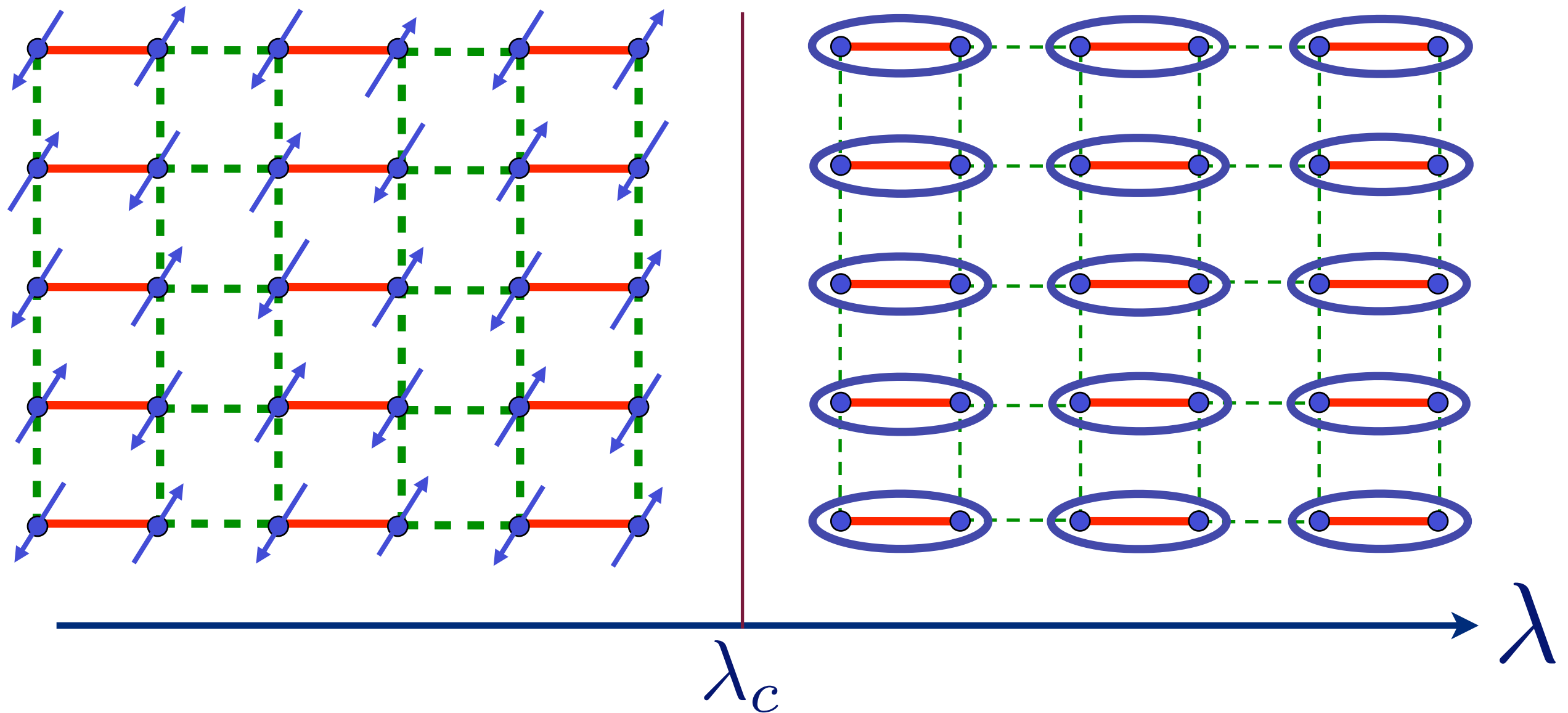
Spin waves

Excitation spectrum in the Néel phase



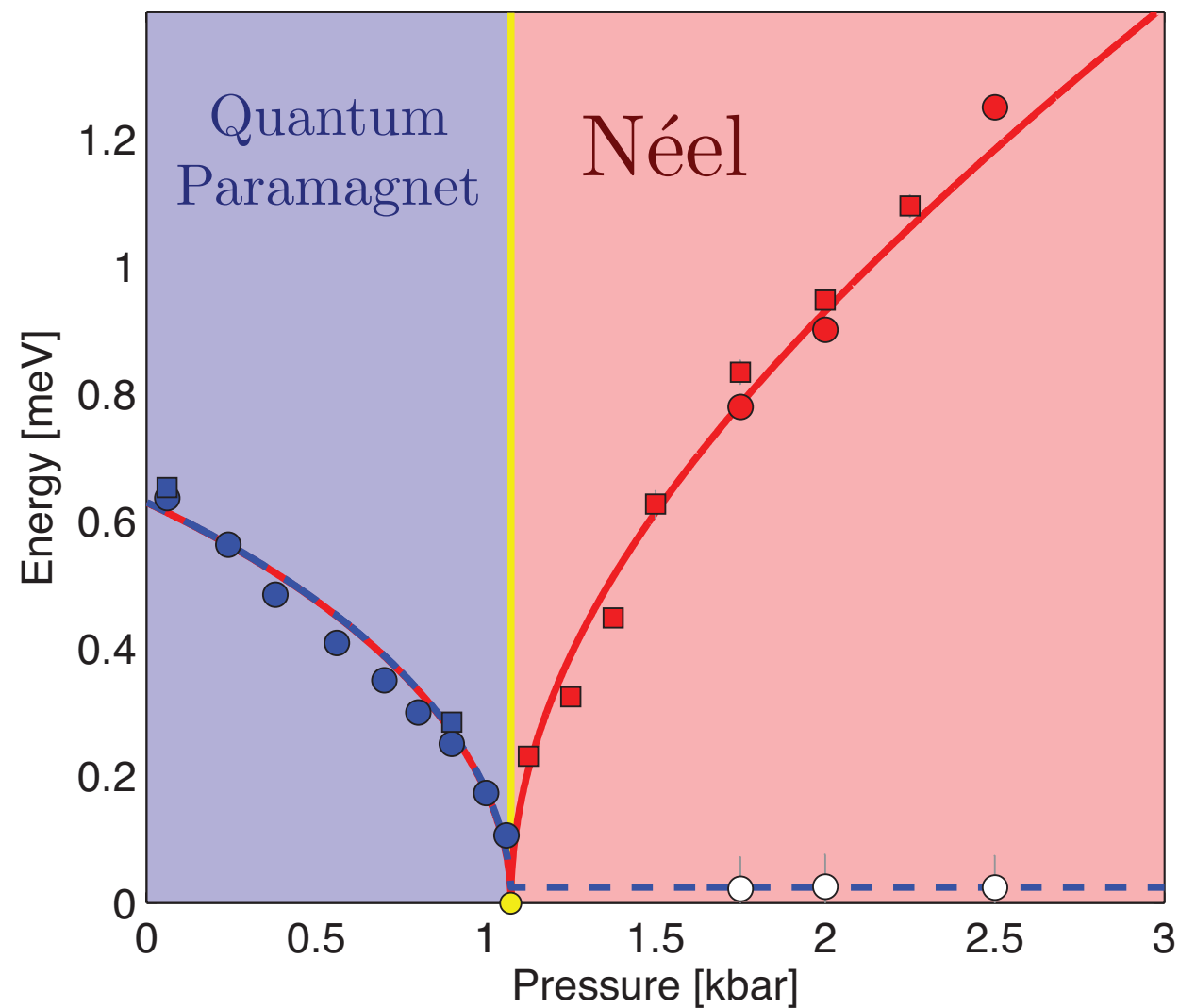
Spin waves

Excitation spectrum in the Néel phase



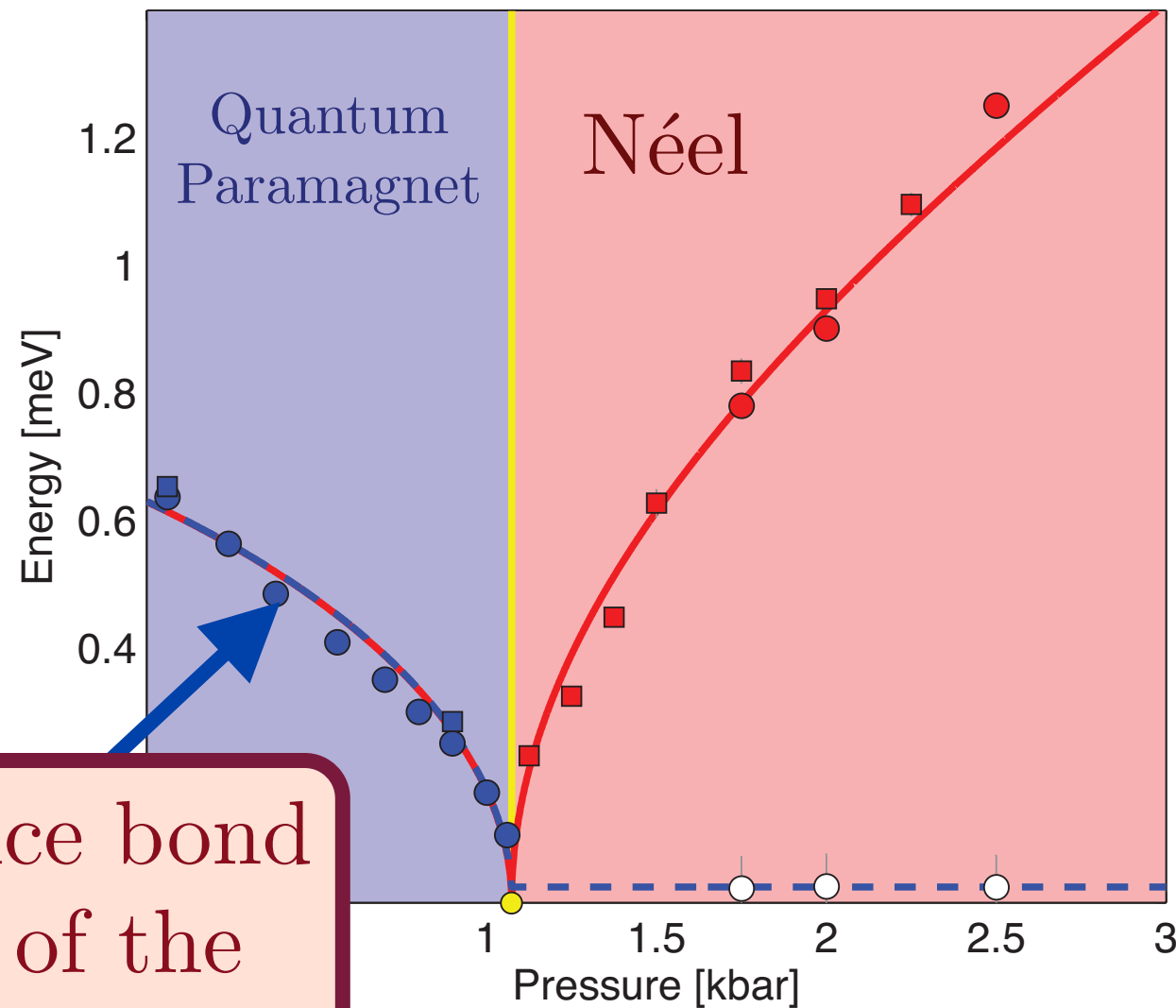
Spin waves

Excitations of TlCuCl_3 with varying pressure



Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

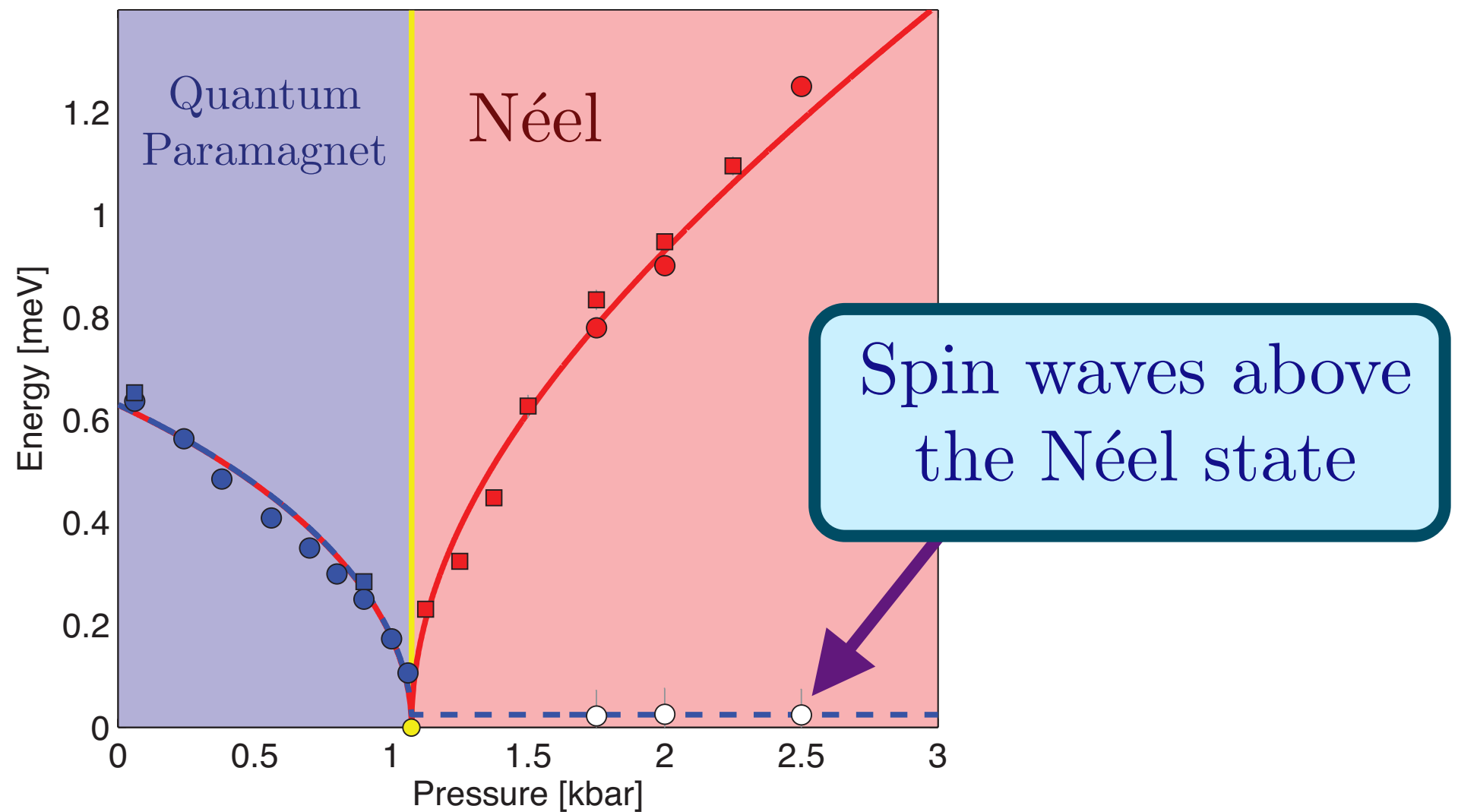
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Broken valence bond excitations of the quantum paramagnet

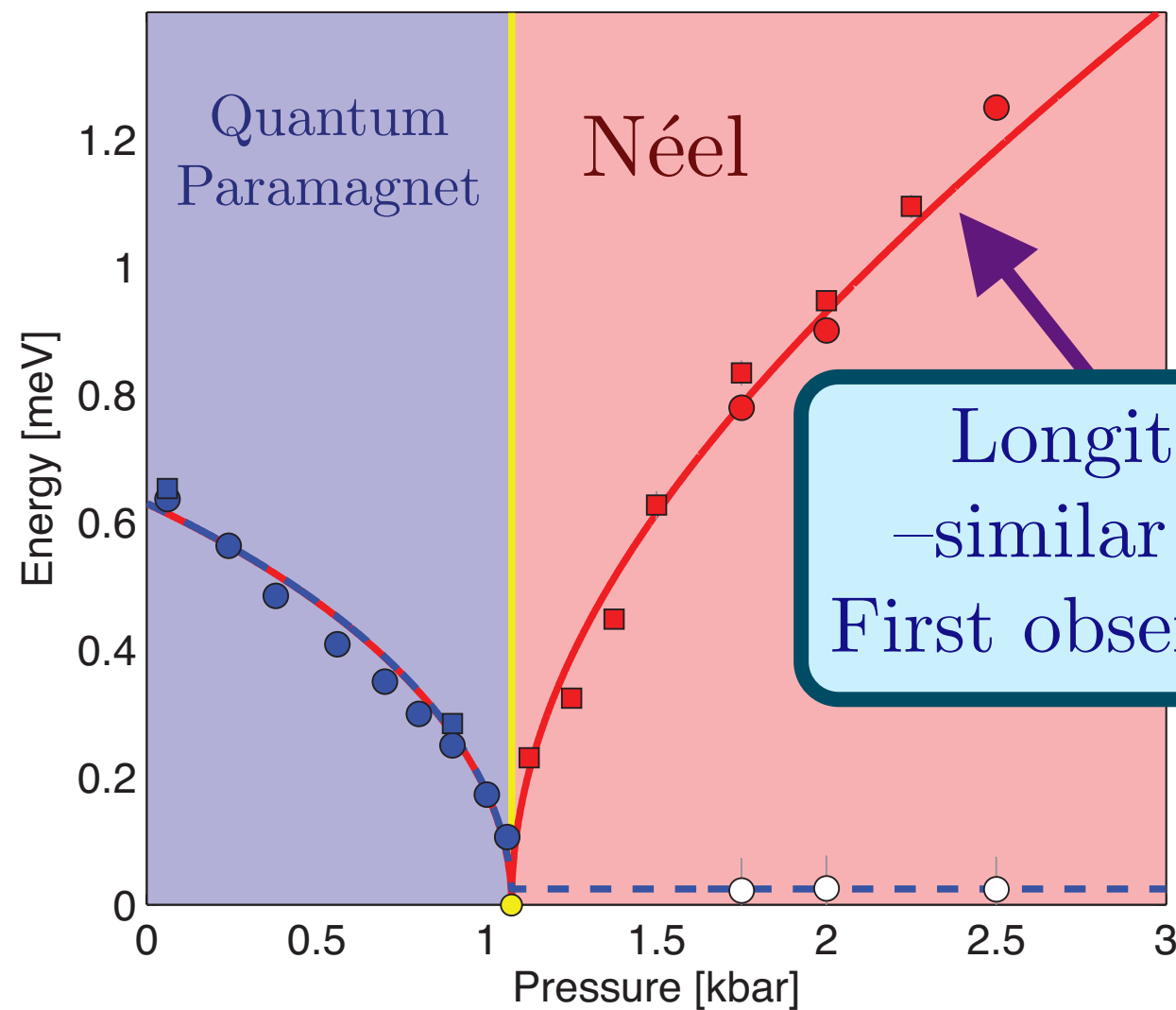
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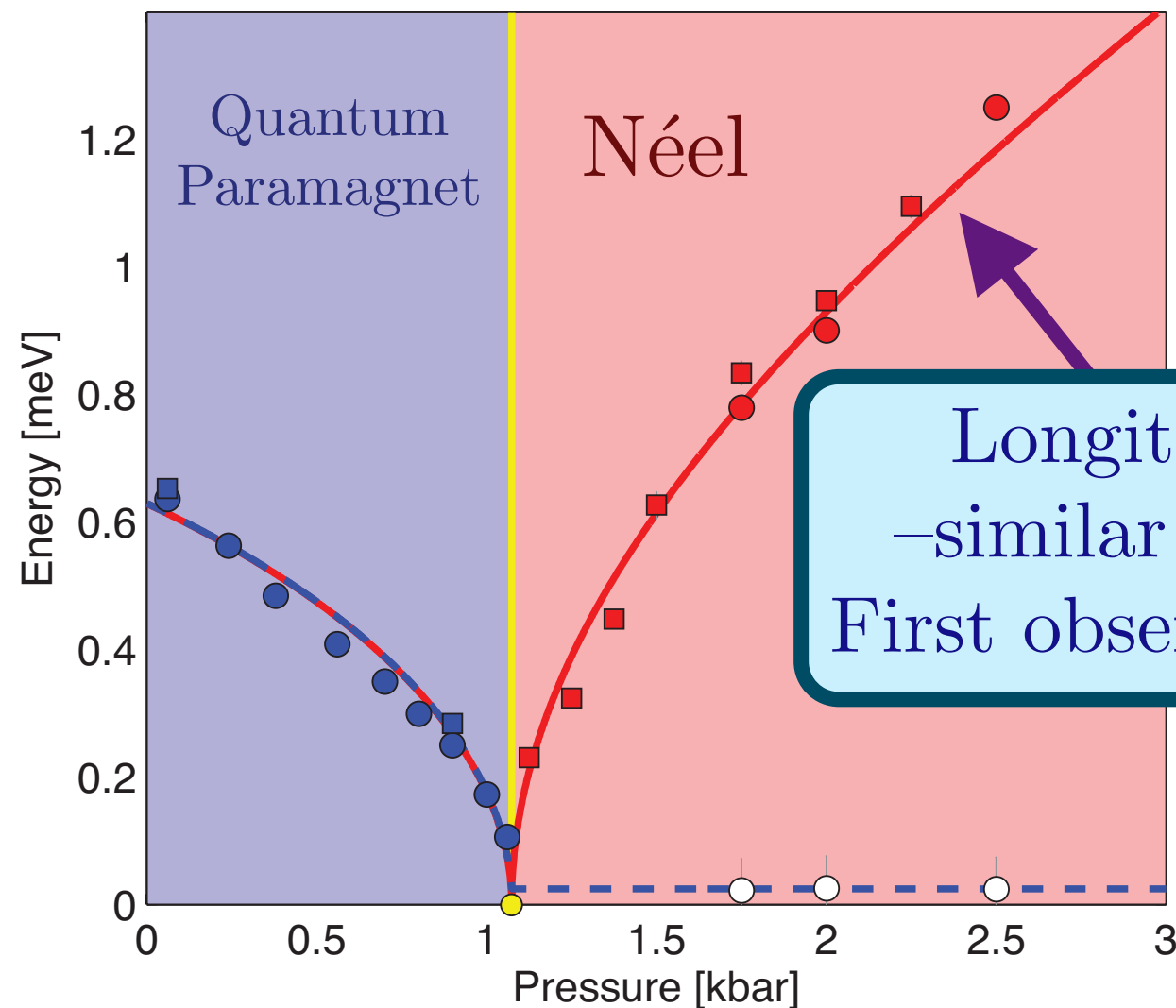
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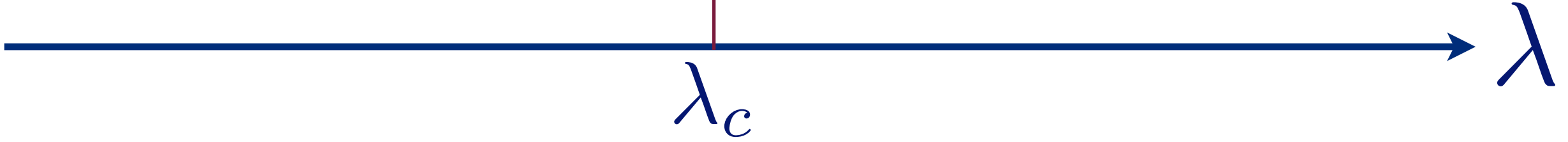
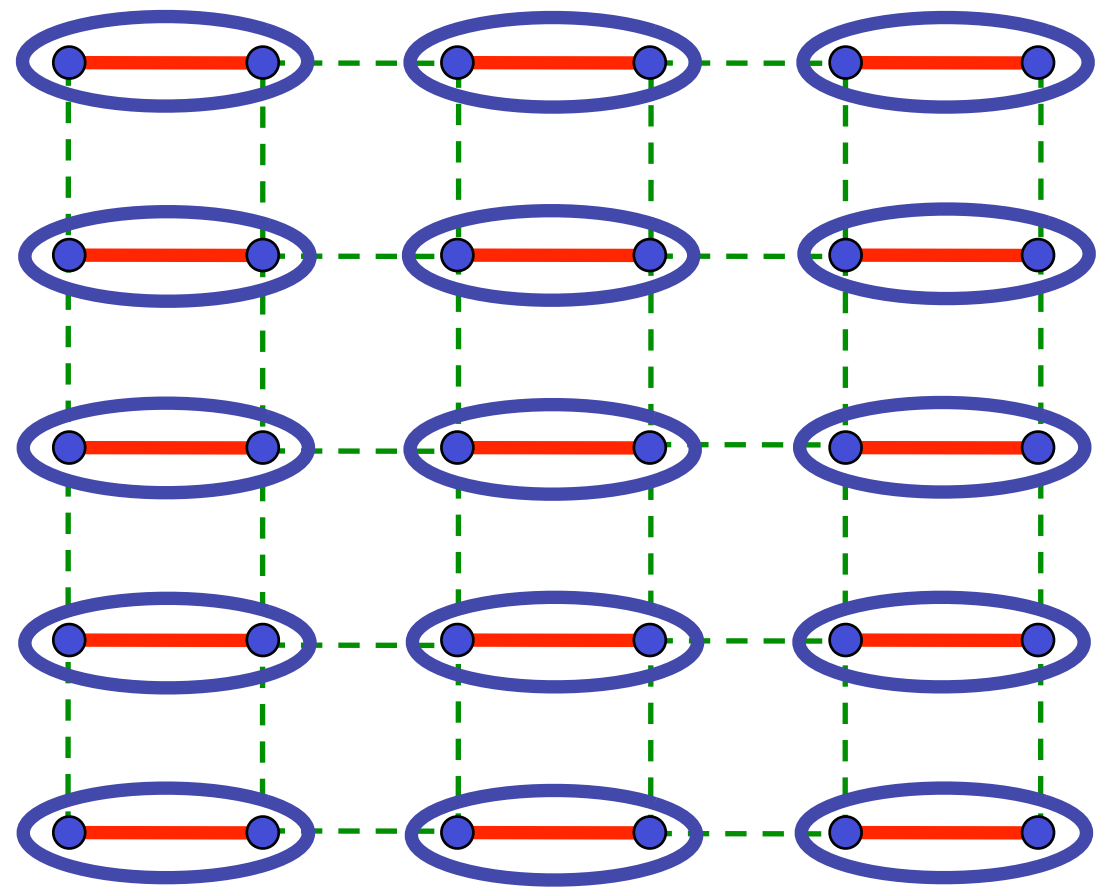
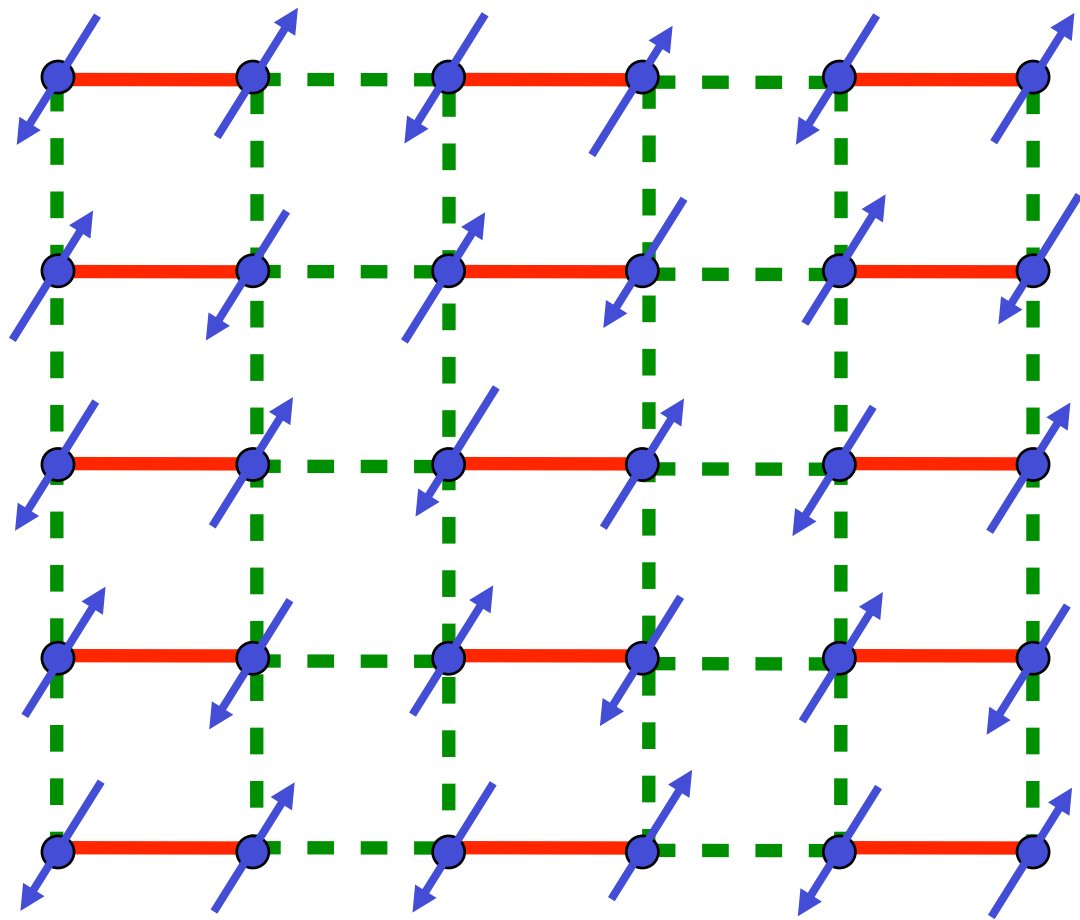
Longitudinal excitations
–similar to the Higgs boson
First observation of the Higgs !

“Higgs” particle appears at theoretically predicted energy

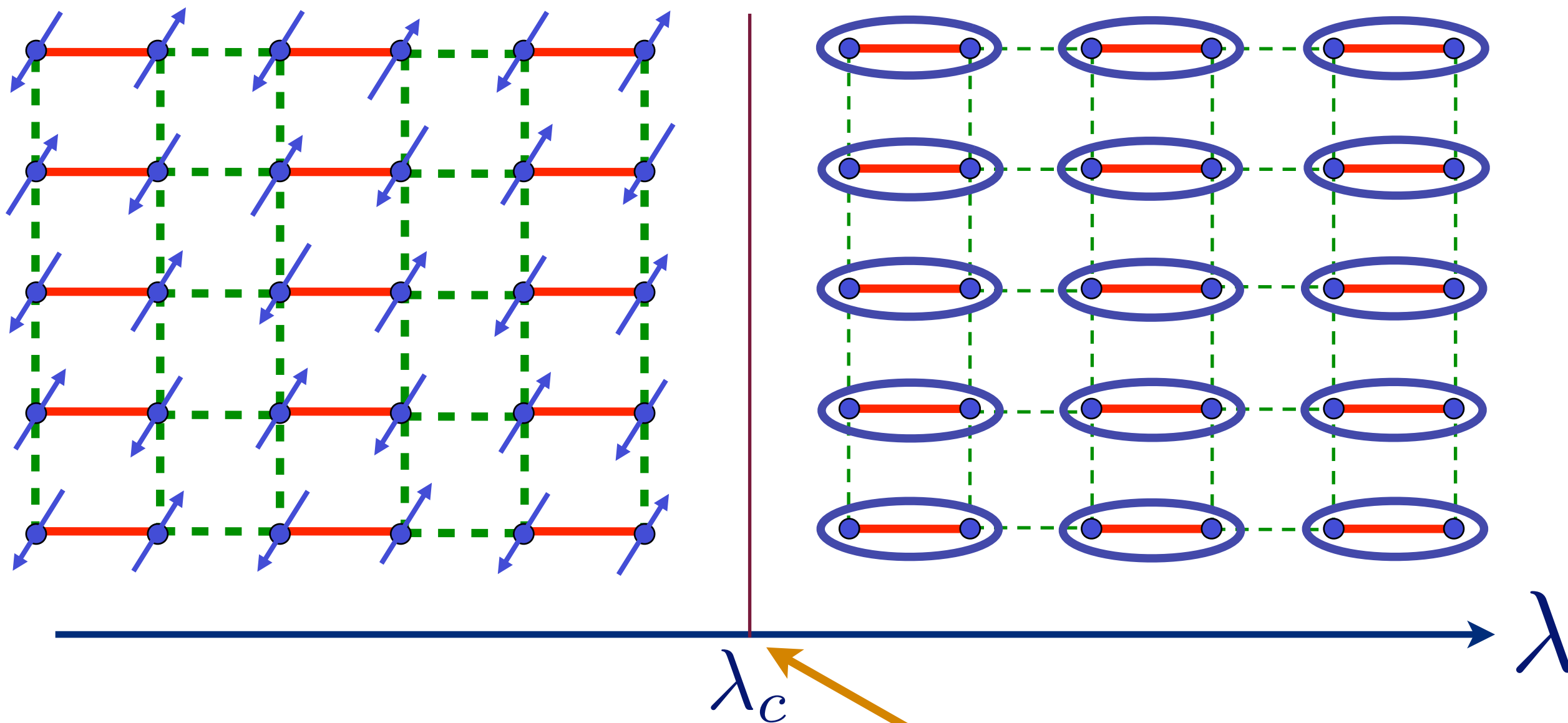
S. Sachdev, arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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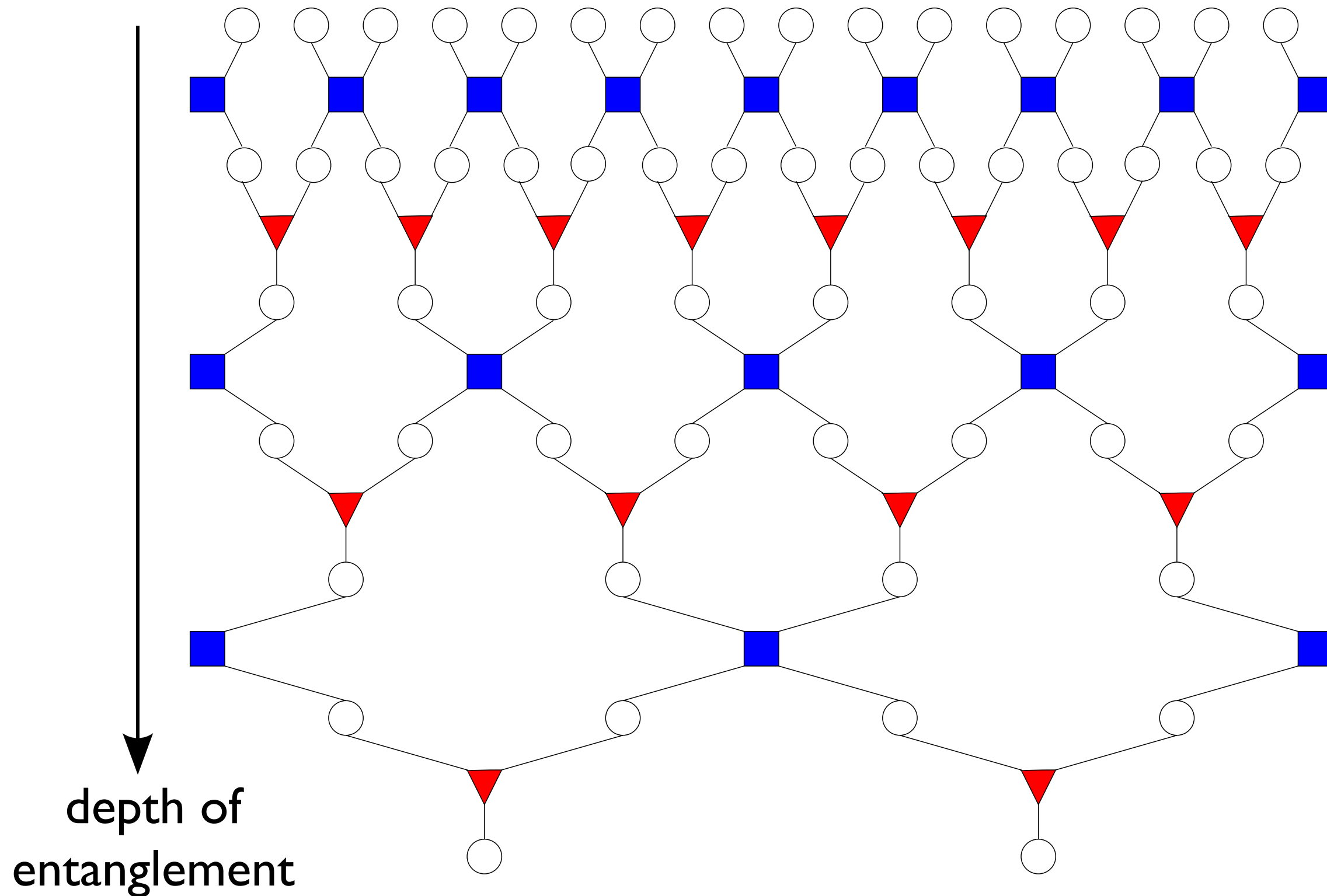
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Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point

D -dimensional
space



Characteristics of quantum critical point

- Long-range entanglement

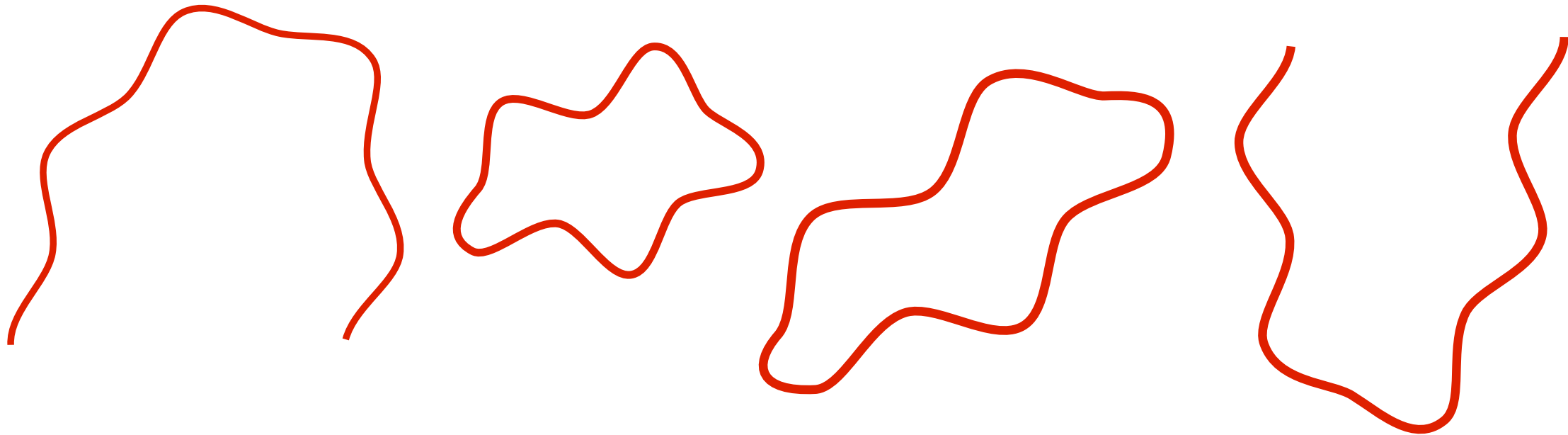
Characteristics of quantum critical point

- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).

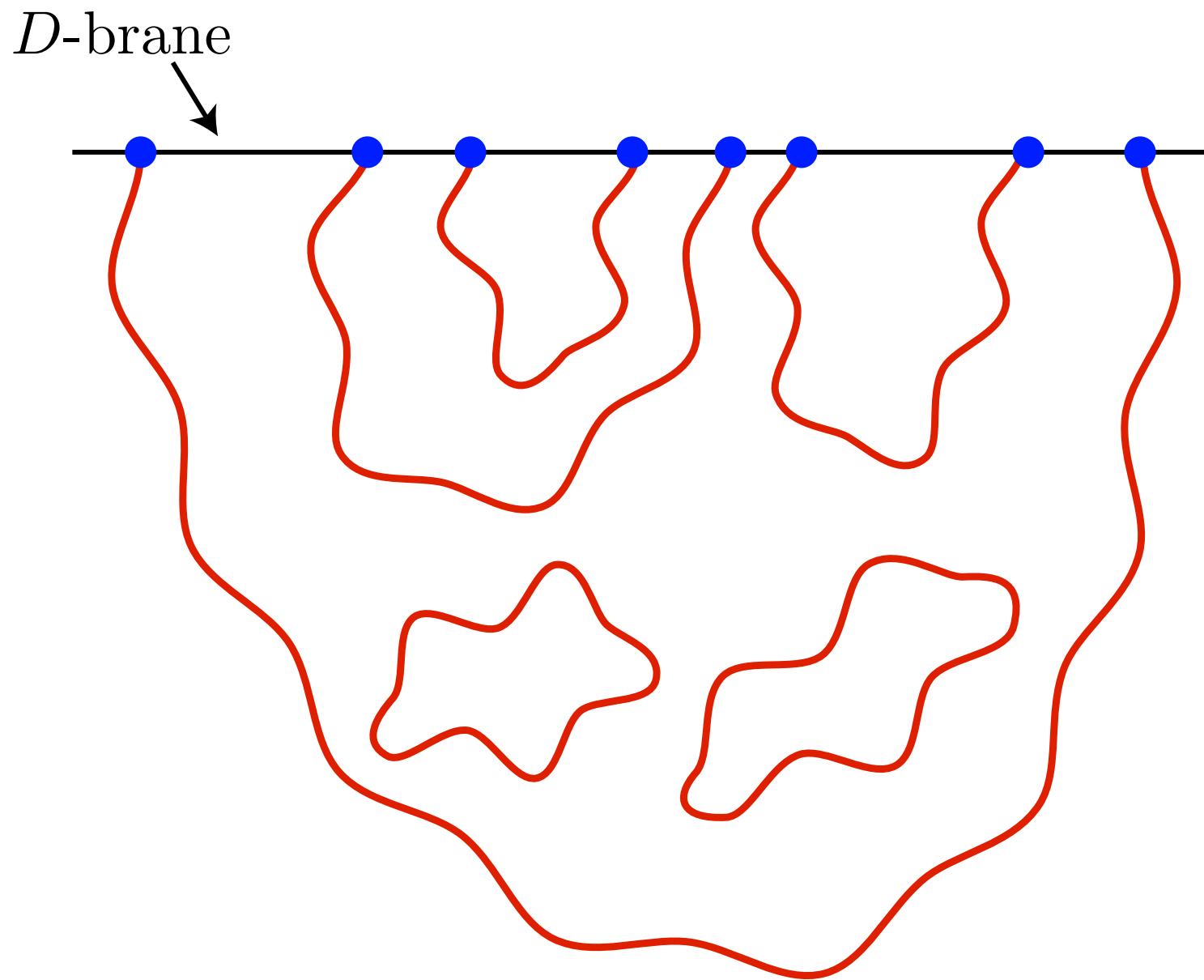
Characteristics of quantum critical point

- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).
- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT₃**

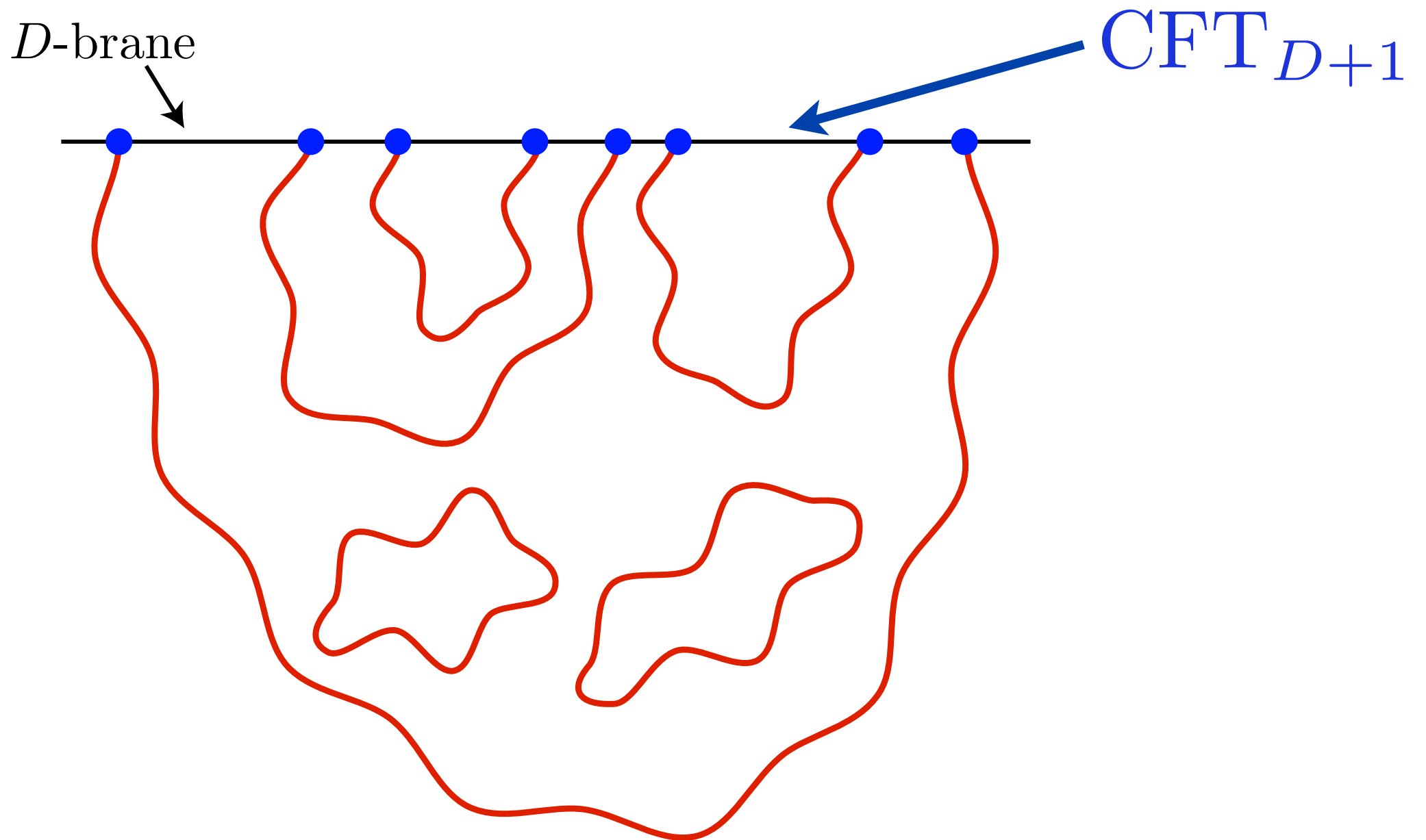
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



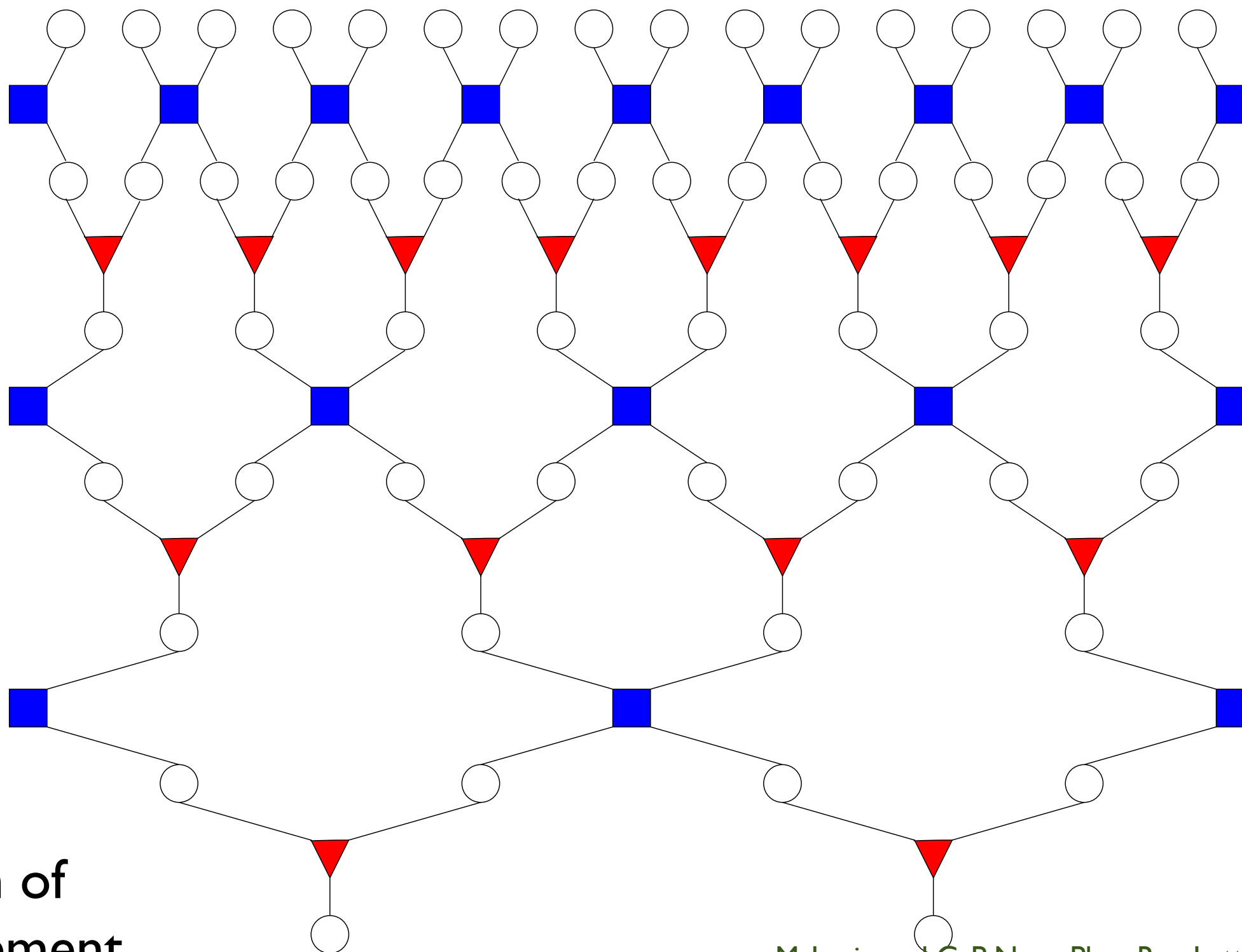
- A D -brane is a D -dimensional surface on which strings can end.
- The low-energy theory on a D -brane is an ordinary quantum field theory with no gravity.



- A D -brane is a D -dimensional surface on which strings can end.
- The low-energy theory on a D -brane is an ordinary quantum field theory with no gravity.
- In $D = 2$, we obtain strongly-interacting **CFT3s**. These are “dual” to string theory on anti-de Sitter space: **AdS4**.

Tensor network representation of entanglement at quantum critical point

D -dimensional
space

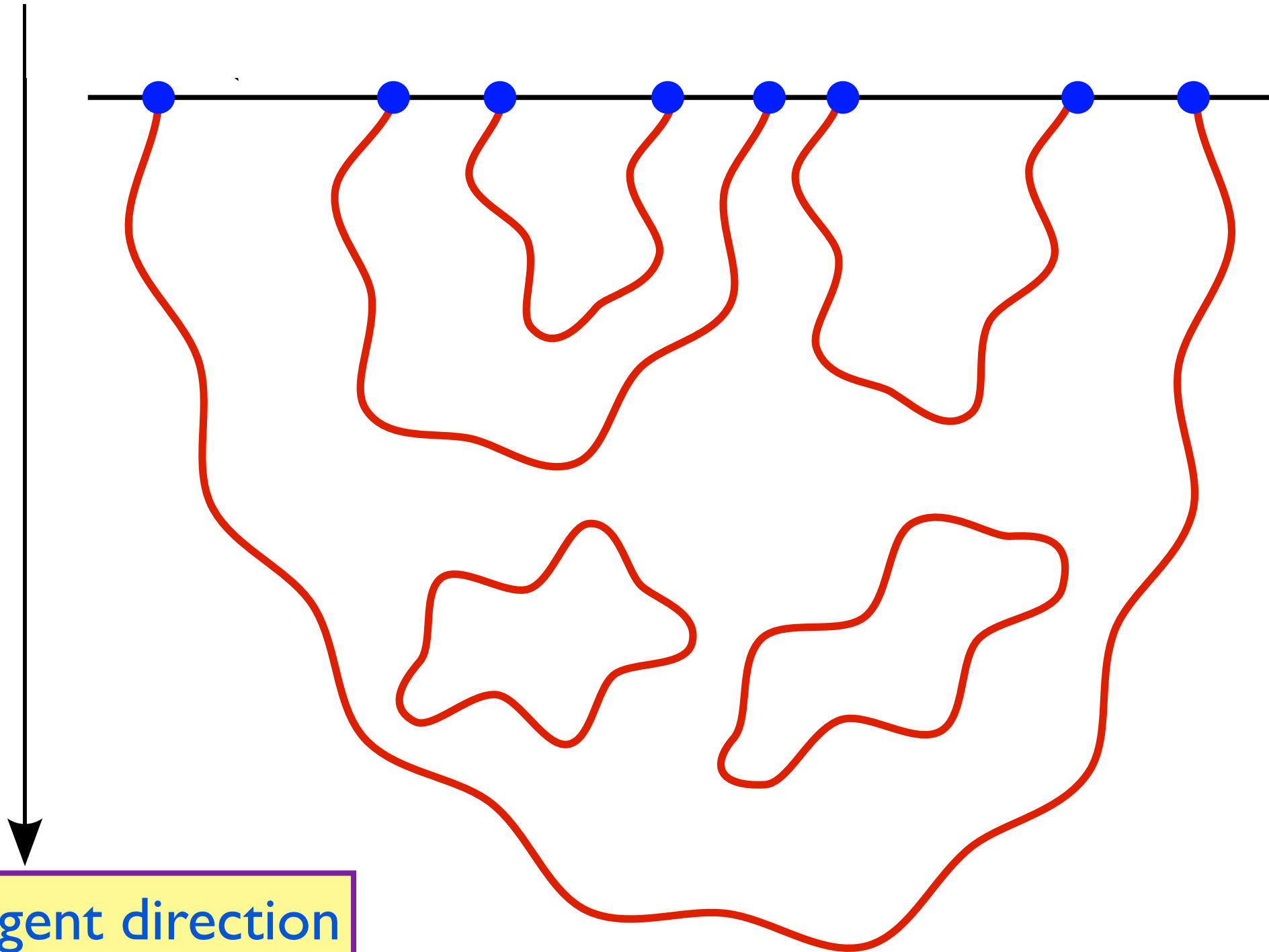


depth of
entanglement

M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

String theory near
a D-brane

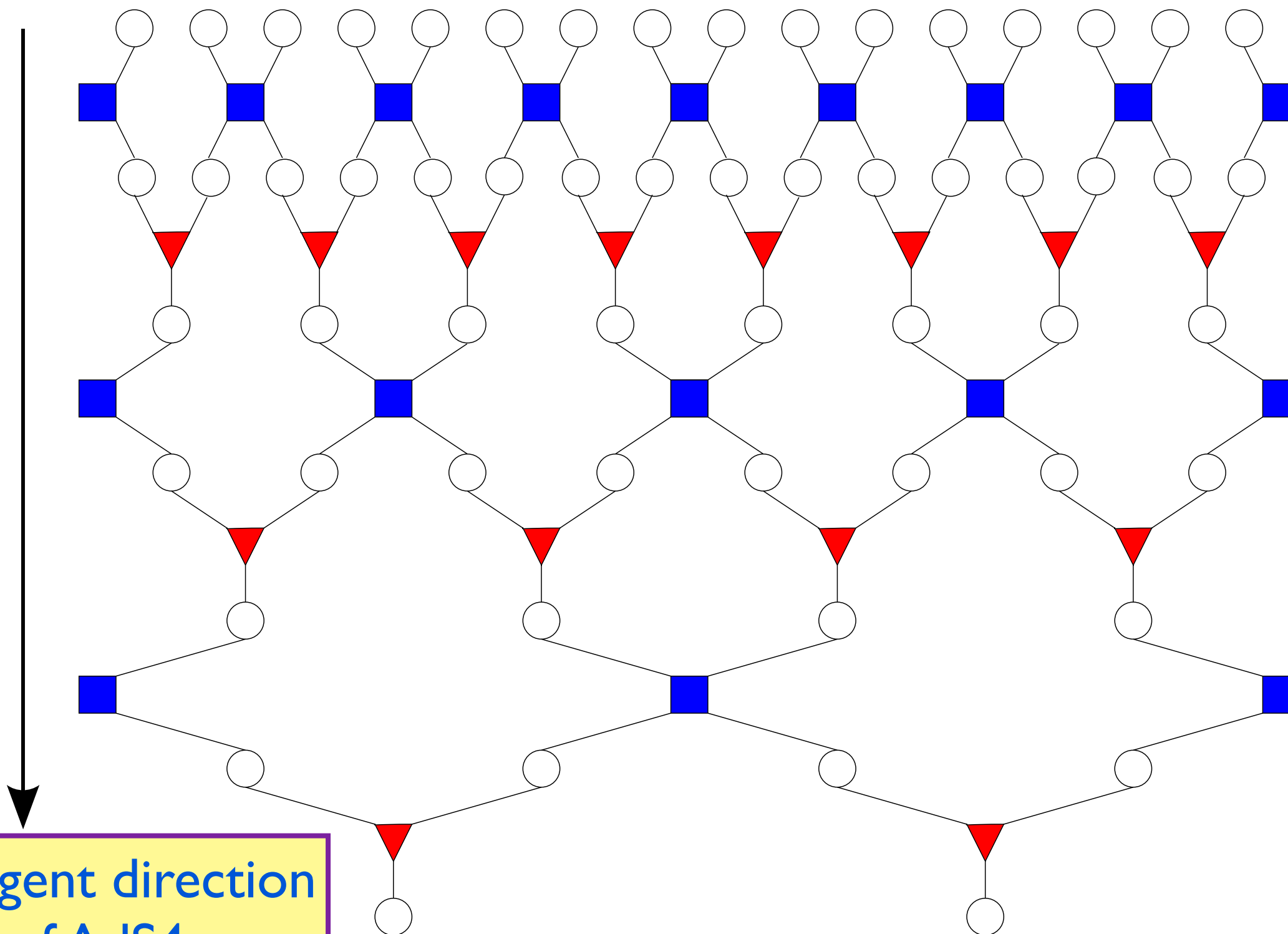
D -dimensional
space



Emergent direction
of AdS4

Tensor network representation of entanglement at quantum critical point

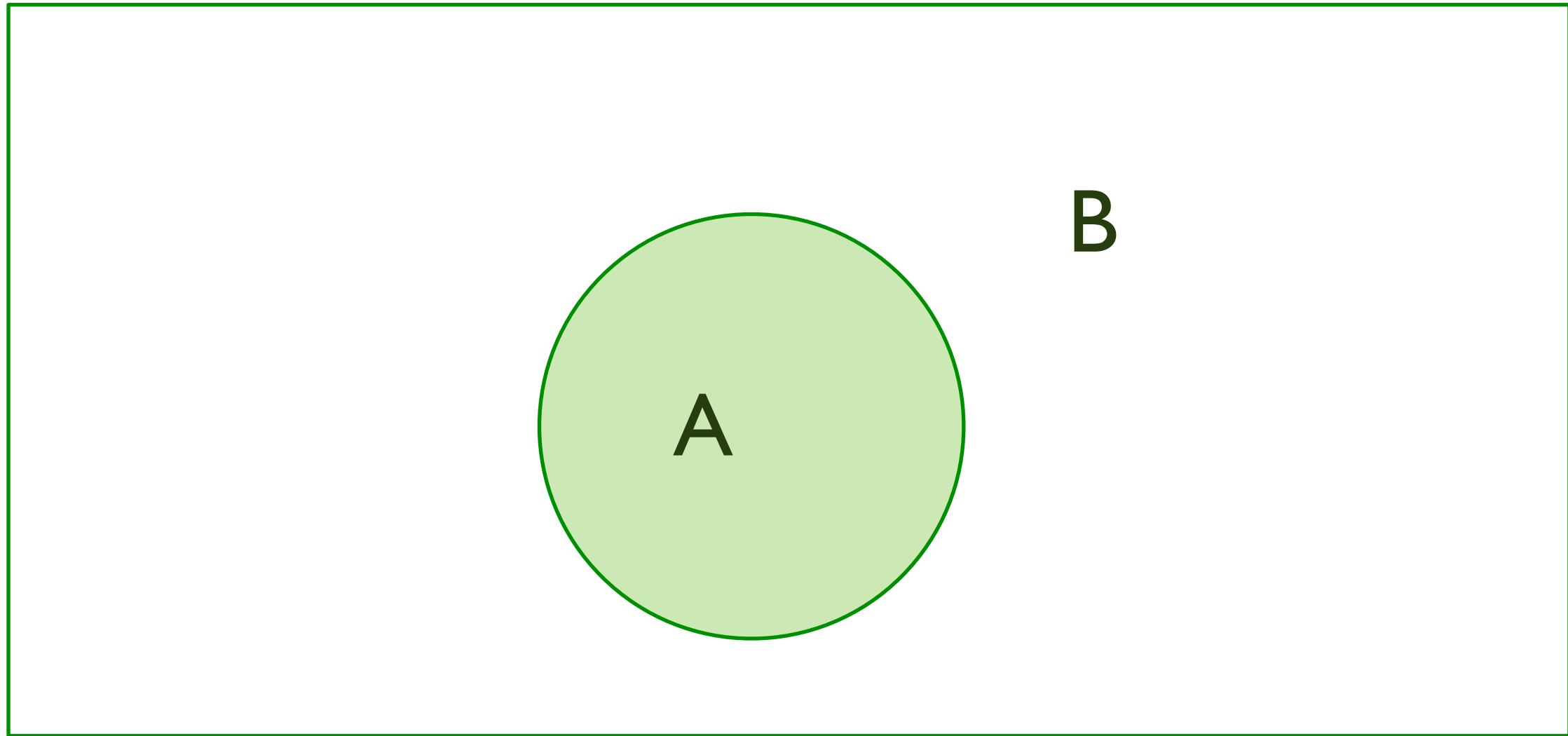
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Emergent direction
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Brian Swingle, arXiv:0905.1317

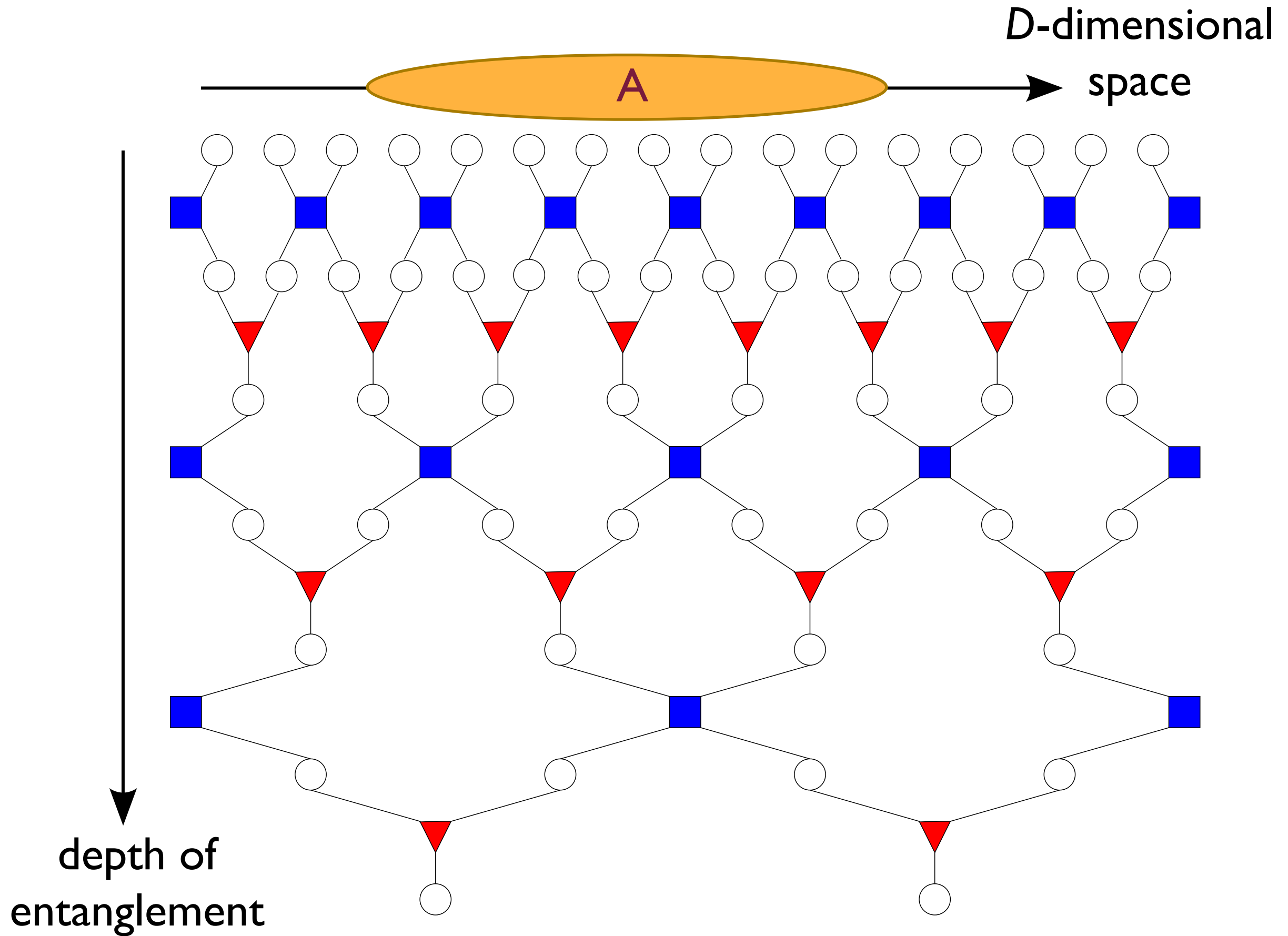
Entanglement entropy



$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

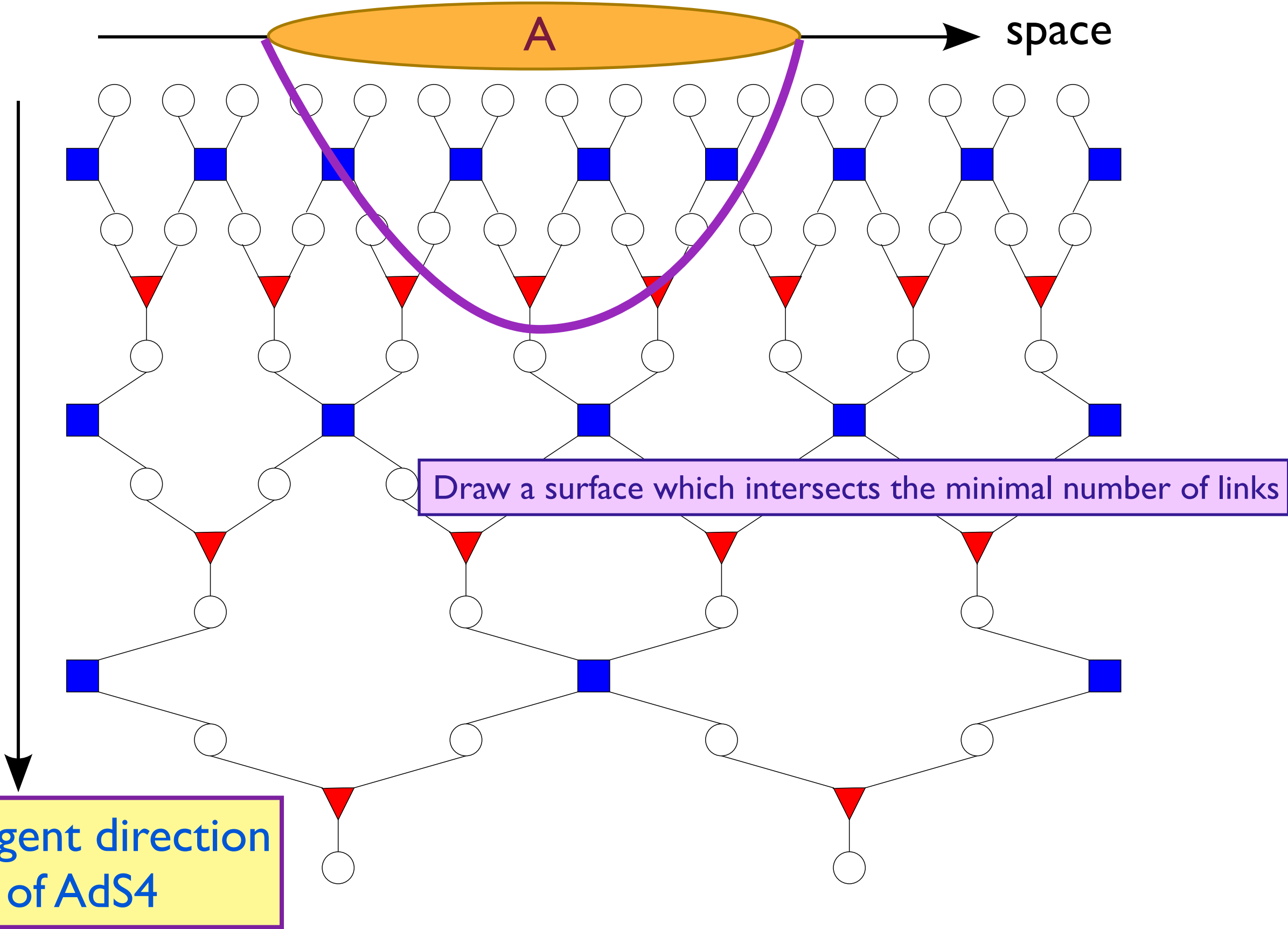
Entanglement entropy $S_{EE} = -\text{Tr}(\rho_A \ln \rho_A)$

Entanglement entropy



Entanglement entropy

D -dimensional
space

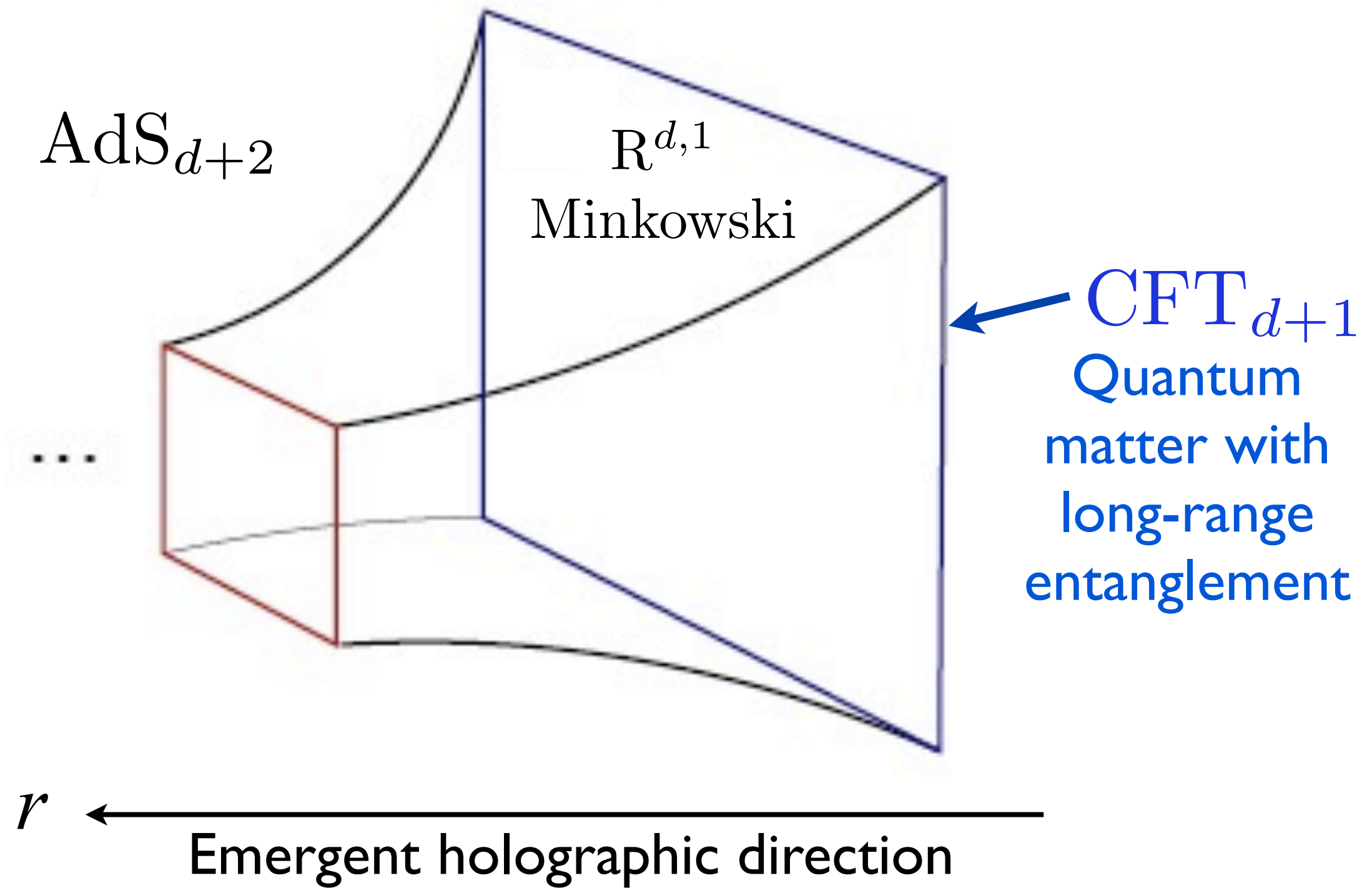


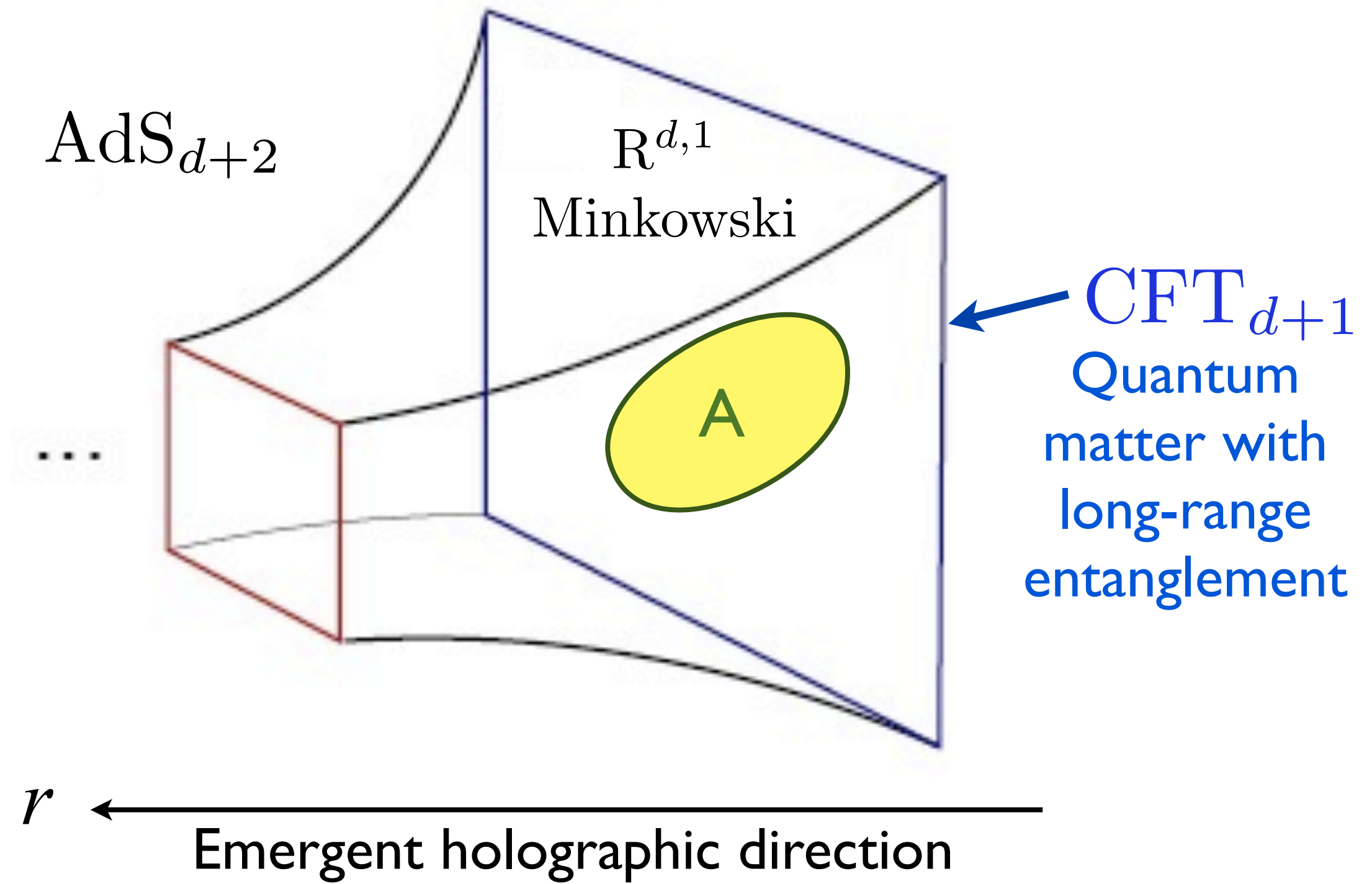
Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A .

This can be seen both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317





AdS_{d+2}

$R^{d,1}$
Minkowski

CFT_{d+1}
Quantum
matter with
long-range
entanglement

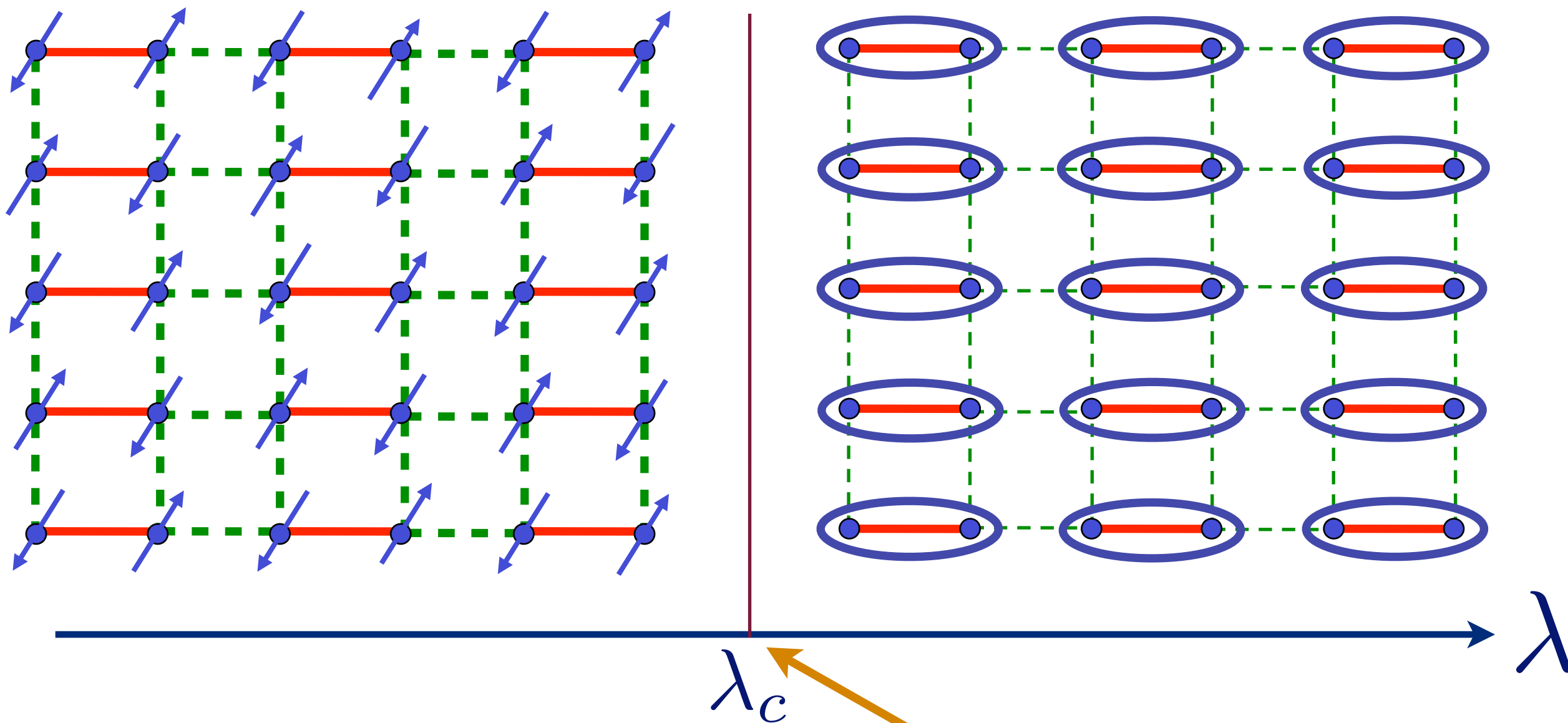
Area
measures
entanglement
entropy

A

r

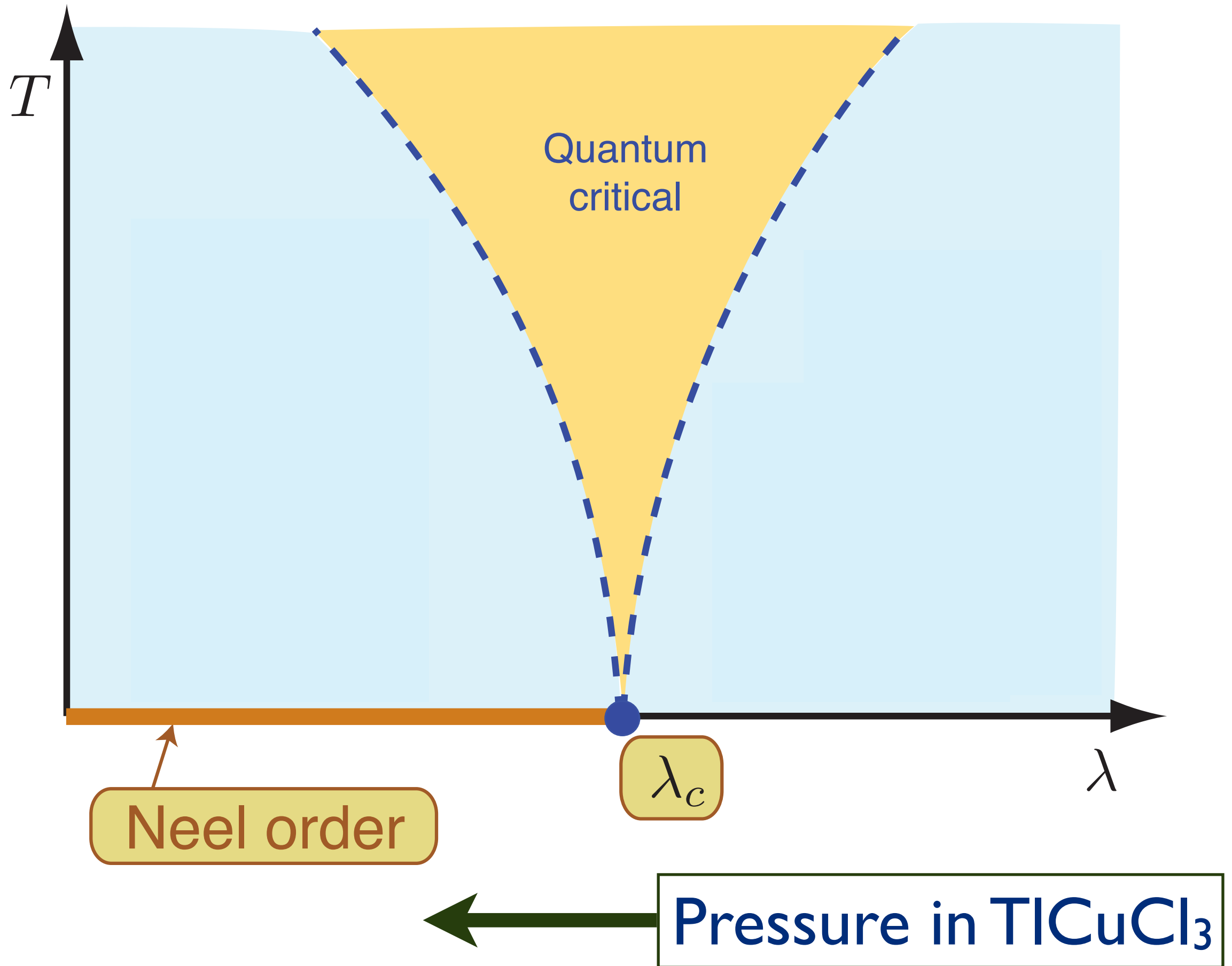
Emergent holographic direction

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

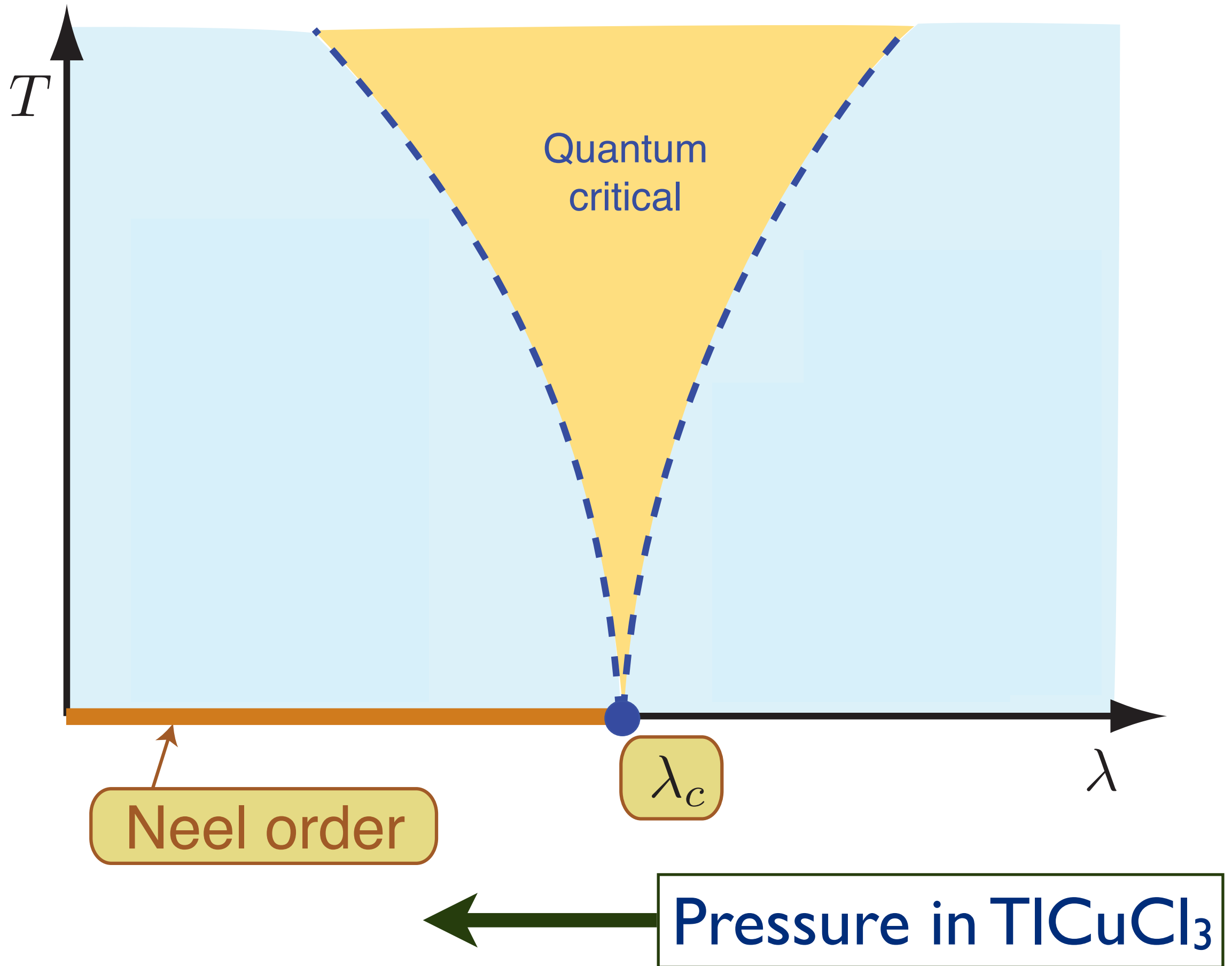


Quantum critical point with non-local entanglement in spin wavefunction

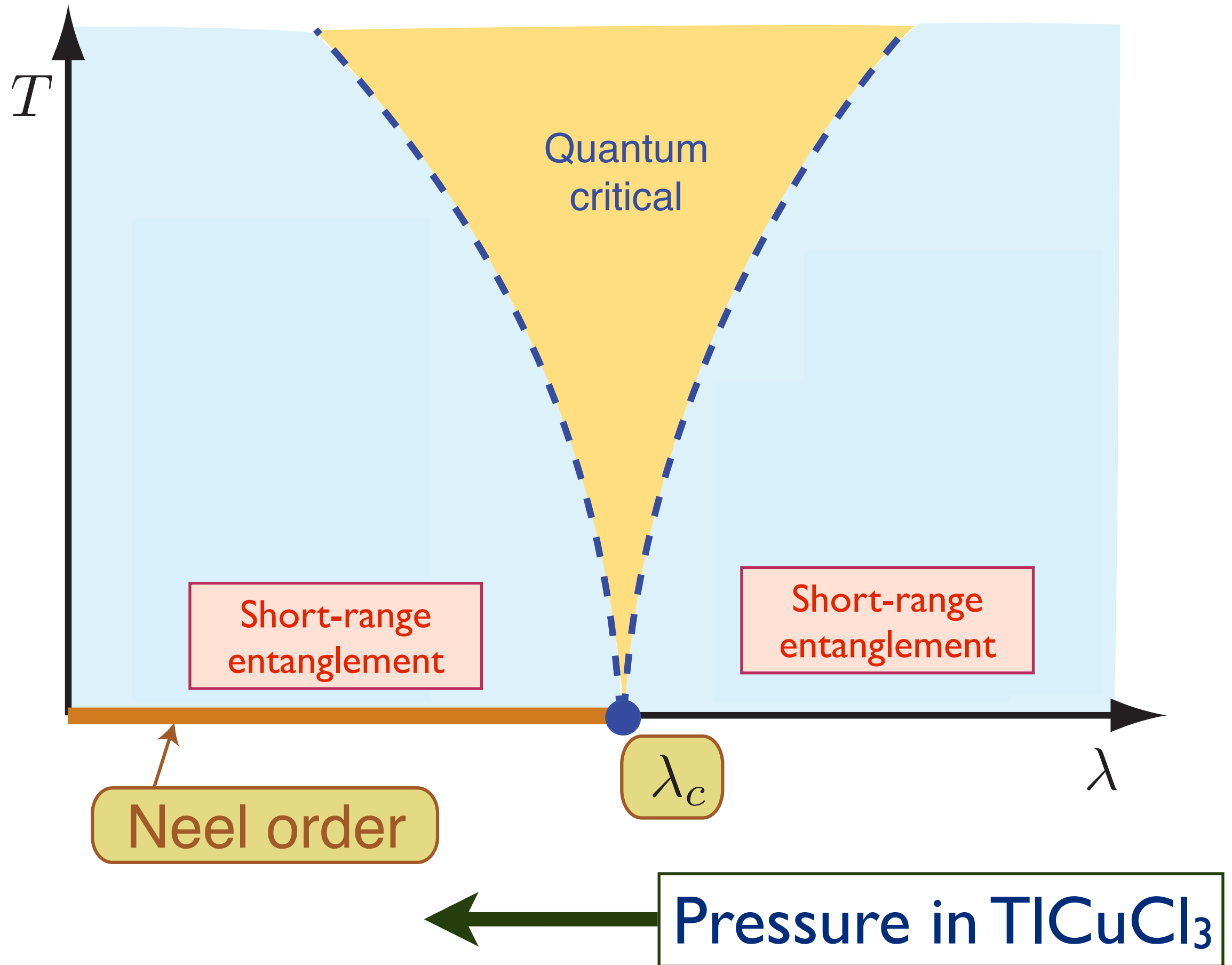
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



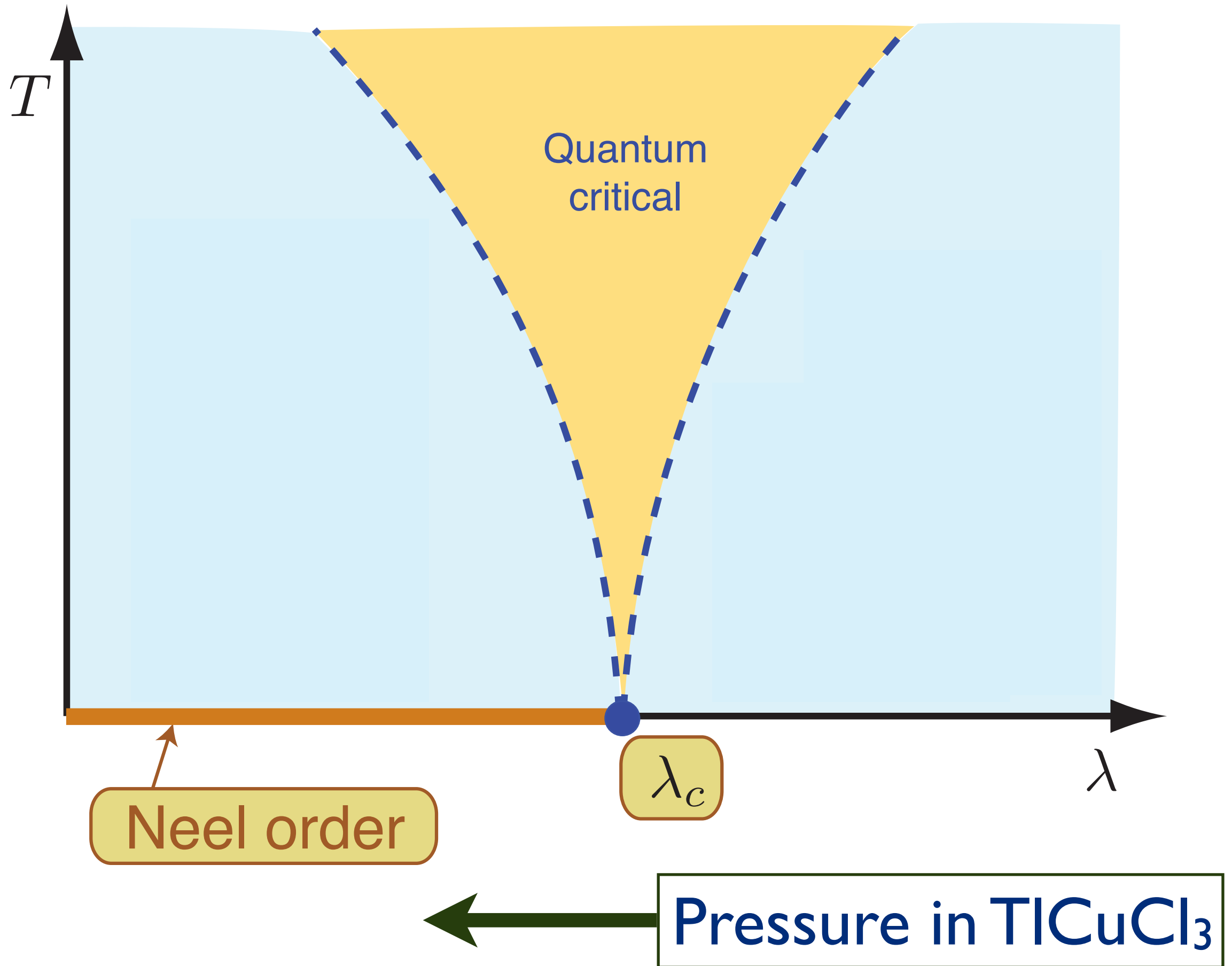
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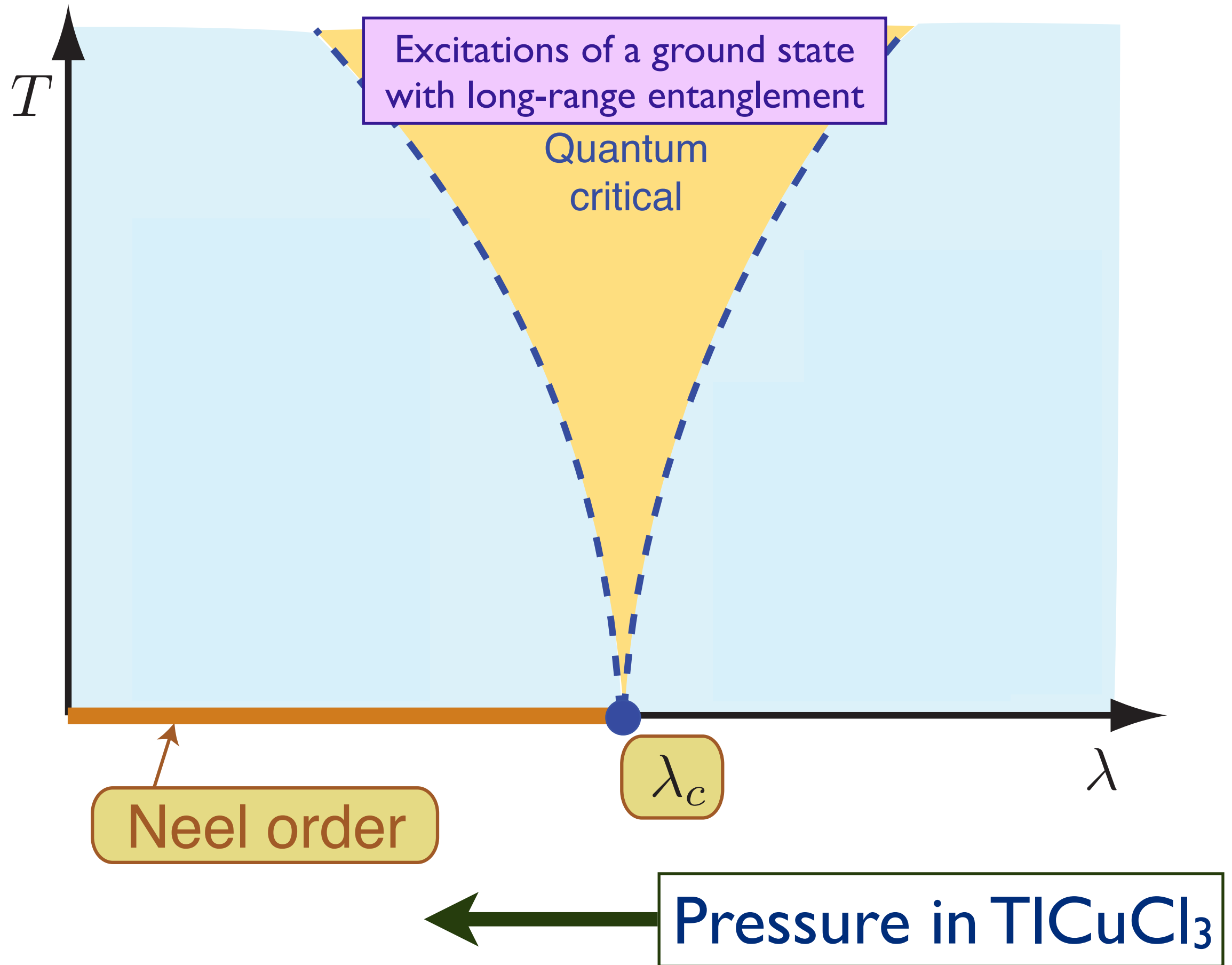
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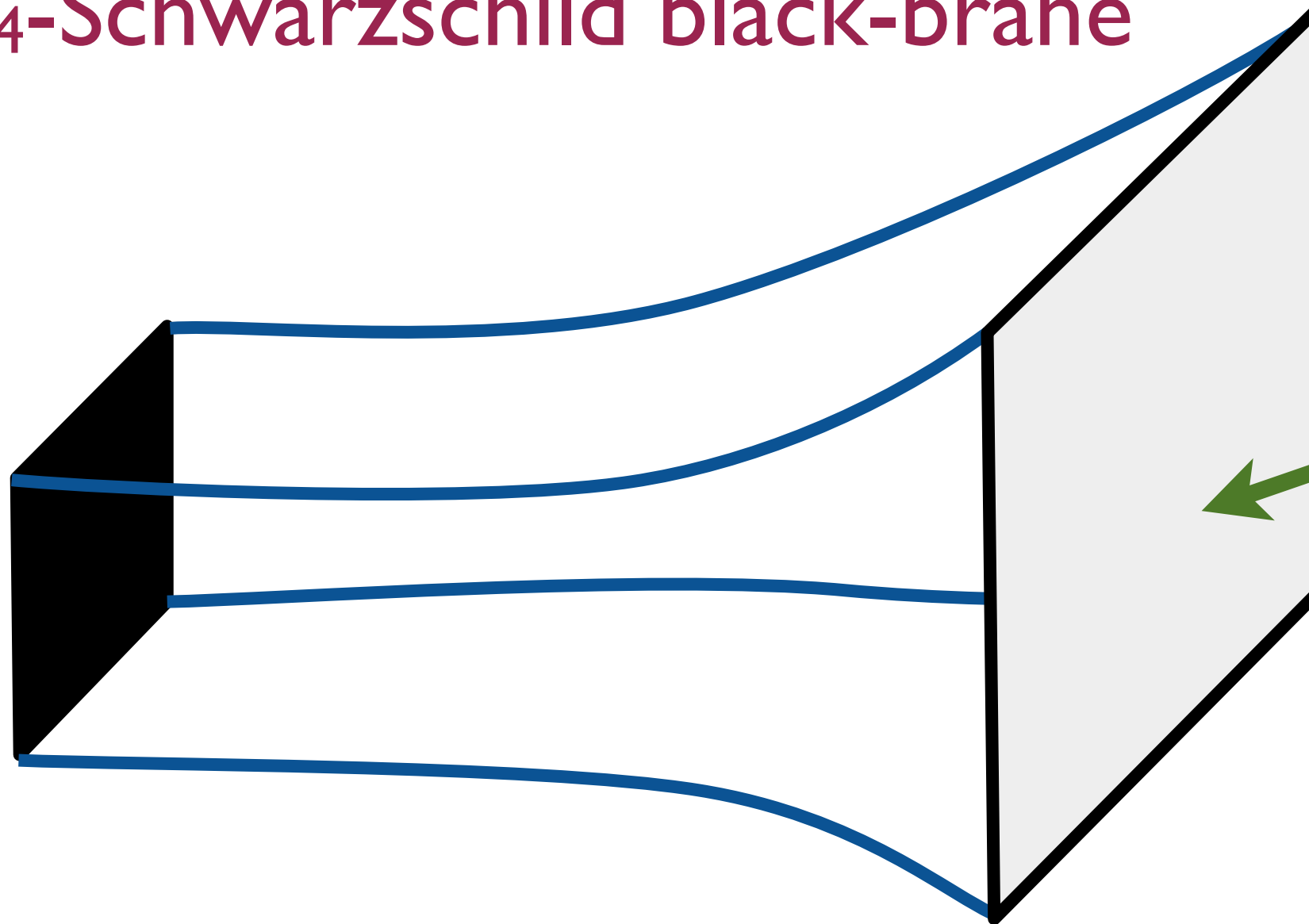


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AdS/CFT correspondence at non-zero temperatures

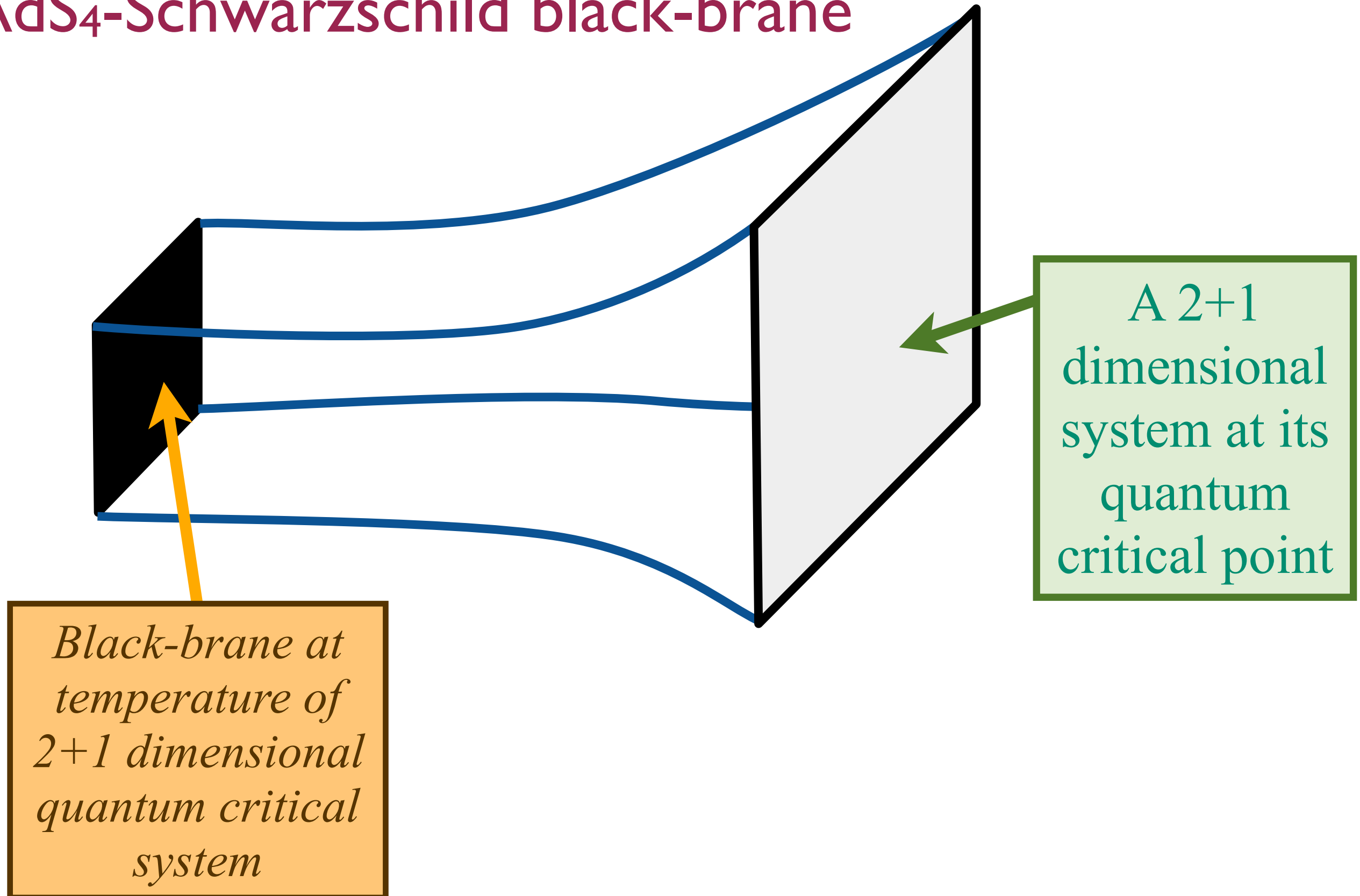
AdS₄-Schwarzschild black-brane



A 2+1
dimensional
system at its
quantum
critical point

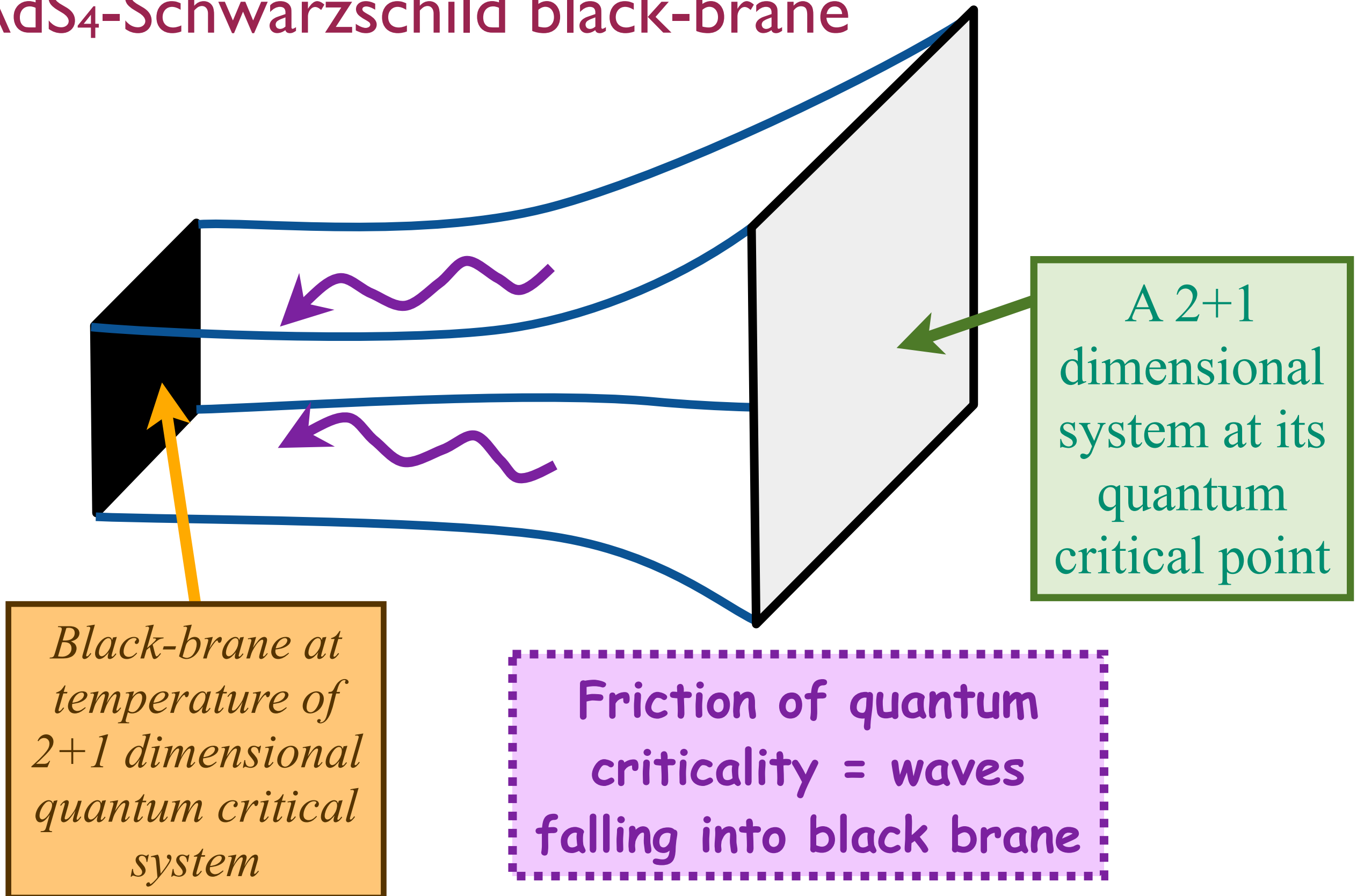
AdS/CFT correspondence at non-zero temperatures

AdS₄-Schwarzschild black-brane



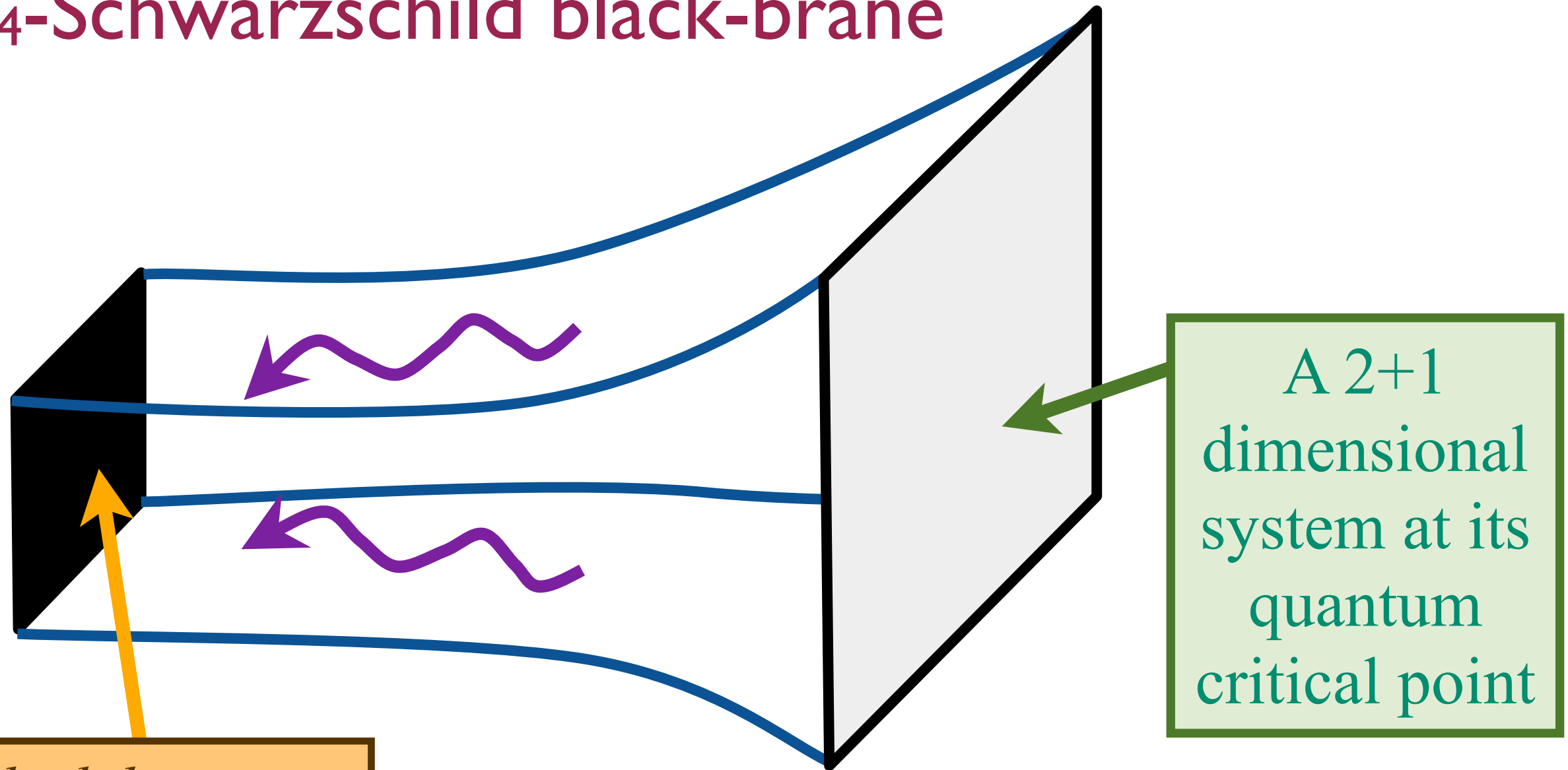
AdS/CFT correspondence at non-zero temperatures

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AdS/CFT correspondence at non-zero temperatures

AdS₄-Schwarzschild black-brane



Black-brane at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point

Provides successful description of many properties of quantum critical points at non-zero temperatures

Outline

1. Quantum critical points and string theory
Entanglement and emergent dimensions

2. High temperature superconductors
and strange metals
Holography of compressible quantum phases

Outline

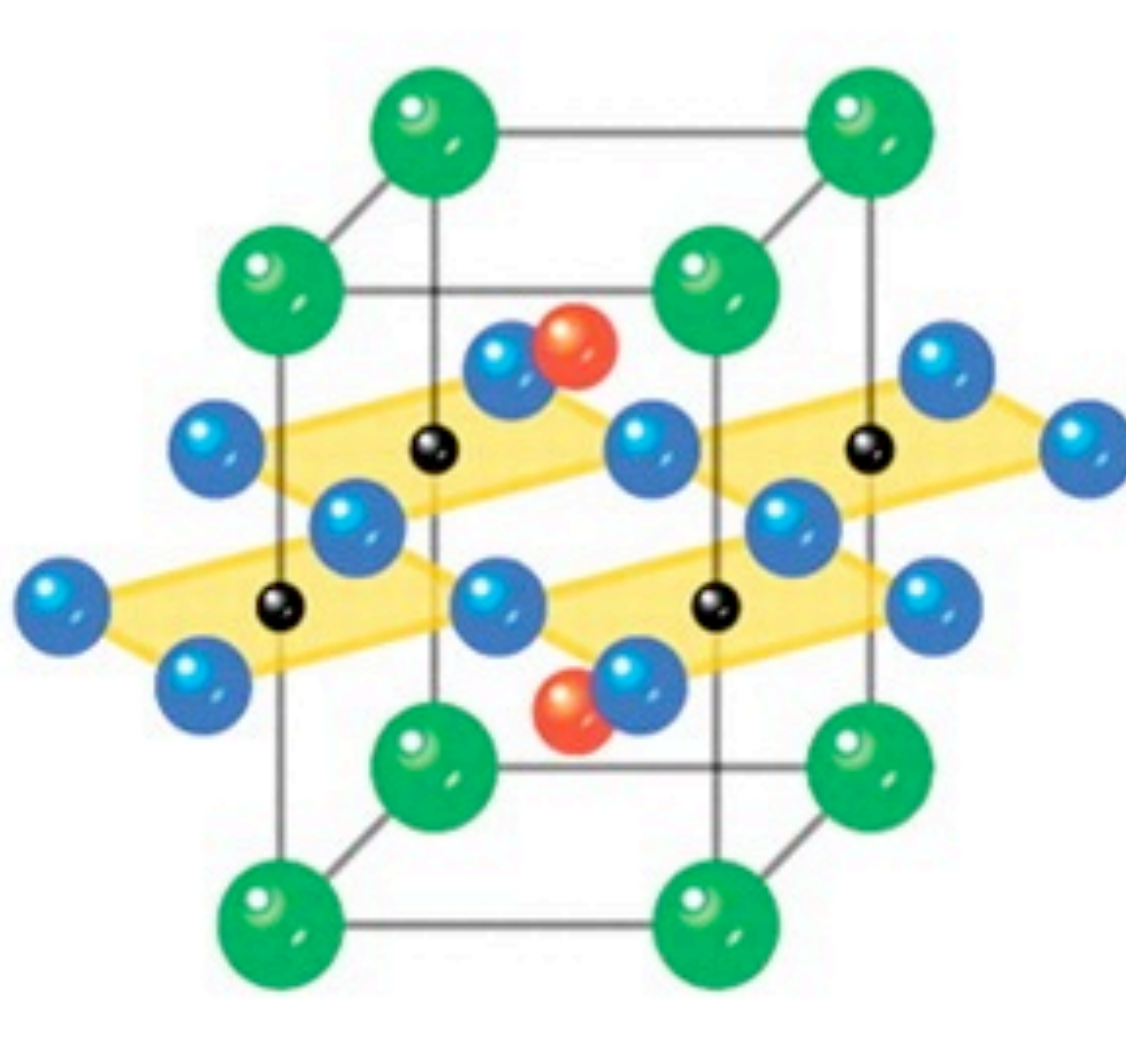
1. Quantum critical points and string theory
Entanglement and emergent dimensions

2. High temperature superconductors
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Holography of compressible quantum phases

The cuprate superconductors

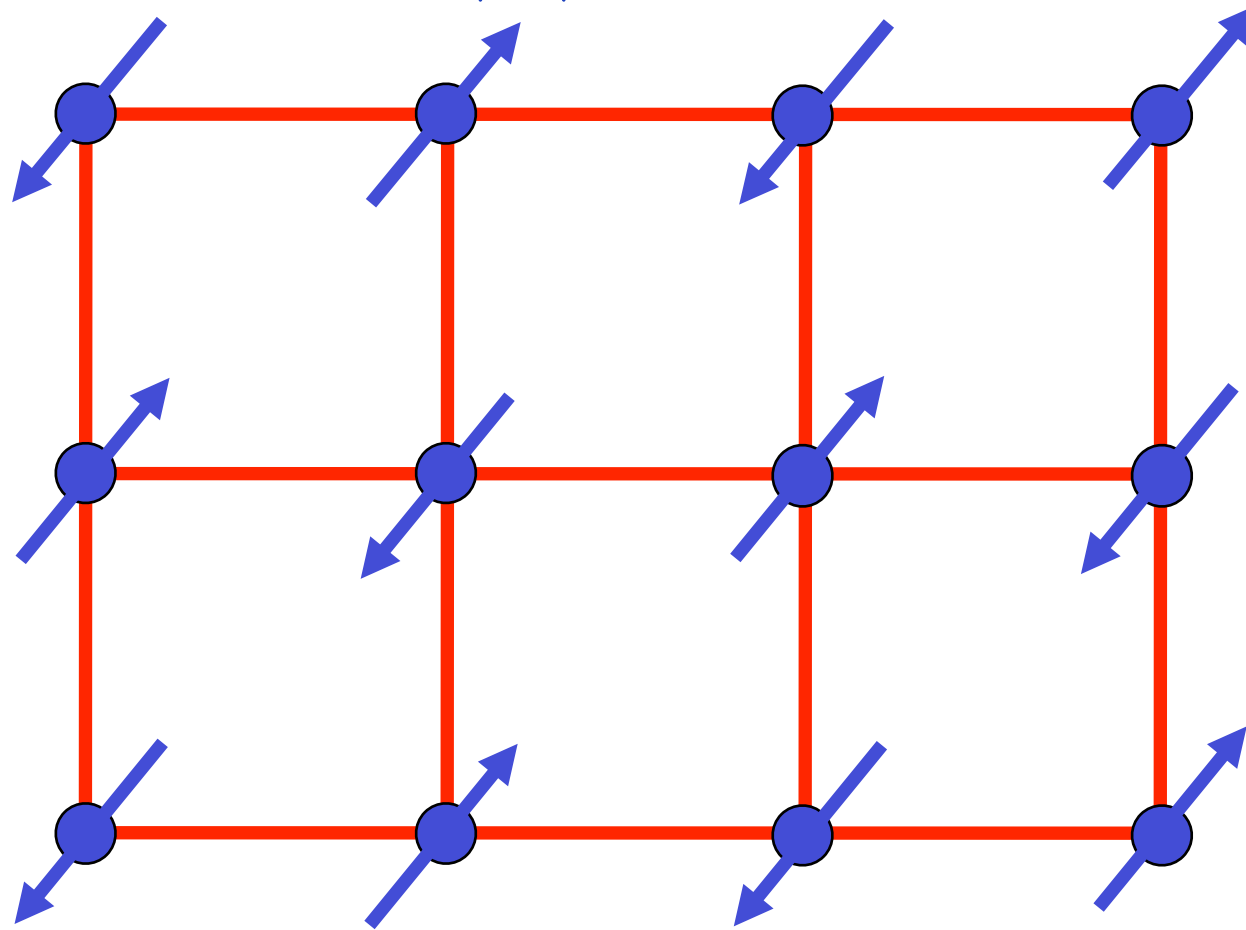
Na-CCOC

- Cu
- Ca/Na
- O
- Cl

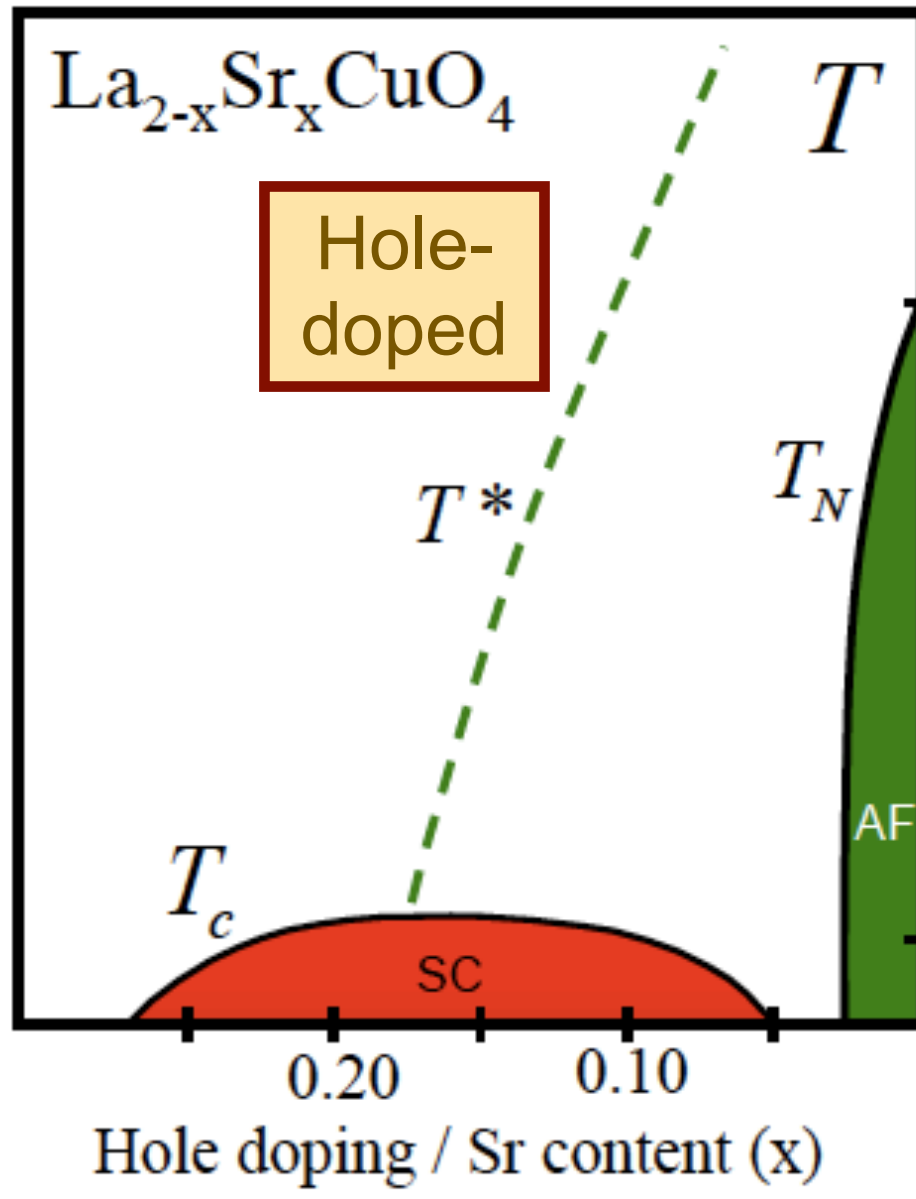


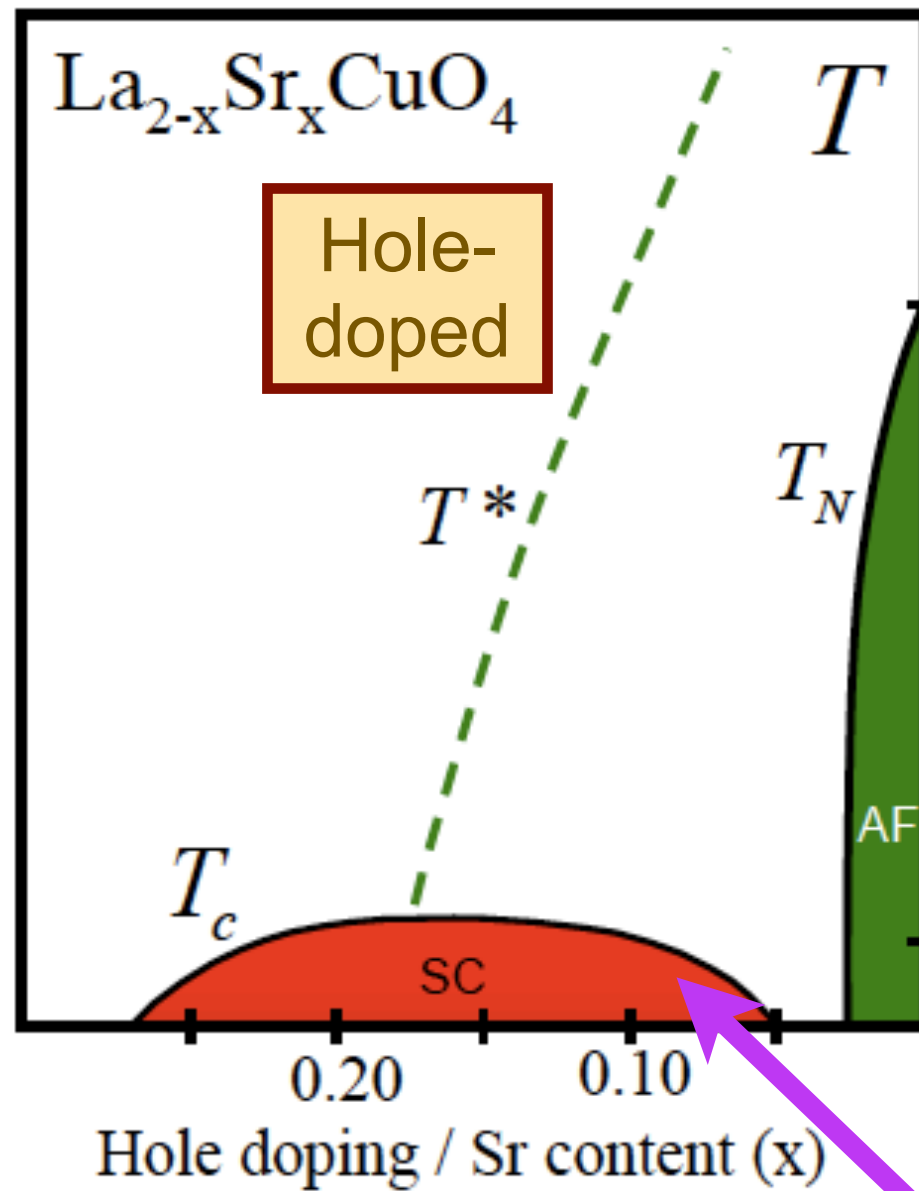
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



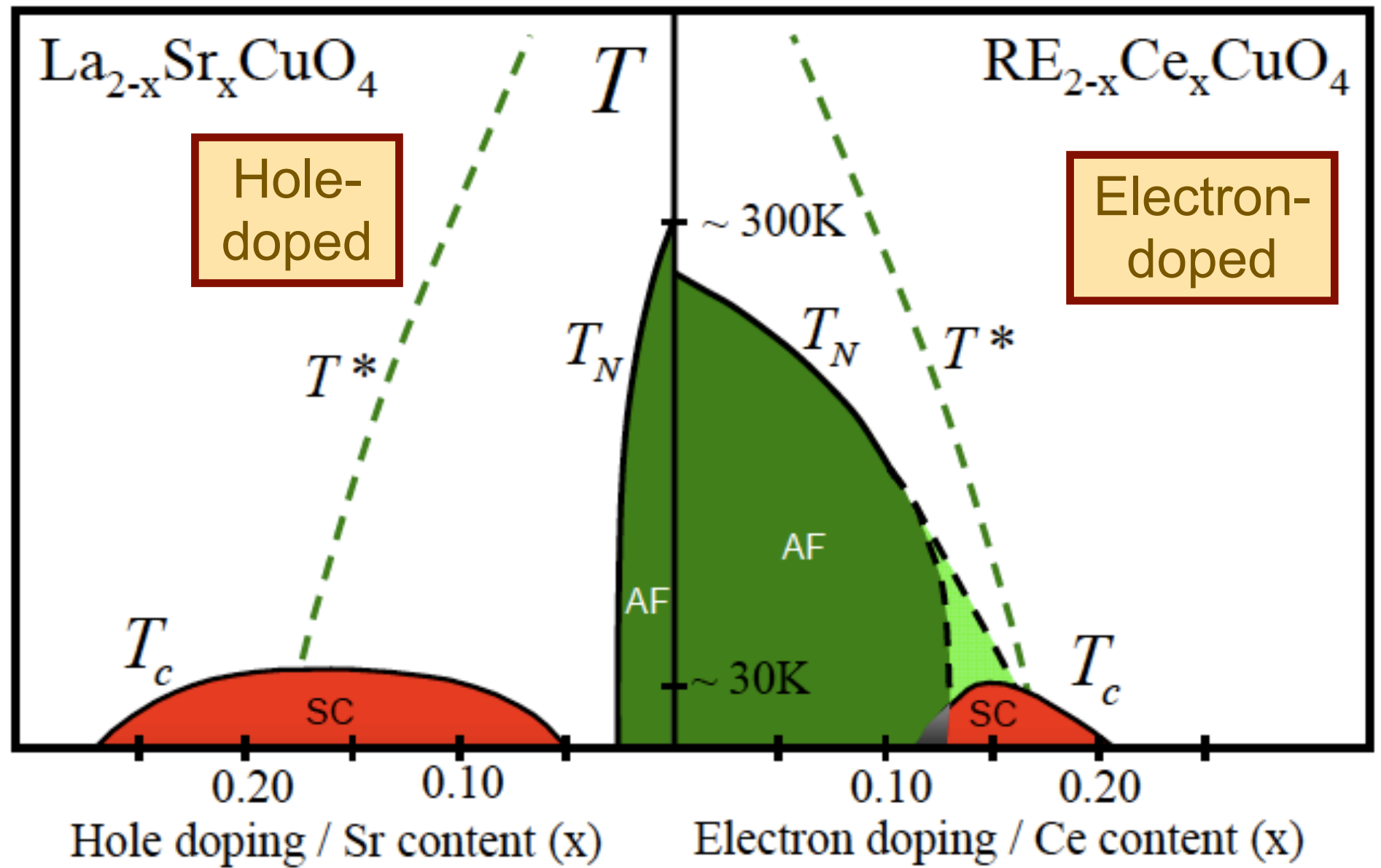
Ground state has long-range Néel order





Superconductor
 Bose condensate of pairs of electrons
 Short-range entanglement

Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

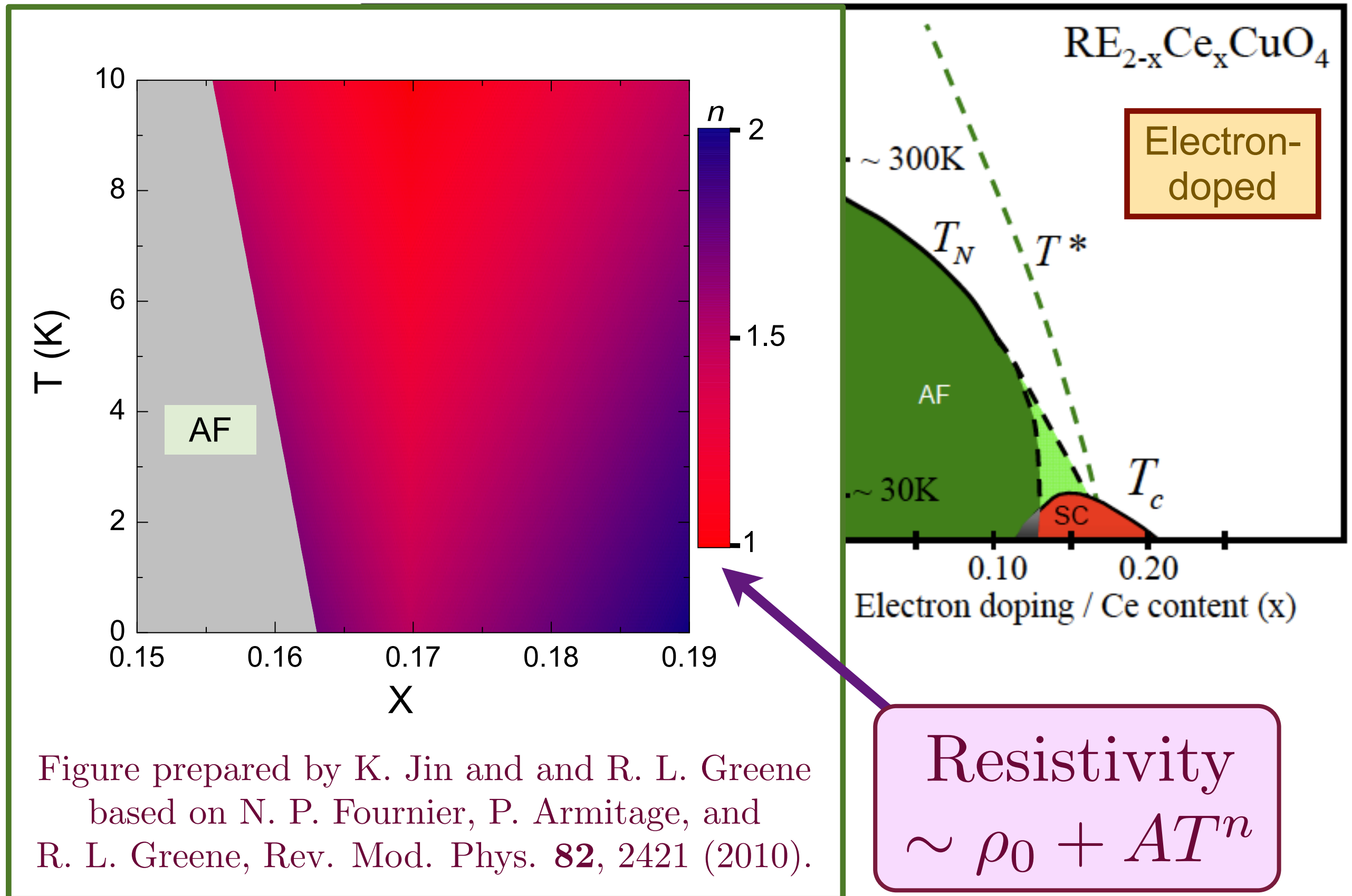
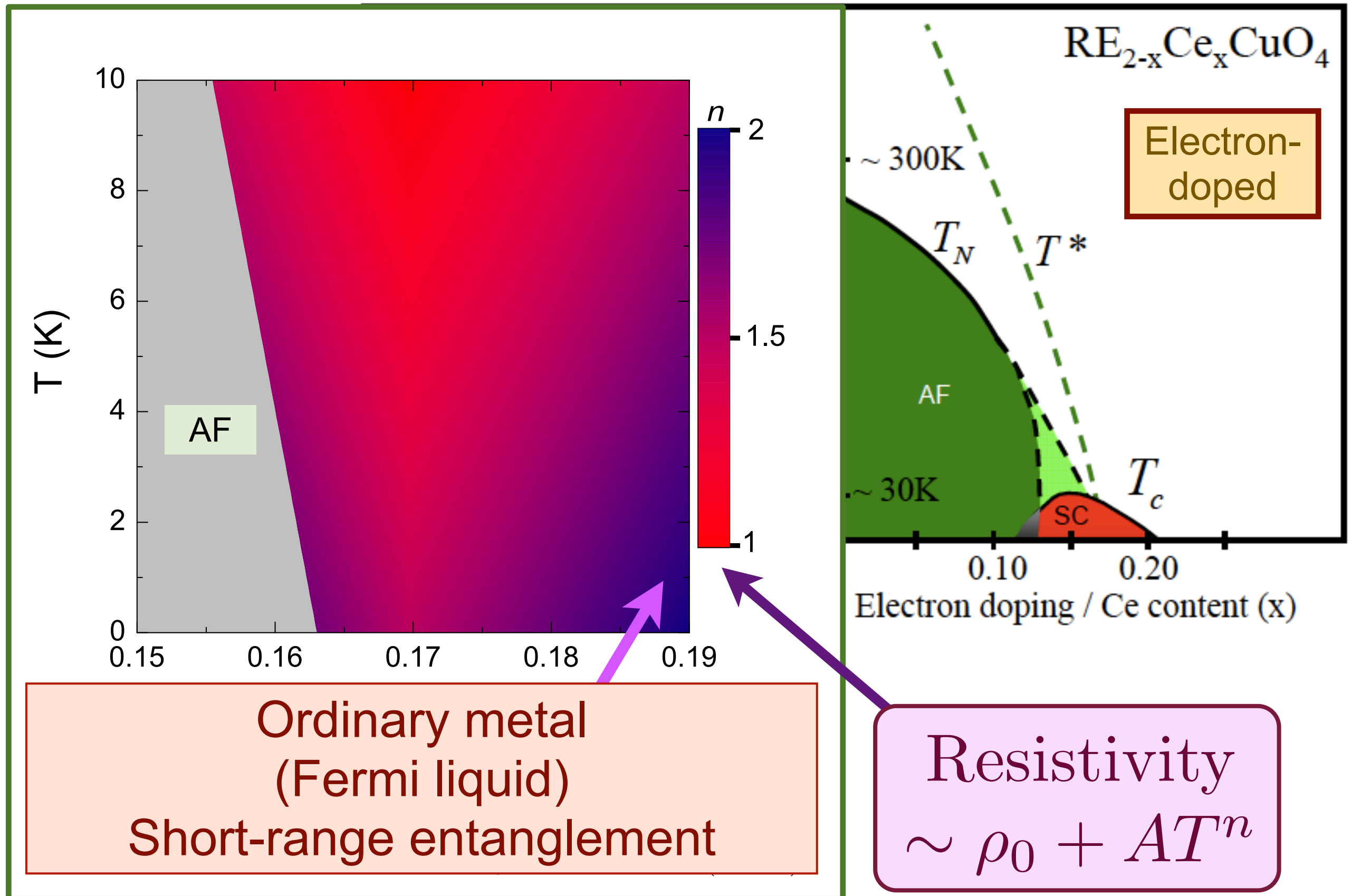


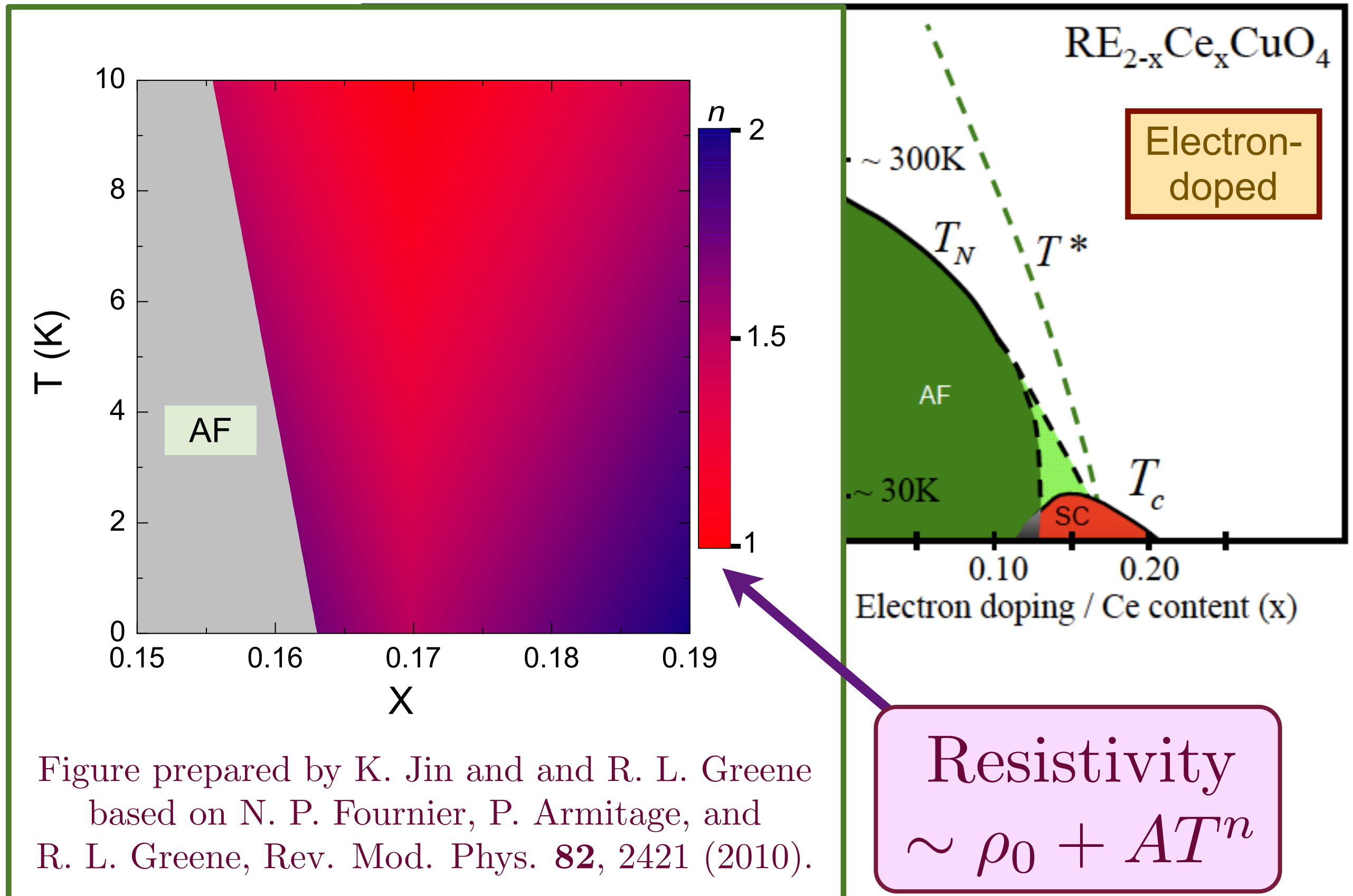
Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

Resistivity
 $\sim \rho_0 + AT^n$

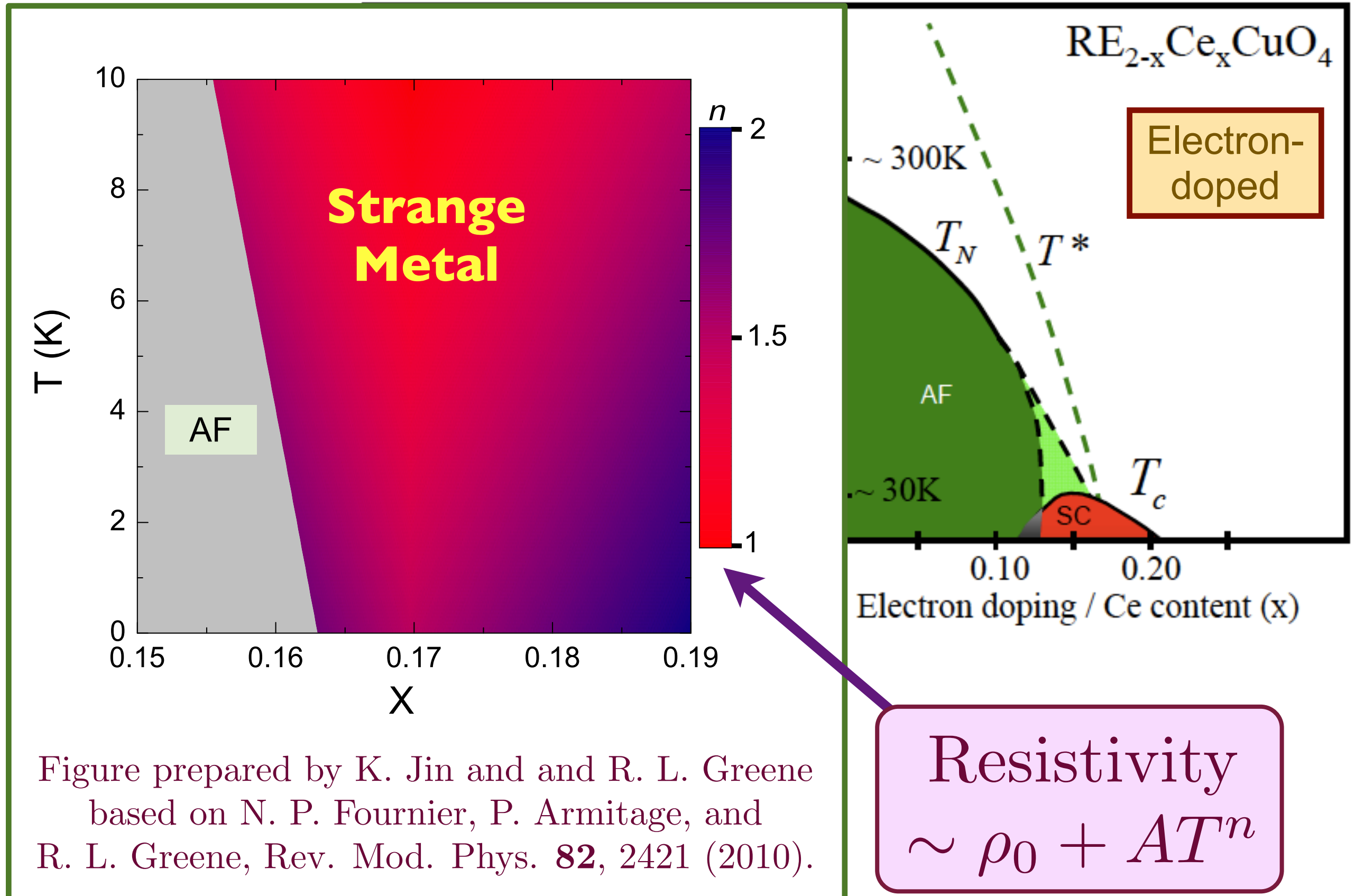
Electron-doped cuprate superconductors



Electron-doped cuprate superconductors



Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

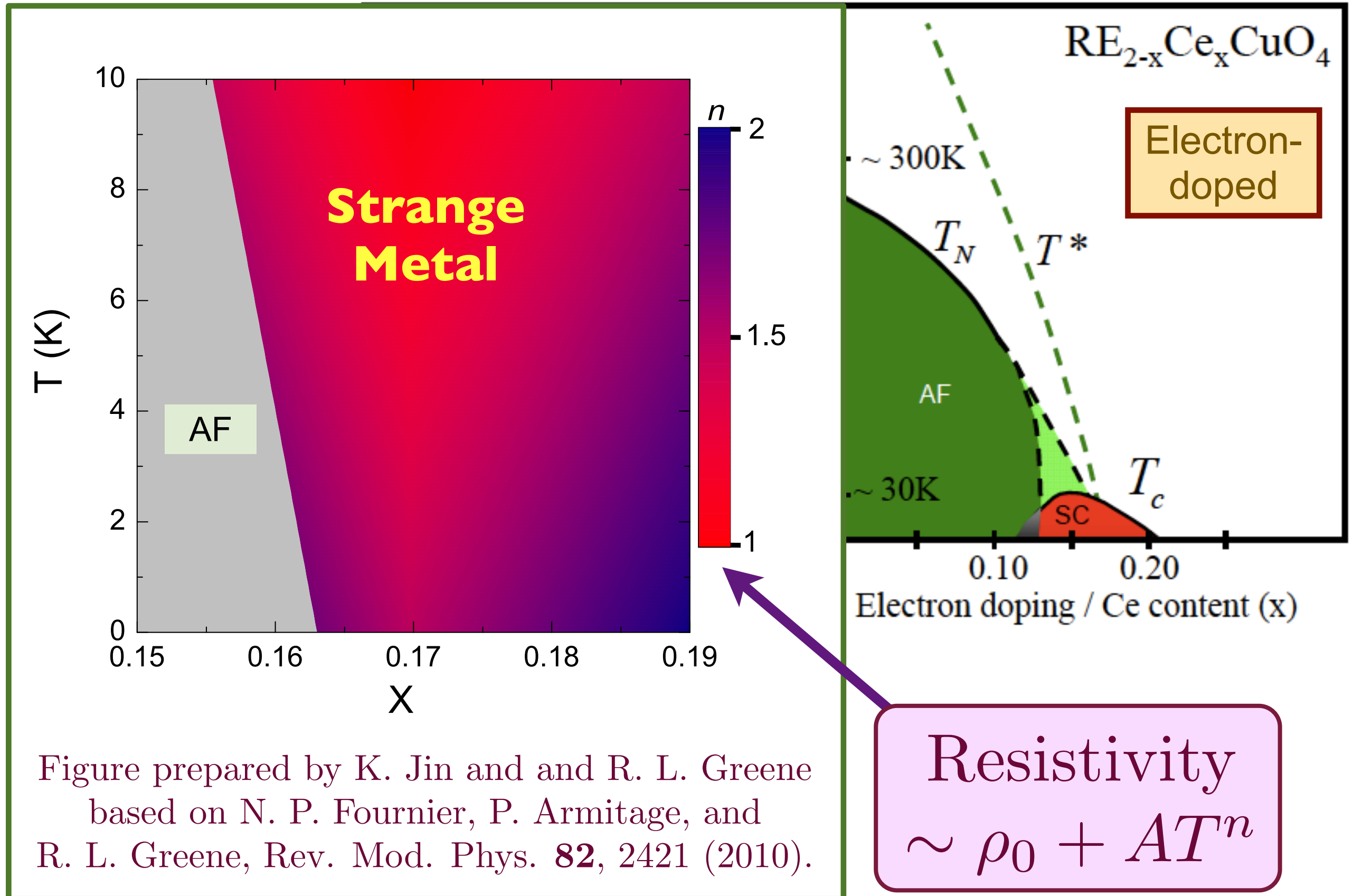
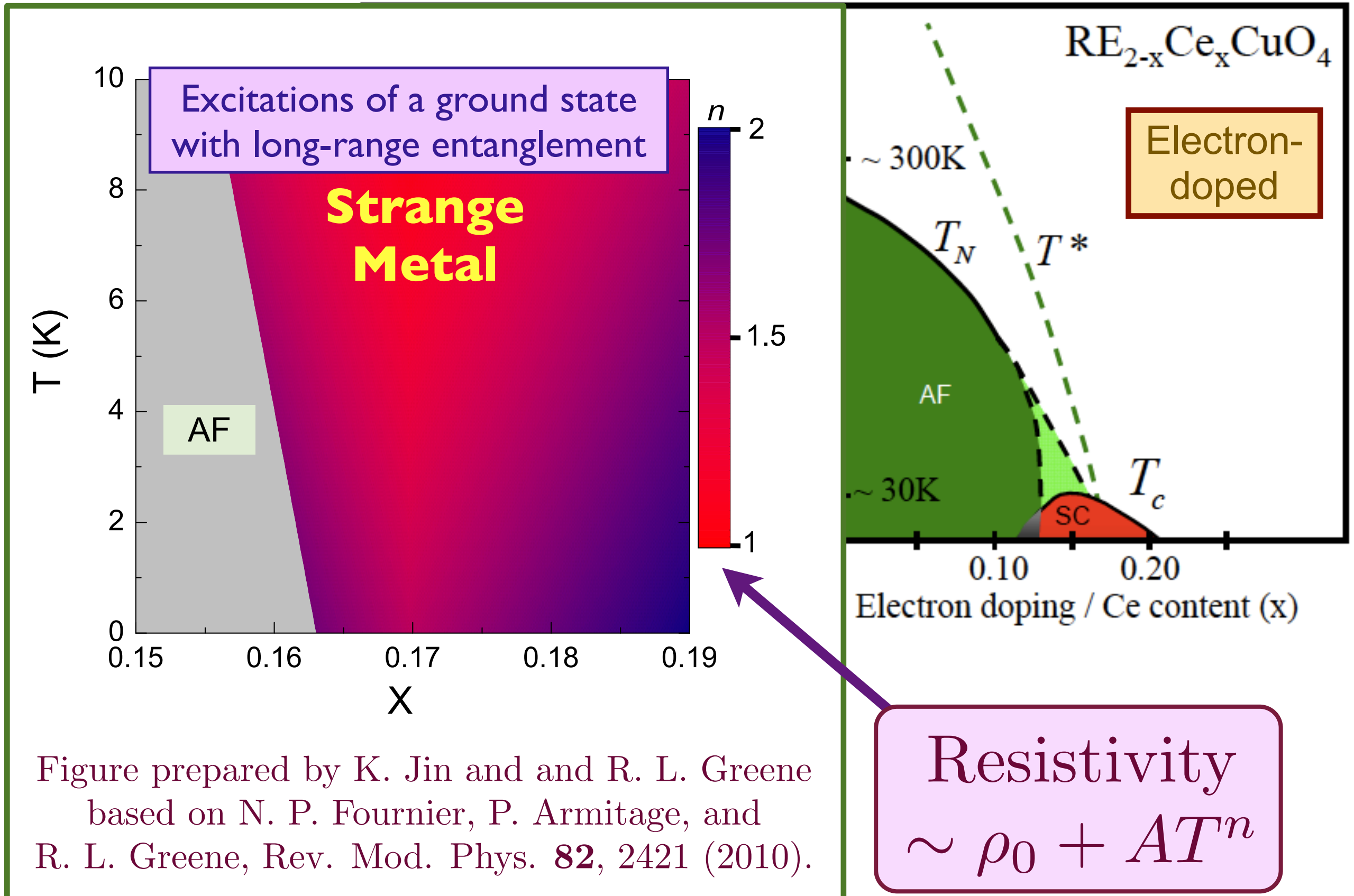


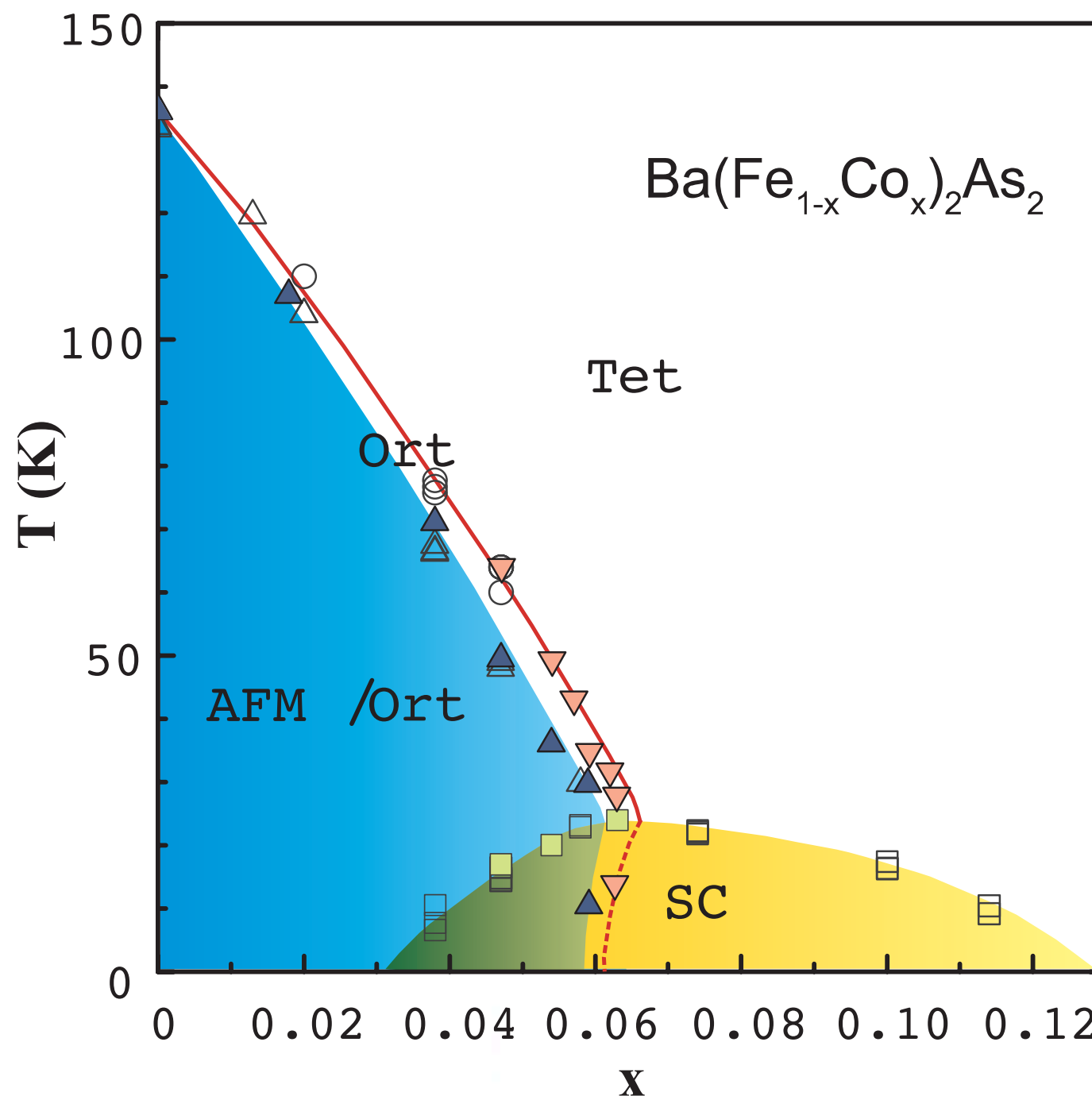
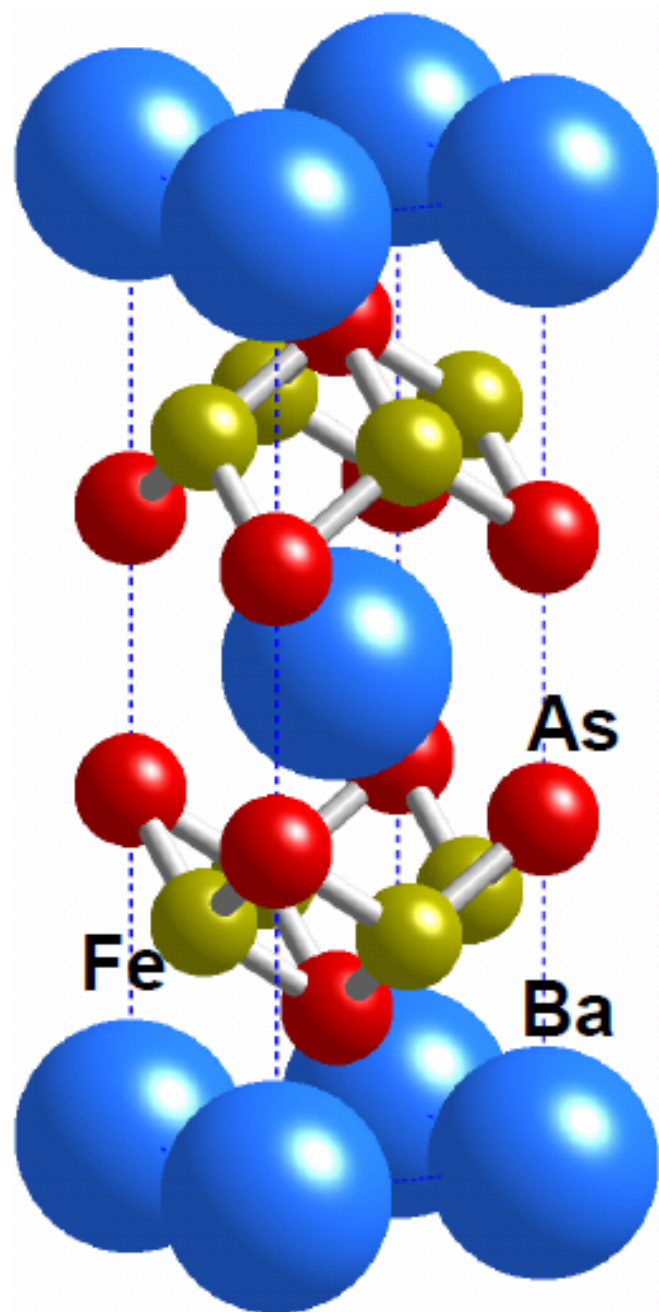
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Electron-doped cuprate superconductors



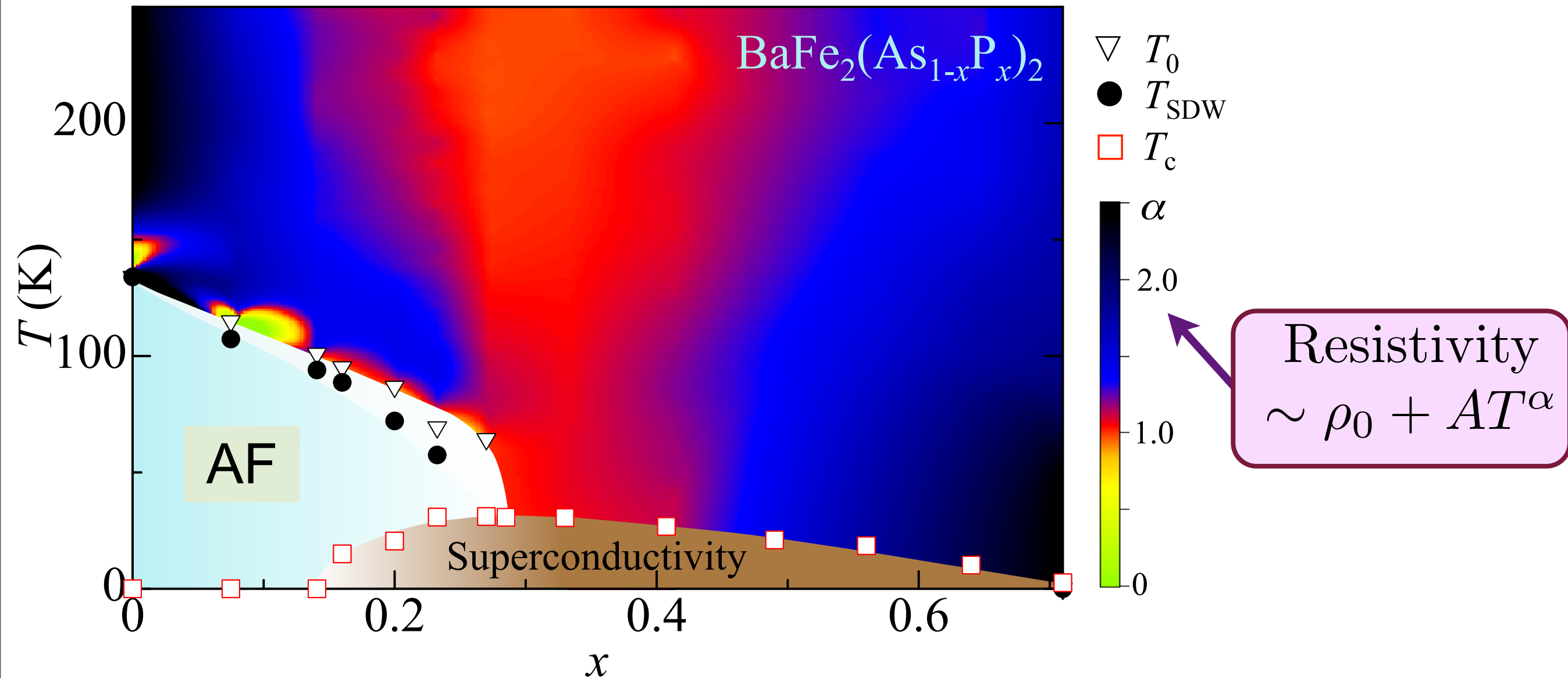
Iron pnictides:

a new class of high temperature superconductors



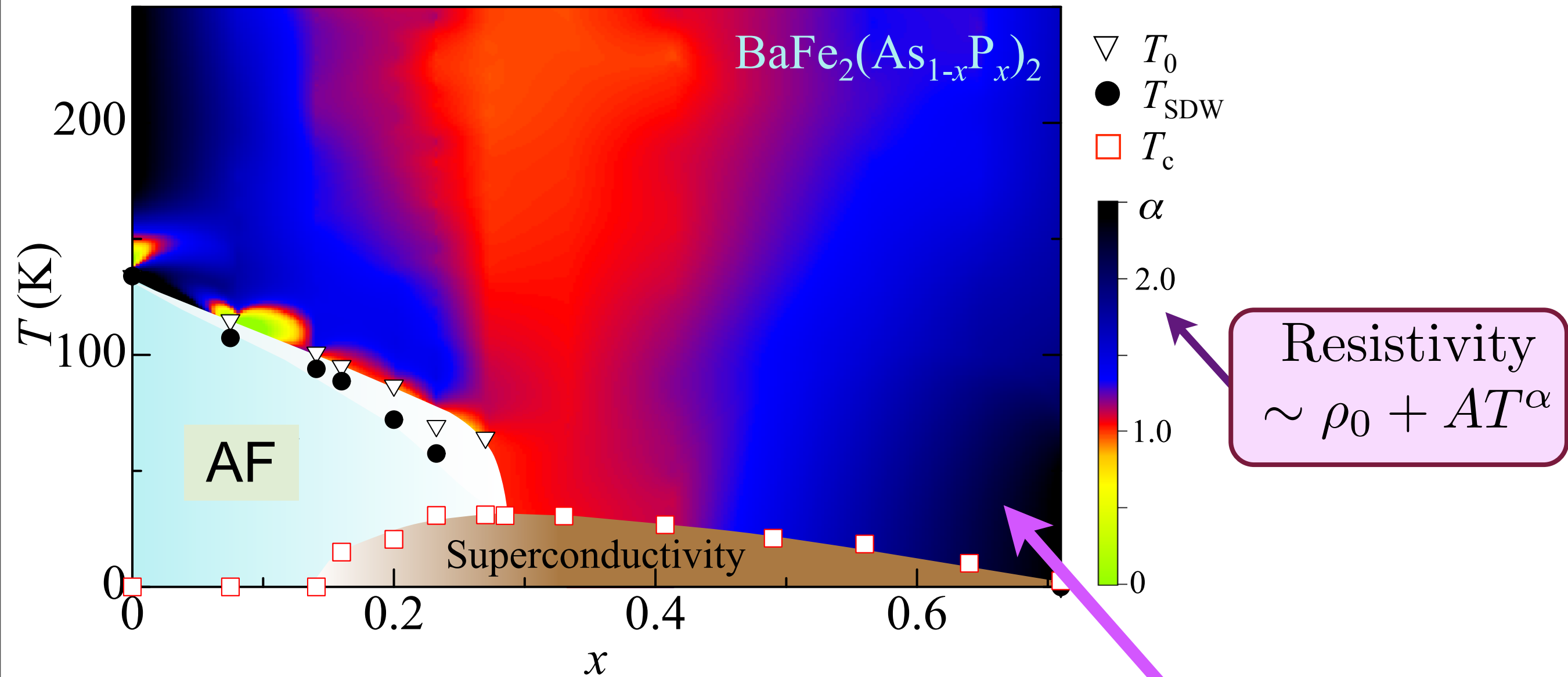
S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

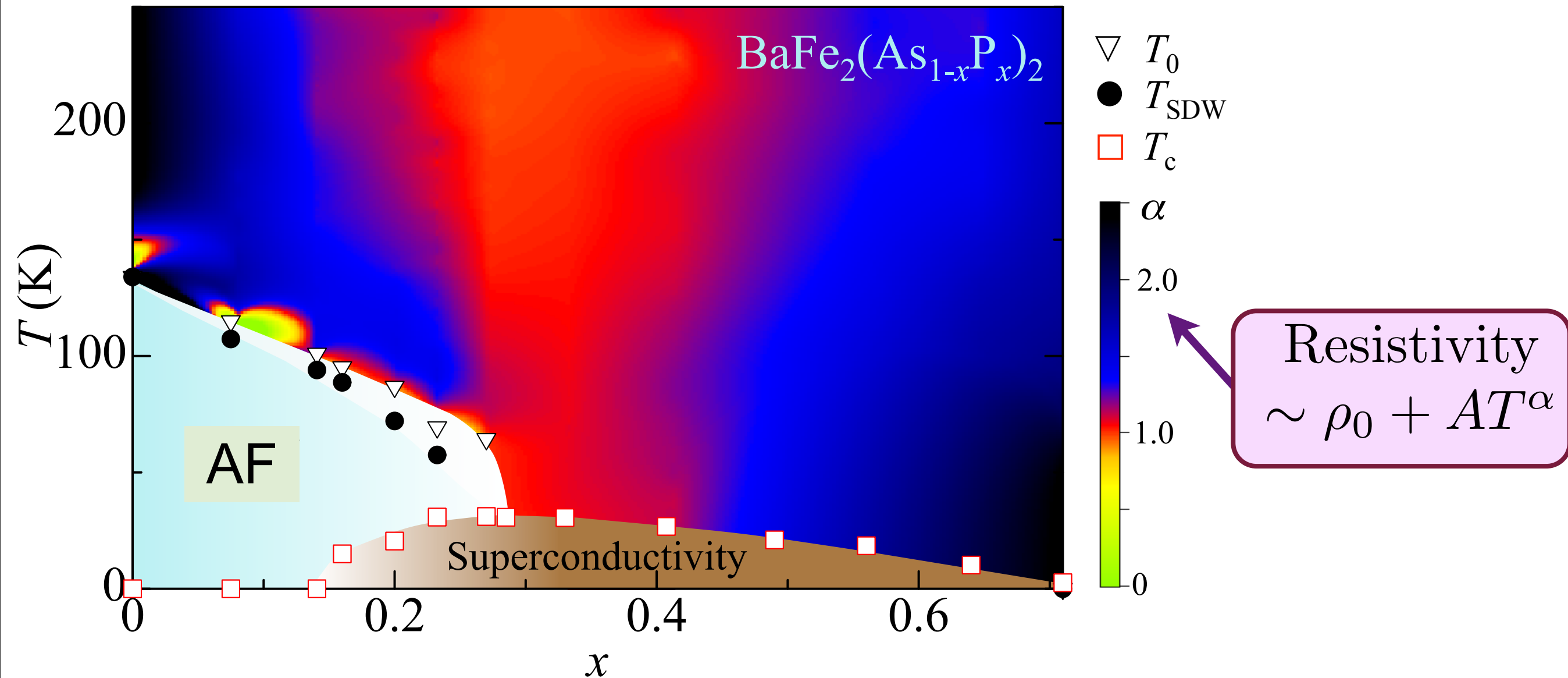
Temperature-doping phase diagram of the iron pnictides:



Ordinary metal
 (Fermi liquid)
 Short-range entanglement

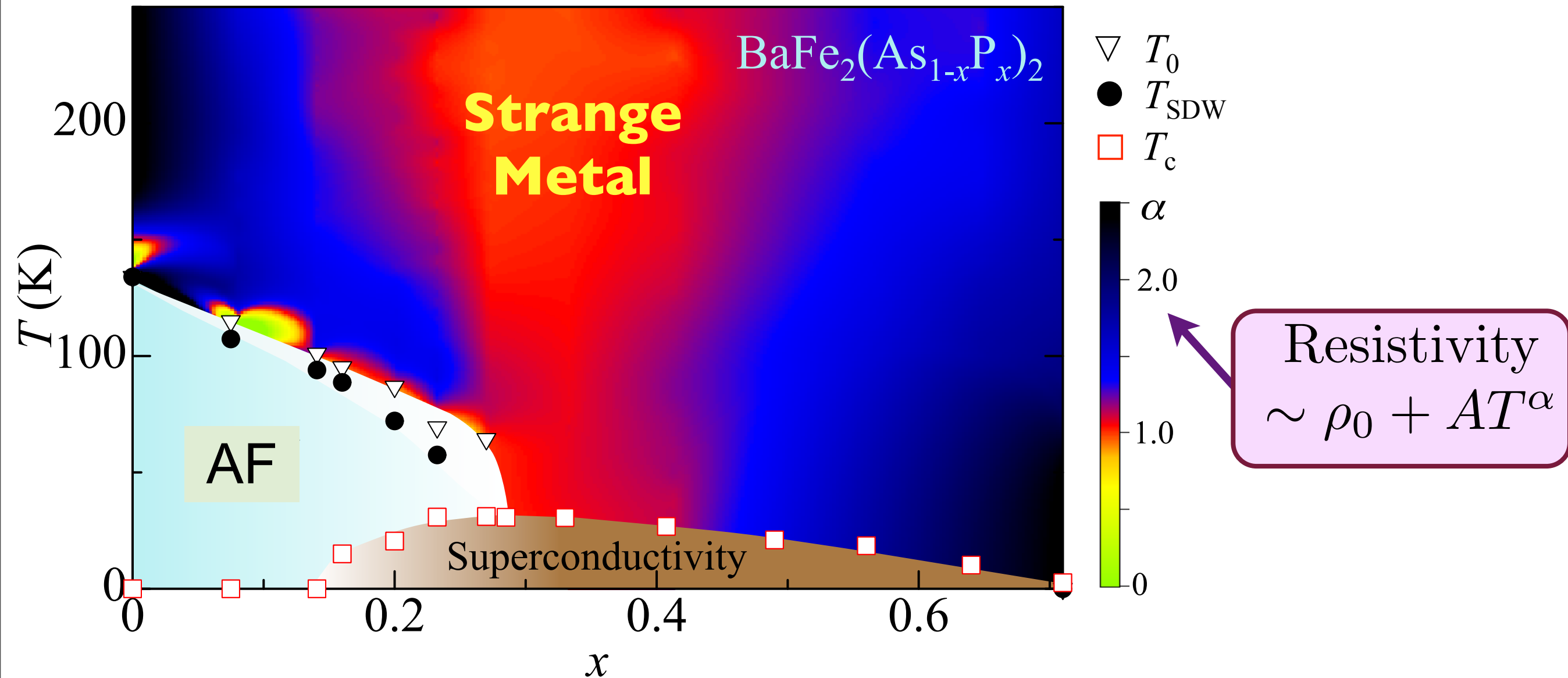
S. Kasahara, T. Shibauchi, K. Hashi
 H. Ikeda, H. Takeya,
Physical R

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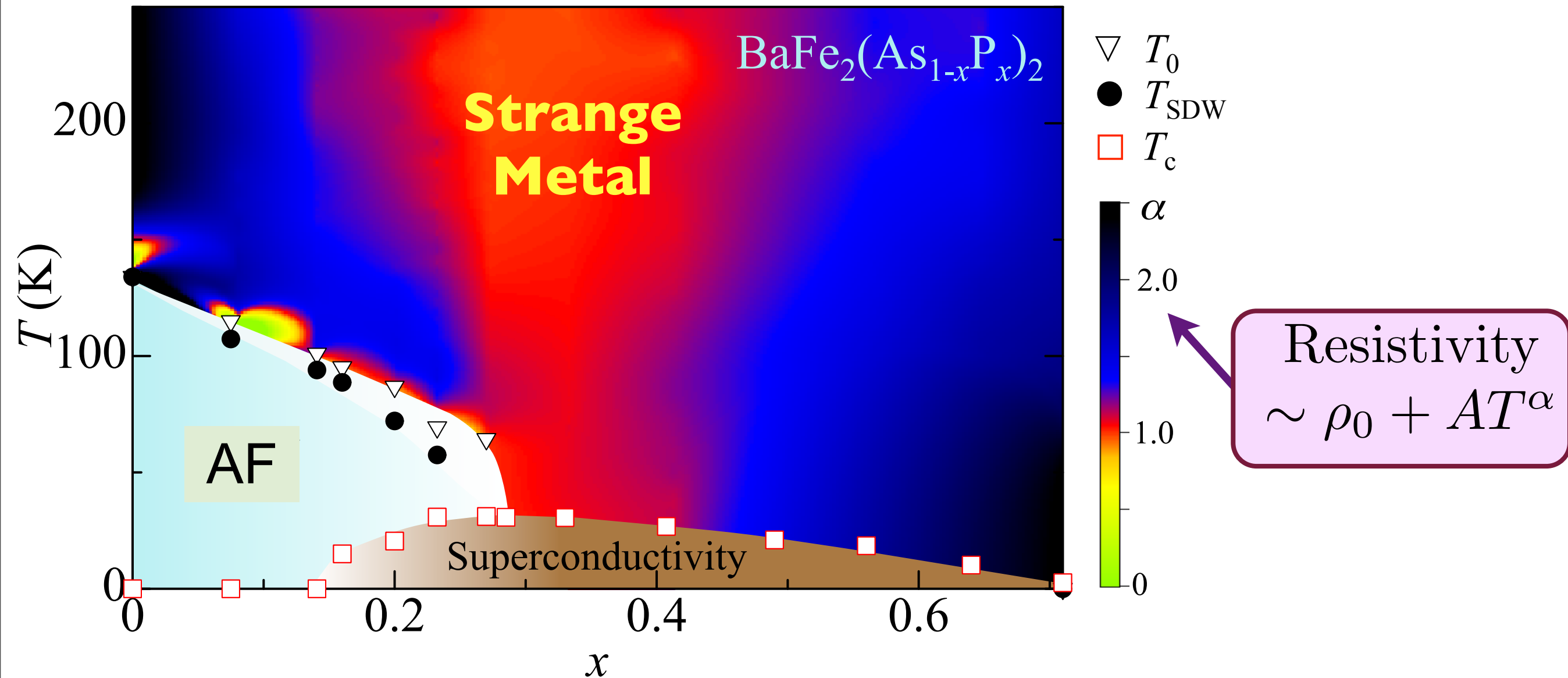
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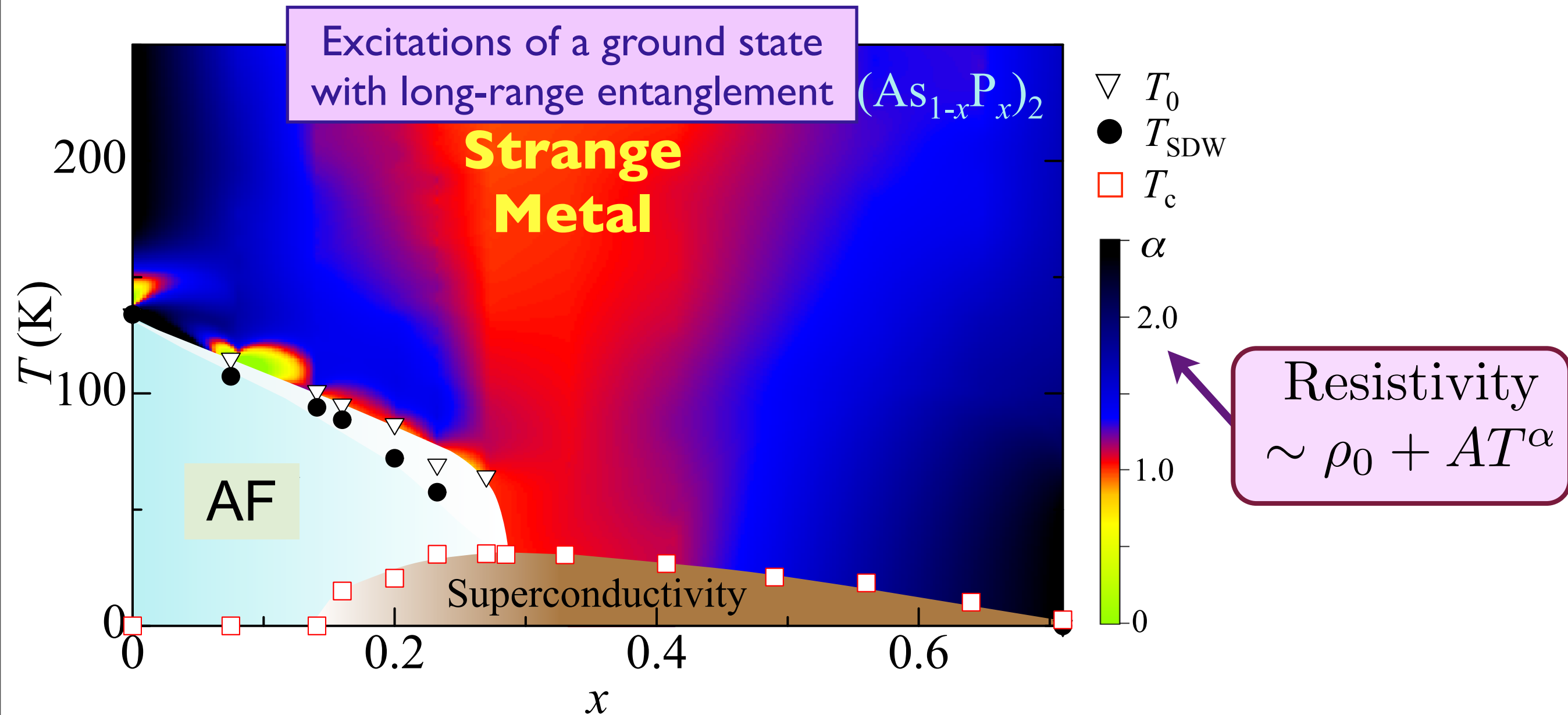
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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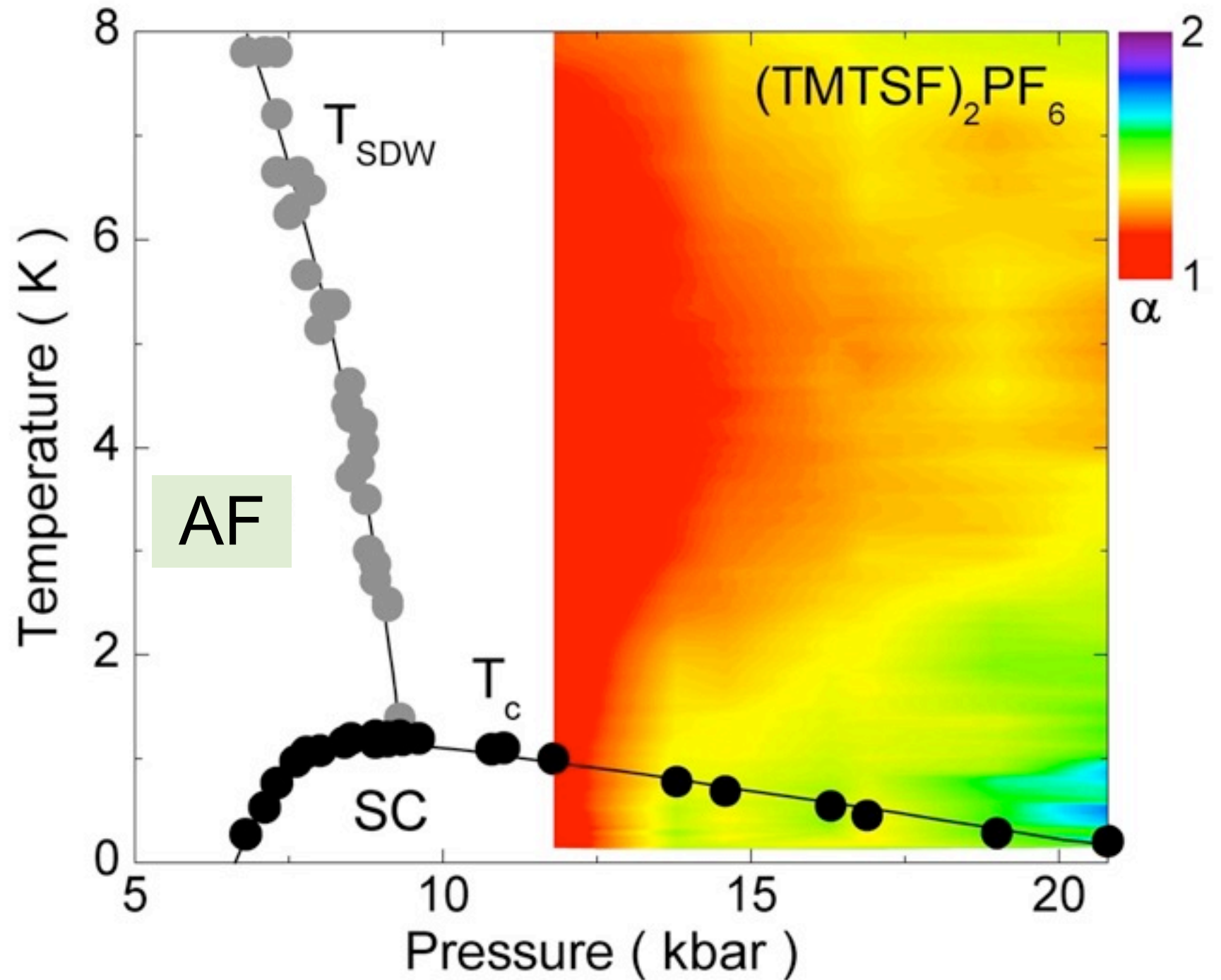
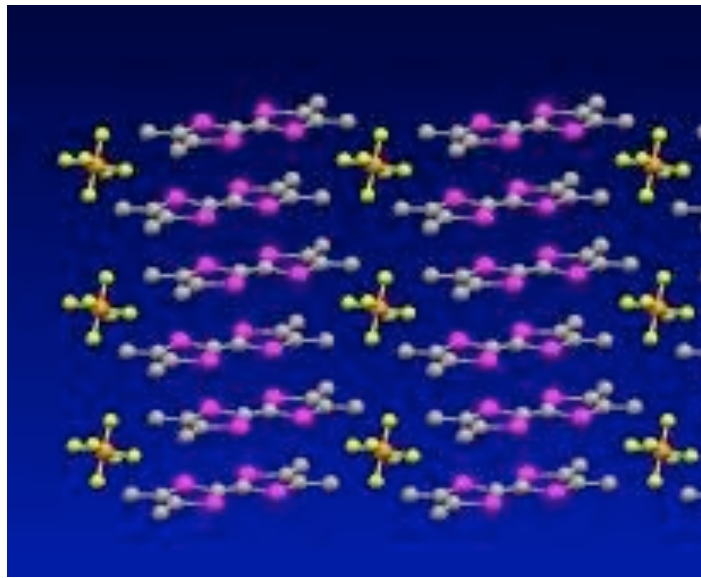
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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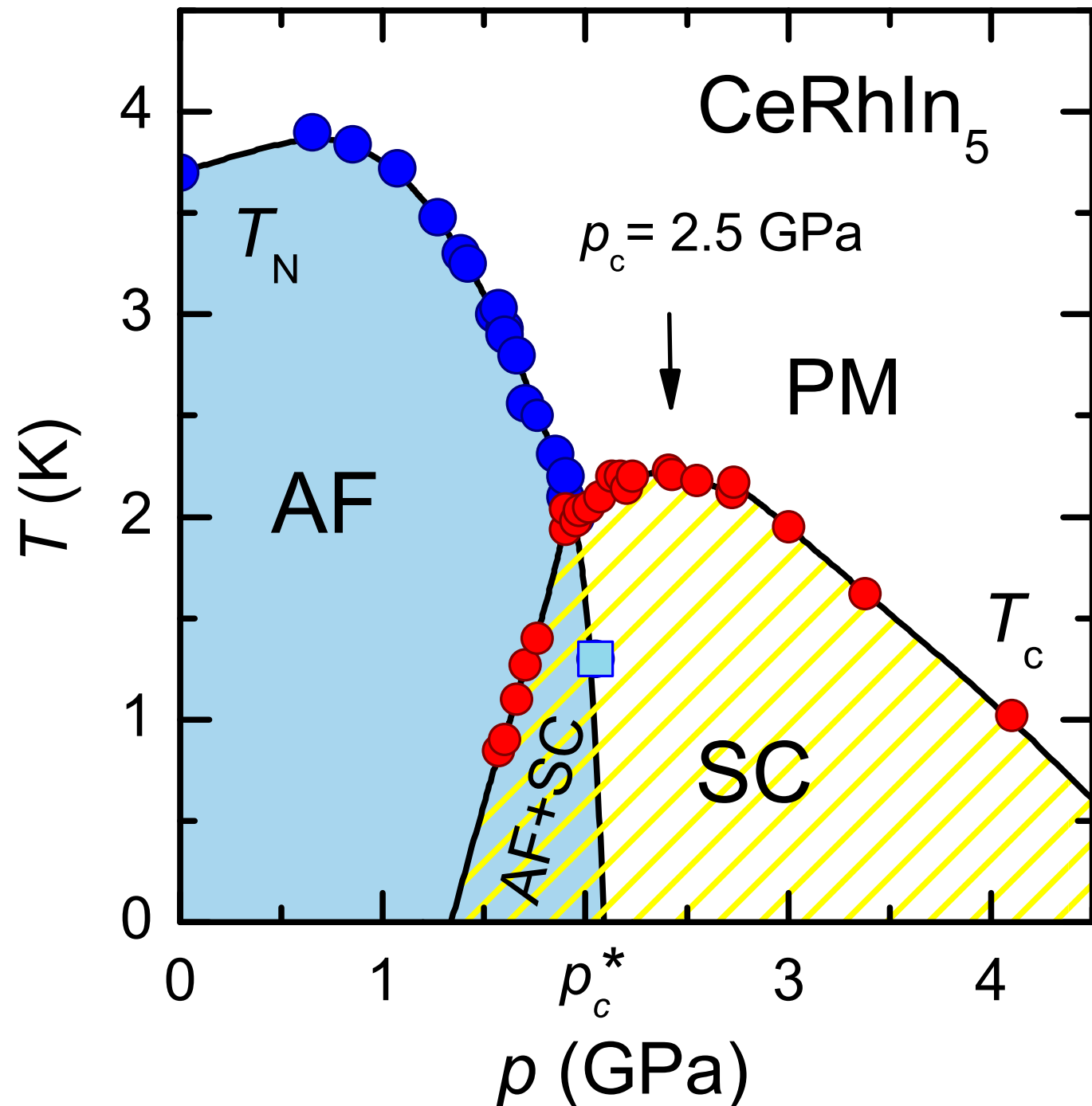
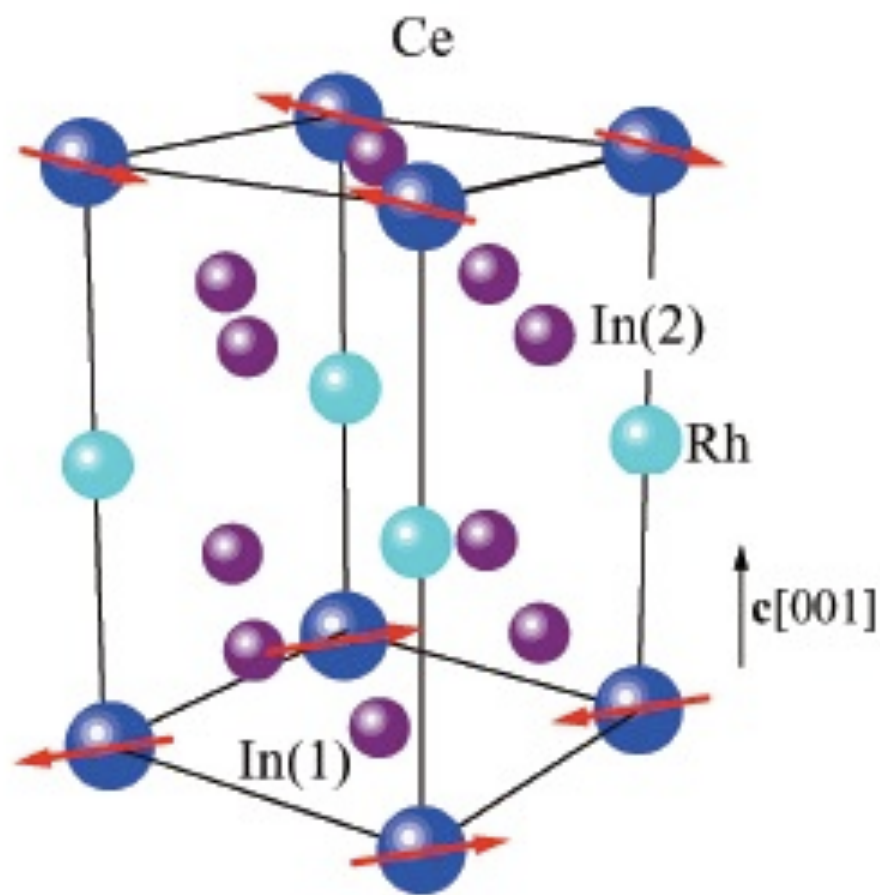
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Temperature-pressure phase diagram of an organic superconductor



N. Doiron-Leyraud, P. Auban-Senzier, S. Rene de Cotret, A. Sedeki, C. Bourbonnais, D. Jerome, K. Bechgaard, and Louis Taillefer, Physical Review B 80, 214531 (2009)

Temperature-pressure phase diagram of an heavy-fermion superconductor

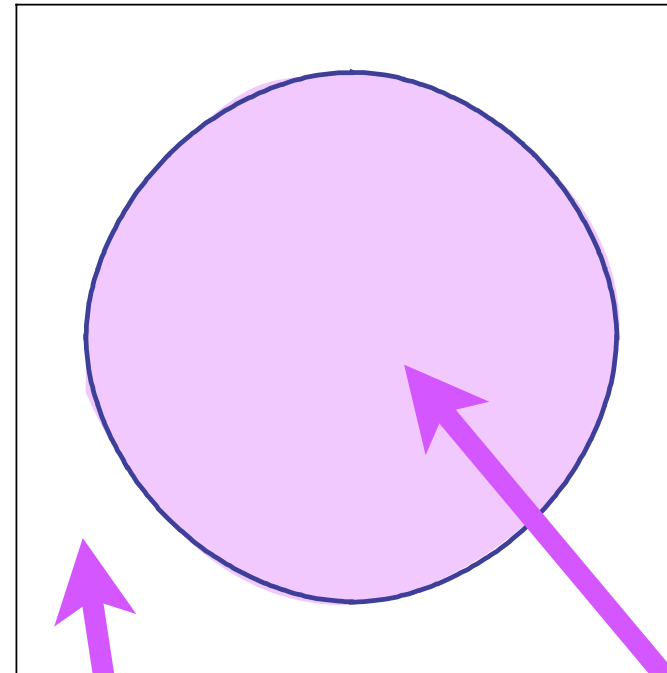


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.

Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

Ordinary metals (Fermi liquids)

Metal with “large”
Fermi surface

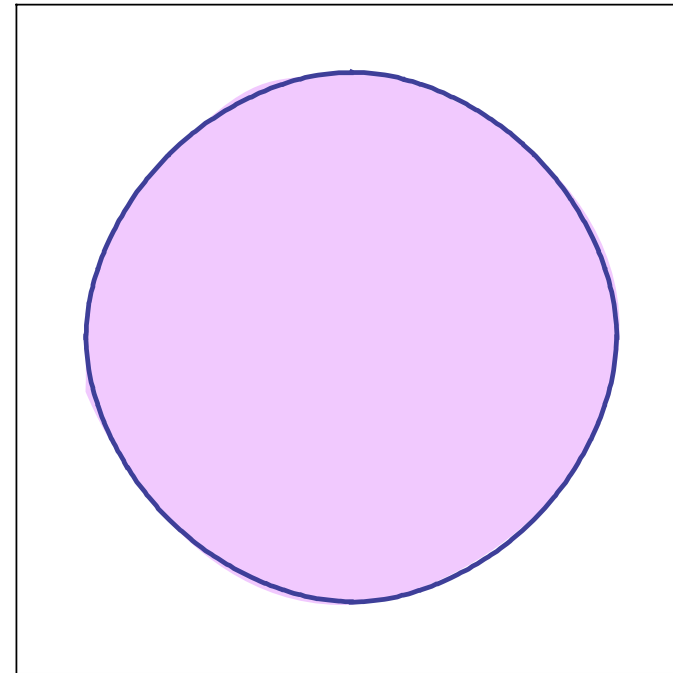


Momenta with
electron states
occupied

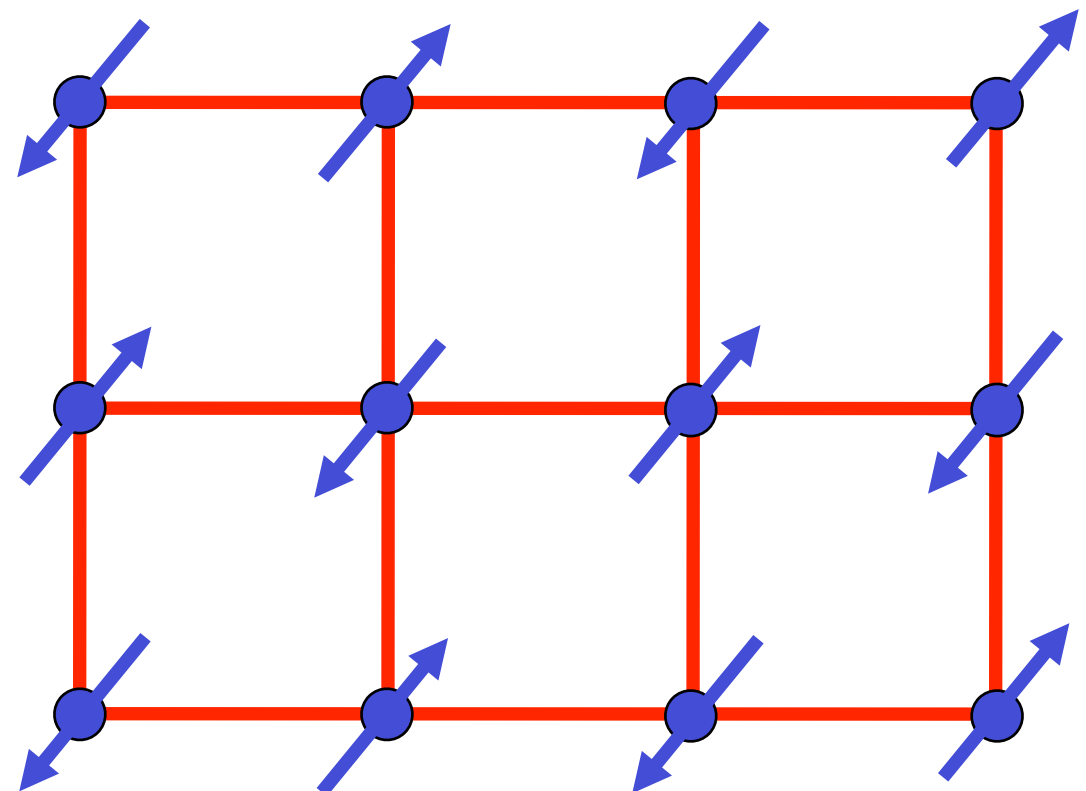
Momenta with
electron states
empty

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



+

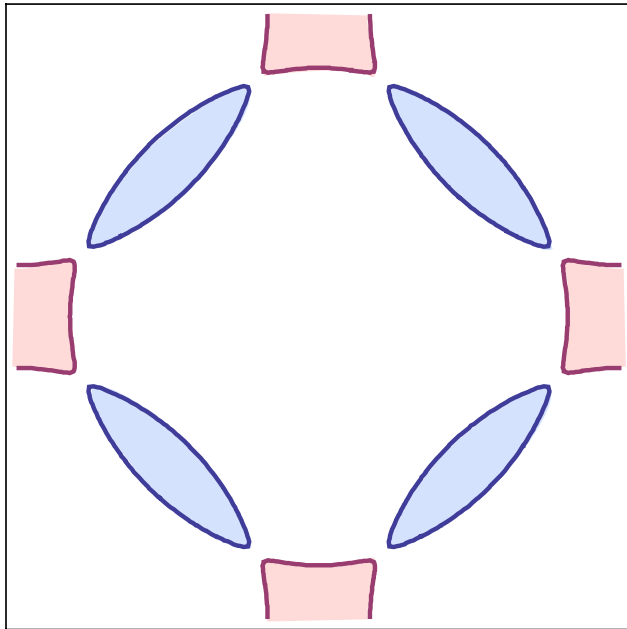


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where \mathbf{K} is the ordering wavevector.

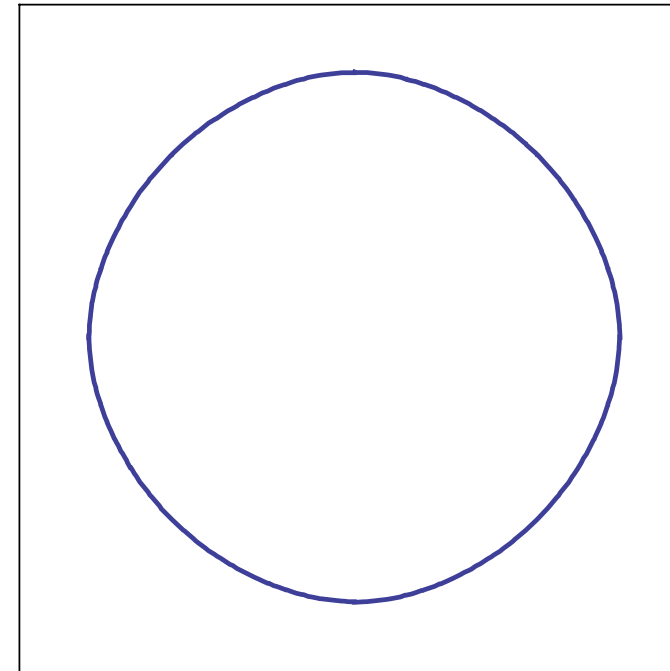
Fermi surface+antiferromagnetism



AF

$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



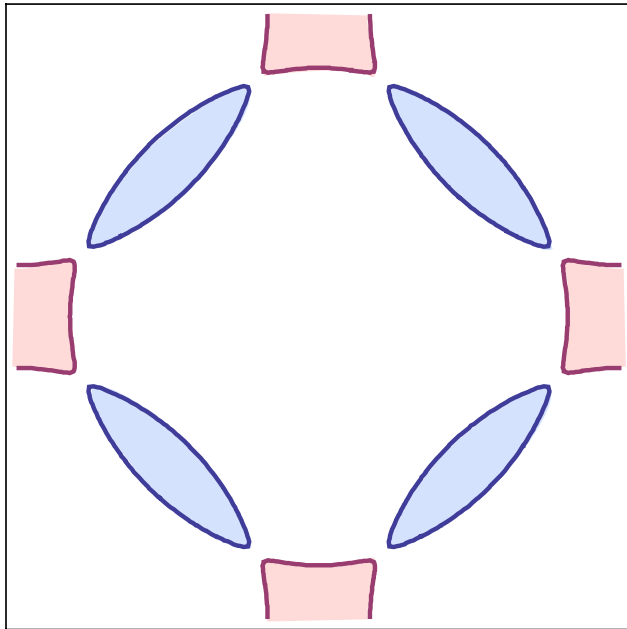
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

← Increasing interaction

Fermi surface reconstruction and
onset of antiferromagnetism

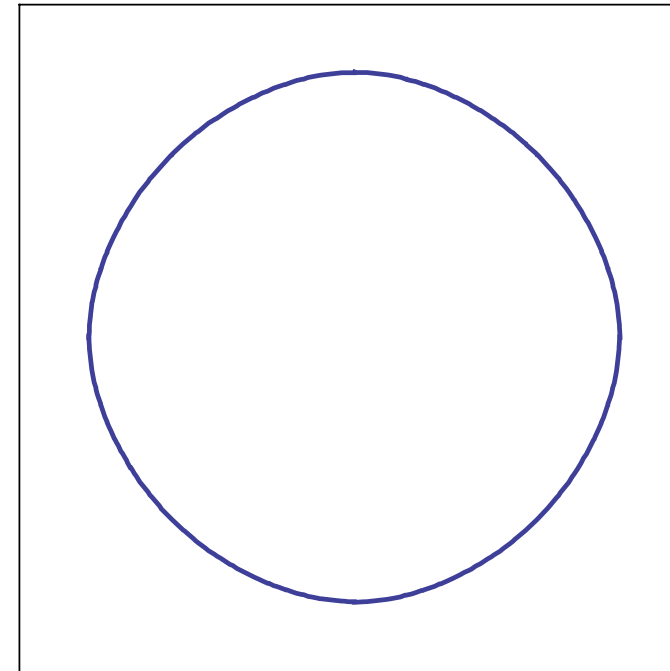
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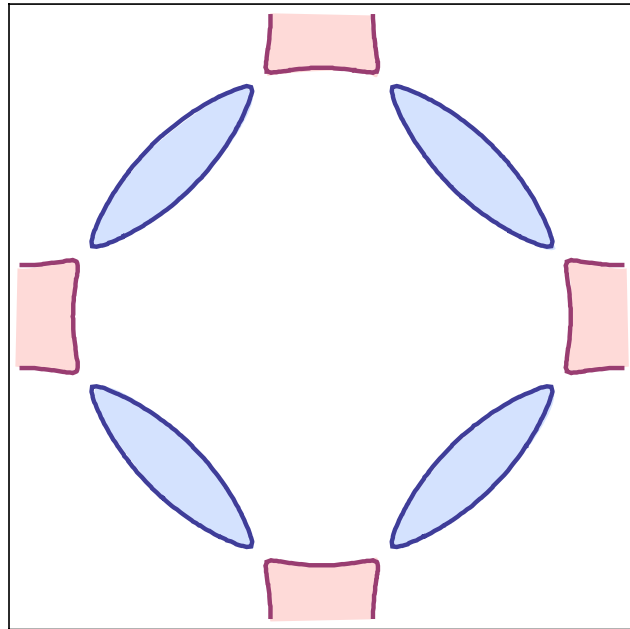
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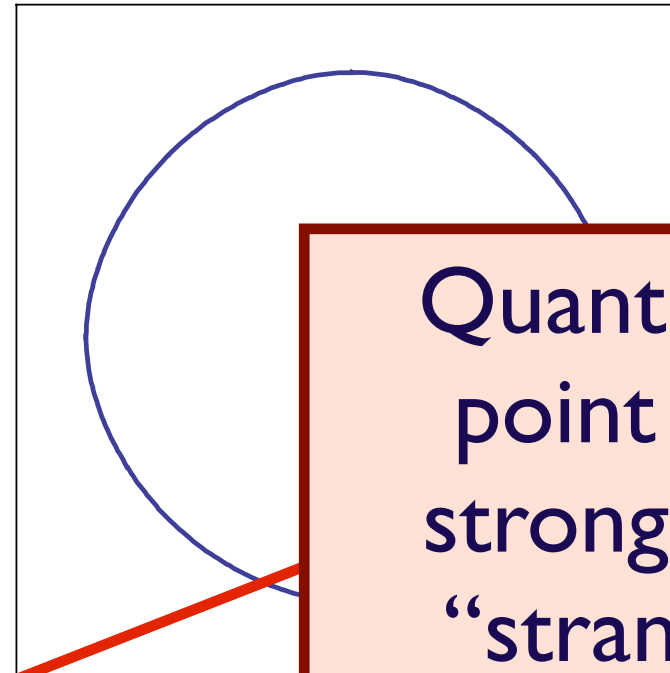
Fermi surface+antiferromagnetism



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and hole pockets



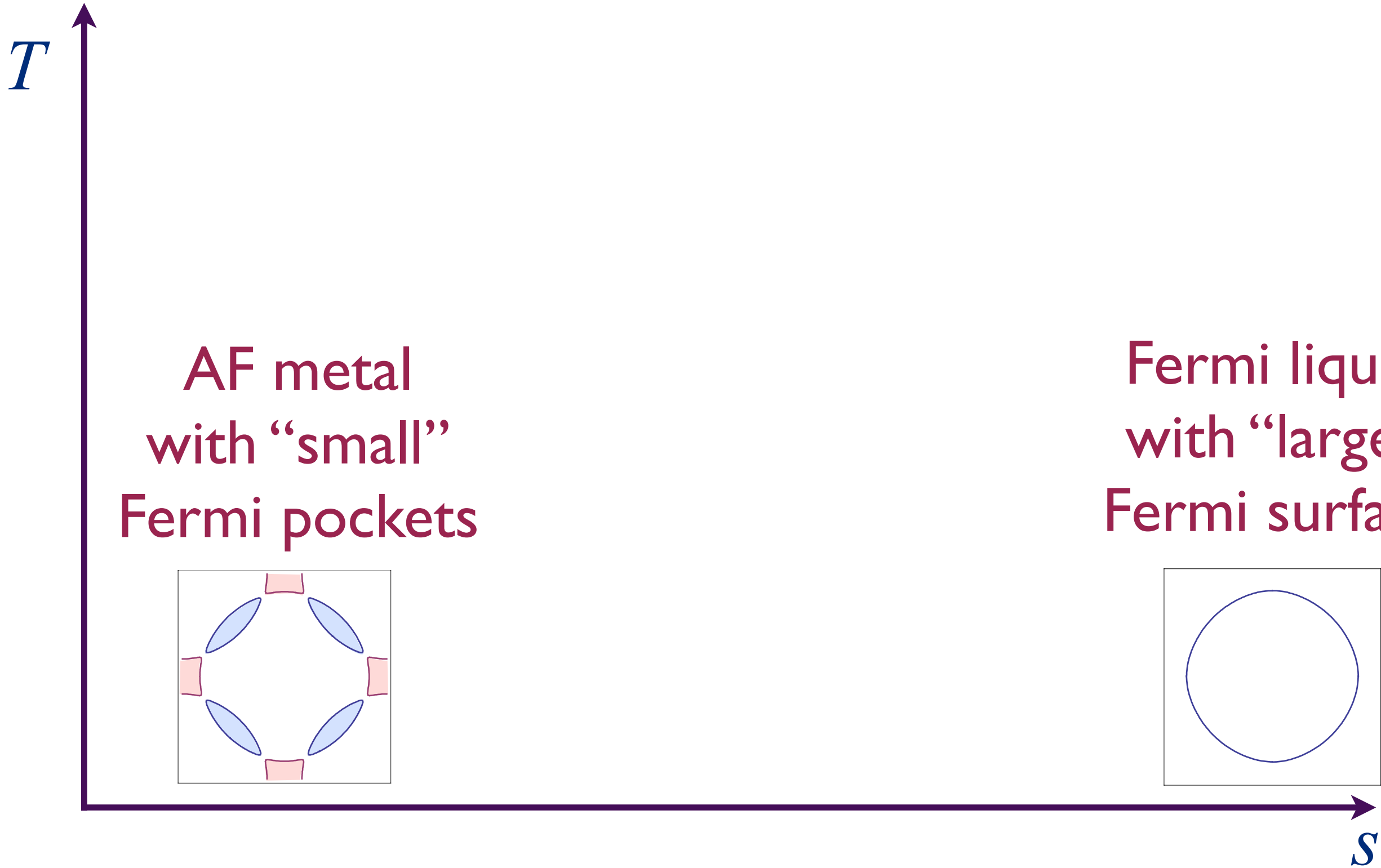
$$\langle \vec{\varphi} \rangle$$

Metal with “large”
Fermi surface

Quantum critical
point realizes a
strongly-coupled
“strange metal”
with long-range
entanglement !

Increasing interaction

Fermi surface reconstruction and
onset of antiferromagnetism

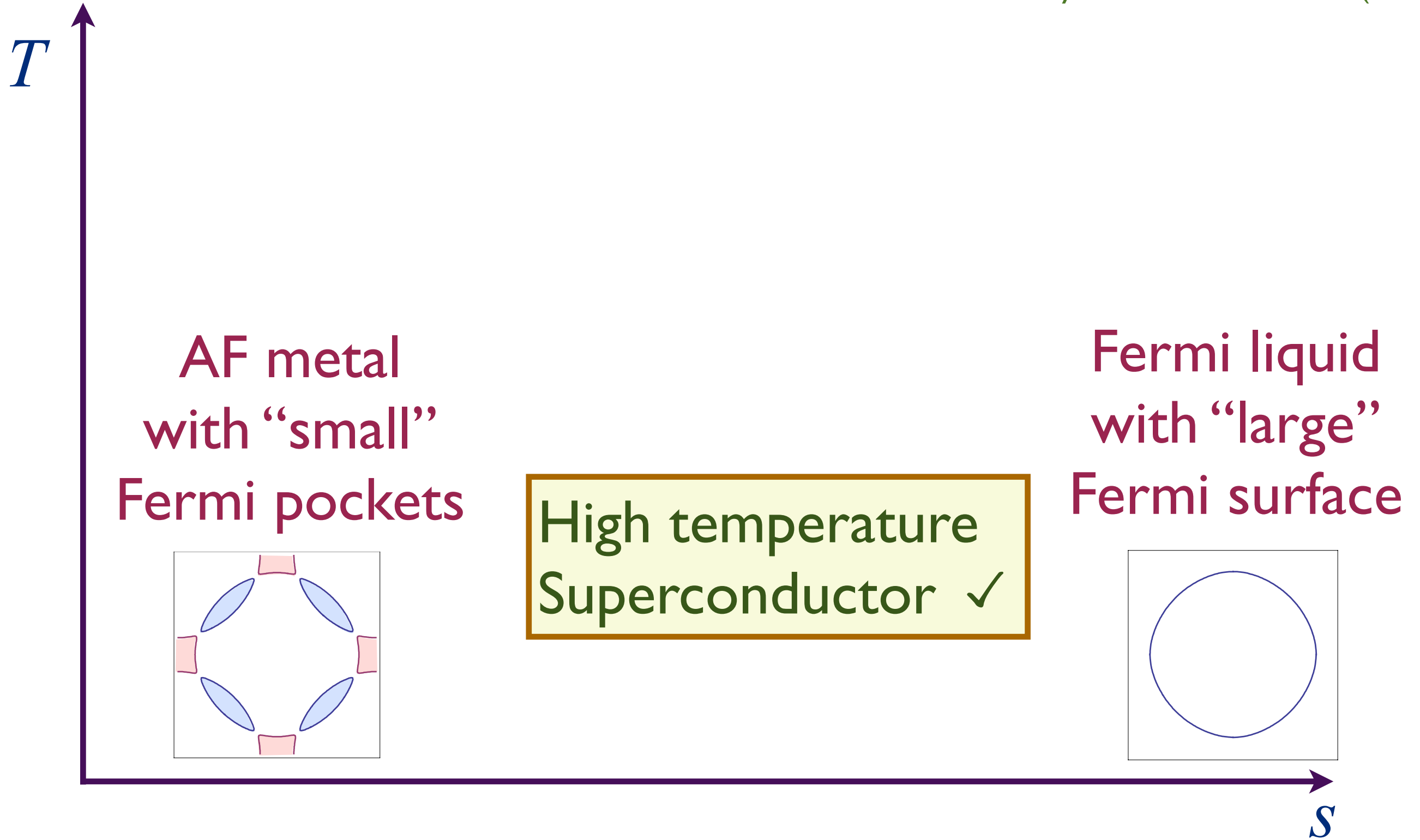


D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)

S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



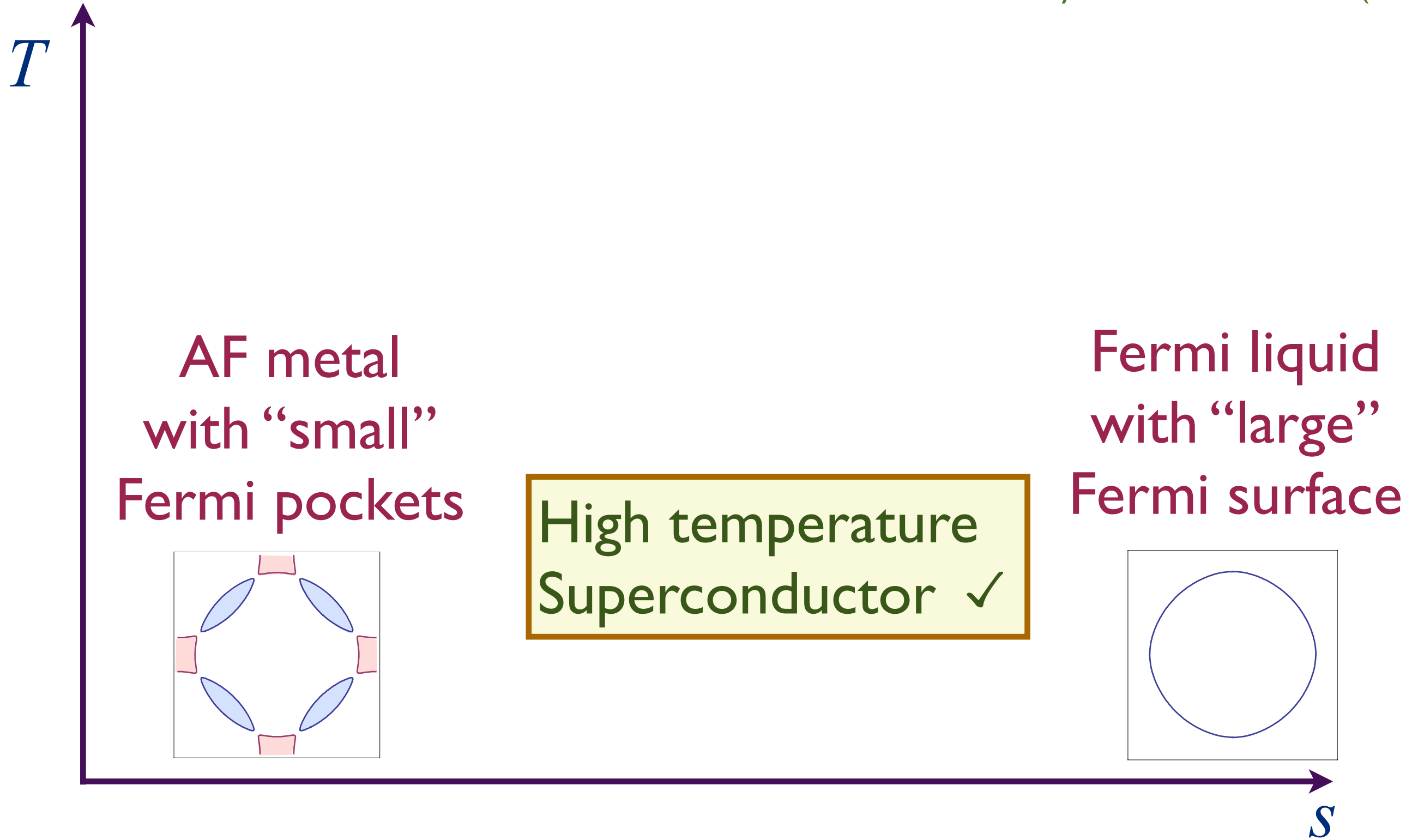
Davis

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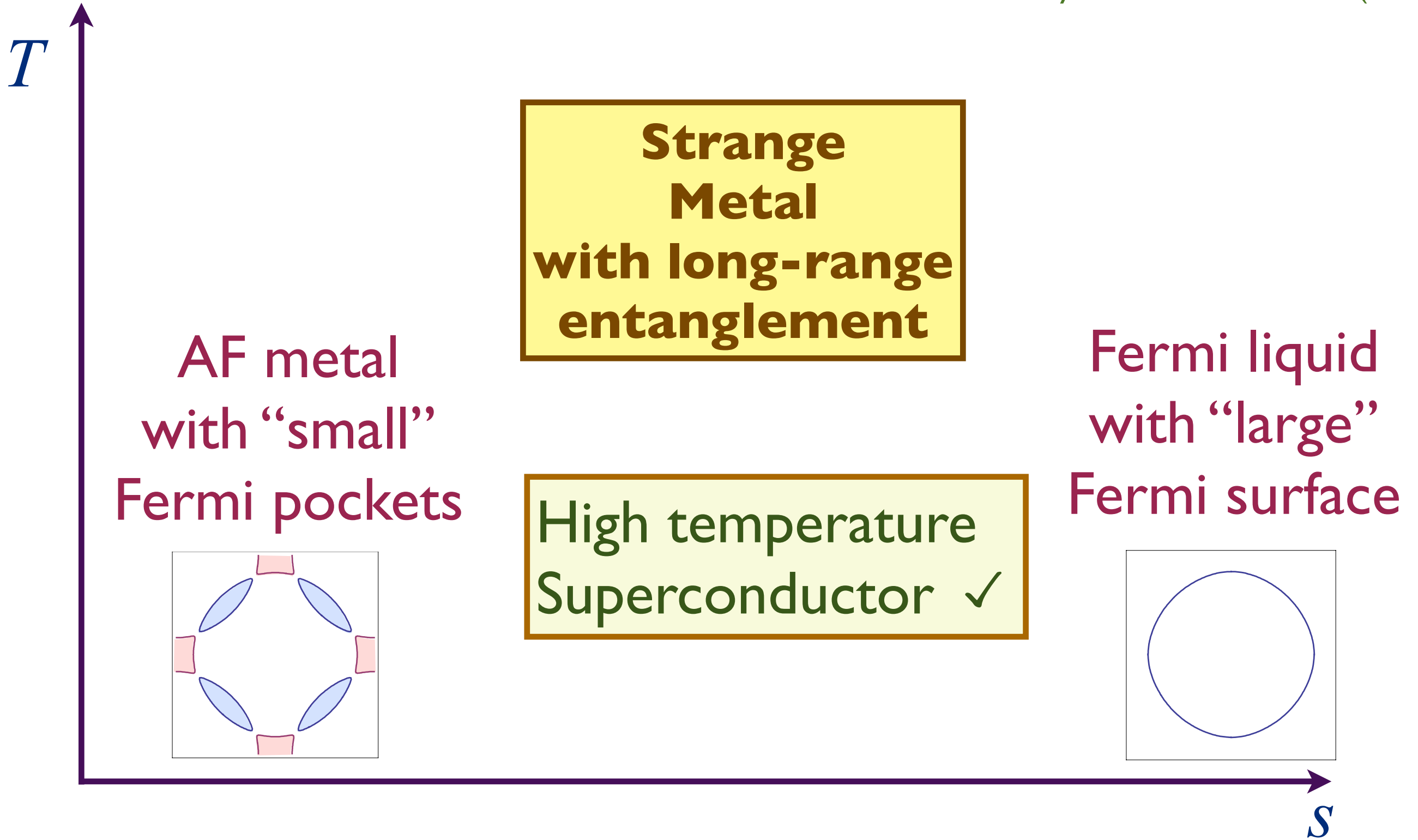
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M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



Davis

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

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Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?

Yes

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals ?

Yes, lots of them, with many “strange” properties !

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic “strange metals” have the correct type of long-range entanglement ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic “strange metals” have the correct type of long-range entanglement ?

Yes, a very select subset has the proper logarithmic violation of the area law of entanglement entropy !!

These are now being studied intensively.....

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”