Competing orders in the cuprate superconductors

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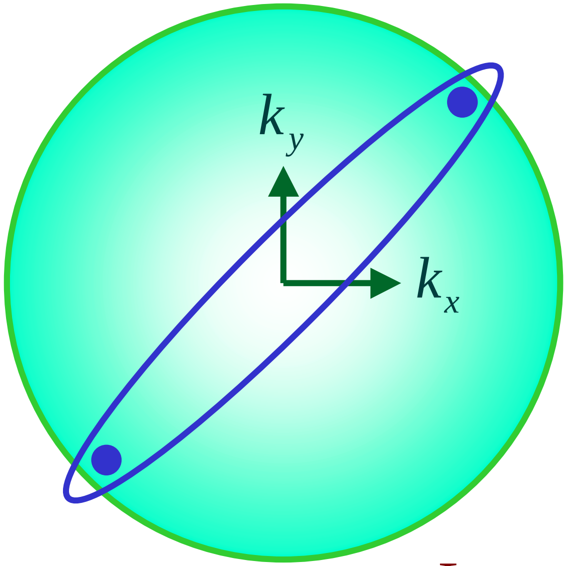
Ying Zhang

Talk online at
http://pantheon.yale.edu/~subir
(Search for "*Sachdev*" on Google)

1701 YALE UNIVERSITY TERCENTENNIAL 2001



Hole-doped cuprates are BCS superconductors with

$\langle c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \rangle \equiv \Delta_{\mathbf{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$ d-wave pairing

$\langle \vec{S} \rangle = 0$ spin-singlet

Low energy excitations:

Superflow: $\Delta_0 \to \Delta_0 e^{i\theta}$

$S = 1/2$ fermionic quasiparticles: $E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$

BCS theory also predicts that the Fermi surface, with gapless quasiparticles, will reveal itself when $\Delta_0 \to 0$, either locally or globally at low temperatures. Δ_0 can be suppressed by a strong magnetic field, and near vortices, impurities and interfaces.

Superconductivity in a doped Mott insulator

Hypothesis: cuprate superconductors have low energy excitations associated with additional order parameters

Theory and experiments indicate that the most likely candidates are spin density waves and associated "charge" order

Superconductivity can be suppressed globally by a strong magnetic field or large current flow.

Competing orders are also revealed when superconductivity is suppressed locally, near impurities or around vortices.

S. Sachdev, *Phys. Rev.* B **45**, 389 (1992);
N. Nagaosa and P.A. Lee, *Phys. Rev.* B **45**, 966 (1992);
D.P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang *Phys. Rev. Lett.* **79**, 2871 (1997);
K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).

<u>**Outline**</u>

I. Experimental introduction

II. Spin density waves (SDW) in LSCO

Tuning order and transitions by a magnetic field.

III. Connection with “charge” order – phenomenological theory

STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

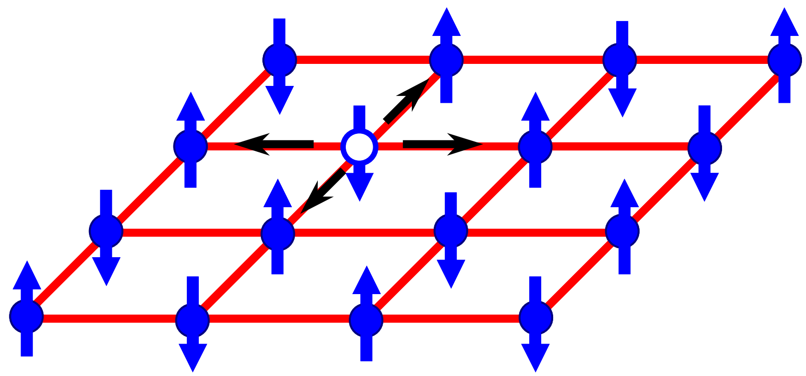
IV. Connection with “charge” order – microscopic theory

Theories of magnetic transitions predict bond-centered modulation of exchange and pairing energies with even periods---a <u>bond order wave</u>

V. Conclusions

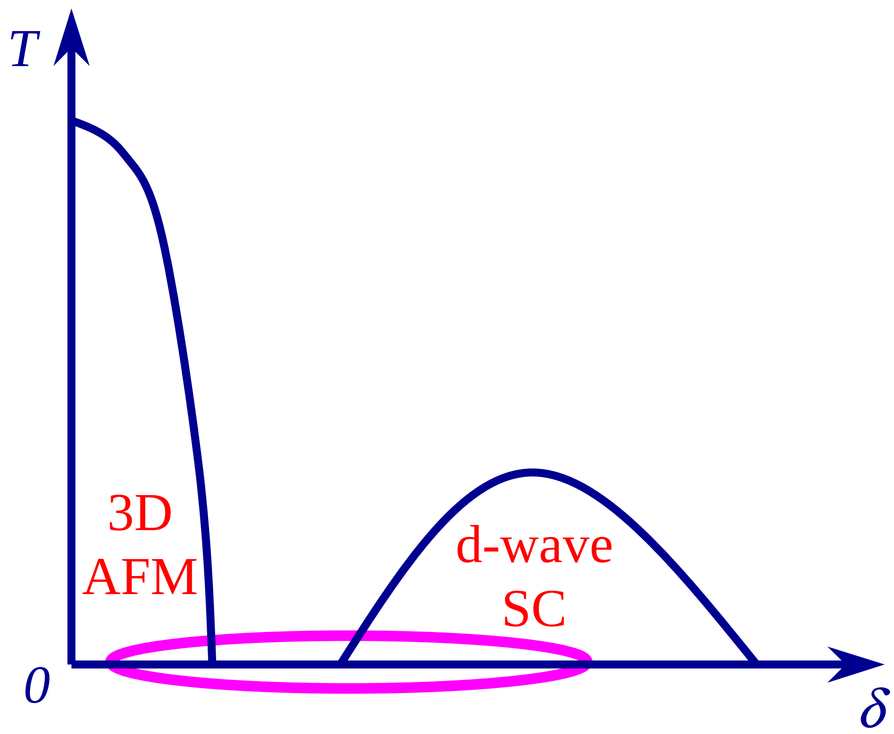
I. Experimental introduction

The doped cuprates

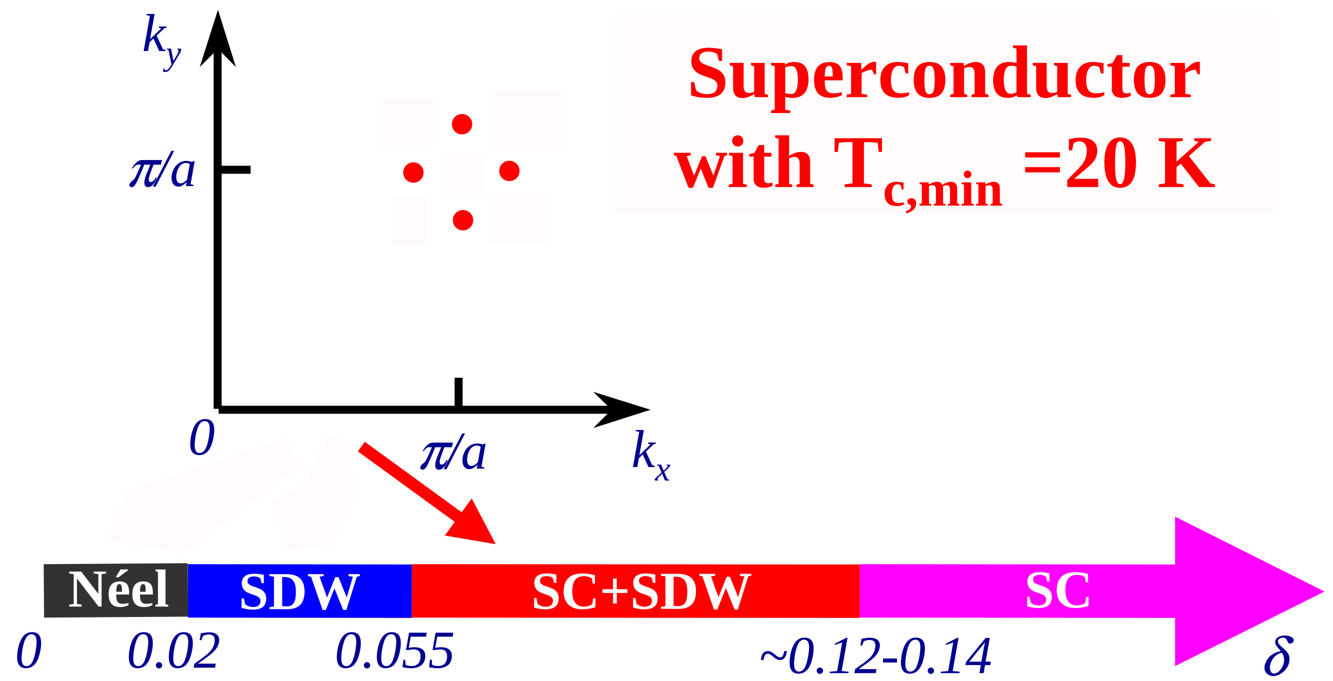


2-D CuO_2 plane
2-D CuO_2 plane
with finite hole doping
Néel ordered ground state at zero doping

Phase diagram of the doped cuprates



T = 0 phases of LSCO



S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).

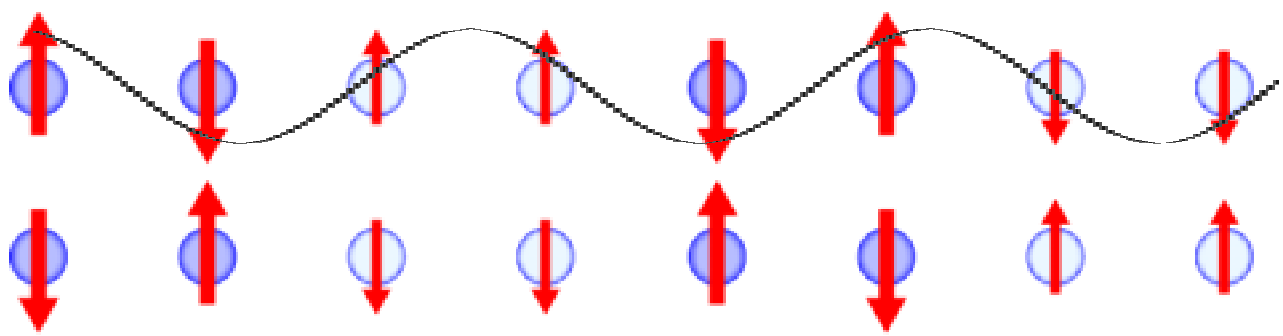
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).

SDW order parameter for general ordering wavevector

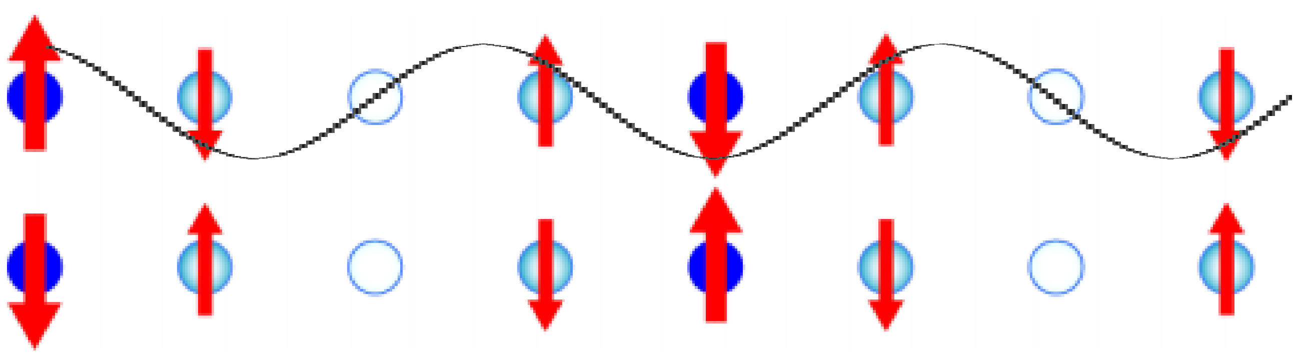
$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K}=(3\pi/4,\pi)$

Spin density wave is ***longitudinal*** (and not spiral): $\Phi_\alpha = e^{i\theta} n_\alpha$

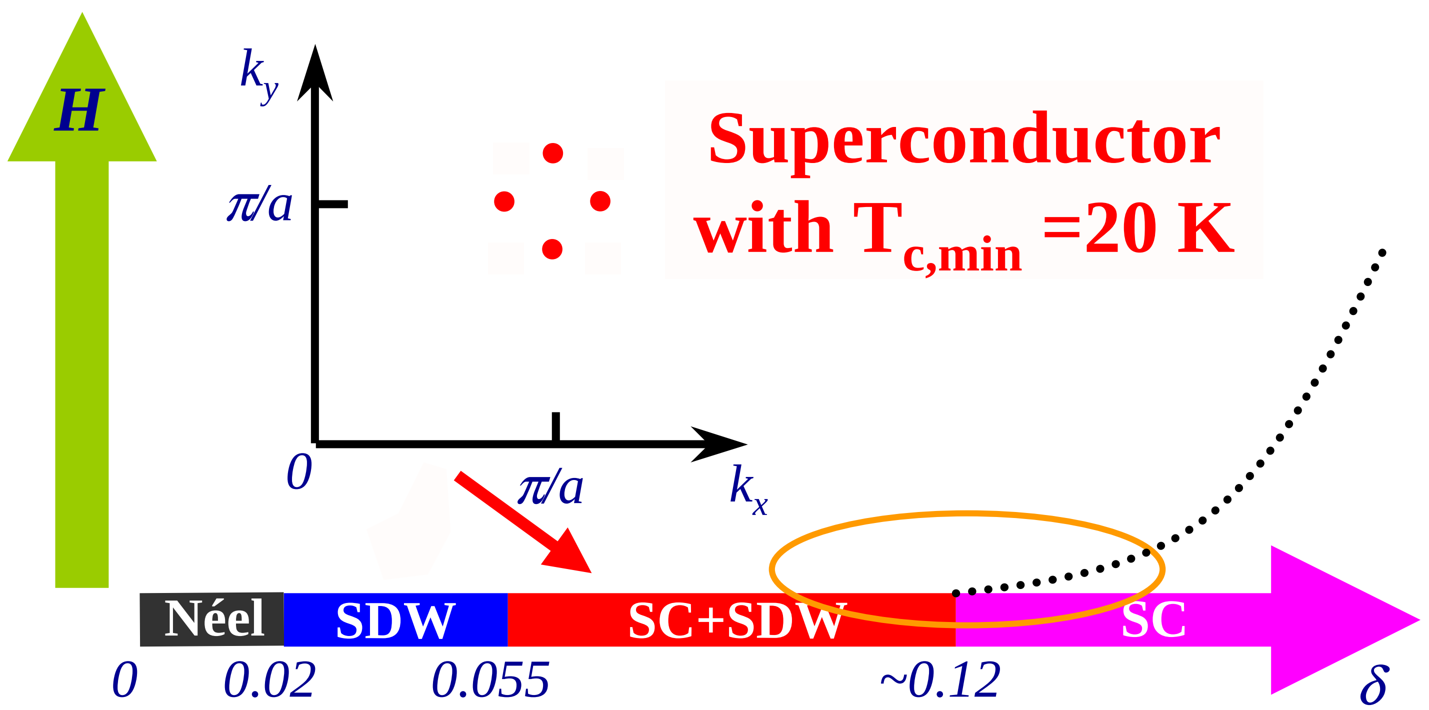


Bond-centered

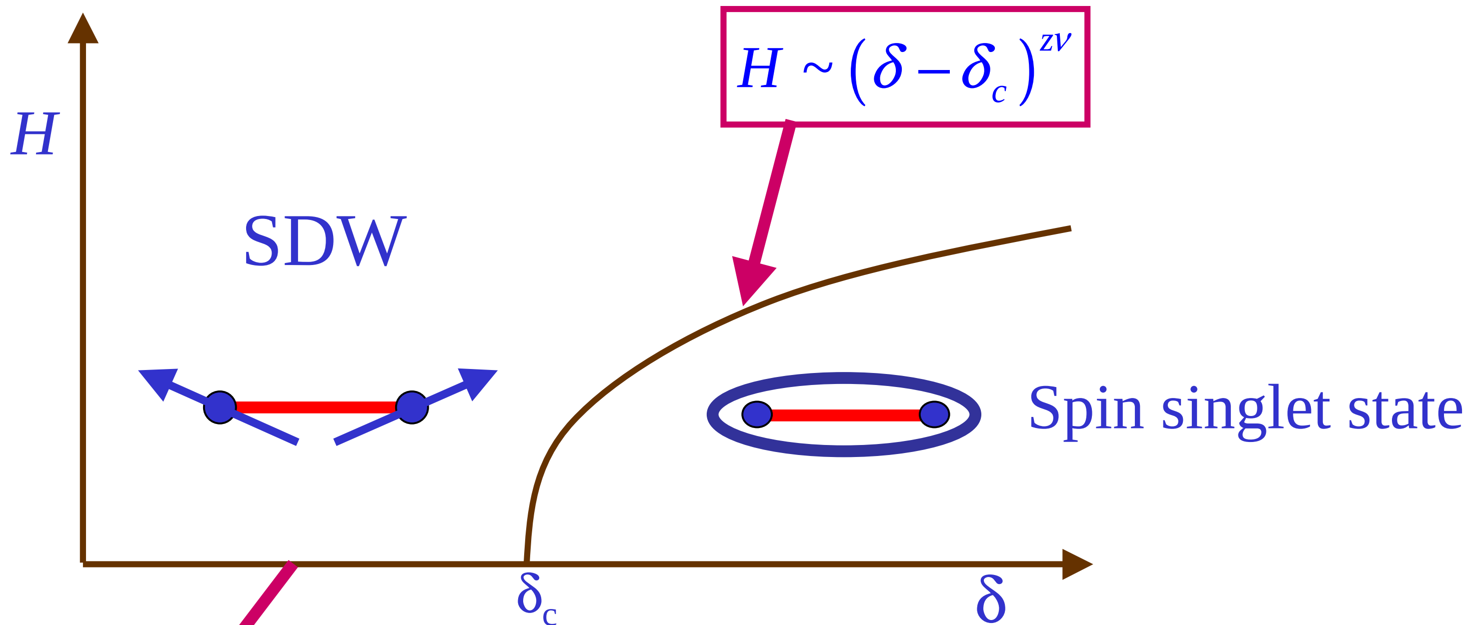


Site-centered

II. Effect of a magnetic field on SDW order with co-existing superconductivity



Effect of the Zeeman term: precession of SDW order about the magnetic field



Elastic scattering intensity

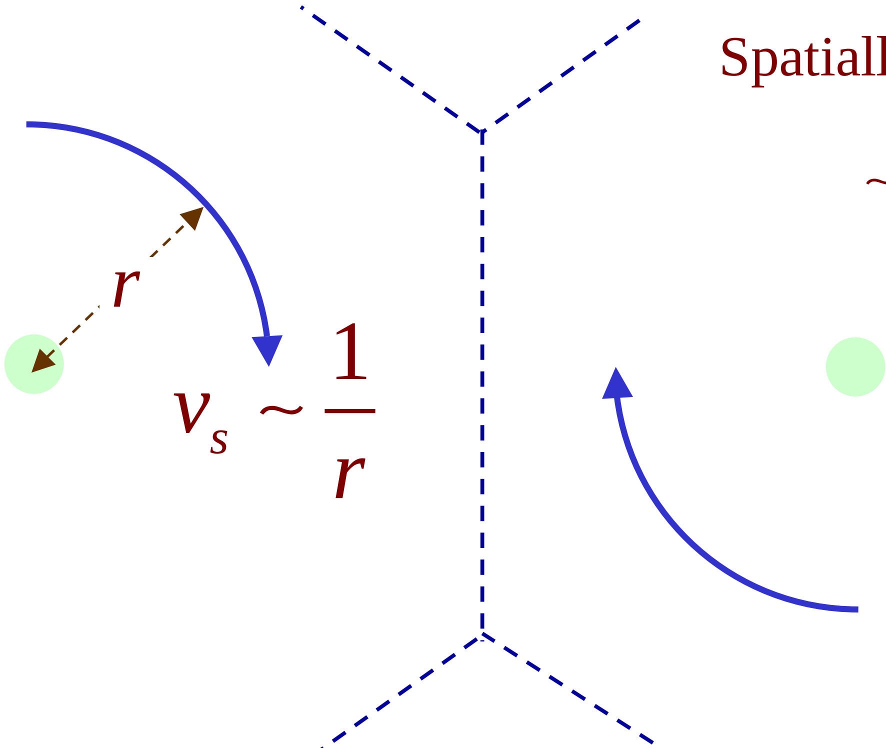
$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Effect is negligible over experimental field scales

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

The presence of the field replaces δ by

$$\delta_{eff}\left(H\right) = \delta - C\frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

Infinite diamagnetic susceptibility of *non-critical* superconductivity leads to a strong effect.

- Theory should account for *dynamic* quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a *static* Ginzburg-Landau theory

$$S_b = \int d^2 r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]$$

$$S_c = \int d^2 r d\tau \left[\frac{v}{2}\left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]$$

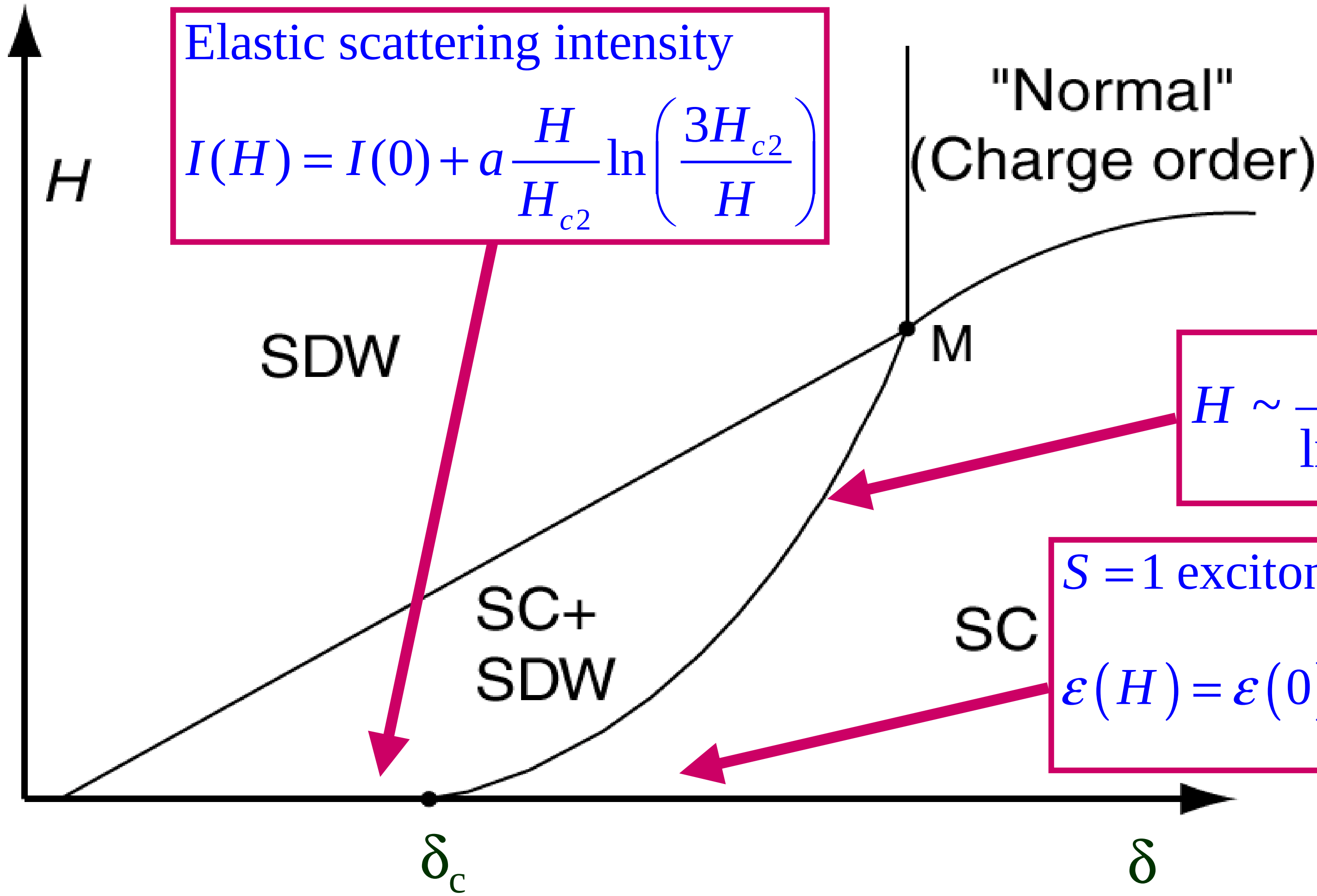
$$F_{GL} = \int d^2 r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]$$

$$Z\left[\psi(r)\right] = \int D\Phi(r,\tau)\, e^{-F_{GL} - S_b - S_c}$$

$$\frac{\delta \ln Z\left[\psi(r)\right]}{\delta \psi(r)} = 0$$

Main results

$T=0$



Elastic scattering intensity

$$I(H) = I(0) + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(\delta-\delta_c)}{\ln\left(1/\left(\delta-\delta_c\right)\right)}$$

$S=1$ exciton energy

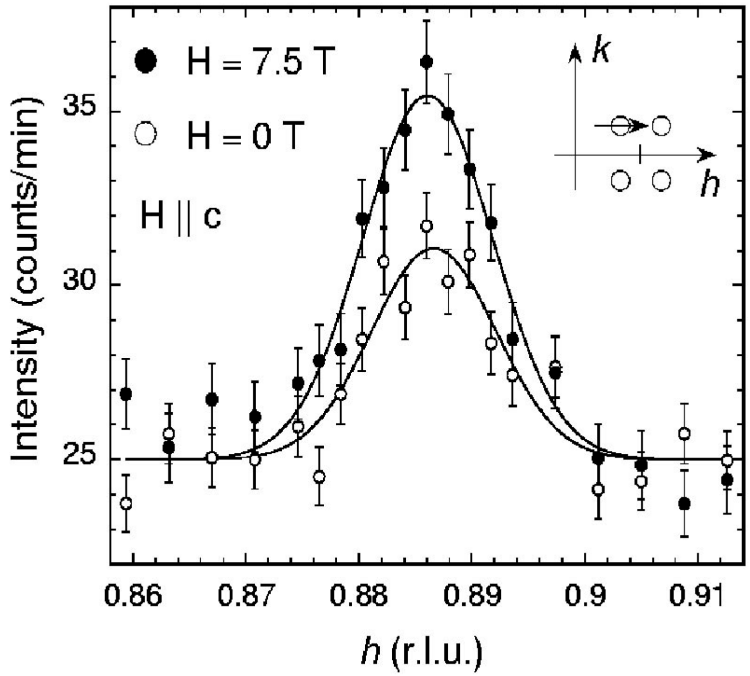
$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

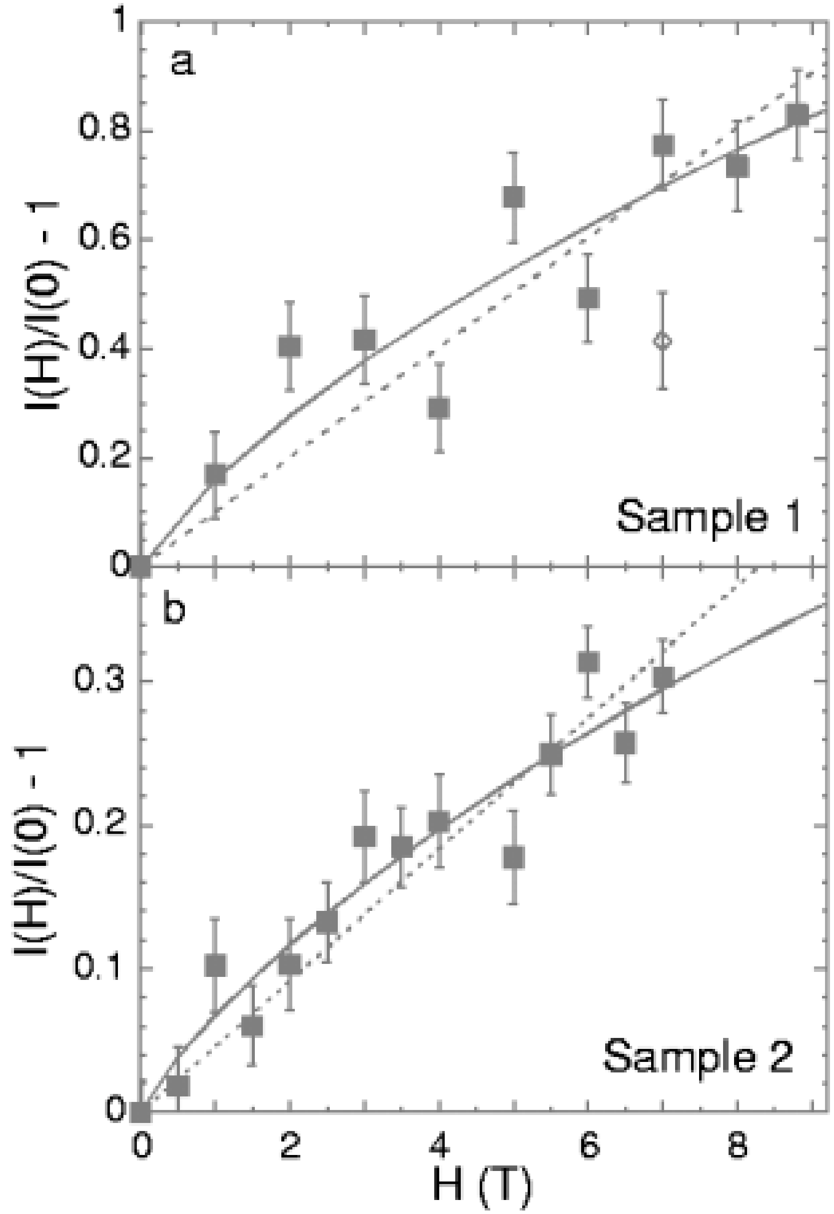
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto,
K. J. Thomas, M. A. Kastner,
and R.J. Birgeneau, cond-mat/0112505.



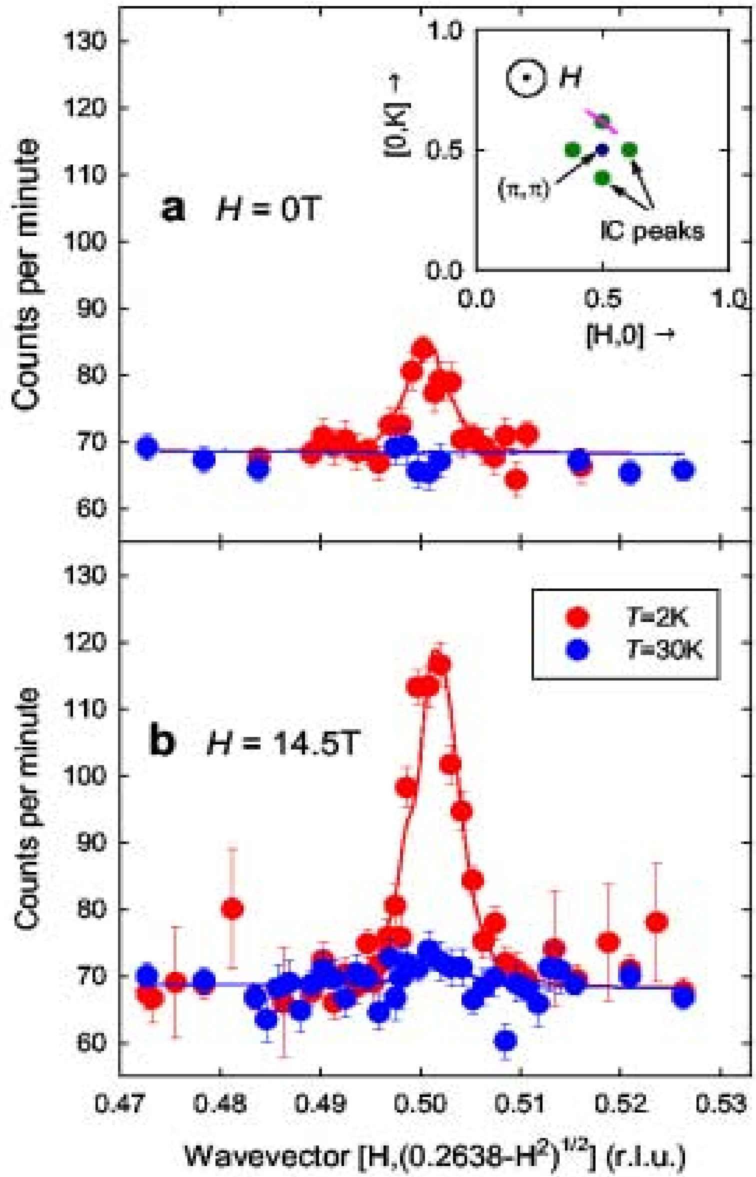


Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

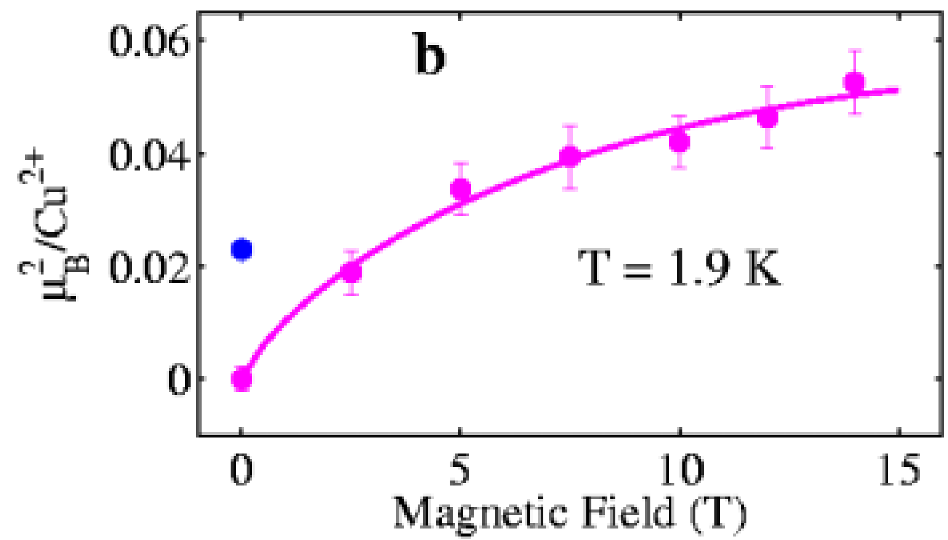
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen,
G. Aeppli, K. Lefmann, D. F. McMorrow,
P. Vorderwisch, P. Smeibidl, N. Mangkorntong,
T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason,
Nature, **415**, 299 (2002).



Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

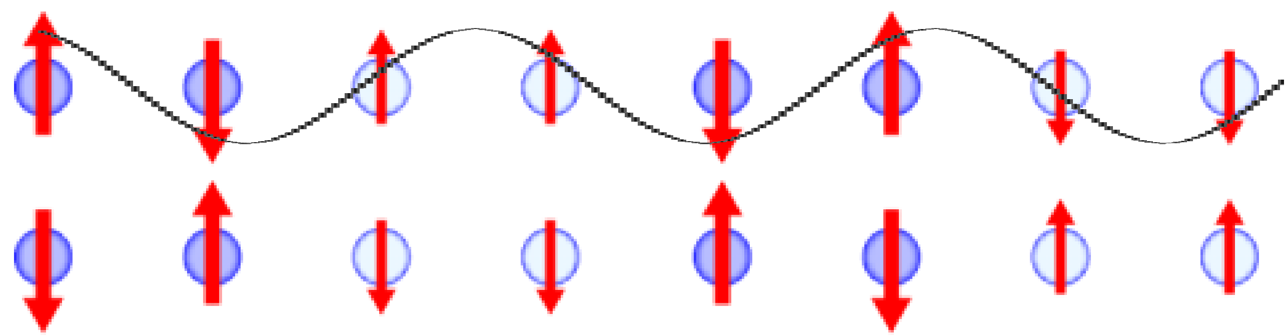
III. Connections with “charge” order – phenomenological theory

Spin density wave order parameter for general ordering wavevector

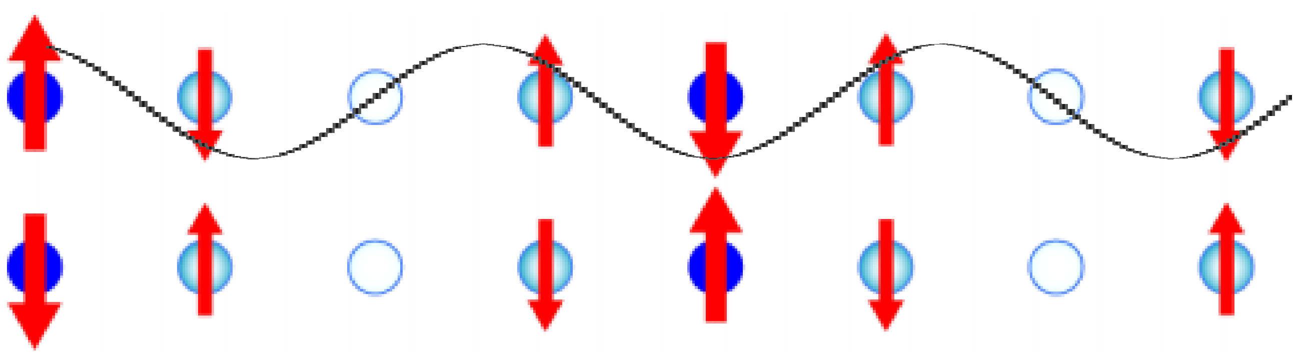
$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K} = (3\pi/4, \pi)$

Spin density wave is ***longitudinal*** (and not spiral): $\Phi_\alpha = e^{i\theta} n_\alpha$



Bond-centered



Site-centered

A longitudinal spin density wave necessarily has an accompanying modulation in the site charge densities, exchange and pairing energy per link etc. at half the wavelength of the SDW

"Charge" order: periodic modulation in local observables invariant under spin rotations and time-reversal.

$$\text{Order parmeter} \sim \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r})$$

$$\delta\rho(\boldsymbol{r}) \propto S_{\alpha}^{2}(\boldsymbol{r}) = \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
K. Machida, Physica **158C**, 192 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

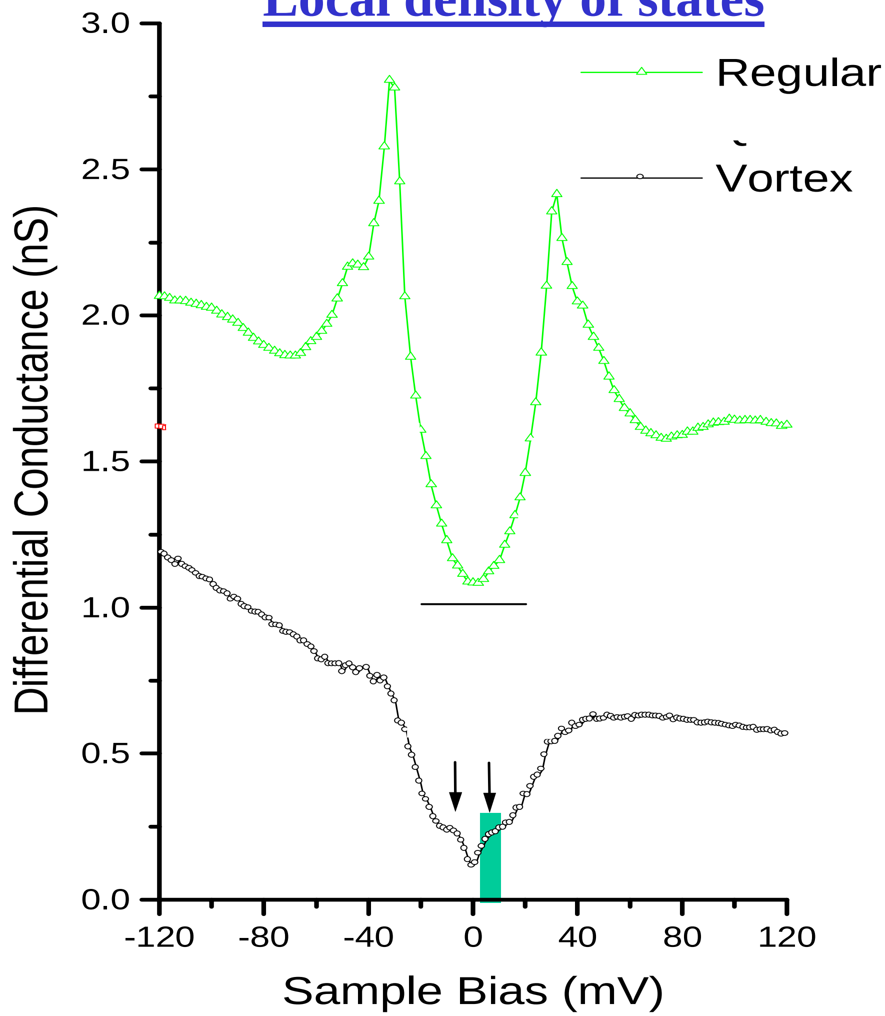
Prediction: Charge order should be pinned in halo around vortex core

K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

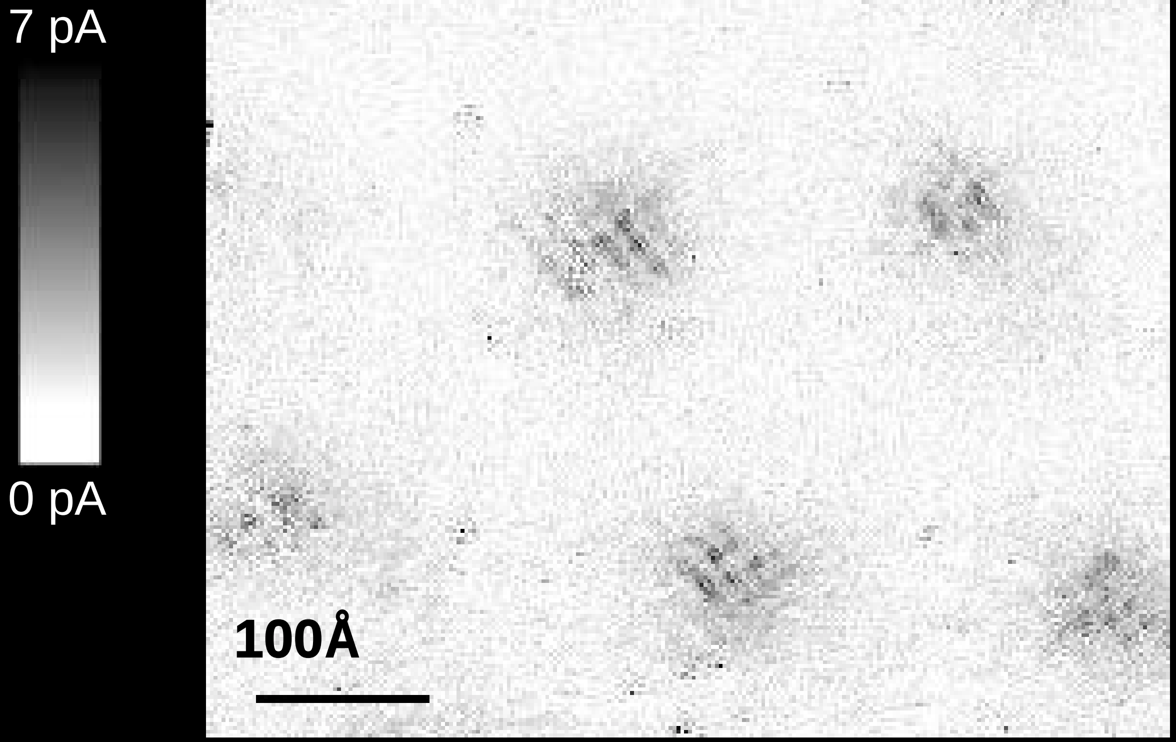


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

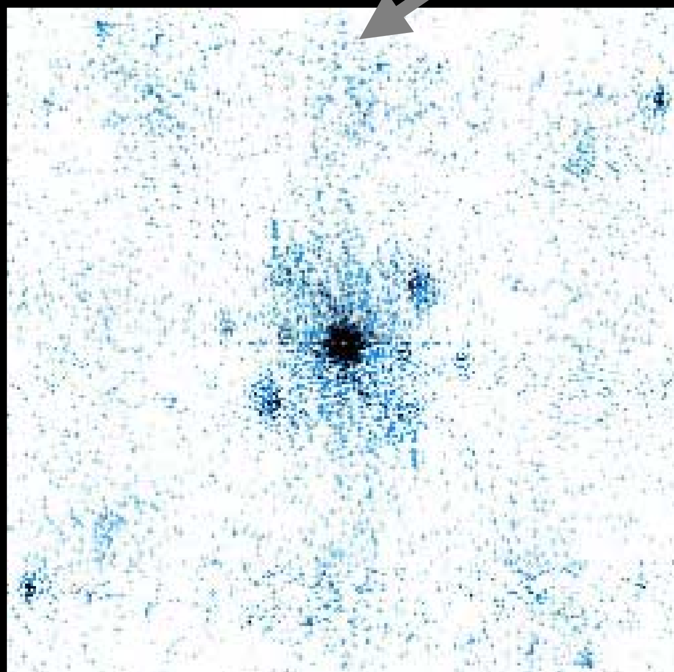
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

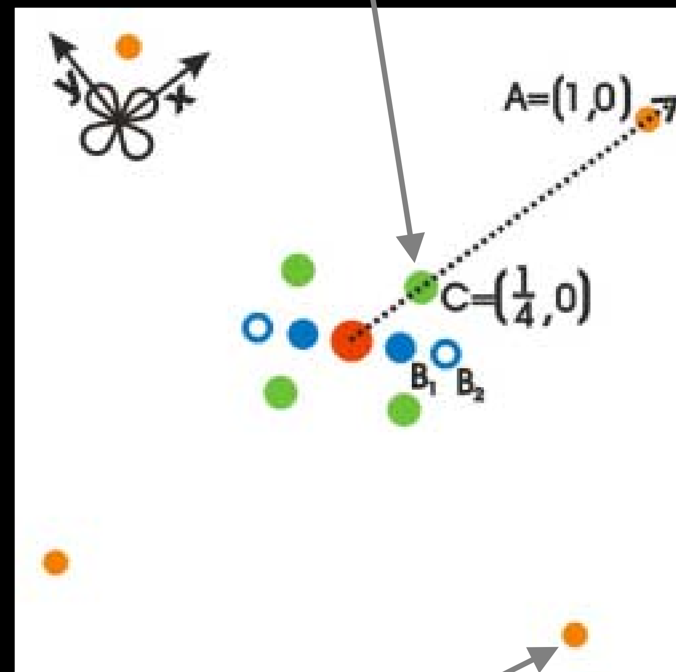


J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Fourier Transform of Vortex-Induced LDOS map

K-space locations of vortex induced LDOS





K-space locations of Bi and Cu atoms

Distances in k –space have units of $2\pi/a_0$

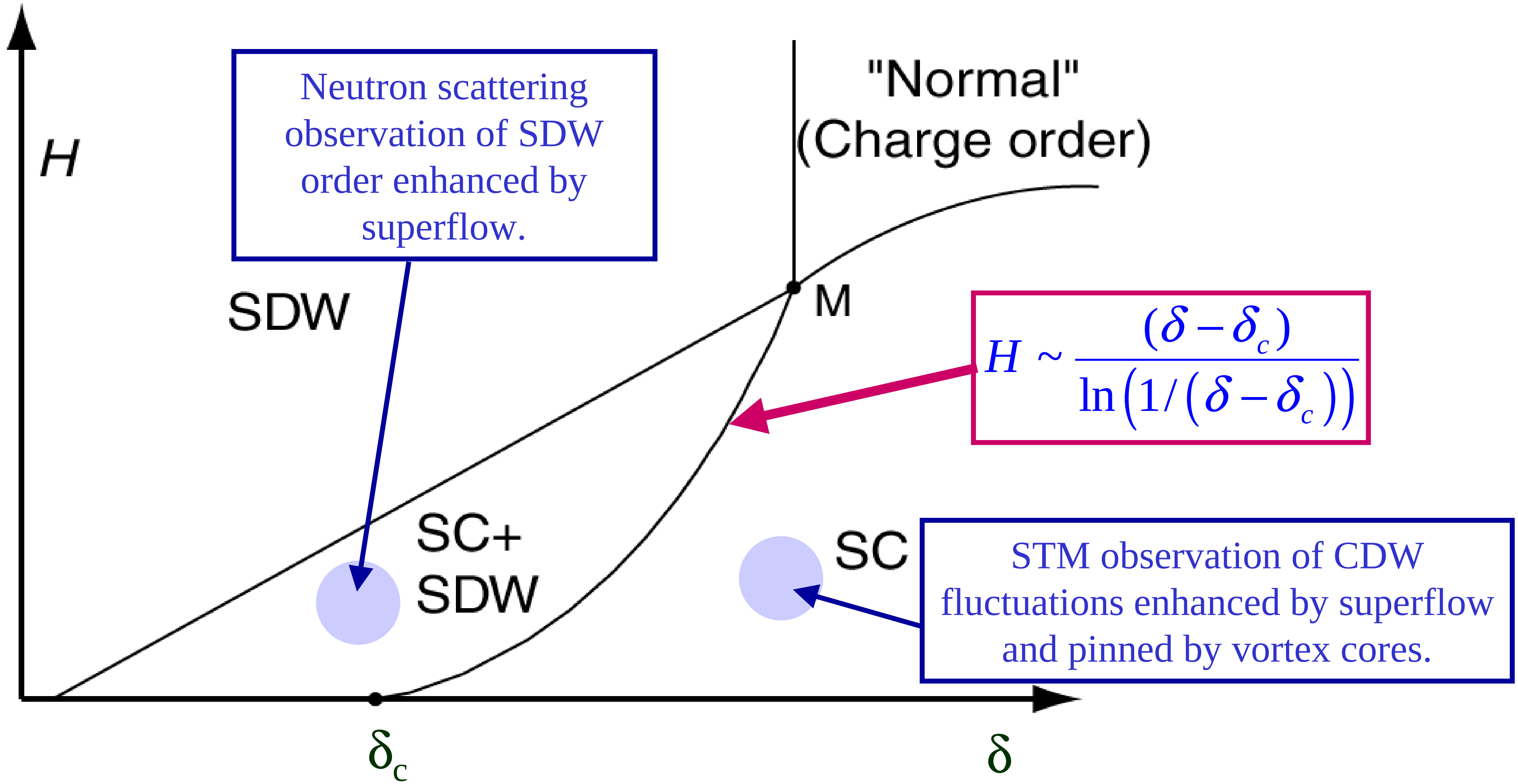
a_0=3.83 Å is Cu-Cu distance

J. Hoffman *et al. Science*, **295**, 466 (2002).

Summary of theory and experiments

(extreme Type II superconductivity)

T=0



E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

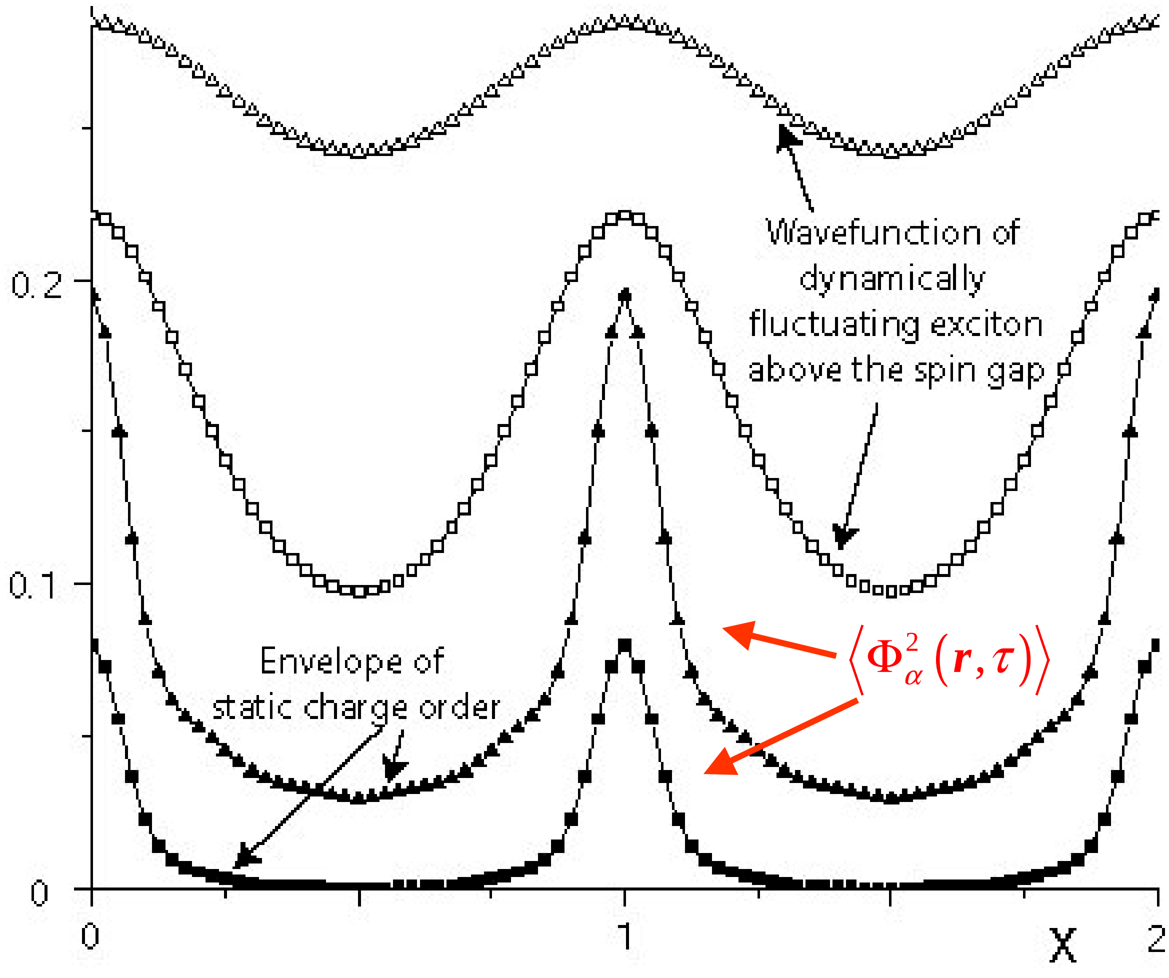
Quantitative connection between the two experiments ?

Pinning of CDW order by vortex cores in SC phase

$$S_{\text{pin}} = \zeta \sum_{\text{All } \boldsymbol{r}_v \text{ where } \psi(\boldsymbol{r}_v)=0} \int_0^{1/T} d\tau \left[\Phi_\alpha^2(\boldsymbol{r}_v) e^{i\theta} + \text{c.c.} \right]$$

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

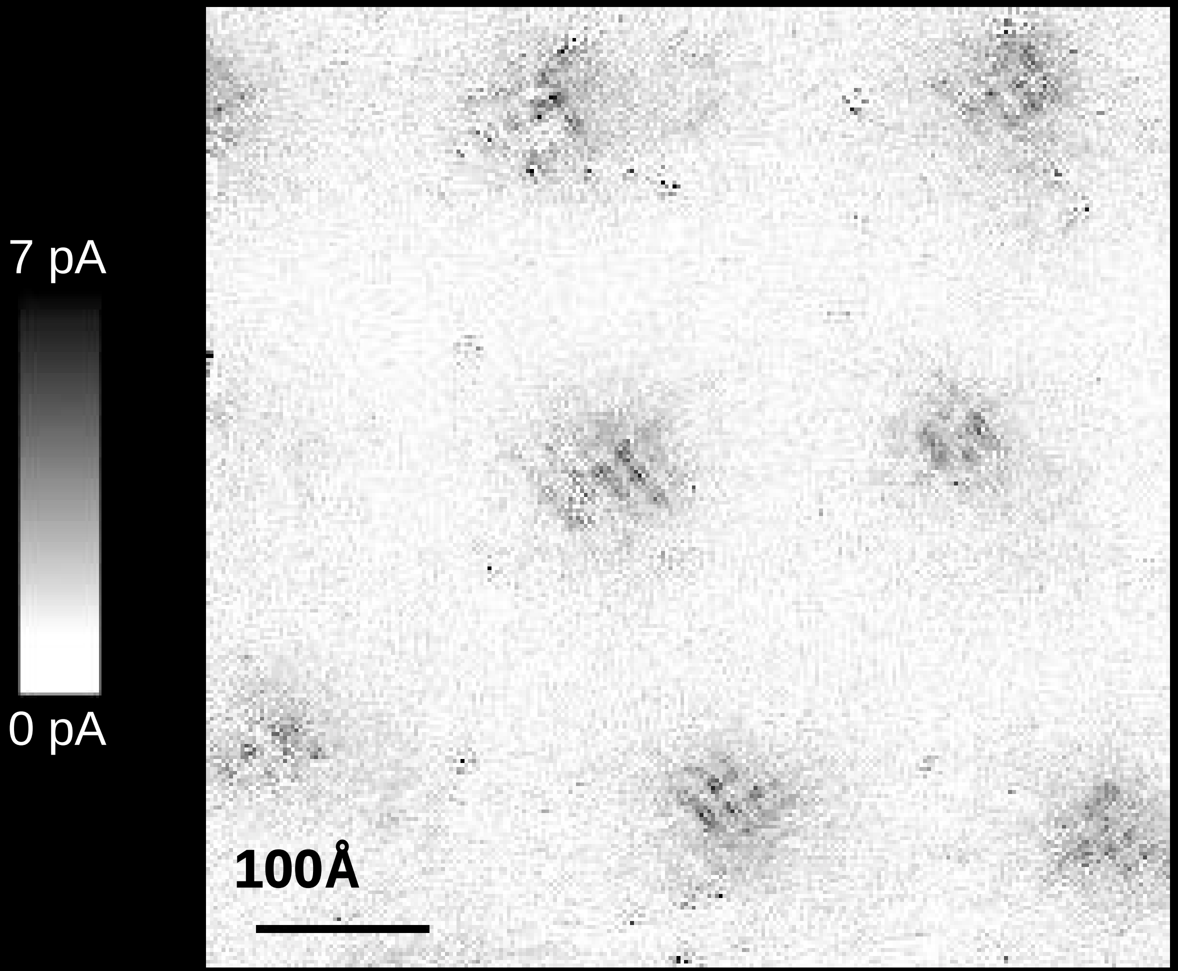
$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau) \right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau) \Phi_\alpha^*(\boldsymbol{r}_v,\tau_1) \right\rangle^2$$



□ → low magnetic field

△ → high magnetic field near the boundary to the SC+SDW phase

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

IV. Microscopic theory of the charge order: magnetic transitions in Mott insulators and superconductors

Magnetic transitions in the coupled ladder antiferromagnet: $H = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$

Action for quantum spin fluctuations in spacetime

Discretize spacetime into a cubic lattice with Néel order orientation $\boldsymbol{n}_a$

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} \right)$$

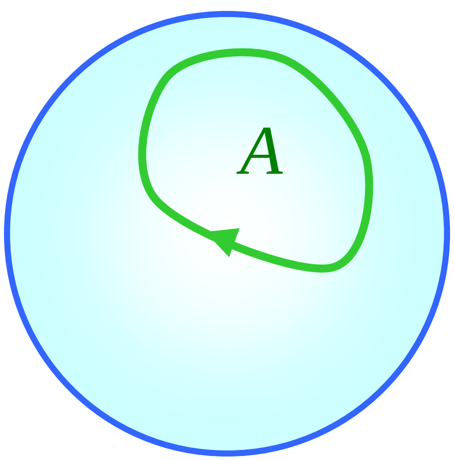
$a \rightarrow$ cubic lattice sites; $\mu \rightarrow x, y, \tau$;

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989).

Quantum path integral for two-dimensional quantum antiferromagnet
$\Leftrightarrow$ Partition function of a classical three-dimensional ferromagnet at a “temperature” g

Missing: Spin Berry Phases

(Can be neglected on the coupled ladder, but not on the square lattice)



$$e^{iSA}$$

Berry phases profoundly modify paramagnetic states with $\langle \boldsymbol{n}_a \rangle = \langle \vec{S} \rangle = 0$

Field theory of paramagnetic ("quantum disordered") phase on the square lattice

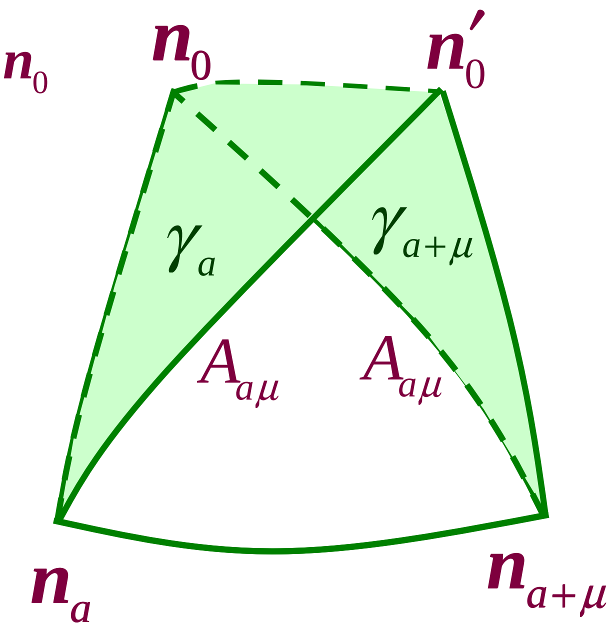
Discretize spacetime into a cubic lattice:

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_a \rightarrow \pm 1$ on two square sublattices ; $\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$



Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).

The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda} \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 2+1 dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is ***always*** in a *confining* phase:

There is an energy gap and the ground state has a **bond order wave**.

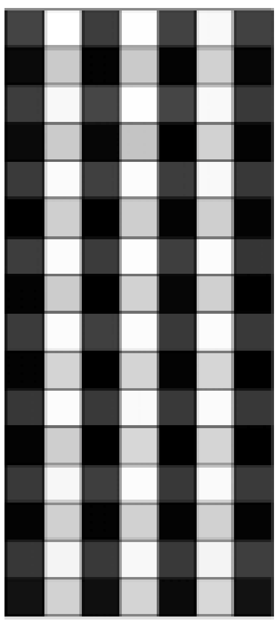
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev*. B **65**, 220405 (2002).

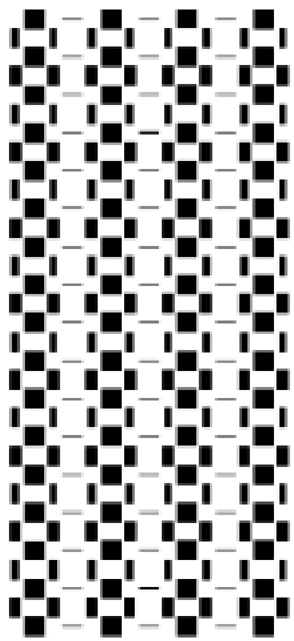
IV. Microscopic theory of the charge order: magnetic transitions in Mott insulators and superconductors

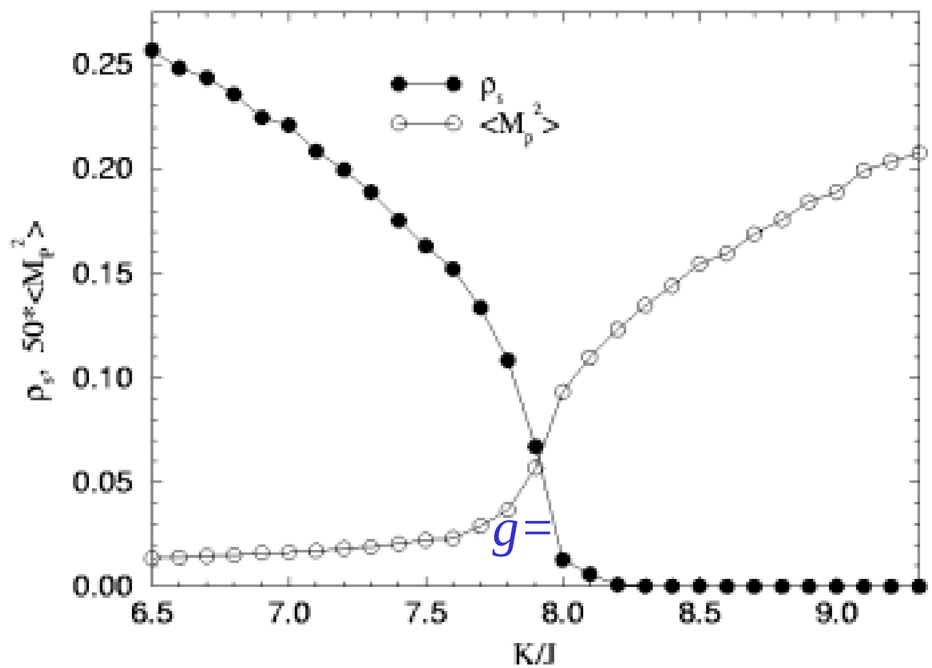
Bond order wave in a frustrated S=1/2 XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in *S=1/2* antiferromagnet with full square lattice symmetry







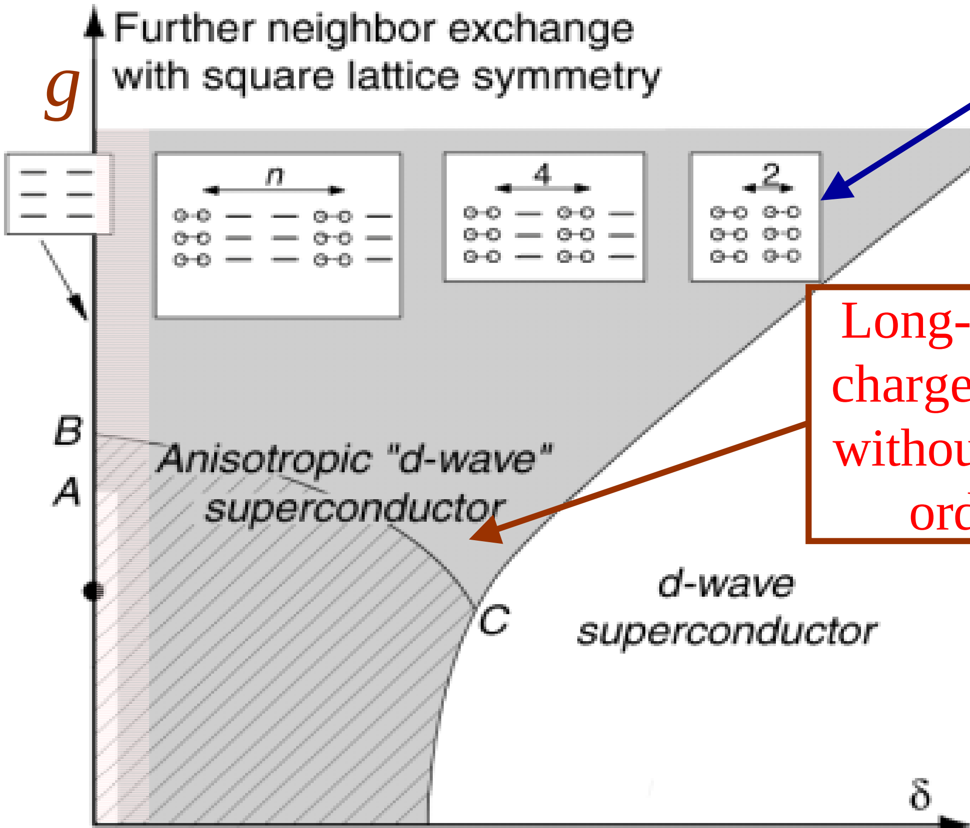
$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989);
S. Sachdev and K. Park, *Annals of Physics* **298**, 58 (2002).

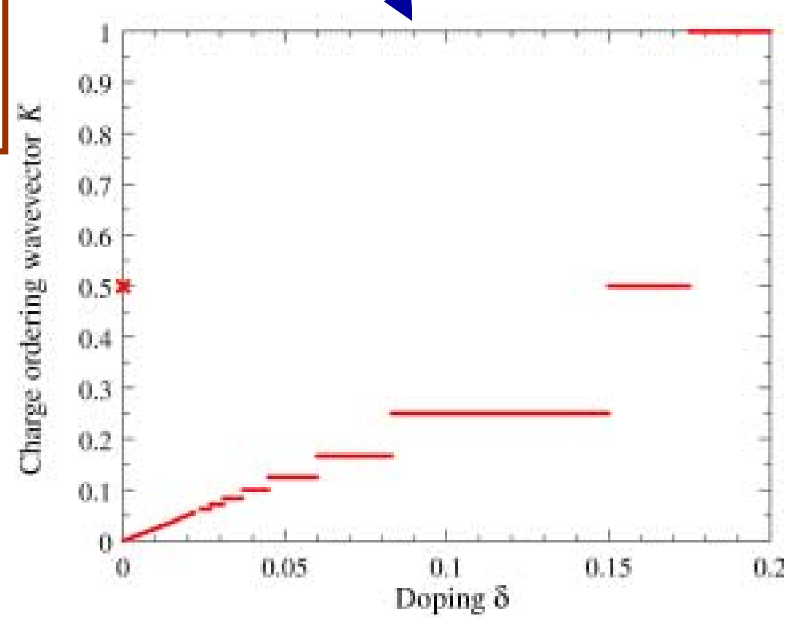
IV. Bond order waves in the superconductor.

“Large *N*” theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).



Long-range charge order without spin order

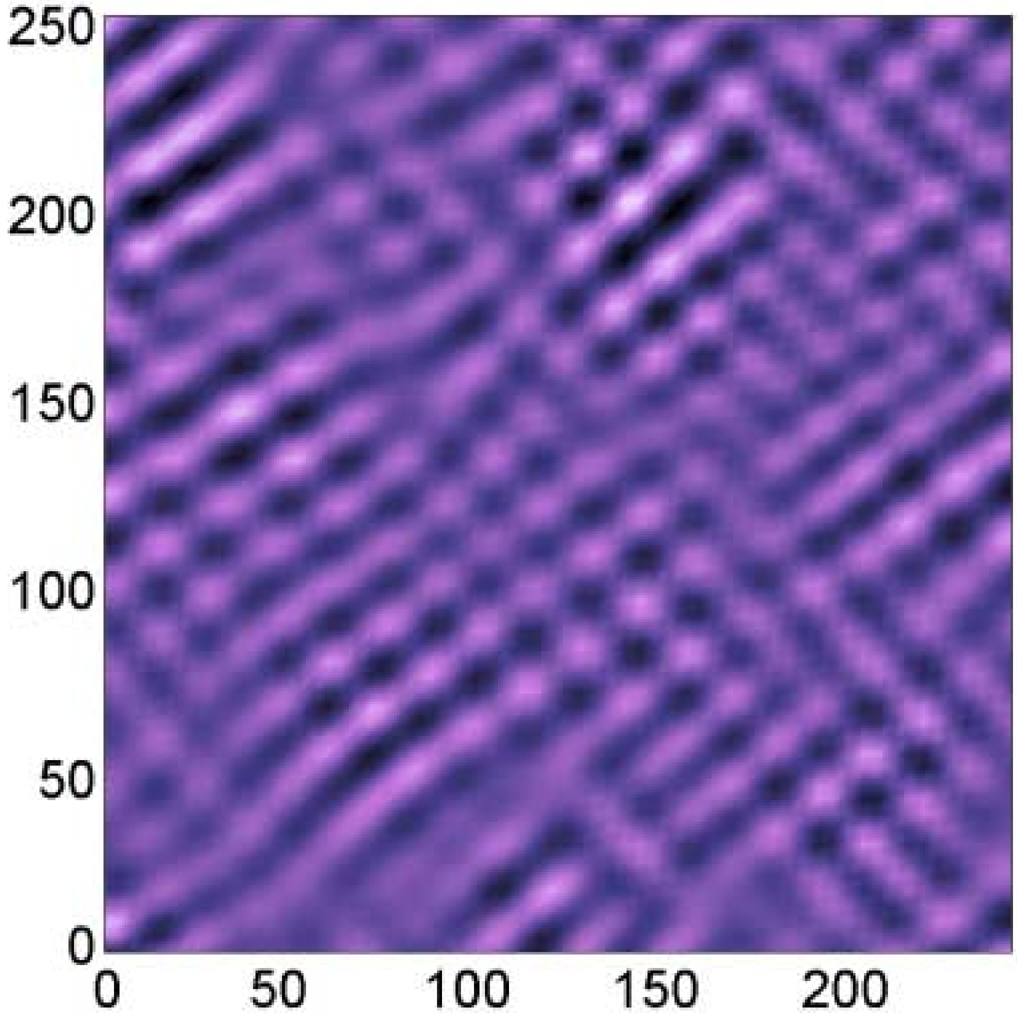


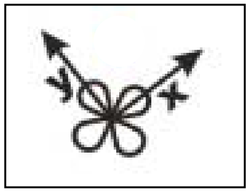
Hatched region --- spin order
Shaded region ---- charge order

Charge order is bond-centered and has an even period.

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett.* **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett.* **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

IV. STM image of pinned charge order in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

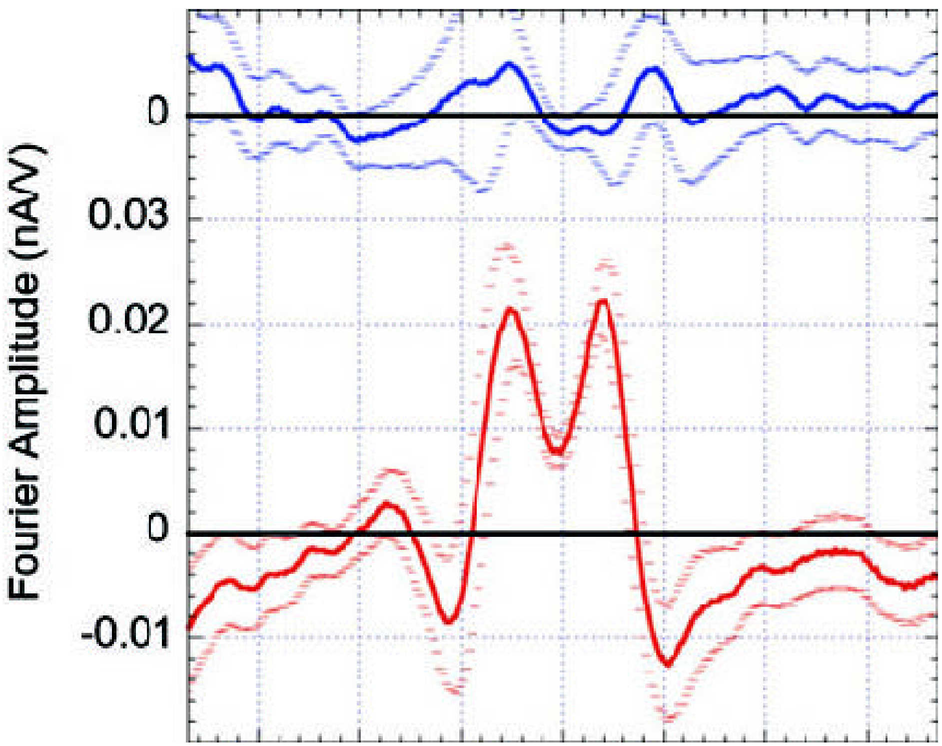




Charge order period
= 4 lattice spacings

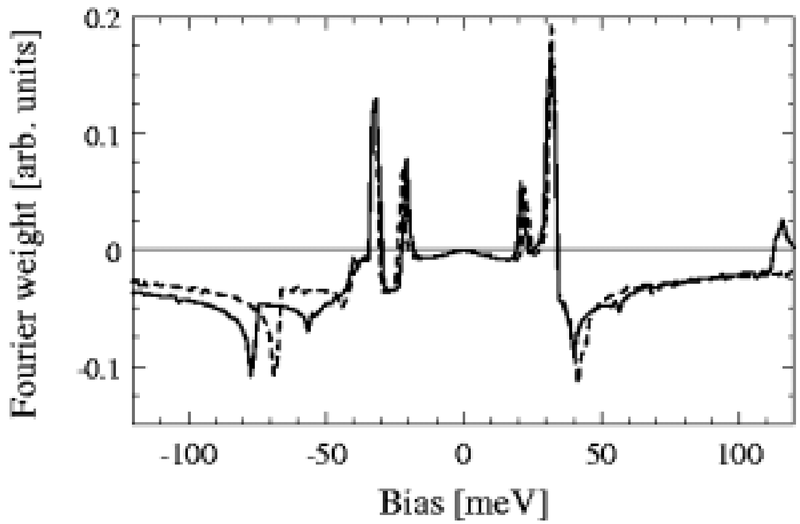
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, cond-mat/0204284.
D. Podolsky, E. Demler,
K. Damle, and B.I. Halperin,
cond-mat/0204011

IV. Neutron scattering observation of static charge order in $YBa_2Cu_3O_{6.35}$

(spin correlations are dynamic)

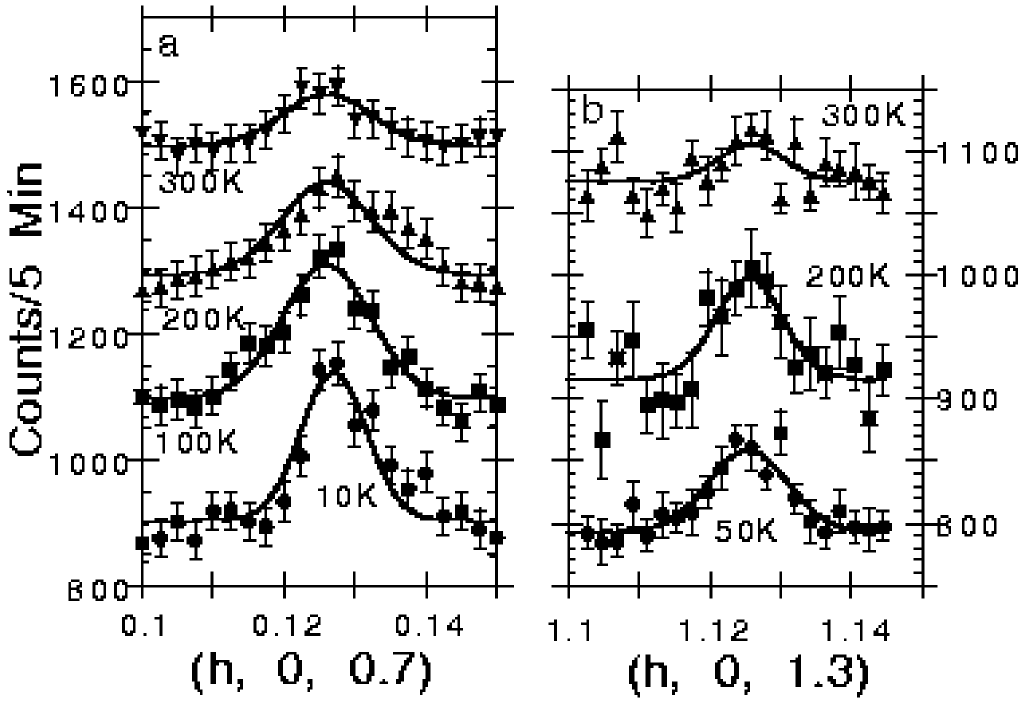
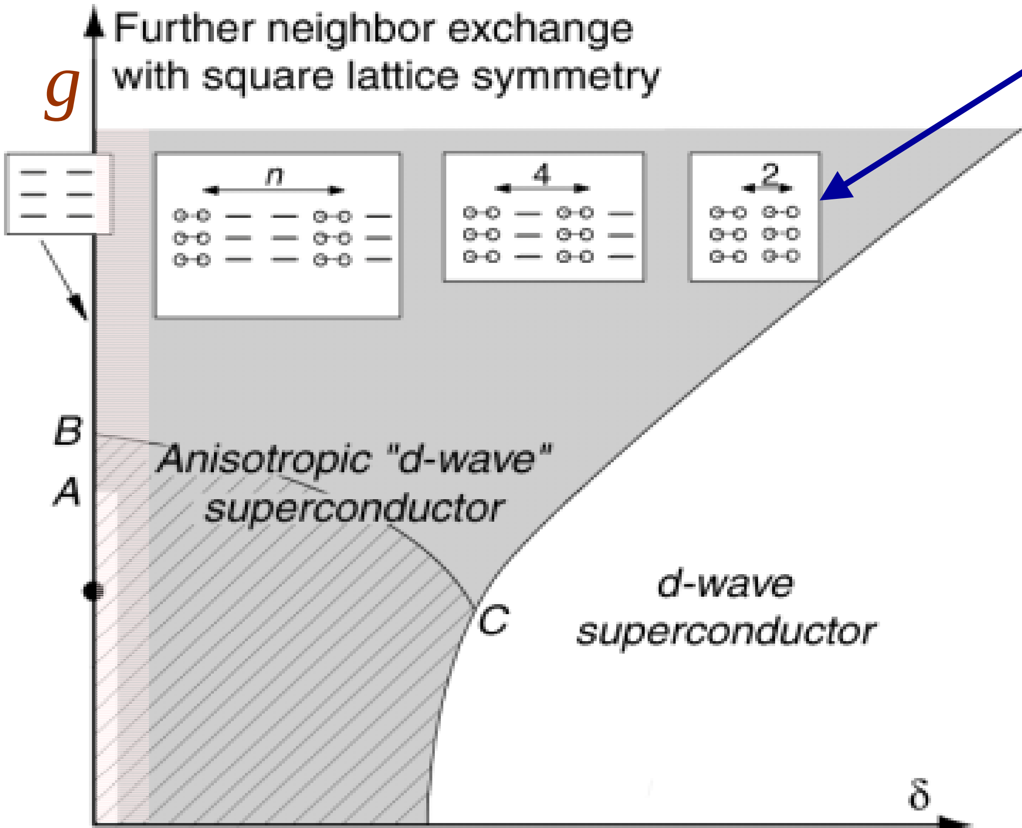


FIG. 1. Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) r.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

Charge order period = 8 lattice spacings

H. A. Mook, Pengcheng Dai, and F. Dogan
Phys. Rev. Lett. **88**, 097004 (2002).

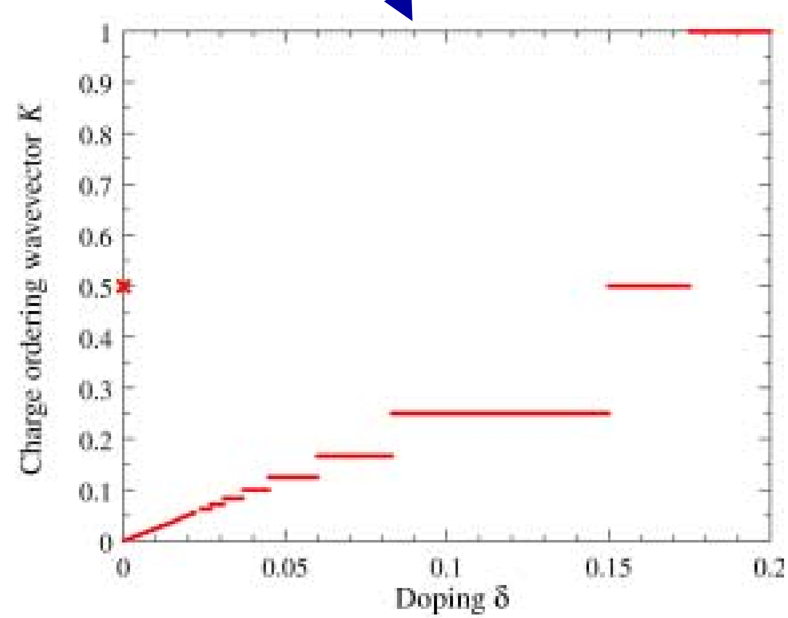
IV. Bond order waves in the superconductor.



Hatched region --- spin order
Shaded region ---- charge order

"Large *N*" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).



See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has bond-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?