Transport without quasiparticles
or
"Electricity without electrons"

Physics of Emergence
University of Pittsburgh
February 1-2, 2014

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PHYSICS
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Foundations of quantum many body theory:

1. *Ground states connected adiabatically to independent electron states*
2. *Quasiparticle structure of excited states*

Modern phases of quantum matter:

1. *Ground states disconnected from independent electron states: many-particle entanglement*
2. *Quasiparticle structure of excited states*

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter:

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2. No quasiparticles

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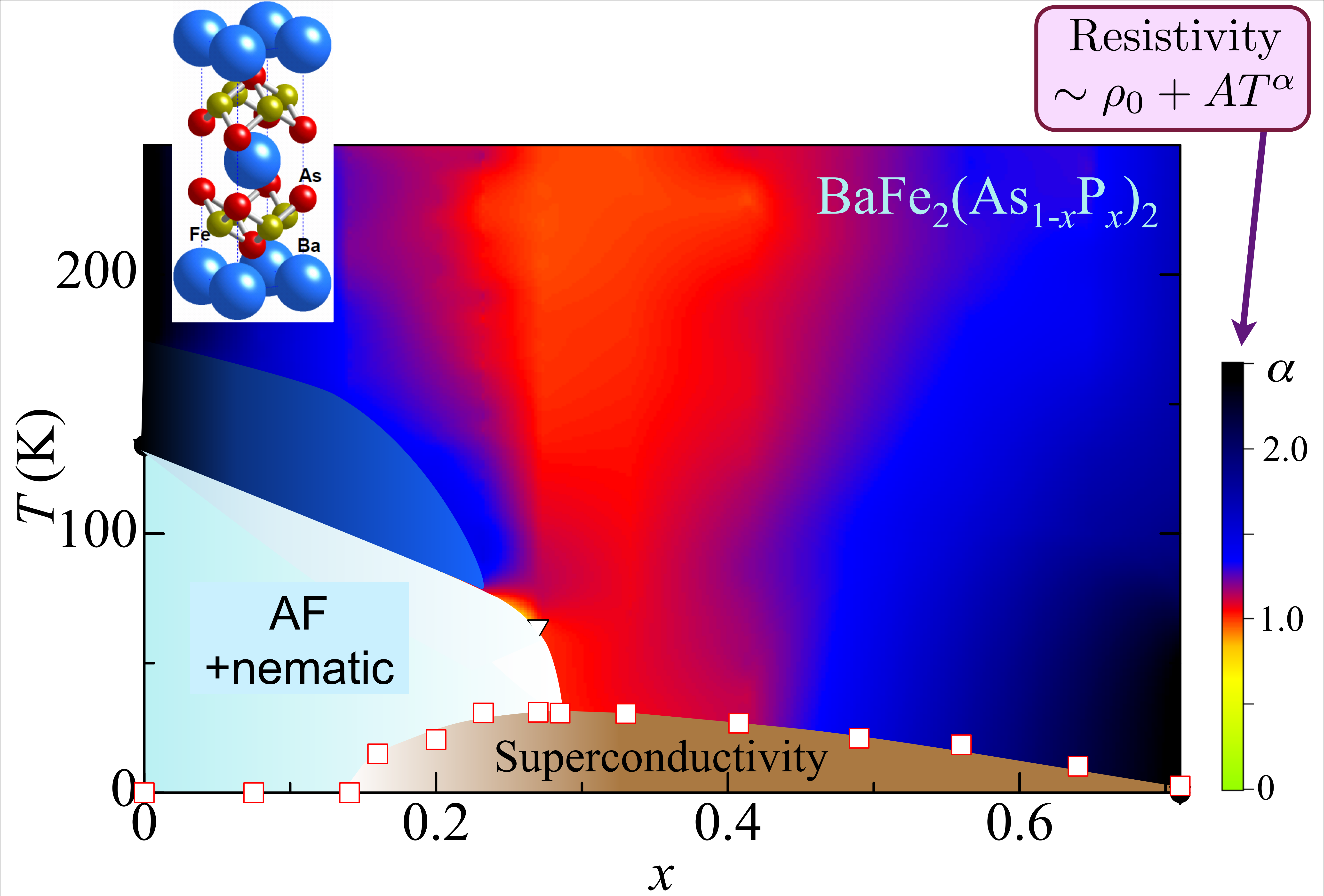
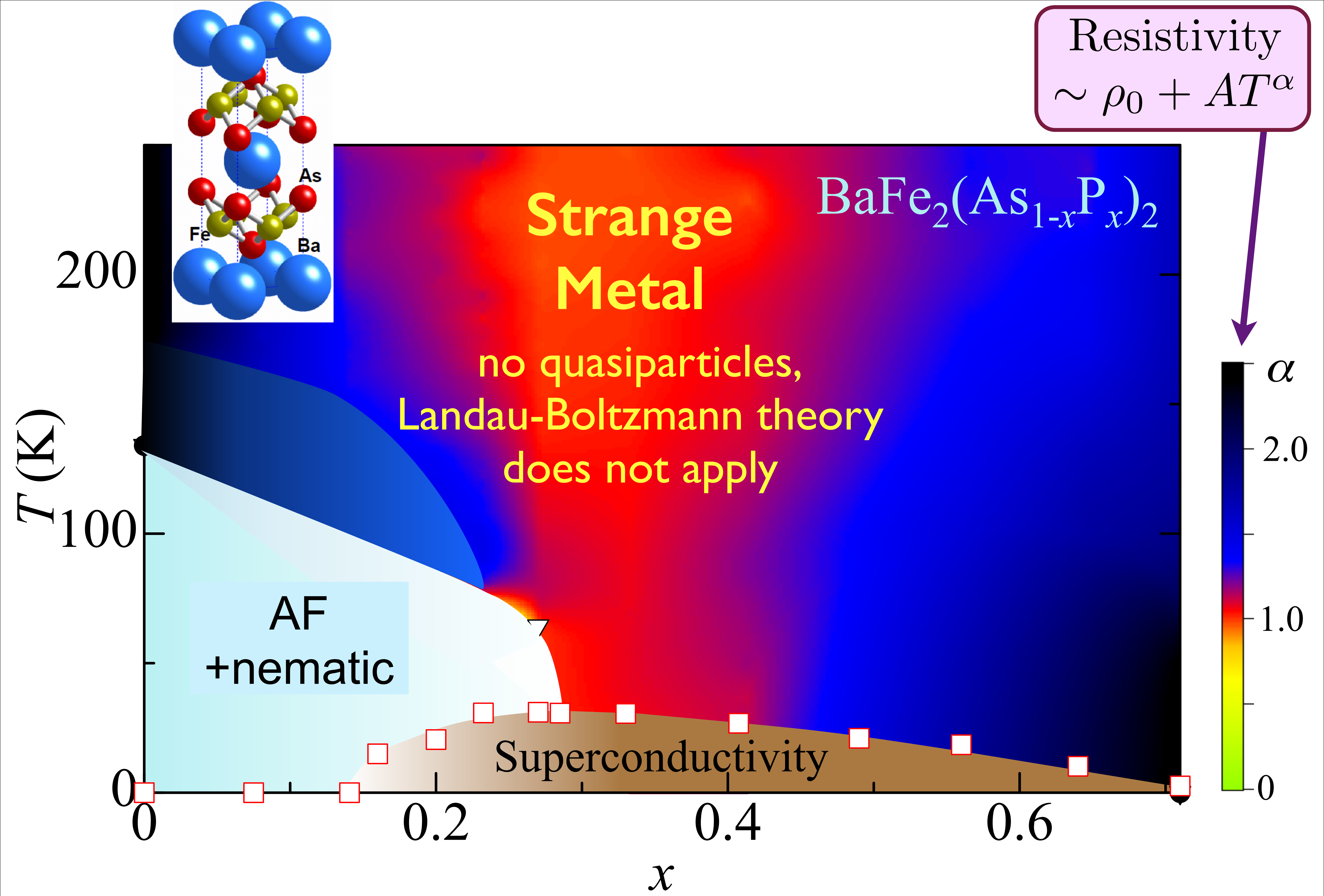
Only 2 examples:

1. Conformal field theories in spatial dimension $d > 1$

2. Quantum critical metals in dimension $d = 2$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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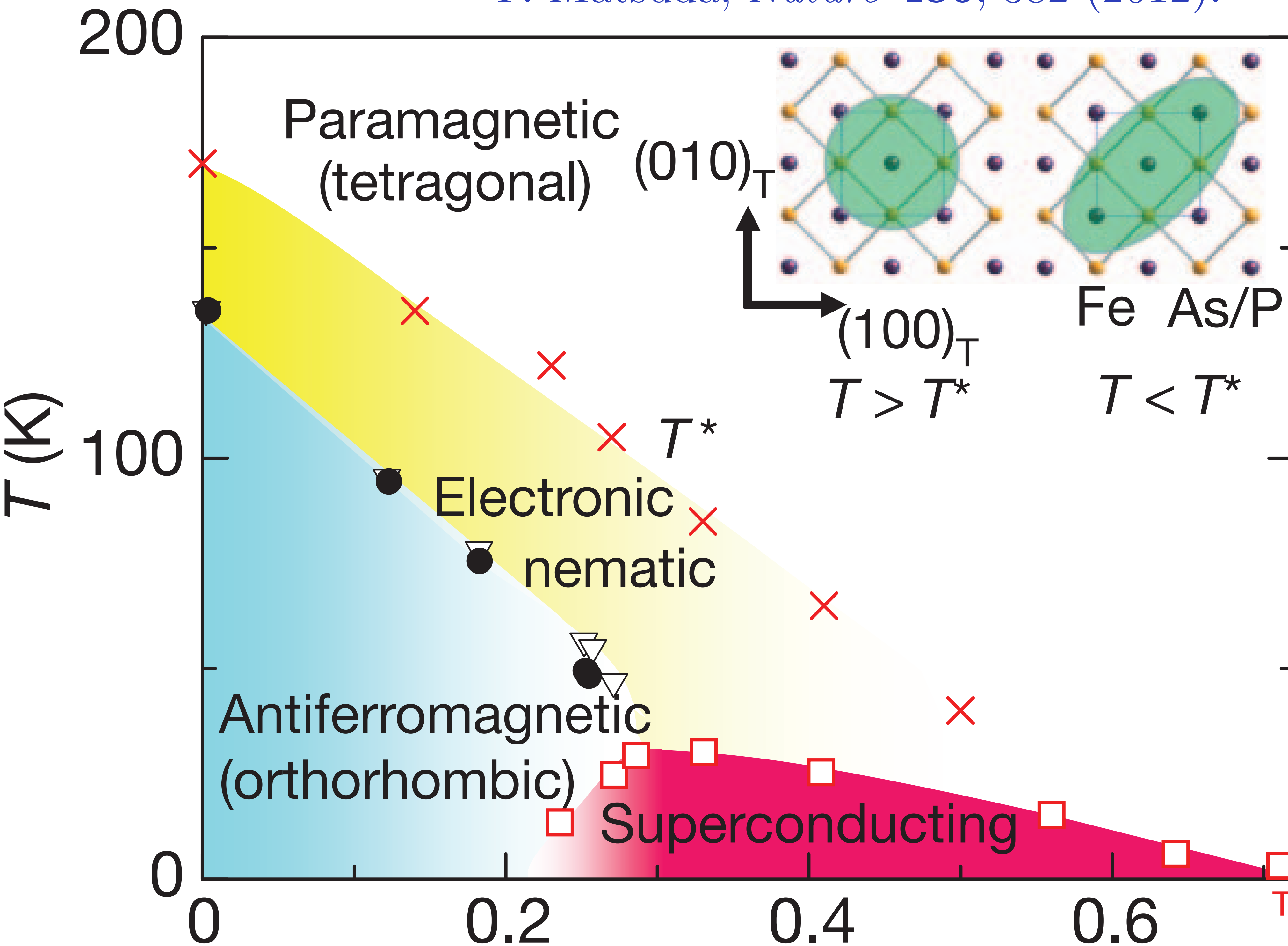
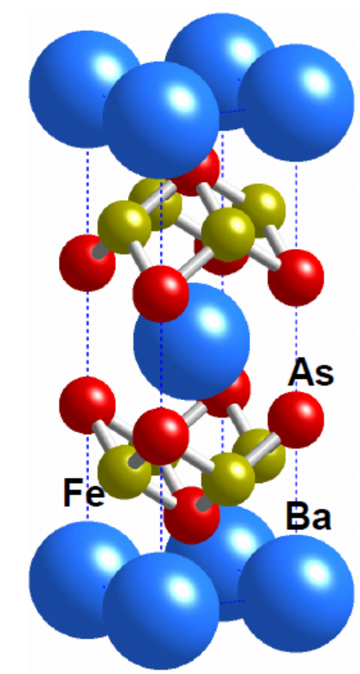
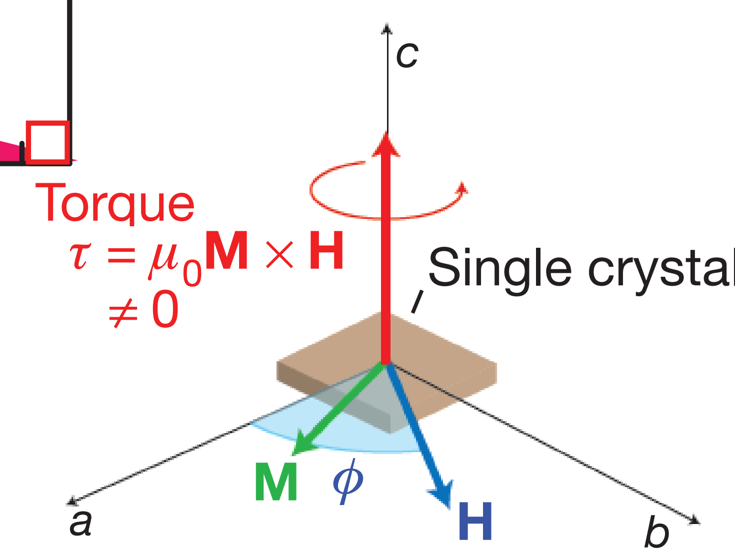


$BaFe_2(As_{1-x}P_x)_2$





Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi
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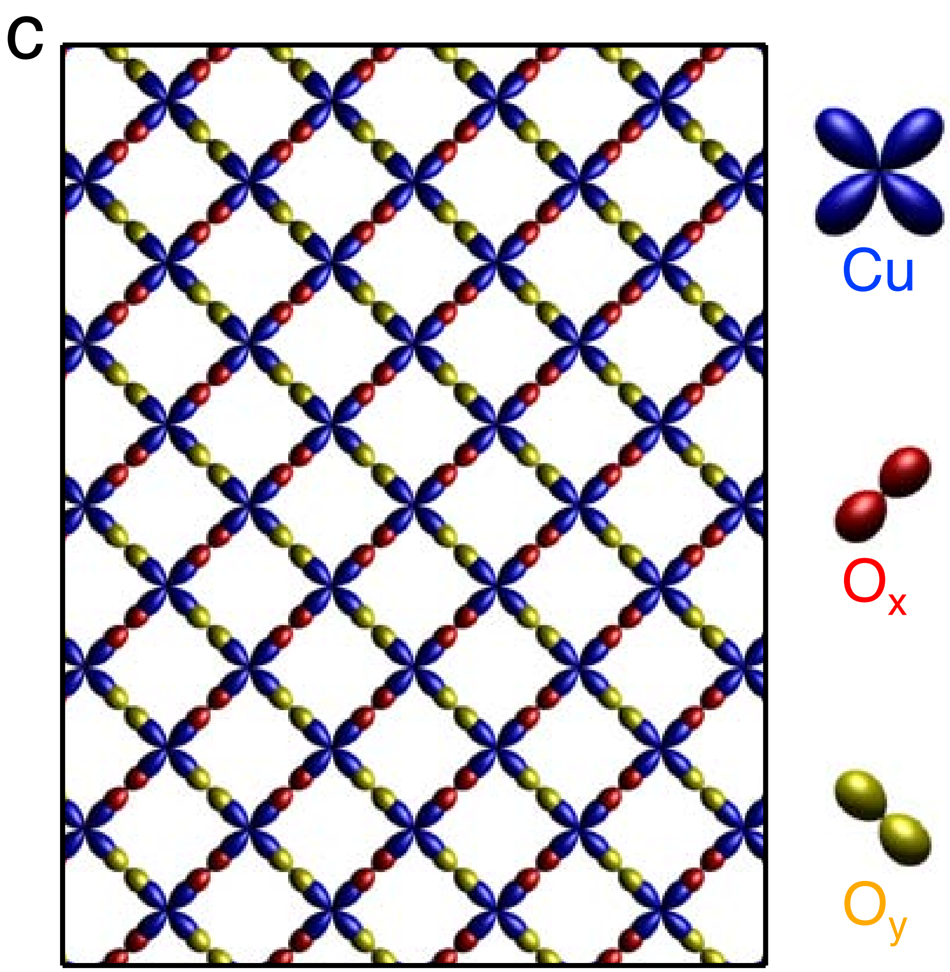

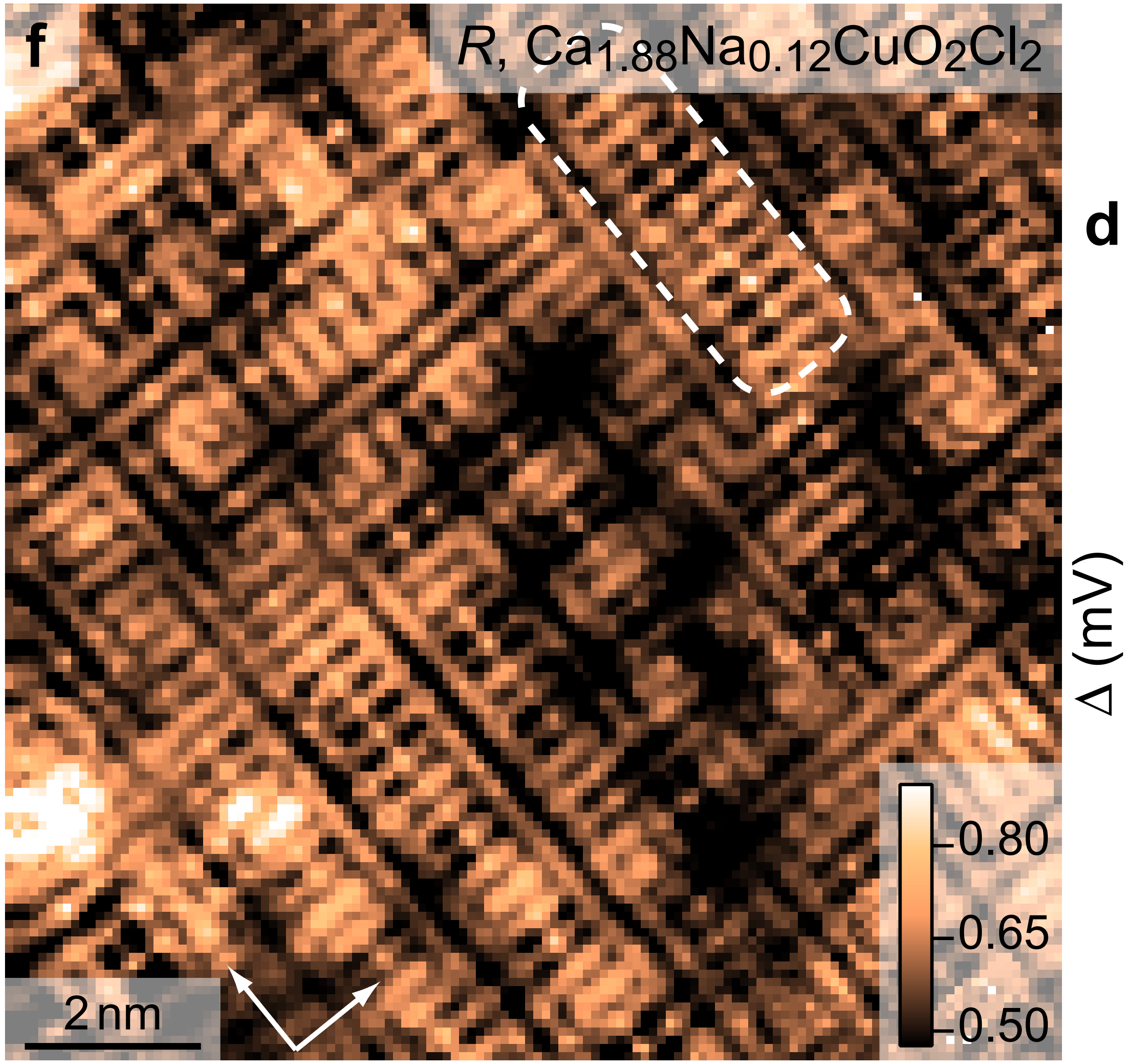

Evidence for "nematic" order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

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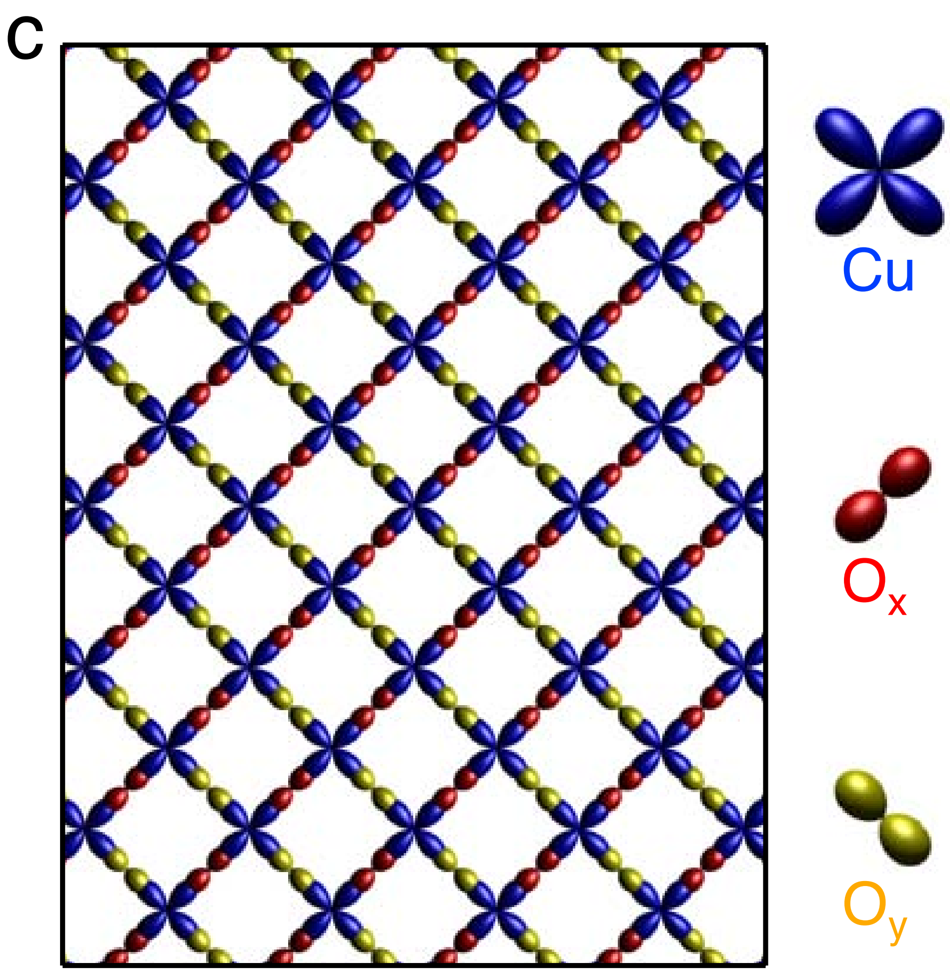

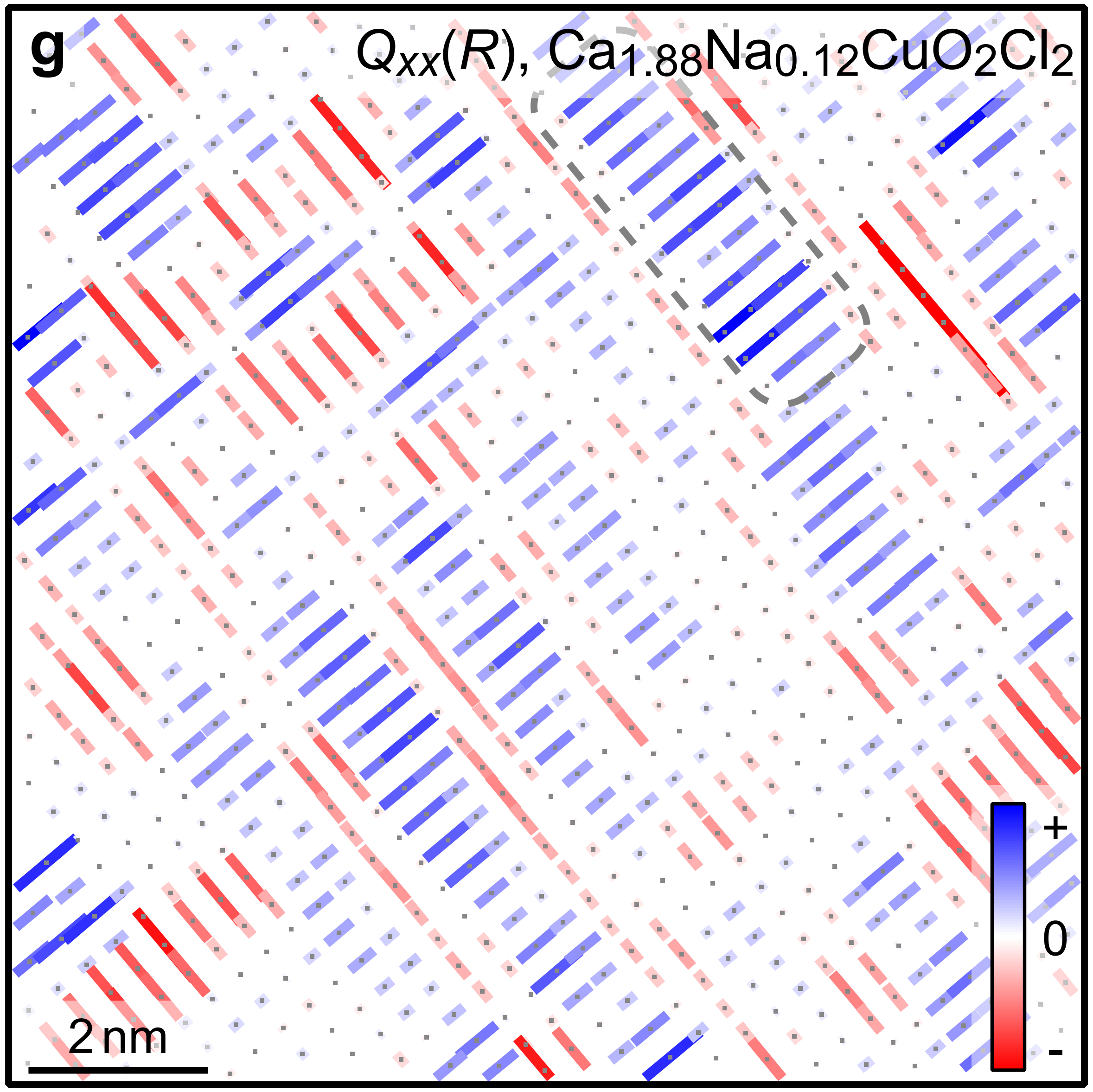

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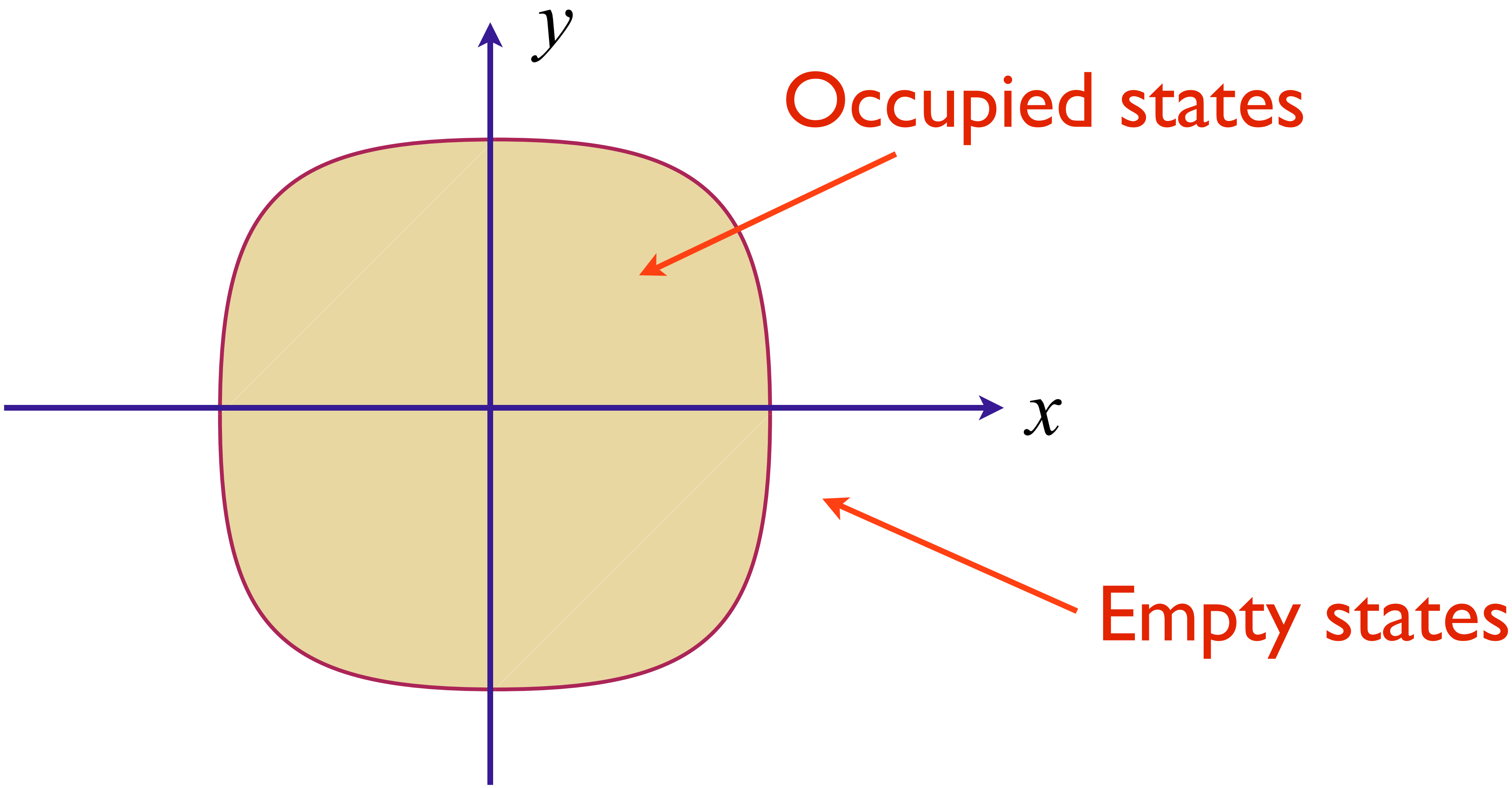
Quantum criticality of Ising-nematic ordering in a metal



A metal with a Fermi surface
with full square lattice symmetry

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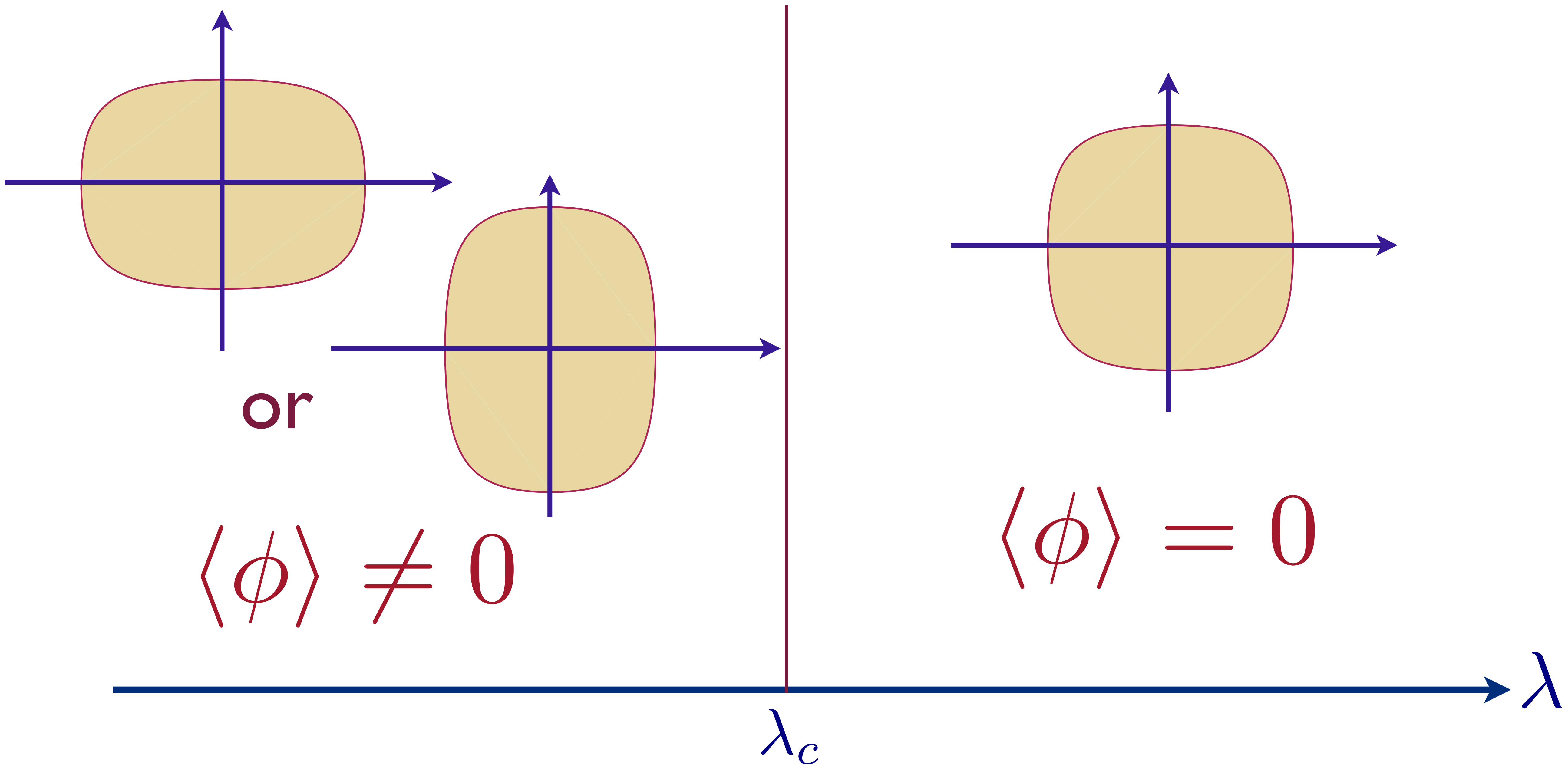

Pomeranchuk instability as a function of coupling λ

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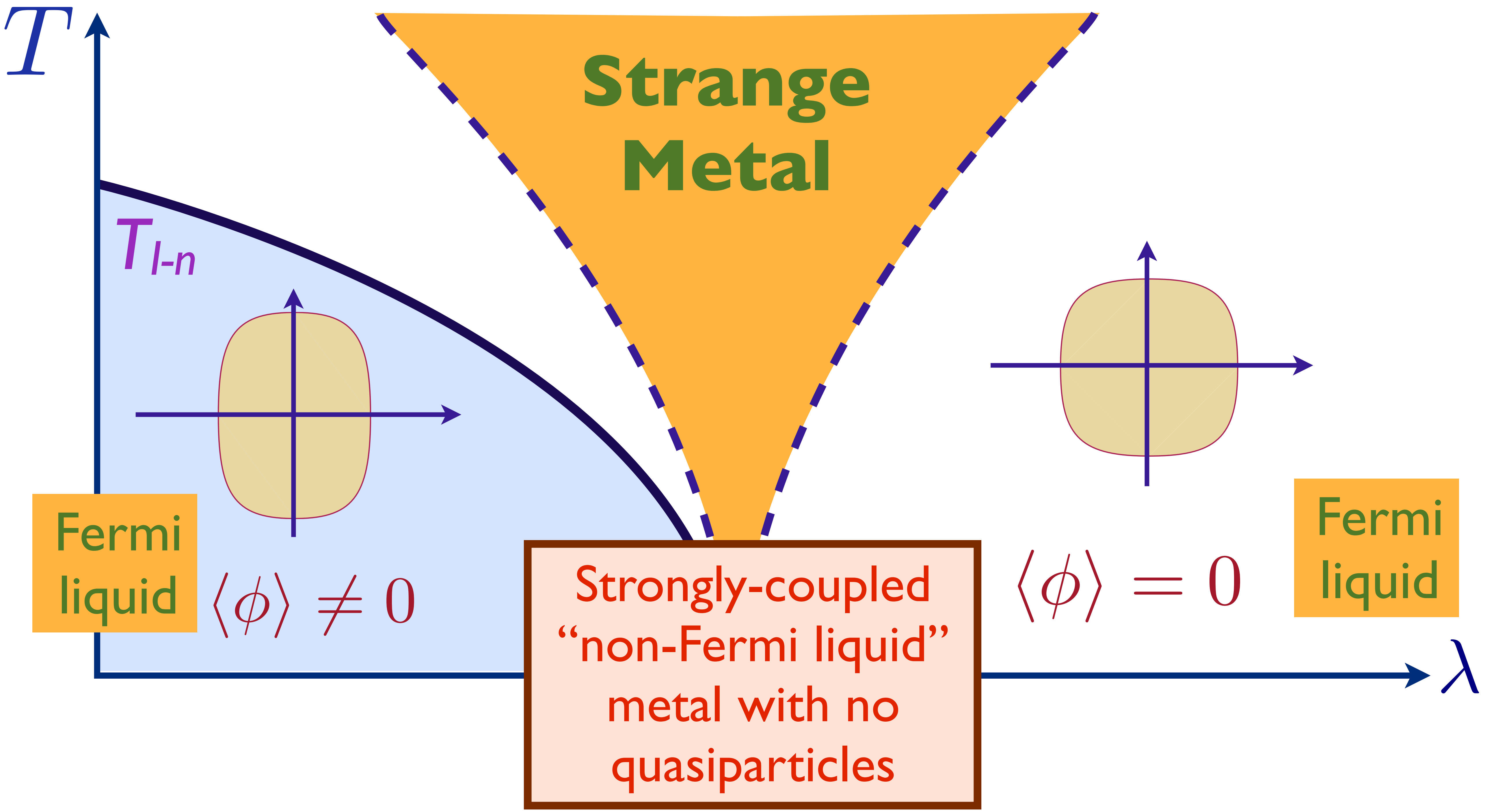

Phase diagram as a function of T and λ

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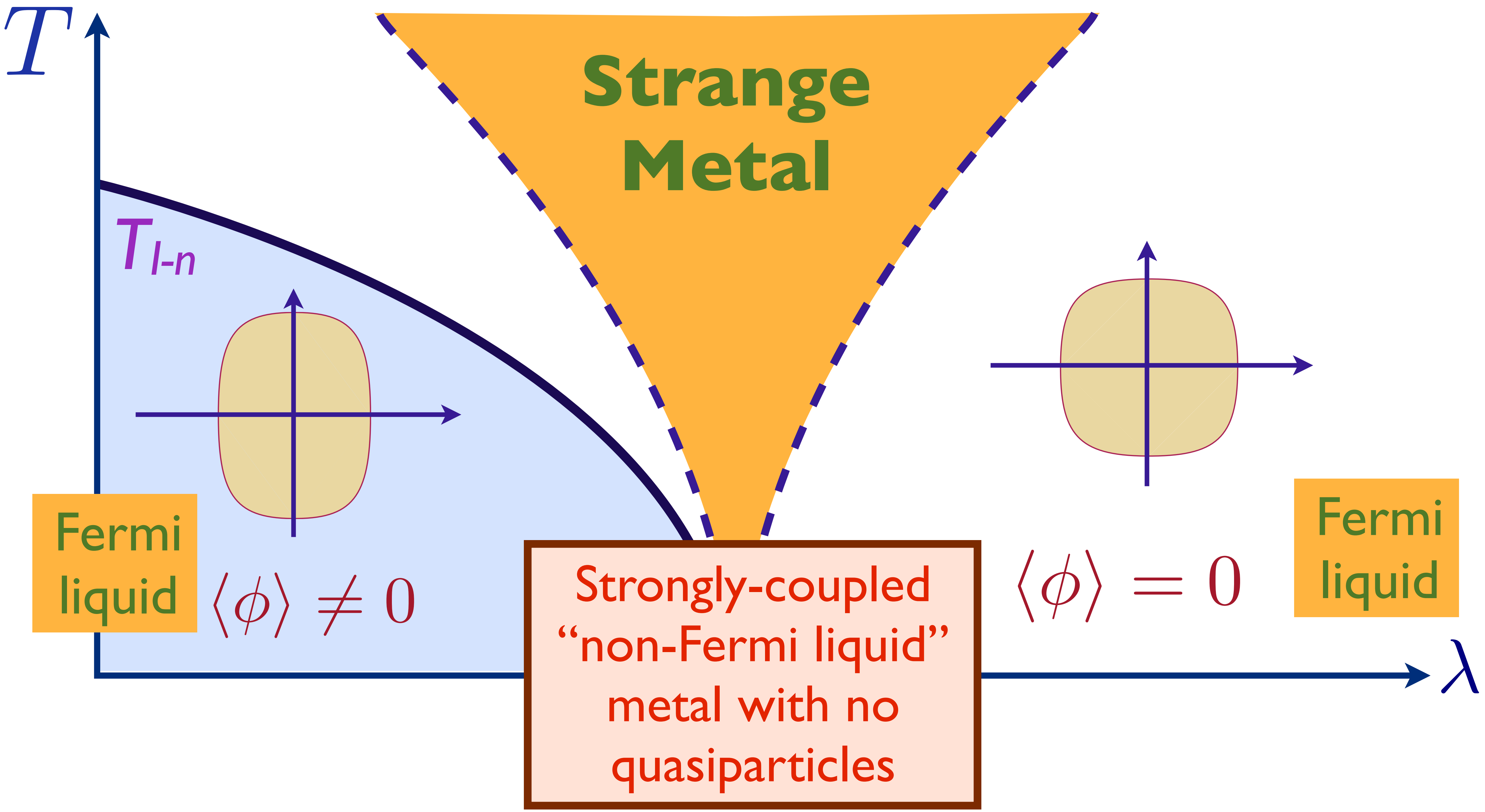

Common theoretical belief: resistivity of strange metal $\rho(T) \sim T^{4/3}$.

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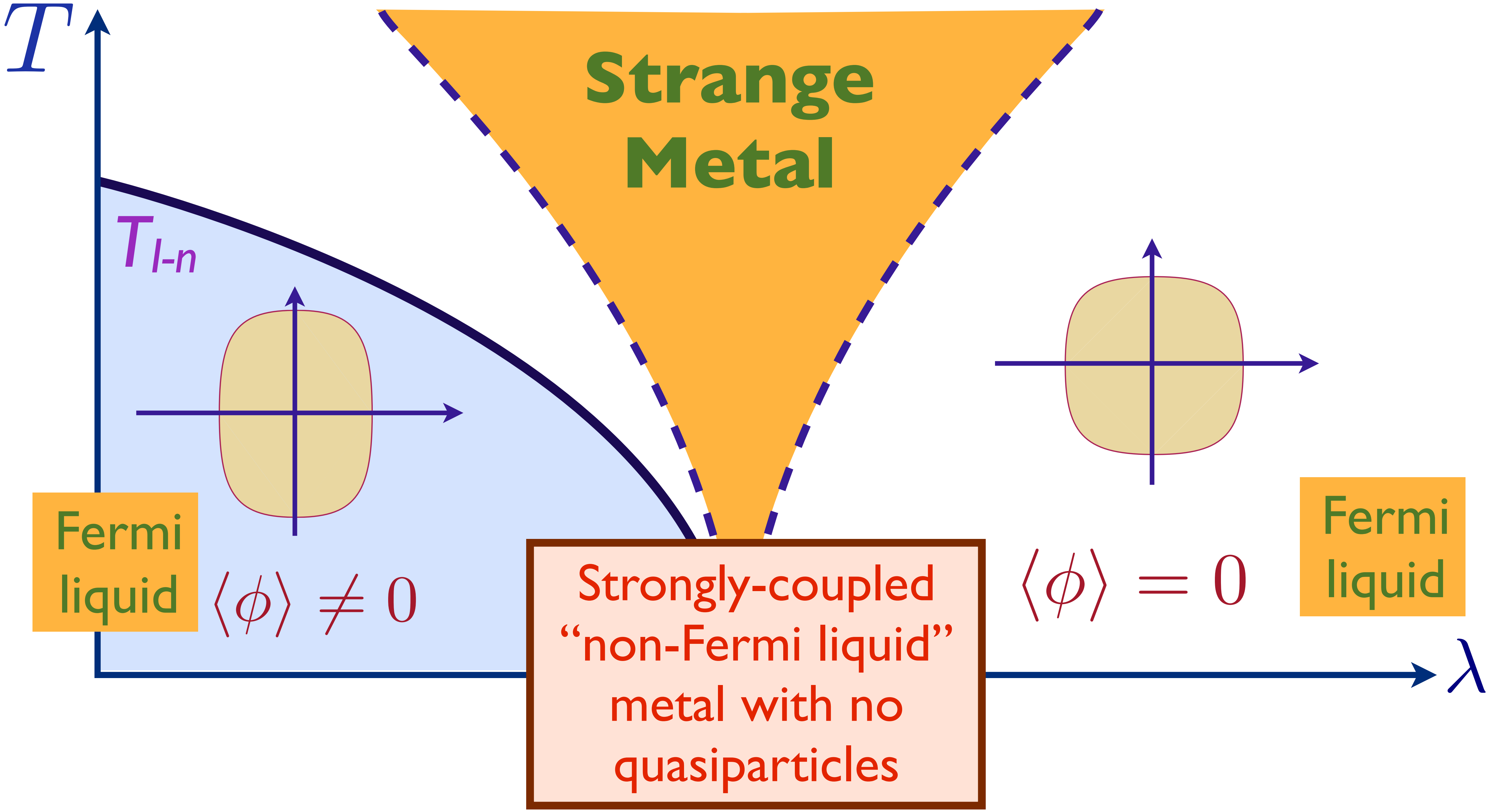

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Ignores conservation of total momentum (analog of "phonon drag")

D. L. Maslov, V. I. Yudson, and A. V. Chubukov, Phys. Rev. Lett. **106**, 106403 (2011).

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

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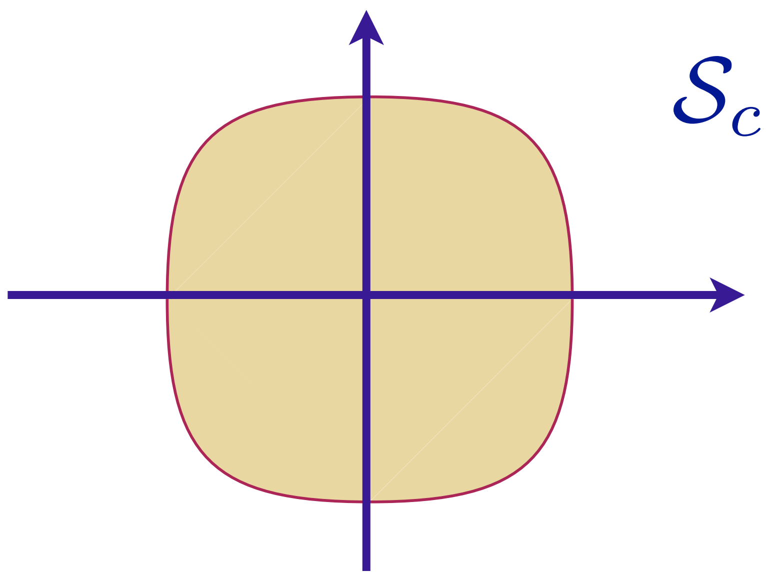
Effective action for electrons:



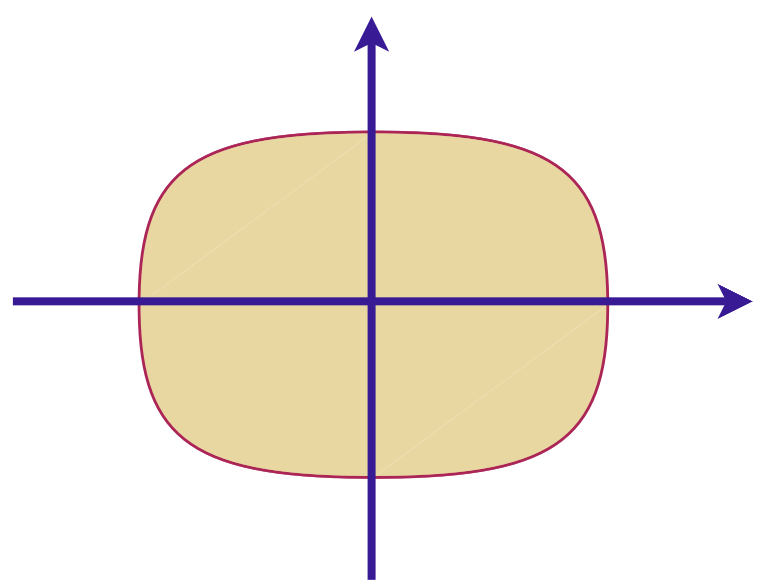
$$\begin{aligned} \mathcal{S}_c &= \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right] \\ &\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \end{aligned}$$

Quantum criticality of Ising-nematic ordering in a metal

Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



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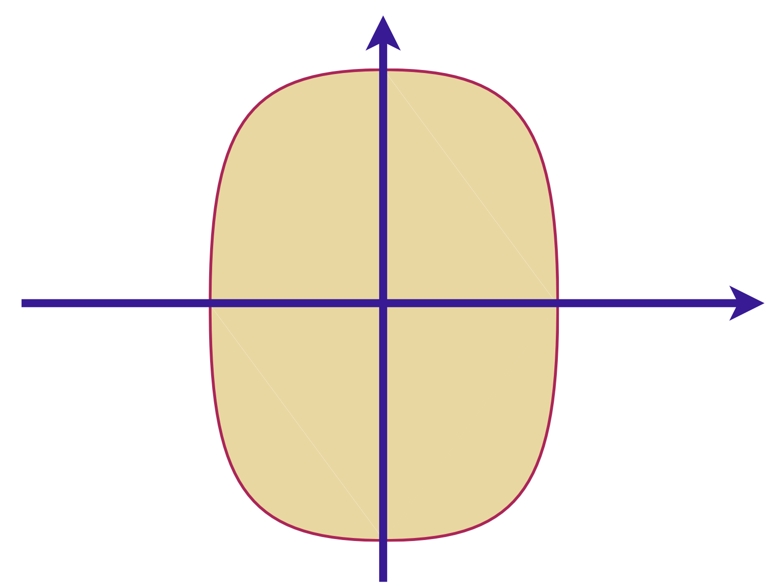

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Memory functions: in the presence of perturbations which lead to $\dot{\mathbf{P}} \neq 0$ (also tested holographically, via "graviton masses"):

$$\rho(T) = \chi_{\mathbf{J},\mathbf{P}}^{-2} \lim_{\omega \to 0} \text{Im} \frac{\chi_{\dot{\mathbf{P}},\dot{\mathbf{P}}}(\omega)}{\omega} .$$

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\rm dis} = \int d^2r d\tau \left[V(\mathbf{r})\, c^\dagger c + h(\mathbf{r})\, \phi \right] \,,$$

$$\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2 \, \delta(\mathbf{r} - \mathbf{r}') \,,$$

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we obtain the resistivity for current along angle ϑ

$$\rho(T) = \frac{1}{\chi^2_{\mathbf{J},\mathbf{P}}} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2} \, k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left(V_0^2 \, \frac{\mathrm{Im}\, \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2 \, \frac{\mathrm{Im}\, D^R_\phi(\omega, \mathbf{k})}{\omega} \right)$$

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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$$

Fermi surface term: Obtain T-dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

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Bosonic term: Dominant contribution:

$$\rho(T) \sim T^{(d-z+\eta)/z}$$

Crosses over from the "relativistic" form ($z = 1$, $\eta \approx 0$) with $\rho(T) \sim T$ at higher T,
to the "Landau-damped" form ($z = 3$, $\eta = 0$) with $\rho(T) \sim (T \ln(1/T))^{-1/2}$ at lower T (subtle corrections to scaling specific to this field theory).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.

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Also obtained in holographic theory of a generalized compressible quantum state (A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.

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- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography