The Superfluid-Insulator transition

Boson Hubbard model

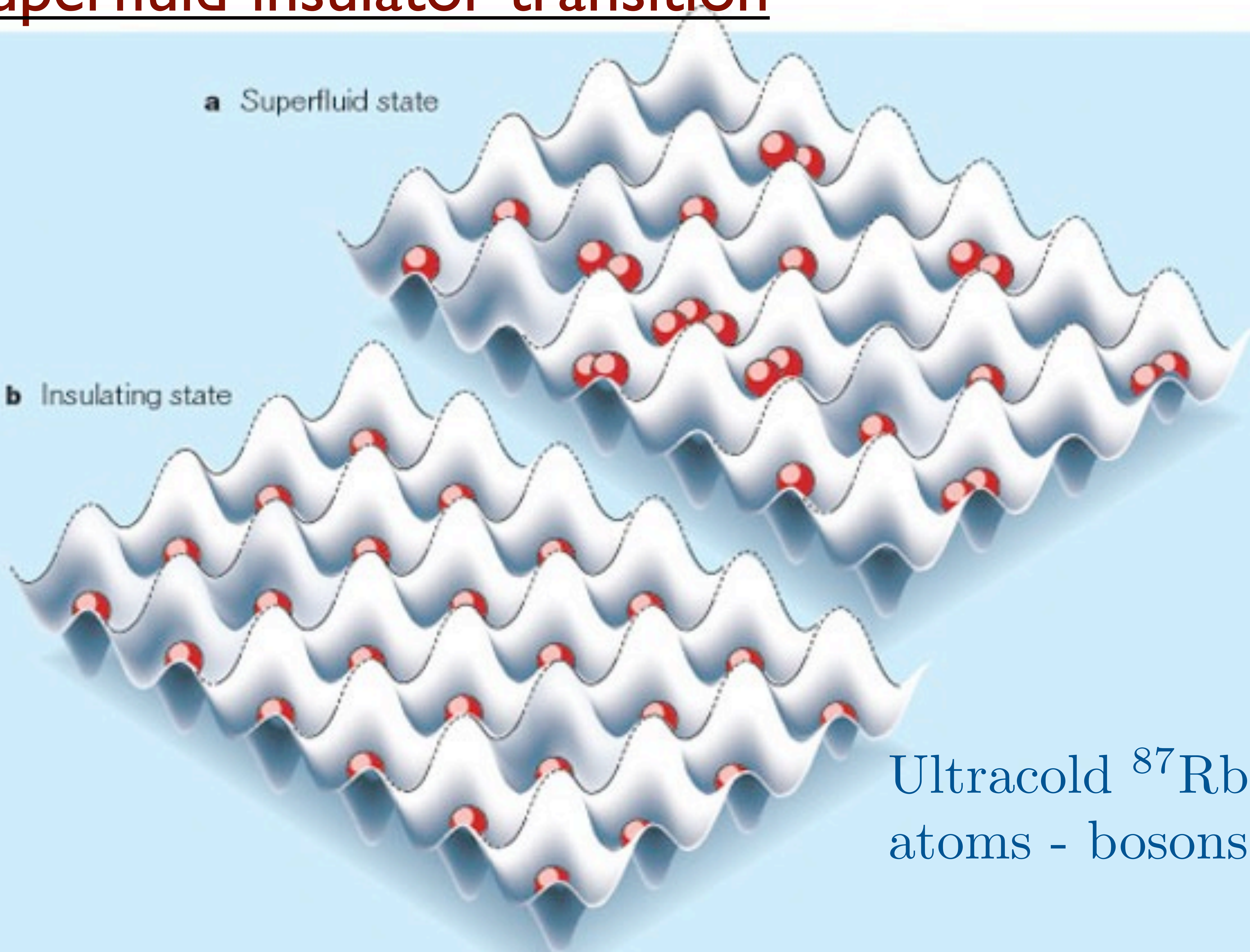
Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

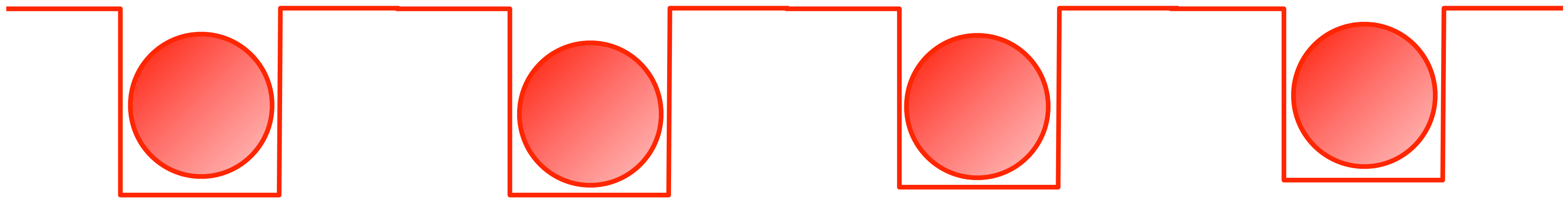
Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

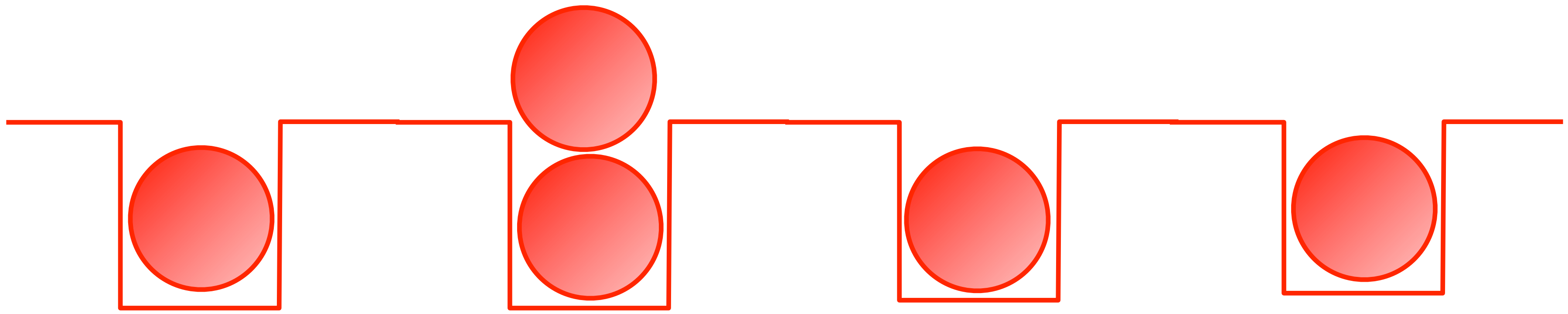
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Friday, June 11, 2010



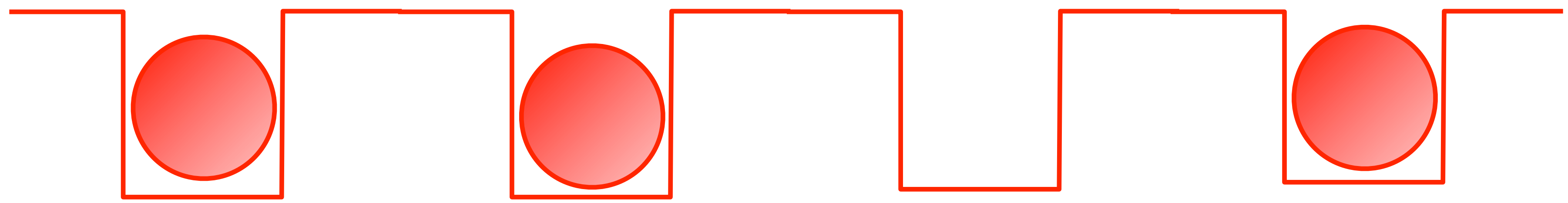
Insulator (the vacuum) at large U

Excitations:



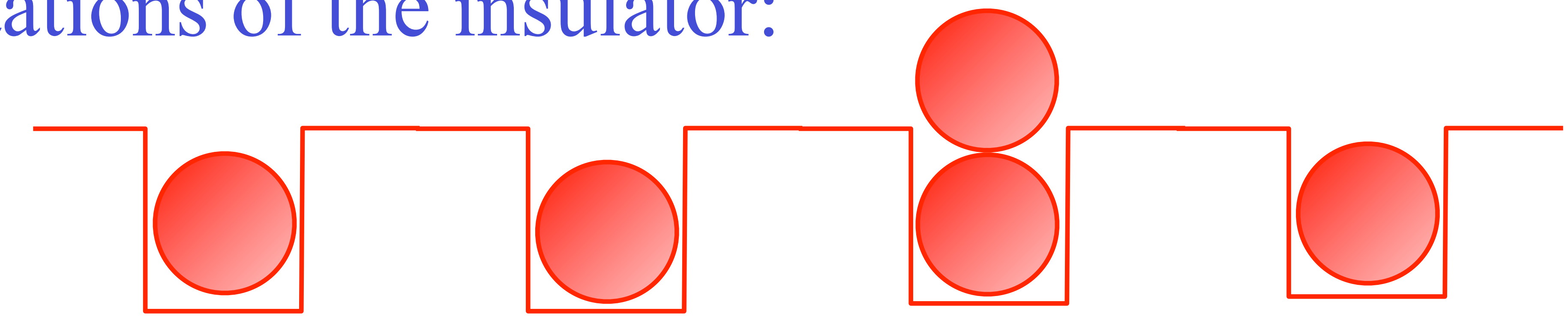
Particles $\sim \psi^\dagger$

Excitations:



Holes ~ ψ

Excitations of the insulator:



Particles $\sim \psi^{\dagger}$

Holes $\sim \psi$

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

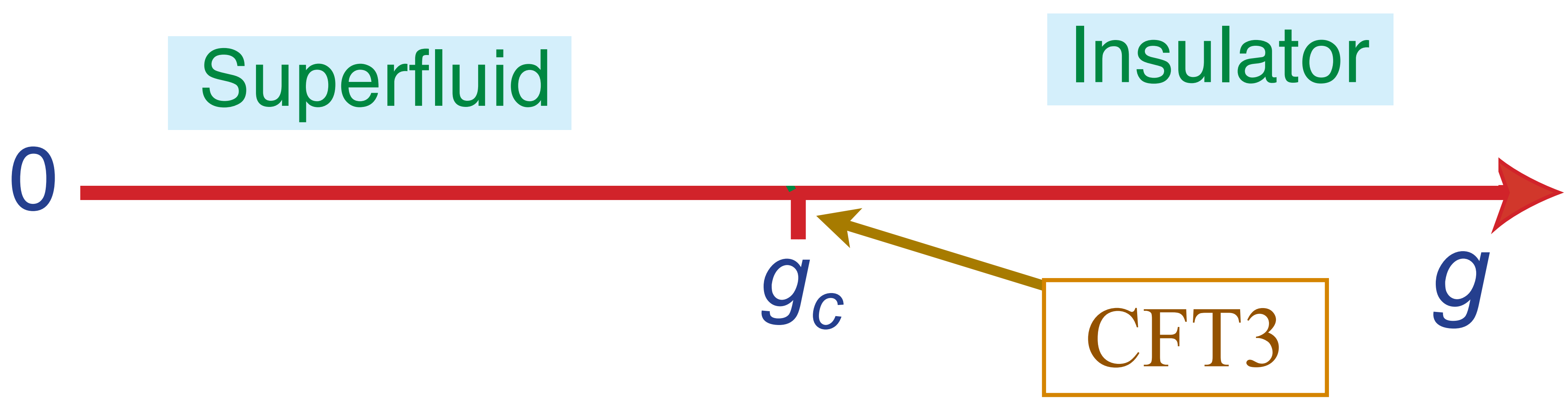
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

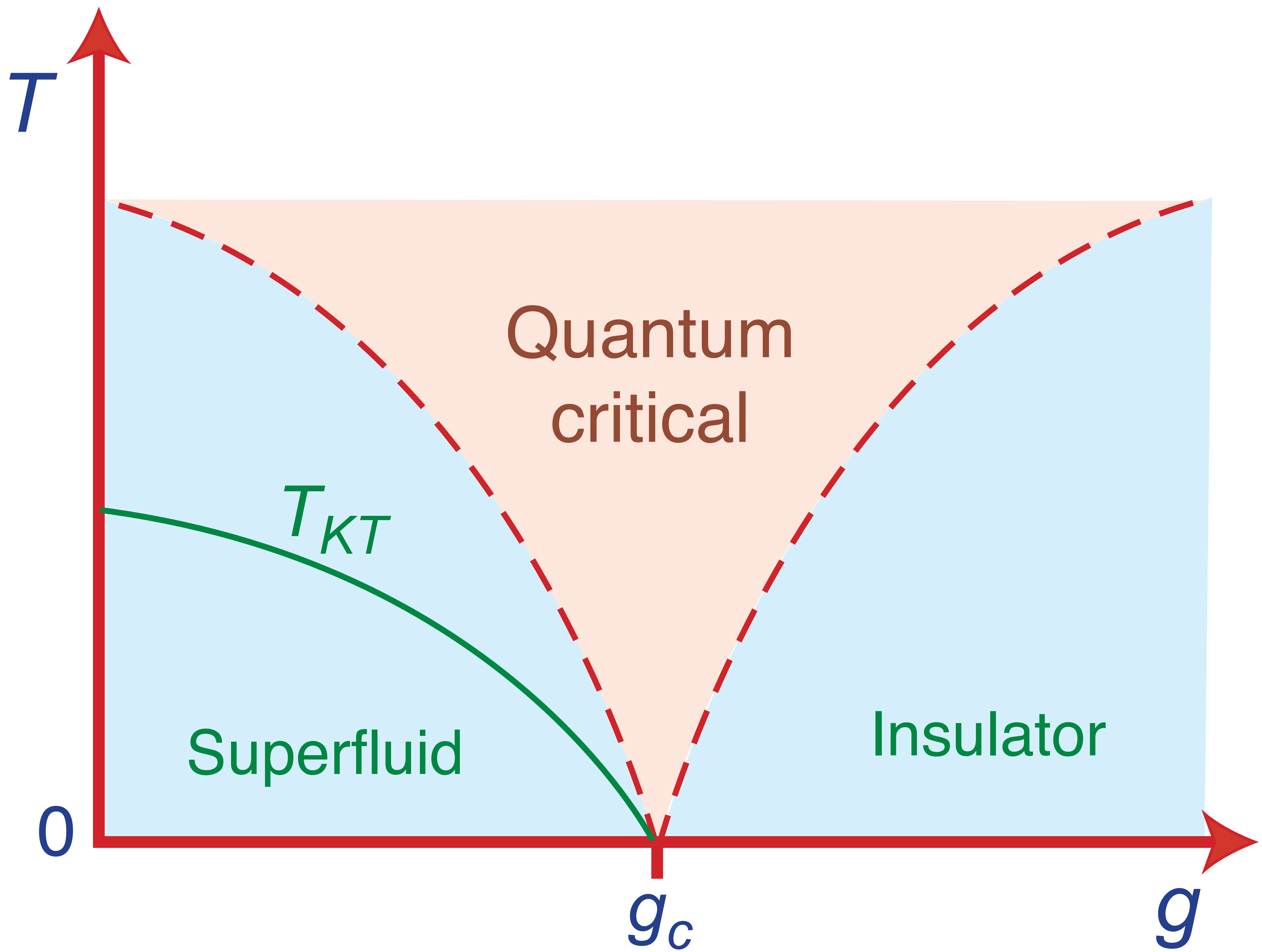
$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$

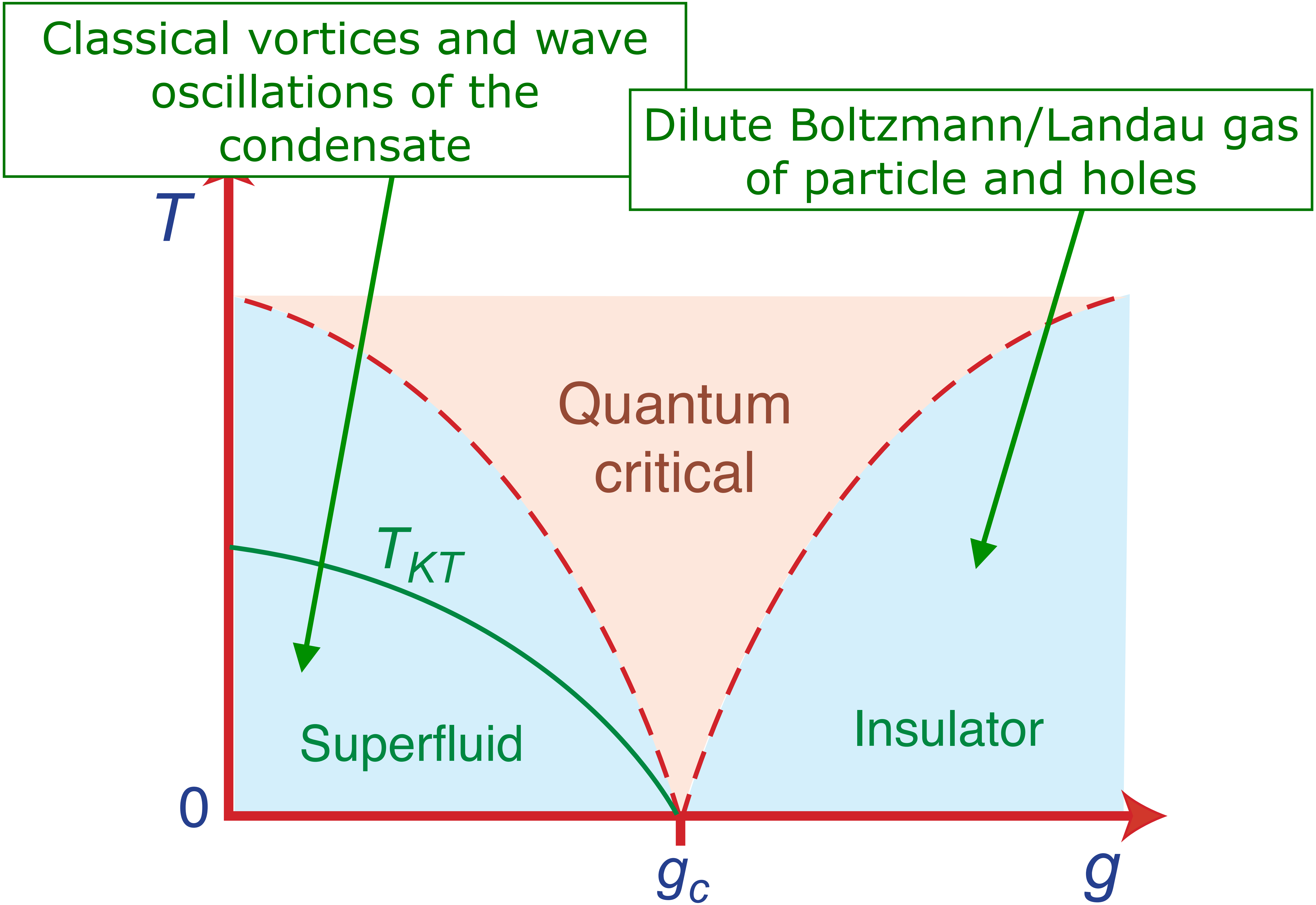
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

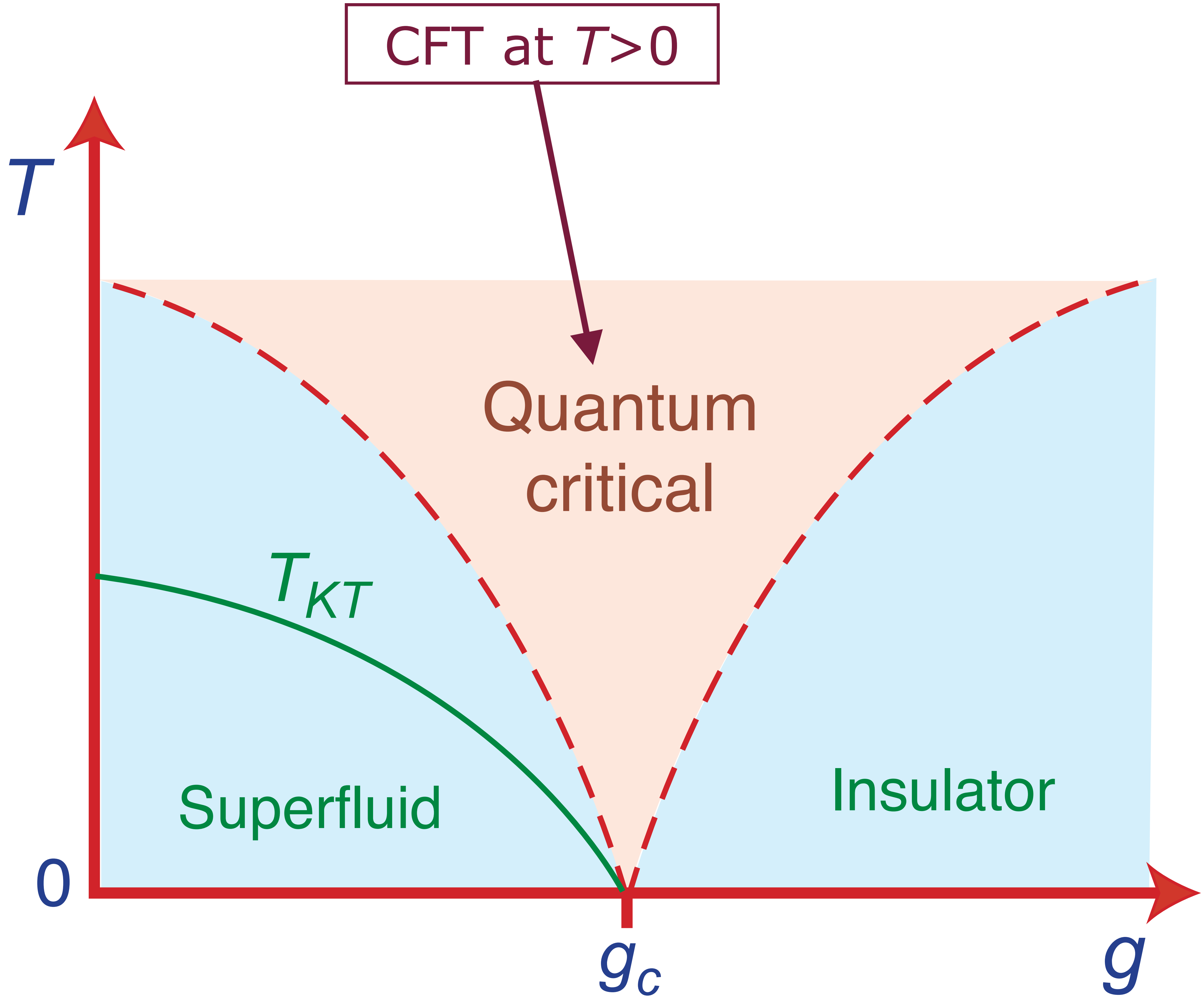
$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$

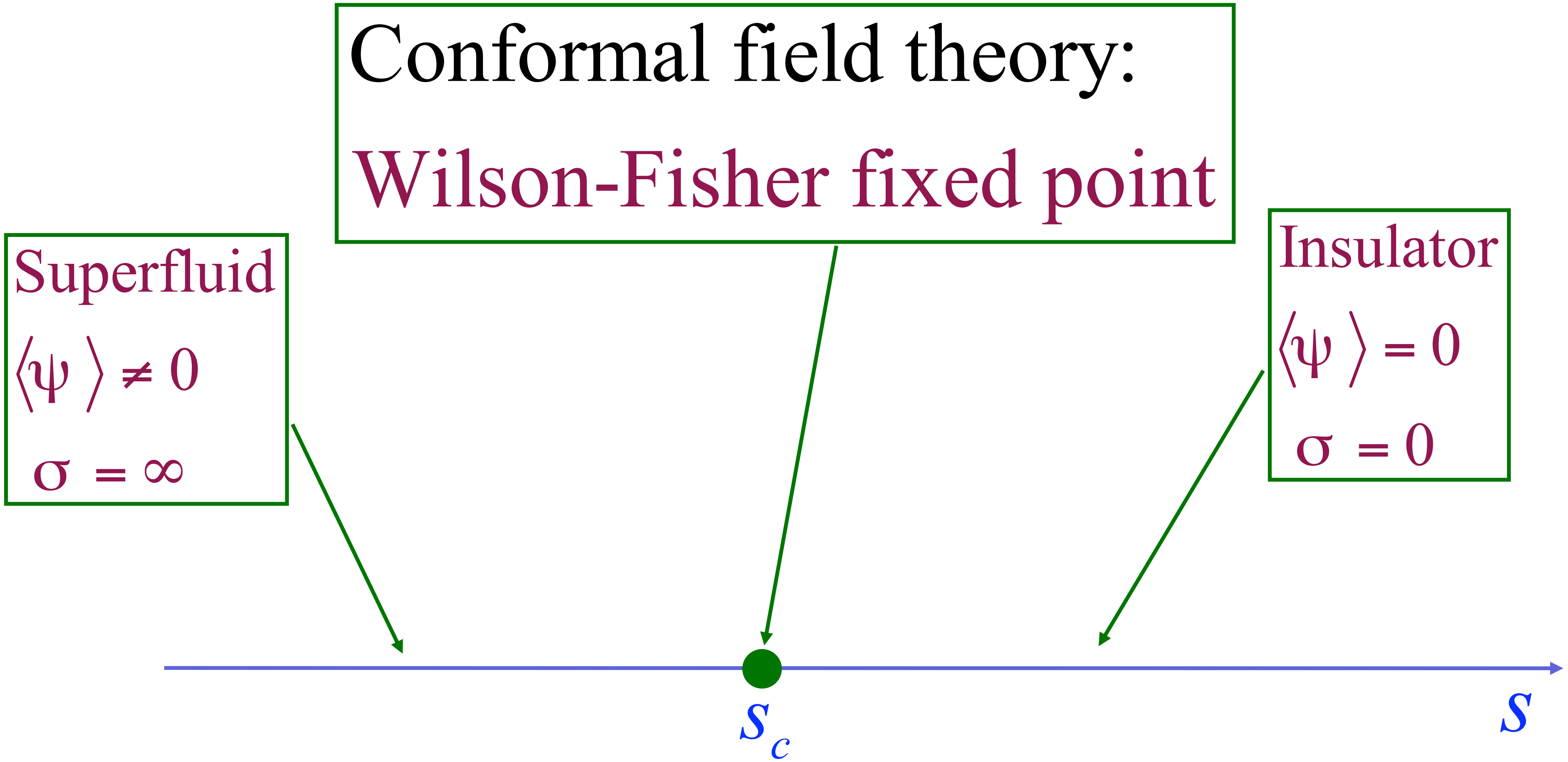




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


Classical vortices and wave oscillations of the condensate
Dilute Boltzmann/Landau gas of particle and holes
T
Quantum critical
T_KT
Superfluid
Insulator
0
g_c
g


CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g



Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett*. **47**, 1556 (1981)

Conformal field theory:
Wilson-Fisher fixed point

Superfluid

$\langle \psi \rangle \neq 0$

$\langle \varphi \rangle = 0$

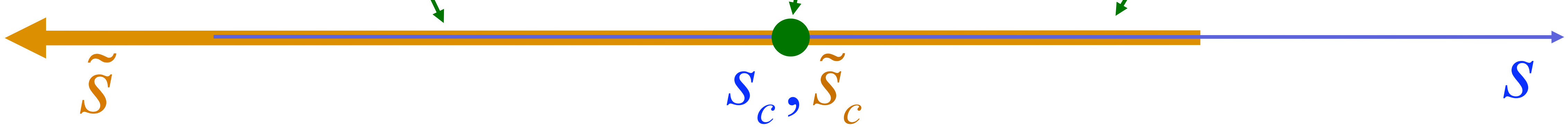
$\sigma = \infty$

Insulator

$\langle \psi \rangle = 0$

$\langle \varphi \rangle \neq 0$

$\sigma = 0$



Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

<u>is dual to</u>

Using the vortex quasiparticle excitations of the superfluid $\sim \varphi$

$$\mathcal{S}_{\rm dual} = \int d^3x \left[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981)

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1. Introduction to the Hubbard model

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2. Coupled dimer antiferromagnet

   CFT3: the Wilson-Fisher fixed point

3. Honeycomb lattice: semi-metal and antiferromagnetism

   CFT3: Dirac fermions and the Gross-Neveu model

4. Quantum critical dynamics

   AdS/CFT and the collisionless-hydrodynamic crossover

5. Hubbard model as a SU(2) gauge theory

   Spin liquids, valence bond solids: analogies with SQED and SYM

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Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

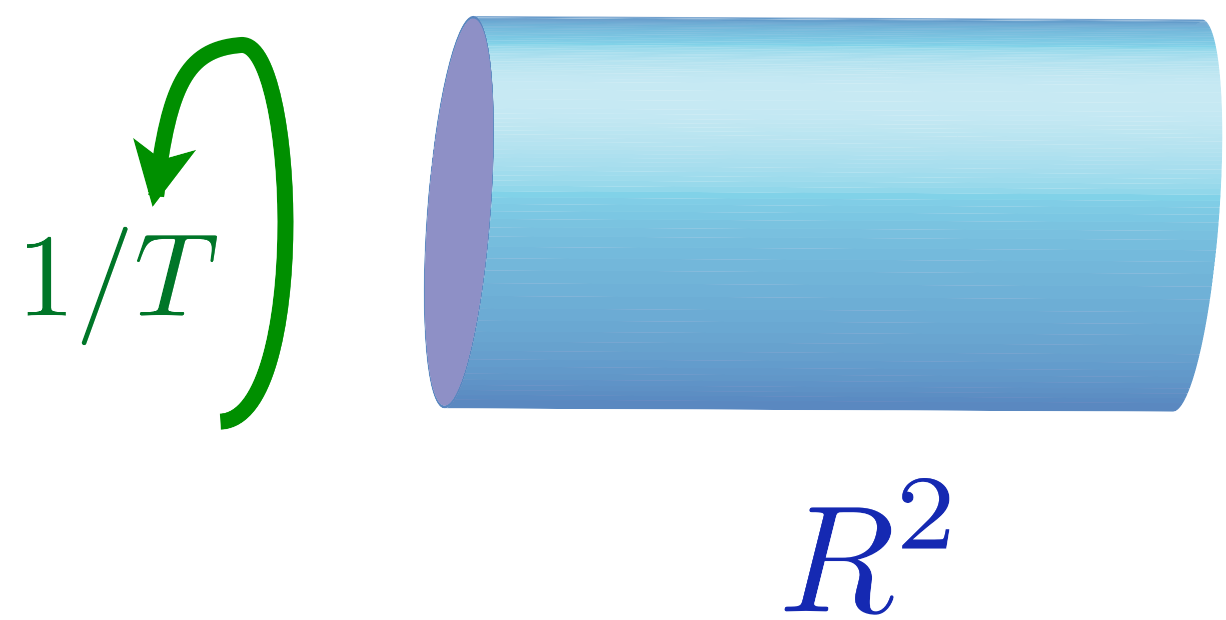
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Euclidean field theory:

Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



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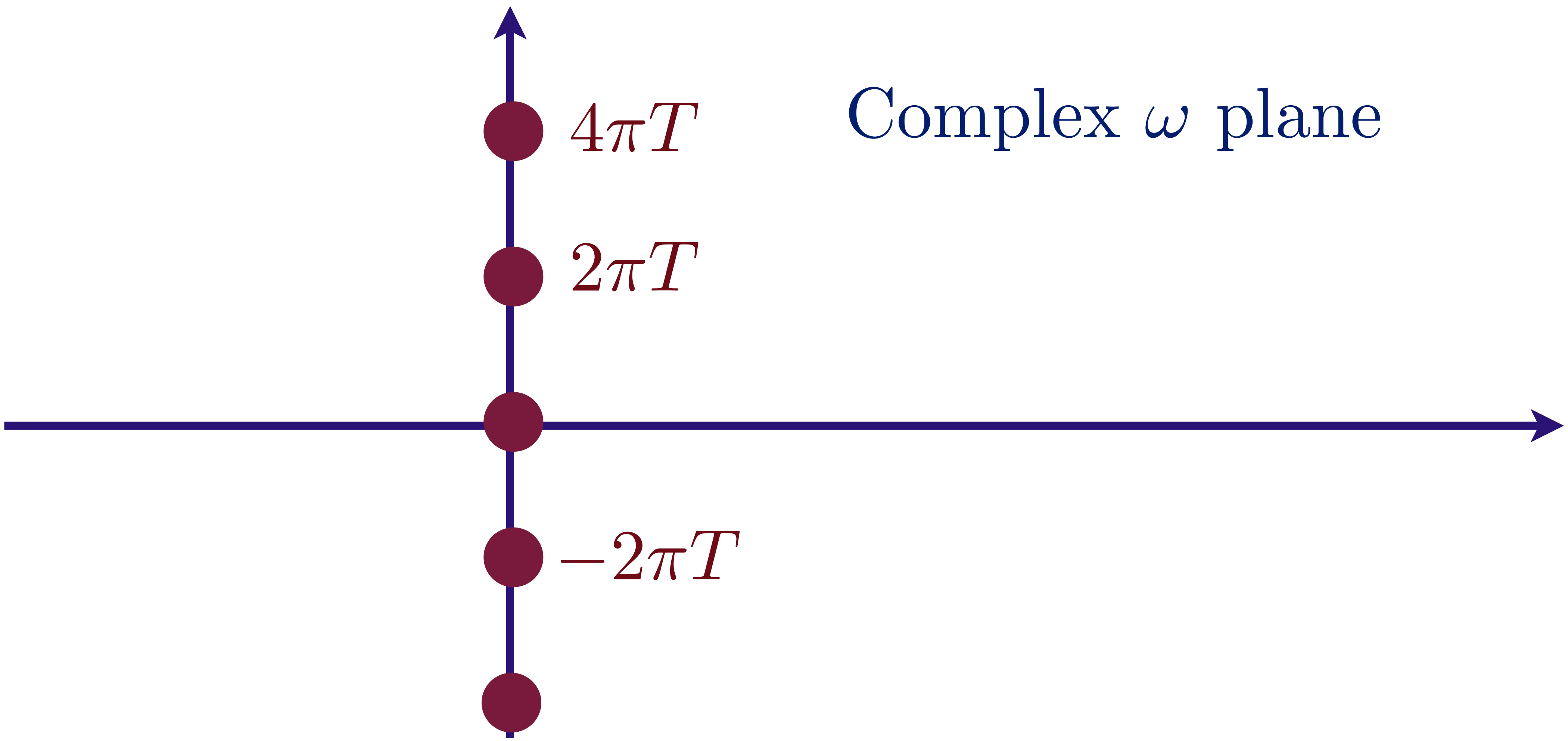

Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer)
or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Quantum critical transport

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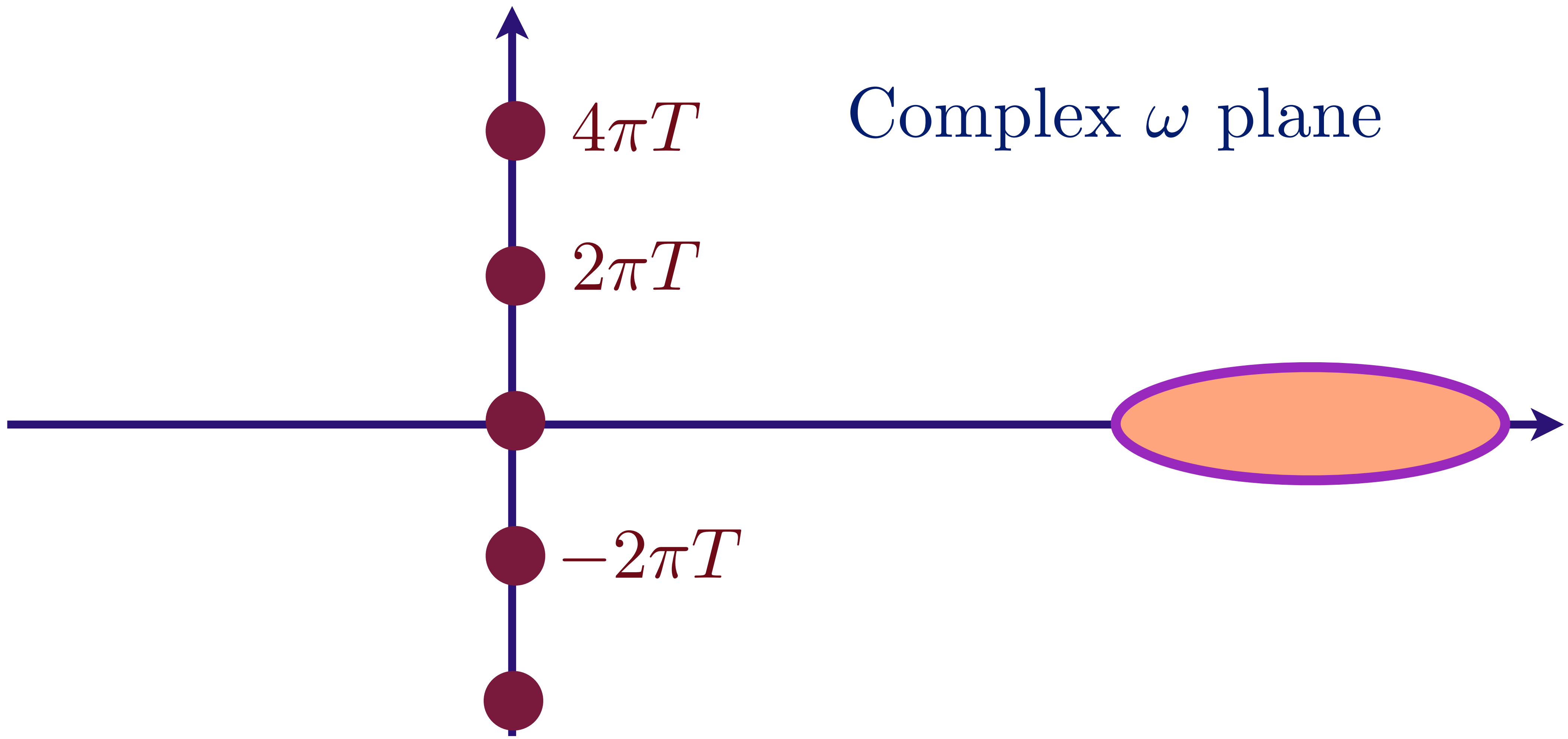


Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi nTi$ (n integer) or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

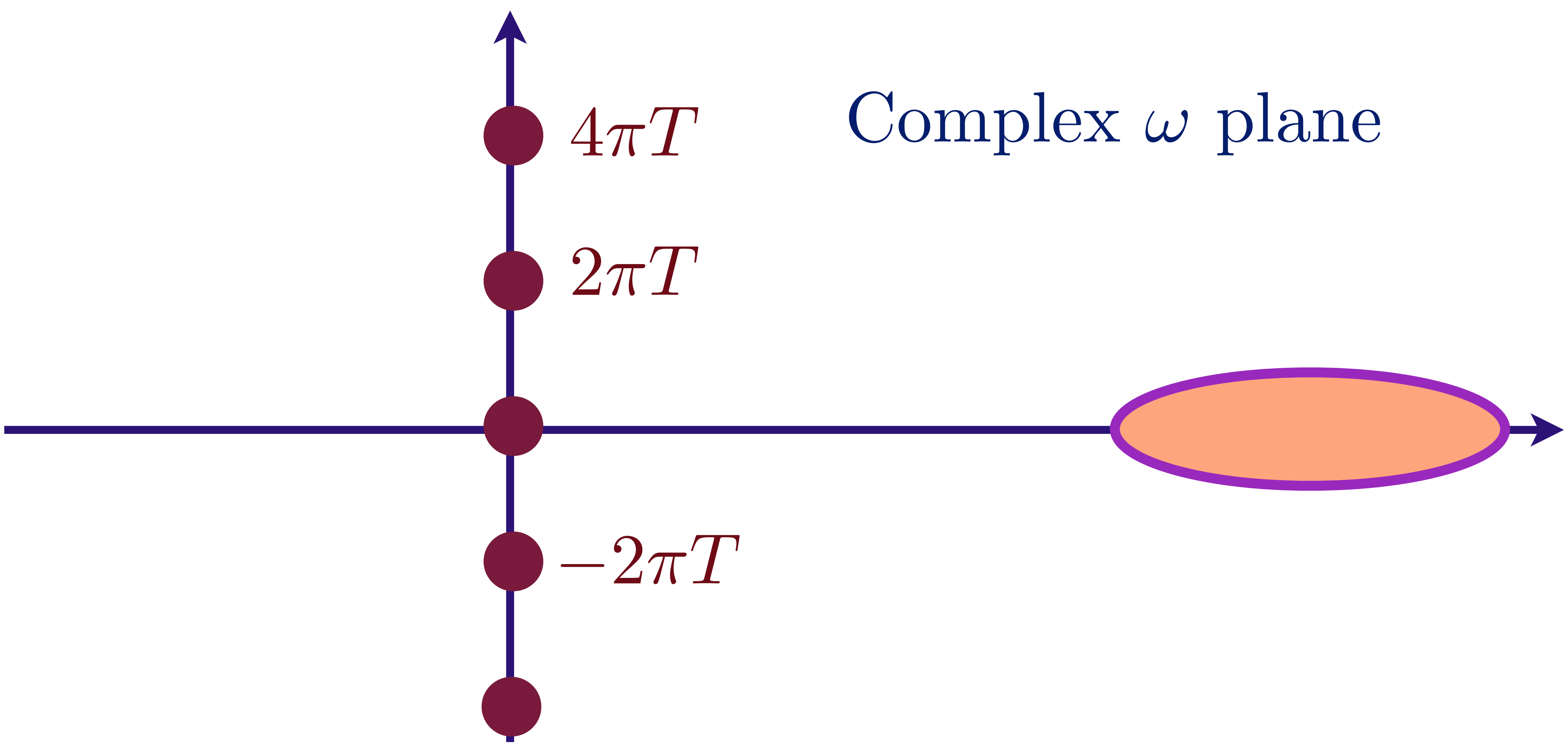
For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) = \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



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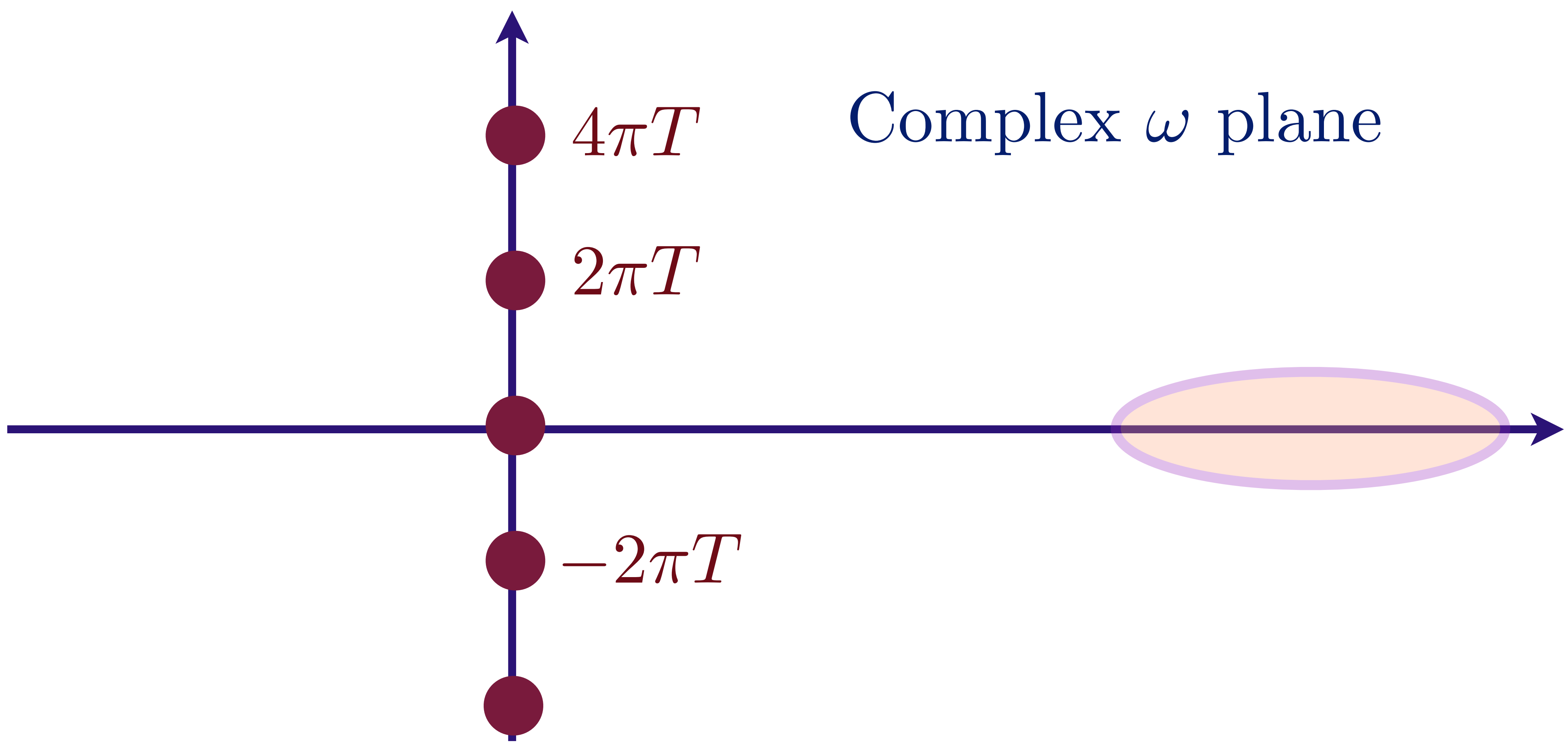
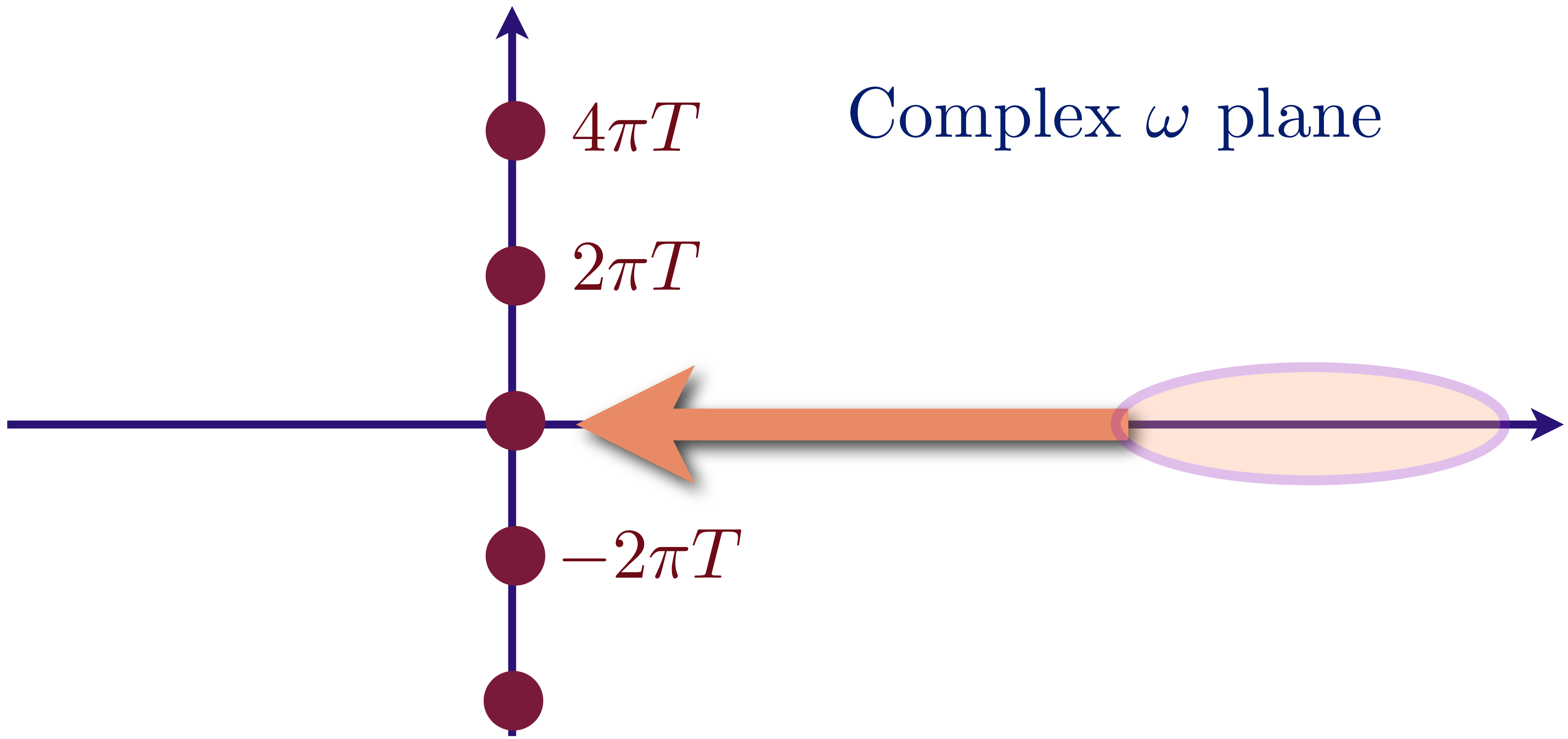


Strong coupling problem:
Correlators at $\hbar\omega \ll k_B T$, along the real axis, in the collision-dominated hydrodynamic regime.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



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Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

For *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k,\omega) \quad = \quad e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = e^2 D \chi_c = \frac{e^2}{h}\Theta_1 \Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T>0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h}\Theta_1\Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1\Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant, e_0, which flows to a strong-coupling fixed point $e_0 = e_0^*$ in the infrared.
- The CFT3 describing this fixed point resembles "critical spin liquid" theories.
- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\text{AdS}_4 \times S_7$.
- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large N limit, even at $T > 0$.

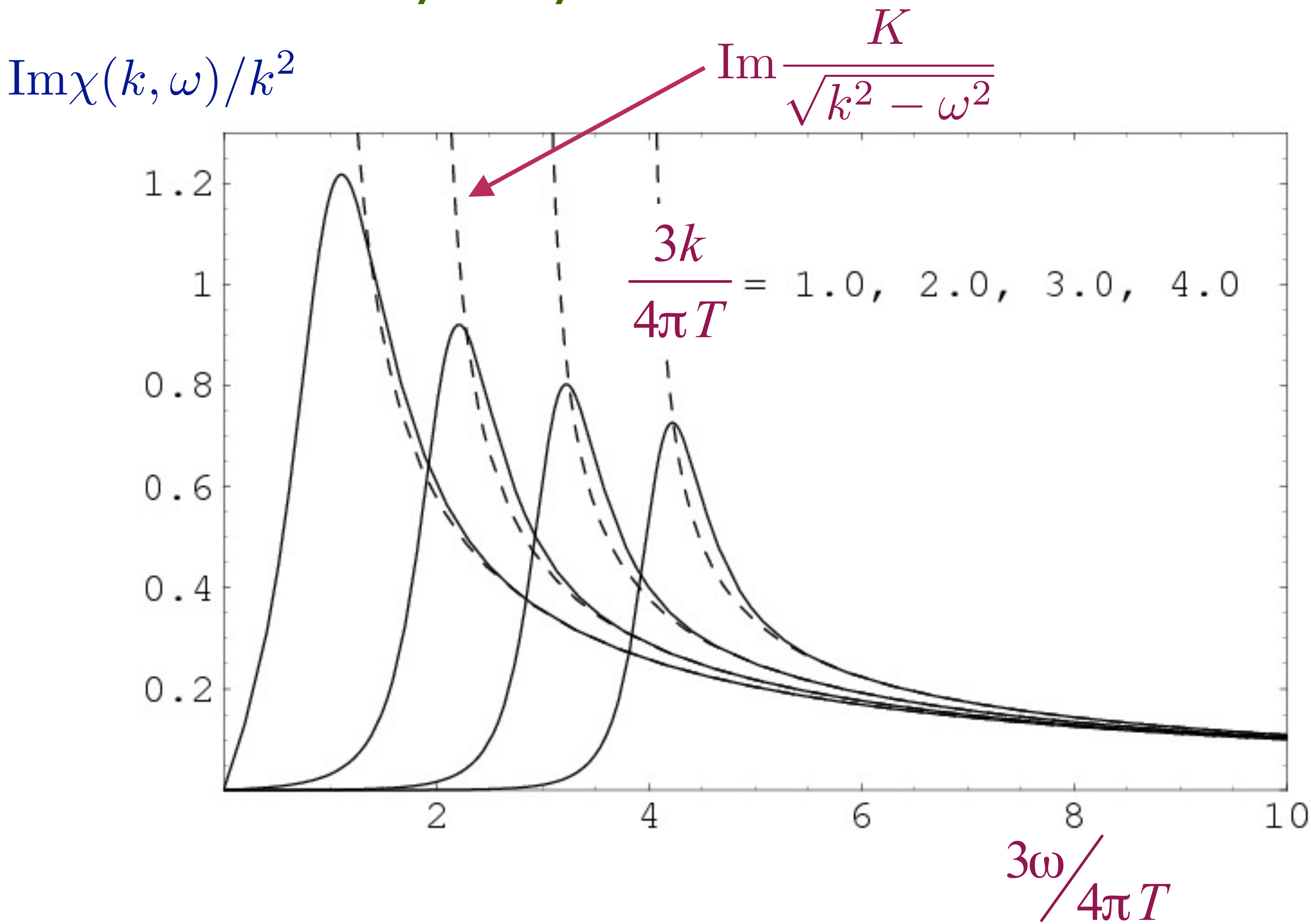
SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- The SO(8) charge correlators of the CFT3 are given by the usual AdS/CFT prescription applied to the following gauge theory on AdS4:

$$\mathcal{S} = -\frac{1}{4g_{4D}^2} \int d^4x \sqrt{-g} g^{MA} g^{NB} F^a_{MN} F^a_{AB}$$

where $a = 1 \ldots 28$ labels the generators of SO(8). Note that in large N theory, this looks like 28 copies of an Abelian gauge theory.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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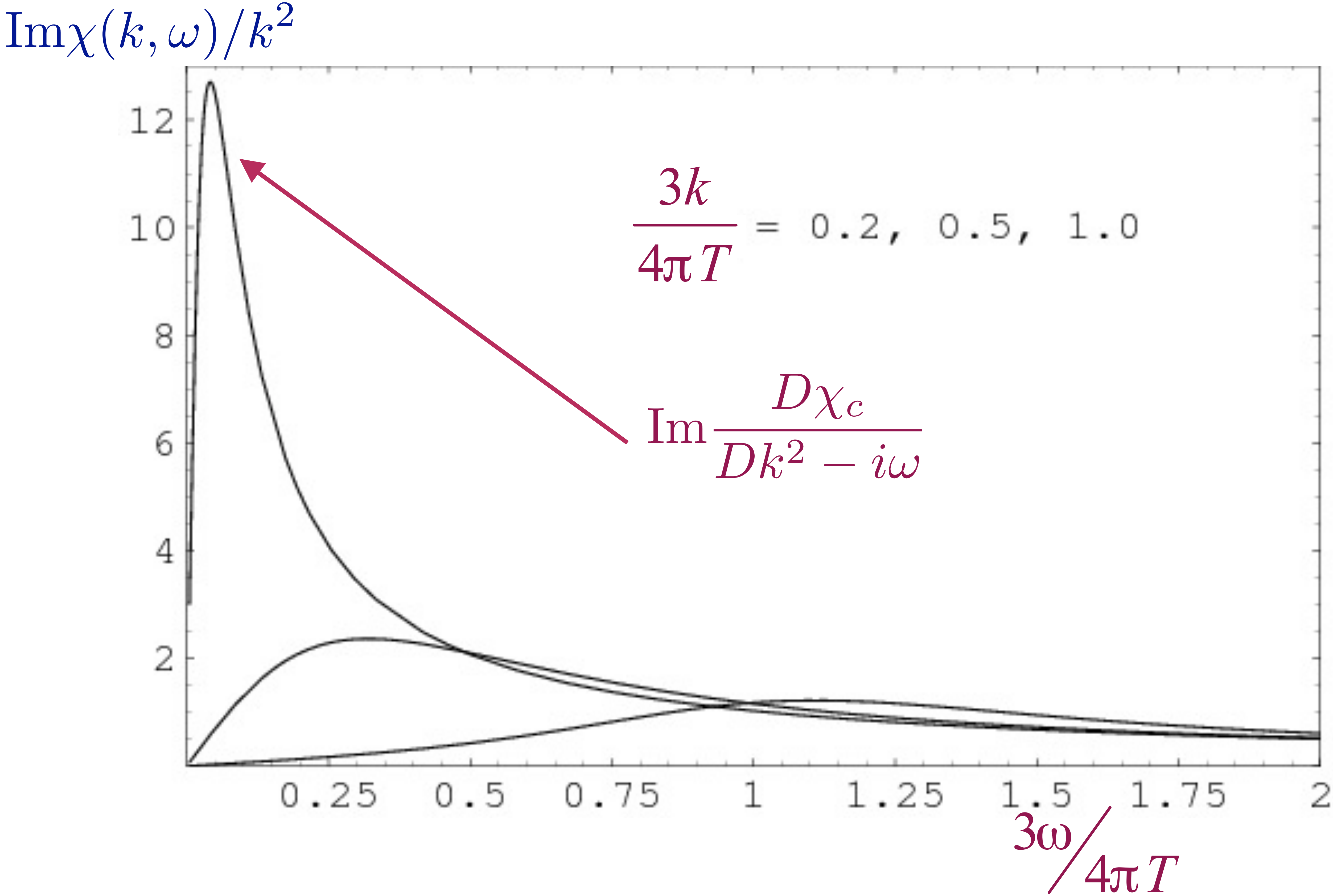

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1$$

$$D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} K & , \quad \hbar\omega \gg k_B T \\ \Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$K = \frac{\sqrt{2}N^{3/2}}{3}$$

$$\Theta_1 = \frac{8\pi^2\sqrt{2}N^{3/2}}{9}$$

$$\Theta_2 = \frac{3}{8\pi^2}$$

C. Herzog, JHEP **0212**, 026 (2002)
P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$, and $\sigma(\omega)$ is frequency $independent$.

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- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on AdS_4. In the large N limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS_4.

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- The conductivity takes the *self-dual* value $\sigma(\omega) = 1/g_{4D}^2$.

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- All these special features are *generic* to theories with a Maxwell-Einstein gravity dual. They are not expected to survive stringy $1/N^2$ corrections

Lessons from AdS/CFT

Strongly interacting quantum-critical systems are nearly "perfect fluids". The AdS description of such perfect fluids suggests that

- All continuous global symmetries are (approximately) self dual.
- We can define a conductivity $\sigma(\omega)$ for each global symmetry: $\sigma(\omega)$ is (nearly) ω independent
- The value of $\sigma(\omega)$ is close to the self dual value

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- We can define a conductivity $\sigma(\omega)$ for each global symmetry: $\sigma(\omega)$ is (nearly) ω independent
- The value of $\sigma(\omega)$ is close to the self dual value

These features are found in experimental and numerical studies

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

- Self-dual value $= 4e^2/h$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

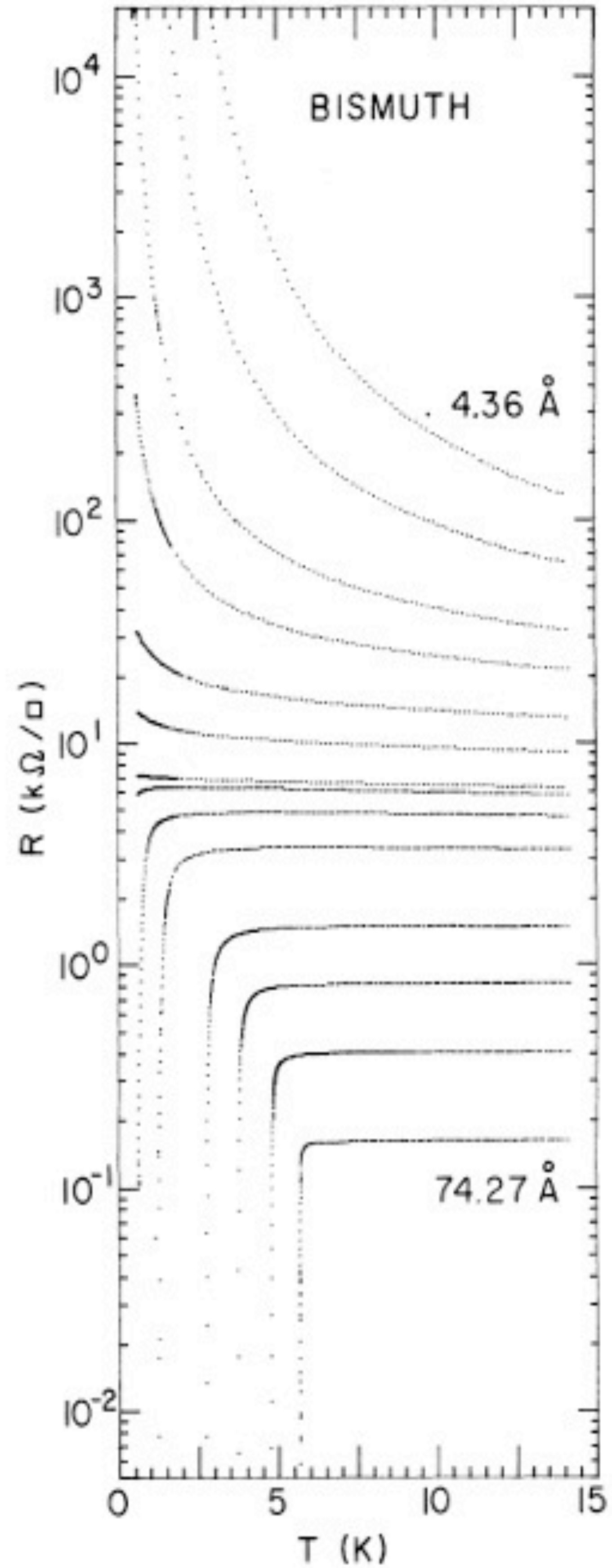

FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Quantum critical transport in graphene

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

$$\frac{\eta}{s} = \frac{\hbar}{k_B \alpha^2(T)} \times 0.130$$

where the "fine structure constant" is

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

L. Fritz, J. Schmalian, M. Müller and S. Sachdev, *Physical Review B* **78**, 085416 (2008)
M. Müller, J. Schmalian, and L. Fritz, *Physical Review Letters* **103**, 025301 (2009)

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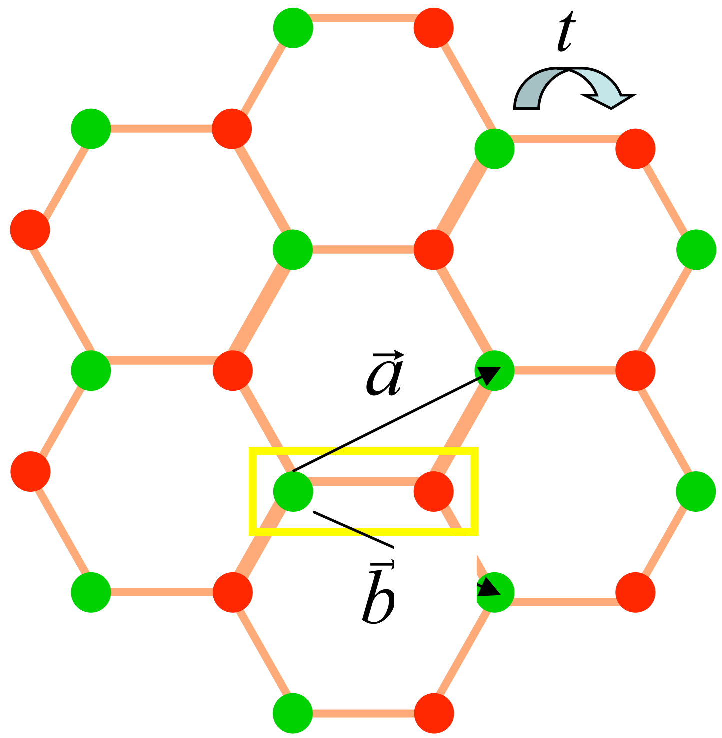
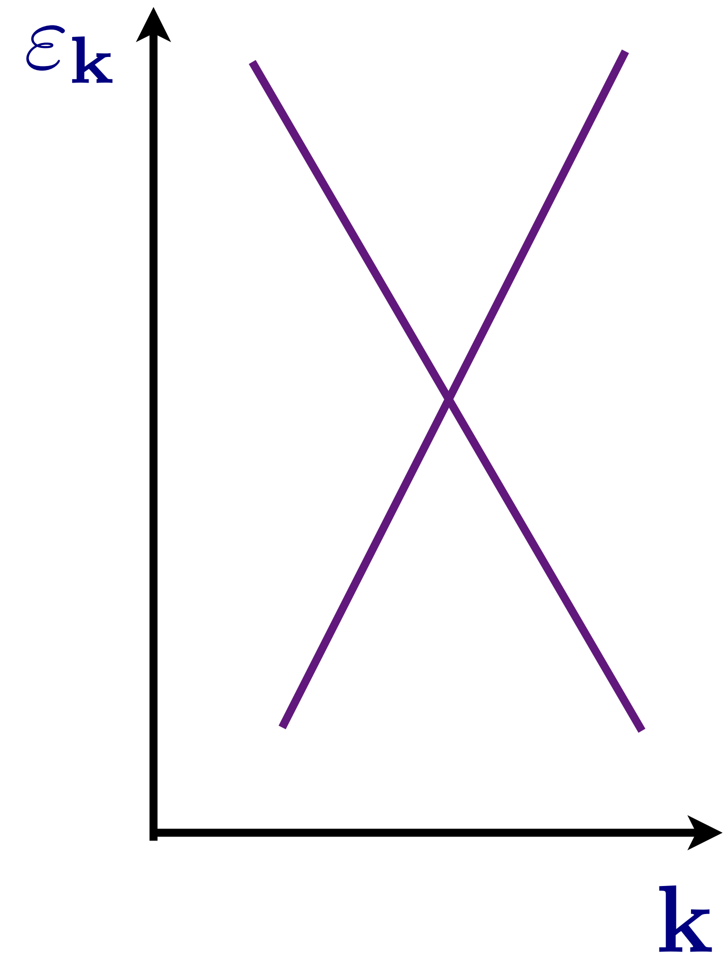
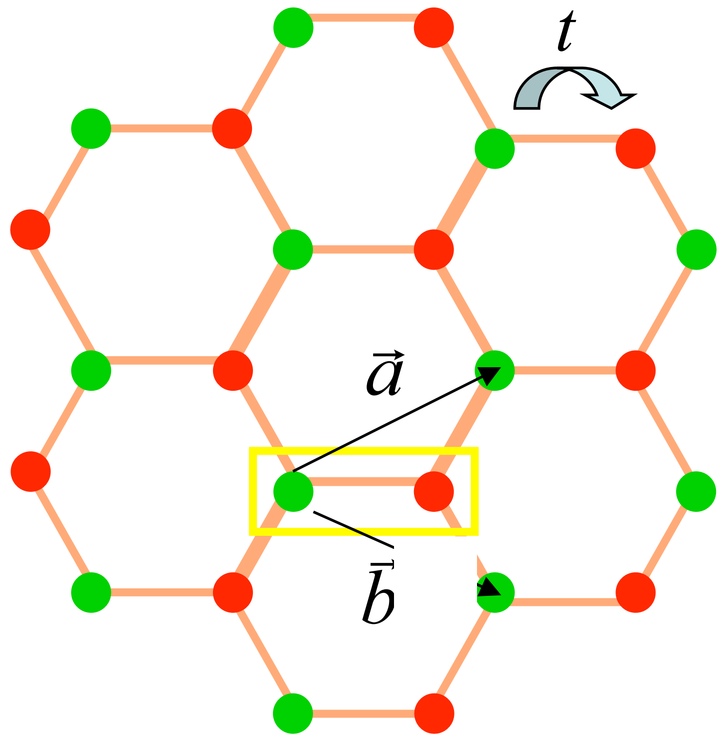
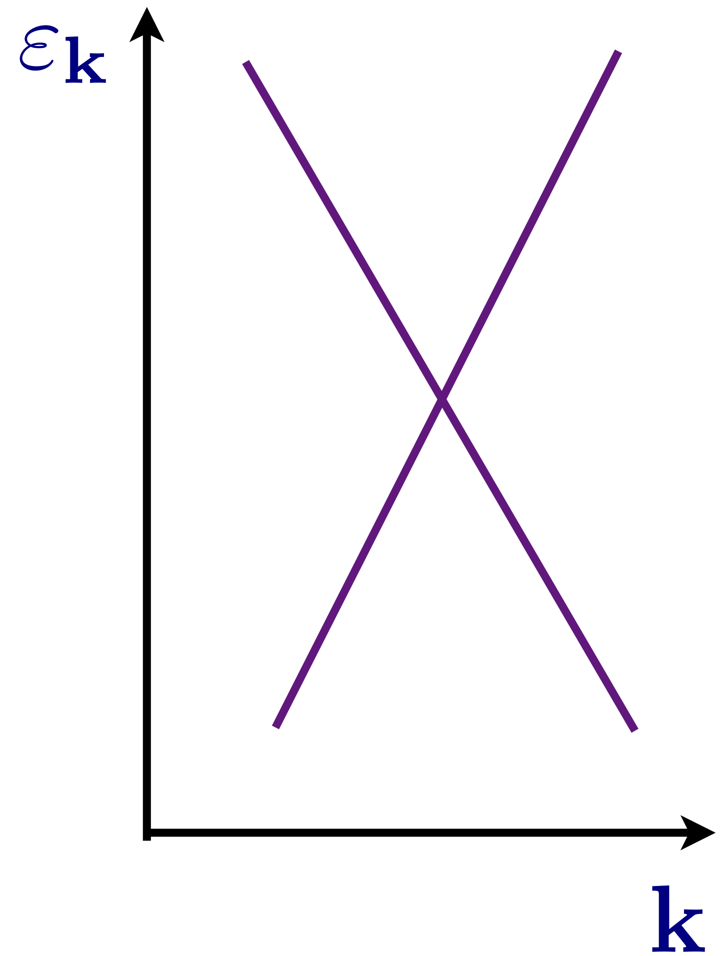




$$H_0 = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{i\alpha}$$

- Begin with free electrons.

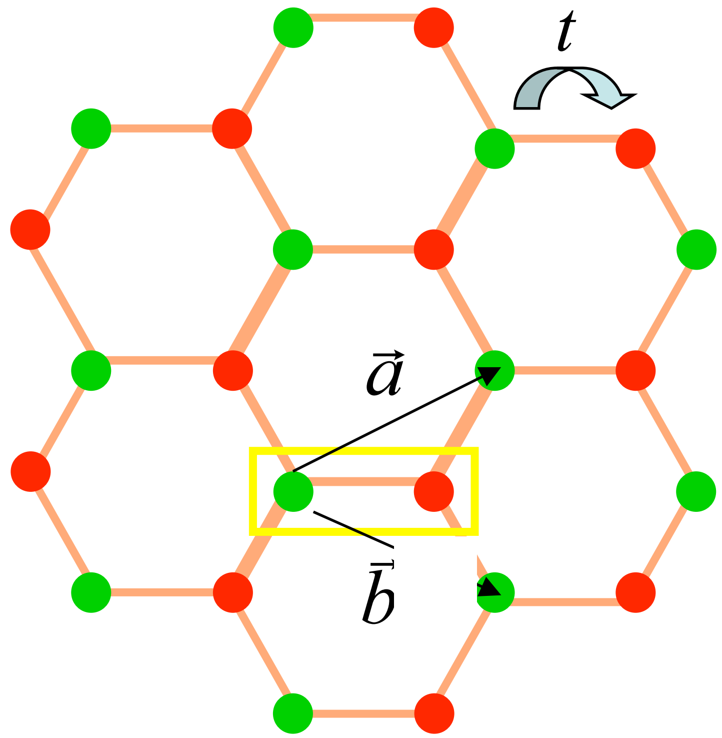


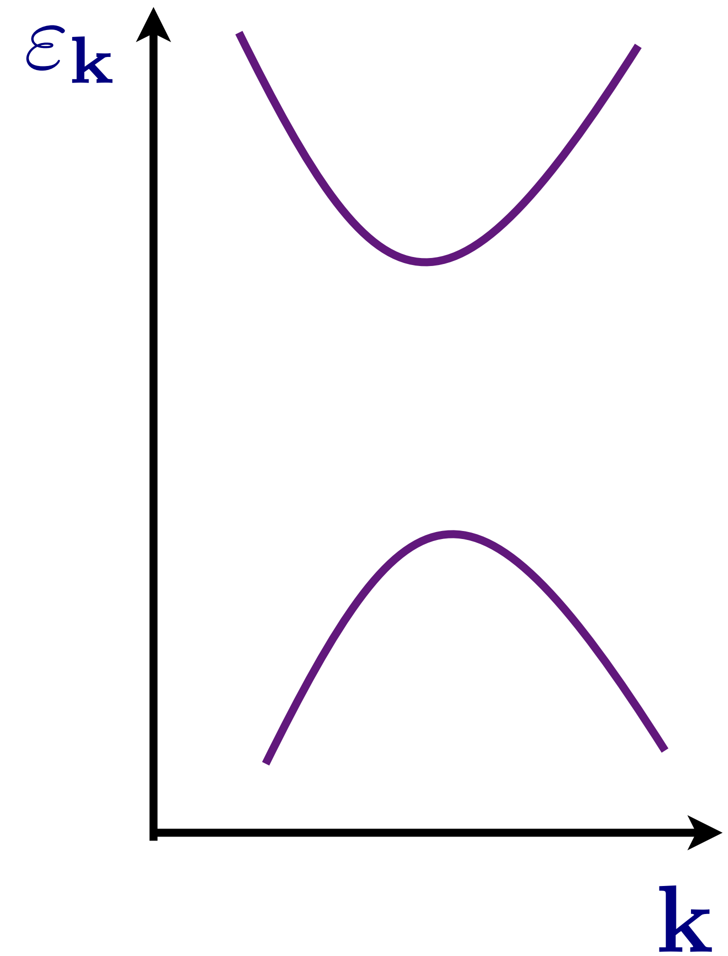


$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha}$$

- Begin with free electrons.
- Add local antiferromagnetism with order parameter $\vec{\varphi}$

$$H_{sdw} = -\sum_{i} \vec{\varphi}(\mathbf{r}_i)(-1)^i c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$





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$$H_{sdw} = -\sum_{i} \vec{\varphi}(\mathbf{r}_i)(-1)^i c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

- The phase with $\langle\vec{\varphi}\rangle \neq 0$ is an insulator with a gap between conduction and valence bands.

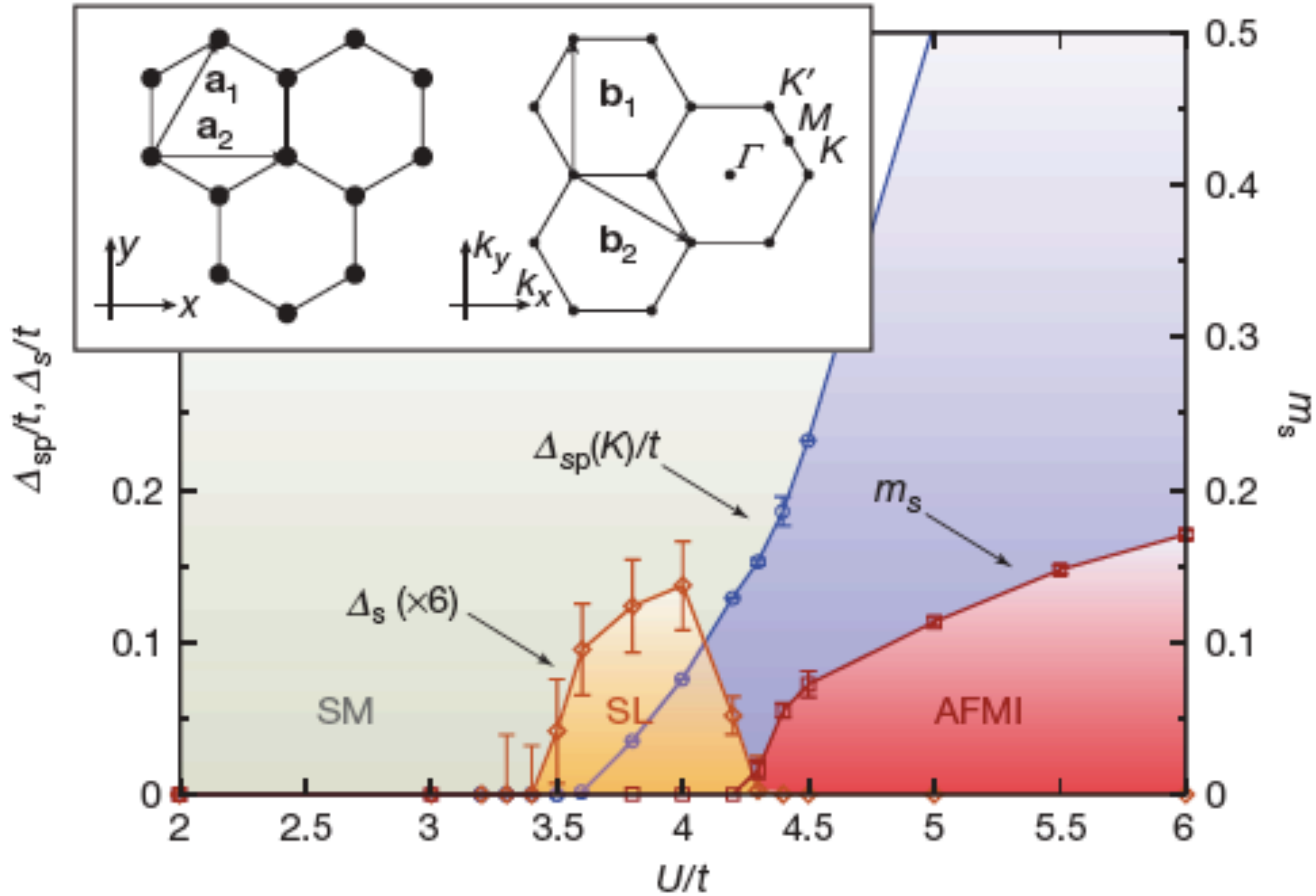


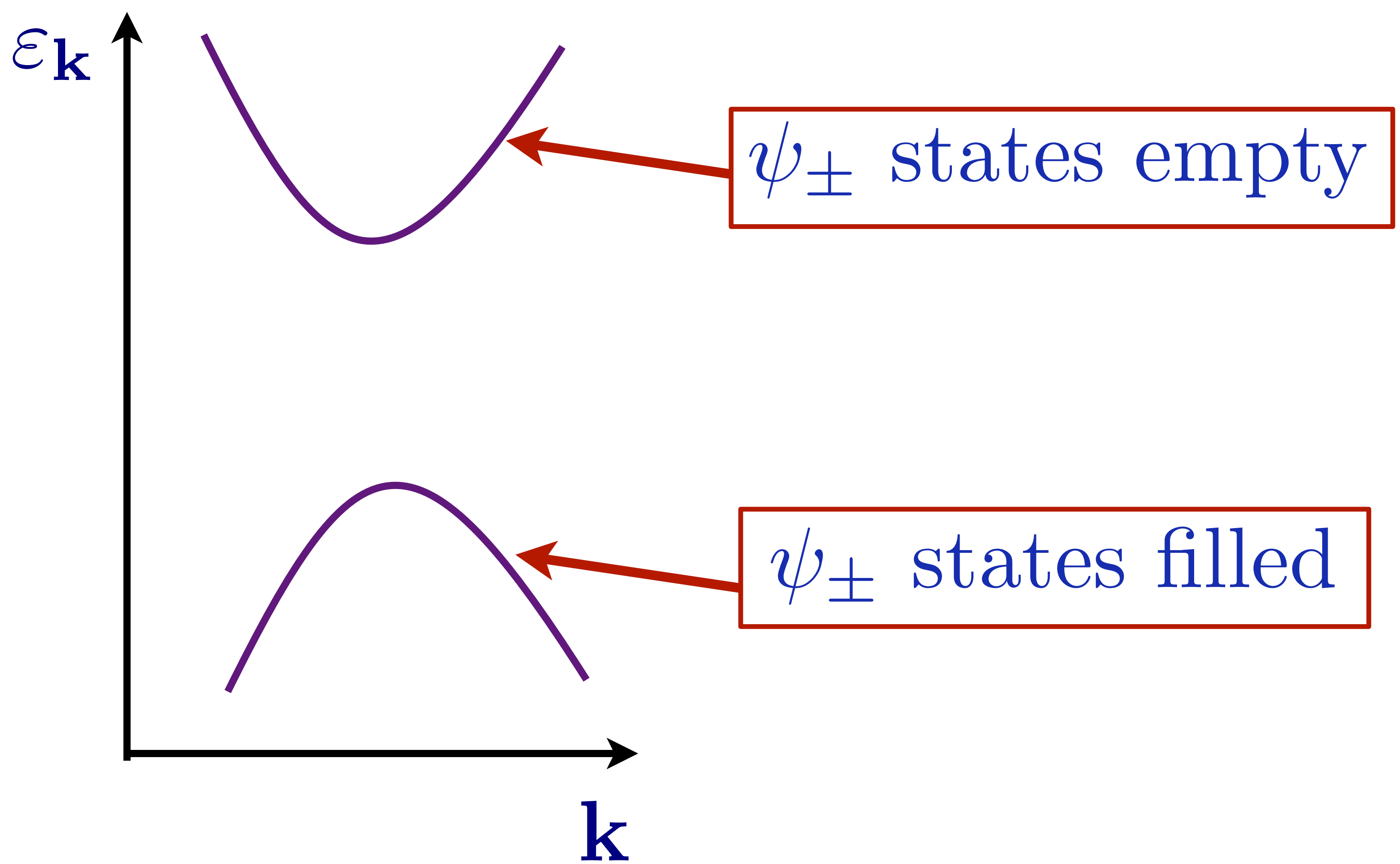
Figure 1 | Phase diagram for the Hubbard model on the honeycomb lattice at half-filling. The semimetal (SM) and the antiferromagnetic Mott insulator (AFMI) are separated by a gapped spin-liquid (SL) phase in an intermediate-coupling regime. $\Delta_{sp}(K)$ denotes the single-particle gap and Δ_s denotes the spin gap; m_s denotes the staggered magnetization, whose saturation value is 1/2. Error bars, s.e.m. Inset, the honeycomb lattice with primitive vectors $\mathbf{a}_1$ and $\mathbf{a}_2$, and the reciprocal lattice with primitive vectors $\mathbf{b}_1$ and $\mathbf{b}_2$. Open and filled sites respectively indicate two different sublattices. The Dirac points K and K' and the M and Γ points are marked.

Z. Y. Meng *et al.*, Nature **464**, 847 (2010).

$$\mathcal{R}_z(x,\tau)\,|\text{Néel}\rangle$$

Perform SU(2) rotation $\mathcal{R}_z$ on filled band of electrons:

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$



Quantum "disordering" magnetic order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

Quantum "disordering" magnetic order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \bullet \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \qquad \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$\mathrm{SU}(2)_{\mathrm{spin}}$

Quantum "disordering" magnetic order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \bullet \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$\mathrm{U}(1)_{\mathrm{charge}}$

Quantum "disordering" magnetic order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \bullet \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$U \times U^{-1}$

$\mathrm{SU(2)}_{\mathrm{s;gauge}}$

S. Sachdev, M. A. Metlitski, Y. Qi, and S. Sachdev *Phys. Rev. B* **80**, 155129 (2009)

Quantum "disordering" magnetic order

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

The Hubbard model can be written *exactly* as a lattice gauge theory with a

$$\mathrm{SU}(2)_{s;g} \times \mathrm{SU}(2)_{\mathrm{spin}} \times \mathrm{U}(1)_{\mathrm{charge}}$$

invariance.

The $\mathrm{SU}(2)_{s;g}$ is a gauge invariance, while $\mathrm{SU}(2)_{\mathrm{spin}} \times \mathrm{U}(1)_{\mathrm{charge}}$ is a global symmetry

Matter context of $SU(2)_{s;g} \times SU(2)_{\text{spin}} \times U(1)_{\text{charge}}$ theory

- Fundamental fermions ψ transforming as $(\mathbf{2}, \mathbf{1}, 1)$,

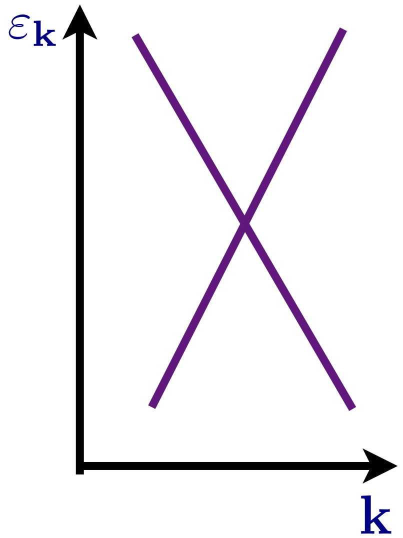
- Fundamental scalar z transforming as $(\bar{\mathbf{2}}, \mathbf{2}, 0)$, connecting local to global Néel order,

- Adjoint scalar $\vec{N}(\mathbf{r}_i) = \psi_i^\dagger \vec{\sigma} \psi_i$ transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring local Néel order.

Phase diagram

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle = 0$	$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle \neq 0$
$\langle z \rangle = 0 \quad , \quad \langle N \rangle = 0$	$\langle z \rangle = 0 \quad , \quad \langle N \rangle \neq 0$

Phase diagram



Semi-metal

$$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle = 0$$

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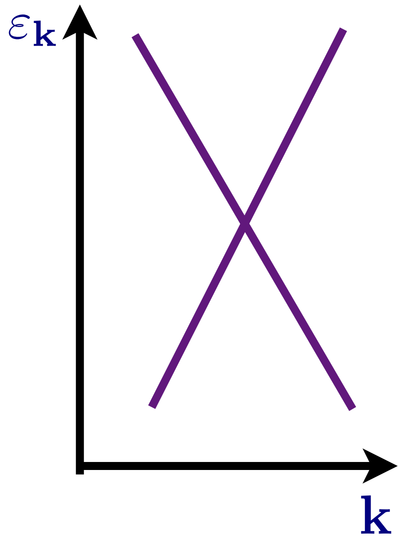
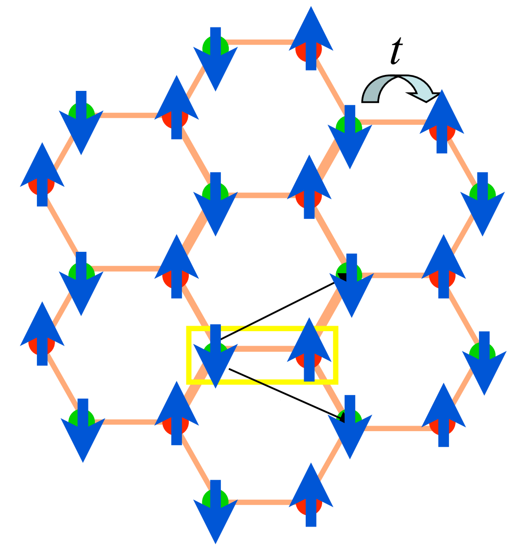
Phase diagram



Semi-metal



Insulator with Neel order

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle = 0$

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle \neq 0$

$\langle z \rangle = 0 \quad , \quad \langle N \rangle = 0$

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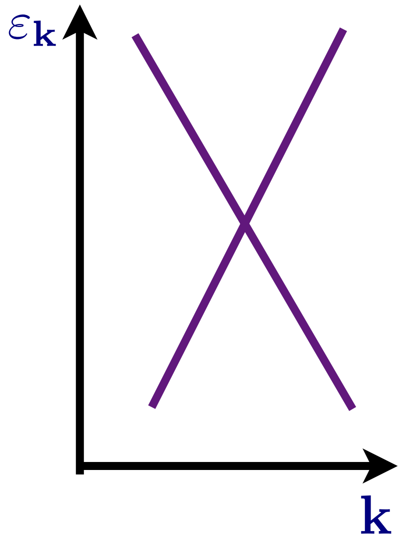
Phase diagram



Semi-metal

or

eel

SU(2) QCD with $N_f = 4$ massless Dirac quarks

$$\mathcal{L} = \frac{1}{g^2} F_{\mu\nu}^2 + \overline{\psi} \gamma_\mu (\partial_\mu - i A_\mu) \psi$$

Could describe a CFT3 like SYM, but could also be unstable to confinement.

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle = 0$

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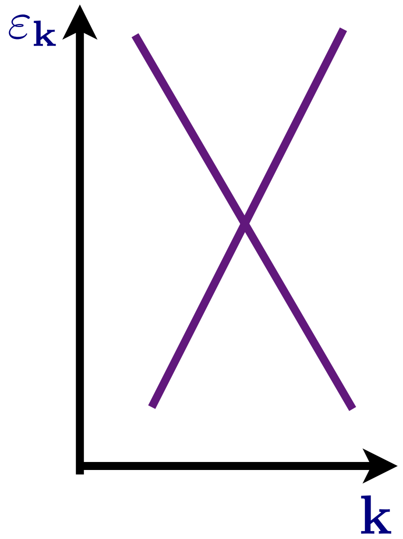
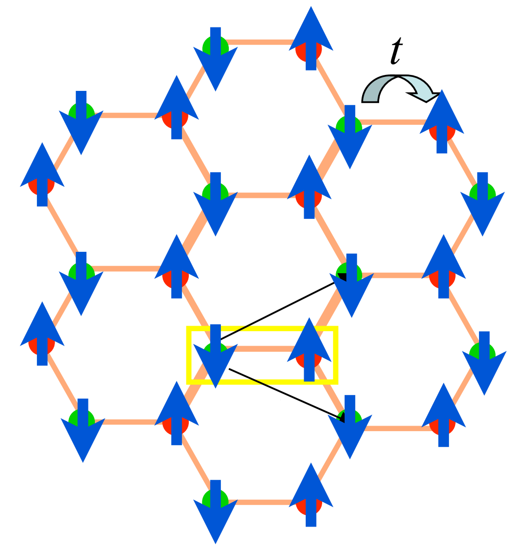
Phase diagram



Semi-metal



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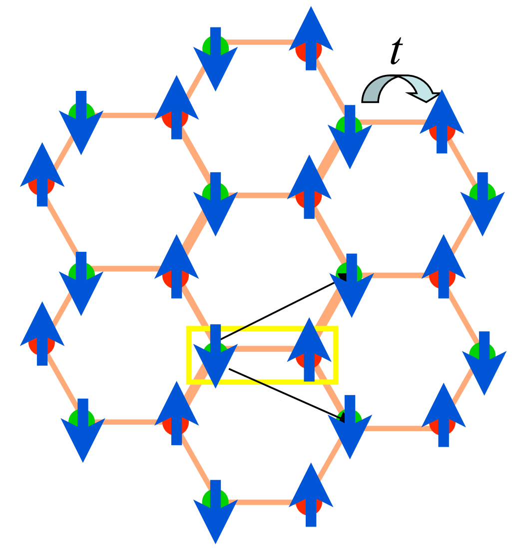
Spin liquid:
CFT of SU(2) QCD
with N_f=4 massless
Dirac quarks

Phase diagram

- SU(2) is Higgsed down to U(1).
- All matter fields are gapped.
- U(1) monopoles drive Polyakov confinement
- Spectral flow in filled fermion bands leads to Berry phases of monopoles, endowing them with crystal momentum.
- Confining state has valence bond solid (VBS) order



Insulator with Neel order

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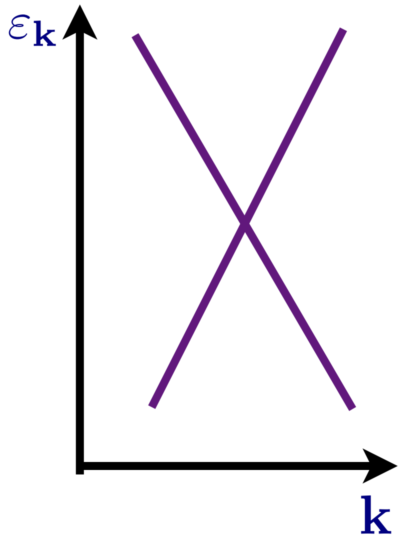
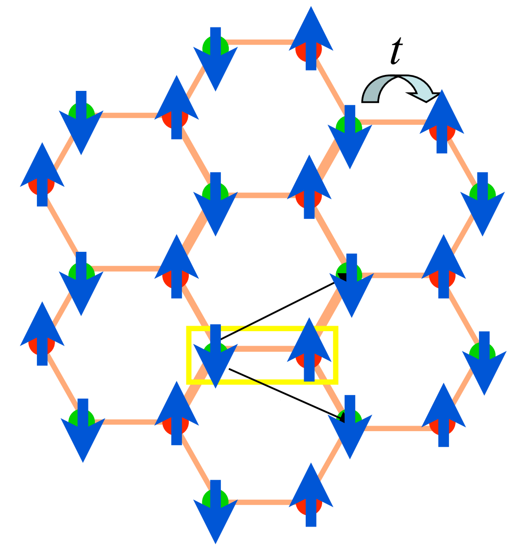
$$\langle z \rangle = 0 \quad , \quad \langle N \rangle \neq 0$$

Phase diagram



Semi-metal



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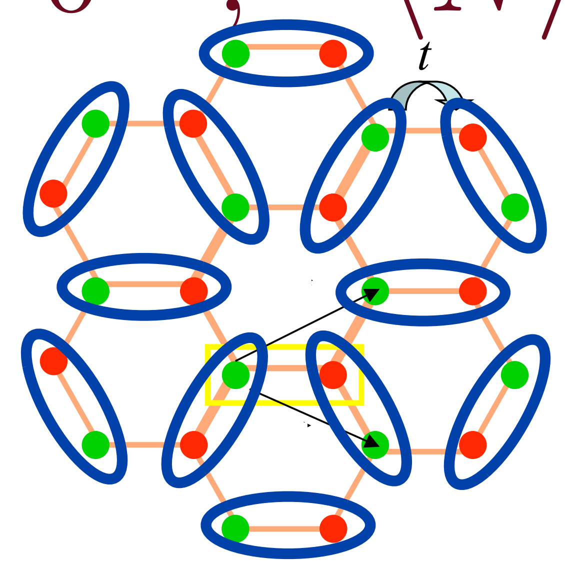
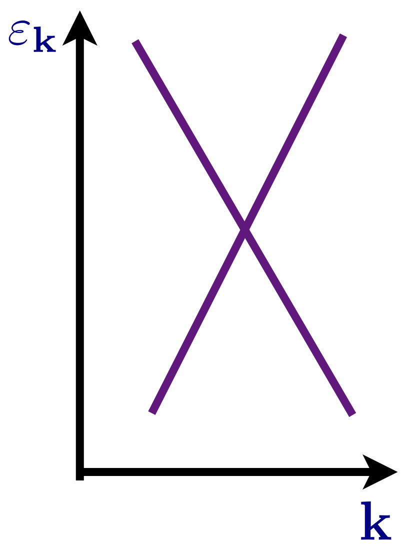
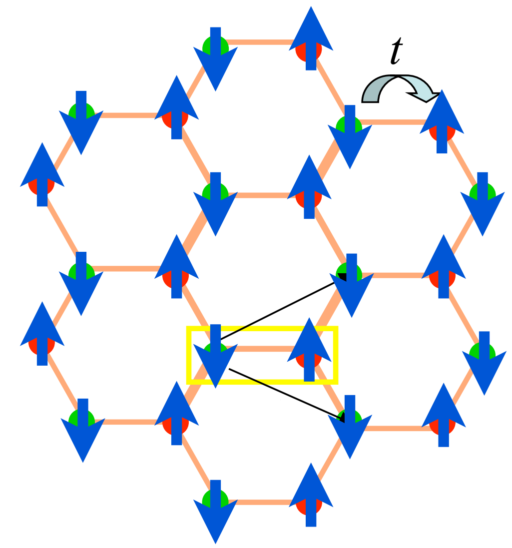


Confining insulator with VBS (kekule) order

Phase diagram



Semi-metal



Insulator with Neel order

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle = 0$

M

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle \neq 0$

$\langle z \rangle = 0 \quad , \quad \langle N \rangle = 0$

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CFT of SU(2) QCD
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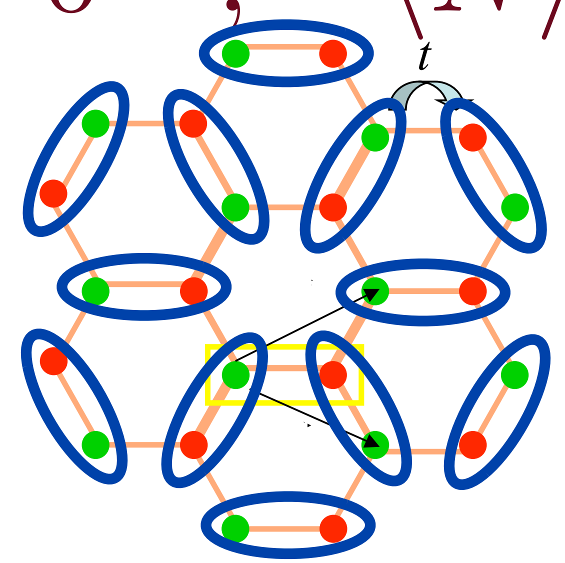
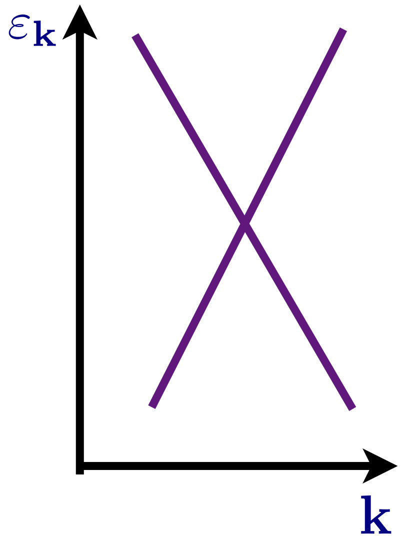


Confining insulator with VBS (kekule) order

Phase diagram



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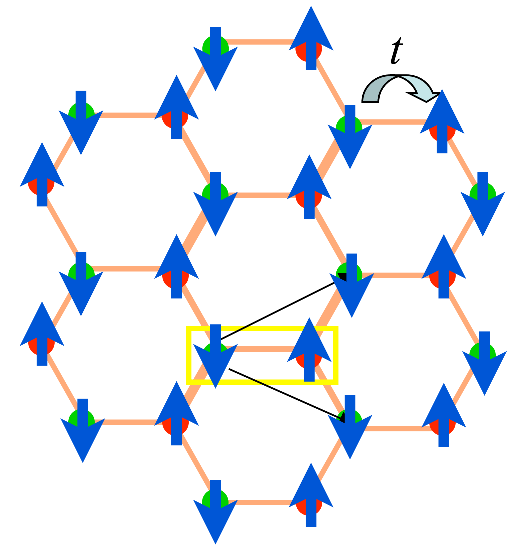

Insulator with Neel order

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M

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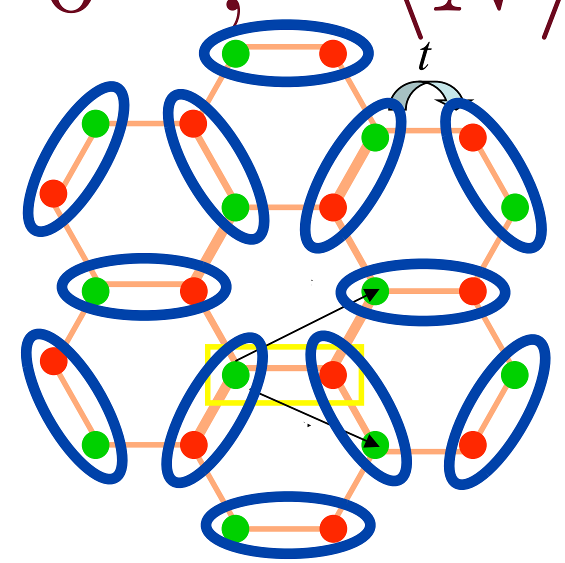
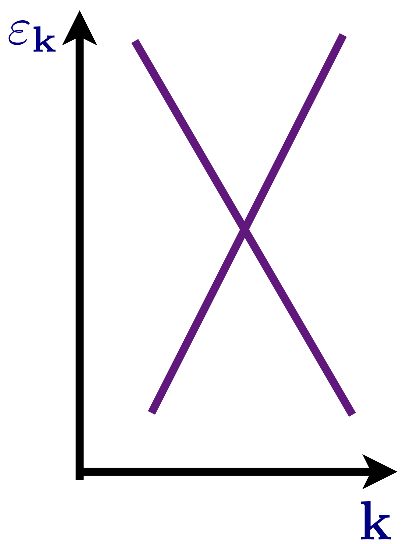
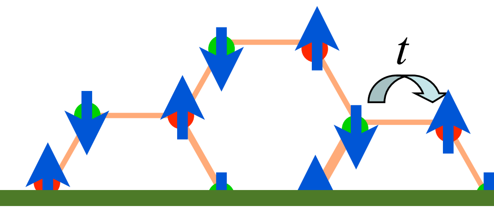
Spin liquid:
CFT of SU(2) QCD with N_f=4 massless Dirac quarks



Confining insulator with VBS (kekule) order

Phase diagram





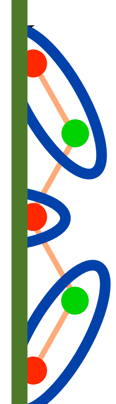
Insulator with Neel order

$\langle z \rangle \neq 0$

V$\rangle \neq 0$

$\langle z \rangle = 0$

V$\rangle \neq 0$



Spi
CFT of
with N_f
Dira

lator with
VBS (Kekule) order

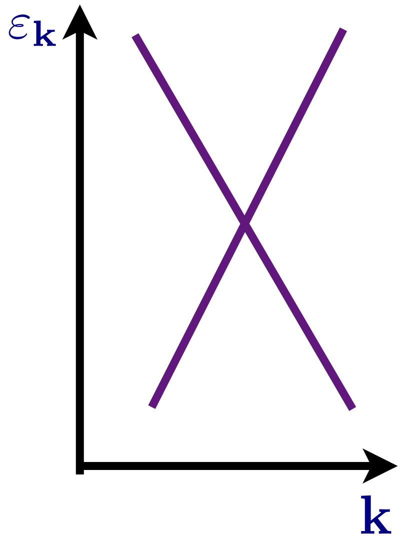
<u>Multicritical point M:</u>
SU(2) QCD with $N_f = 4$ massless Dirac quarks, 4 real fundamental scalars, and 3 real adjoint scalars.

$$\mathcal{L} = \frac{1}{g^2} F_{\mu\nu}^2 + \overline{\psi} \gamma_\mu (\partial_\mu - i A_\mu) \psi$$

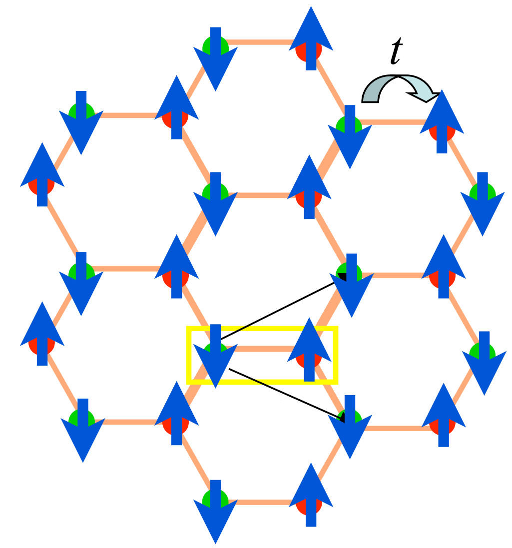
$$+ |(\partial_\mu - i A_\mu) z|^2 + ((\partial_\mu - i A_\mu) \phi)^2$$

Could describe a CFT3 like SYM, but could also be unstable to confinement.

Phase diagram



Semi-metal



Insulator with Neel order

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle = 0$

M

$\langle z \rangle \neq 0 \quad , \quad \langle N \rangle \neq 0$

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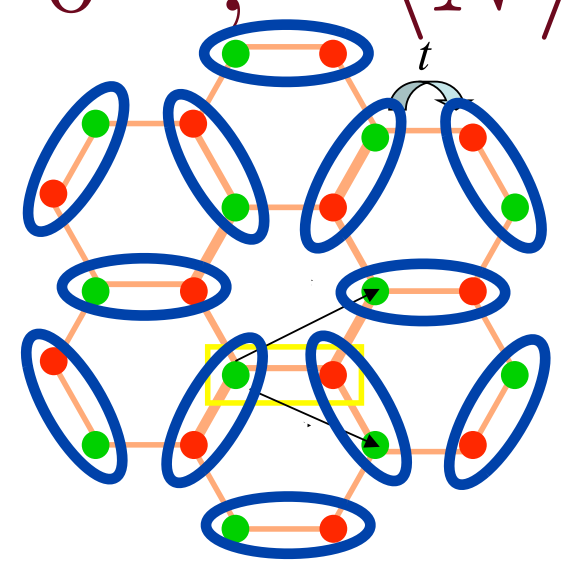

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