Strange metals and black holes

Distinguished Lecture
Texas A&M University,
College Station, November 9, 2017

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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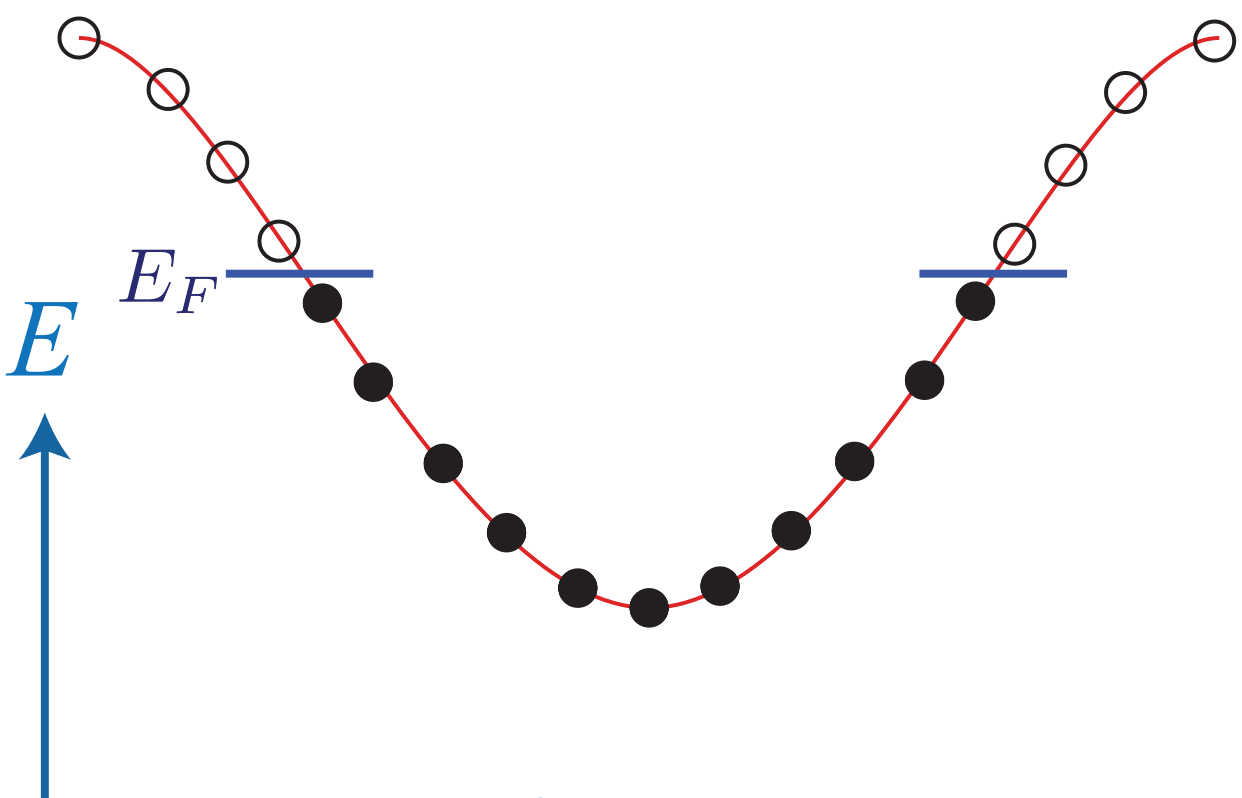
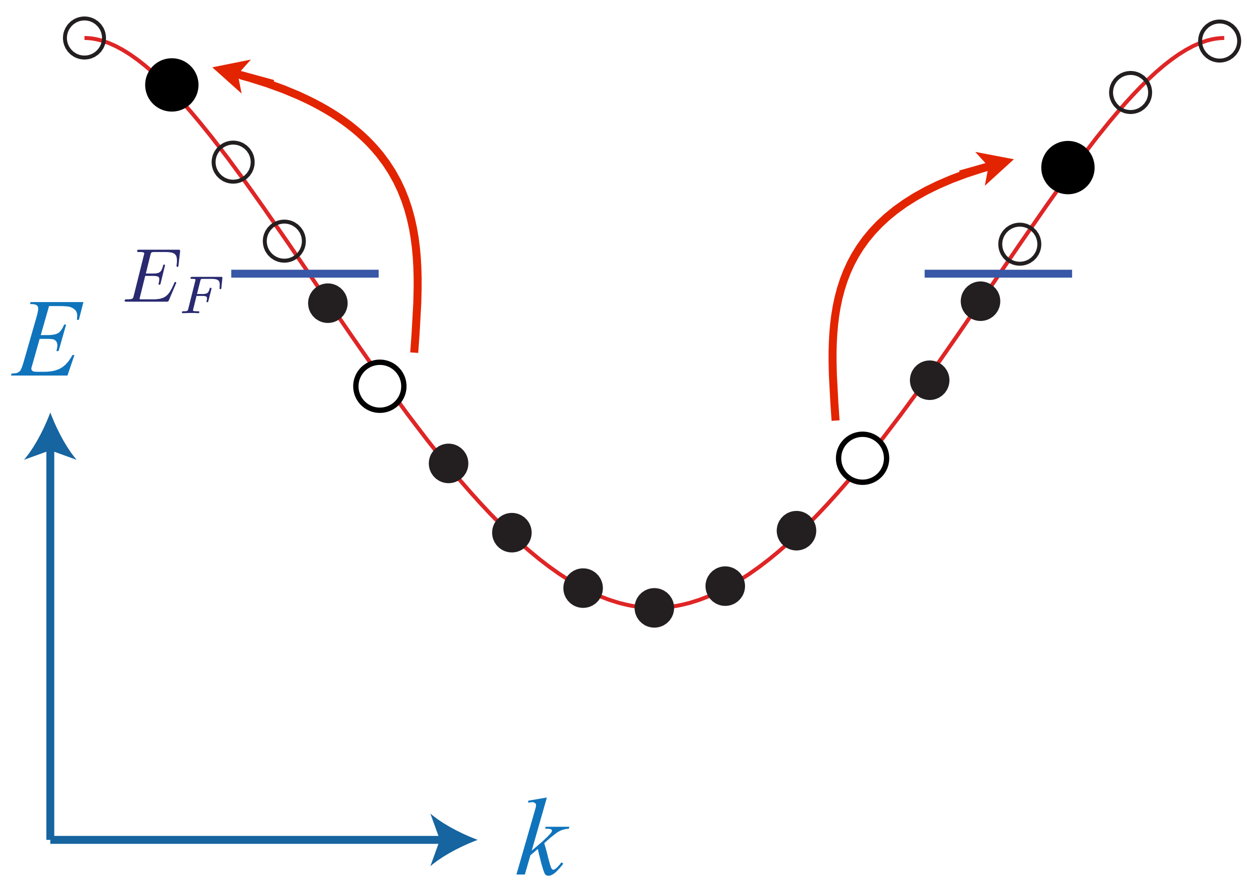
Theory of (ordinary) metals

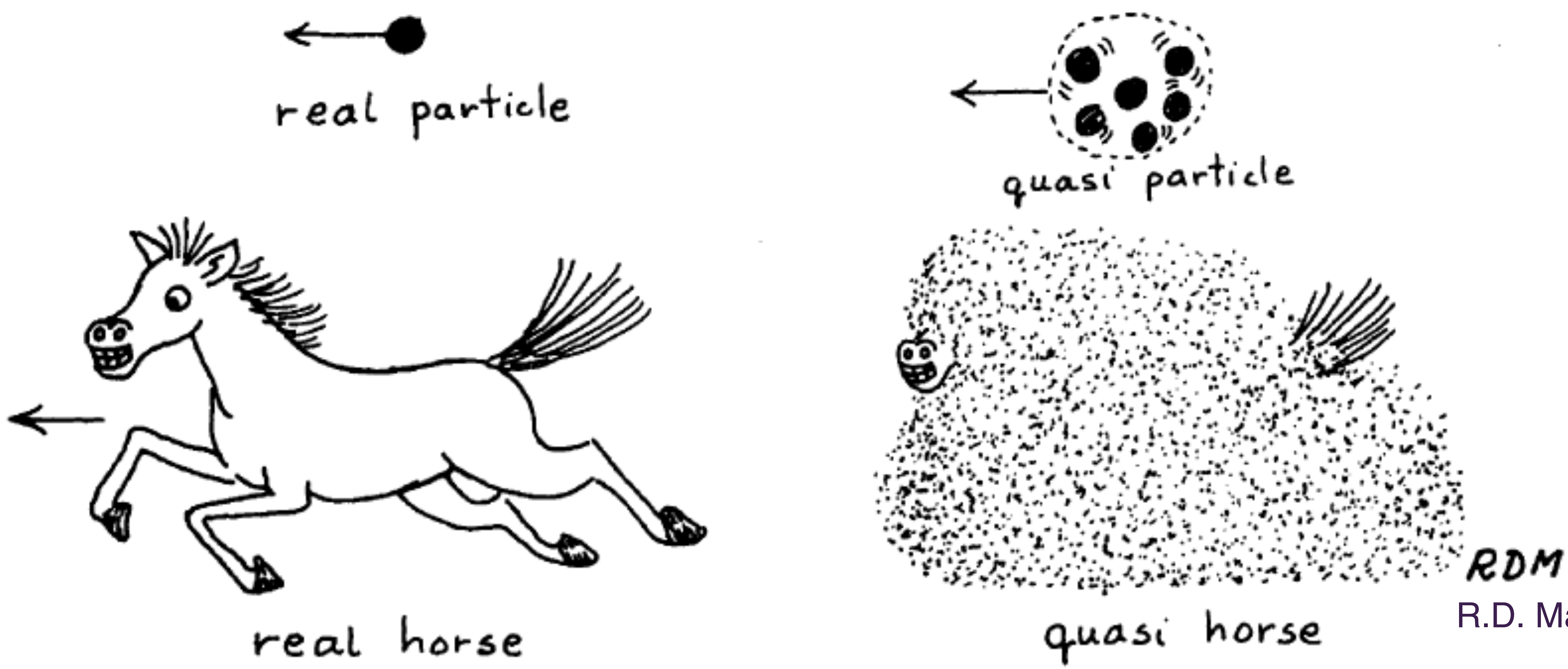


Theory of (ordinary) metals



Quasiparticles:

A quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

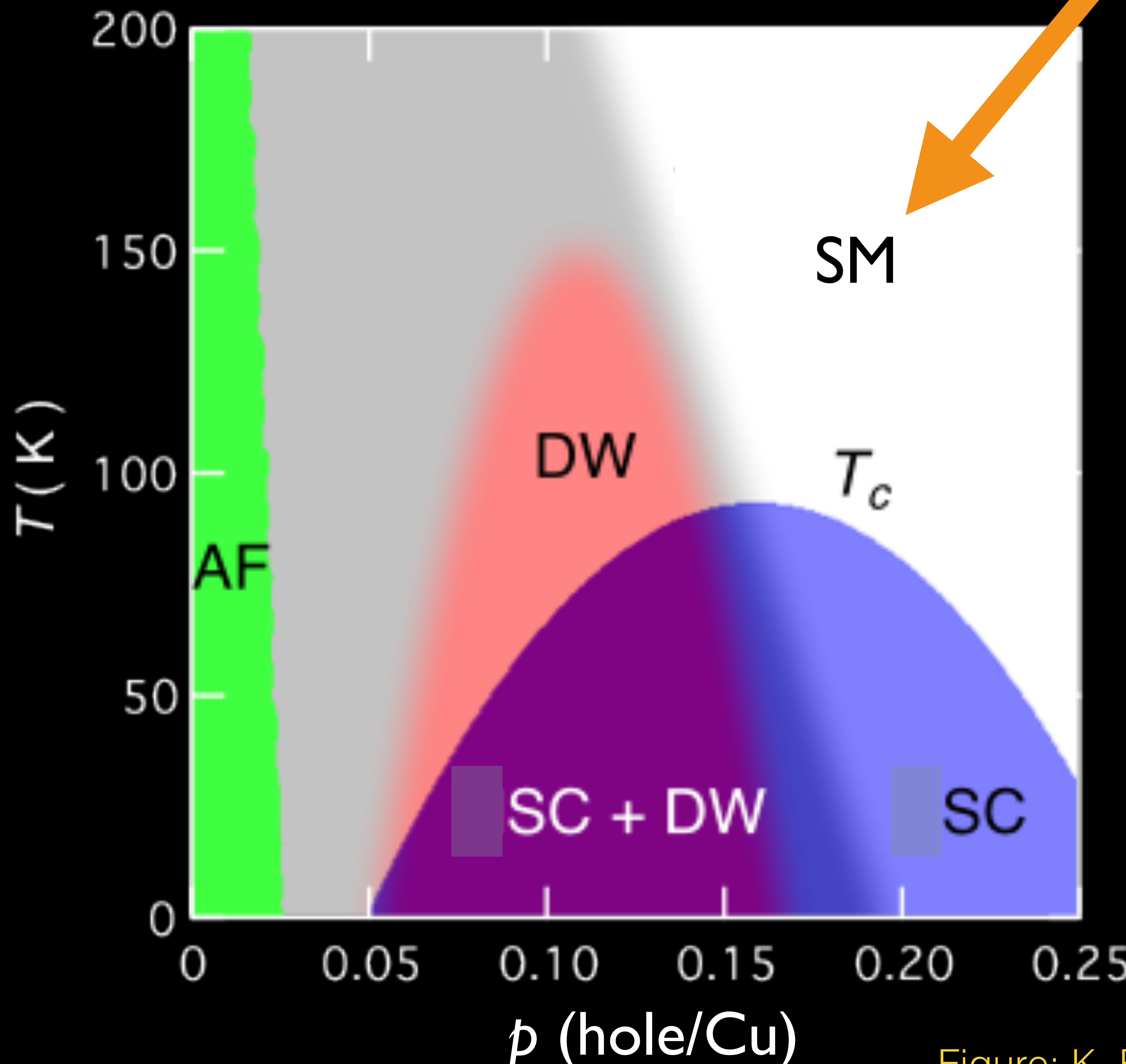
Quantum matter <u>with</u> quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Quantum matter without quasiparticles

Strange metal



Entangled electrons lead to "strange" temperature dependence of resistivity and other properties

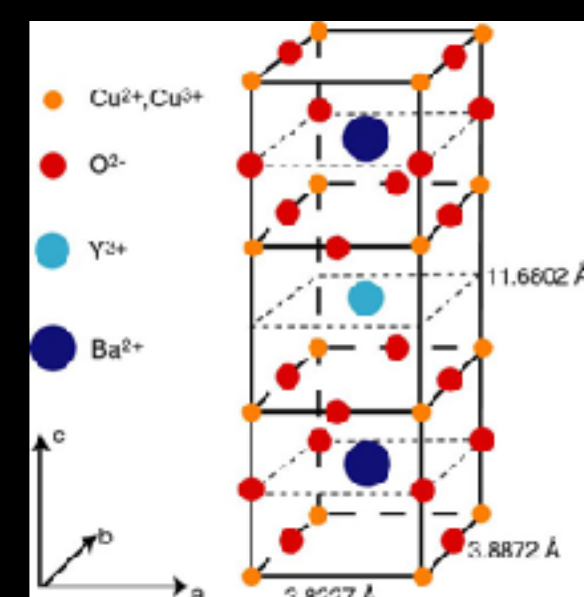


Figure: K. Fujita and J. C. Seamus Davis

"Strange",

"Bad",

or "Incoherent",

metal has a resistivity, ρ, which obeys

$$\rho \sim T,$$

and

$$\rho \gg h/e^2$$

(in two dimensions),
where h/e^2 is the quantum unit of resistance.

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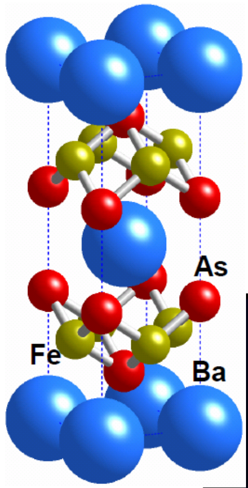
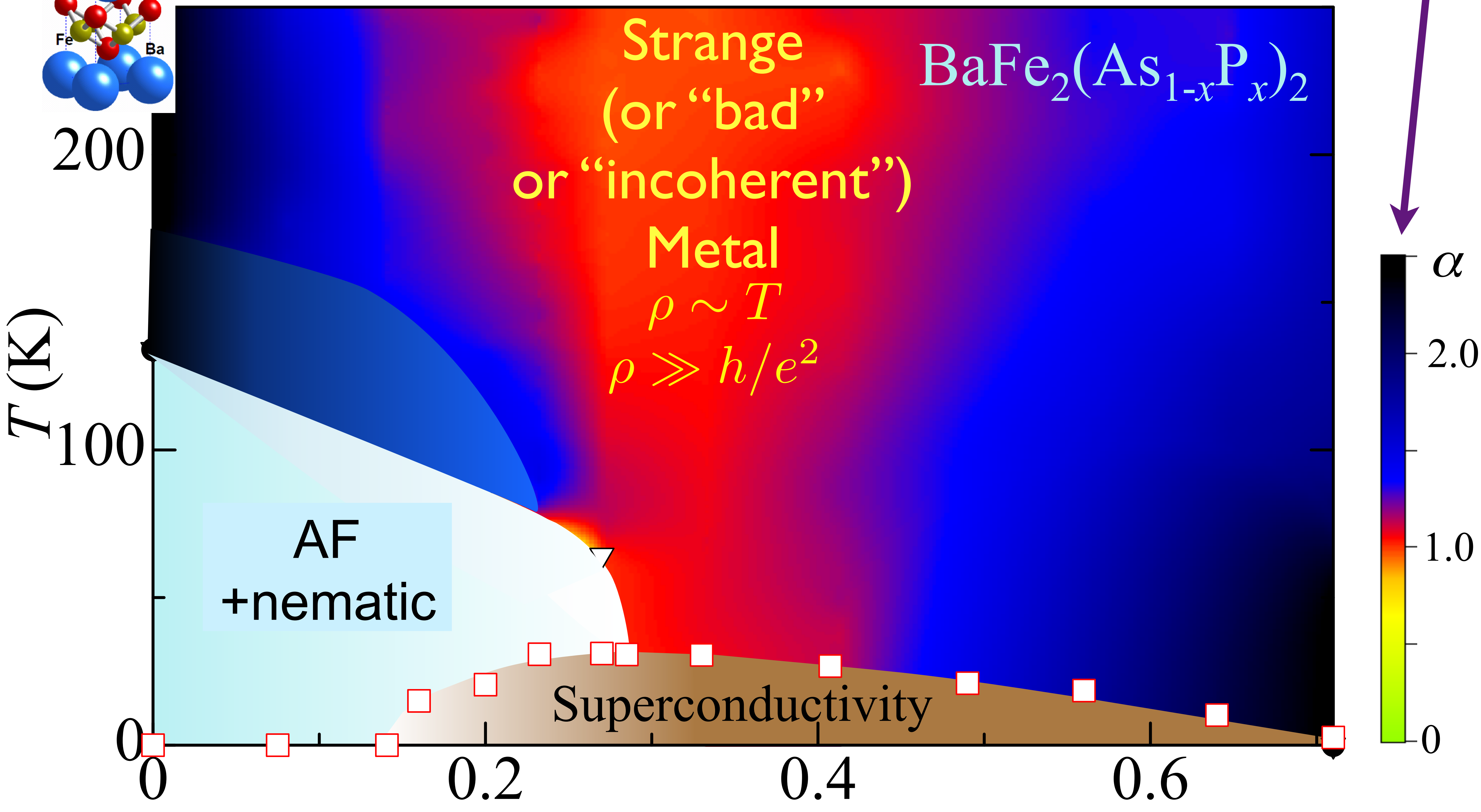


Quantum matter without quasiparticles

Resistivity
$\sim \rho_0 + AT^{\alpha}$



Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* **81**, 184519 (2010)

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

Quantum matter with quasiparticles:

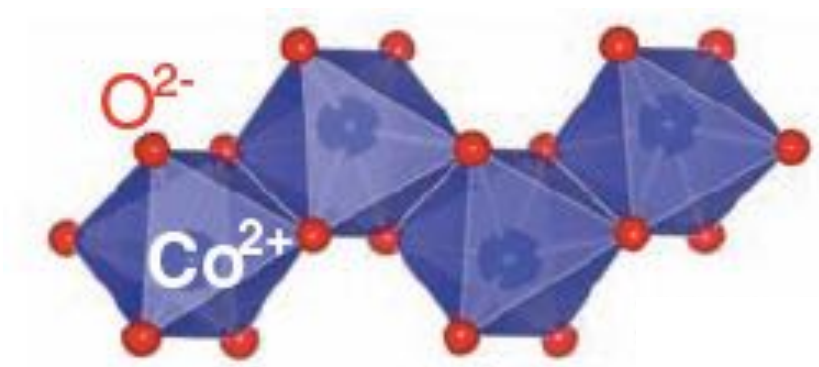
- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

Quantum Ising models

Qubits with states $|\uparrow\rangle_i$, $|\downarrow\rangle_i$, on the sites, i, of a regular lattice.



$CoNb_2O_6$

$$\sigma^z |\uparrow\rangle = |\uparrow\rangle \quad , \quad \sigma^z |\downarrow\rangle = -|\downarrow\rangle$$
$$\sigma^x |\uparrow\rangle = |\downarrow\rangle \quad , \quad \sigma^x |\downarrow\rangle = |\uparrow\rangle$$

$$H = -J \left(\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + g \sum_i \sigma_i^x \right)$$

For $g = 0$, ground state is a ferromagnet:

$$|G\rangle = |\cdots \uparrow\uparrow\uparrow\uparrow\uparrow \cdots\rangle \quad \text{or} \quad |\cdots \downarrow\downarrow\downarrow\downarrow\downarrow \cdots\rangle$$

For $g \gg 1$, unique 'paramagnetic' ground state:

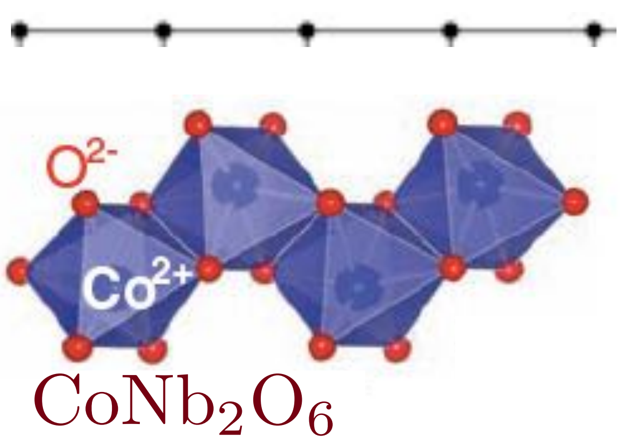
$$|G\rangle = |\cdots \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\rangle$$

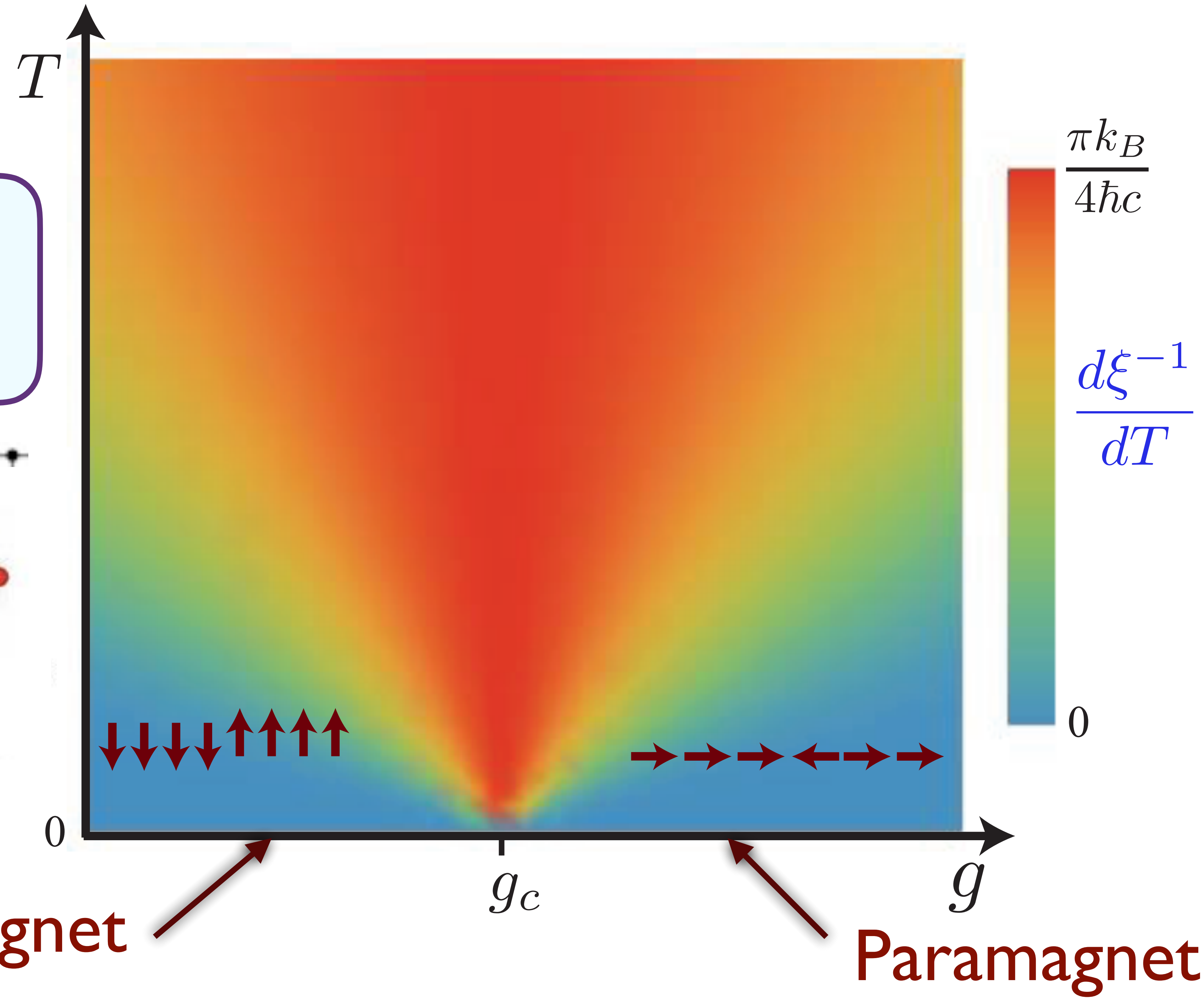
where

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle + |\downarrow\rangle \right) \quad , \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle - |\downarrow\rangle \right)$$

Quantum Ising models

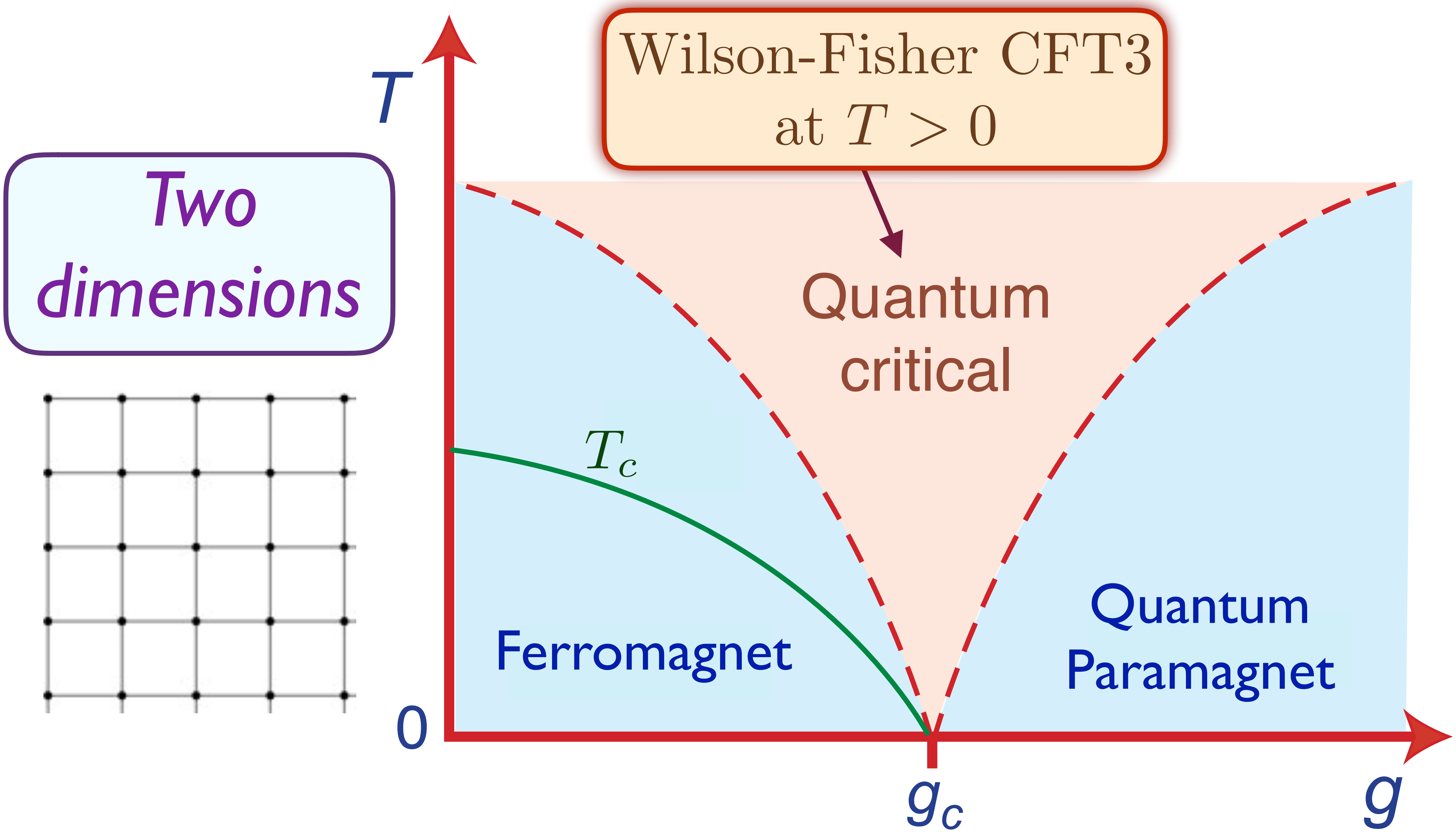
One dimension





- In one dimension, quasiparticles exist even at the quantum critical point: there is a non-local transformations from the qubits to a system of free fermions.

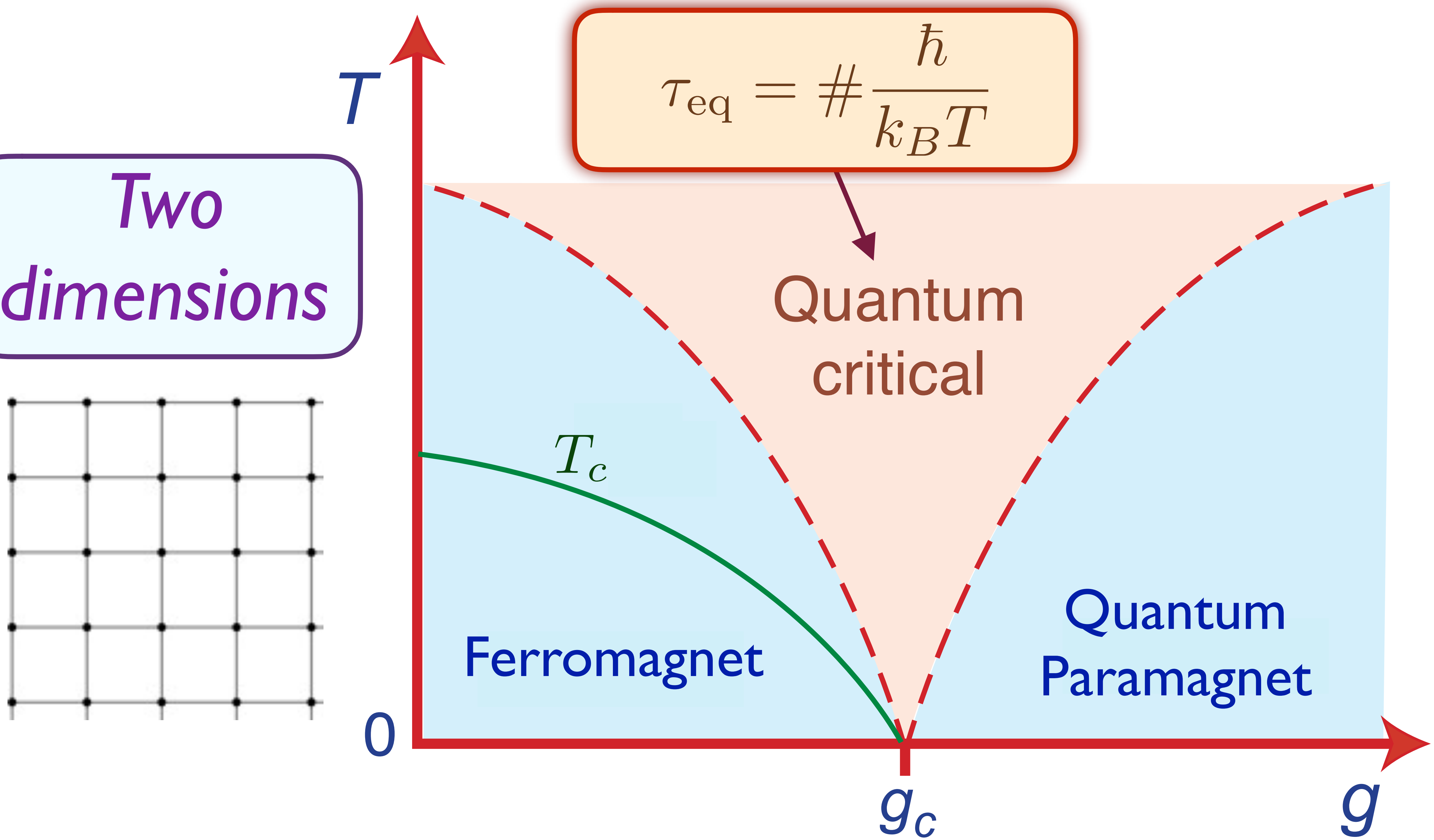
Quantum Ising models



- In two dimensions, the "quantum critical" region provides us the first example of a system without a quasiparticle description. This is described by a strongly-coupled conformal field theory (CFT) in 2+1 dimensions, and dynamic properties cannot be computed accurately.

Quantum Ising models

Two dimensions



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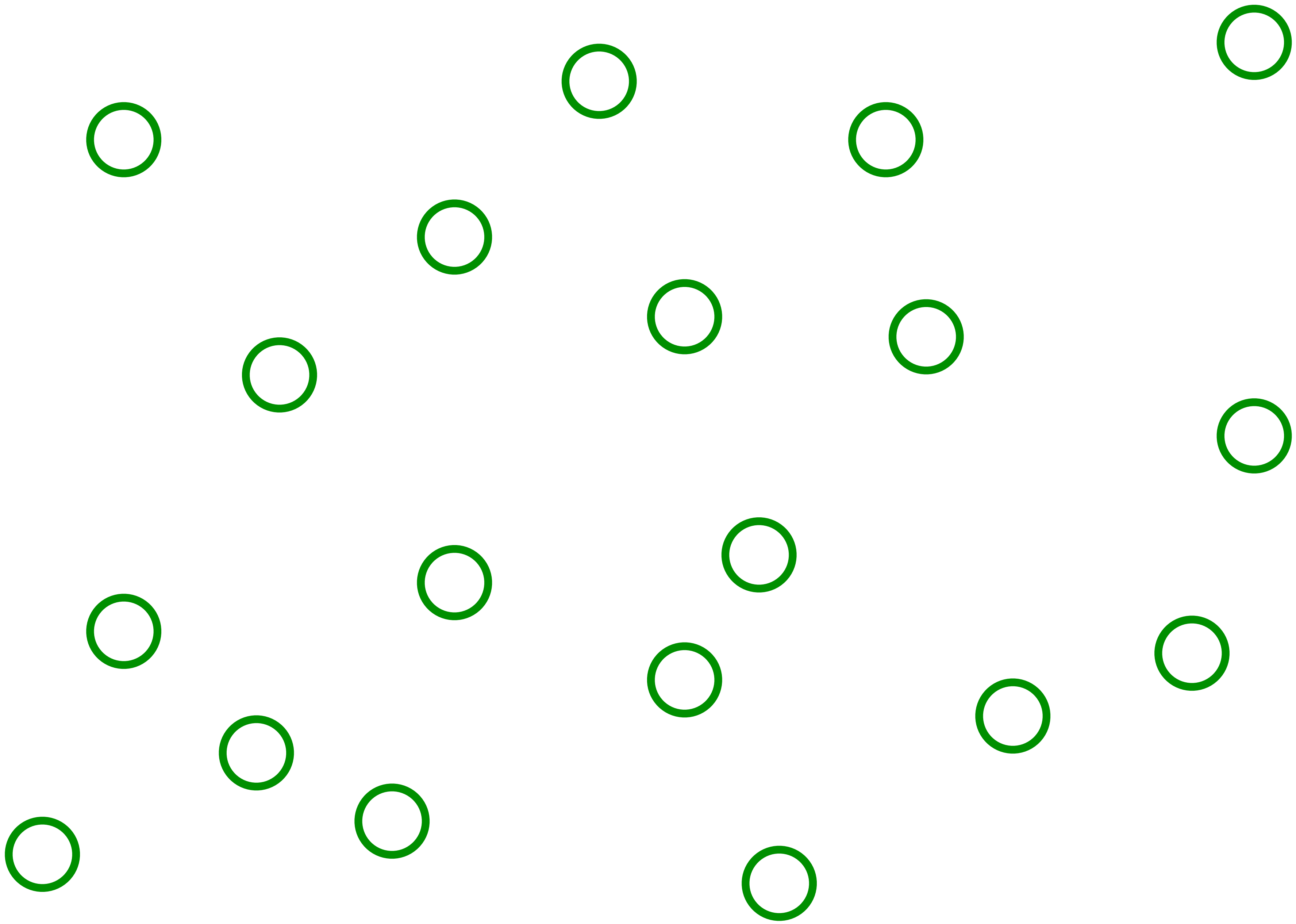
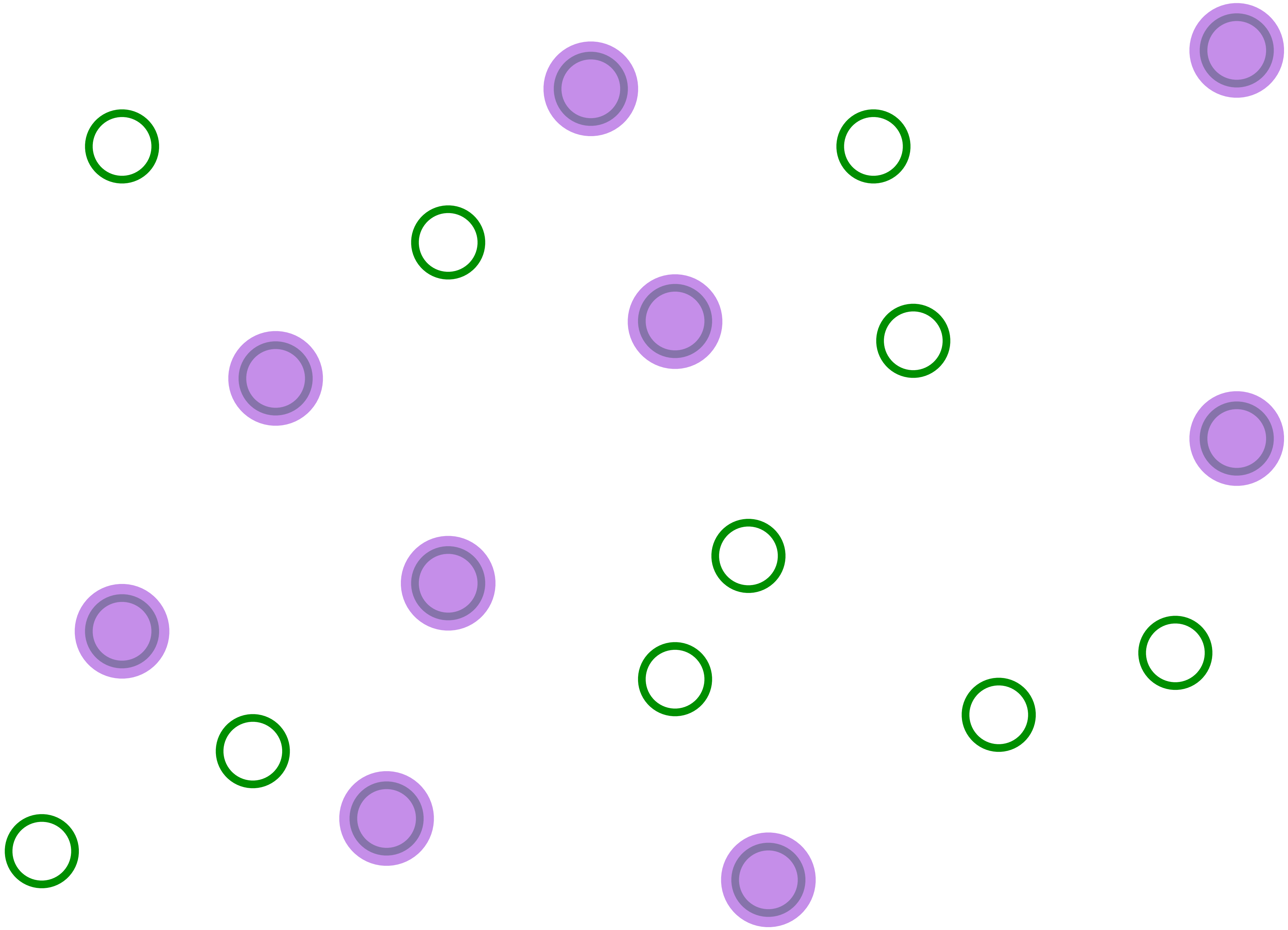
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A simple model of a metal with quasiparticles



Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

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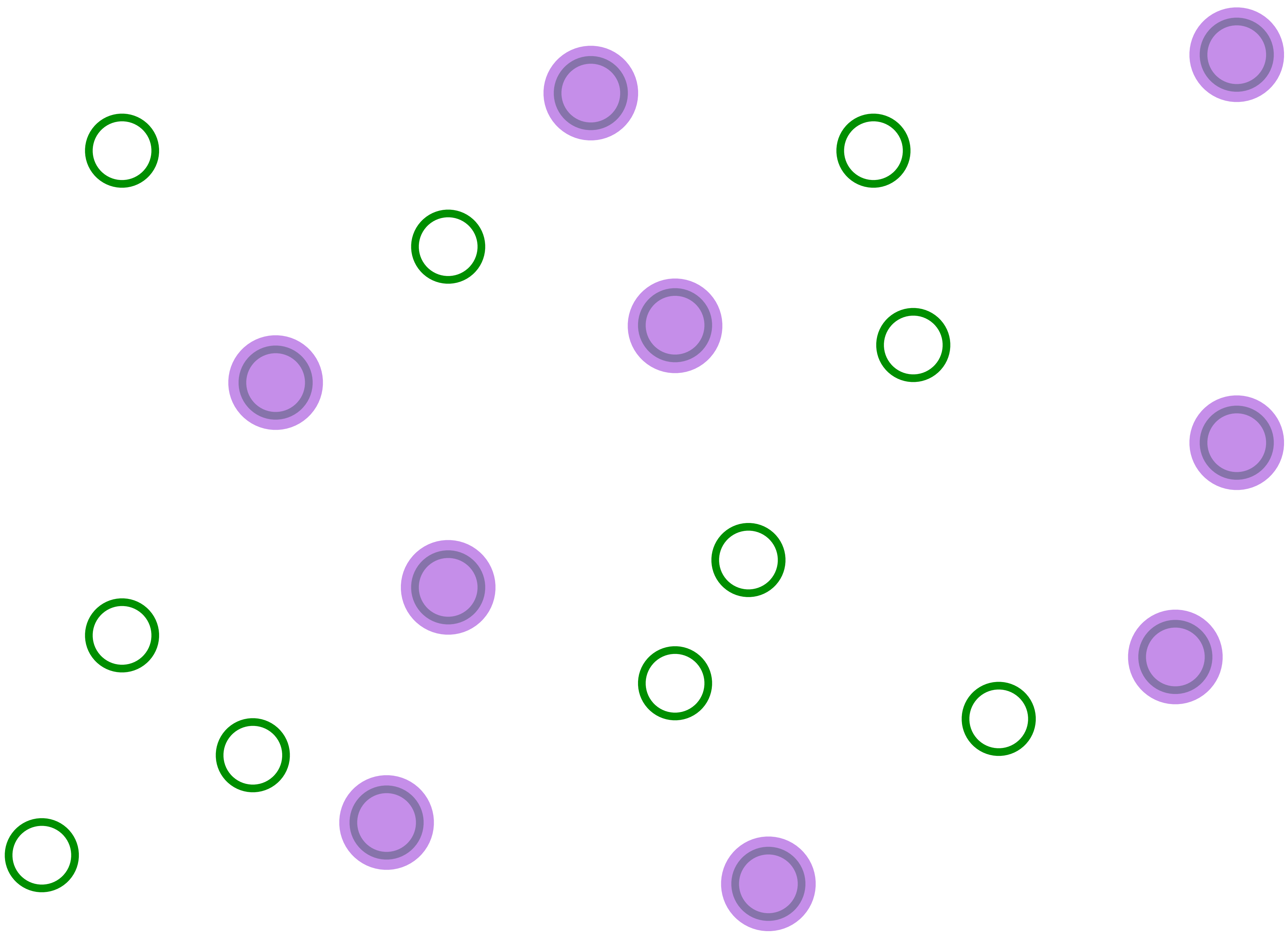

Electrons move one-by-one randomly

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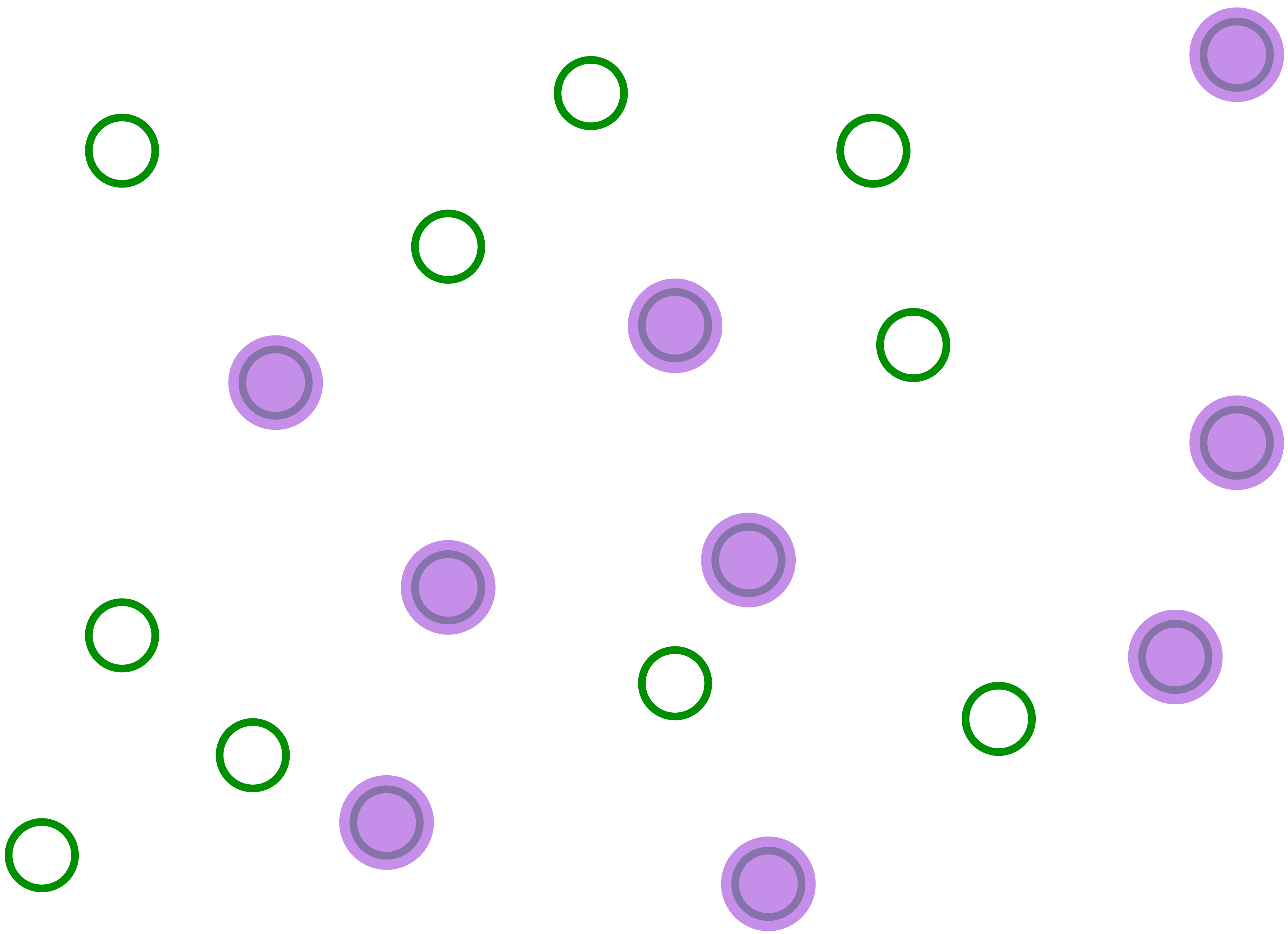

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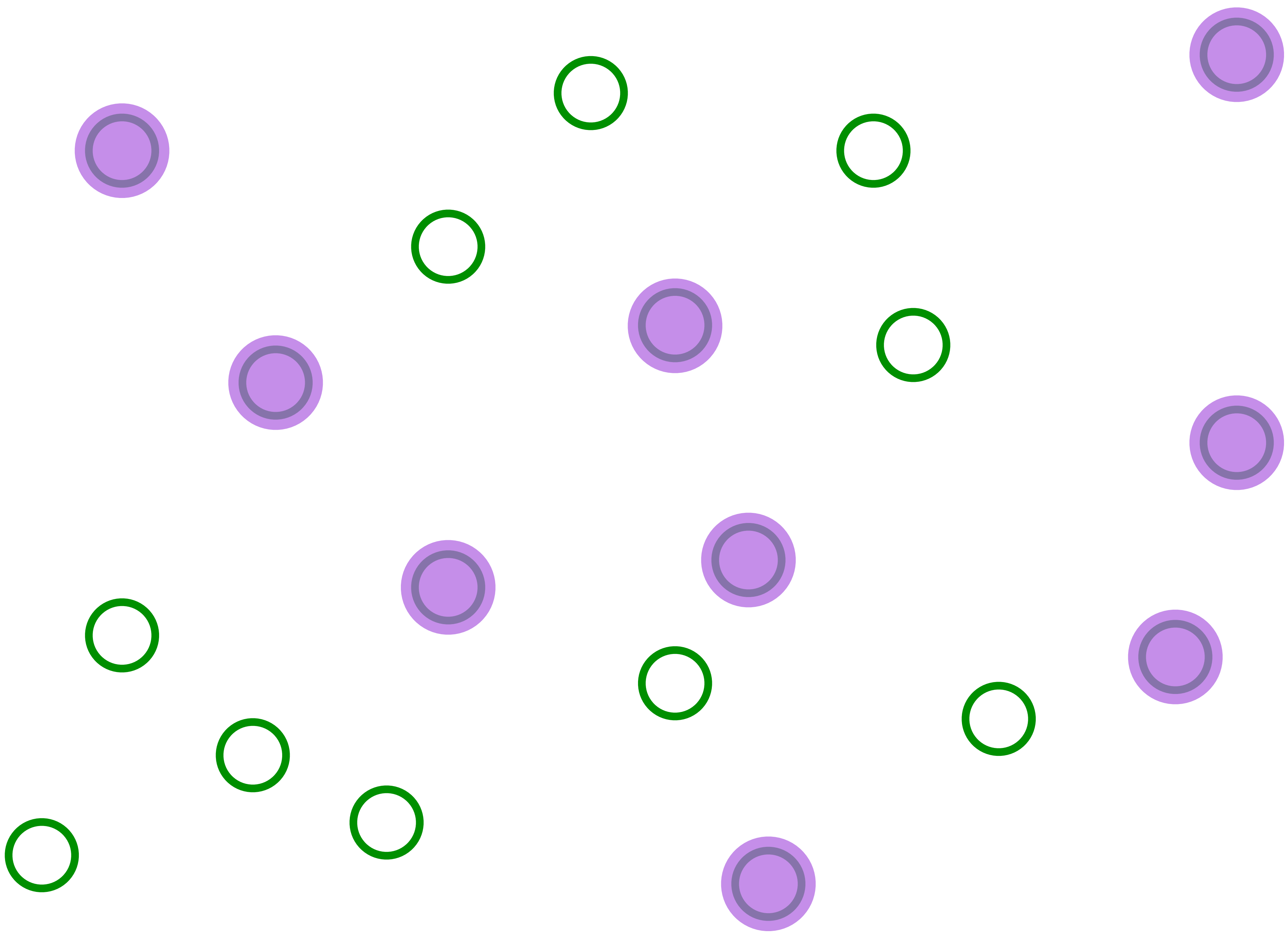

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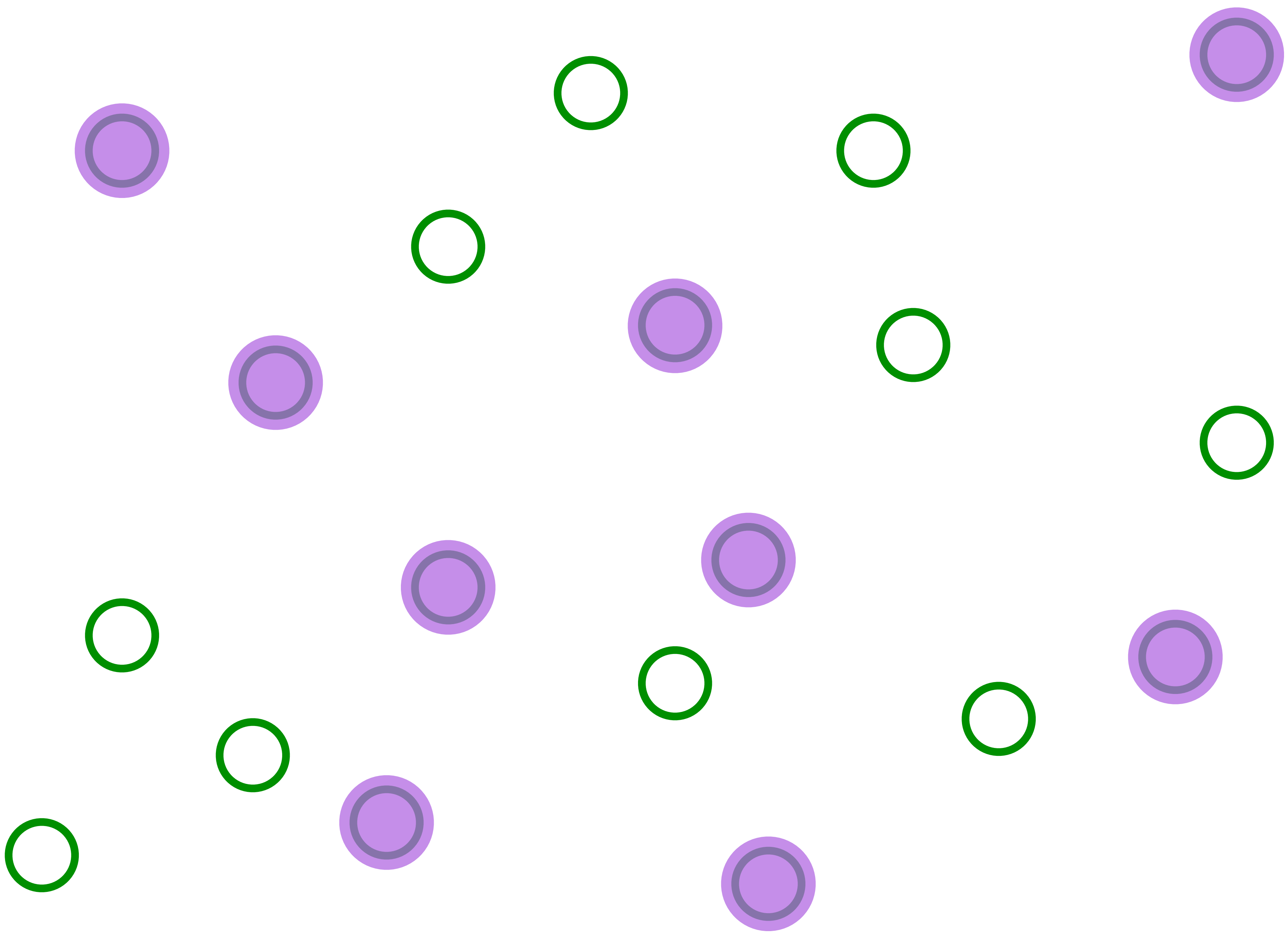


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

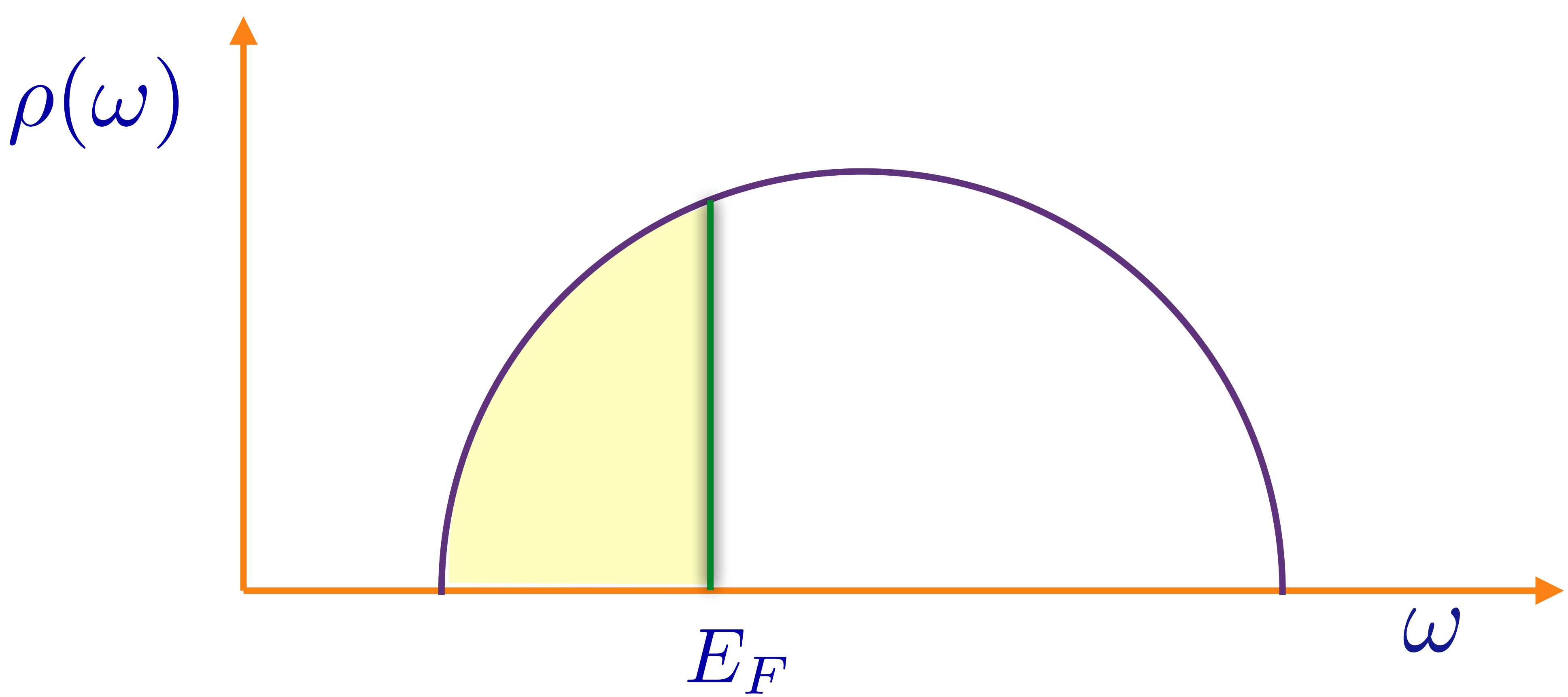
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



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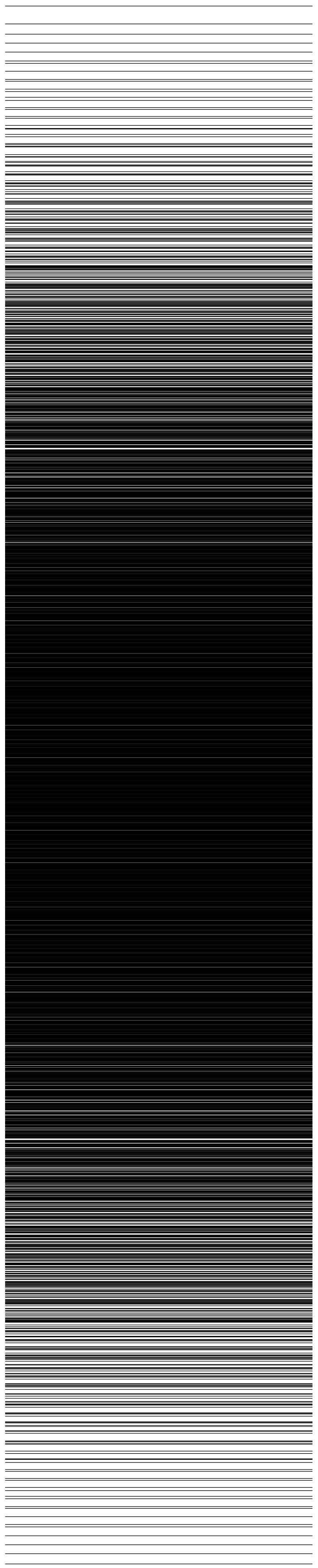

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha ,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

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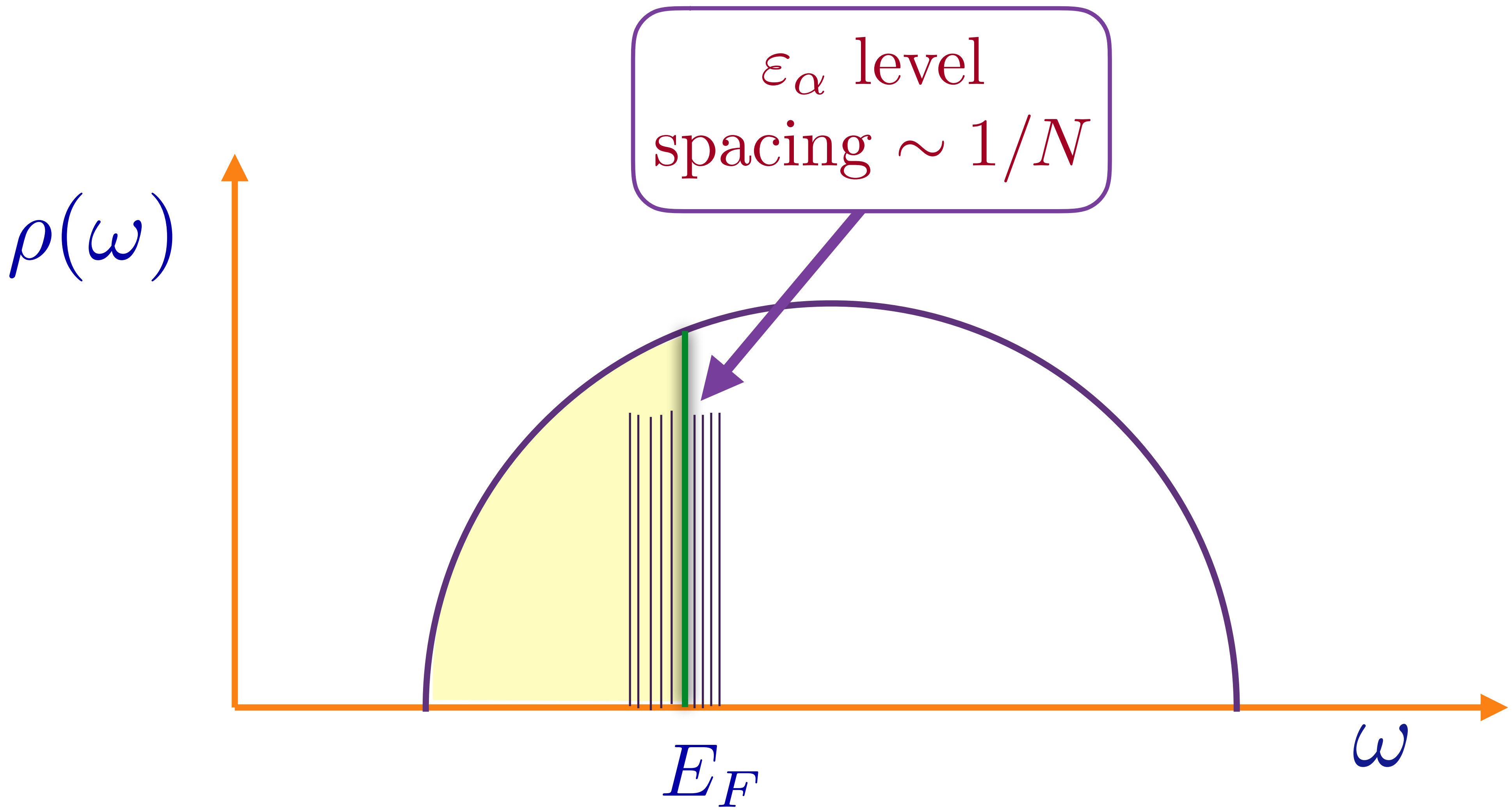

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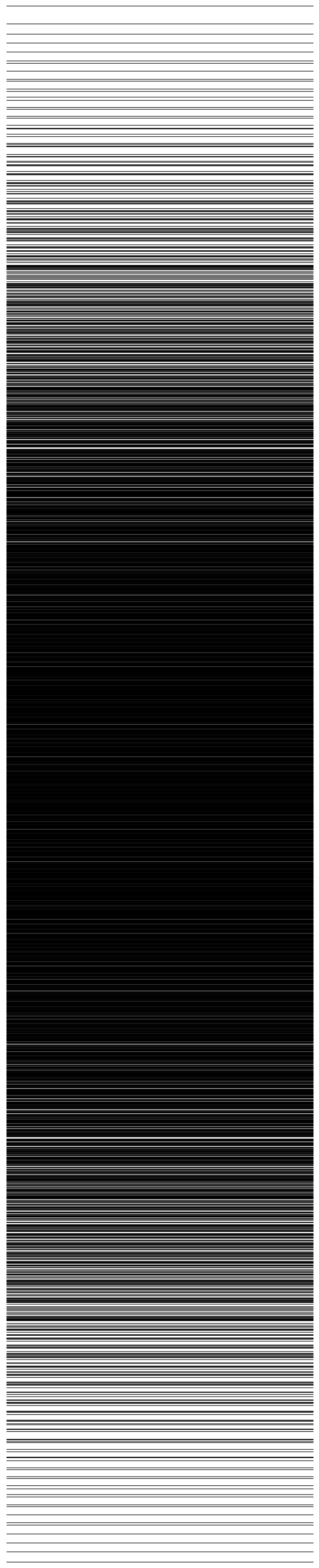

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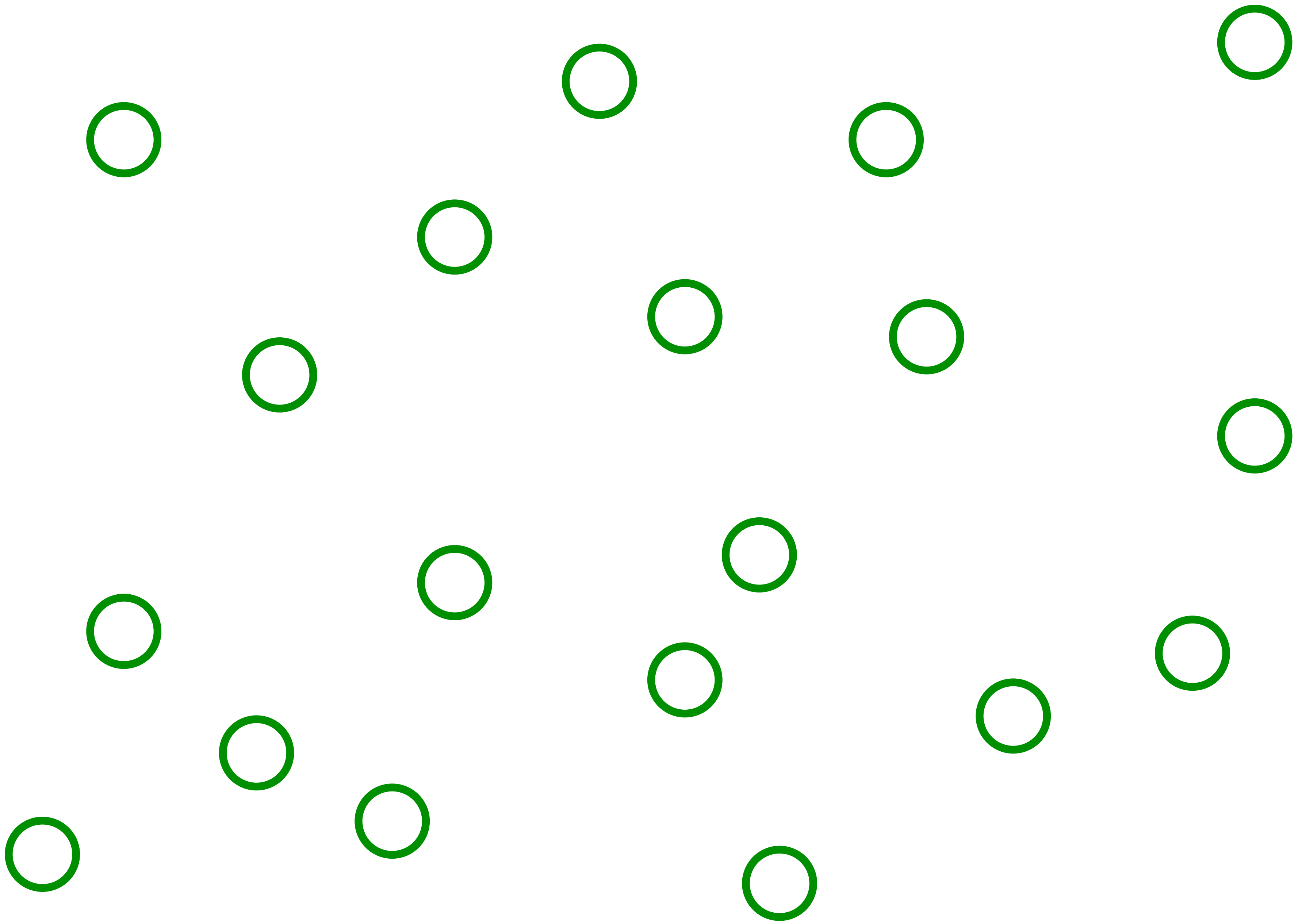
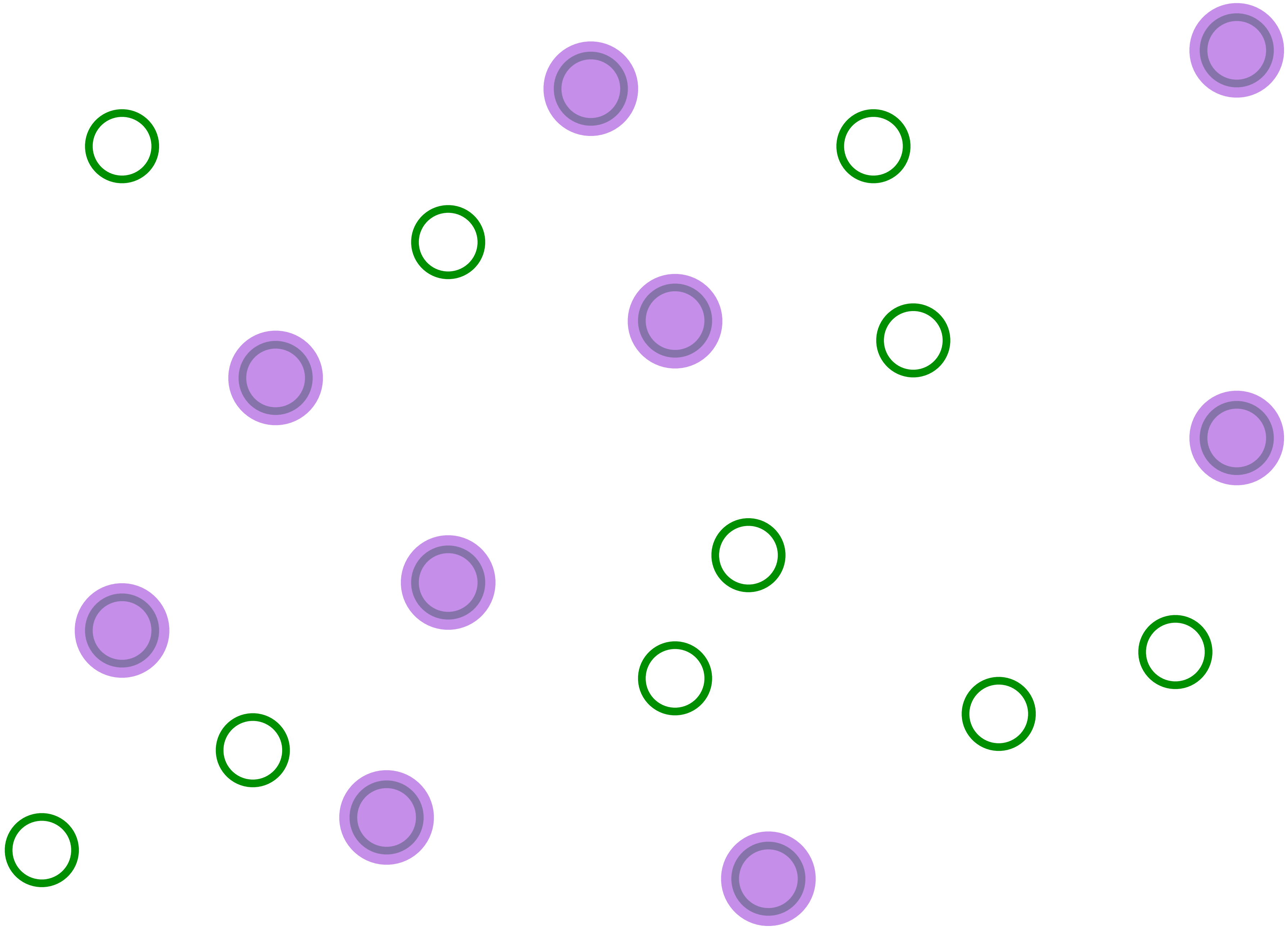
The Sachdev-Ye-Kitaev (SYK) model



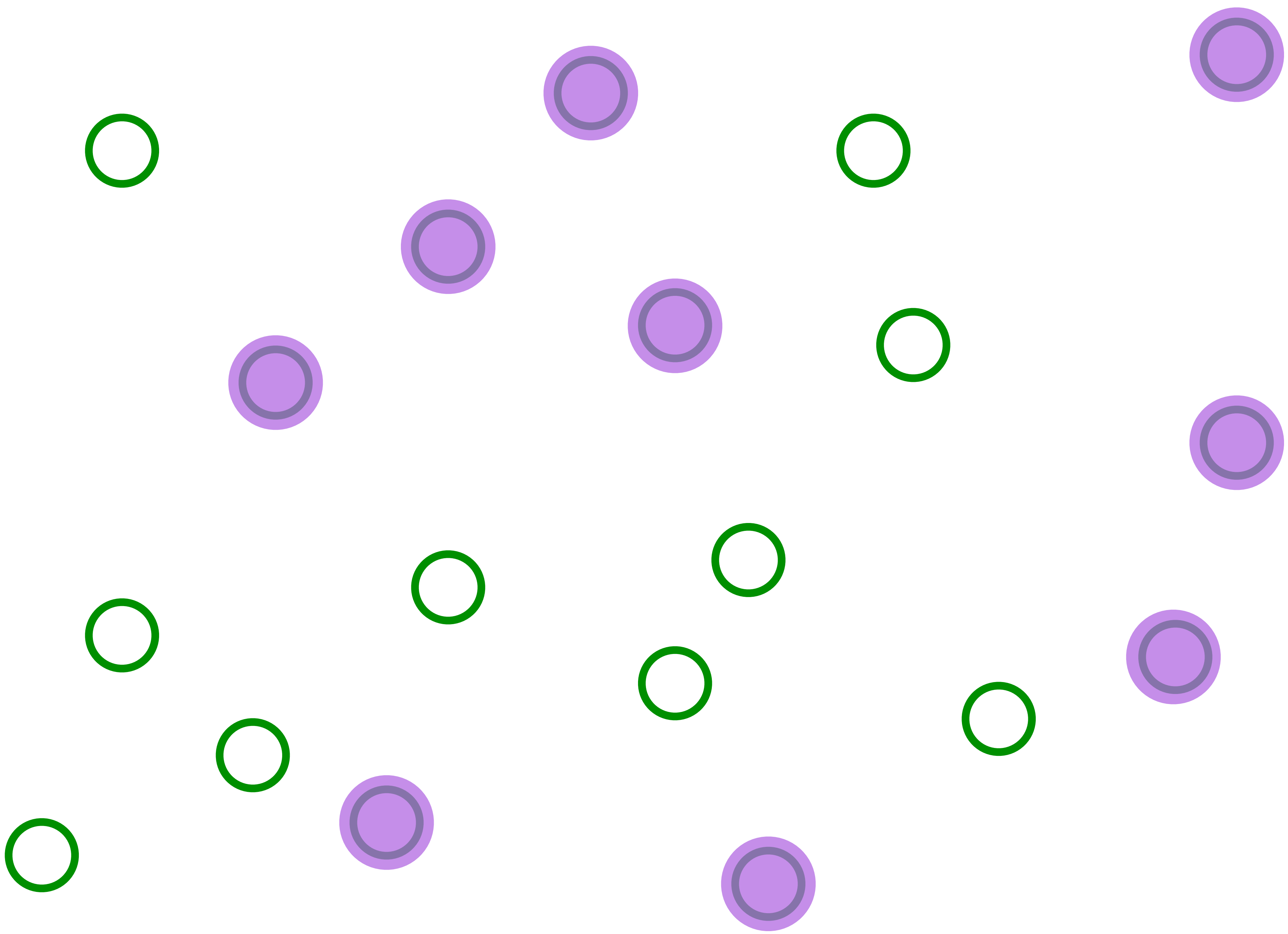
Pick a set of random positions

The SYK model



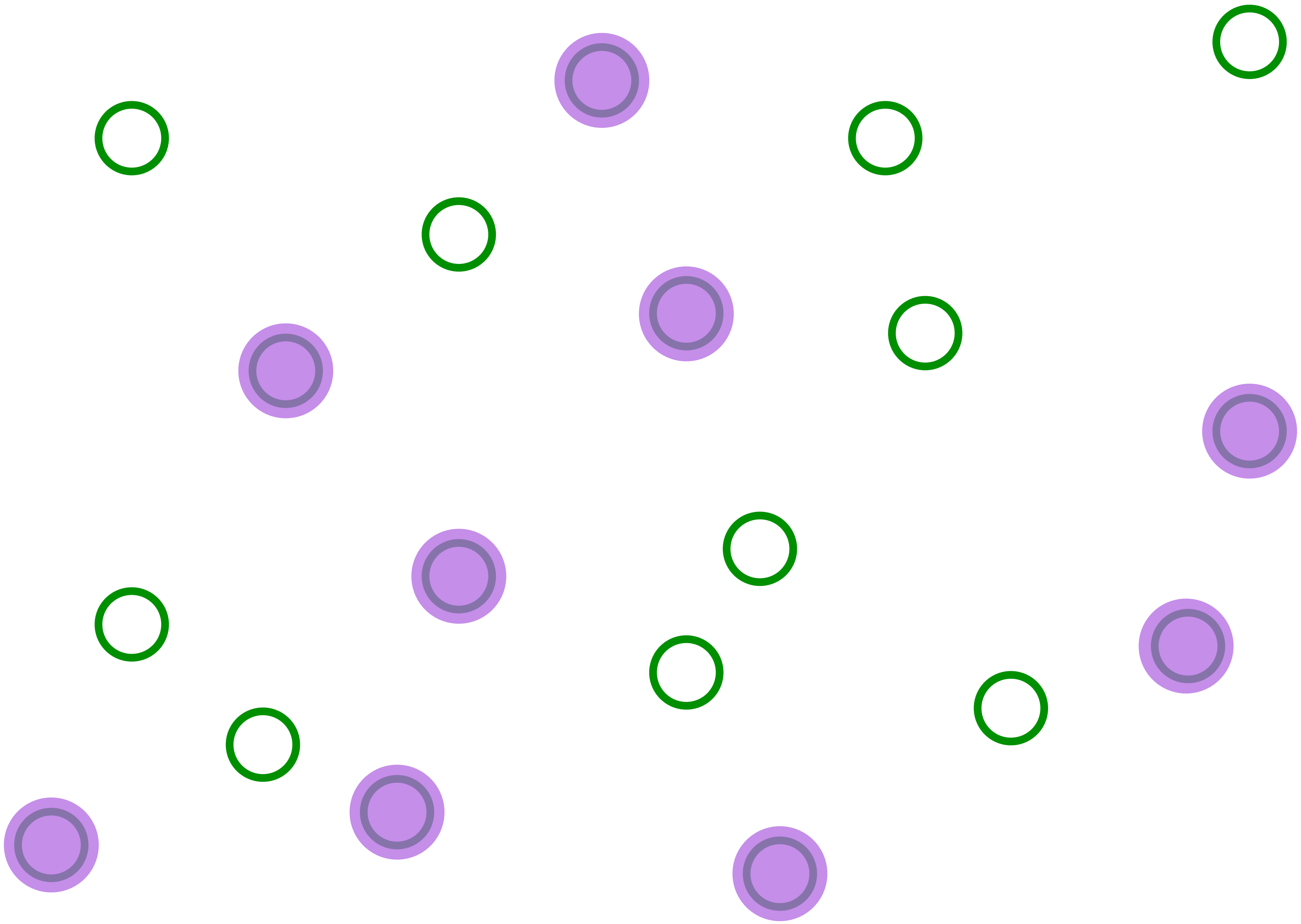
Place electrons randomly on some sites

The SYK model



Entangle electrons pairwise randomly

The SYK model



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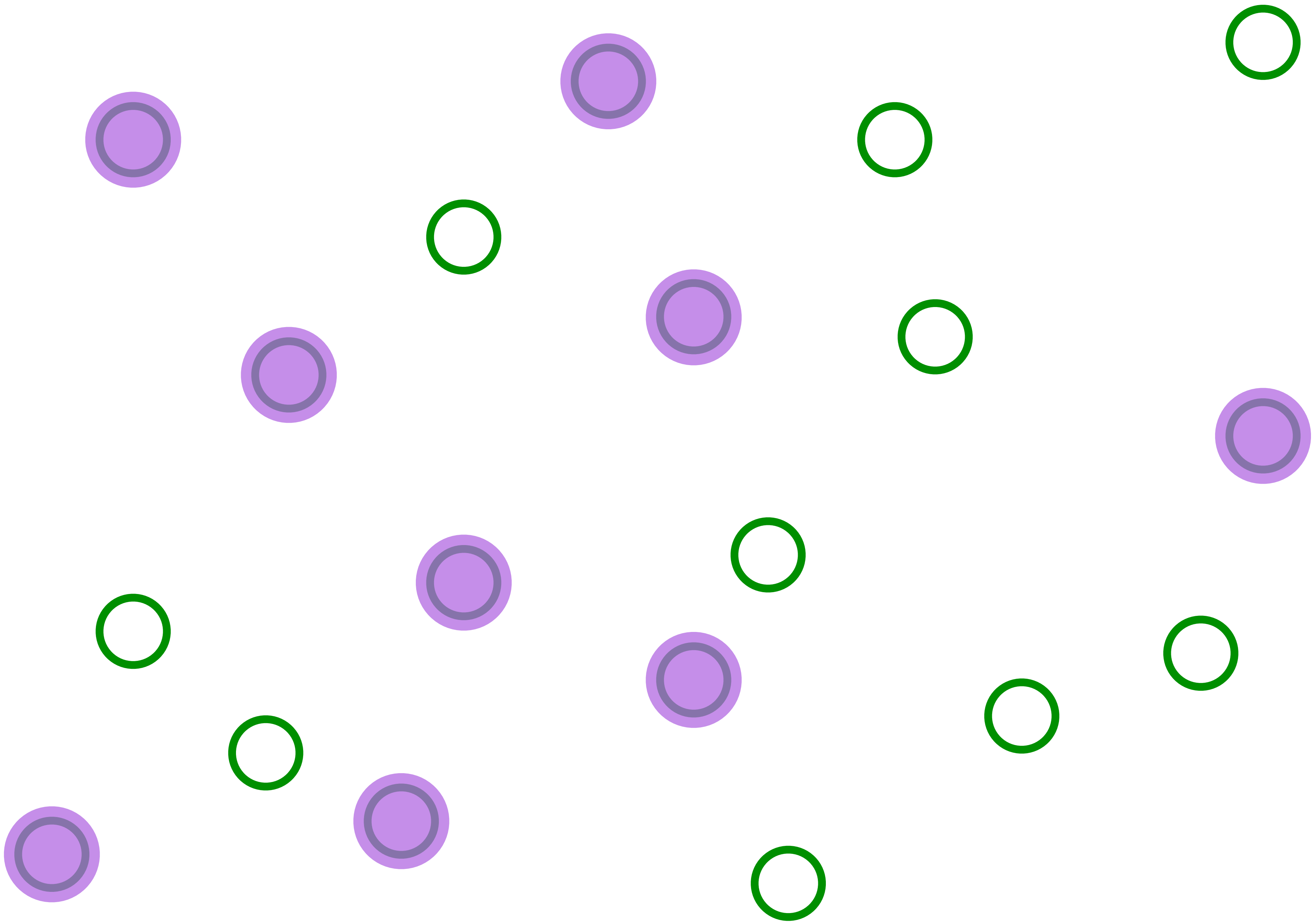

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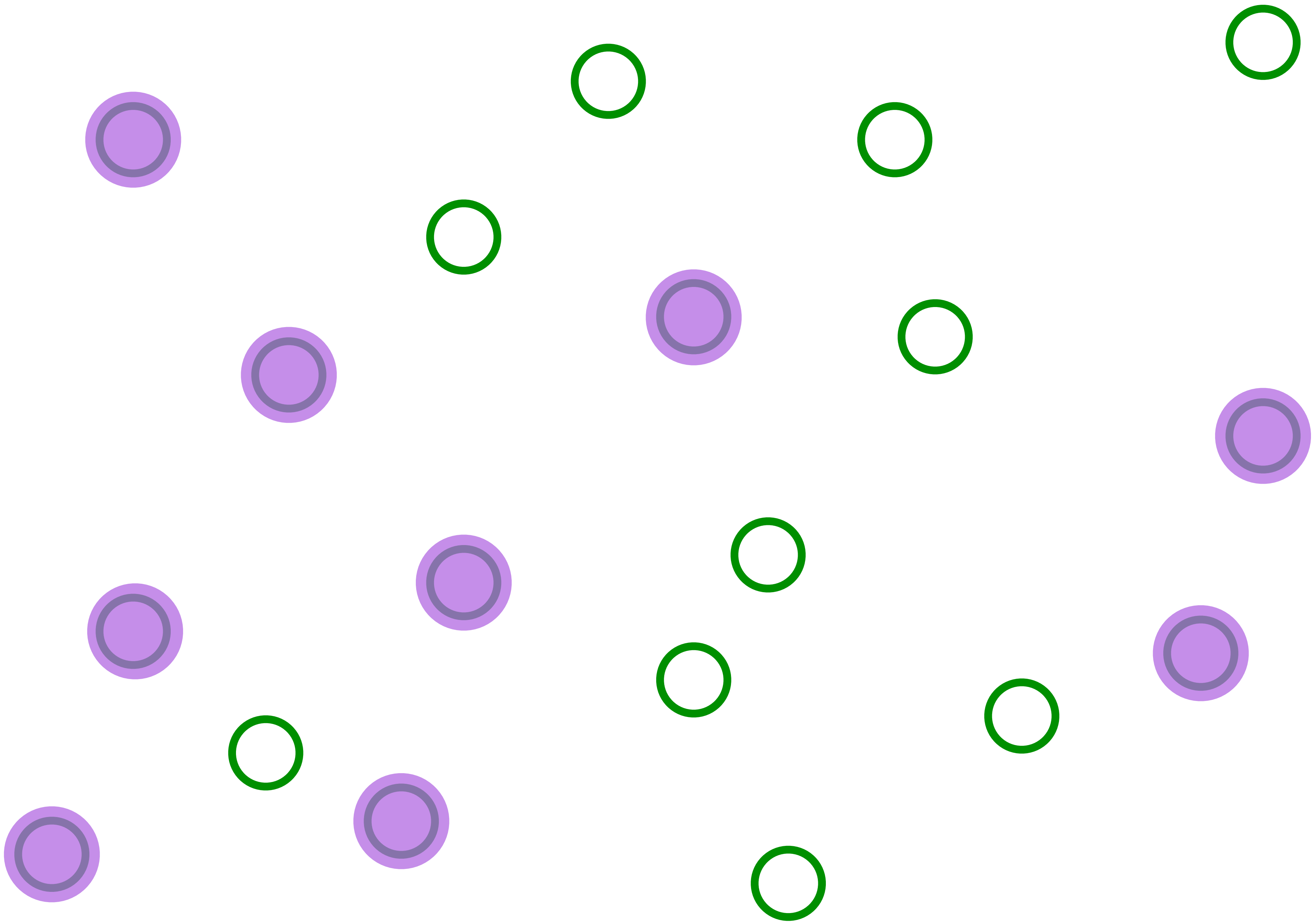

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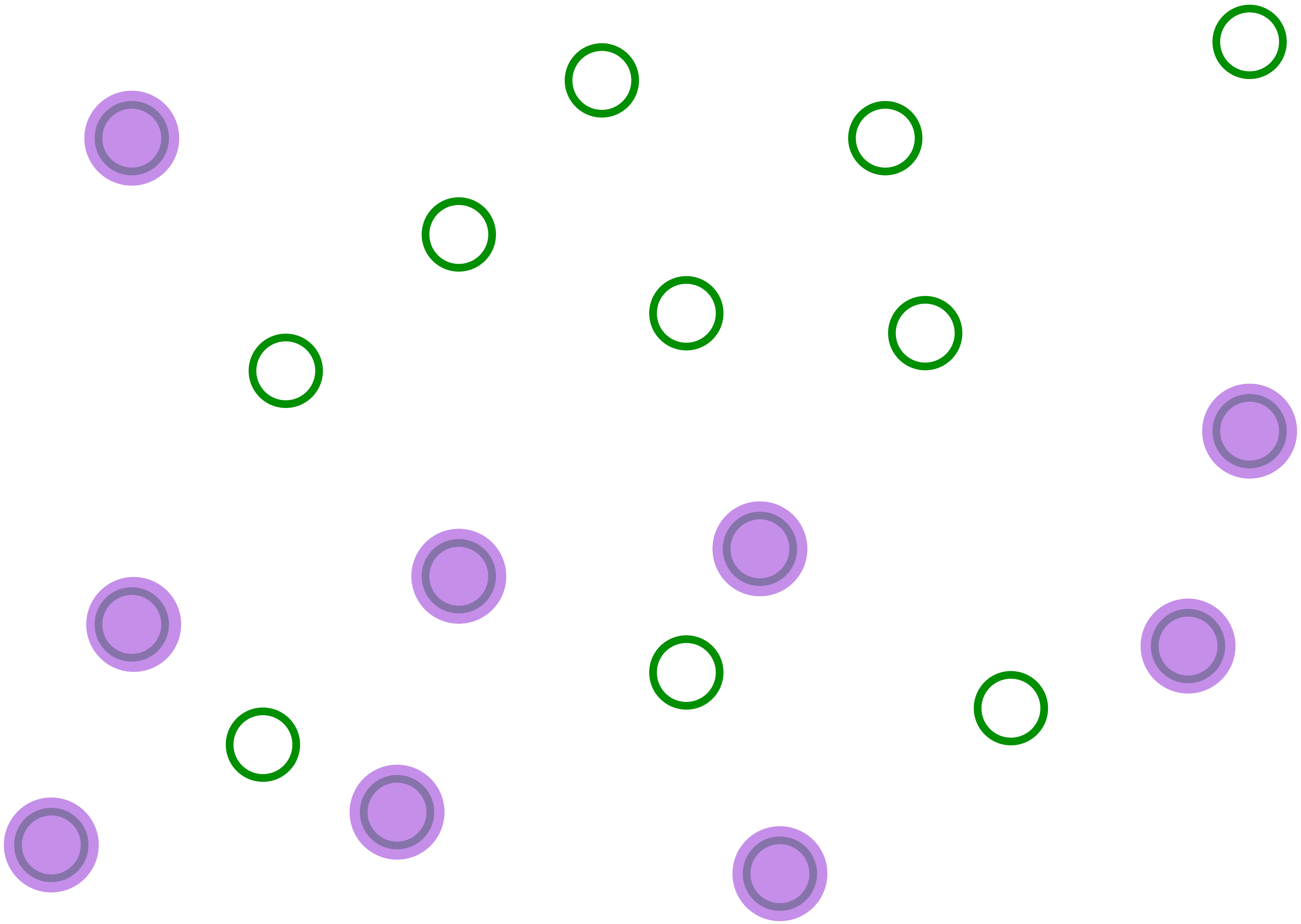

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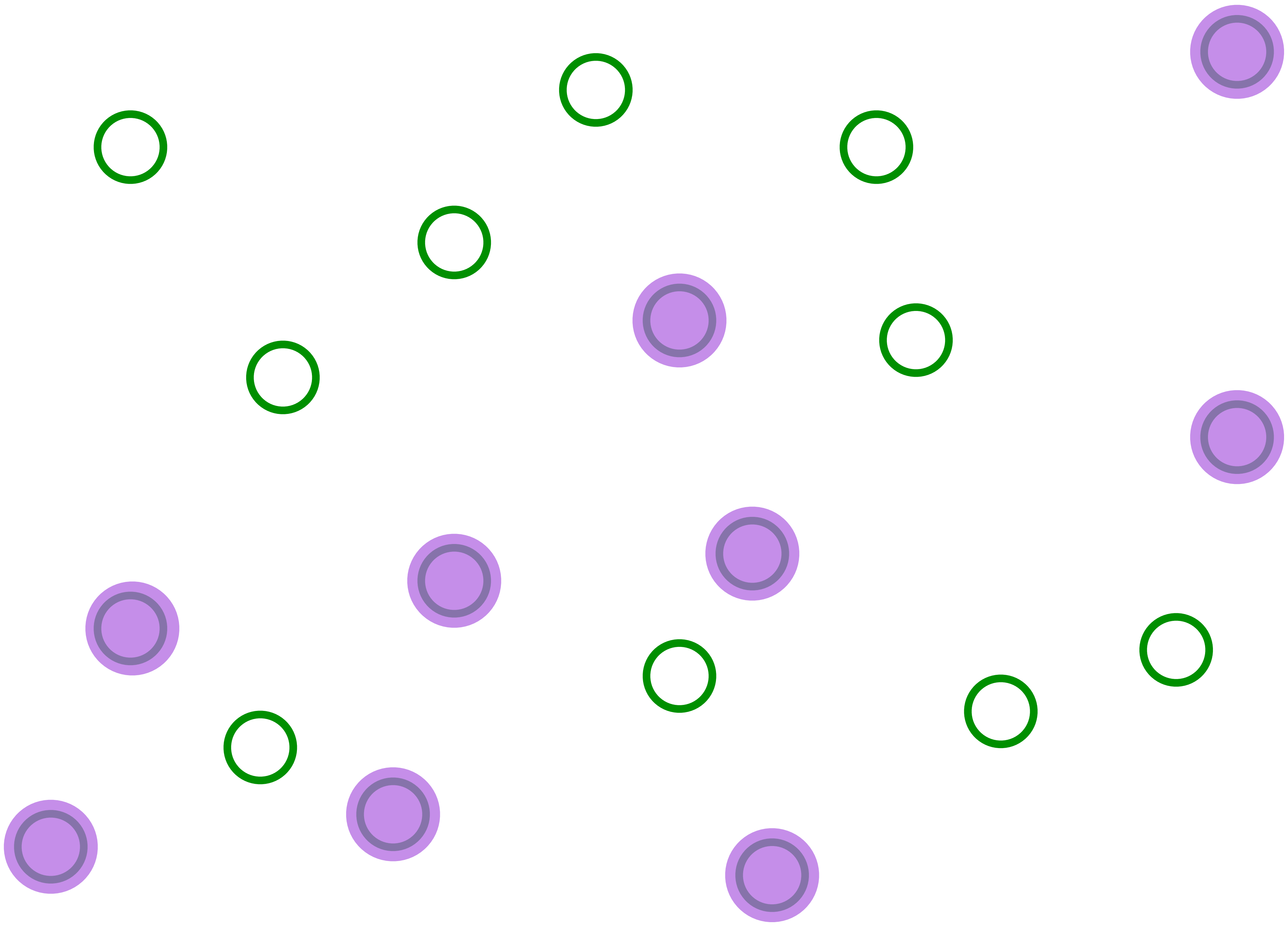

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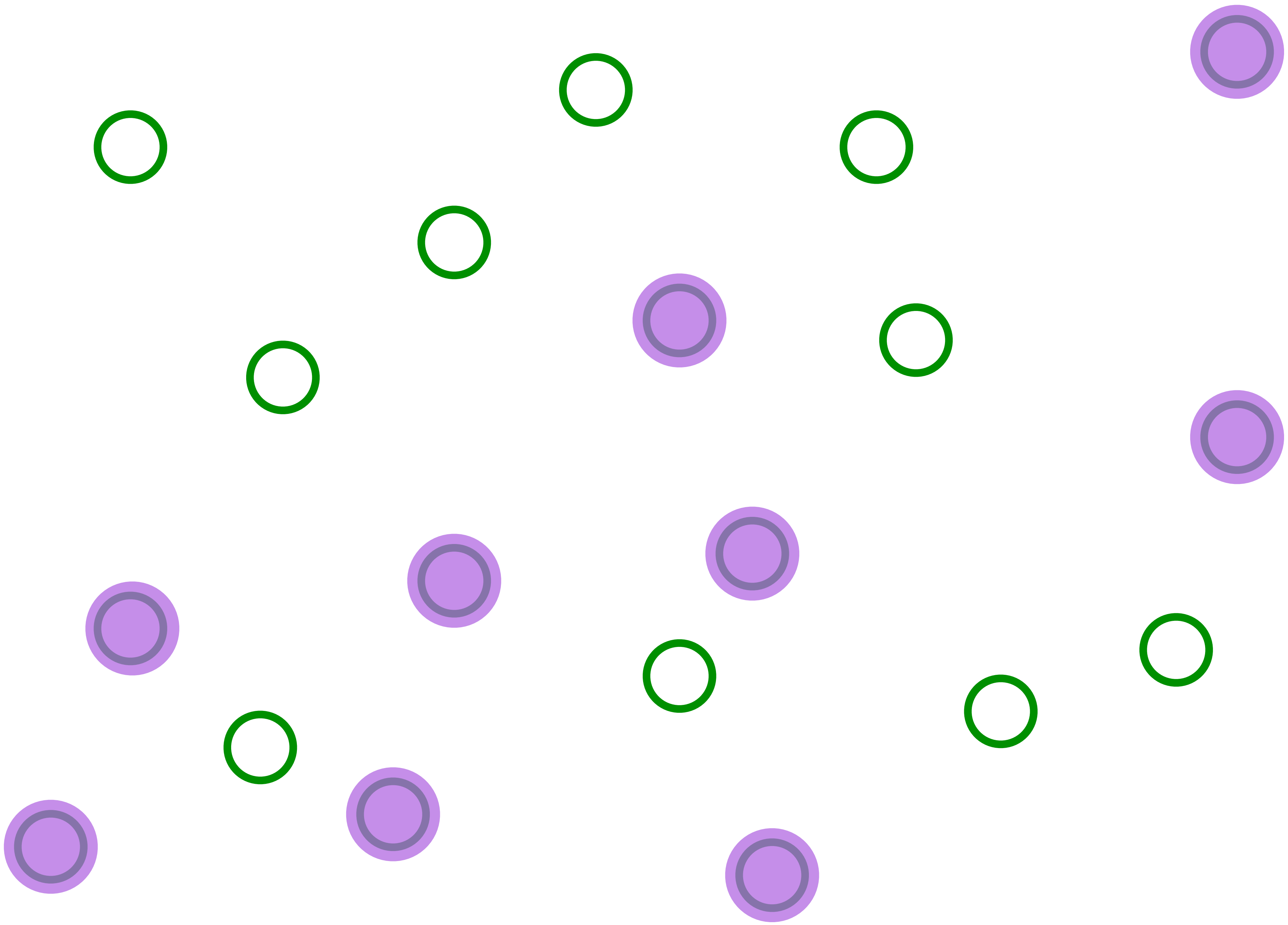


This describes both a strange metal and a black hole!

The SYK model

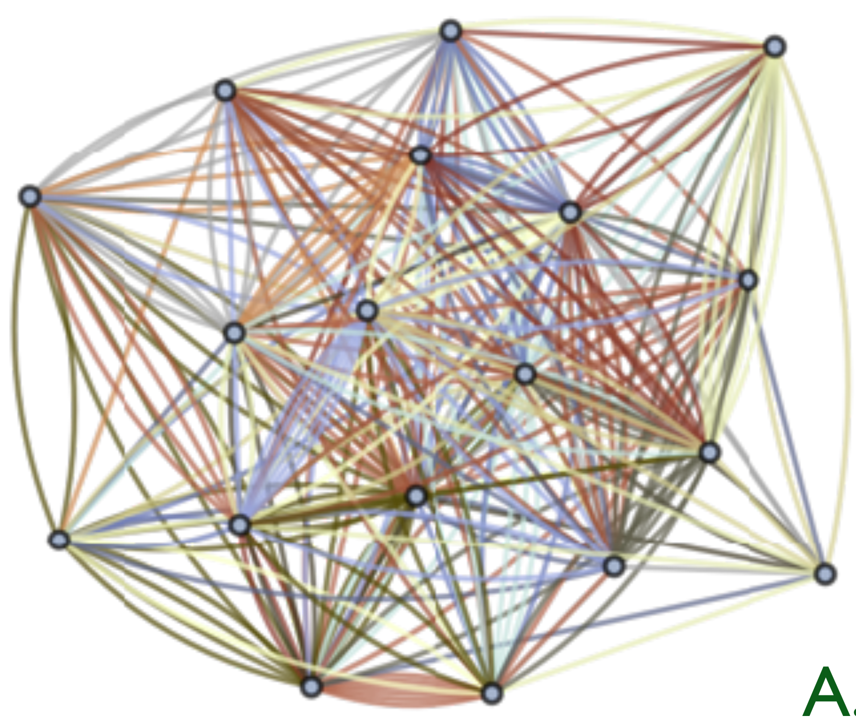
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

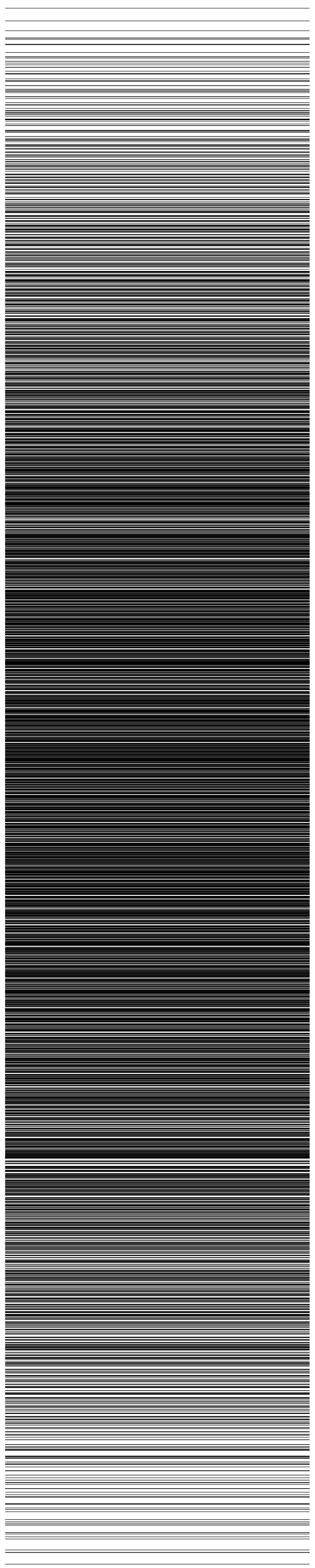
$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

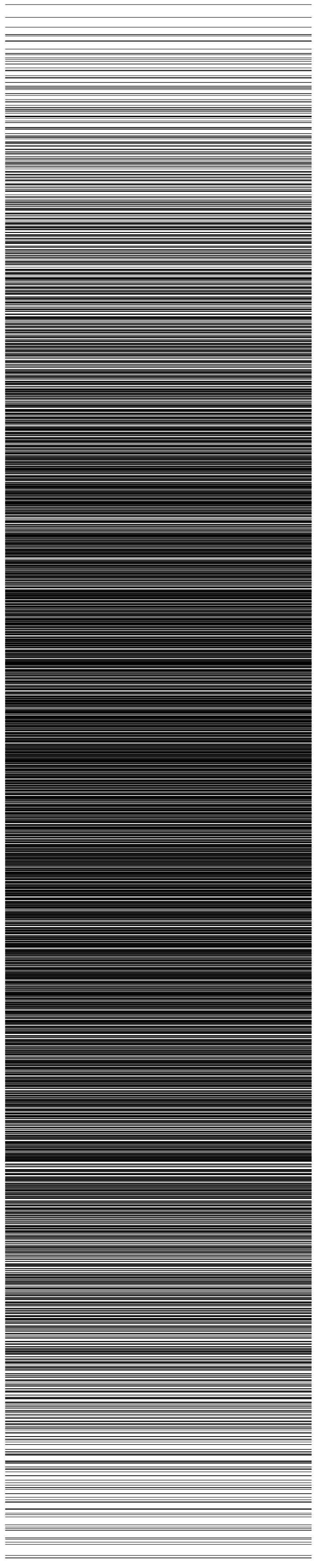
$$
\begin{aligned}
s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\
&< \ln 2
\end{aligned}
$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model



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Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

No quasiparticles !
$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$

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PRB **63**, 134406 (2001)

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The SYK model

- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

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- The last property indicates $\tau_{\rm eq} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Quantum matter without quasiparticles:

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- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T}$$

- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} .$$

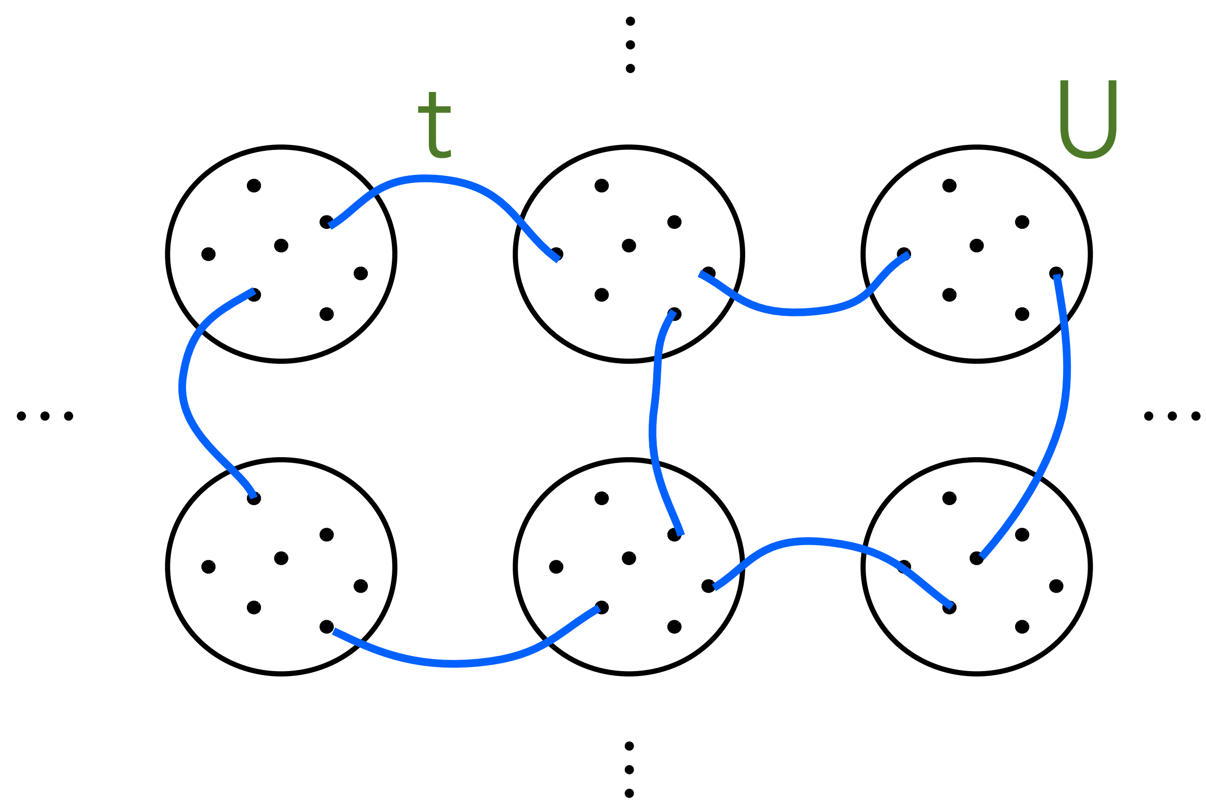
S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

A strongly correlated metal built from Sachdev-Ye-Kitaev models

Xue-Yang Song, Chao-Ming Jian, and L. Balents, arXiv:1705.00117

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

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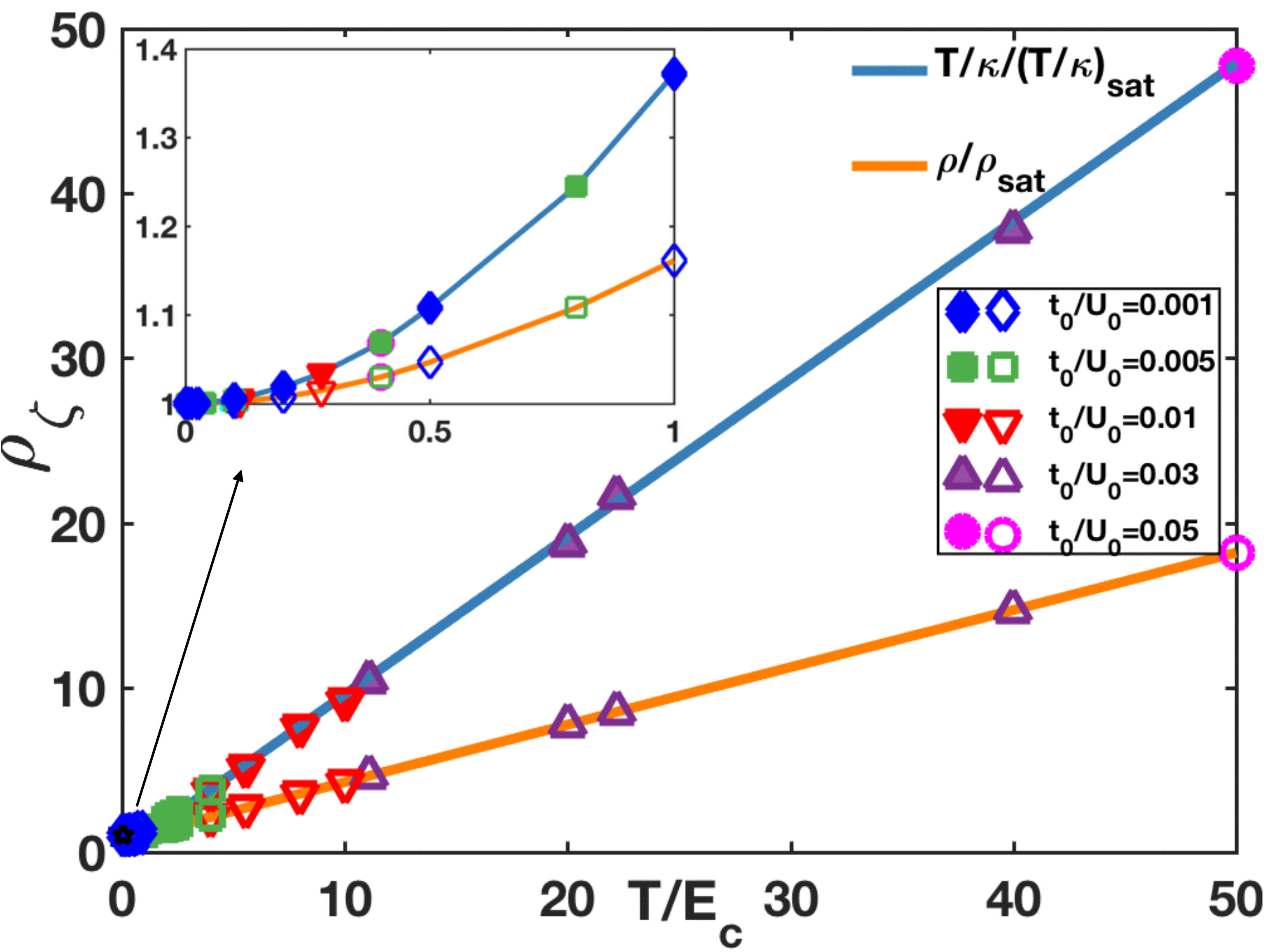

Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$

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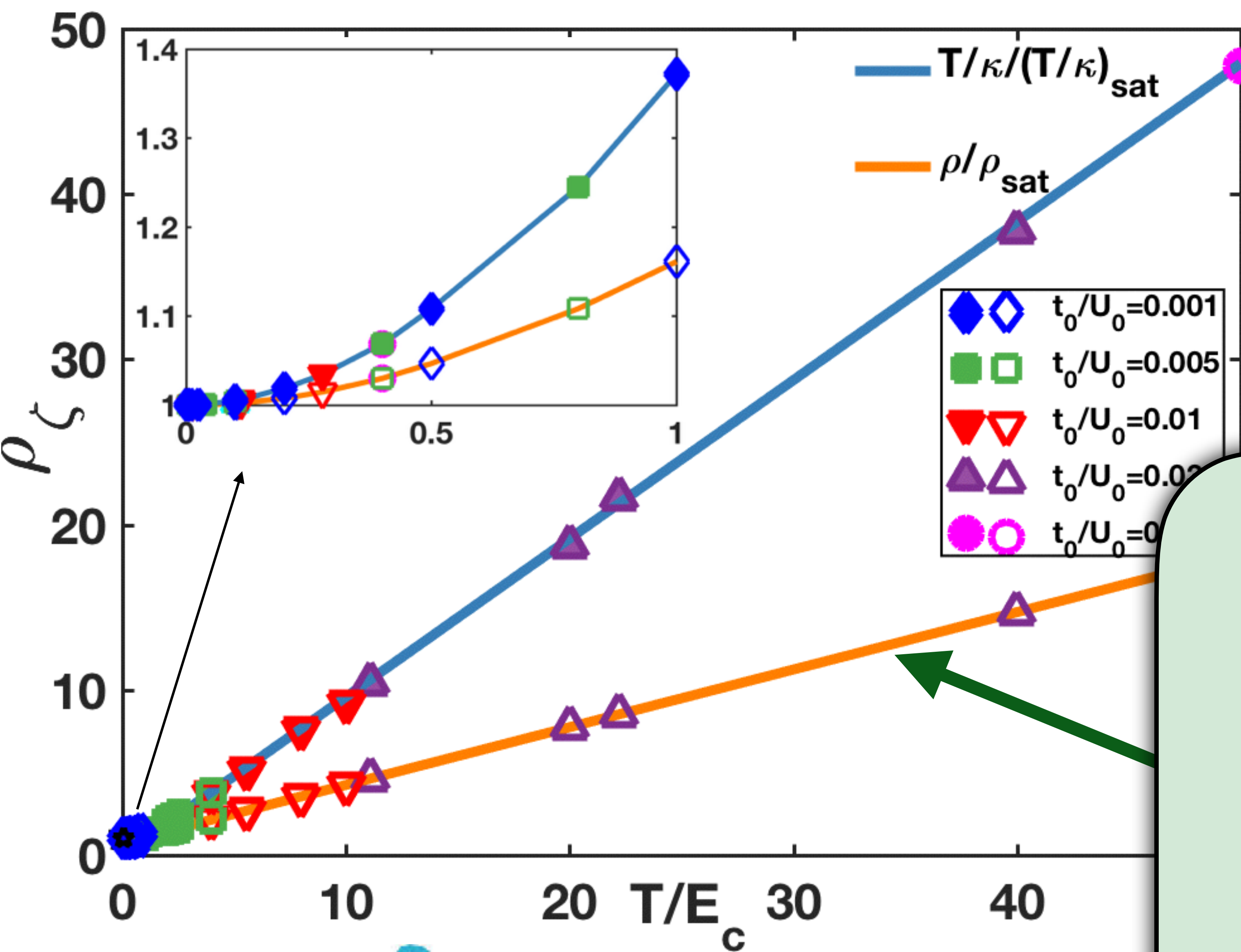
Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$

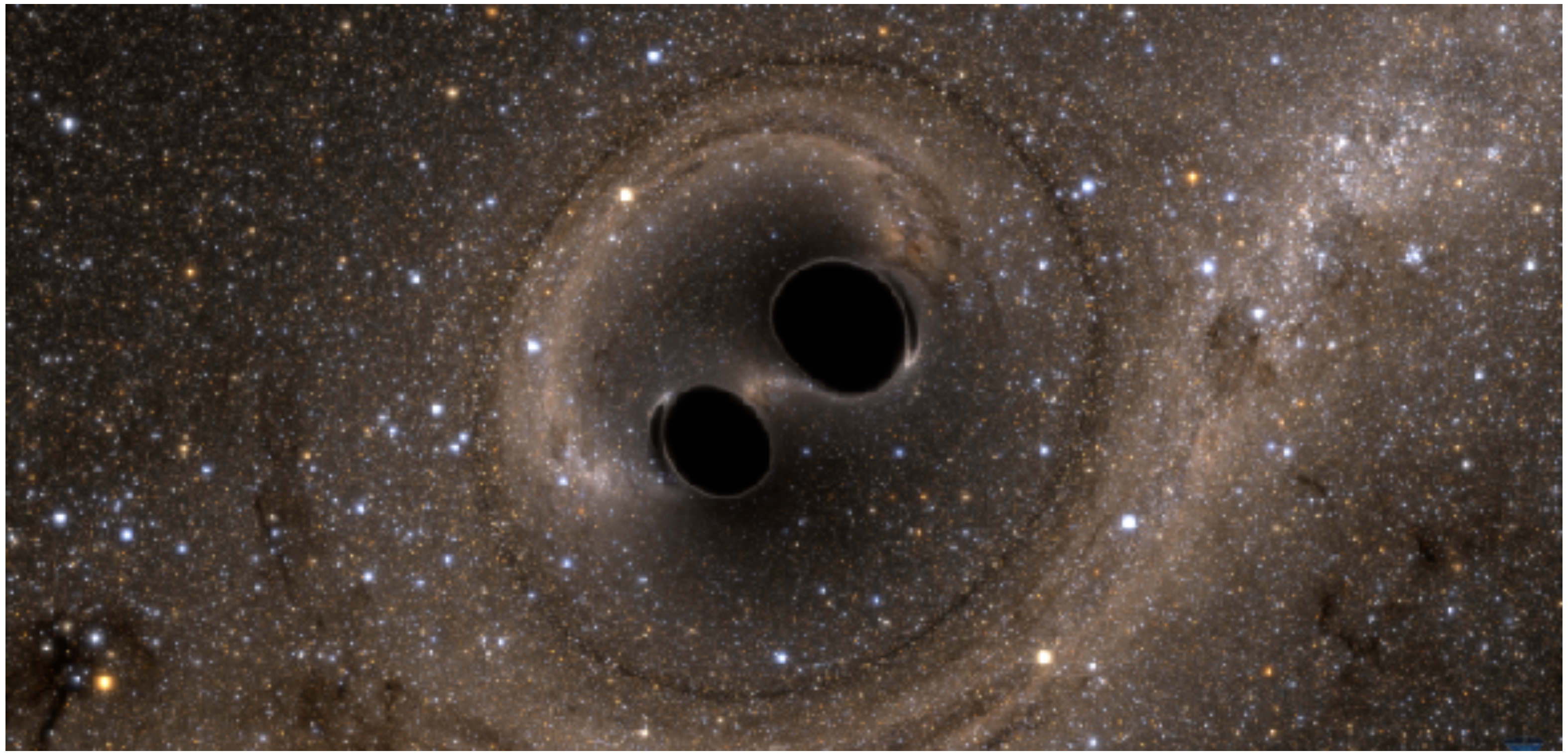


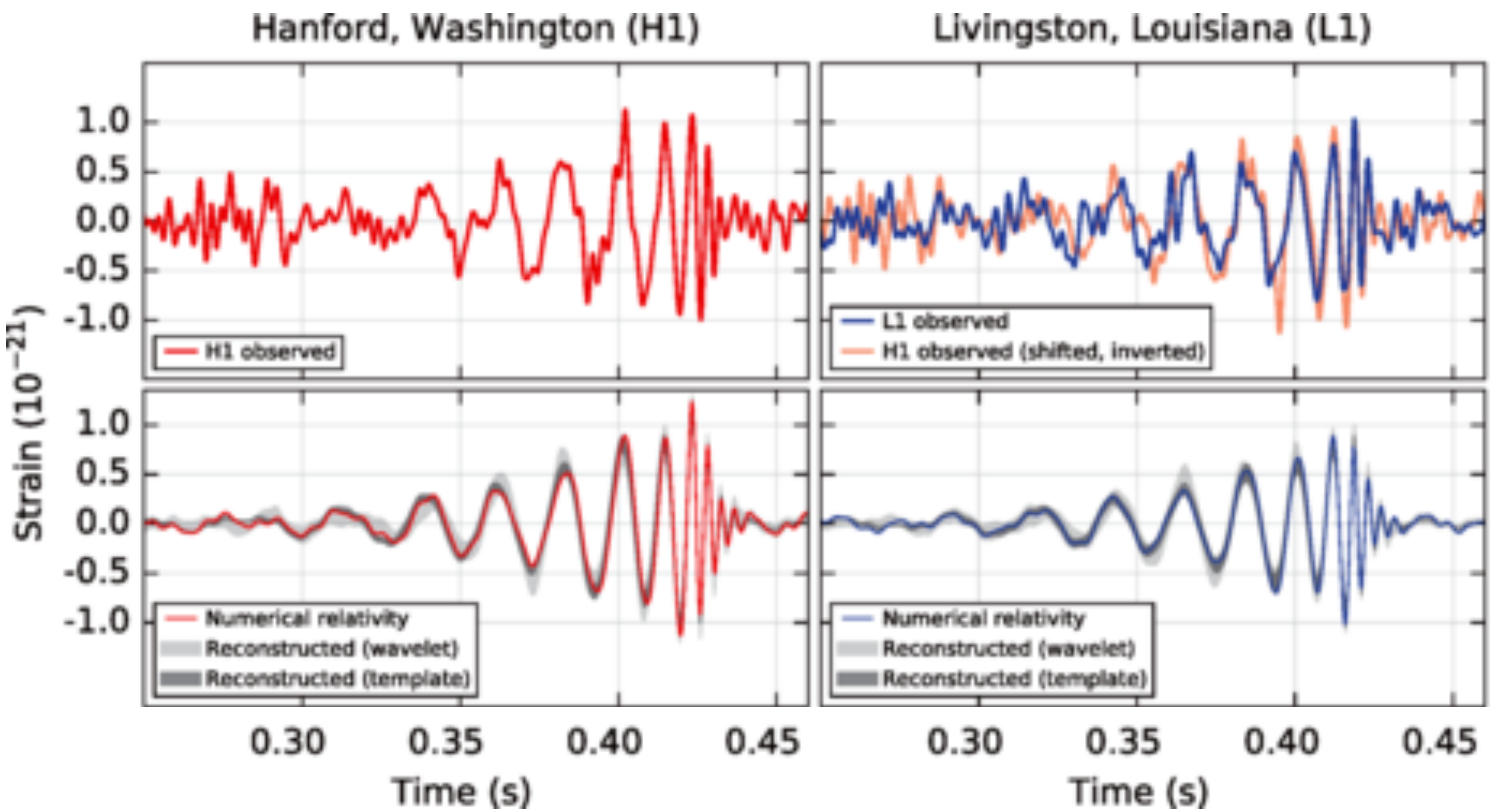
For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

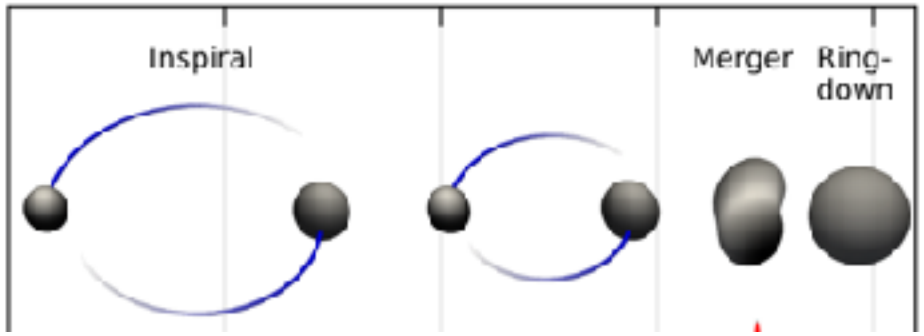
$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \qquad s = s_0$$

Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy is proportional to their surface area.





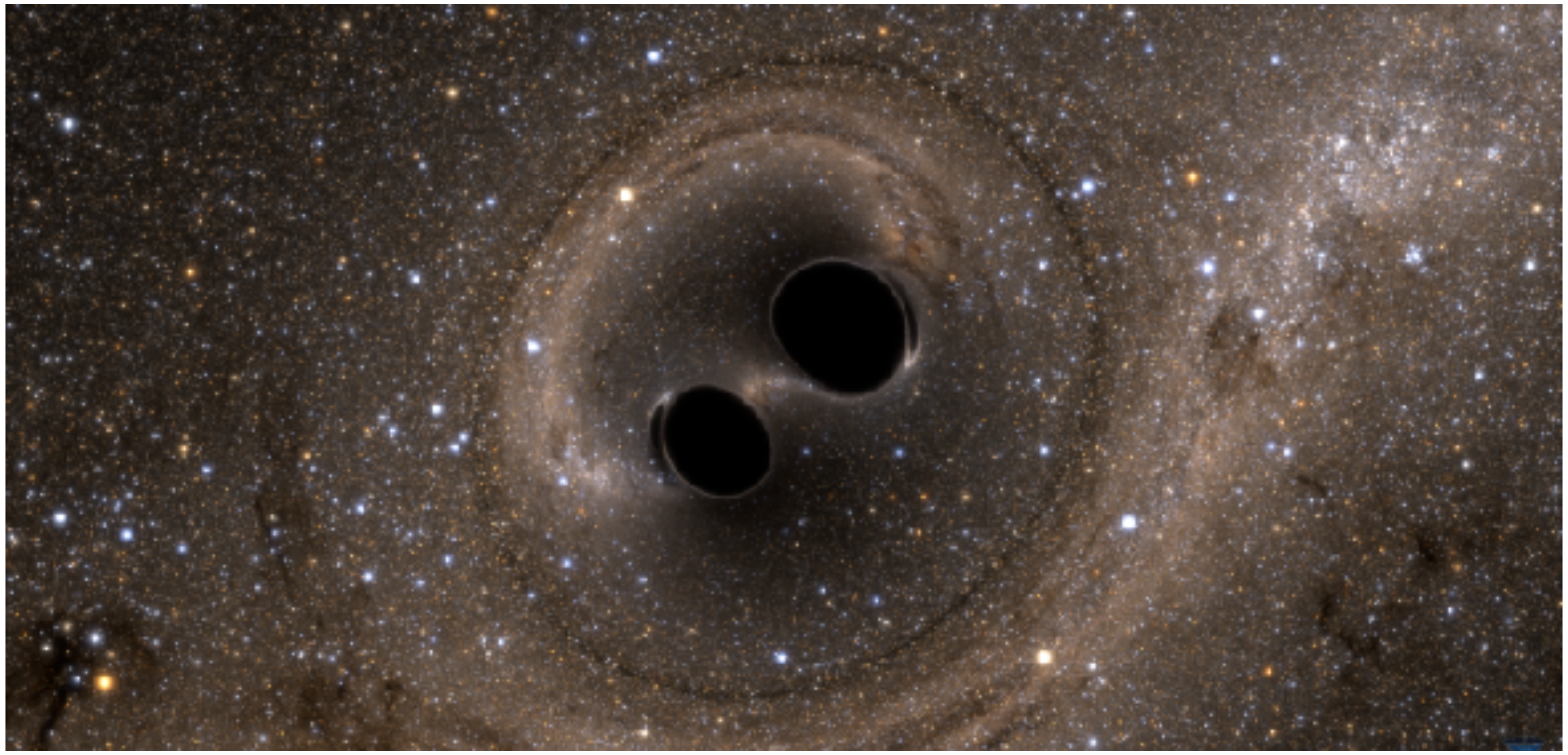


LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\frac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate.

Black holes

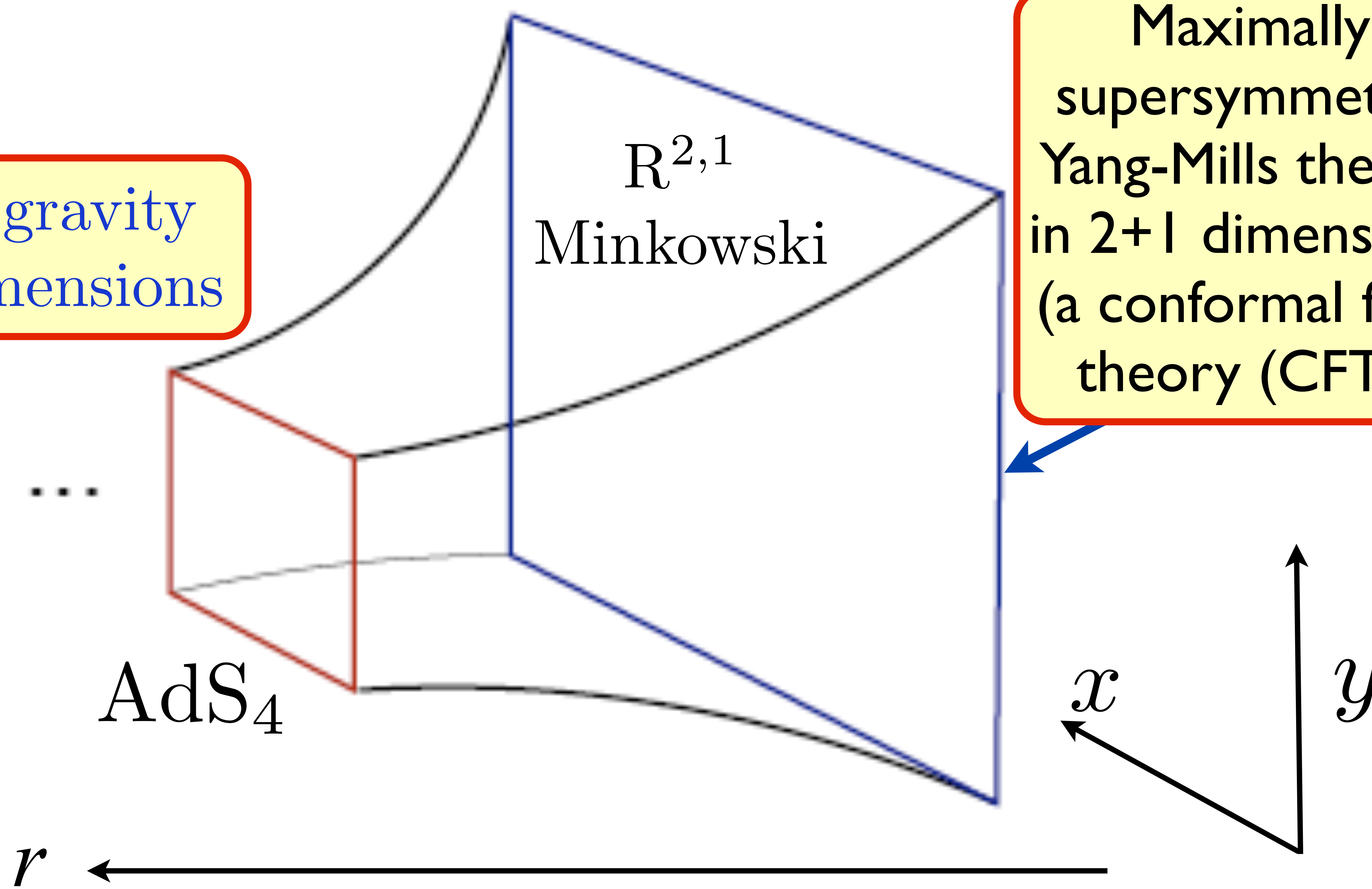
- Black holes have an entropy and a temperature, T_H.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.



AdS/CFT correspondence at zero temperature

Quantum gravity in 3+1 dimensions

Maximally supersymmetric Yang-Mills theory in 2+1 dimensions (a conformal field theory (CFT))



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + dx^2 + dy^2\right]$$

Maldacena, Gubser, Klebanov, Polyakov, Witten

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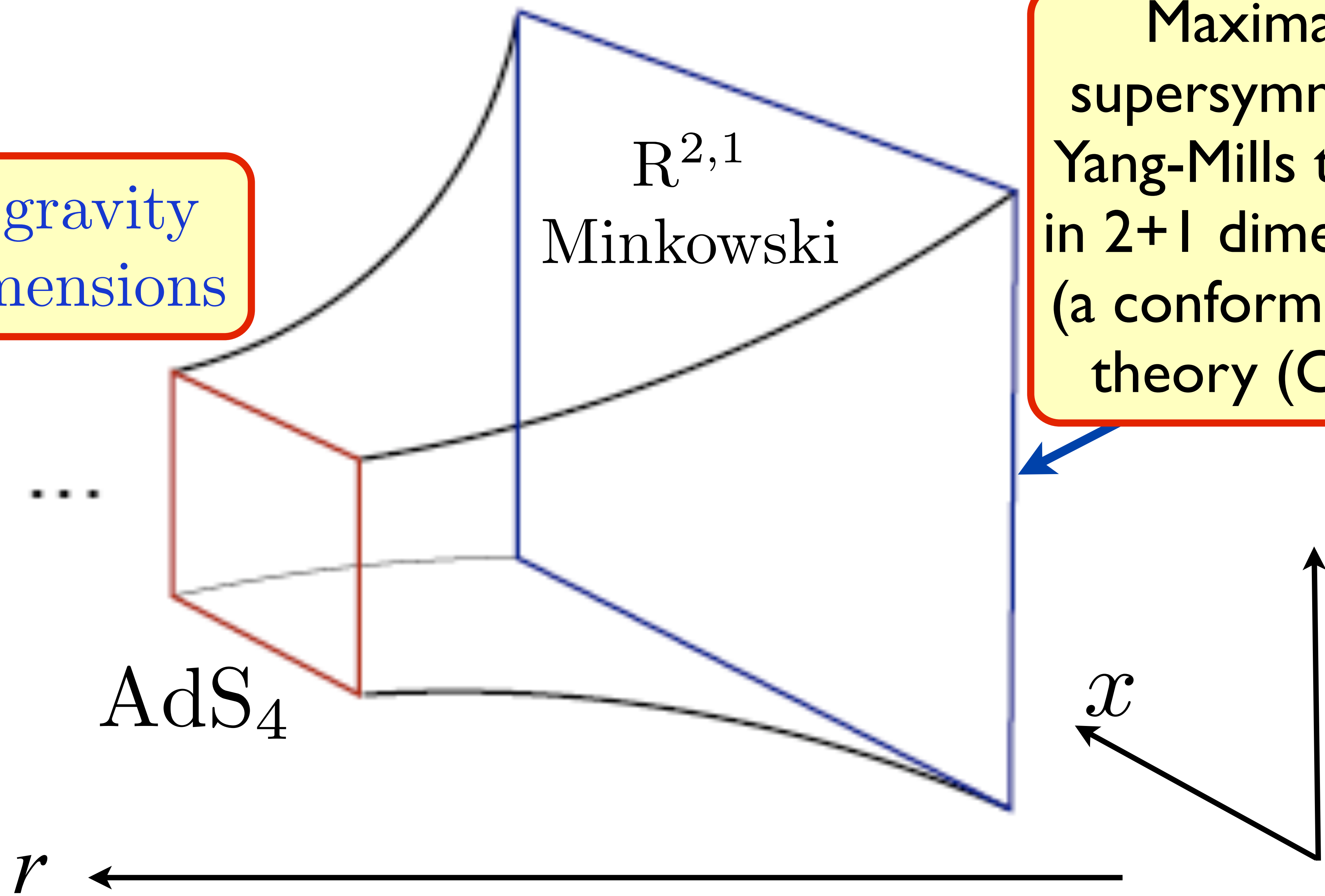


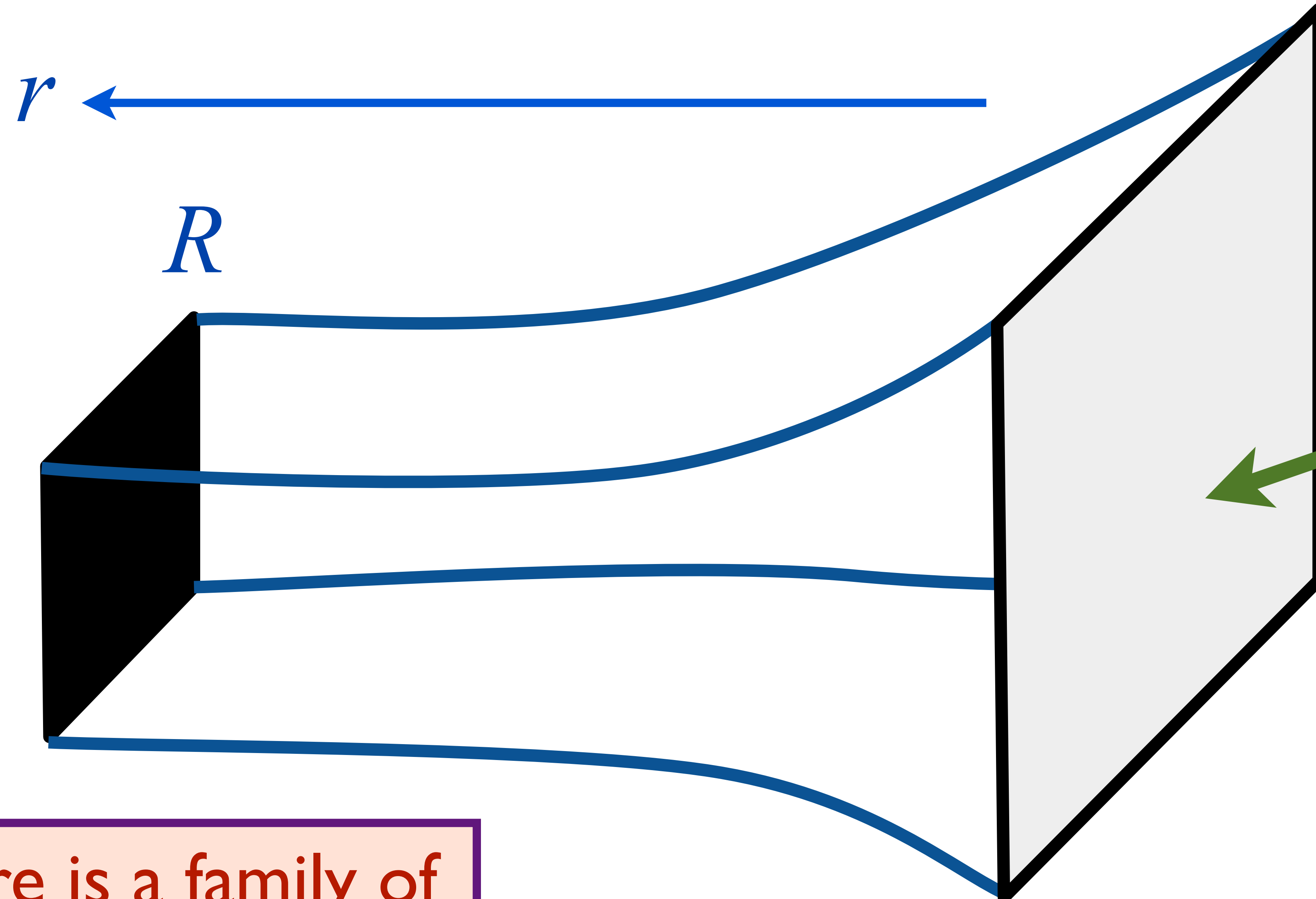
This spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



Maximally supersymmetric Yang-Mills at a temperature

$$k_B T = \frac{3\hbar}{4\pi R}.$$

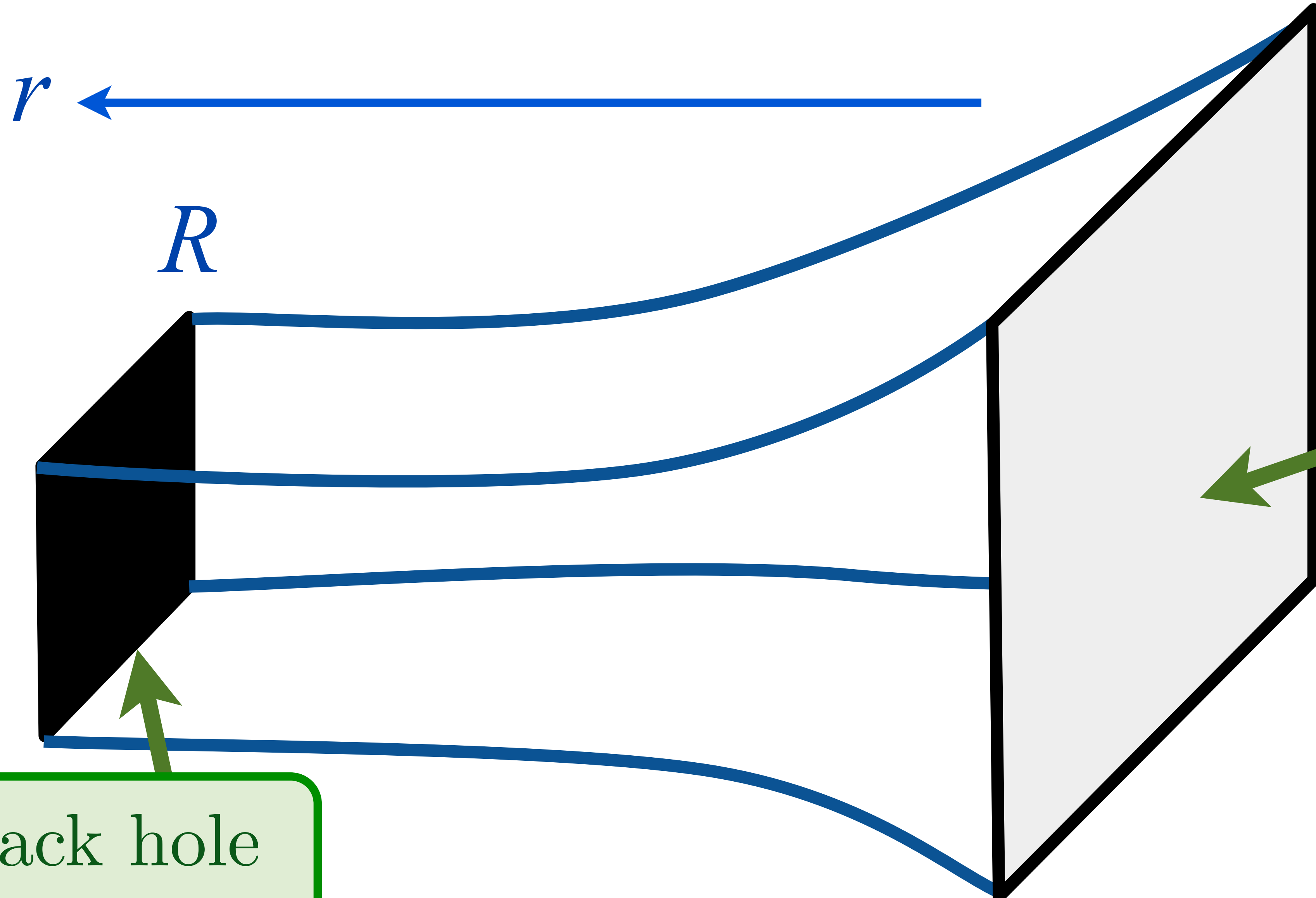
There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS_4-Schwarzschild black-brane



Maximally supersymmetric Yang-Mills at a temperature $k_B T = \frac{3\hbar}{4\pi R}$.

Black hole horizon at a Hawking temperature $T_H = T$

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

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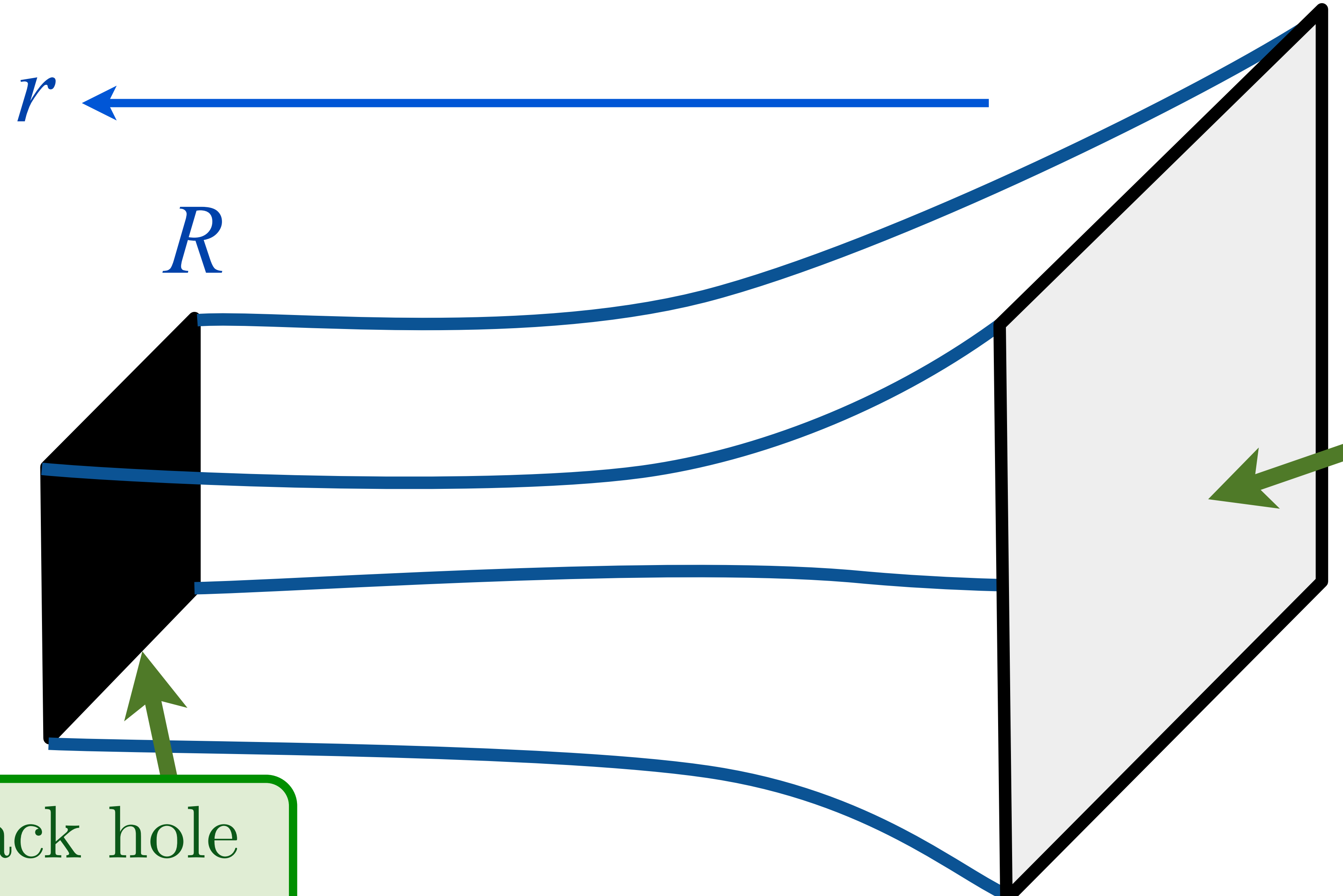

Maximally supersymmetric Yang-Mills at a temperature $k_B T = \dfrac{3\hbar}{4\pi R}$.

Black hole entropy = Entropy of Yang-Mills theory

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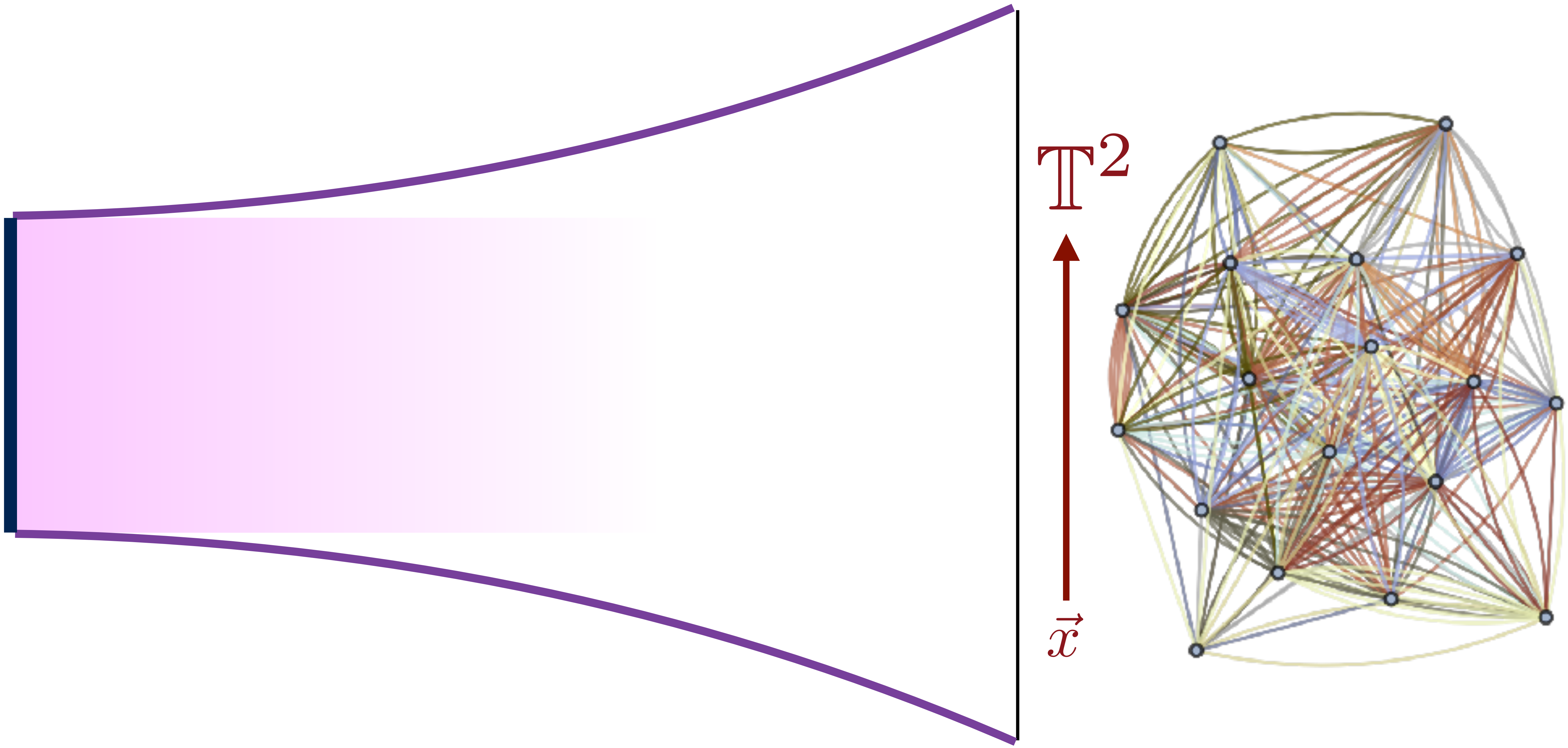
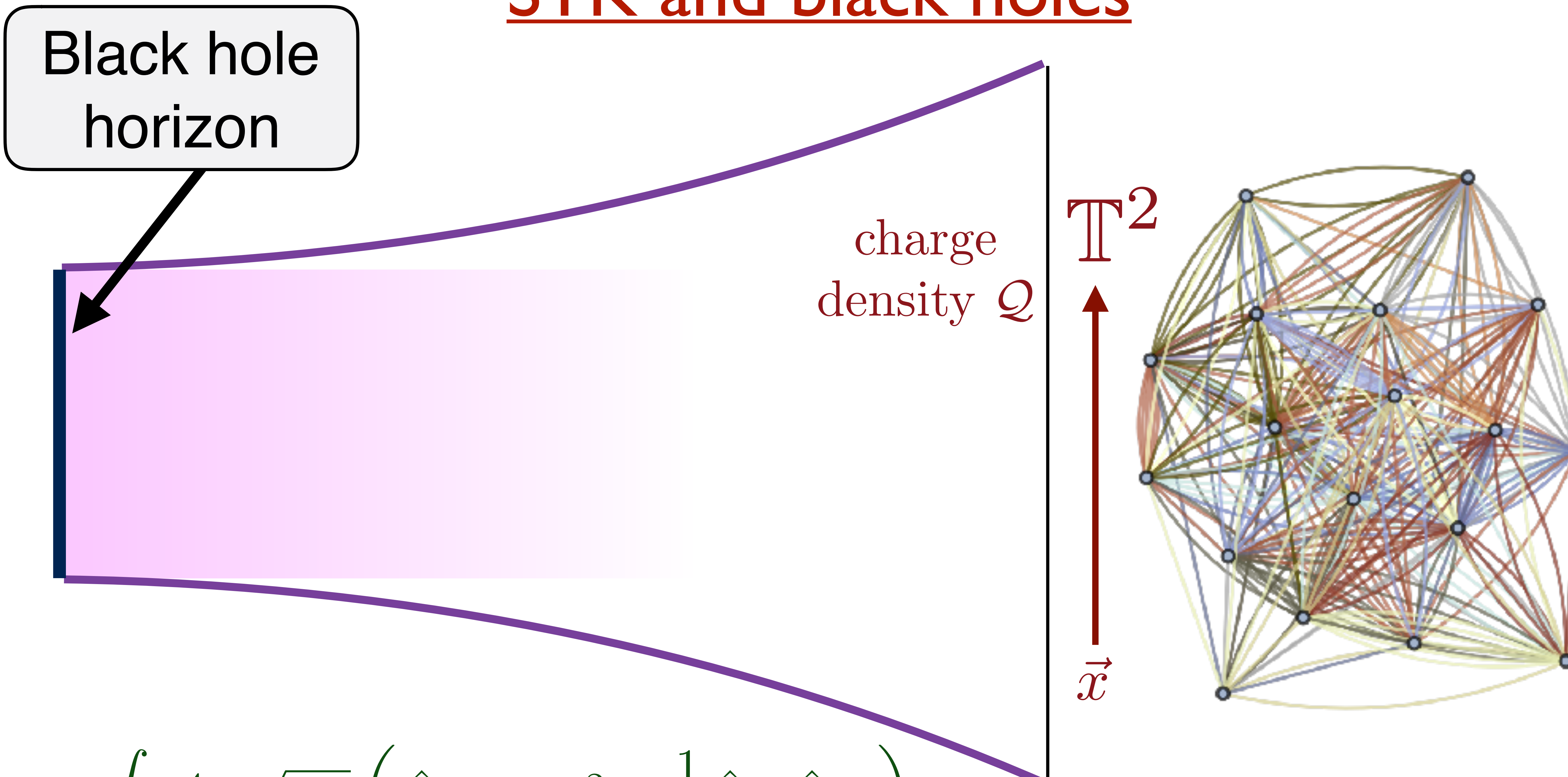
SYK and black holes



Is there a holographic quantum gravity dual of the SYK model ?

SYK and black holes

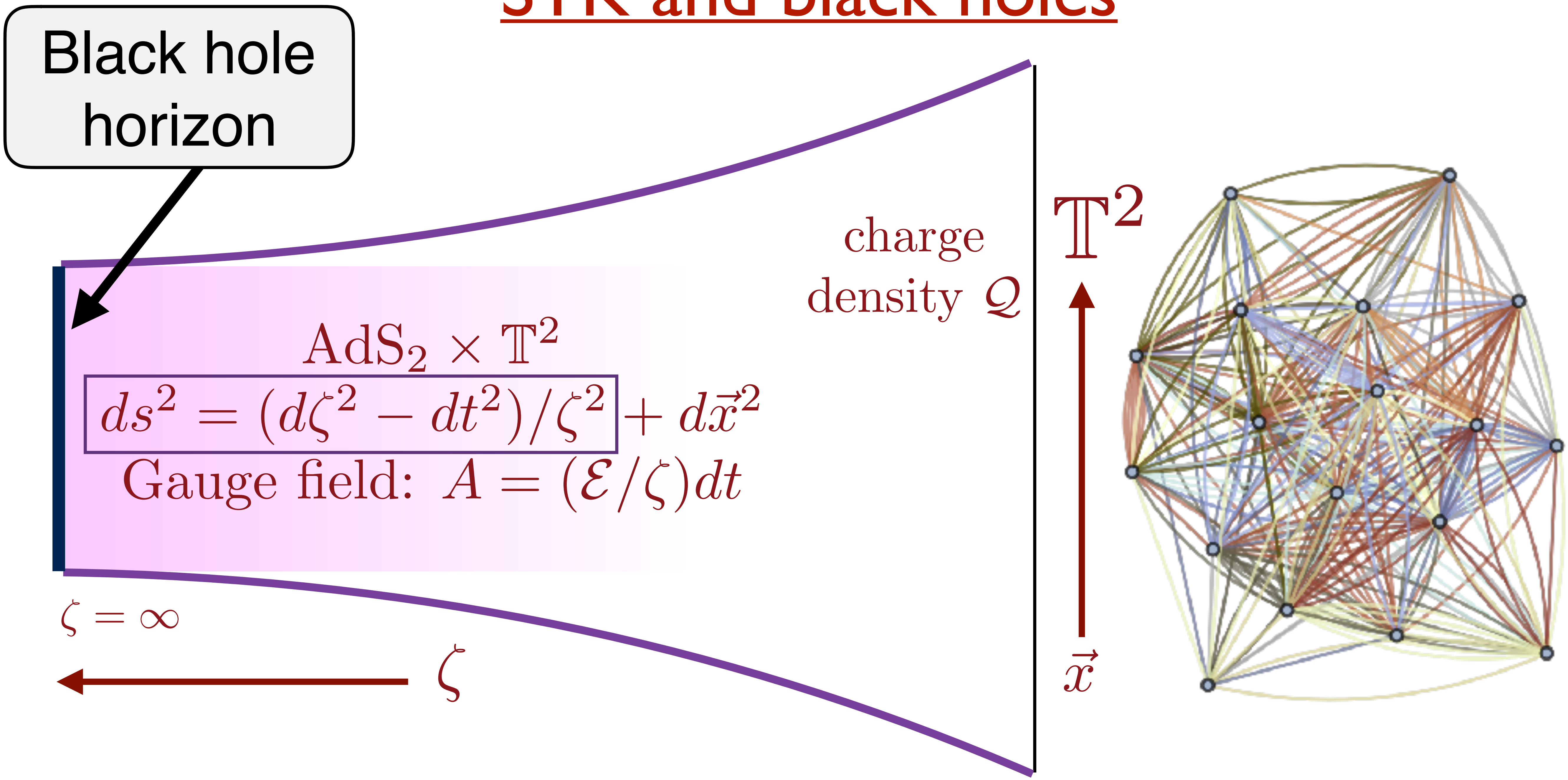


$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

Yes, the properties of a charged black hole in Einstein-Maxwell theory holographically match those of the SYK model !

S. Sachdev, PRL **105**, 151602 (2010)

SYK and black holes



Quantum gravity on the 1+1 dimensional spacetime AdS_2 (when embedded in AdS_4) is holographically matched to the 0+1 dimensional SYK model

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (unpublished); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

Many-body quantum chaos

- Using holographic analogies, Shenker and Stanford introduced the "Lyapunov time", τ_L, the time over which a generic many-body quantum system loses memory of its initial state.

S. Shenker and D. Stanford, arXiv:1306.0622

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J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

- The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

A. Kitaev, unpublished
J. Maldacena and D. Stanford, arXiv:1604.07818

Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states: $E \neq \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$

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- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.
- These are also characteristics of black holes in quantum gravity.