Dynamics and Transport in Strange Metals

Stanford University
October 16, 2015

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Talk online: sachdev.physics.harvard.edu

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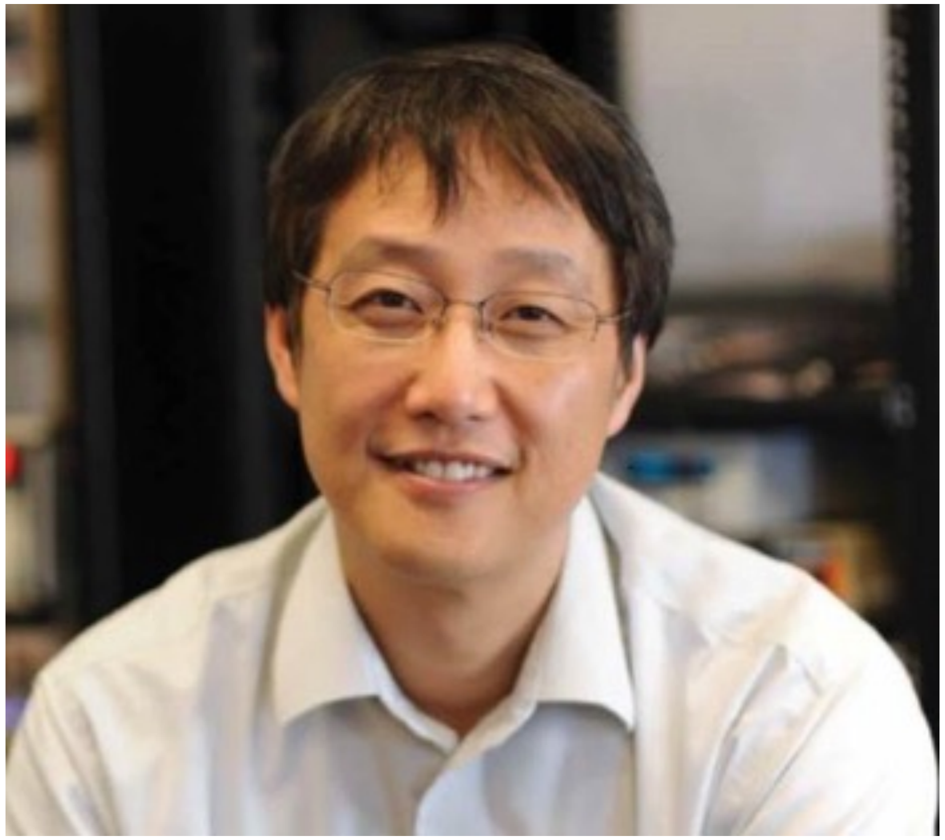
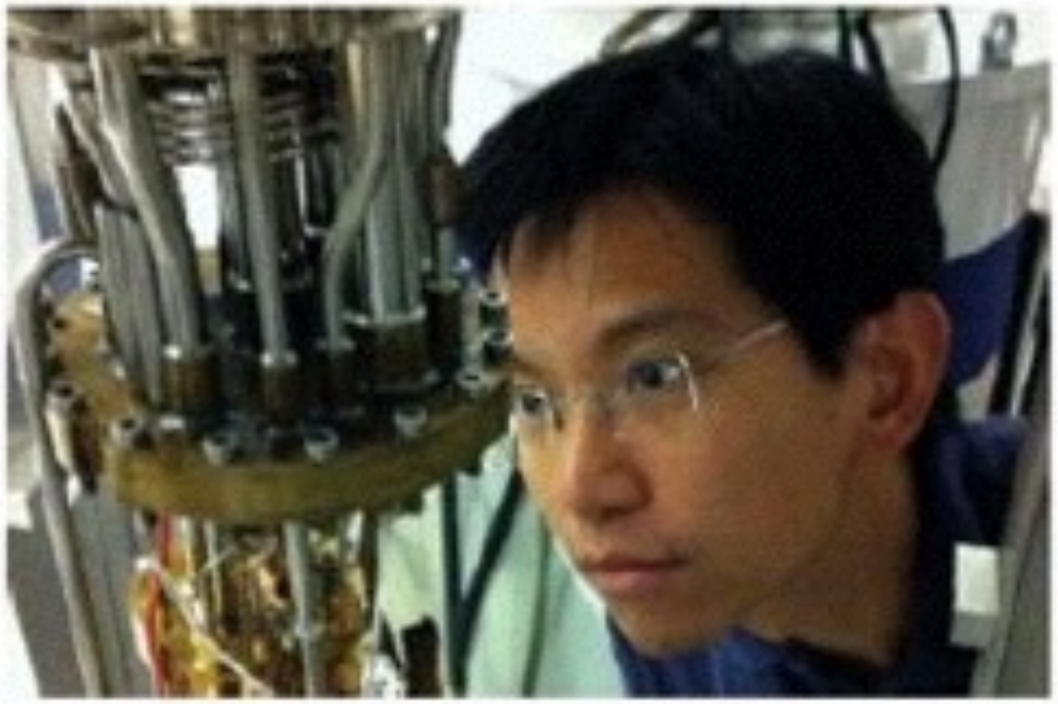
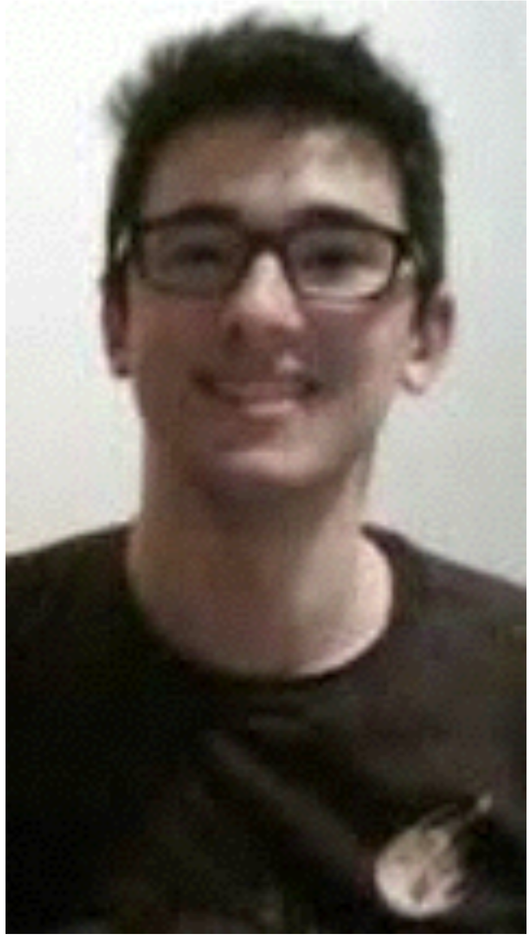


Philip Kim

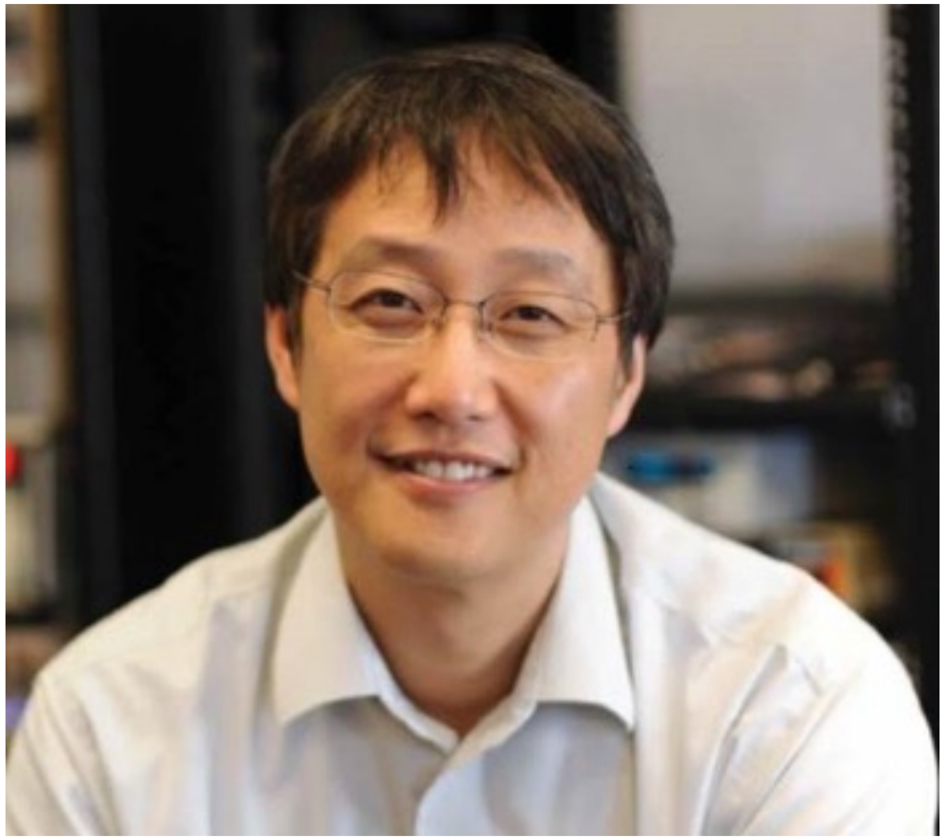
Jesse Crossno



Kin Chung Fong



Andrew Lucas



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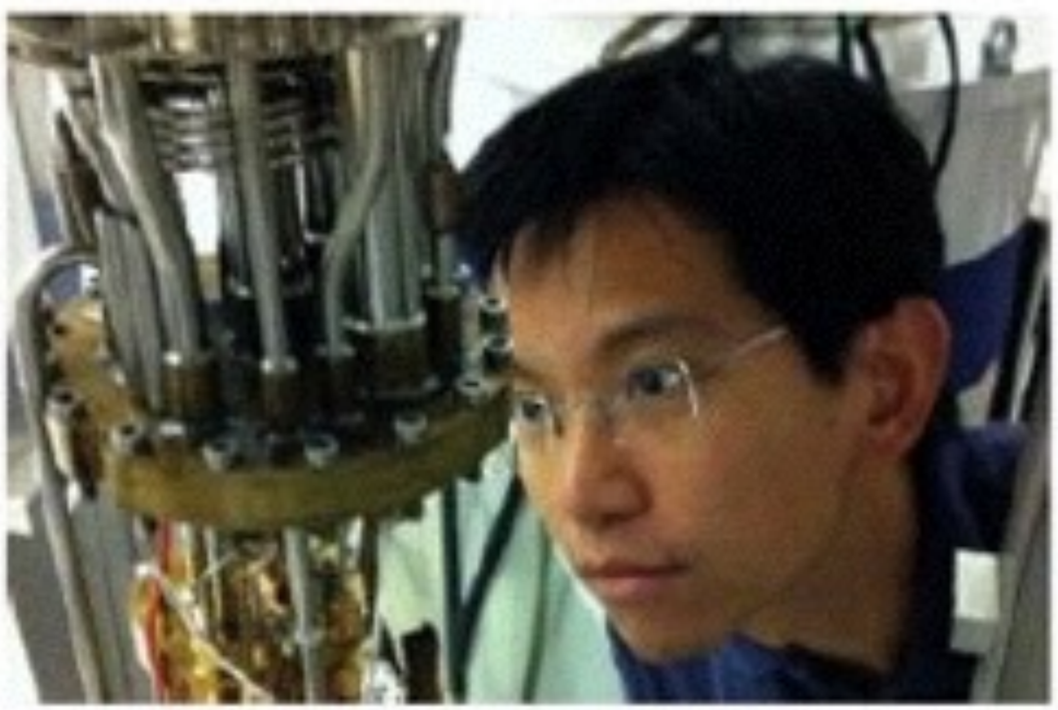

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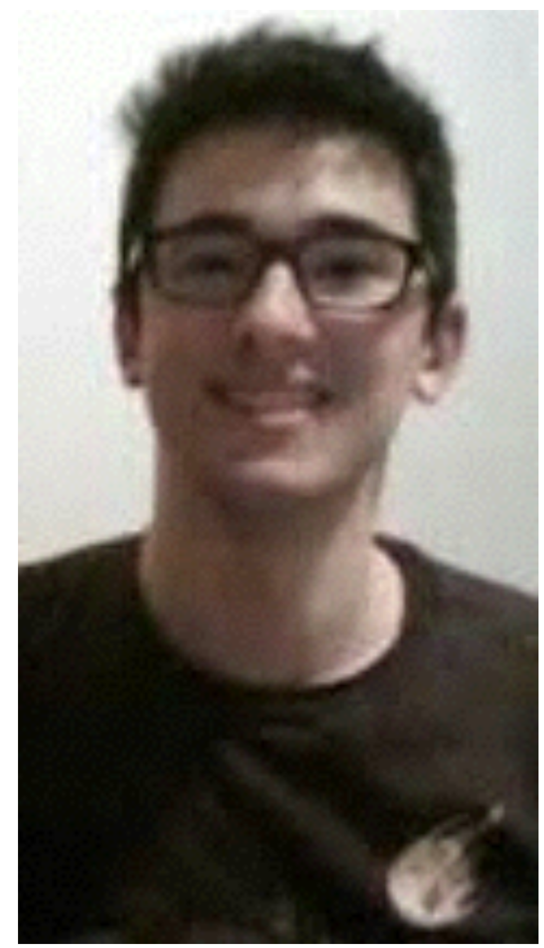
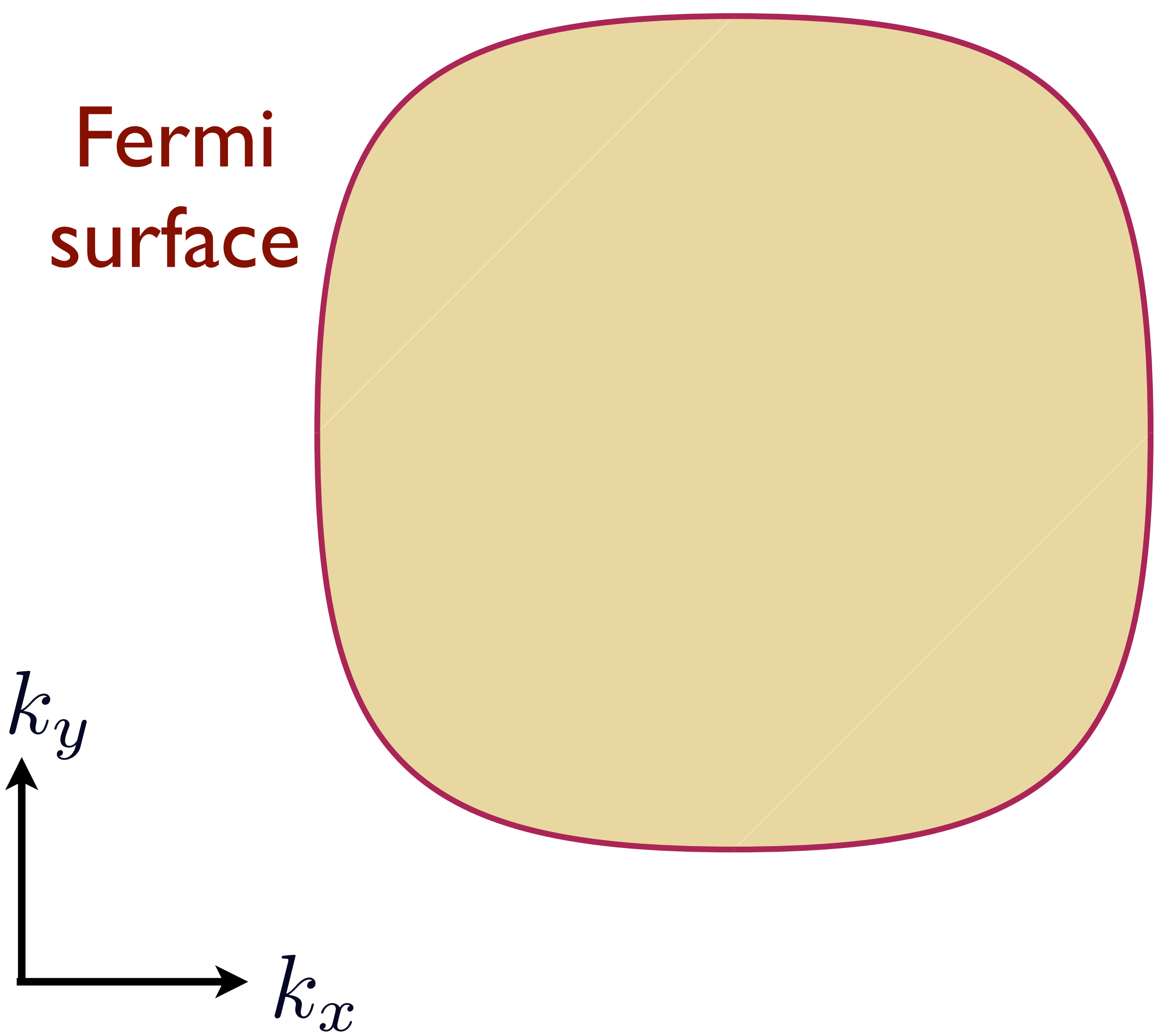


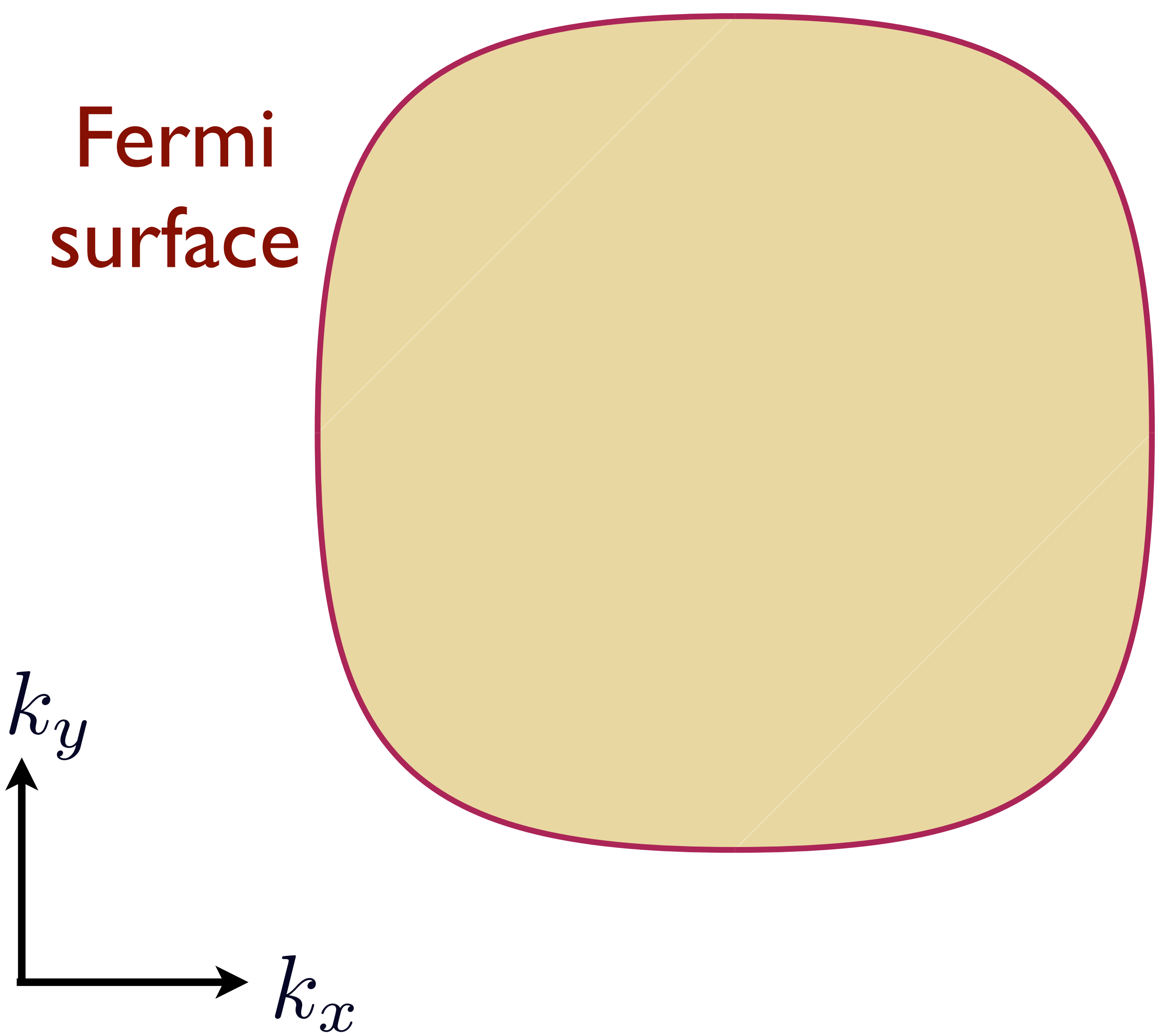
Andrew Lucas

Ordinary metals: the Fermi liquid



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = $\mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

Ordinary metals: the Fermi liquid



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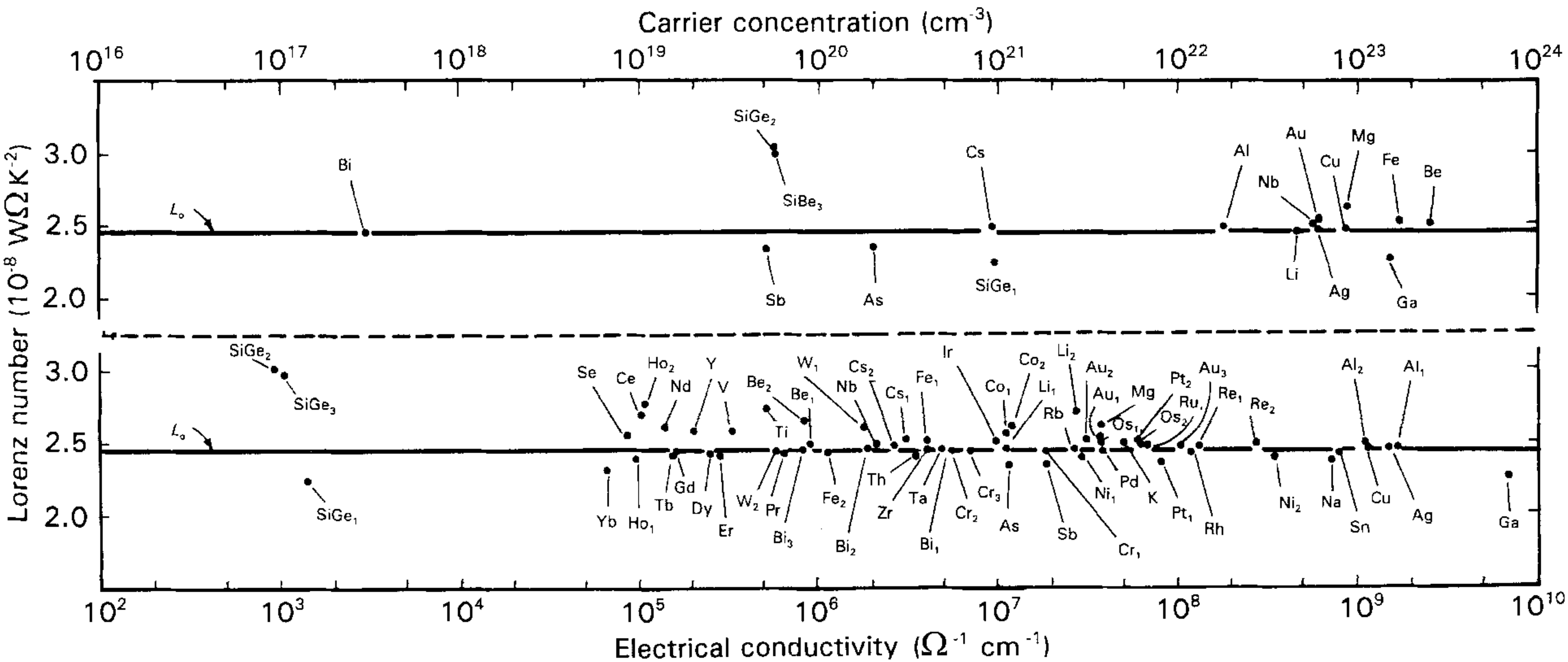
- Area enclosed by Fermi surface = $\mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

- $$\frac{(\text{Thermal conductivity})}{T\,(\text{Electrical conductivity})} = \frac{\pi^2 k_B^2}{3e^2}$$

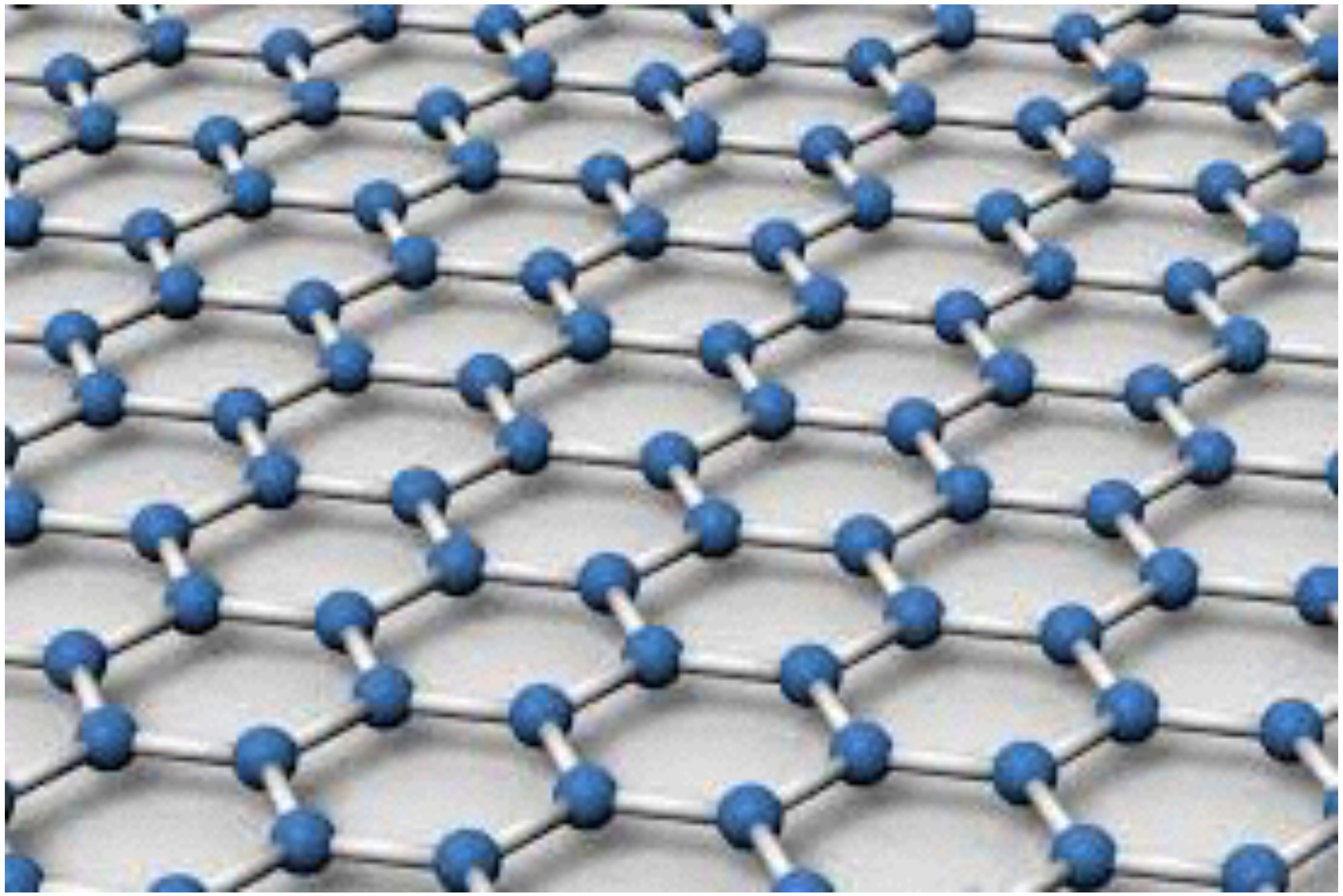
- Wiedemann-Franz law in a Fermi liquid:

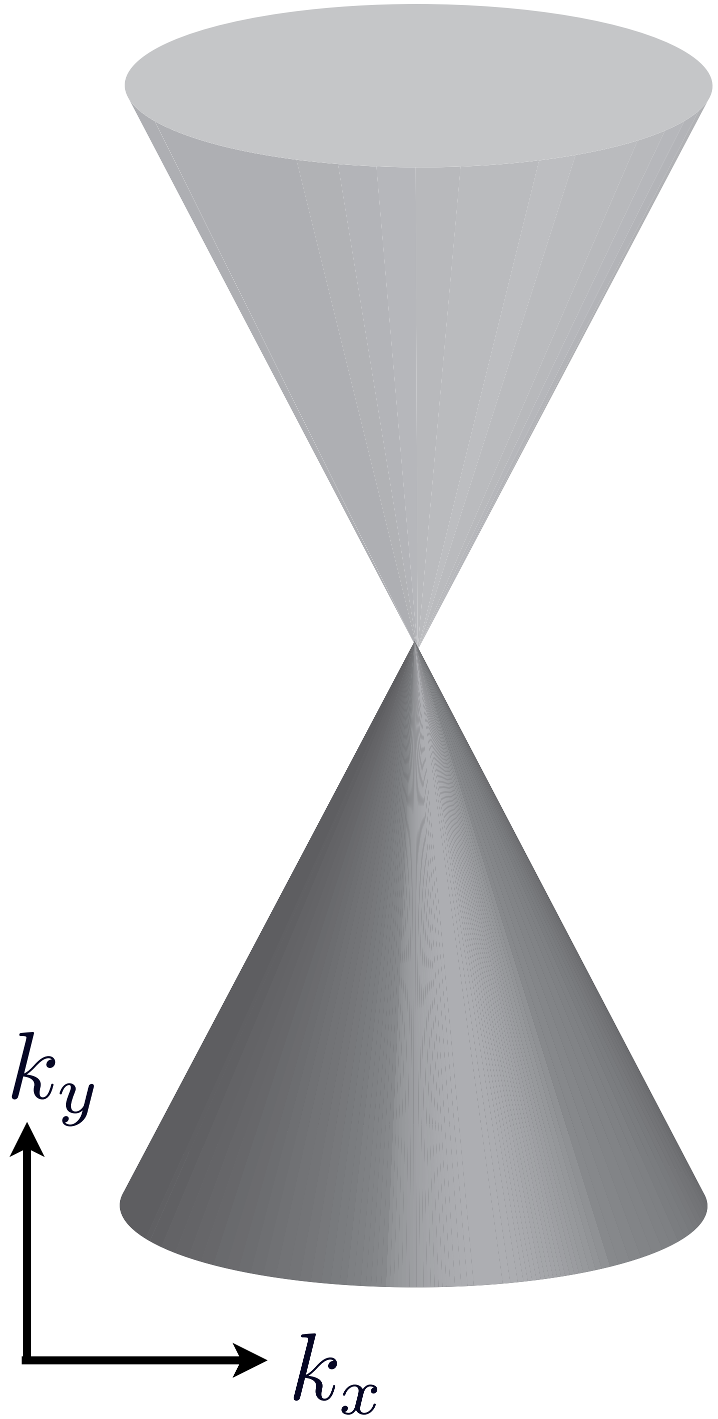
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



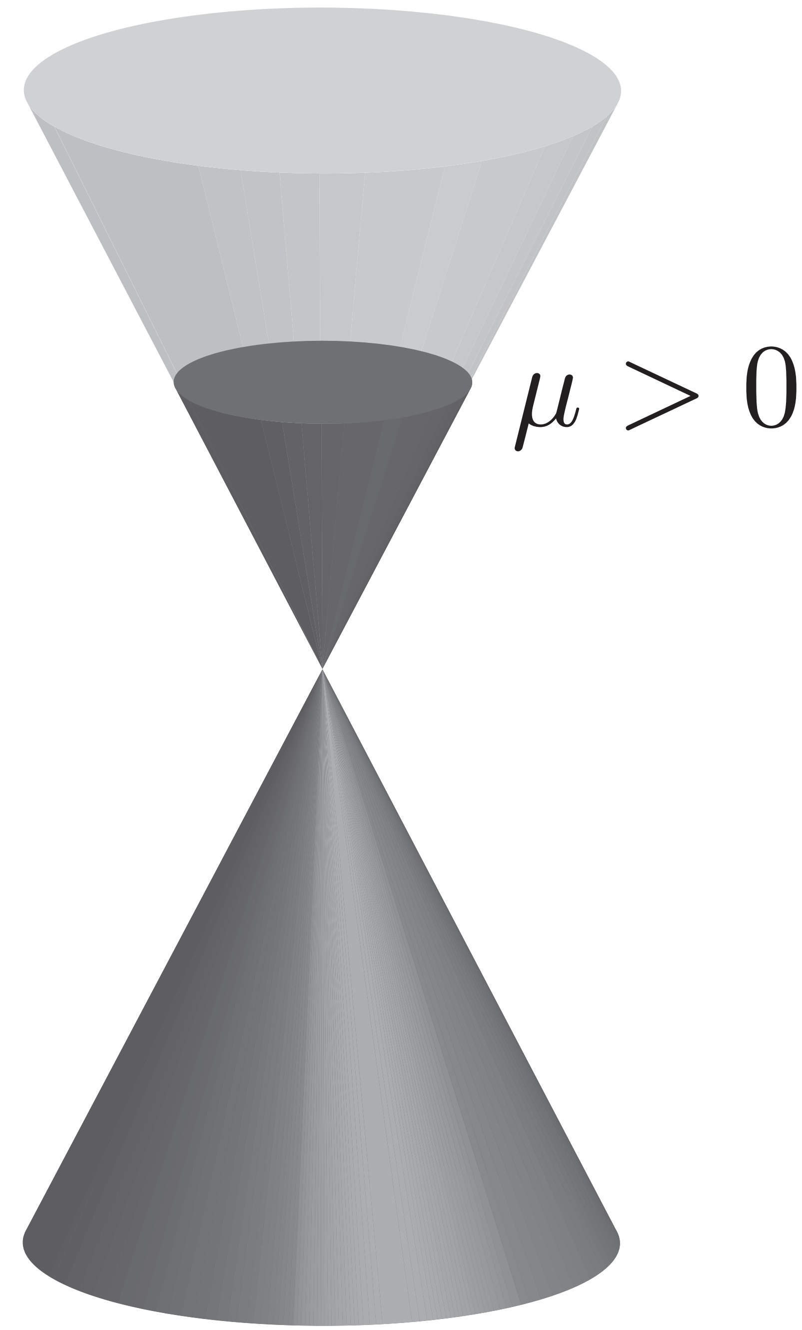
G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Graphene





Graphene



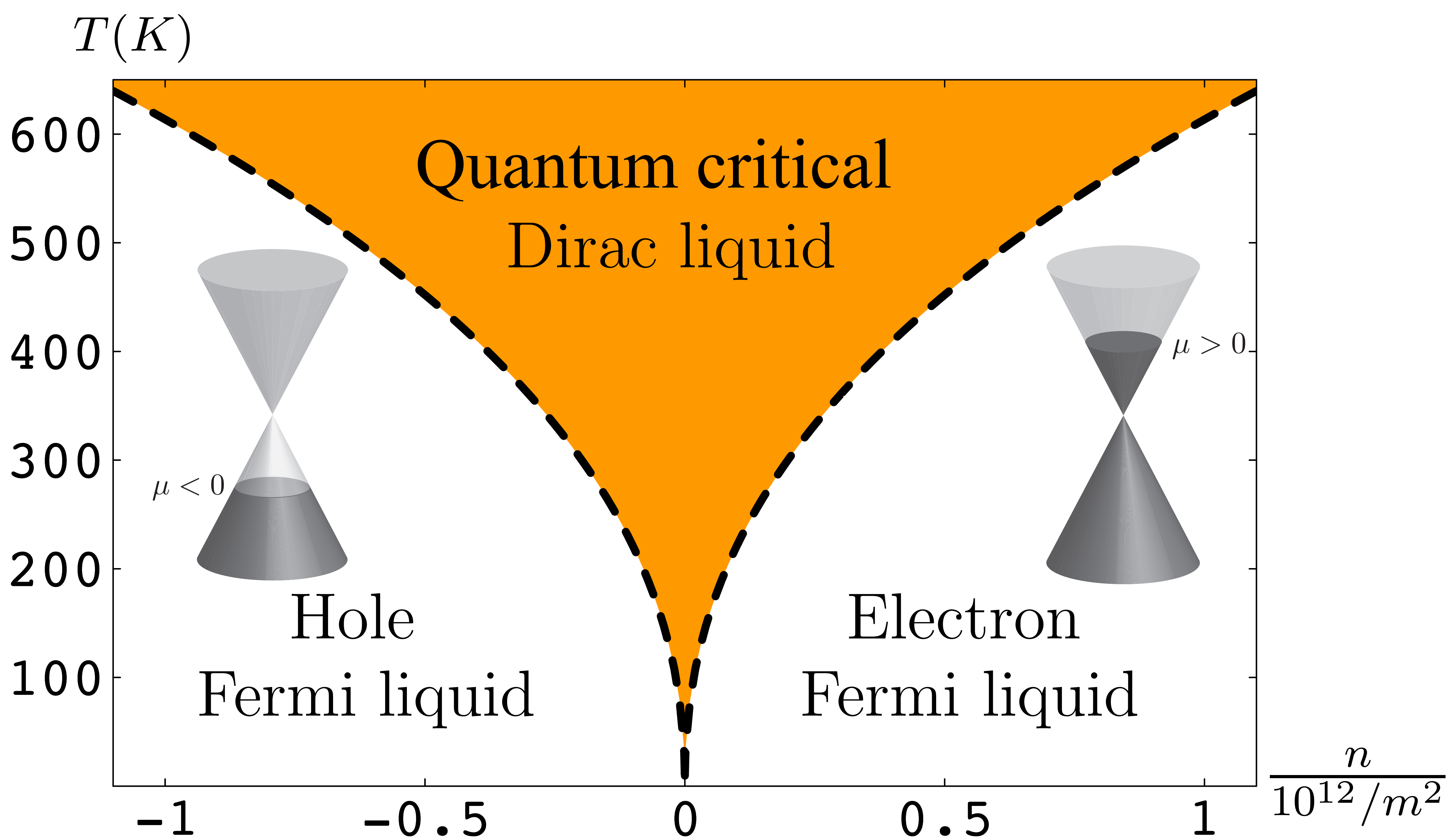
Electron
Fermi surface

Graphene

Hole
Fermi surface

Electron
Fermi surface

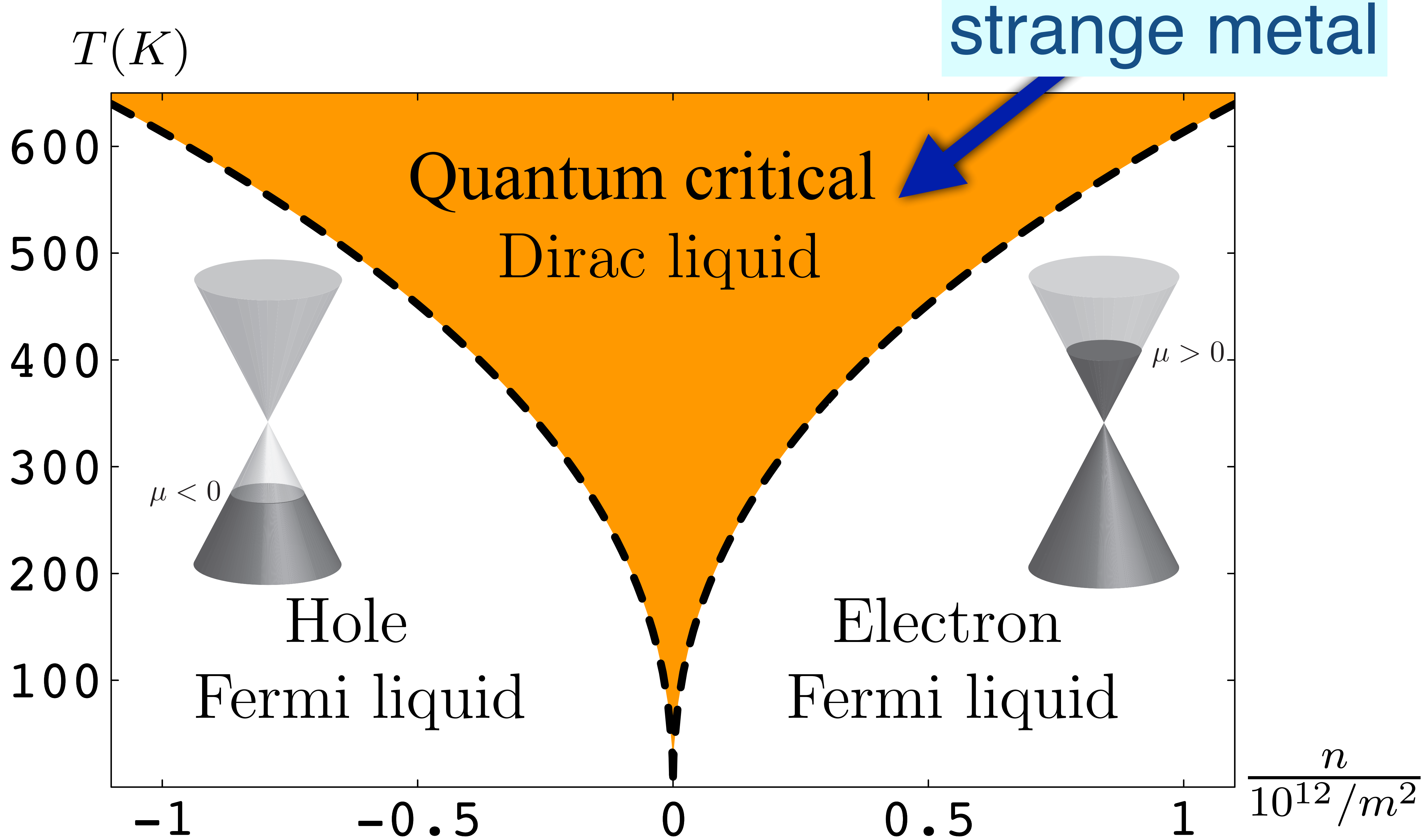
Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Predicted strange metal



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
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Key properties of a strange metal

- No quasiparticle excitations

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- Shortest possible “collision time”, or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

S. Sachdev, Quantum Phase Transitions (1999)

Key properties of a strange metal

- No quasiparticle excitations

- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

- Continuously variable density, $\mathcal{Q}$ (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H}\tau}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H}\tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\mathcal{Q} \to 0$ and $\tau_{\rm imp} \to \infty$ do not commute.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]

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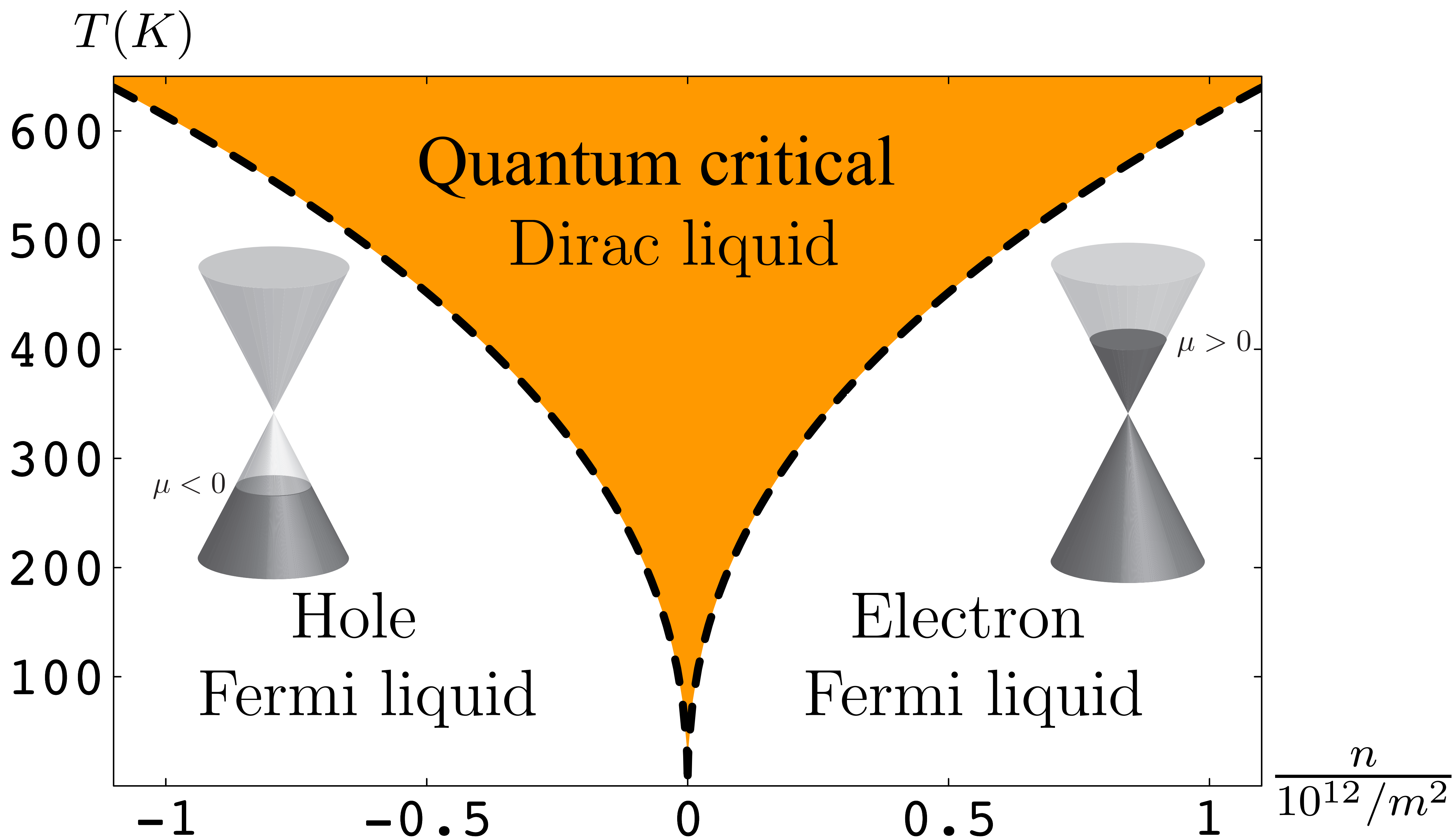
[5] *Raytheon BBN Technologies, Quantum Information Processing Group, Cambridge, Massachusetts 02138, USA*

(Dated: September 28, 2015)

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, due to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report the breakdown of the Wiedemann-Franz law in graphene, with a thermal conductivity an order of magnitude larger than the value predicted by Fermi liquid theory. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.

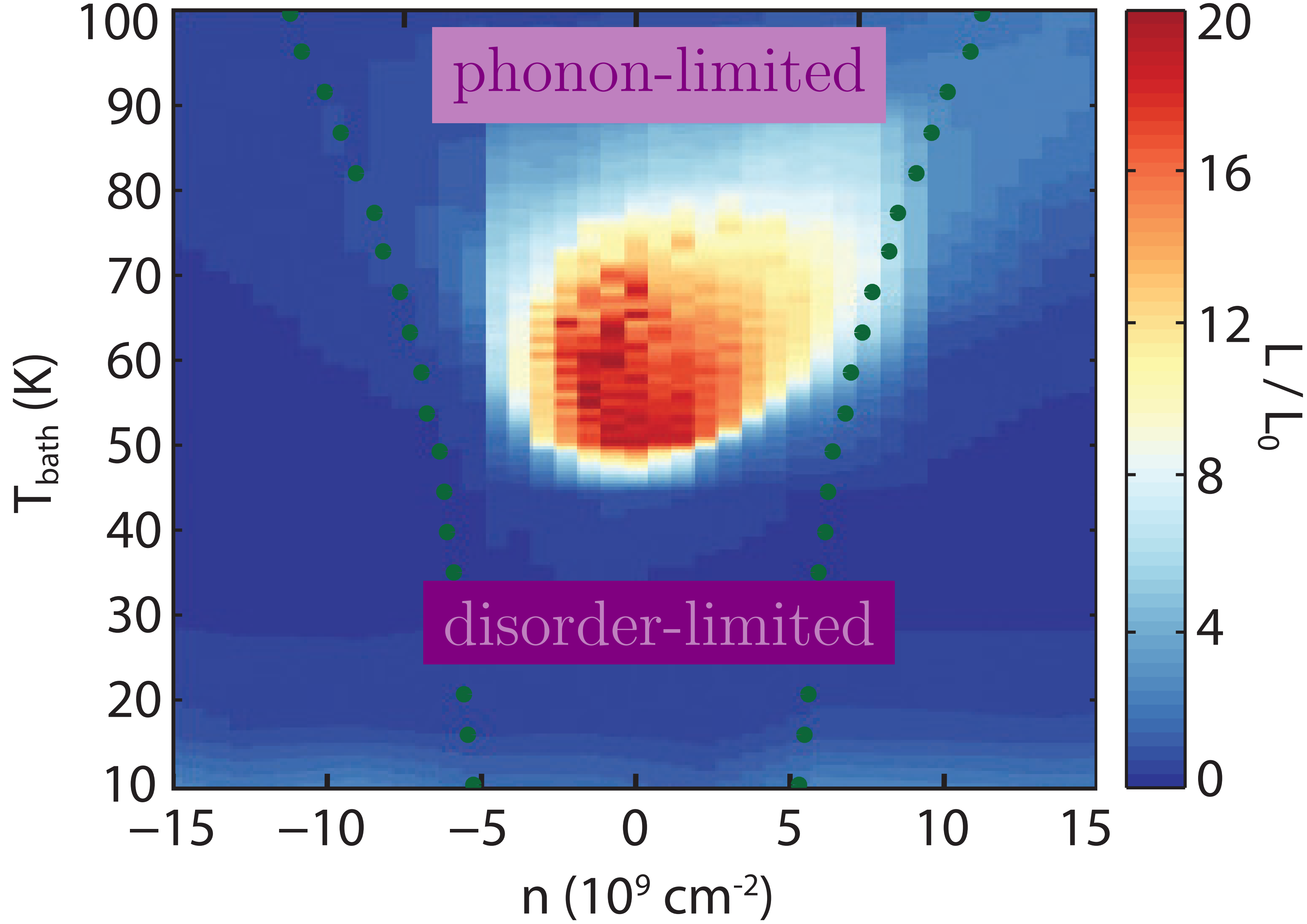
arXiv:1509.04713

Graphene



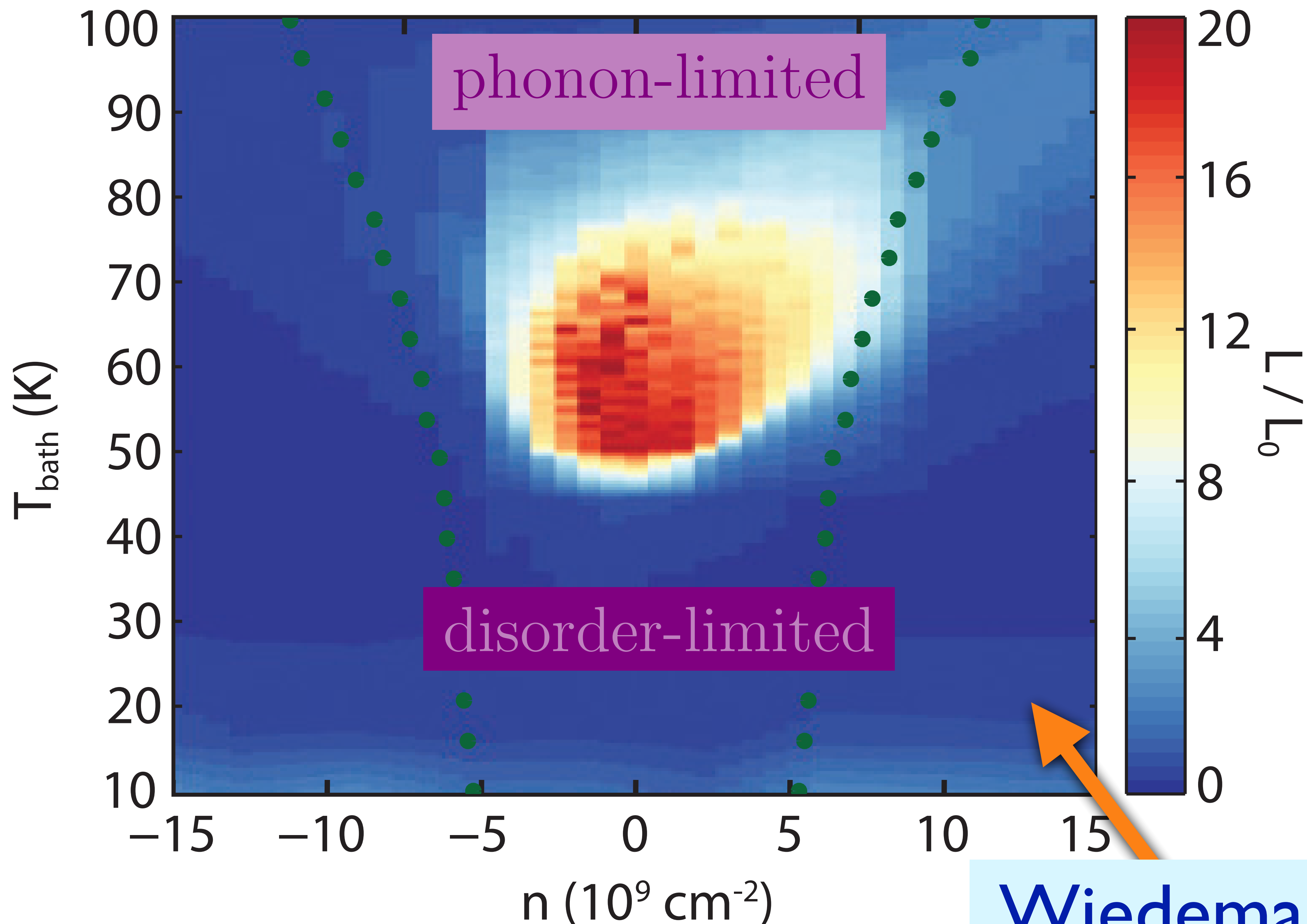
D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Strange metal in graphene



J. Crossno *et al.* arXiv:1509.04713

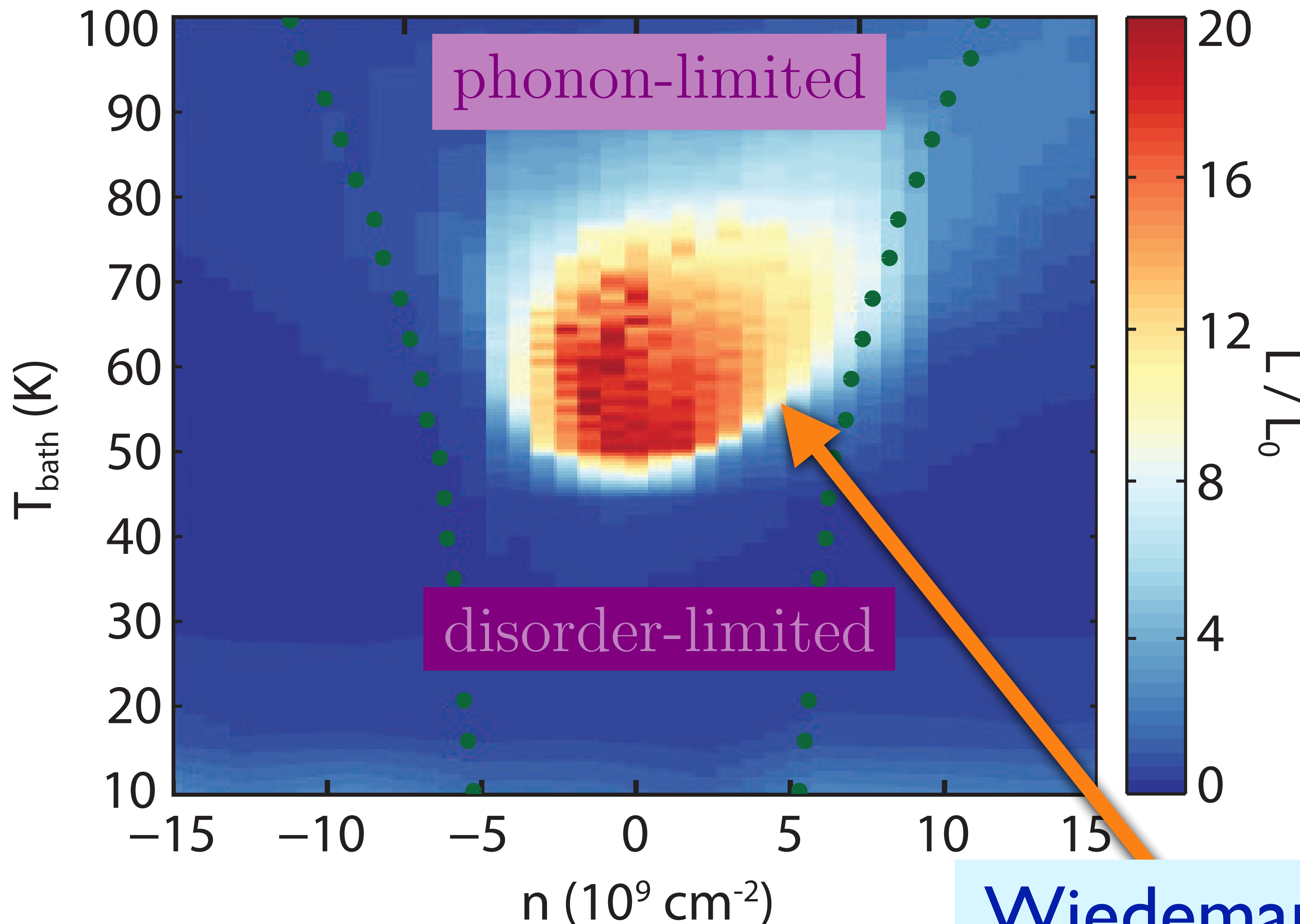
Strange metal in graphene



Wiedemann-Franz obeyed

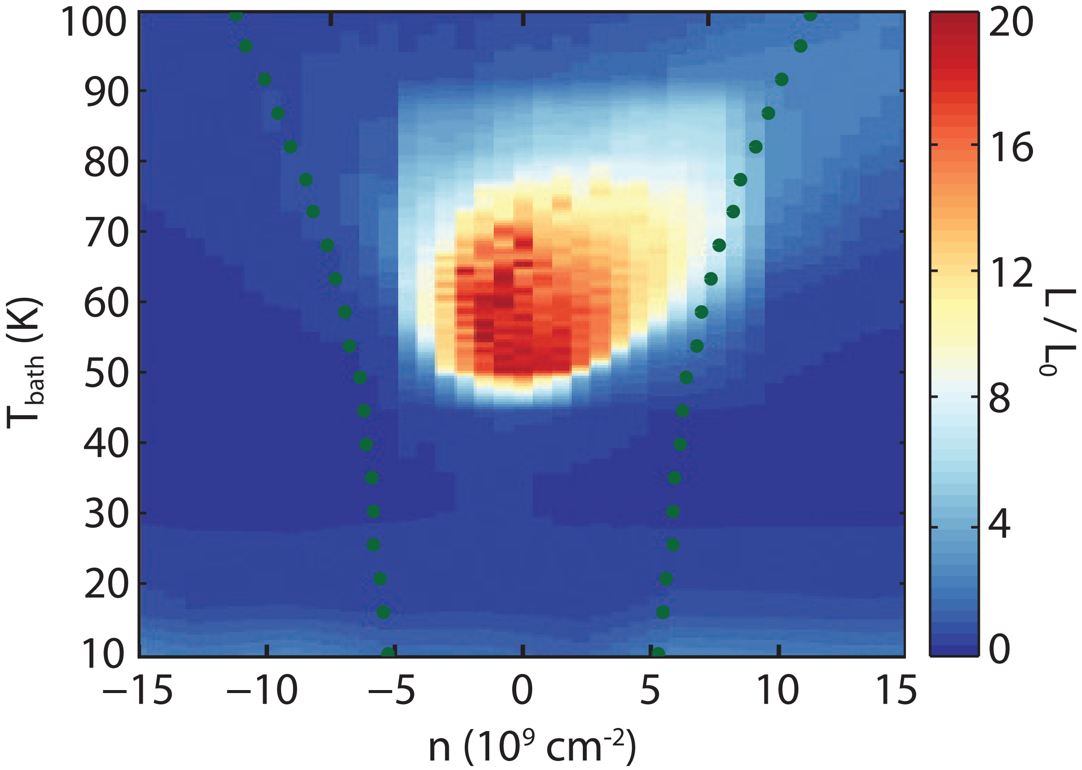
J. Crossno *et al.* arXiv:1509.04713

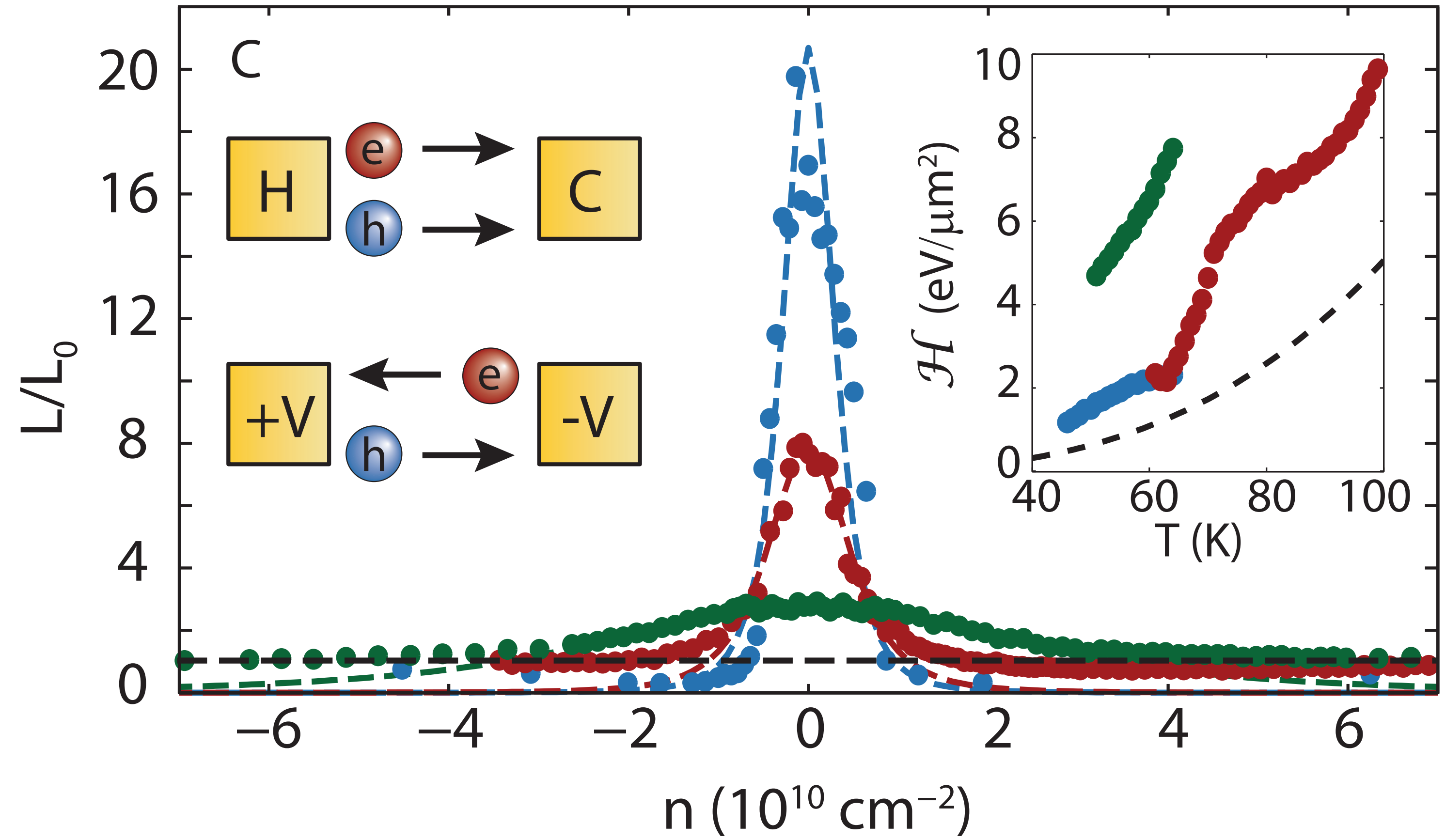
Strange metal in graphene

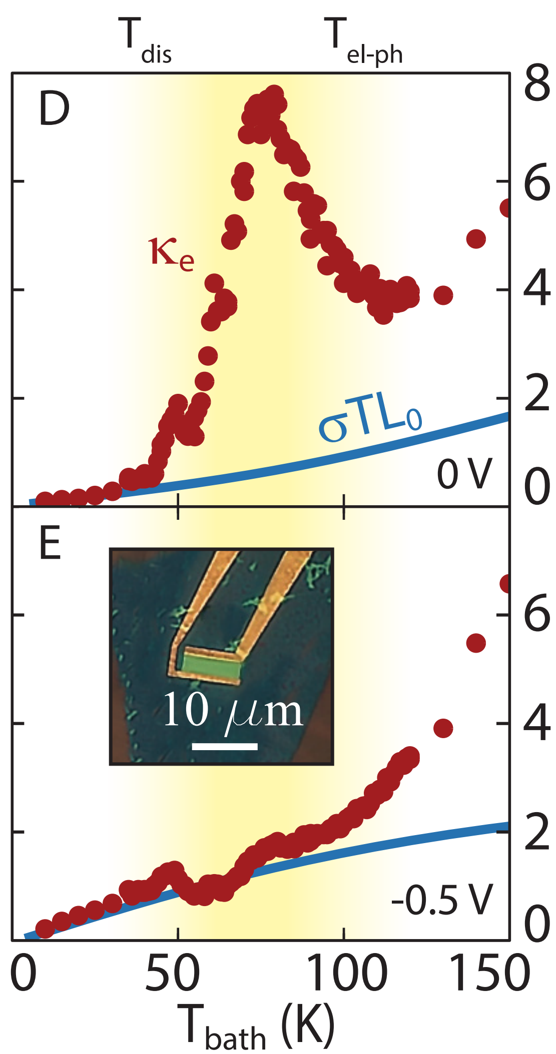


J. Crossno *et al.* arXiv:1509.04713

Wiedemann-Franz violated !



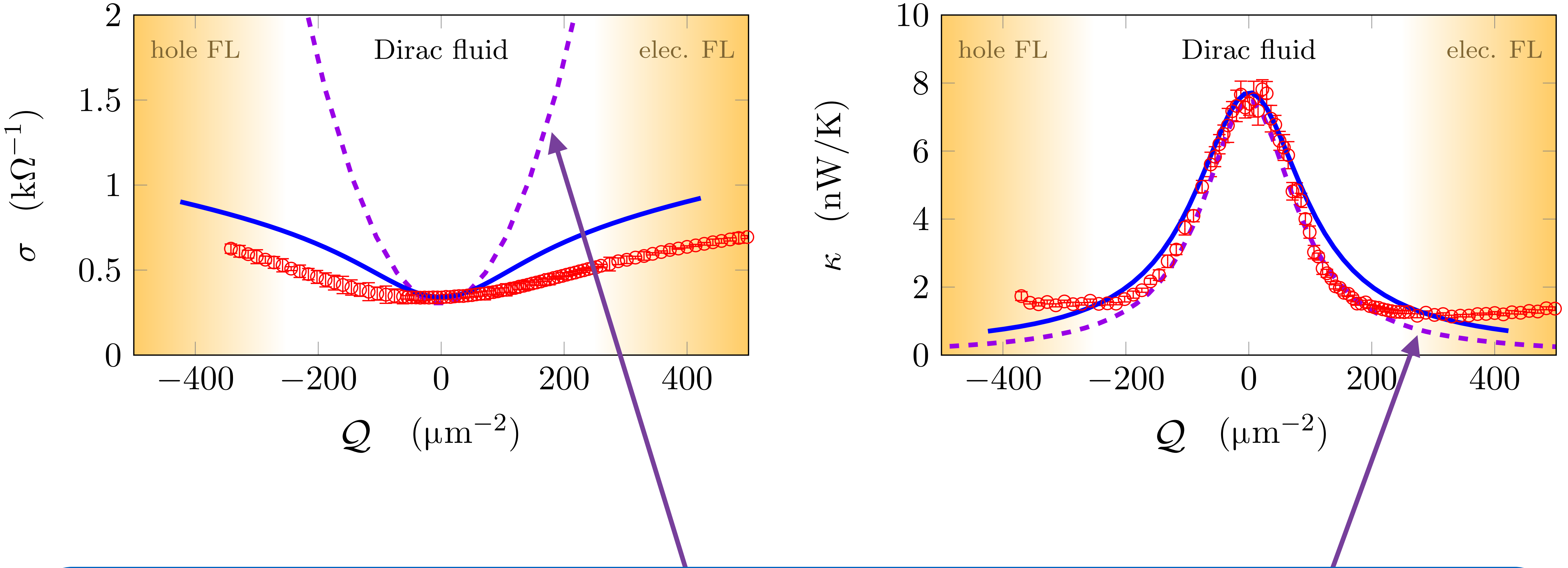




$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$

$$= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}/(\mathcal{H}\sigma_Q)\right)^2}$$

J. Crossno *et al.* arXiv:1509.04713



Comparison to theory with a single momentum relaxation time τ_{imp}. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

Non-perturbative treatment of disorder

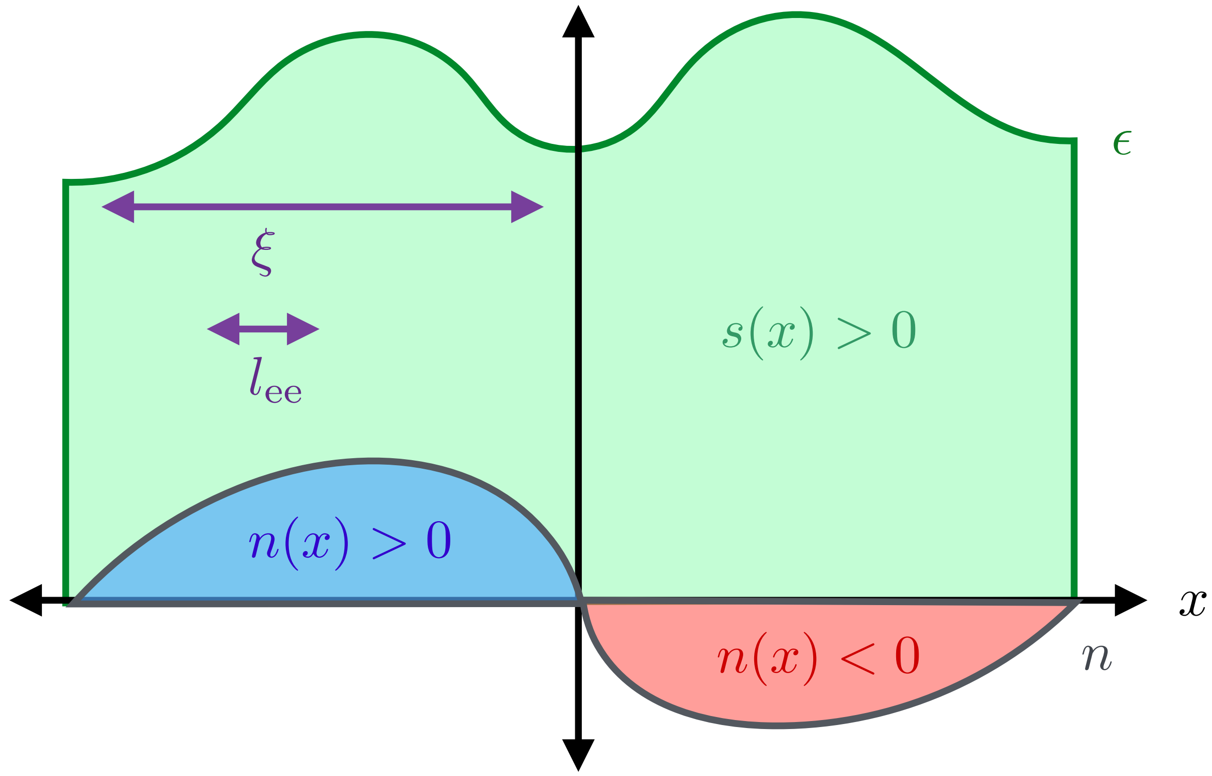
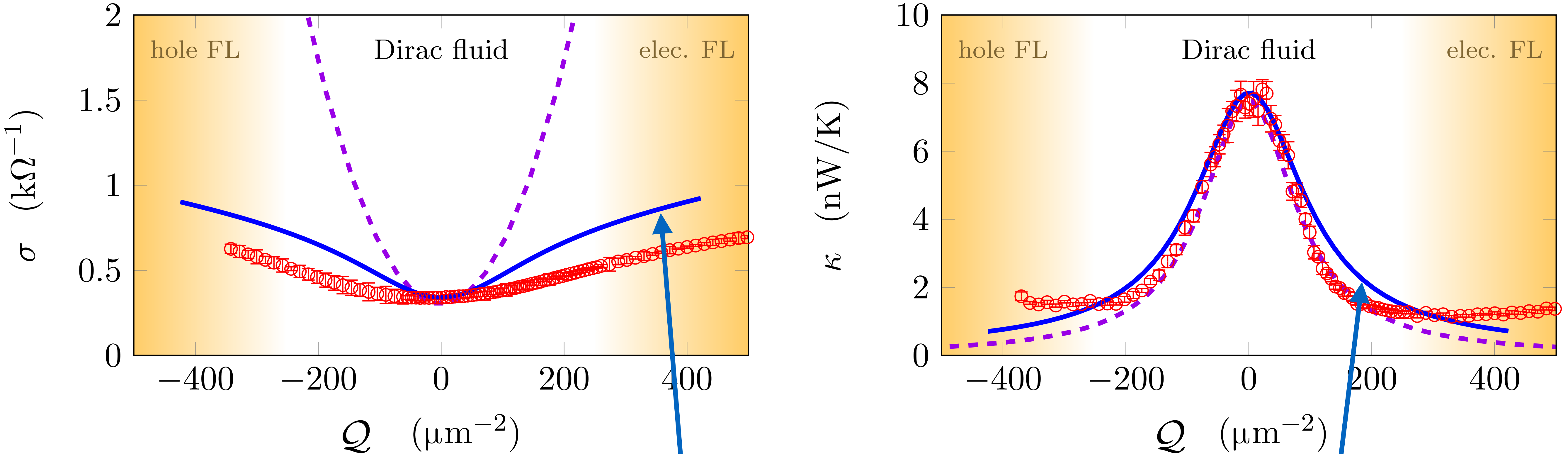


Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\rm B}T$, and so the entropy density $s/k_{\rm B}$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than $l_{\rm ee}$, the electron-electron interaction length.

Note
$n \equiv \mathcal{Q}$

Numerically solve the hydrodynamic equations of Hartnoll, Kovtun, Müller, Sachdev (PRB 76, 144502 (2007)) but in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738



Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

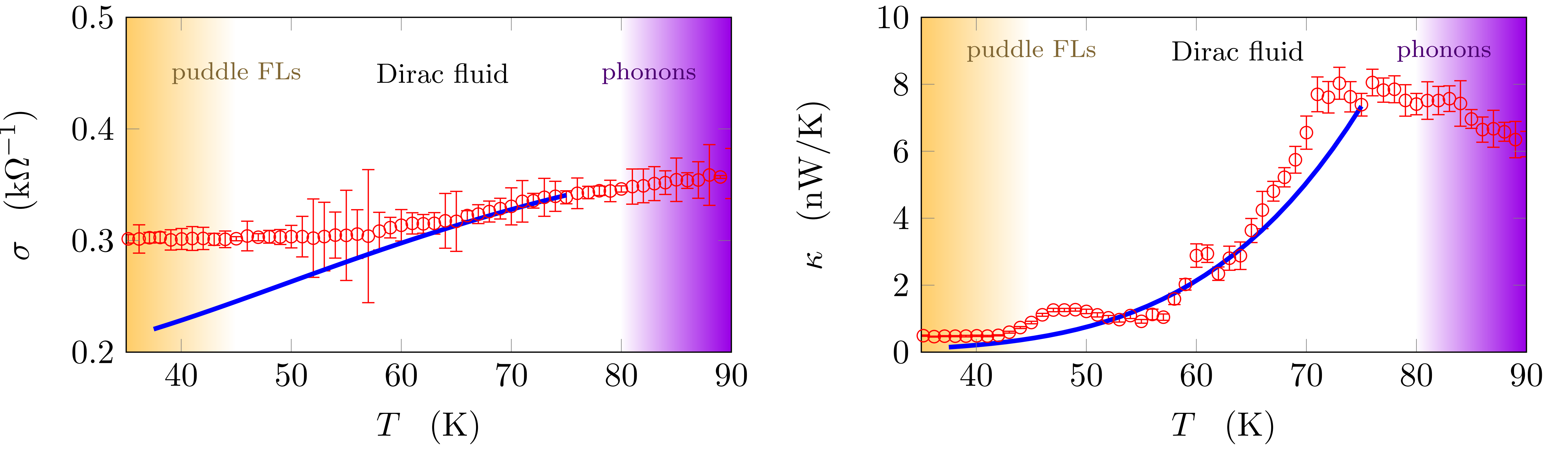


Figure 2: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at the charge neutrality point ($n = 0$). We use no new fit parameters co $\mathcal{Q}$ red to Figure 1. The yellow shaded region denotes $\mathcal{Q}$ re Fermi liquid behavior is observed; the purple shaded region denotes the likely onset of electron-phonon coupling.

Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
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A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

Strange metal in graphene

arXiv:1509.04165

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]

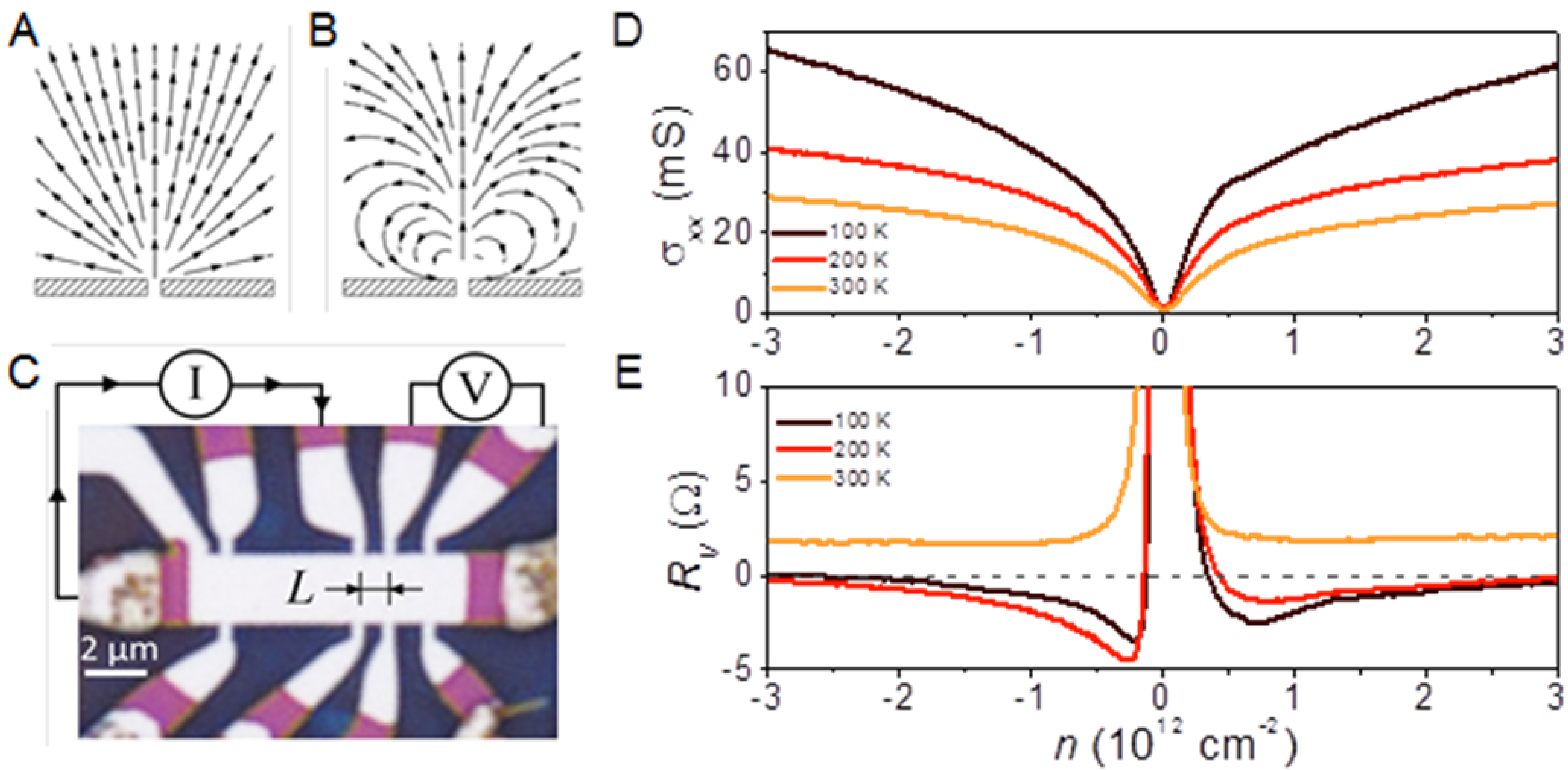

Figure 1. Viscous backflow in doped graphene. (**a,b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d,e**) Longitudinal conductivity σ_{xx} and $R_{\rm V}$ for this device as a function of n induced by applying gate voltage. $I = 0.3$ μA; $L = 1$ μm. For more detail, see Supplementary Information.

Quantum matter without quasiparticles

1. Experiment and theory in graphene

2. A solvable model of a strange metal

3. Holography and charged black holes

4. Transport in strange metals

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Infinite-range model of a Fermi liquid

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

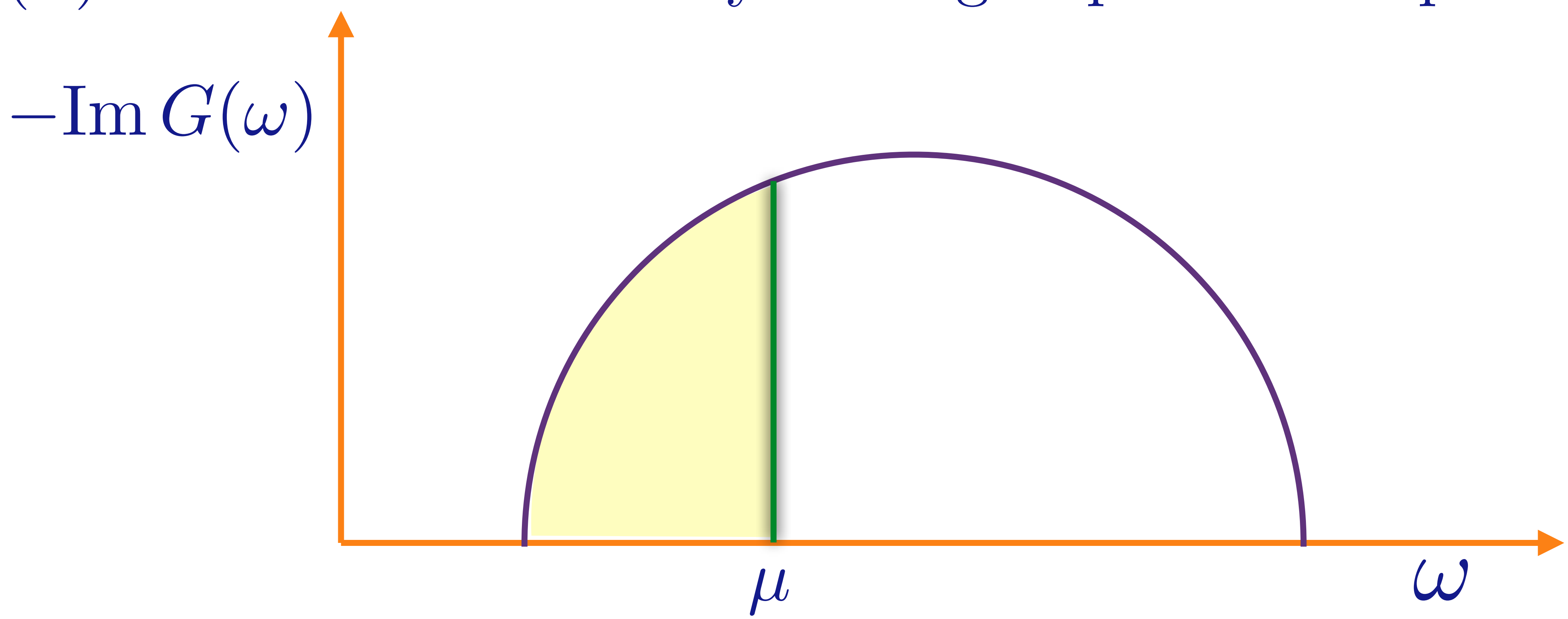
Infinite-range model of a Fermi liquid

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Fermions occupying eigenstates with a "semi-circular" density of states

Infinite-range model of a strange metal

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c^\dagger_{i\alpha} c_{i\beta} c^\dagger_{j\beta} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c^\dagger_{j\beta} + c^\dagger_{j\beta} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c^\dagger_{i\alpha} c_{i\alpha} = \mathcal{Q}$$

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$
$N \to \infty$ at $M = 2$ yields spin-glass ground state.
$N \to \infty$ and then $M \to \infty$ yields critical strange metal

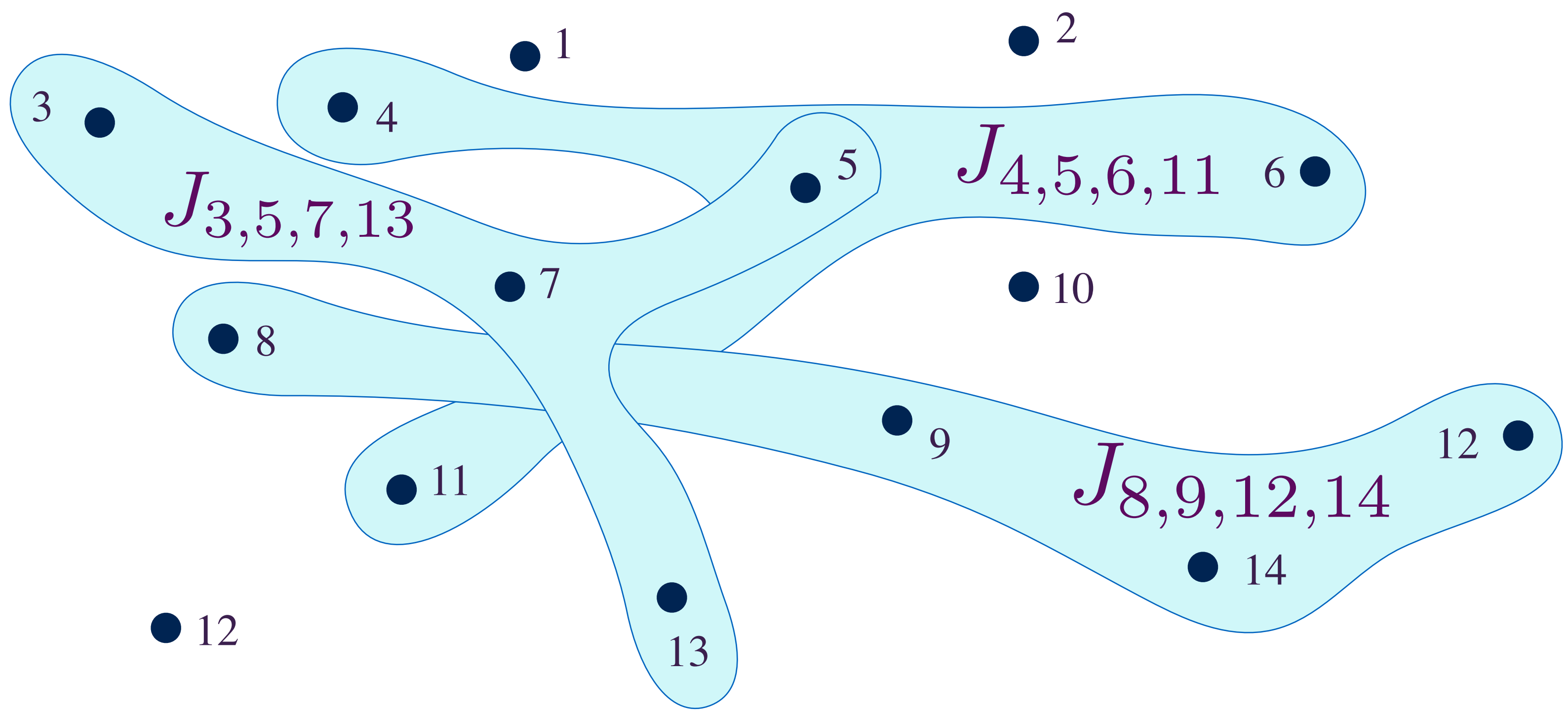
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields <u>same</u> critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, arXiv:1506.05111

Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\mathrm{Im}\, G(\omega) \sim \begin{cases} \omega^{-1/2} & , \; \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2} & , \; \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

While $\mathcal{E}$ determines the *low* energy spectrum, it is determined by the *total* fermion density $\mathcal{Q}$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

Analog of the relationship between $\mathcal{Q}$ and k_F in a Fermi liquid.

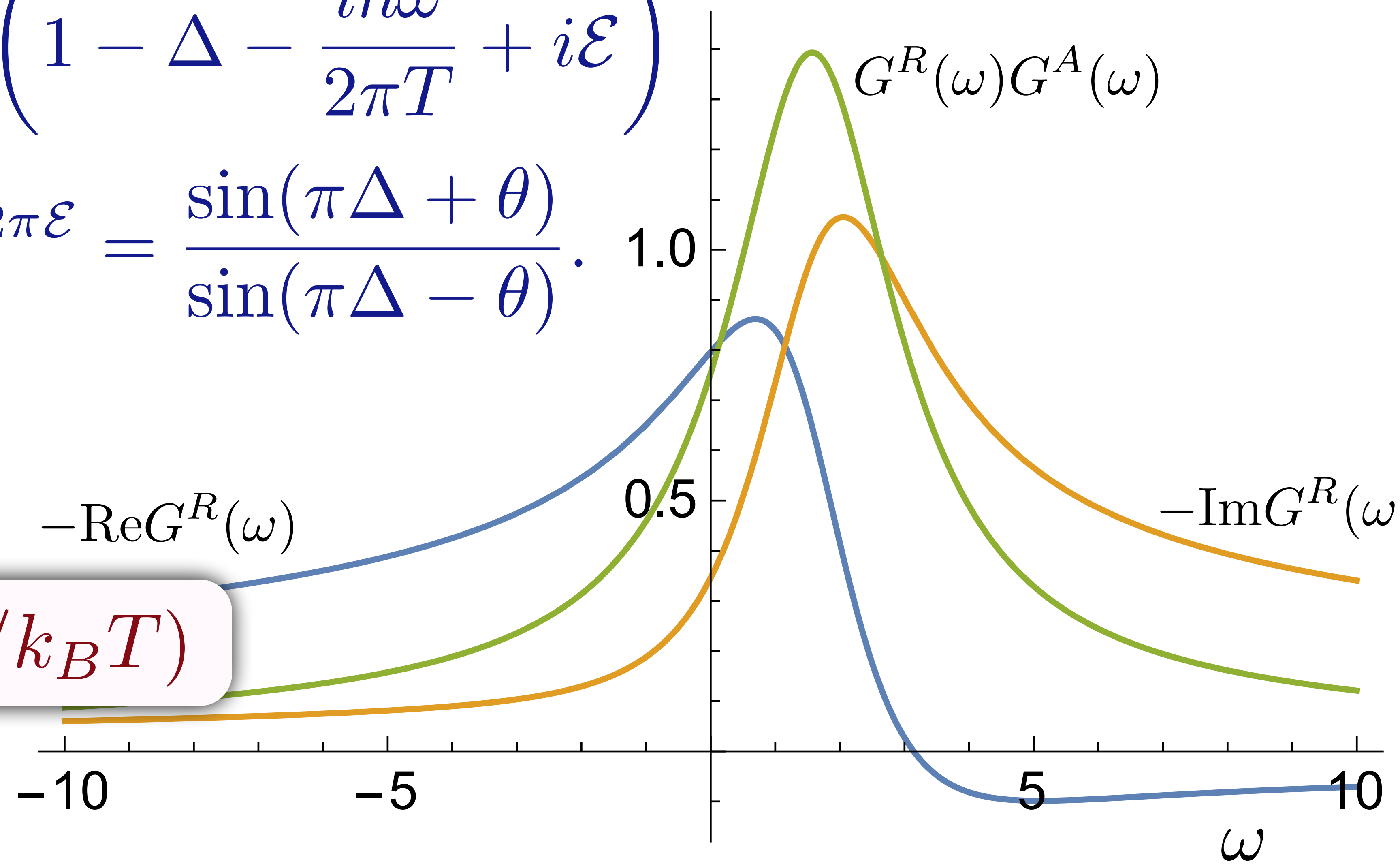
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

At non-zero temperature, T, the Green's function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

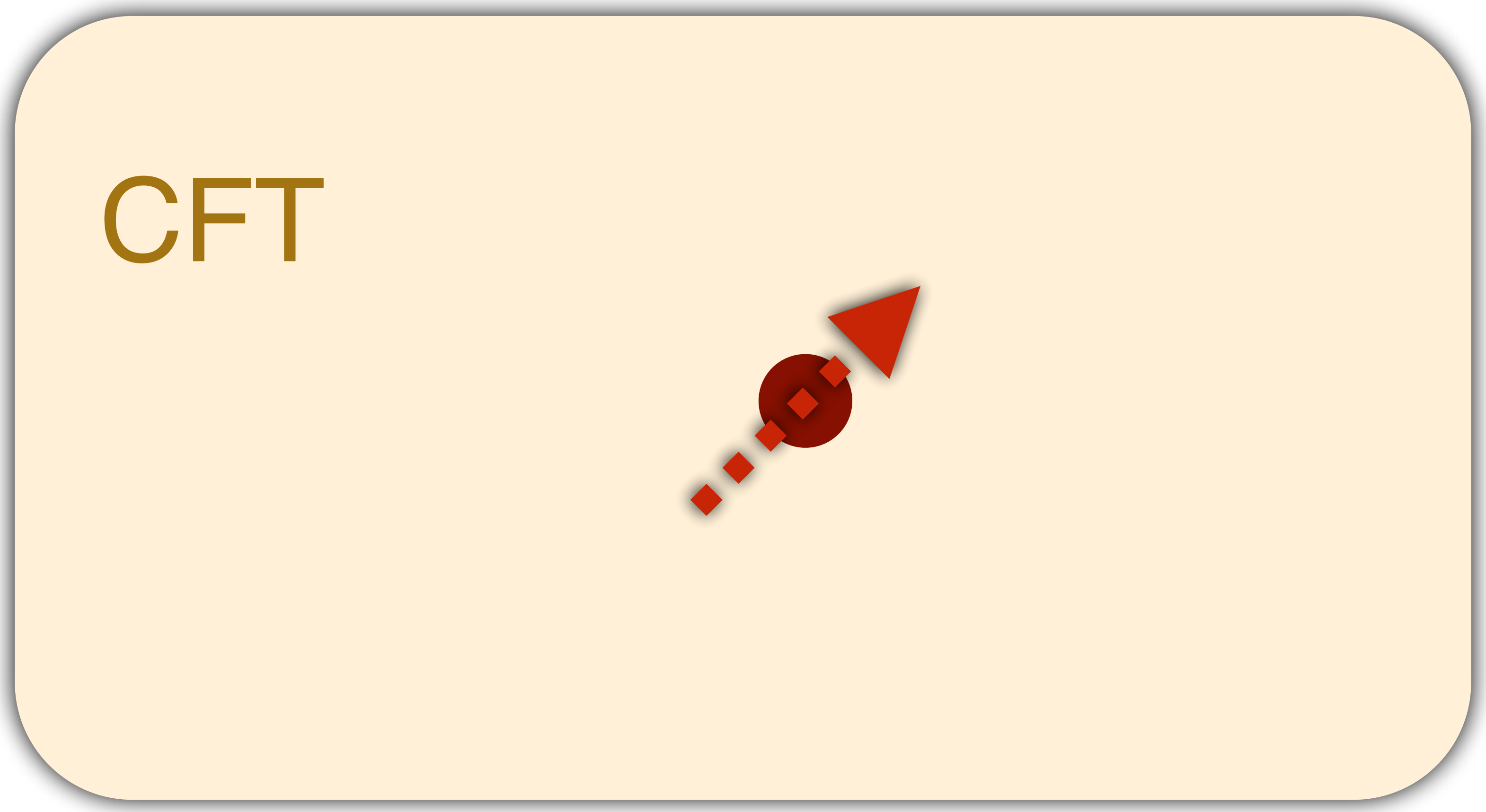


Note $G(\omega) \equiv f(\hbar\omega/k_B T)$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).
S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

"Critically-screened" spin has "irrational" entropy



Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).
S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

This entropy obeys

$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

Note that $\mathcal{S}$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

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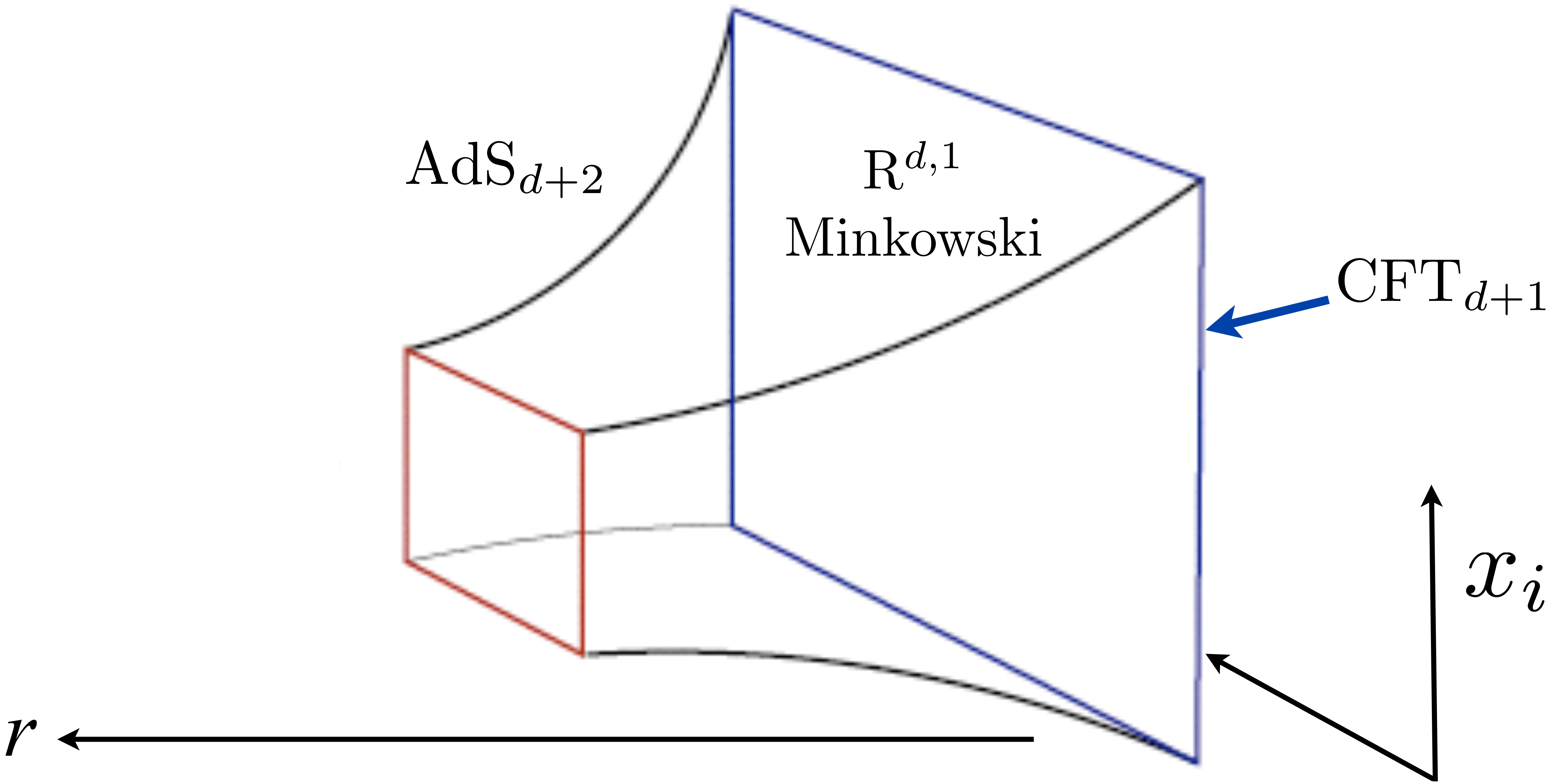
4. Transport in strange metals

AdS/CFT correspondence at zero temperature

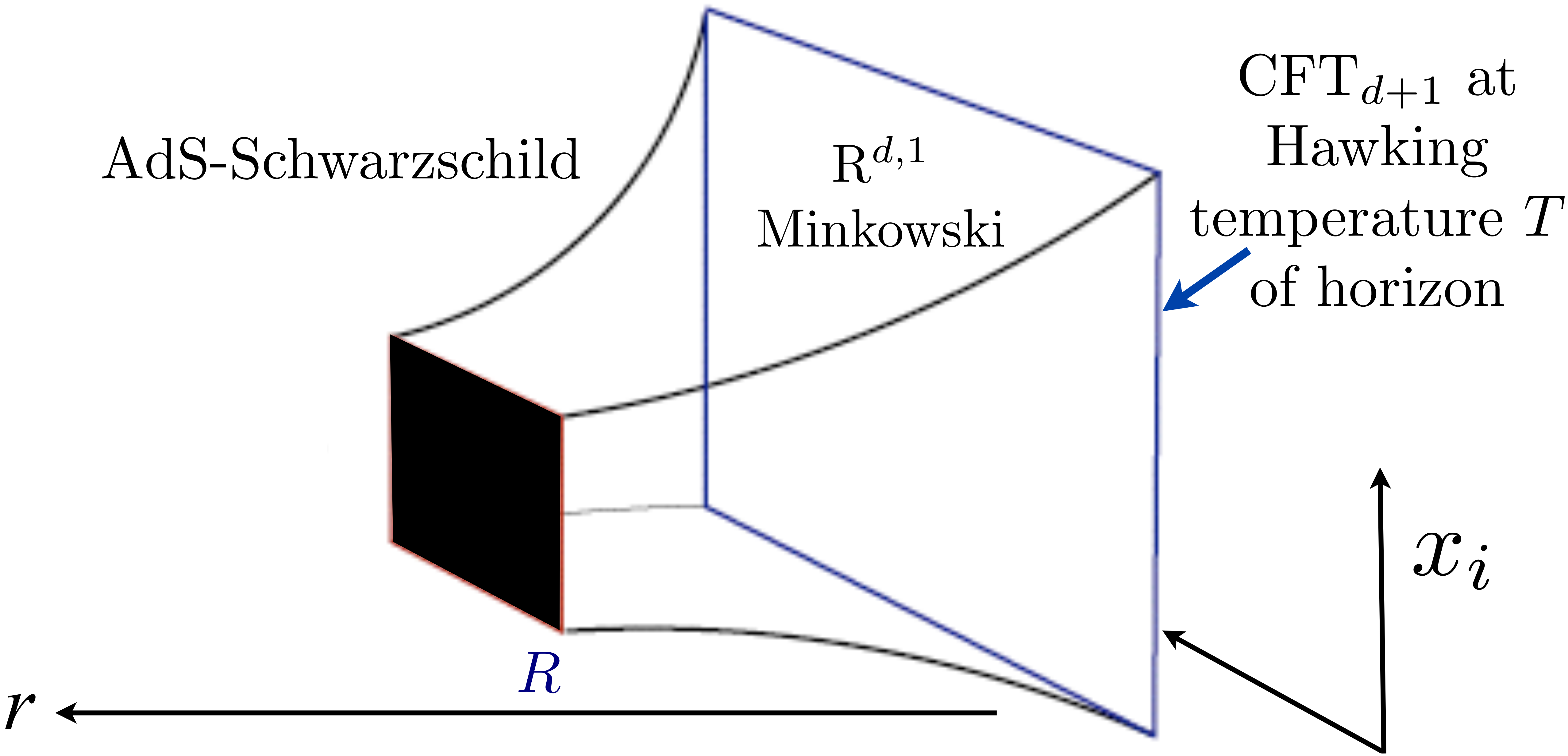
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$

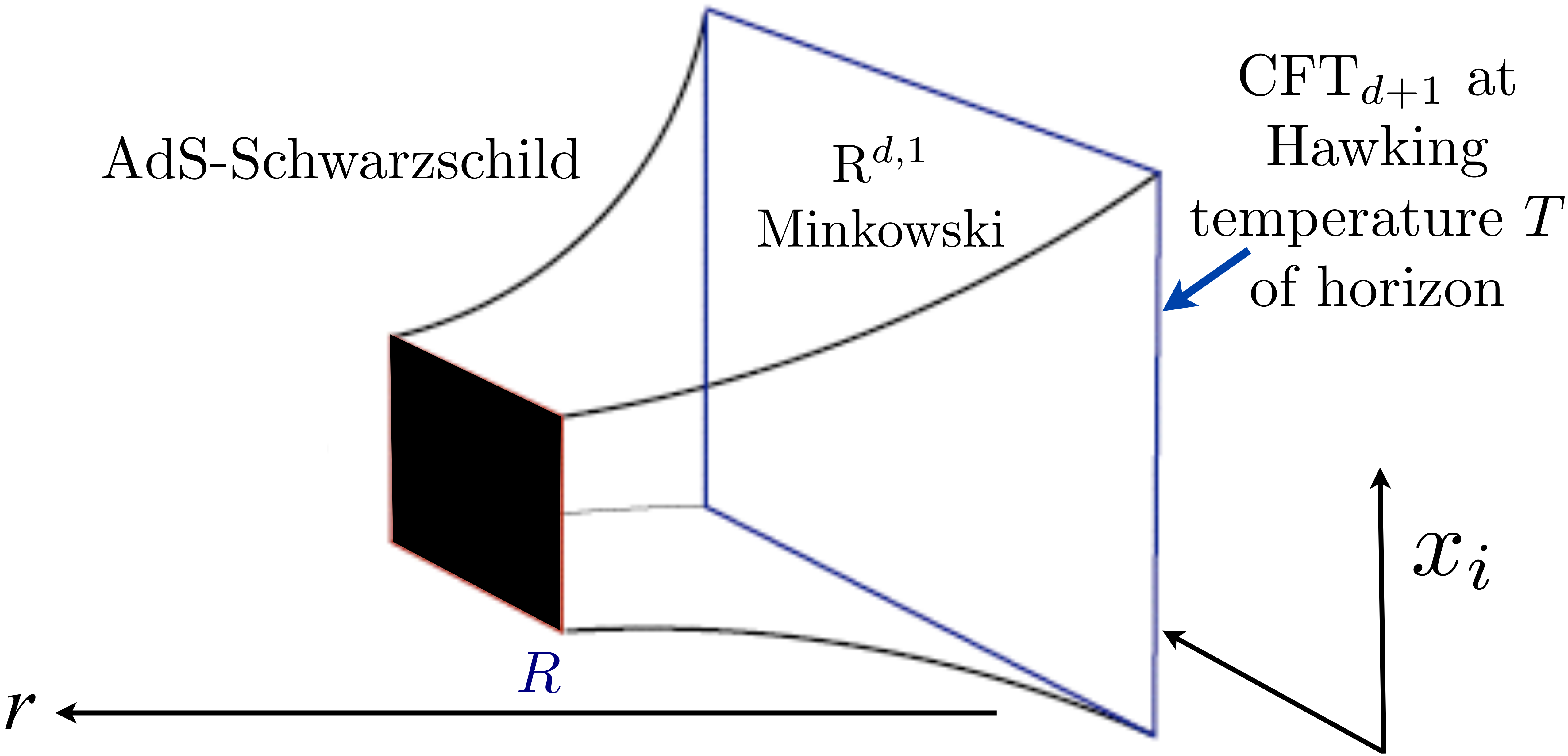


Entropy density of CFT_{d+1}, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\mathrm{BH}} \sim T^d$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$

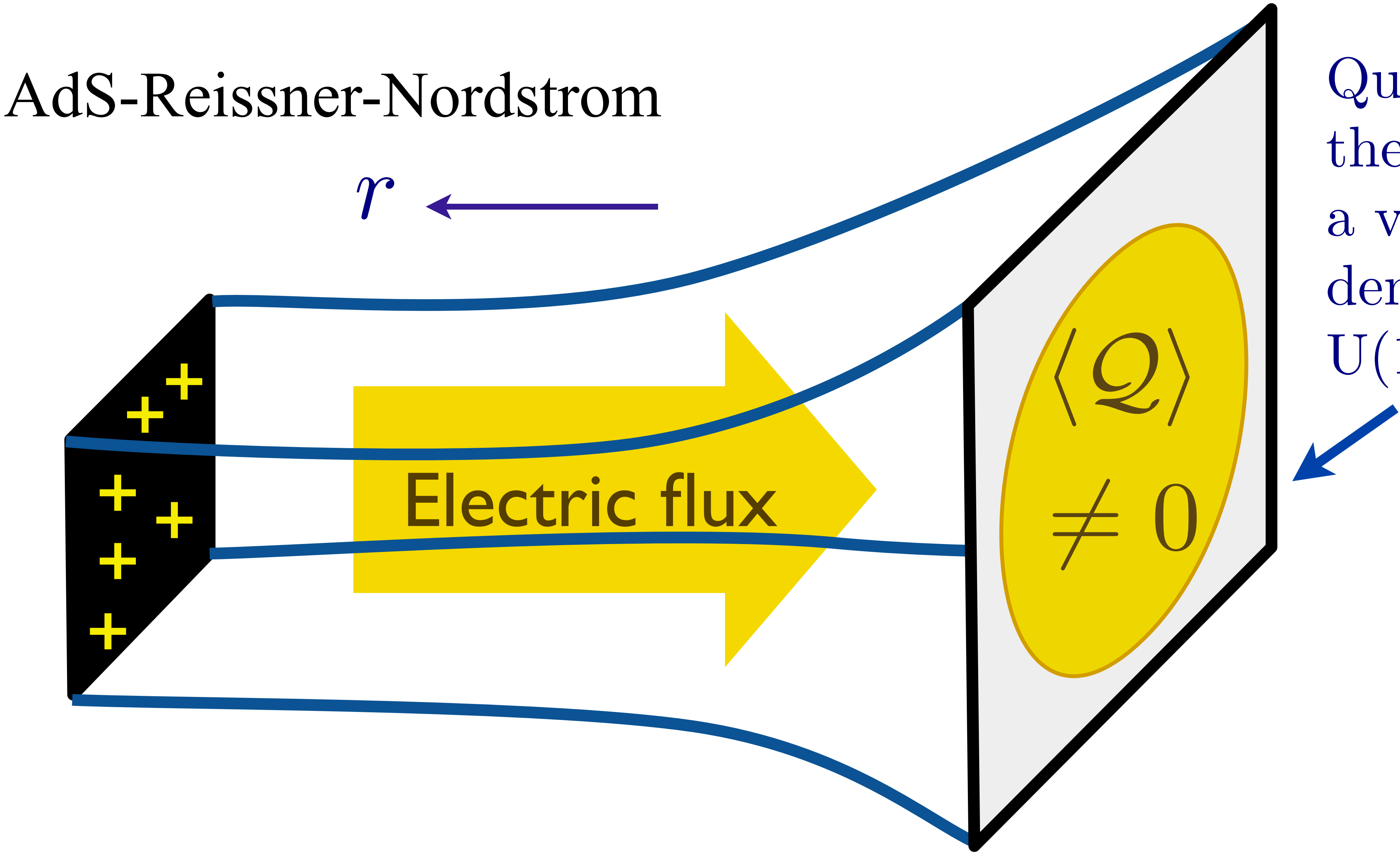


For SU(N) SYM in $d=3$, $\mathcal{S}_{\rm BH} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

Gubser, Klebanov, Peet 96

Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$

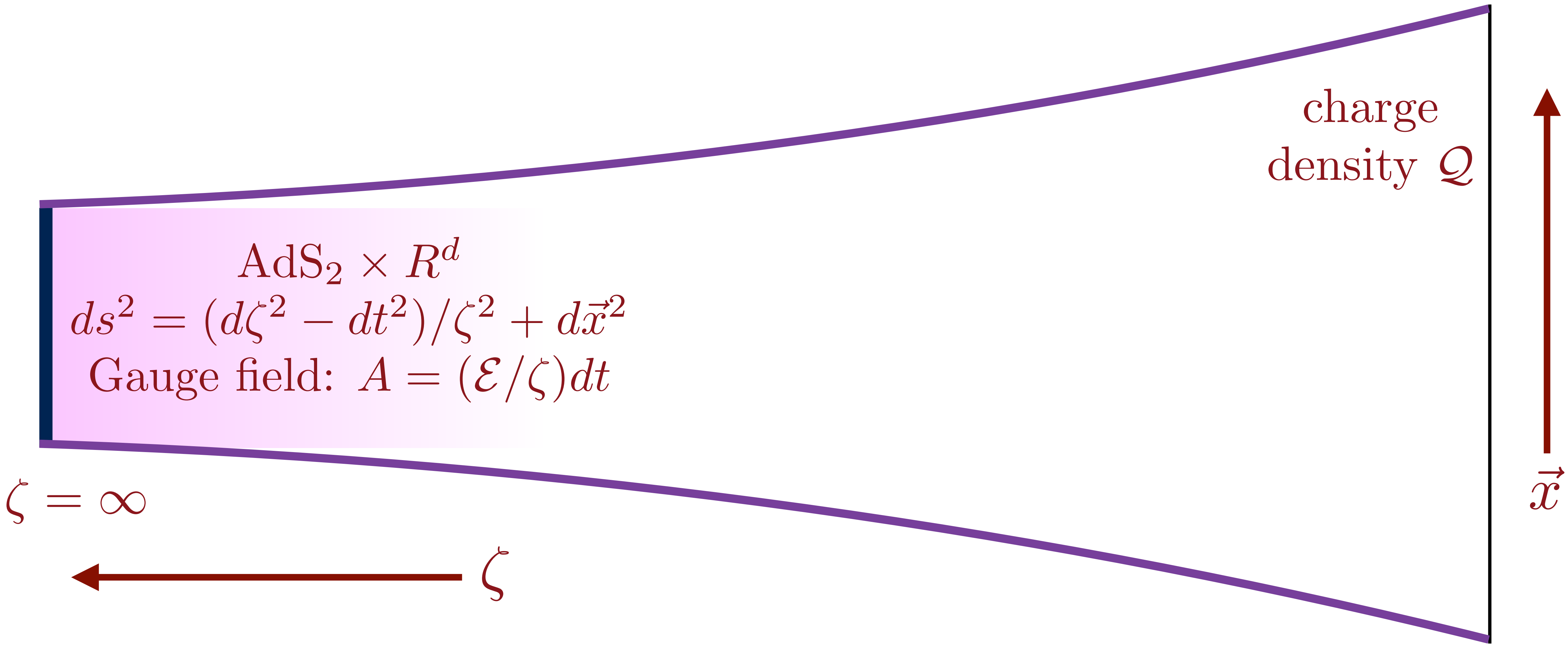


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

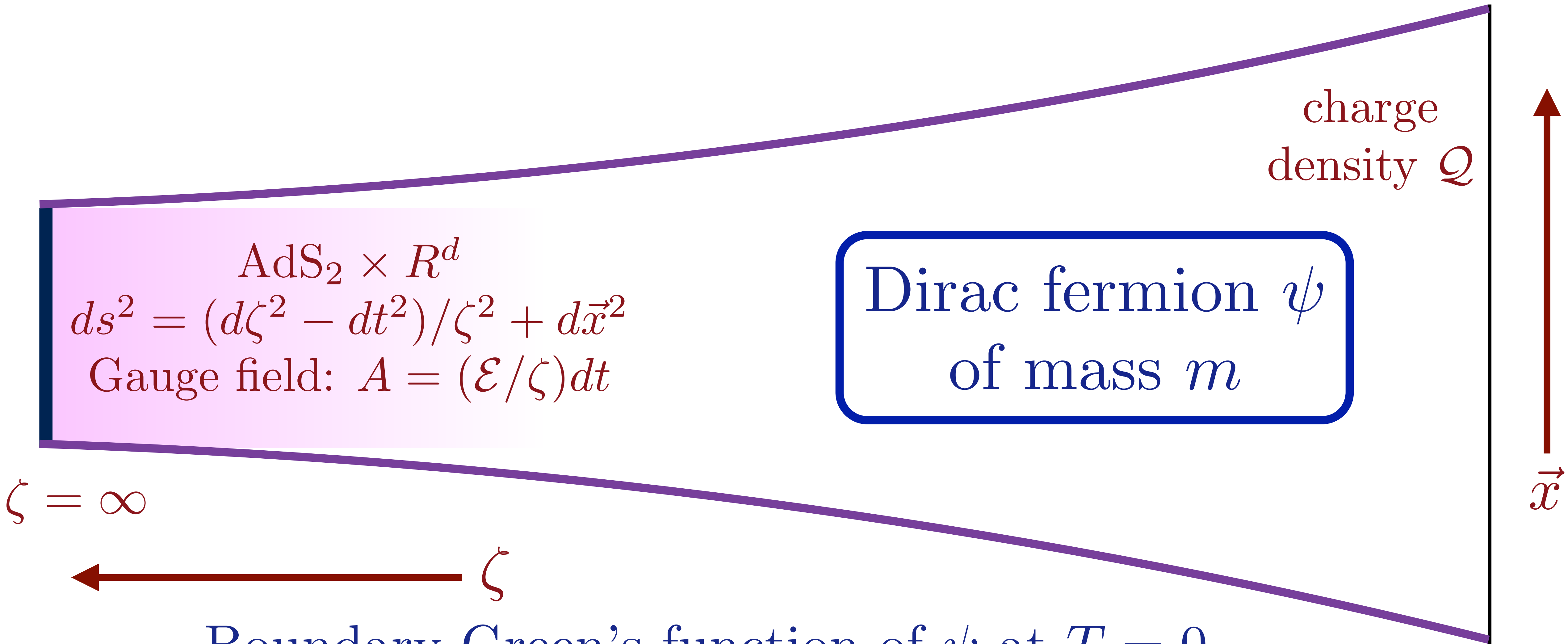
A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

General Relativity of charged black branes



- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.

Quantum fields on charged black branes



Boundary Green's function of ψ at $T = 0$

$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-(1-2\Delta)} & , \ \omega < 0. \end{cases}$$

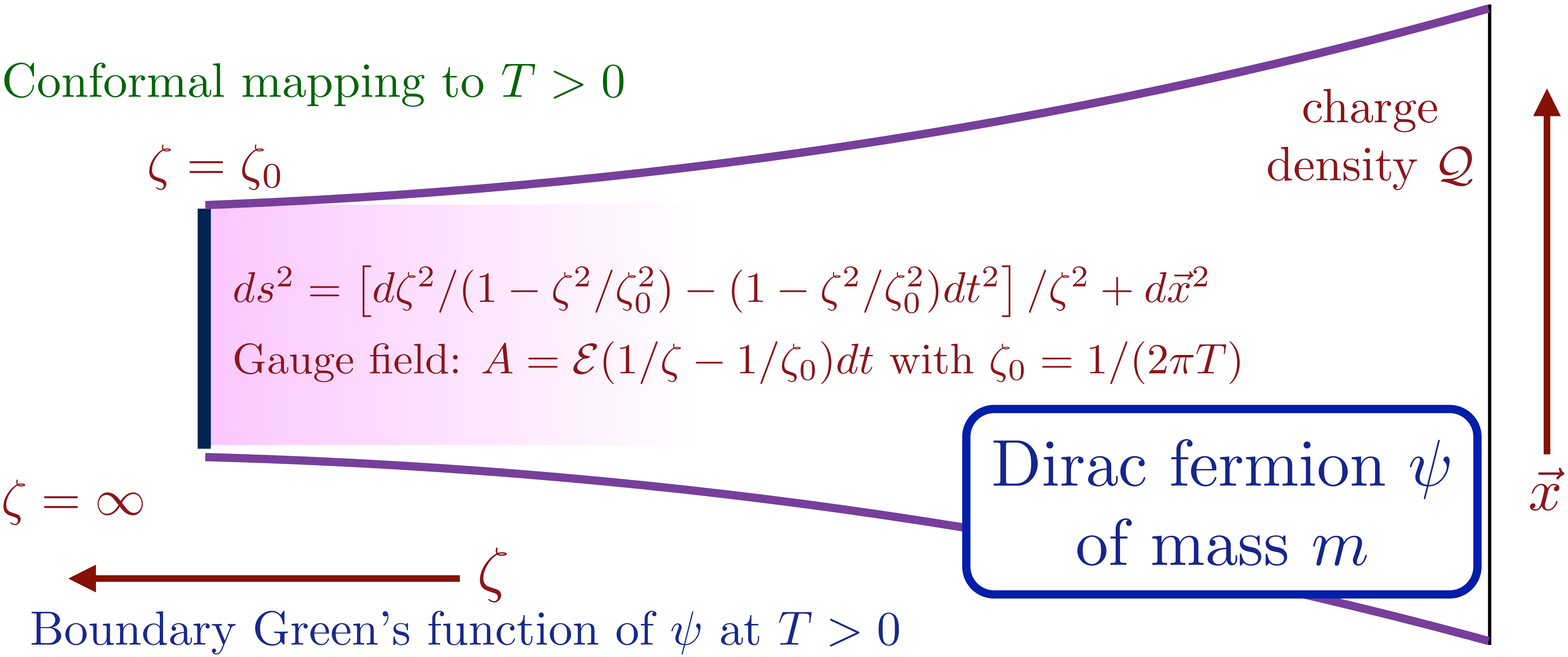
where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

Quantum fields on charged black branes

Conformal mapping to $T > 0$



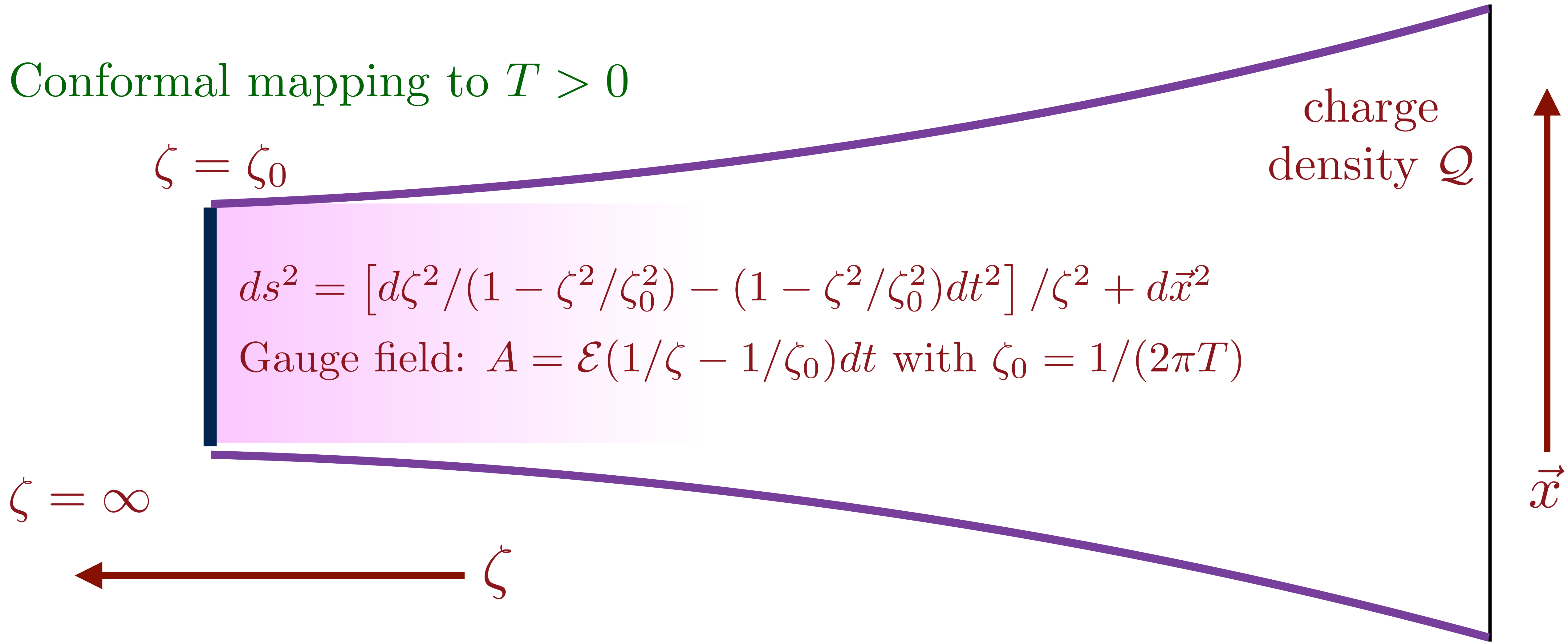
Boundary Green's function of ψ at $T > 0$ is fully determined by $\mathcal{E}$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

General Relativity of charged black branes

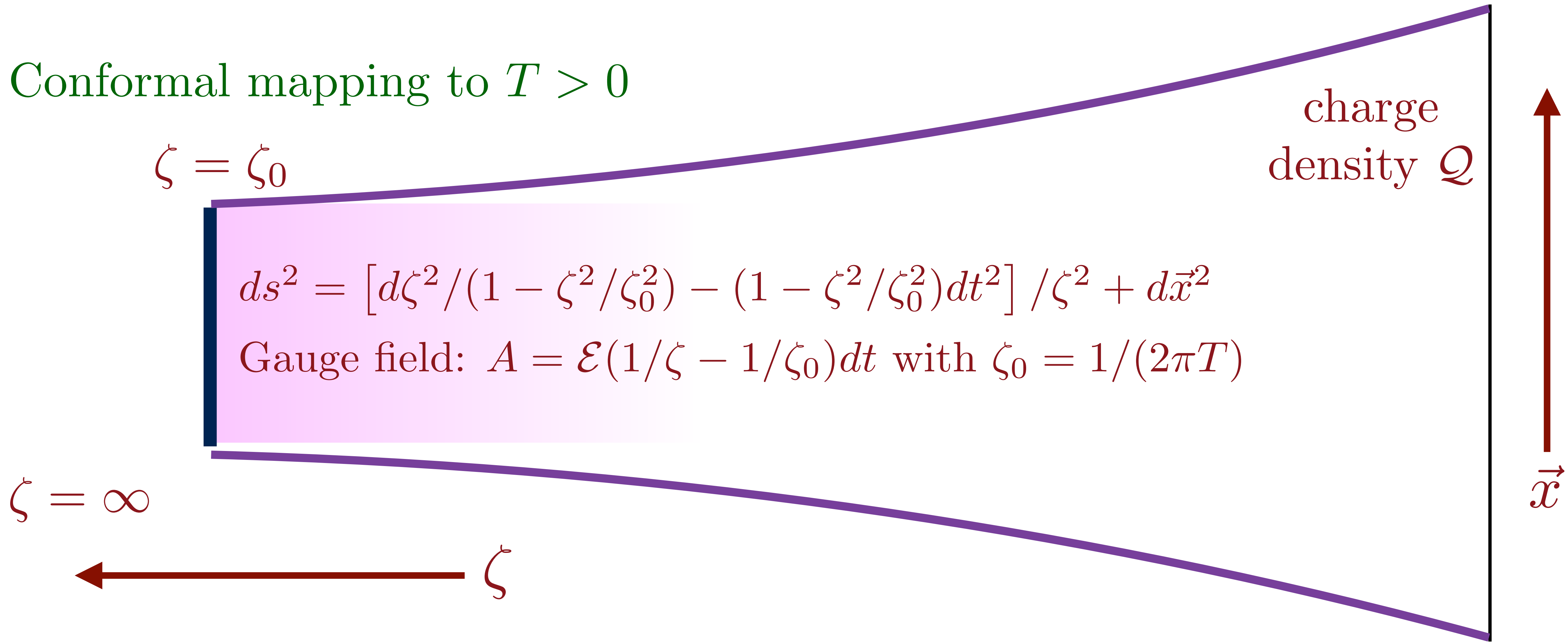


- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{BH}$.
- Using Gauss's Law, it can be shown that $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

General Relativity of charged black branes



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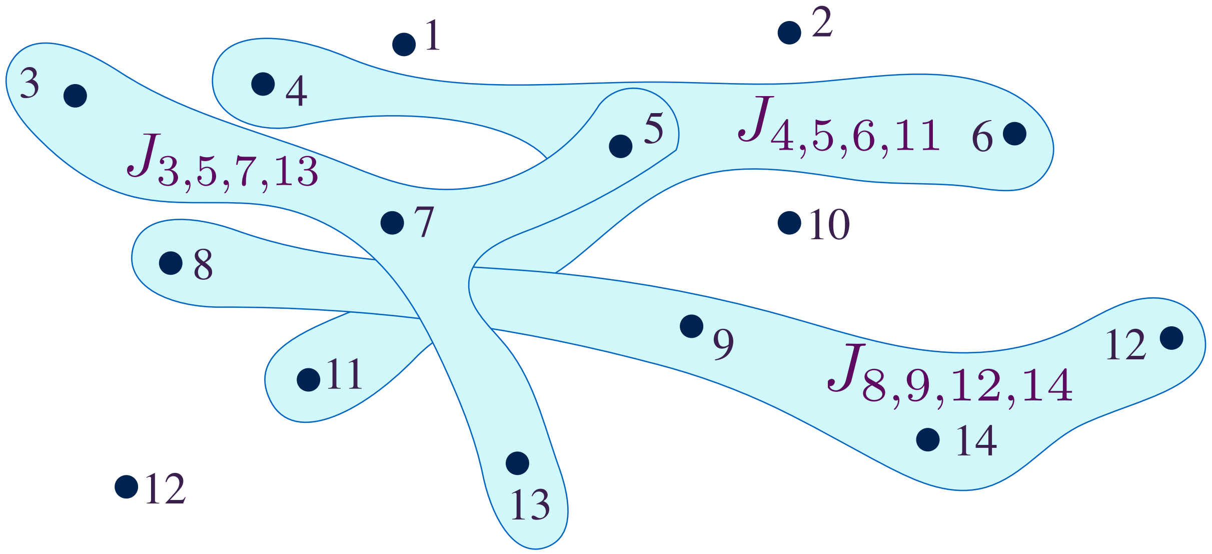
Also obeyed by Wald entropy in higher-derivative gravity.

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2} & , \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

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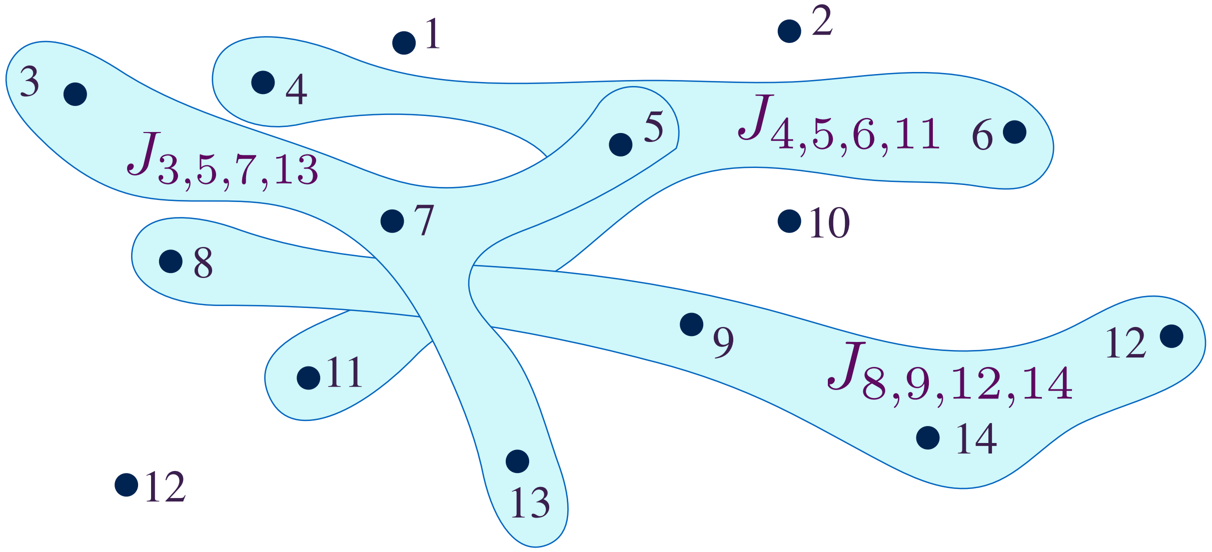

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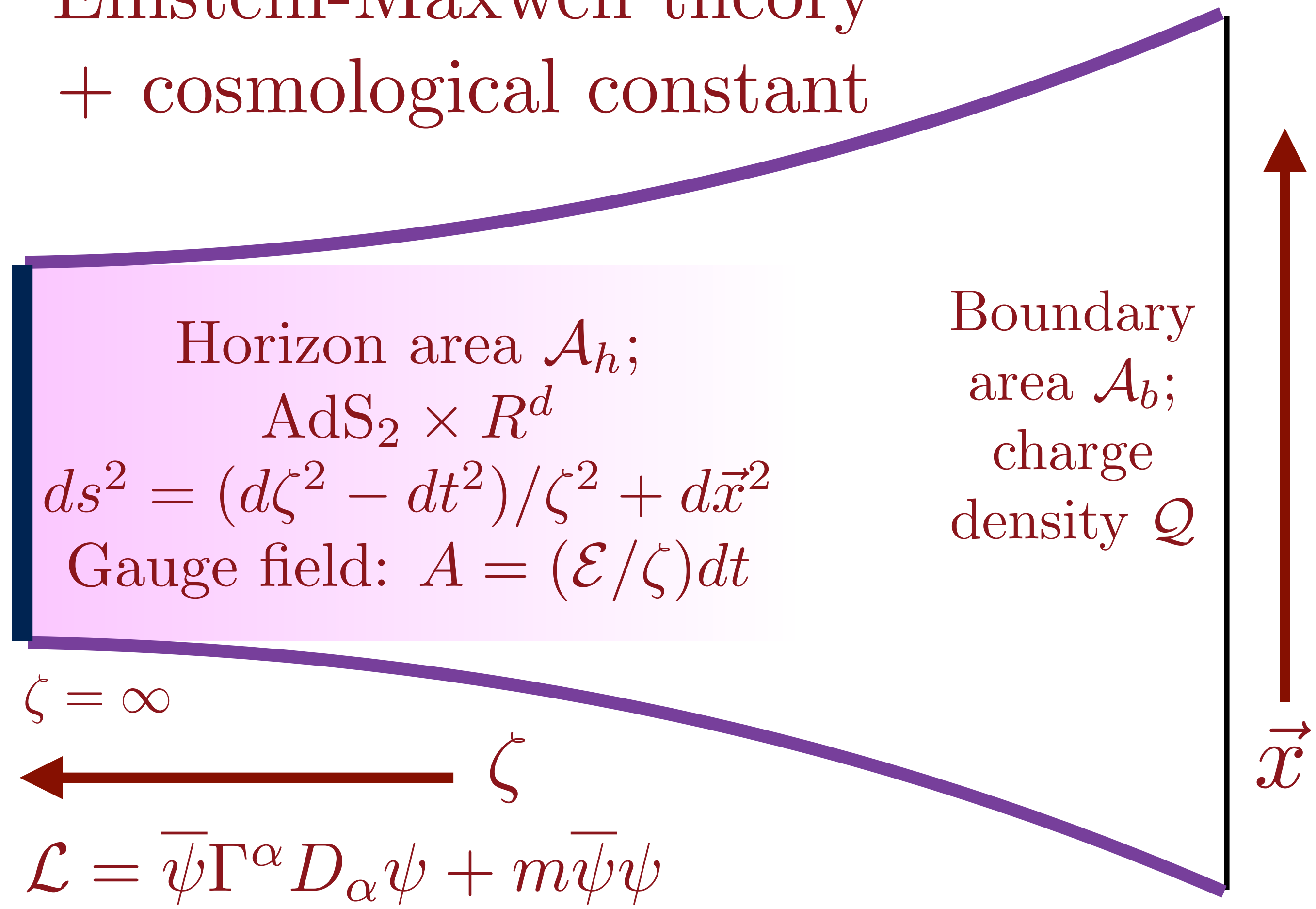
Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

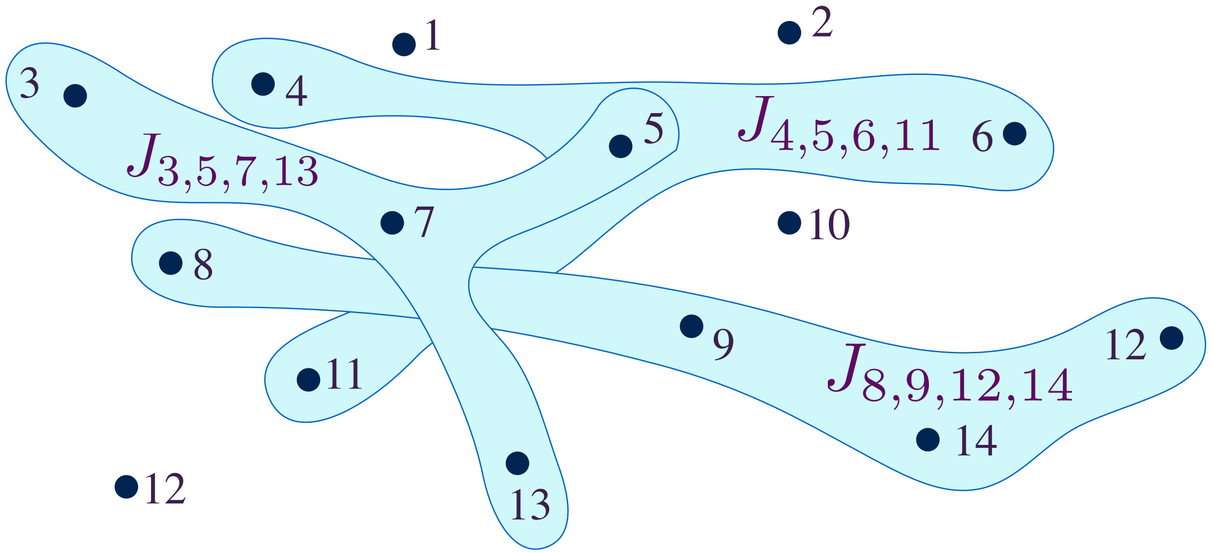
'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, arXiv:hep-th/0506177; S. Sachdev, arXiv:1506.0511

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



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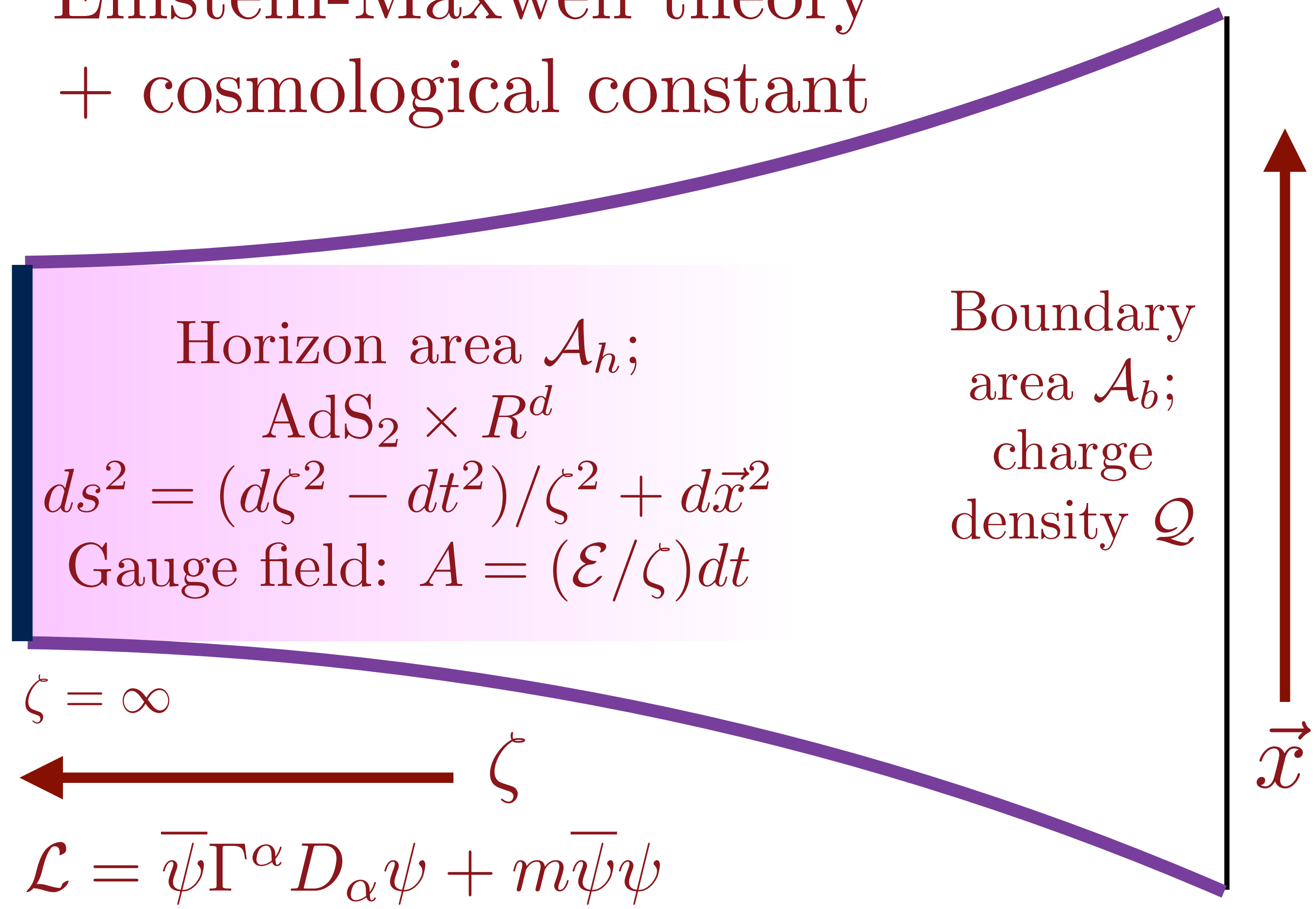

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entropy density, $\mathcal{S}$, obeys

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Evidence for AdS_2 gravity dual of H

lack hole thermodynamics
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$$\frac{\partial \mathcal{S}_{BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

S. Sachdev, arXiv:1506.05111

Quantum matter without quasiparticles

1. Experiment and theory in graphene
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4. Transport in strange metals

Transport in Strange Metals

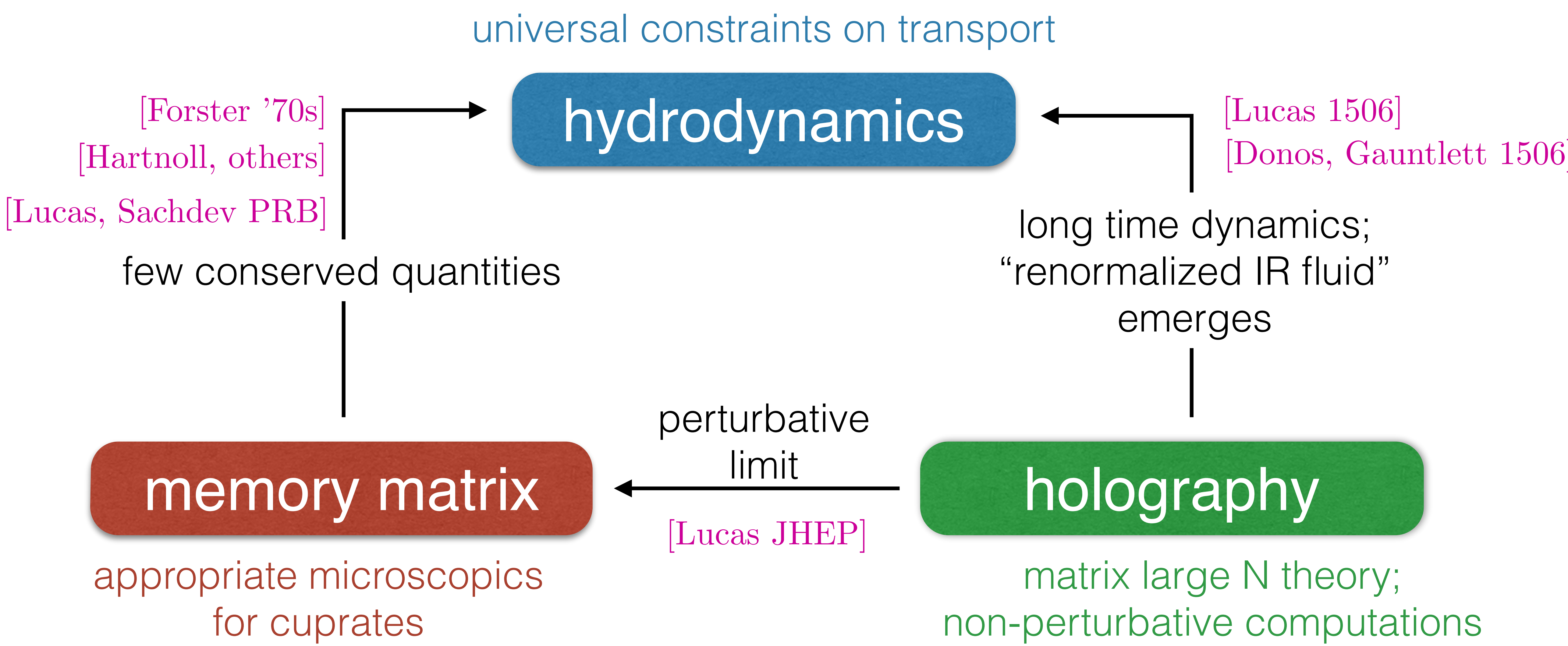
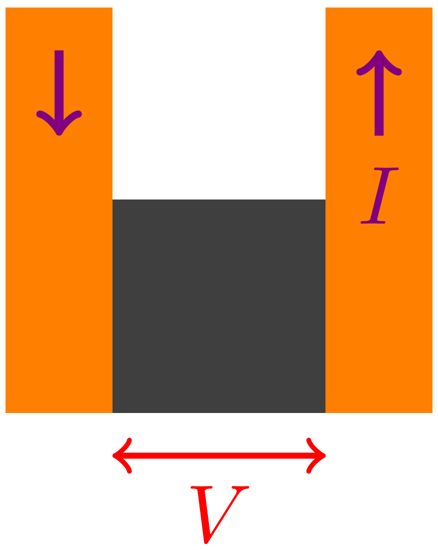


figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)



$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

- σ = easy experiment; related to QFT correlators:

$$\sigma_{ij}(\omega) = \frac{\mathrm{i}}{\omega}\langle J_i(-\omega) J_j(\omega)\rangle, \quad \text{etc.}$$

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \pi\delta(\omega) + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \pi\delta(\omega) + \alpha_Q(\omega)$$

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with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

Obtained in hydrodynamics, holography, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Relativistic hydrodynamics

- hydrodynamics when $l \gg l_{\text{ee}}$, $t \gg t_{\text{ee}}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\text{ext}}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, J^μ in perturbative parameter $l_{\text{ee}}\partial_\mu$:

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu - 2\mathcal{P}^{\mu\rho}\mathcal{P}^{\nu\sigma}\eta\partial_{(\rho}u_{\sigma)} - \mathcal{P}^{\mu\nu}\left(\zeta - \frac{2\eta}{d}\right)\partial_\rho u^\rho + \cdots,$$

$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\text{Q}}\mathcal{P}^{\mu\rho}\left(\partial_\rho\mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\text{ext}}_{\rho\nu}\right) + \cdots,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = J^i - \mu T^{ti}$$

- quantum physics $\to$ values of P, σ_{Q}, etc...

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

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with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

In theories which are relativistic at high energies (including graphene), $T\alpha_Q(\omega) = -\mu\sigma_Q(\omega)$, $T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega)$, $\mathcal{M} = T\mathcal{S} + \mu\mathcal{Q} = \mathcal{H}$ the enthalpy density, and $\mathcal{Q} = n$ the electron density

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

$$\frac{\mathcal{M}}{\tau} = \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\text{Im} \left(G^{\text{R}}_{\mathcal{O}\mathcal{O}}(q,\omega) \right)_{H_0}}{\omega} + \text{higher orders in } h$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Magneto-electric transport

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

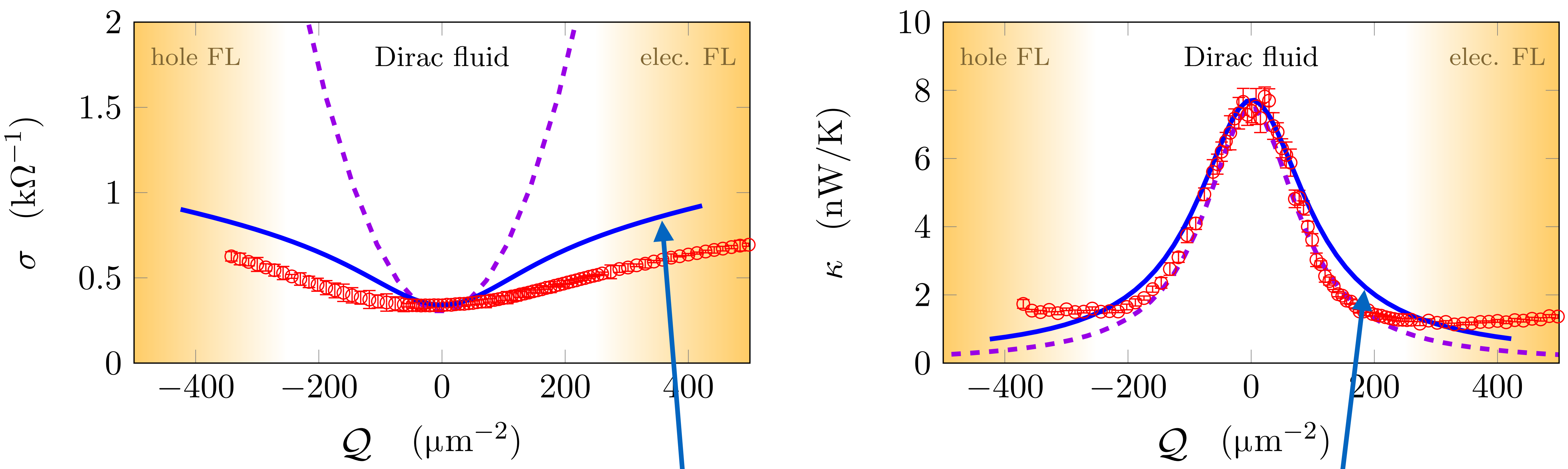
$$\sigma_{xx} = \frac{(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau} - i\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} B\mathcal{Q}.$$

Electrical and thermal magnetotransport in a magnetic field *B* with no additional parameters

(assuming σ_Q is field-independent)

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)



Solution of the HKMS equations in the presence of a space-dependent chemical potential. Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738

Quantum matter without quasiparticles

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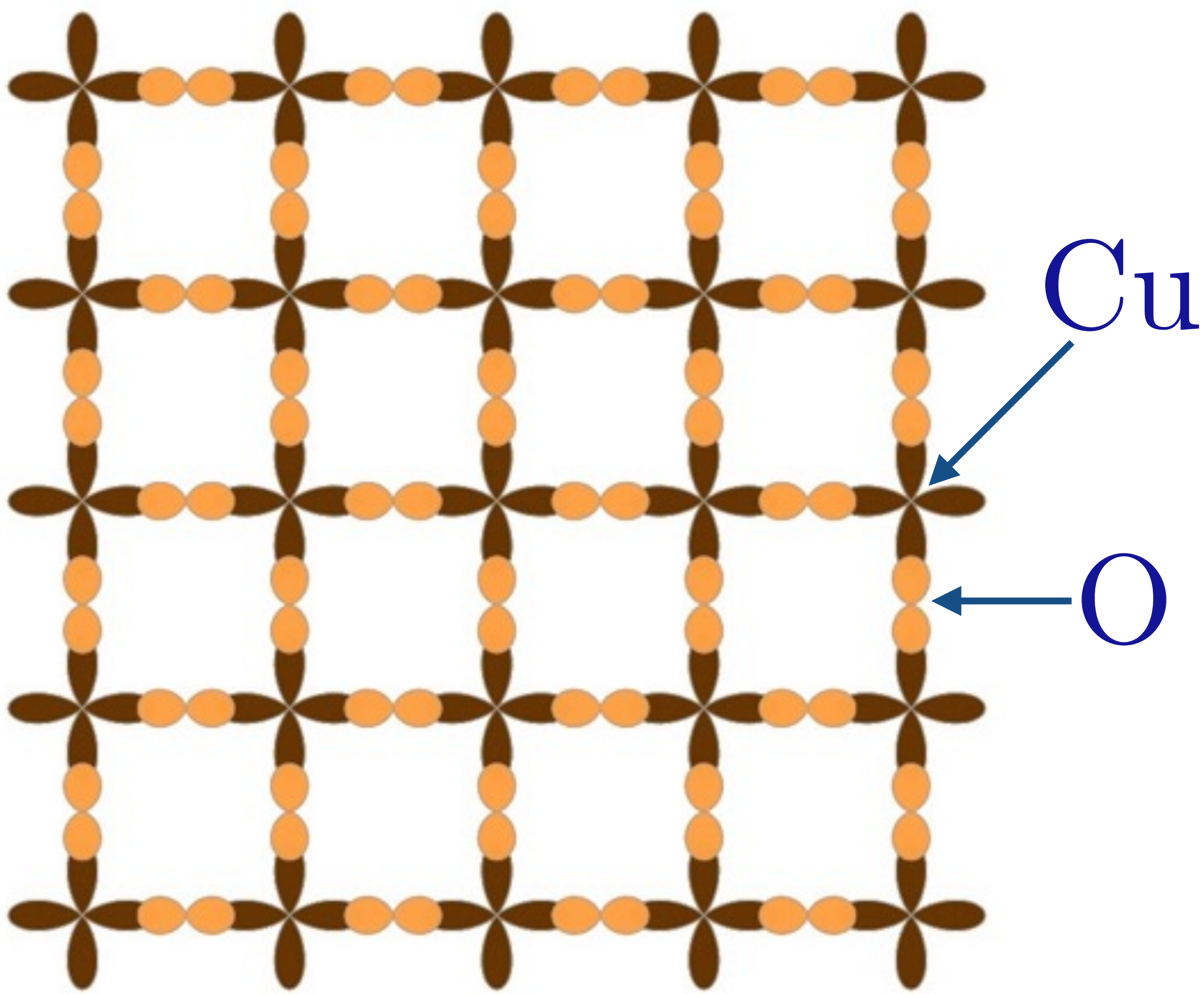
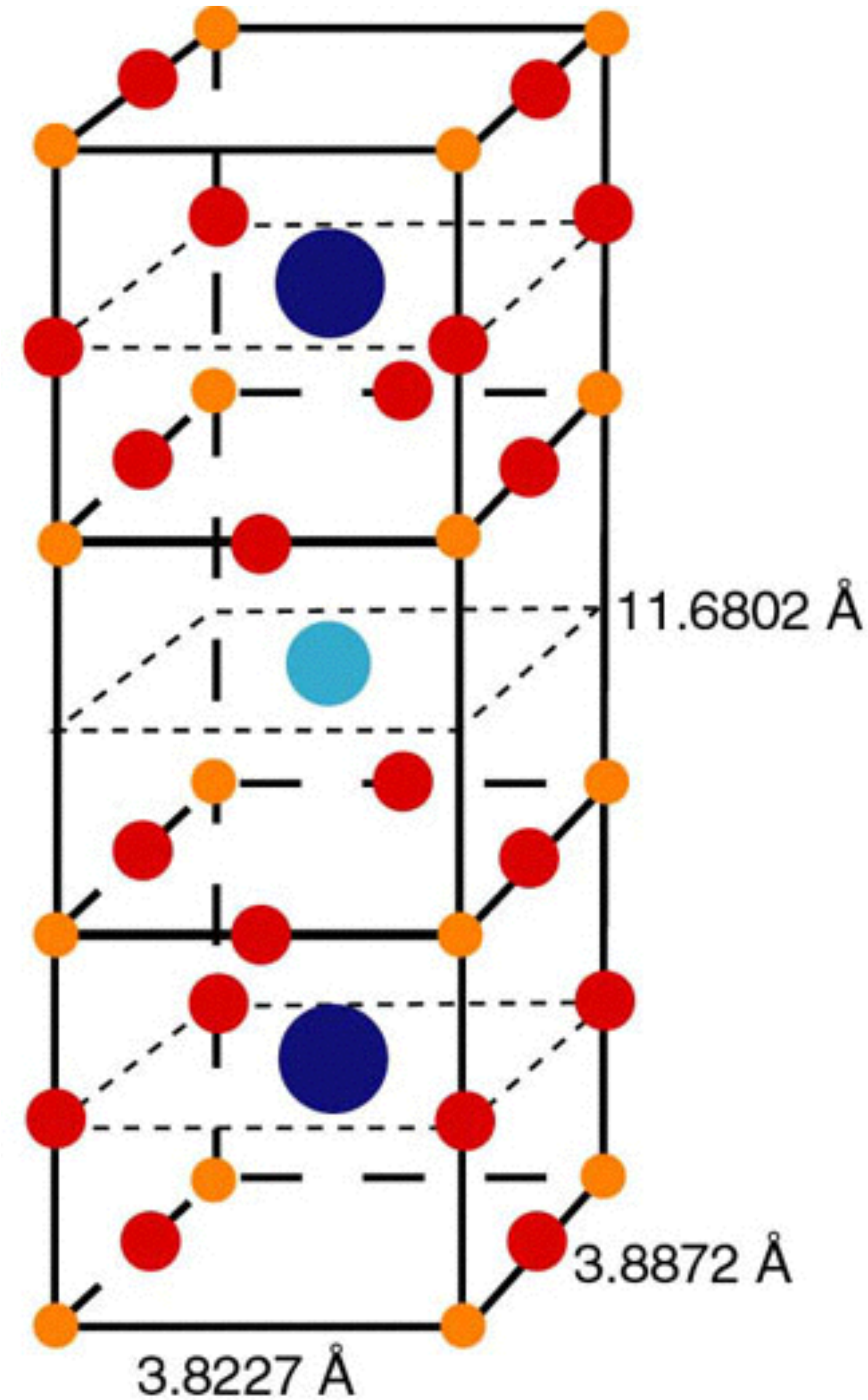
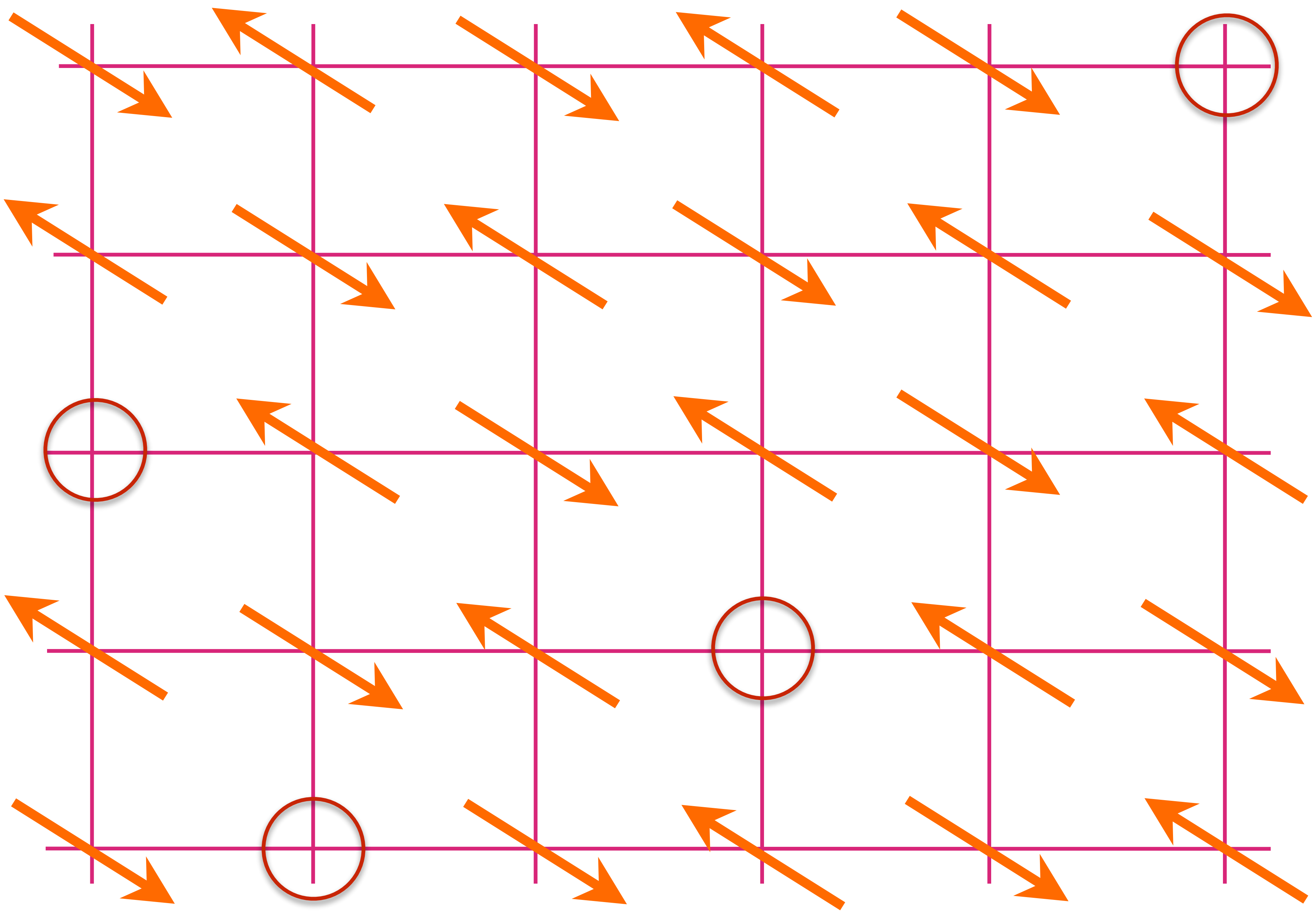
High temperature superconductors



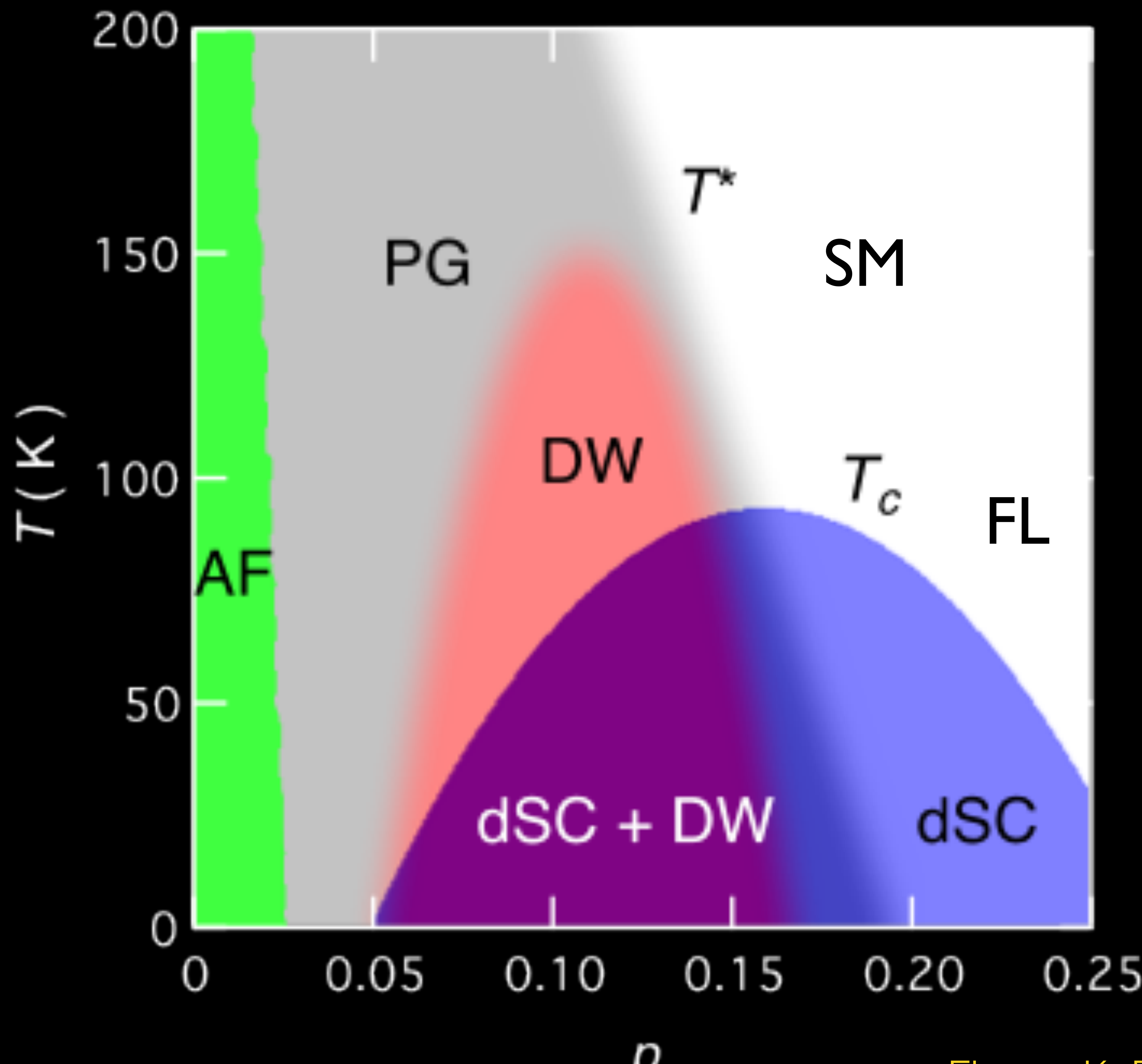
CuO_2 plane

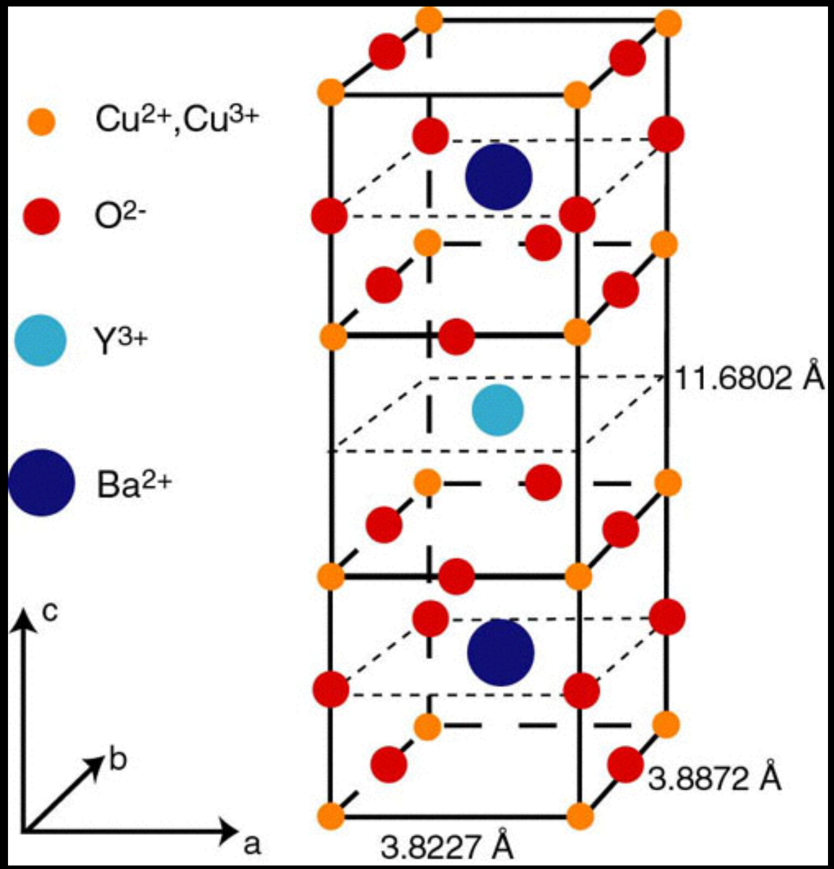


$YBa_2Cu_3O_{6+x}$



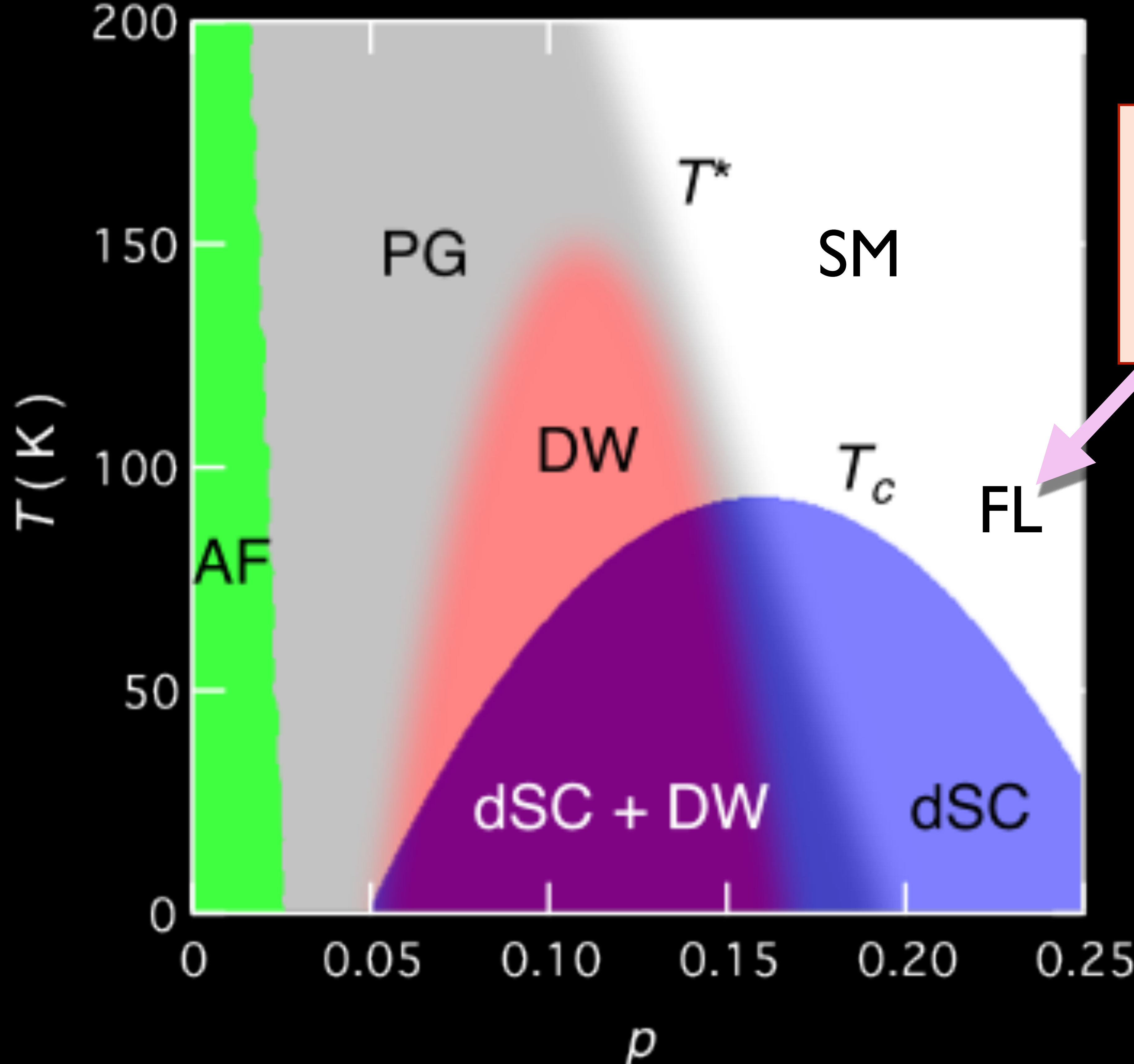
Anti-ferromagnet with p holes per square





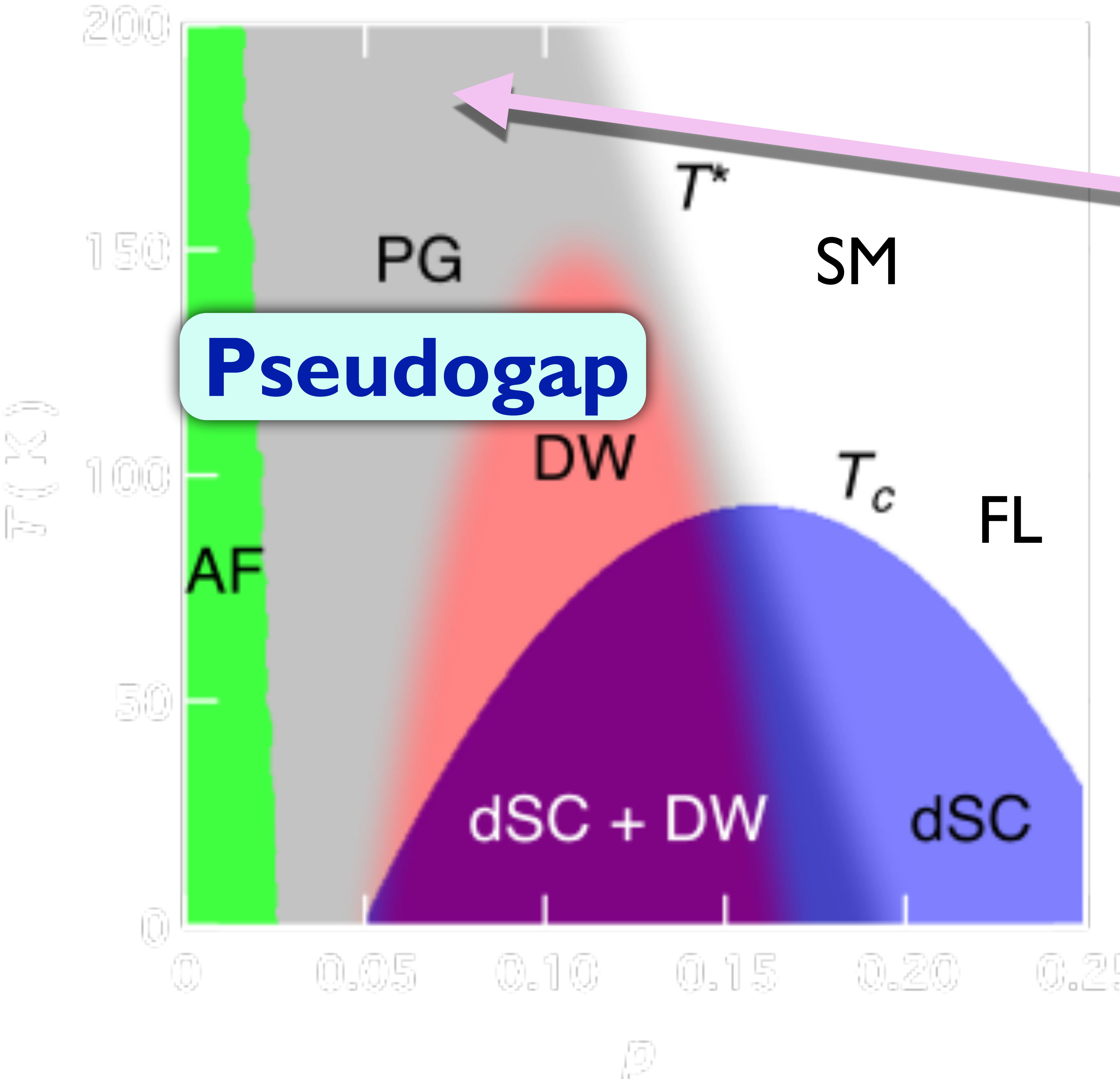
$YBa_2Cu_3O_{6+x}$

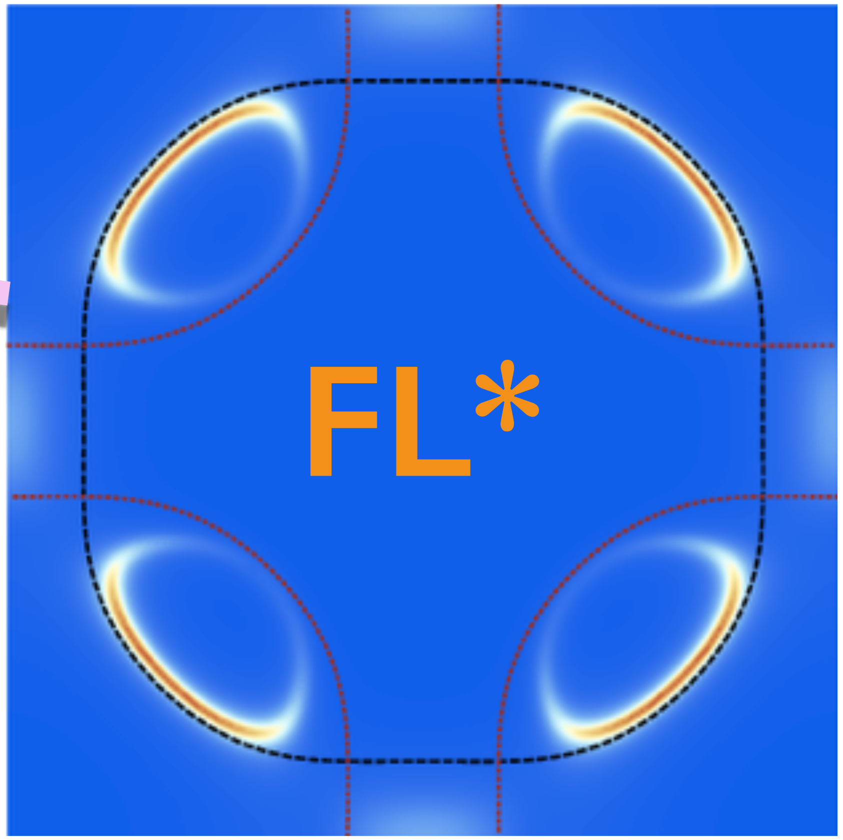
Figure: K. Fujita and J. C. Seamus Davis



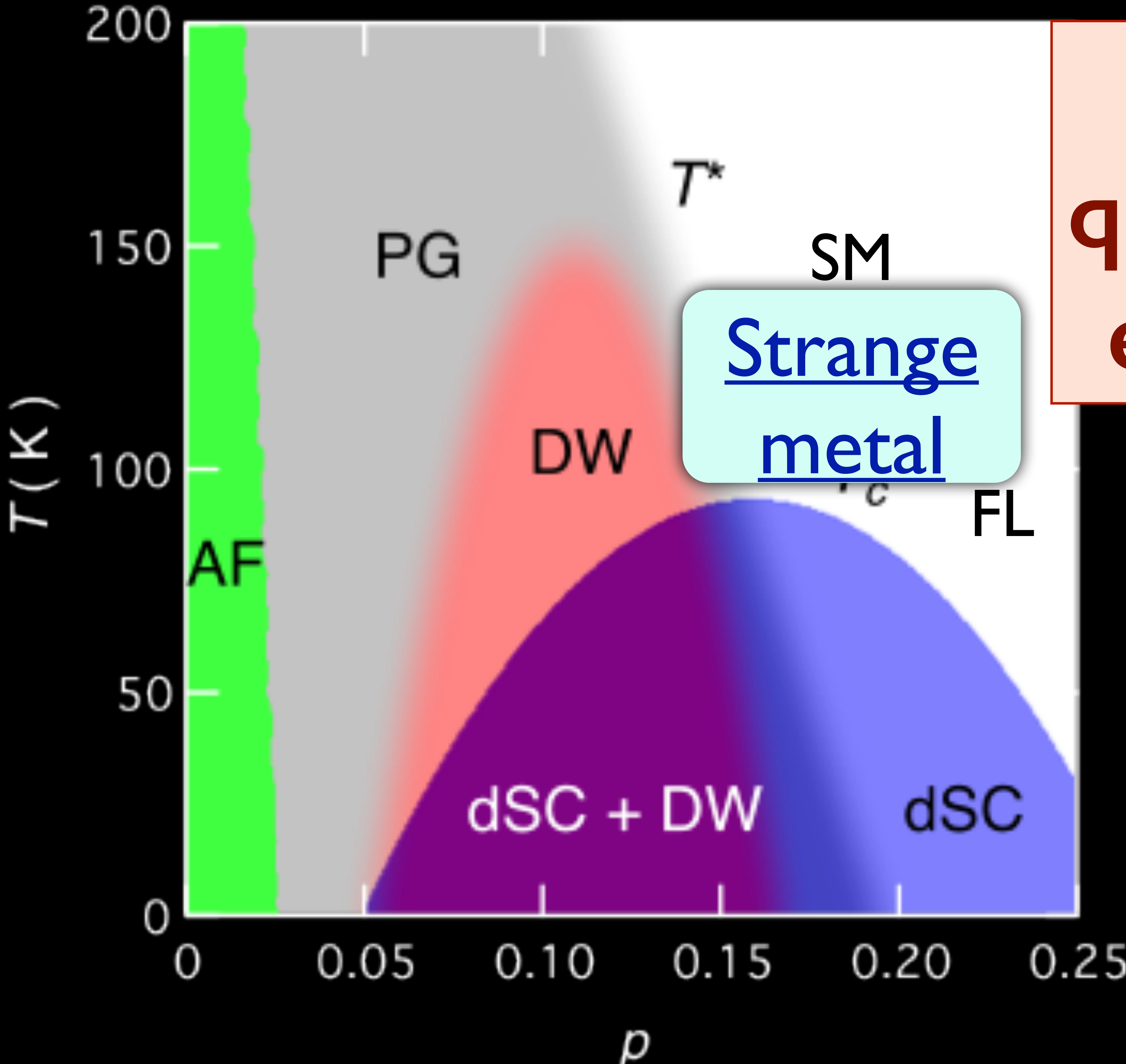
Fermi liquid (FL): a conventional metal

S. Sachdev, Phys. Rev. B **49**, 6770 (1994)
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
D. Chowdhury and S. Sachdev, Phys. Rev. B **90**, 245136 (2014)





A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p



No quasiparticle excitations

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S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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Magneto-electric transport

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma_{xx} = \frac{(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau} - i\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}B\mathcal{Q}.$$

Electrical and thermal magnetotransport in a magnetic field *B* with no additional parameters

(assuming σ_Q is field-independent)

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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Blake and Donos: With $\sigma_Q \sim 1/T$, $\tau_L \sim 1/T^2$ and assuming momentum drag dominates longitudinal transport, we obtain $\sigma_{xx} \sim 1/T$ and $\tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2$, in agreement with strange metal data on cuprates (such data cannot be explained in a quasiparticle model).

M. Blake and A. Donos, PRL **114**, 021601 (2015)

Computations on non-Fermi liquids

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

Spin density wave critical point

Momentum drag conductivity: $\sigma \sim T^{-1}$ from disorder-induced shifts in the position of the critical point

A.A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

Quantum critical conductivity: $\sigma \sim T^0$

S.A. Hartnoll, D. M. Hofman, M.A. Metlitski and S. Sachdev, PRB **84**, 125115 (2011)
A.A. Patel, P. Strack and S. Sachdev, PRB **92**, 165105 (2015)

Computations on non-Fermi liquids

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

Ising-nematic critical point

Momentum drag conductivity: $\sigma \sim T^{-2/3}$ from disorder-induced shifts in the position of the critical point

A. A. Patel and S. Sachdev, PRB **90**, 165146 (2014)

Momentum drag conductivity: $\sigma \sim T^{1/2}$ from random-field disorder

S. A. Hartnoll, R. Mahajan, M. Punk, and S. Sachdev, PRB **89**, 155130 (2014)

Quantum critical conductivity: $\sigma \sim ??$, in progress...

Quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$
  (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)
- Theory built from hydrodynamics/holography
  /memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.