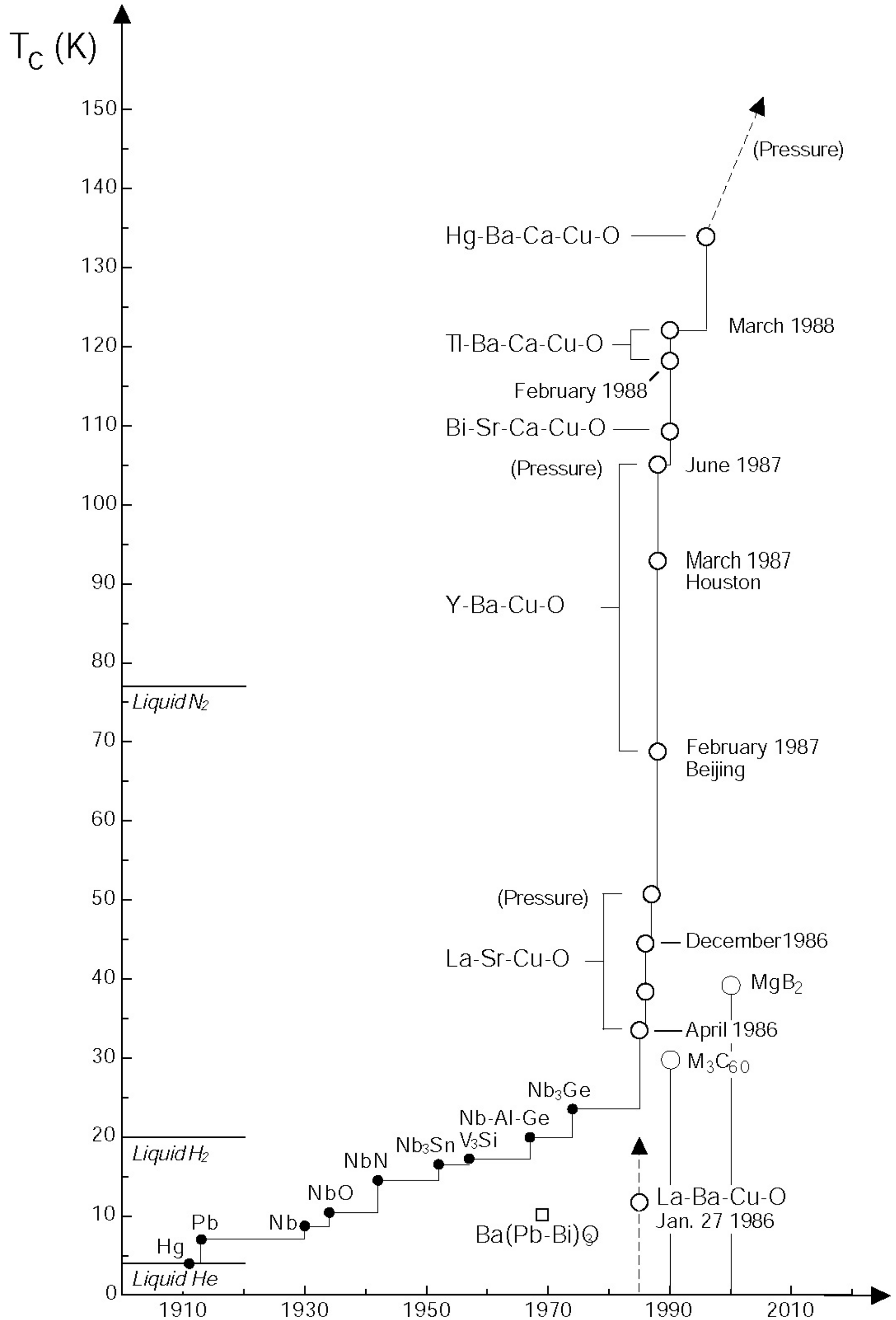
Superfluids and their vortices

Subir Sachdev

Talk online:
http://pantheon.yale.edu/~subir

Superfluidity/superconductivity occur in:

- liquid ^4He
- metals Hg, Al, Pb, Nb, Nb_3Sn…..
- liquid ^3He
- neutron stars
- cuprates $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{6+y}$….
- M_3C_{60}
- ultracold trapped atoms
- MgB_2

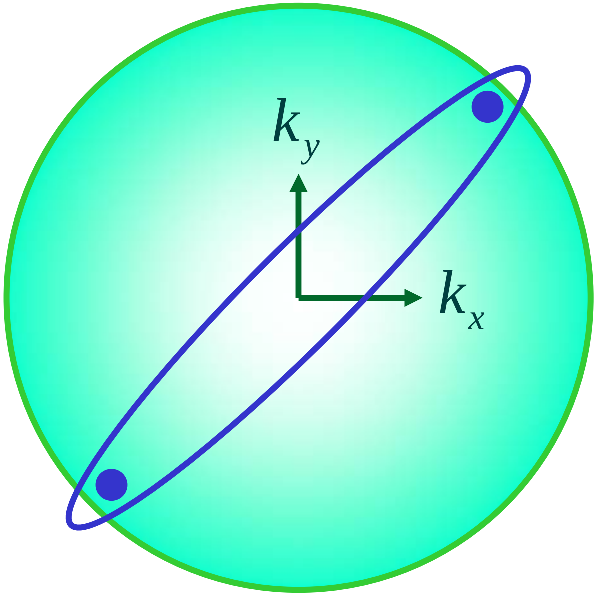


The reason for superflow:

The Bose-Einstein condensate:

A macroscopic number of bosons occupy the lowest energy quantum state

Such a condensate also forms in systems of fermions, where the bosons are Cooper pairs of fermions:

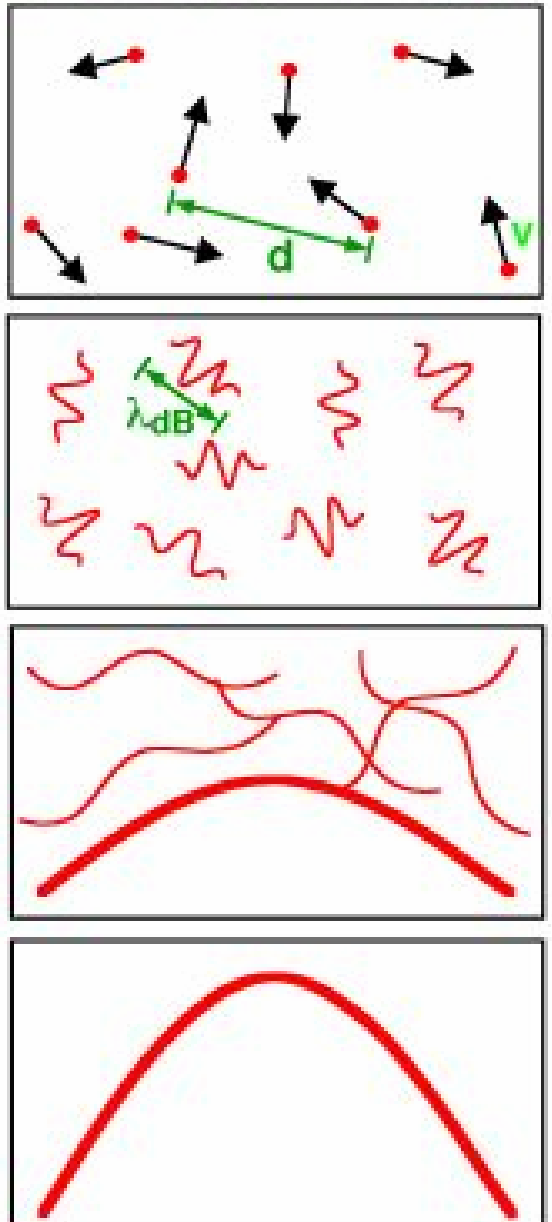


Pair wavefunction in cuprates:

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

What is Bose-Einstein condensation (BEC)?



High Temperature T:

thermal velocity v
density d^{-3}

"Billiard balls"

Low Temperature T:

De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$

"Wave packets"

$T=T_{crit}$: Bose-Einstein Condensation

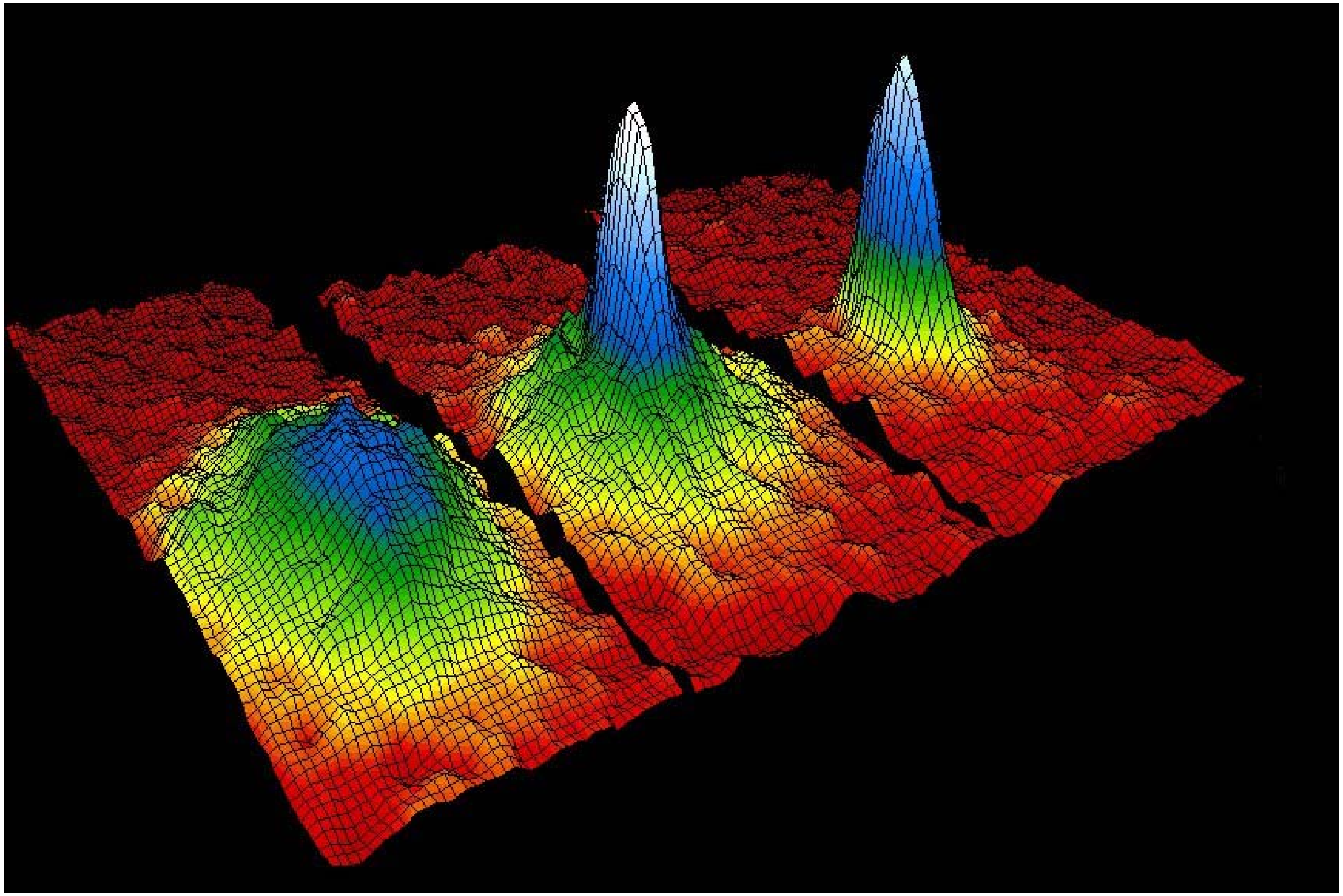
$\lambda_{dB} \approx d$

"Matter wave overlap"

T=0: Pure Bose condensate

"Giant matter wave"

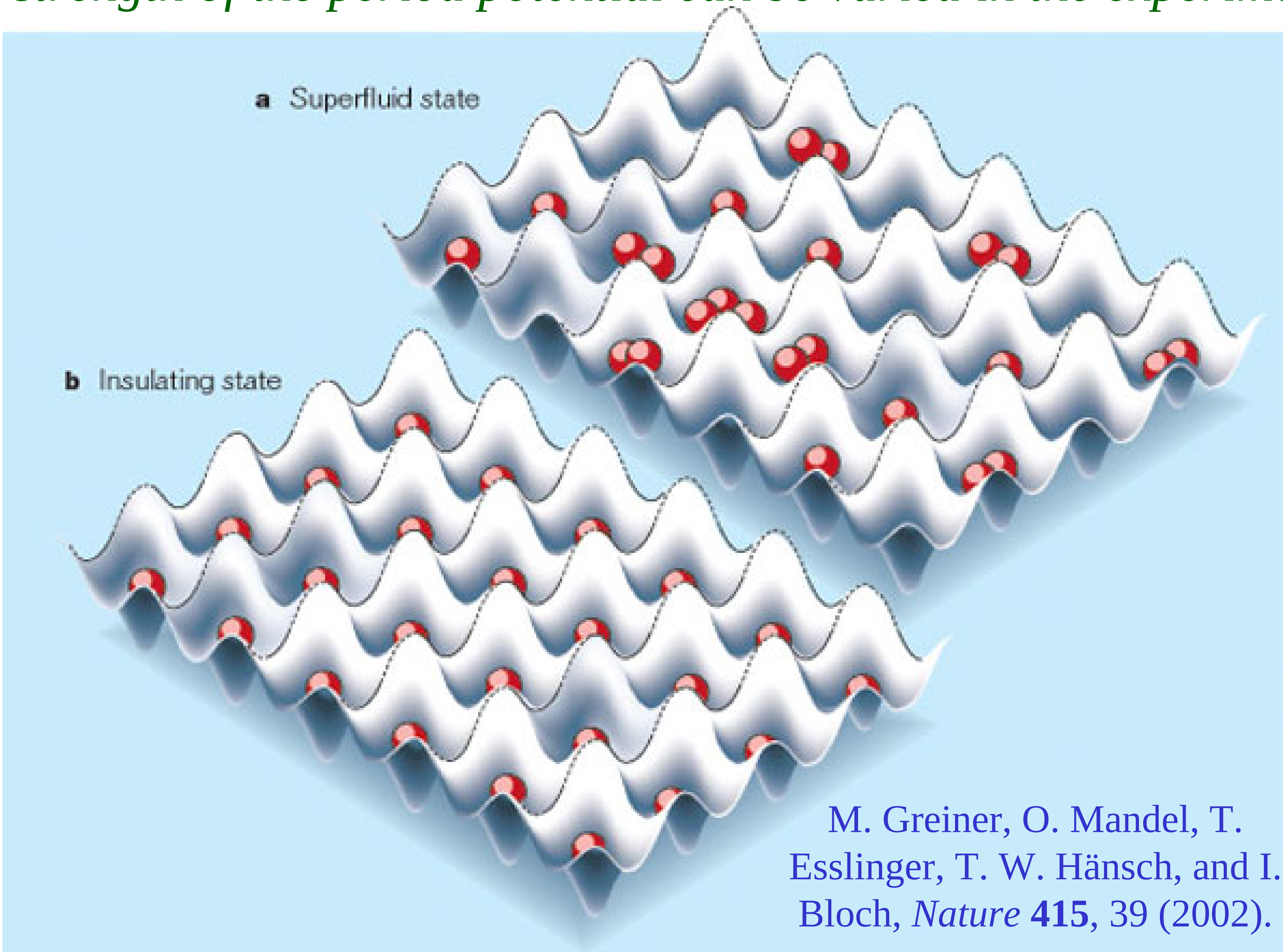
Velocity distribution function of ultracold ^{87}Rb atoms



M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

^{87}Rb bosonic atoms in a magnetic trap and an optical lattice potential

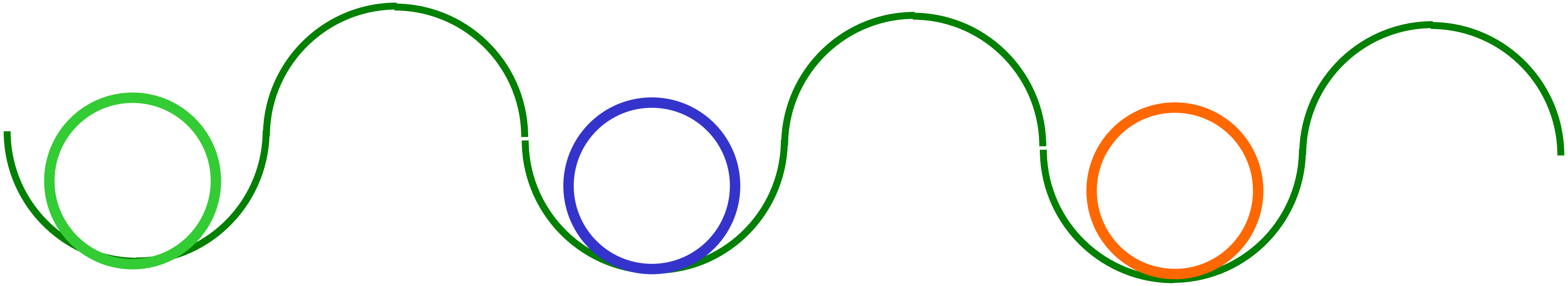
The strength of the period potential can be varied in the experiment



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Strong periodic potential

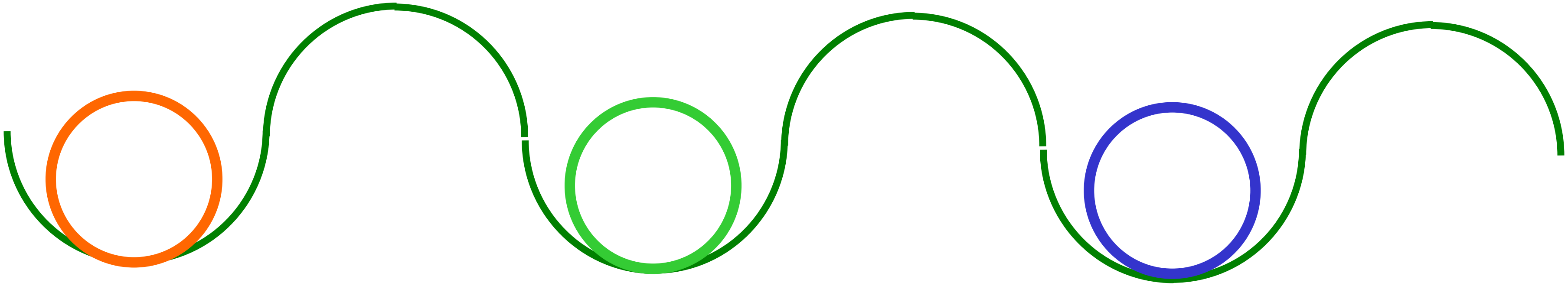
"Eggs in an egg carton"



Tunneling between neighboring minima is negligible and atoms remain localized in a well. However, the total wavefunction must be symmetric between exchange

Strong periodic potential

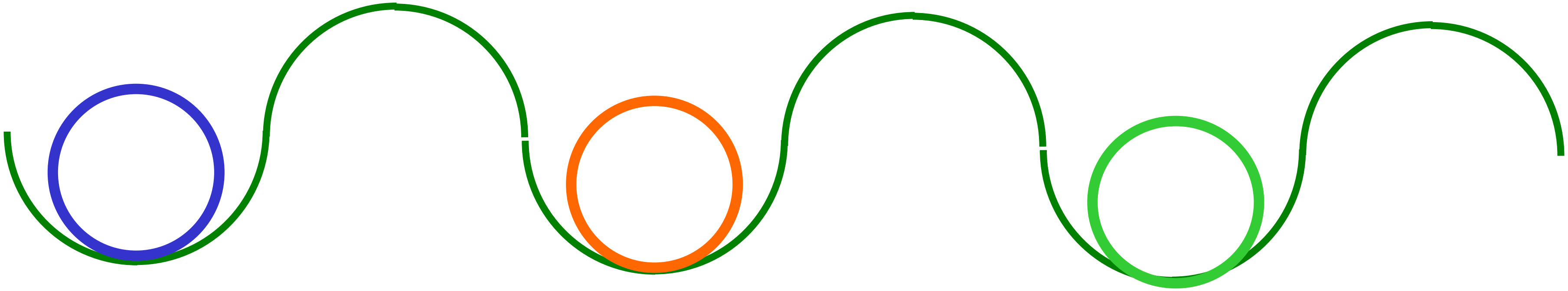
"Eggs in an egg carton"



Tunneling between neighboring minima is negligible and atoms remain localized in a well. However, the total wavefunction must be symmetric between exchange

Strong periodic potential

"Eggs in an egg carton"

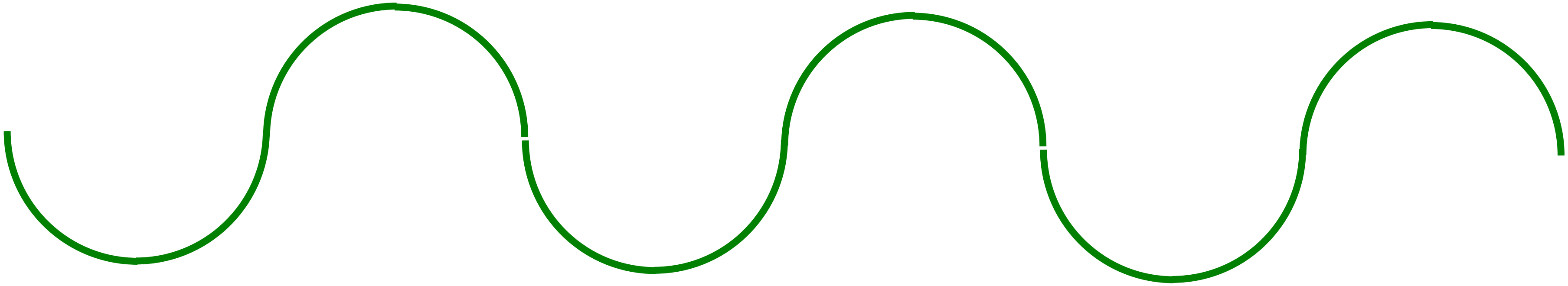


Tunneling between neighboring minima is negligible and atoms remain localized in a well. However, the total wavefunction must be symmetric between exchange

$$|\text{Insulator}\rangle = \big||\bigcirc|\bigcirc|\bigcirc|\big\rangle + \big||\bigcirc|\bigcirc|\bigcirc|\big\rangle + \big||\bigcirc|\bigcirc|\bigcirc|\big\rangle$$

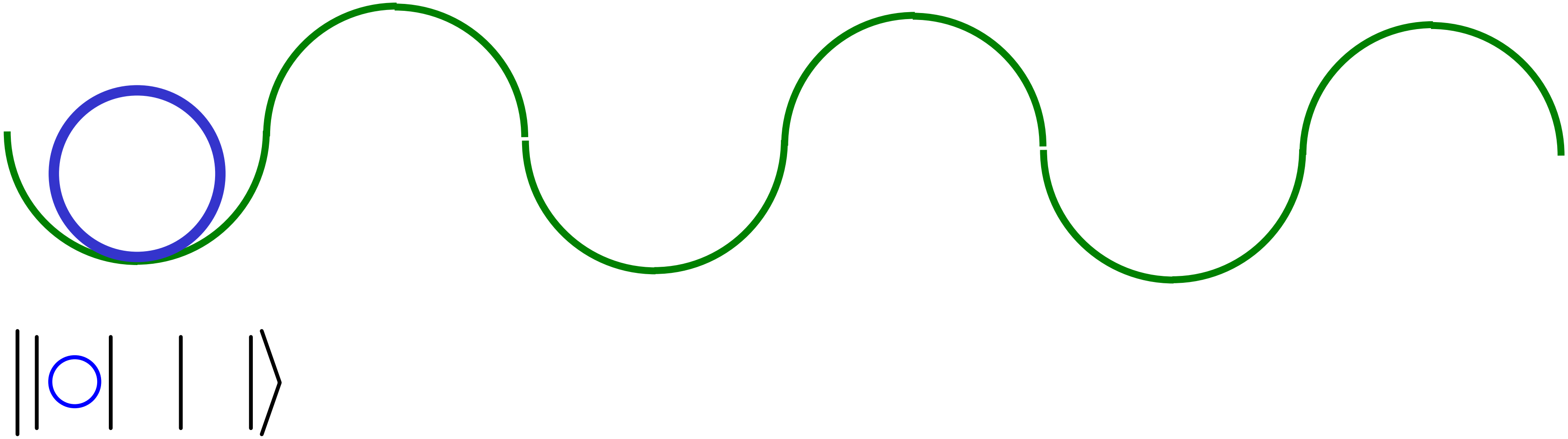
$$+ \big||\bigcirc|\bigcirc|\bigcirc|\big\rangle + \big||\bigcirc|\bigcirc|\bigcirc|\big\rangle + \big||\bigcirc|\bigcirc|\bigcirc|\big\rangle$$

Weak periodic potential



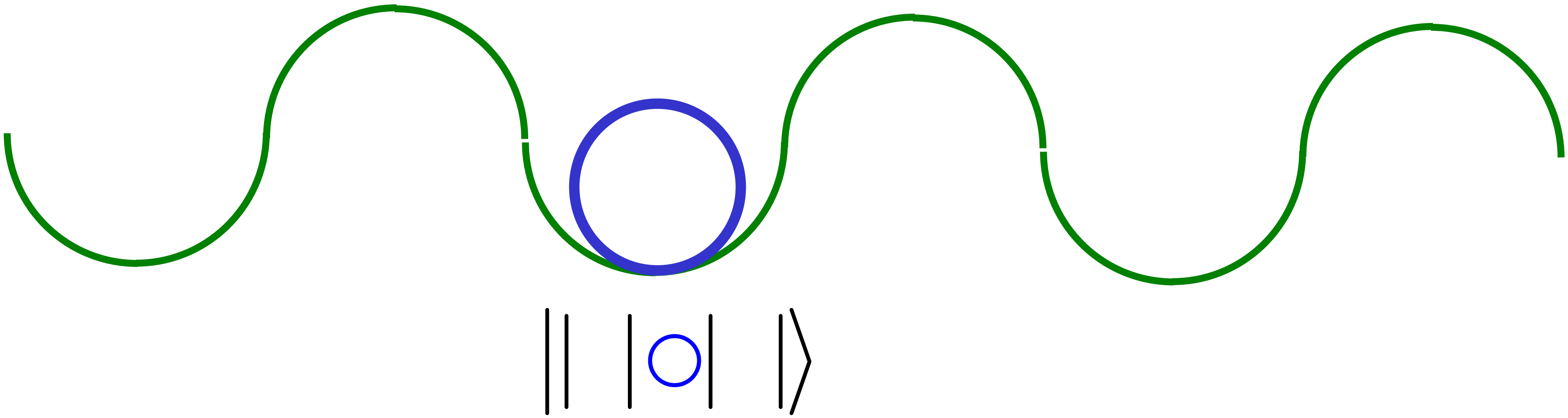
A single atom can tunnel easily between neighboring minima

Weak periodic potential



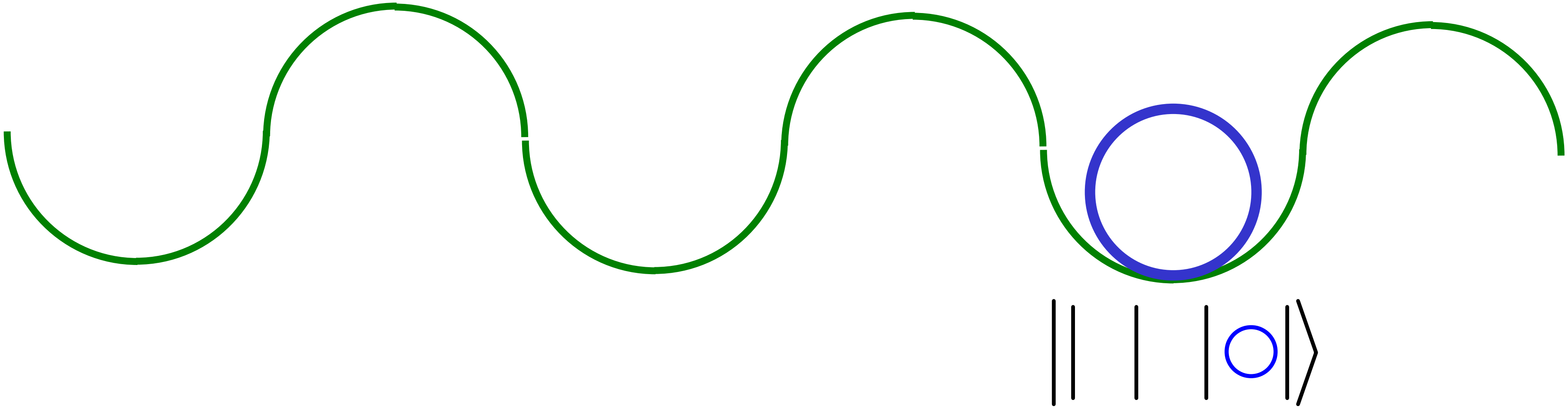
A single atom can tunnel easily between neighboring minima

Weak periodic potential



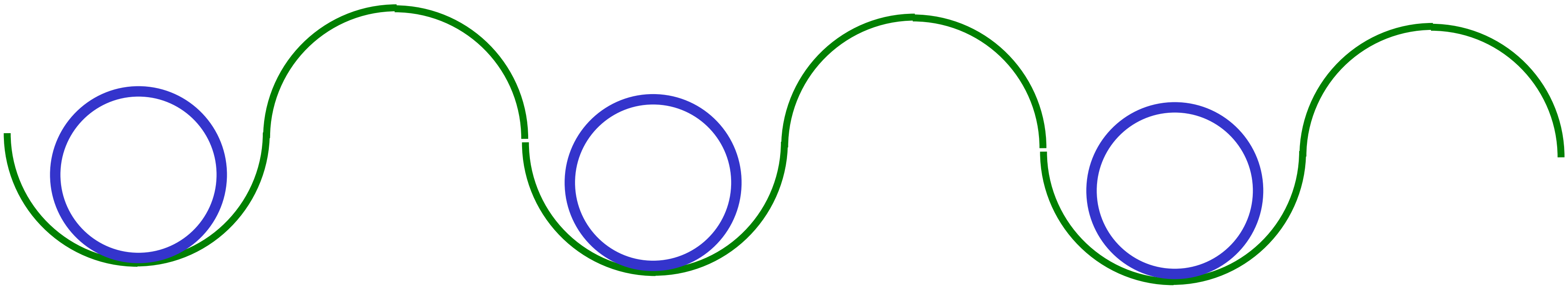
A single atom can tunnel easily between neighboring minima

Weak periodic potential



A single atom can tunnel easily between neighboring minima

Weak periodic potential



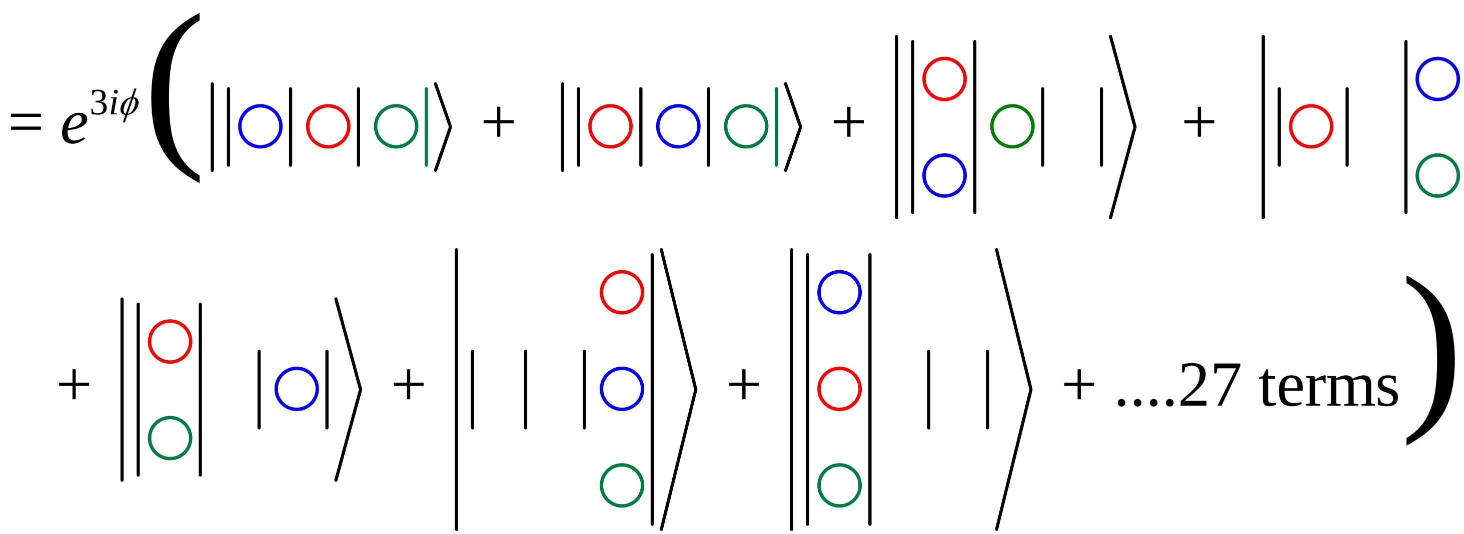
$$|G\rangle = e^{i\phi}\left(\big|\,|\bigcirc|\;\;|\;\;|\big\rangle + \big|\,|\;\;|\bigcirc|\;\;|\big\rangle + \big|\,|\;\;|\;\;|\bigcirc|\big\rangle \right)$$

The ground state of a single particle is a zero momentum state, which is a quantum superposition of states with different particle locations.

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms

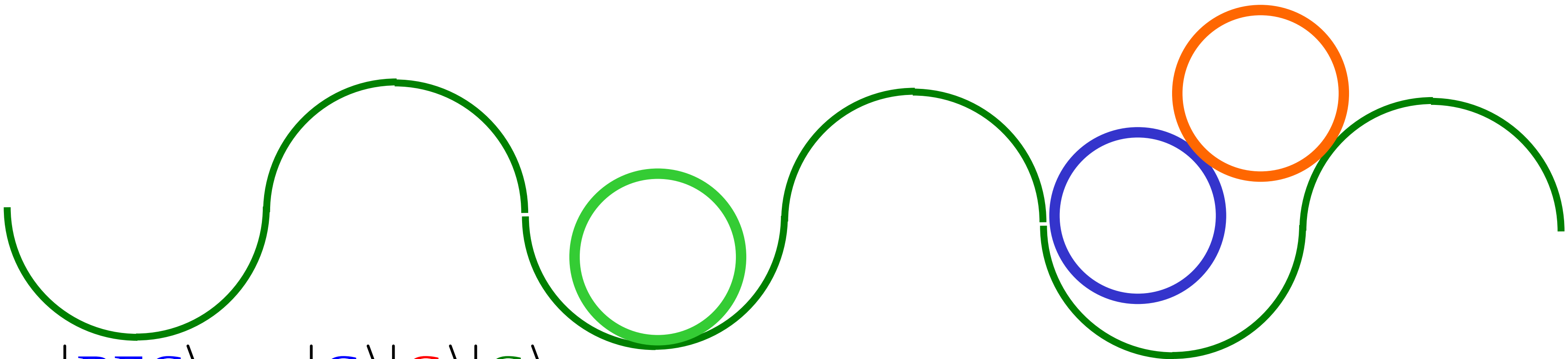
$$|\mathrm{BEC}\rangle = |\mathrm{G}\rangle|\mathrm{G}\rangle|\mathrm{G}\rangle$$



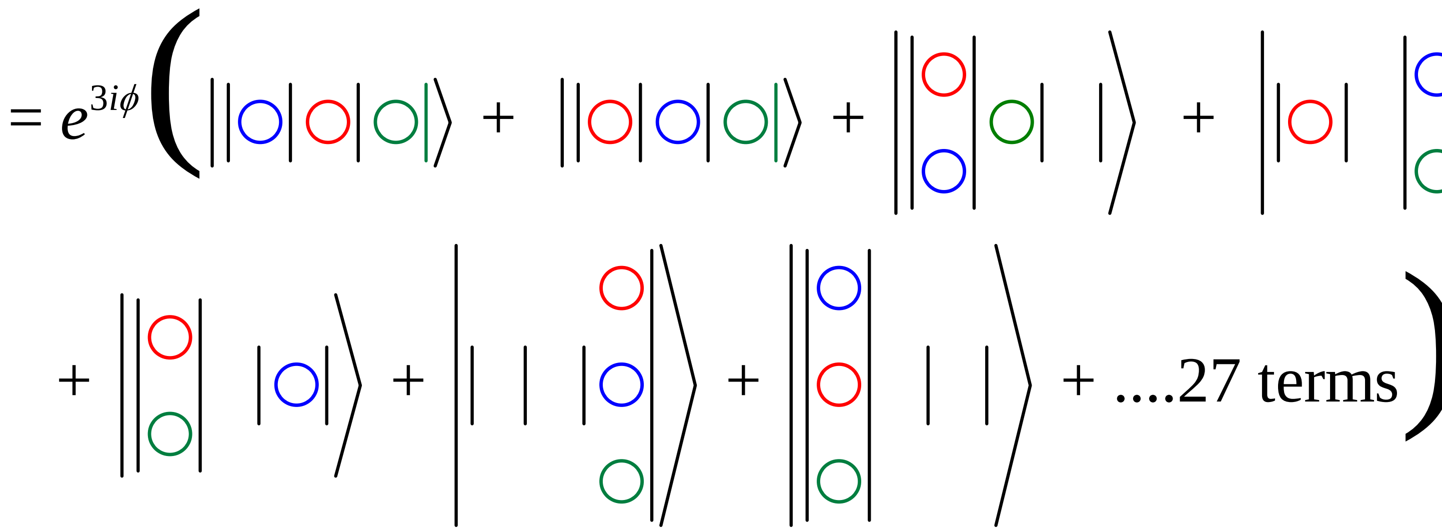
Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



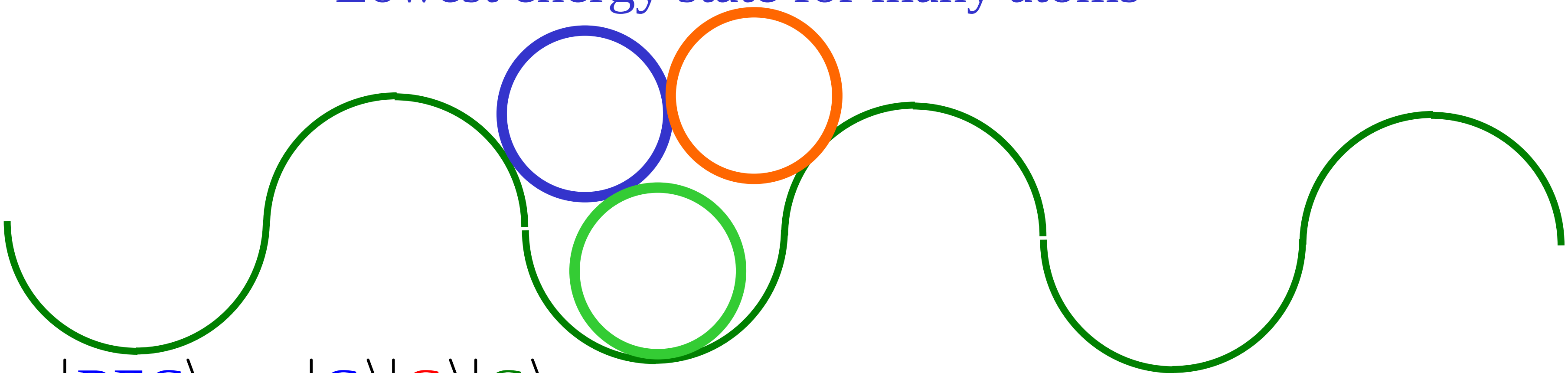
$$|\mathrm{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$



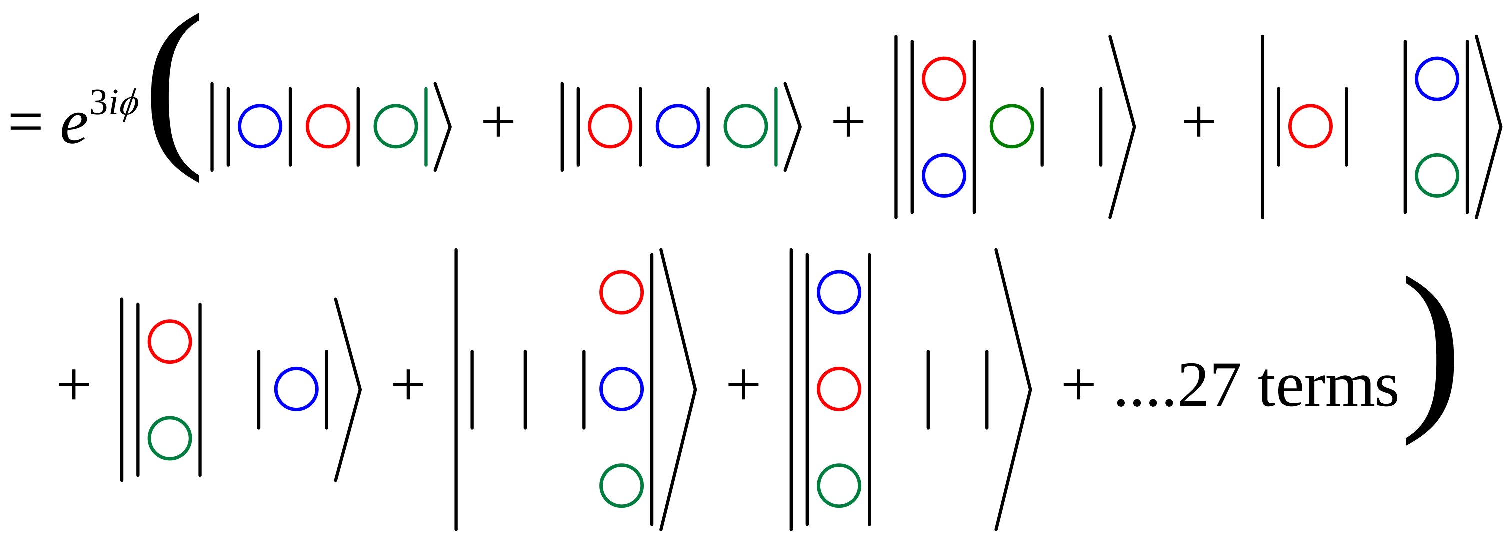
Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can "flow" without dissipation)

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



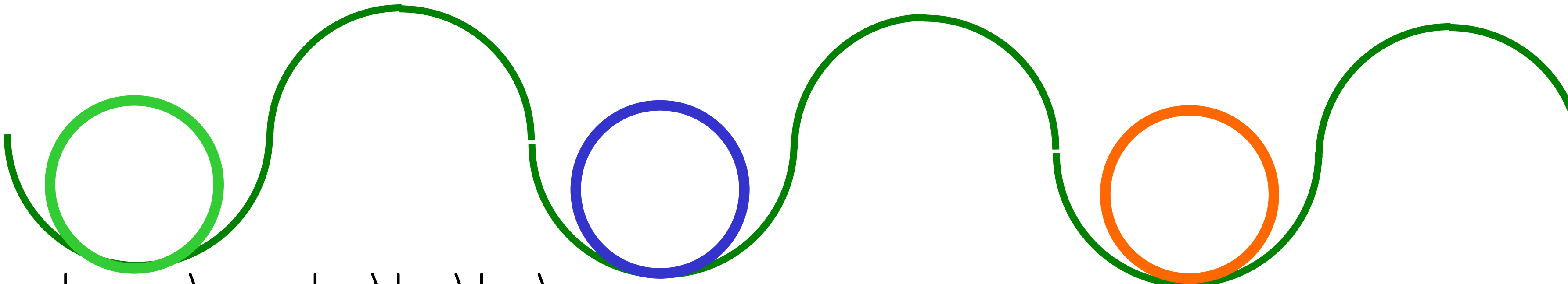
$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$



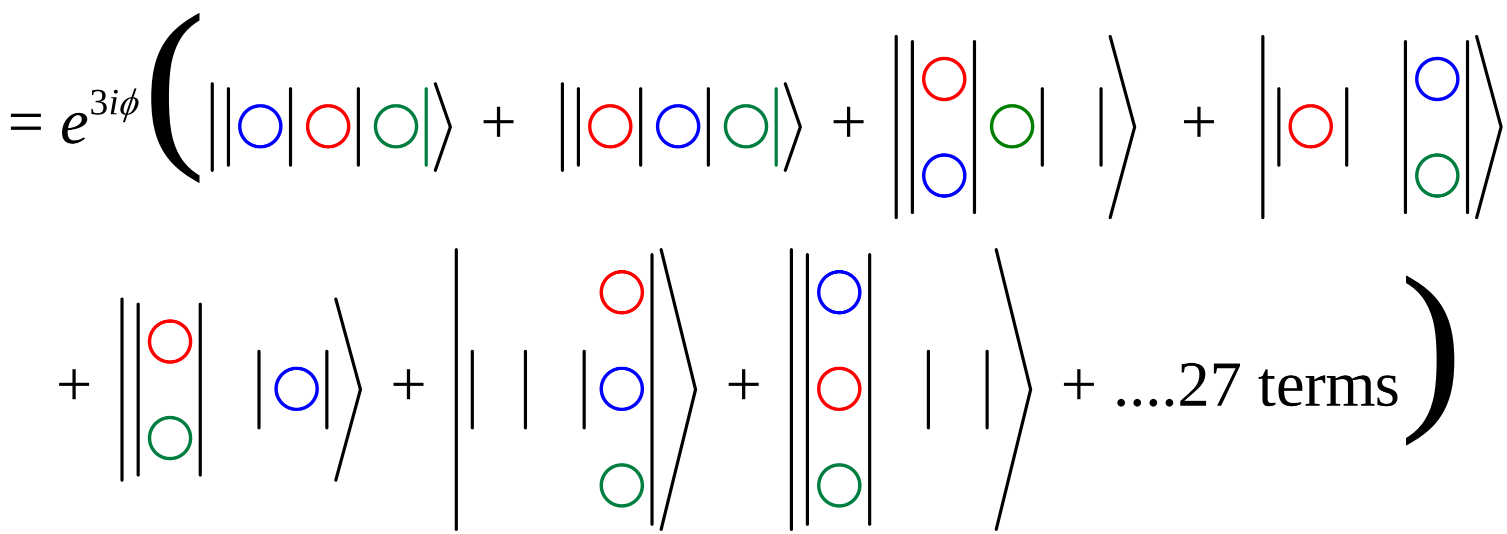
Large fluctuations in number of atoms in each potential well
– *superfluidity* (atoms can "flow" without dissipation)

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



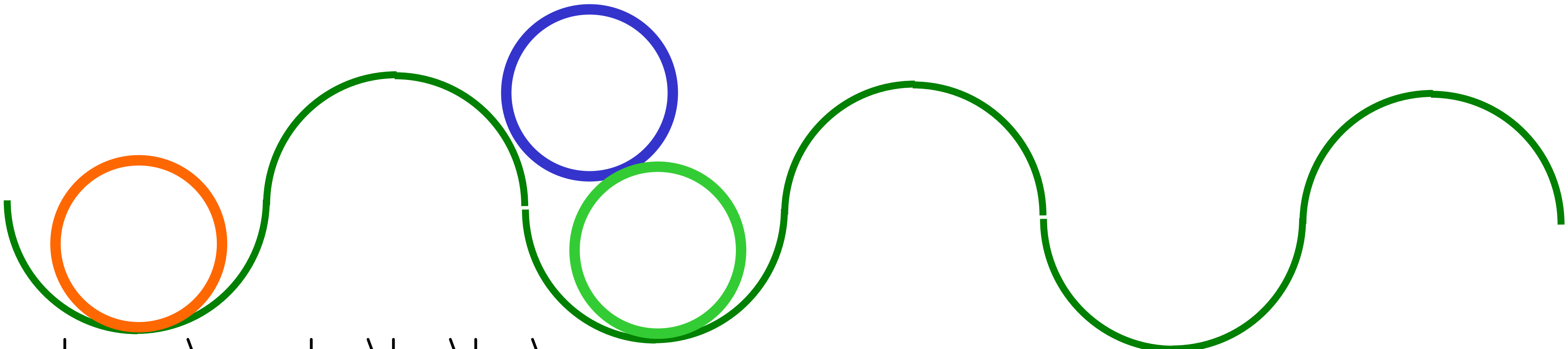
$$|\mathrm{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$



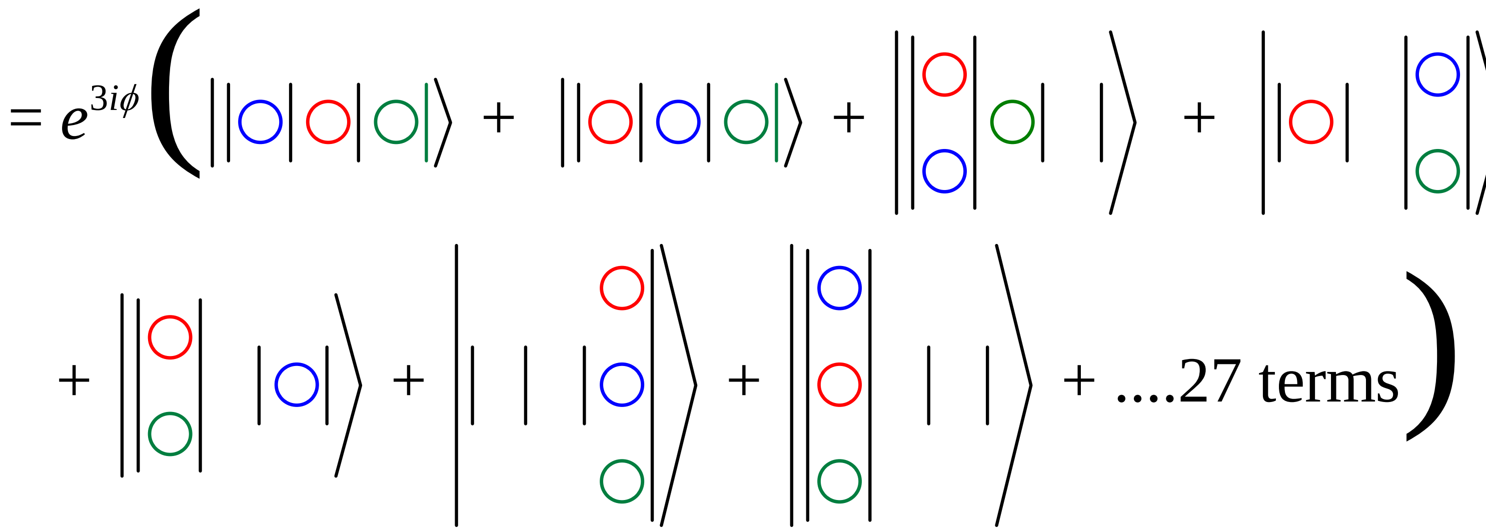
Large fluctuations in number of atoms in each potential well
– *superfluidity* (atoms can “flow” without dissipation)

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



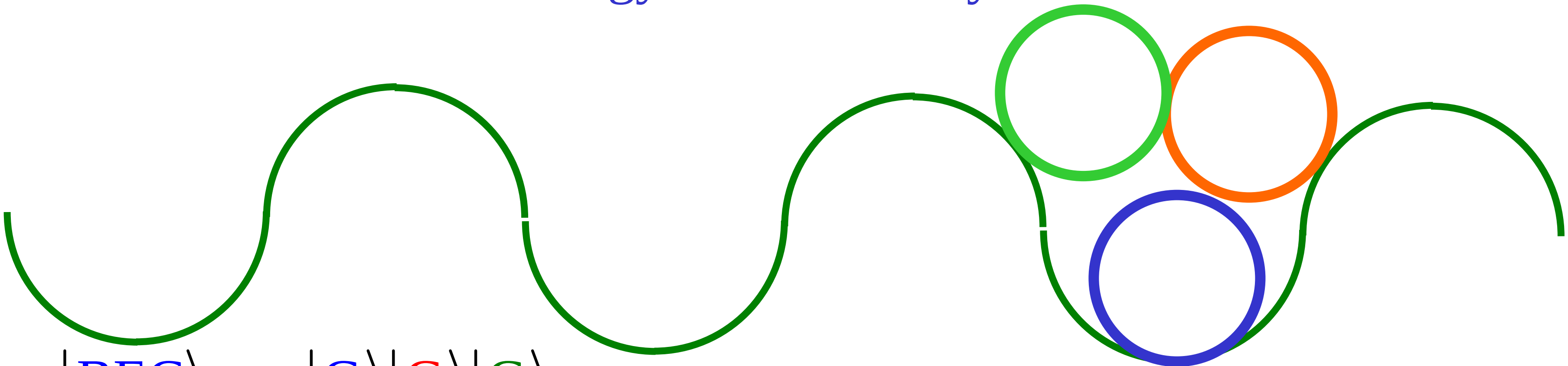
$$|\text{BEC}\rangle = |\text{G}\rangle|\text{G}\rangle|\text{G}\rangle$$



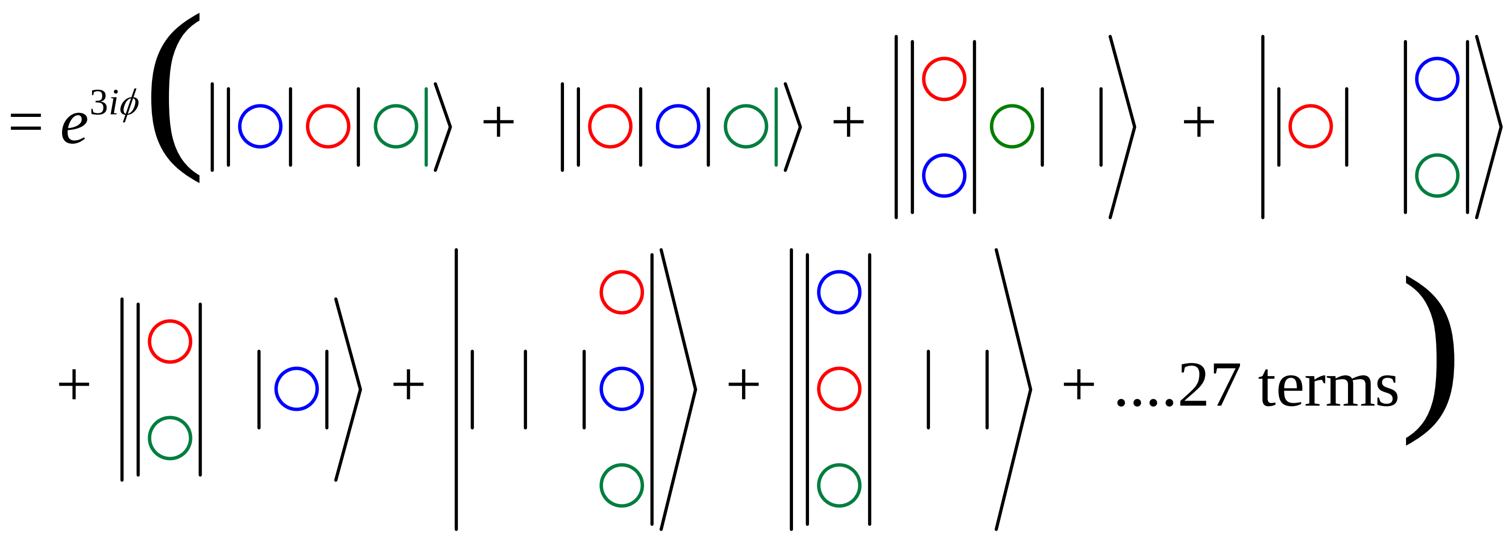
Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can "flow" without dissipation)

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



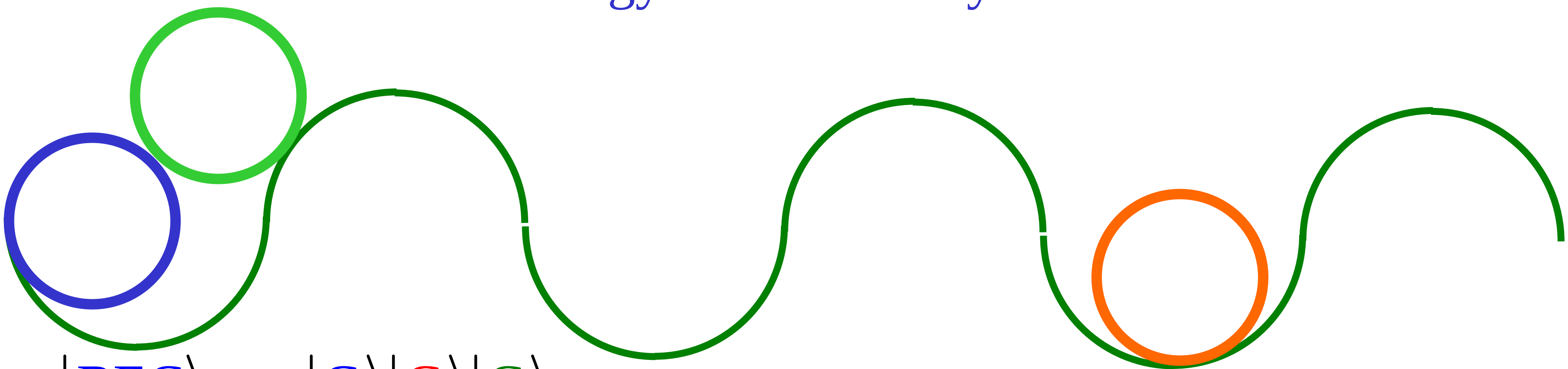
$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$



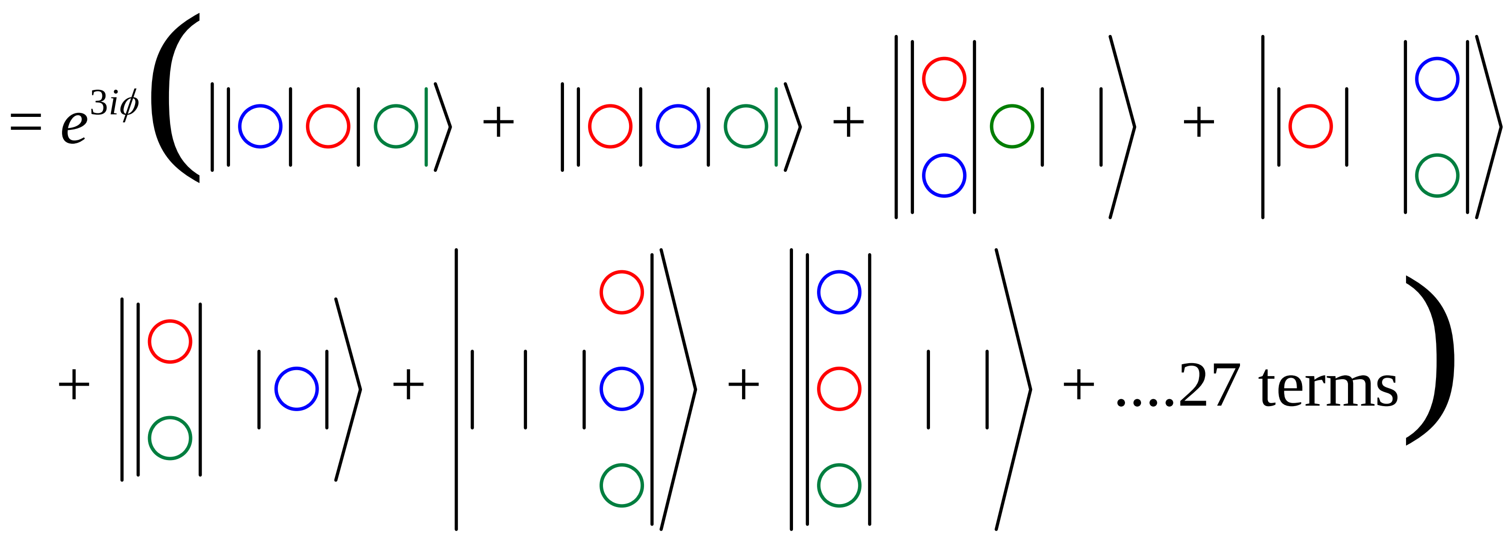
Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



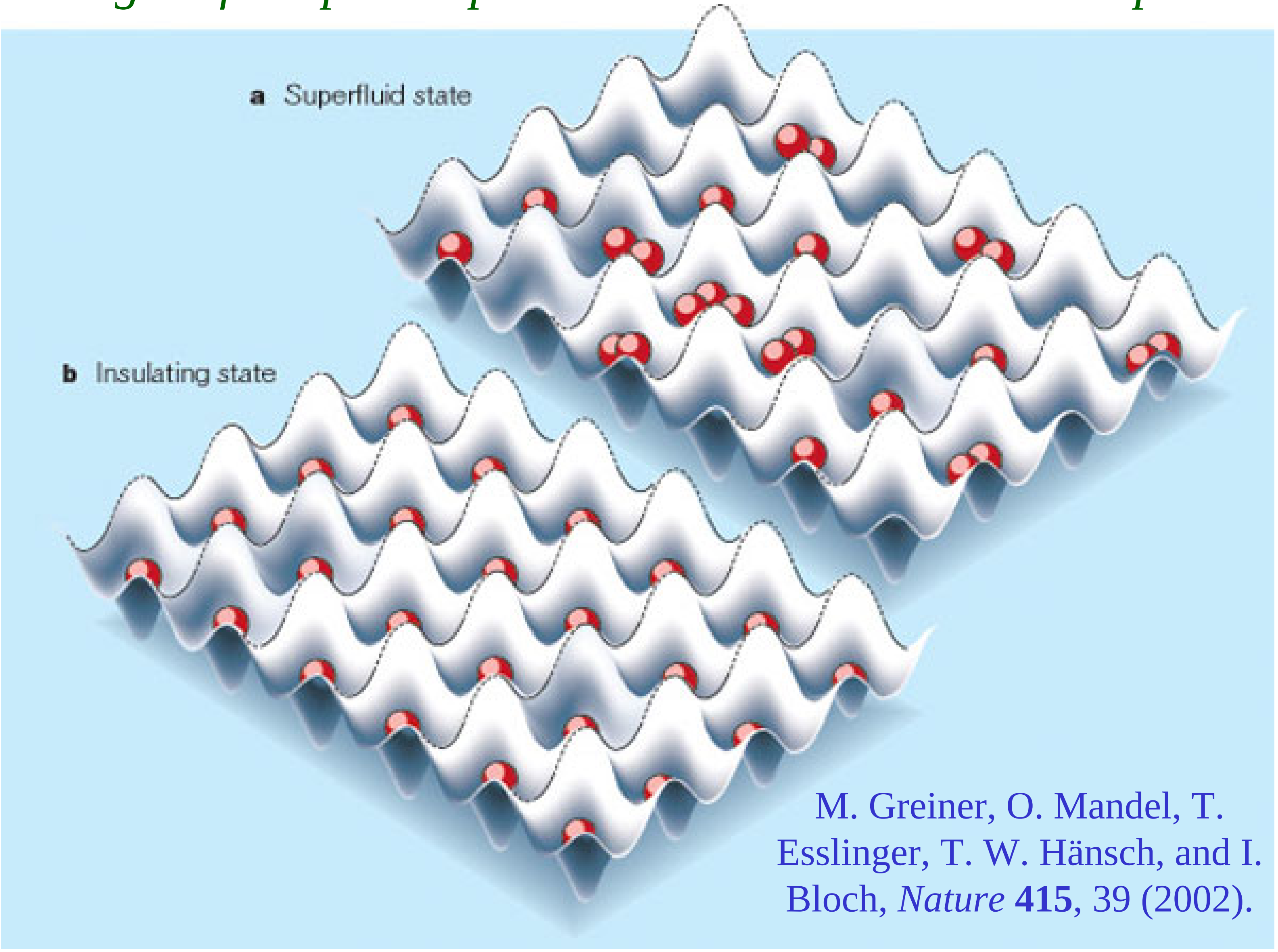
$$|\mathrm{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$



Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can "flow" without dissipation)

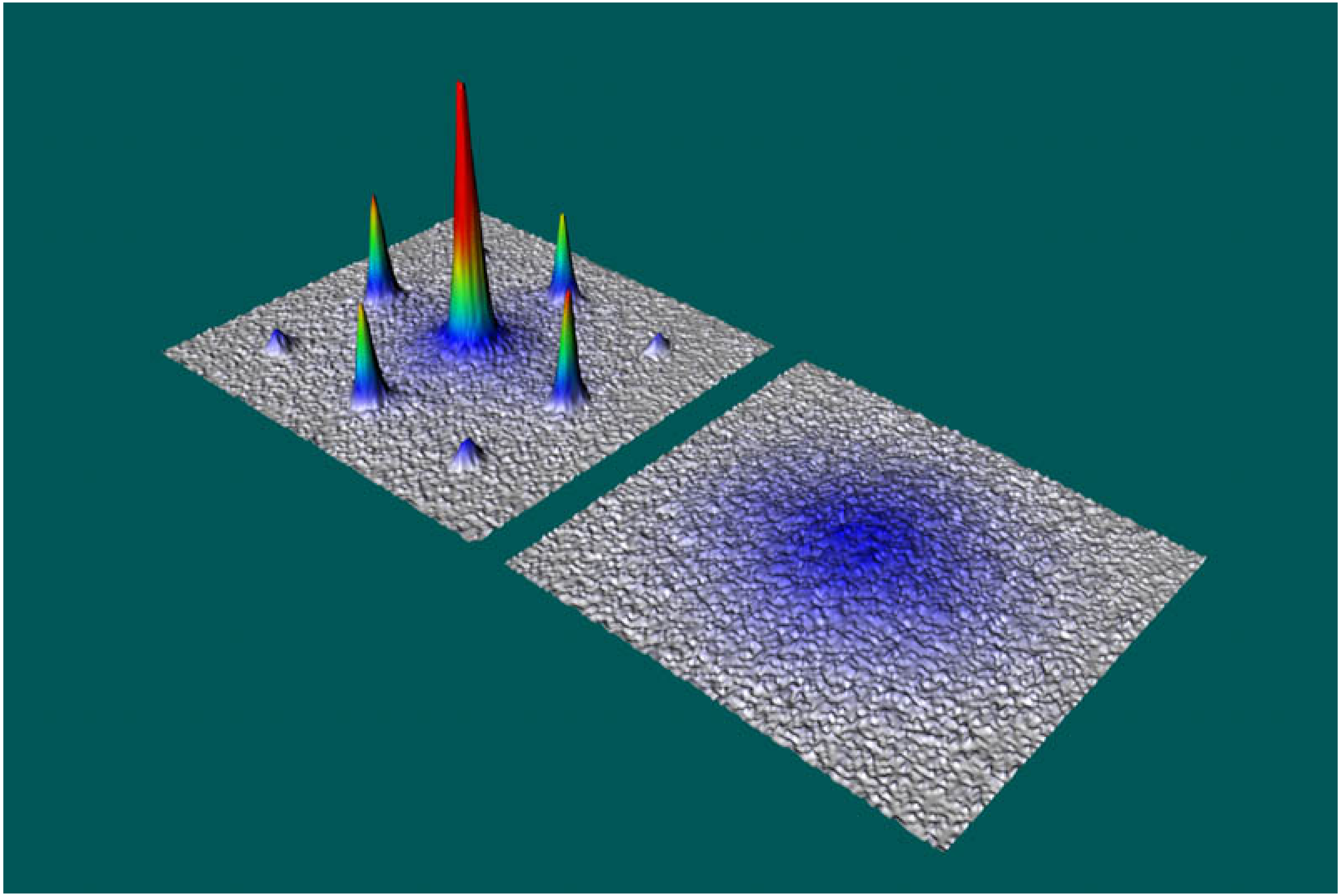
^{87}Rb bosonic atoms in a magnetic trap and an optical lattice potential

The strength of the period potential can be varied in the experiment



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Superfluid-insulator quantum phase transition at T=0



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Understanding superflow

The wavefunction of bosons at rest

$$\Psi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots \boldsymbol{r}_N\right)$$

The wavefunction of bosons flowing with velocity $\boldsymbol{v}_s$

$$e^{i\frac{m}{\hbar}\boldsymbol{v}_s \bullet \left(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 + \ldots + \boldsymbol{r}_N\right)}\Psi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots \boldsymbol{r}_N\right)$$

$$= e^{i\theta(\boldsymbol{r}_1) + i\theta(\boldsymbol{r}_2) + i\theta(\boldsymbol{r}_3) + \ldots + i\theta(\boldsymbol{r}_N)}\Psi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots \boldsymbol{r}_N\right)$$

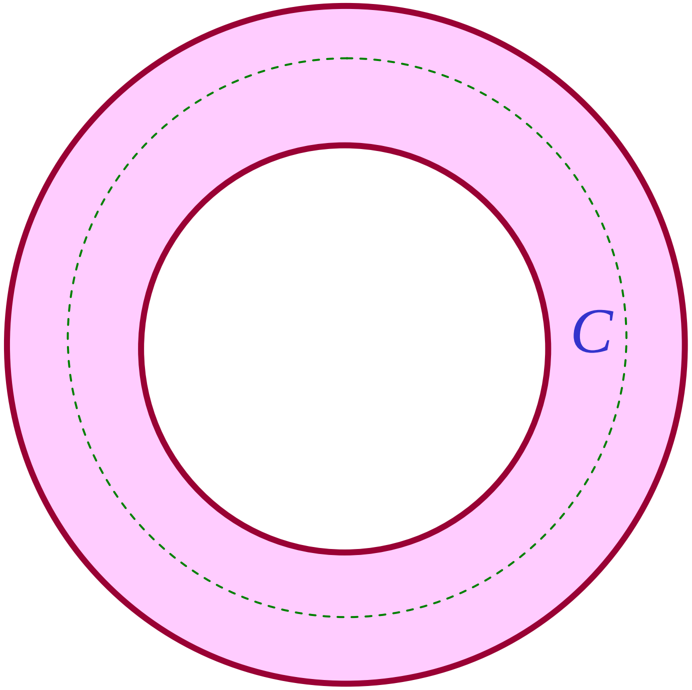
where

$$\boldsymbol{v}_s = \frac{\hbar}{m}\nabla\theta$$

(for non-Galilean invariant superfluids, the co-efficient of $\nabla\theta$ is modified)

Understanding superflow

Persistent currents



$$\oint_C \nabla \theta \, dx = 2\pi n$$

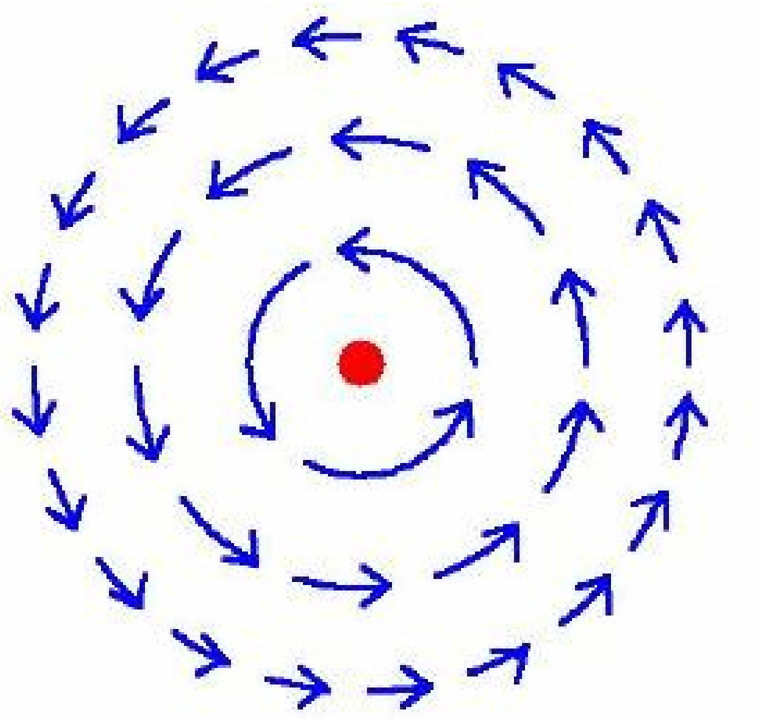
No local change of the wavefunction can change the value of *n*

Supercurrent flows "forever"

Vortices in the superfluid

Magnus forces, duality, and point vortices as dual "electric" charges

Excitations of the superfluid: **Vortices**

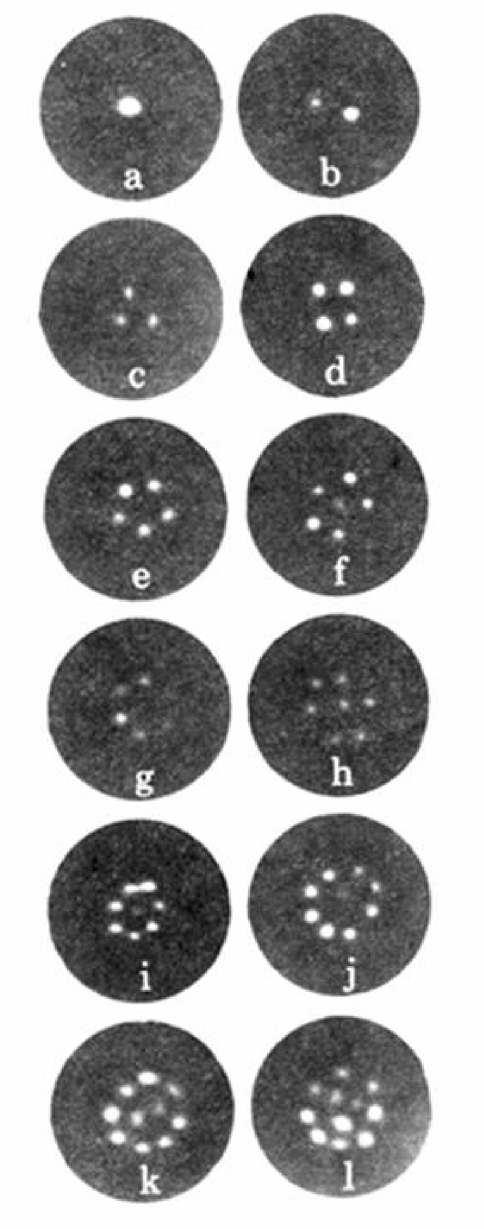


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

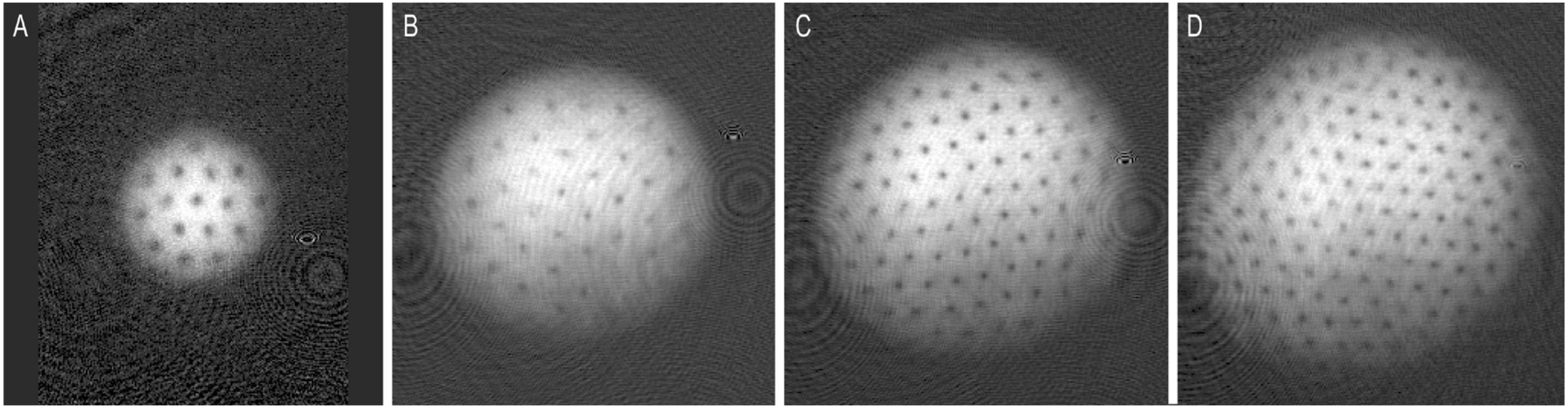
where n is an integer.

Observation of quantized vortices in rotating ^{4}He



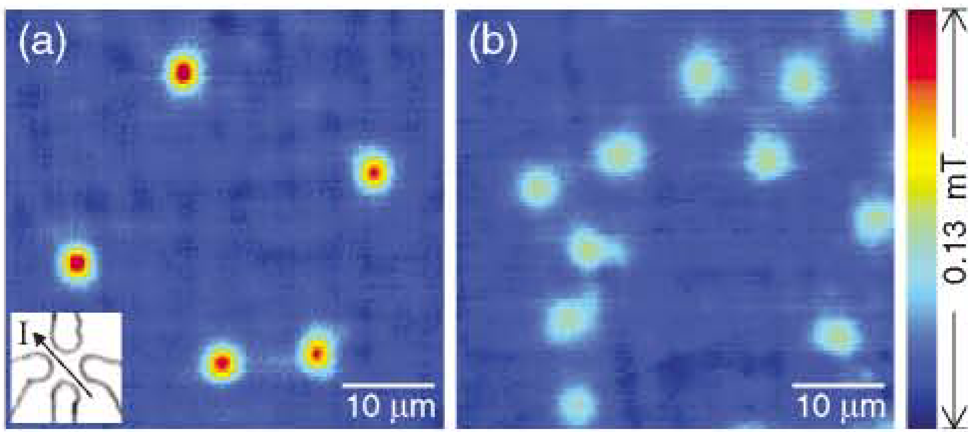
E.J. Yarmchuk, M.J.V. Gordon, and R.E. Packard, *Observation of Stationary Vortex Arrays in Rotating Superfluid Helium*, Phys. Rev. Lett. **43**, 214 (1979).

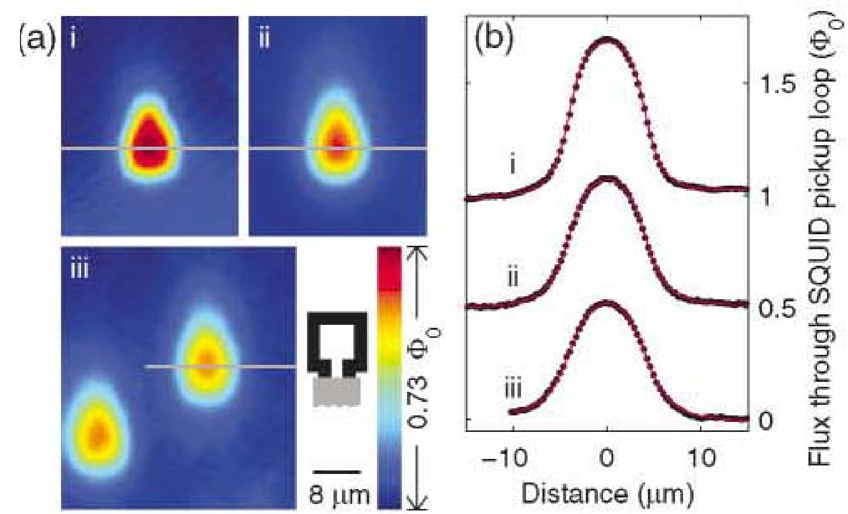
Observation of quantized vortices in rotating ultracold Na



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle,
Observation of Vortex Lattices in Bose-Einstein Condensates,
Science **292**, 476 (2001).

Quantized fluxoids in $YBa_2Cu_3O_{6+y}$



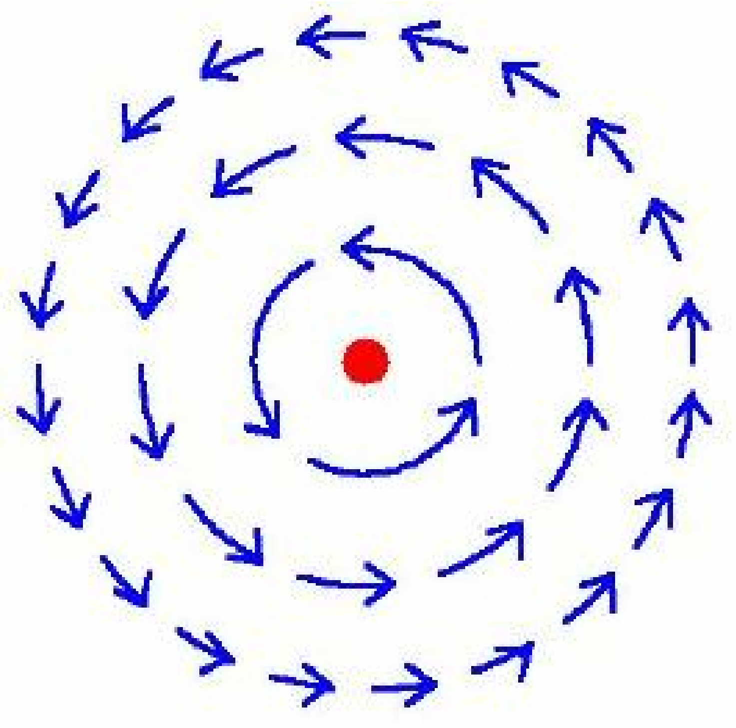


J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett*. **87**, 197002 (2001).

In superconductors, vortices carry quantized magnetic flux:

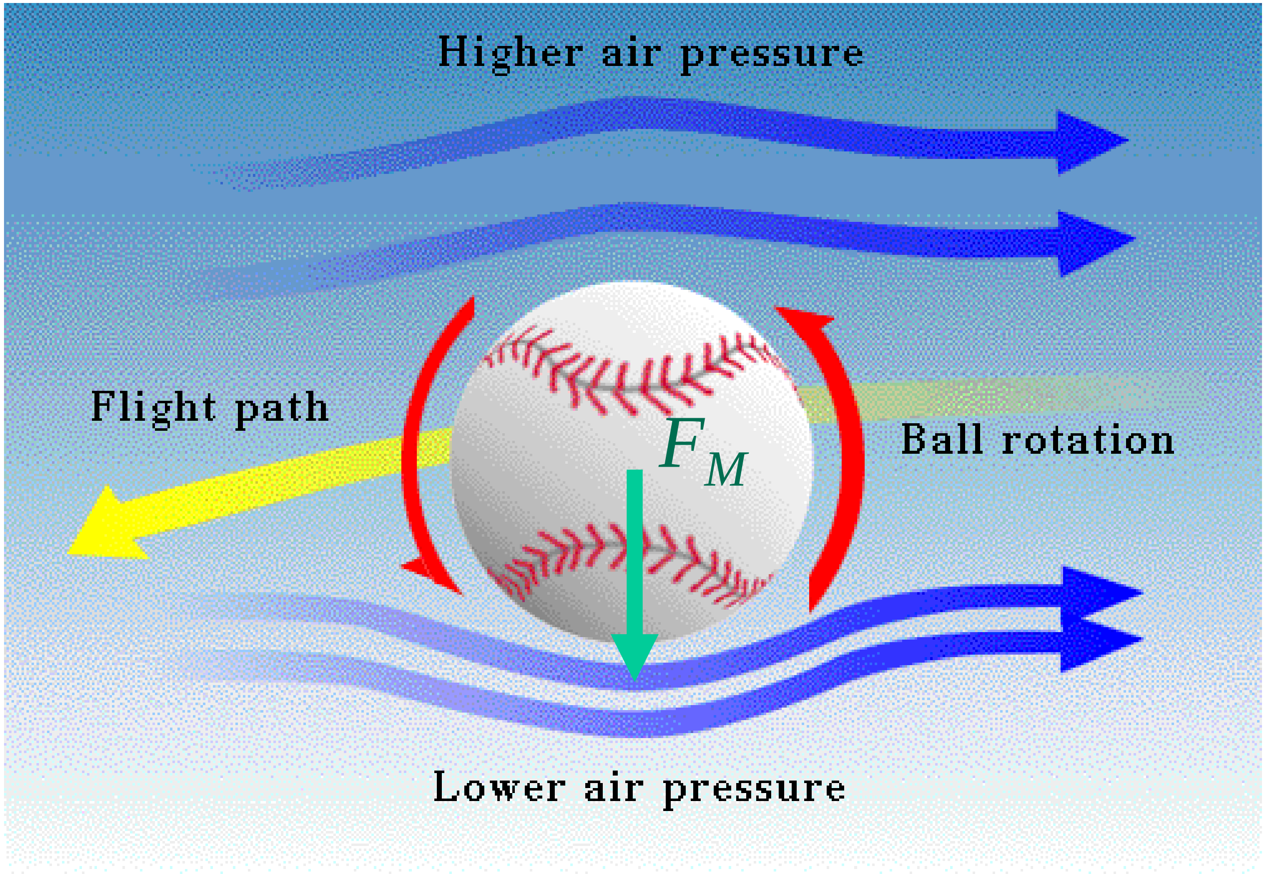
$$\int \mathbf{B} \cdot d\mathbf{S} = n\frac{hc}{2e}$$

Excitations of the superfluid: **Vortices**



<u>Central question</u>:
In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these "particles" ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \bullet (\text{velocity of ball}) \bullet (\text{circulation})$$

For a vortex in a superfluid, this is

$$
\begin{aligned}
\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right) \\
&= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}
\end{aligned}
$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vortex in a superfluid, this is

$$
\begin{aligned}
\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s\cdot d\mathbf{r}\right) \\
&= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}} \\
&= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt}\times\mathbf{B}\right)
\end{aligned}
$$

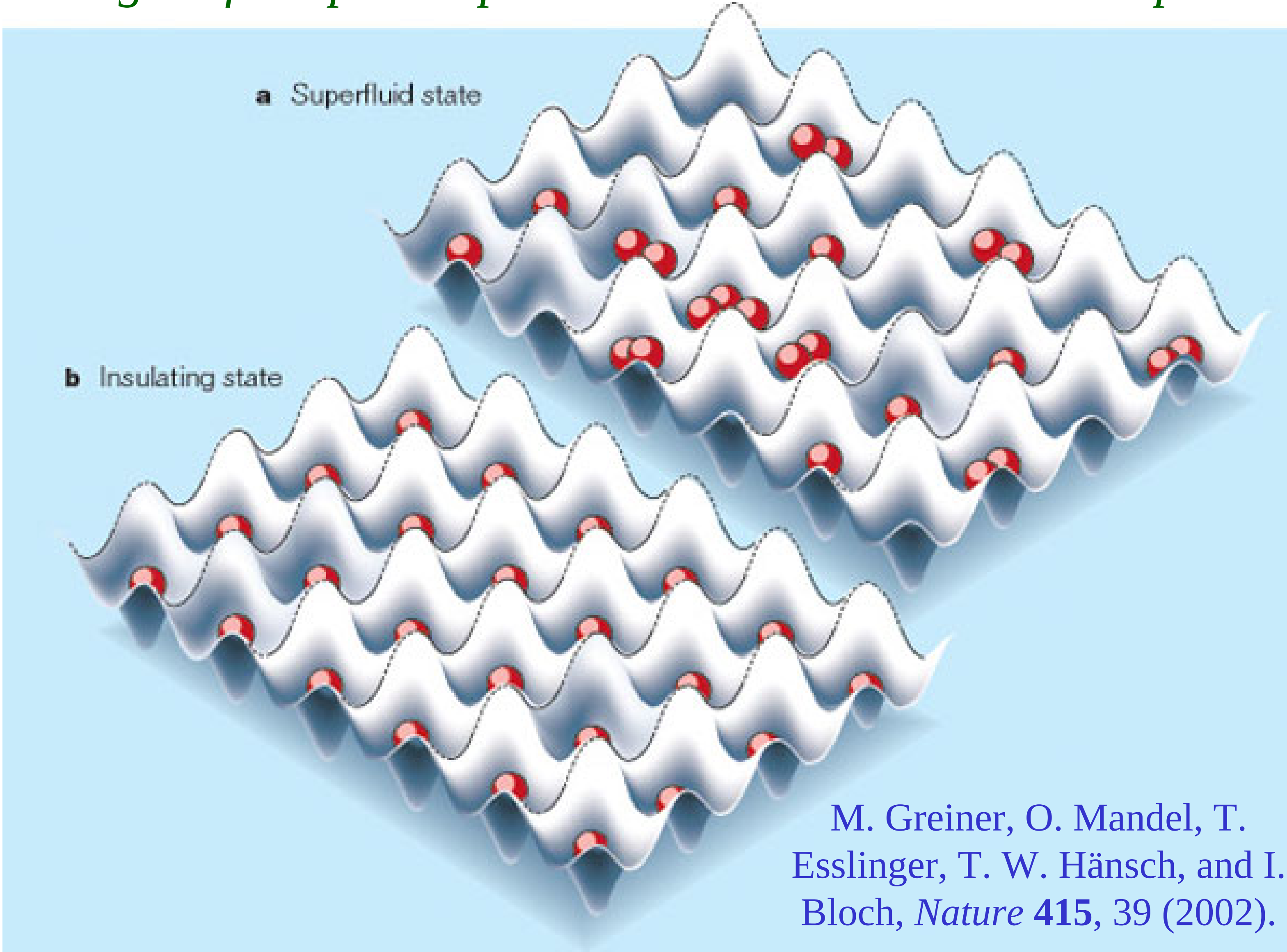
where $\mathbf{E} = \rho\mathbf{v}_s\times\hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

Dual picture:

The vortex is a quantum particle with dual "electric" charge n, moving in a dual "magnetic" field of strength = h×(number density of Bose particles)

^{87}Rb bosonic atoms in a magnetic trap and an optical lattice potential

The strength of the period potential can be varied in the experiment



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Upon approaching the insulator, the phase of the condensate becomes “uncertain”.

Vortices cost less energy and vortex-anti-vortex pairs proliferate.

The quantum mechanics of vortices plays a central role in the superfluid-insulator quantum phase transition.