What can gauge-gravity duality teach us about condensed matter physics ?

Twelfth Arnold Sommerfeld Lecture Series
January 31 - February 3, 2012

sachdev.physics.harvard.edu

PHYSICS
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Rob Myers

Ajay Singh

Liza Huijse

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Brian Swingle

1. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

2. Strange metals

A. Field theory

B. Holography

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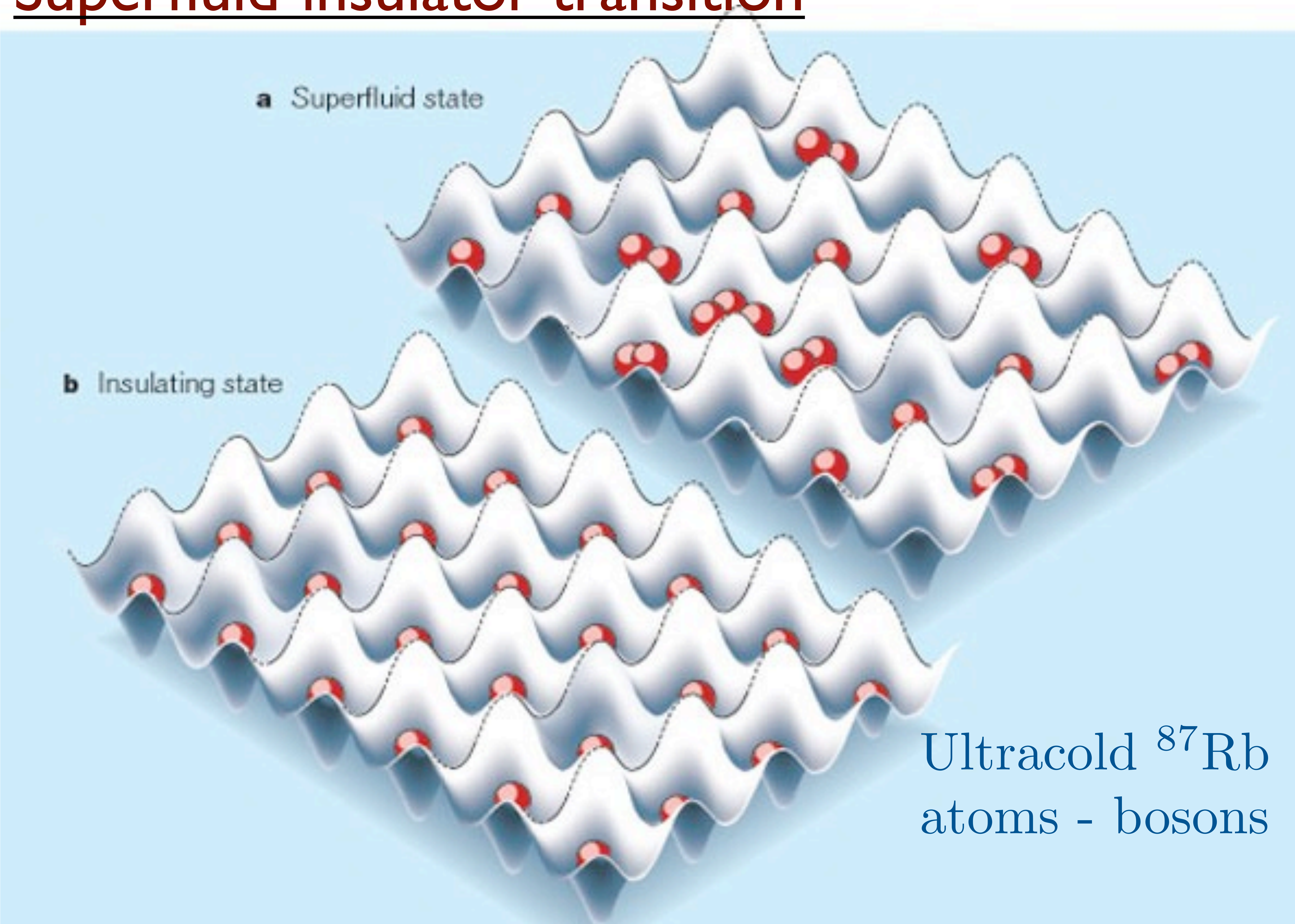
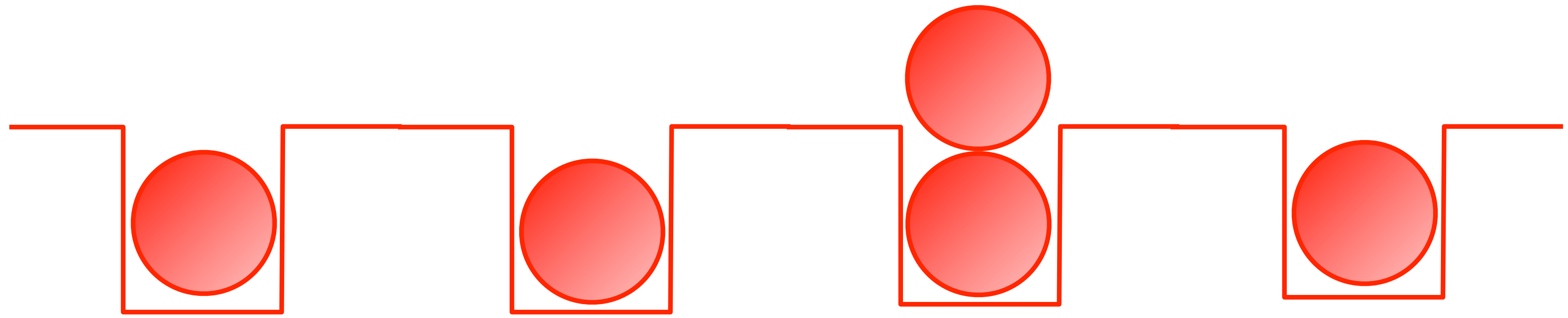
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Excitations of the insulator:



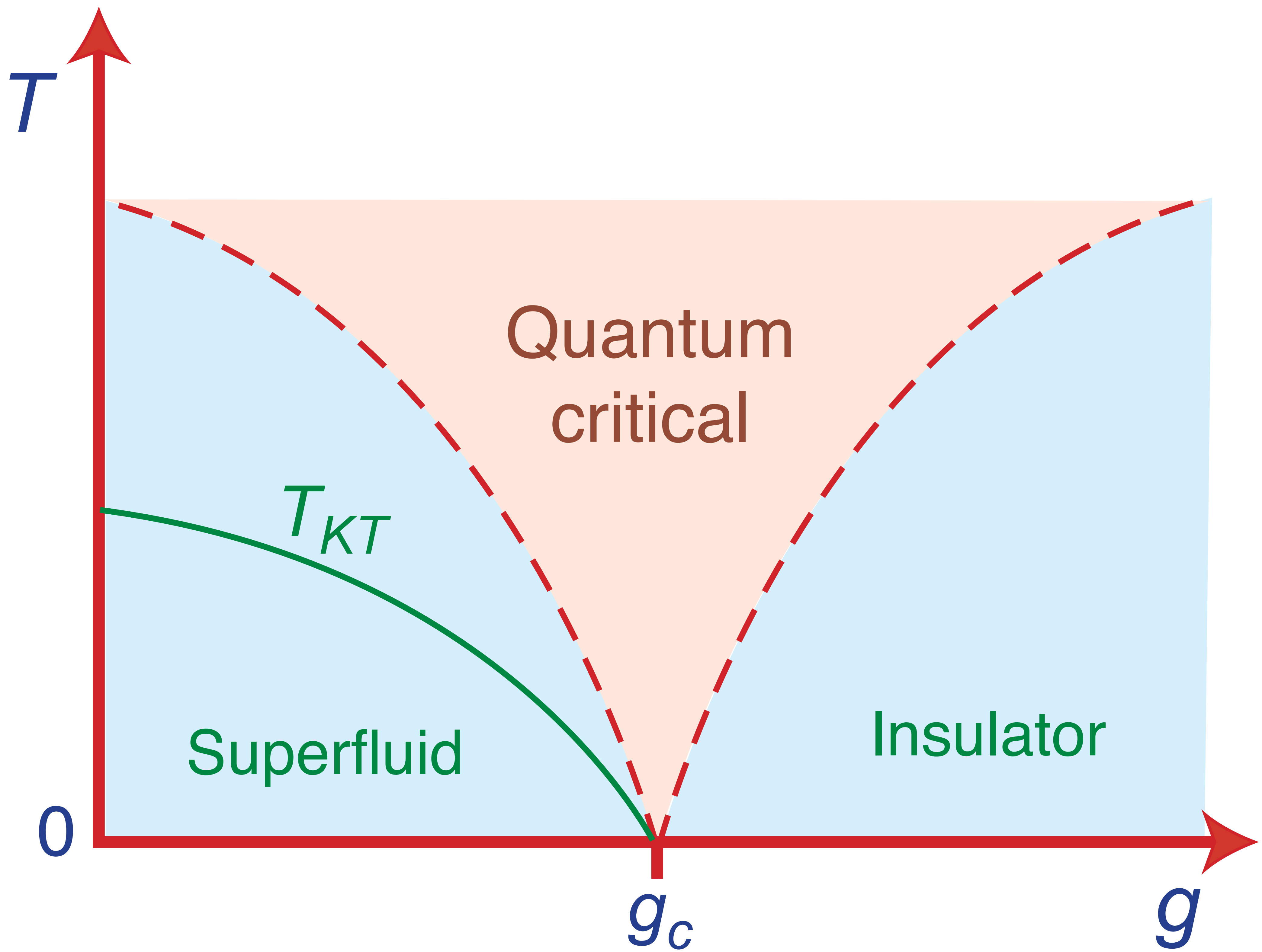
Particles $\sim \psi^\dagger$

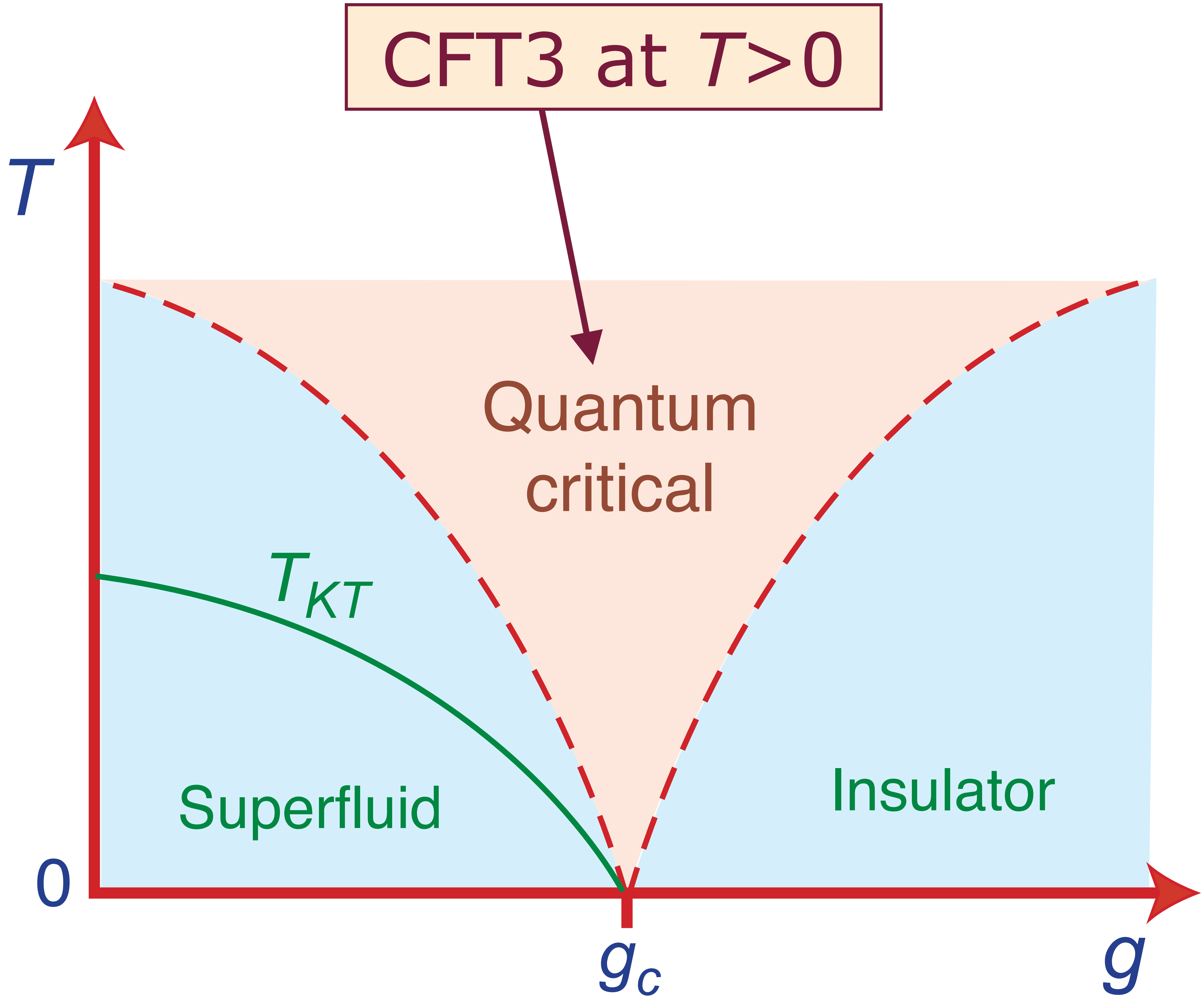
Holes $\sim \psi$

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT3 at $T>0$
T
Quantum
critical
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Superfluid
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0
g_c
g

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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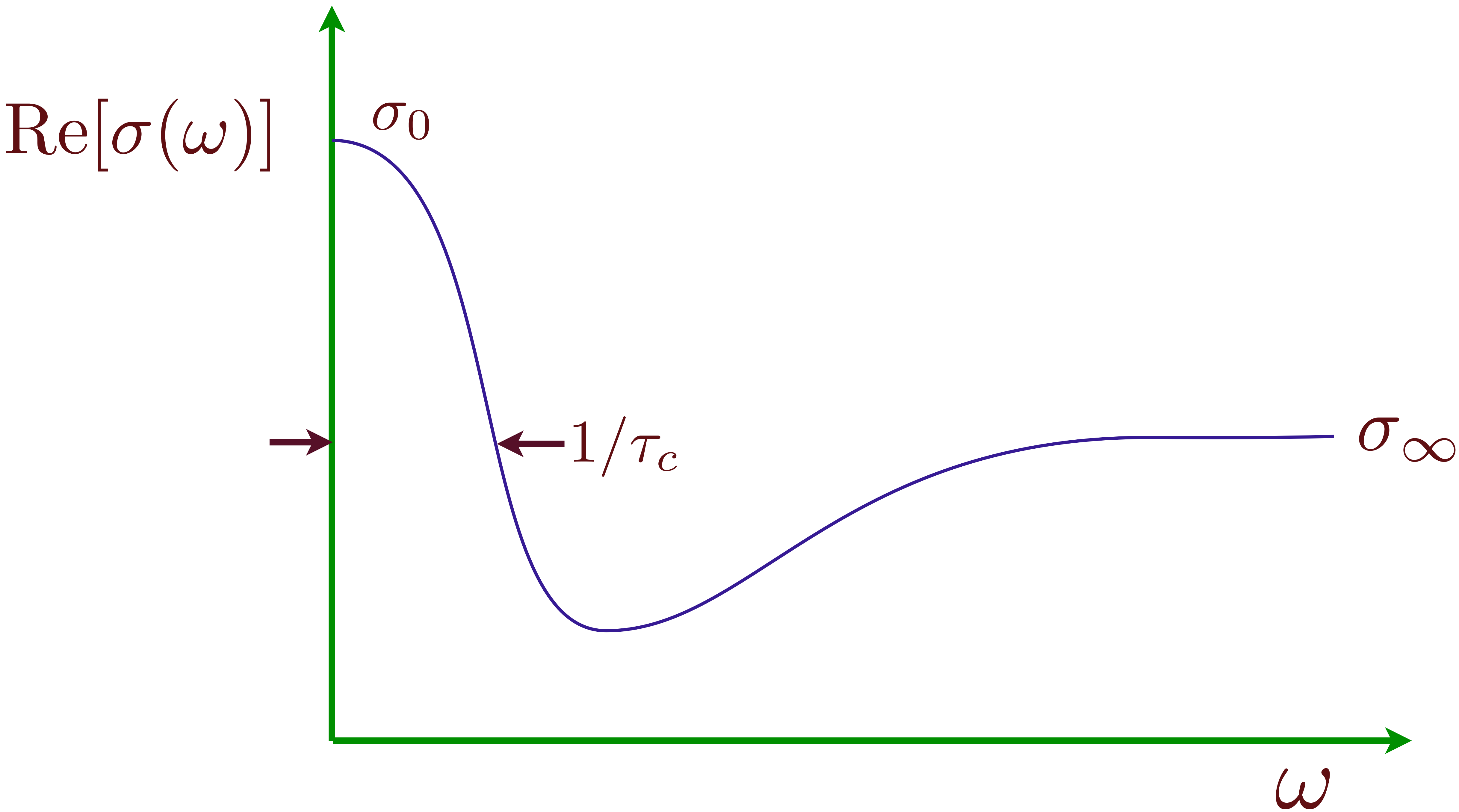
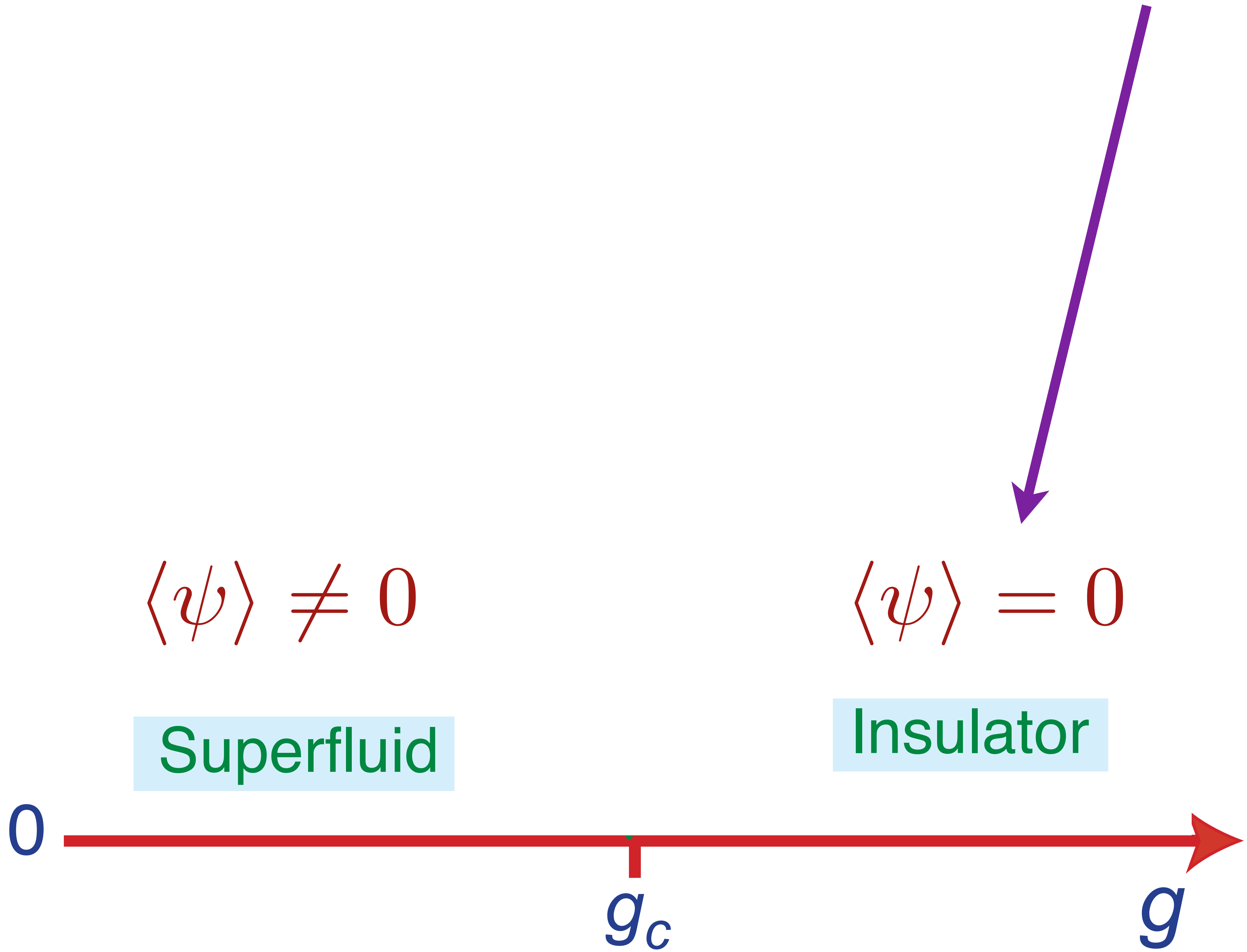
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

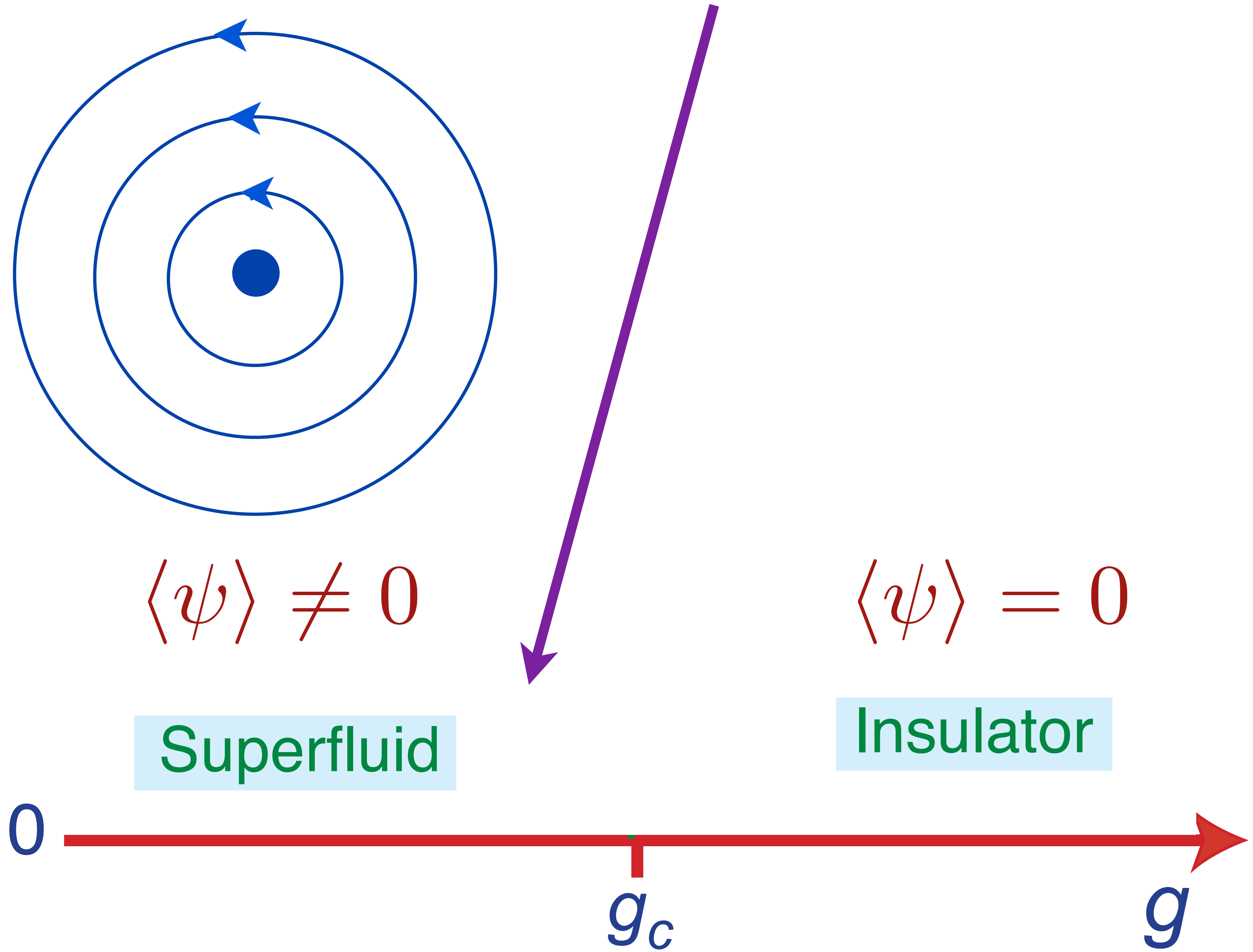
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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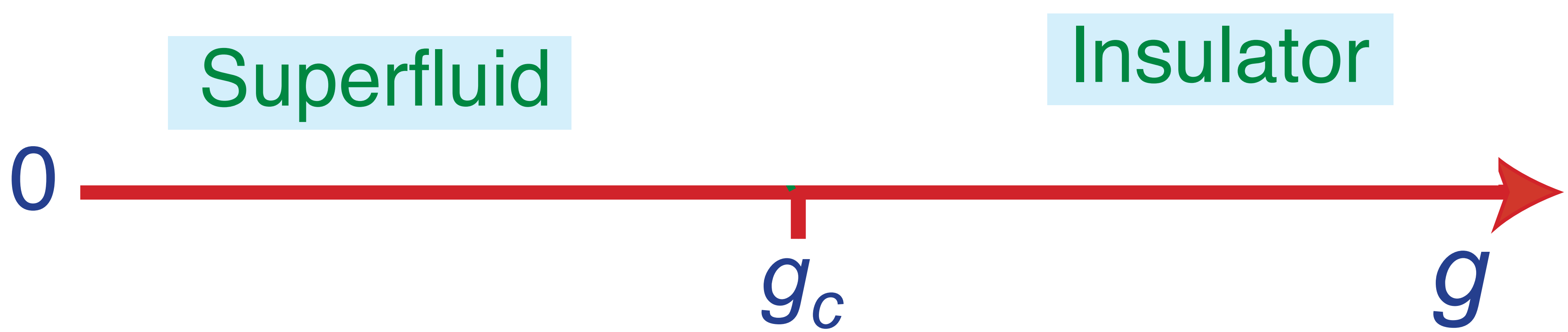
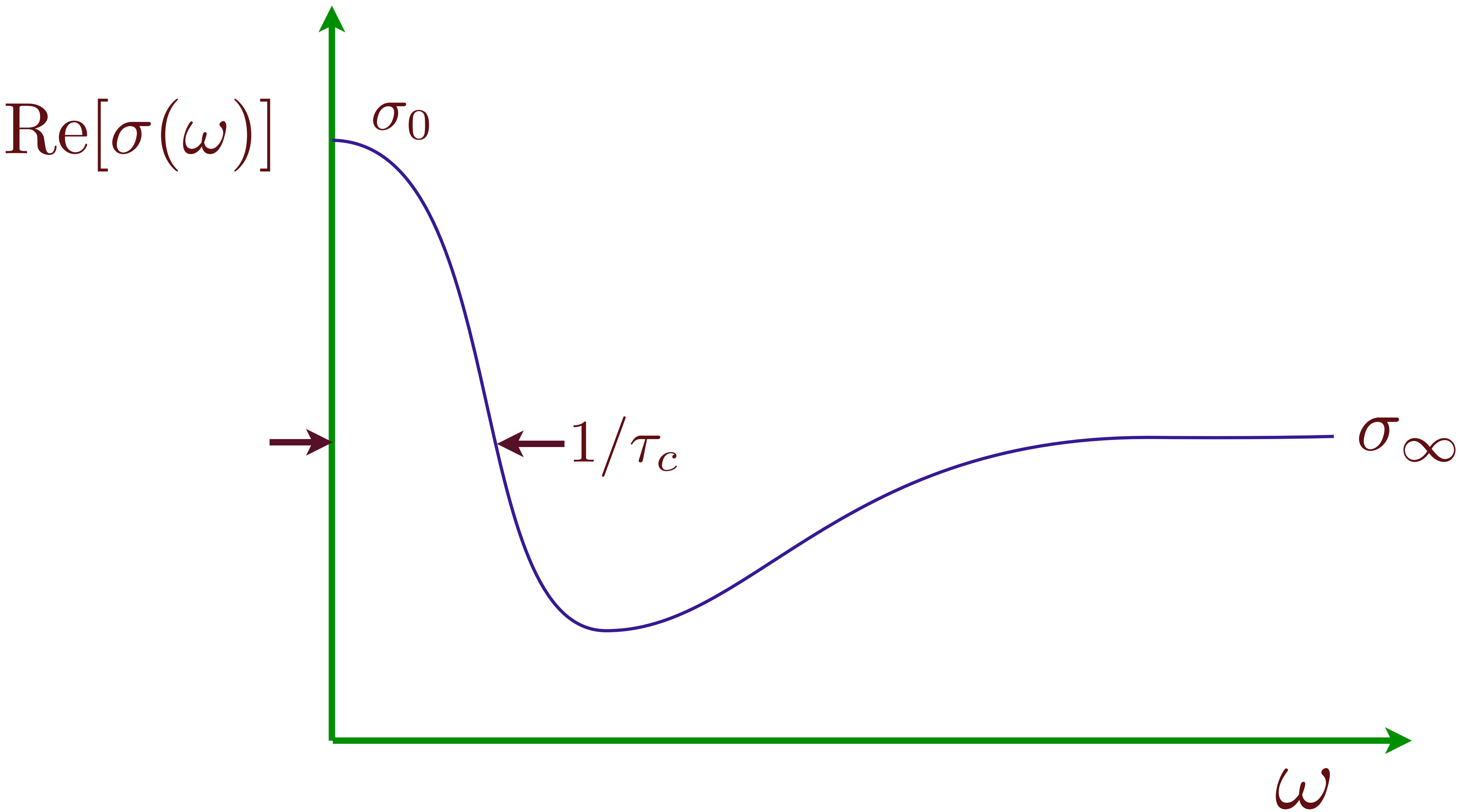
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) "dual" CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

Conductivity = Resistivity of vortices

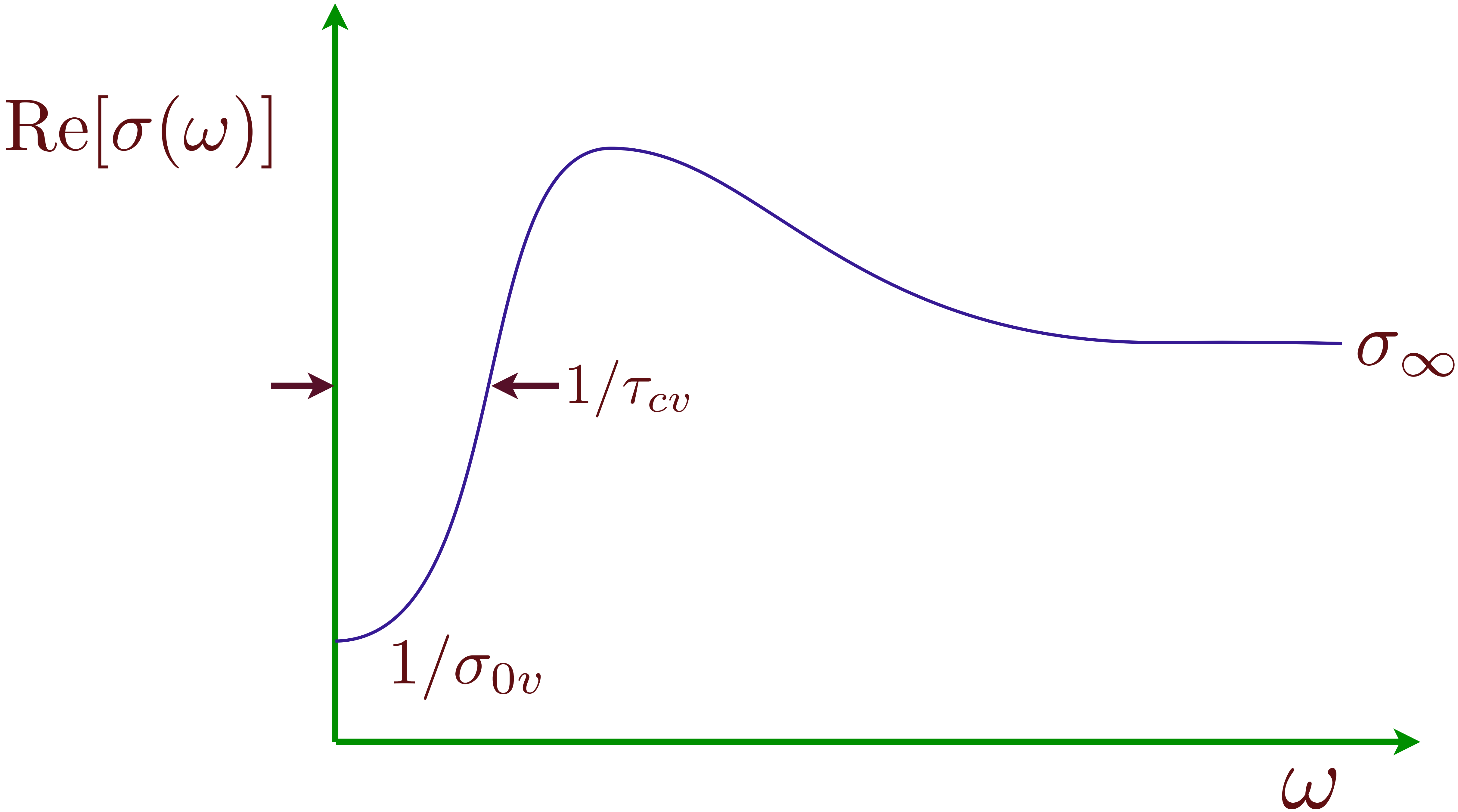
$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$



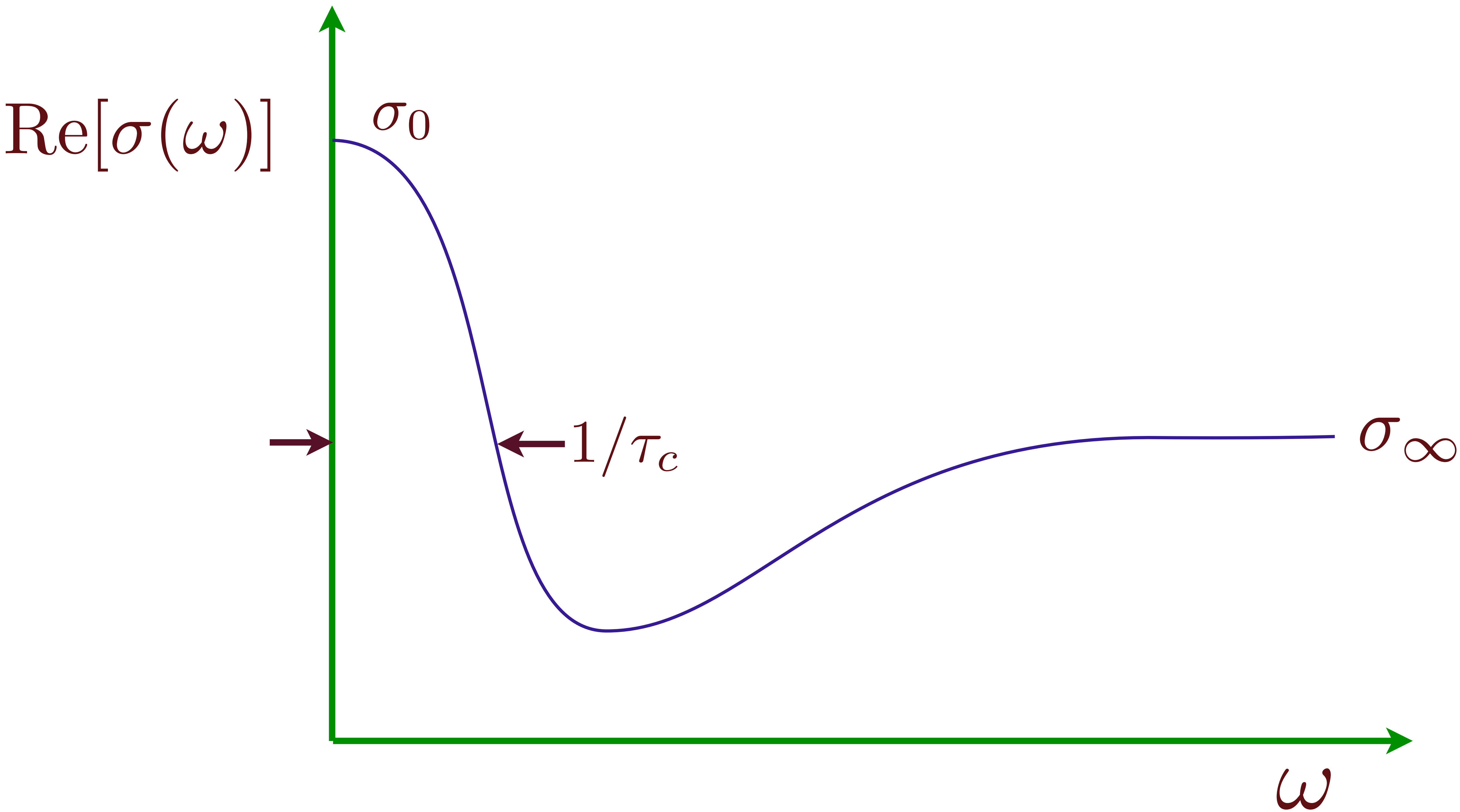
Boltzmann theory of bosons



Boltzmann theory of vortices



Boltzmann theory of bosons



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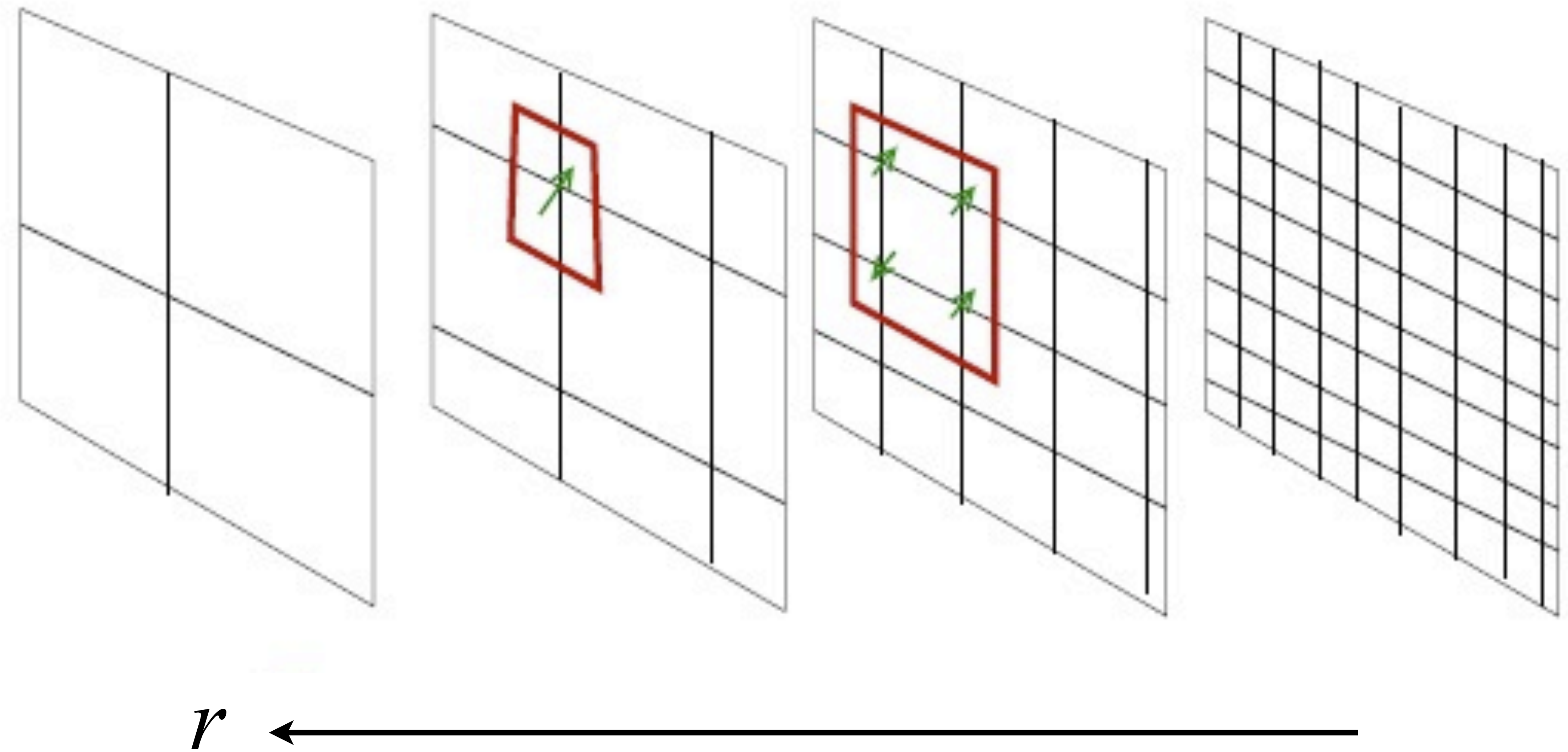
Why AdS_{d+2} ?



J. McGreevy, arXiv0909.0518

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For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS$_{d+2}$.

AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

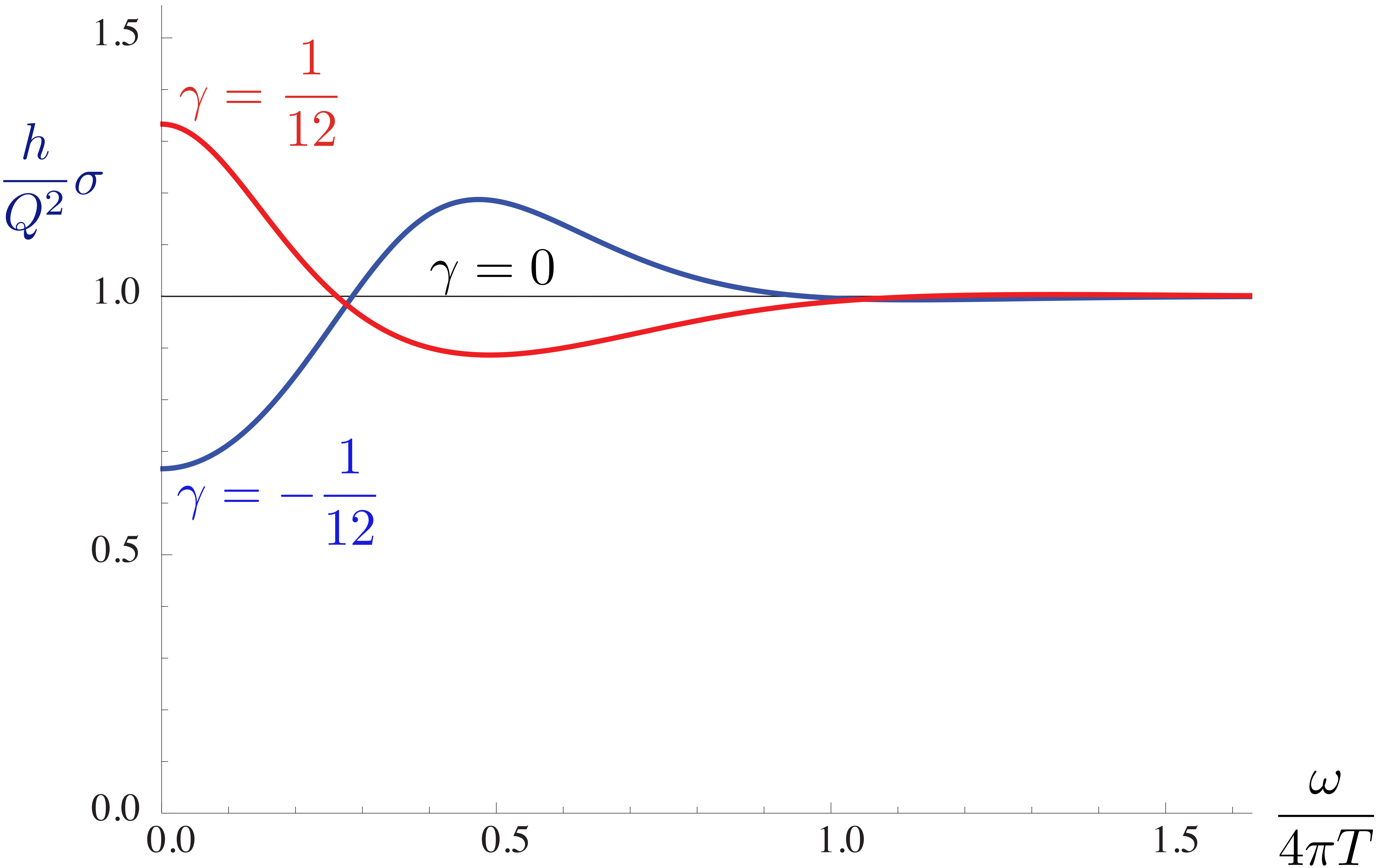
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S}_{EM} = \int d^4x\sqrt{-g}\left[-\frac{1}{4e^2}F_{ab}F^{ab} + \frac{\gamma L^2}{e^2}C_{abcd}F^{ab}F^{cd}\right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.
The parameters e^2 and γ can be determined from OPE coefficients of CFT of interest.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



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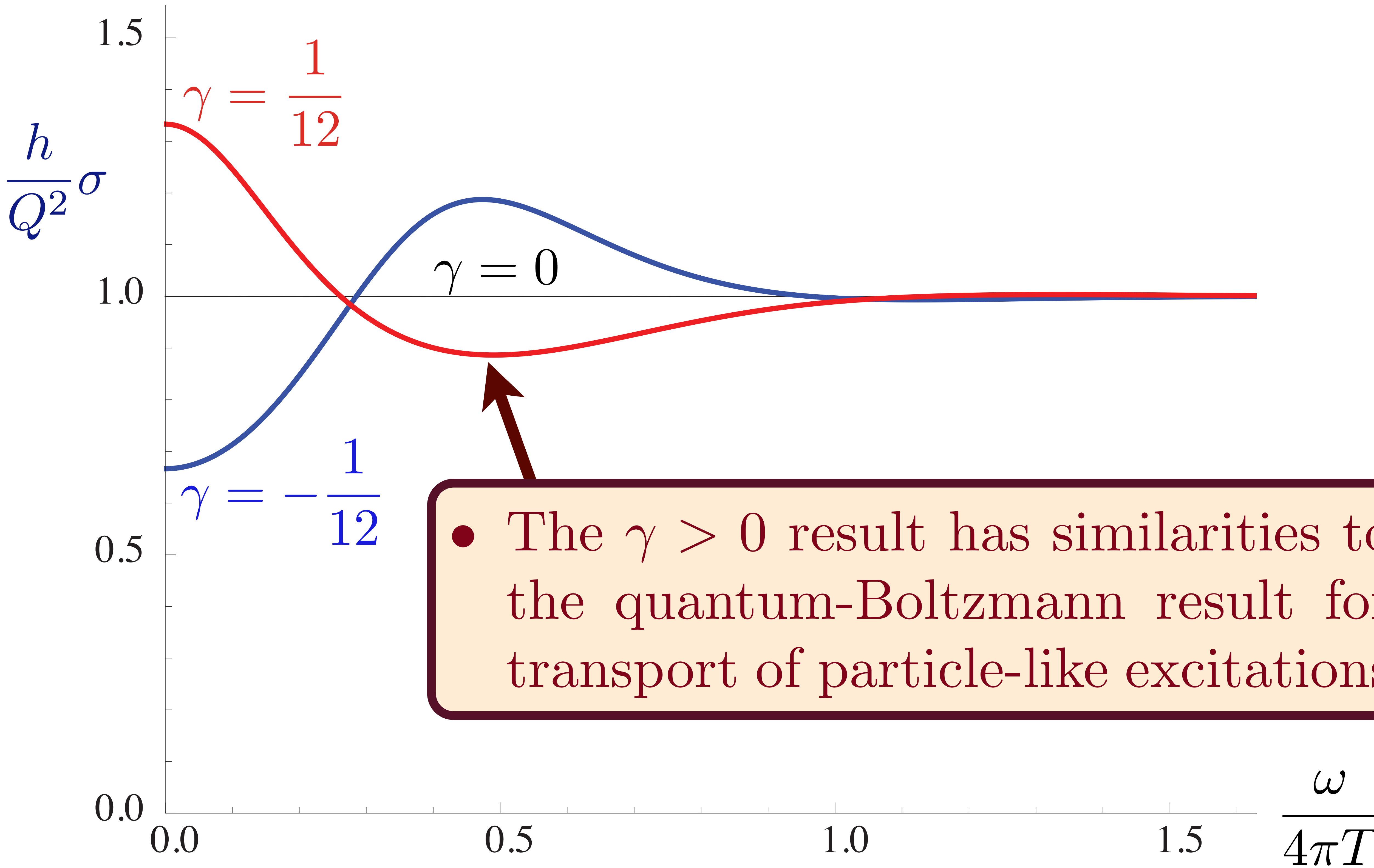

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

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Thursday, February 2, 2012

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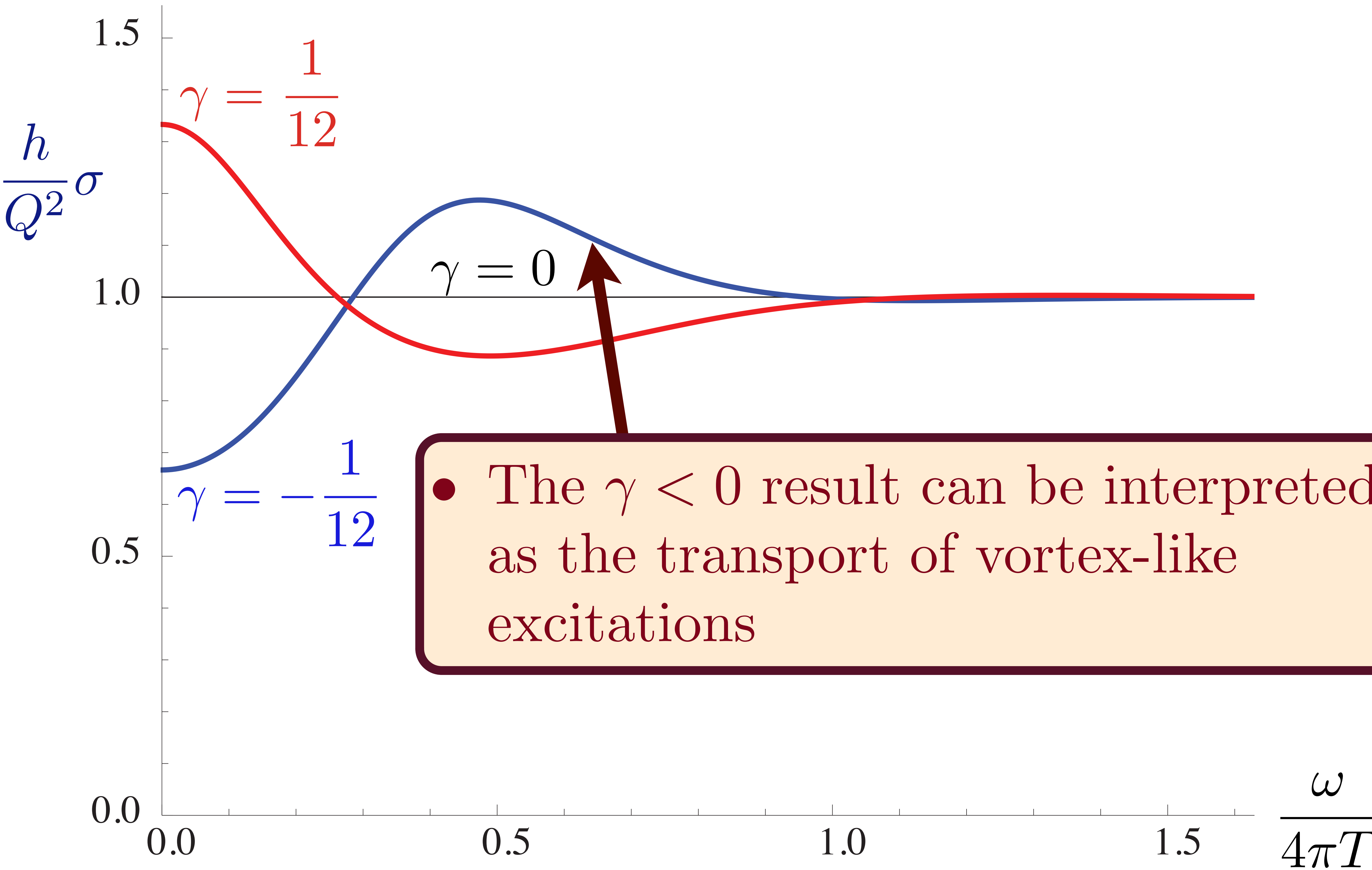
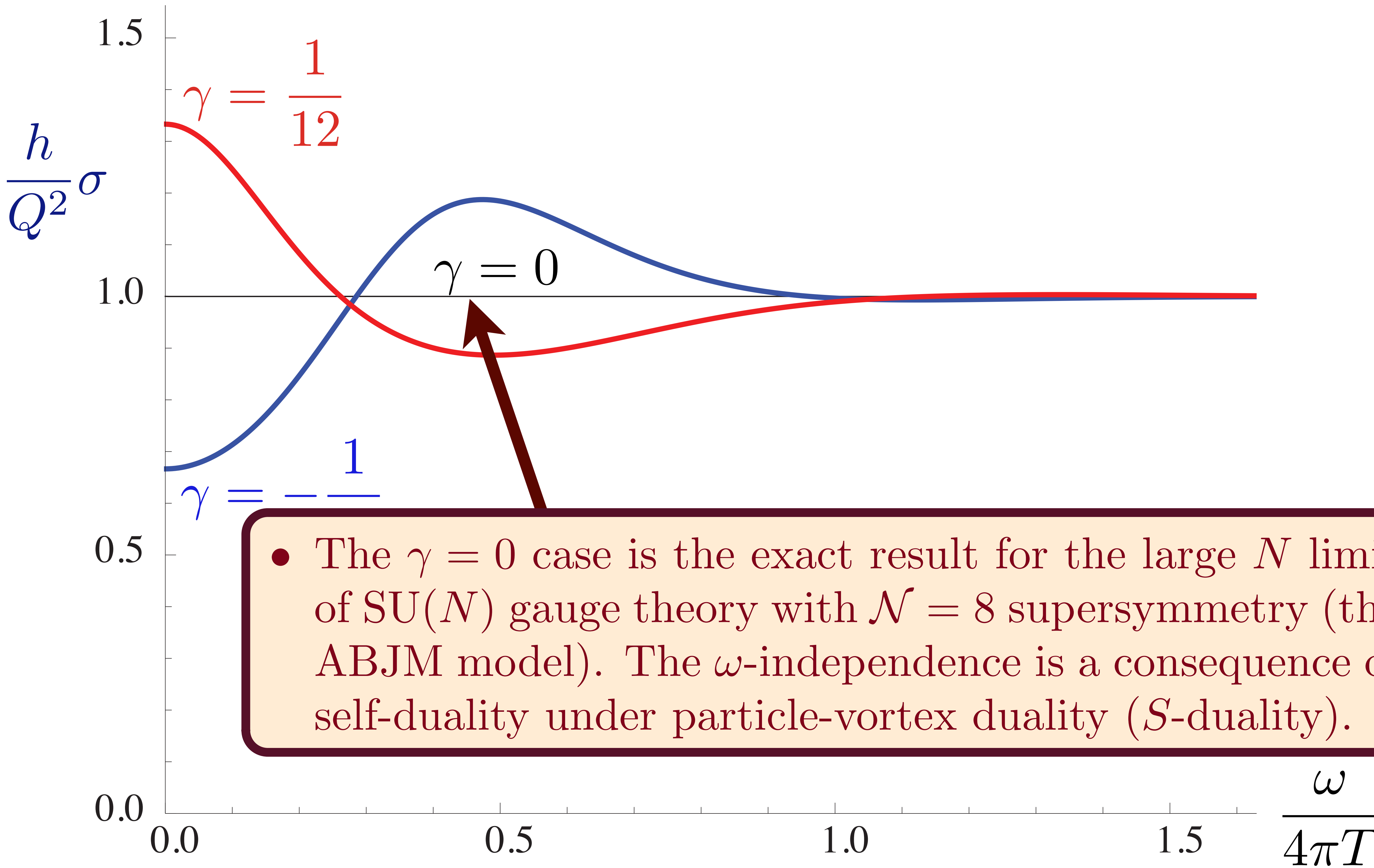


- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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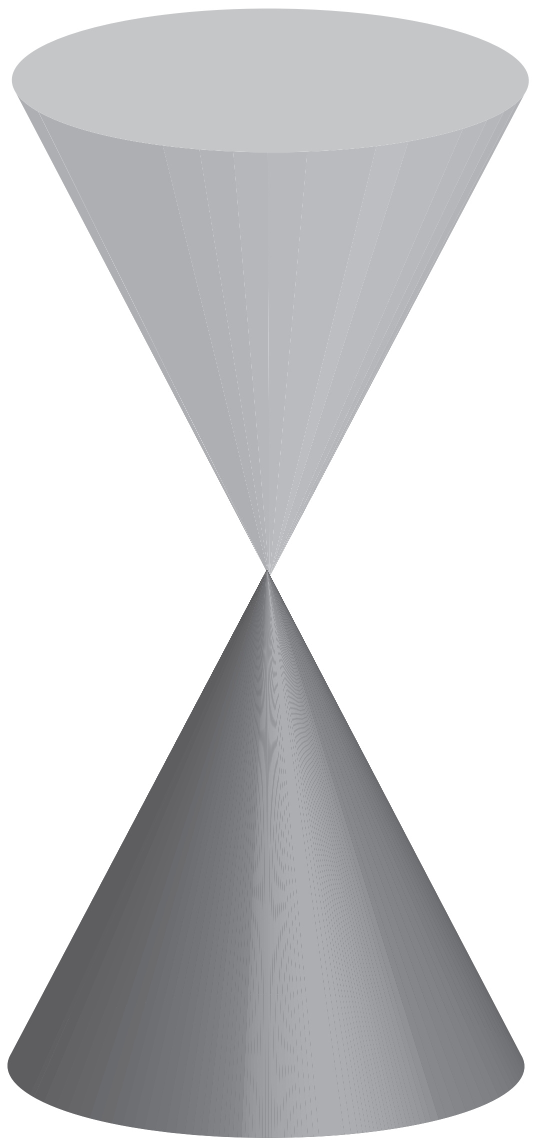
- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe <u>*zero temperature*</u> phases where $d\langle\mathcal{Q}\rangle/d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu\mathcal{Q}$.

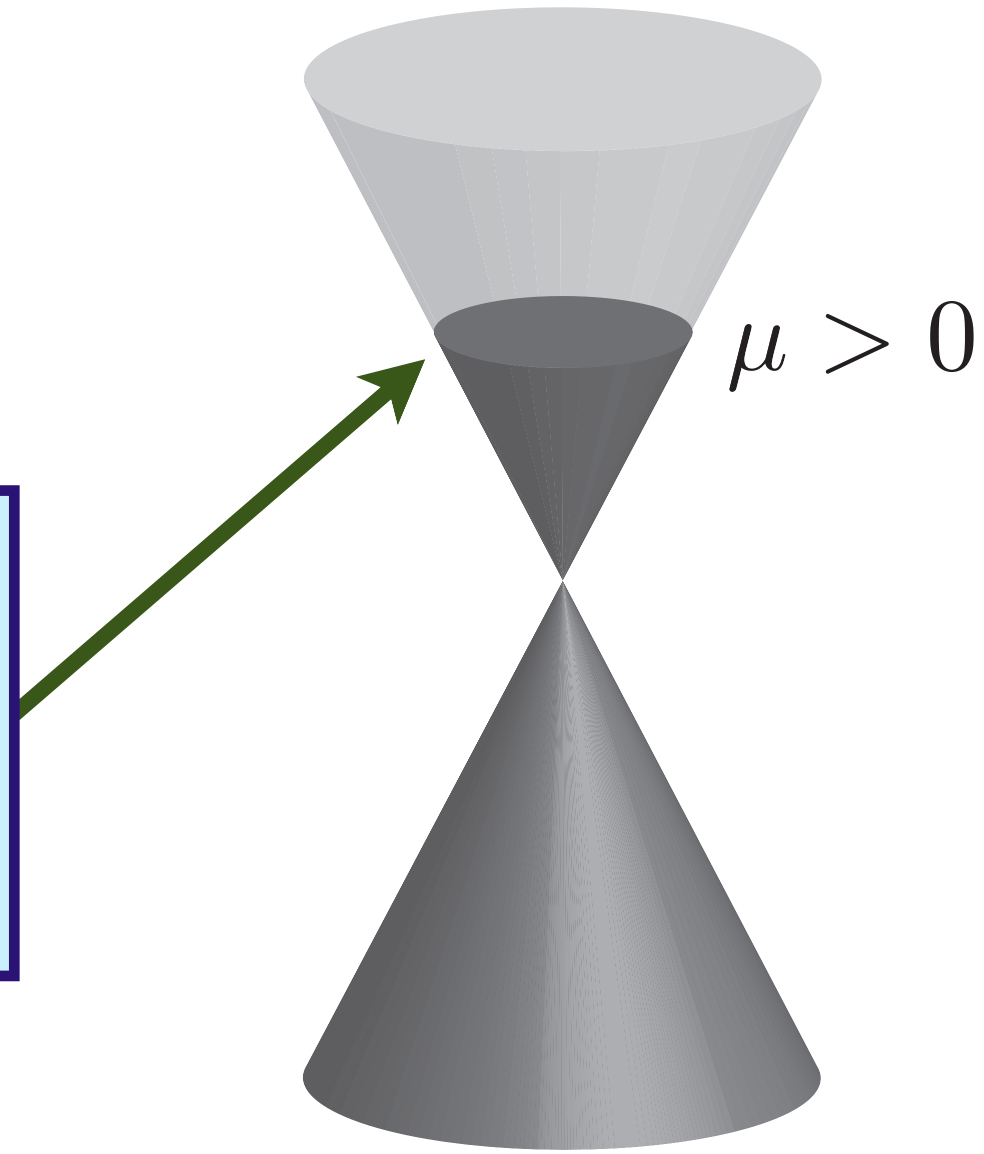
The only compressible phase of traditional condensed matter physics which does not break the translational or $\mathrm{U}(1)$ symmetries is the Landau Fermi liquid

Conformal quantum matter



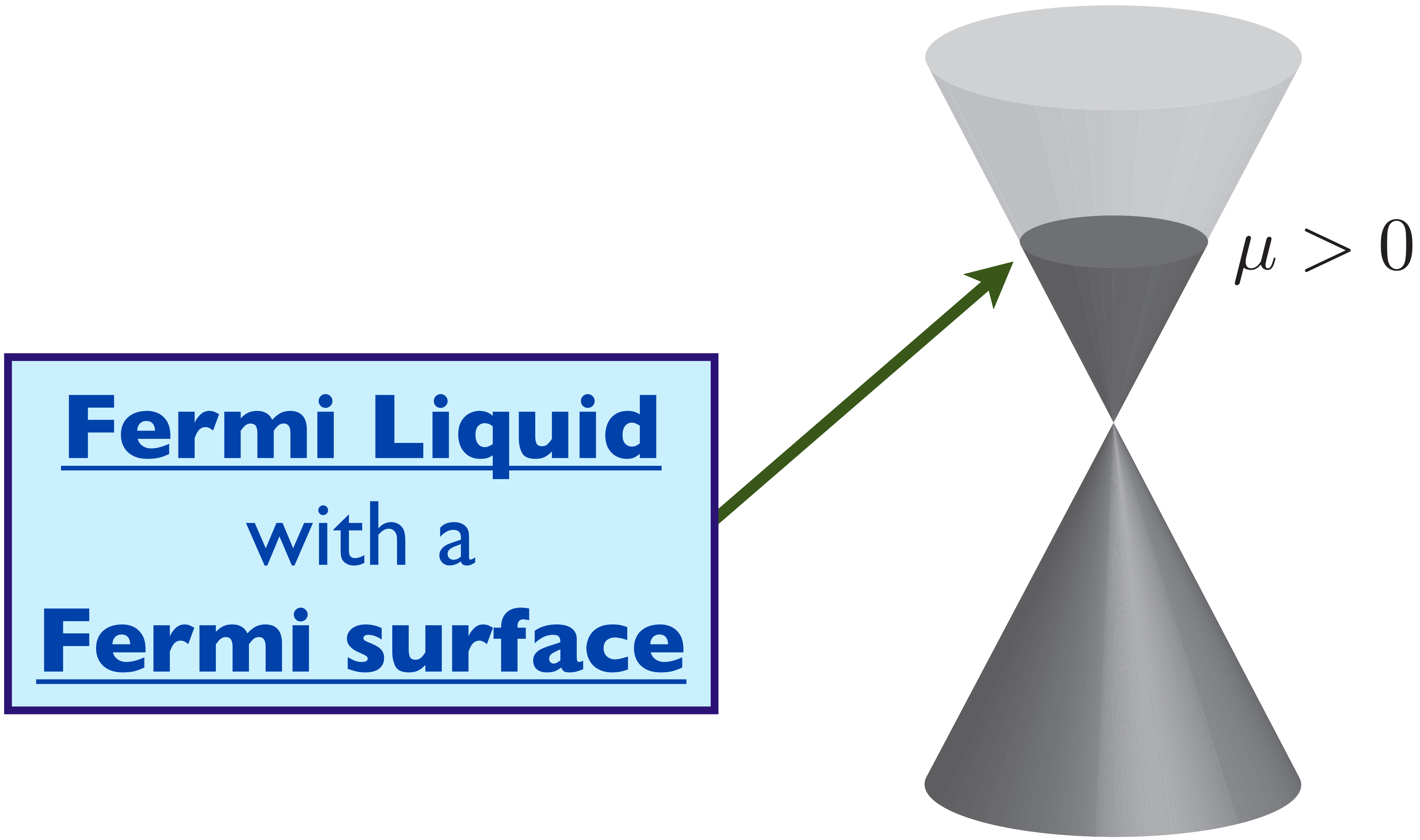
Graphene

Compressible quantum matter



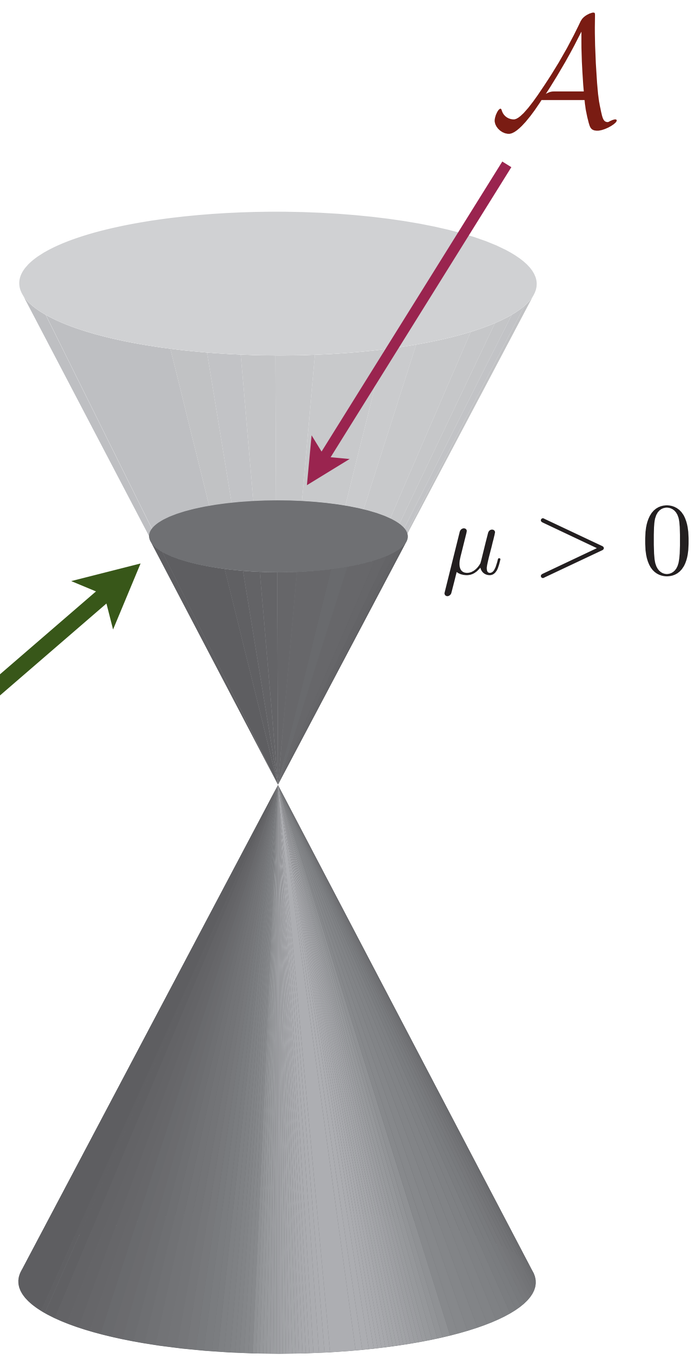
Fermi Liquid
with a
Fermi surface

Compressible quantum matter



- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

Compressible quantum matter



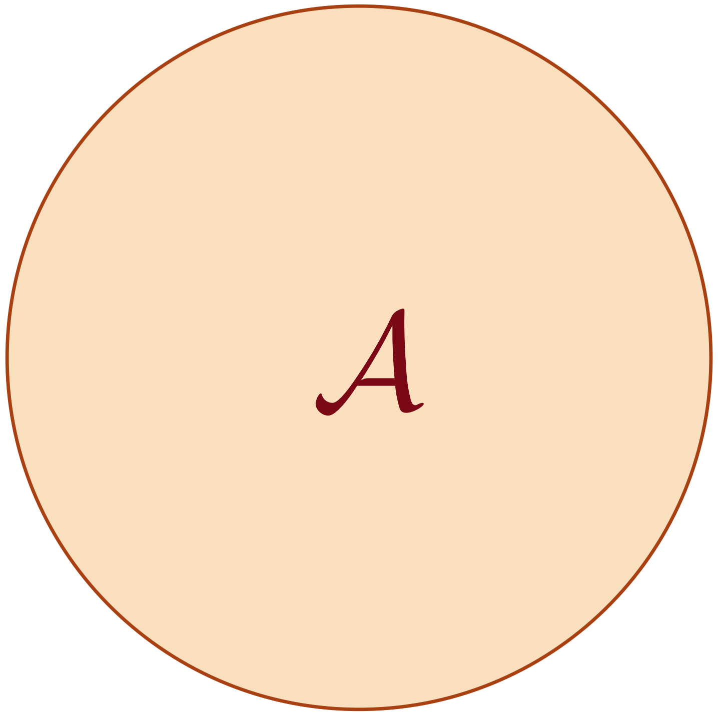
Fermi Liquid
with a
Fermi surface

- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

The Fermi Liquid (FL)

$$\mathcal{A} = \langle f_\sigma^\dagger f_\sigma \rangle = \langle \mathcal{Q}_\sigma \rangle$$

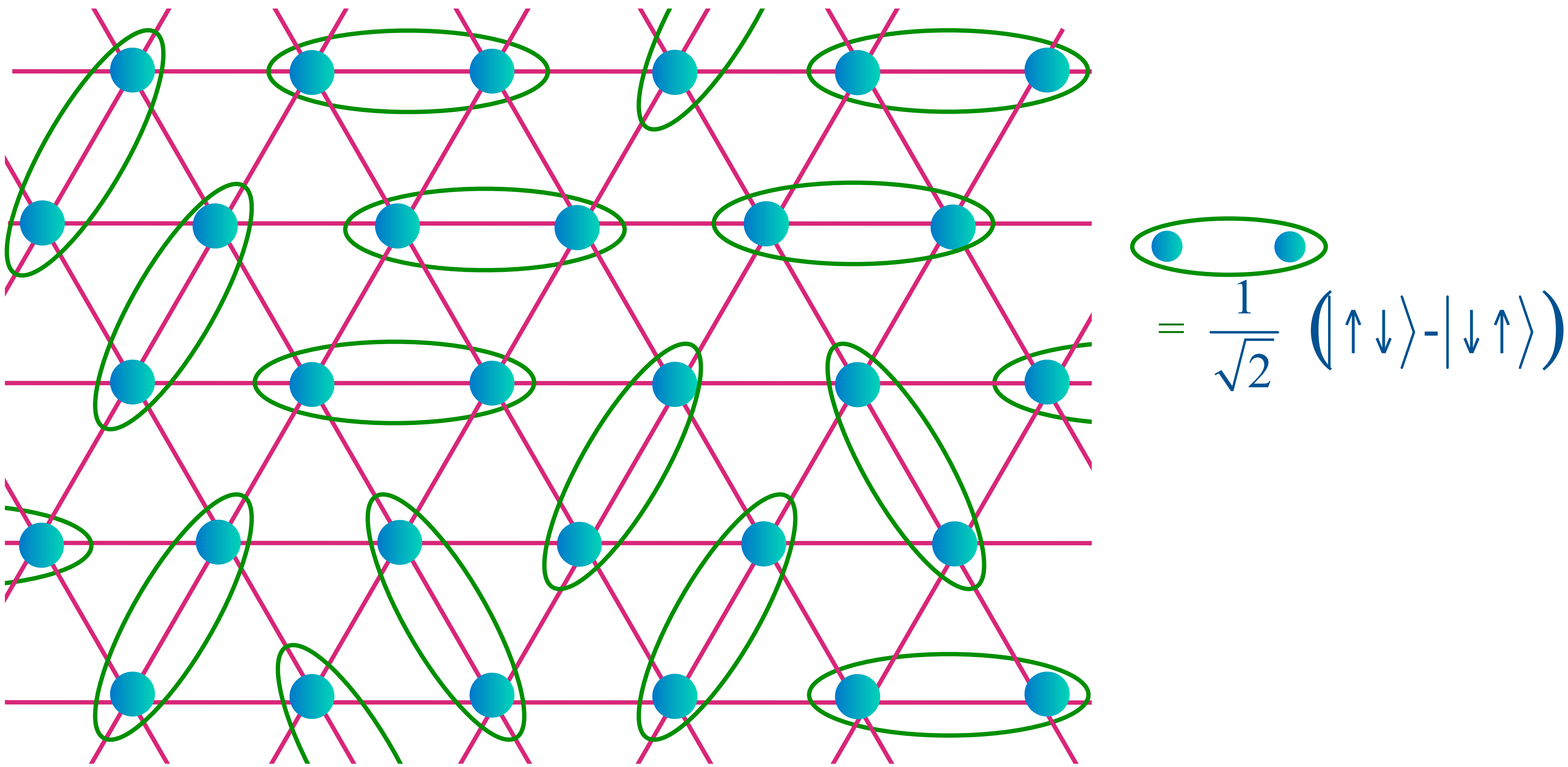
$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + 4 \text{ Fermi terms}$$

The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

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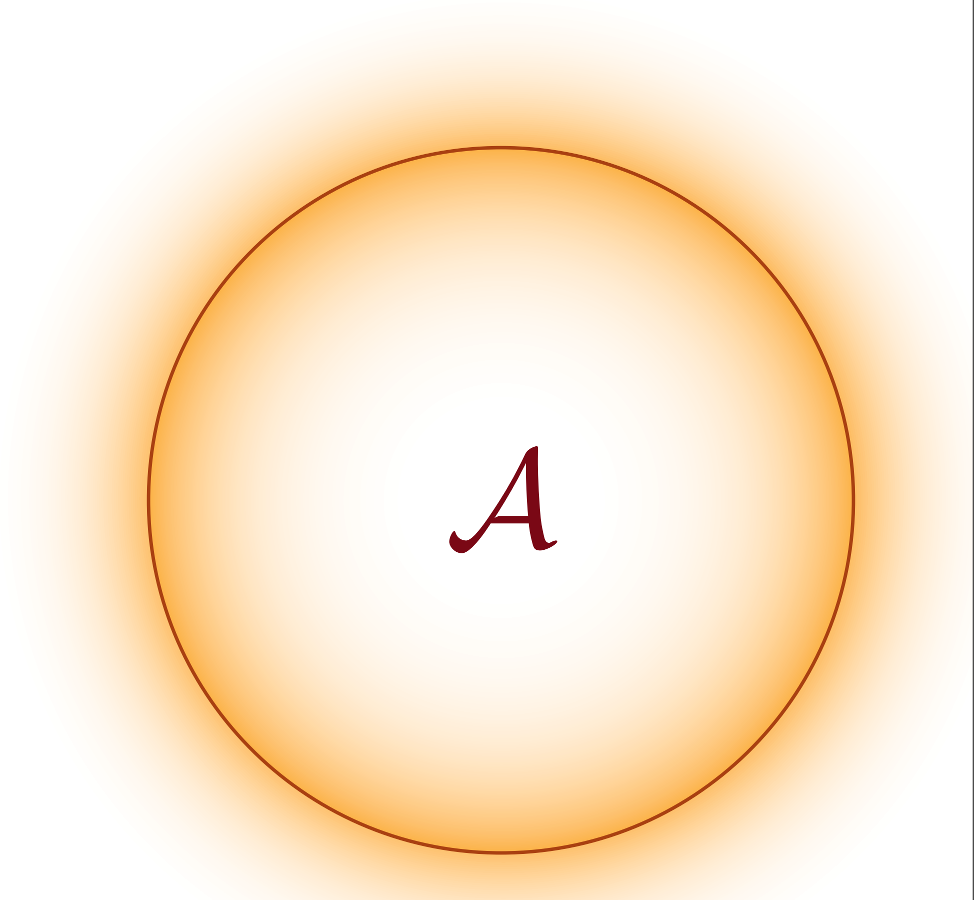
The theory of this strange metal is strongly coupled in two spatial dimensions, and the traditional field-theoretic expansion methods break down.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

The Non-Fermi Liquid (NFL)

- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*



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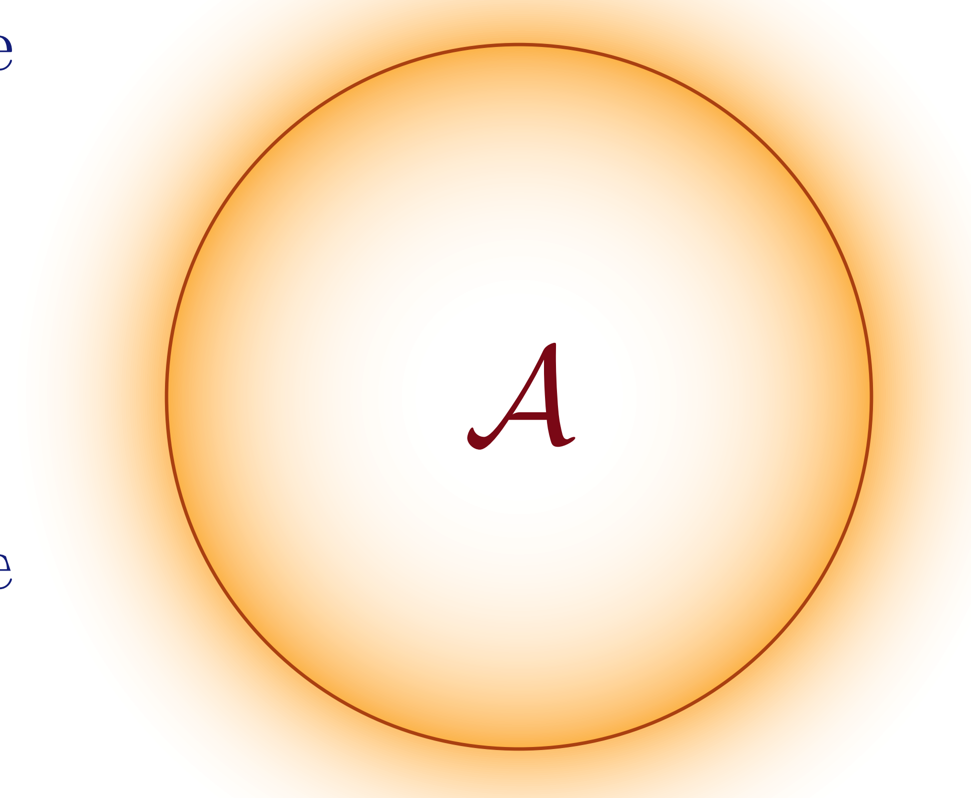
- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*

- The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

$$G_f^{-1} = q^{1-\eta} F(\omega/q^z)$$

where $q = |k| - k_F$ is the distance from the Ferm surface. So fermions disperse transverse to the Fermi surface with dynamic exponent z.



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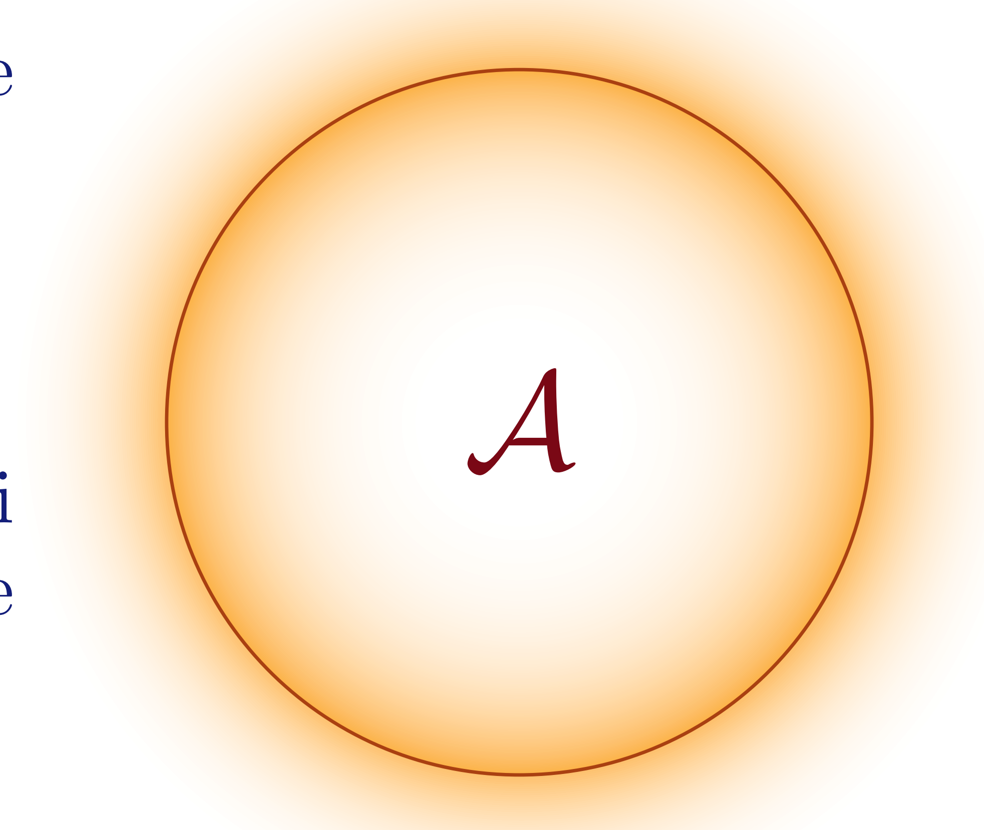
- To three-loop order, we find $\eta \neq 0$ and $z = 3/2$.



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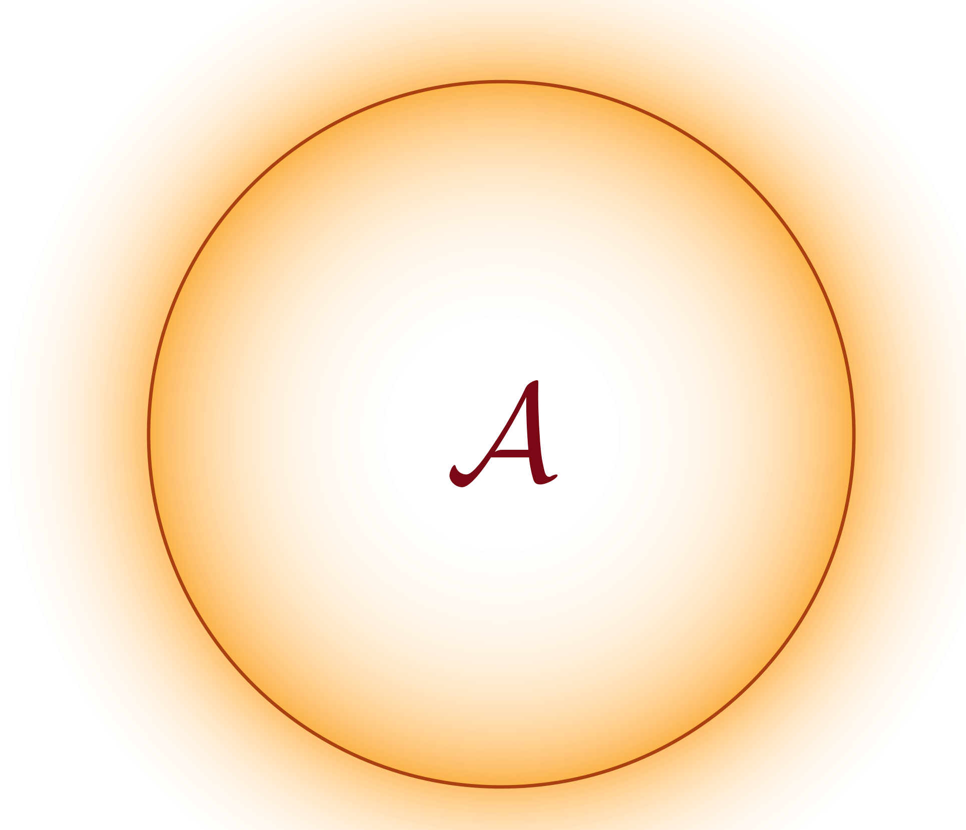
- The entropy density of the non-Fermi liquid $S \sim T^{1/z}$, because the density of fermion states is effectively one dimensional.



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M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Study the large N limit of a SU(N) gauge field coupled to adjoint (matrix) fermions at a non-zero chemical potential

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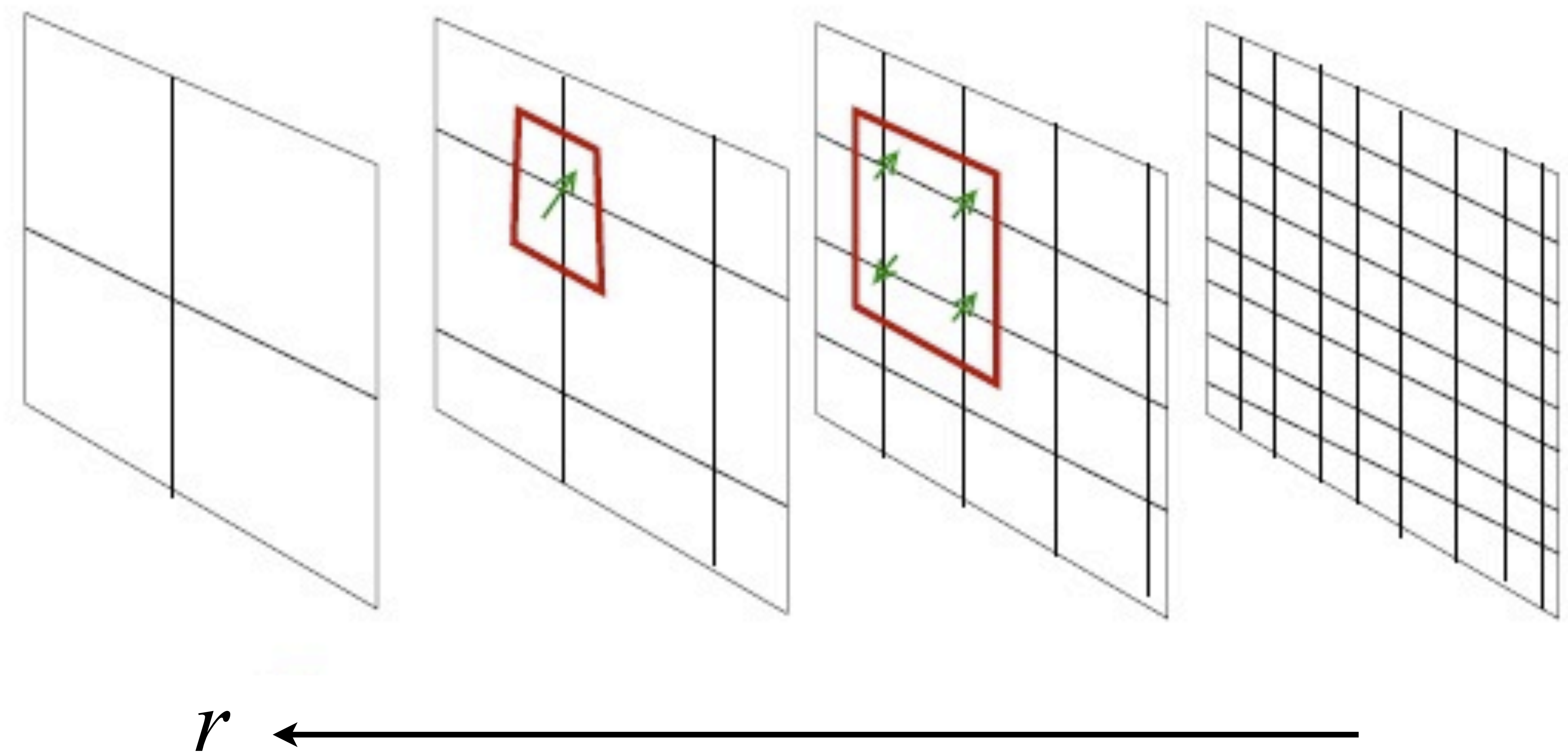
Holography of non-Fermi liquids



J. McGreevy, arXiv0909.0518

Holography of non-Fermi liquids

Consider the following (most) general metric for the holographic theory

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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This metric transforms under rescaling as

$$\begin{aligned} x_i &\to \zeta\, x_i \\ t &\to \zeta^z\, t \\ ds &\to \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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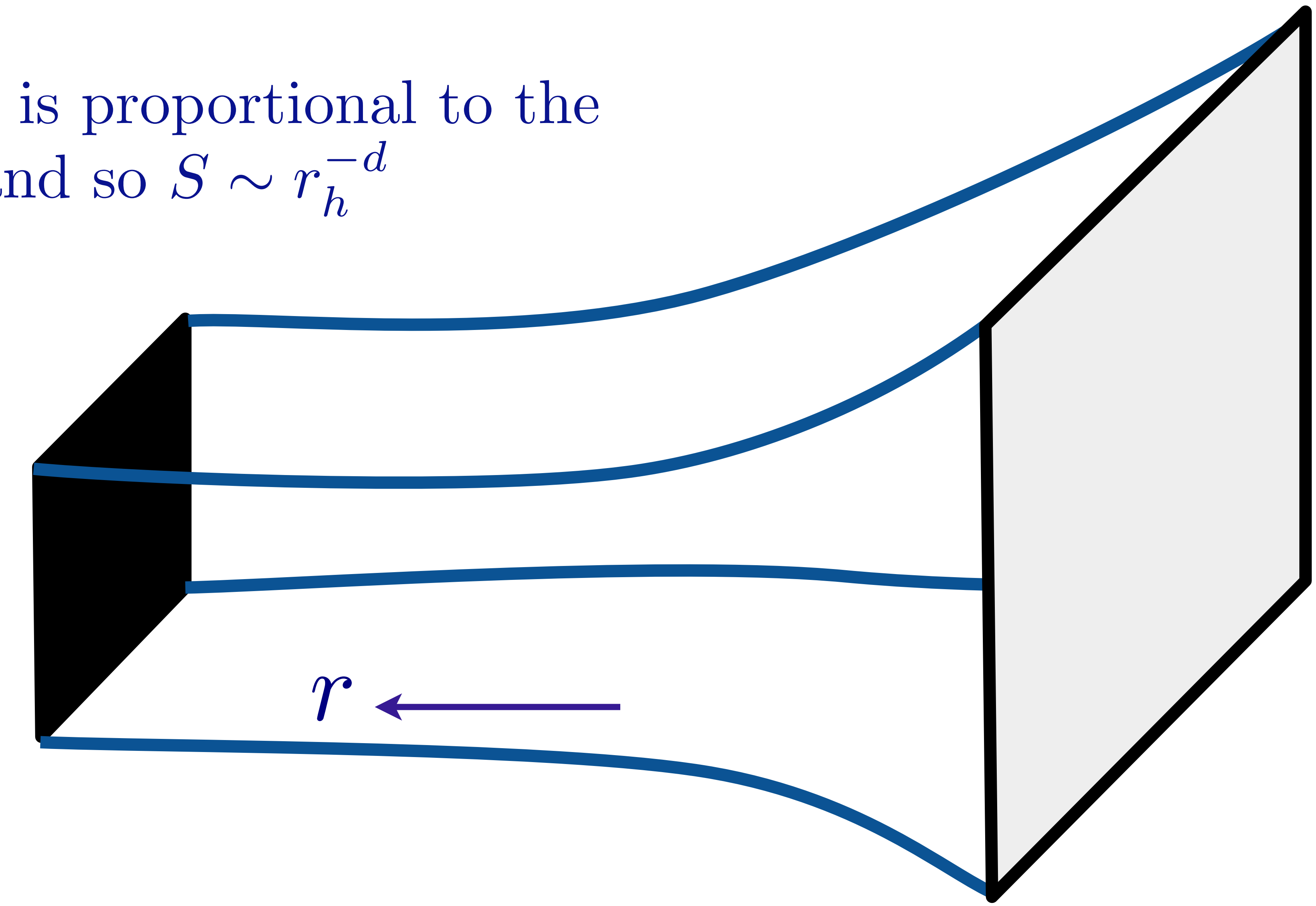
What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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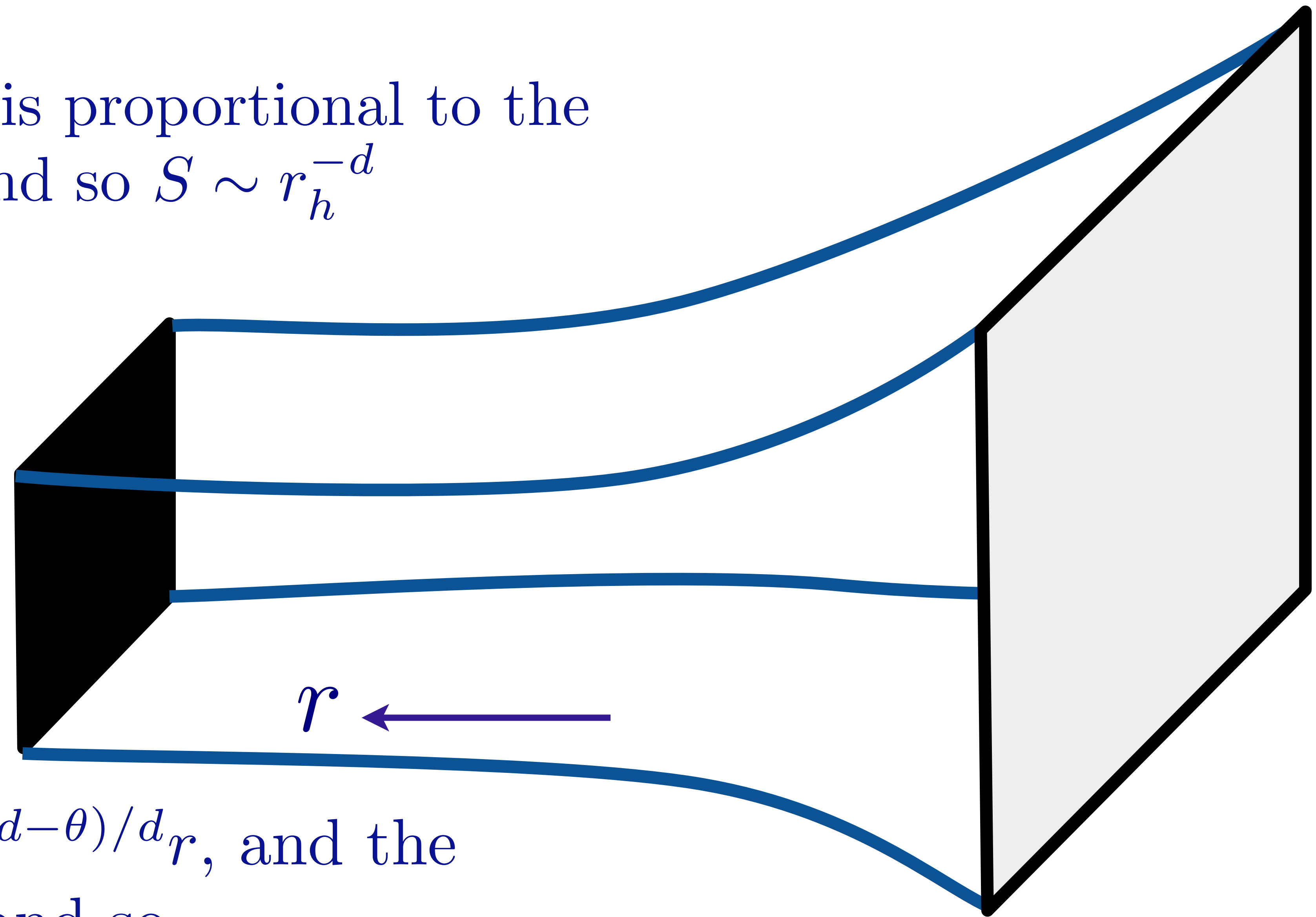

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z}$$

So θ is the "violation of hyperscaling" exponent.

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z, with entropy density $\sim T^{1/z}$. So we expect compressible quantum states to have an effective dimension $d - \theta$ with

$$\theta = d - 1$$

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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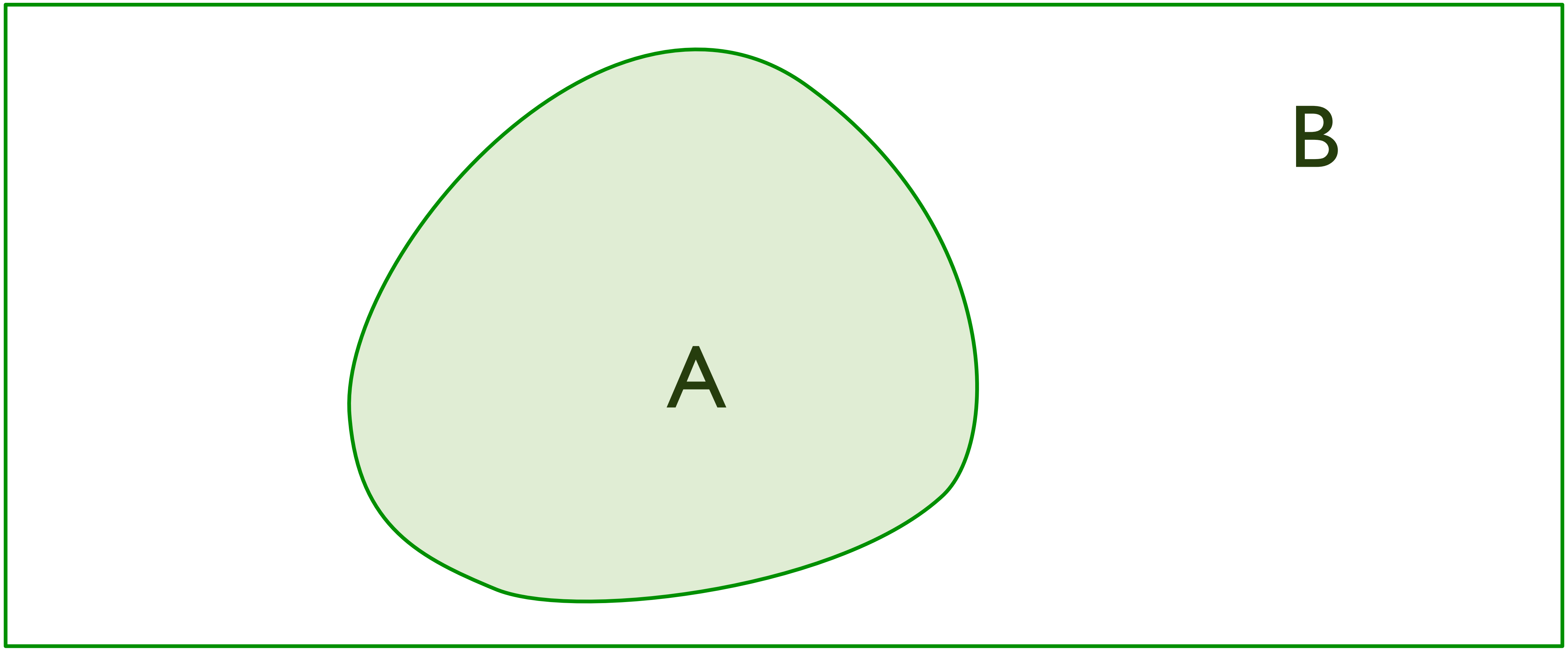
Entanglement entropy of Fermi surfaces

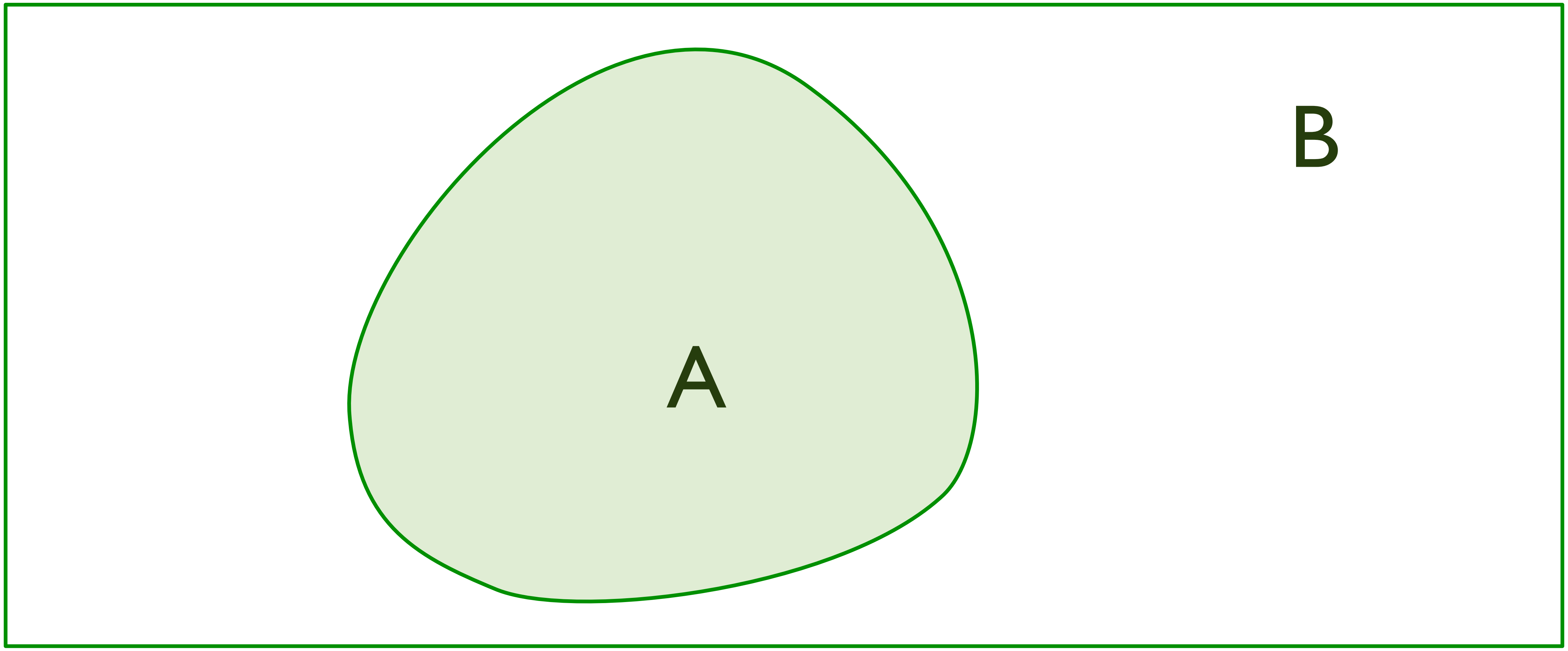


Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of Fermi surfaces



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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

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N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- The metric can be realized as the solution of a Einstein-Maxwell-Dilaton theory with no explicit fermions. The density of the "hidden Fermi surfaces" of the boundary gauge-charged fermions can be deduced from the electric flux leaking to $r \to \infty$.

K. Goldstein, S. Kachru, S. Prakash, and S. P. Trivedi JHEP **1008**, 078 (2010)

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L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- These two methods of deducing with fermion density are consistent with the Luttinger relation !

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Inequalities

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The area law of entanglement entropy is obeyed for

$$\theta \leq d - 1.$$

The "null energy condition" of the gravity theory yields

$$z \geq 1 + \frac{\theta}{d}.$$

Remarkably, for $d = 2$, $\theta = d - 1$ and $z = 1 + \theta/d$, we obtain $z = 3/2$, the same value associated with the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Conclusions

Non-Fermi liquid metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and holography offer a remarkable new approach to describing "strange metal" states with long-range quantum entanglement.

Conclusions

Presented evidence for holographic dual of a Fermi surface coupled to Abelian or non-Abelian guage fields