INTERNATIONAL SOLVAY INSTITUTES
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Exotic phases and quantum phase transitions in model systems

Subir Sachdev
Harvard University

Outline

1. Landau-Ginzburg criticality

   Coupled-dimer antiferromagnets

2. Quantum "disordering" magnetic order

   Z_2 spin liquids and valence bond solids

3. Critical spin liquids

   Deconfined criticality; fermionic spinons near the Mott transition

4. Triangular, kagome, and hyperkagome lattices

   Connections to experiments

[[[5. Correlated boson model

   Supersolids and stripes]]]

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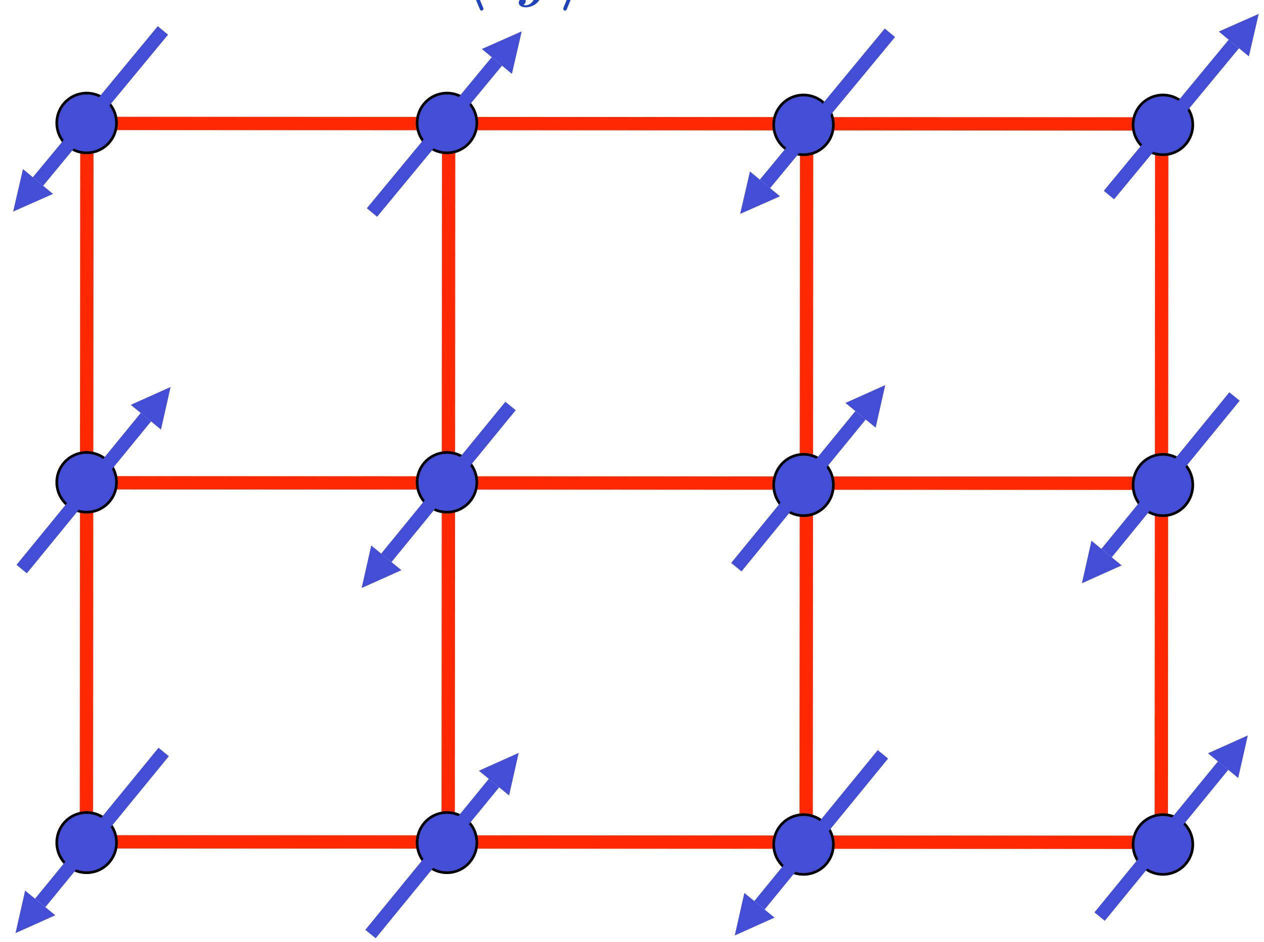
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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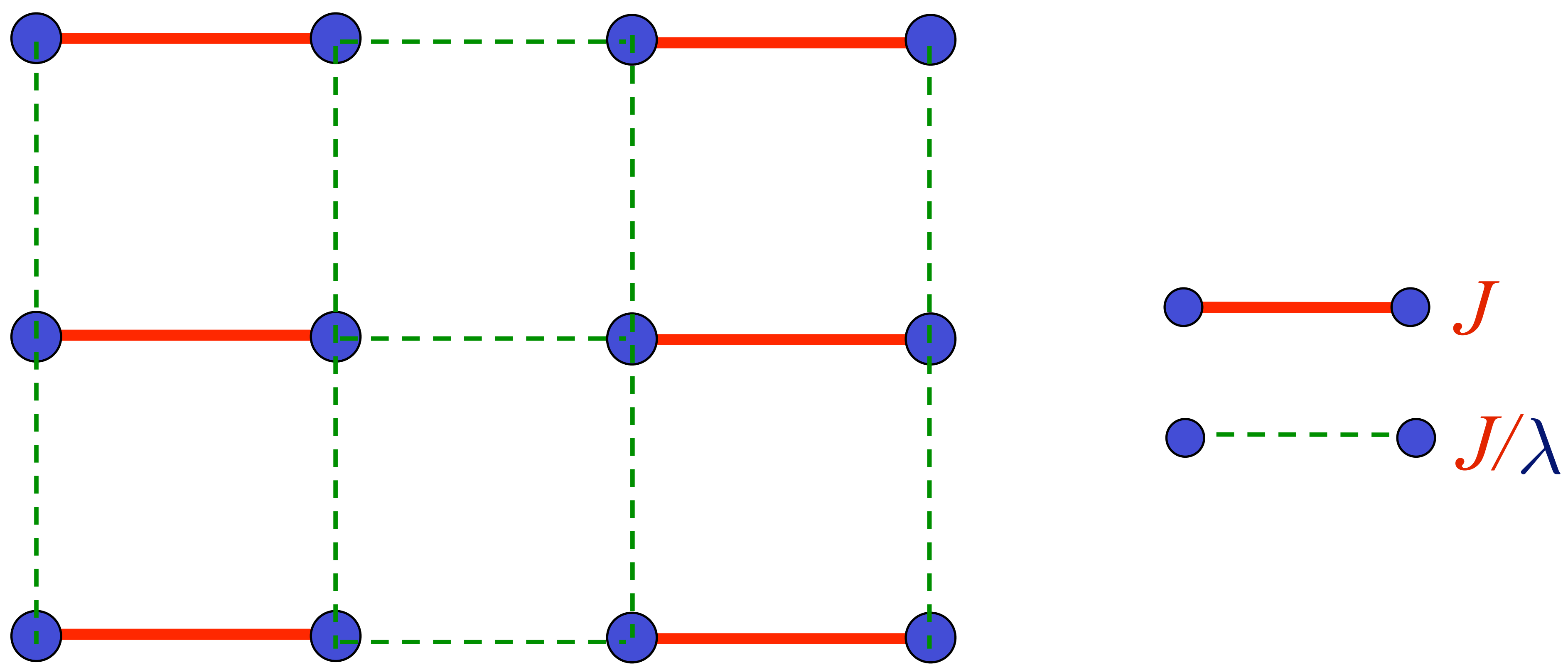
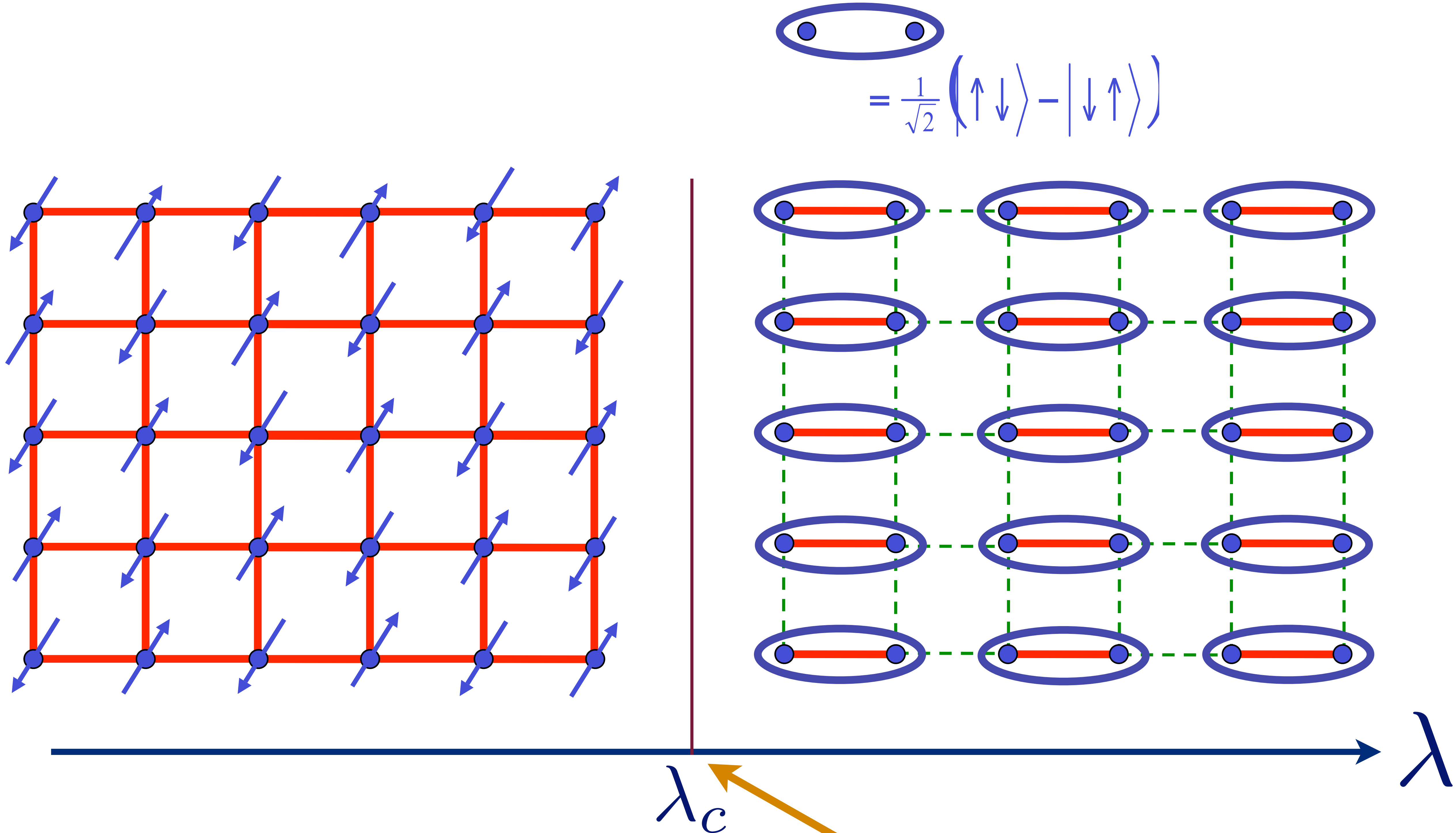
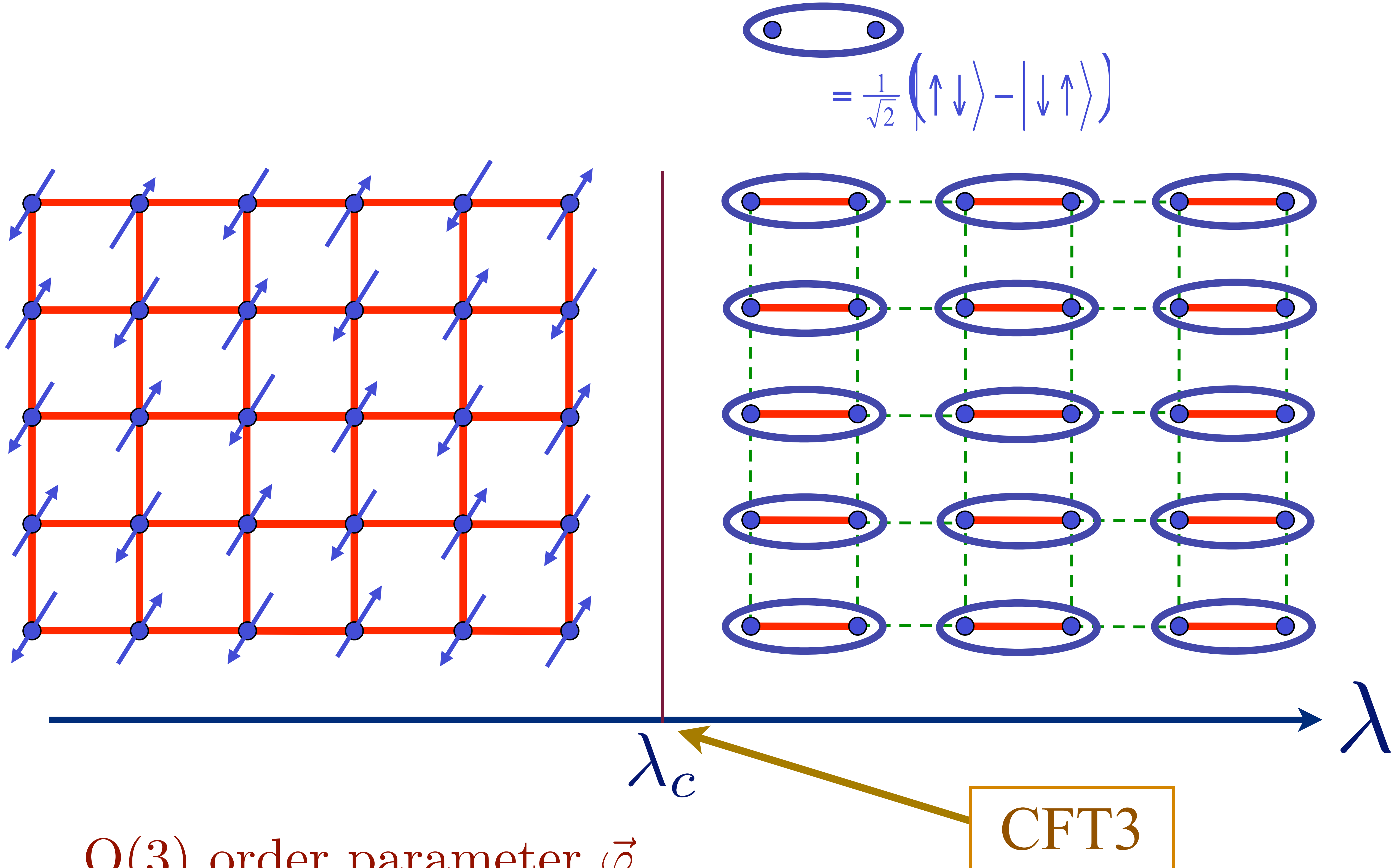


Weaken some bonds to induce spin entanglement in a new quantum phase



Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [...] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096 - \sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096 + \sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230 - \sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230 + \sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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Field-theoretic RG of CFT3 E. Vicari *et al.*

S. Wenzel and W. Janke, arXiv:0808.1418
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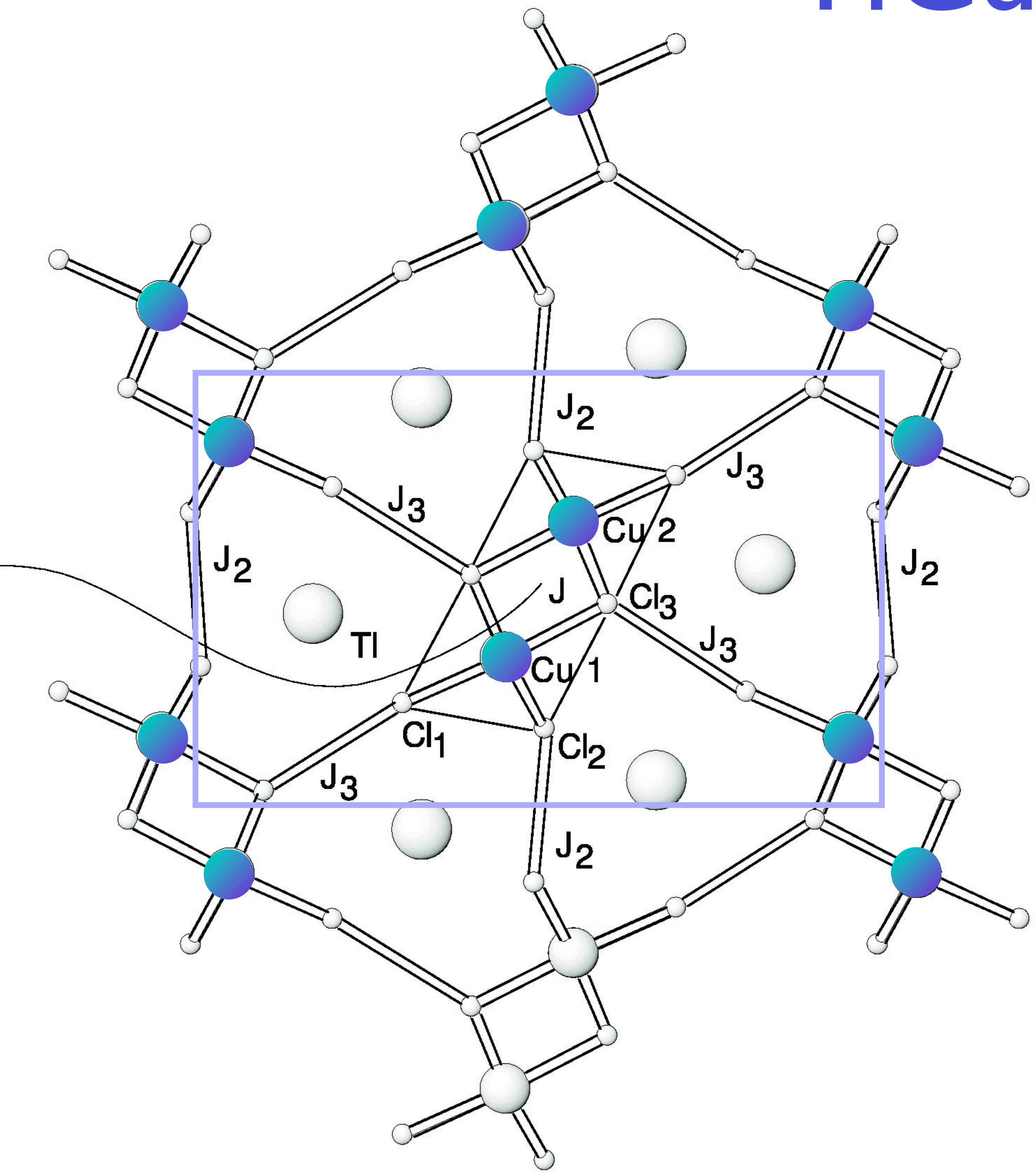
$TlCuCl_3$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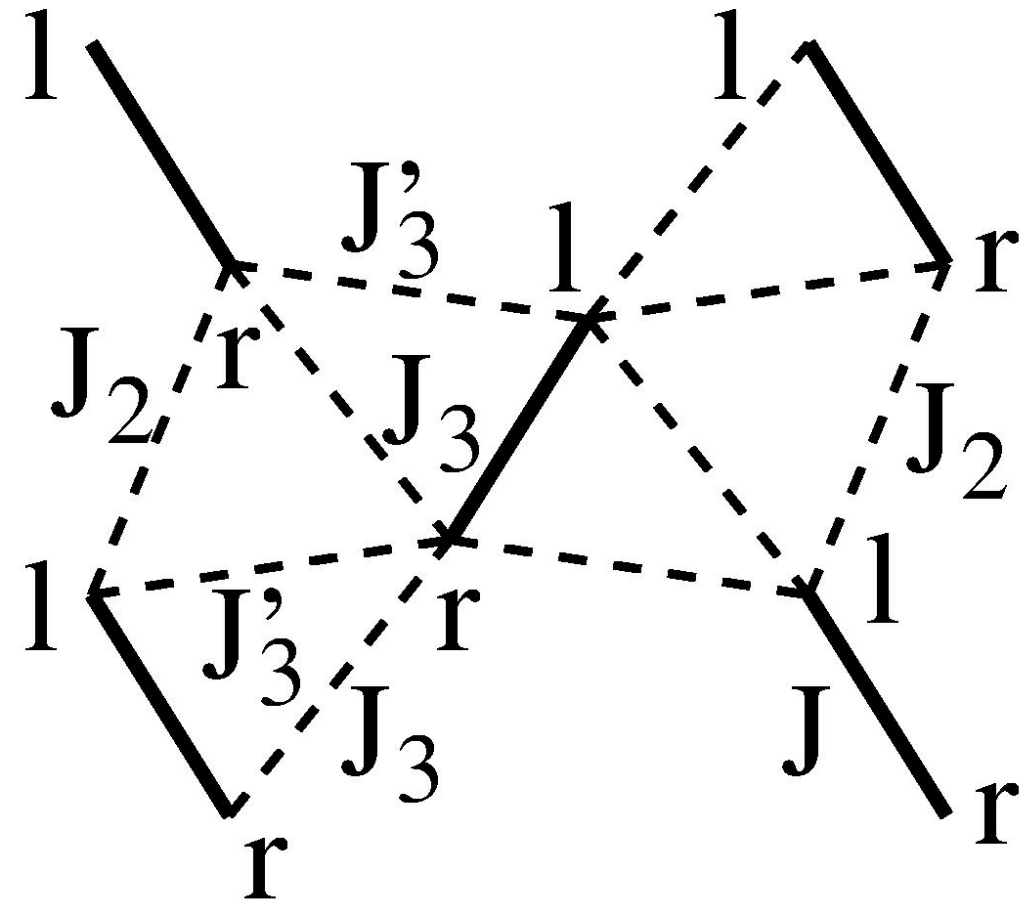
J2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J'3
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J'3
J3
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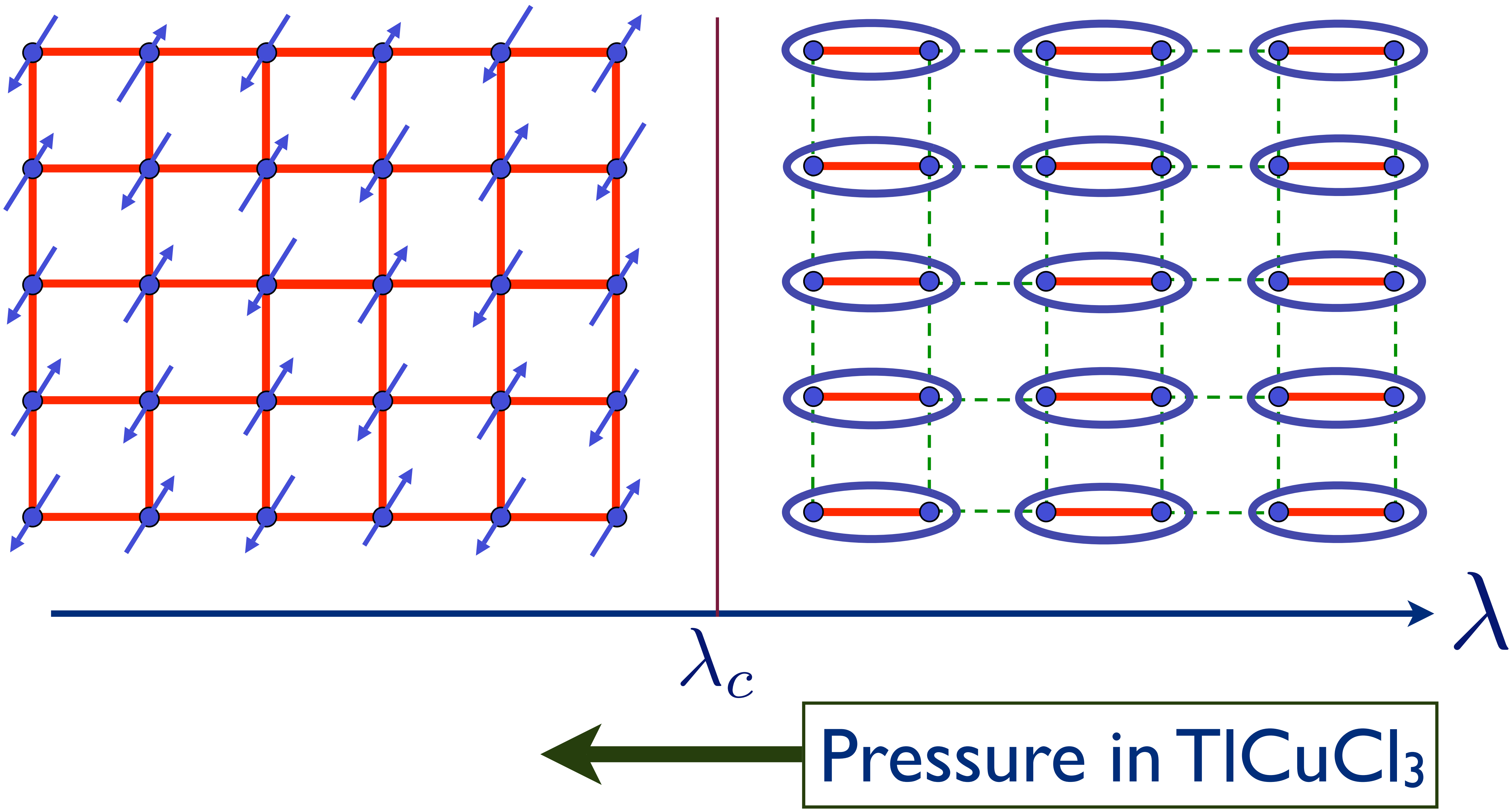
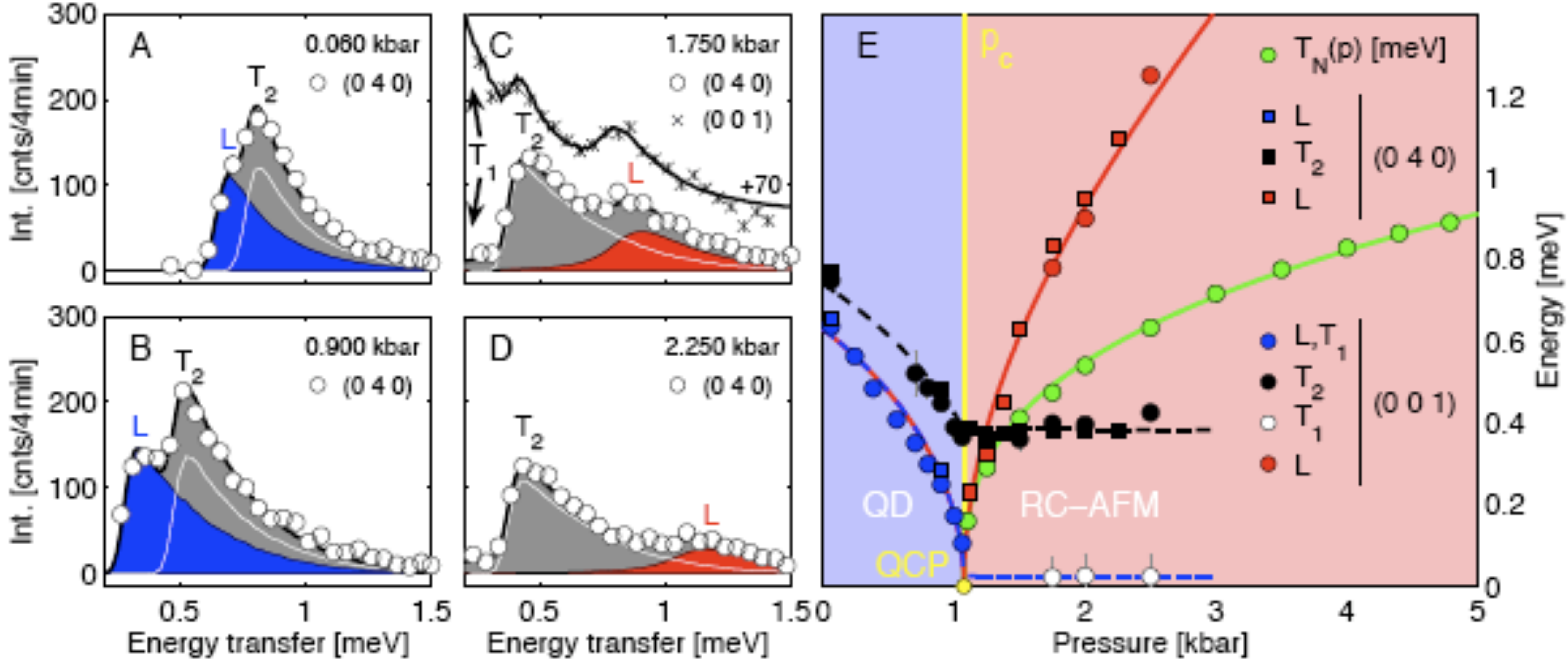
r

Phase diagram as a function of the ratio of exchange interactions, λ



$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode (the "Higgs boson") in Néel phase, and vanishing of Néel temperature at quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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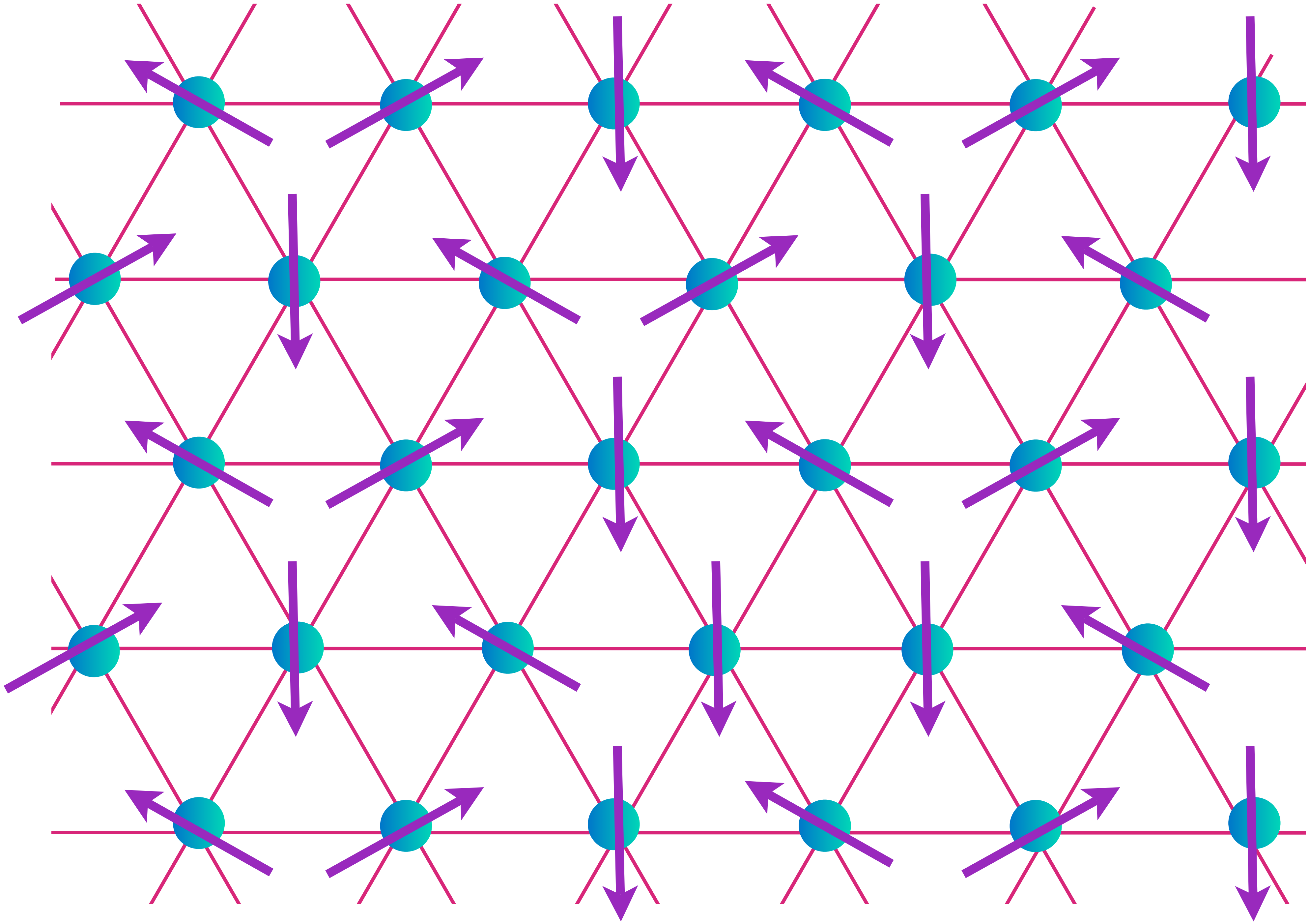
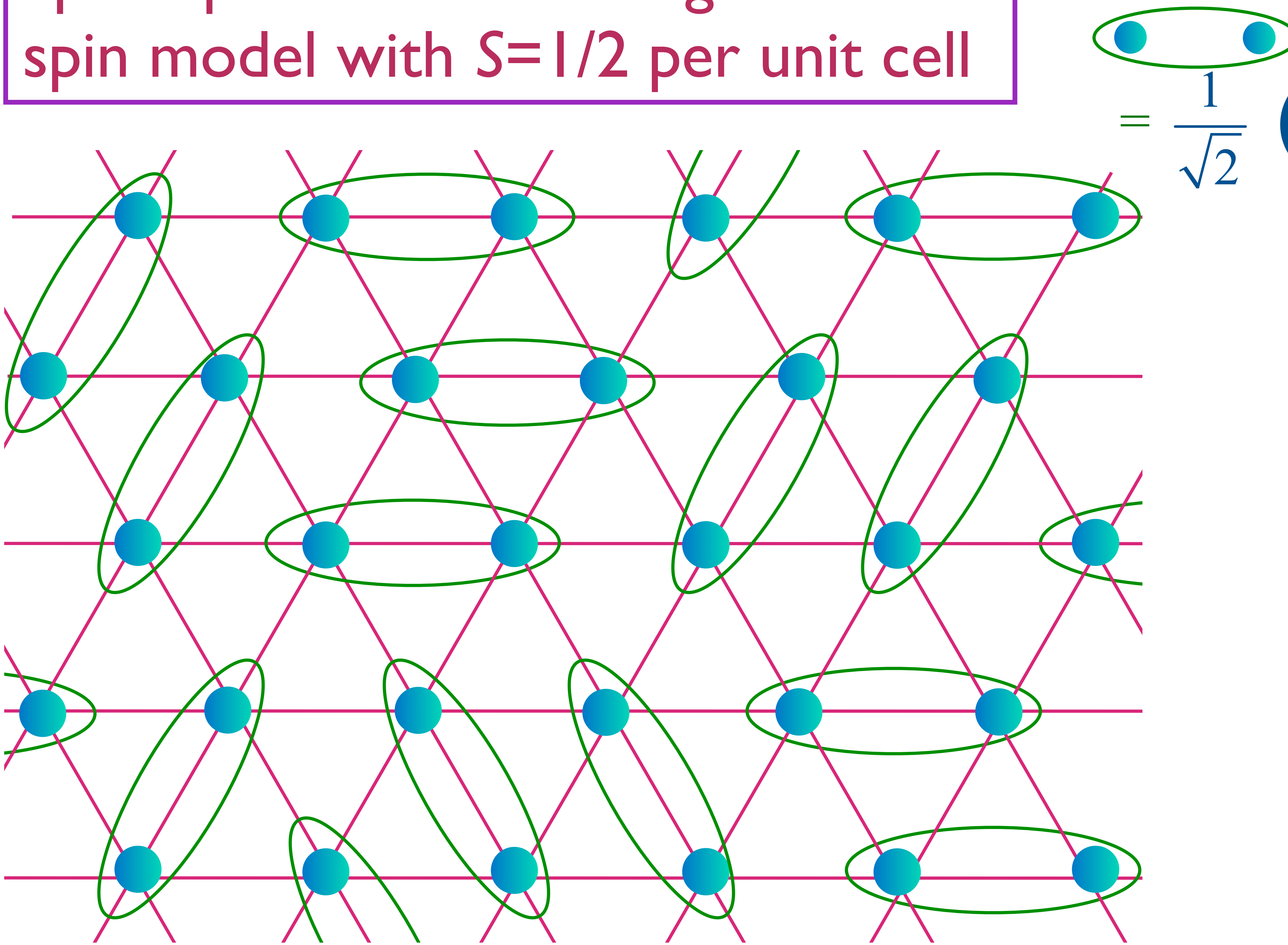


Nearest-neighbor model has non-collinear Neel order

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with S=1/2 per unit cell

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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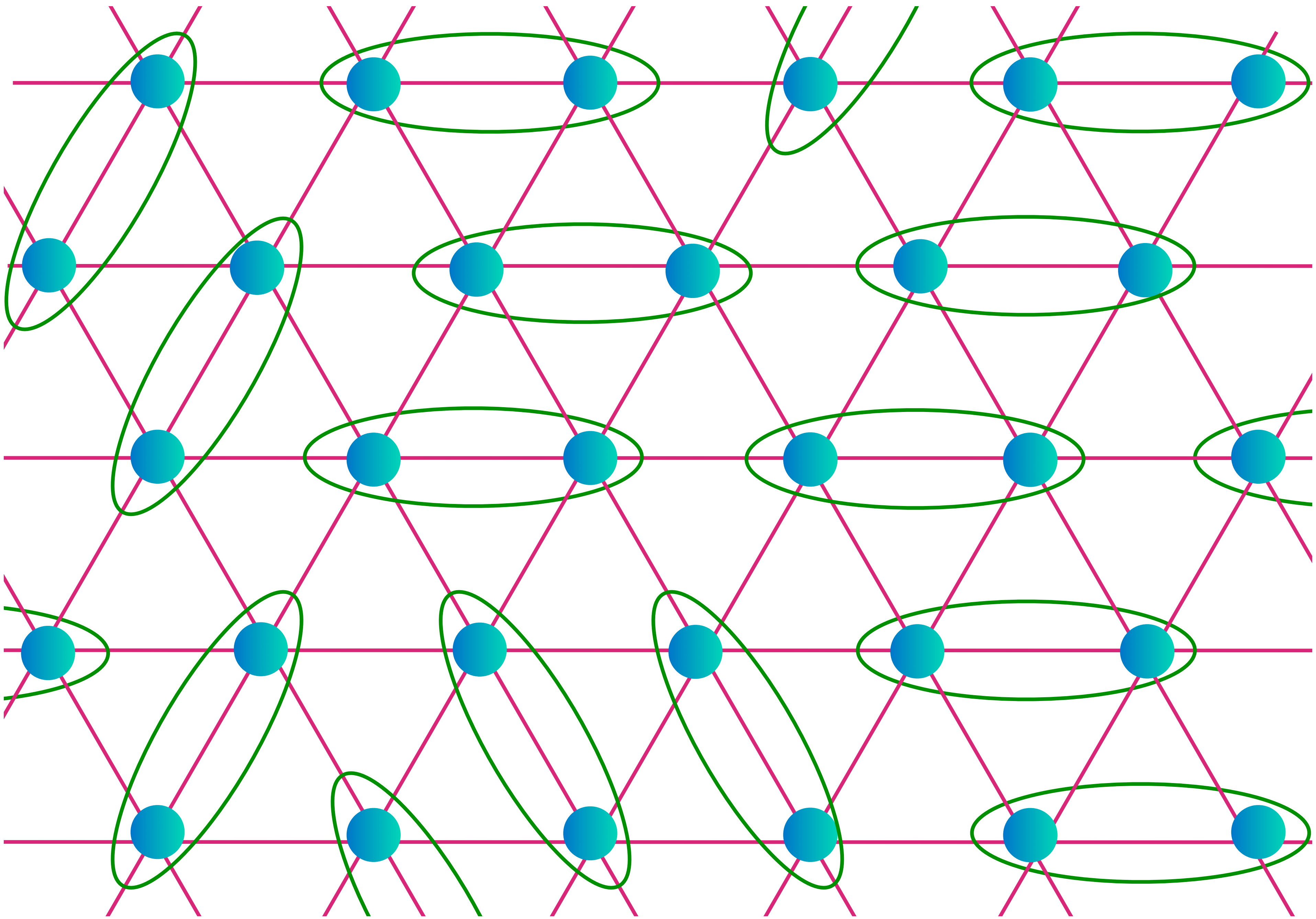

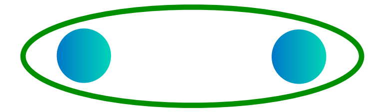

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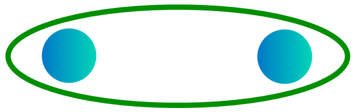

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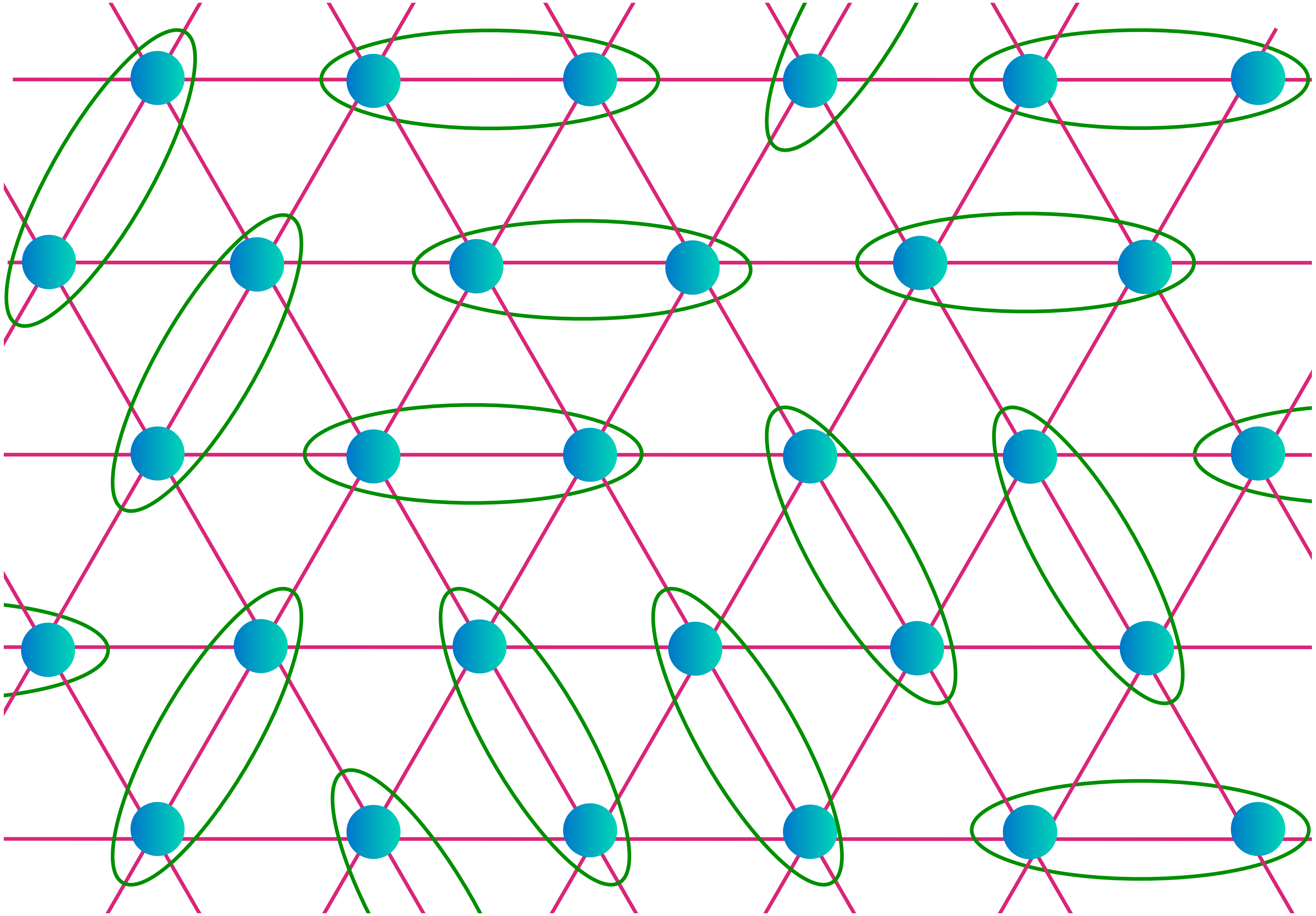

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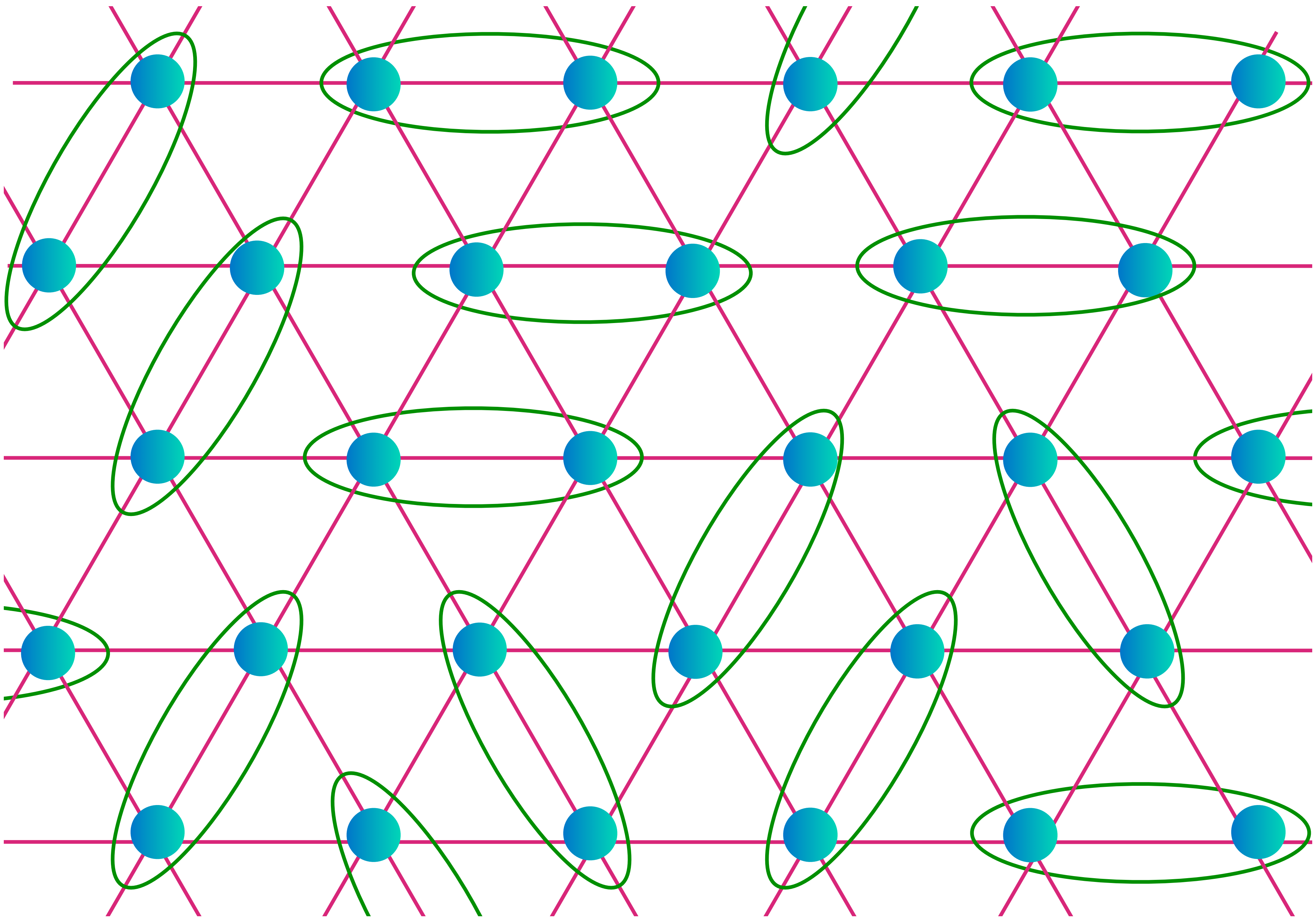

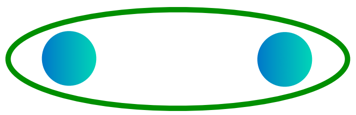

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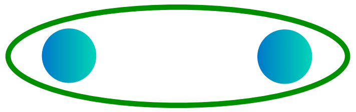

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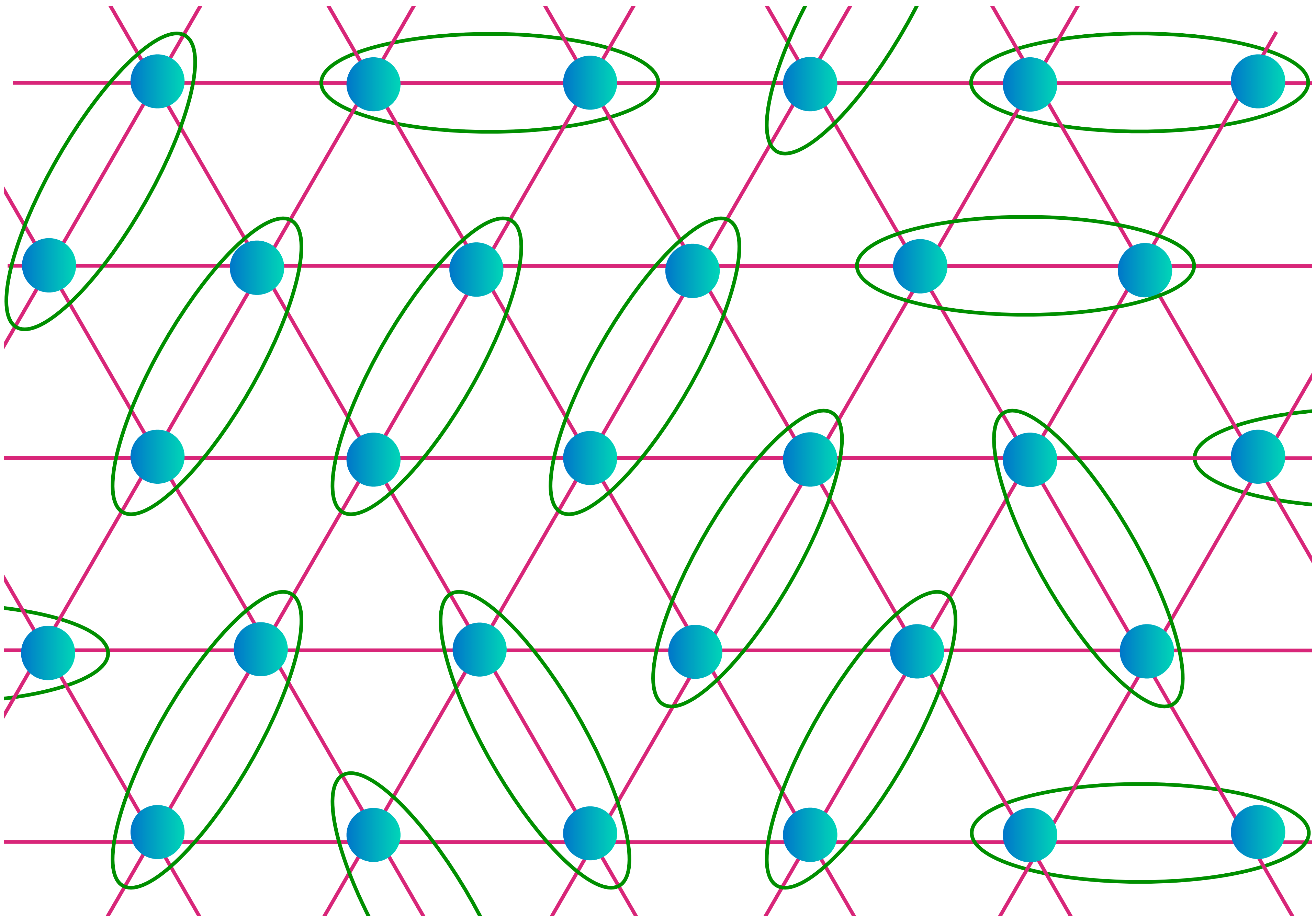

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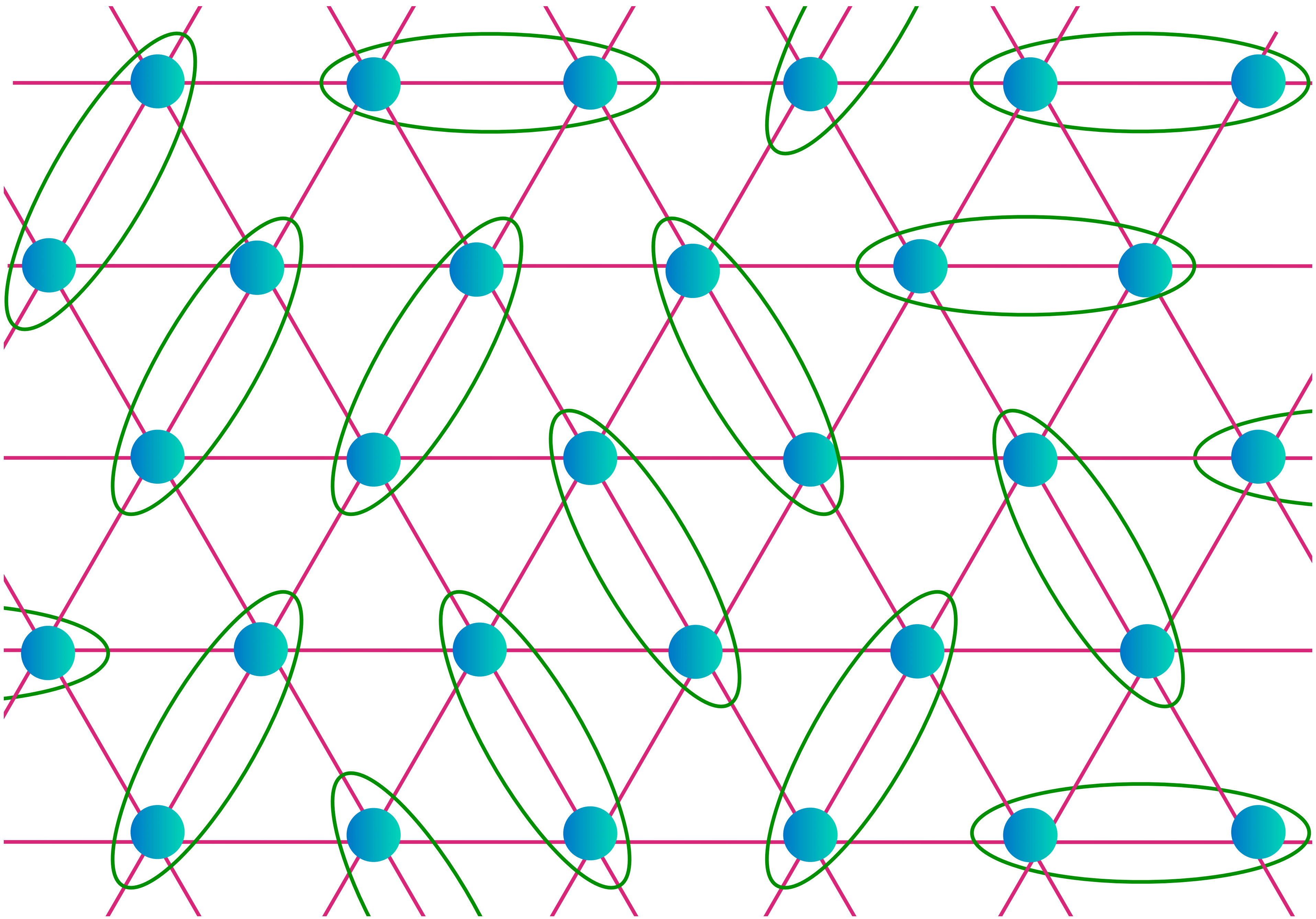

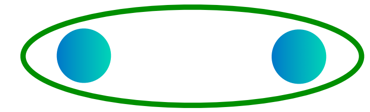

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First approach

Look for spin liquids across continuous (or weakly first-order) quantum transitions from antiferromagnetically ordered states

D. P. Arovas and A. Auerbach, *Phys. Rev. B* **38**, 316 (1988).

Second approach

Look for spin liquids across continuous (or weakly first-order) quantum transitions to an insulator from a metal

S. Burdin, D.R. Grempel, and A. Georges, *Phys. Rev. B* **66**, 045111 (2002).
T. Senthil, M. Vojta and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).
S. Florens and A. Georges, *Phys. Rev. B* **70**, 035114 (2004).
T. Senthil, *Phys. Rev. B* **78** 045109 (2008).

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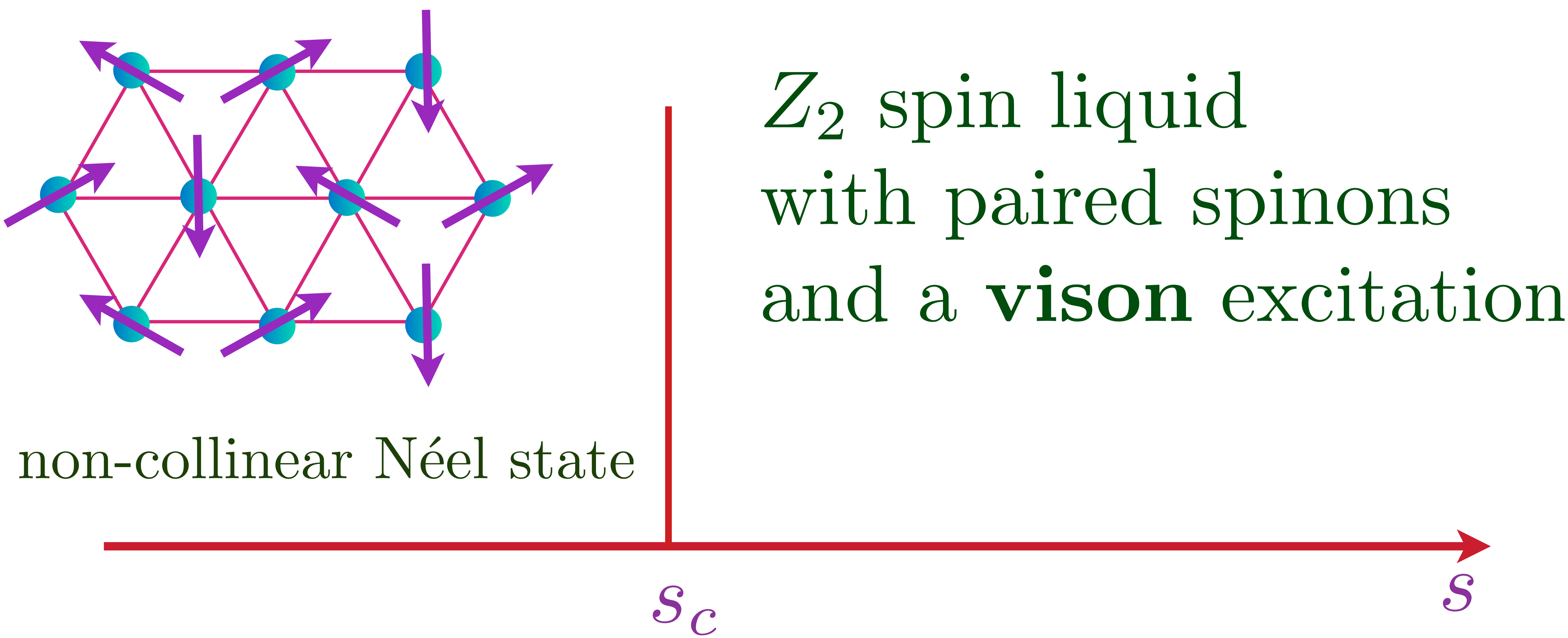
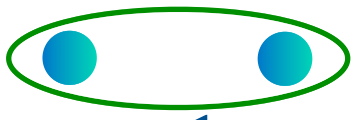


N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Excitations of the Z_2 Spin liquid

A spinon



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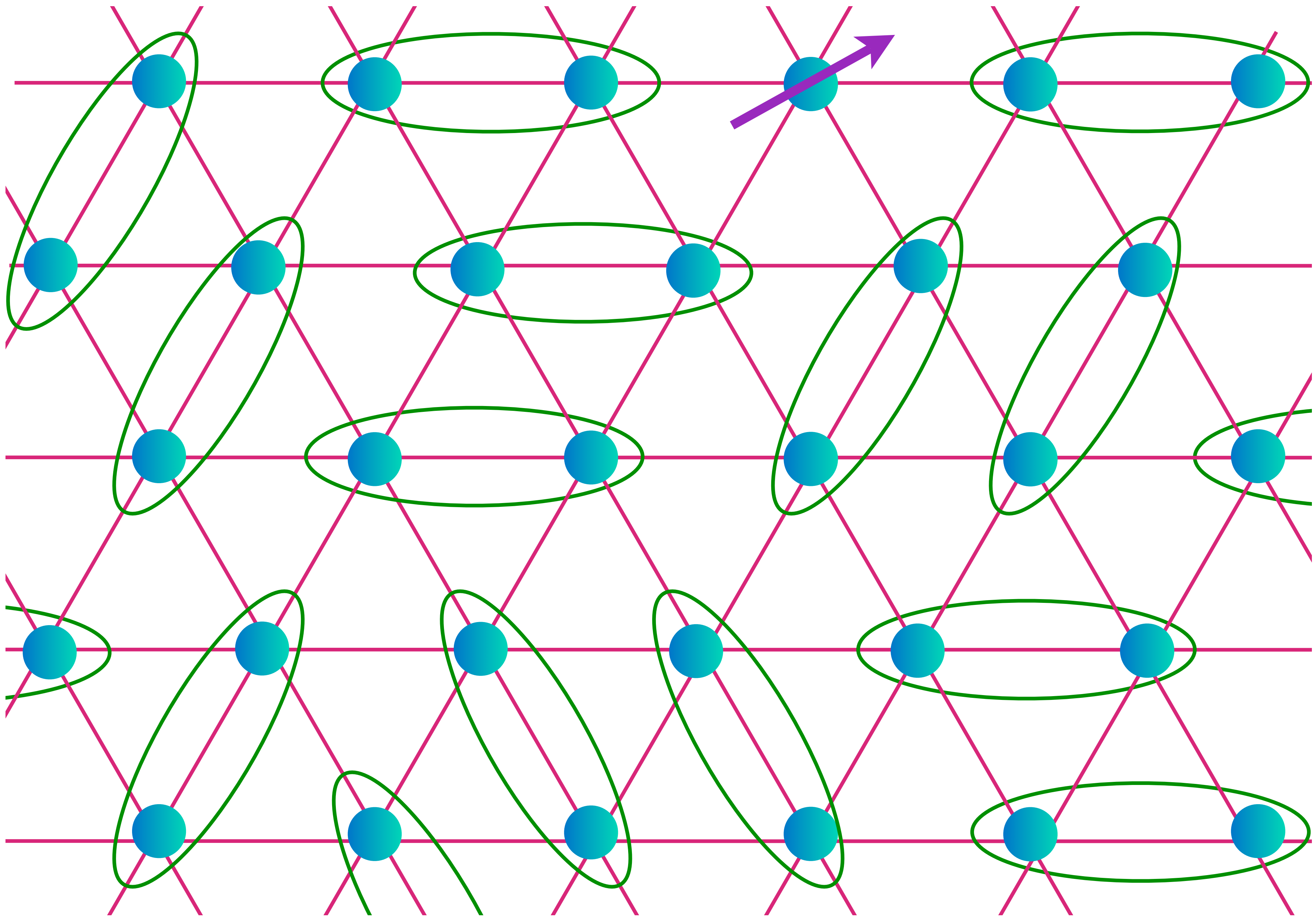

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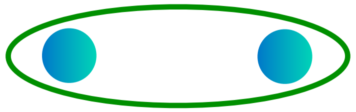

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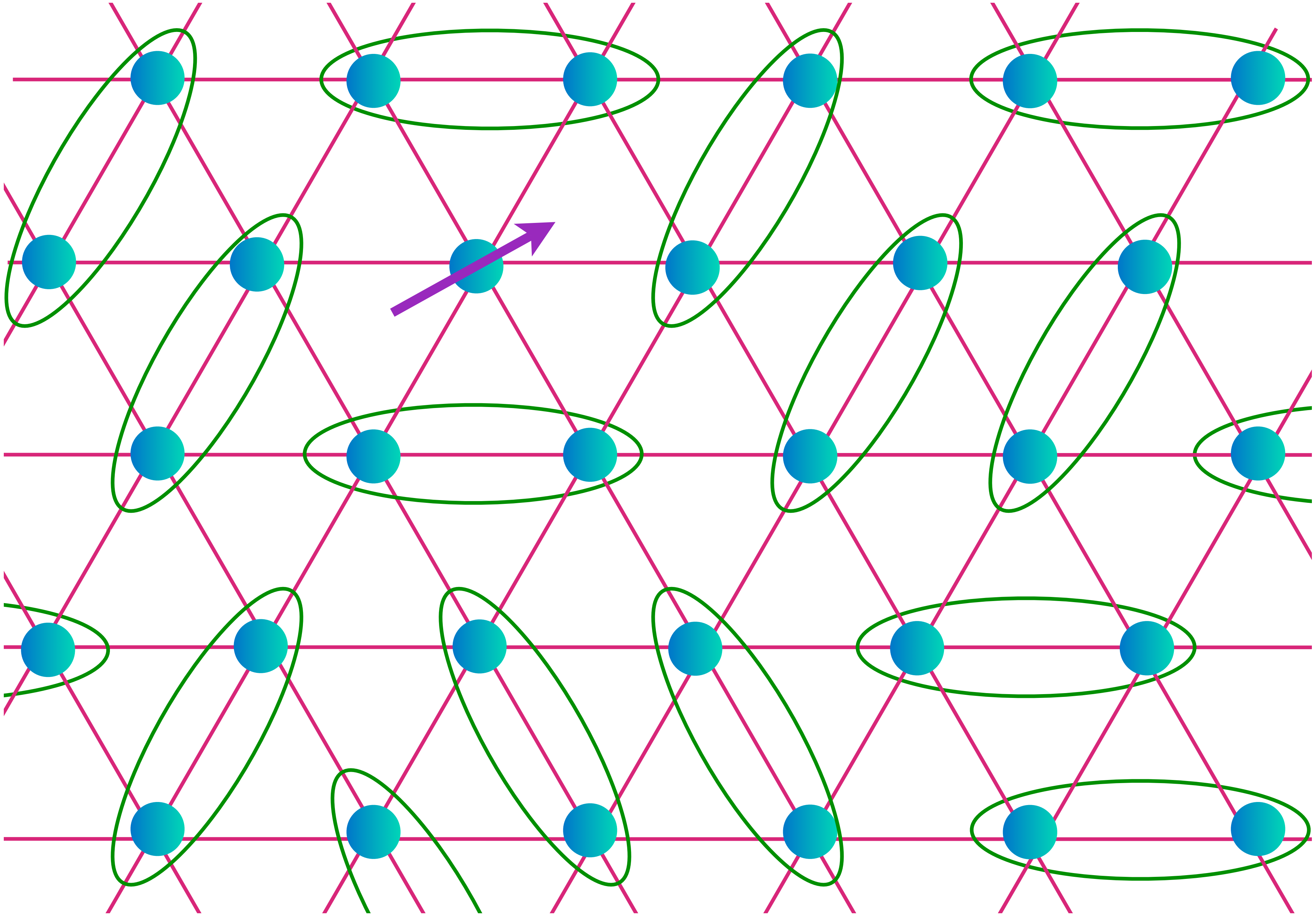

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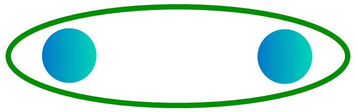

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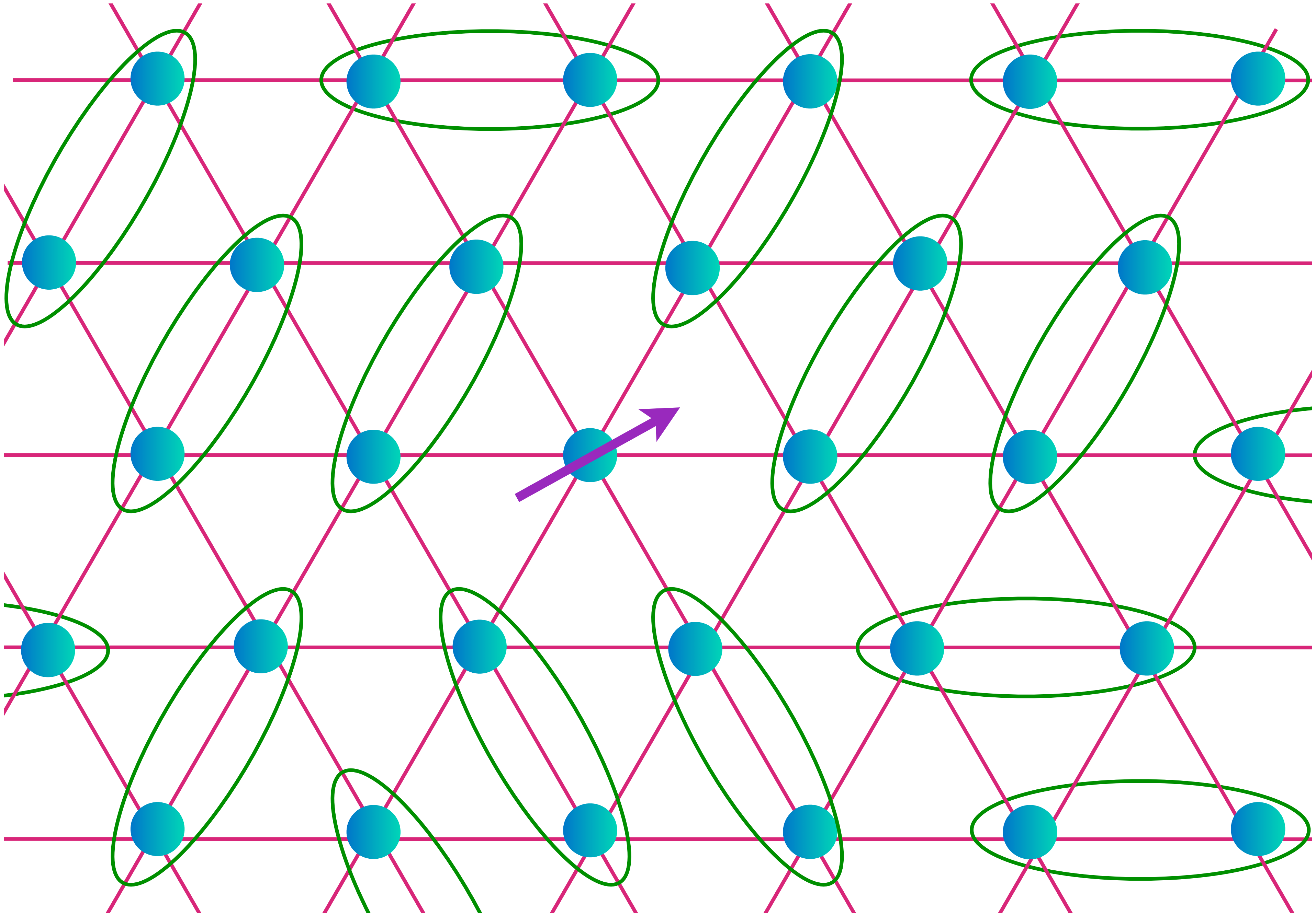

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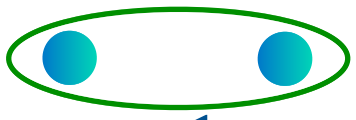

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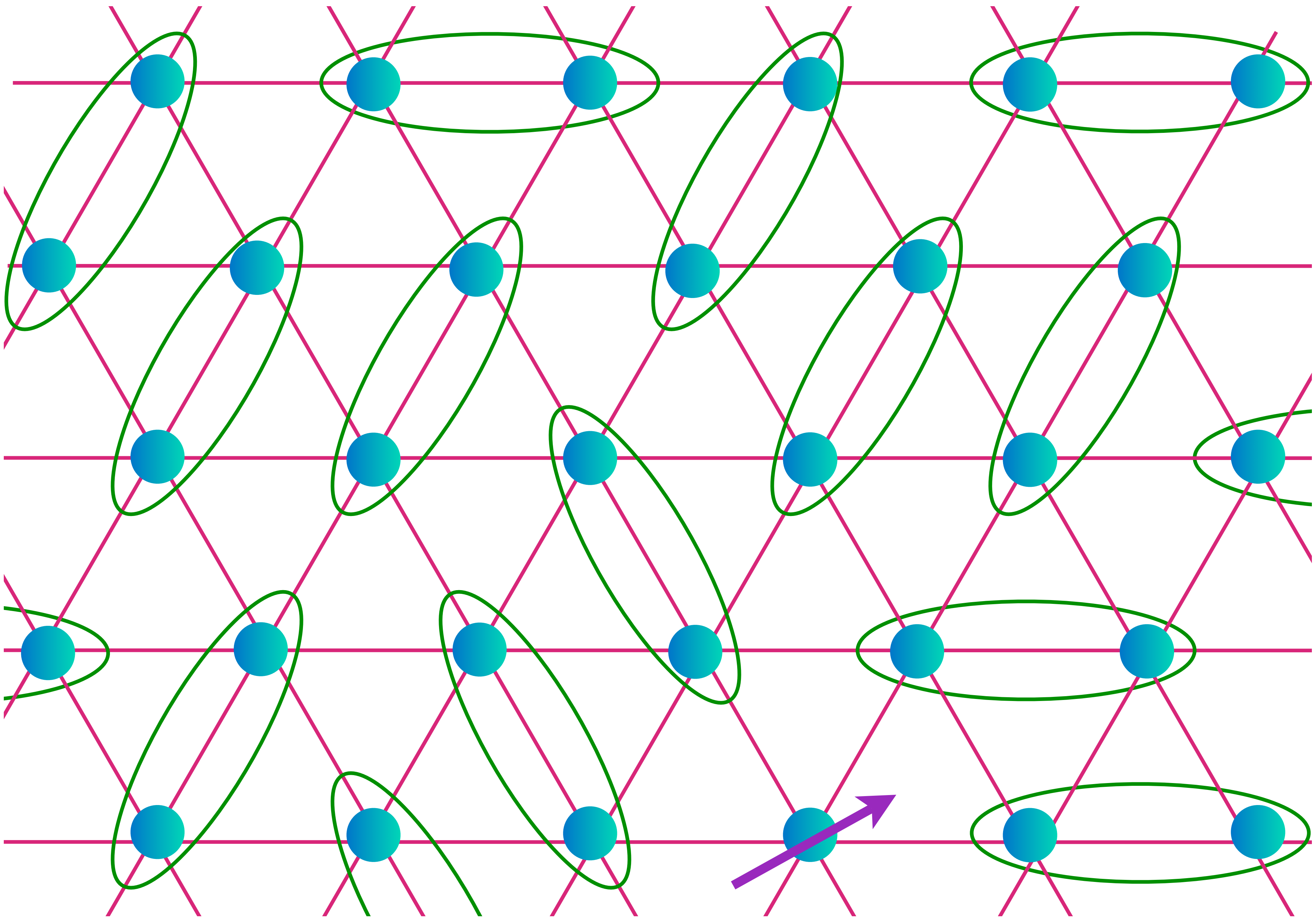

Excitations of the Z_2 Spin liquid

A vison

- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate
- A vison is an Abrikosov vortex in the pair condensate of spinons
- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and B. Chakraborty, *Phys. Rev. B* **40,** 7133 (1989)
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

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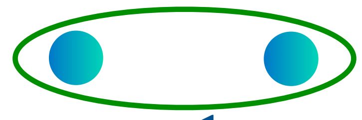

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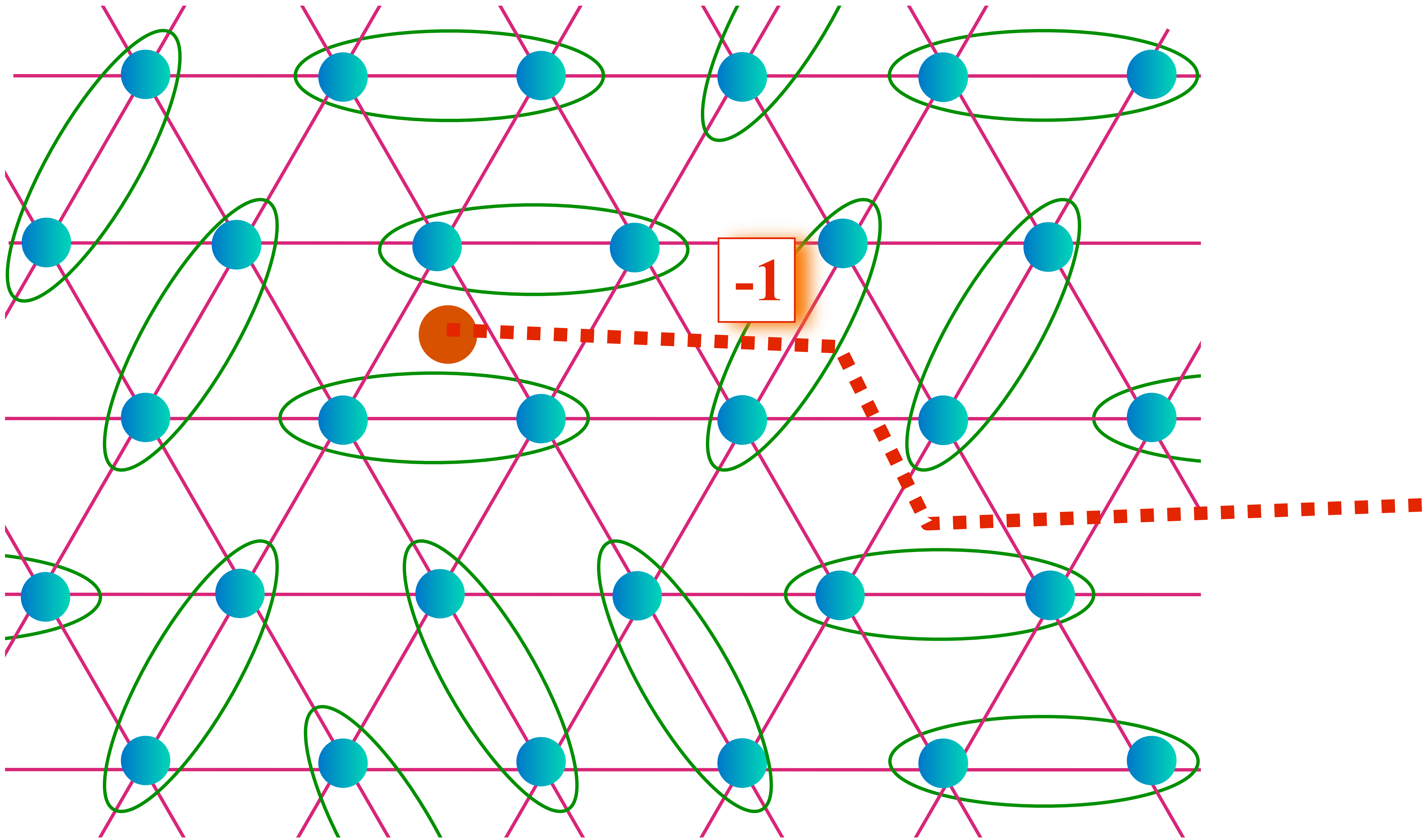

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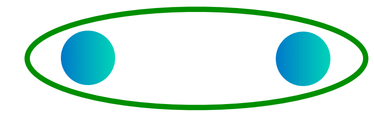

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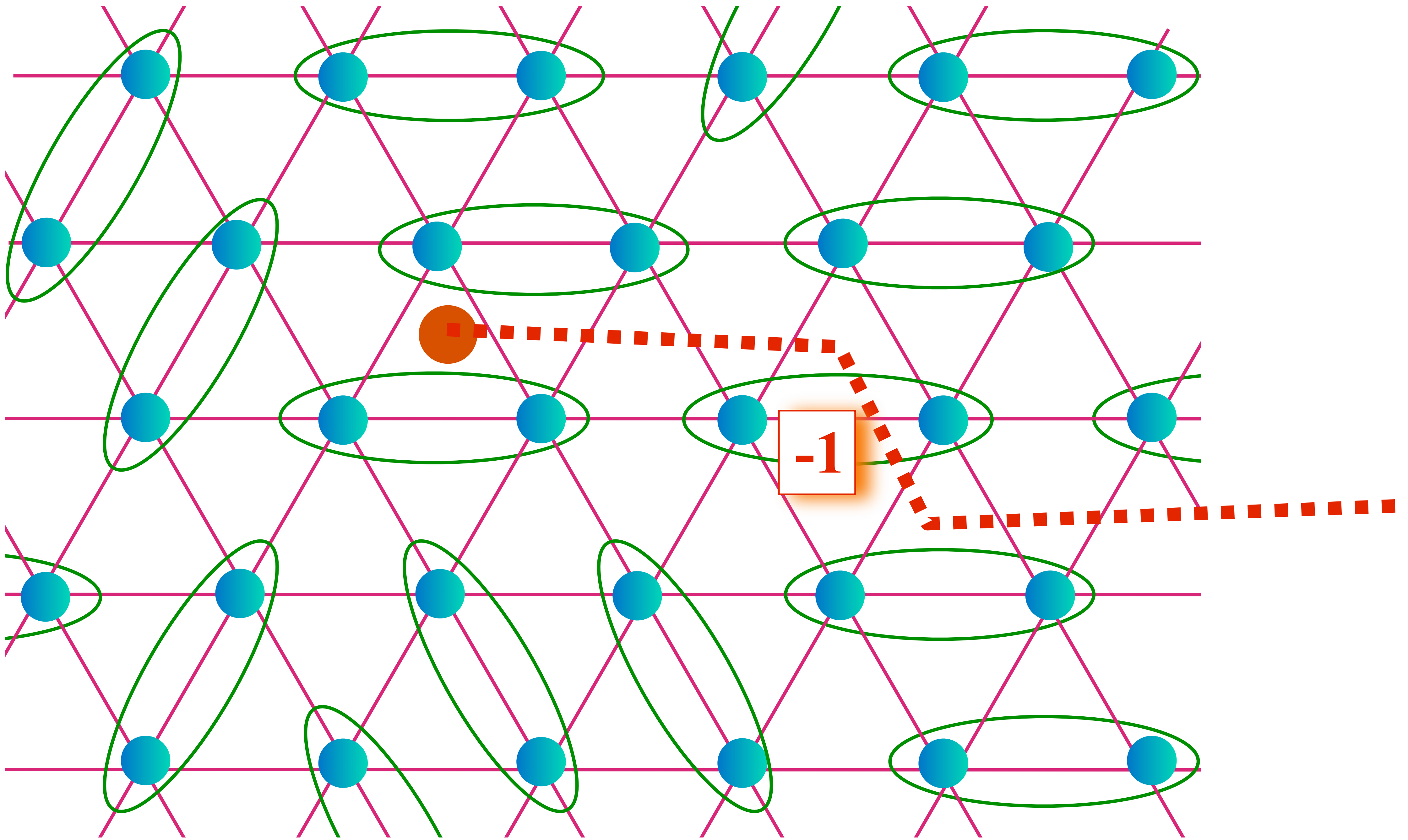

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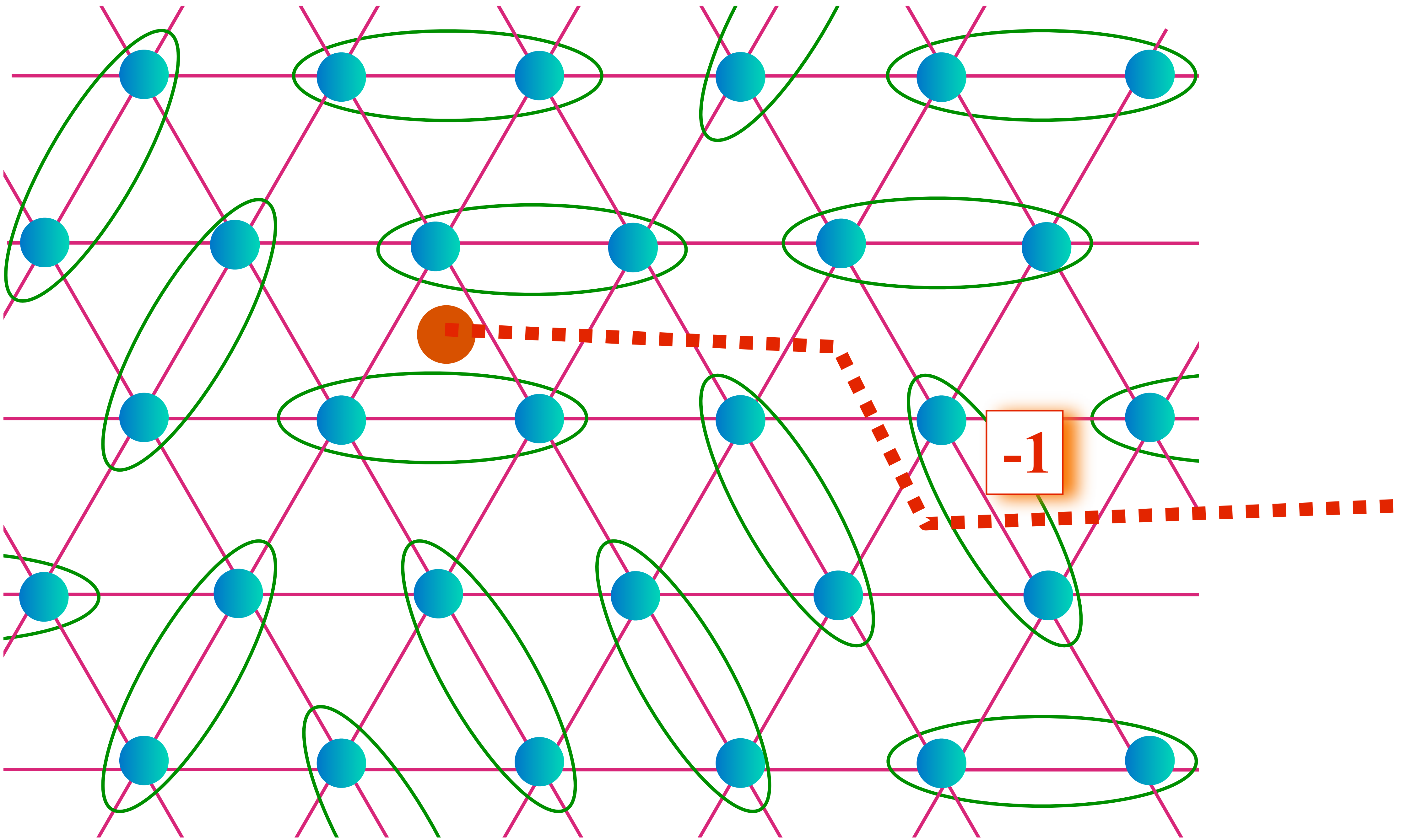

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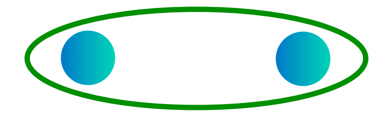

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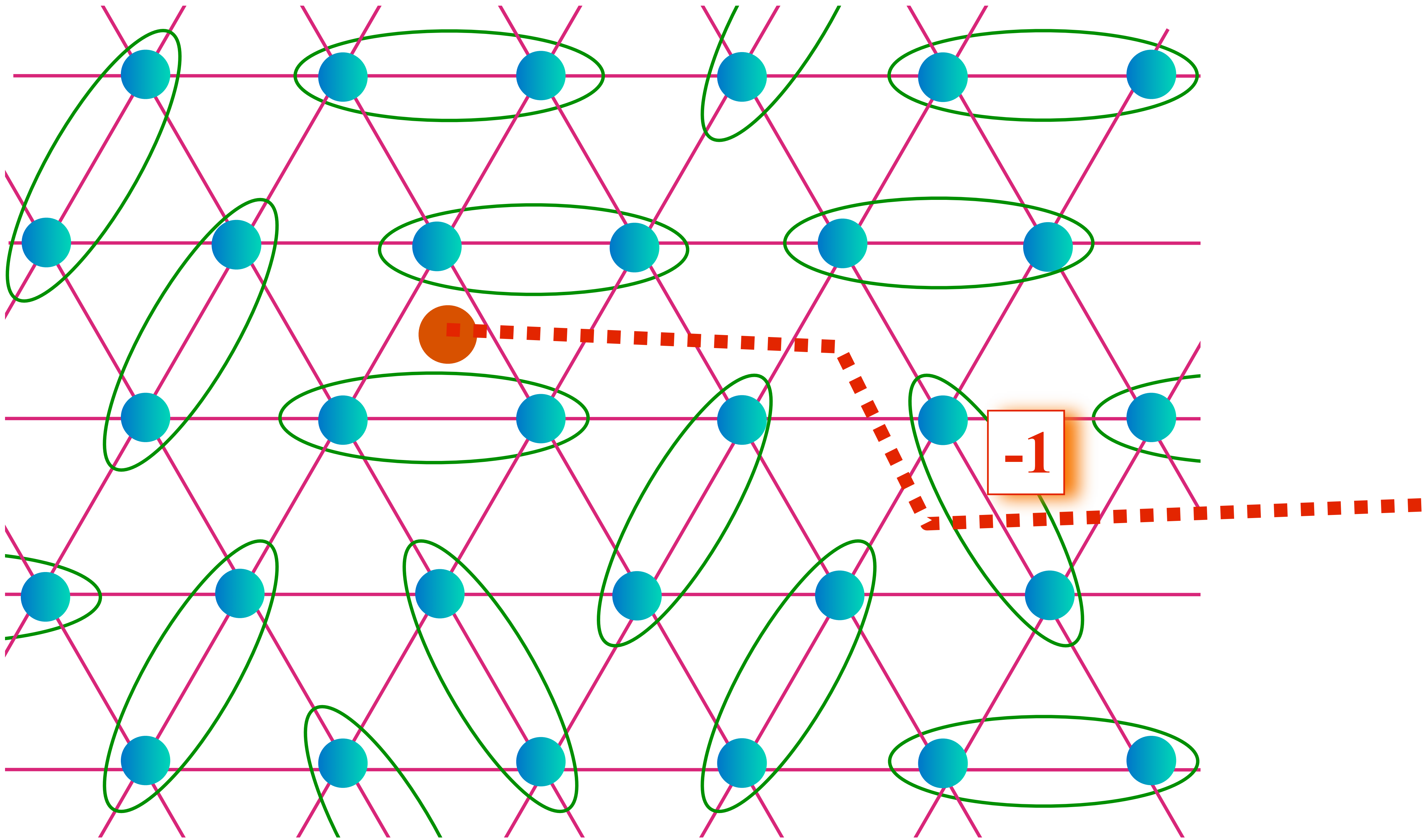

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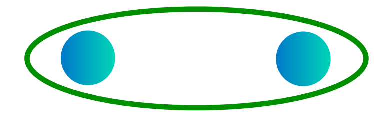

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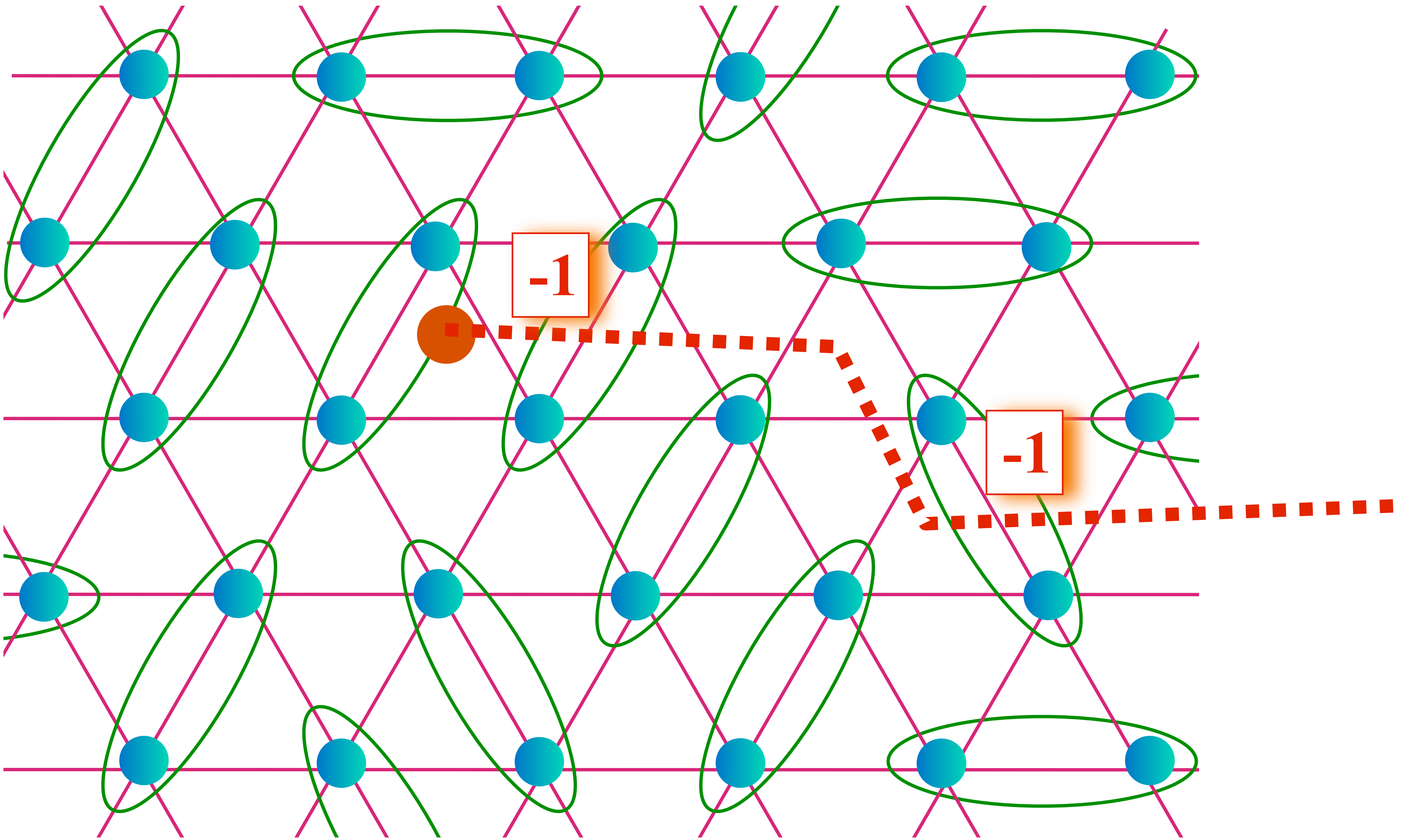

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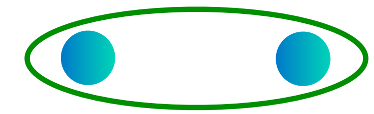

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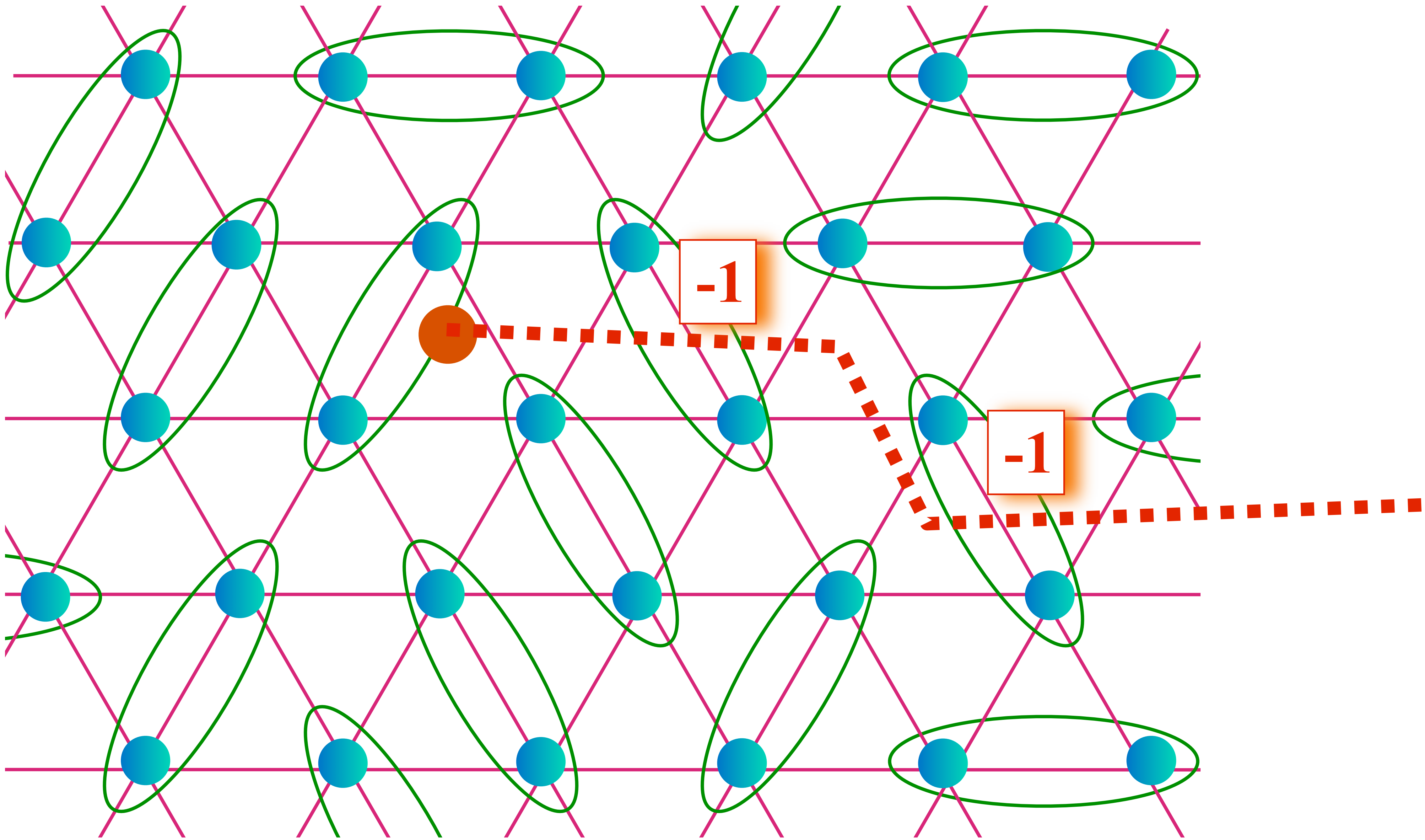

N. Read and B. Chakraborty, *Phys. Rev. B* **40,** 7133 (1989)
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Solvable models of Z_2 spin liquids

- Z_2 gauge theory - T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001).

- Quantum dimer models - D. Rokhsar and S. A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988); R. Moessner and S. L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001).

- Integrable Z_2 gauge theory - A. Y. Kitaev, *Annals of Physics* **303**, 2 (2003).

- Majorana fermion models - X.-G. Wen, *Phys. Rev. Lett.* **90**, 016803 (2003).

A Simple Toy Model (A. Kitaev, 1997)

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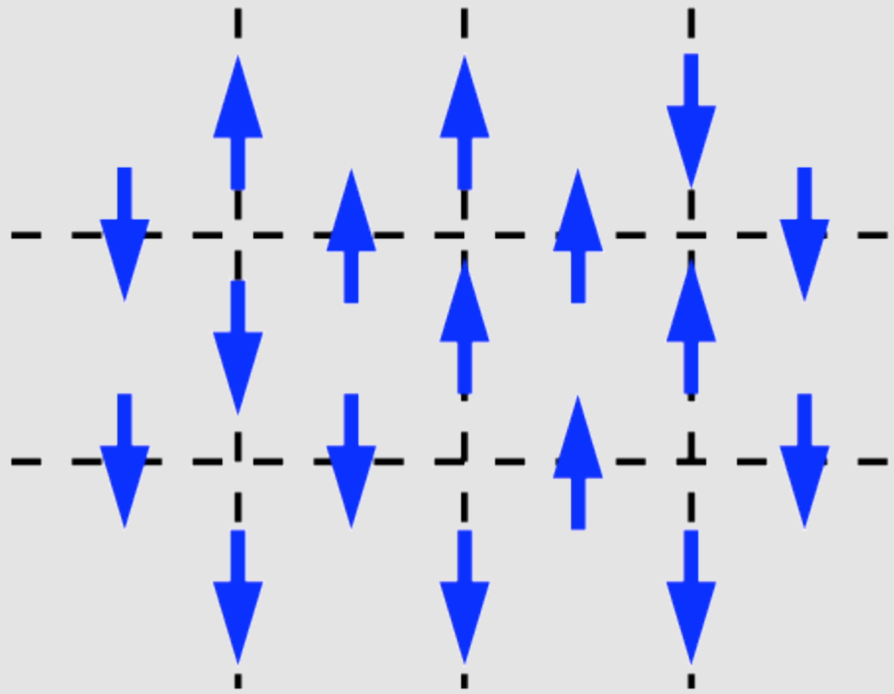

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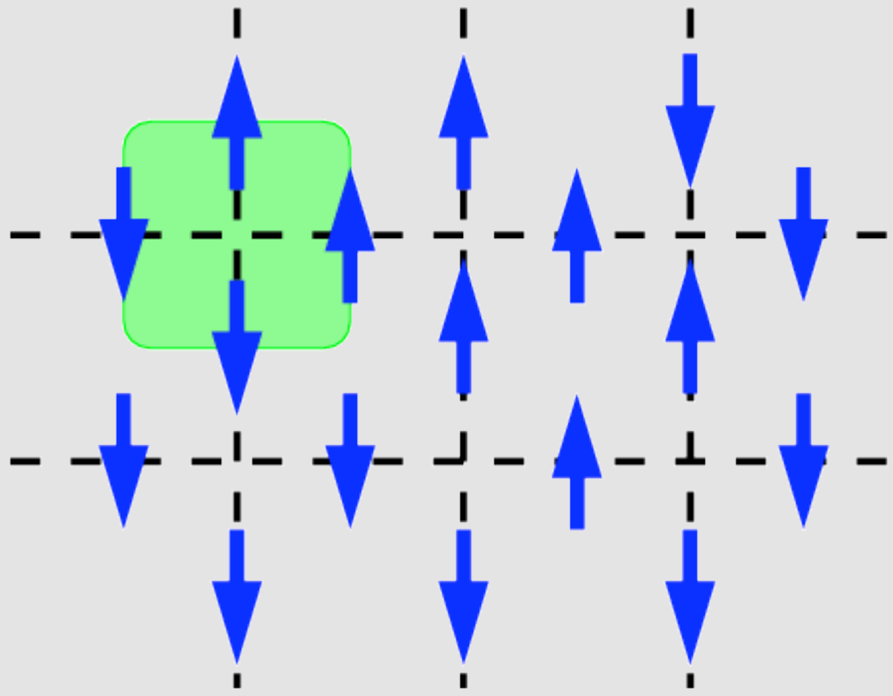

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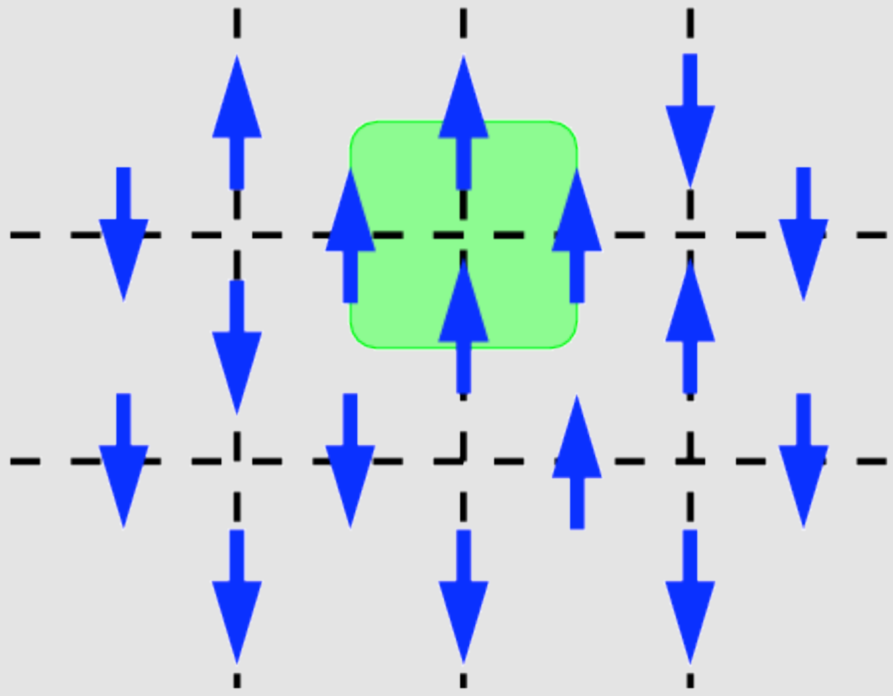

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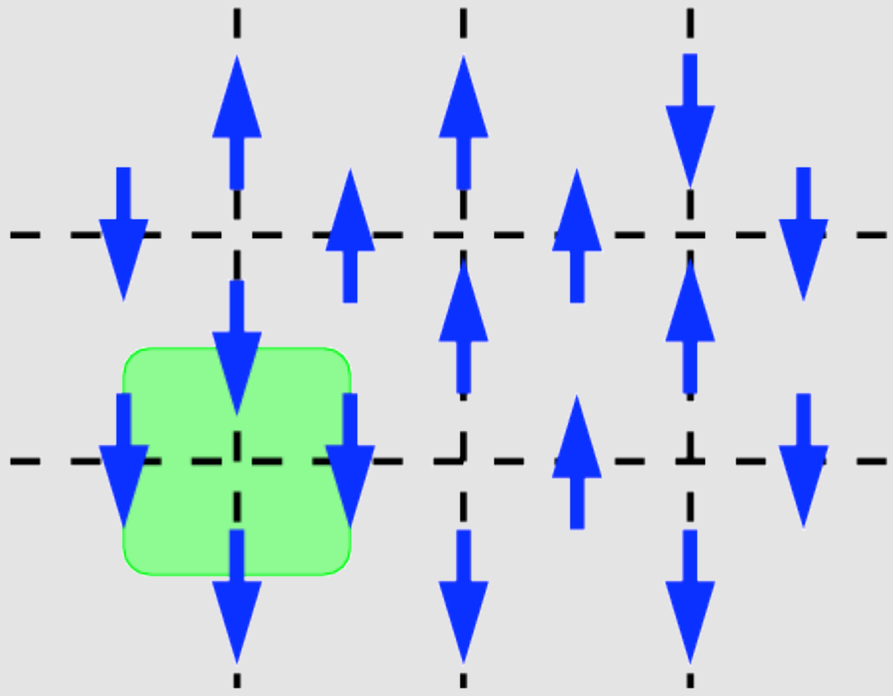

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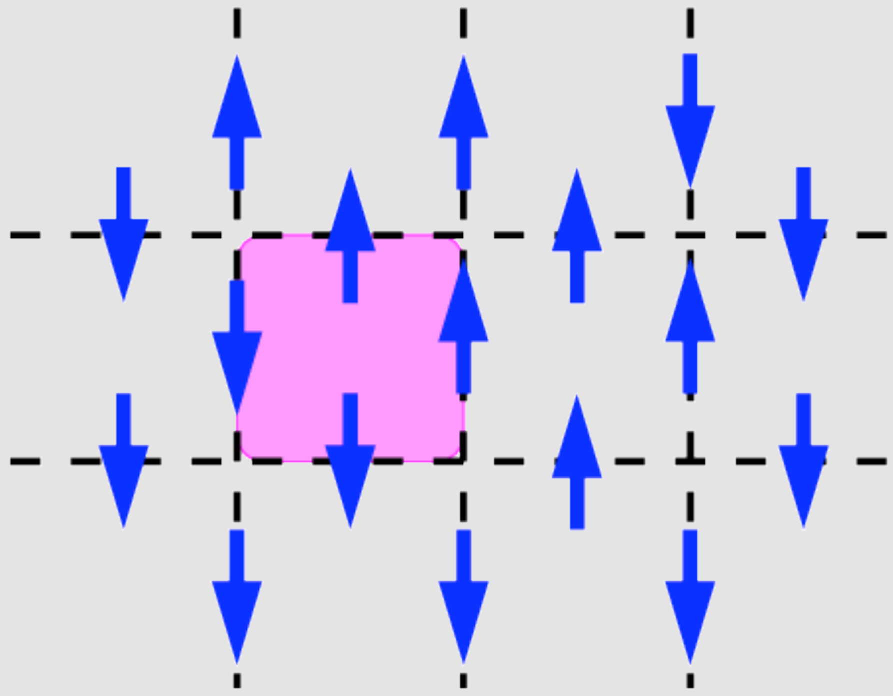

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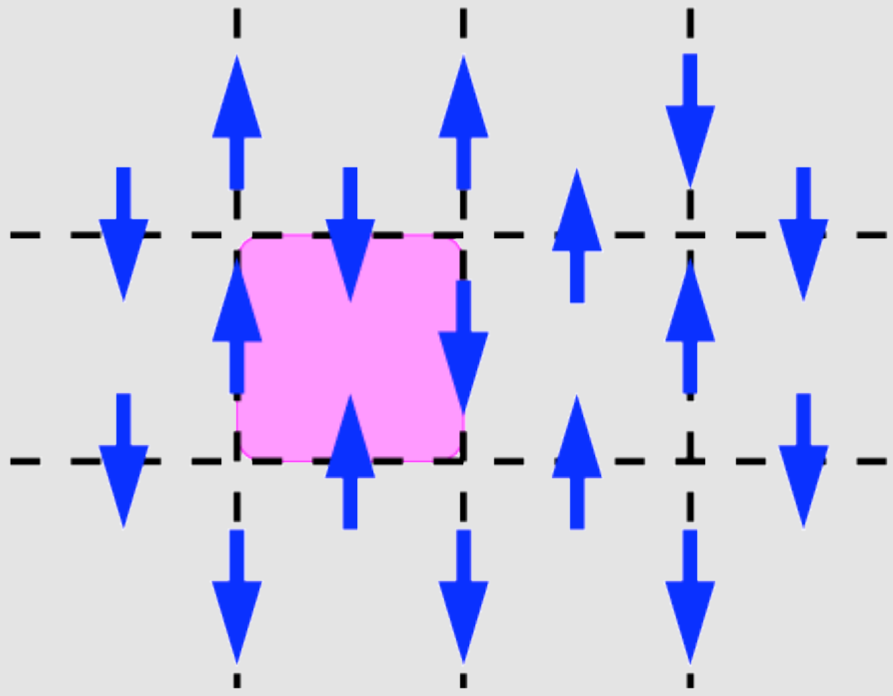

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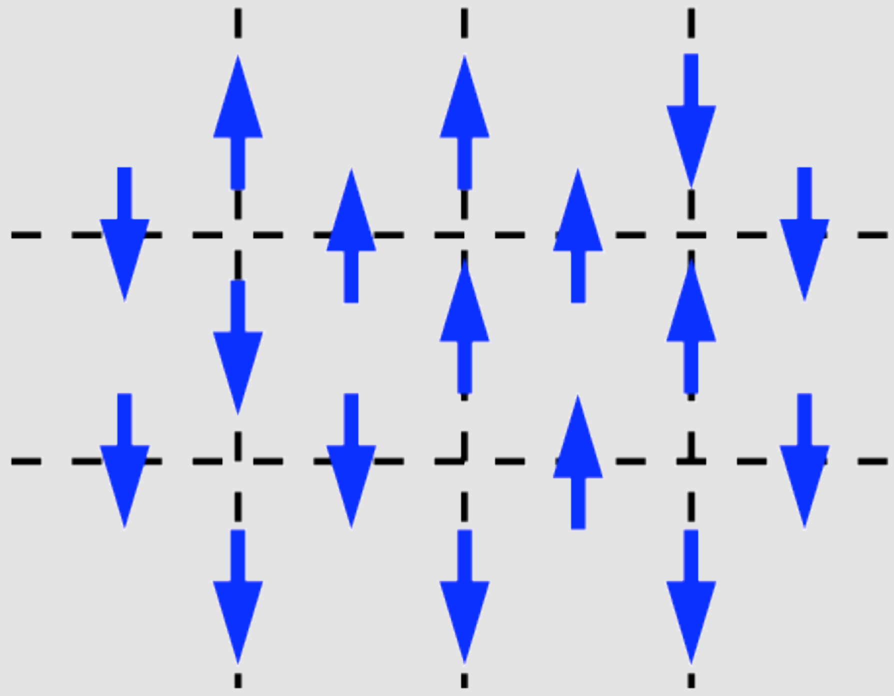

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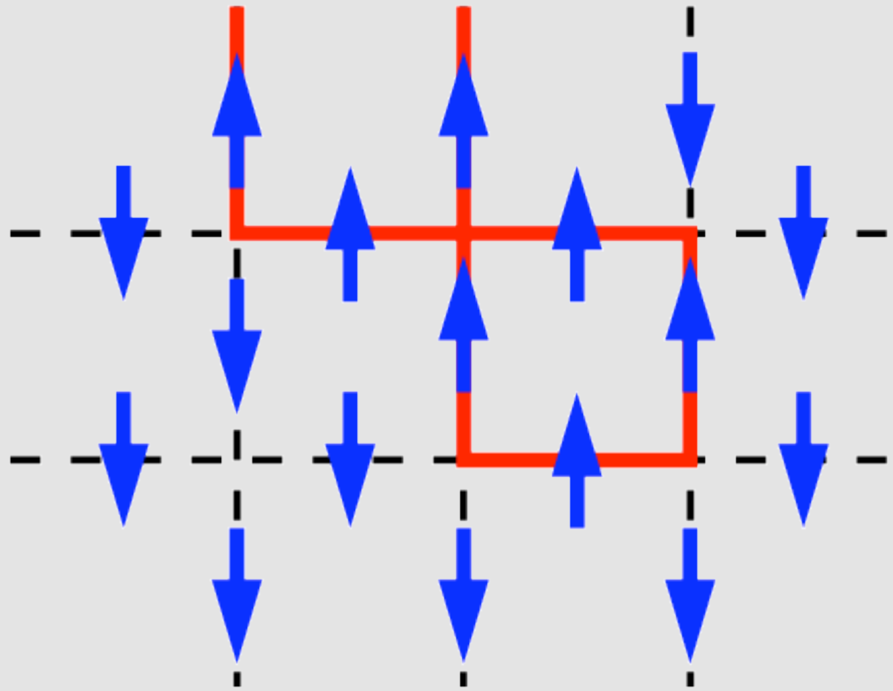

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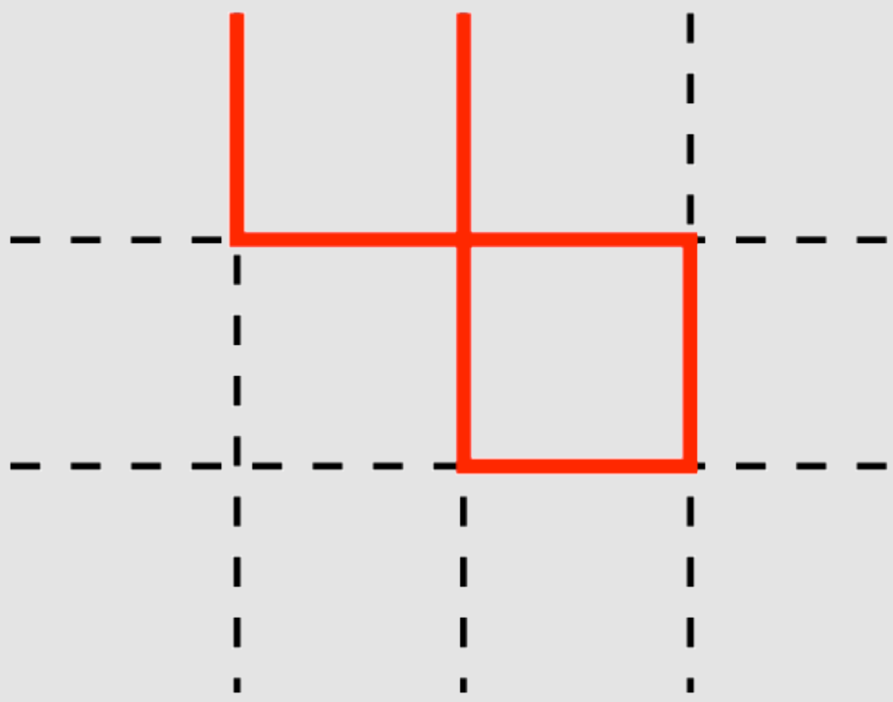

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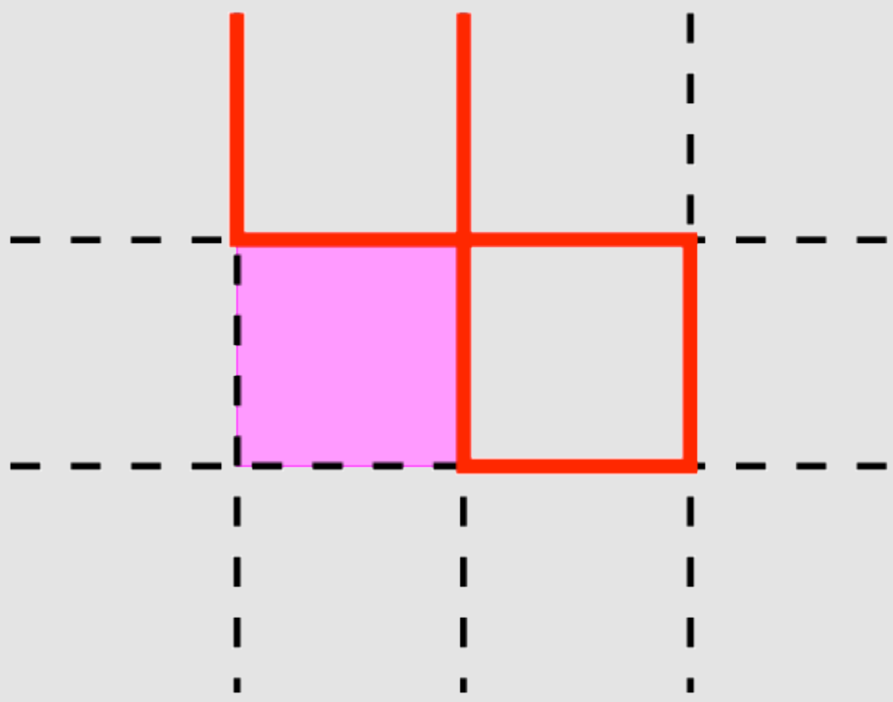

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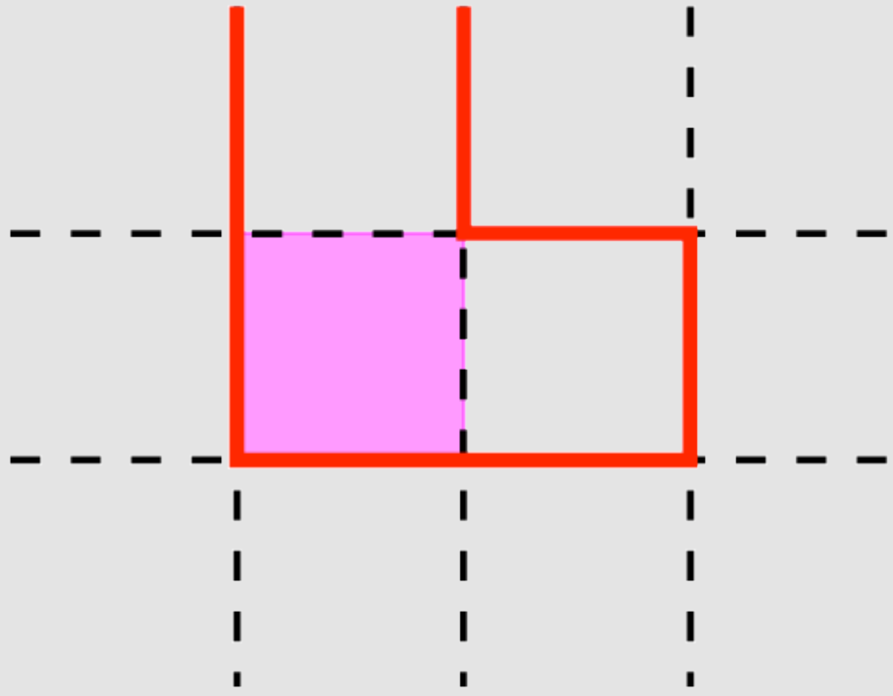

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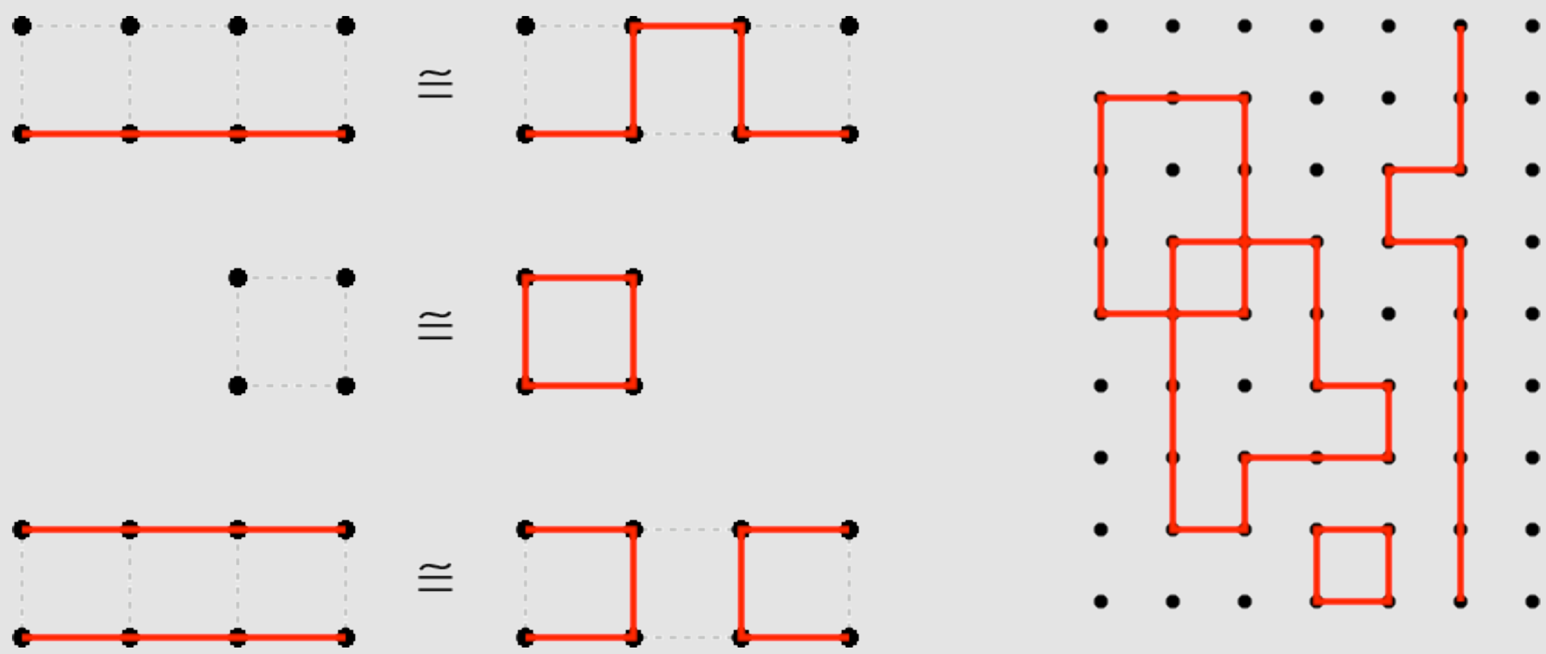
Properties of the Ground State

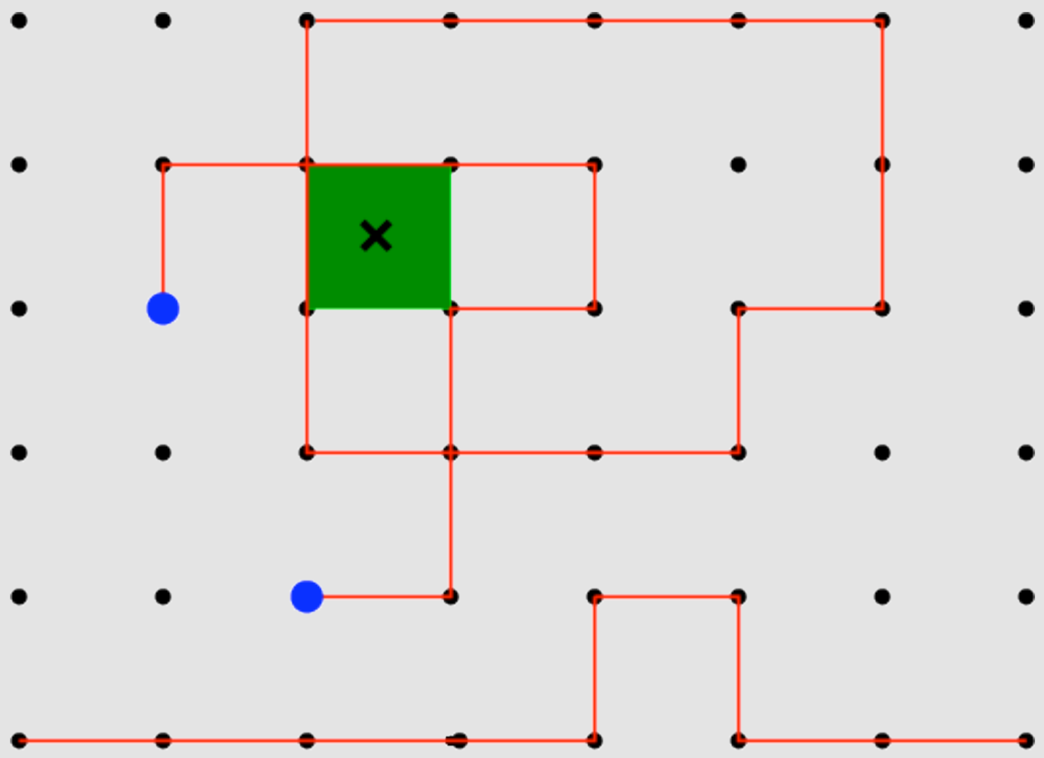
- Ground state: all A_i=1, F_p=1
- Pictorial representation: color each link with an up-spin.
- A_i=1 : closed loops.
- F_p=1 : every plaquette is an equal-amplitude superposition of inverse images.



The GS wavefunction takes the same value on configurations connected by these operations. It does not depend on the geometry of the configurations, only on their topology.

Properties of Excitations

- "Electric" particle, or $A_i = -1$ – endpoint of a line
- "Magnetic particle", or *vortex*: $F_p = -1$ – a "flip" of this plaquette changes the sign of a given term in the superposition.
- Charges and vortices interact via topological Aharonov-Bohm interactions.



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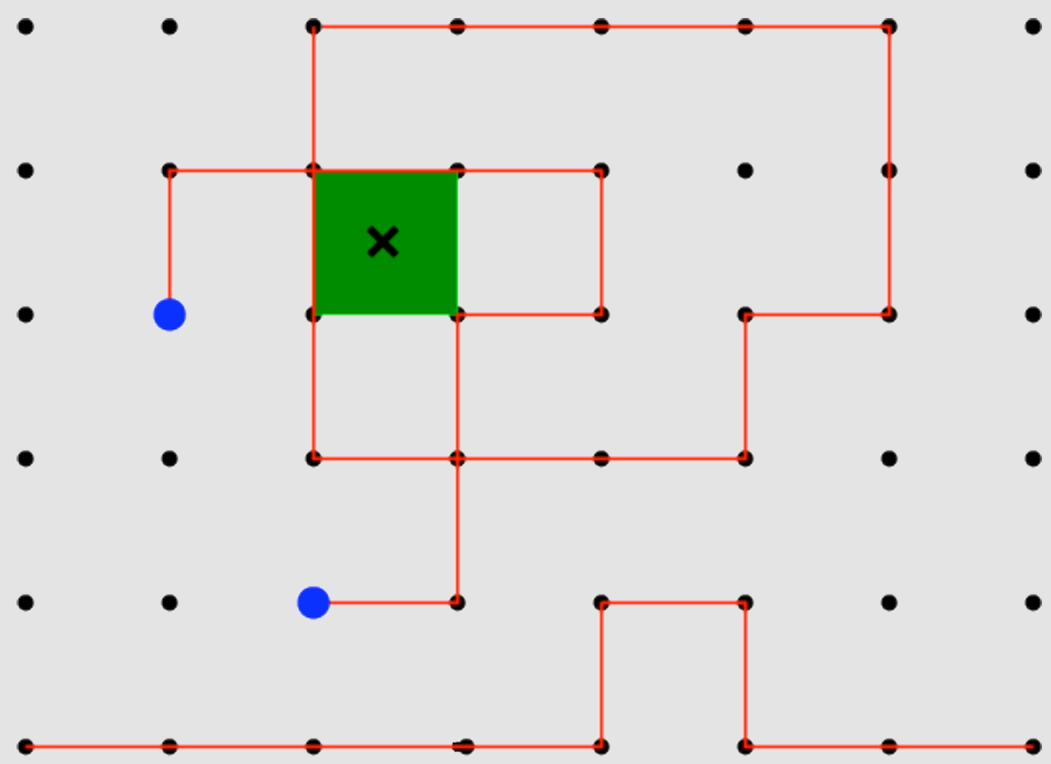

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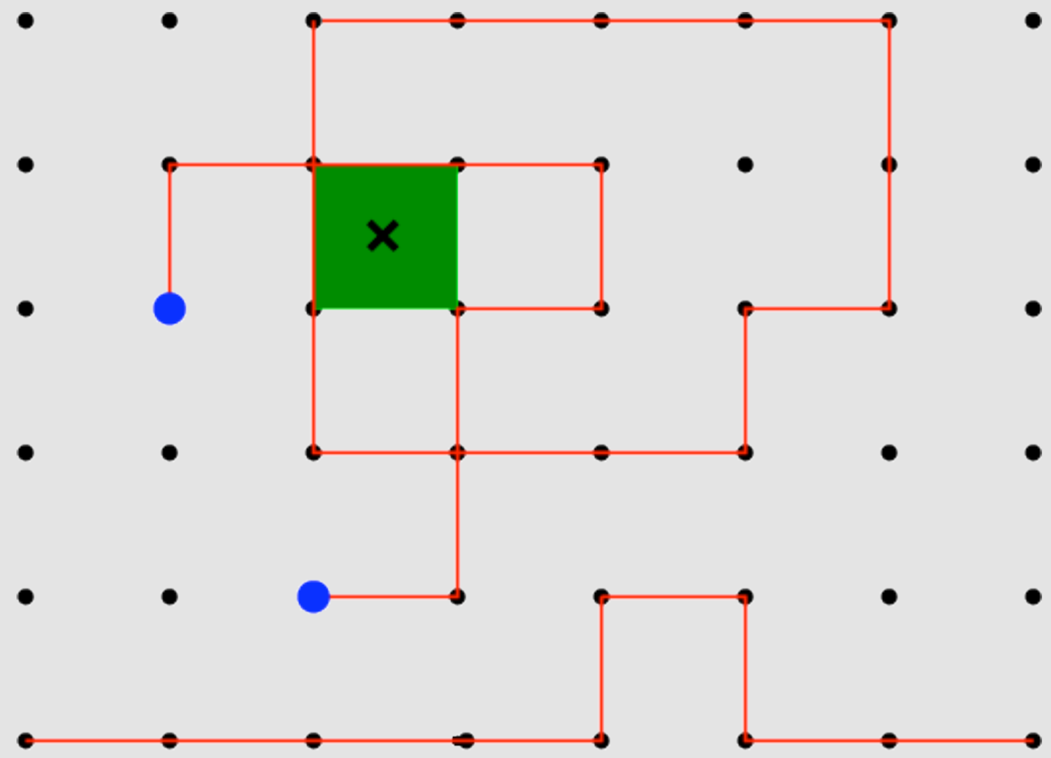

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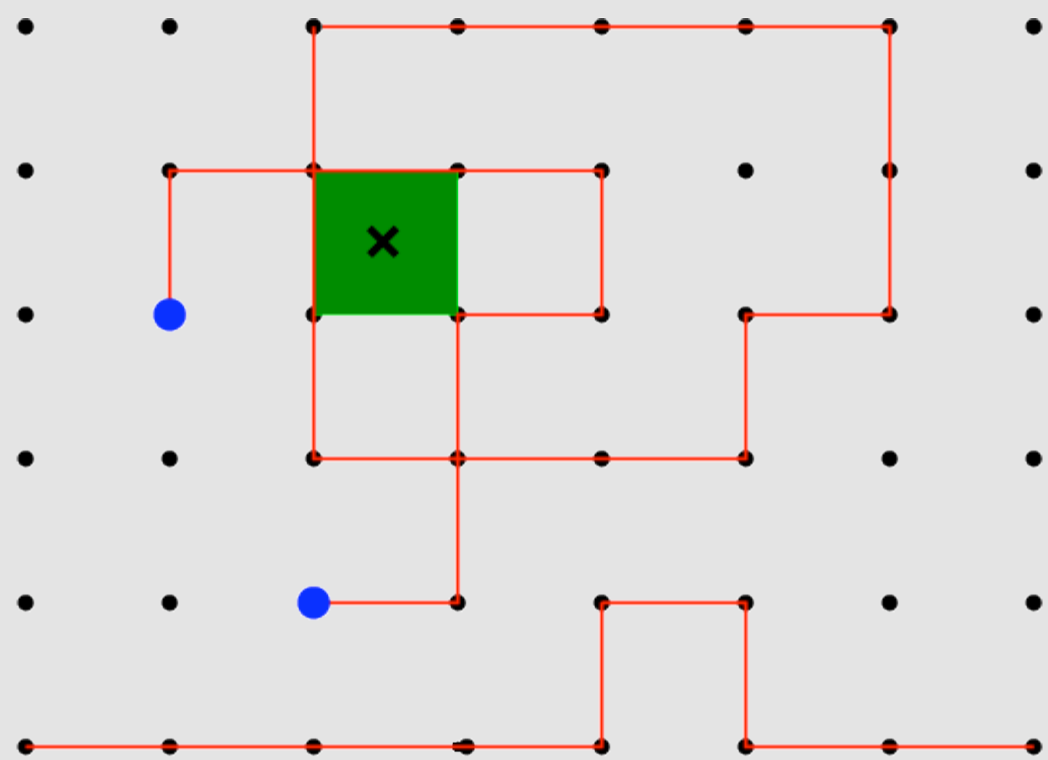


- *Spinons and visons are mutual semions*

Beyond Z_2 spin liquids

Quantum spin liquids with anyonic excitations

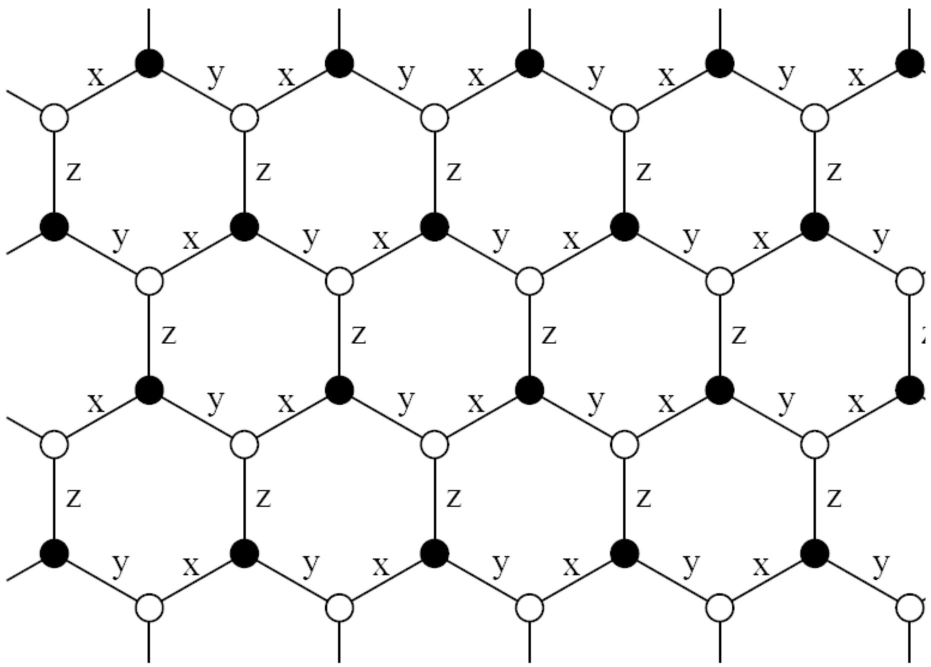
- Doubled $SU(2)_k$ Chern-Simons theory - M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, *Annals of Physics* **310**, 428 (2004).

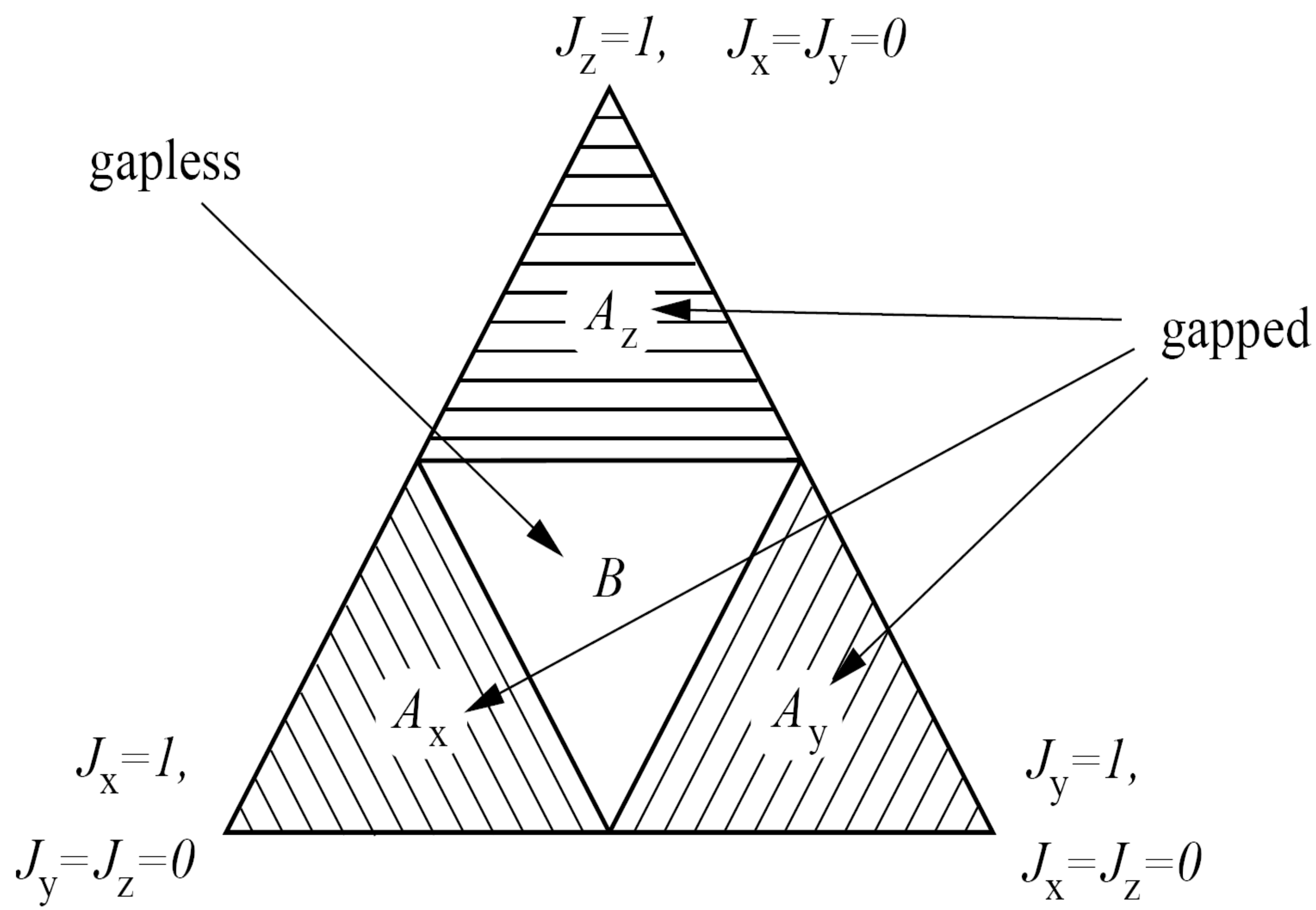
- String nets - M. A. Levin and X.-G. Wen, *Phys. Rev. B* **71**, 045110 (2005).

- Spin model on the honeycomb lattice - A. Y. Kitaev, *Annals of Physics* **321**, 2 (2006).

Kitaev Model

$$H = J_1 \sum_{x-link} \sigma_n^x \sigma_m^x + J_2 \sum_{y-link} \sigma_n^y \sigma_m^y + J_3 \sum_{z-link} \sigma_n^z \sigma_m^z$$





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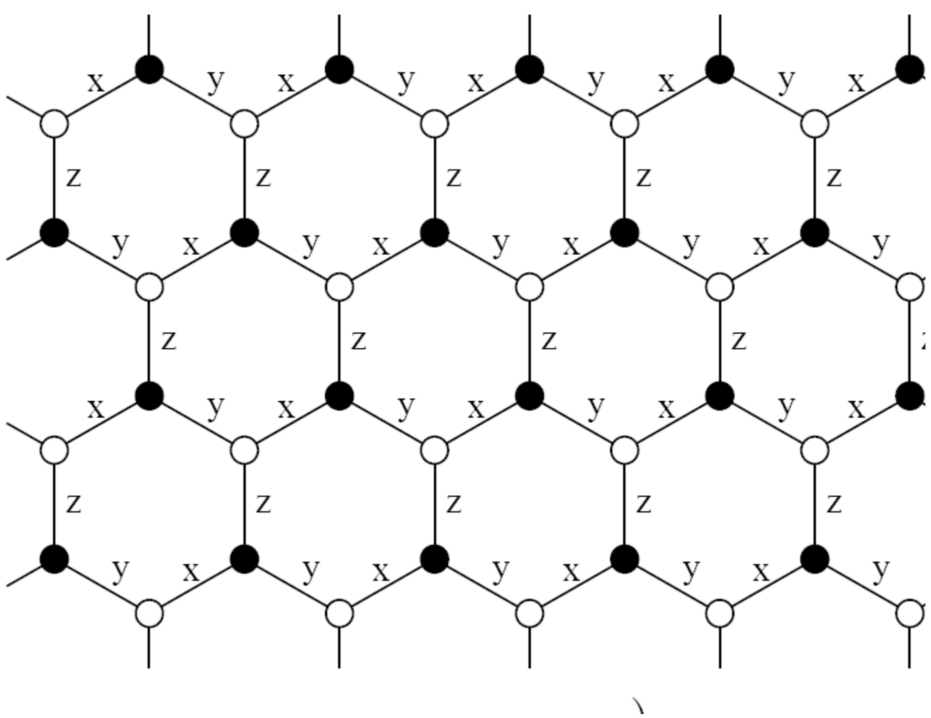

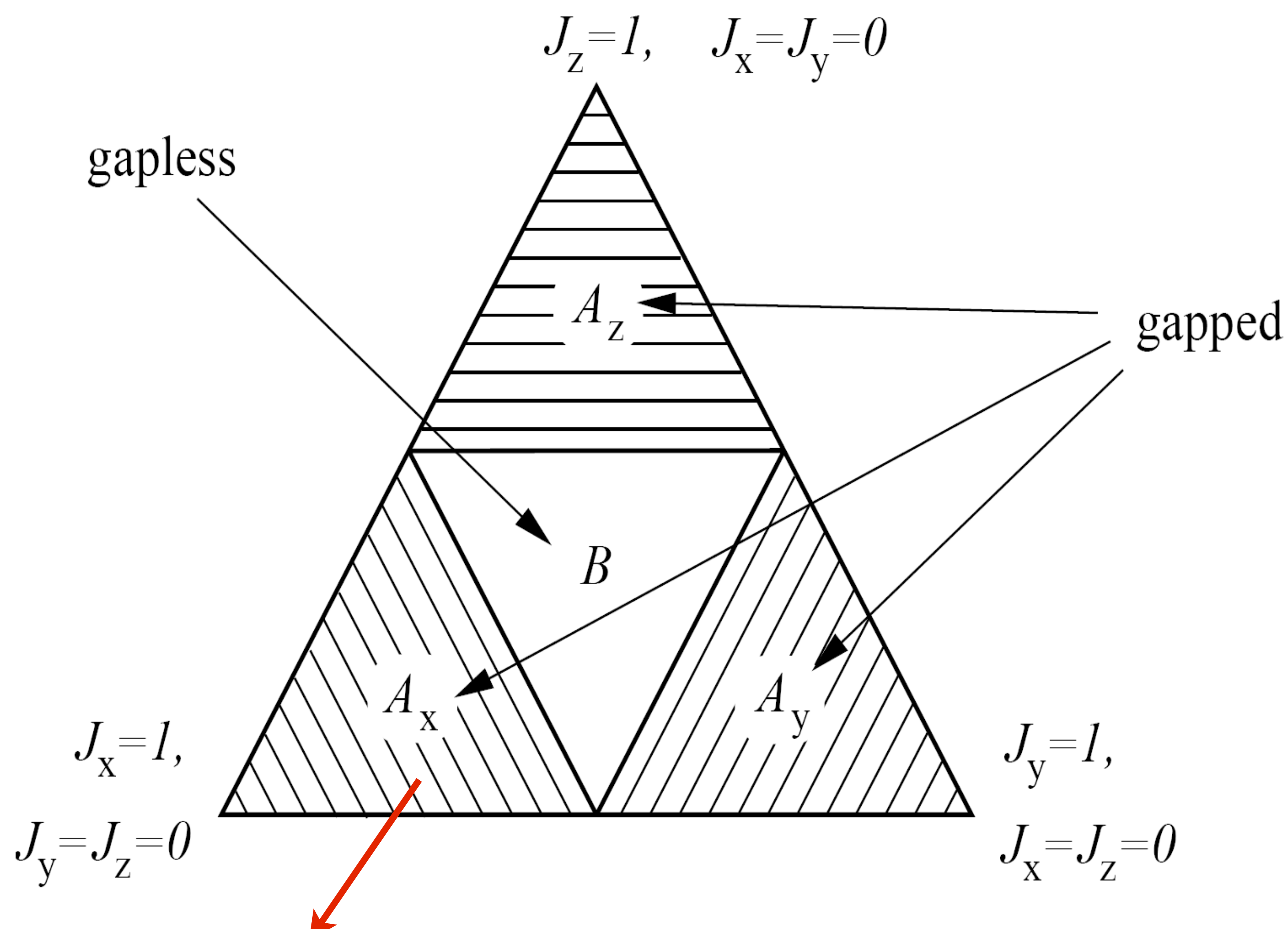

Z_2 spin liquids

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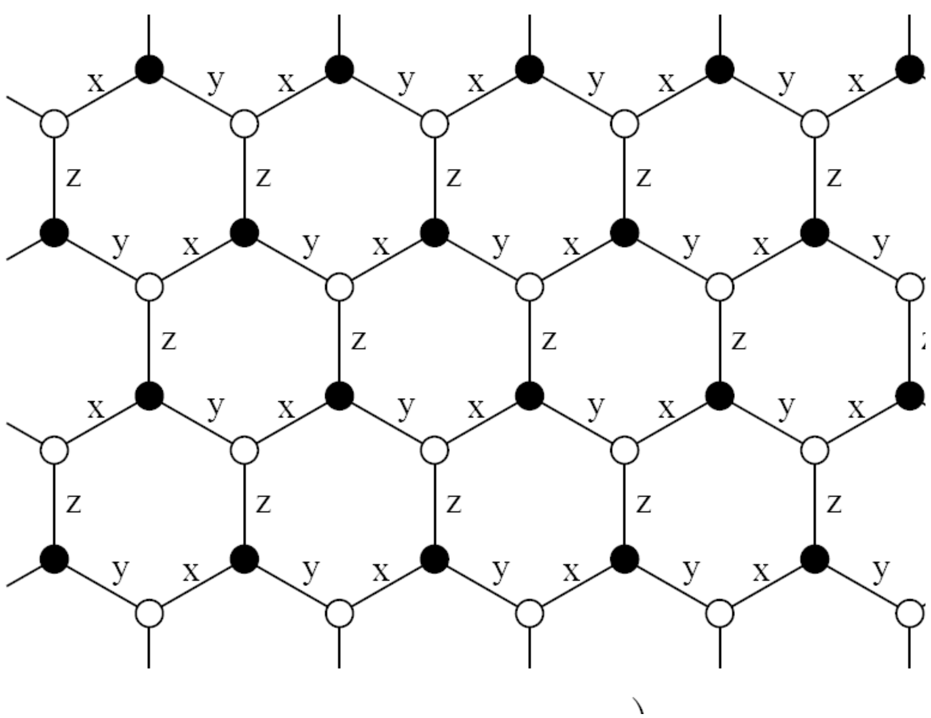
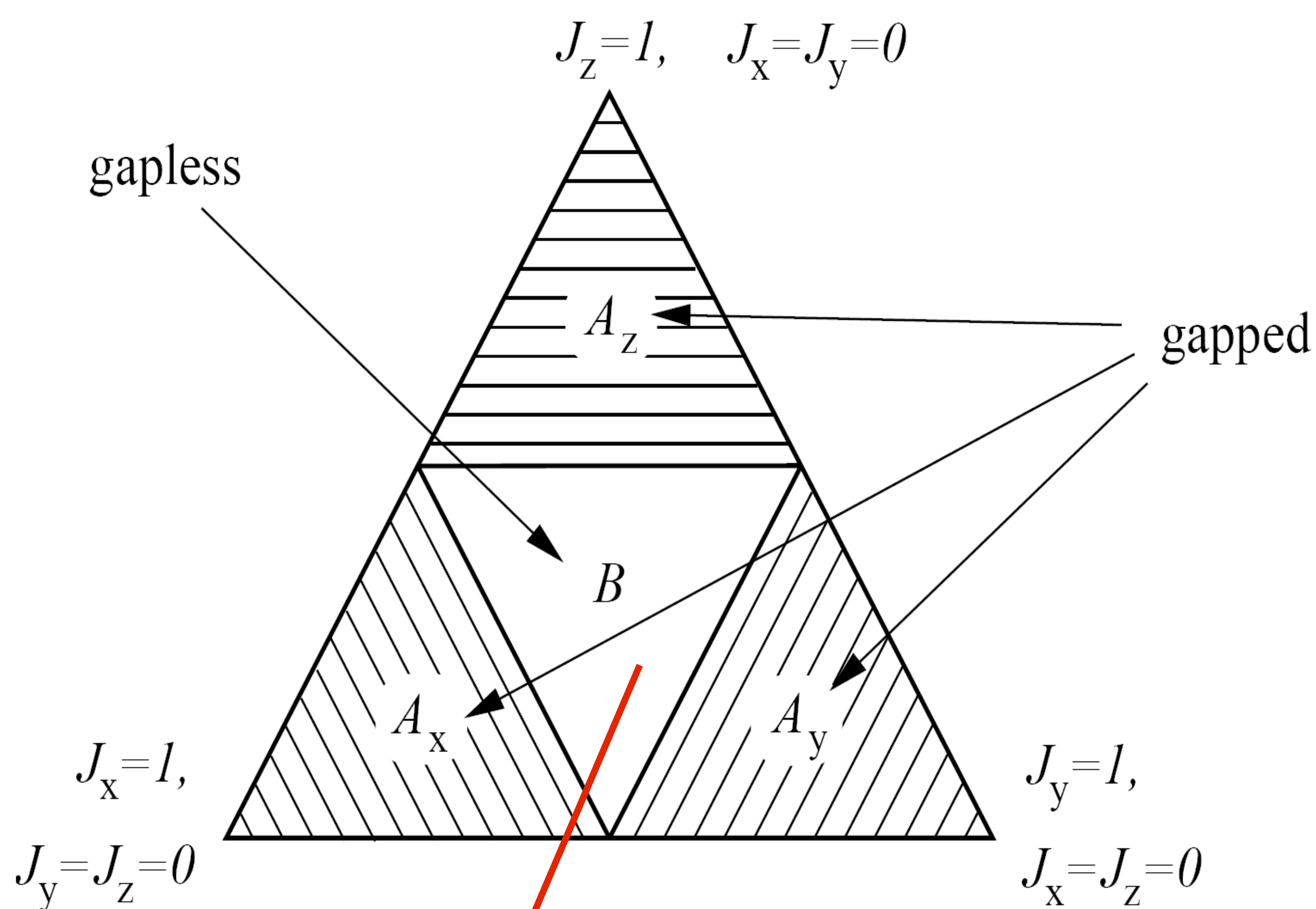
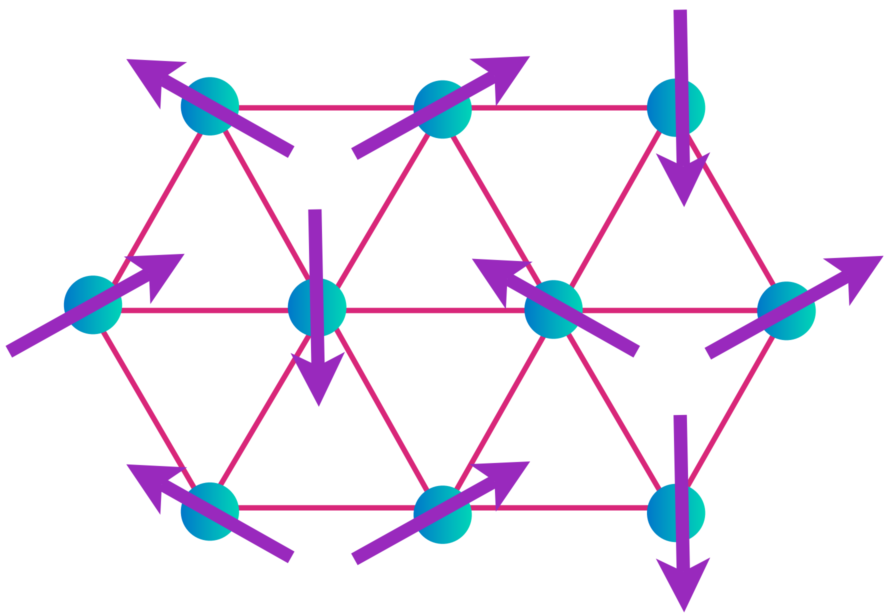




Non-abelian anyons in a magnetic field

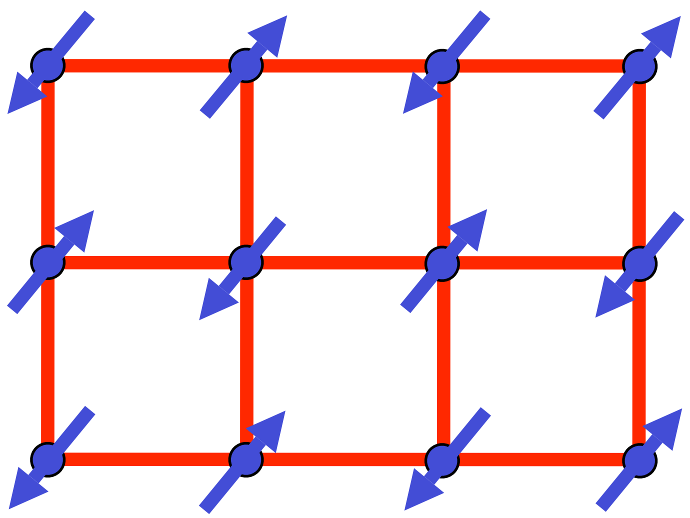
Quantum "disordering" magnetic order



Z_2 spin liquid
with paired spinons
and a **vison** excitation

non-collinear Néel state

Quantum “disordering” magnetic order



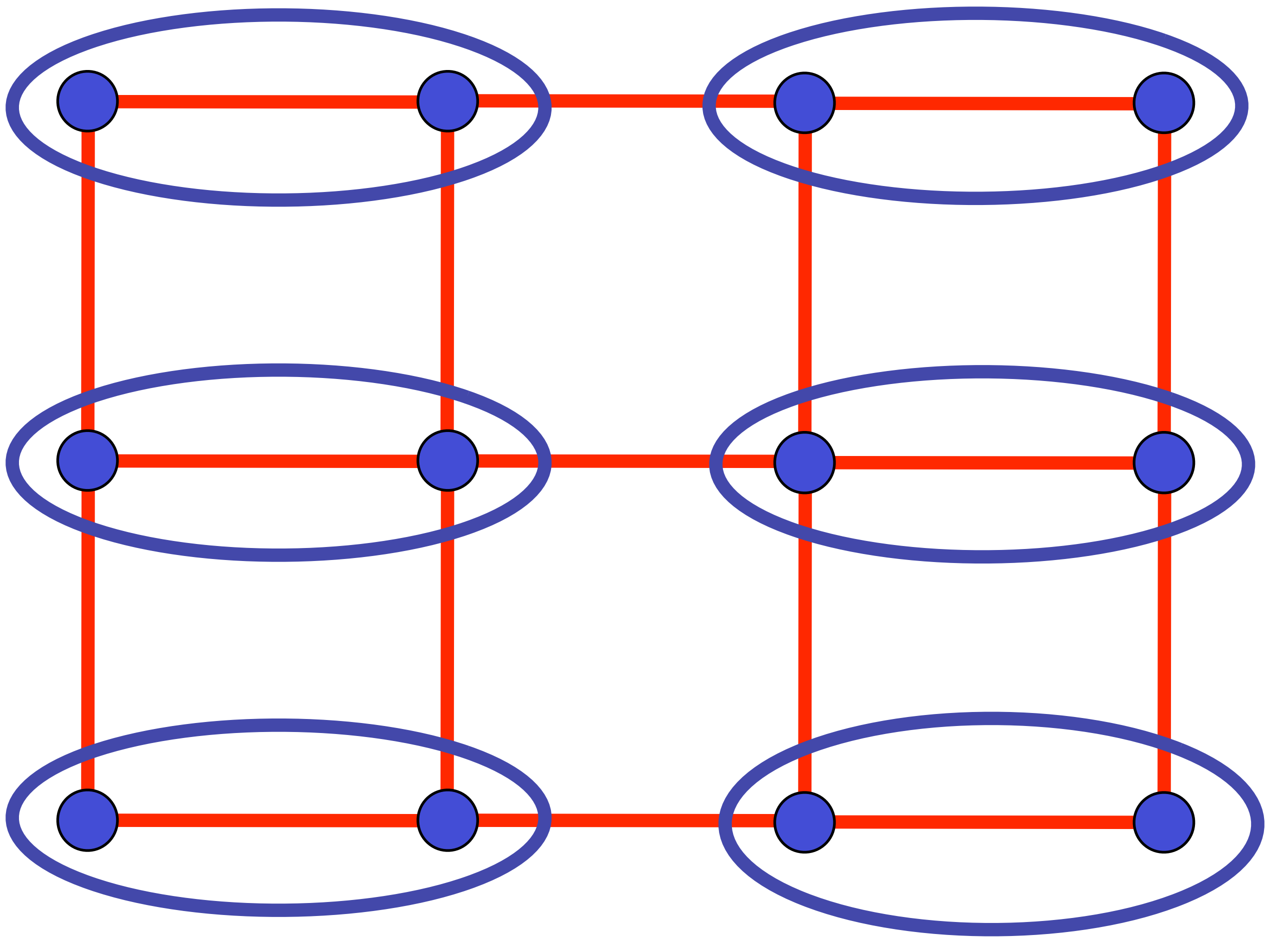
Spin liquid with a **“photon”**, which is unstable to the appearance of valence bond solid (VBS) order

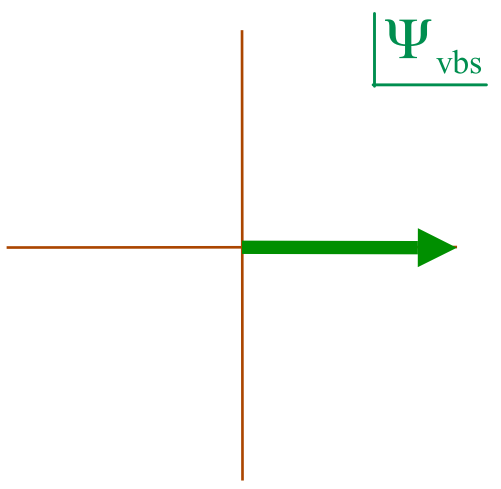
collinear Néel state

s_c $\qquad$ s

D. P. Arovas and A. Auerbach, *Phys. Rev. B* **38**, 316 (1988).
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

Order parameter of VBS state





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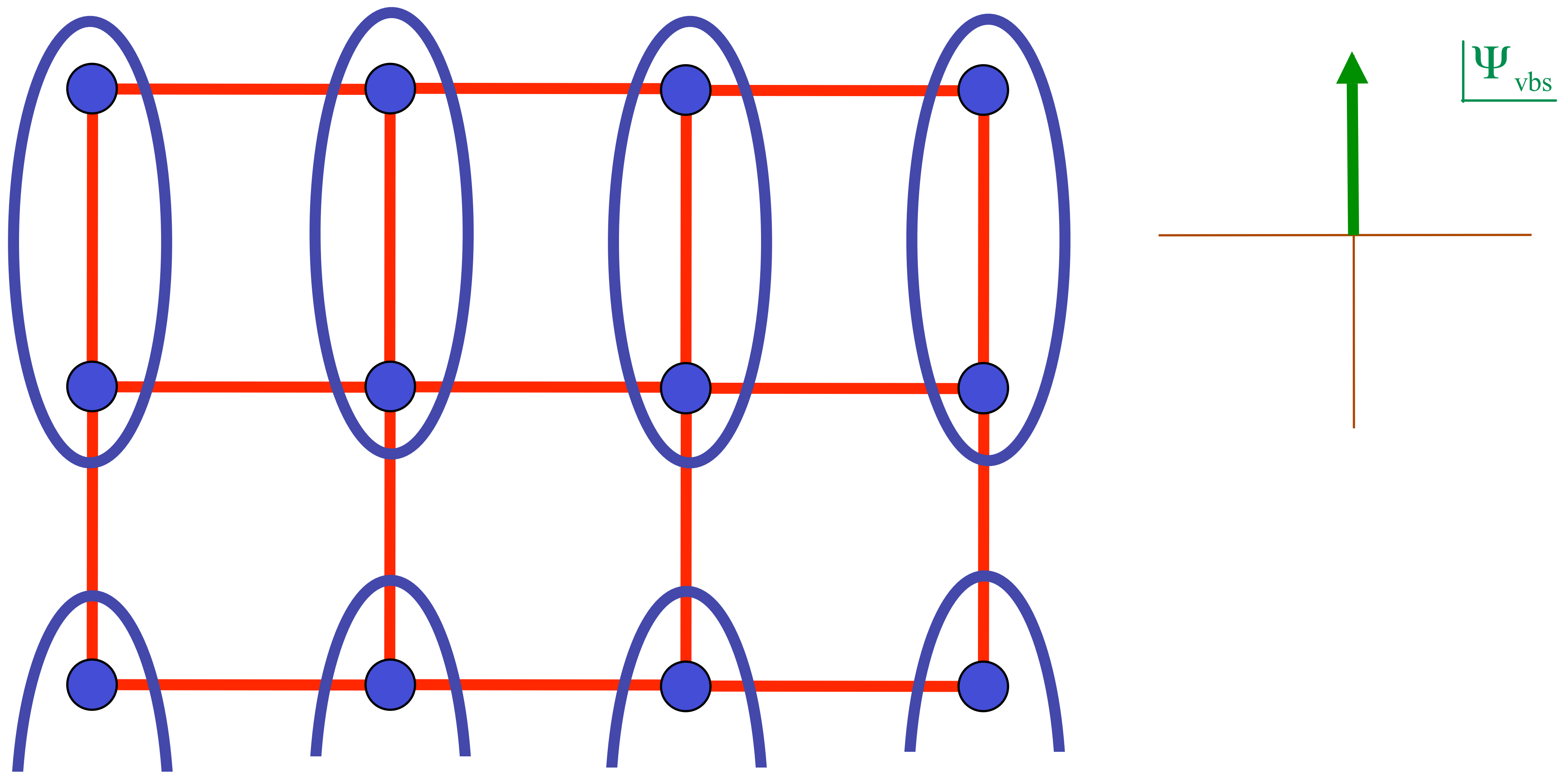
Order parameter of VBS state



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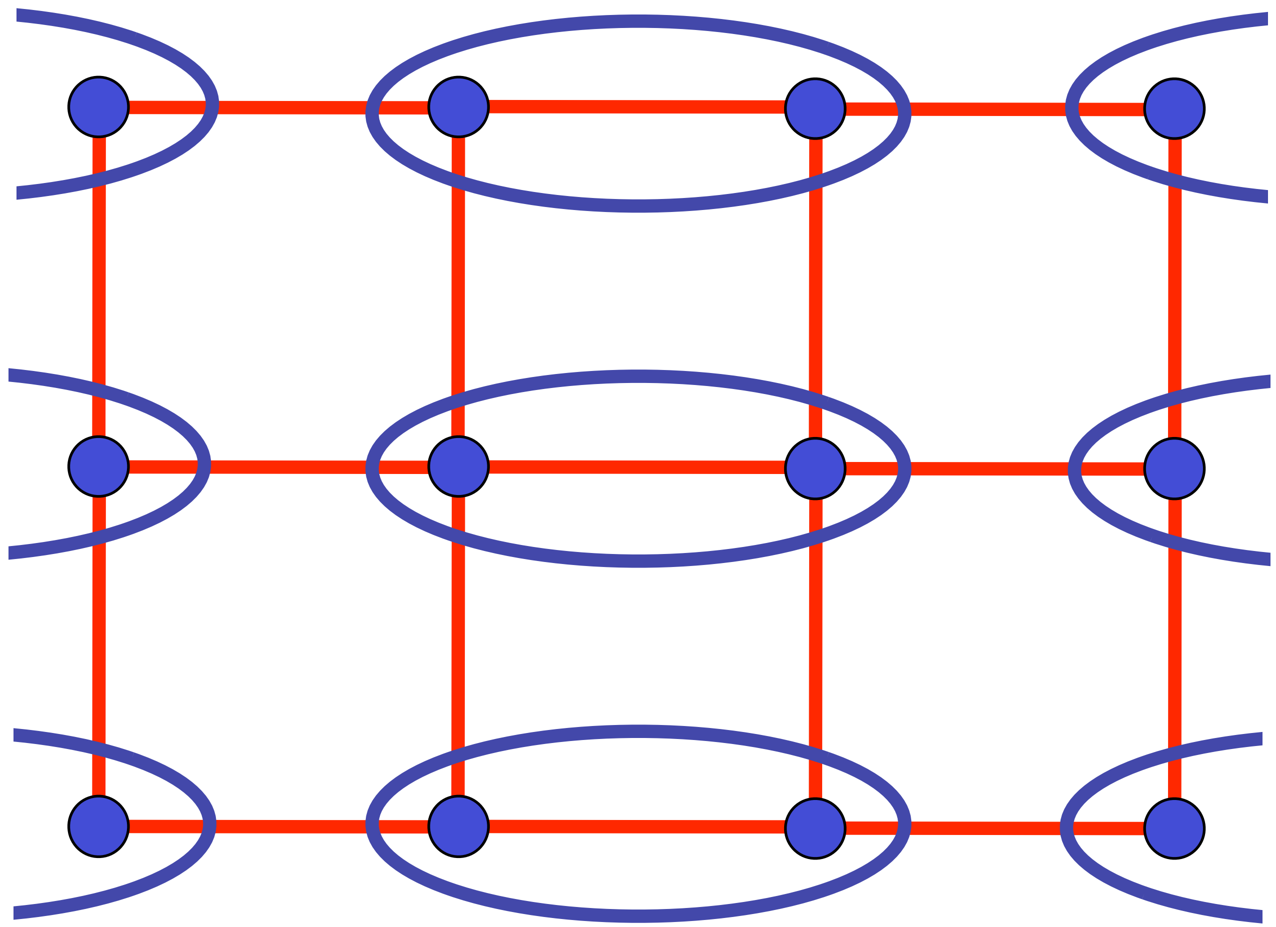

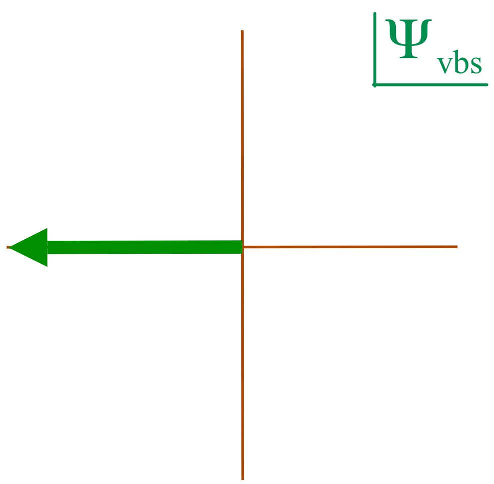

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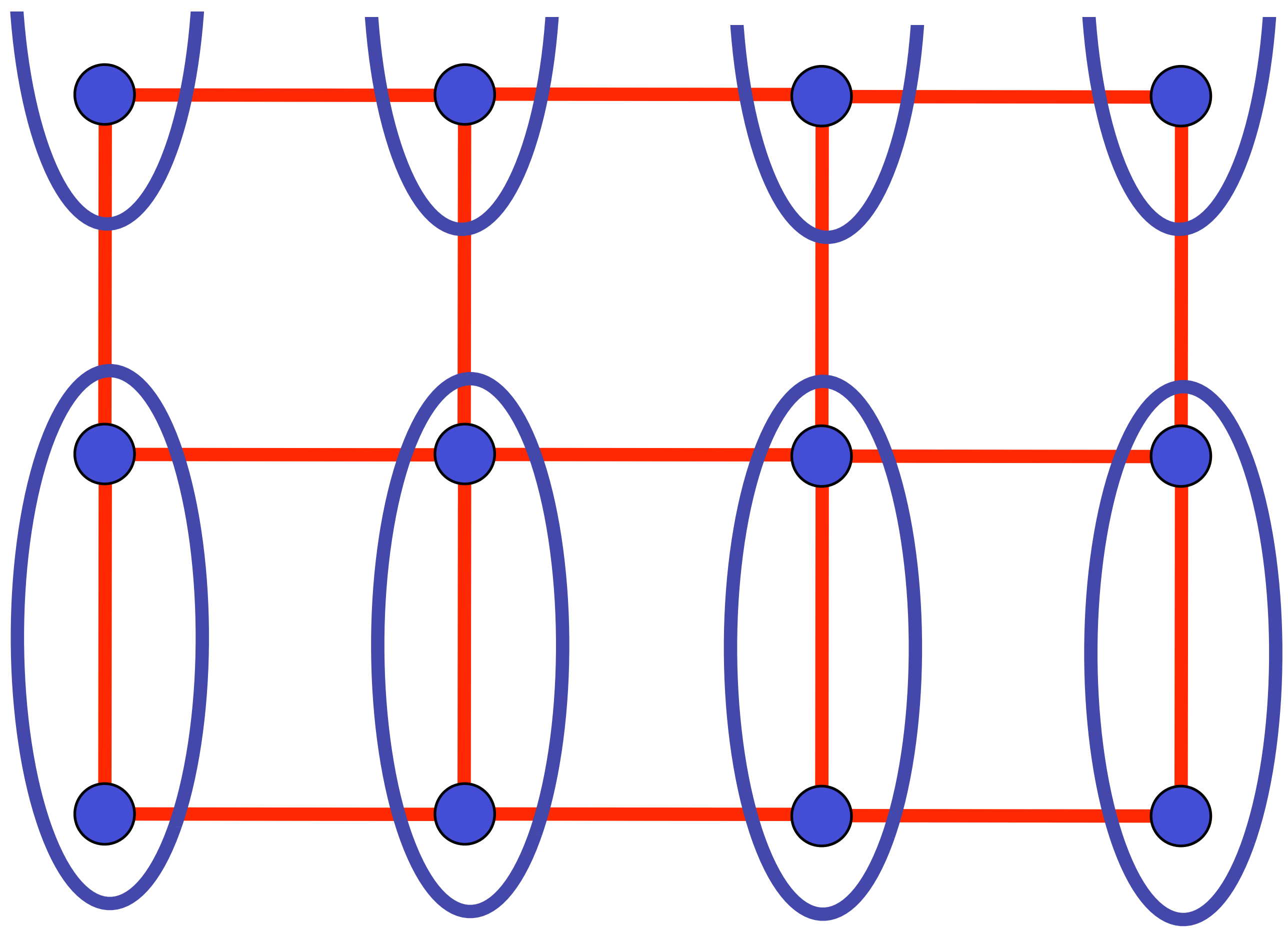

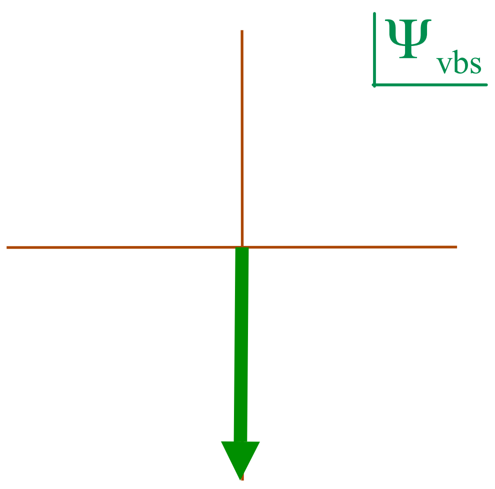

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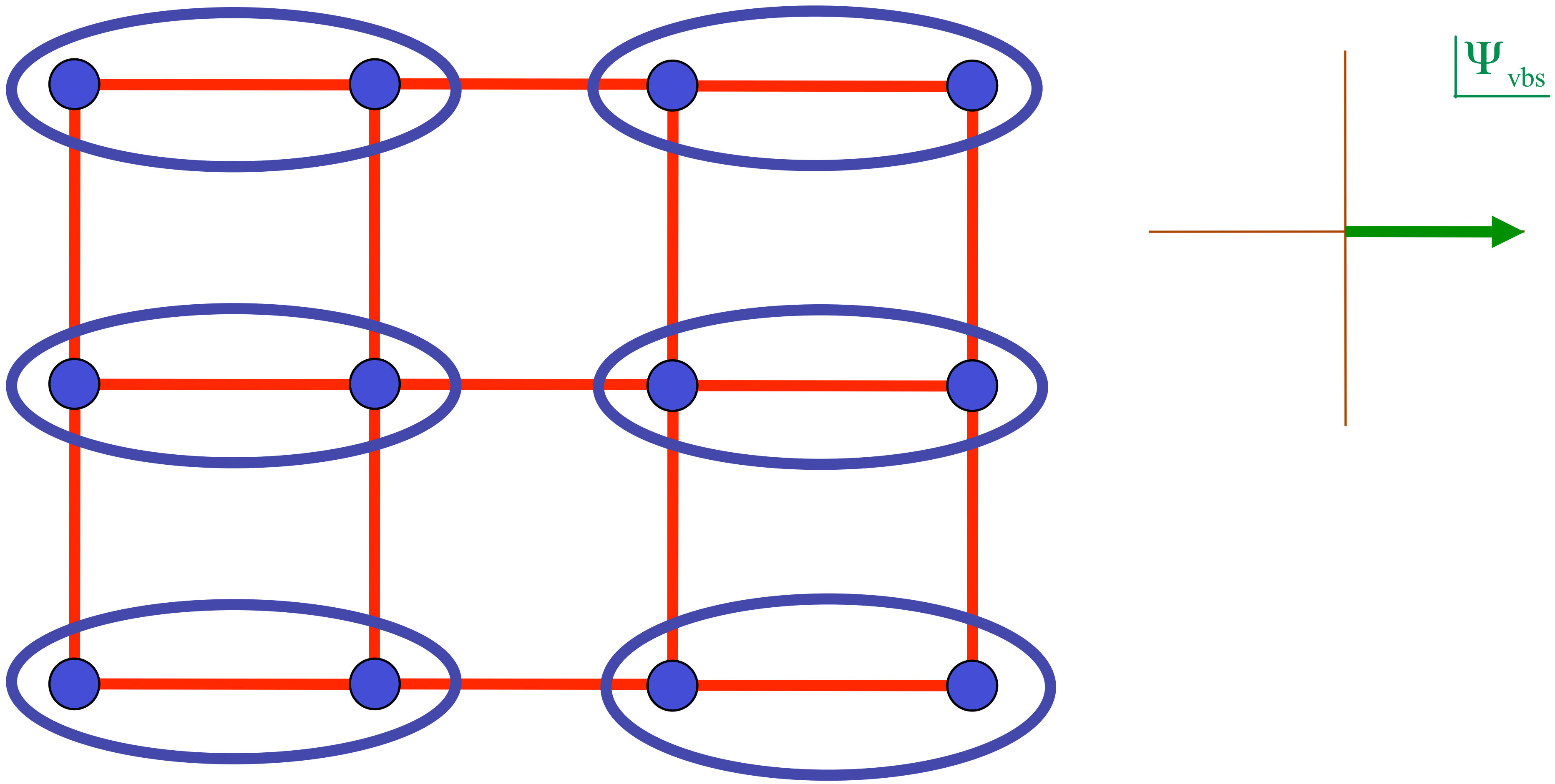

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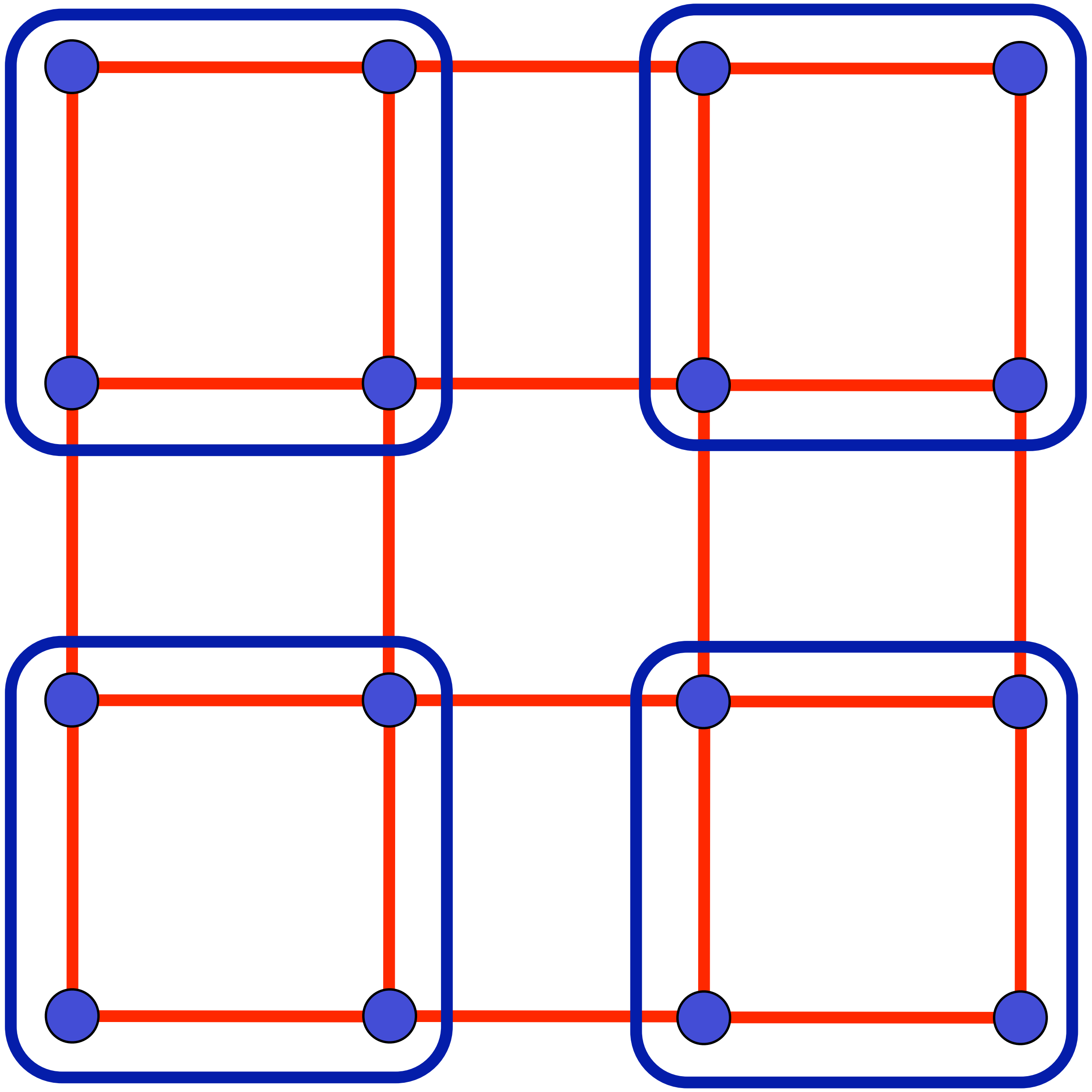

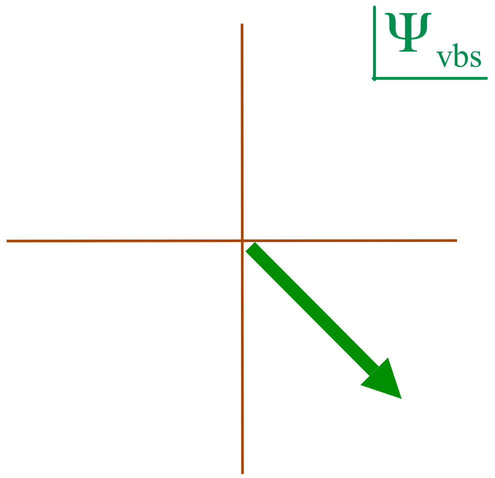

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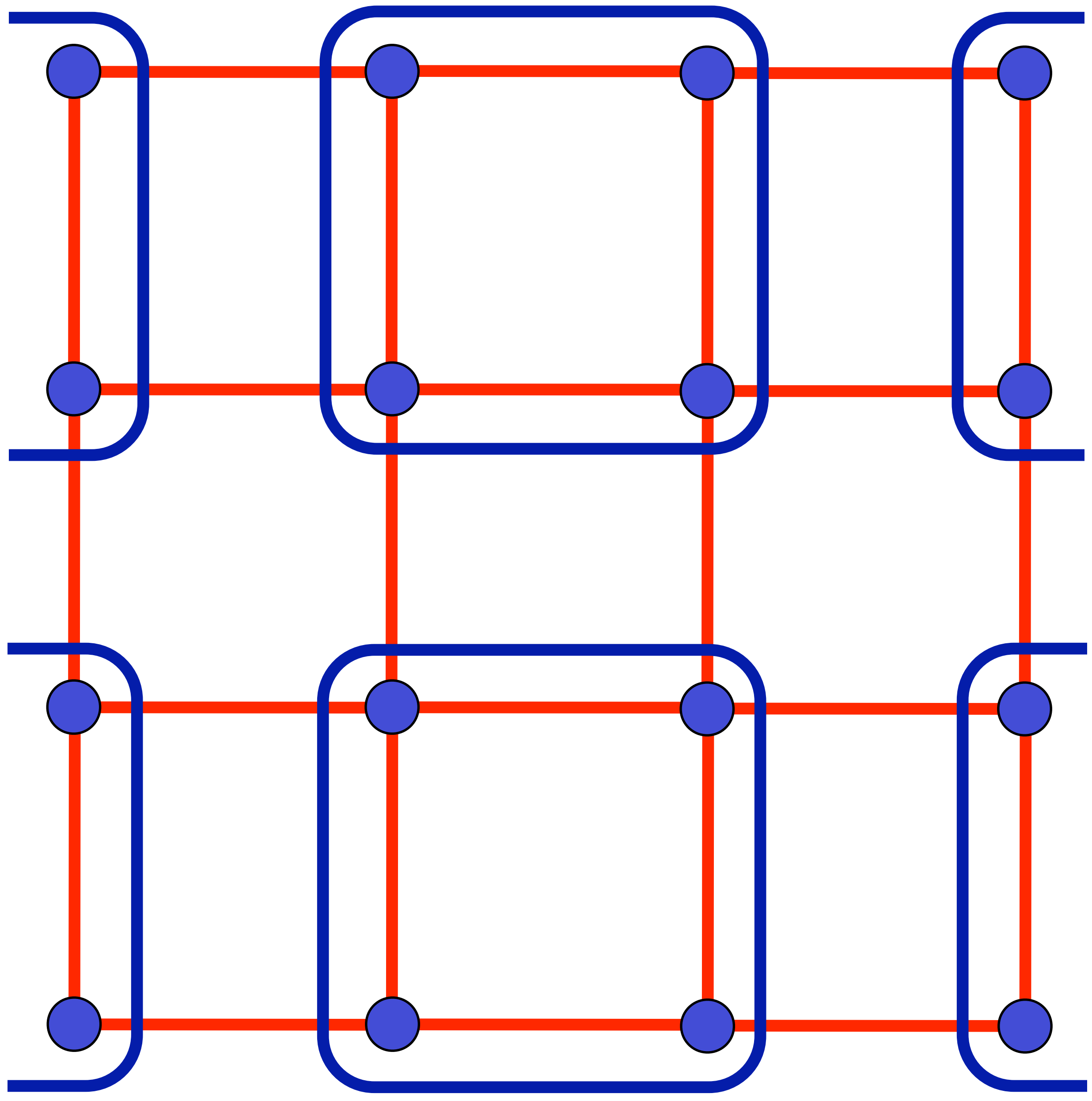

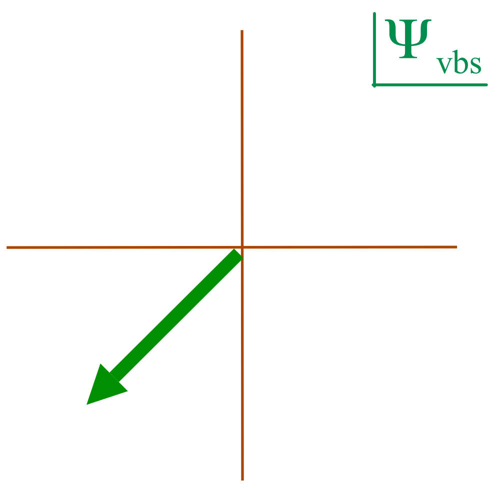

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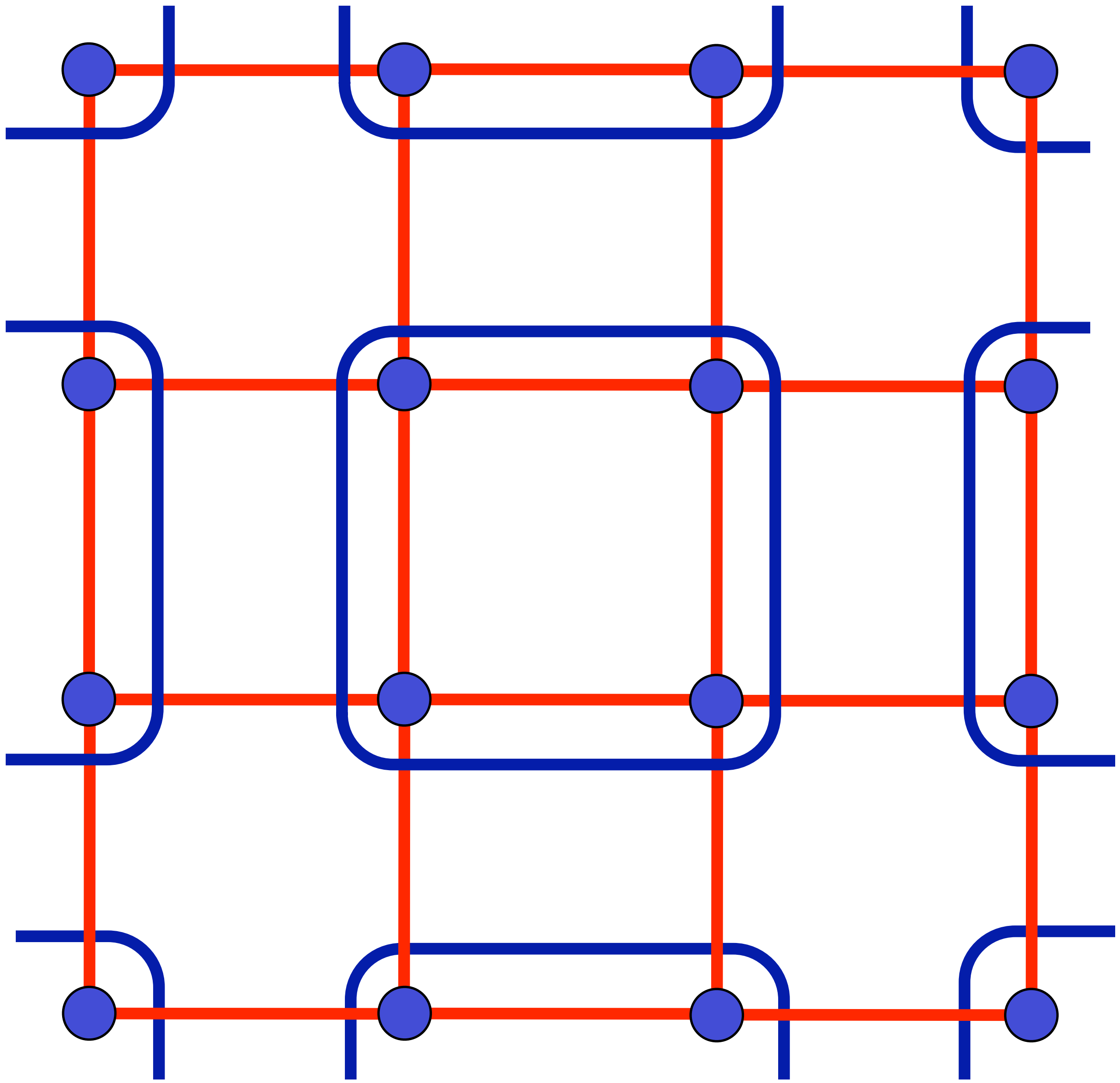

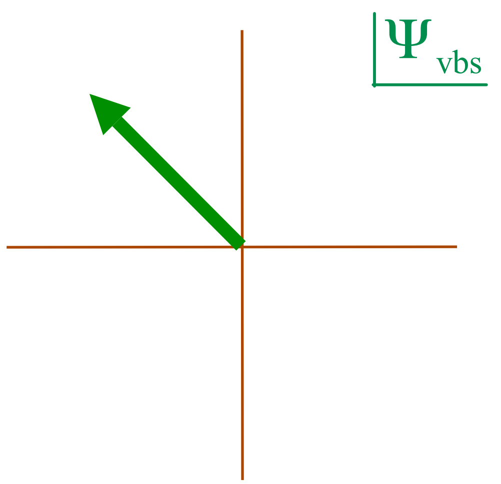

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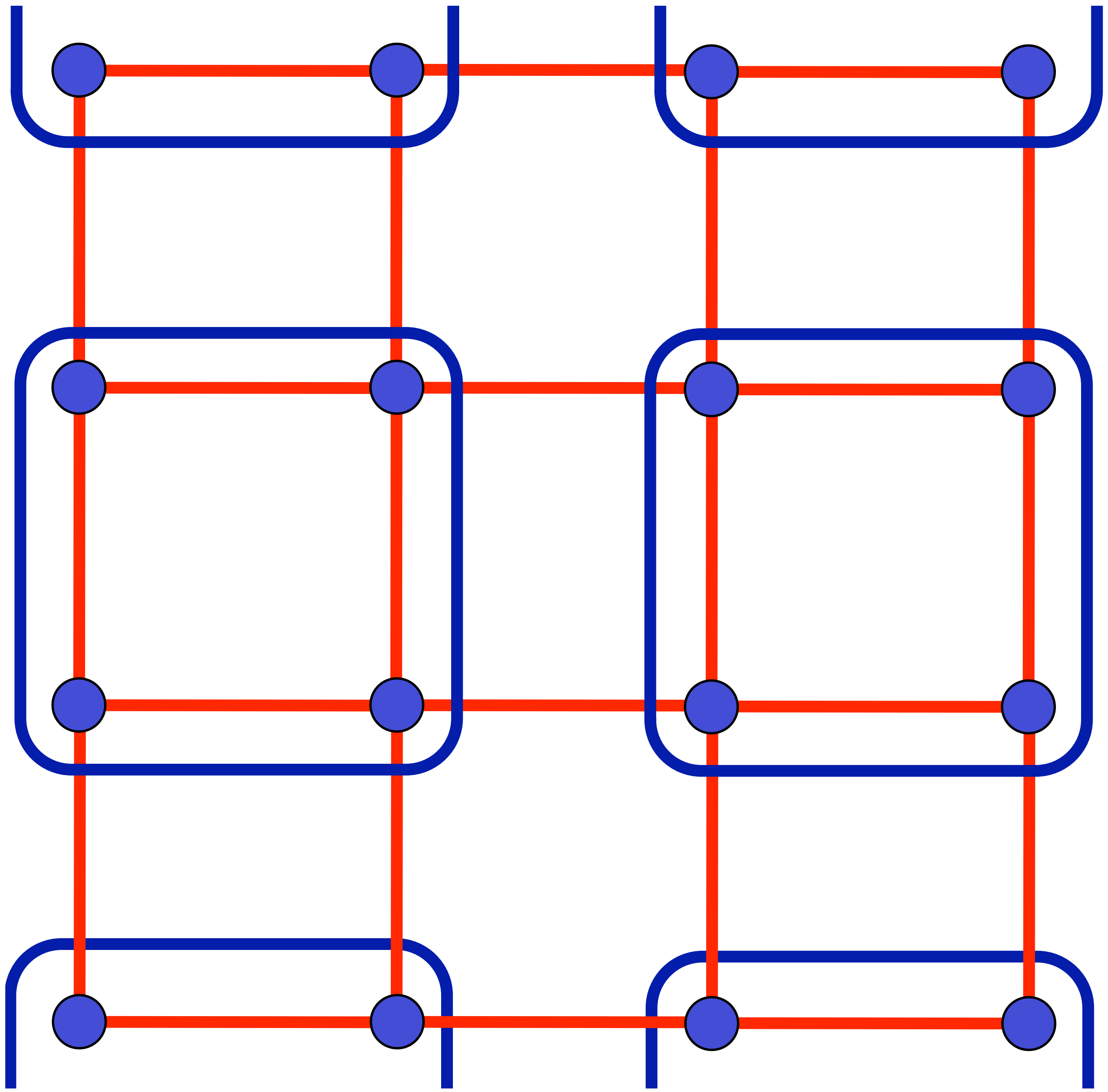

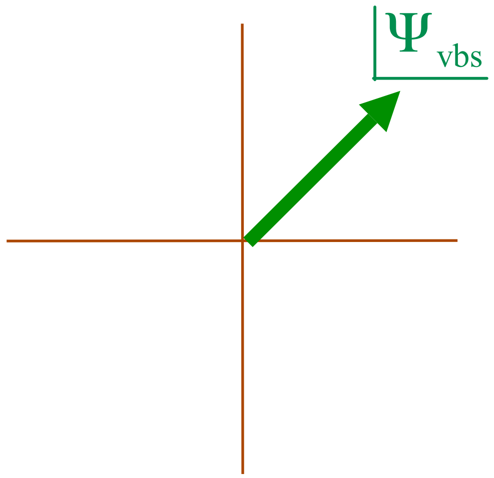

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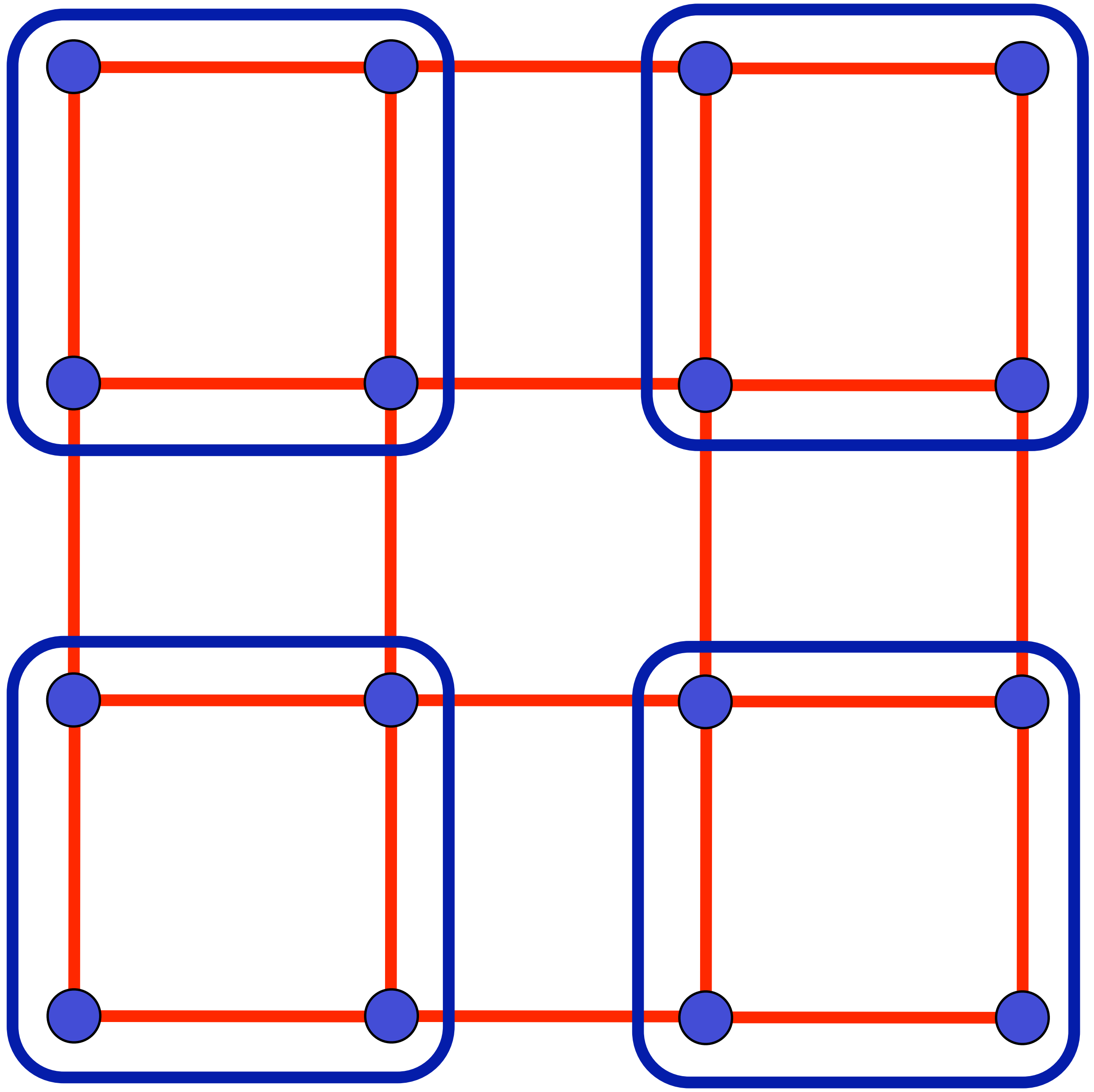
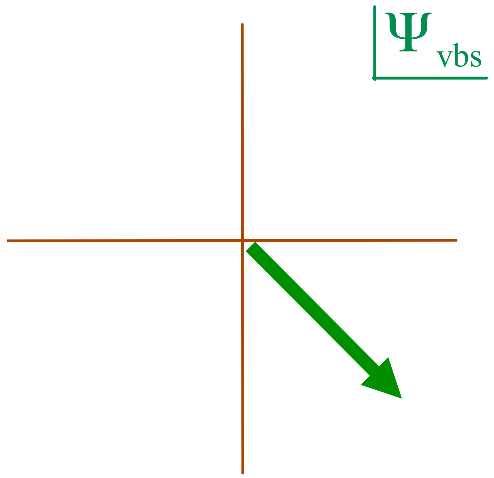
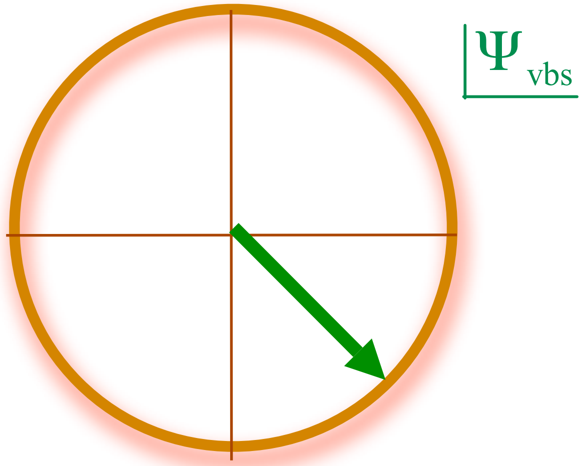
Order parameter of VBS state





$$\Psi_{\rm vbs}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i\,\arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

- Near the Néel-VBS transition, the (nearly) gapless photon can be identified with the Goldstone mode associated with an emergent circular symmetry



$$\Psi_{\rm vbs} \rightarrow \Psi_{\rm vbs} e^{i\theta}.$$

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)
O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

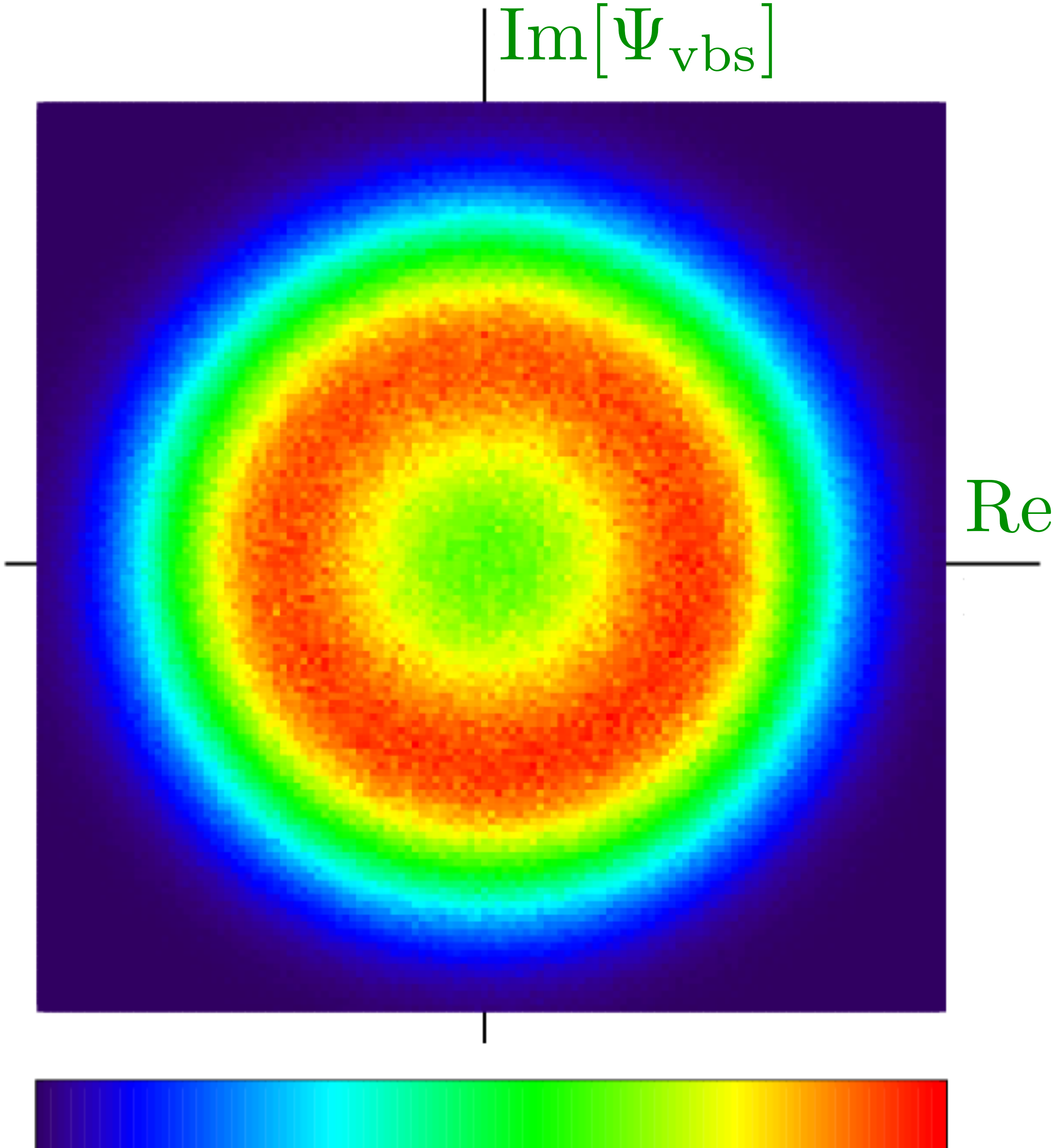
$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

Quantum Monte Carlo simulations display convincing evidence for a transition from a

Neel state at small Q
to a
VBS state at large Q

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).
F.-J. Jiang, M. Nyfeler, S. Chandrasekharan, and U.-J. Wiese, arXiv:0710.3926

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

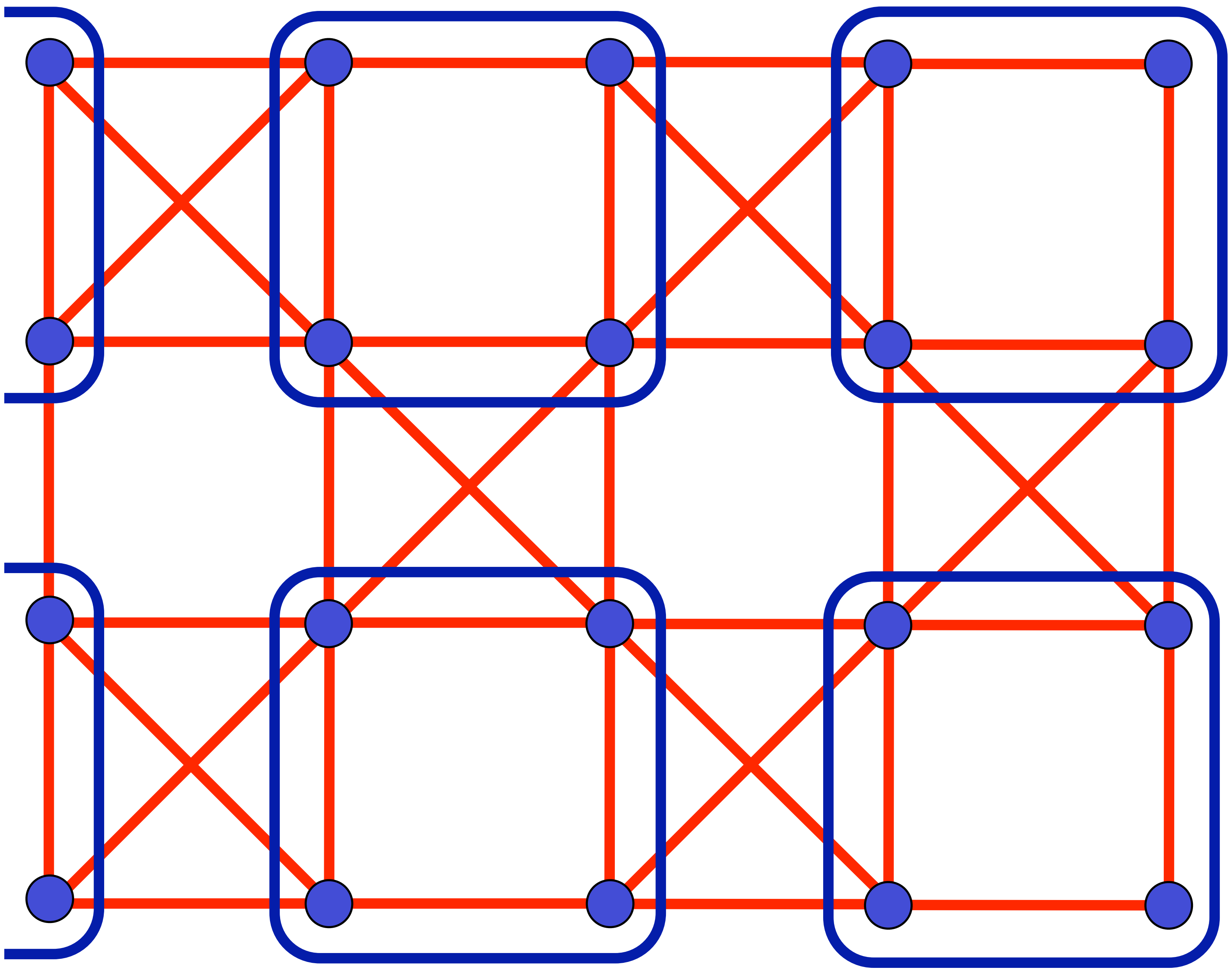


Distribution of VBS order Ψ_{vbs} at large Q

Emergent circular symmetry is evidence for U(1) photon and topological order

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Many other models display VBS order: e.g. the planar pyrochlore lattice



C.H. Chung, J.B. Marston, and S. Sachdev, *Phys. Rev. B* **64**, 134407 (2001)
J.-B. Fouet, M. Mambrini, P. Sindzingre, and C. Lhuillier, *Phys. Rev. B* **67**, 054411 (2003)
R. Moessner, O. Tchernyshyov, and S. L. Sondhi, *J. Stat. Phys.* **116**, 755 (2004).
O.A. Starykh, A. Furusaki, and L. Balents, *Phys. Rev. B* **72**, 094416 (2005)
M. Raczkowski and D. Poilblanc, arXiv:0810.0738

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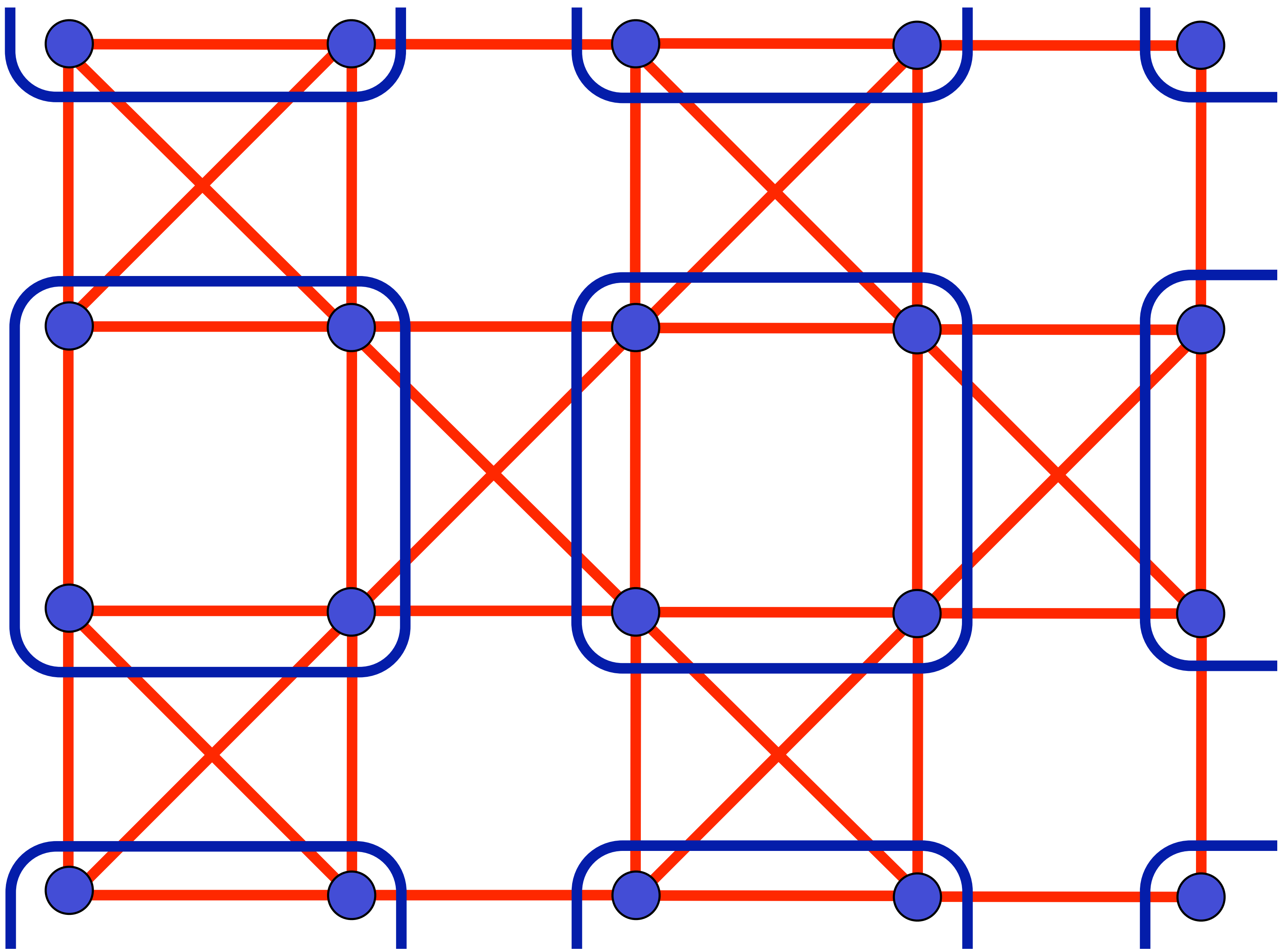

C.H. Chung, J.B. Marston, and S. Sachdev, *Phys. Rev. B* **64**, 134407 (2001)
J.-B. Fouet, M. Mambrini, P. Sindzingre, and C. Lhuillier, *Phys. Rev. B* **67**, 054411 (2003)
R. Moessner, O. Tchernyshyov, and S. L. Sondhi, *J. Stat. Phys.* **116**, 755 (2004).
O.A. Starykh, A. Furusaki, and L. Balents, *Phys. Rev. B* **72**, 094416 (2005)
M. Raczkowski and D. Poilblanc, arXiv:0810.0738

Outline

1. Landau-Ginzburg criticality

   Coupled-dimer antiferromagnets

2. Quantum "disordering" magnetic order

   Z_2 spin liquids and valence bond solids

3. Critical spin liquids

   Deconfined criticality; fermionic spinons near the Mott transition

4. Triangular, kagome, and hyperkagome lattices

   Connections to experiments

[[[5. Correlated boson model

   Supersolids and stripes]]]

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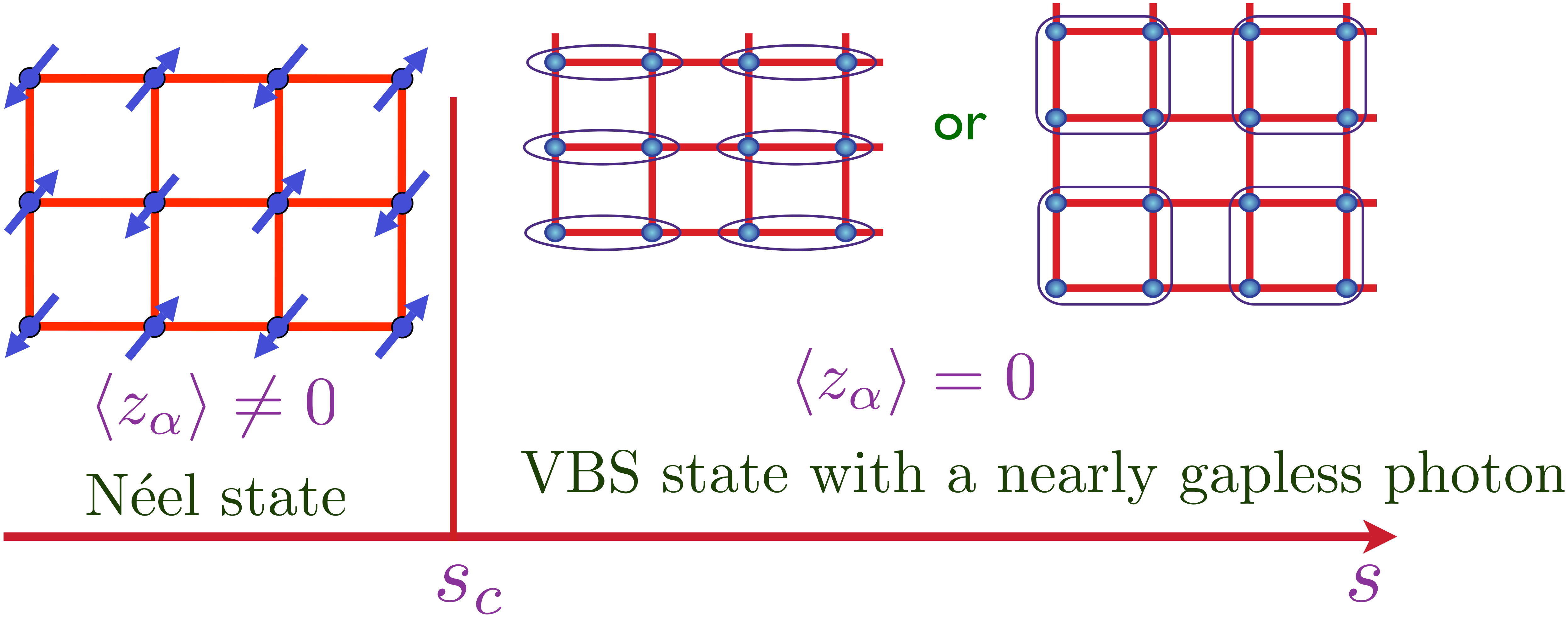
Neel-VBS quantum transition

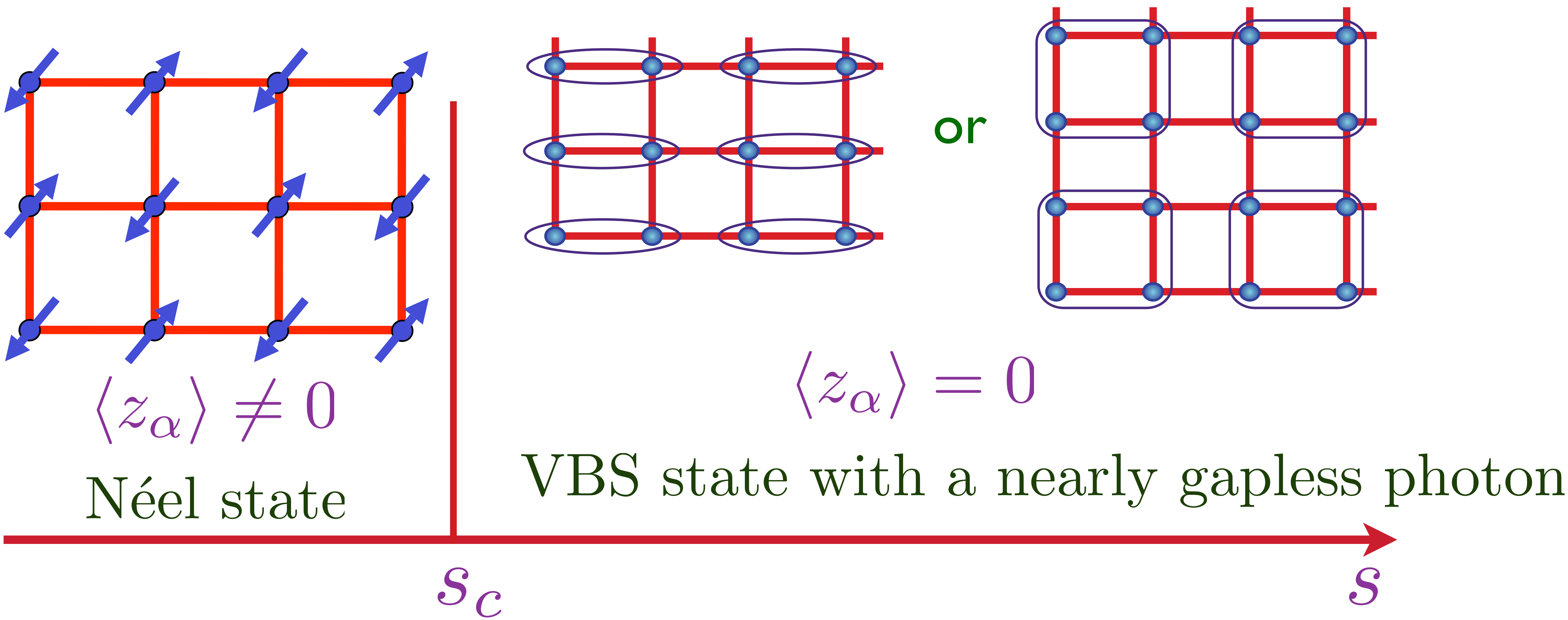


Critical theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Neel-VBS quantum transition



Monte Carlo studies support CP^1 field theory and presence of emergent photon. At system sizes larger than $L \sim 50$, and most studies show weakly first-order behavior at larger scales.

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).

A. B. Kuklov, M. Matsumoto, N. V. Prokof 'ev, B. V. Svistunov, and M. Troyer, *Phys. Rev. Lett.* **101**, 050405 (2008)

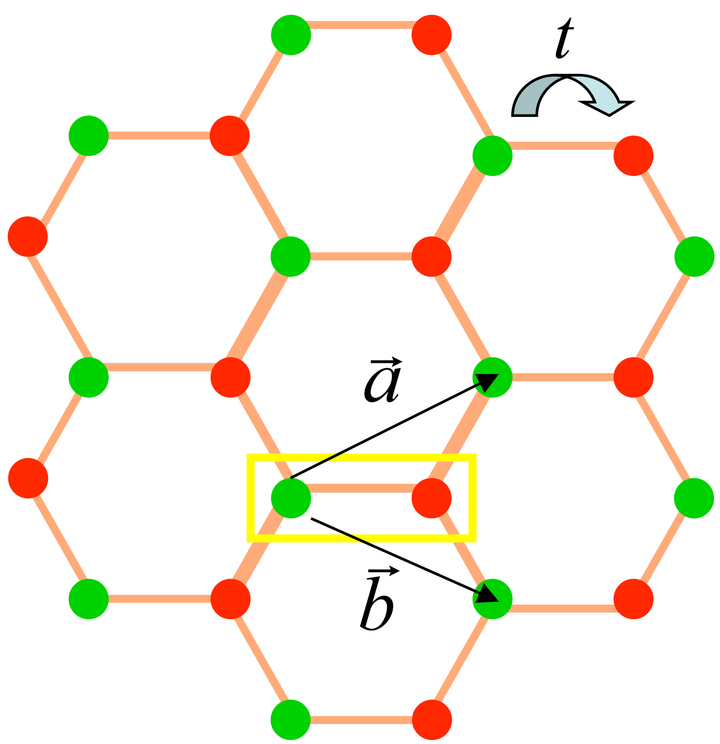
F.-J. Jiang, M. Nyfeler, S. Chandrasekharan, and U.-J. Wiese, arXiv:0710.3926

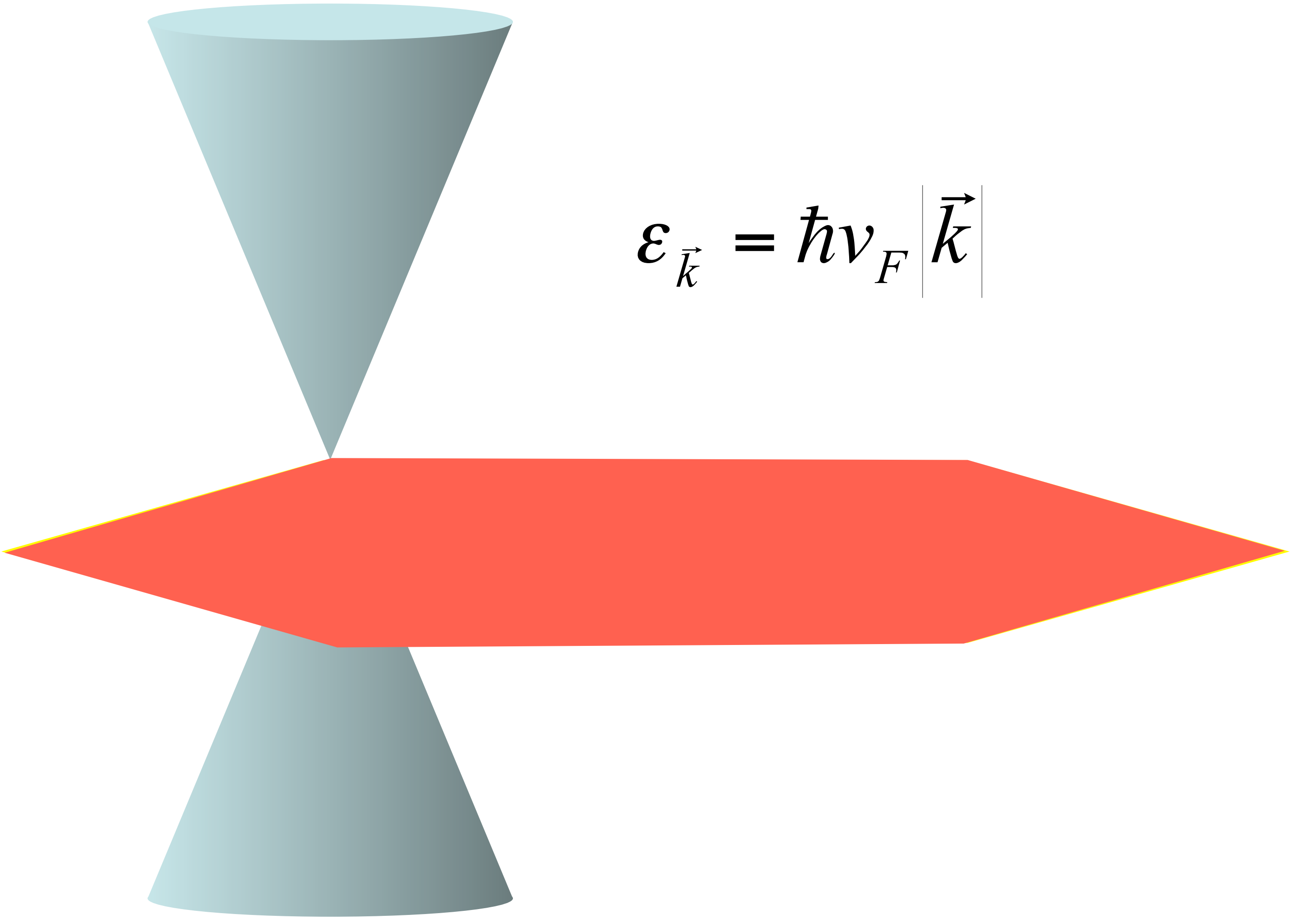
Second approach

Look for spin liquids across continuous (or weakly first-order) quantum transitions to an insulator from a metal

S. Burdin, D.R. Grempel, and A. Georges, *Phys. Rev. B* **66**, 045111 (2002).
T. Senthil, M. Vojta and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).
S. Florens and A. Georges, *Phys. Rev. B* **70**, 035114 (2004).
T. Senthil, *Phys. Rev. B* **78** 045109 (2008).

Mott transition on the honeycomb lattice

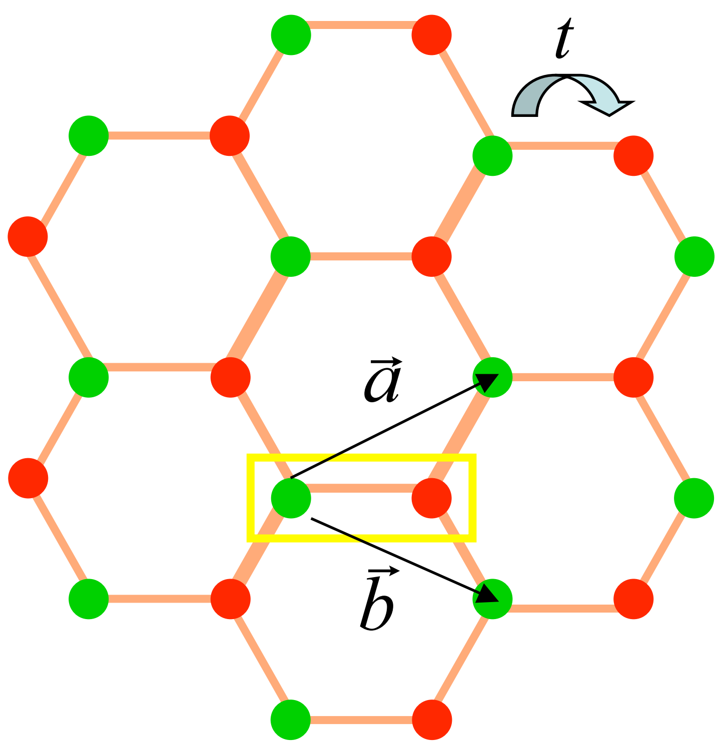


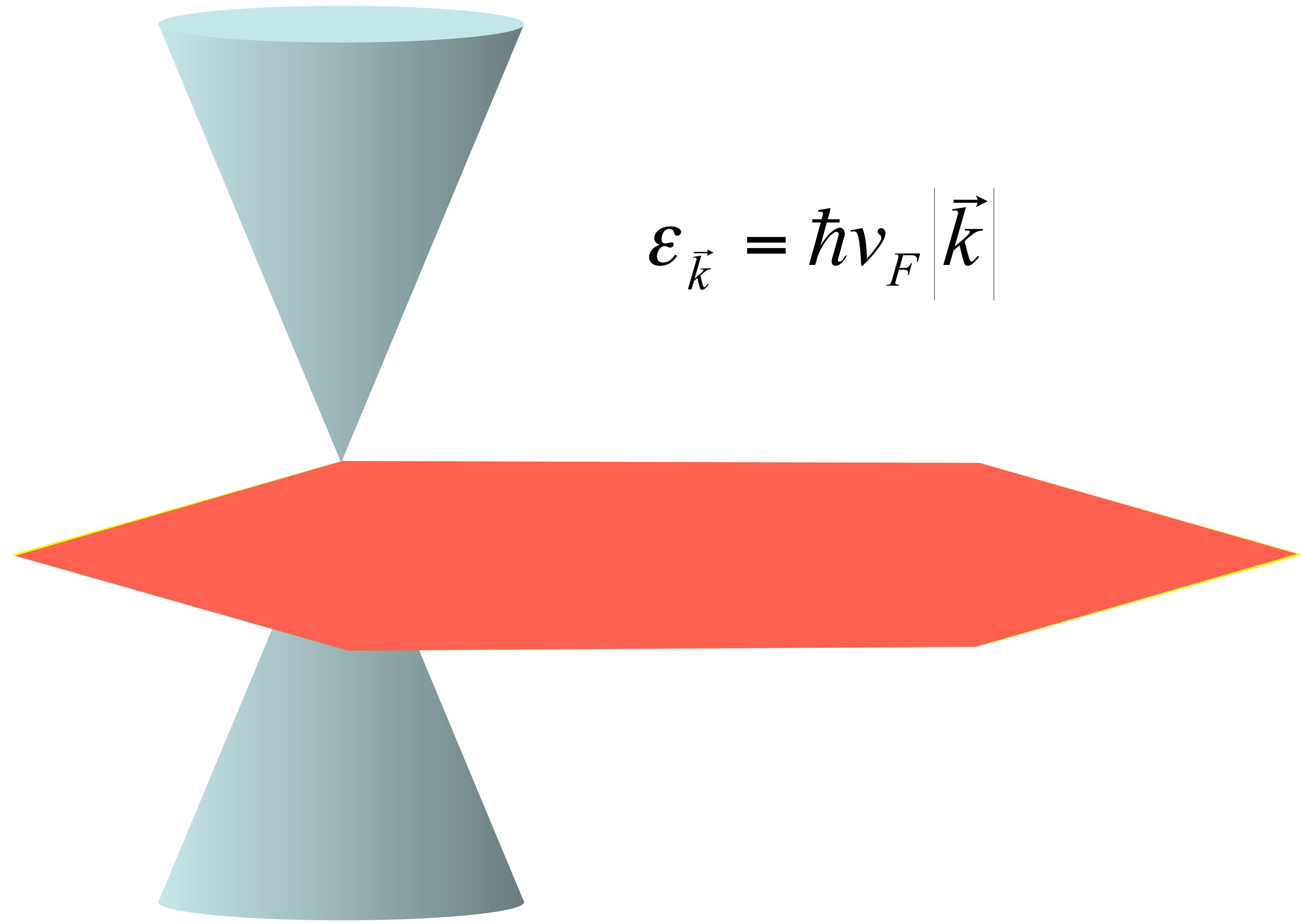


$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

M. Hermele, *Phys. Rev. B* **76**, 035125 (2007).

Mott transition on the honeycomb lattice



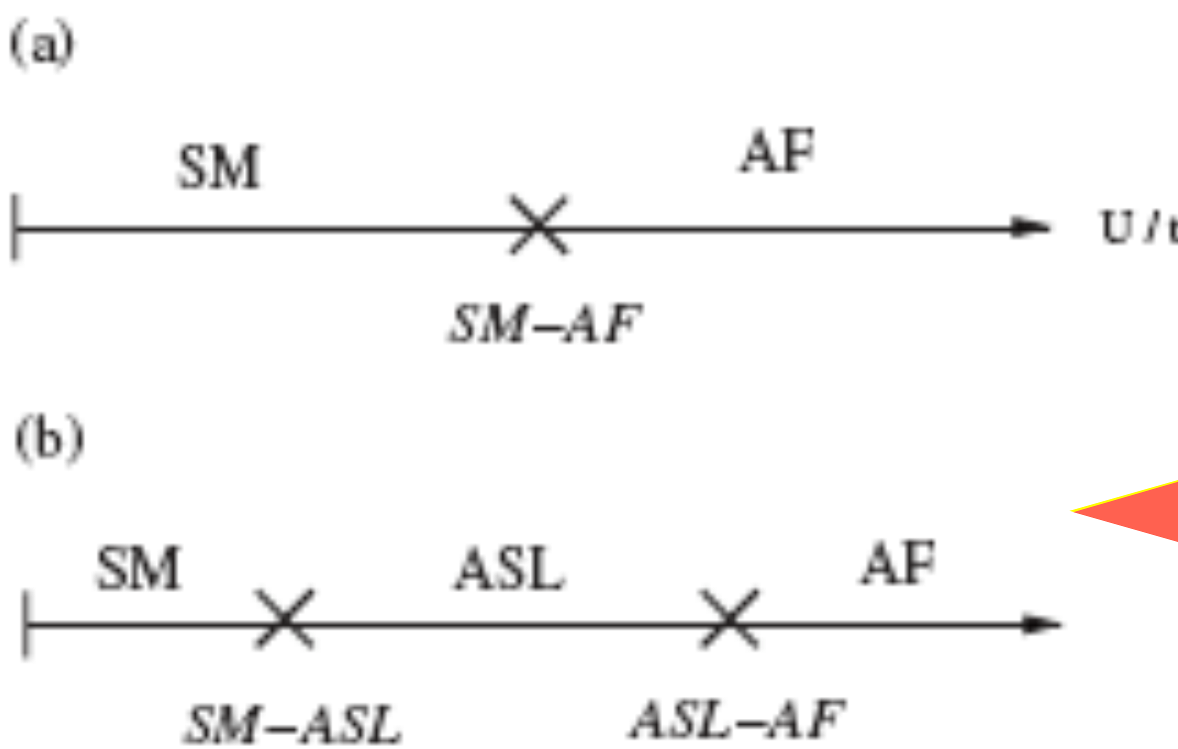


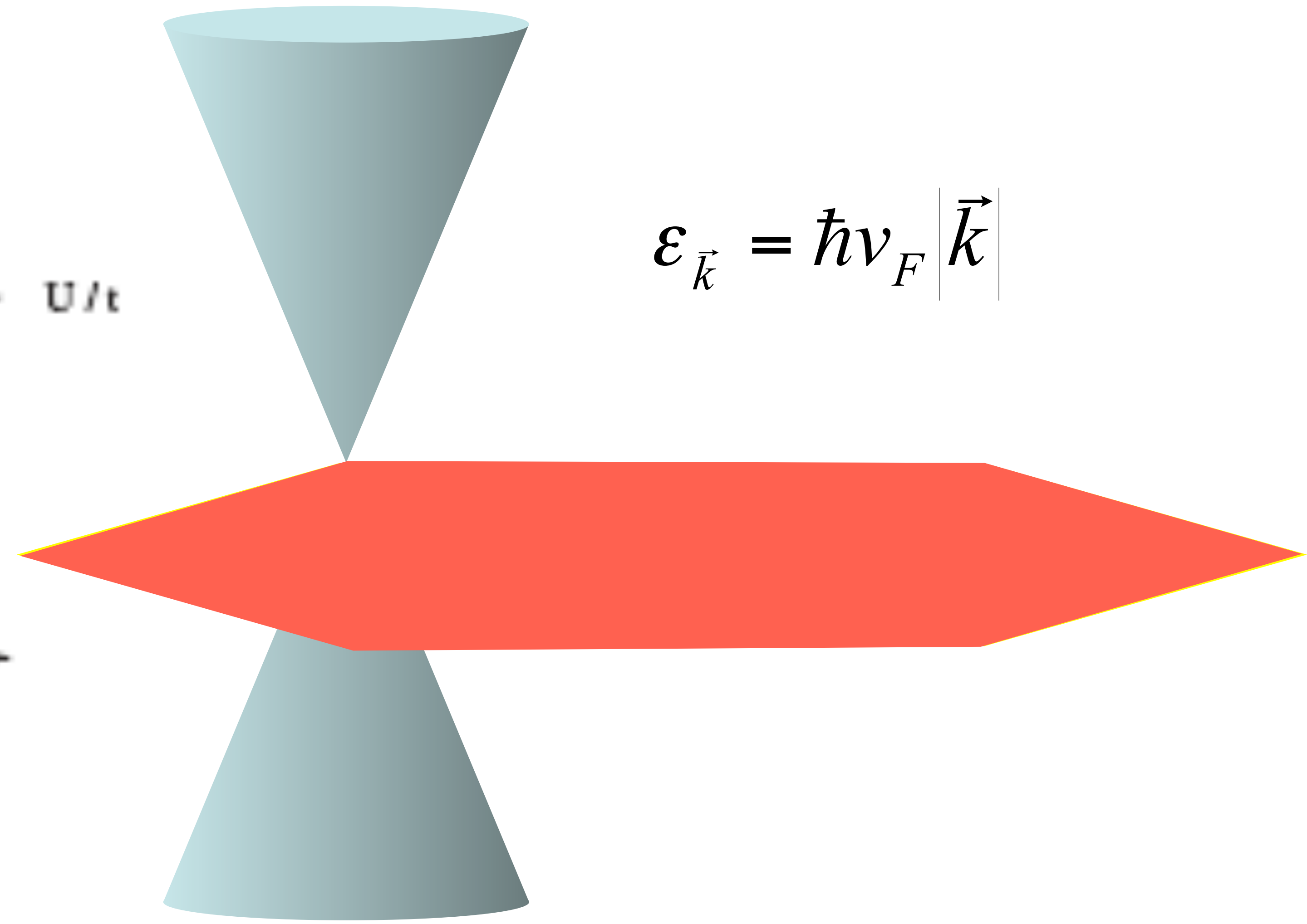
$$\varepsilon_{\vec{k}} = \hbar v_F |\vec{k}|$$

Turn up interactions to transform semi-metal into an insulator with a charge gap

M. Hermele, *Phys. Rev. B* **76**, 035125 (2007).

Mott transition on the honeycomb lattice





$$\varepsilon_{\vec{k}} = \hbar v_F |\vec{k}|$$

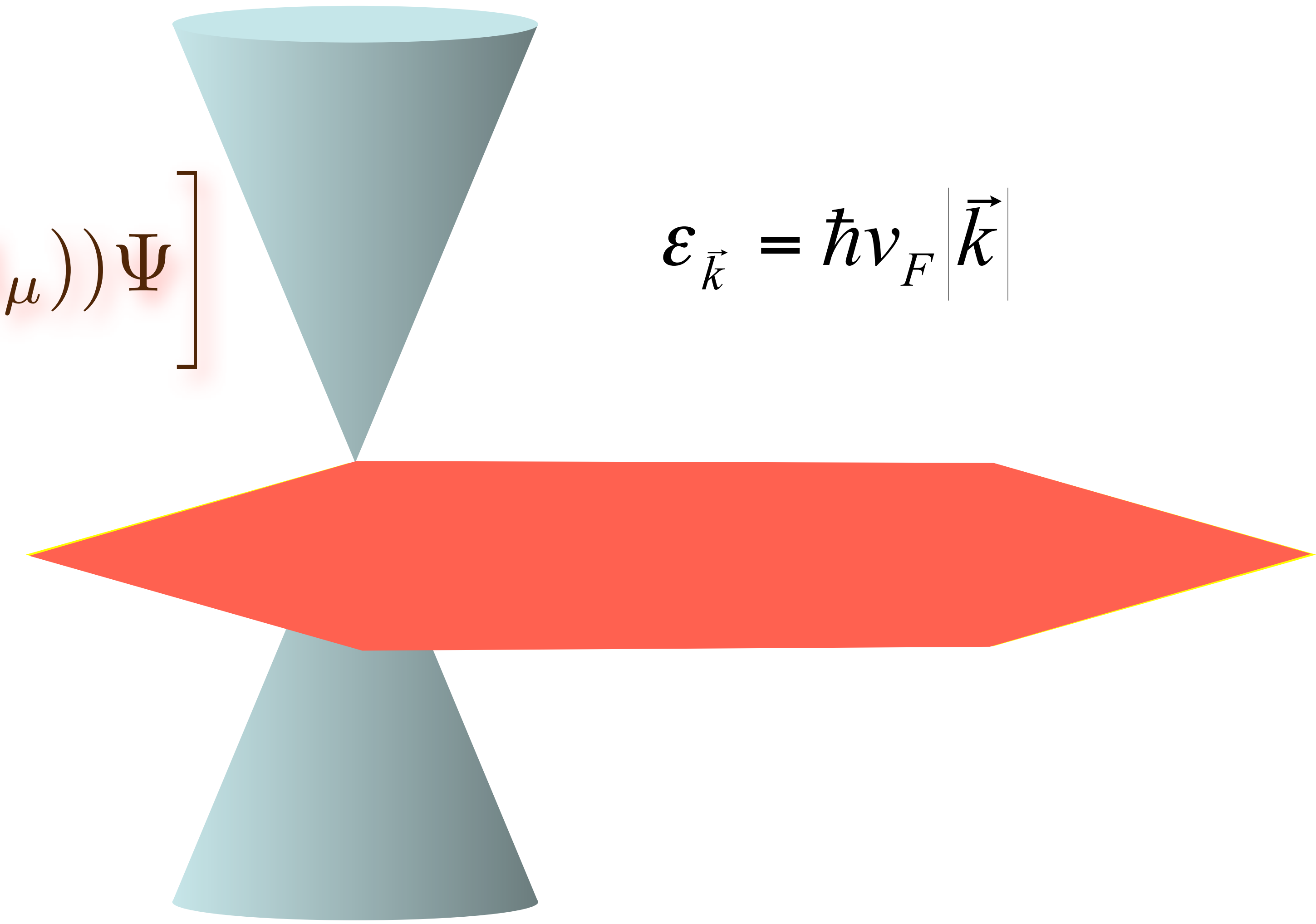
Possible intermediate critical (or algebraic) spin liquid (ASL) phase with neutral fermionic Dirac spinons, which are remnants of the electrons of the semi-metal

M. Hermele, *Phys. Rev. B* **76**, 035125 (2007).

Fermionic critical spin liquids

$$\mathcal{S}_{\text{ASL}} = \int d^2r d\tau \left[\overline{\Psi} \gamma^{\mu} (\partial_{\mu} - i A_{\mu})) \Psi \right]$$

$$\varepsilon_{\vec{k}} = \hbar v_F |\vec{k}|$$



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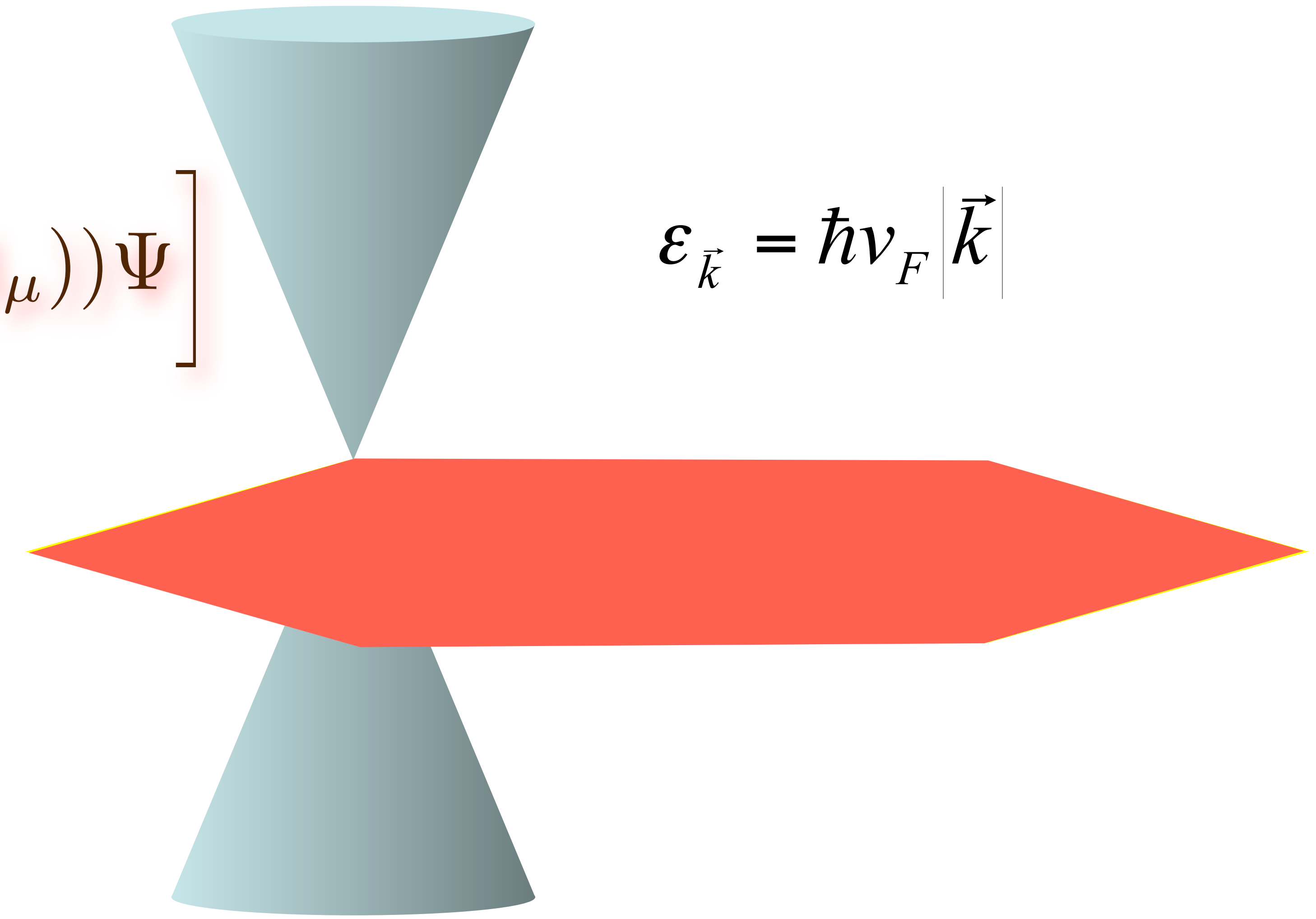


$$\varepsilon_{\vec{k}} = \hbar v_F |\vec{k}|$$

Similar states have been proposed for the square and kagome lattices

I. Affleck and J. B. Marston, *Phys. Rev. B* **37**, 3774-3777 (1988).

W. Rantner and X.-G. Wen, *Phys. Rev. Lett.* **86**, 3871-3874 (2001).

M. Hermele, T. Senthil, and M.P.A. Fisher, *Phys. Rev. B* **72** 104404 (2005).

Y. Ran, M. Hermele, P.A. Lee, and X.-G. Wen, *Phys. Rev. Lett*. **98**, 117205 (2007).

Fermionic critical spin liquids

PHYSICAL REVIEW B 71, 075103 (2005)

Phase diagram of the half-filled two-dimensional SU(N) Hubbard-Heisenberg model: A quantum Monte Carlo study

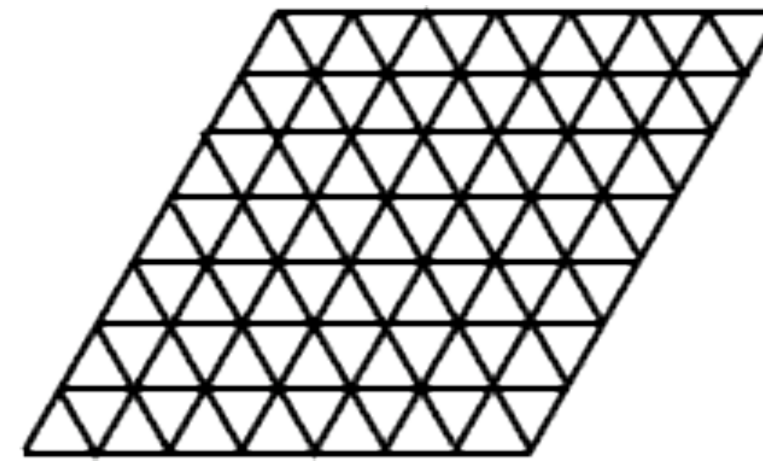
F. F. Assaad
Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland D-97074 Würzburg, Germany
(Received 1 June 2004; revised manuscript received 7 September 2004; published 4 February 2005)

We investigate the phase diagram of the half-filled SU(N) Hubbard-Heisenberg model with hopping t, exchange J, and Hubbard U, on a two-dimensional square lattice. In the large-N limit, and as a function of decreasing values of t/J, the model shows a transition from a d-density wave state to a spin dimerized insulator. A similar behavior is observed at $N=6$ whereas at $N=2$ a spin density wave insulating ground state is stabilized. The $N=4$ model, has a d-density wave ground state at large values of t/J which as a function of decreasing values of t/J becomes unstable to an insulating state with no apparent lattice and spin broken symmetries. In this state, the staggered spin-spin correlations decay as a power law, resulting in gapless spin excitations at $\vec{q}=(\pi,\pi)$. Furthermore, low lying spin modes with small spectral weight are apparent around the wave vectors $\vec{q}=(0,\pi)$ and $\vec{q}=(\pi,0)$. This gapless spin liquid state is equally found in the SU(4) Heisenberg ($U/t\rightarrow\infty$) model in the self-adjoint antisymmetric representation. An interpretation of this state in terms of a π-flux phase is offered. Our results stem from projective ($T=0$) quantum Monte Carlo simulations on lattice sizes ranging up to 24×24.

Evidence critical spin liquid is stable for SU(4) but not SU(2)

Hubbard model on triangular lattice

$$\mathcal{H} = -t \sum_{\langle ij \rangle} [c^\dagger_{i\alpha} c_{j\alpha} + h.c.] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

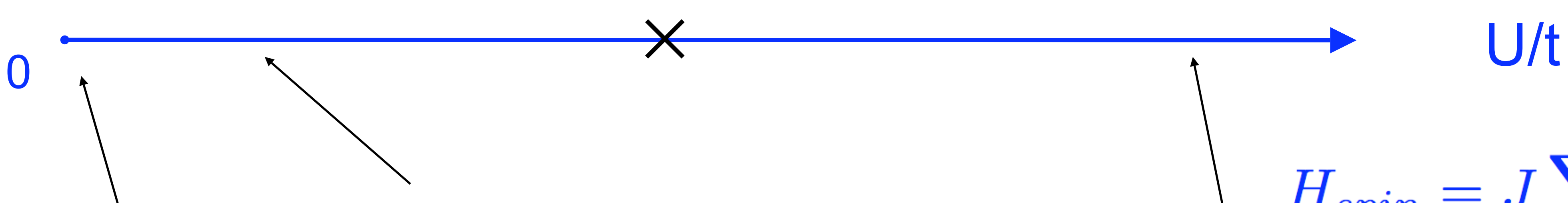


Phase diagram at Half filling?

Weak coupling

Strong coupling

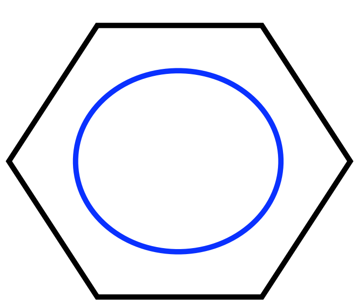
metal ????? insulator



$$H_{spin} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Free Fermi gas,
un-nested

Fermi liquid

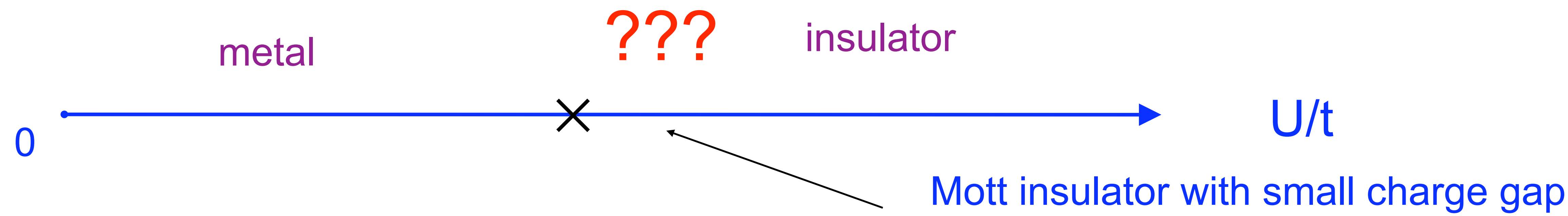


Heisenberg n.n. AFM
3-sublattice Neel state

$J = t^2/U$

O.I. Motrunich, *Phys. Rev. B* **72**, 045105 (2005).
D.N. Sheng, O.I. Motrunich, S. Trebst, E. Gull, and M.P.A. Fisher, *Phys. Rev. B* **78**, 054520 (2008).

"Weak" Mott insulator - Ring exchange



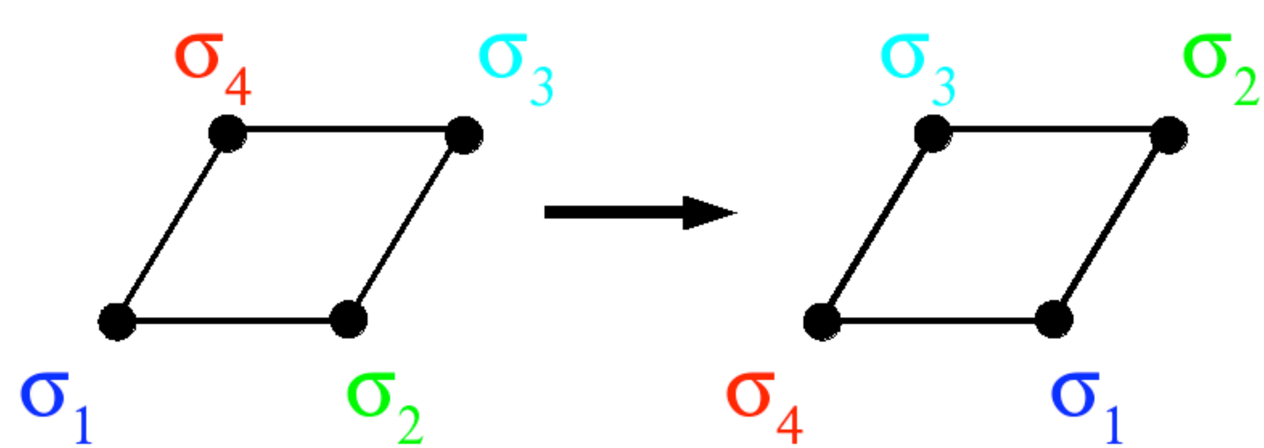
$$\hat{H}_{\text{Hubbard}} = -t \sum_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Insulator --> effective spin model

$$\hat{H}_{\text{eff}} = \frac{2t^2}{U} \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{20t^4}{U^3} \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.}) + \ldots$$

Ring exchange:
(mimics charge fluctuations)

P_{1234} :



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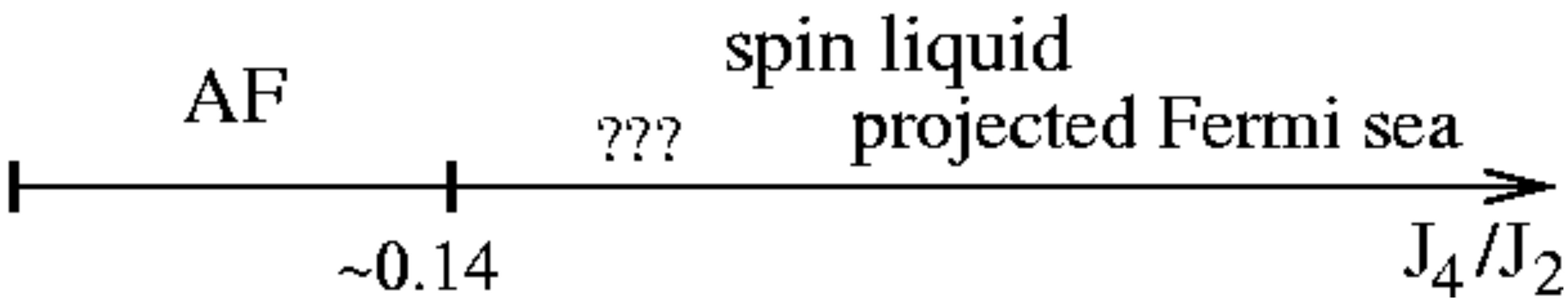
D.N. Sheng, O.I. Motrunich, S. Trebst, E. Gull, and M.P.A. Fisher, *Phys. Rev. B* **78**, 054520 (2008)

Is a Spinon Fermi sea actually the ground state of Triangular ring model (or Hubbard model)?

$$\hat{H}_{\text{ring}} = J_2 \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_4 \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.})$$

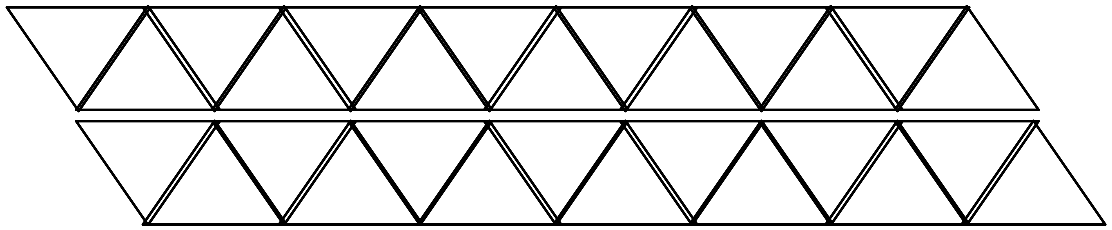
Variational Monte Carlo analysis suggests it might be for $J_4/J_2 > 0.3$



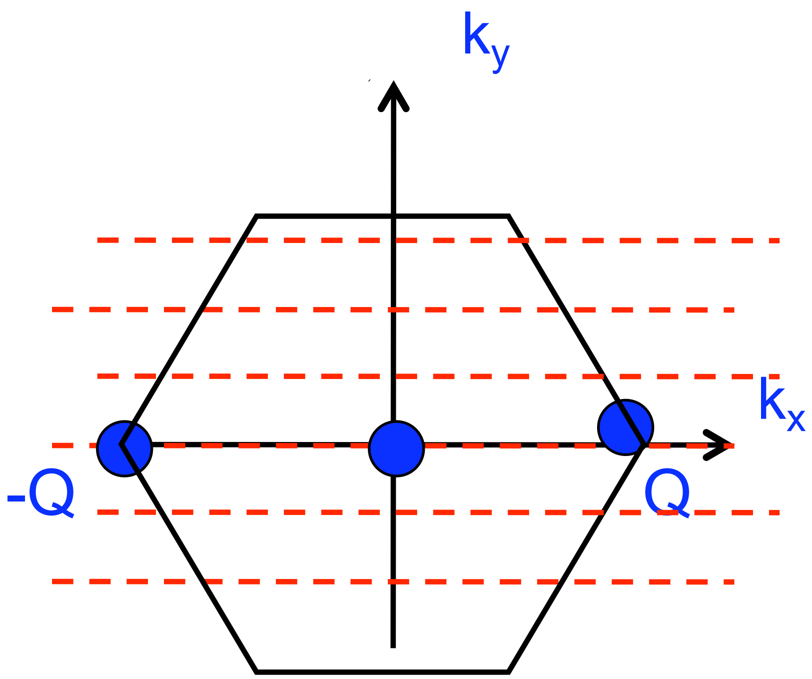
O.I. Motrunich, *Phys. Rev. B* **72**, 045105 (2005).
D.N. Sheng, O.I. Motrunich, S. Trebst, E. Gull, and M.P.A. Fisher, *Phys. Rev. B* **78**, 054520 (2008)

Quasi-1d route to "Spin-Metals"

Triangular strips:

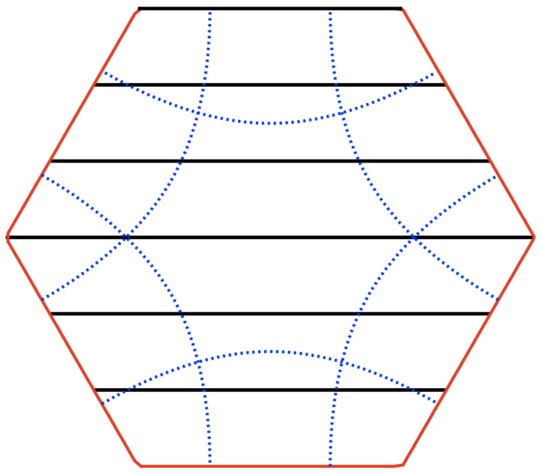


Neel or Critical Spin liquid



Few gapless 1d modes

Spin-Metal

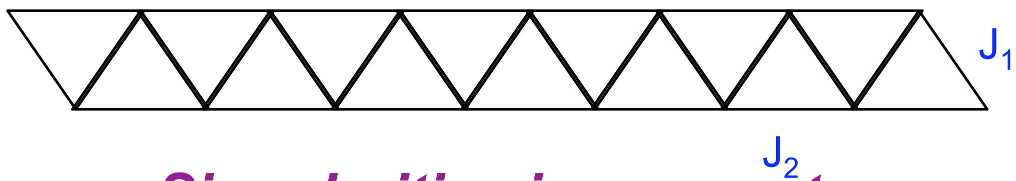


Fingerprint of 2d singular surface - many gapless 1d modes, of order N

New spin liquid phases on quasi-1d strips, each a descendent of a 2d spin-metal

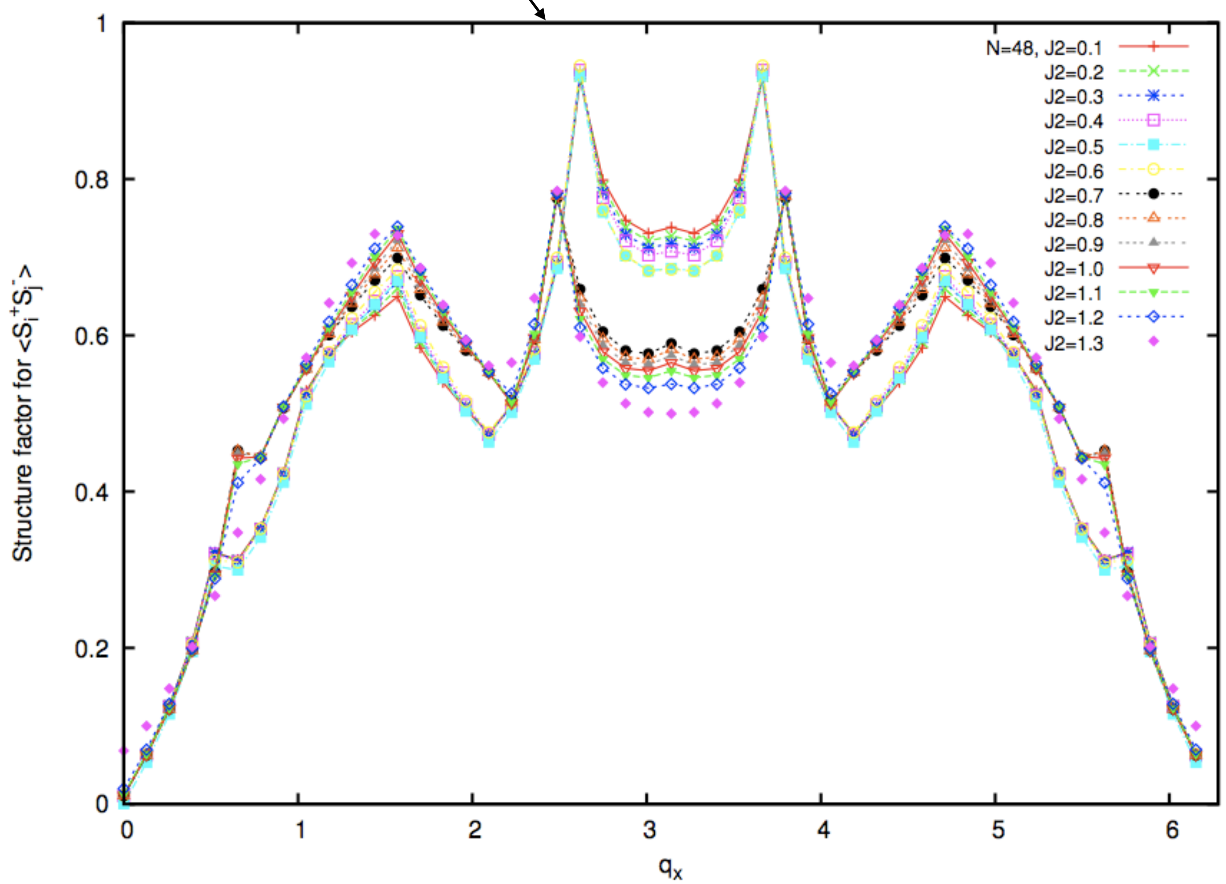
D.N. Sheng, O.I. Motrunich, S. Trebst, E. Gull, and M.P.A. Fisher, *Phys. Rev. B* **78**, 054520 (2008)

$$\mathcal{H}_\Delta = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle 1234 \rangle} [\mathcal{P}_{1234} + \mathcal{P}_{1234}^{-1}]$$

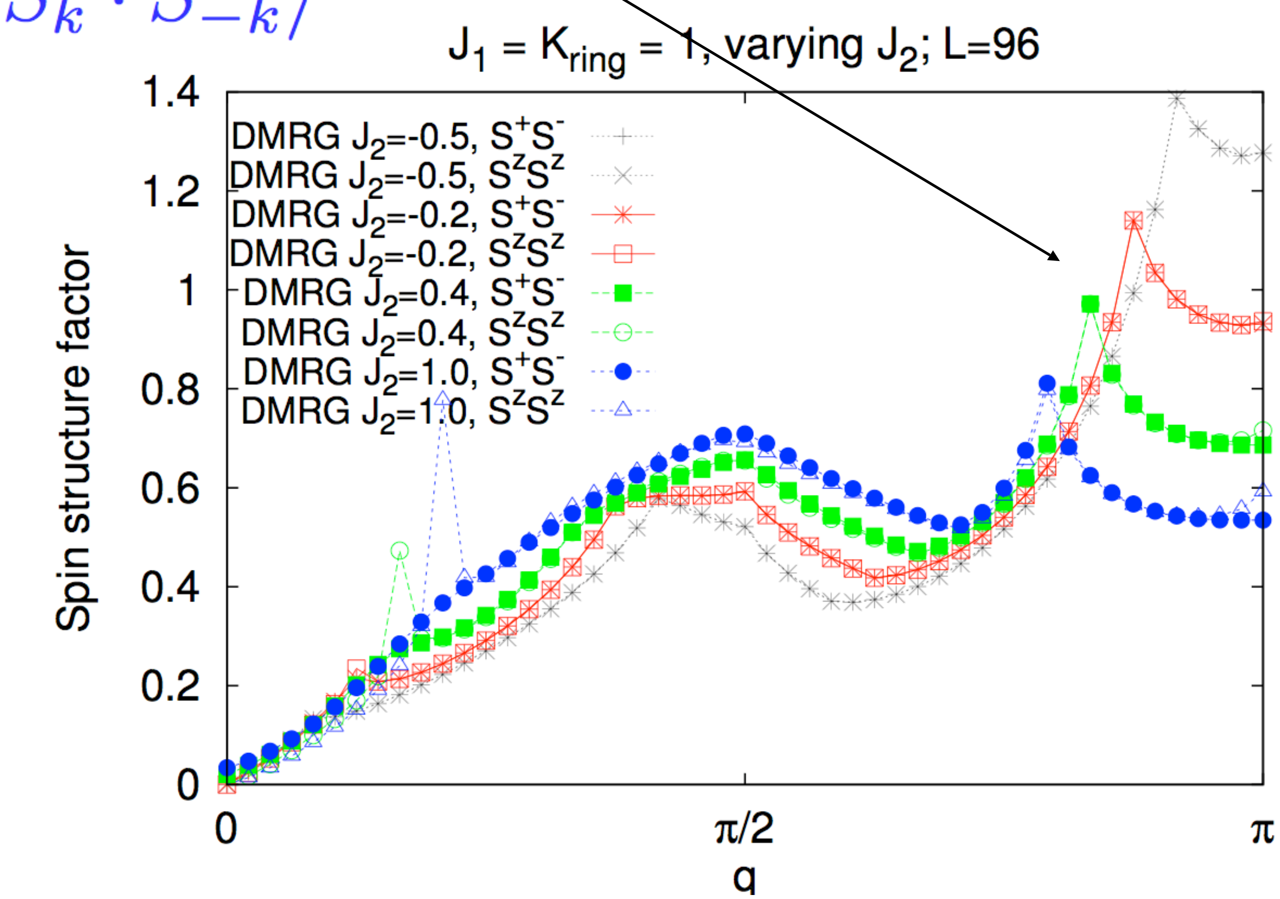


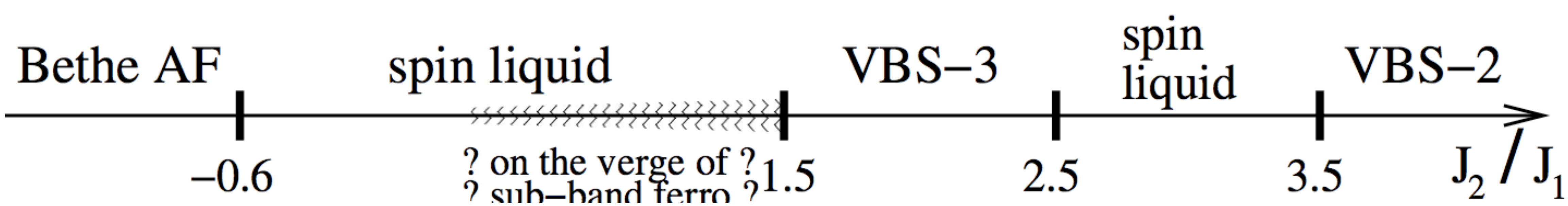
Singularities in momentum space locate the "Bose" surface (points in 1d)

$\langle \vec{S}_k \cdot \vec{S}_{-k} \rangle$



$\langle \vec{S}_k \cdot \vec{S}_{-k} \rangle$





D. N. Sheng, O. I. Motrunich, S. Trebst, E. Gull, and M. P. A. Fisher, Phys. Rev. B 78, 054520 (2008)

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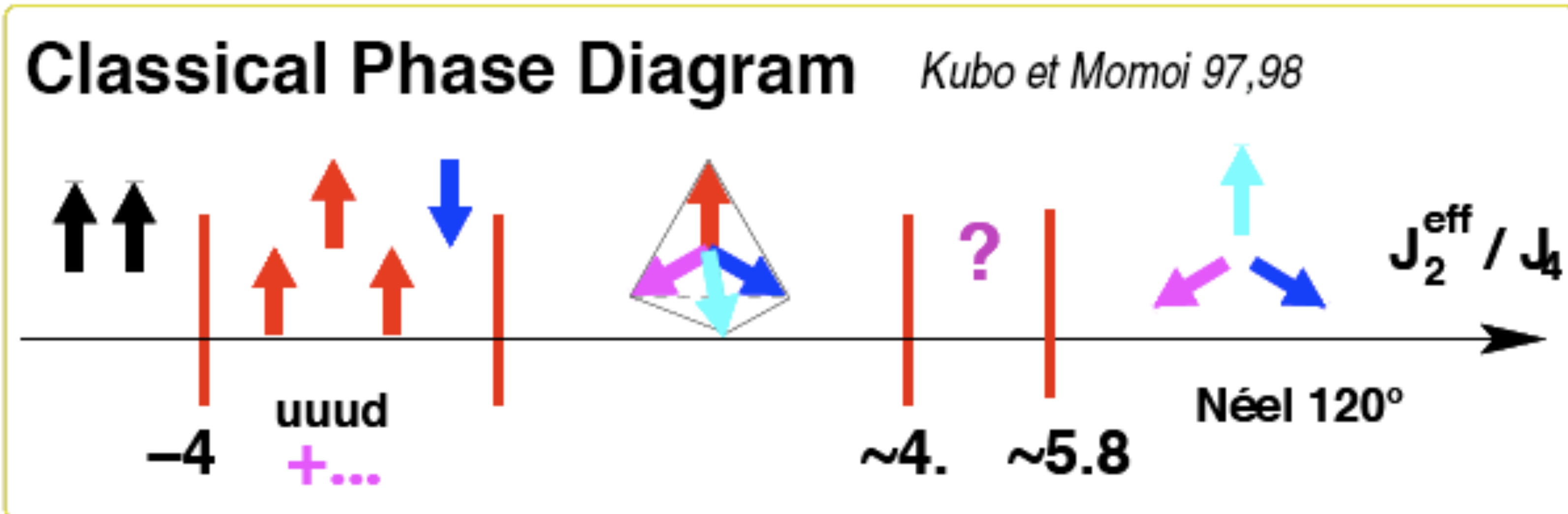
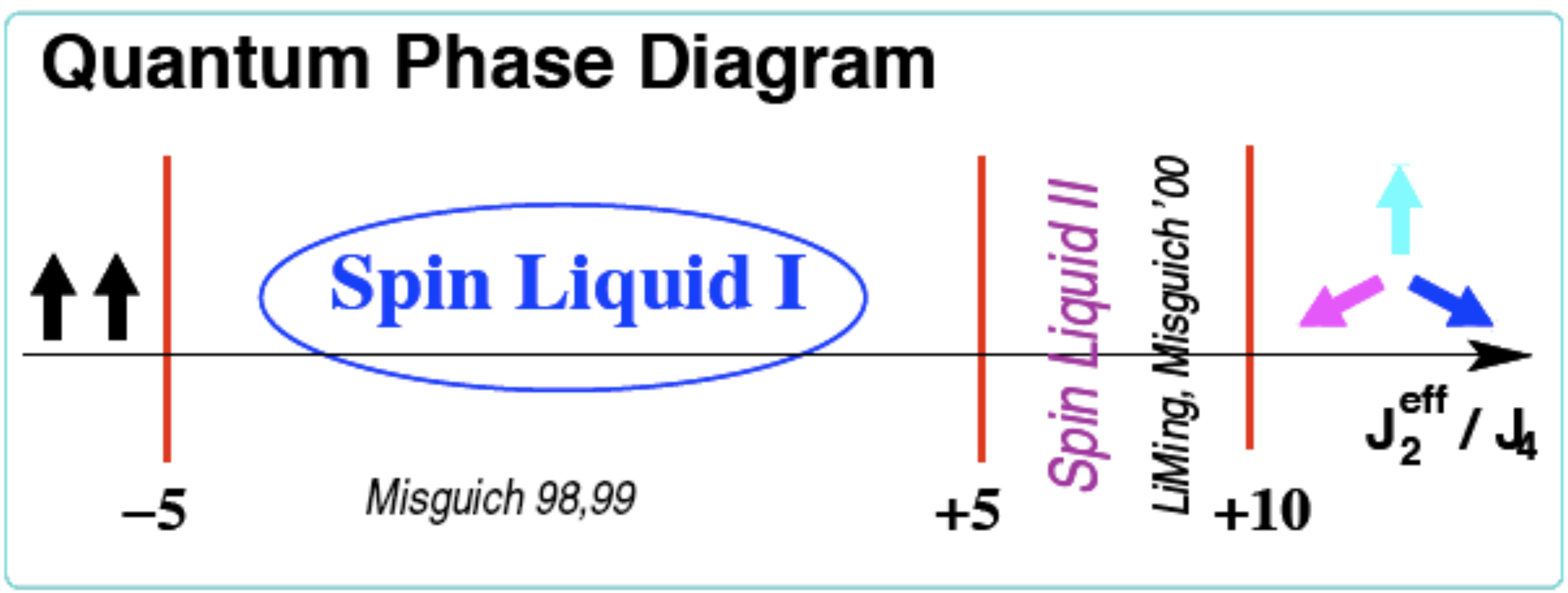
Triangular lattice antiferromagnet: exact diagonalization

$$\hat{H}_{\rm ring} = J_2 \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_4 \sum_{\rm rhombi} (P_{ijkl} + {\rm h.c.})$$





G. Misguich, C. Lhuillier, B. Bernu, and C. Waldtmann, Phys. Rev. B 60, 1064 (1999).
W. LiMing, G. Misguich, P. Sindzingre, and C. Lhuillier, Phys. Rev. B 62, 6372,6376 (2000).

Triangular lattice antiferromagnet: exact diagonalization

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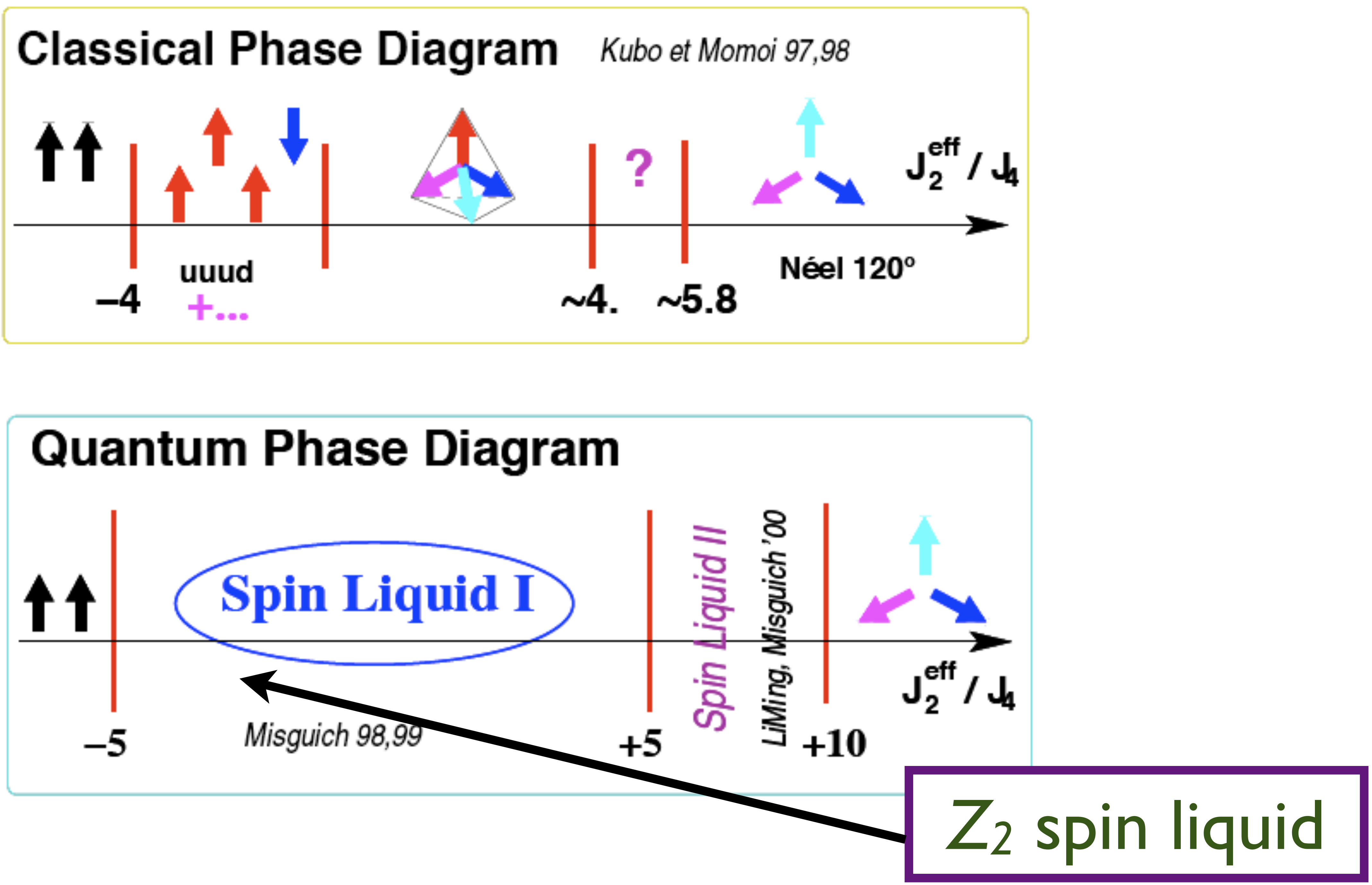

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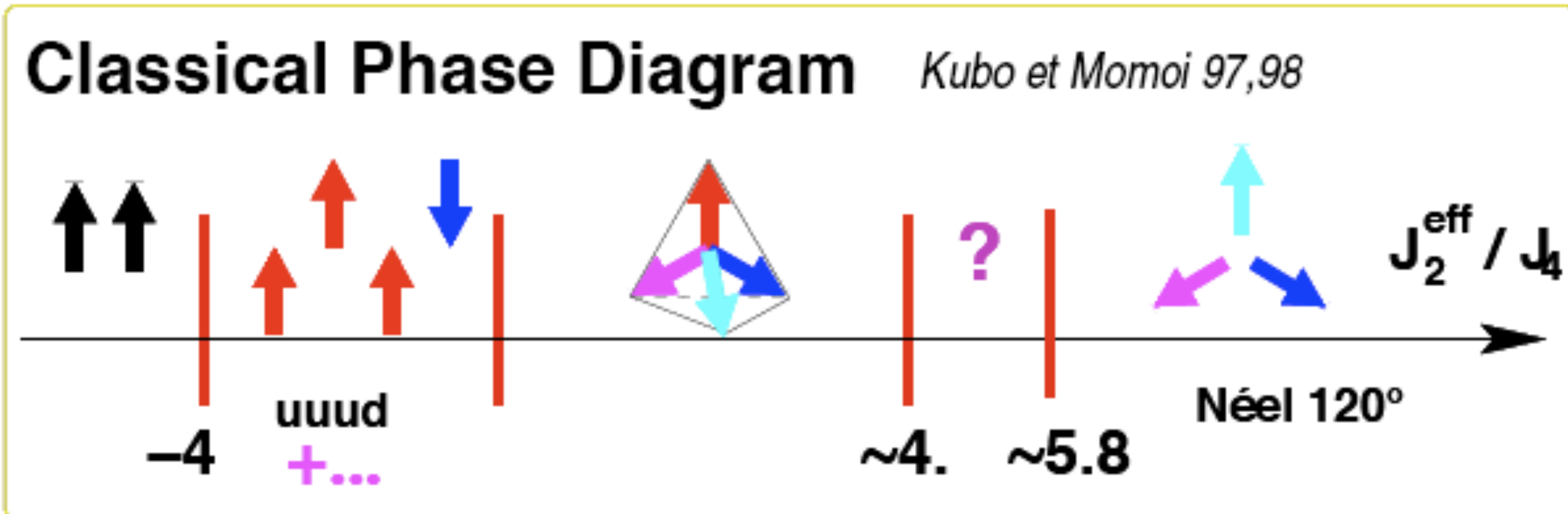
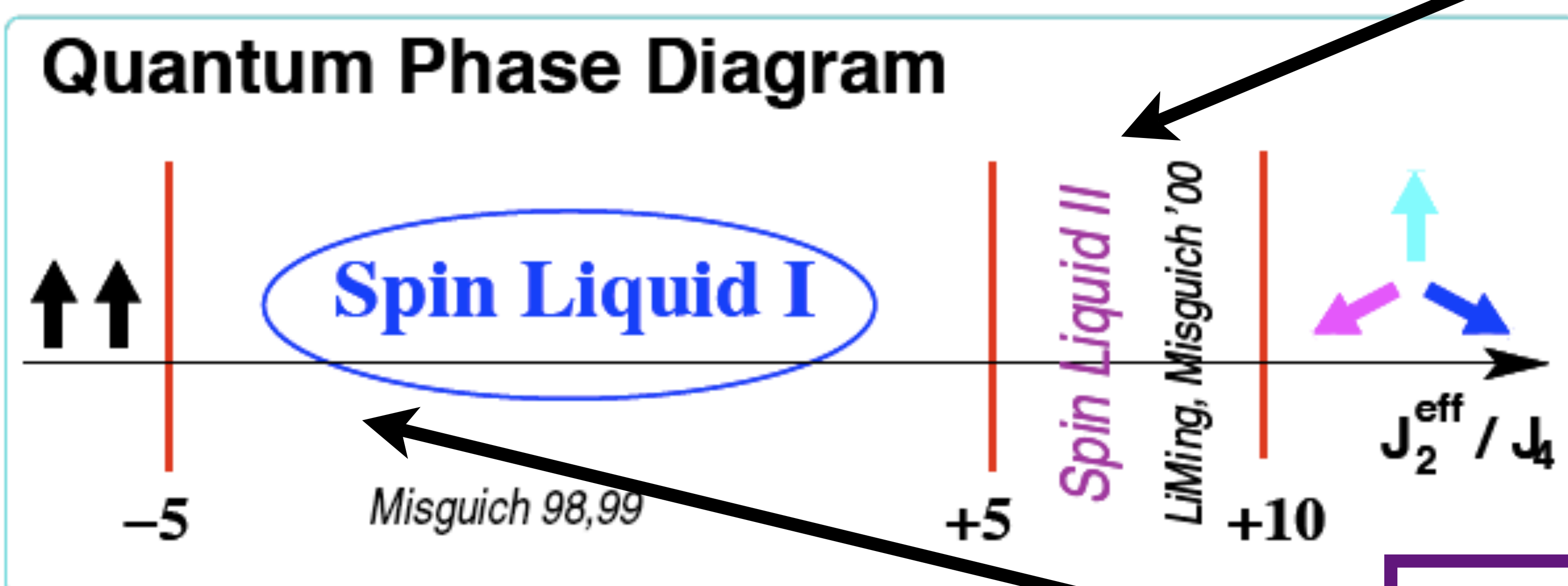




Z_2 spin liquid
or
spinon Fermi
surface
or both ?

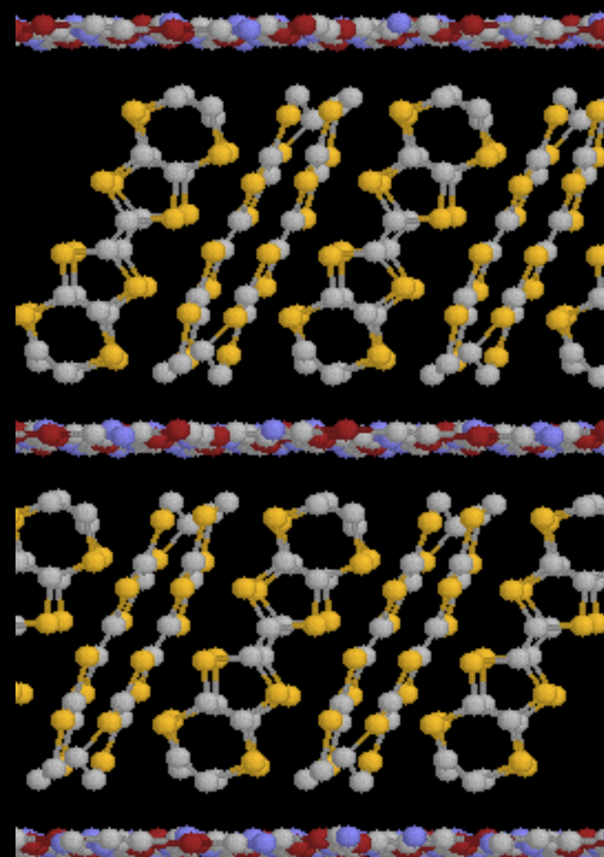
Z_2 spin liquid

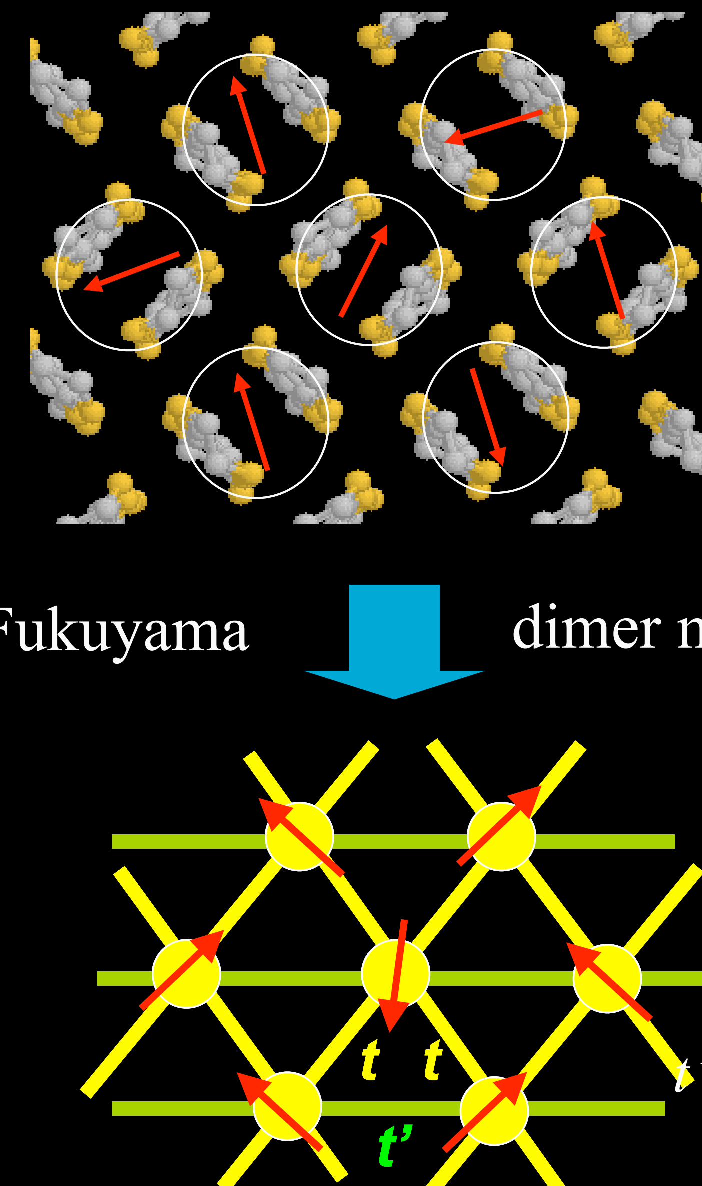
G. Misguich, C. Lhuillier, B. Bernu, and C. Waldtmann, Phys. Rev. B 60, 1064 (1999).
W. LiMing, G. Misguich, P. Sindzingre, and C. Lhuillier, Phys. Rev. B 62, 6372,6376 (2000).

Q2D organics κ-$(ET)_2X$; spin-1/2 on triangular lattice

ET layer

X layer





Triangular lattice
Half-filled band

X^-	Ground State	t'/t
$Cu_2(CN)_3$	Mott insulator	1.06
$Cu[N(CN)_2]Cl$	Mott insulator	0.75
$Cu[N(CN)_2]Br$	SC	0.68
$Cu(NCS)_2$	SC	0.84

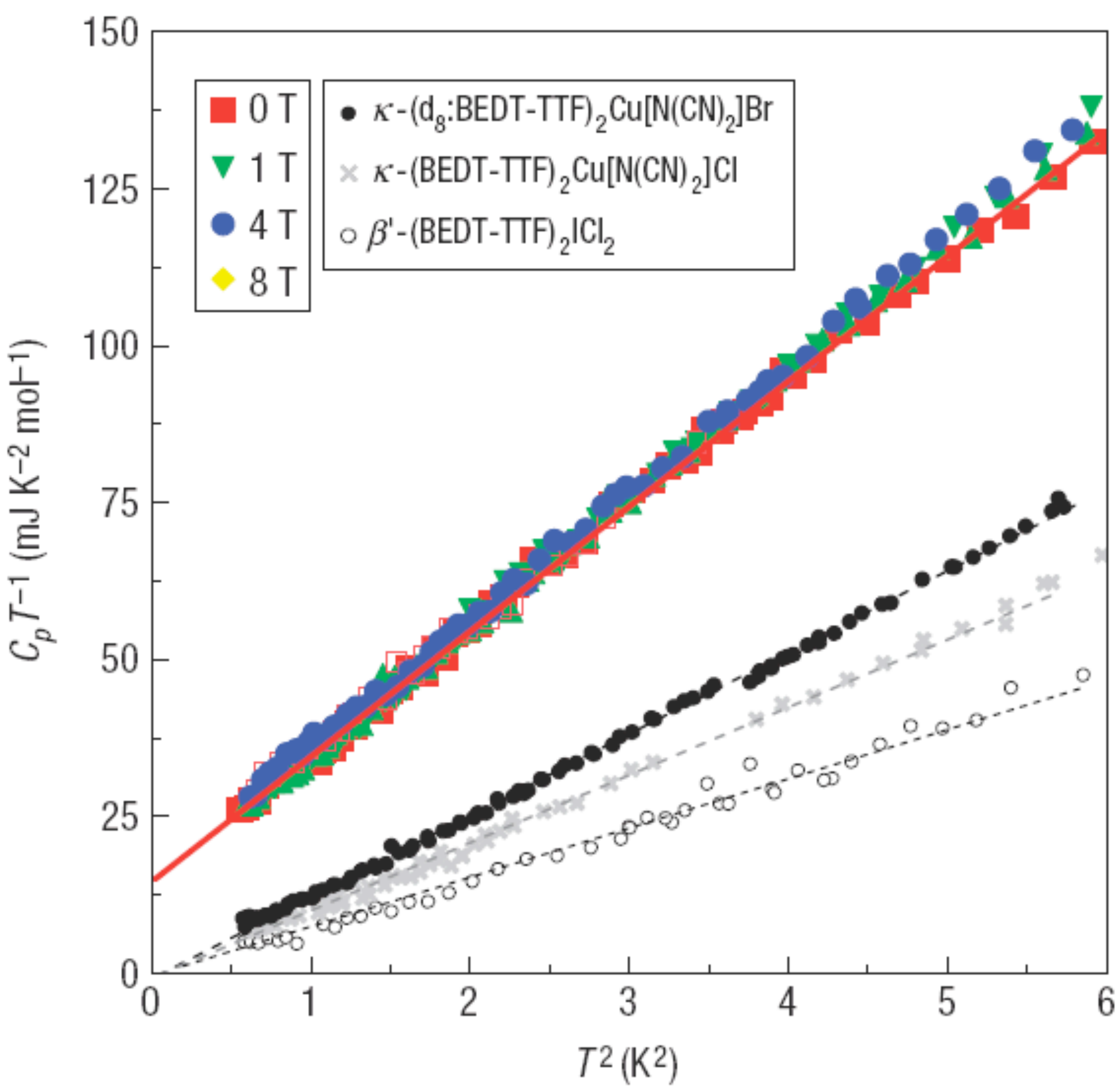
Heat capacity measurements

Thermodynamic properties of a spin-1/2 spin-liquid state in a κ-type organic salt

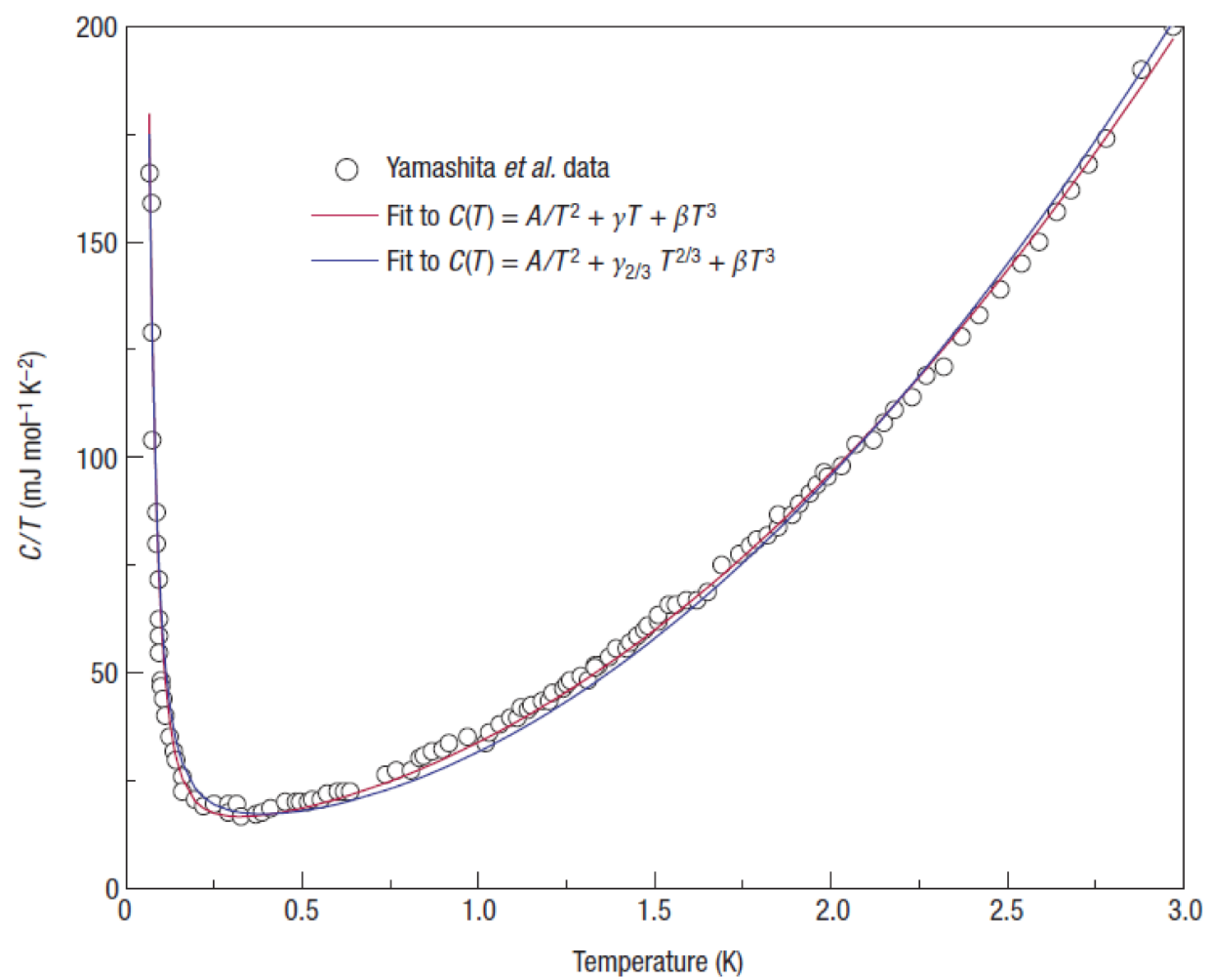
SATOSHI YAMASHITA[1], YASUHIRO NAKAZAWA[1,2*], MASAHARU OGUNI[3], YUGO OSHIMA[2,4], HIROYUKI NOJIRI[2,4], YASUHIRO SHIMIZU[5], KAZUYA MIYAGAWA[2,6] AND KAZUSHI KANODA[2,6]

$$\gamma = 15 \, {}^{\mathrm{mJ}}\!/_{\mathrm{K^2 mol}}$$

Evidence for Gapless spinon?



S. Yamashita, et al., Nature Physics **4**, 459 - 462 (2008)



A. P. Ramirez, Nature Physics **4**, 442 (2008)

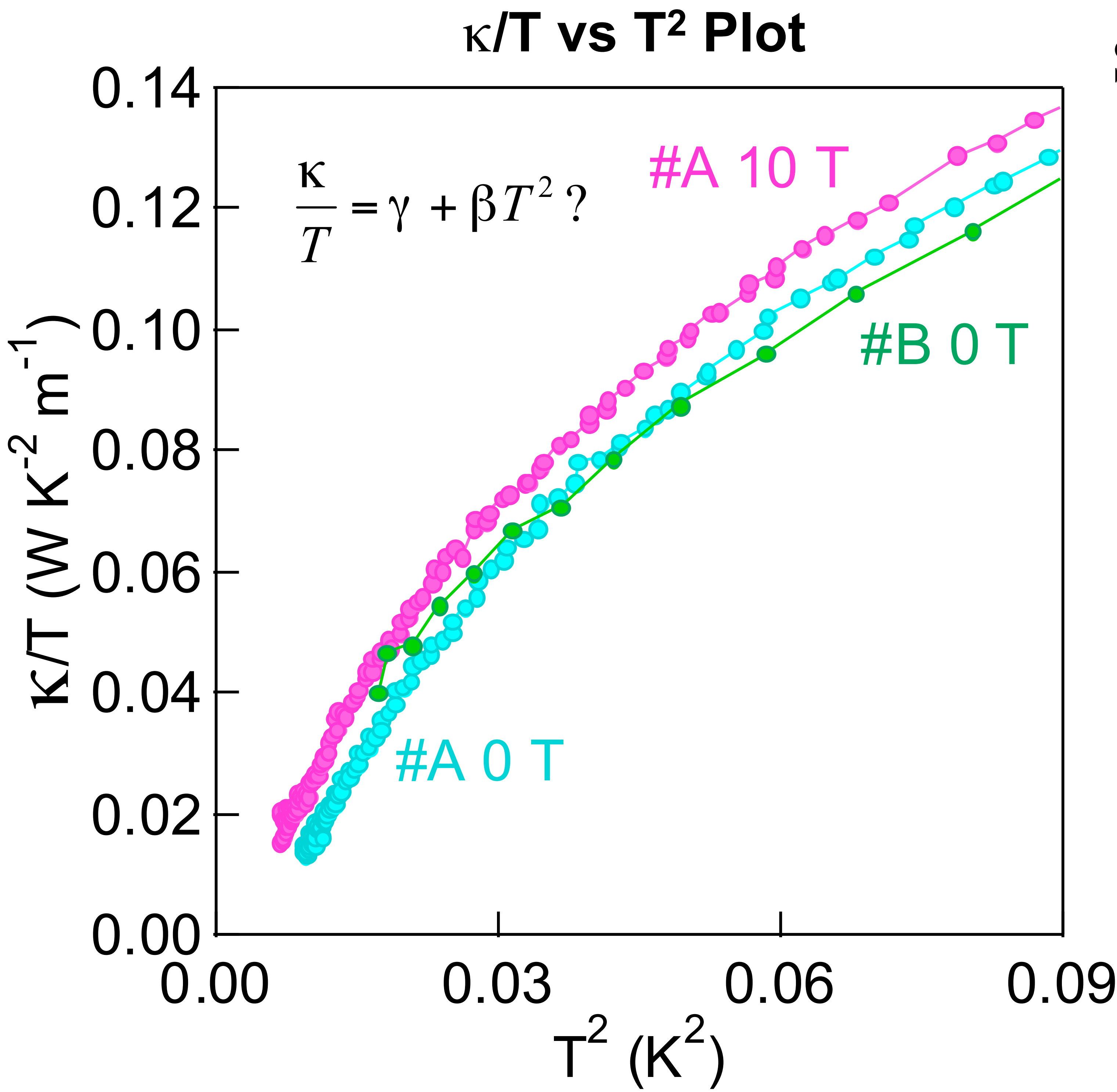
Thermal Conductivity below 300 mK

- Convex, non-T^3 dependence in κ
- Magnetic fields enhance κ

$\kappa = \kappa_{phonon} + \kappa_{mag}$

($\kappa_{phonon} \propto T^3$ in low T)

Substantial portion of κ_{mag} in κ

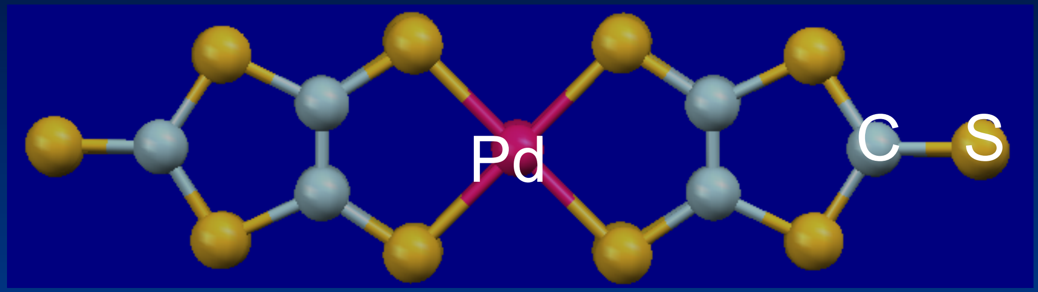


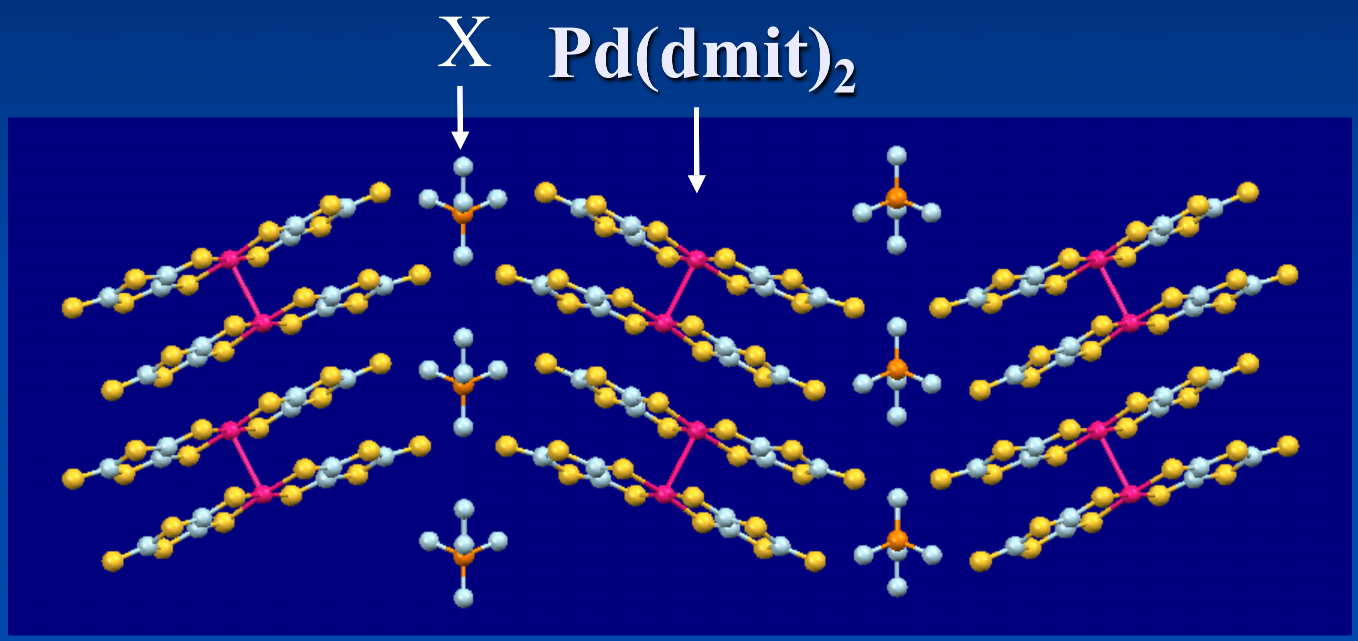
$\gamma = 0$

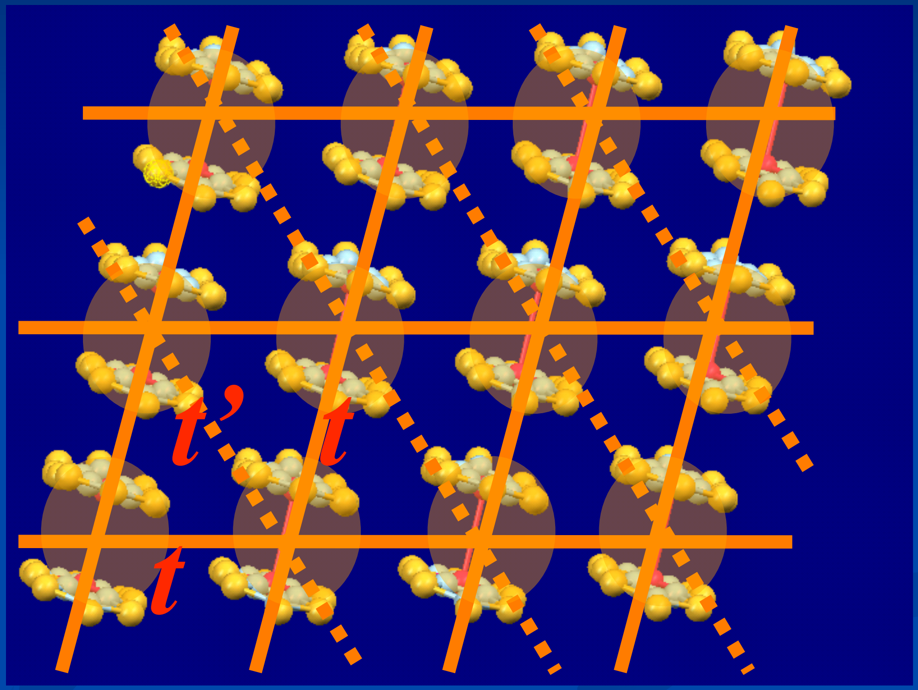
Note: phonon contribution has no effect on this conclusion.

M. Yamashita, H. Nakata, Y. Kasahara, S. Fujimoto, T. Shibauchi, Y. Matsuda, T. Sasaki, N. Yoneyama, and N. Kobayashi, preprint and 25th International Conference on Low Temperature Physics, SaM3-2, Amsterdam, August 9, 2008.

X[Pd(dmit)$_2$]$_2$





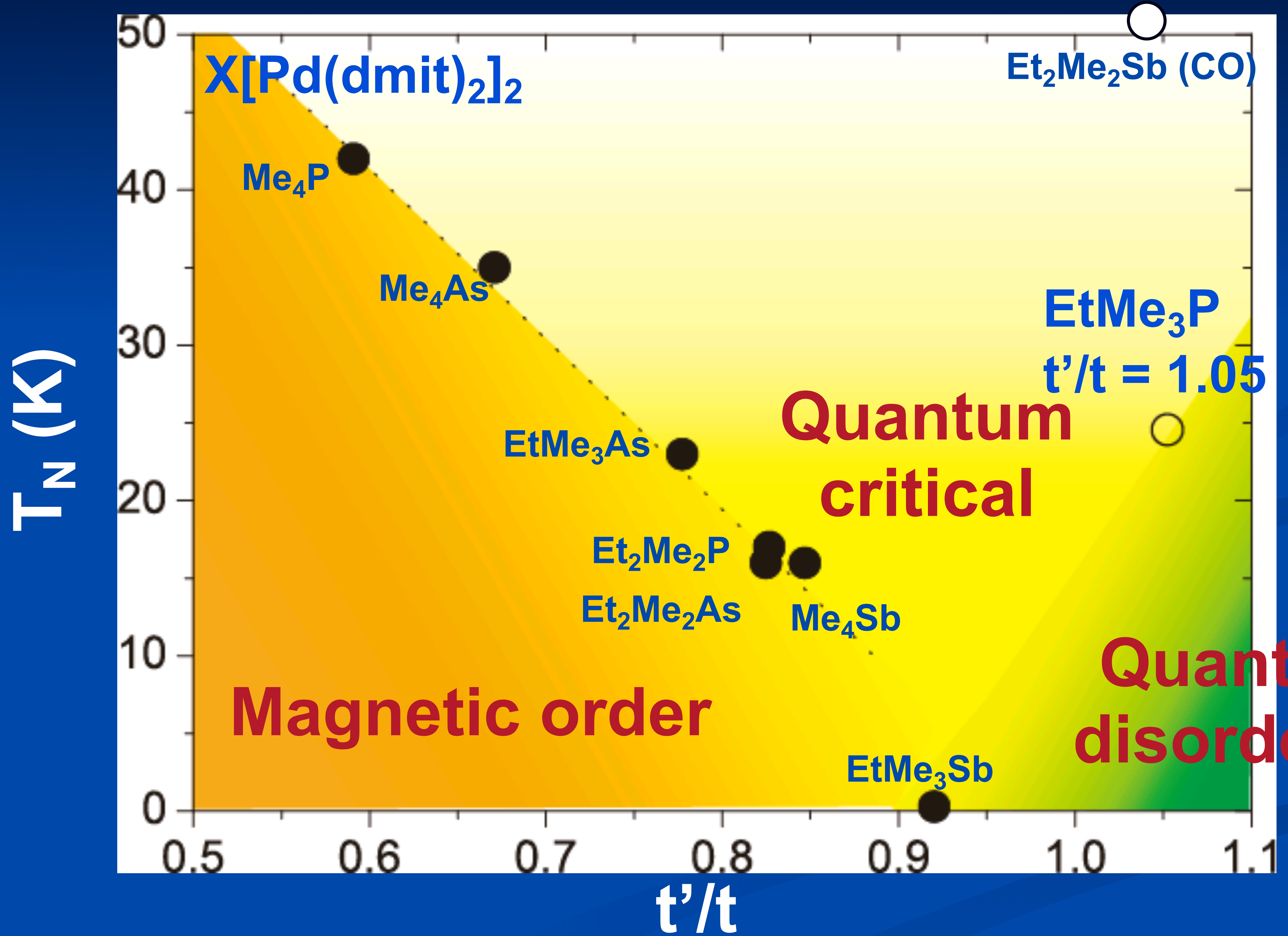


Half-filled band → Mott insulator with spin S = 1/2

Triangular lattice of [Pd(dmit)$_2$]$_2$

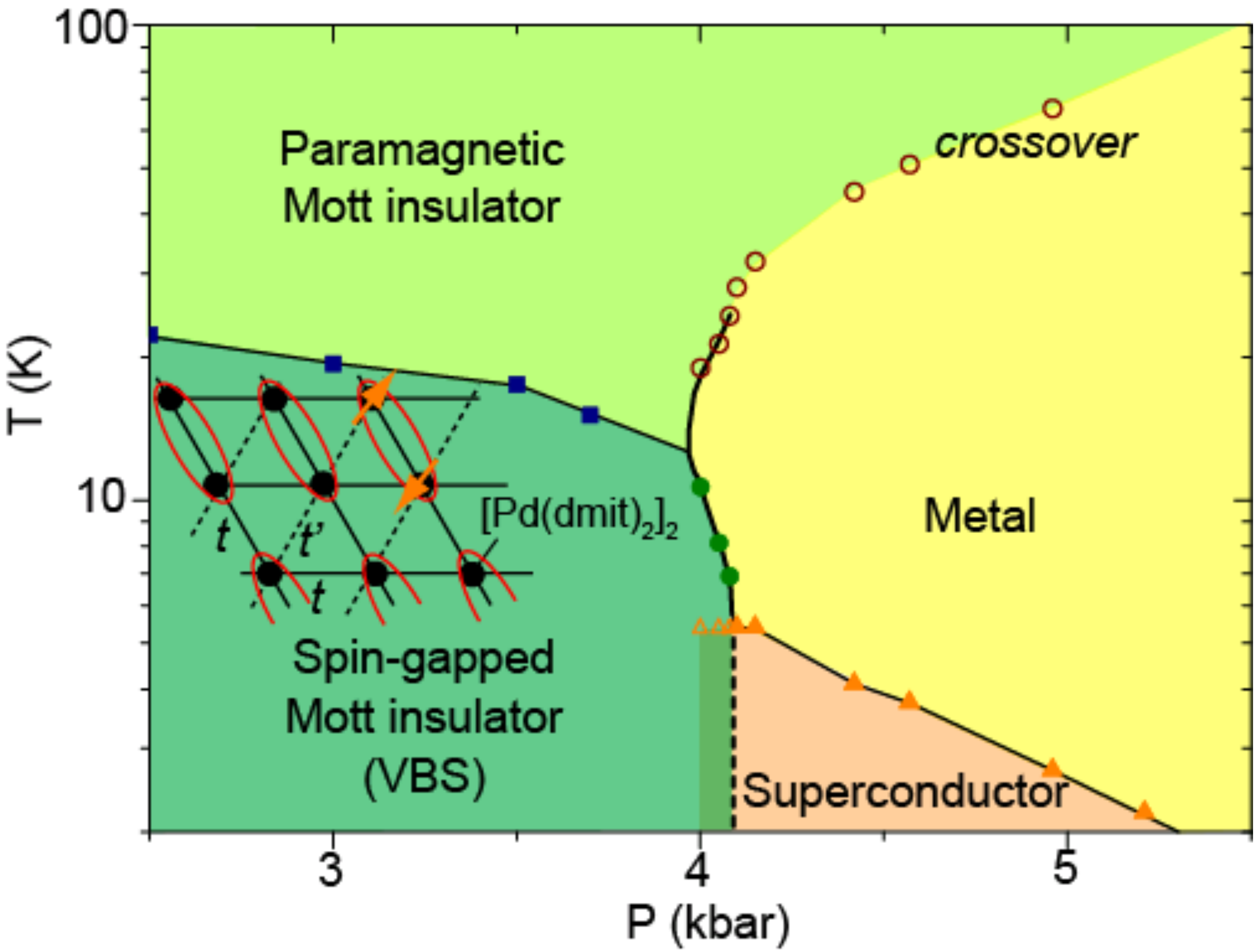
→ frustrated quantum spin system

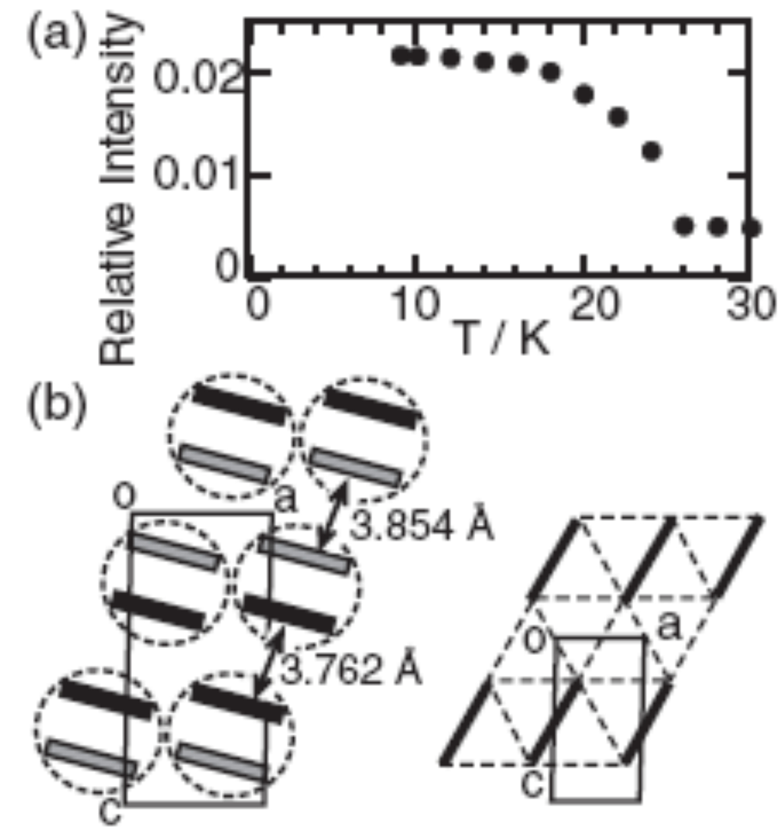
Magnetic Criticality



Y. Shimizu *et al. J.Phys. Conden. Matter,* in press.

Observation of a valence bond solid (VBS)





Spin gap ~ 40 K

J ~ 250 K

Pressure-temperature phase diagram of $ETMe_3P[Pd(dmit)_2]_2$

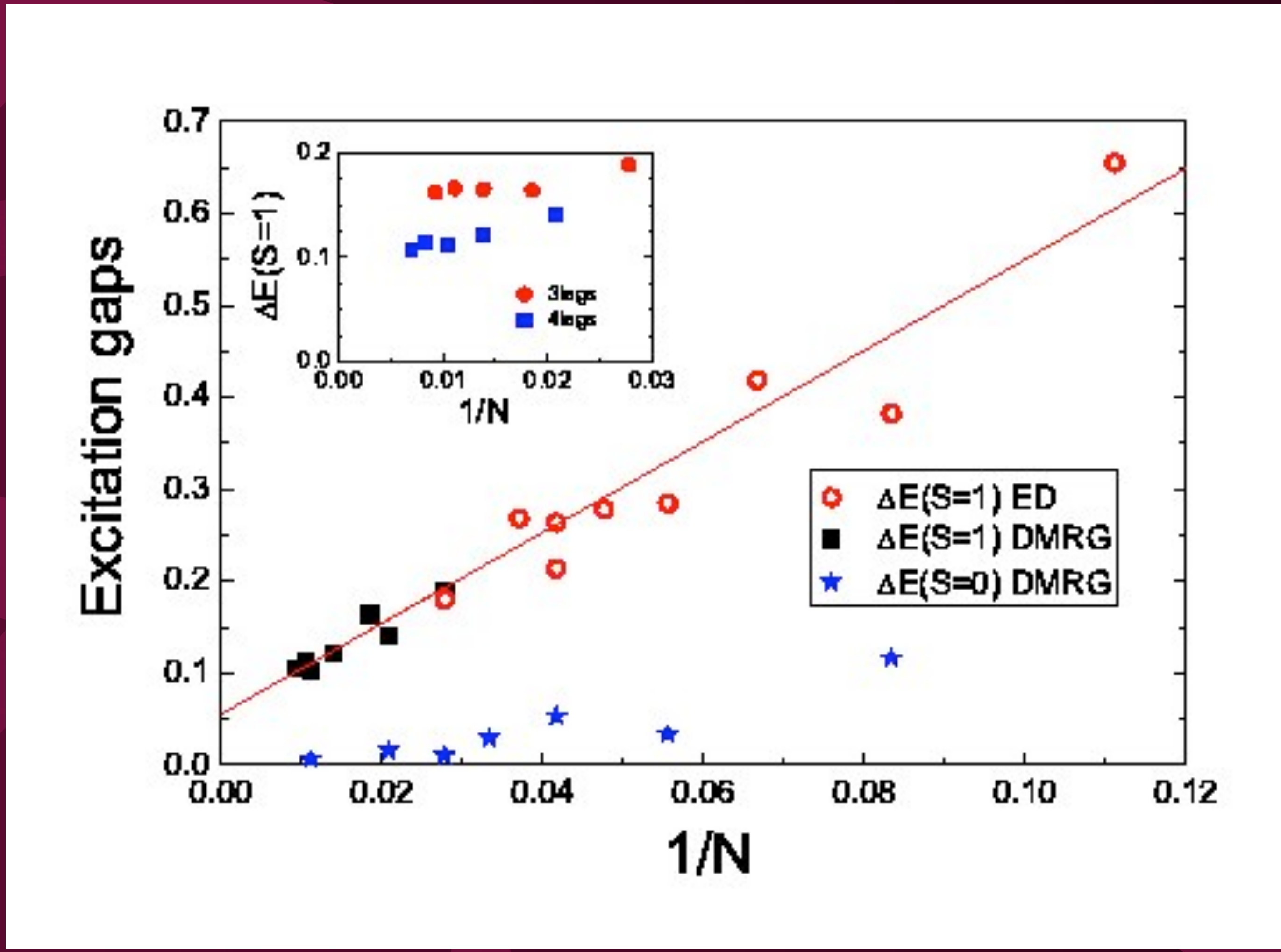
M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Kagome antiferromagnet

Recent DMRG

- Large systems (up to 108 spins)
- Spin Gap 0.055(5)
- Singlet gap Tiny
- E=-0.433



H.C. Jiang, Z.Y. Weng, and D.N. Sheng, *Phys. Rev. Lett.* **101**, 117203 (2008)

Kagome antiferromagnets

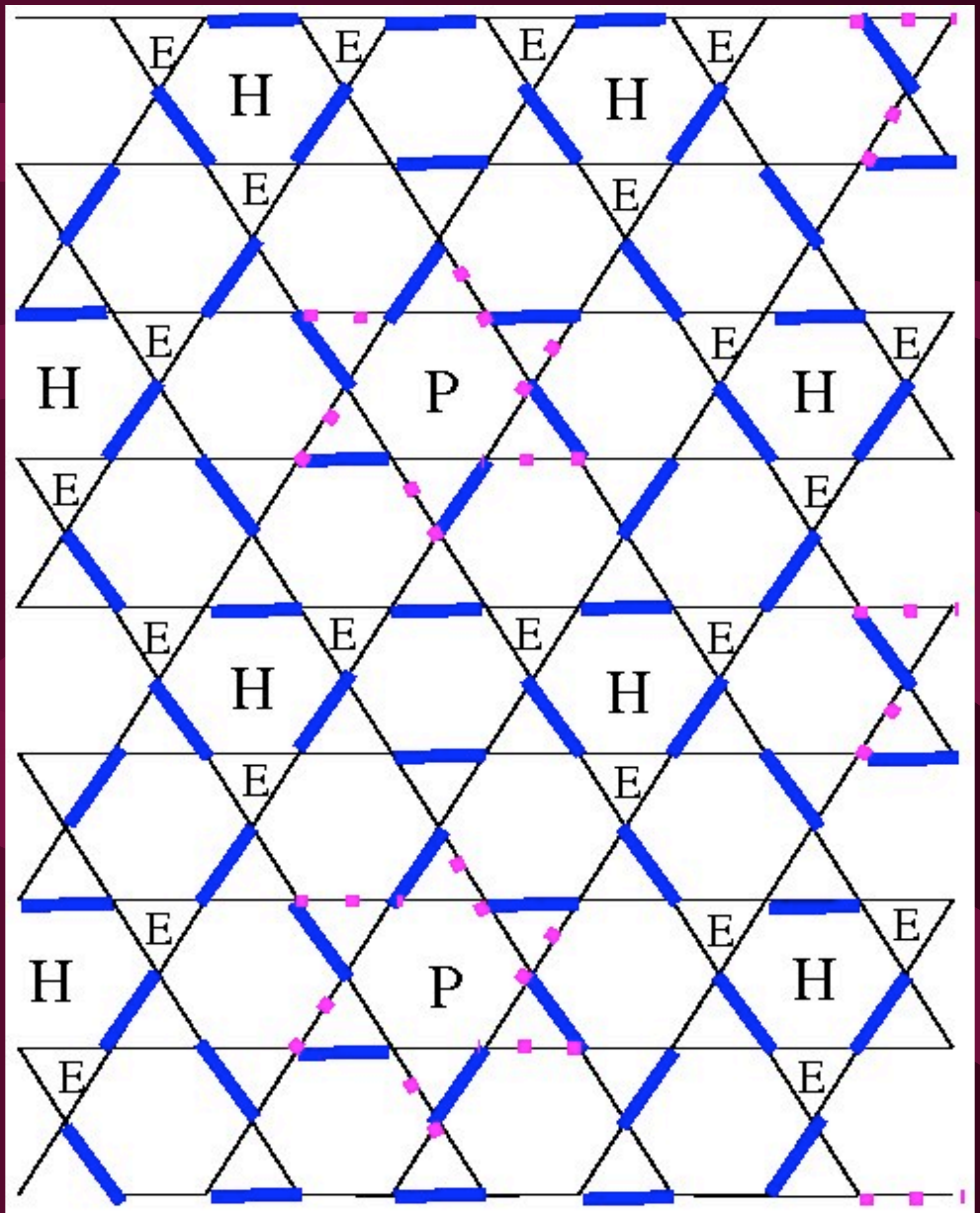
Proposed VBS ground state

3^{rd} Order: Bind 3Es into H—maximize H

4^{th} Order: Honeycomb Lattice

Leftover: Pinwheels

24*2^(N/36) Low energy states



J. Marston and C. Zeng, *J. Appl. Phys.* **69** 5962 (1991)
P. Nikolic and T. Senthil, *Phys. Rev. B* **68**, 214415 (2003)
R. R. P. Singh and D.A. Huse, *Phys. Rev. B* **76**, 180407(R) (2007)

Kagome antiferromagnet

Series show excellent Convergence

Order	Honeycomb	Stripe VBC	36-site PBC
0	-0.375	-0.375	-0.375
1	-0.375	-0.375	-0.375
2	-0.421875	-0.421875	-0.421875
3	-0.42578125	-0.42578125	-0.42578125
4	-0.431559245	-0.43101671	-0.43400065
5	-0.432088216	-0.43153212	-0.43624539

Ground State Energy per site

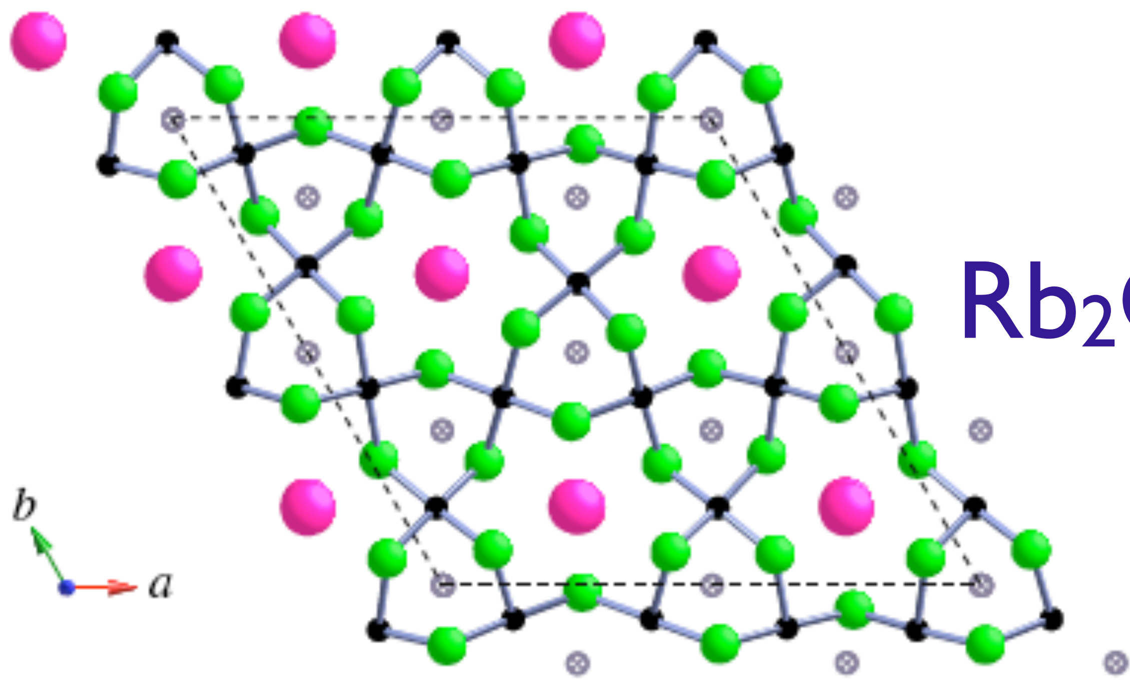
Estimated H-VBC energy: -0.433(1) (ED, DMRG)

36-site PBC: Energy=-0.43837653

Variational state of Ran et al -0.429

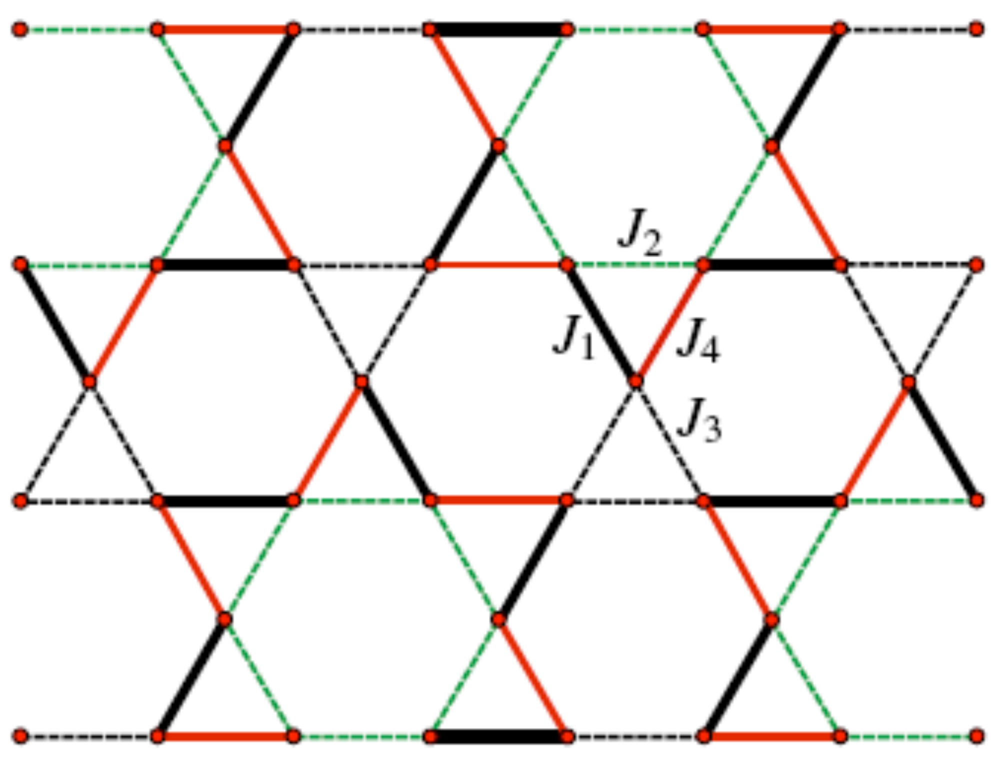
R. R. P. Singh and D.A. Huse, *Phys. Rev. B* **76**, 180407(R) (2007)

Kagome antiferromagnet



(a)

$Rb_2Cu_3SnF_{12}$



(b)

Spin gap ~ 20 K
J ~ 200 K

Distorted kagome (or VBS order in kagome spin-phonon model ?)

K. Morita, M. Yano. T. Ono, H. Tanaka, K. Fujii, H. Uekusa, Y. Narumi, and K. Kindo, *J. Phys. Soc. Japan* **77**, 043707 (2008)

Kagome antiferromagnet

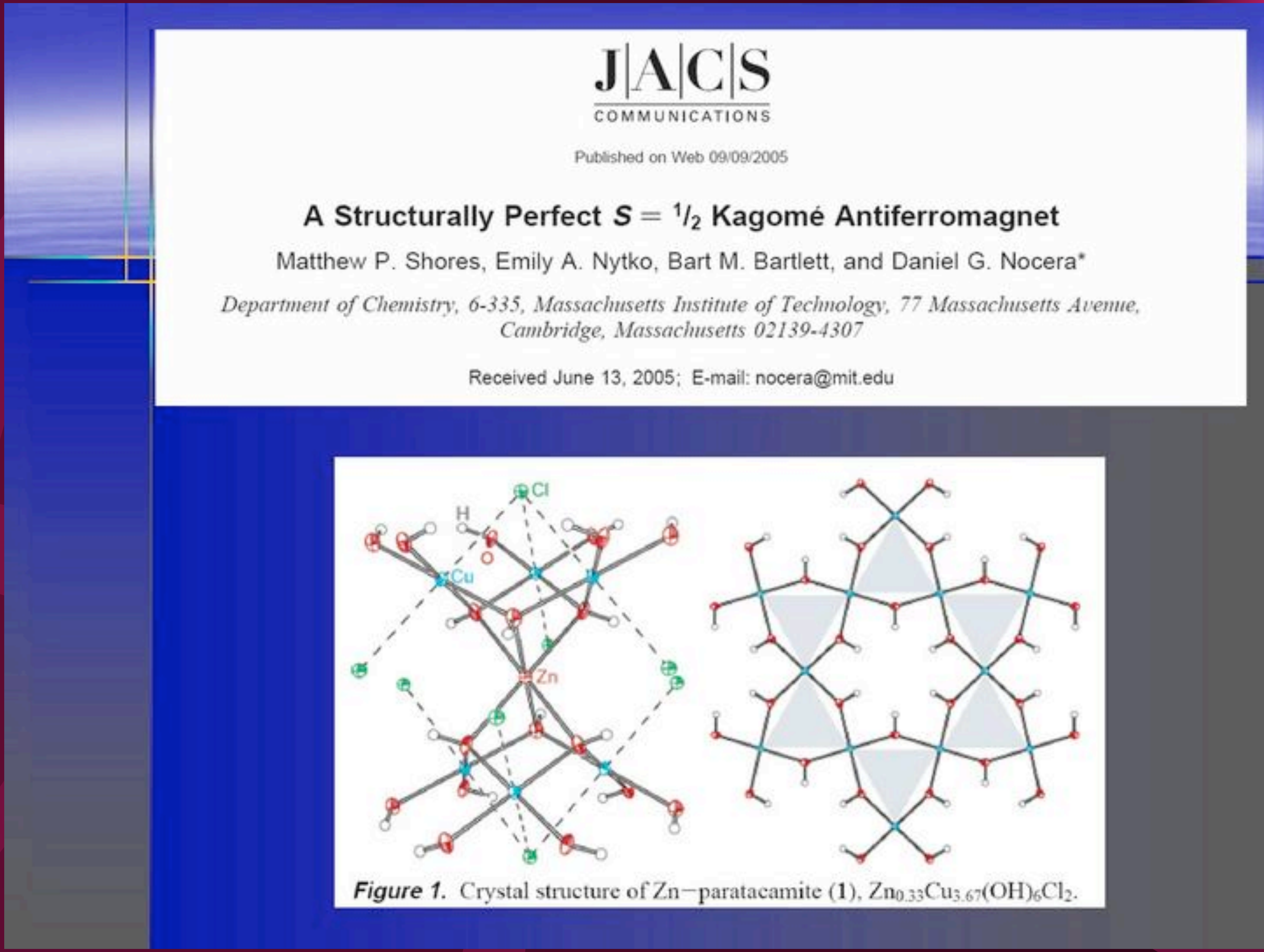
- New material: Herbertsmithite

  ZnCu_3(OH)_6Cl_2

  Cu atoms carry spin-half

  Kagome-layers of Cu

  Separated by layers of Zn



J|A|C|S
COMMUNICATIONS

Published on Web 09/09/2005

A Structurally Perfect $S = {}^{1}/_{2}$ Kagomé Antiferromagnet

Matthew P. Shores, Emily A. Nytko, Bart M. Bartlett, and Daniel G. Nocera*

Department of Chemistry, 6-335, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139-4307

Received June 13, 2005; E-mail: nocera@mit.edu

Figure 1. Crystal structure of Zn−paratacamite (**1**), $Zn_{0.33}Cu_{3.67}(OH)_6Cl_2$.

Kagome antiferromagnet

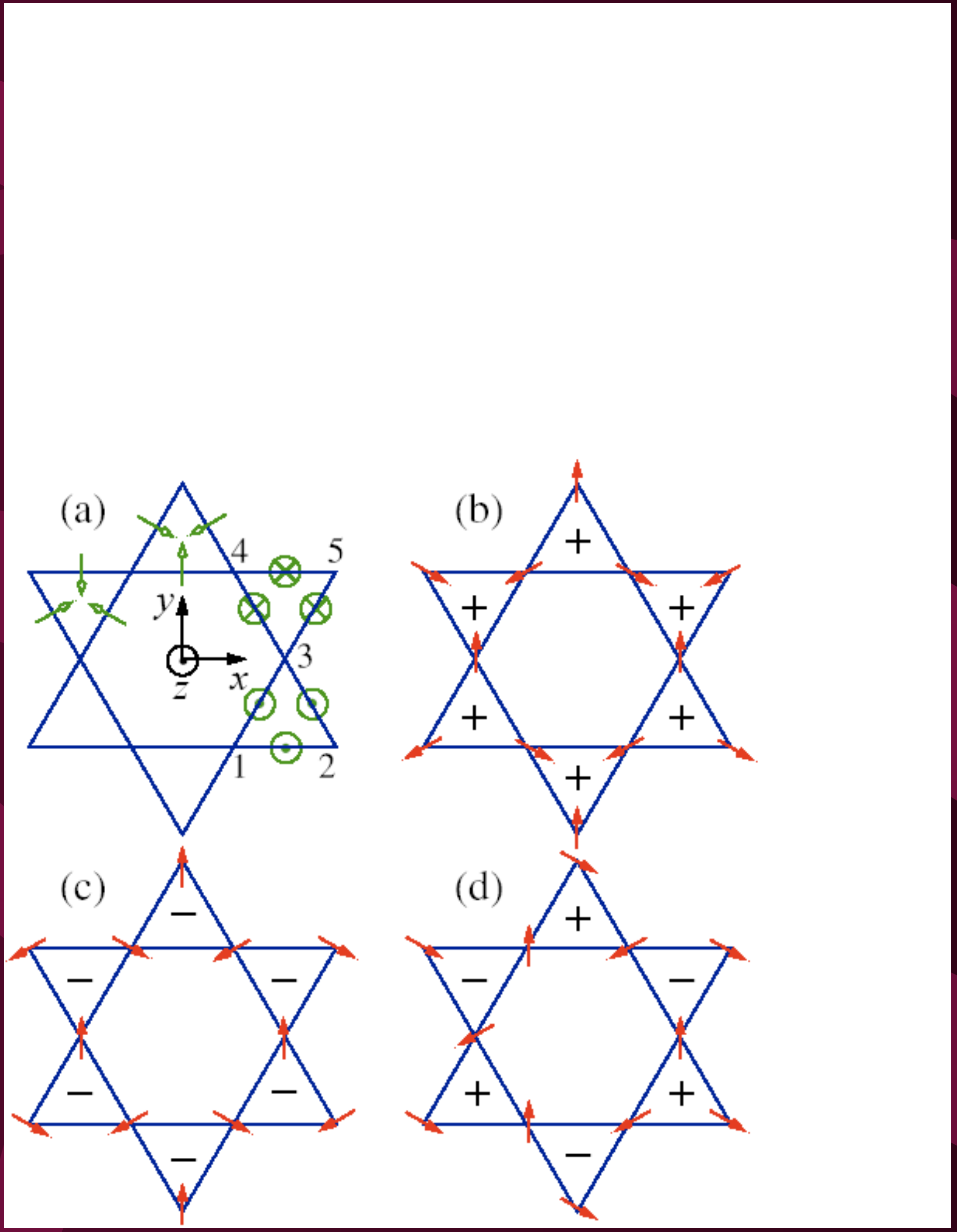
Two factors complicate interpretation of experiments on herbertsmithite

- Substitutional Impurities (Zn for Cu)

  Causes extra spins and dilution

- Anisotropy

  Dzyaloshinski-Moria Interaction can cause weak ferromagnetism

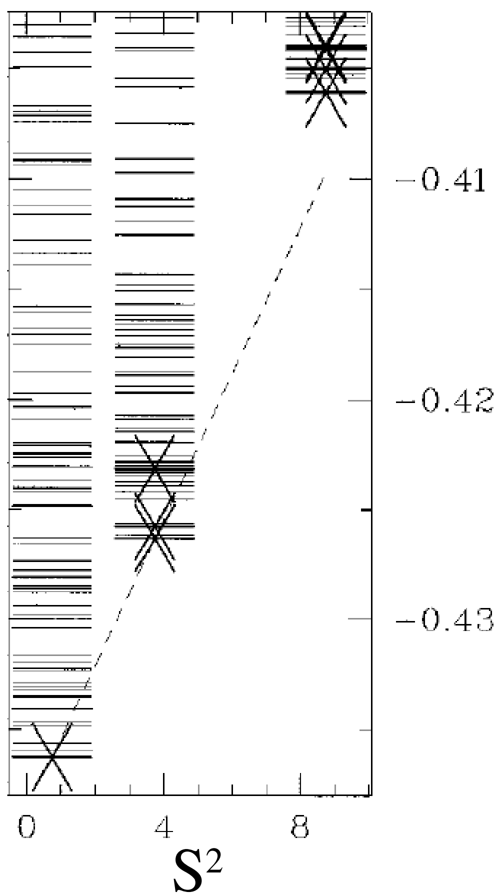


Dz and Dp

Signature of a broken-symmetry state :

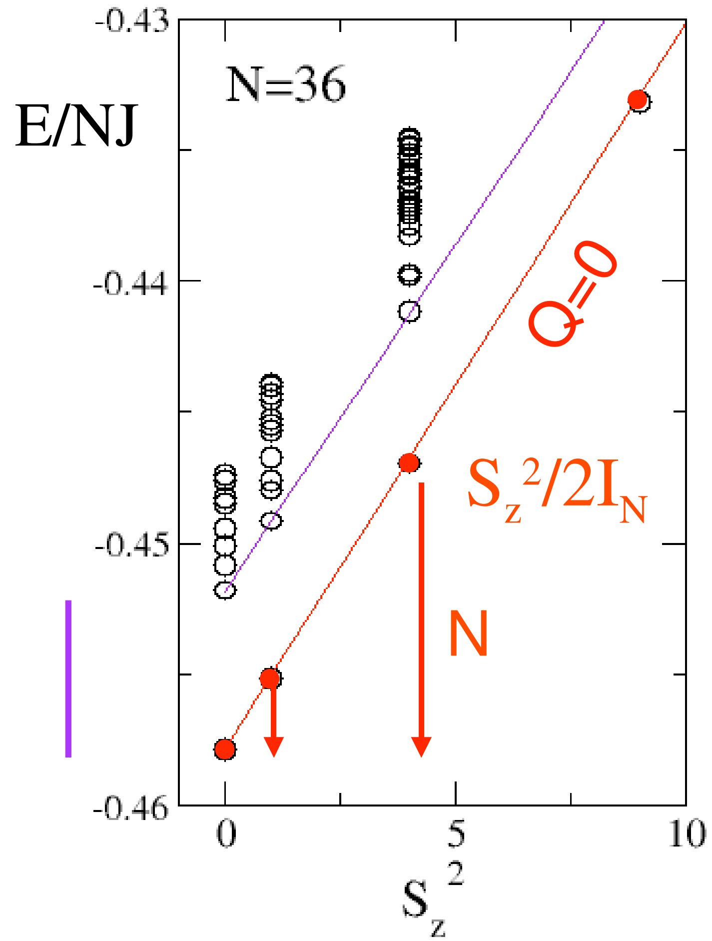
Energy levels of $H = \sum_{nn} [J\mathbf{S}_i.\mathbf{S}_j + \mathbf{D}\mathbf{e}^z_{ij}.(\mathbf{S}_i \times \mathbf{S}_j)]$ by exact diagonalization of N=21, 24, 27, 30, 36 sites

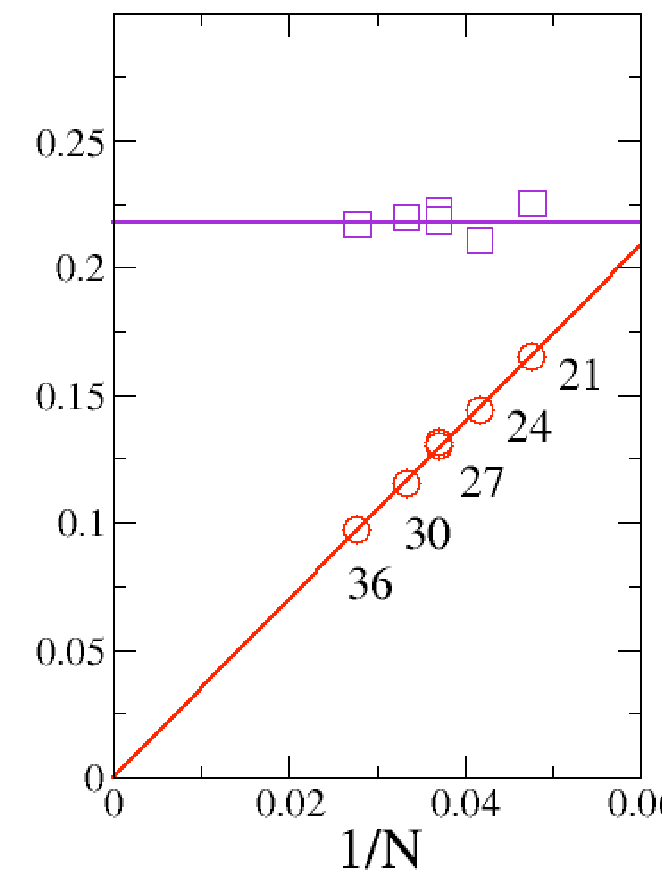
D=0



Lechemintant et al. PRB 1997
Waldtman et al.

D/J=0.1





Here is a clear separation of energy scales

Collapse for N infinity : onset of a broken symmetry state

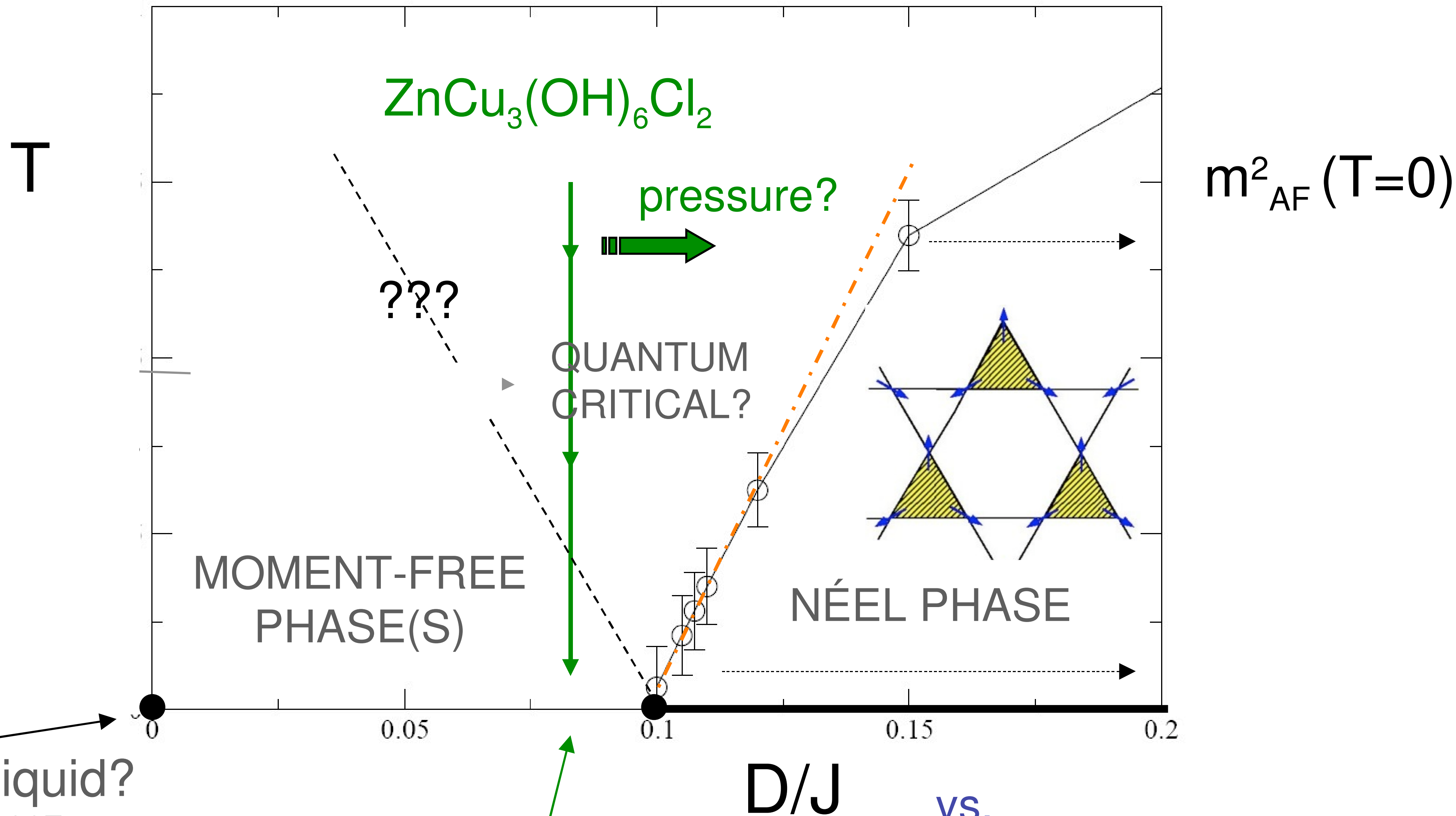
($\Delta S_z \, \Delta\varphi > h$)

Is there a critical D/J ?

O. Cepas, C.M. Fong, P.W. Leung, and C. Lhuillier, arXiv:0806.0393

Quantum critical point at $D_c \approx 0.1J$

Olivier Cépas, C.M. Fong, P.W. Leung, and C.Lhuillier, cond-mat/0806.0393v2



VBC, spin liquid?
Singh and Huse 2007
Budnik and Auerbach 2004
Syromyatnikov and Maleyev 2004
Nikolic and Senthil 2003
Marston and Zeng 1991

D = 0.08J (from ESR)
Zorko et al. PRL 2008

vs.
SWT : $D_c = 0.03J$ Elhajal et al. 2002
ASL : $D_c = 0$ Hermele et al. 2008

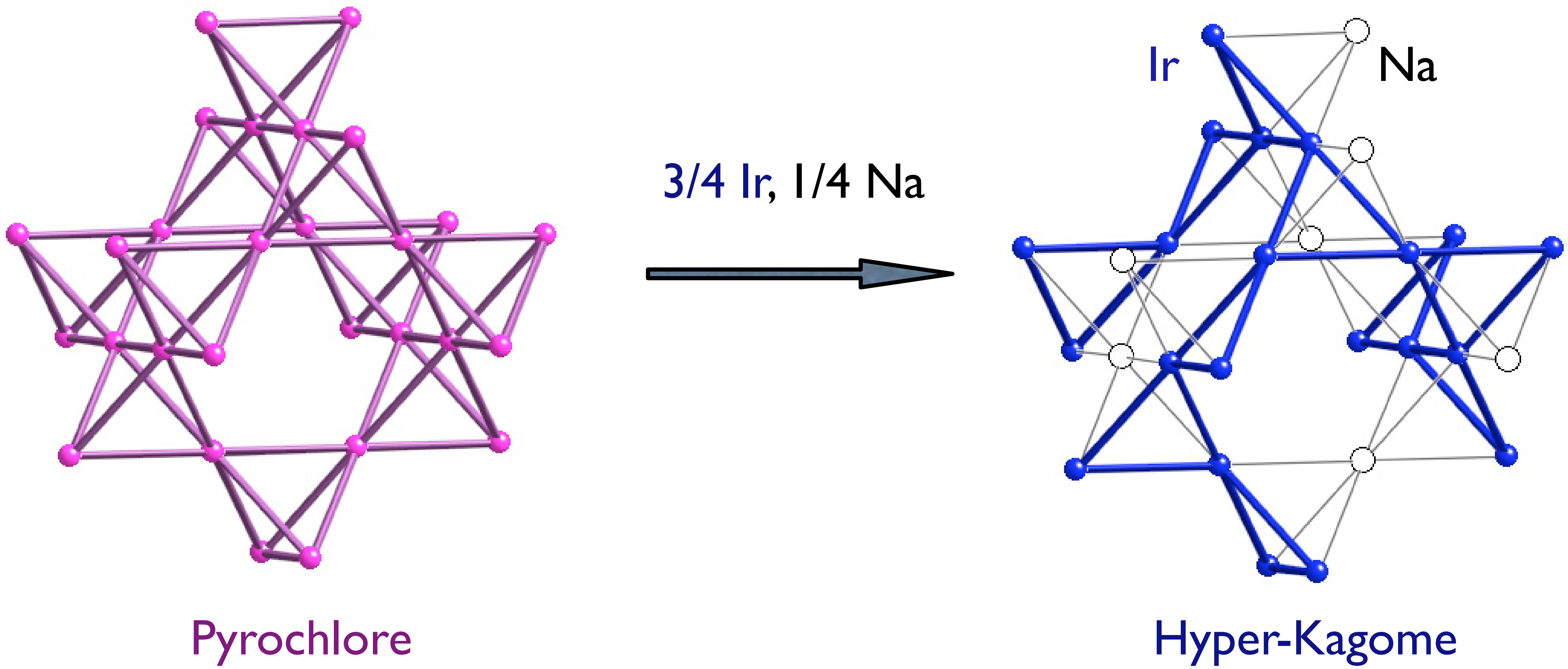
algebraic spin liquid
Hastings 2000; Ran et al. 2007;
Ryu et al. 2007; Hermele et al. 2008

Power law behaviour?

- Algebraic spin liquid
- Disorder : Rozenberg and Chitra, PRB 2008
- or proximity to this quantum critical point ???

Three-dimensional S=1/2 Frustrated Magnet

$Na_4Ir_3O_8$ has a Hyper-Kagome sublattice of Ir ions



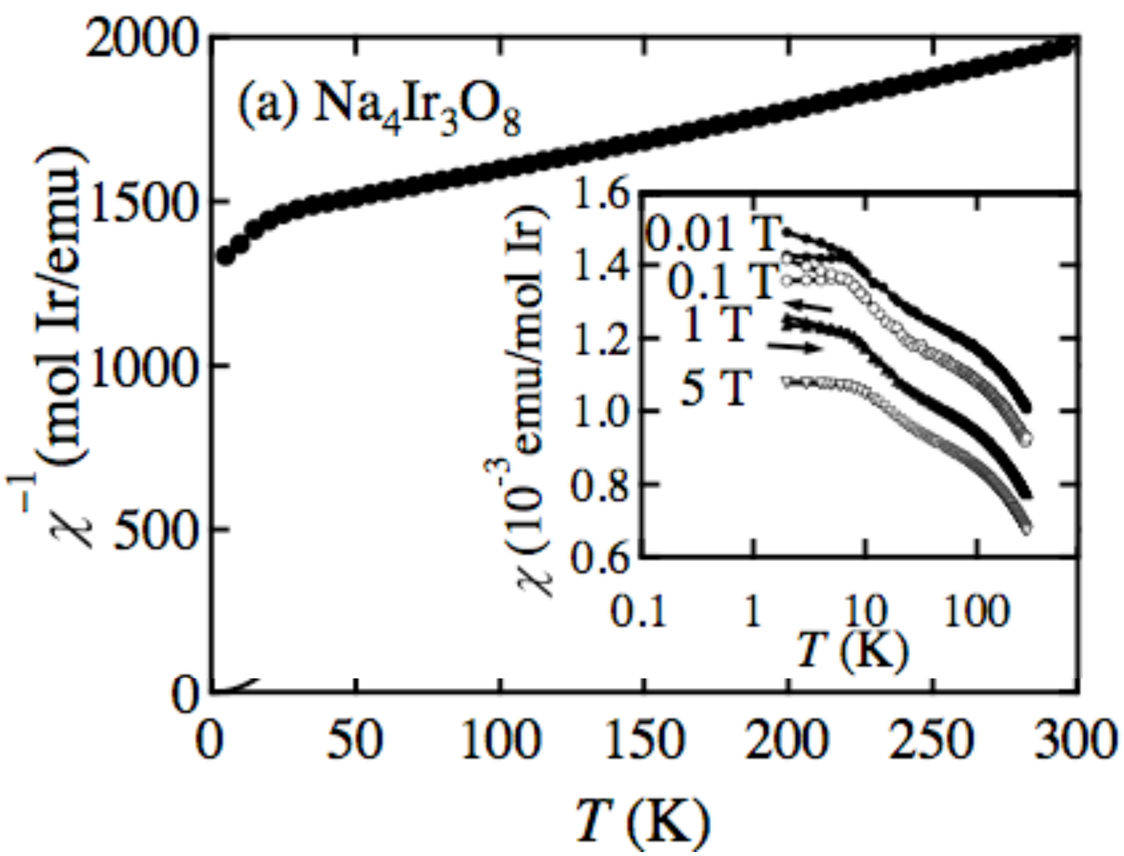
All Ir-Ir bonds are equivalent

Ir^{4+} $(5d^5)$ carries "S=1/2" moment ?

Y. Okamoto, M. Nohara, H. Agura-Katrori, and H. Takagi, PRL 99, 137207 (2007)

Inverse Spin Susceptibility; Strong Spin Frustration

Y. Okamoto, M. Nohara, H. Agura-Katrori, and H. Takagi, PRL 99, 137207 (2007)

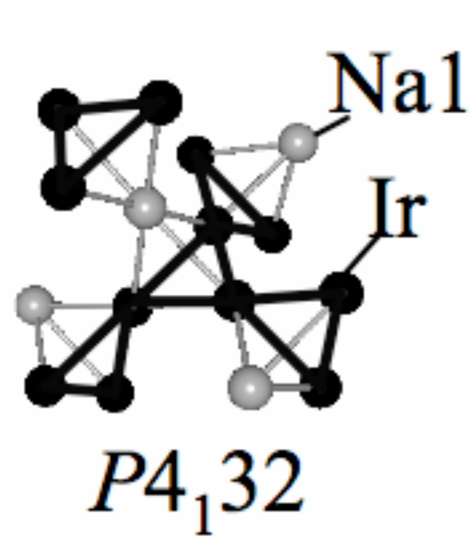


Curie-Weiss fit

$$\Theta_{\rm CW} = -650K$$

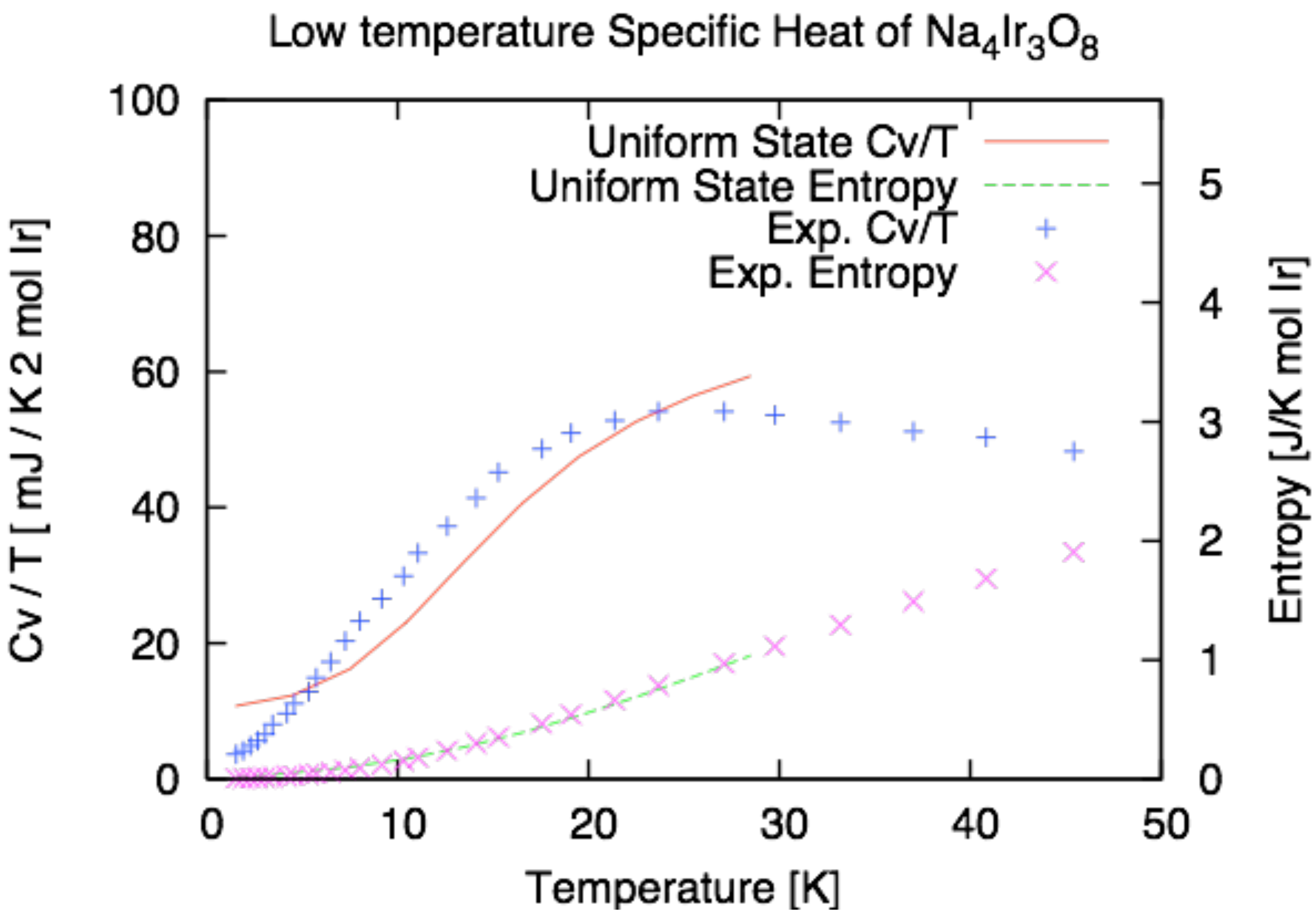
No magnetic ordering down to $|\Theta_{\rm CW}|/300$

Large Window of Cooperative Paramagnet



Strong Frustration - Macroscopic degeneracy of classical ground states

Theory of Spin Liquid with Gapless Fermionic Spinons



J = 304K obtained from ED and Gutzwiller factor g=3

(can only fix the overall scale, but it is a zero-parameter fit !)

C/T looks linear for 5K < T < 20K

M. J. Lawler, A. Paramekanti, Y. B. Kim, L. Balents, arXiv:0806.4395
Y. Zhou, P.A. Lee, T.-K. Ng, F.-C. Zhang, arXiv:0806.3323

Outline

1. Landau-Ginzburg criticality

   Coupled-dimer antiferromagnets

2. Quantum "disordering" magnetic order

   Z_2 spin liquids and valence bond solids

3. Critical spin liquids

   Deconfined criticality; fermionic spinons near the Mott transition

4. Triangular, kagome, and hyperkagome lattices

   Connections to experiments

[[[5. Correlated boson model

   Supersolids and stripes]]]

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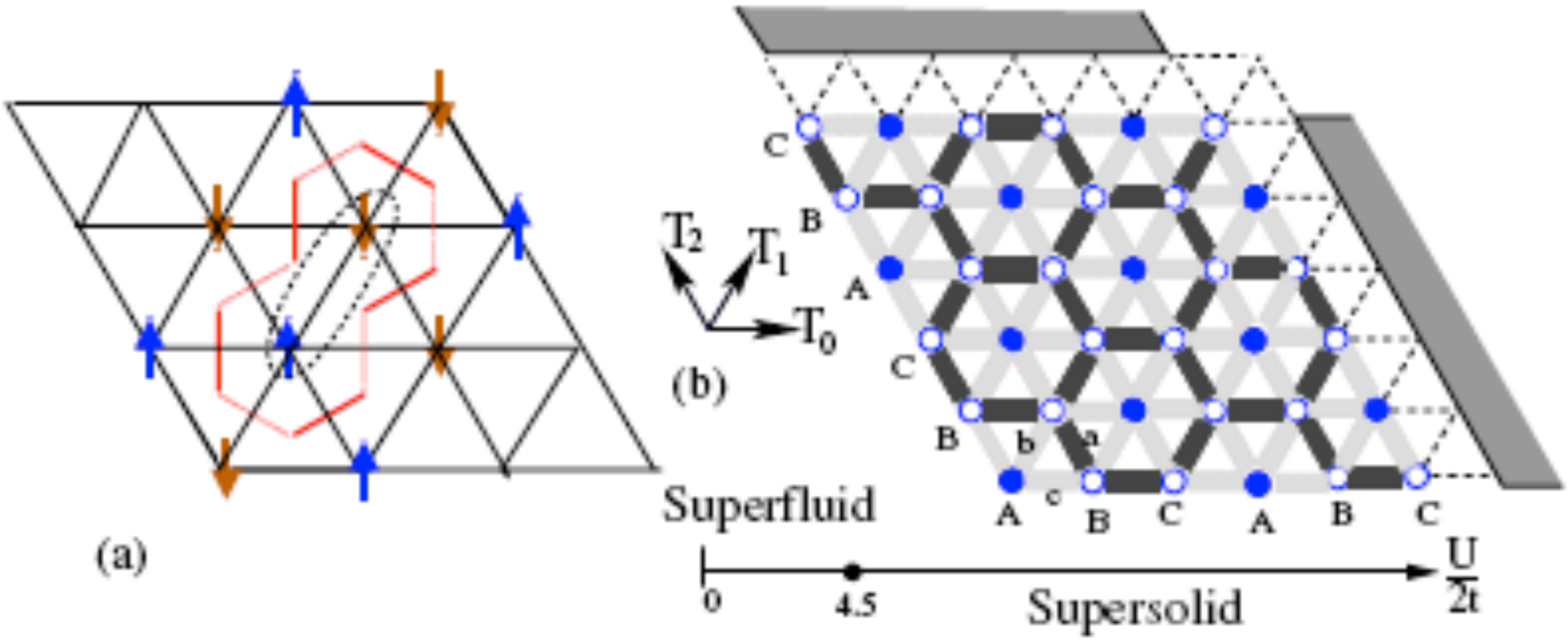
[[[5. Correlated boson model

Supersolids and stripes]]]

Bosons with repulsive interactions on the triangular lattice near half-filling

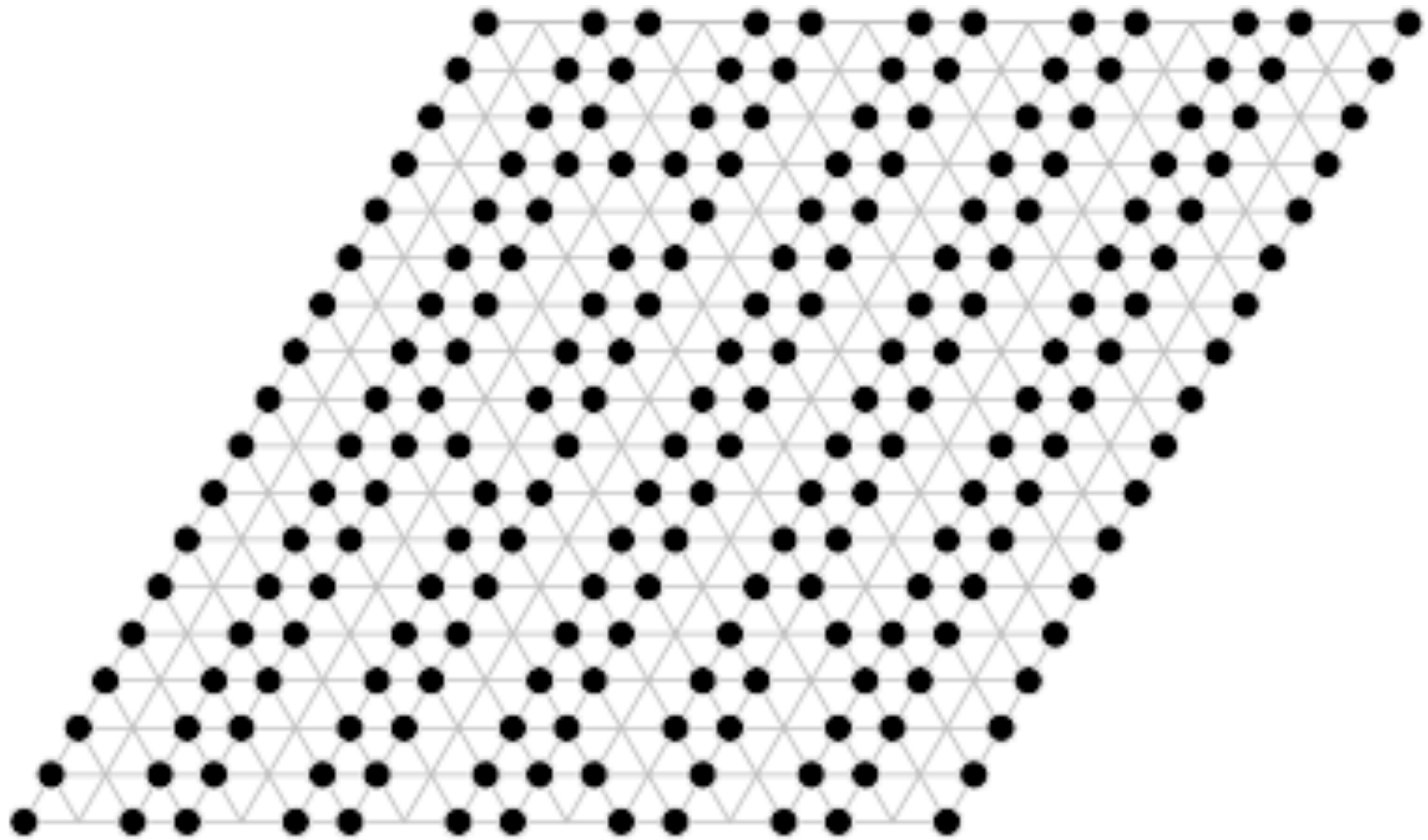


S. Wessel and M. Troyer, *Phys. Rev. Lett.* **95**, 127205 (2005)

D. Heidarian and K. Damle, *Phys. Rev. Lett.* **95**, 127206 (2005)

R. G. Melko, A. Paramekanti, A. A. Burkov, A. Vishwanath, D. N. Sheng and L. Balents, *Phys. Rev. Lett.* **95**, 127207 (2005)

Bosons with (longer range) repulsive interactions on the triangular lattice near half-filling



Striped supersolid with very small anisotropy in superfluid stiffness

R. G. Melko, A. Del Maestro, and A. A. Burkov, *Phys. Rev. B* **74**, 214517 (2006)

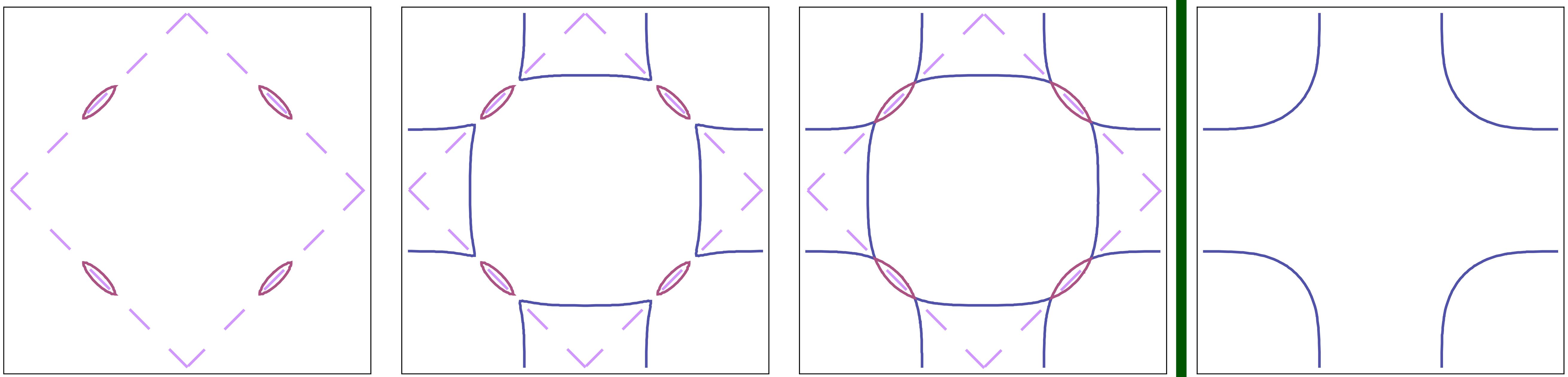
Open questions:
phase transitions with Fermi surfaces/points

- Slave-particle gauge theories from conventional phases with Fermi surfaces/points lead to exotic phases with “ghosts” of Fermi surfaces/points

- Is it possible to have direct transitions between conventional Fermi liquid phases but with distinct topologies of Fermi surfaces/points ?

Spin density wave theory for Fermi surface evolution in the hole-doped cuprates

Increasing AFM order

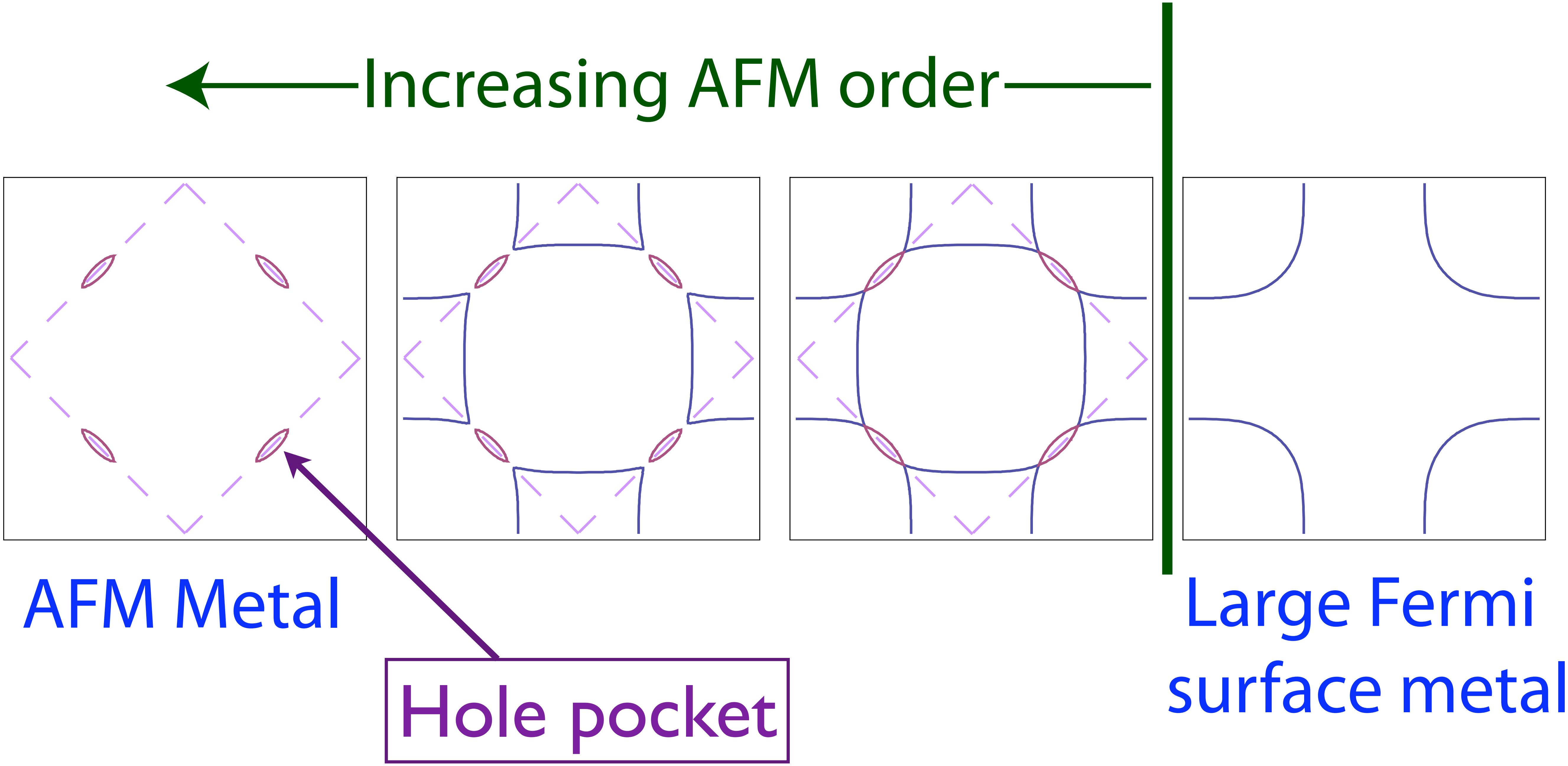


AFM Metal

Large Fermi surface metal

A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997)

Spin density wave theory for Fermi surface evolution in the hole-doped cuprates



A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997)

Spin density wave theory for Fermi surface evolution in the hole-doped cuprates

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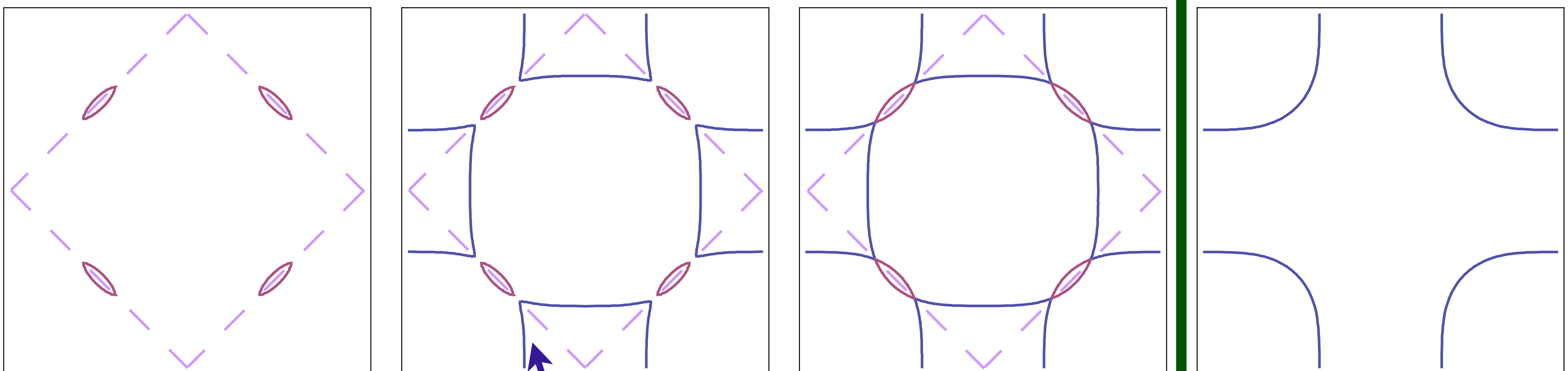


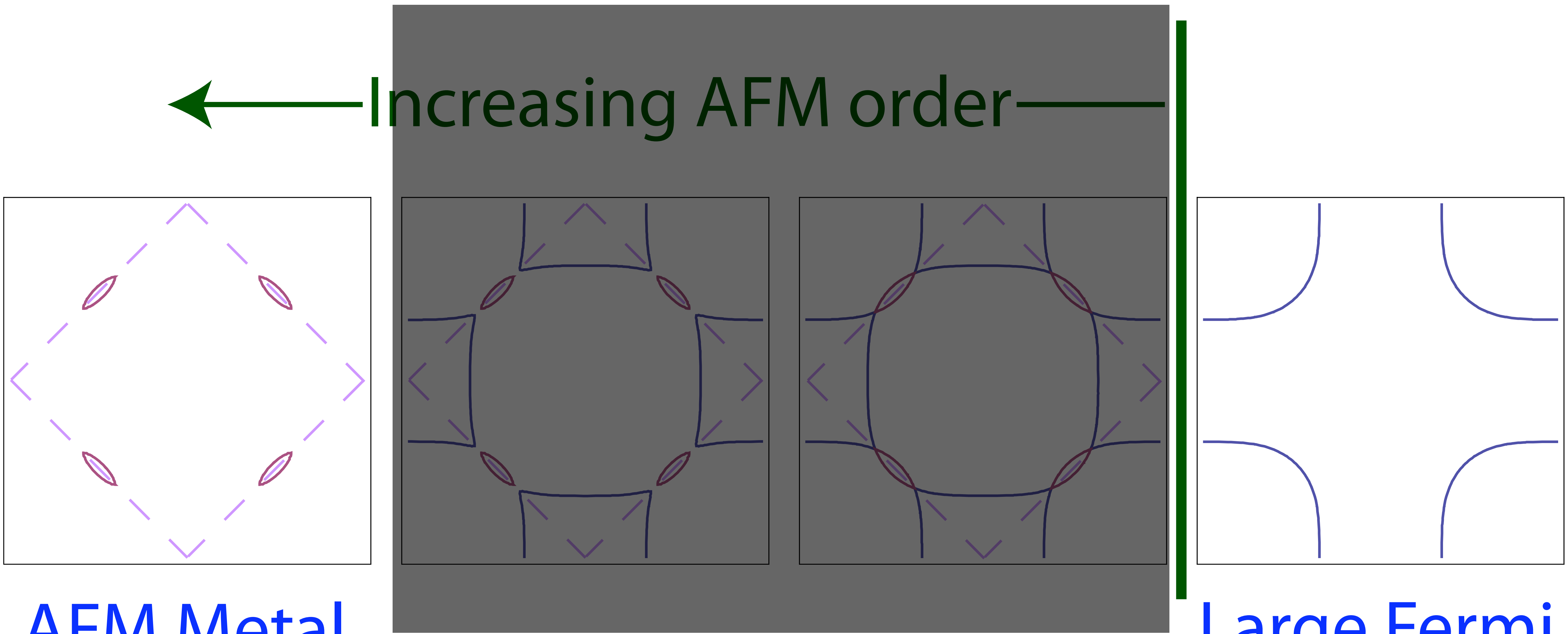
AFM Metal

Large Fermi surface metal

Electron pocket
(seen in high magnetic fields ?)

A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997)

Direct transition between small and large Fermi surfaces ?



AFM Metal

Large Fermi surface metal

Direct transition between small and large Fermi surfaces ?

- Is there a critical theory for such a transition ?

- Does such a critical point/phase control strange metal behavior in the cuprates ?

- Beyond slave particle gauge theories:
help from the AdS/CFT correspondence ?
(Sung-Sik Lee, arXiv:0809.3402)