Vortices in the cuprate superconductors

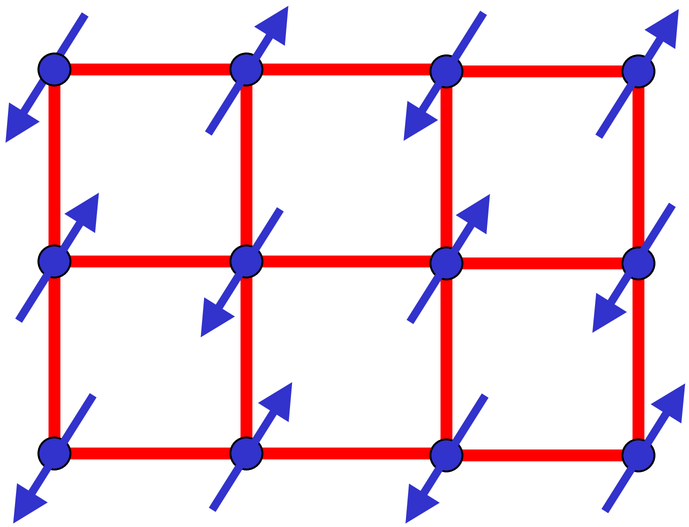
Eugene Demler (Harvard)
Kwon Park
Anatoli Polkovnikov
Subir Sachdev
Matthias Vojta (Augsburg)
Ying Zhang

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

Parent compound of the high temperature superconductors: La_2CuO_4

Mott insulator: square lattice antiferromagnet



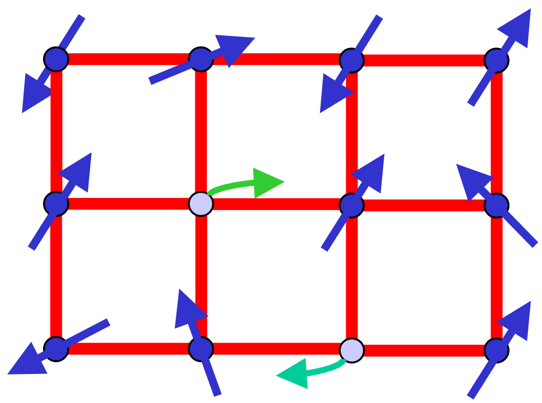
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic Néel order, or a "spin density wave (SDW)"

Néel order parameter: $\vec{\phi} = (-1)^{i_x + i_y} \vec{S}_i$

$$\langle \vec{\phi} \rangle \neq 0 \quad ; \quad \langle \vec{S}_i \rangle \neq 0$$

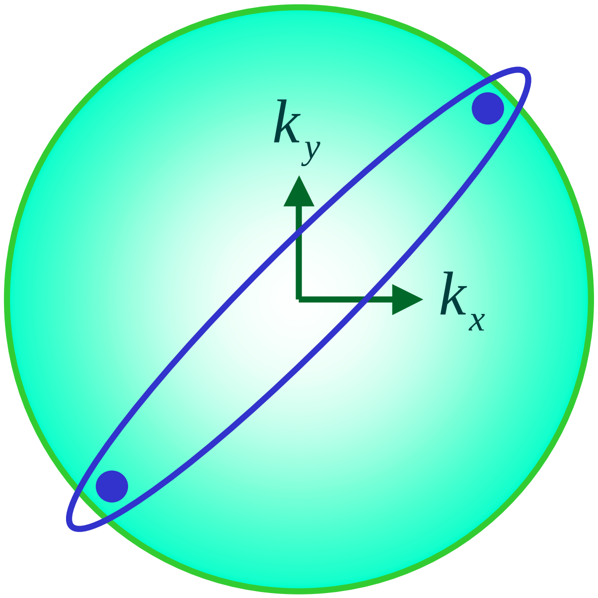
Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$



Exhibits superconductivity below a high critical temperature T_c

Superconductivity in a doped Mott insulator

BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid



Pair wavefunction

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

Observed *low* temperature properties of the cuprate superconductors appear to be qualitatively similar to those predicted by BCS theory.

Many experiments above T_c are not described quantitatively by BCS theory: this is probably due to strong-coupling "crossover" effects, and I will not discuss this issue further.

Superconductivity in a doped Mott insulator

$\Updownarrow$?

BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid

Quantum numbers of ground state and low energy quasiparticles are the same, but characteristics of the Mott insulator are revealed near the vortices (and near impurities).

S. Sachdev, *Phys. Rev*. B **45**, 389 (1992); K. Park and S. Sachdev *Phys. Rev*. B **64**, 184510 (2001).

STM measurement of J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Outline

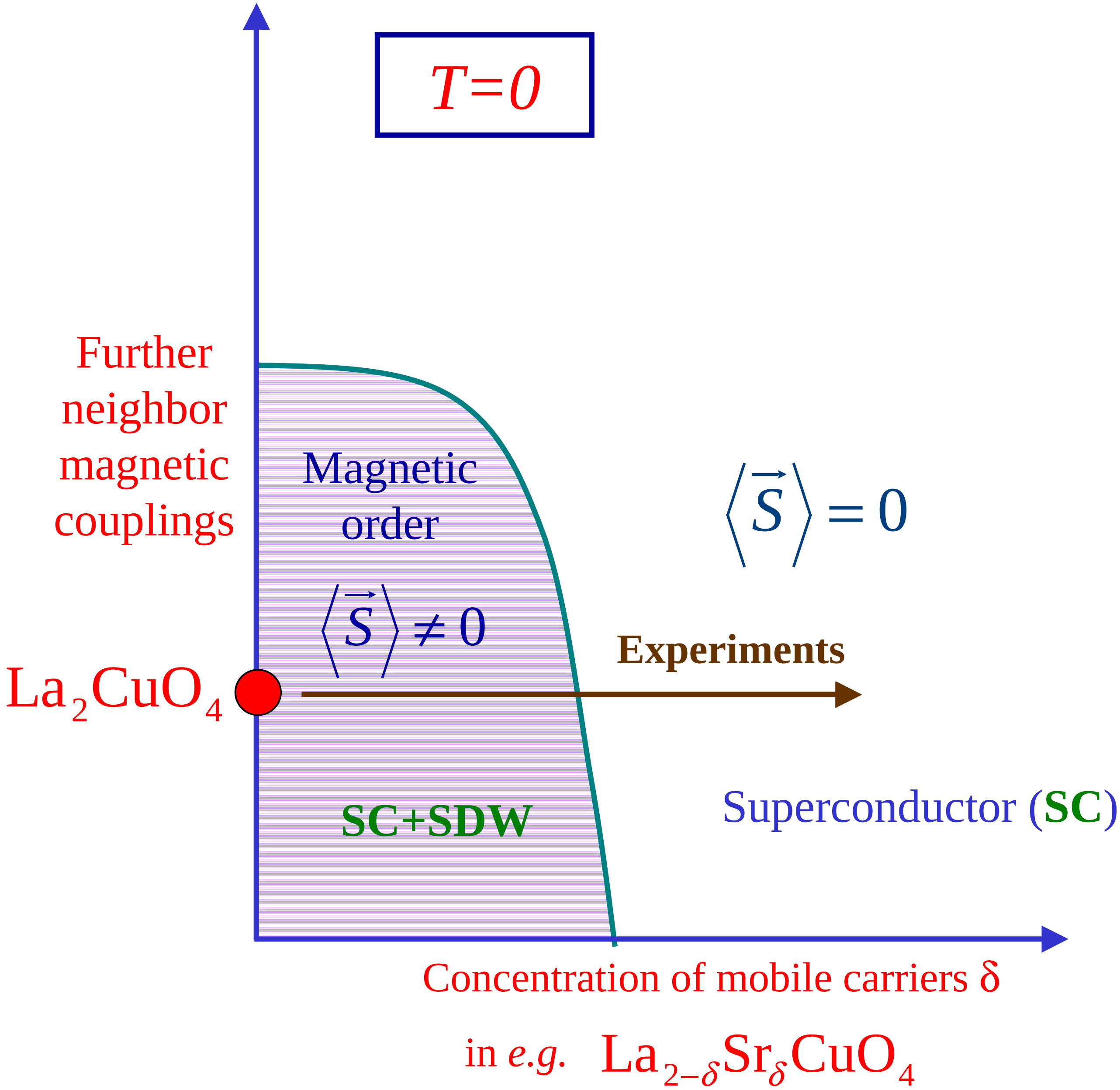
I. Mott insulators – with and without magnetic order.

II. Theory of doped paramagnetic Mott insulators.

III. Experiments on
   A. Vortices
   B. Zn/Li impurities

IV. Competition between co-existing magnetism and superconductivity at low carrier concentrations: theory and neutron scattering experiments.

V. Conclusions

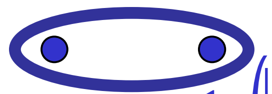


P.W. Anderson, *Science*, **235**, 1196 (1987).
S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev*. B **49**, 11919 (1994)

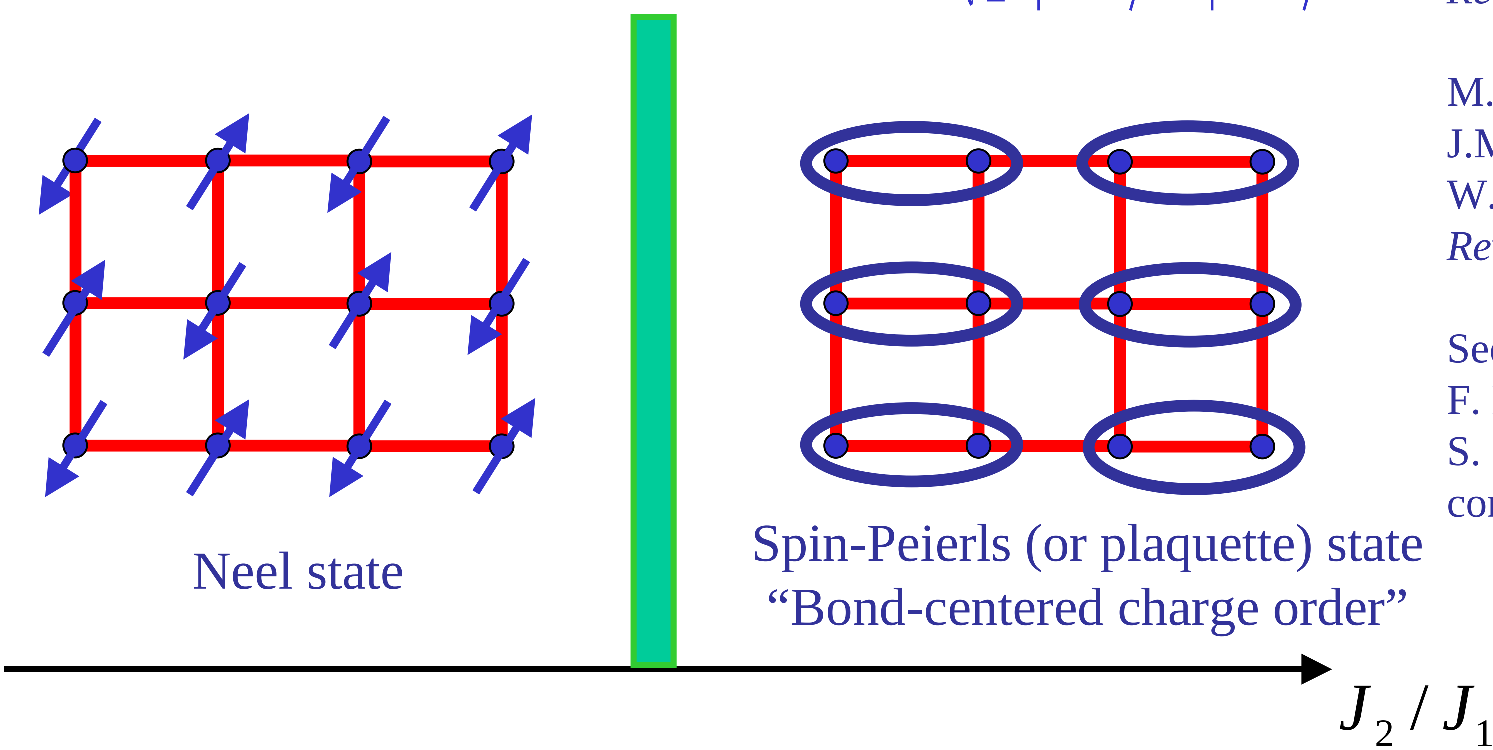
I. Magnetic ordering transitions in the Mott insulator

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions (say)

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$



$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \Big)$$



N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

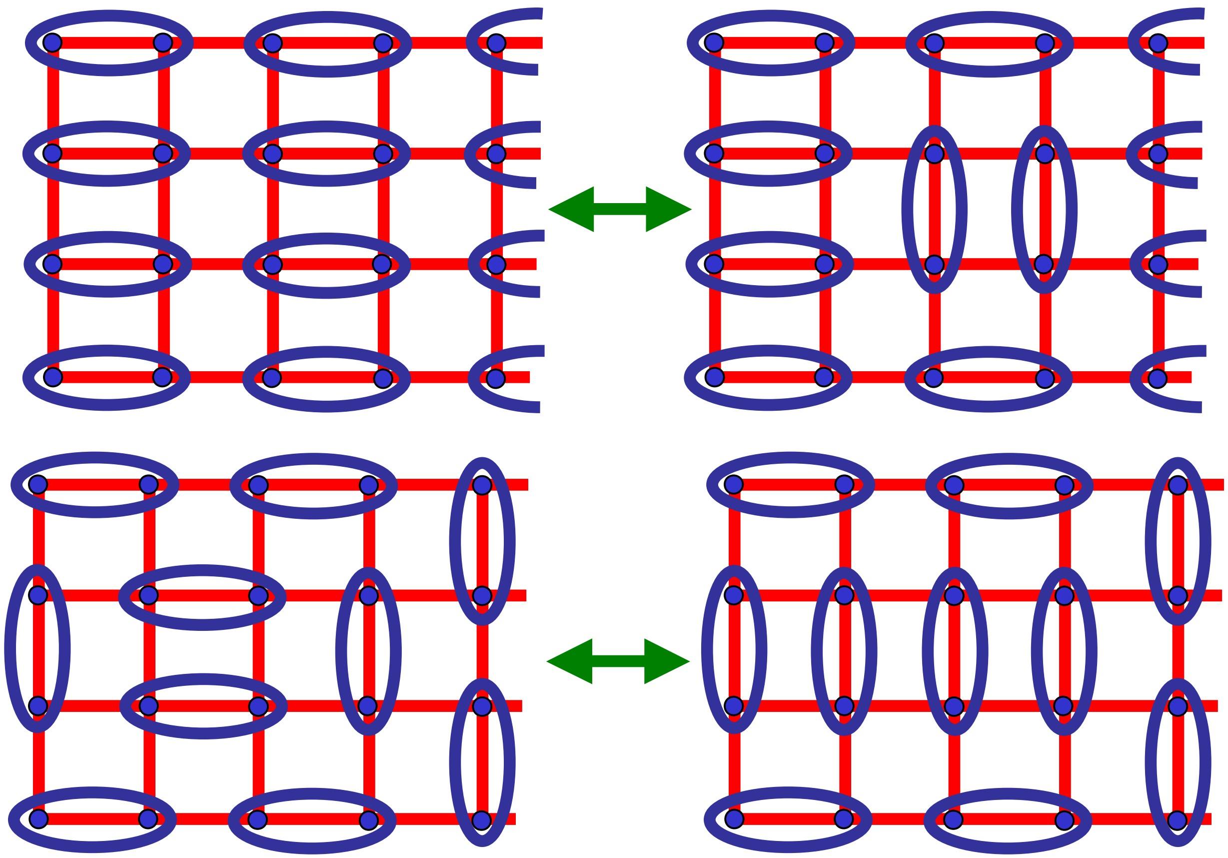
O. P. Sushkov, J. Oitmaa, and Z. Weihong, *Phys. Rev*. B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, *Phys. Rev*. B **62**, 14844 (2000).

See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .

Quantum dimer model –

D. Rokhsar and S. Kivelson *Phys. Rev. Lett.* **61**, 2376 (1988)



Quantum "entropic" effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

These effects lead to a broken square lattice symmetry near the transition to the Neel state.

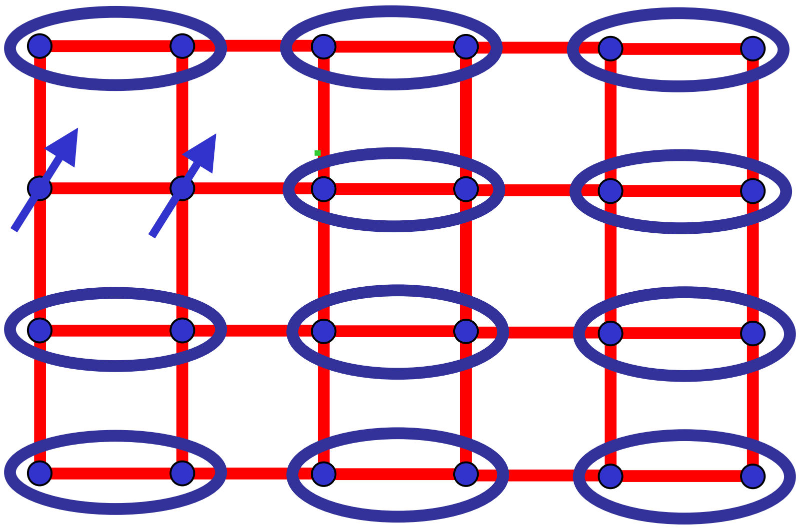
N. Read and S. Sachdev *Phys. Rev. B* **42**, 4568 (1990).

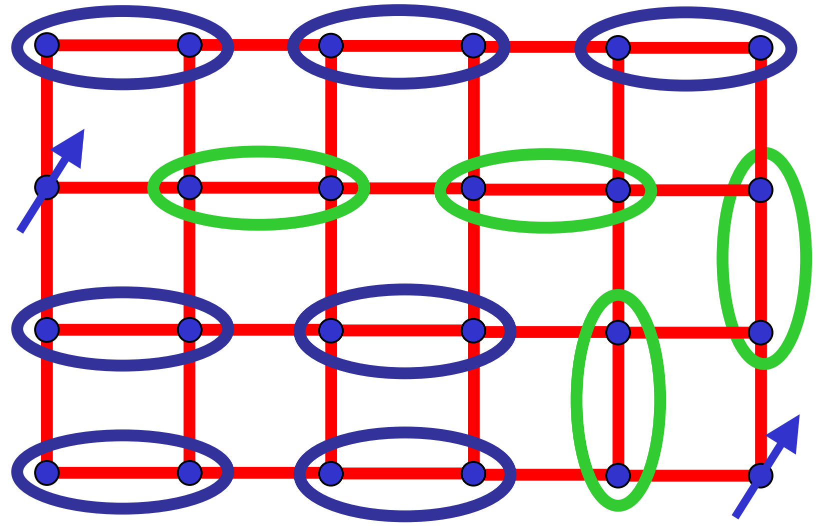
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

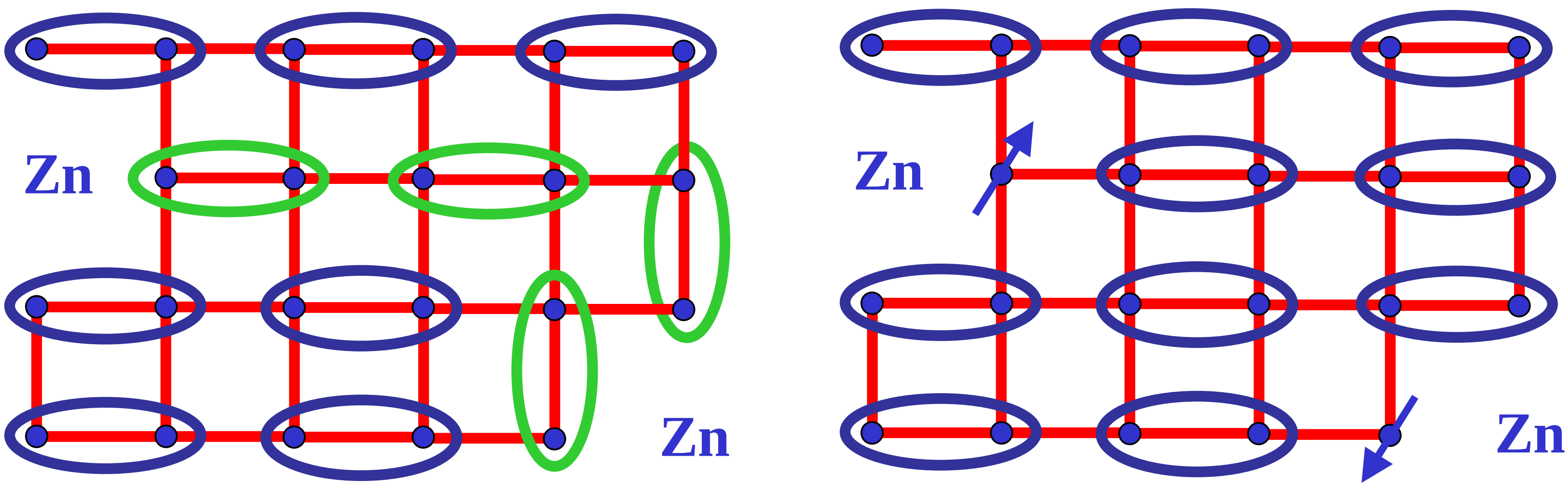
$\Delta \longrightarrow$ Spin gap





S=1/2 spinons are ***confined*** by a linear potential.

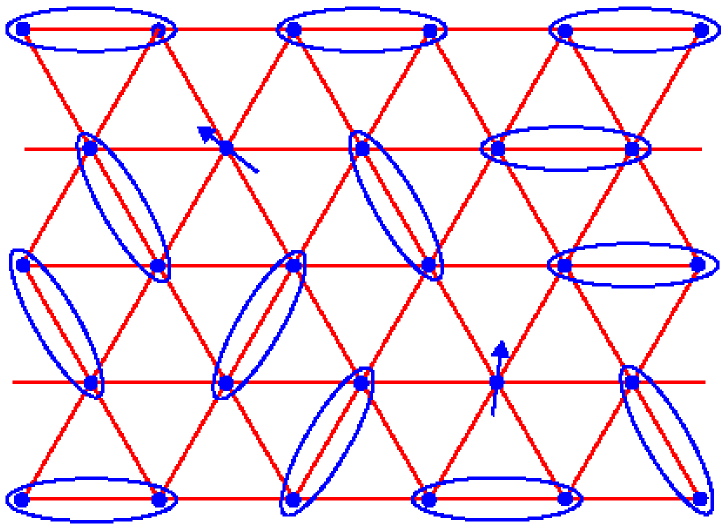
Effect of static non-magnetic impurities (Zn or Li)



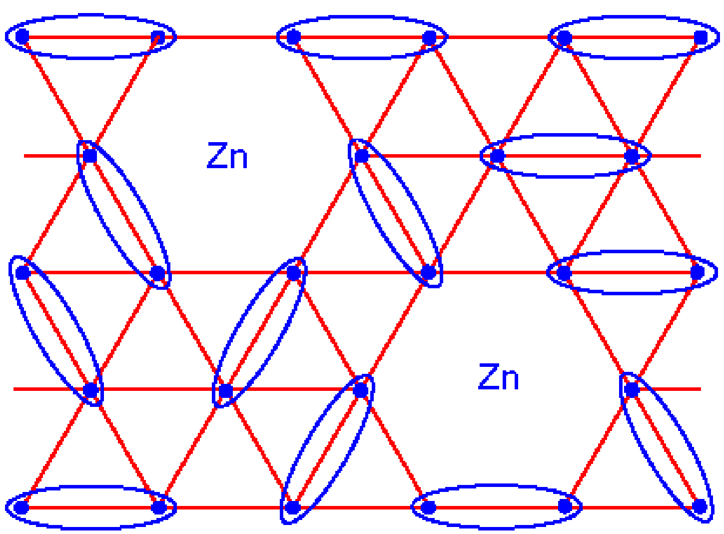
Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \rightarrow 0) = \frac{S(S+1)}{3k_B T}$$

Impurities in paramagnets with spinon deconfinement



Spinons are deconfined



Free S=1/2 moments need not be present near the impurities

"Spin Liquid"

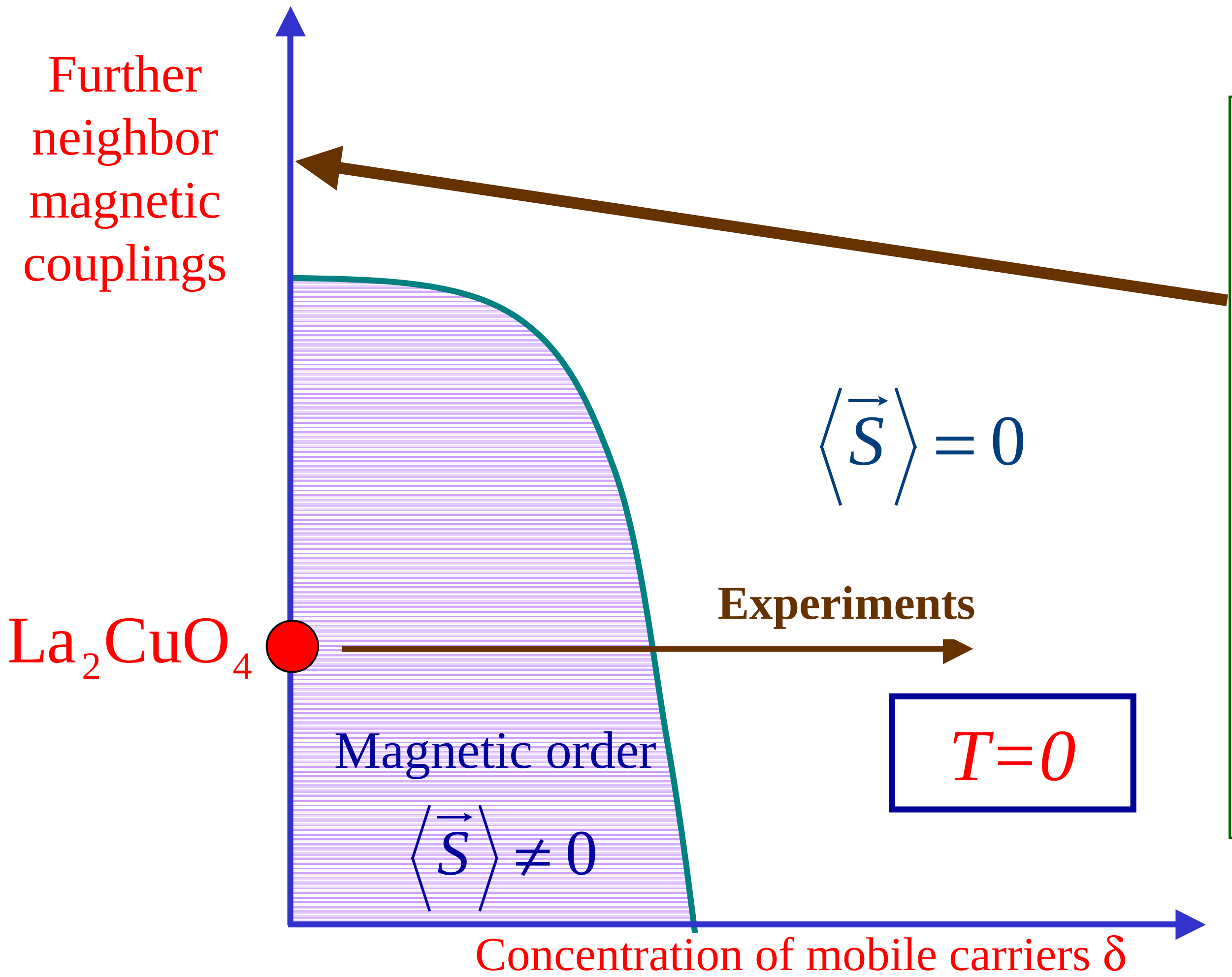
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

S. Sachdev, *Phys. Rev*. B **45**, 12377 (1992).

G. Misguich and C. Lhuillier, cond-mat/0002170.

R. Moessner and S.L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001).

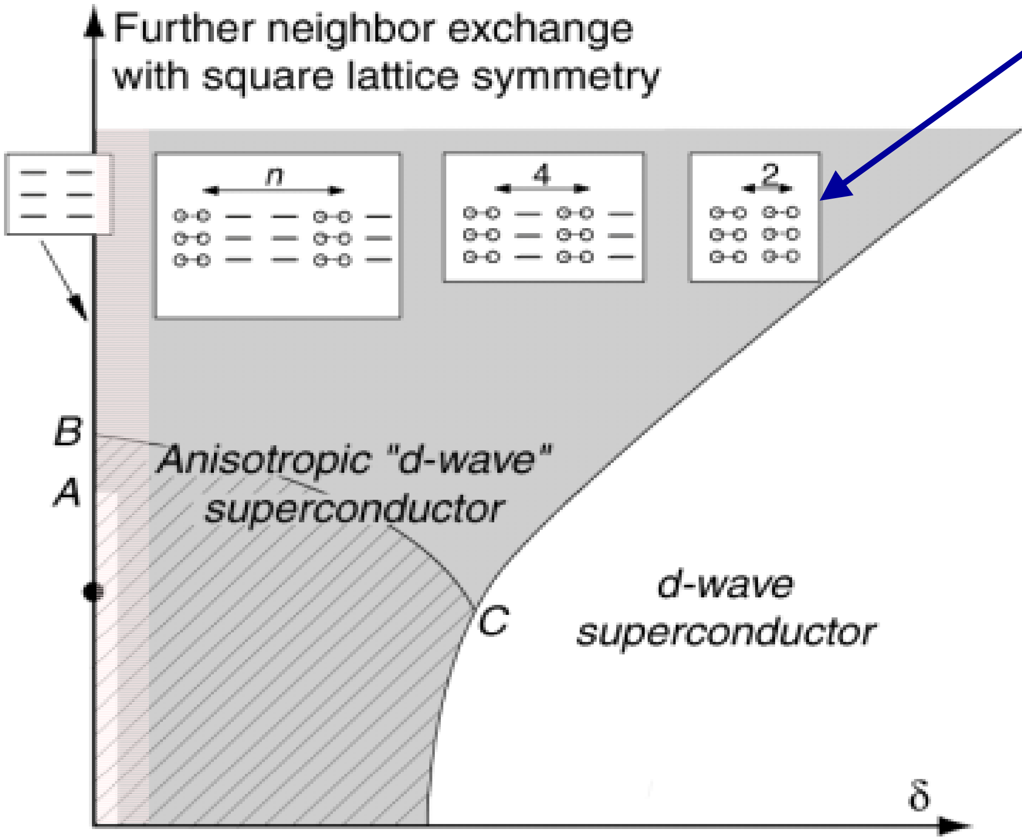
Framework for spin/charge order in cuprate superconductors



Confined, paramagnetic Mott insulator has

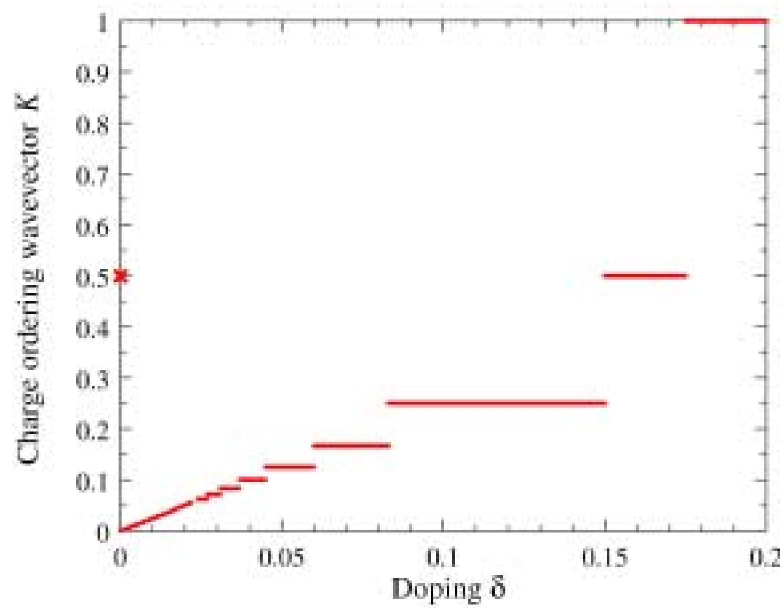
1. Stable S=1 spin exciton.
2. Broken translational symmetry:- bond-centered charge order.
3. S=1/2 moments near non-magnetic impurities

II. Doping the Mott insulator



"Large *N*" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).

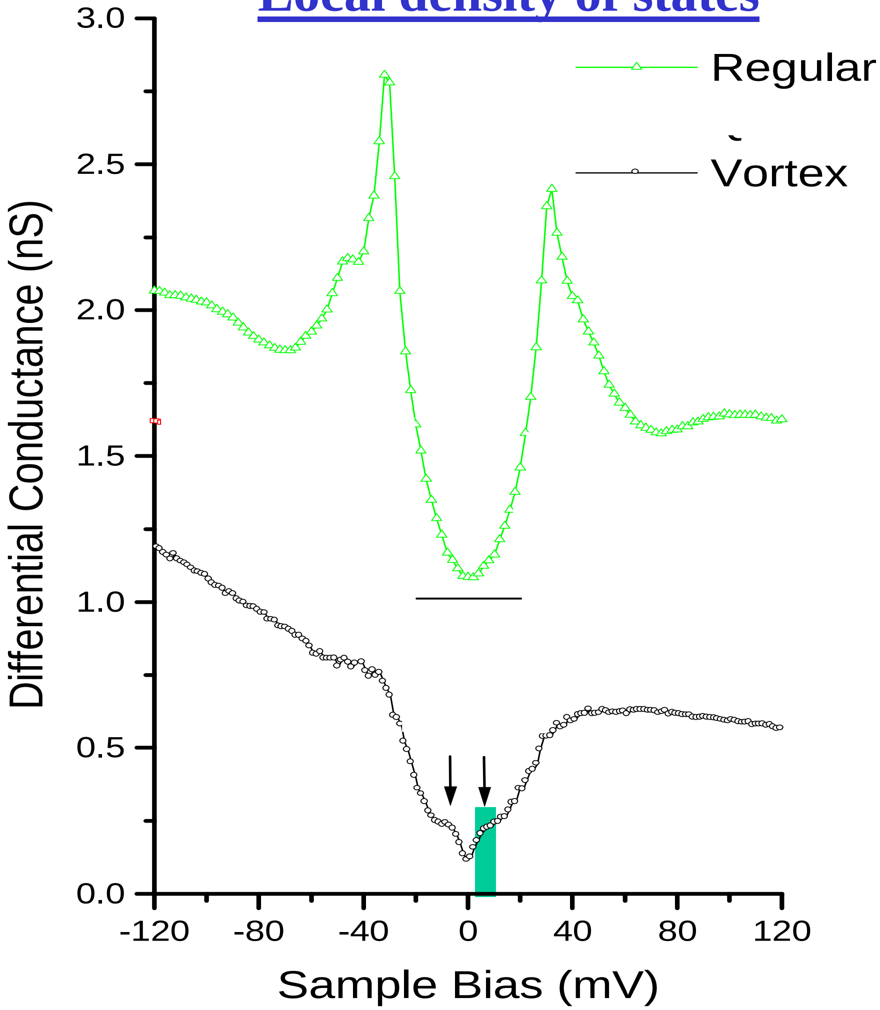


See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

III.A STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

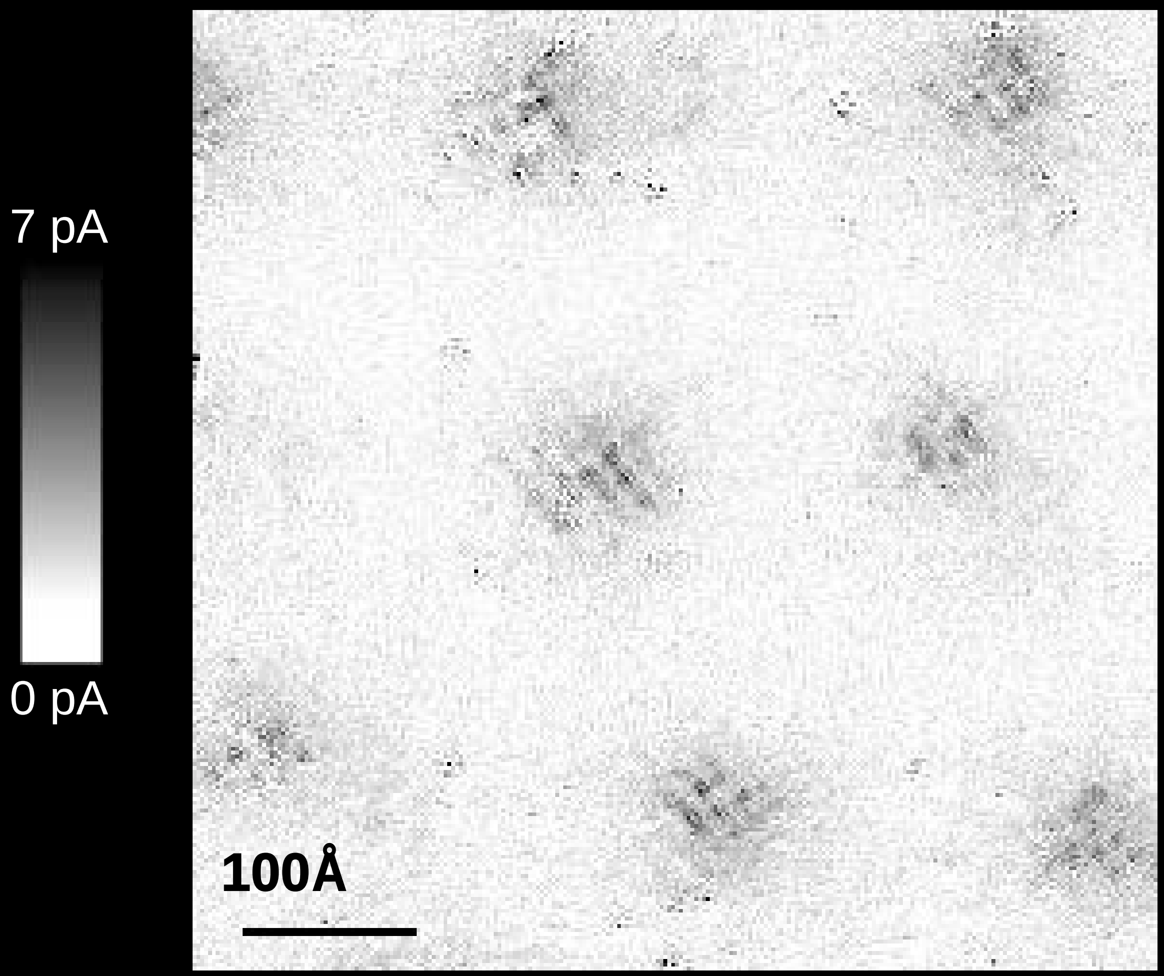


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

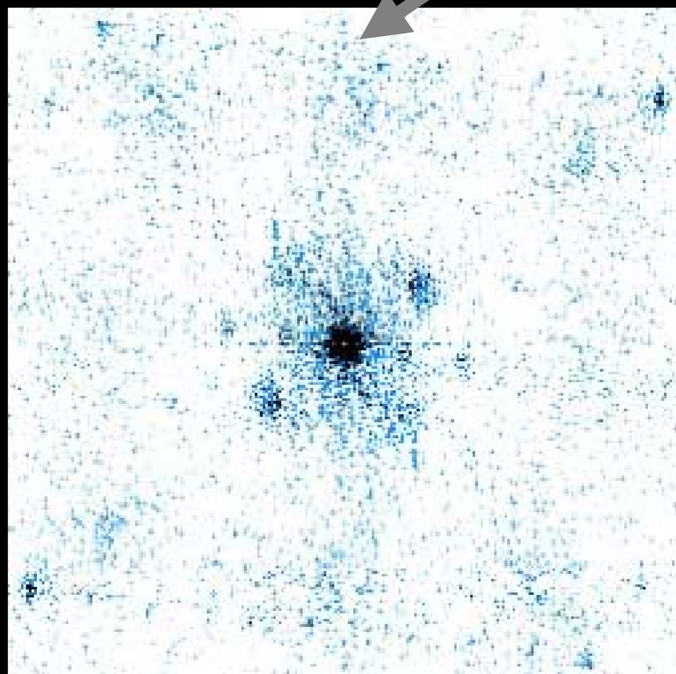
Vortex-induced LDOS integrated from 1meV to 12meV

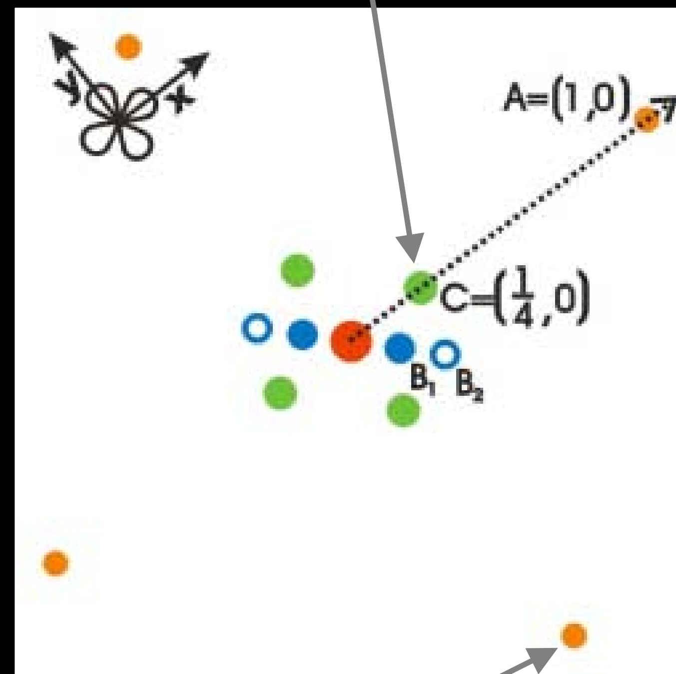


J. Hoffman *et al. Science*, **295**, 466 (2002).

Fourier Transform of Vortex-Induced LDOS map

K-space locations of vortex induced LDOS





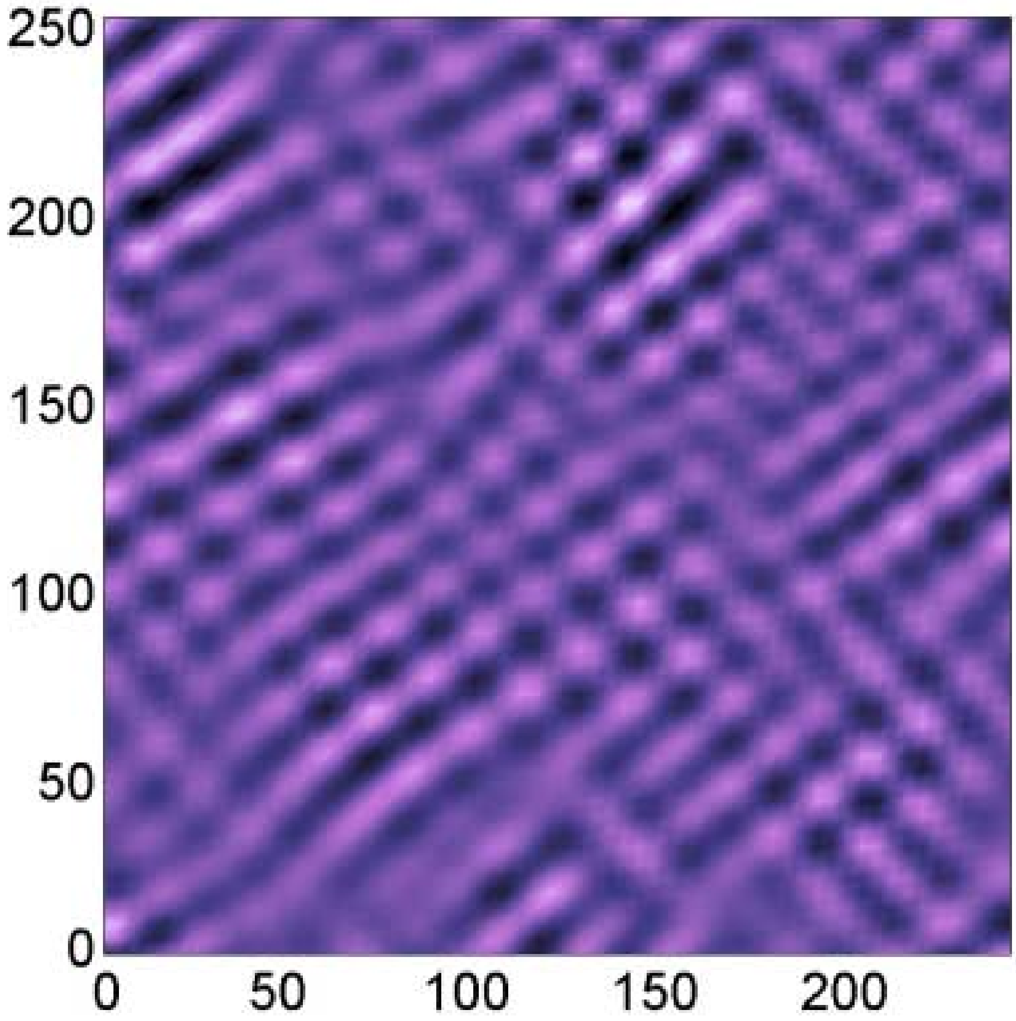
K-space locations of Bi and Cu atoms

Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

J. Hoffman *et al. Science*, **295**, 466 (2002).

Two recent experiments

1. STM image of pinned charge order in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field



Charge order period = 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

2. Observation of static charge order in $YBa_2Cu_3O_{6.35}$
(spin correlations remain dynamic)

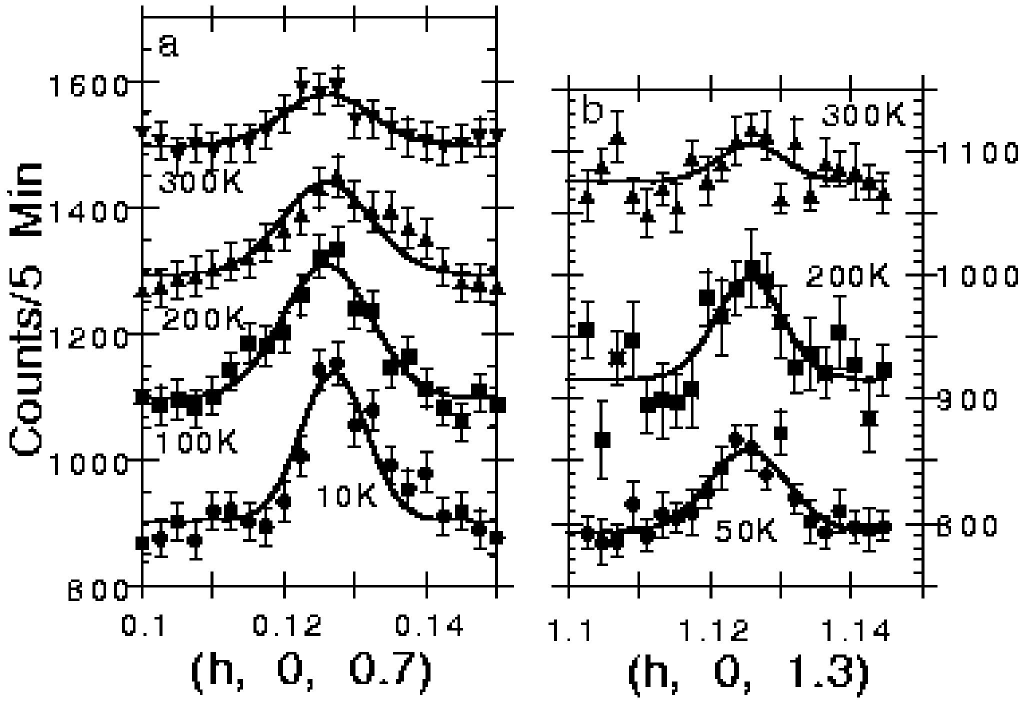
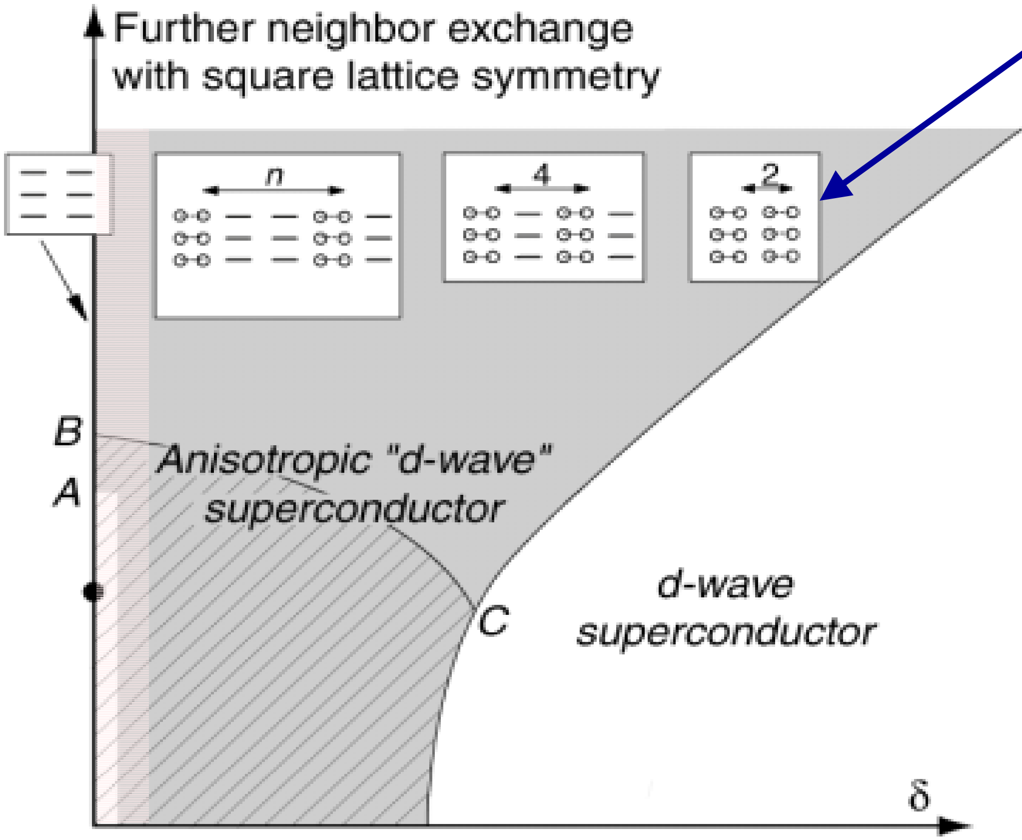


FIG. 1. Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) r.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

Charge order period = 8 lattice spacings

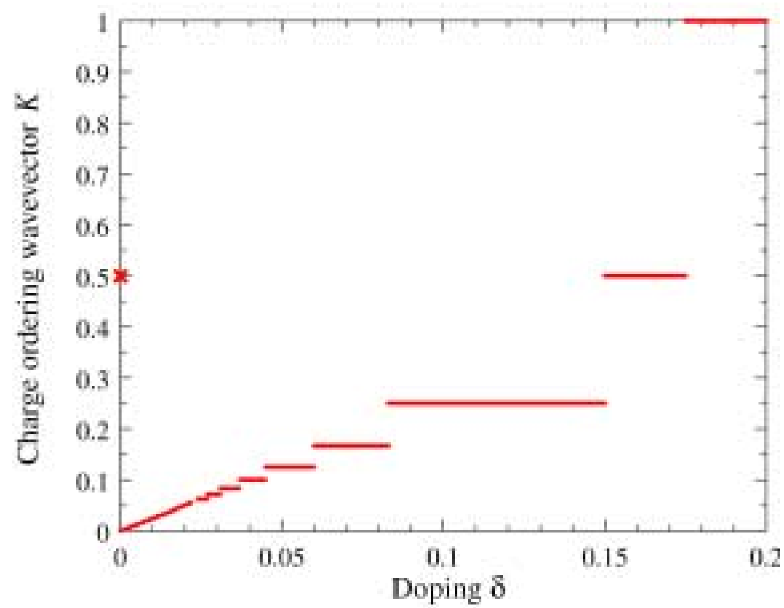
H. A. Mook, Pengcheng Dai, and F. Dogan
Phys. Rev. Lett. **88**, 097004 (2002).

II. Doping the Mott insulator



"Large *N*" theory in region with preserved spin rotation symmetry

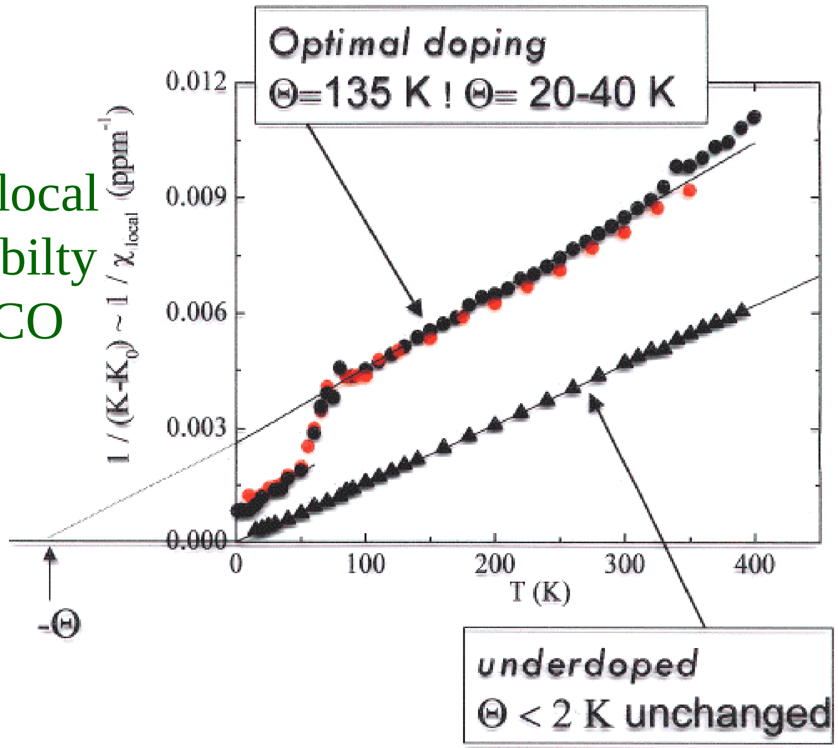
S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).

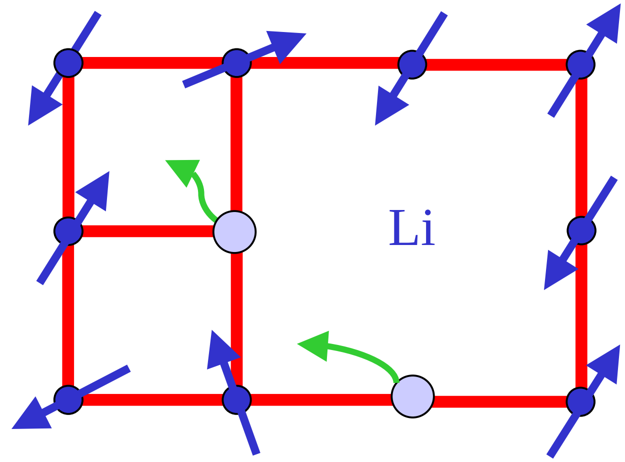


See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

III.B Spatially resolved NMR of Zn/Li impurities in the superconducting state

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

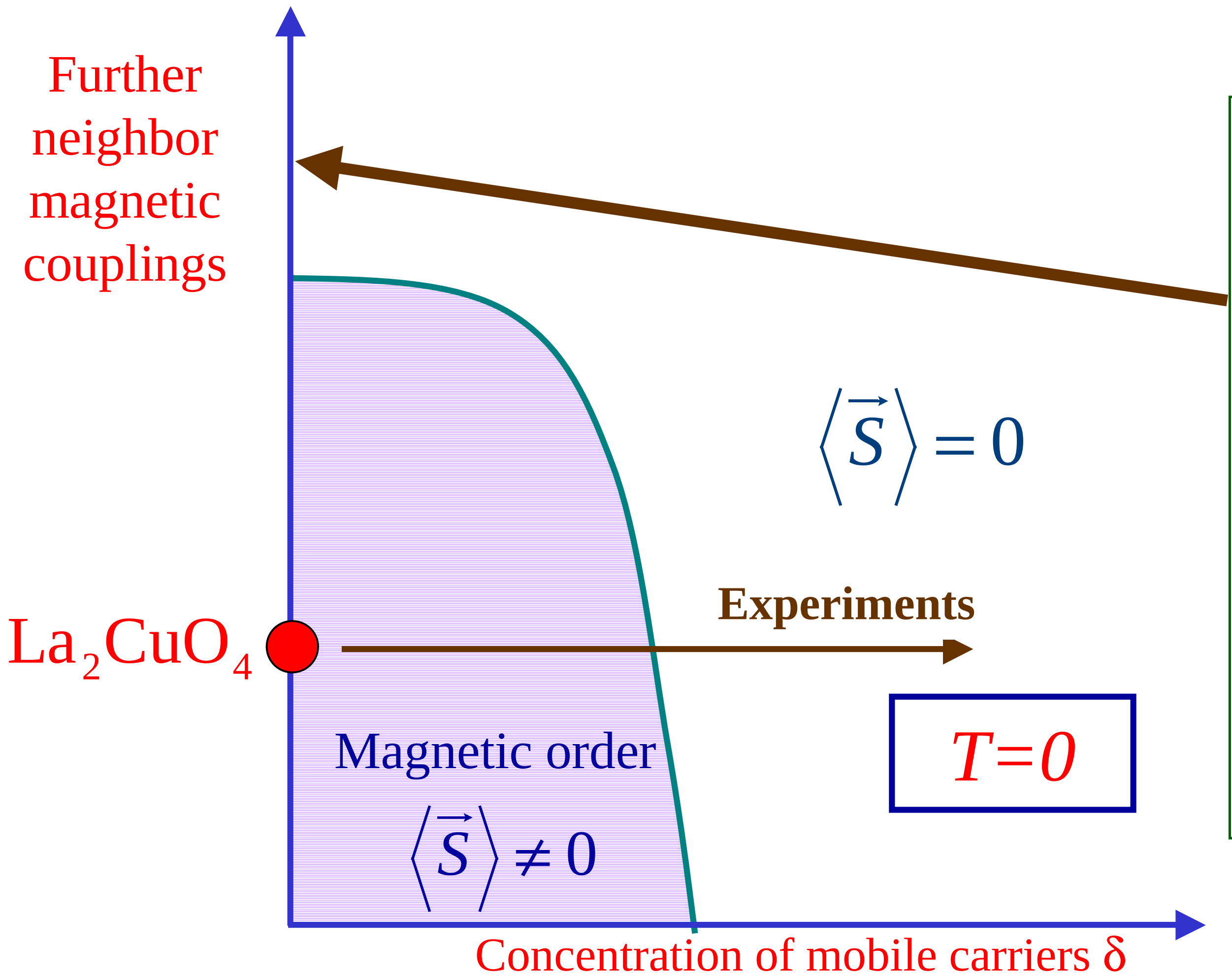
J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, *Phys. Rev. Lett*. **86**, 4116 (2001).

Measured $\chi_{impurity}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

This behavior does not emerge out of BCS theory.

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, *Physica* C **168**, 370 (1990).

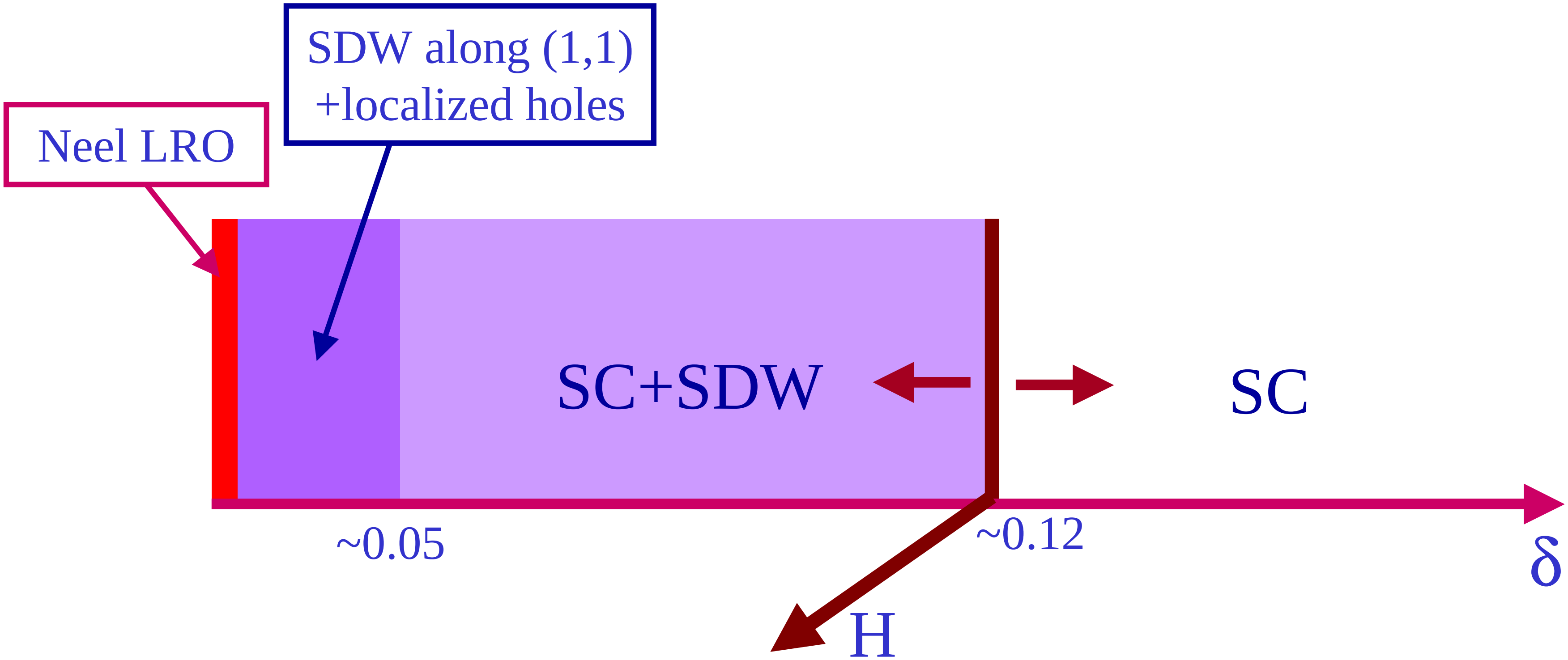
Framework for spin/charge order in cuprate superconductors



Confined, paramagnetic Mott insulator has

1. Stable *S*=1 spin exciton.
2. Broken translational symmetry:- bond-centered charge order.
3. *S*=1/2 moments near non-magnetic impurities

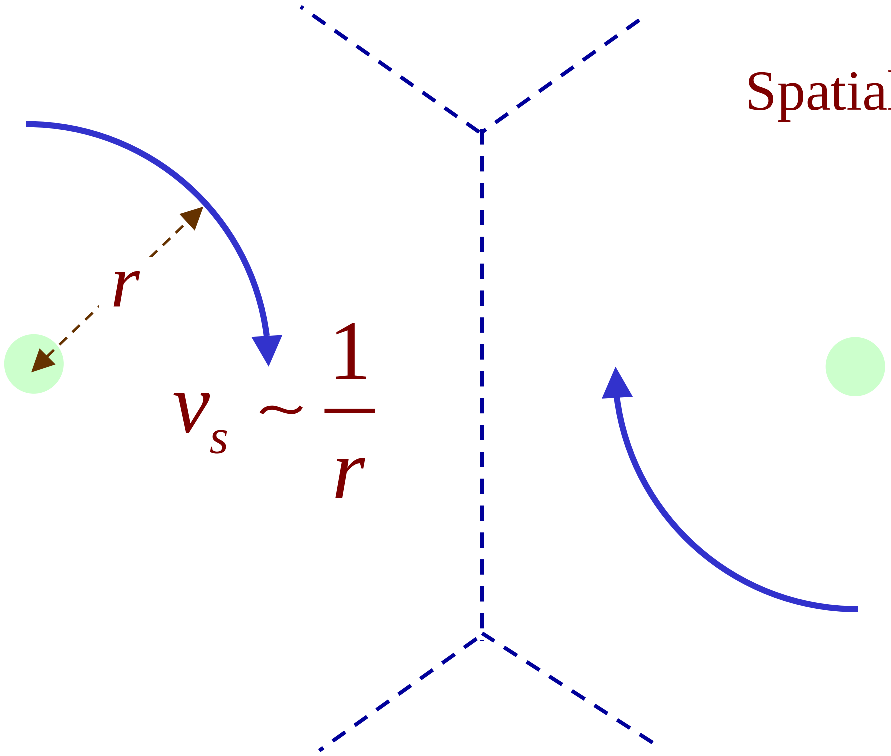
IV. Competition between magnetism and superconductivity as a function of hole density



Theory for a system with strong interactions:
describe SC and SC+SDW phases by expanding in the deviation from the quantum critical point between them.

B. Keimer *et al*. *Phys. Rev.* B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev.* B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev.* B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

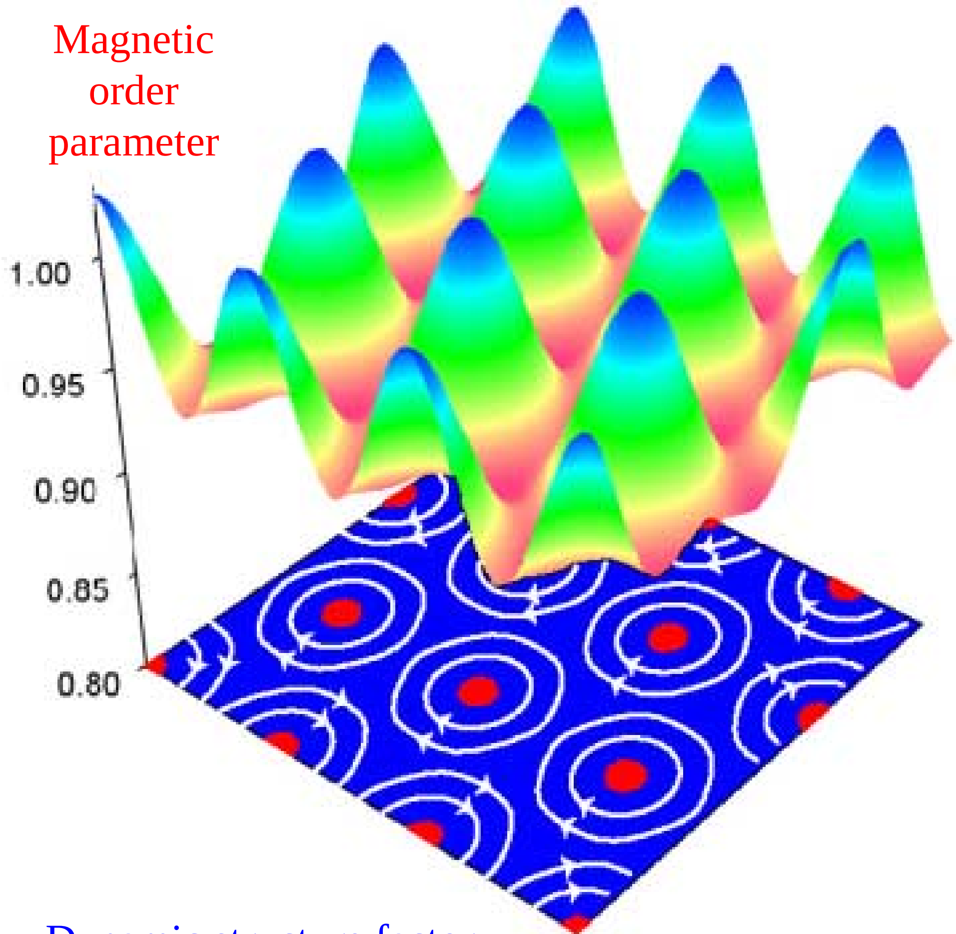
$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Coupling determining spin excitation energy, s,

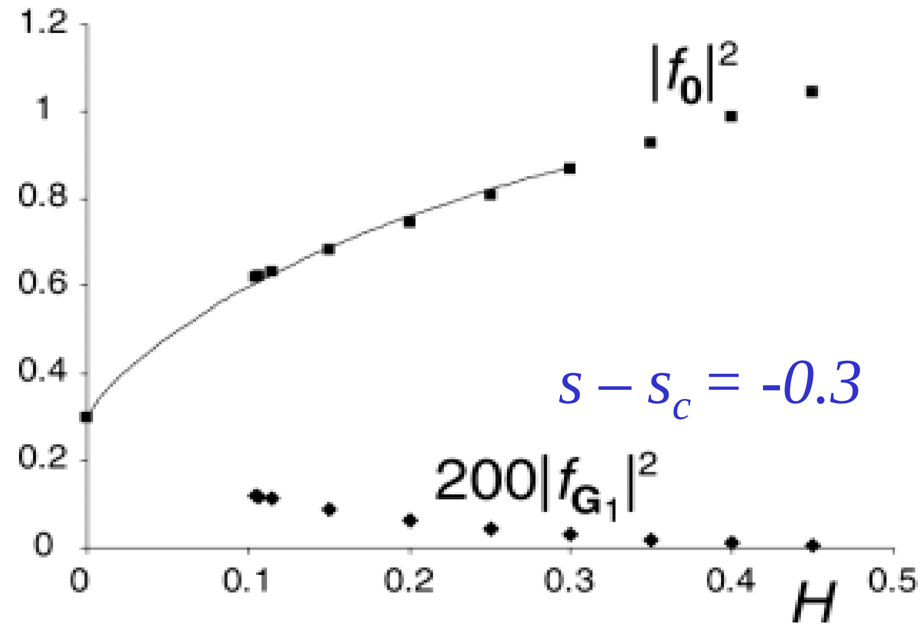
replaced by $s_{eff}\left(H\right) = s - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$

Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



$$\delta\left|f_0\right|^2 \propto H\ln(1/H)$$



$s - s_c = -0.3$

Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3\,\delta(\omega)\sum_{\boldsymbol{G}}\left|f_{\boldsymbol{G}}\right|^2\delta(\boldsymbol{k}-\boldsymbol{G}) + \cdots$$

$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

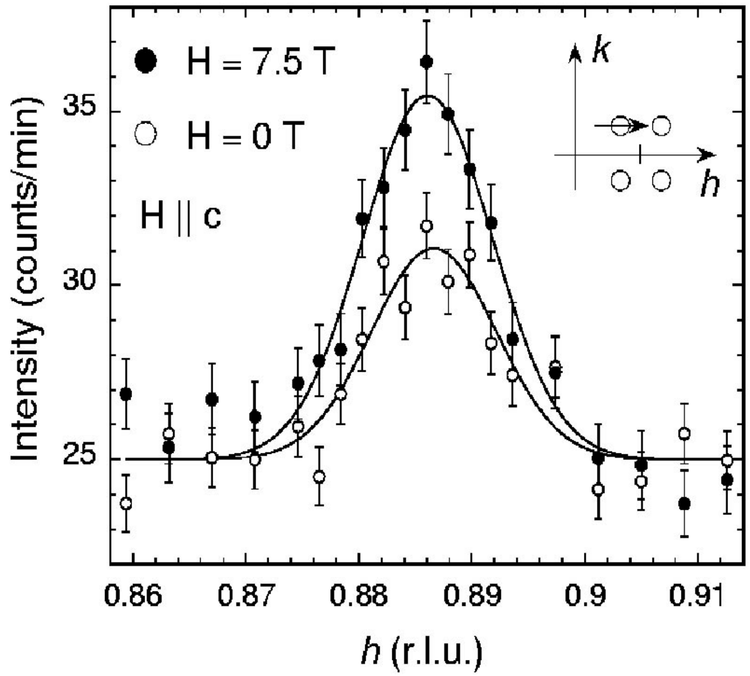
$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

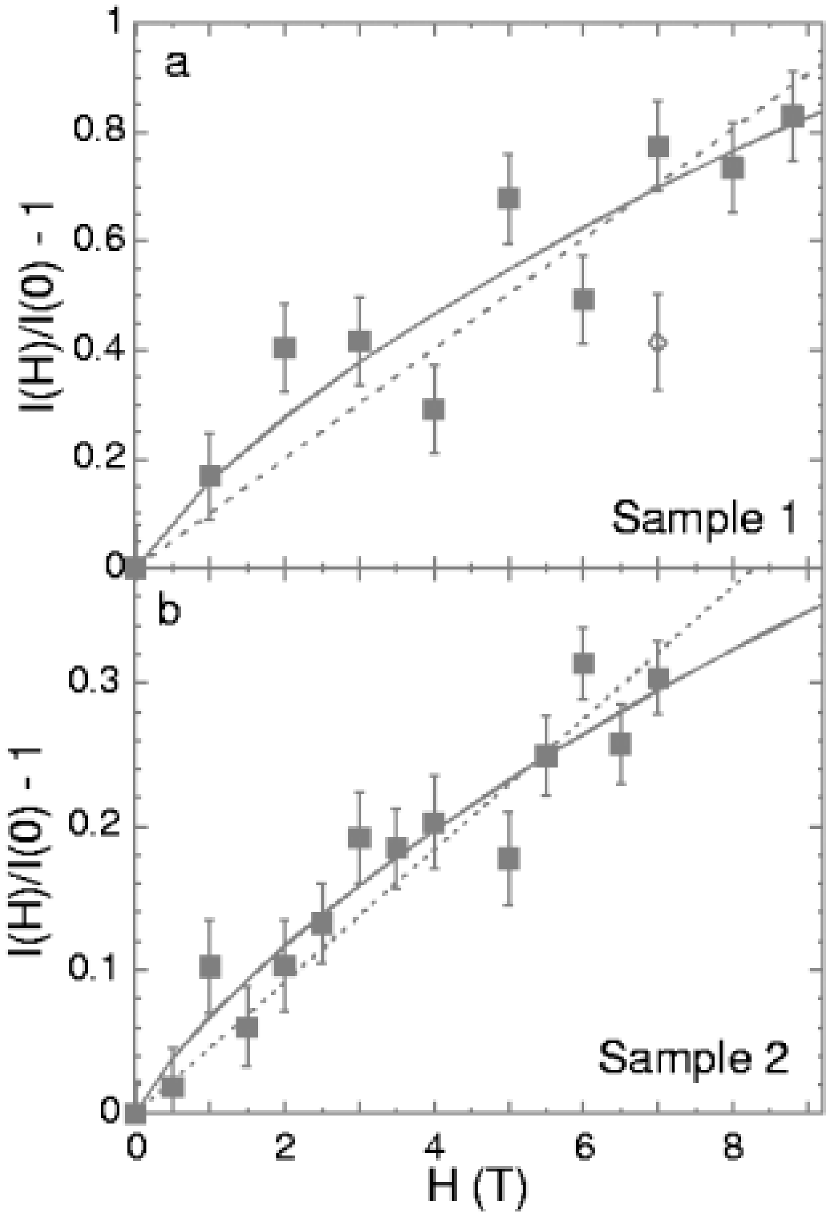
D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) discussed static magnetism in the vortex cores.

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, cond-mat/0112505.





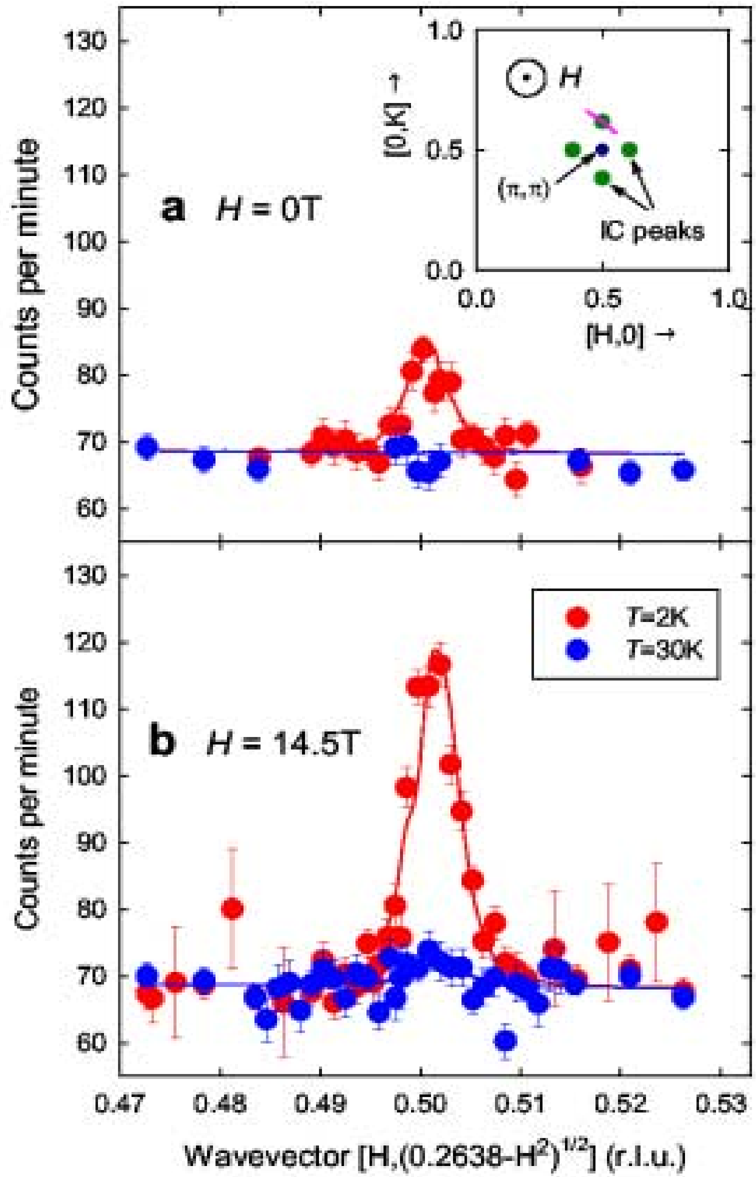
Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

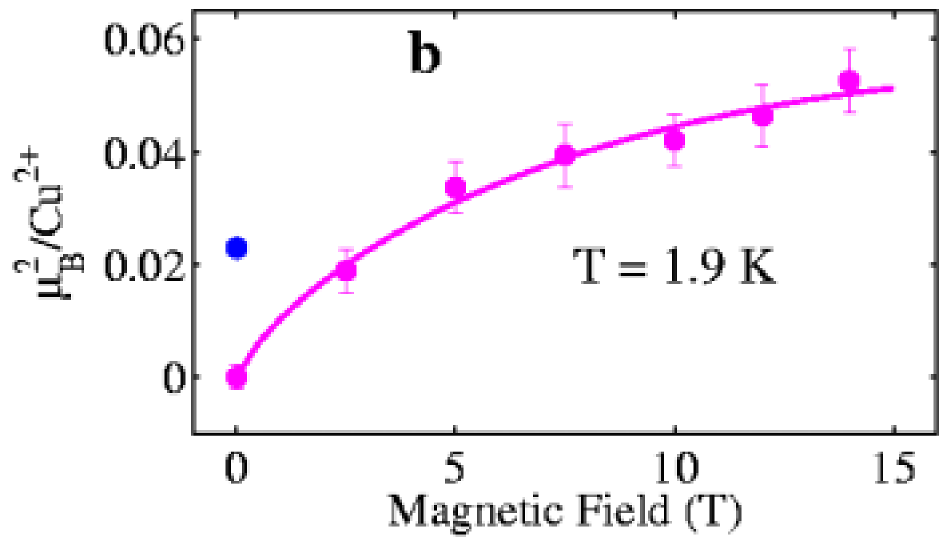
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

B. Lake, G. Aeppli, *et al.*, *Nature*, **415**, 299 (2002).





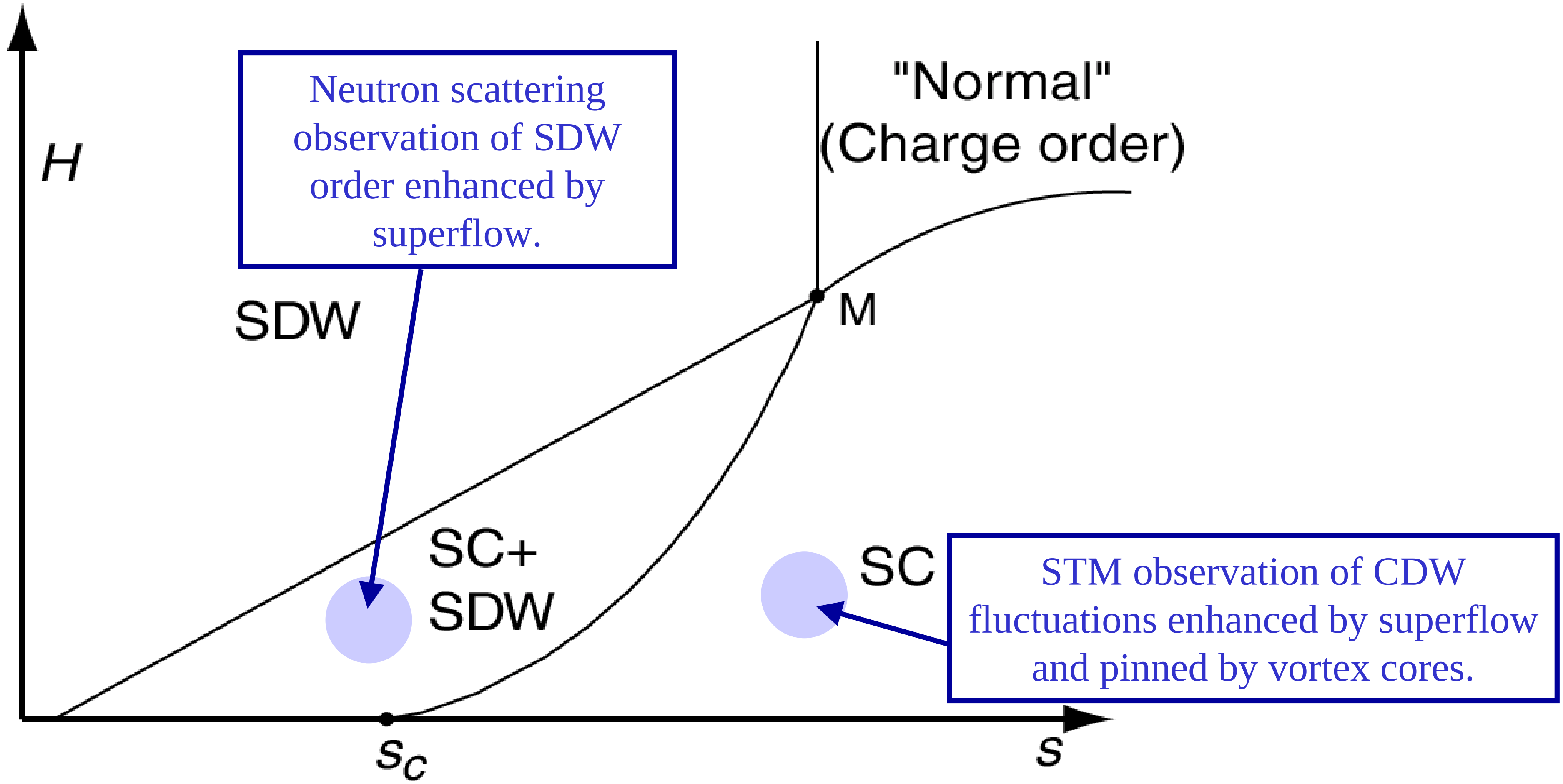
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

T=0

Main results



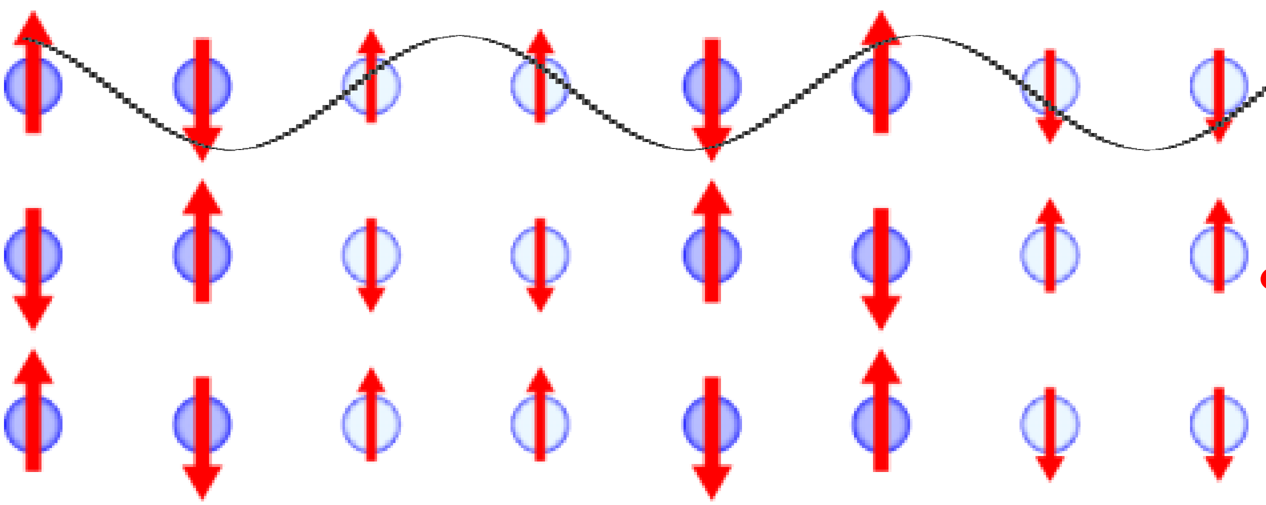
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Theory of SC+SDW to SC quantum transition

Spin density wave order parameter for general ordering wavevector

$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a complex field $\left(\text{except for } \boldsymbol{K}=(\pi,\pi) \text{ when } e^{i\boldsymbol{K}\cdot\boldsymbol{r}} = (-1)^{r_x+r_y}\right)$



Wavevector $\boldsymbol{K}=(3\pi/4,\pi)$

Exciton wavefunction $\Phi_\alpha(\boldsymbol{r})$ describes *envelope* of this order. Phase of $\Phi_\alpha(\boldsymbol{r})$ represents sliding degree of freedom

Associated "charge" density wave order

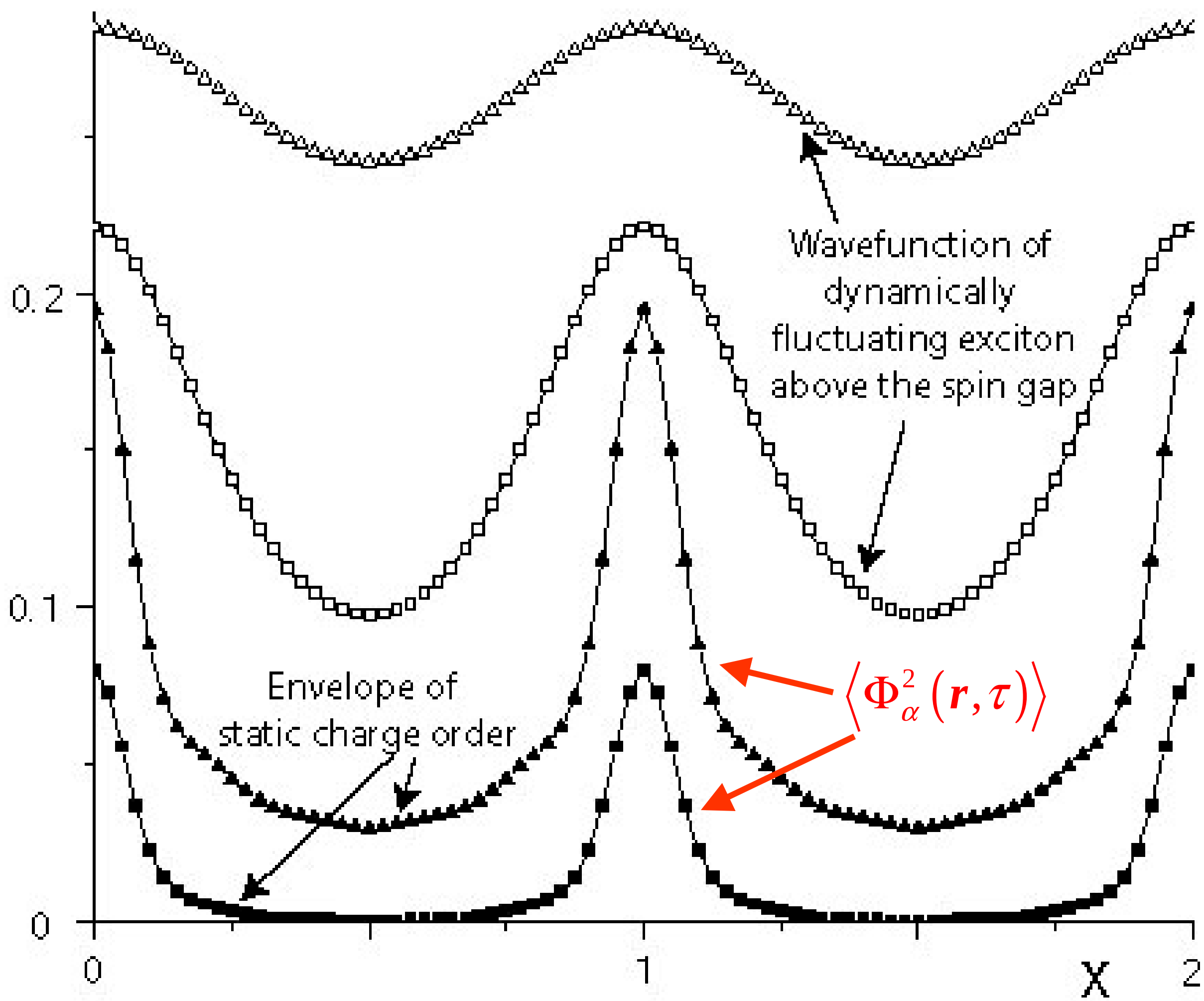
$$\delta\rho(\boldsymbol{r}) \propto S_\alpha^2(\boldsymbol{r}) = \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

Pinning of CDW order by vortex cores in SC phase

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

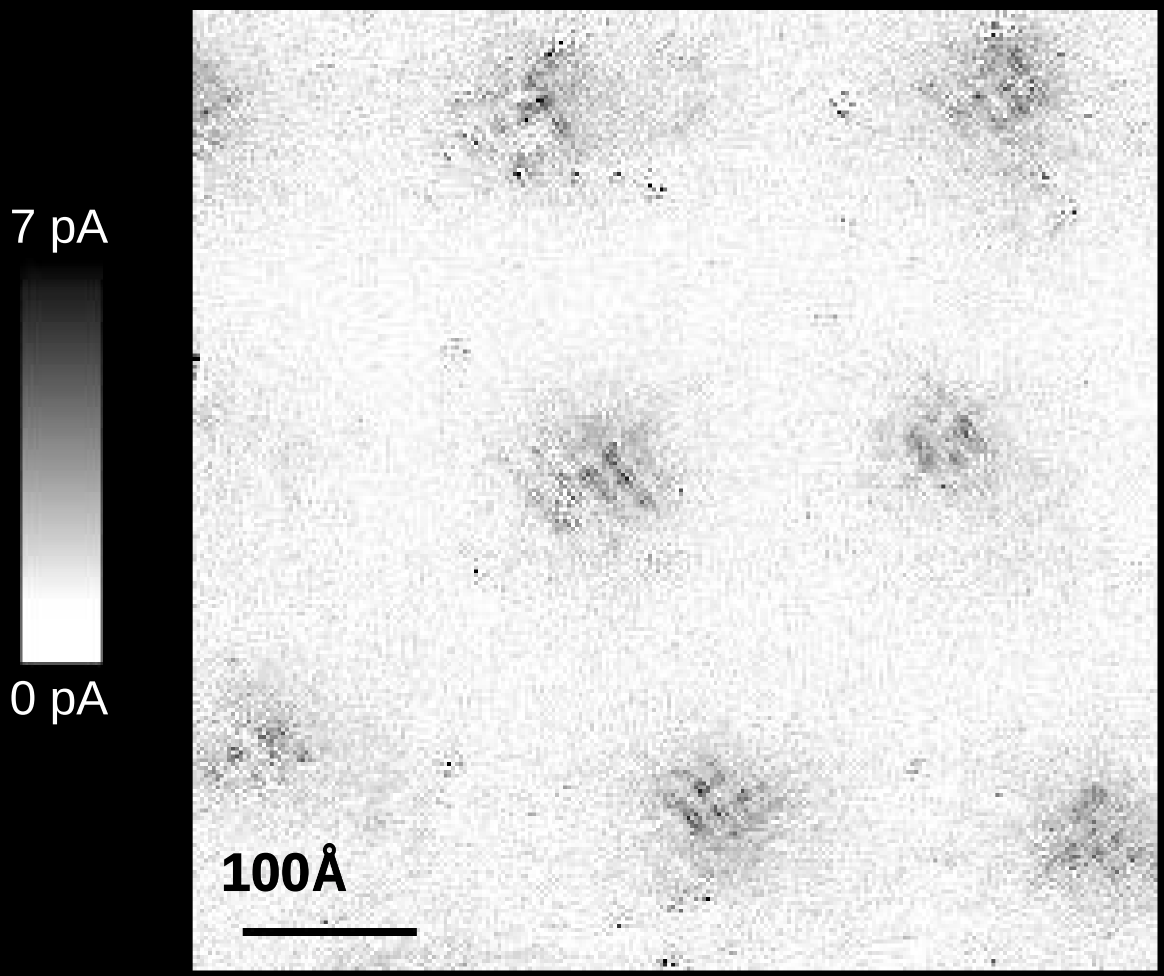
$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\,\Phi_\alpha^*(\boldsymbol{r}_v,\tau_1)\right\rangle^2$$



□ → low magnetic field

△ → high magnetic field near the boundary to the SC+SDW phase

Vortex-induced LDOS integrated from 1meV to 12meV



J. Hoffman *et al.* **295**, 466 (2002).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has charge-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?