Competing orders: beyond Landau-Ginzburg-Wilson theory

Colloquium article in *Reviews of Modern Physics* **75**, 913 (2003)

Leon Balents (UCSB)
Lorenz Bartosch (Yale)
Anton Burkov (UCSB)
Eugene Demler (Harvard)
Matthew Fisher (UCSB)
Anatoli Polkovnikov (Harvard)
Krishnendu Sengupta (Yale)
T. Senthil (MIT)
Ashvin Vishwanath (MIT)
Matthias Vojta (Karlsruhe)

Talk online: Google Sachdev

Putting competing orders in their place near the Mott transition

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Possible origins of the pseudogap in the cuprate superconductors:

- "Phase fluctuations", "preformed pairs"

  Complex order parameter: Ψ_{sc}

  $\Psi_{sc} \rightarrow \Psi_{sc} e^{i\theta}$ symmetry encodes number conservation

- "Charge/valence-bond/pair-density/stripe" order

  Order parameters: $\rho(\boldsymbol{r}) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$

  (density ρ represents any observable invariant under spin rotations, time-reversal, and spatial inversion)

  $\rho_{\boldsymbol{Q}} \rightarrow \rho_{\boldsymbol{Q}} e^{i\vartheta}$ encodes space group symmetry

- "Spin liquid"...............

Order parameters are not independent

Ginzburg-Landau-Wilson approach to competing order parameters (combine order parameters into a "superspin"):

$$F = F_{sc}\left[\Psi_{sc}\right] + F_{\text{charge}}\left[\rho_{\mathbf{Q}}\right] + F_{\text{int}}$$

$$F_{sc}\left[\Psi_{sc}\right] = r_1\left|\Psi_{sc}\right|^2 + u_1\left|\Psi_{sc}\right|^4 + \cdots$$

$$F_{\text{charge}}\left[\rho_{\mathbf{Q}}\right] = r_2\left|\rho_{\mathbf{Q}}\right|^2 + u_2\left|\rho_{\mathbf{Q}}\right|^4 + \cdots$$

$$F_{\text{int}} = v\left|\Psi_{sc}\right|^2\left|\rho_{\mathbf{Q}}\right|^2 + \cdots$$

Distinct symmetries of order parameters permit couplings only between their energy densities (there are no symmetries which "rotate" two order parameters into each other)

S. Sachdev and E. Demler, *Phys. Rev.* B **69**, 144504 (2004).

Predictions of LGW theory

Superconductor

$\langle \Psi_{sc} \rangle \neq 0, \langle \rho_{\mathbf{Q}} \rangle = 0$

First order transition

Charge-ordered insulator

$\langle \Psi_{sc} \rangle = 0, \langle \rho_{\mathbf{Q}} \rangle \neq 0$

$r_1 - r_2$

Superconductor

$\langle \Psi_{sc} \rangle \neq 0, \langle \rho_{\mathbf{Q}} \rangle = 0$

Coexistence

(Supersolid)

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"*Disordered*"

($\neq$ topologically ordered)

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Non-superconducting quantum phase must have
some other "order":

- Charge order in an insulator
- Fermi surface in a metal
- "Topological order" in a spin liquid
- ……………

This requirement is *not* captured by LGW theory.

Outline

A. Superfluid-insulator transitions of bosons
on the square lattice at fractional filling
Dual vortex theory and
the magnetic space group.

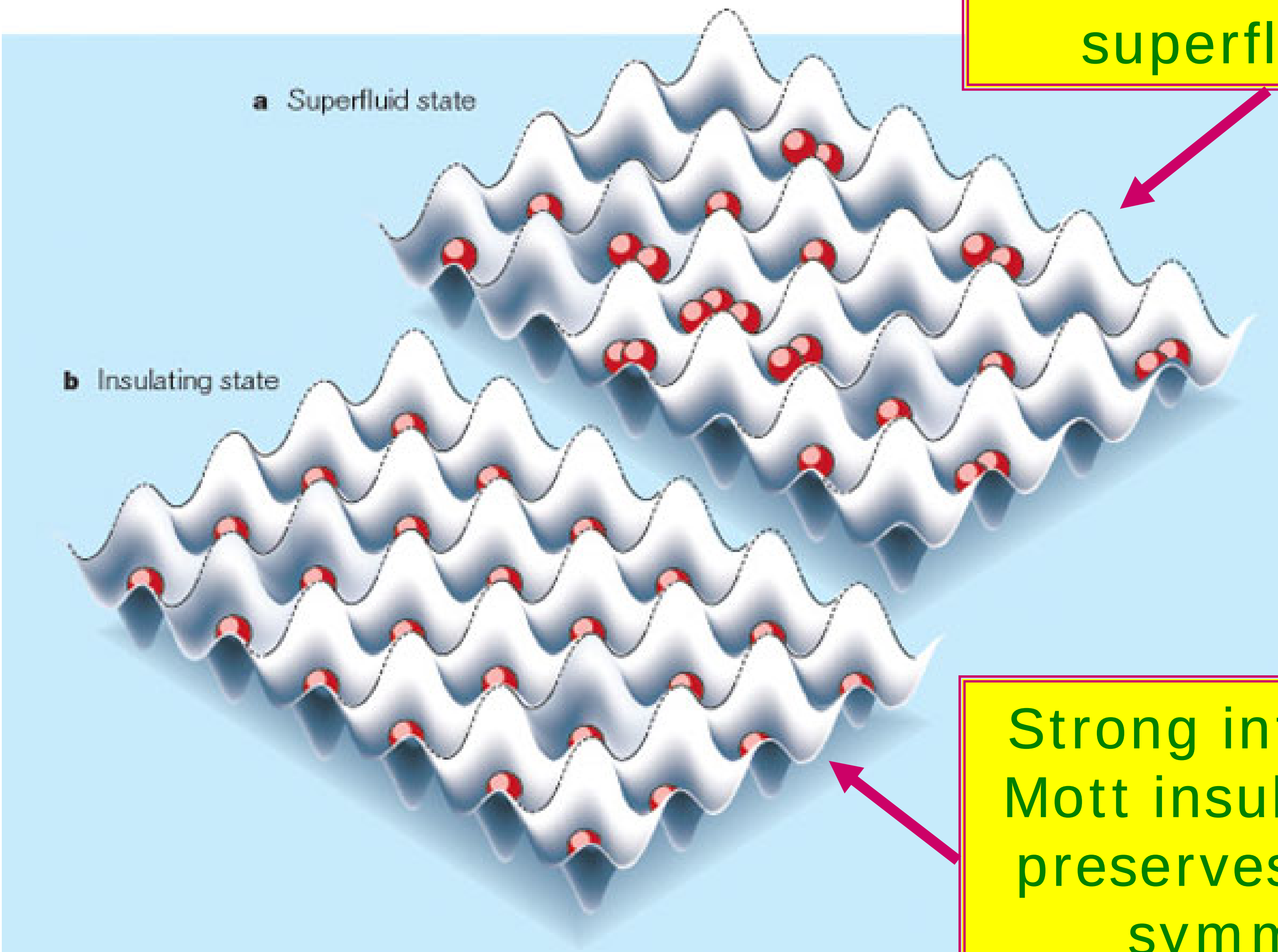
B. Application to a short-range pairing model for
the cuprate superconductors
Charge order and d-wave superconductivity
in an effective theory for the spin S=0 sector.

C. Implications for STM

A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling

Dual vortex theory and
the magnetic space group.

Bosons at density f = 1



Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

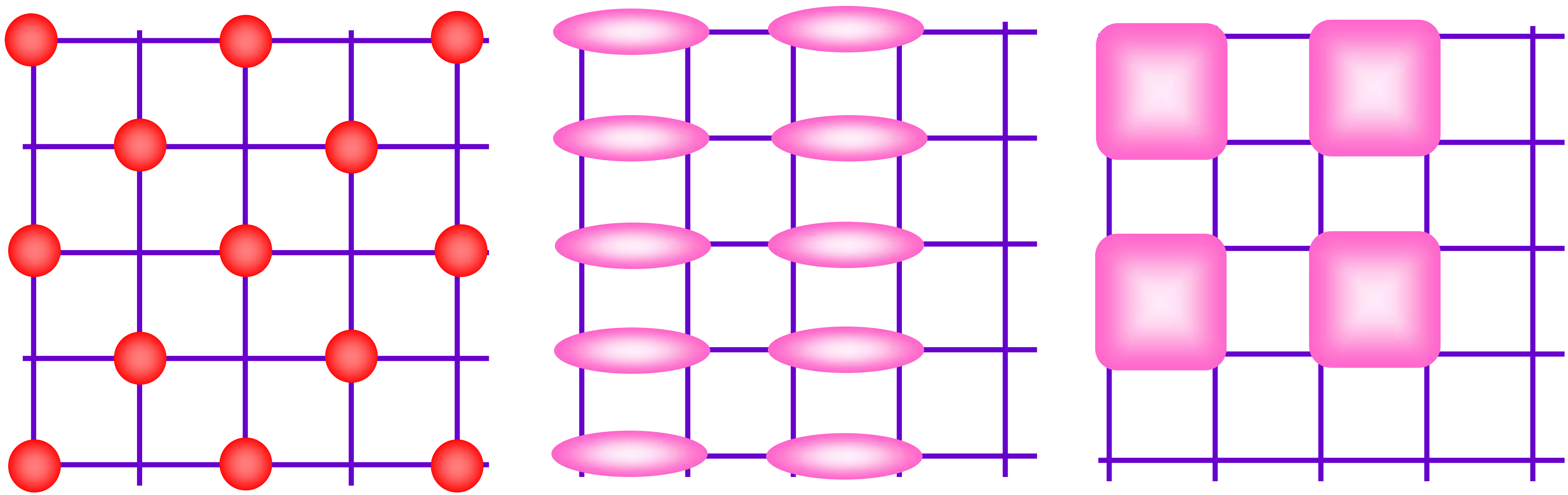
LGW theory: continuous quantum transitions between these states

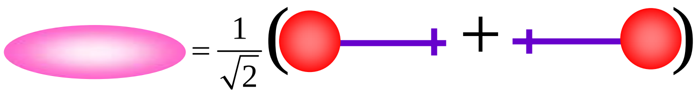
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Bosons at density f = 1/2 (equivalent to S=1/2 AFMs)

Weak interactions: superfluidity $\langle \Psi_{sc} \rangle \neq 0$

Strong interactions: Candidate insulating states





All insulating phases have density-wave order $\rho(\boldsymbol{r}) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$ with $\langle \rho_{\boldsymbol{Q}} \rangle \neq 0$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Predictions of LGW theory

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$$\langle \Psi_{sc} \rangle \neq 0, \langle \rho_{\mathbf{Q}} \rangle = 0$$

First order transition

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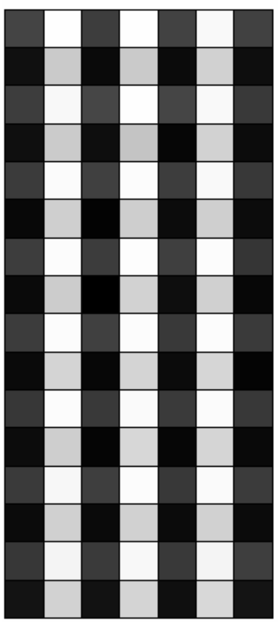
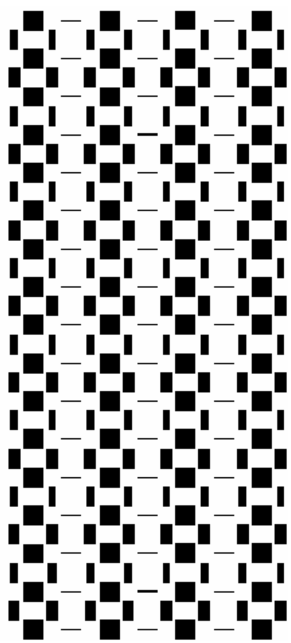
Superfluid-insulator transition of hard core bosons at f=1/2 (Neel-valence bond solid transition of S=1/2 AFM)

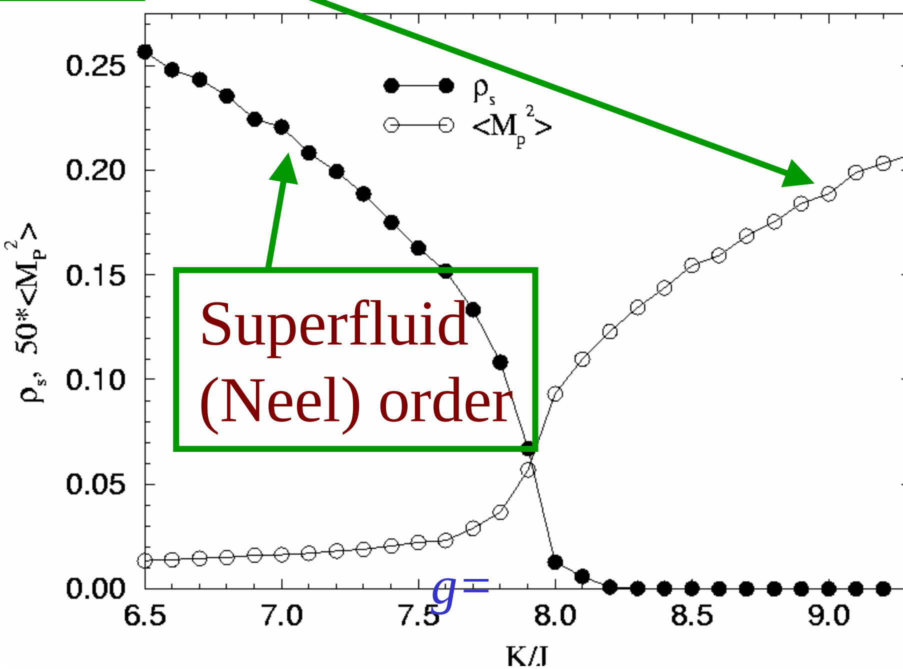
A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

Large scale (> 8000 sites) numerical study of the destruction of superfluid (i.e. magnetic Neel) order at half filling with full square lattice symmetry

VBS insulator





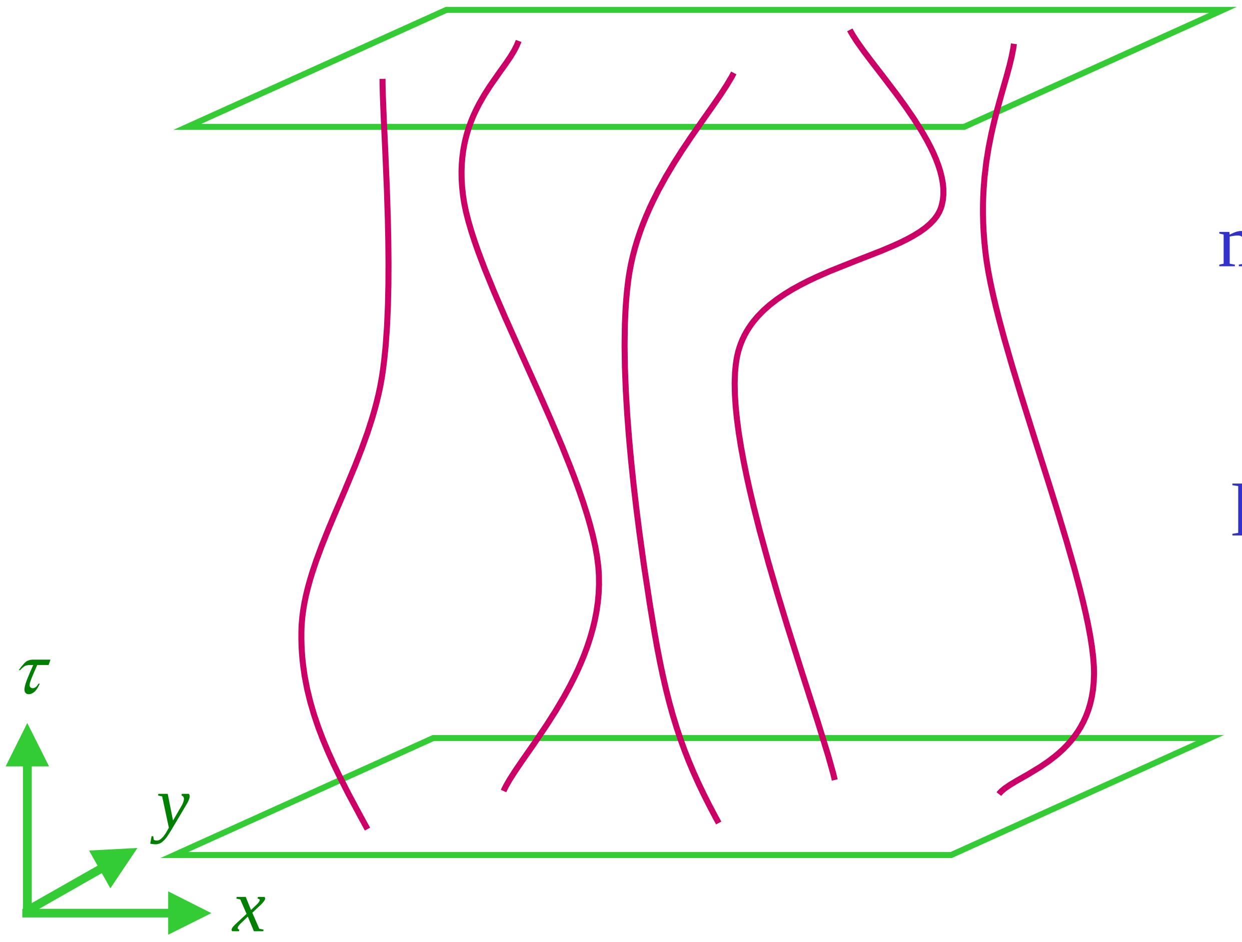


Superfluid (Neel) order

$g=$

$$H = J\sum_{\langle ij\rangle}\left(S_i^+S_j^- + S_i^-S_j^+\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+S_j^-S_k^+S_l^- + S_i^-S_j^+S_k^-S_l^+\right)$$

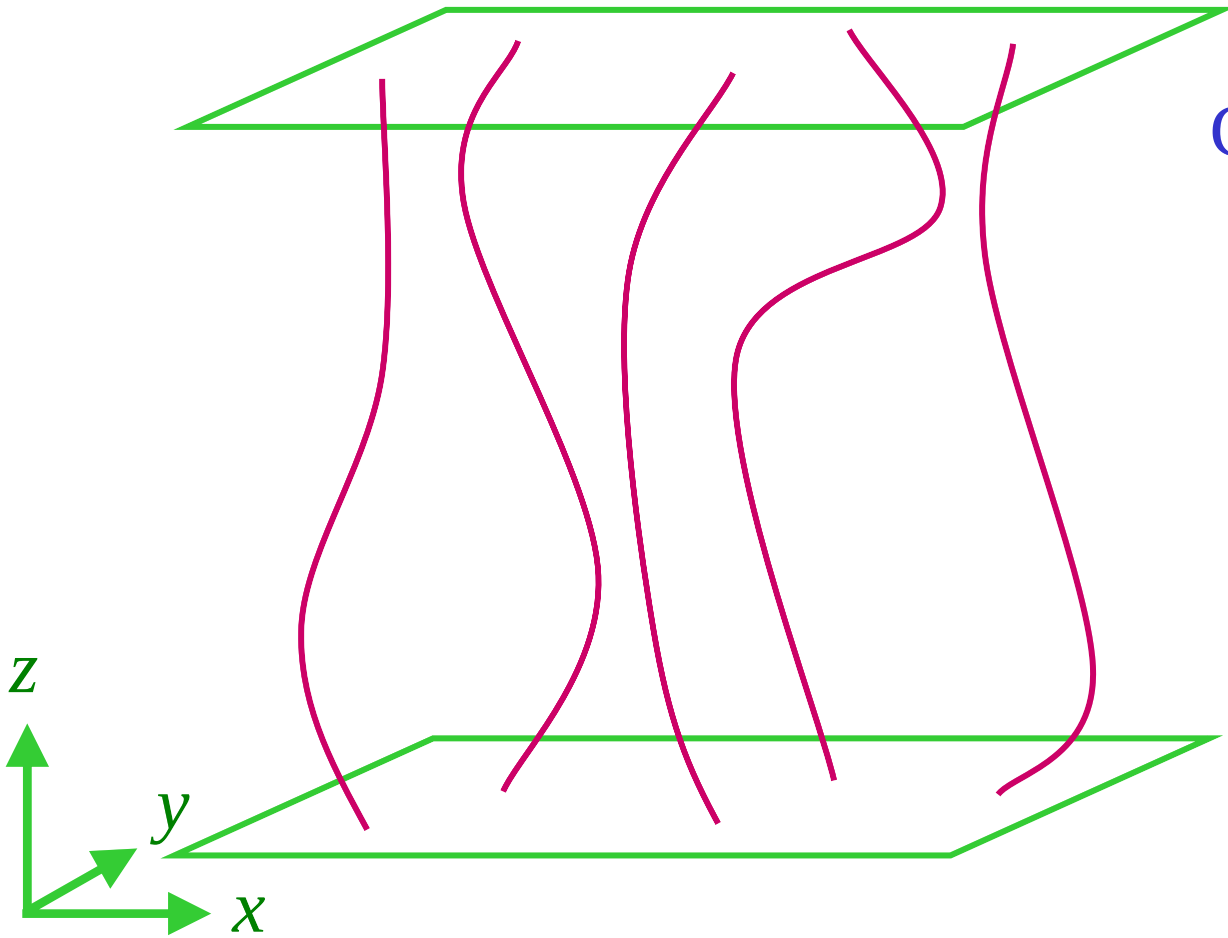
Boson-vortex duality



Quantum mechanics of two-dimensional bosons: world lines of bosons in spacetime

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

Boson-vortex duality



Classical statistical mechanics of a "dual" three-dimensional superconductor: vortices in a "magnetic" field

Strength of "magnetic" field = density of bosons
= f flux quanta per plaquette

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

Boson-vortex duality

Statistical mechanics of dual superconductor is invariant under the square lattice space group:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

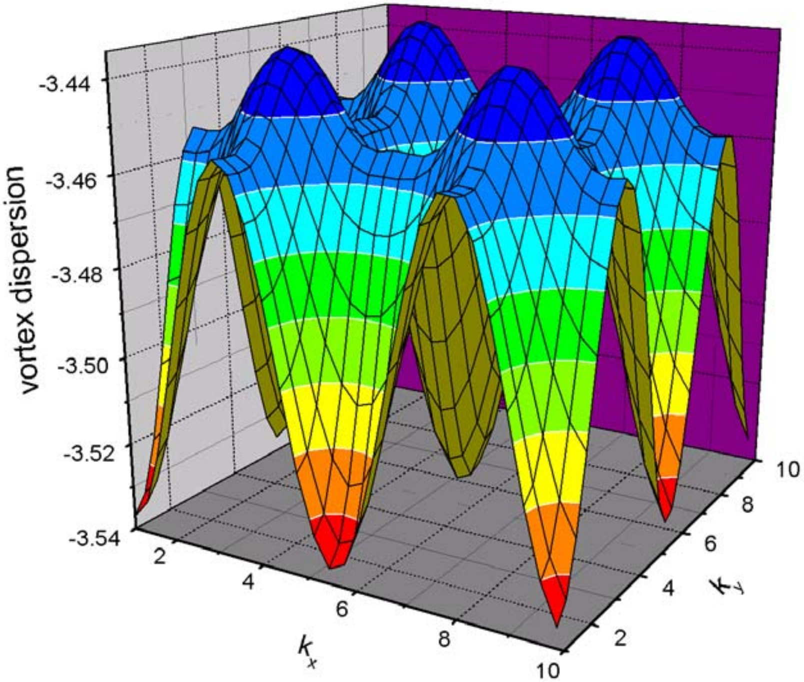
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Strength of "magnetic" field = density of bosons
= f flux quanta per plaquette

Boson-vortex duality

Hofstäder spectrum of dual "superconducting" order



At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Boson-vortex duality

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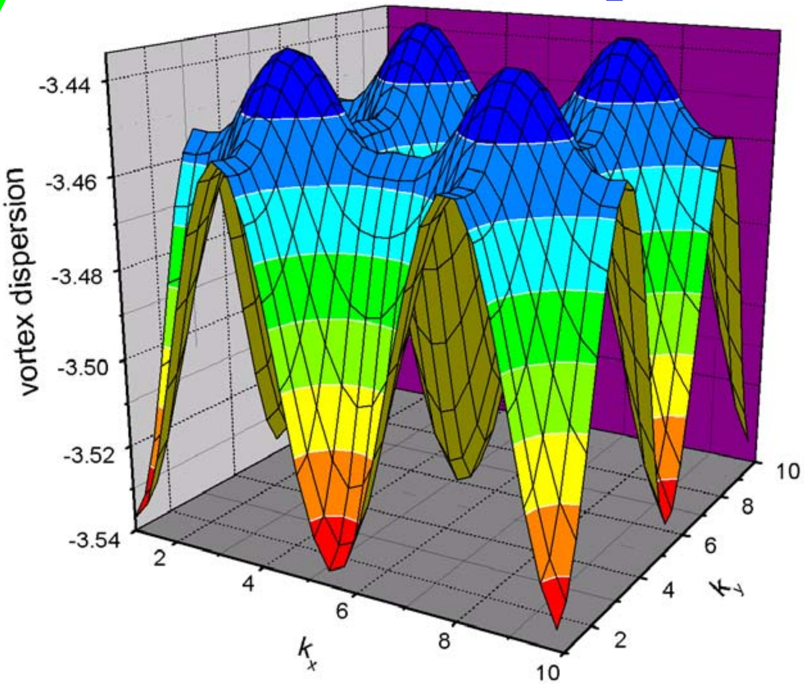

At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

The q vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

See also X.-G. Wen, *Phys. Rev.* B **65**, 165113 (2002)

Boson-vortex duality

The φ_ℓ fields characterize *both* superconducting and charge order

Superconductor/insulator : $\langle \varphi_\ell \rangle = 0 / \langle \varphi_\ell \rangle \neq 0$

Charge order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

$$R : \rho(\mathbf{Q}) \to \rho(R\mathbf{Q})$$

Boson-vortex duality

The φ_ℓ fields characterize *both* superconducting and charge order

Competition between superconducting and charge orders:

"*Extended* LGW" theory of the φ_ℓ fields with the action invariant under the projective transformations:

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

Immediate benefit: There is no intermediate "disordered" phase with neither order

(or without "topological" order).

Analysis of "extended LGW" theory of projective representation

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First order transition

Charge-ordered insulator

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Analysis of "extended LGW" theory of projective representation

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First order transition

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$\langle \varphi_\ell \rangle = 0, \langle \rho_{mn} \rangle = 0$

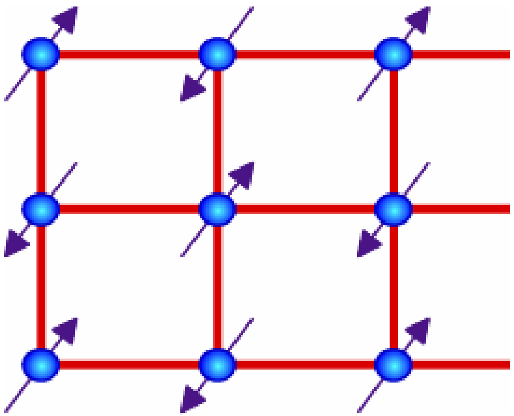
Second order transition

Charge-ordered insulator

$\langle \varphi_\ell \rangle \neq 0, \langle \rho_{mn} \rangle \neq 0$

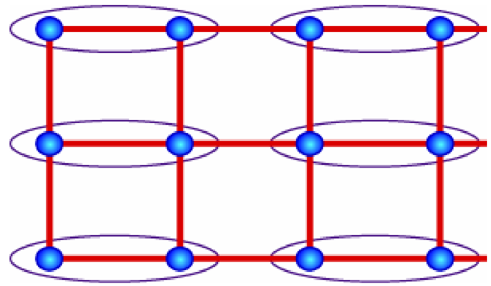
$r_1 - r_2$

Phase diagram of S=1/2 square lattice antiferromagnet

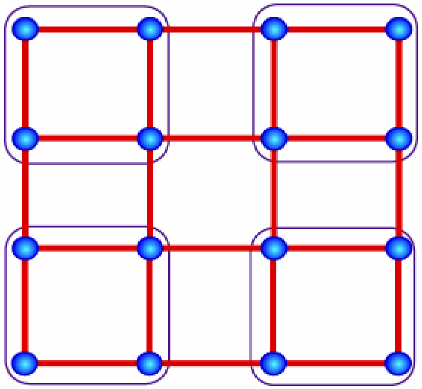


Neel order

$\vec{\varphi} \sim z_{\alpha}^{*} \vec{\sigma}_{\alpha\beta} z_{\beta} \neq 0$



or



Bond order $\Psi \neq 0$,

$S = 1/2$ spinons z_{α} confined,

$S = 1$ triplon excitations

g

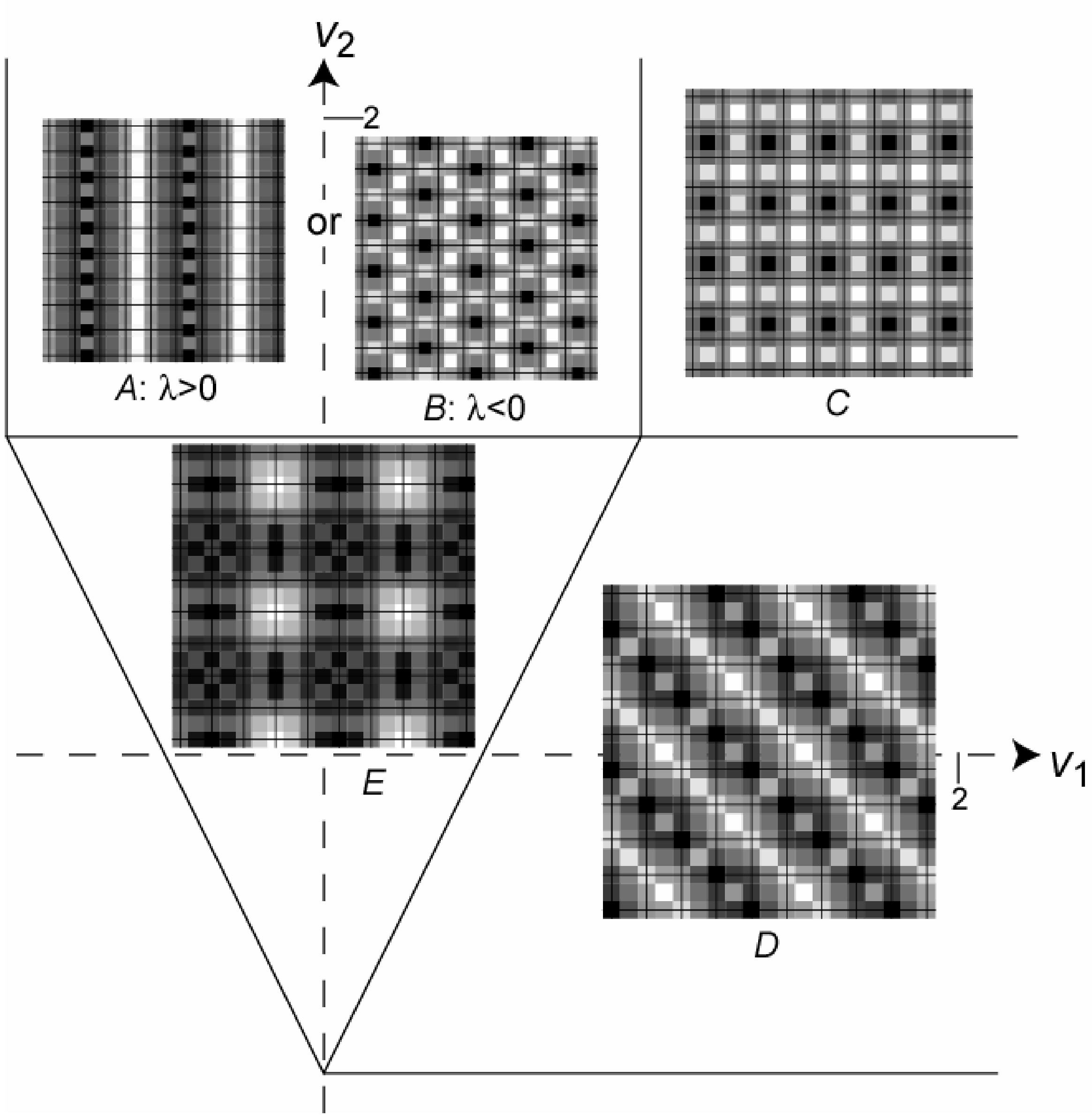
Second-order critical point described by a theory of deconfined spinons

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + r\, |z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right]$$

where the U(1) gauge field A_μ is *non-compact*. Deconfinement also happens in critical *phases* (Hermele *et al.* cond-mat/0404751) and monopole screening arguments (Herbut, Sachdev *et al.*) fail at critical points/phases.

Analysis of "extended LGW" theory of projective representation

Spatial structure of insulators for q=4 (f=1/4 or 3/4)



$a \times b$ unit cells;

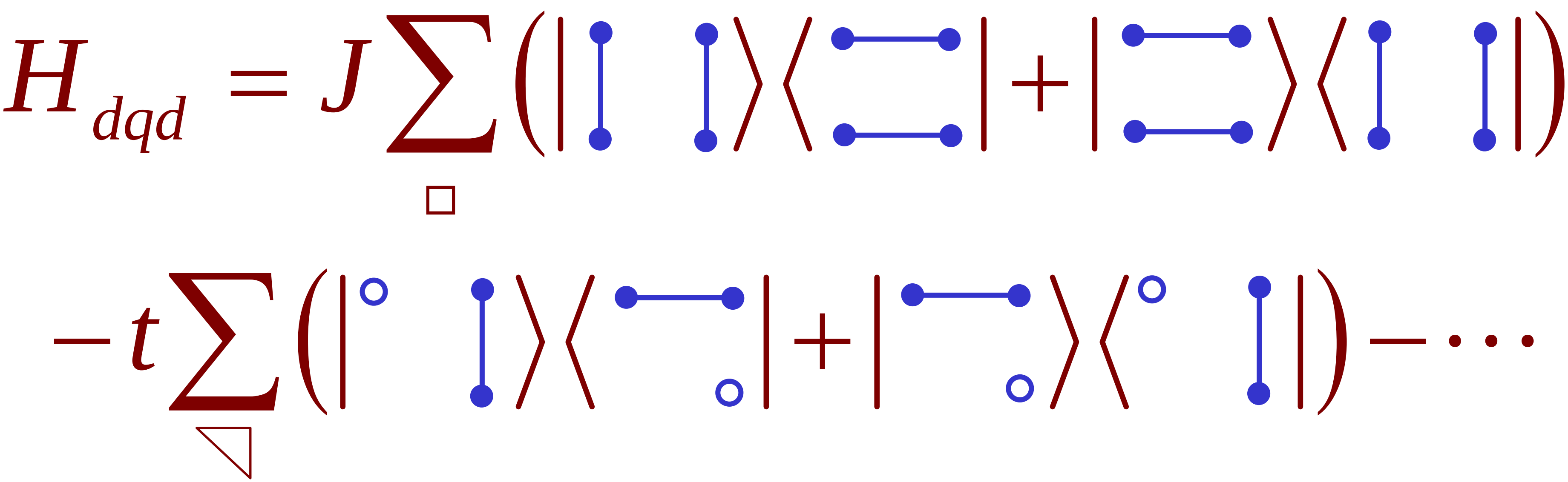
q/a, q/b, ab/q,

all integers

B. Application to a short-range pairing model for the cuprate superconductors

Charge order and d-wave superconductivity in an effective theory for the spin S=0 sector.

A convenient derivation of the effective theory of short-range pairs is provided by the doped quantum dimer model



Density of holes = δ

E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett.* B **4**, 225 (1990).

Duality mapping of doped dimer model shows:

(a) Superfluid, insulator, and supersolid ground states of a theory which obeys the magnetic algebra

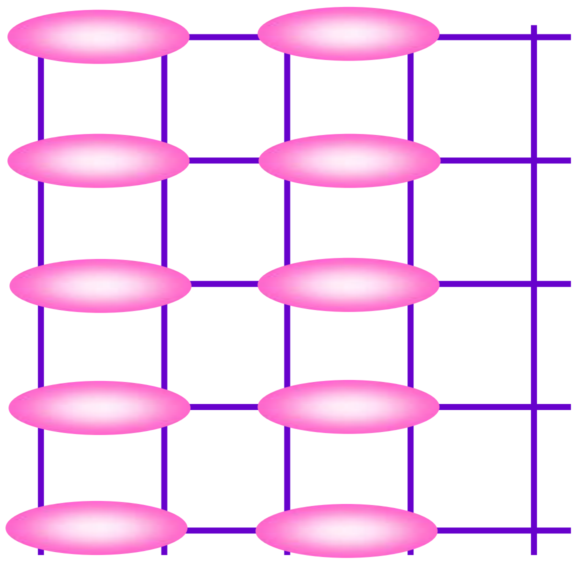
$$T_x T_y = e^{2\pi i f} T_y T_x$$

$$\text{with} \quad f = \frac{1-\delta}{2}$$

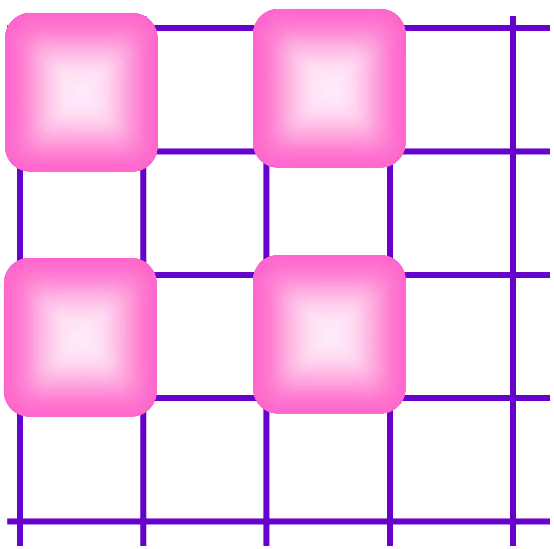
Duality mapping of doped dimer model shows:

(b) At $\delta = 0$, the ground state is a Mott insulator with valence-bond-solid (VBS) order. This associated with f=1/2 and the algebra

$$T_x T_y = -T_y T_x$$



or



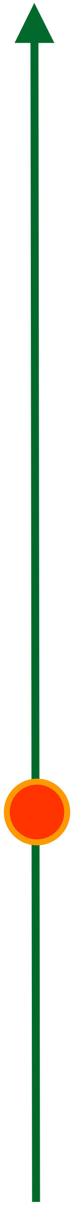
Duality mapping of doped dimer model shows:

(c) At larger δ, the ground state is a *d*-wave superfluid. The structure of the "extended LGW" theory of the competition between superfluid and solid order is identical to that of bosons on the square lattice with density *f*. These bosons can therefore be viewed as *d*-wave Cooper pairs of electrons. The phase diagrams of part (A) can therefore be applied here.

$$T_x T_y = e^{2\pi i f} T_y T_x$$

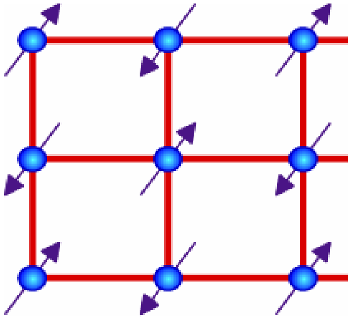
$$\text{with} \quad f = \frac{1-\delta}{2}$$

Global phase diagram



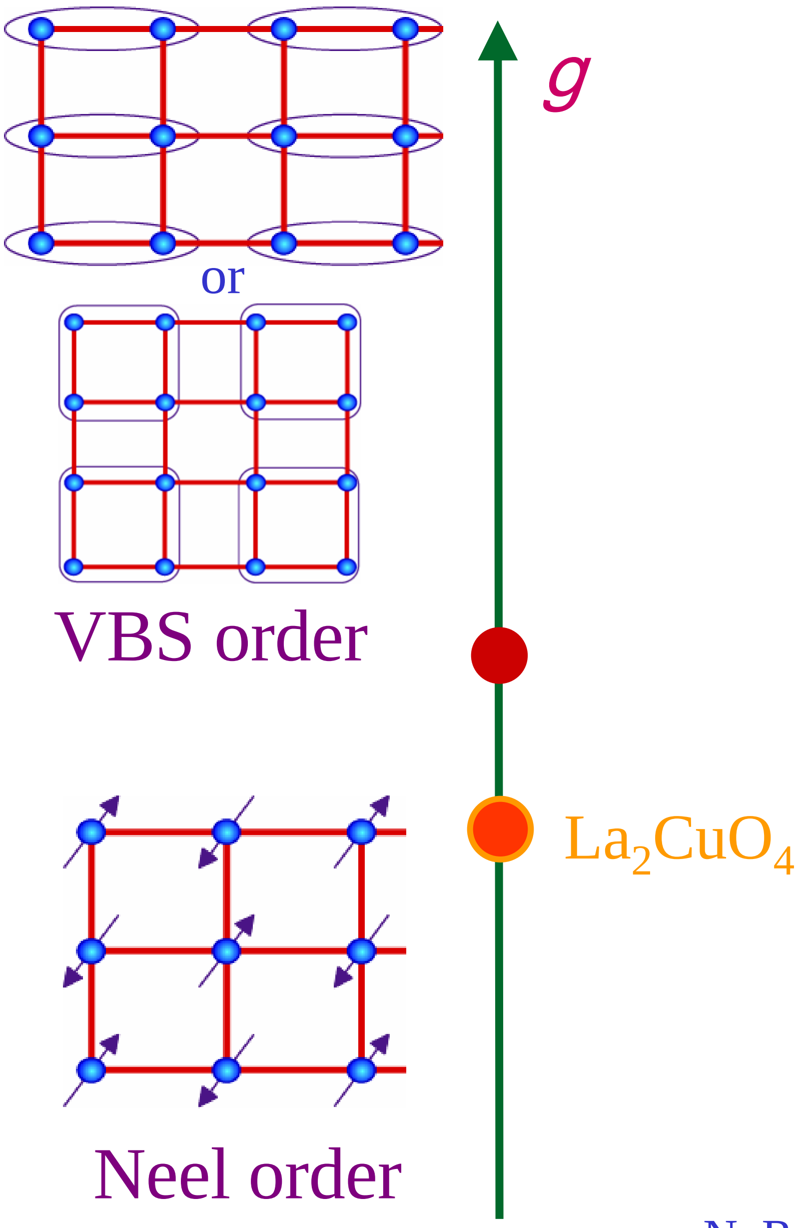
g = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

La_2CuO_4



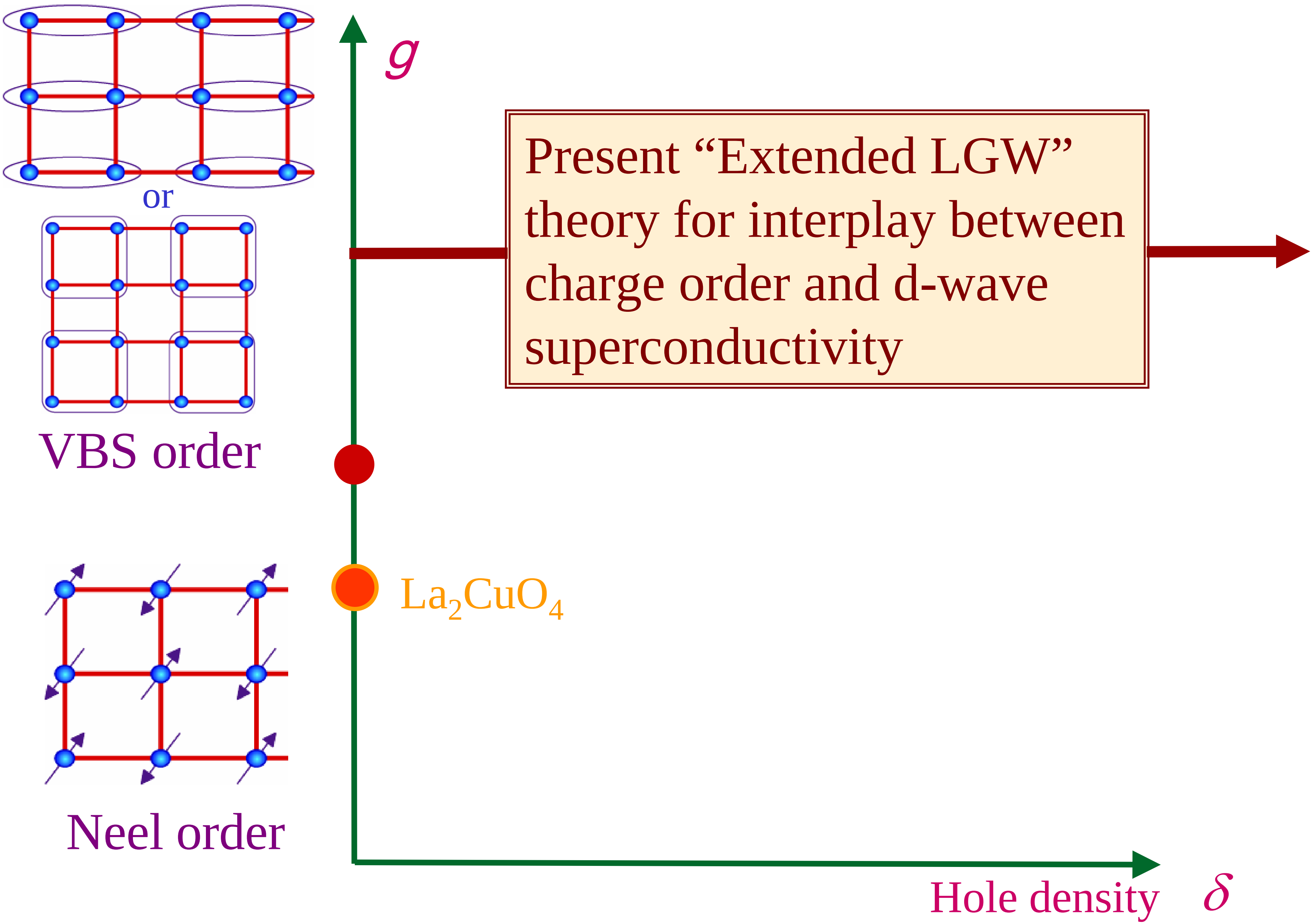
Neel order

Global phase diagram



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

Global phase diagram



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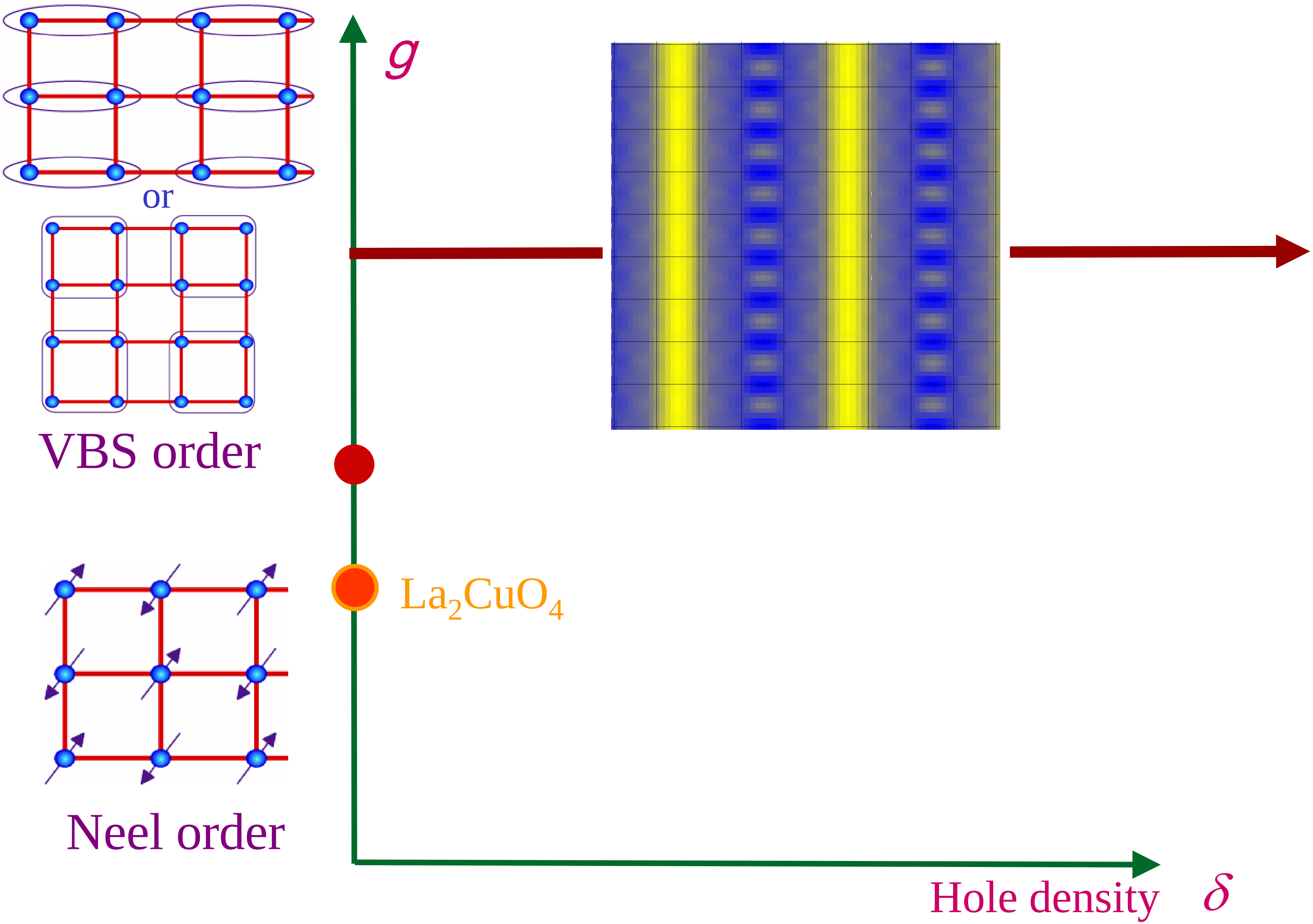

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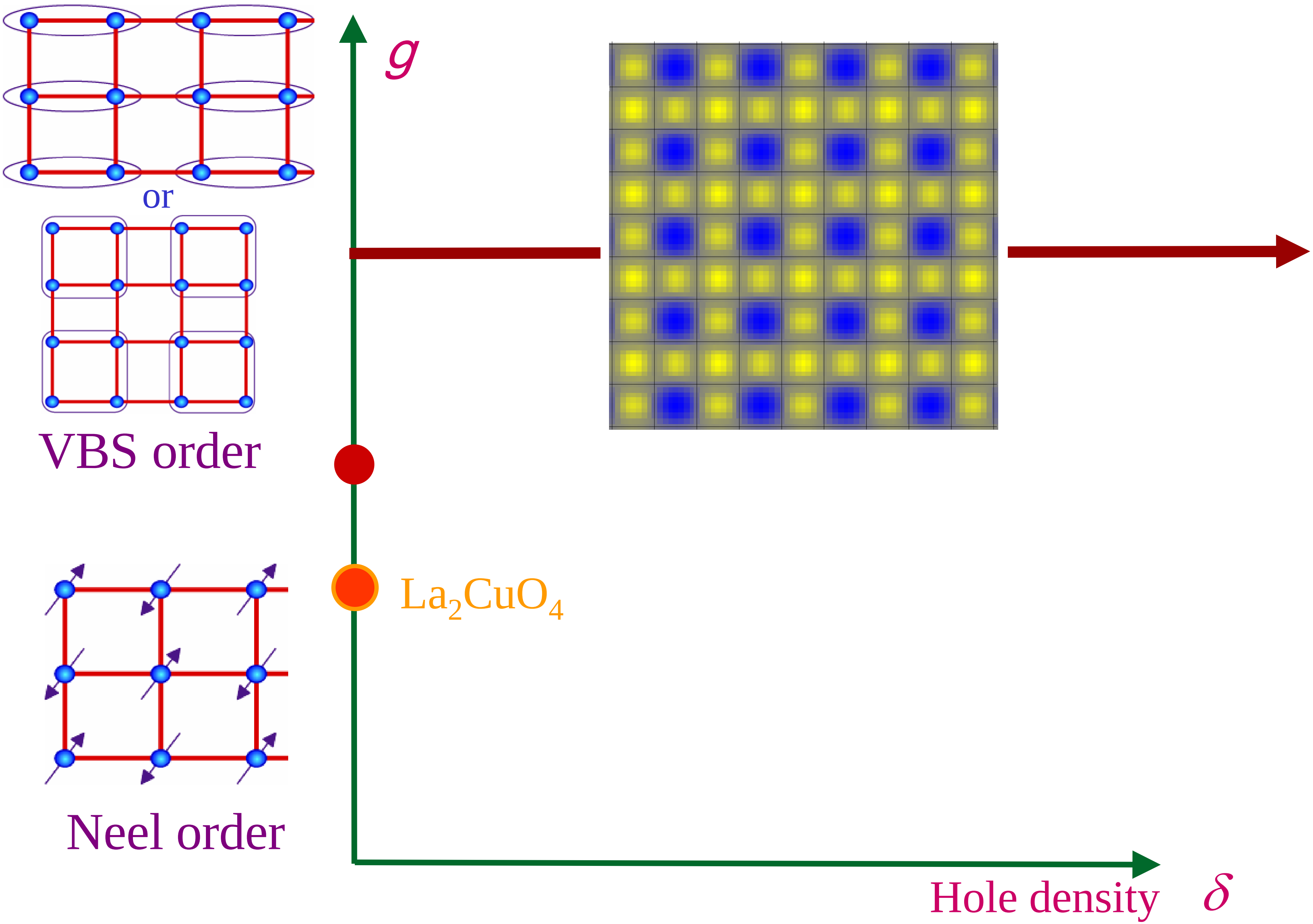

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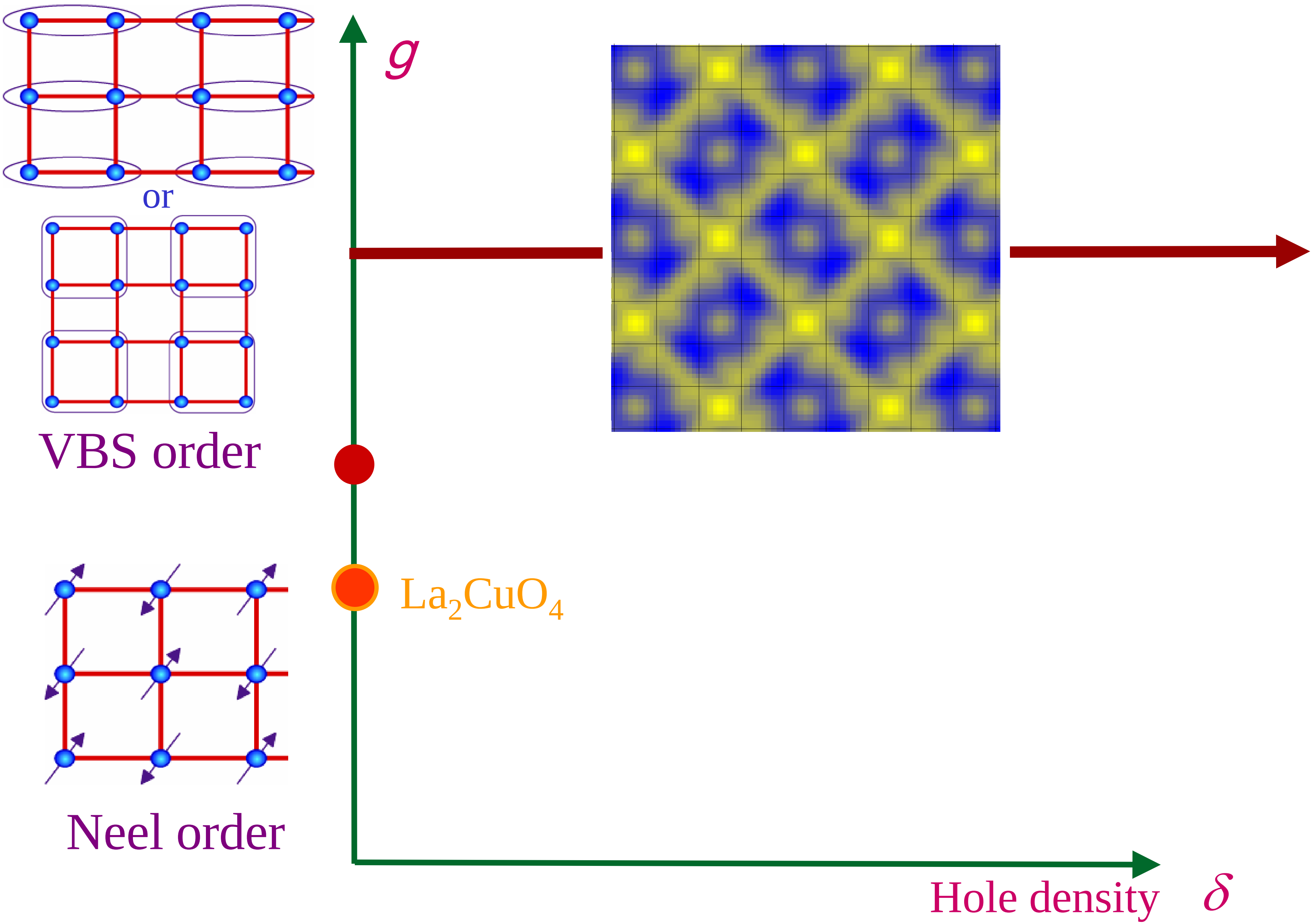

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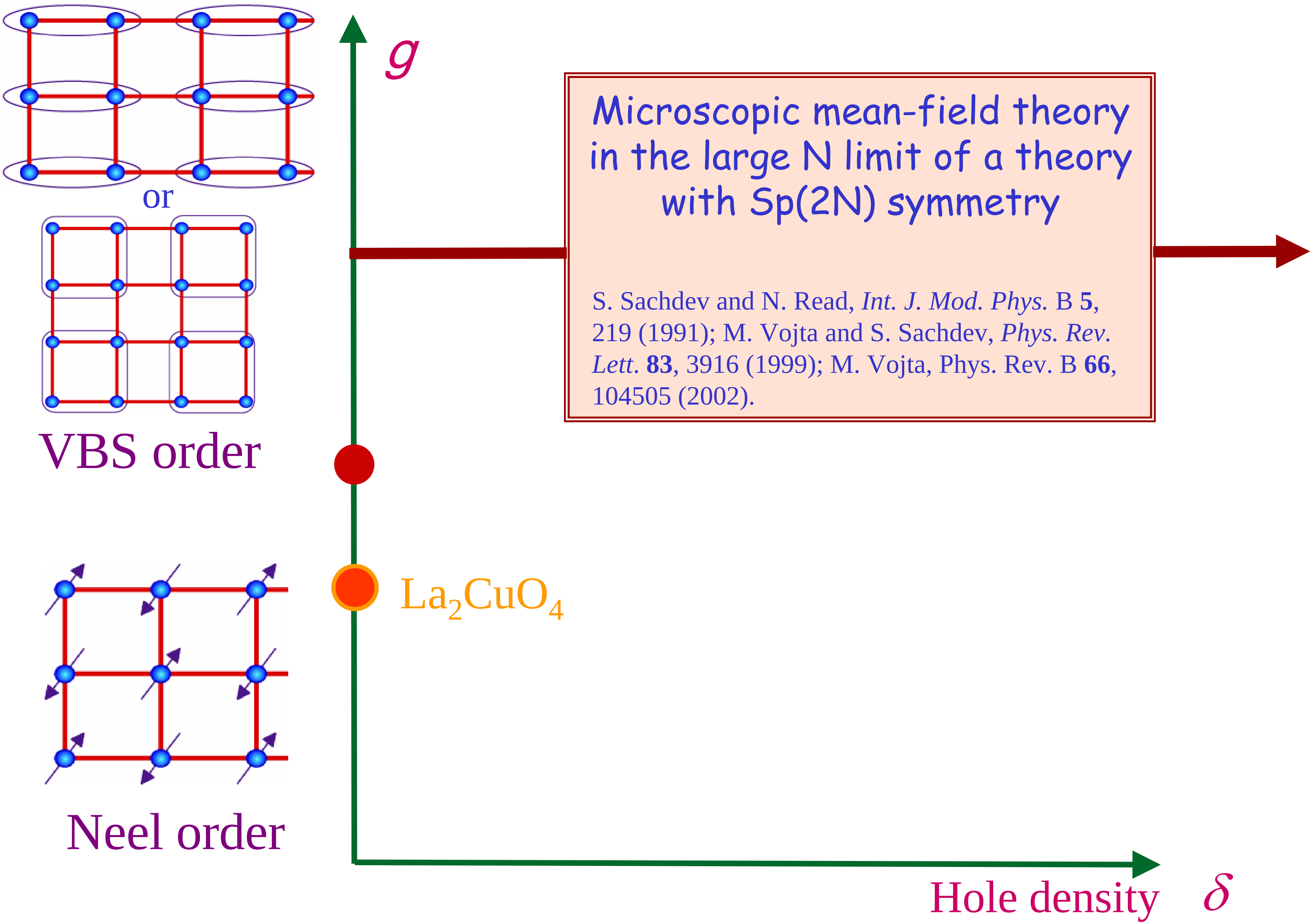

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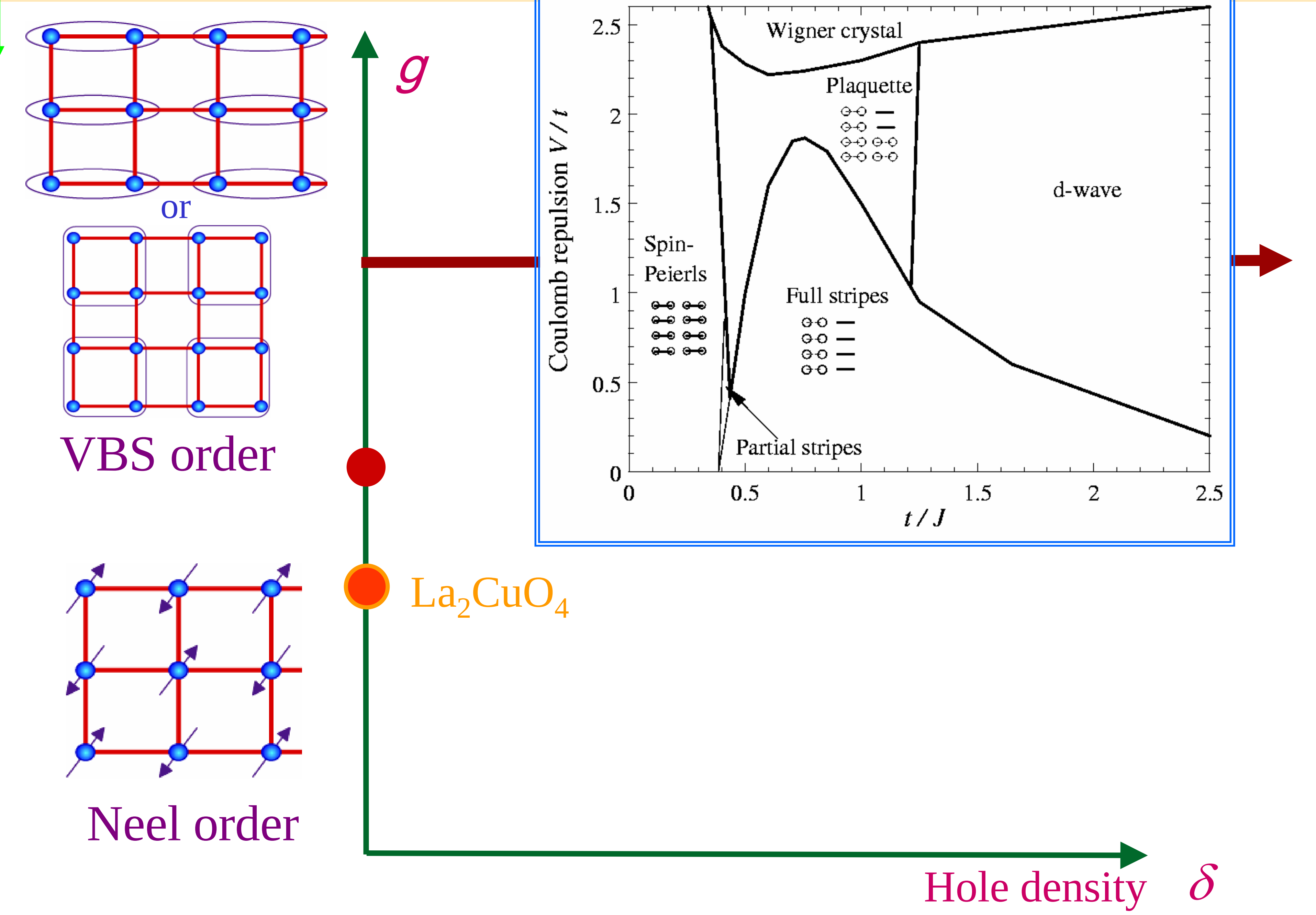
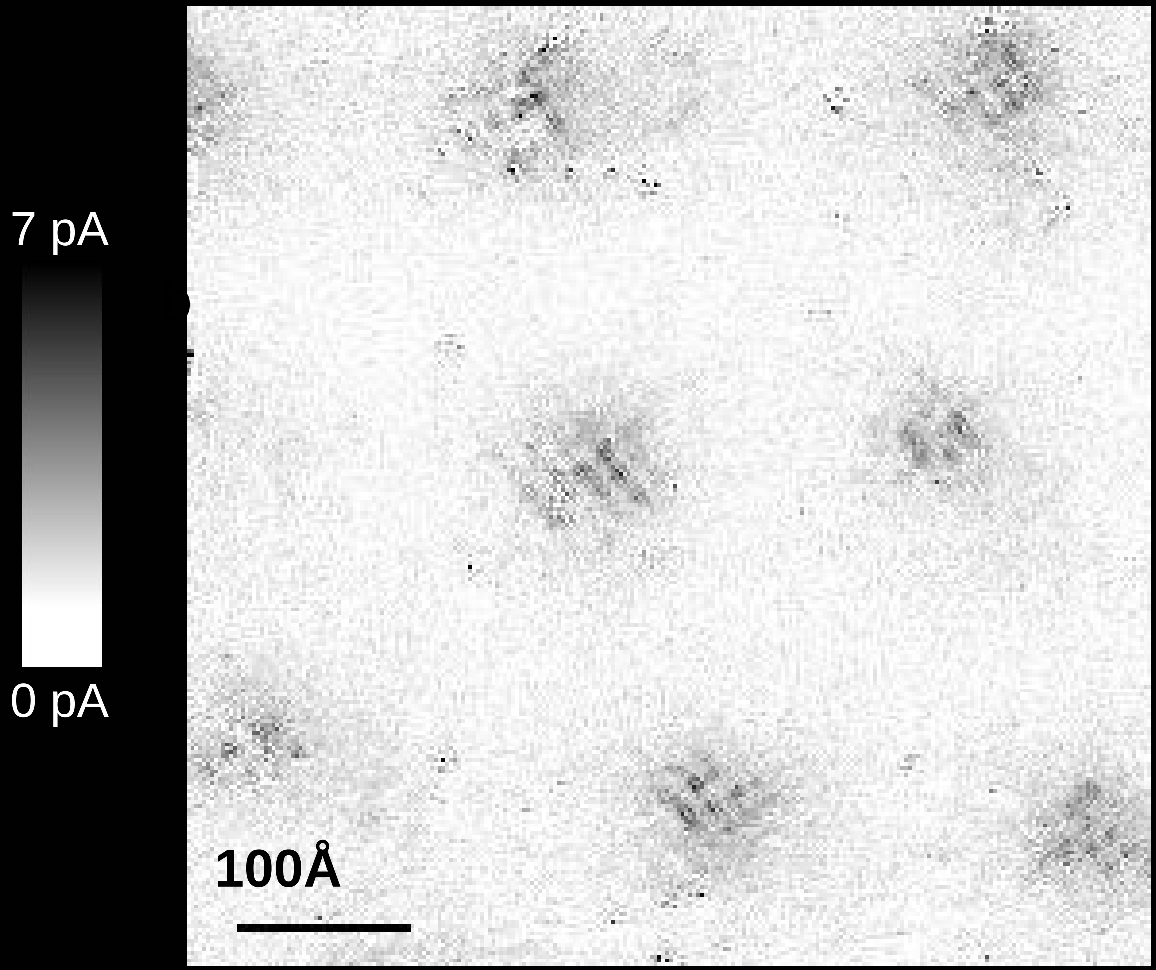
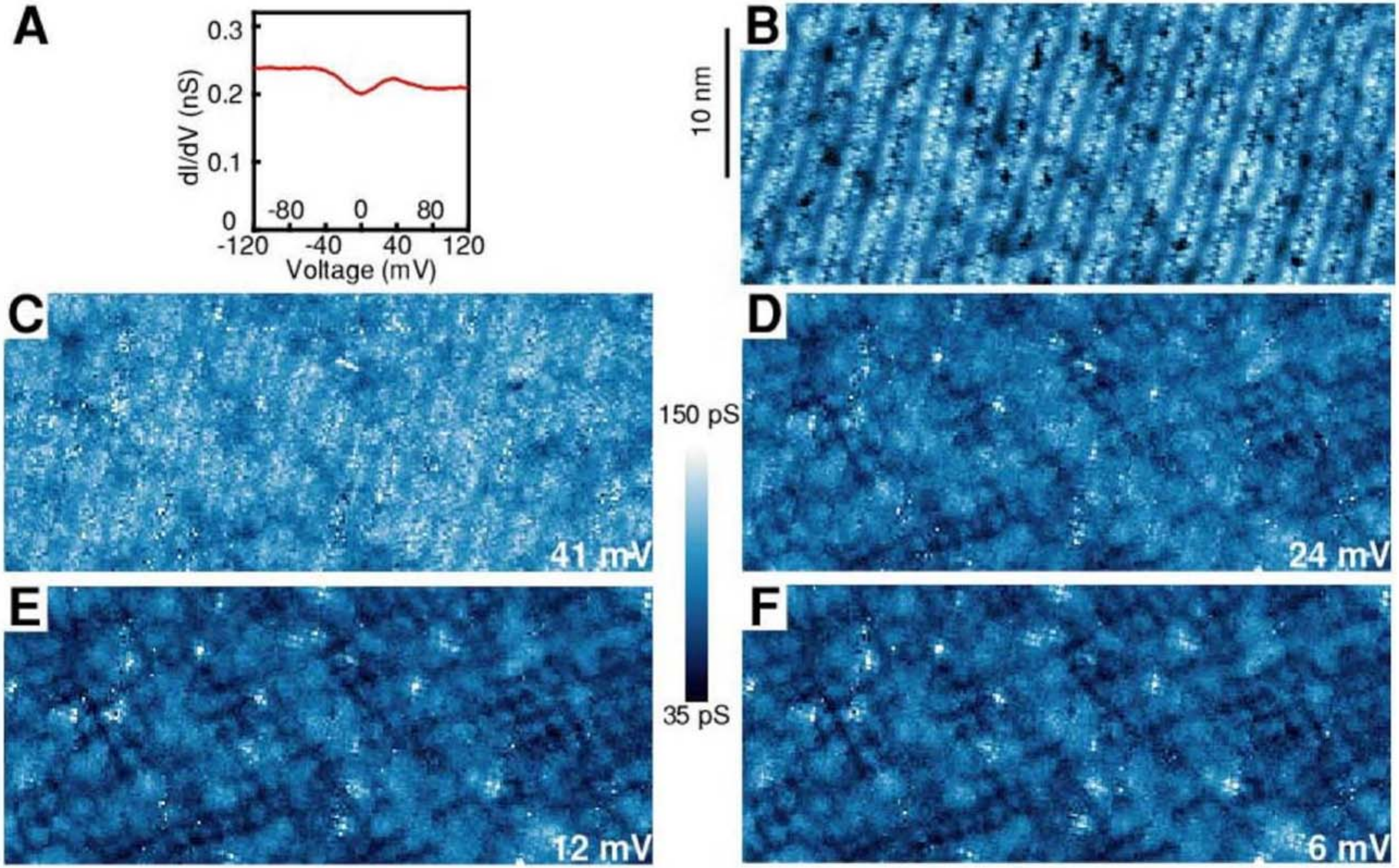


C. Implications for STM

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

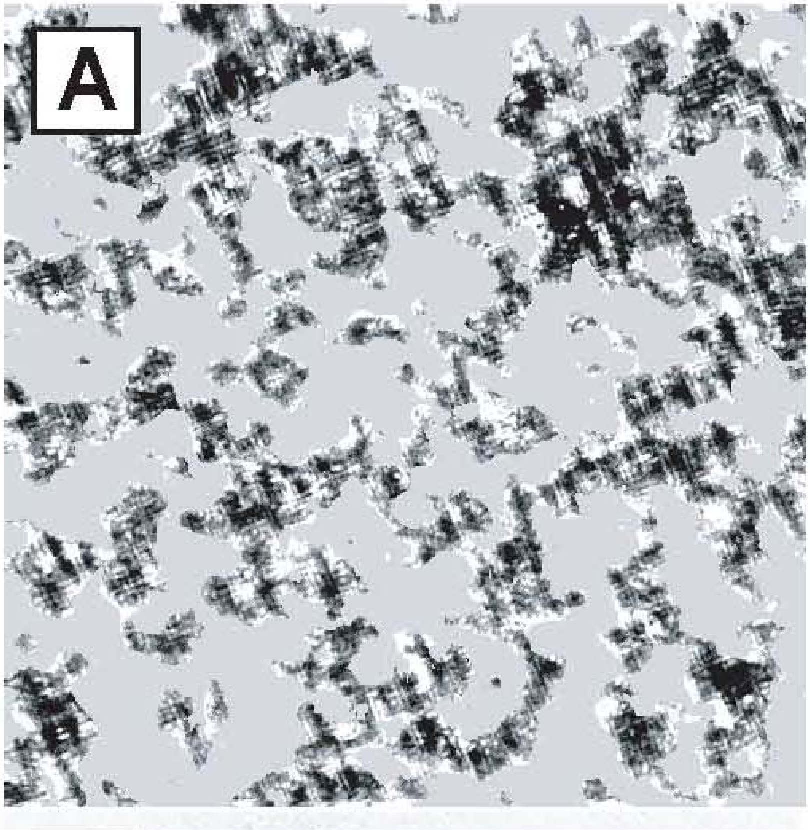


J. Hoffman E. W. Hudson, K. M. Lang,
V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida,
and J. C. Davis, *Science* 295, 466 (2002).



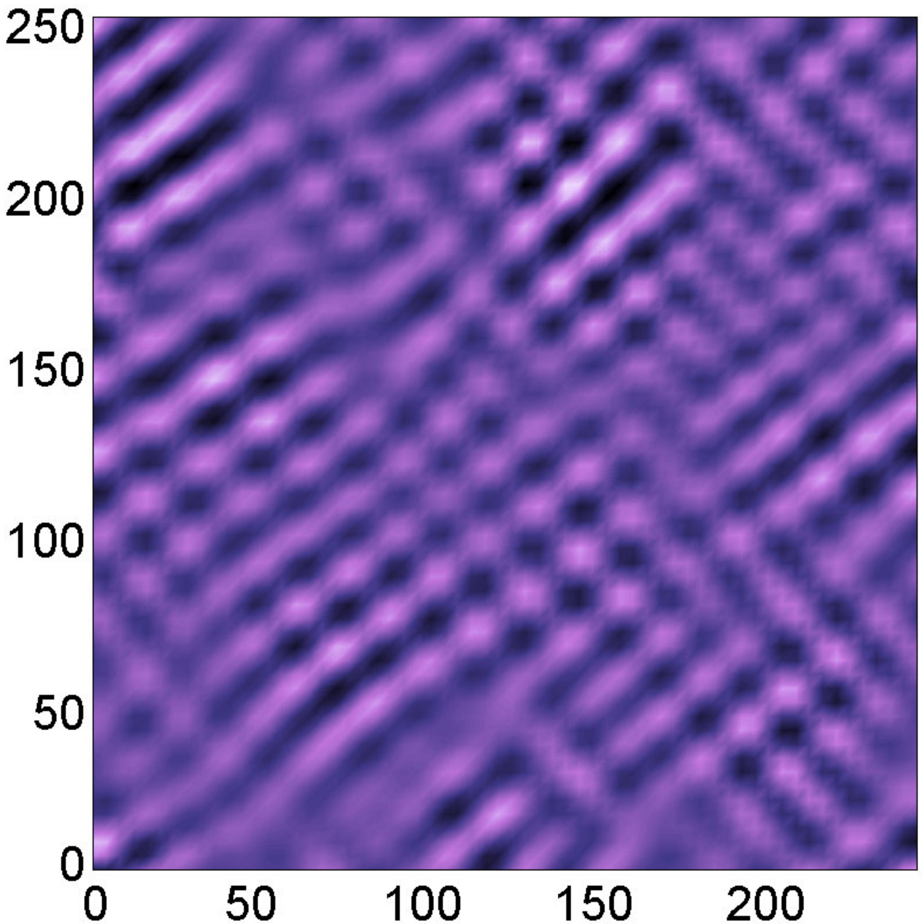
LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

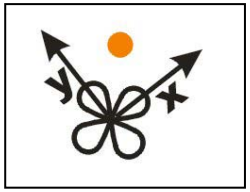
M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).



Energy integrated LDOS (between 65 and 150 meV) of strongly underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ at low temperatures, showing only regions without superconducting "coherence peaks"

K. McElroy, D.-H. Lee, J. E. Hoffman, K. M Lang, E. W. Hudson, H. Eisaki, S. Uchida, J. Lee, J.C. Davis, cond-mat/0404005.





STM of LDOS modulations (filtered) in $Bi_2Sr_2CaCu_2O_{8+\delta}$

C. Howald, H. Eisaki, N. Kaneko, M. Greven,and A. Kapitulnik,
Phys. Rev. B **67**, 014533 (2003).

Pinning of charge order in a superconductor

The φ_ℓ vortex fields are fluctuating, and we can use the action

$$S = \int d^2 r d\tau \left(L_0 \left[\varphi_\ell \right] + L_{\text{pin}} \left[\varphi_\ell \right] \right)$$

$$L_0 \left[\varphi_\ell \right] = \sum_\ell \left(\left| \partial_\mu \varphi_\ell \right|^2 + s \left| \varphi_\ell \right|^2 \right) + \cdots$$

$$L_{\text{pin}} \left[\varphi_\ell \right] = V_{\text{pin}} \left(r \right) \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i \boldsymbol{Q} \bullet r} \quad \text{with } \rho_{\boldsymbol{Q}_{mn}} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f}$$

The projective transformation properties of vortices imply that each vortex carries the quantum numbers of density wave order. The vacuum fluctuations of vortex-anti-vortex produce density wave modulations which are observable near pinning sites at wavevectors

$$\boldsymbol{Q}_{mn} = \frac{2\pi p}{q} \left(m, n \right) = 2\pi f \left(m, n \right)$$

Charge order in a magnetic field

The φ_ℓ vortex fields are fluctuating, and we can use the action

$$S = \int d^2 r d\tau \left(L_0 \left[\varphi_\ell \right] + L_{\text{pin}} \left[\varphi_\ell \right] \right)$$

$$L_0 \left[\varphi_\ell \right] = \sum_\ell \left(\left| \partial_\mu \varphi_\ell \right|^2 + s \left| \varphi_\ell \right|^2 \right) + \cdots$$

$$L_{\text{pin}} \left[\varphi_\ell \right] = V_{\text{pin}} \left(r \right) \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \mathbf{r}}$$ with $\rho_{\mathbf{Q}_{mn}} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$

Recompute modulation in *same* theory but in sector with φ_ℓ "charge" = number of vortices. Additional density wave order parameter appears as a halo around pinned vortices.

Conclusions

I. Description of the competition between superconductivity and charge order in term of defects (vortices). Theory naturally excludes “disordered” phase with no order.

II. Vortices carry the quantum numbers of *both* superconductivity *and* the square lattice space group (in a projective representation).

III. Vortices carry halo of charge order, and pinning of vortices/anti-vortices leads to a unified theory of STM modulations in zero and finite magnetic fields.

IV. Conventional picture: density wave order is responsible for the transport energy gap, and for the appearance of the Mott insulator. New picture: Mott localization of charge carriers is more fundamental, and (weak) density wave order emerges naturally in theory of the Mott transition.