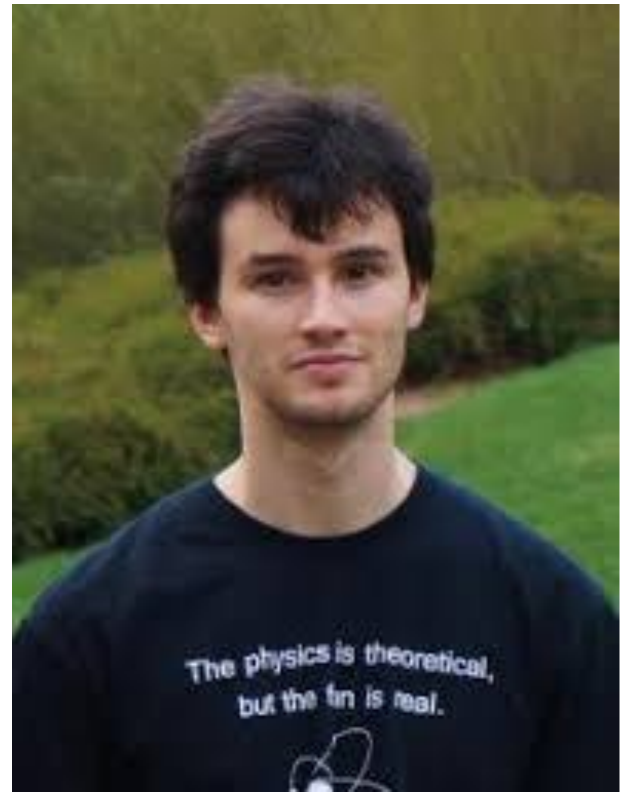
Gauge theory for the cuprates near optimal doping

Developments in Quantum Field Theory and Condensed Matter Physics
Simons Center for Geometry and Physics,
Stony Brook University
November 7, 2018

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Mathias Scheurer



Grigory Tarnopolsky

Harley Scammell

arXiv:1811.xxxxx

PHYSICS

VE RI TAS

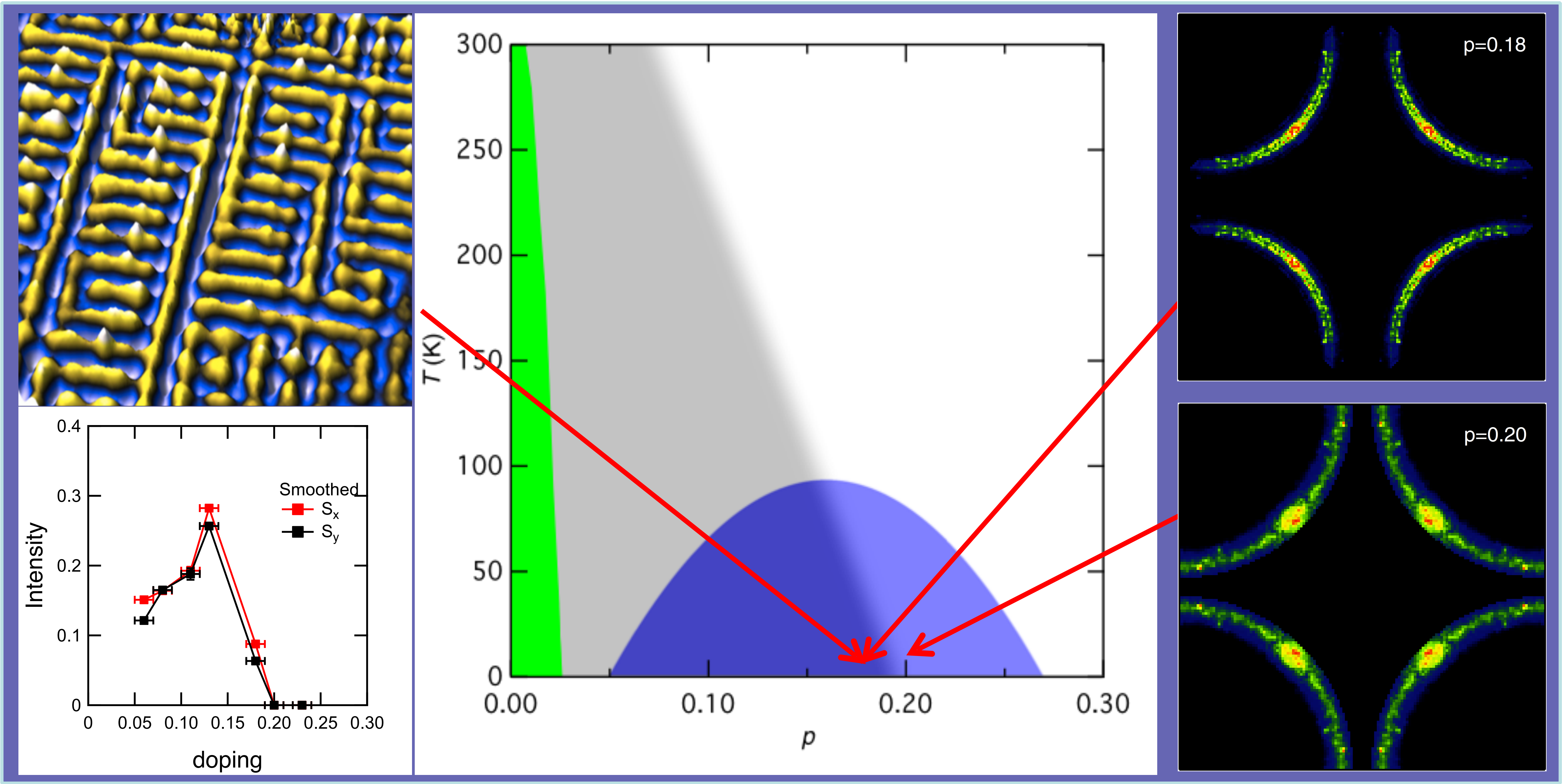
HARVARD

Fermi Surface and Pseudogap Evolution in a Cuprate Superconductor

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

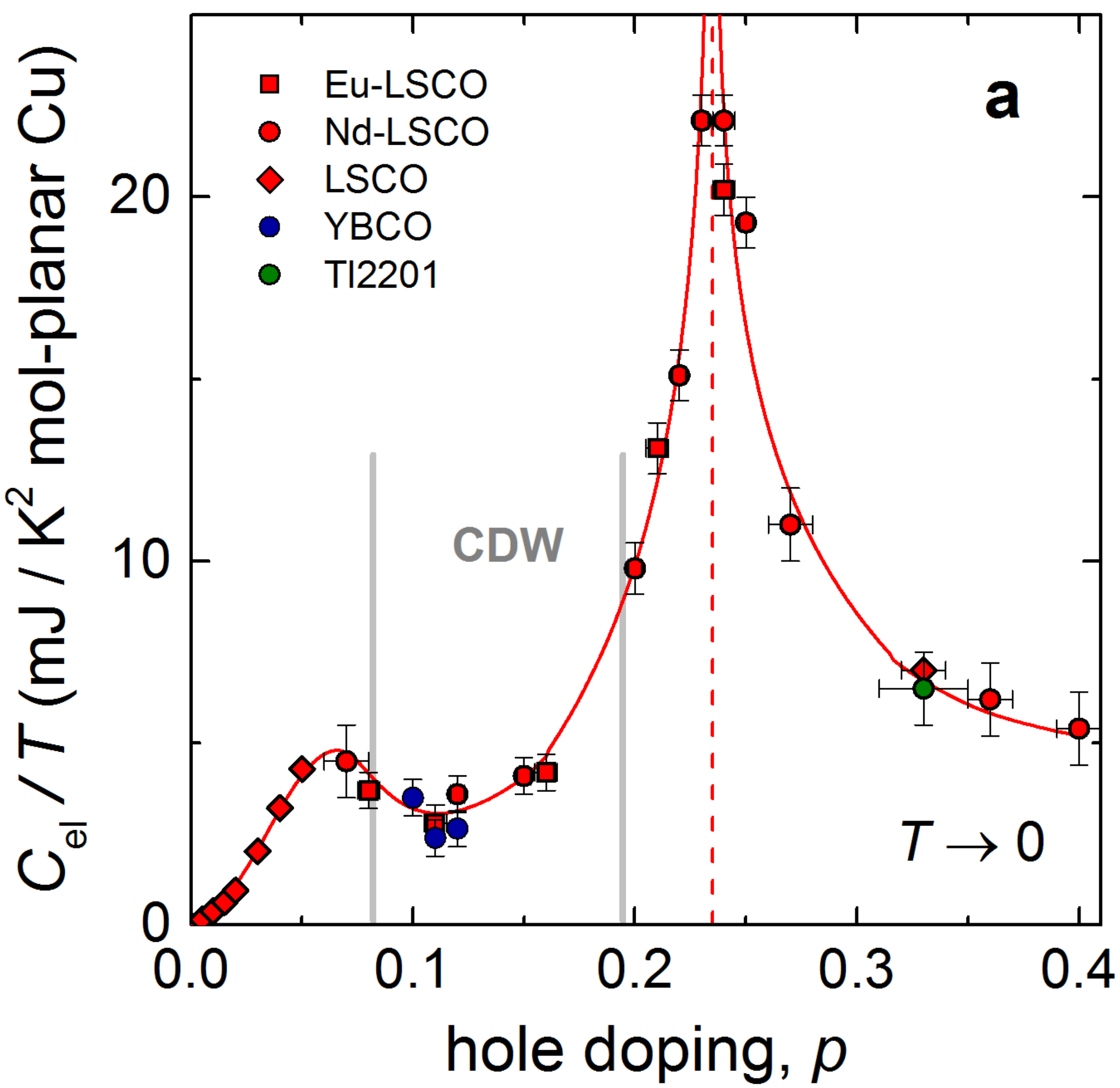
Simultaneous Transitions in Cuprate Momentum-Space Topology and Electronic Symmetry Breaking

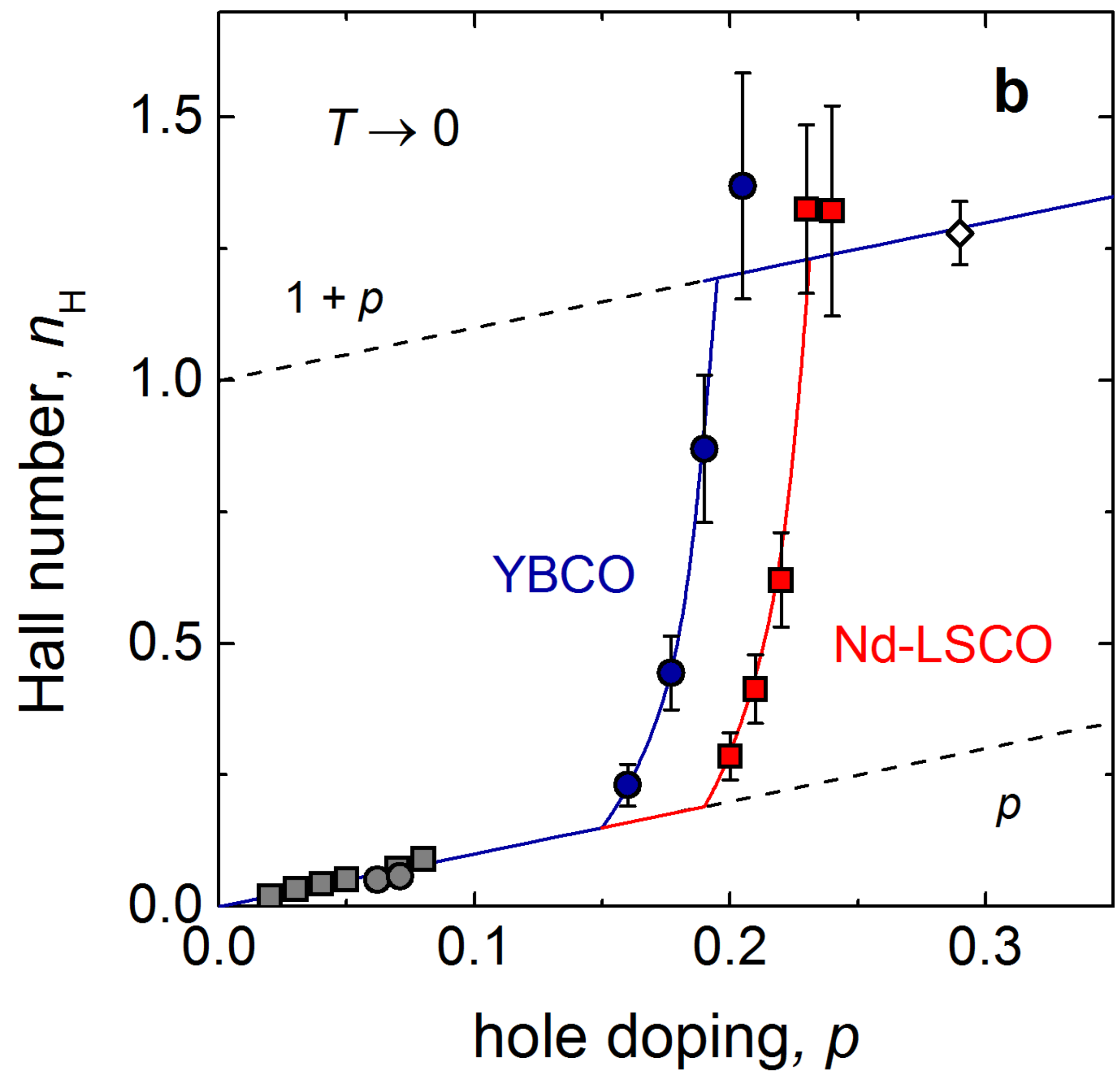
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)



The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507





1. SU(2) gauge theory with adjoint scalars

Confining and Higgs phases in 2+1 dimensions

2. SU(2) gauge theory of fluctuating antiferromagnetism

3. Role of fermions

Pseudogap as a FL phase*

SU(2) gauge theory

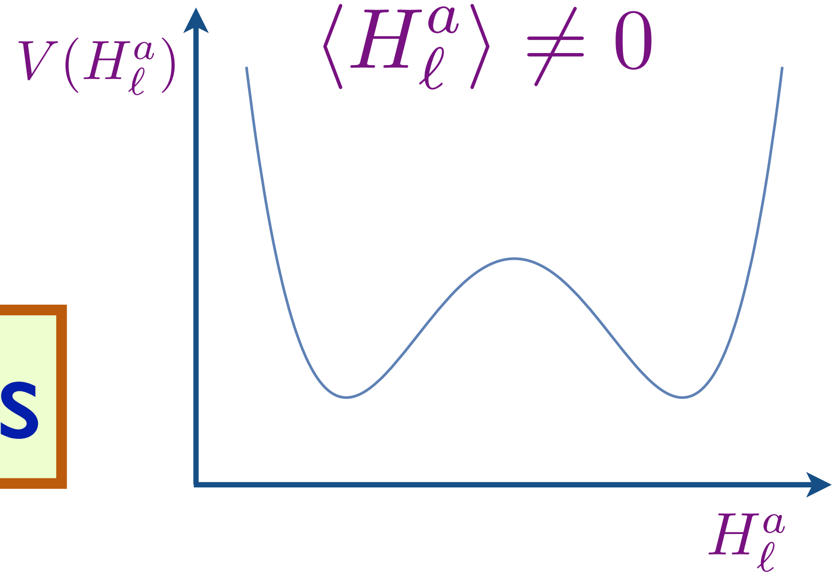
SU(2) gauge theory with N_h adjoint Higgs fields H_ℓ^a ($a = 1, 2, 3,\ \ell = 1 \ldots N_h$), with potential $V(H_\ell^a)$ and SU(2) gauge field A_μ^a. There is O(N_h) global flavor symmetry.

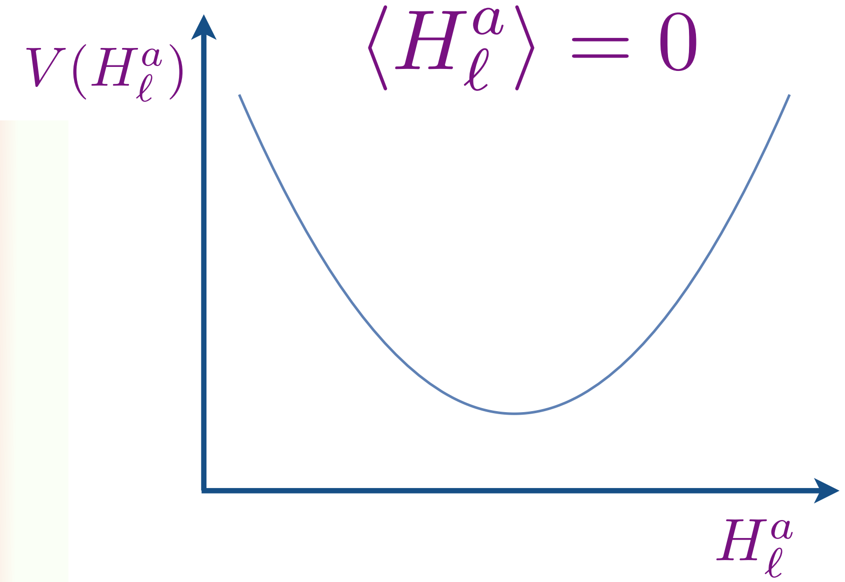
$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} \left(\partial_\mu H_\ell^a - \epsilon_{abc} A_\mu^b H_\ell^c\right)^2 + V(H_\ell^a)$$

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(H_\ell^a) &= s\, H_\ell^a H_\ell^a + u_0\, H_\ell^a H_\ell^a H_m^b H_m^b \\ &\quad + u_1 \left(H_\ell^a H_m^a H_\ell^b H_m^b - \frac{1}{N_h} H_\ell^a H_\ell^a H_m^b H_m^b \right) \end{aligned}$$

$N_h = 1$

Phase diagrams of SU(2) gauge theory





Higgs

- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles
- Monopole action $\sim \sqrt{-s}$, leading to an exponentially large confinement scale

Crossover

Confinement

s

A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

$N_h = 2$

- There is a global $\mathrm{O}(N_h)$ symmetry, and we can define a gauge-invariant order parameter

$$Q_{\ell m} = H^a_\ell H^a_m - \frac{\delta_{\ell m}}{N_h} H^a_n H^a_n$$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H^a_1 = (H_0, 0, 0) \quad , \quad H^a_2 = (0, 0, 0)$$

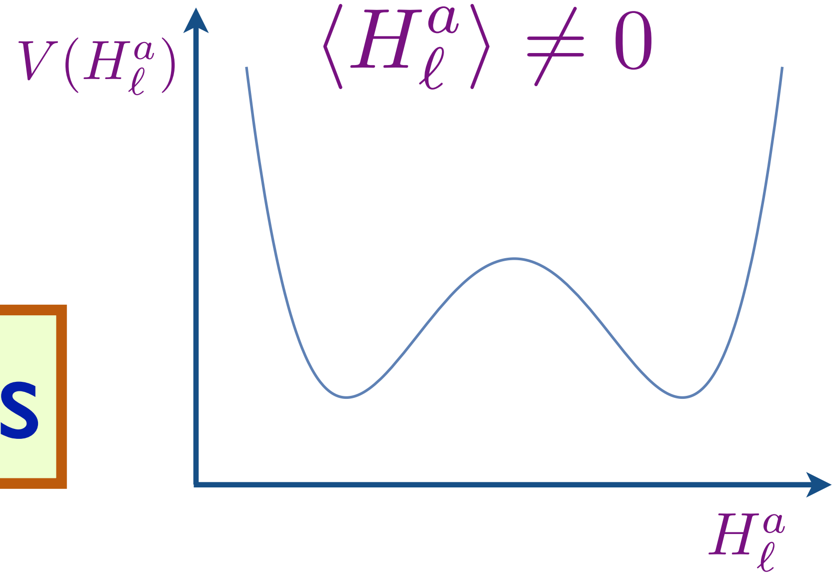
This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1$. But the global O(2) is broken because

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h = 2$

Phase diagrams of SU(2) gauge theory

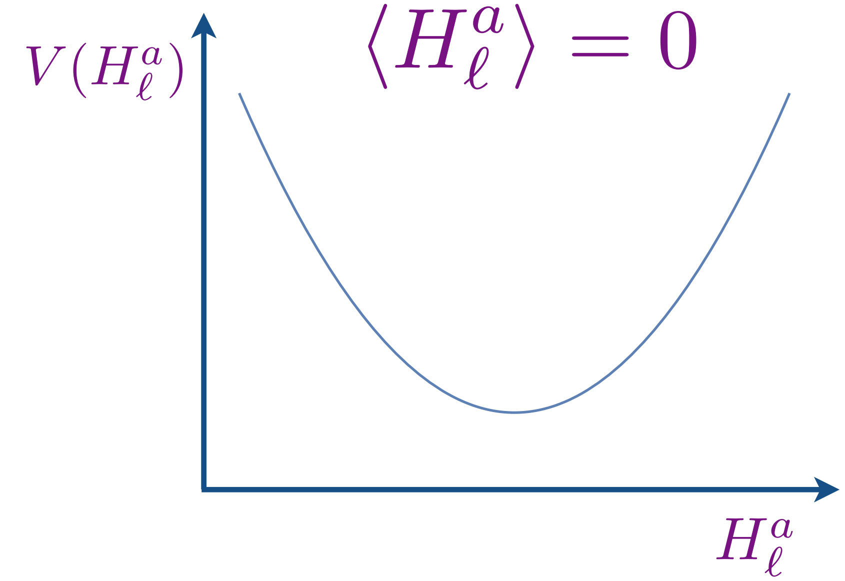
Higgs



$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines
- Broken O(2) symmetry, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

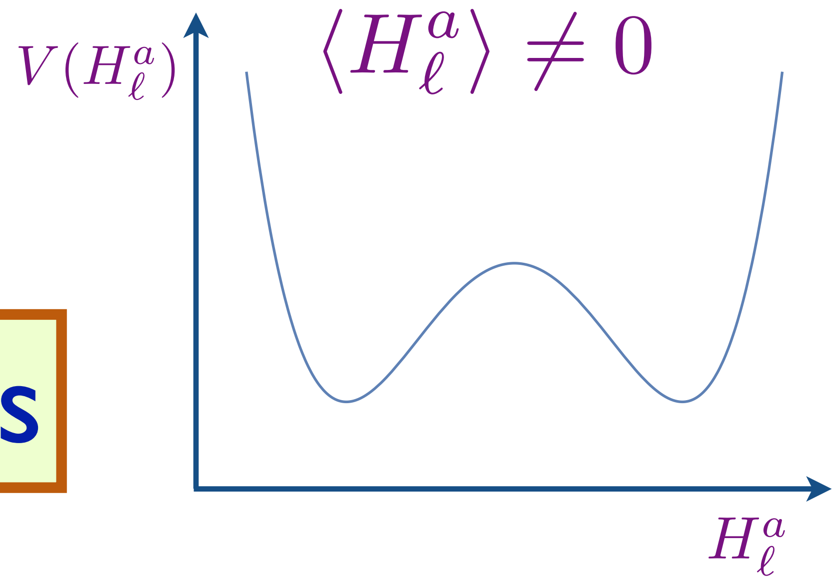


Confinement

s

$N_h = 2$

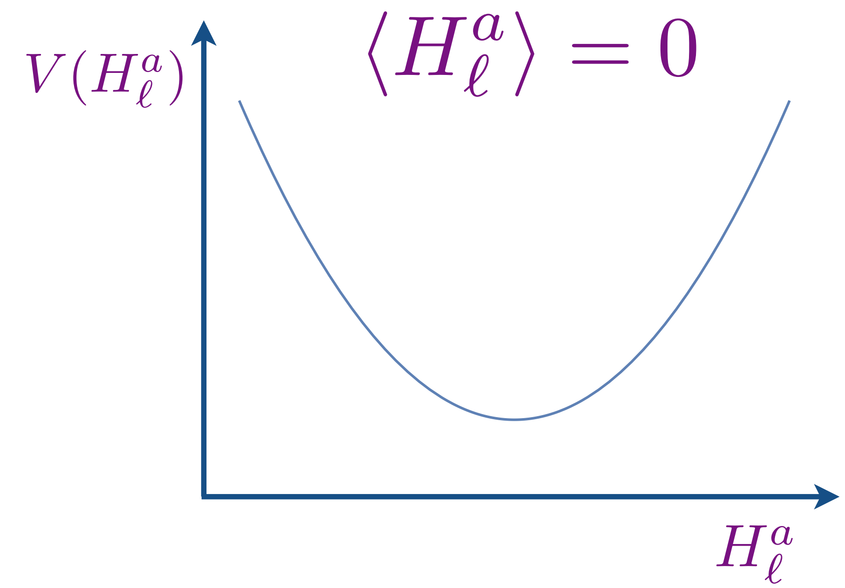
Phase diagrams of SU(2) gauge theory



Higgs

$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines
- Broken O(2) symmetry, $\langle Q_{\ell m} \rangle \neq 0$.



Confinement

Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition ?

s

$$\boxed{N_h = 2}$$

- There is a global O(N_h) symmetry, and we can define a gauge-invariant order parameter

$$Q_{\ell m} = H_\ell^a H_m^a - \frac{\delta_{\ell m}}{N_h} H_n^a H_n^a$$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0)$$

  This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1$. But the global O(2) is broken because

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

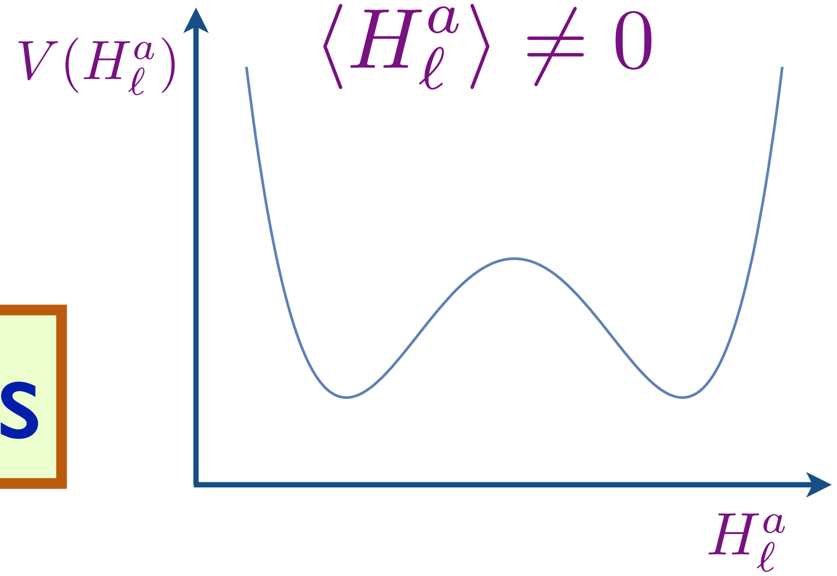
$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0)$$

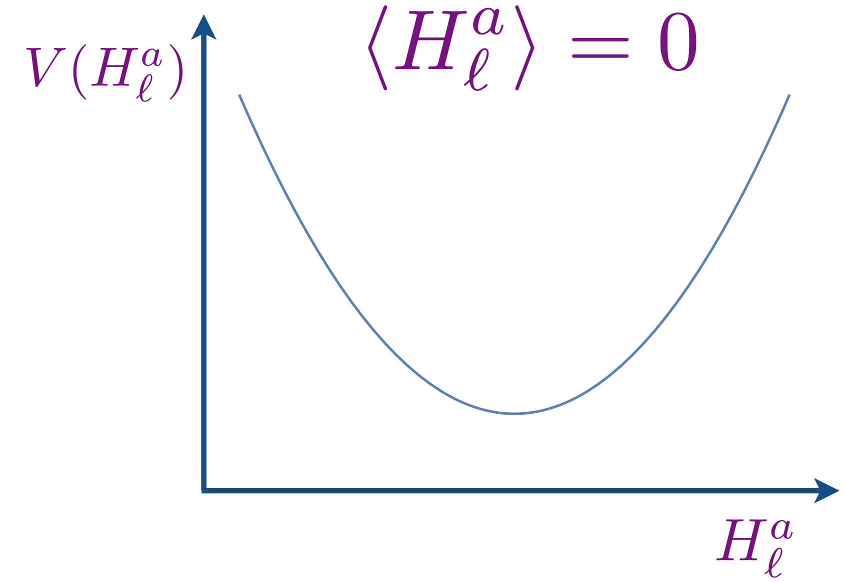
  This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the 'toric code'). But now the global O(2) is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$

$N_h = 2$

Phase diagrams of SU(2) gauge theory





Higgs

$u_1 > 0$

- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.

- Unbroken O(2) symmetry, $\langle Q_{\ell m} \rangle = 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement

s

$N_h = 3$

- For $u_1 < 0$, the Higgs condensate is of the form

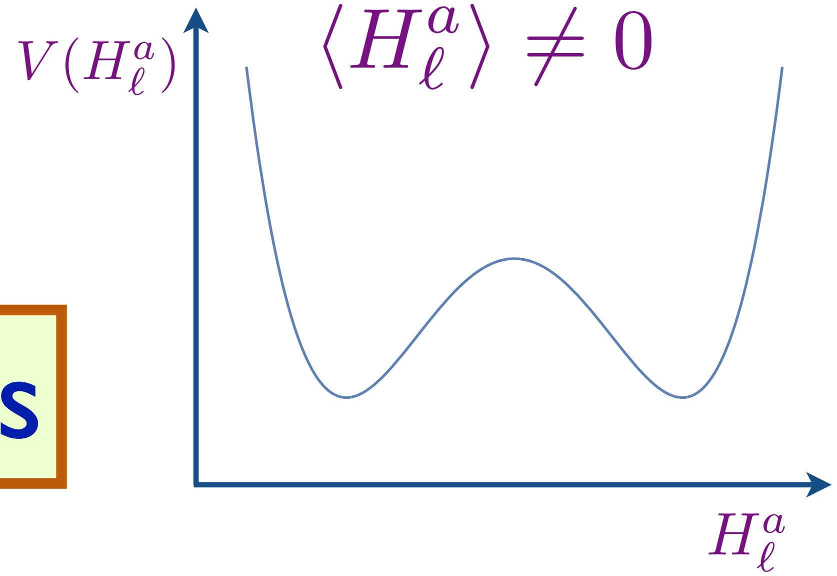
$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0) \quad , \quad H_3^a = (0, 0, 0)$$

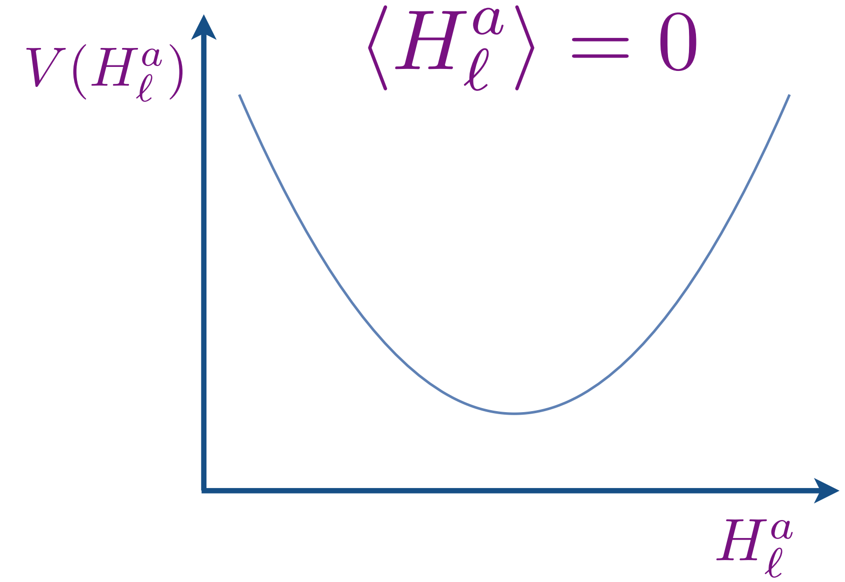
  This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(3) is broken to O(2) and

$$\langle Q_{\ell m} \rangle \neq 0$$

$N_h = 3$

Phase diagrams of SU(2) gauge theory





Higgs

$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines
- O(3) symmetry broken to O(2), $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

Confinement

Multi-universality of a Landau-allowed transition ?

s

$N_h = 3$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, 0, 0) \quad , \quad H_3^a = (0, 0, 0)$$

  This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(3) is broken to O(2) and

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0) \quad , \quad H_3^a = (0, 0, H_0)$$

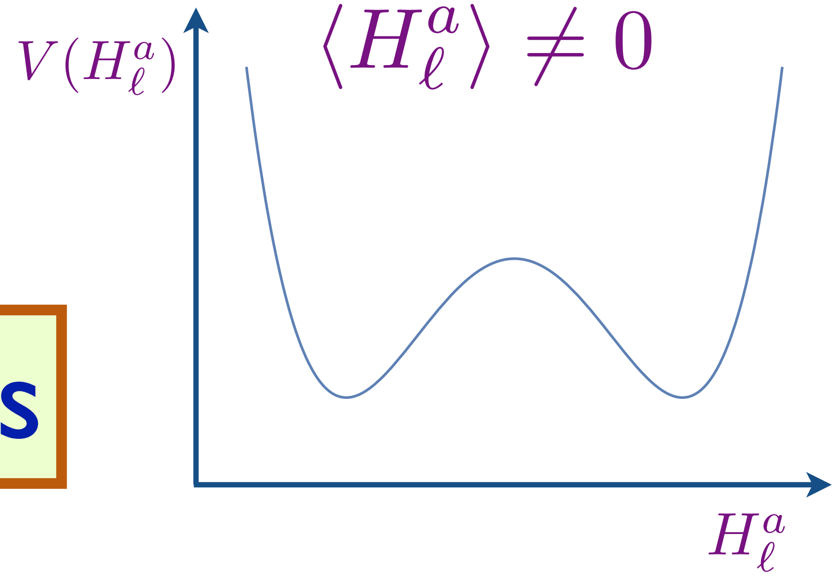
  This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the 'toric code'). But now the global O(3) is unbroken because

$$\langle Q_{\ell m} \rangle = 0$$

$N_h = 3$

Phase diagrams of SU(2) gauge theory

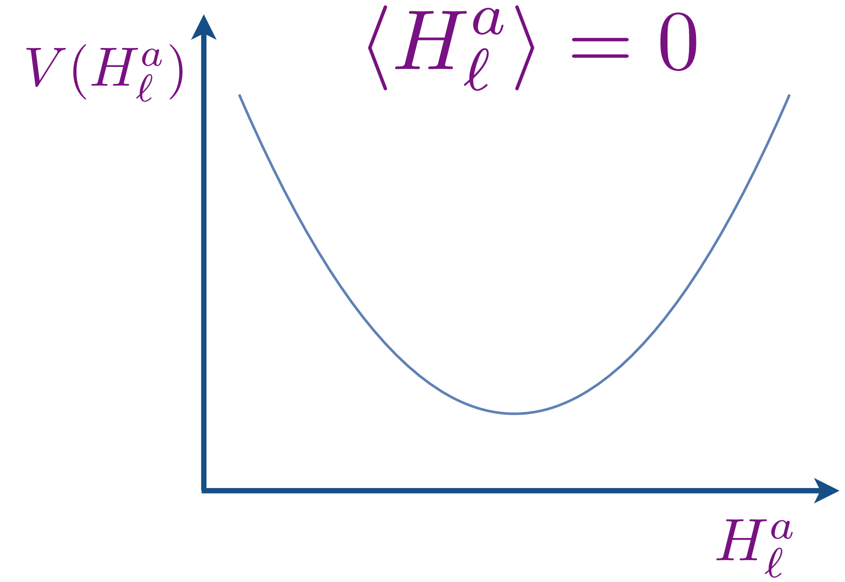
Higgs



$u_1 > 0$

- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.
- Unbroken O(3) symmetry, $\langle Q_{\ell m} \rangle = 0$.

Possible Deconfined critical SU(2) gauge theory



Confinement

s

$\boxed{N_h \geq 4}$

$N_h \geq 4$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_{\ell>1}^a = (0, 0, 0)$$

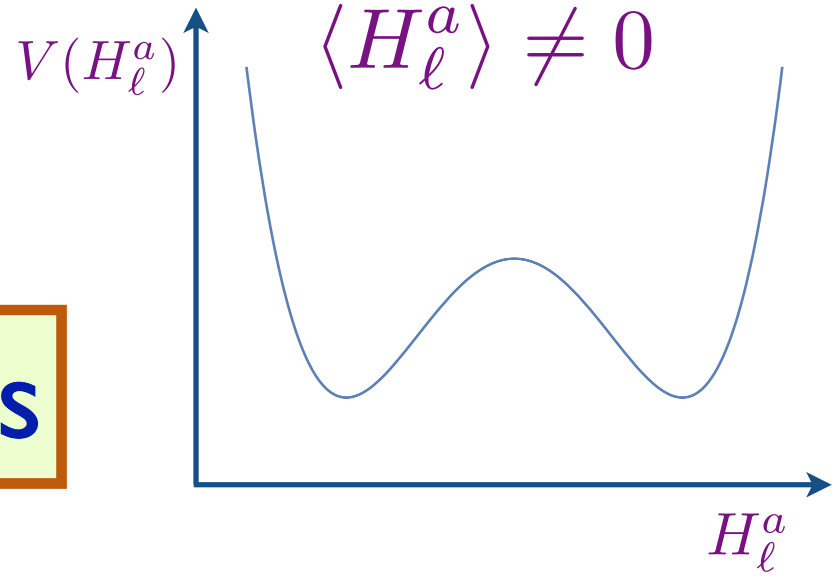
This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global $\mathrm{O}(N_h)$ is broken to $\mathrm{O}(N_h - 1)$ and

$$\langle Q_{\ell m} \rangle \neq 0$$

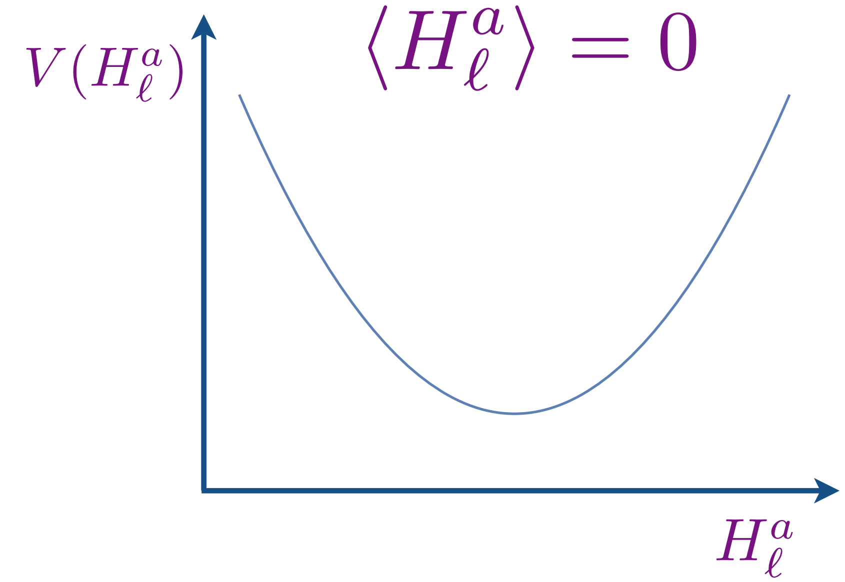
$\boxed{N_h \geq 4}$

Phase diagrams of SU(2) gauge theory

$\langle H^a_\ell \rangle \neq 0$



$\langle H^a_\ell \rangle = 0$



Higgs

Confinement

$u_1 < 0$

- Condensation of H^a breaks SU(2) to U(1), which confines

- O(N_h) symmetry broken to O($N_h - 1$), $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

Multi-universality of a Landau-allowed transition ?

s

$\boxed{N_h \geq 4}$

$N_h \geq 4$

- For $u_1 < 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_{\ell>1}^a = (0, 0, 0)$$

This breaks gauge SU(2) to U(1), and the U(1) confines, as for $N_h = 1, 2$. The global O(N_h) is broken to O($N_h - 1$) and

$$\langle Q_{\ell m} \rangle \neq 0$$

- For $u_1 > 0$, the Higgs condensate is of the form

$$H_1^a = (H_0, 0, 0) \quad , \quad H_2^a = (0, H_0, 0) \quad , \quad H_3^a = (0, 0, H_0) \quad , \quad H_{\ell>3}^a = (0, 0, 0)$$

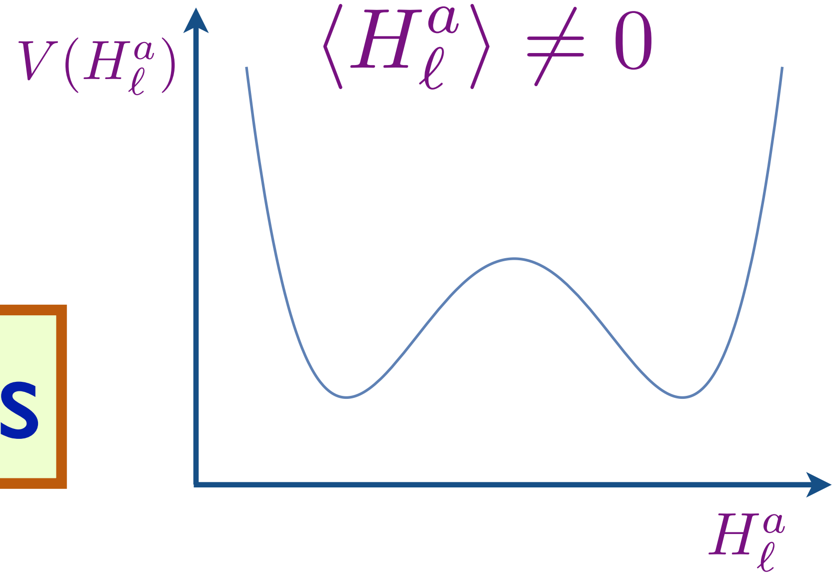
This breaks gauge SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order (as in the 'toric code'). But now the global O(N_h) is broken to O($N_h - 3$)$\times$O(3), and

$$\langle Q_{\ell m} \rangle \neq 0$$

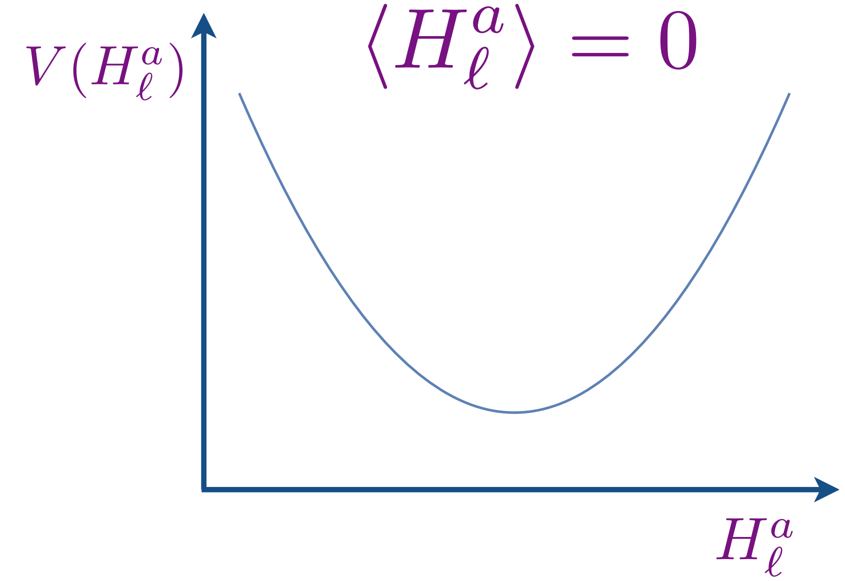
$N_h \geq 4$

Phase diagrams of SU(2) gauge theory

$\langle H_\ell^a \rangle \neq 0$



$\langle H_\ell^a \rangle = 0$



Higgs

$u_1 > 0$

Confinement

- Condensation of H^a breaks SU(2) to $\mathbb{Z}_2$, leading to $\mathbb{Z}_2$ topological order.

- $\mathrm{O}(N_h)$ symmetry broken to $\mathrm{O}(N_h - 3)\times\mathrm{O}(3)$, $\langle Q_{\ell m} \rangle \neq 0$.

Possible Deconfined critical SU(2) gauge theory

s

1. SU(2) gauge theory with adjoint scalars

Confining and Higgs phases in 2+1 dimensions

2. SU(2) gauge theory of fluctuating antiferromagnetism

3. Role of fermions

Pseudogap as a FL phase*

Gauge theory of fluctuating antiferromagnetism

We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned} \mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\rm int} \end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^p(i)$, $p = x, y, z$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^p c_{i,\beta} + V_\Phi$$

Gauge theory of fluctuating antiferromagnetism

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^p \Phi^p(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

Field	Symbol	Statistics	$\mathrm{SU(2)_{gauge}}$	$\mathrm{SU(2)_{spin}}$	$\mathrm{U(1)_{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \to V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \to R_i V_i^\dagger$$

$$\sigma^a H^a(i) \to V_i\, \sigma^b H^b(i)\, V_i^\dagger .$$

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Field	Symbol	Statistics	$\mathrm{SU}(2)_{\mathrm{gauge}}$	$\mathrm{SU}(2)_{\mathrm{spin}}$	$\mathrm{U}(1)_{\mathrm{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Begin with an effective theory for the Higgs field alone

Gauge theory of fluctuating antiferromagnetism

We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):
$N_h = 1$,
$\boldsymbol{K} = (\pi, \pi)$,
$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$

Bidirectional incommensurate correlations (hole doped cuprates):
$N_h = 4$,
$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,
$H^a(i) = \mathrm{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$

Gauge theory of fluctuating antiferromagnetism

For the hole-doped cuprates, $N_h = 4$, we define complex Higgs fields

$$\mathcal{H}_x^a = H_1^a + i H_2^a \quad , \quad \mathcal{H}_y^a = H_3^a + i H_4^a \, .$$

The SU(2) gauge theory is

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left|\partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c\right|^2 + \frac{1}{2}\left|\partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c\right|^2 \\
&\quad + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}_{x,y}^a) \\
F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\
V(\mathcal{H}_{x,y}^a) &= s\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right) + u_0\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 \\
&\quad \frac{u_1}{4}\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 + \frac{u_2}{2}\left[\left|\mathcal{H}_x^a\mathcal{H}_x^a\right|^2 + \left|\mathcal{H}_y^a\mathcal{H}_y^a\right|^2\right] \\
&\quad + u_3\left(\left|\mathcal{H}_x^a\mathcal{H}_y^a\right|^2 + \left|\mathcal{H}_x^a\mathcal{H}_y^{a*}\right|^2\right) .
\end{aligned}$$

Gauge theory of fluctuating antiferromagnetism

There are multiple order parameters for different broken symmetries (Note: spin rotations are preserved and there is no SDW order)

- Ising nematic order

$$\phi = \mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $2\boldsymbol{K}_{x,y}$

$$\Phi_x = \mathcal{H}_x^a\mathcal{H}_x^a \quad , \quad \Phi_y = \mathcal{H}_y^a\mathcal{H}_y^a$$

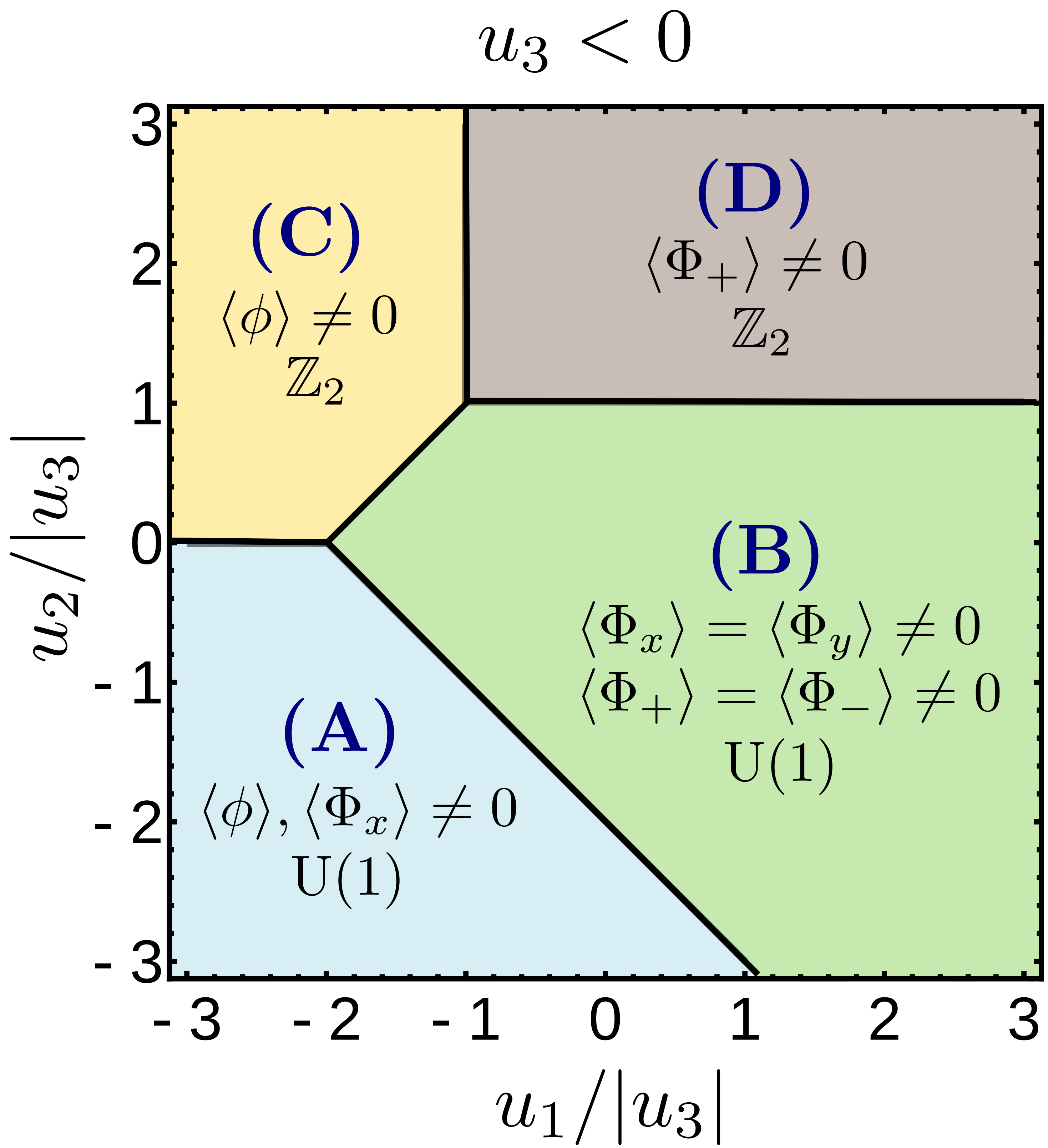
- Charge density wave (CDW) order at wavevectors $\boldsymbol{K}_x \pm \boldsymbol{K}_y$

$$\Phi_+ = \mathcal{H}_x^a\mathcal{H}_y^a \quad , \quad \Phi_- = \mathcal{H}_x^a\mathcal{H}_y^{a*}$$

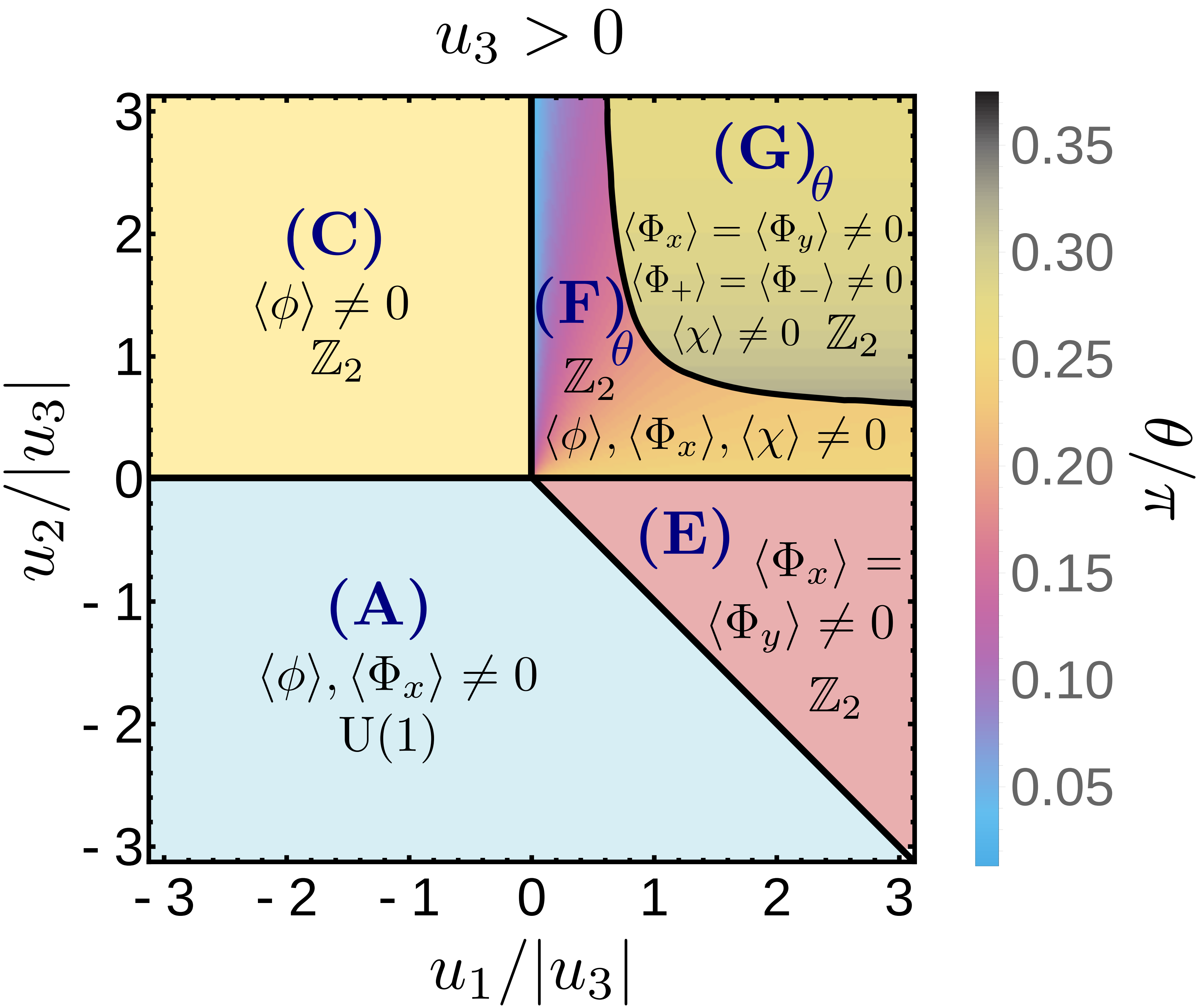
- (Modulated) scalar spin chirality

$$\chi_{ijk} = \epsilon_{abc}H^a(\boldsymbol{r}_i)H^b(\boldsymbol{r}_j)H^c(\boldsymbol{r}_k)$$

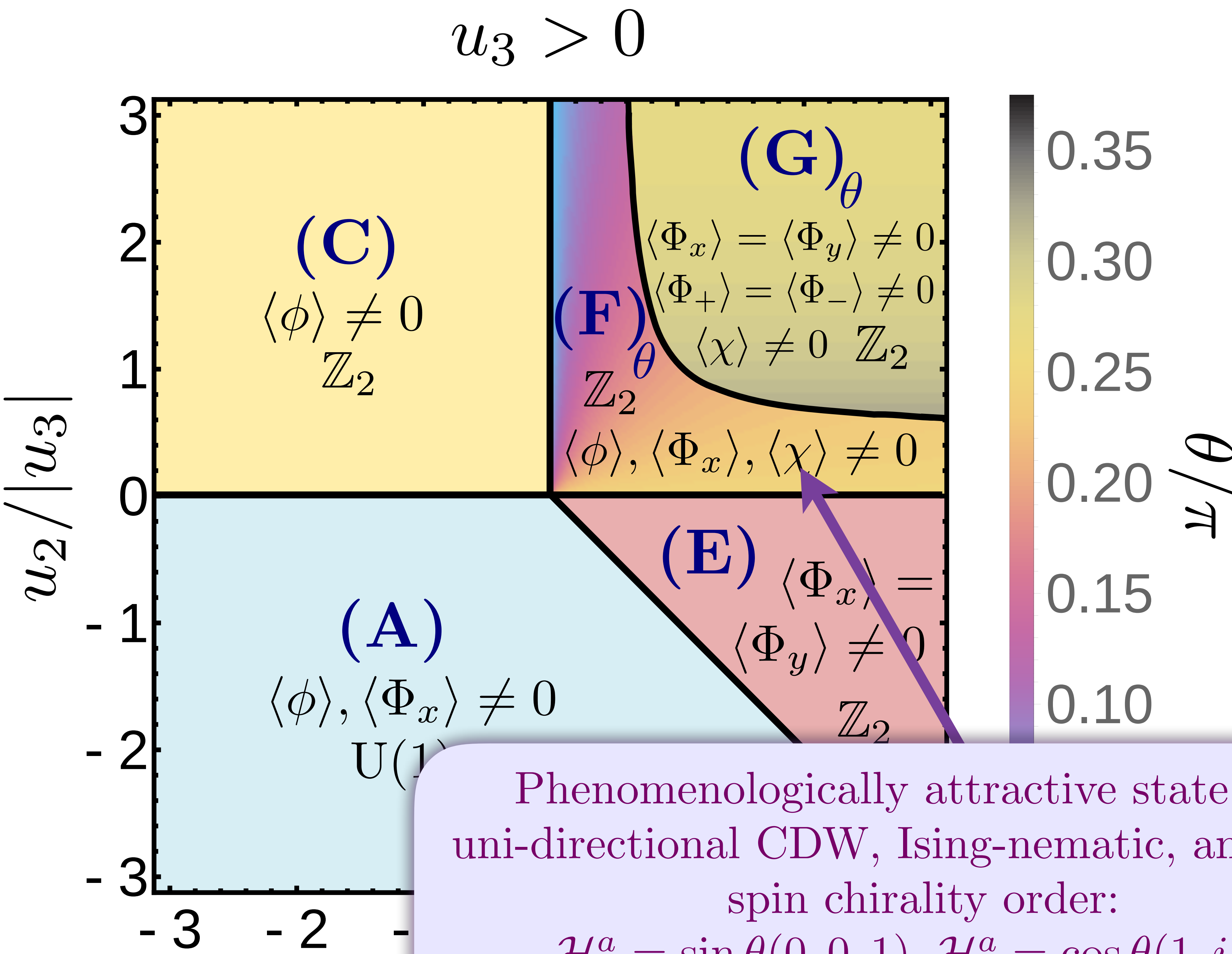
Broken symmetries and topological order in the Higgs phase



Broken symmetries and topological order in the Higgs phase



Broken symmetries and topological order in the Higgs phase



Phenomenologically attractive state with uni-directional CDW, Ising-nematic, and scalar spin chirality order:

$$\mathcal{H}_x^a = \sin\theta(0,0,1),\ \mathcal{H}_y^a = \cos\theta(1,i,0)$$

i.e. short-range collinear spin correlations along x, and short-range spiral spin correlations along y.

1. SU(2) gauge theory with adjoint scalars

Confining and Higgs phases in 2+1 dimensions

2. SU(2) gauge theory of fluctuating antiferromagnetism

3. Role of fermions

Pseudogap as a FL phase*

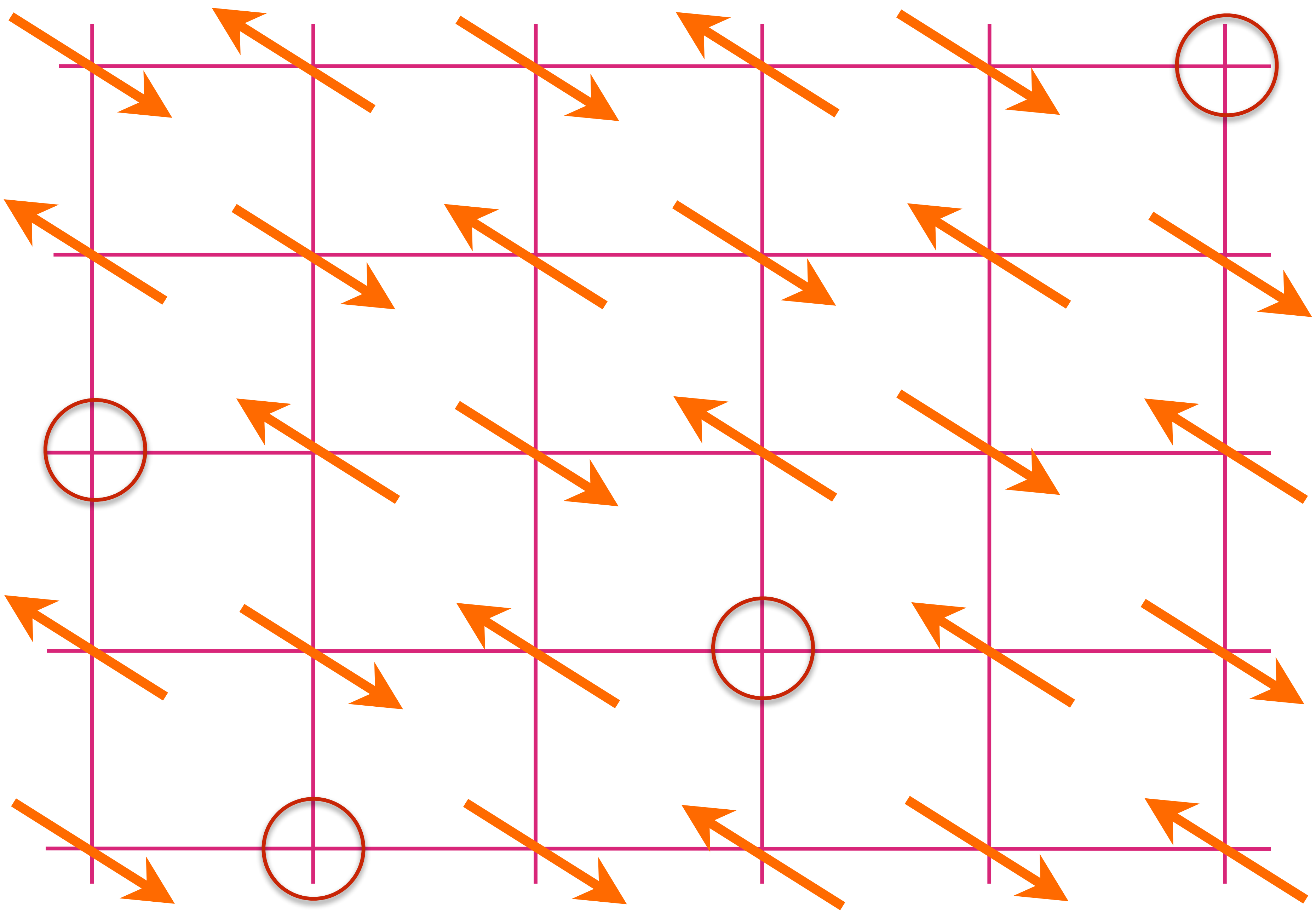
Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{\rm gauge}$	$SU(2)_{\rm spin}$	$U(1)_{\rm e.m.charge}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

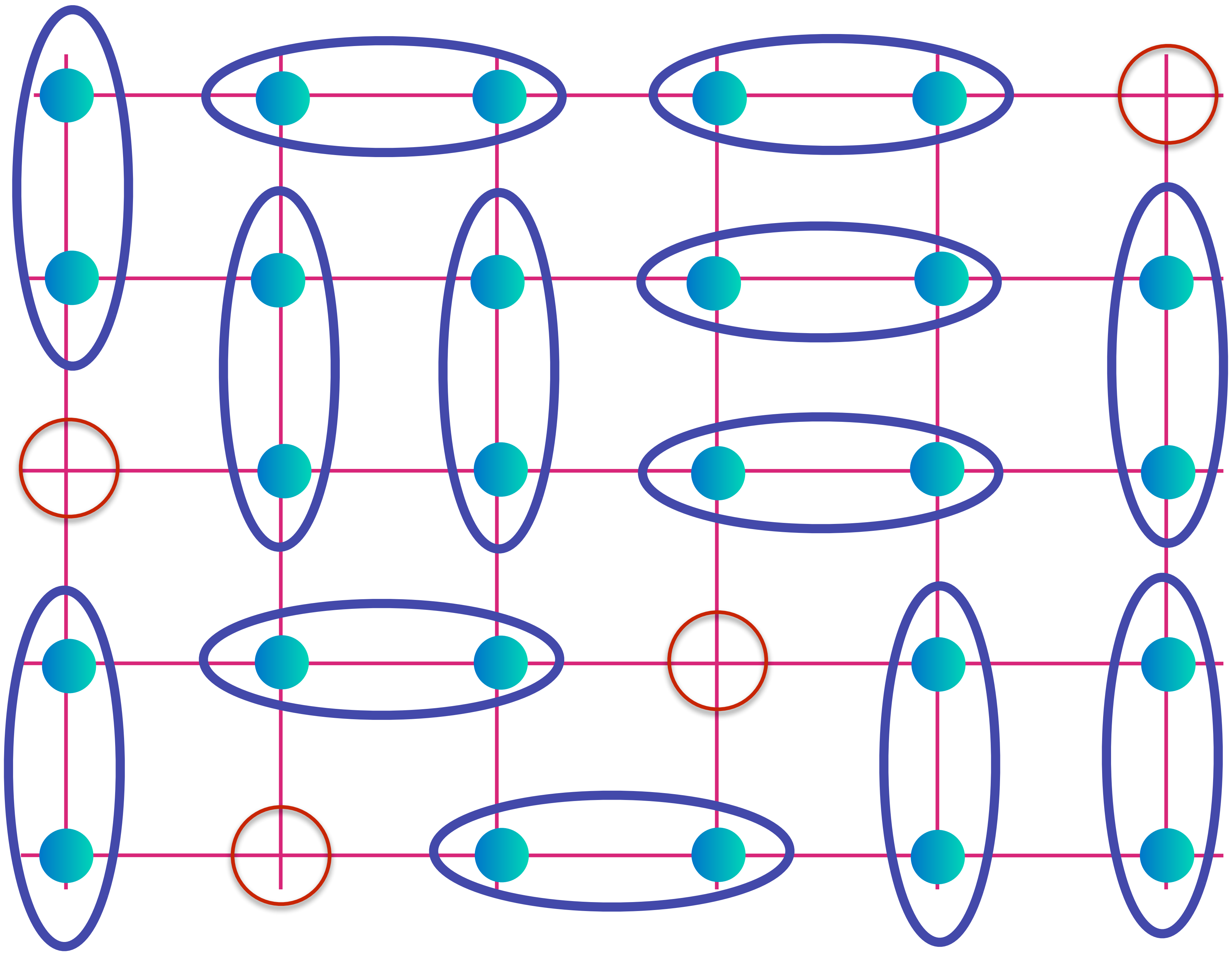
Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

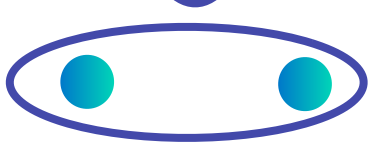
SU(2) gauge theory: fractionalize the SDW order parameter into the Higgs field (H) and the spinons (R); fractionalize the electron (c) into chargons (ψ) and spinons (R). When the Higgs field is condensed, the ψ fermions and R bosons are deconfined particles in an algebraic charge liquid (ACL) state, but with a strong residual attractive interaction from the t_{ij}.



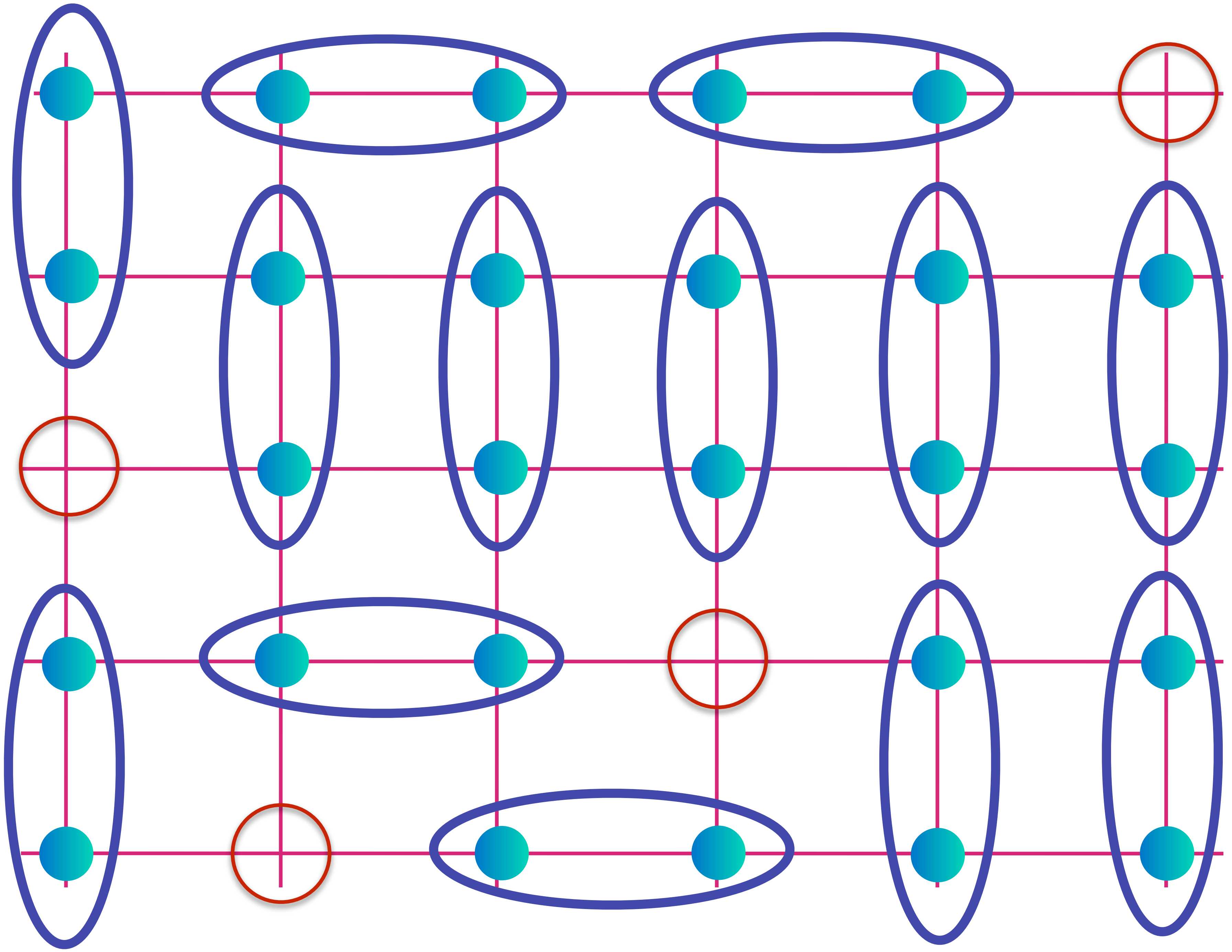
Anti-ferromagnet with p holes per square

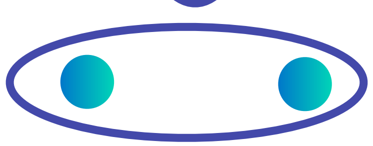


ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



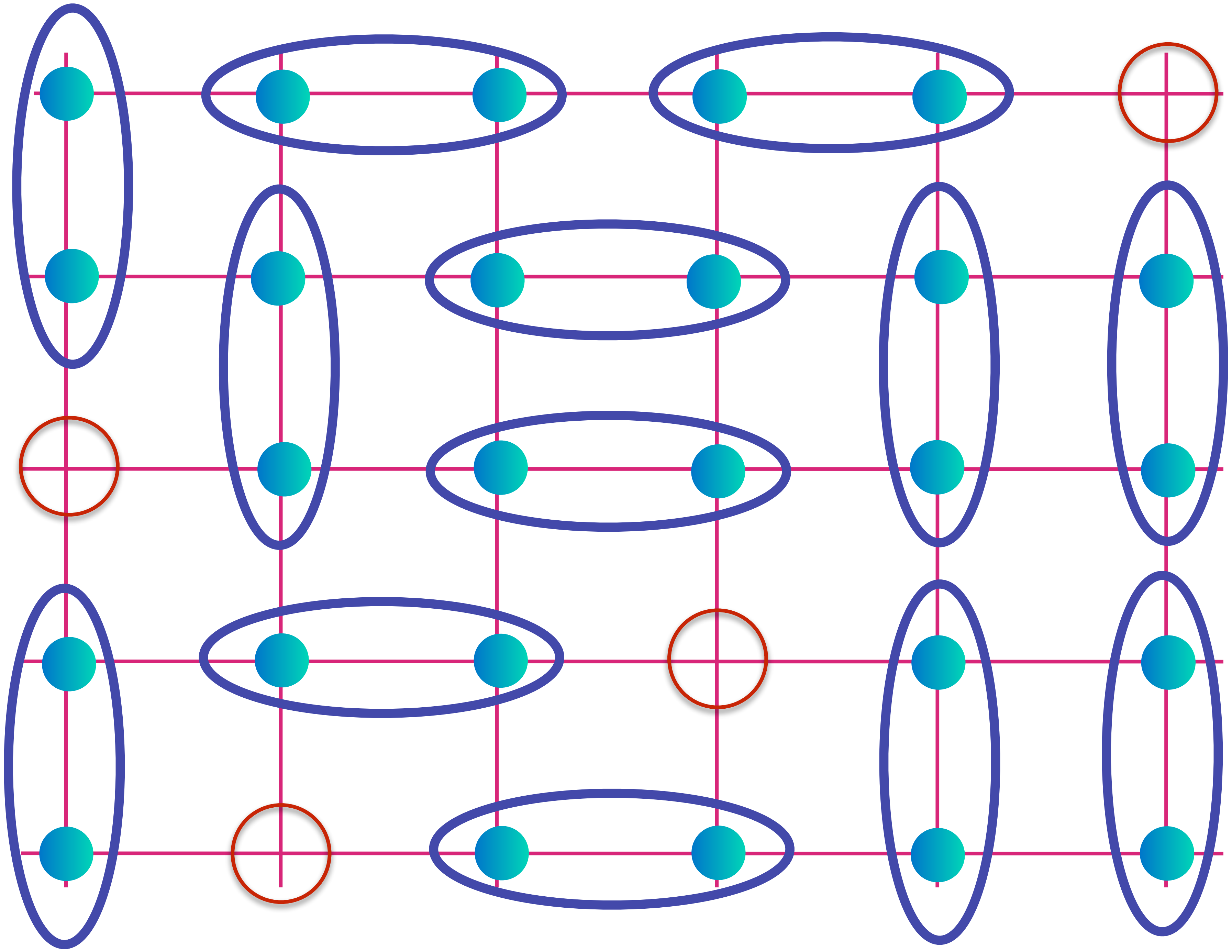
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



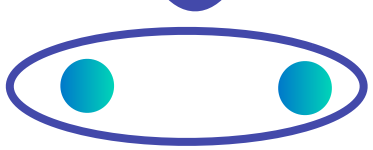


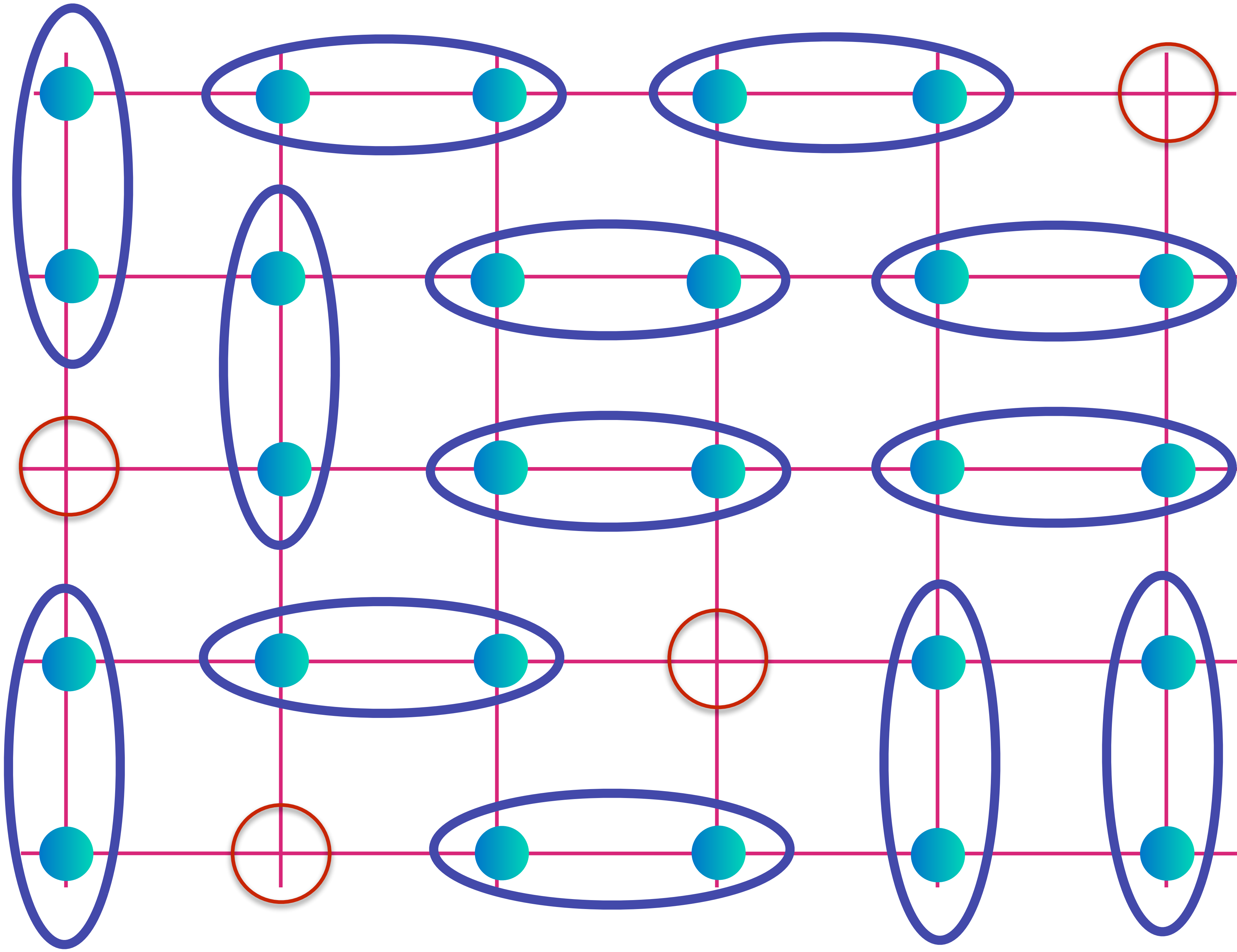
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

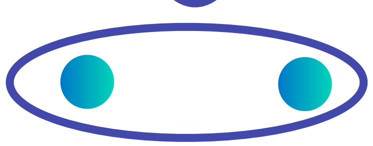


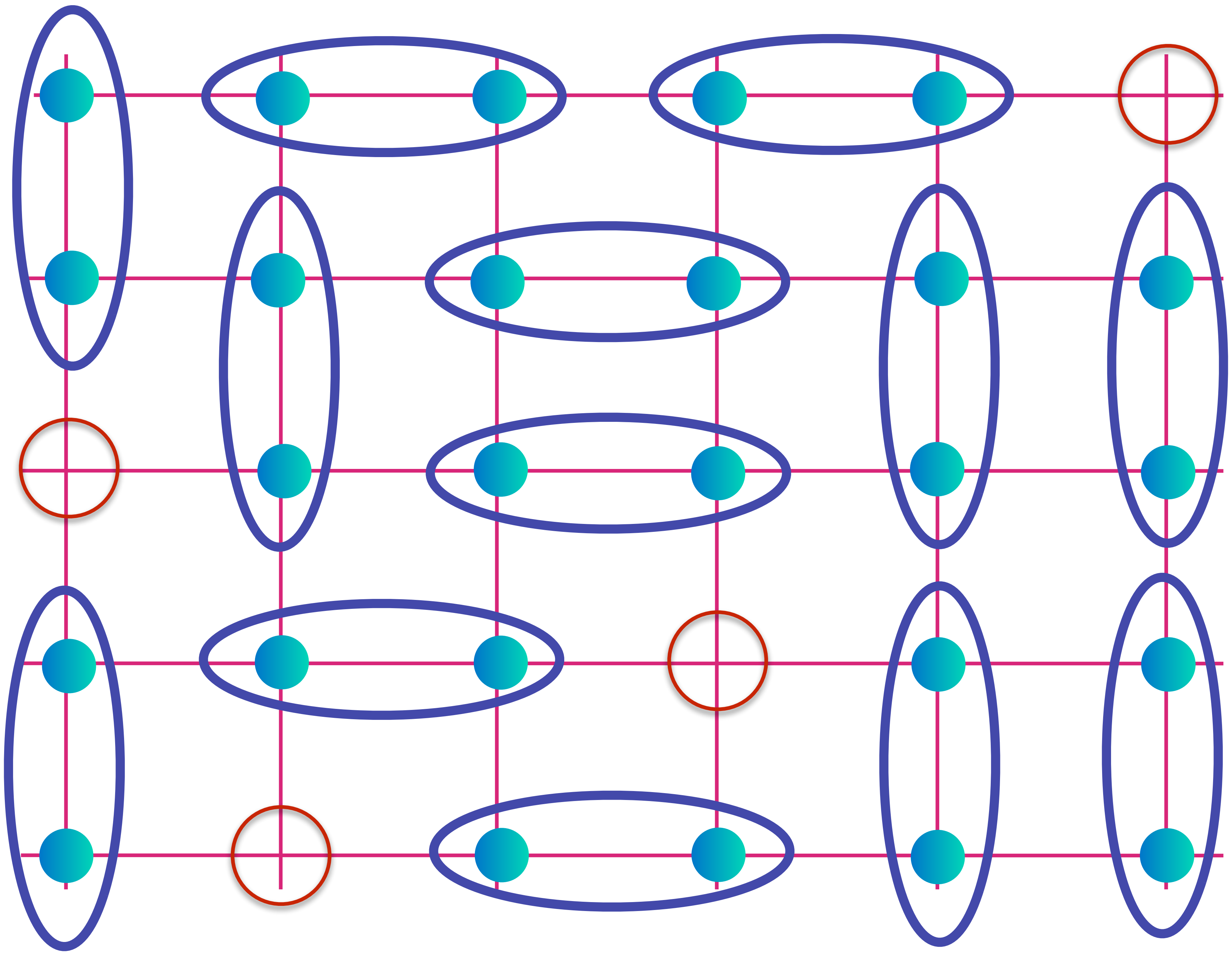
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

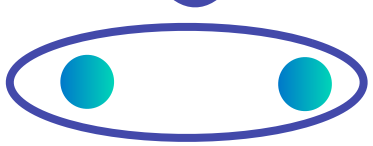


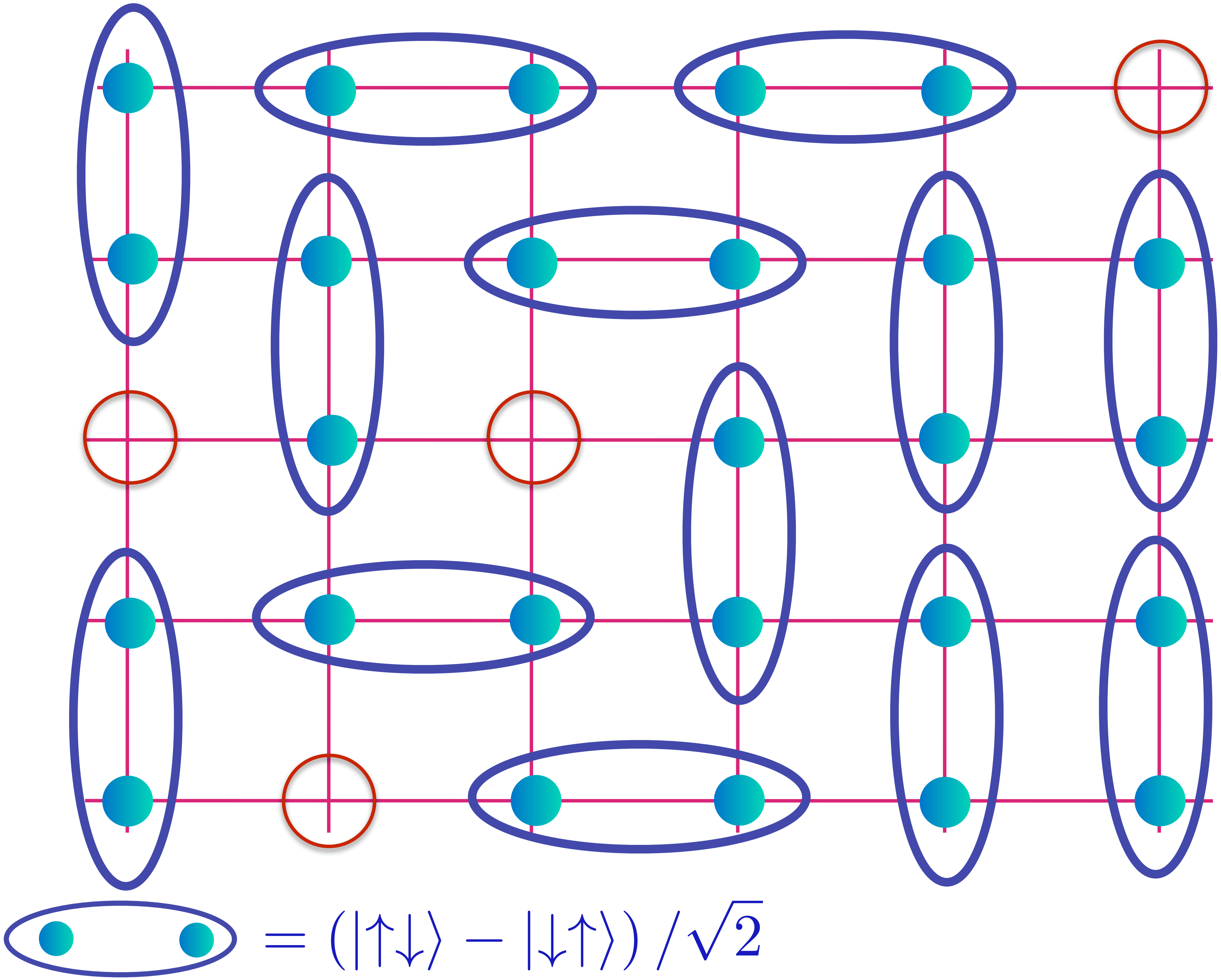
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



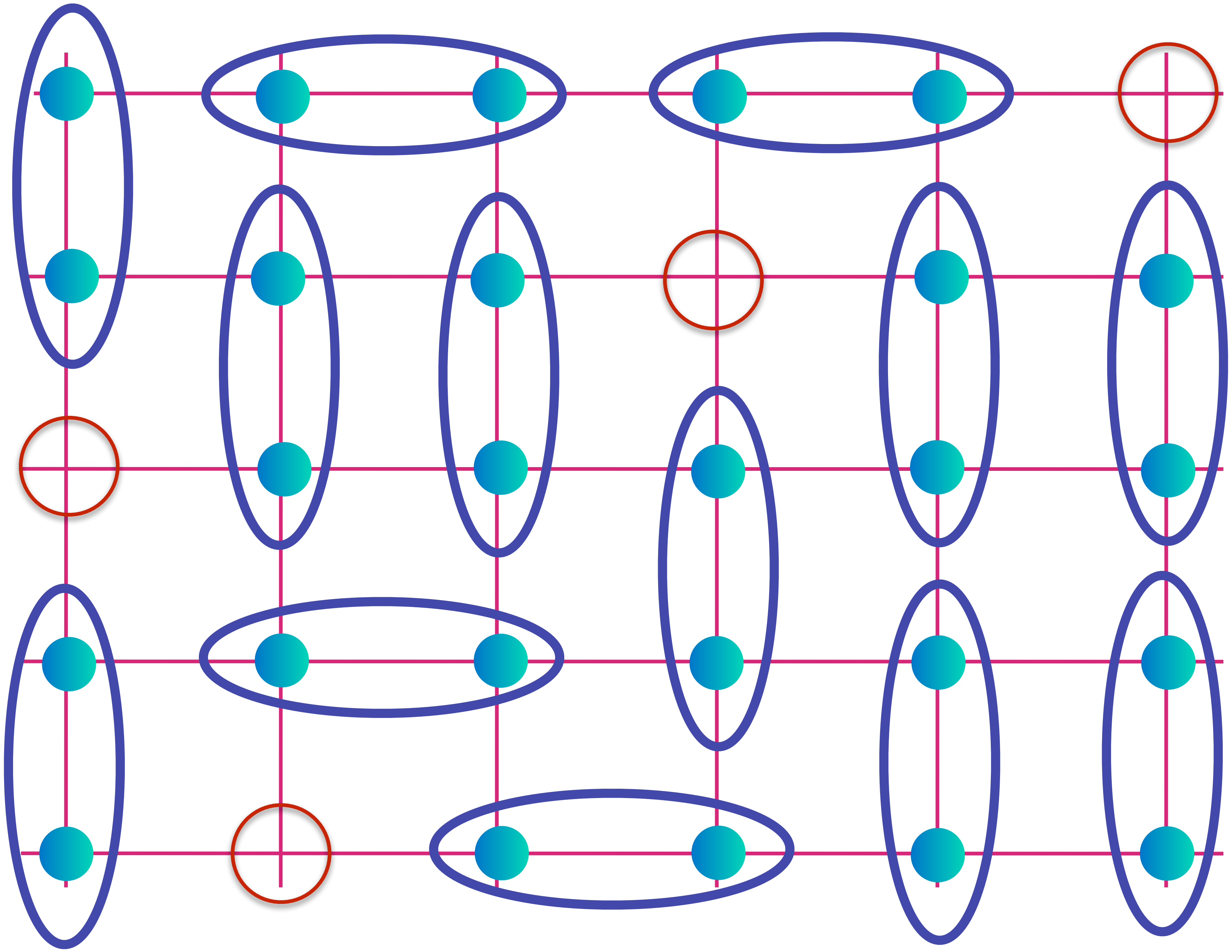
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



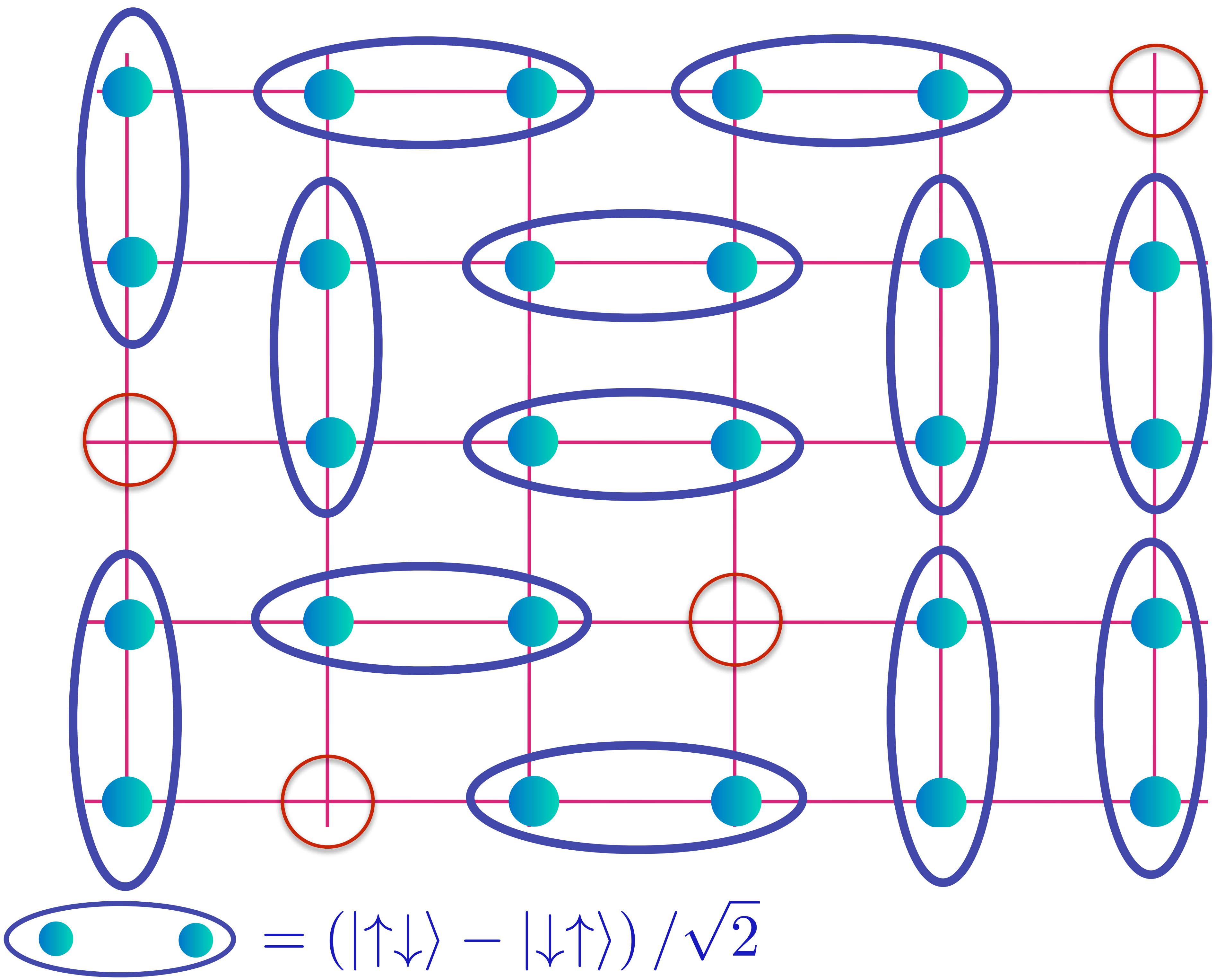
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

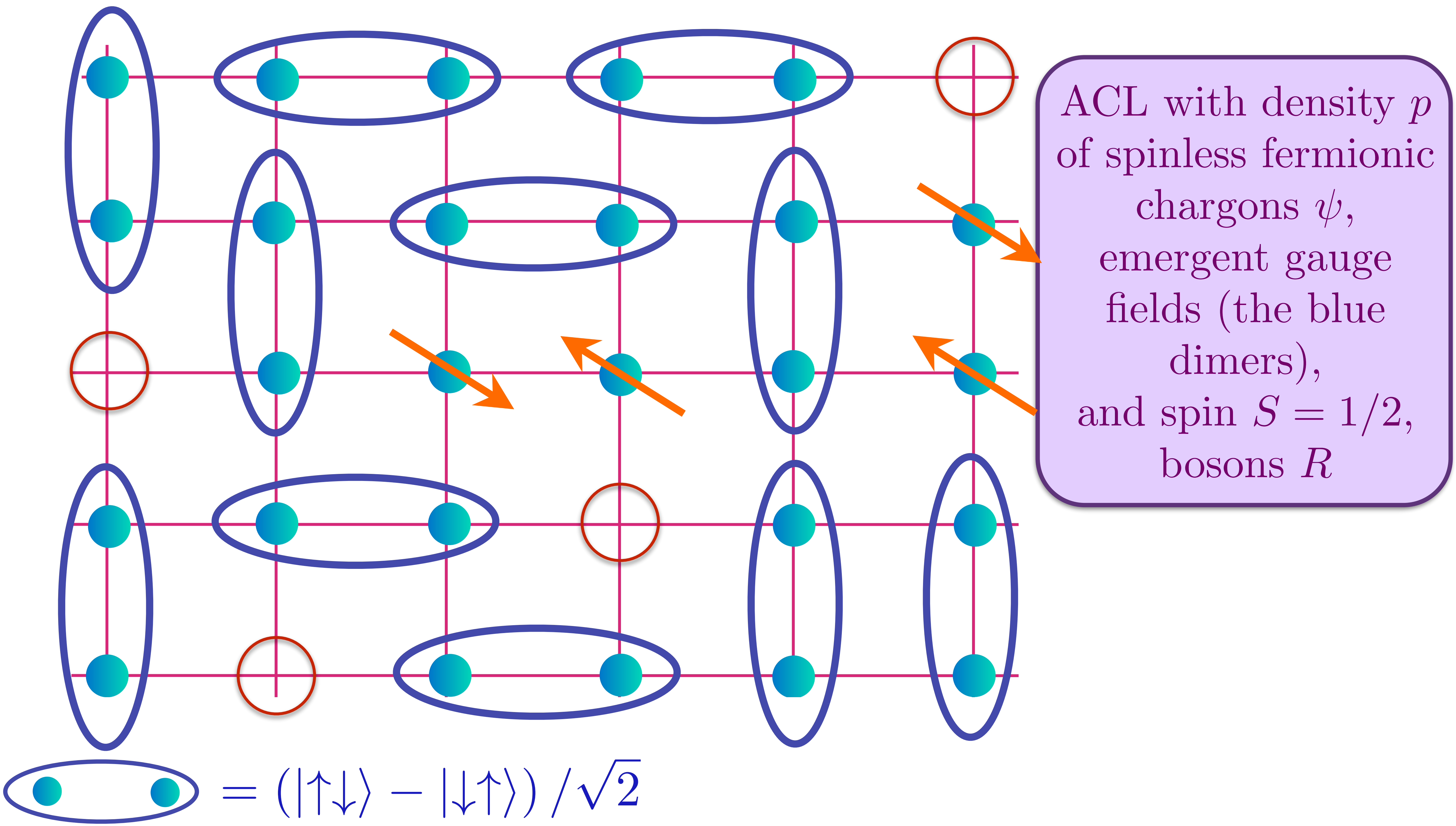


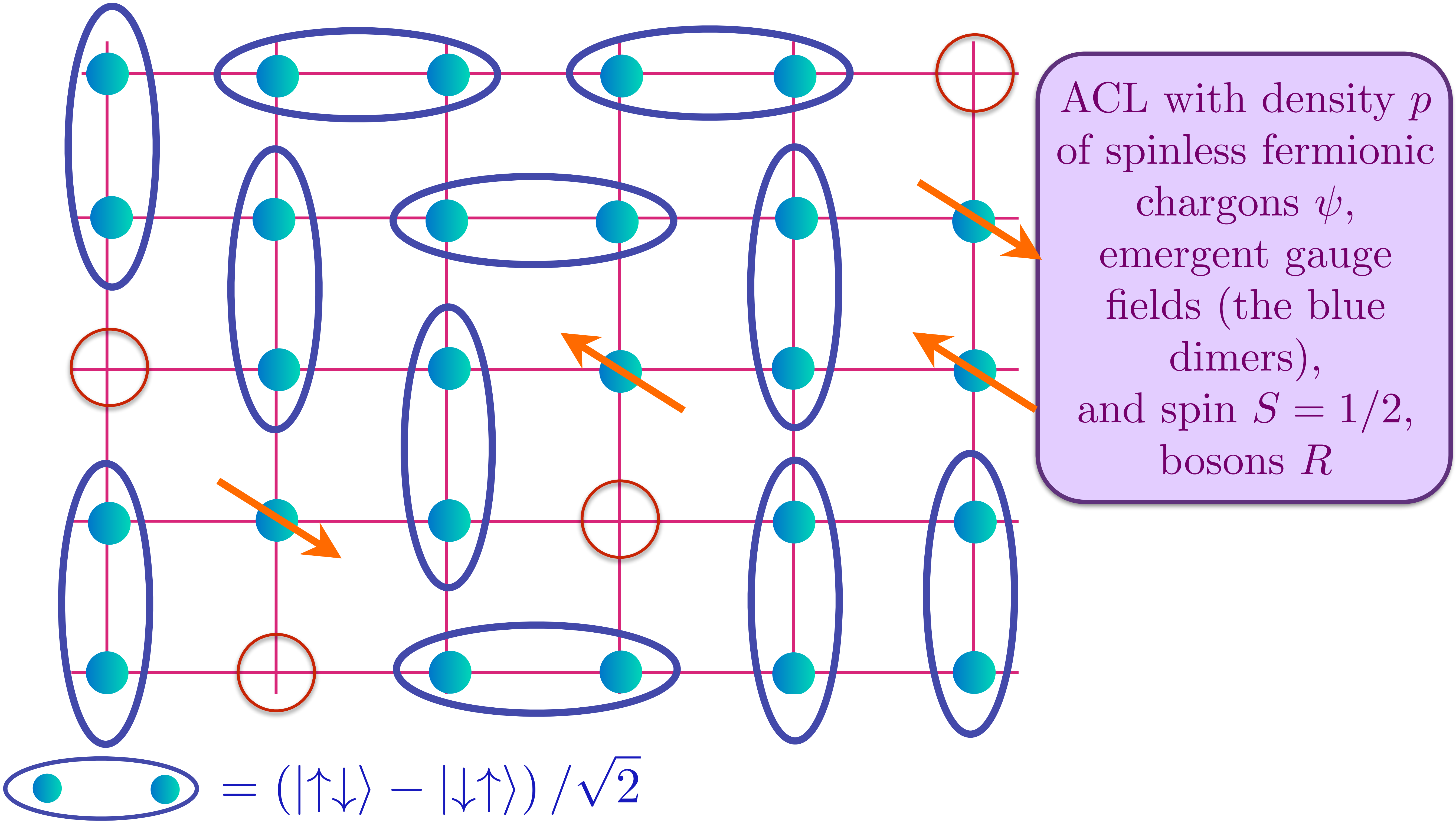
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

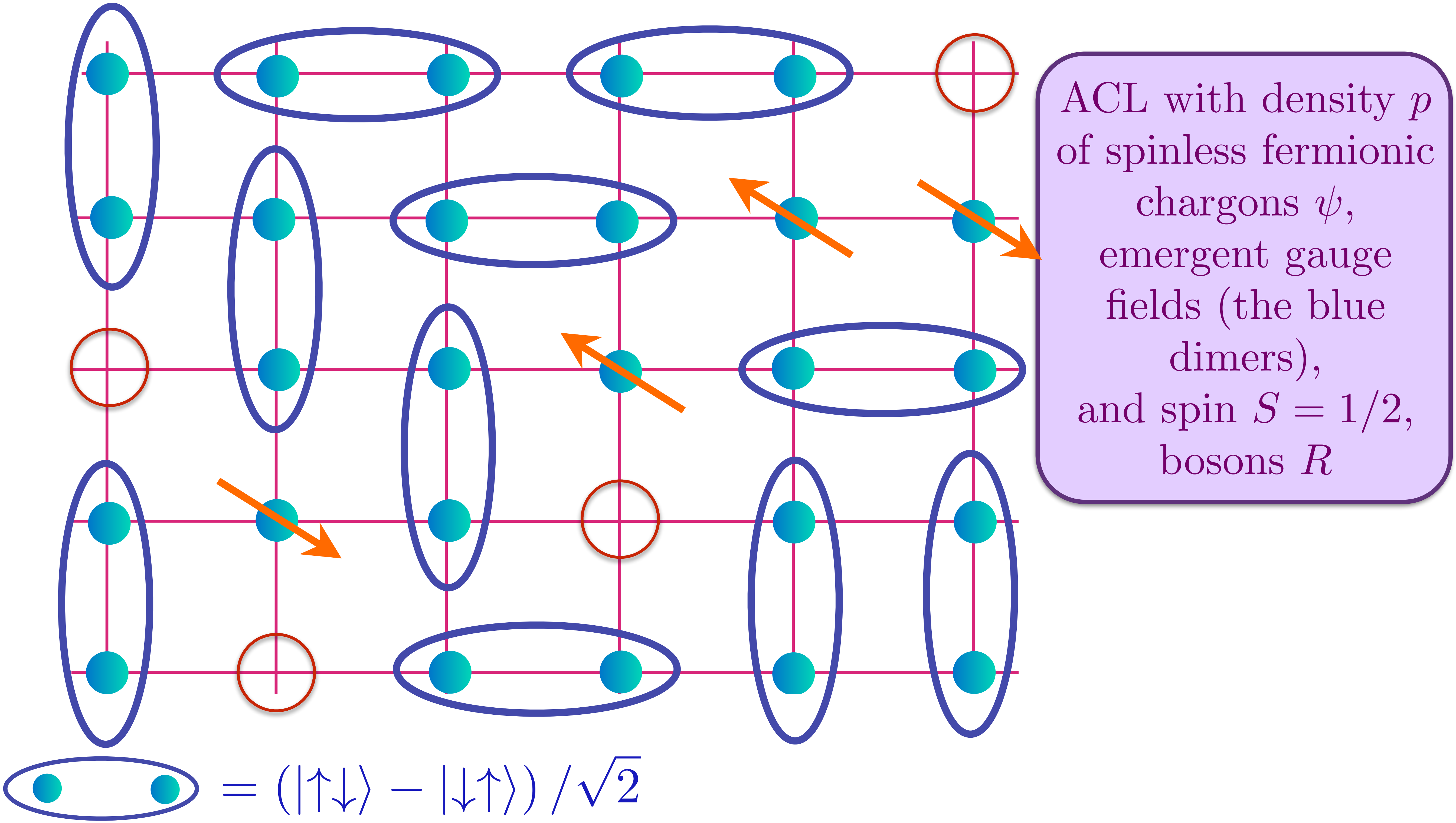
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

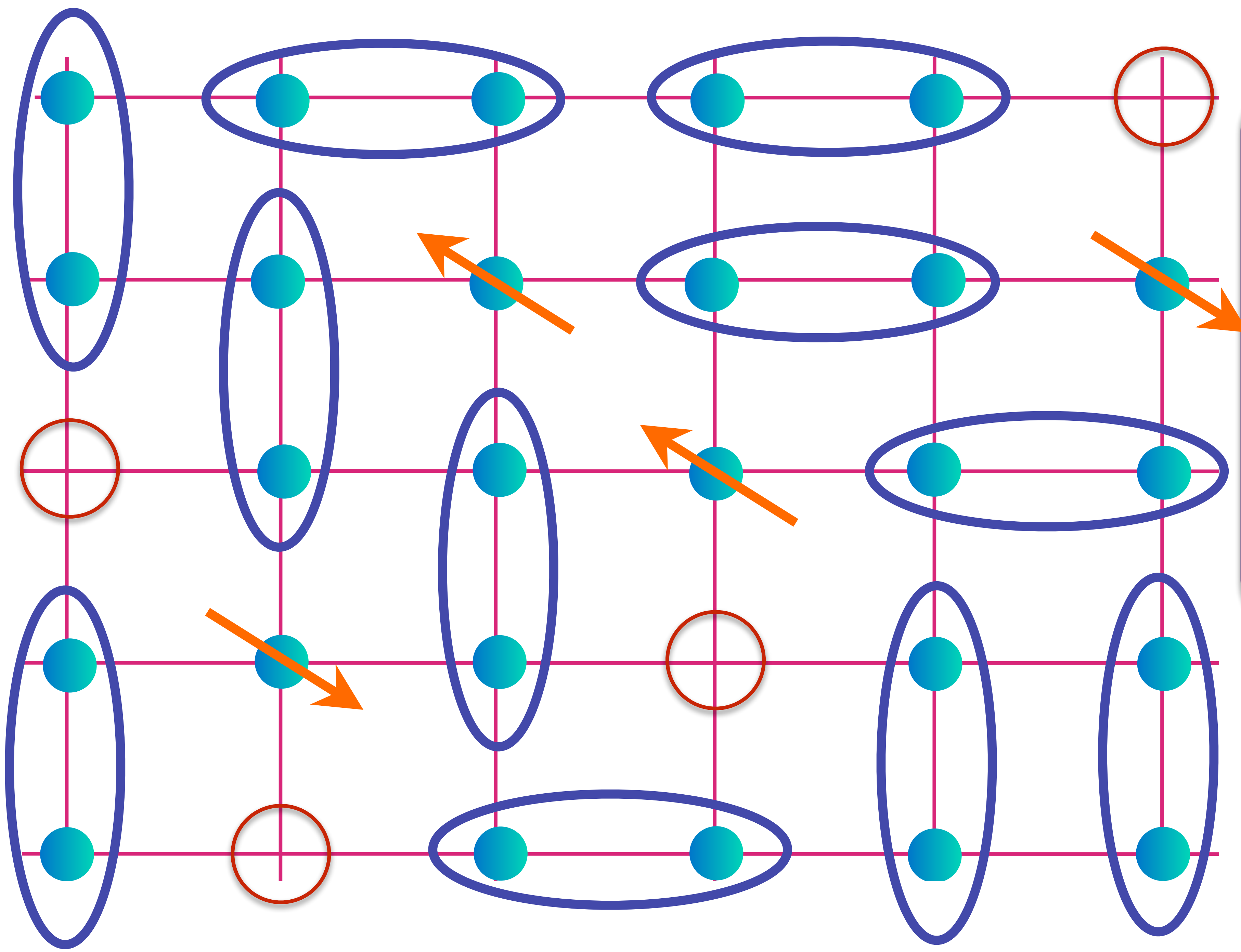


ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

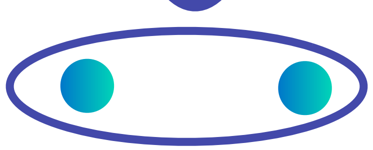

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S=1/2$, bosons R
$= \left(\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle\right)/\sqrt{2}$

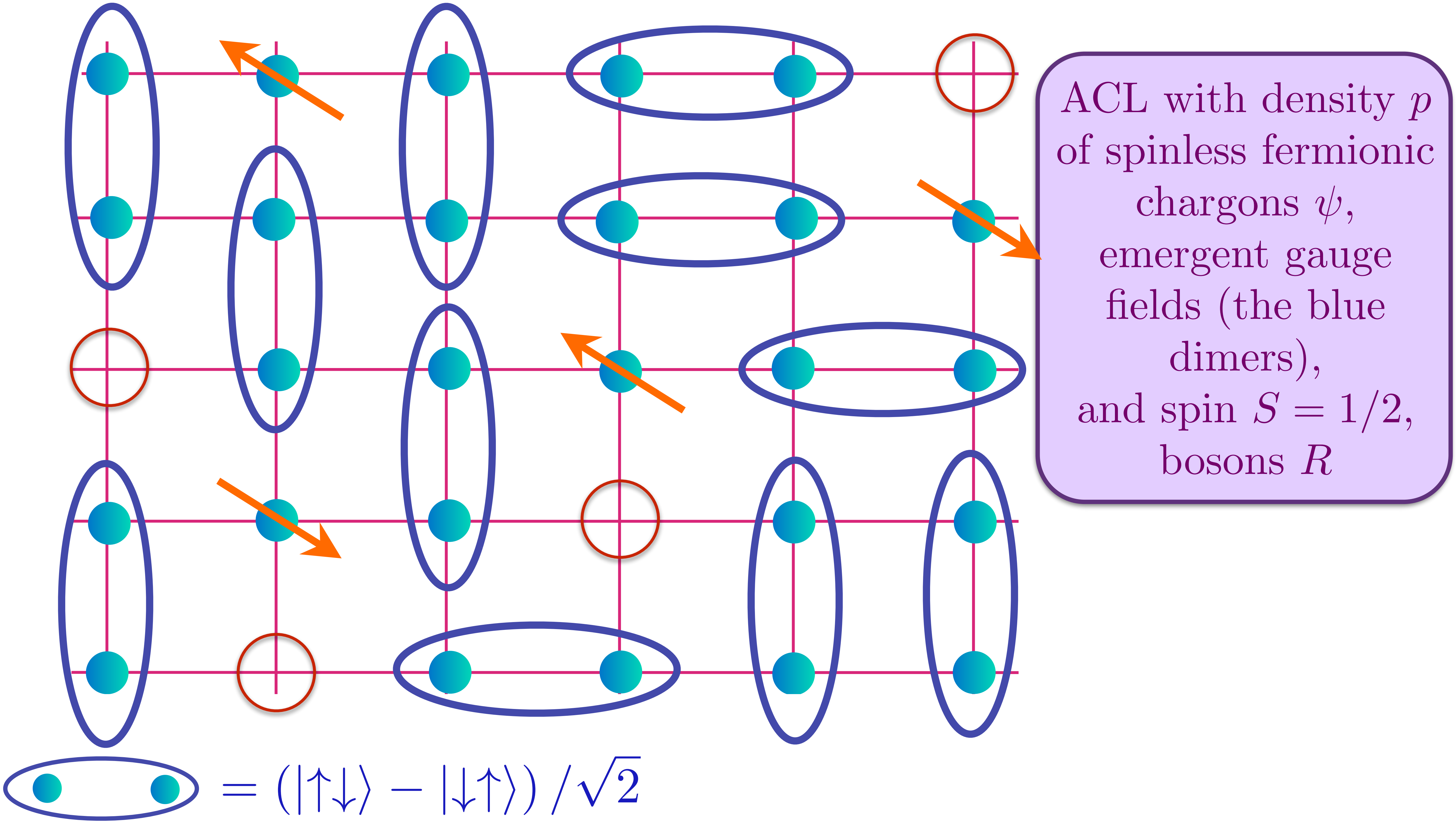

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$


ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

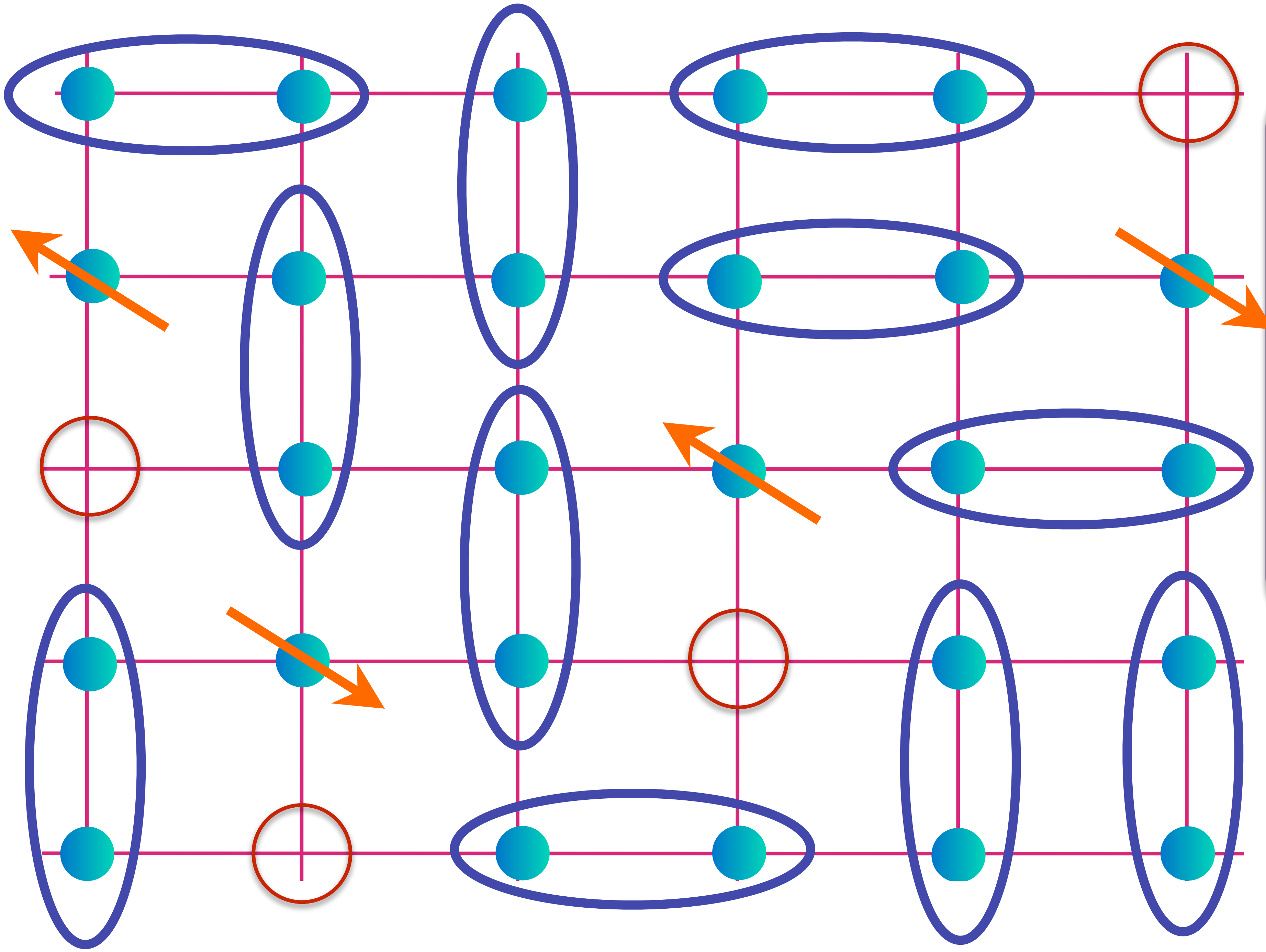


ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R

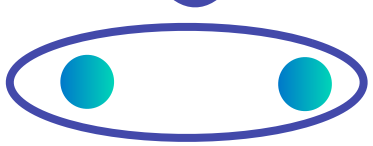
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



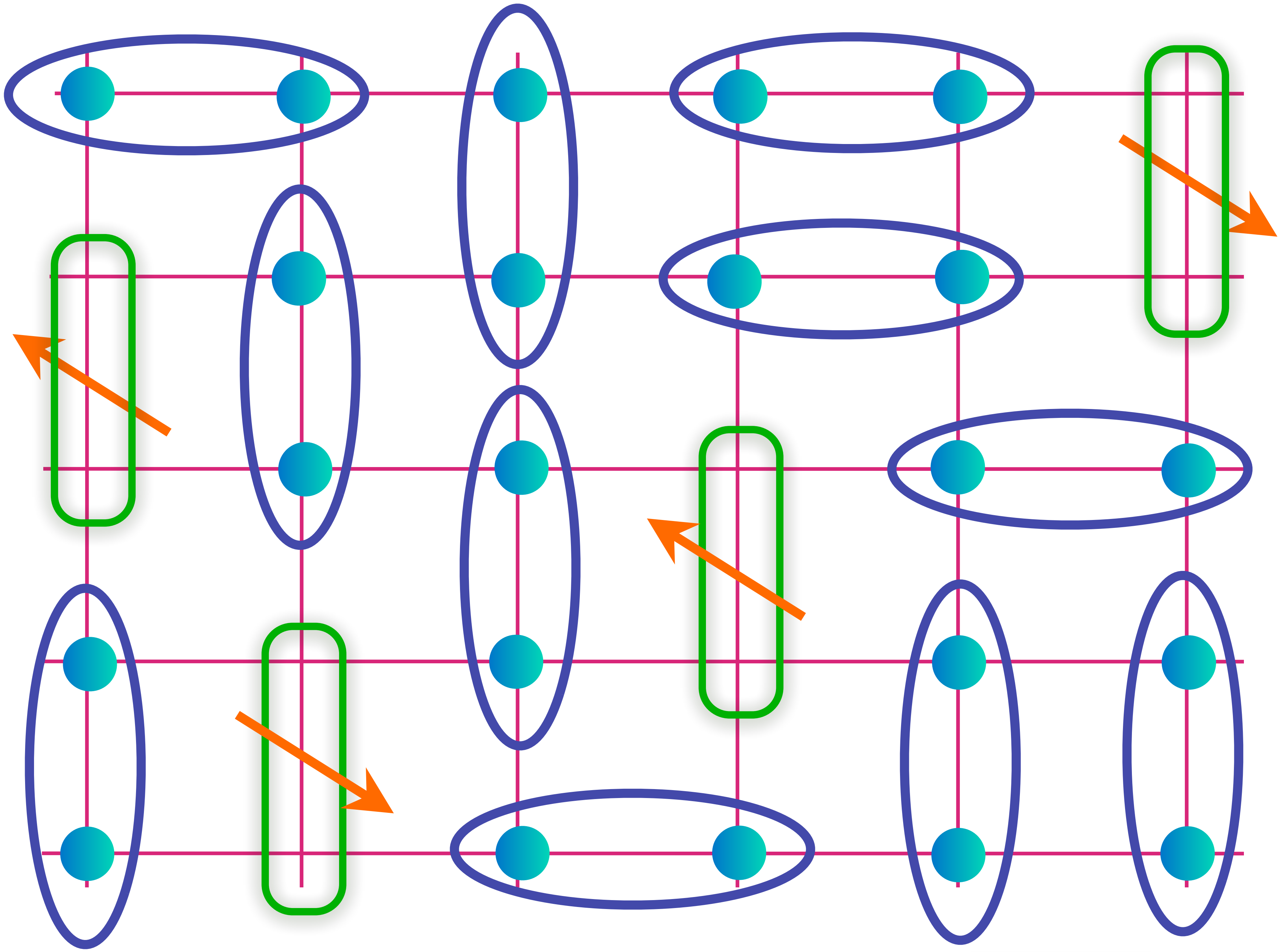
ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S=1/2$, bosons R

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

FL*



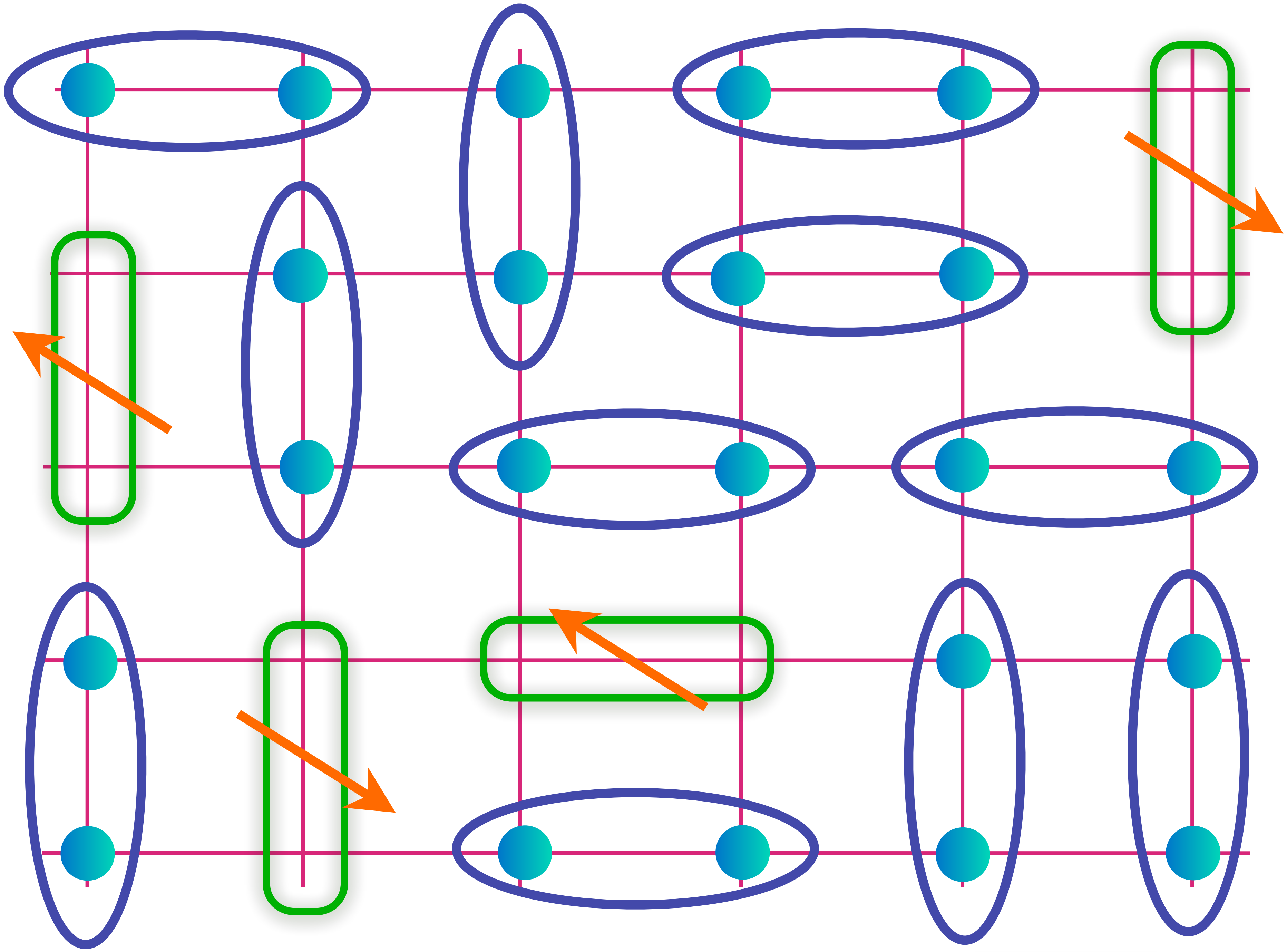
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

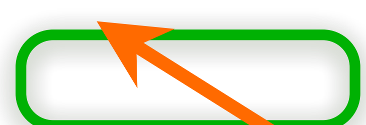
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



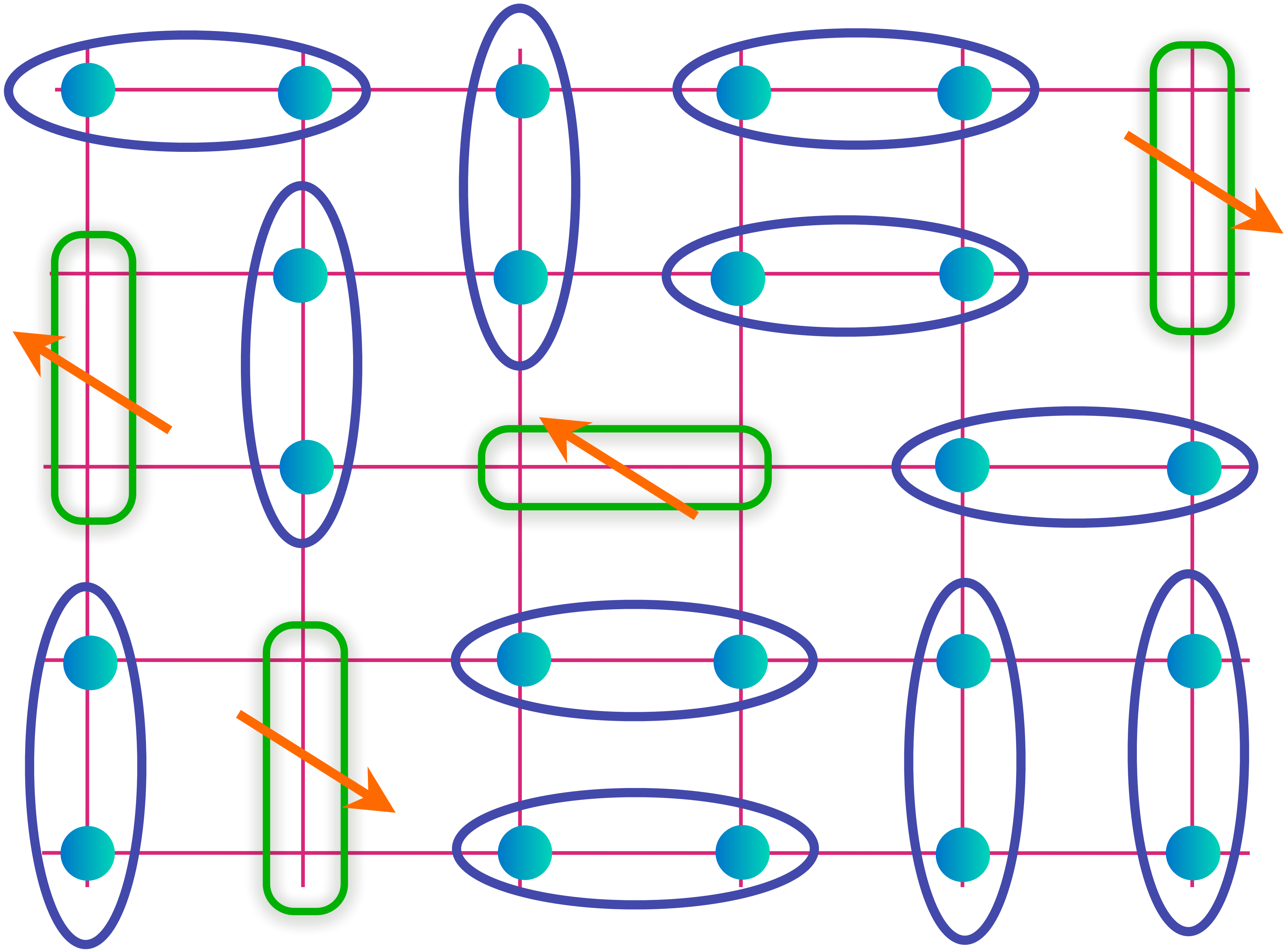
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

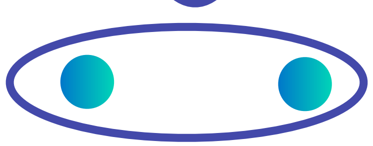
 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

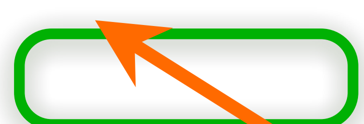
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



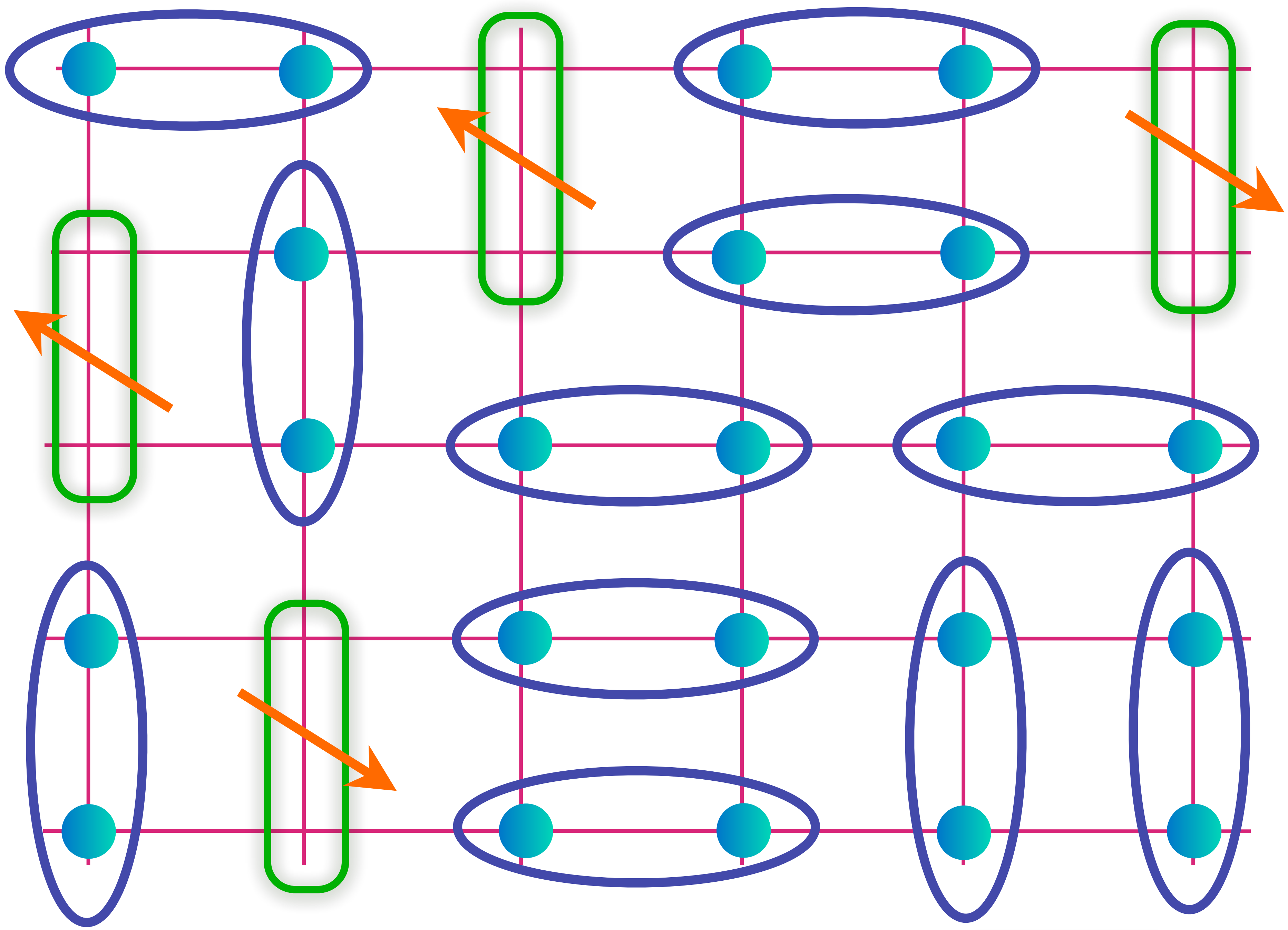
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

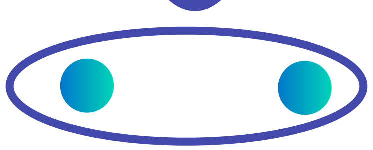
 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

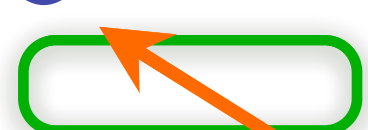
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



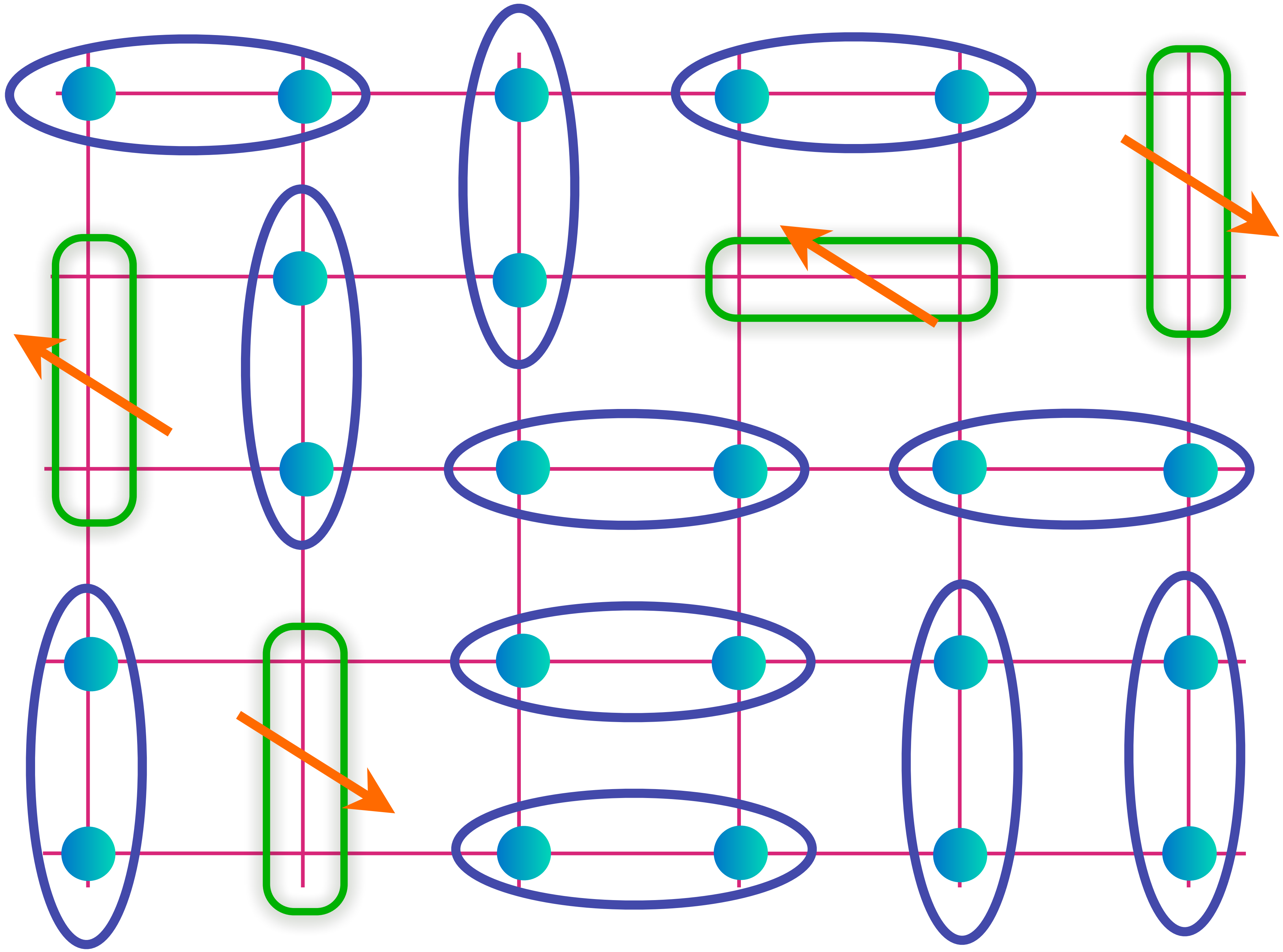
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



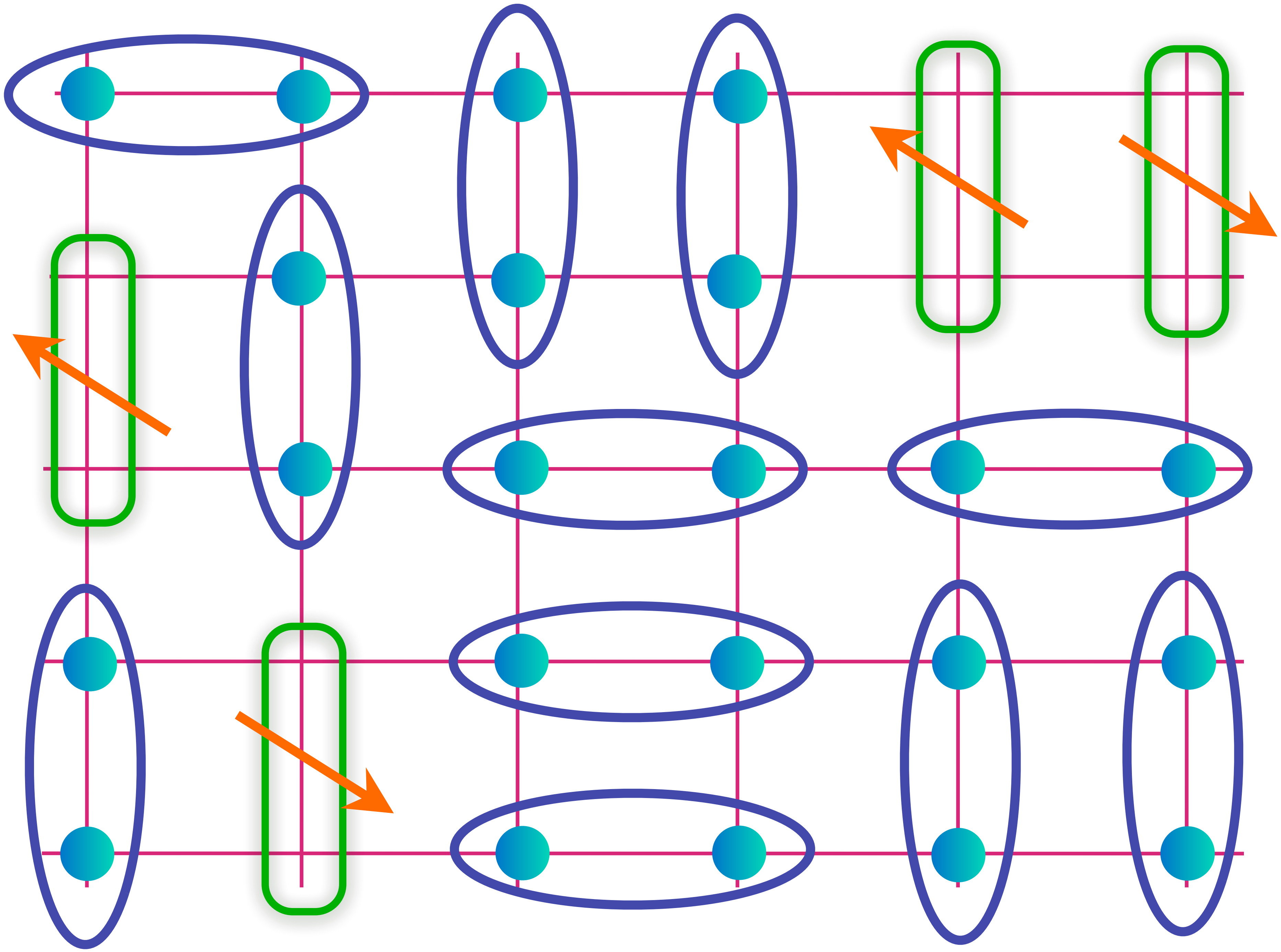
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

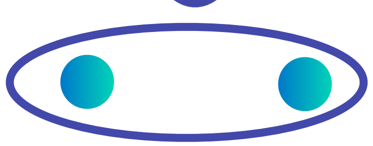
$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

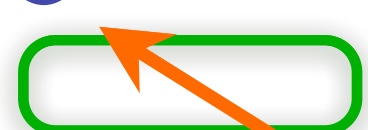
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



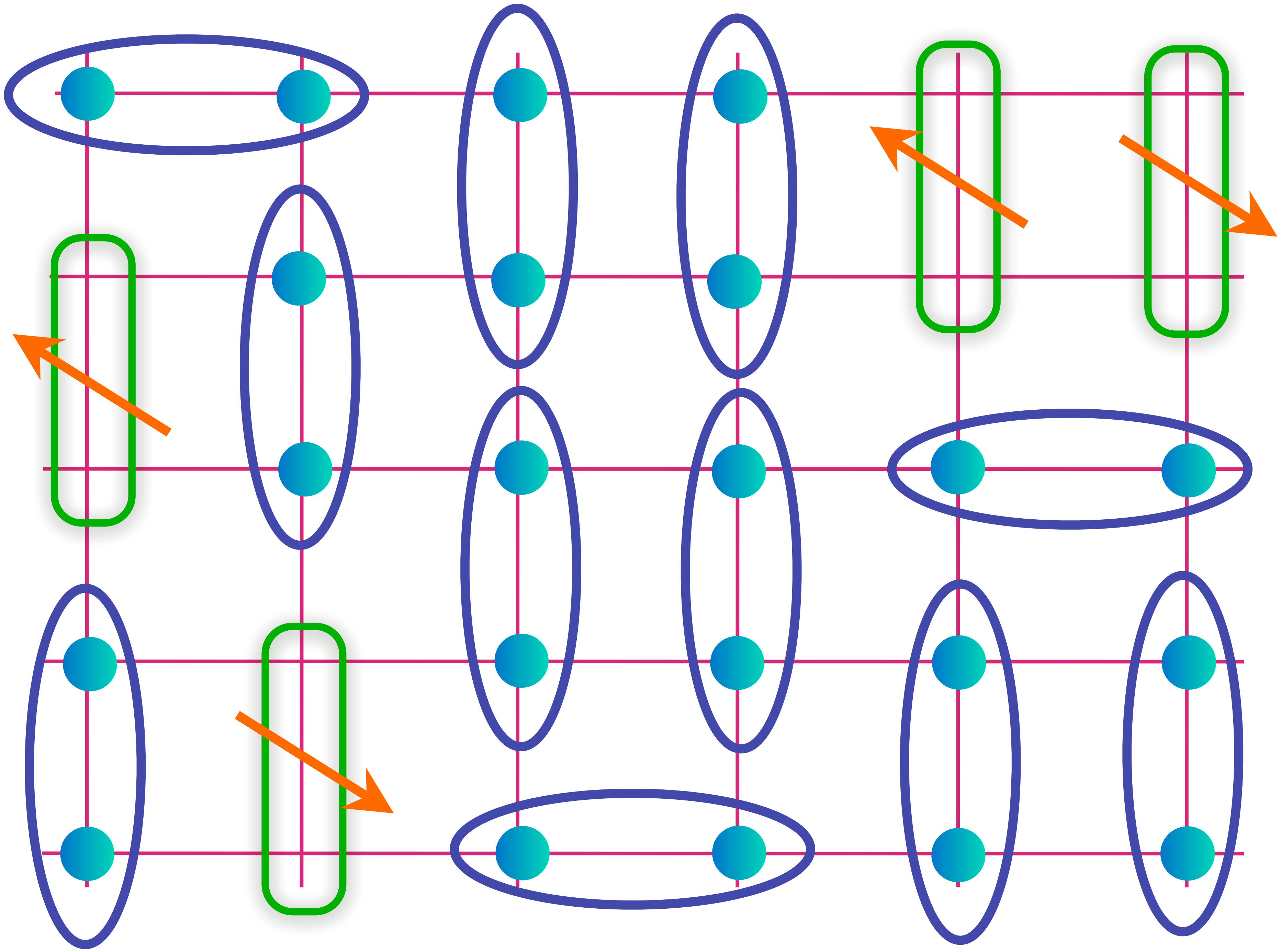
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



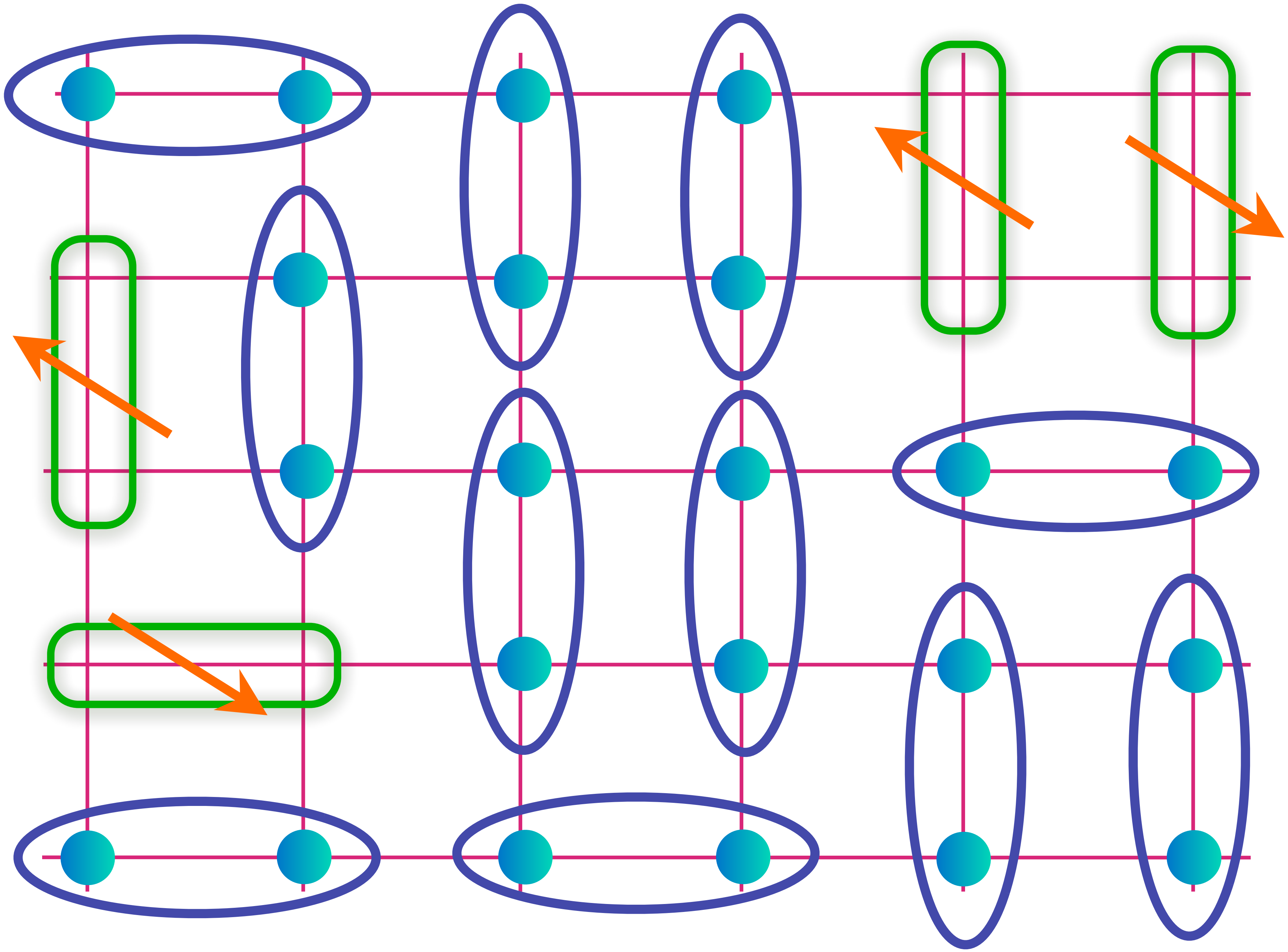
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

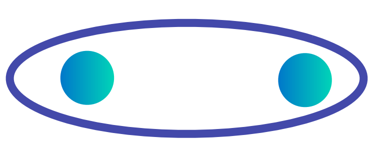
$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

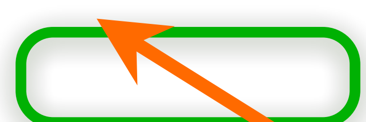
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Constraints on volume enclosed by the Fermi surface

- In a conventional Fermi liquid state, Fermi volume must equal *(1-p) (mod 2)*.
- When the unit cell is doubled by SDW order, total Fermi volume must equal *(1-p) (mod 1)*.
- A state with Fermi volume *(-p) (mod 2)*, but no translational symmetry breaking, must have non-quasiparticle excitations with vanishing energy on a torus *i.e.* emergent gauge fields (bulk topological order)

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

SU(2) gauge theory: The ACL was used to model the photoemission and the cluster DMFT results in the intermediate phase.

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

Theory of optimal doping criticality

Field	Symbol	Statistics	$SU(2)_{\rm gauge}$	$SU(2)_{\rm spin}$	$U(1)_{\rm e.m.charge}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

SU(2) gauge theory for **quantum criticality***:* fractionalize the SDW order parameter into the Higgs field (H) and the spinons (R); but do *not* fractionalize the electron (c) into chargons (ψ) and spinons. In the FL* state, the low energy fermionic excitations have the quantum numbers of an electron

Theory of optimal doping criticality

SU(2) gauge theory

SU(2) gauge theory with adjoint complex Higgs fields $\mathcal{H}^a_{x,y}$ $(a = 1, 2, 3)$, and gauge-invariant, electron-like fermions c_α with a large Fermi surface.

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}\left|\partial_\mu \mathcal{H}^a_x - \epsilon_{abc} A^b_\mu \mathcal{H}^c_x\right|^2 + \frac{1}{2}\left|\partial_\mu \mathcal{H}^a_y - \epsilon_{abc} A^b_\mu \mathcal{H}^c_y\right|^2 \\
& + \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + V(\mathcal{H}^a_{x,y}) \\
& - \sum_{j,\rho} t_\rho \left(c^\dagger_{j,\alpha} c_{j+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{j+\boldsymbol{v}_\rho,\alpha} c_{j,\alpha} \right) - \mu \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} \\
& + \lambda \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} H^a(j) H^a(j)
\end{aligned}
$$

The fermions do not have Yukawa coupling to the Higgs fields, or a minimal coupling to the gauge fields: both are prohibited by gauge invariance. We treat the quartic coupling, λ perturbatively.

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

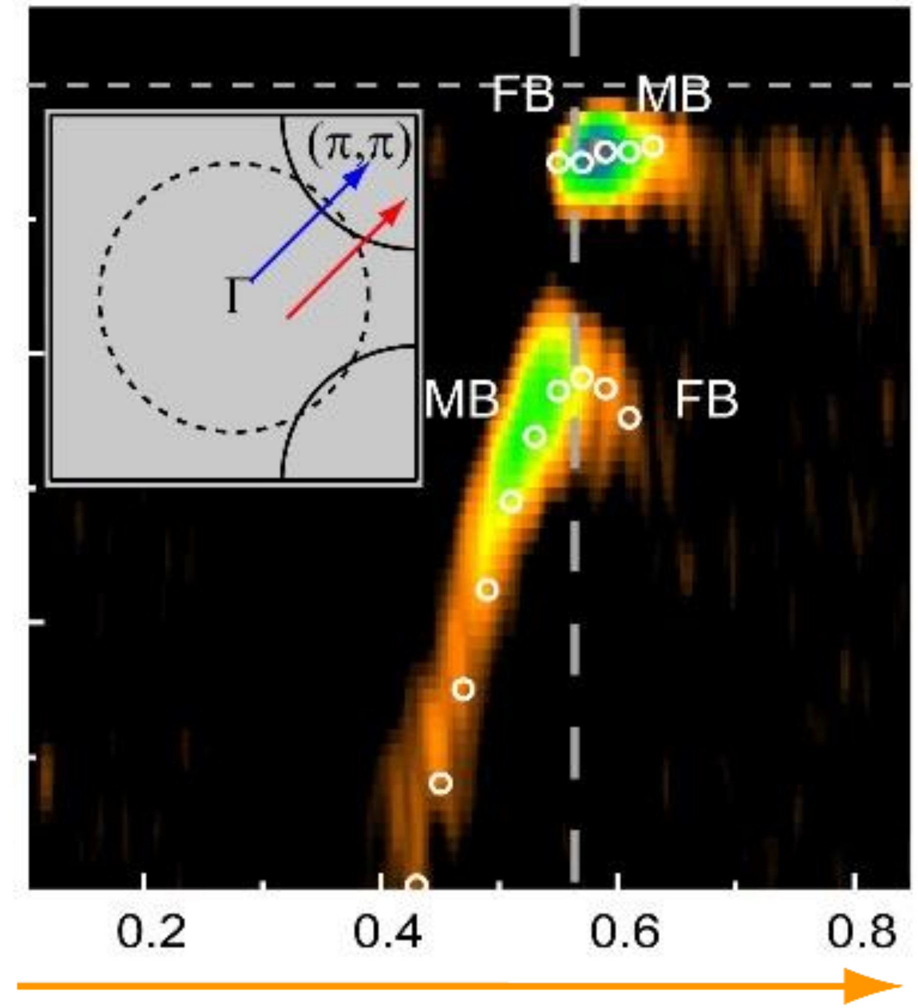
Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

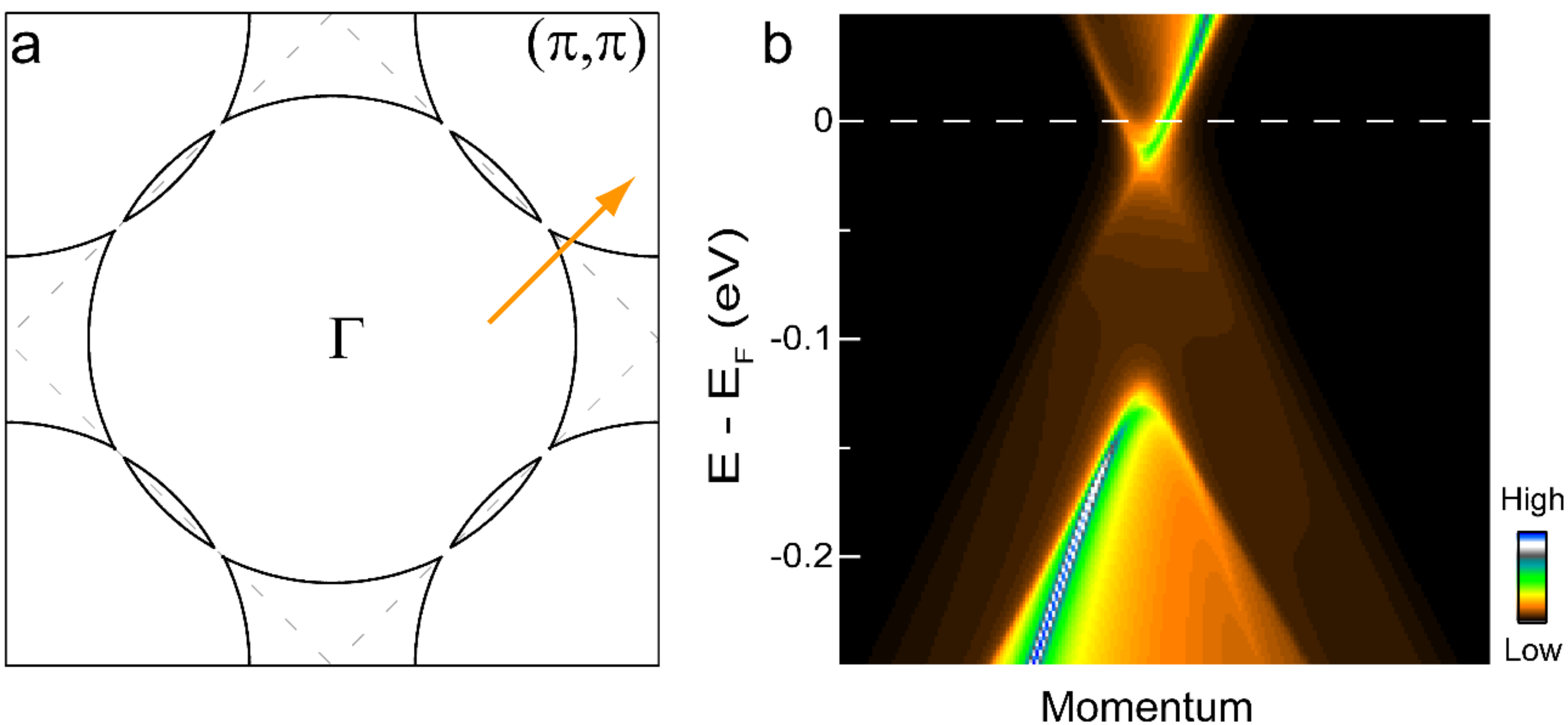
- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

Mathias Scheurer



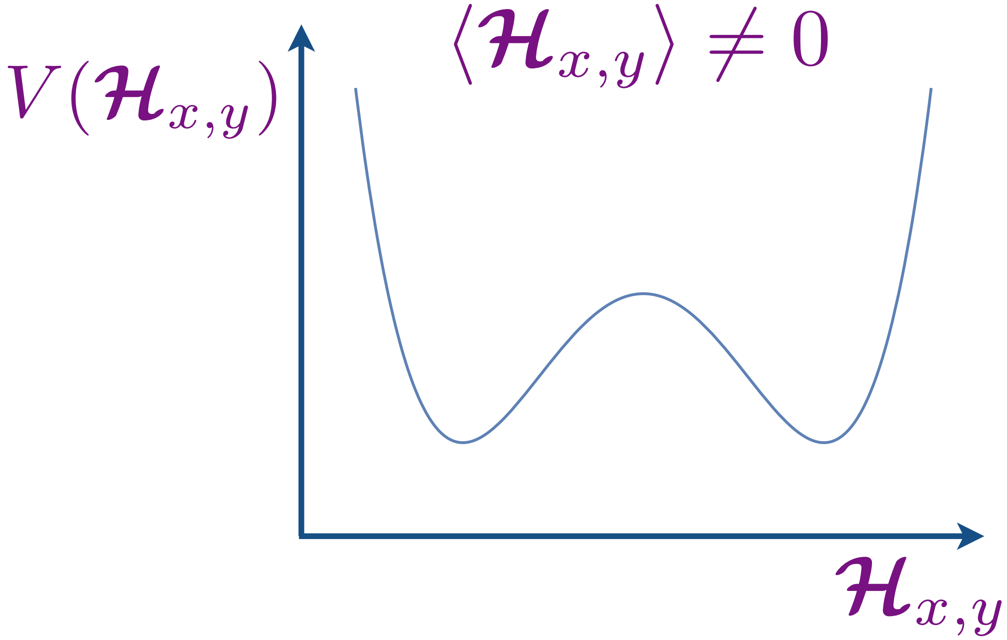
S. Sachdev, Topological order and Fermi surface reconstruction, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

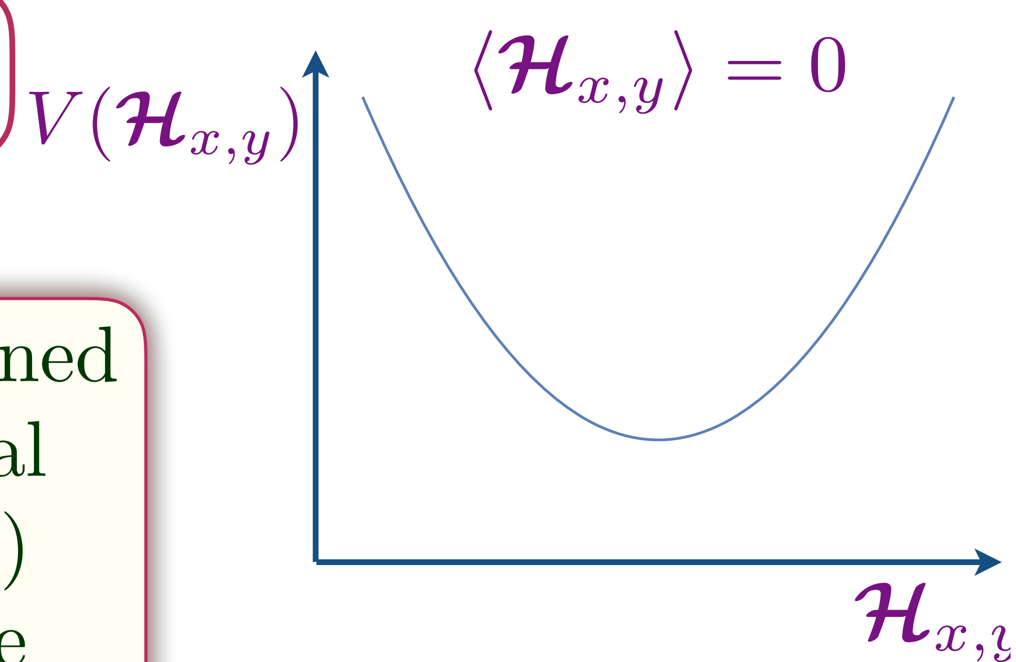


Hole-doped cuprates

$N_h = 4$



Deconfined critical SU(2) gauge theory



Higgs

One or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in the U(1) cases

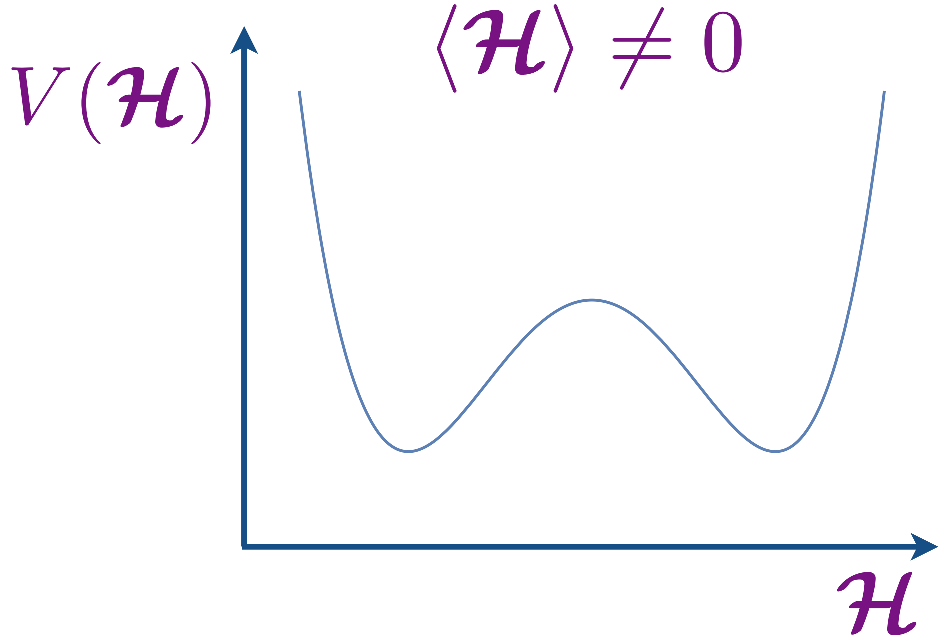
Confinement

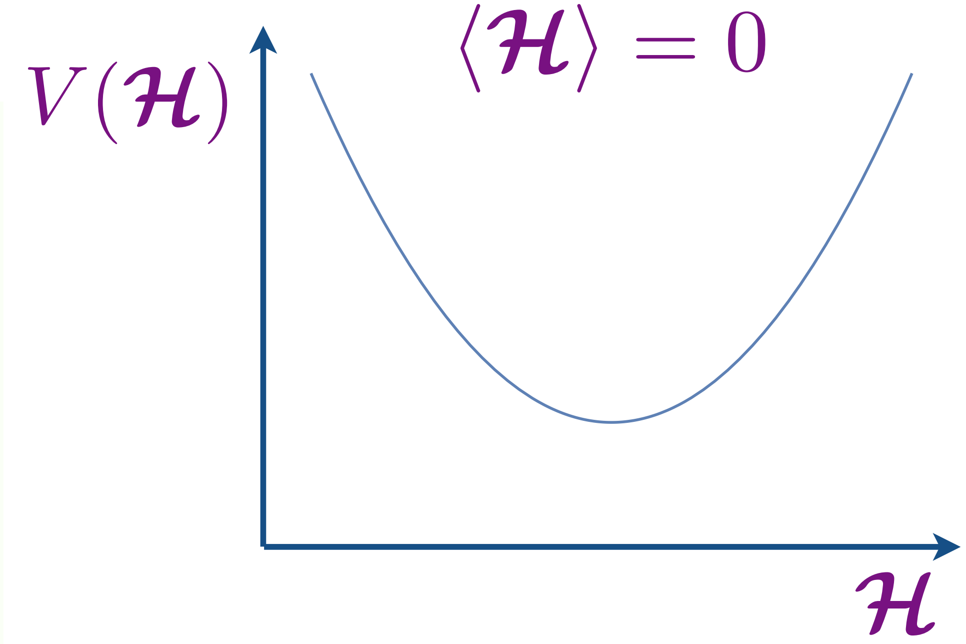
Fermi liquid with large Fermi surface

s

Electron-doped cuprates

$\boxed{N_h = 1}$





Higgs

Crossover

Confinement

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in a U(1) gauge theory

Fermi liquid with large Fermi surface

s