

Theory of Quantum Matter: from Quantum Fields to Strings

Salam Distinguished Lectures

The Abdus Salam International Center for Theoretical Physics

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Talk online: sachdev.physics.harvard.edu

PHYSICS



HARVARD

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

*B. Conformal field theories in $2+1$ dimensions and
the AdS/CFT correspondence*

2. Metals without quasiparticles

A. Review of Fermi liquid theory

*B. A “non-Fermi” liquid: the Ising-nematic
quantum critical point*

C. Holography, entanglement, and strange metals

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C. Holography, entanglement, and strange metals

(a) Entanglement

(b) Holography, entanglement, and CFTs

(c) Generalized holography beyond CFTs

(d) Holography of strange metals

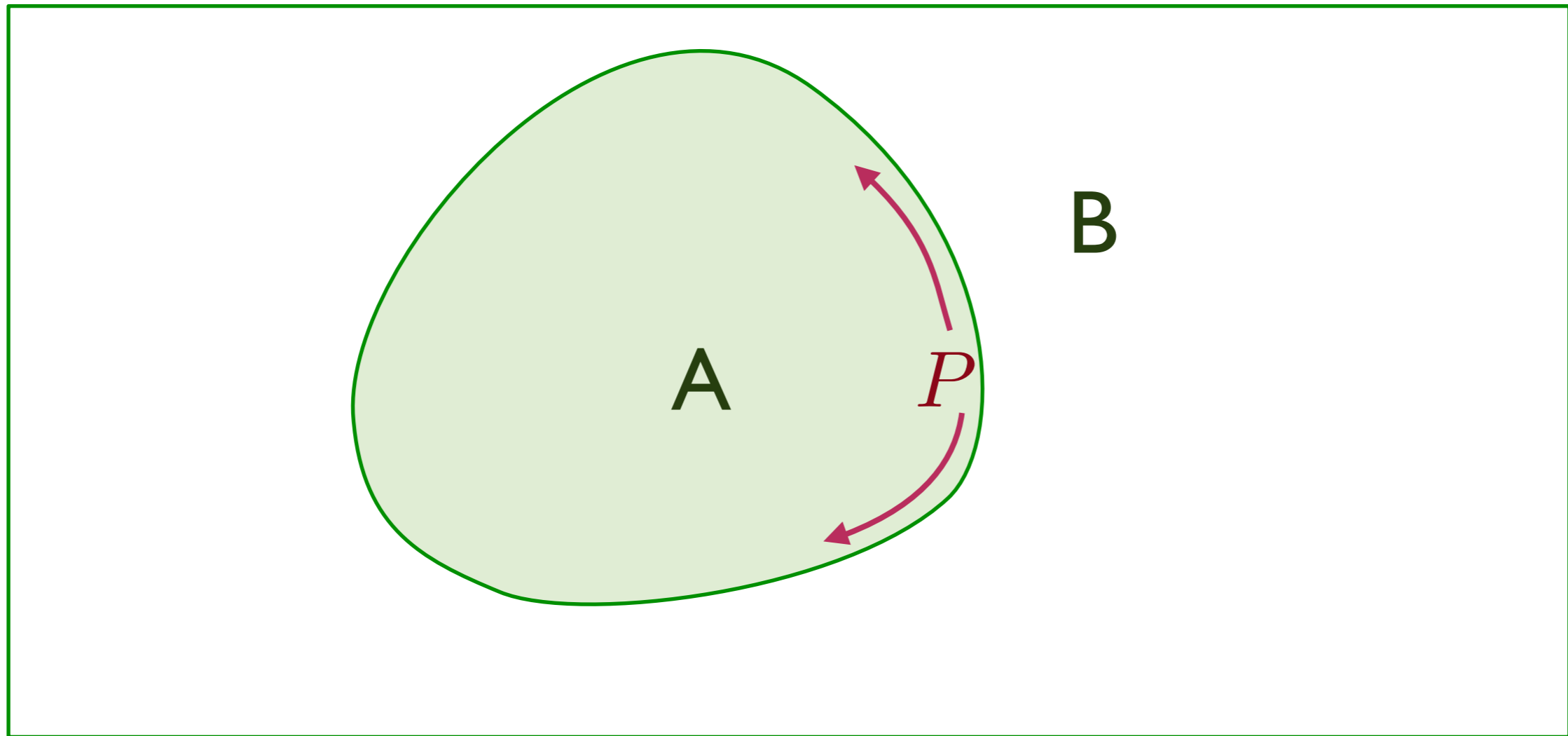
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Entanglement entropy



$|\Psi\rangle \Rightarrow$ Ground state of entire system,
 $\rho = |\Psi\rangle\langle\Psi|$

$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_E = -\text{Tr}(\rho_A \ln \rho_A)$

Entanglement entropy

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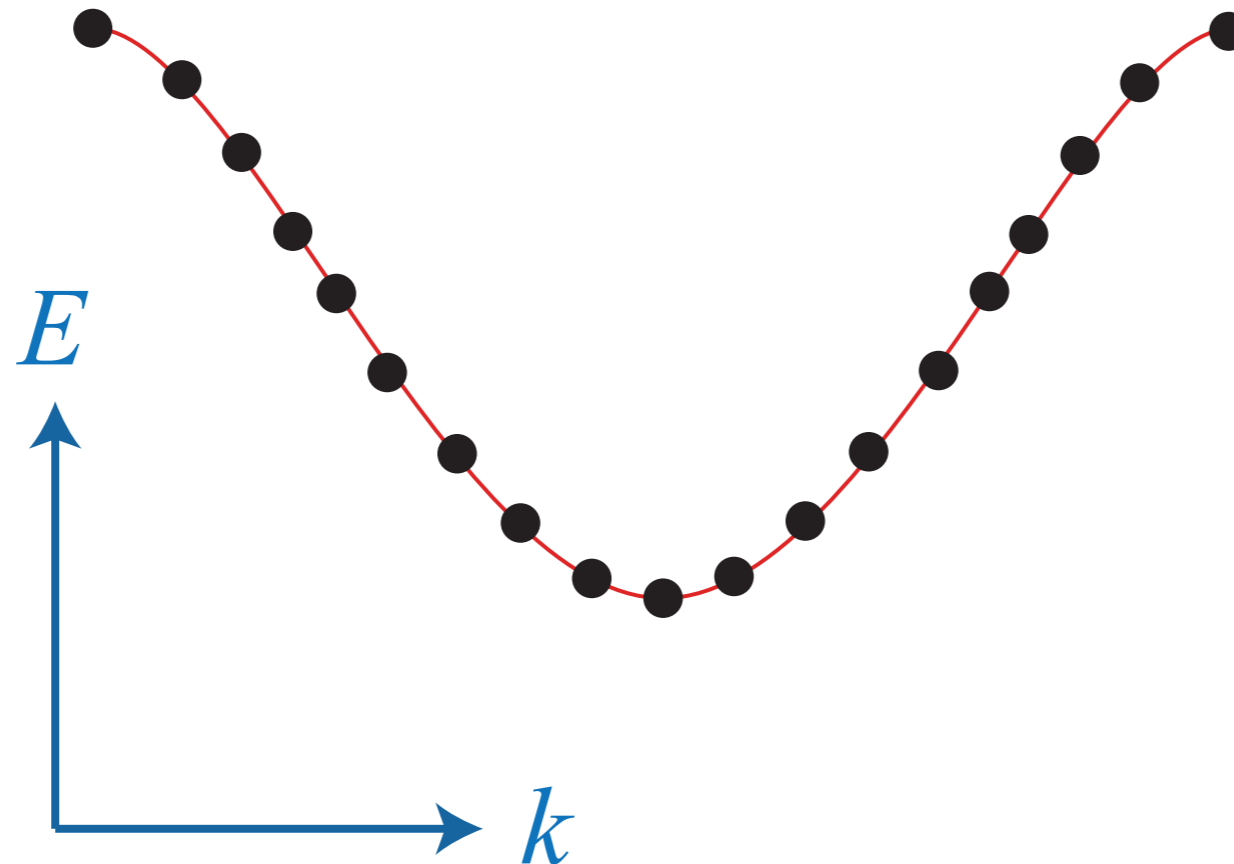
$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B)$$

Then $\rho_A = \text{Tr}_B \rho =$ density matrix of region A
 $= \frac{1}{2} (|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A)$

Entanglement entropy $S_E = -\text{Tr} (\rho_A \ln \rho_A)$
 $= \ln 2$

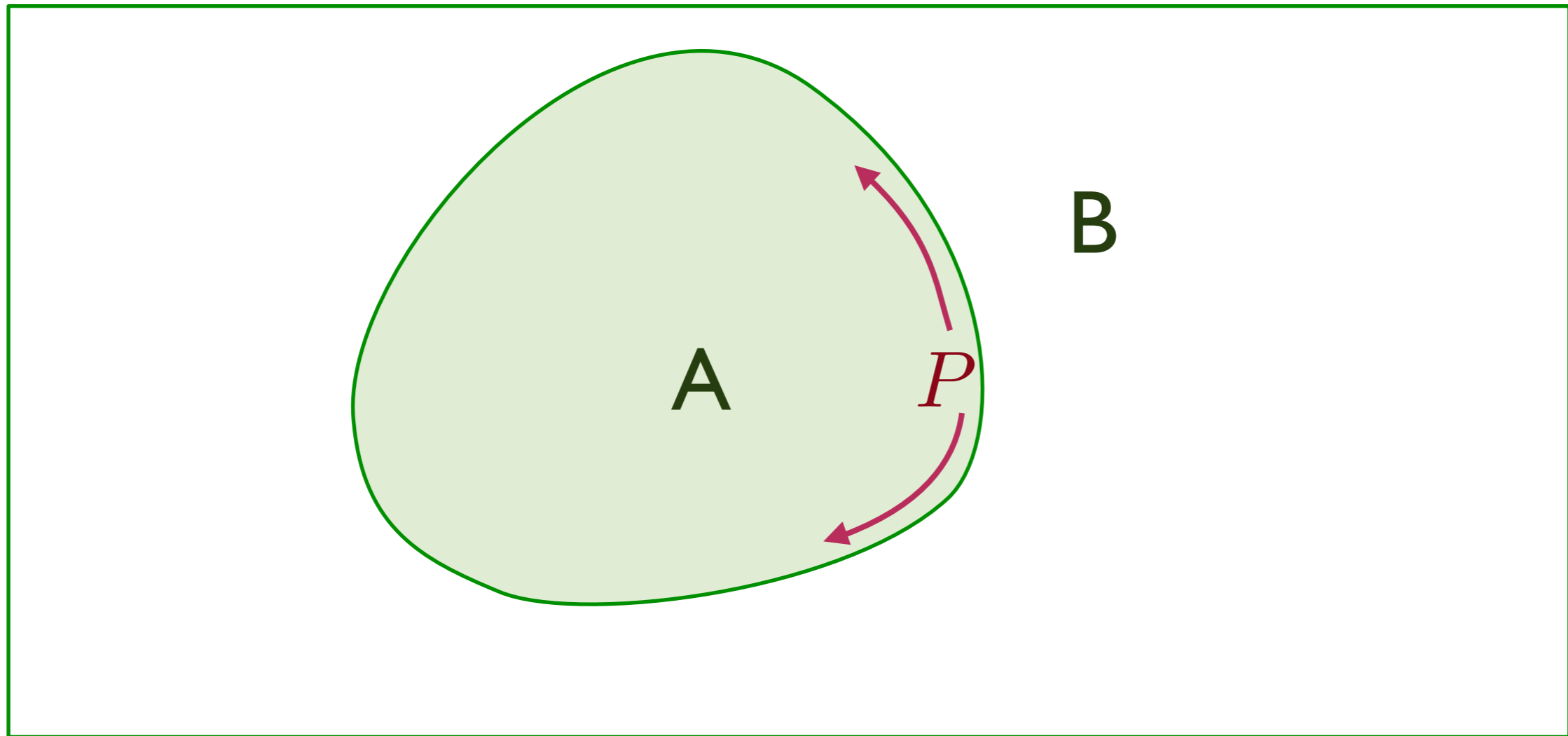
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

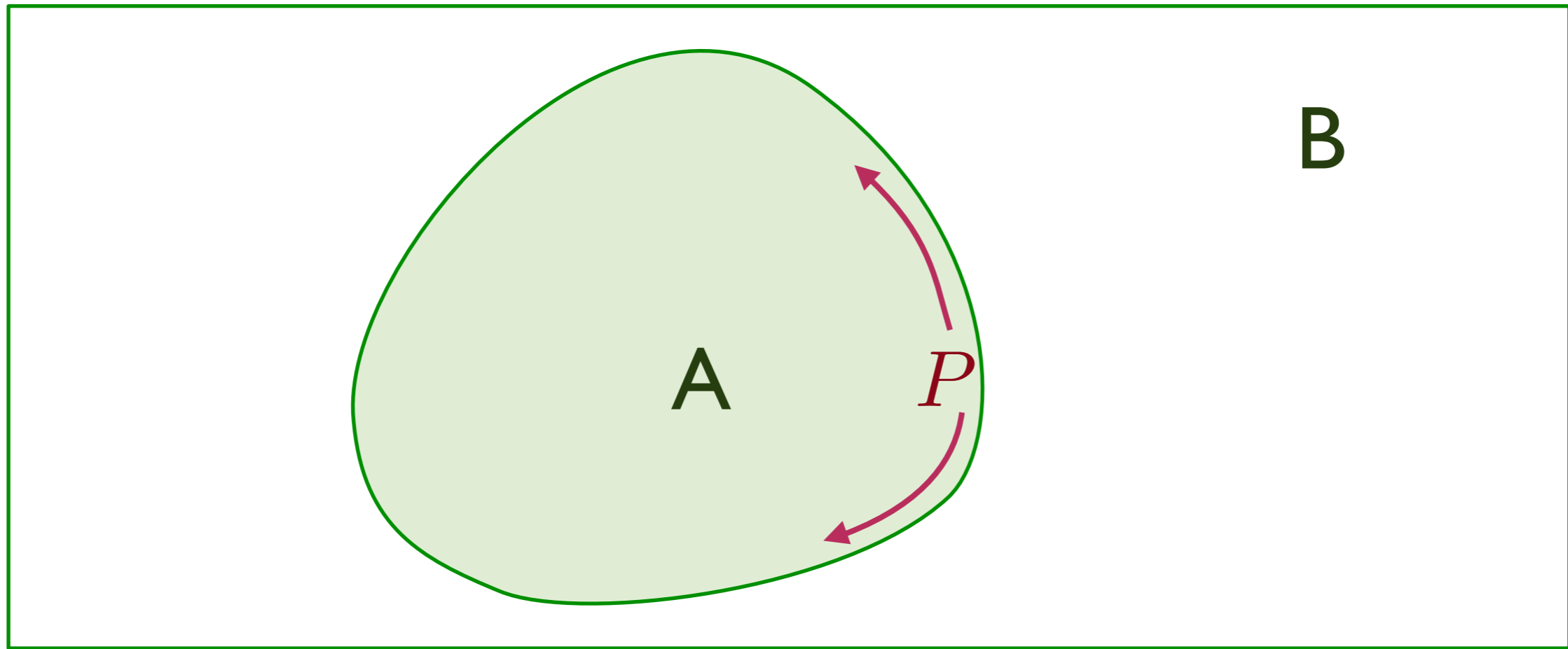
Entanglement entropy of a band insulator



$$S_E = aP - b \exp(-cP)$$

where P is the surface area (perimeter) of the boundary between A and B.

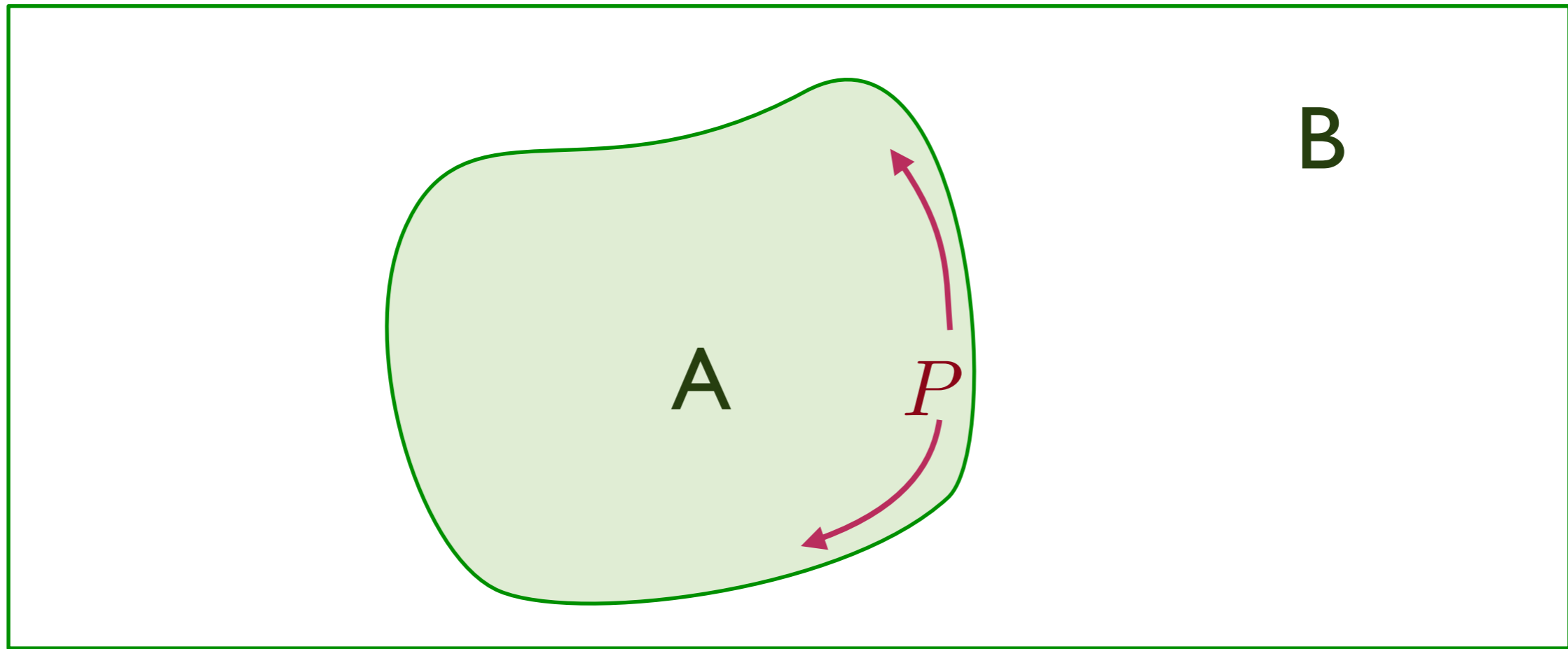
Entanglement entropy of Fermi surfaces



Logarithmic violation of “area law”: $S_E = \mathcal{C}_E k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape. The prefactor \mathcal{C}_E is independent of the shape of the entangling region, and dependent only on IR features of the theory.

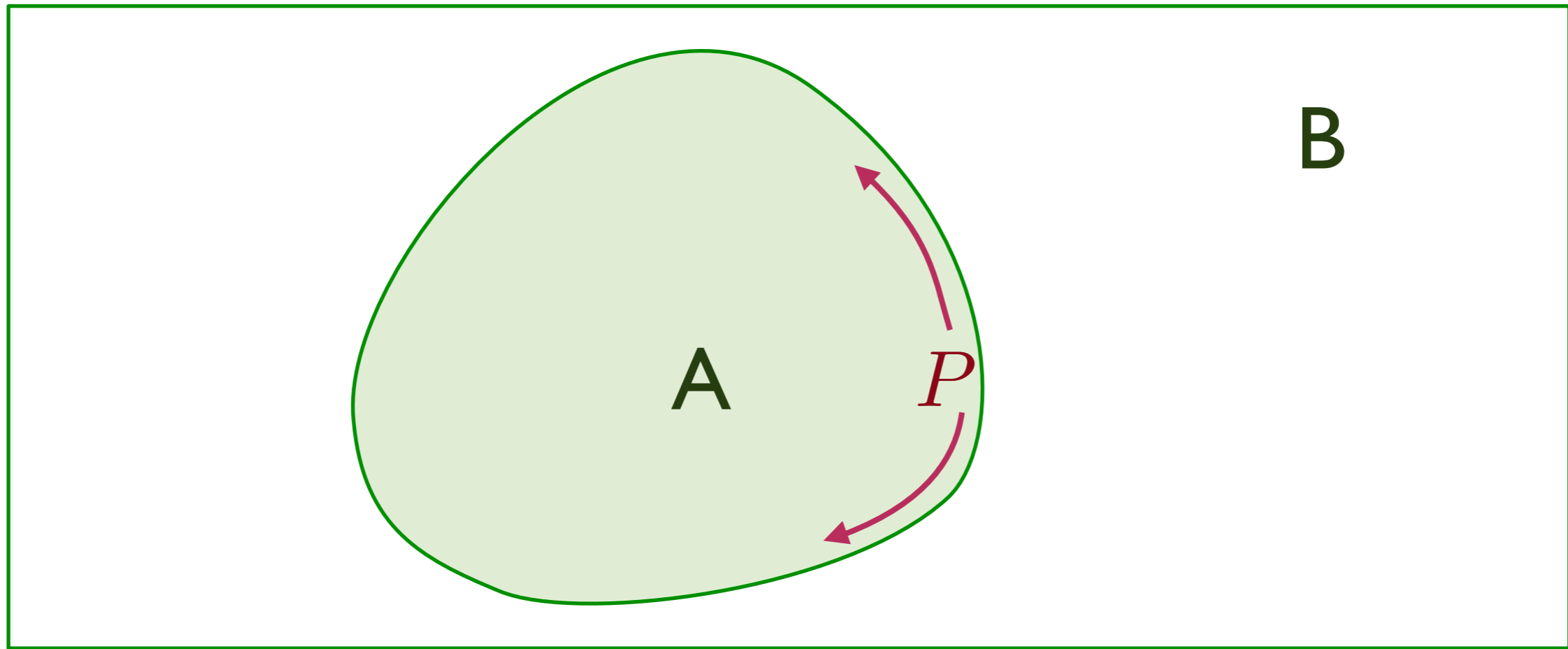
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Entanglement entropy of Fermi surfaces



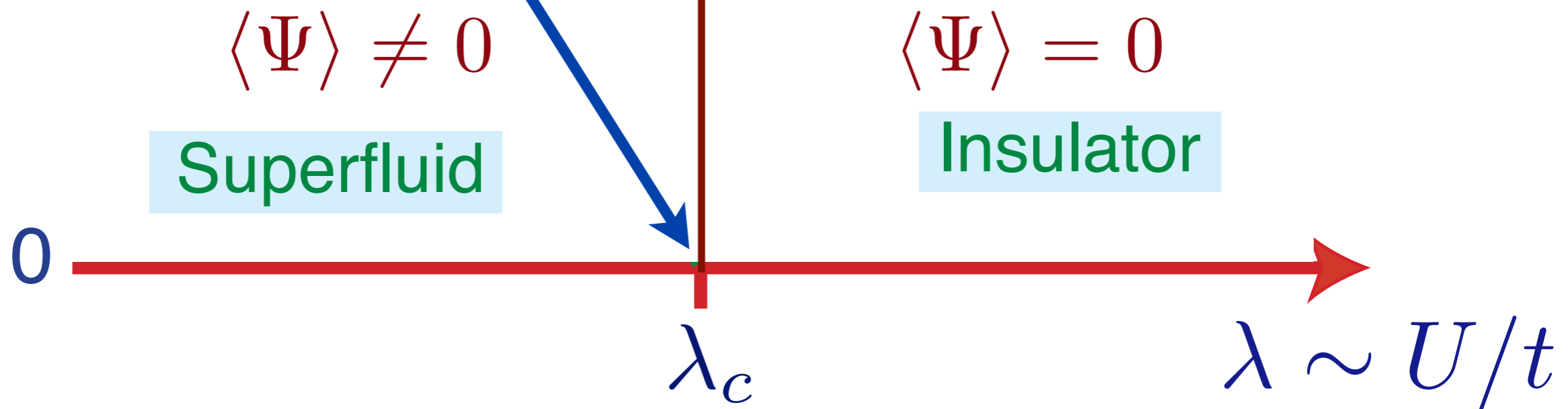
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$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

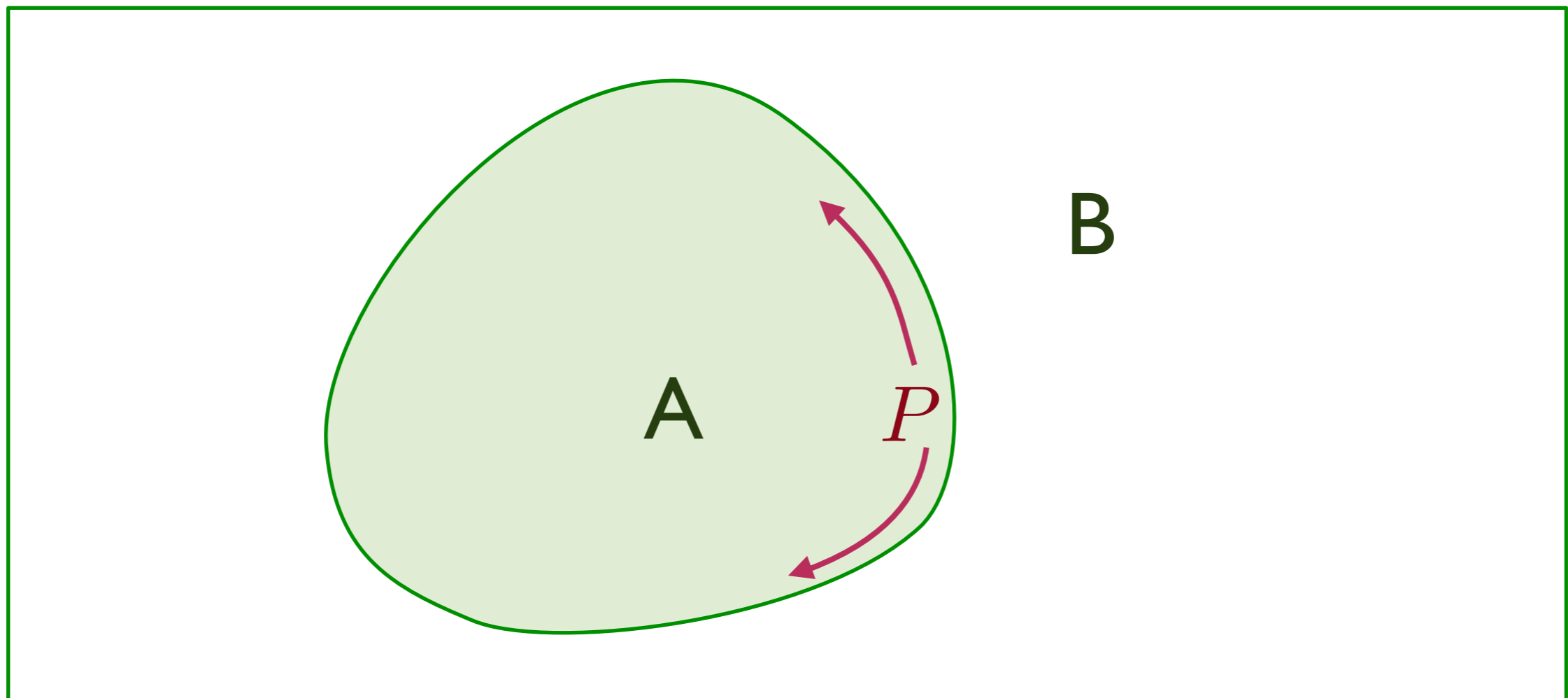
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

Quantum state with
complex, many-body,
“long-range” quantum entanglement

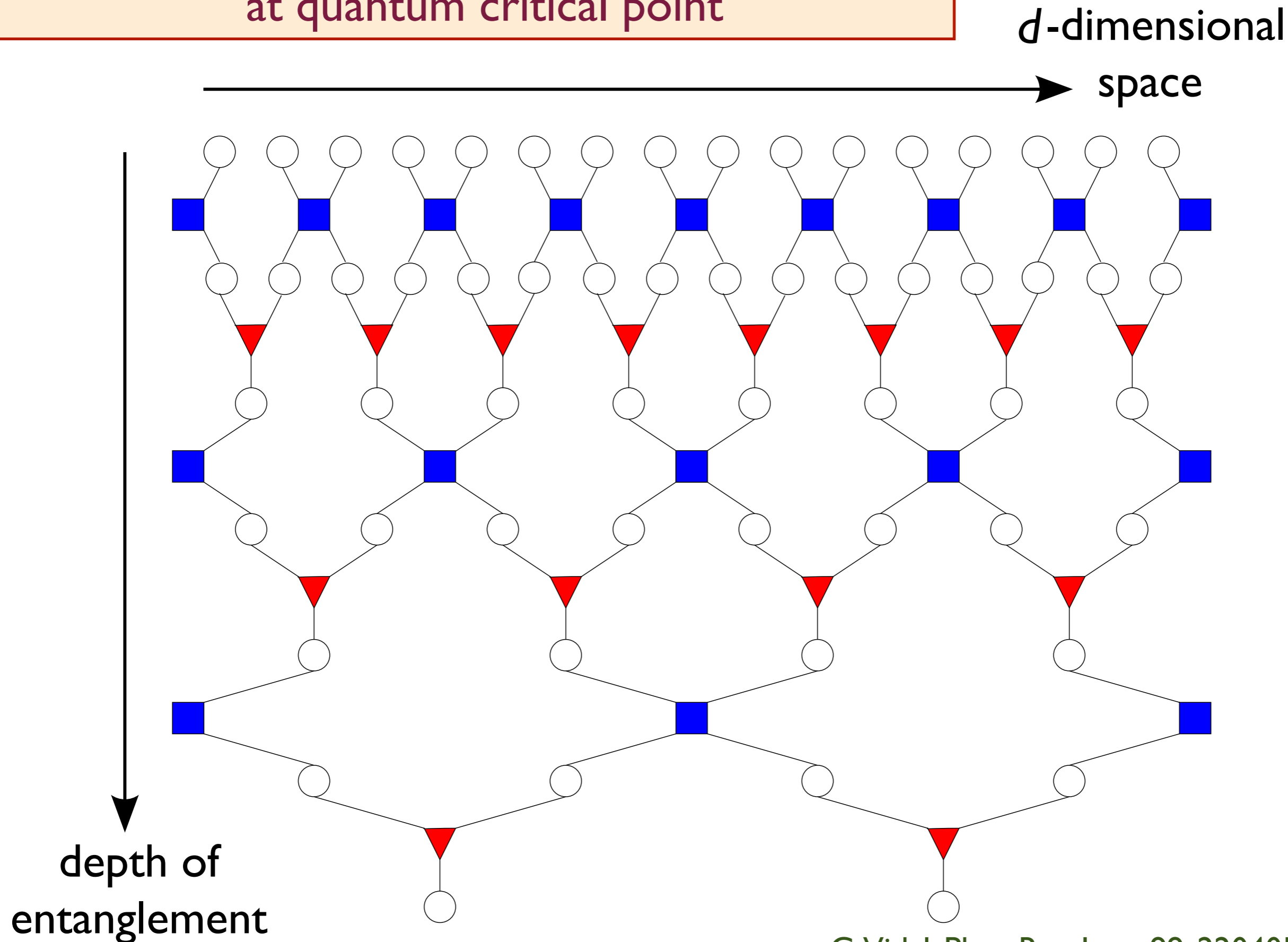


Entanglement at the quantum critical point

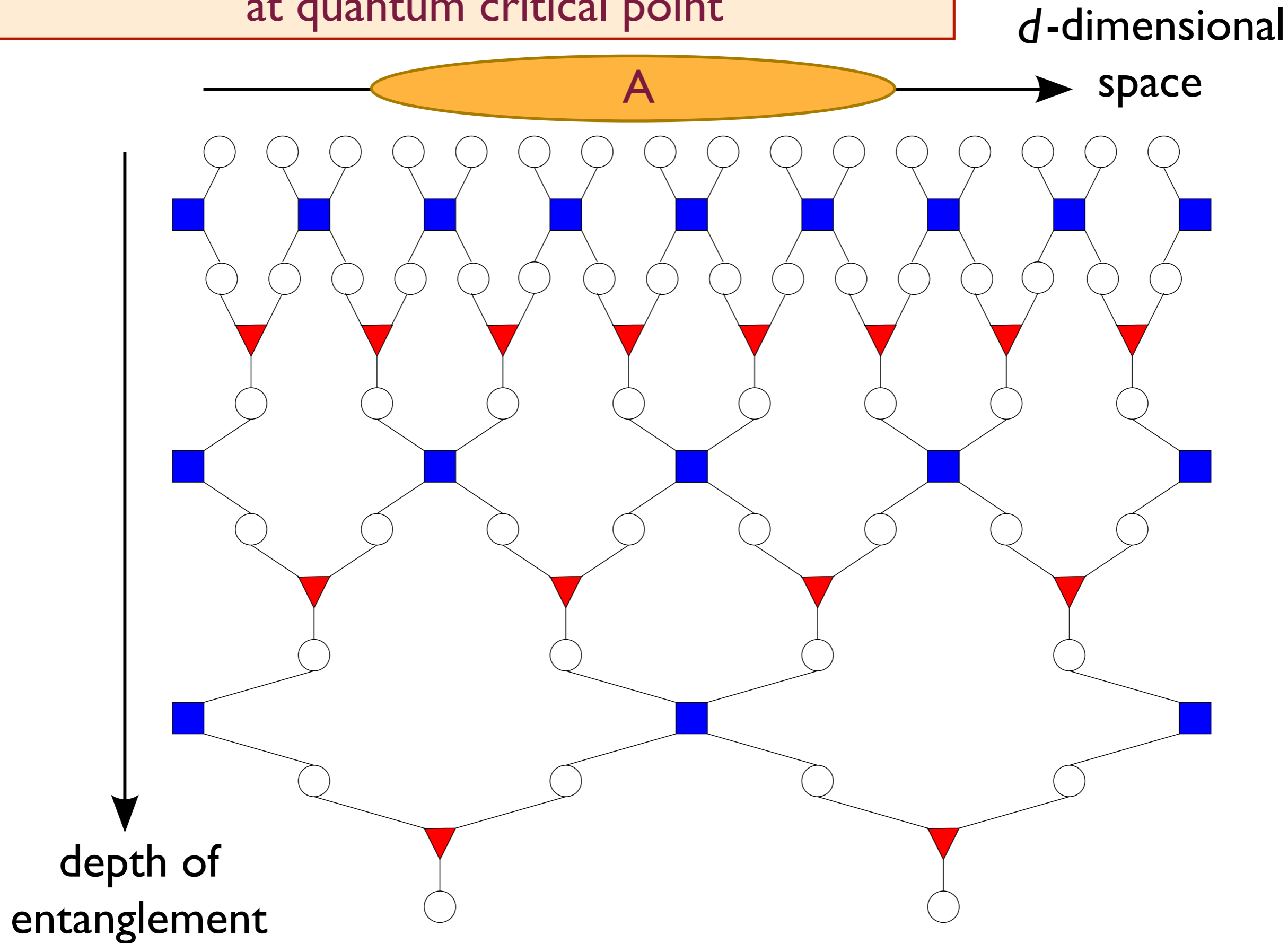
- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



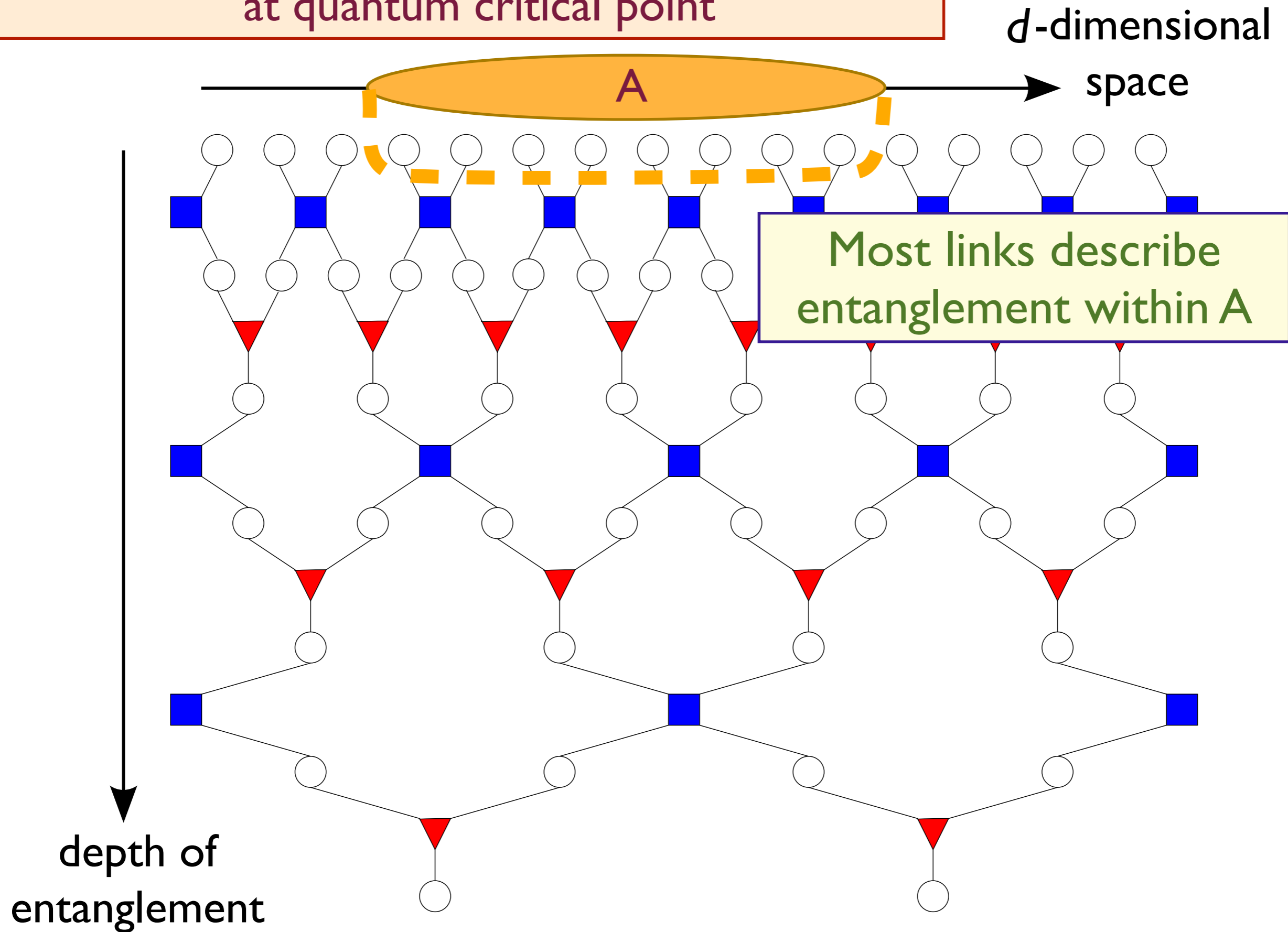
Tensor network representation of entanglement at quantum critical point



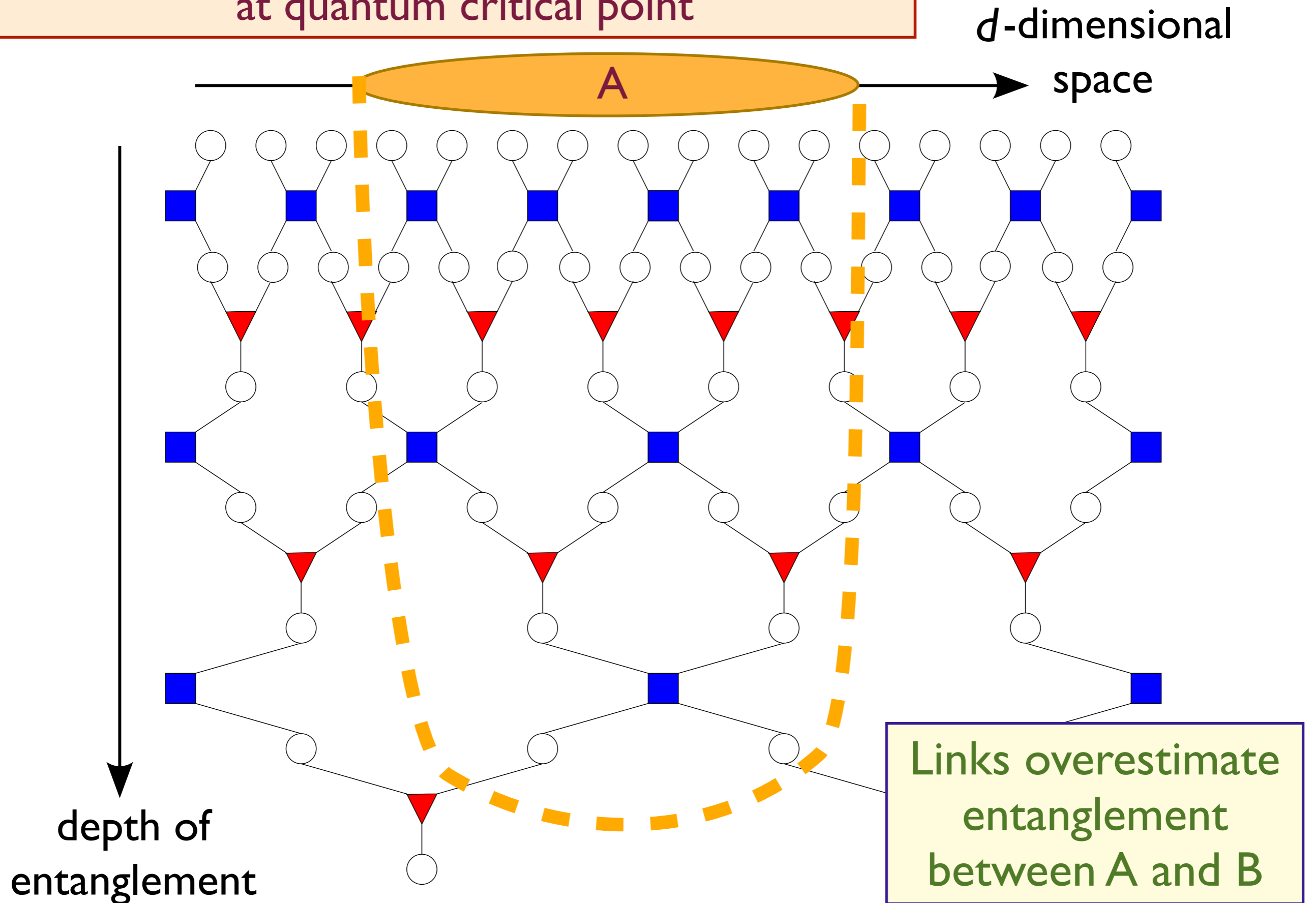
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Tensor network representation of entanglement at quantum critical point

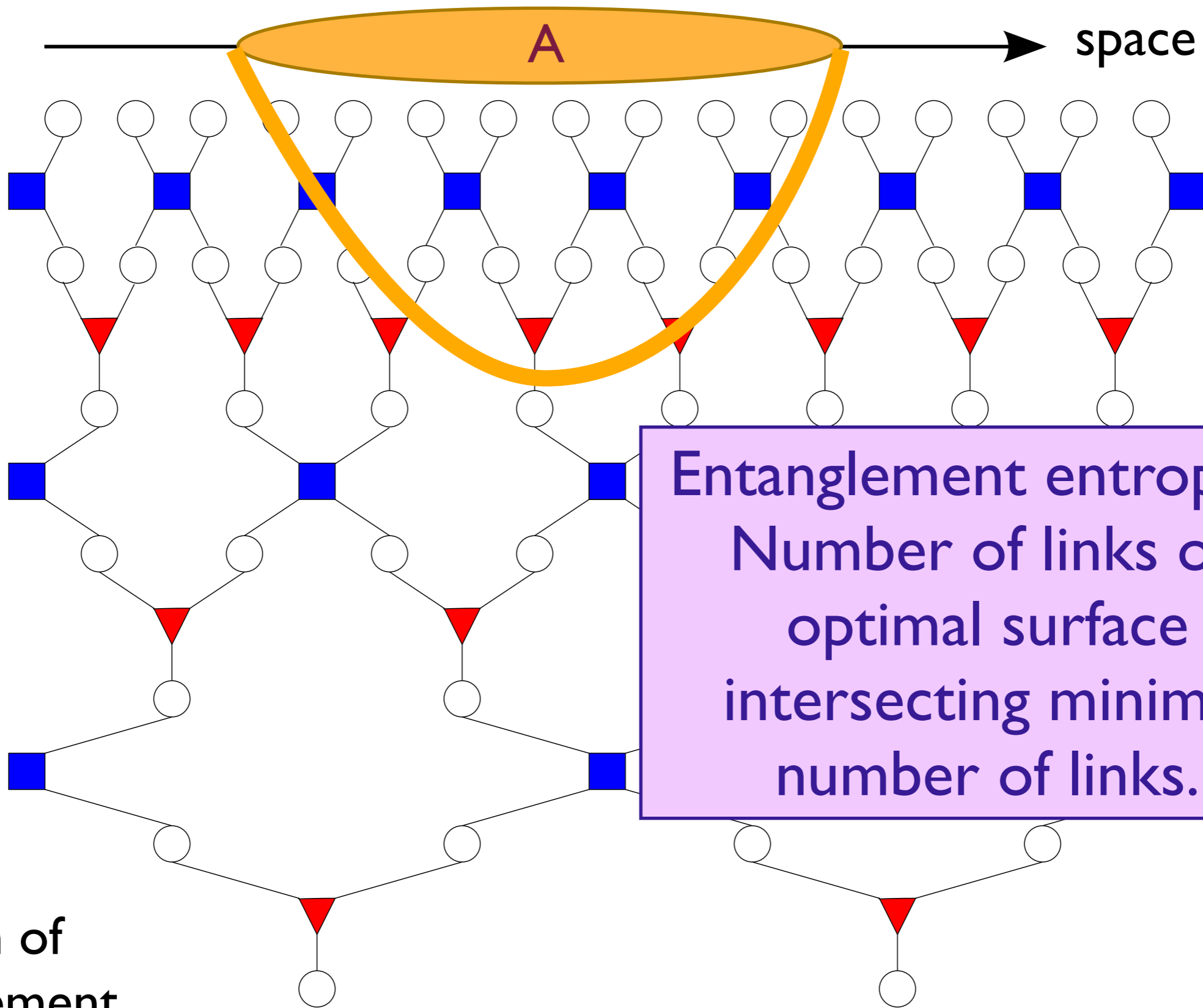


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Tensor network representation of entanglement at quantum critical point

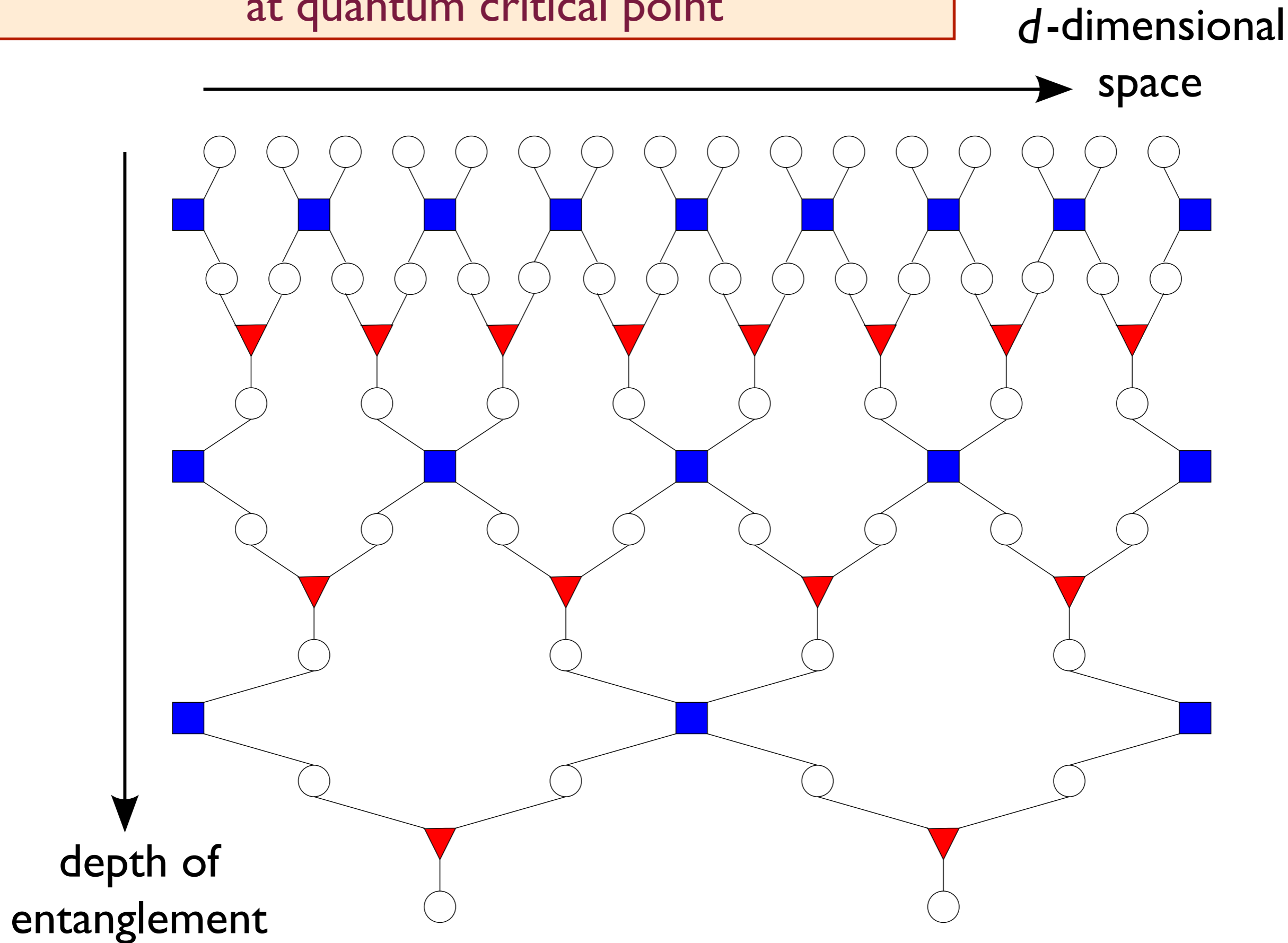
d -dimensional space



Entanglement entropy =
Number of links on
optimal surface
intersecting minimal
number of links.

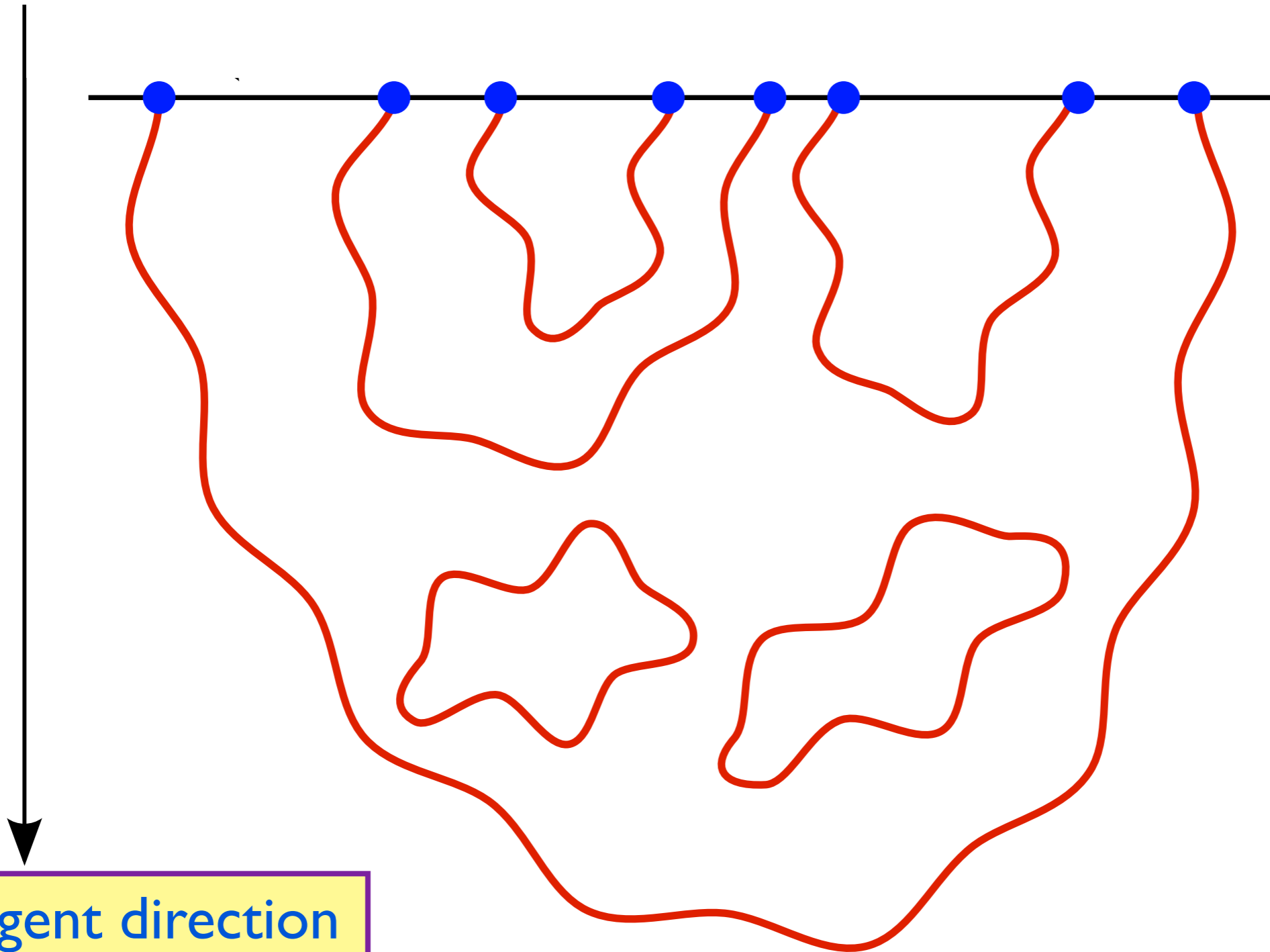
depth of entanglement

Tensor network representation of entanglement at quantum critical point



String theory near
a D-brane

d -dimensional
space

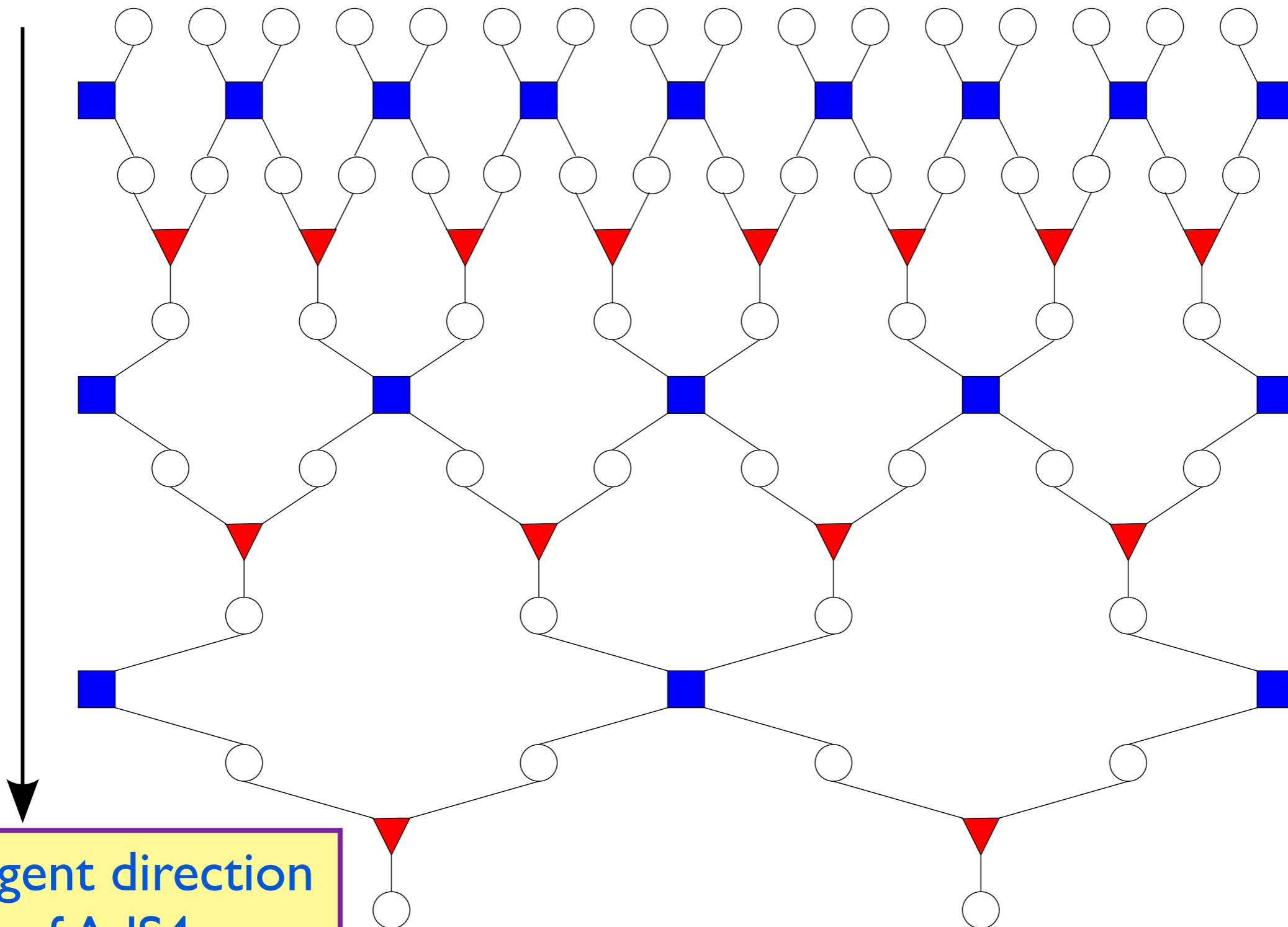


Emergent direction
of AdS4

Tensor network representation of entanglement at quantum critical point

d -dimensional

space



Emergent direction
of AdS4

(a) Entanglement

(b) Holography, entanglement, and CFTs

(c) Generalized holography beyond CFTs

(d) Holography of strange metals

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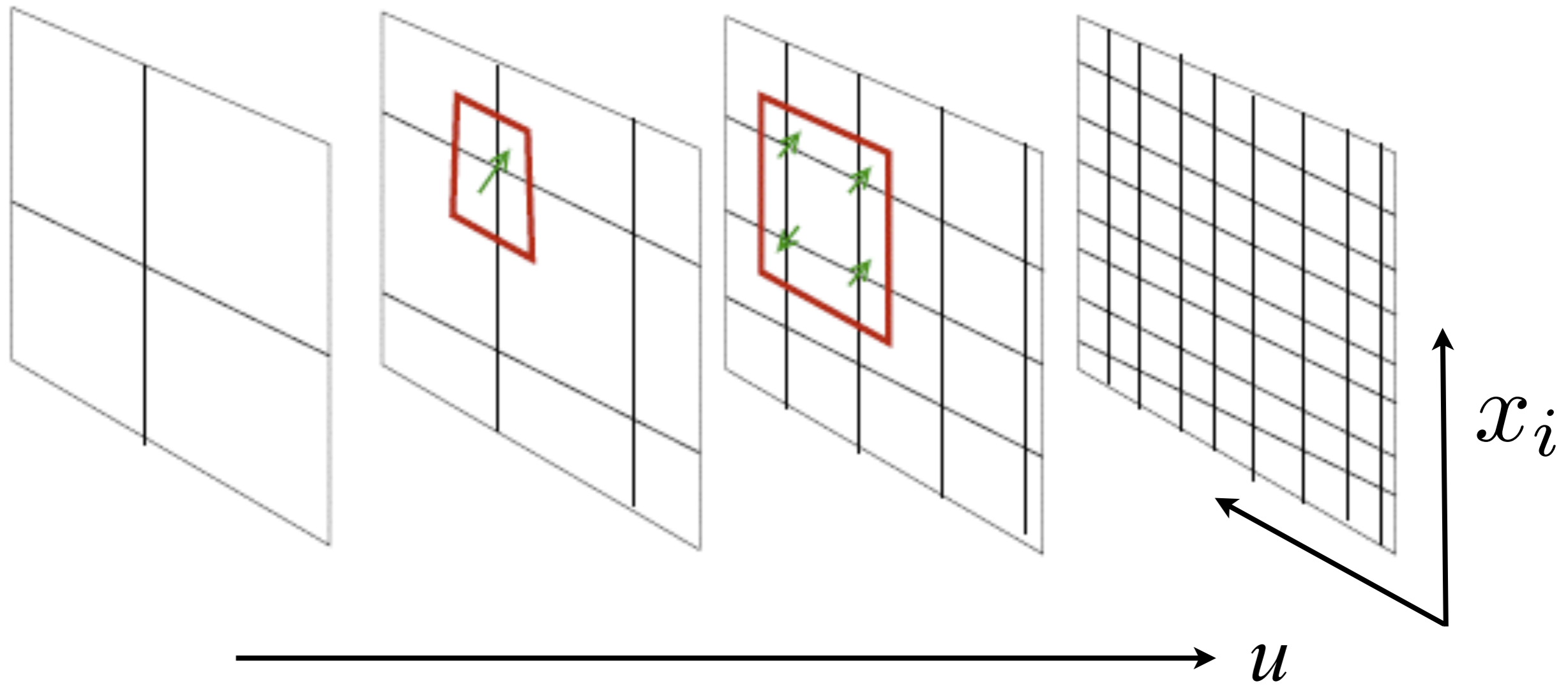
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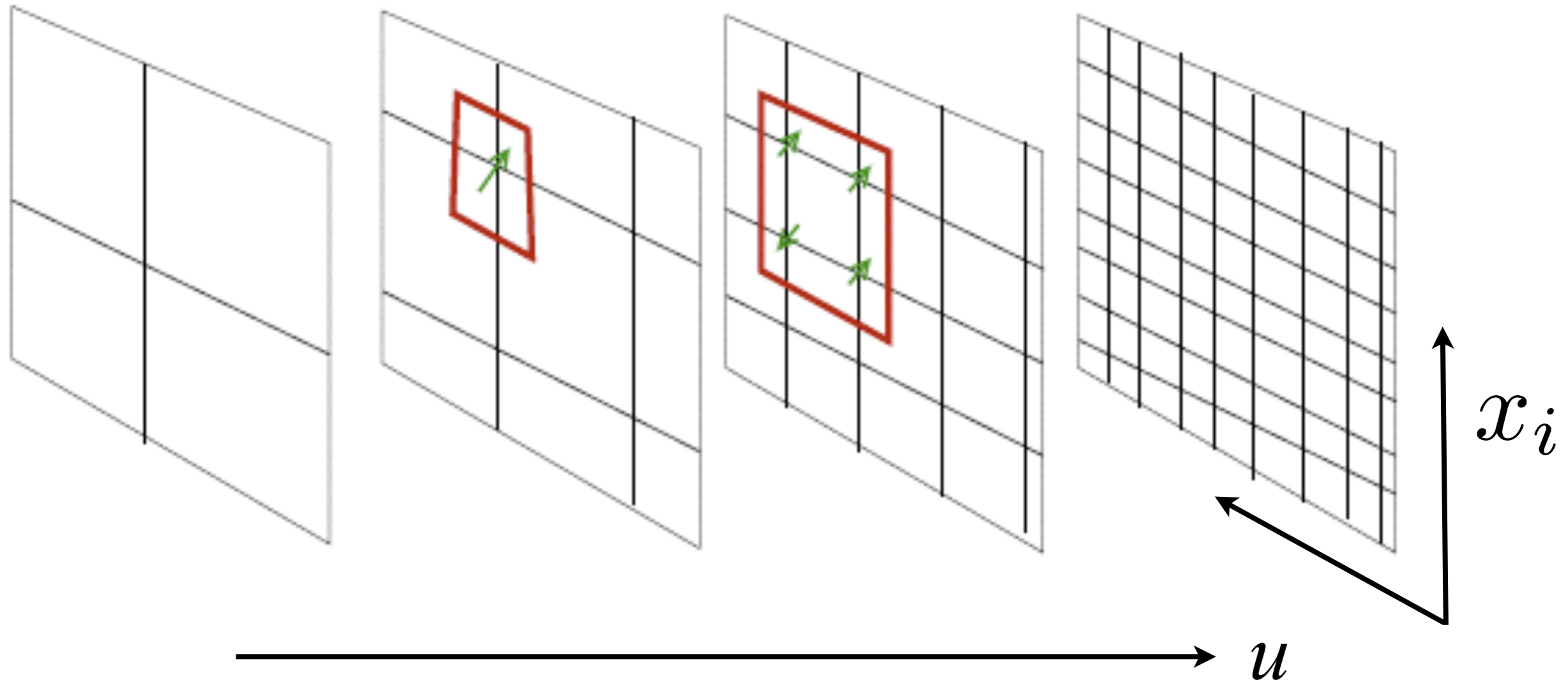
Field theories in $d + 1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u .

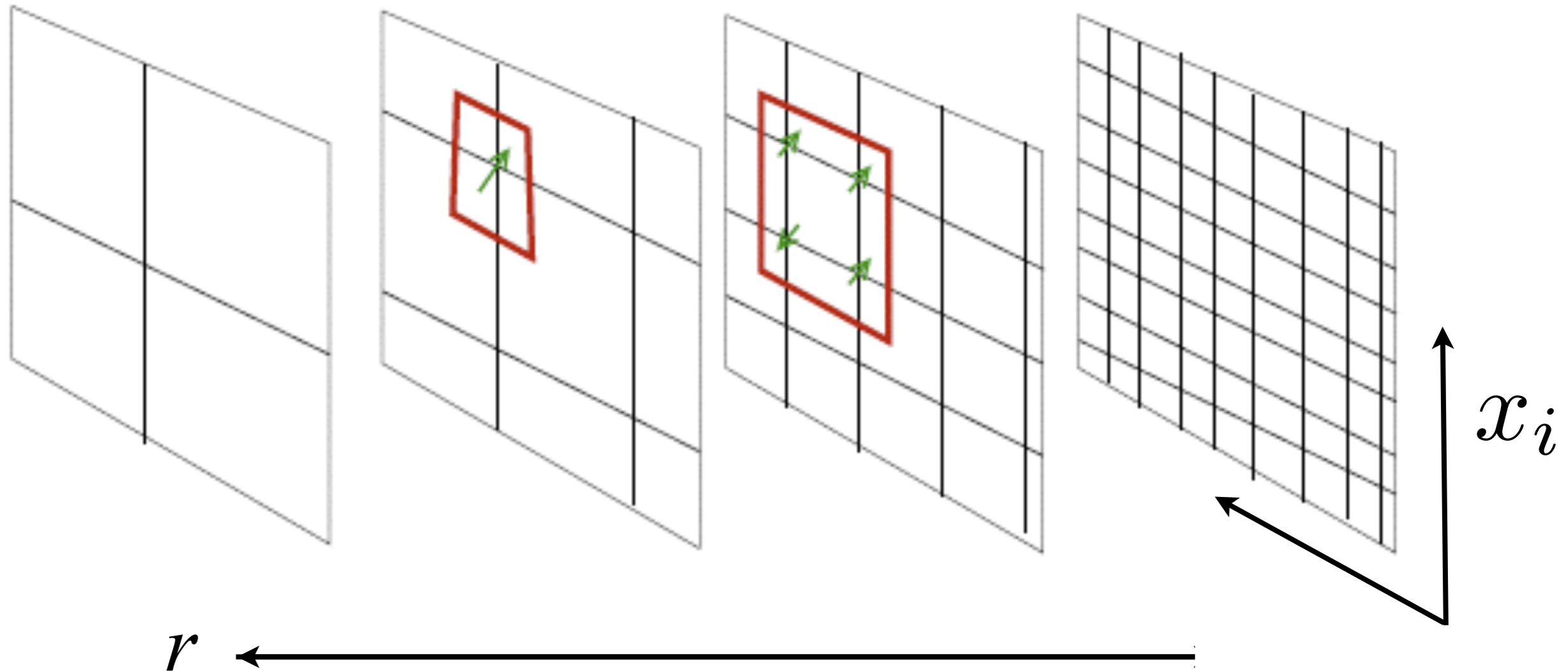


Holography



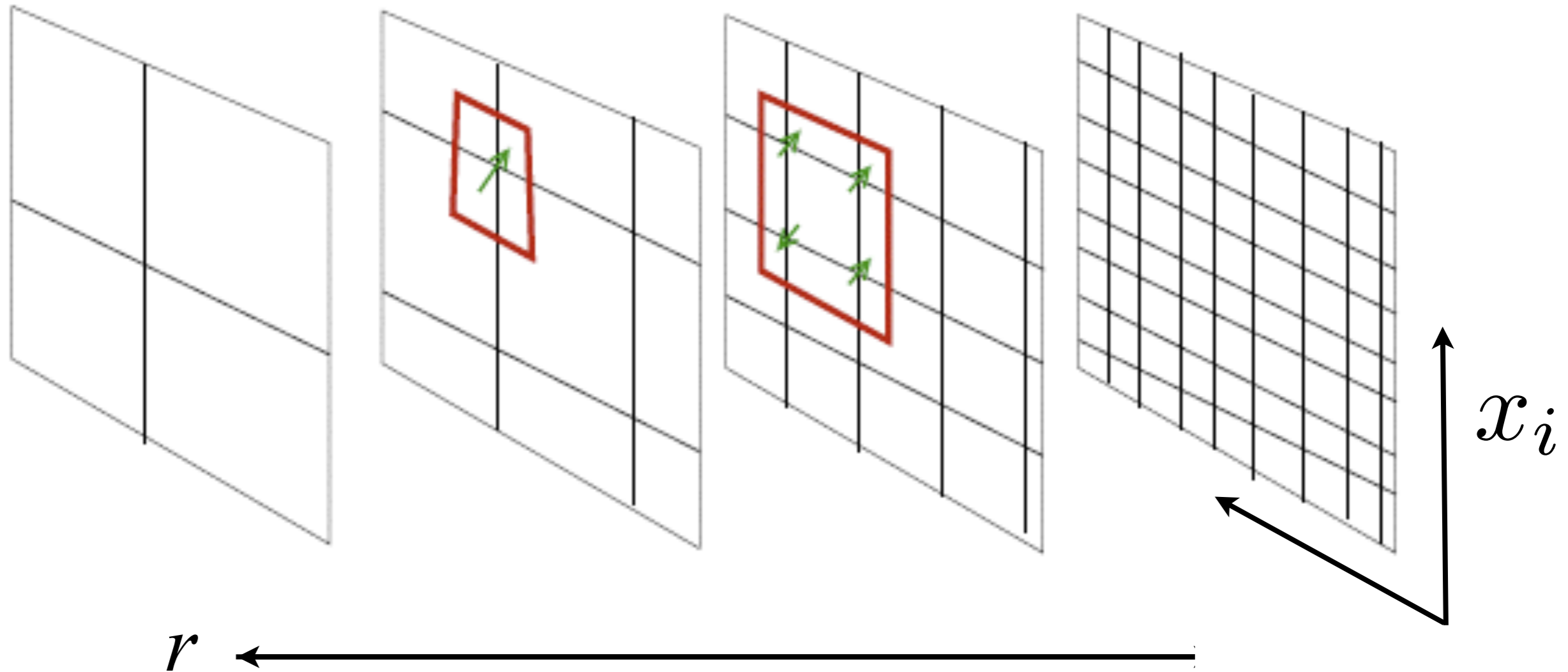
Key idea: \Rightarrow Implement u as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

Holography



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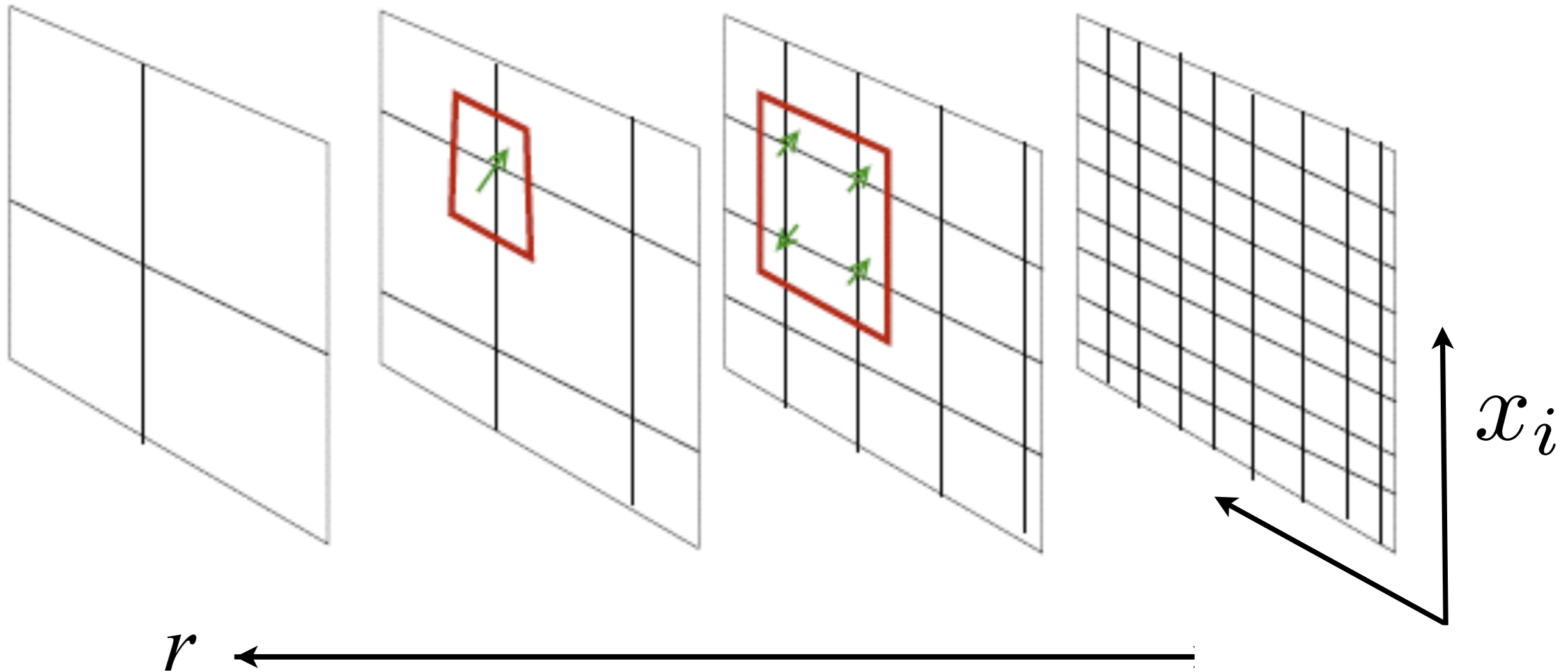
Holography



For a relativistic CFT in d spatial dimensions, the proper length, ds , in the holographic space is fixed by demanding the scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

Holography

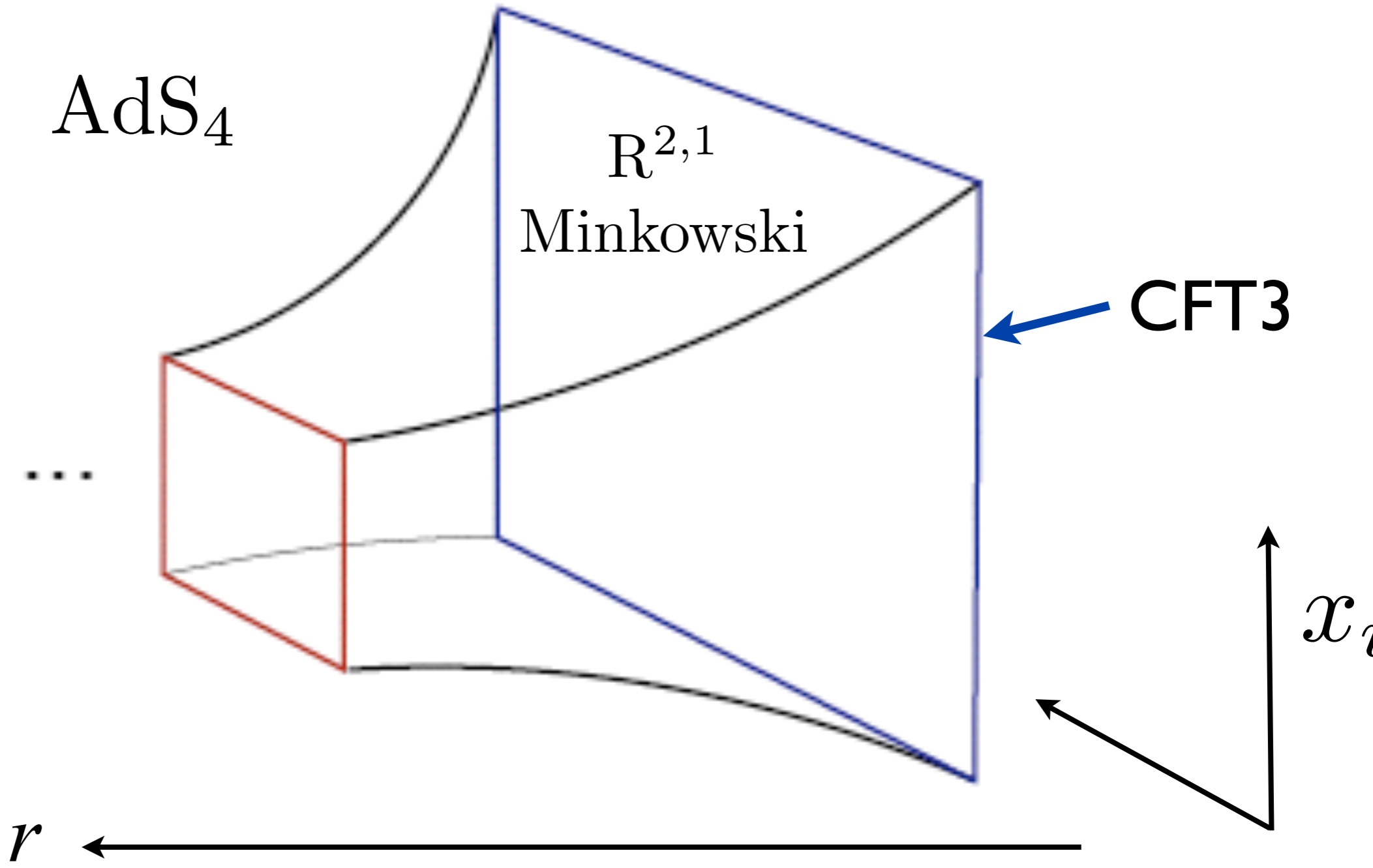


This gives the unique metric

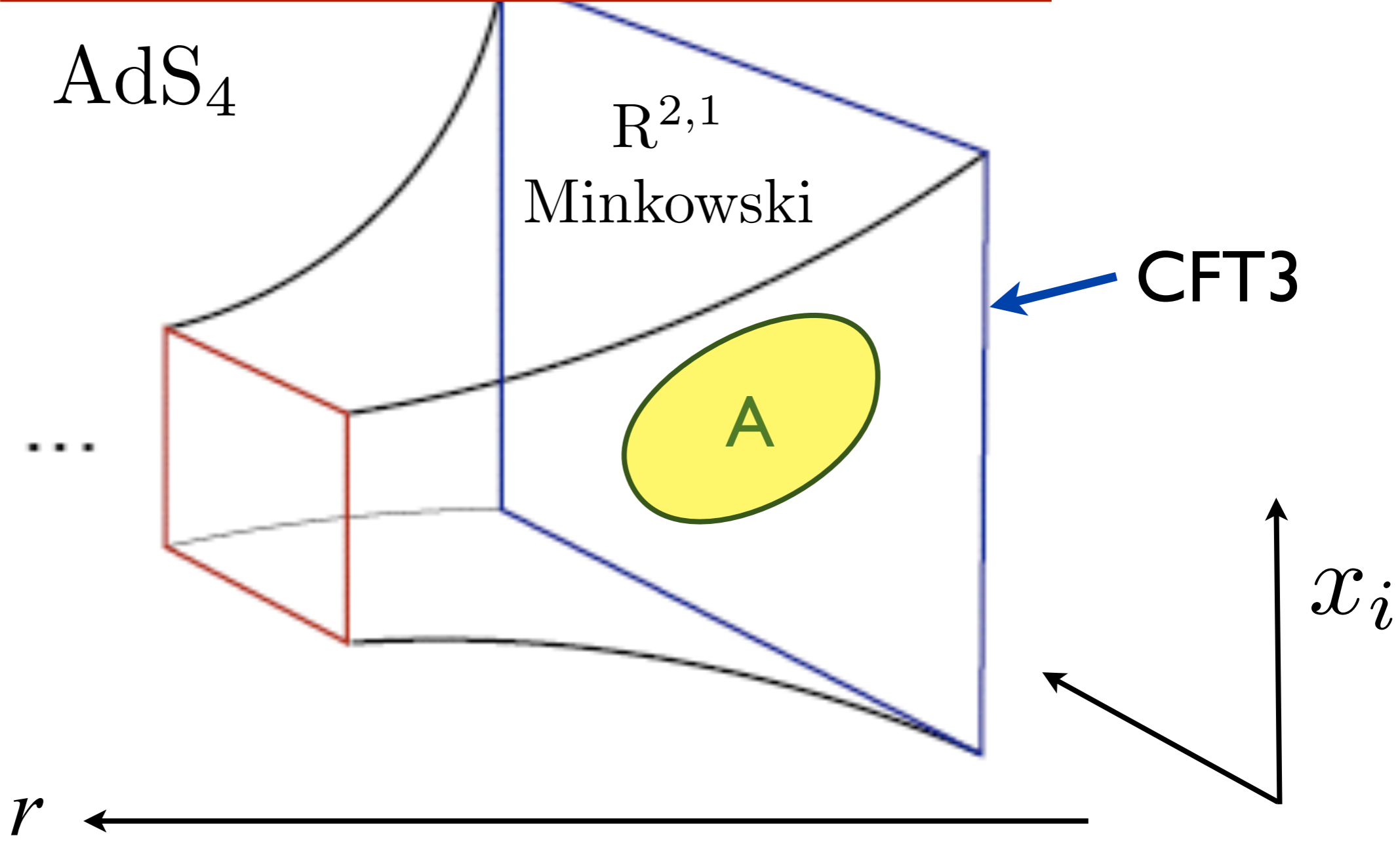
$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

This is the metric of anti-de Sitter space AdS_{d+2} .

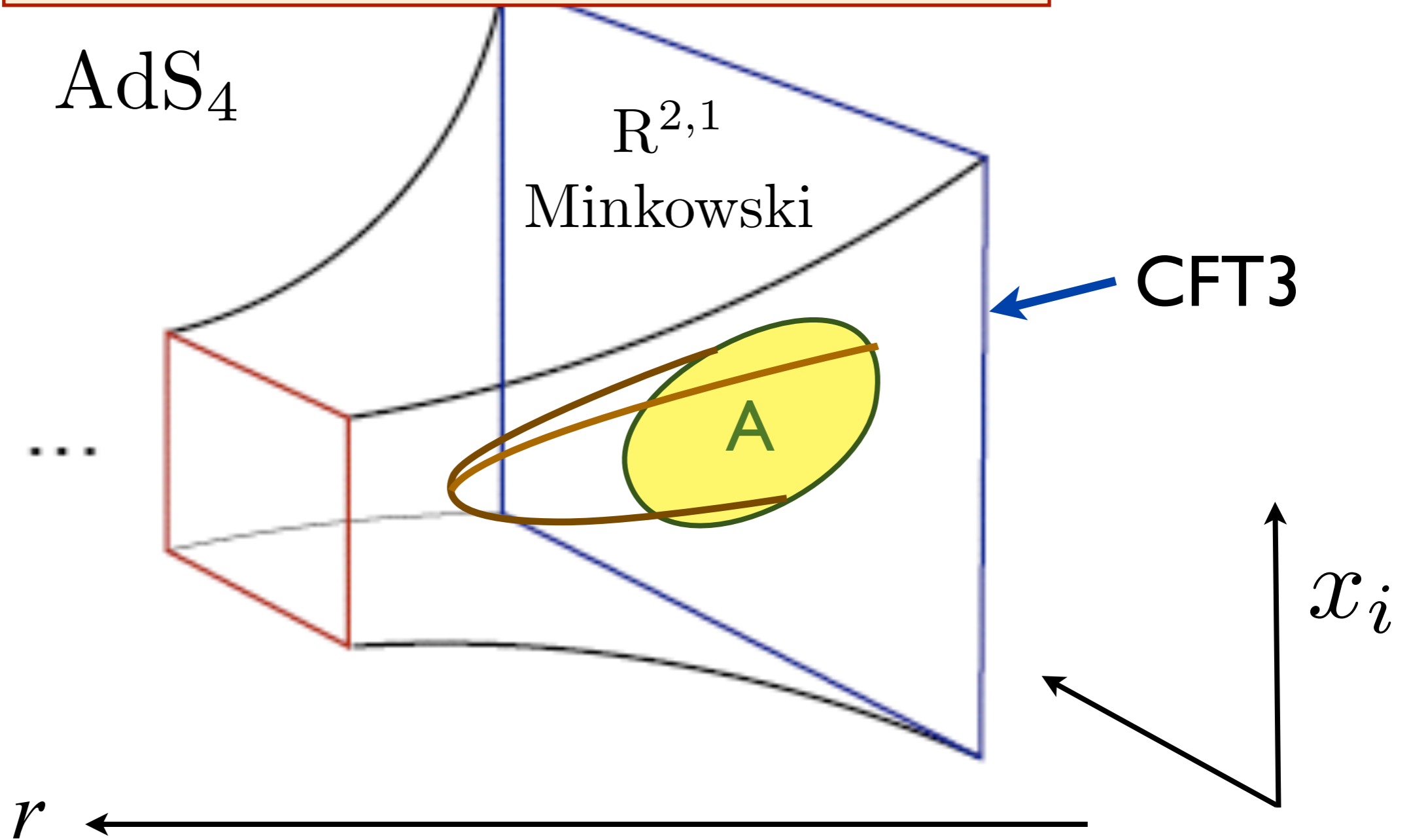
AdS/CFT correspondence



Holography and Entanglement

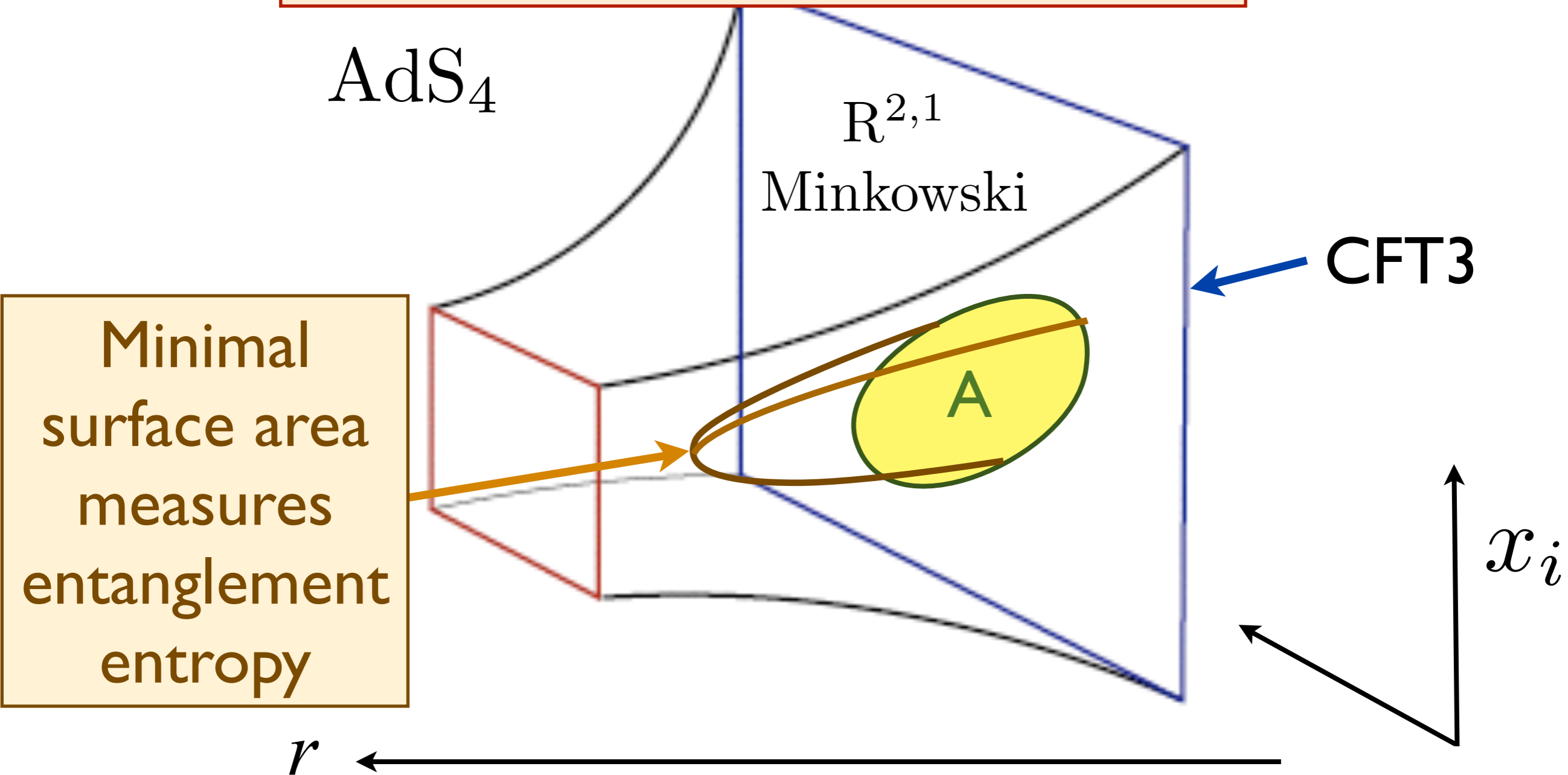


Holography and Entanglement

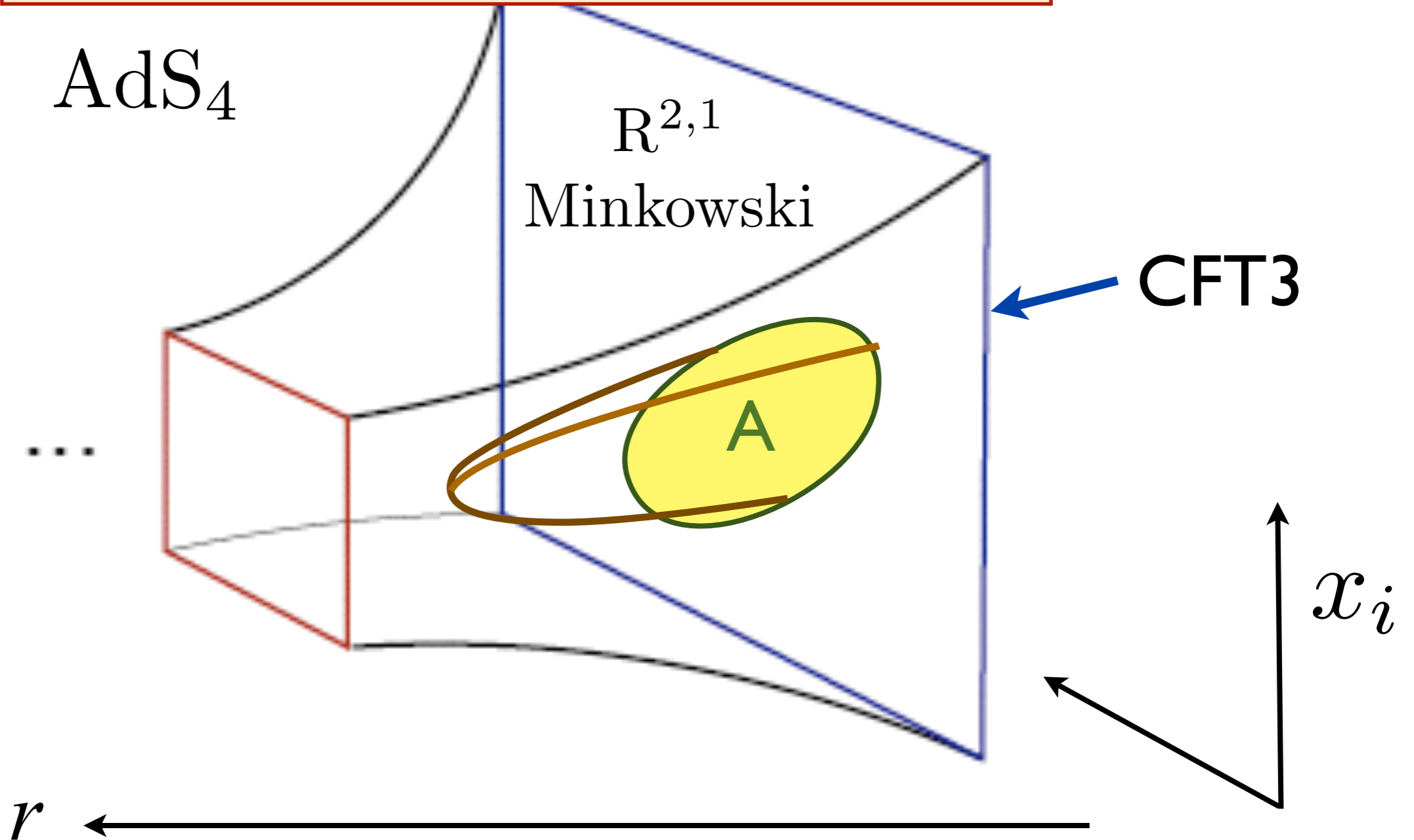


Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe “outside” : *i.e.* the region is surrounded by an imaginary horizon.

Holography and Entanglement



Holography and Entanglement



- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

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(d) Holography of strange metals

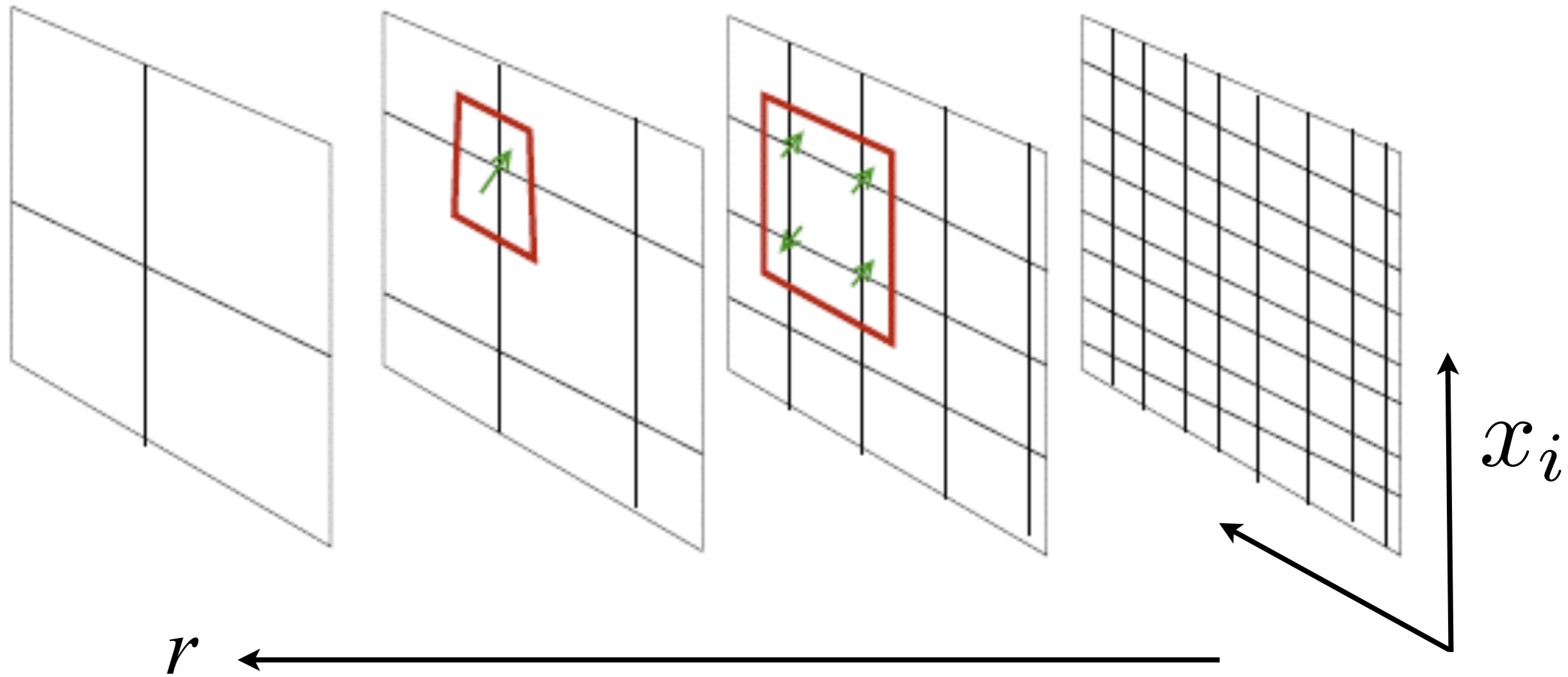
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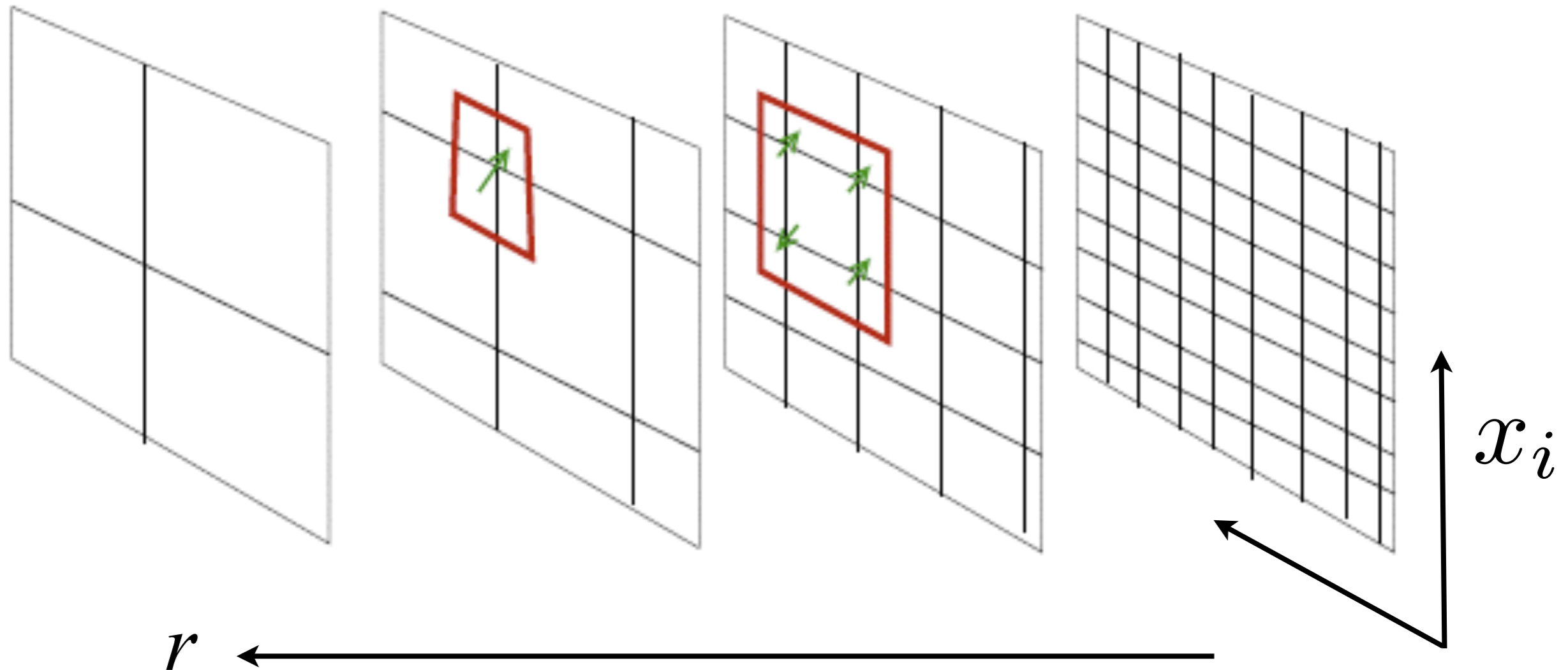
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Generalized holography



Generalized holography

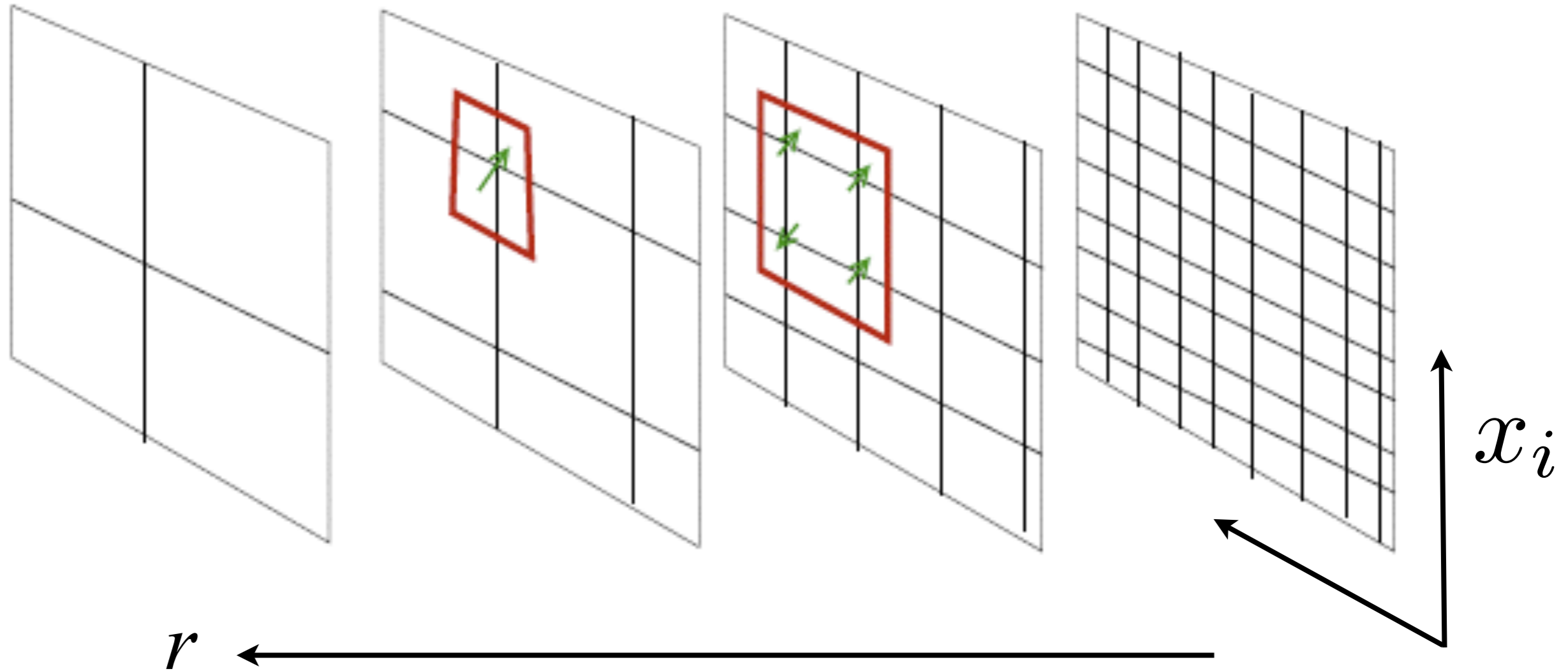


Consider a metric which transforms under rescaling as

$$x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Generalized holography



The most general such metric is

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

This is the most general metric which is invariant under the scale transformation

$$\begin{aligned} x_i &\rightarrow \zeta x_i \\ t &\rightarrow \zeta^z t \\ ds &\rightarrow \zeta^{\theta/d} ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points). We will see shortly that θ is the violation of hyperscaling exponent.

We have used reparametrization invariance in r to define it so that it scales as

$$r \rightarrow \zeta^{(d-\theta)/d} r.$$

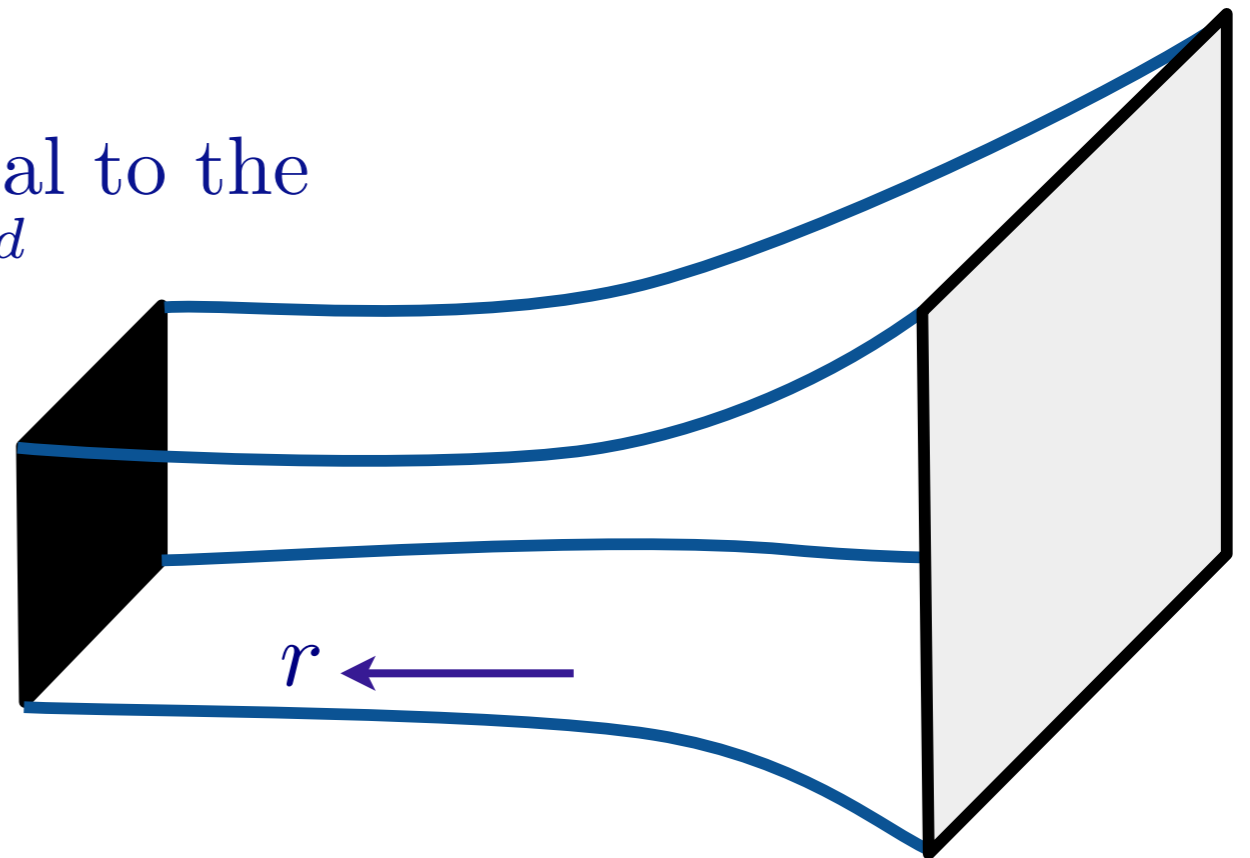
Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



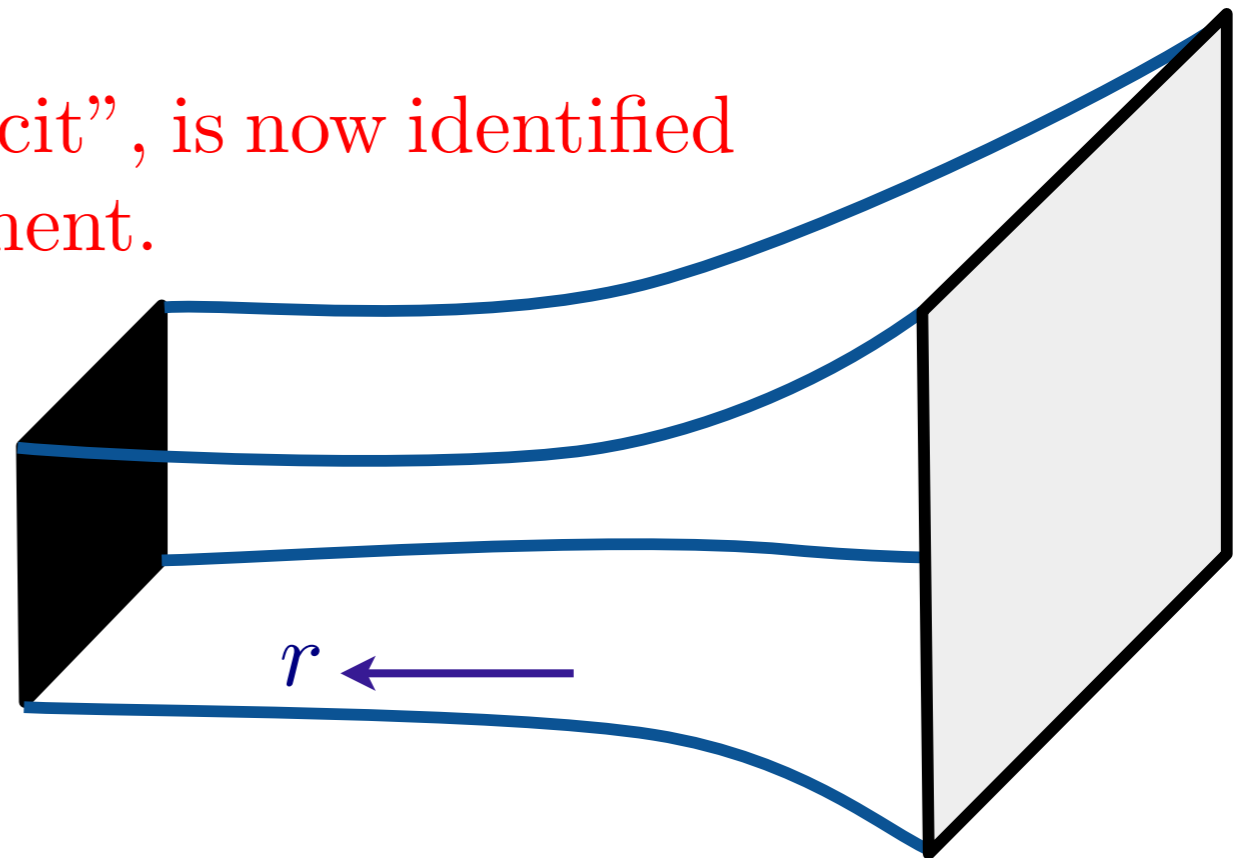
Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$, the “dimension deficit”, is now identified as the violation of hyperscaling exponent.



Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

Generalized holography

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$$z \geq 1 + \frac{\theta}{d}$$

The non-Fermi liquid in $d = 2$ has $\theta = d - 1$, and this implies $z \geq 3/2$. So the lower bound is precisely the value obtained for the non-Fermi liquid!

Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases} .$$

Generalized holography

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The non-Fermi liquid has log-violation of “area law”, and this appears precisely at the correct value $\theta = d - 1$!

Generalized holography

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The non-Fermi liquid has log-violation of “area law”, and this appears precisely at the correct value $\theta = d - 1$!

Moreover, the co-efficient of $P \ln P$ computed holographically is independent of the shape of the entangling region just as expected for a circular Fermi surface!!

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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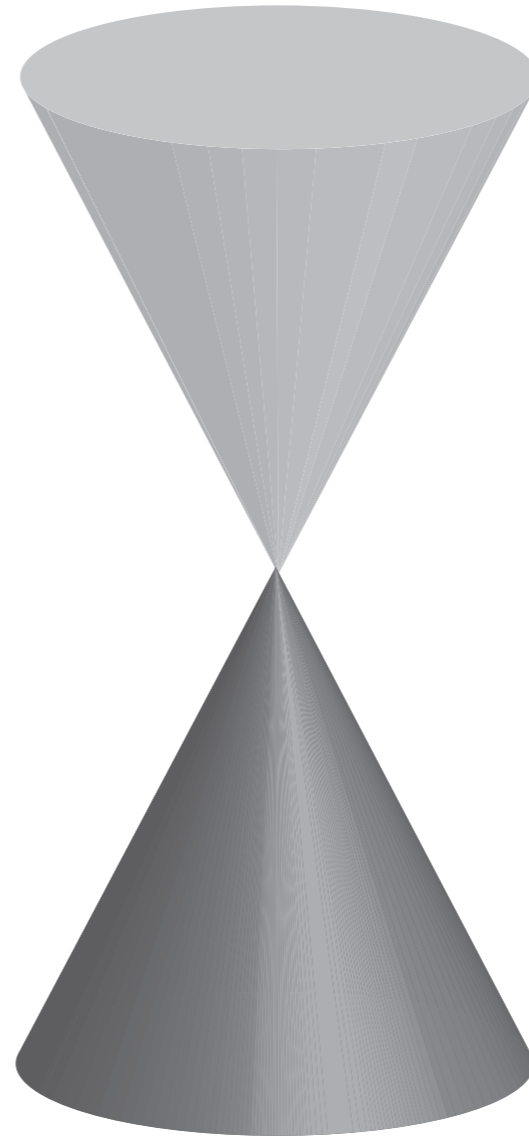
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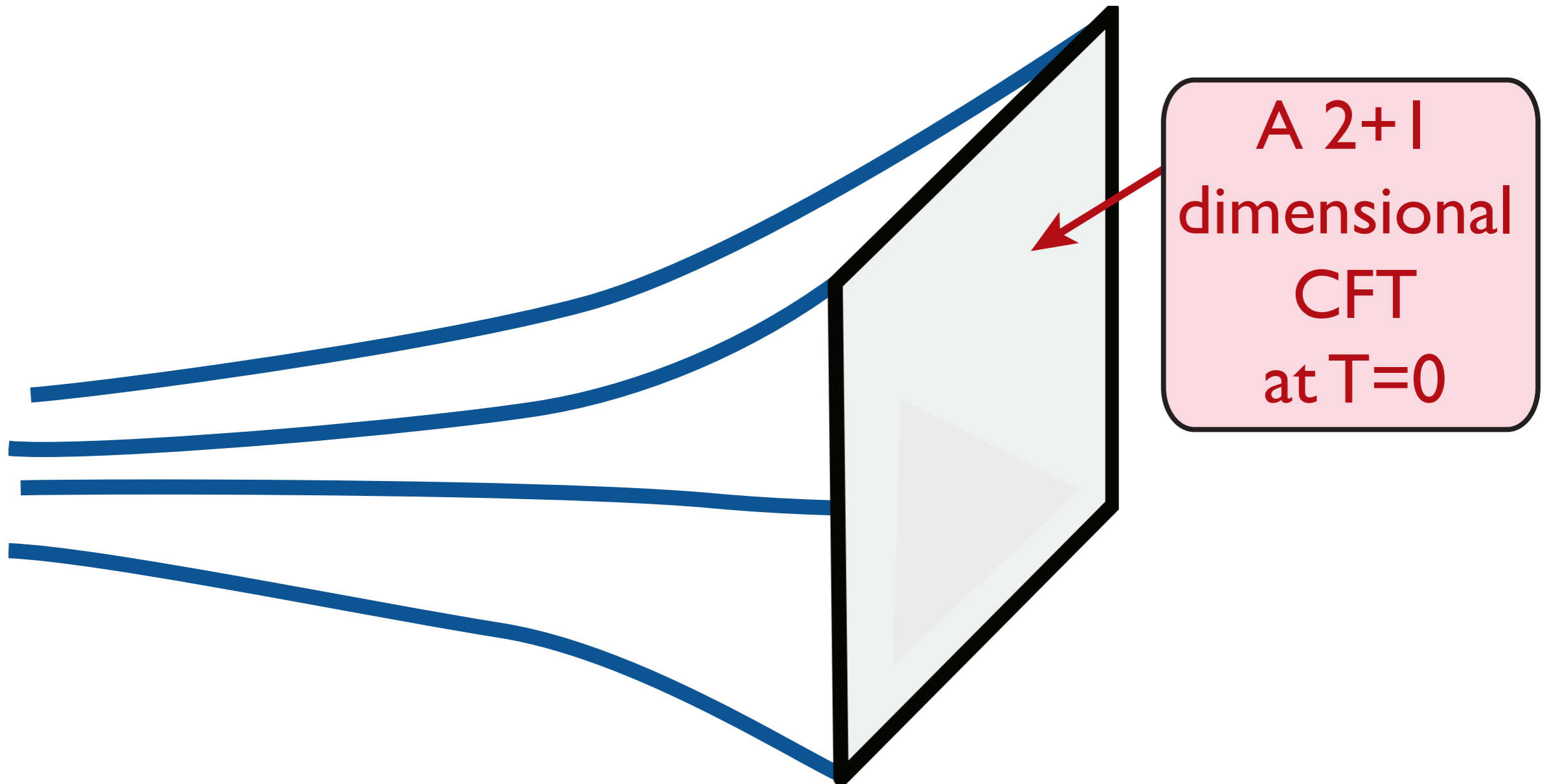
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Begin with a CFT

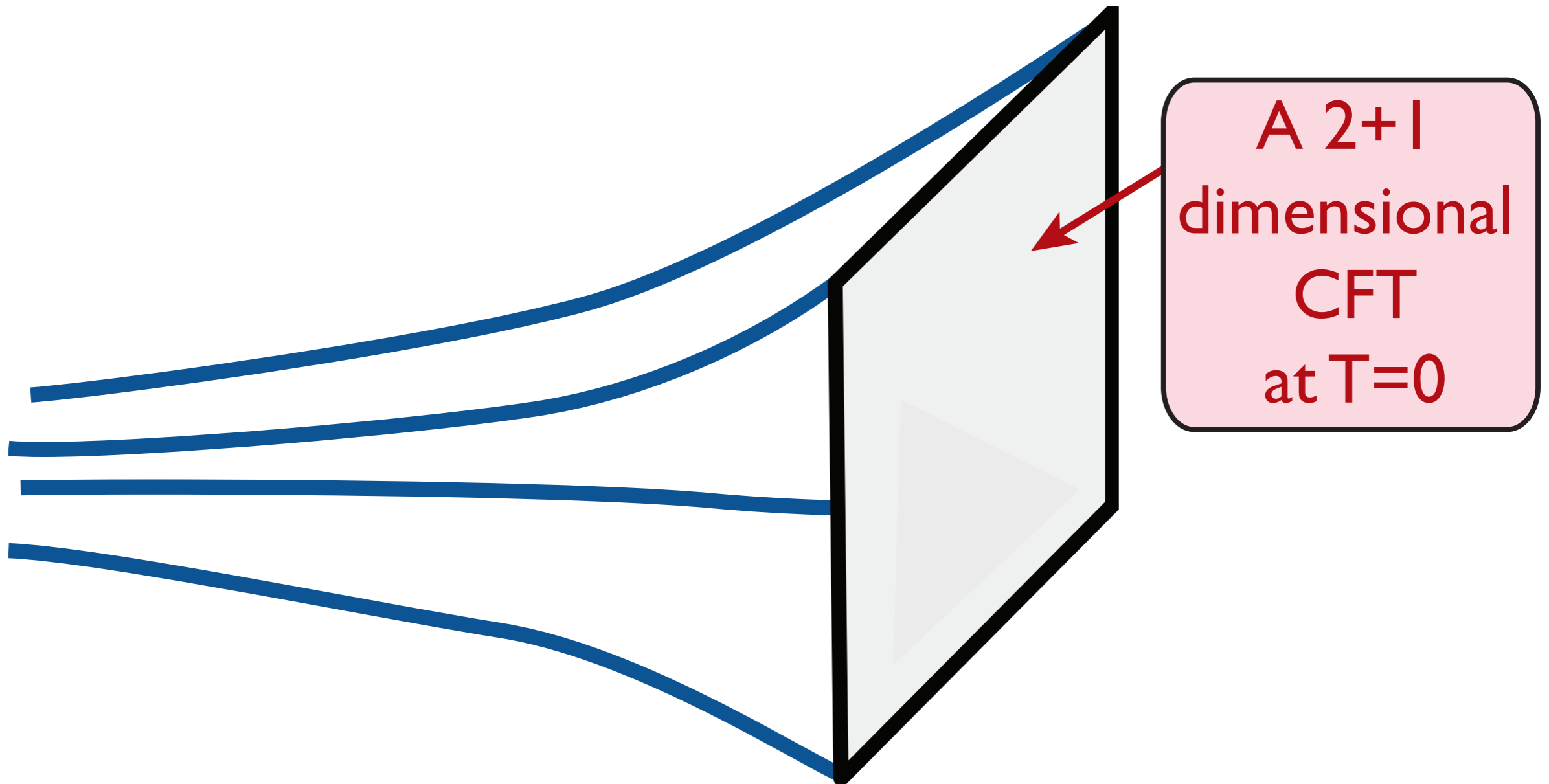


Holographic representation: AdS₄



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

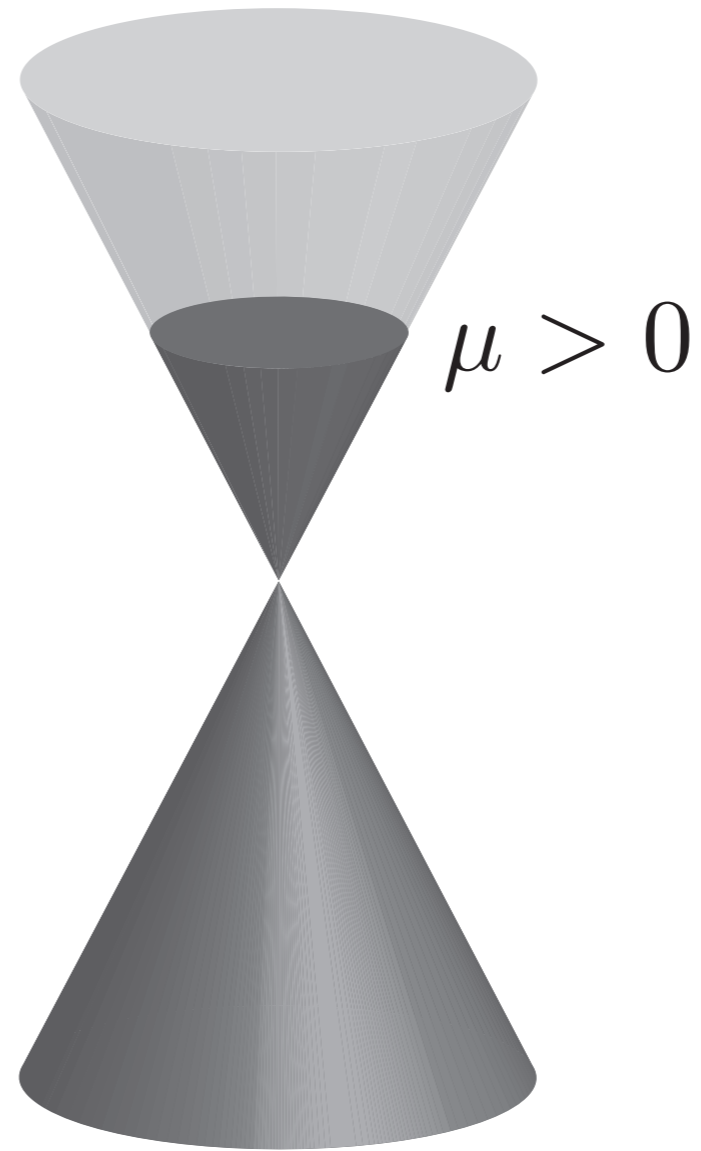
Holographic representation: AdS₄



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1$

Apply a chemical potential



AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right].$$

This is to be solved subject to the constraint

$$A_\mu(r \rightarrow 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where \mathcal{A}_μ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

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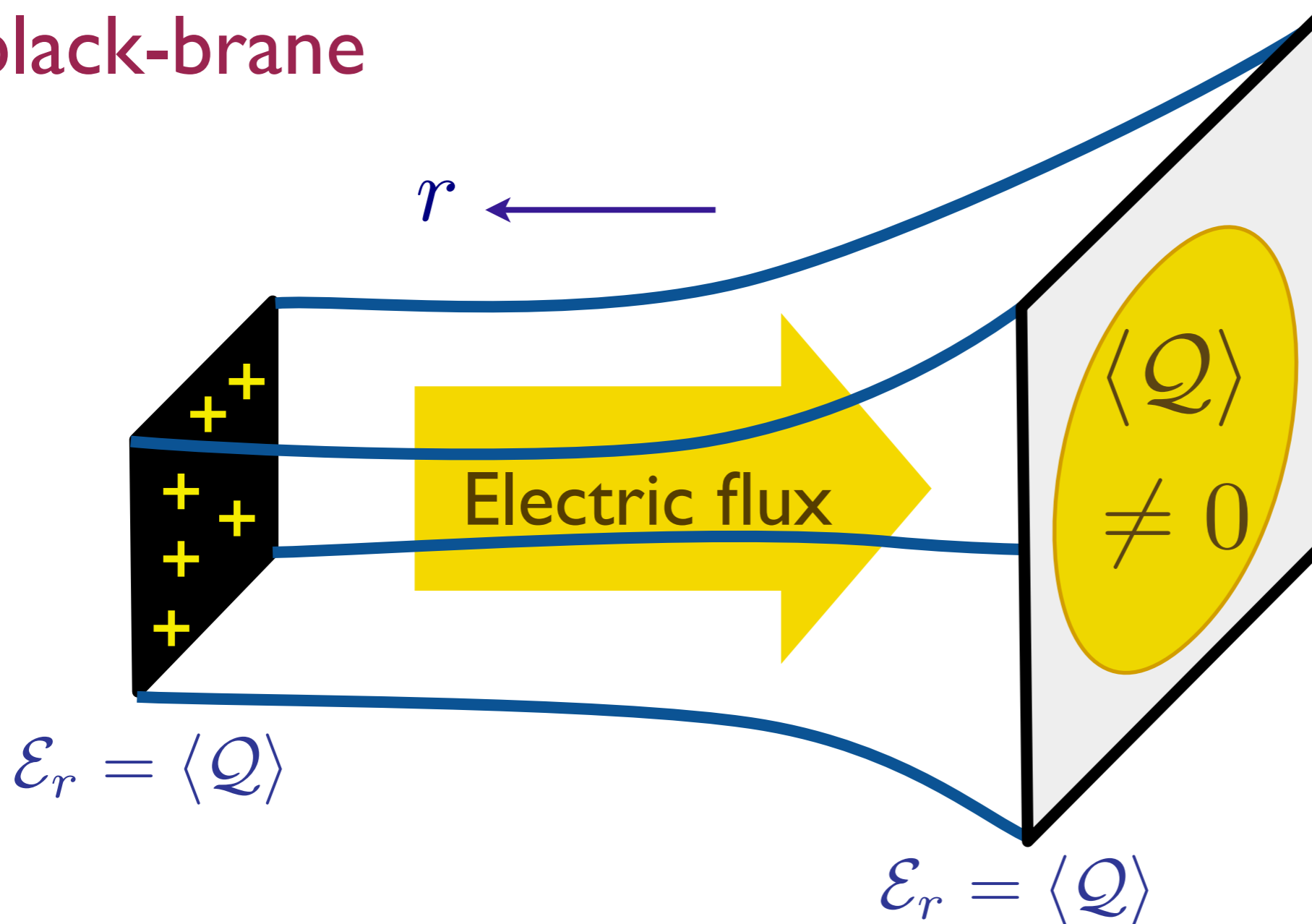
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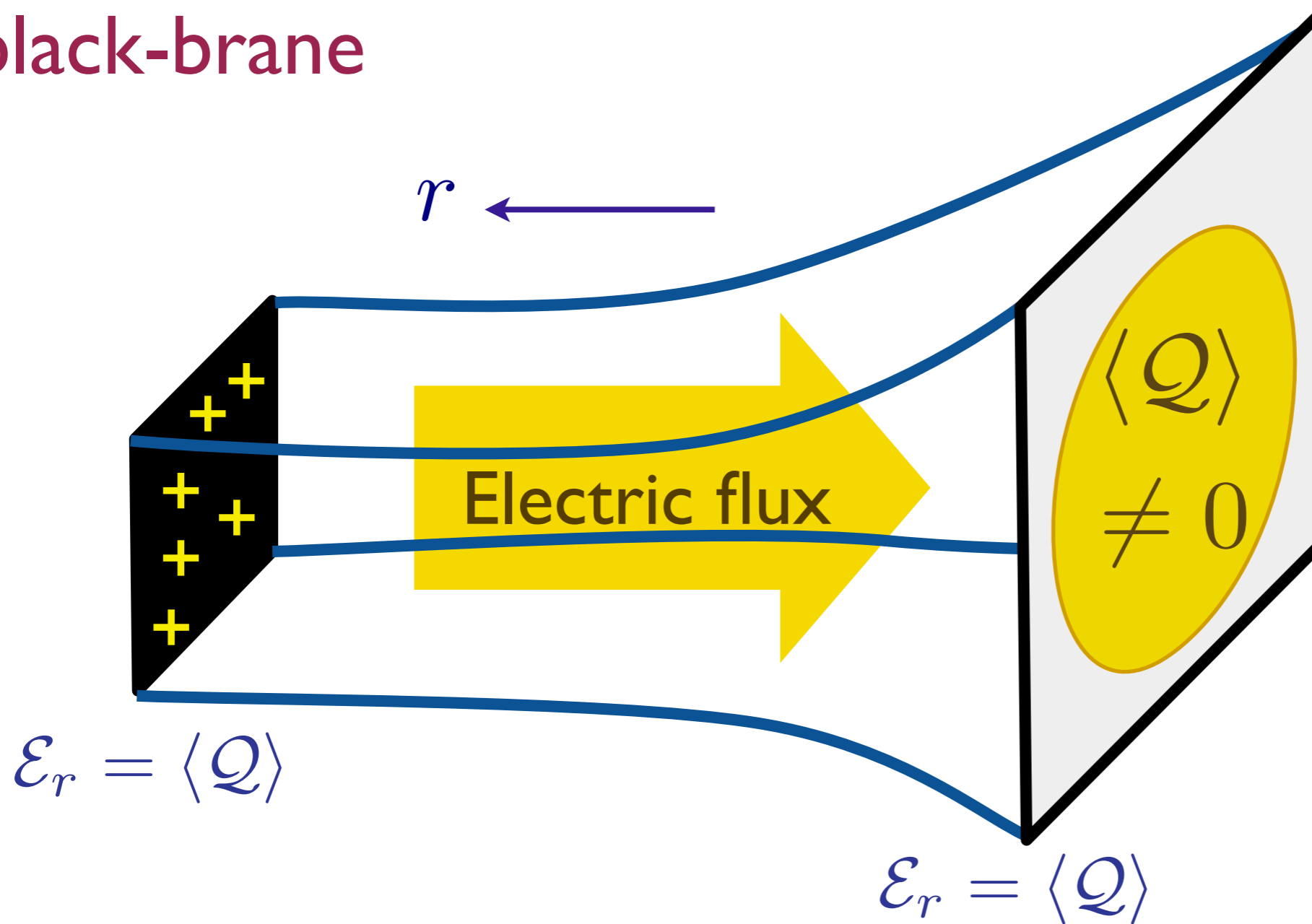
At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

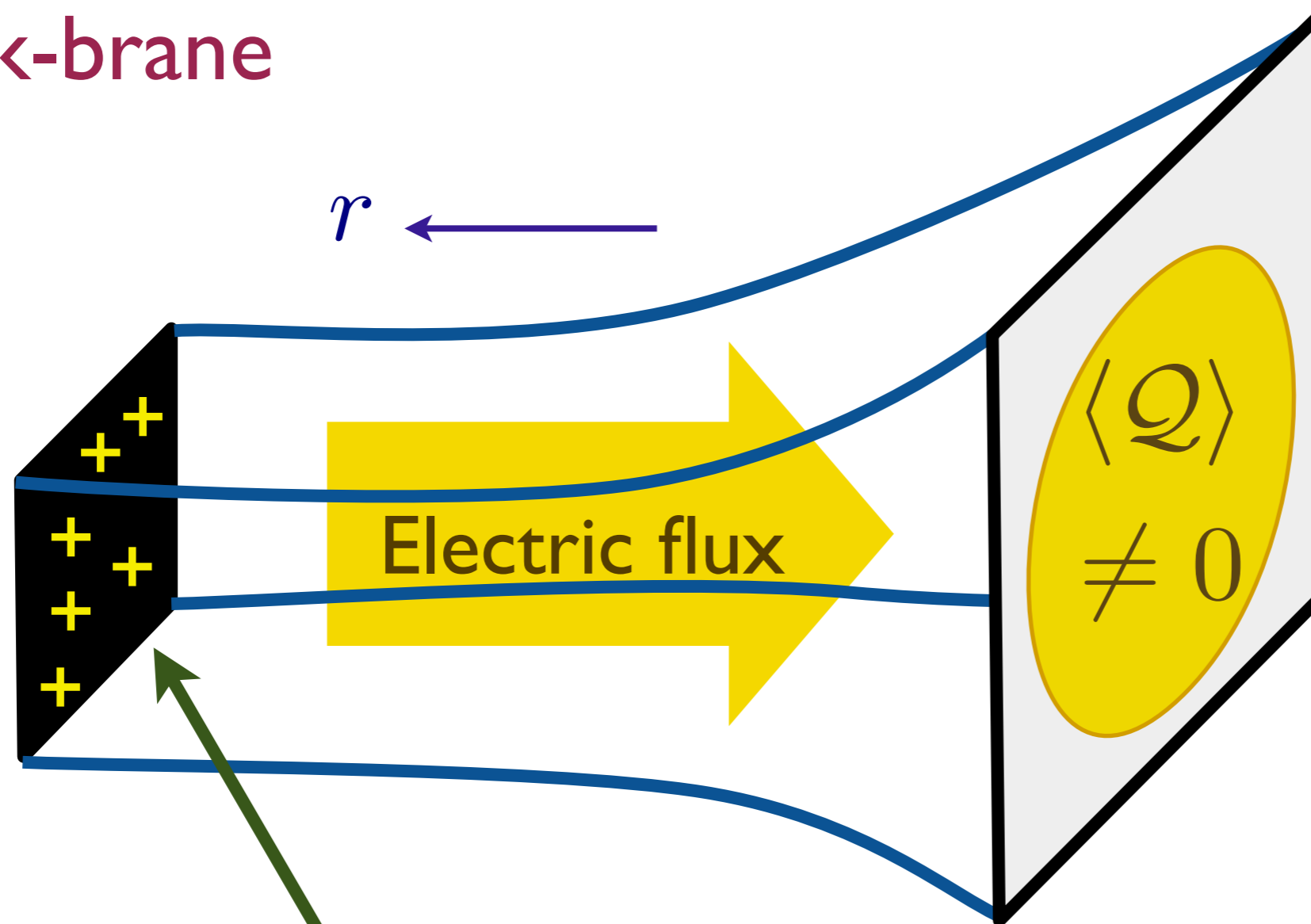
The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

$$\text{with } f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right) \text{ and } R = \frac{\sqrt{6}Lg_4}{\kappa\mu}, \text{ and } A_\tau = \mu \left(1 - \frac{r}{R}\right)$$

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



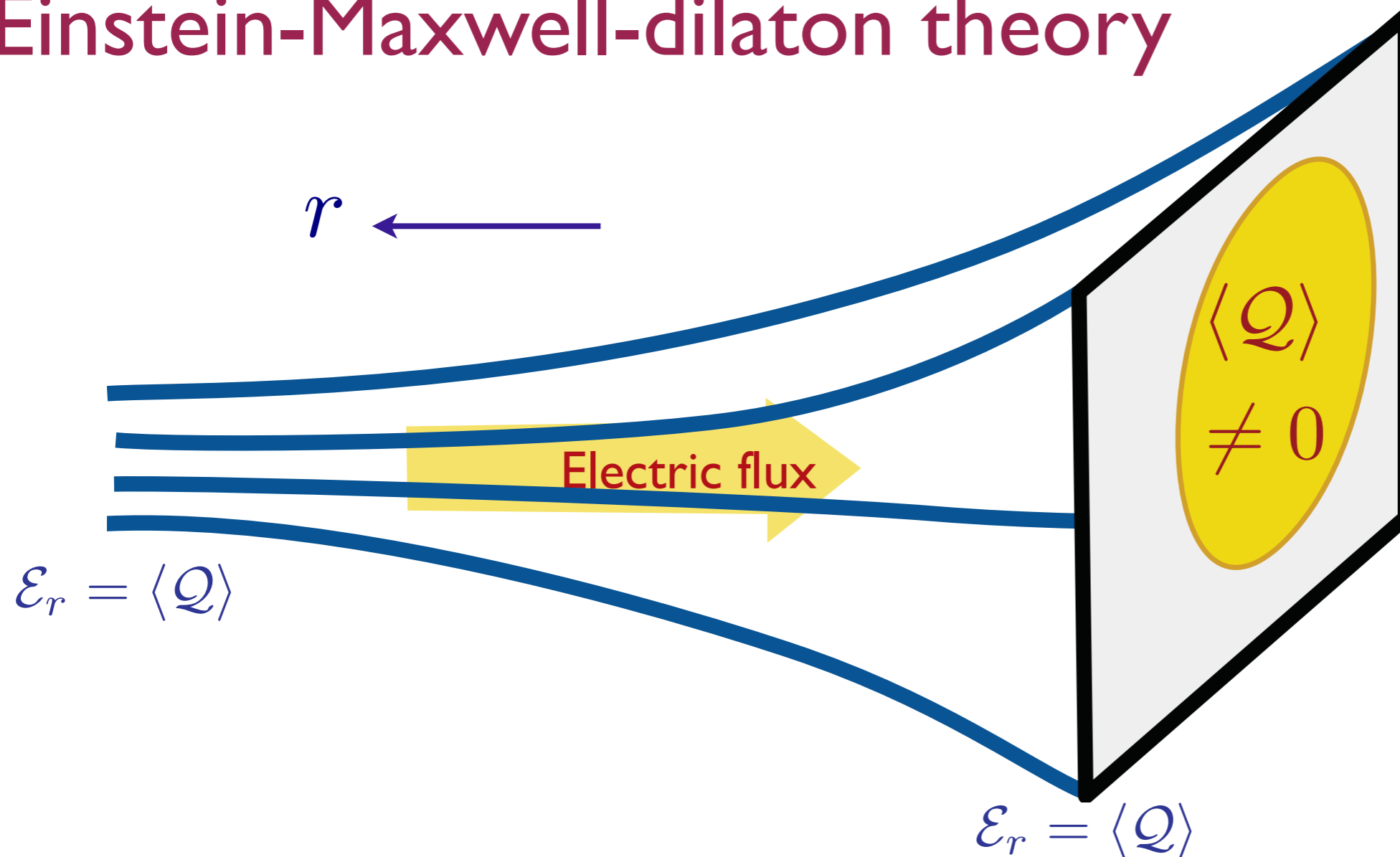
At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\text{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6} \left(\frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$

T. Faulkner, H. Liu,
J. McGreevy,
and D. Vegh,
arXiv:0907.2694

Holography of a non-Fermi liquid

Einstein-Maxwell-dilaton theory



$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).

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Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

The $r \rightarrow \infty$ limit of the metric of the Einstein-Maxwell-dilaton (EMD) theory has the most general form with

$$\theta = \frac{d^2 \beta}{\alpha + (d-1)\beta}$$
$$z = 1 + \frac{\theta}{d} + \frac{8(d(d-\theta) + \theta)^2}{d^2(d-\theta)\alpha^2}.$$

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Computation of the entanglement entropy in the EMD theory via the Ryu-Takayanagi formula for $\theta = d - 1$ yields

$$\mathcal{S}_E = \mathcal{C}_E Q^{(d-1)/d} P \ln P$$

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This is precisely as expected for a Fermi surface, when combined with the Luttinger theorem!

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- Why is k_F not observable as Friedel oscillations in correlators of the density, and other gauge-neutral operators ?

Answer: need non-perturbative effects of monopole operators
T. Faulkner and N. Iqbal, arXiv:1207.4208;
S. Sachdev, Phys. Rev. D **86**, 126003 (2012)

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- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography