Theory of Quantum Matter: from Quantum Fields to Strings

Salam Distinguished Lectures
The Abdus Salam International Center for Theoretical Physics
Trieste, Italy
January 27-30, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
HARVARD

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. Holography, entanglement, and strange metals

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

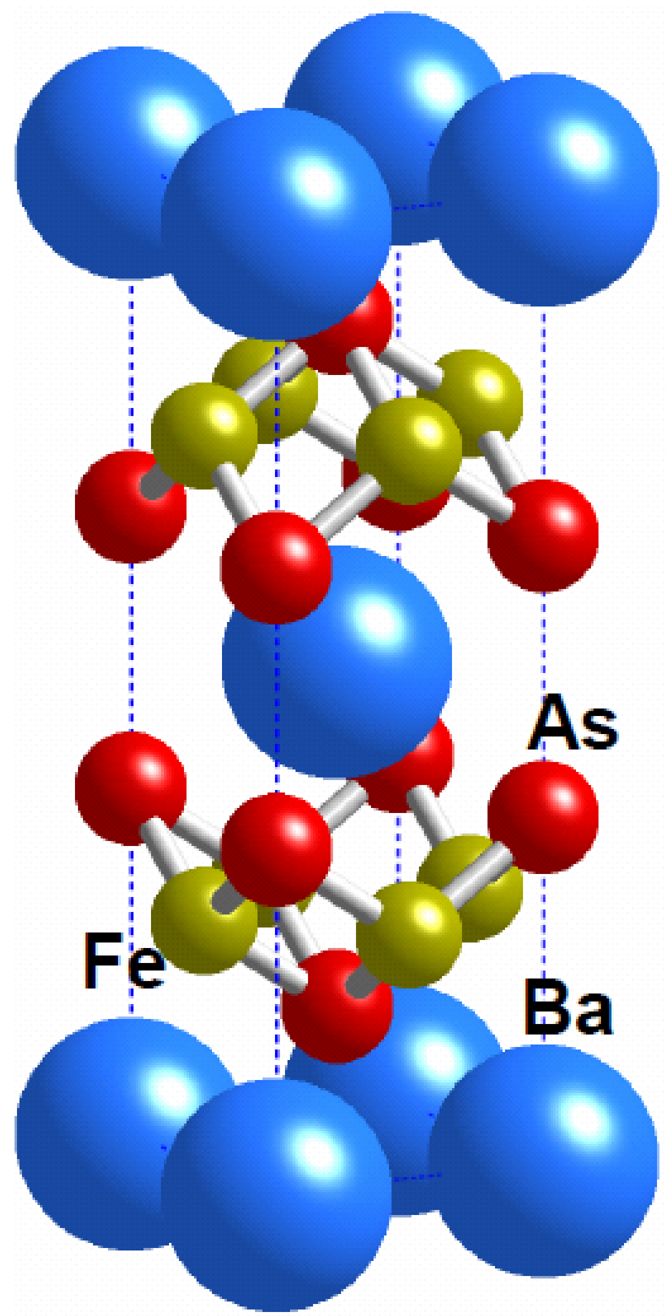
B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

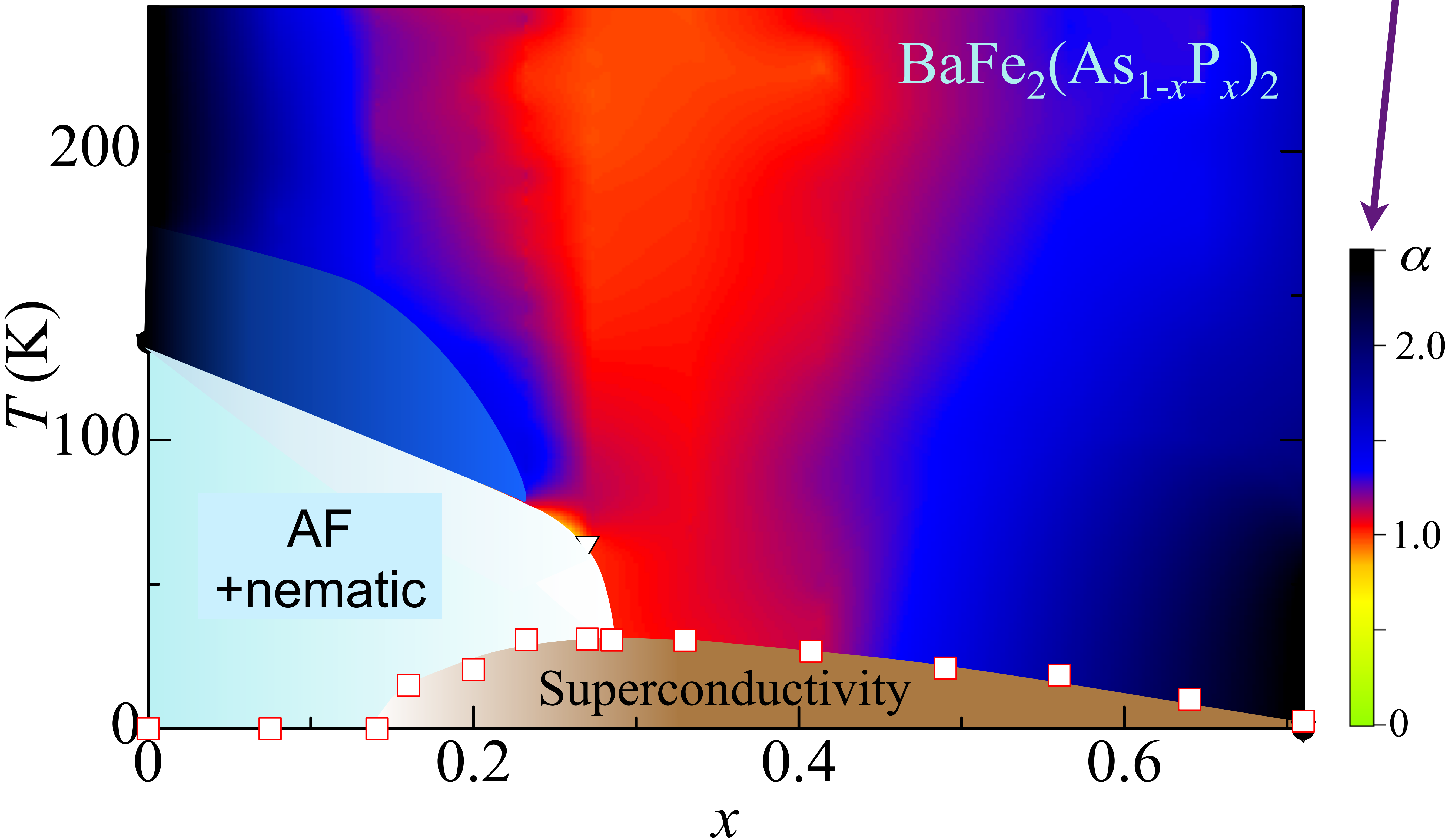
C. Holography, entanglement, and strange metals

Iron pnictides:

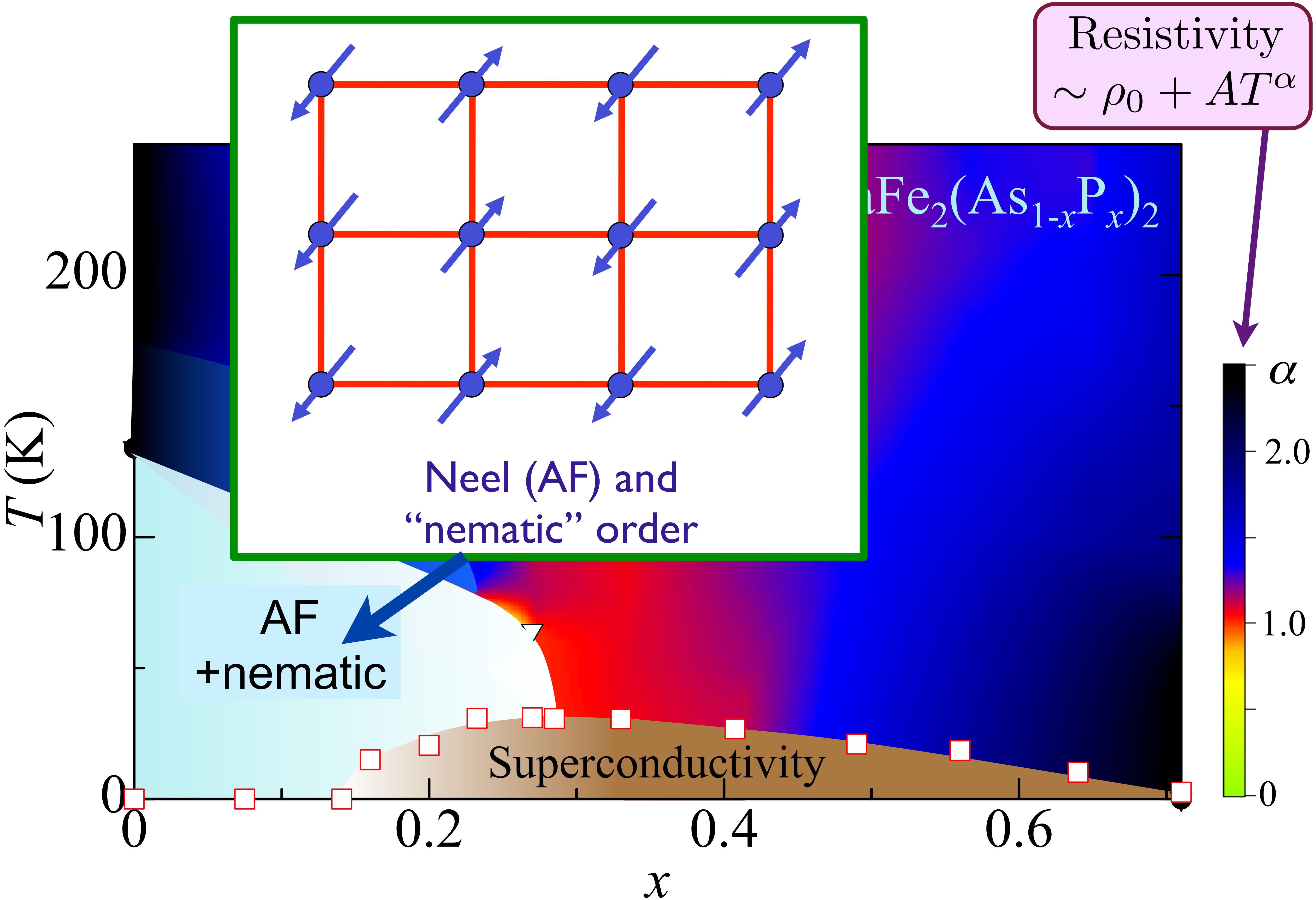
a new class of high temperature superconductors



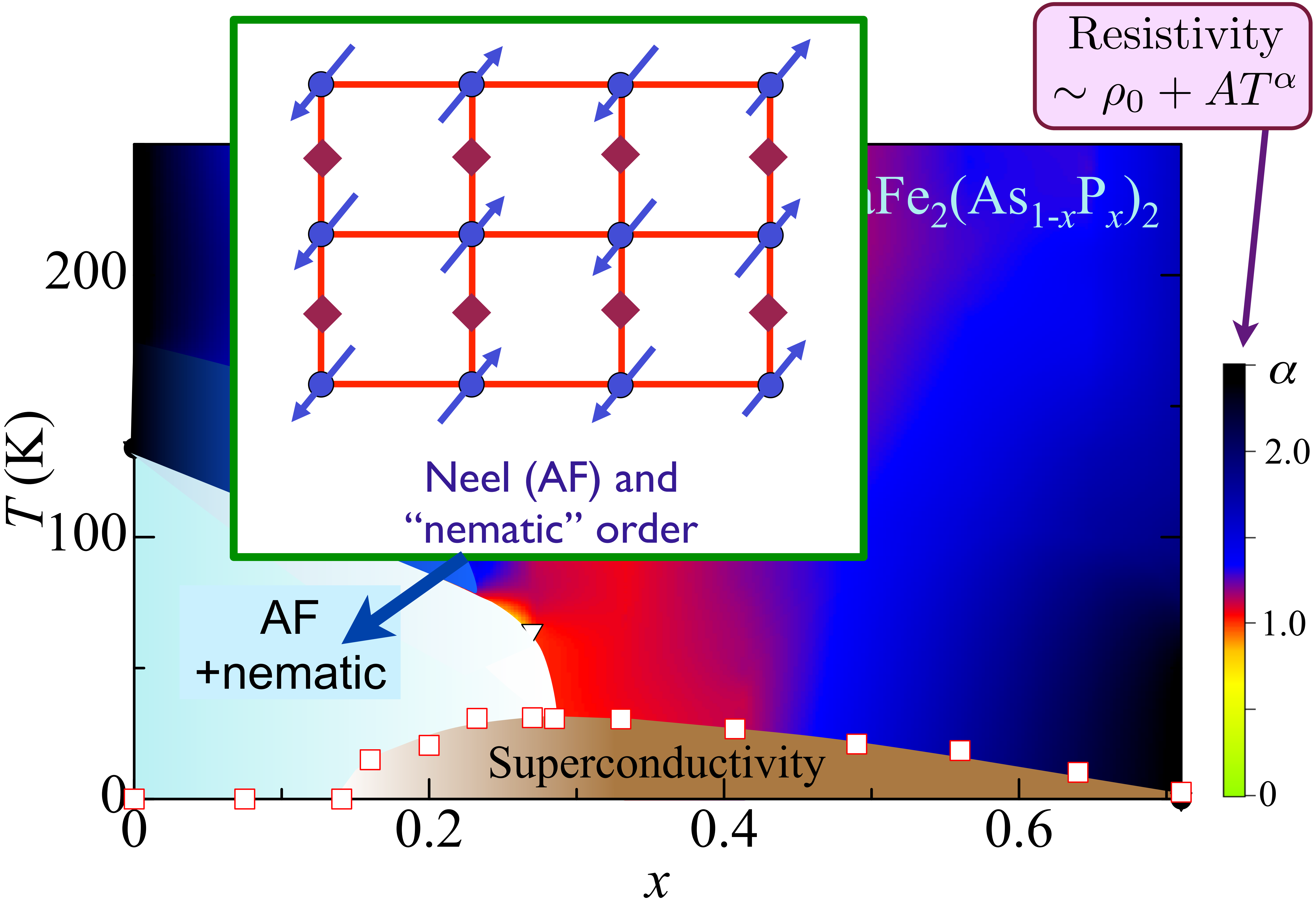
Resistivity
$\sim \rho_0 + AT^{\alpha}$



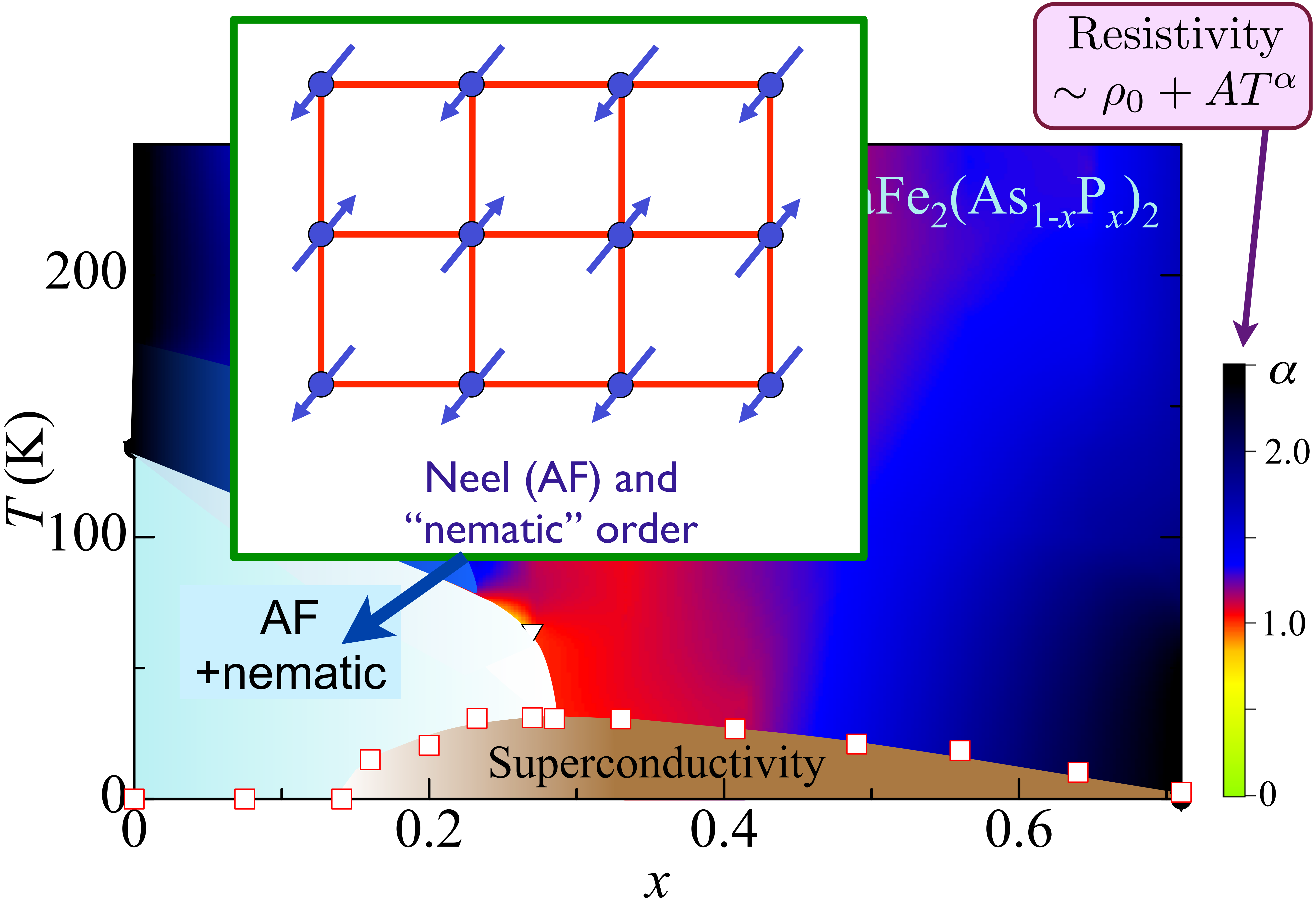
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



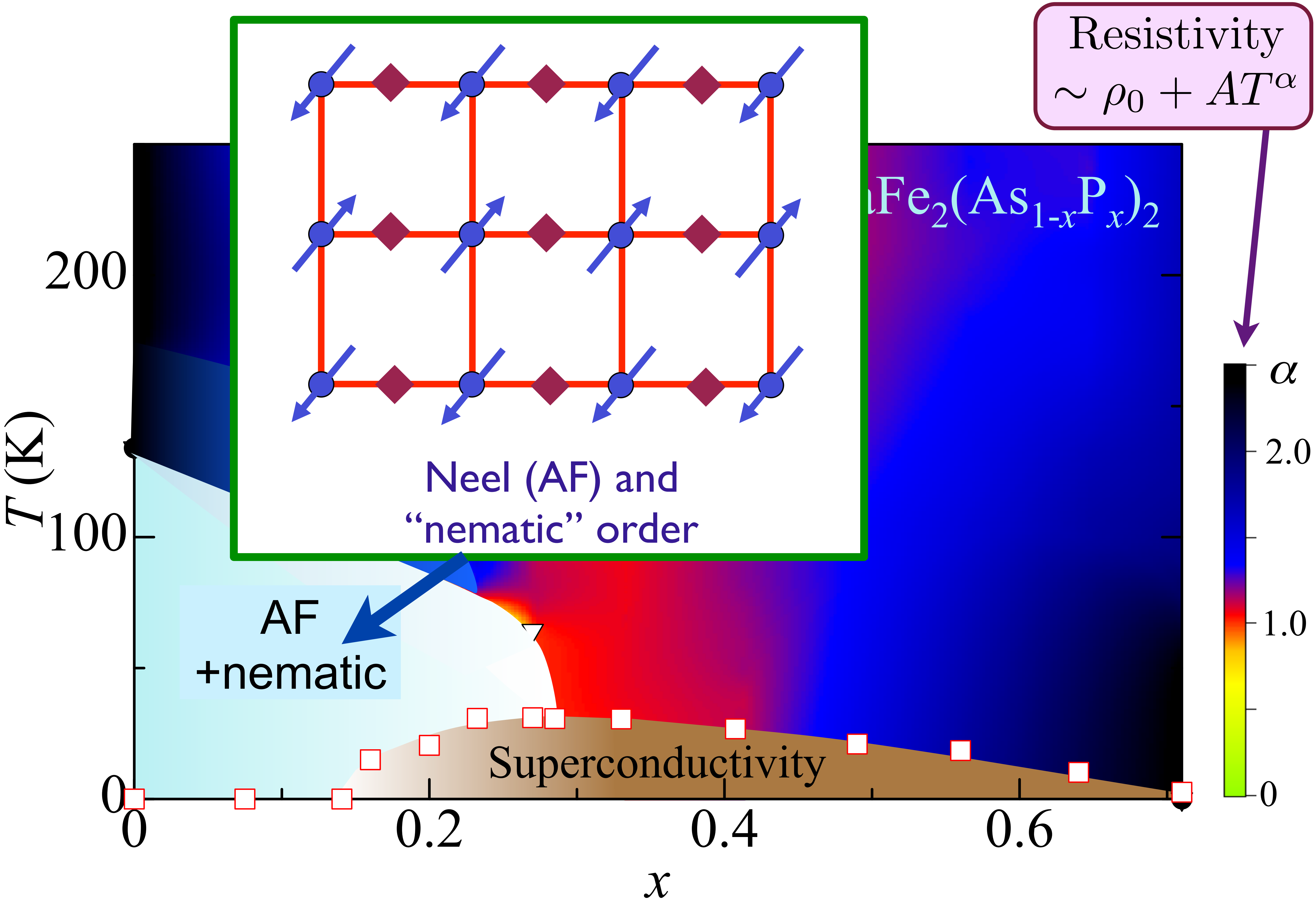
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

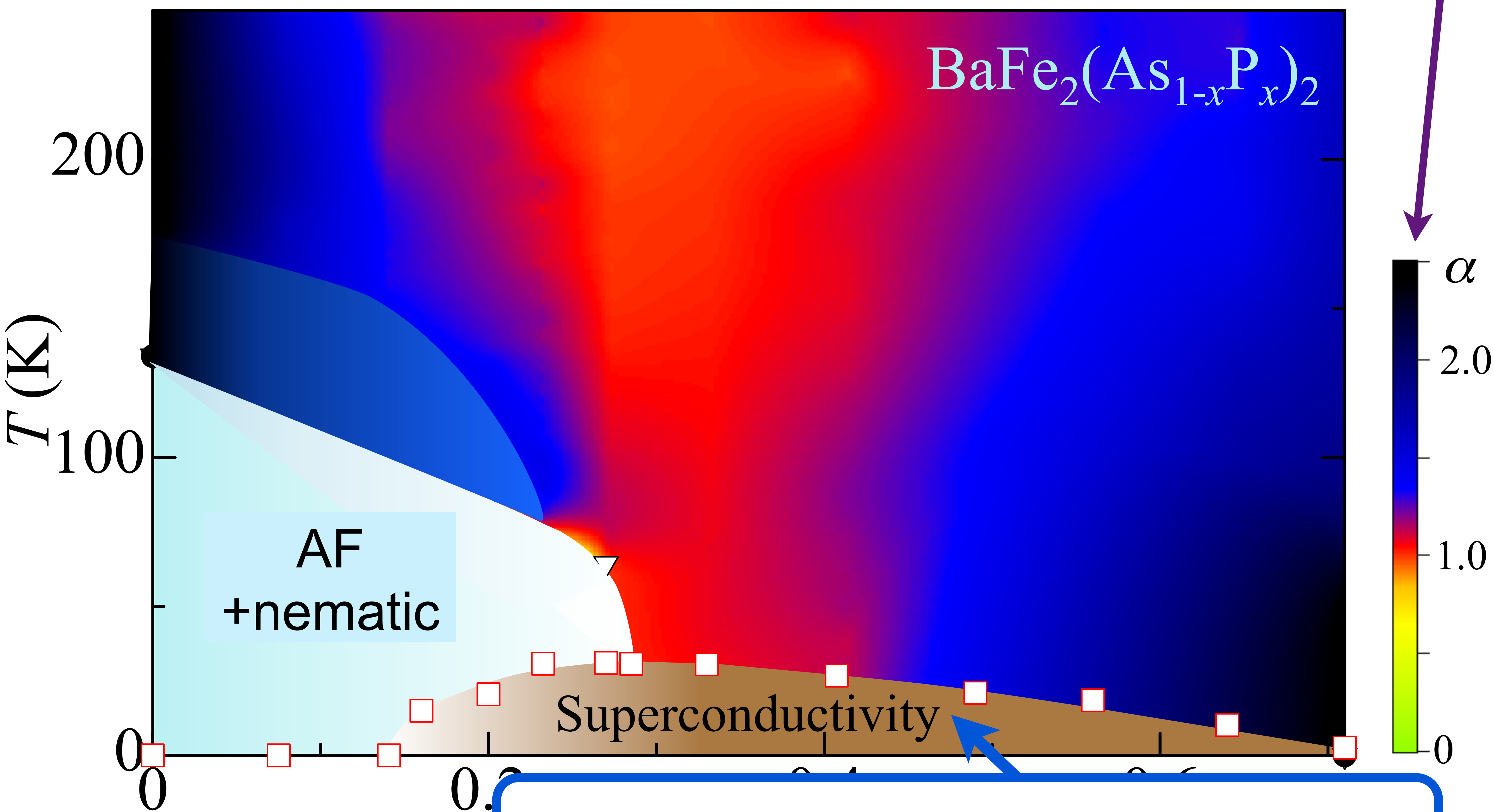


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

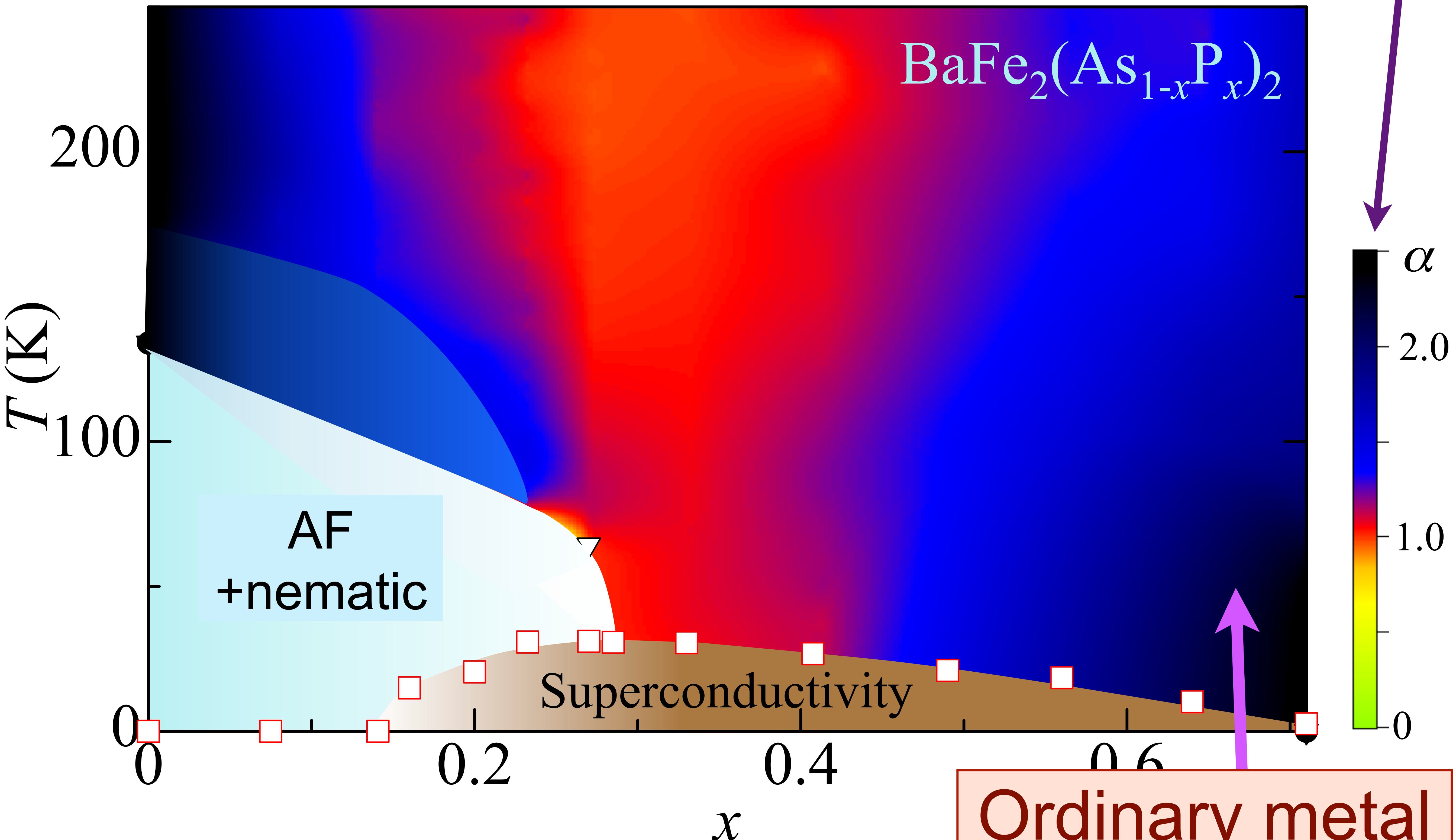
Resistivity
$\sim \rho_0 + AT^{\alpha}$



Superconductor
Bose condensate of pairs of electrons

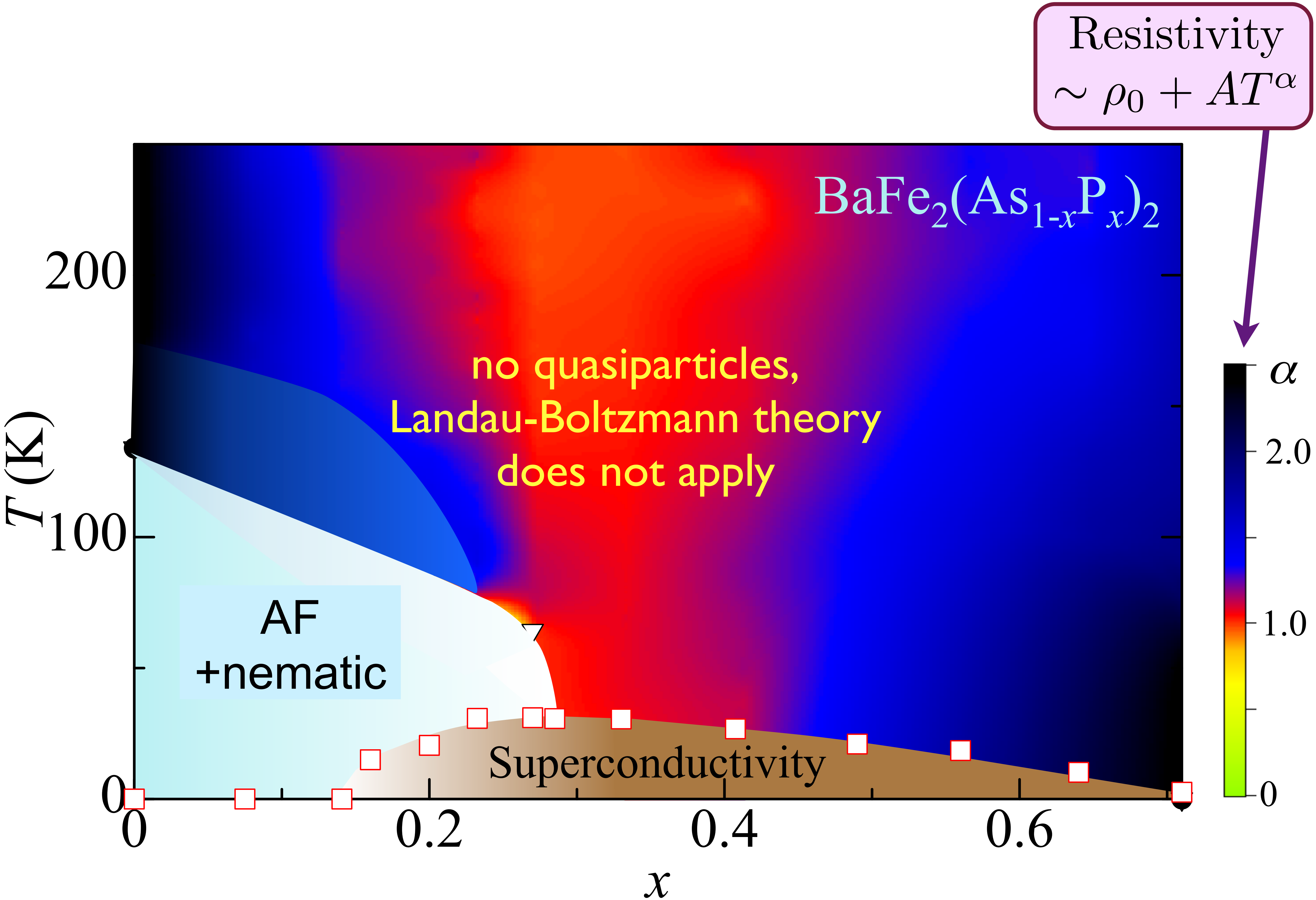
S. Kasahara, T. Shiba
H. Ike

Resistivity
$\sim \rho_0 + AT^{\alpha}$

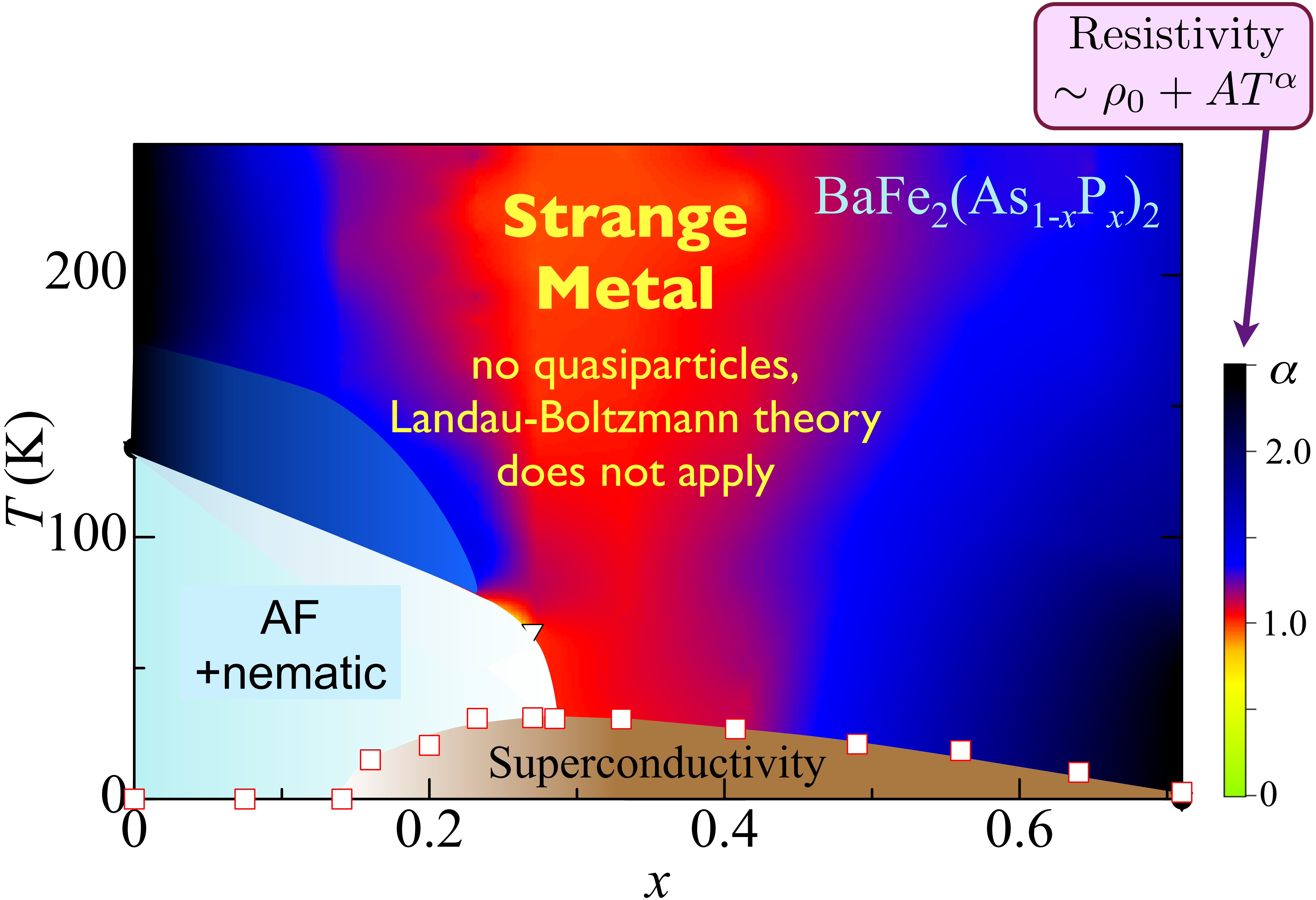


Ordinary metal
(Fermi liquid)

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Mat
Physical Review B **81**, 184519 (2010)

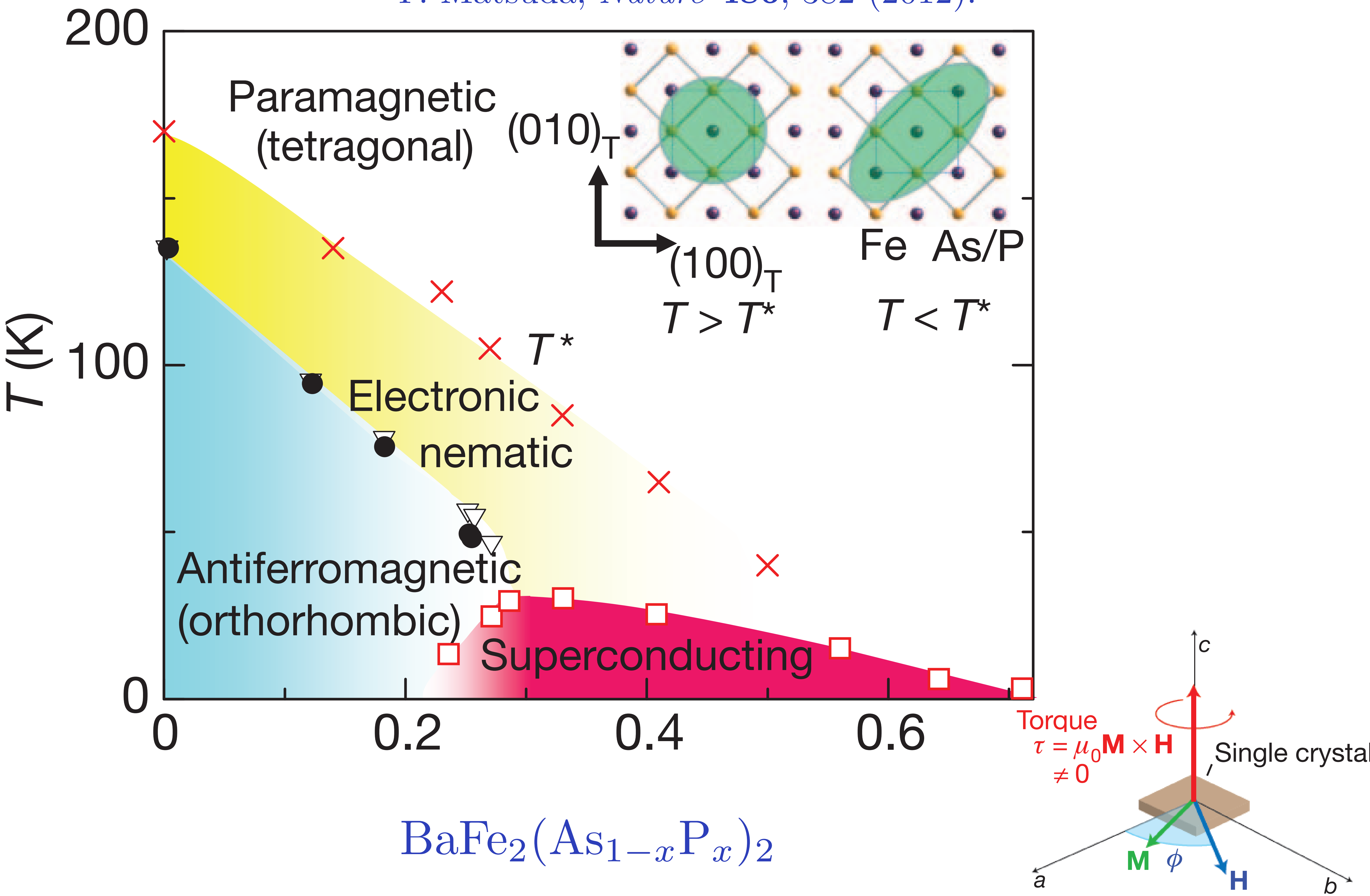


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

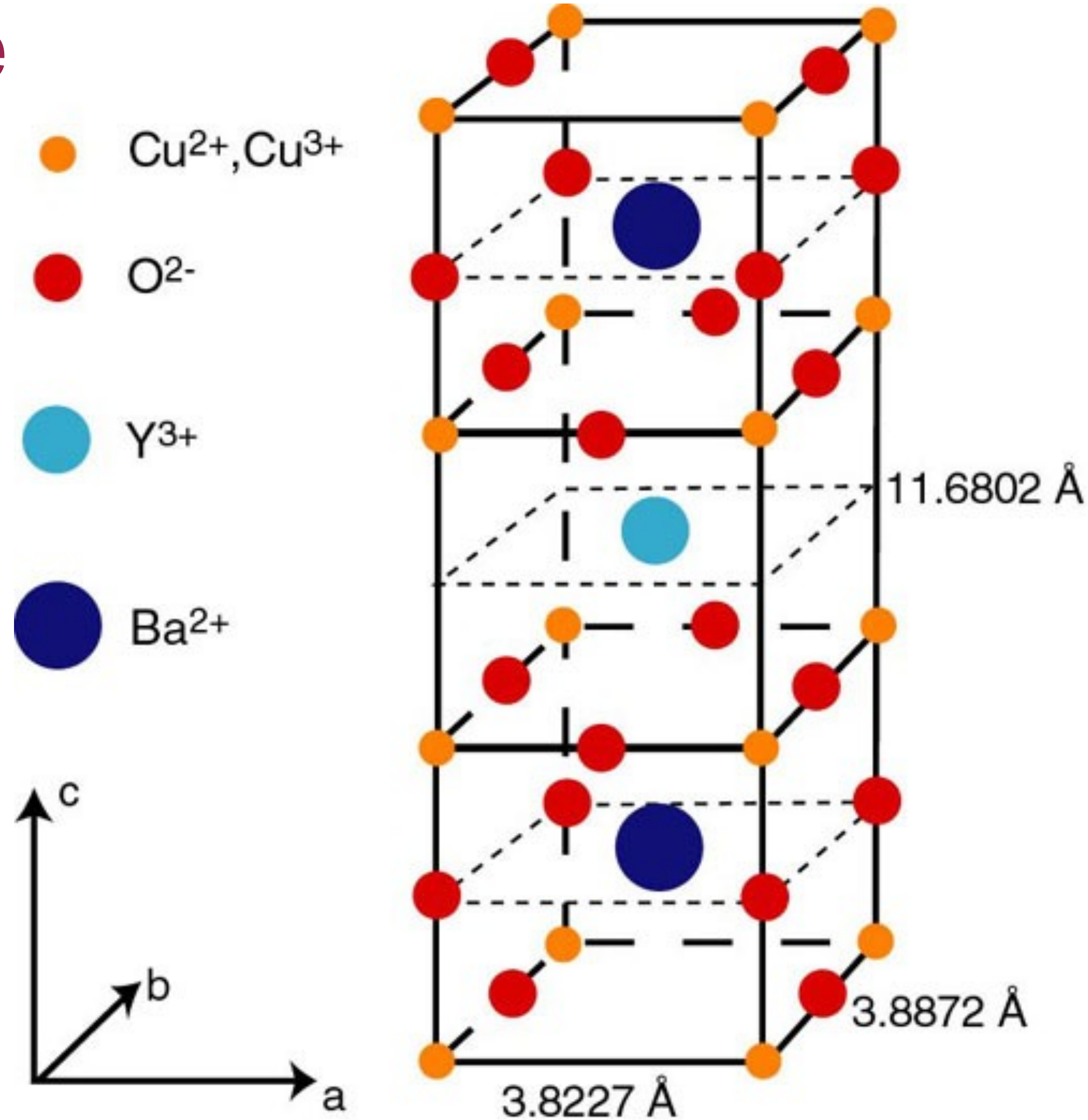


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

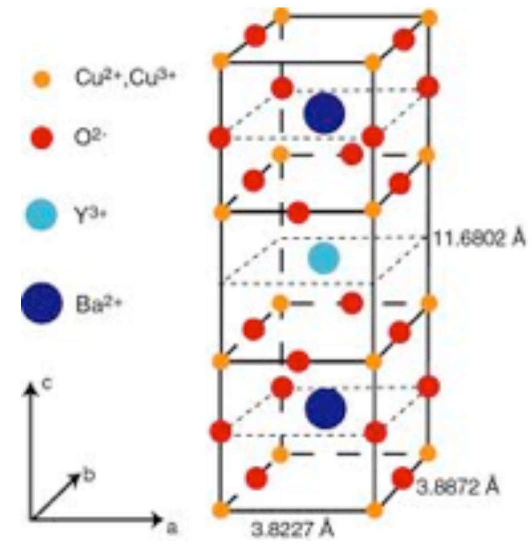
S. Kasahara, H.J. Shi, K. Hashimoto, S. Tonegawa, Y. Mizukami, T. Shibauchi, K. Sugimoto, T. Fukuda, T. Terashima, A.H. Nevidomskyy, and Y. Matsuda, *Nature* **486**, 382 (2012).

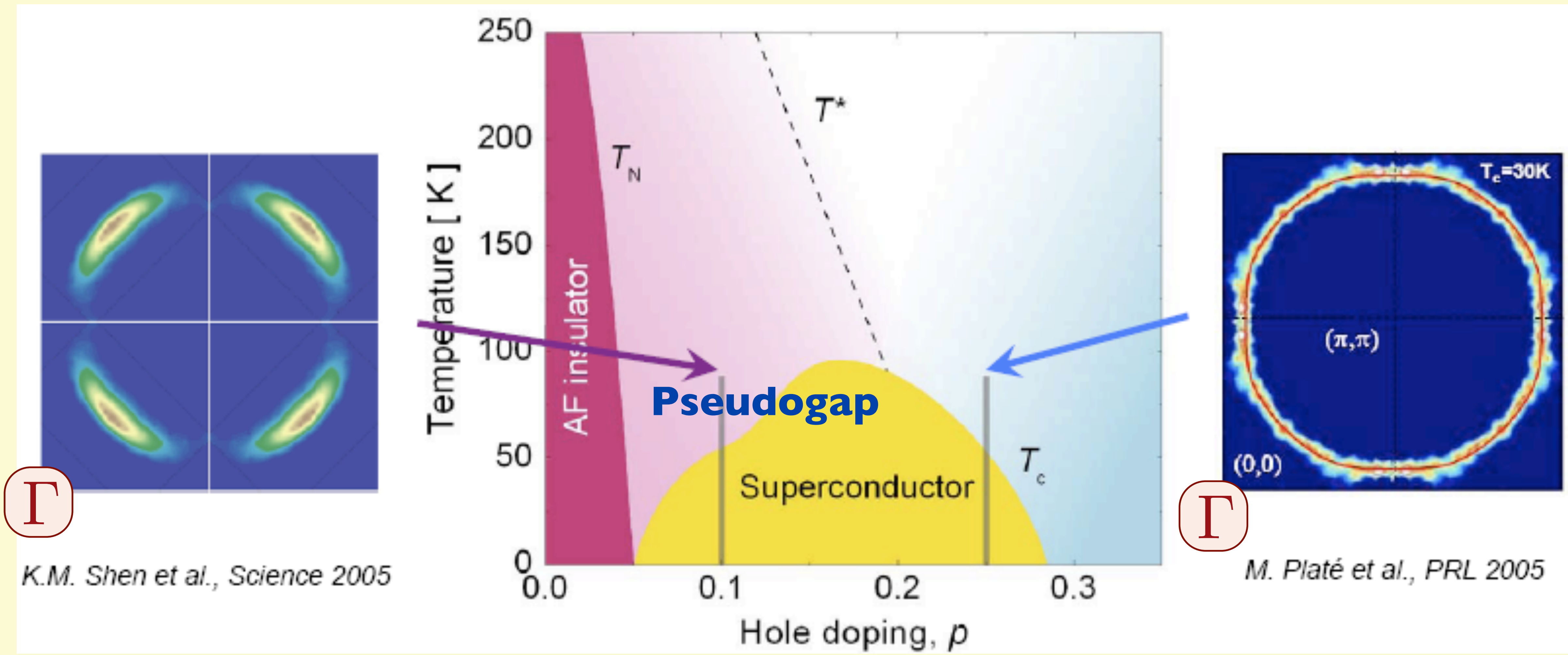


High temperature superconductors



$$\mathrm{YBa_2Cu_3O_{6+x}}$$



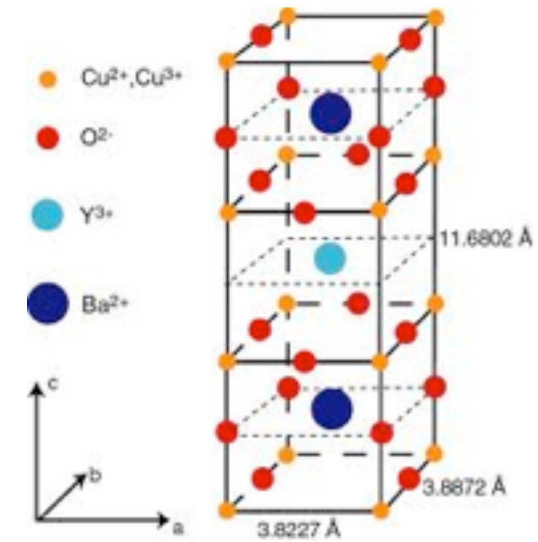


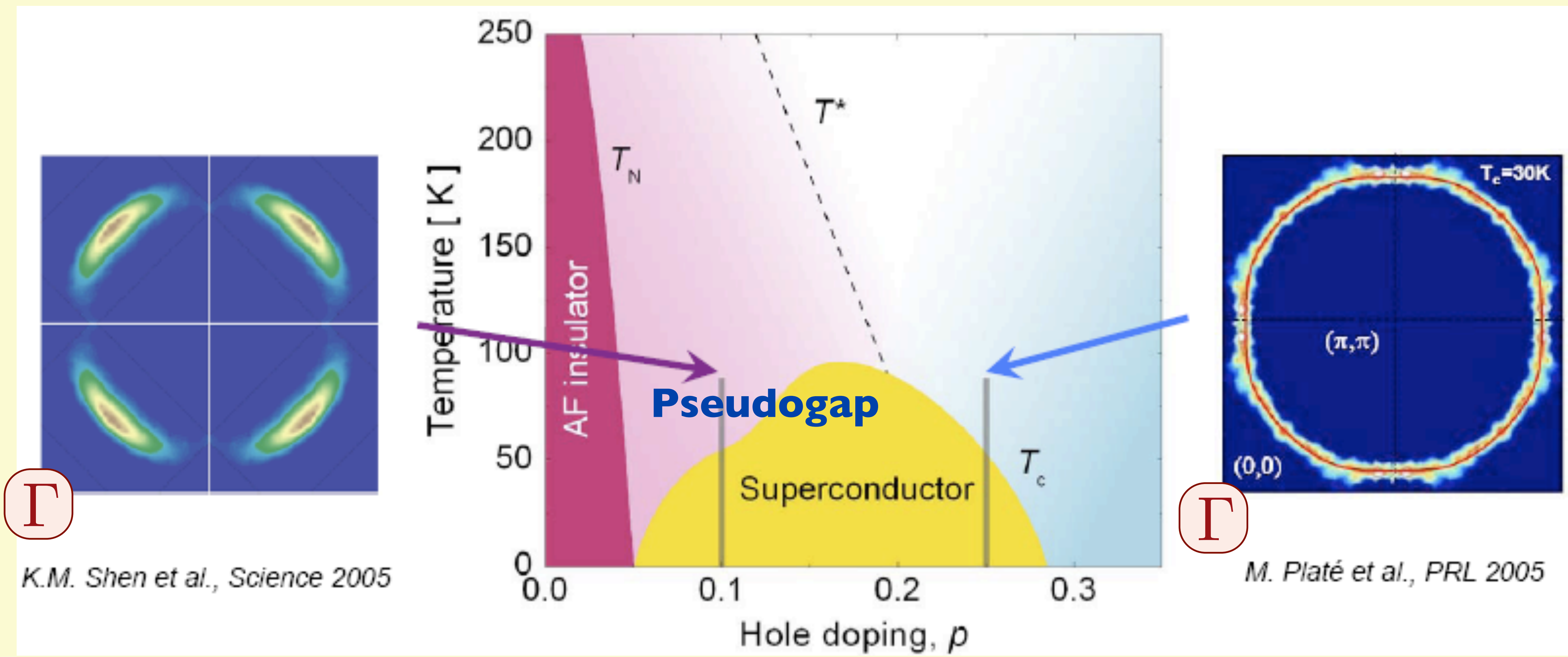
K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

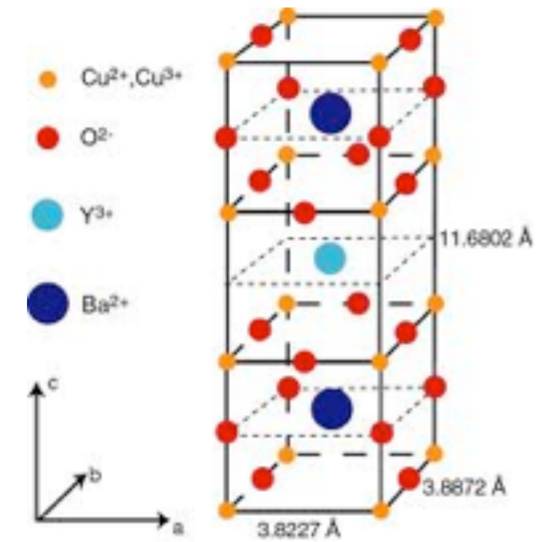
Large hole Fermi surface

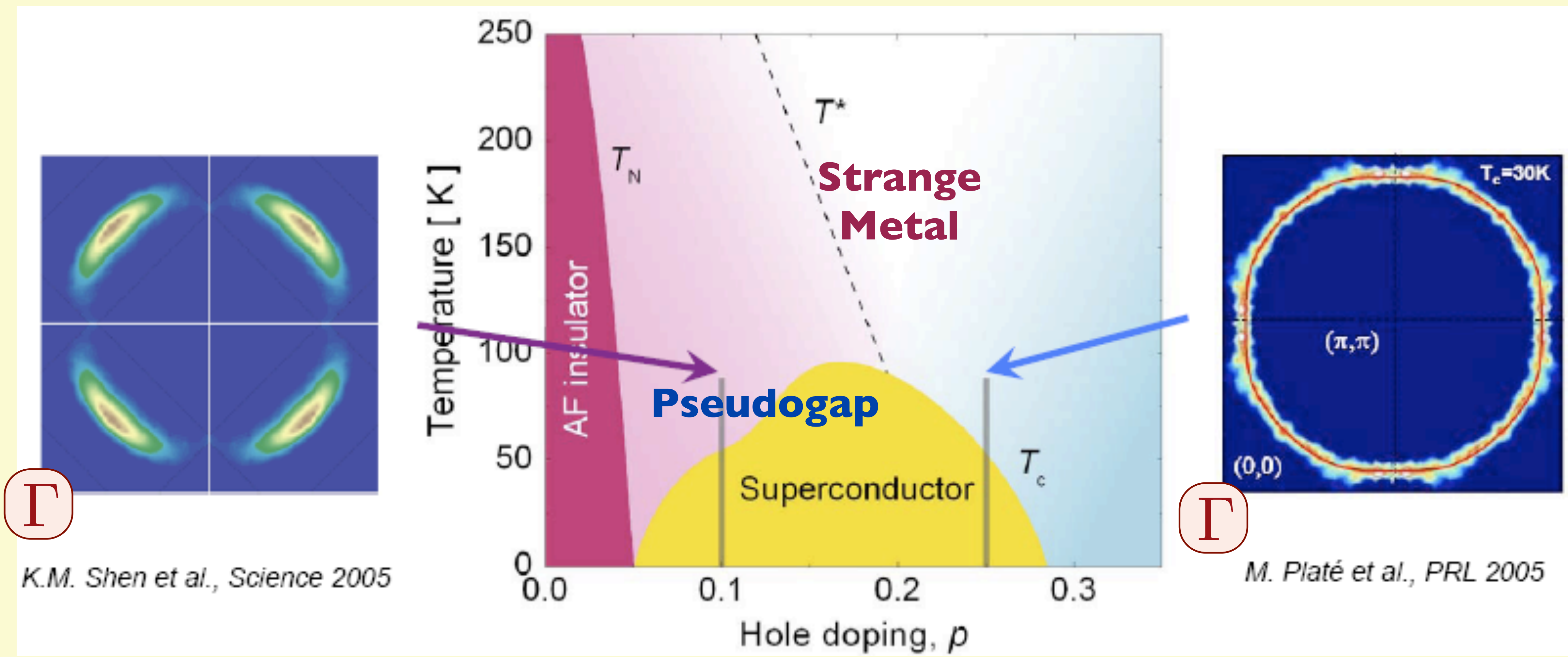




Smaller hole Fermi-pockets

Large hole Fermi surface





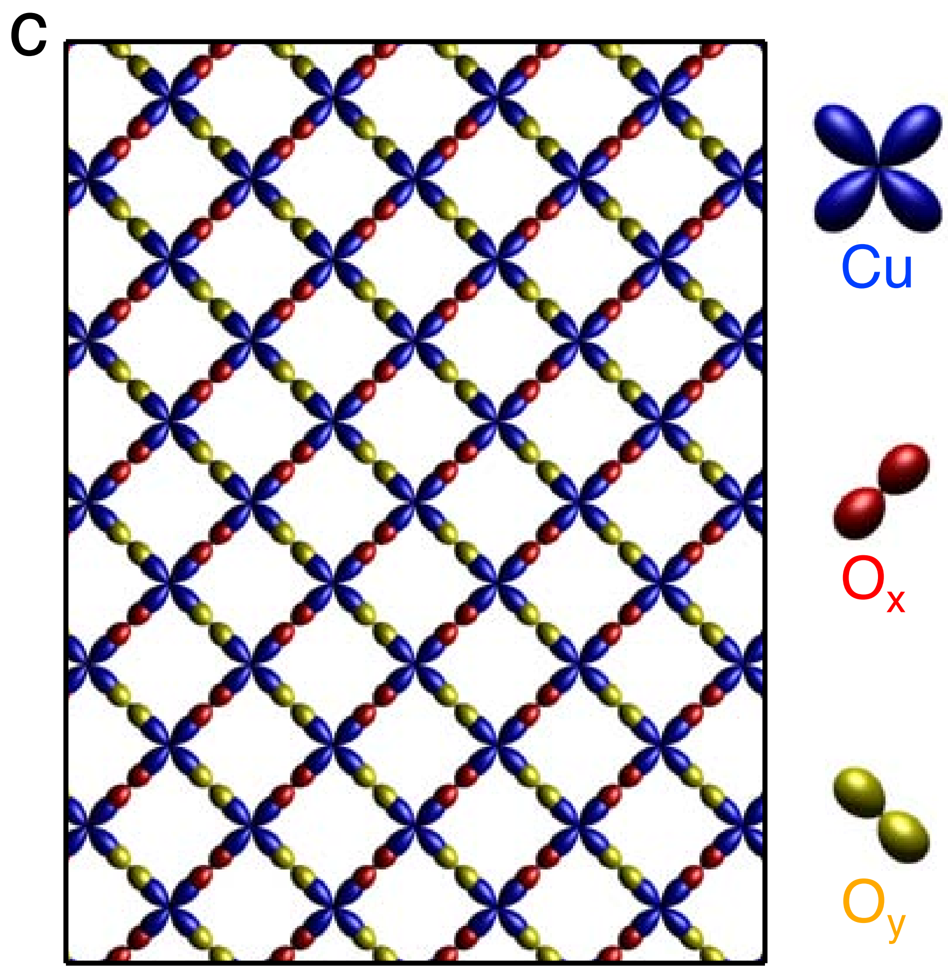
Smaller hole Fermi-pockets

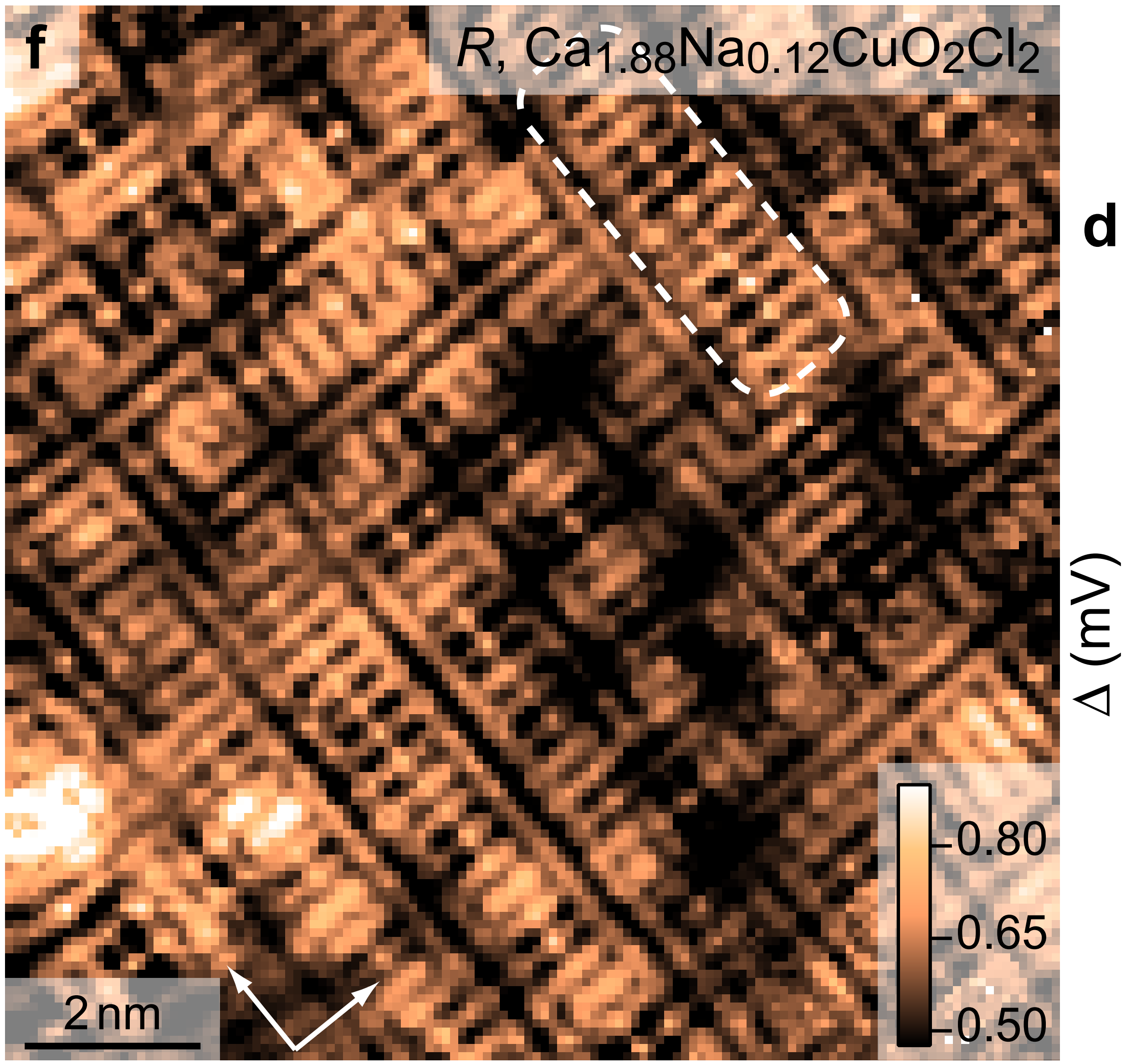
Large hole Fermi surface

Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi

Nature Physics, **8**, 534 (2012).

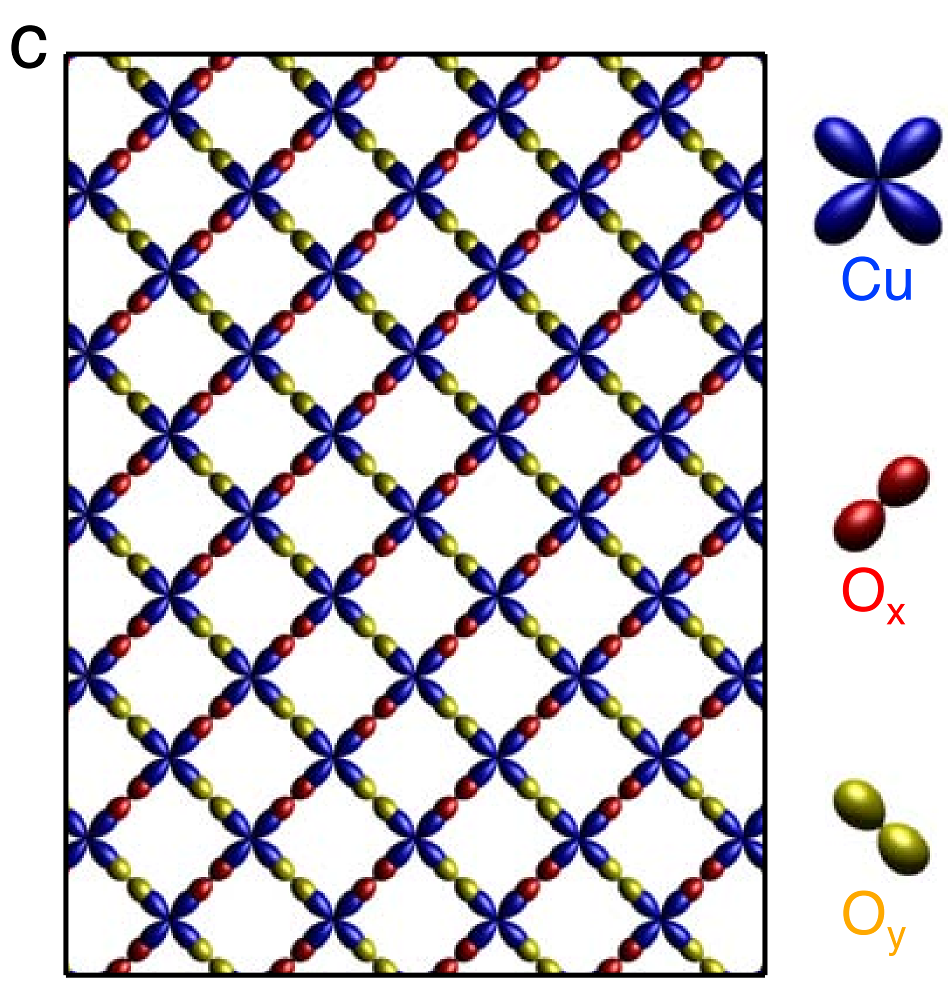


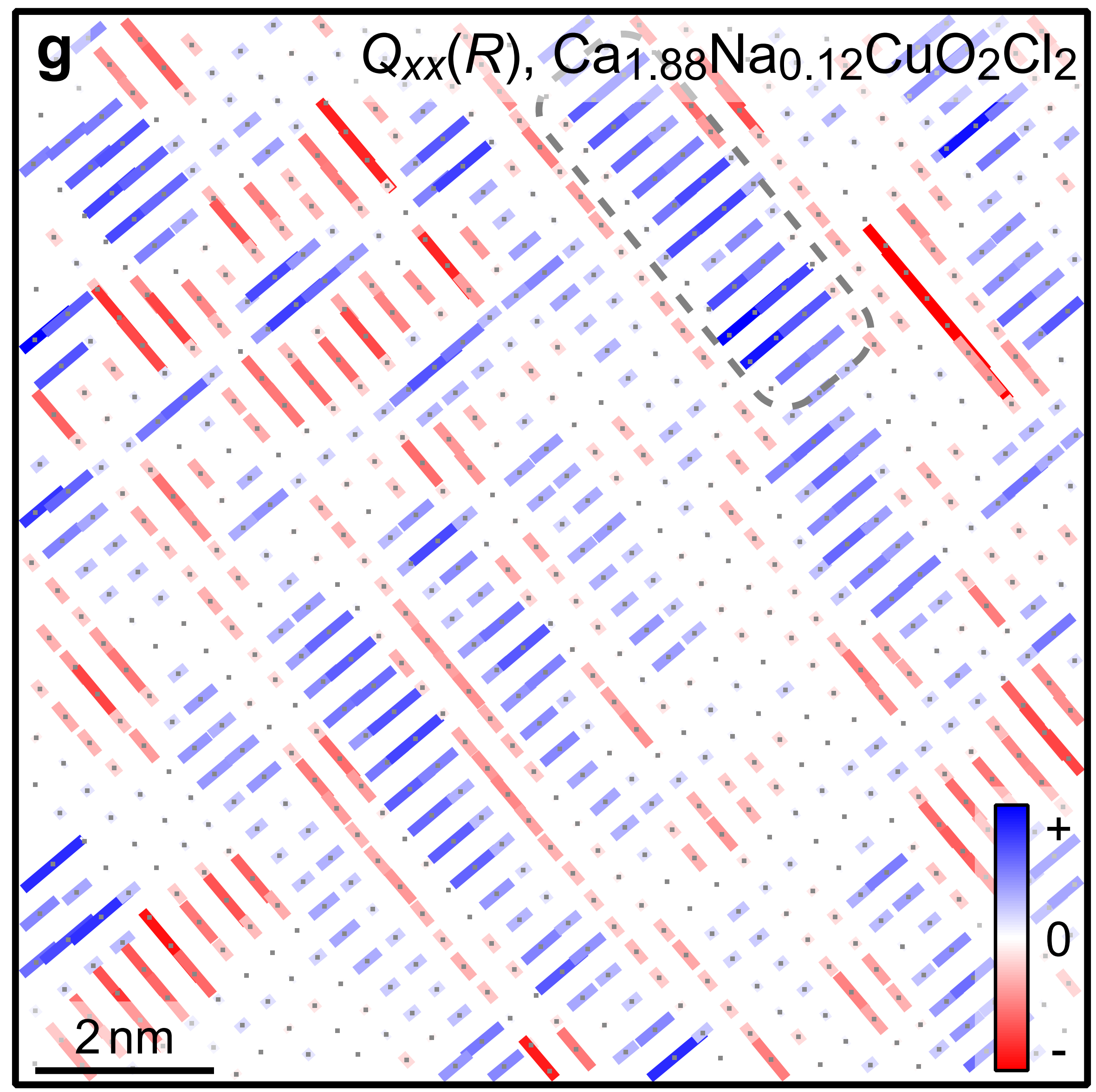


Evidence for "nematic" order (*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi
Nature Physics, **8**, 534 (2012).

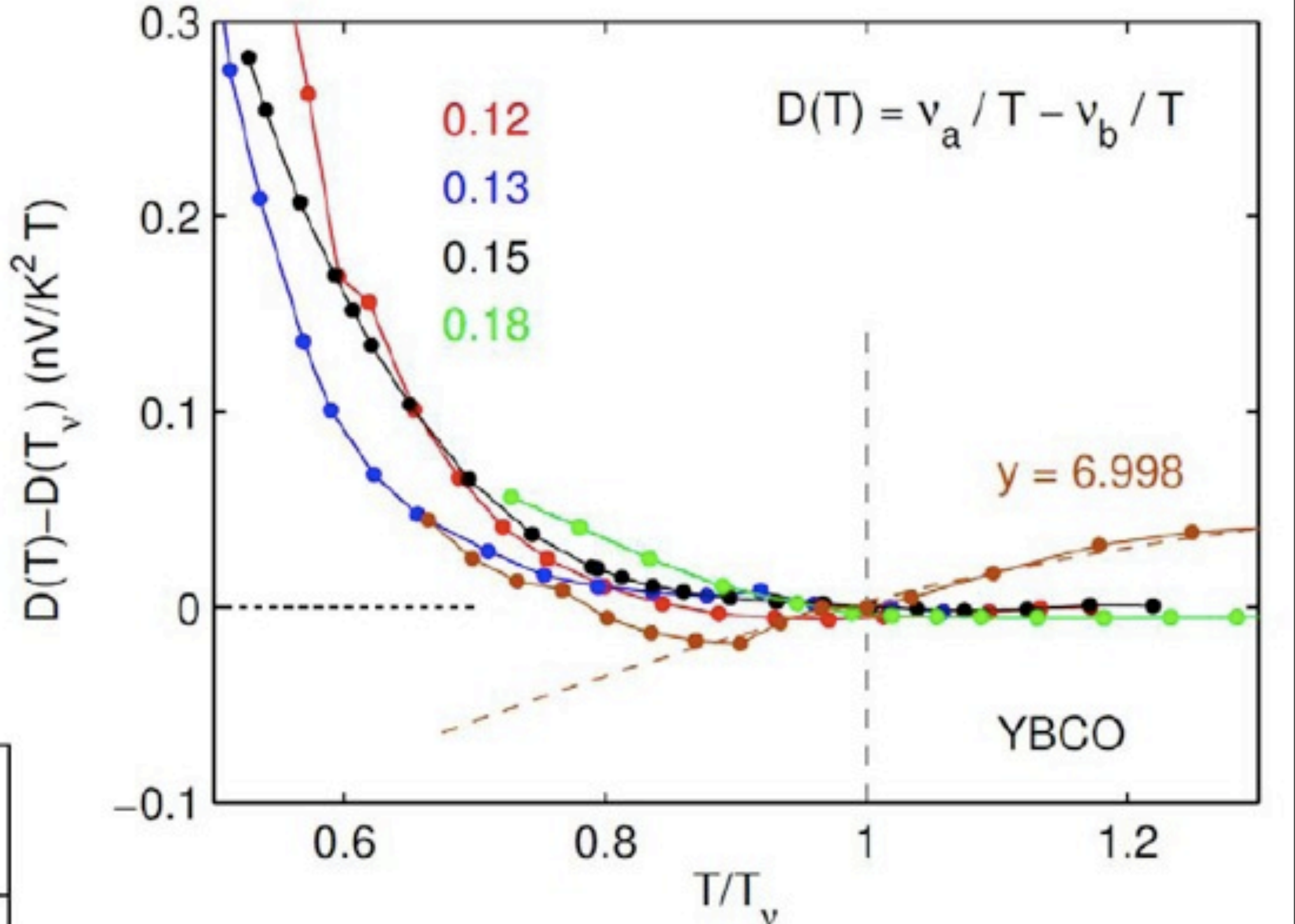


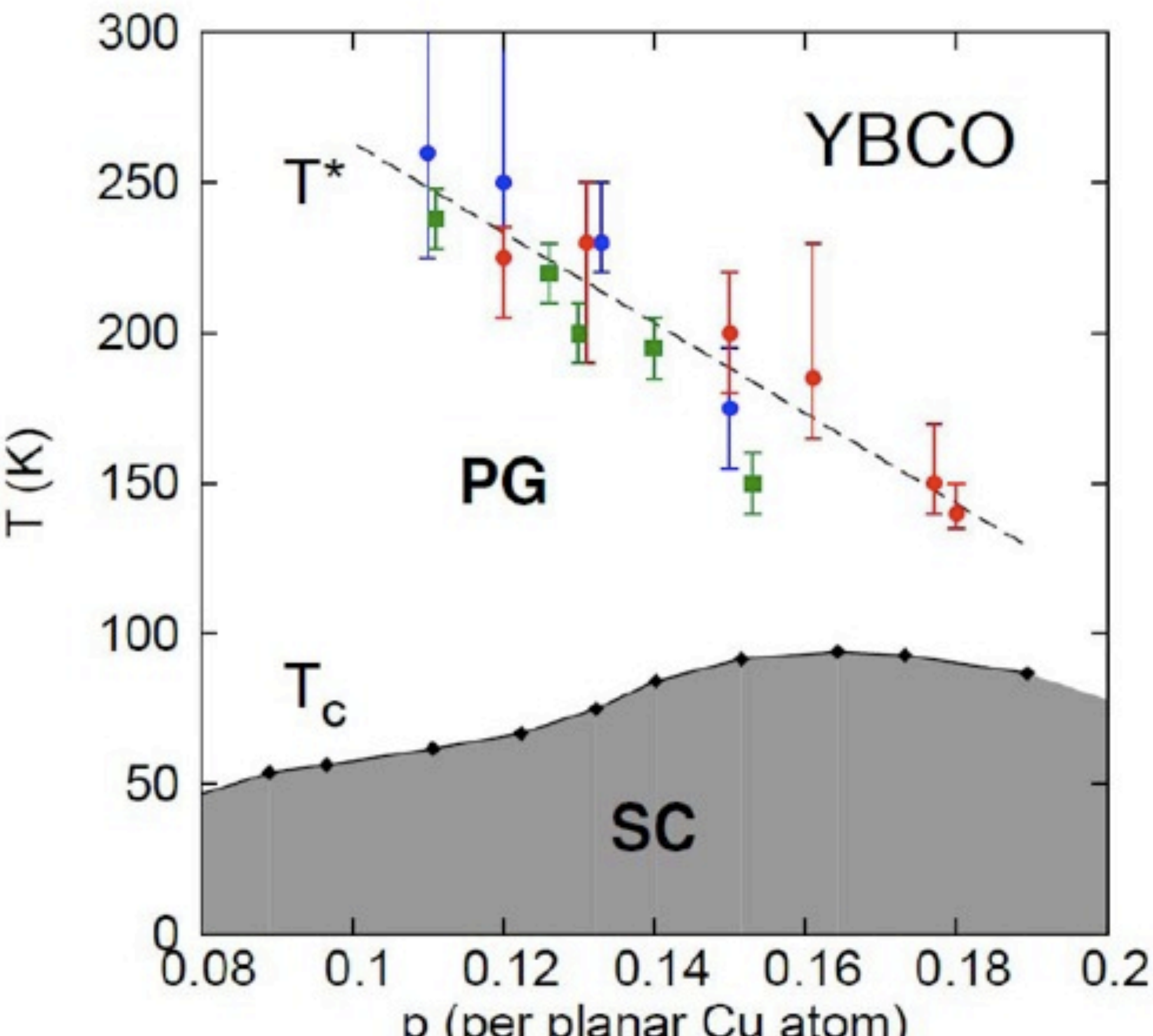


Evidence for “nematic” order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

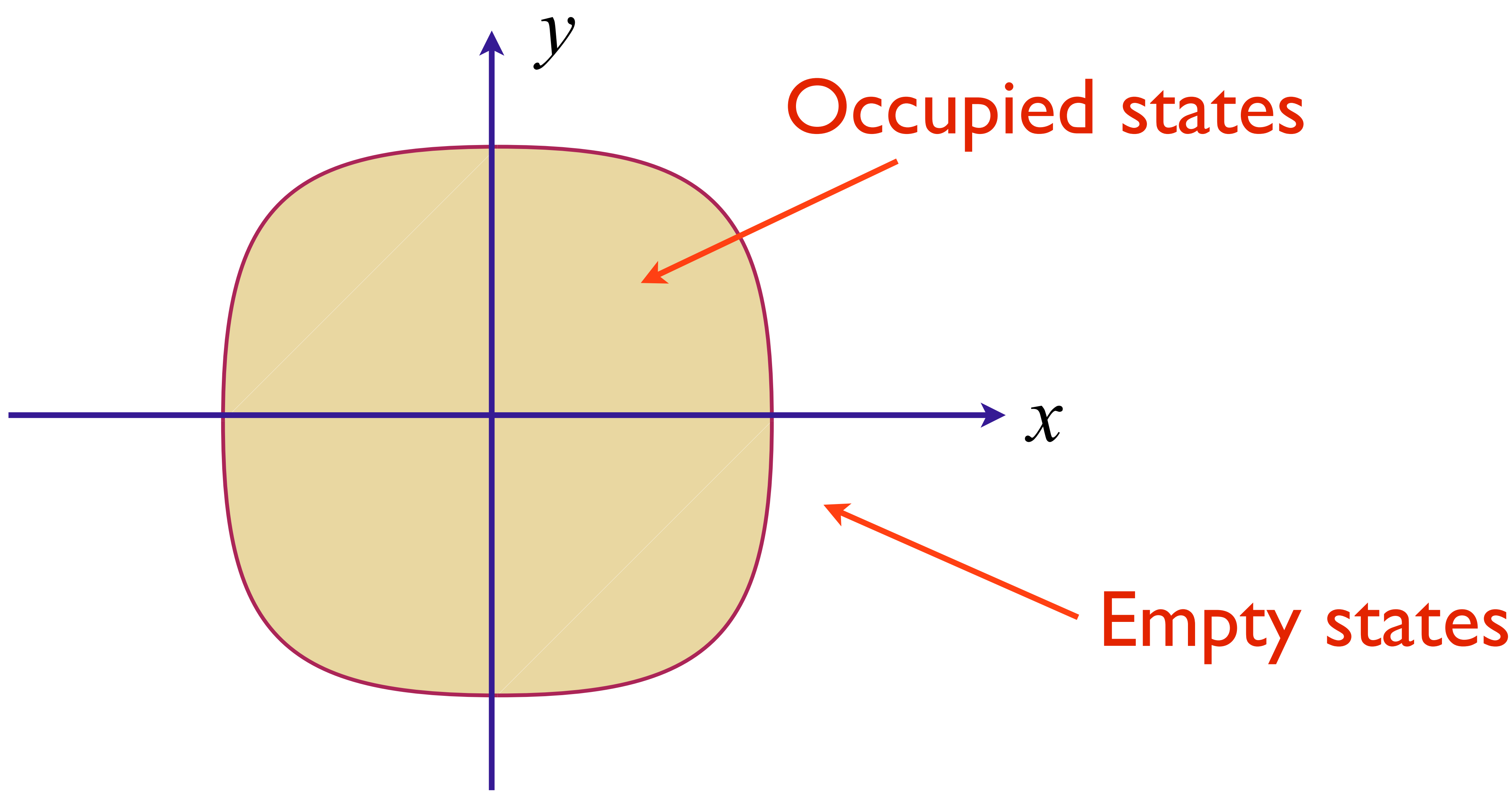
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).



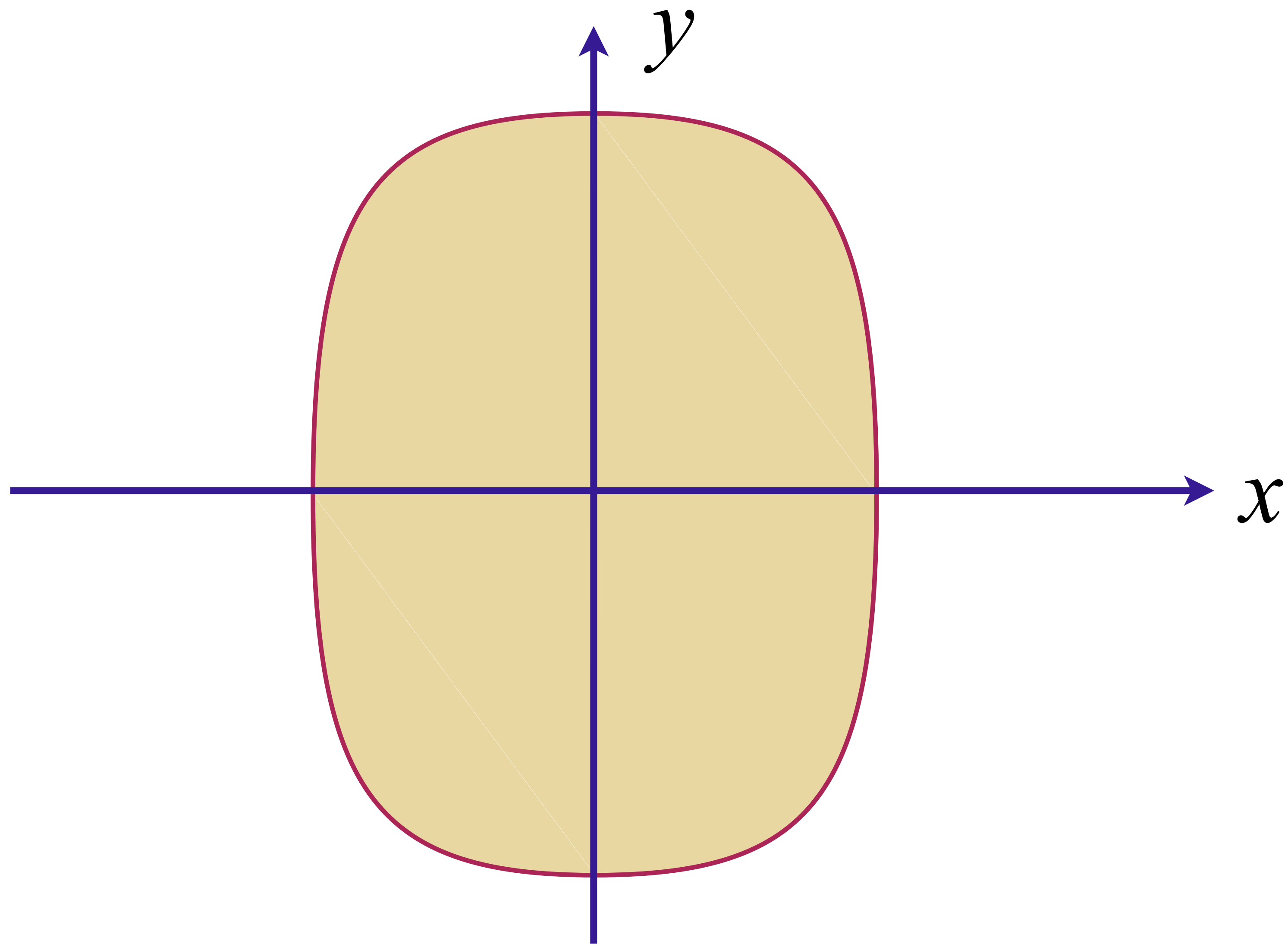


Quantum criticality of Ising-nematic ordering in a metal



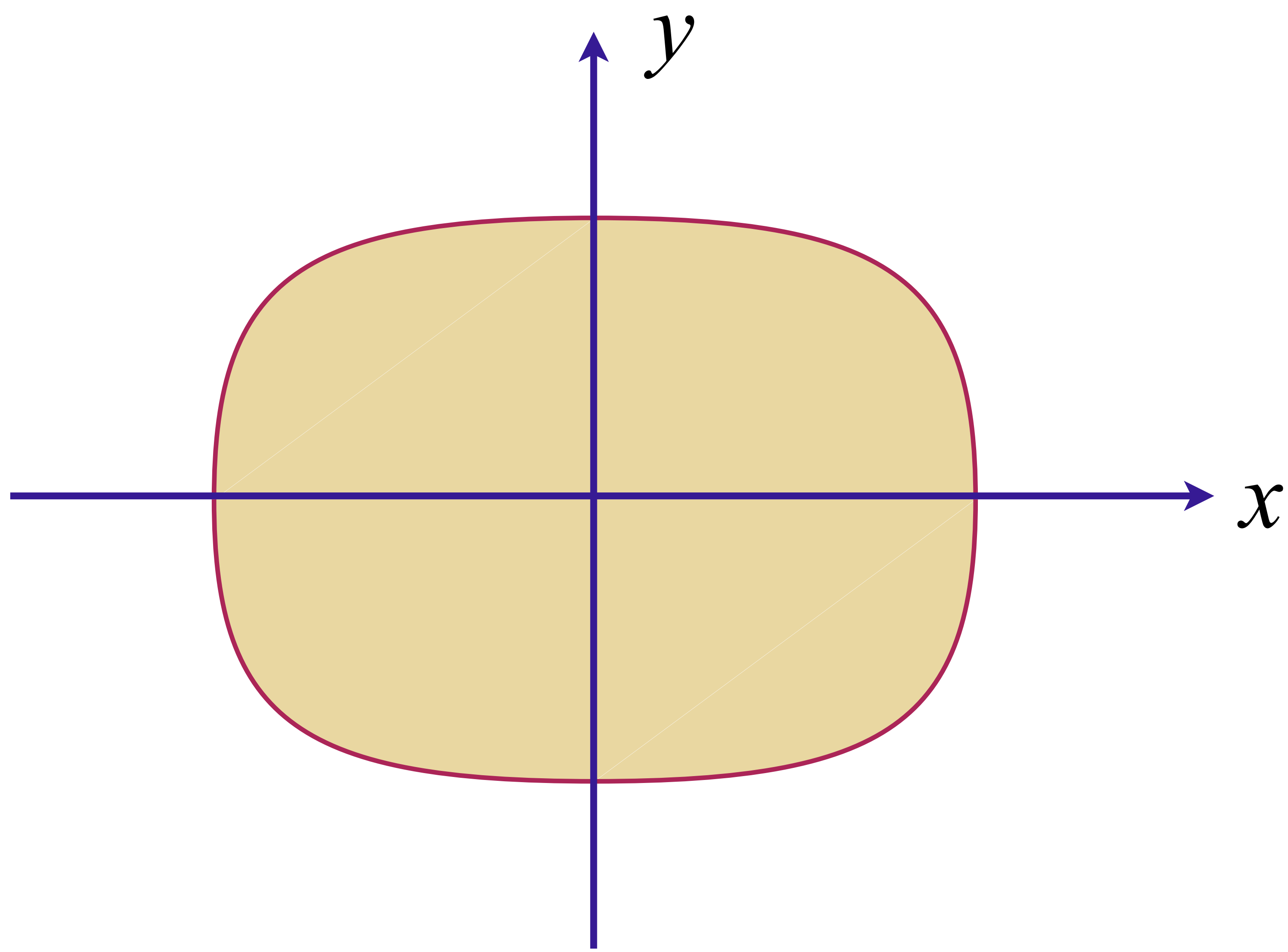
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Spontaneous elongation along y direction:

Quantum criticality of Ising-nematic ordering in a metal



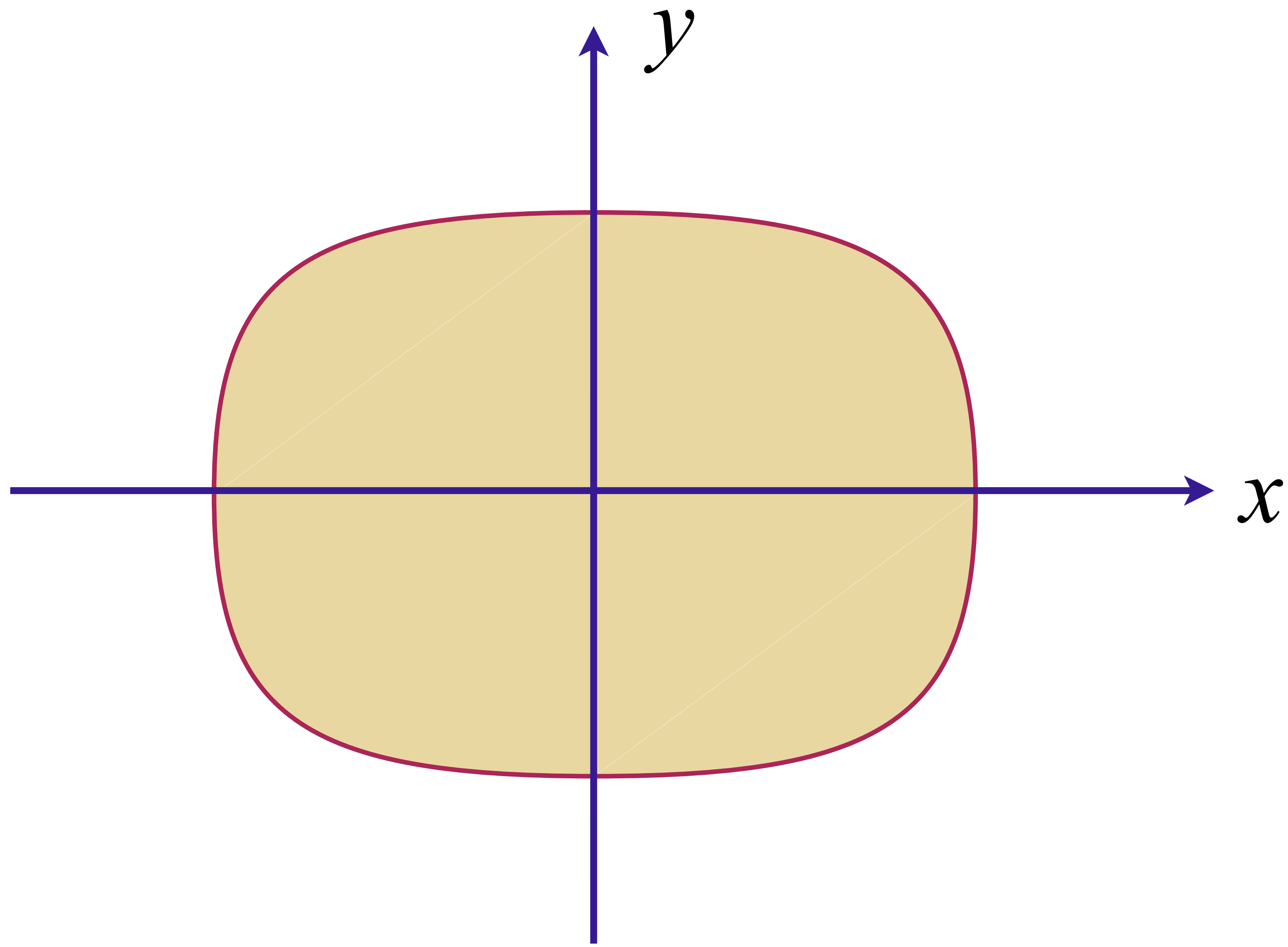
Spontaneous elongation along x direction:

Ising-nematic order parameter

$$\phi \sim \int d^2k \, (\cos k_x - \cos k_y) \, c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

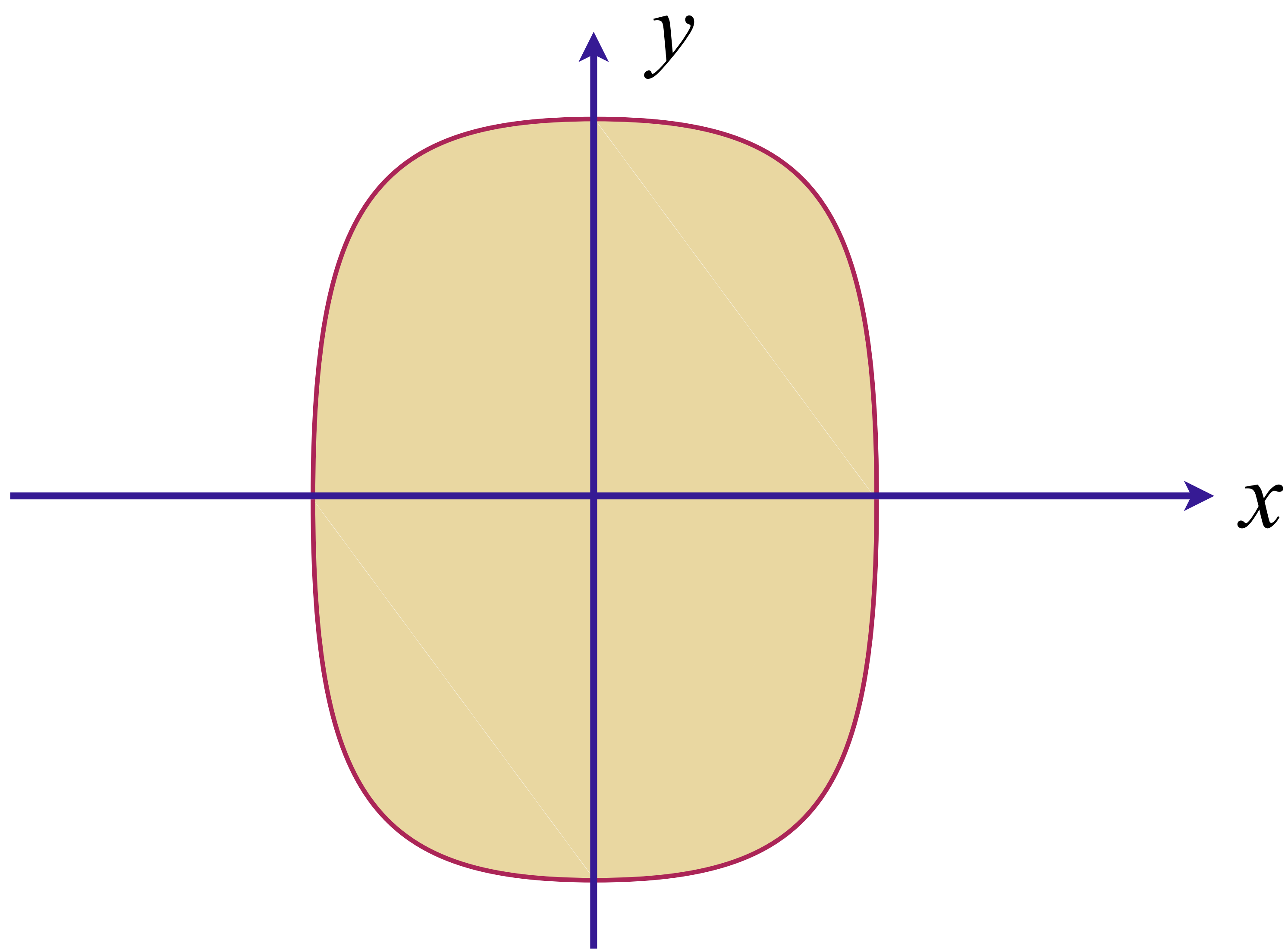
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering in a metal



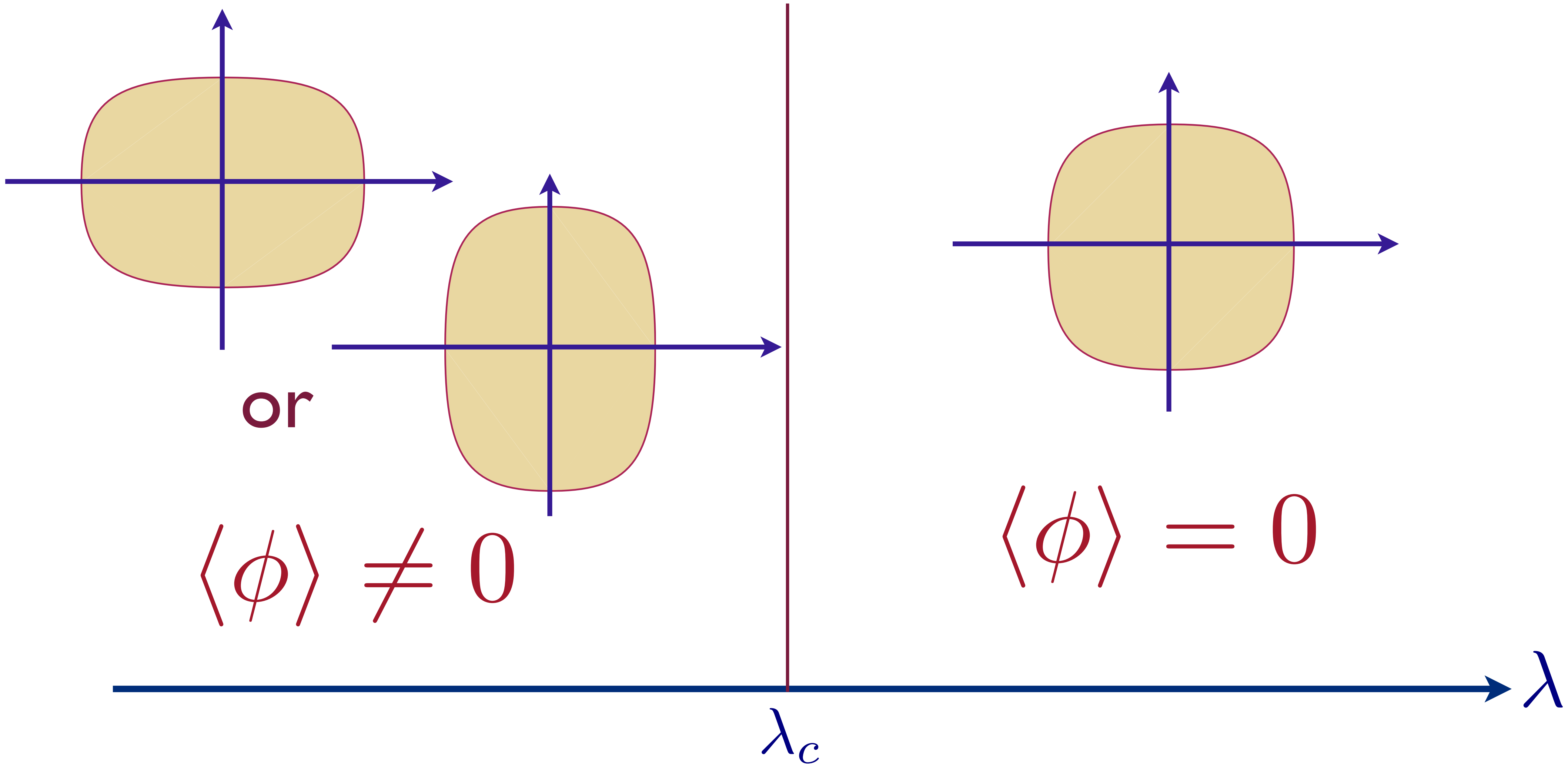
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Ising-nematic ordering in a metal



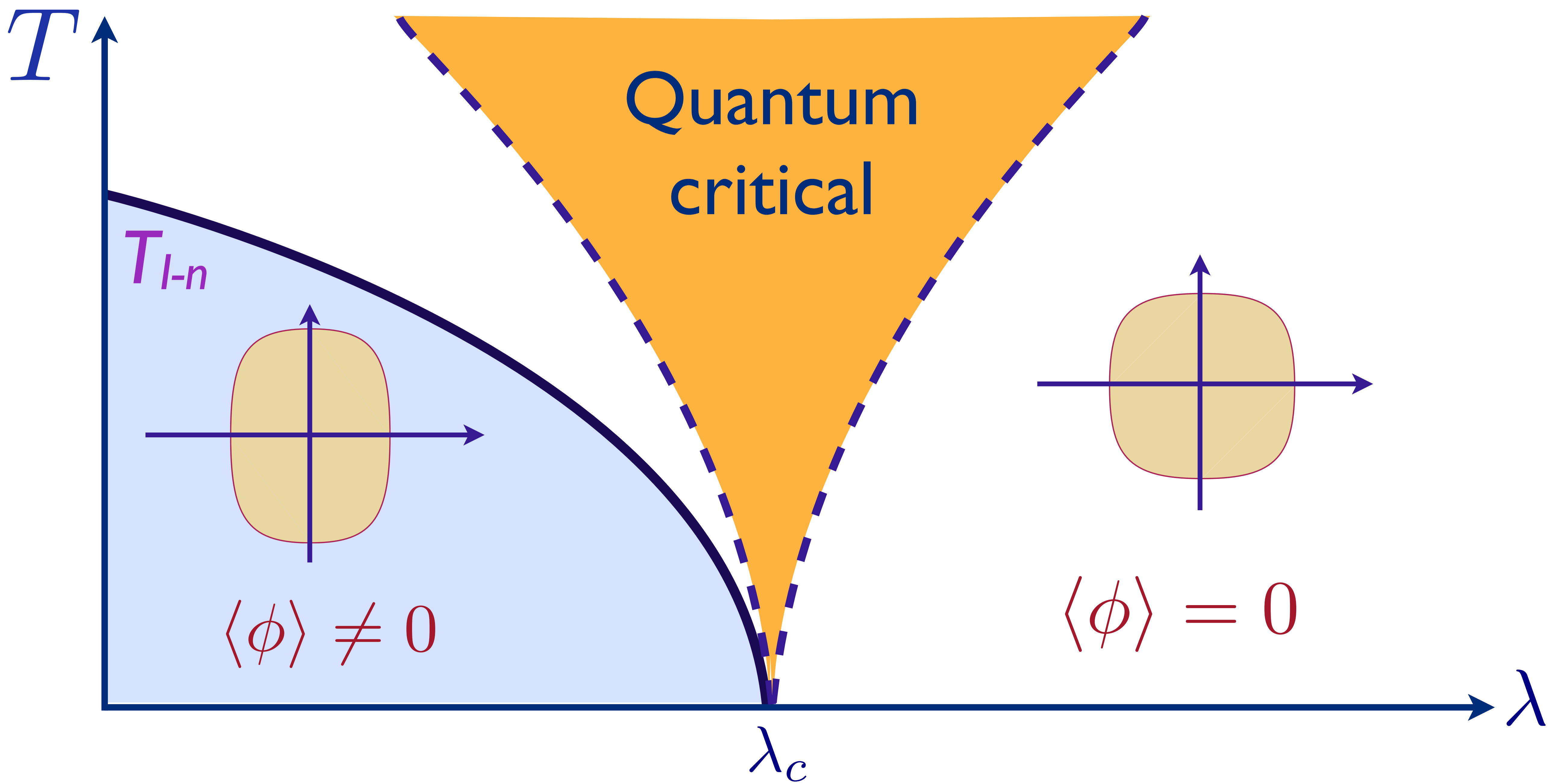
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Ising-nematic ordering in a metal



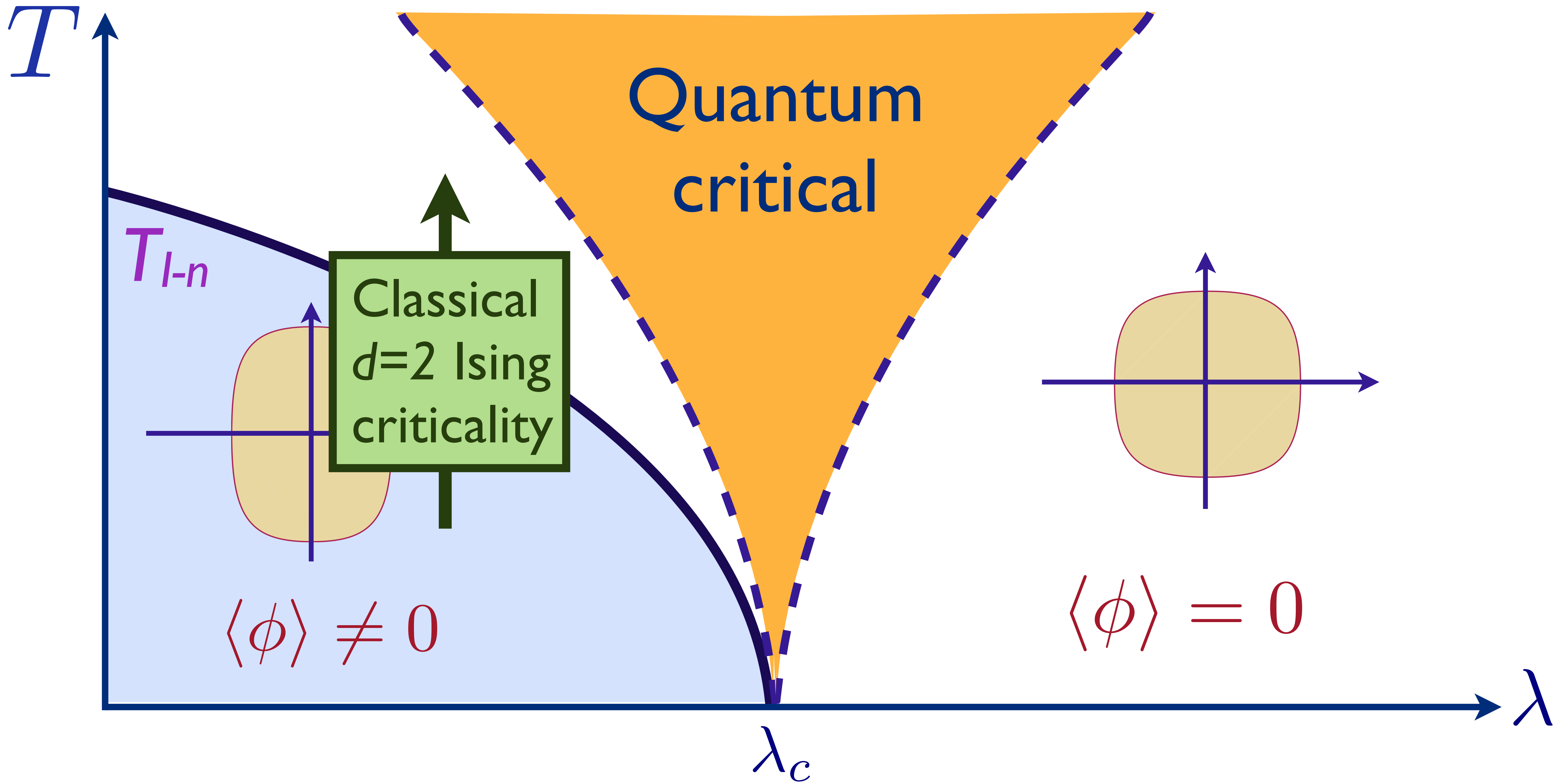
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



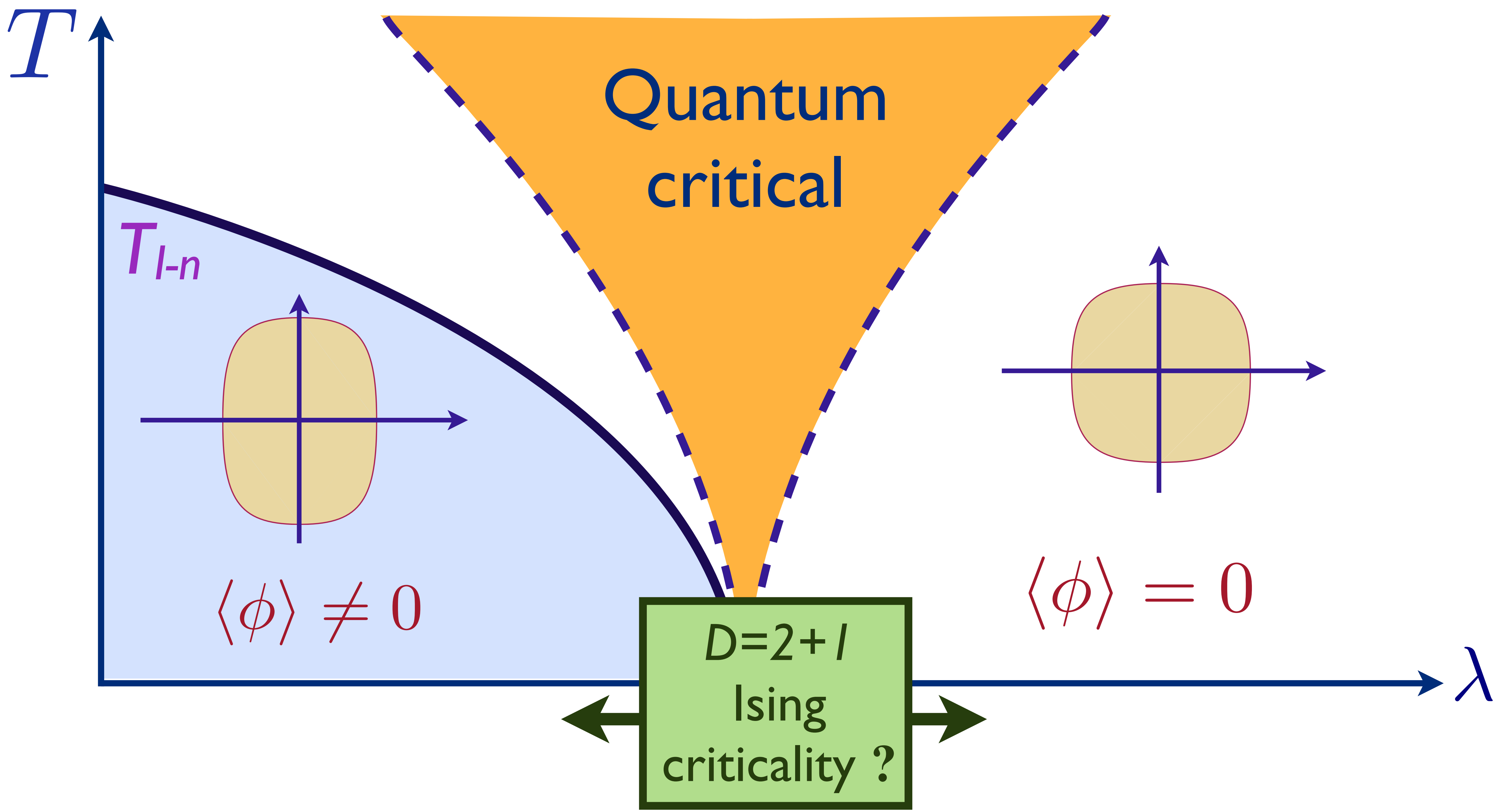
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



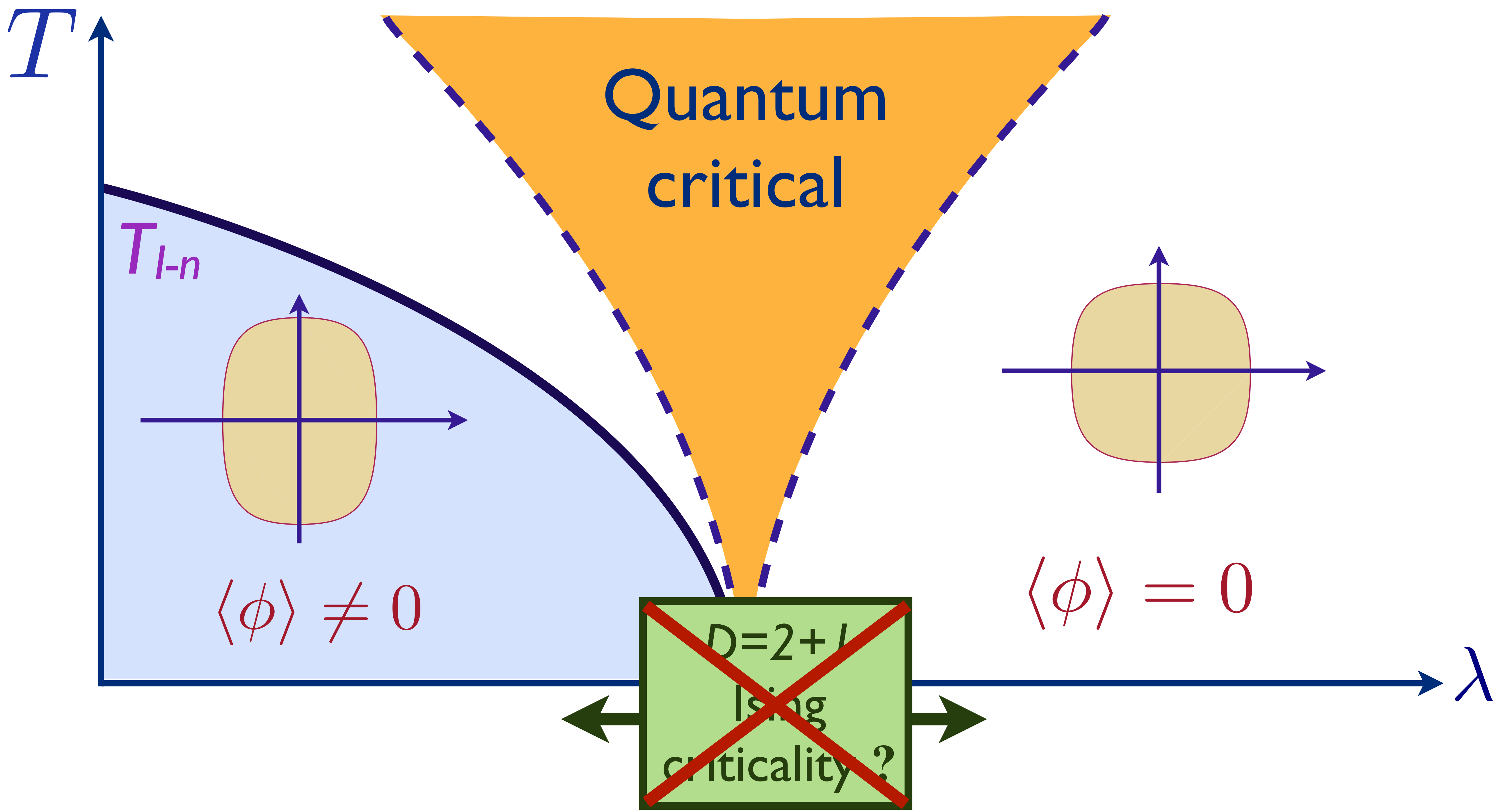
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



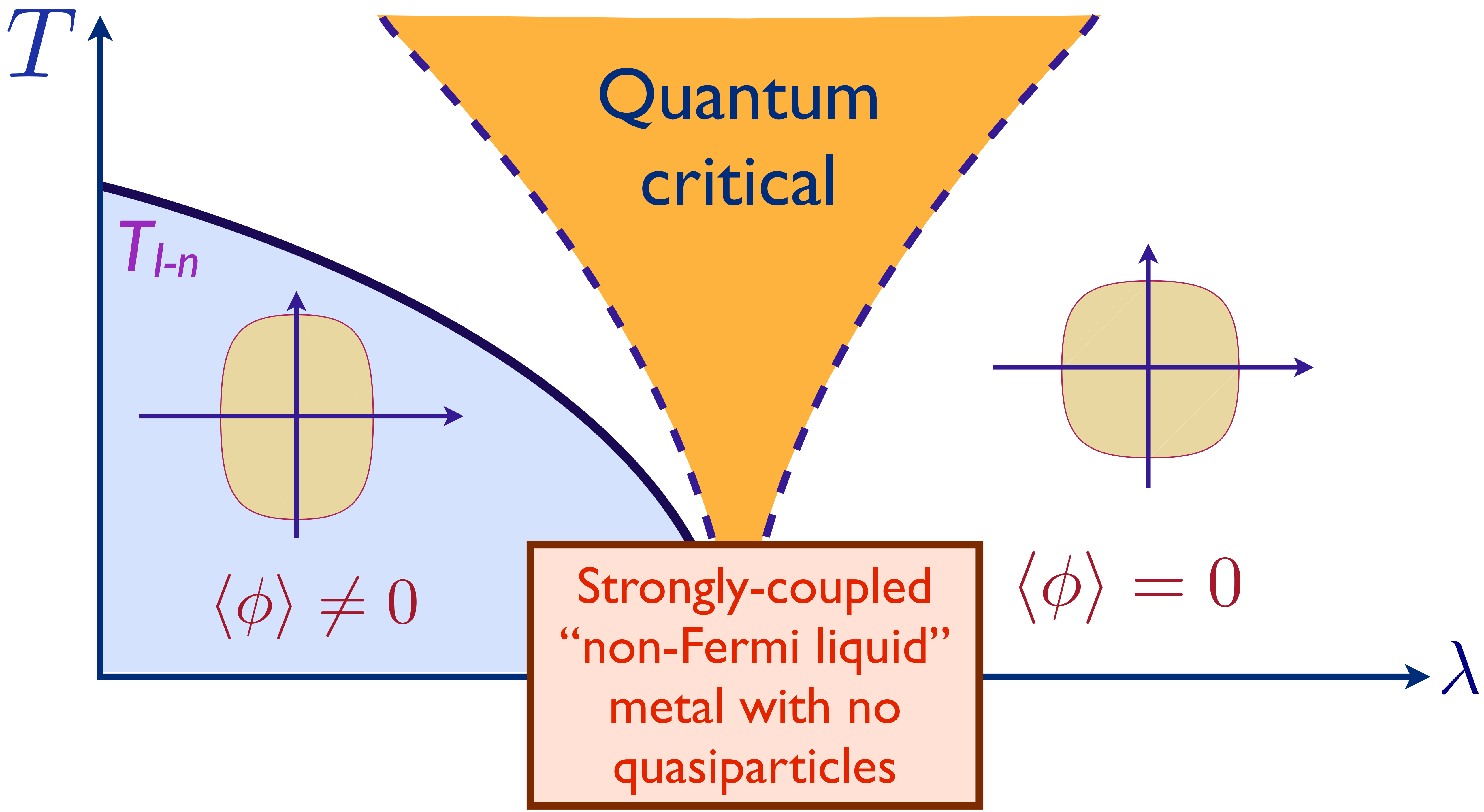
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



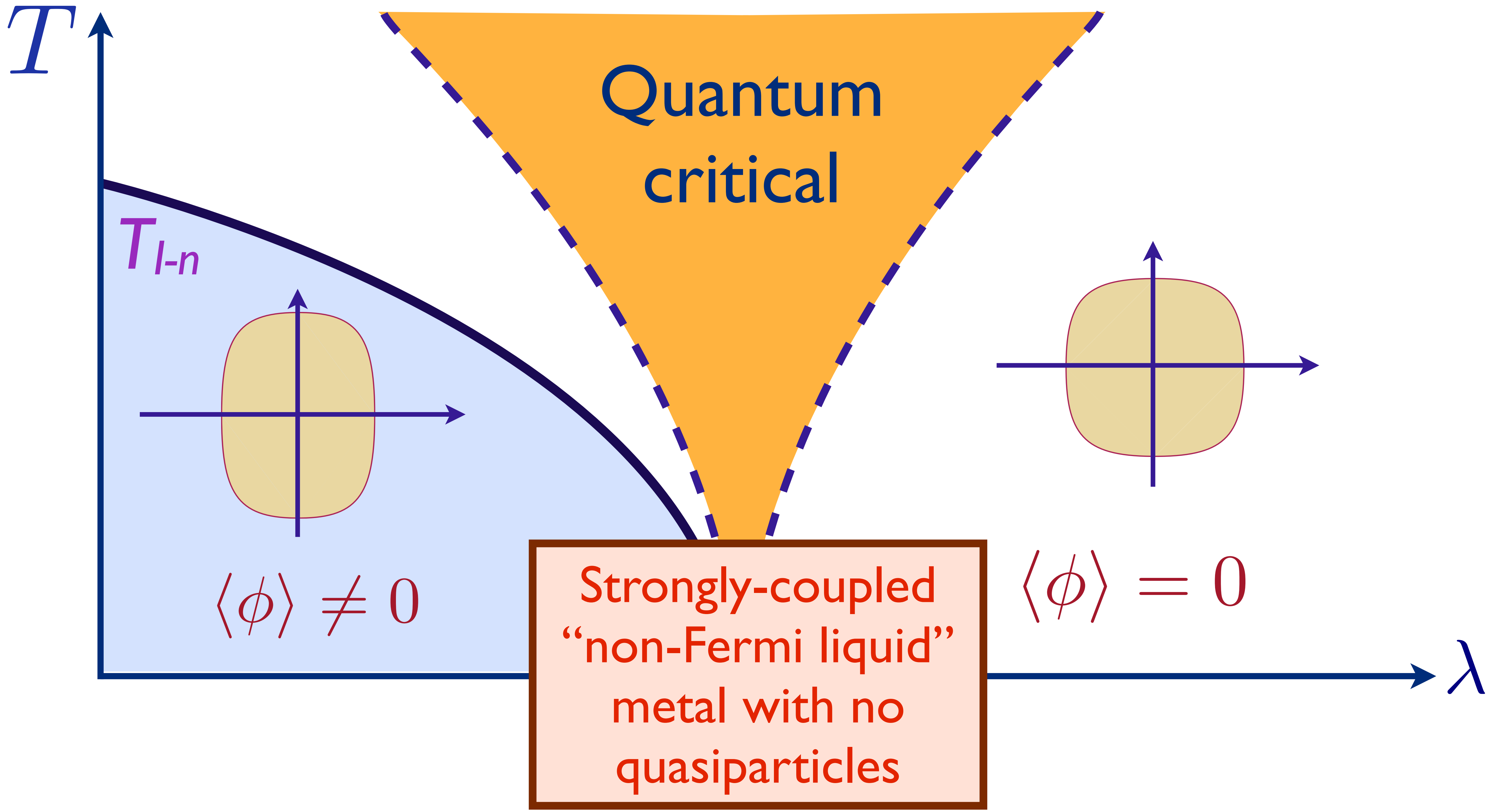
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



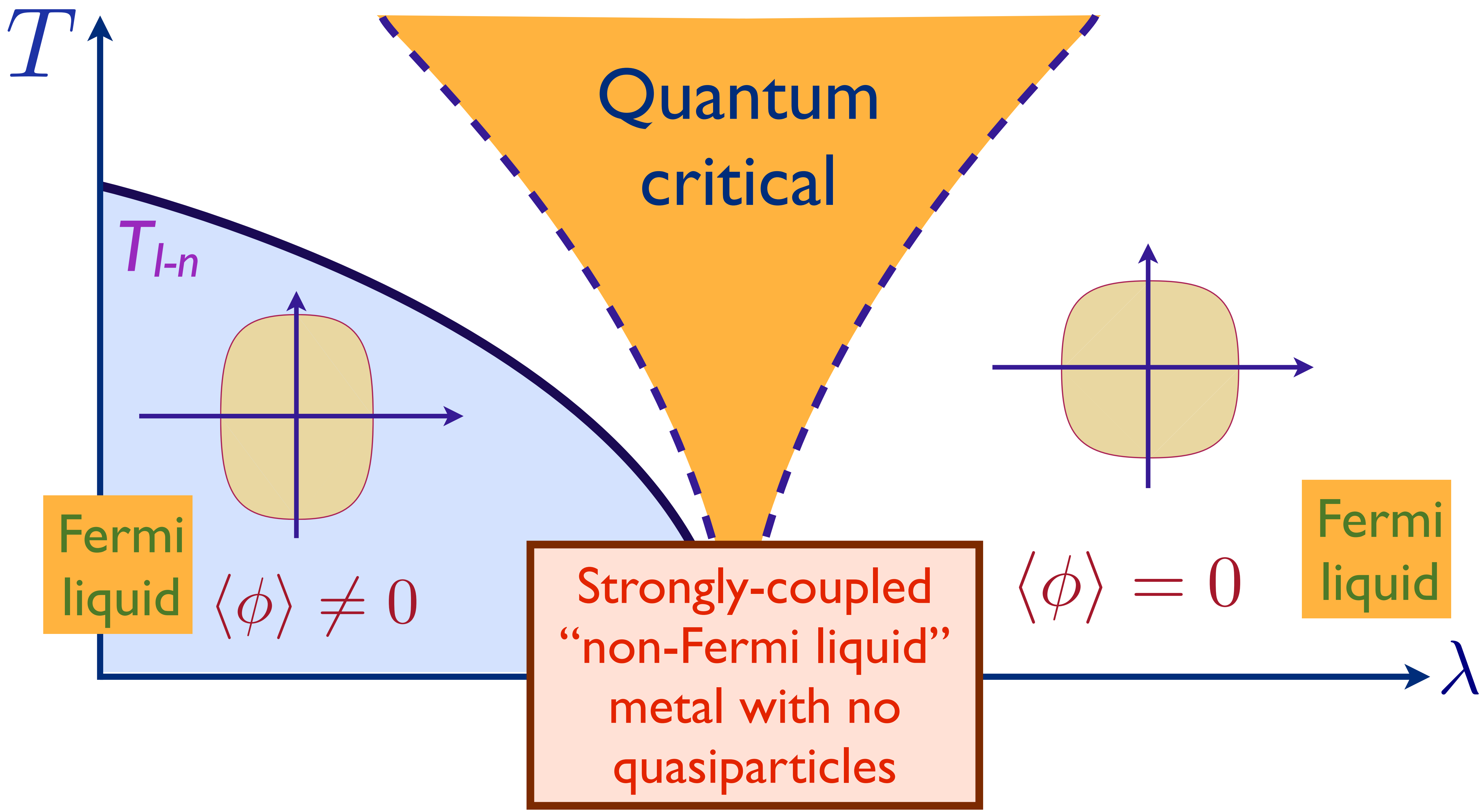
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



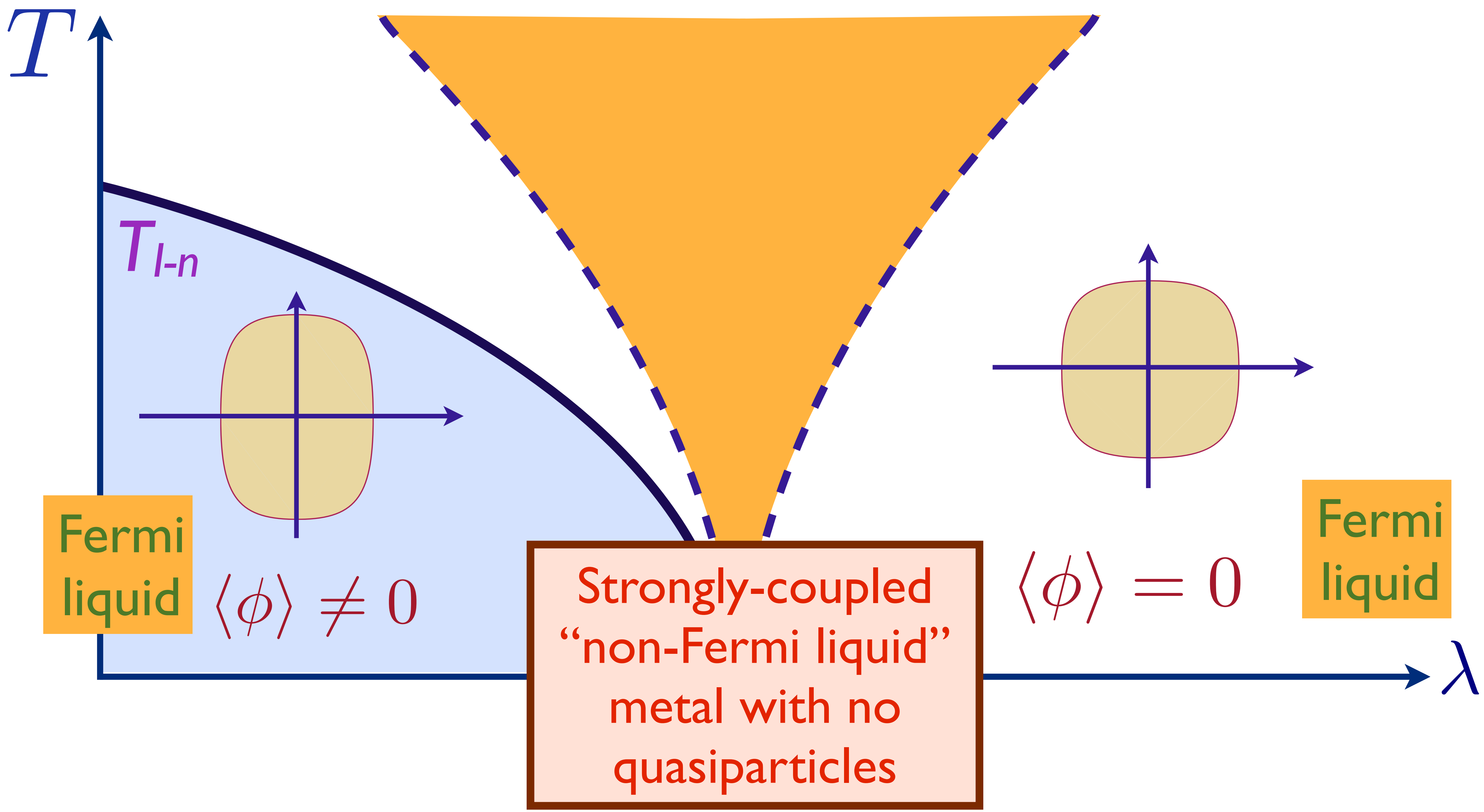
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



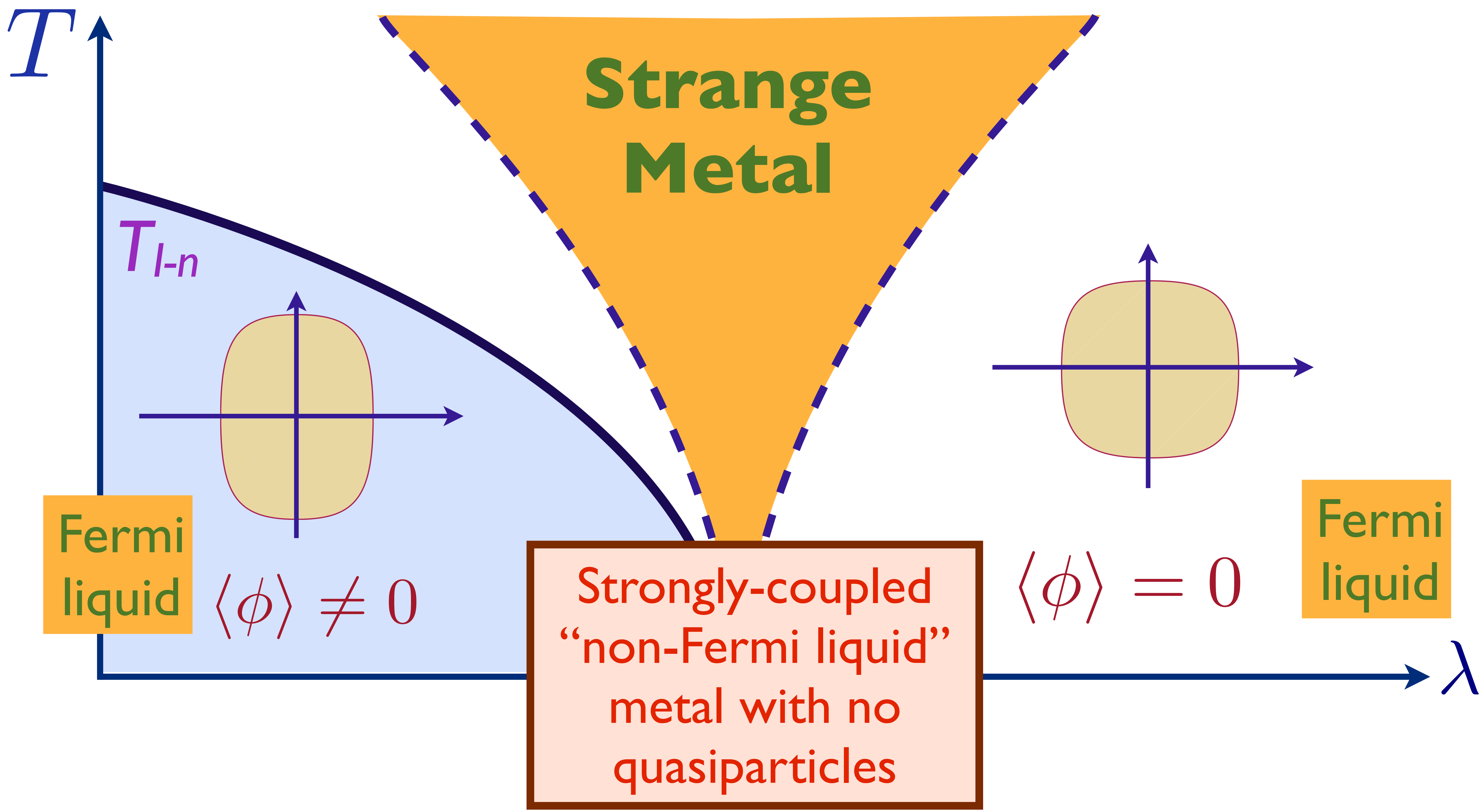
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. Holography, entanglement, and strange metals

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

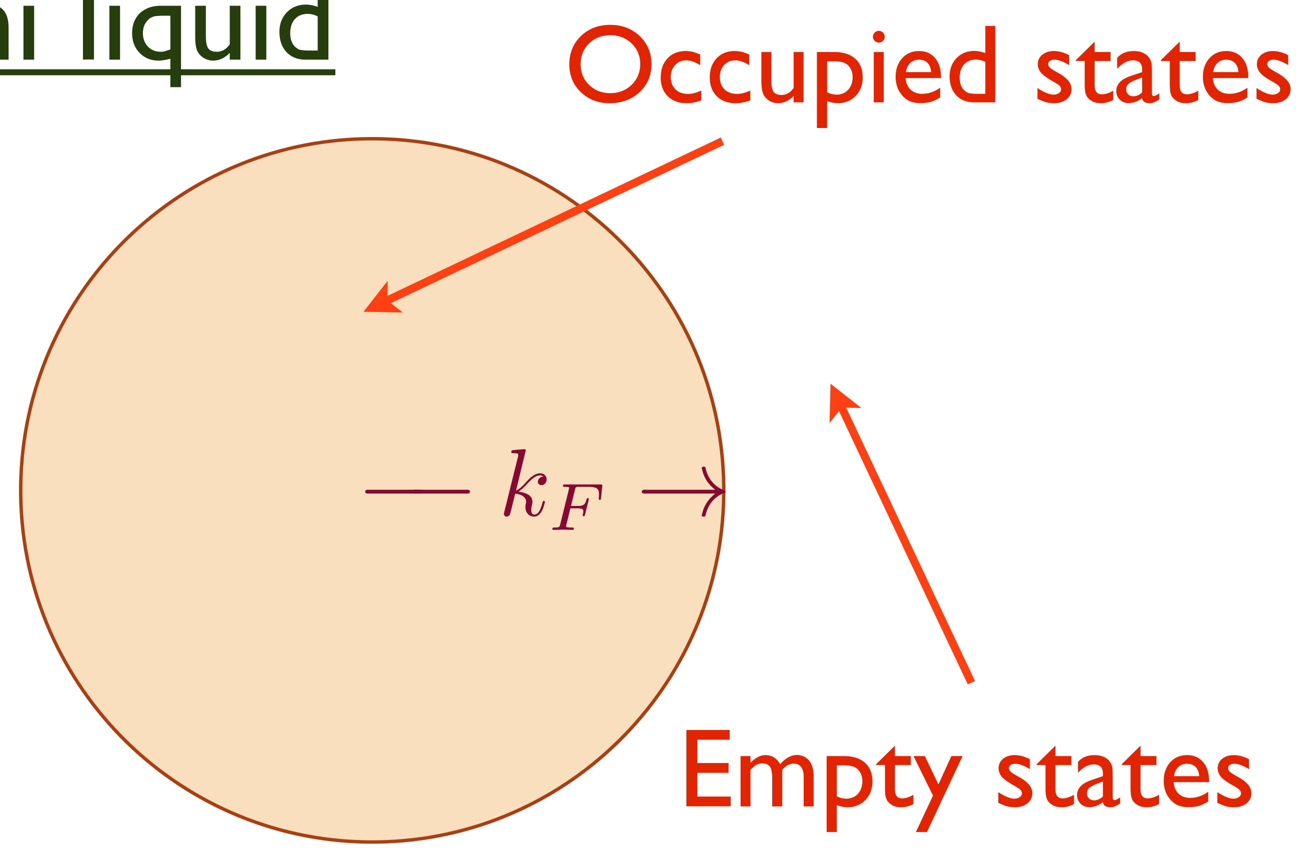
B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. Holography, entanglement, and strange metals

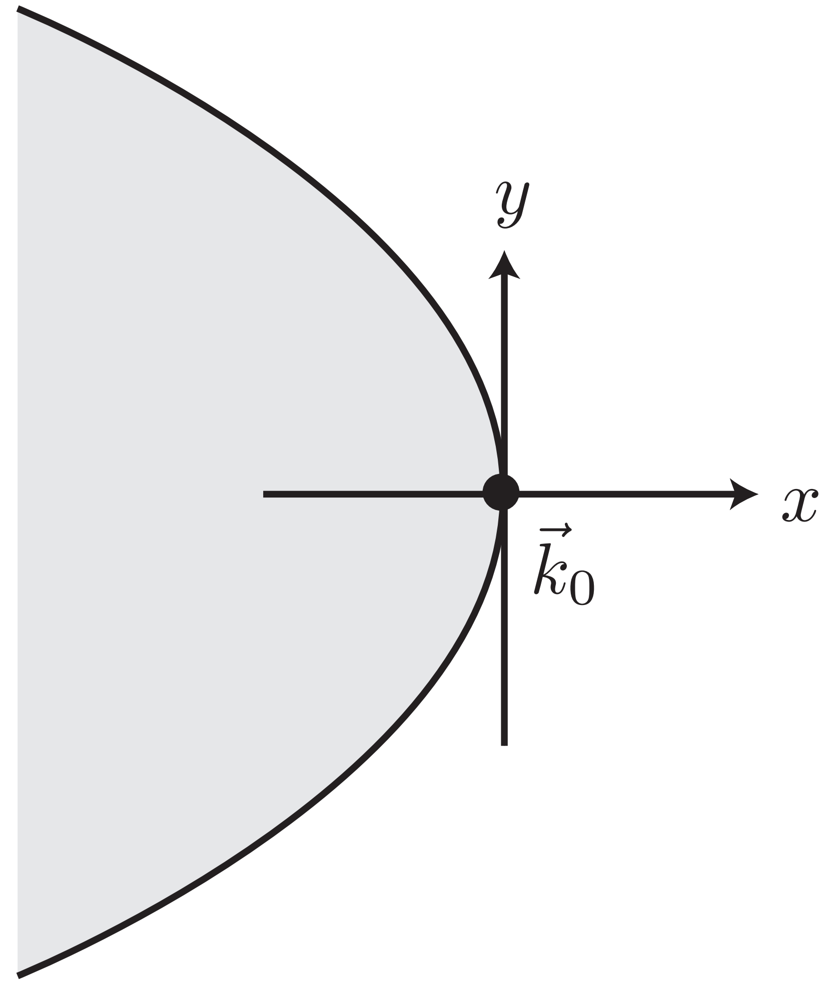
The Fermi liquid

$$\mathcal{L} = f_\alpha^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\alpha + u\, f_\alpha^\dagger f_\beta^\dagger f_\beta f_\alpha$$



The Fermi liquid: RG

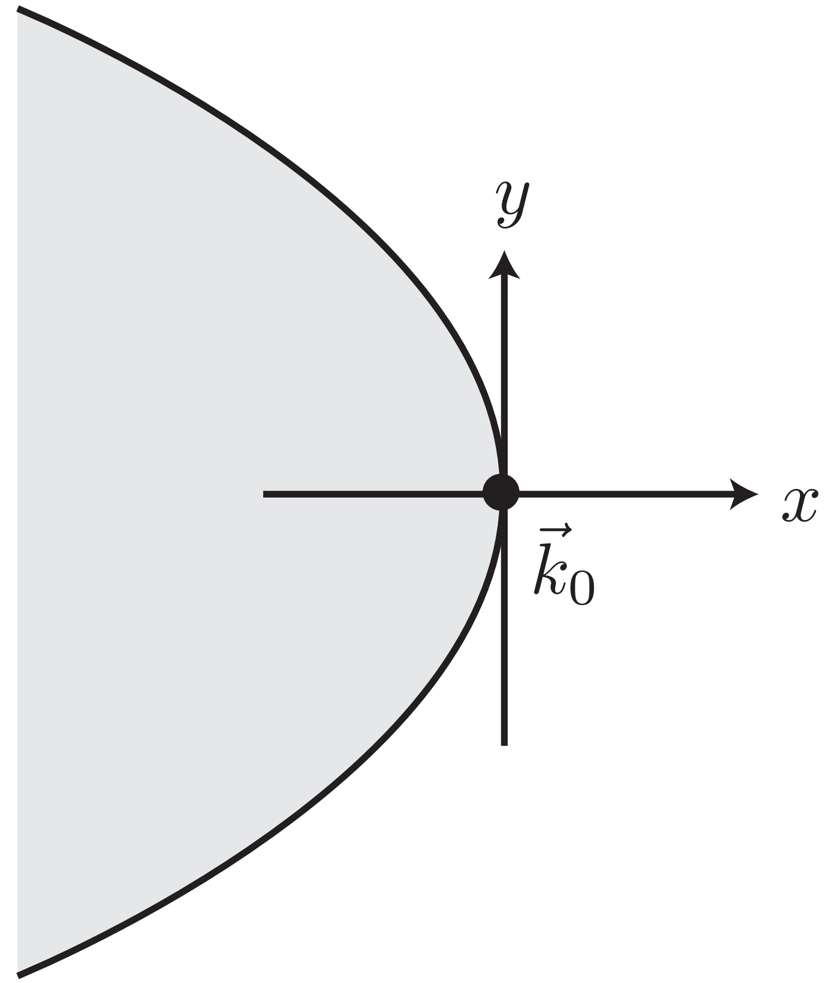
$$\mathcal{L} = f_\alpha^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\alpha$$
$$+ u\, f_\alpha^\dagger f_\beta^\dagger f_\beta f_\alpha$$



- Expand fermion kinetic energy at wavevectors about $\vec{k}_0$, by writing $f_\alpha(\vec{k}_0 + \vec{q}) = \psi_\alpha(\vec{q})$

The Fermi liquid: RG

$$\mathcal{L} = f_\alpha^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\alpha$$
$$+ u \, f_\alpha^\dagger f_\beta^\dagger f_\beta f_\alpha$$



- Expand fermion kinetic energy at wavevectors about $\vec{k}_0$, by writing $f_\alpha(\vec{k}_0 + \vec{q}) = \psi_\alpha(\vec{q})$

$$\mathcal{L}[\psi_\alpha] = \psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2 \right) \psi_\alpha + u \, \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha$$

The Fermi liquid: RG

$$\boxed{\mathcal{S}[\psi_\alpha] = \int d^{d-1}y\, dx\, d\tau \left[\psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_\alpha + u\, \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha\right]}$$

The Fermi liquid: RG

$$\mathcal{S}[\psi_\alpha] = \int d^{d-1}y \, dx \, d\tau \left[\psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2 \right) \psi_\alpha + u \, \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha \right]$$

The kinetic energy is invariant under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, provided $z = 1$ and

$$\psi \quad \to \quad \psi \, s^{(d+1)/4}.$$

The Fermi liquid: RG

$$\mathcal{S}[\psi_\alpha] = \int d^{d-1}y\, dx\, d\tau \left[\psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_\alpha + u\,\psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha\right]$$

The kinetic energy is invariant under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, provided $z = 1$ and

$$\psi \quad \to \quad \psi\, s^{(d+1)/4}.$$

Then we find $u \to u s^{(1-d)/2}$, and so we have the RG flow

$$\frac{du}{d\ell} \quad = \quad \frac{(1-d)}{2} u$$

Interactions are *irrelevant* in $d = 2$!

The Fermi liquid: RG

$$\mathcal{S}[\psi_\alpha] = \int d^{d-1}y\, dx\, d\tau \left[\psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_\alpha + u\,\psi_\alpha^\dagger\psi_\beta^\dagger\psi_\beta\psi_\alpha\right]$$

The fermion Green's function to order u^2 has the form (upto logs)

$$G(\vec{q},\omega) = \frac{\mathcal{A}}{\omega - q_x - q_y^2 + ic\,\omega^2}$$

So the quasiparticle pole is sharp.

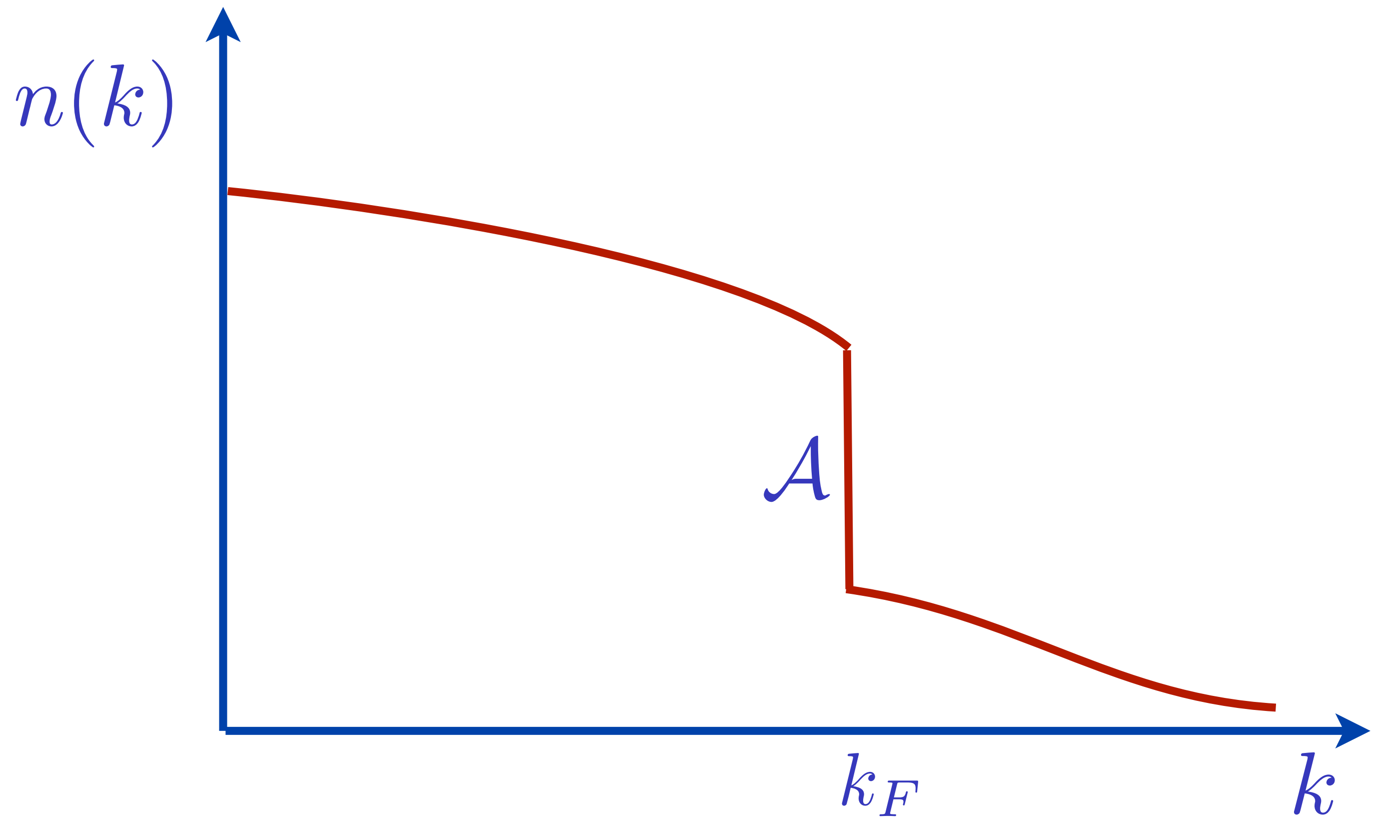
The Fermi liquid: RG

$$\mathcal{S}[\psi_\alpha] = \int d^{d-1}y\, dx\, d\tau \left[\psi_\alpha^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_\alpha + u\,\psi_\alpha^\dagger\psi_\beta^\dagger\psi_\beta\psi_\alpha\right]$$

The fermion Green's function to order u^2 has the form (upto logs)

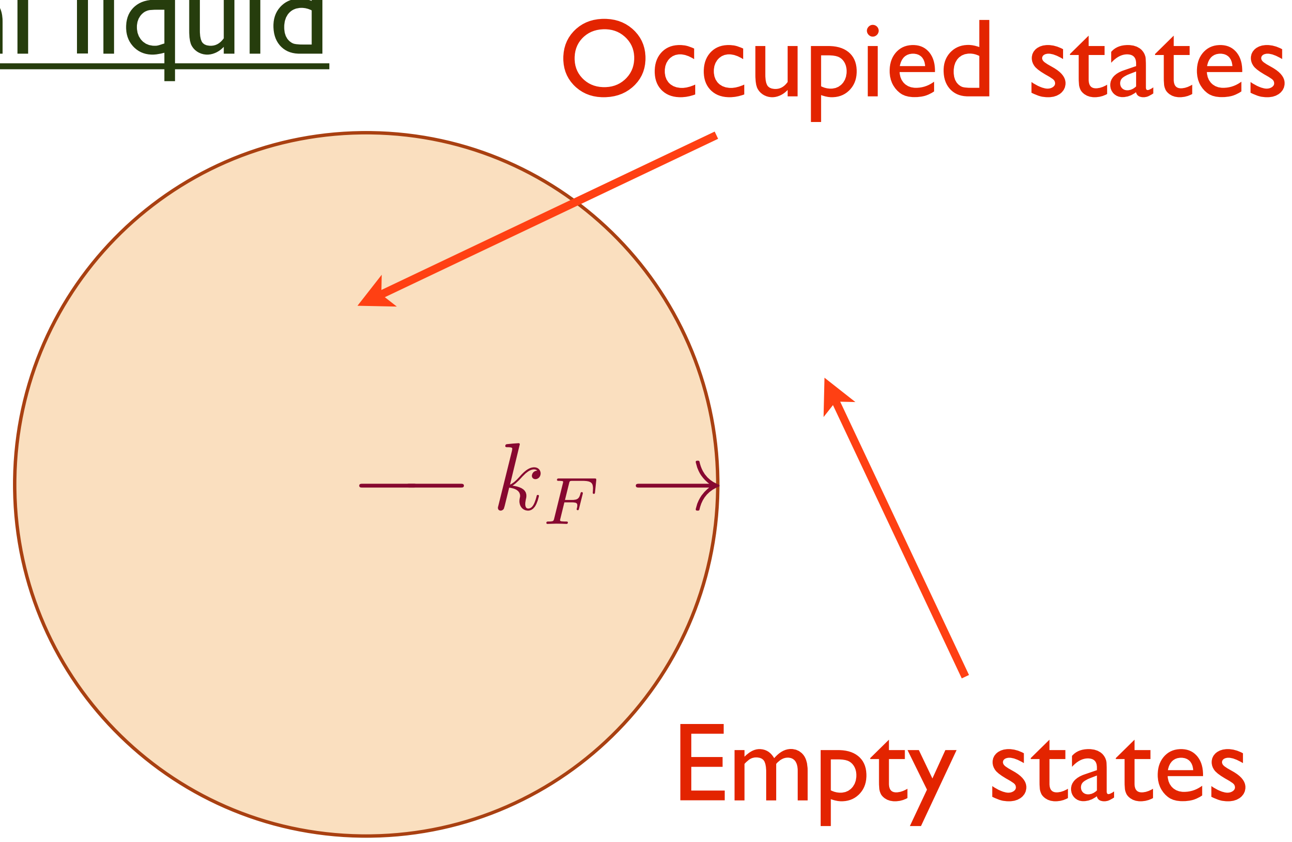
$$G(\vec{q},\omega) = \frac{\mathcal{A}}{\omega - q_x - q_y^2 + ic\,\omega^2}$$

So the quasiparticle pole is sharp. And fermion momentum distribution function $n(\vec{k}) = \left\langle f_\alpha^\dagger(\vec{k}) f_\alpha(\vec{k})\right\rangle$ had the following form:



The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$
$$+ \, 4 \text{ Fermi terms}$$

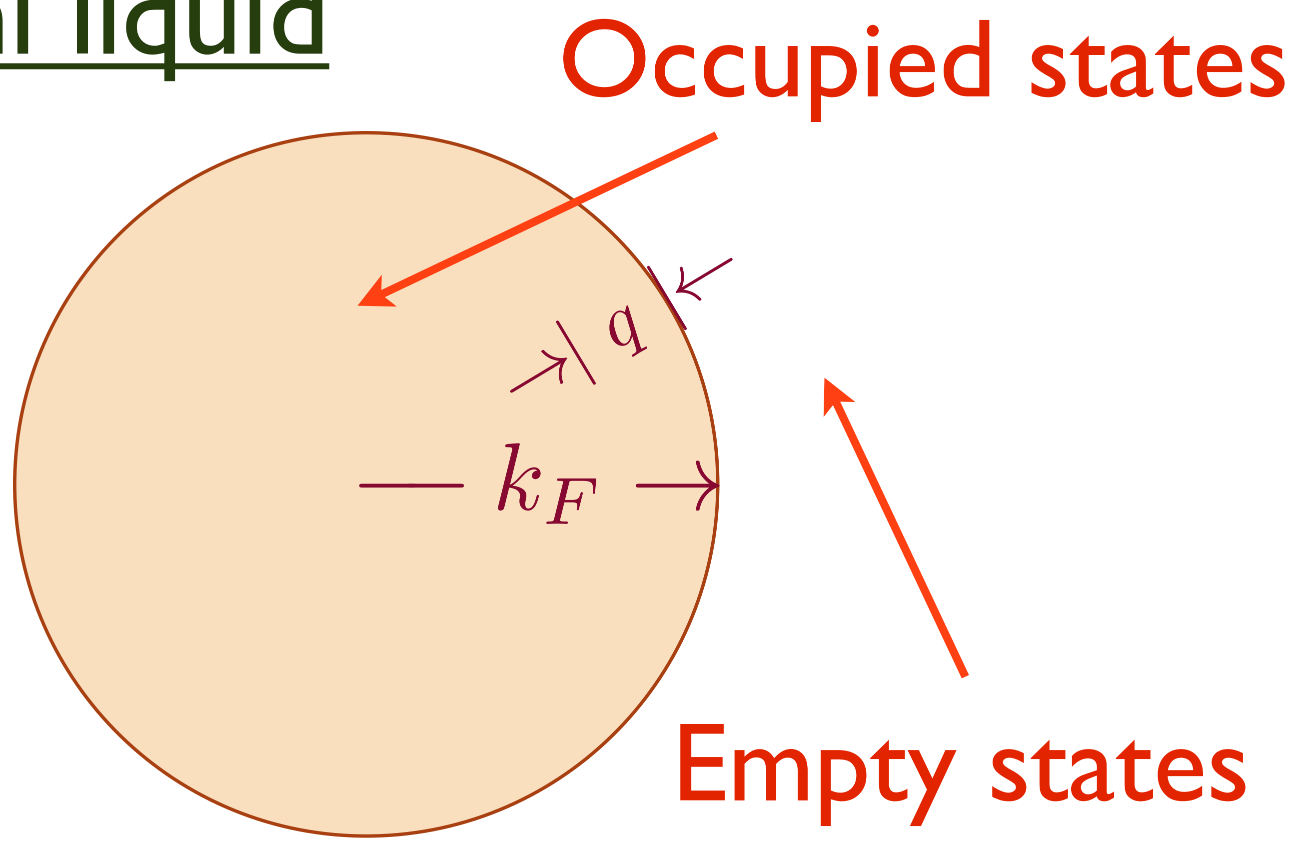


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density

The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$

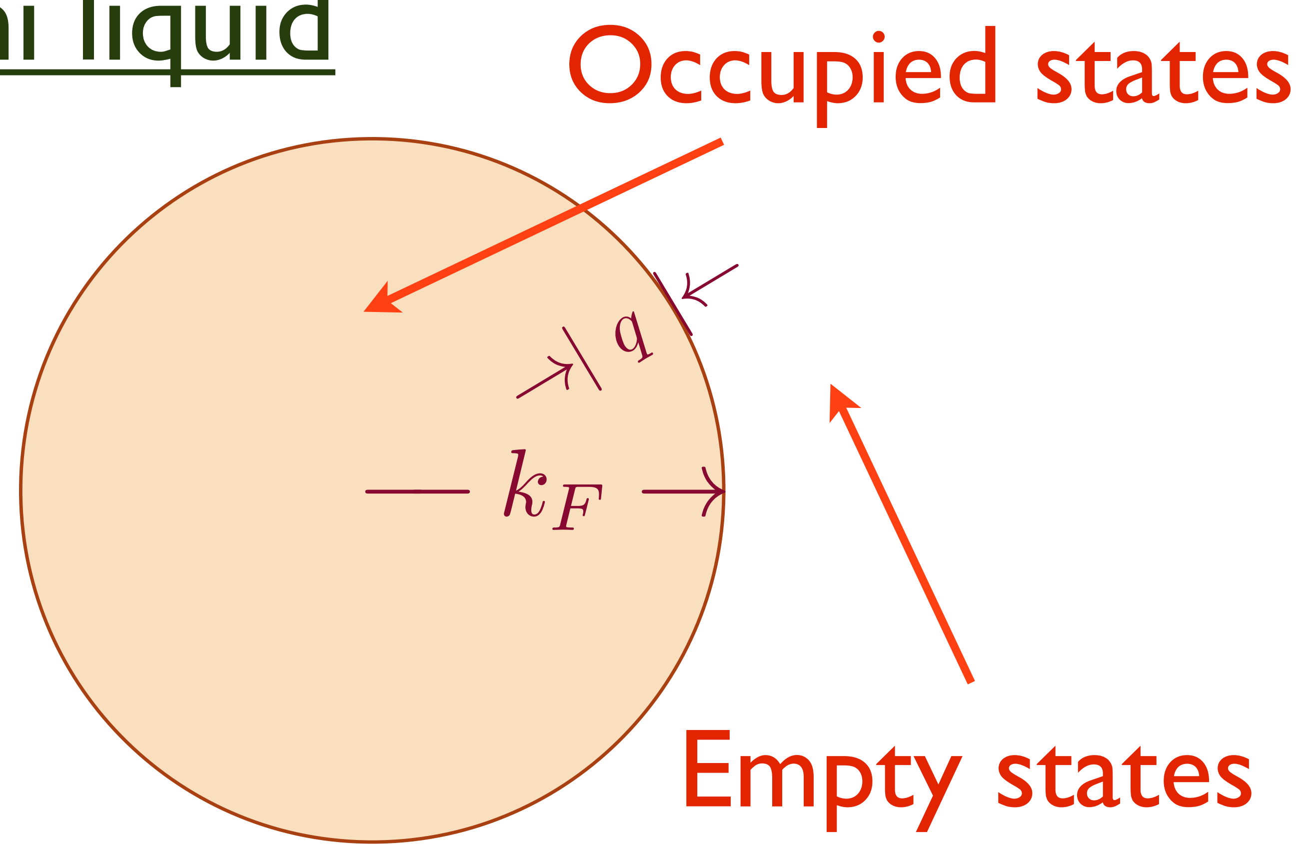


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

The Fermi liquid

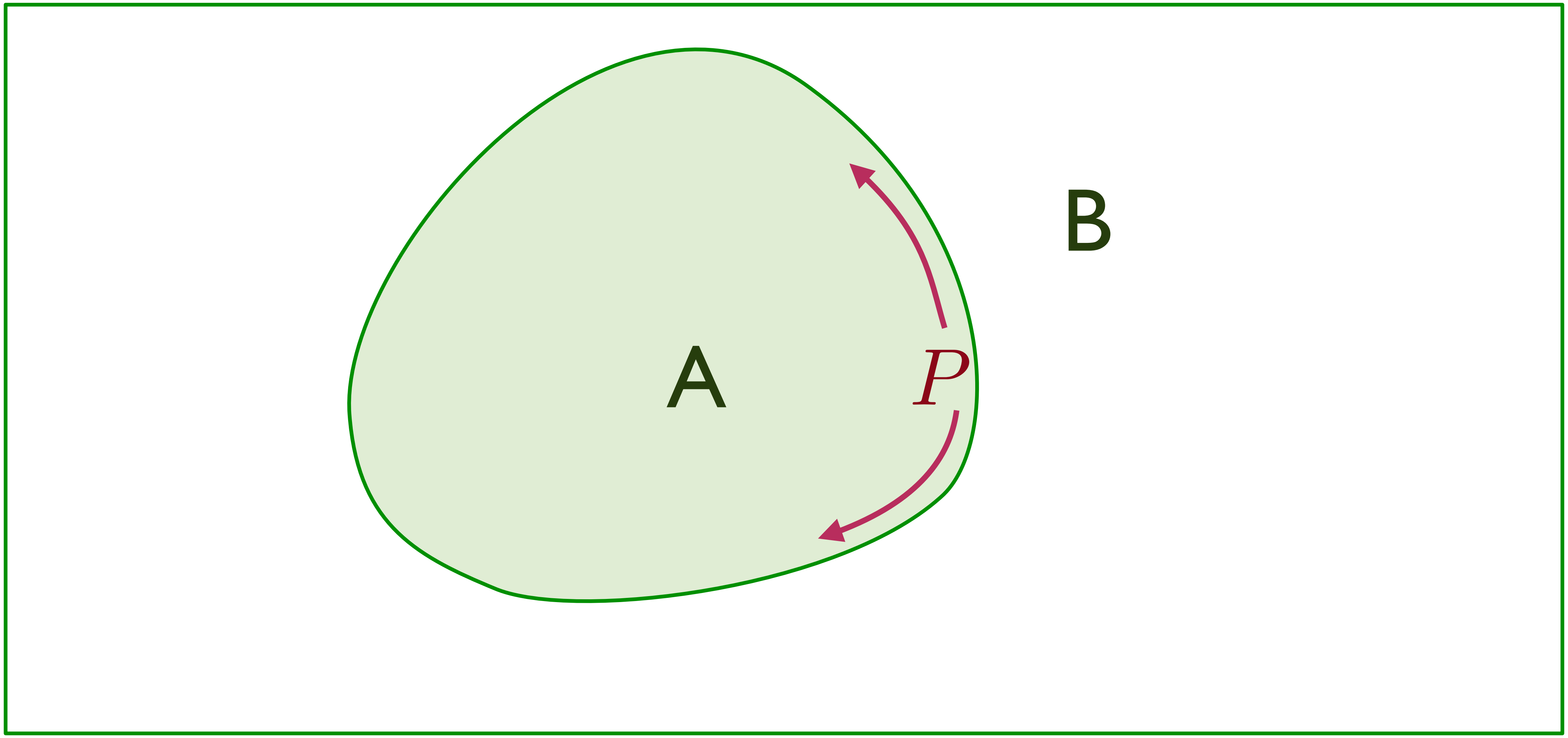
$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$



- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this is as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy

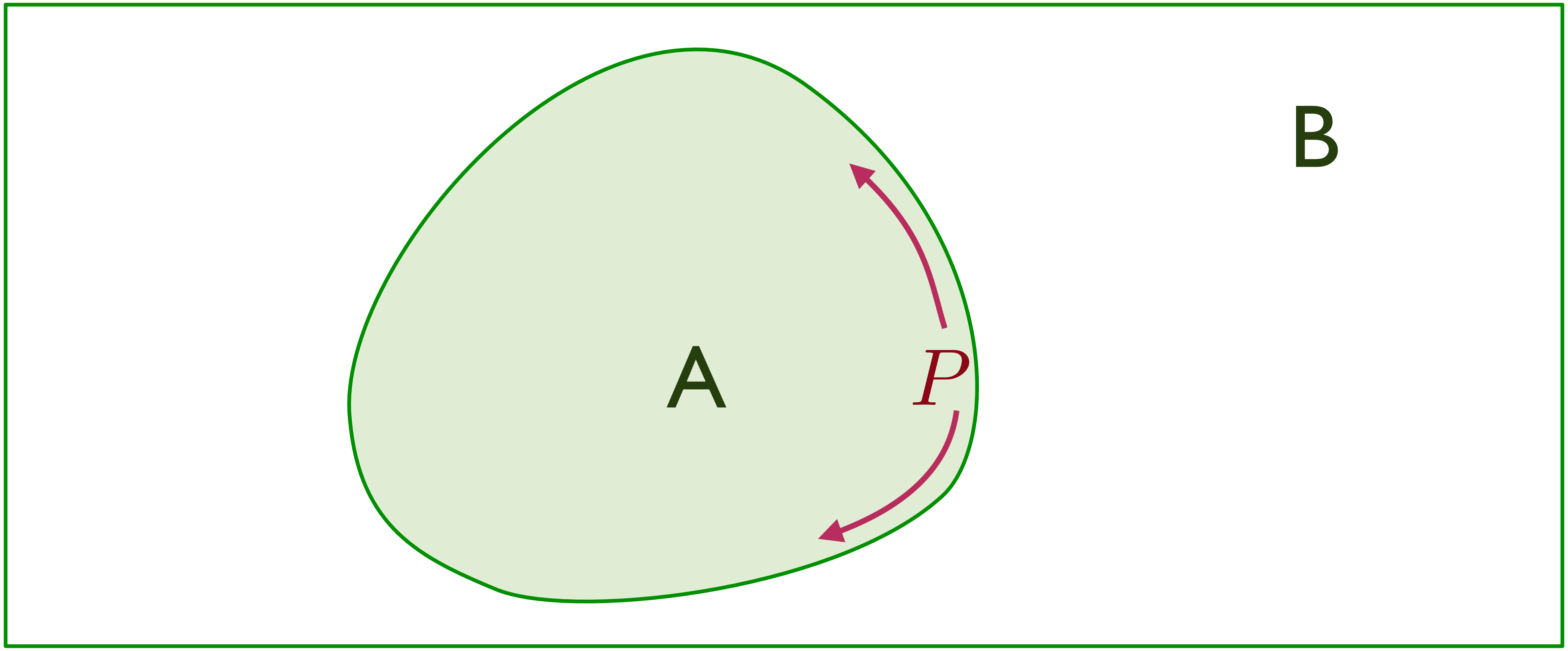
$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

$$\begin{aligned}\text{Then } \rho_A = \mathrm{Tr}_B \rho &= \text{density matrix of region } A \\ &= \tfrac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A\right)\end{aligned}$$

$$\begin{aligned}\textbf{Entanglement entropy } S_E &= -\mathrm{Tr}\left(\rho_A \ln \rho_A\right) \\ &= \ln 2\end{aligned}$$

Entanglement entropy of the Fermi liquid

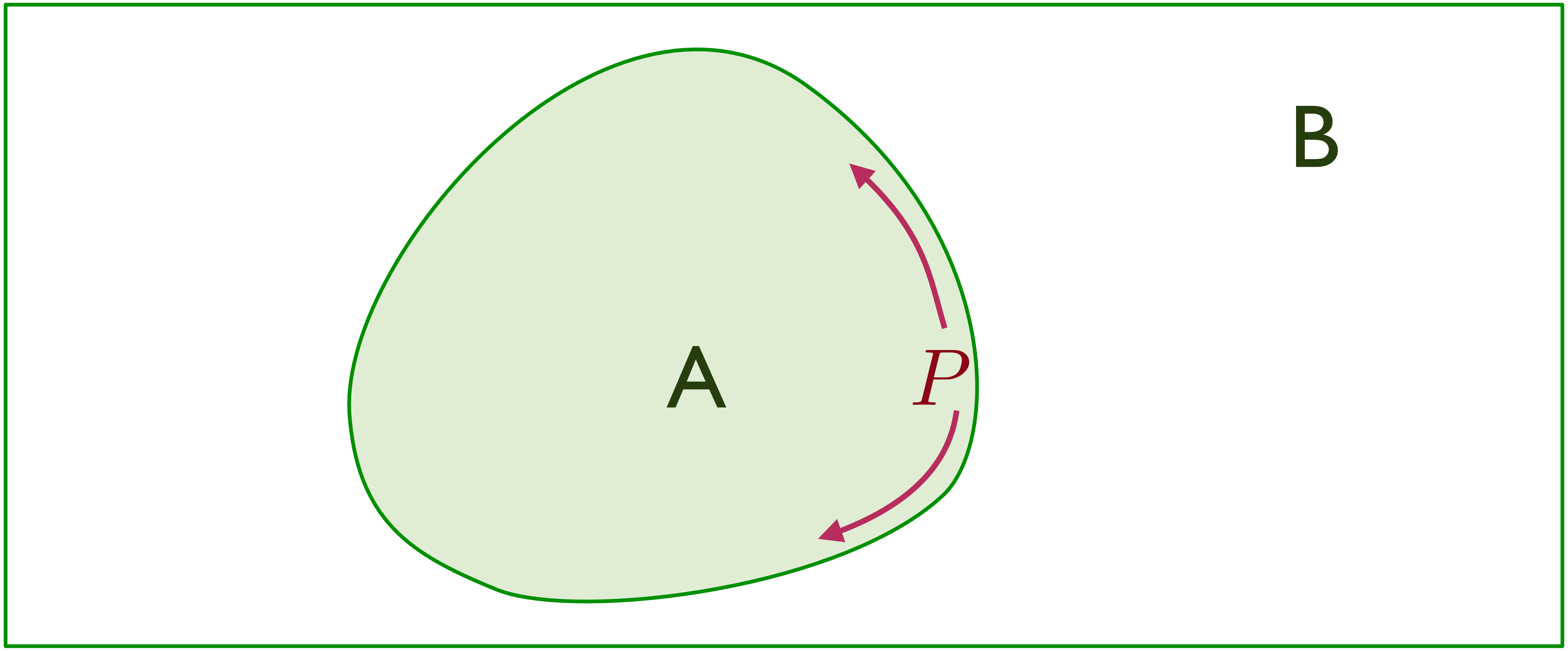


Logarithmic violation of "area law": $S_E = \dfrac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of the Fermi liquid

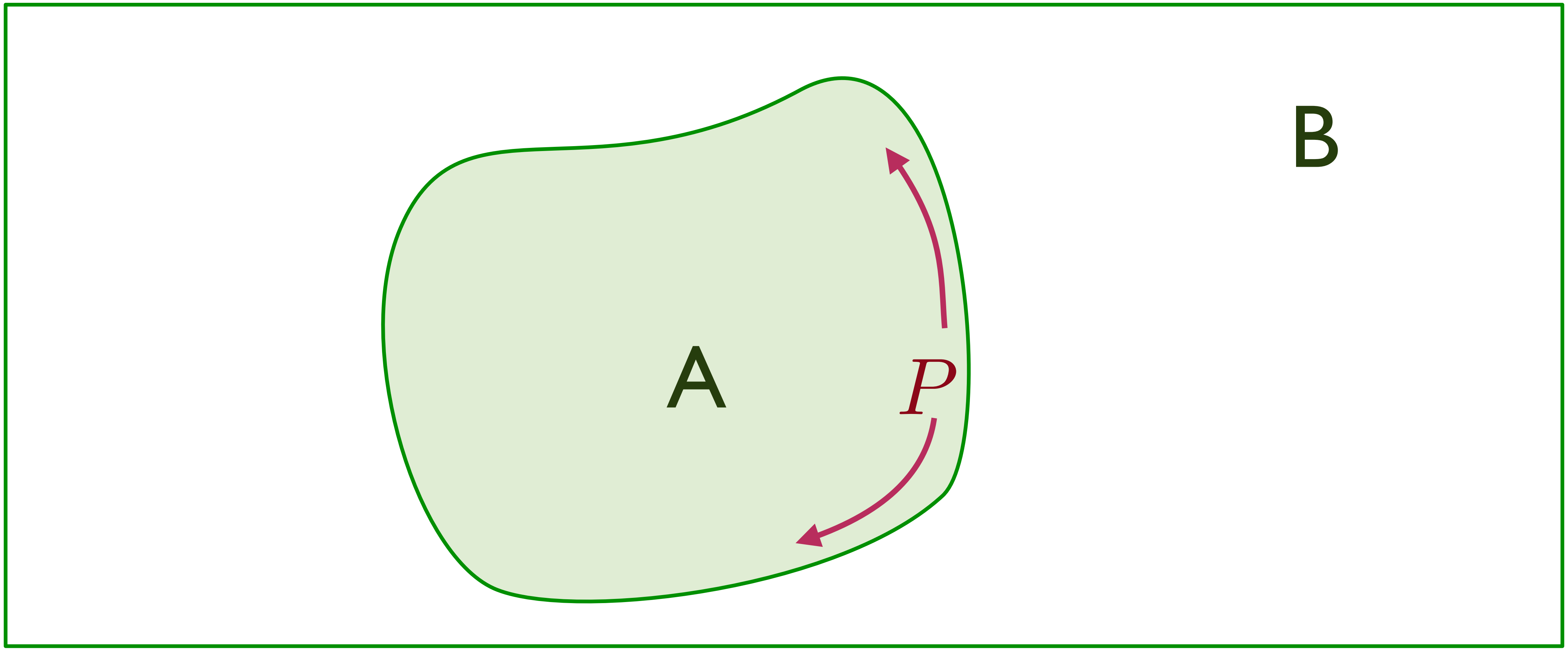


Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $1/12$ is *universal*: it is independent of the shape of the entangling region, and of the strength of the interactions.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of the Fermi liquid

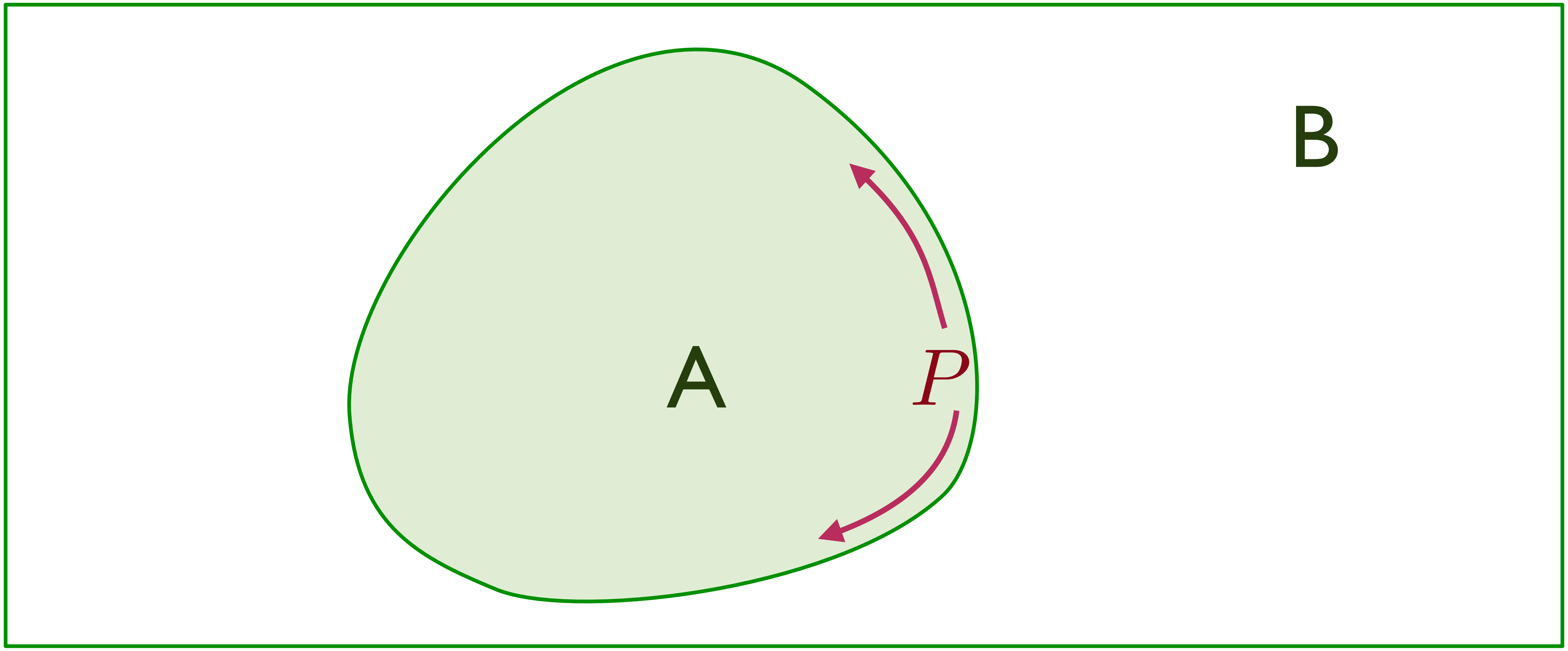


Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F, where P is the perimeter of region A with an arbitrary smooth shape. The prefactor $1/12$ is *universal*: it is independent of the shape of the entangling region, and of the strength of the interactions.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of the Fermi liquid

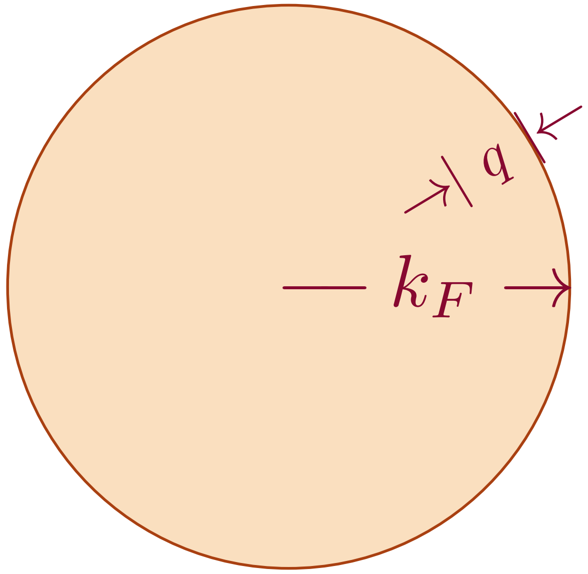


Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $1/12$ is *universal*: it is independent of the shape of the entangling region, and of the strength of the interactions.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

FL Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. Holography, entanglement, and strange metals

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

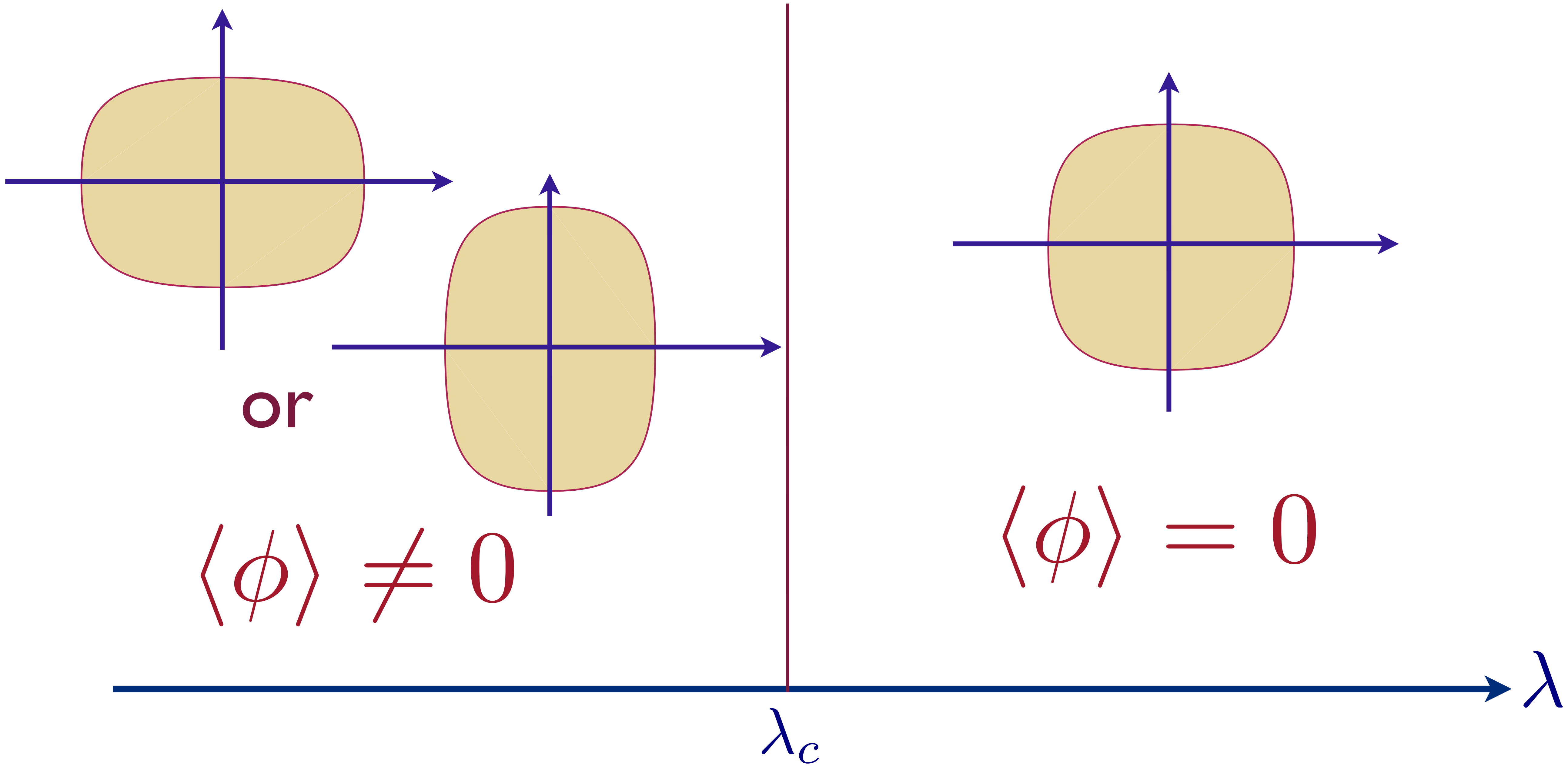
A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. Holography, entanglement, and strange metals

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

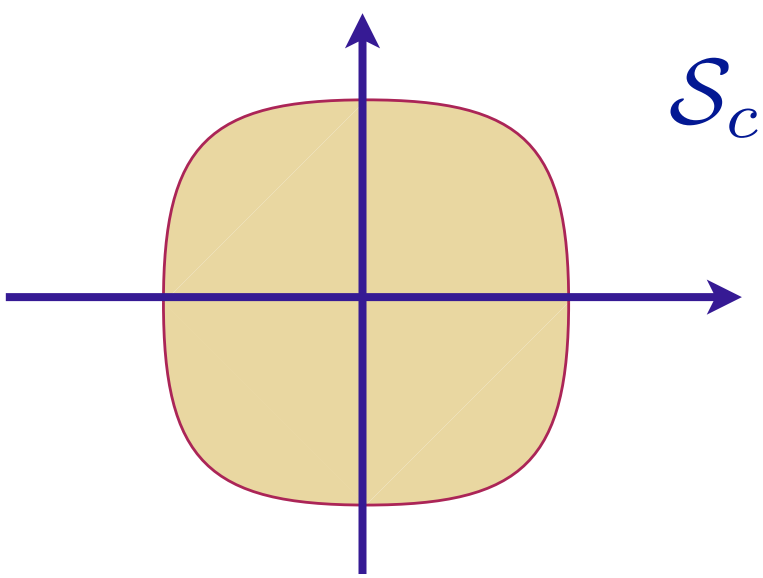
$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

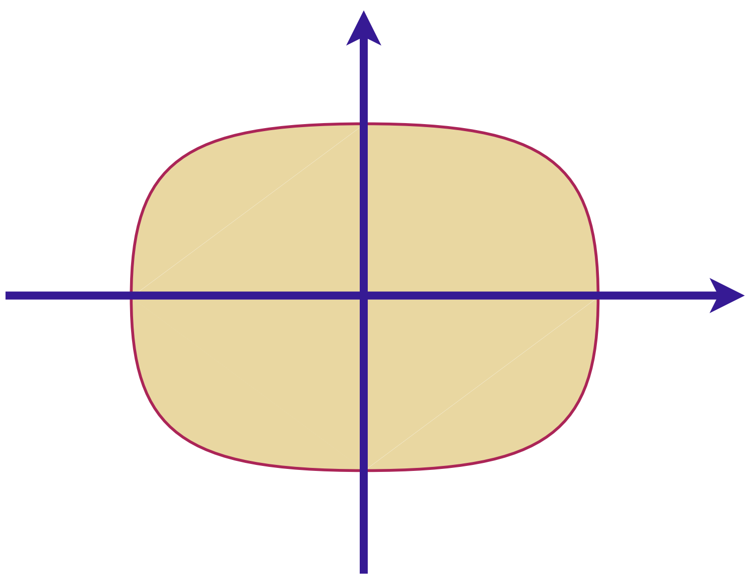
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal

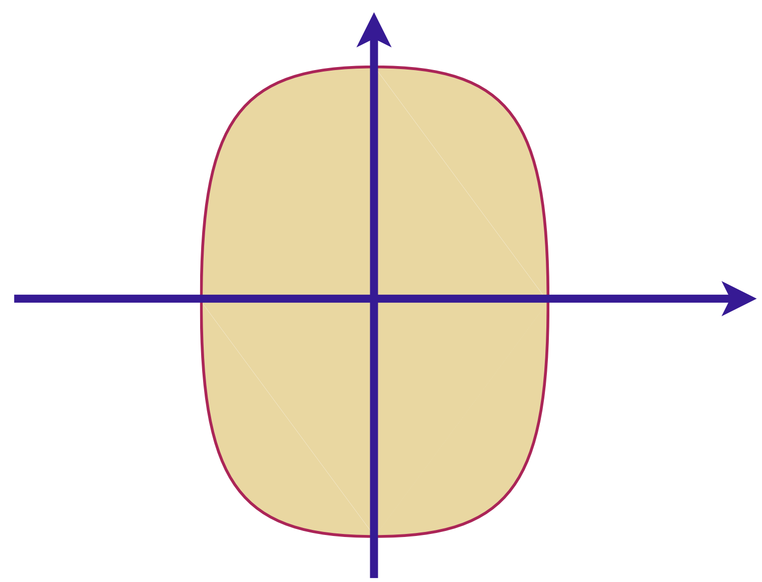
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



$\langle \phi \rangle < 0$

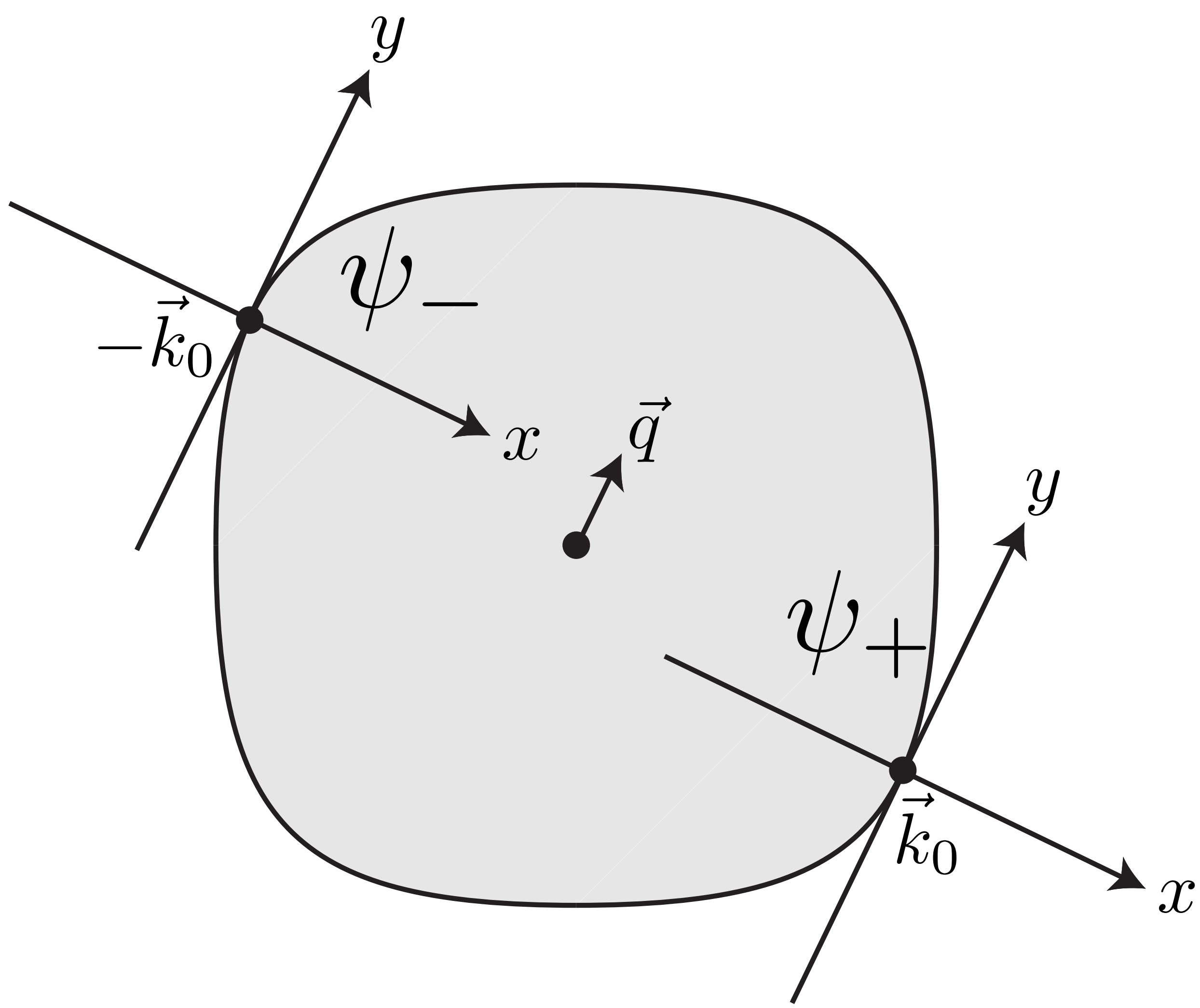
Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

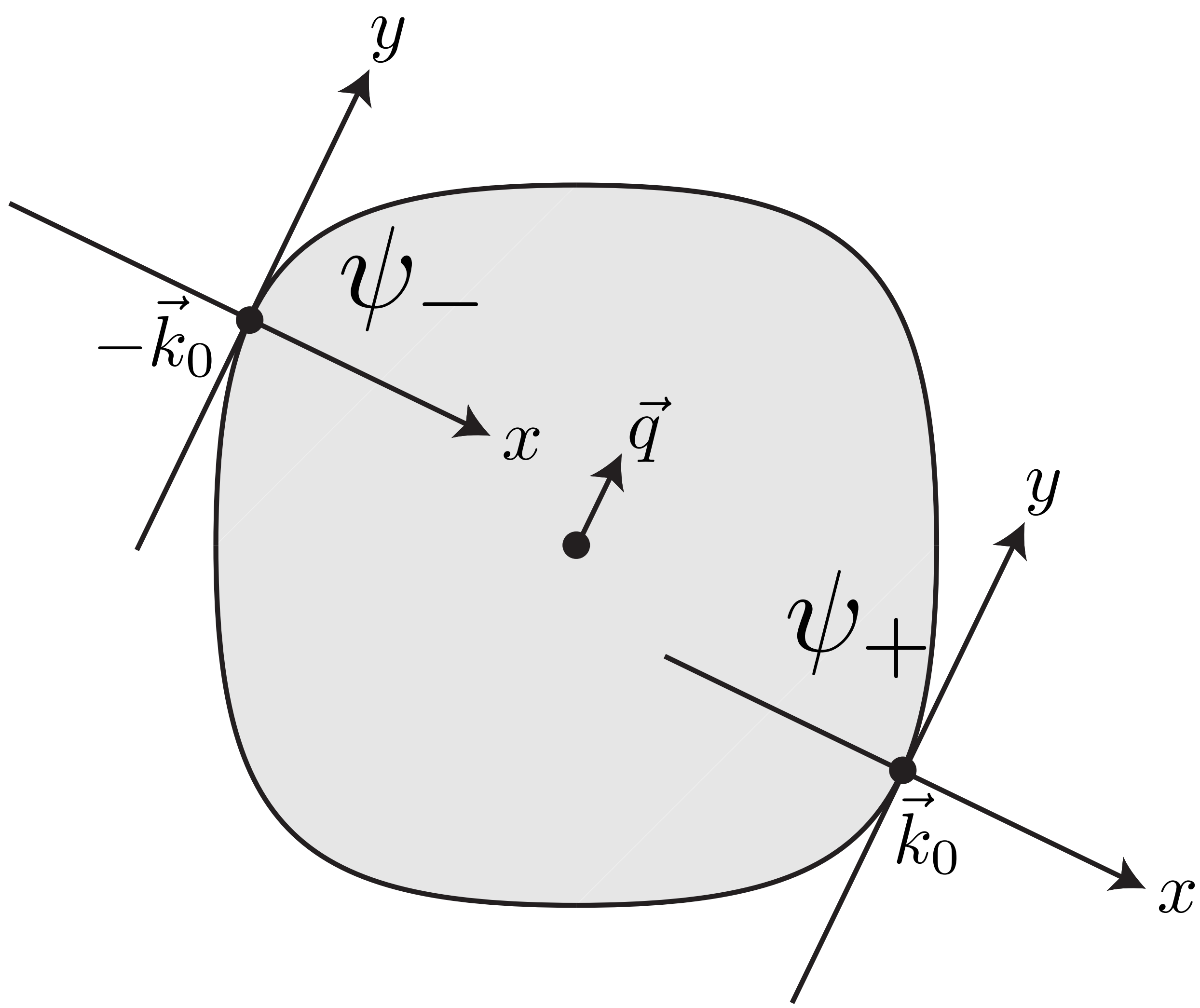
$$\mathcal{S}_{\phi c} = - g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal



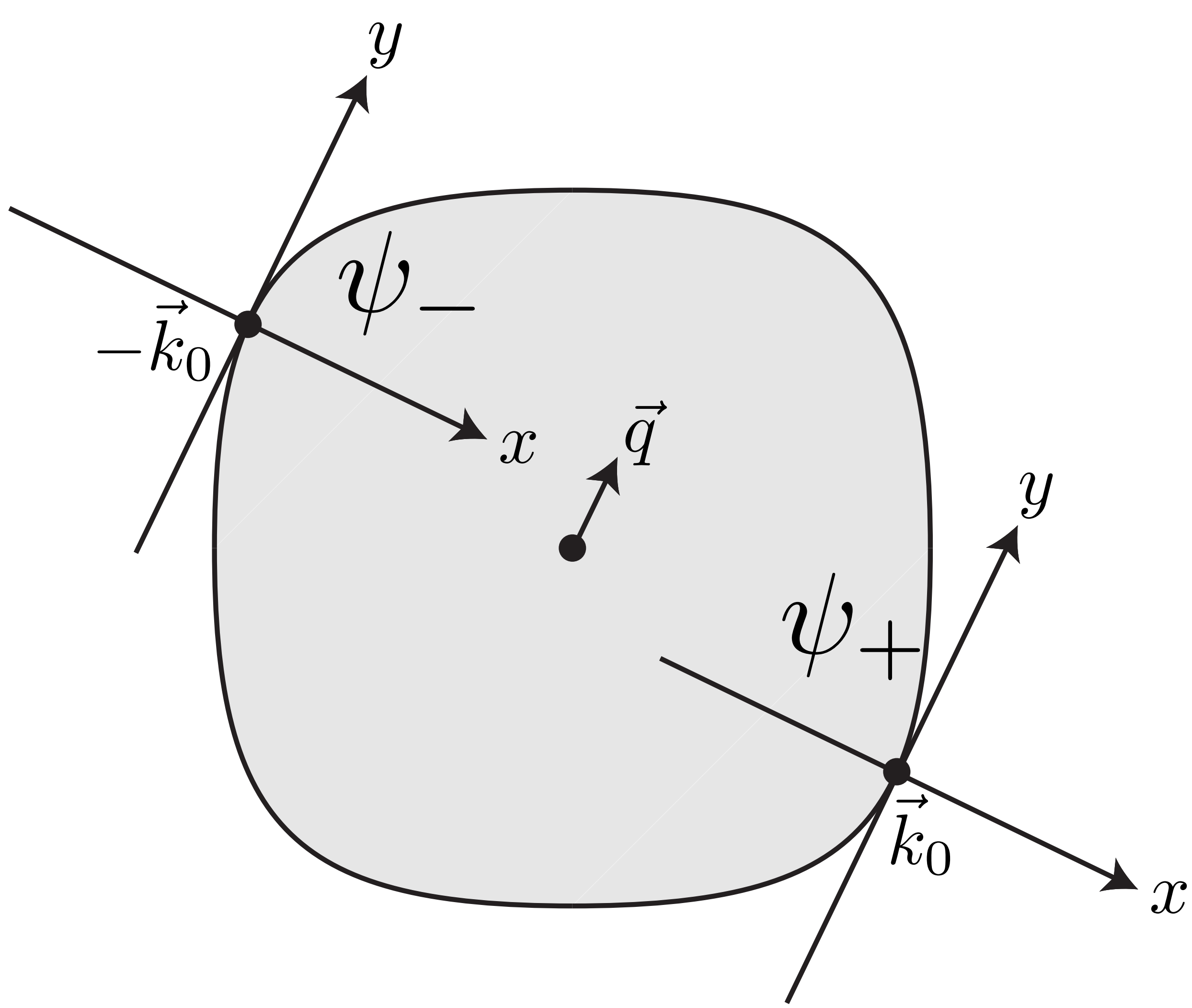
- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering in a metal



$$\mathcal{L}[\psi_\pm, \phi] =$$

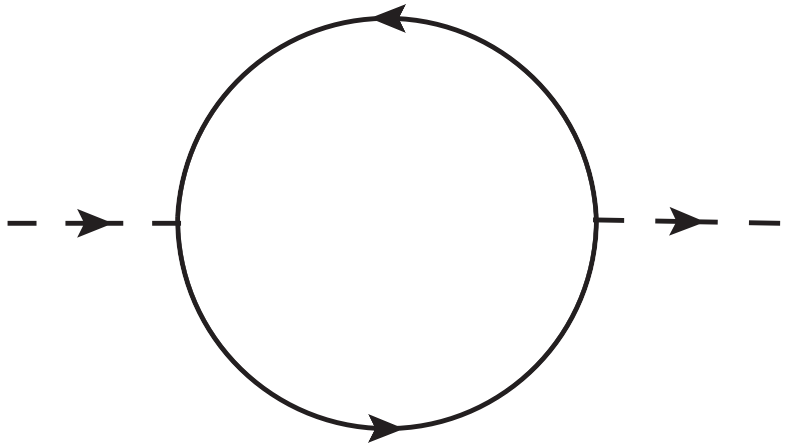
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-\phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



One loop ϕ self-energy with N_f fermion flavors:

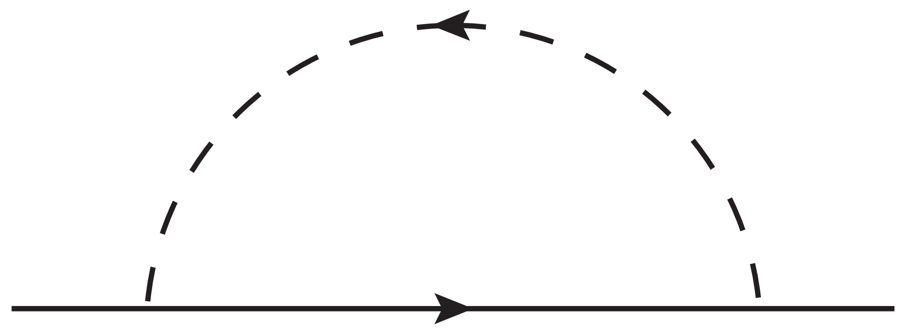
$$\Sigma_\phi(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]}$$

$$= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering in a metal

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2\end{aligned}$$

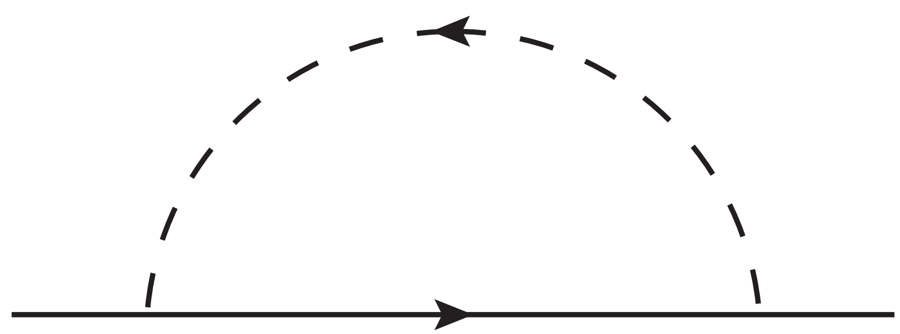


Electron self-energy at order $1/N_f$:

$$\begin{aligned}\Sigma(\vec{k},\Omega) &= -\frac{1}{N_f}\int\frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\dfrac{q_y^2}{g^2}+\dfrac{|\omega|}{|q_y|}\right]} \\ &= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}\end{aligned}$$

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$
$$- \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$



Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int \frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\dfrac{q_y^2}{g^2}+\dfrac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

$\boxed{\sim |\Omega|^{d/3} \text{ in dimension } d.}$

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_\perp^2 + \dfrac{|\omega|}{|q_\perp|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_\perp^2 - i\mathrm{sgn}(\omega)|\omega|^{d/3}/N_f}$$

In the boson case, $q_\perp^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.
In the fermion case, $q_x \sim q_\perp^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2$ $\Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.
Strongly-coupled theory in $d = 2$.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Schematic form of ϕ and fermion Green's functions in $d = 2$

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i\,\text{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering in a metal

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - \phi\left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y \phi\right)^2\end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering in a metal

$$
\begin{aligned}
\mathcal{L} &= \psi_+^\dagger \left(\cancel{\partial_\tau} - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\cancel{\partial_\tau} + i\partial_x - \partial_y^2\right) \psi_- \\
&\quad - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2
\end{aligned}
$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\cancel{\partial_\tau} - i\partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left(\cancel{\partial_\tau} + i\partial_x - \partial_y^2 \right) \psi_-$$
$$- \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left(\partial_y \phi \right)^2$$

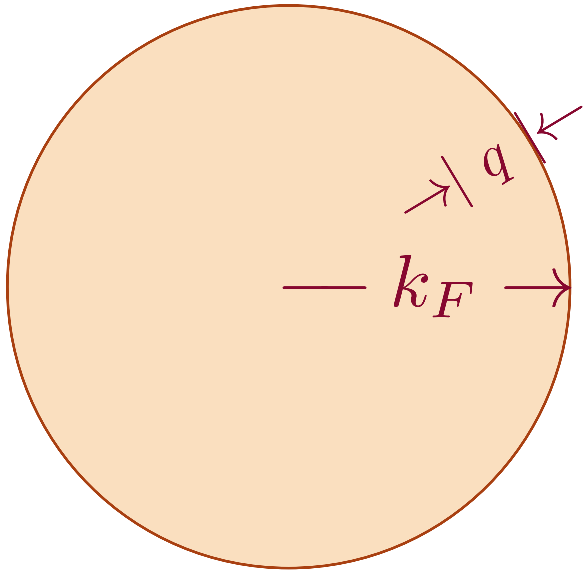
Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\to \phi\, s \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

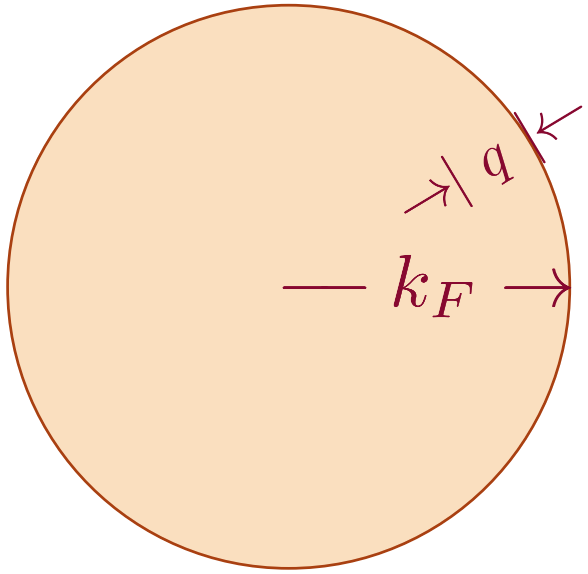
So the action is invariant provided $z = 3/2$.

FL Fermi liquid



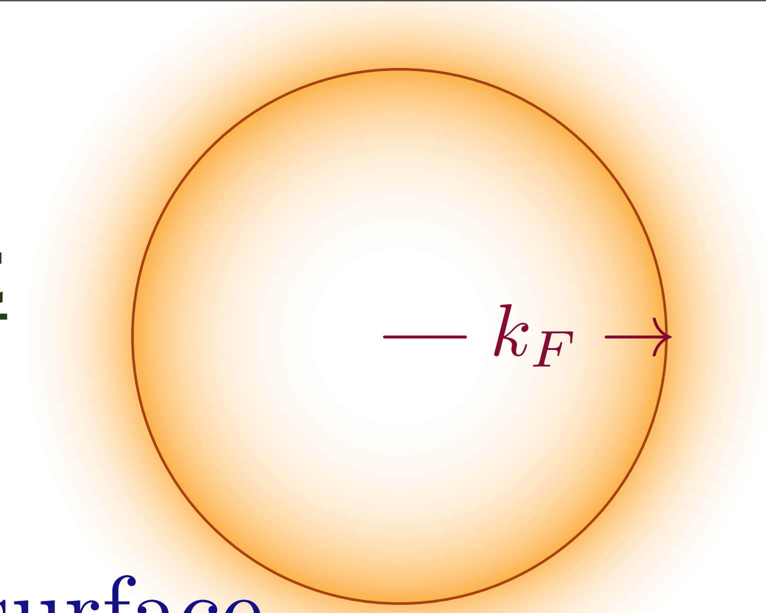
- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density

NFL Nematic QCP

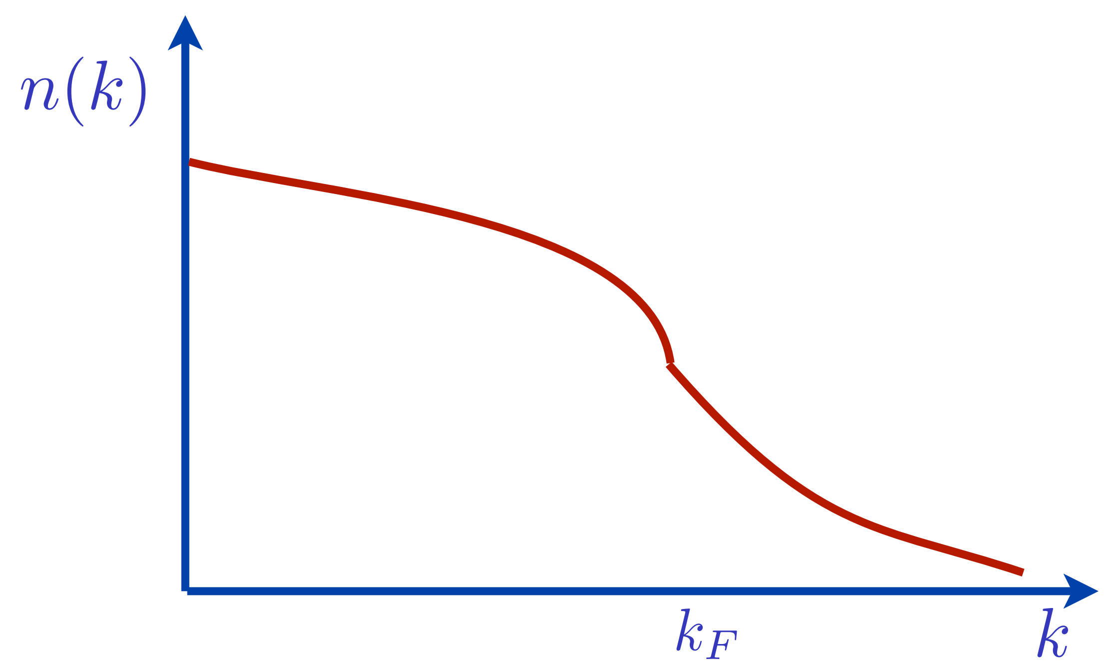


- Fermi surface with $k_F^d \sim \mathcal{Q}$.

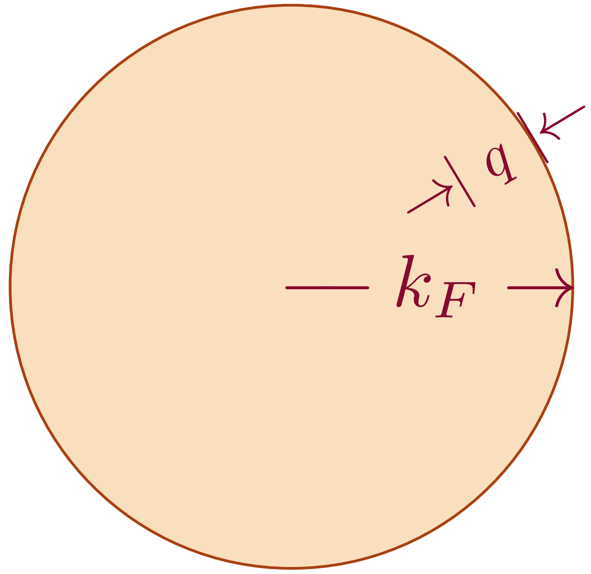
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

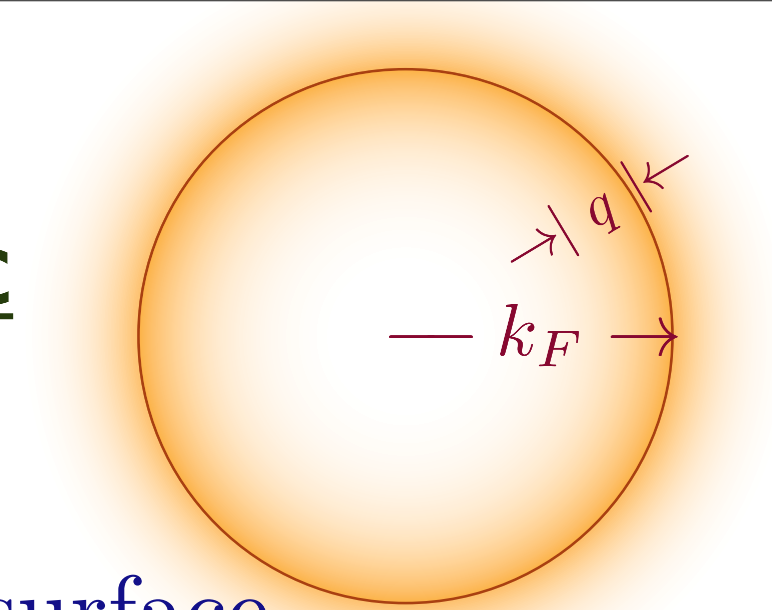


FL
Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

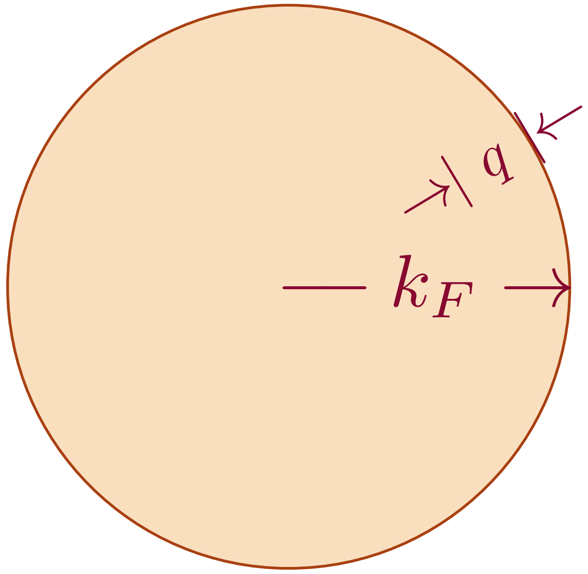
NFL
Nematic
QCP



- Fermi surface with $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

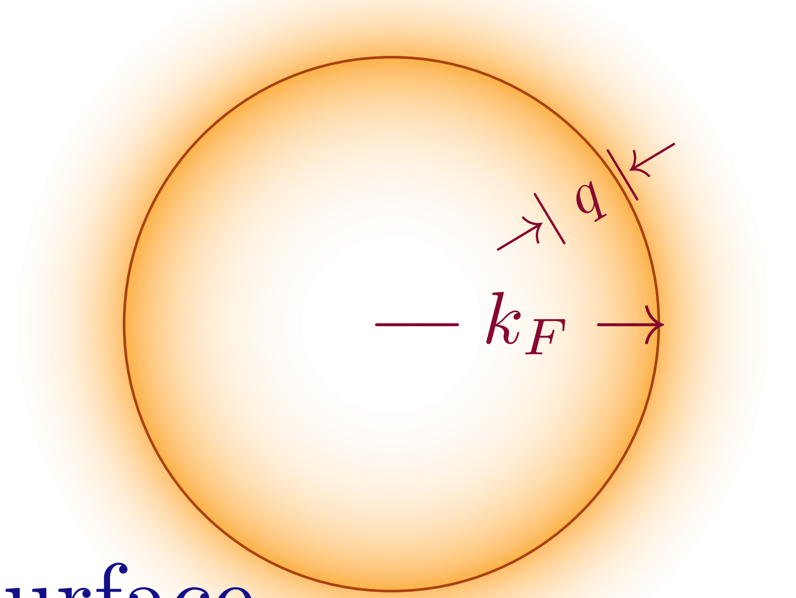
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

FL Fermi liquid

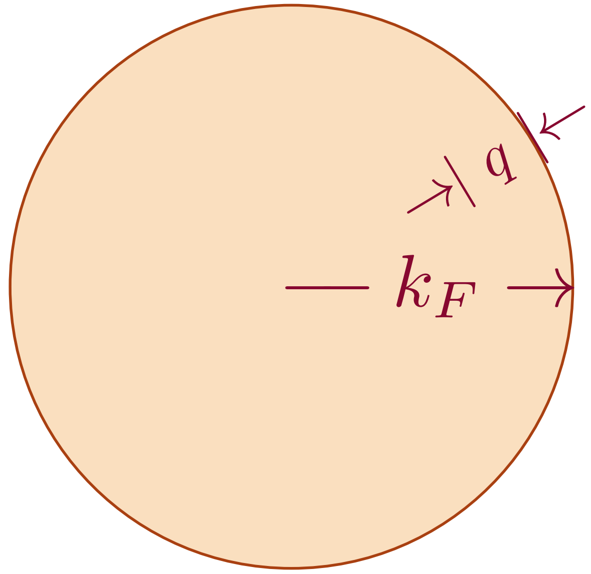


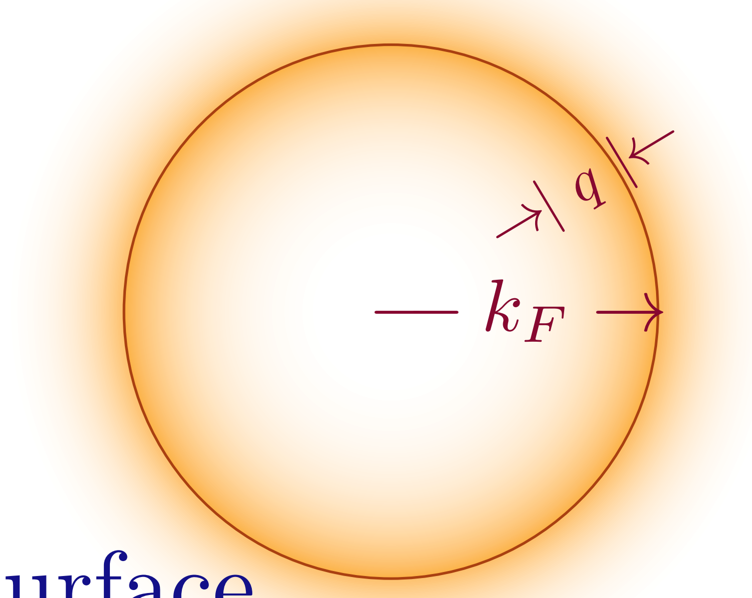
- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

NFL Nematic QCP



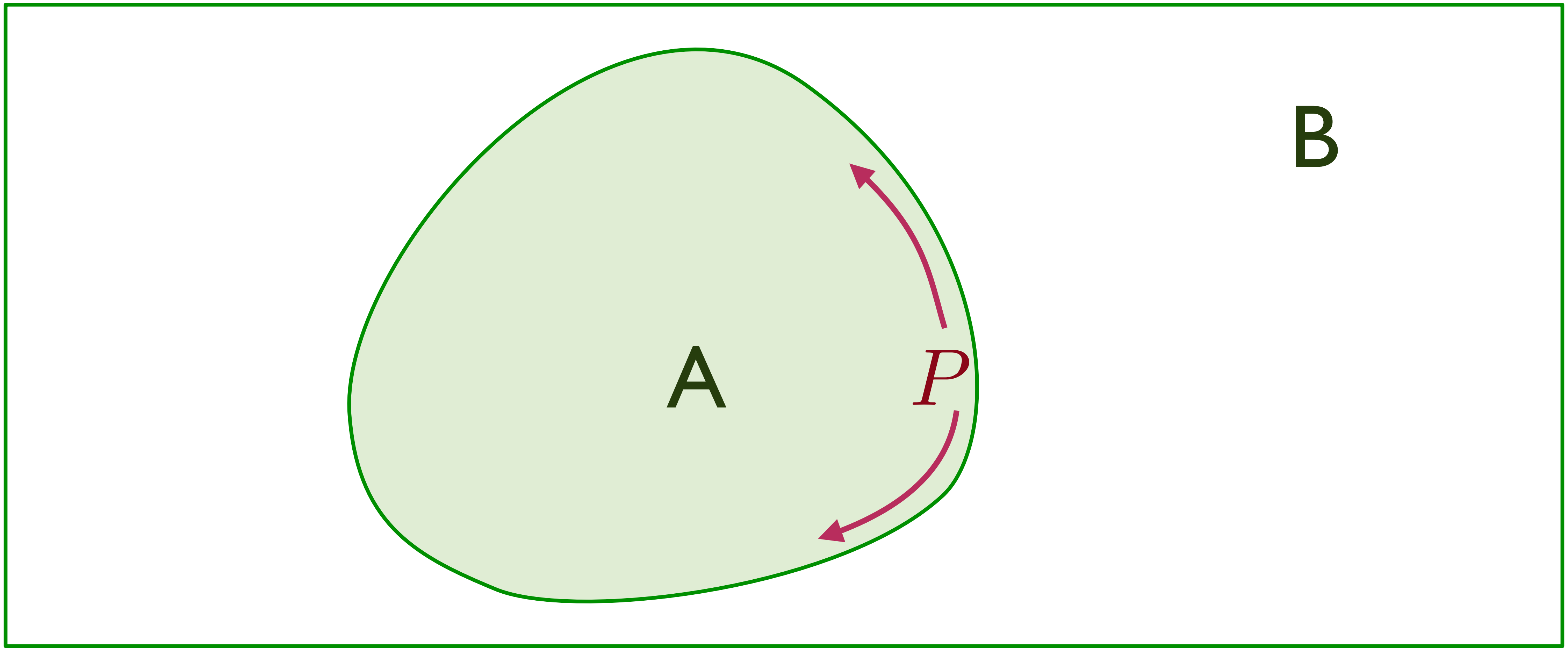
- Fermi surface with $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.
- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.





FL Fermi liquid	NFL Nematic QCP
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \|q\|^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	• $S_E \sim k_F^{d-1} P \ln P$.

Entanglement entropy of the non-Fermi liquid

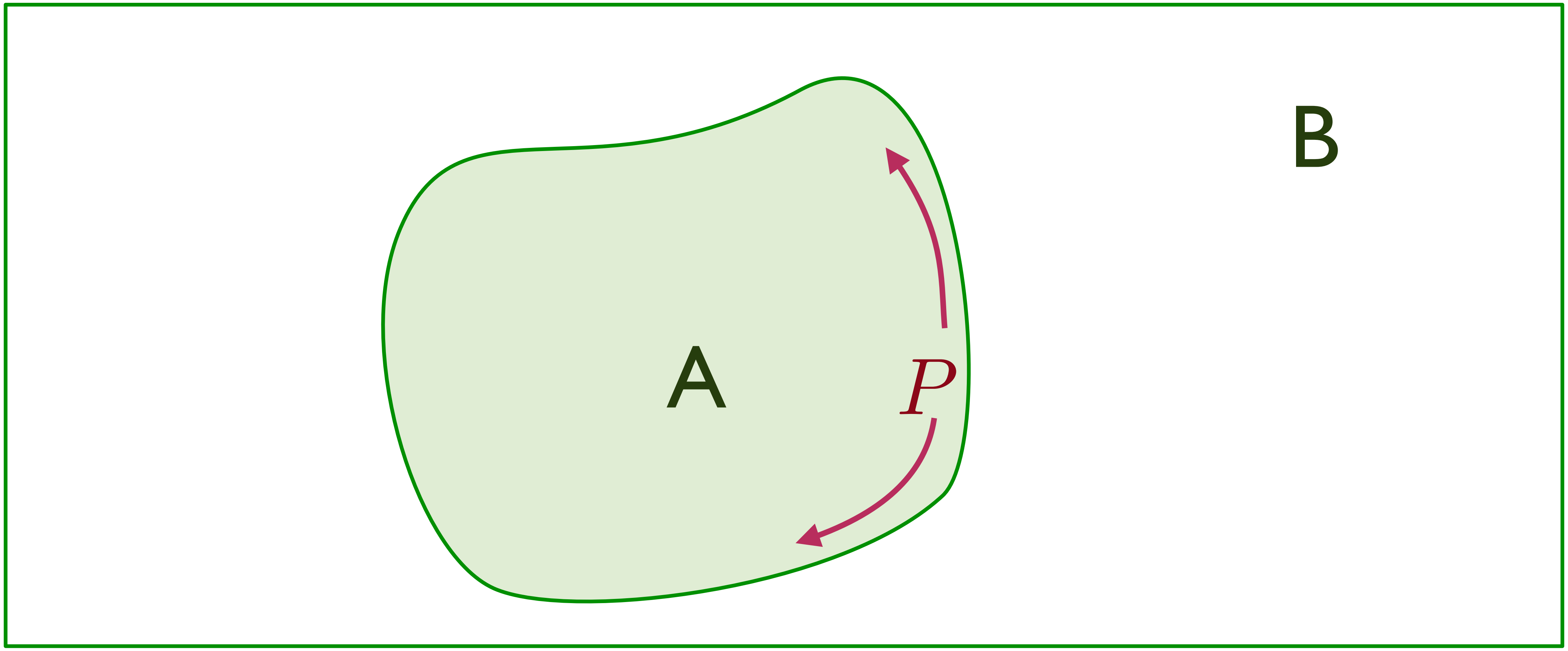


Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Entanglement entropy of the non-Fermi liquid

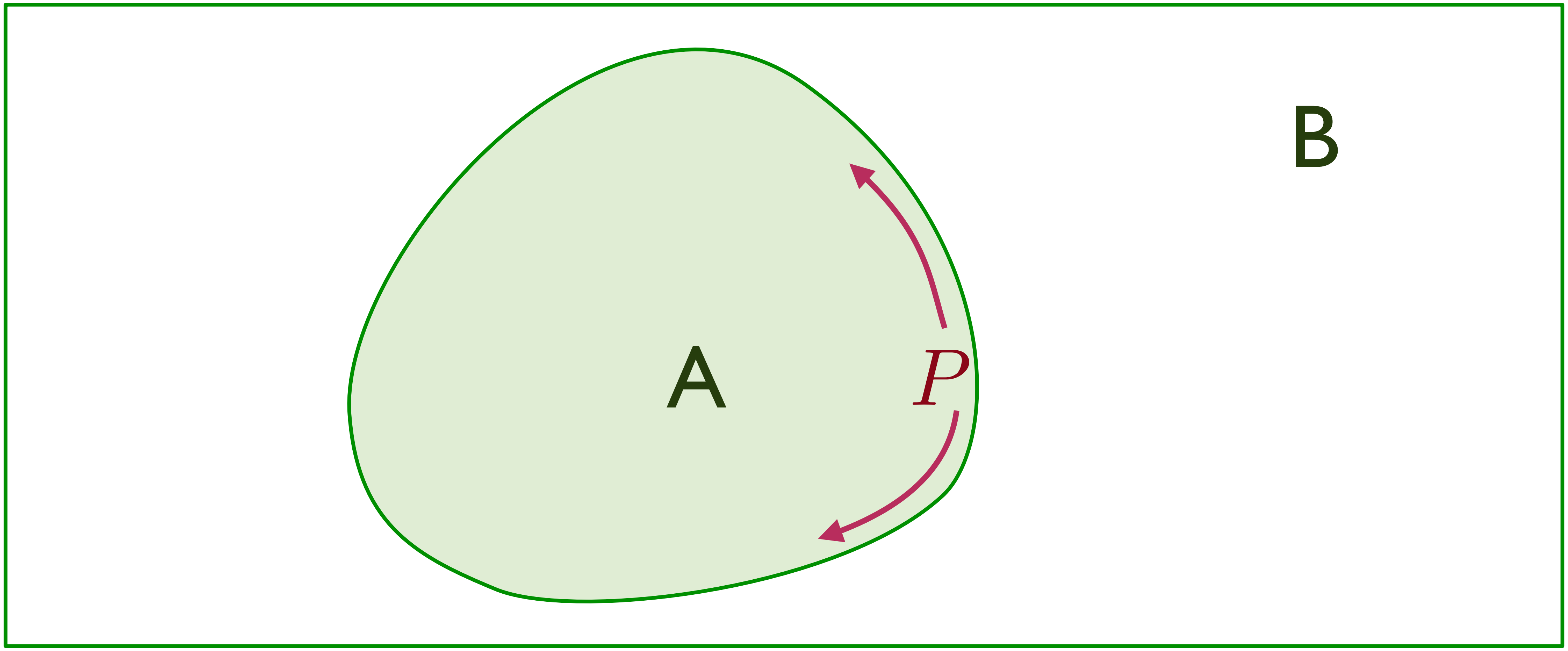


Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

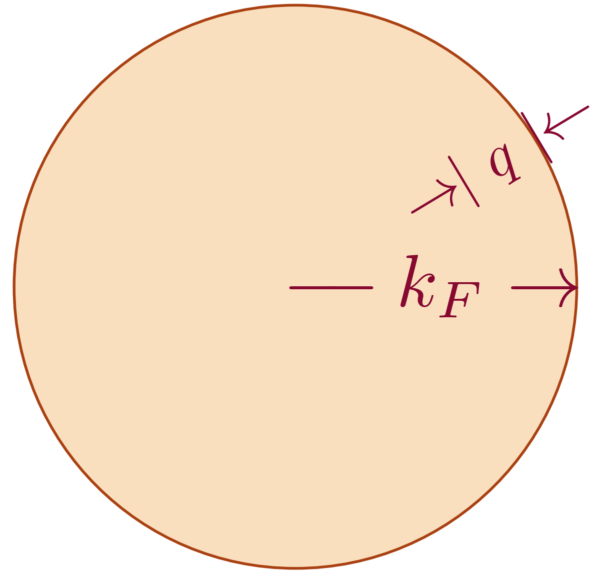
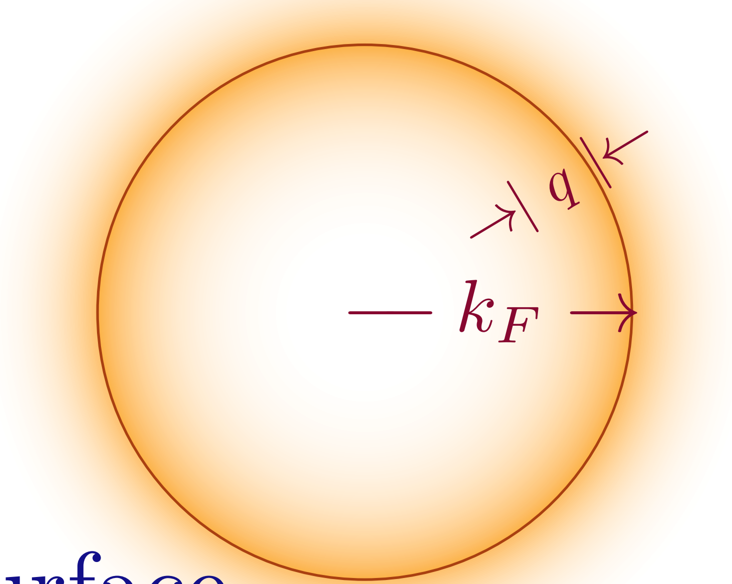
Entanglement entropy of the non-Fermi liquid



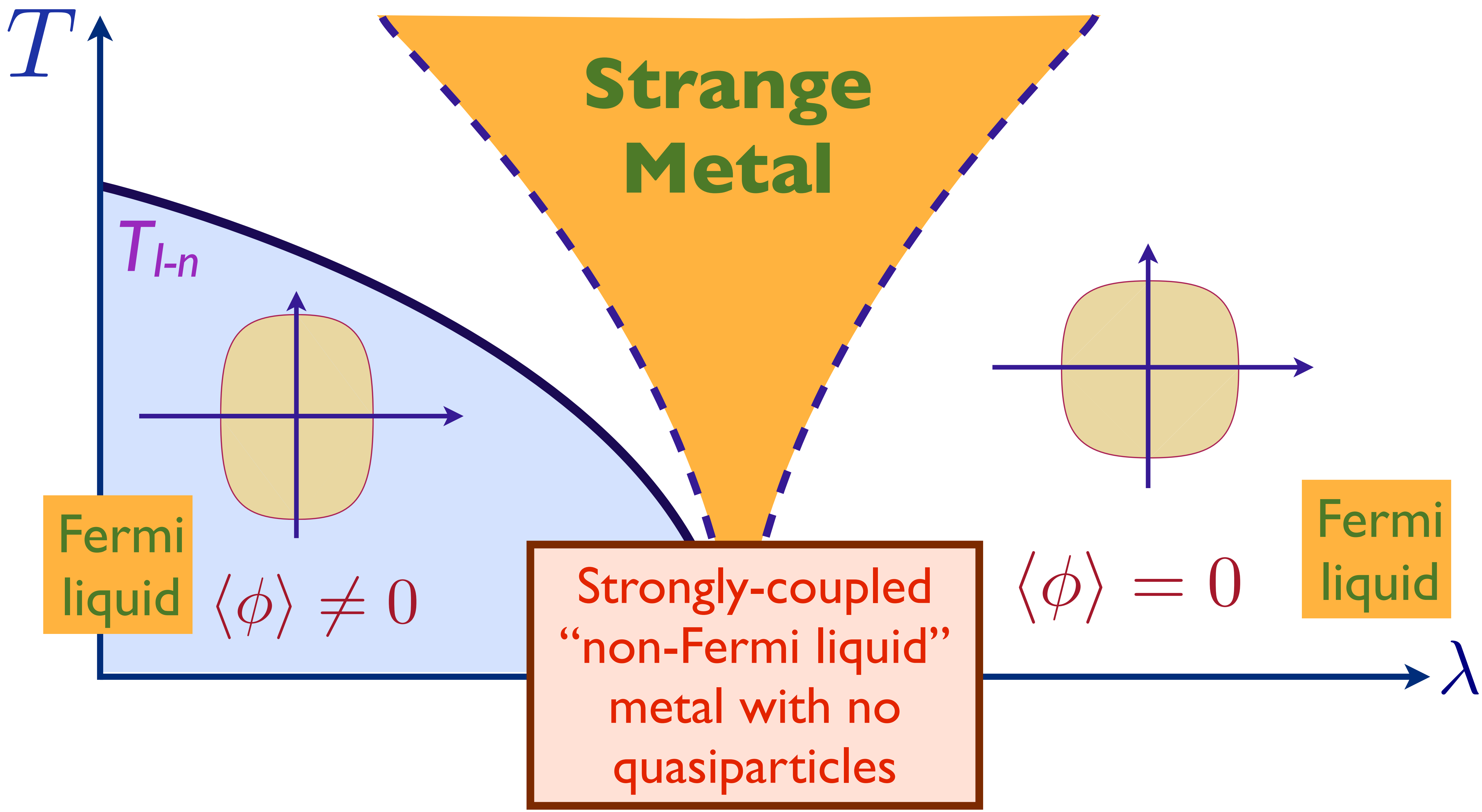
Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

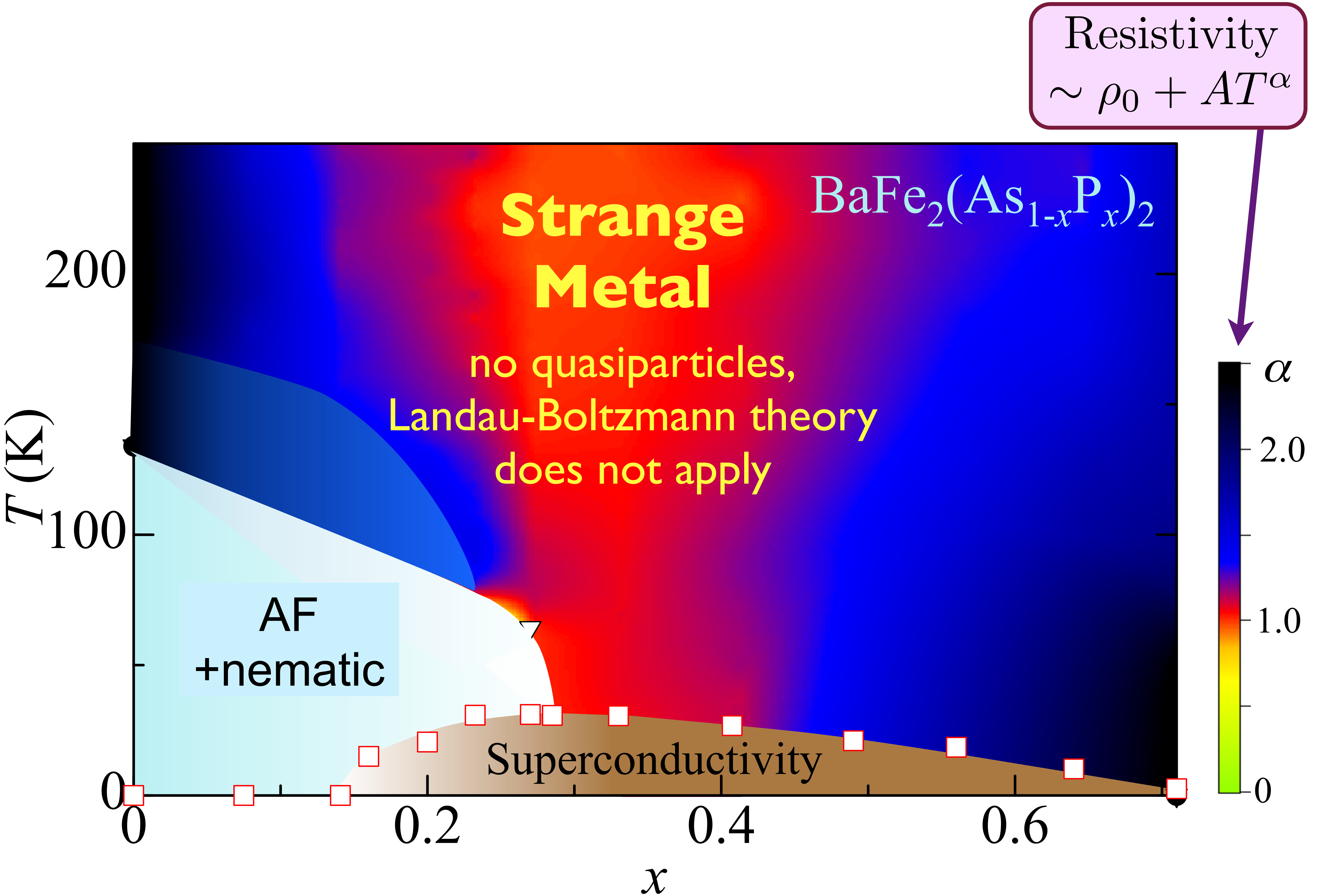
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

FL Fermi liquid	NFL Nematic QCP
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	• $S_E \sim k_F^{d-1} P \ln P$.

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)