Theory of Quantum Matter: from Quantum Fields to Strings

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Talk online: sachdev.physics.harvard.edu

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic quantum critical point

C. Holography, entanglement, and strange metals

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Basic characteristics of CFTs

Ordinary quantum field theories are characterized by their particle spectrum, and the S-matrices describing interactions between the particles. The analog of these concepts for CFTs are the *primary operators* $O_a(x)$ and their *operator product expansions* (OPEs). Each primary operator is associated with a scaling dimension Δ_a, defined by the ($T = 0$) expectation value (for the simplest case of scalar operators):

$$\langle O_a(x) O_b(0) \rangle = \frac{\delta_{ab}}{|x|^{2\Delta_a}}$$

Basic characteristics of CFTs

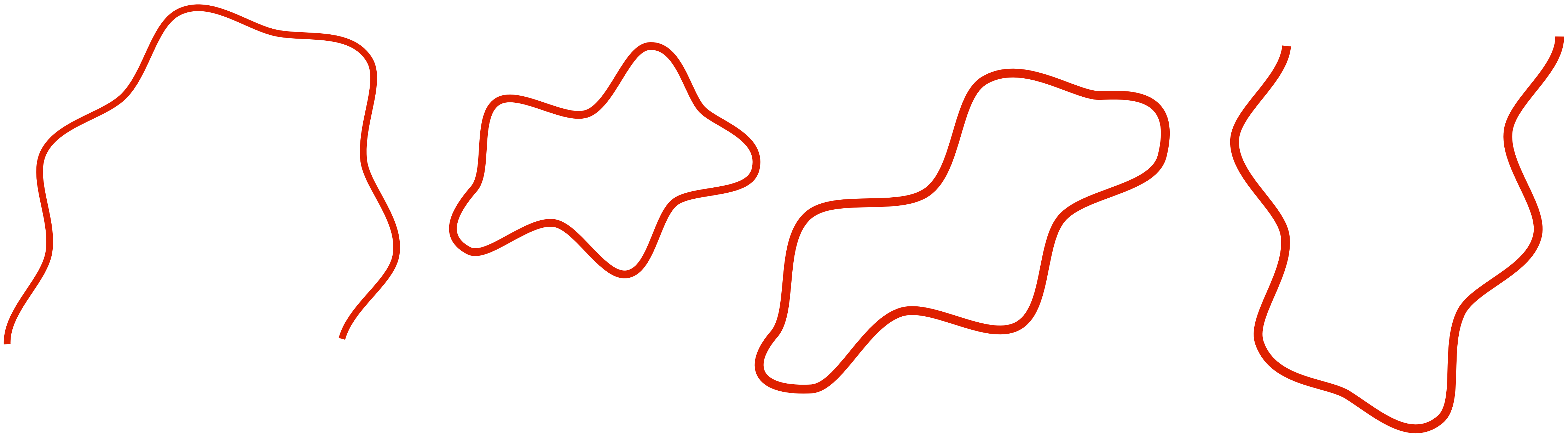
The OPE describes what happens when two operators come together at a single spacetime point (considering scalar operators only)

$$\lim_{x' \to x} \langle \phi_a(x') \phi_b(x) \phi_c(0) \rangle = \frac{f_{abc}}{|x|^{\Delta_a + \Delta_b + \Delta_c}}$$

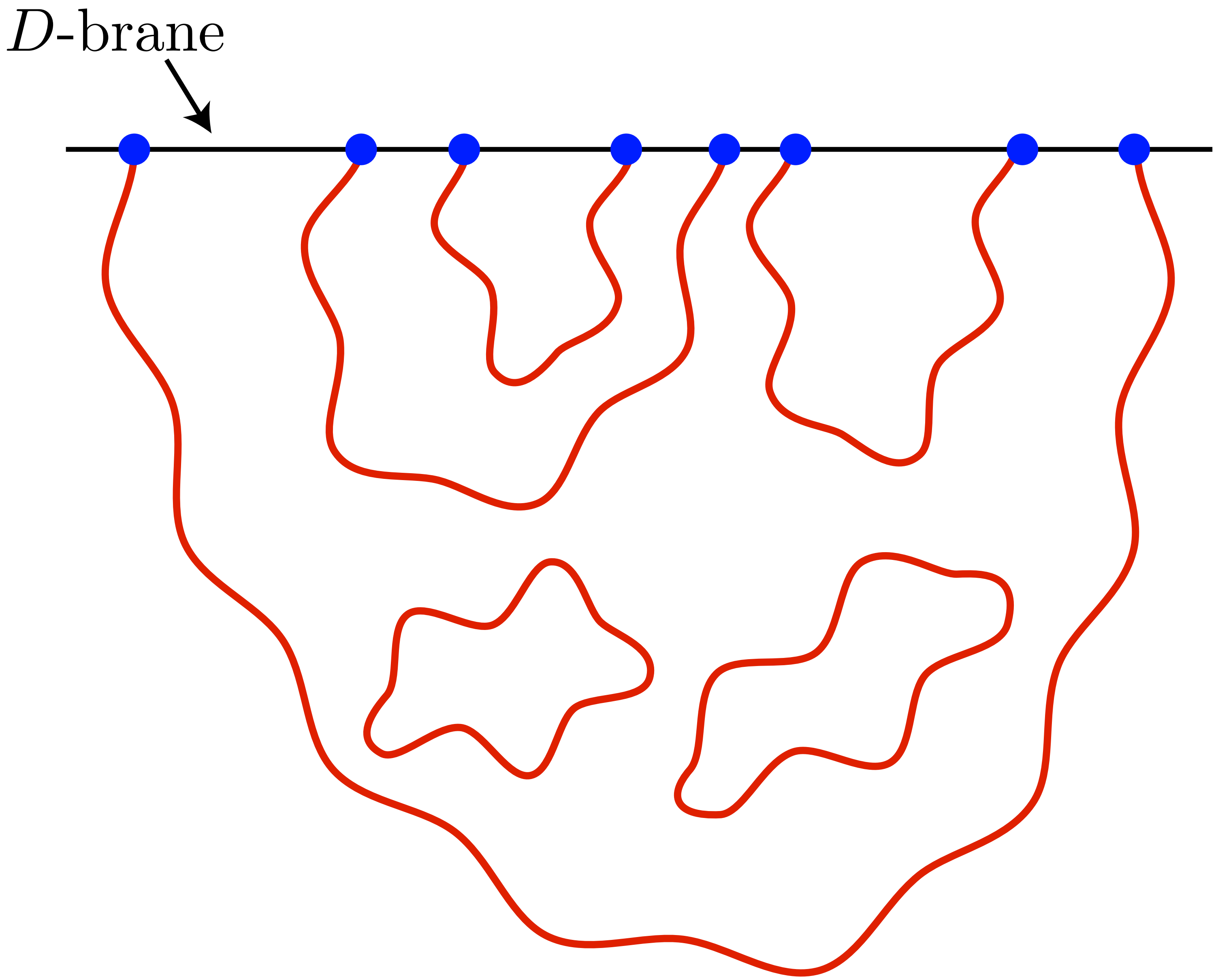
The values of $\{\Delta_a, f_{abc}\}$ determine (in principle) all observable properties of the CFT, as constrained by a complex set of conformal Ward identities.

For the Wilson-Fisher CFT3, systematic methods exist to compute (in principle) all the $\{\Delta_a, f_{abc}\}$, and we will assume this data is *known*. This knowledge will be taken as an *input* to the holographic analysis.

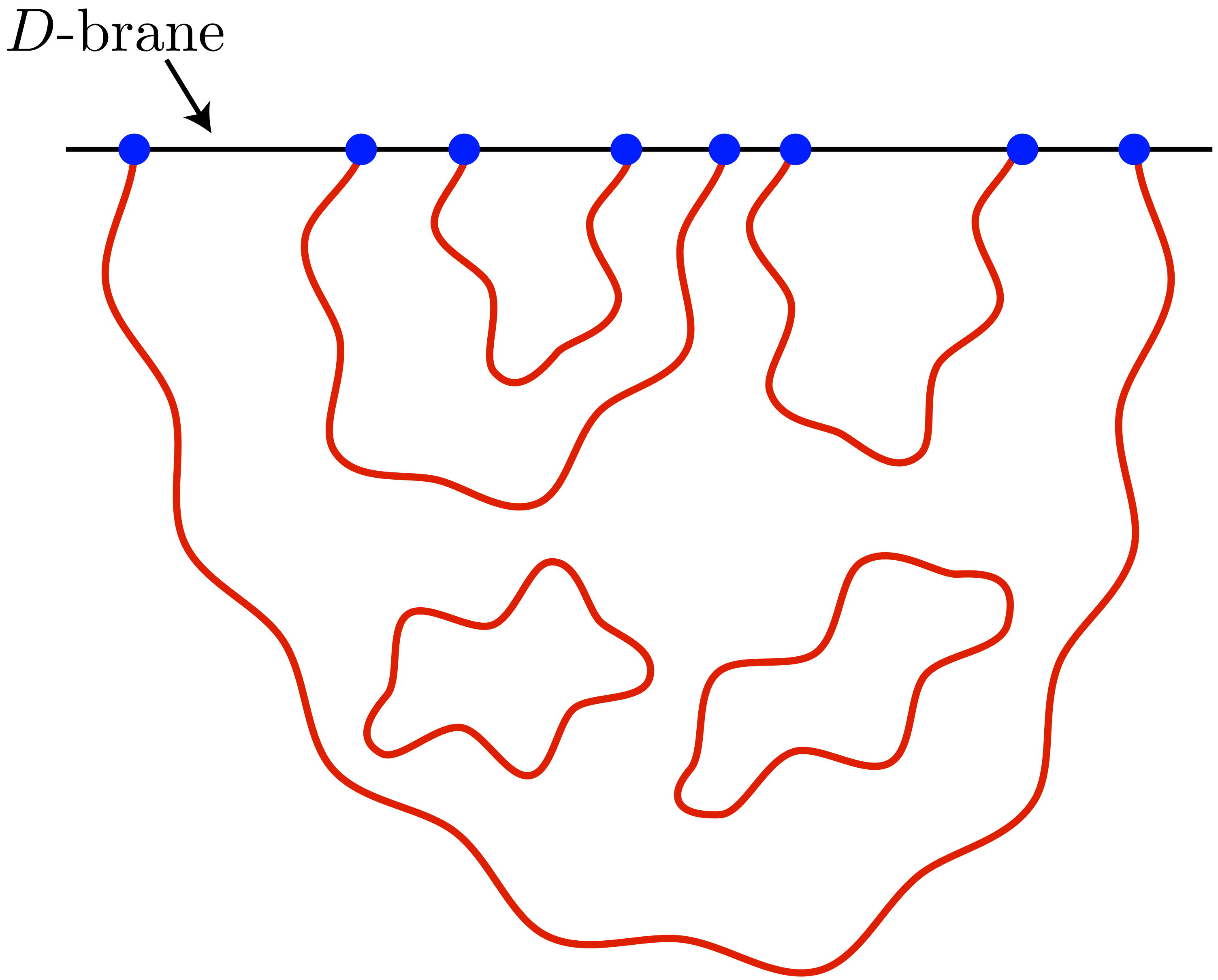
String theory



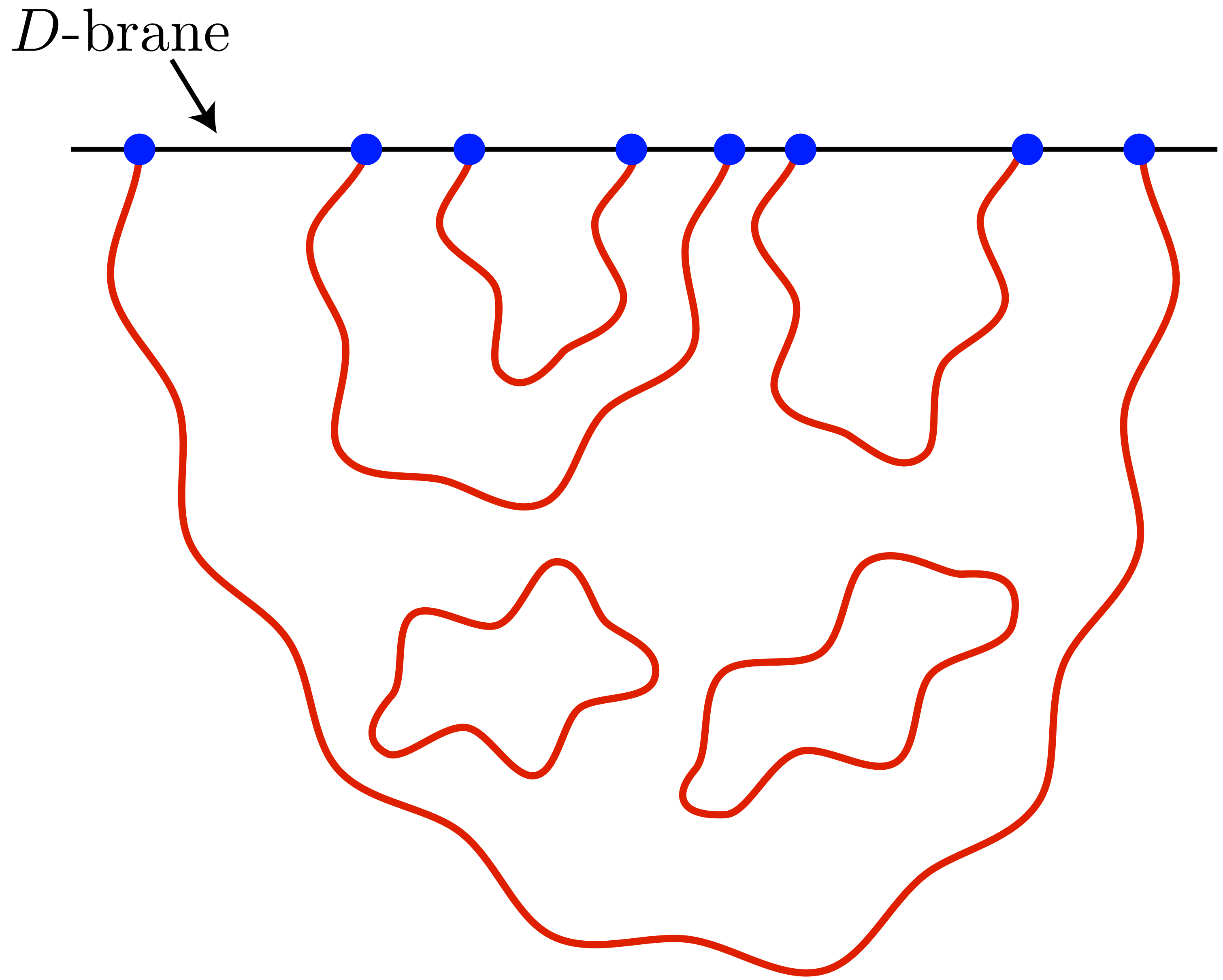
- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a d-dimensional surface on which strings can end.



- A D-brane is a d-dimensional surface on which strings can end.
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.

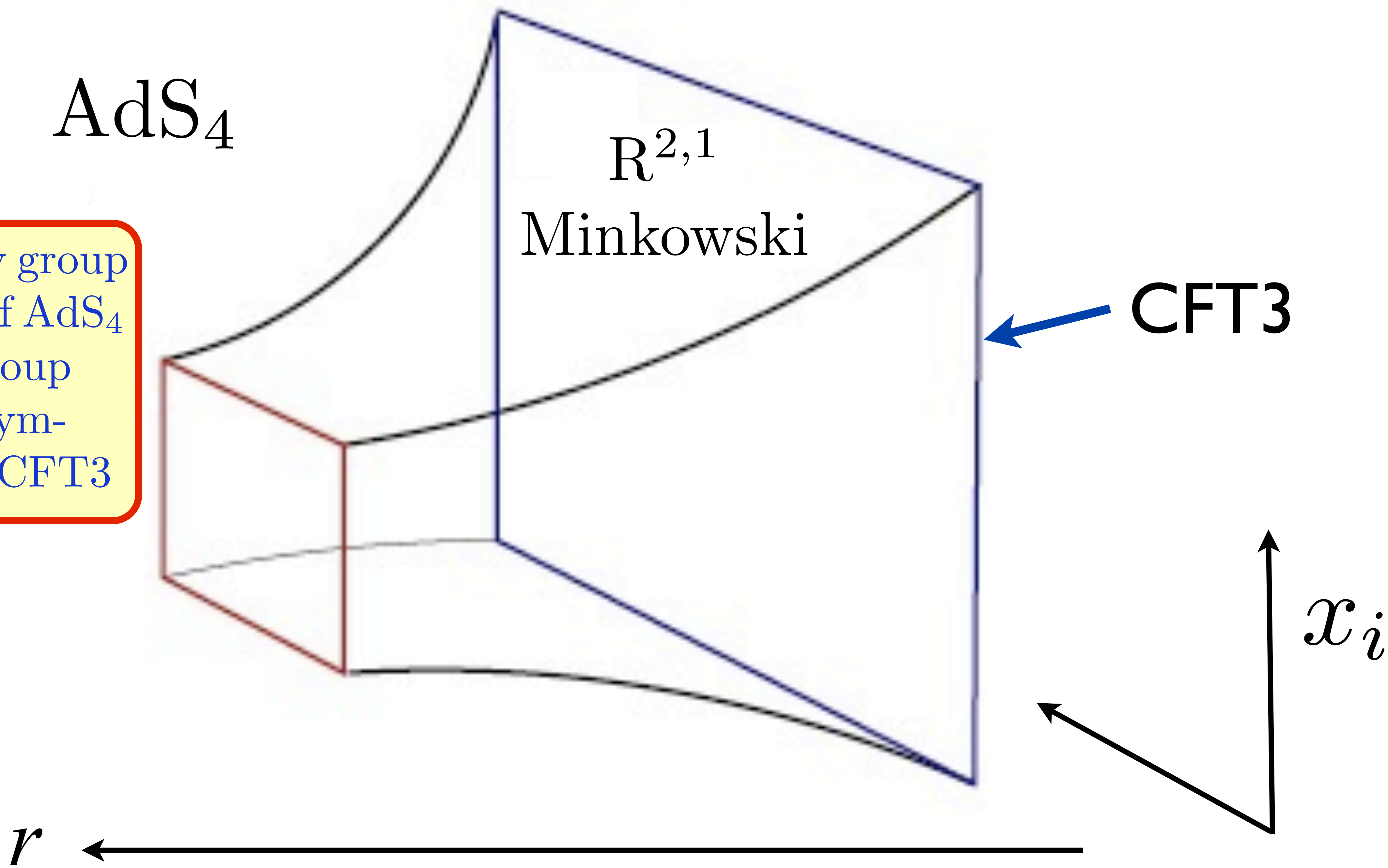


- A *D*-brane is a *d*-dimensional surface on which strings can end.

- The low-energy theory on a *D*-brane has no gravity, similar to theories of entangled electrons of interest to us.

- In $d = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

AdS/CFT correspondence at zero temperature

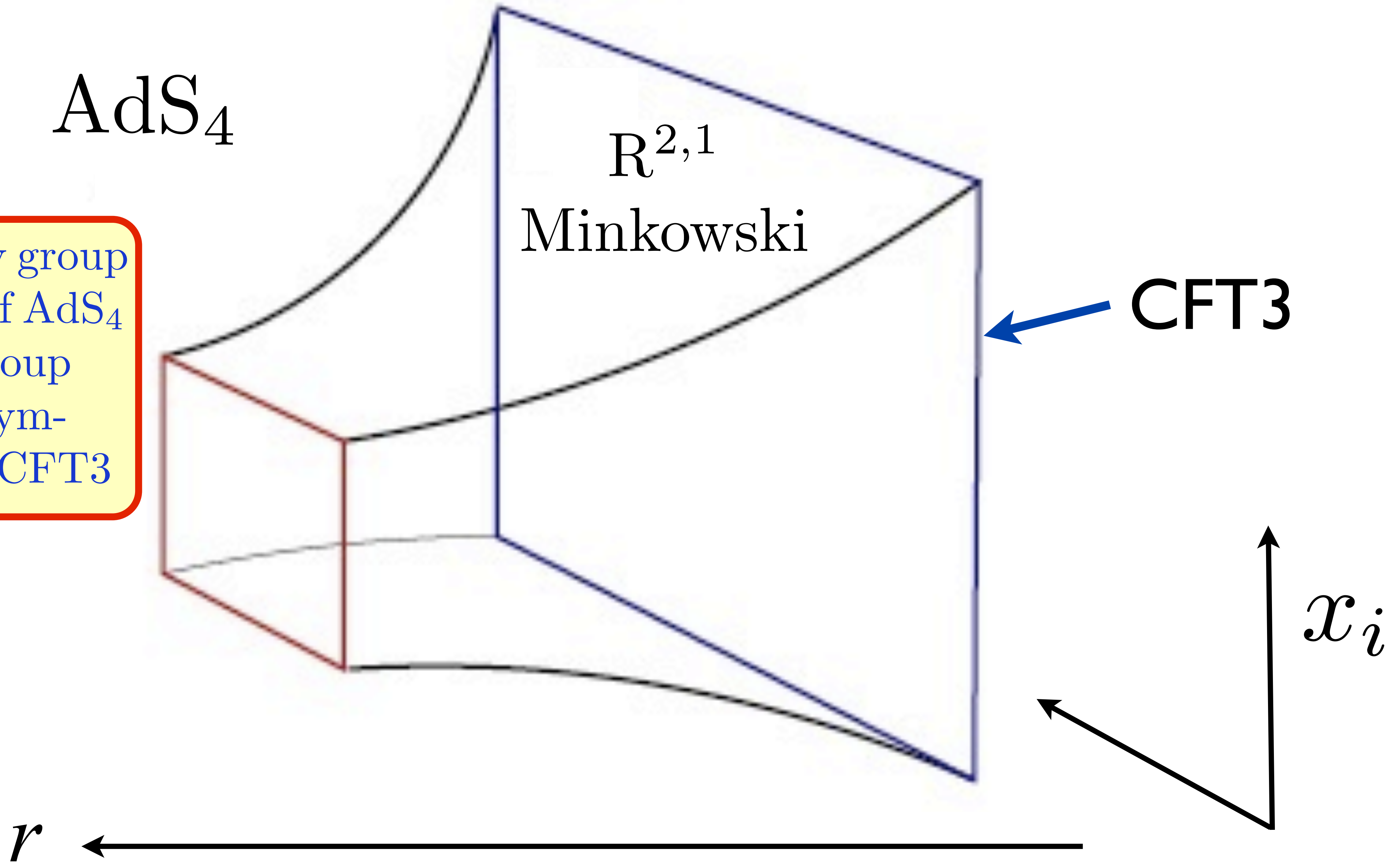


The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3

This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence at zero temperature



The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + dx^2 + dy^2\right]$$

AdS/CFT correspondence at zero temperature

Consider a CFT in D space-time dimensions with primary operators $O_a(\boldsymbol{x})$ with scaling dimension Δ_a. This is presumed to be equivalent to a dual gravity theory on AdS$_{D+1}$ with action $\mathcal{S}_{\text{bulk}}$. The bulk theory has fields $\phi_a(\boldsymbol{x}, r)$ corresponding to each primary operator. The CFT and the bulk theory are related by the GKPW ansatz

$$\int \mathcal{D}\phi_a \exp\left(-\mathcal{S}_{\text{bulk}}\right)\Big|_{\text{bdy}} = \left\langle \exp\left(\int d^D x \, \phi_{a0}(\boldsymbol{x}) O_a(\boldsymbol{x})\right)\right\rangle_{\text{CFT}}$$

where the boundary condition is

$$\lim_{r\to 0} \phi_a(\boldsymbol{x}, r) = r^{D-\Delta} \phi_{a0}(\boldsymbol{x}).$$

AdS/CFT correspondence at zero temperature

For every primary operator $O(\boldsymbol{x})$ in the CFT, there is a corresponding field $\phi(\boldsymbol{x}, r)$ in the bulk (gravitational) theory. For a scalar operator $O(\boldsymbol{x})$ of dimension Δ, the correlators of the boundary and bulk theories are related by

$$\langle O(\boldsymbol{x}_1) \ldots O(\boldsymbol{x}_n) \rangle_{\text{CFT}} = Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(\boldsymbol{x}_1, r_1) \ldots \phi(\boldsymbol{x}_n, r_n) \rangle_{\text{bulk}}$$

where the "wave function renormalization" factor $Z = (2\Delta - D)$.

AdS/CFT correspondence at zero temperature

For a U(1) conserved current J_μ of the CFT, the corresponding bulk operator is a U(1) *gauge* field A_μ. With a Maxwell action for the gauge field

$$\mathcal{S}_M = \frac{1}{4g_M^2}\int d^{D+1}x\,\sqrt{g}F_{ab}F^{ab}$$

we have the bulk-boundary correspondence

$$\langle J_\mu(\boldsymbol{x}_1)\ldots J_\nu(\boldsymbol{x}_n)\rangle_{\rm CFT} = \\ (Zg_M^{-2})^n \lim_{r\to 0} r_1^{2-D}\ldots r_n^{2-D}\,\langle A_\mu(\boldsymbol{x}_1,r_1)\ldots A_\nu(\boldsymbol{x}_n,r_n)\rangle_{\rm bulk}$$

with $Z = D-2$.

AdS/CFT correspondence at zero temperature

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\boldsymbol{x}_1)\ldots T_{\rho\sigma}(\boldsymbol{x}_n)\rangle_{\text{CFT}} = \left(\frac{ZL^2}{\kappa^2}\right)^n \lim_{r\to 0} r_1^{-D}\ldots r_n^{-D}\,\langle\chi_{\mu\nu}(\boldsymbol{x}_1,r_1)\ldots\chi_{\rho\sigma}(\boldsymbol{x}_n,r_n)\rangle_{\text{bulk}}\,,$$

with $Z = D$.

AdS/CFT correspondence at zero temperature

So the minimal bulk theory for a CFT with a conserved U(1) current is the *Einstein-Maxwell* theory with a cosmological constant

$$\mathcal{S} = \frac{1}{4g_M^2}\int d^4x\,\sqrt{g}F_{ab}F^{ab} + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2, which are related to the conductivity $\sigma(\omega) = \mathcal{K}$ and the central charge of the CFT.

AdS/CFT correspondence at zero temperature

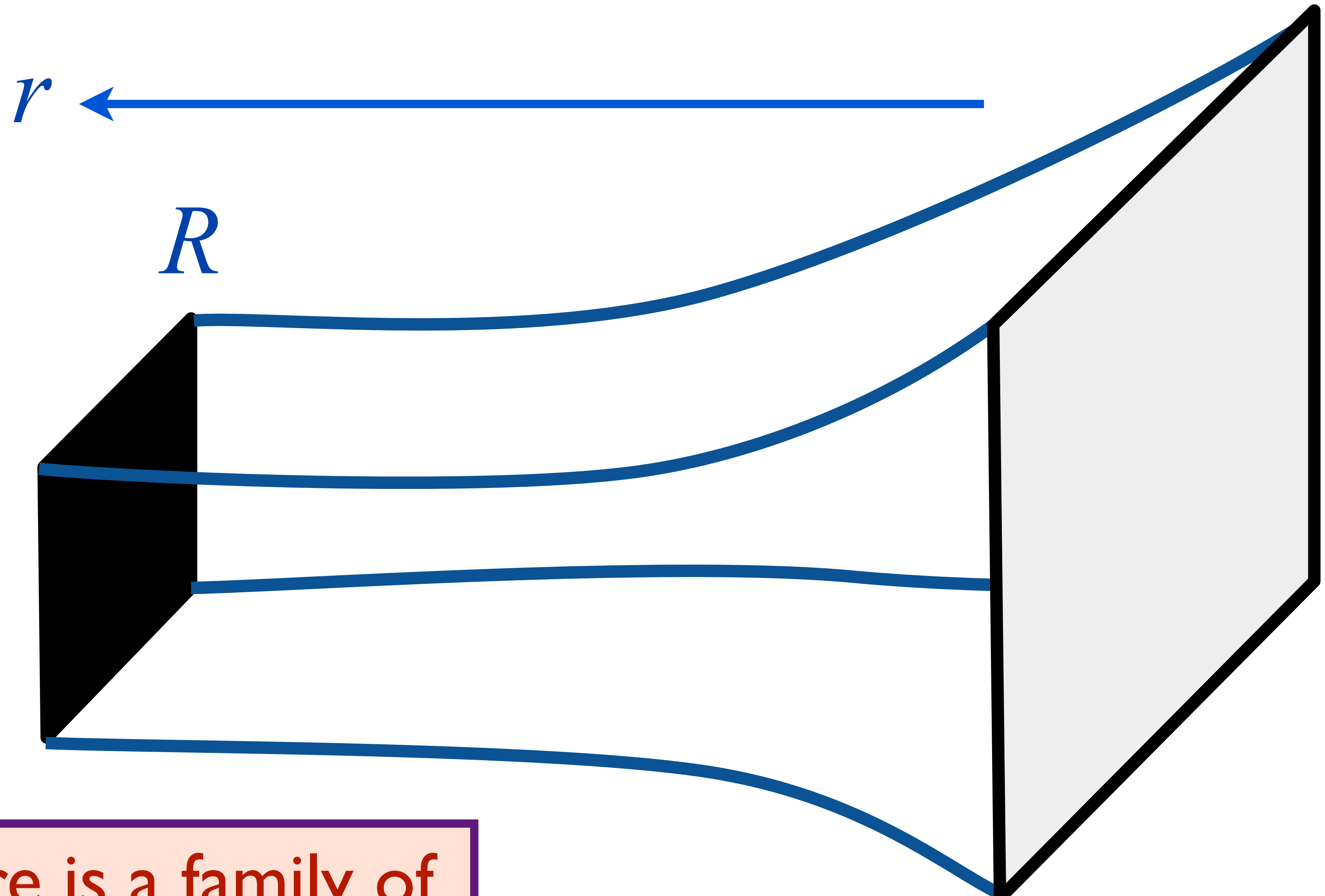
This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[- \frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

where C_{abcd} is the Weyl tensor. The parameter γ can be related to 3-point correlators of J_μ and $T_{\mu\nu}$. Both boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* **87**, 085138 (2013).

AdS_4-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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AdS/CFT correspondence at non-zero temperatures

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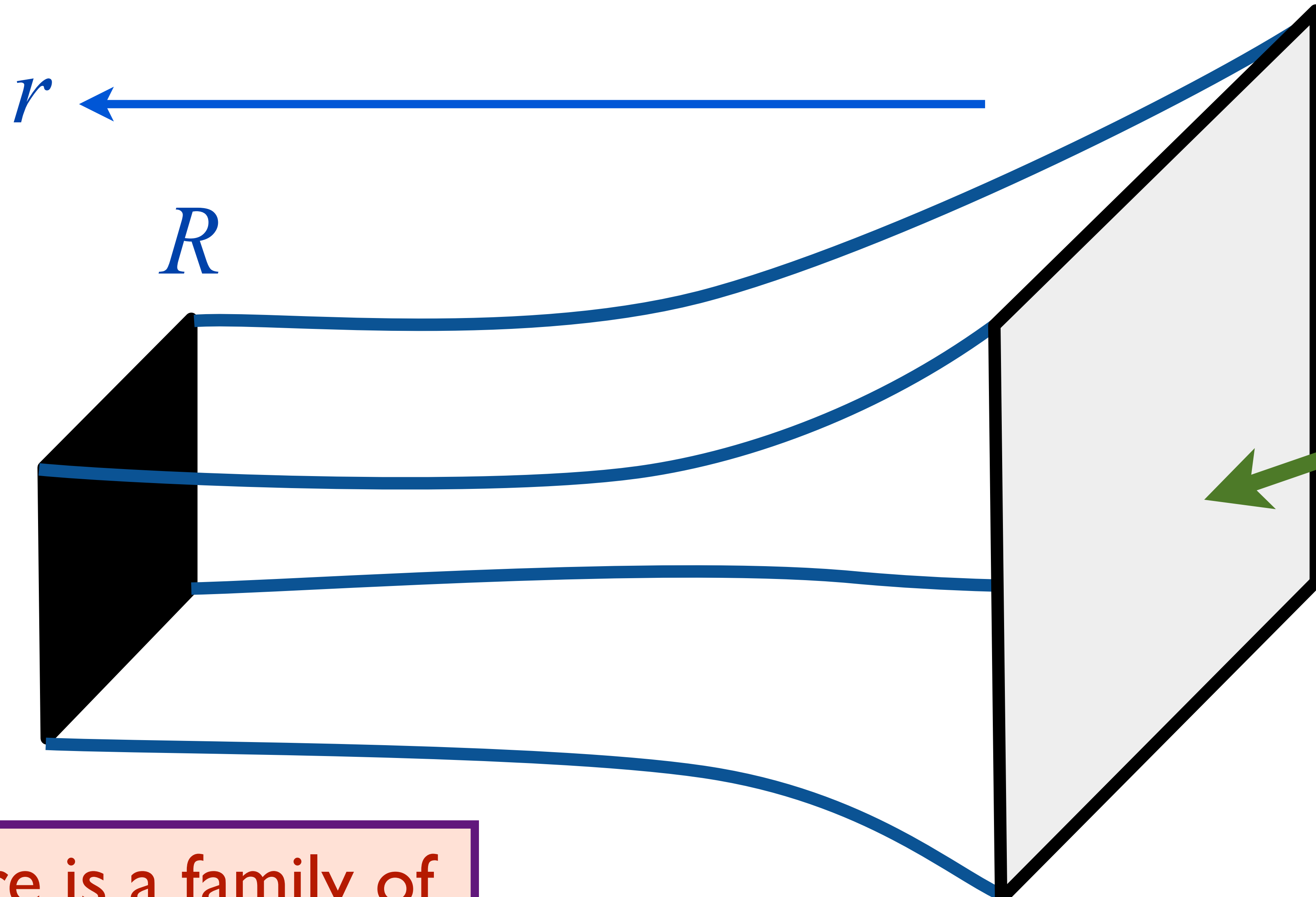

A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R}.$$

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$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS/CFT correspondence at non-zero temperatures

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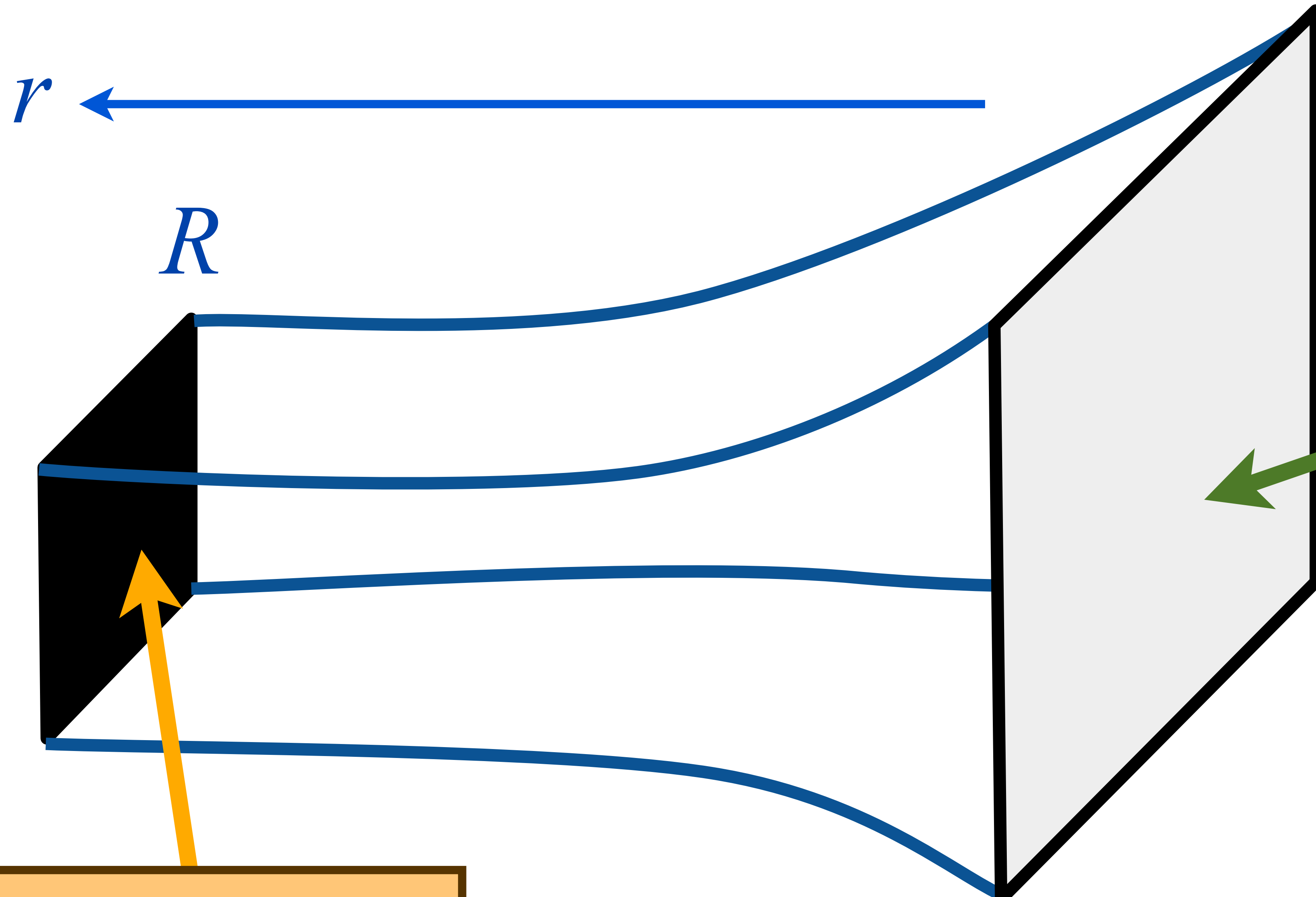

A 2+1 dimensional system at its quantum critical point: $k_B T = \dfrac{3\hbar}{4\pi R}$.

Black-brane at temperature of 2+1 dimensional quantum critical system

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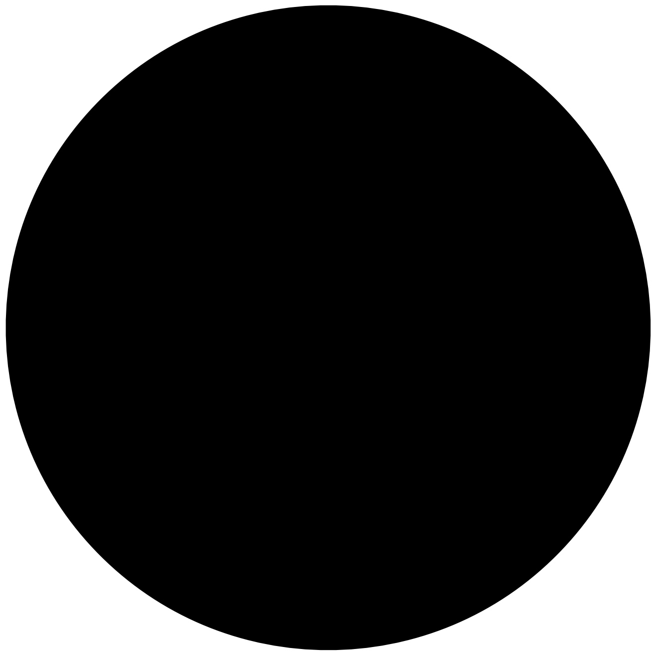
Black Holes

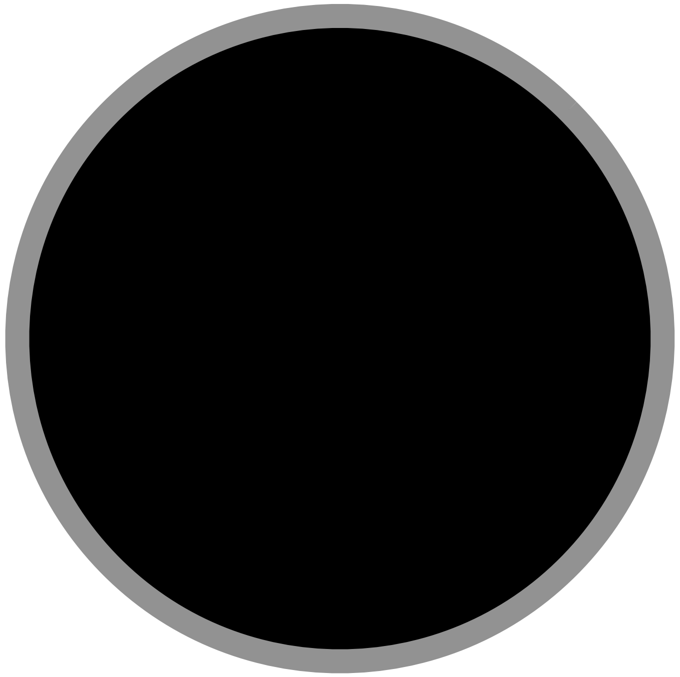
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.

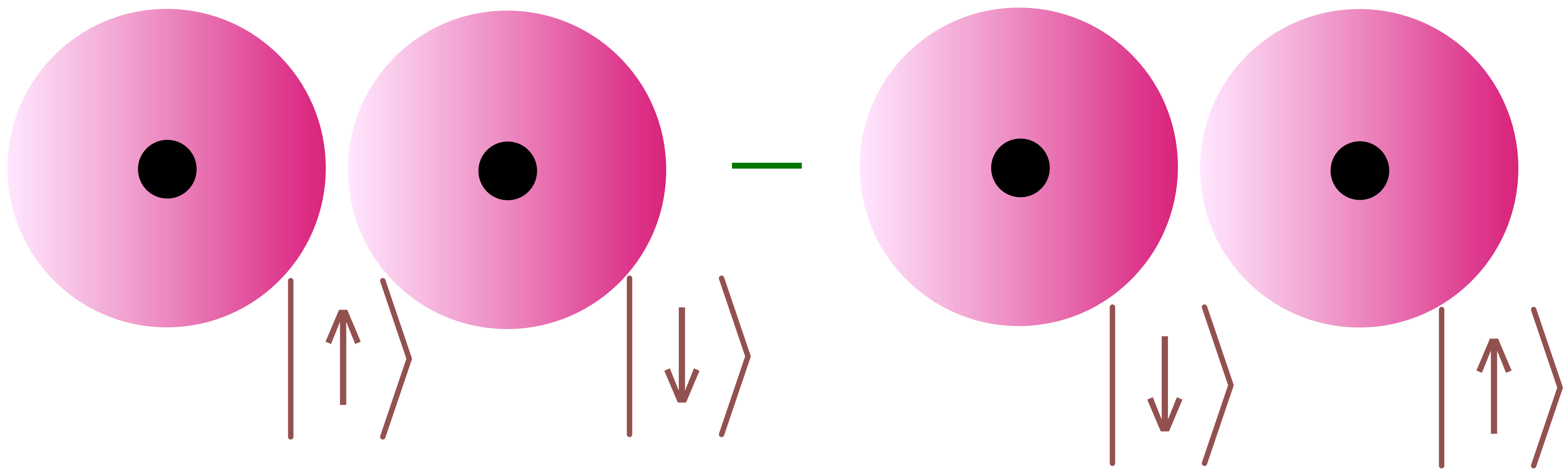


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



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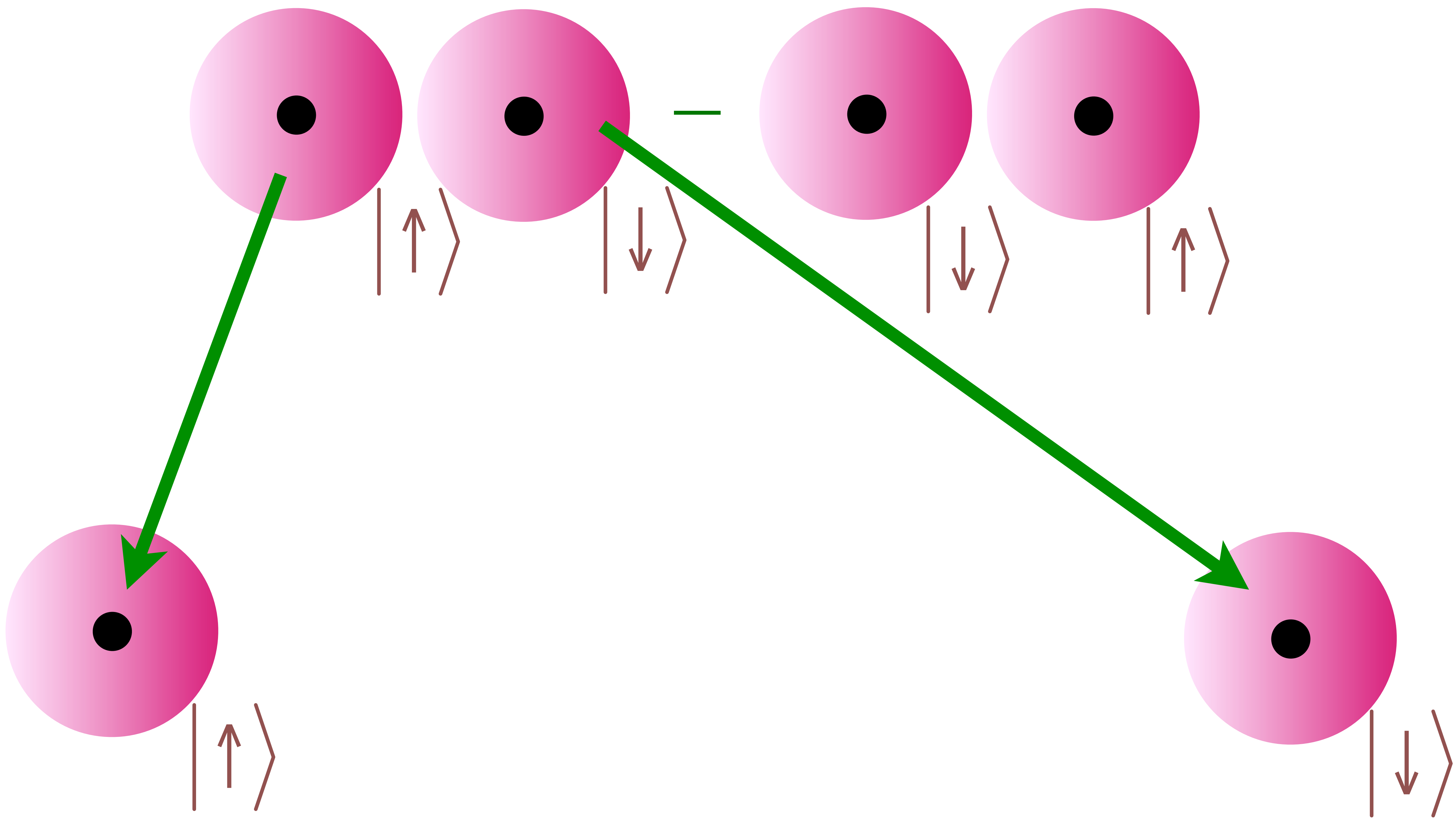

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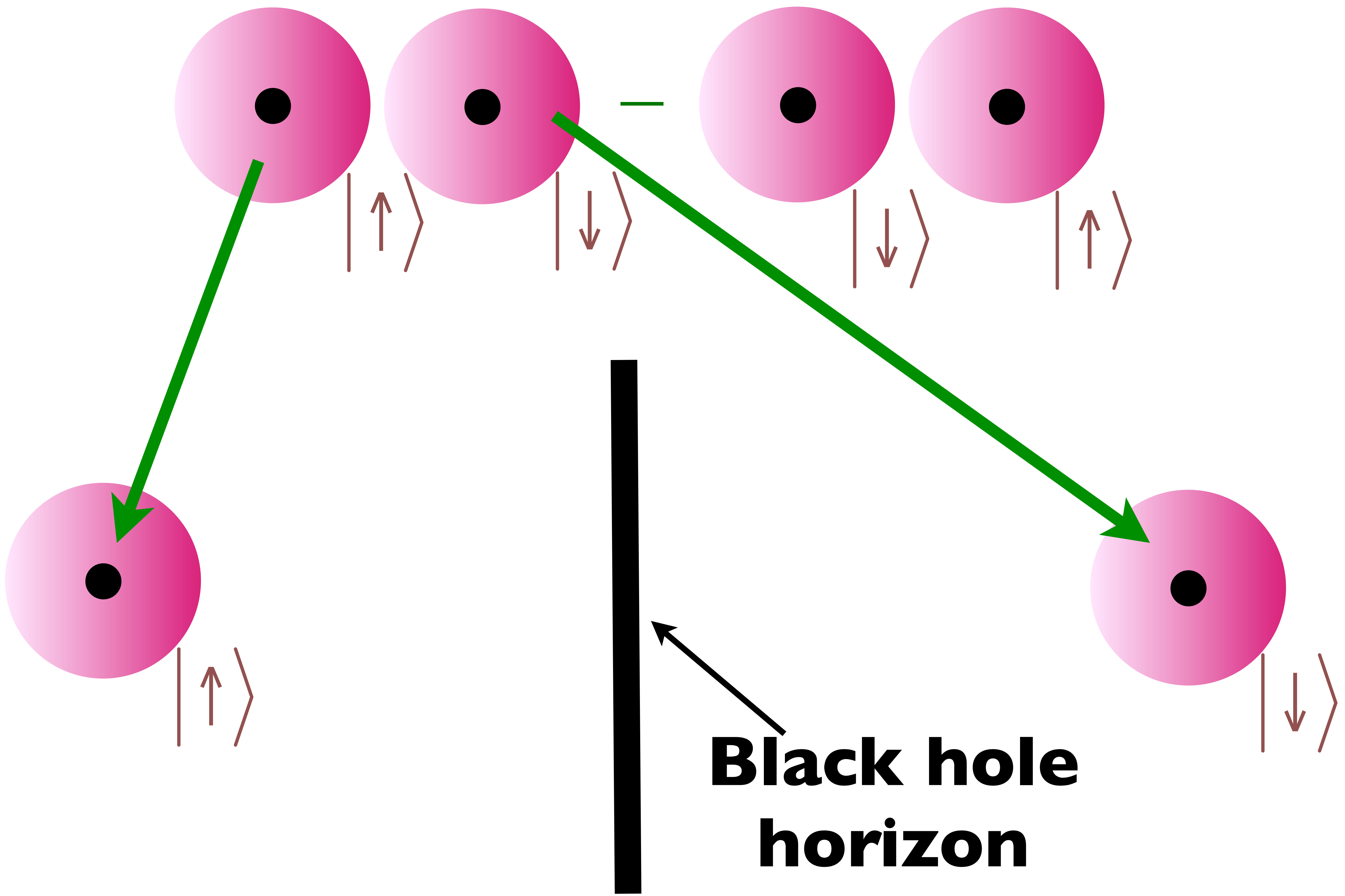

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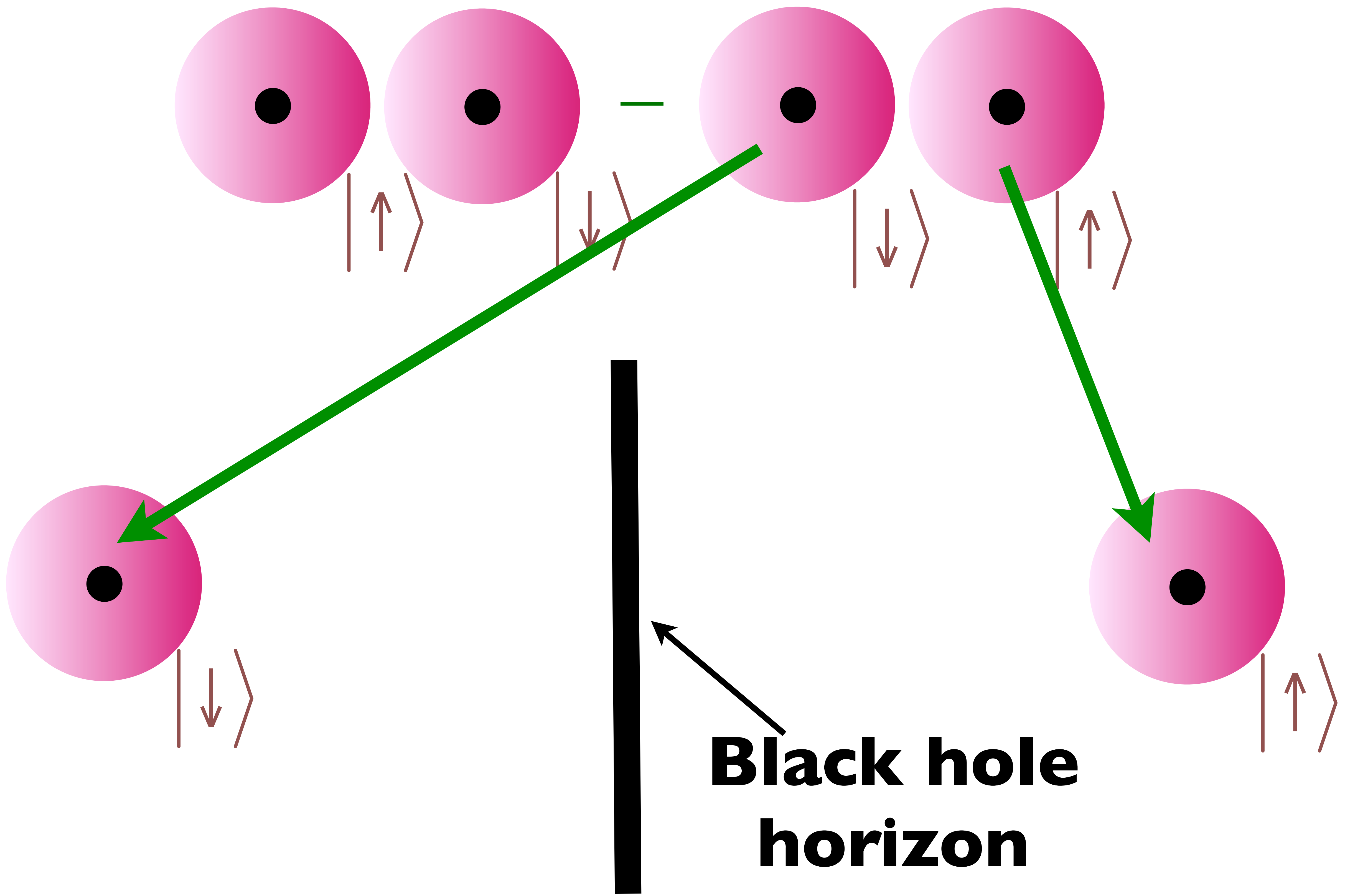
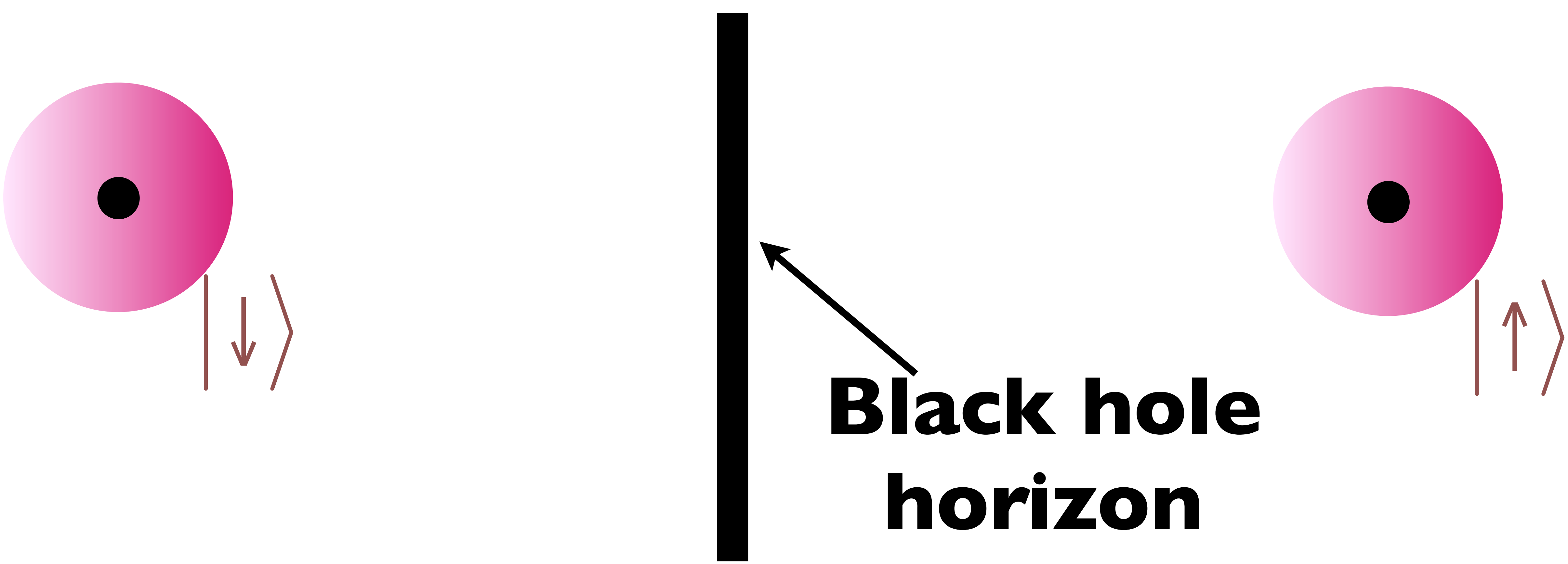


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



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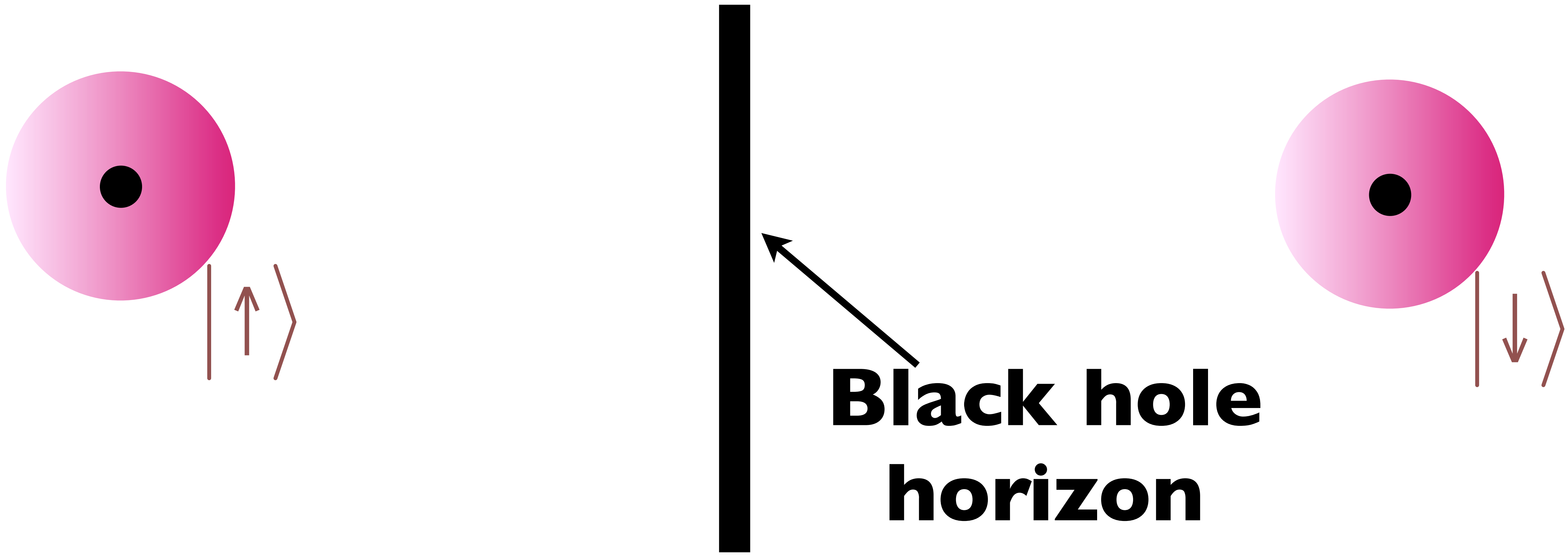


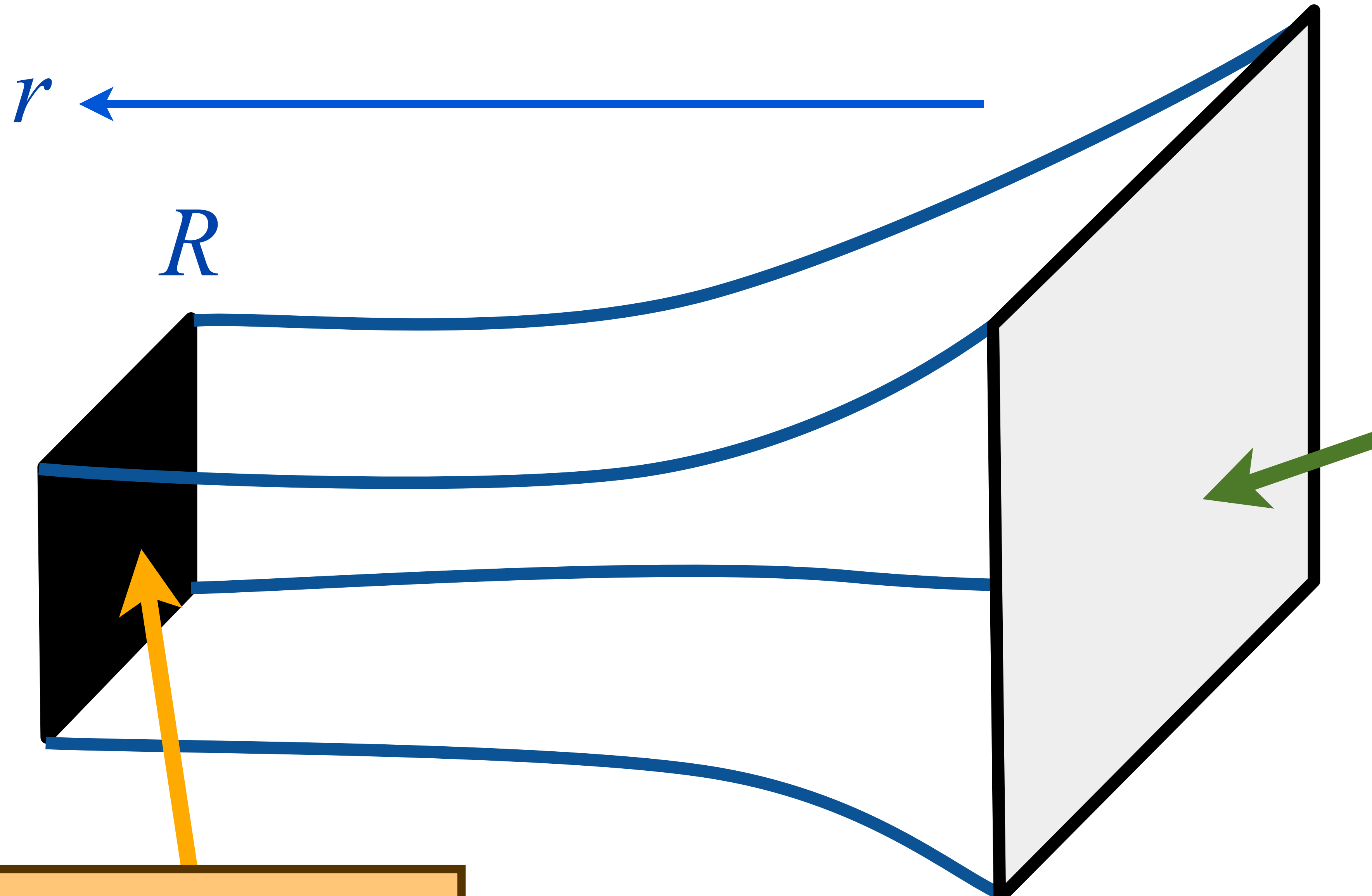
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point:

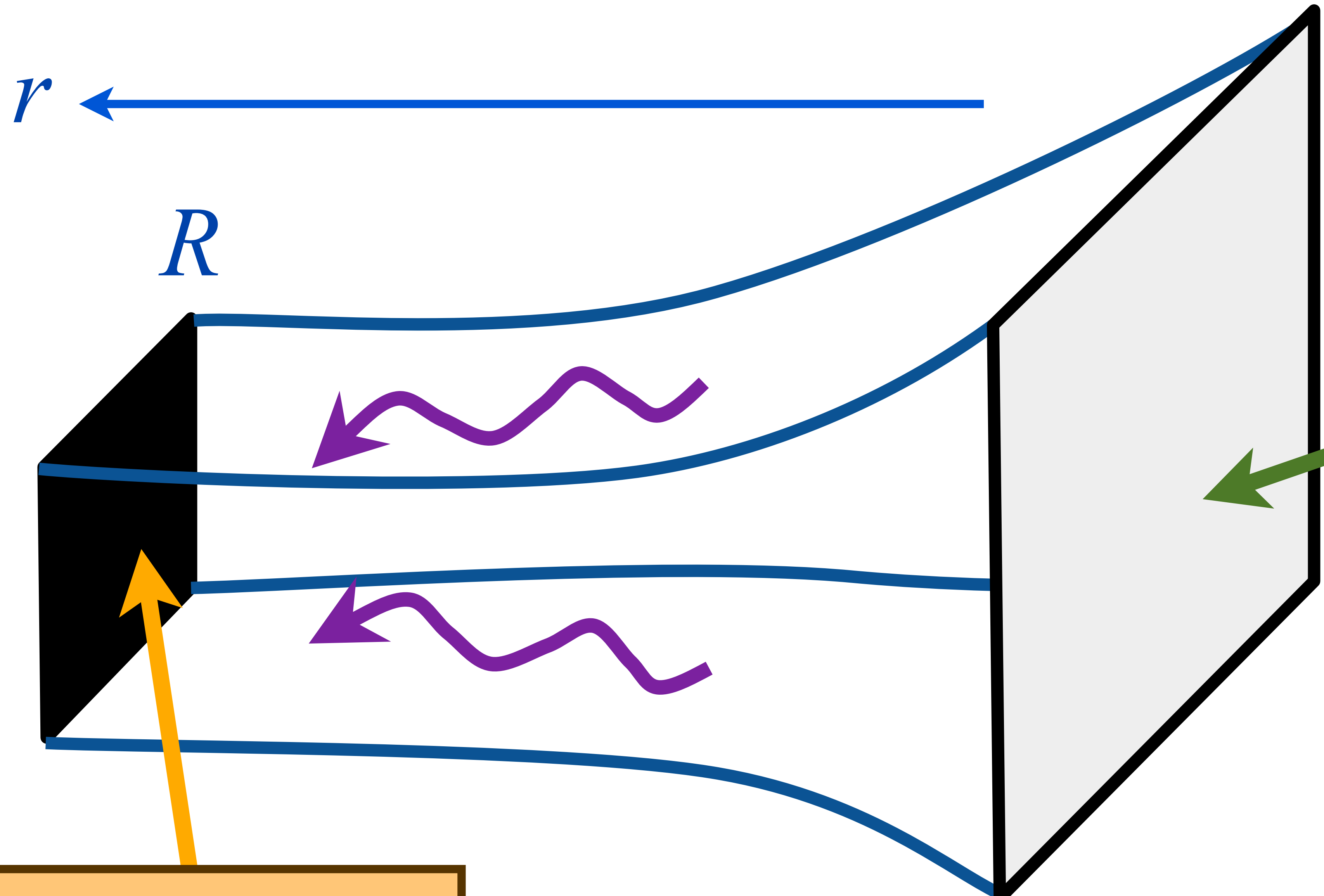
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

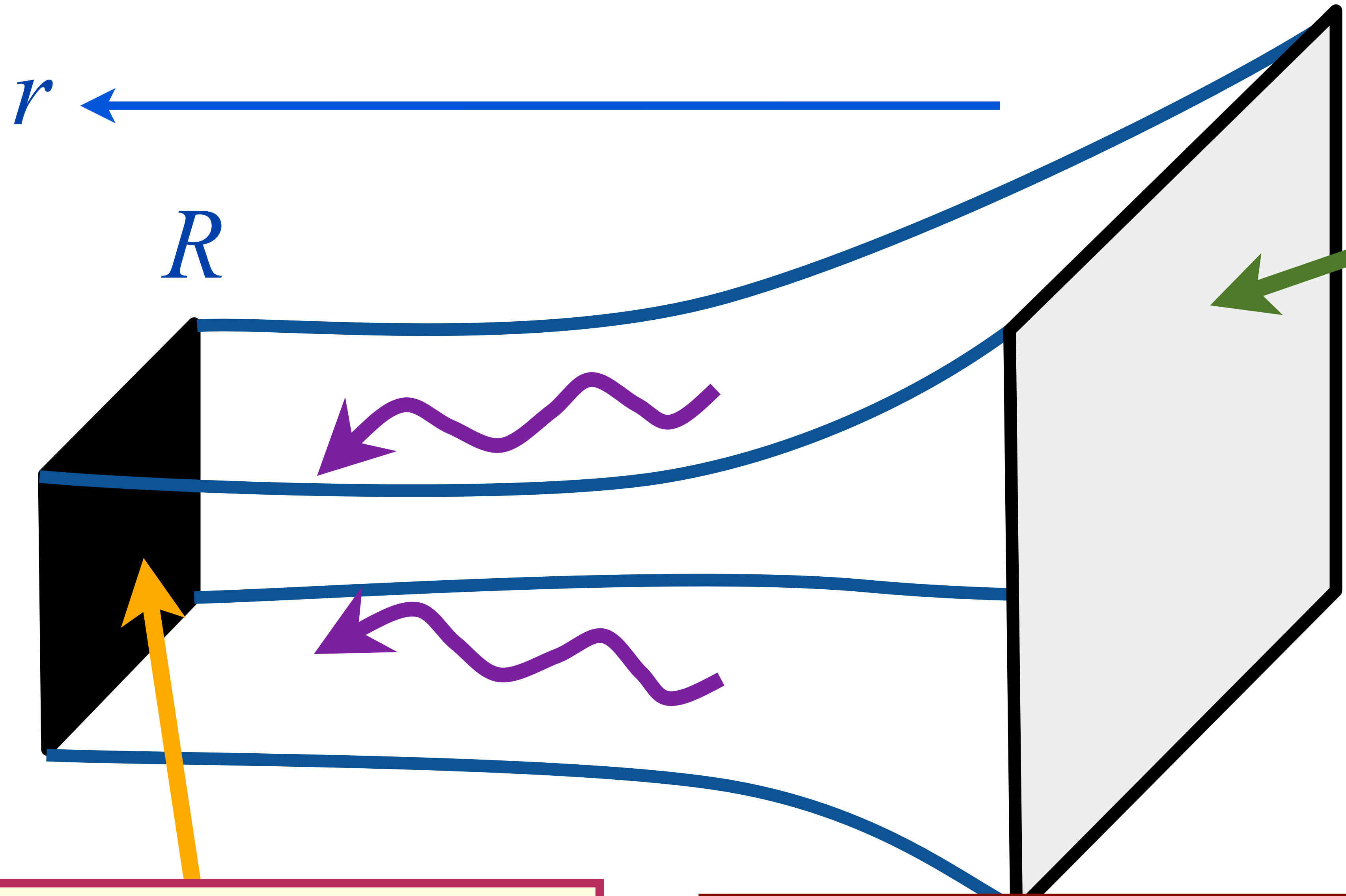


A 2+1 dimensional system at its quantum critical point:
$k_B T = \frac{3\hbar}{4\pi R}$.

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

AdS/CFT correspondence at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Characteristic damping time of quasi-normal modes: $(k_B/\hbar)\times$ Hawking temperature

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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Holography and black-branes

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Traditional CMT	Holography and black-branes
Identify quasiparticles and their dispersions	Start with strongly interacting CFT without particle- or wave-like excitations
Compute scattering matrix elements of quasiparticles (or of collective modes)	Compute OPE co-efficients of operators of the CFT
These parameters are input into a quantum Boltzmann equation	Relate OPE co-efficients to couplings of an effective graviational theory on AdS
Deduce dissipative and dynamic properties at non-zero temperatures	Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

AdS$_4$ theory of quantum criticality

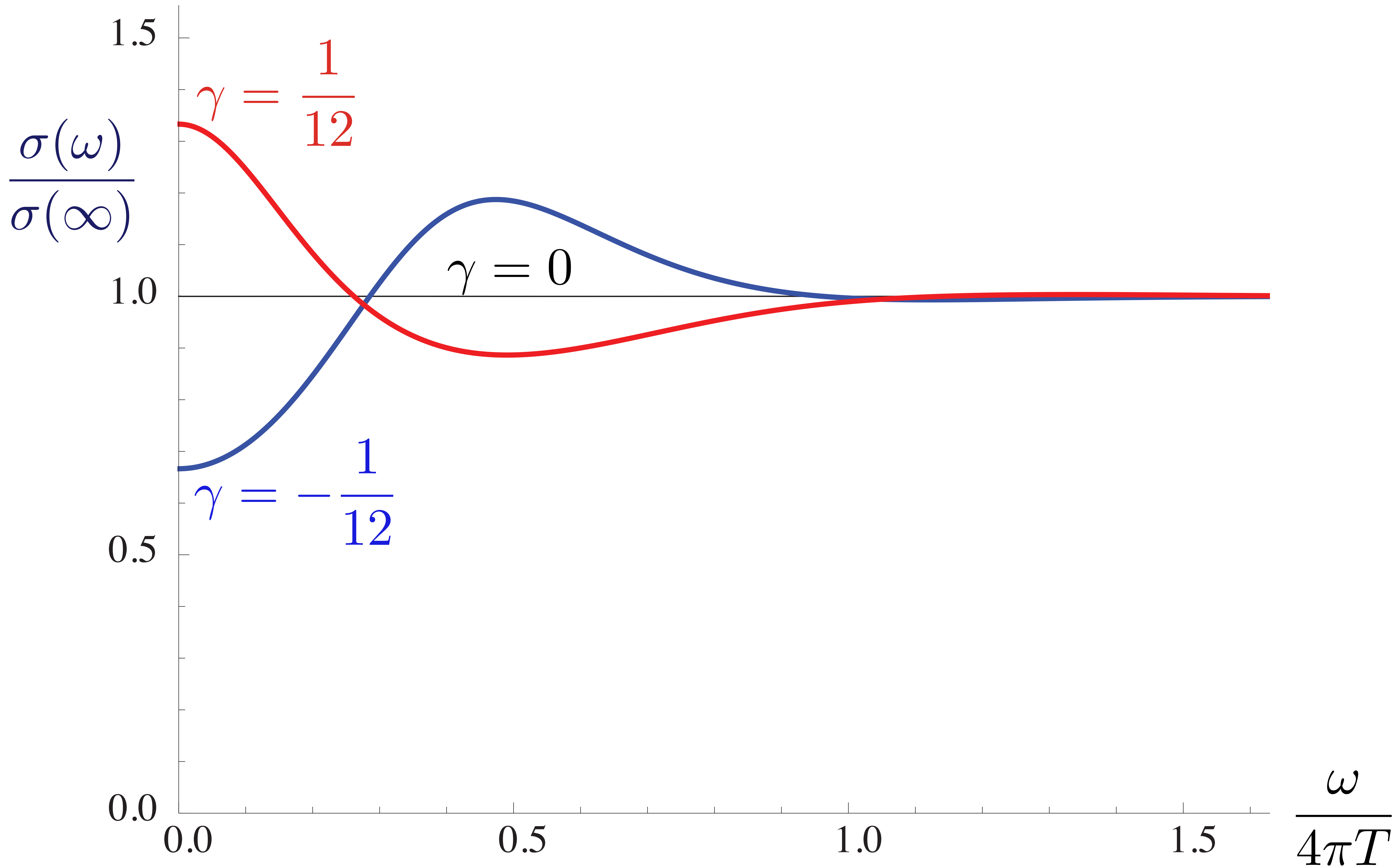
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833$..

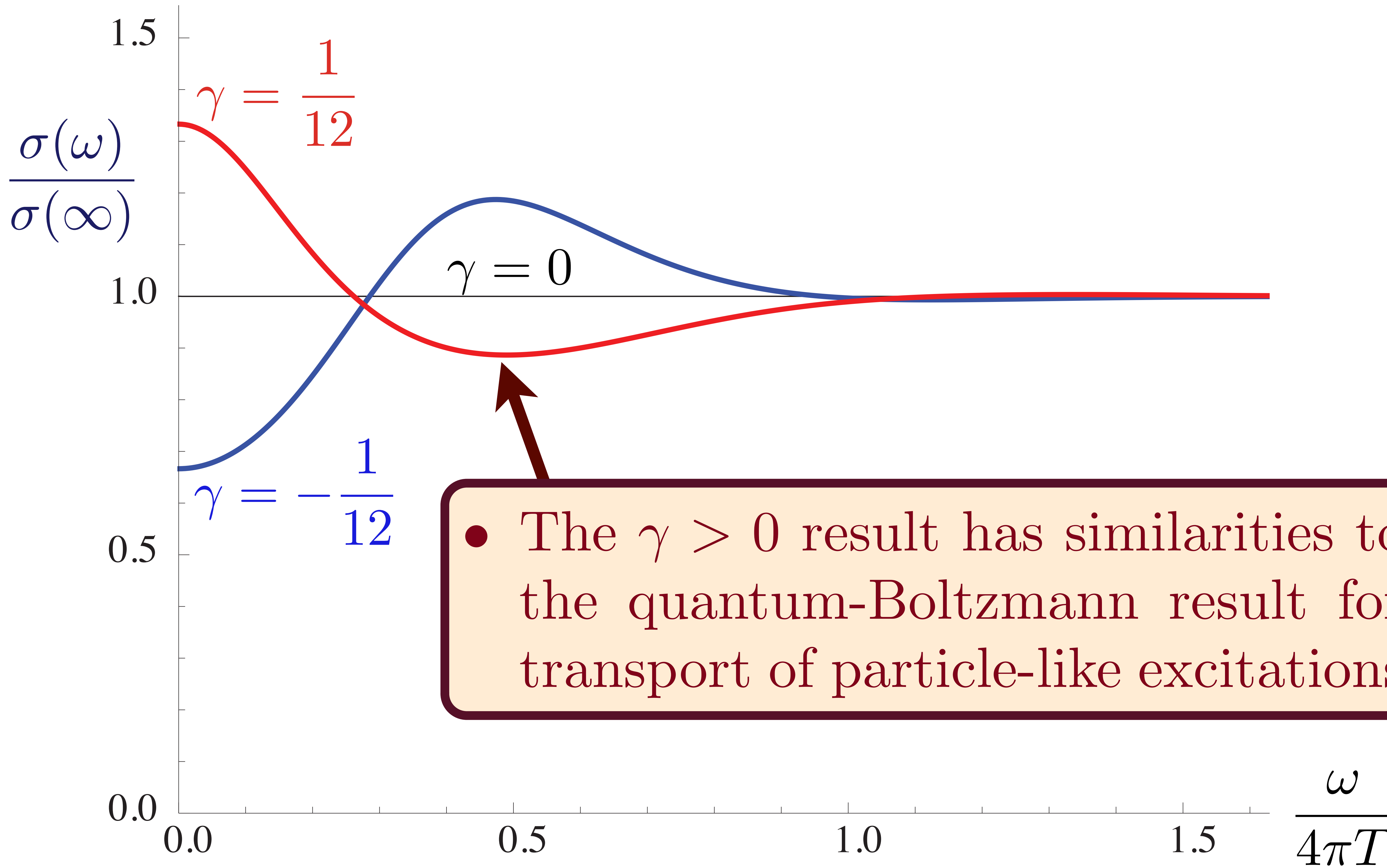
R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS_4 theory of quantum criticality



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

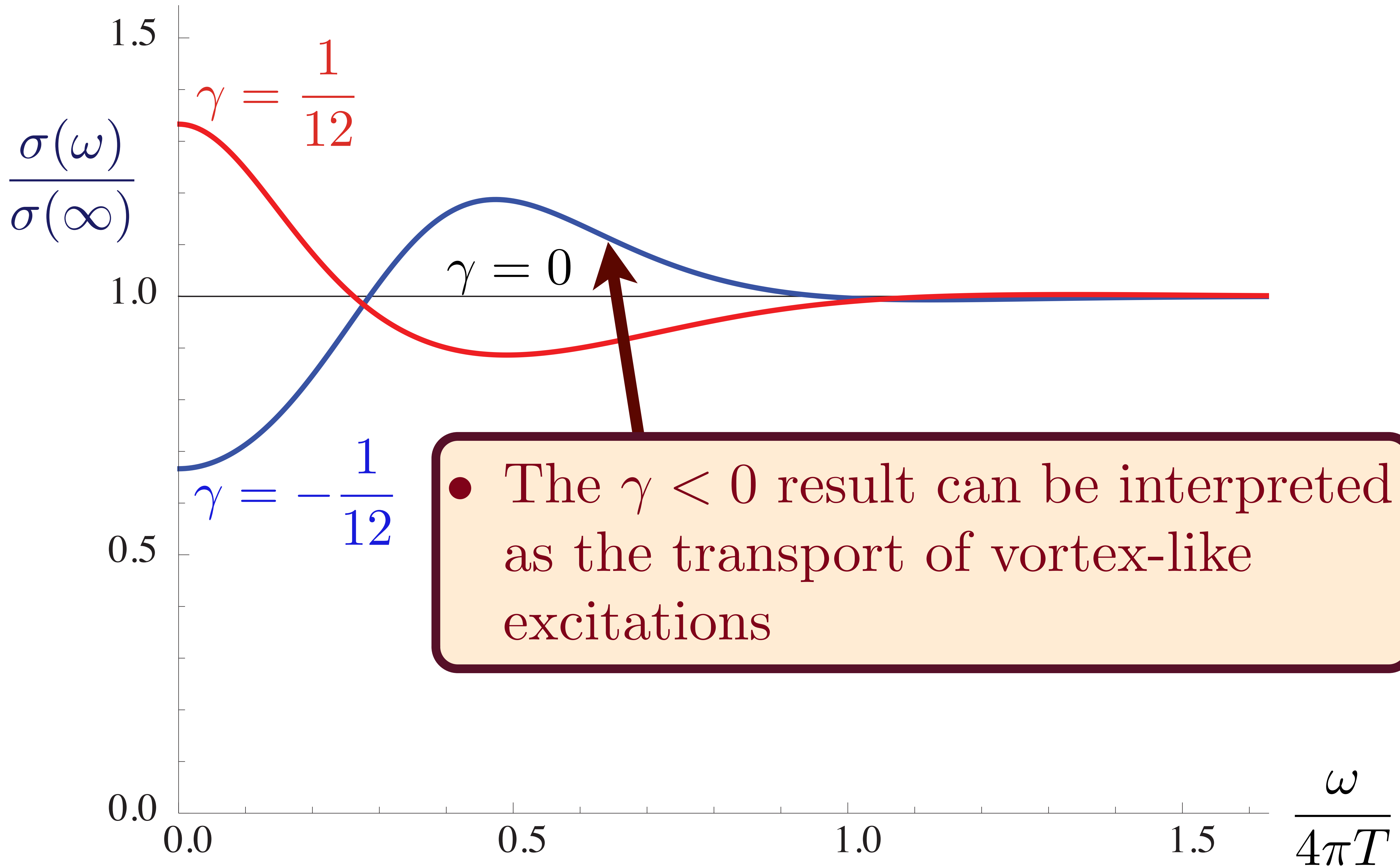
AdS$_4$ theory of quantum criticality



- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

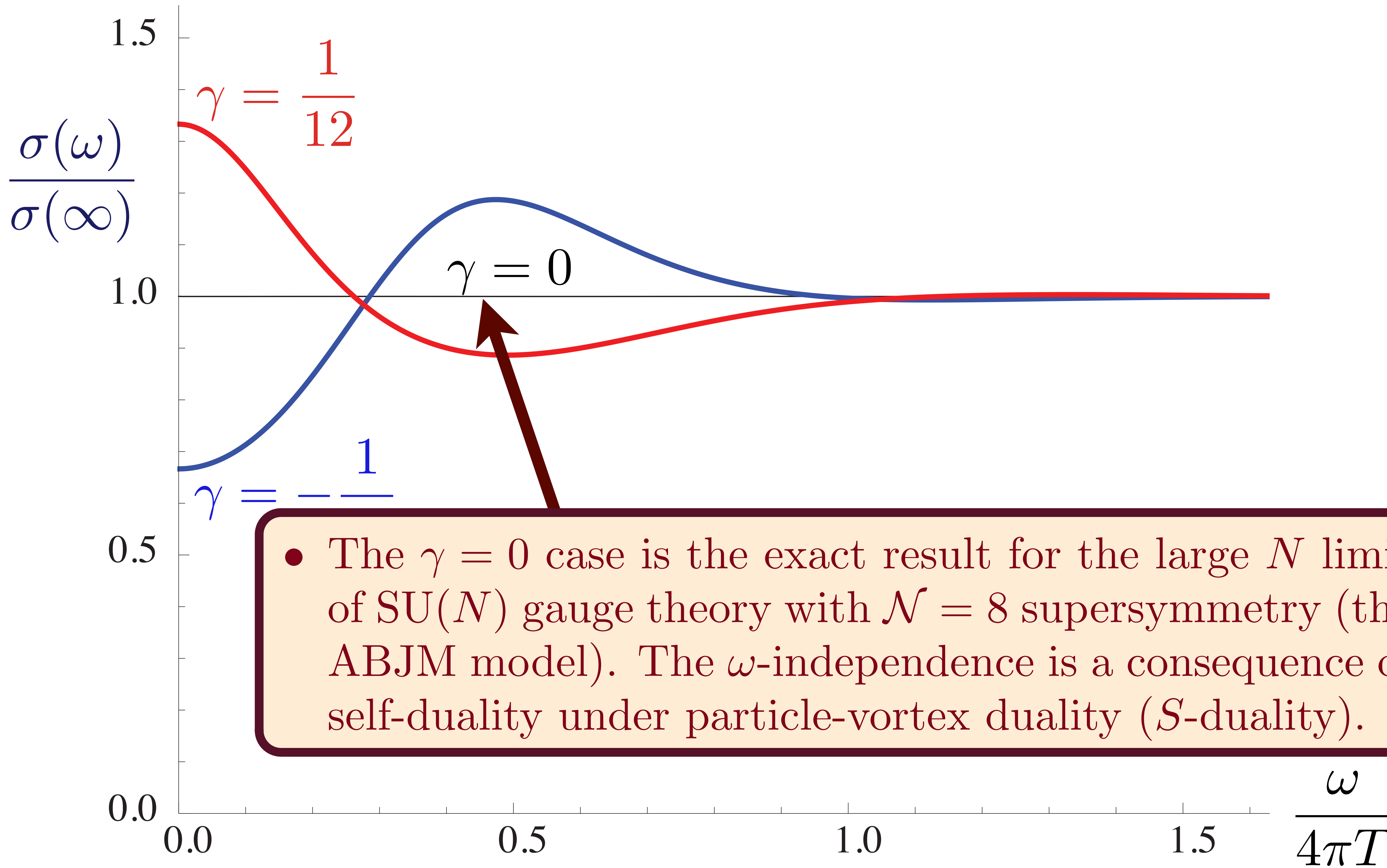
AdS$_4$ theory of quantum criticality



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

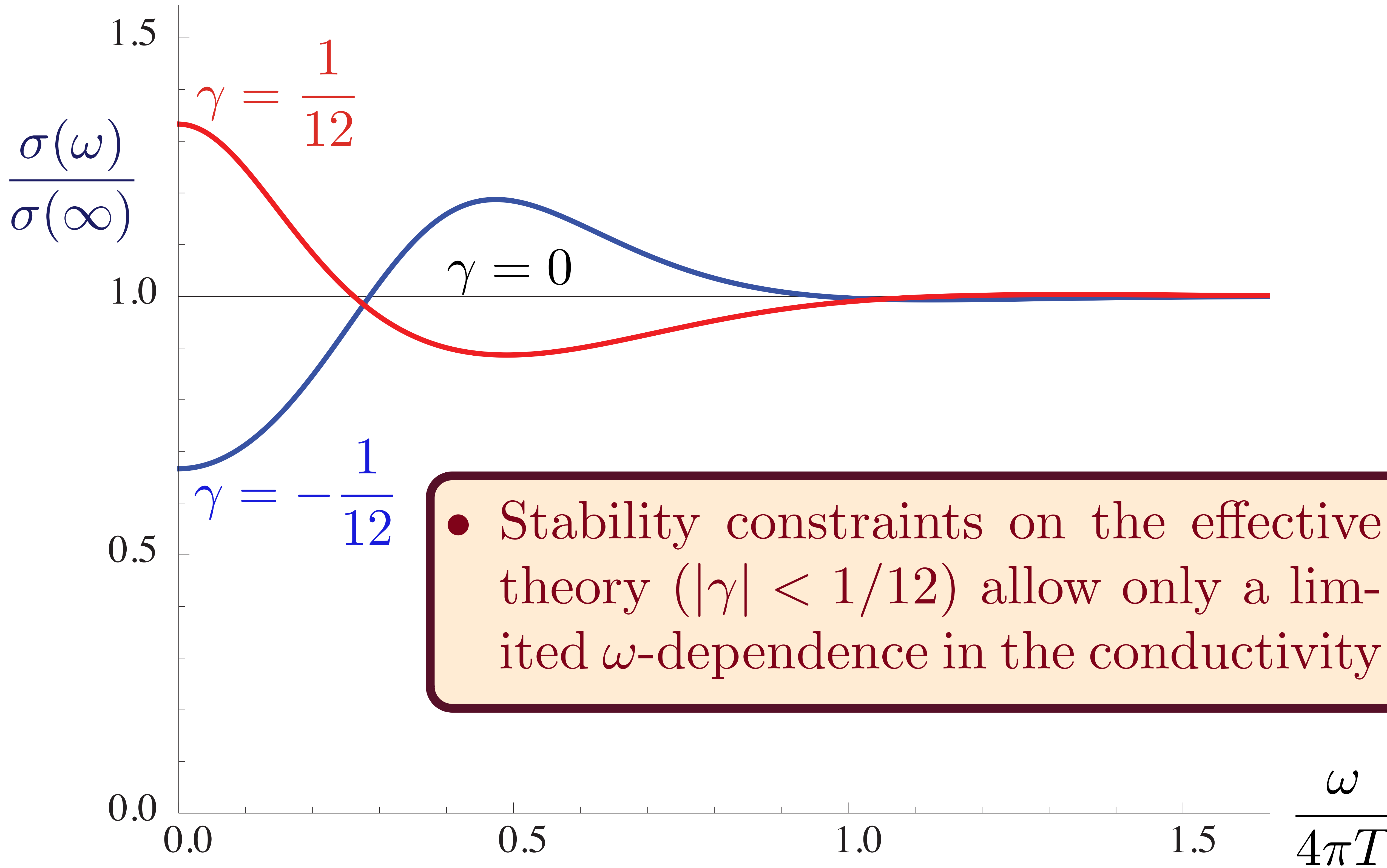
AdS$_4$ theory of quantum criticality



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

arXiv.org > cond-mat > arXiv:1309.2941

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Condensed Matter > Strongly Correlated Electrons

The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

arXiv.org > cond-mat > arXiv:1309.5635

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Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional J-current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau, $\sigma(\infty) = 0.359(4)\sigma_Q$ with σ_Q the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our $\sigma(i\omega_n) - \sigma(\infty)$ function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.

Quantum Monte Carlo for lattice bosons

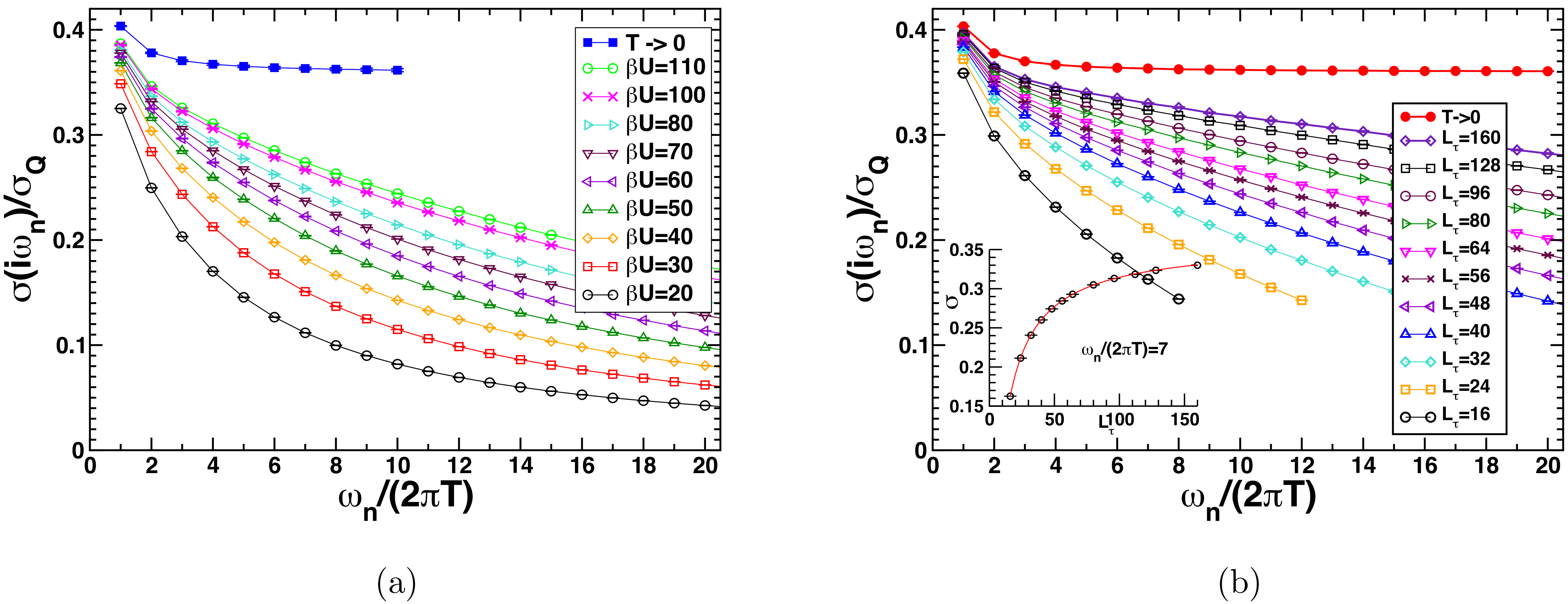
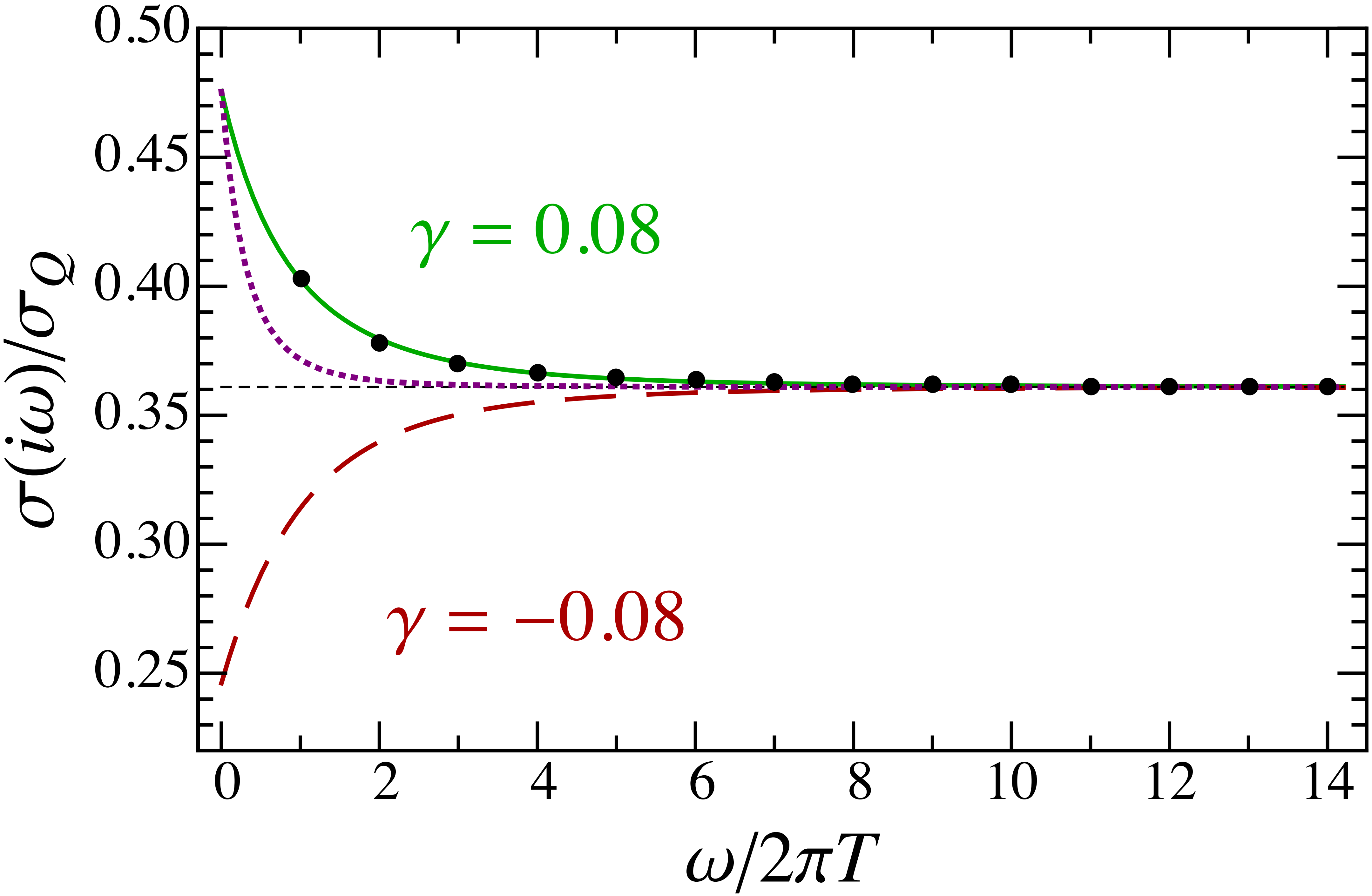


FIG. 2. **Quantum Monte Carlo data** (a) Finite-temperature conductivity for a range of βU in the $L \to \infty$ limit for the quantum rotor model at $(t/U)_c$. The solid blue squares indicate the final $T \to 0$ extrapolated data. (b) Finite-temperature conductivity in the $L \to \infty$ limit for a range of L_τ for the Villain model at the QCP. The solid red circles indicate the final $T \to 0$ extrapolated data. The inset illustrates the extrapolation to $T = 0$ for $\omega_n/(2\pi T) = 7$. The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS$_4$ theory of quantum criticality

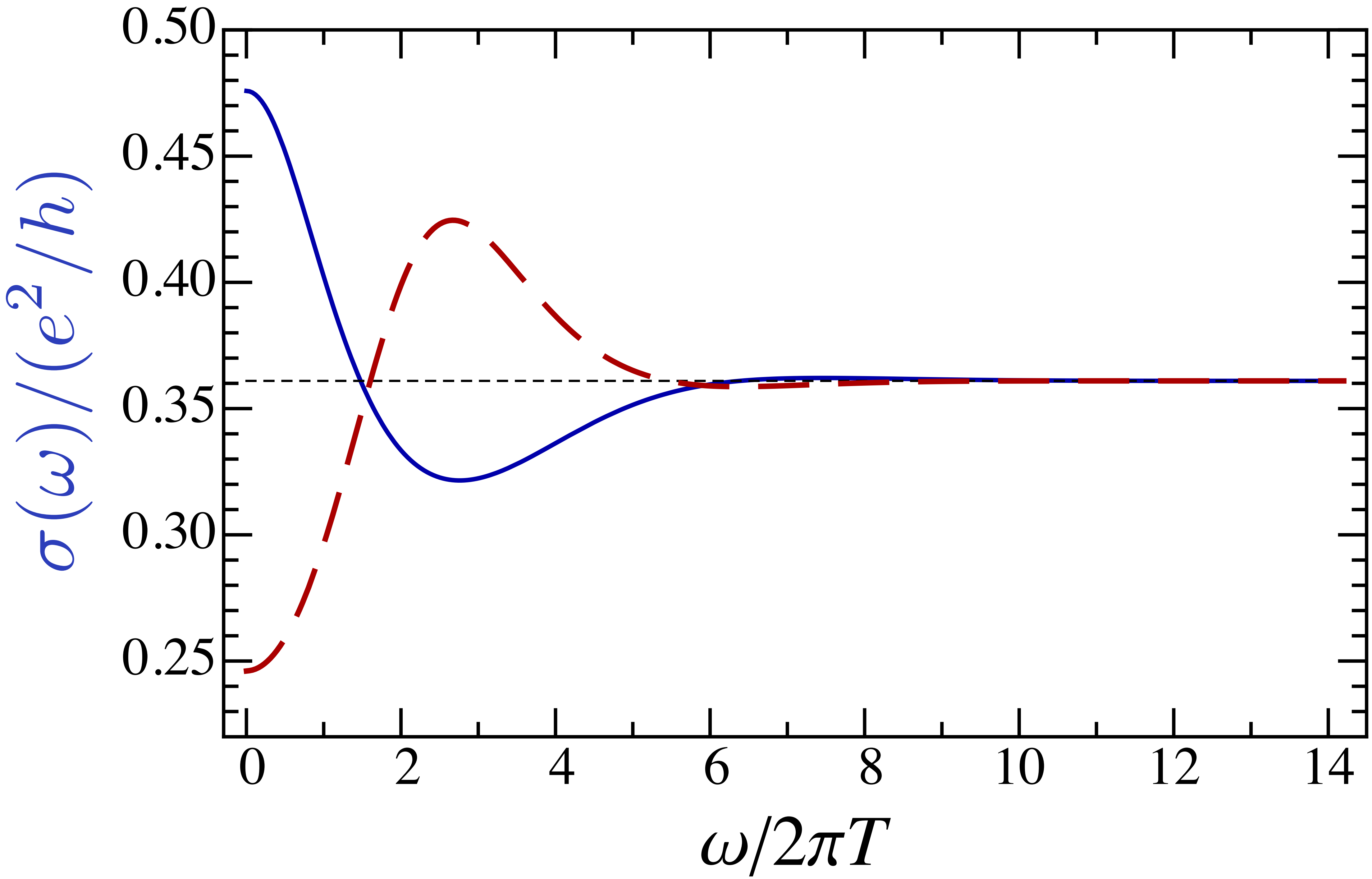


Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective T and taking $\sigma_Q = 1/g_M^2$.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS_4 theory of quantum criticality



Predictions of holographic theory,
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635