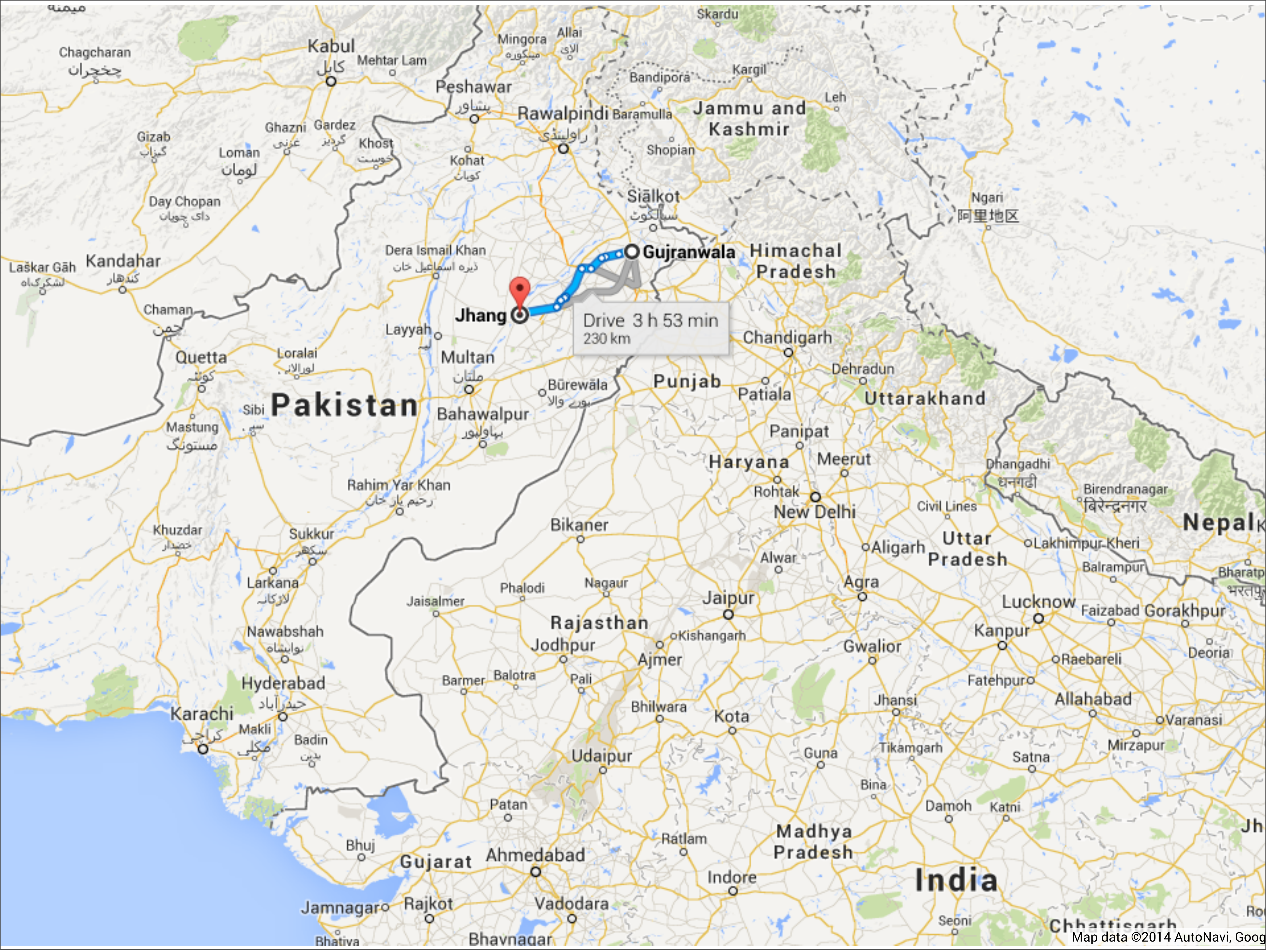
Theory of Quantum Matter: from Quantum Fields to Strings

Salam Distinguished Lectures
The Abdus Salam International Center for Theoretical Physics
Trieste, Italy
January 27-30, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD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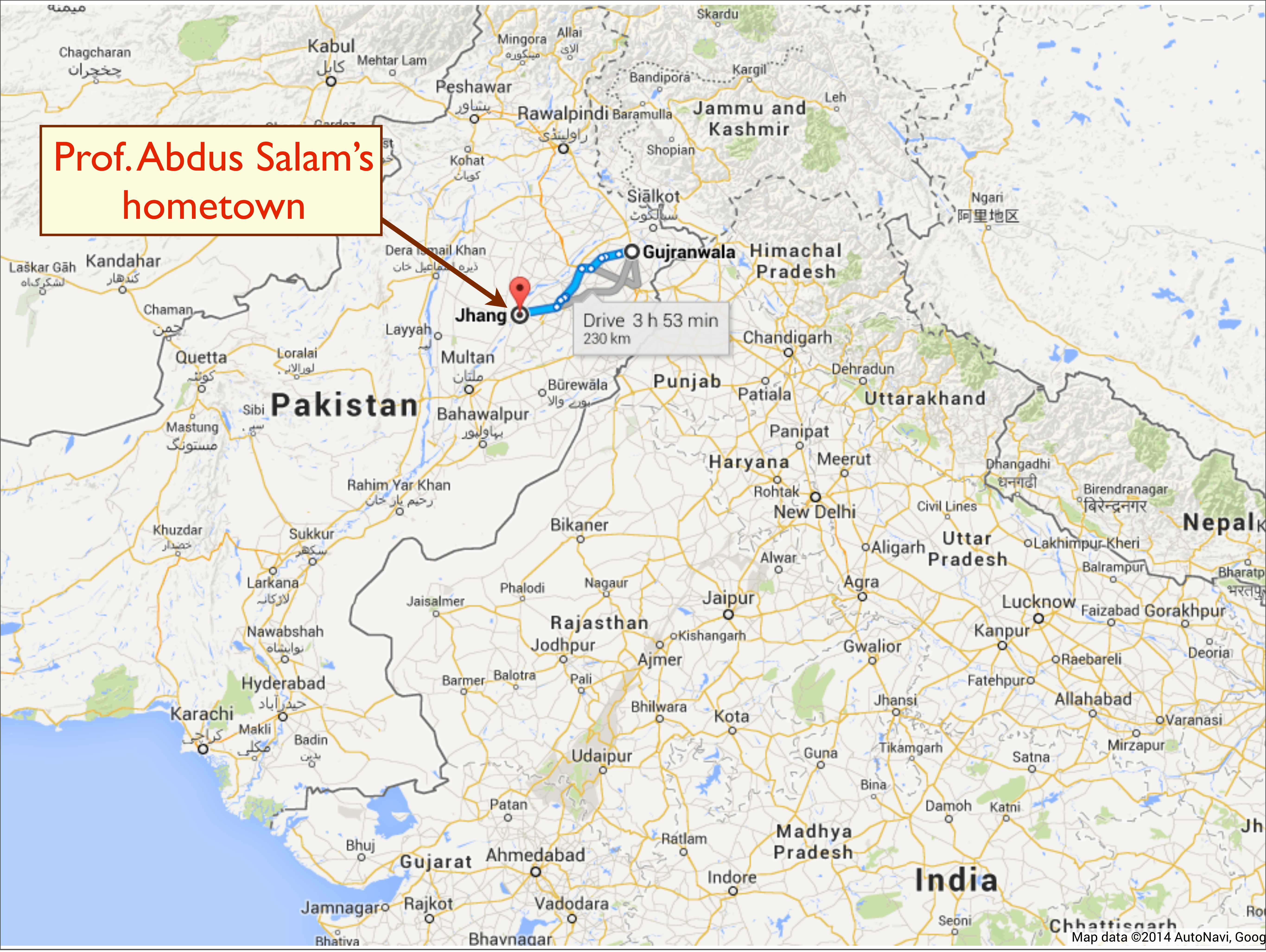

Prof. Abdus Salam's hometown
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Layyah
Quetta
Loralai
Multan
Chandigarh
Dehradun
Punjab
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Mastung
Bahawalpur
Panipat
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Nawabshah
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Bhilwara
Kota
Jhansi
Varanasi
Badin
Mirzapur
Tikamgarh
Guna
Satna
Udaipur
Bina
Damoh
Katni
Patan
Madhya Pradesh
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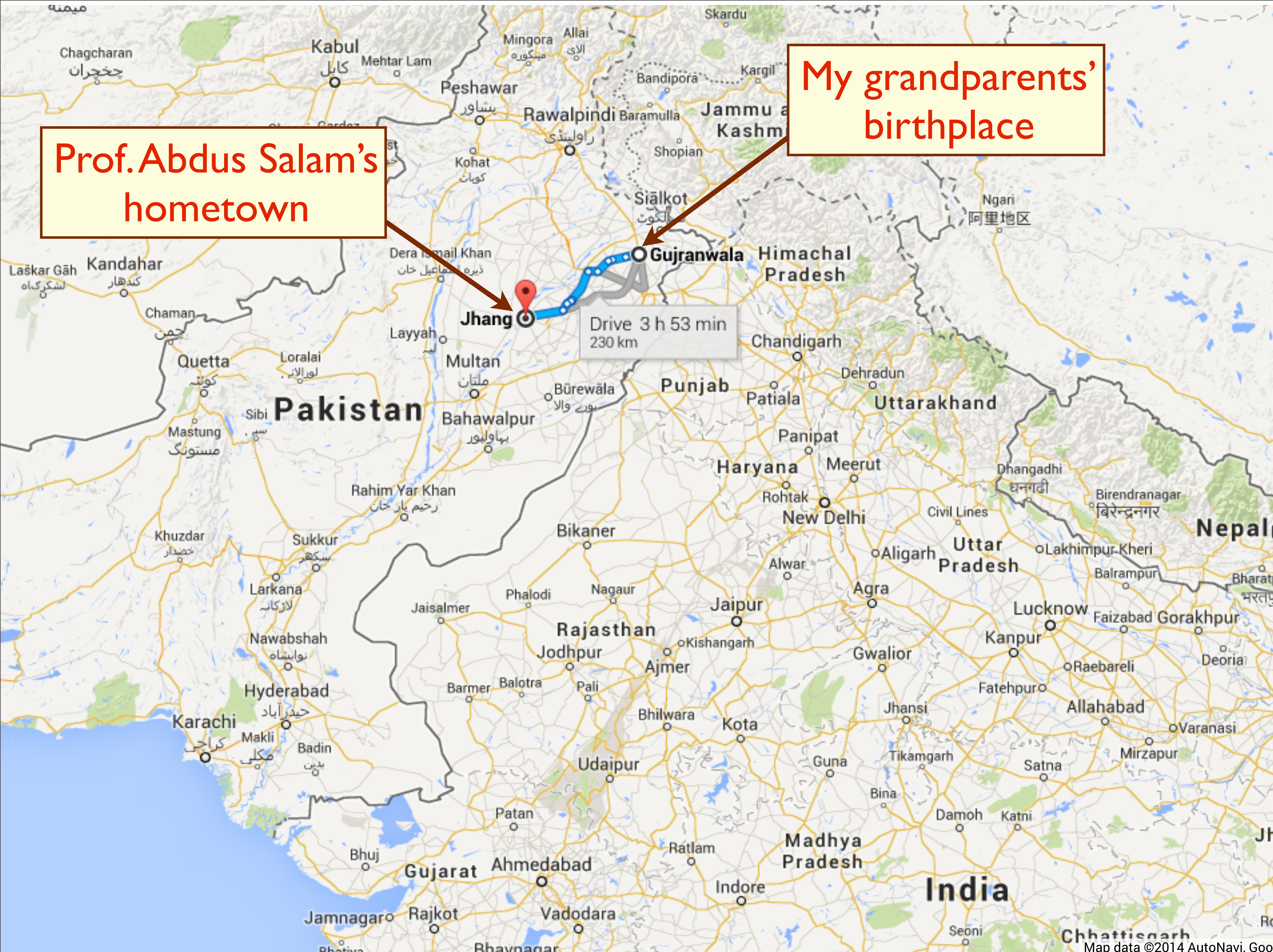
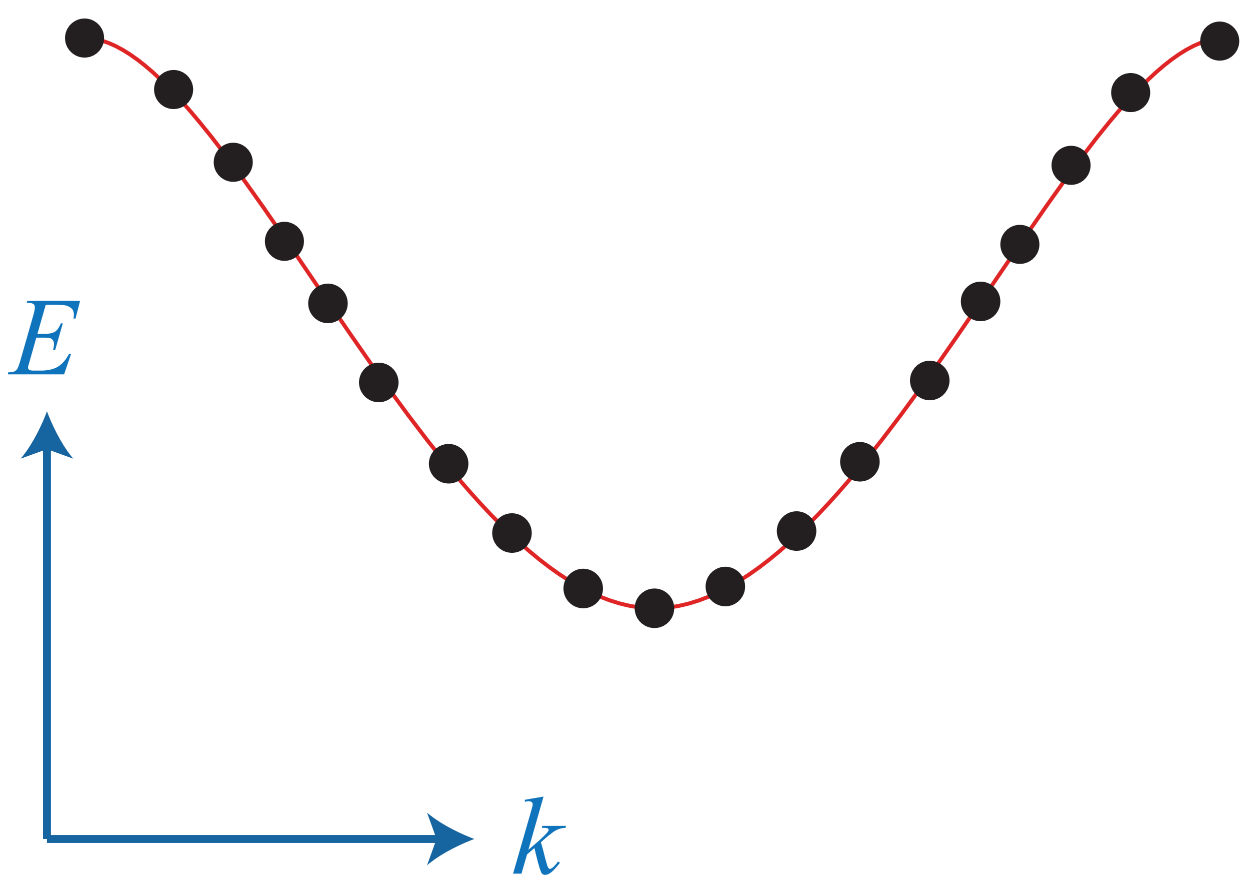


Thursday, January 30, 14

Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of
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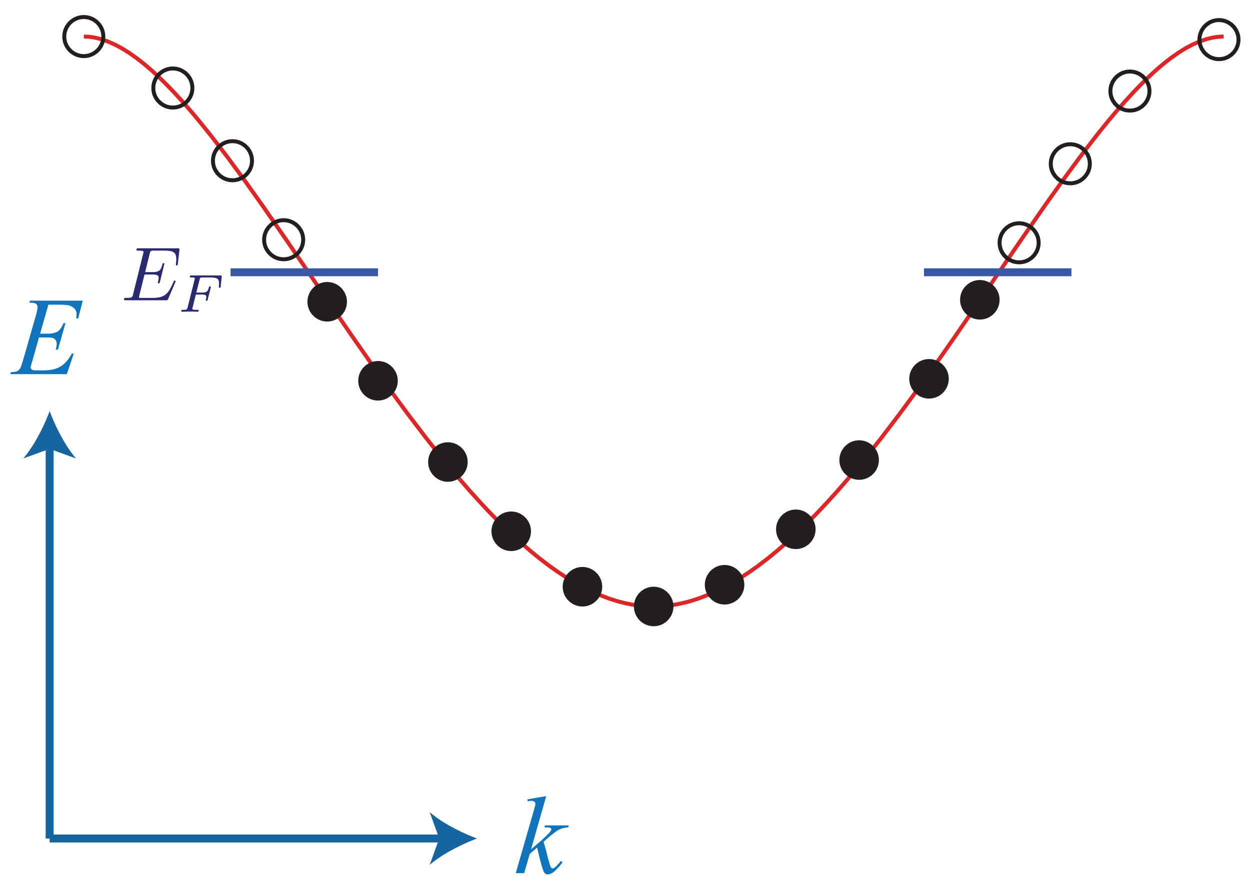
Metals



Sommerfeld-Bloch theory of
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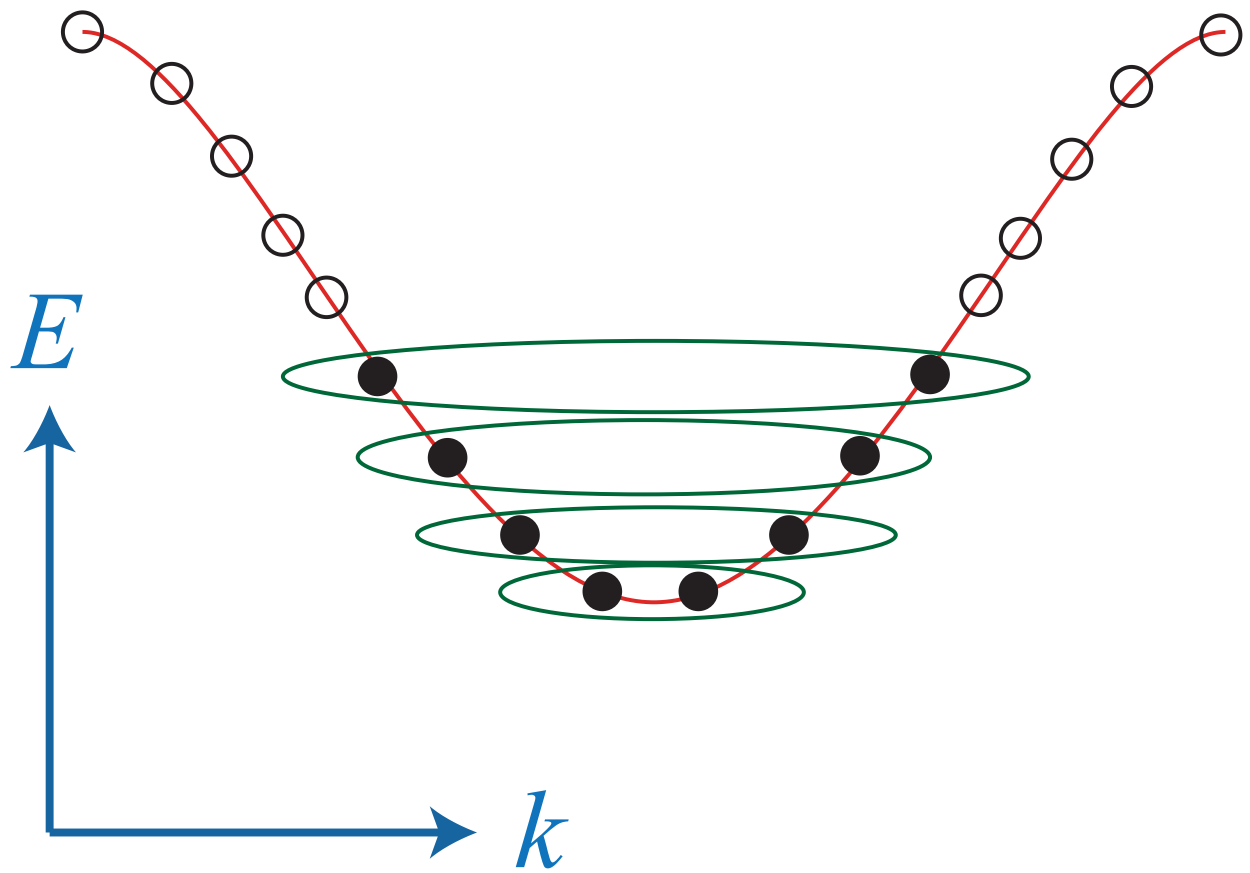
Superconductors



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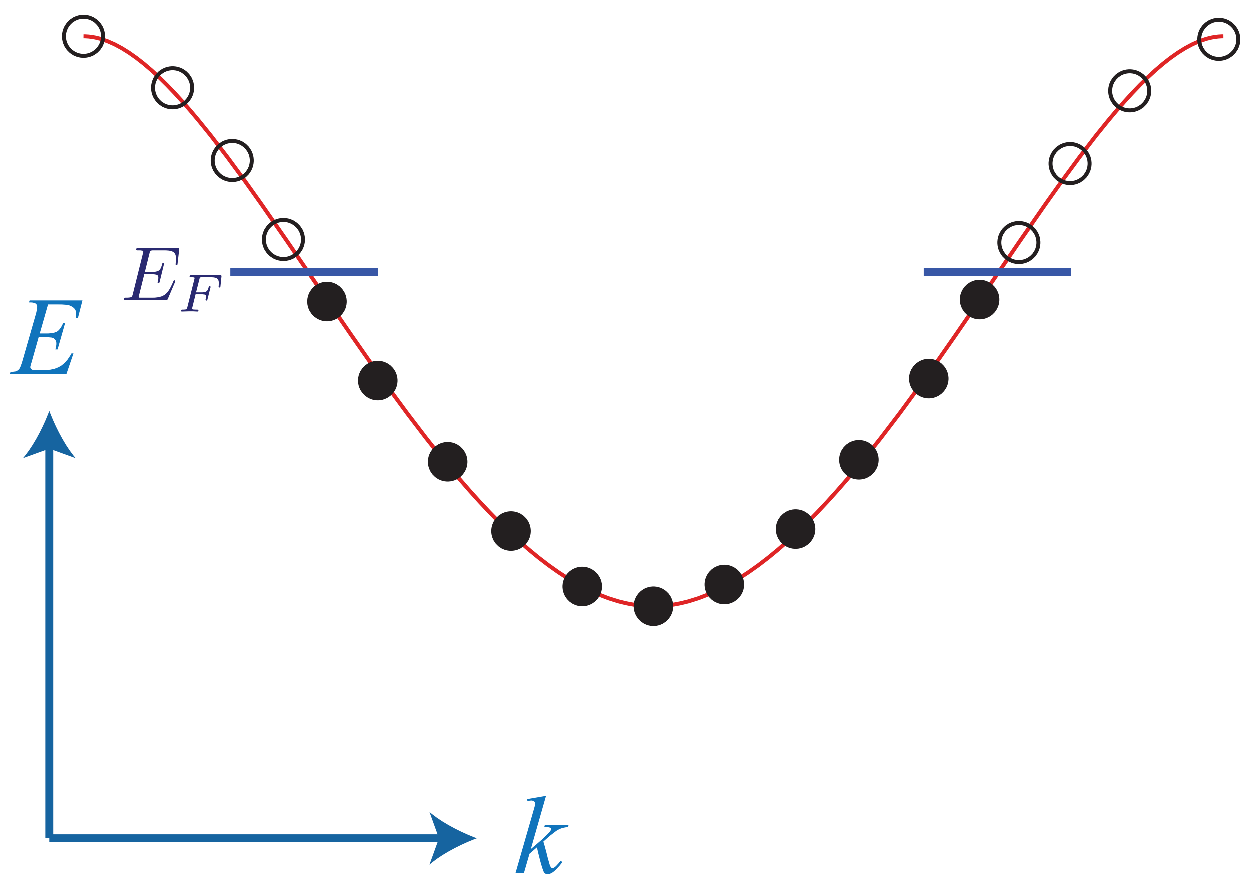
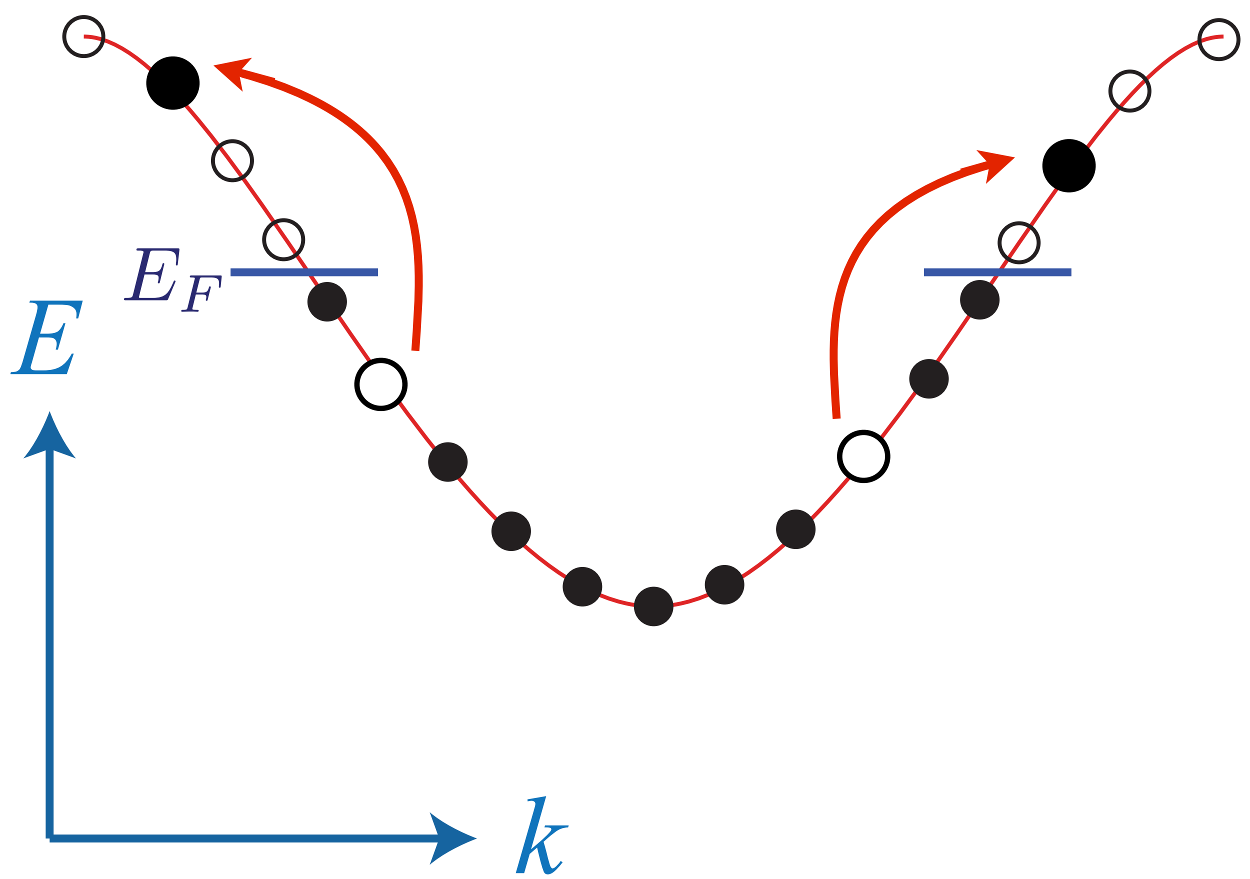
Metals



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

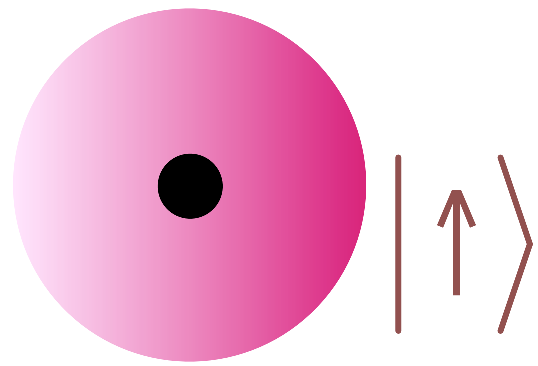
Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

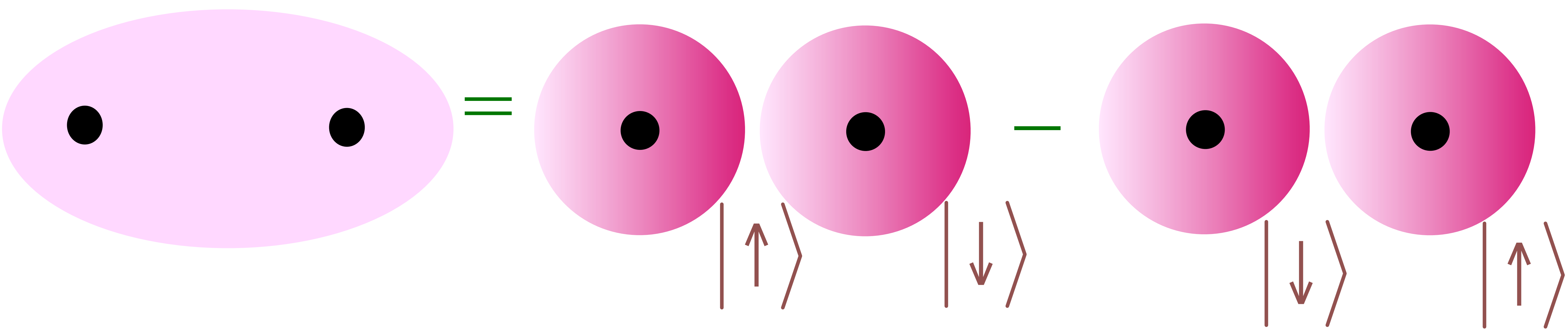
<u>Quantum Entanglement:</u> quantum superposition with more than one particle

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

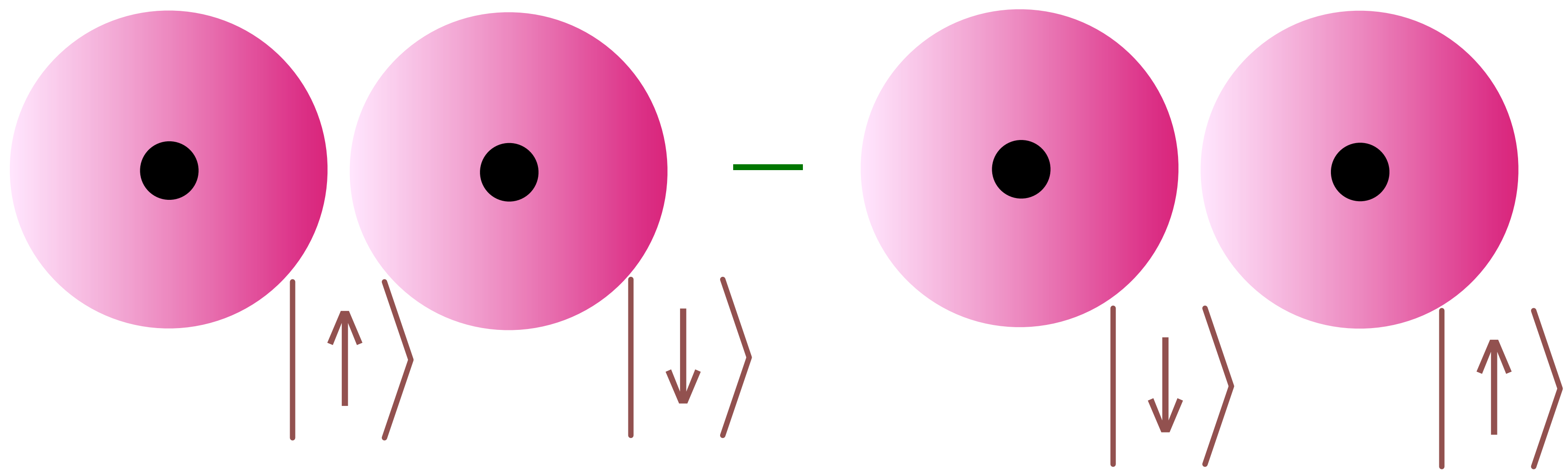


Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



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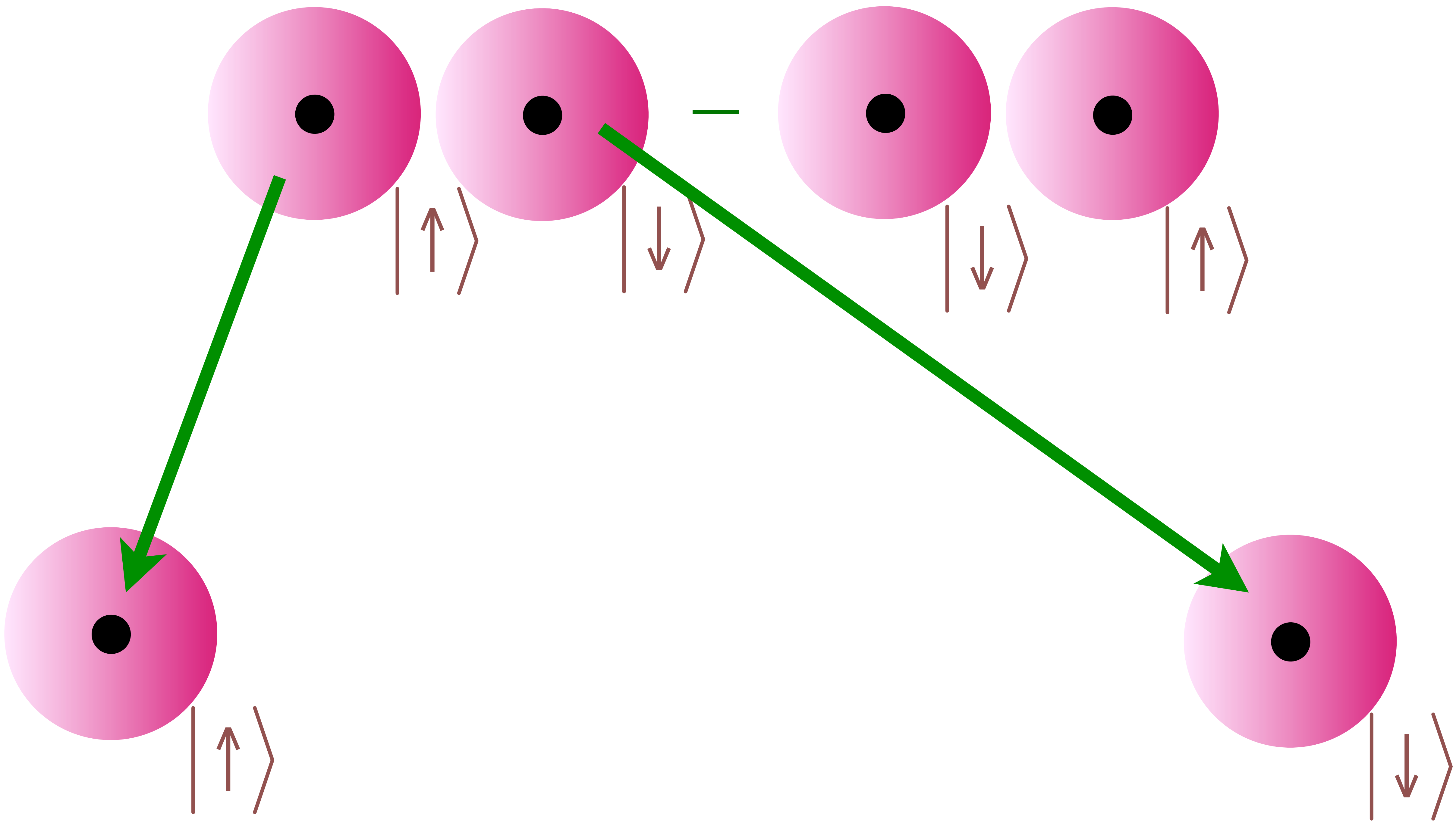

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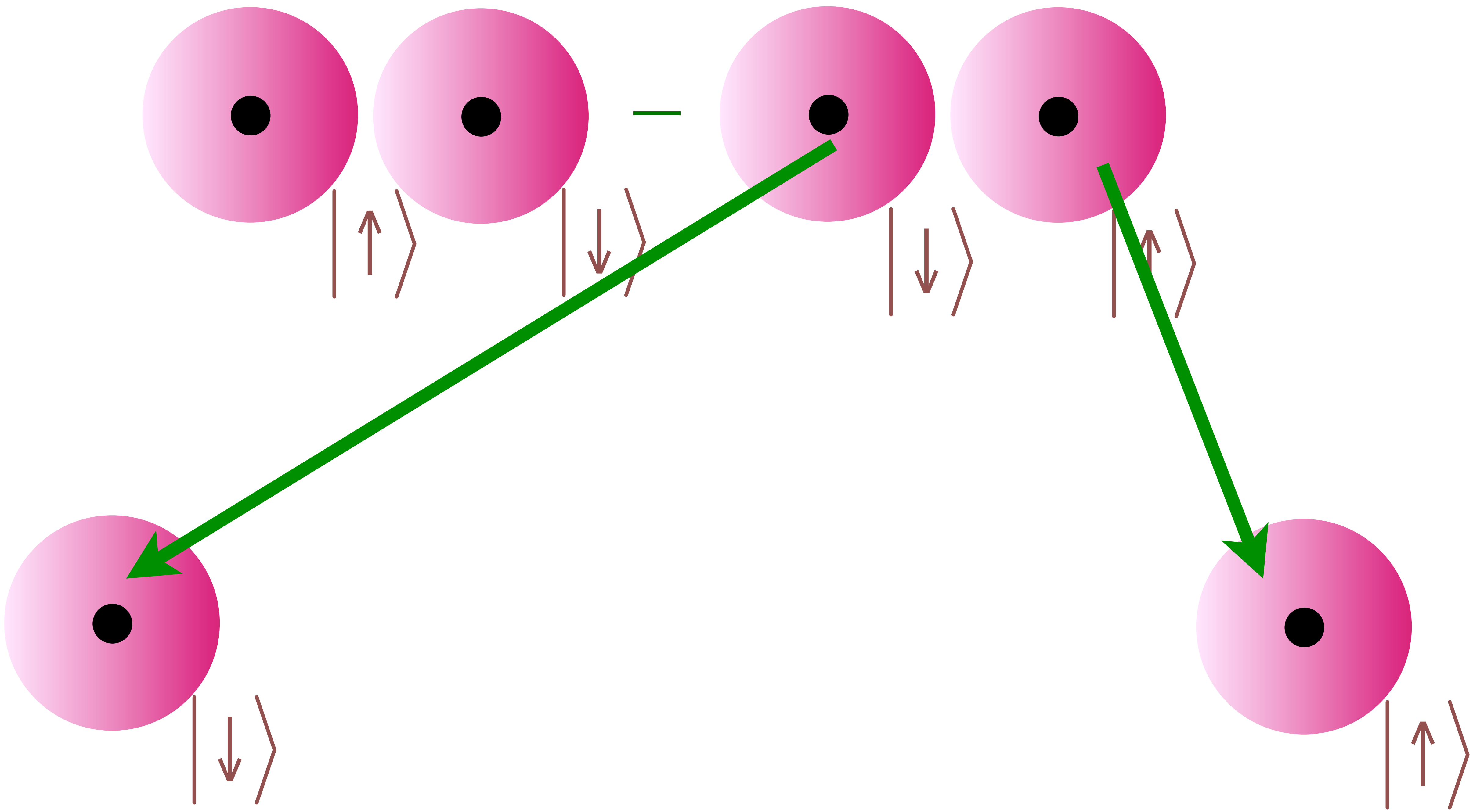

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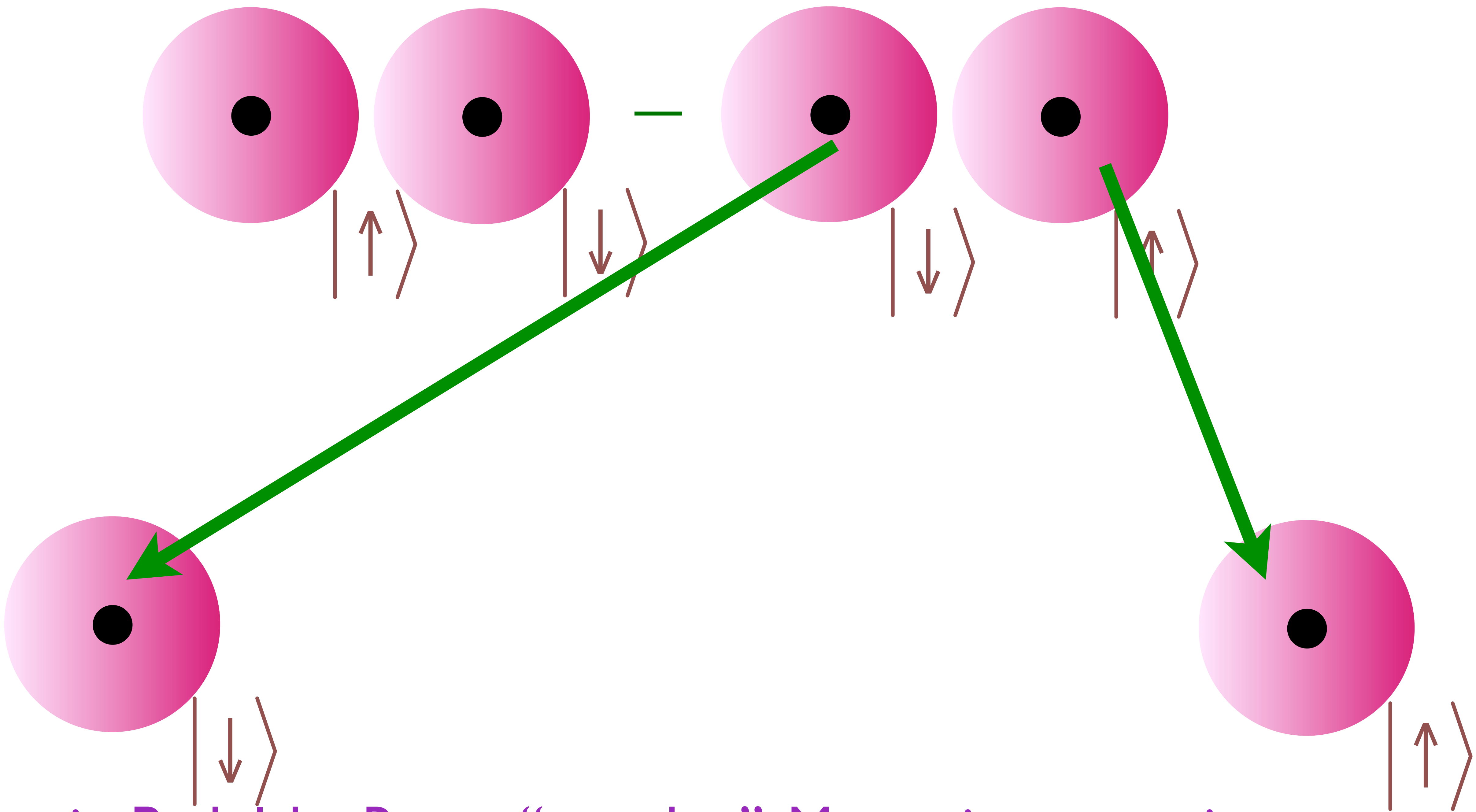

Einstein-Podolsky-Rosen “paradox”: Measuring one spin instantaneously effects the state of another electron far away

Modern phases of quantum matter

Not adiabatically connected
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many-particle
quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
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Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter

Not adiabatically connected

to independent electron states:

many-particle

quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,
and no quasiparticles

Outline

1. The simplest models without quasiparticles

A. Superfluid-insulator transition

of ultracold bosons in an optical lattice

B. Conformal field theories in 2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

A. Review of Fermi liquid theory

B. A "non-Fermi" liquid: the Ising-nematic

quantum critical point

C. Holography, entanglement, and strange metals

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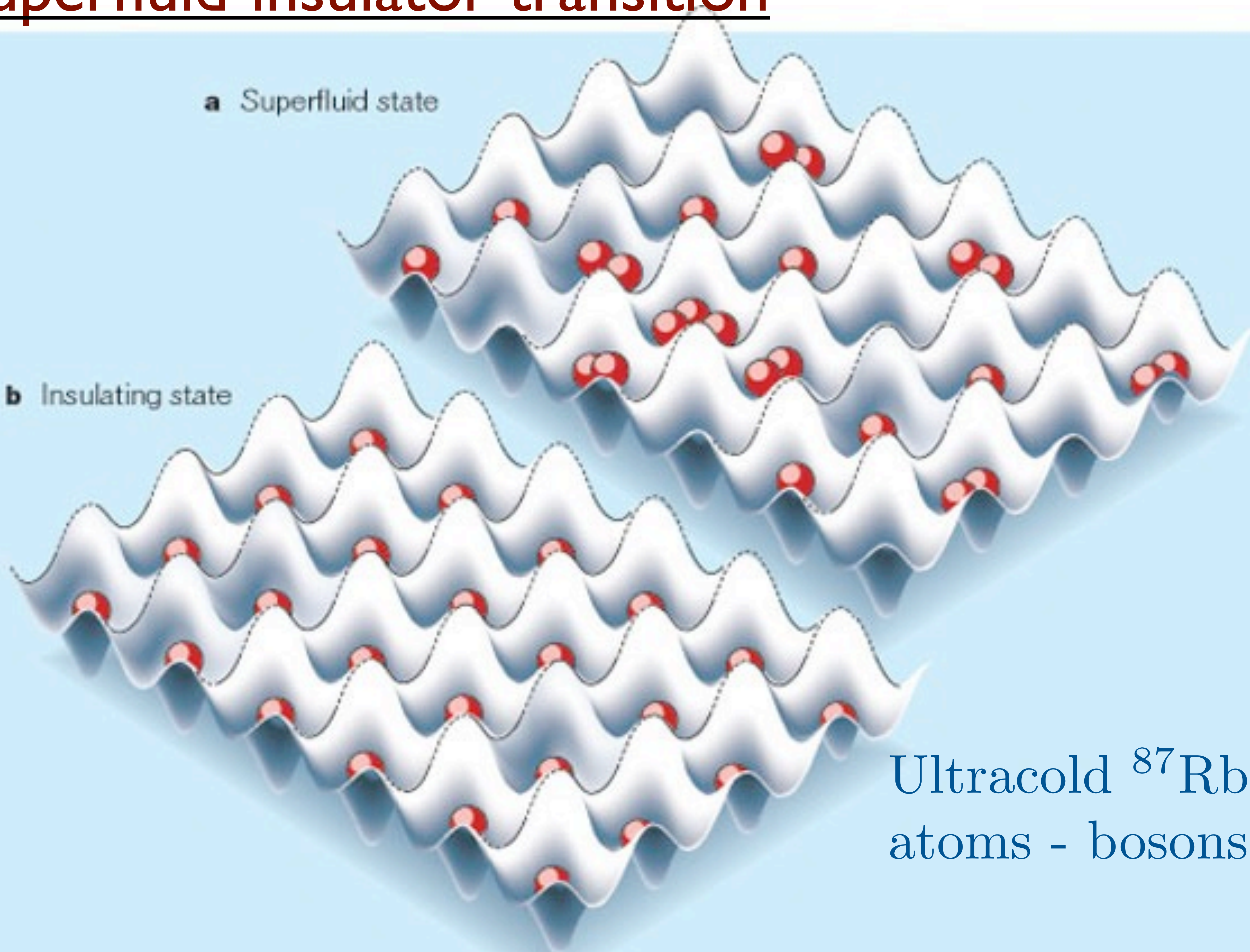
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

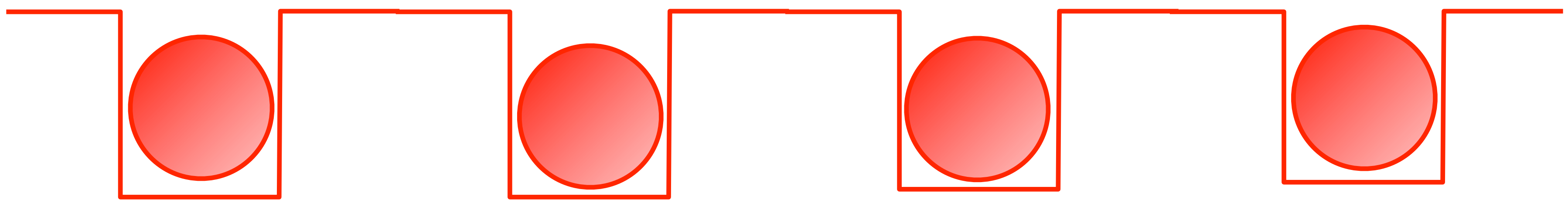
$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

$U \gg t$

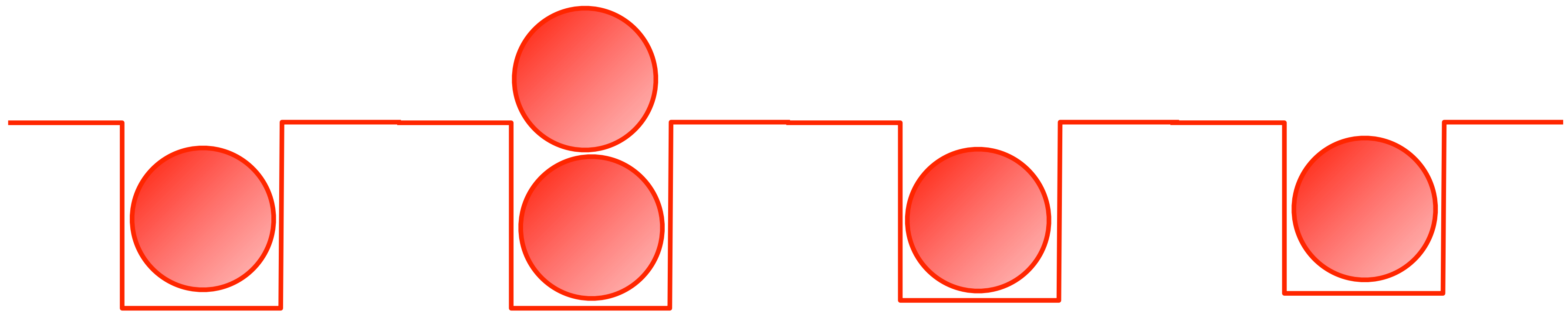


Insulator (the vacuum)
at large repulsion between bosons

$$|\text{Ground state}\rangle = \prod_i b_i^\dagger \, |0\rangle$$

$U \gg t$

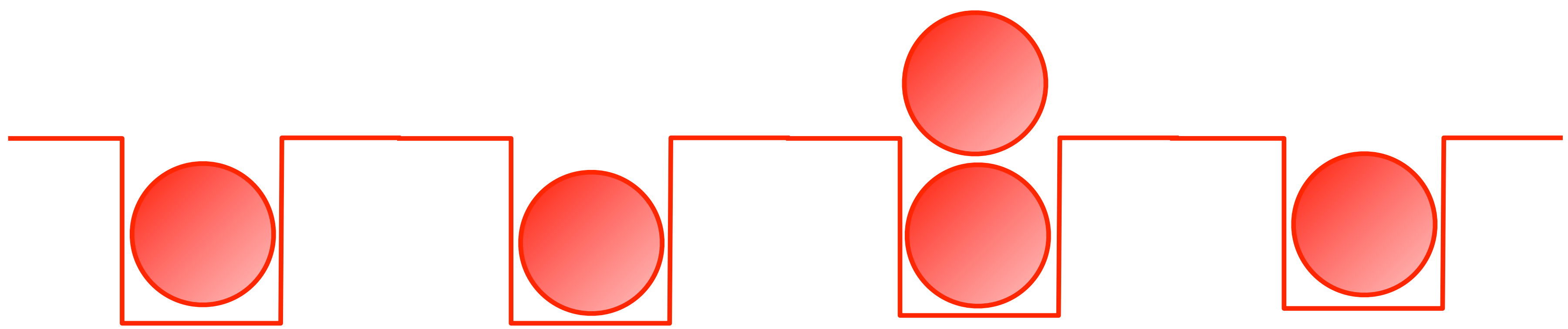
Excitations of the insulator:



Particles $\sim \psi^\dagger$

$U \gg t$

Excitations of the insulator:



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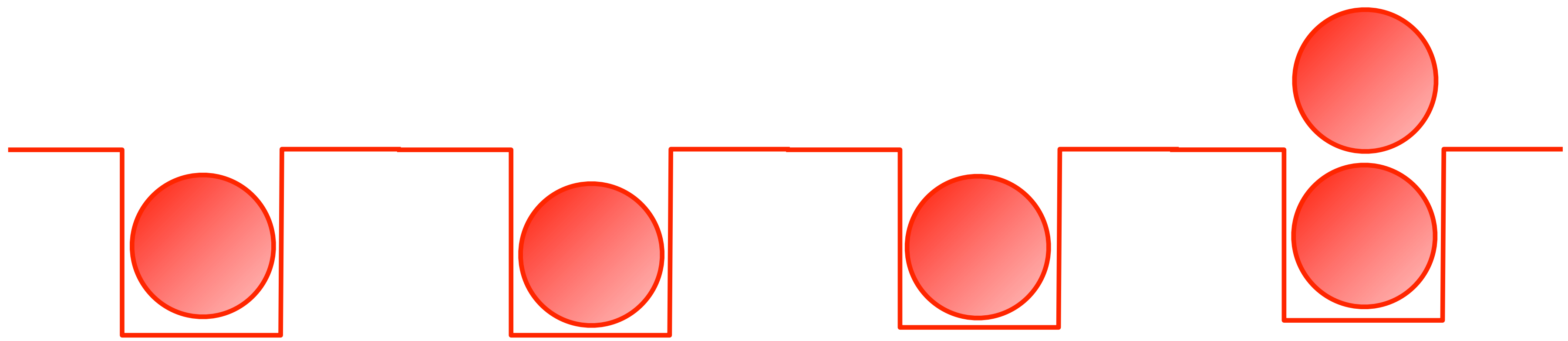

Particles $\sim \psi^{\dagger}$

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Excitations of the insulator:

Holes $\sim \psi$

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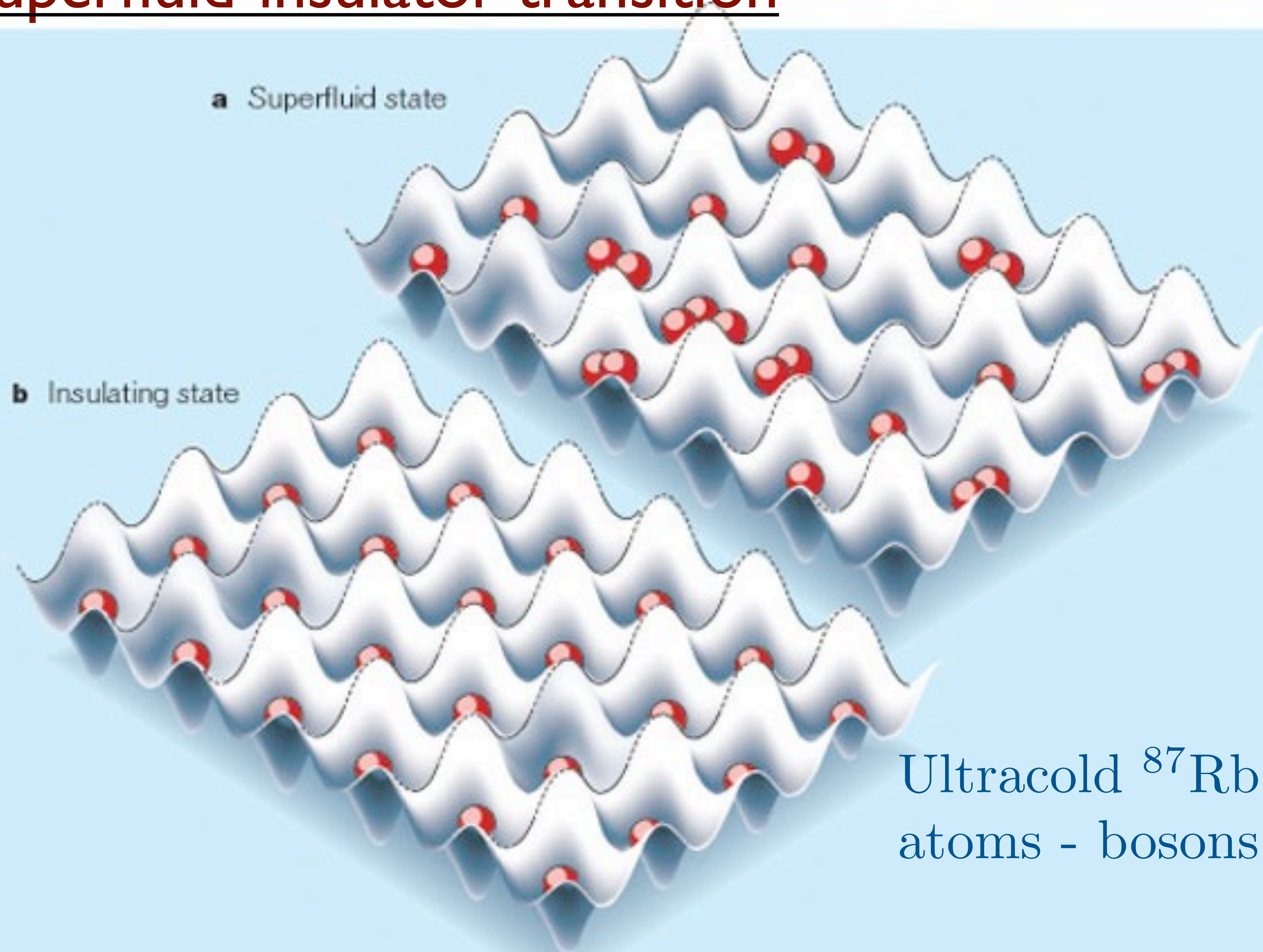
Excitations of the insulator:

Holes $\sim \psi$

Superfluid-insulator transition



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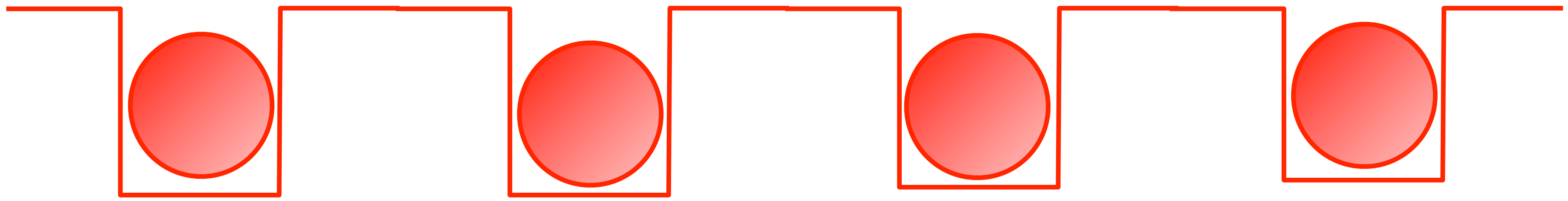
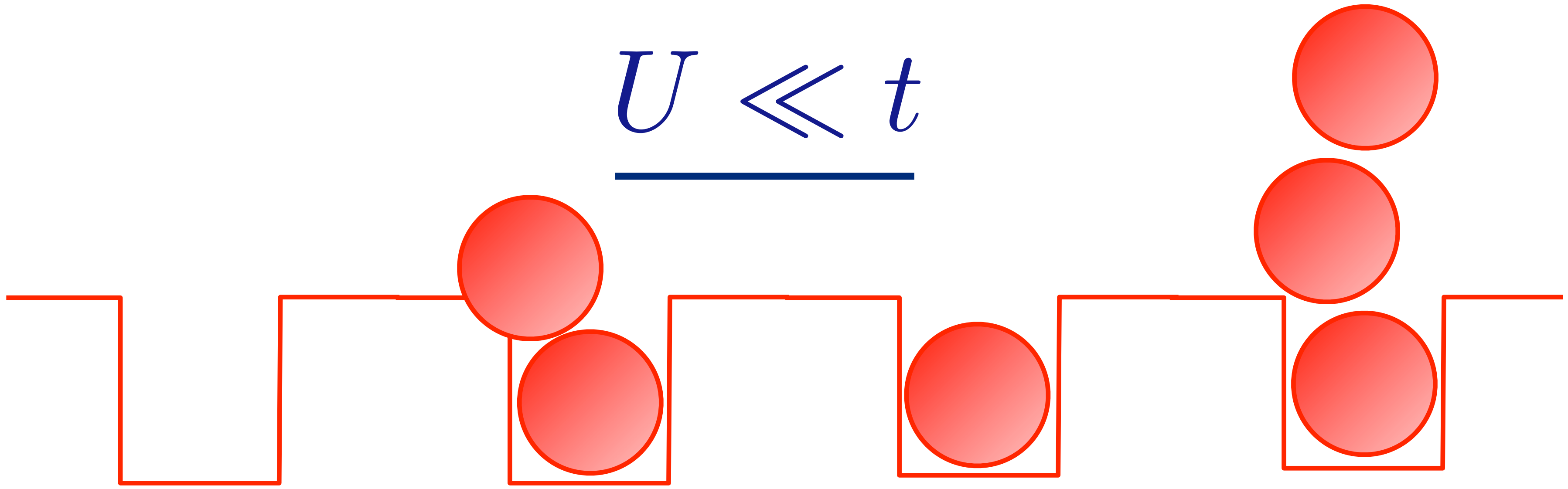


Insulator (the vacuum)
at large repulsion between bosons

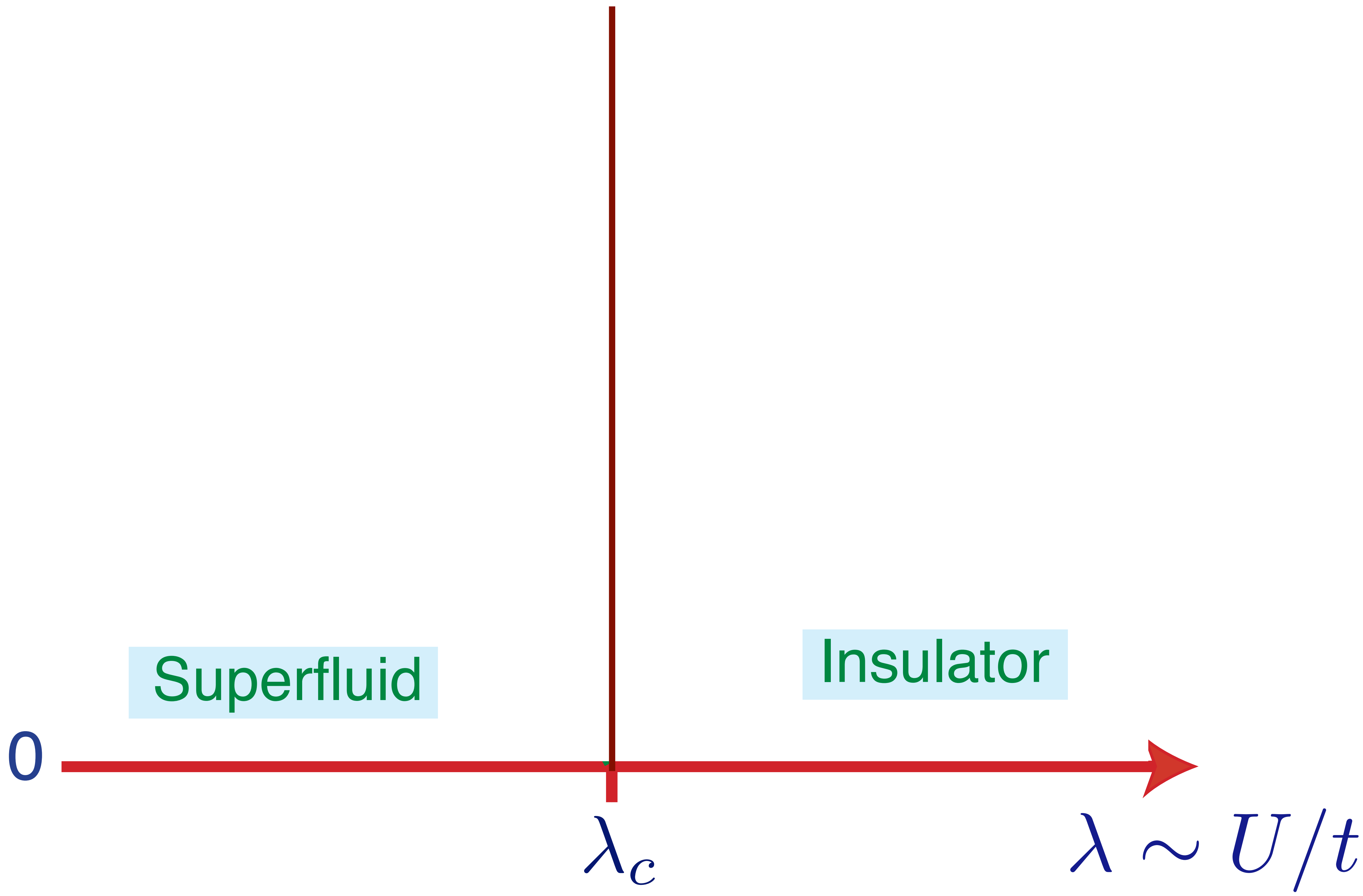
$$|\text{Ground state}\rangle = \prod_i b_i^\dagger \, |0\rangle$$

$U \ll t$

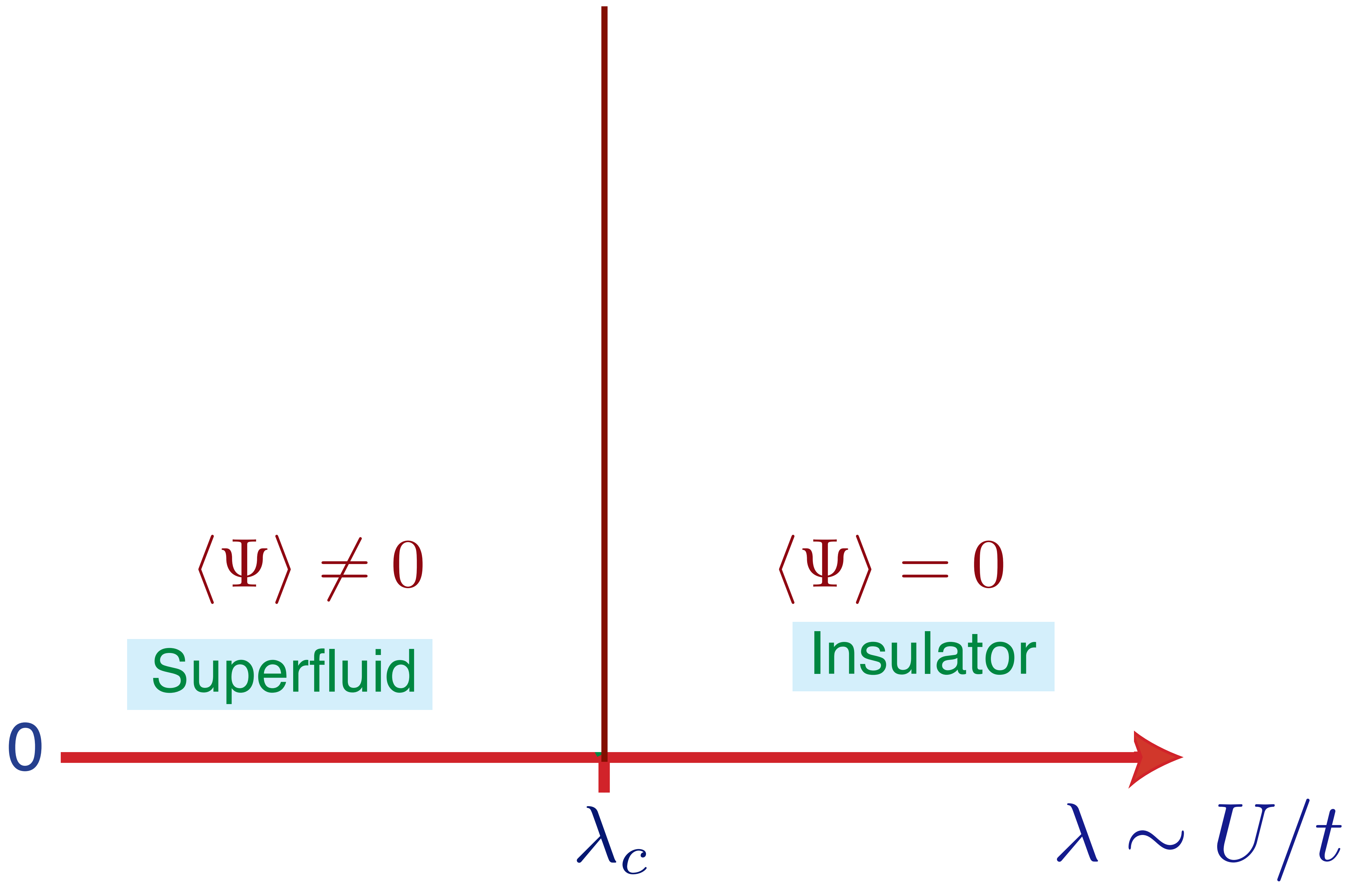


Superfluid
at small repulsion between bosons

$$|\text{Ground state}\rangle = \left[\sum_i b_i^\dagger\right]^N |0\rangle$$

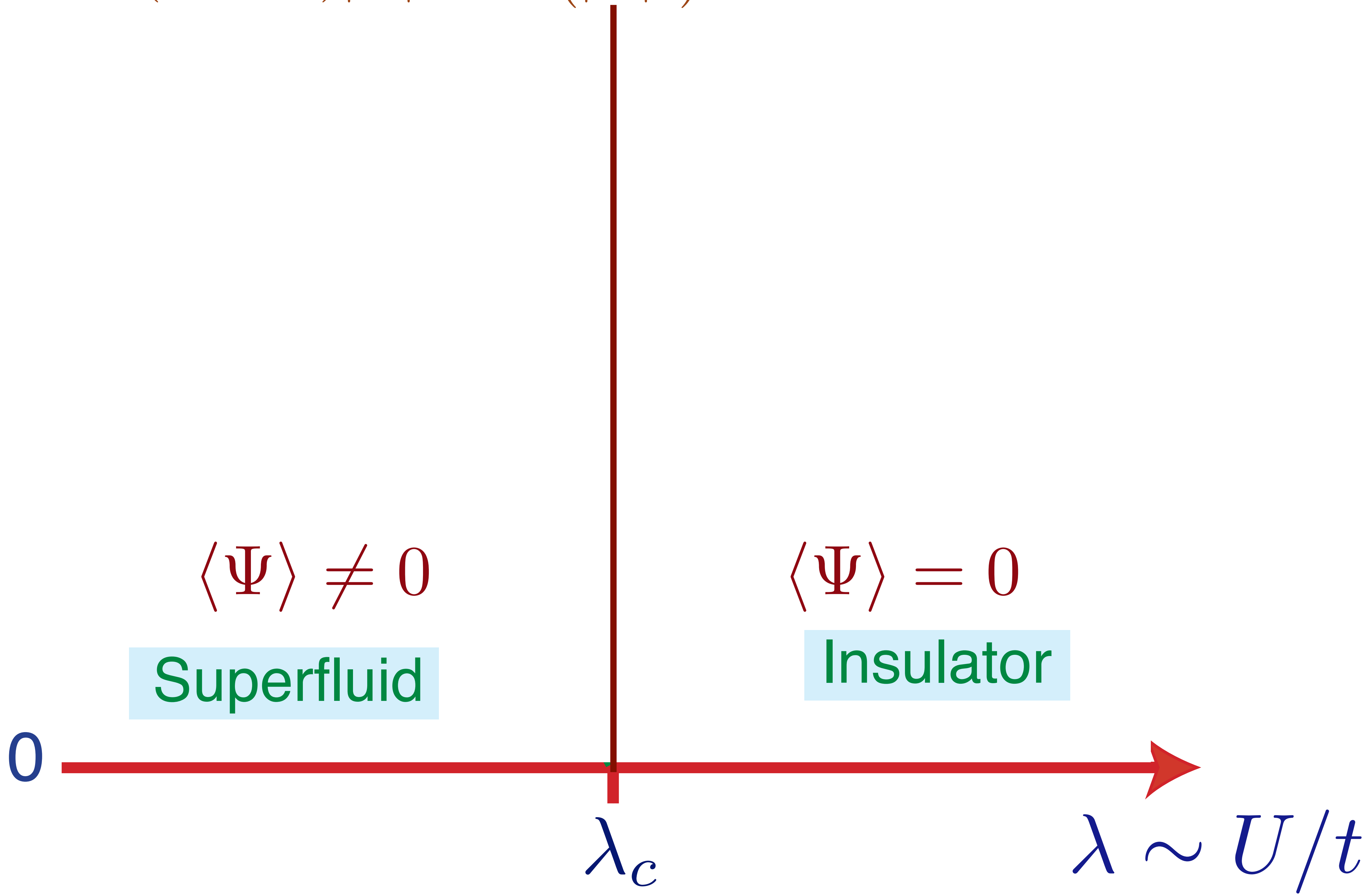

Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$

$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



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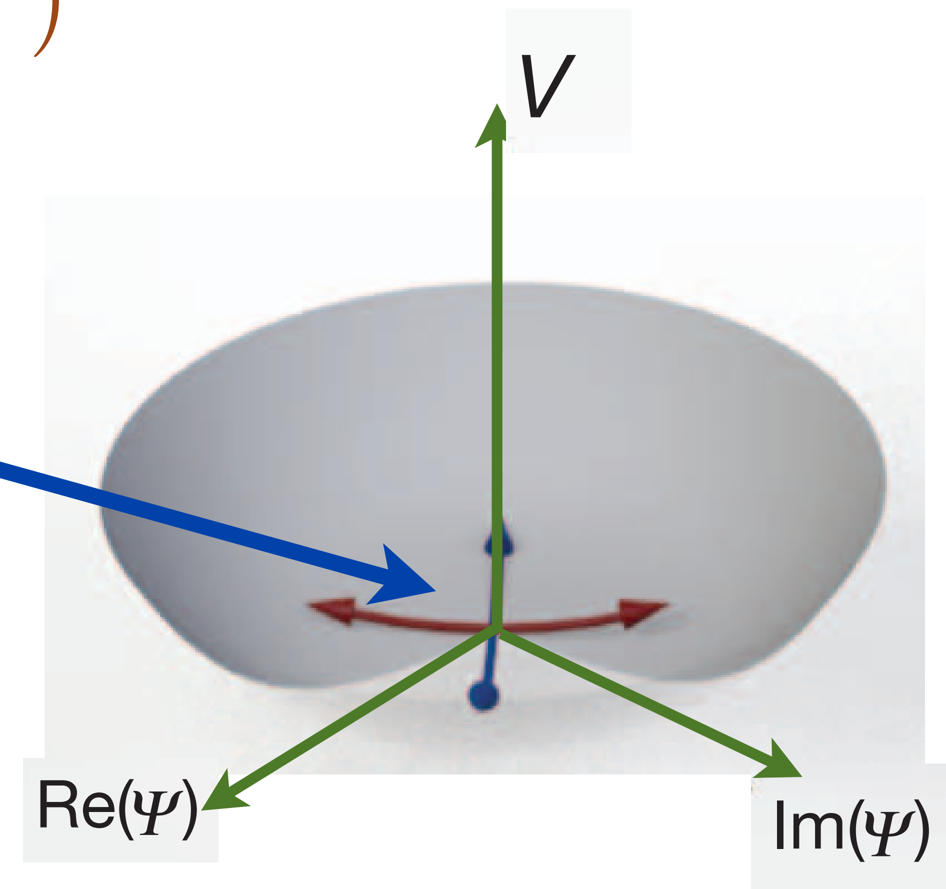
Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0

λ_c

$\lambda \sim U/t$

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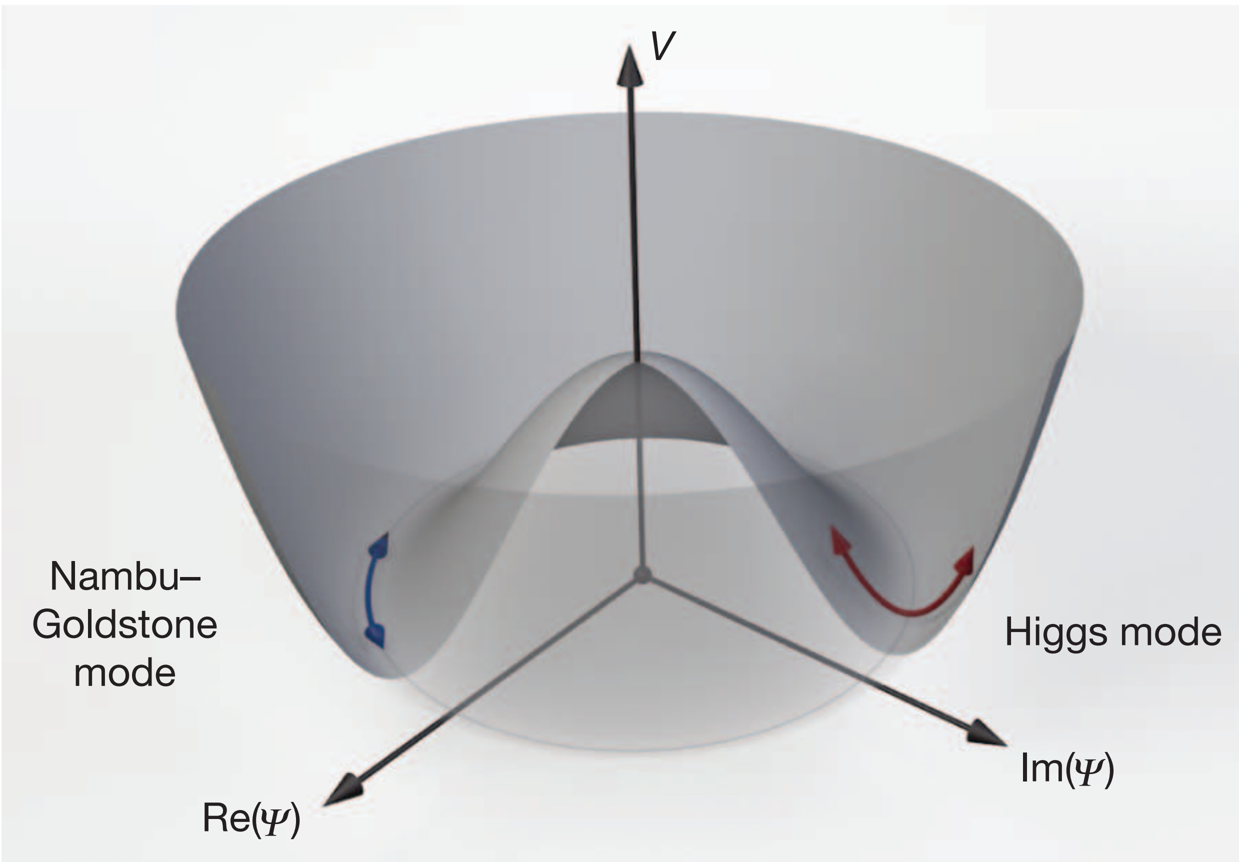

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0

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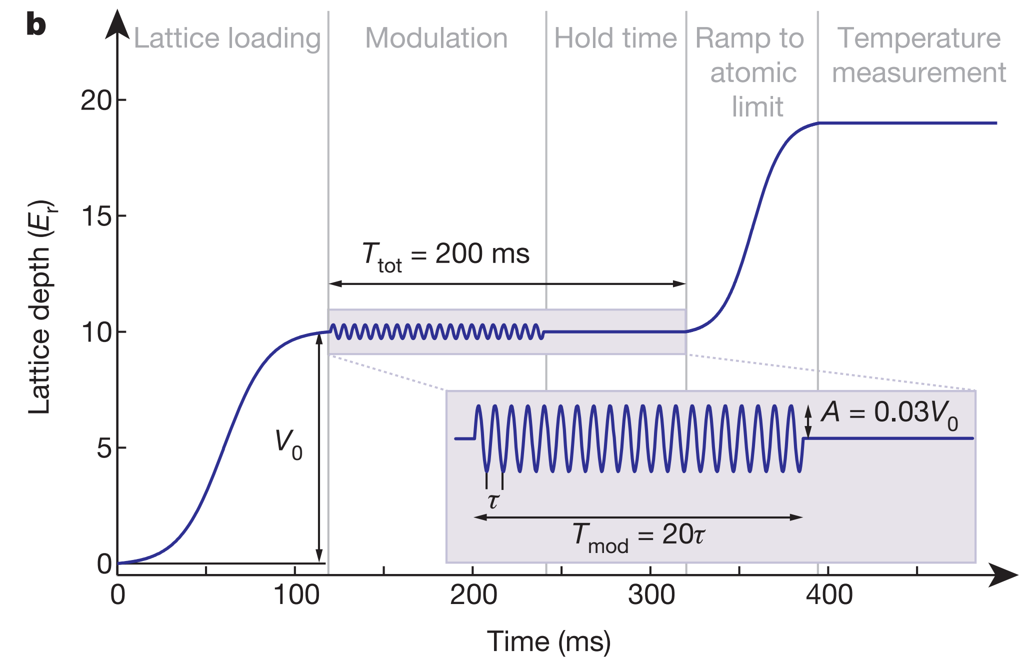
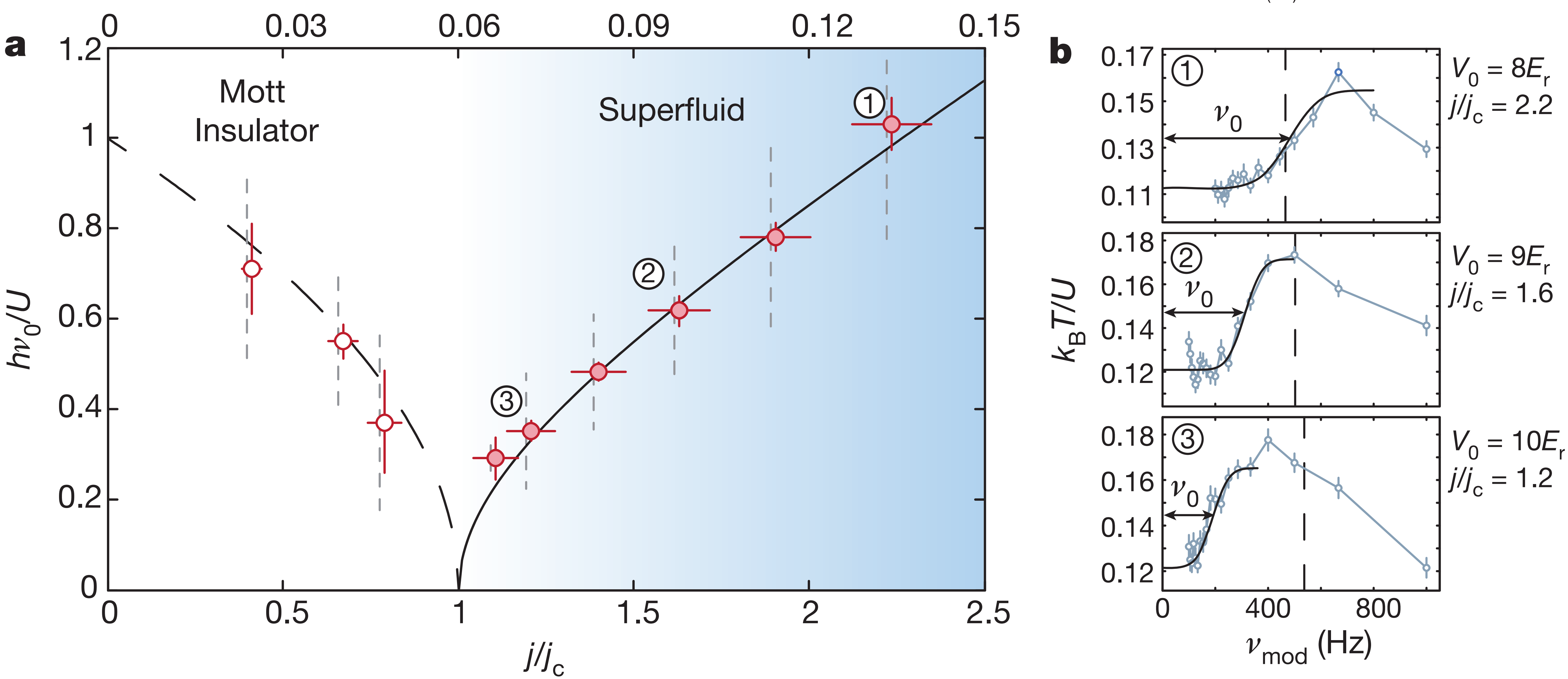
$\lambda \sim U/t$

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole





Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

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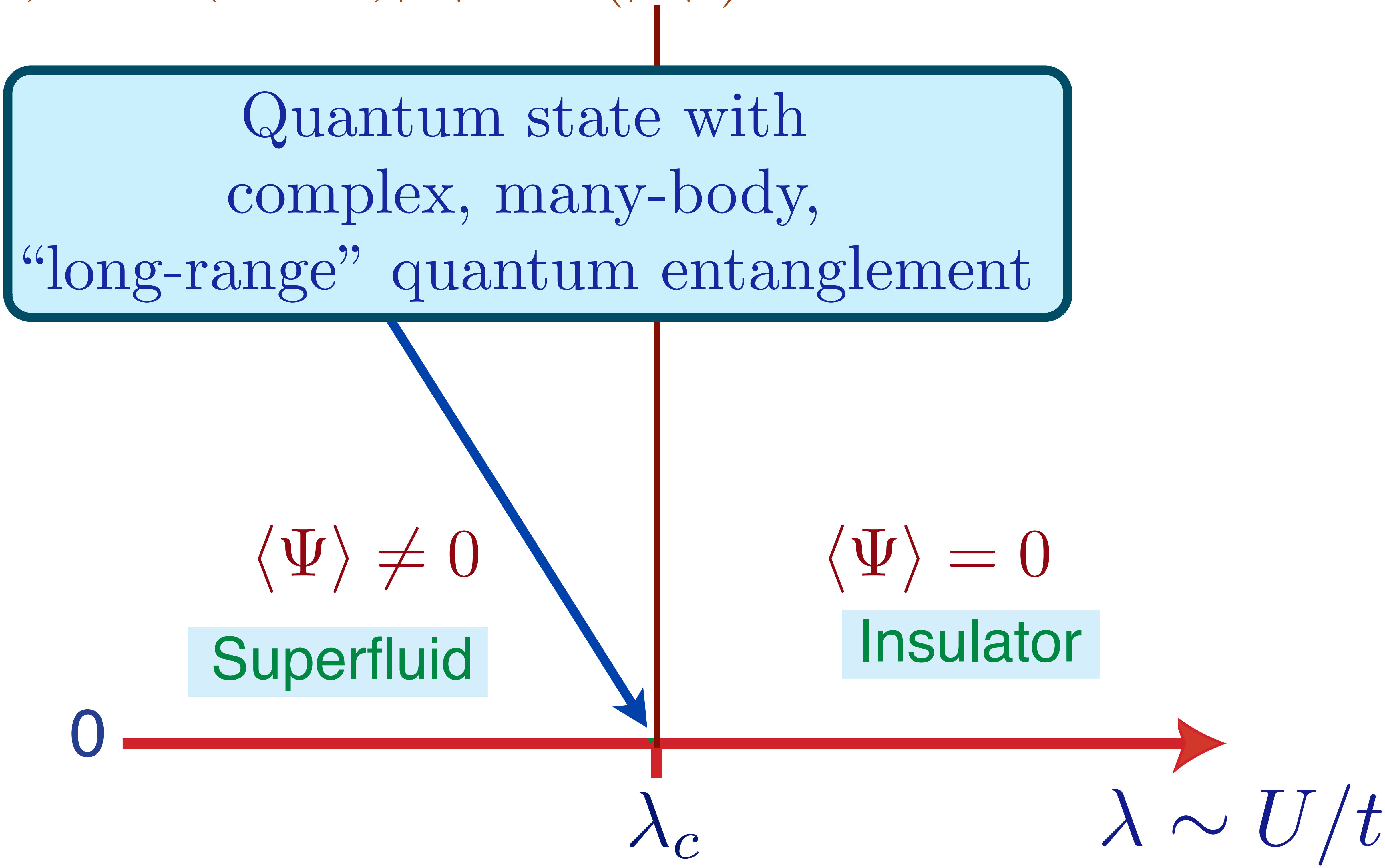

Characteristics of quantum critical point

- Long-range entanglement

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- No quasiparticles - no simple description of excitations.

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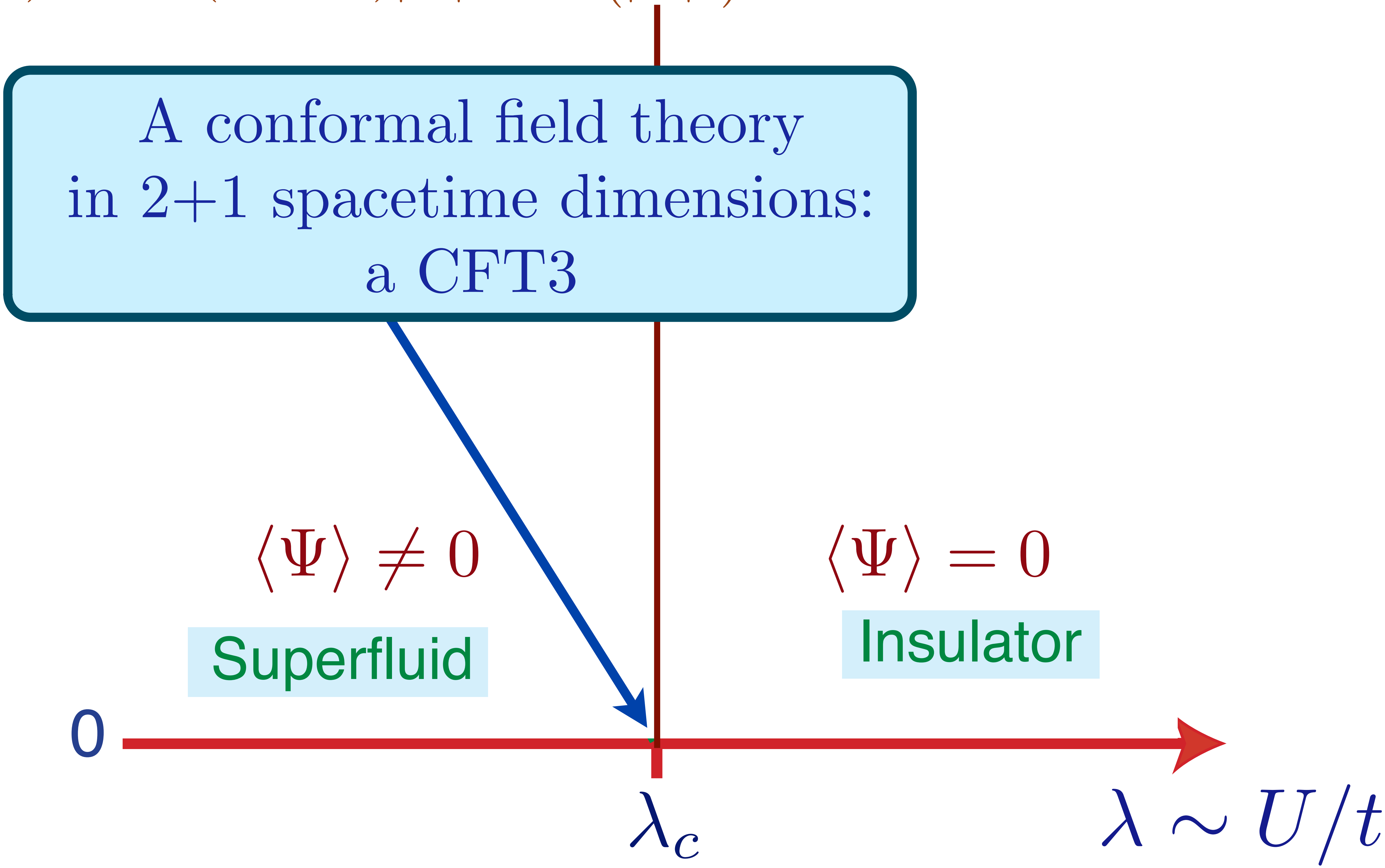
- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

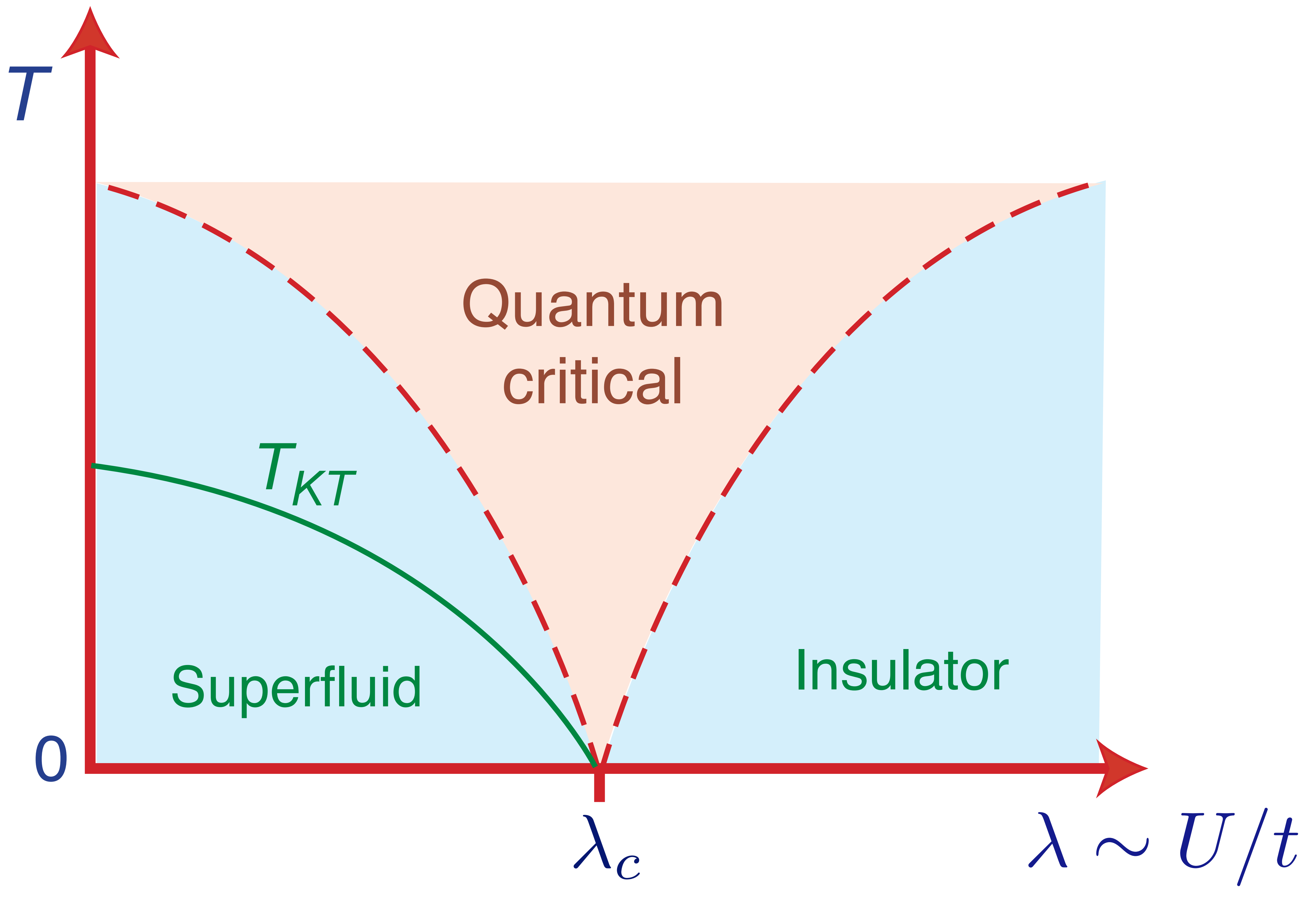
Characteristics of quantum critical point

- Long-range entanglement
- No quasiparticles - no simple description of excitations.
- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
- The theory of the critical point is strongly-coupled because the quartic-coupling u flows to a renormalization group fixed point (the Wilson-Fisher fixed point). This fixed point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a **CFT3**

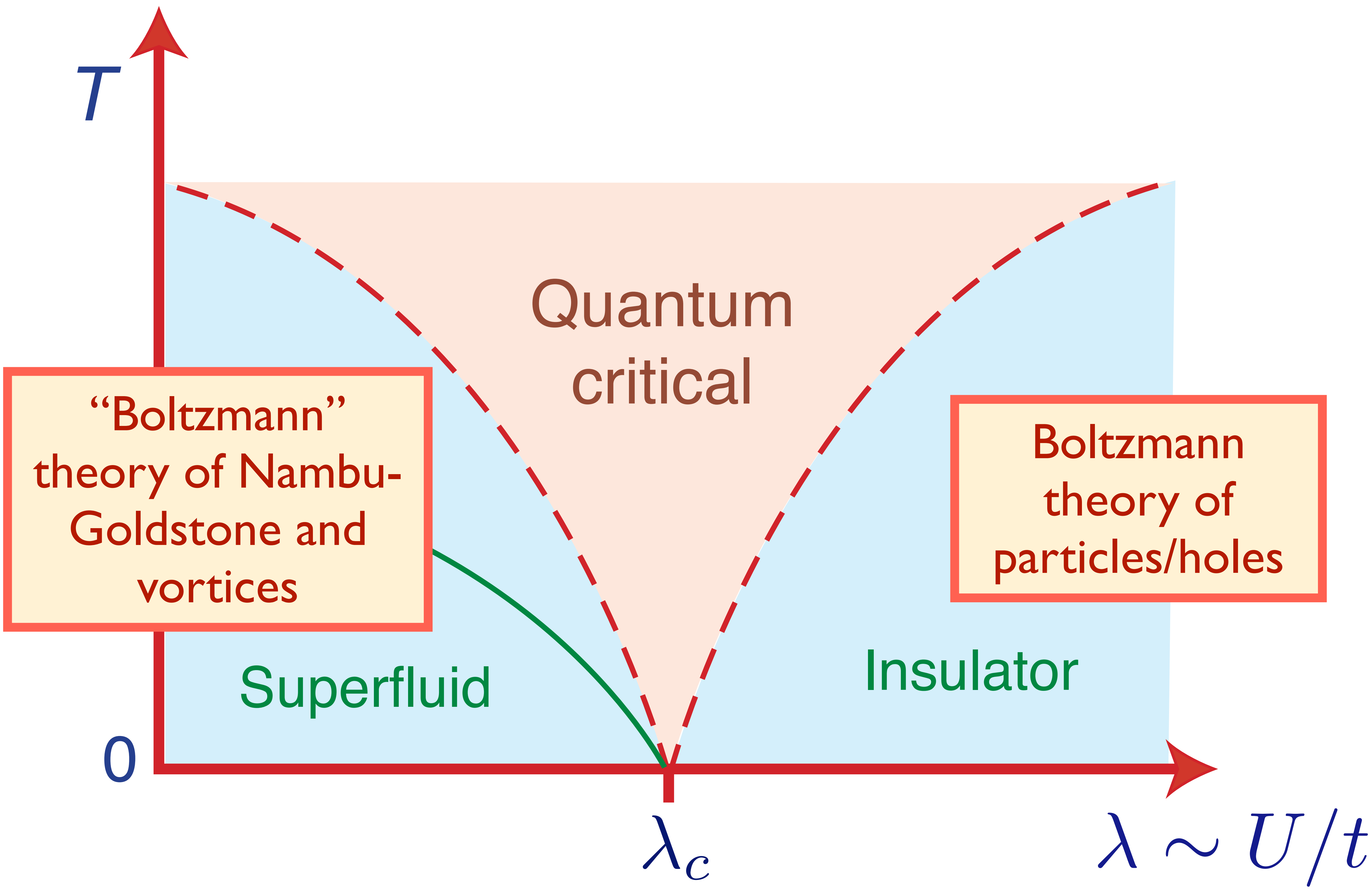
$$\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$

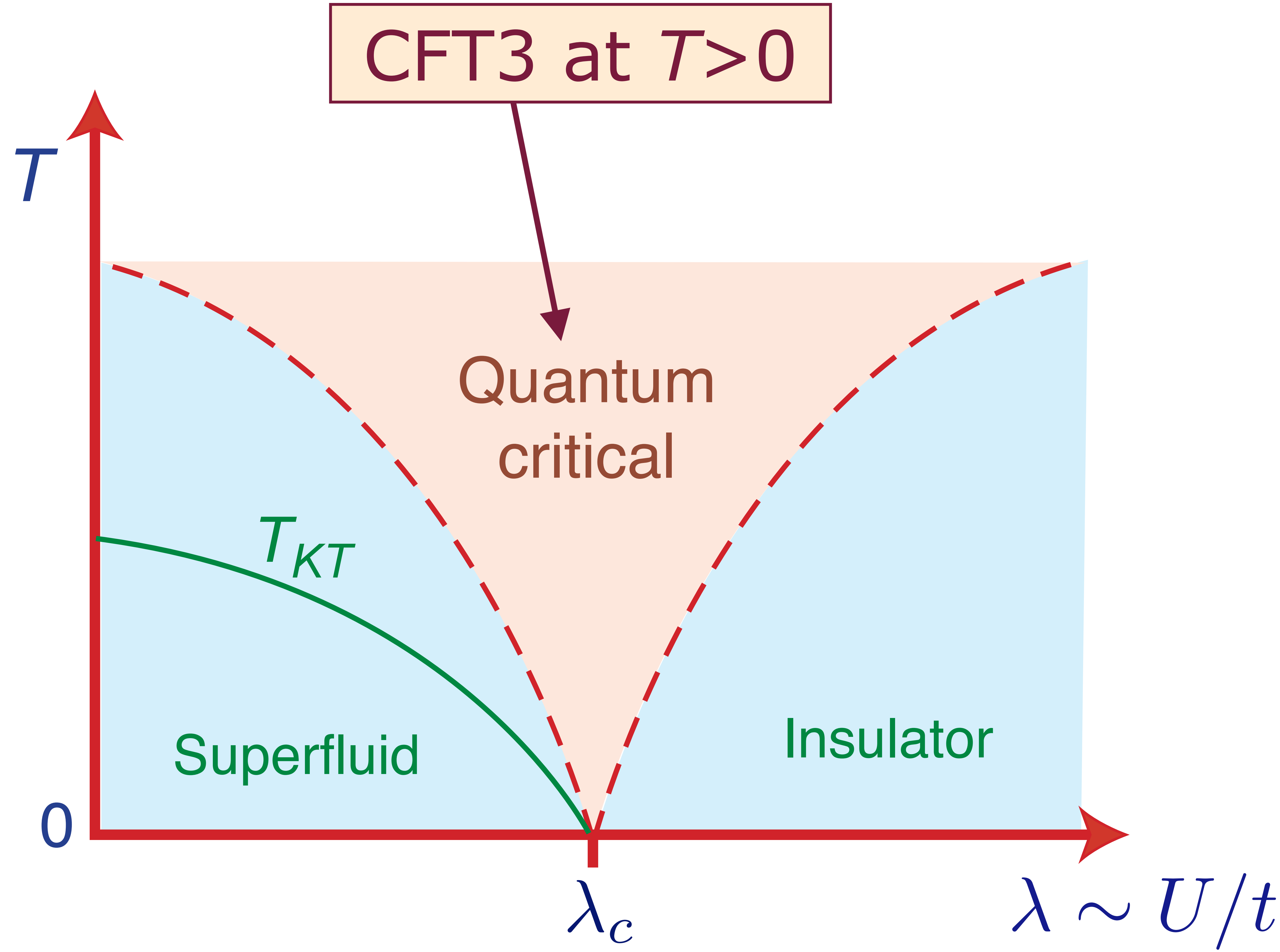




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$



T
“Boltzmann” theory of Nambu-Goldstone and vortices
Quantum critical
Boltzmann theory of particles/holes
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$


CFT3 at $T>0$
T
Quantum
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T_{KT}
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0
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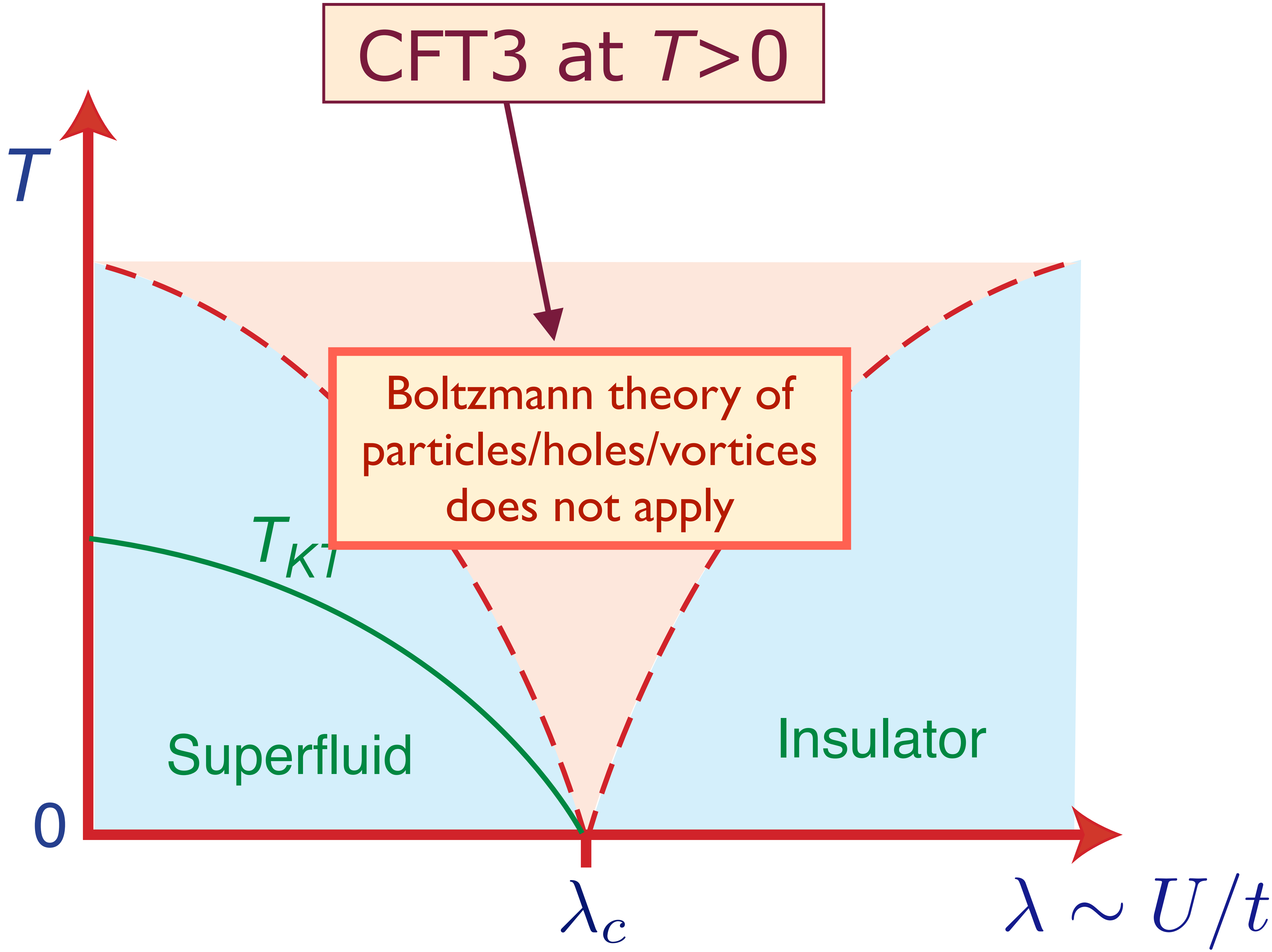
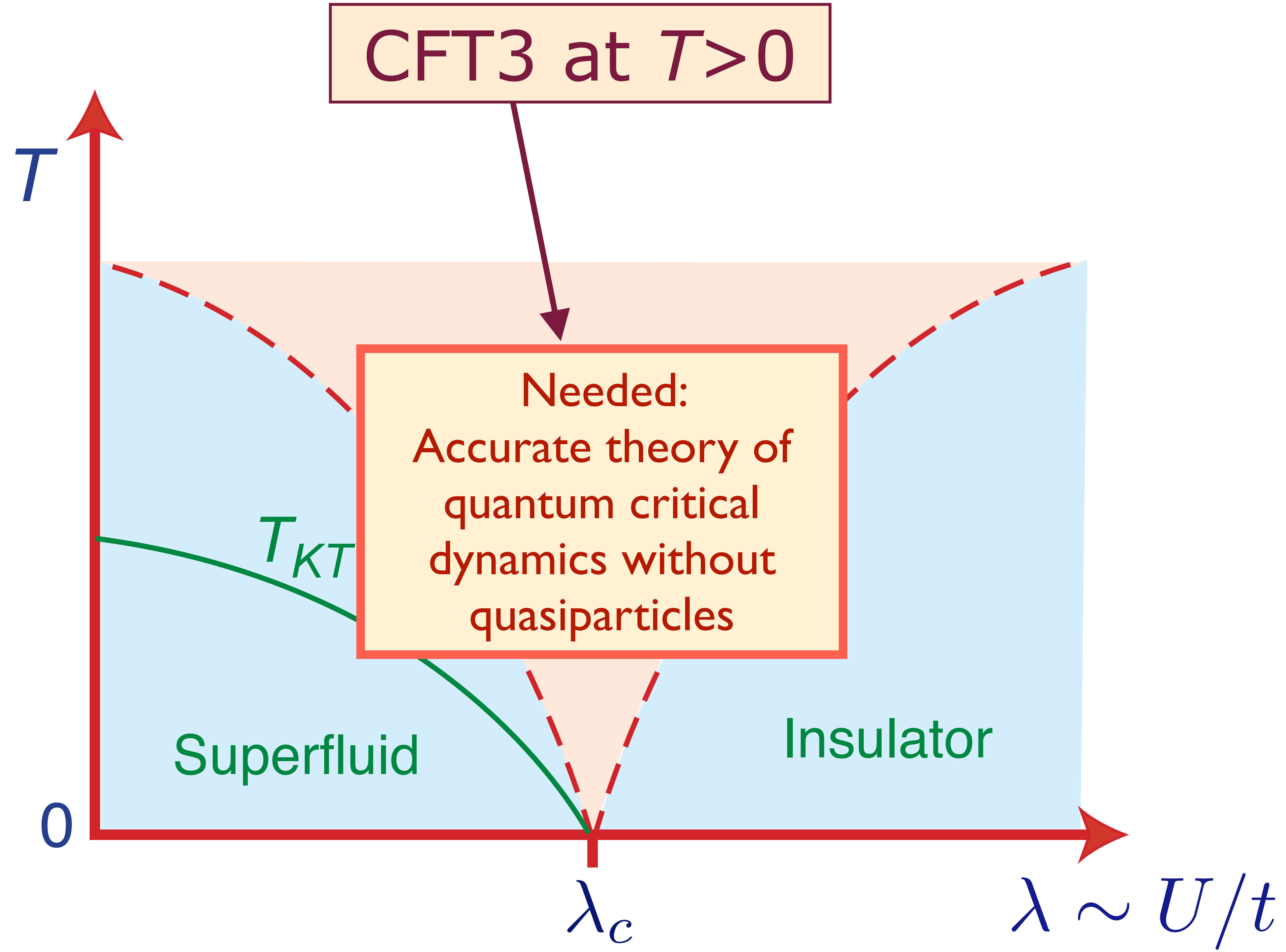
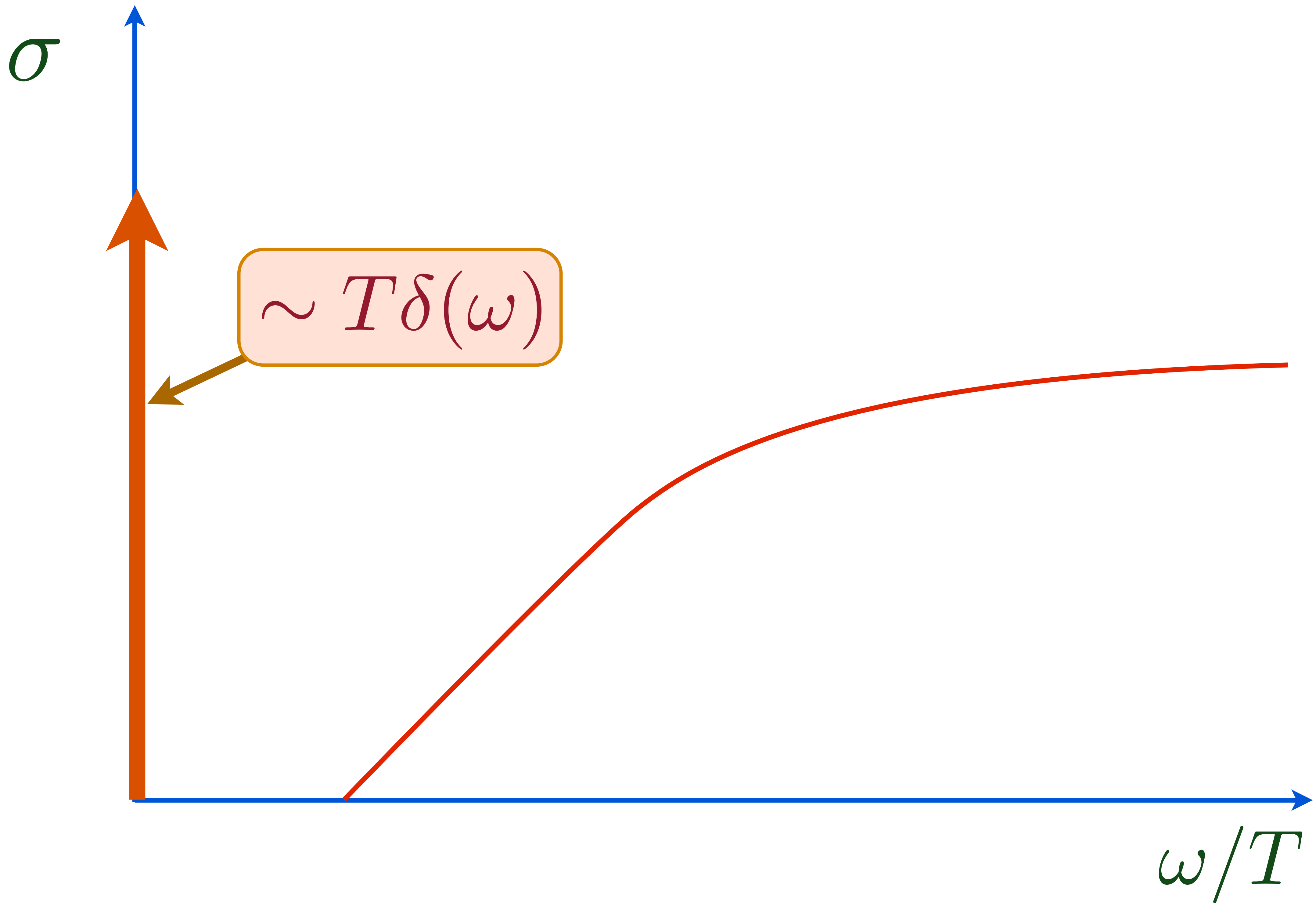
$\lambda \sim U/t$



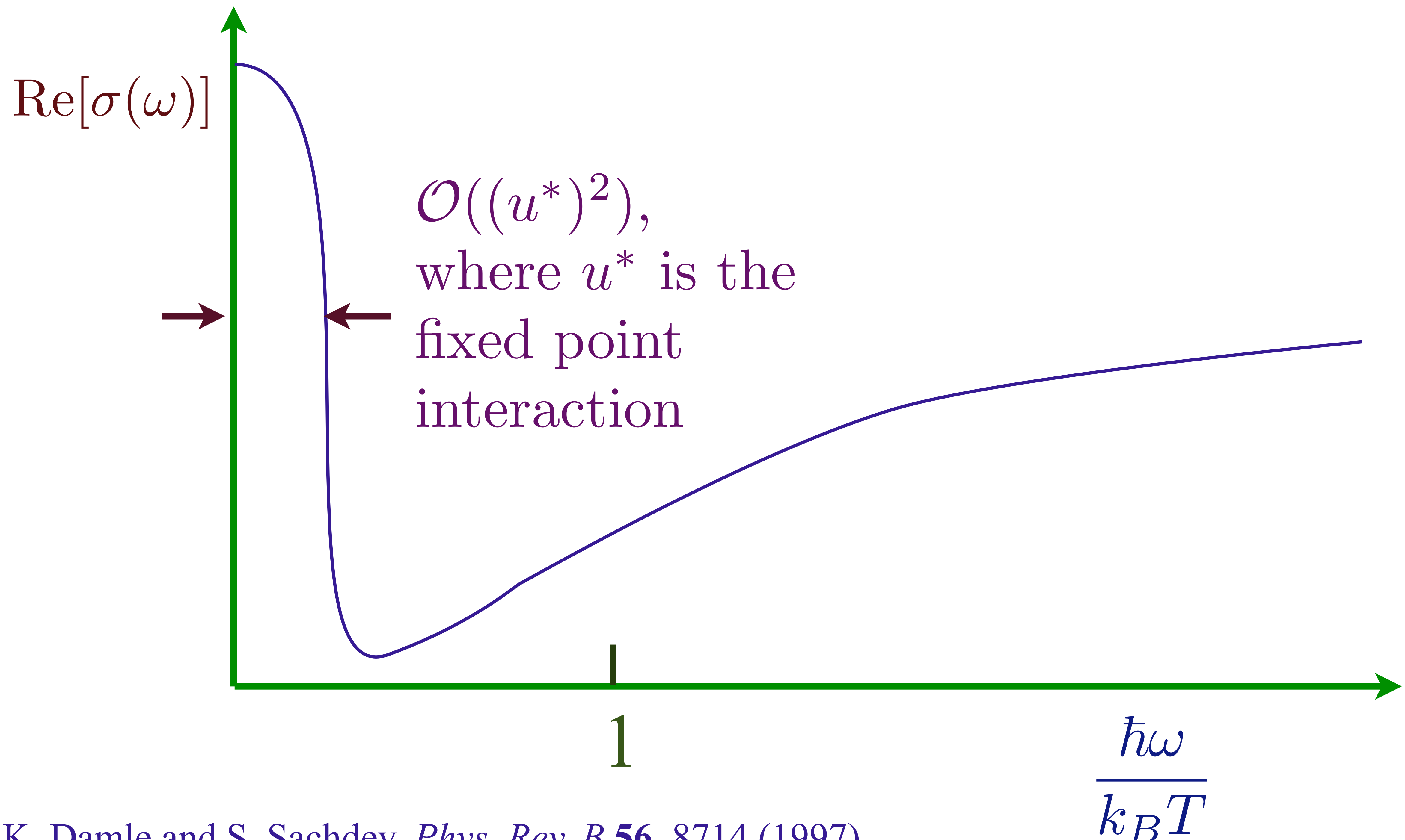
CFT3 at $T > 0$
T
Boltzmann theory of particles/holes/vortices does not apply
T_{KT}
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$


CFT3 at T>0
T
Needed:
Accurate theory of
quantum critical
dynamics without
quasiparticles
T_{KT}
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$

Electrical transport in a free quasiparticle CFT3 for *T > 0*

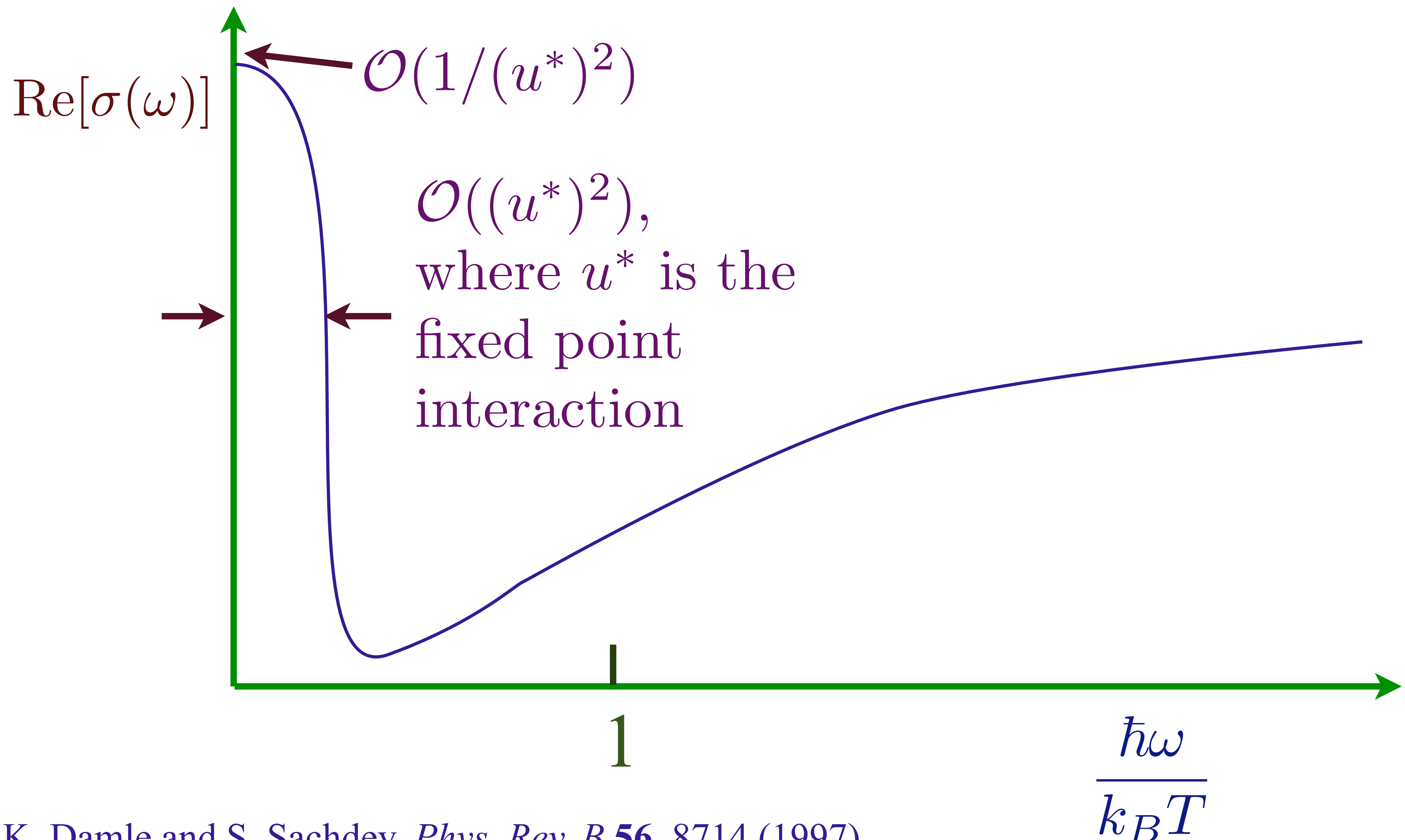


Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



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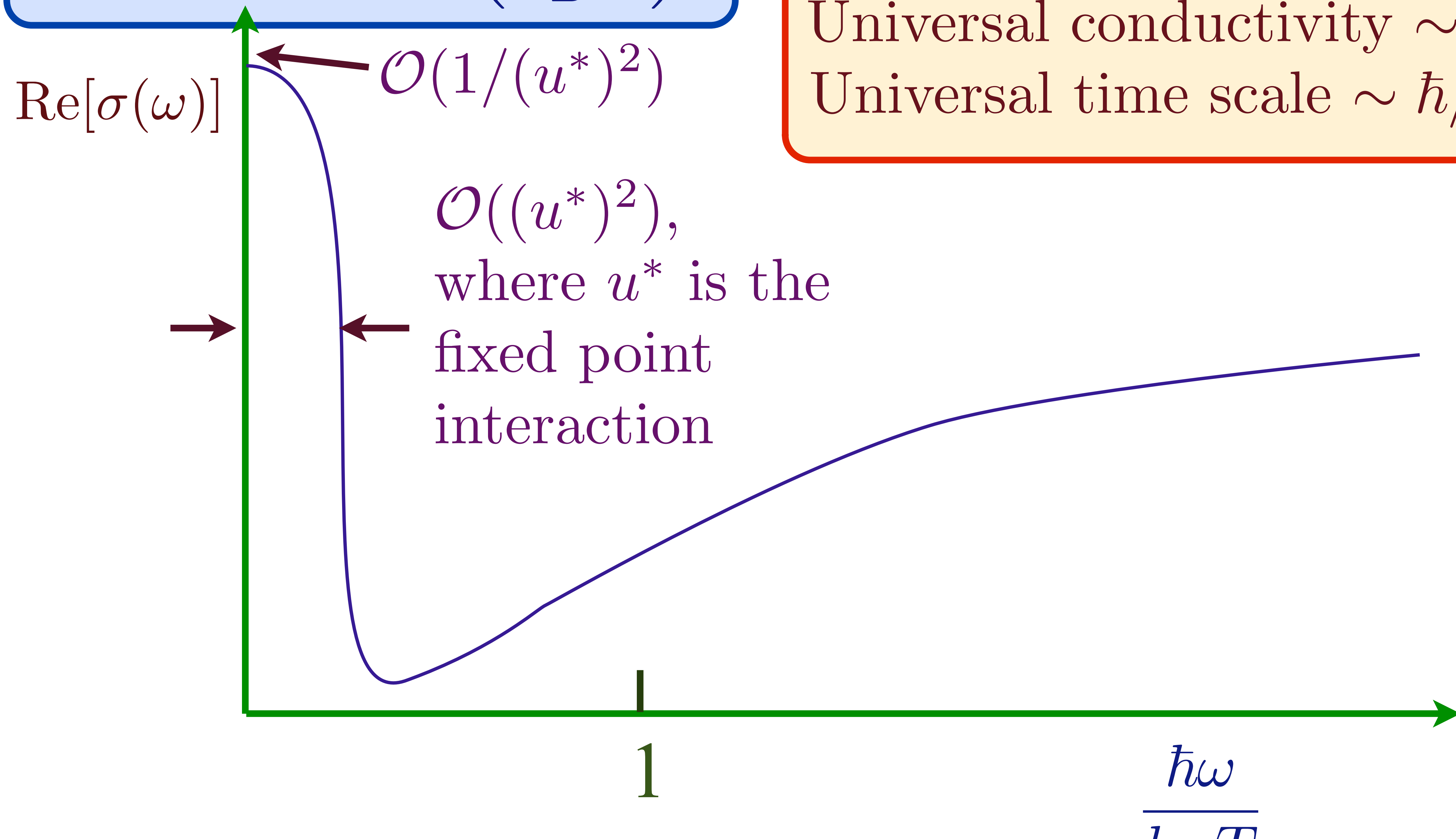
$\Sigma \to$ a universal function

Universal conductivity $\sim e^2/h$

Universal time scale $\sim \hbar/k_B T$



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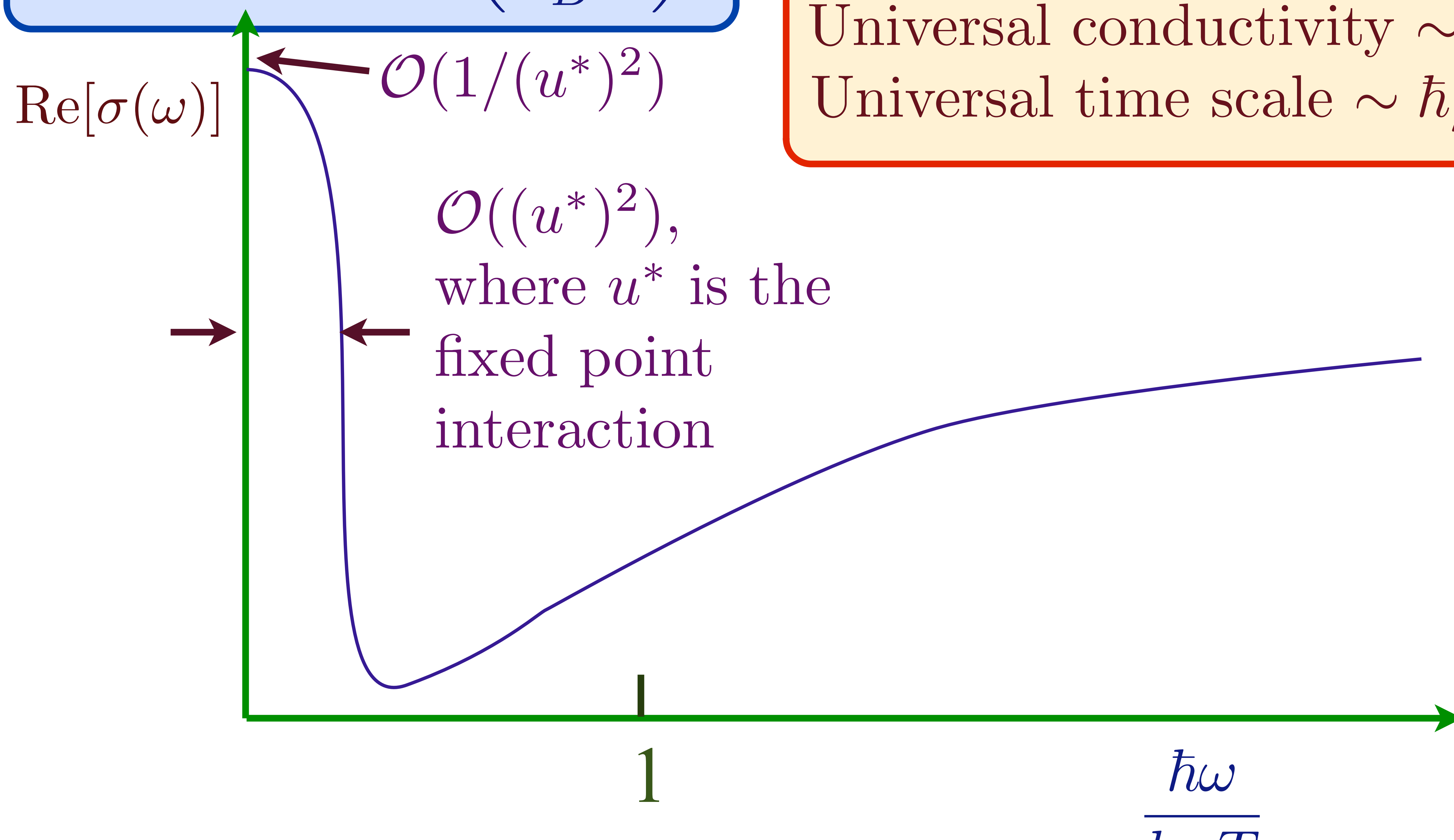

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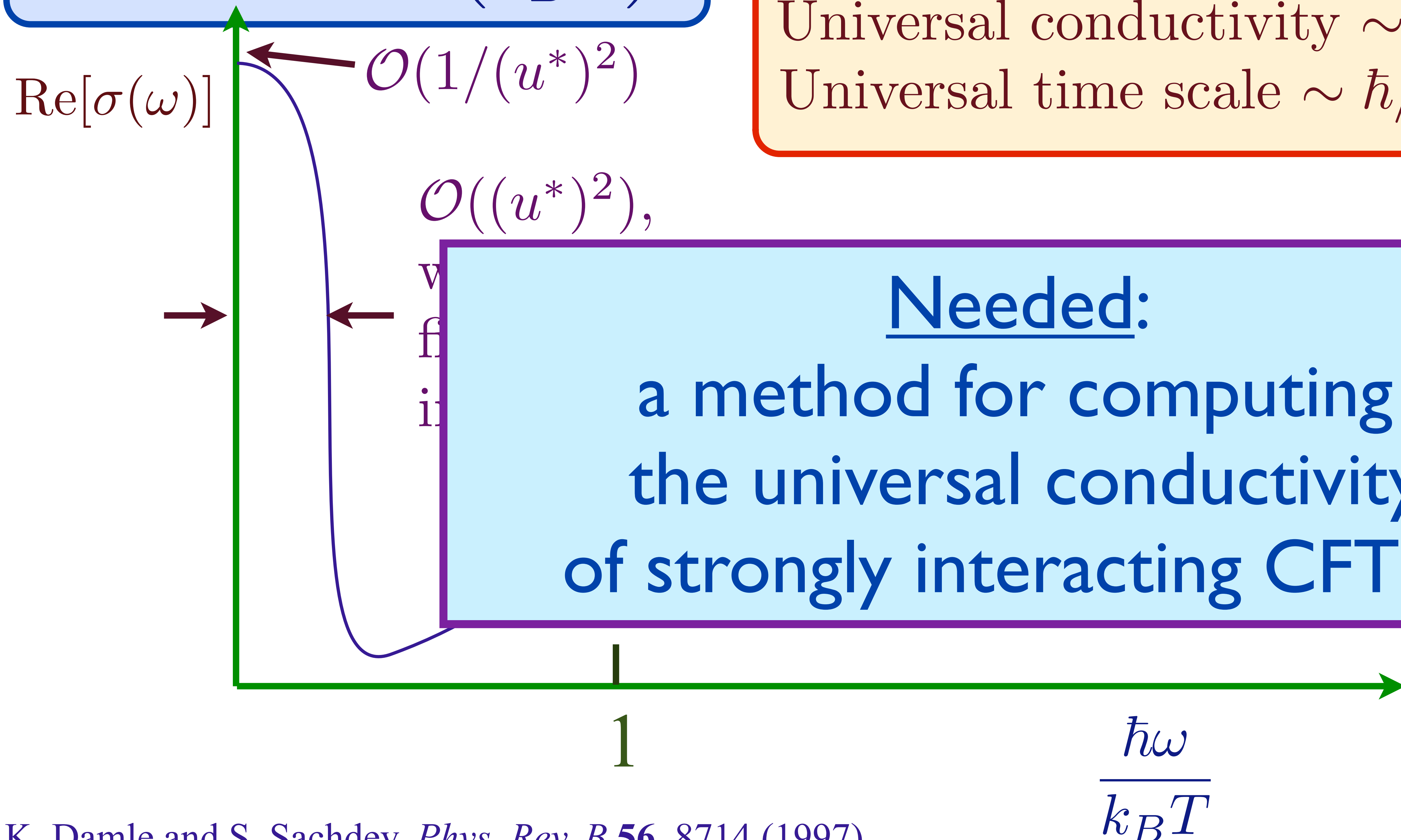

Needed:

a method for computing the universal conductivity of strongly interacting CFT3s

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