Planckian metals and the time reparameterization soft-mode

25th Rencontres Itzykson
Many Body Chaos, Scrambling and Thermalization in Interacting Quantum Systems
Institut de Physique Theorique, Saclay
June 3, 2021

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PHYSICS
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1. SYK models
2. Time reparameterization soft mode
3. Random t-J model

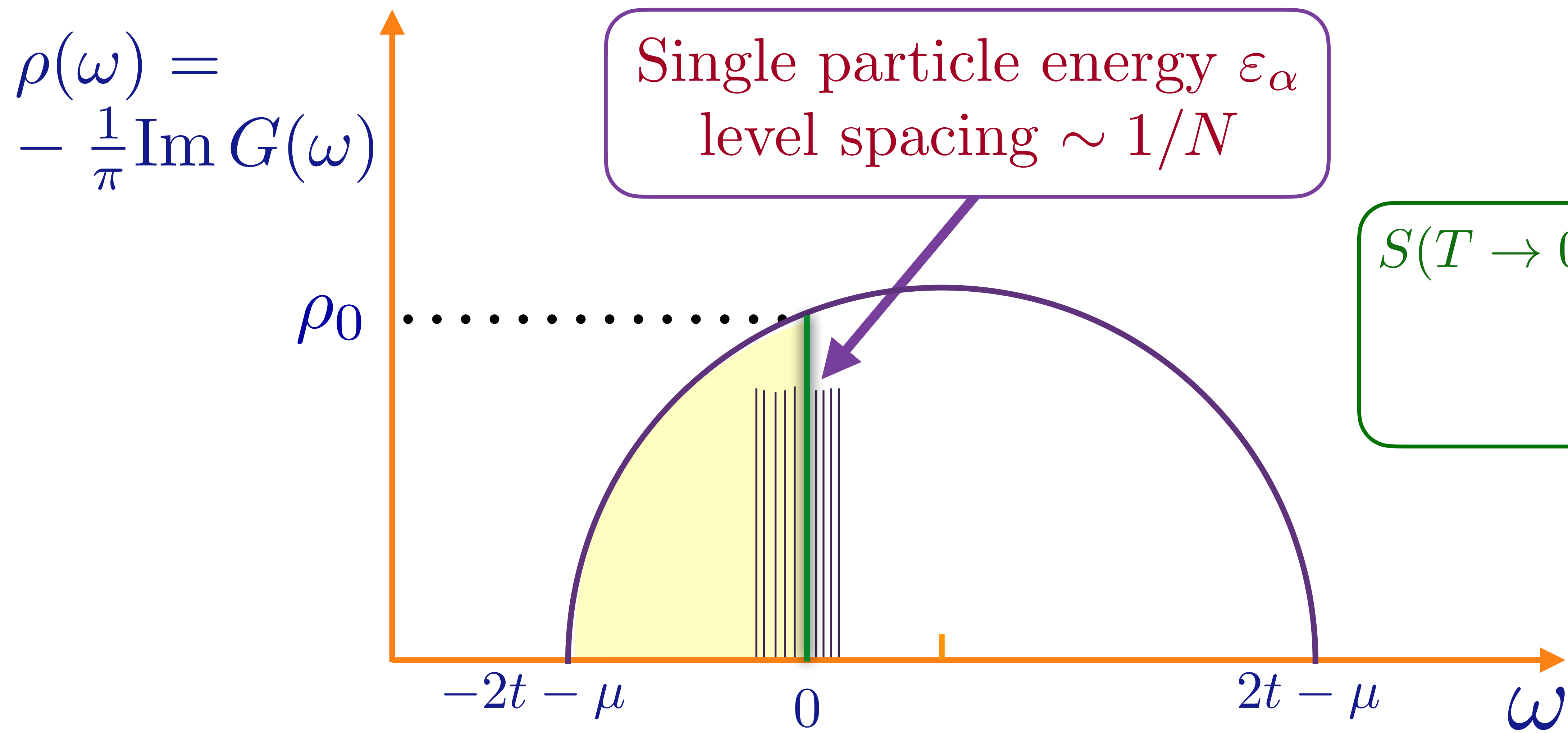
A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$



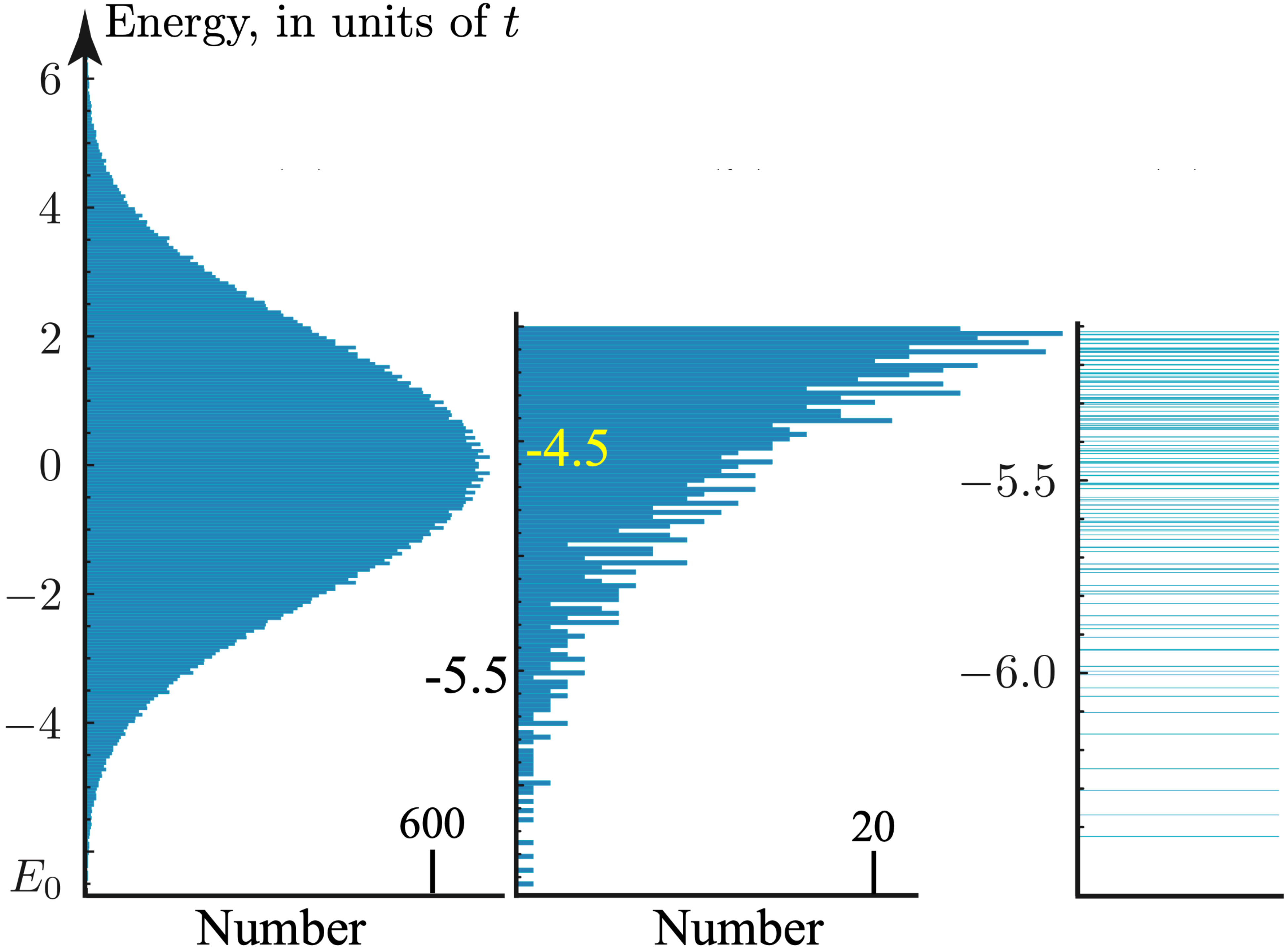
$S(T \to 0) = N\gamma T$

$\gamma = \frac{\pi^2}{3}\rho_0$

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number



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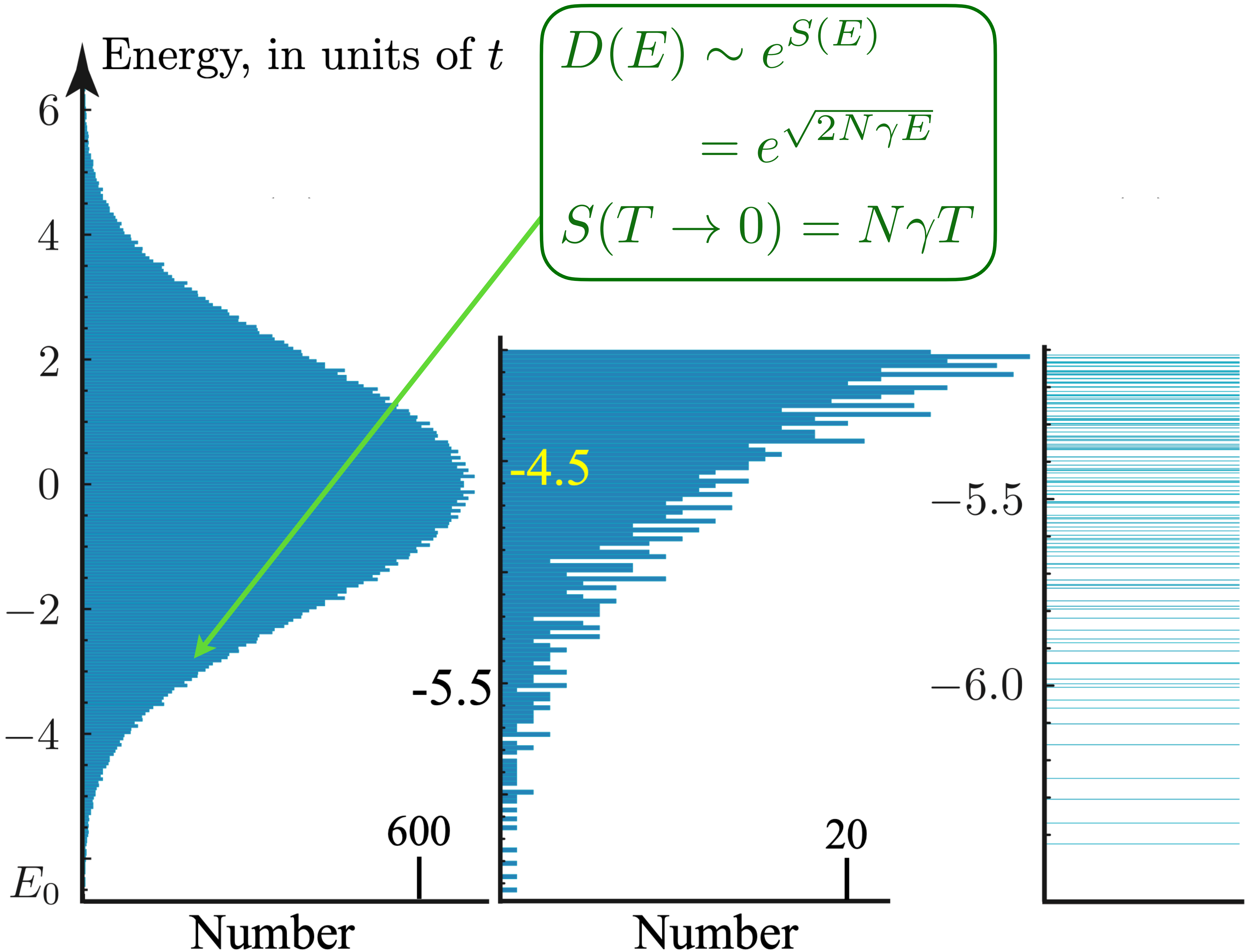

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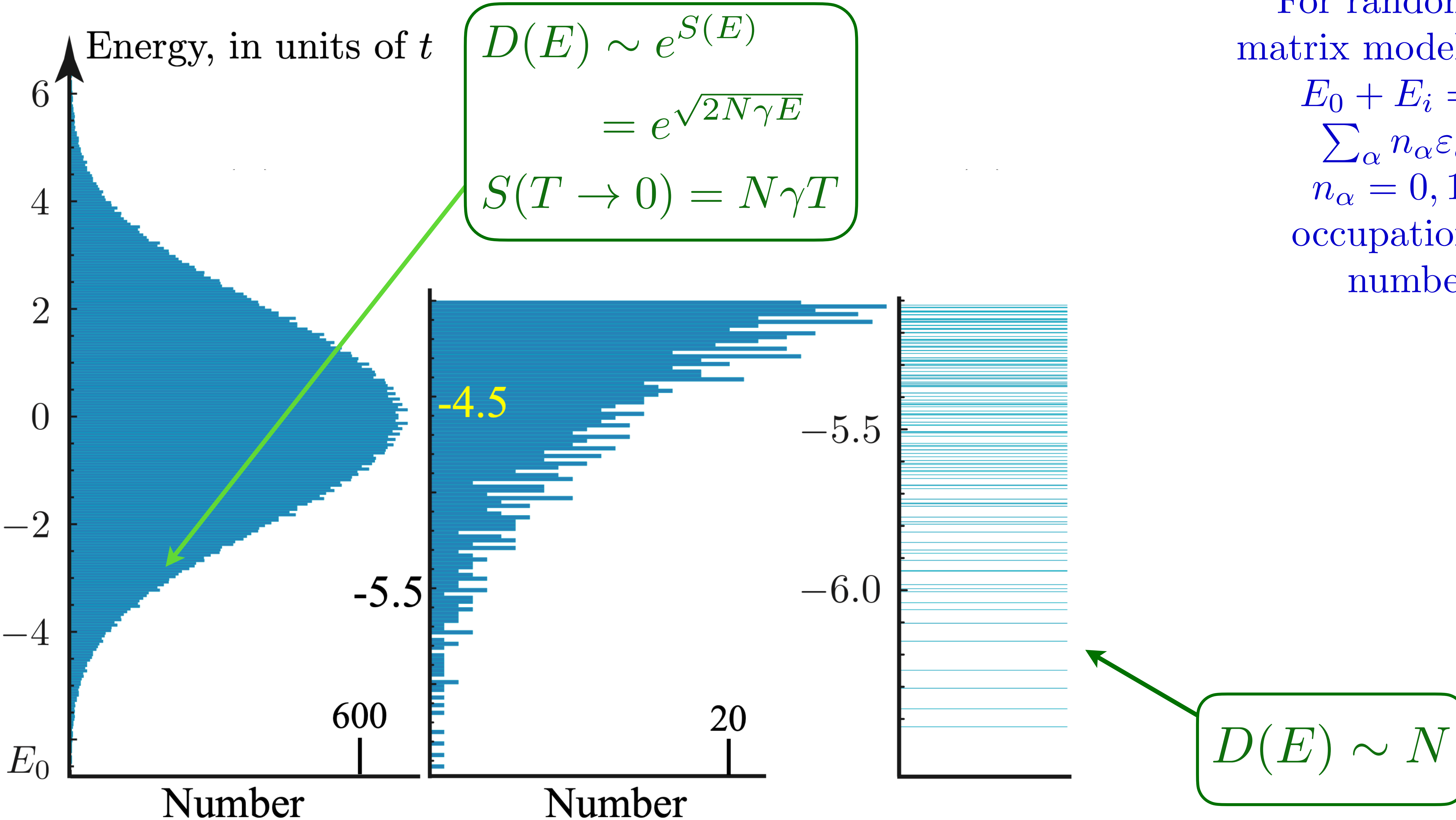


Random matrix model

The Sachdev-Ye-Kitaev (SYK) model

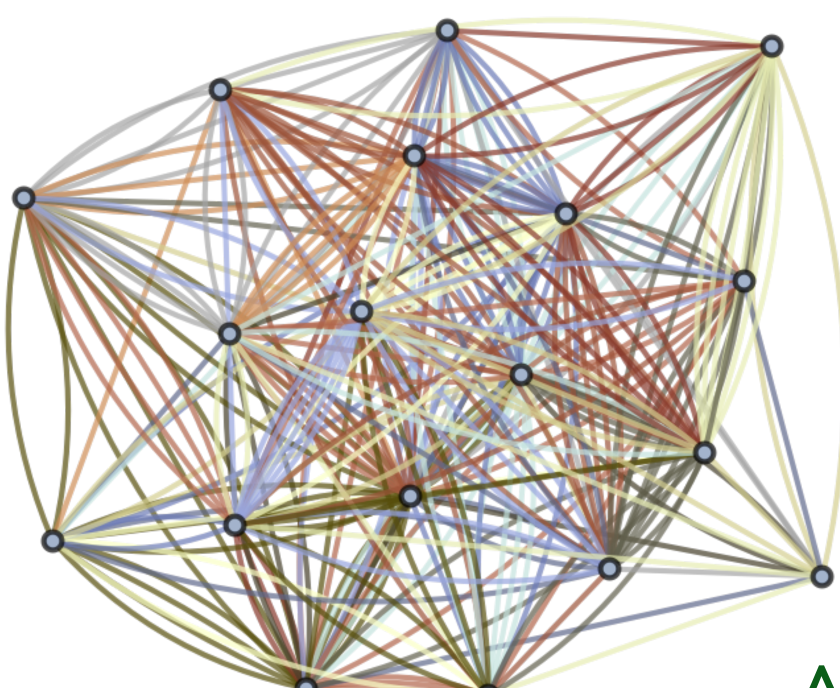
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

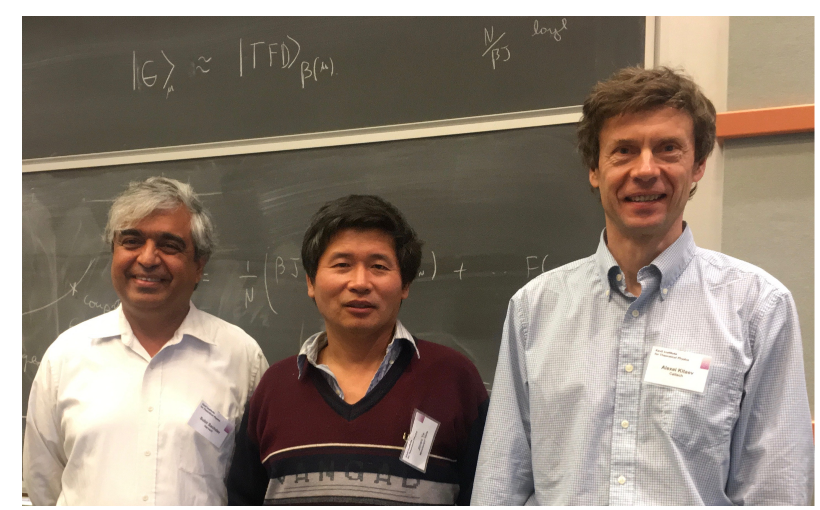
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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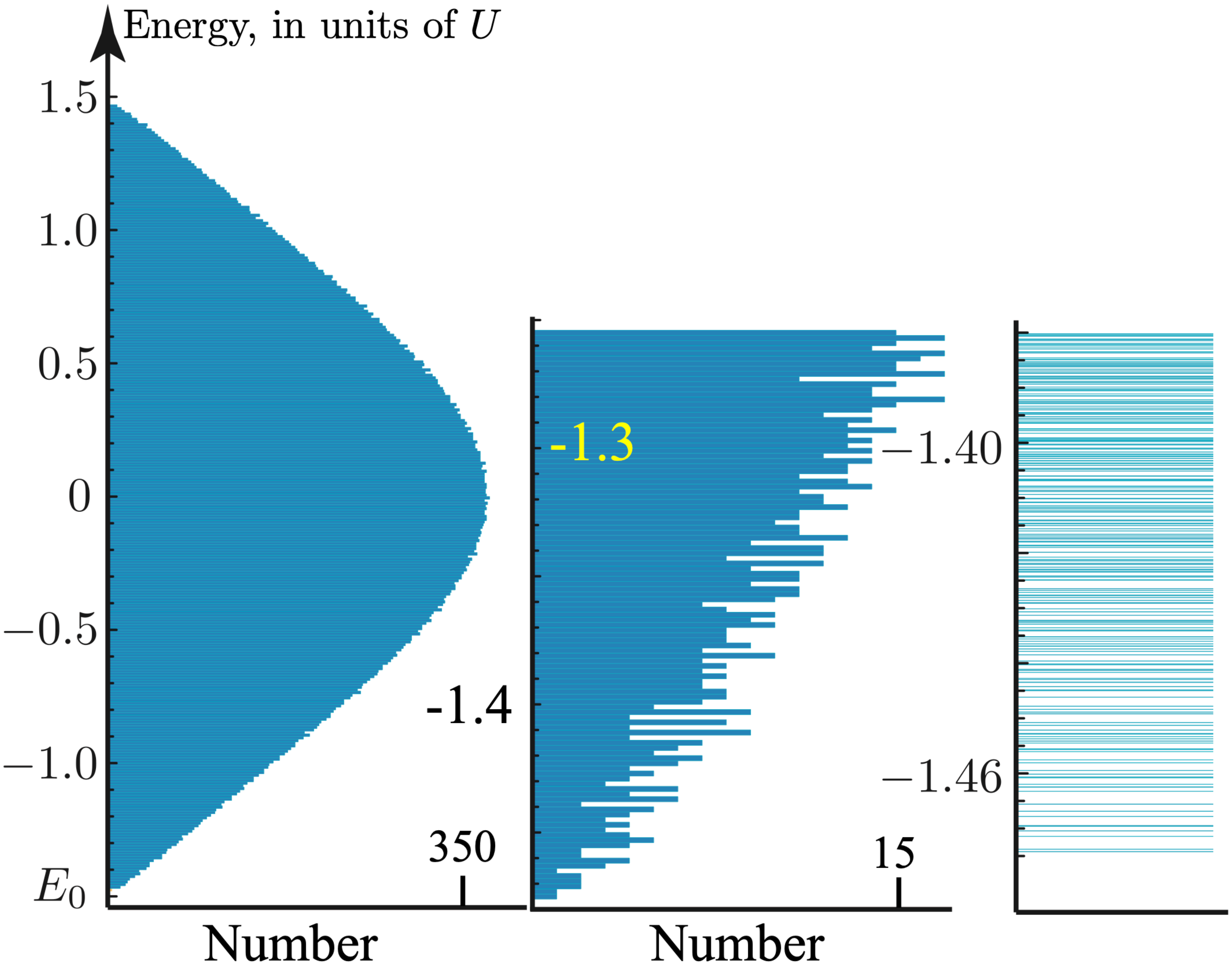
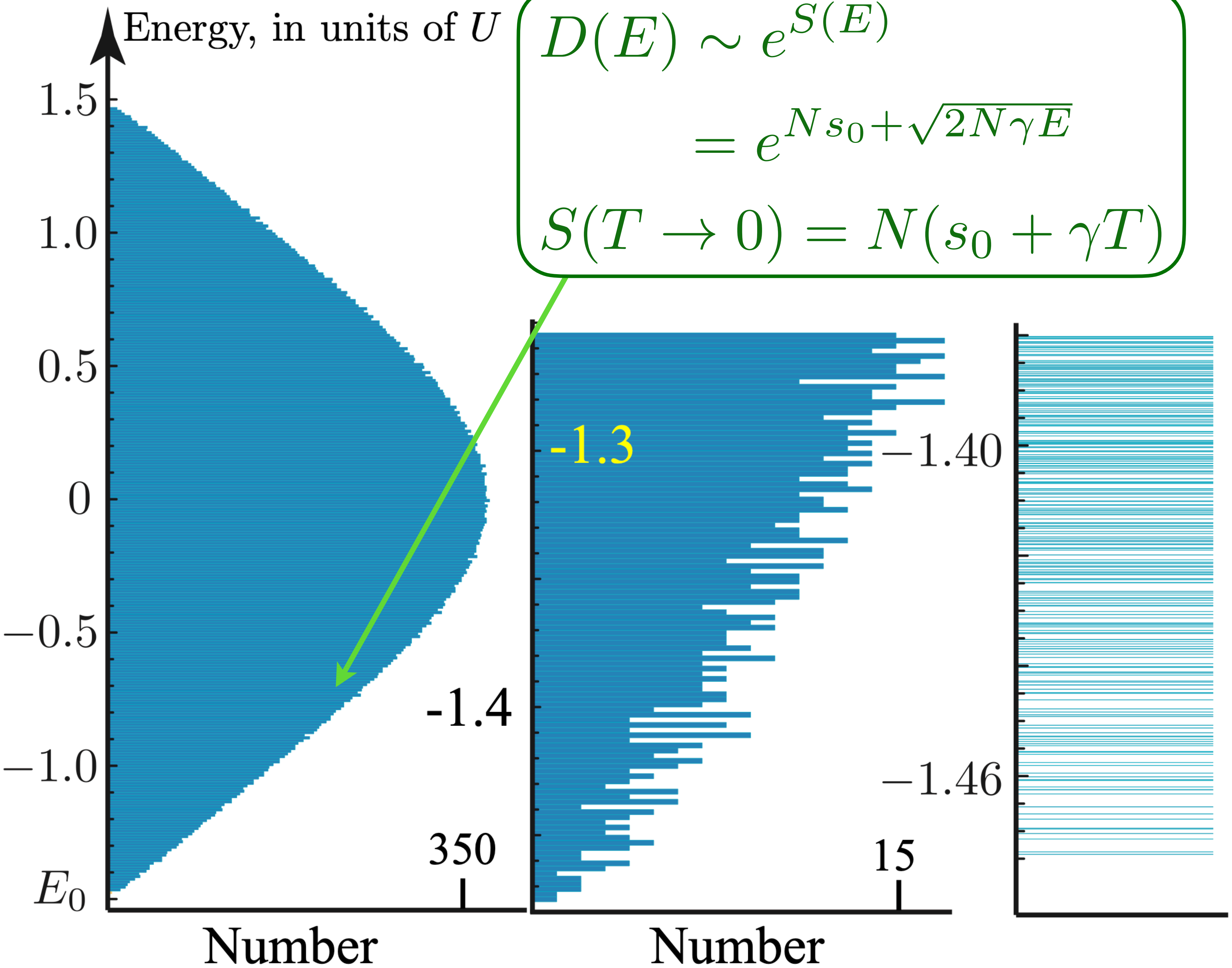


Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2 N \gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$



$s_0 = 0.464848\ldots$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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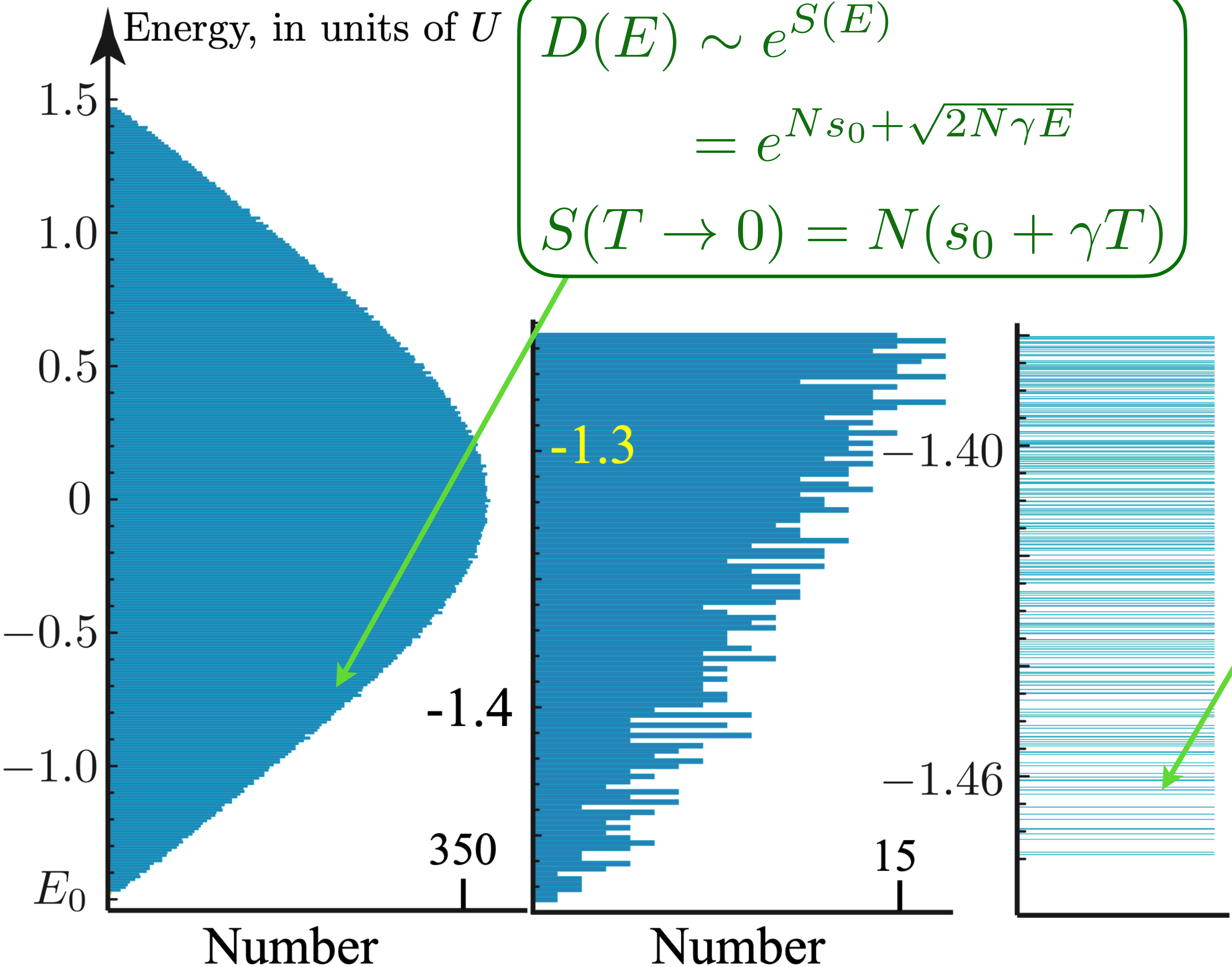

$$D(E) \sim e^{S(E)} = e^{Ns_0 + \sqrt{2N\gamma E}}$$

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$$D(E) \sim 2\, e^{Ns_0} \sqrt{2N\gamma E}$$

No quasiparticle decomposition of many-body states

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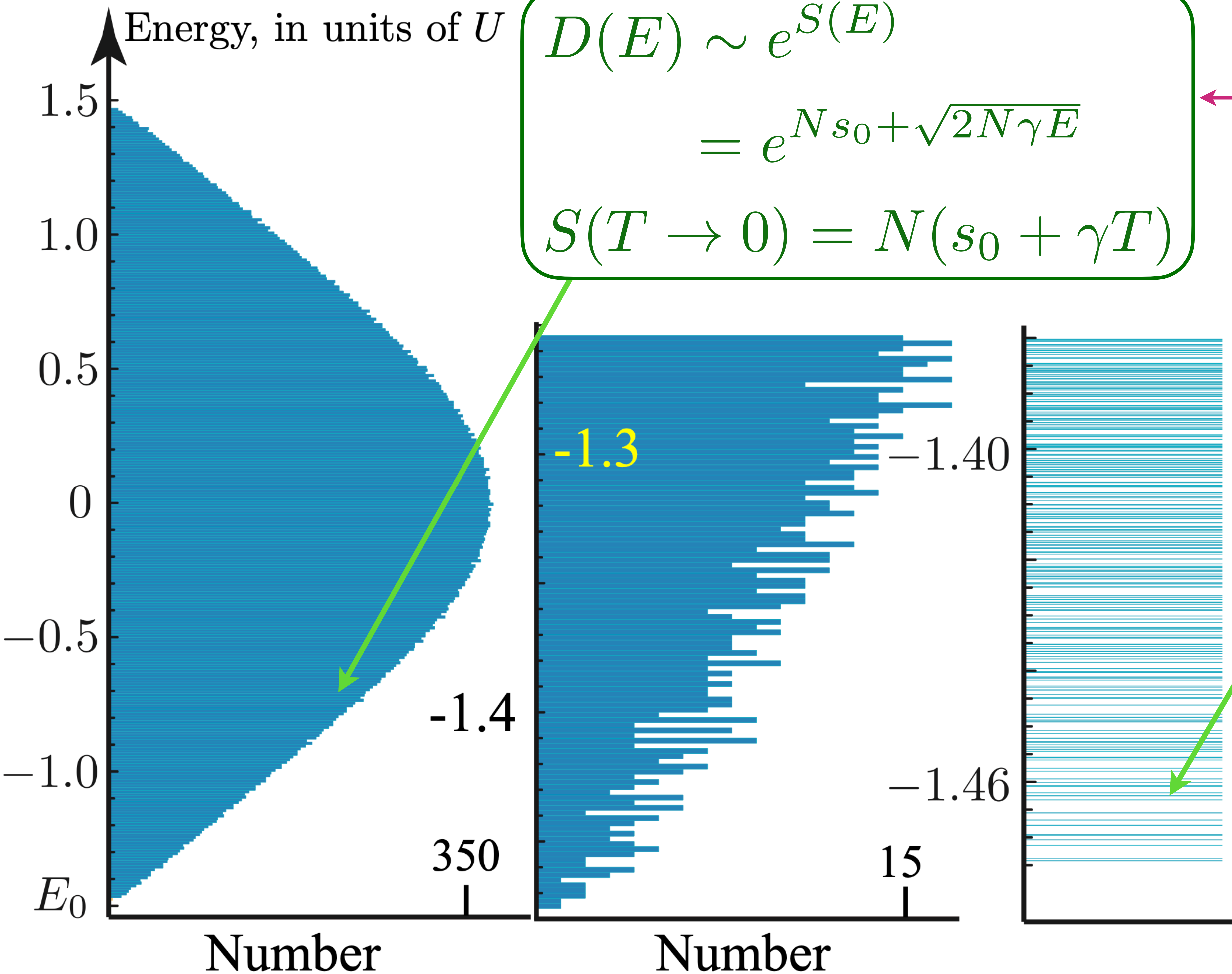

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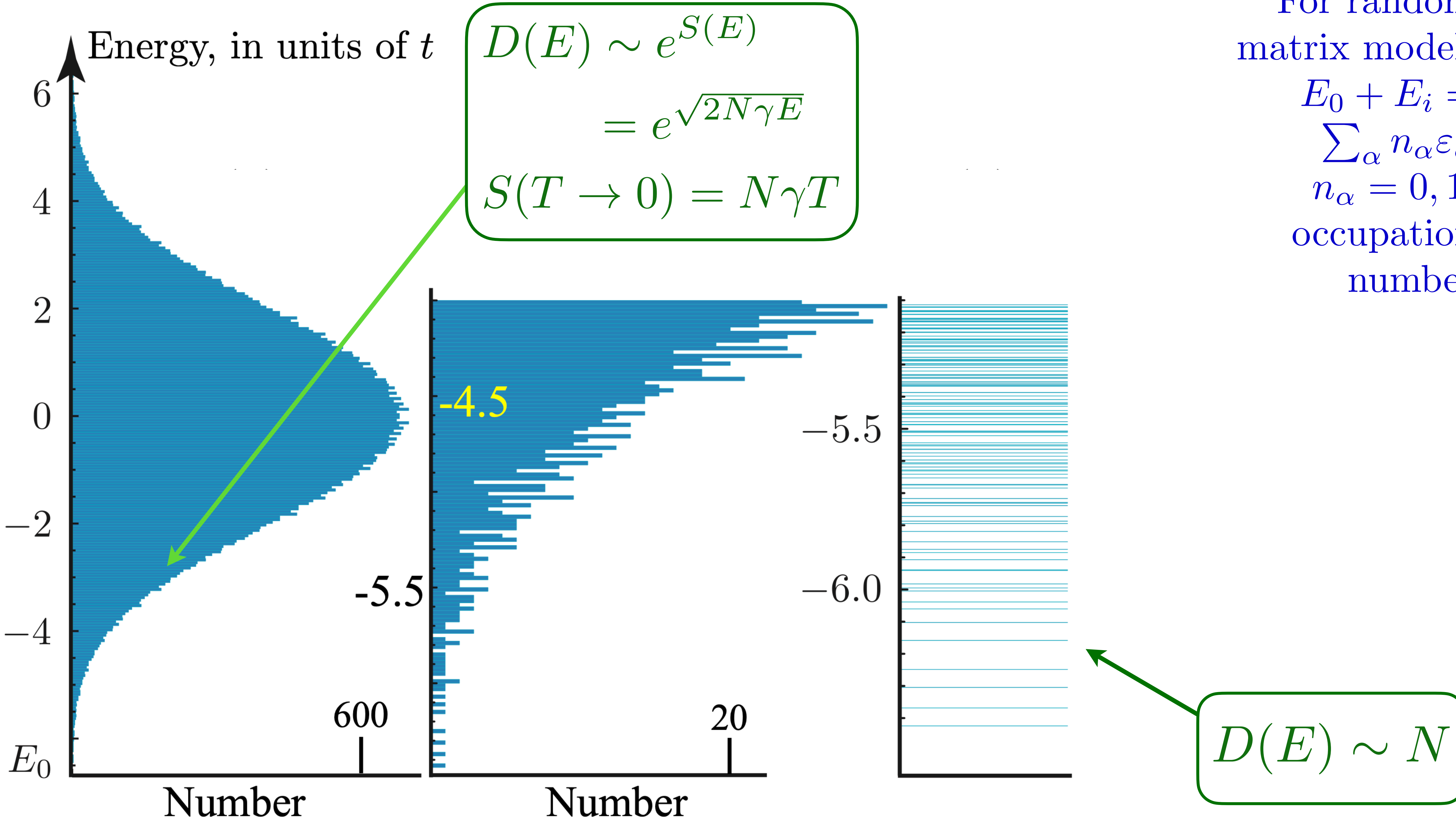

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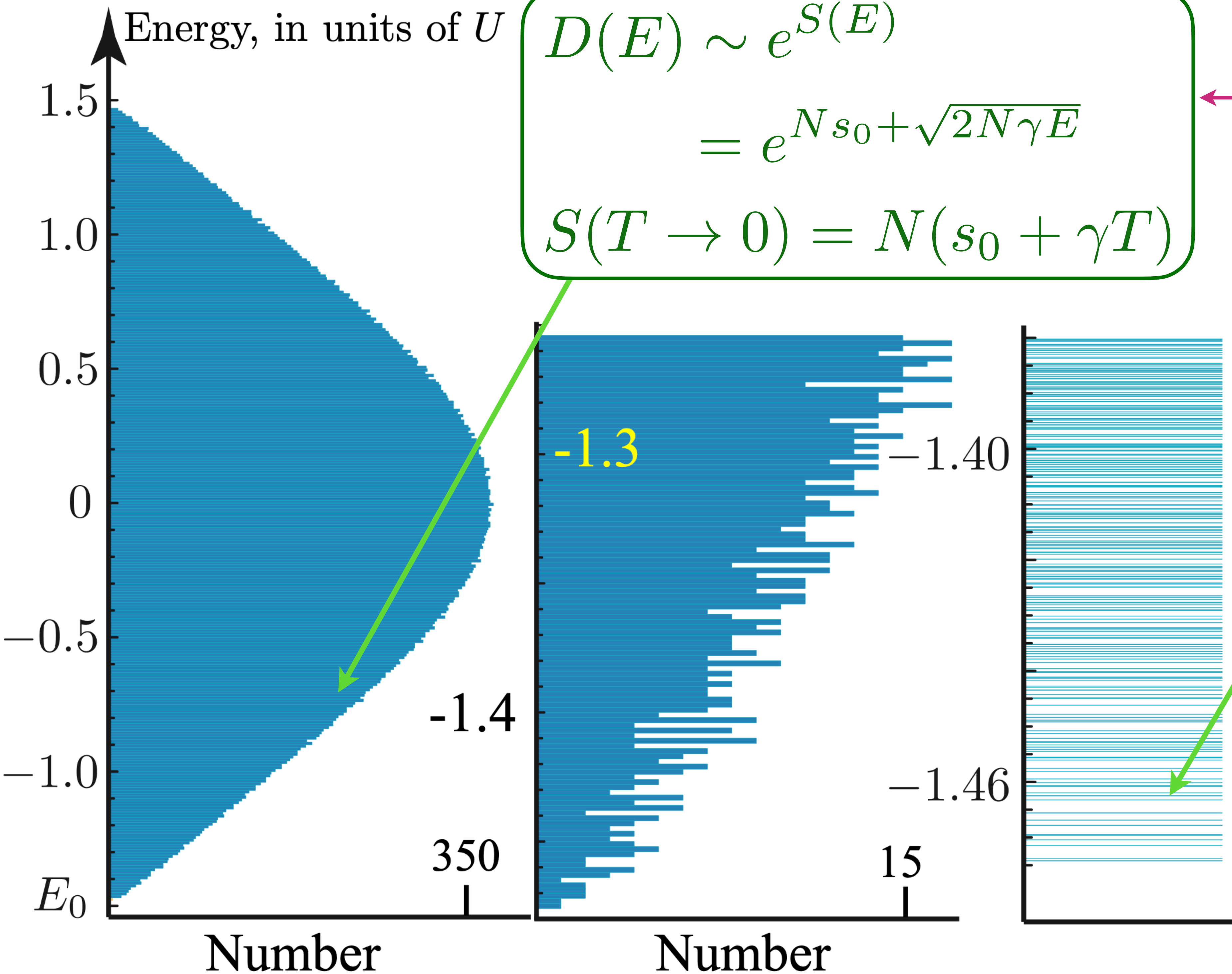

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Complex SYK model

Thermodynamics of quantum black holes:

$$\int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\, \mathcal{S}^{(d+1)}_{\text{Einstein gravity}}[g_{\mu\nu}]\right)$$

$$= \exp\left(S_{BH}\right) \times \left(\quad ....????.... \quad \right)$$

Metric of spacetime

Gibbons, Hawking (1977)

$$S_{BH} = \frac{Ac^3}{4G\hbar}$$

($\hbar/(k_B T)$ is the length of the Euclidean time circle)
A is the area of the black hole horizon.
Interpretation: Black hole entropy is entanglement entropy across the horizon.

SYK model and charged black holes

Thermodynamics of charged quantum black holes

$$\int \mathcal{D} g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein−Maxwell theory}}[g_{\mu\nu}]\right) \quad T \to 0,$$

$$\approx \int \mathcal{D} g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\text{Gravity of AdS}_2\text{ and boundary}}[g_{\mu\nu}]\right)$$

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$$= \exp\left(S_{BH}\right) \times \exp\left(-\frac{1}{T} \times \text{Free energy of SYK model}\right)$$

The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016) ;
Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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$$S(T \to 0, \mathcal{Q}) = S_{BH} - \frac{3}{4} \ln\left(\frac{\hbar c^5}{G T^2}\right)$$

$$S_{BH} = \frac{A c^3}{4 G \hbar}\left(1 + \frac{2(\pi A)^{1/2} T}{\hbar c}\right)$$

A is the area of the charged black hole horizon at $T = 0$, $\mathcal{Q}$ is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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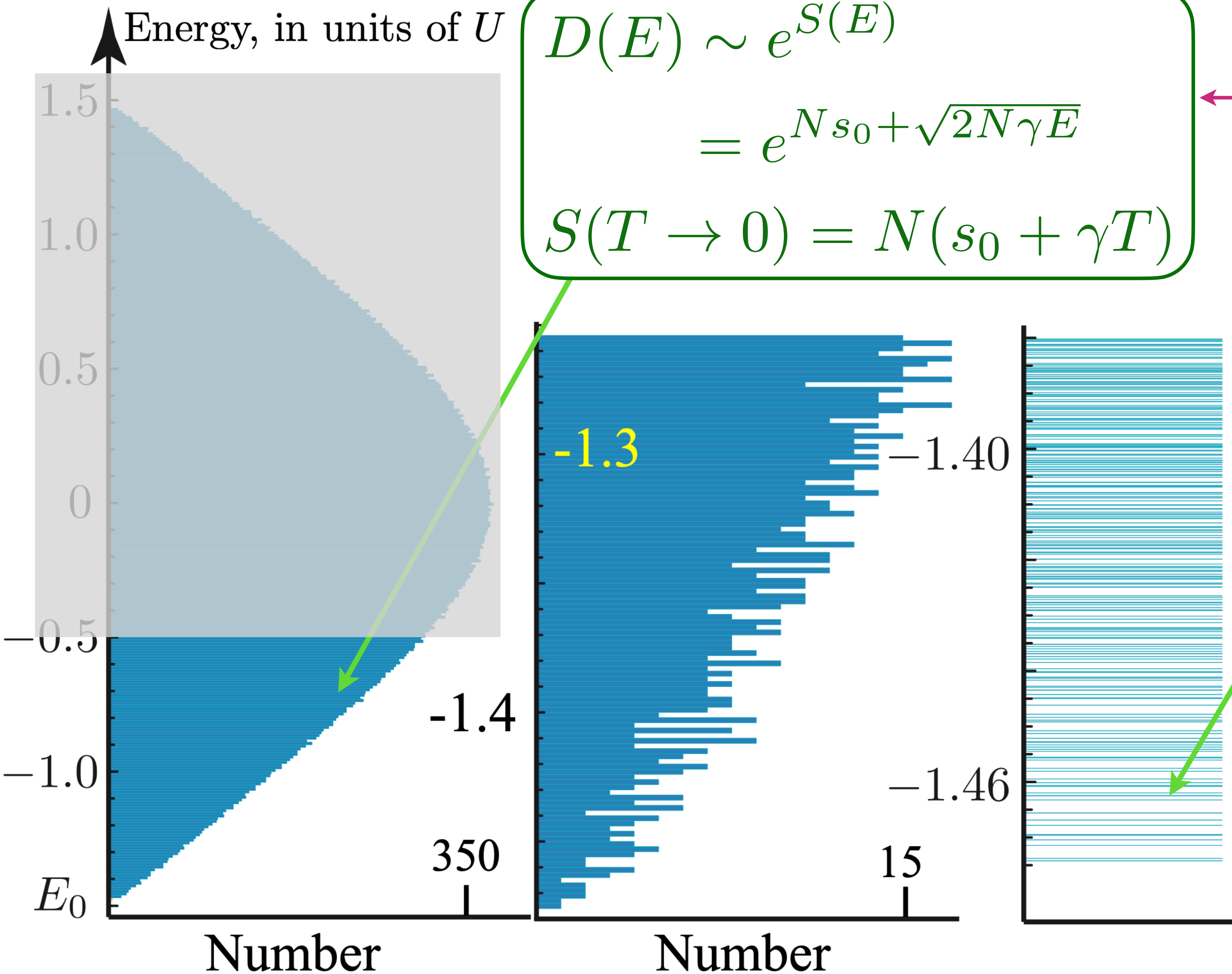

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Time reparameterization symmetry and 2D gravity

After introducing replicas $a = 1 \dots n$, and integrating out the disorder, the partition function can be written as

$$
\begin{aligned}
Z = \int \mathcal{D}c_{\alpha a}(\tau) \exp \Bigg[& - \sum_{ia} \int_0^\beta d\tau \, c^\dagger_{\alpha a} \left(\frac{\partial}{\partial \tau} - \mu \right) c_{\alpha a} \\
& - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c^\dagger_{\alpha a}(\tau) c_{\alpha b}(\tau') \right|^4 \Bigg] .
\end{aligned}
$$

For simplicity, we neglect the replica indices, and introduce the identity

$$
\begin{aligned}
1 = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \Bigg[& -N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \Bigg(G(\tau_2, \tau_1) \\
& + \frac{1}{N} \sum_i c_\alpha(\tau_2) c^\dagger_\alpha(\tau_1) \Bigg) \Bigg] .
\end{aligned}
$$

Time reparameterization symmetry and 2D gravity

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2)\exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

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At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4}\frac{g(\sigma_1)}{g(\sigma_2)}\widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4}\frac{g(\sigma_1)}{g(\sigma_2)}\widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

Time reparameterization symmetry and 2D gravity

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) e^{-E_0/T + NS(E_0) - NS_{\text{eff}}[f,\phi]},$$

where $E_0 \propto N$ is the ground state energy.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Time reparameterization symmetry and 2D gravity

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau \, \{\tan(\pi T f(\tau)), \tau\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\text{eff}}\left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
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The same effective action is obtained for the boundary graviton of 2D gravity on AdS_2.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

- Exact evaluation of the path integral over $f(\tau)$ and $\phi(\tau)$ leads to the many-body density of states

$$D(E) \sim 2e^{S_0} \sinh(\sqrt{2N\gamma E})$$

- Saddle-point shift leads to a correction to the Green's function:

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}} \left(1 + \frac{\alpha_G}{|\tau|} + \ldots\right)$$

  From this, we can compute the susceptibility $\chi(\tau) \sim G(\tau)G(-\tau)$

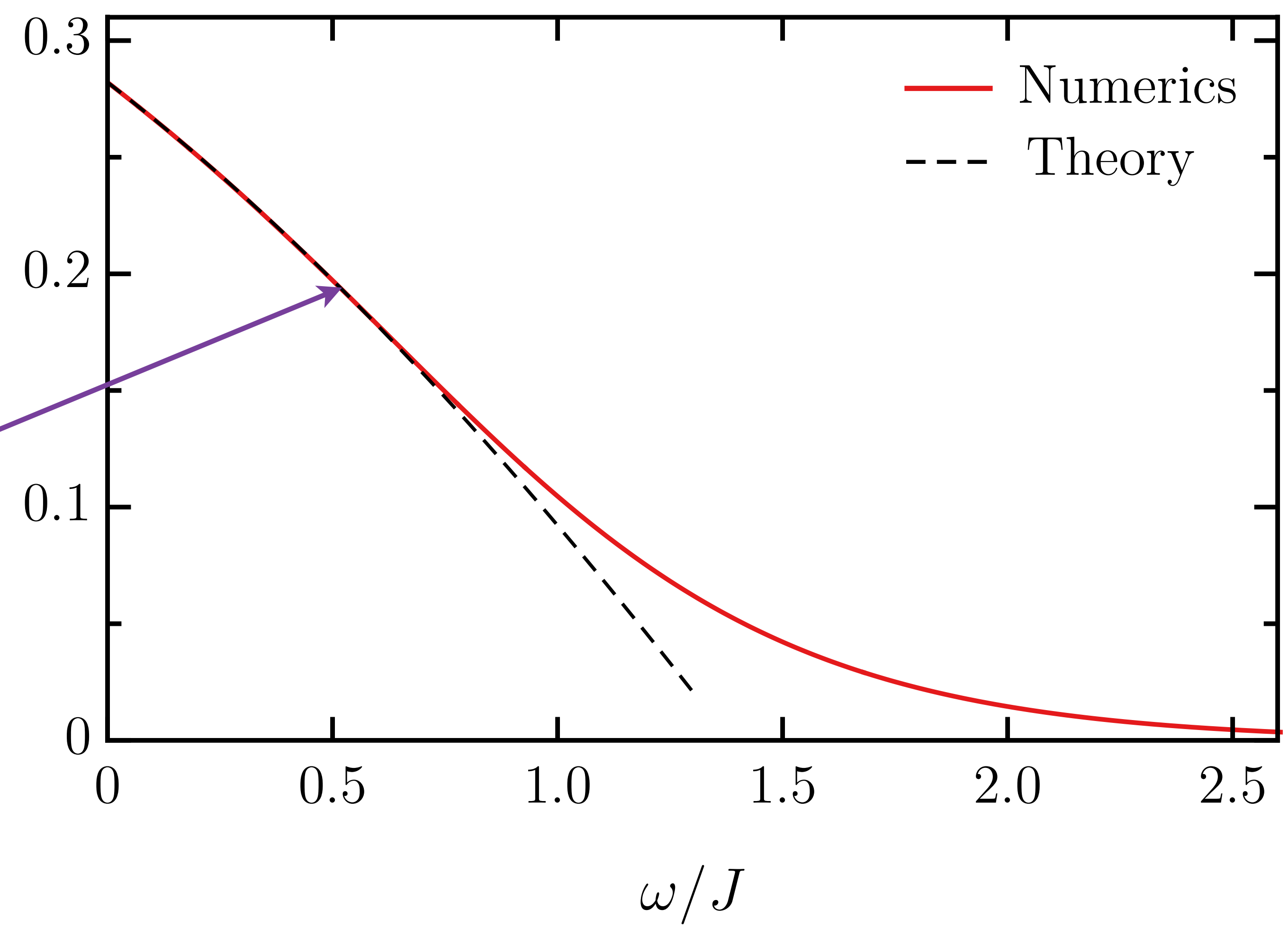
Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

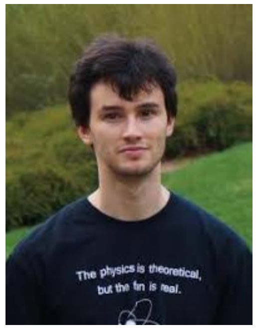
$$\chi_L(\omega) = \sum_n \left|\langle 0| S_{+i} |n\rangle\right|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T=0)$$

$$\mathrm{Im}\chi_L(\omega) \sim \mathrm{sgn}(\omega)\left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots\right]$$

Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory.
$\mathcal{C}$ is the co-efficient of the action for the ‘boundary graviton’ in holographic dual.

Correction from the boundary graviton





Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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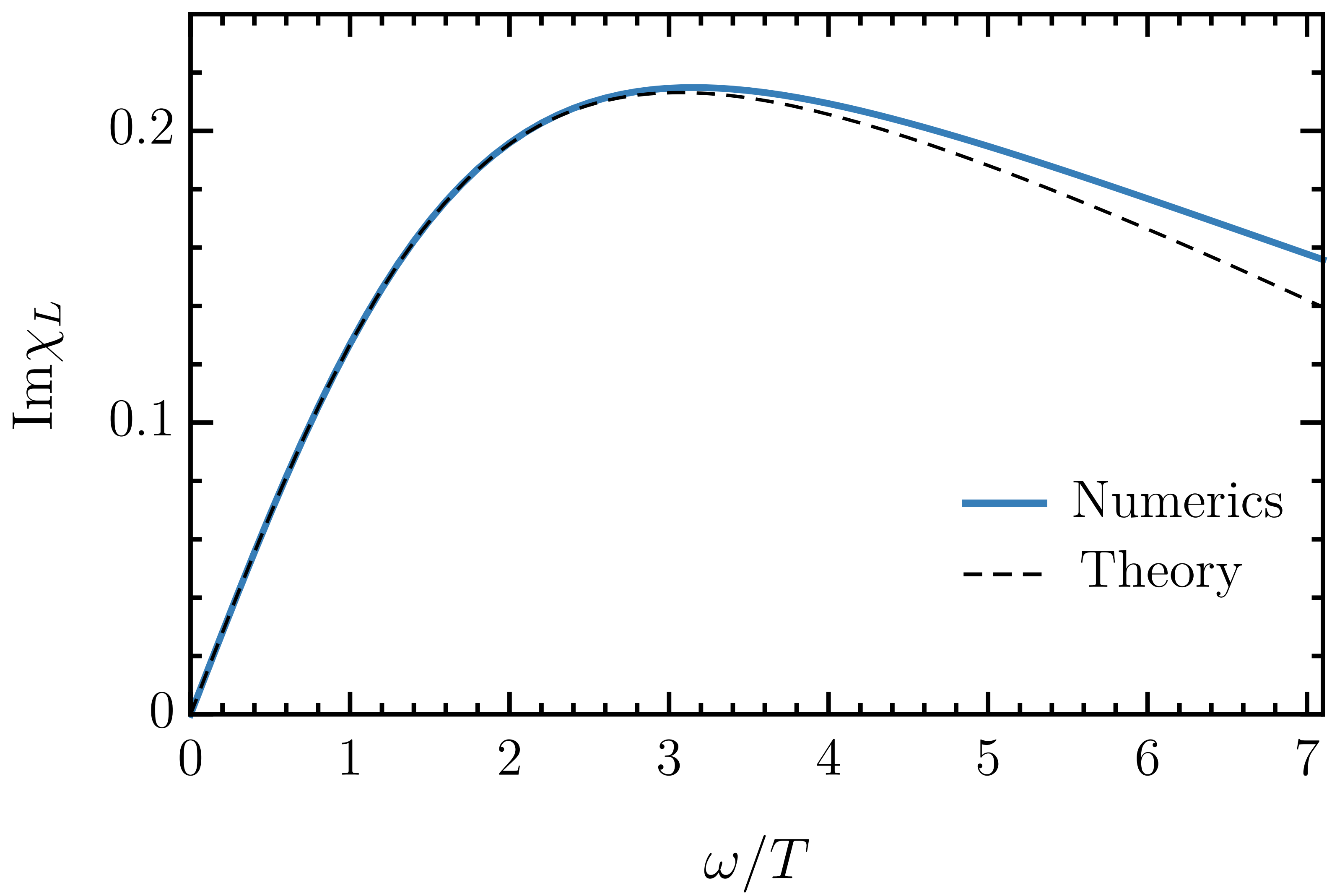

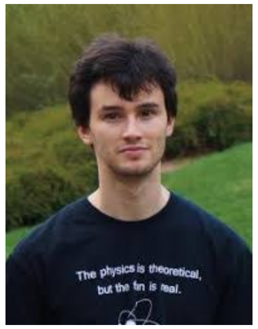

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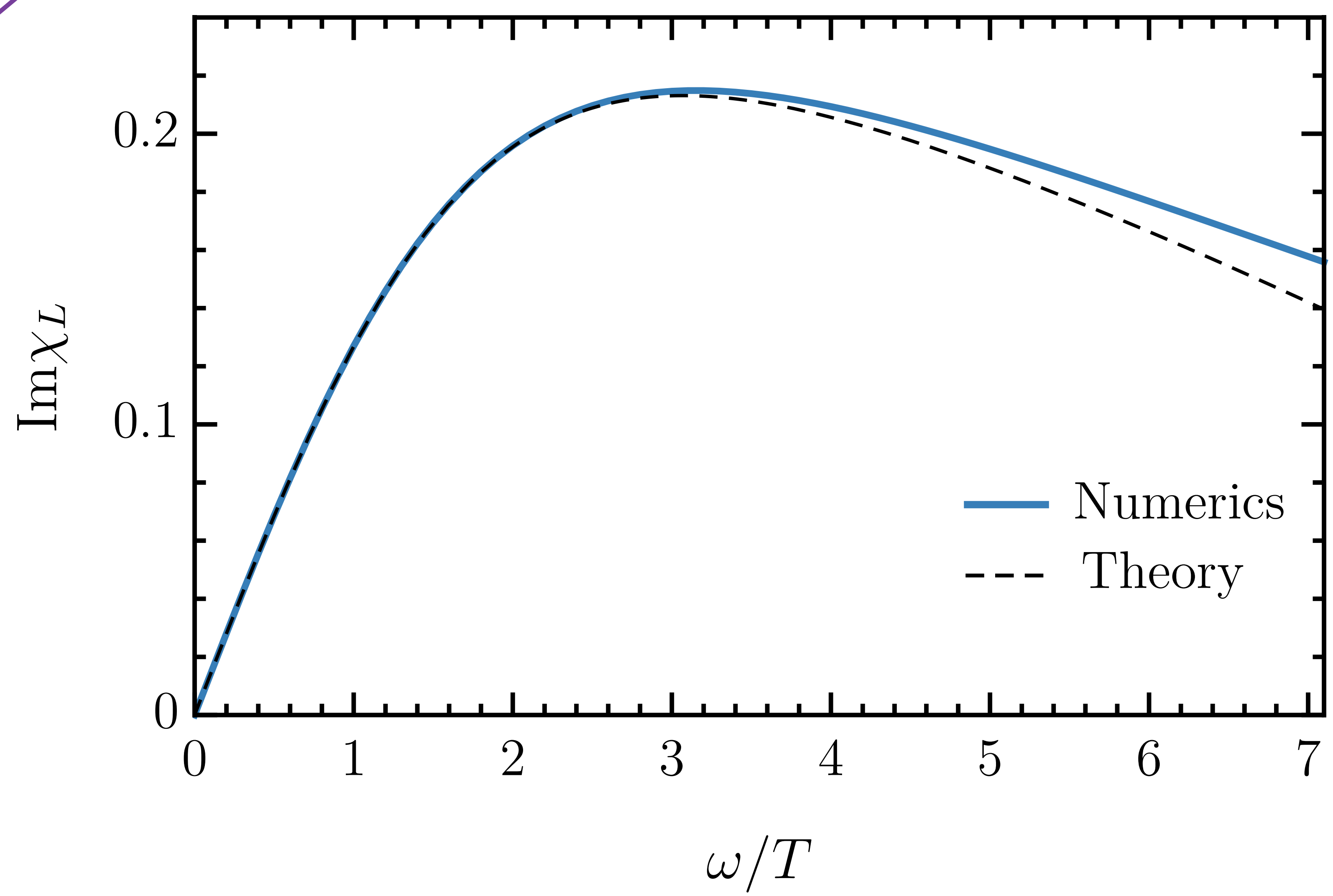
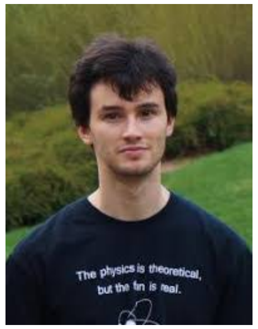
Conformally (SL(2,R)) invariant result with characteristic dissipative time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)





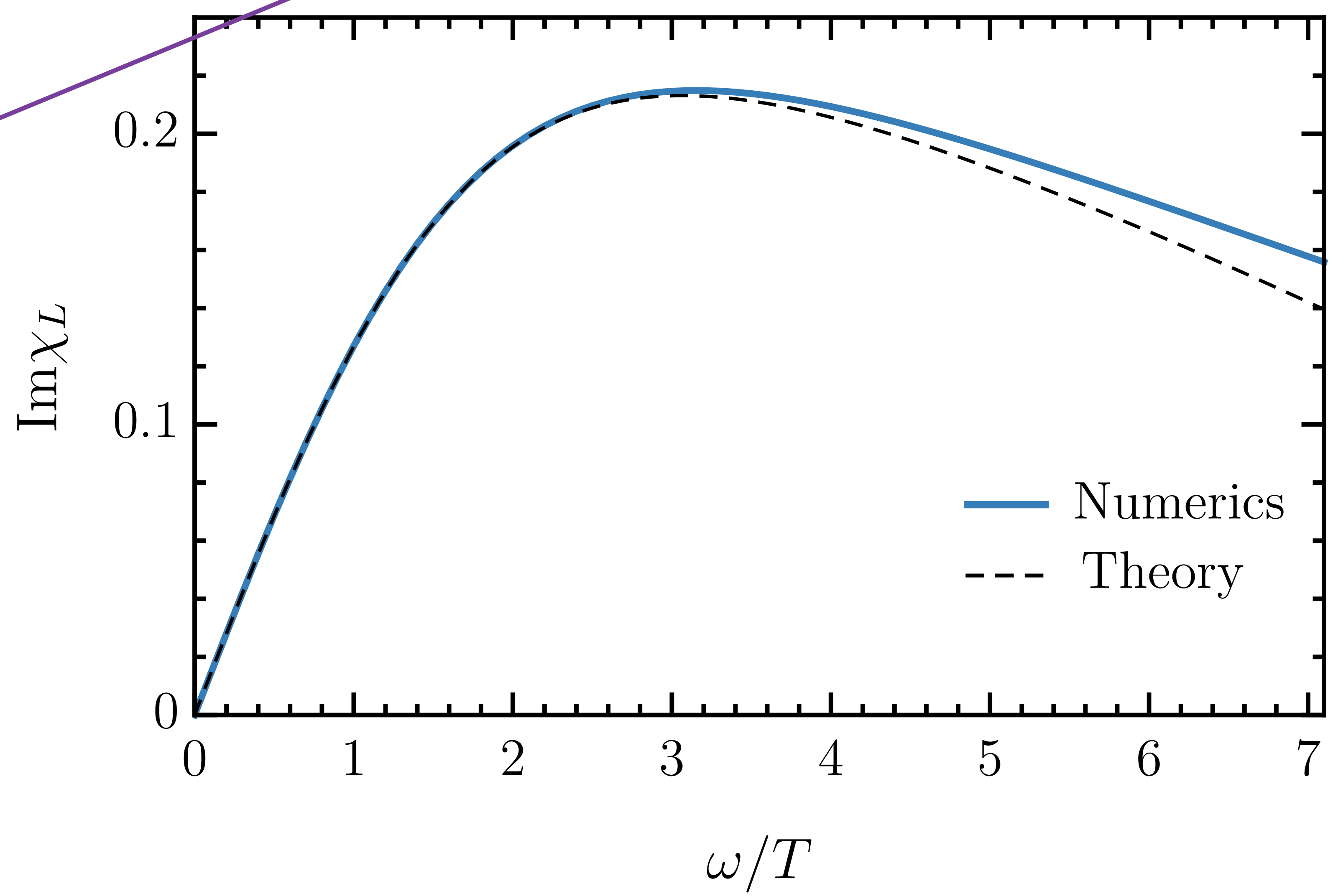
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

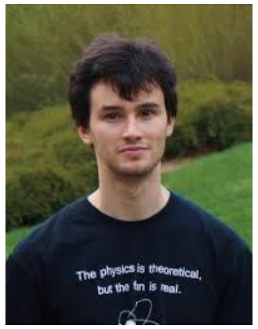
Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0| S_{+i} |n\rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right)\left[1 - \mathcal{C}\gamma\,\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \dots\right]$$

Correction from the boundary graviton



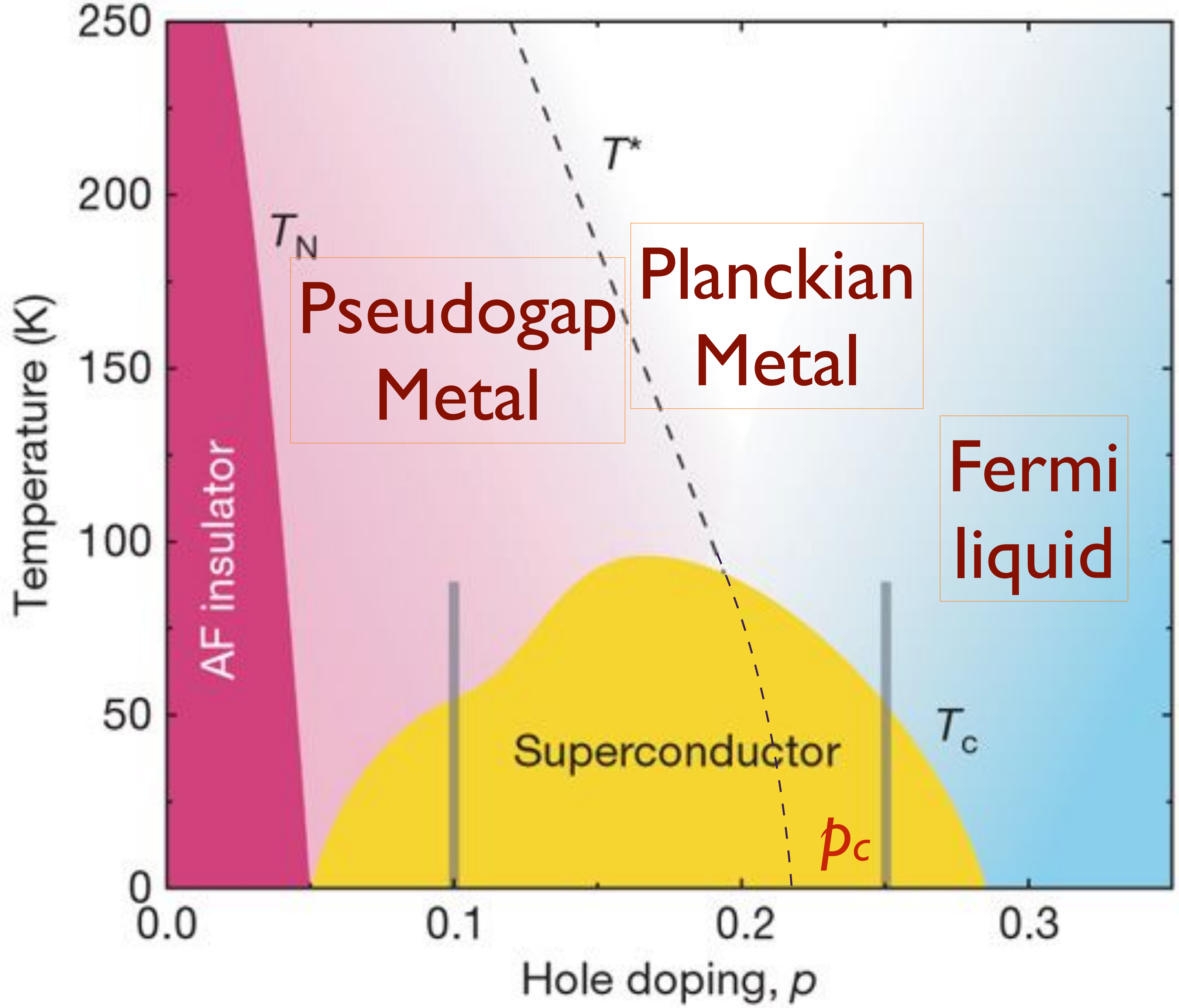


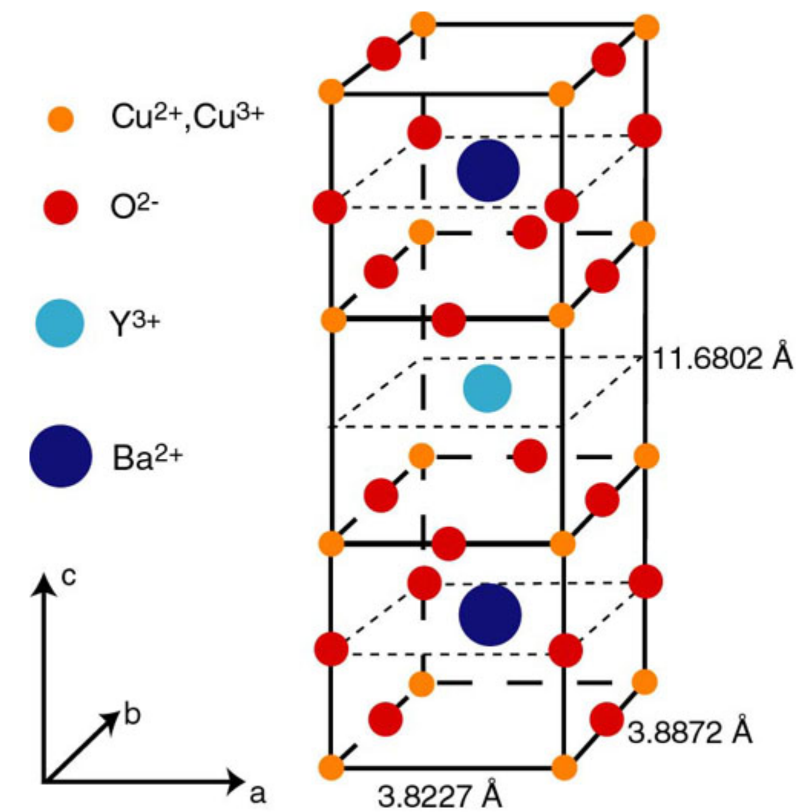
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

1. SYK models

2. Time reparameterization soft mode

3. Random t-J model



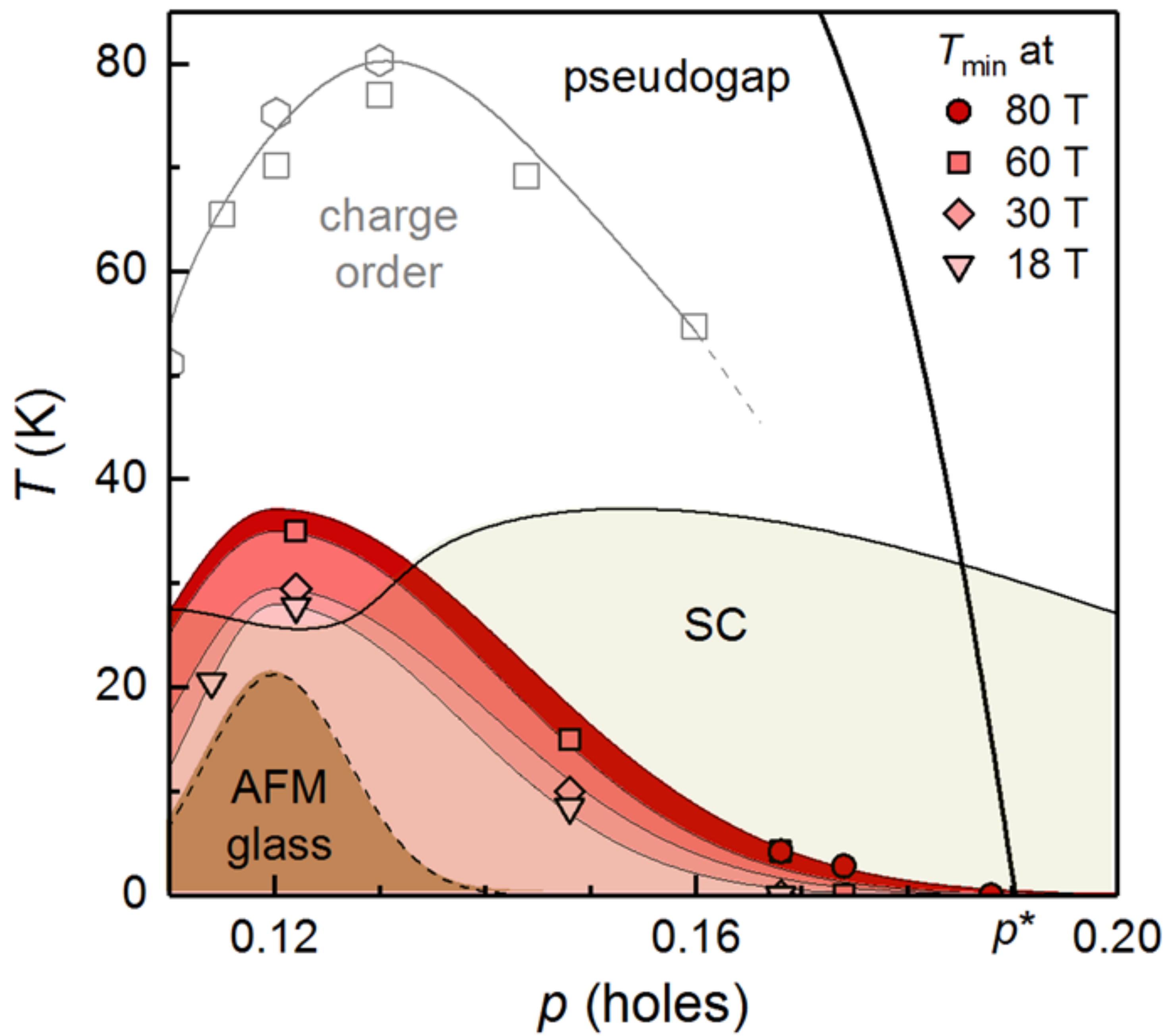


(Extended) SYK models have "Planckian" behavior, but cannot obtain pseudogap metal or insulator at any *U*: missing local "Mott" repulsion

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics **16**, 1064 (2020)

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



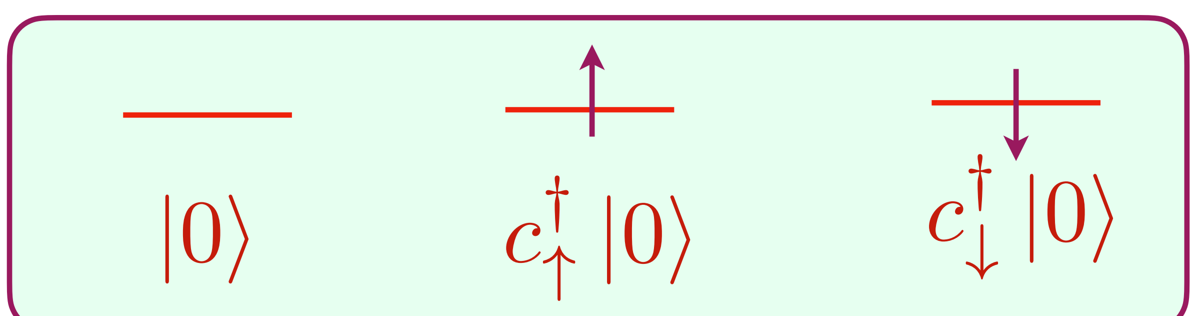
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \boxed{\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1,} \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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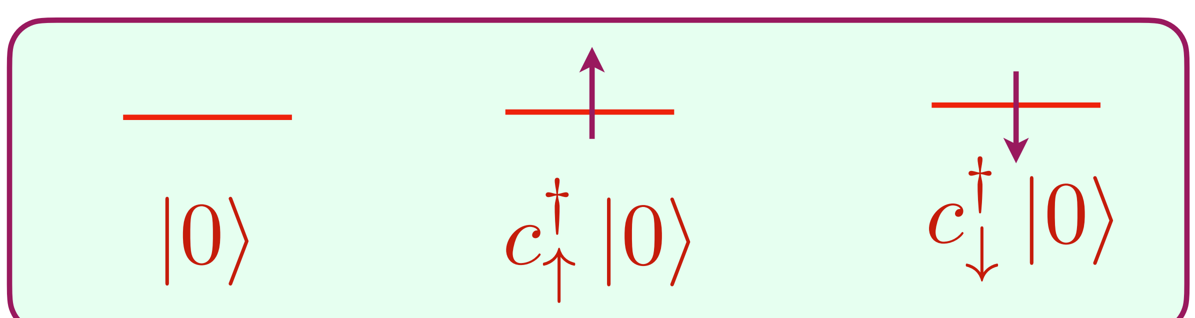
$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

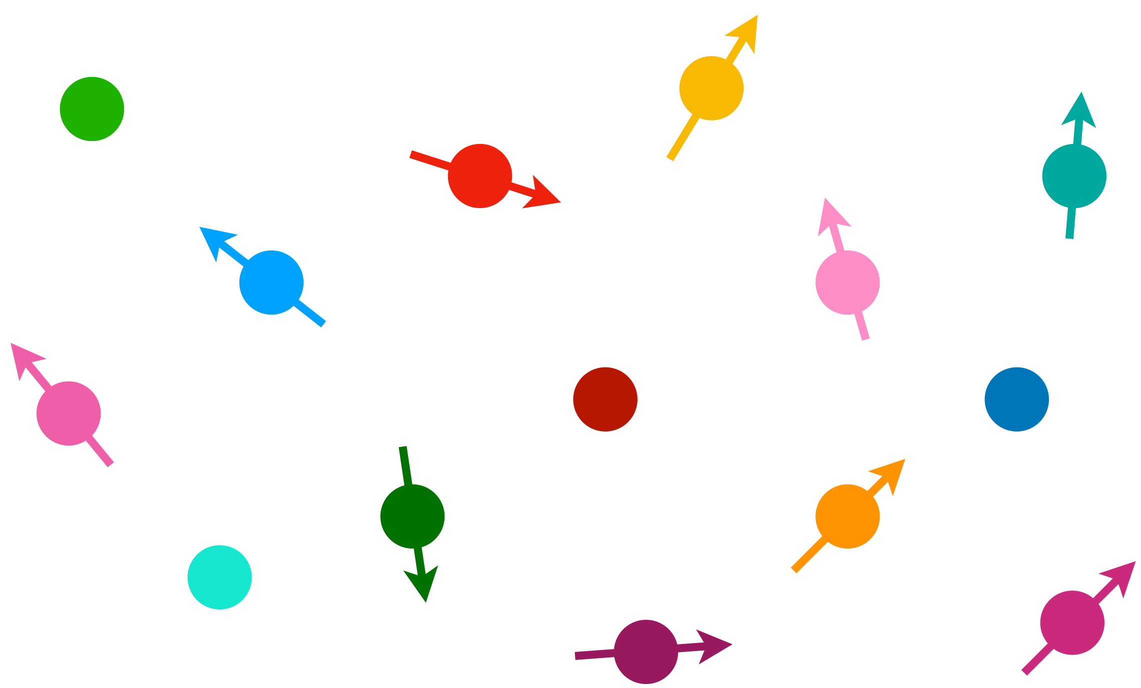
$$t_{ij} \text{ random}, \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$

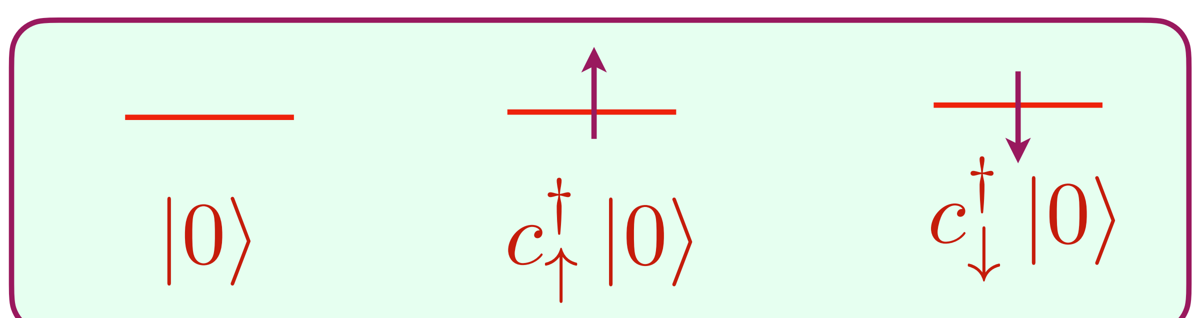


Random t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

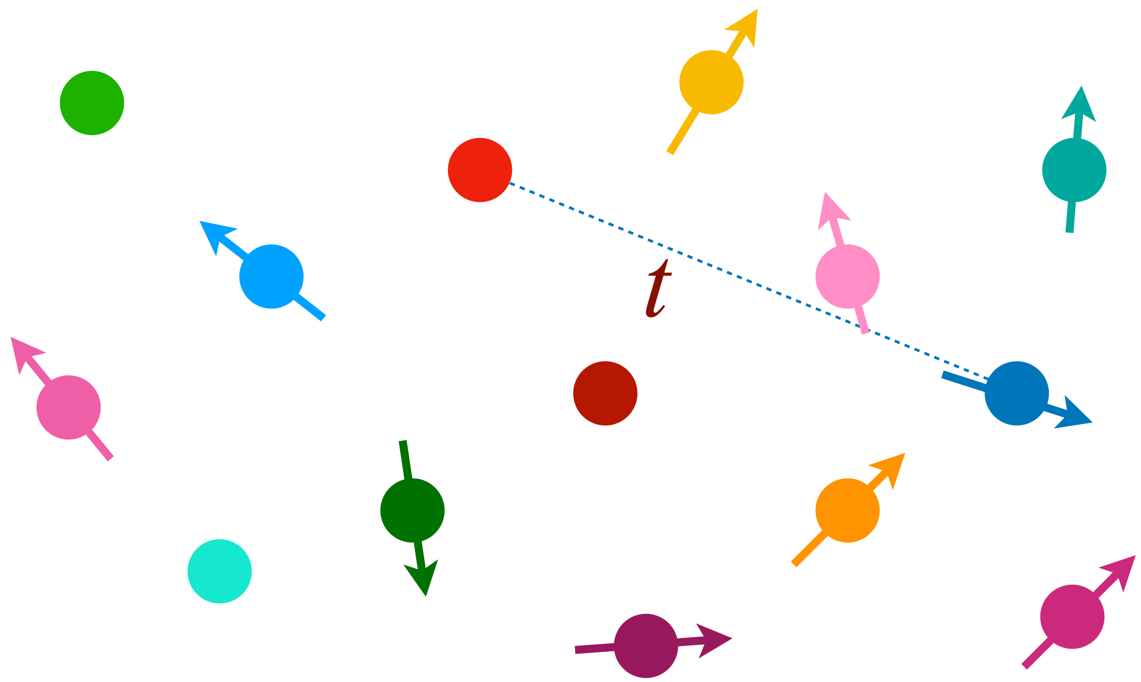


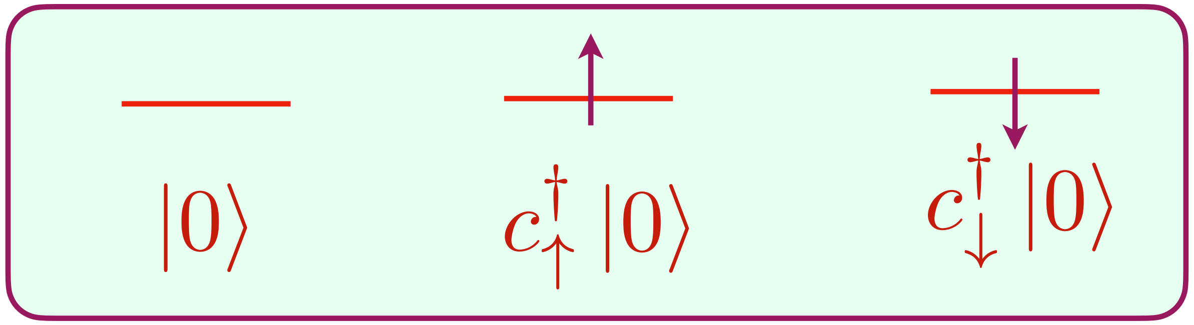


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

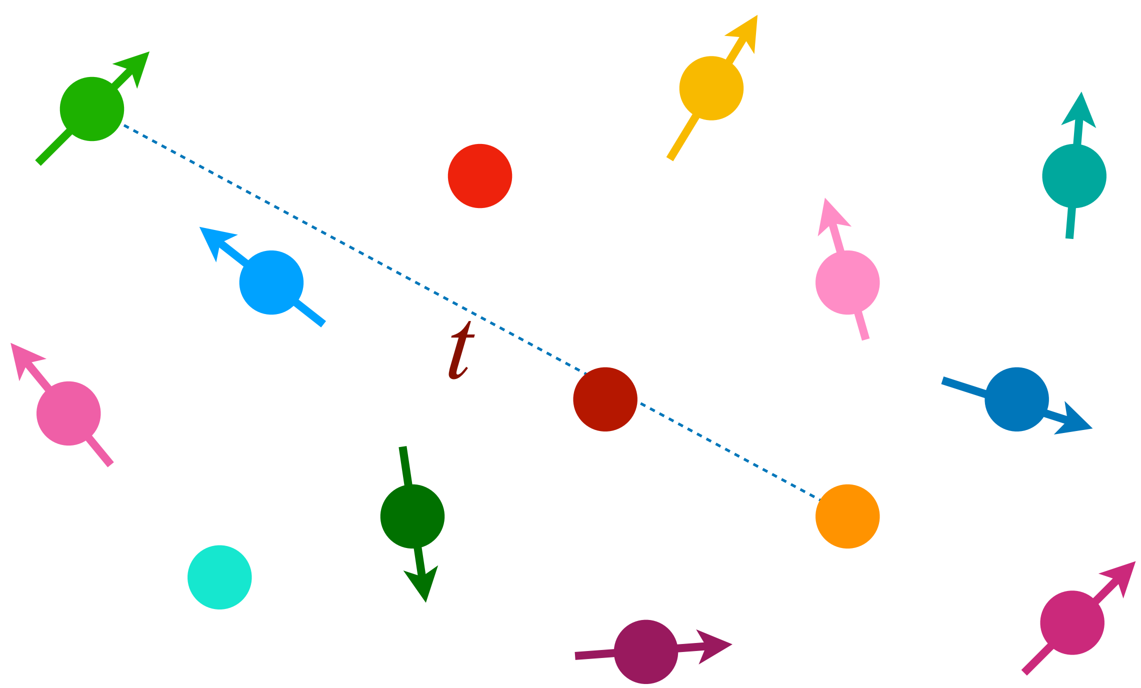


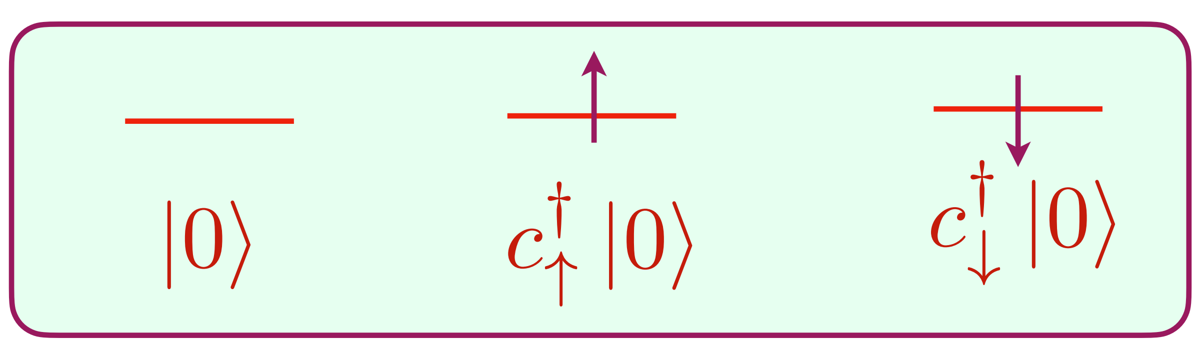


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

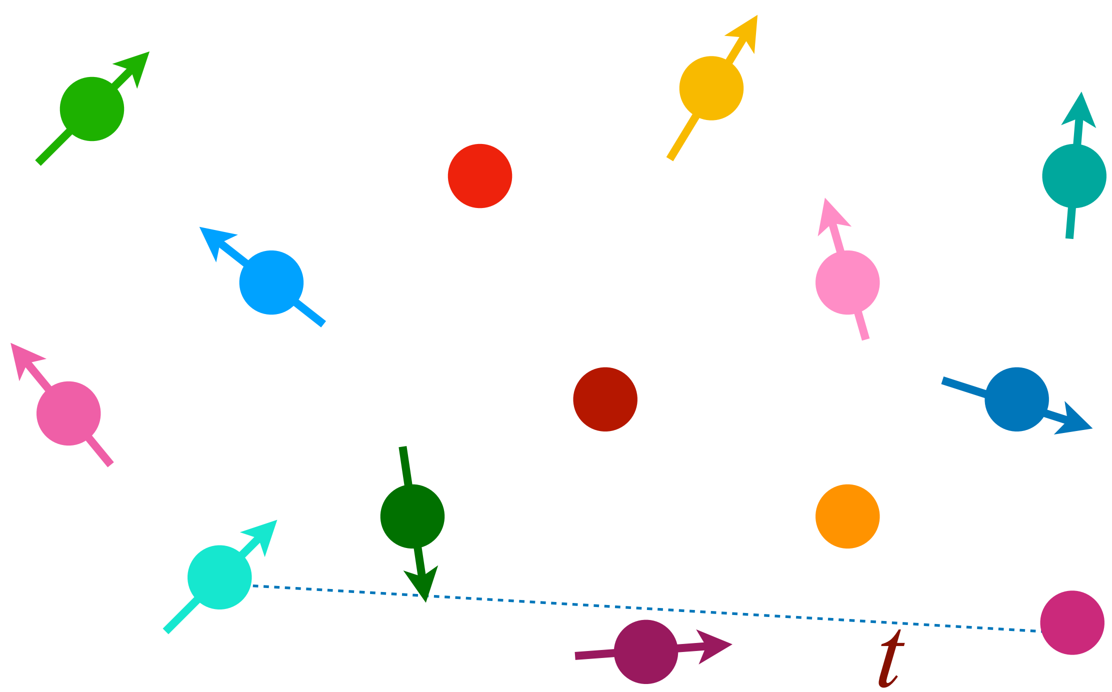


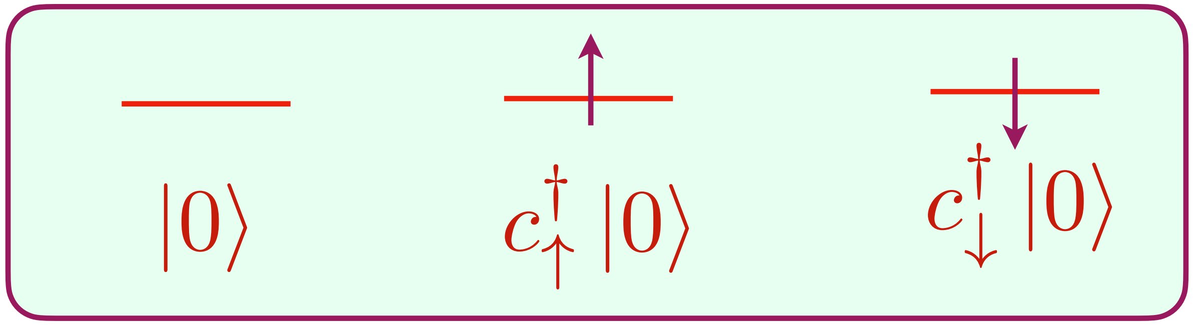


Random *t*-*J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

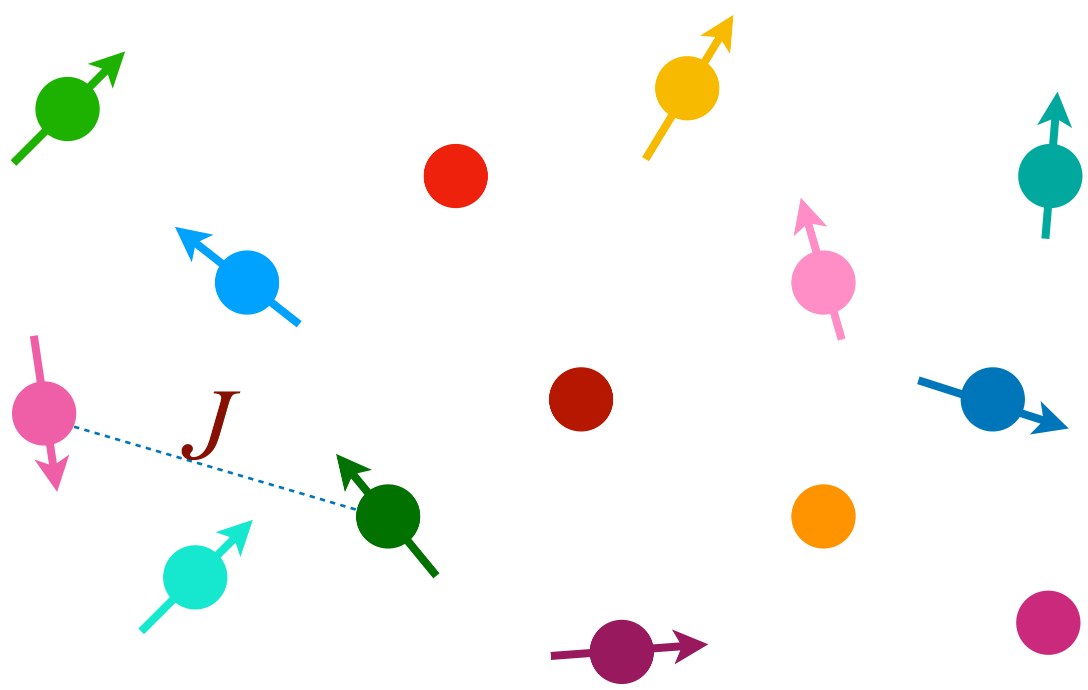


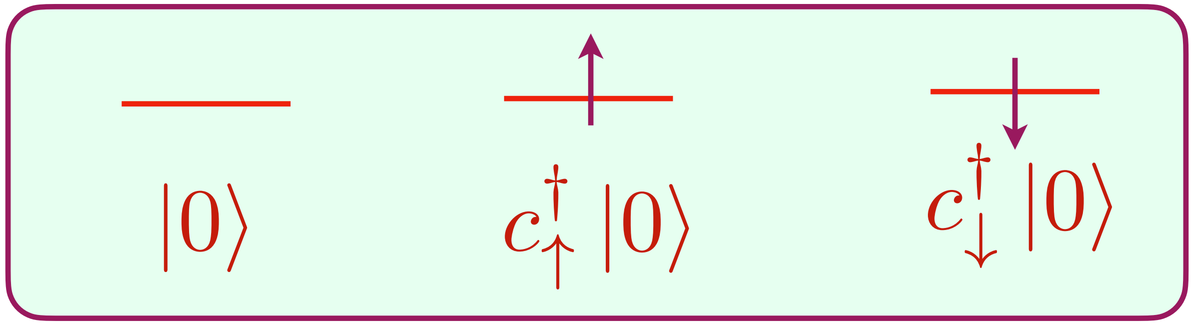


Random *t*-*J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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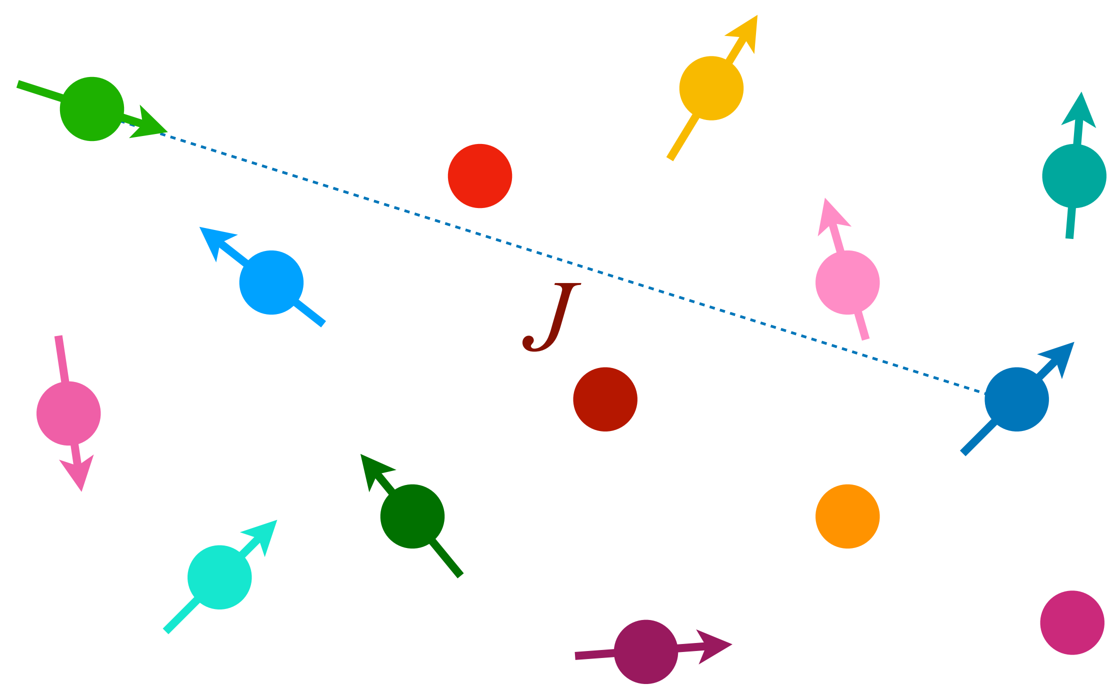
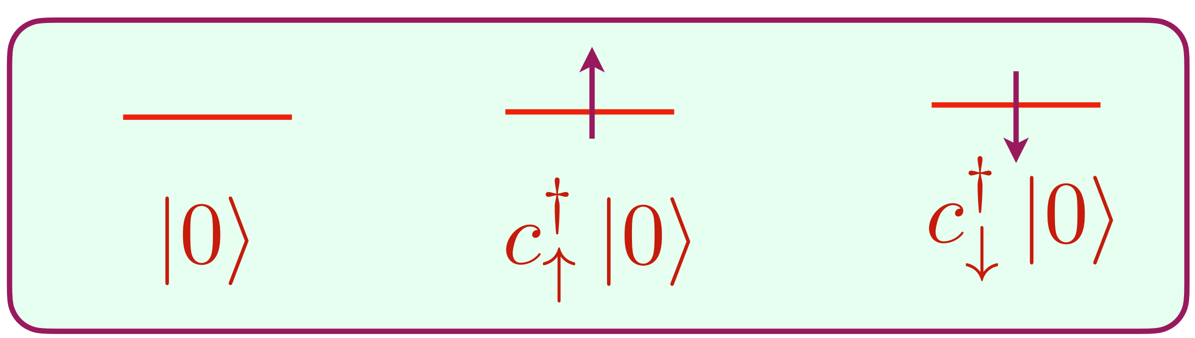
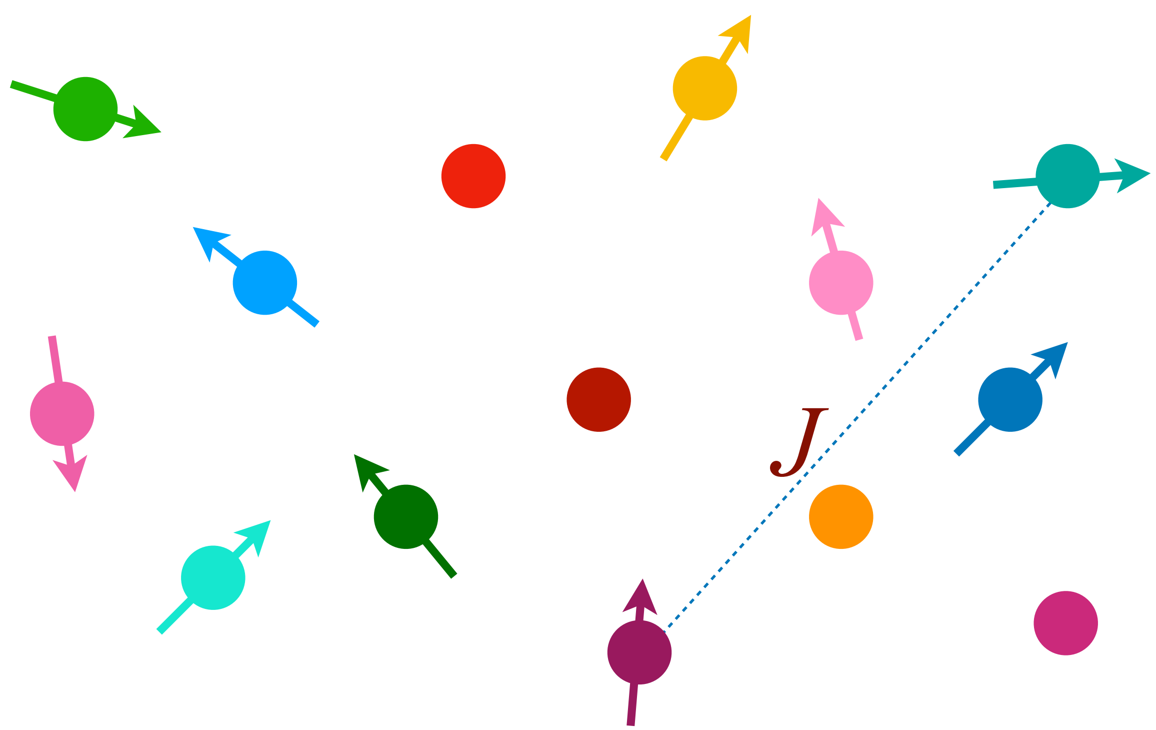


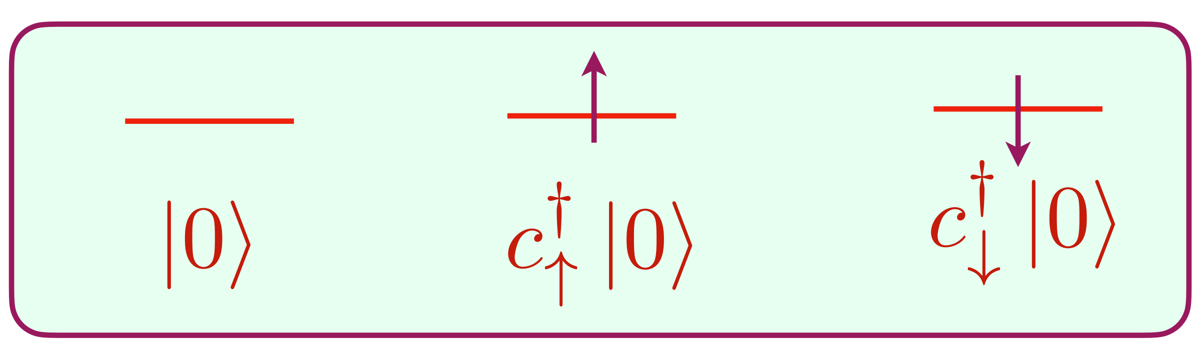


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

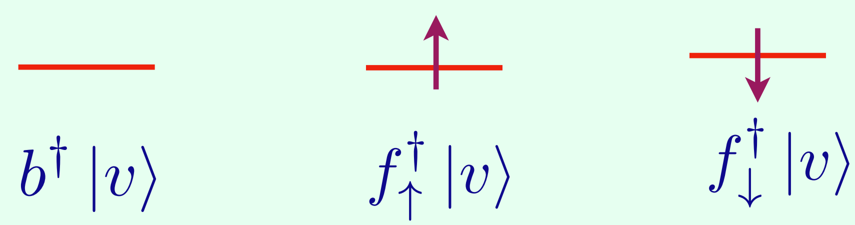




Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

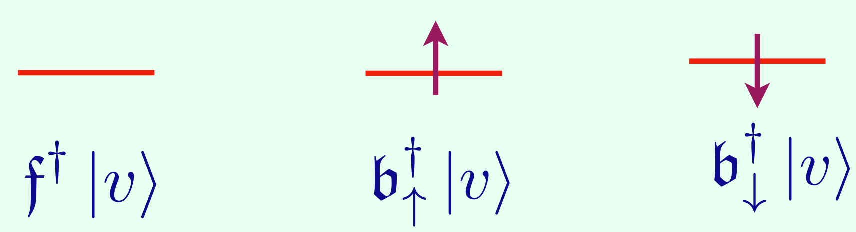
U(1) gauge invariance, $\quad b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

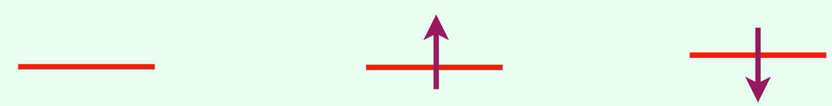
U(1) gauge invariance, $\mathfrak{f} \rightarrow \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \rightarrow \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random *t-J* model

$$H=-\frac{1}{\sqrt{N}}\sum_{i,j=1}^{N}t_{ij}\,c_{i\alpha}^{\dagger}c_{j\alpha}+\frac{1}{\sqrt{N}}\sum_{i<j=1}^{N}J_{ij}\,\vec{S}_i\cdot\vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$\mathfrak{f}^\dagger\left|v\right\rangle$ $\quad$ $\mathfrak{b}_\uparrow^\dagger\left|v\right\rangle$ $\quad$ $\mathfrak{b}_\downarrow^\dagger\left|v\right\rangle$

SU(1|2) ≡ SU(2|1)

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha\mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2}\mathfrak{b}_\alpha^\dagger\sigma_{\alpha\beta}\mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger\mathfrak{b}_\alpha+\mathfrak{f}^\dagger\mathfrak{f} &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad \mathfrak{f}\rightarrow\mathfrak{f}e^{i\phi}\,,\quad \mathfrak{b}_\alpha\rightarrow\mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random *t-J* model and cuprate phase diagram

Metallic
spin glass.

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau)c^{\dagger}_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

Disordered
Fermi liquid.

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau)c^{\dagger}_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

p_c

p

Random *t-J* model and cuprate phase diagram

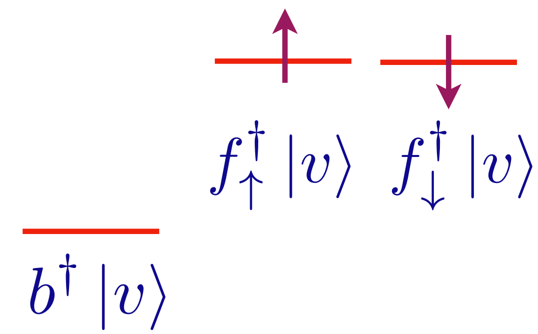
Metallic spin glass.

$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau)c^\dagger_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

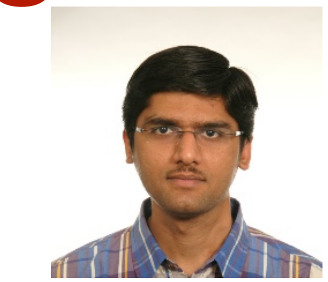
Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$

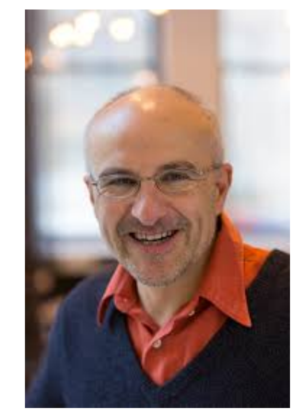


$$\left\langle \vec{S}_i(\tau)\cdot\vec{S}_i(0)\right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau)c^\dagger_{i\alpha}(0)\right\rangle \sim \frac{1}{\tau}$$

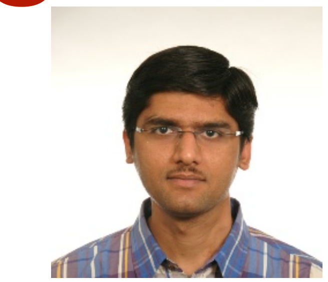
p_c $\quad$ p



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random *t-J* model and cuprate phase diagram



SU(2|1) theory

Metallic
spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger \left| v \right\rangle$

$\mathfrak{b}_\uparrow^\dagger \left| v \right\rangle$ $\mathfrak{b}_\downarrow^\dagger \left| v \right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

Disordered
Fermi liquid.
Condense holon b,
f_α carrier density $1 + p$

$f_\uparrow^\dagger \left| v \right\rangle$ $f_\downarrow^\dagger \left| v \right\rangle$

$b^\dagger \left| v \right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

p_c p

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Random *t-J* model

SU(2|1) theory

Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality of fractionalized excitations

SU(1|2) theory

Disordered Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

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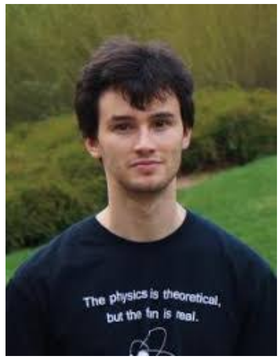
$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$



p_c $\quad$ p

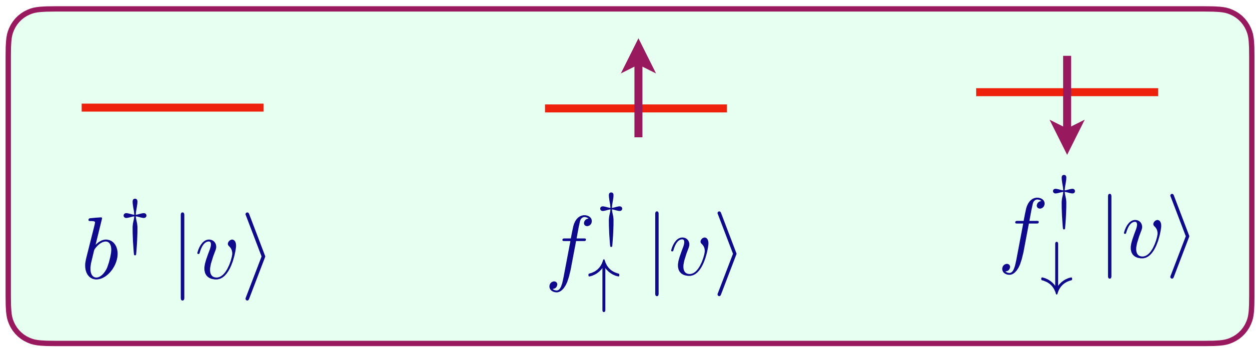
SU($M \to \infty$) theory of SYK liquids of fractionalized holons and spinons

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Fractionalization in the t-J model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} f_{i\alpha}^{\dagger} f_{j\alpha} b_i b_j^{\dagger} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} \frac{J_{ij}}{4} f_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{i\beta} \cdot f_{j\gamma}^{\dagger} \vec{\sigma}_{\gamma\delta} f_{j\delta}$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$\begin{aligned} c_\alpha &= f_\alpha b^{\dagger} \\ \vec{S} &= \frac{1}{2} f_\alpha^{\dagger} \sigma_{\alpha\beta} f_\beta \\ f_\alpha^{\dagger} f_\alpha + b^{\dagger} b &= 1 \\ \text{U(1) gauge invariance,} \quad & b \rightarrow b e^{i\phi}, \quad f_\alpha \rightarrow f_\alpha e^{i\phi} \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space; both t and J terms in H are *quartic* in terms of fractionalized particles.

Random *t-J* model

SU(2|1) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SYK criticality of fractionalized excitations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

Disordered Fermi liquid. Condense holon b, f_α carrier density $1 + p$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

p_c $\quad$ p

Zeroth order, $p_c = 1/3$

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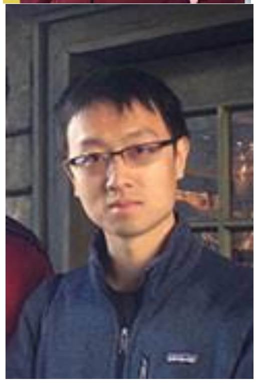
Time reparameterization soft mode and linear-T resistivity

The time reparameterization soft-mode now leads to corrections to the Green's functions of the partons

$$G_{f,b}(\tau) \sim \frac{\pm 1}{\sqrt{|\tau|}} \left(1 + \frac{\alpha_{f,b}}{|\tau|} + \ldots \right)$$



We can compute the resistivity from this in a large-d model, and find

$$\rho(T) = \rho(0) \left(1 + 8\alpha_G \frac{T}{J} + \ldots \right) .$$

The α_G term arises from the contribution of the boundary graviton!

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

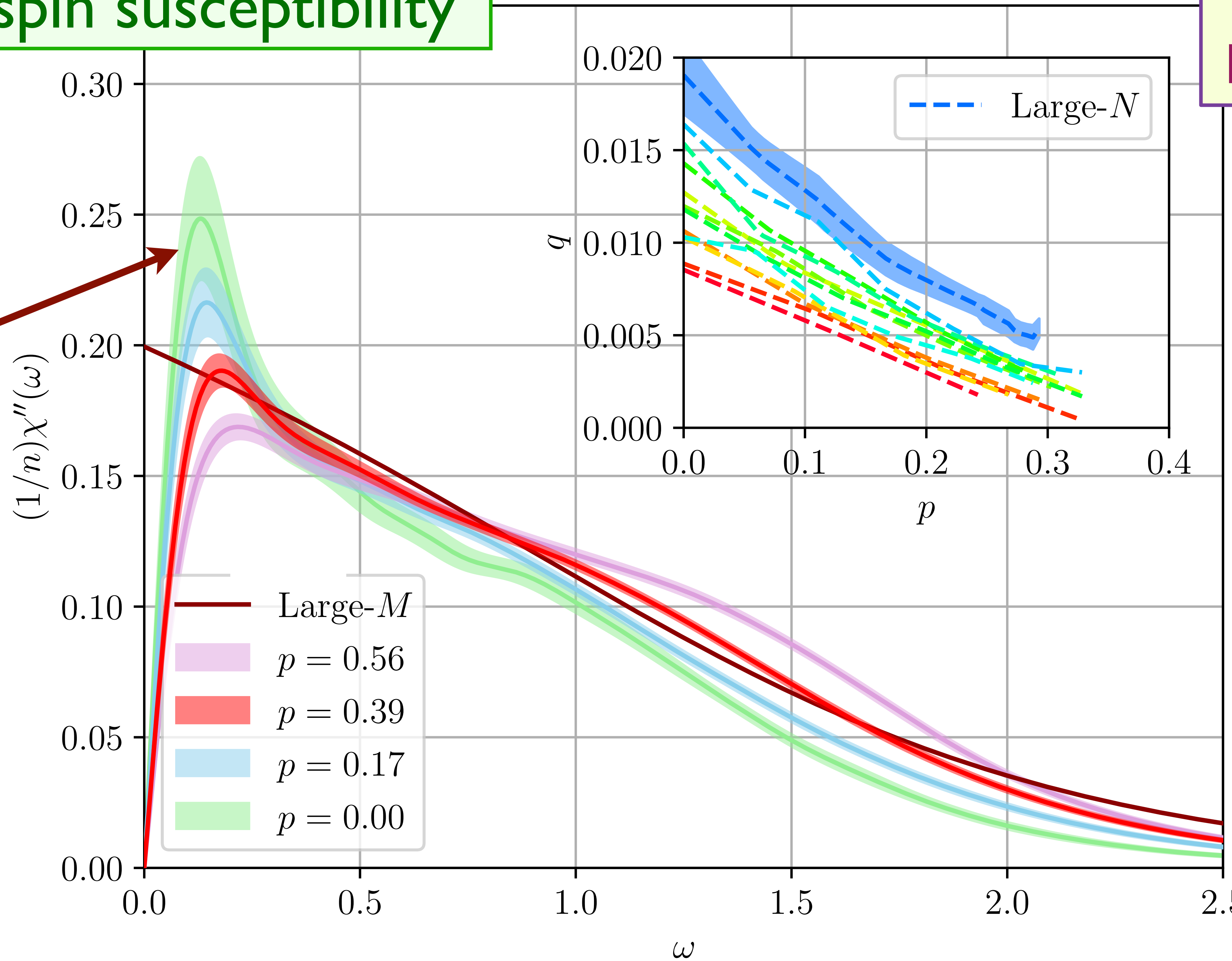
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Numerics on the SU(2) random t-J model

See next talk by Olivier Parcollet

Dynamic spin susceptibility

Spin glass order



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Evidence for a quantum critical point at $p = p_c \approx 0.3$.
Spin glass order q non-zero for $p < p_c$

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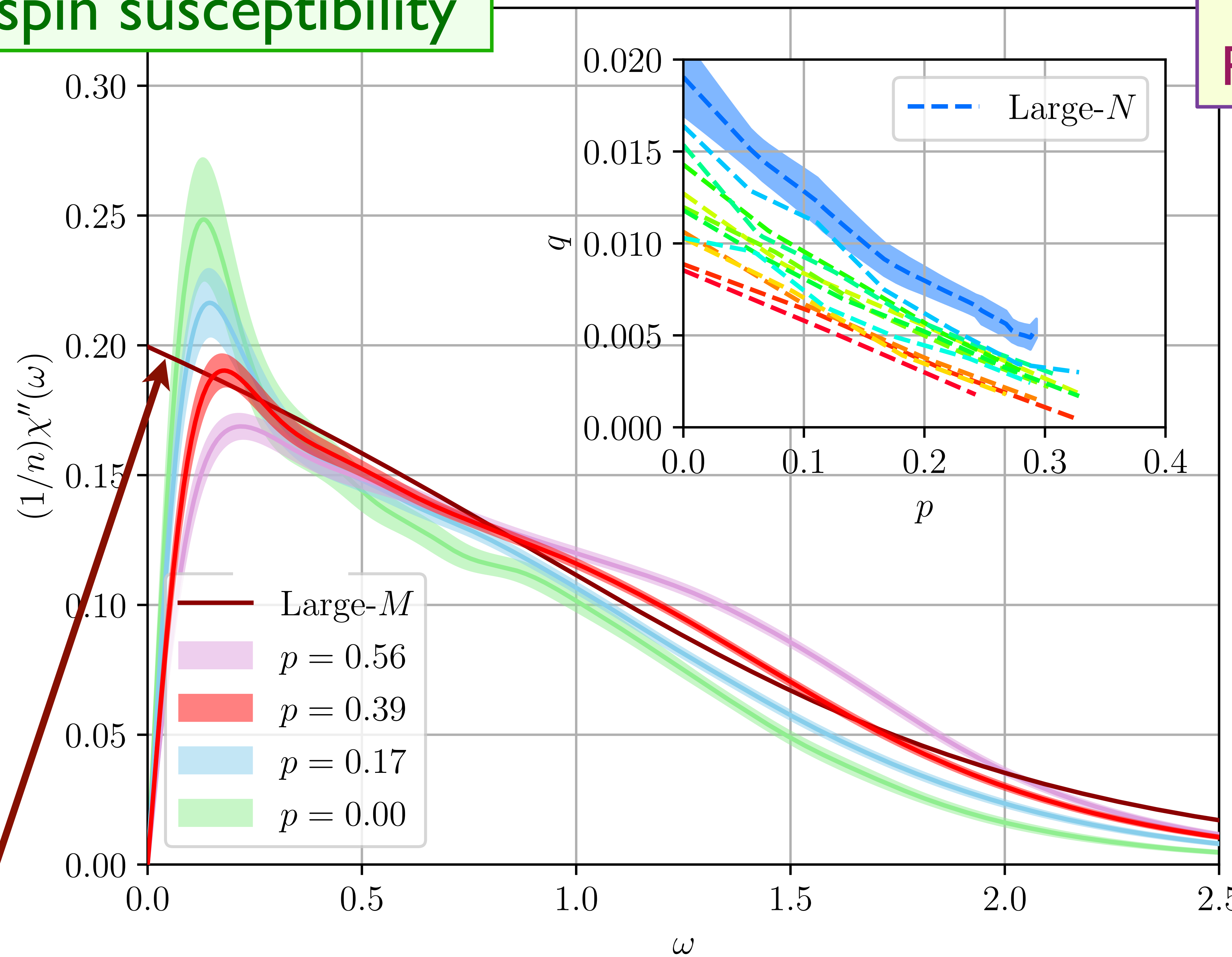

See next talk by Olivier Parcollet

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Critical spin susceptibility matches the large M SU(M) SYK model.
$\chi''(\omega) \sim \text{sgn}(\omega)\,[1 - \mathcal{C}\gamma|\omega| + \ldots]$: the $|\omega|$ is the boundary graviton.
Shown is the numerical solution of SYK equations (SY, PRL 1993), after rescaling J.

Summary

- SYK: a solvable model without quasiparticle excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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  - $-3/2\,\ln(1/T)$ correction to Bekenstein-Hawking entropy of low T charged black holes in Einstein gravity.