Where is the quantum critical point in the cuprate superconductors ?

arXiv:0907.0008

PHYSICS

HARVARD

Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
Phys. Rev. B **75**, 235122 (2007).

Algebraic charge liquids
R. K. Kaul, Yong-Baek Kim, S. Sachdev, and T. Senthil,
Nature Physics **4**, 28 (2008).

Destruction of Neel order in the cuprates by electron doping,
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu,
Physical Review B **78**, 045110 (2008).

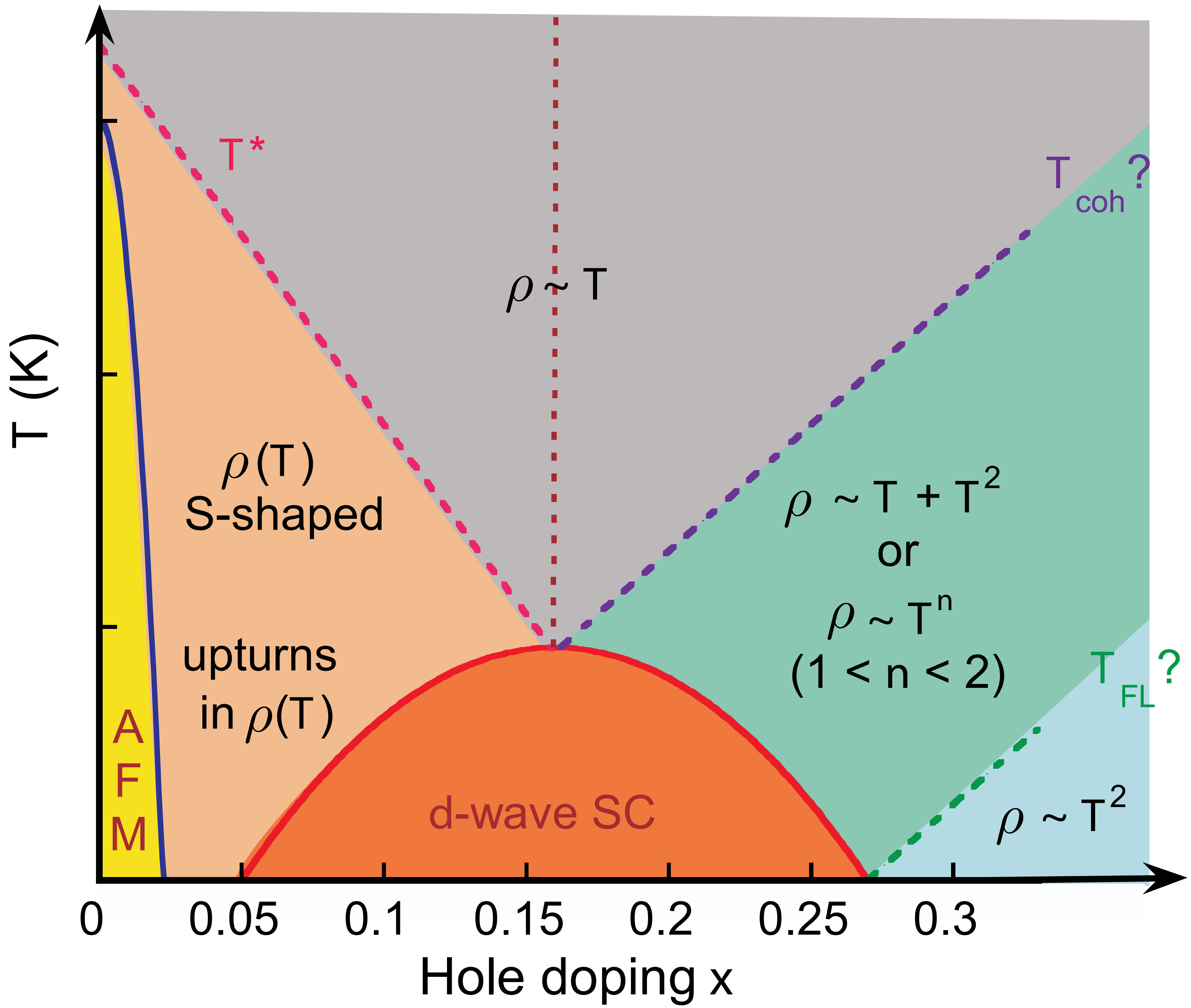
Paired electron pockets in the underdoped cuprates,
V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Competition between spin density wave order and superconductivity in the underdoped cuprates,
Eun Gook Moon and S. Sachdev,
arXiv:0905.2608

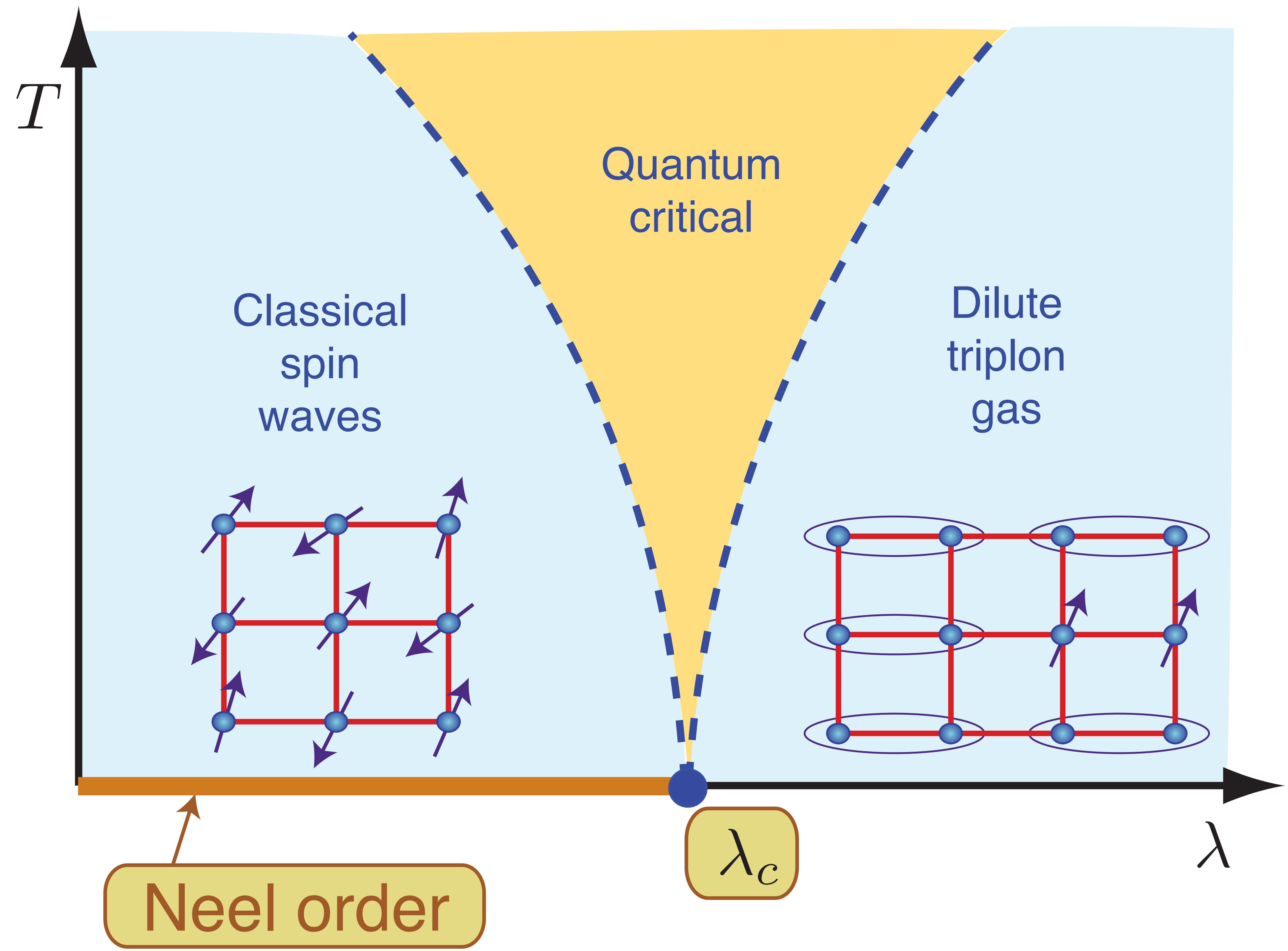
Fluctuating spin density waves in metals
S. Sachdev, M. Metlitski, Yang Qi, and Cenke Xu
arXiv:0907.3732

PHYSICS
HARVARD

Crossovers in transport properties of hole-doped cuprates

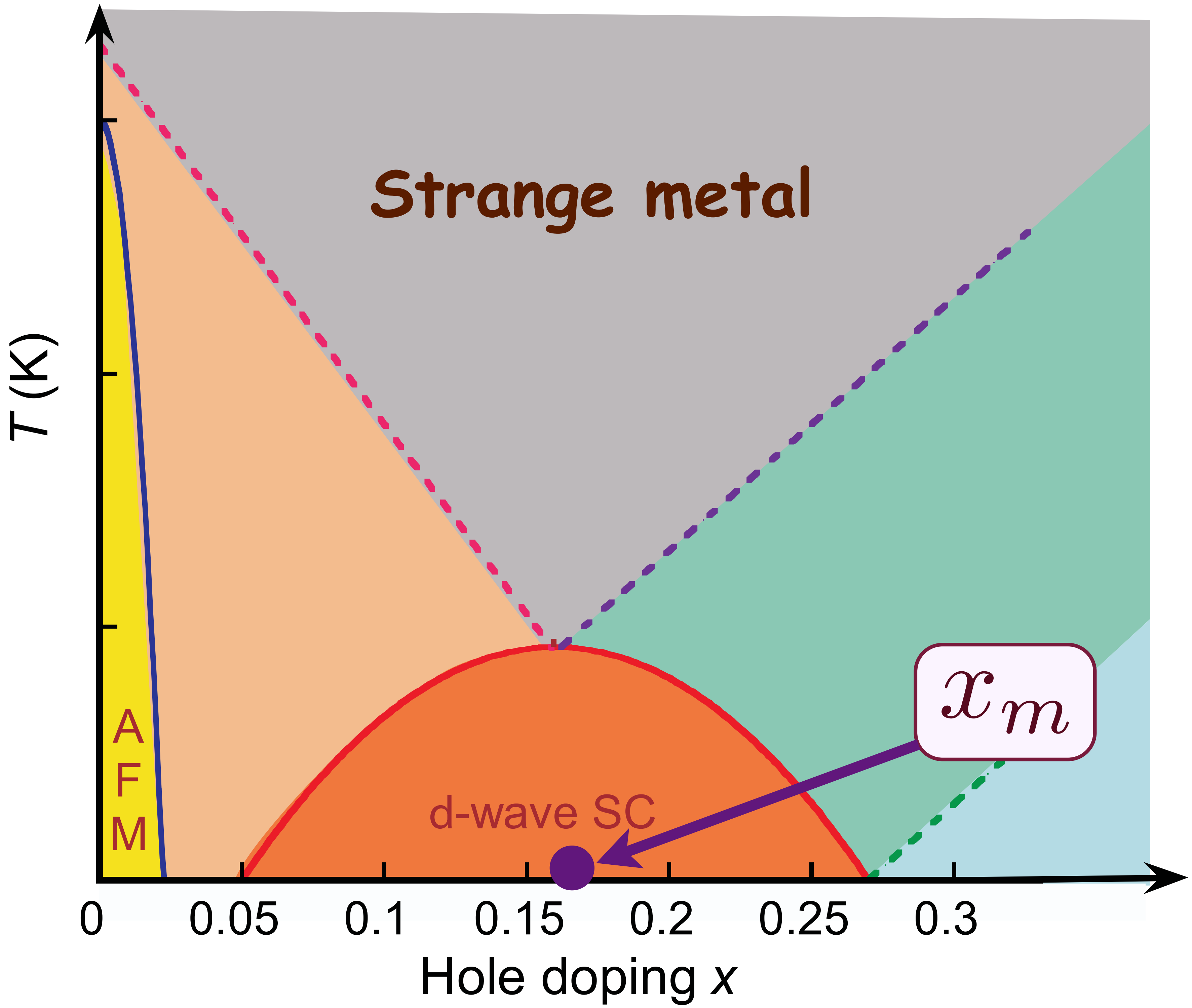


N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)



T
Quantum critical
Classical spin waves
Dilute triplon gas
λ
λ_c
Neel order

Crossovers in transport properties of hole-doped cuprates

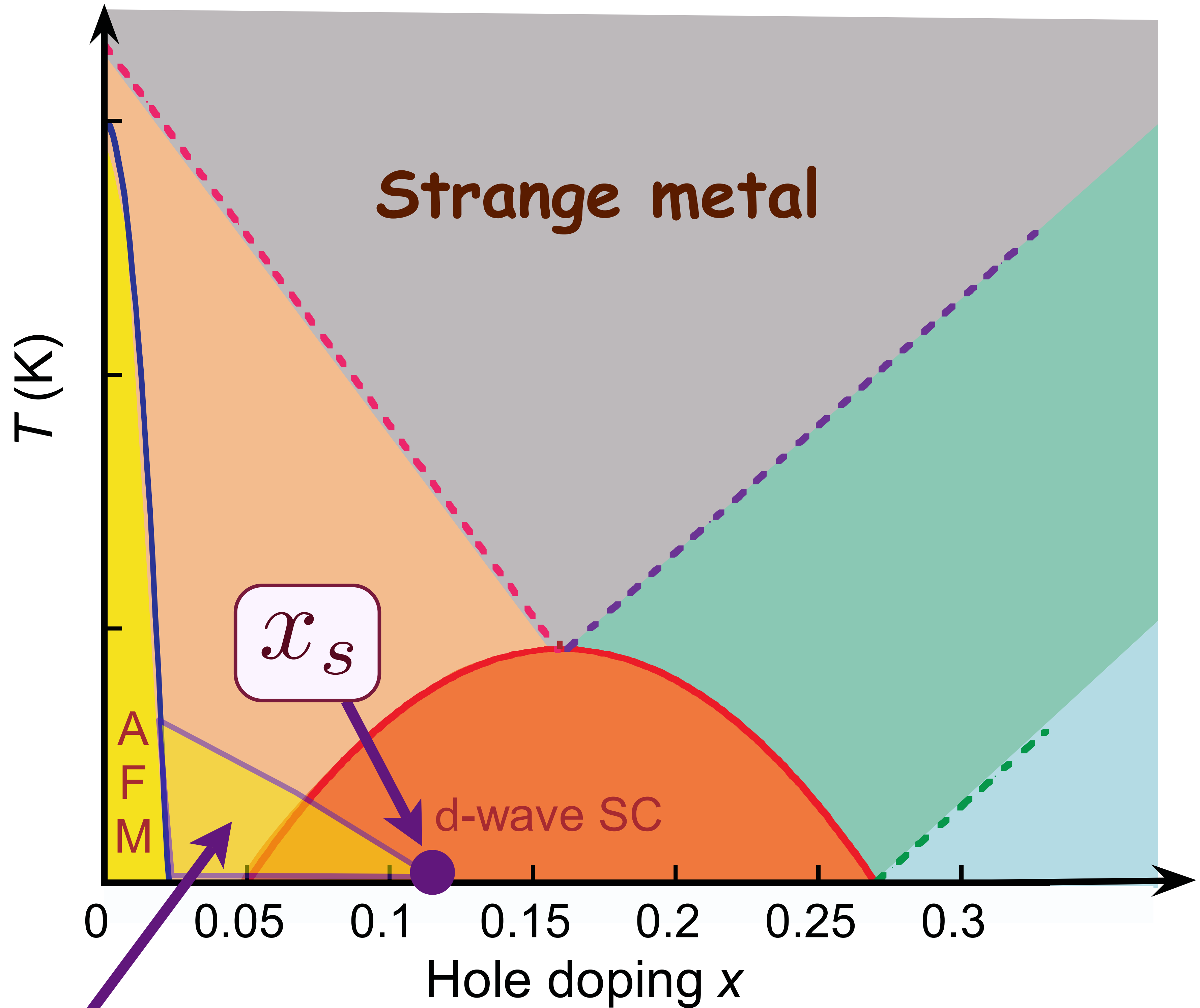


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

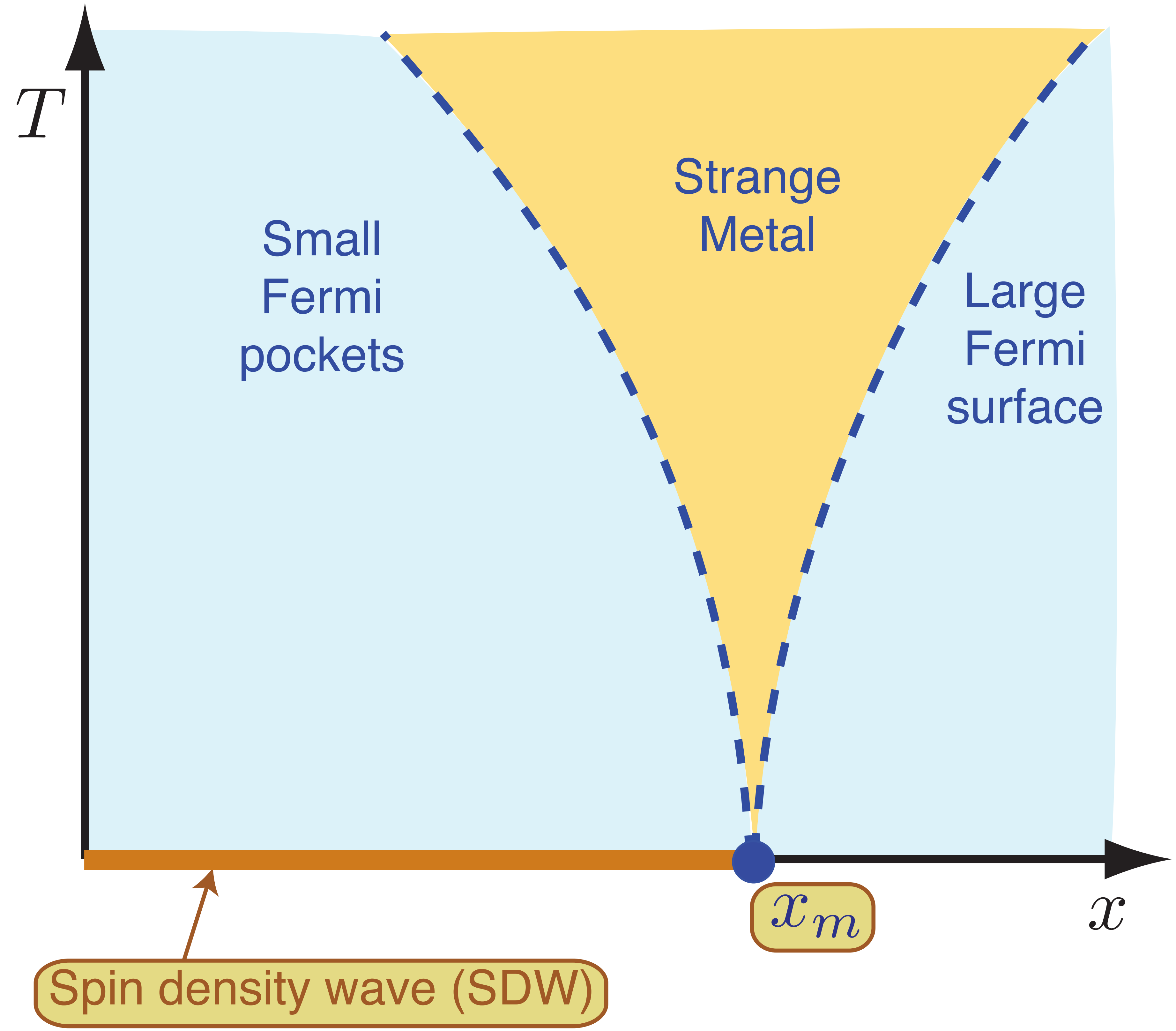
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Theory of quantum criticality in the cuprates



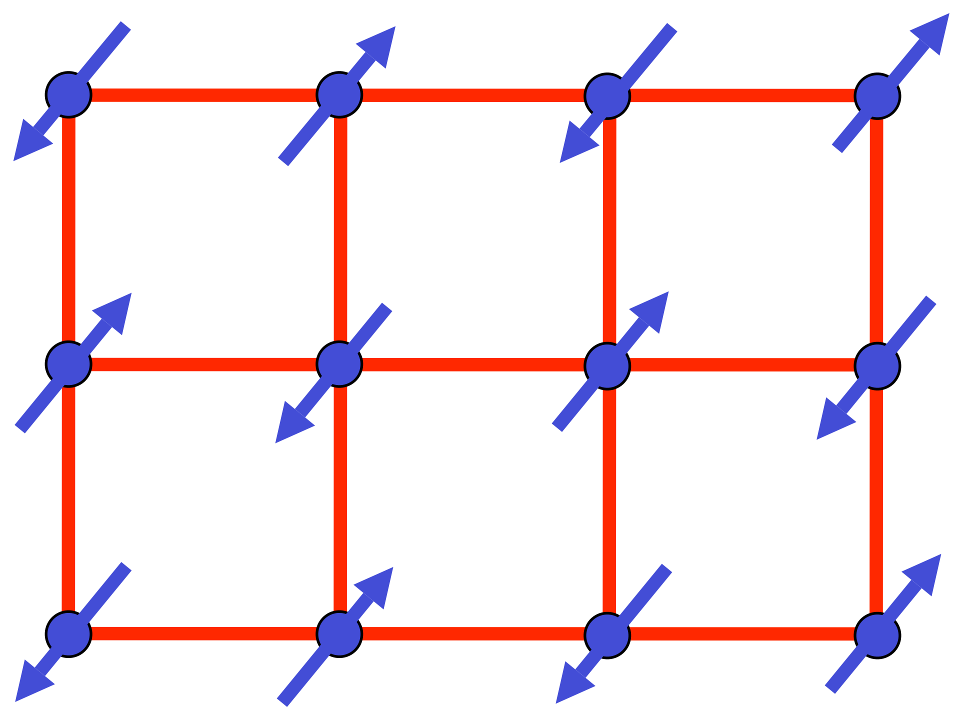
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Spin density wave theory

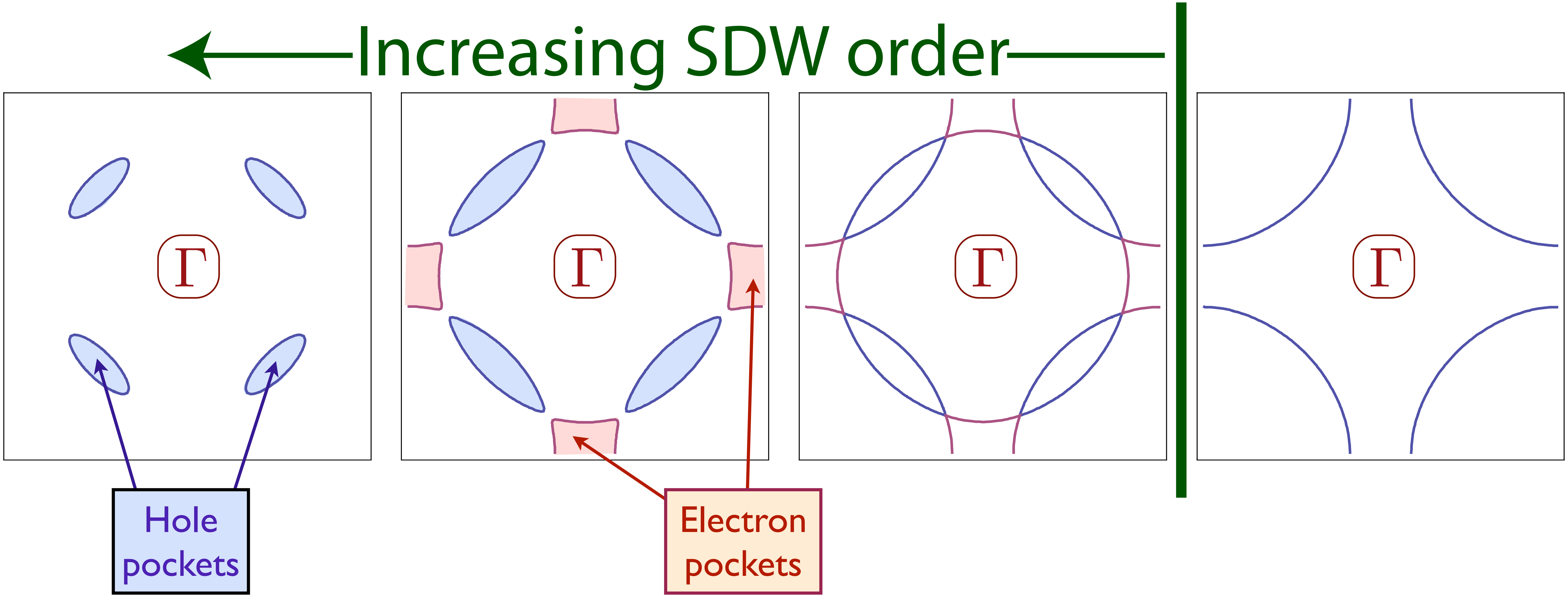
A spin density wave (SDW) is the spontaneous appearance of an oscillatory spin polarization. The electron spin polarization is written as

$$\vec{S}(\mathbf{r},\tau) = \vec{\varphi}(\mathbf{r},\tau)e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the SDW order parameter, and $\mathbf{K}$ is a fixed ordering wavevector. For simplicity we will consider the case of $\mathbf{K} = (\pi,\pi)$, but our treatment applies to general $\mathbf{K}$.



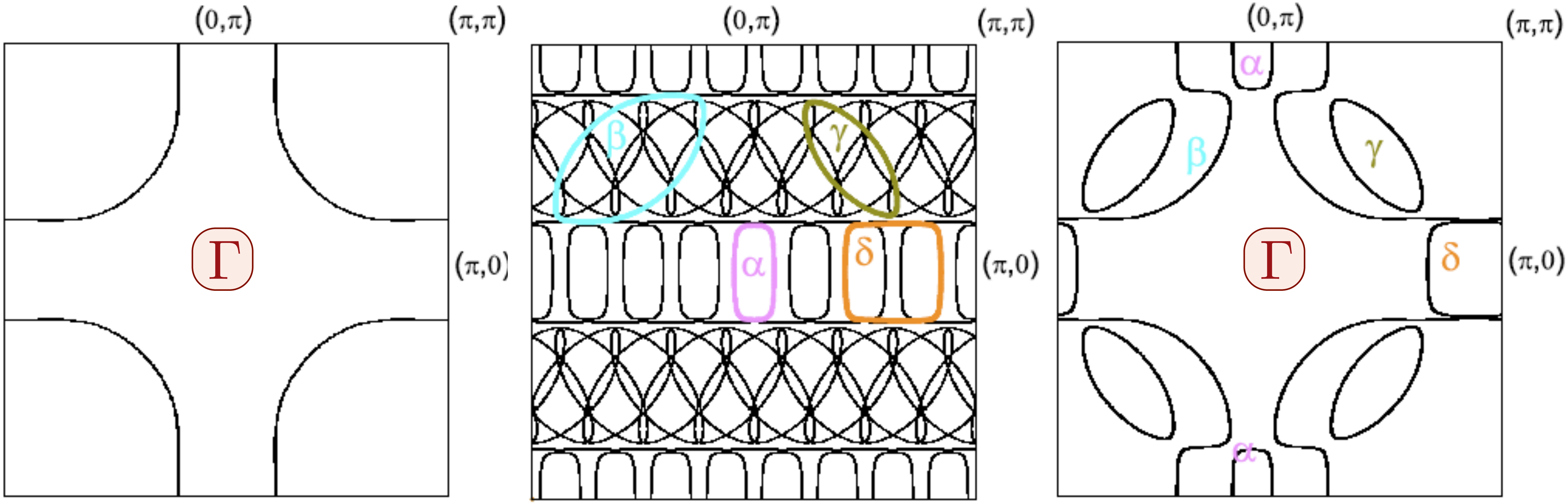
Spin density wave theory in hole-doped cuprates



The amplitude of the SDW order parameter $\vec{\varphi}$ vanishes at the transition to the Fermi liquid.

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

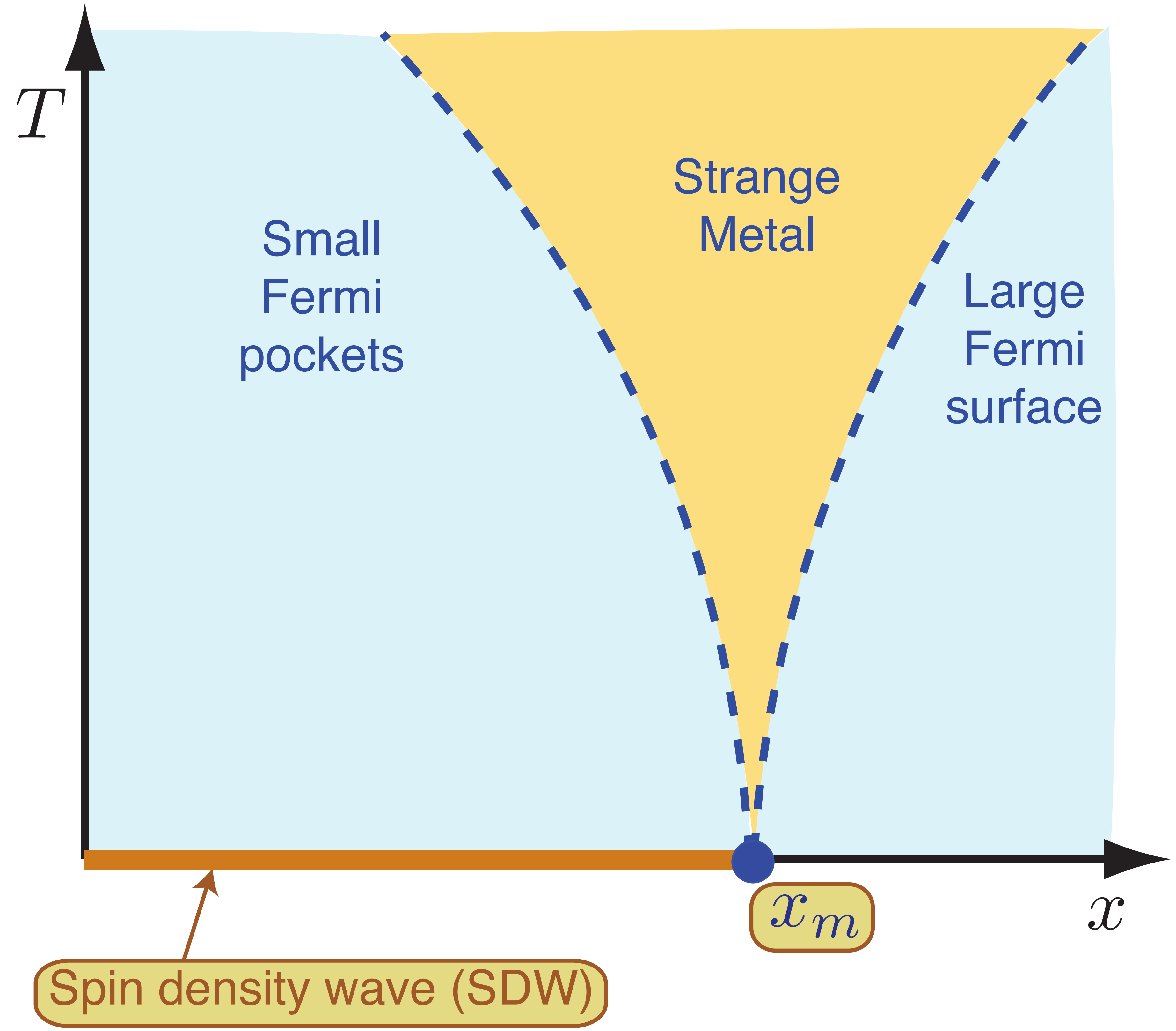
Spin density wave theory in hole-doped cuprates



Incommensurate order in $YBa_2Cu_3O_{6+x}$

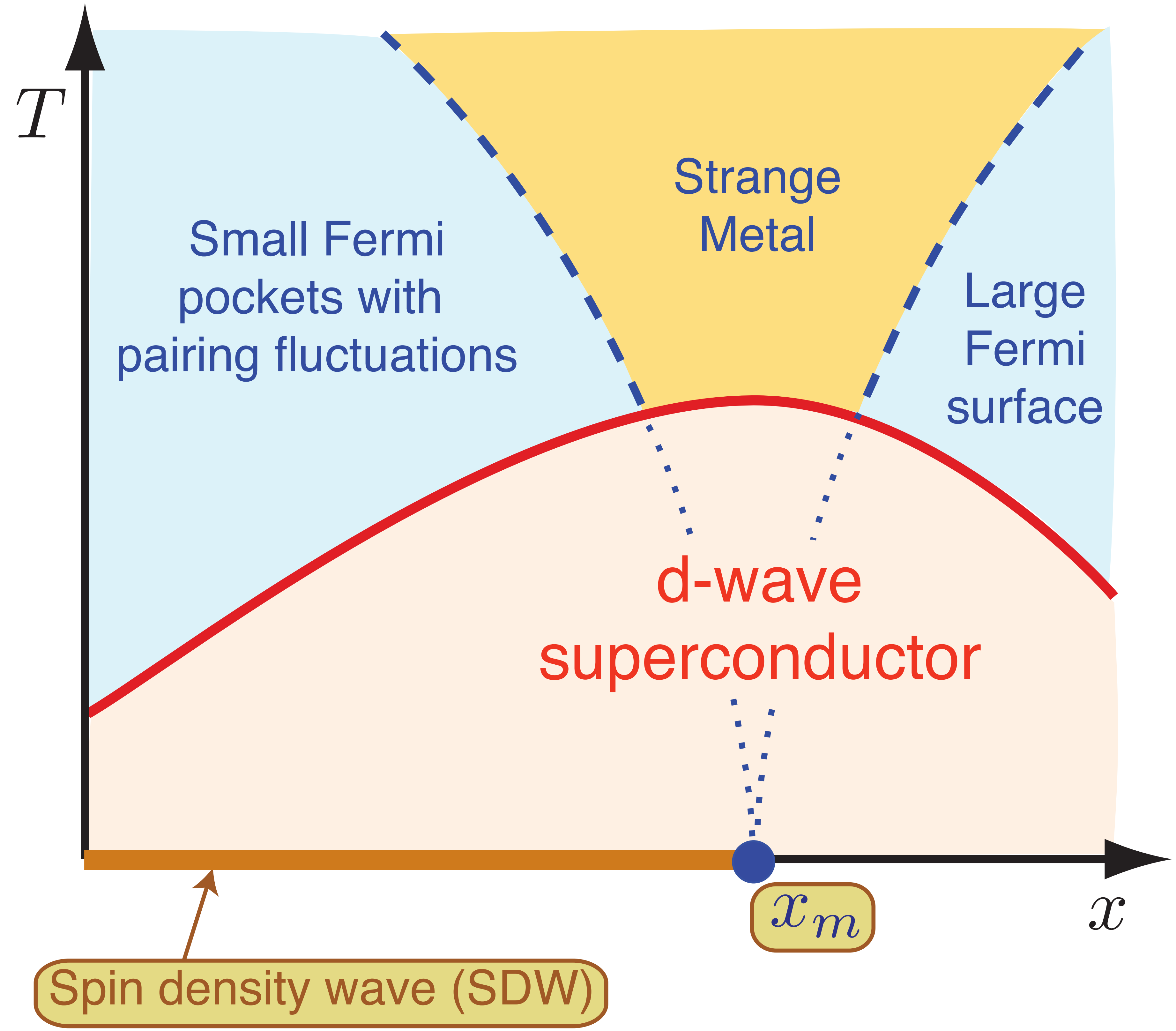
A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Theory of quantum criticality in the cuprates



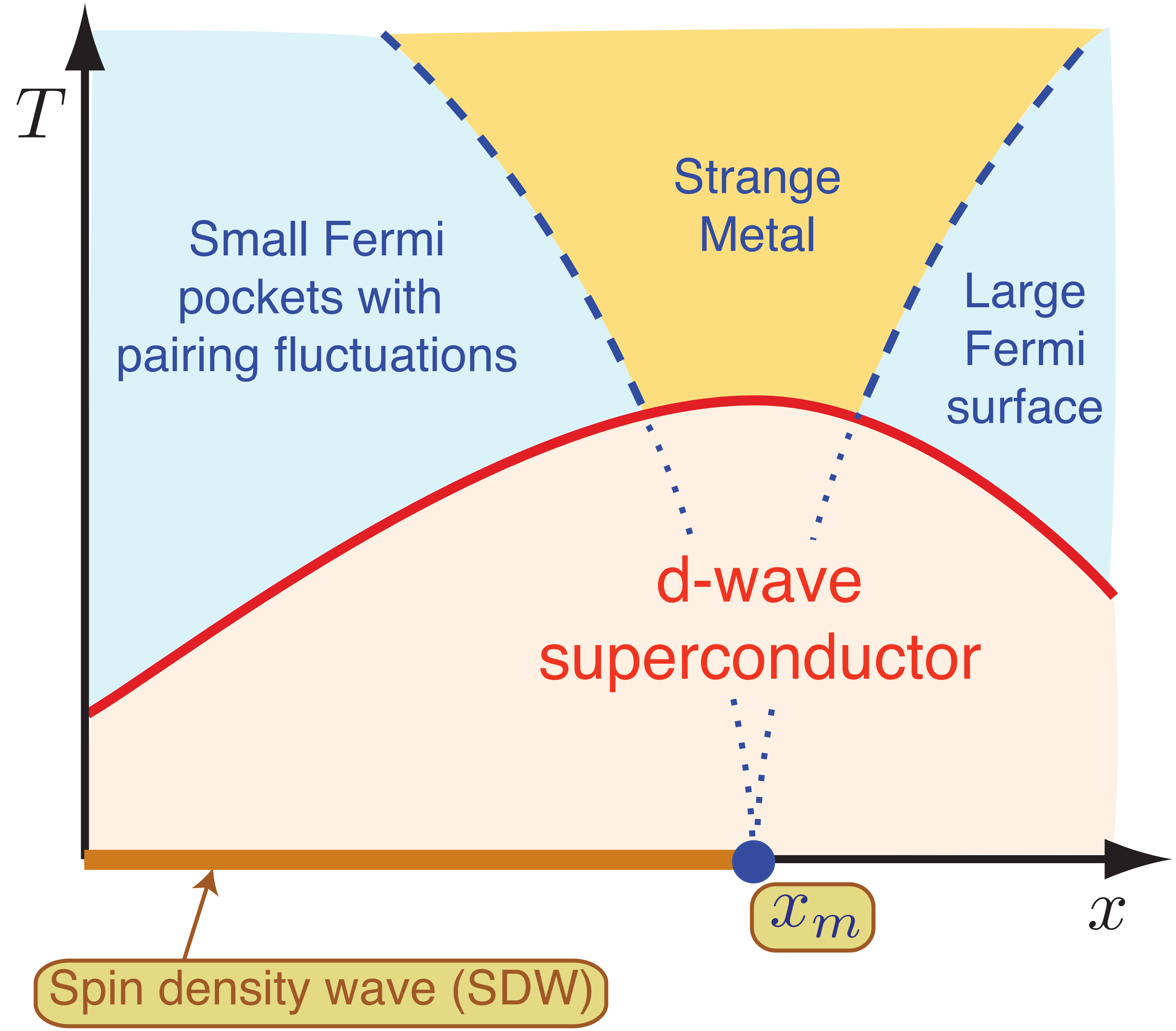
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



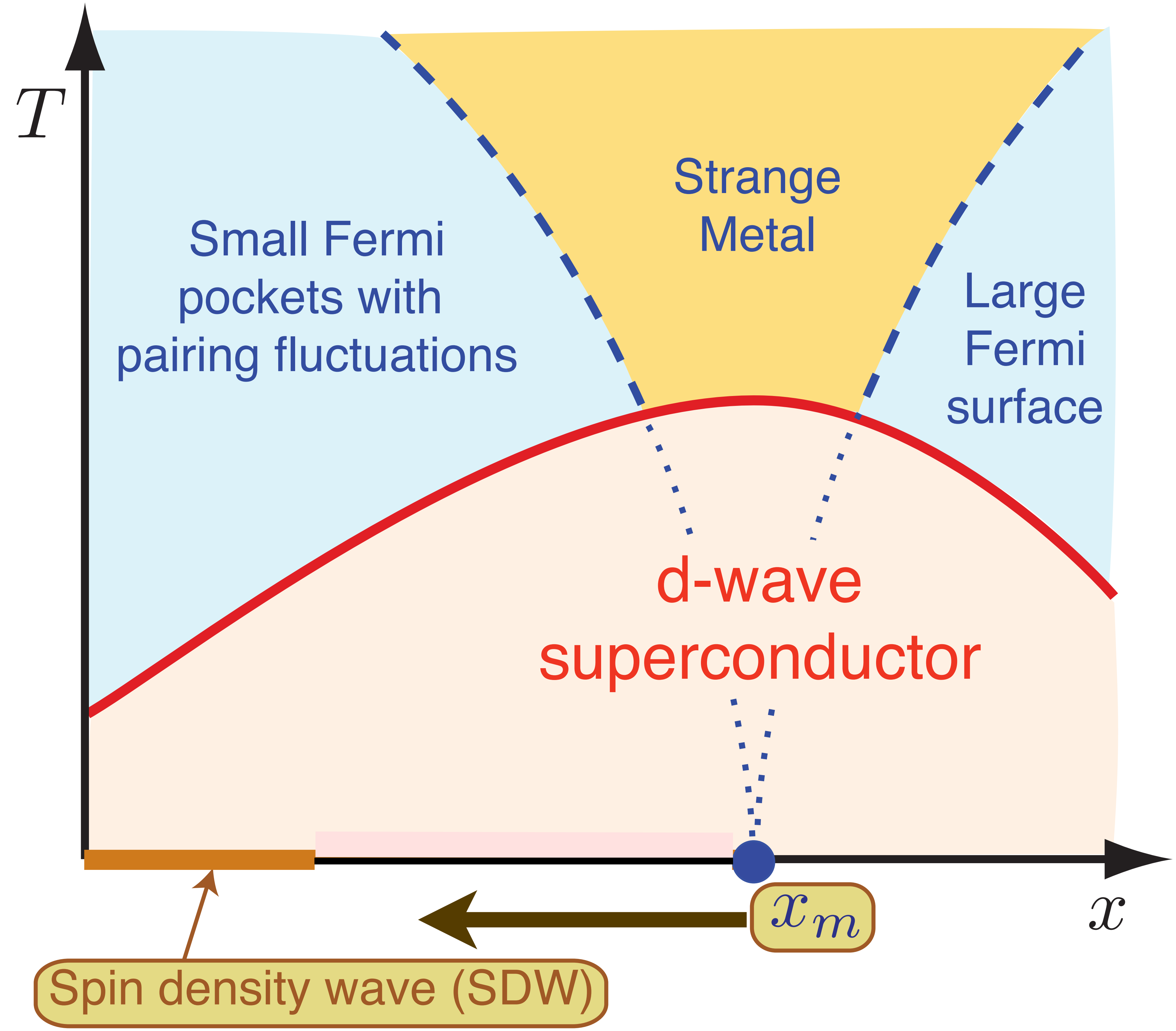
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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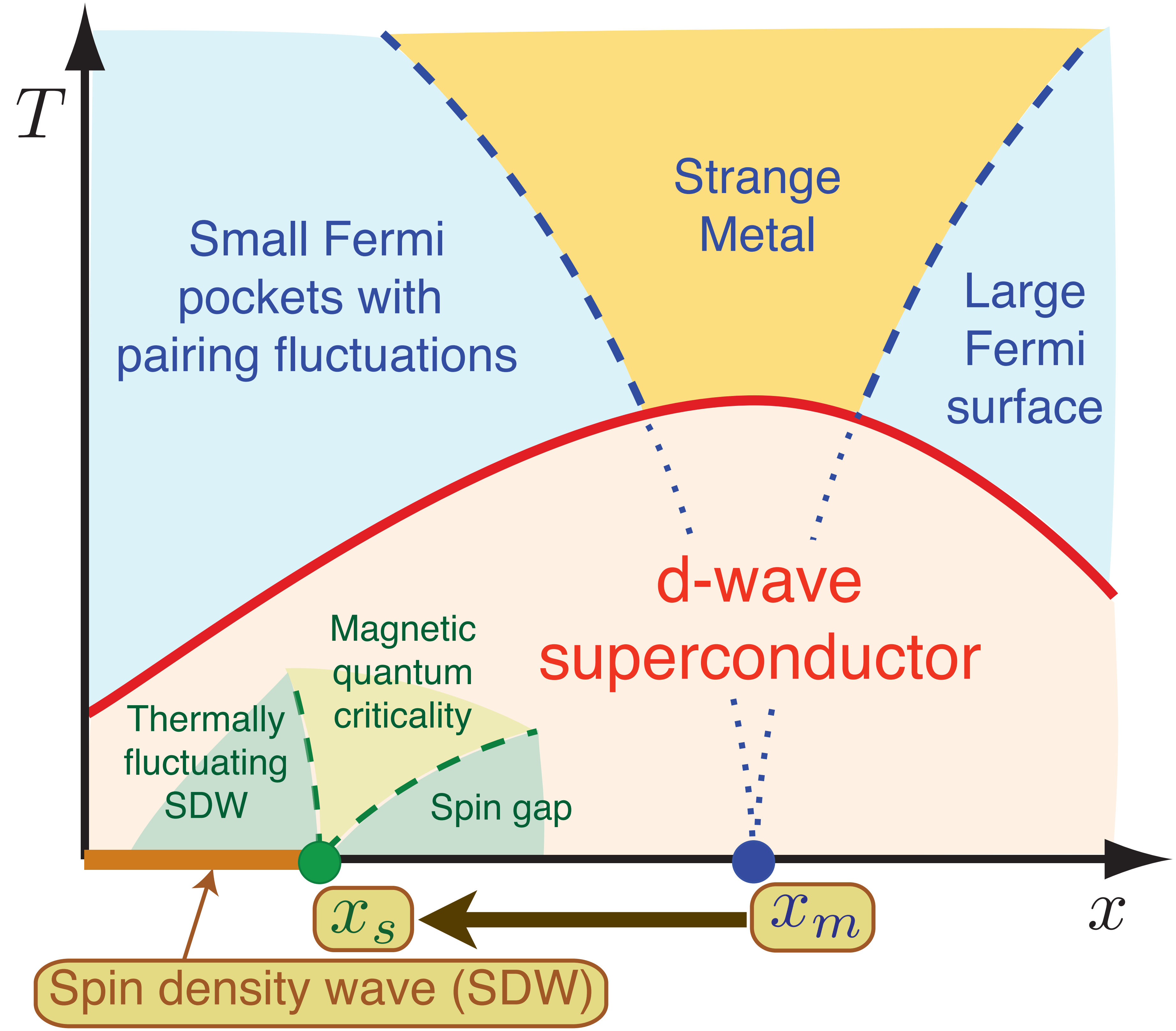

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Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order

4. A unified theory
   SU(2) gauge theory of transition from Fermi pockets to a large Fermi surface

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Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \frac{u}{4} \left(\vec{\varphi}^2 \right)^2 \right.$$
$$\left. + \kappa \, \vec{\varphi}^2 \left| \psi \right|^2 \right]$$
$$+ \int d^2 r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516 (2002).

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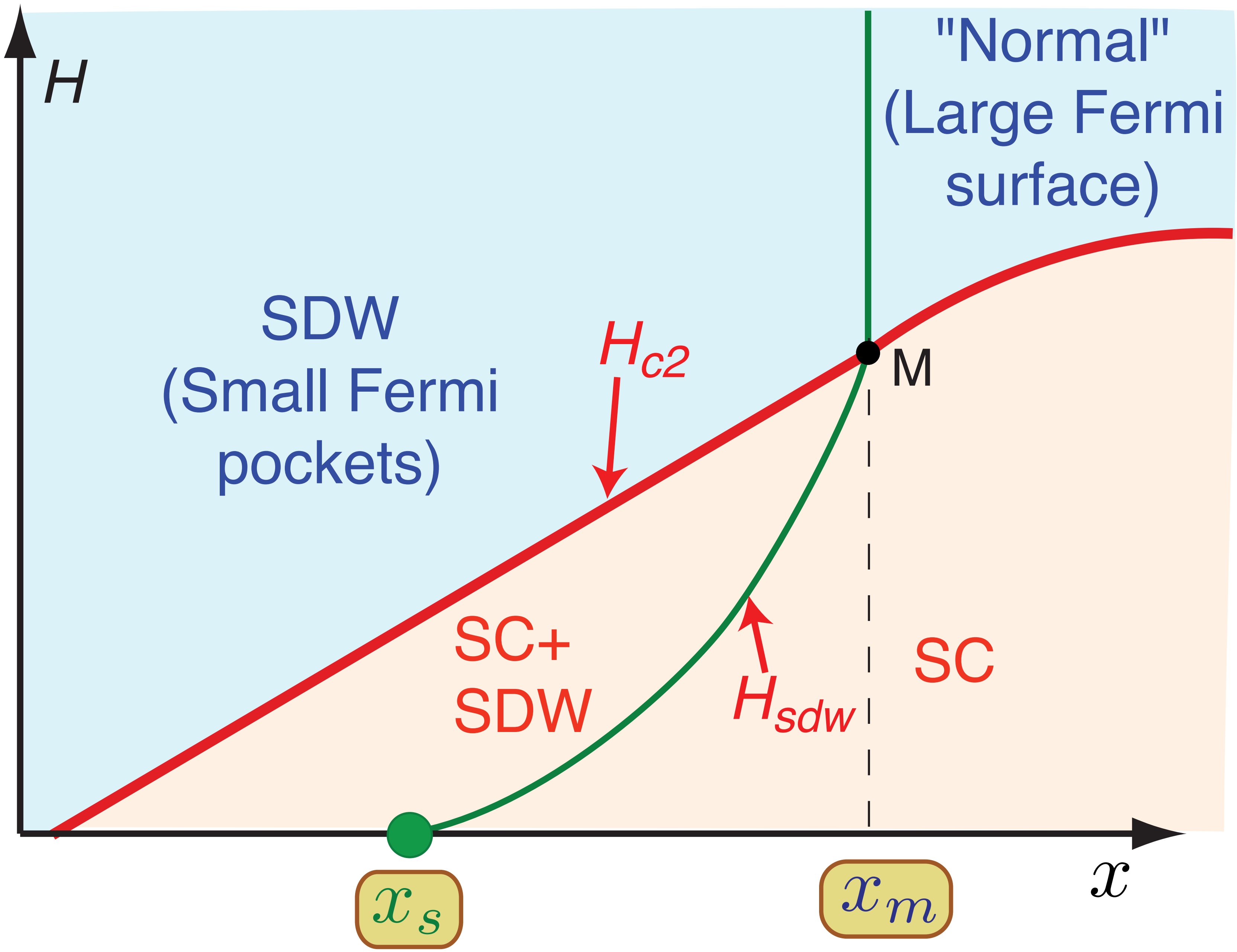

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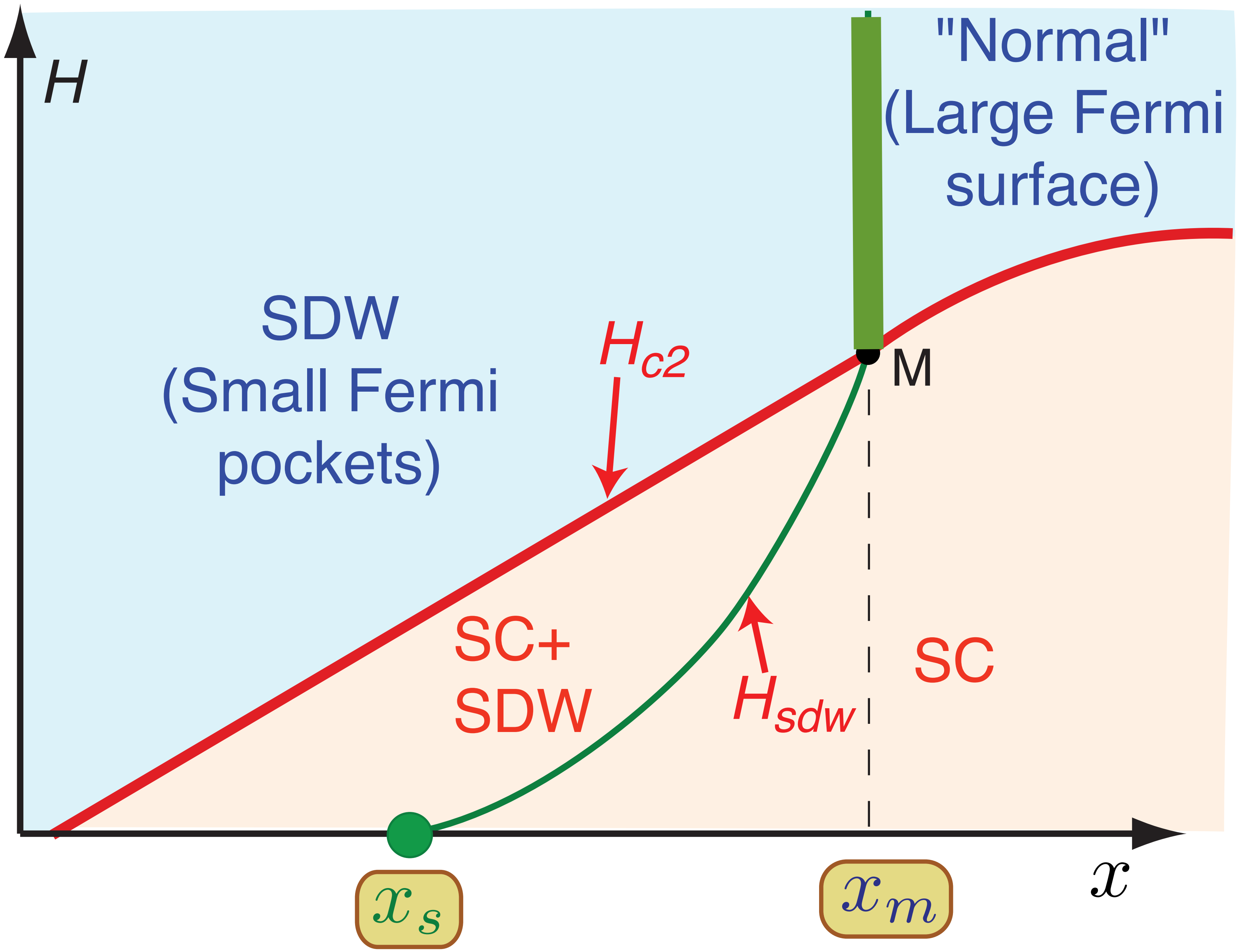

- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

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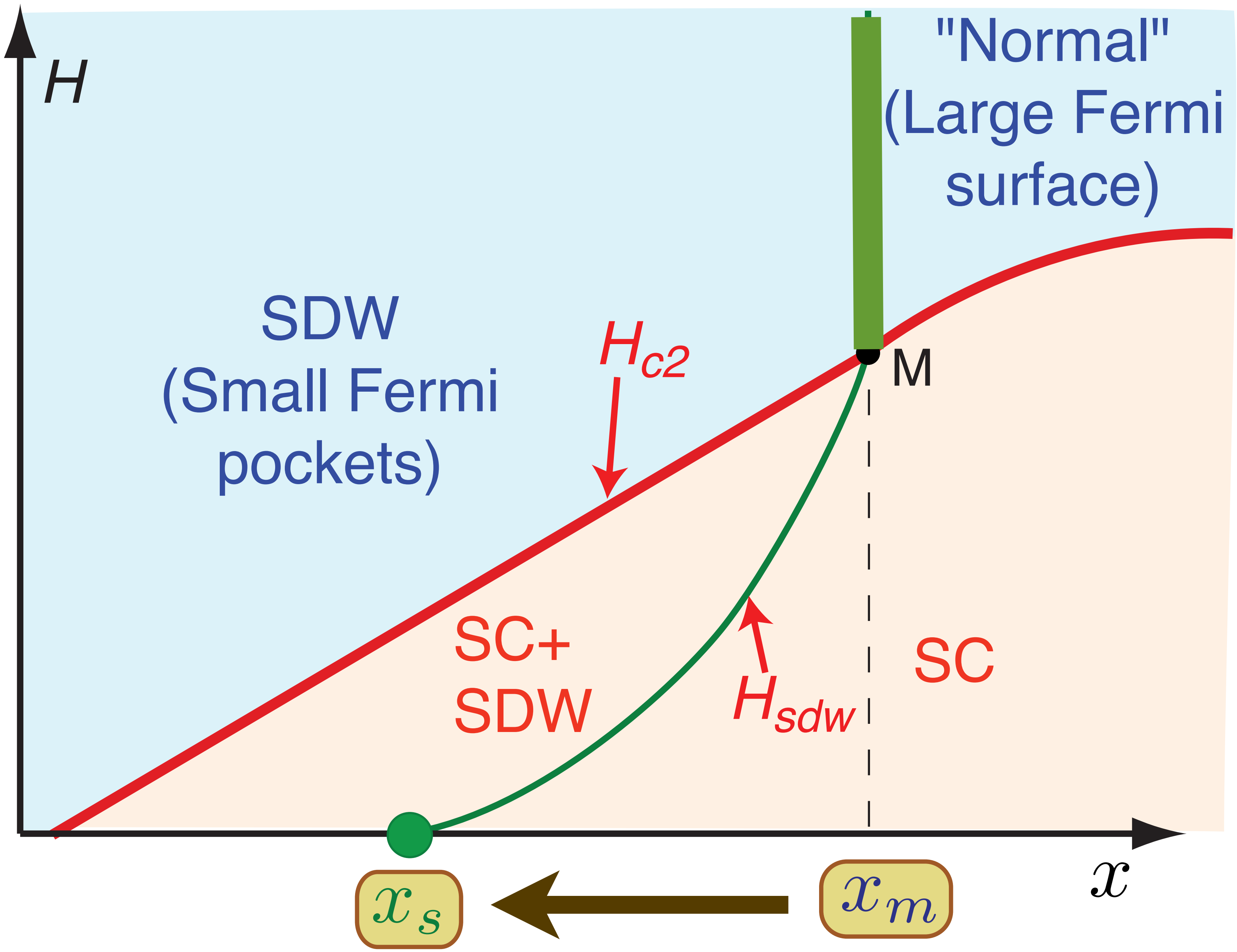

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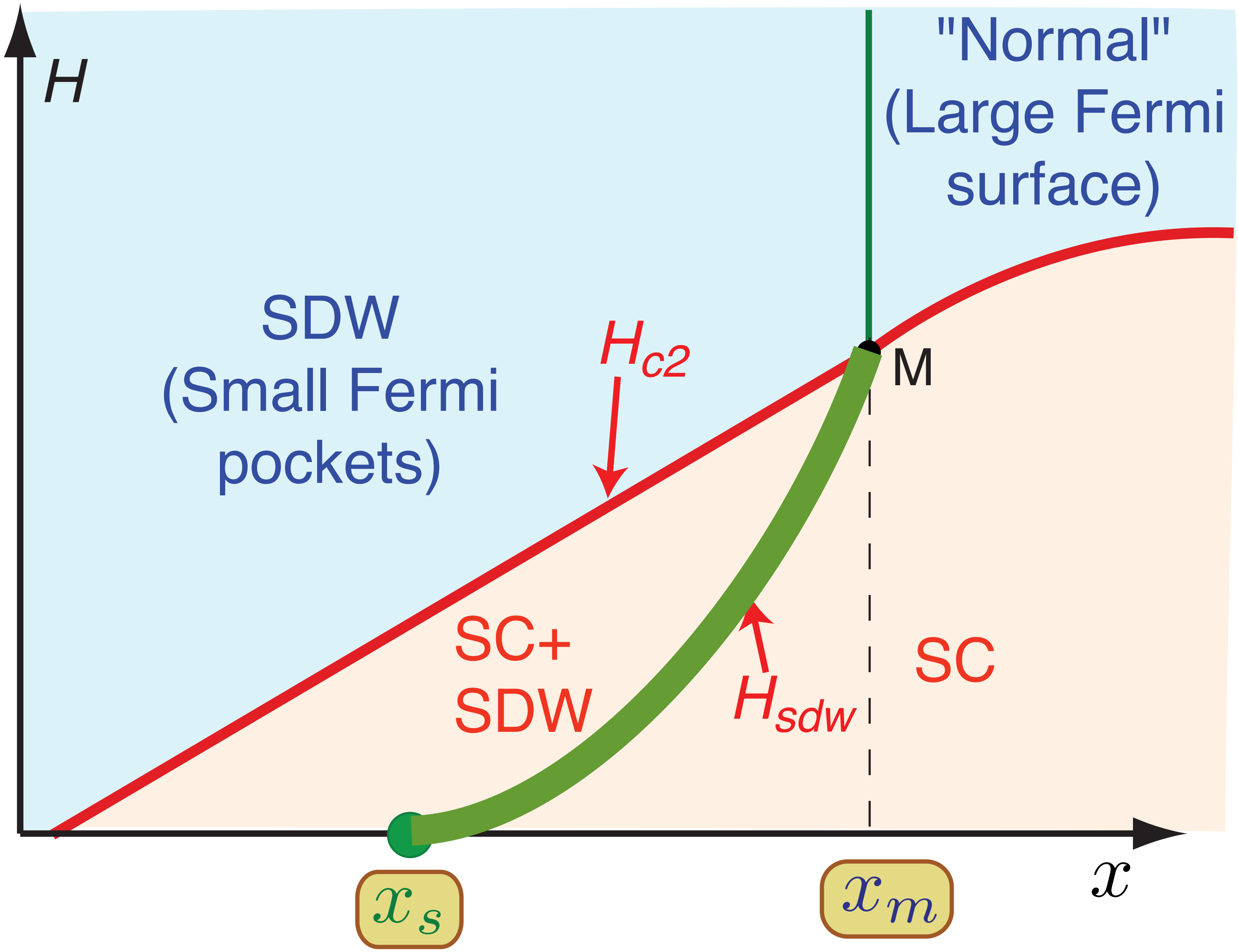

- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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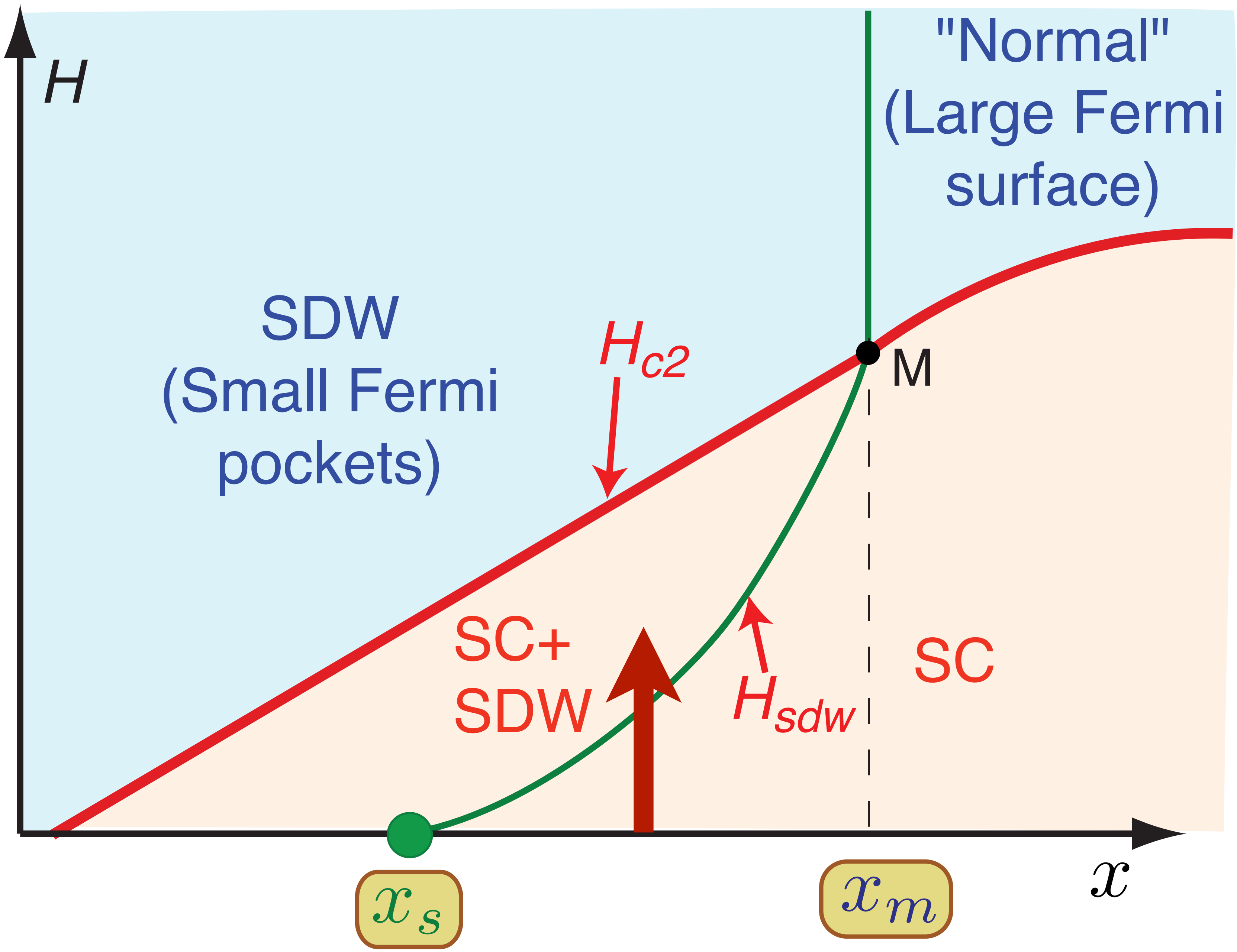

Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, *Phys. Rev. Lett.* 102, 177006 (2009).

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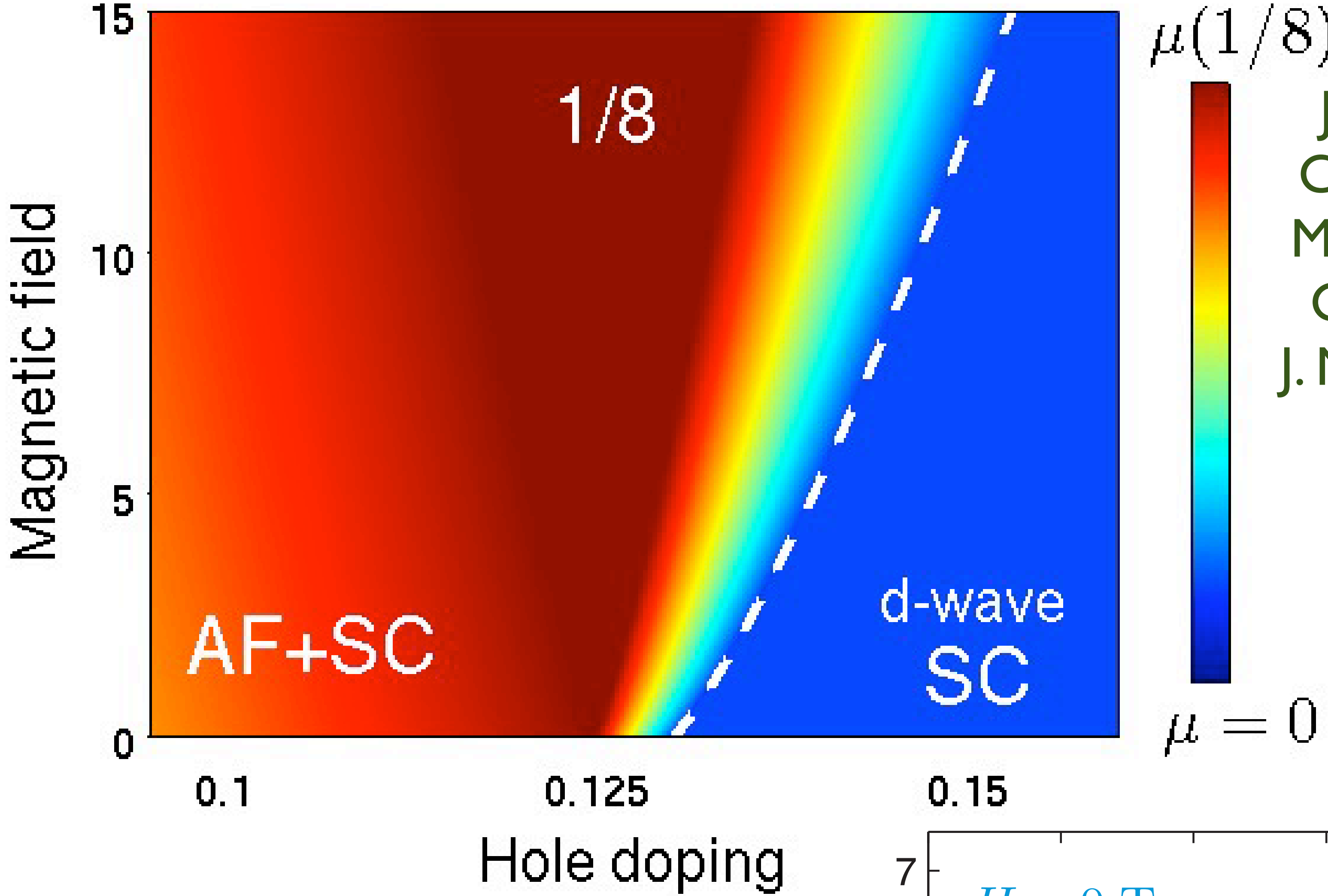
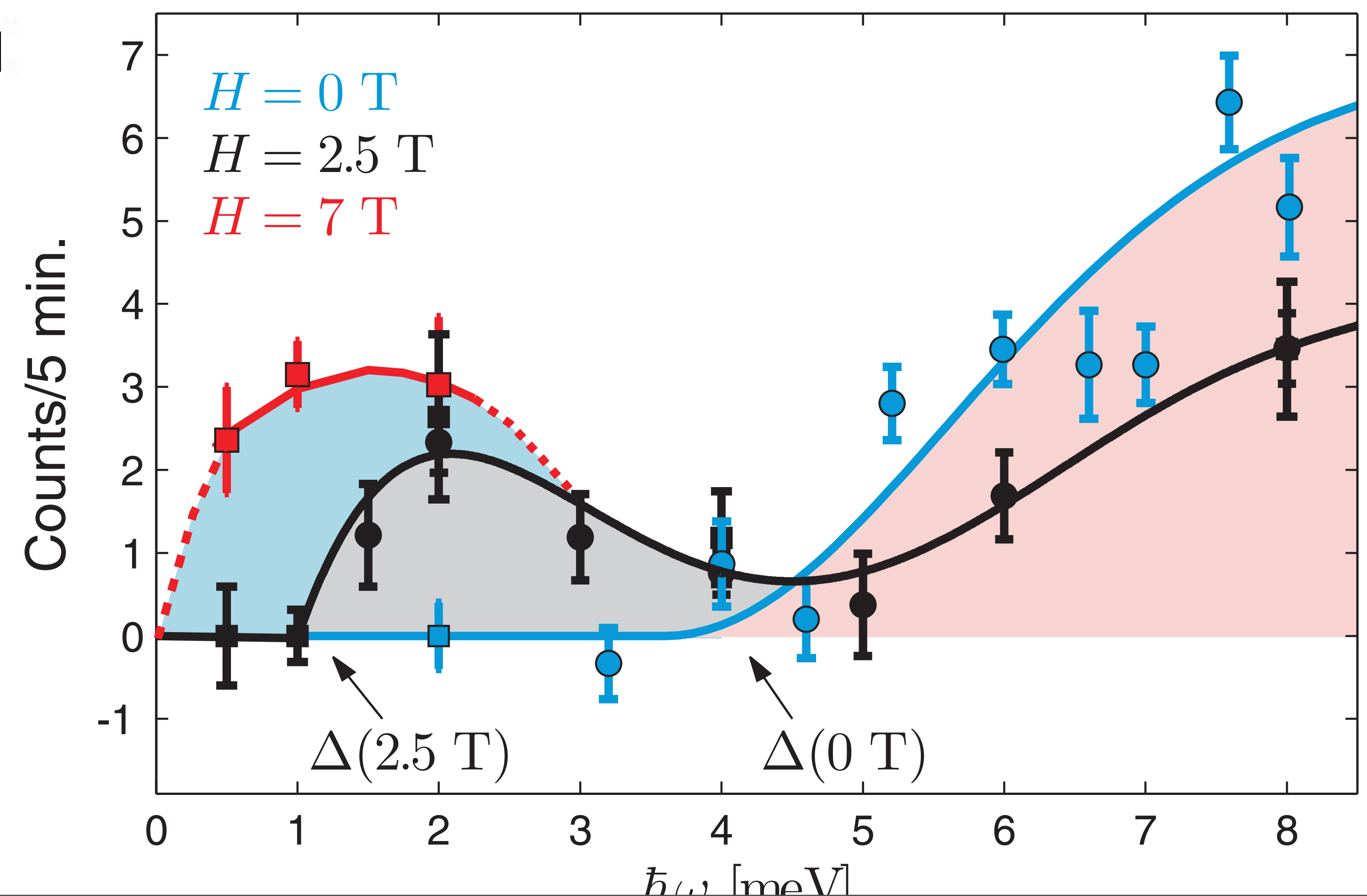


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

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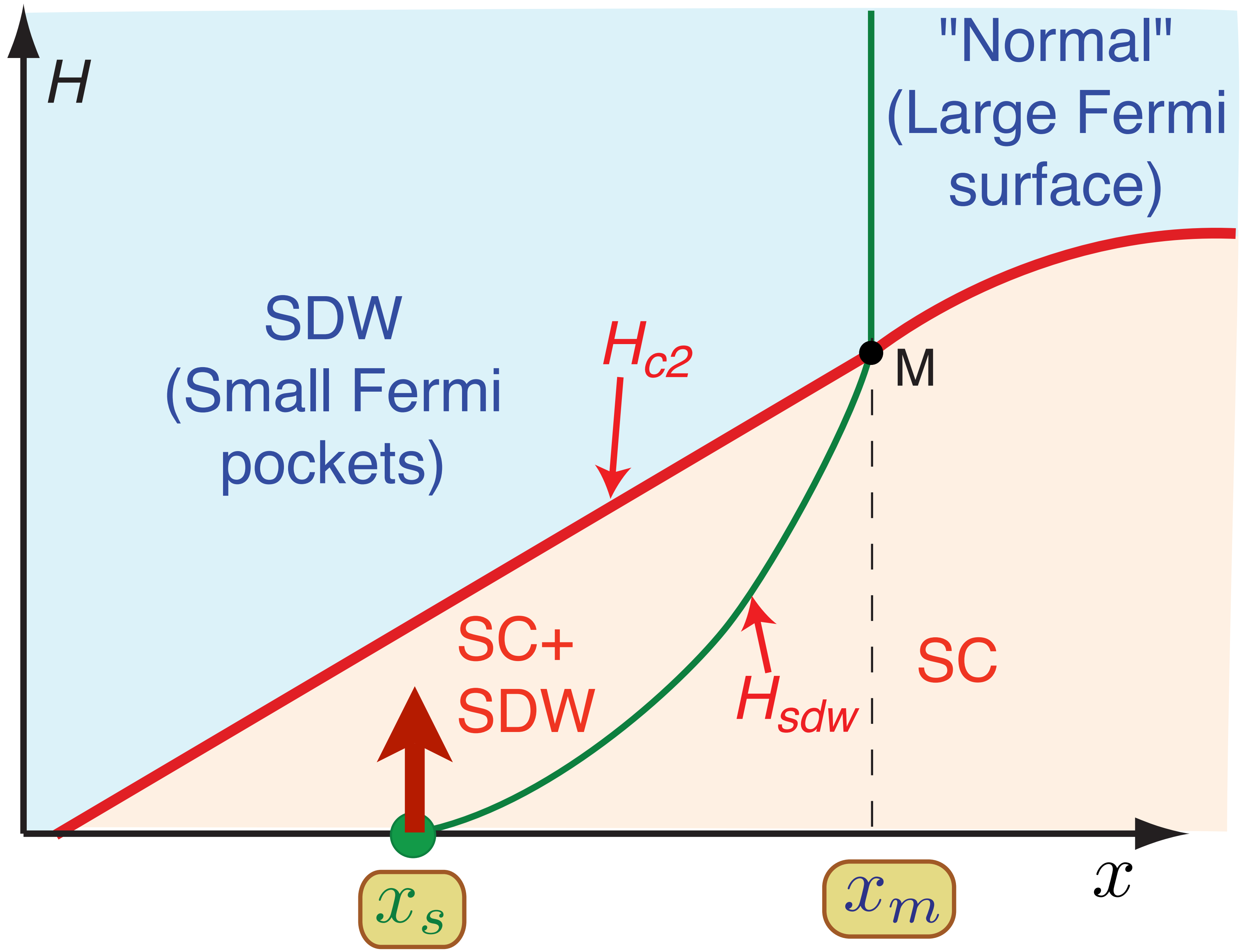

Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, *Phys. Rev. Lett.* **103**, 017001 (2009).

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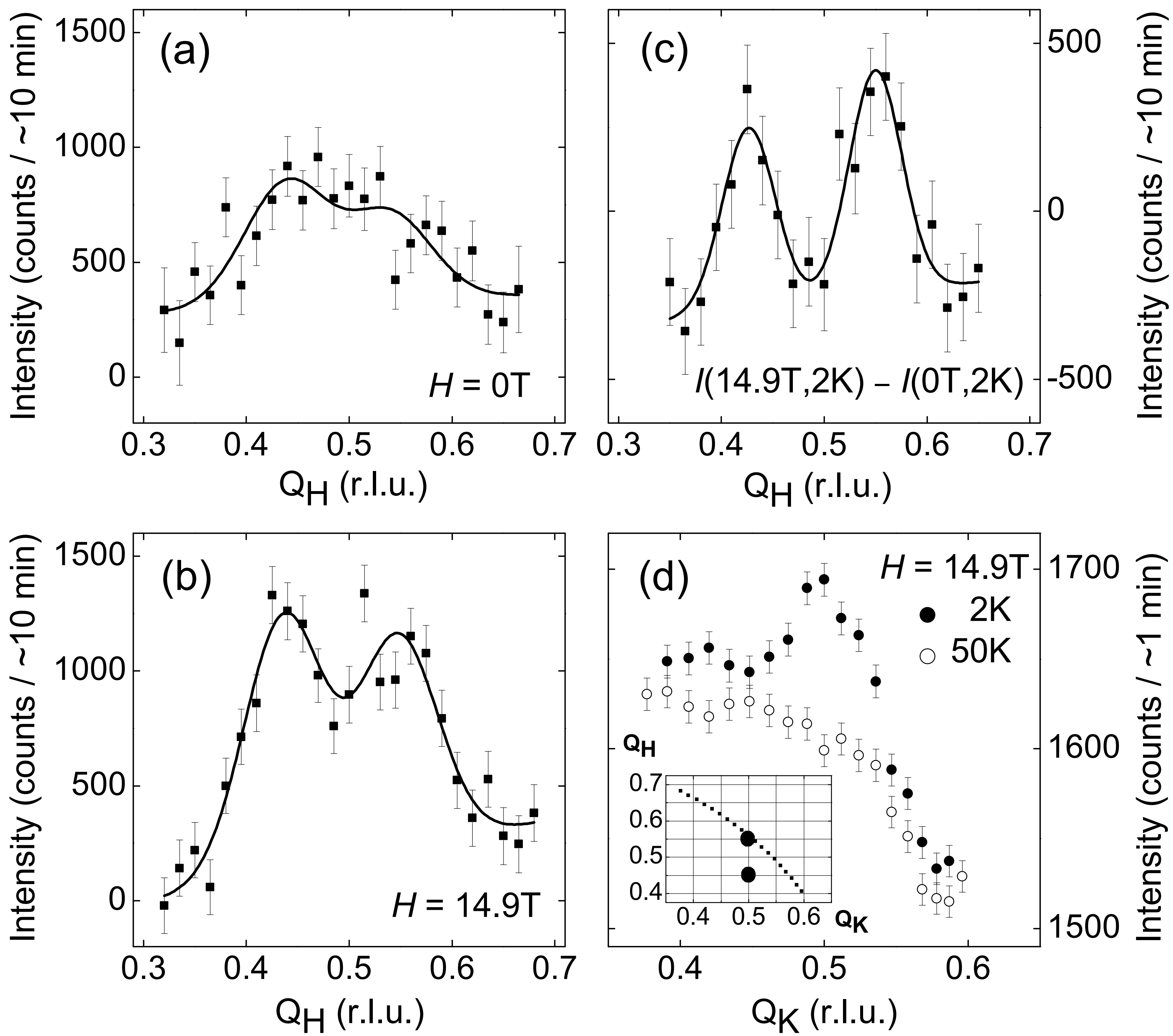

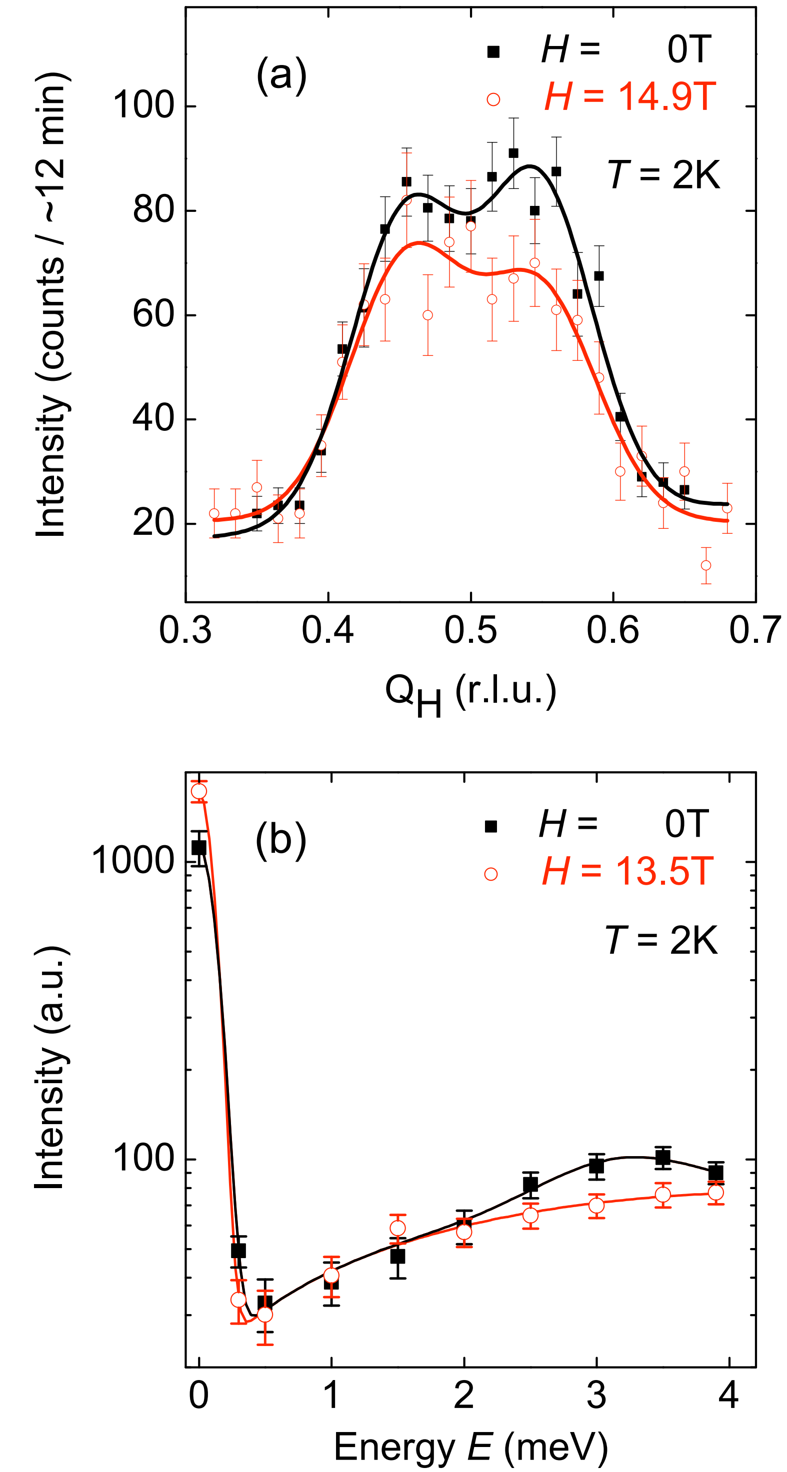

D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

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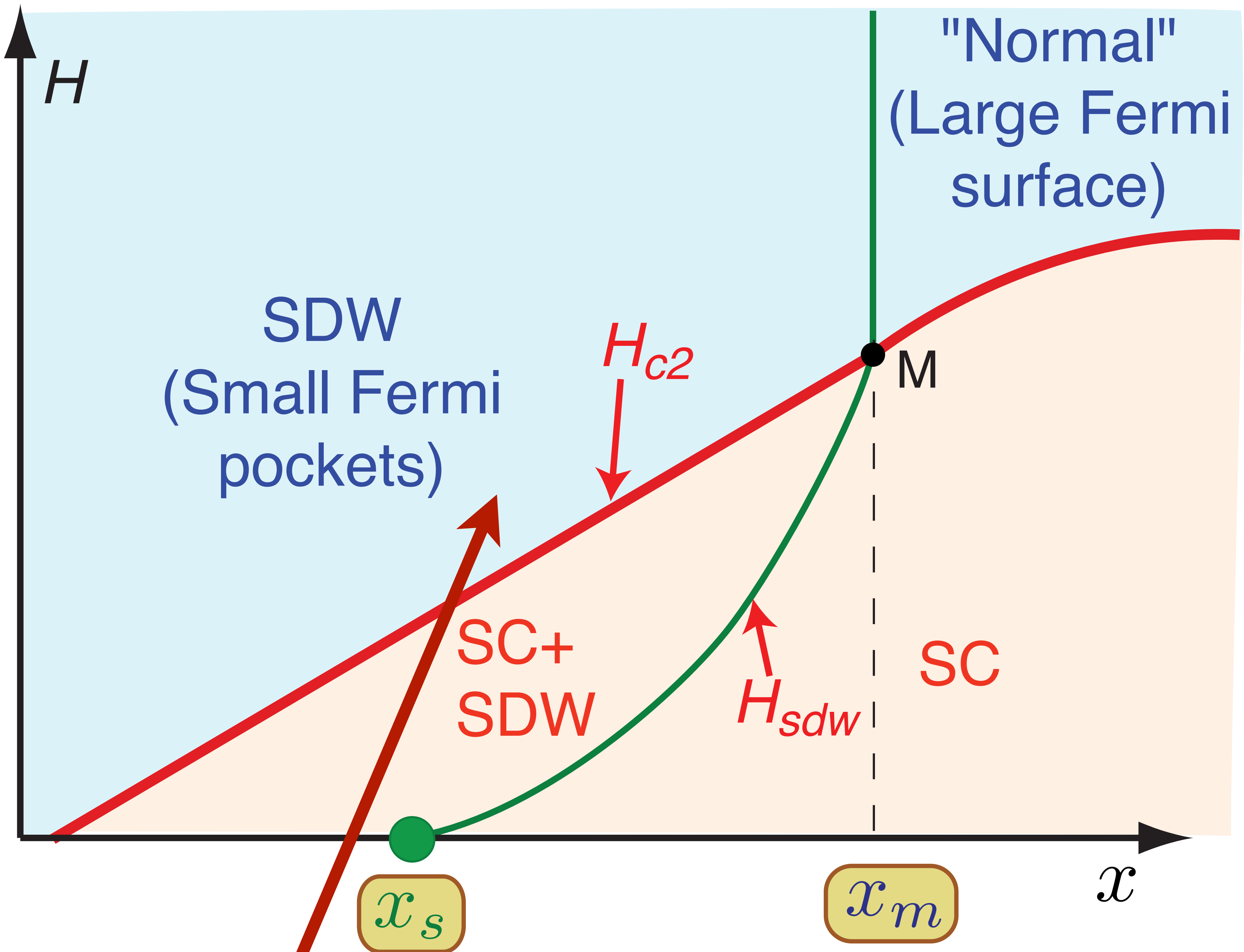


Quantum oscillations without Zeeman splitting

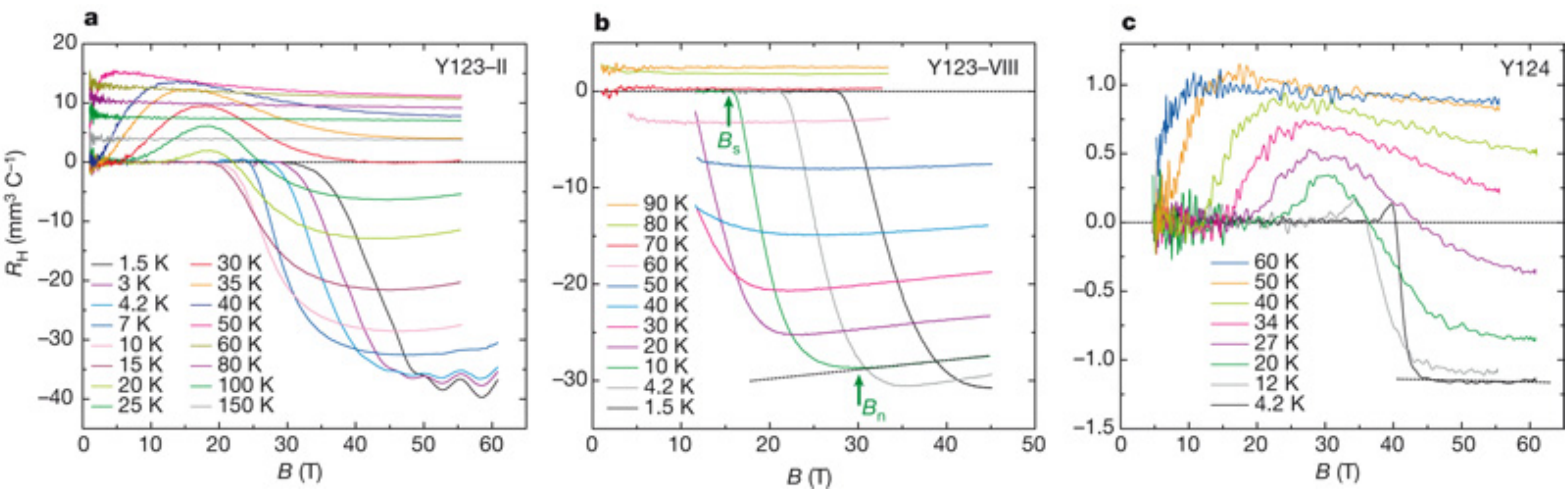
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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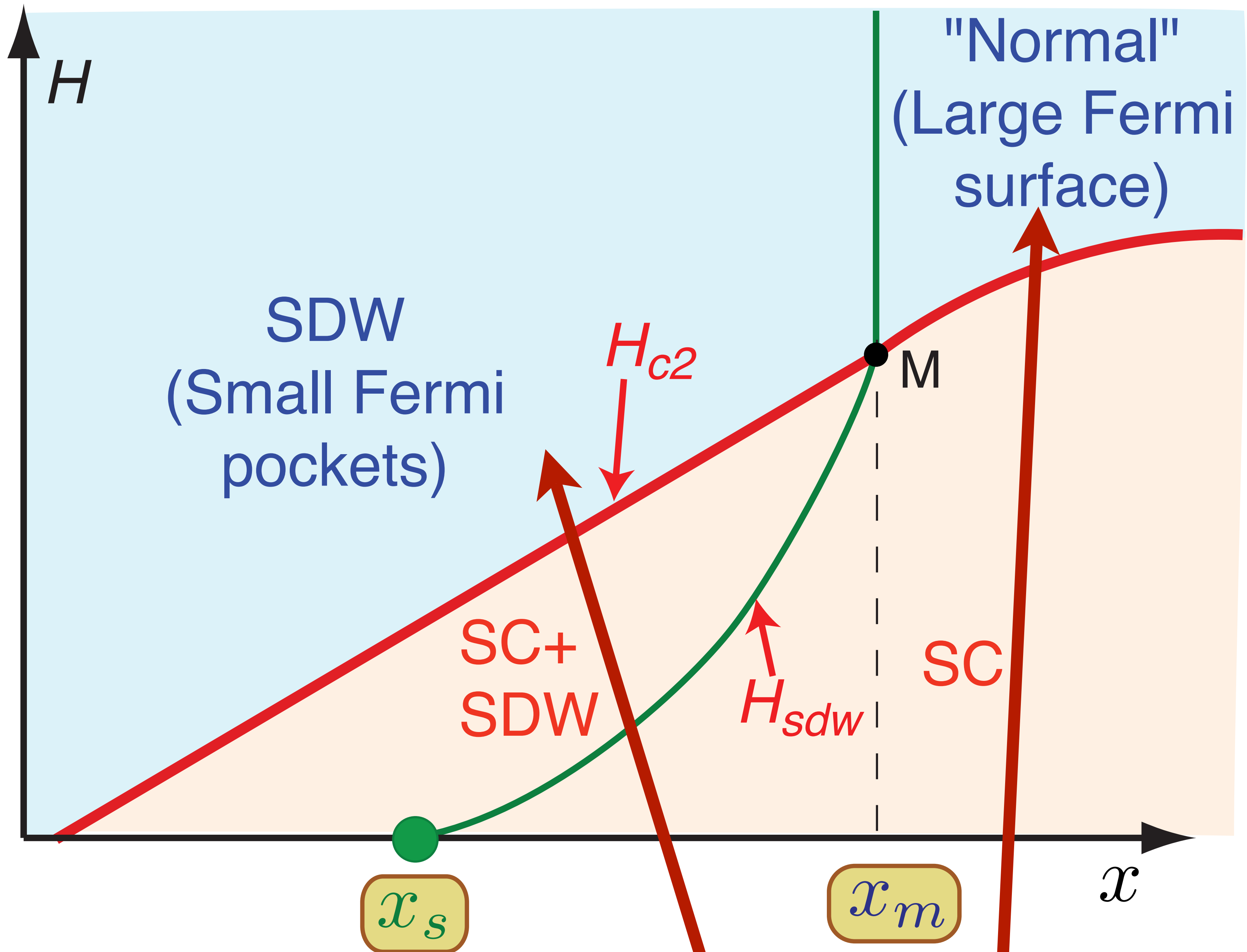
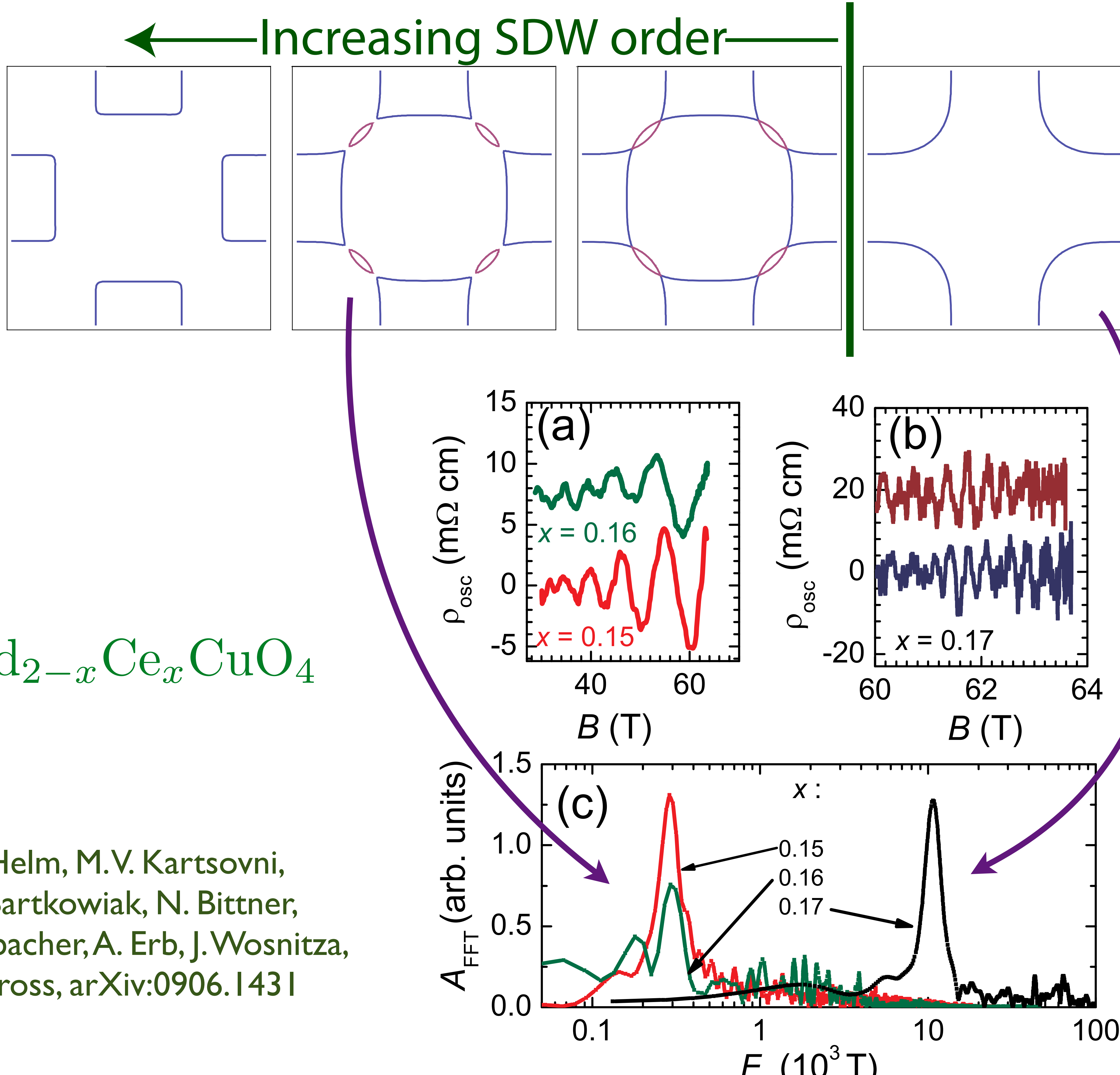


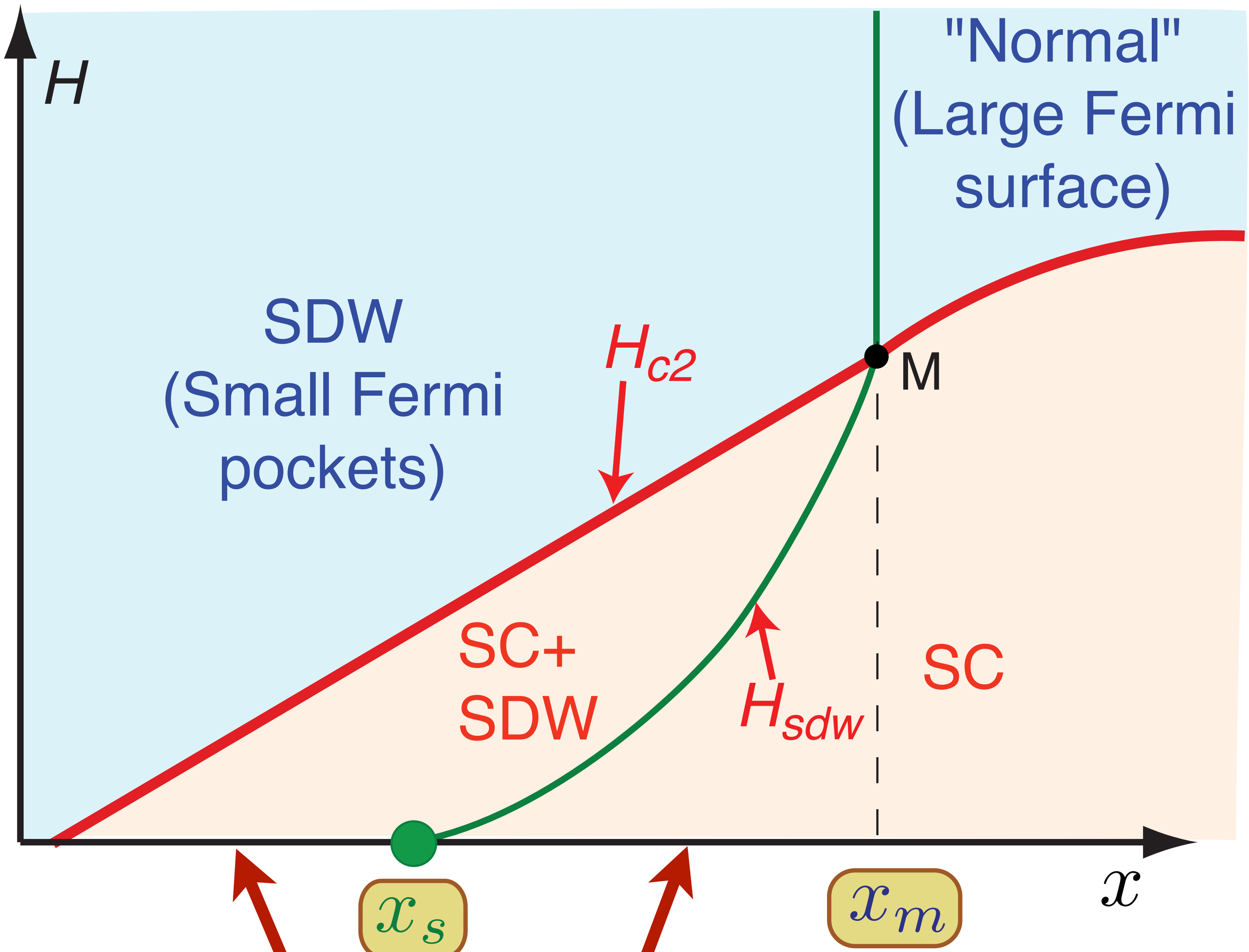
Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$



$Nd_{2-x}Ce_xCuO_4$

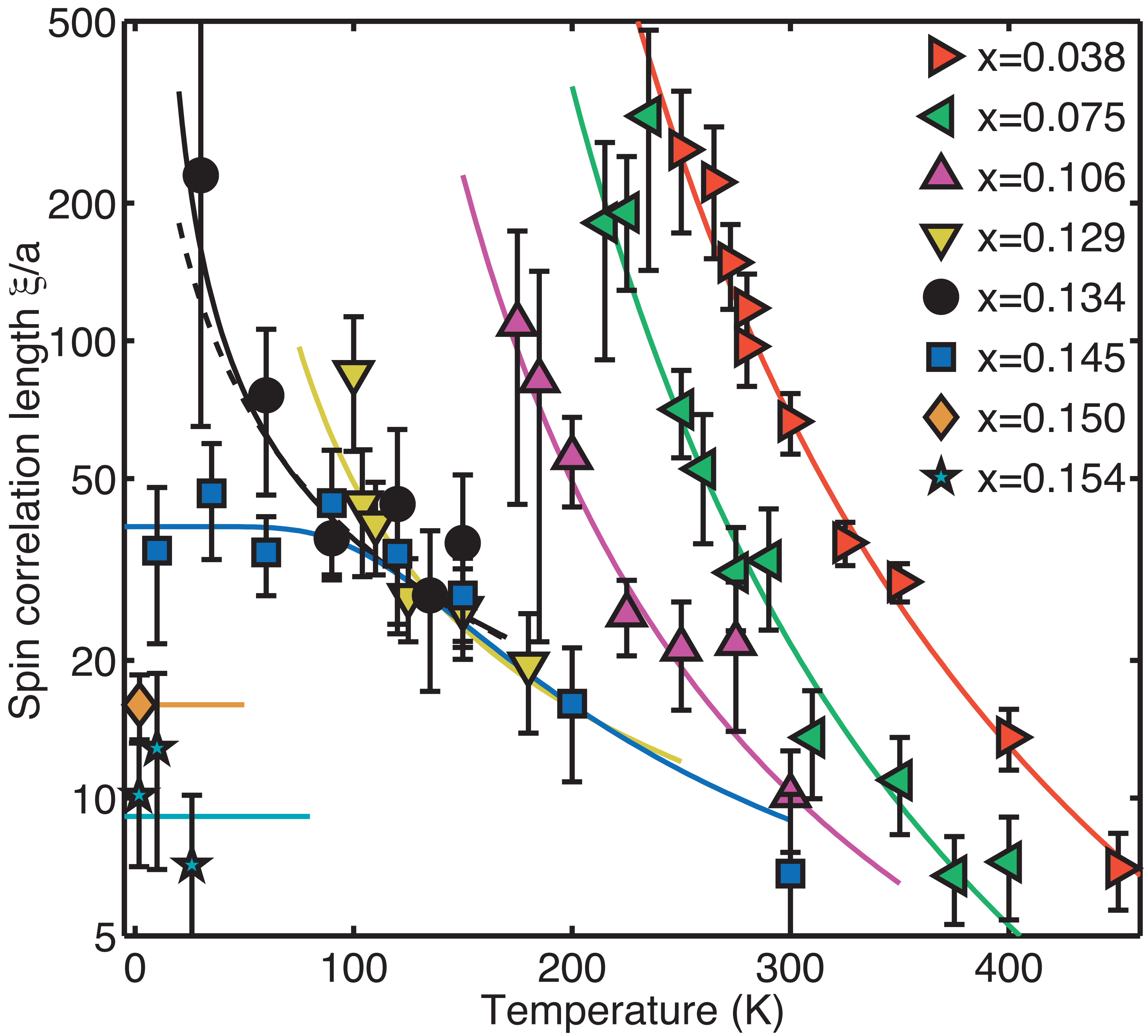
T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

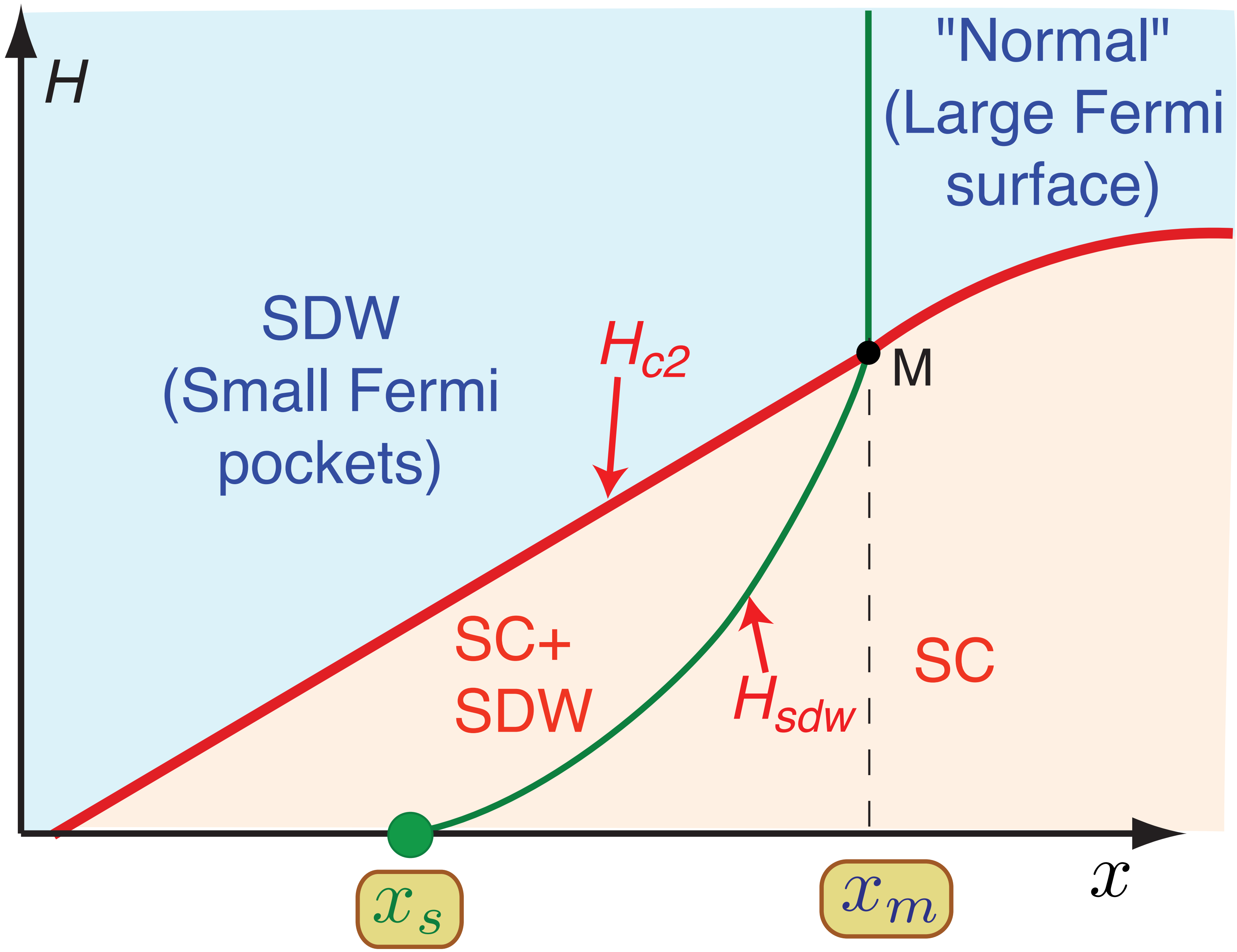


Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$

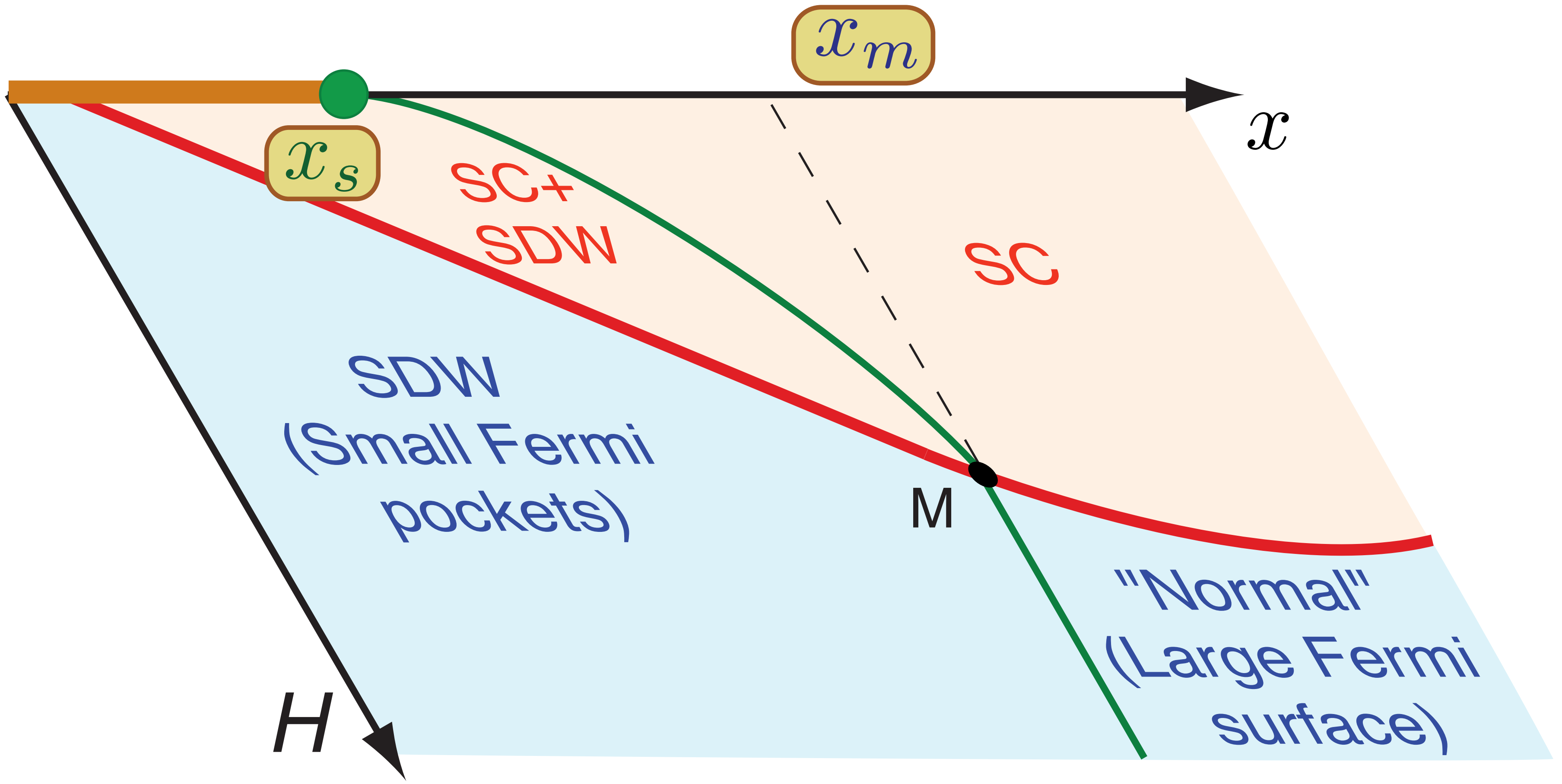
$Nd_{2-x}Ce_xCuO_4$



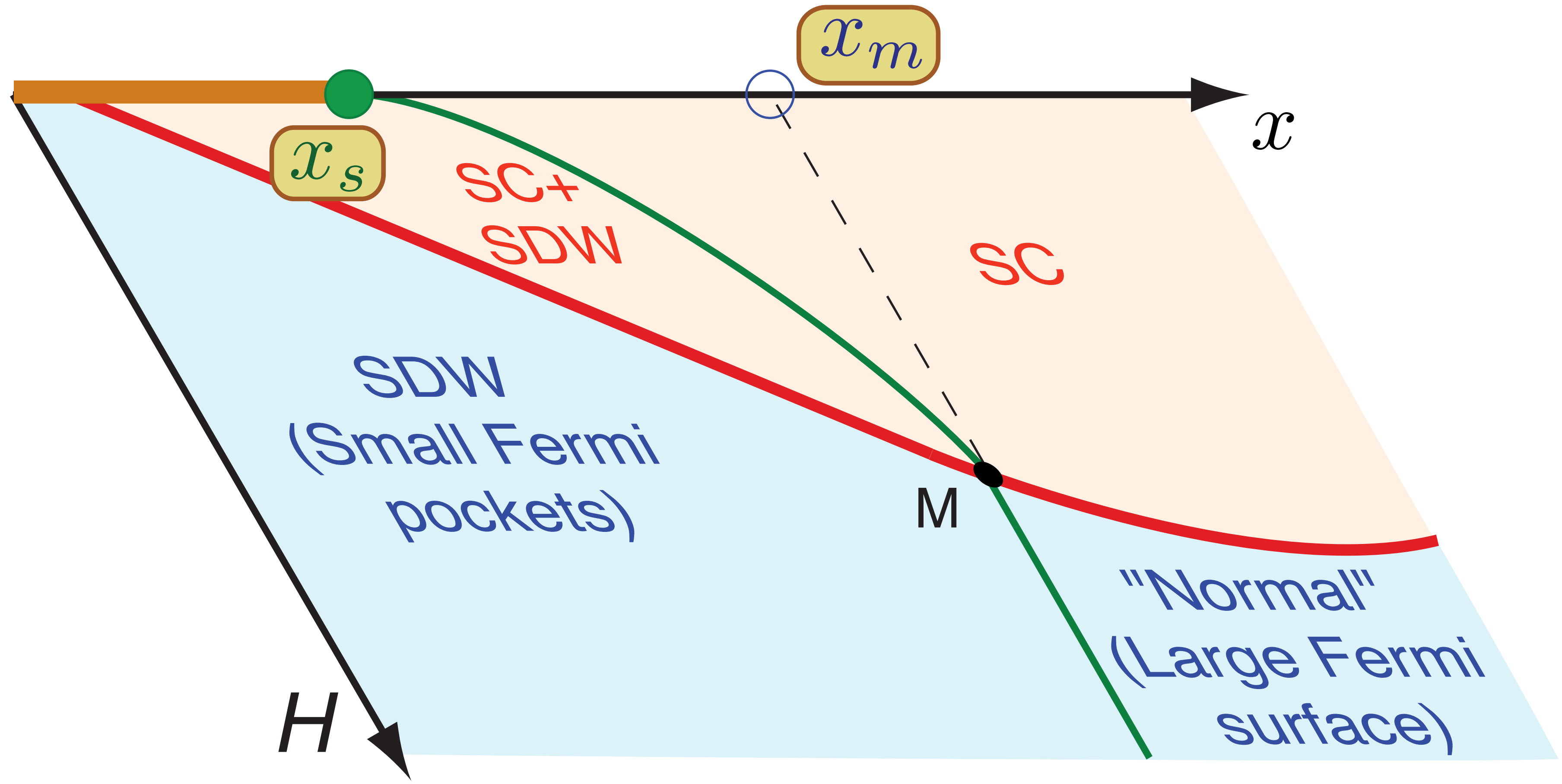
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).



H
"Normal"
(Large Fermi
surface)
SDW
(Small Fermi
pockets)
H_{c2}
M
SC+
SDW
H_{sdw}
SC
x_s
x_m
x



x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H


x_m
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SC+
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(Small Fermi
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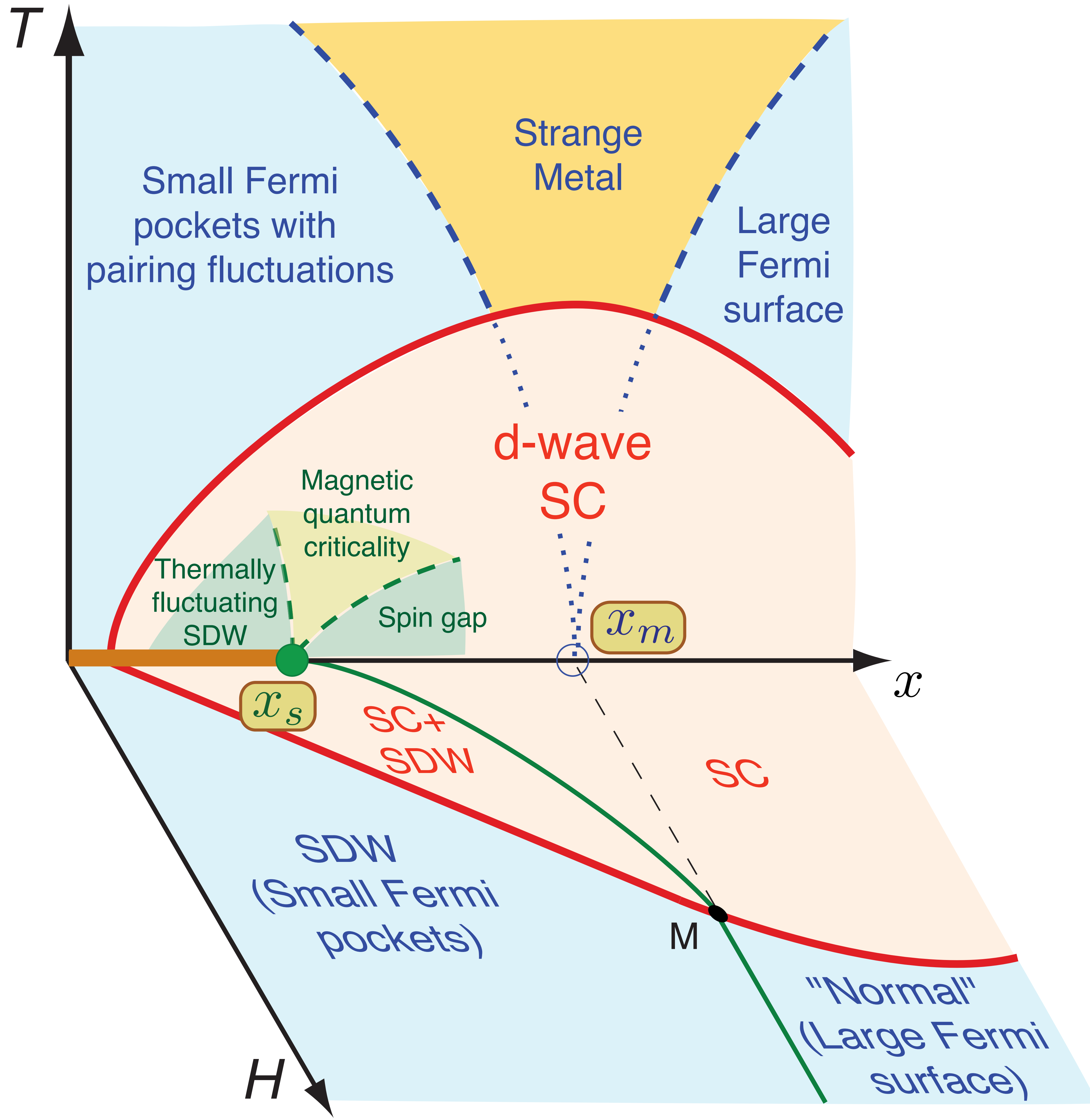
T
Small Fermi
pockets with
pairing fluctuations
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
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H

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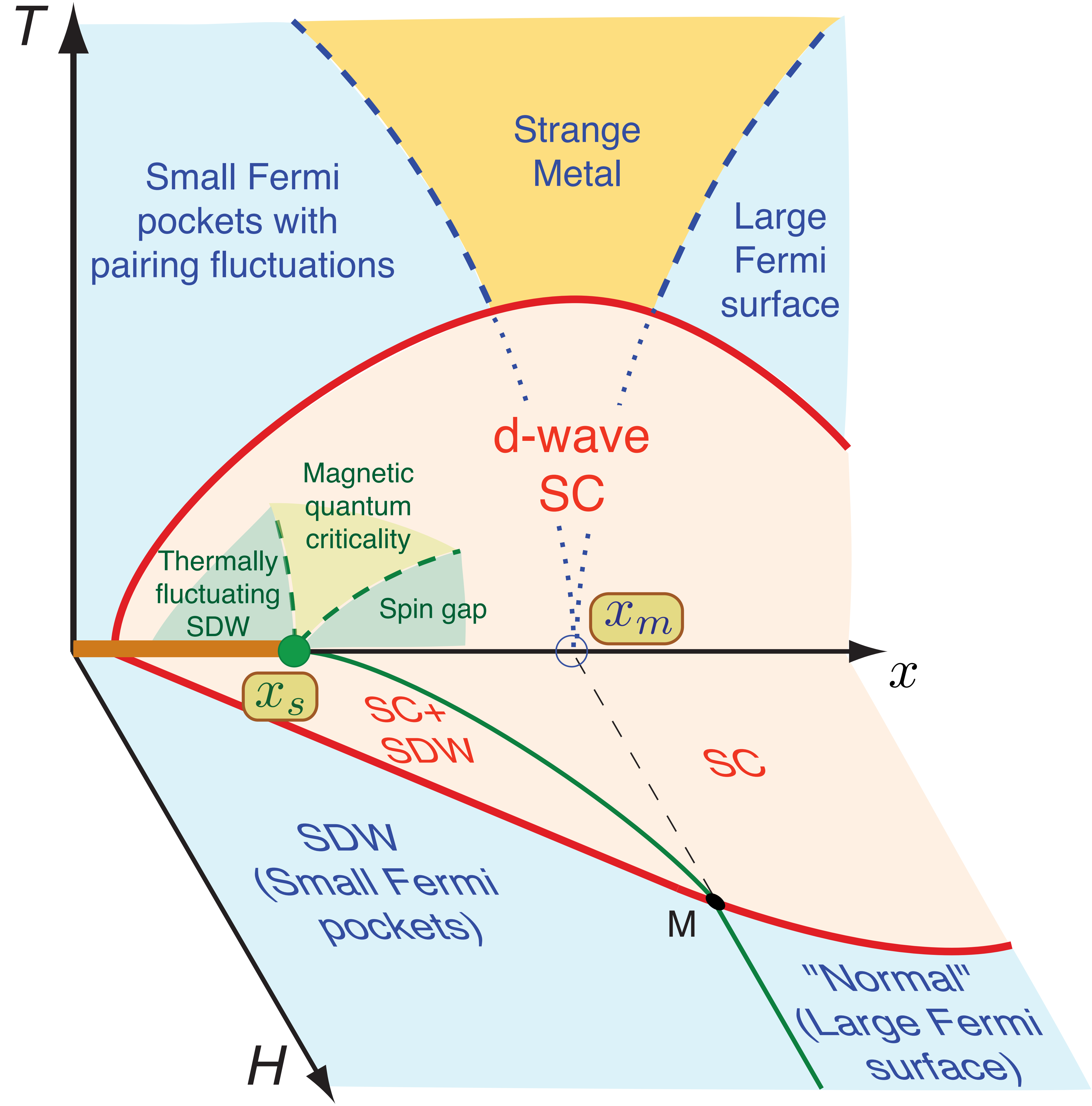
T
Small Fermi
pockets with
pairing fluctuations
Strange
Metal
Large
Fermi
surface
d-wave
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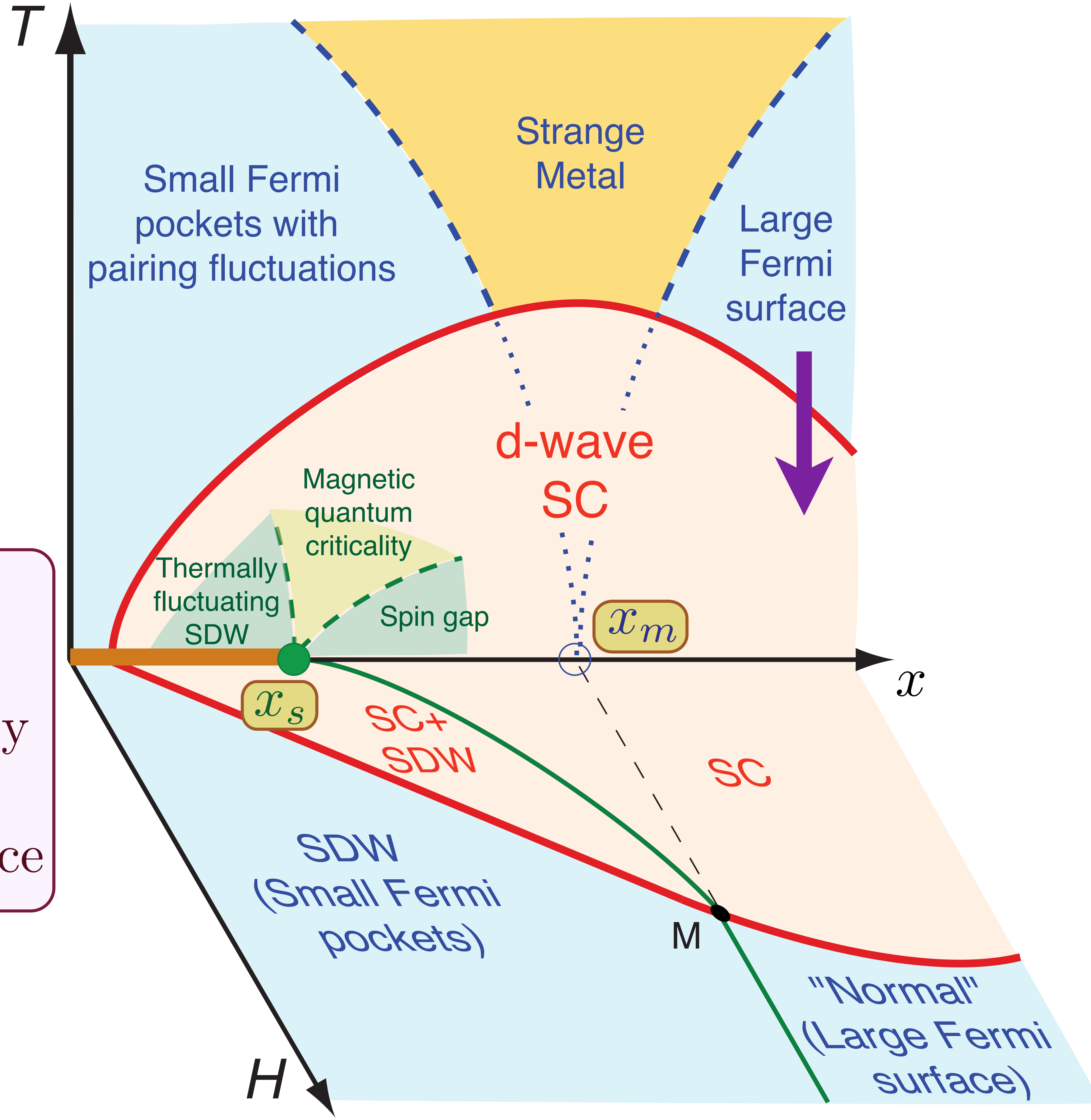
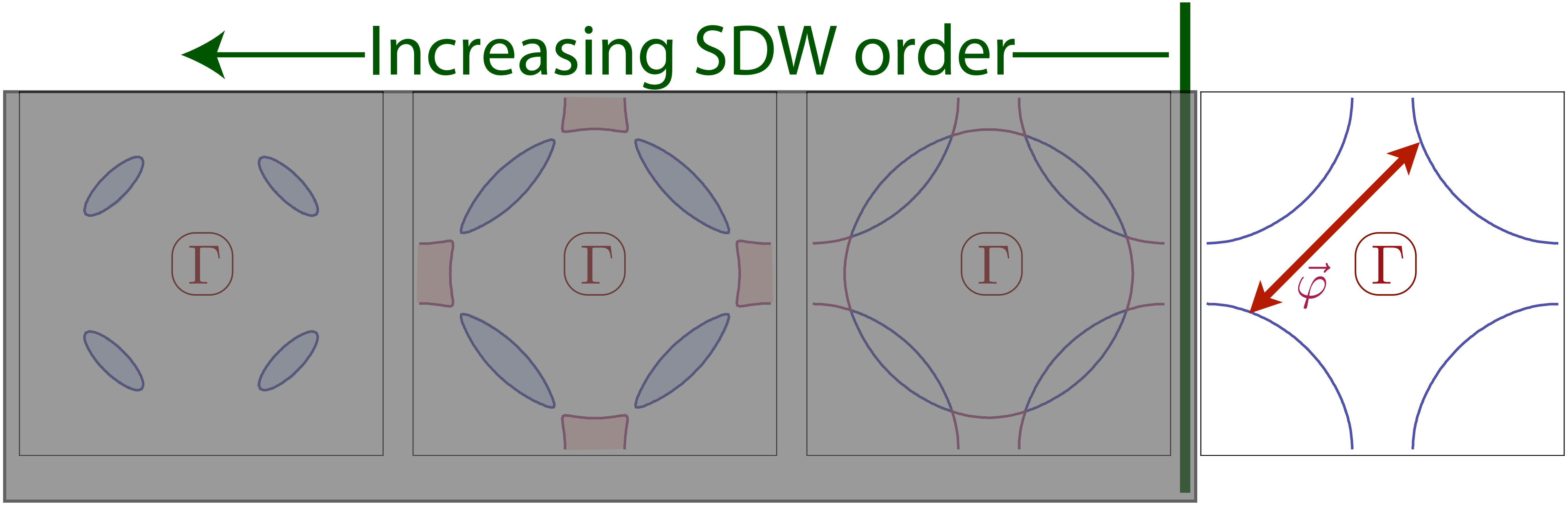
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

Theory of the onset of d-wave superconductivity from a large Fermi surface



Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = - \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta} .$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^\dagger_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

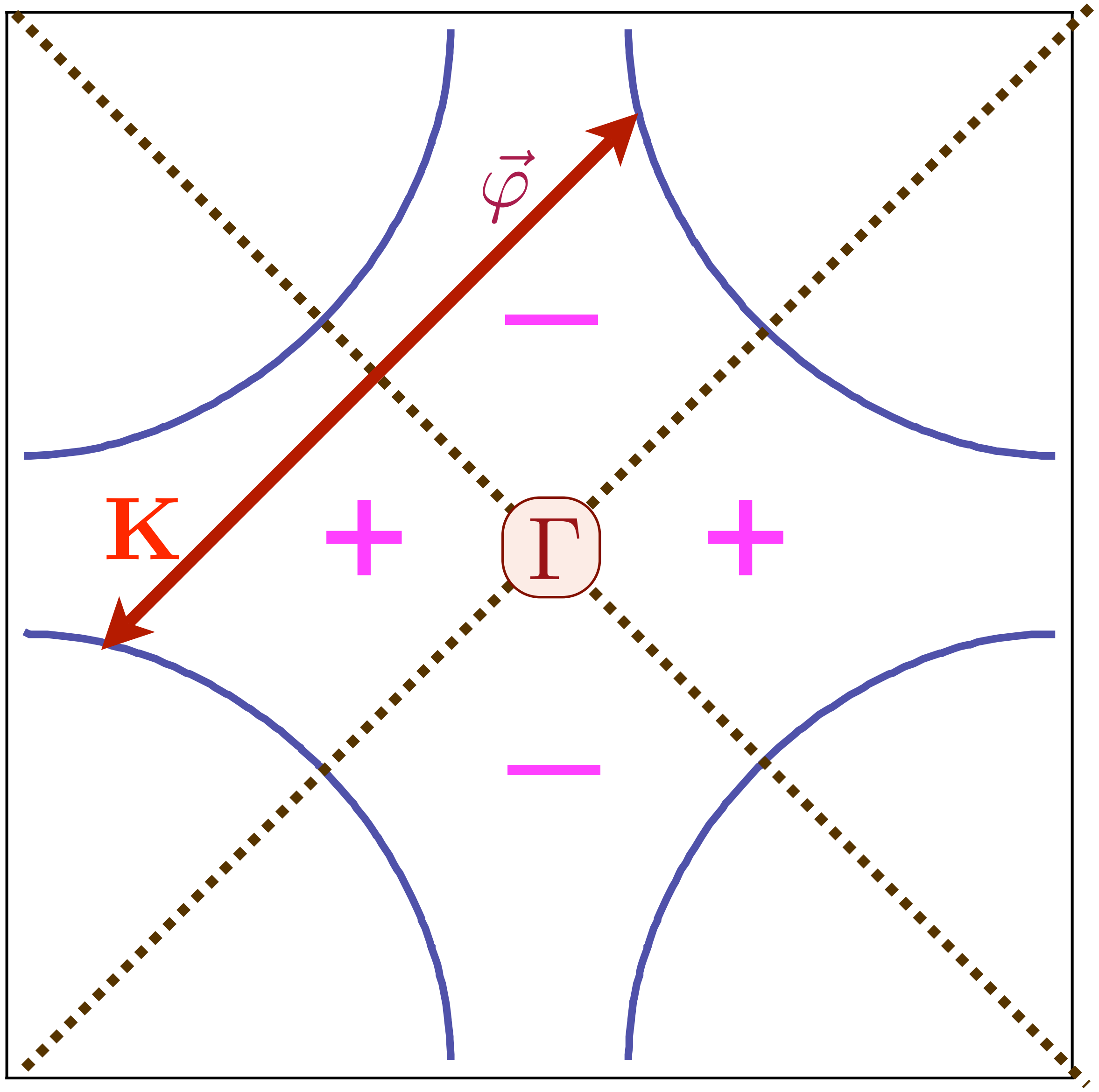
where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

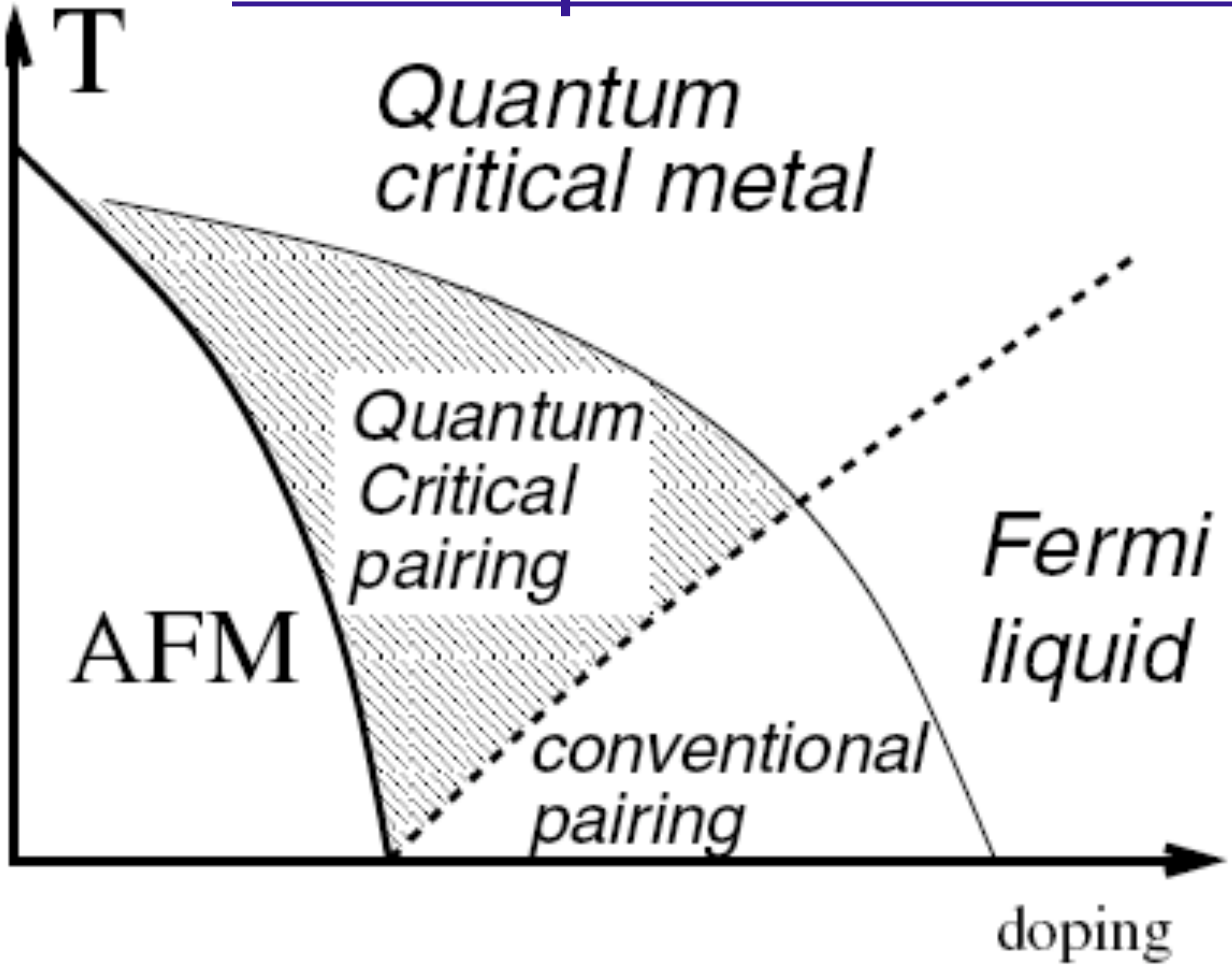
d-wave pairing of the large Fermi surface



$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

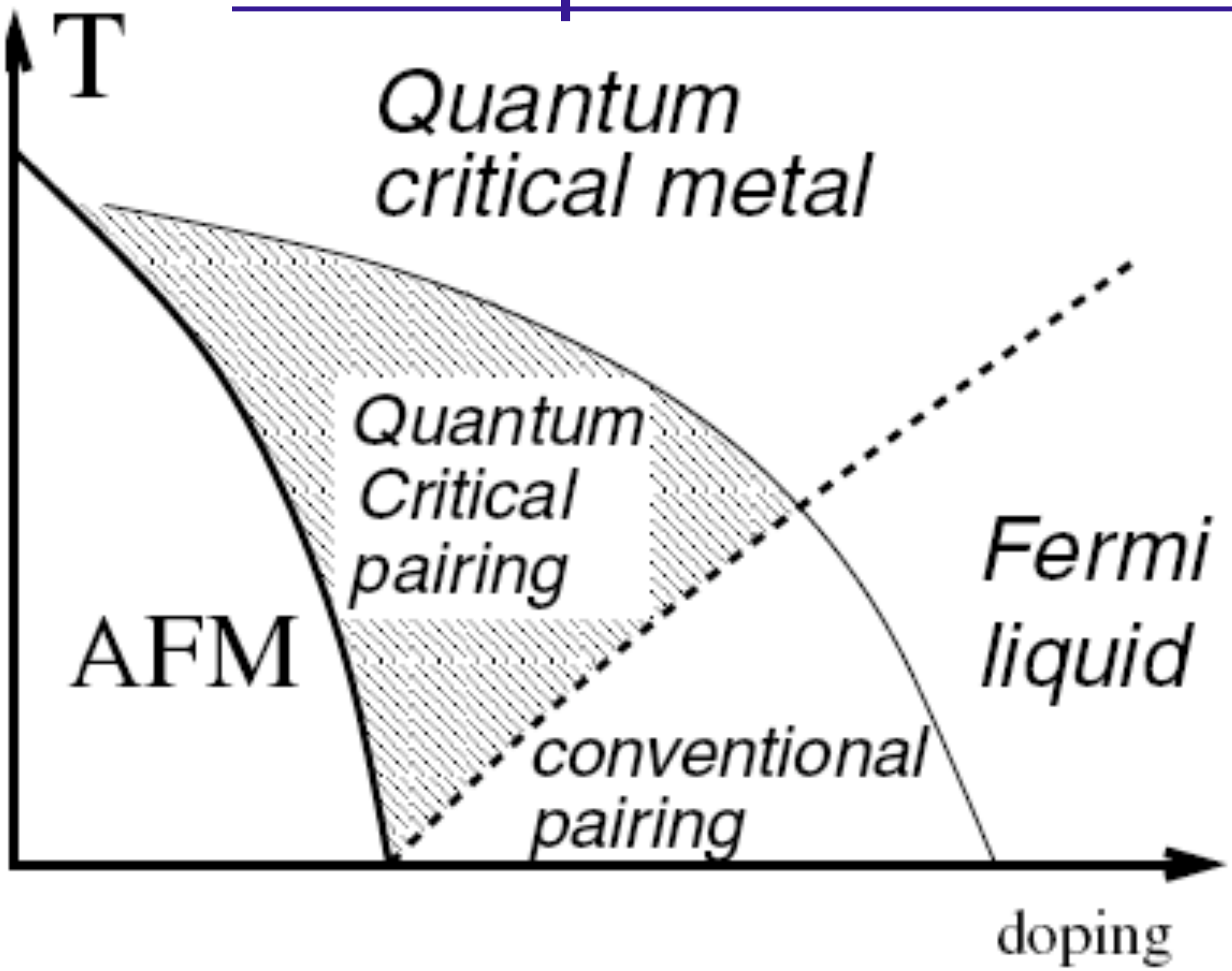
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.

- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

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   Survey of recent experiments

2. Superconductivity in the overdoped regime

   BCS pairing by spin fluctuation exchange

3. Superconductivity in the underdoped regime

   U(1) gauge theory of fluctuating SDW order

4. A unified theory

   SU(2) gauge theory of transition from Fermi pockets to a large Fermi surface

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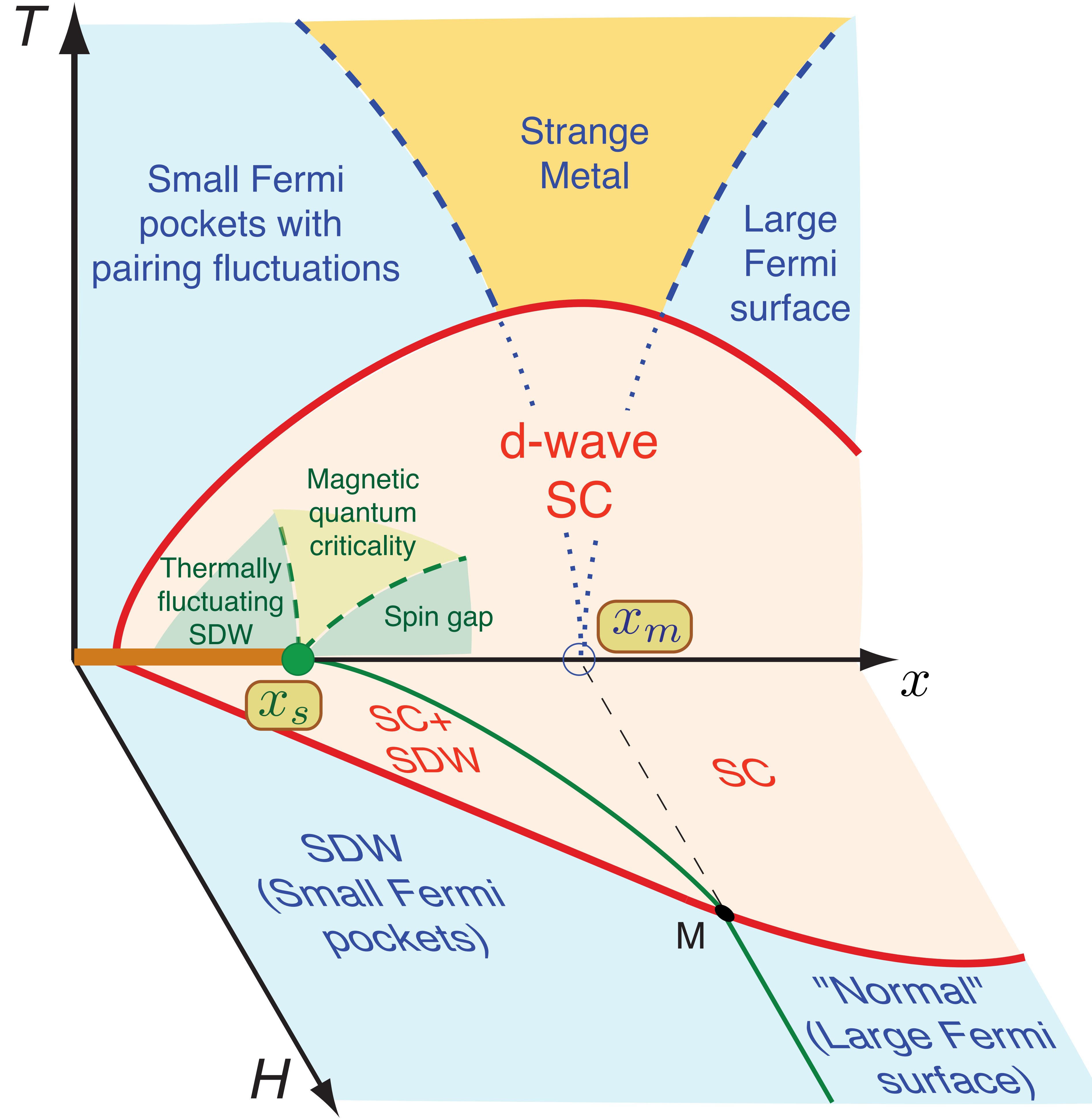
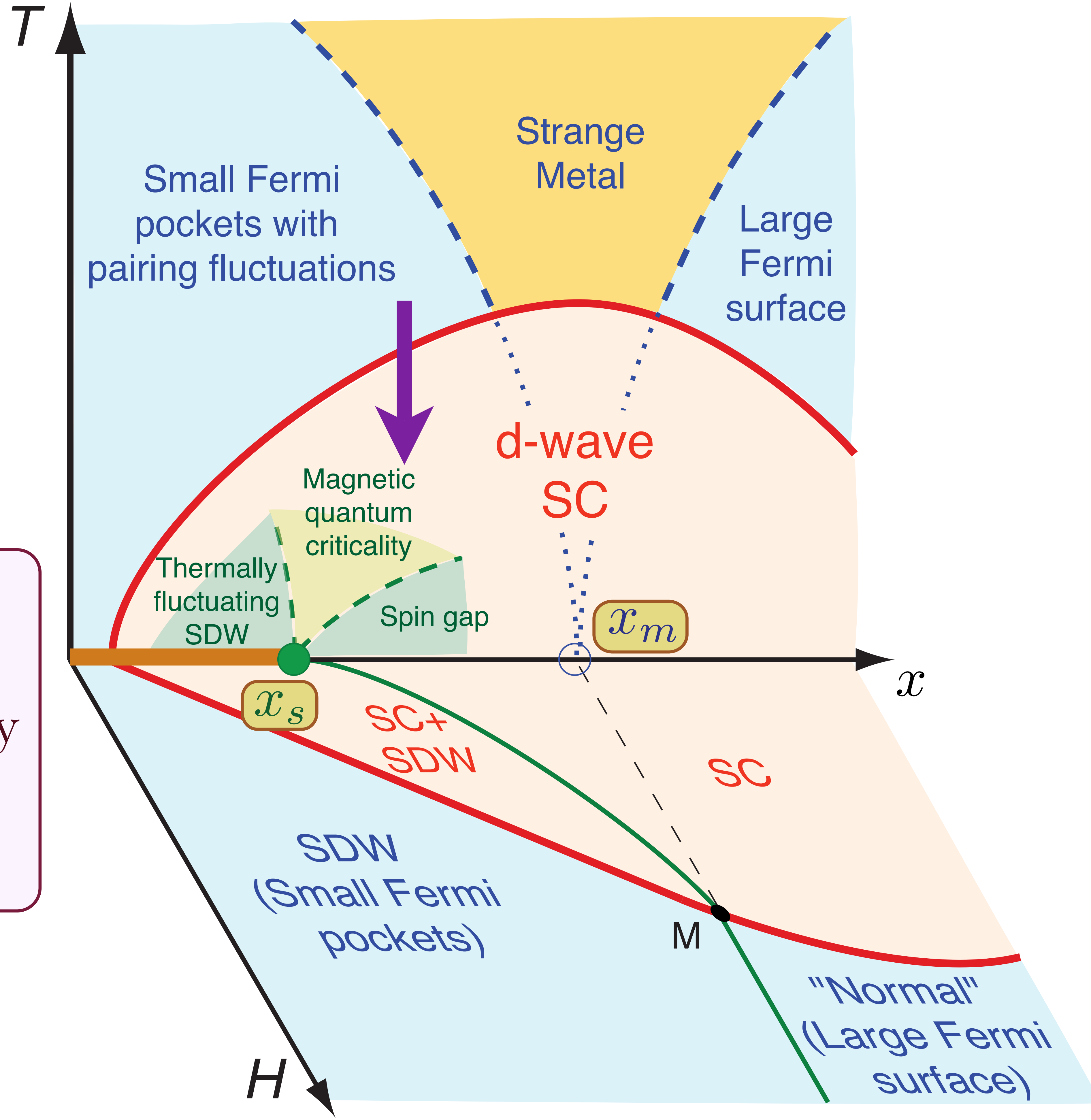
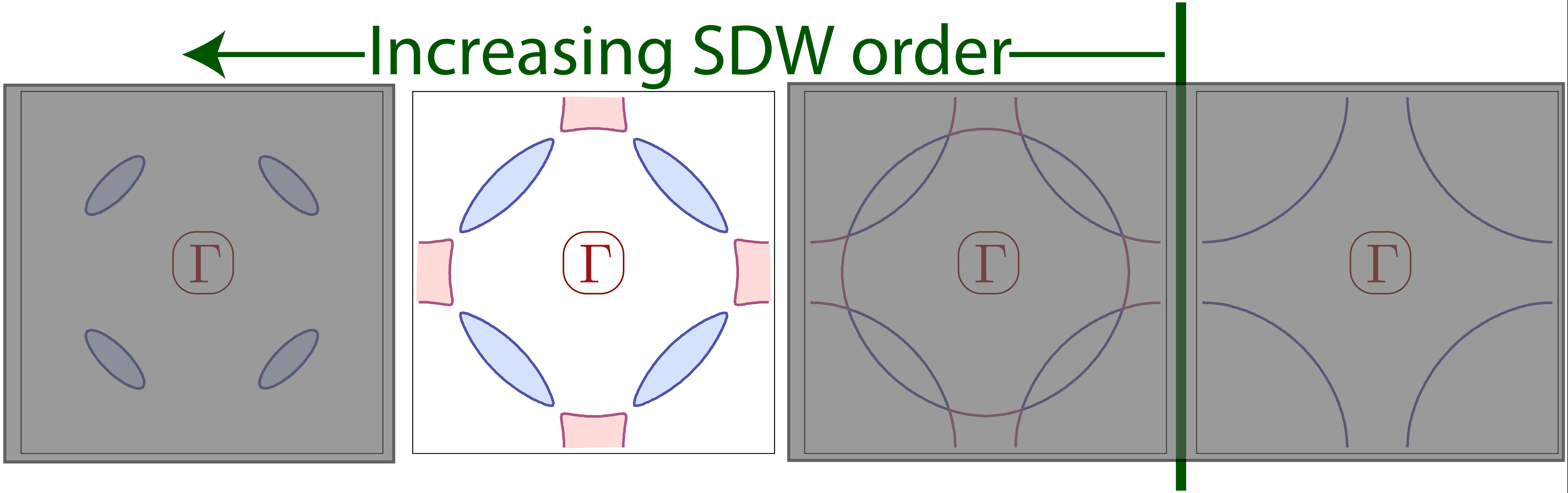


T
Small Fermi
pockets with
pairing fluctuations
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

Theory of the onset of d-wave superconductivity from small Fermi pockets



Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\left(\begin{array}{c} c_\uparrow \\ c_\downarrow \end{array} \right) = R \left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \quad ; \quad \psi_+ \rightarrow e^{-i\theta} \psi_+ \quad ; \quad \psi_- \rightarrow e^{i\theta} \psi_-$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

Theory of underdoped cuprates

Starting from the "SDW-fermion" model with Lagrangian

$$\mathcal{L} = \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} - E_{sdw} \sum_{i} c^{\dagger}_{i\alpha} \hat{\vec{\varphi}}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} + \frac{1}{2t} \left(\partial_\mu \hat{\vec{\varphi}} \right)^2$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

$$\mathcal{L} = \sum_{\mathbf{k},p=\pm} \Bigg[\psi^\dagger_{\mathbf{k}p} \left(\frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \Bigg]$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order

$$\mathcal{L} = \sum_{\mathbf{k}, p=\pm} \left[\psi^\dagger_{\mathbf{k}p} \left(\frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} \right.$$

$$\left. - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \right]$$

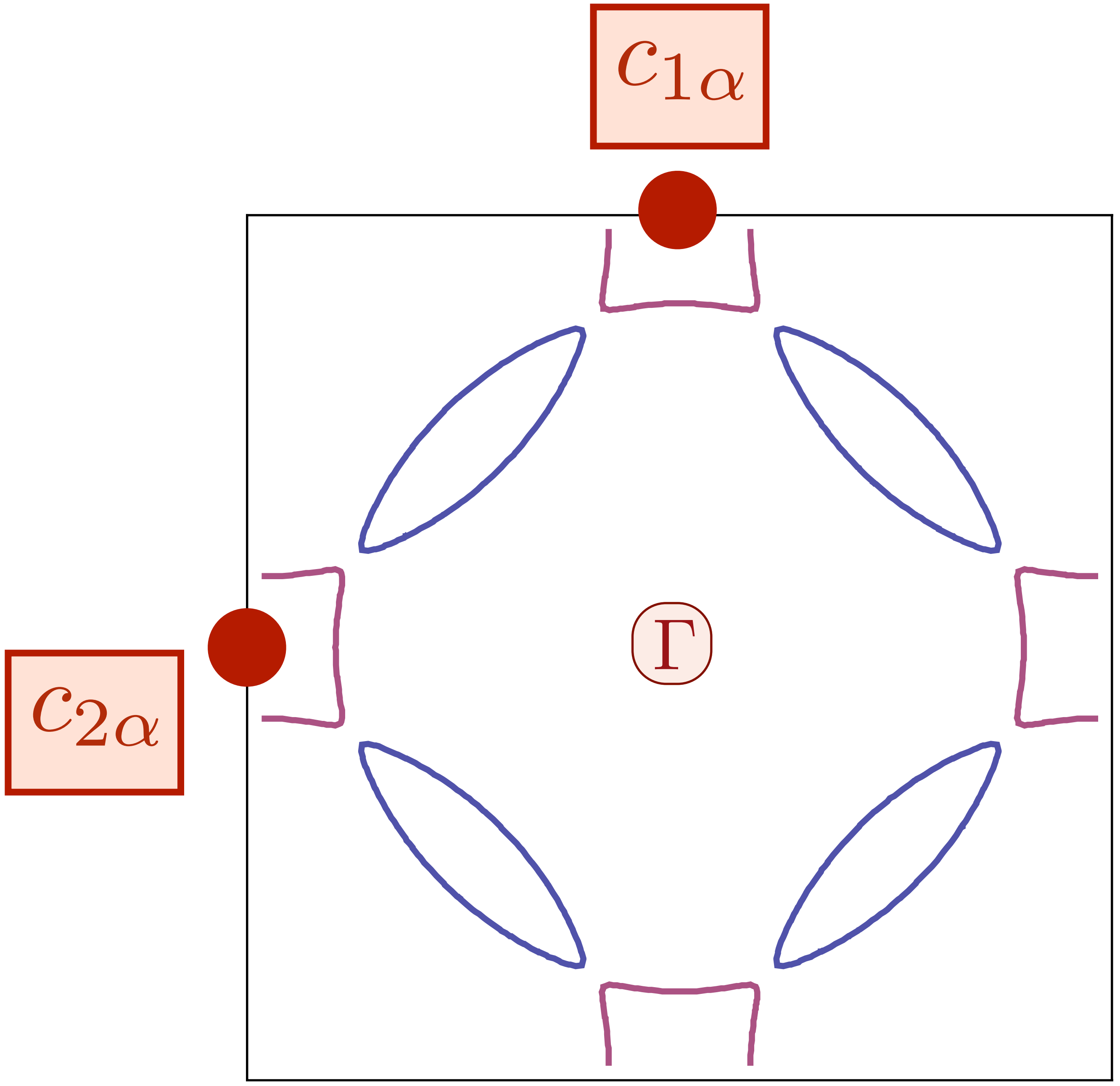
$$+ \frac{1}{t} \left[|(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - i\mathbf{A}) z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1) \right]$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order
- Monopoles carrying Berry phases; onset of superconductivity leads to confinement via condensation of monopoles, which induces charge order.

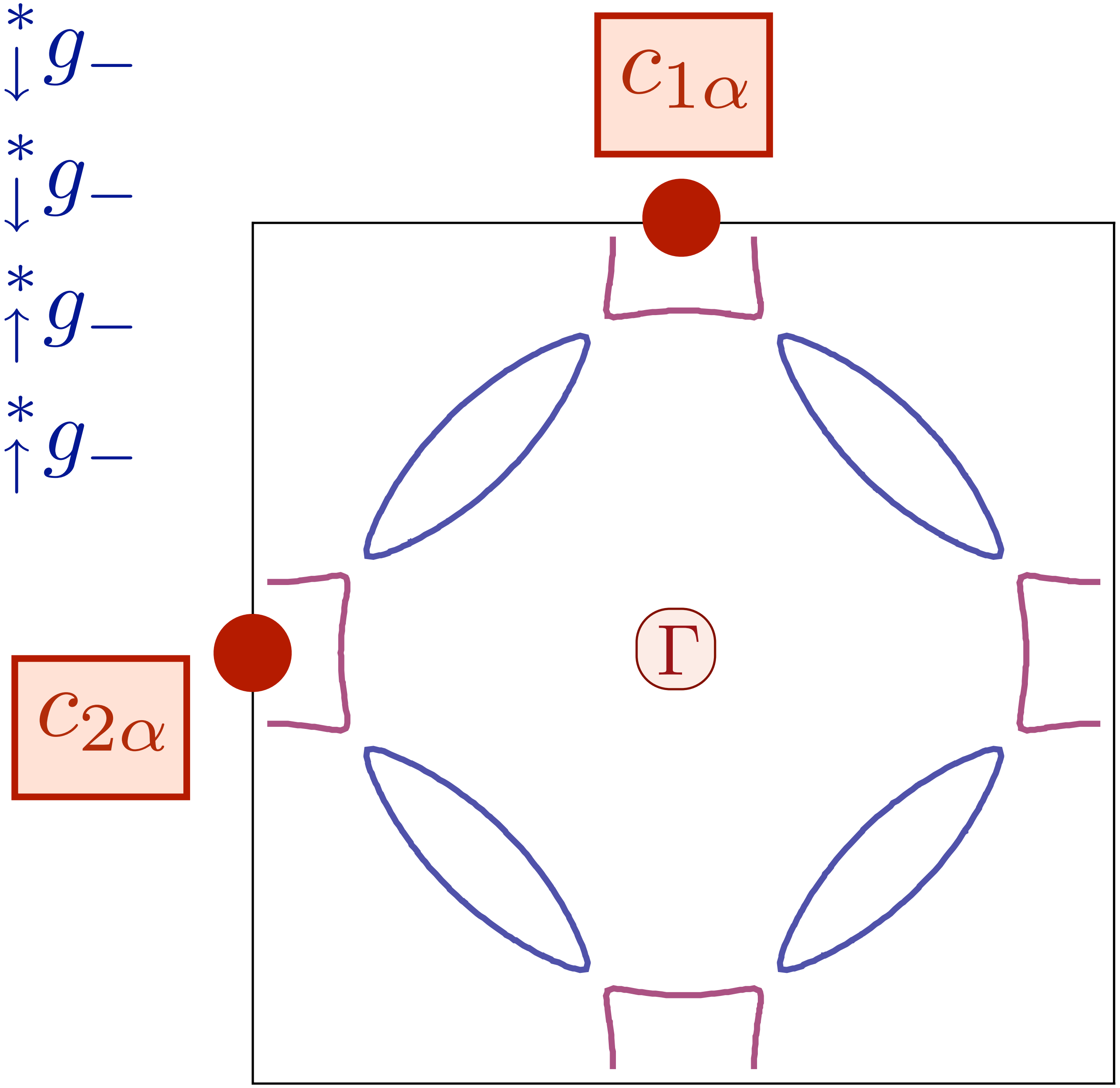
Electrons near $(0,\pi)$ and $(\pi,0)$



Electrons near $(0, \pi)$ and $(\pi, 0)$

These electrons are represented by the low energy limit of a single $\psi_\pm$ fermion $= g_\pm$.

$$
\begin{aligned}
c_{1\uparrow} &= z_\uparrow g_+ - z_\downarrow^* g_- \\
c_{2\uparrow} &= z_\uparrow g_+ + z_\downarrow^* g_- \\
c_{1\downarrow} &= z_\downarrow g_+ + z_\uparrow^* g_- \\
c_{2\downarrow} &= z_\downarrow g_+ - z_\uparrow^* g_-
\end{aligned}
$$



Electrons near $(0,\pi)$ and $(\pi,0)$

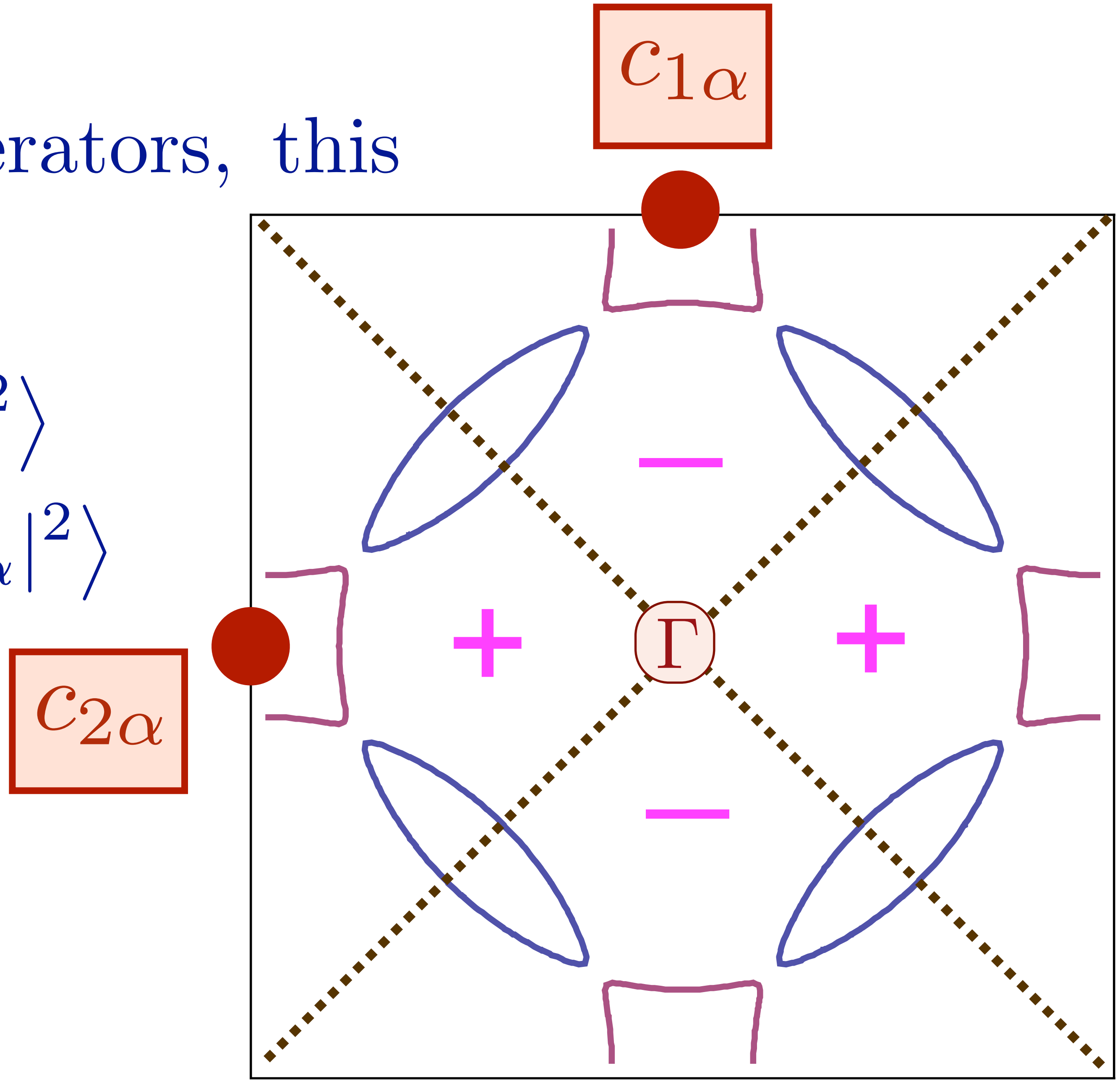
In the theory, the $g_\pm$ are unstable to a simple s-wave pairing with

$$\langle g_+ g_- \rangle = \Delta$$

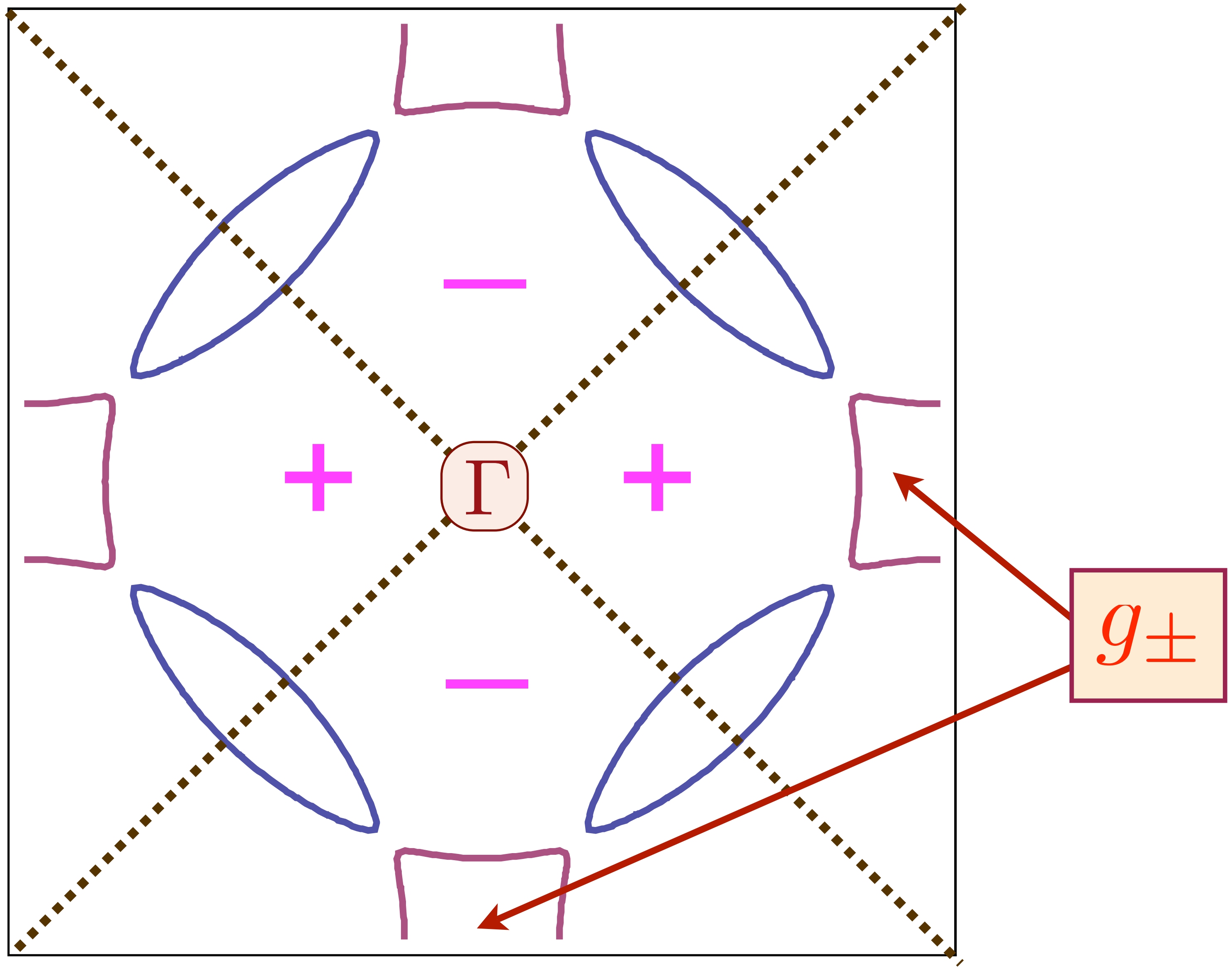
For the physical electron operators, this pairing implies

$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

i.e. d-wave pairing !

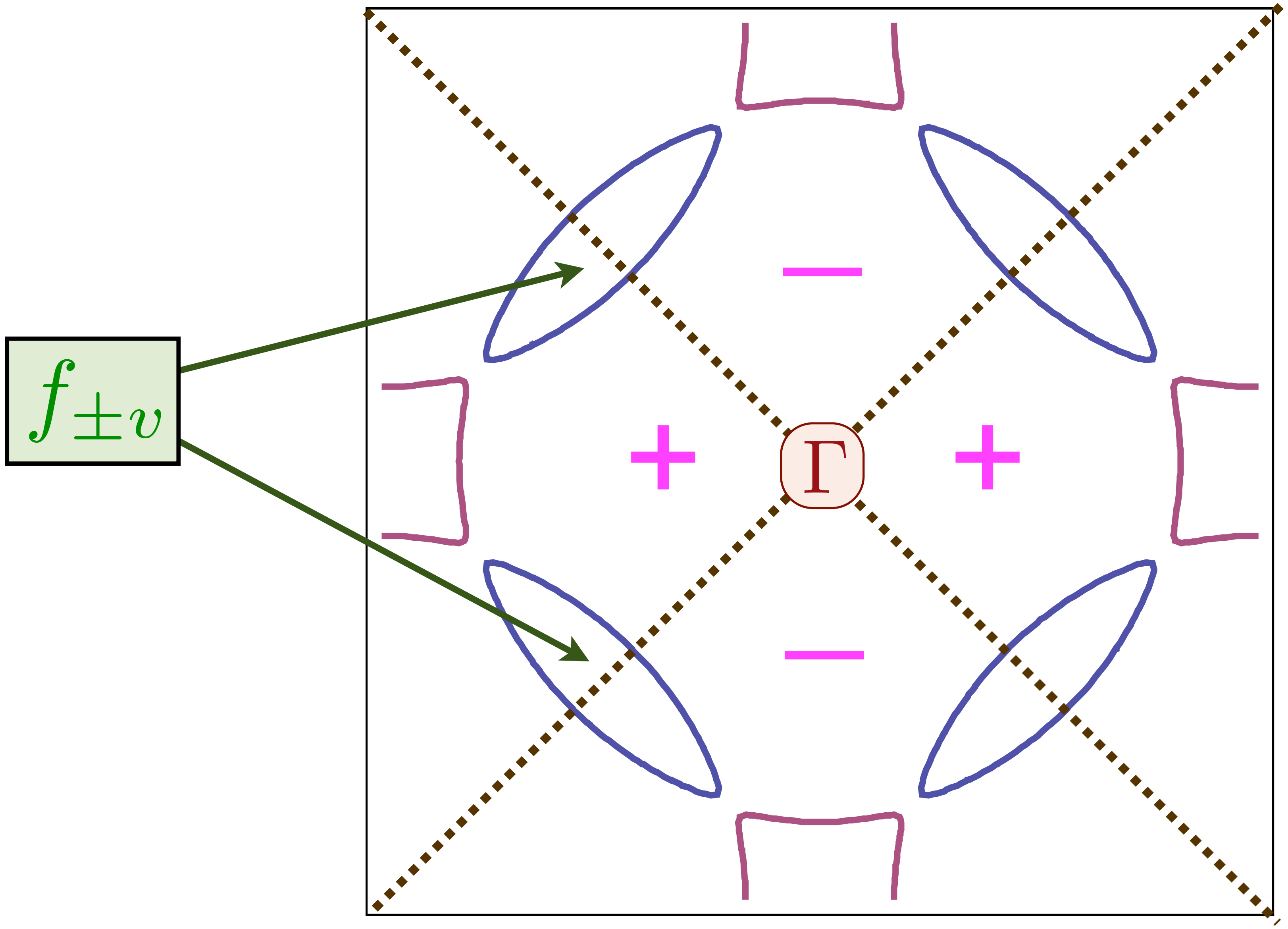


Strong s-wave pairing of the $g_\pm$ electron pockets



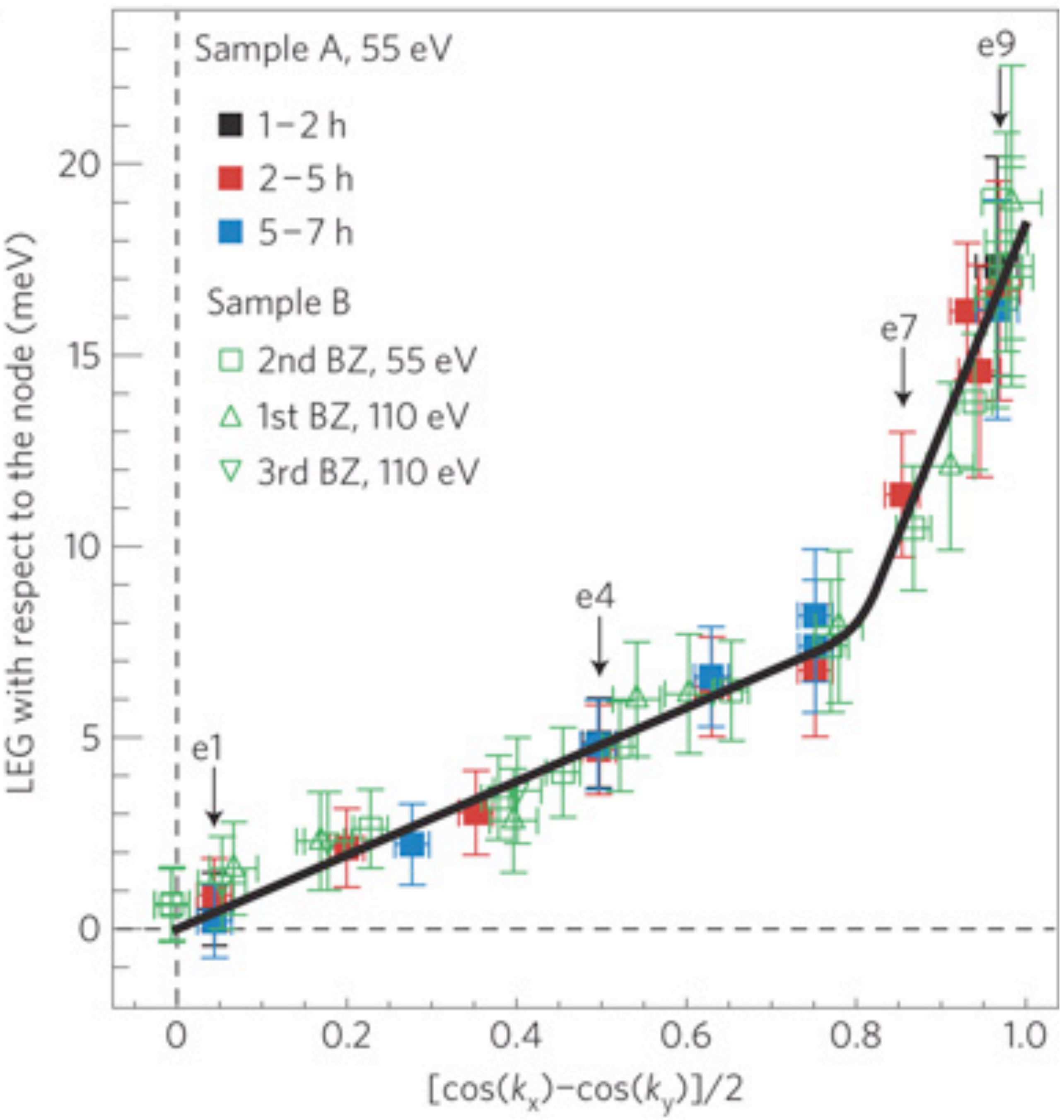
$$\langle g_+ g_- \rangle = \Delta$$

Weak p-wave pairing of the hole pockets



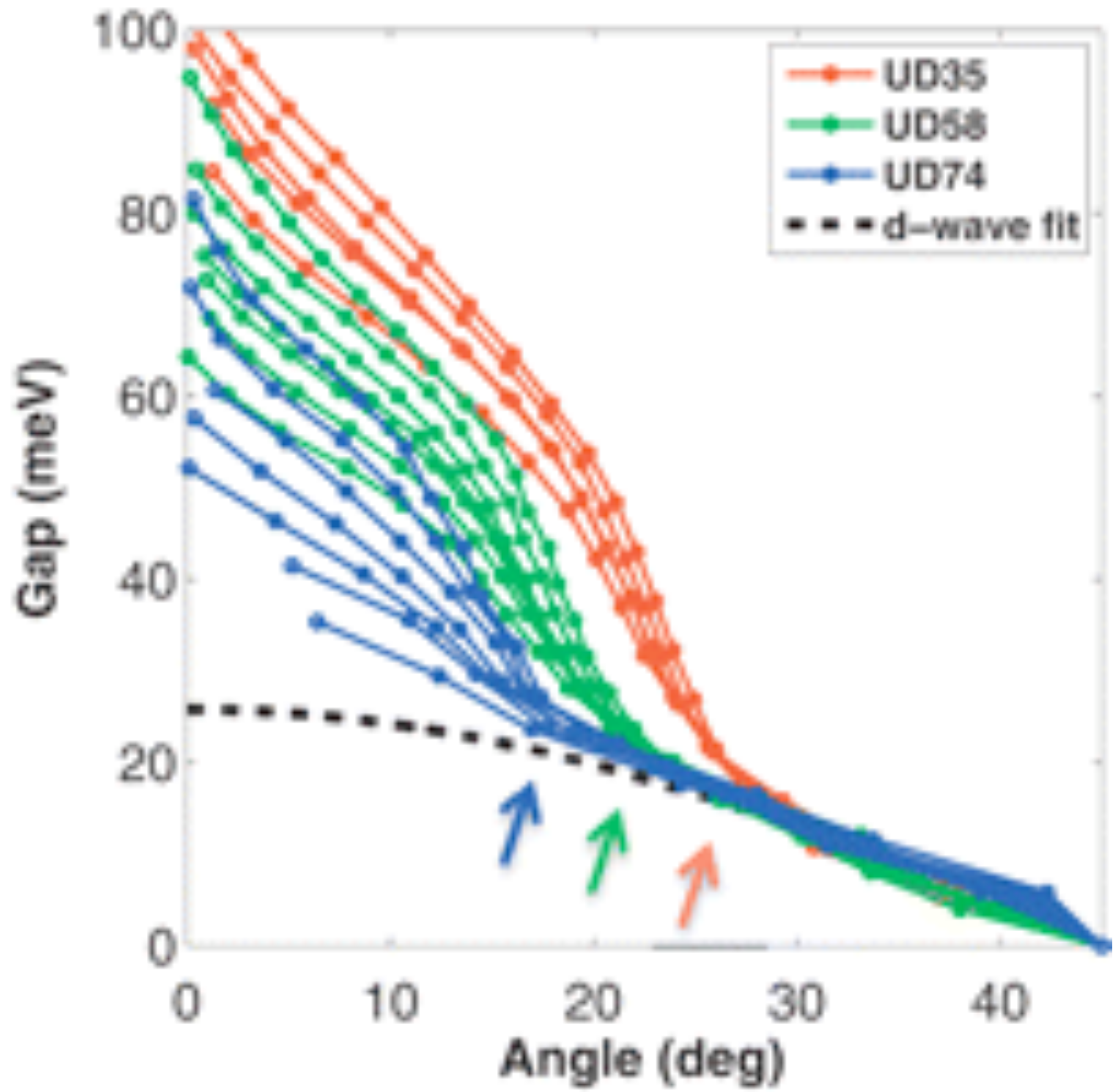
$$
\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,
\end{aligned}
$$

Photoemission in LBCO

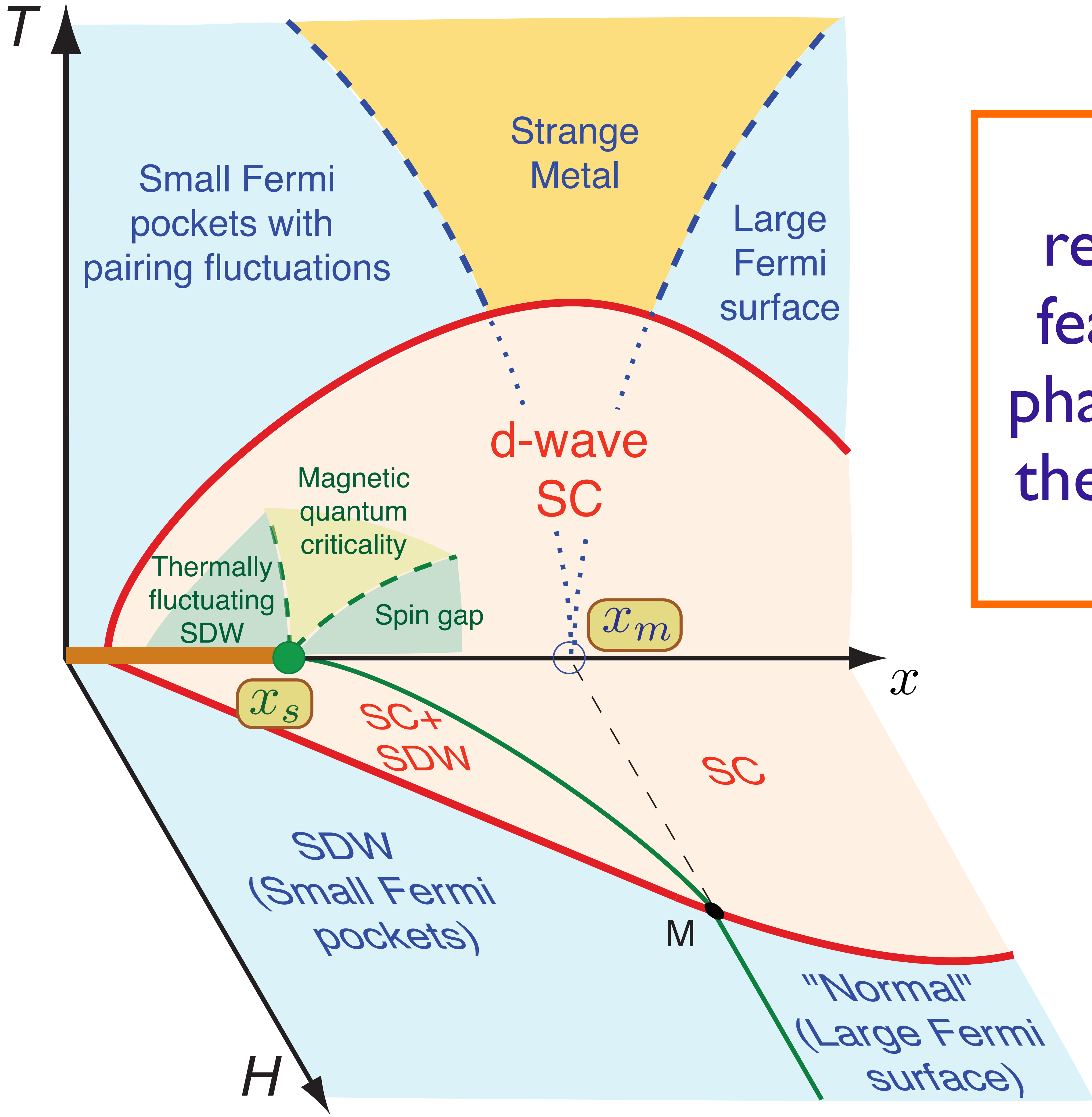


R.-H. He, K. Tanaka, S.-K. Mo, T. Sasagawa, M. Fujita, T. Adachi, N. Mannella, K. Yamada, Y. Koike, Z. Hussain and Z.-X. Shen, *Nature Physics* **5**, 119 (2008)

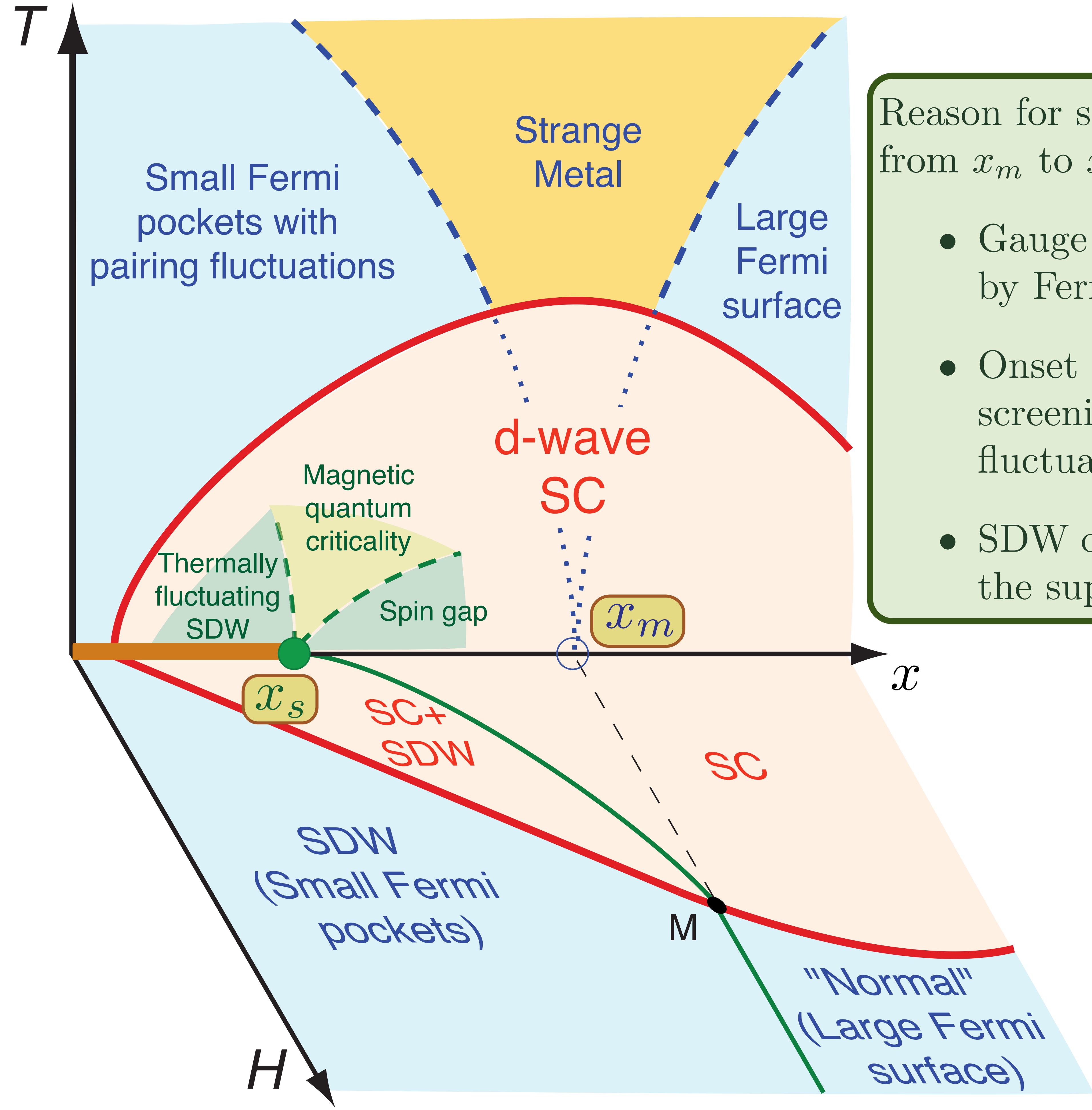
STM in BSCCO



A. Pushp, C.V. Parker, A. N. Pasupathy, K. K. Gomes, S. Ono, J. Wen, Z. Xu, G. Gu, and A. Yazdani, *Science* **324**, 1689 (2009)



Theory reproduces all features of the phase diagram in the underdoped regime



Reason for shift in onset of SDW from x_m to x_s:

- Gauge fluctuations are screened by Fermi surface in metal
- Onset of pairing suppresses screening, and enhances gauge fluctuations
- SDW order is suppressed in the superconductor

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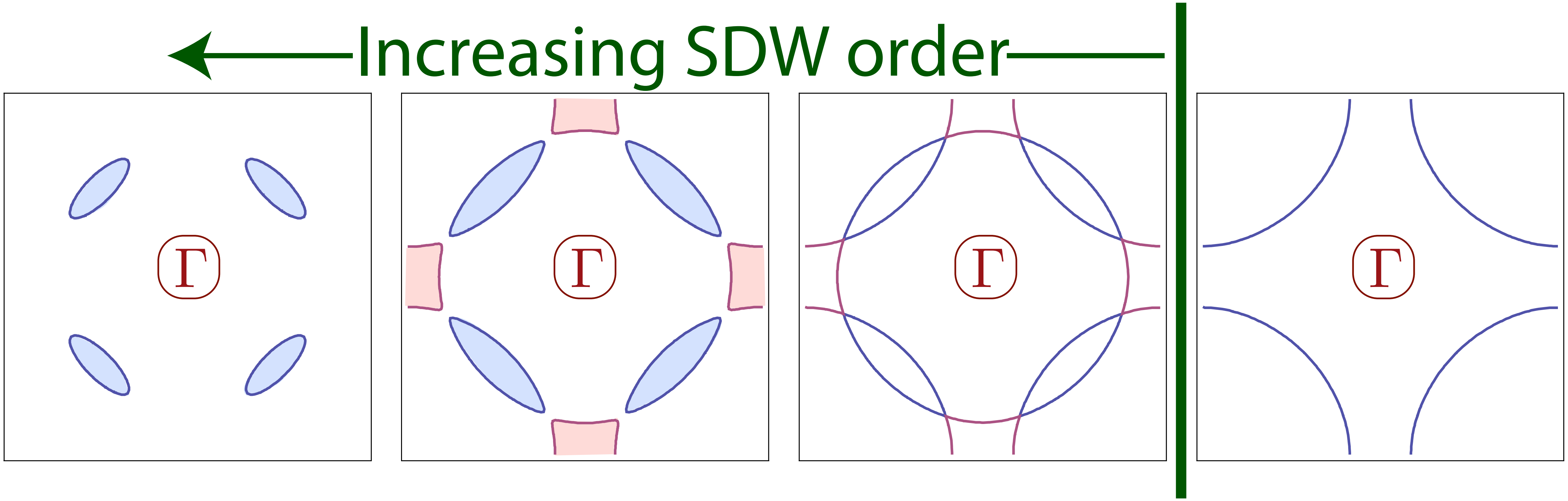
Unified theory



The parameterization $\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$

is actually invariant under a SU(2) gauge transformation

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow U \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R \rightarrow R U^\dagger$$

Unified theory

The theory has $\mathrm{SU(2)_{gauge}} \otimes \mathrm{SU(2)_{spin}} \otimes \mathrm{U(1)_{em\ charge}} \otimes \mathrm{(lattice\ space\ group)}$ invariance,

Unified theory

The theory has $SU(2)_{\rm gauge} \otimes SU(2)_{\rm spin} \otimes U(1)_{\rm em\ charge}$ $\otimes$(lattice space group) invariance,
and matter content

- fermion ψ transforming as $(\mathbf{2}, \mathbf{1}, 1)$, and with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure,

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- relativistic real scalar N transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring the local SDW amplitude,

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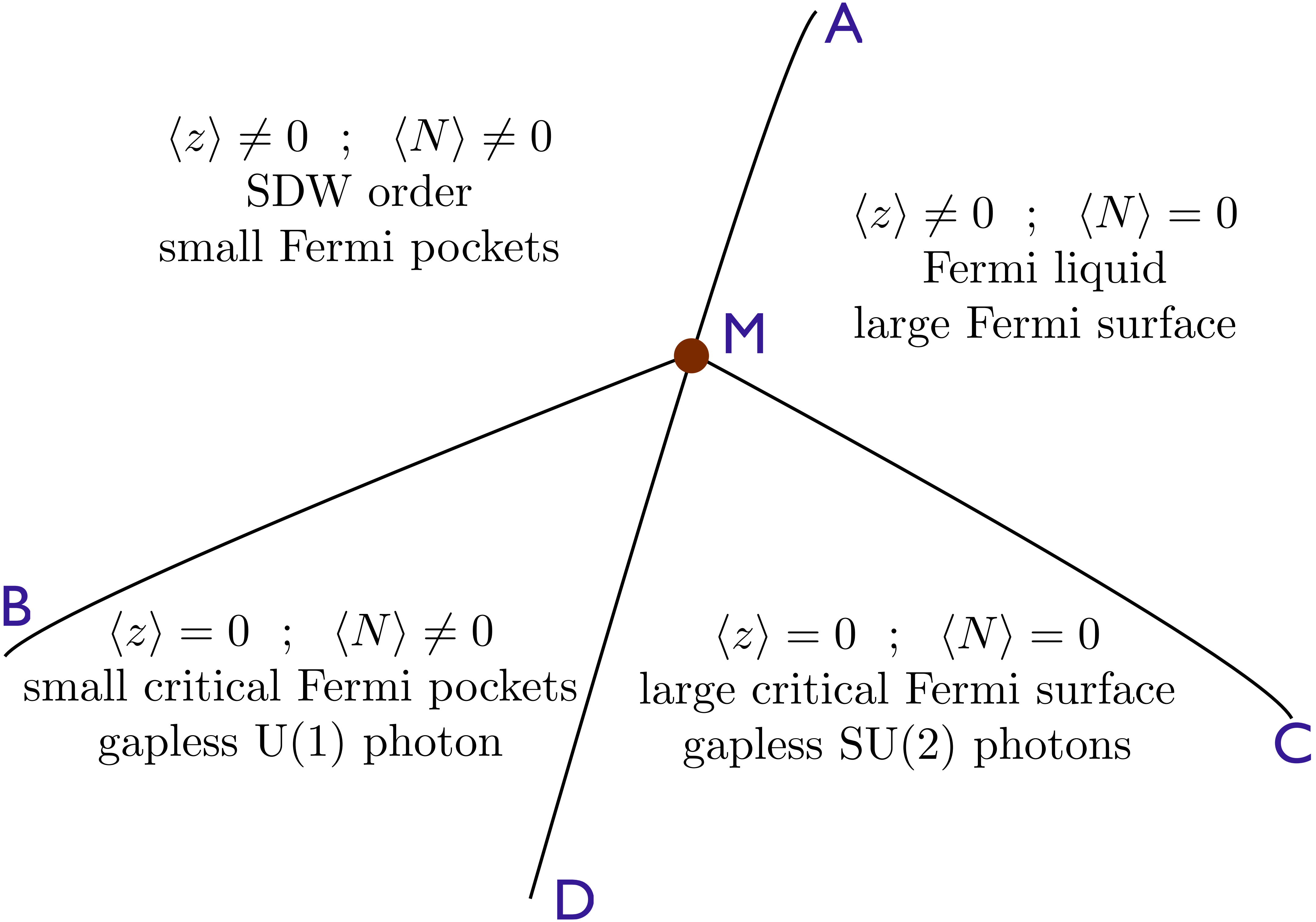
- relativistic real scalar N transforming as $(\mathbf{3}, \mathbf{1}, 0)$, measuring the local SDW amplitude,

- a Yukawa coupling between N and ψ, which $\sim e^{i\mathbf{K}\cdot\mathbf{r}}$ because of space group transformations.

Unified theory

Conjectured phase diagram (assuming a phase with gapless SU(2) photons is possible)



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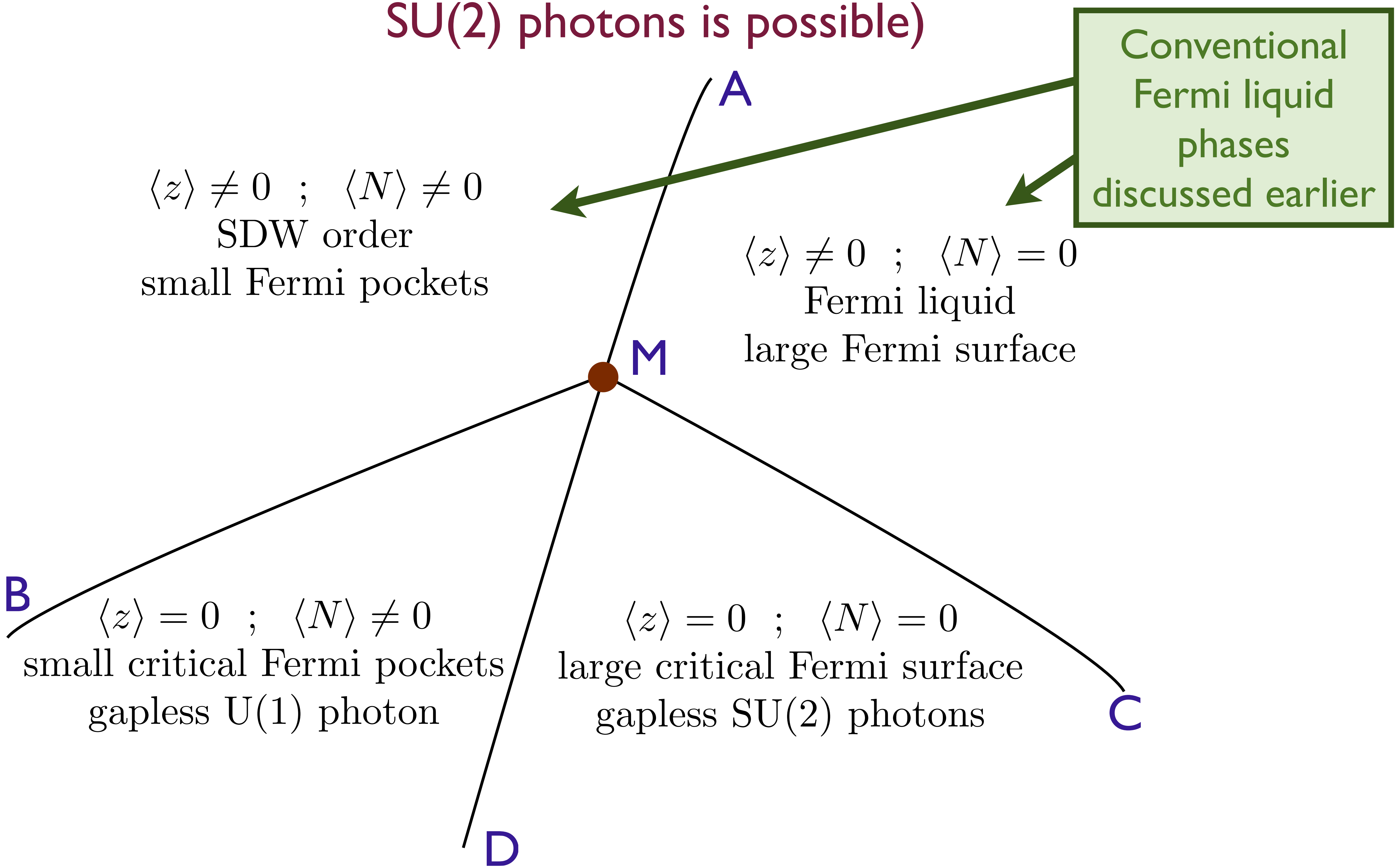

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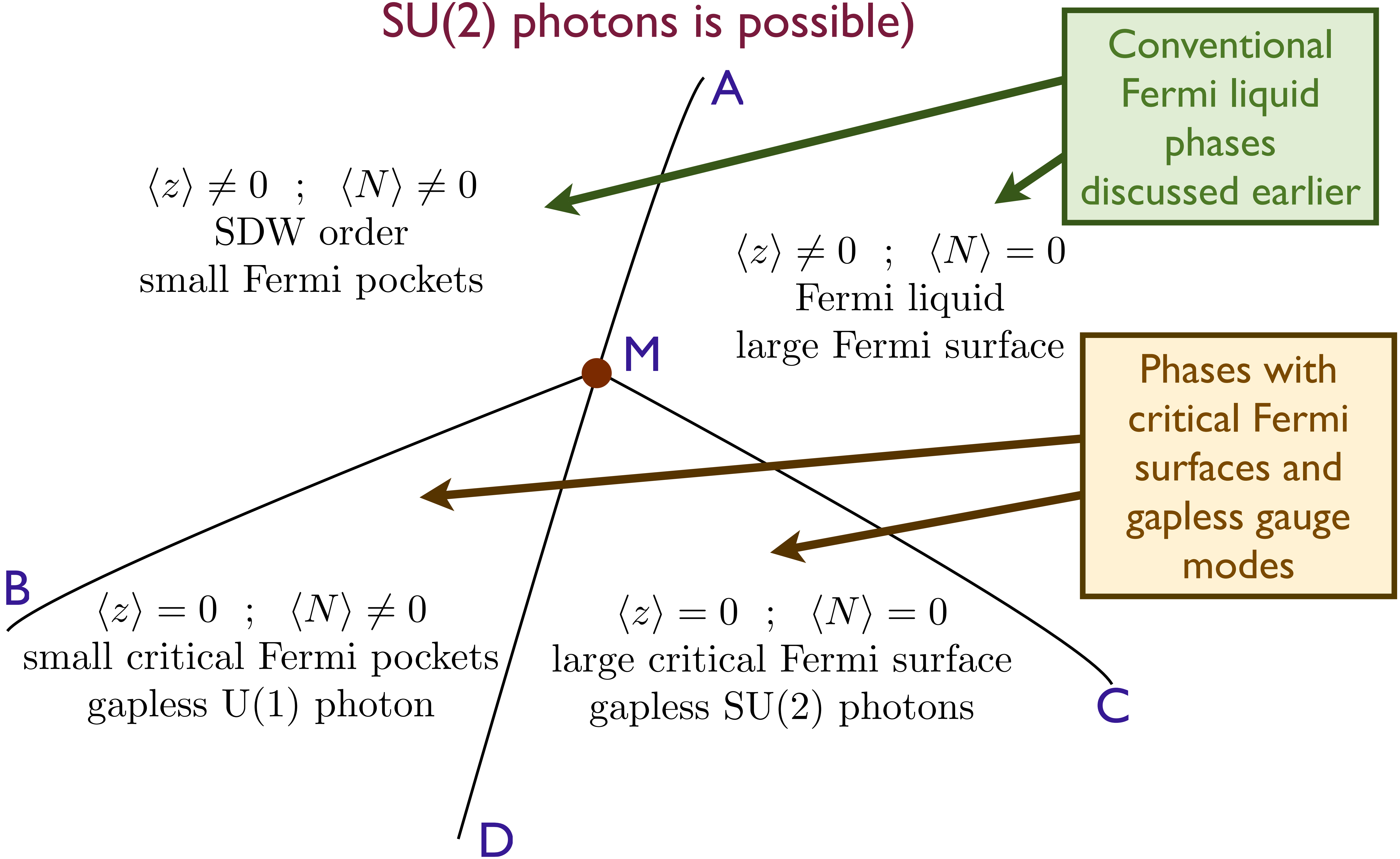

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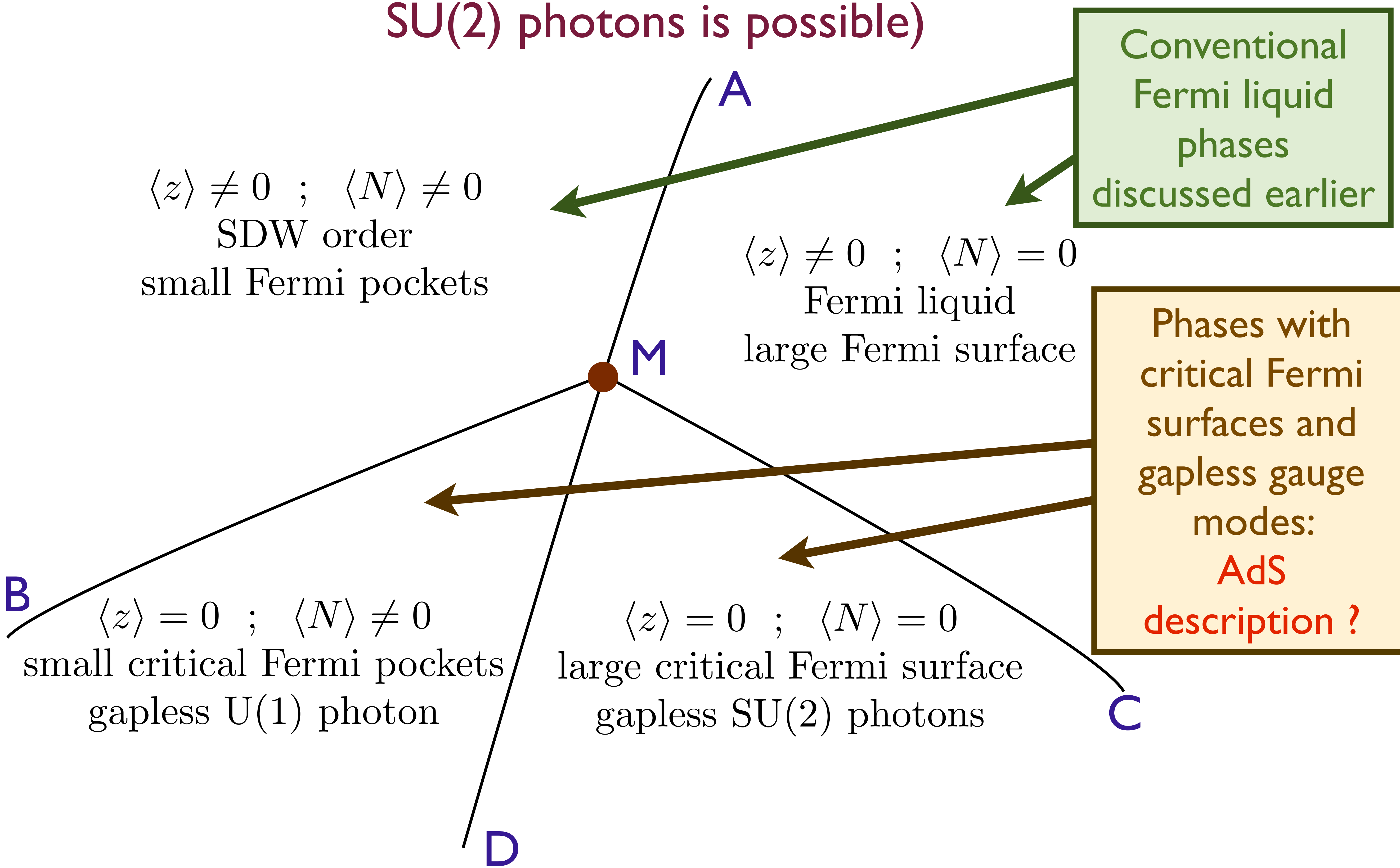
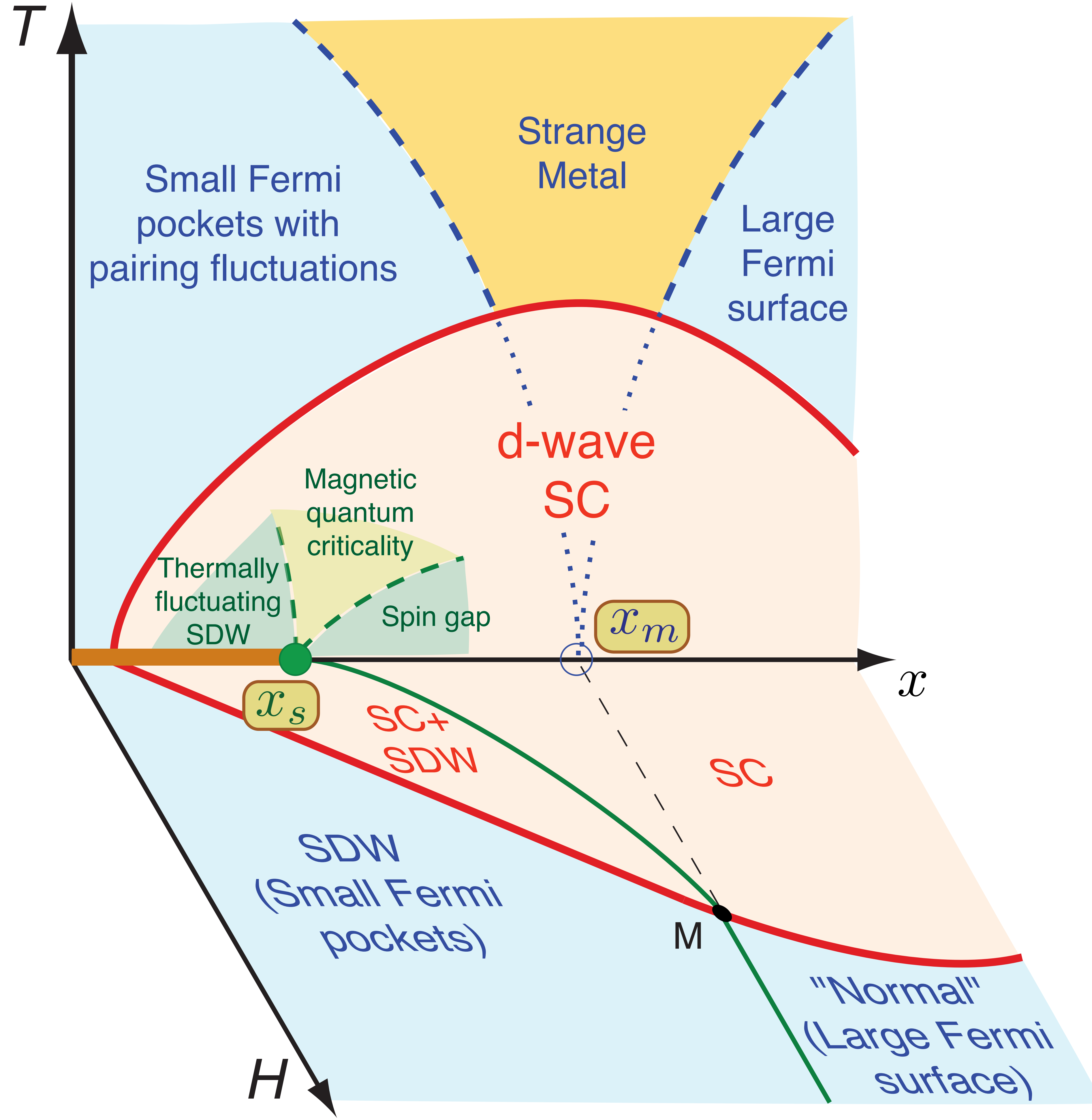
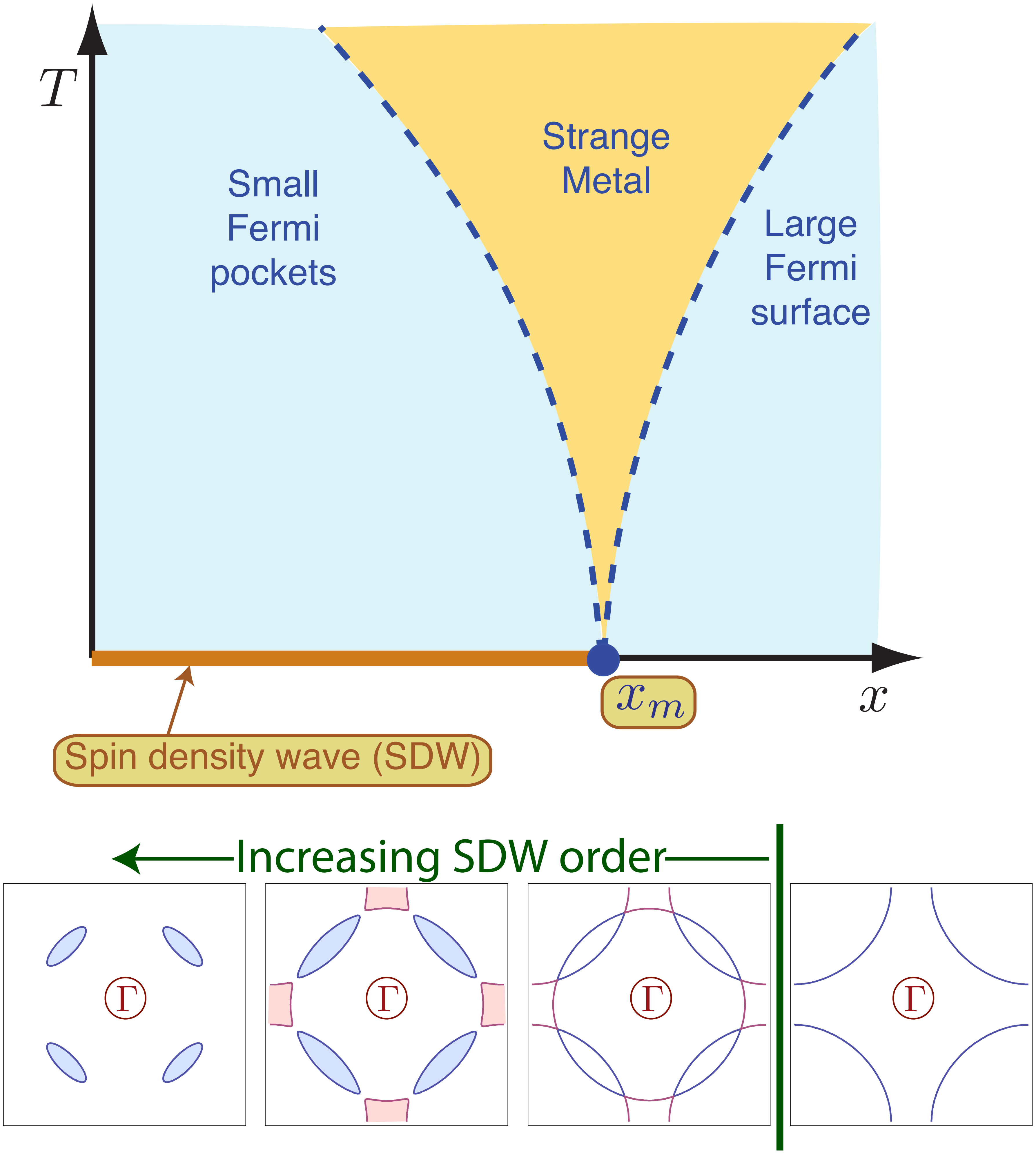


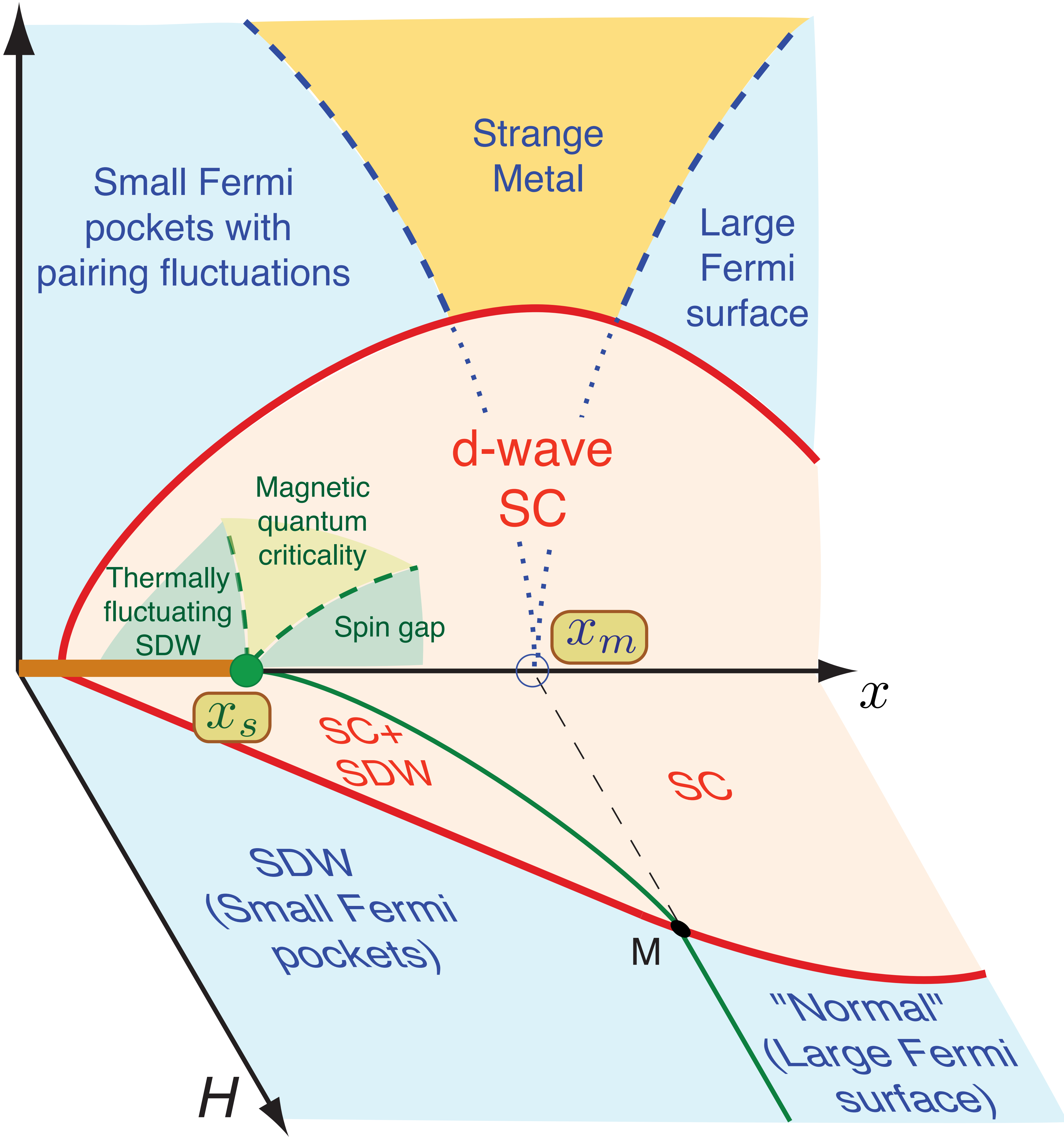


T
Small Fermi pockets with pairing fluctuations
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x
x_s
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H



T
Small Fermi pockets
Strange Metal
Large Fermi surface
x_m
x
Spin density wave (SDW)
Increasing SDW order
Γ
Γ
Γ
Γ

U(1) theory reproduces all features of the phase diagram in the underdoped regime



Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity