Emergent gauge fields and the high temperature superconductors

Unifying physics and technology in light of Maxwell's equations
The Royal Society, London
November 16, 2015

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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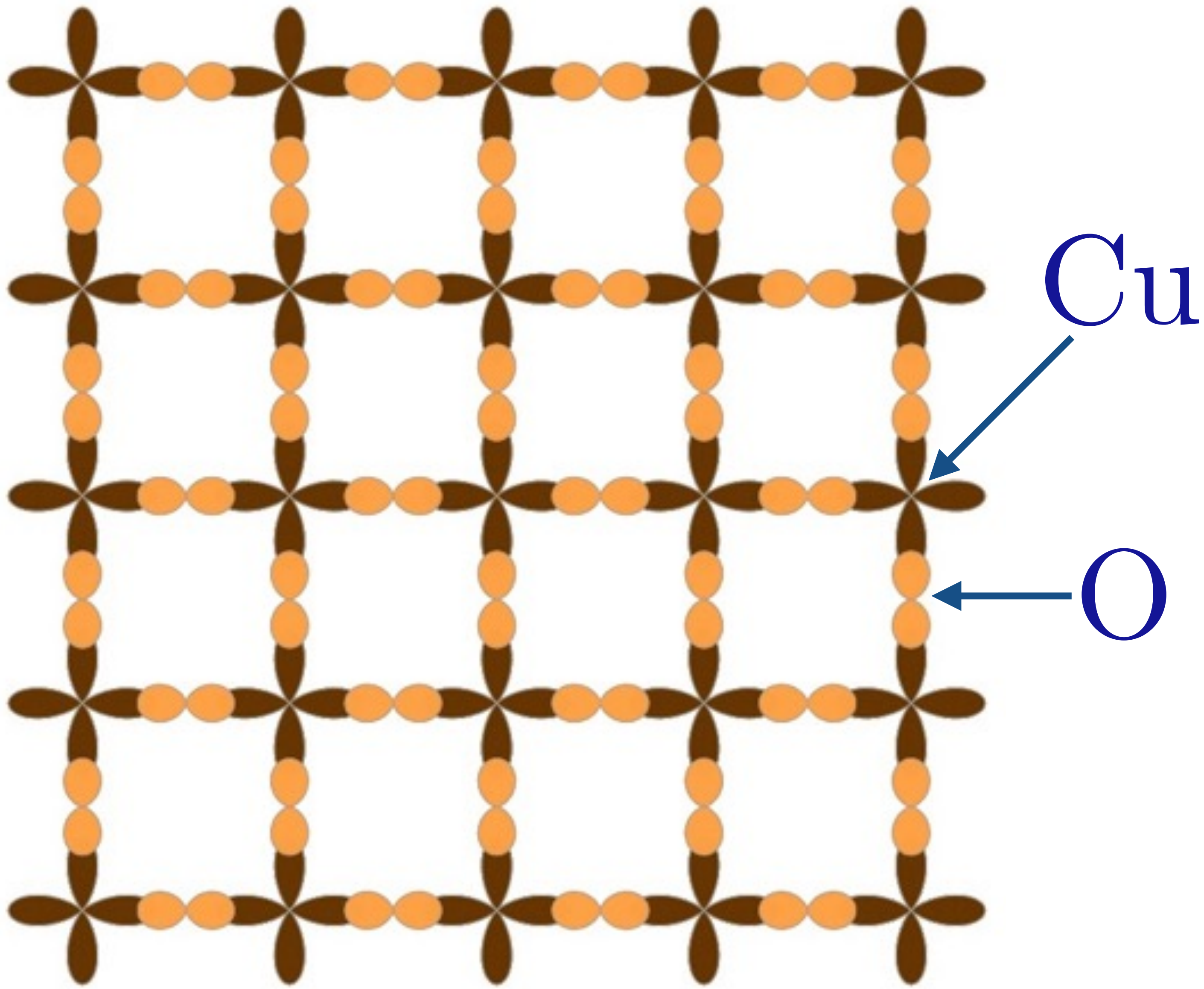
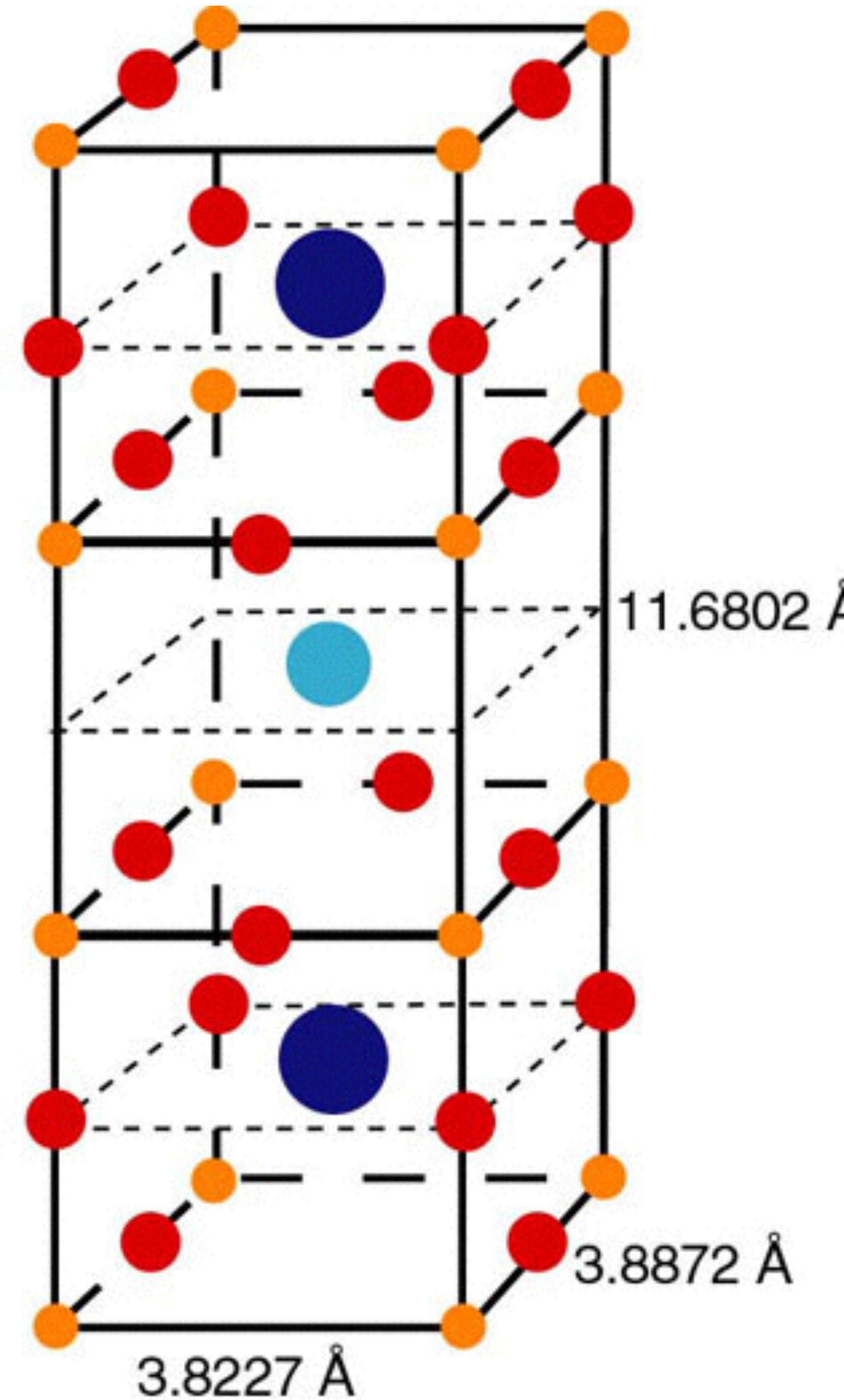
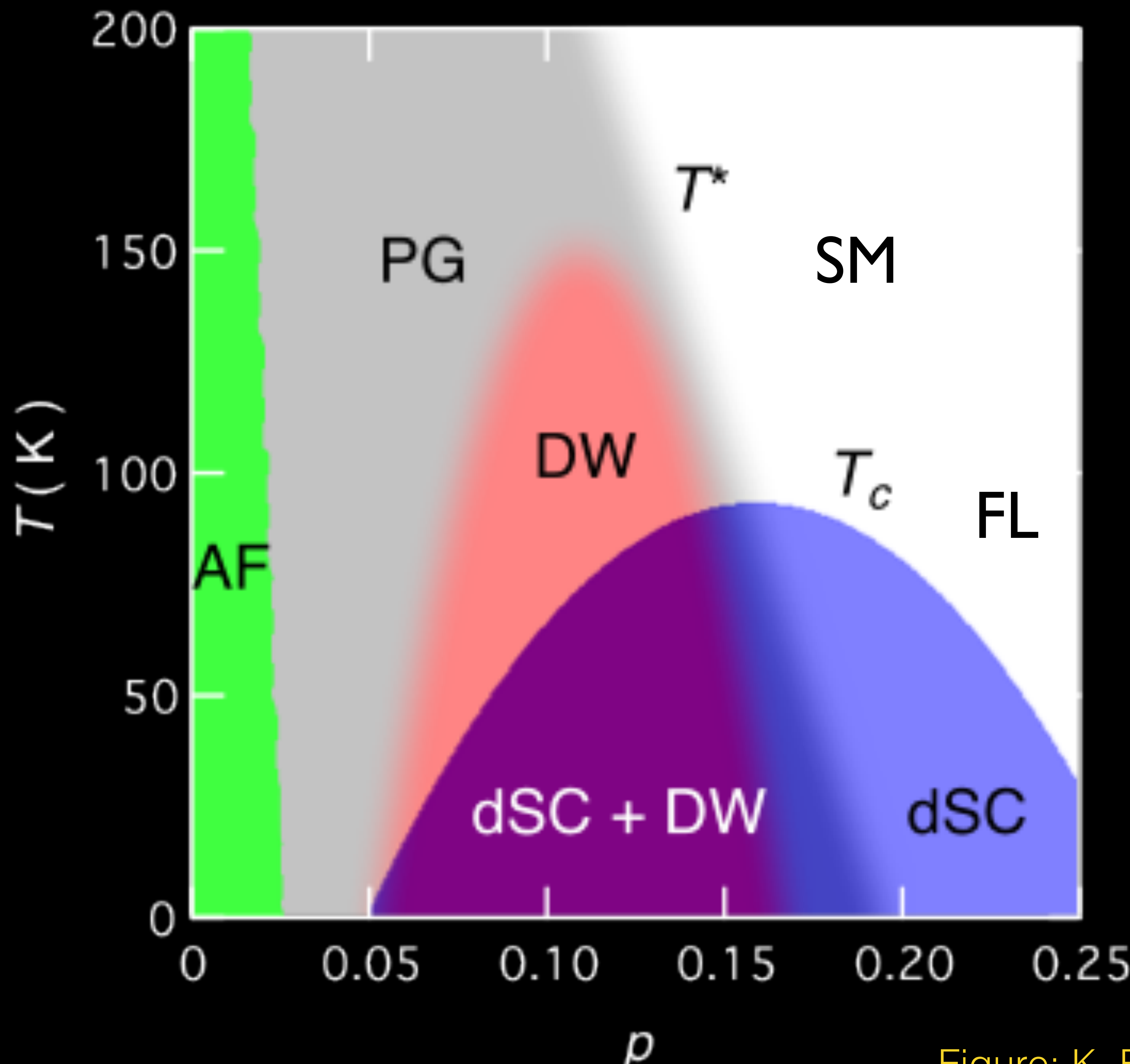
High temperature superconductors



CuO_2 plane



$YBa_2Cu_3O_{6+x}$



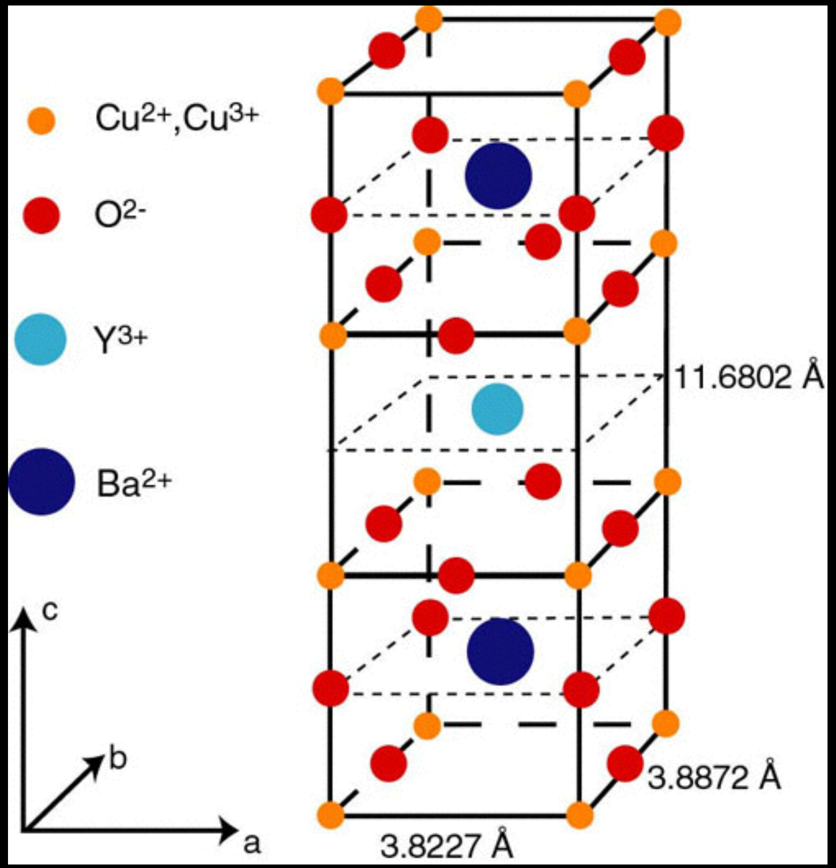


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Figure: K. Fujita and J. C. Seamus Davis

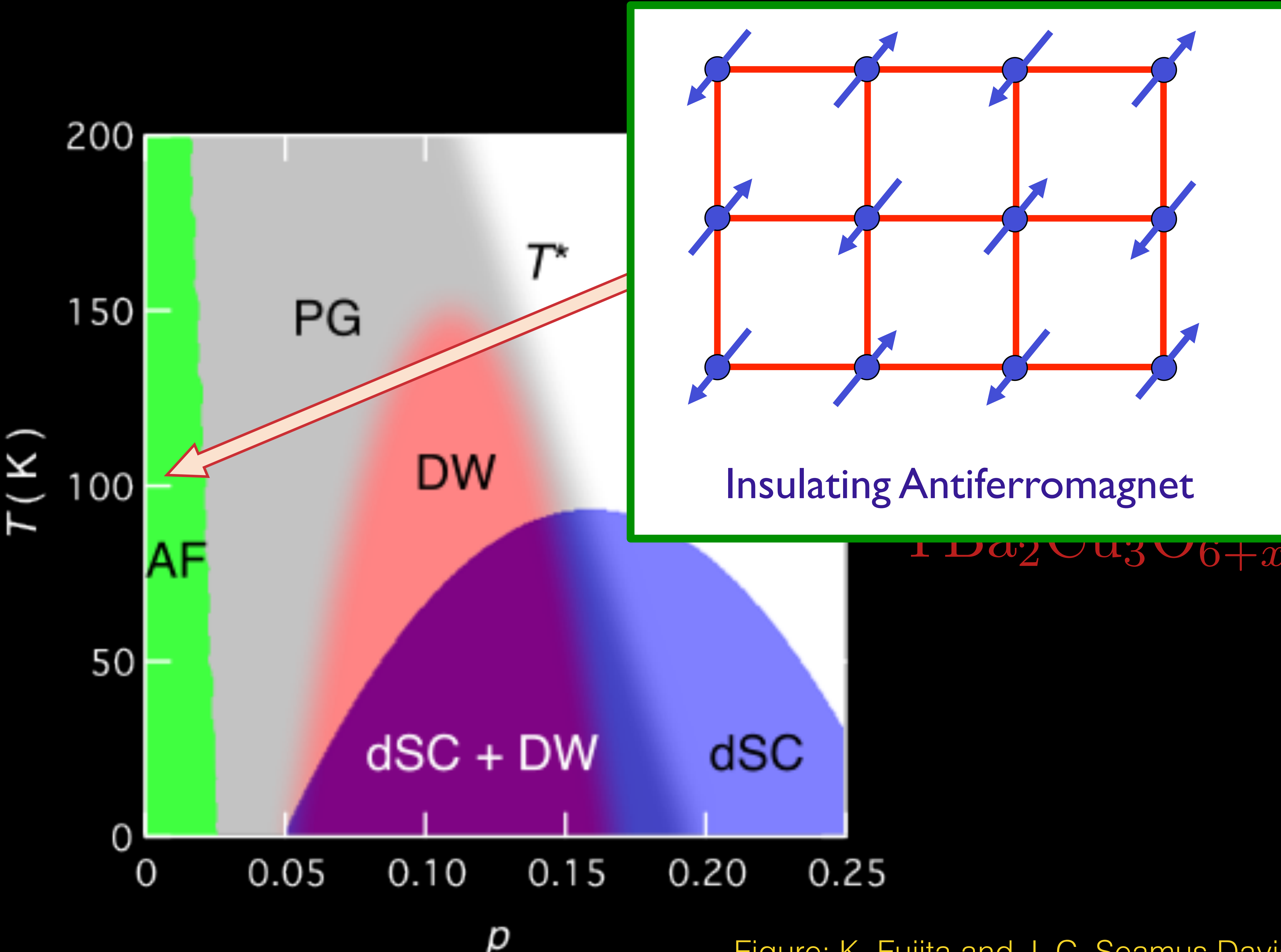
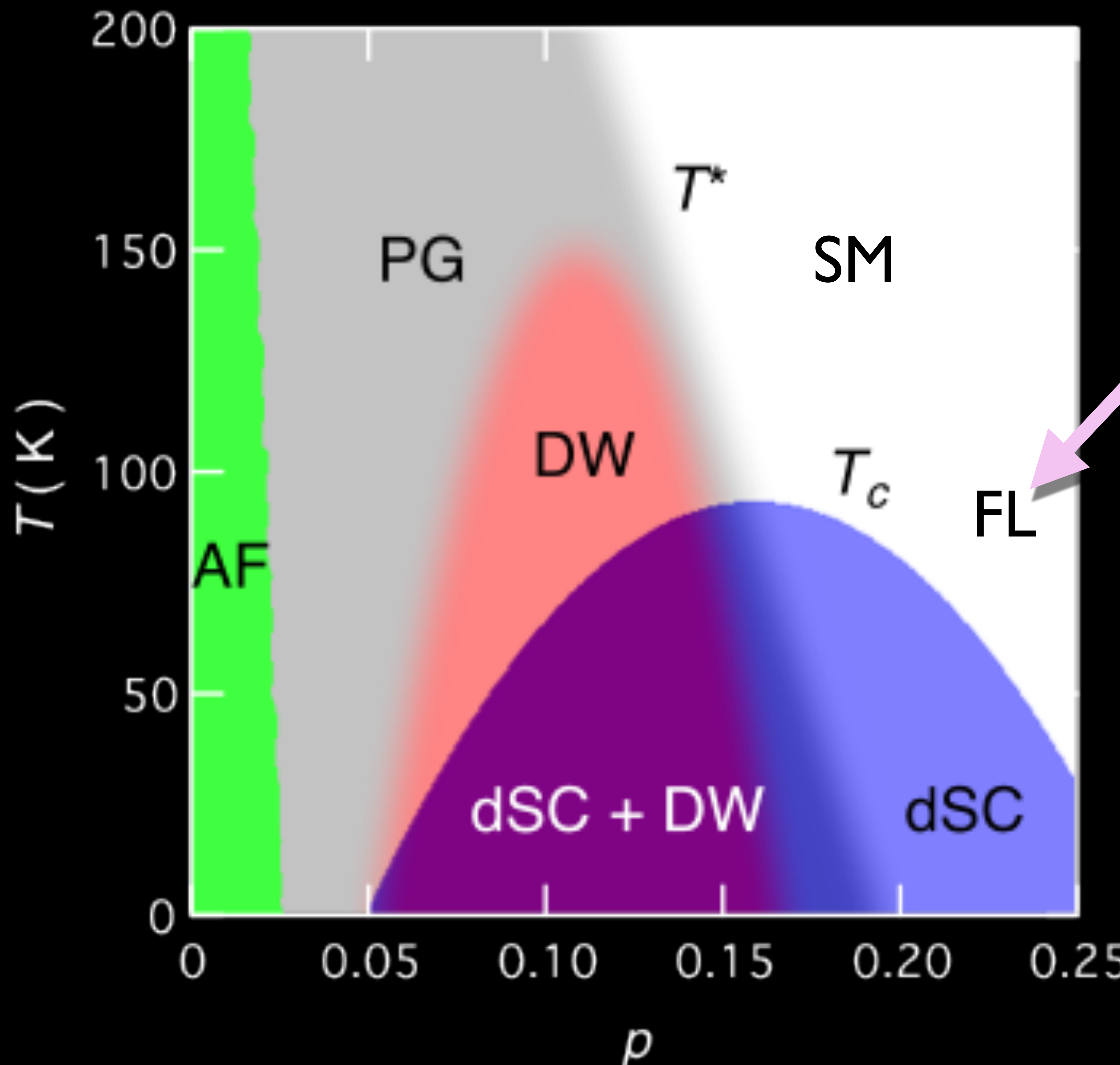
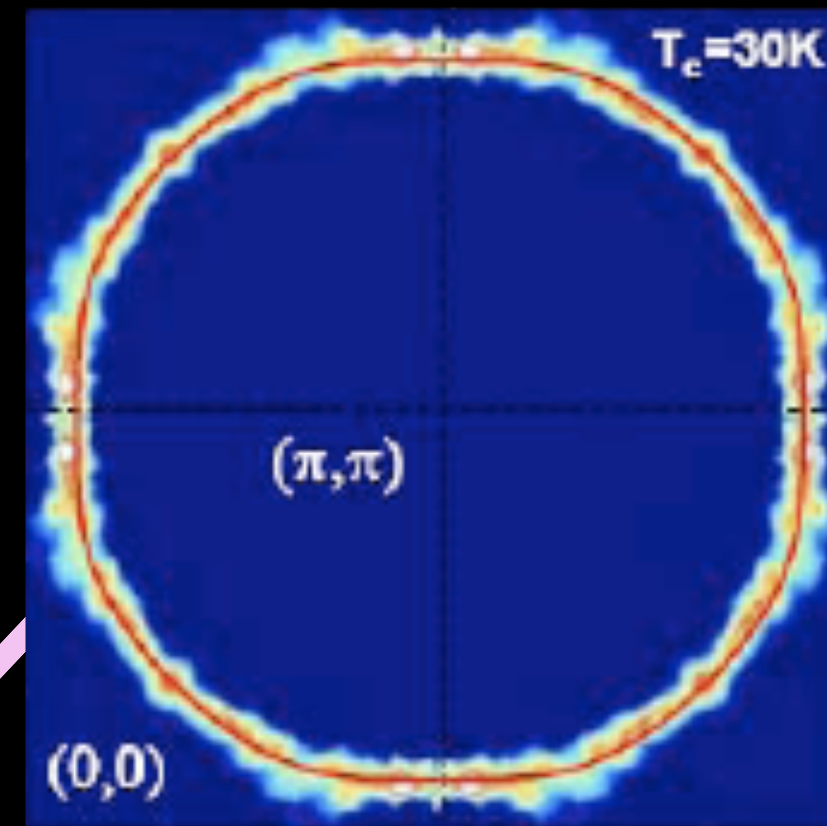


Figure: K. Fujita and J. C. Seamus Davis

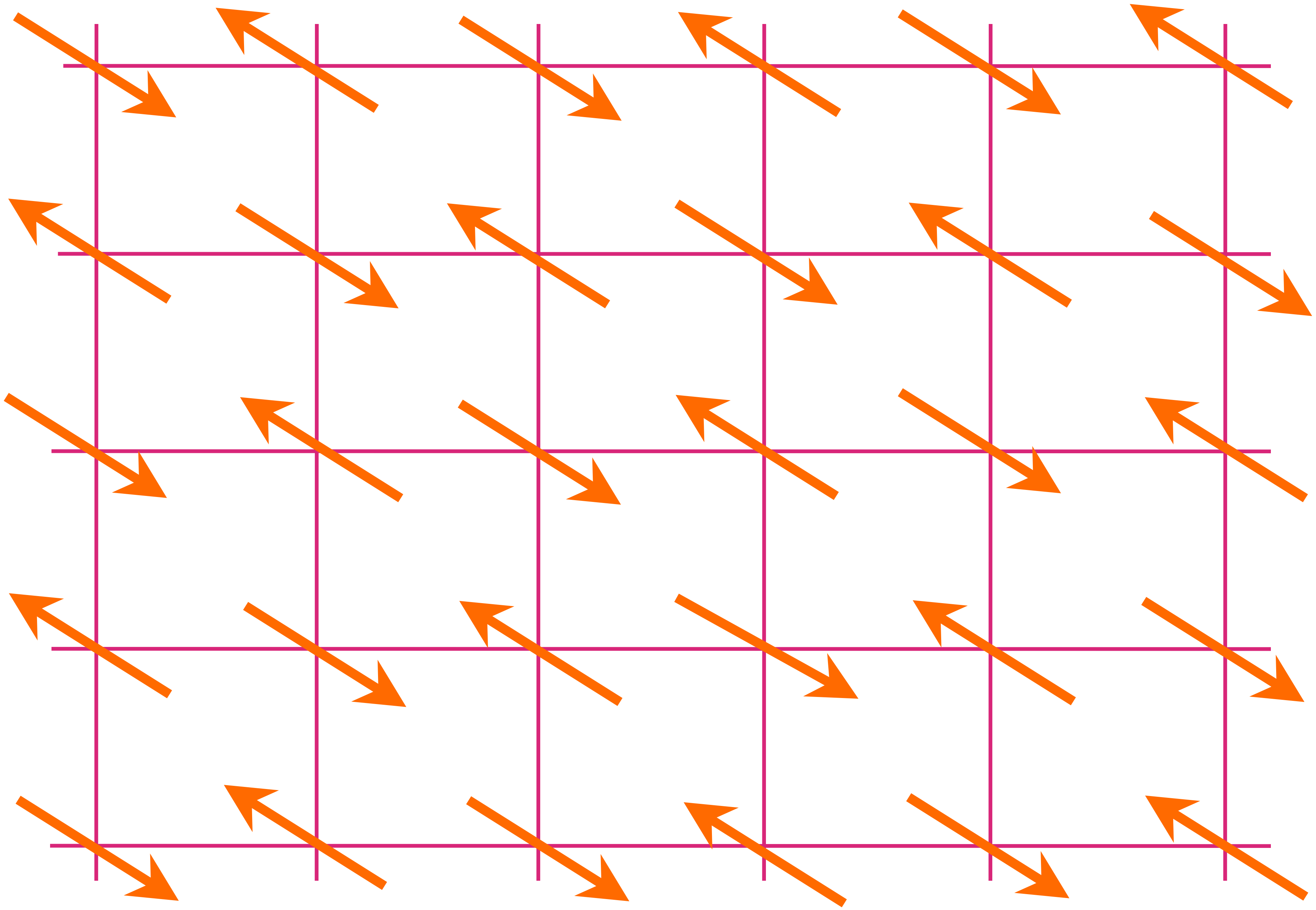
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





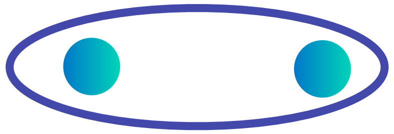
A conventional metal:
the Fermi liquid

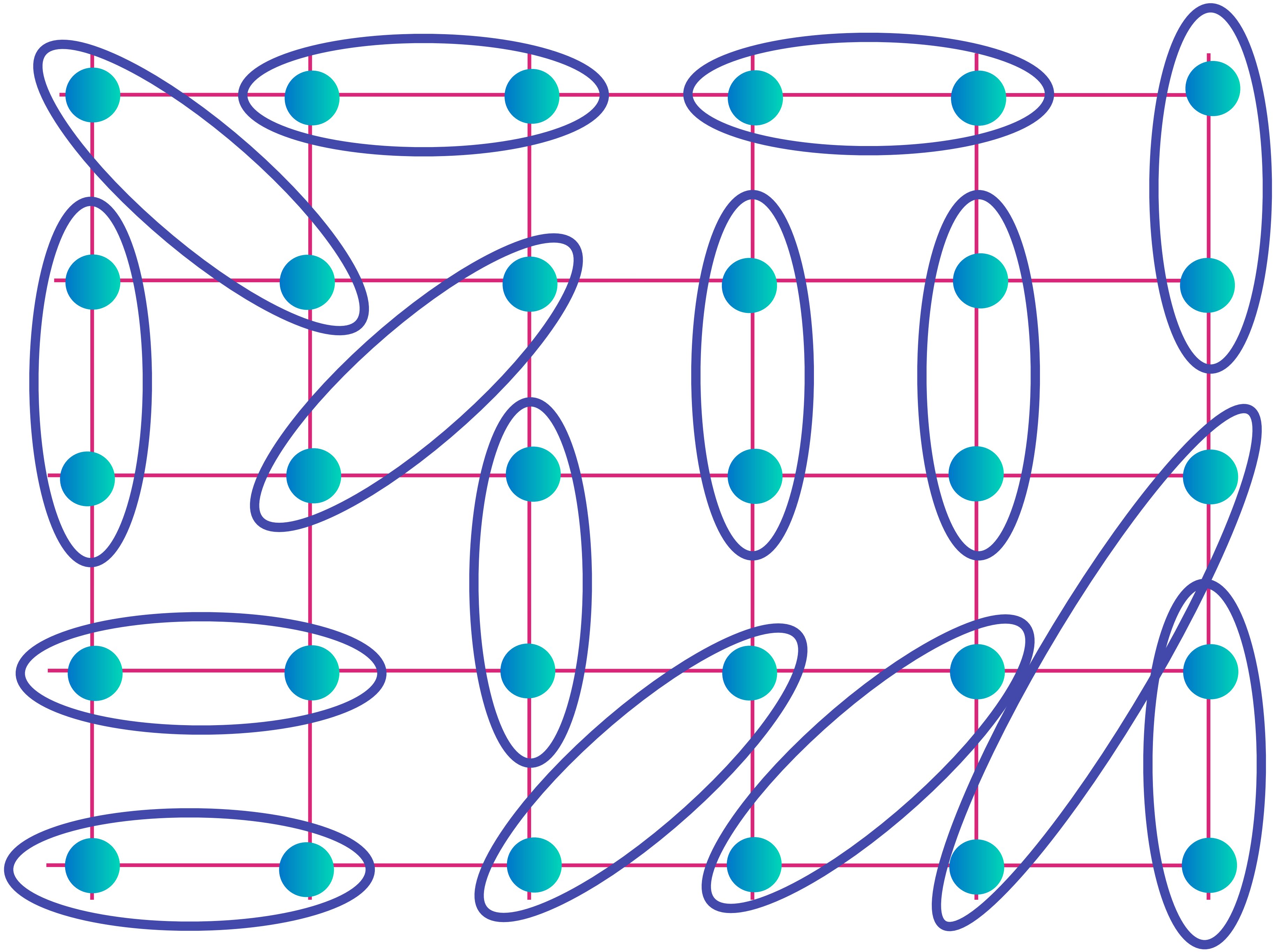
1. Emergent gauge fields and long-range entanglement in insulators



“Undoped” Anti-ferromagnet

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



An insulator with emergent gauge fields and long-range entanglement

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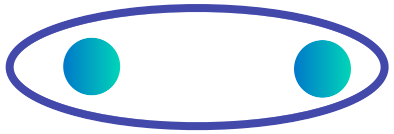

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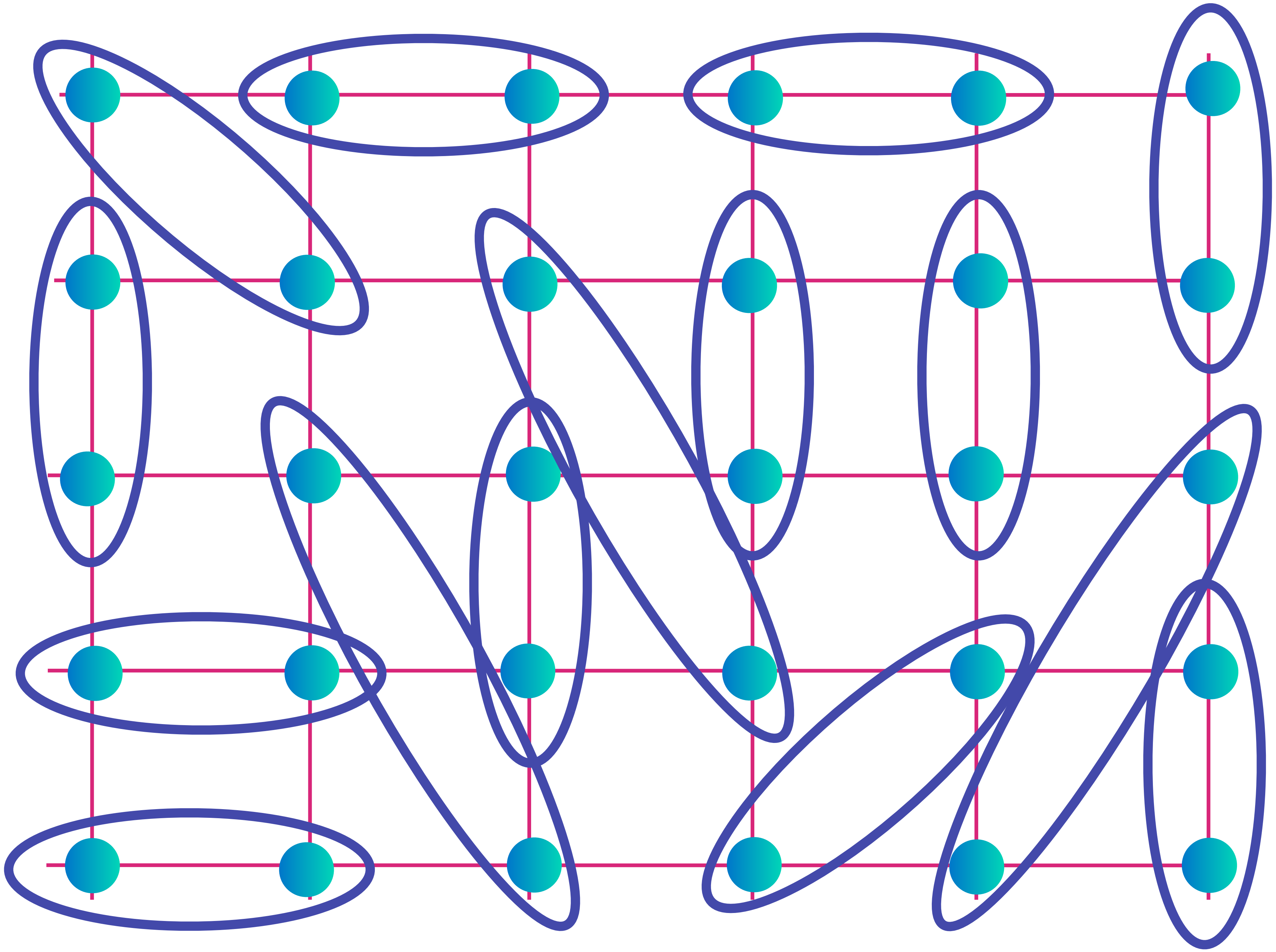

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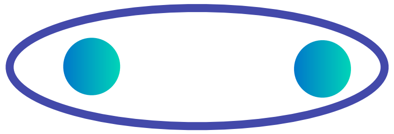

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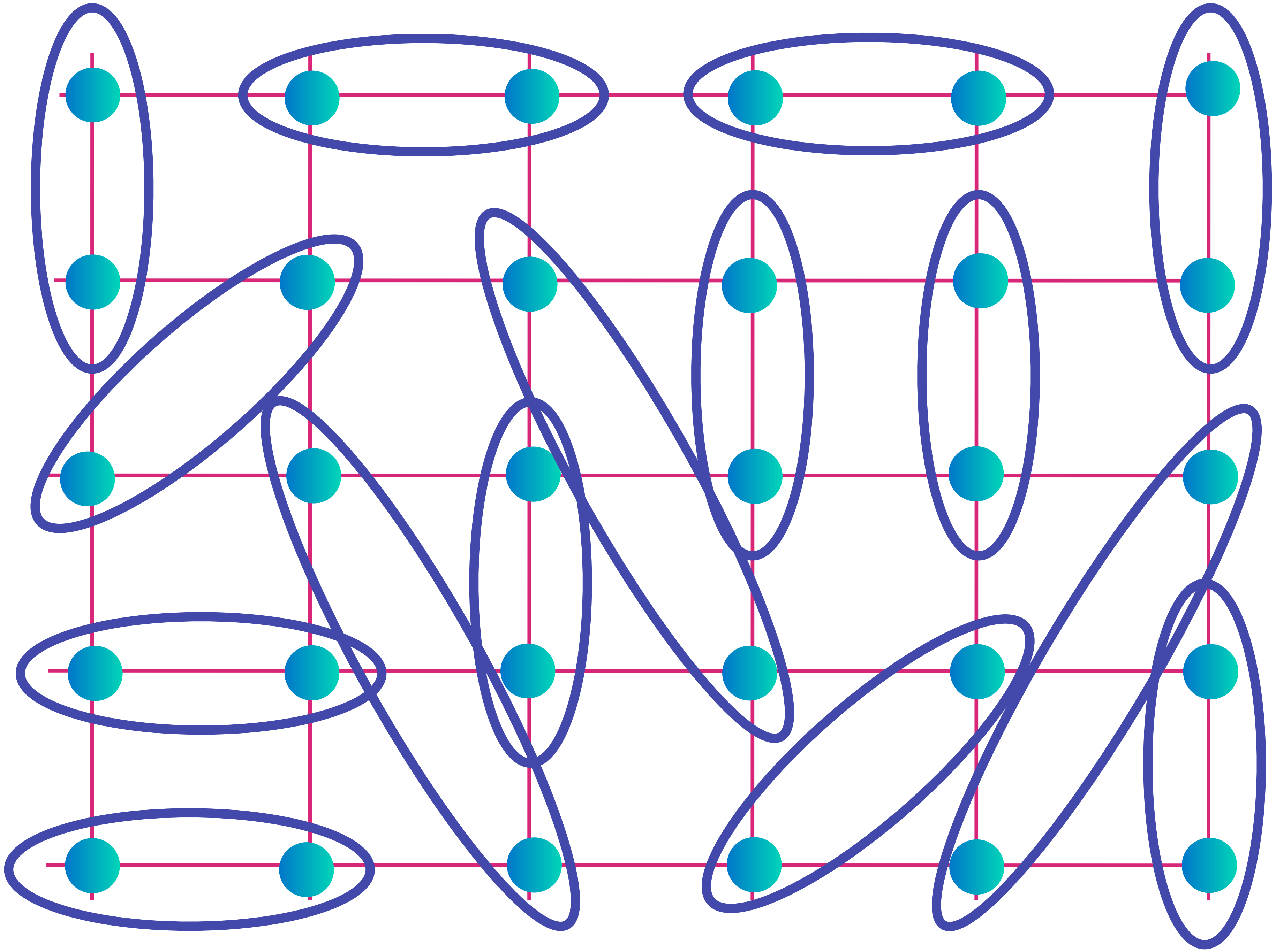

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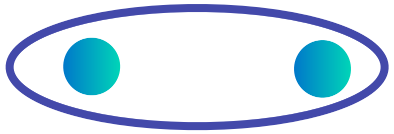
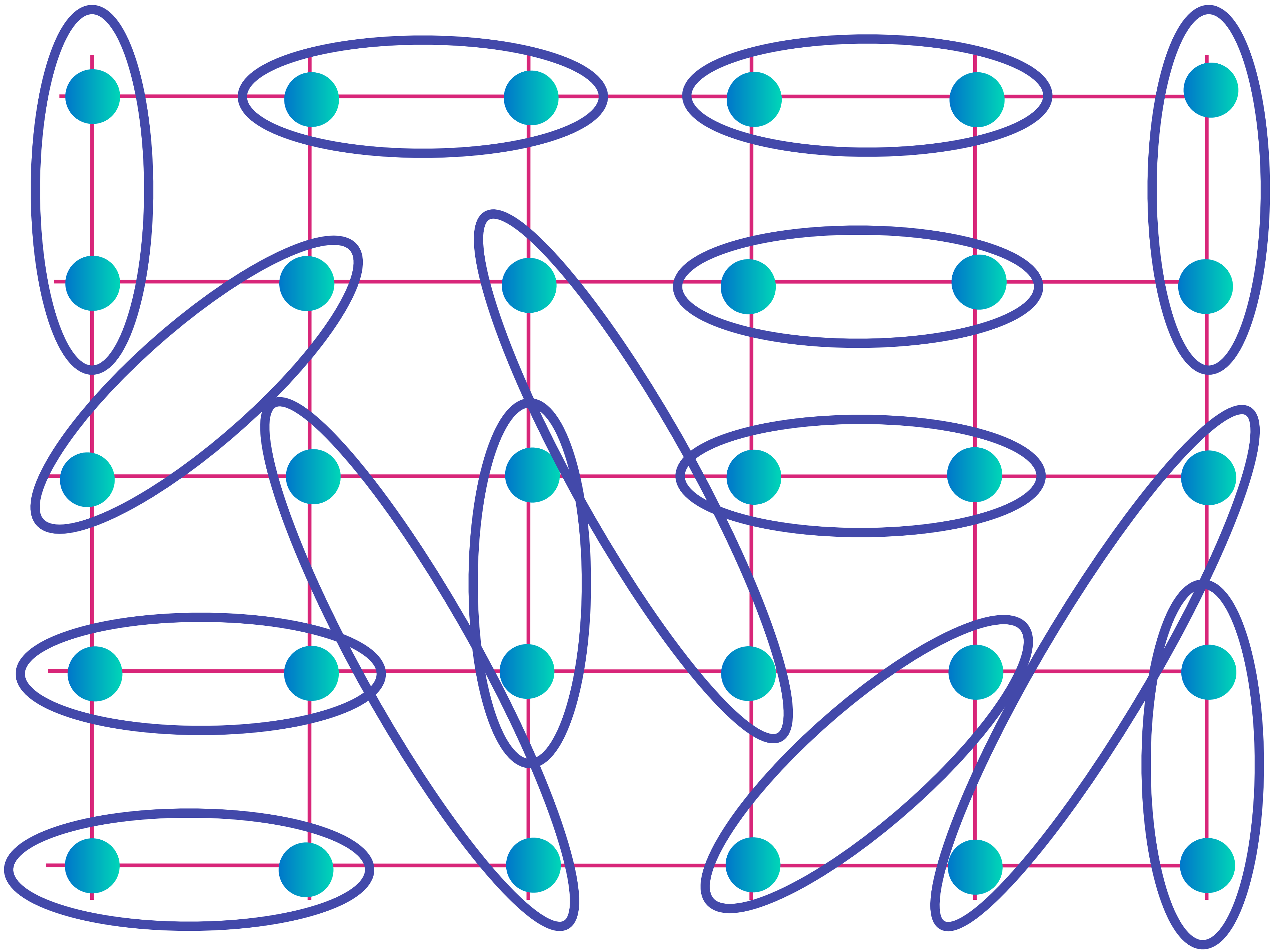
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



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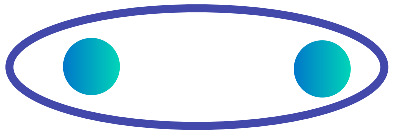
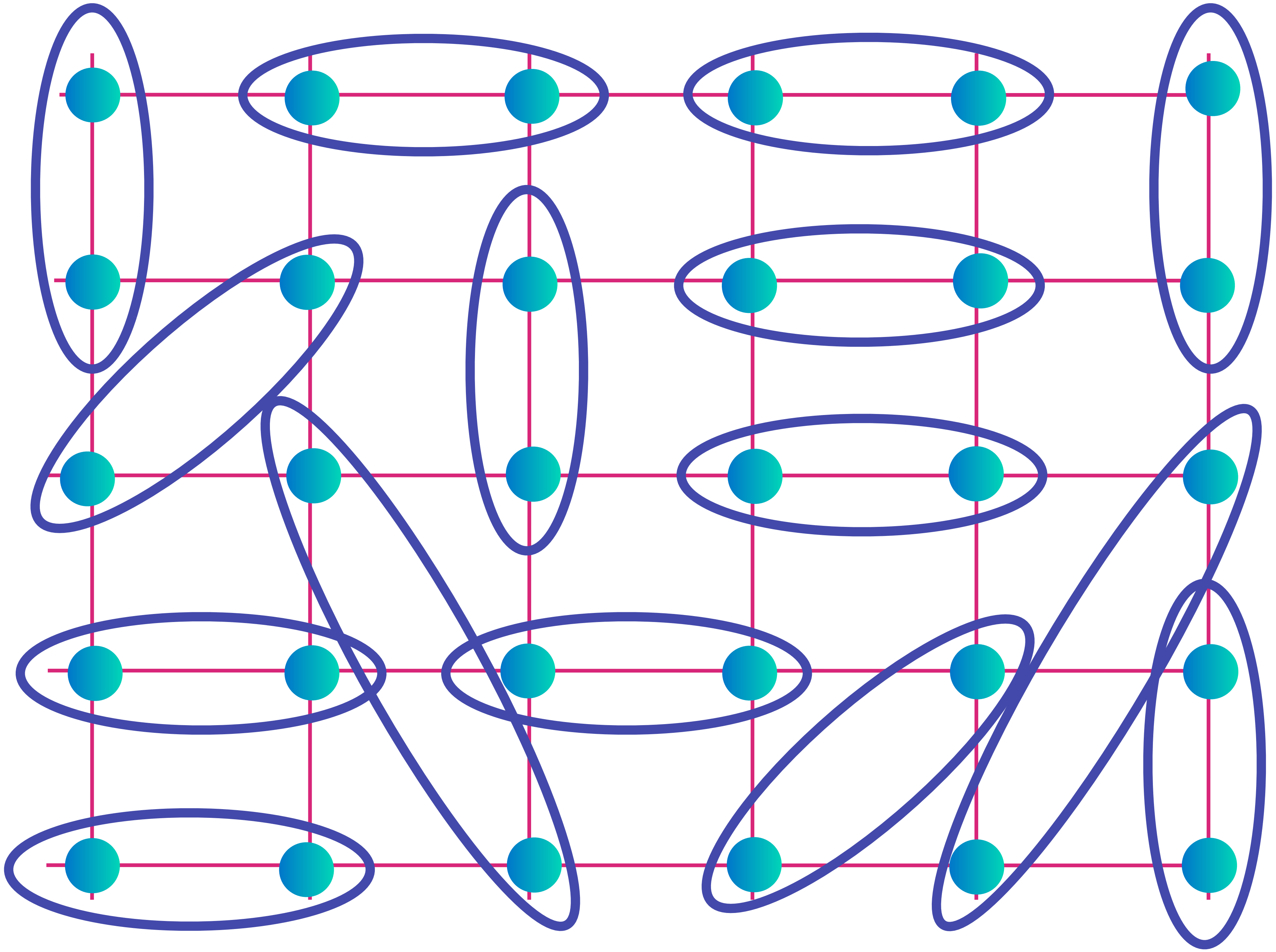
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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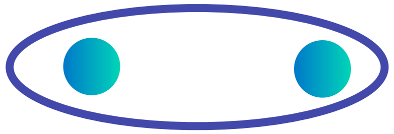

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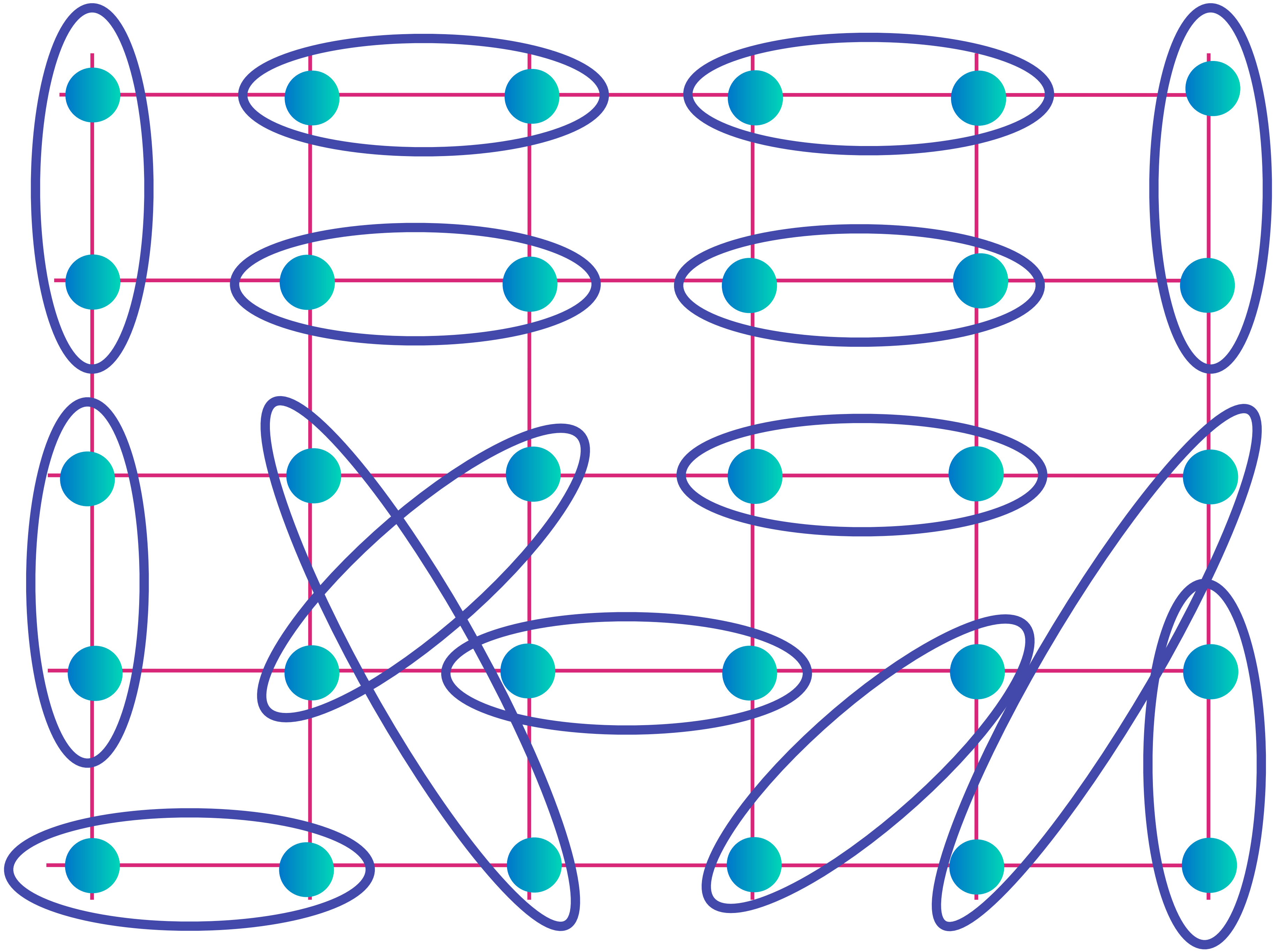

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Ground state degeneracy

Place insulator on a torus;

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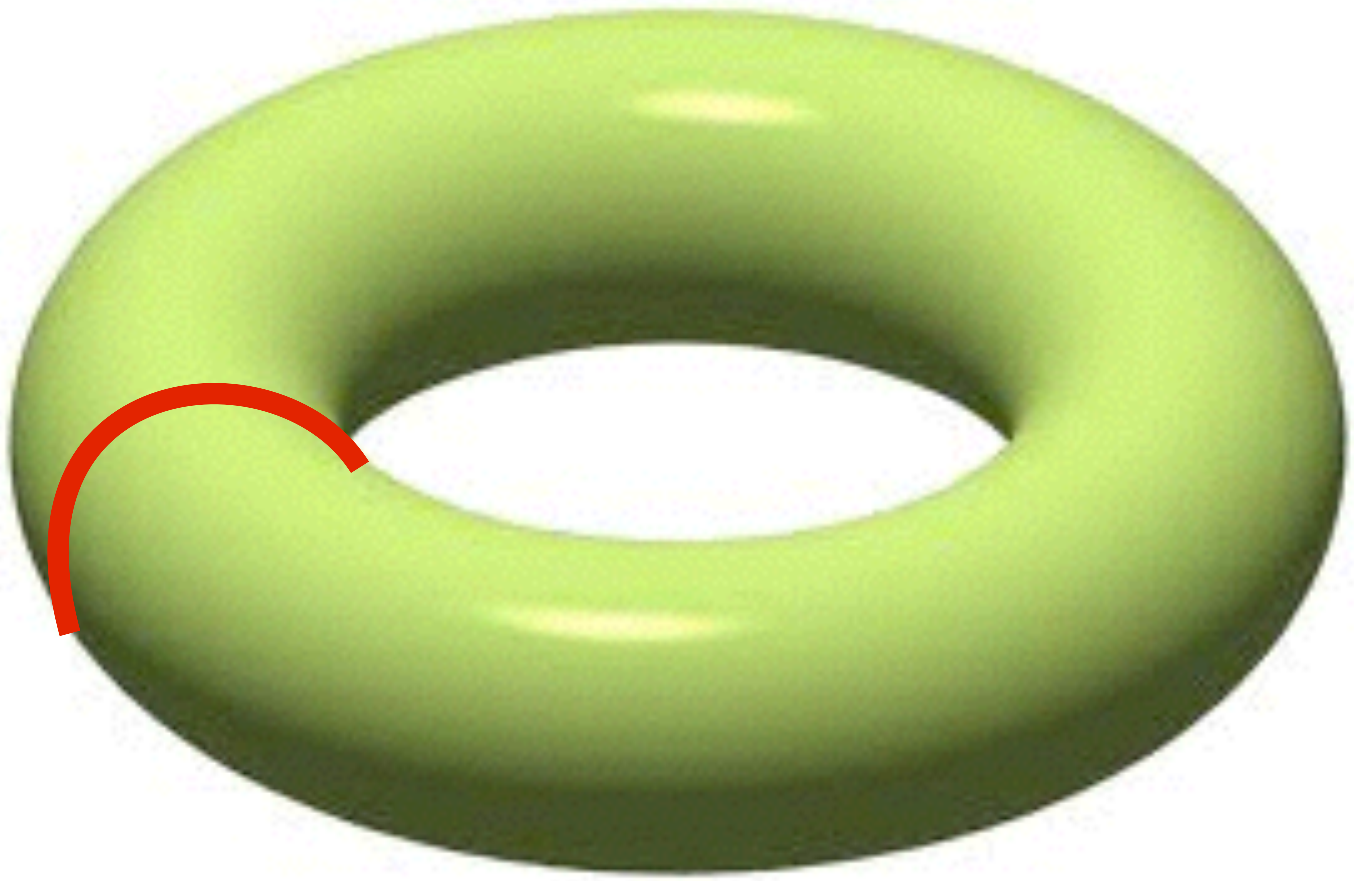

Place insulator on a torus;

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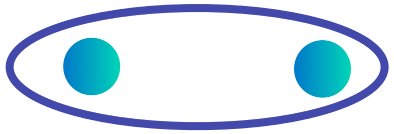
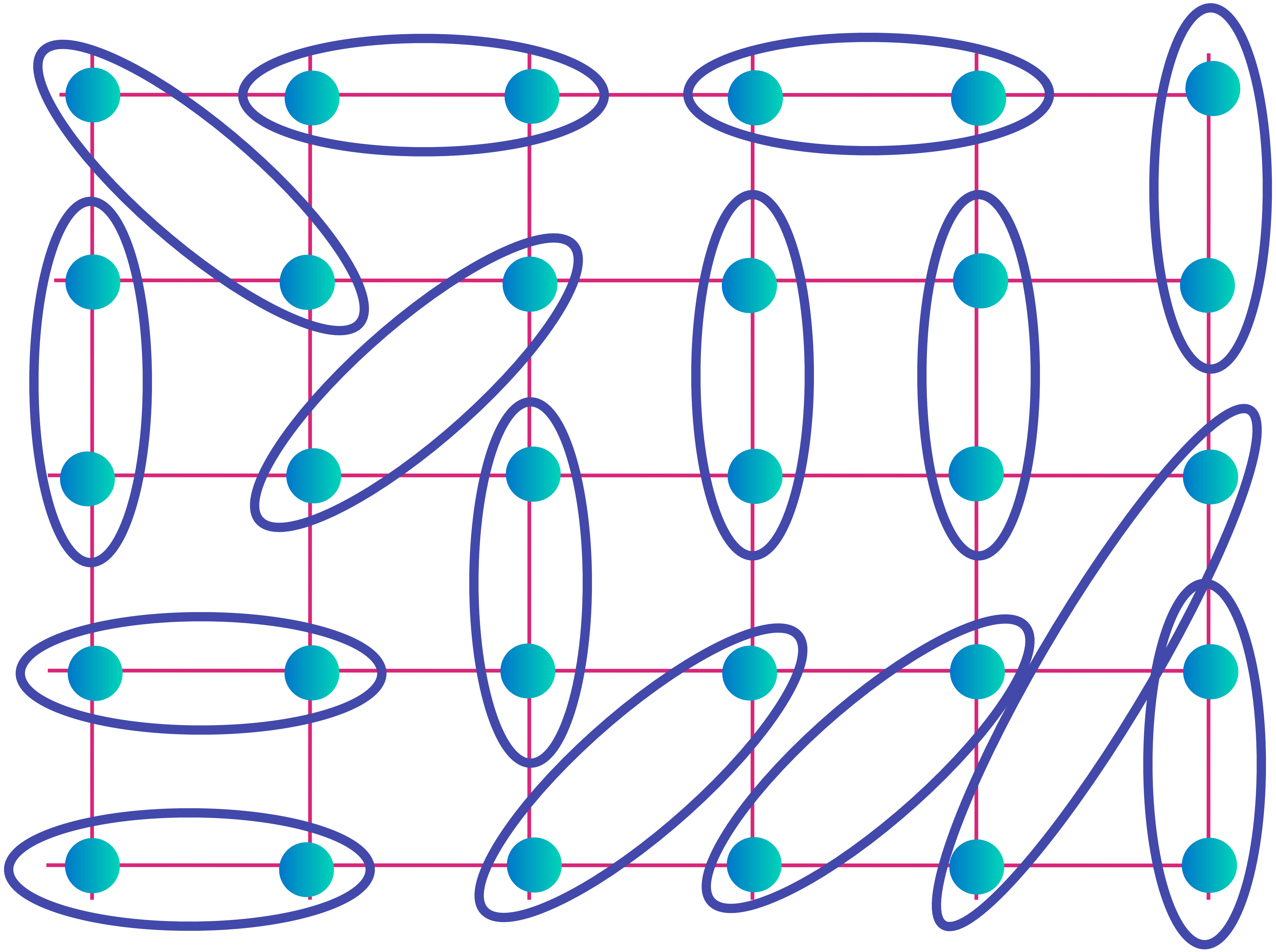
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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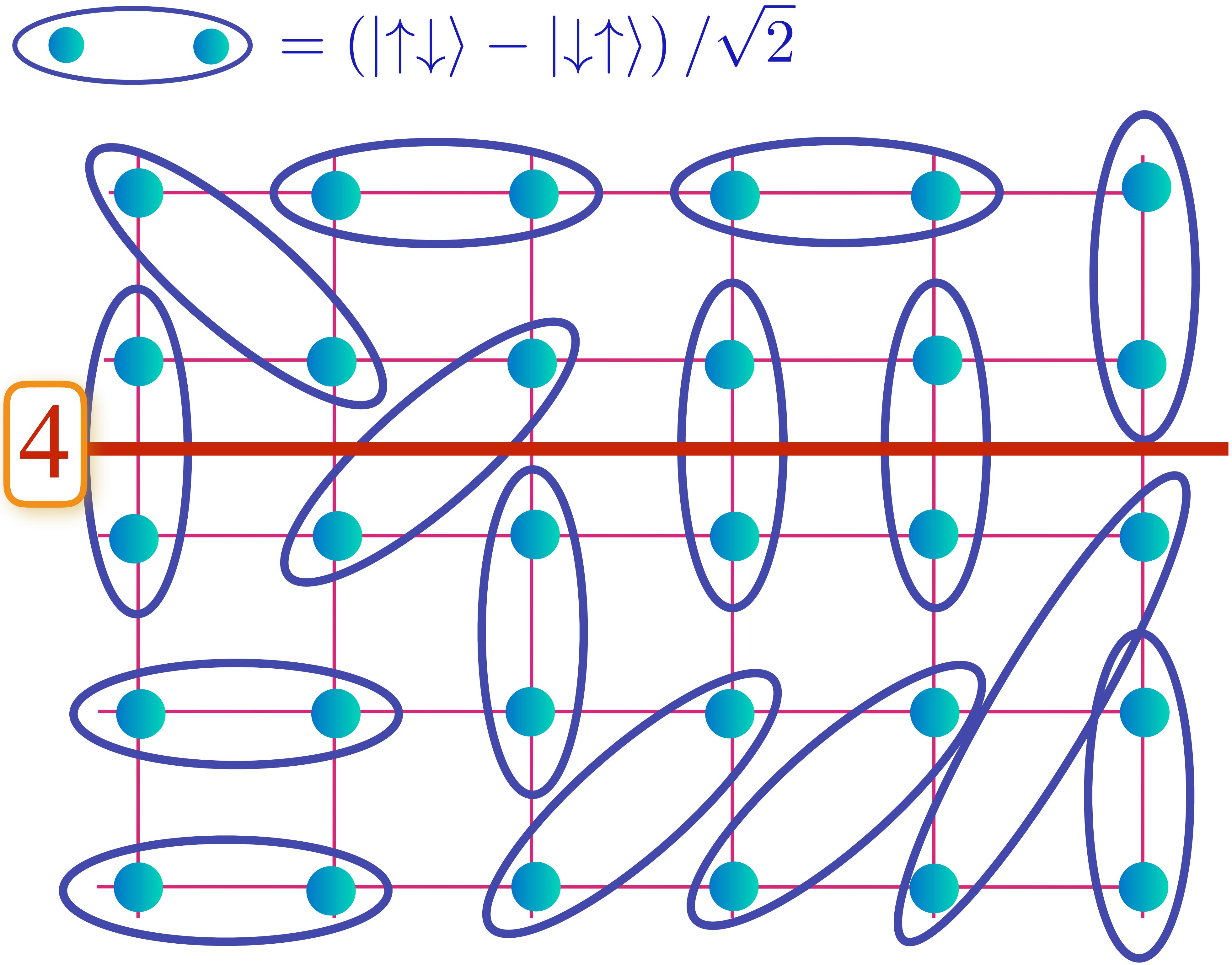

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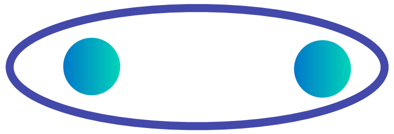
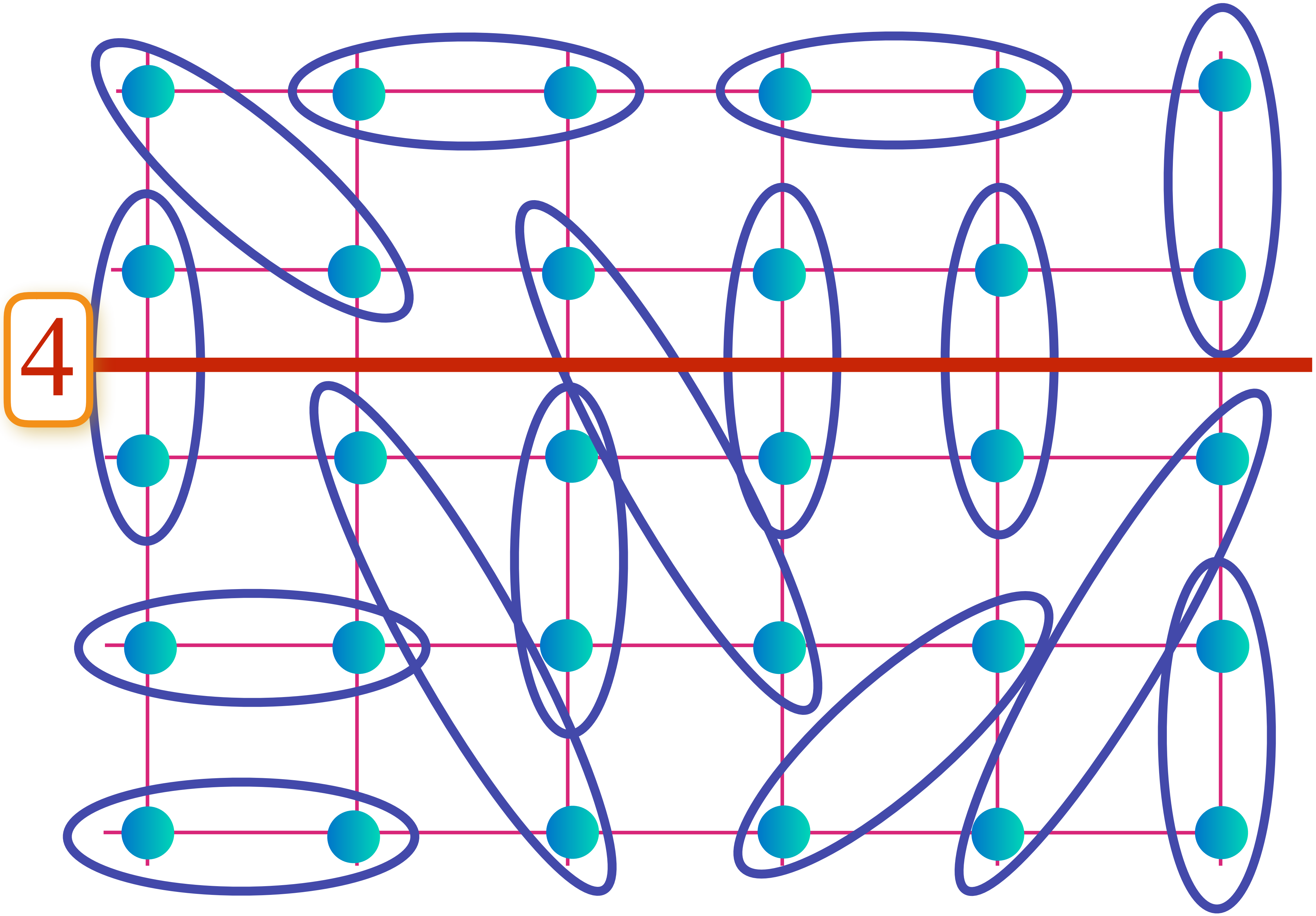
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



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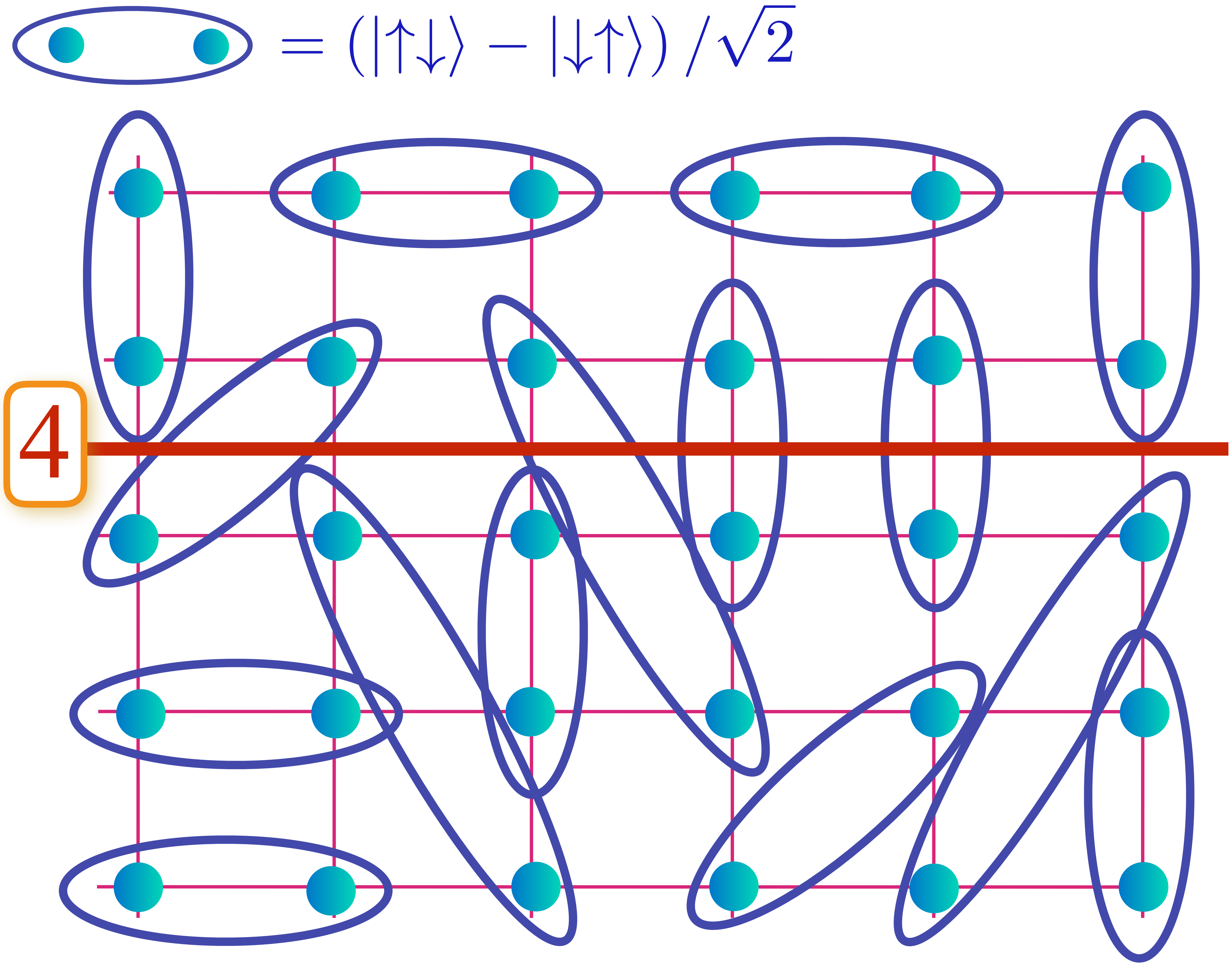

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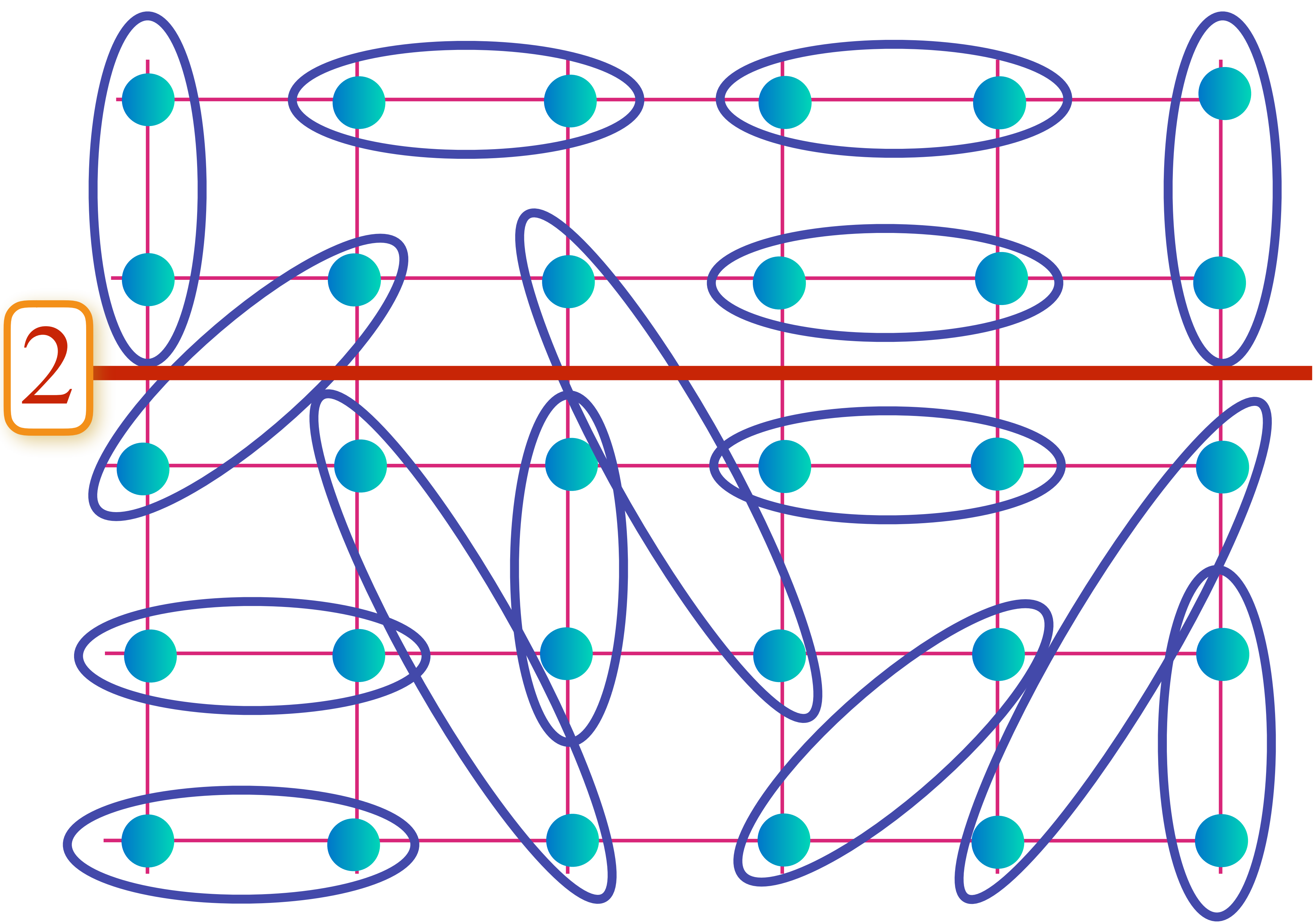

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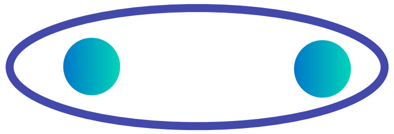
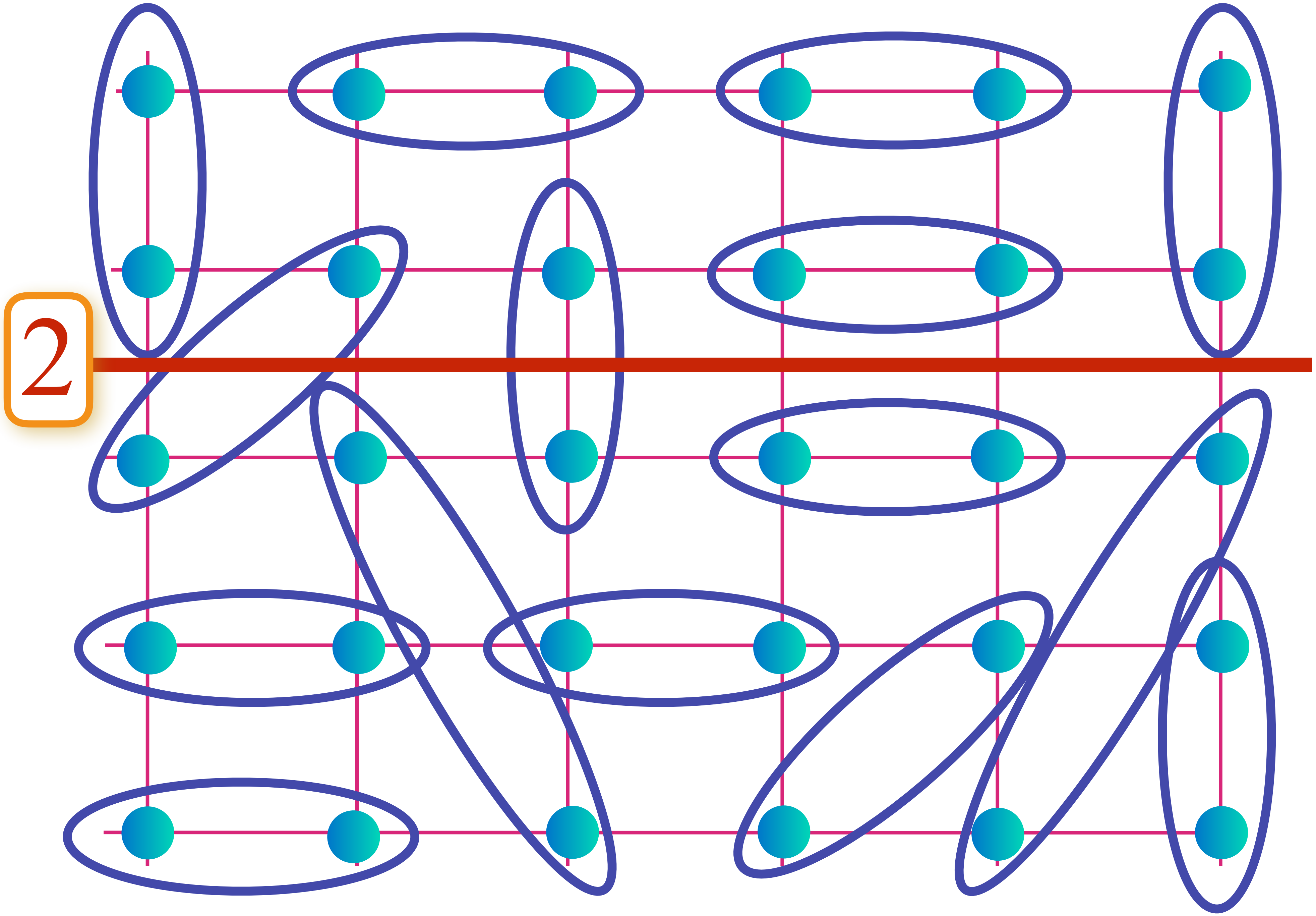
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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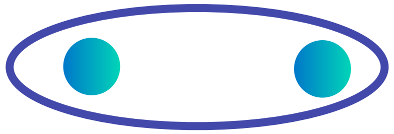

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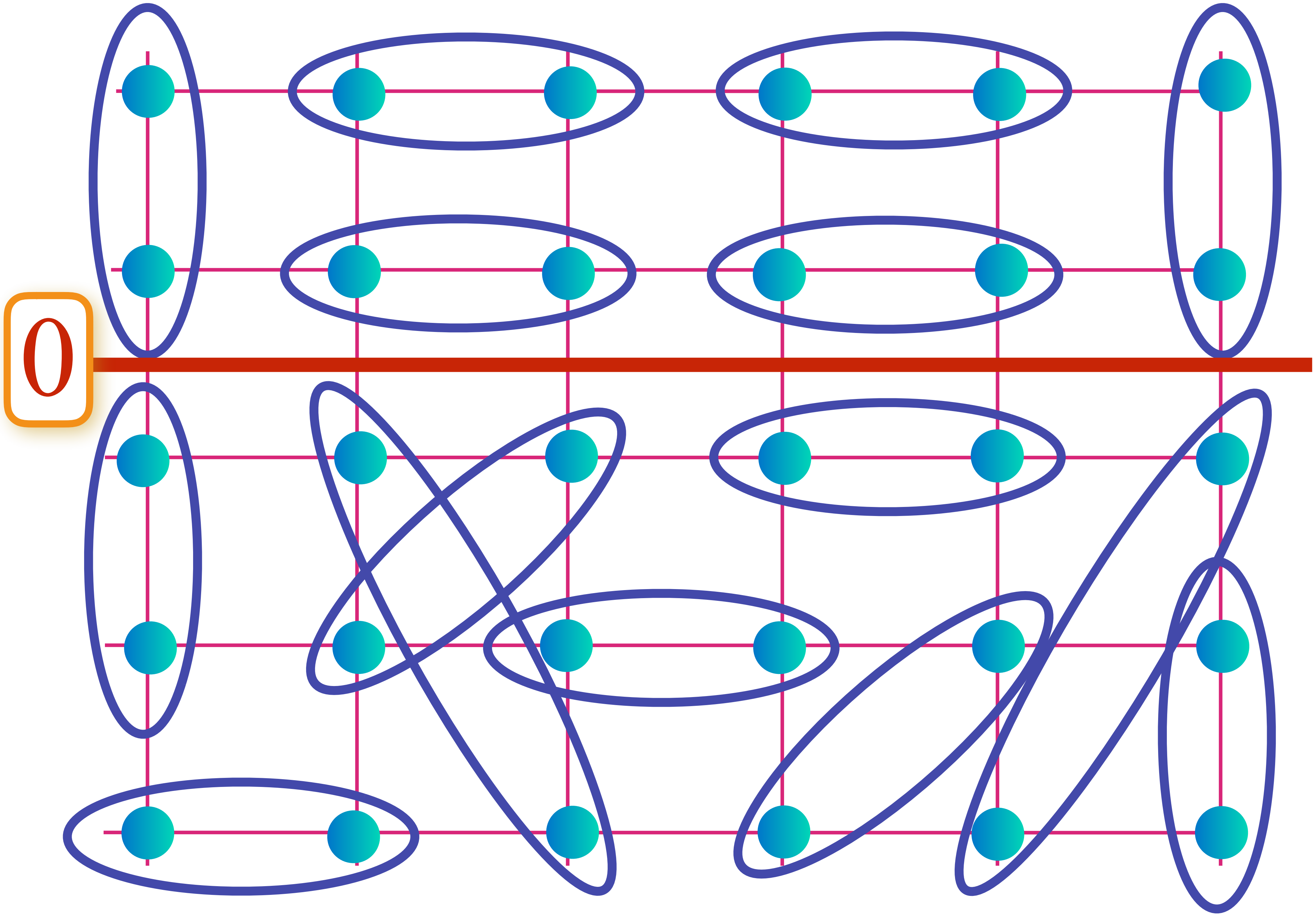

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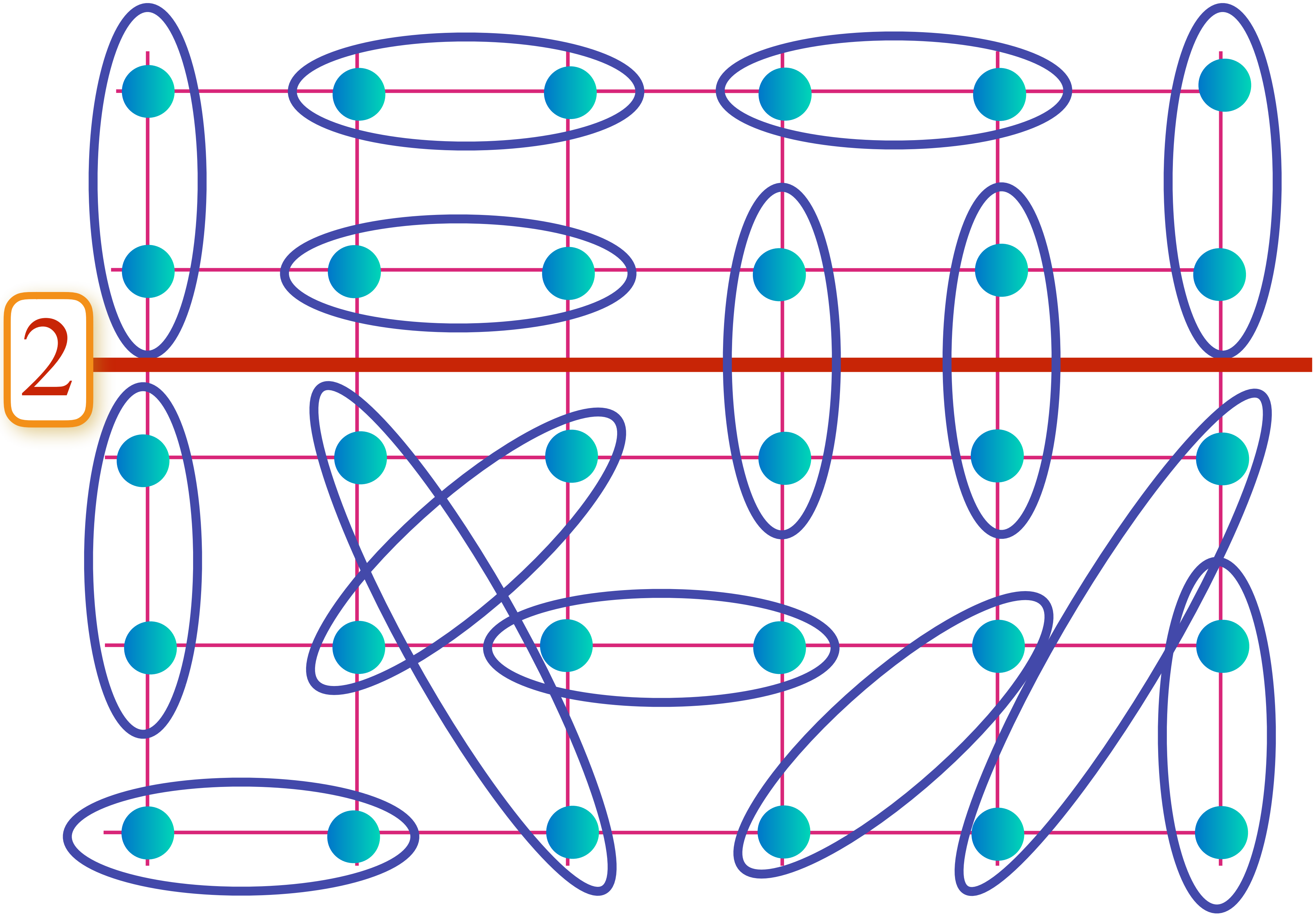

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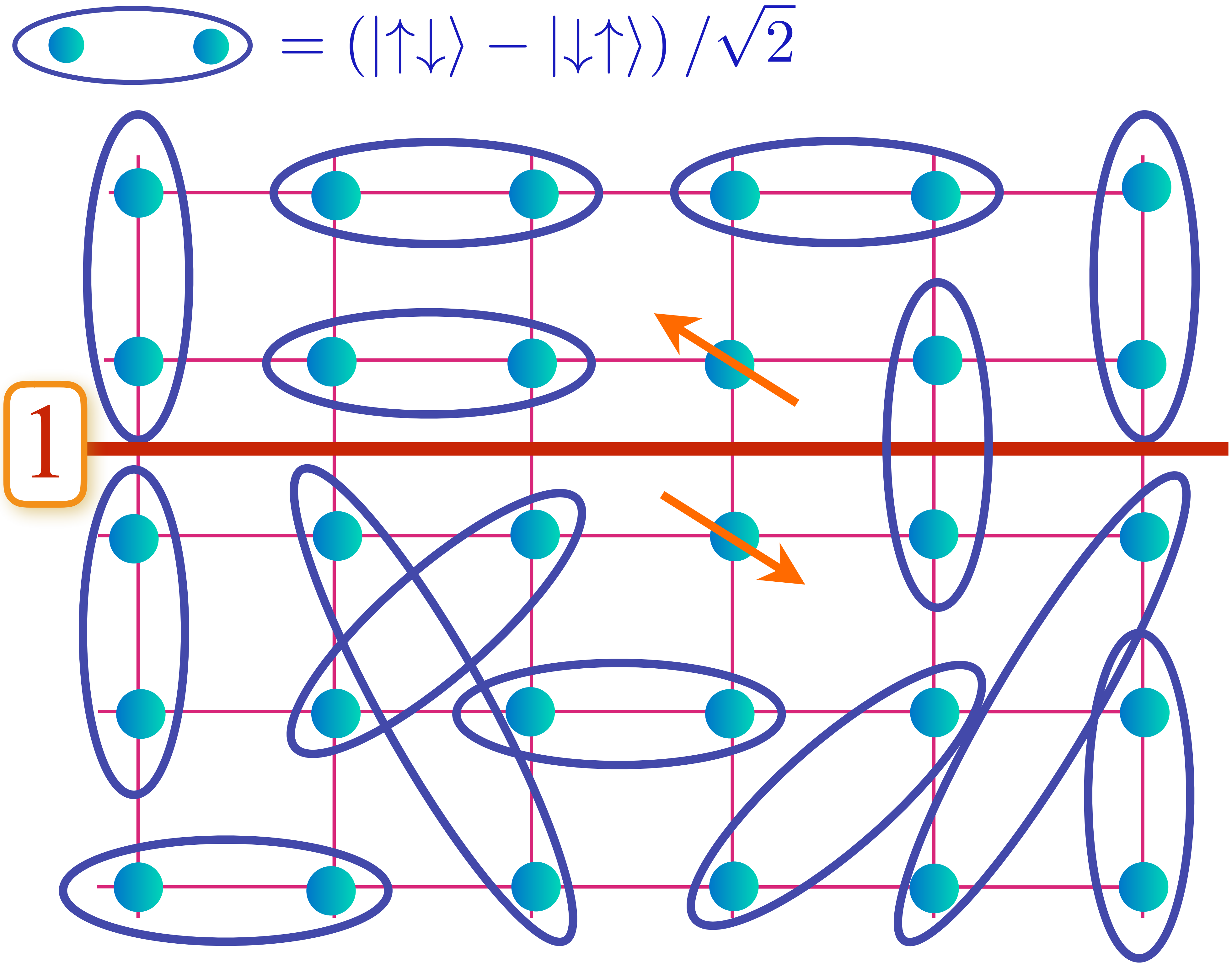

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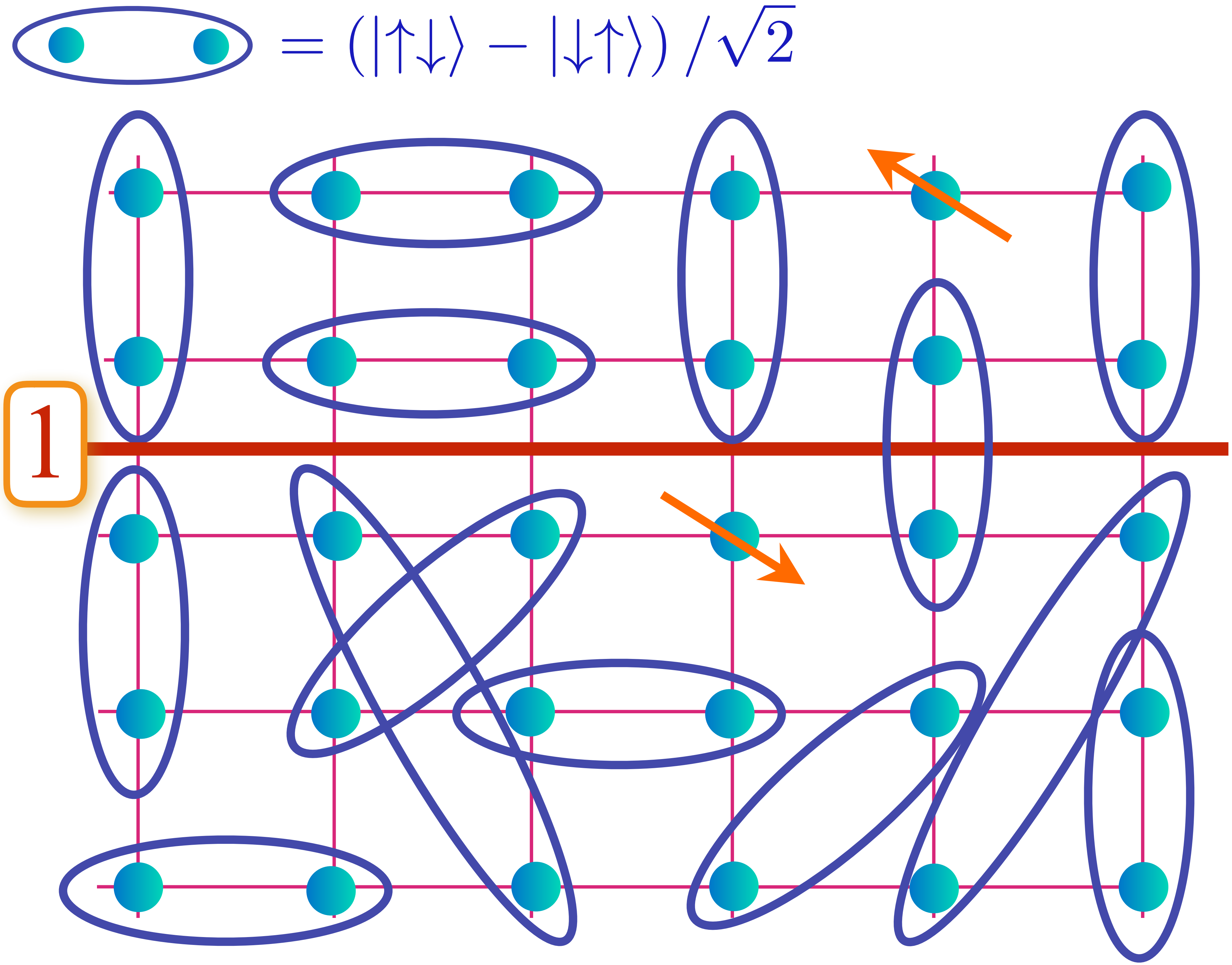

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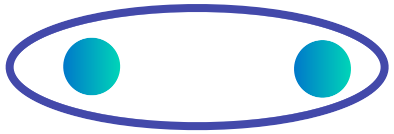
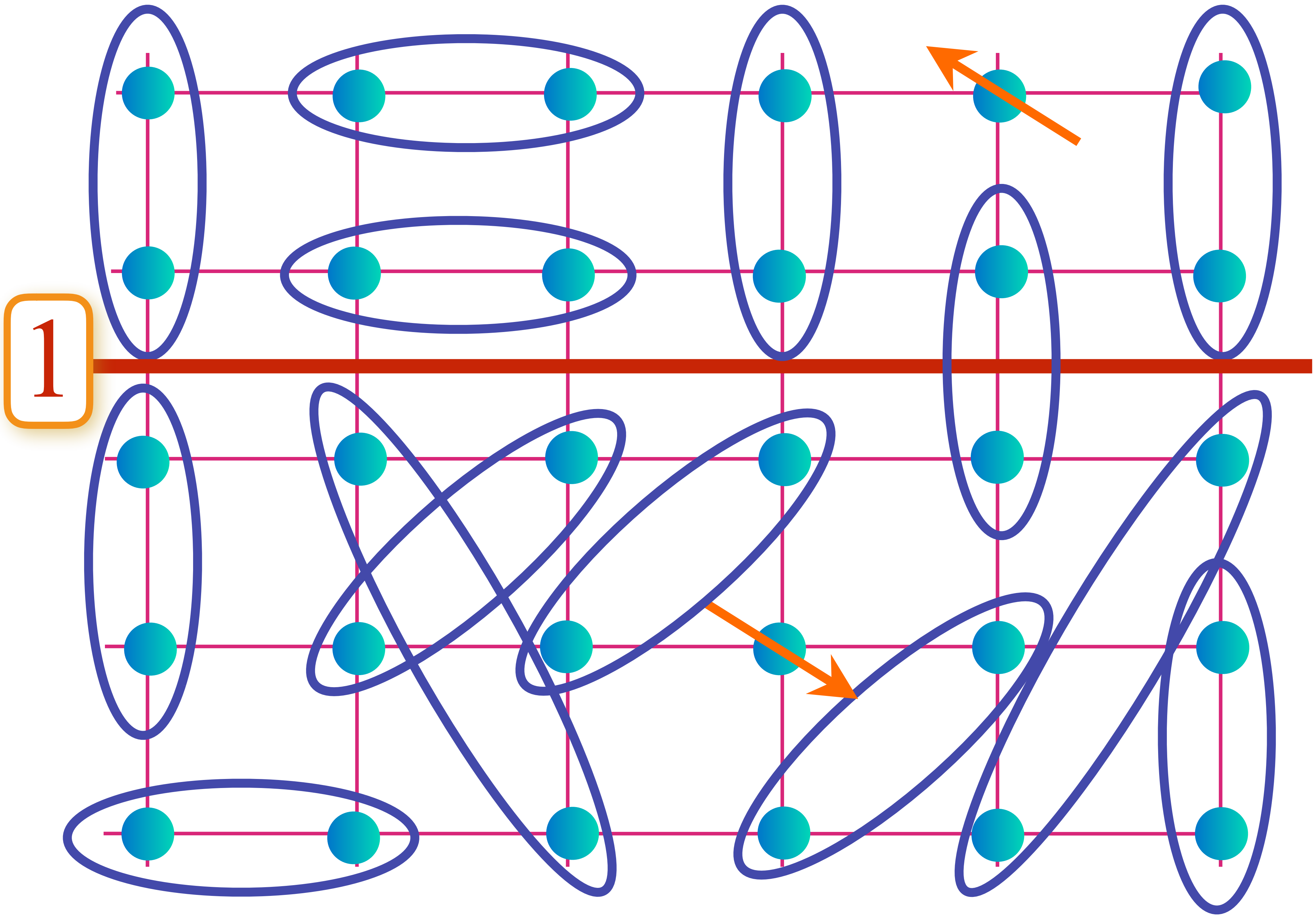
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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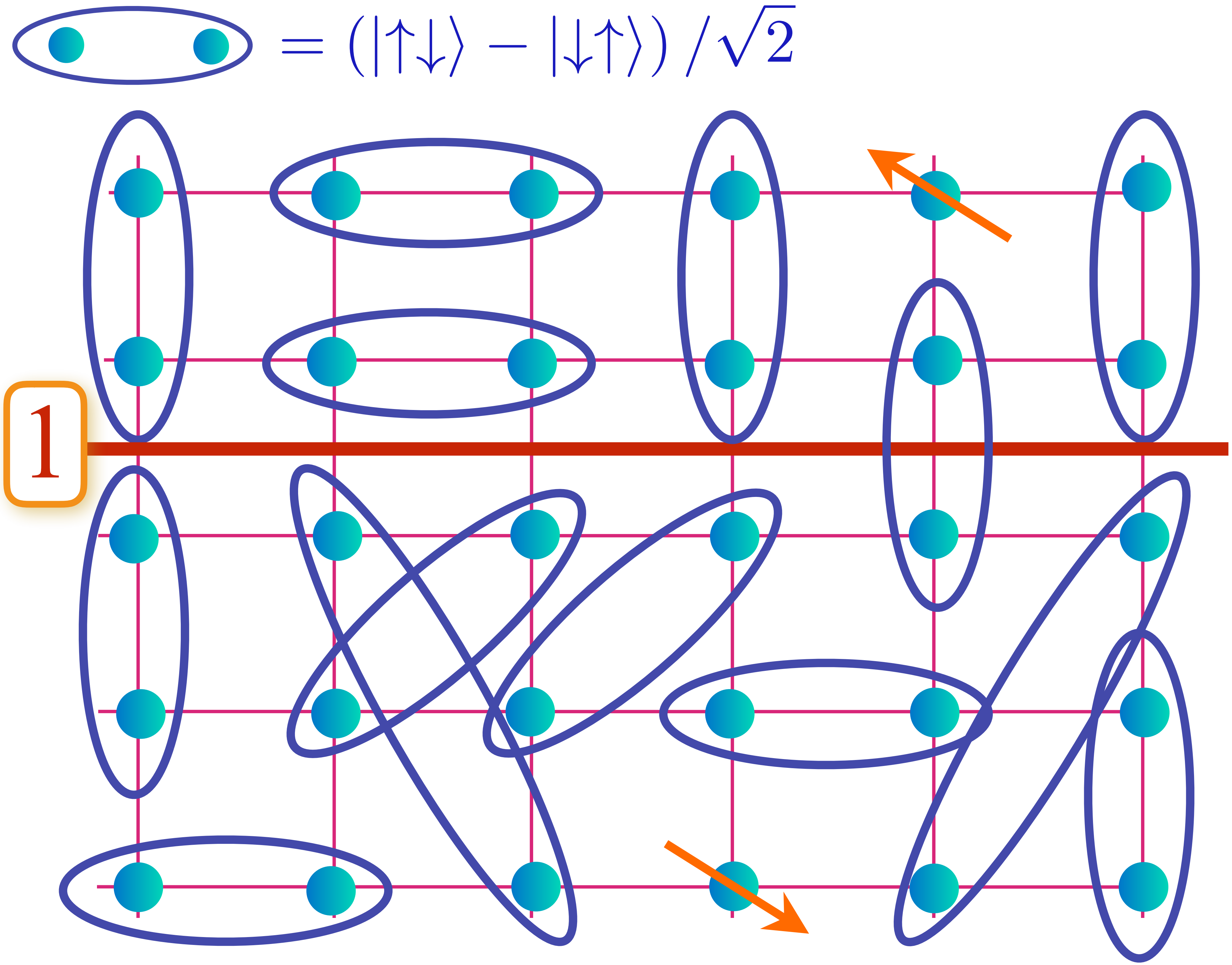

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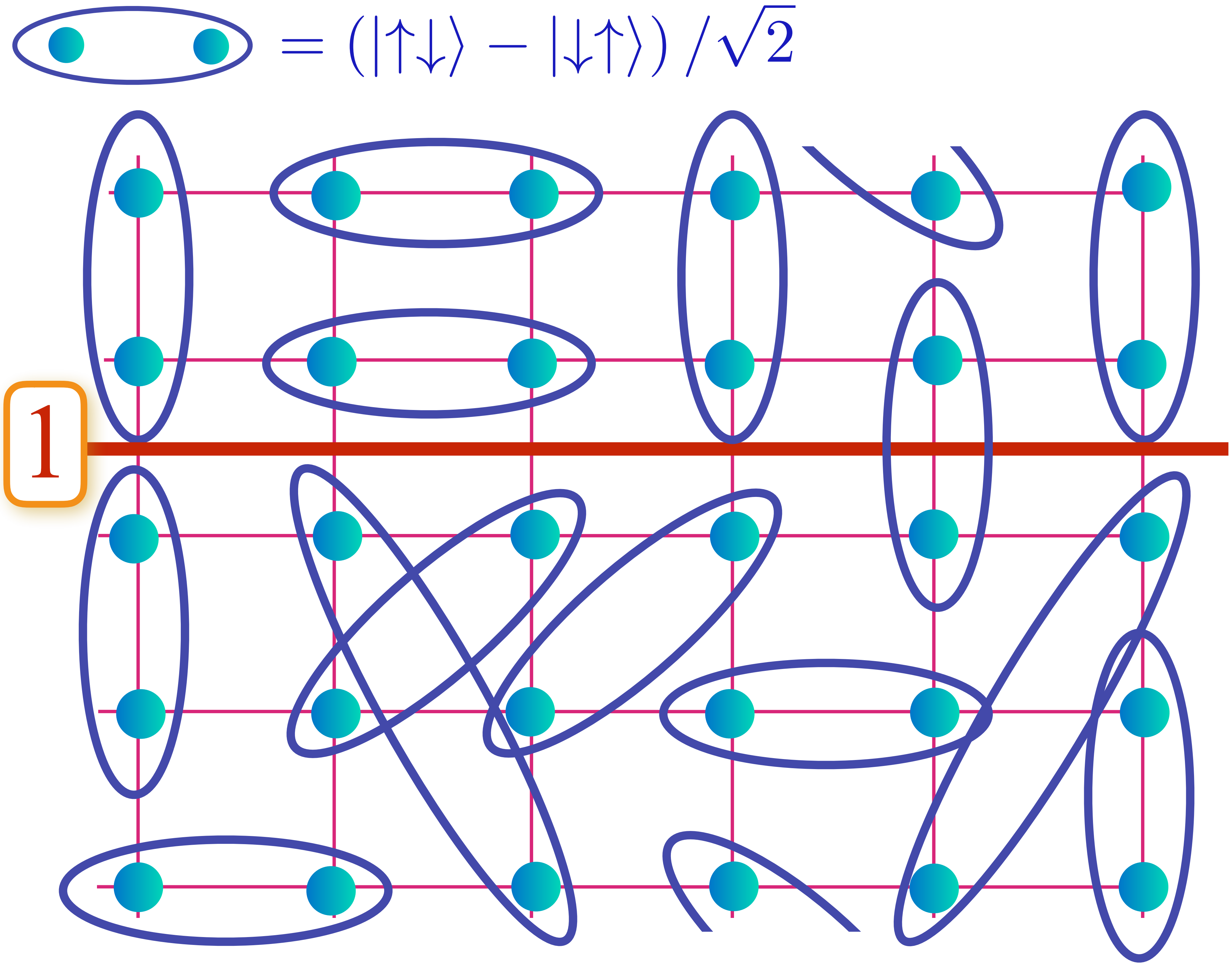

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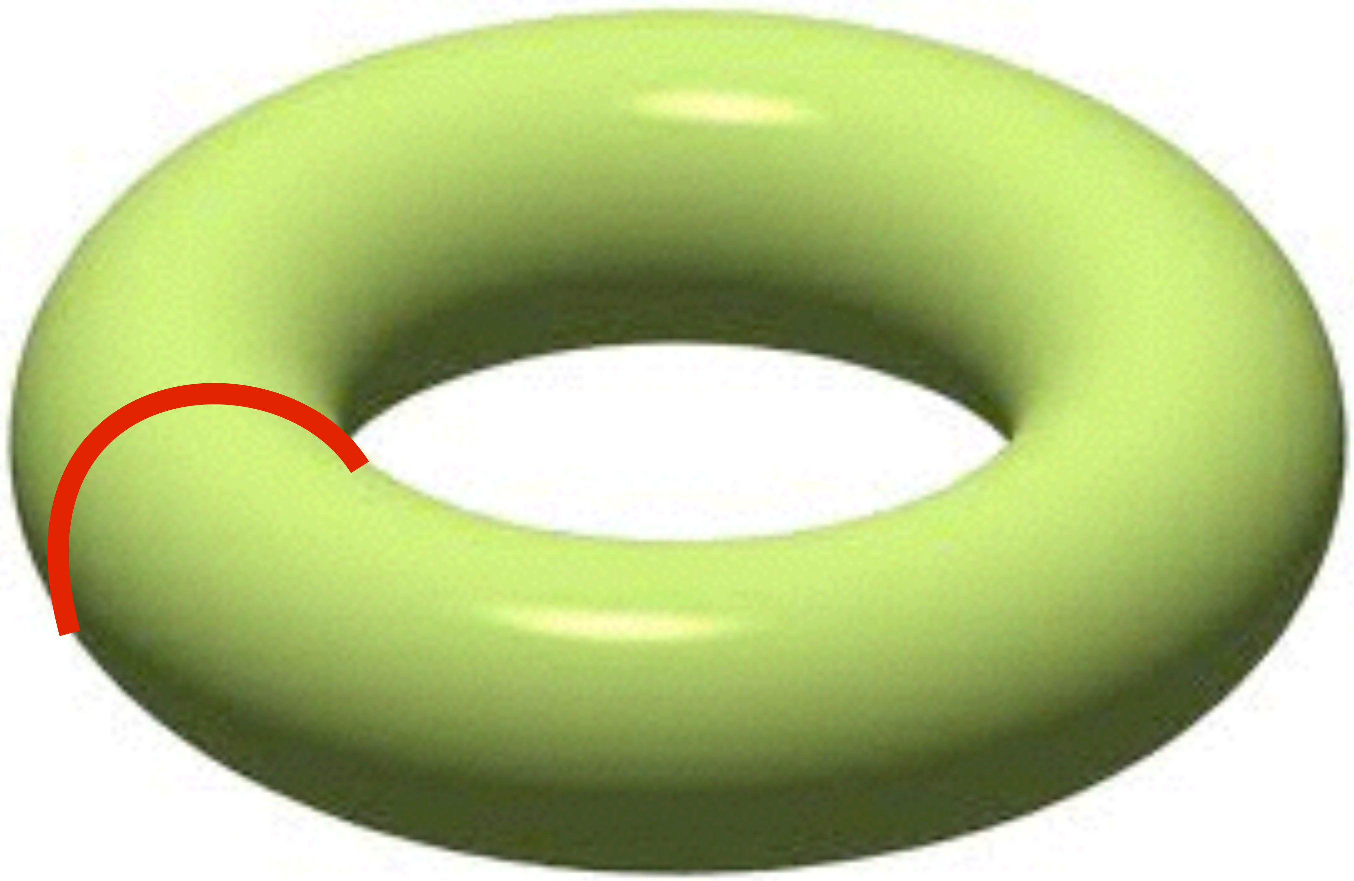

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The sensitivity of the degeneracy to the global topology indicates long-range quantum entanglement

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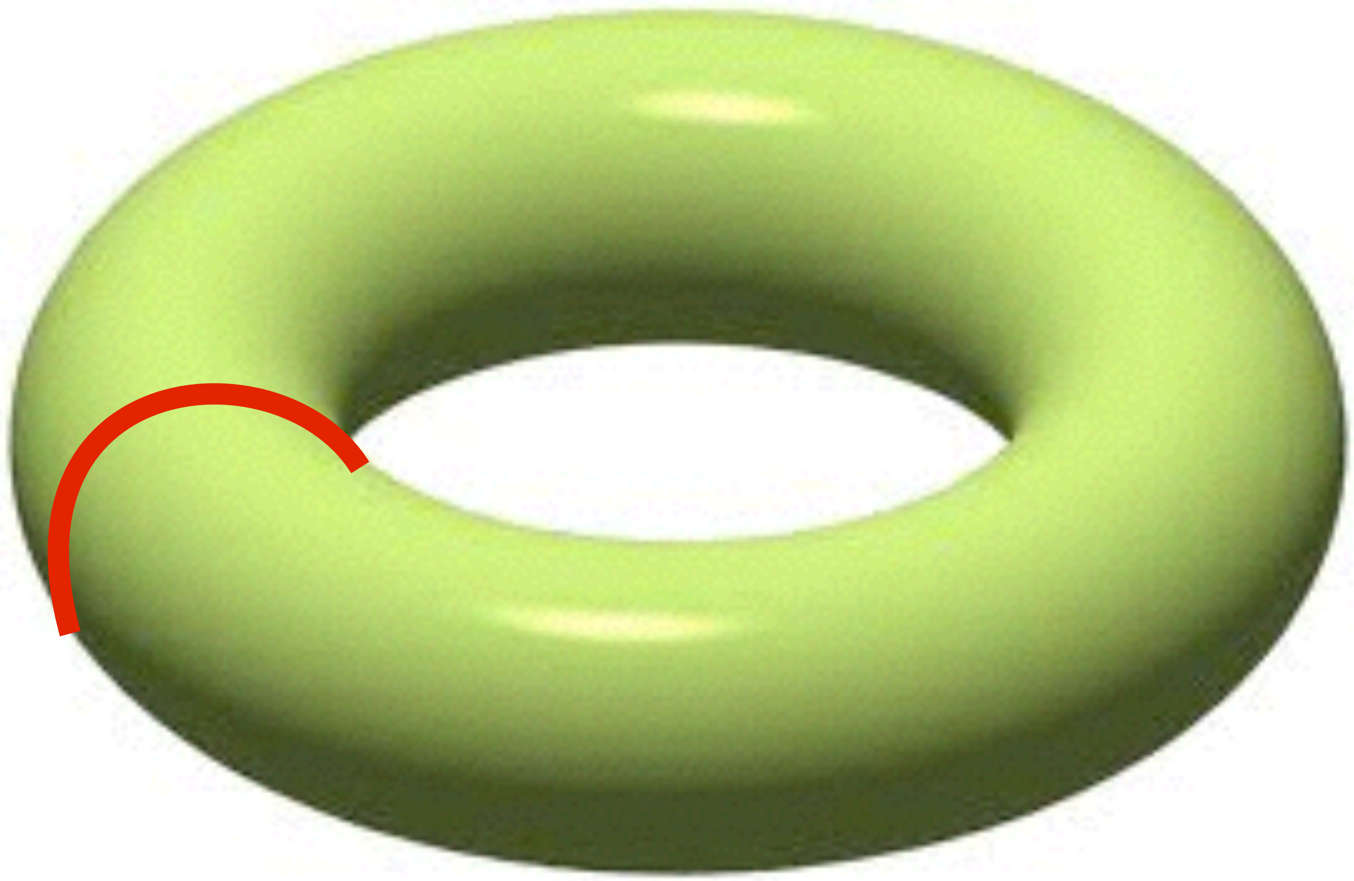

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The degenerate states are conjugate to the flux of an emergent gauge field piercing the cycles of the torus

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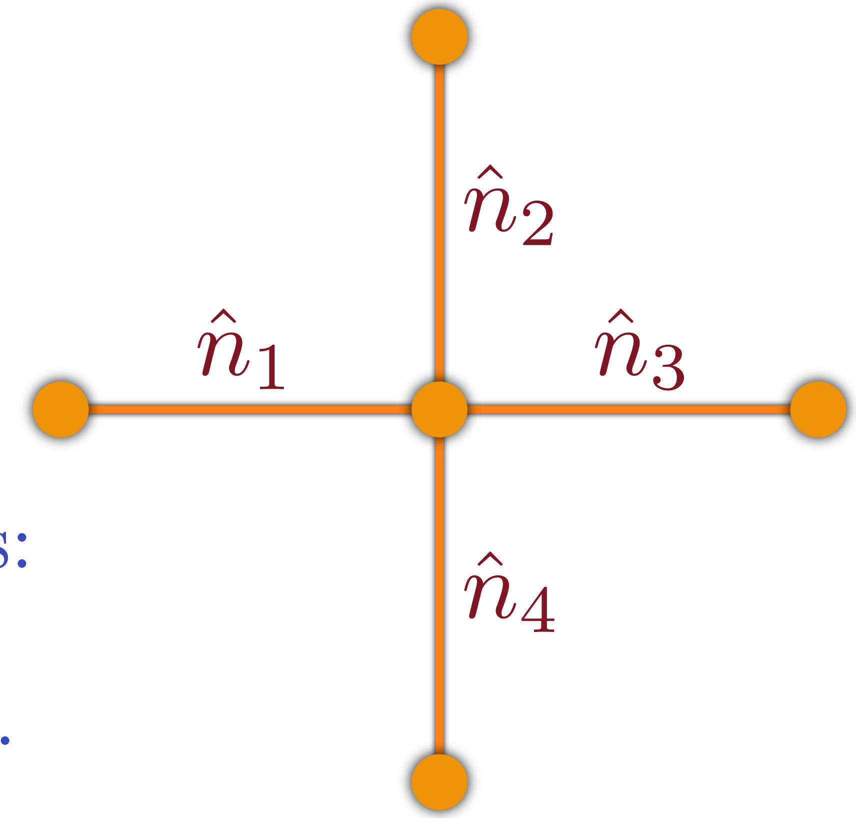
Emergent gauge fields



Local constraint on dimer number operators:

$$\hat{n}_1 + \hat{n}_2 + \hat{n}_3 + \hat{n}_4 = 1.$$

Identify dimer number with an 'electric' field, $\hat{E}_{i\alpha} = (-1)^{i_x+i_y}\,\hat{n}_{i\alpha}$, $(\alpha = x, y)$; the constraint becomes 'Gauss's Law':

$$\Delta_\alpha \hat{E}_{i\alpha} = (-1)^{i_x+i_y}.$$

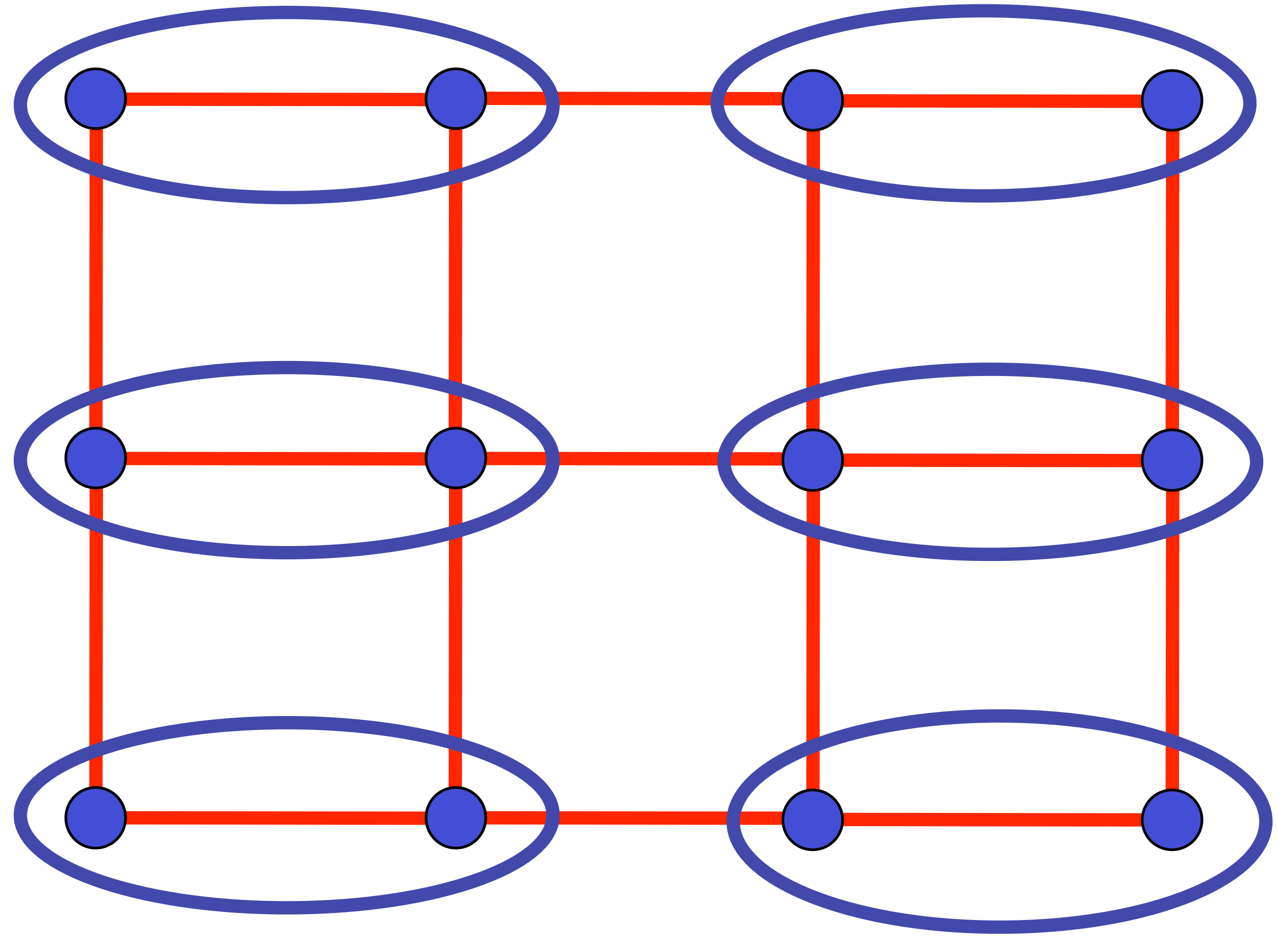
The theory of the dimers is *compact* U(1) quantum electrodynamics in the presence of static background charges. The *compact* theory allows the analog of Dirac's magnetic monopoles as tunneling events/excitations.

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

E. Fradkin and S. A. Kivelson, Mod. Phys. Lett. B **4**, 225 (1990)

Emergent gauge fields

Valence
Bond
Solid



Monopoles proliferate in compact U(1) QED in 2+1 dimensions, and the spin liquidd ultimately confines into a valence bond solid, except at *deconfined quantum critical points*. Consequently, emergent Maxwell electromagnetism is ultimately stable only at such critical points (or phases).

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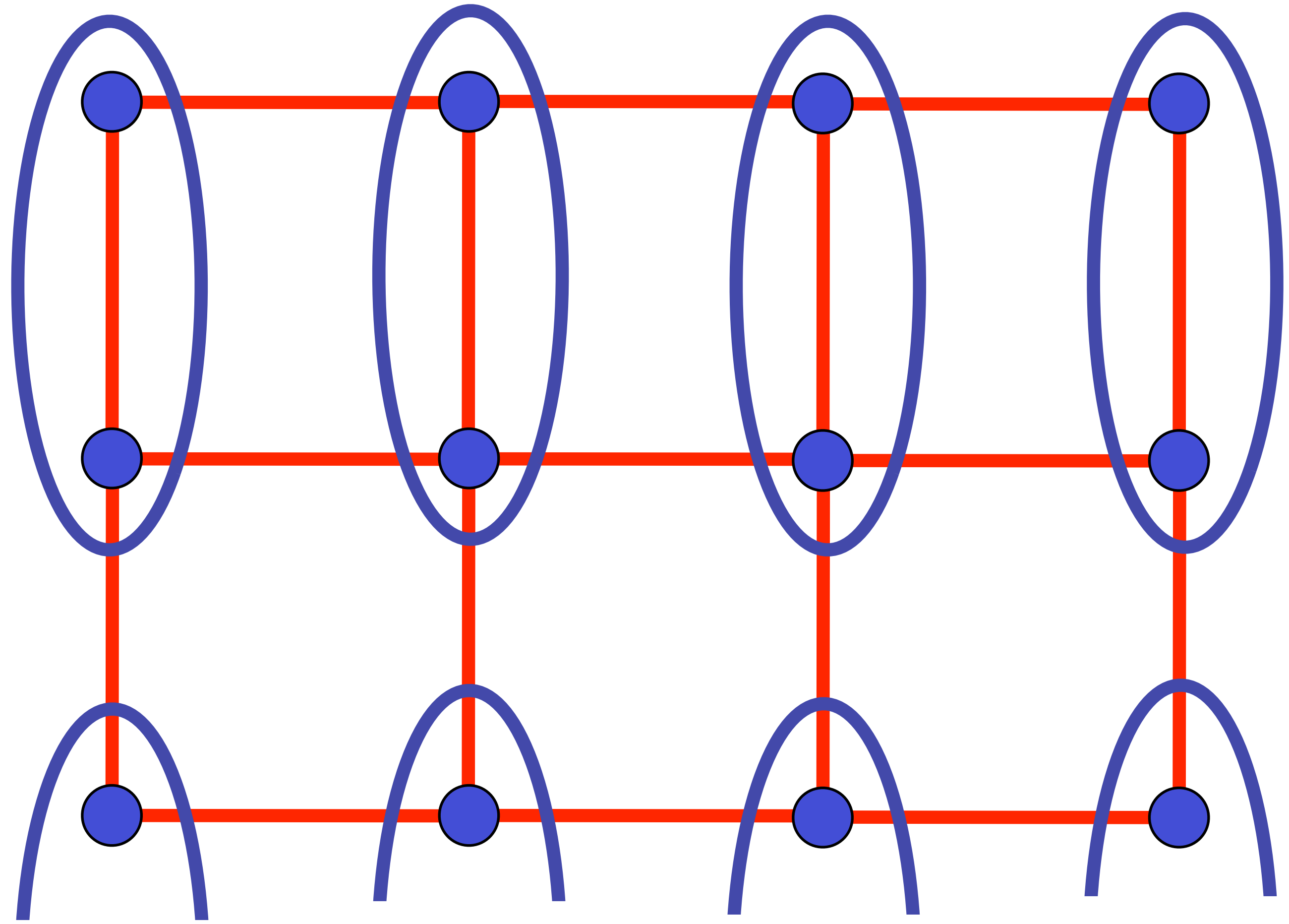

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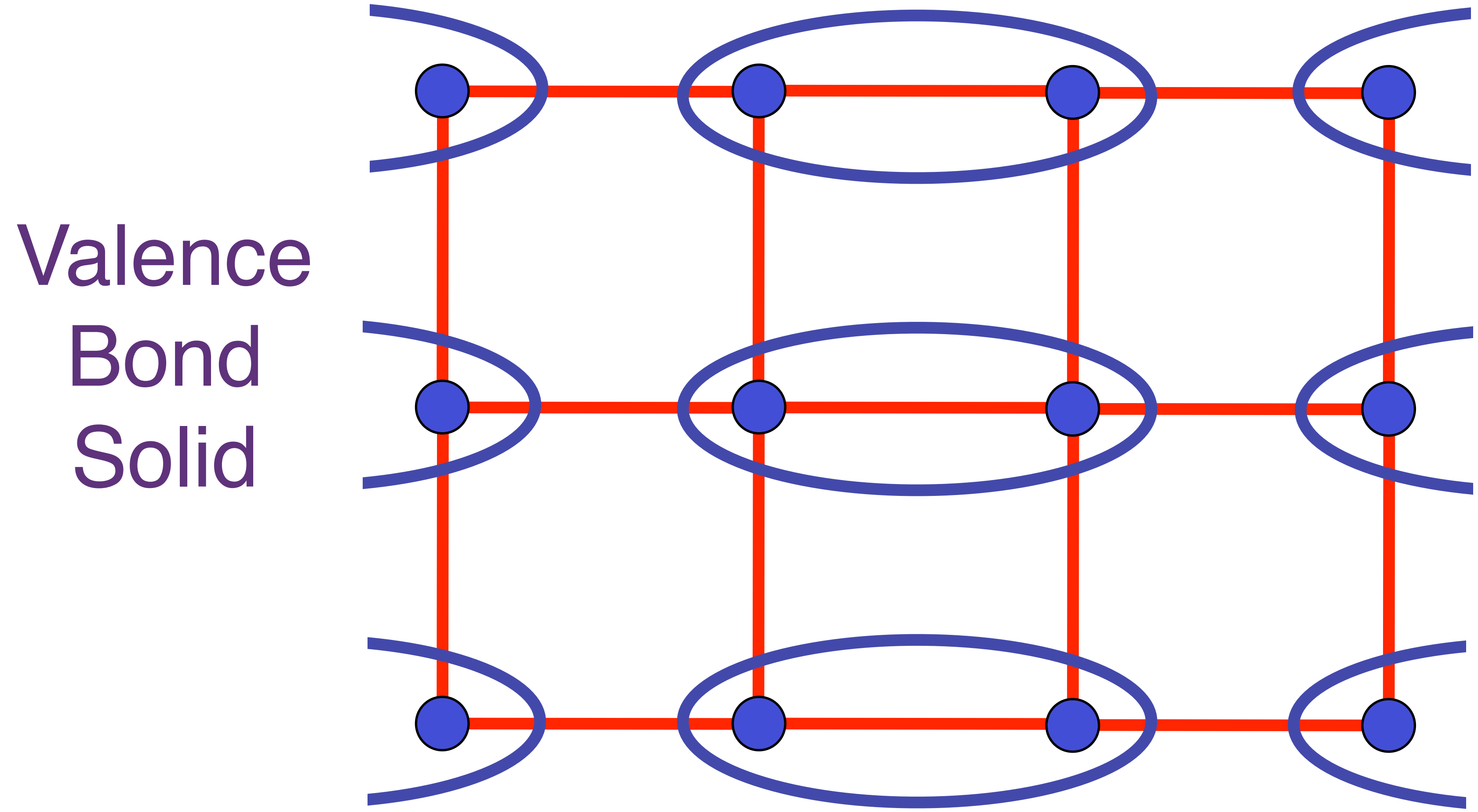

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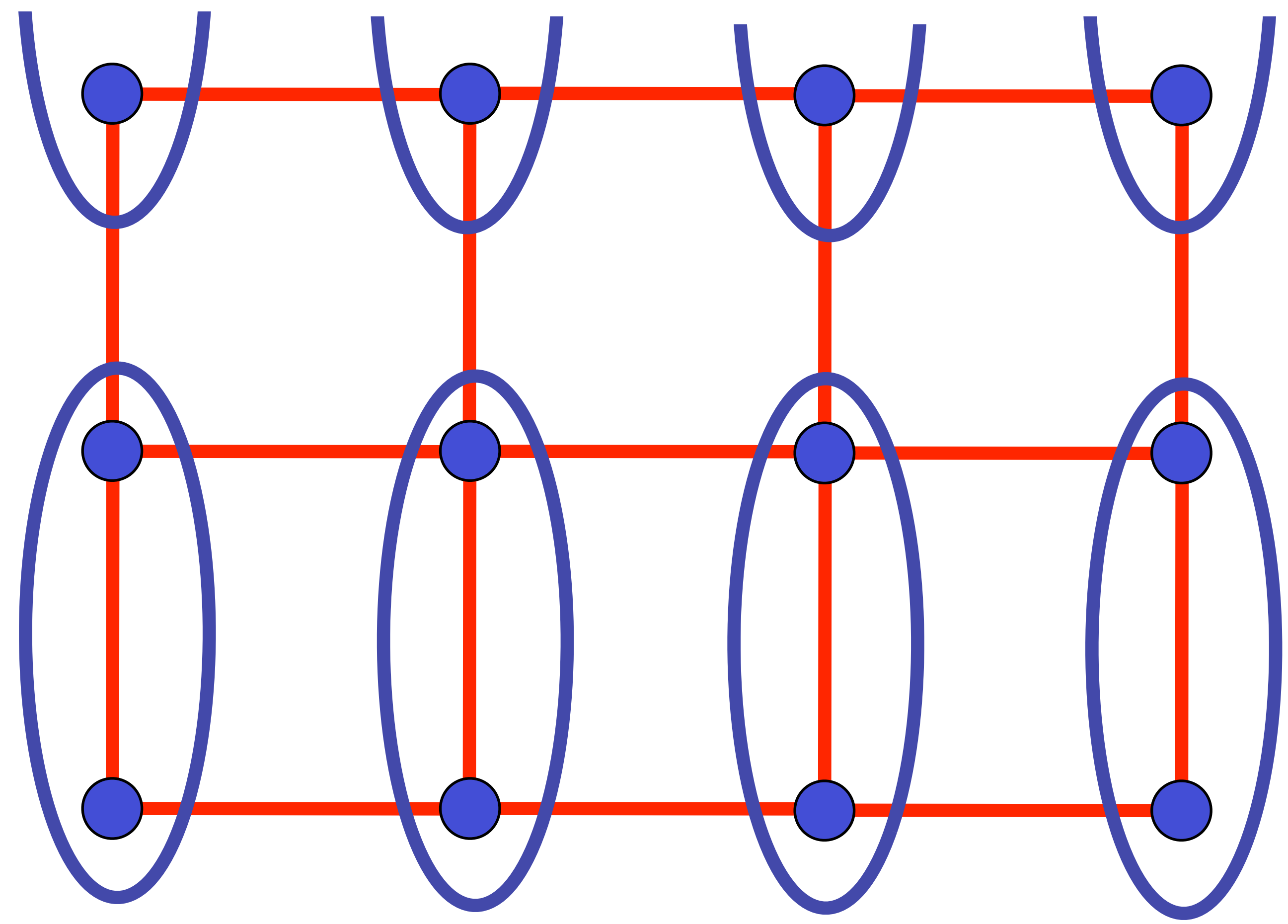
Emergent gauge fields

Valence Bond Solid



Monopoles proliferate in compact U(1) QED in 2+1 dimensions, and the spin liquidd ultimately confines into a valence bond solid, except at *deconfined quantum critical points*. Consequently, emergent Maxwell electromagnetism is ultimately stable only at such critical points (or phases).

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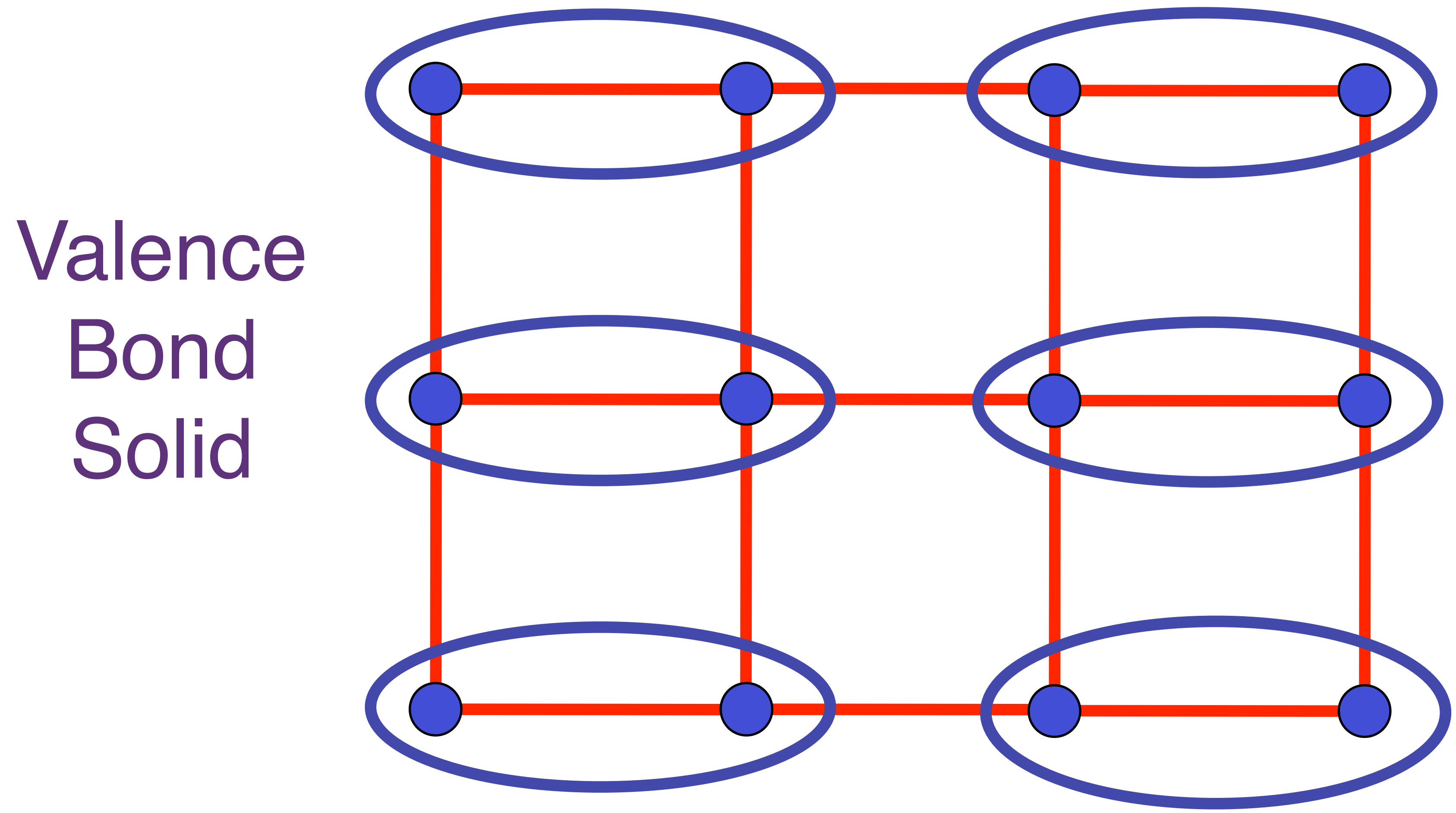

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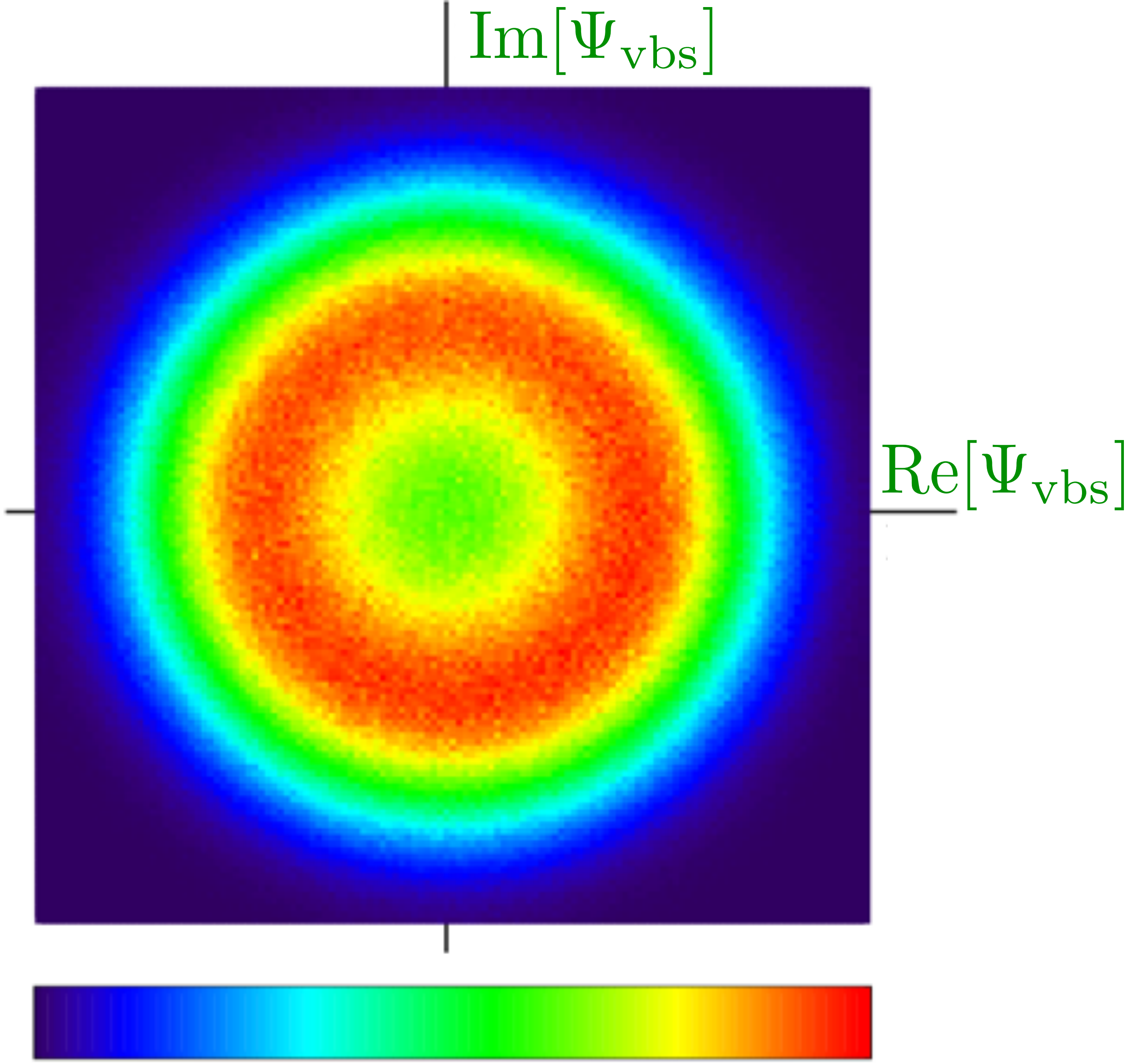
Emergent gauge fields

Emergent U(1) symmetry of distribution function of valence bond solid order parameter $\Psi_{\rm vbs}$ at the critical point of a spin model is evidence for the emergence of a gapless photon.

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007).



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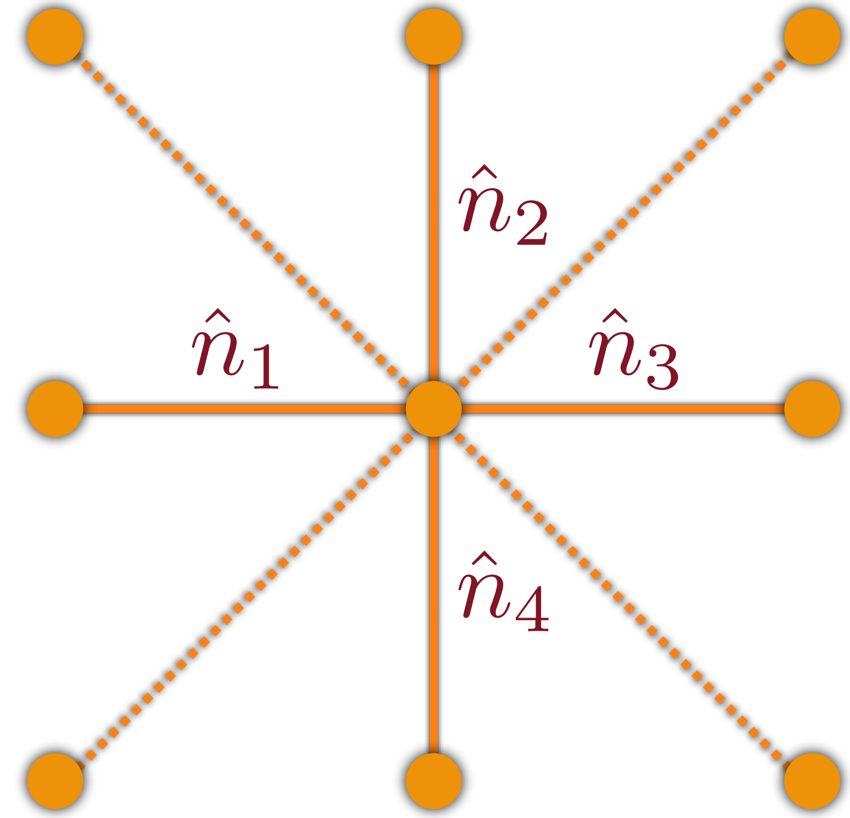


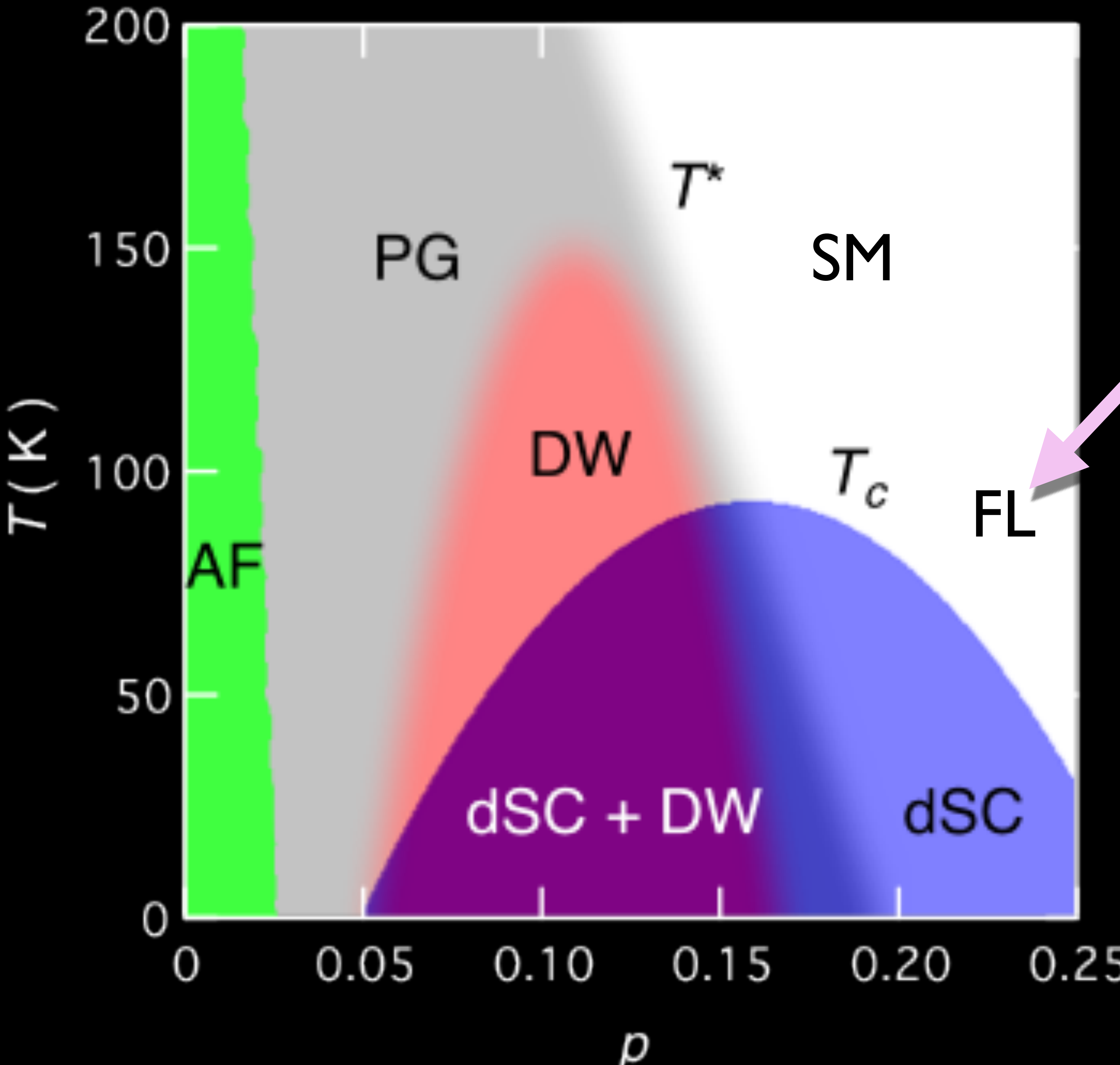
Including dimers connecting the same sublattice leads to a $\mathbb{Z}_2$ gauge theory in the presence of Berry phases of static background charges. This has a stable deconfined phase in 2+1 dimensions. By varying parameters it can undergoes a confinement transition to a valence bond solid, described by a frustrated Ising model.

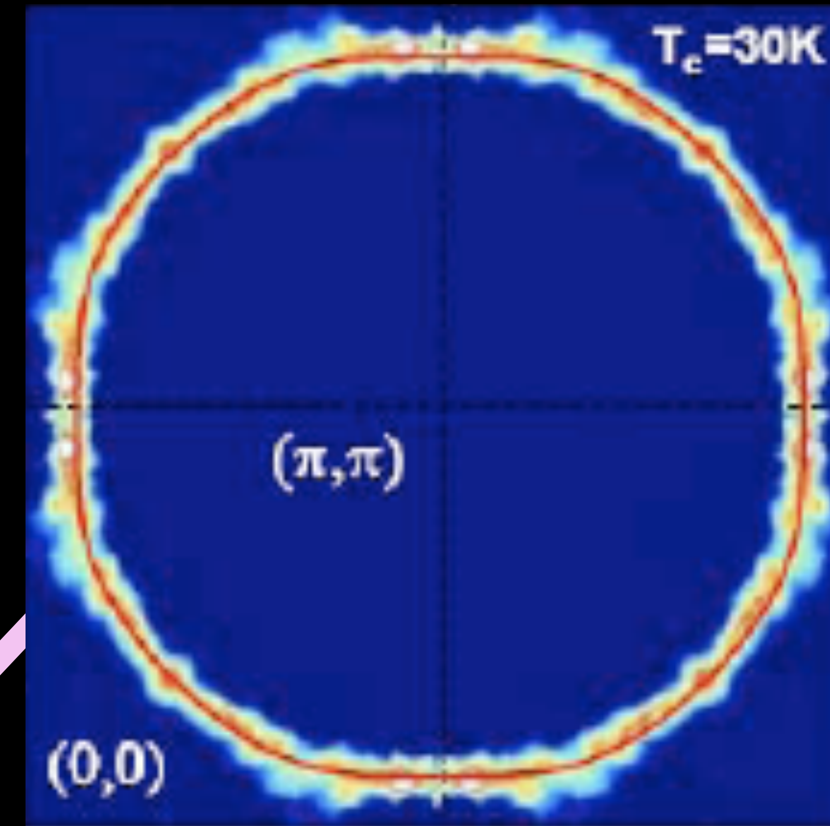
R. A. Jalabert and S. Sachdev, Phys. Rev. B **44**, 686 (1991)

S. Sachdev and M. Vojta, J. Phys. Soc. Jpn **69**, Supp. B, 1 (1999)

1. Emergent gauge fields and long-range entanglement in insulators

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



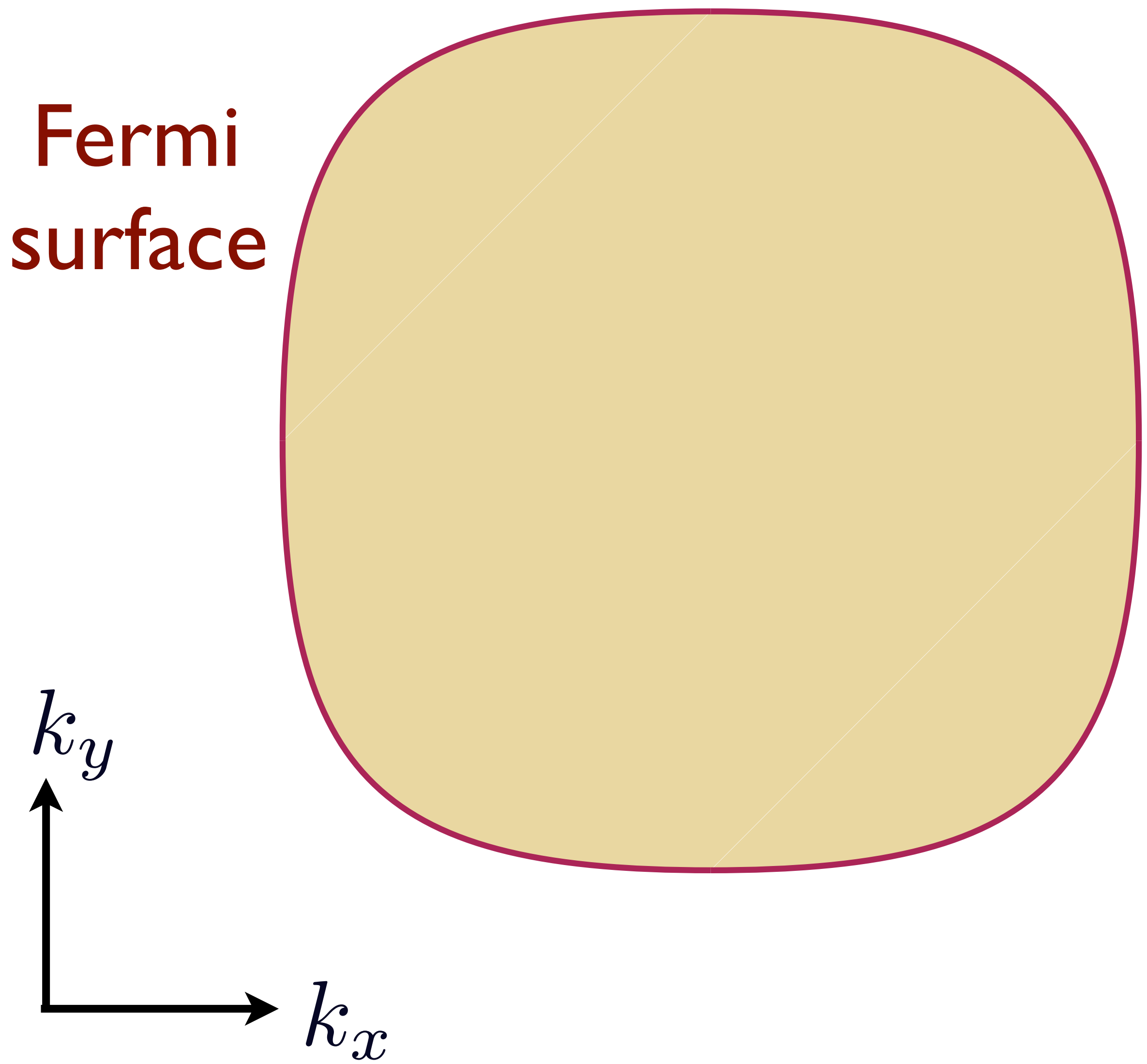


A conventional metal:
the Fermi liquid

1. Emergent gauge fields and long-range entanglement in insulators

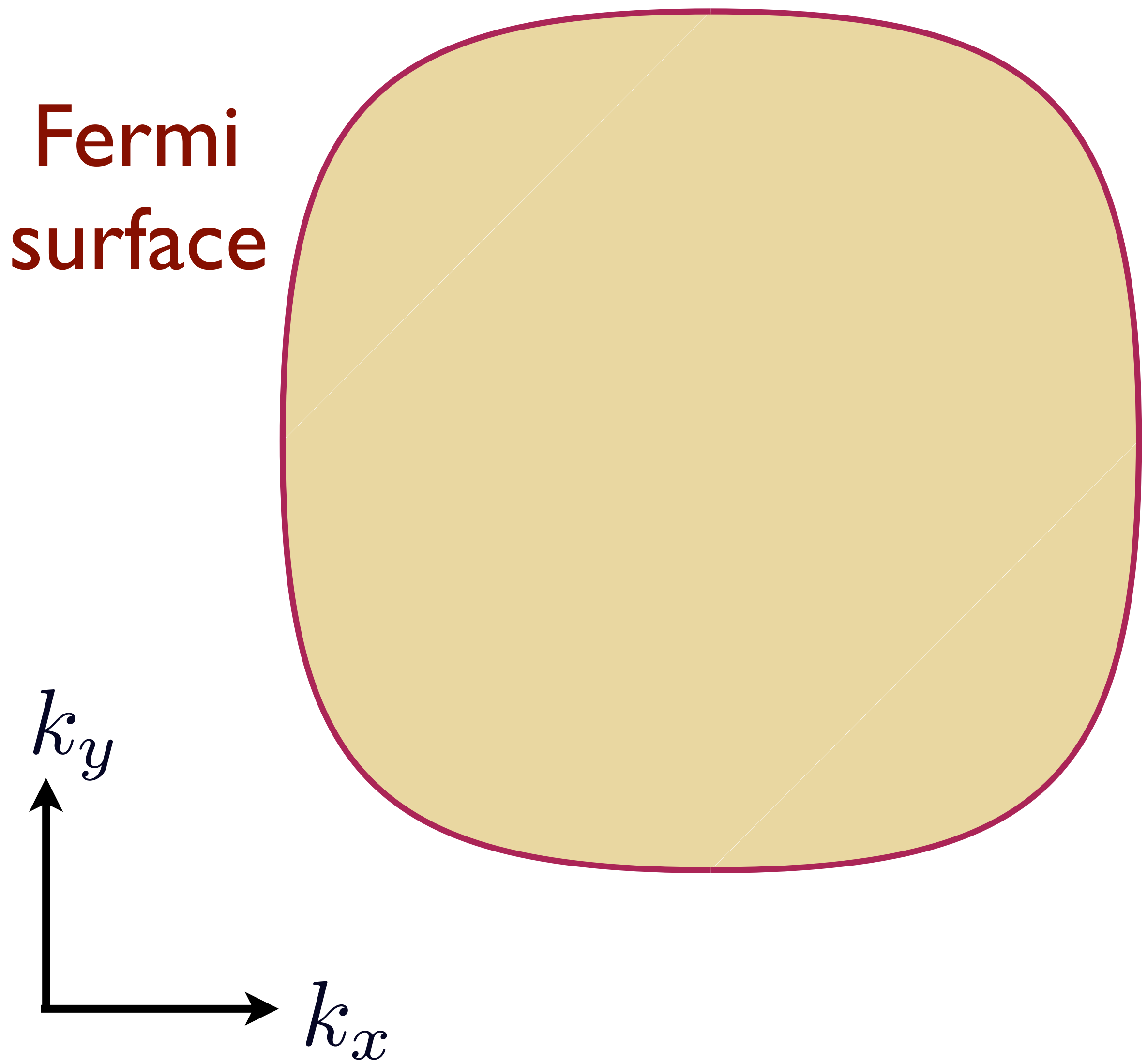
2. Theory of ordinary metals: Fermi liquids (FL)
   (a) Quasiparticles
   (b) Luttinger theorem for volume enclosed by Fermi surface

Ordinary metals: the Fermi liquid



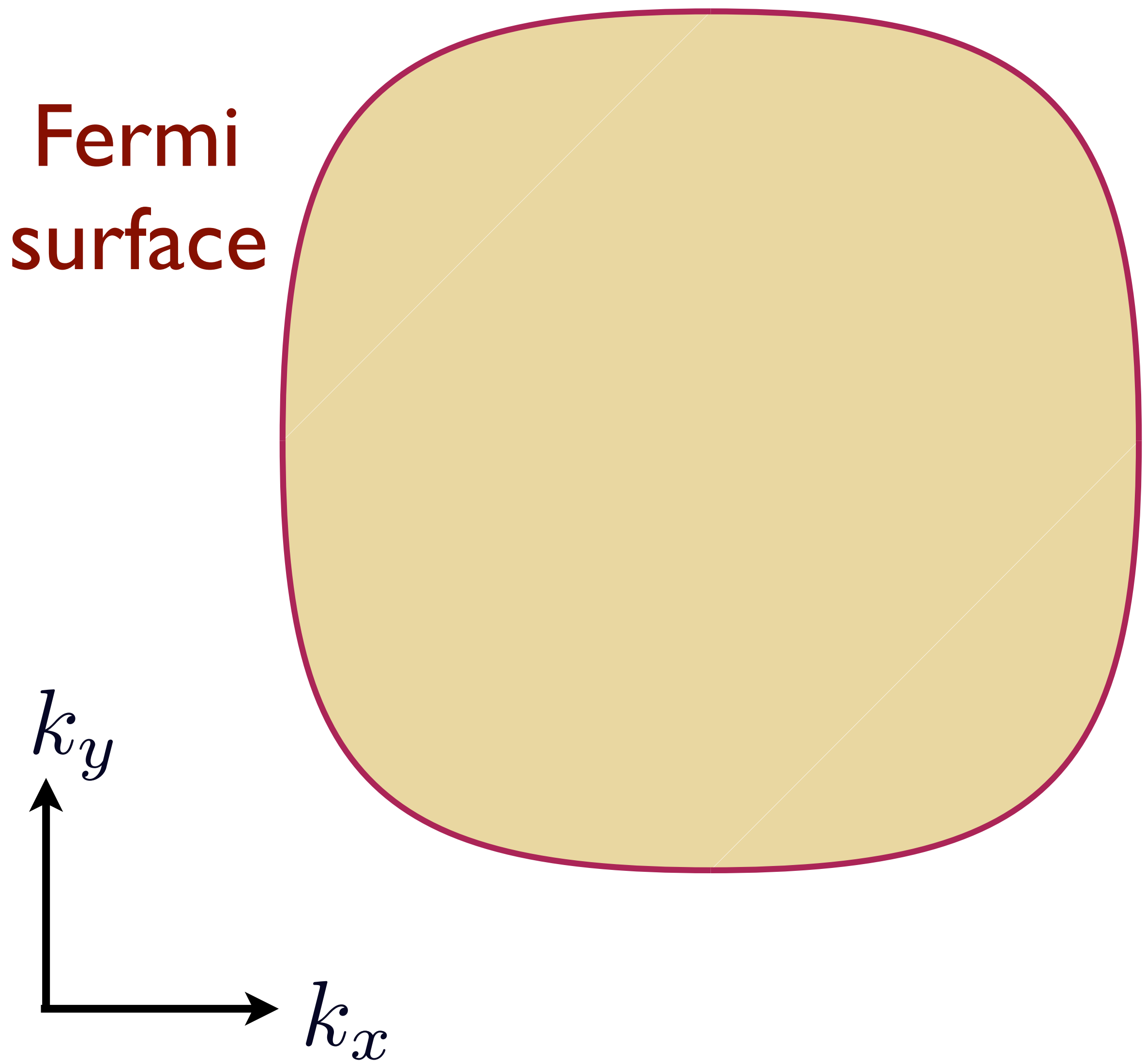
- Fermi surface separates empty and occupied states in momentum space.

Ordinary metals: the Fermi liquid

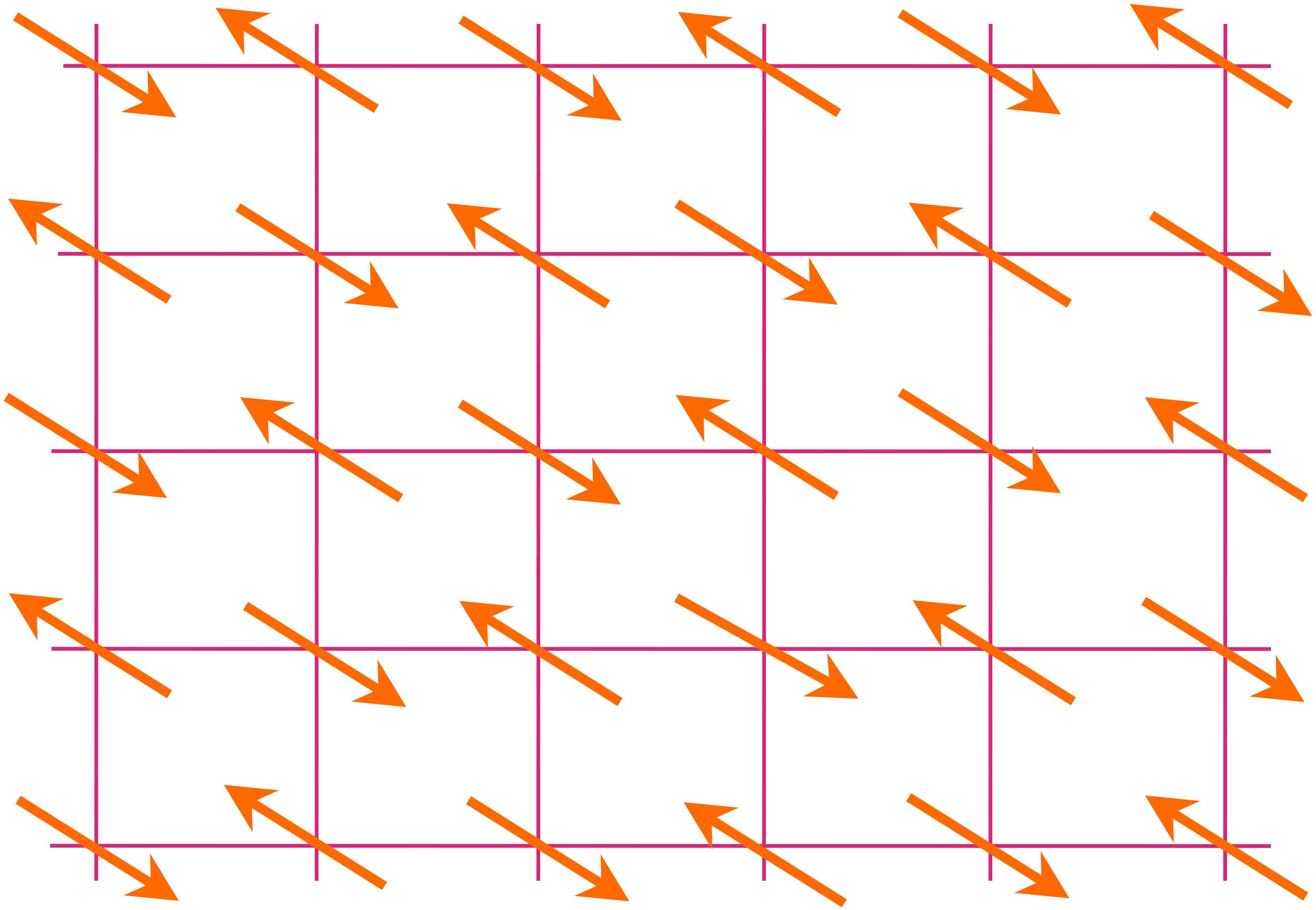


- Fermi surface separates empty and occupied states in momentum space.
- *Luttinger Theorem*: volume (area) enclosed by Fermi surface = the electron density.

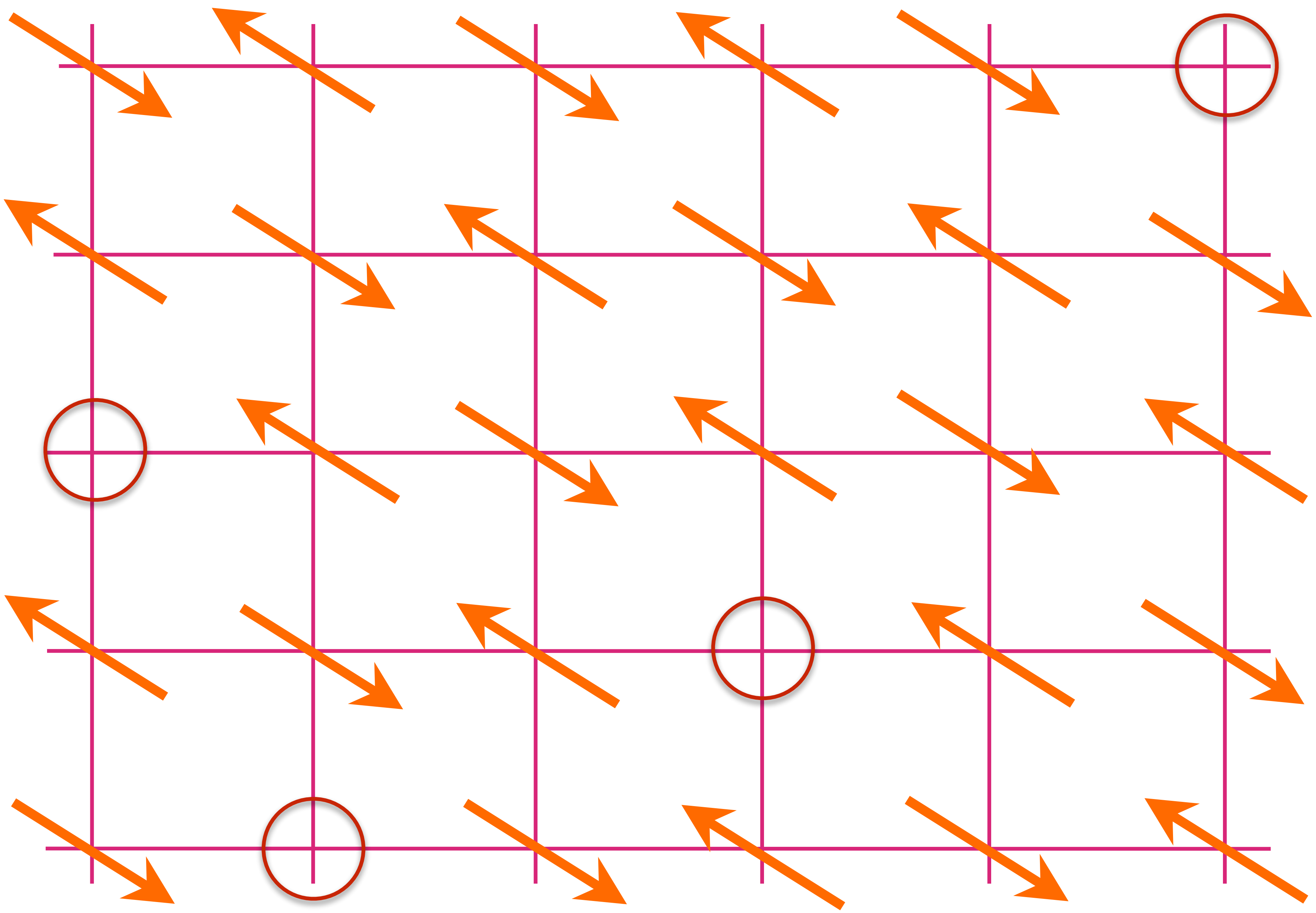
Ordinary metals: the Fermi liquid



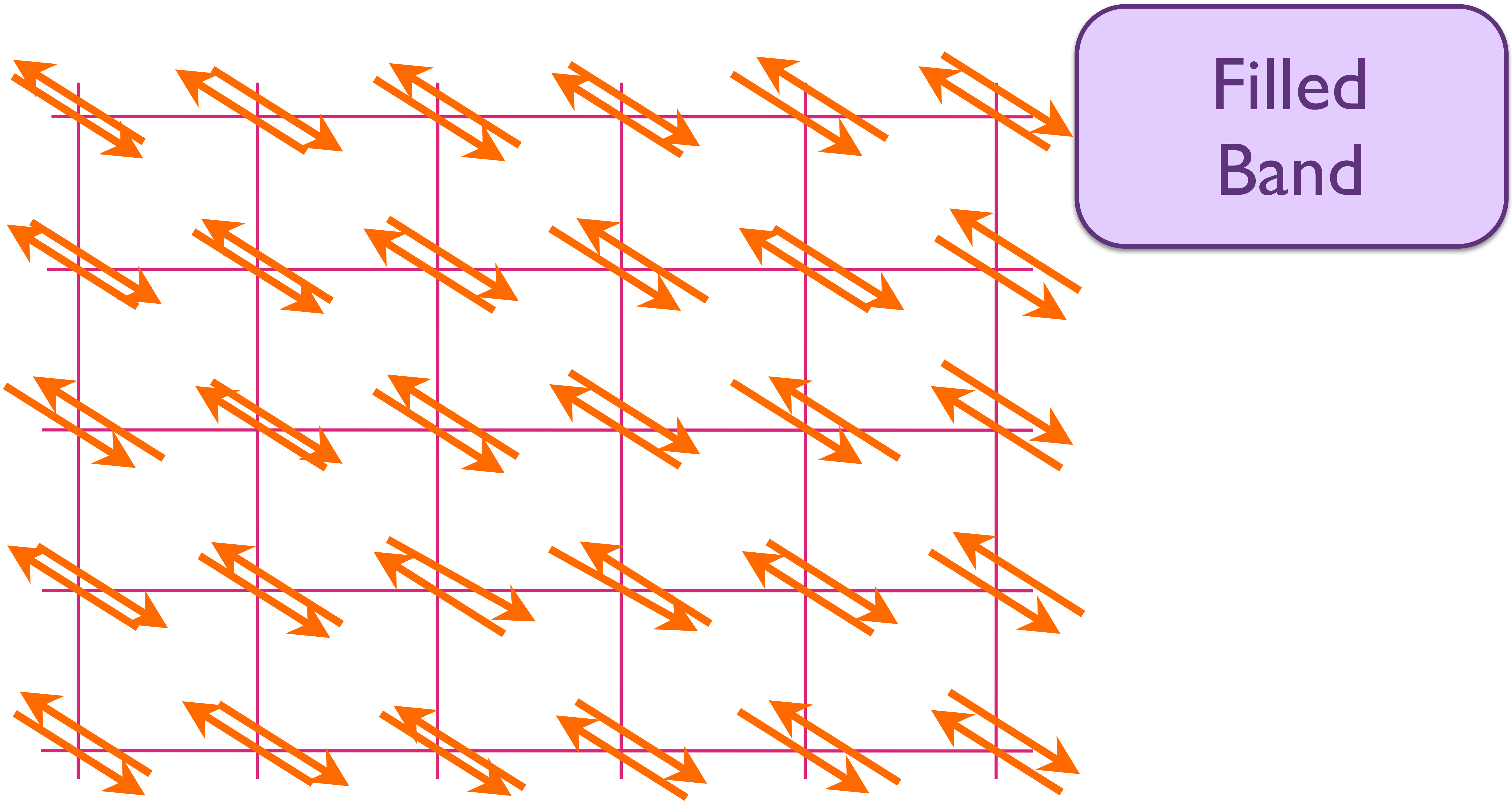
- Fermi surface separates empty and occupied states in momentum space.
- *Luttinger Theorem*: volume (area) enclosed by Fermi surface = the electron density.
- Hall co-efficient $R_H = -1/((\text{Fermi volume}) \times e)$.

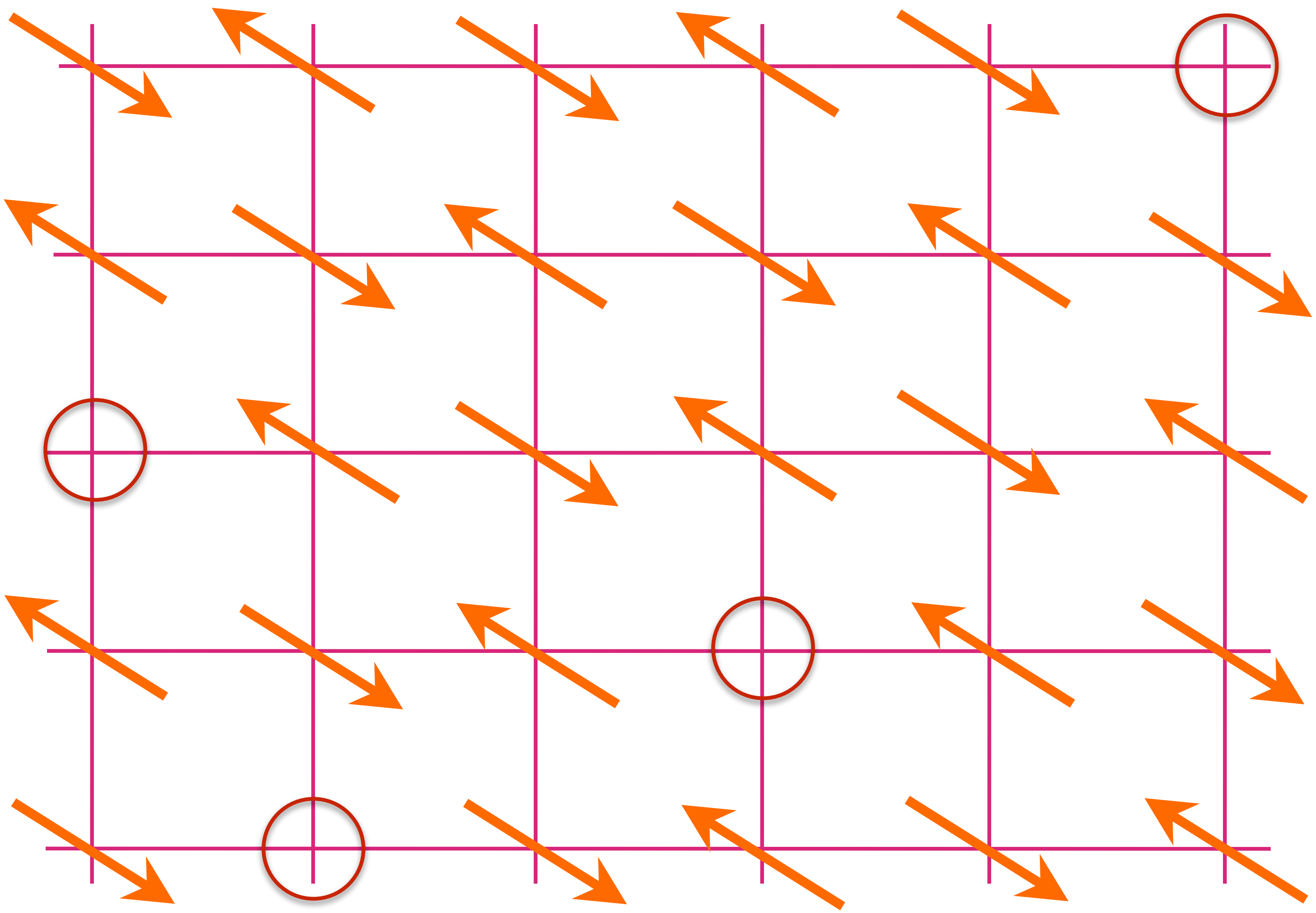


"Undoped" Anti-ferromagnet



Anti-ferromagnet with p holes per square

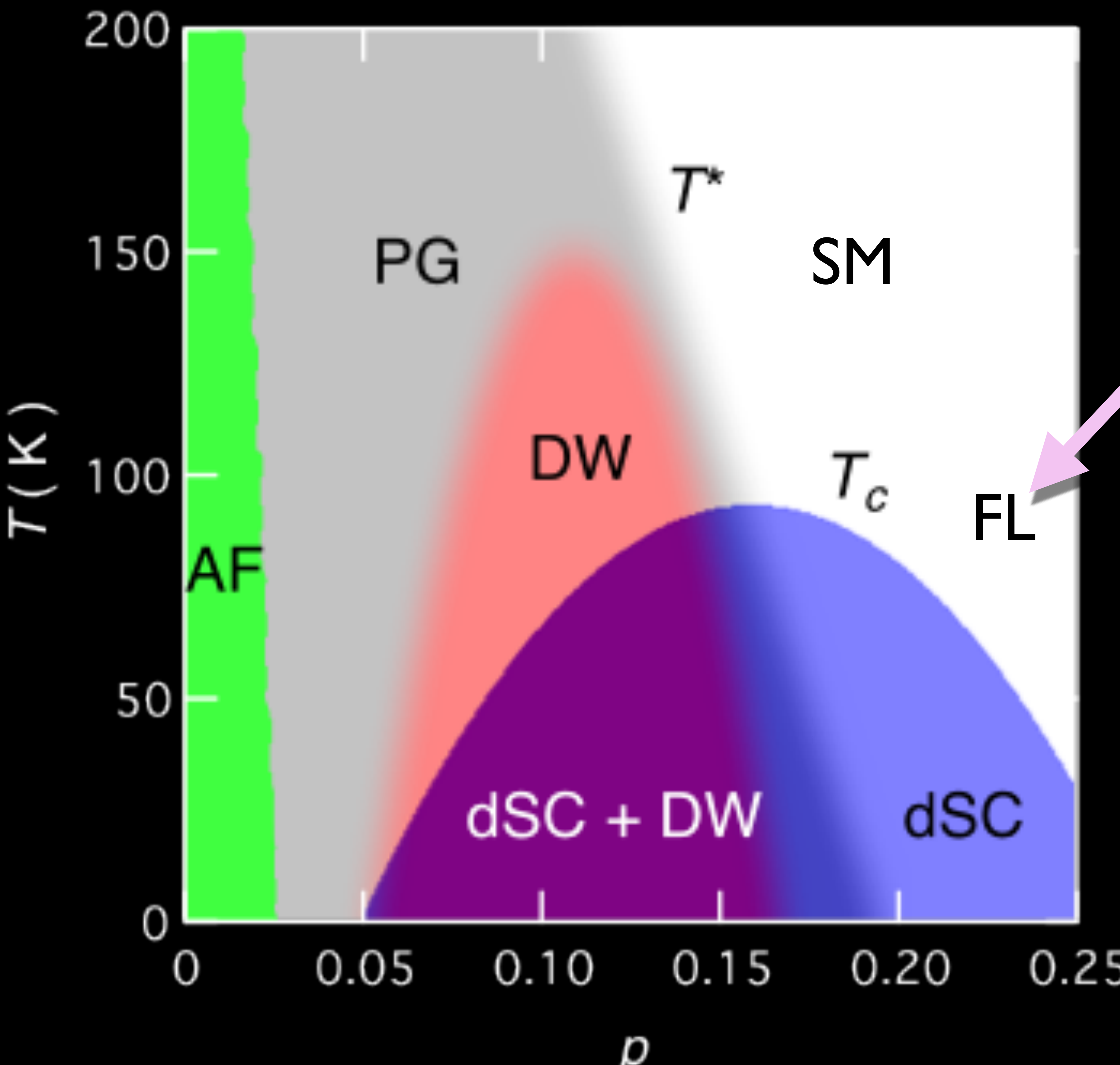

Filled
Band

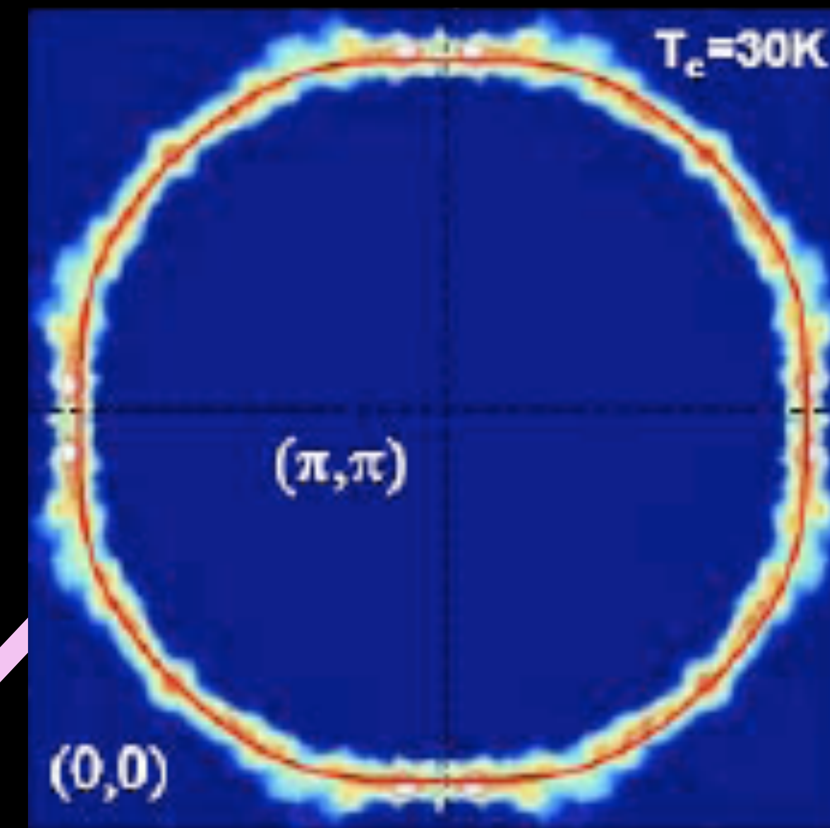


Anti-ferromagnet with p holes per square

But relative to the band insulator, there are $1+p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1+p$

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

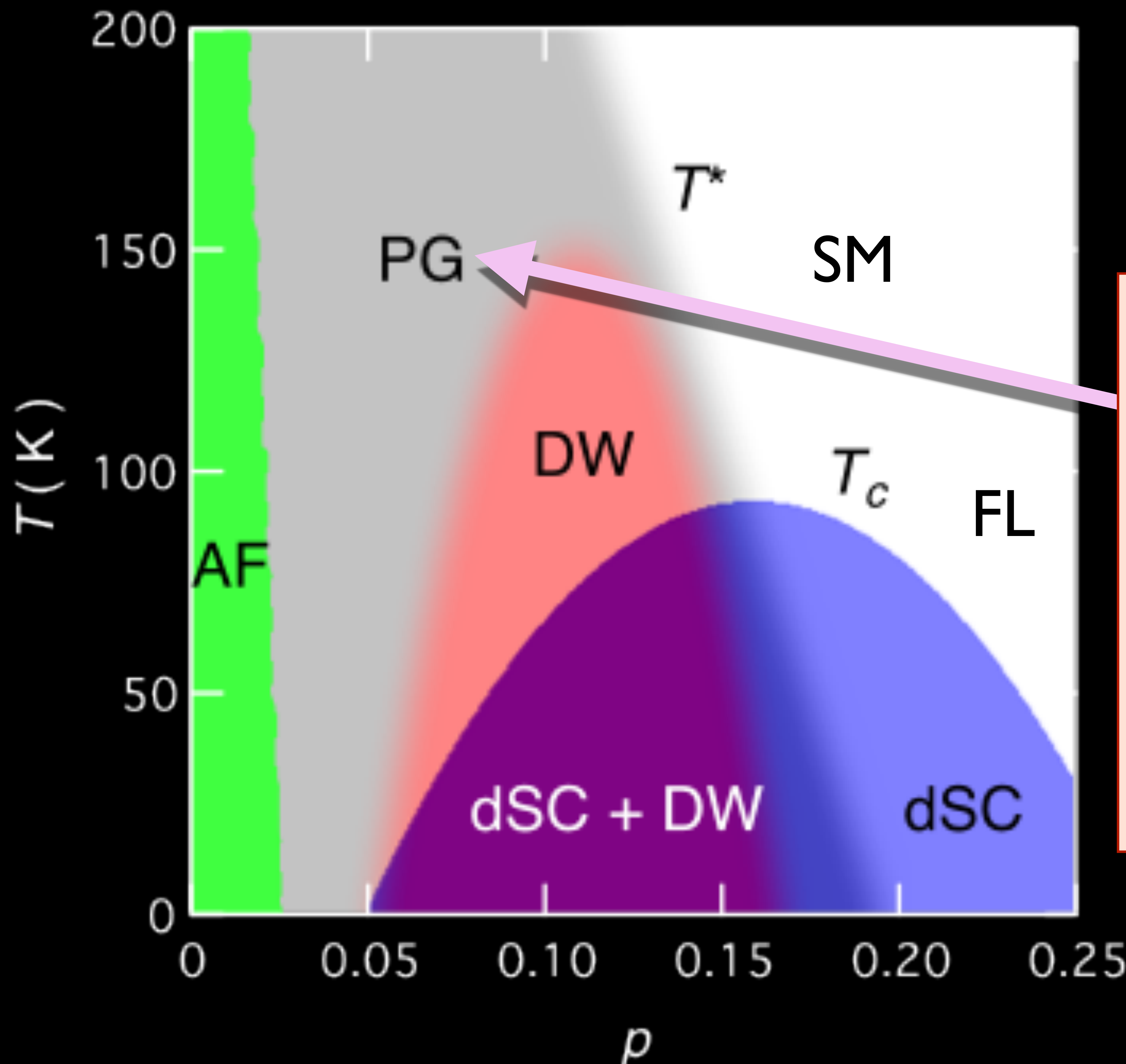




Fermi liquid

Area enclosed by Fermi surface =1+p

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



The PG regime behaves in many respects like a Fermi liquid, but with a Fermi surface size of p and not $1+p$

1. Emergent gauge fields and long-range entanglement in insulators

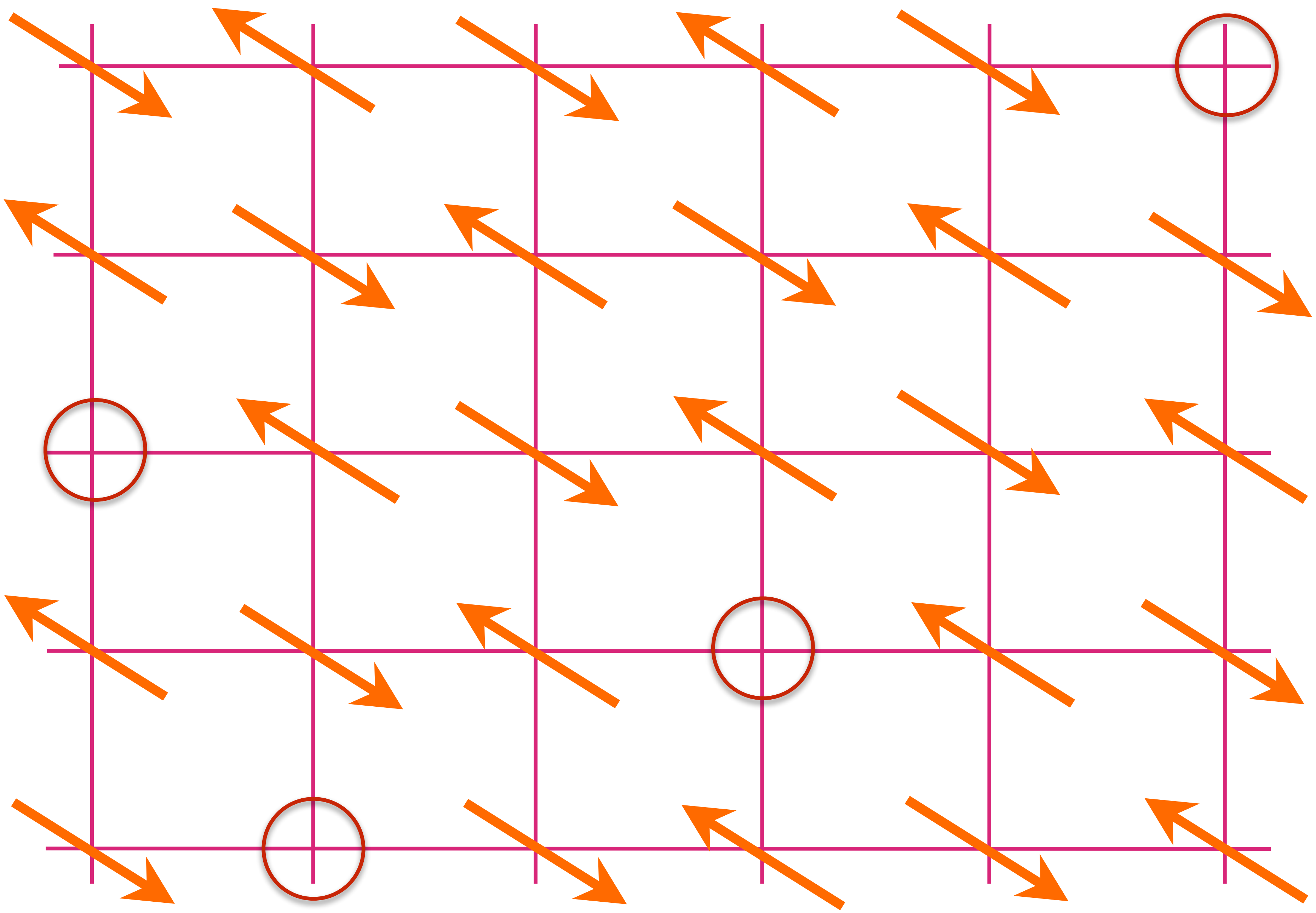
2. Theory of ordinary metals: Fermi liquids (FL)

   (a) Quasiparticles

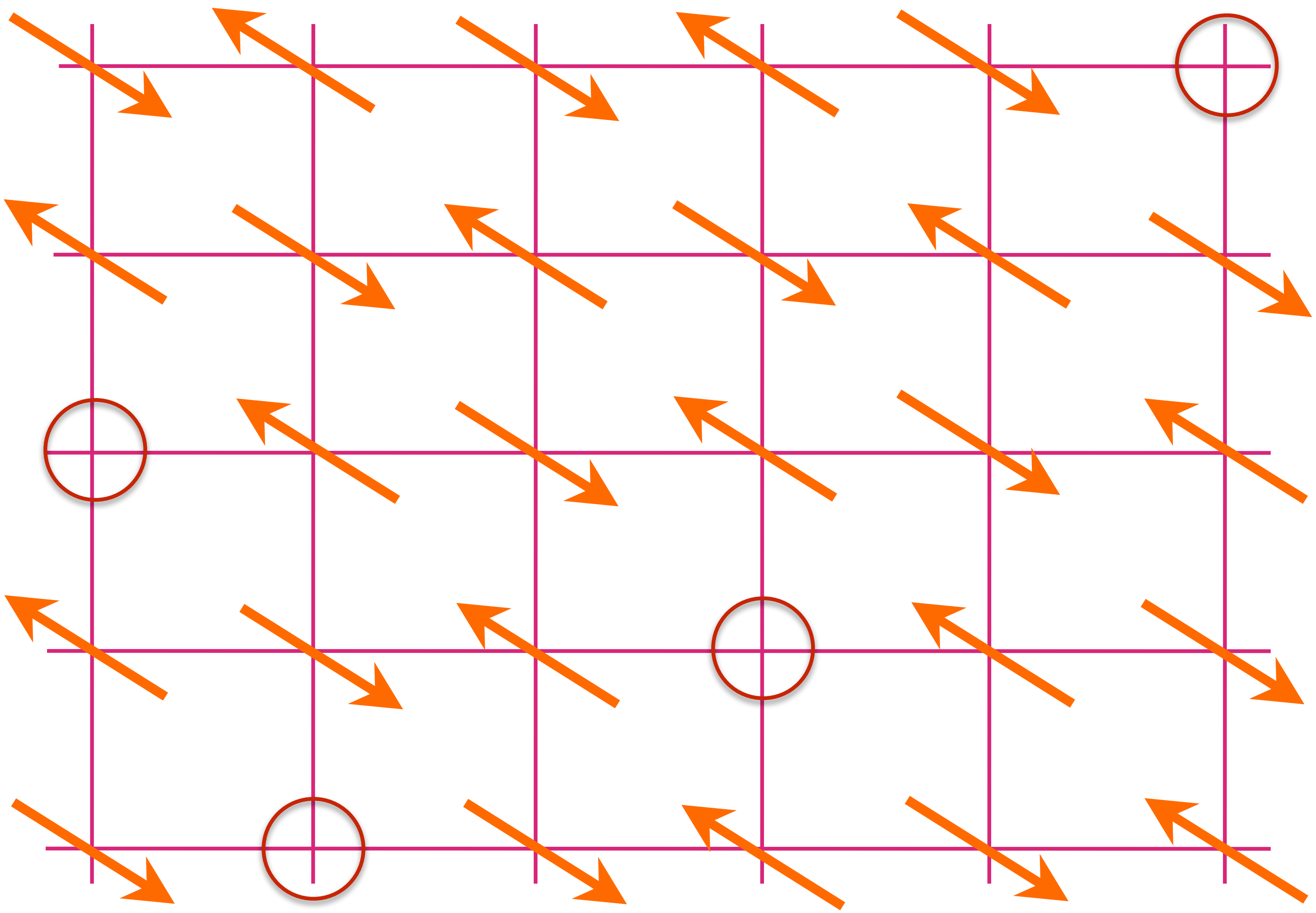
   (b) Luttinger theorem for volume enclosed by Fermi surface

3. Fractionalized Fermi liquids (FL*)

   Quasiparticles with a non-Luttinger volume, and emergent gauge fields



Anti-
ferromagnet
with p holes
per square

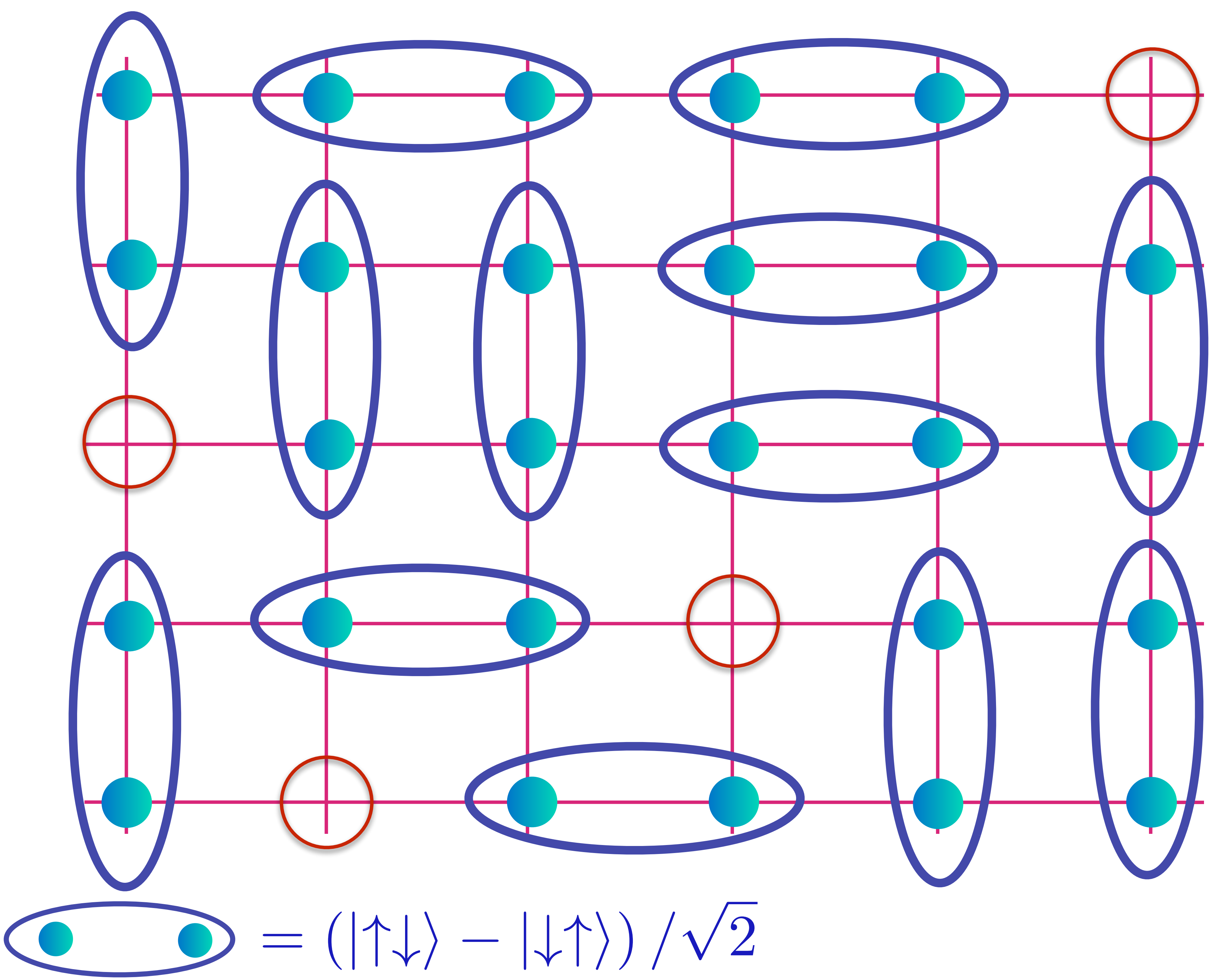


Anti-ferromagnet with p holes per square

Can we get a Fermi surface of size p?
(and full square lattice symmetry)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but this is not visible in electron photo-emission

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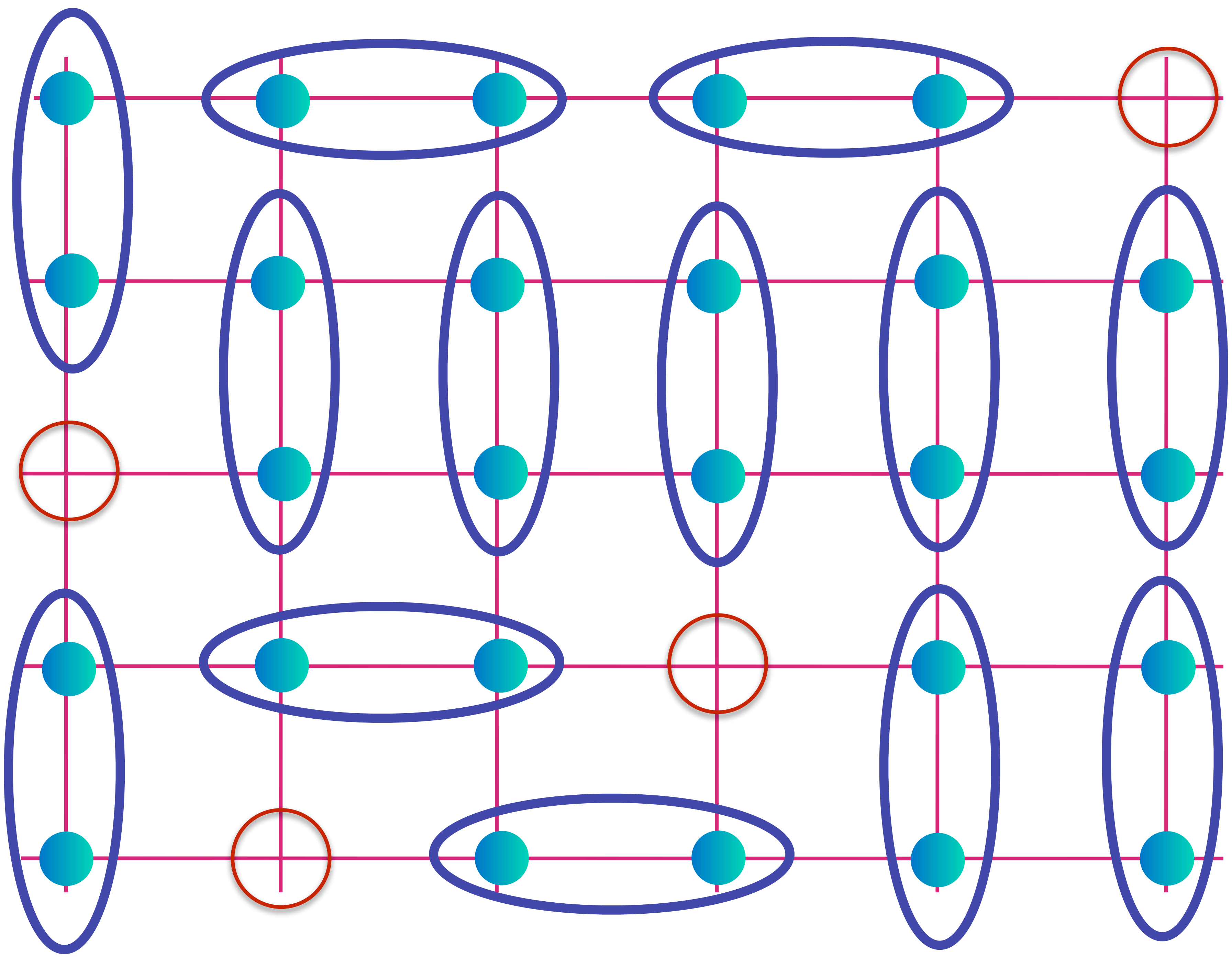

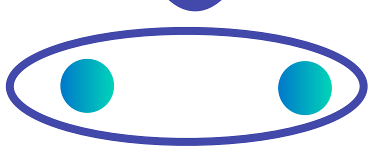

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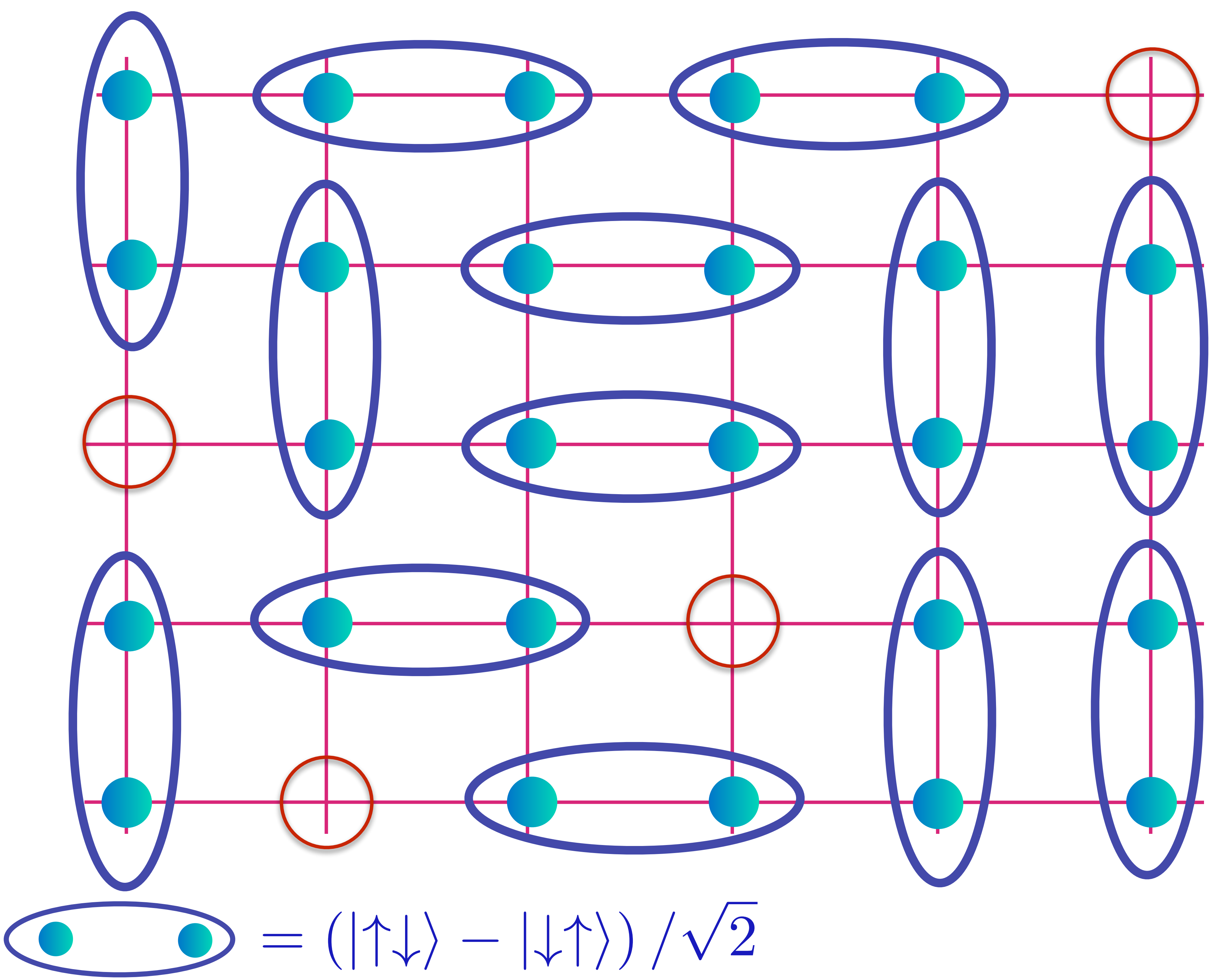

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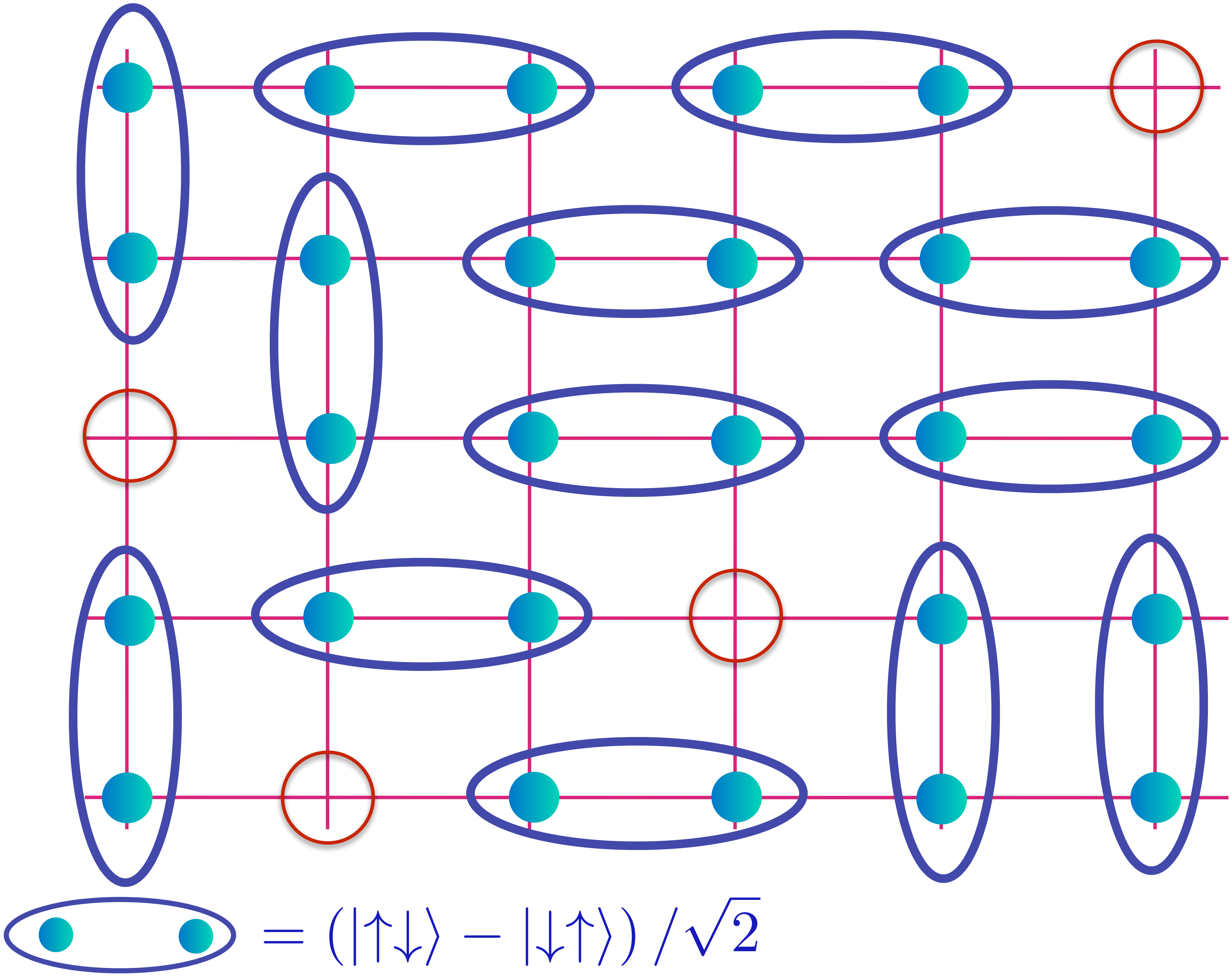

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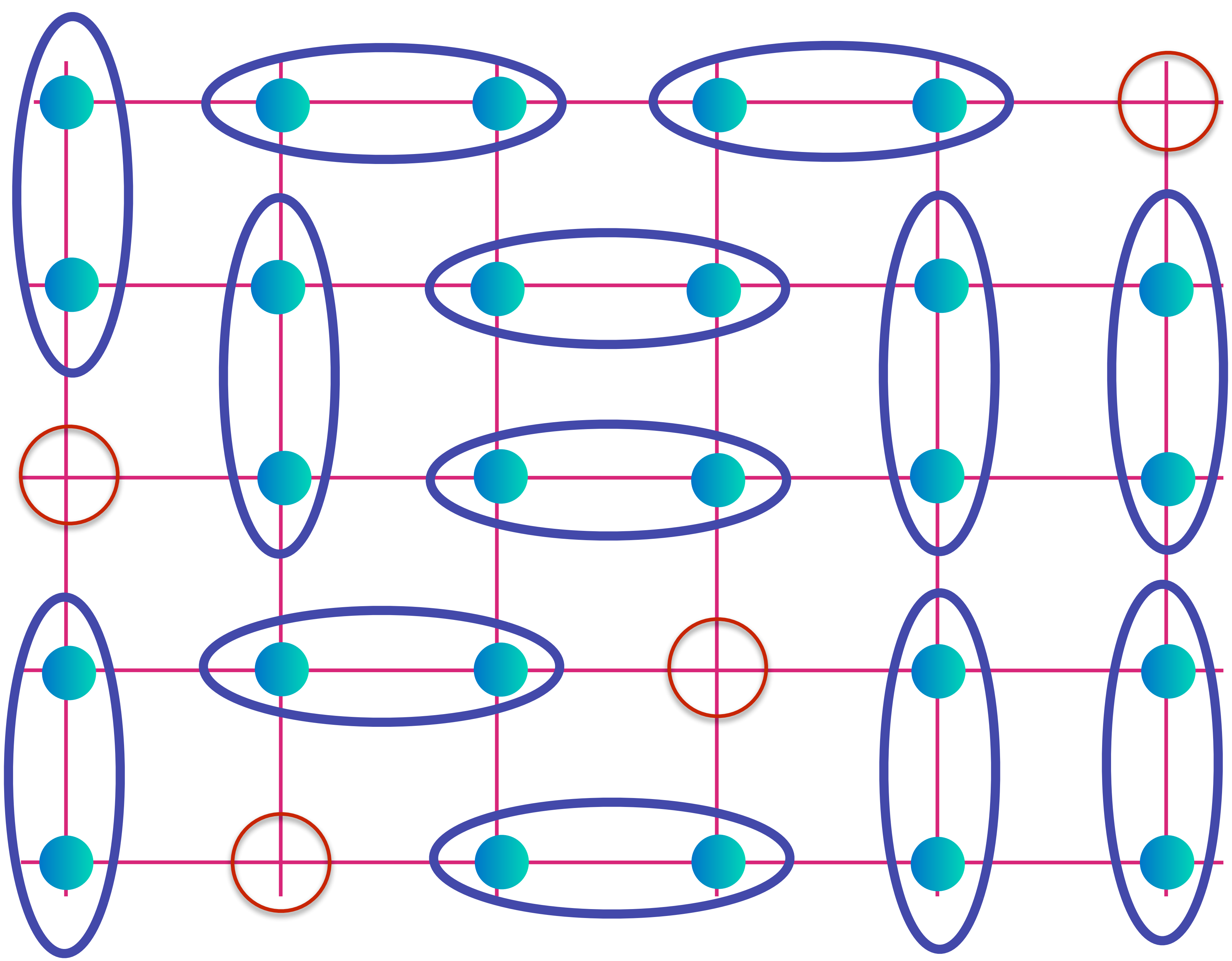

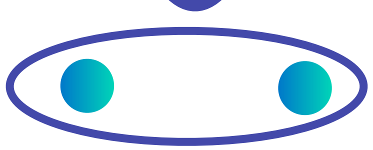 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

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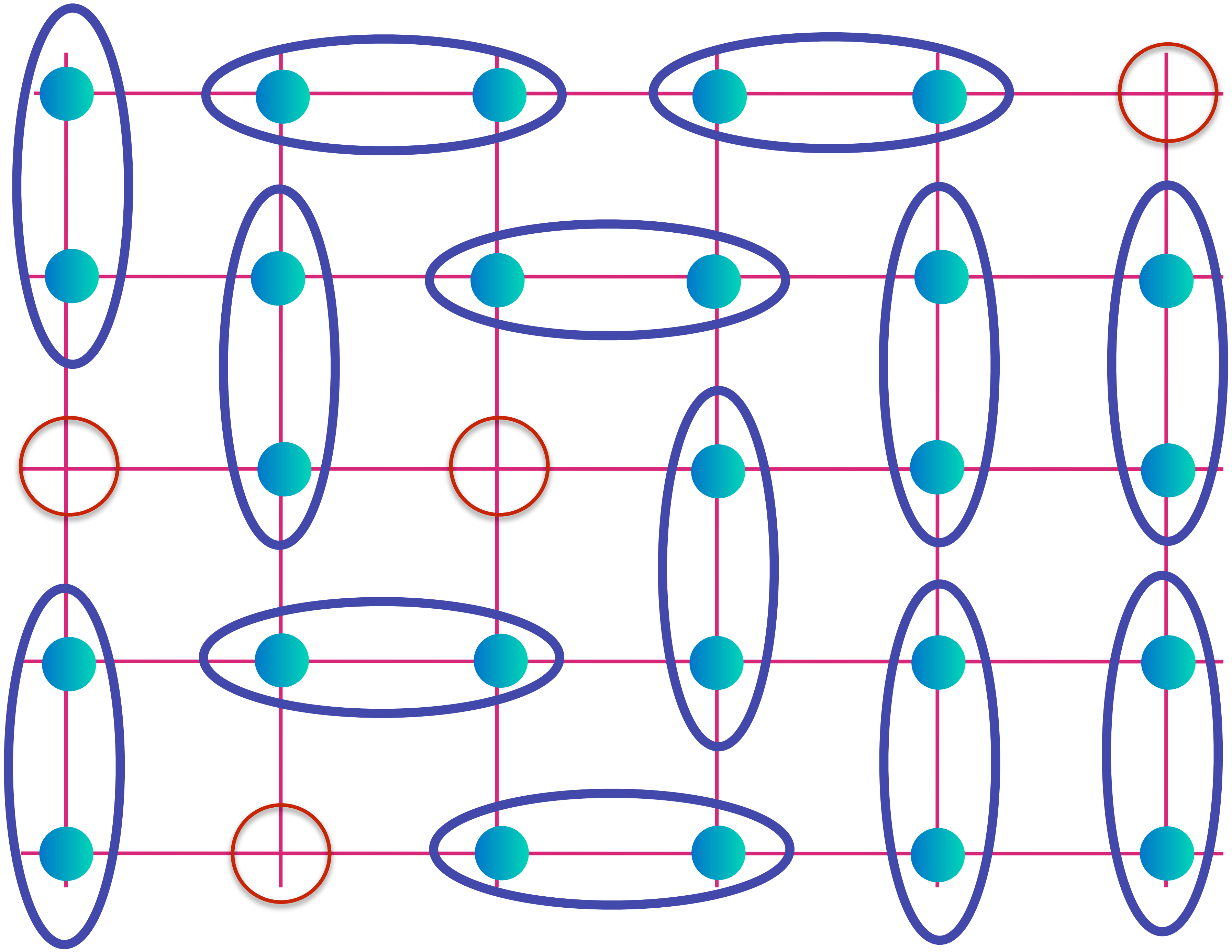

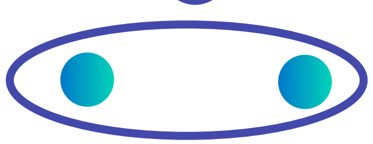

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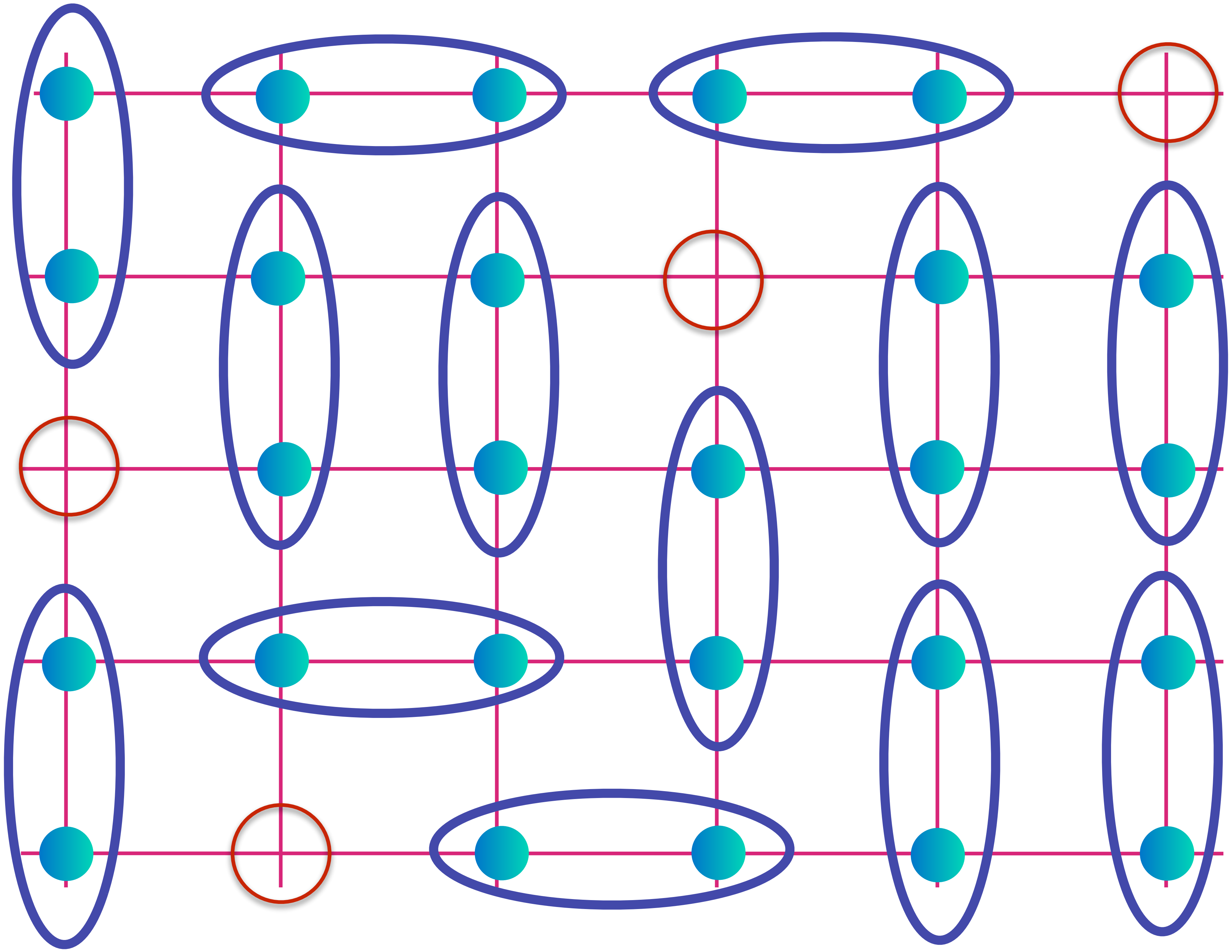

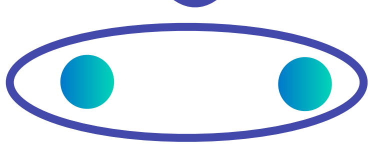 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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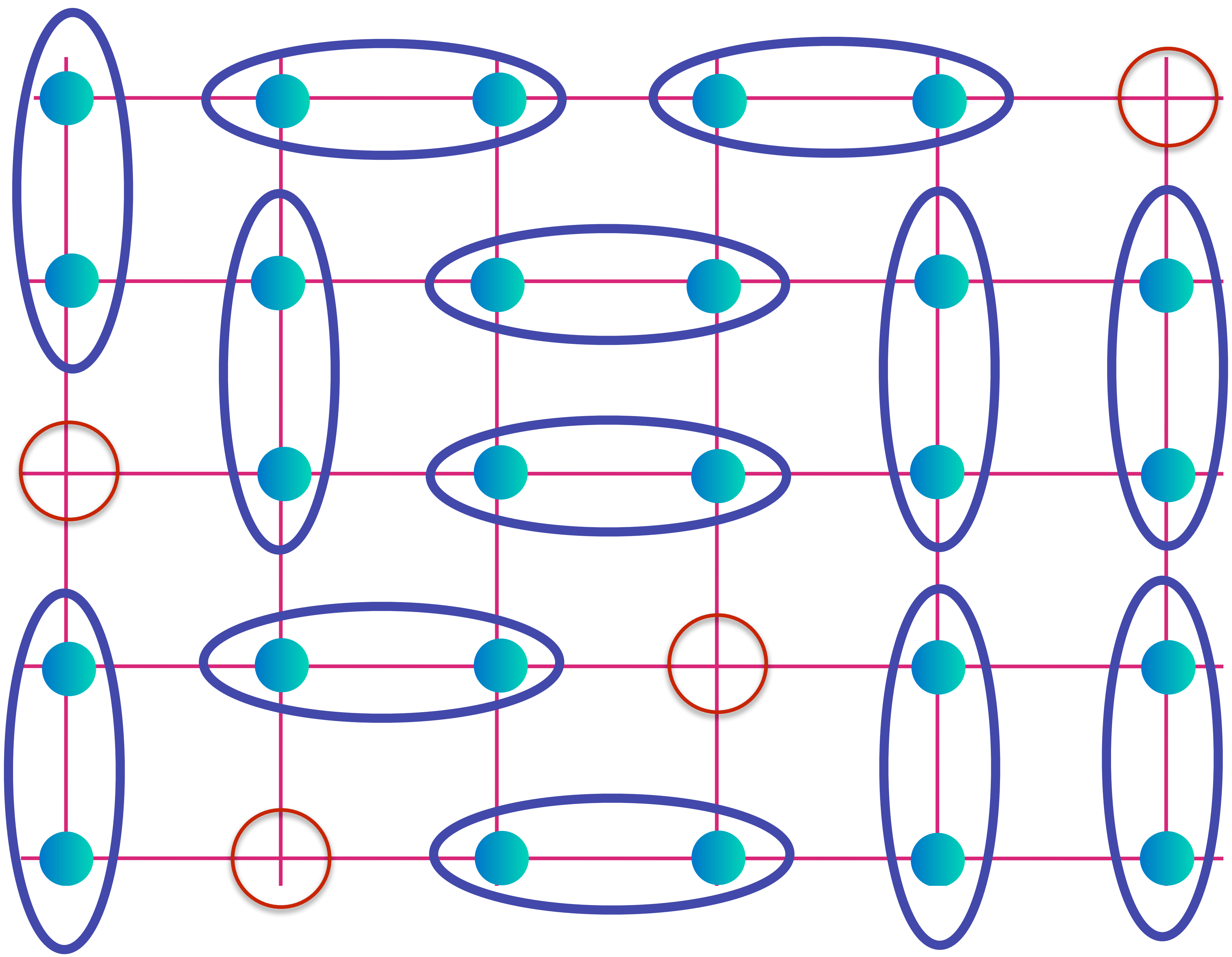

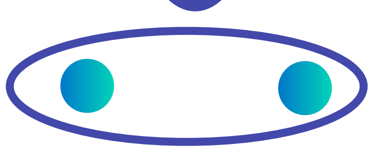

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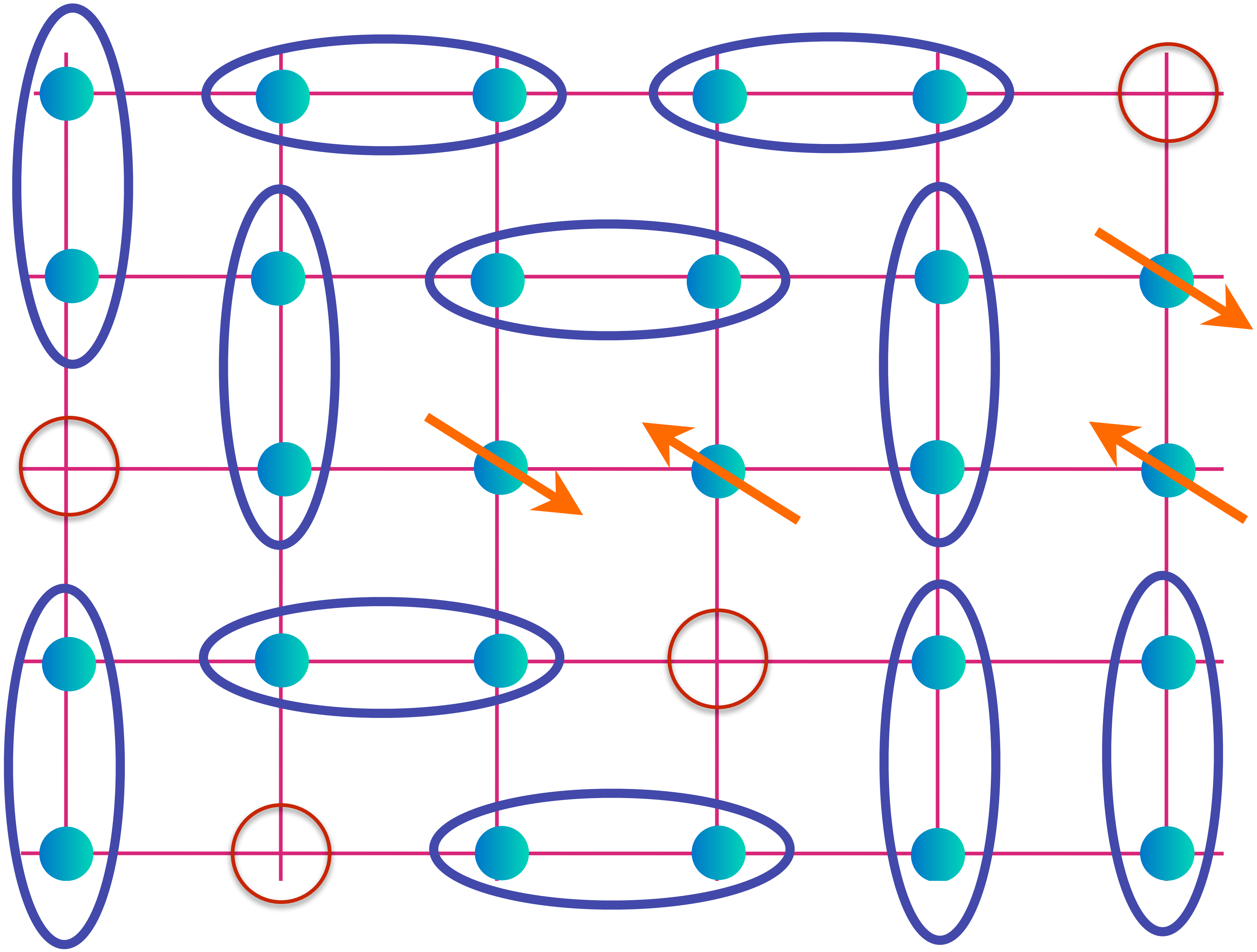

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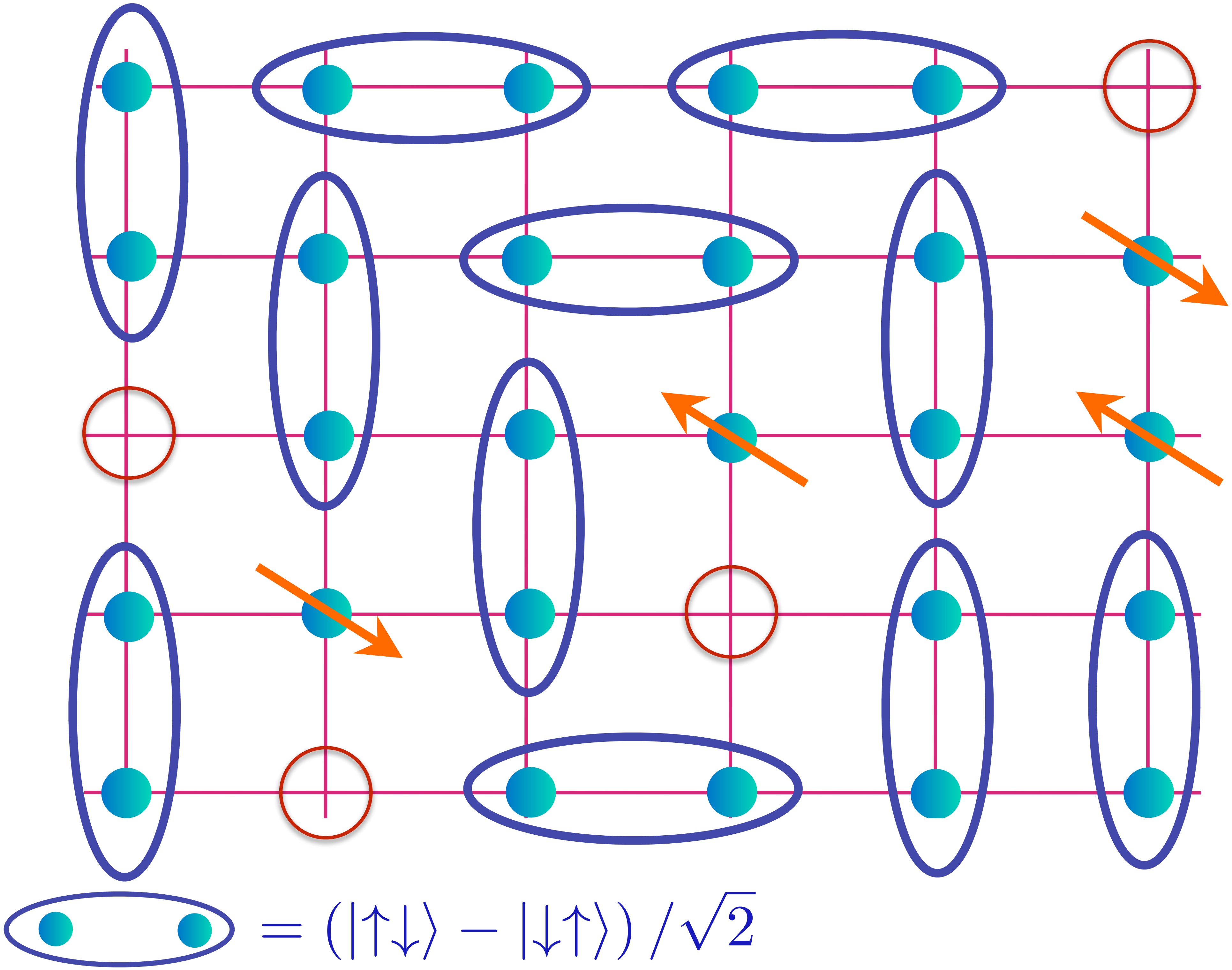
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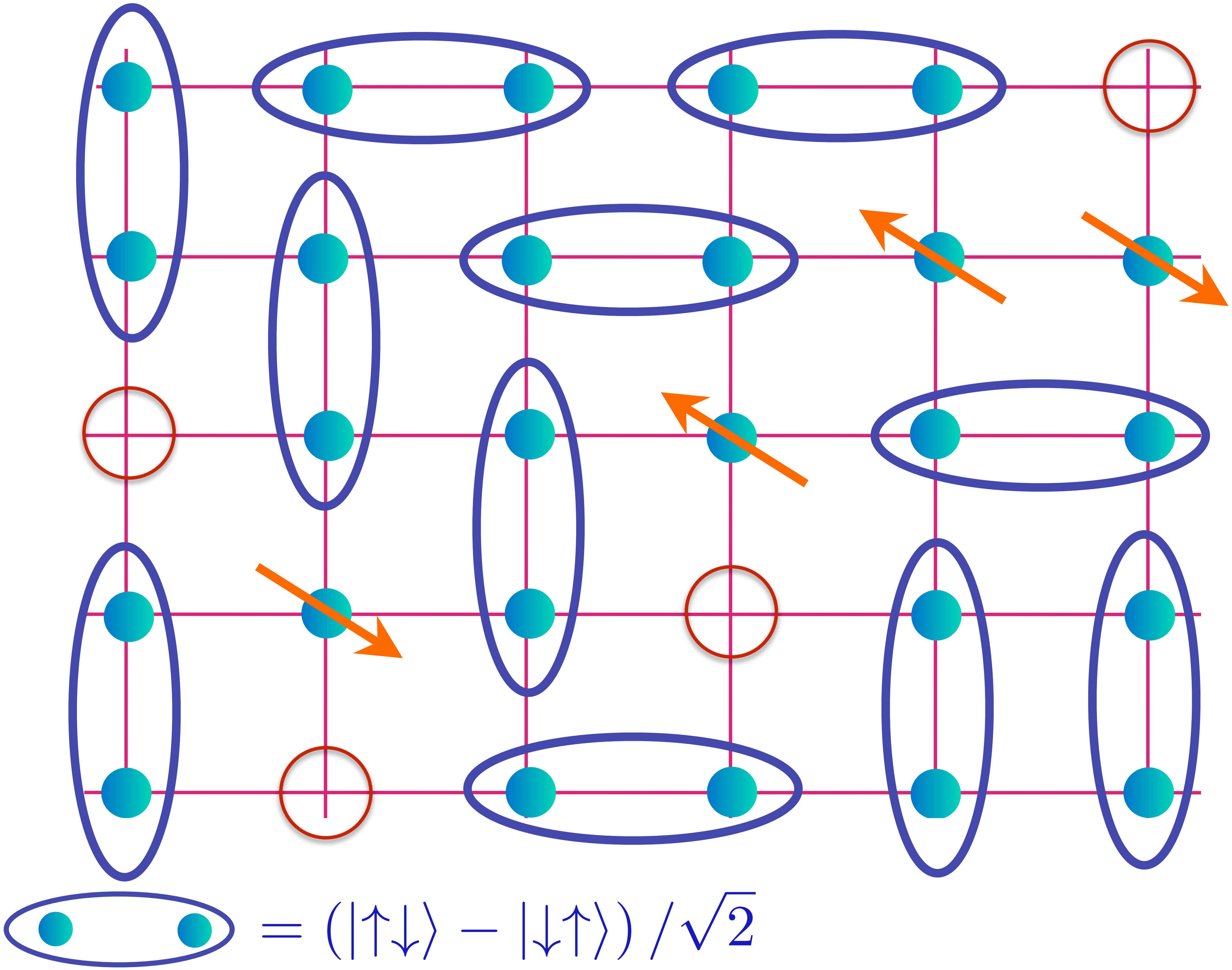
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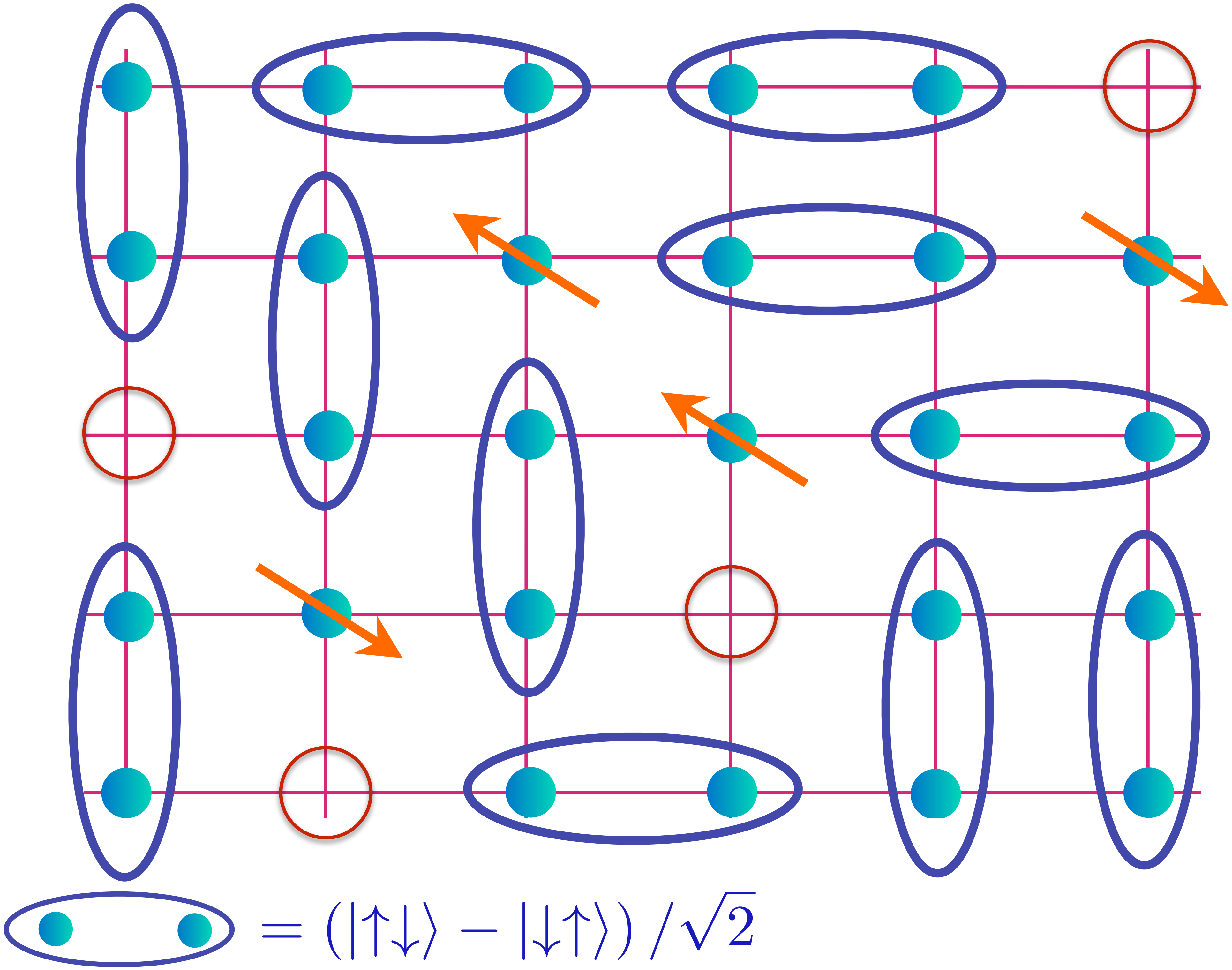
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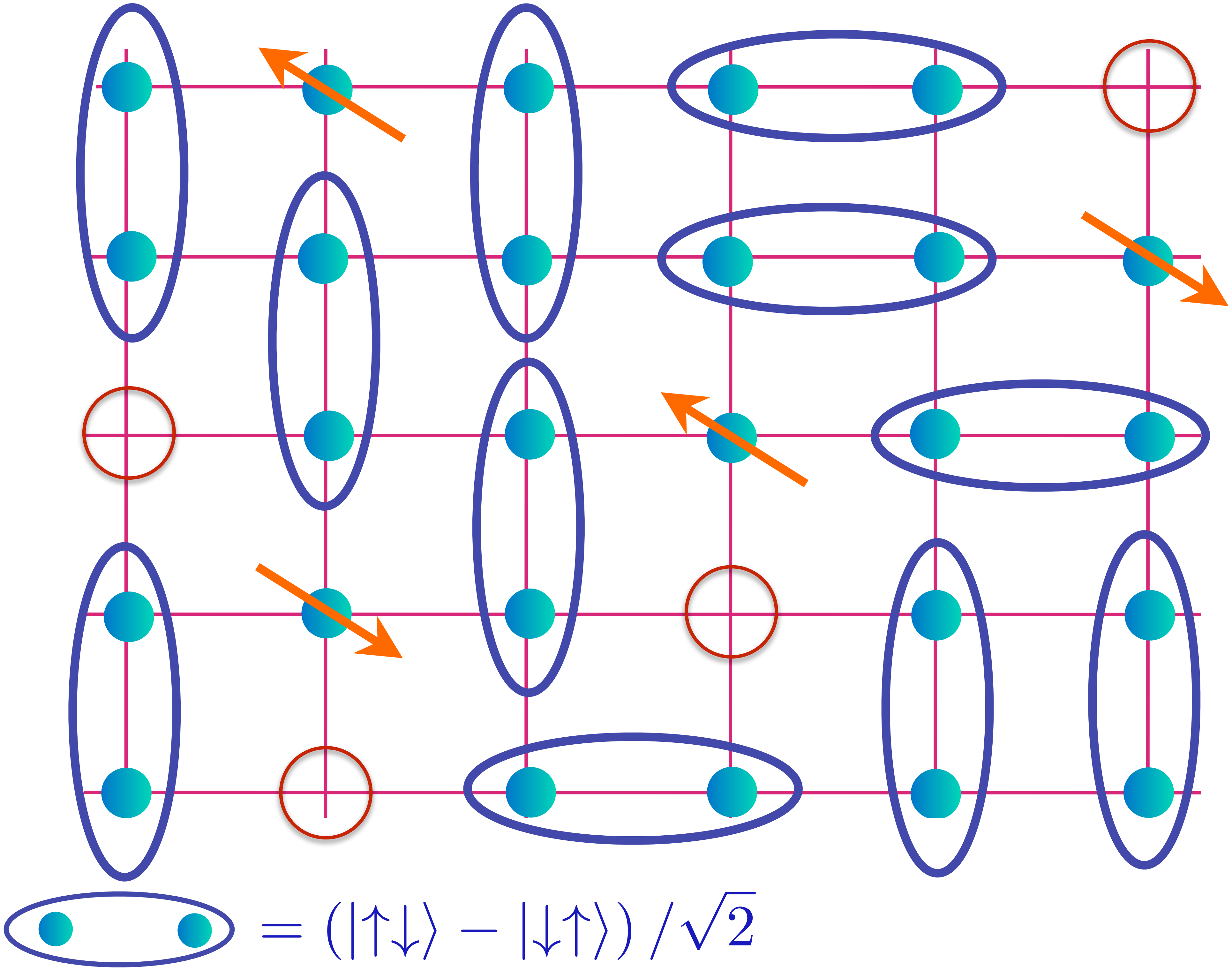

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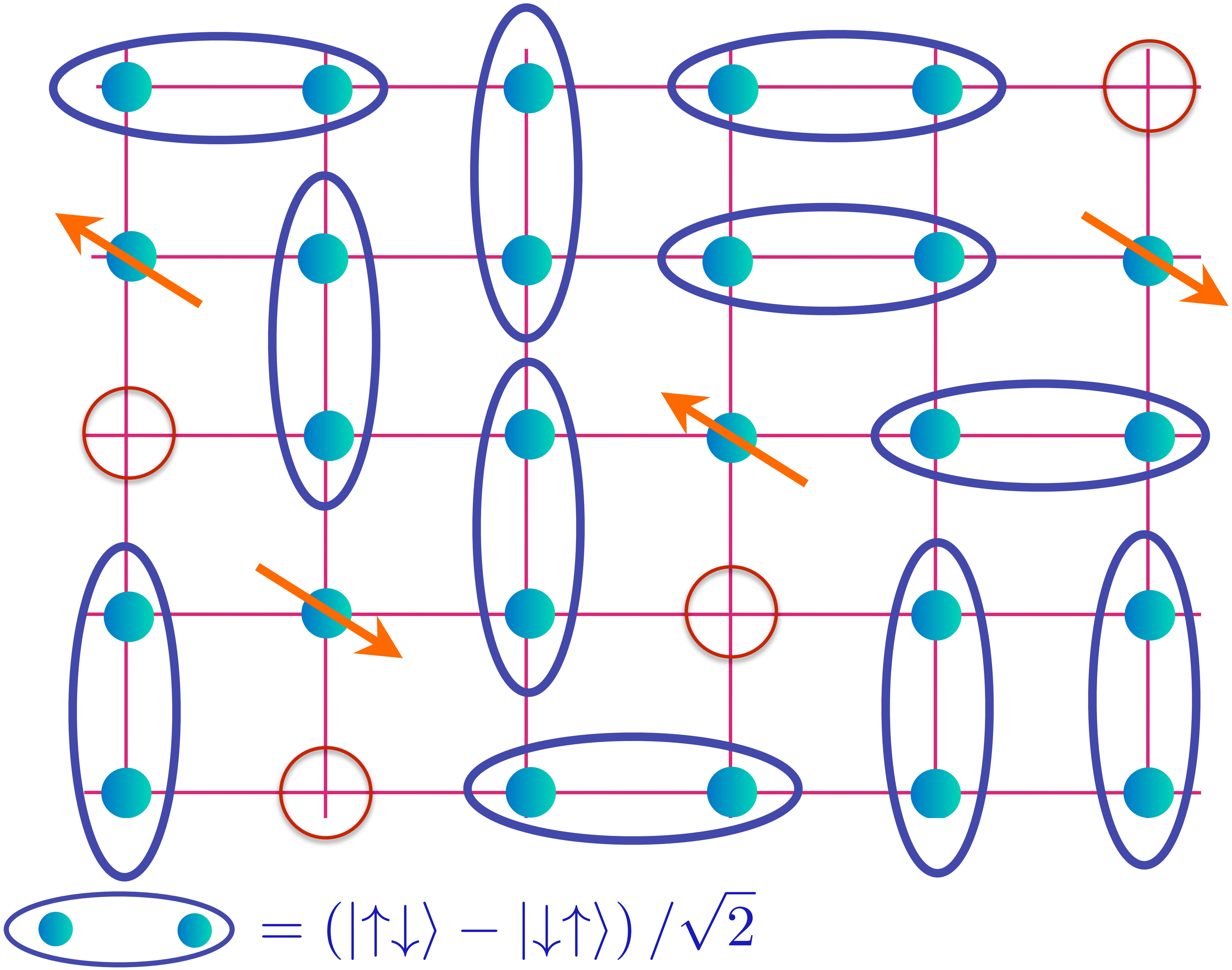
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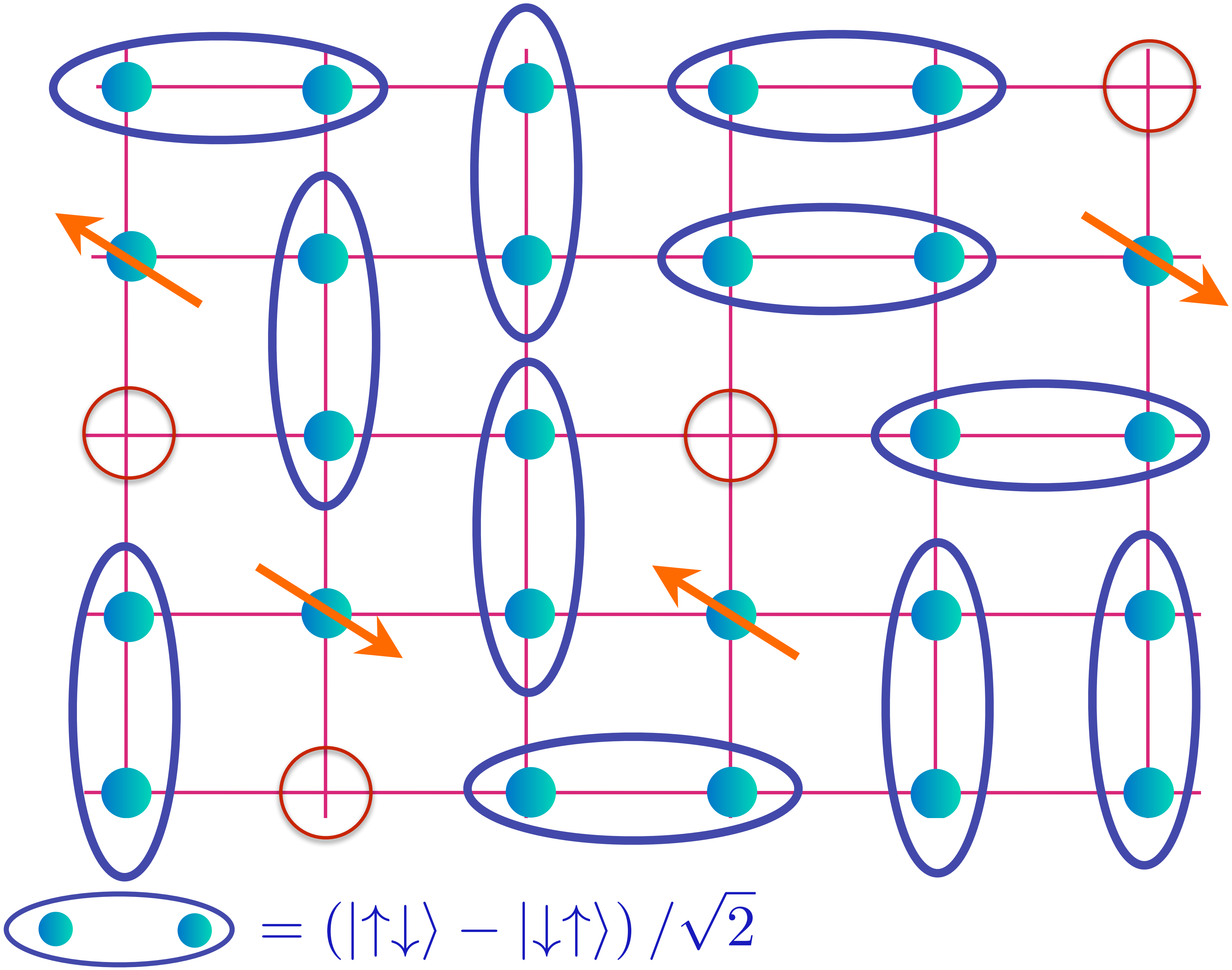

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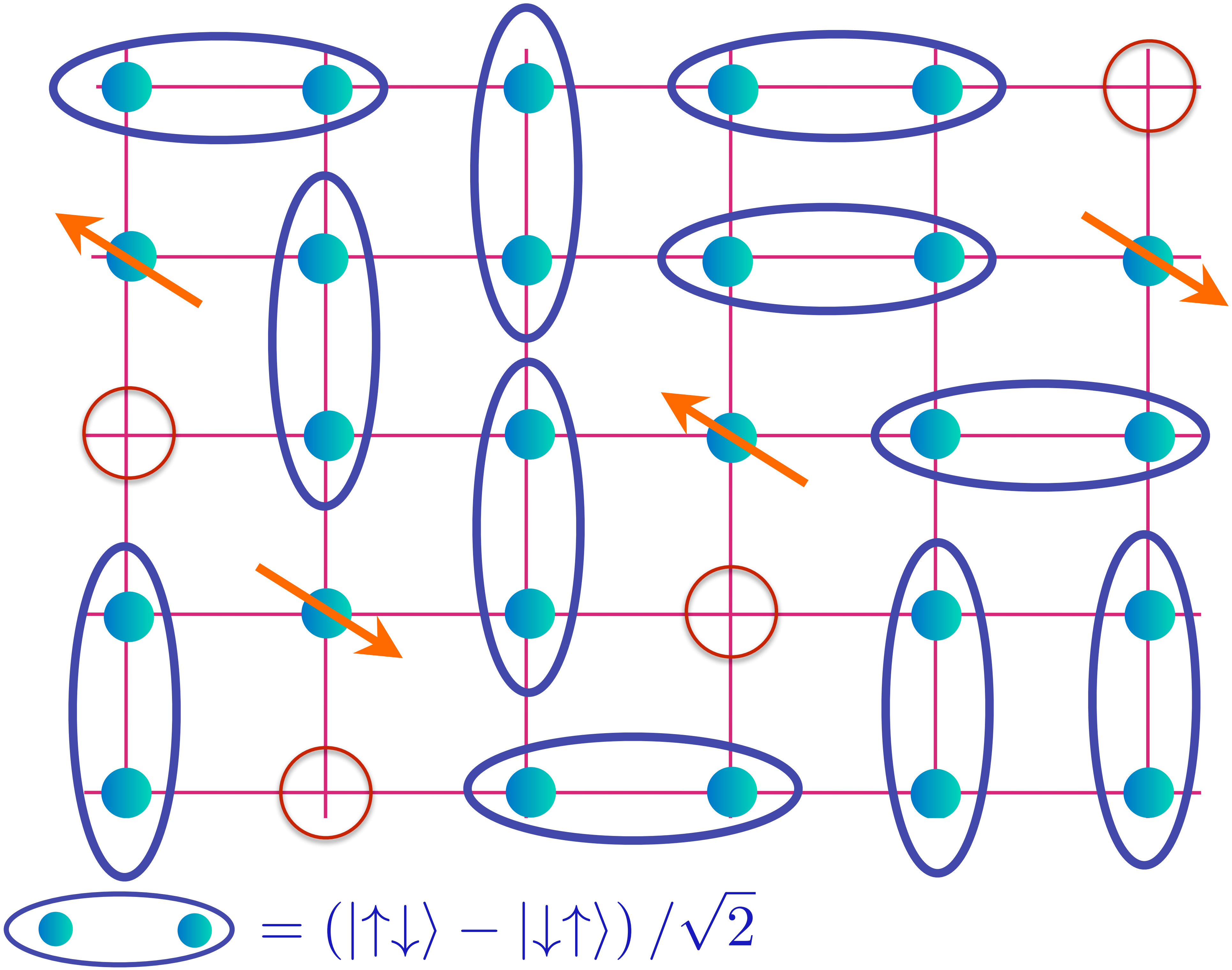
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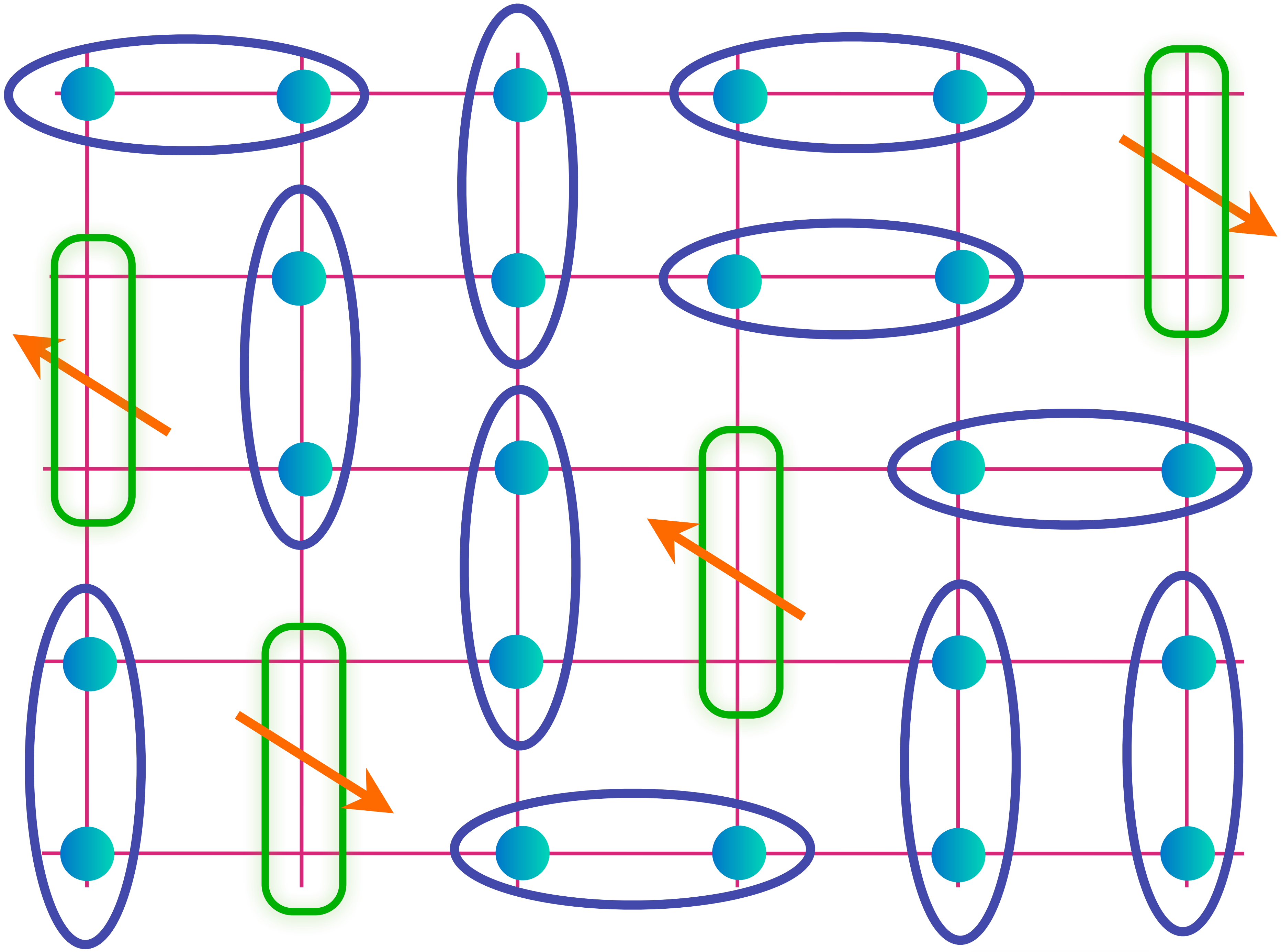
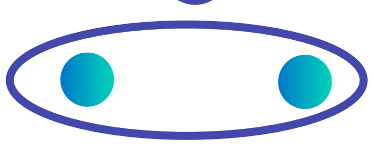
Fractionalized Fermi liquid (FL*)

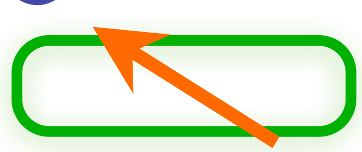
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p visible in electron photo-emission

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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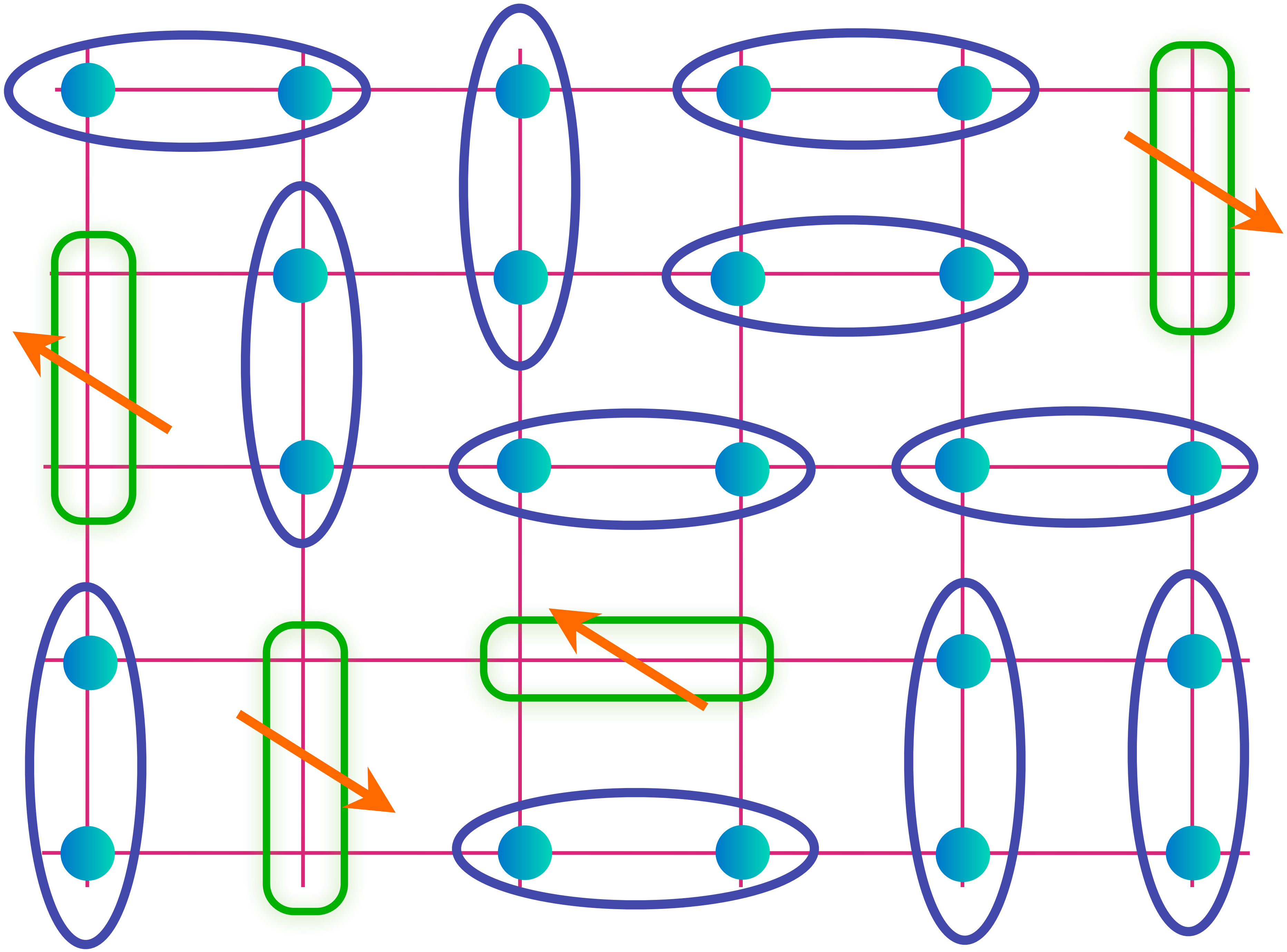

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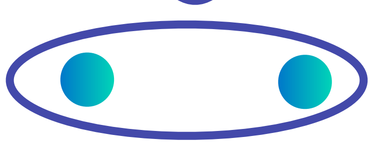
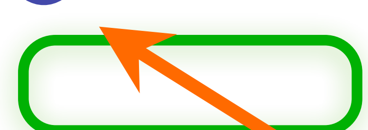
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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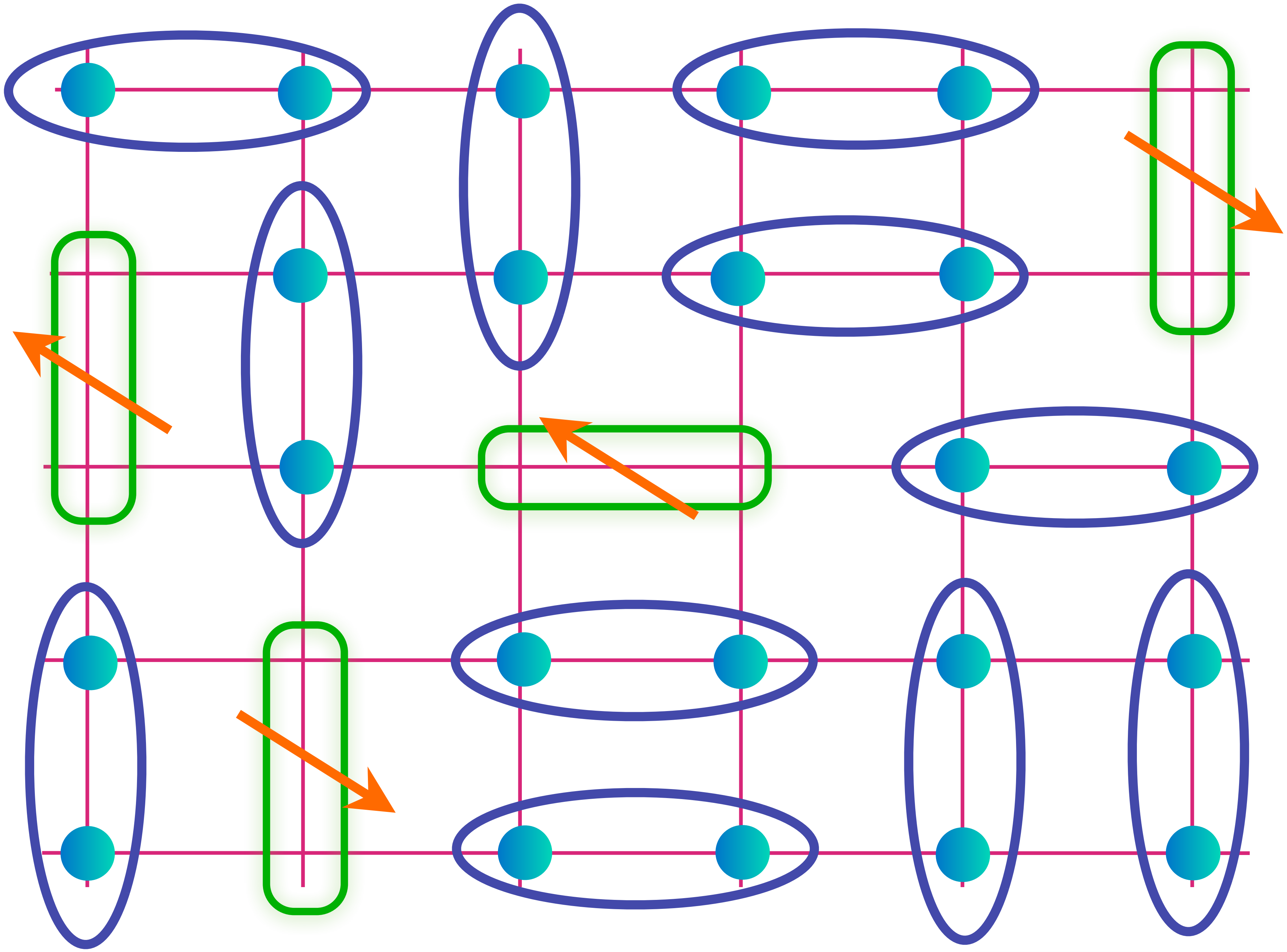

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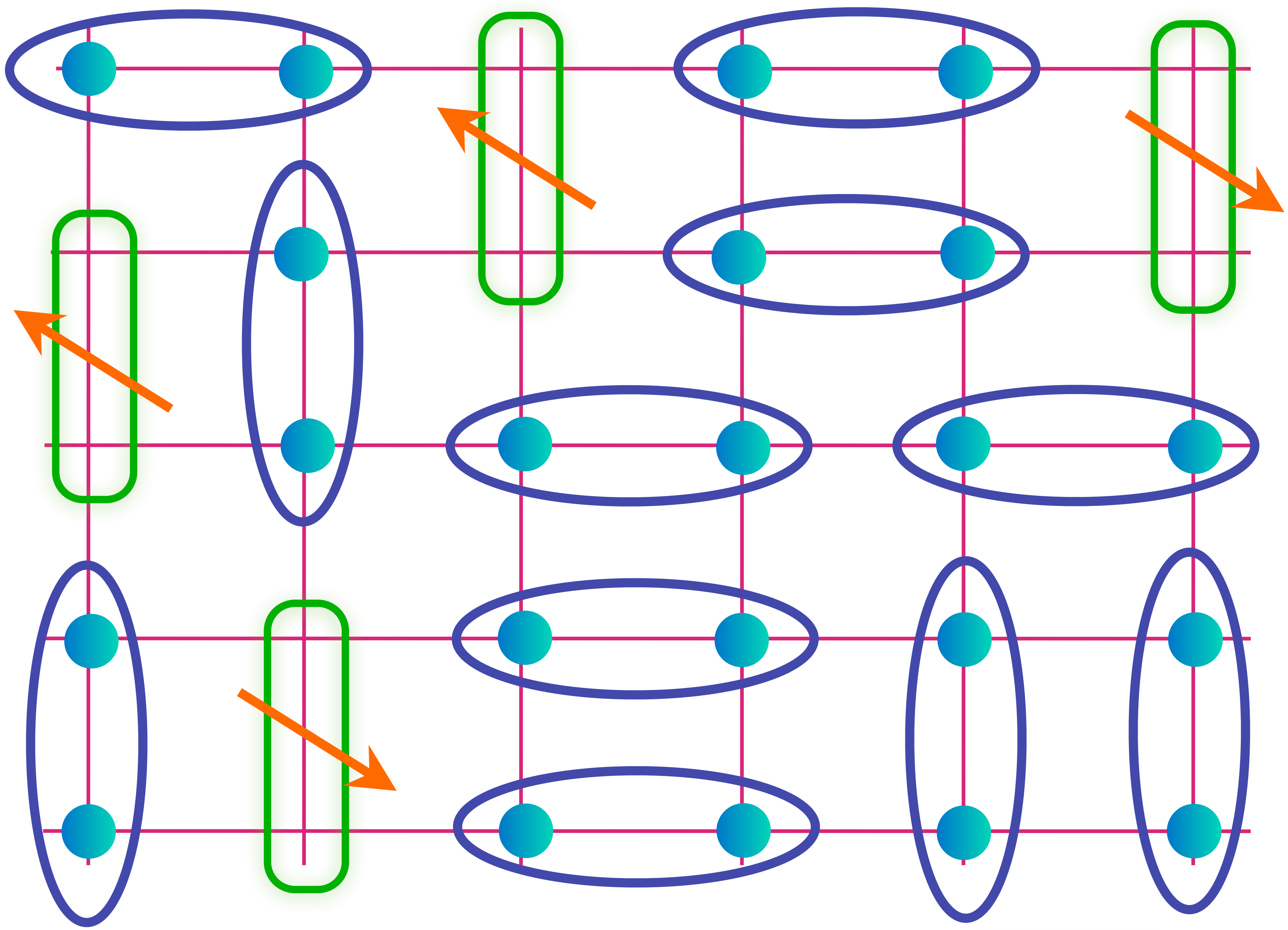
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Fractionalized Fermi liquid (FL*)

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



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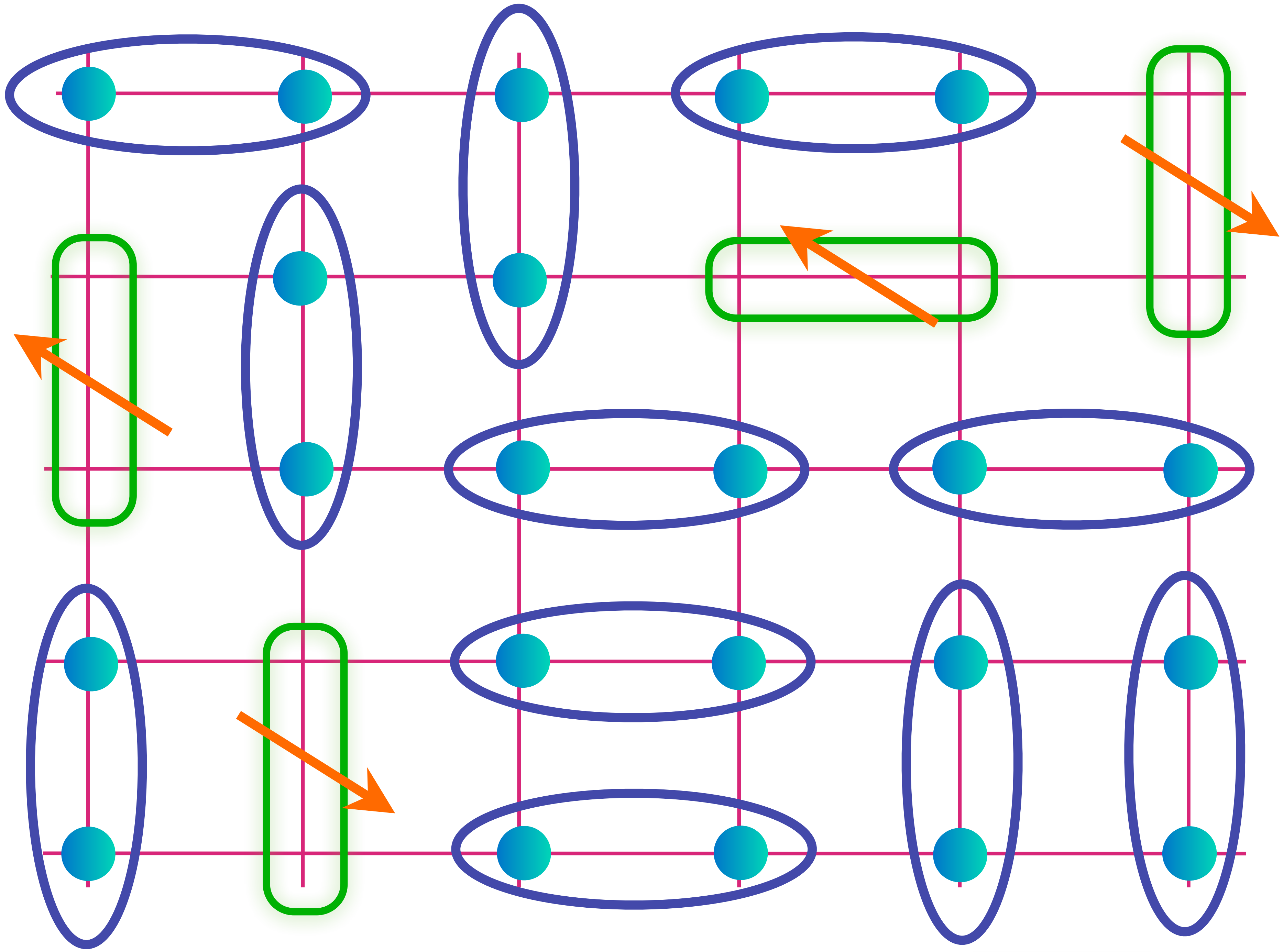

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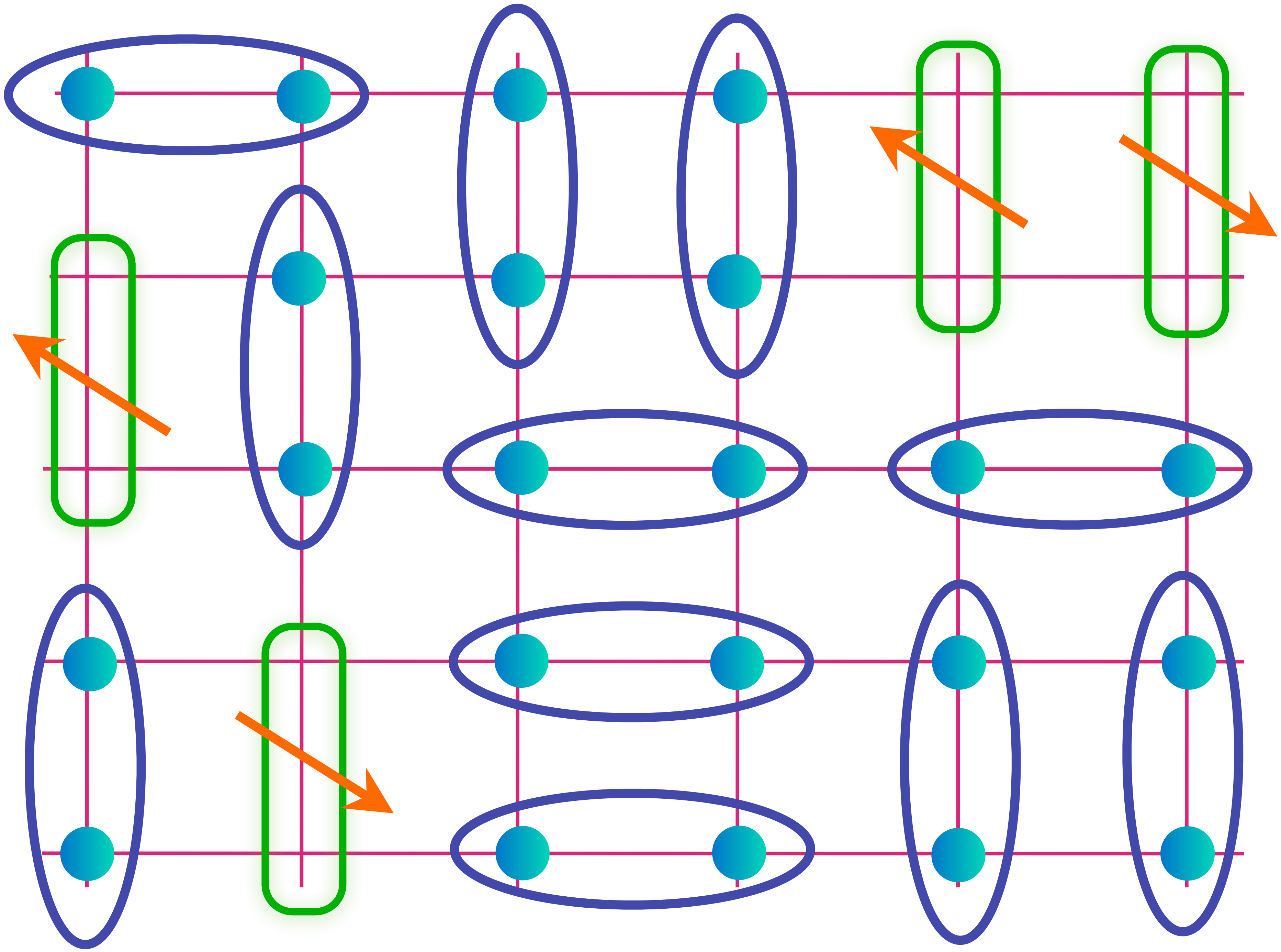

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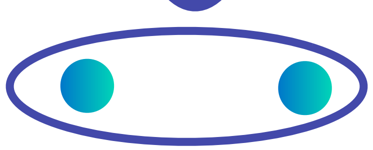
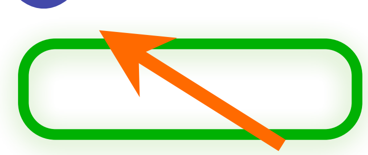
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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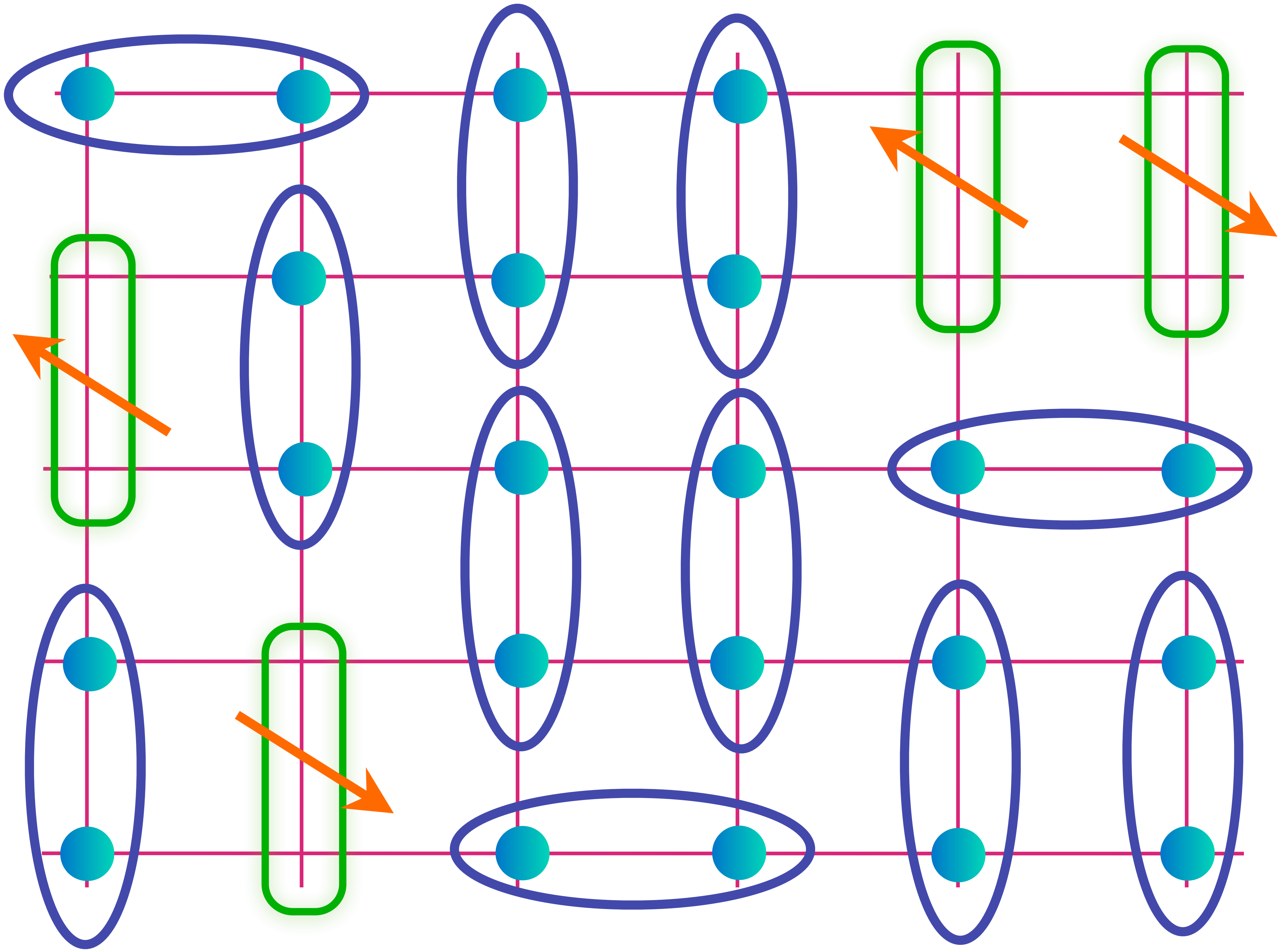

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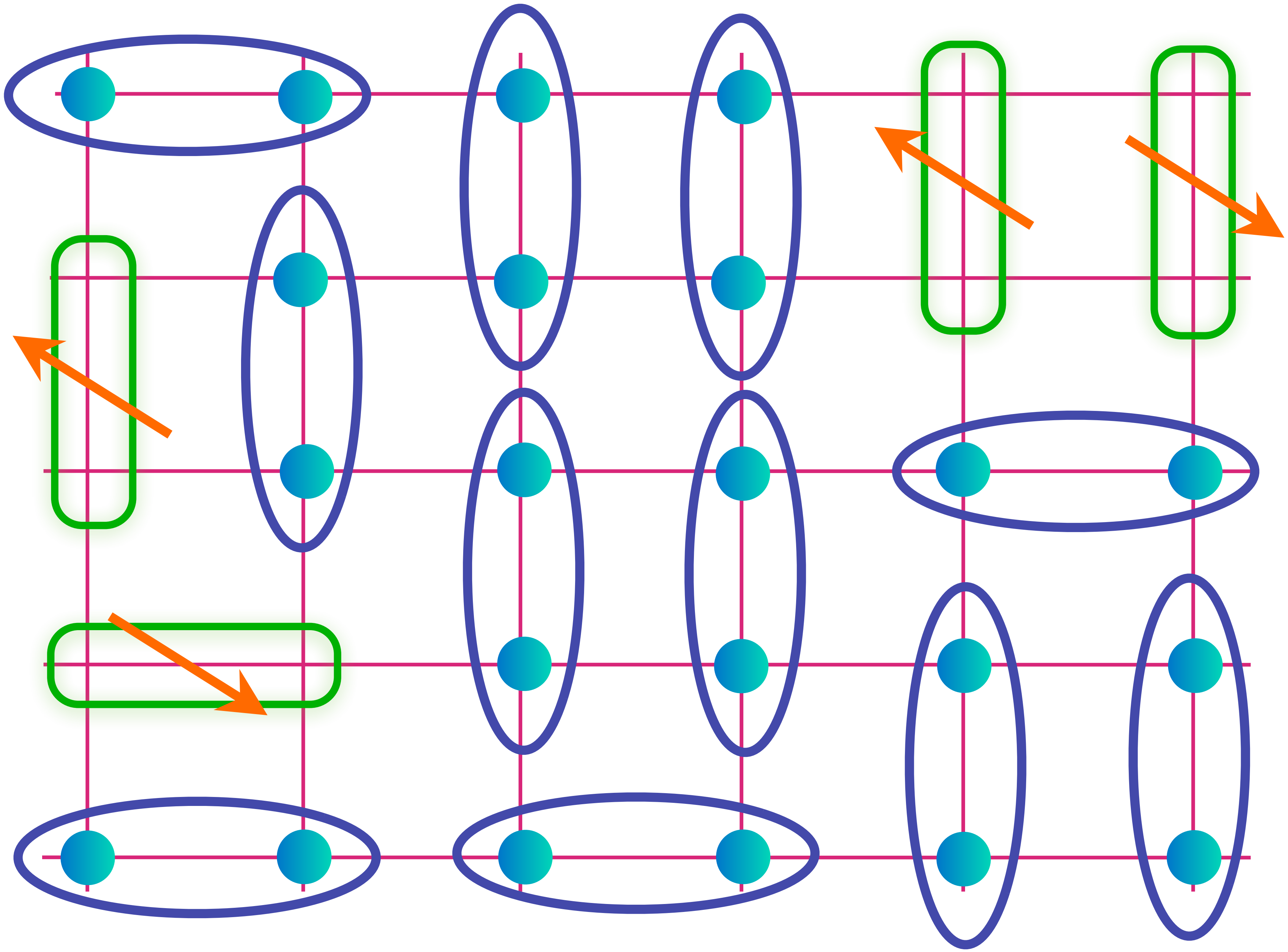

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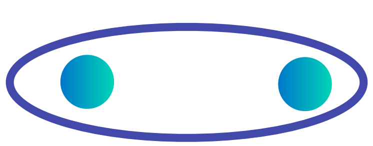
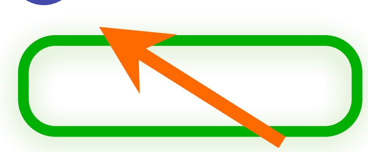
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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Ground state degeneracy

Place FL*
on a torus:

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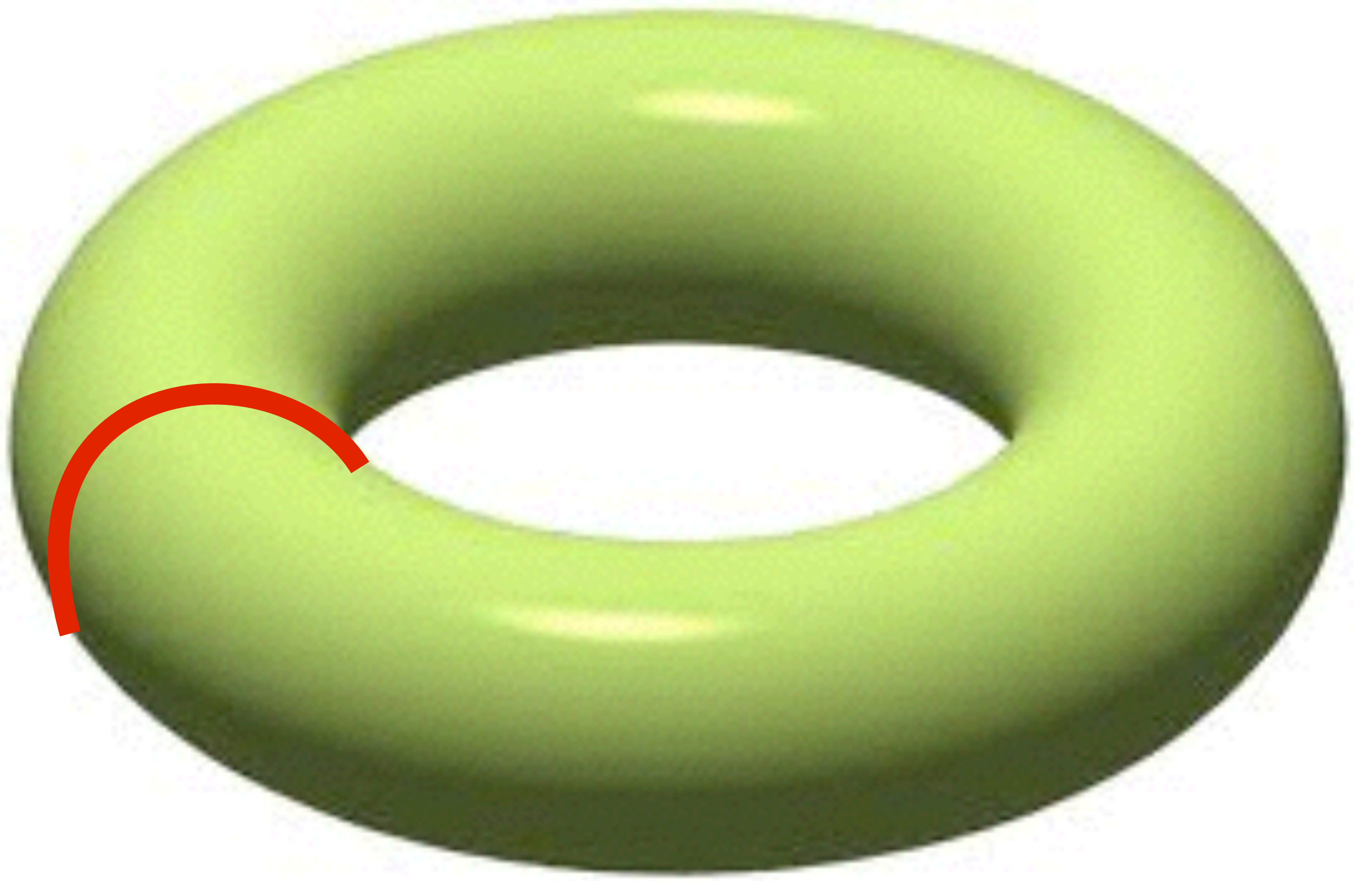

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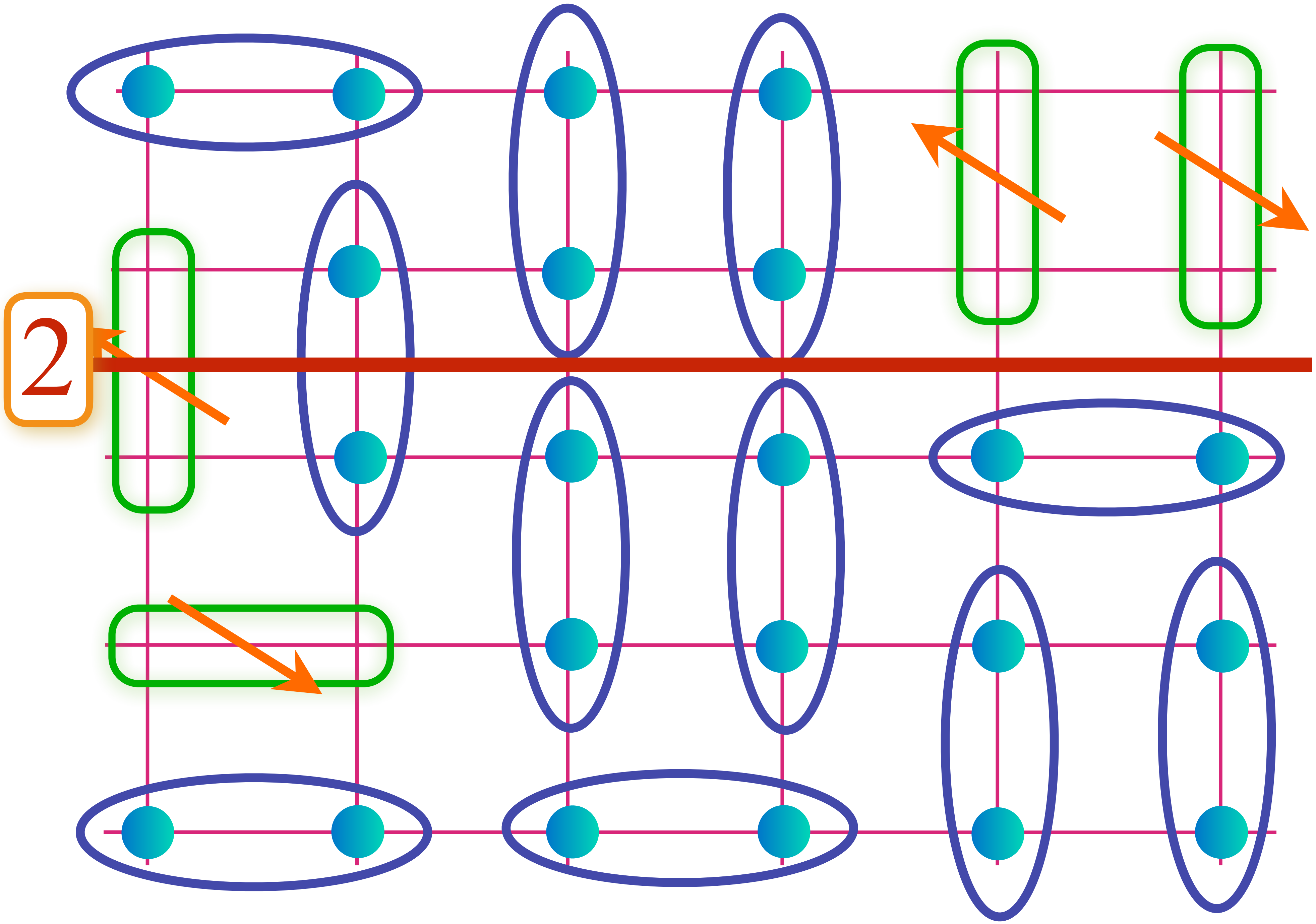

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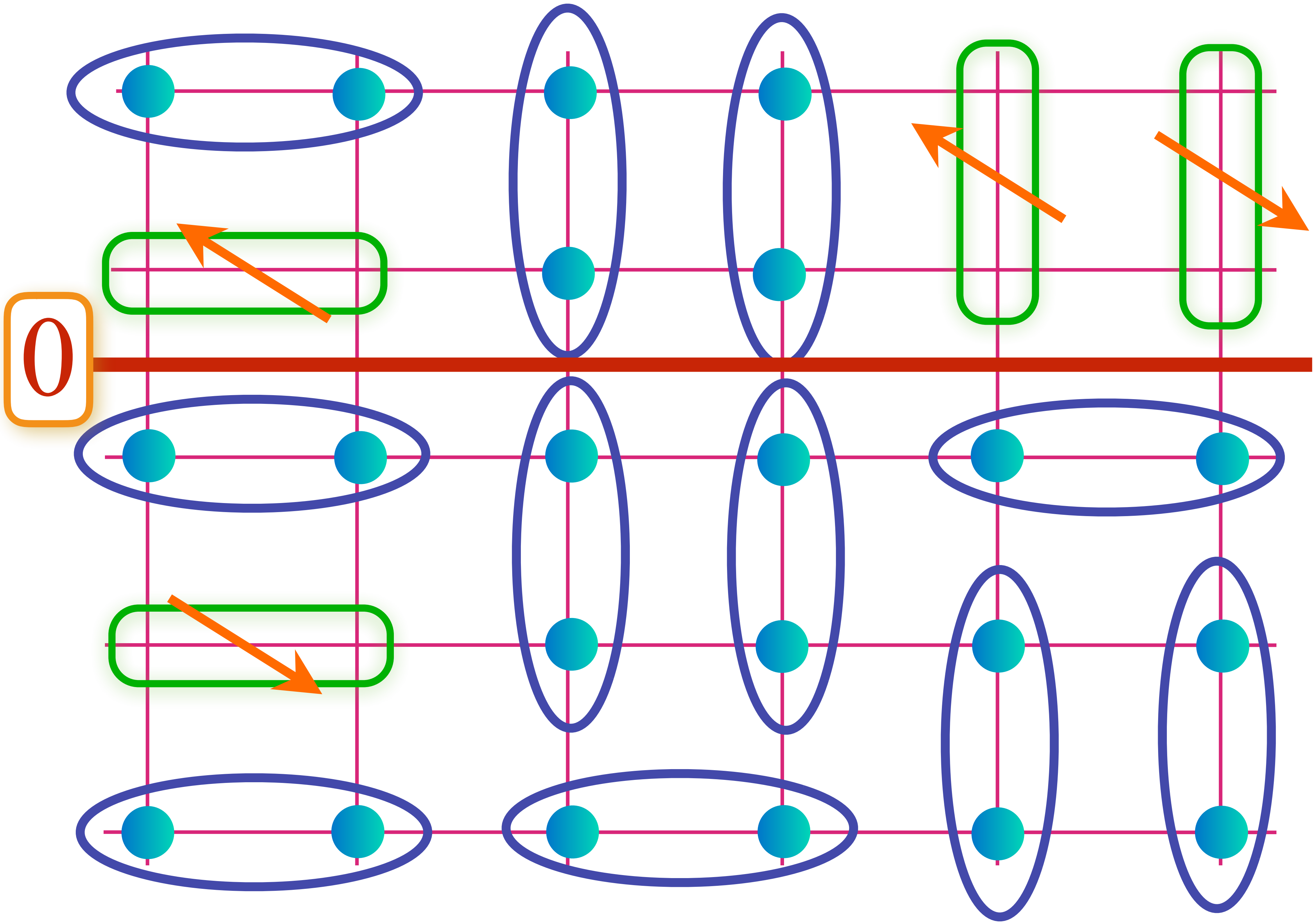

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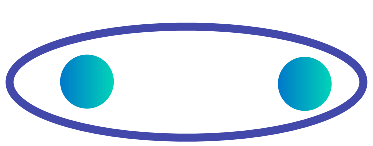
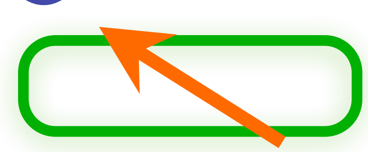
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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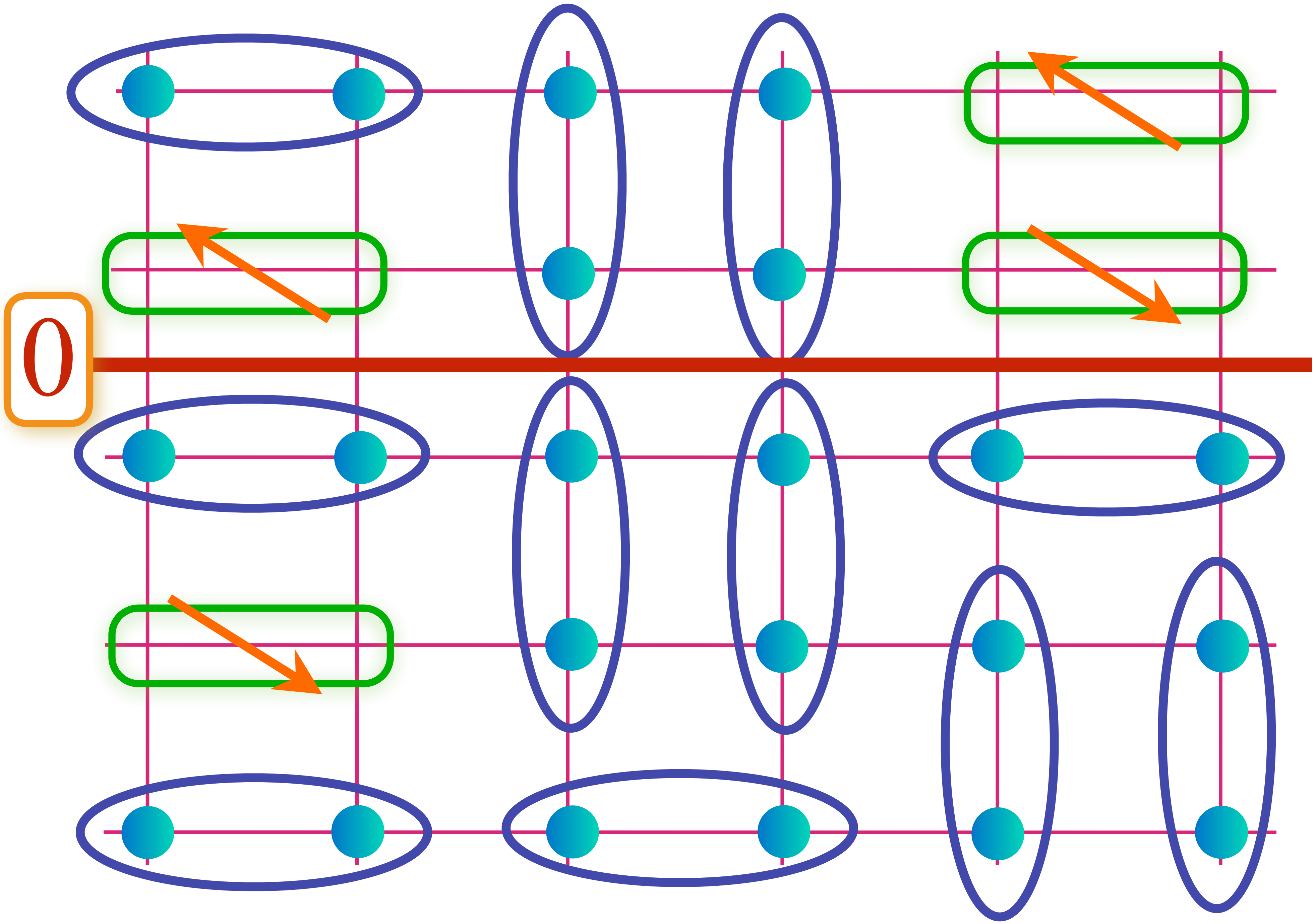

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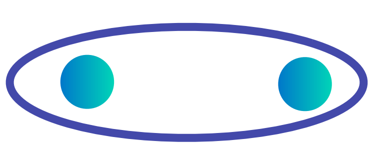

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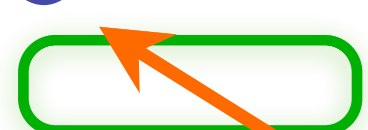

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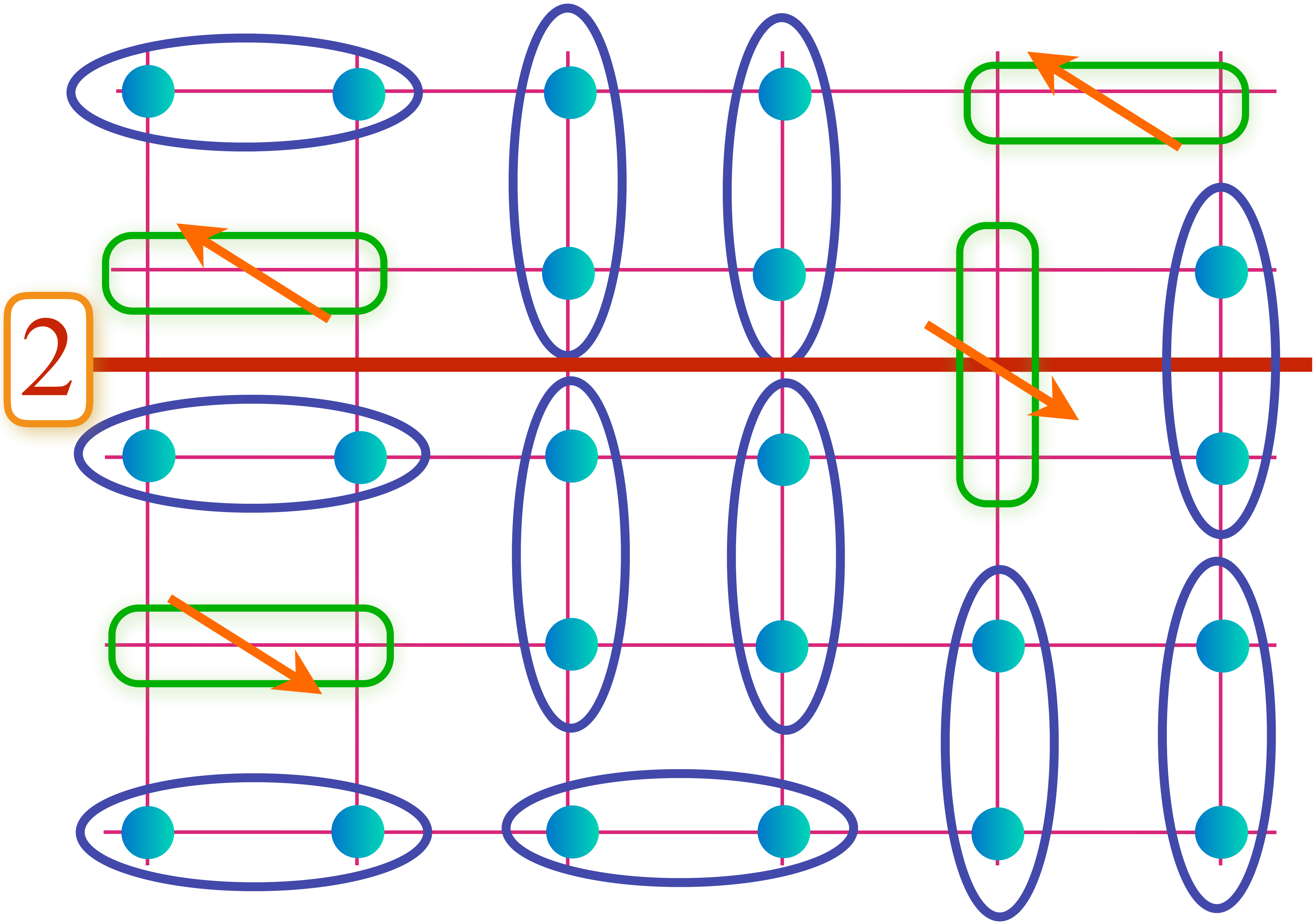

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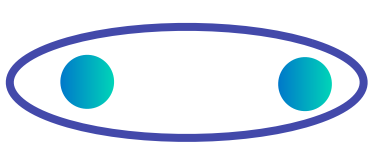
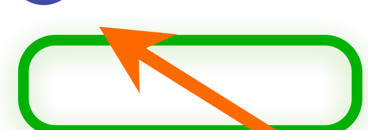
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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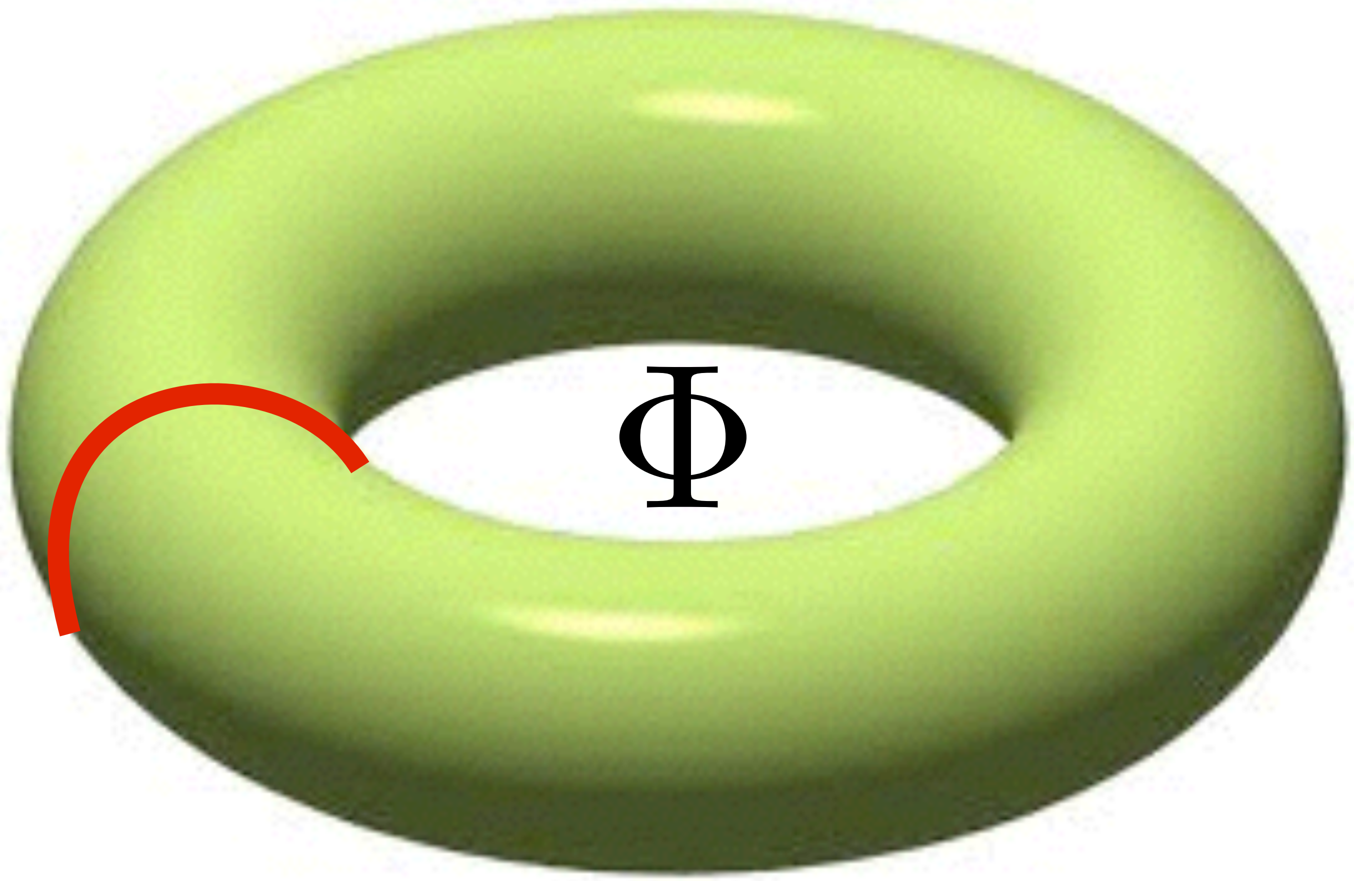

Following the evolution of the quantum state under adiabatic insertion of a flux quantum leads to a non-perturbative argument for the volume enclosed by the Fermi surface

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1. Emergent gauge fields and long-range entanglement in insulators

2. Theory of ordinary metals: Fermi liquids (FL)

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   Quasiparticles with a non-Luttinger volume,
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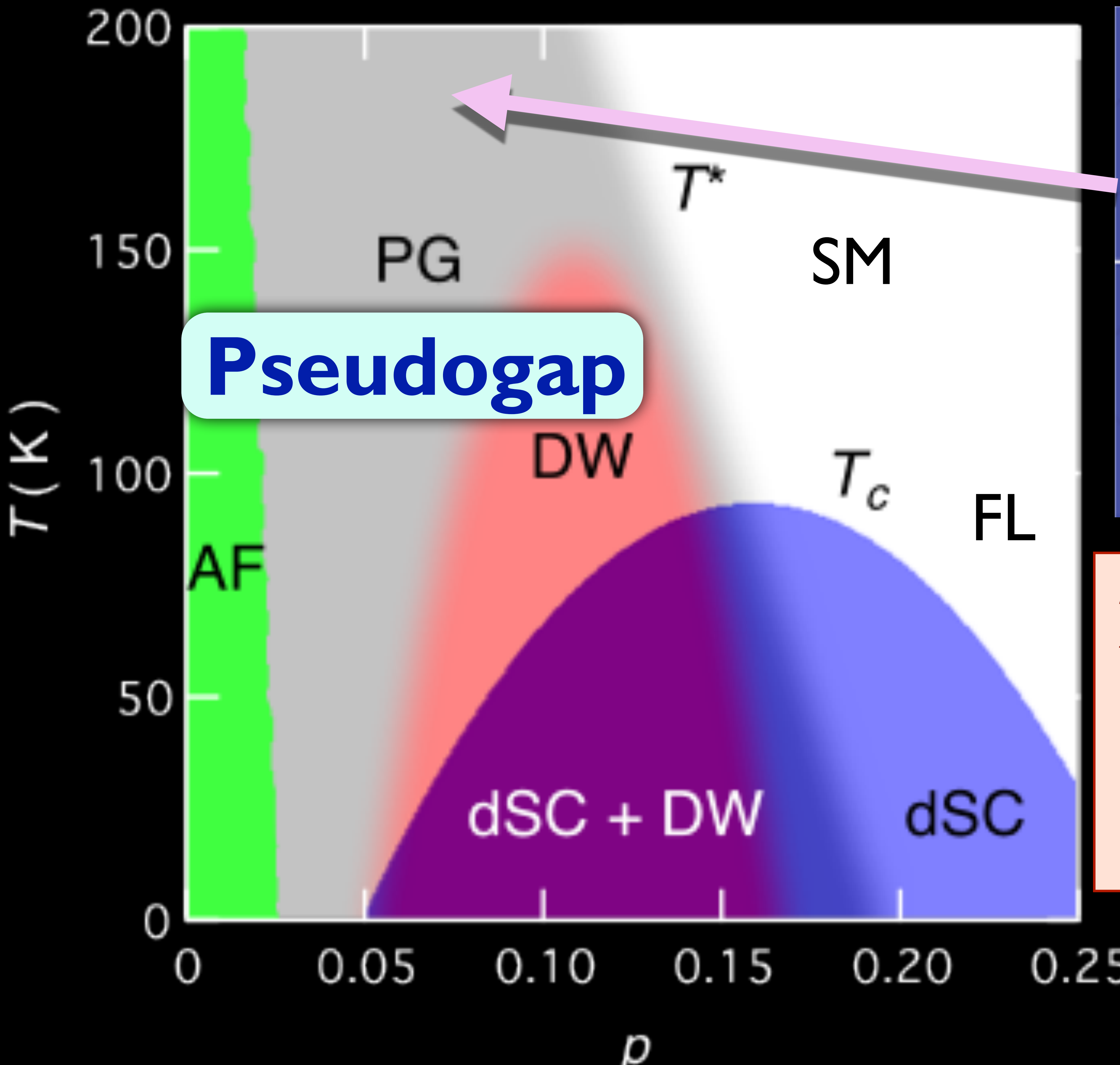
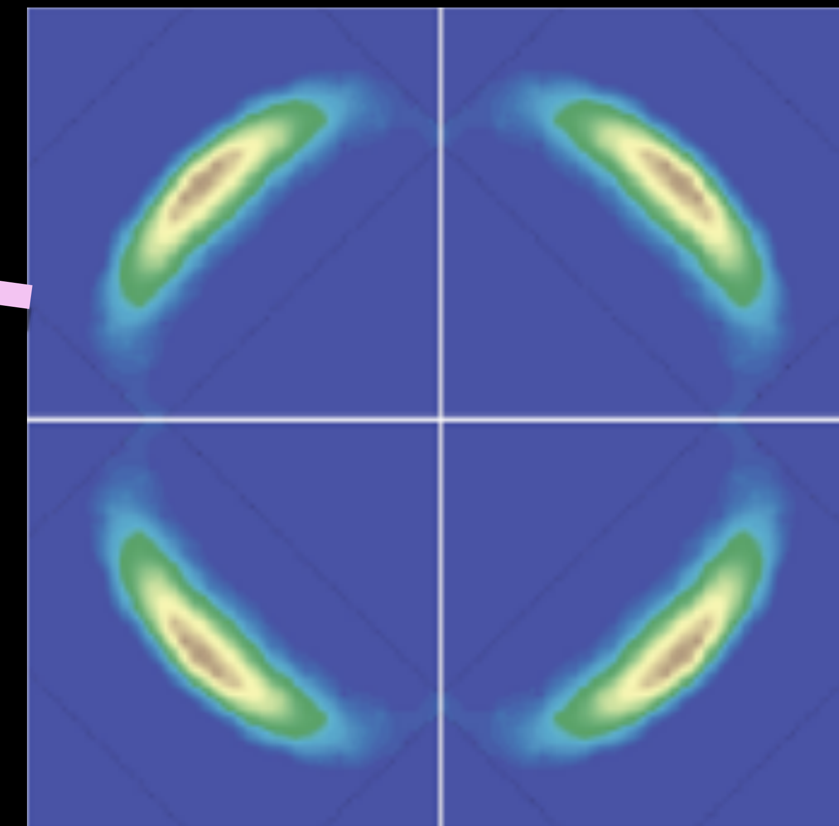
4. The pseudogap metal of the cuprate superconductors

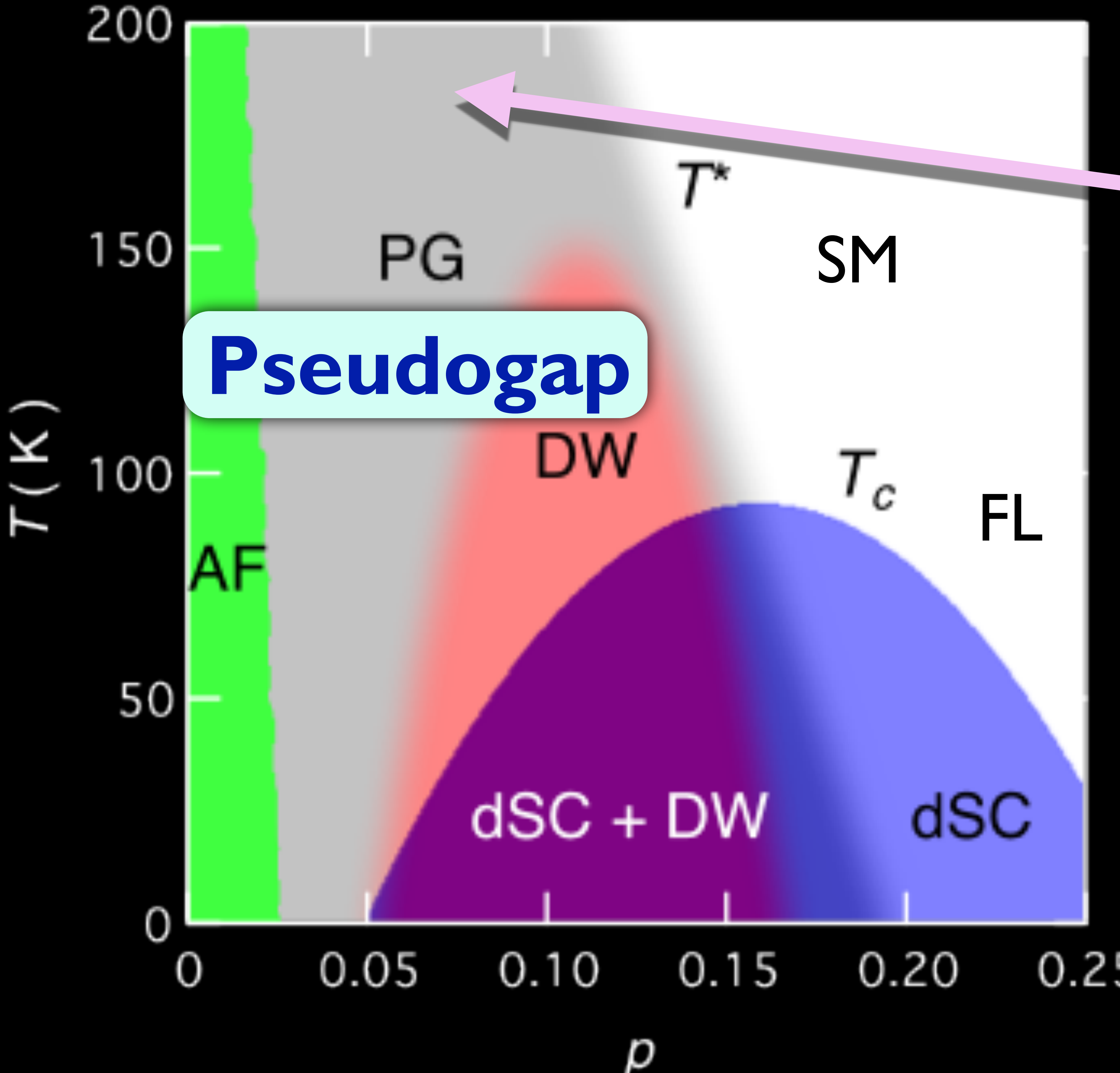
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

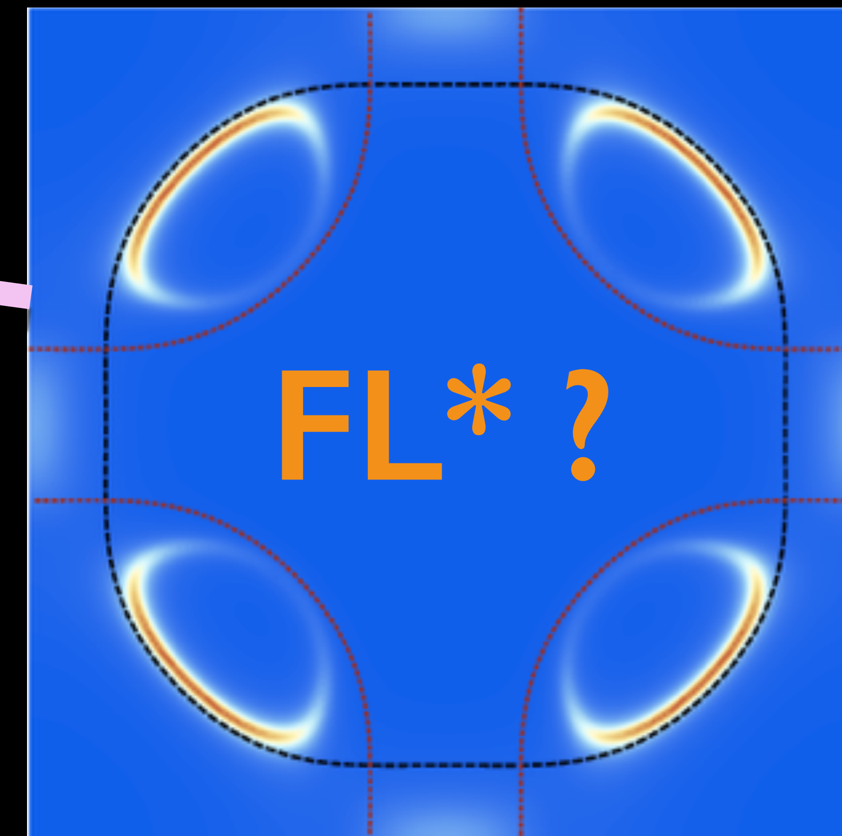




2. Pseudogap metal at low p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)





A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p

Recent evidence for pseudogap metal as FL*

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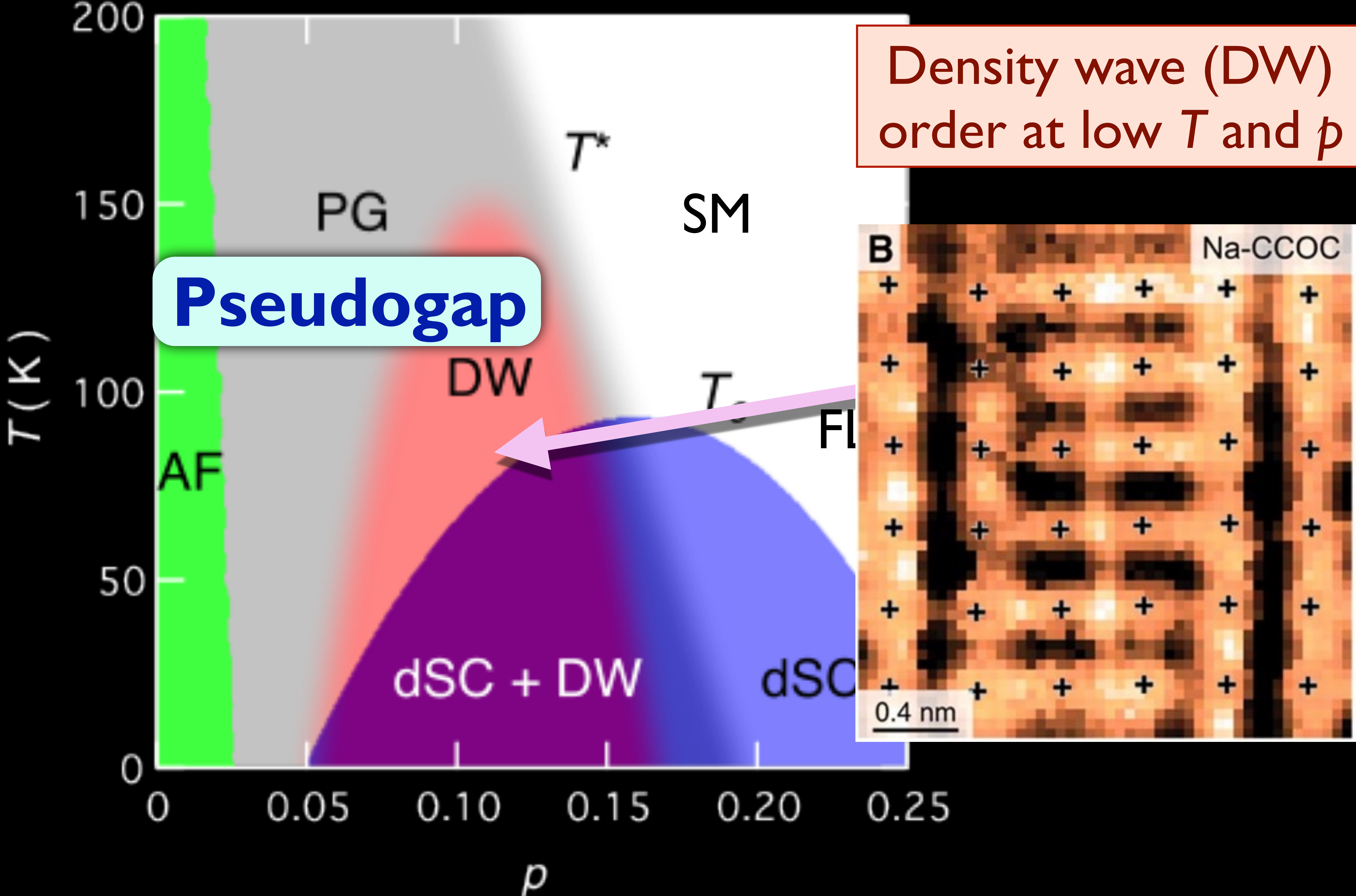
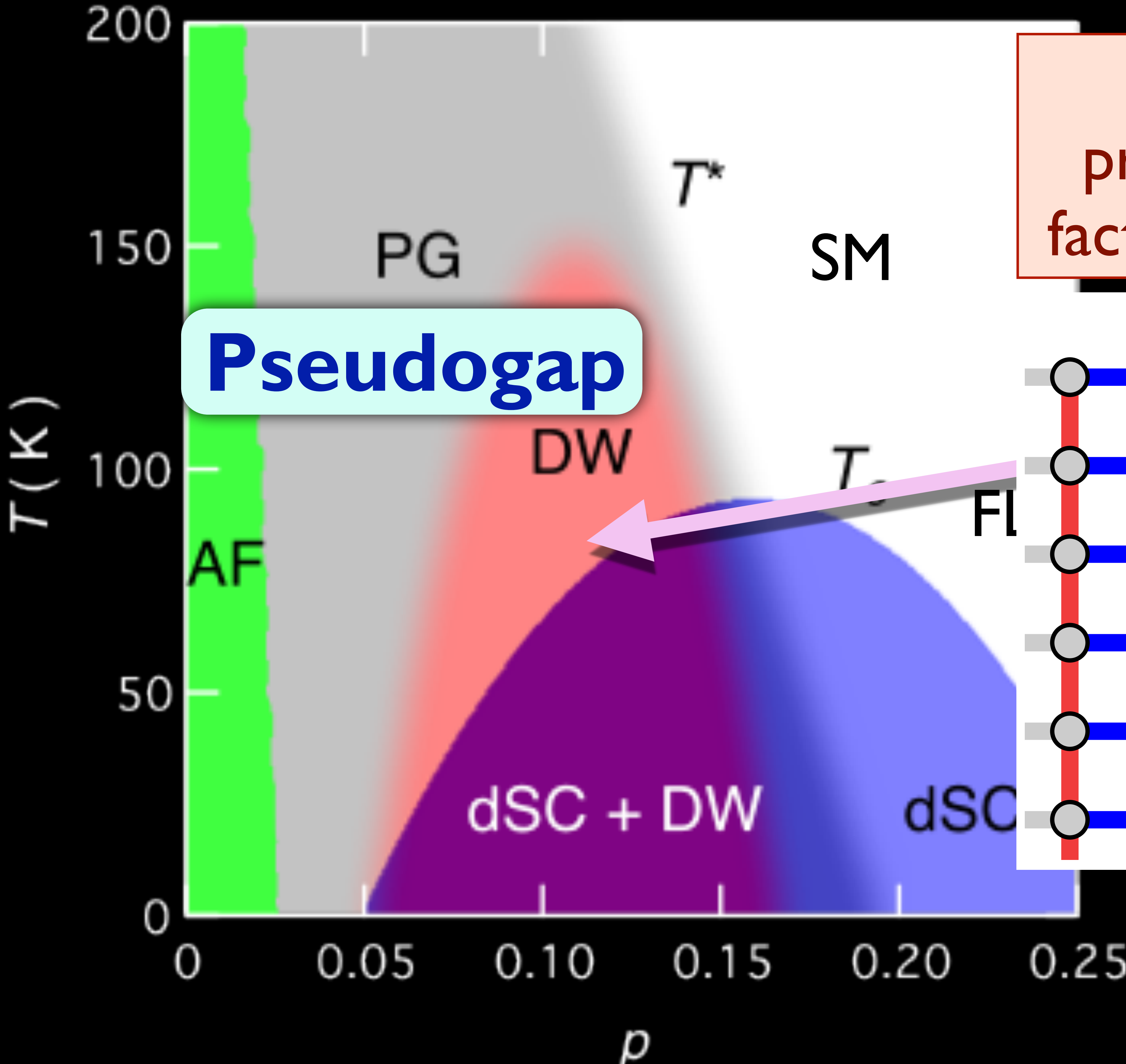
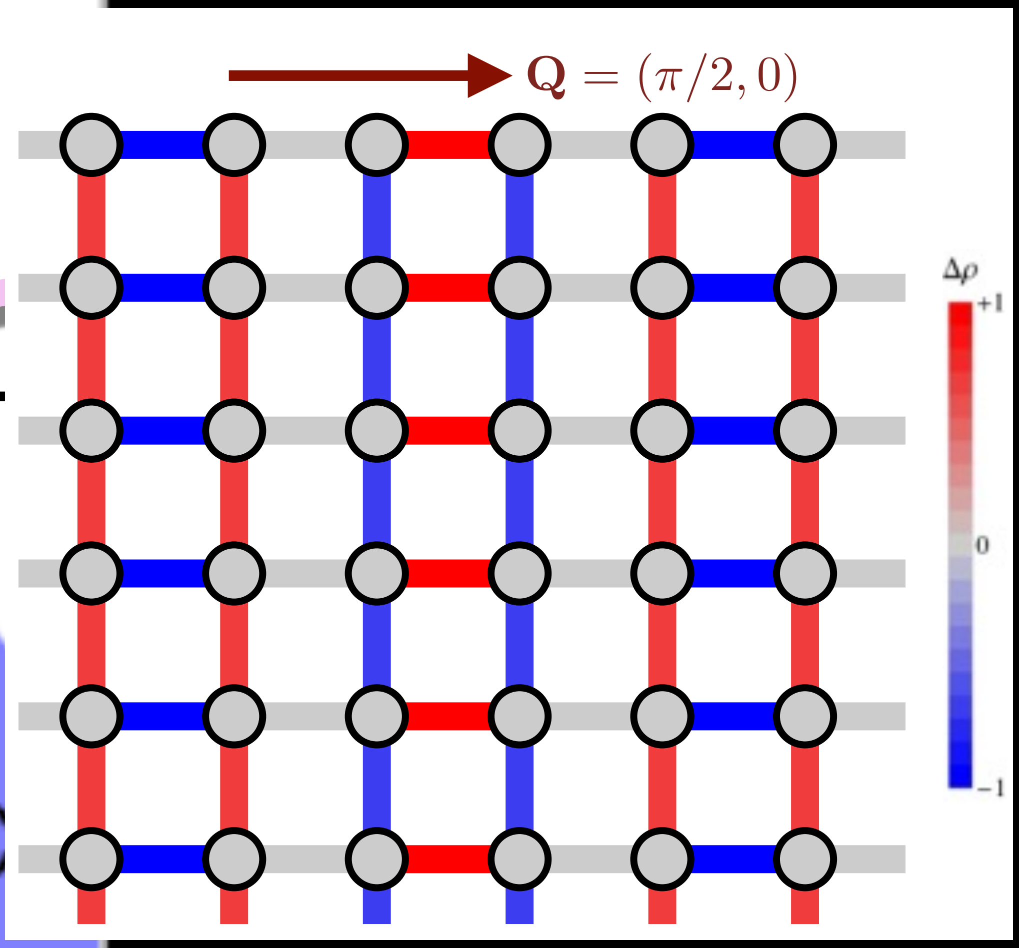
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



Identified as a predicted "*d*-form factor density wave"



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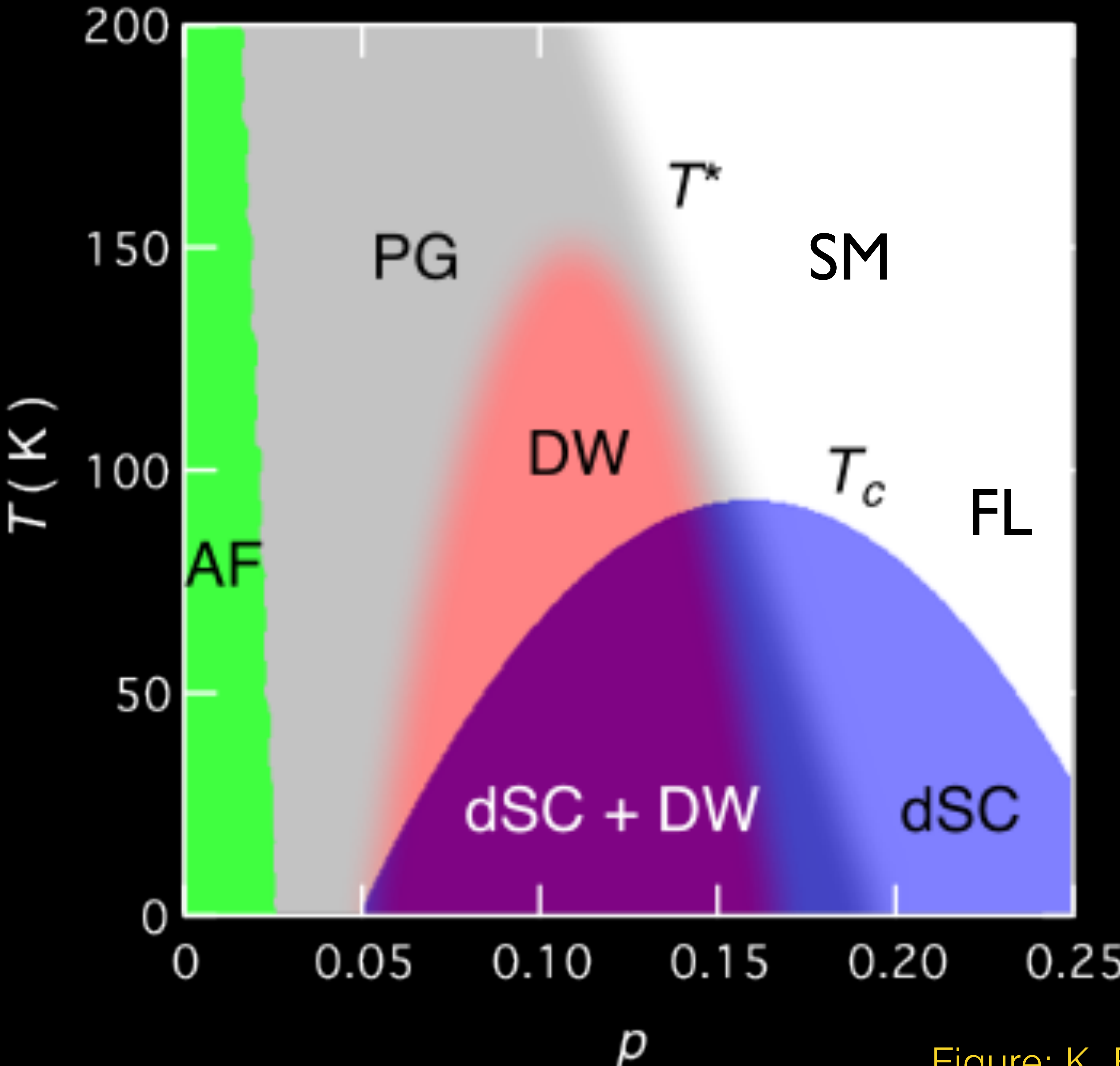

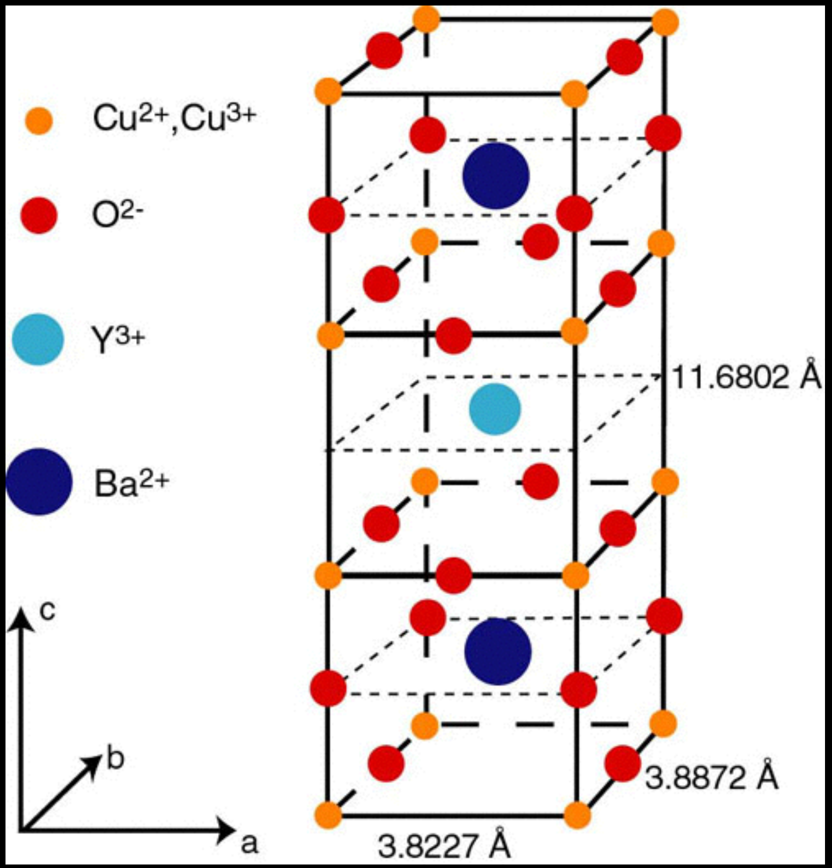

$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

High field, low T measurements show a positive Hall co-efficient corresponding to carriers of density $1 + p$

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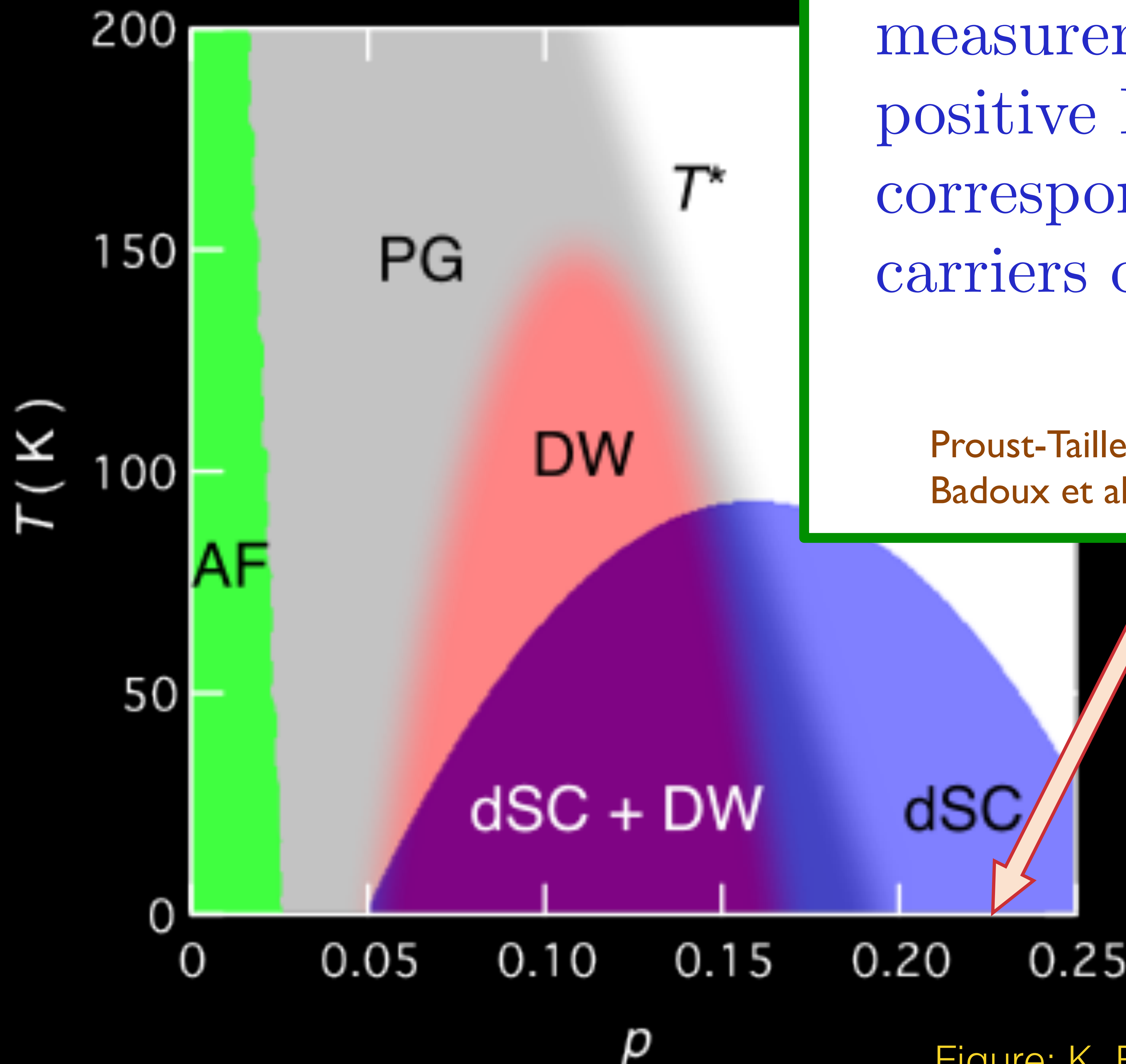

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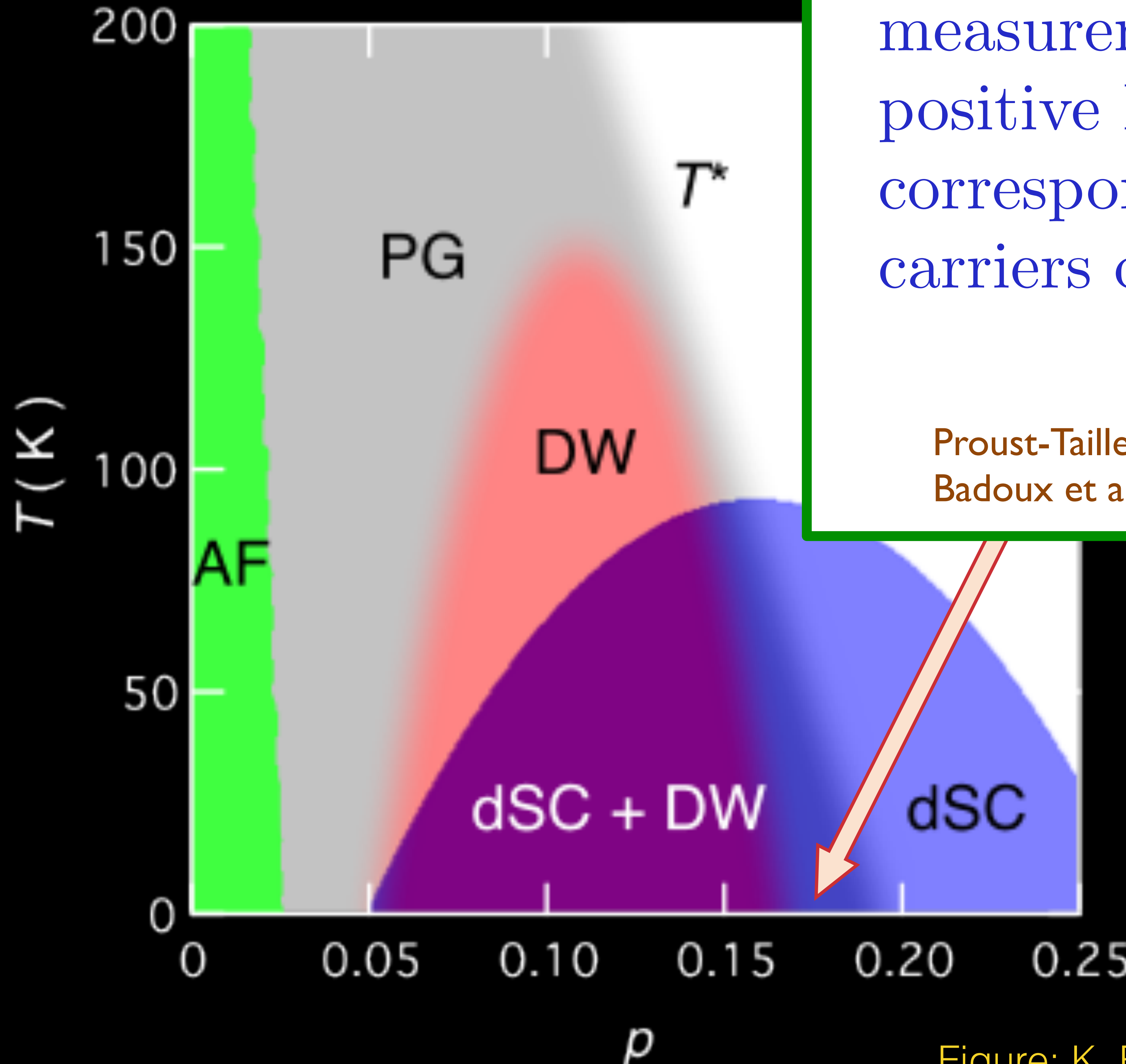

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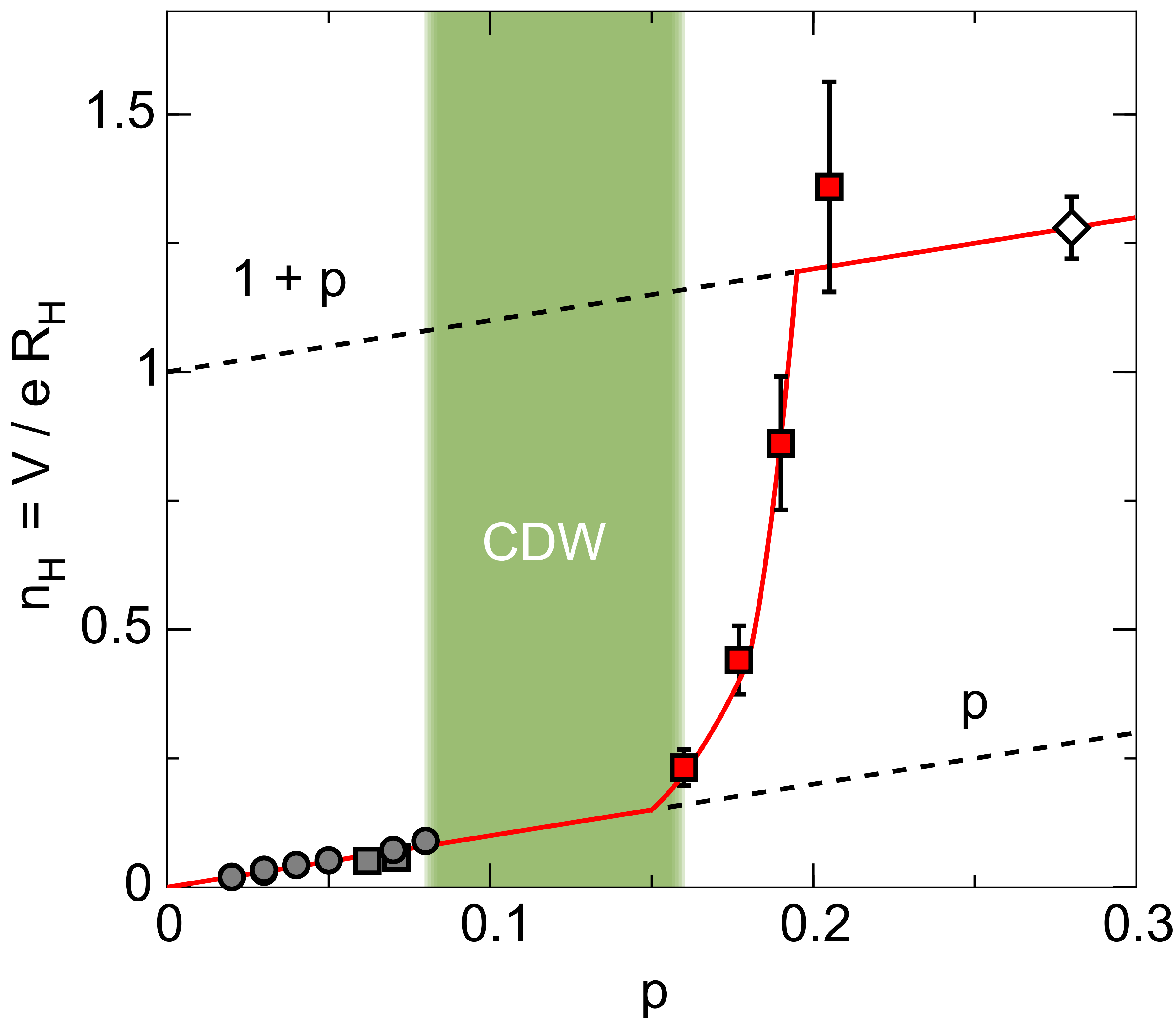

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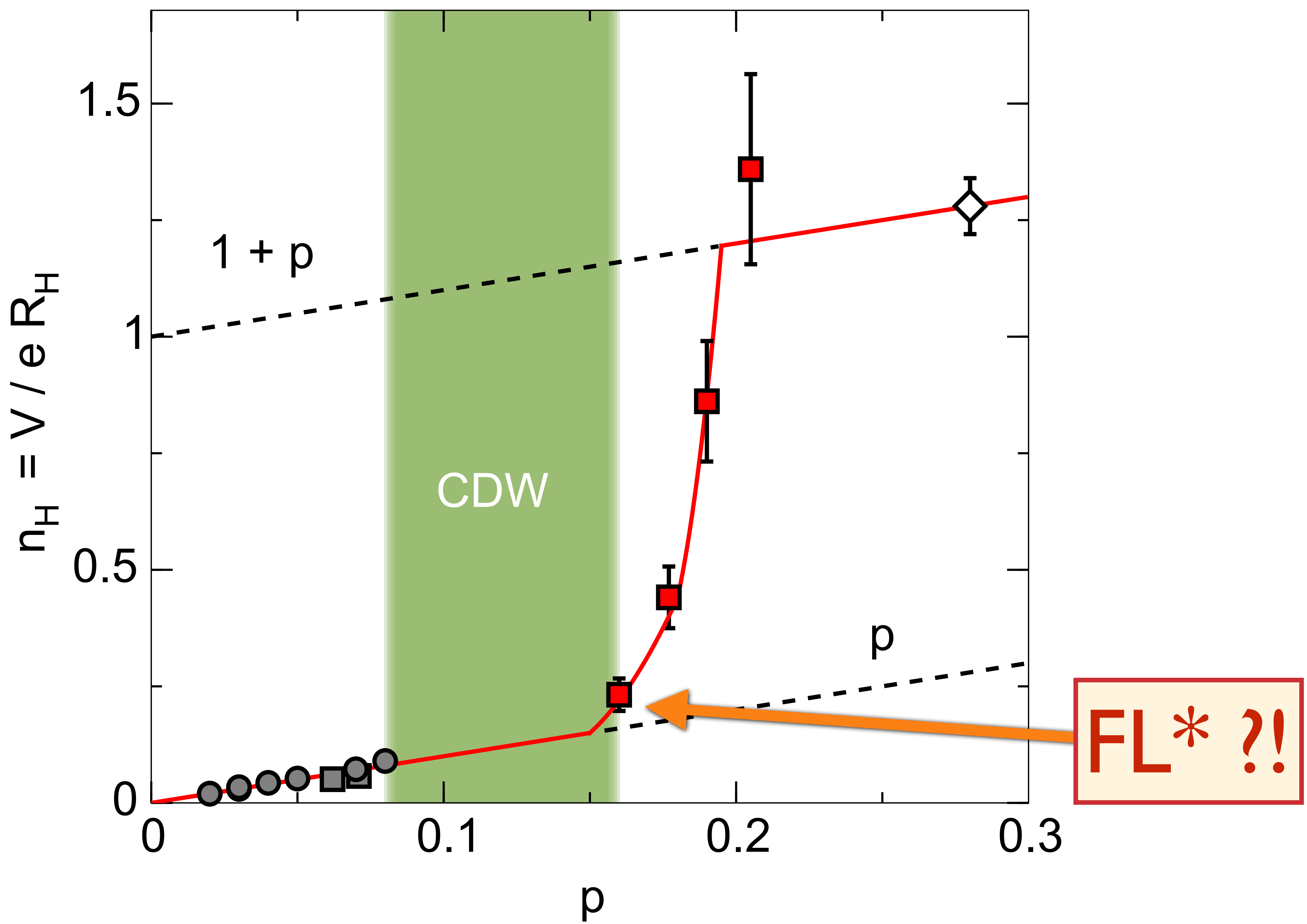

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