The phase diagrams of the high temperature superconductors

Talk online: sachdev.physics.harvard.edu

PHYSICS
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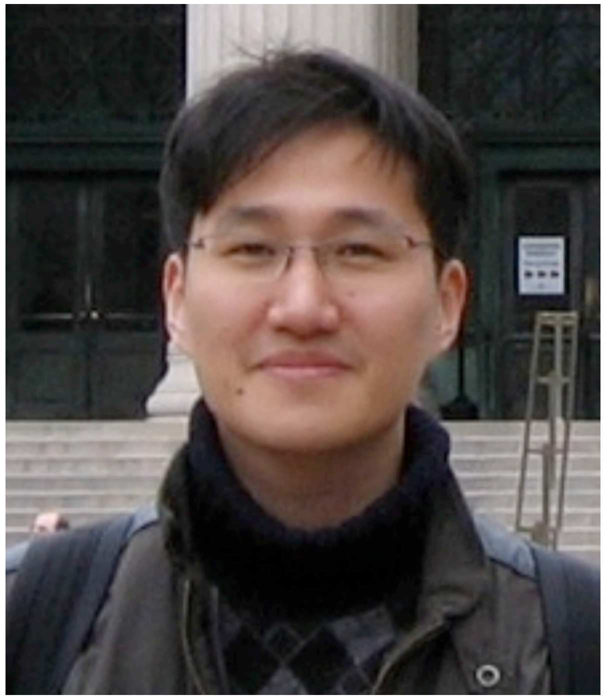
Max Metlitski, Harvard



Eun Gook Moon, Harvard

PHYSICS

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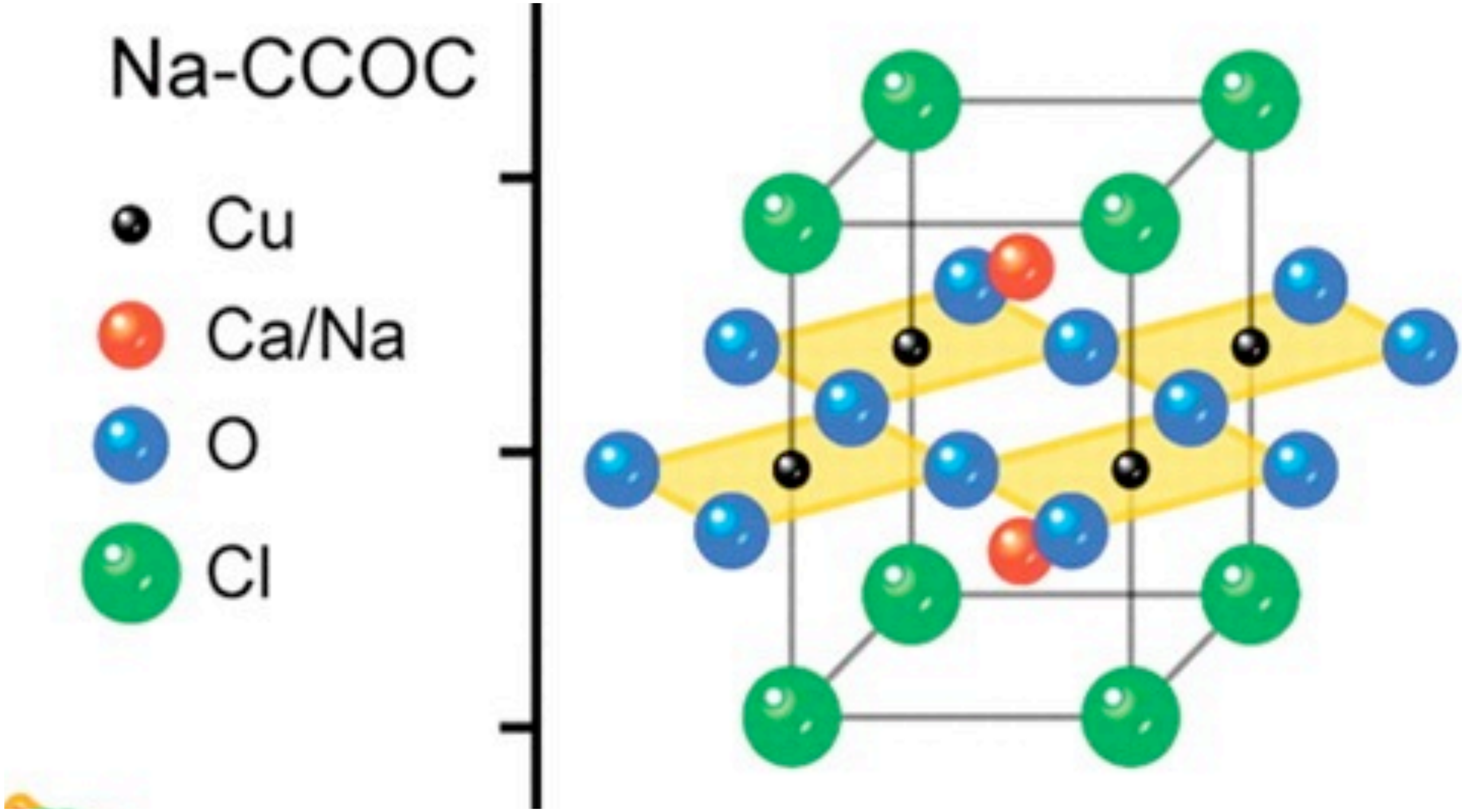
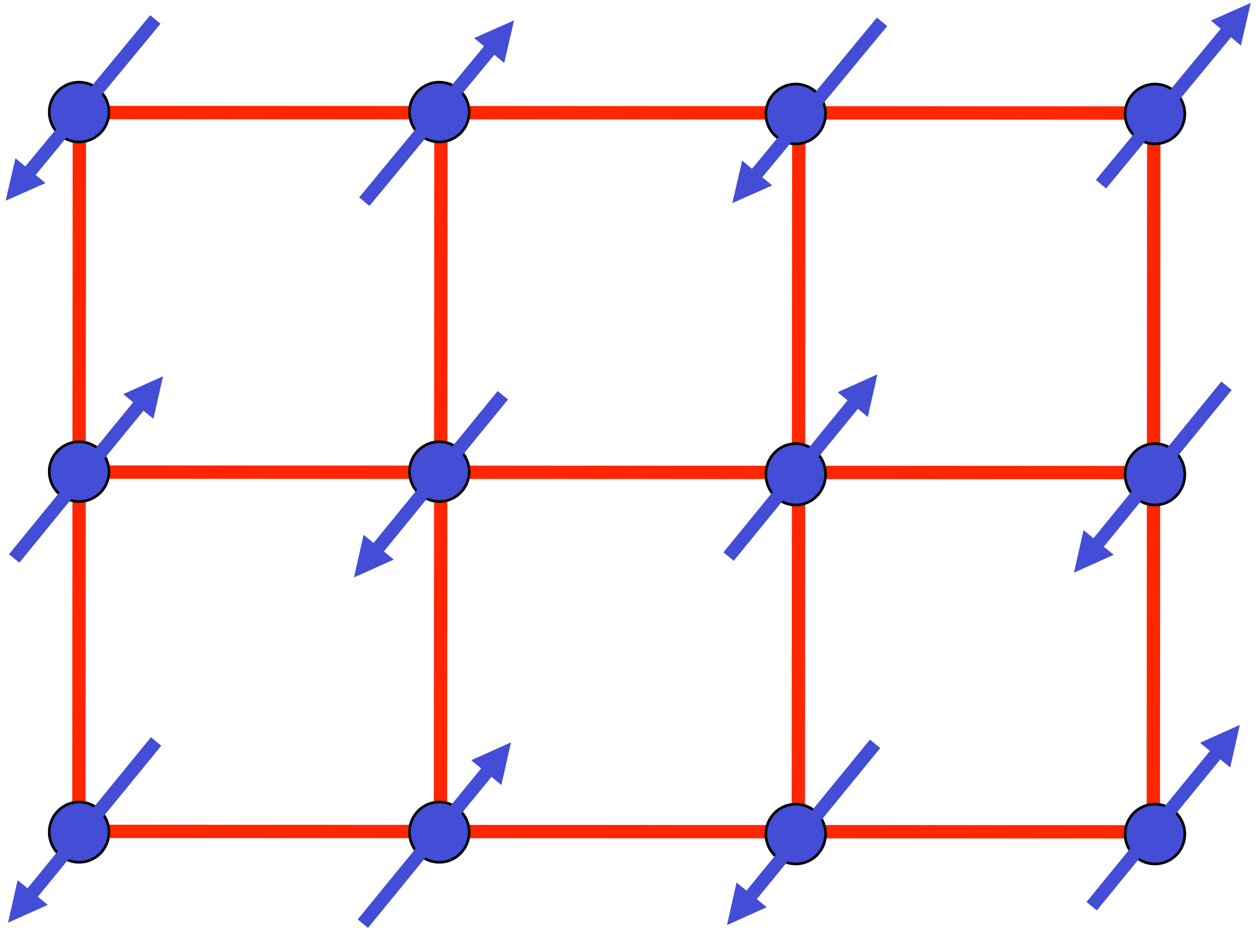
The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



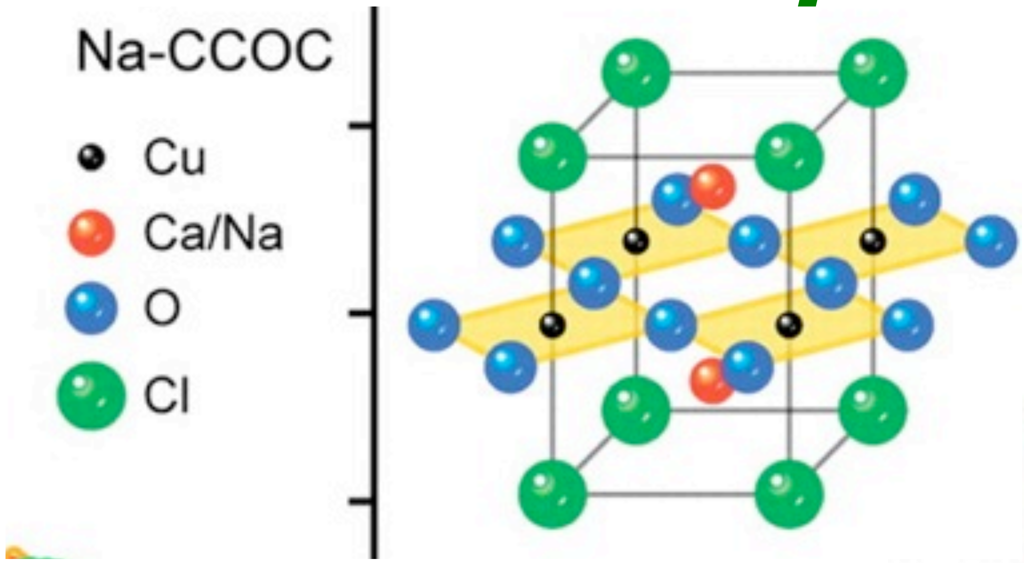
Ground state has long-range Néel order

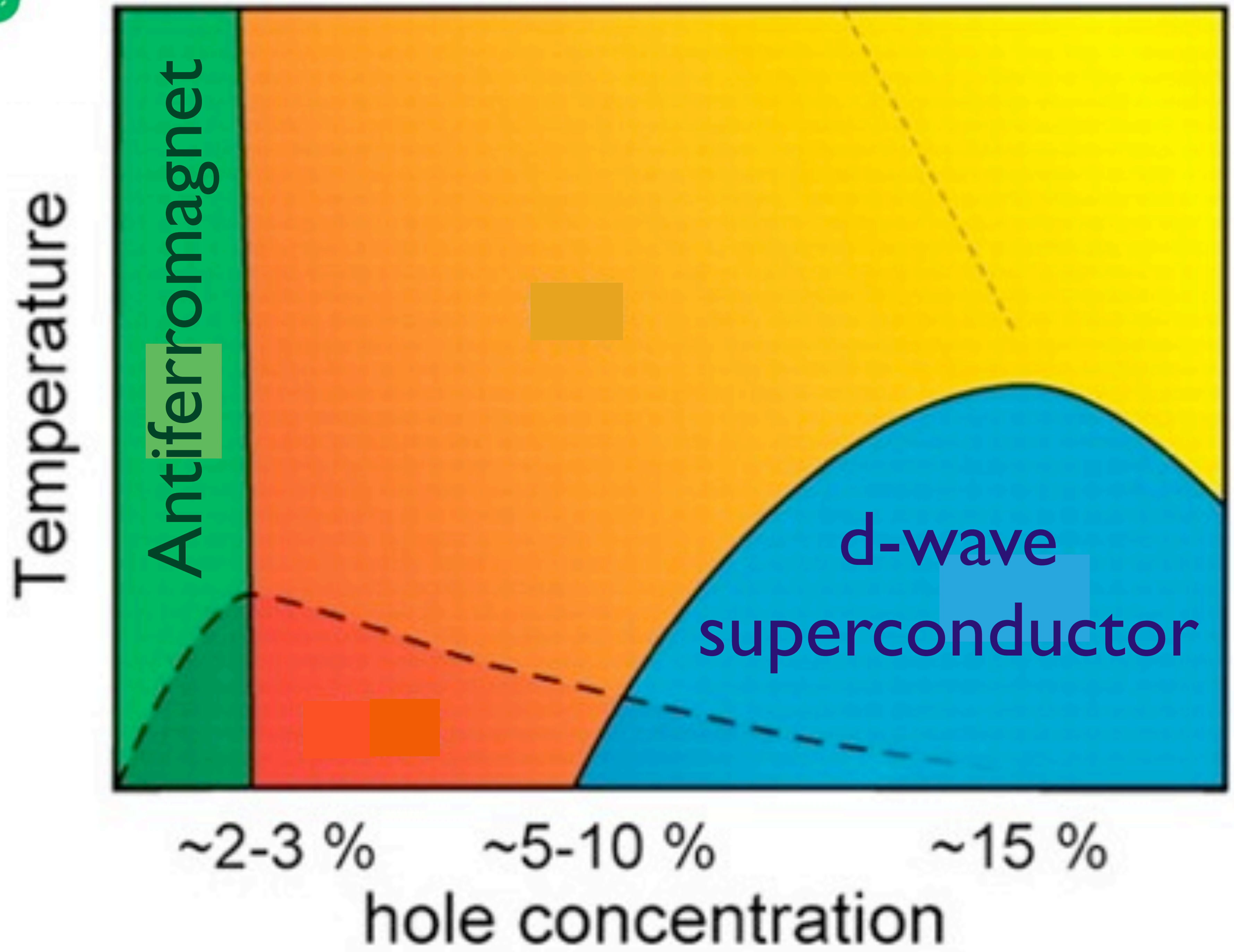
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

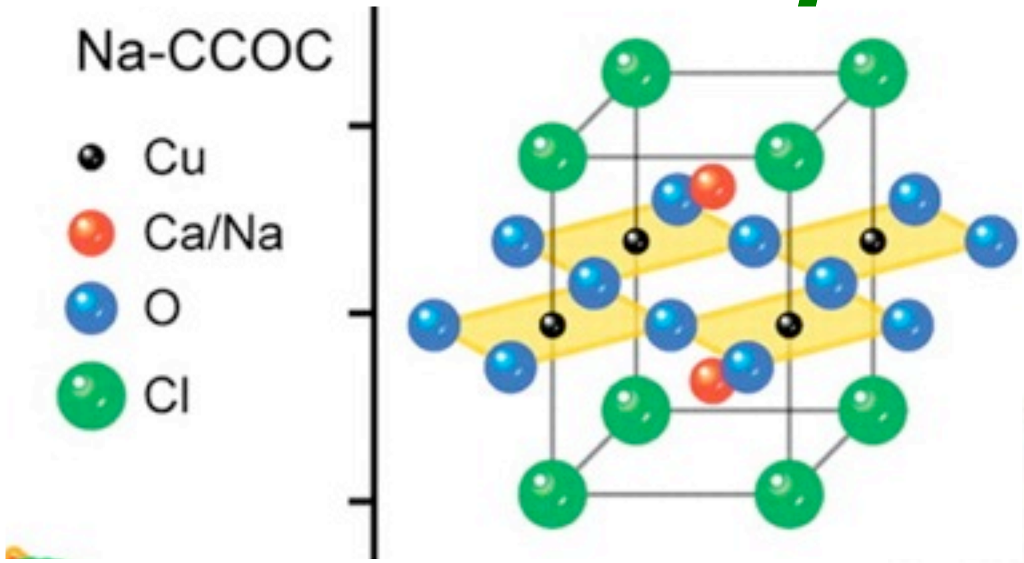
$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

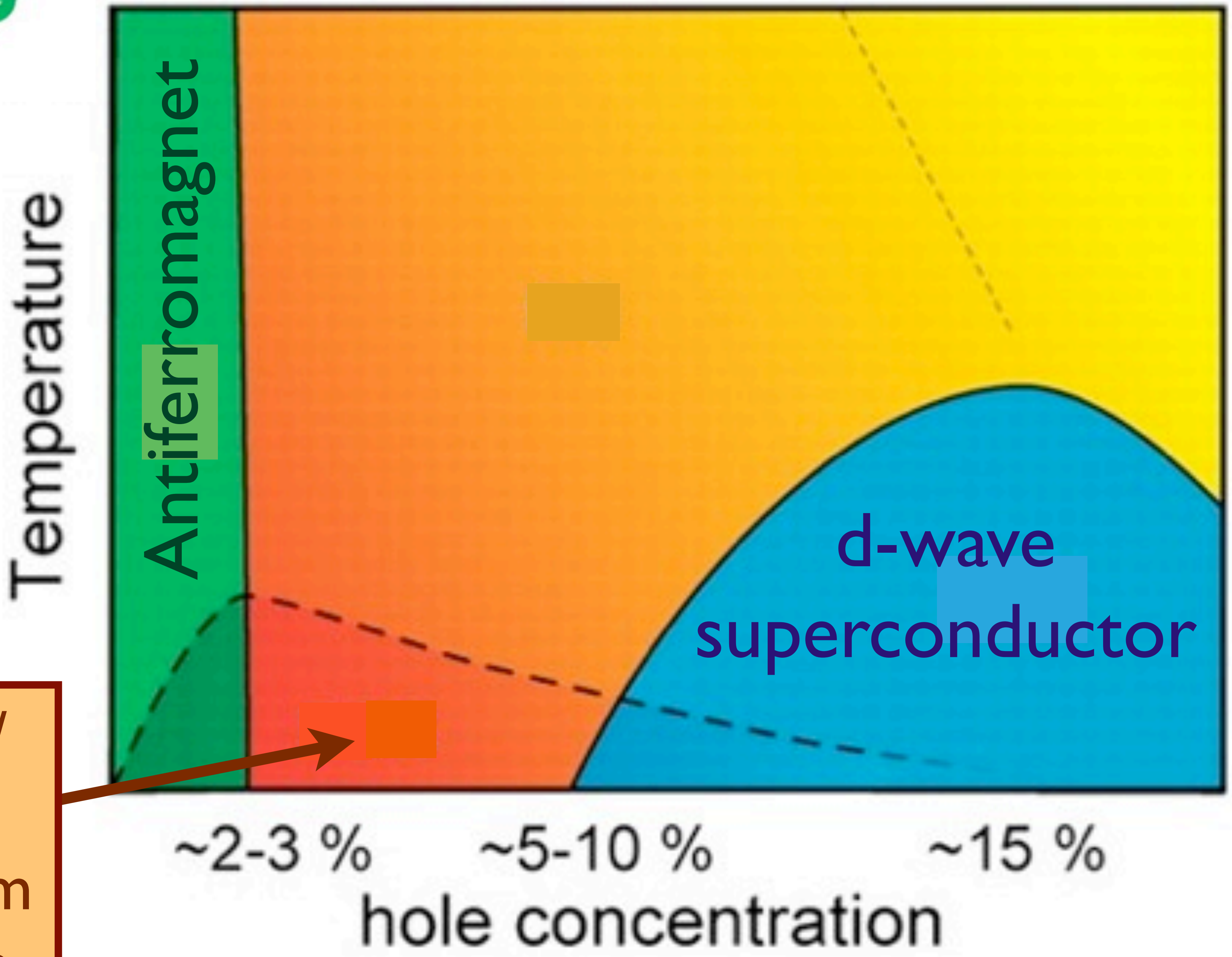
The cuprate superconductors





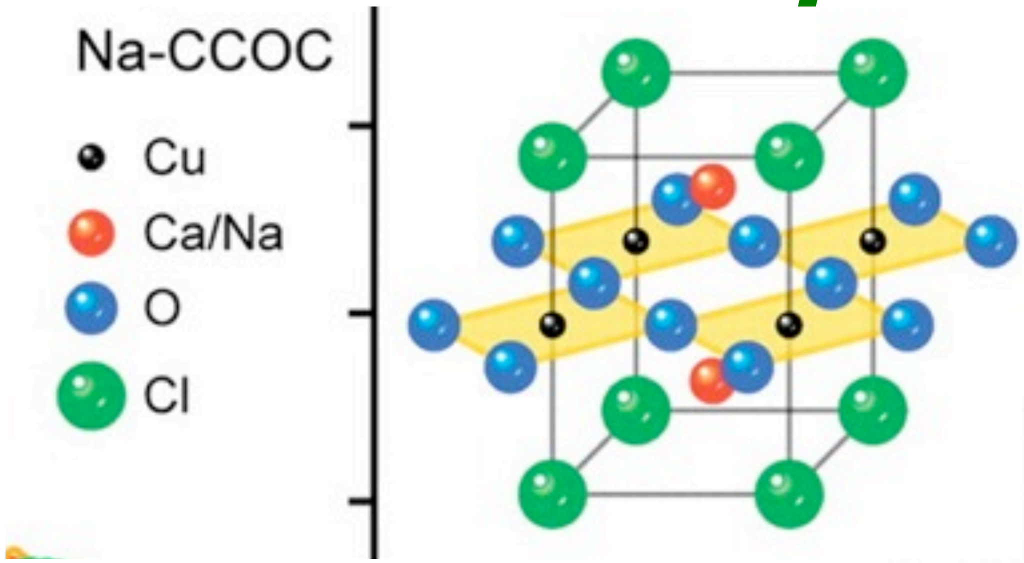
The cuprate superconductors

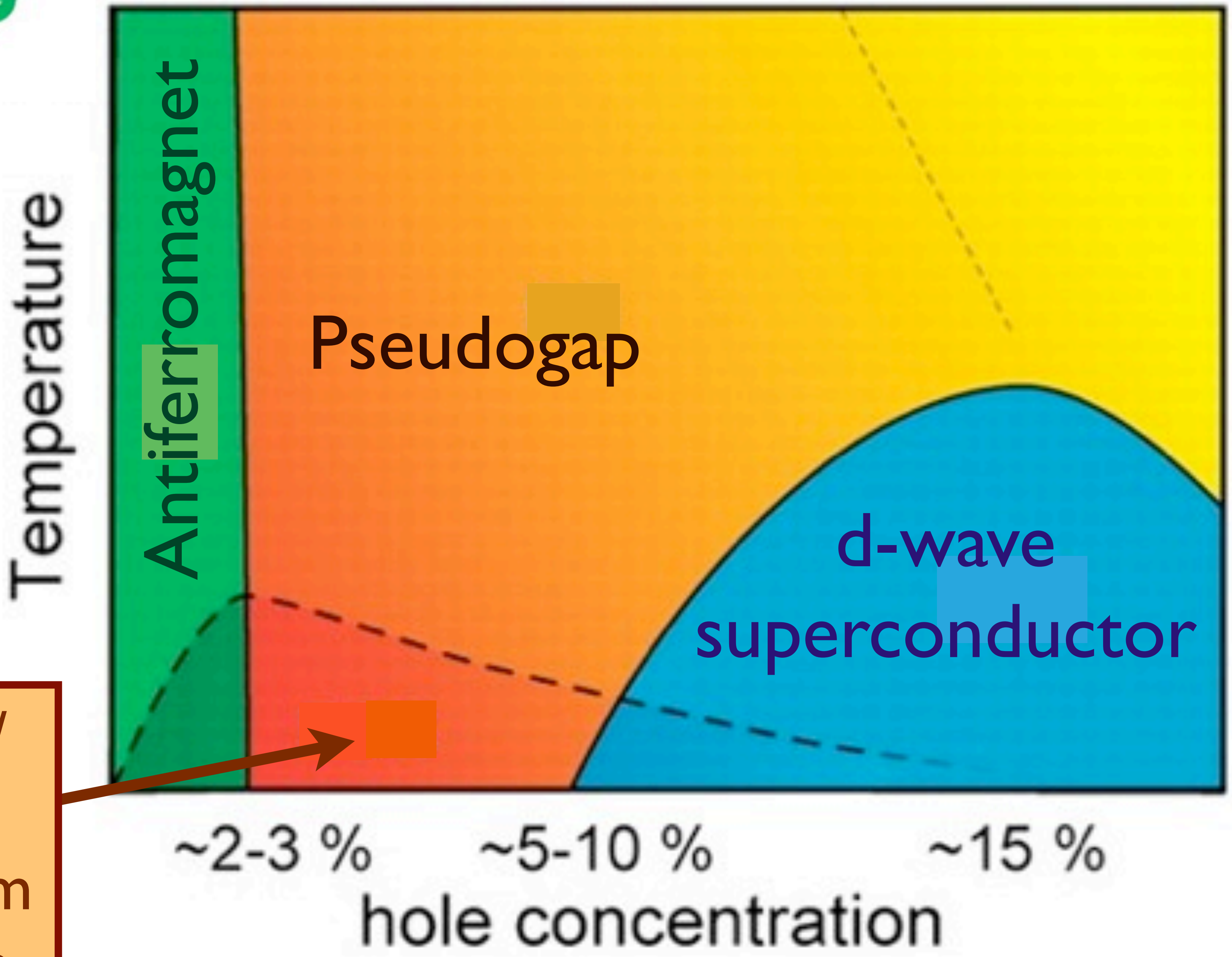




Incommensurate/ disordered antiferromagnetism and charge order

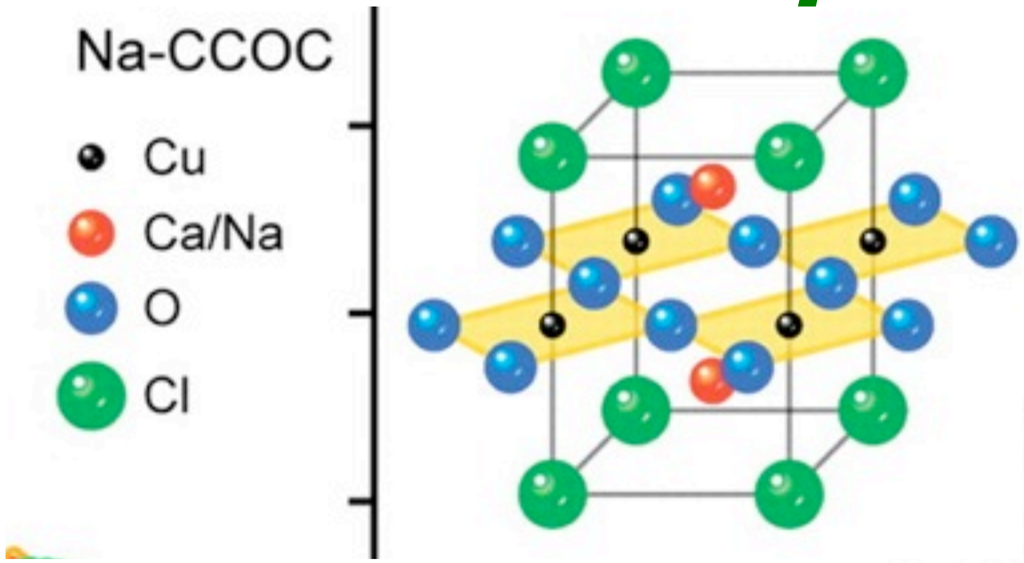
The cuprate superconductors

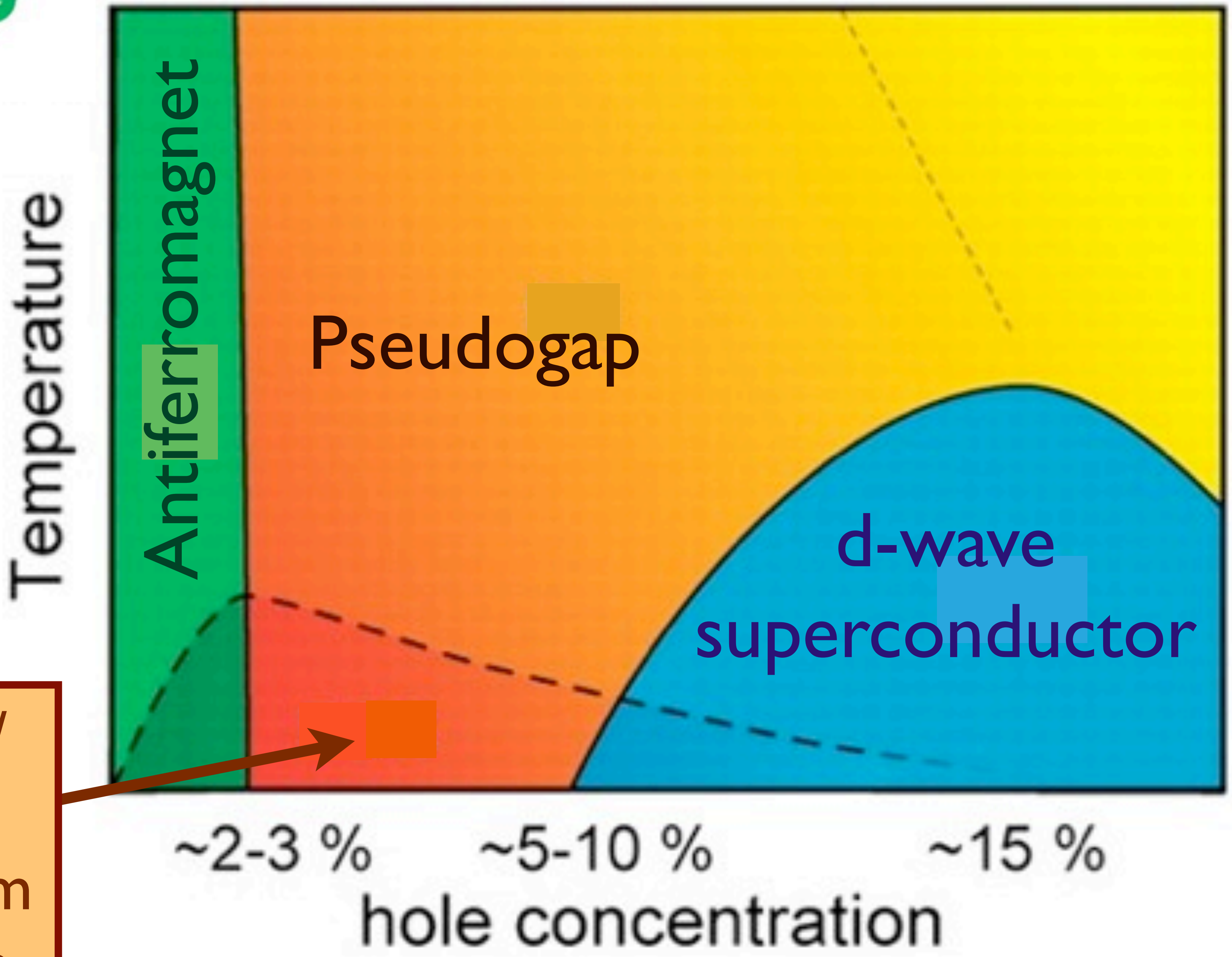




Incommensurate/
disordered
antiferromagnetism
and charge order

The cuprate superconductors





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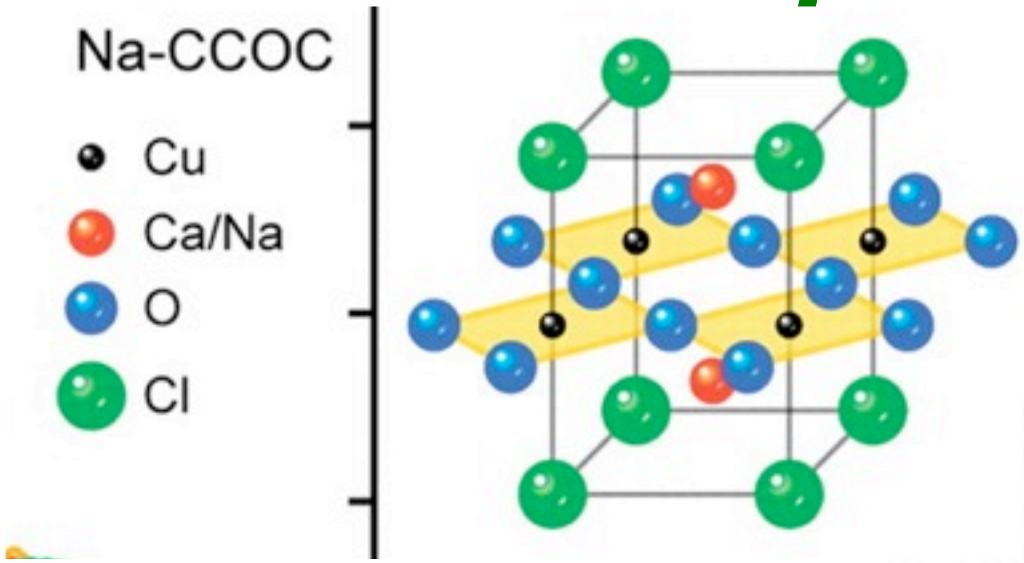

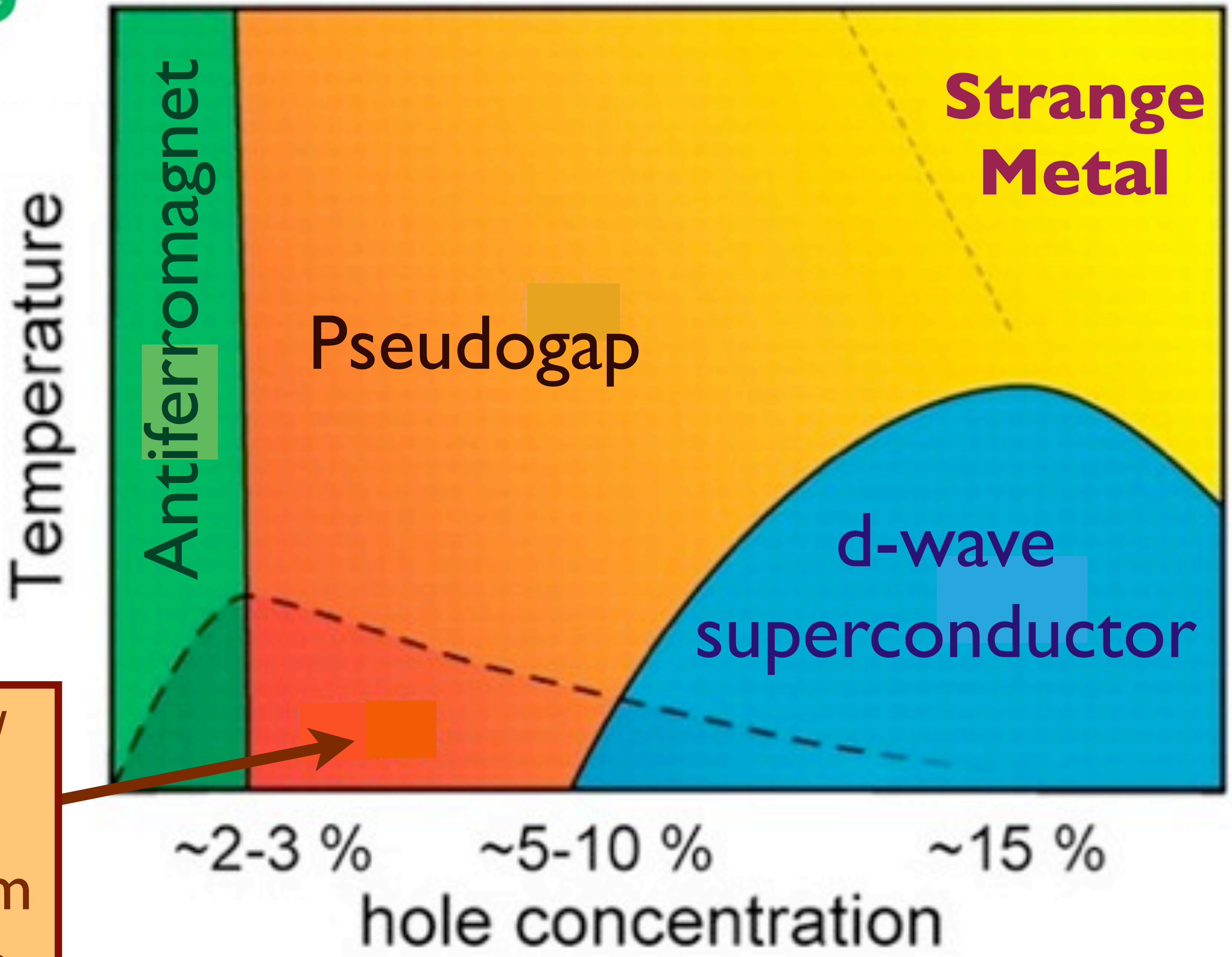

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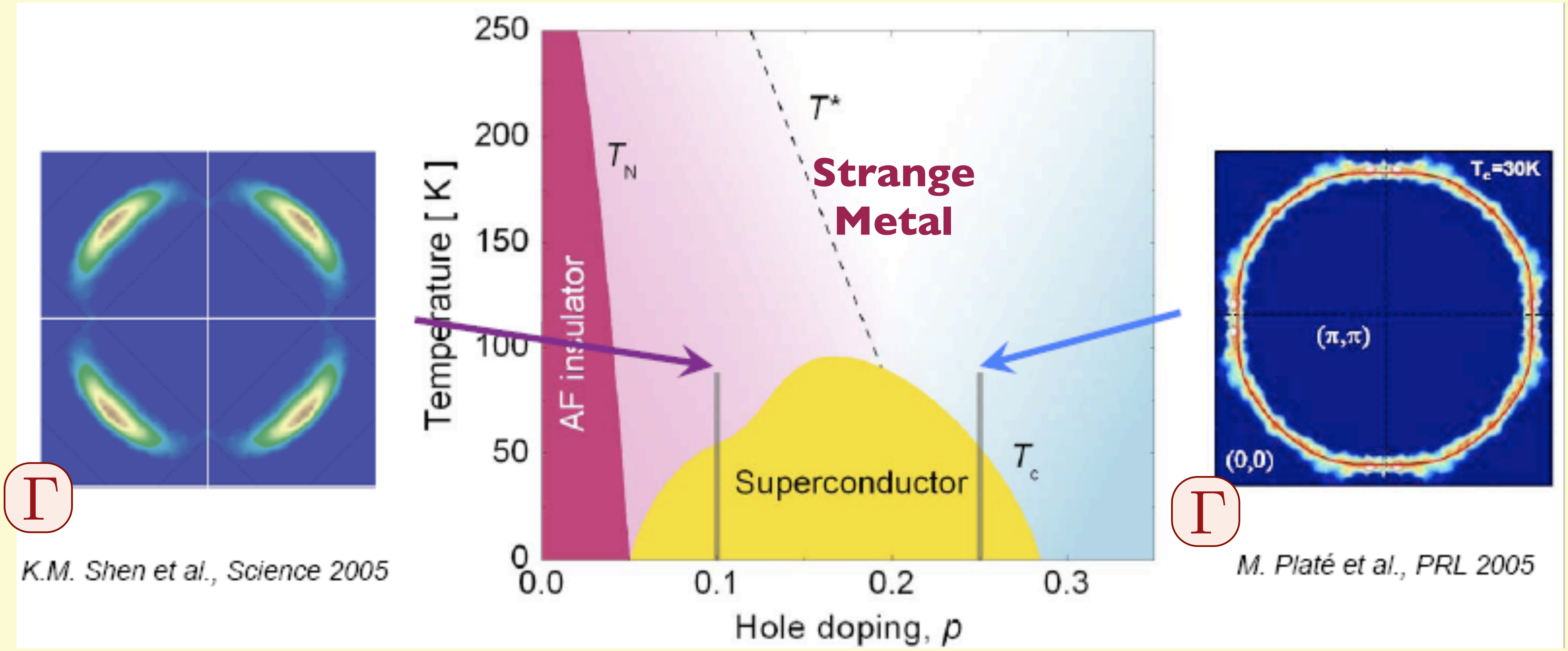


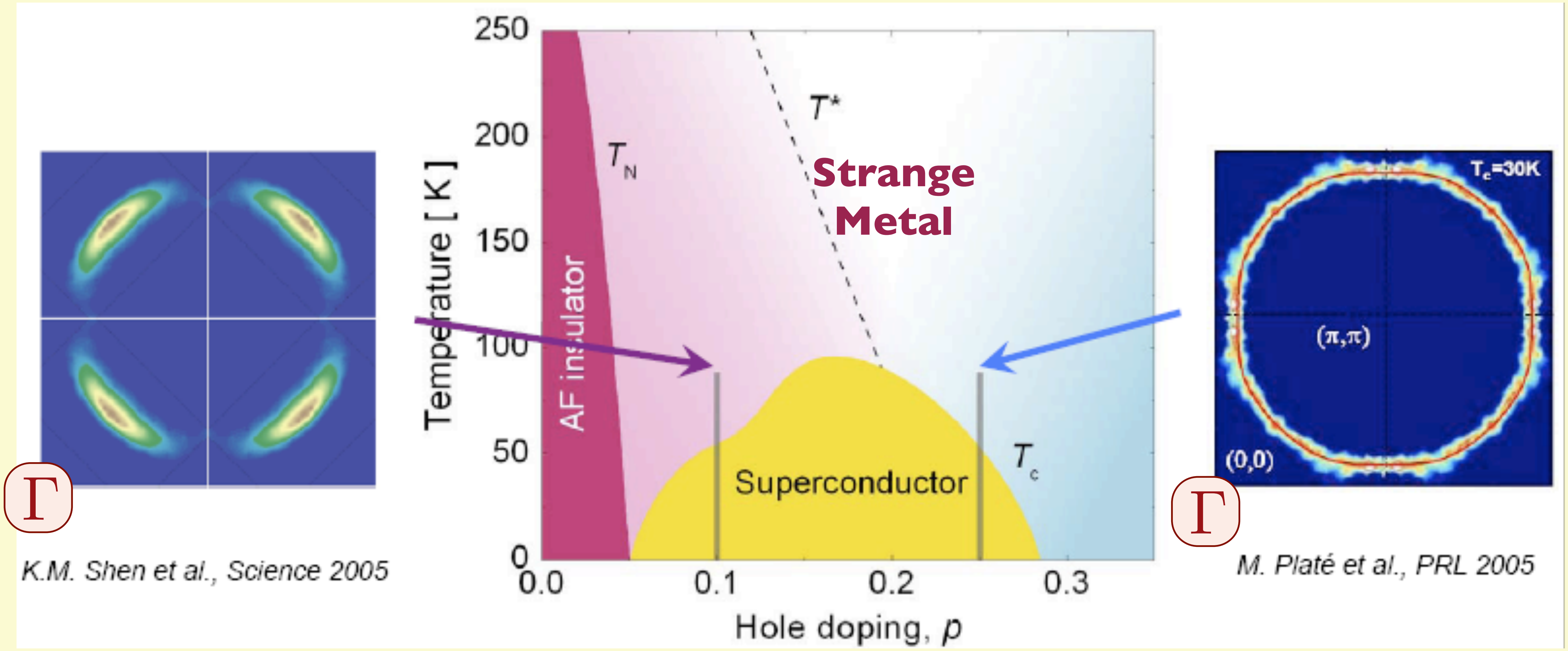
K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface

Key Ingredients:



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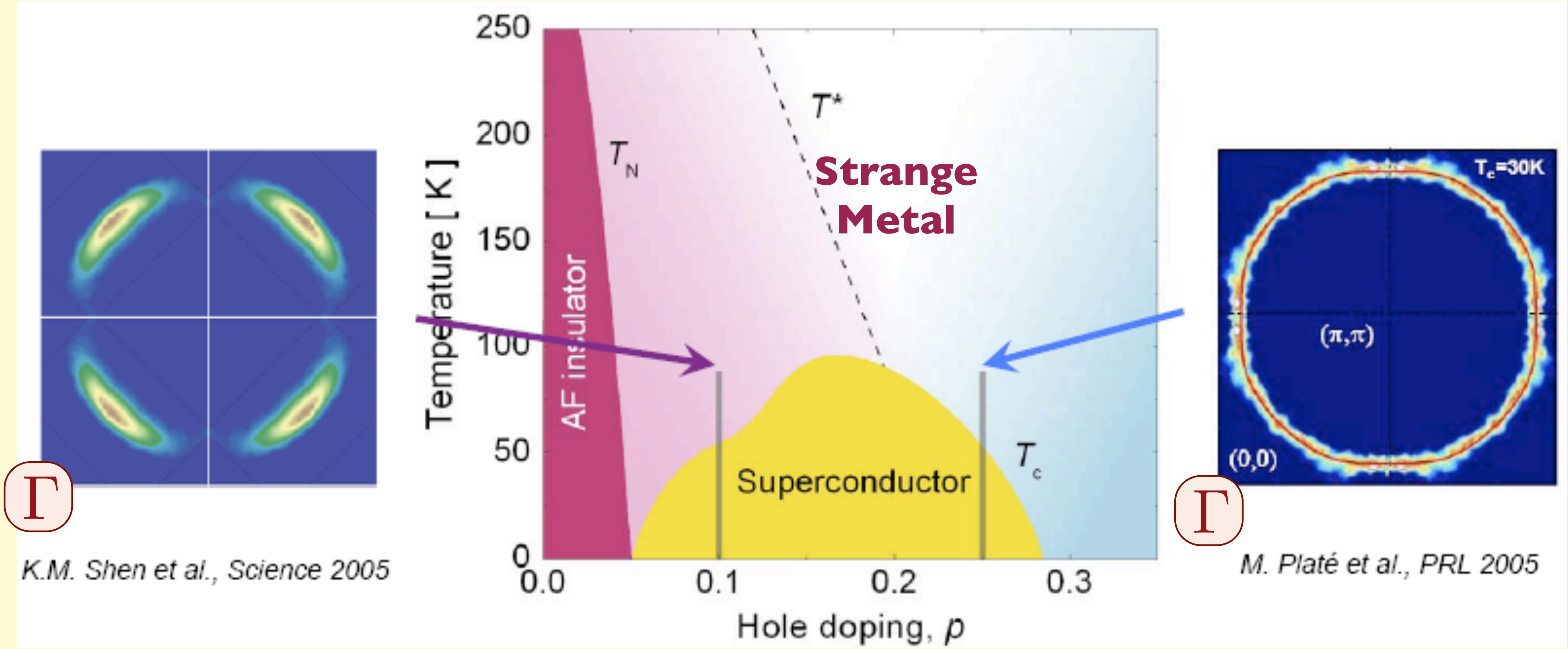
Key Ingredients:

Antiferromagnetism (AF)
Spin density wave (SDW)



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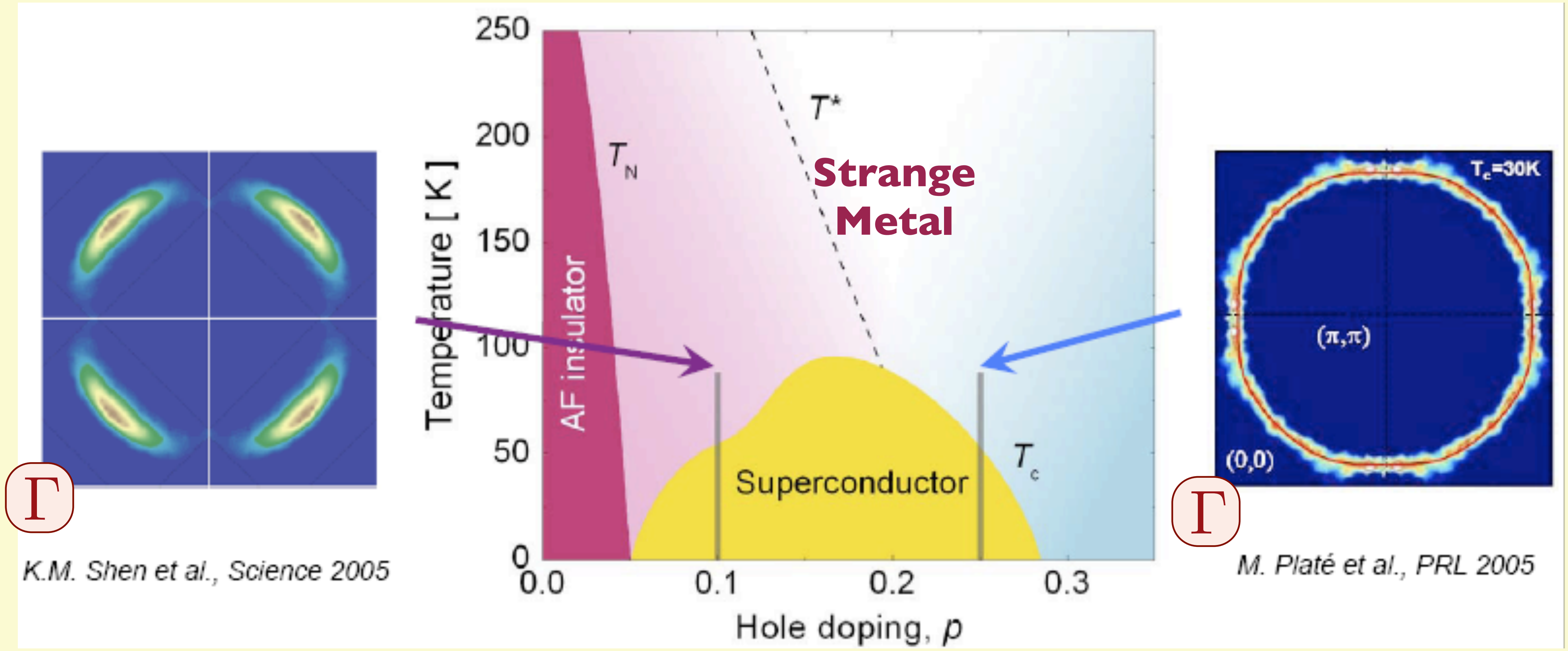

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Key Ingredients:

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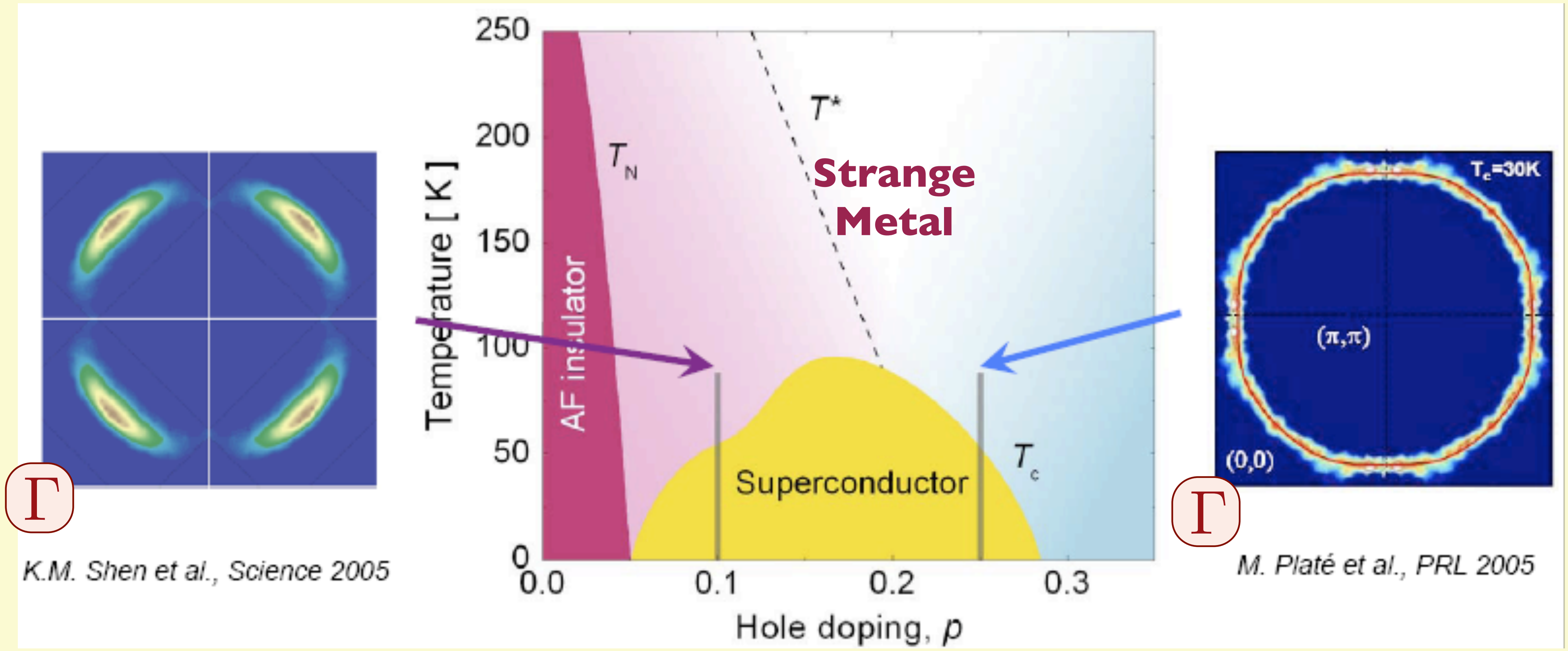
Fermi surface reconstruction



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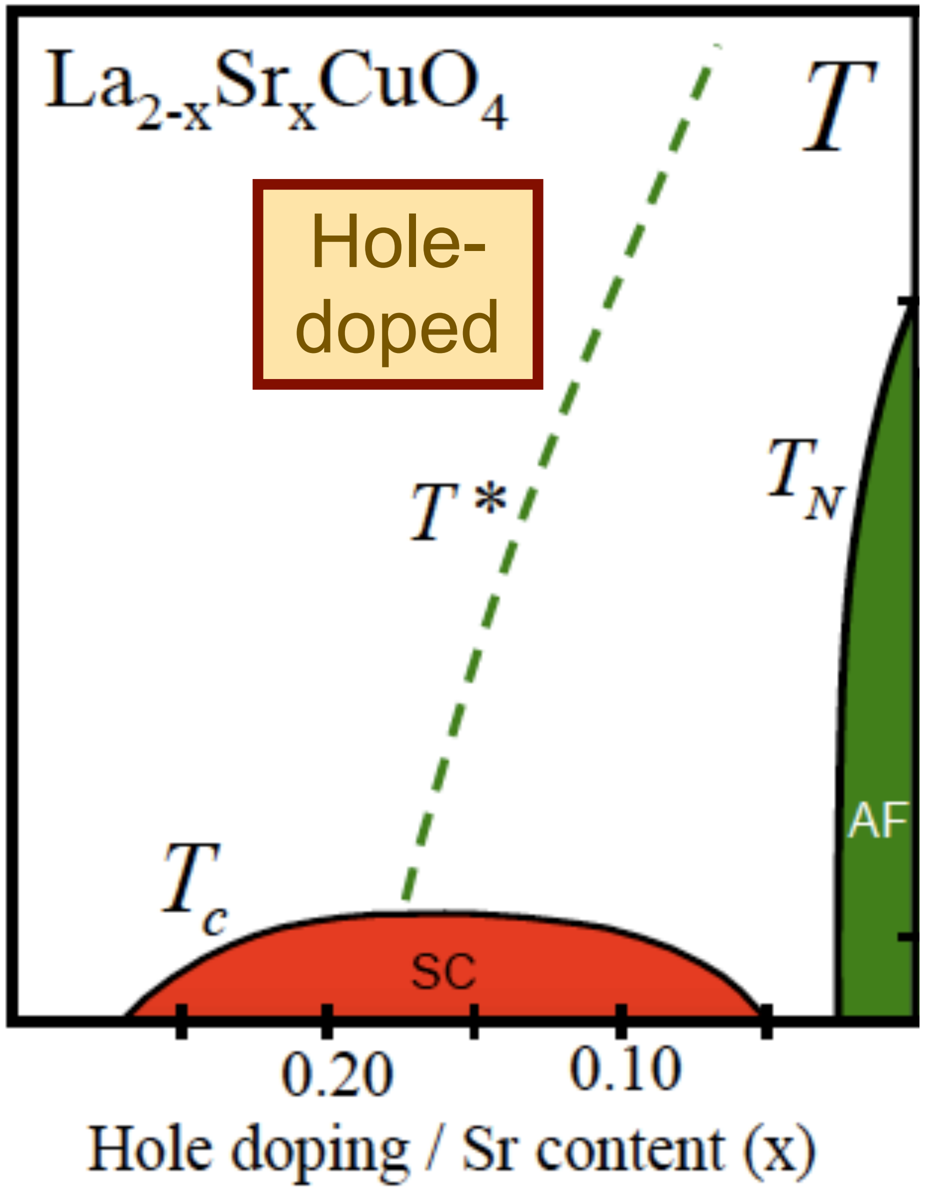
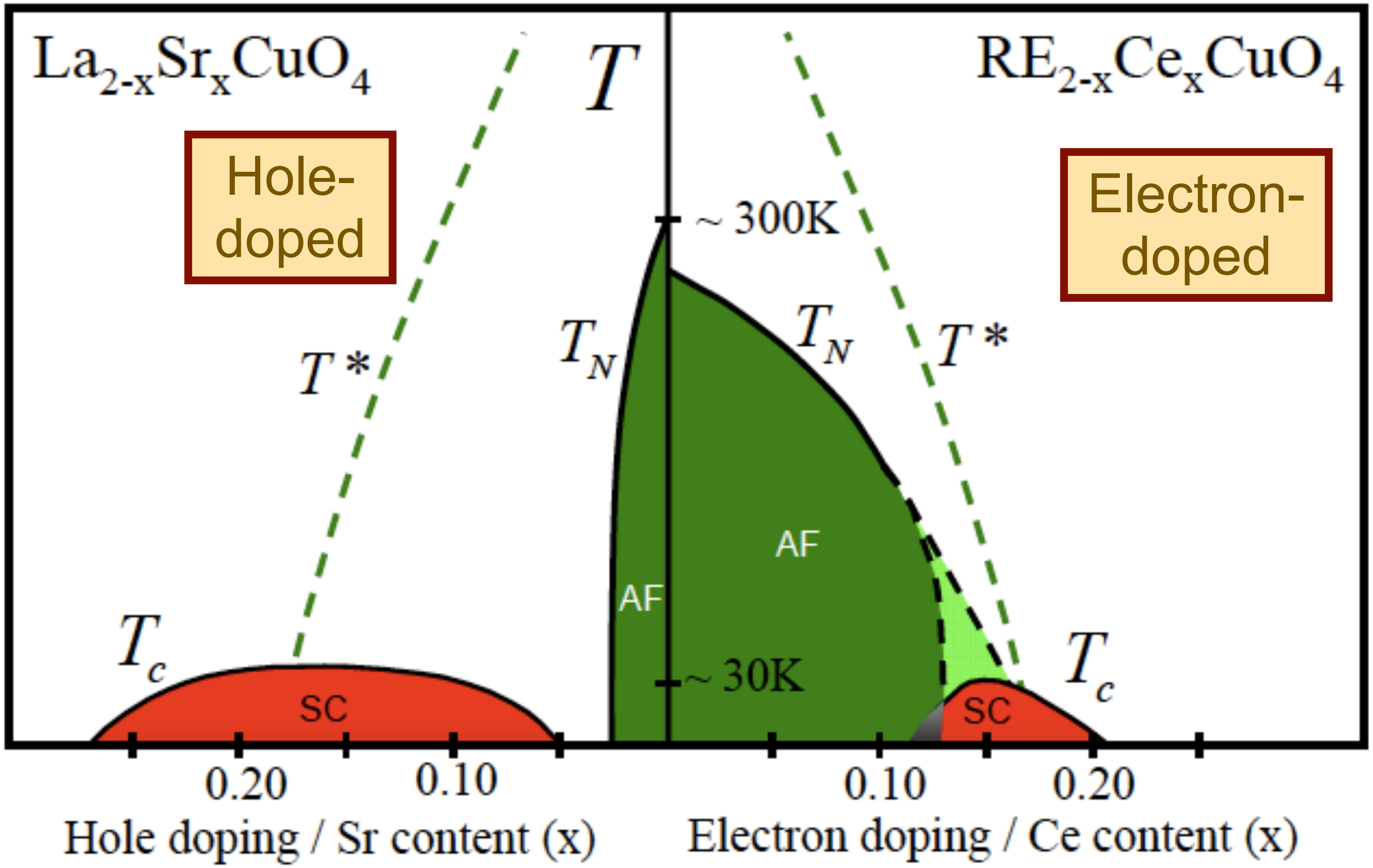

$La_{2-x}Sr_xCuO_4$
Hole-doped
T
T^*
T_N
T_c
SC
AF
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors



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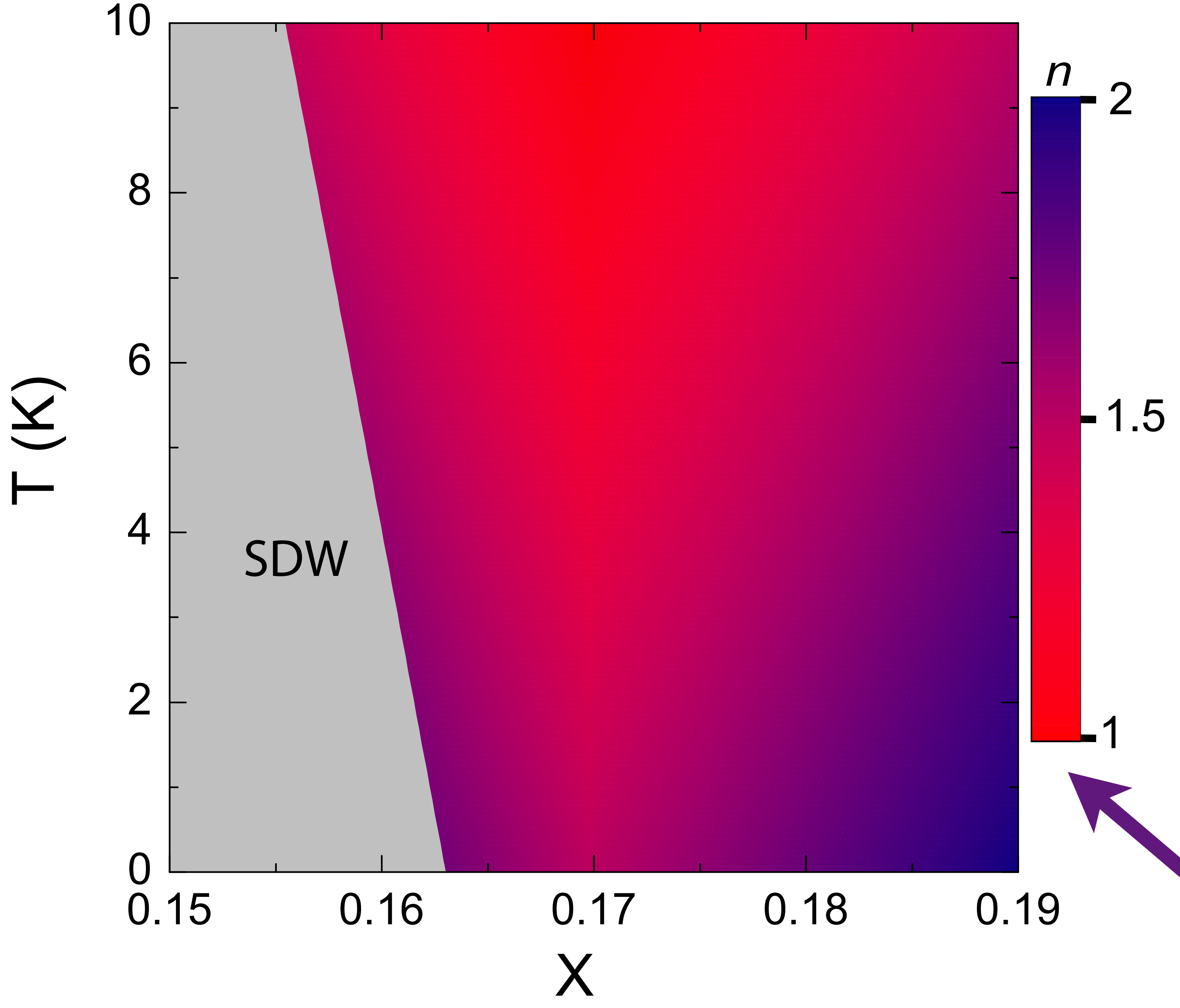
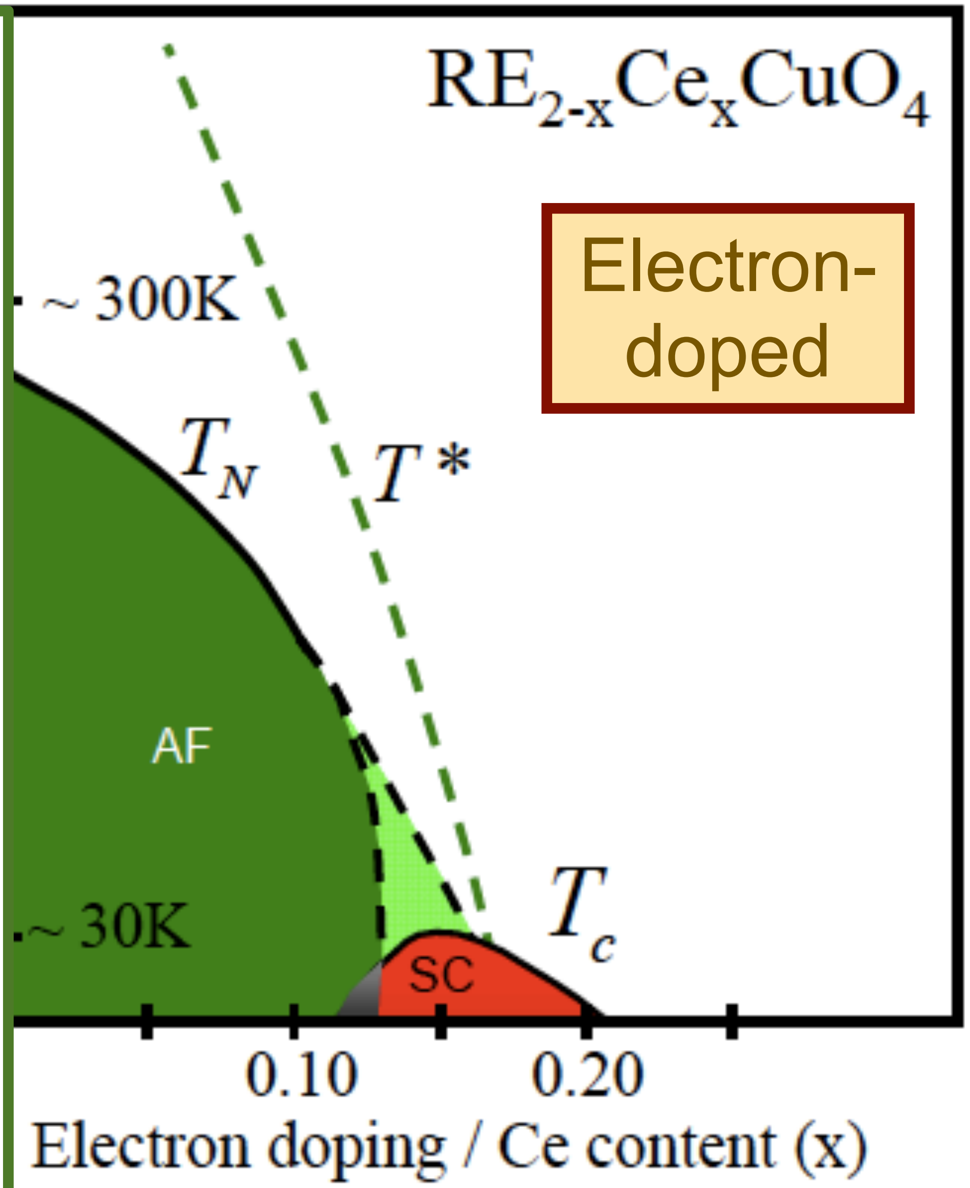


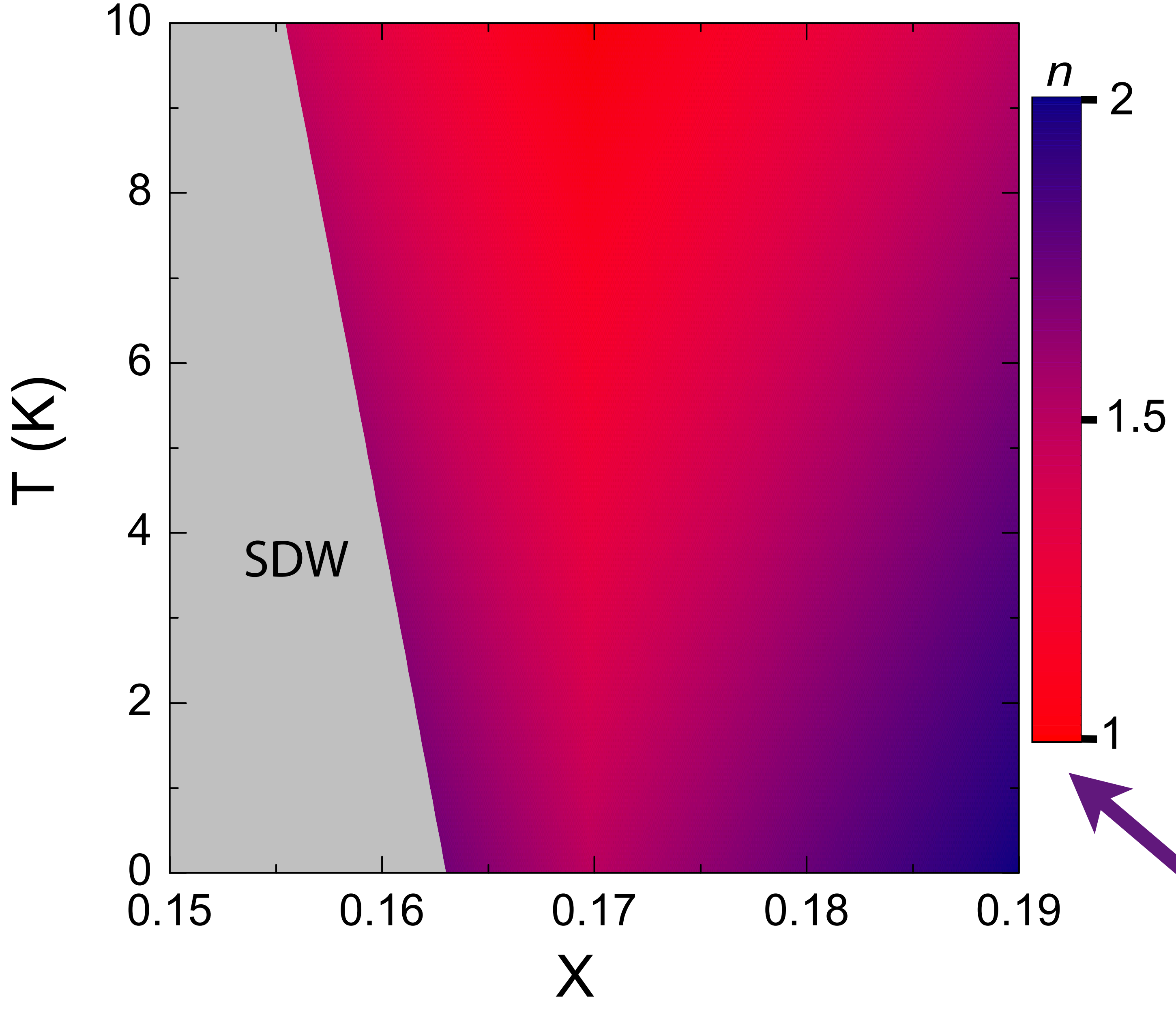
Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity

$$\sim \rho_0 + AT^n$$

Electron-doped cuprate superconductors



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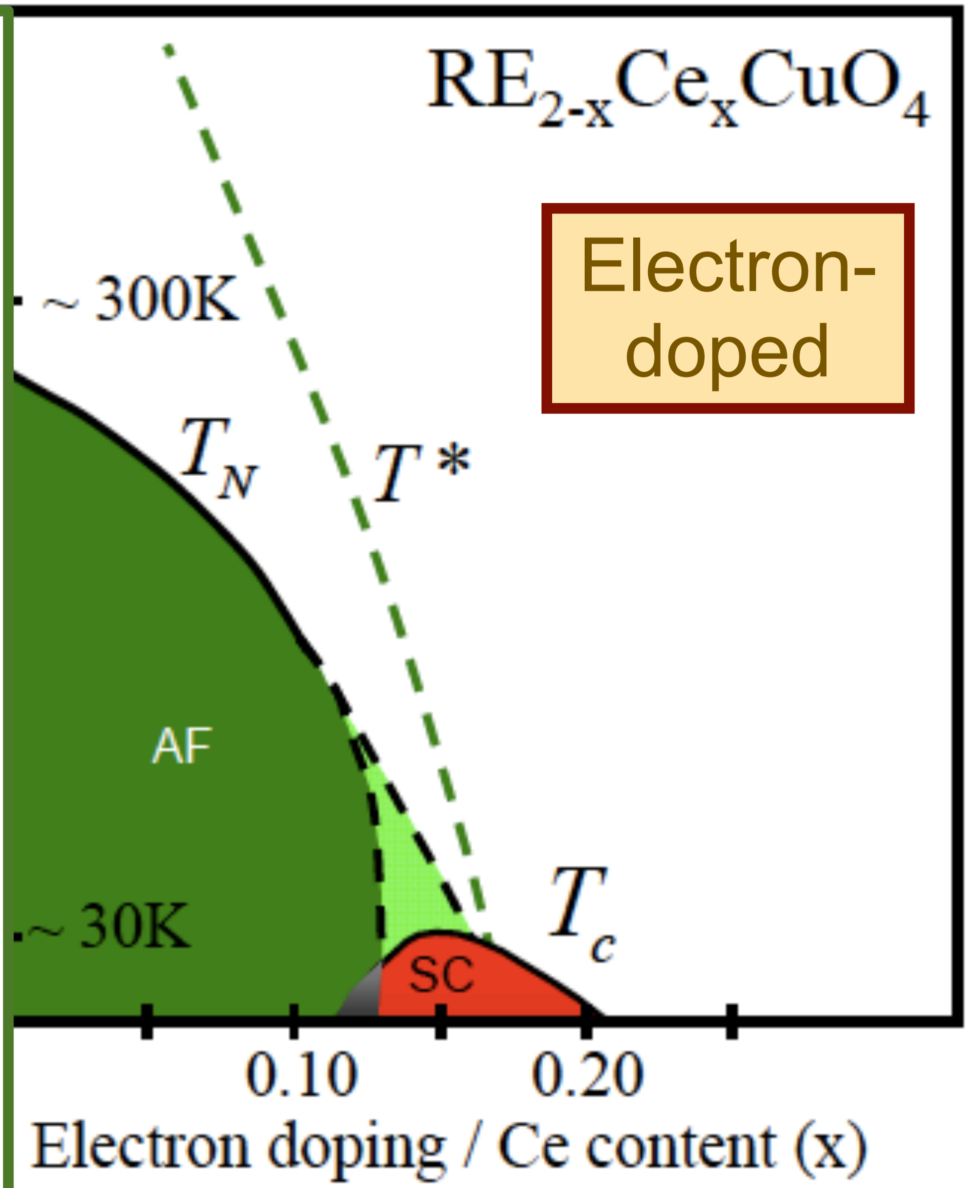

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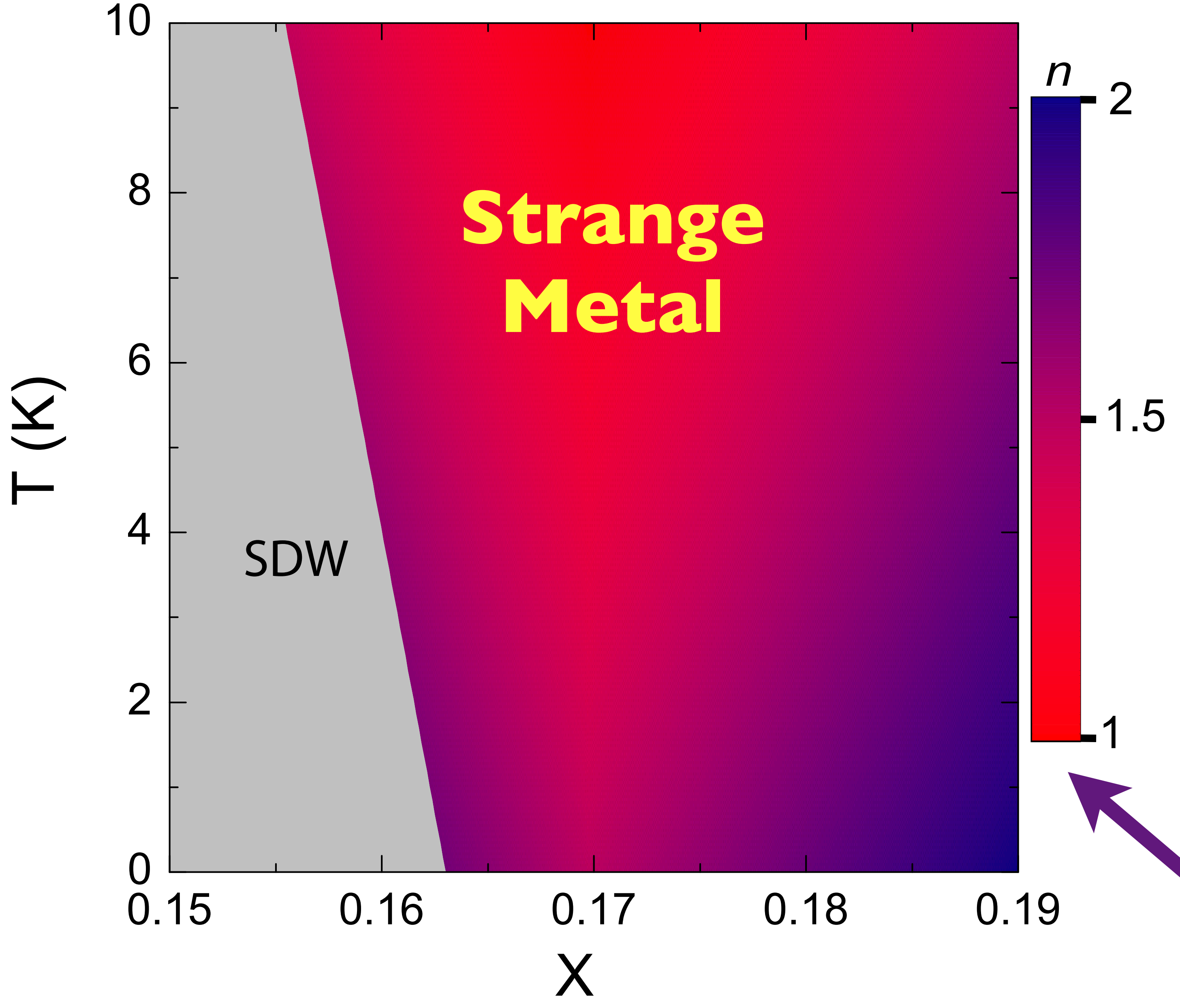

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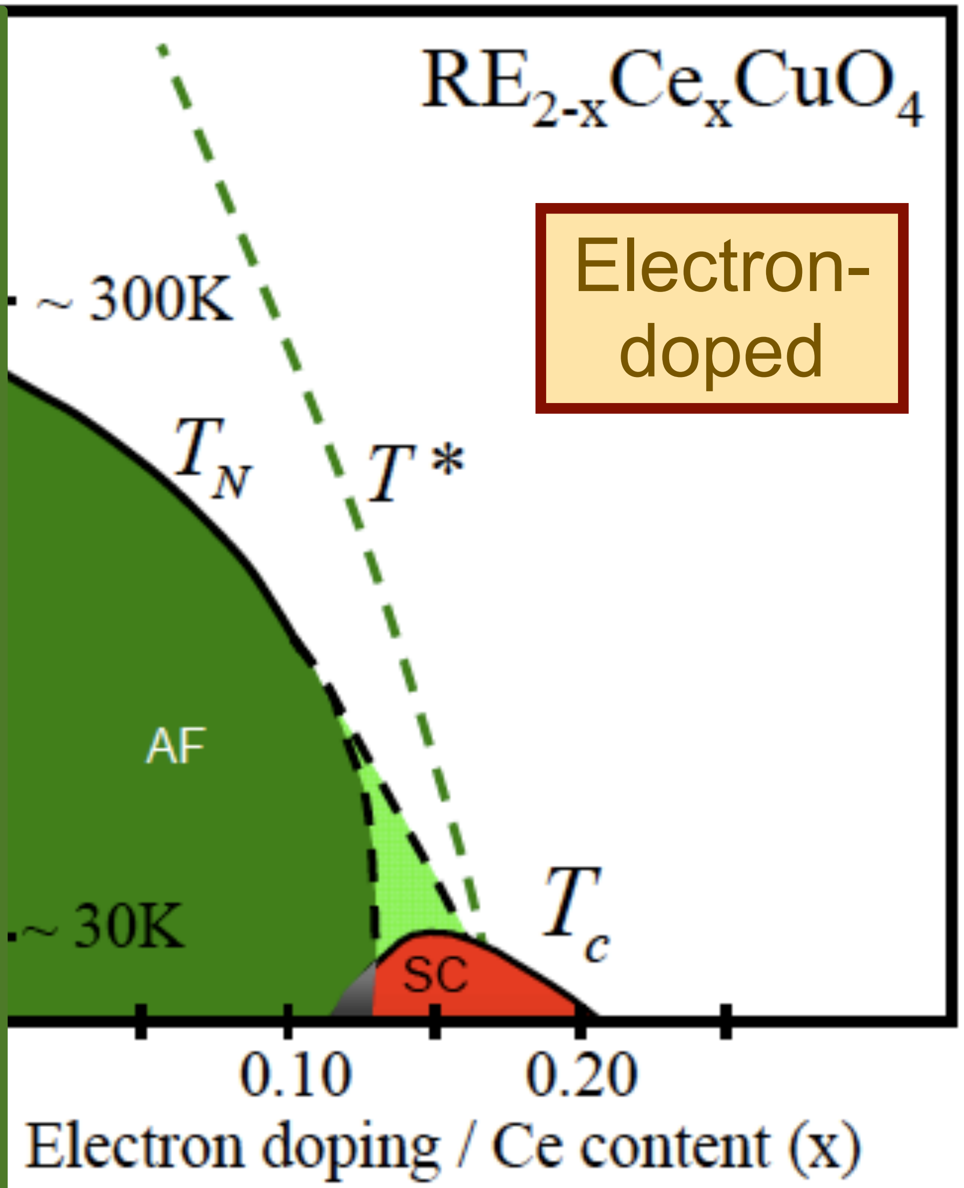

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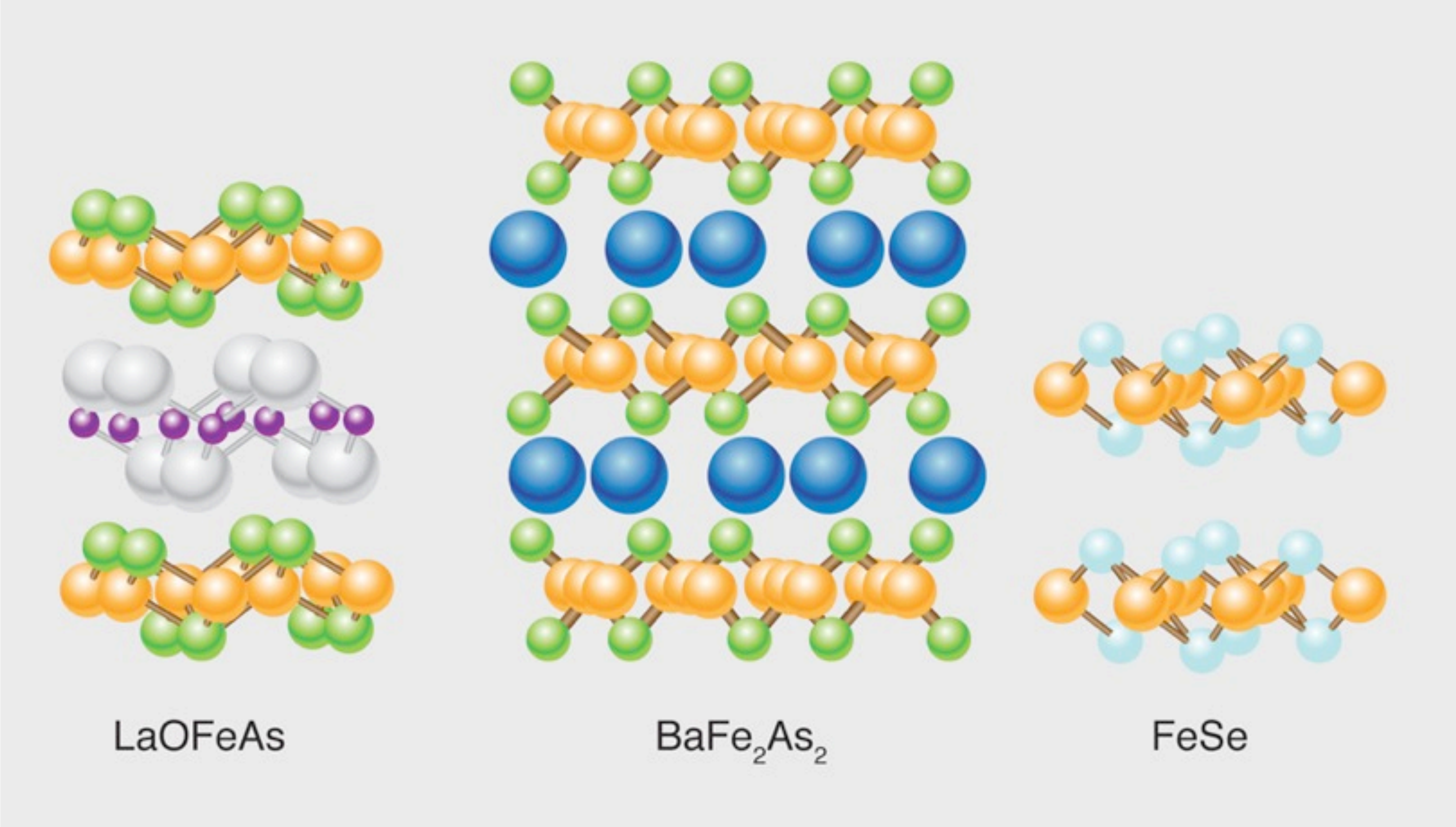
Iron pnictides:

a new class of high temperature superconductors



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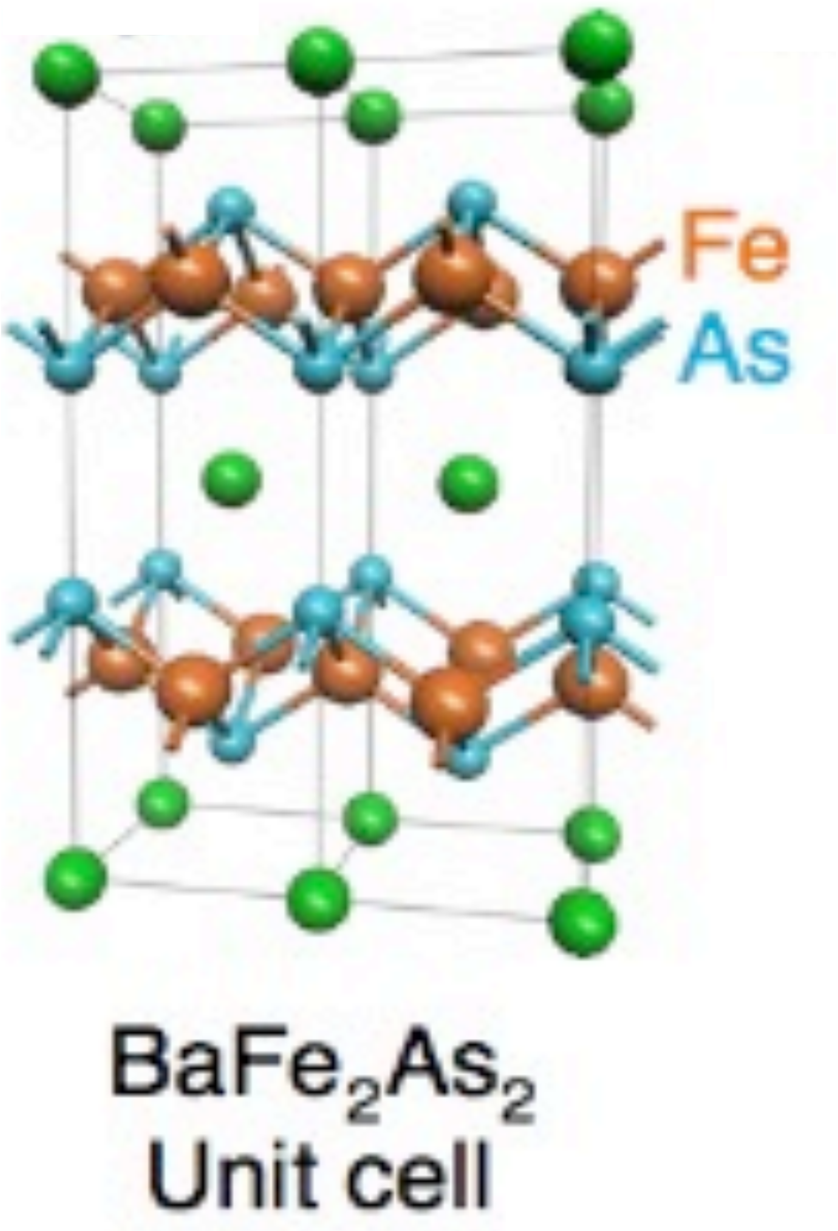
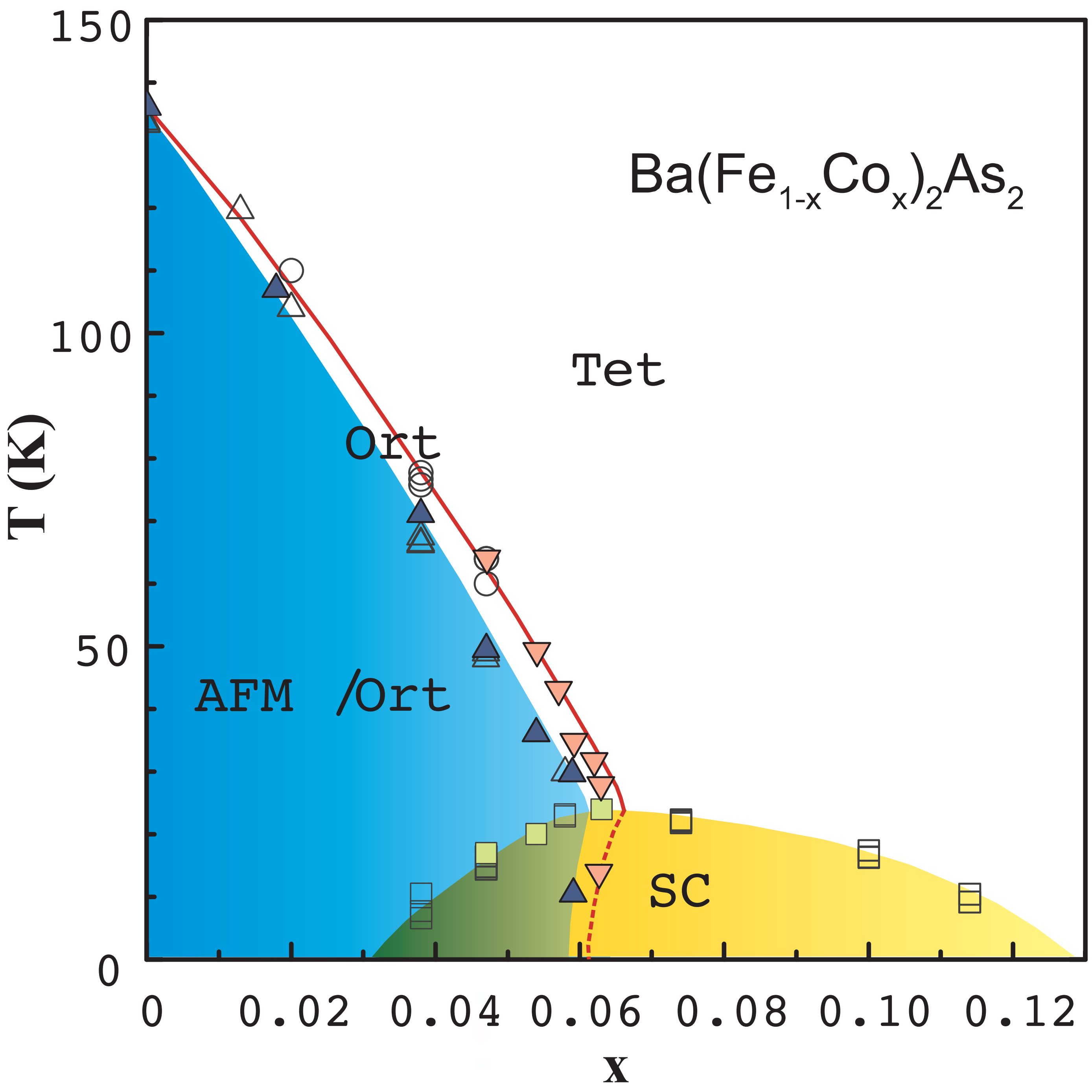
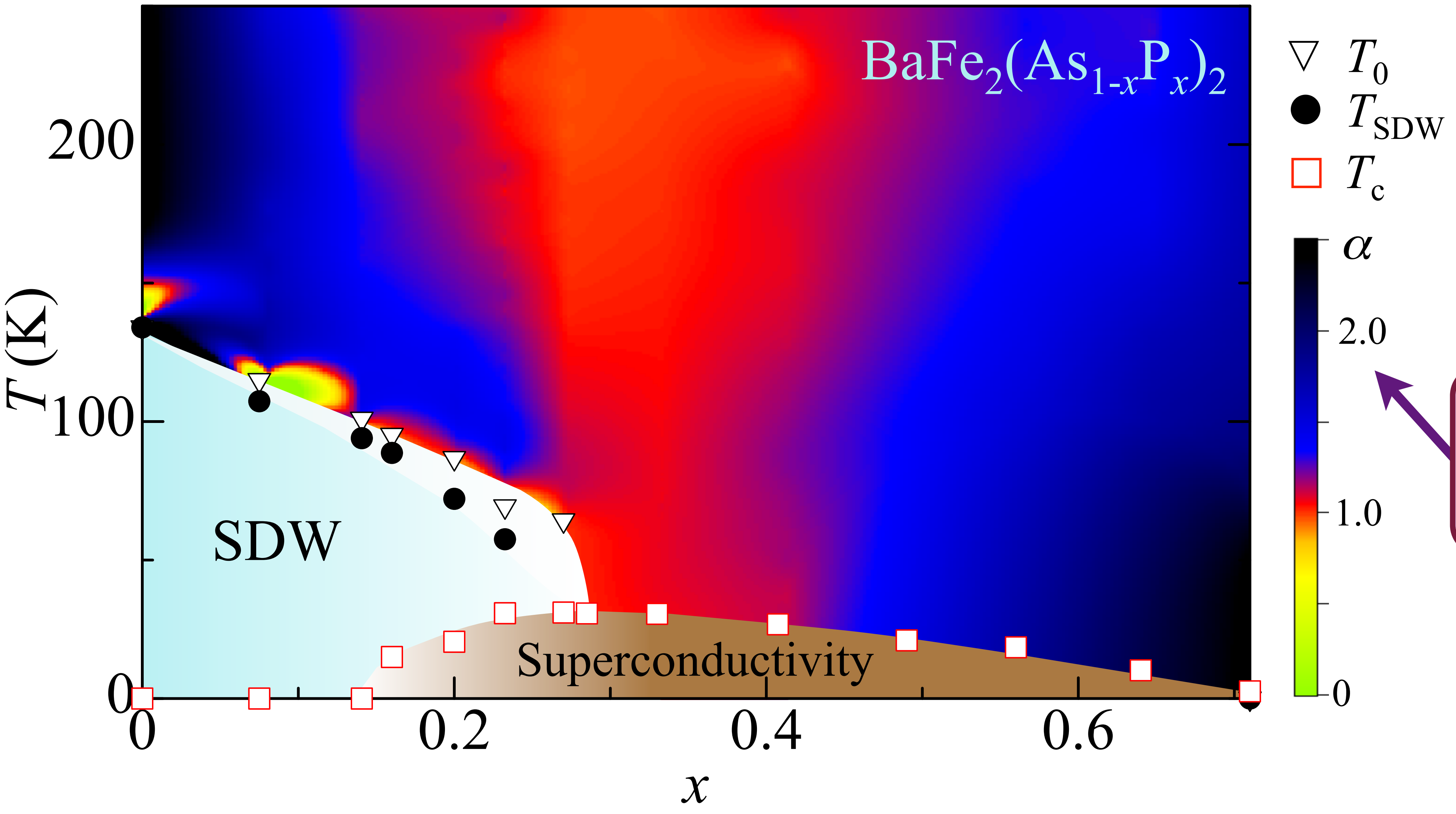




S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



Resistivity $\sim \rho_0 + AT^{\alpha}$

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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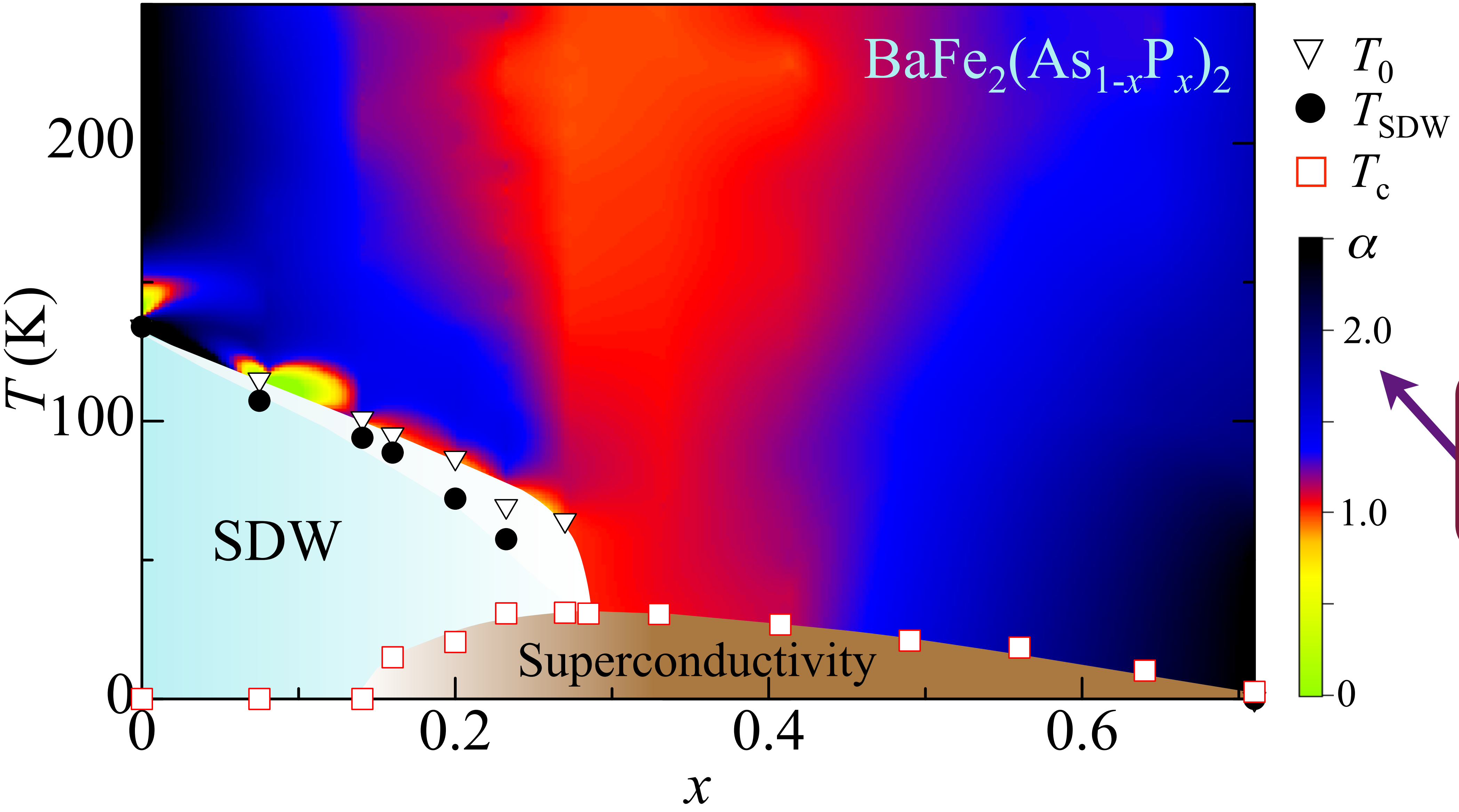

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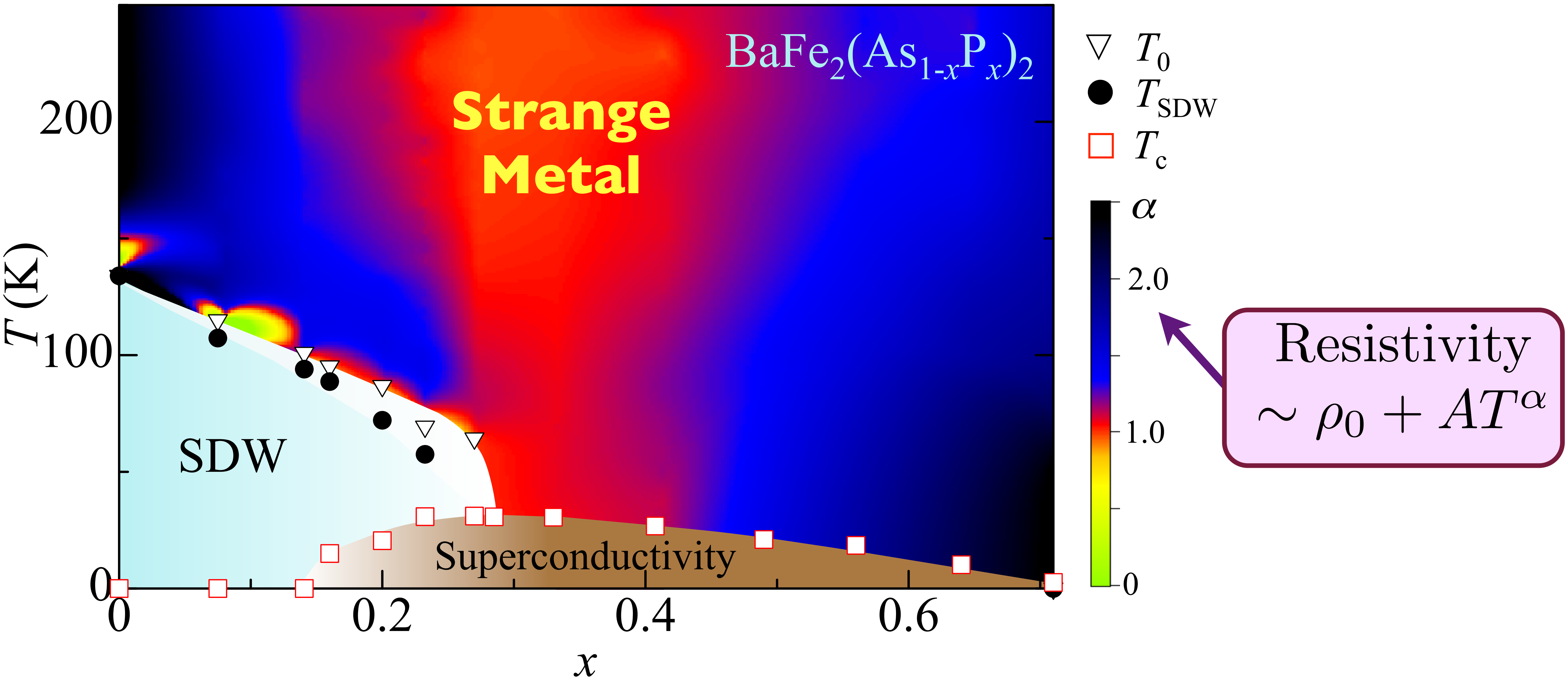

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Questions

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- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   "Mechanism" of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

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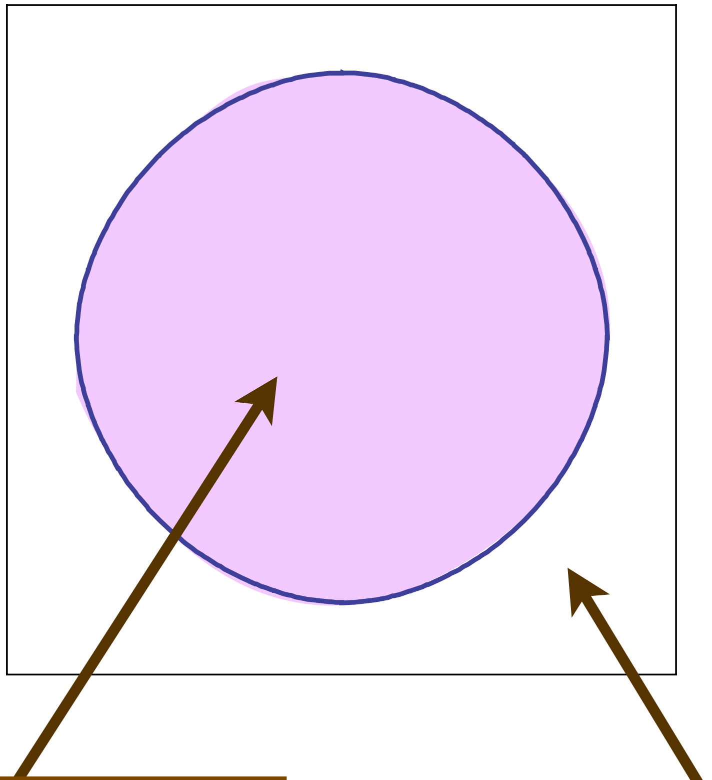
Fermi surface

Metal with “large”
Fermi surface

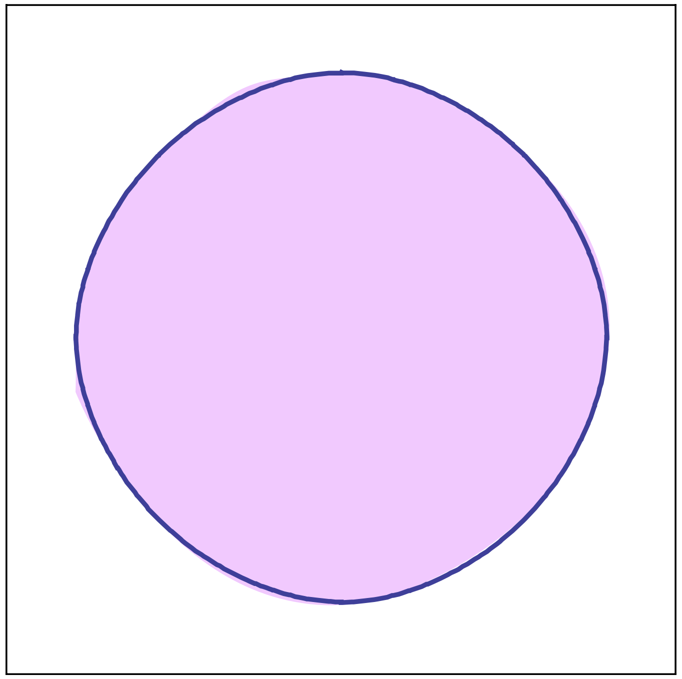


Momenta with
electronic
states empty

Momenta with
electronic
states
occupied

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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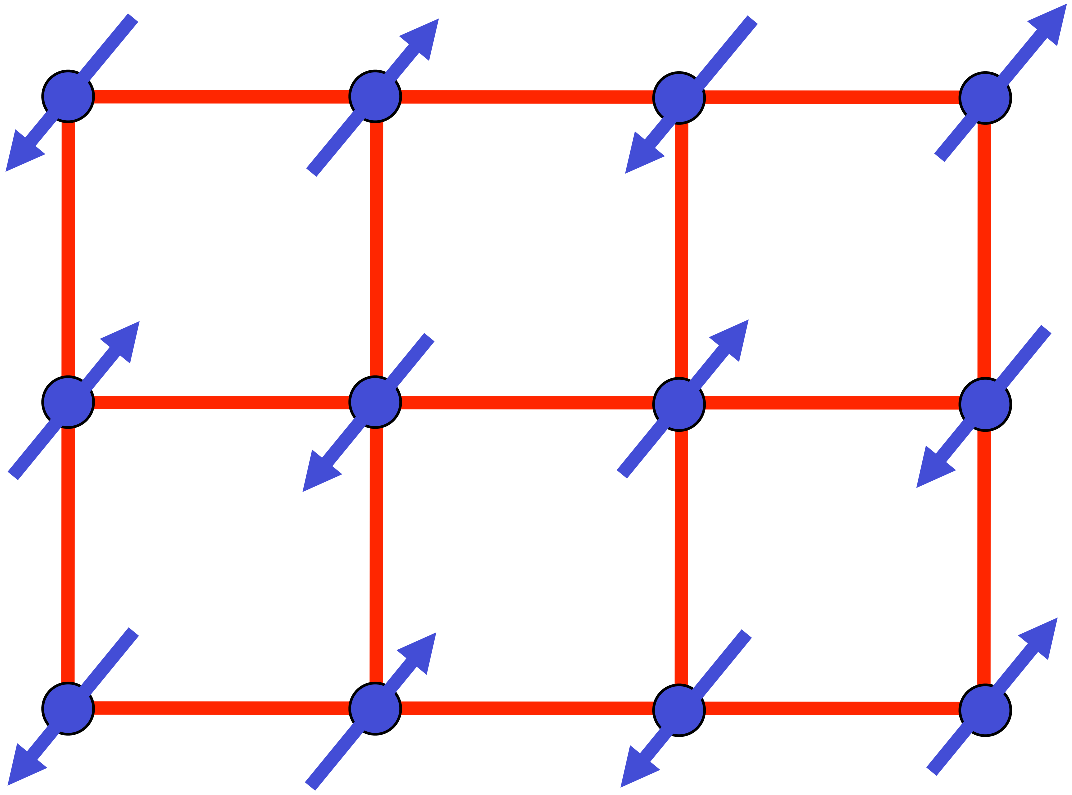
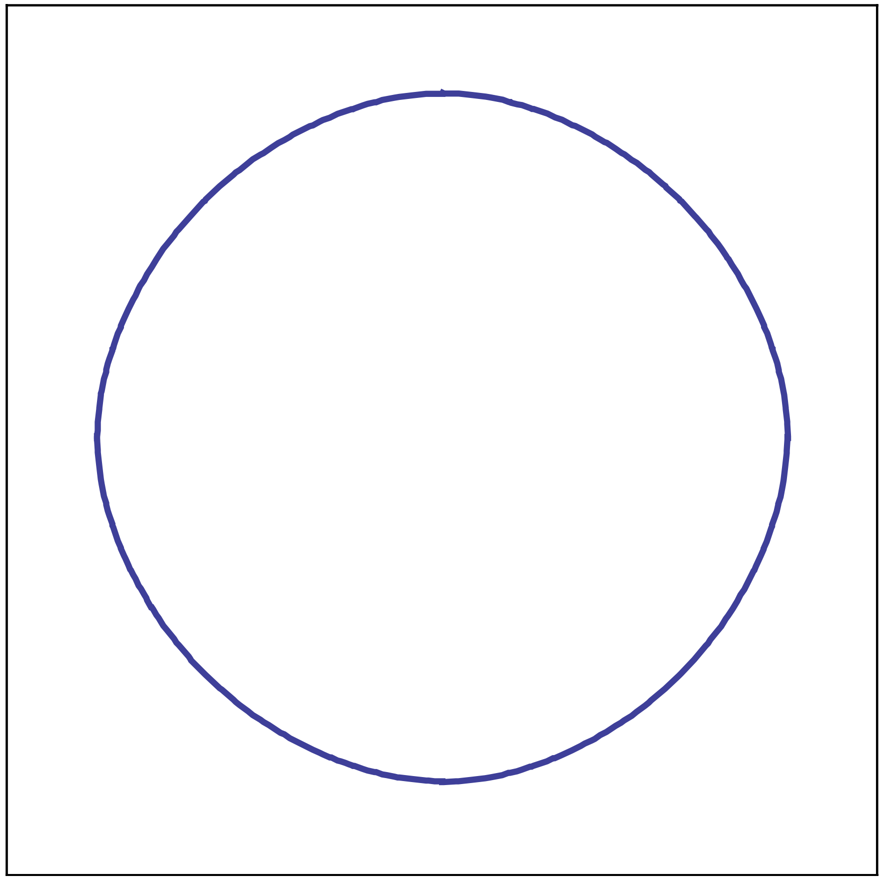


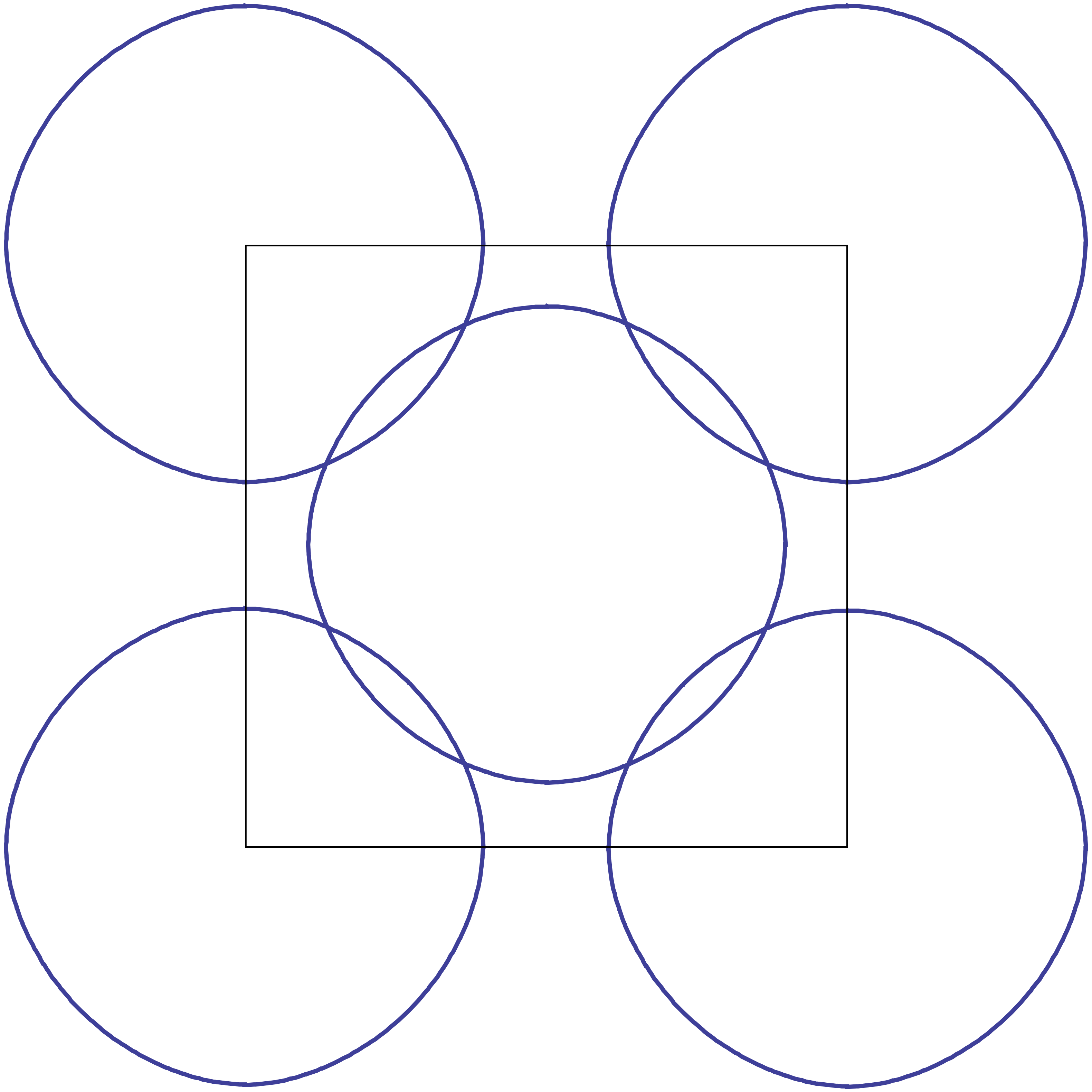
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

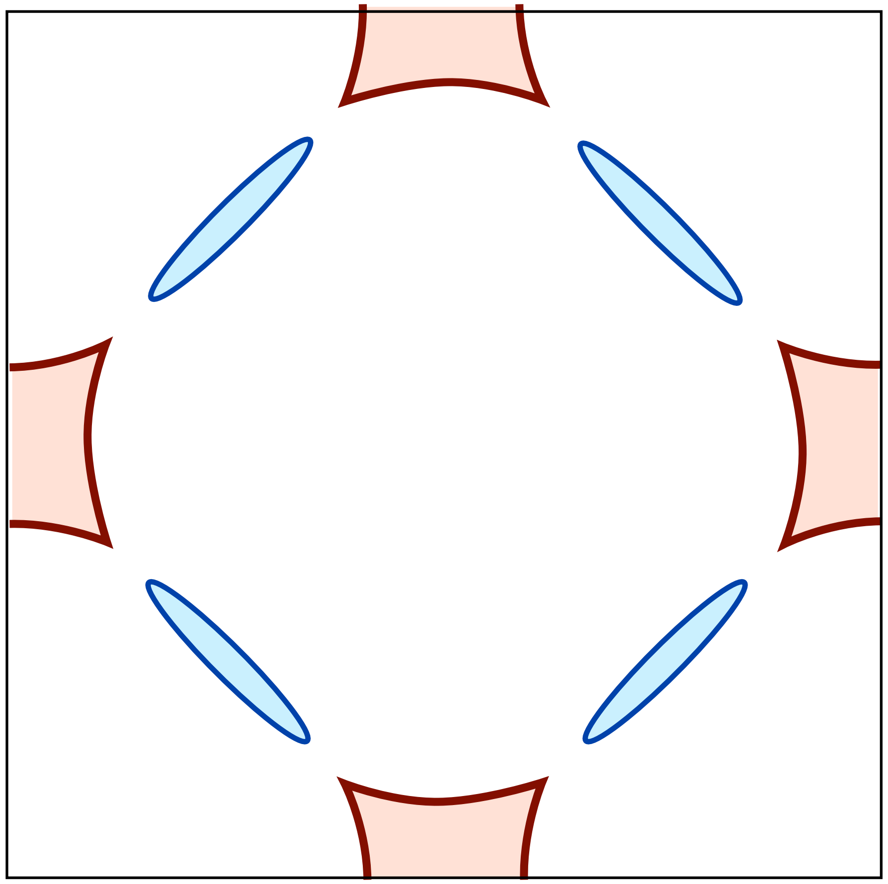
where $\mathbf{K}$ is the ordering wavevector.



Metal with “large” Fermi surface

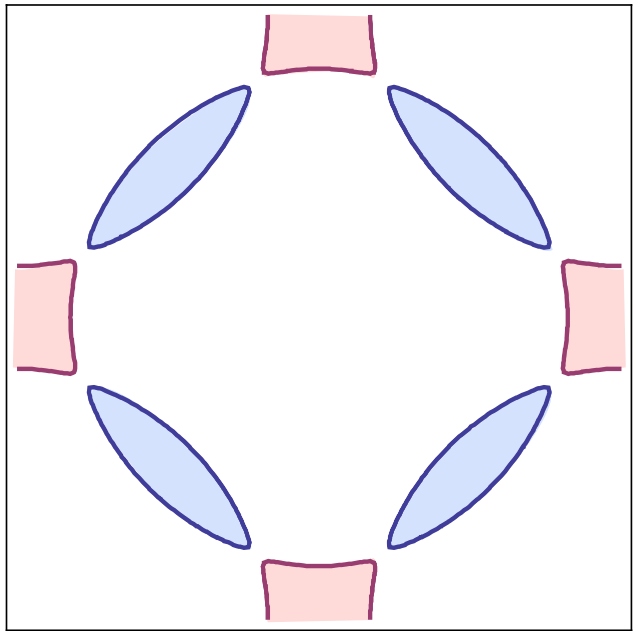


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



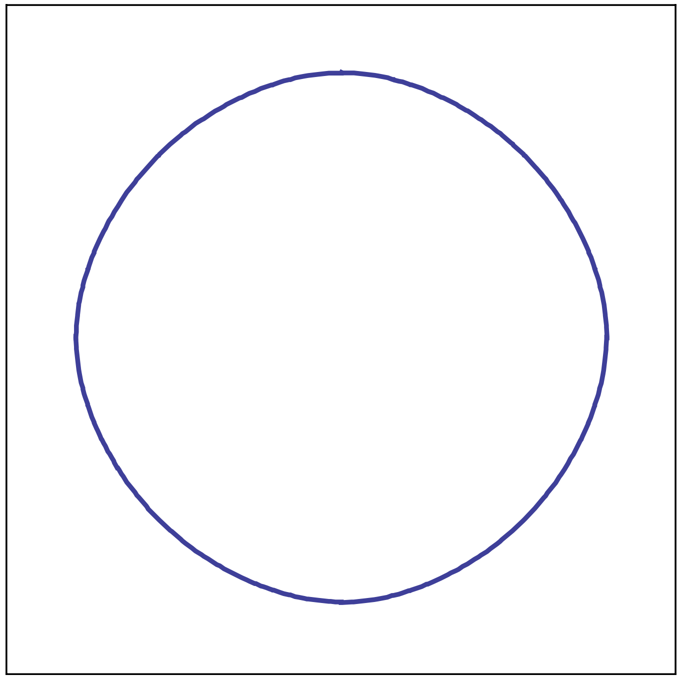
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



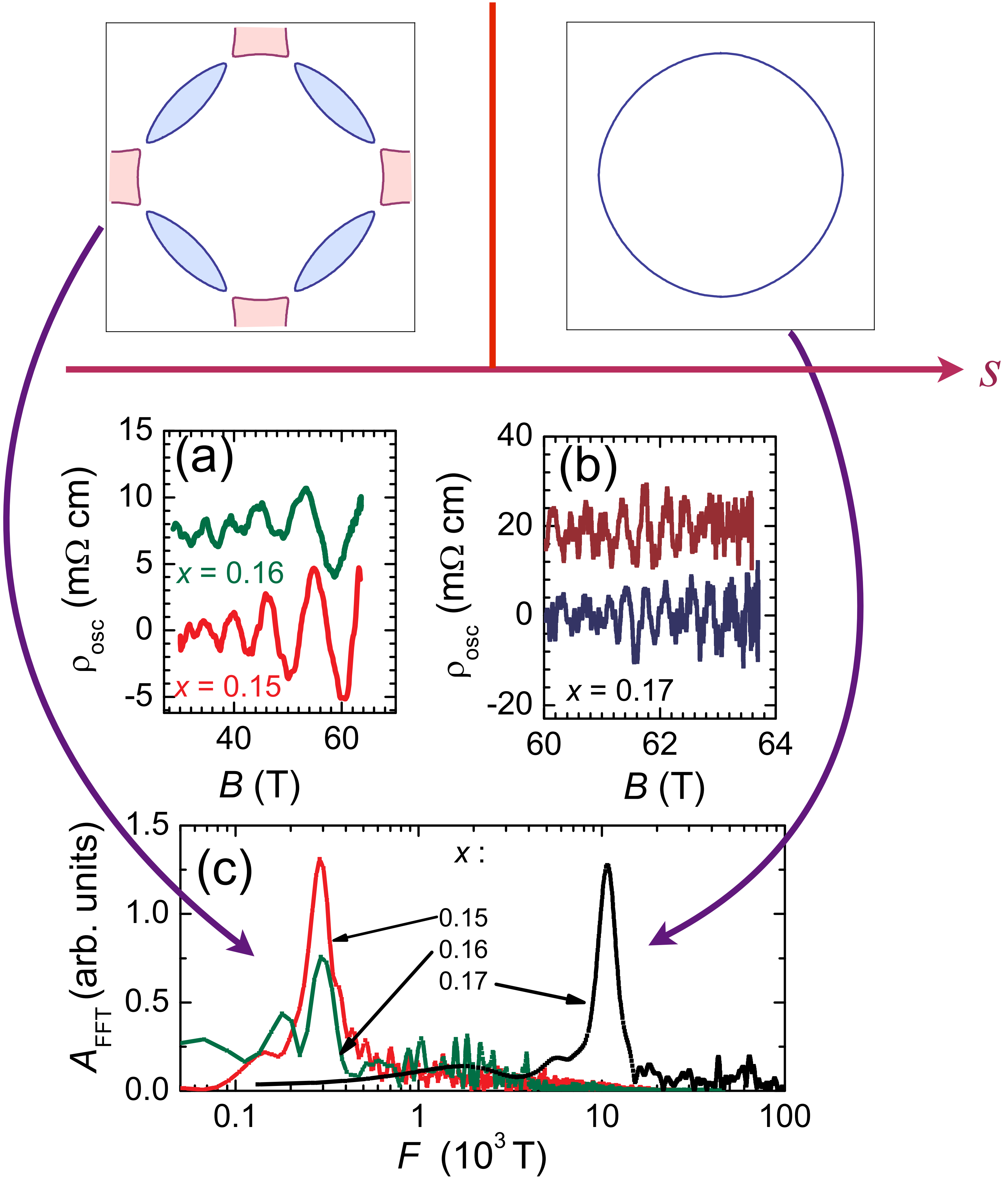
$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large"
Fermi surface

Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

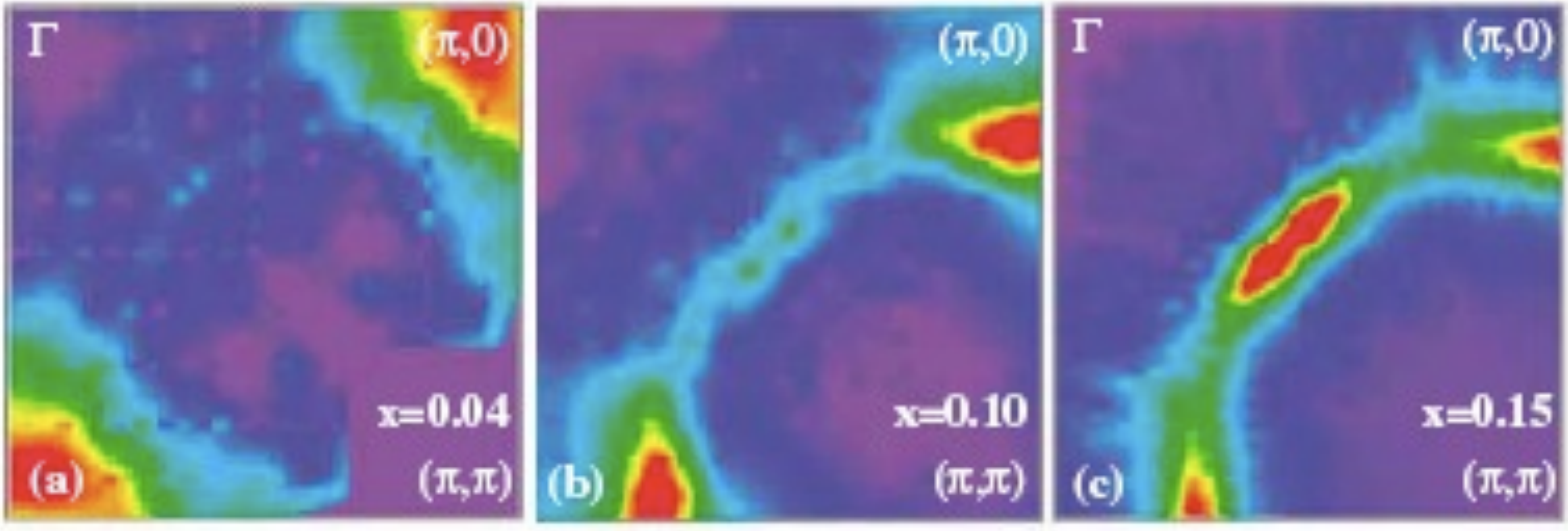
Quantum oscillations



$Nd_{2-x}Ce_xCuO_4$

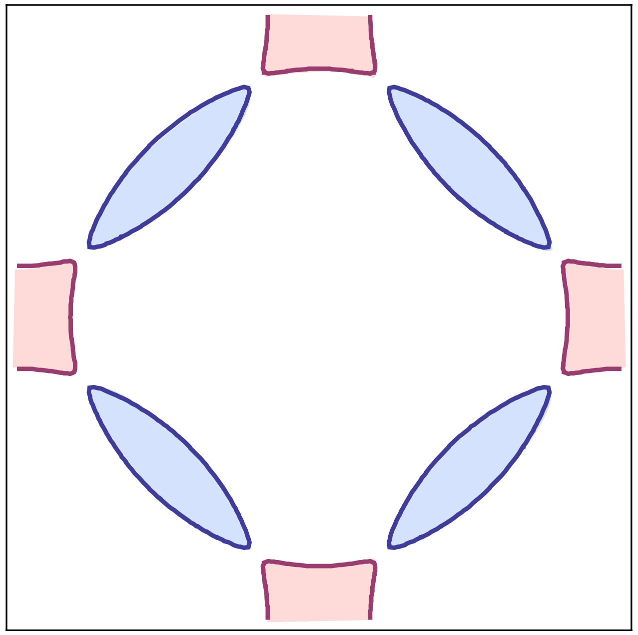
T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

Photoemission in $Nd_{2-x}Ce_xCuO_4$



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Fermi surface+antiferromagnetism



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Metal with electron
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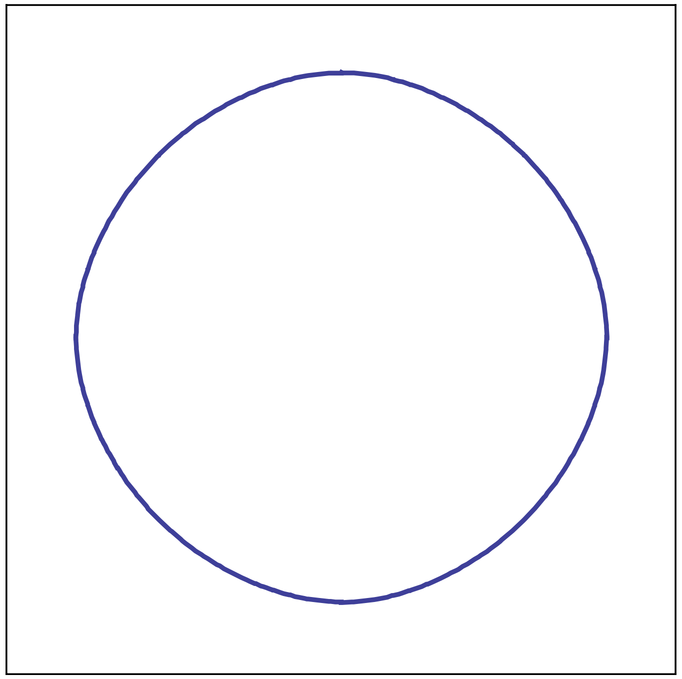

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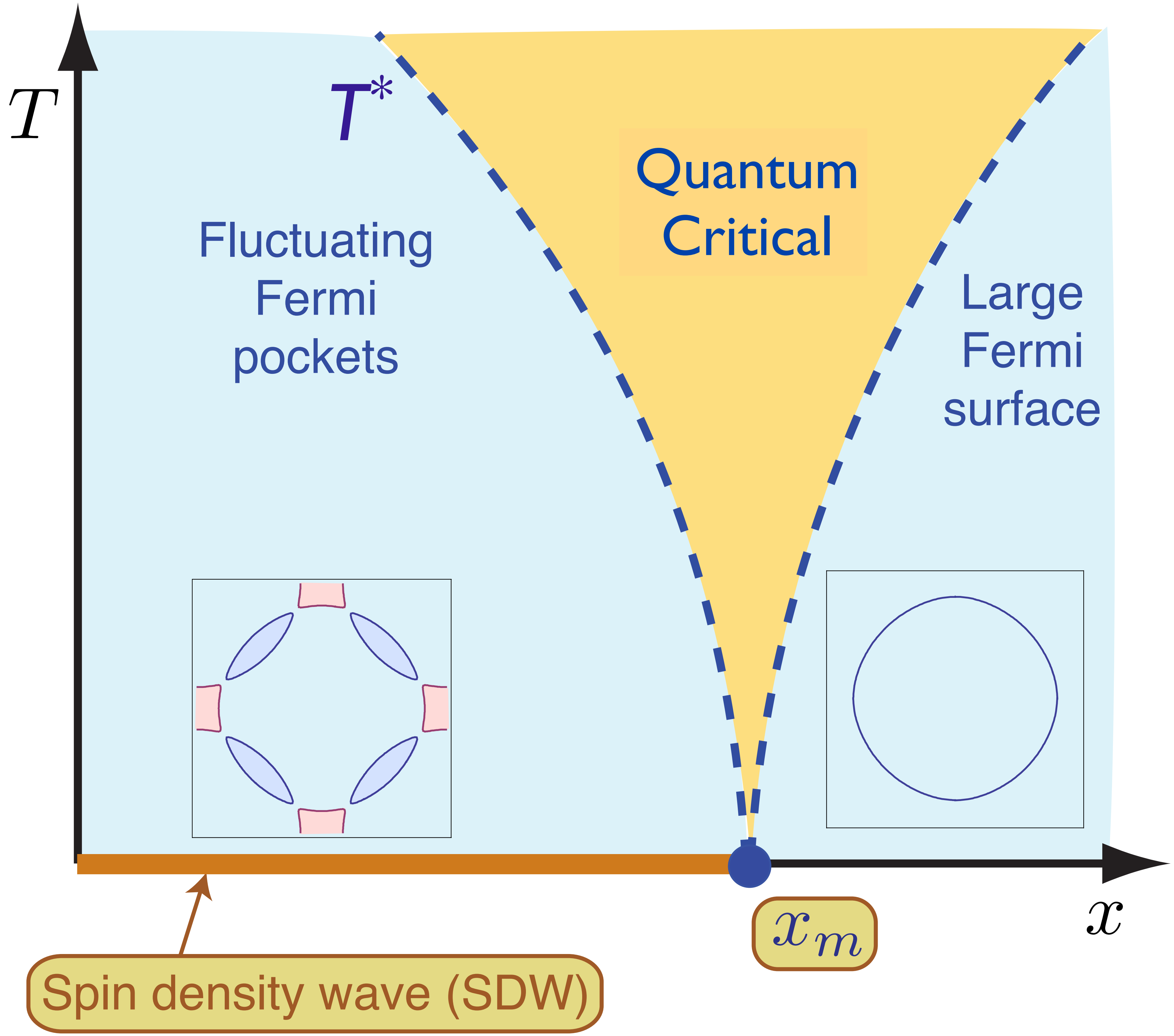
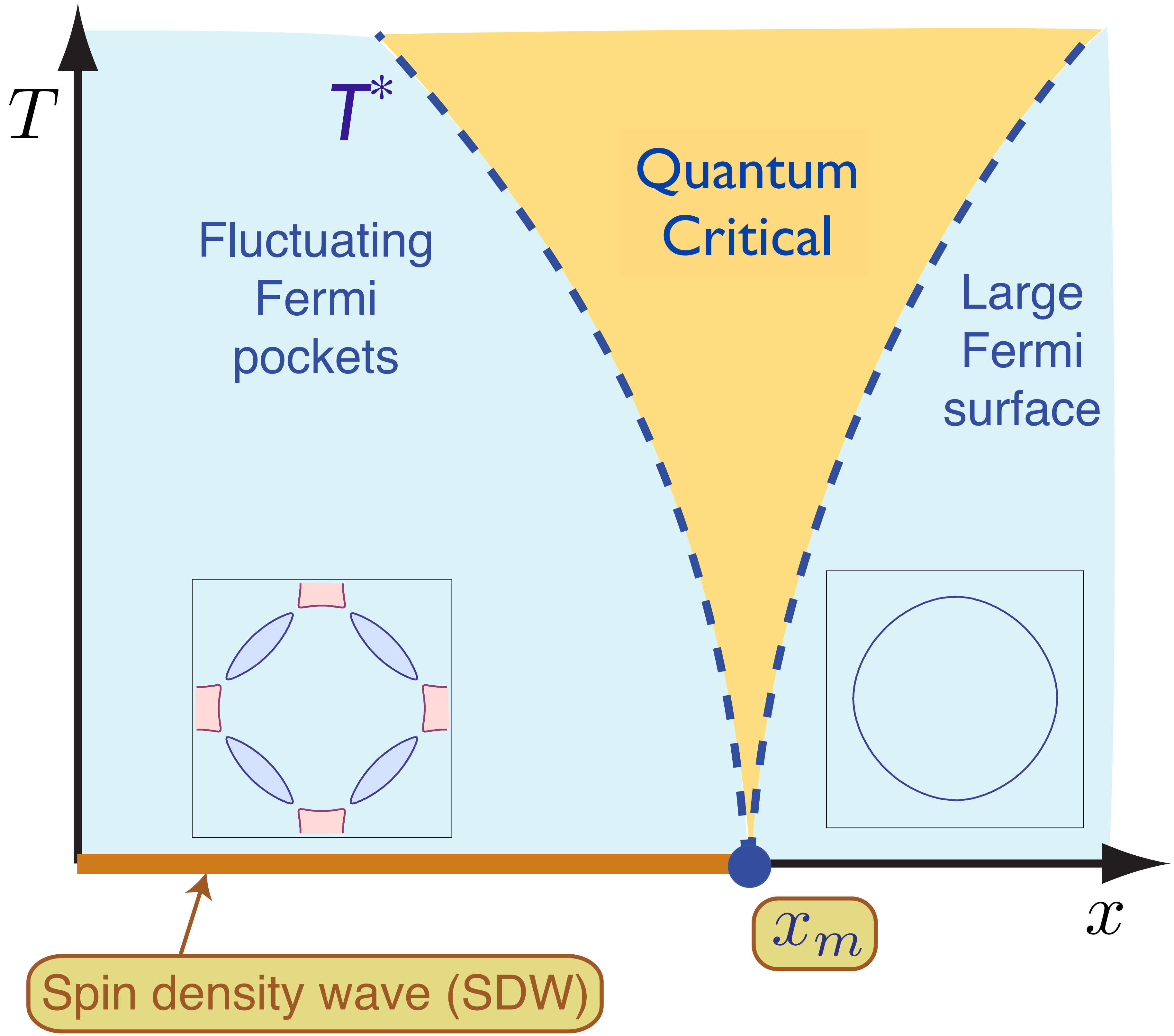
Theory of quantum criticality in the cuprates



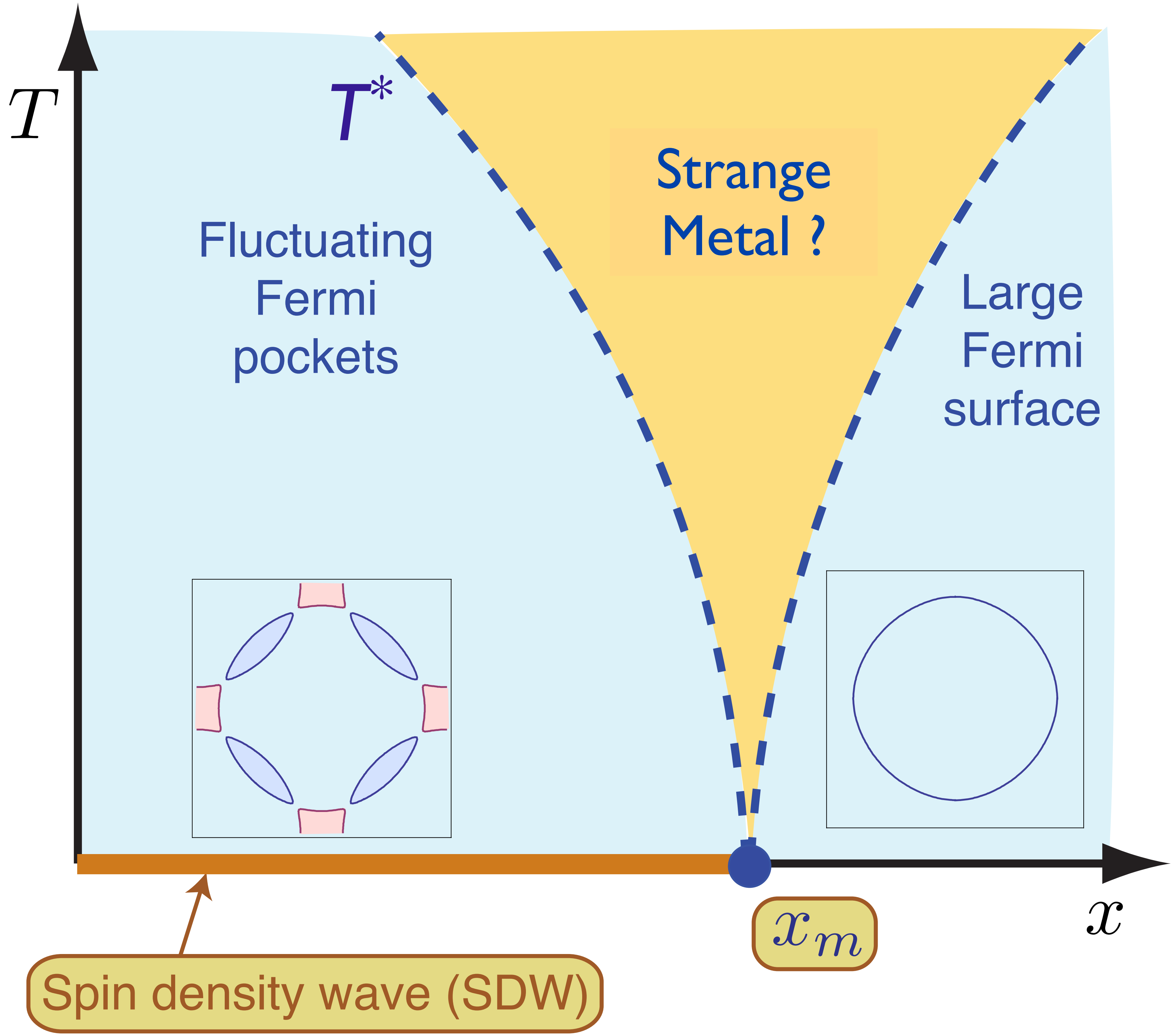
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_BT$ are robust properties of strongly-coupled quantum criticality

Theory of quantum criticality in the cuprates



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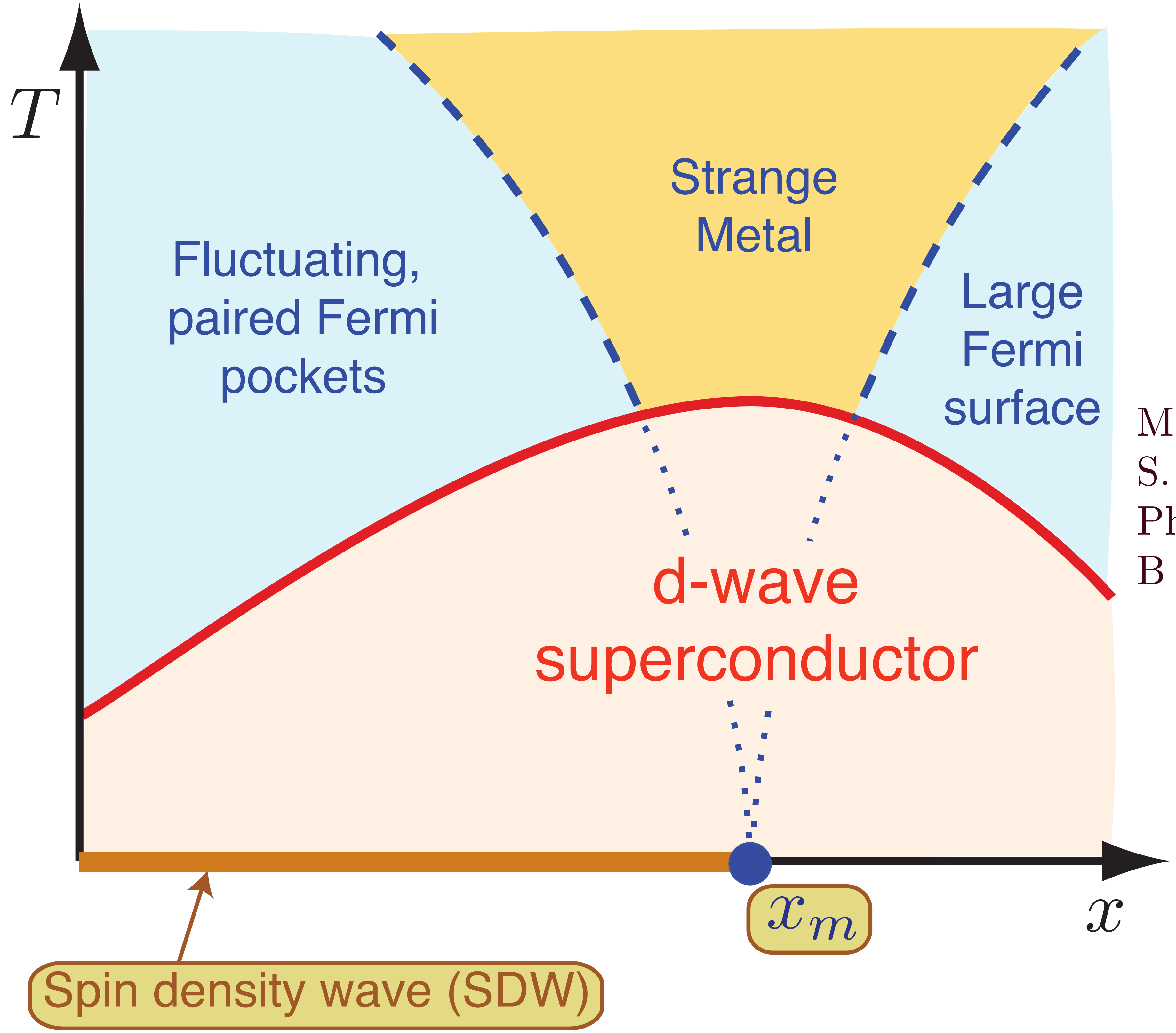
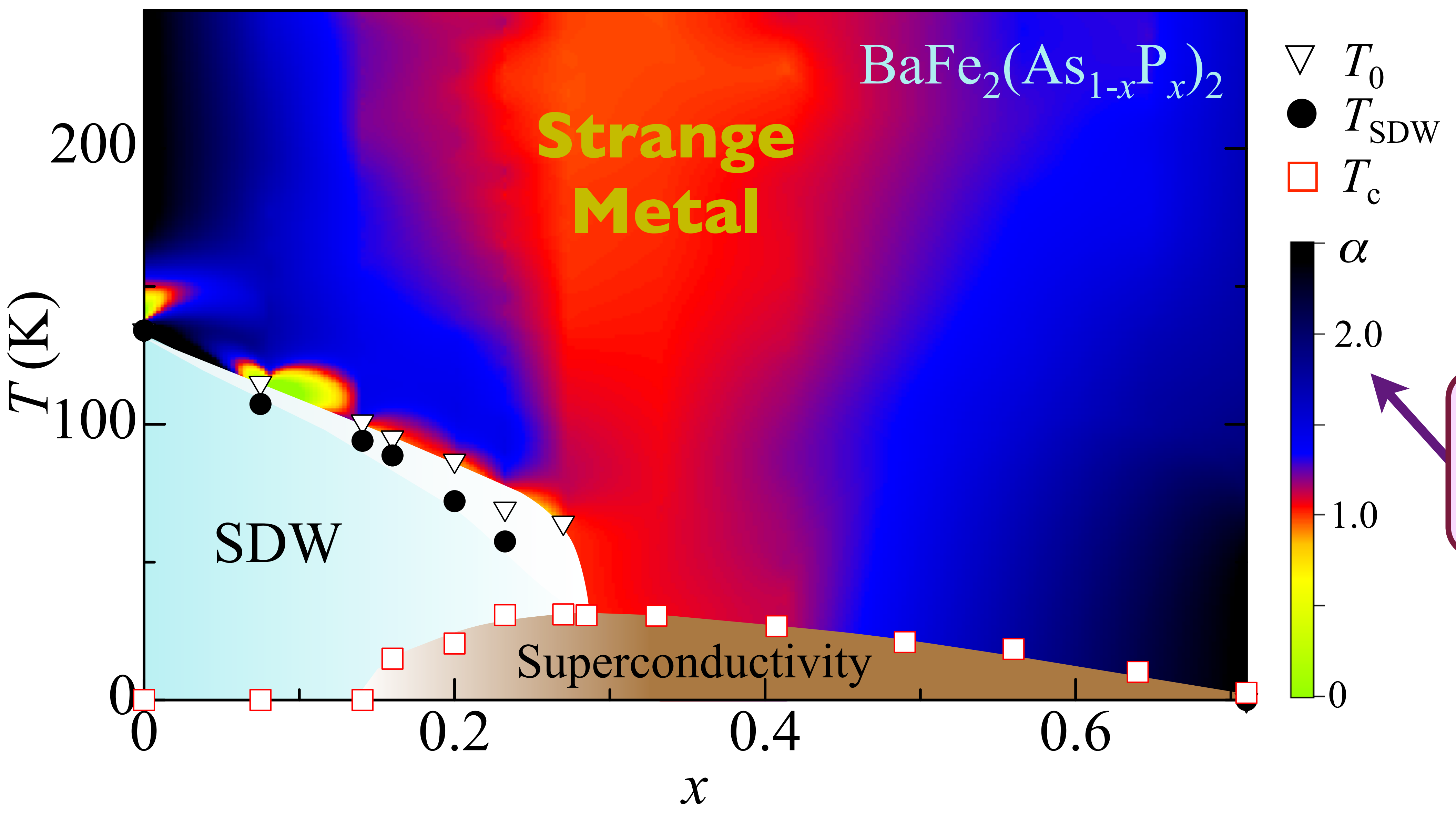


M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

Temperature-doping phase diagram of the iron pnictides:



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Physical Review B **81**, 184519 (2010)

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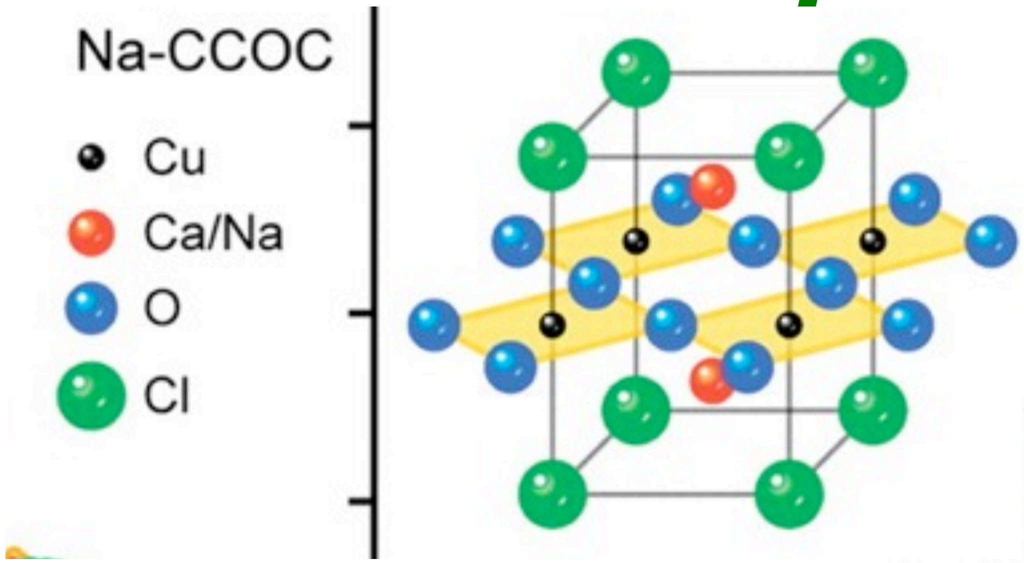
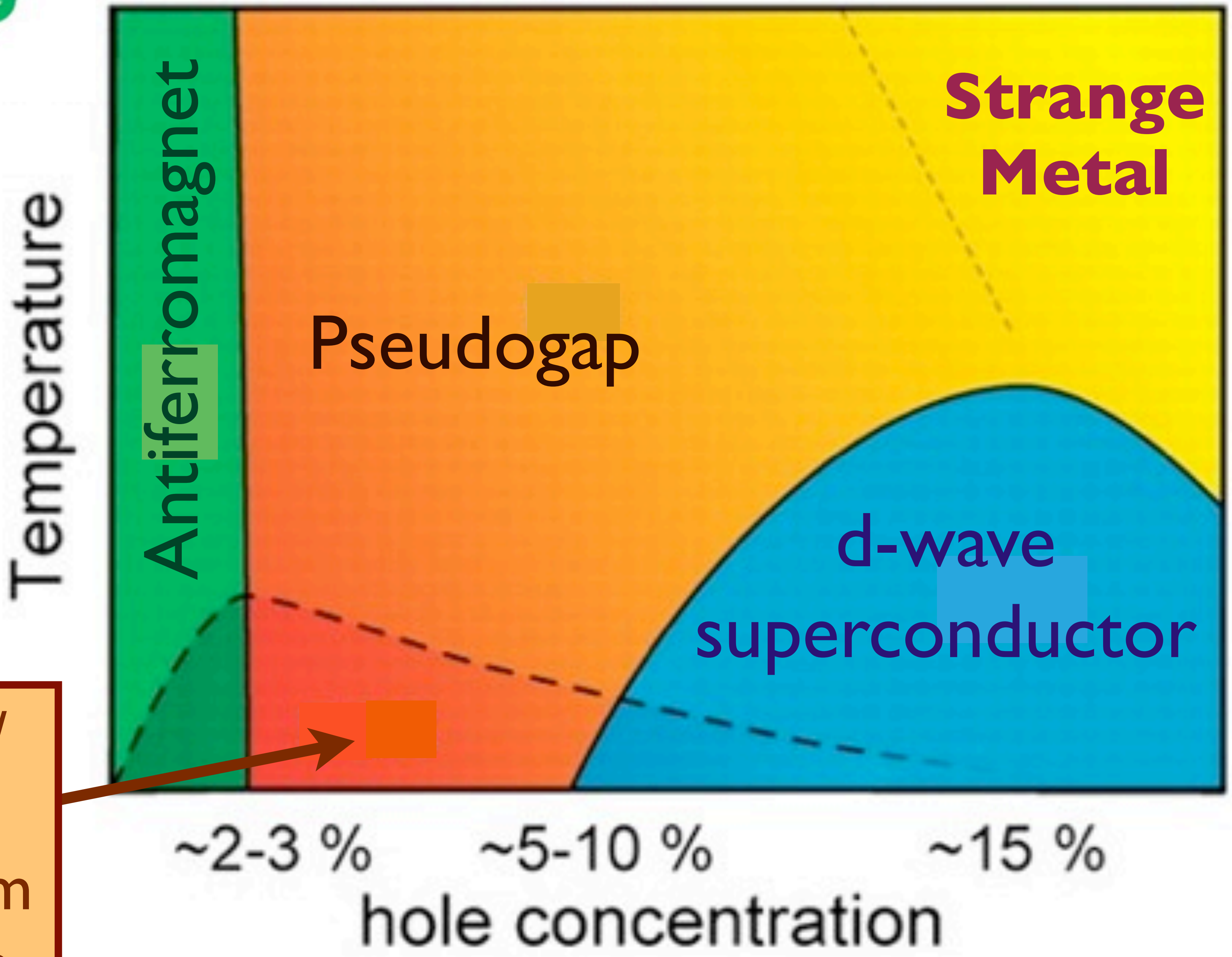
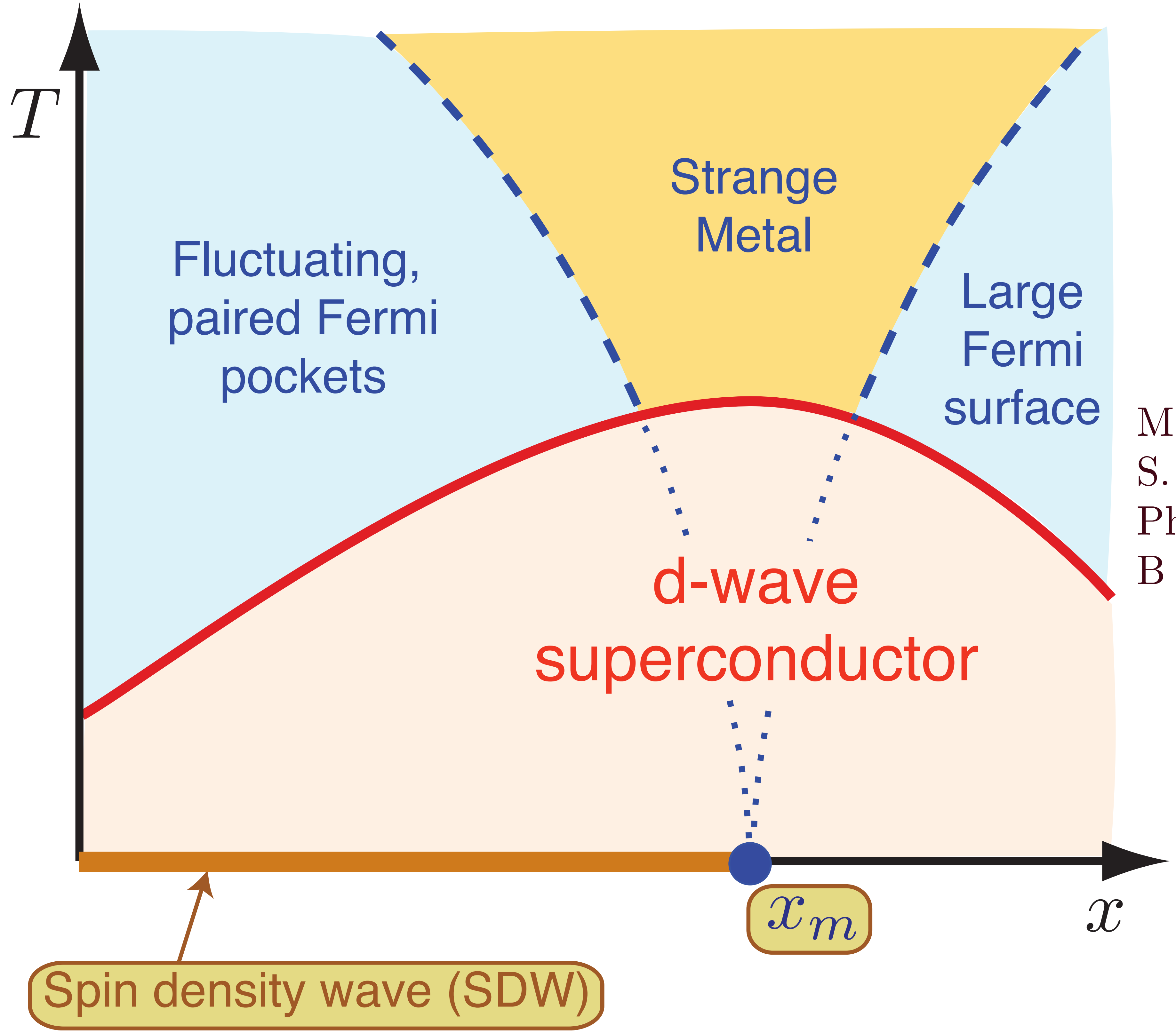




Incommensurate/
disordered
antiferromagnetism
and charge order

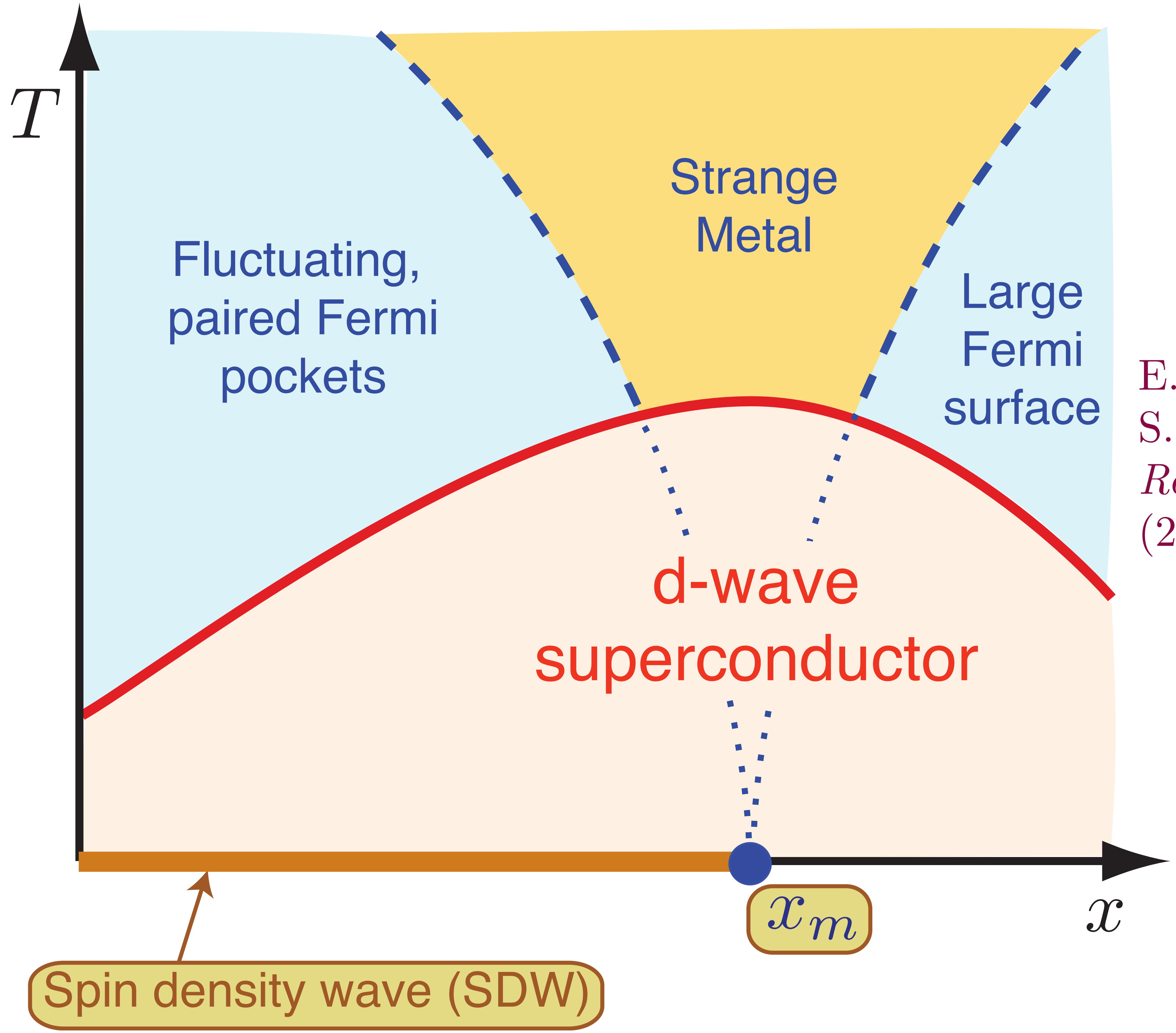
Theory of quantum criticality in the cuprates



M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to d-wave superconductivity
This instability is stronger than that in the BCS theory

Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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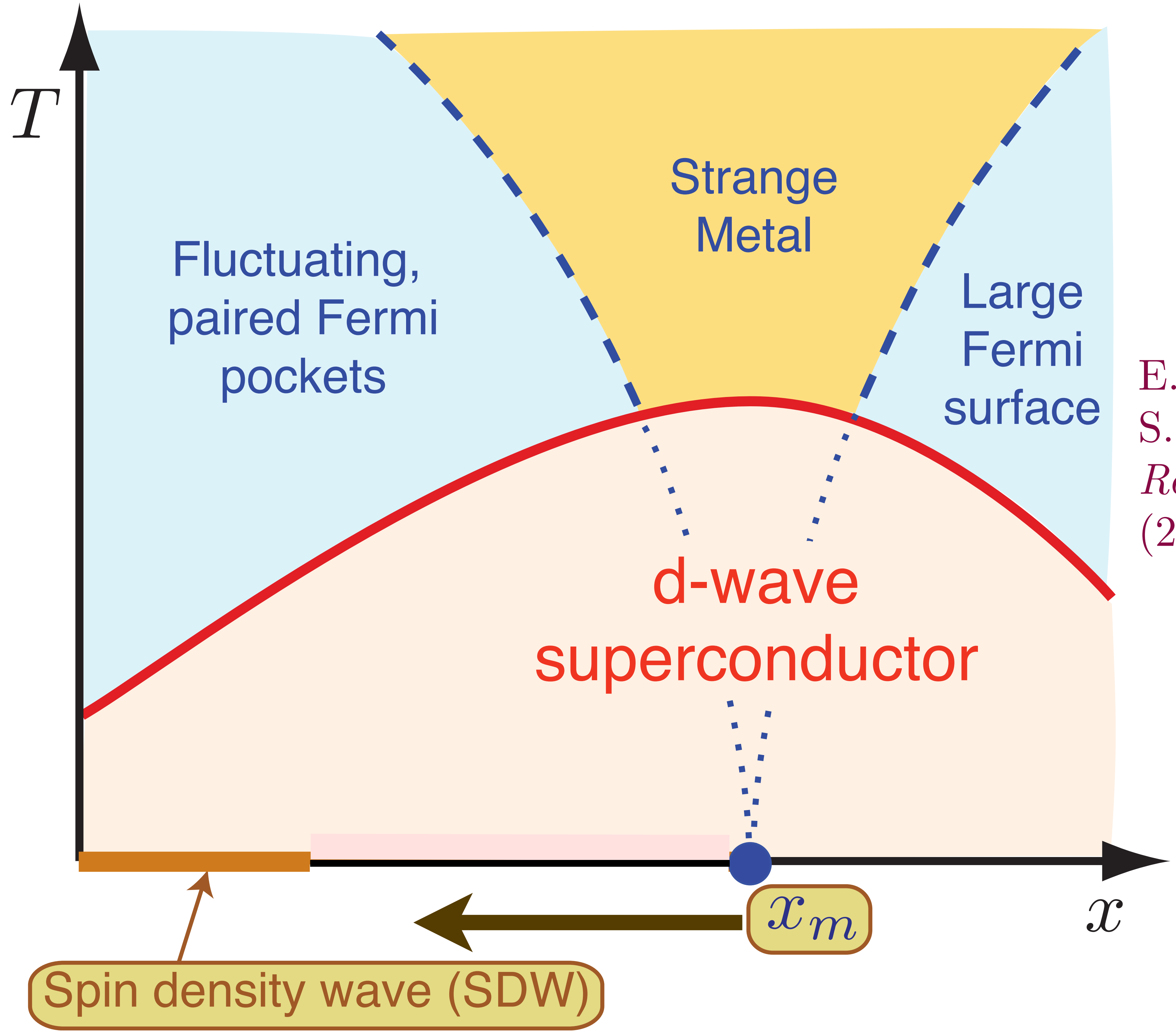

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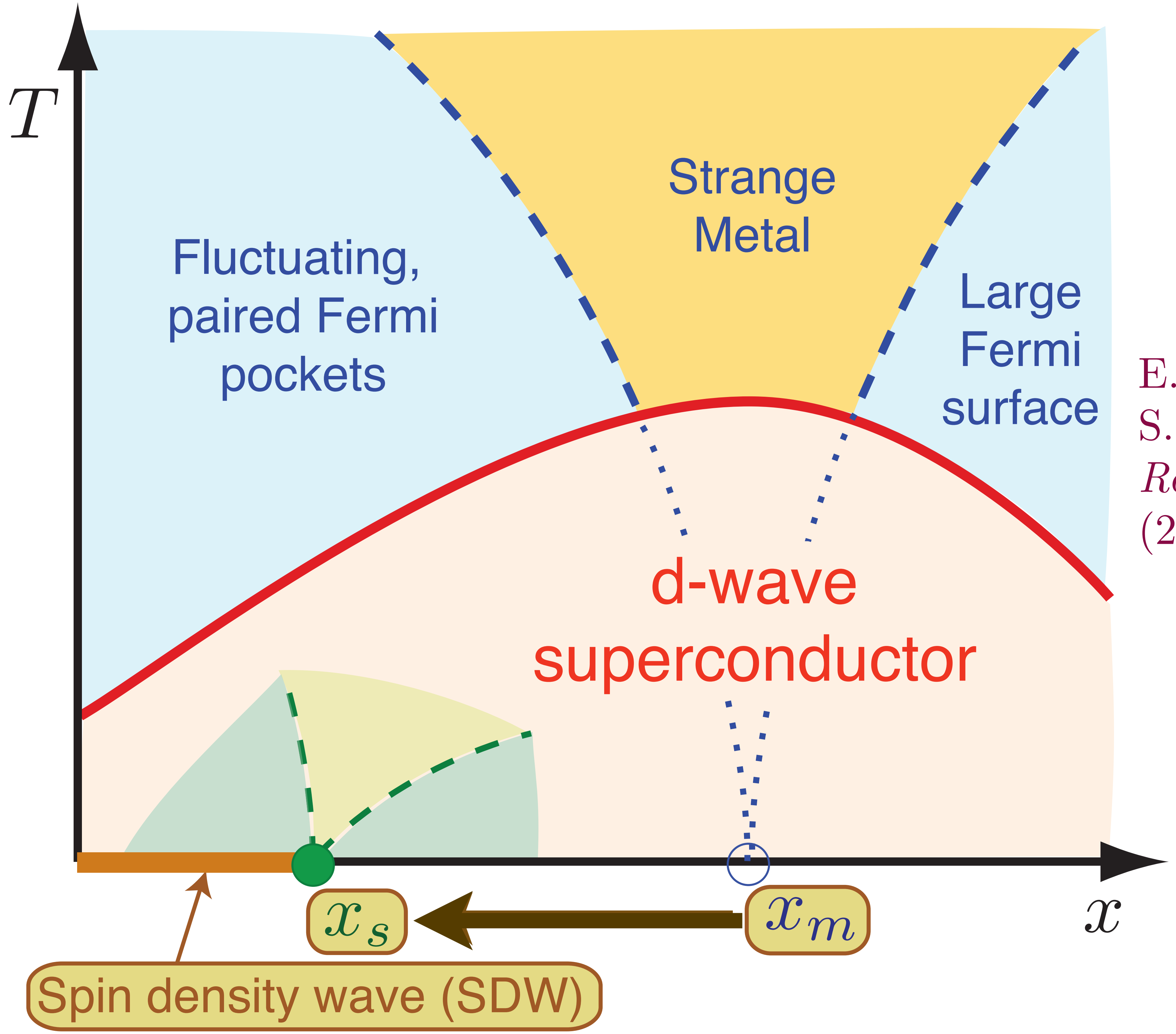

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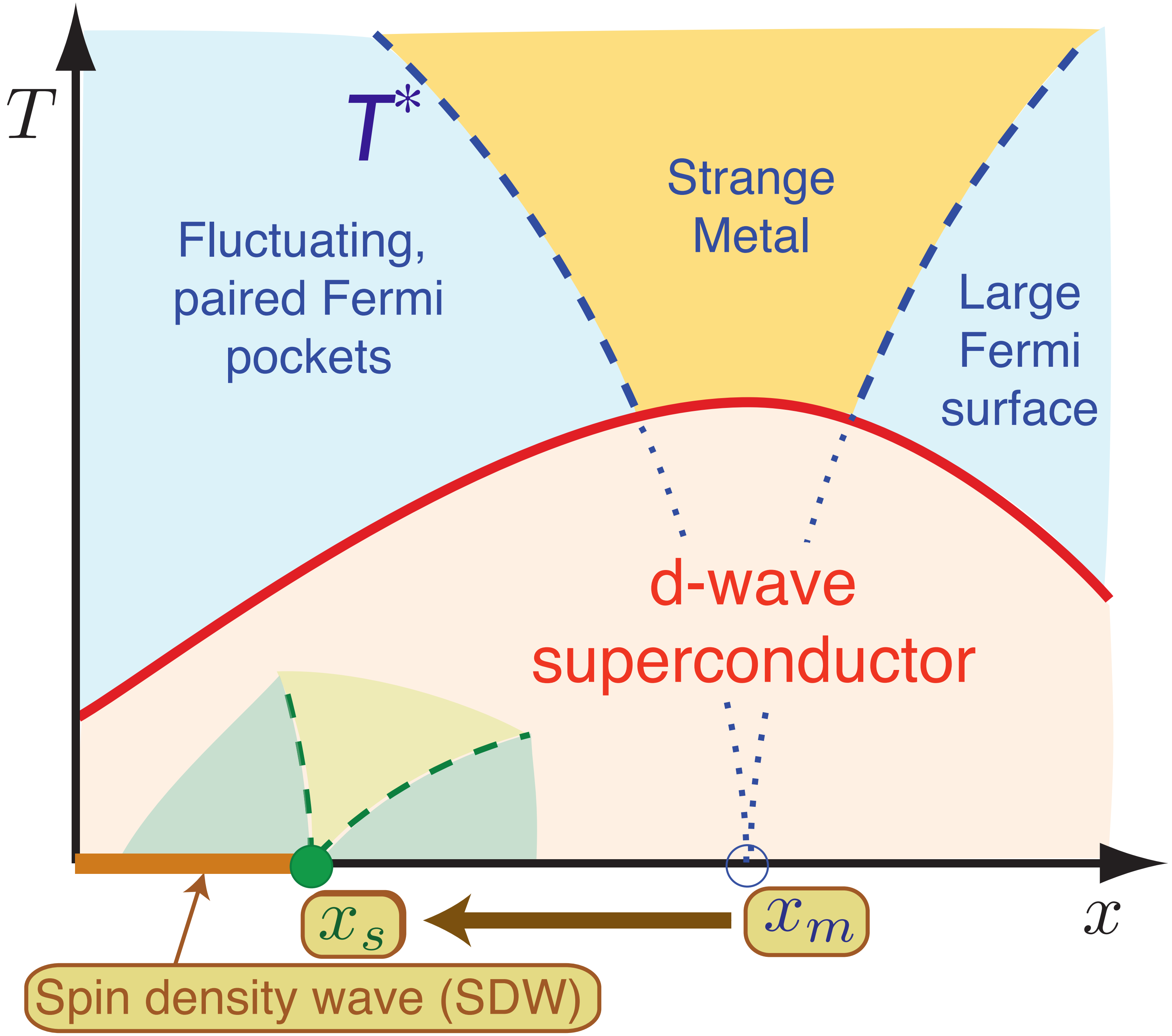
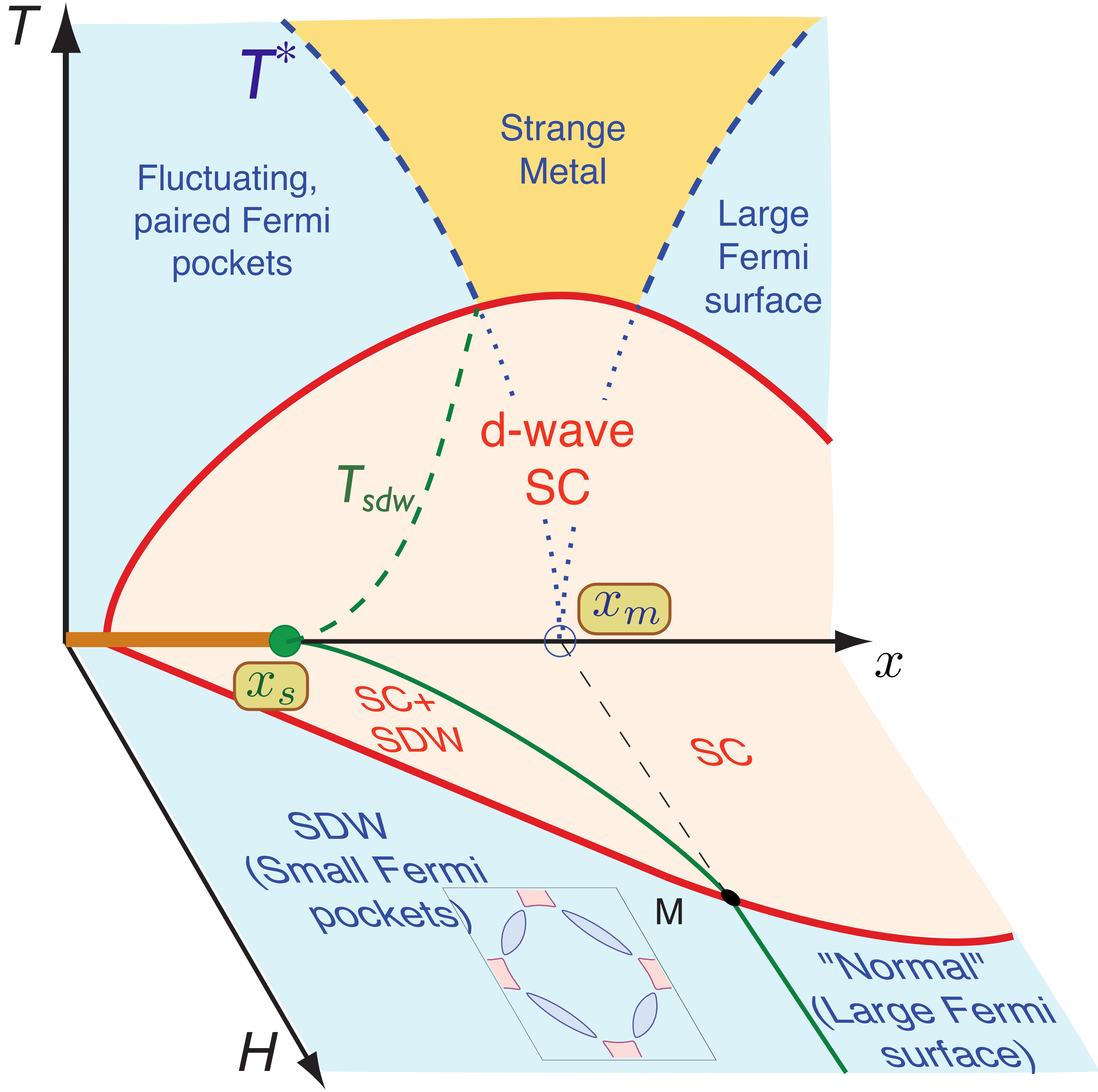
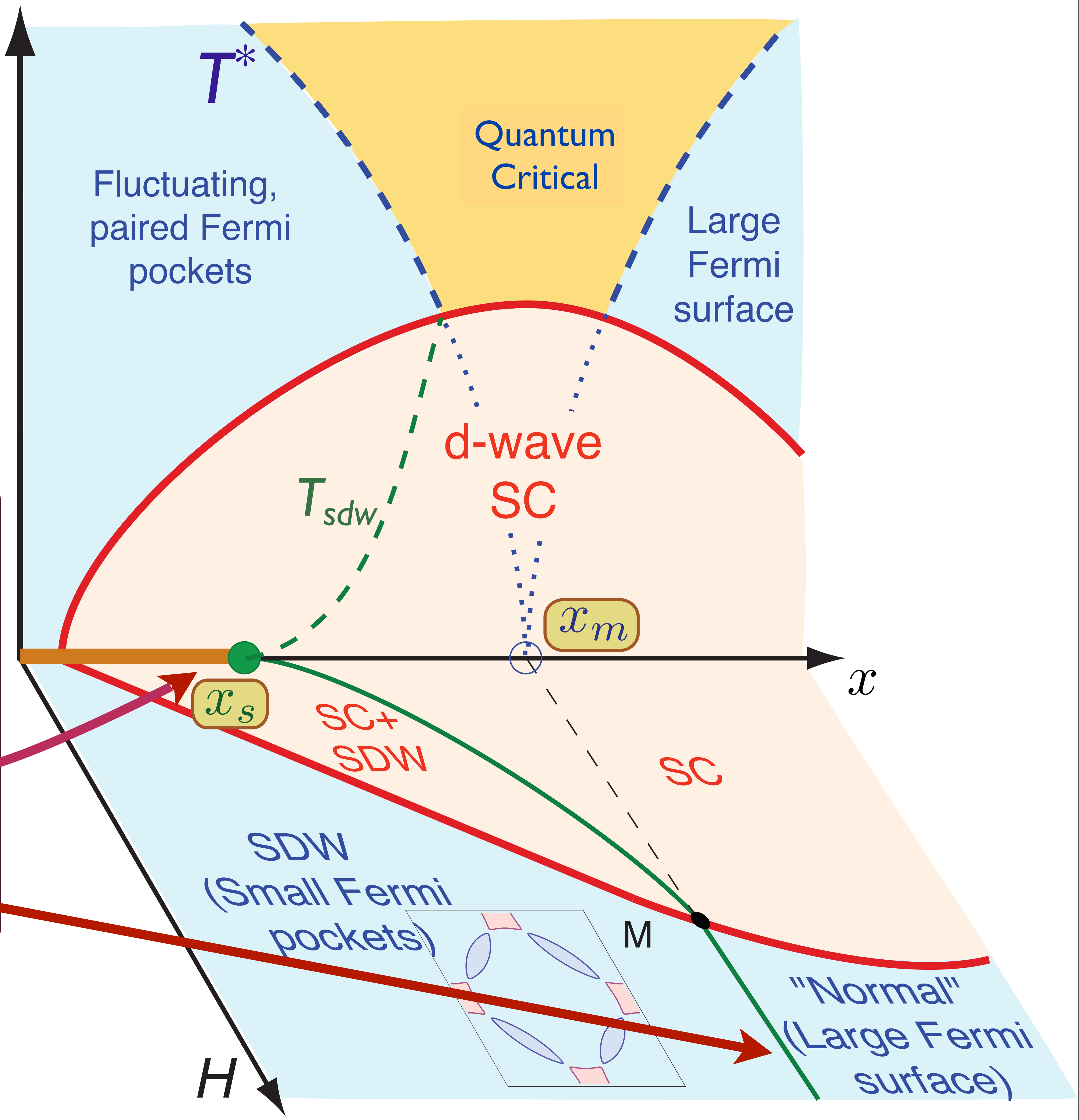

T
T^*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave superconductor
x_s
x_m
x
Spin density wave (SDW)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

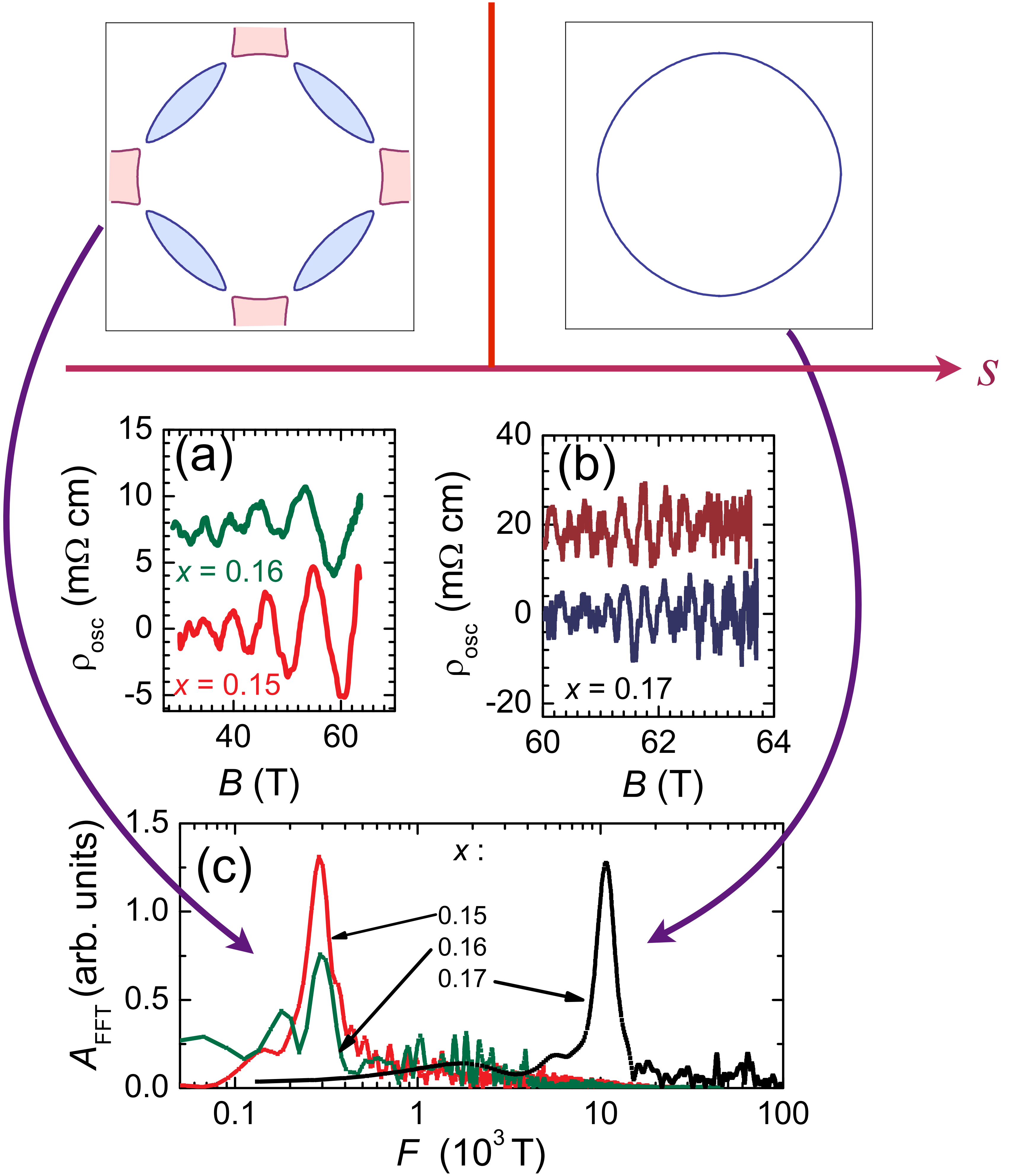


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $\underline{x_s = 0.14}$, while quantum oscillations at high fields show that $\underline{x_m = 0.165}$.



Quantum oscillations



$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

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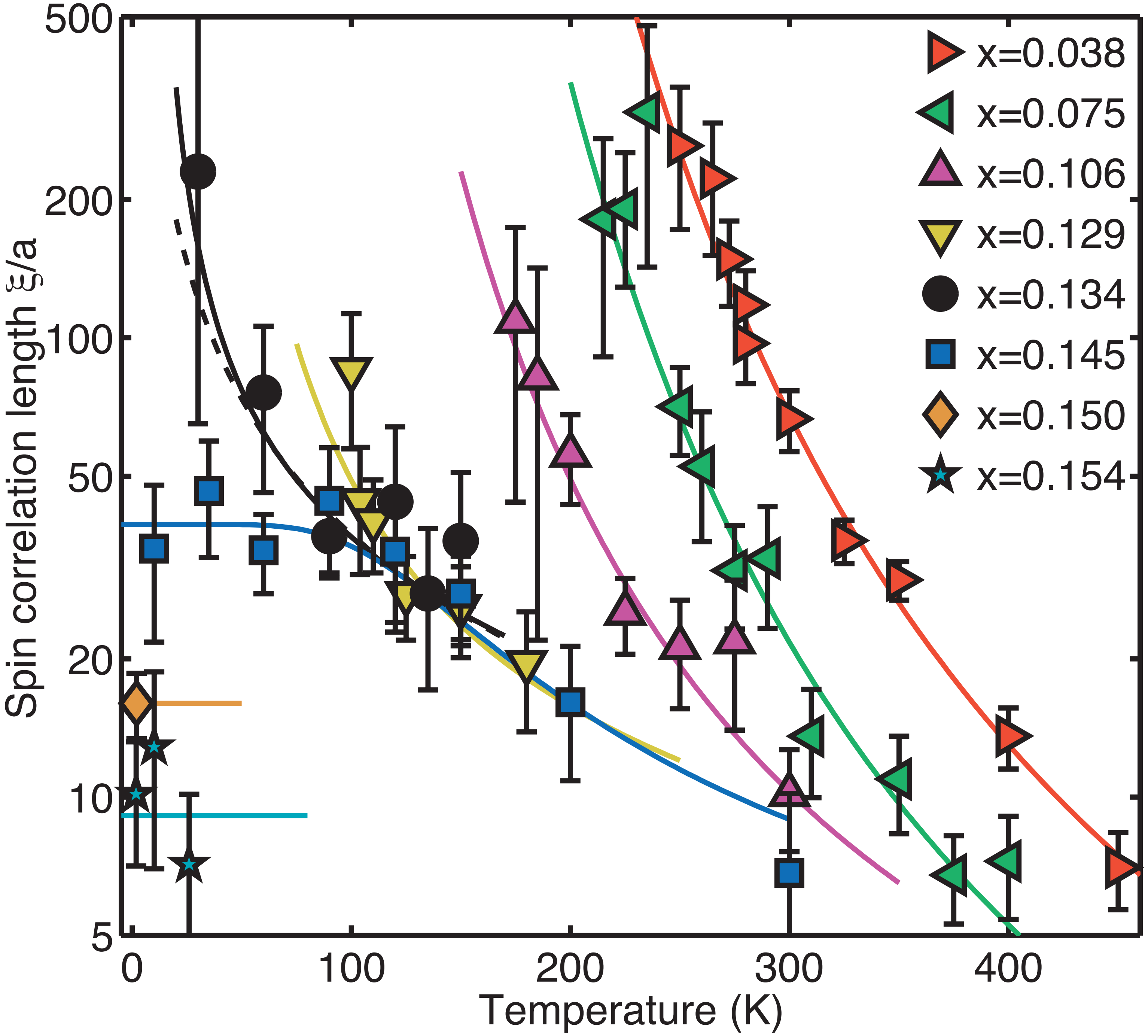

E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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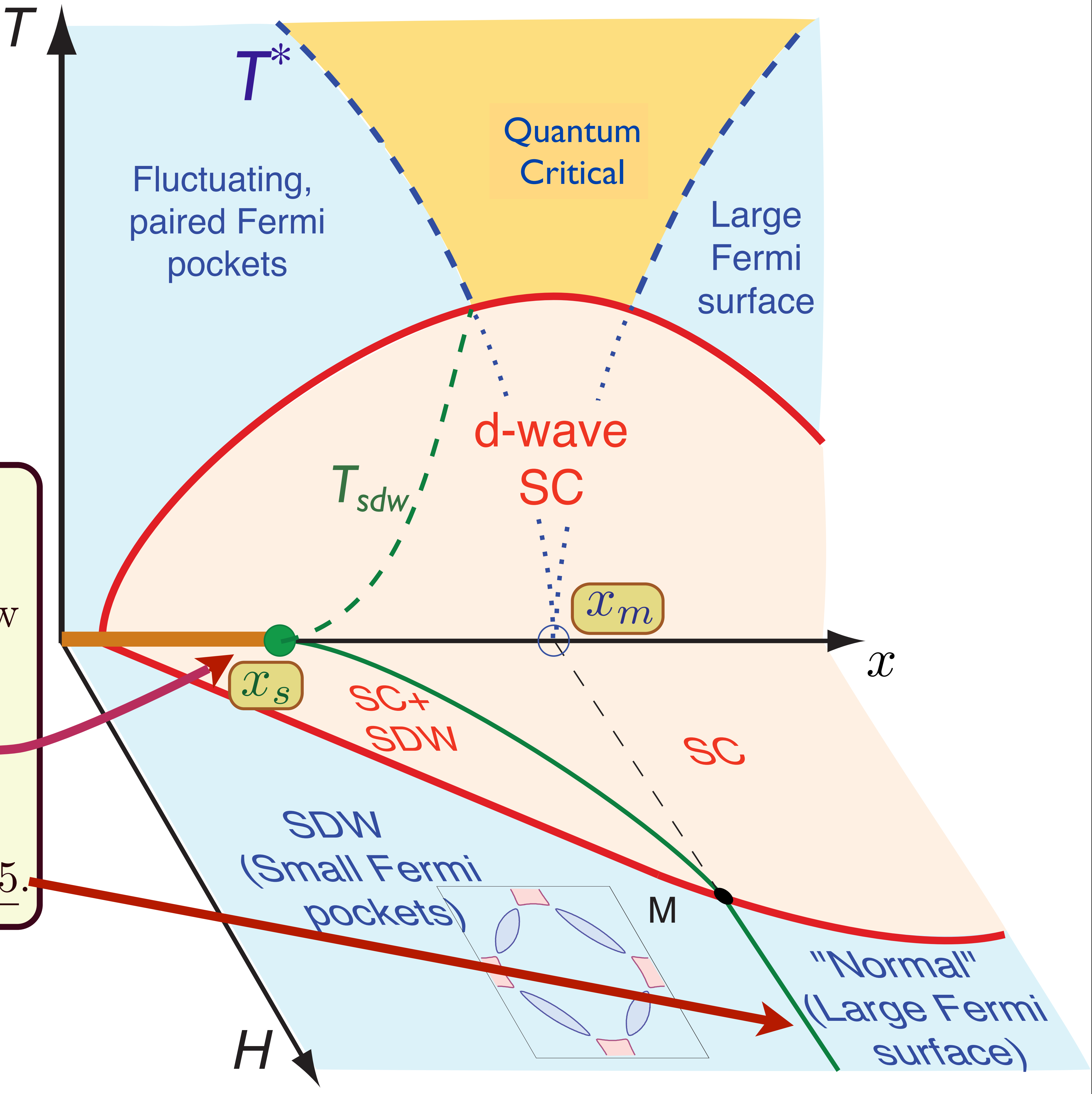
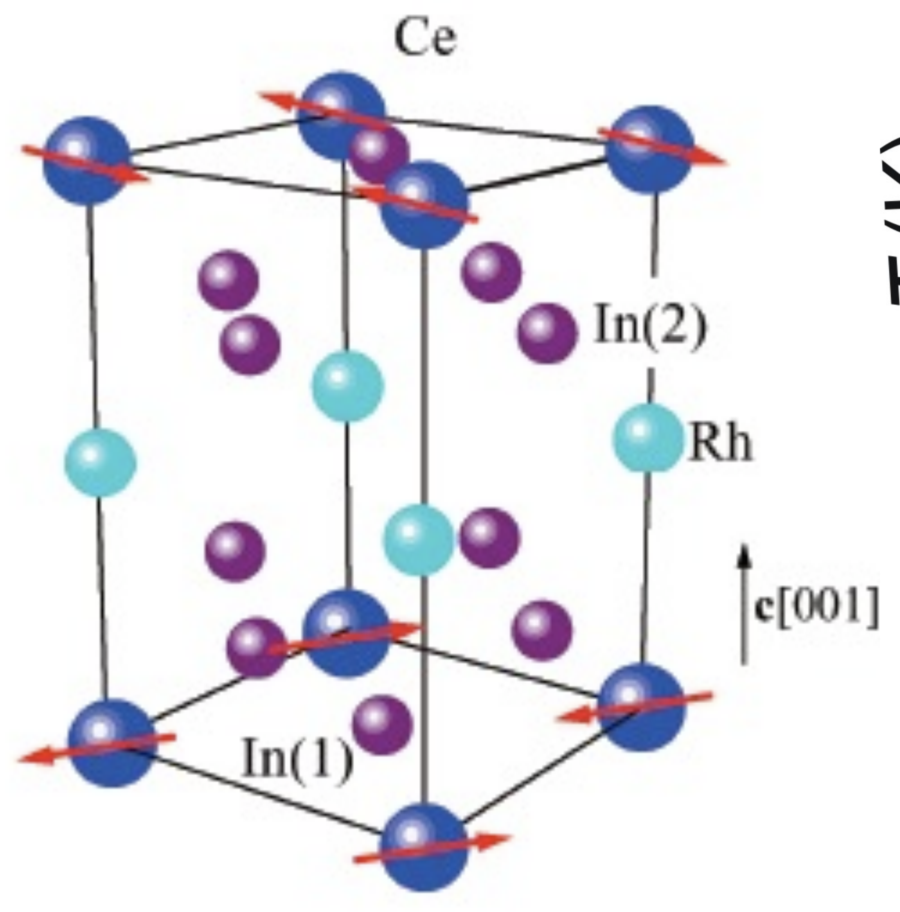
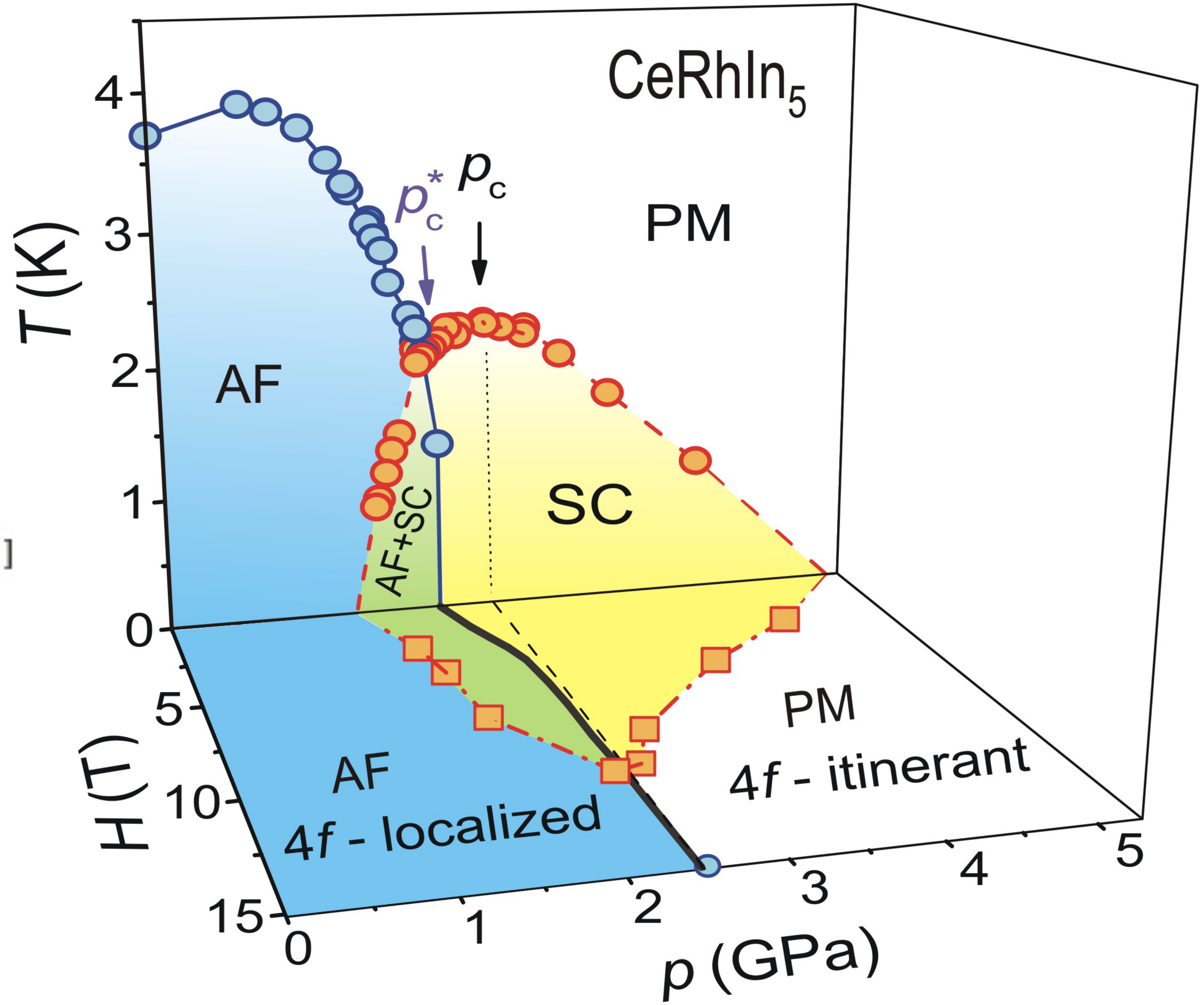


Similar phase diagram for $\mathrm{CeRhIn_5}$

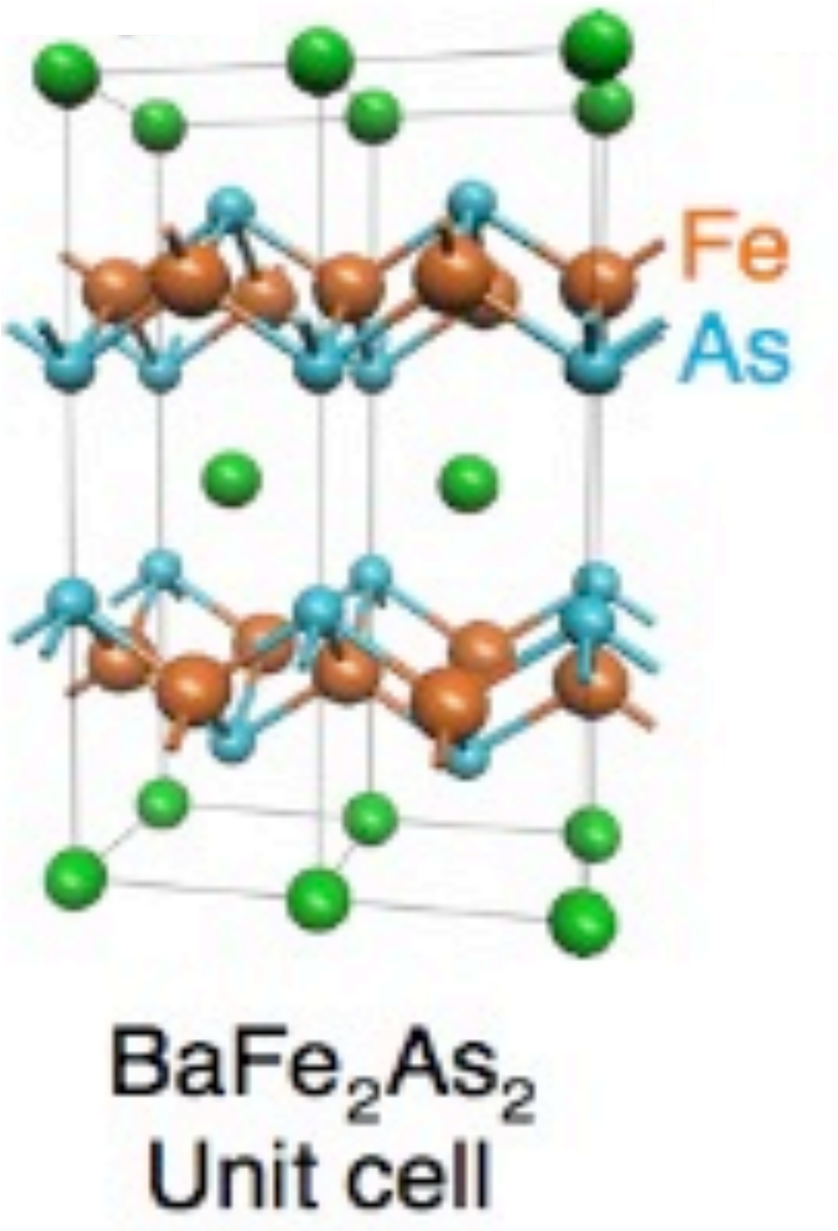


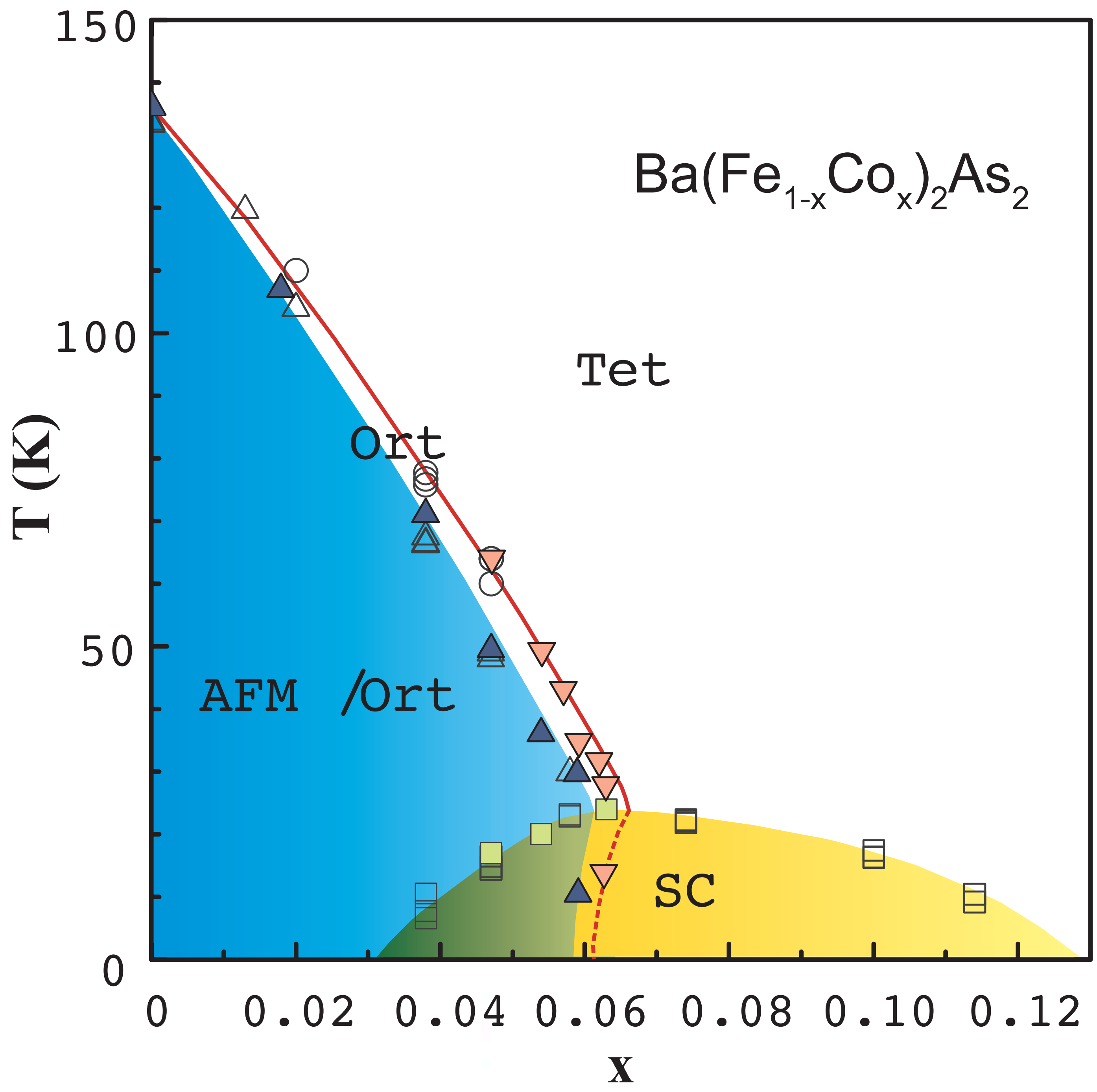


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

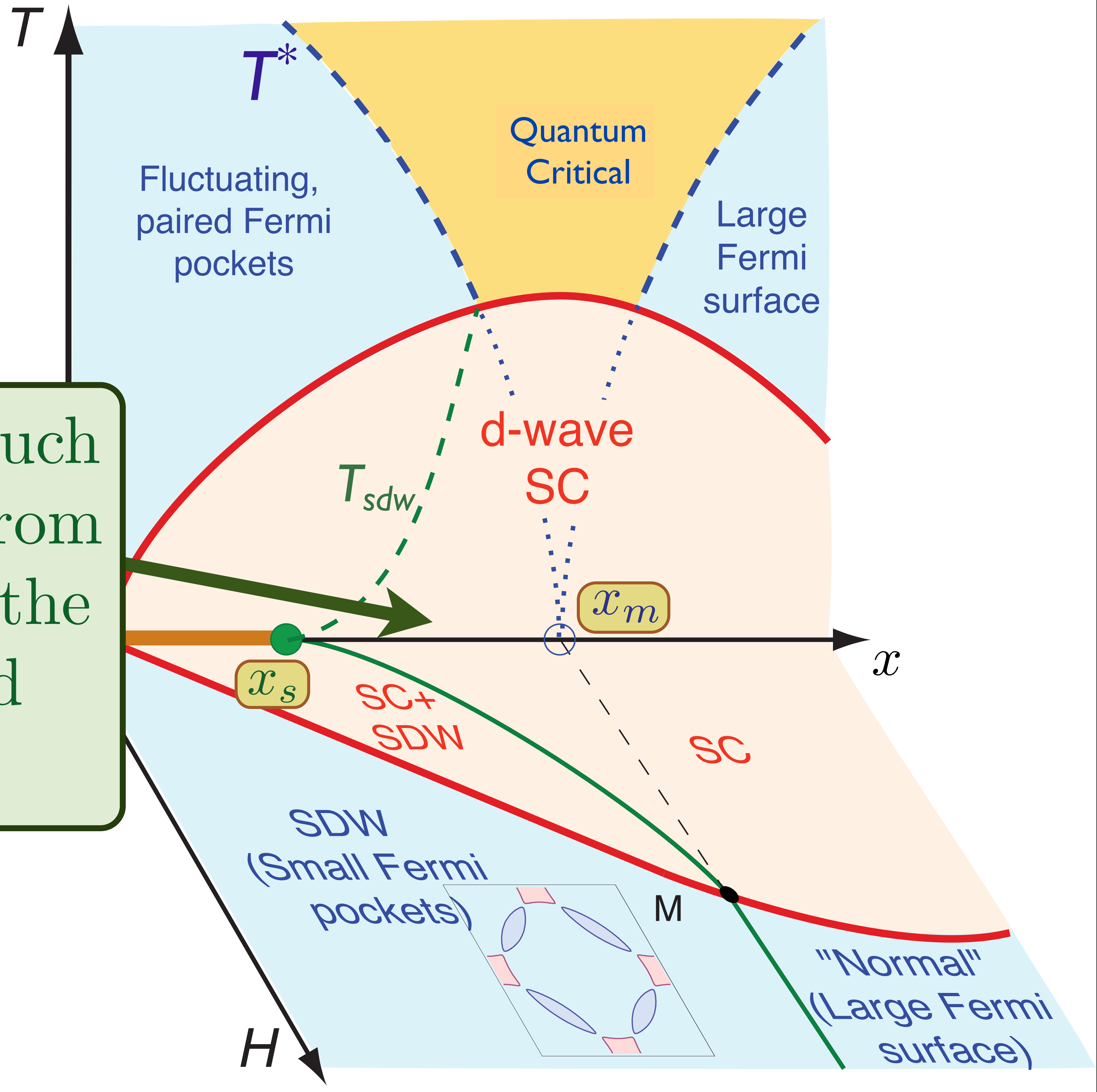
Iron pnictides:

a new class of high temperature superconductors



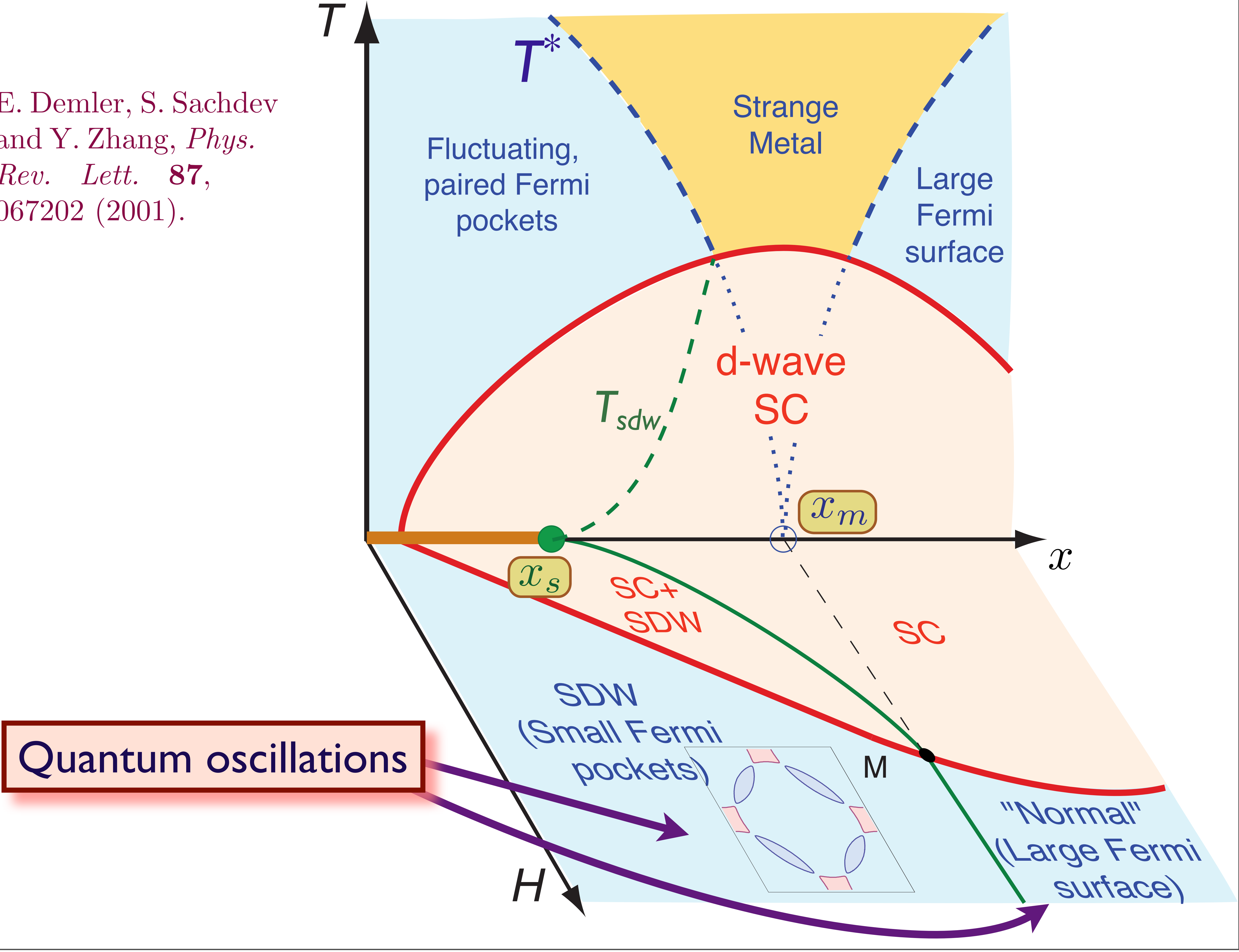


S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).



There is a much larger shift from x_m to x_s in the hole-doped cuprates.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Evidence for small Fermi pockets in hole-doped cuprates

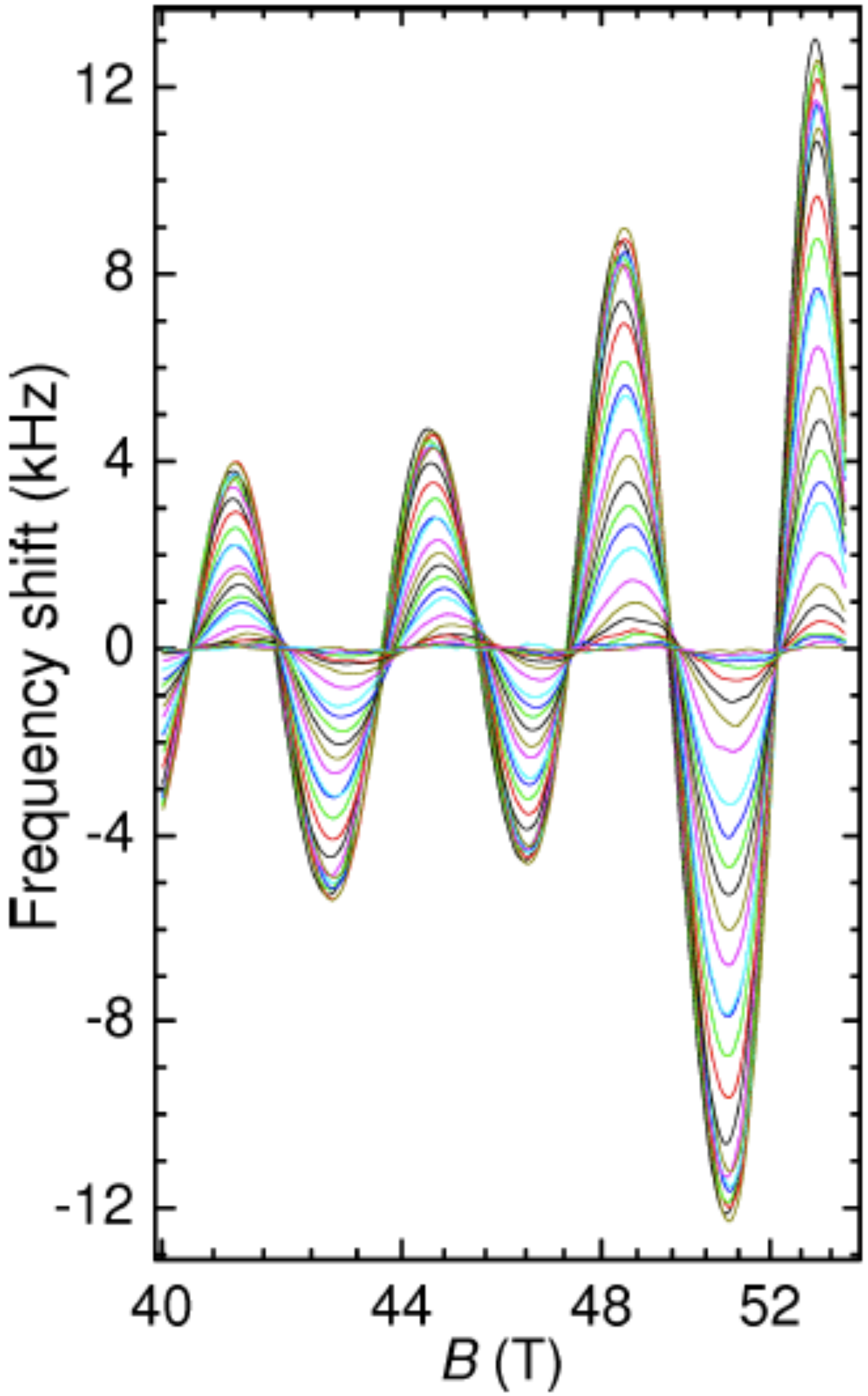
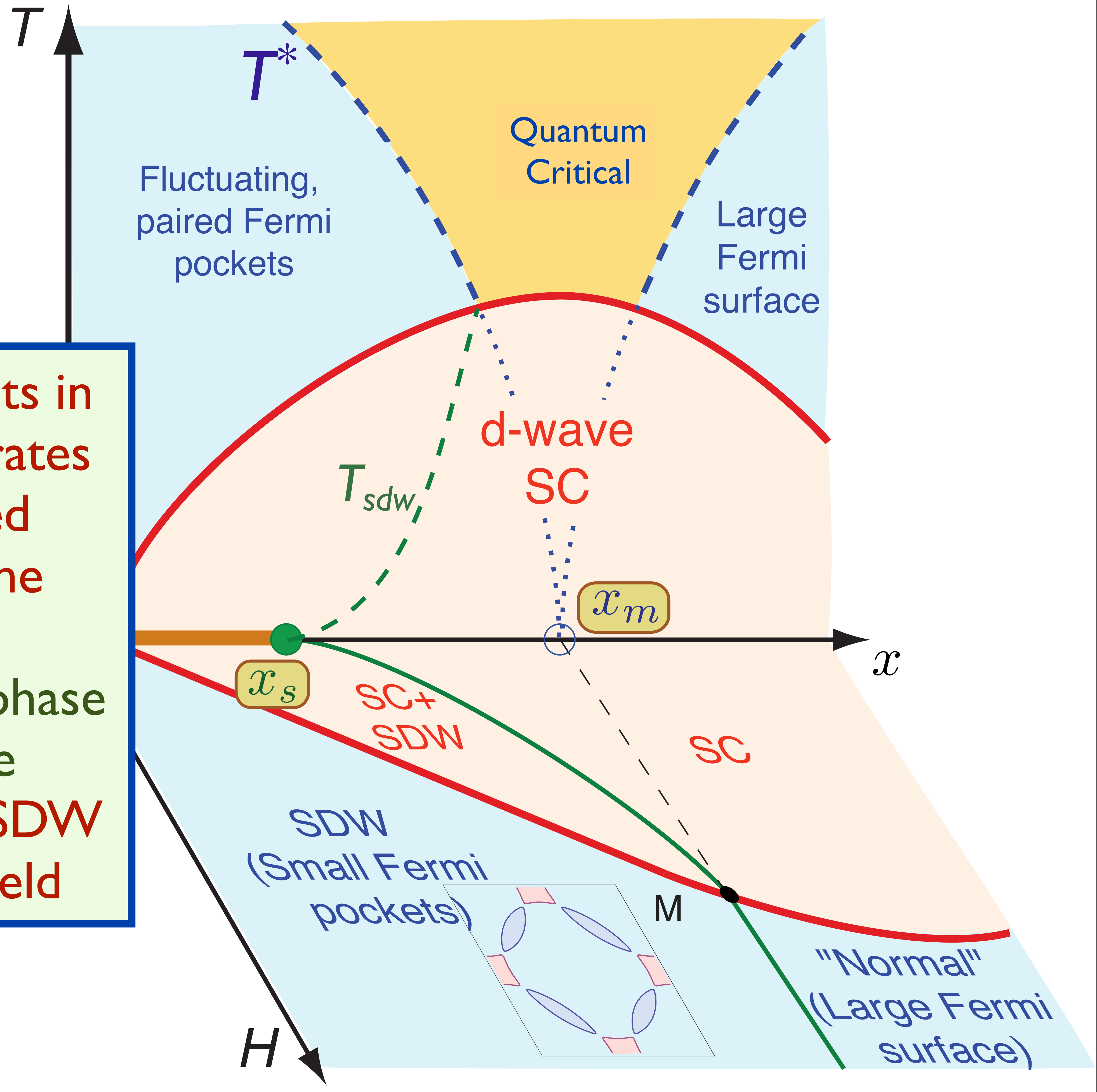


FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between T = 1 and 18 K. The actual T values are provided in Fig. 3.

Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D. A. Bonn,
W. N. Hardy, and G. G. Lonzarich
Physical Review B **81**, 140505(R) (2010)

Original observation:
N. Doiron-Leyraud, C. Proust,
D. LeBoeuf, J. Levallois,
J.-B. Bonnemaison, R. Liang,
D. A. Bonn, W. N. Hardy,
and L. Taillefer,
Nature **447**, 565 (2007)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Many experiments in hole-doped cuprates have presented evidence for the predicted green quantum phase transition line from SC to SC+SDW in a magnetic field

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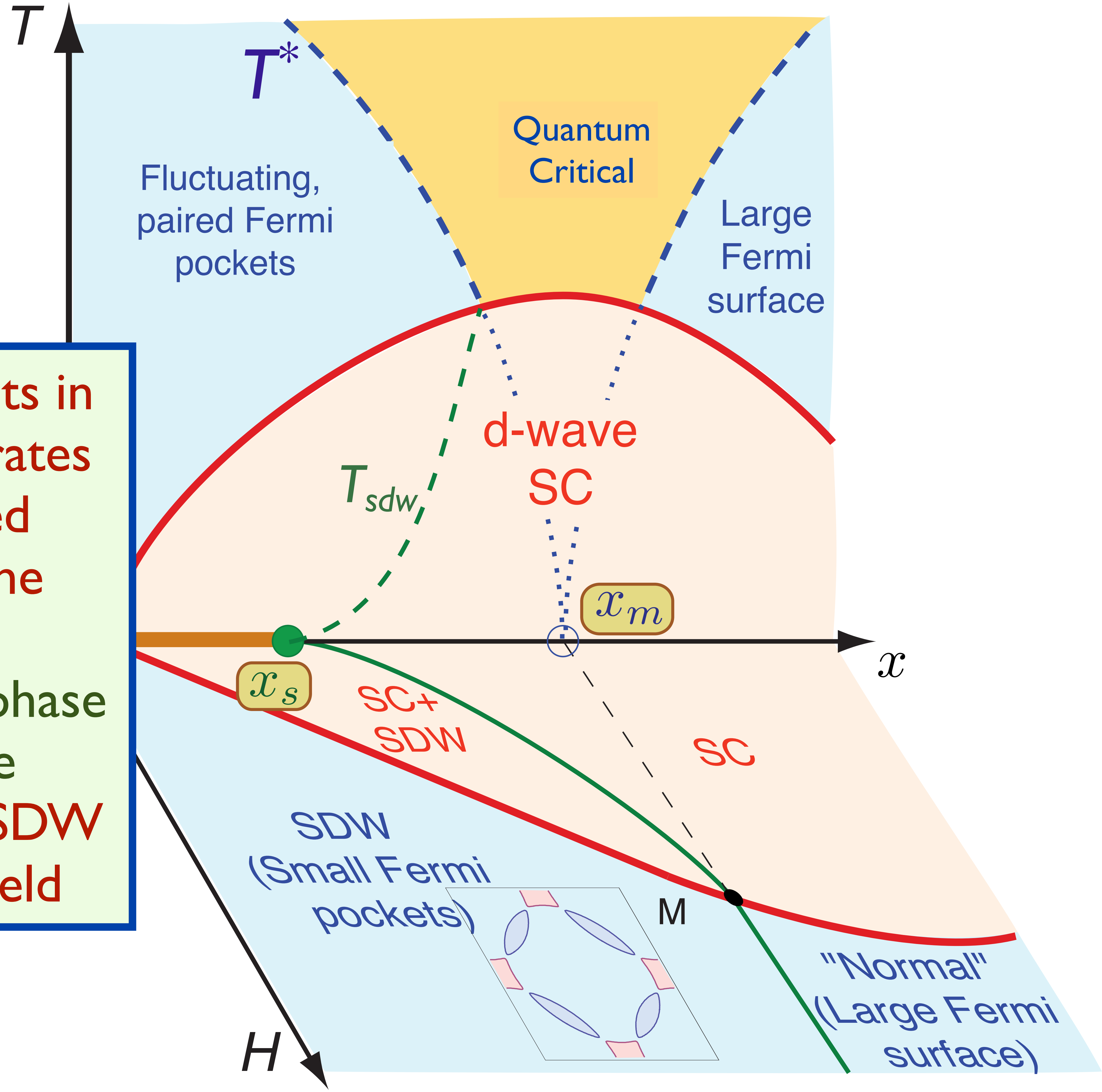

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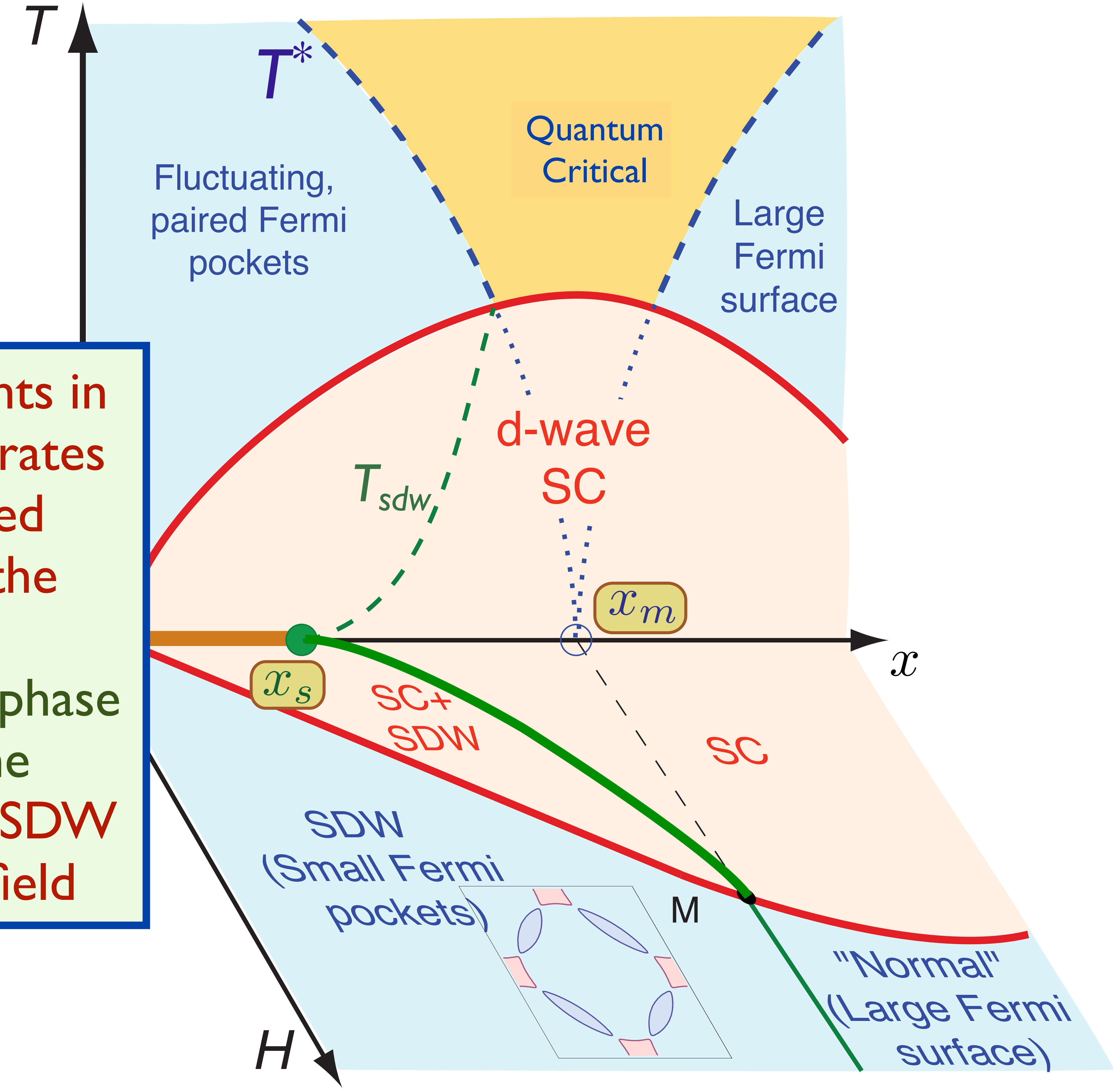
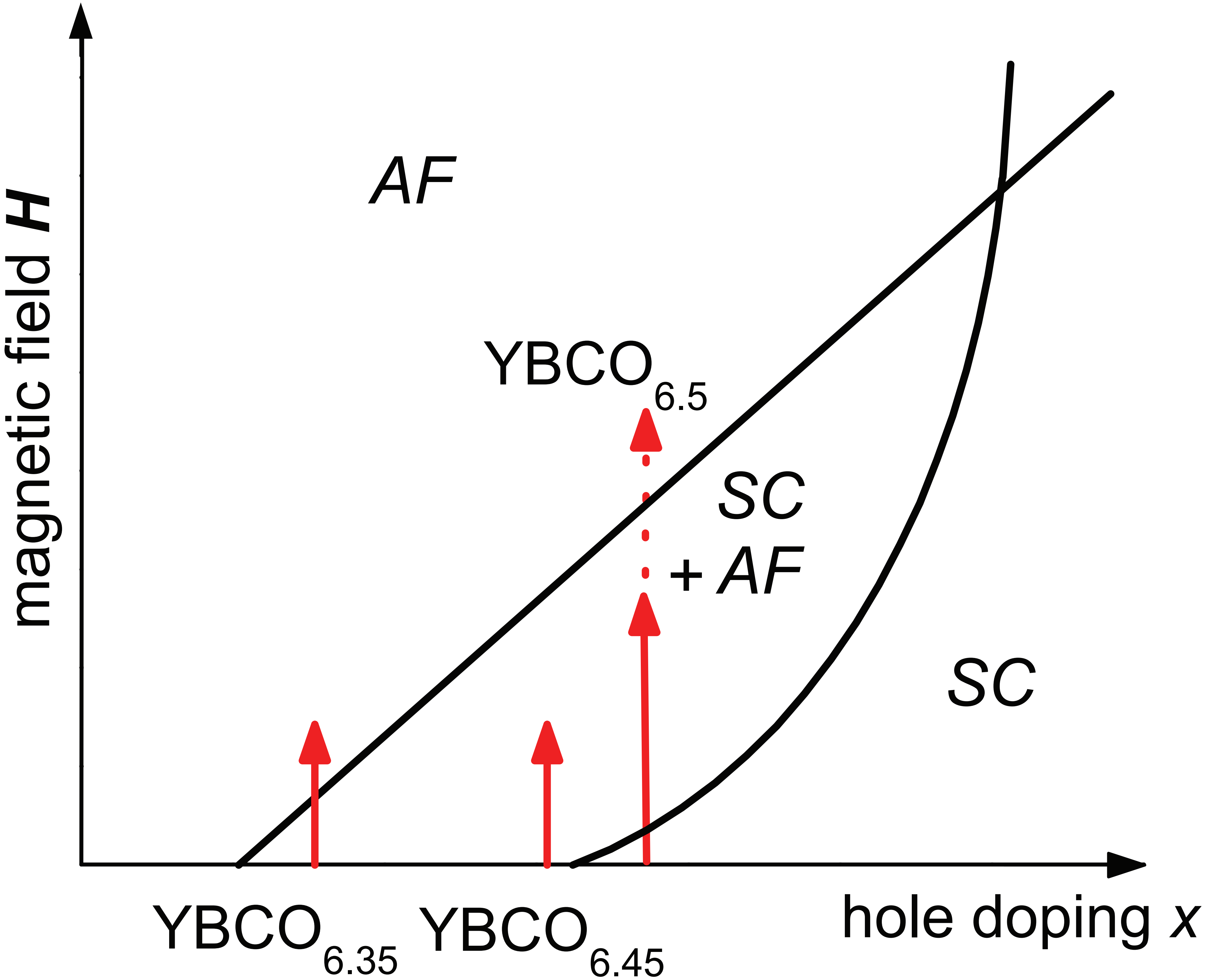
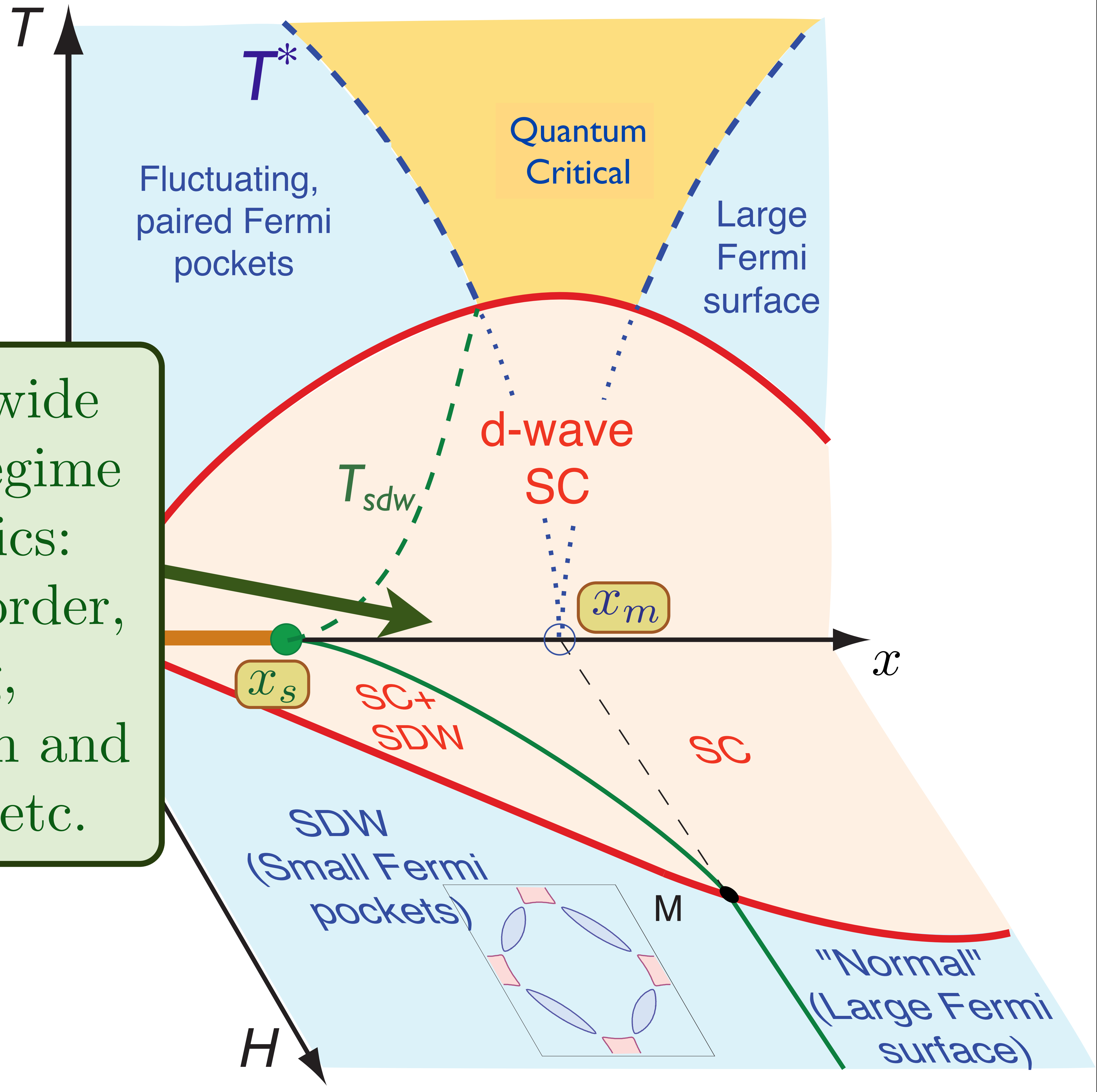




D. Haug, V. Hinkov, Y. Sidis, P. Bourges, N. B. Christensen, A. Ivanov, T. Keller, C. T. Lin, and B. Keimer, *New J. Phys.* **12**, 105006 (2010)



This opens a wide intermediate regime for new physics: bond-nematic order, T-breaking, fractionalization and Mott physics etc.

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

"Mechanism" of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

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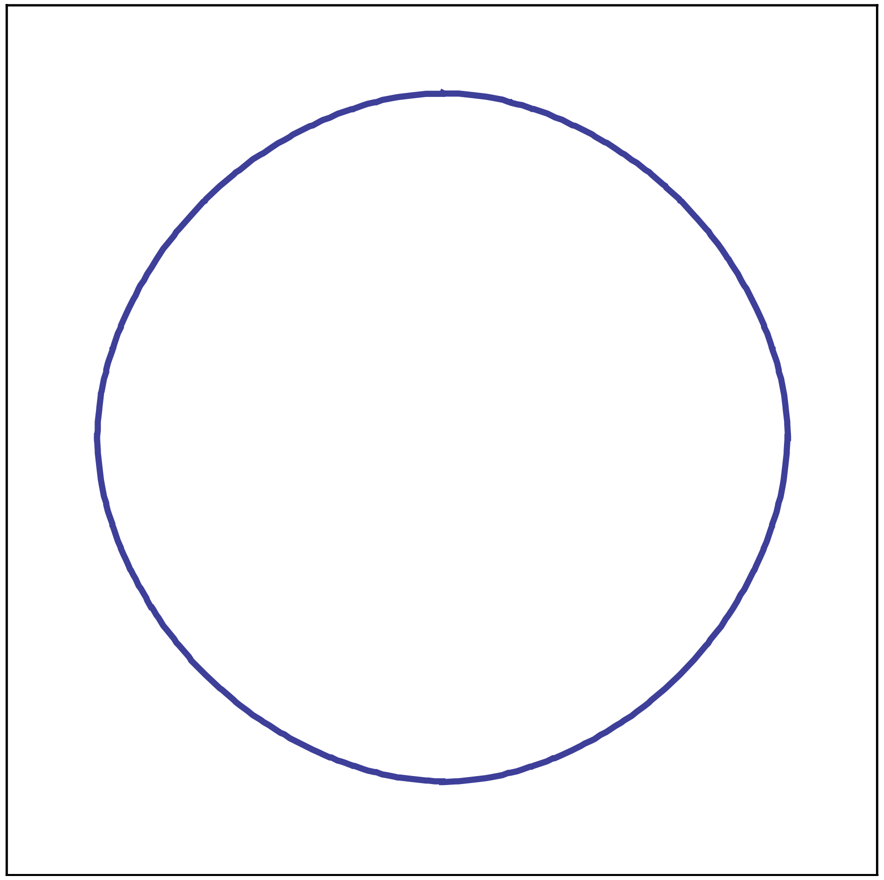
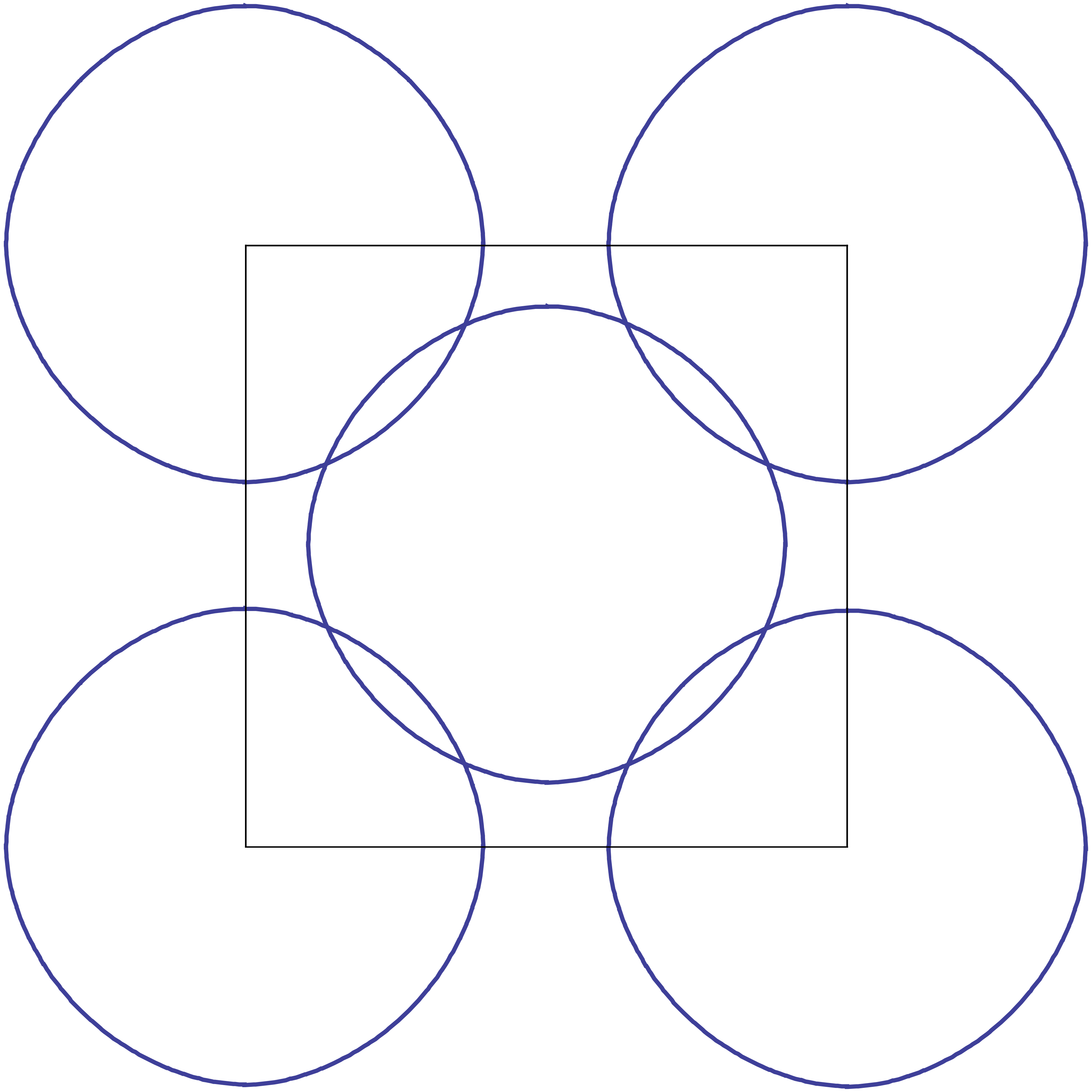
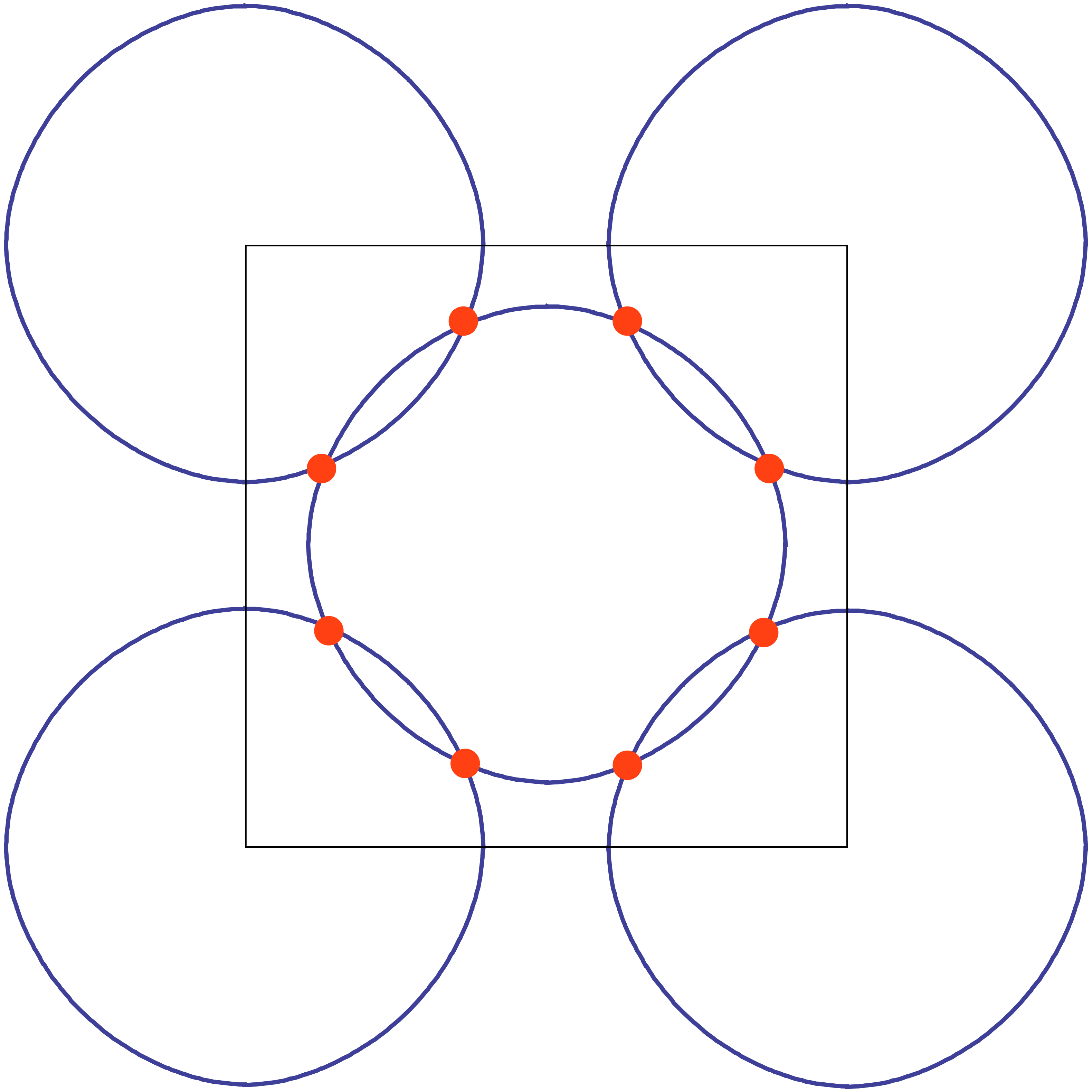


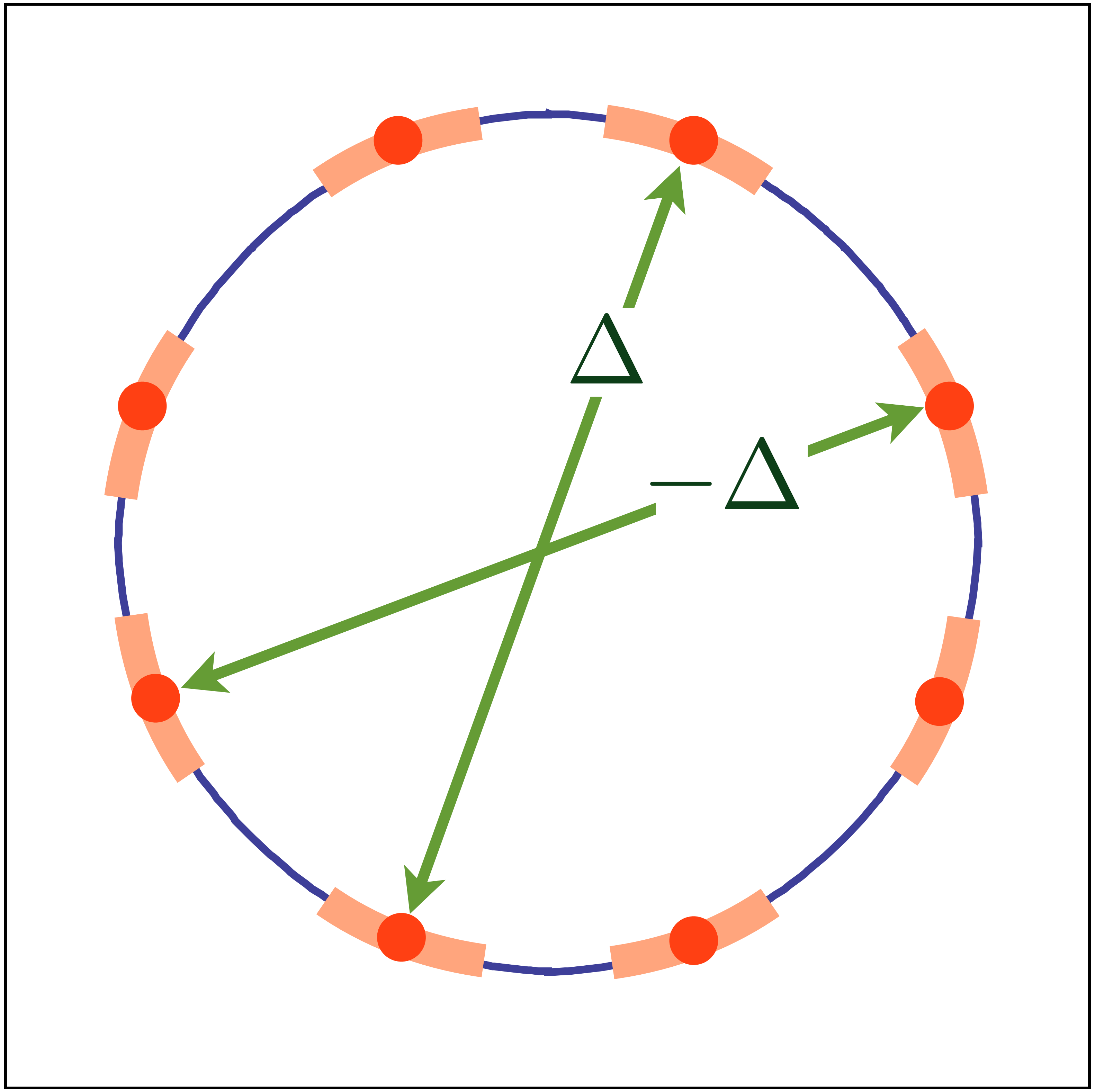
Metal with “large” Fermi surface



Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

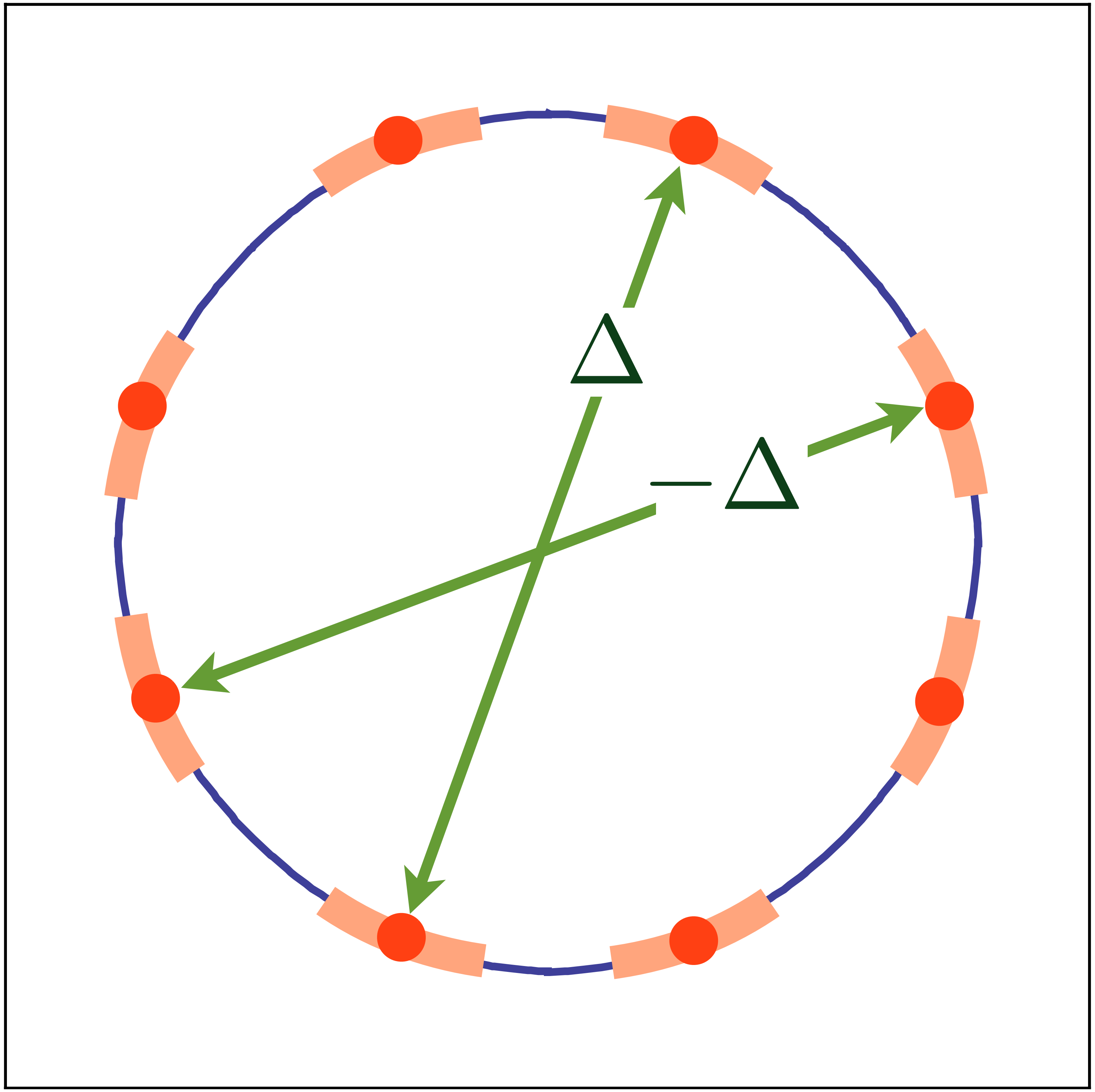


“Hot” spots



Unconventional pairing at *and near* hot spots

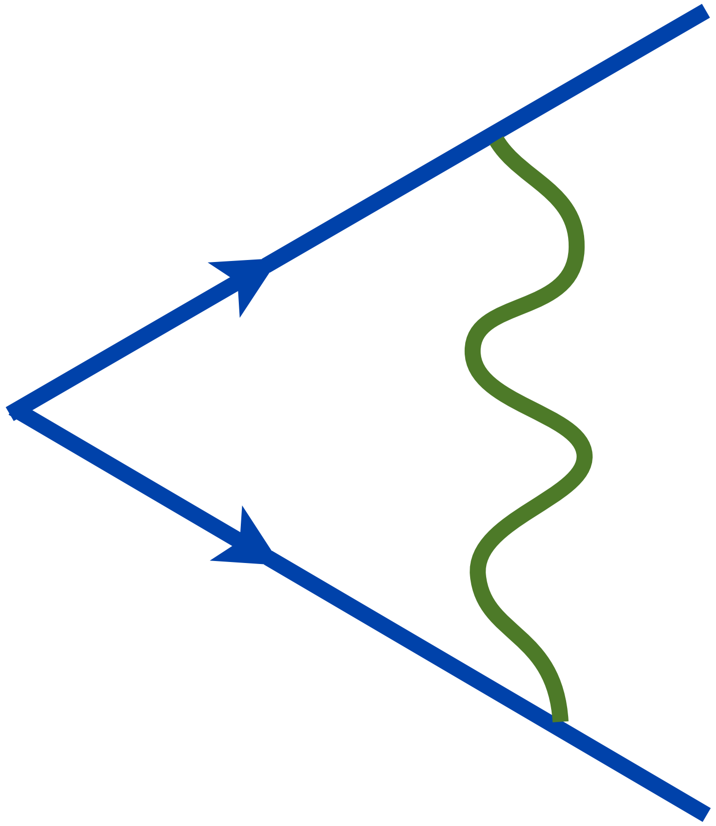
$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

BCS theory

$$1+\lambda_{\text{e-ph}}\underbrace{\log\left(\frac{\omega_D}{\omega}\right)}_{\boxed{\text{Cooper logarithm}}}$$

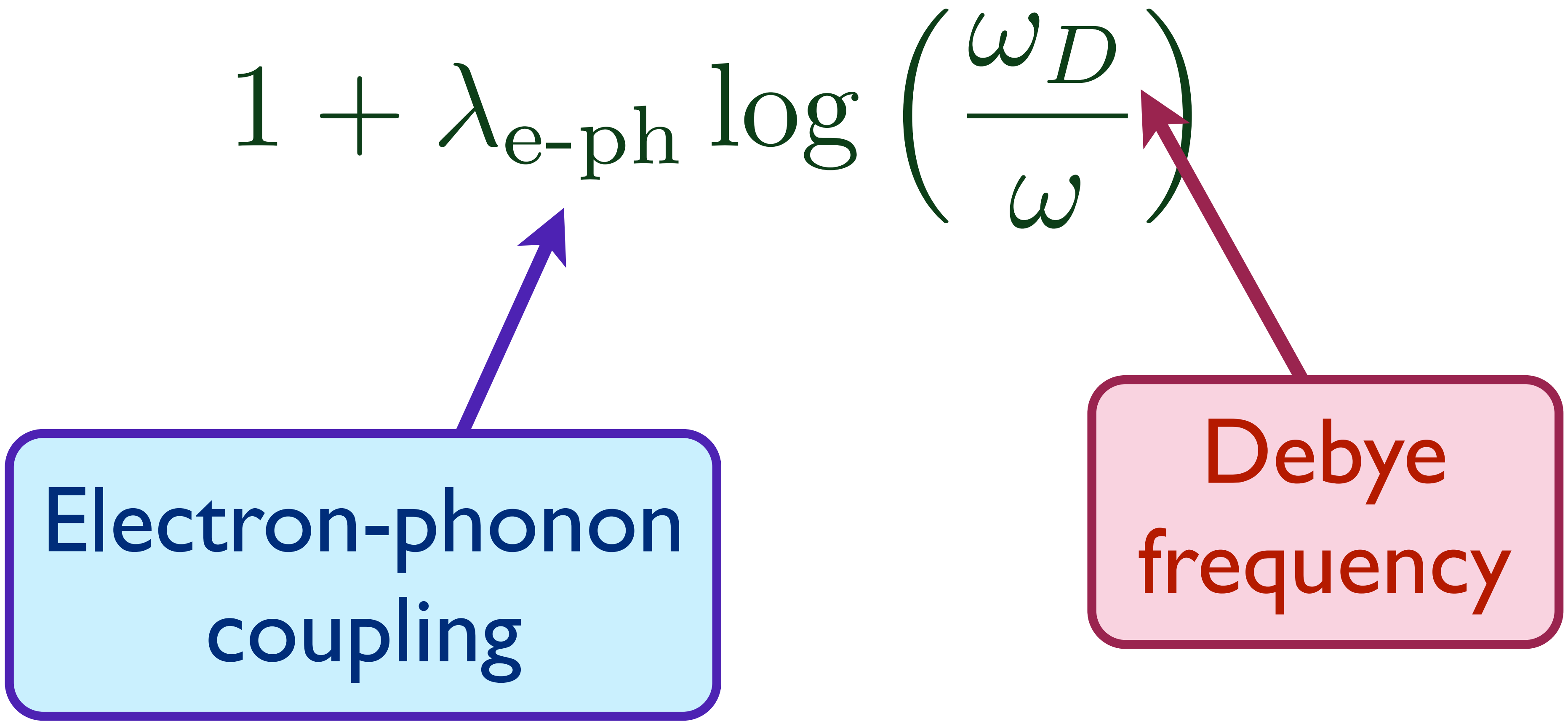


BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

BCS theory



$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

Debye frequency

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Electron-phonon coupling

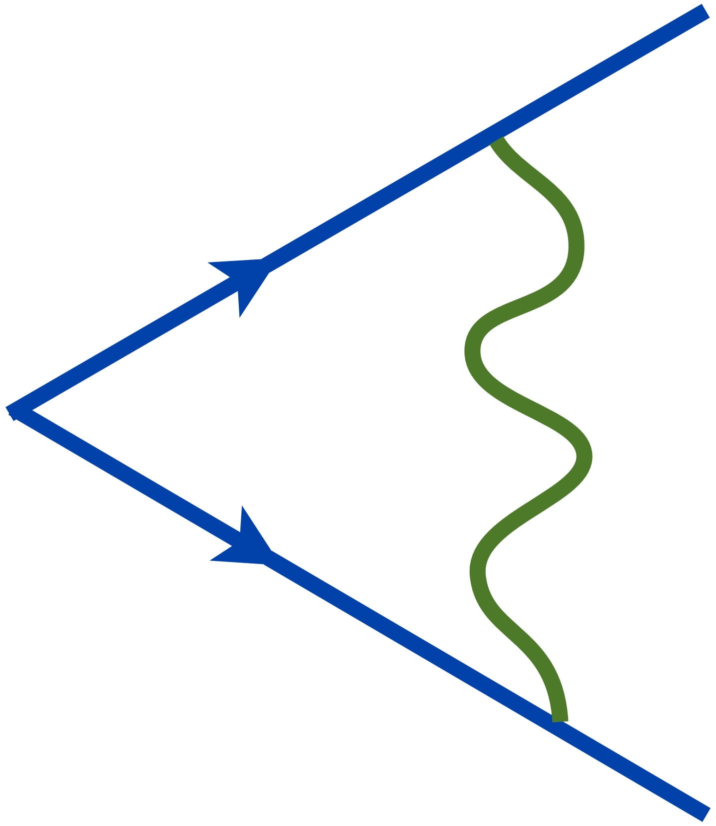
Debye frequency

Implies
$T_c \sim \omega_D \exp\left(-1/\lambda\right)$

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1+\left(\frac{U}{t}\right)^2 \underbrace{\log\left(\frac{E_F}{\omega}\right)}_{\boxed{\text{Cooper logarithm}}}$$



V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

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$$1 + \left(\frac{U}{t}\right)^2 \log\left(\frac{E_F}{\omega}\right)$$

Fermi energy

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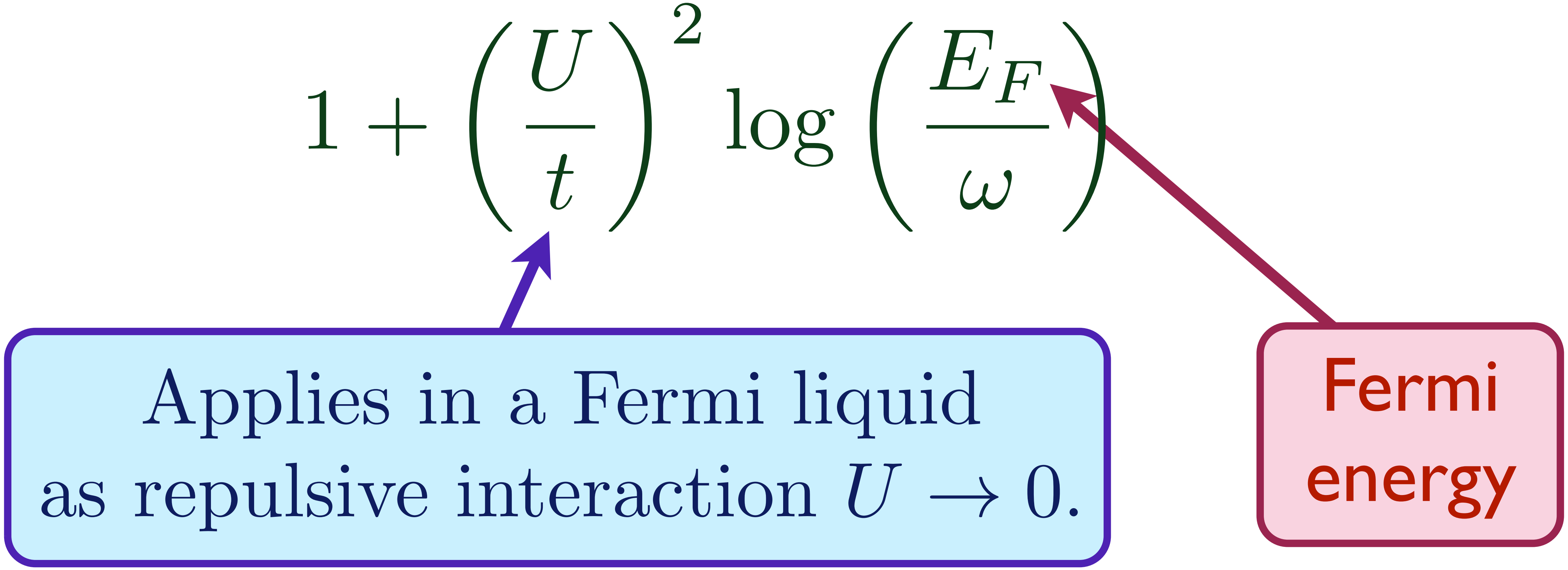

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Applies in a Fermi liquid
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Fermi energy

Implies
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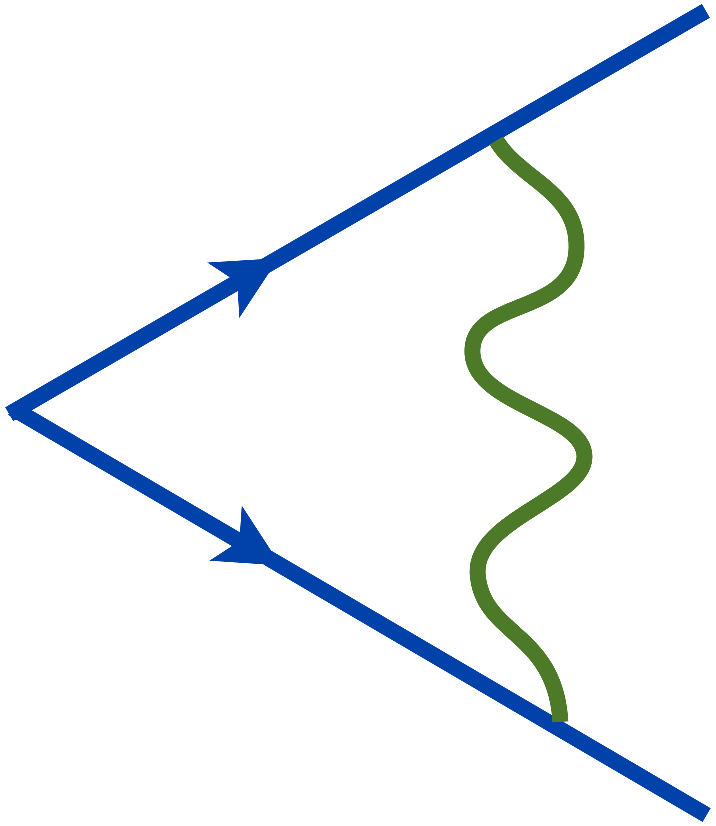
Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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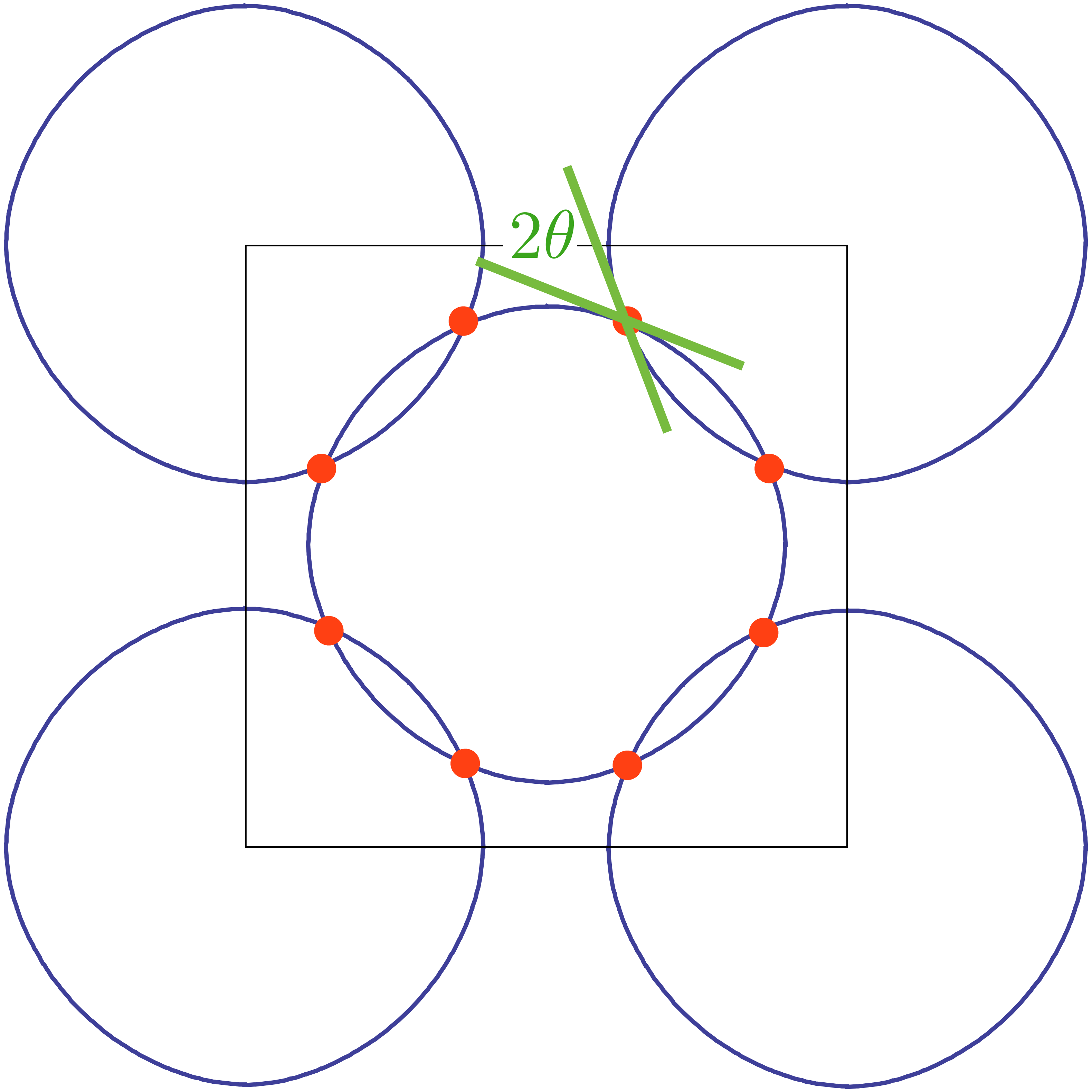
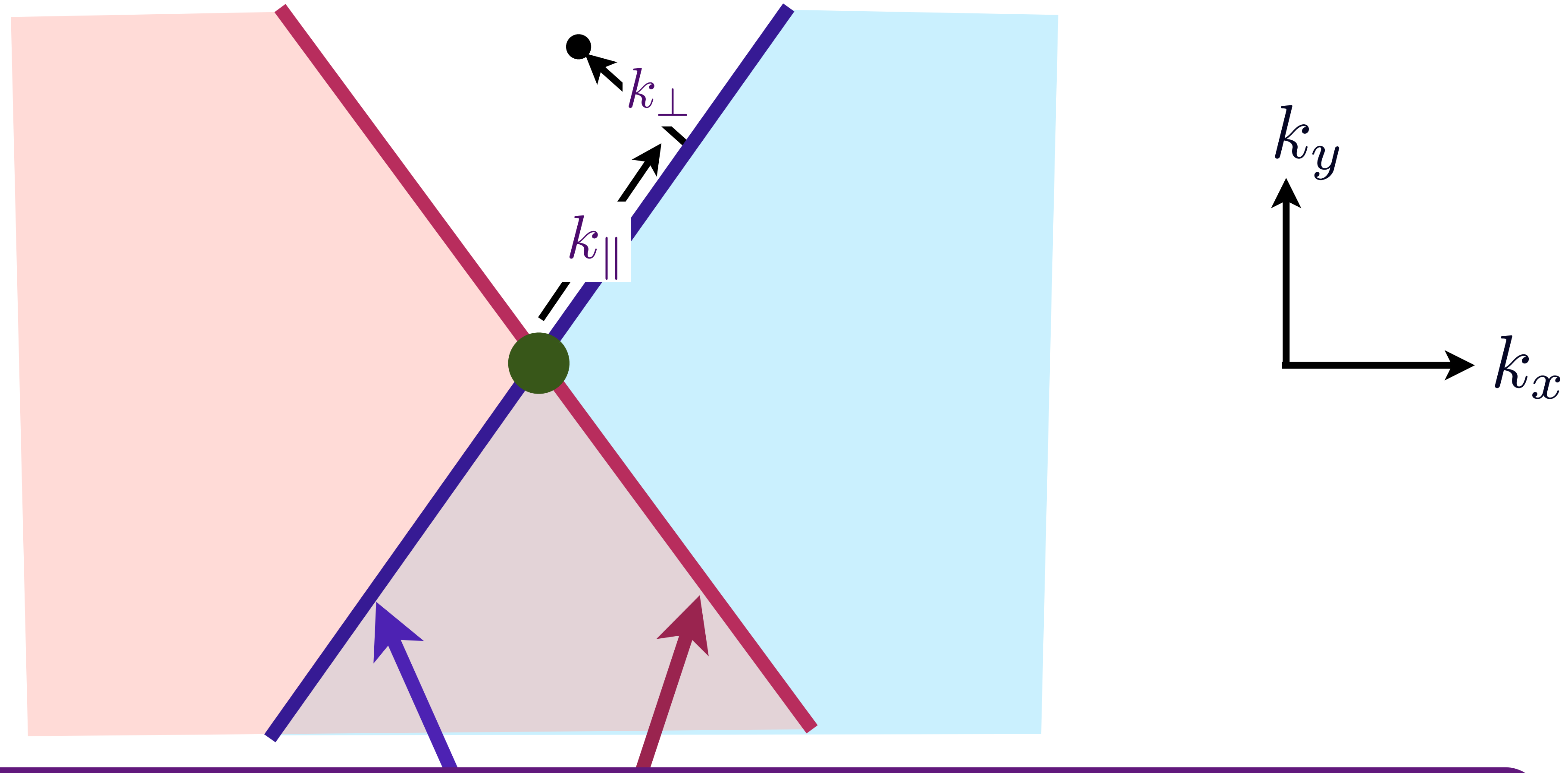
Fermi energy

$\alpha = \tan\theta$, where 2θ is the angle between Fermi lines. *Independent* of interaction strength U in 2 dimensions.

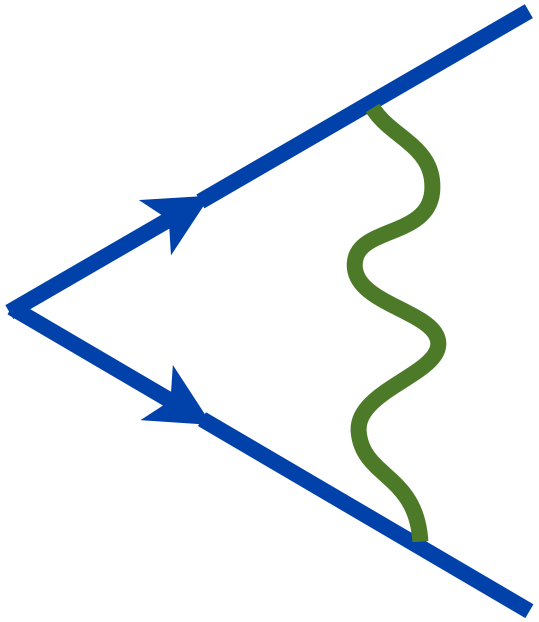
(see also Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)


2θ

M.A. Metlitski
and S. Sachdev,
Phys. Rev. B **85**,
075127 (2010)



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{i\omega - v_F(k_\parallel) k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$



$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

M. A. Metlitski
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Phys. Rev. B **85**,
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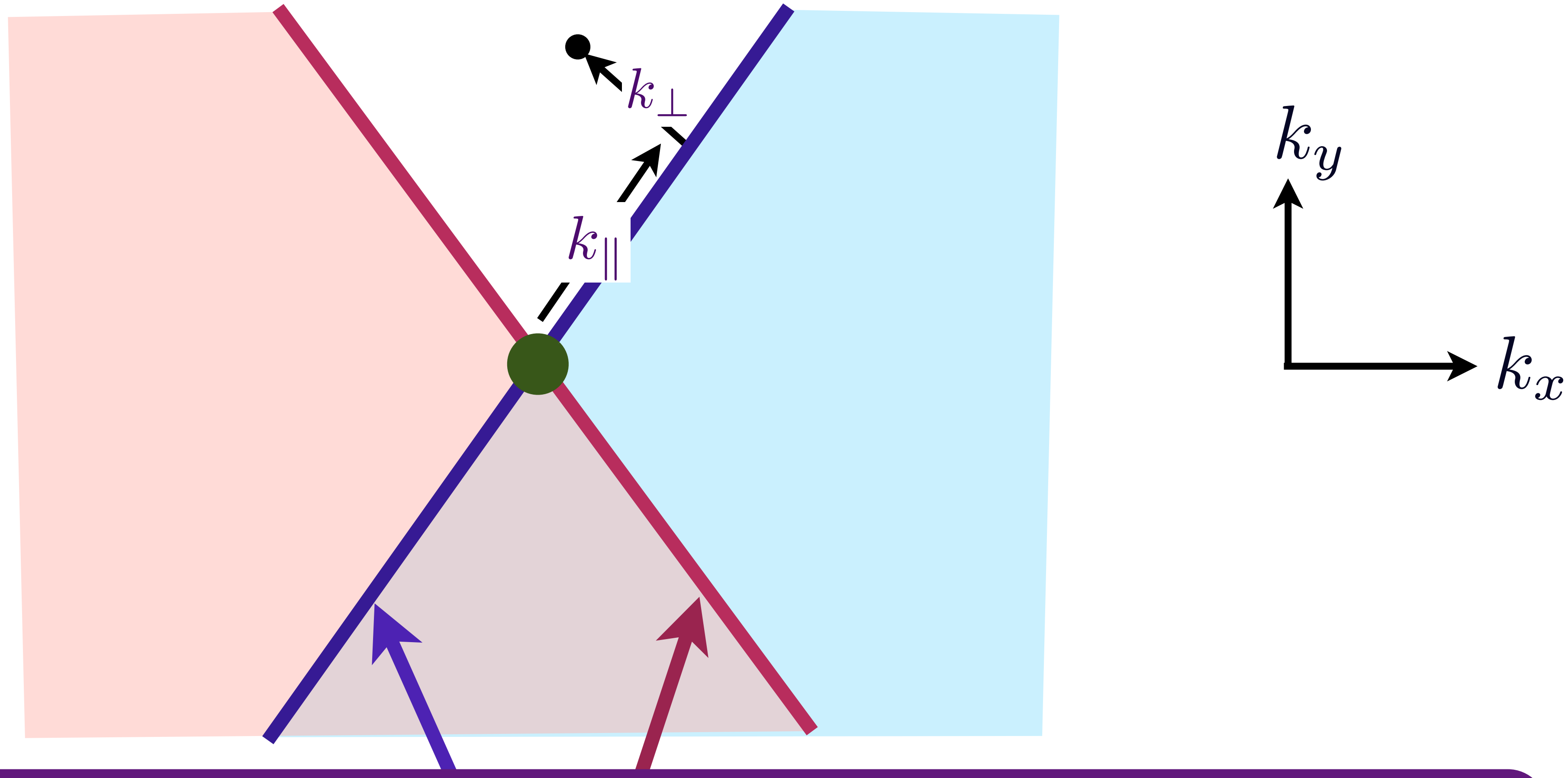

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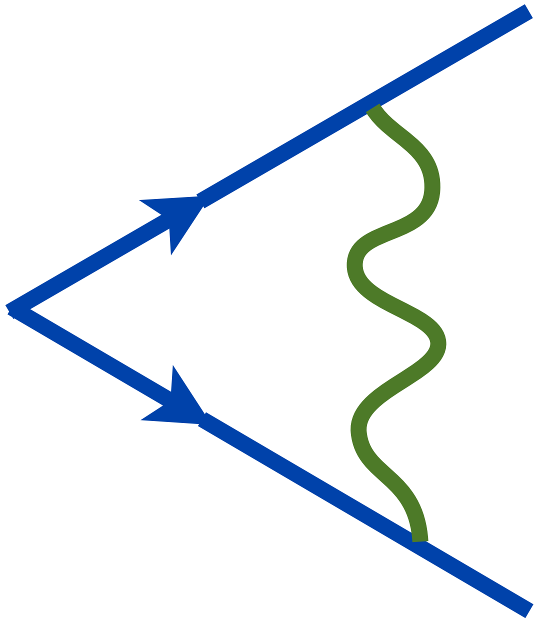
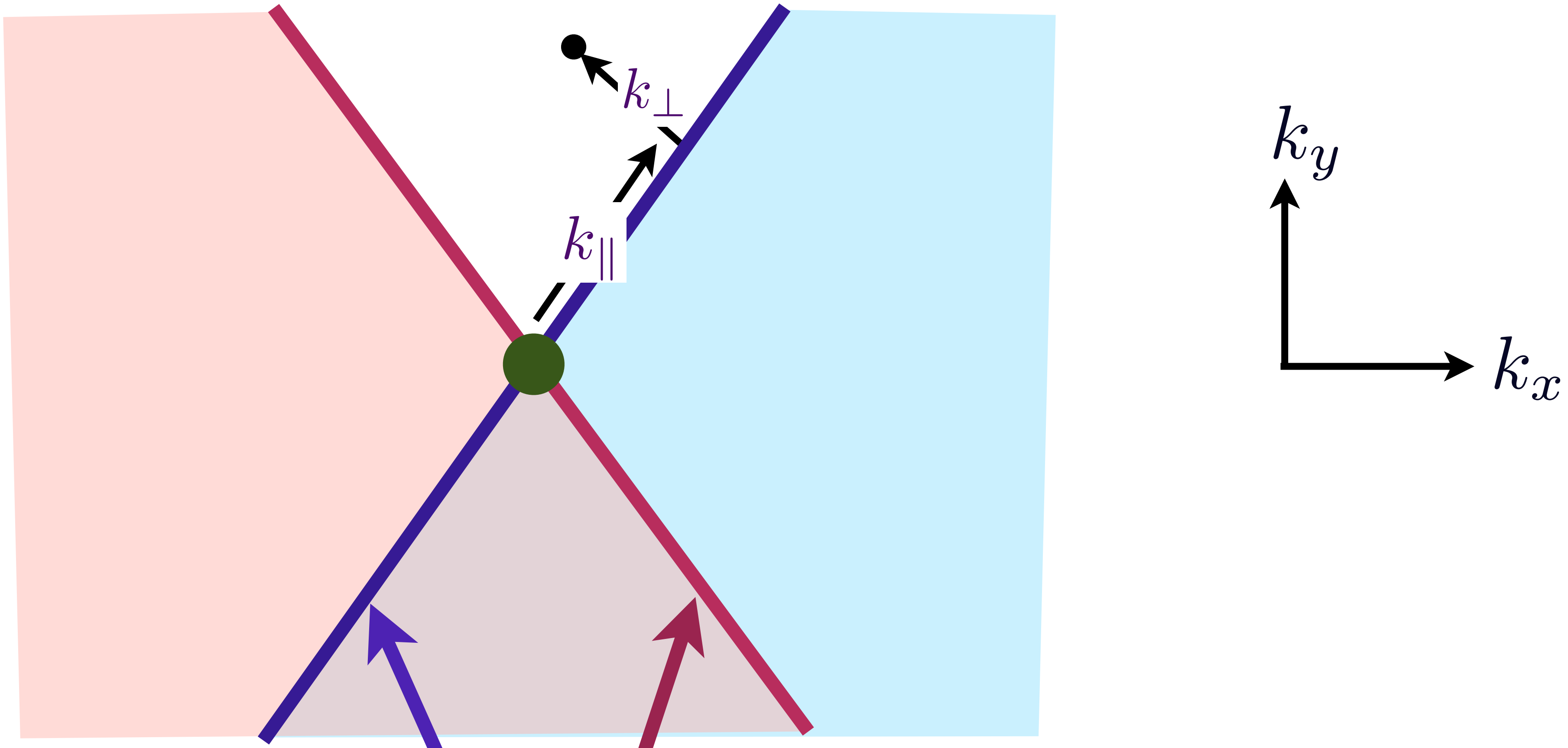


$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \underbrace{\left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)}\right) \log \frac{k_\parallel^2}{\omega}}_{\boxed{\text{Cooper logarithm}}}$$

M.A. Metlitski
and S. Sachdev,
Phys. Rev. B **85**,
075127 (2010)



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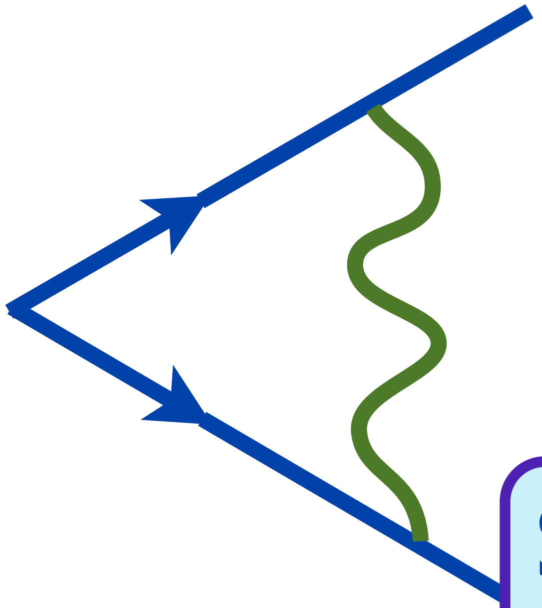

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Spin fluctuation propagator

Cooper logarithm

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

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$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

   Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

   "Mechanism" of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

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Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (Δ):

$$\begin{aligned}\mathcal{S} &= \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 \right. \\ &\qquad \left. + \kappa\, \vec{\varphi}^2 \left|\Delta\right|^2 \right] \\ &\quad + \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta|^2 - |\Delta|^2 + \frac{|\Delta|^4}{2} \right]\end{aligned}$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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where $\boxed{\kappa > 0 \text{ is the repulsion between the two order parameters,}}$ and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

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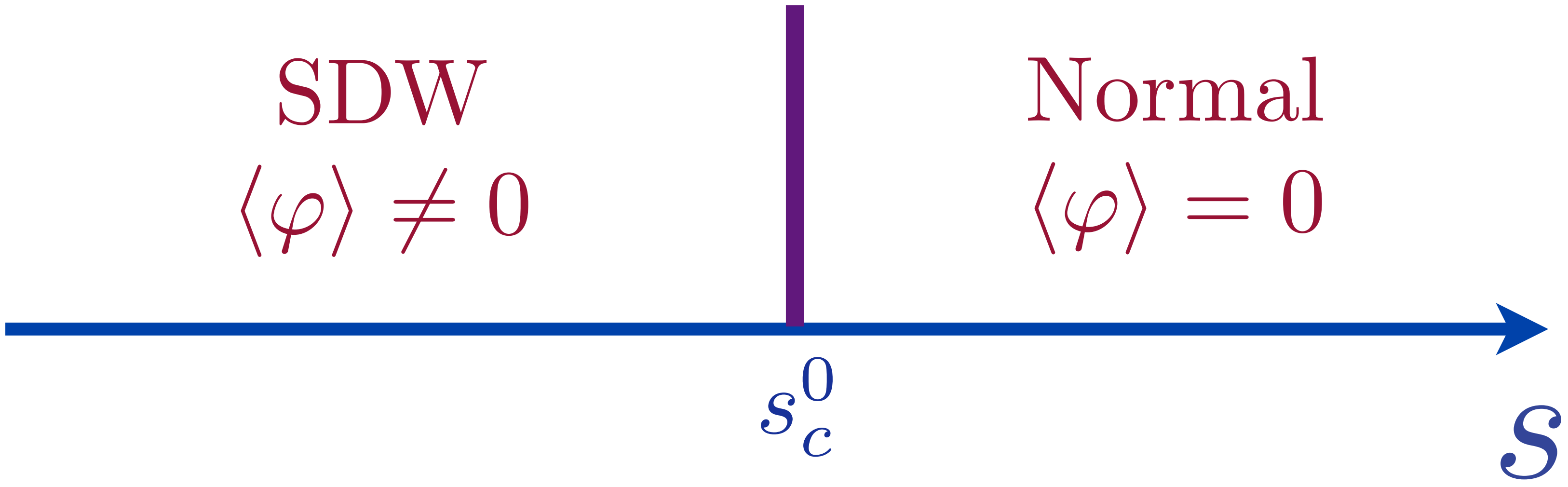


SDW

$\langle \varphi \rangle \neq 0$

Normal

$\langle \varphi \rangle = 0$

s_c^0

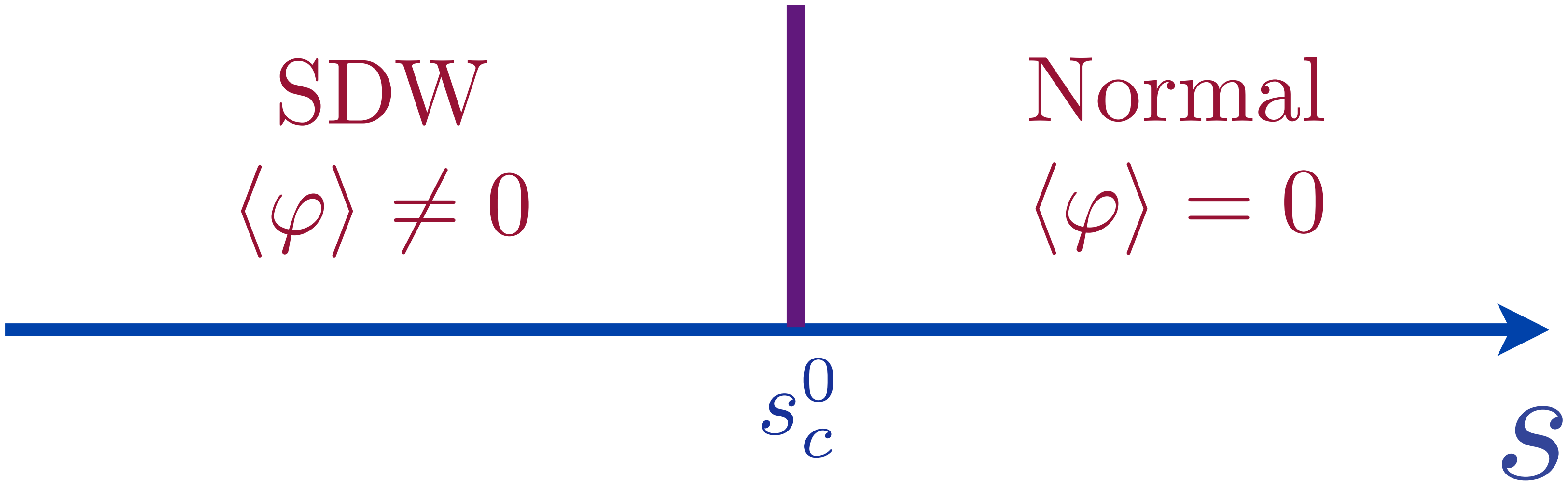
s

No SC

$\Delta = 0$

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



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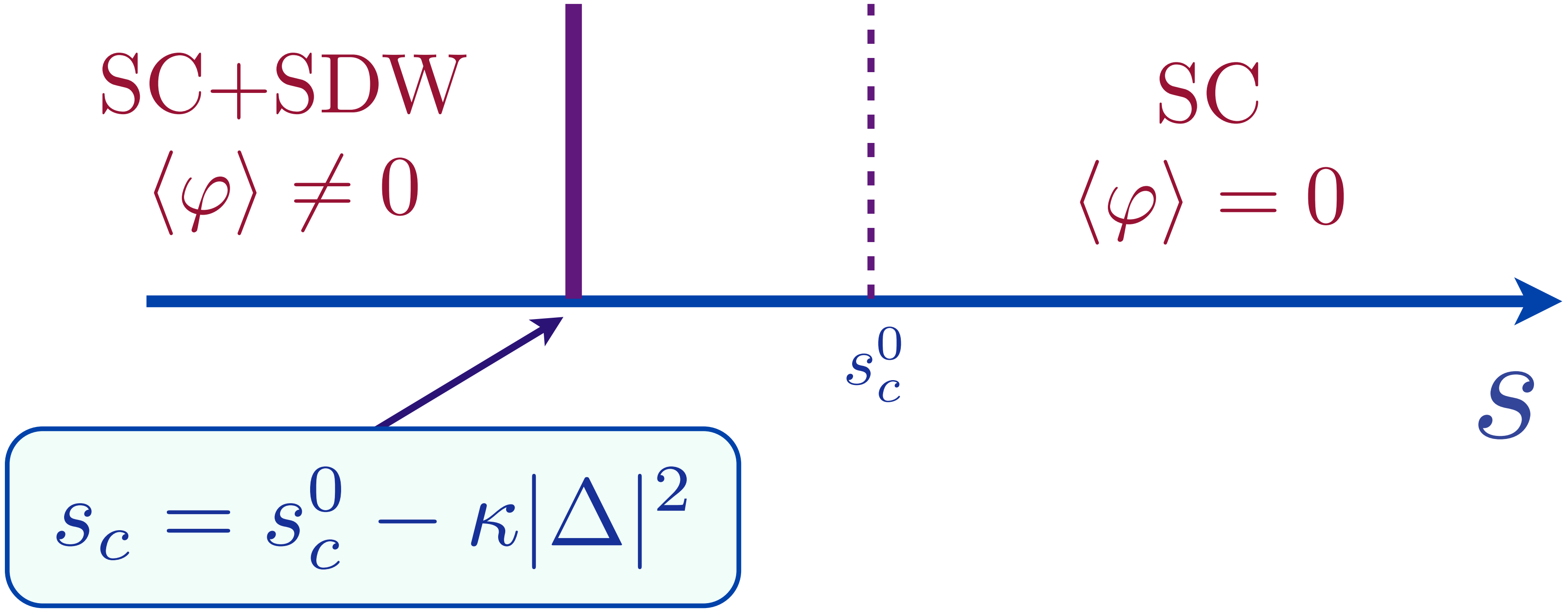


SC+SDW

$\langle \varphi \rangle \neq 0$

SC

$\langle \varphi \rangle = 0$

s_c^0

s

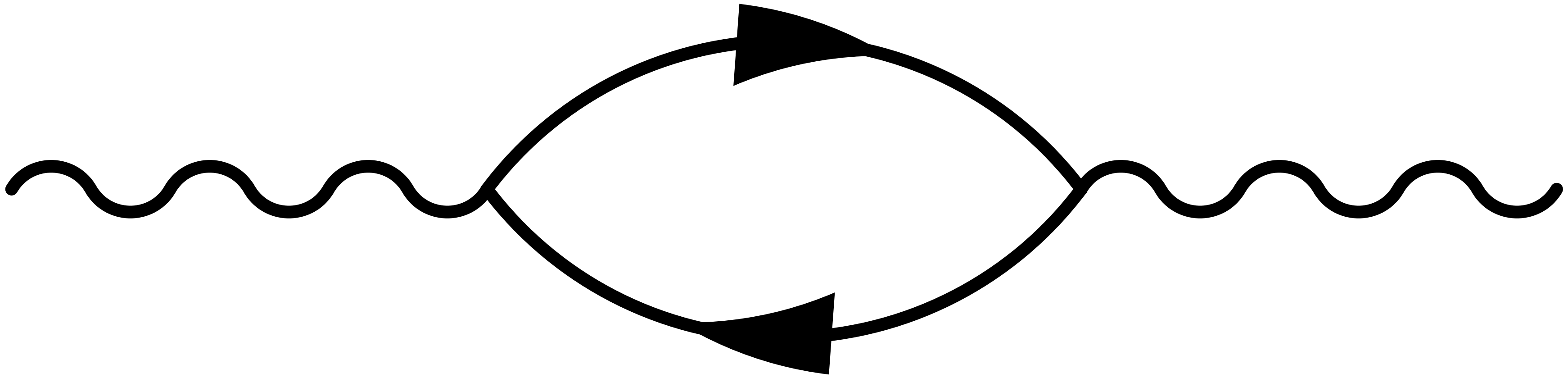
$$s_c = s_c^0 - \kappa |\Delta|^2$$

SC

$\Delta \neq 0$

Fermi surface theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



Compute the SDW susceptibility, χ, in the superconducting state. As $\Delta \to 0$, we find

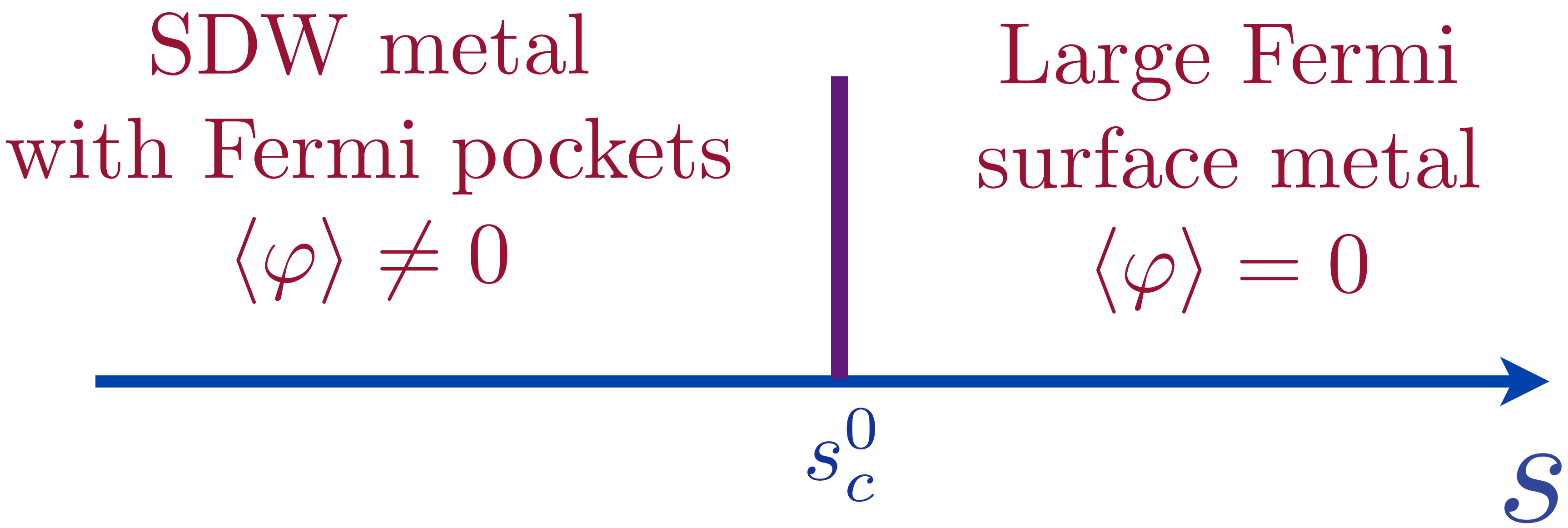
$$\chi(\Delta) = \chi(0) - C|\Delta|$$

where C is a universal constant dominated by the vicinity of the hot spots.

E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

Fermi surface theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



SDW metal
with Fermi pockets
$\langle \varphi \rangle \neq 0$

Large Fermi
surface metal
$\langle \varphi \rangle = 0$

s_c^0

s

No SC
$\Delta = 0$

E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

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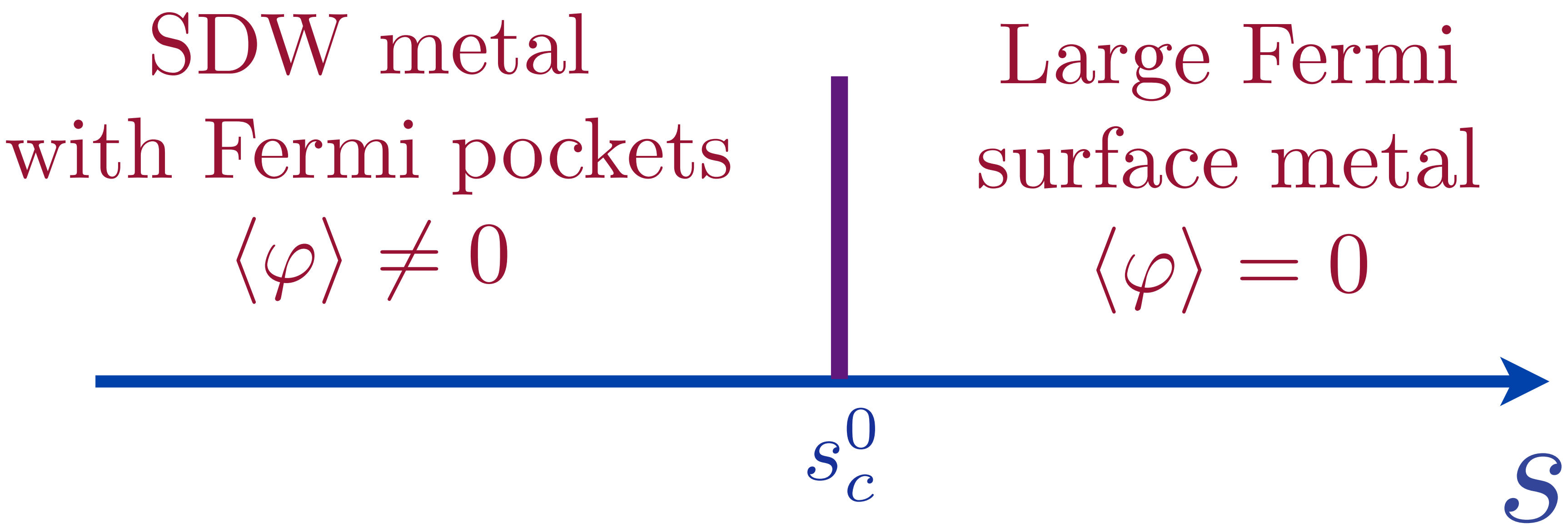

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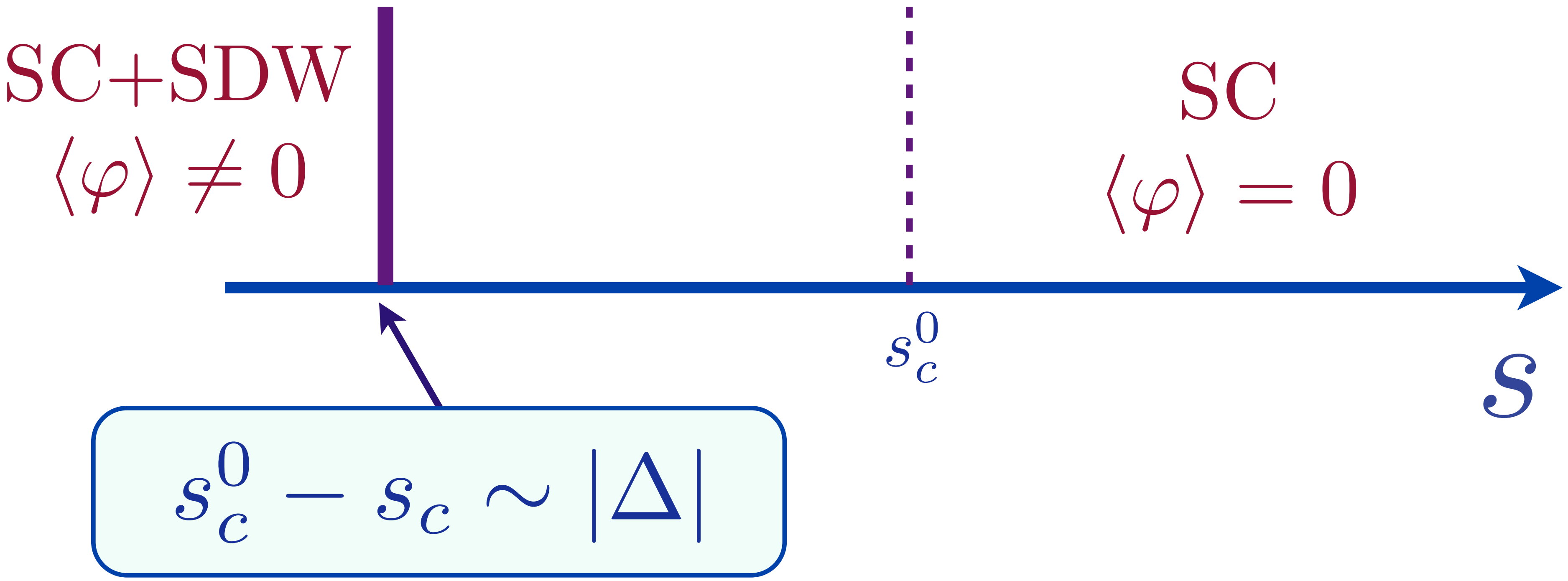

SC+SDW
$\langle \varphi \rangle \neq 0$

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s_c^0

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$s_c^0 - s_c \sim |\Delta|$

SC
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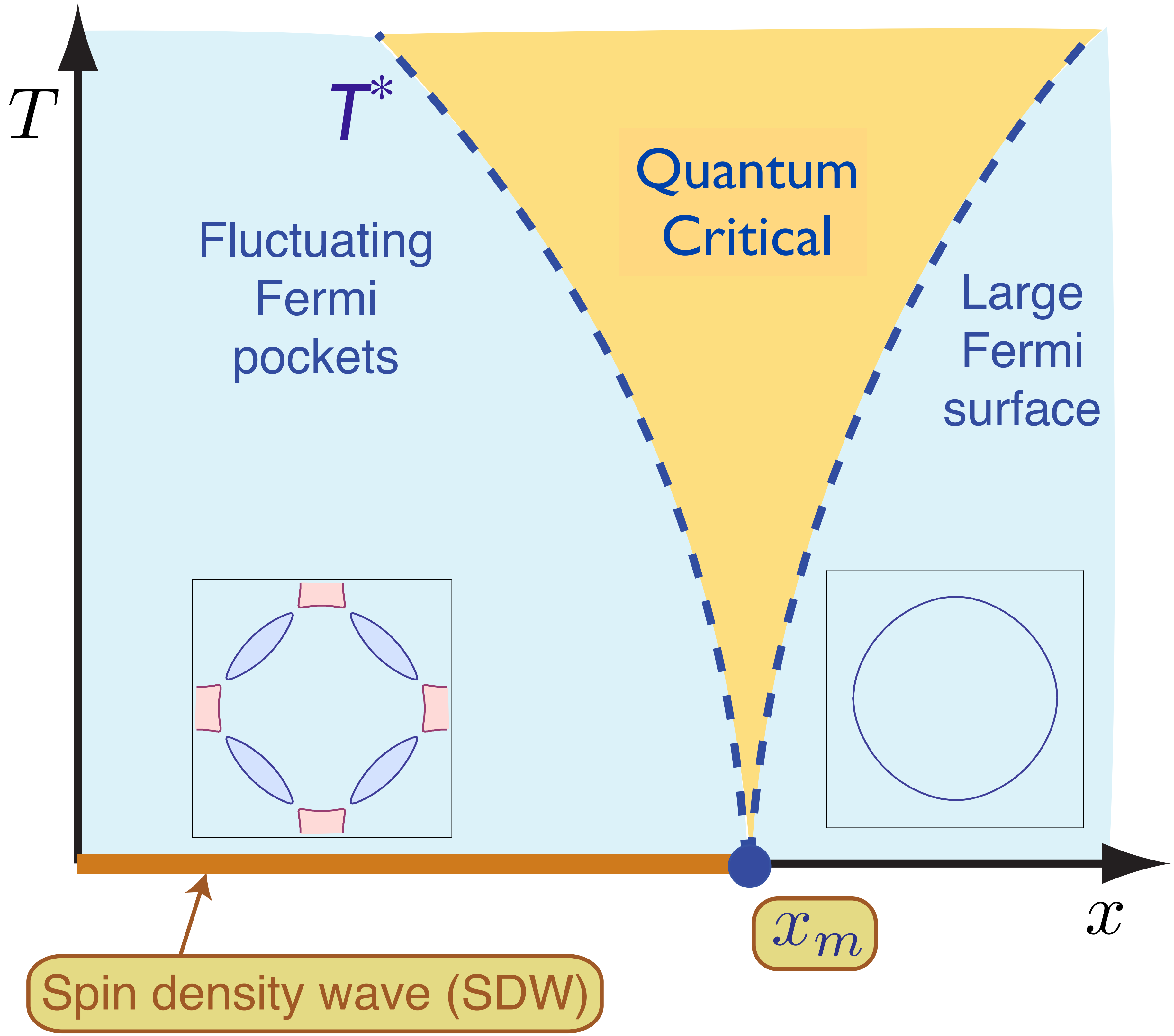
E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

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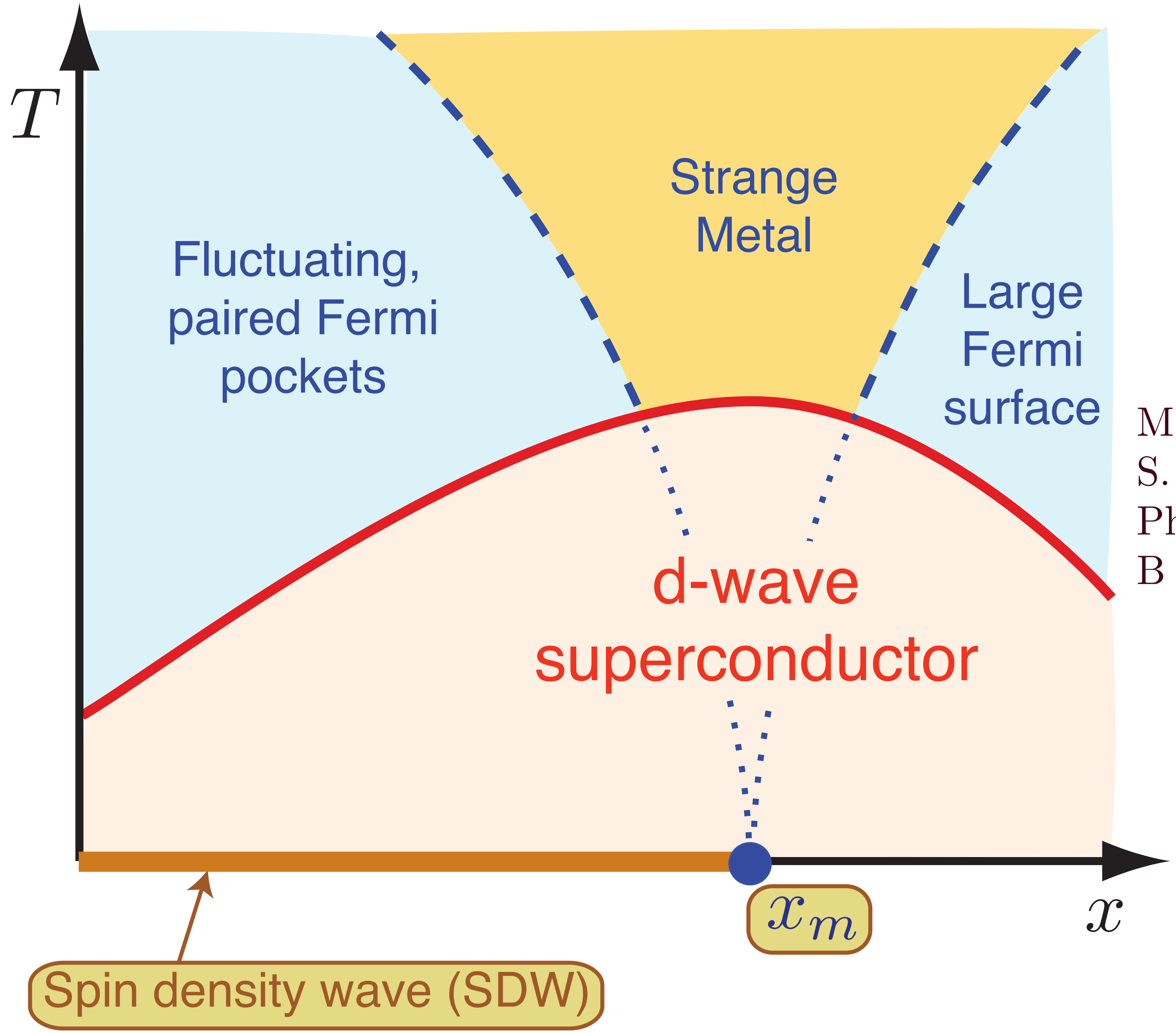
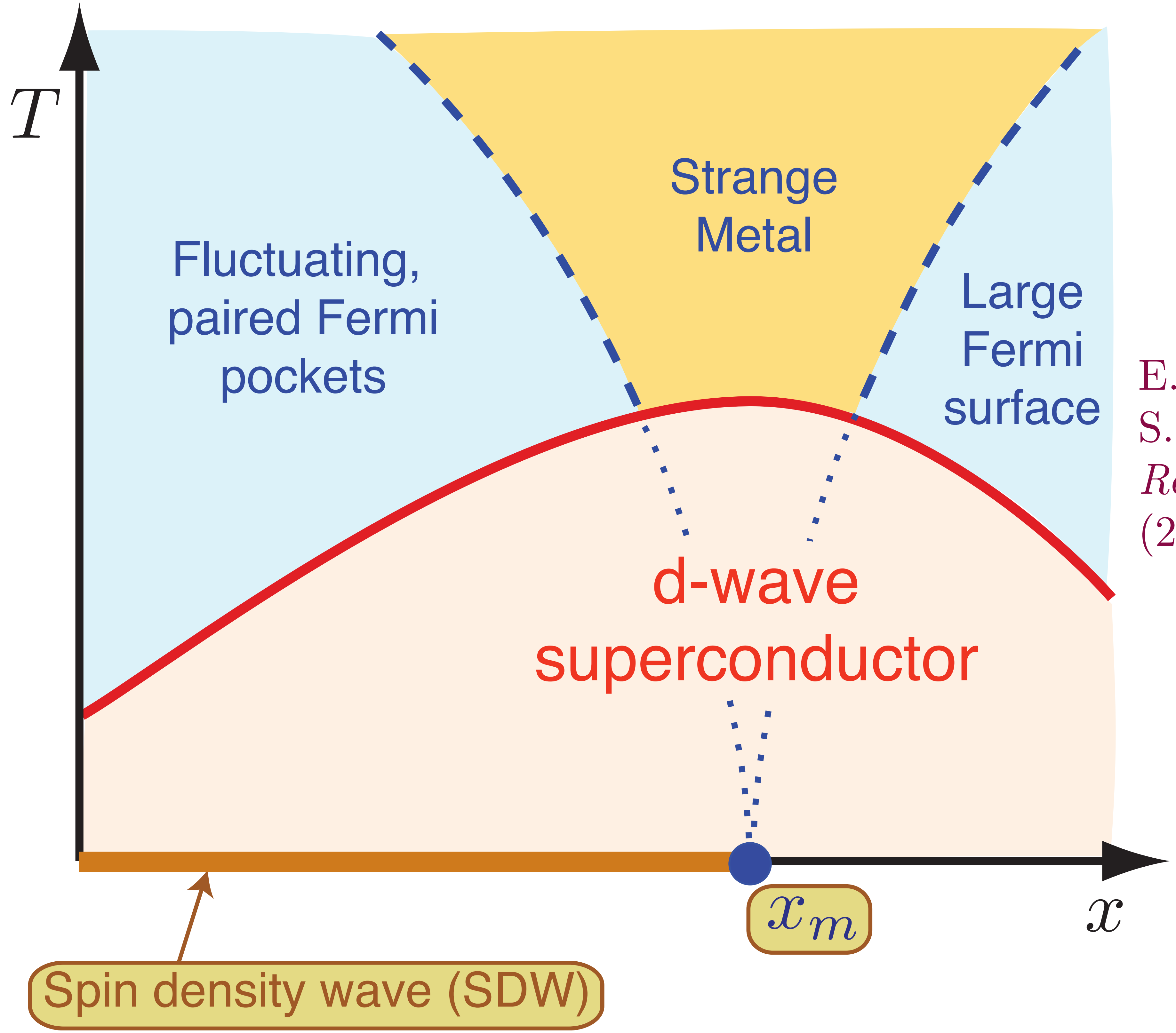


M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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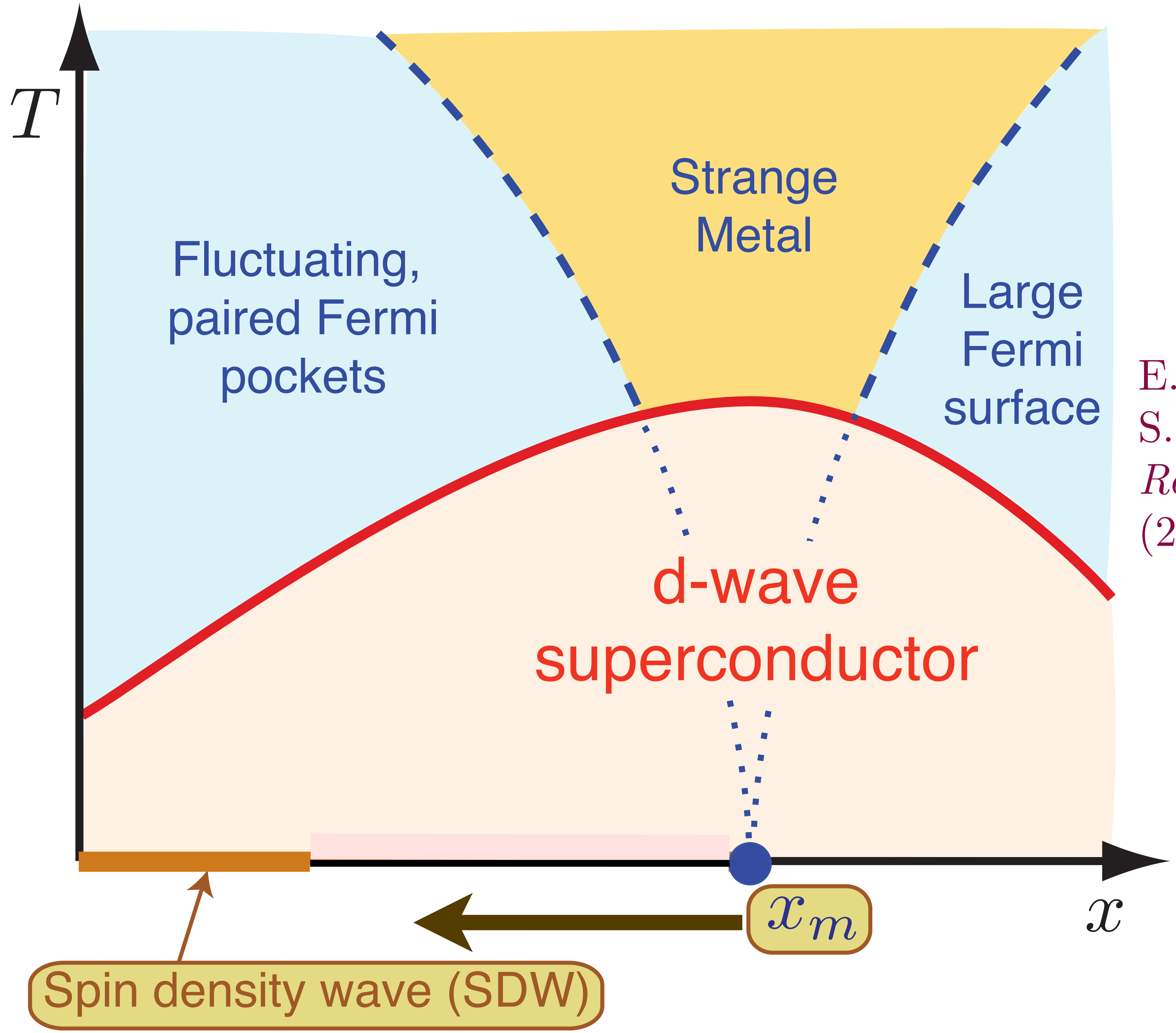

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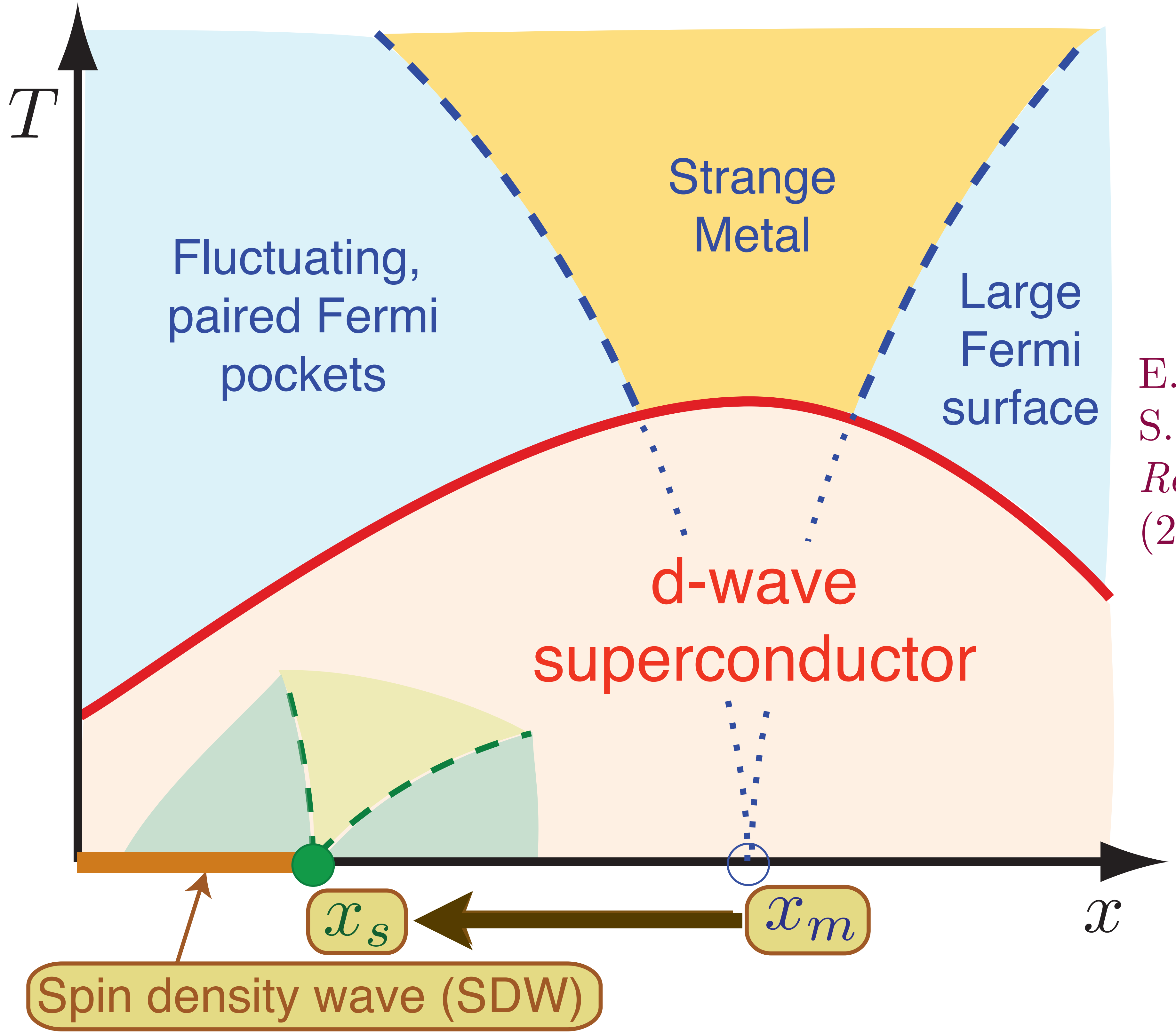

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

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Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

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