Strange metals:
strongly correlated quantum matter with spatially random interactions

Subir Sachdev

PHYSICS

HARVARD
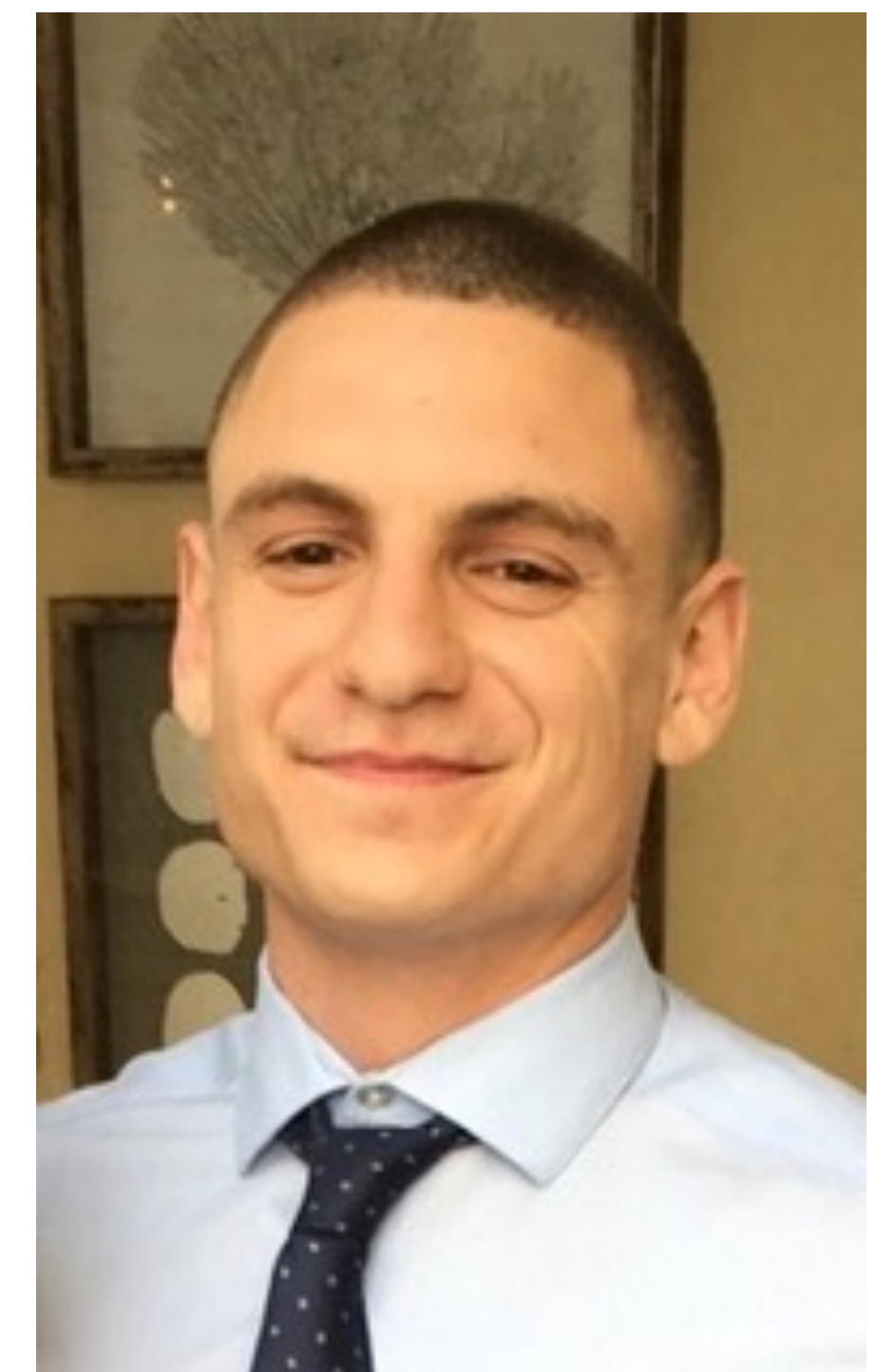
Talk online: sachdev.physics.harvard.edu

Aavishkar Patel
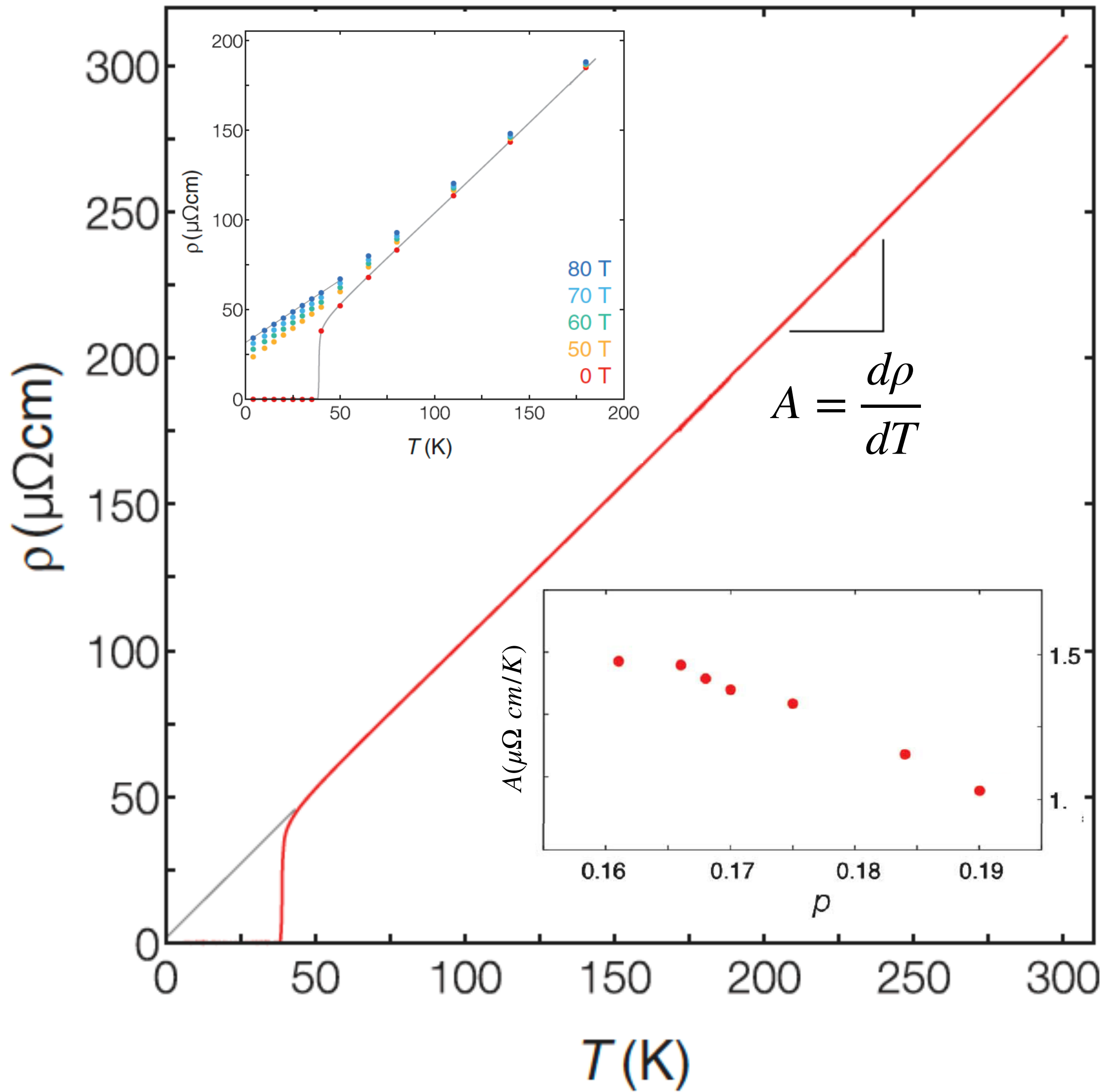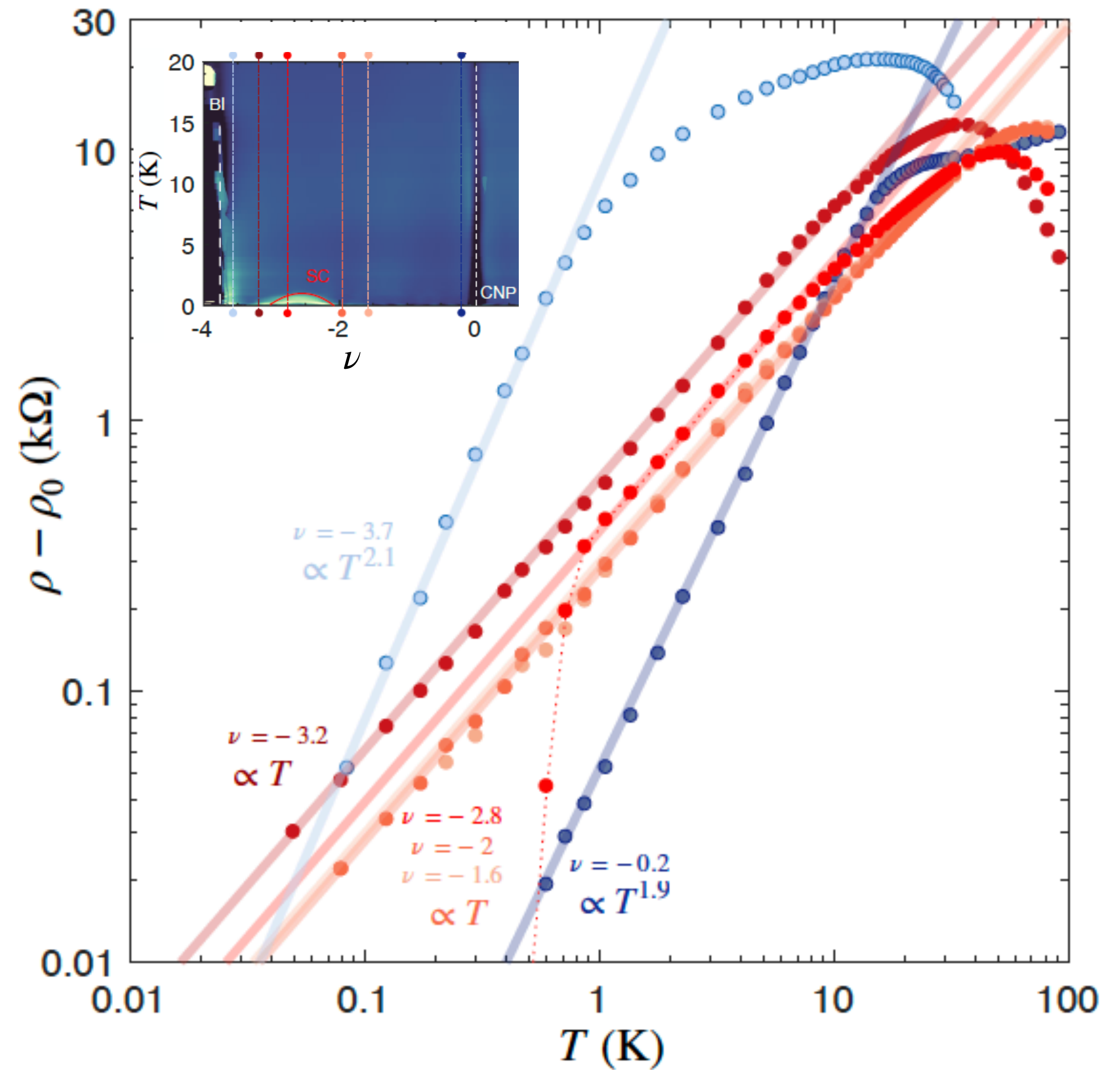
Flatiron Institute, NYC

Haoyu Guo

Harvard

Ilya Esterlis

Harvard ➡ Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

$$A = \frac{d\rho}{dT}$$

LSCO: Giraldo-Gallo et al. 2018

MATBG: Jaoui et al. 2021

$\nu = -3.7$
$\propto T^{2.1}$

$\nu = -3.2$
$\propto T$

$\nu = -2.8$
$\nu = -2$
$\nu = -1.6$
$\propto T$

$\nu = -0.2$
$\propto T^{1.9}$

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

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2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

- Optical conductivity

D. van der Mar
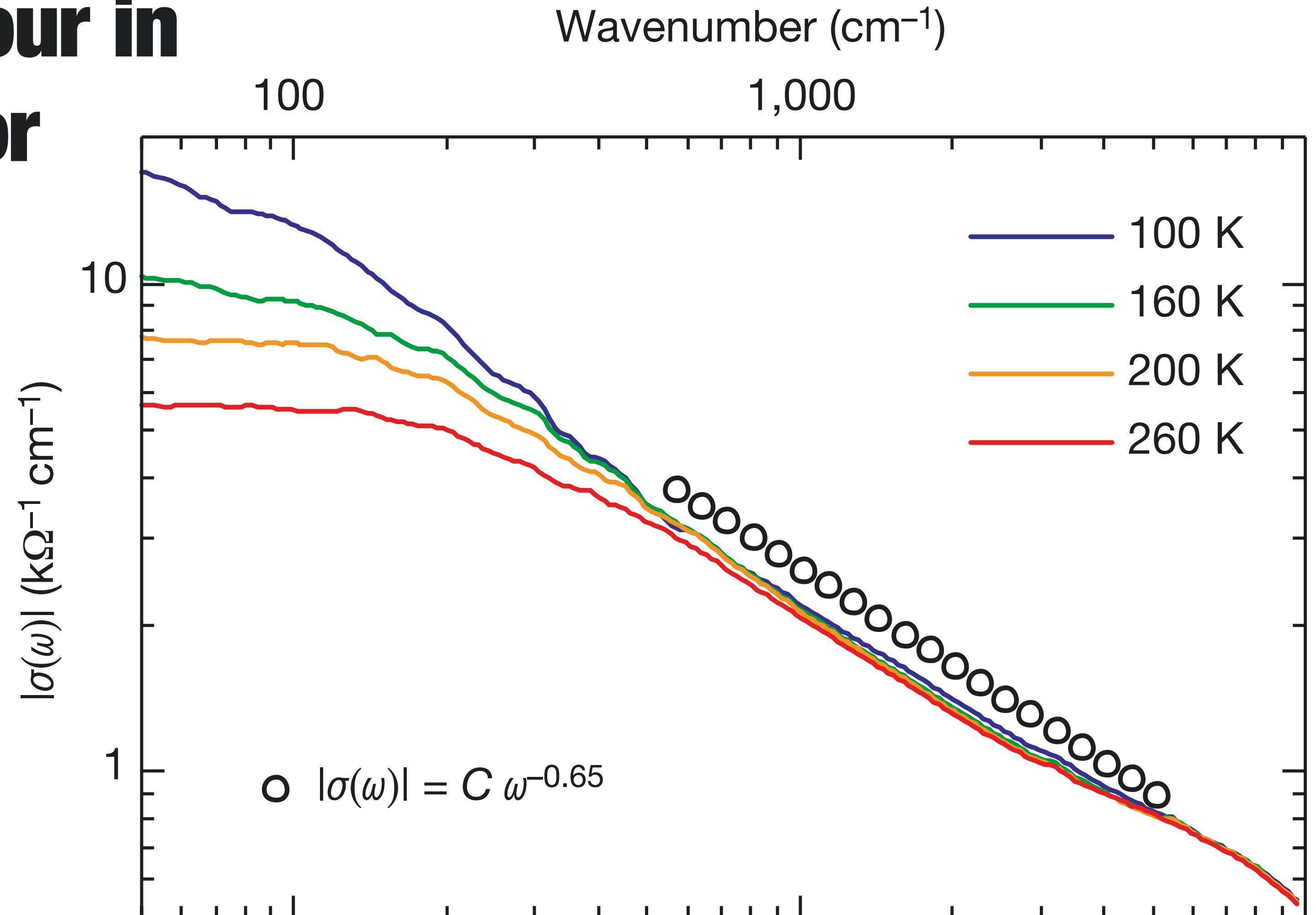F. Carbone[1]*, A

Quantum critical behaviour in a high-T_c superconductor

But no $\hbar\omega/(k_B T)$ scaling.

Properties of a strange metal:

- ## Optical conductivity

B. Michon,[1,2,3] C. Berthod,[3] C. W. Rischau,[3] A. Ataei,[4] L. Chen,[4] S. Komiya,[5] S. Ono,[5] L. Taillefer,[4,6] D. van der Marel,[3] and A. Georges[7,8,3,
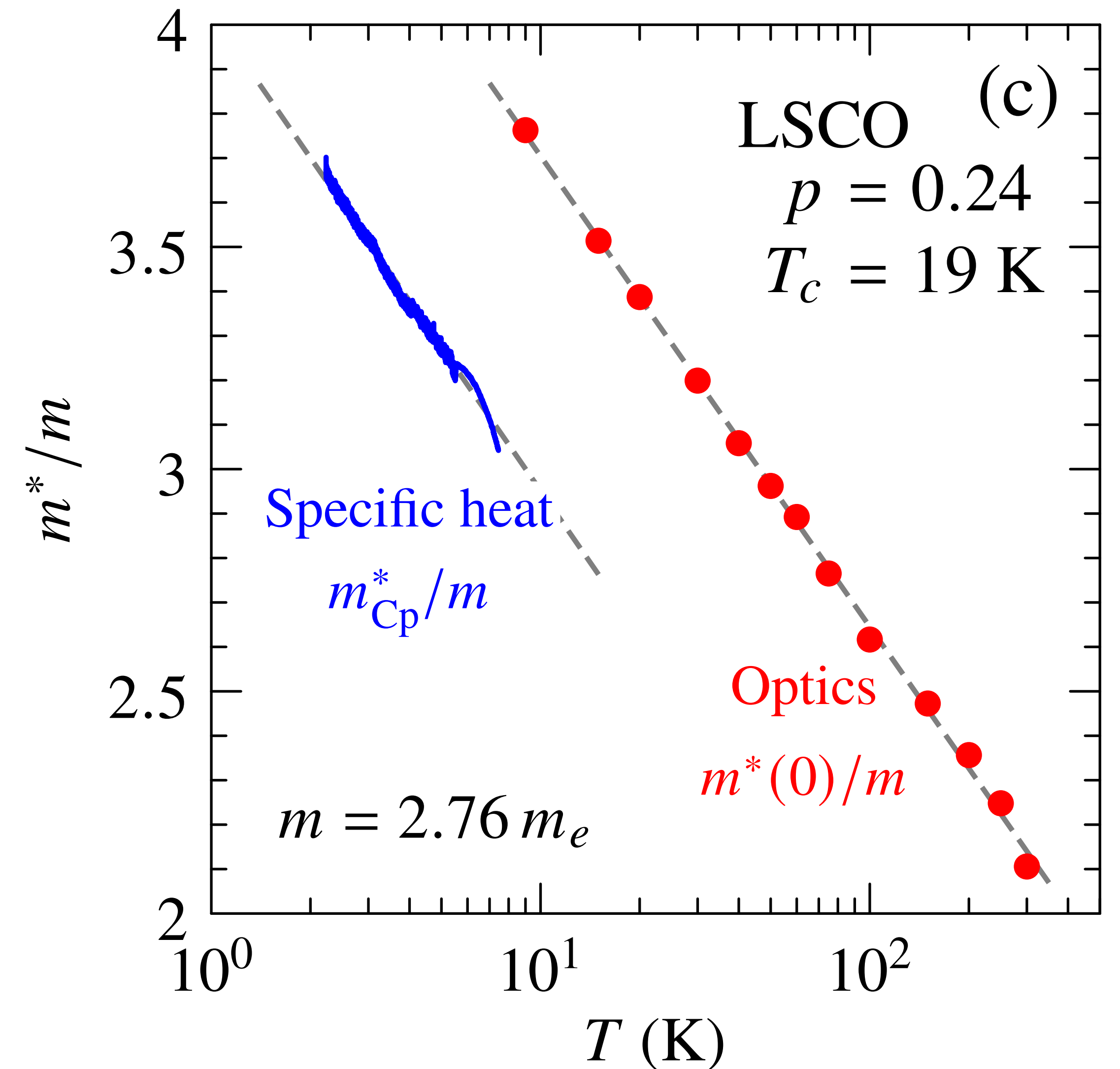
Planckian Behavior of Cuprate Superconductors: Reconciling the Scaling of Optical Conductivity with Resistivity and Specific Heat

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

$$\text{Causality:} \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \ln\left(\frac{\Lambda}{\text{Max}(\hbar\omega, k_B T)}\right)$$

Properties of a strange metal:

- ## Optical conductivity

B. Michon,[1,2,3] C. Berthod,[3] C. W. Rischau,[3] A. Ataei,[4] L. Chen,[4]
S. Komiya,[5] S. Ono,[5] L. Taillefer,[4,6] D. van der Marel,[3] and A. Georges[7,8,3,

(b)

$\hbar\omega \leqslant 0.4$ eV

40 K $\leqslant T \leqslant 300$ K

$\dfrac{\hbar/\tau}{k_{\mathrm{B}}T}$

$\hbar\omega/k_{\mathrm{B}}T$

- 40 K
- 50 K
- 60 K
- 75 K
- 100 K
- 150 K
- 200 K
- 250 K
- 300 K

(c)

LSCO
$p = 0.24$
$T_c = 19$ K

m^*/m

Specific heat
m^*_{Cp}/m

Optics

$m^*(0)/m$

$m = 2.76\, m_e$

T (K)
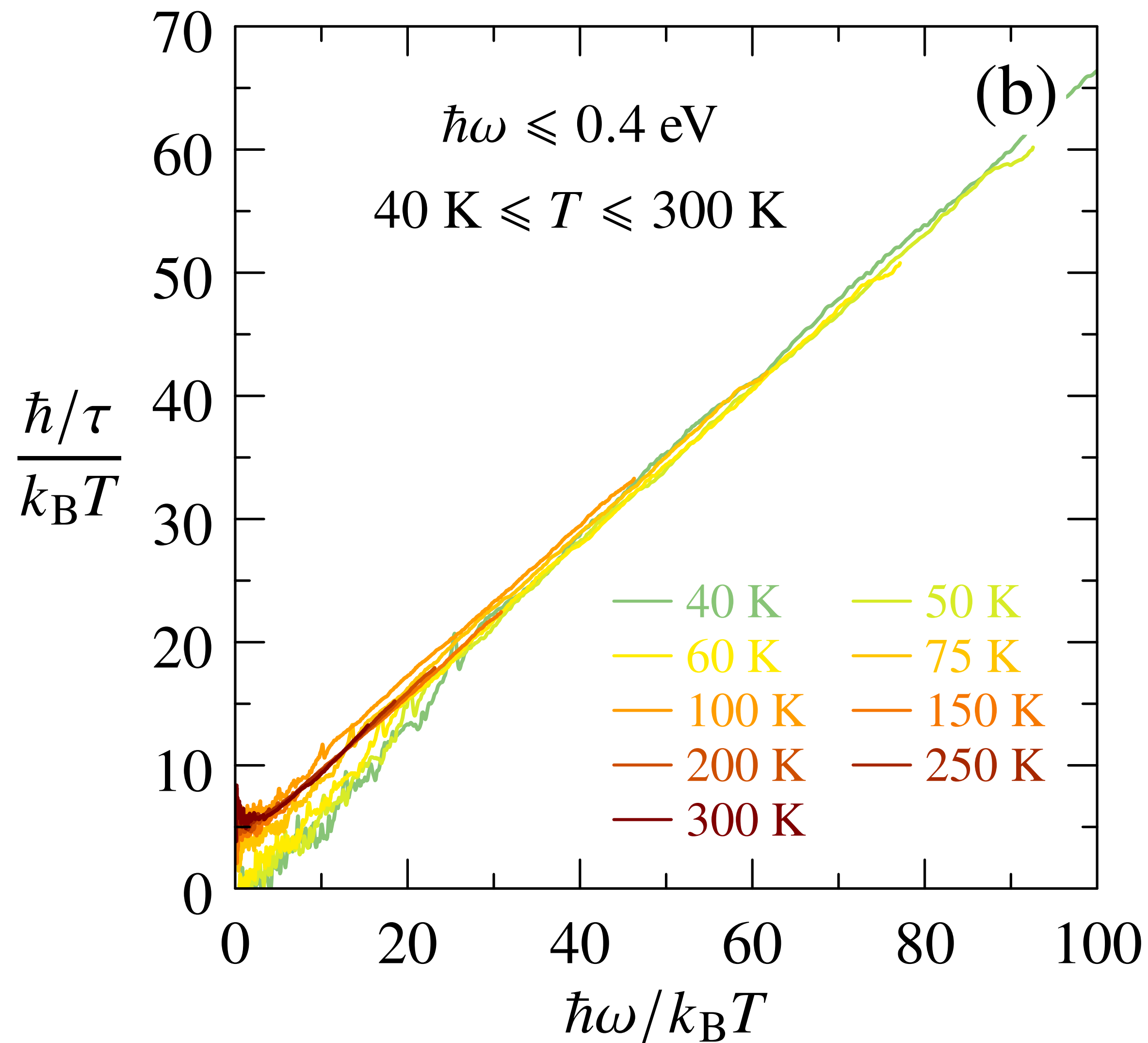
Properties of a strange metal:

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   <u>and</u> $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are <u>bad metals</u>.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802
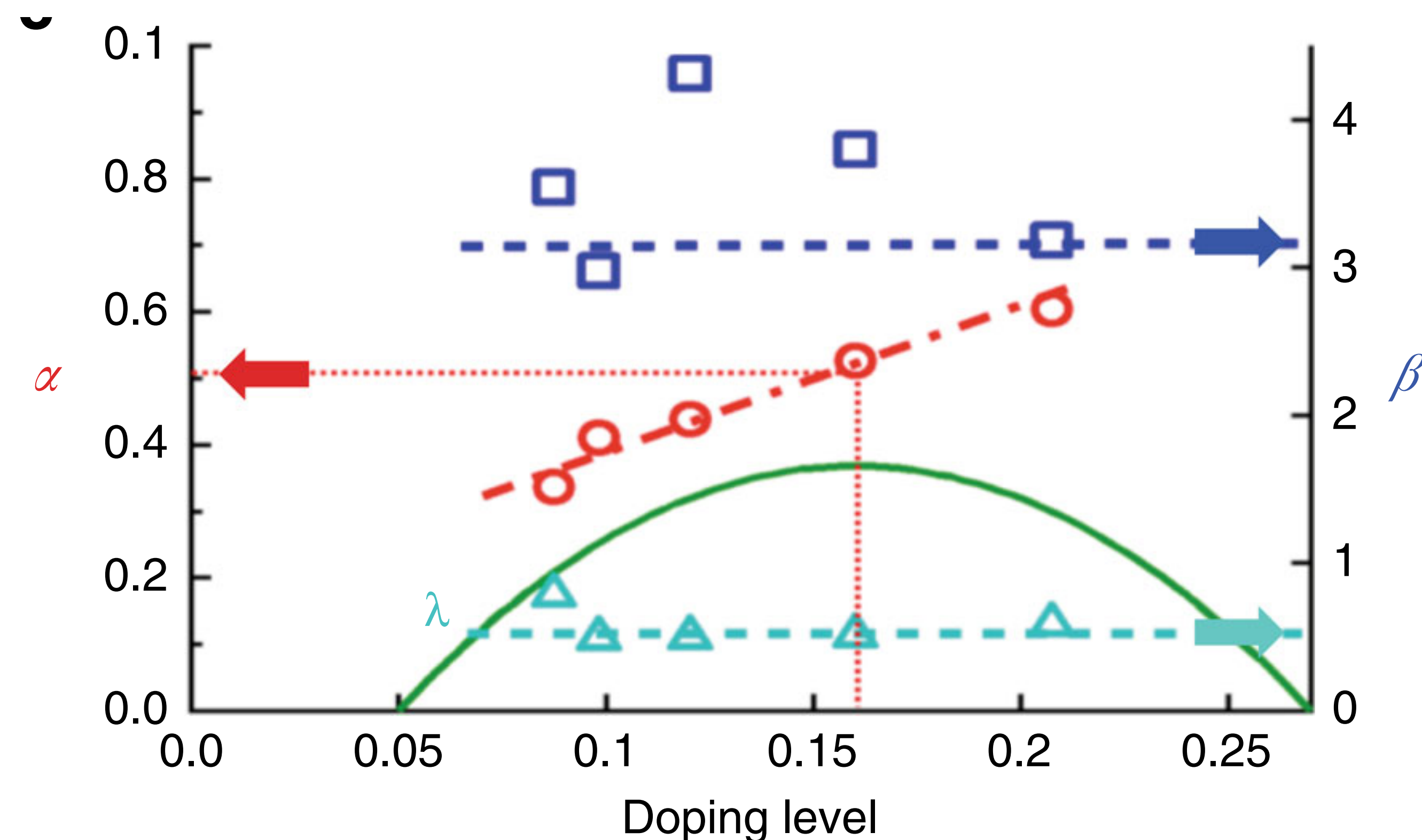
3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon......A. Georges, arXiv:2205.04030

- Photoemission

A unified form of low-energy nodal electronic interactions in hole-doped cuprate superconductors

T.J. Reber[1,5*], X. Zhou [1*], N.C. Plumb [1,6], S. Parham[1], J.A. Waugh[1], Y. Cao[1], Z. Sun[1,7], H. Li[1], Q. Wang[1], J.S. Wen [2], Z.J. Xu[2], G. Gu[2], Y. Yoshida[3], H. Eisaki[3], G.B. Arnold[1] & D.S. Dessau[1,4*]

$$\Sigma''_{\mathrm{PLL}}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_{\mathrm{B}} T)^2\right]^\alpha}{(\hbar\omega_N)^{2\alpha-1}}$$

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

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S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$
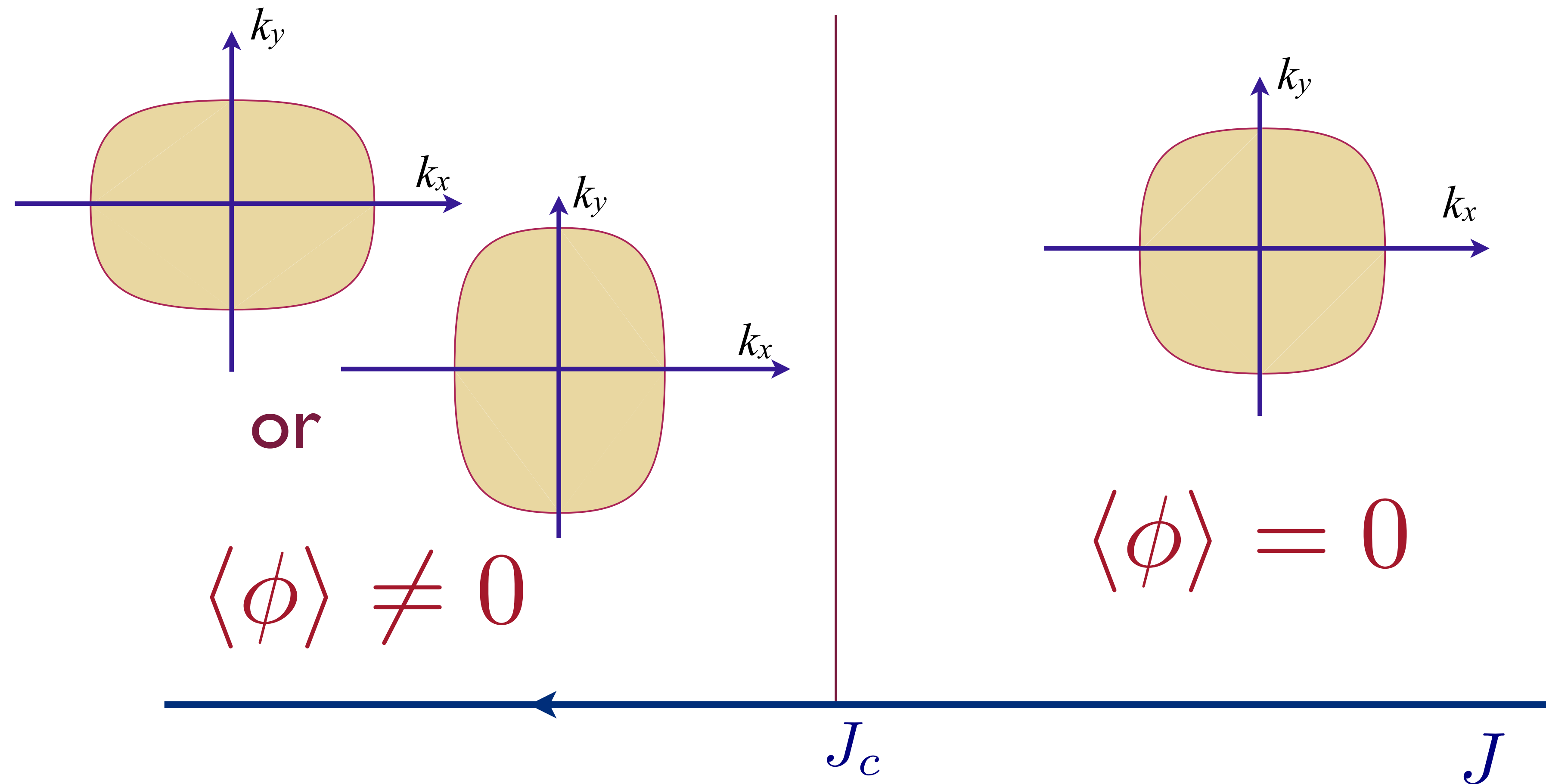
B. Michon......A. Georges, arXiv:2205.04030

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$
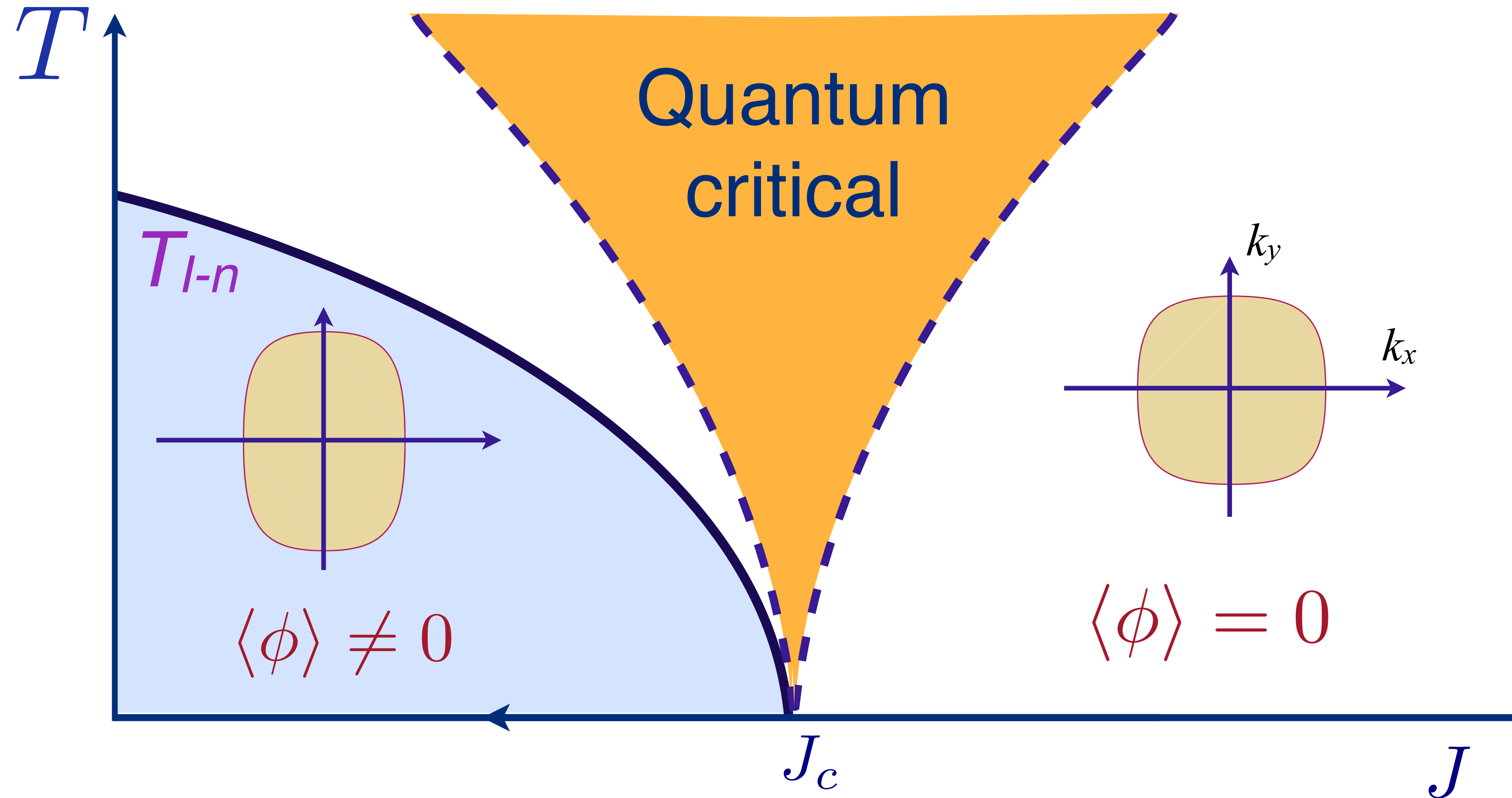
T.J. Reber....D. Dessau, Nature Communications **10**, 5737 (2019)

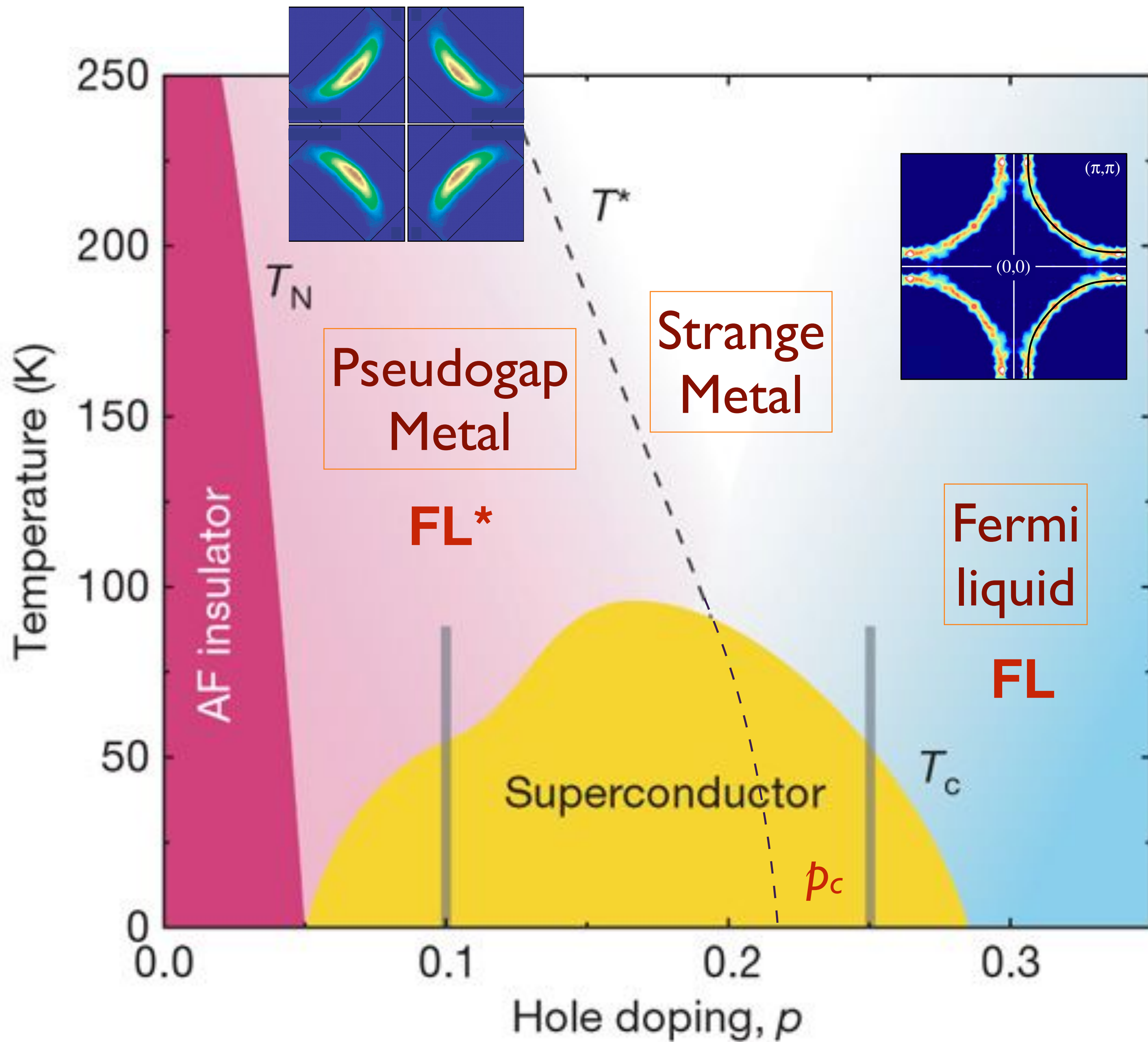# Quantum criticality of Ising-nematic ordering in a metal

Pomeranchuk instability as a function of coupling J

Quantum criticality of Ising-nematic ordering in a metal
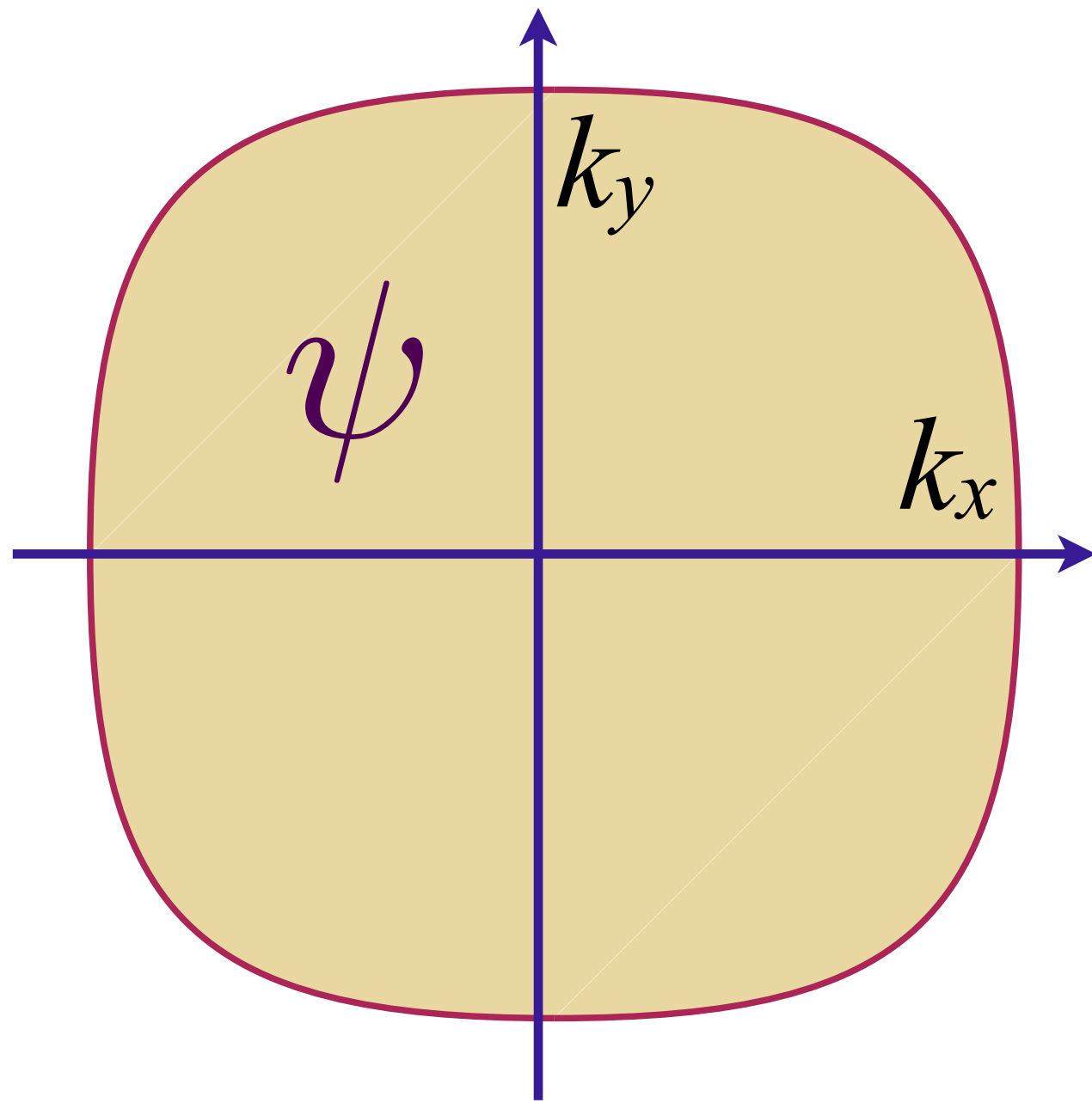

Phase diagram as a function of T and J

Our results also apply to the transition from a

fractionalized Fermi liquid (FL*) to a Fermi liquid (FL)

in a single-band Hubbard model, or in a Kondo lattice model;

a Higgs field ("slave boson") takes the place of ϕ.
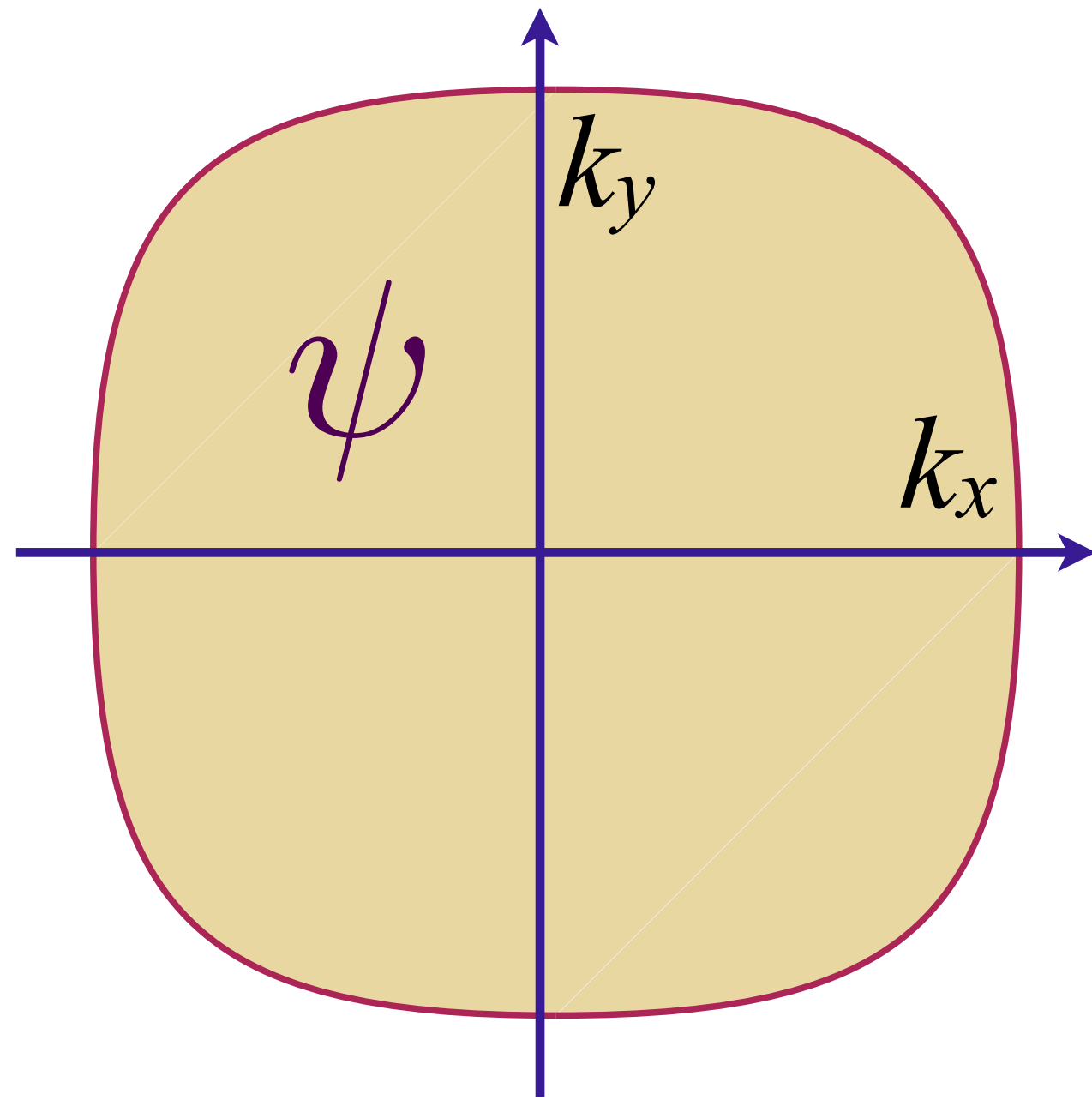
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

ψ

k_y

k_x

$$-J\,\psi^\dagger(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\psi(\boldsymbol{r})$$

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

k_y

k_x

ψ

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$
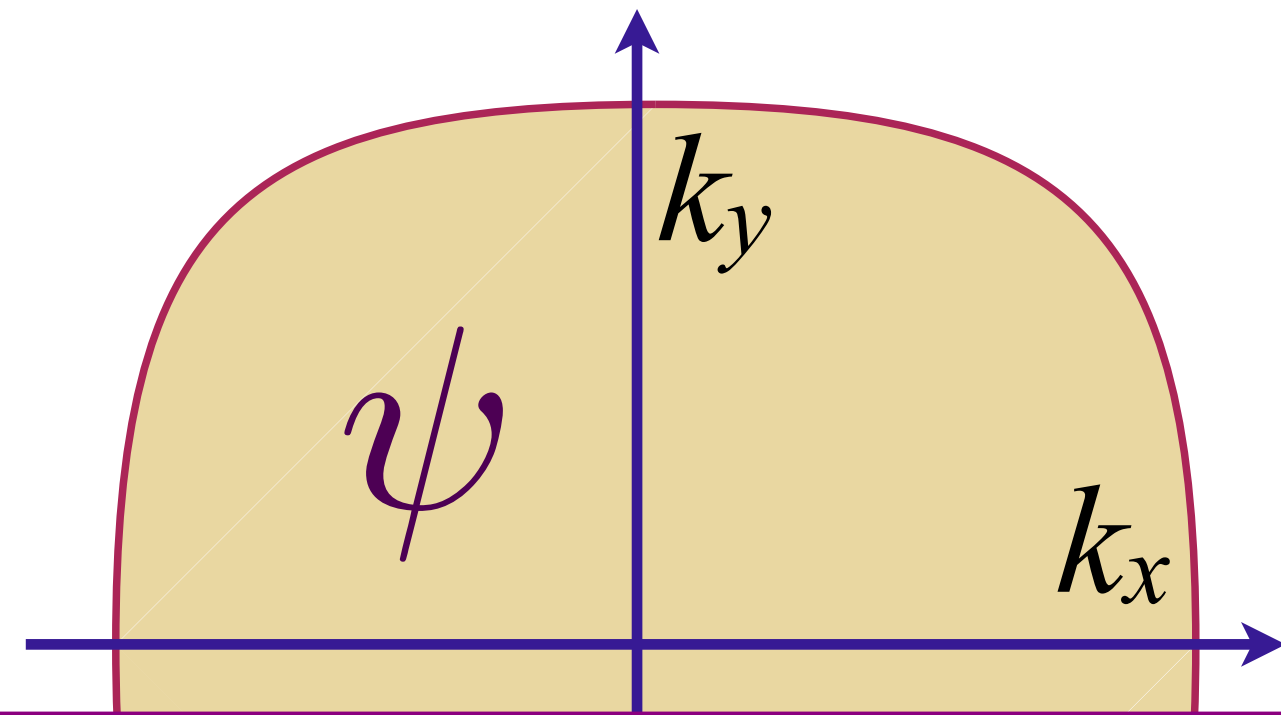
a critical boson ϕ
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$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Solve in a large N limit with Yukawa coupling

$$\frac{g_{ij\ell}}{N} \int d^2 r\, d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:
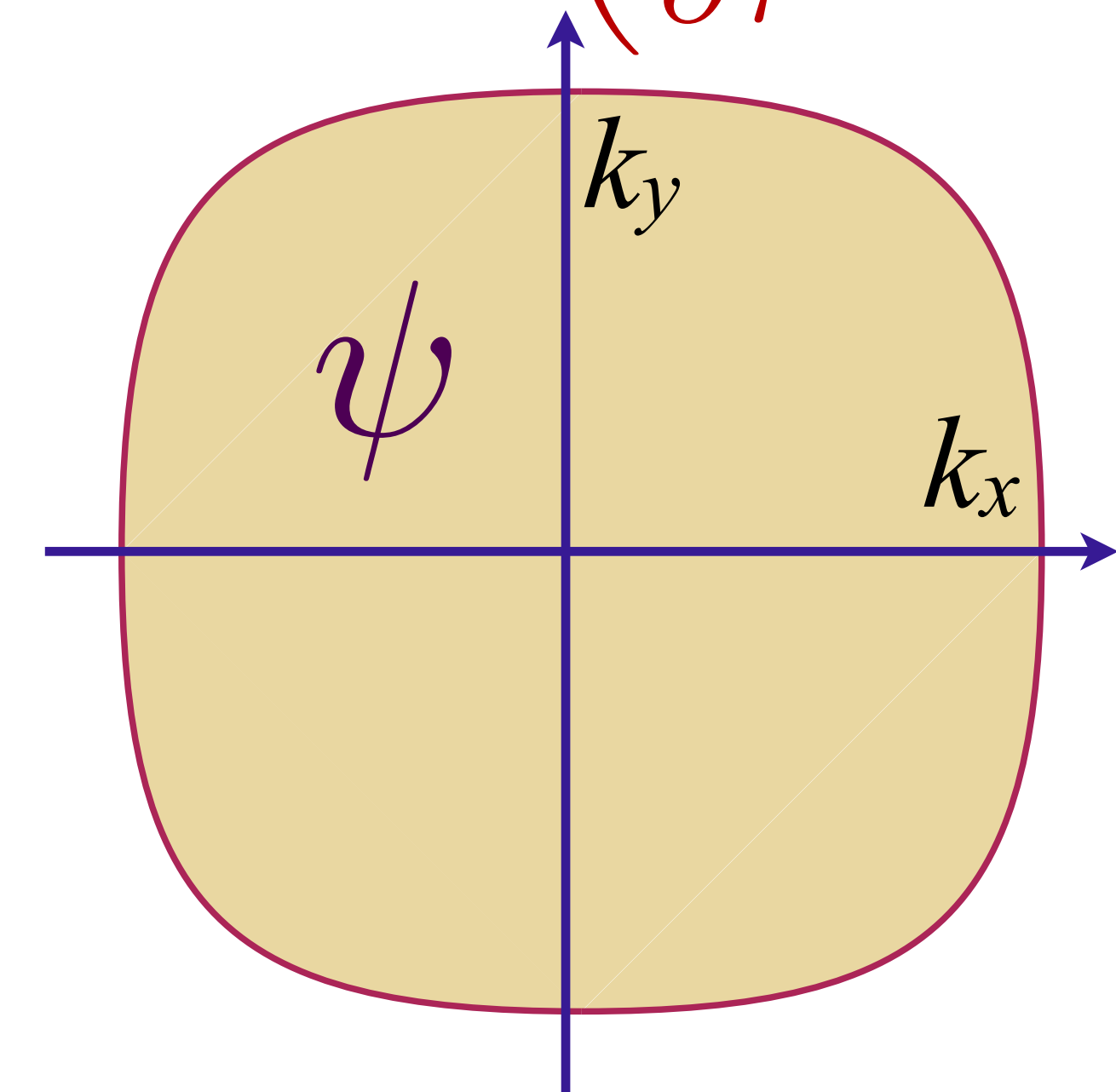
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\mathrm{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

ψ

k_y

k_x

a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Transport—a perfect metal!
Conservation of momentum and
fermion-boson drag imply:

$$\mathrm{Re}\,[\sigma(\omega)] = D\delta(\omega) + \dots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
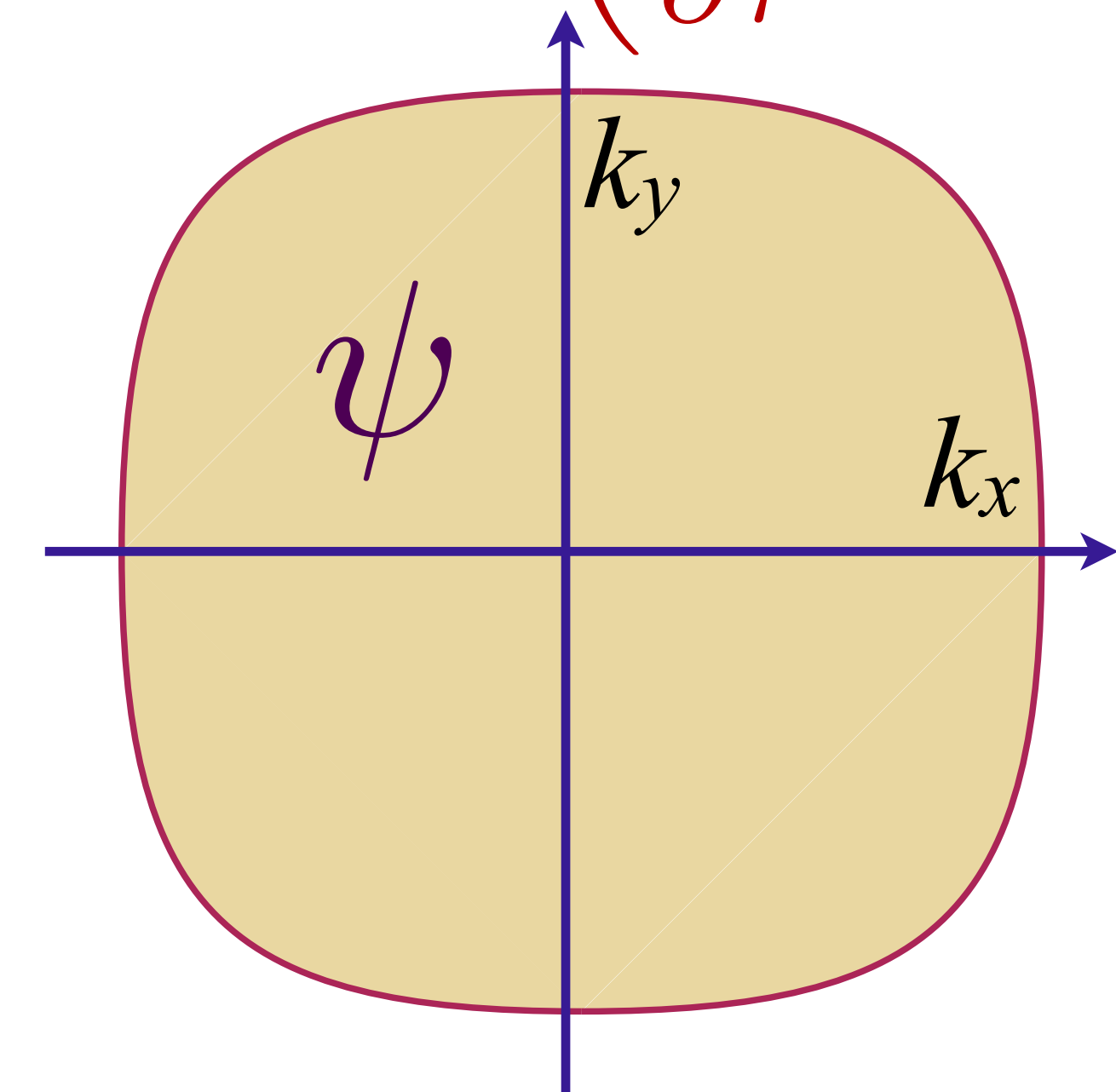S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

ψ

k_y

k_x

Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \cdots \qquad (\omega^{-2/3} \text{ term has vanishing co-efficient})$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

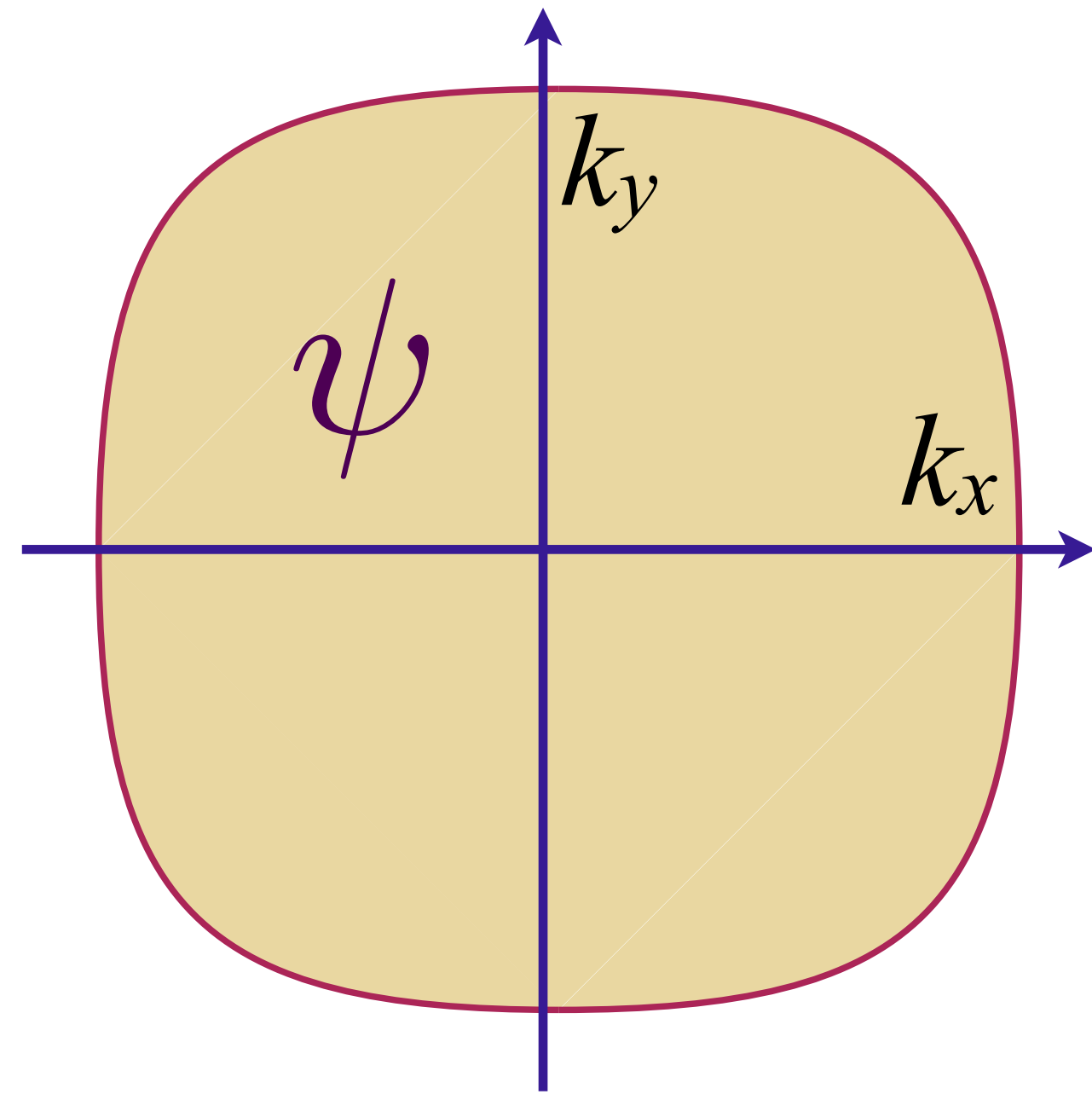See also Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order
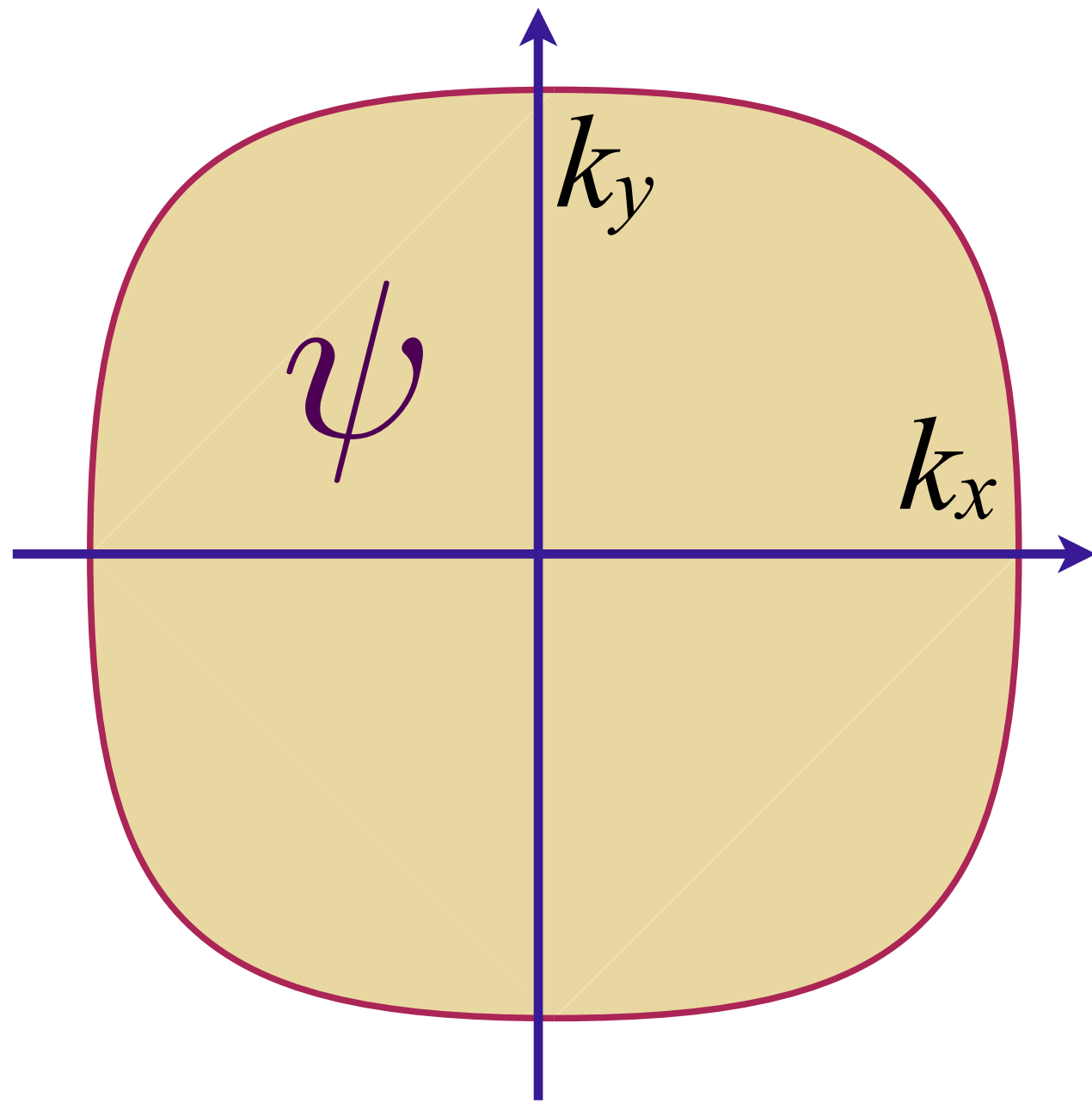
$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order
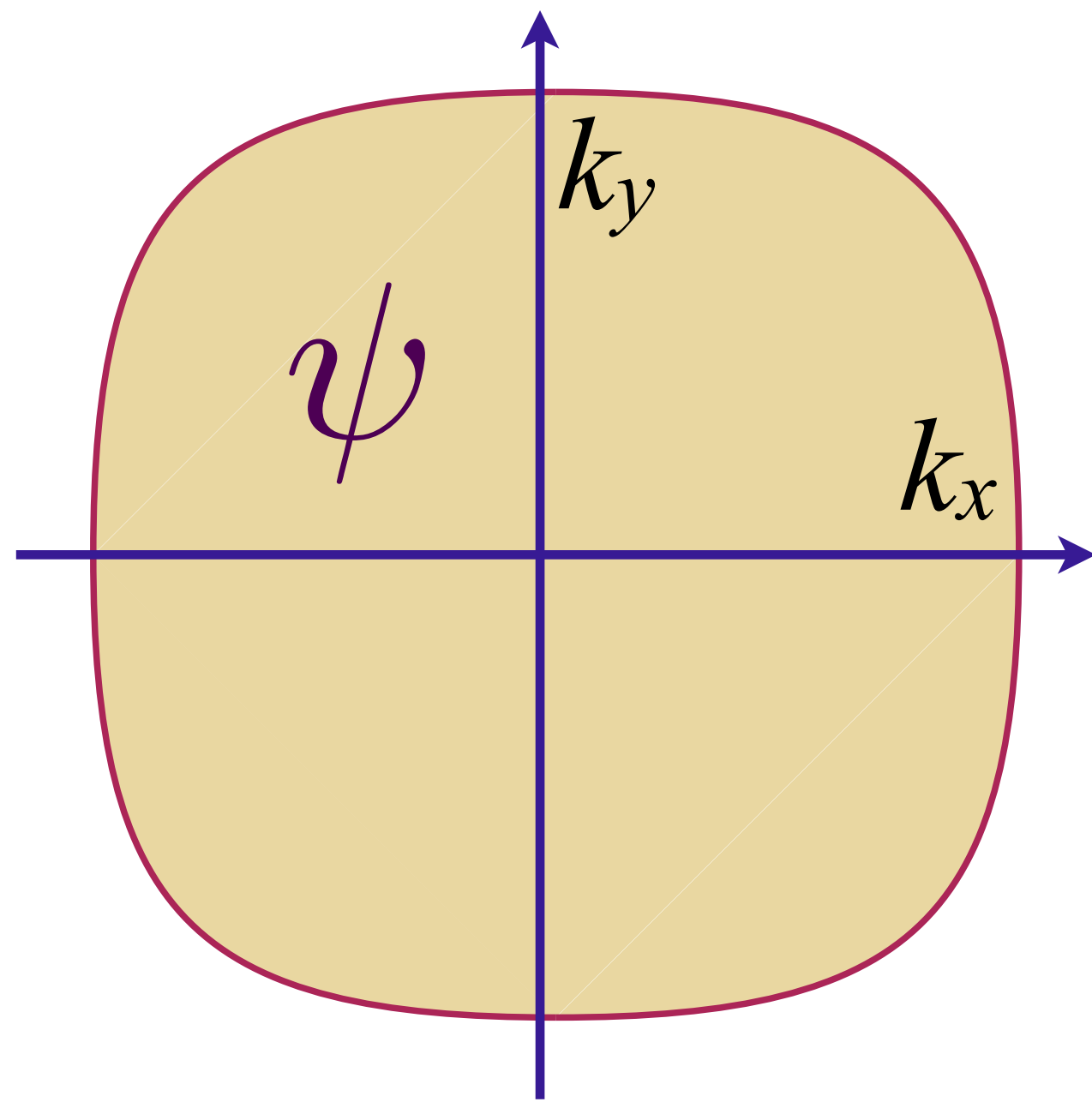
$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})] \, \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

# Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge,
Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos,
Amber Vervloet, Steef Smit, Erik van Heumen,
Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin,
Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen,
Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740
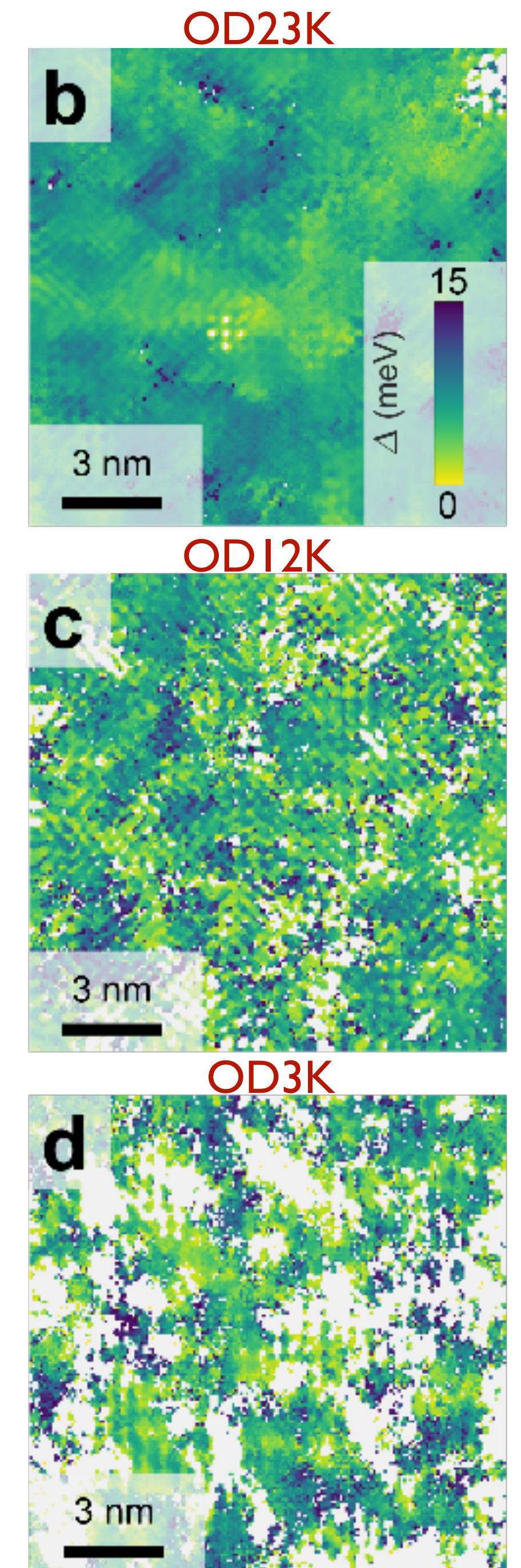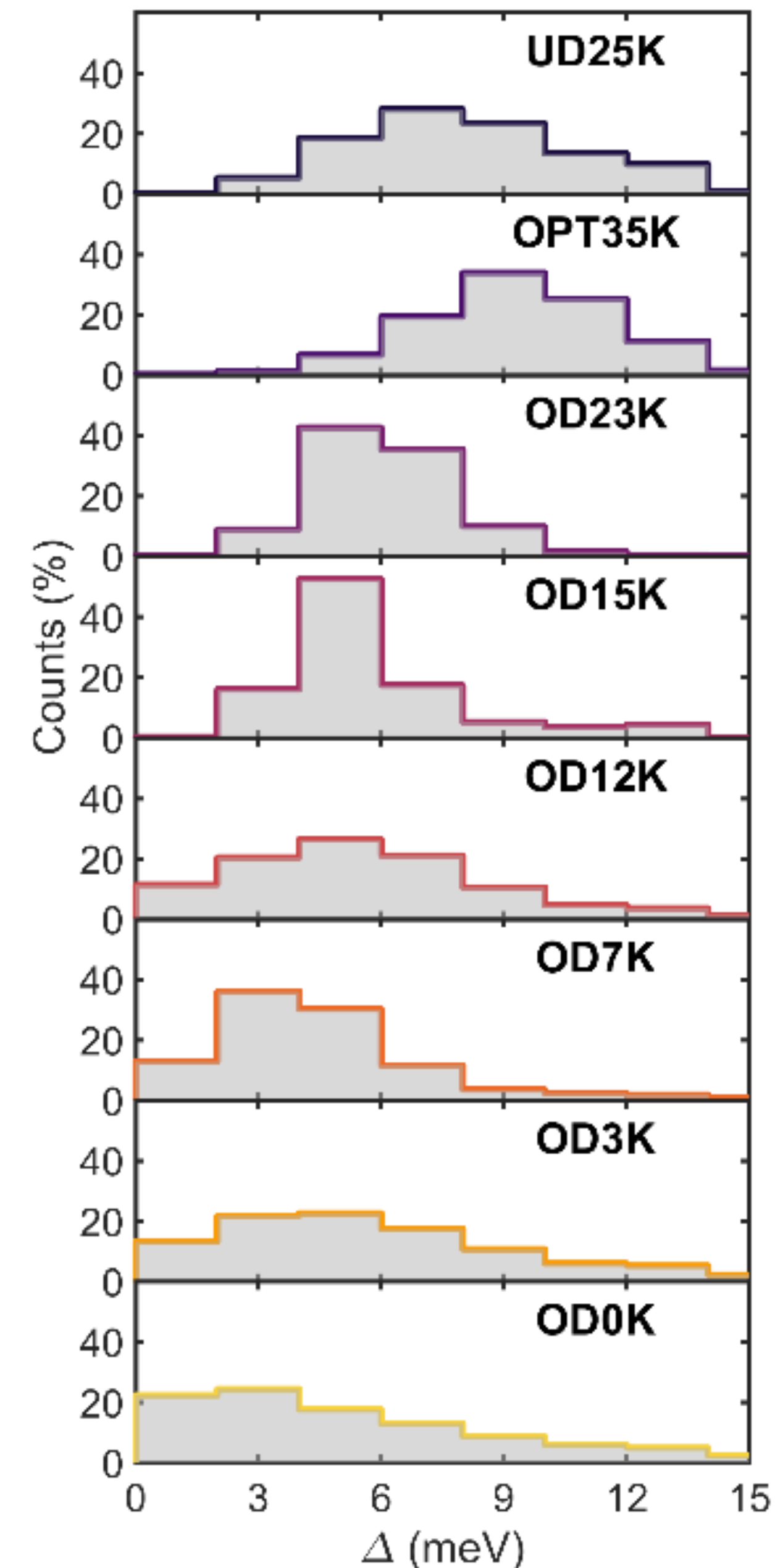
Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G \, \mathcal{D}\Sigma \, \mathcal{D}D \, \mathcal{D}\Pi \, \exp(-N S_{\mathrm{all}})$$

$$S_{\mathrm{all}} = -\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi)$$

$$+ \int d\tau d^2 r \int d\tau' d^2 r' \left[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r})G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r})D(\tau, \mathbf{r}; \tau', \mathbf{r}') \right.$$

$$+ \frac{g^2}{2}G(\tau, \mathbf{r}; \tau', \mathbf{r}')G(\tau', \mathbf{r}'; \tau, \mathbf{r})D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2}G(\tau, \mathbf{r}; \tau', \mathbf{r}')G(\tau', \mathbf{r}'; \tau, \mathbf{r})\delta(\mathbf{r} - \mathbf{r}')$$

$$\left. + \frac{g'^2}{2}G(\tau, \mathbf{r}; \tau', \mathbf{r}')G(\tau', \mathbf{r}'; \tau, \mathbf{r})D(\tau, \mathbf{r}; \tau', \mathbf{r}')\delta(\mathbf{r} - \mathbf{r}') \right] .$$

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi\,\exp(-NS_{\text{all}})$$

Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r})G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r})\delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r})D(\tau, \mathbf{r})\delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r})G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r})G(\tau, \mathbf{r})\delta^2(\mathbf{r}),$$
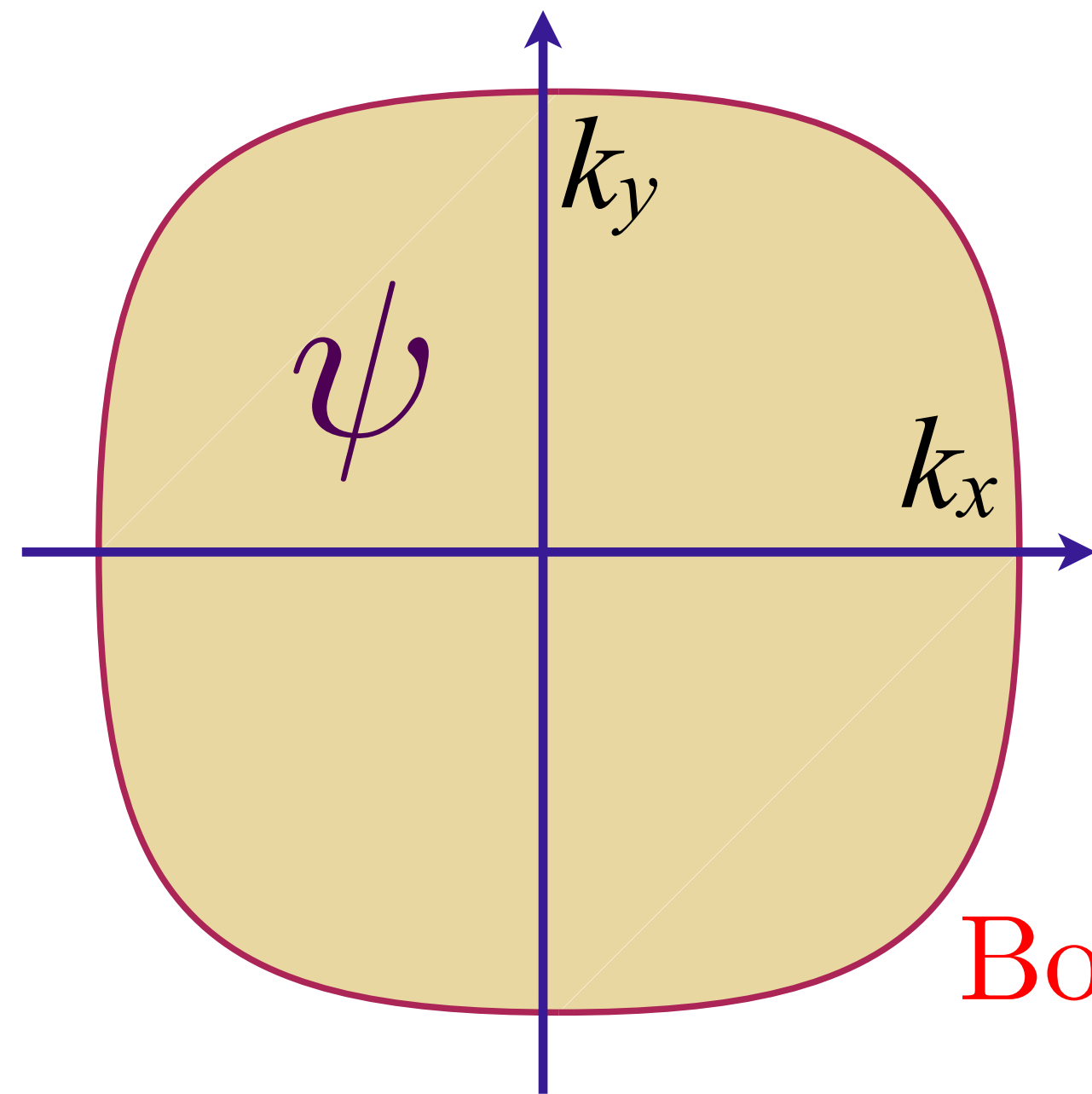
$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$
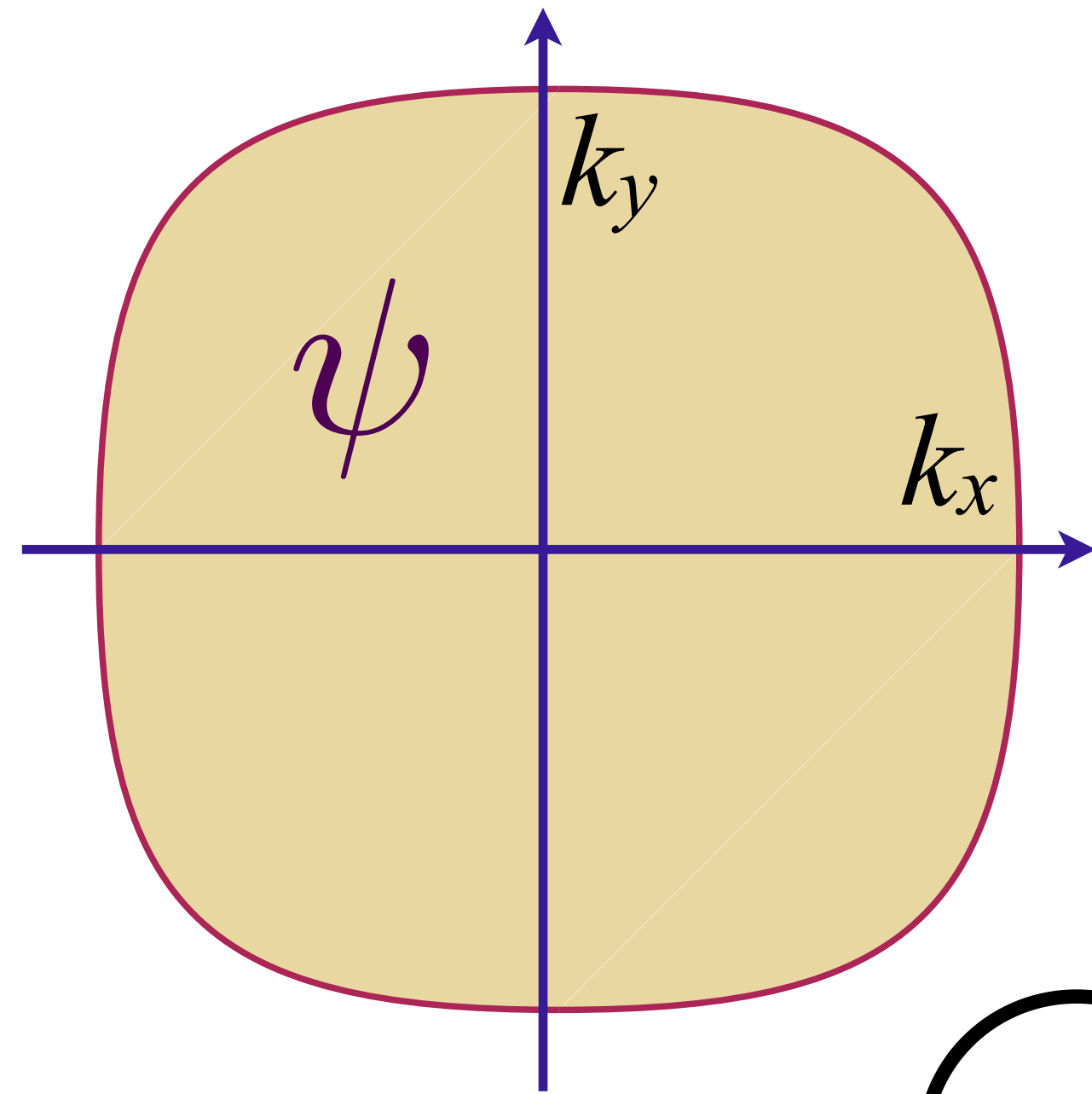Fermion self energy:

$$\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\left(\frac{g^2}{v^2} + g'^2\right)\omega \ln(1/|\omega|); \quad \frac{1}{\tau_{\mathrm{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega|$$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})] \, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$
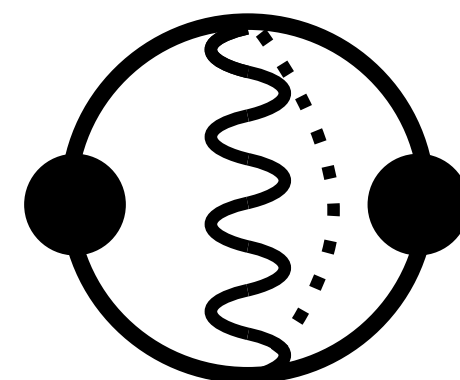
Conductivity:
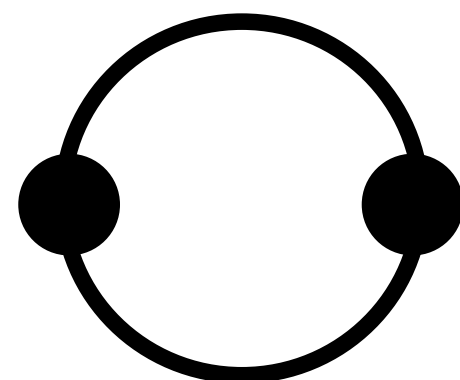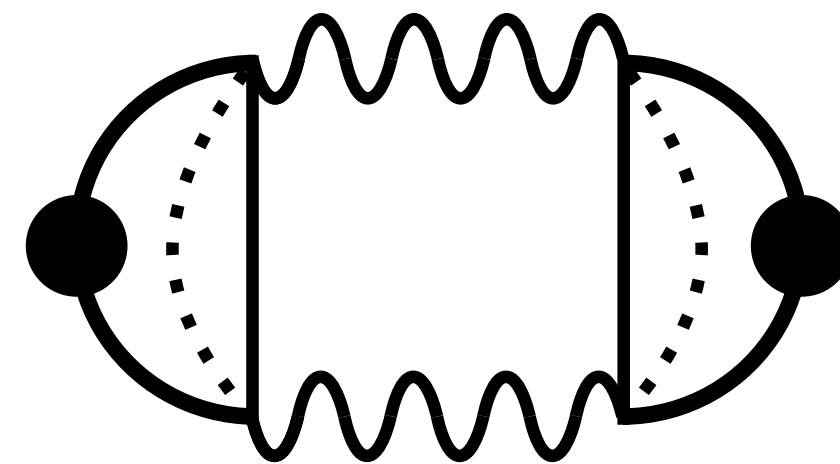

(a) σ_v
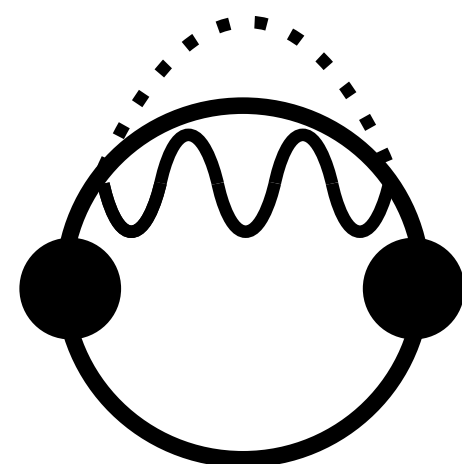
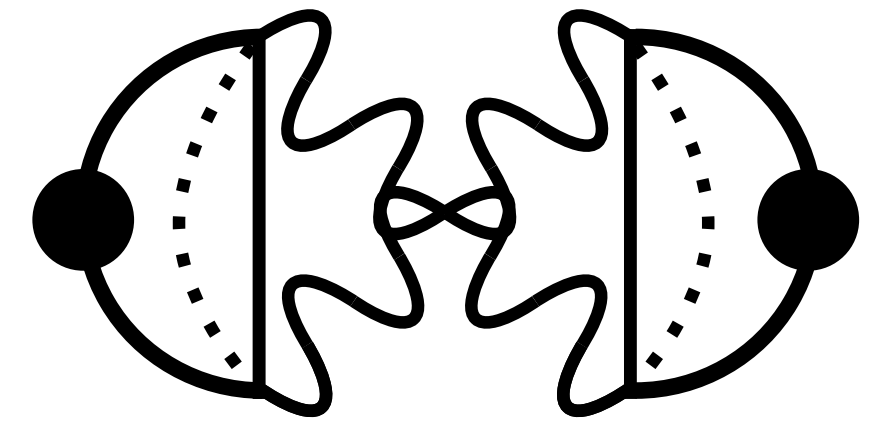(b) $\dfrac{\sigma_{\Sigma,g}}{2}, \ \dfrac{\sigma_{\Sigma,g'}}{2}$

(c) $\sigma_{V,g}$

(d)

(e)

+ all ladders and bubbles.....

Fermi surface coupled to a critical boson with disorder

Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega\, \dfrac{m^*_{\text{trans}}(\omega)}{m}}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g

Fermi surface coupled to a critical boson with disorder

Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\text{in}}(\omega)}\right)\text{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:
No spatial disorder
A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:
<u>No spatial disorder</u>
A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:
<u>Potential disorder v</u>

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:
Potential disorder v

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:
Interaction disorder g'

A marginal Fermi liquid AND a strange metal