“Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Z_2 Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

S. Sachdev, 100th anniversary Solvay conference (2011), arXiv:1203.4565

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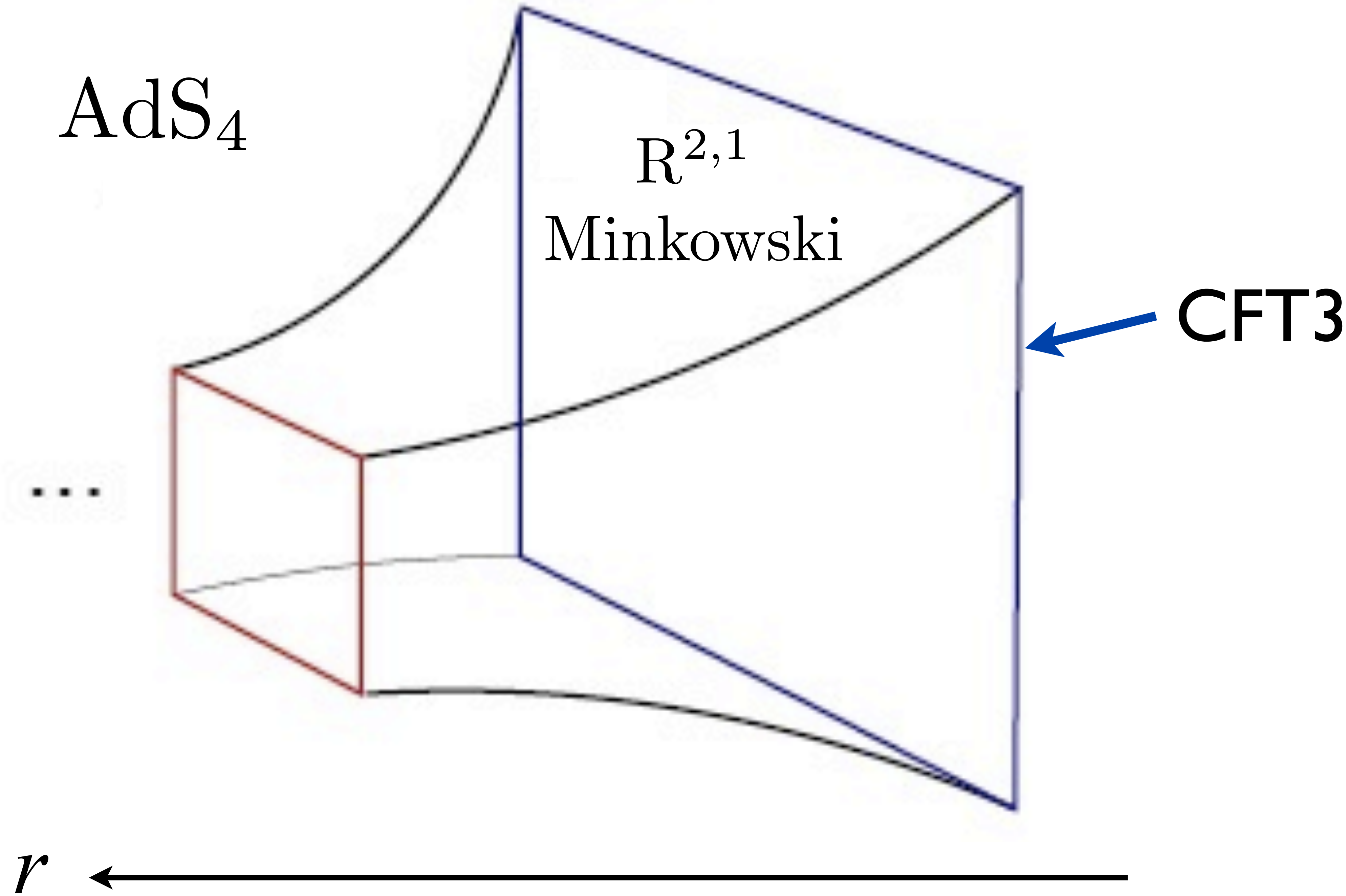
Conformal quantum matter

A. Field theory:
Honeycomb lattice
Hubbard model

B. Gauge-gravity duality

AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

Consider a CFT in D space-time dimensions with a scalar operator $O(\boldsymbol{x})$ with scaling dimension Δ. This is presumed to be equivalent to a dual gravity theory on AdS_{D+1} with action $\mathcal{S}_{\text{bulk}}$. The CFT and the bulk theory are related by the GKPW ansatz

$$\int \mathcal{D}\phi \exp\left(-\mathcal{S}_{\text{bulk}}\right)\Big|_{\text{bdy}} = \left\langle \exp\left(\int d^D x\, \phi_0(\boldsymbol{x}) O(\boldsymbol{x})\right)\right\rangle_{\text{CFT}}$$

where the boundary condition is

$$\lim_{r\to 0} \phi(\boldsymbol{x}, r) = r^{D-\Delta}\phi_0(\boldsymbol{x}).$$

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We consider the simplest case of a single scalar field, where the bulk action is

$$\mathcal{S}_{\text{bulk}} = \frac{1}{2}\int d^{D+1}x\sqrt{g}\left[g^{ab}\partial_a\phi\partial_b\phi + m^2\phi^2\right]$$

where g_{ab} is the AdS_{D+1} metric (we are working with a Euclidean signature, and a, b extend over $D+1$ dimensions) and $g = \det(g_{ab})$. After Fourier transforming space-time co-ordinates to momenta $\boldsymbol{k}$, the saddle-point equation for $\phi(\boldsymbol{k}, r)$ is

$$-r^{D-1}\frac{d}{dr}\left(\frac{1}{r^{D-1}}\frac{d\phi}{dr}\right) + \left(k^2 + \frac{m^2}{r^2}\right)\phi = 0.$$

Suvrat Raju

This equation has two solutions as $r \to 0$, with $\phi \sim r^{\Delta}$ or $\phi \sim r^{D-\Delta}$ where

$$\Delta = \frac{D}{2} \pm \sqrt{\frac{D^2}{4} + m^2}.$$

We will choose the positive sign in the manipulations below, but the final results hold for both signs. The complete solution of the saddle-point equation with the needed boundary condition can be written as

$$\phi(\boldsymbol{k}, r) = G_{\text{bulk-bdy}}(k, r)\phi_0(\boldsymbol{k})$$

where

$$G_{\text{bulk-bdy}}(k, r) = \frac{2^{1-\Delta+D/2}}{\Gamma(\Delta - D/2)} k^{\Delta - D/2} r^{D/2} K_{\Delta - D/2}(kr)$$

where $K_{\Delta-D/2}$ is a modified Bessel function.

Next, it is useful to obtain the bulk-bulk Green's function by inverting the operator in the equation of motion. A standard computation yields

$$G_{\text{bulk-bulk}}(k, r_1, r_2) = (r_1 r_2)^{D/2} I_{\Delta-D/2}(kr_<) K_{\Delta-D/2}(kr_>)$$

where $r_<$ ($r_>$) is the smaller (larger) or $r_{1,2}$. This bulk-bulk Green's function is evaluated in the absence of any sources on the boundary, and so we have to impose the boundary condition $\phi(\boldsymbol{k}, r) \sim r^{\Delta}$ as $r \to 0$ in solving the saddle-point equation. The utility of this bulk-bulk Green's function is that it now allows us to extend our results to include interactions in $\mathcal{S}_{\rm bulk}$ by the usual Feynman graph expansion. We can account for the presence of the boundary source $\phi_0(\boldsymbol{k})$ in the CFT by imagining there is a corresponding bulk source field $J_0(\boldsymbol{k}, r)$ which is localized at very small values of r. Then this bulk source field will generate a bulk $\phi(\boldsymbol{k}, r)$ via the propagator $G_{\rm bulk-bulk}$. We now note that

$$\lim_{r_2 \to 0} G_{\rm bulk-bulk}(k, r_1, r_2) = \frac{r_2^{\Delta}}{(2\Delta - D)} G_{\rm bulk-bdy}(k, r_1).$$

This is a key relation which shows us that functional derivatives of the full action w.r.t. $J_0(\boldsymbol{k}, r)$ (which yield bulk-bulk correlation functions) are the *same* as functional derivatives w.r.t. $\phi_0(\boldsymbol{k})$ (which yield correlators of the CFT). This yields the second statement of the equivalence between the bulk and boundary theories

$$\langle O(\boldsymbol{x}_1) \ldots O(\boldsymbol{x}_n) \rangle_{\rm CFT} = Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(\boldsymbol{x}_1, r_1) \ldots \phi(\boldsymbol{x}_n, r_n) \rangle_{\rm bulk}$$

where the "wave function renormalization" factor $Z = (2\Delta - D)$. Note that this relationship holds for arbitrary bulk actions, and permits full quantum fluctuations in the bulk theory. Also, both correlators are evaluated in the absence of external sources; for the bulk theory this means that we have the boundary condition

AdS/CFT correspondence at zero temperature

Suvrat Raju

$\phi(\boldsymbol{k}, r) \sim r^{\Delta}$ as $r \to 0$. From this general relation we can evaluate the two-point correlator of the CFT for the case of a bulk Gaussian action:

$$\begin{aligned}\langle O(\boldsymbol{k}) O(-\boldsymbol{k})\rangle_{\rm CFT} &= Z^2 \lim_{r\to 0} (r)^{-2\Delta} G_{\rm bulk-bulk}(k, r, r) \\ &= \lim_{r\to 0} (2\Delta - D) r^{-(2\Delta - D)} - (2\Delta - D) \left(\frac{k}{2}\right)^{2\Delta - D} \frac{\Gamma(1 - \Delta + D/2)}{\Gamma(1 + \Delta - D/2)}\end{aligned}$$

The first term is divergent, but it is independent of k: so it does not contribute to the long-distance correlations of the CFT, and can be dropped. The second term has the singular dependence $\sim k^{2\Delta - D}$, which is just as expected for a field with scaling dimension Δ, for the Fourier transformation yields

$$\langle O(\boldsymbol{x}_1) O(\boldsymbol{x}_2)\rangle_{\rm CFT} \sim |\boldsymbol{x}_1 - \boldsymbol{x}_2|^{-2\Delta}.$$

AdS/CFT correspondence at zero temperature

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The final formulation of the bulk-boundary correspondence appears by using the above relations for arbitrary multi-point correlators in the absence of a source, to make a statement for the one-point function in the presence of a source, working to all orders in the source and all bulk interactions. As we noted earlier, the CFT source $\phi_0(\boldsymbol{k})$ can be simulated by a source $J_0(\boldsymbol{k}, r)$ which is localized near the boundary but acts on the bulk theory. Because of the identity above between the source-free correlators, we can conclude that $\langle O(\boldsymbol{x})\rangle$ equals $Z \lim_{r\to 0} r^{-\Delta}\langle\phi(\boldsymbol{x}, r)\rangle$. However, we have to remember that the source $J_0(\boldsymbol{k}, r)$ is actually realized by a boundary condition on $\phi(\boldsymbol{x}, r)$, and so the complete statement is

$$\lim_{r\to 0}\langle\phi(\boldsymbol{x}, r)\rangle = r^{D-\Delta}\phi_0(\boldsymbol{x}) + \frac{r^{\Delta}}{Z}\langle O(\boldsymbol{x})\rangle,$$

in the presence of the source $\phi_0(\boldsymbol{x})$. Note that this result is not just linear response, and holds to all orders in the source; it also allows for arbitrary bulk interactions and quantum fluctuations. It can be checked that it is indeed obeyed by the correlators above of the Gaussian theory. This relationship is frequently used in practice, because it is often straightforward to implement, especially when we are using the classical saddle-point approximation for the bulk theory. Then we simply have to extract the subleading behavior in $\phi(\boldsymbol{x}, r)$ as $r \to 0$ to extract the full non-linear response function to the perturbation $\phi_0(\boldsymbol{x})$ to the CFT.

AdS/CFT correspondence at zero temperature

Suvrat Raju

A similar analysis can be applied to operators of the CFT with non-zero Lorentz spin. Of particular interest to are correlators of a conserved current, J_μ, associated with a global 'flavor' symmetry, and the conserved stress energy tensor $T_{\mu\nu}$.

We couple the conserved current to a source a_μ and so are interested in evaluating

$$\mathcal{Z}(a_\mu) = \left\langle \exp\left(\int d^D x \, a_\mu(\boldsymbol{x}) J_\mu(\boldsymbol{x}) \right) \right\rangle_{\text{CFT}} .$$

The conservation law $\partial_\mu J_\mu = 0$ now implies that this partition function is invariant under the gauge transformation $a_\mu \to a_\mu + \partial_\mu \alpha$. So the bulk field dual to a (say) U(1) conserved current J_μ is a U(1) gauge field, which we denote $A_a(\boldsymbol{x}, r)$. We assume the gauge field has a Maxwell action

$$\mathcal{S}_M = \frac{1}{4 g_M^2} \int d^{D+1} x \, \sqrt{g} F_{ab} F^{ab}$$

plus other possible gauge couplings to the bulk fields. By an analysis very similar to the scalar field, we can establish the following bulk-boundary correspondences

$$\begin{aligned}
\lim_{r \to 0} \langle A_\mu(\boldsymbol{x}, r) \rangle &= a_\mu(\boldsymbol{x}) + \frac{r^{D-2}}{Z g_M^{-2}} \langle J_\mu(\boldsymbol{x}) \rangle \\
\lim_{r \to 0} \langle A_r(\boldsymbol{x}, r) \rangle &= 0 \\
\langle J_\mu(\boldsymbol{x}_1) \ldots J_\nu(\boldsymbol{x}_n) \rangle_{\text{CFT}} &= (Z g_M^{-2})^n \lim_{r \to 0} r_1^{2-D} \ldots r_n^{2-D} \, \langle A_\mu(\boldsymbol{x}_1, r_1) \ldots A_\nu(\boldsymbol{x}_n, r_n) \rangle_{\text{bulk}}
\end{aligned}$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

with $Z = D - 2$. Working with only the Maxwell action these relations yield

$$\langle J_\mu(\boldsymbol{k}) J_\nu(\boldsymbol{k}) \rangle_{\rm CFT} = \frac{(D-2)}{g_M^2} \frac{\Gamma(2 - D/2)}{\Gamma(D/2)} \left(\frac{k}{2}\right)^{D-2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

This is precisely the expected form for the correlator of a conserved current in a CFT in D space-time dimensions. For the case $D = 3$ it has the expected form

$$\langle J_\mu(\boldsymbol{k}) J_\nu(\boldsymbol{k}) \rangle_{\rm CFT} = \mathcal{K}\, k \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

with

$$\mathcal{K} = \frac{1}{g_M^2}.$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the metric of the bulk theory. Now the needed partition function is

$$\mathcal{Z}(\chi_{\mu\nu}) = \left\langle \exp\left(\int d^D x \, \chi_{\mu\nu}(\boldsymbol{x}) T_{\mu\nu}(\boldsymbol{x})\right) \right\rangle_{\rm CFT}$$

and the source is related to the metric g_{ab} via

$$\begin{aligned} \lim_{r\to 0} g_{rr} &= \frac{L^2}{r^2} \\ \lim_{r\to 0} g_{r\mu} &= 0 \\ \lim_{r\to 0} g_{\mu\nu} &= \frac{L^2}{r^2}\left(\delta_{\mu\nu} + \chi_{\mu\nu}\right) \end{aligned}$$

The bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\boldsymbol{x}_1) \ldots T_{\rho\sigma}(\boldsymbol{x}_n)\rangle_{\rm CFT} = \left(\frac{Z L^2}{\kappa^2}\right)^n \lim_{r\to 0} r_1^{-D} \ldots r_n^{-D} \left\langle \chi_{\mu\nu}(\boldsymbol{x}_1, r_1) \ldots \chi_{\rho\sigma}(\boldsymbol{x}_n, r_n)\right\rangle_{\rm bul}$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

with $Z = D$. Applying this prescription to the Einstein action, we obtain in $D = 3$

$$\langle T_{\mu\nu}(\boldsymbol{k}) T_{\rho\sigma}(-\boldsymbol{k})\rangle_{\rm CFT} = \mathcal{C}_T |k|^3 \Big(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\nu}\frac{k_\rho k_\sigma}{k^2} + \delta_{\rho\sigma}\frac{k_\mu k_\nu}{k^2} - \delta_{\mu\rho}\frac{k_\nu k_\sigma}{k^2} - \delta_{\nu\rho}\frac{k_\mu k_\sigma}{k^2} - \delta_{\mu\sigma}\frac{k_\nu k_\rho}{k^2} - \delta_{\nu\sigma}\frac{k_\mu k_\rho}{k^2} + \frac{k_\mu k_\nu k_\rho k_\sigma}{k^4} \Big)$$

This is the most-general form expected for any CFT, and the "central charge" is related to a dimensionless combination of the gravitational constant and the AdS radius

$$\mathcal{C}_T \propto \frac{L^2}{2\kappa^2}.$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

So, to recapitulate, we have equated the correlators of the CFT3 to a bulk theory on AdS_4 with the Einstein-Hilbert action

$$\mathcal{S} = \frac{1}{4g_M^2}\int d^4x\,\sqrt{g}F_{ab}F^{ab} + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2. These parameters determine, respectively, the two-point correlators of a conserved U(1) current J_μ and the stress-energy tensor $T_{\mu\nu}$.

However, this action is non-linear, and it also implies non-zero multipoint correlators of these operators, even at tree-level in the bulk theory. For the simplest 3-point correlator, a lengthy computation from the bulk theory yields

$$\langle J_\mu(\boldsymbol{k}_1)J_\nu(\boldsymbol{k}_2)T_{\rho\sigma}(-\boldsymbol{k}_1-\boldsymbol{k}_2)\rangle \sim \frac{k_1k_2}{(k_1+k_2)^5}k_{1\mu}k_{1\nu}k_{1\rho}k_{1\sigma} + 175 \text{ terms}$$

with co-efficients determined by g_M and L^2/κ^2.

We can now compare this 3-point correlator with that obtained by direct computation on a CFT3. A general analysis of the constraints from conformal invariance (Osborn and Petkou, 1993) shows that this 3-point correlator is fully determined by the values $\mathcal{K}$, C_T, and *one* additional dimensionless constant which is characteristic of the CFT3.

AdS/CFT correspondence at zero temperature

Suvrat Raju

To fix this additional constant by the bulk theory, we have to go beyond the Einstein-Maxwell action. This action is the simplest action with up to 2 derivatives of the bulk fields. So, in the spirit of effectively field theory, let us now include all terms up to 4 derivatives. We want to work in linear response for the conserved current, and so we exclude terms which have more than 2 powers of F_{ab}. Then, up to some field redefinitions, there turns out to be a unique 4 derivative term, and the extend action of the bulk theory now becomes

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

where C_{abcd} is the Weyl tensor. Now we have a new dimensionless parameter, γ; stability constraints on this action restrict $|\gamma| < 1/12$. The Weyl tensor vanishes on the AdS metric, and consequently γ does not modify the previous results on the 2-point correlators of J_μ and $T_{\mu\nu}$. However, γ does change the structure of the 3-point correlator. We found that for a suitable choice of γ it is possible to reproduce the 3-point correlator of free conformal fields; we expect we can match the properties of any CFT3 for a suitable γ.

It is clear that similar results apply at higher orders: matching higher multipoint correlators of the CFT requires higher derivative terms in the effective bulk theory.

AdS/CFT correspondence at zero temperature

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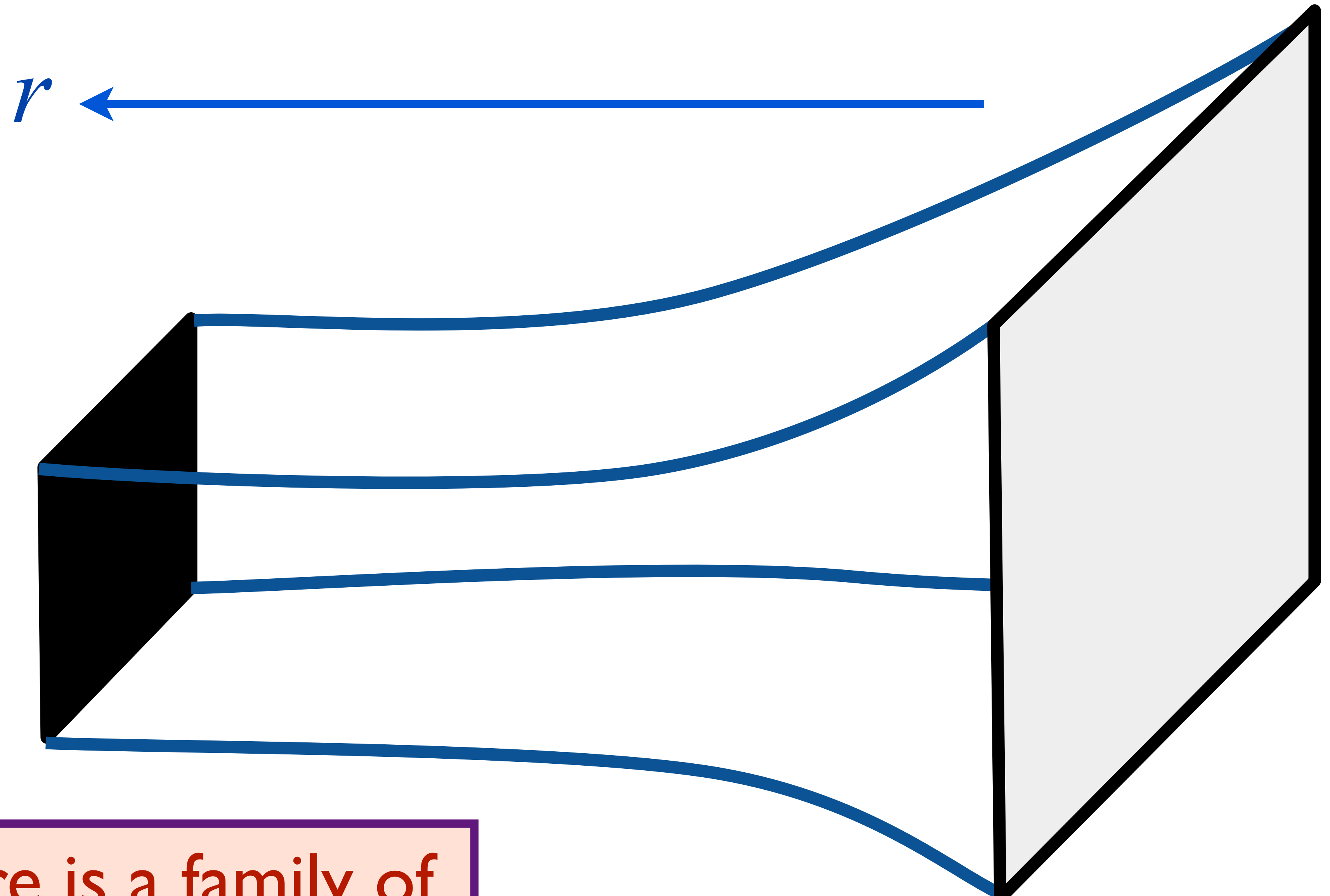
AdS$_4$-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

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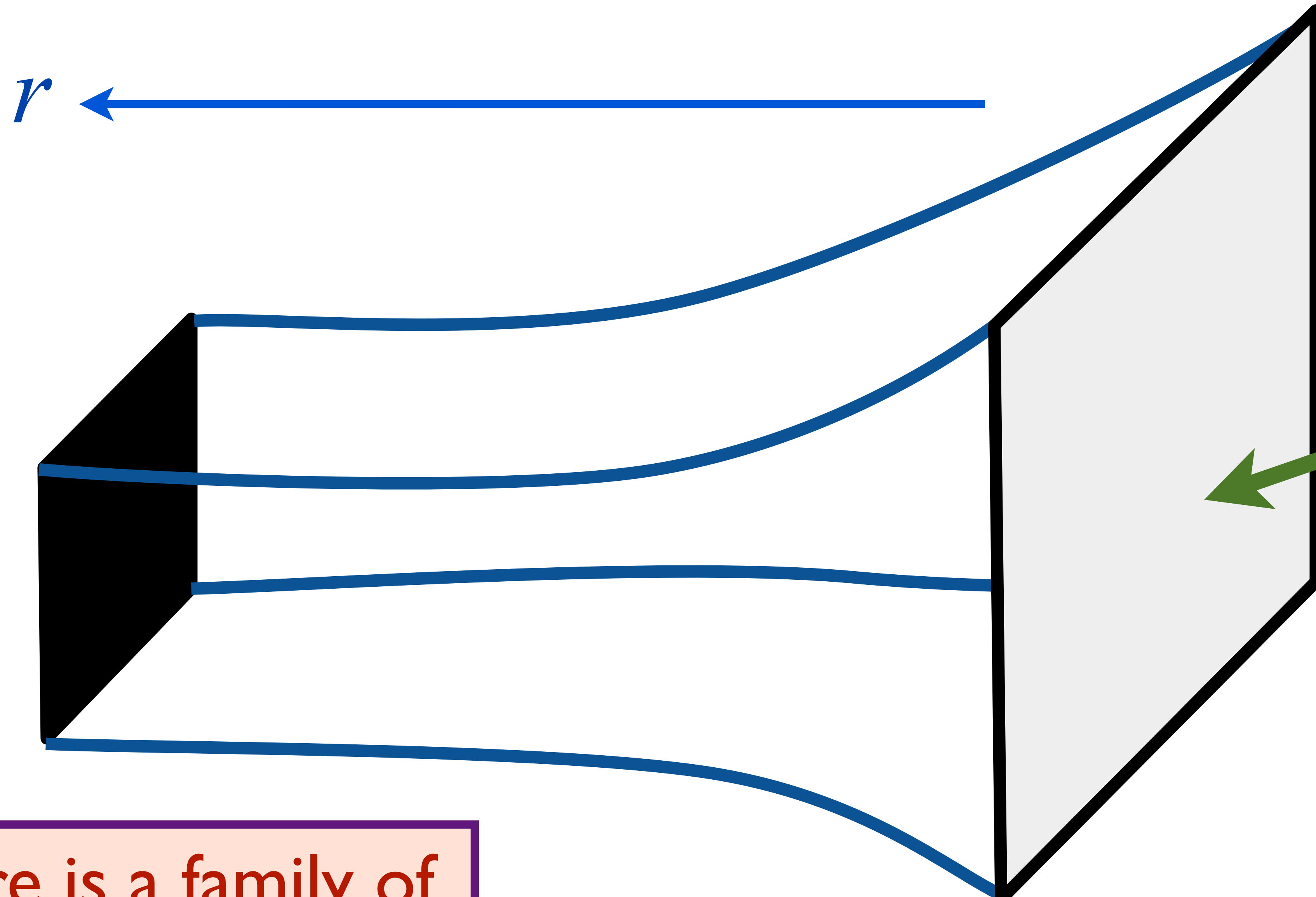

A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R}.$$

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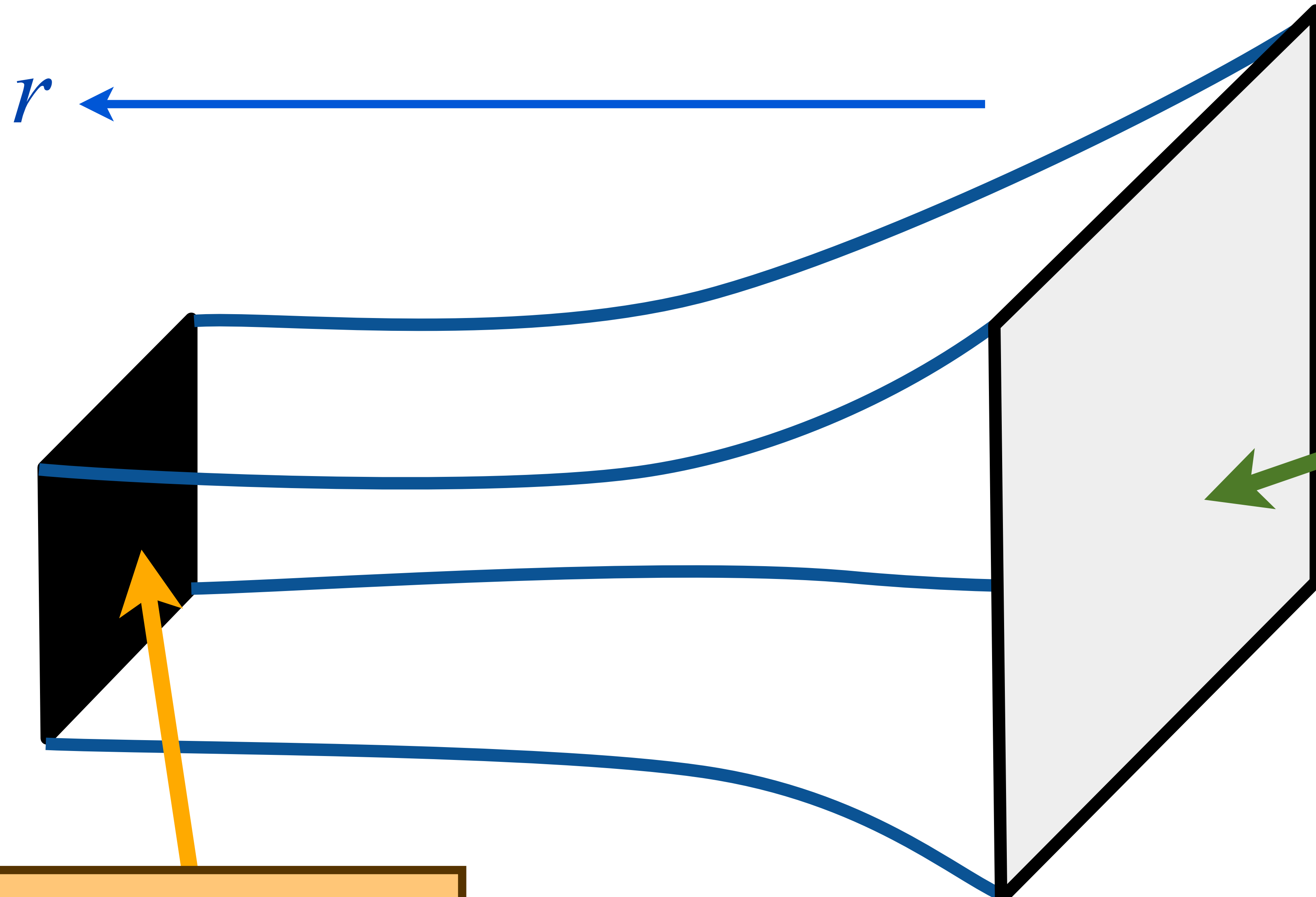
$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



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Black-brane at temperature of 2+1 dimensional quantum critical system

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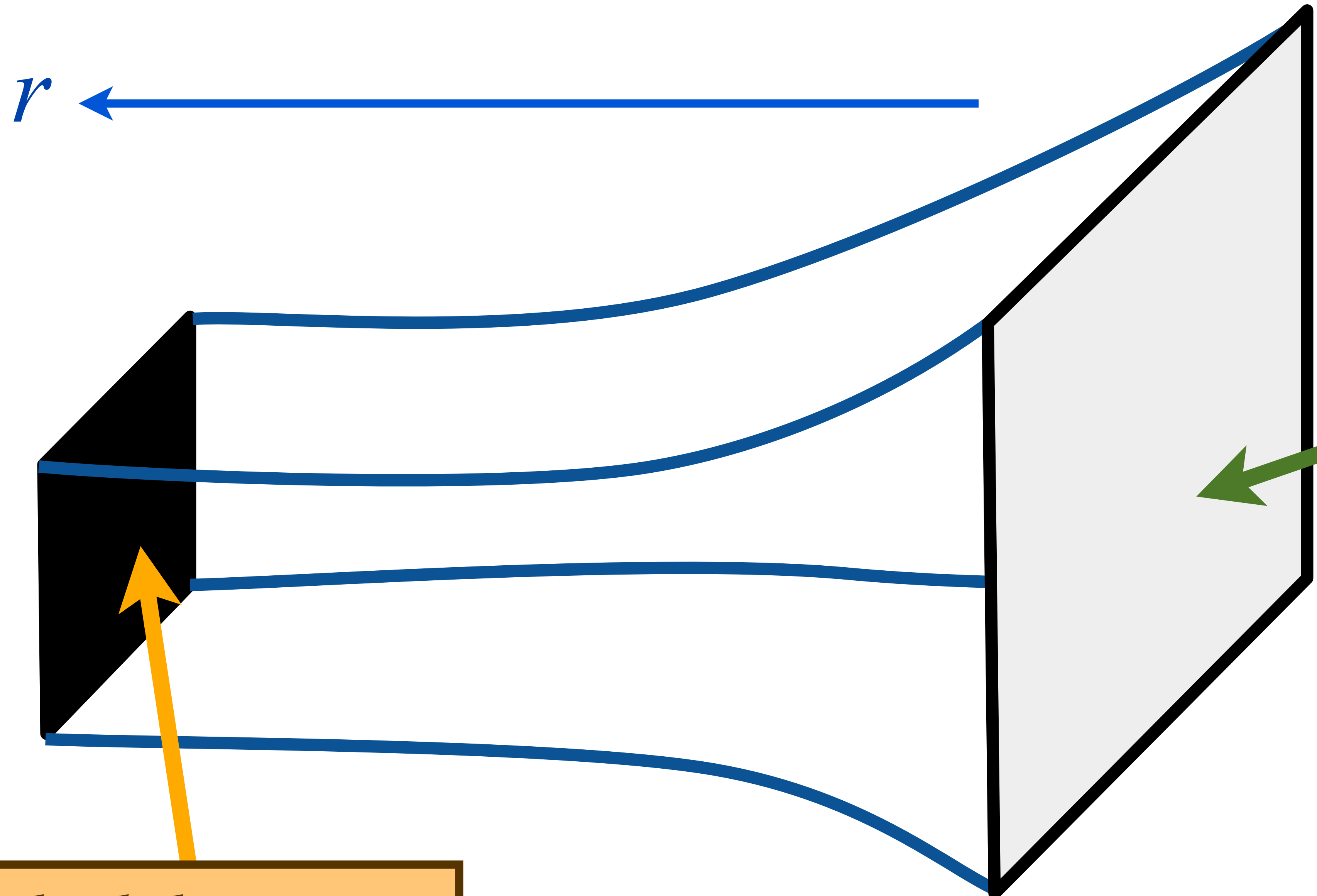


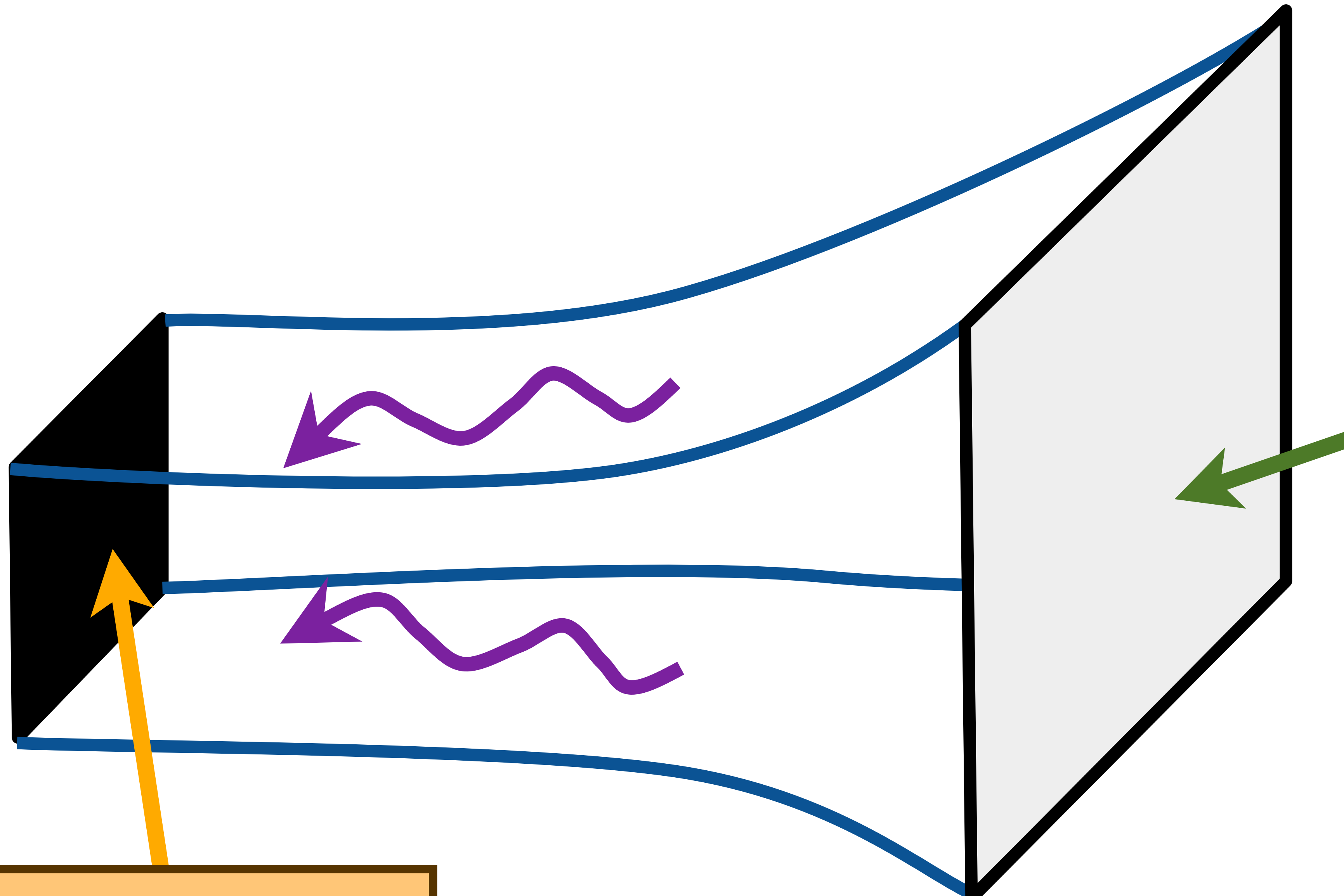
A 2+1 dimensional system at its quantum critical point: $k_B T = \frac{3\hbar}{4\pi R}$.

Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point:
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

AdS/CFT correspondence at non-zero temperatures

At non-zero temperatures, we consider a Euclidean metric with a horizon at $r = R$:

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} + f(r)d\tau^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$; note $f(R) = 0$. In the near horizon region we define $z = R - r$ and write this metric as

$$ds^2 = \left(\frac{L}{R}\right)^2 \left[\frac{dz^2}{|f'(R)|z} + |f'(R)|z\, d\tau^2 + dx^2 + dy^2\right]$$

Now we introduce co-ordinates $\rho = 2\sqrt{z/|f'(R)|}$ and $\theta = 2\pi T\tau$, and then the metric is

$$ds^2 = \left(\frac{L}{R}\right)^2 \left[d\rho^2 + \left(\frac{f'(R)}{4\pi T}\right)^2 \rho^2 d\theta^2 + dx^2 + dy^2\right]$$

Now if we choose the Hawking temperature

$$T = \frac{|f'(R)|}{4\pi}$$

then the spacetime is periodic under $\tau \to \tau + 1/T$, and there is no singularity at the horizon.

Computing conductivity at non-zero temperatures

In Euclidean signature, all the correspondences between the bulk and boundary correlators remain exactly the same as before. We need only add the additional requirement that the bulk solutions remain integrable at the horizon.

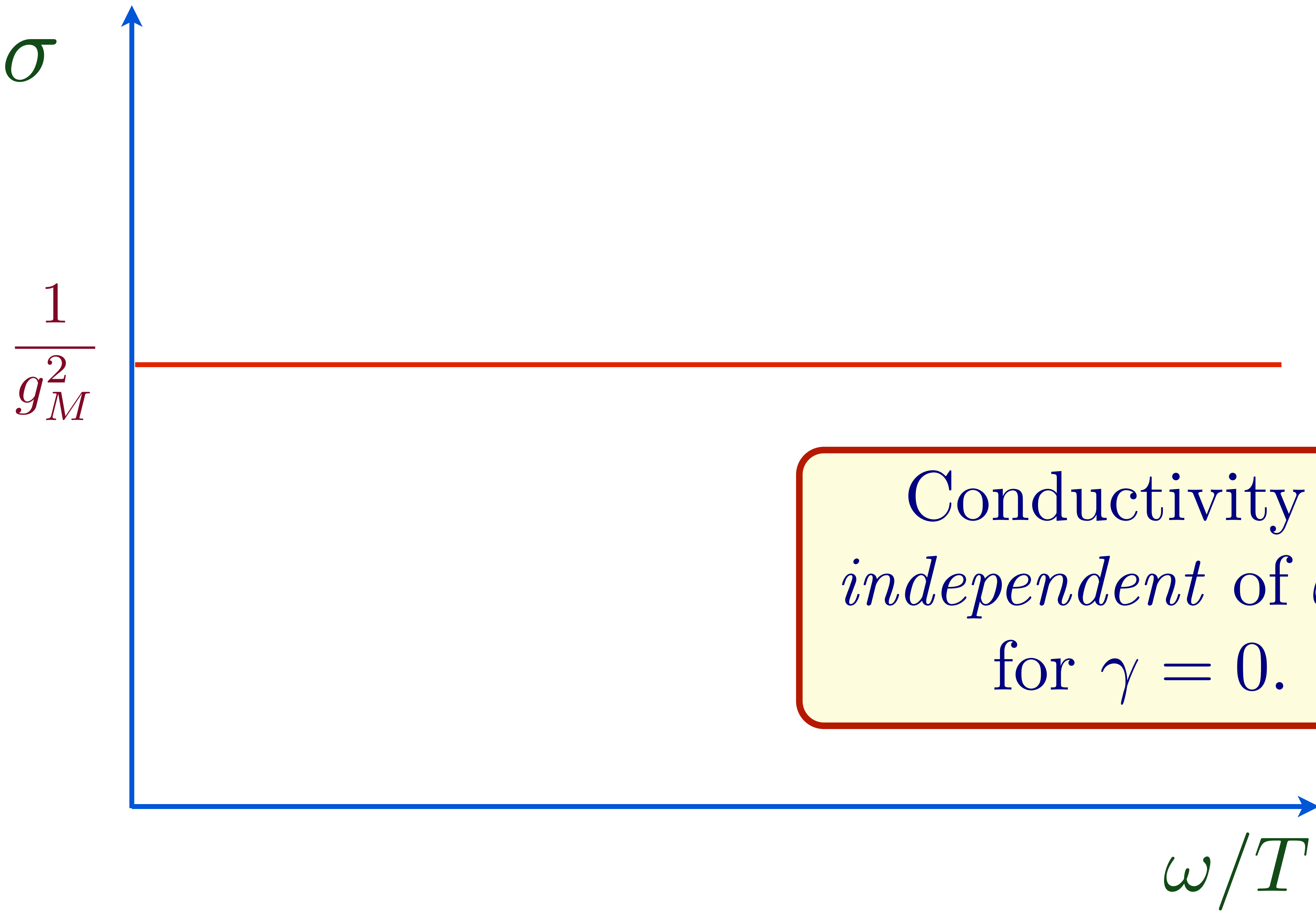
However, it is often convenient to work directly in real time and frequencies, and obtain the corresponding response functions directly, rather than by analytic continuation. It can be shown that the process of analytic continuation translates into the requirement of *in-going waves* at the horizon. The only other change in the equations is due to the change in the metric from AdS_4 to AdS_4-Schwarzschild, via the factor $f(r)$.

In terms of the co-ordinate $u = r/R$, the equation for $A_x(u)$ in the presence of a probe oscillating at frequency ω is

$$A_x'' + \frac{f'(3 - 2u^2\gamma f'') - 2u\gamma f(2f'' + uf''')}{f(3 - 2u^2\gamma f'')} A_x' + \frac{L^4}{R^2}\frac{\omega^2}{f^2} A_x = 0,$$

where the primes are derivatives w.r.t u. Solution of this equation, subject to the boundary conditions discussed earlier yields the conductivity.

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*



Conductivity is *independent* of ω/T for $\gamma = 0$.

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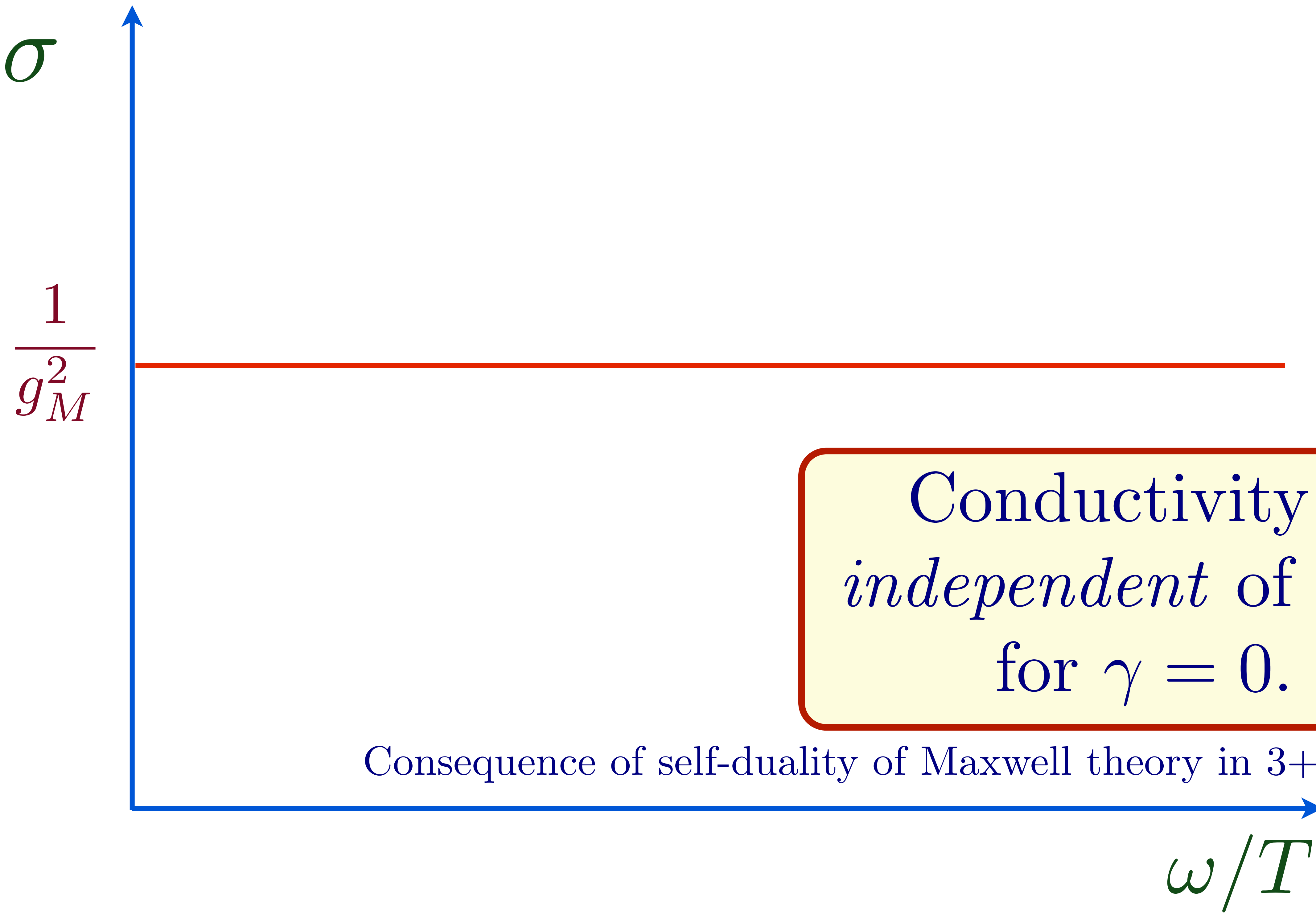
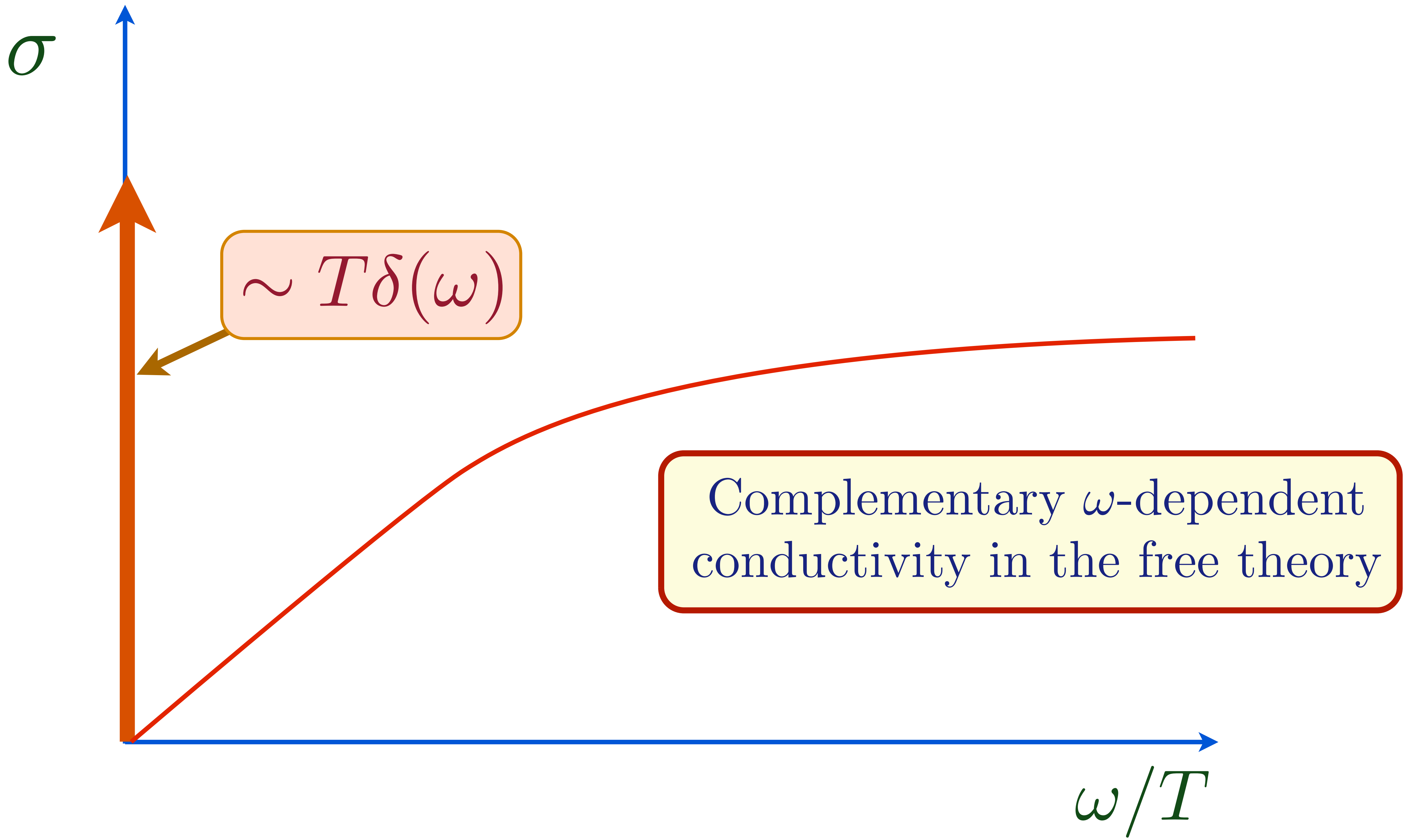


Conductivity is *independent* of ω/T for $\gamma = 0$.

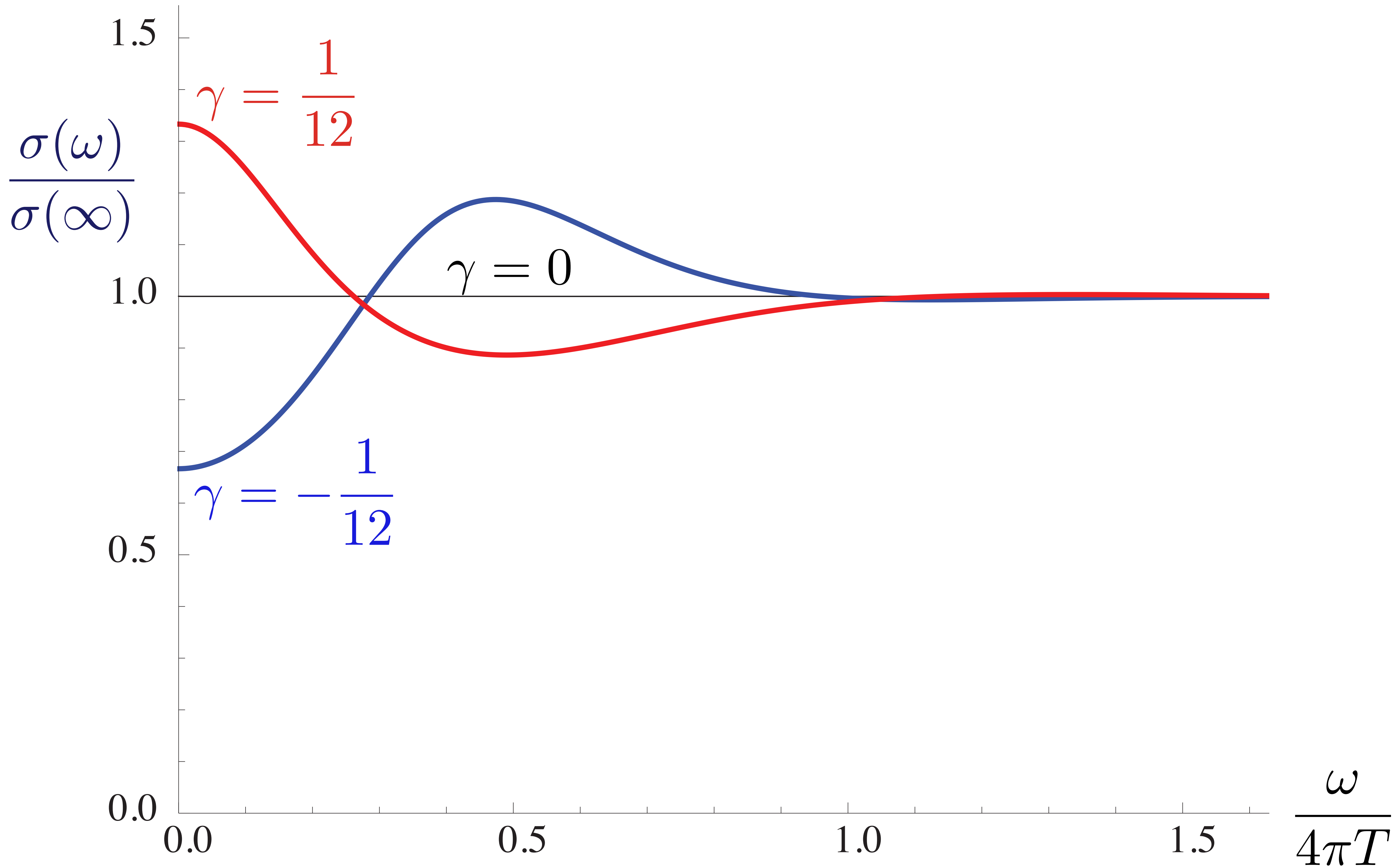
Consequence of self-duality of Maxwell theory in 3+1 dimensions

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

Electrical transport in a free CFT3 for *T* > *0*



AdS$_4$ theory of "nearly perfect fluids"



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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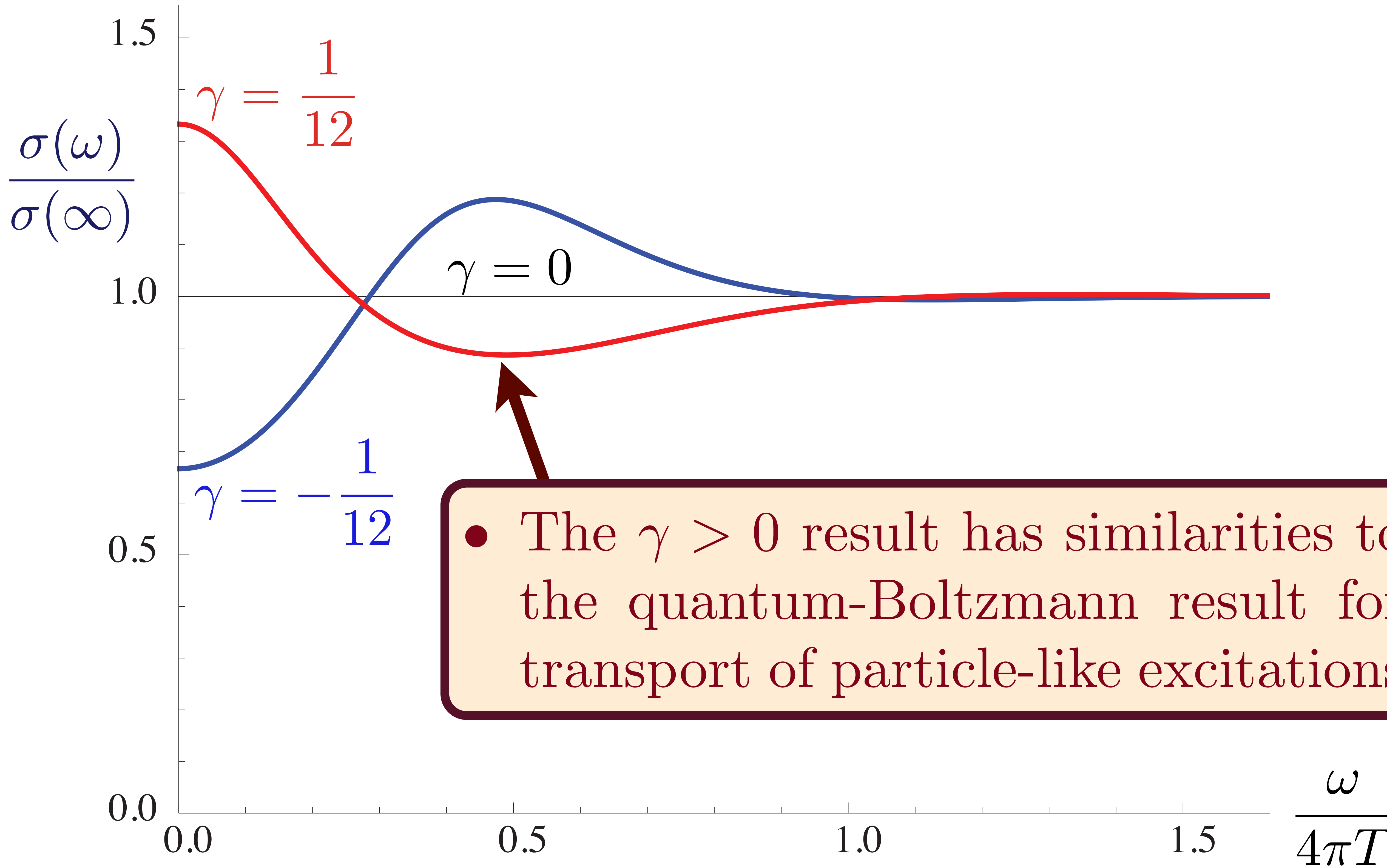

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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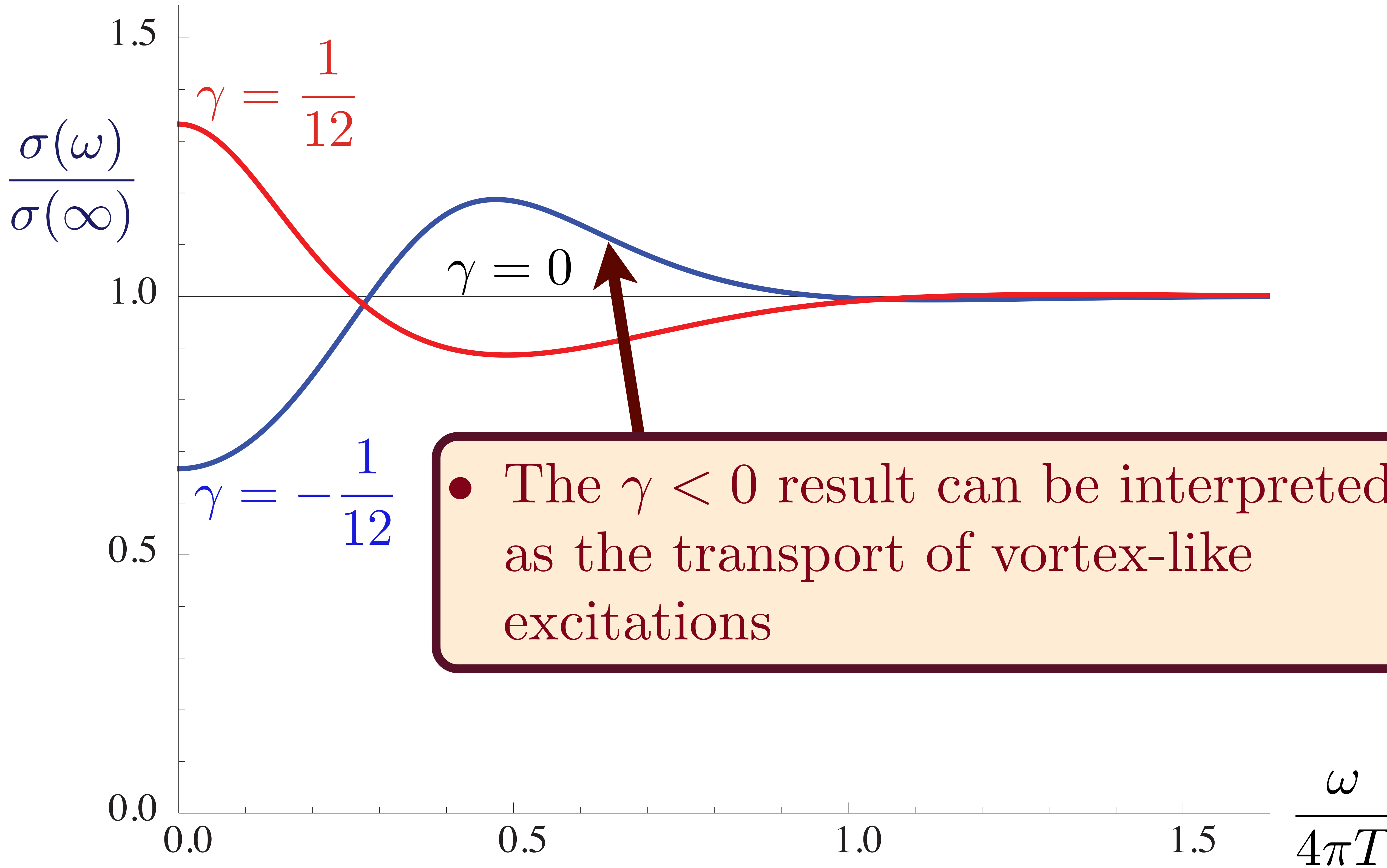
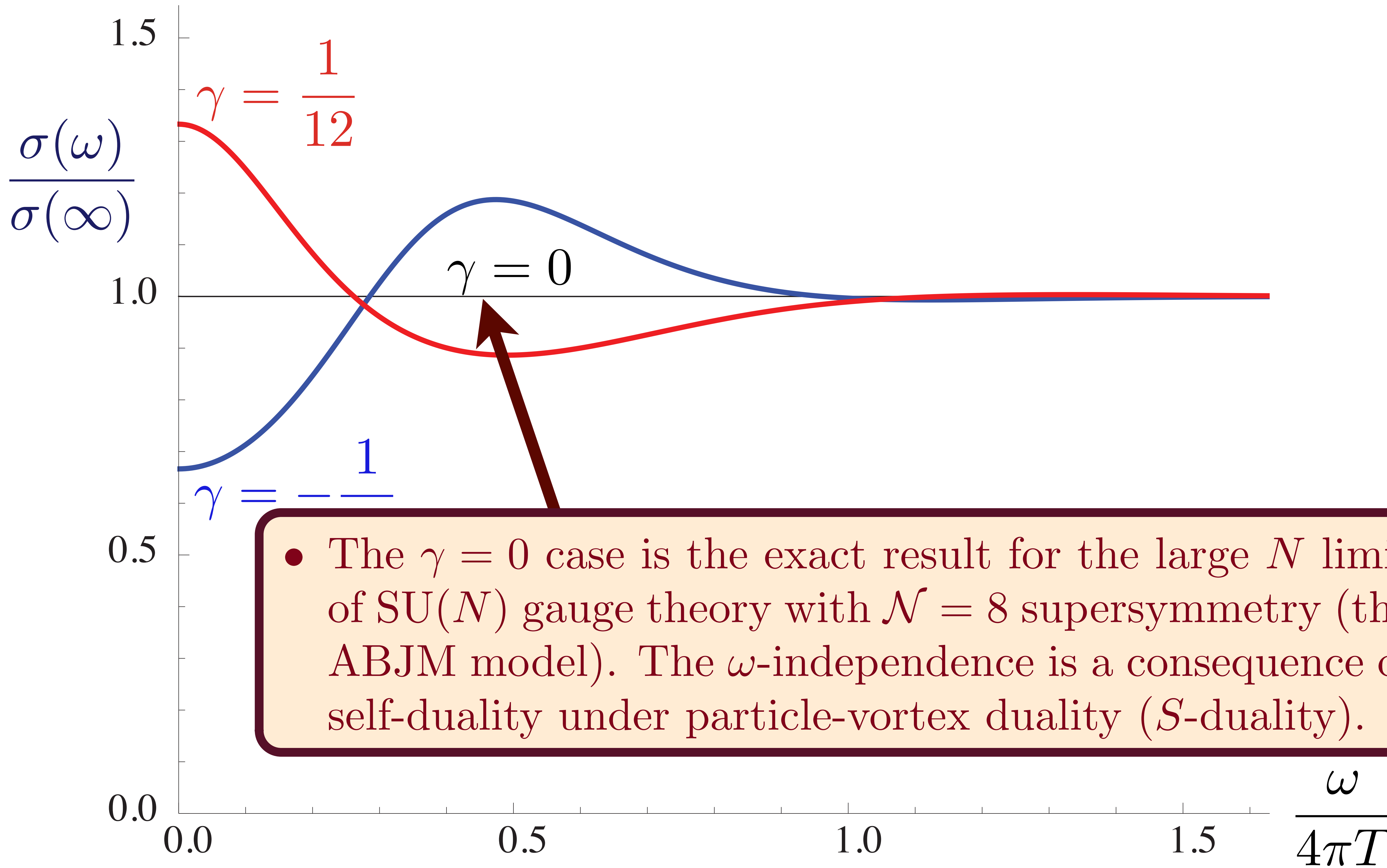


- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

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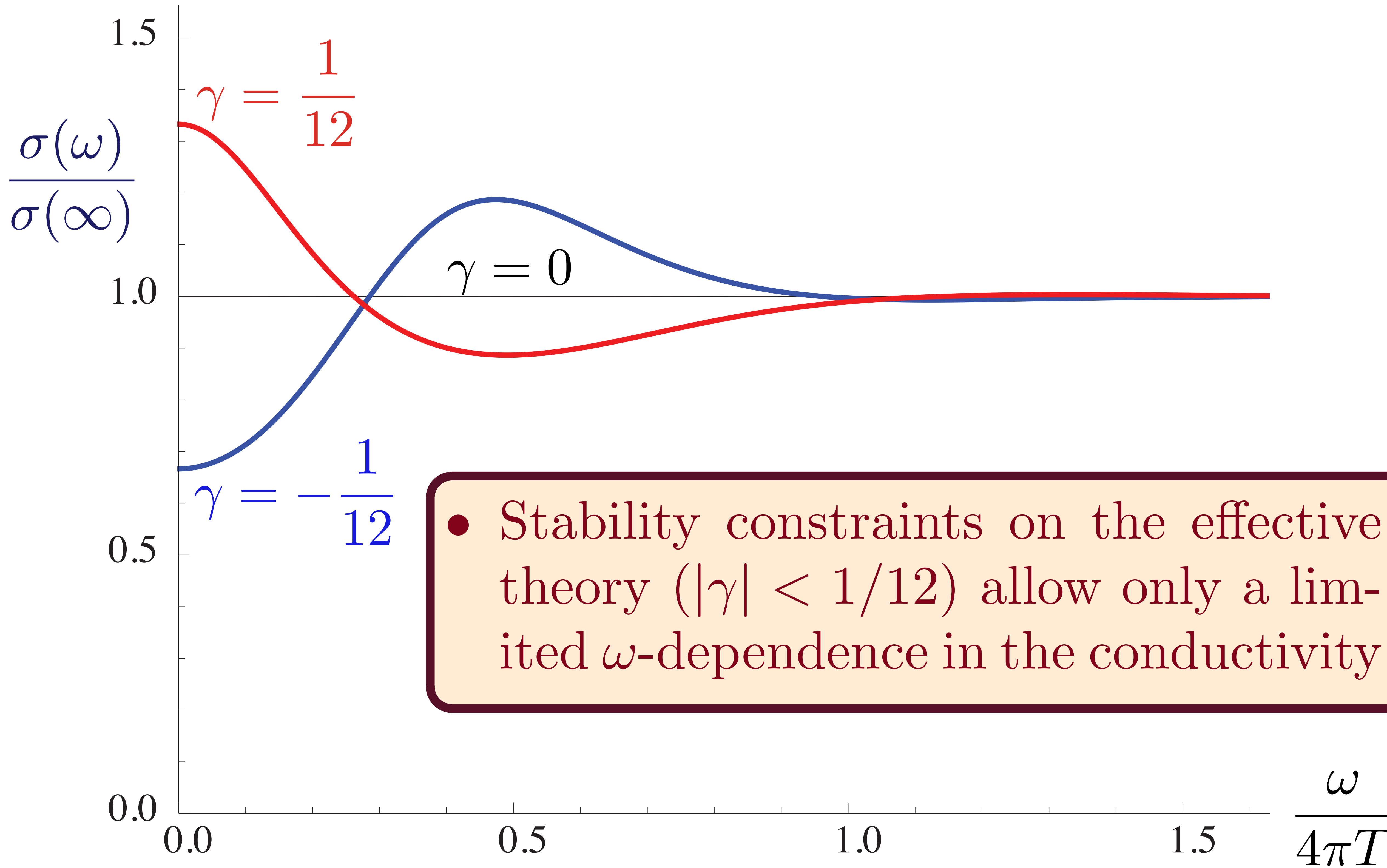

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

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The holographic solutions for the conductivity are found to satisfy two sum rules (W. Witzack-Krempa and S. Sachdev, arXiv:1210.4166)

$$\begin{aligned} \int_0^\infty d\omega \text{Re}\left[\sigma(\omega) - \sigma(\infty)\right] &= 0 \\ \int_0^\infty d\omega \text{Re}\left[\frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)}\right] &= 0 \end{aligned}$$

These sum rules are expected to be obeyed by all CFT3s. The second sum rule follows from the existence of a S-dual CFT3, whose conductivity is the inverse of the conductivity of the original CFT3.

Traditional Boltzmann theory computations make a choice of a "particle" basis at the outset: consequently they satisfy only *one* sum rule but not the other.

The holographic computation is unique in that it satisfies *both* sum rules. It does not bias the theory to either the particle or vortex basis: it starts from the self-dual point, and is able to expand away from it using the small value of the parameter γ.

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- Solve Einsten-Maxwell-... equations, allowing for a horizon at non-zero temperatures.