A simple model of entangled qubits, describing black holes and superconductors

QM^3 Quantum Matter meets Maths
University of Lisbon
January 4, 2021

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

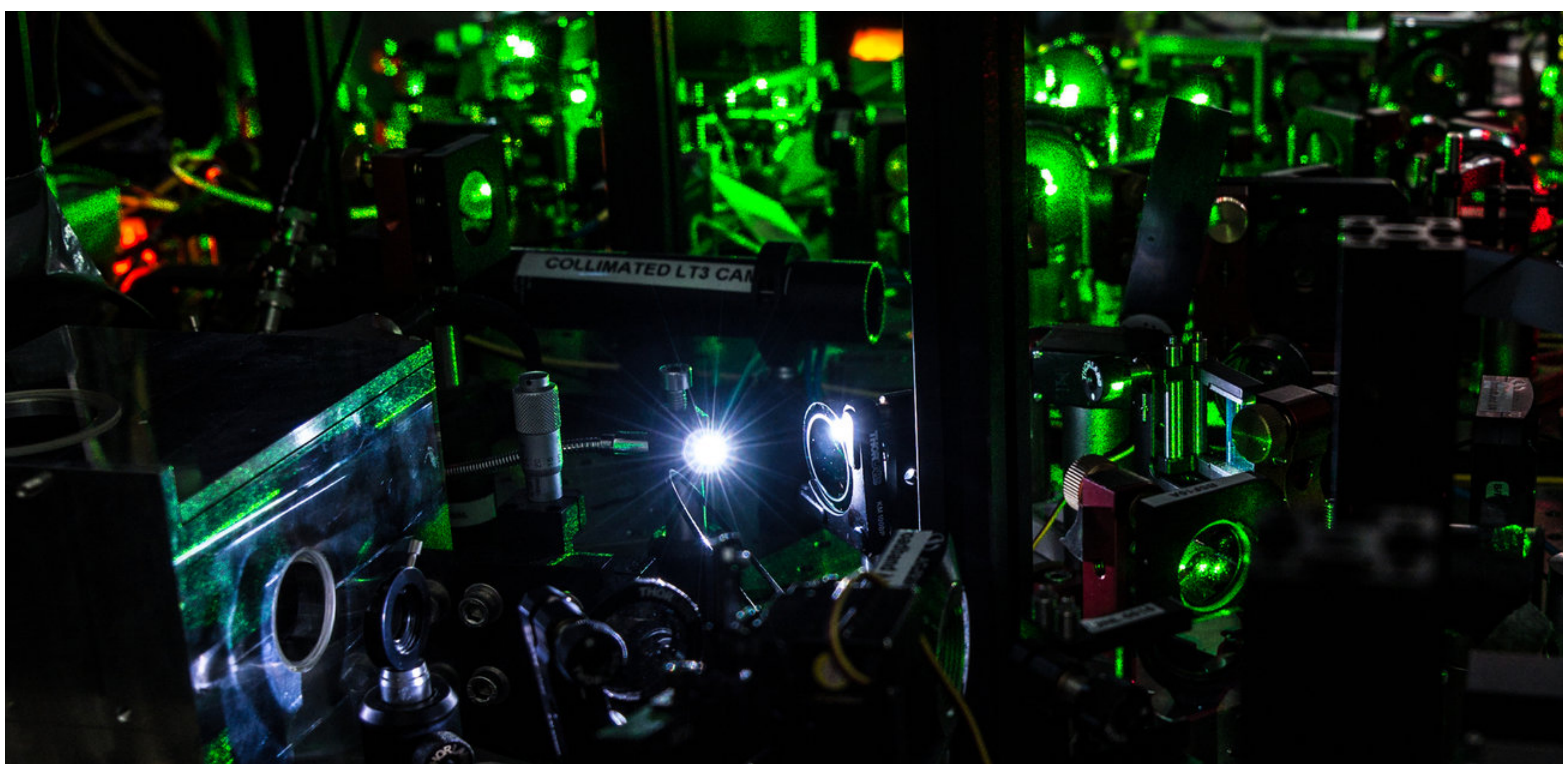
PHYSICS
HARVARD

The New York Times

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

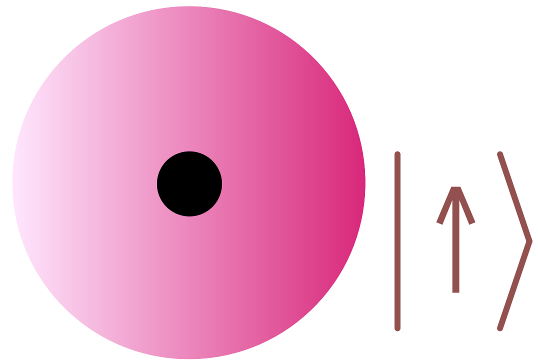


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

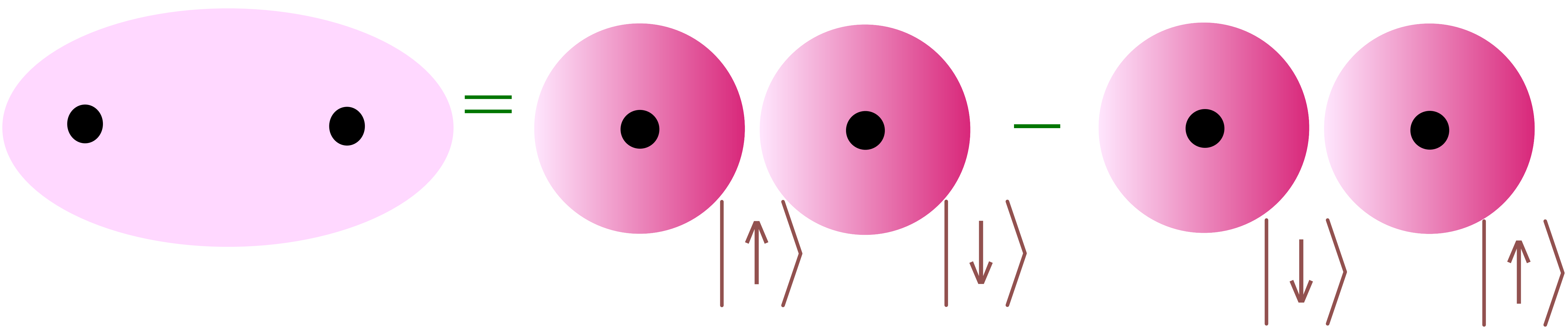
Quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

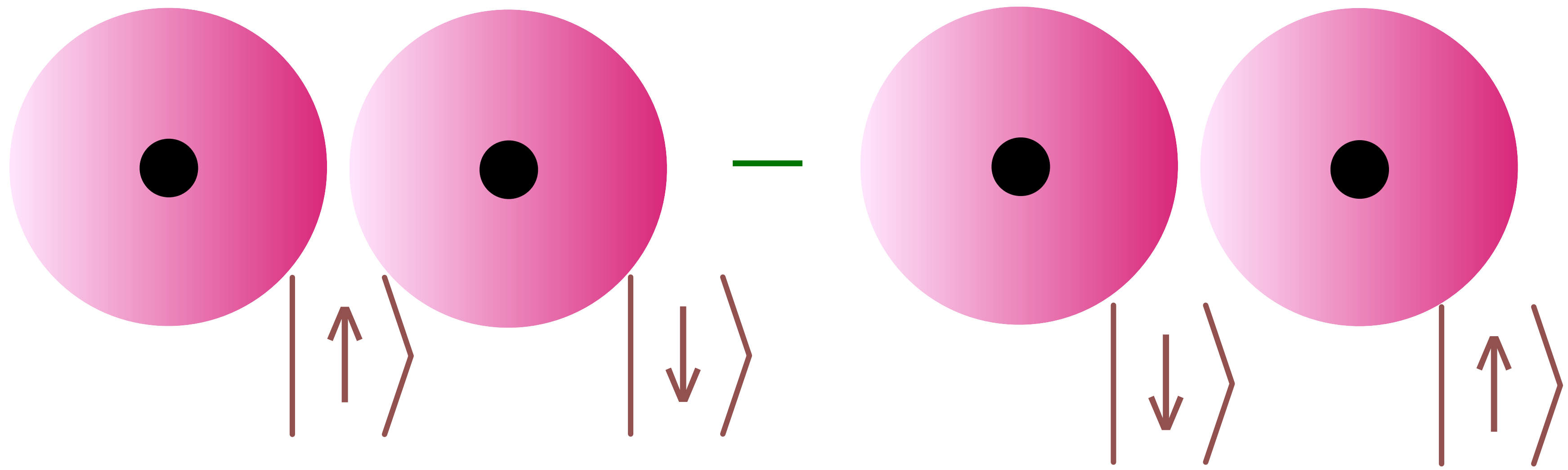


Hydrogen molecule:

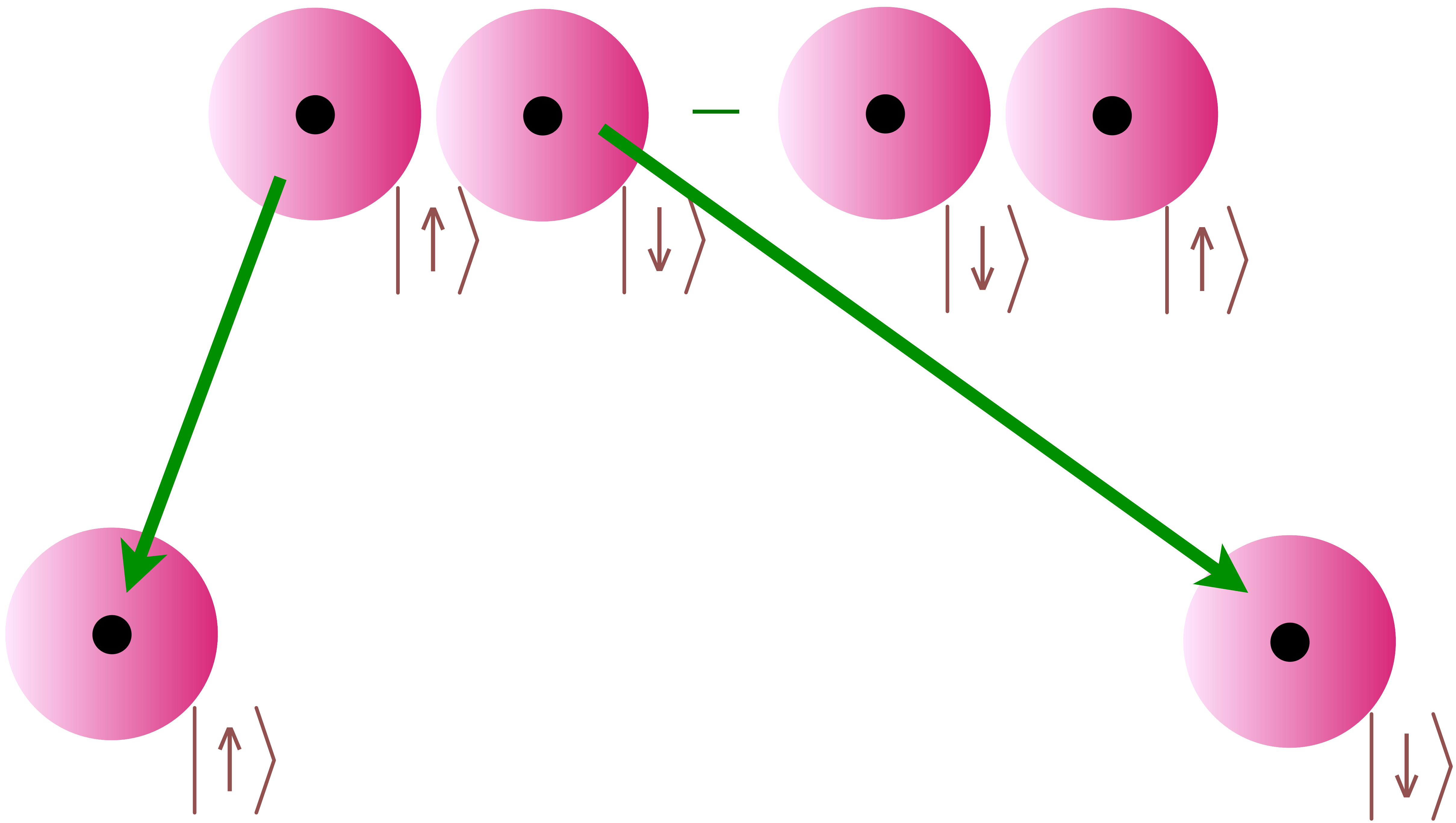


$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

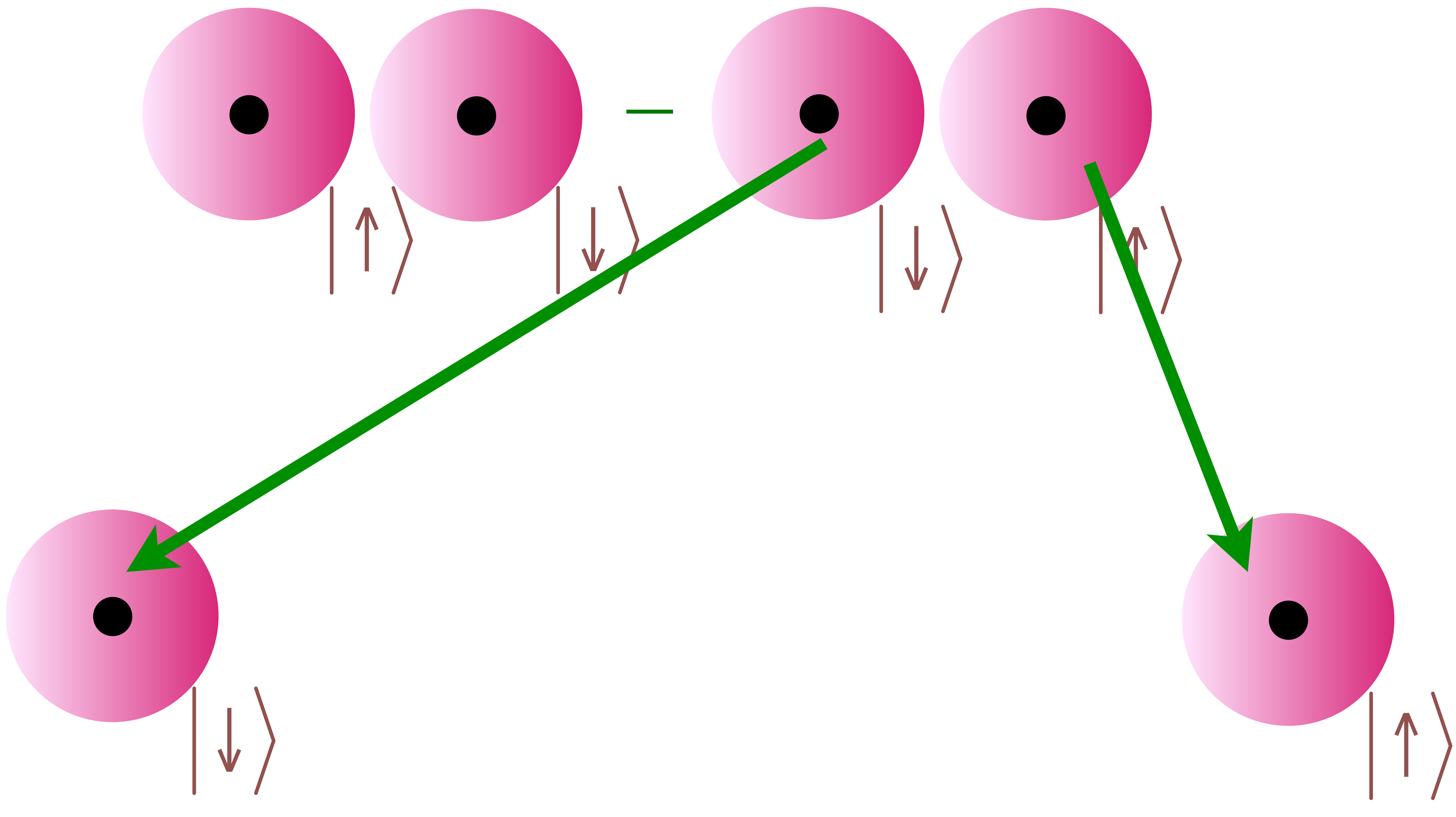
Quantum Entanglement: quantum superposition with more than one particle



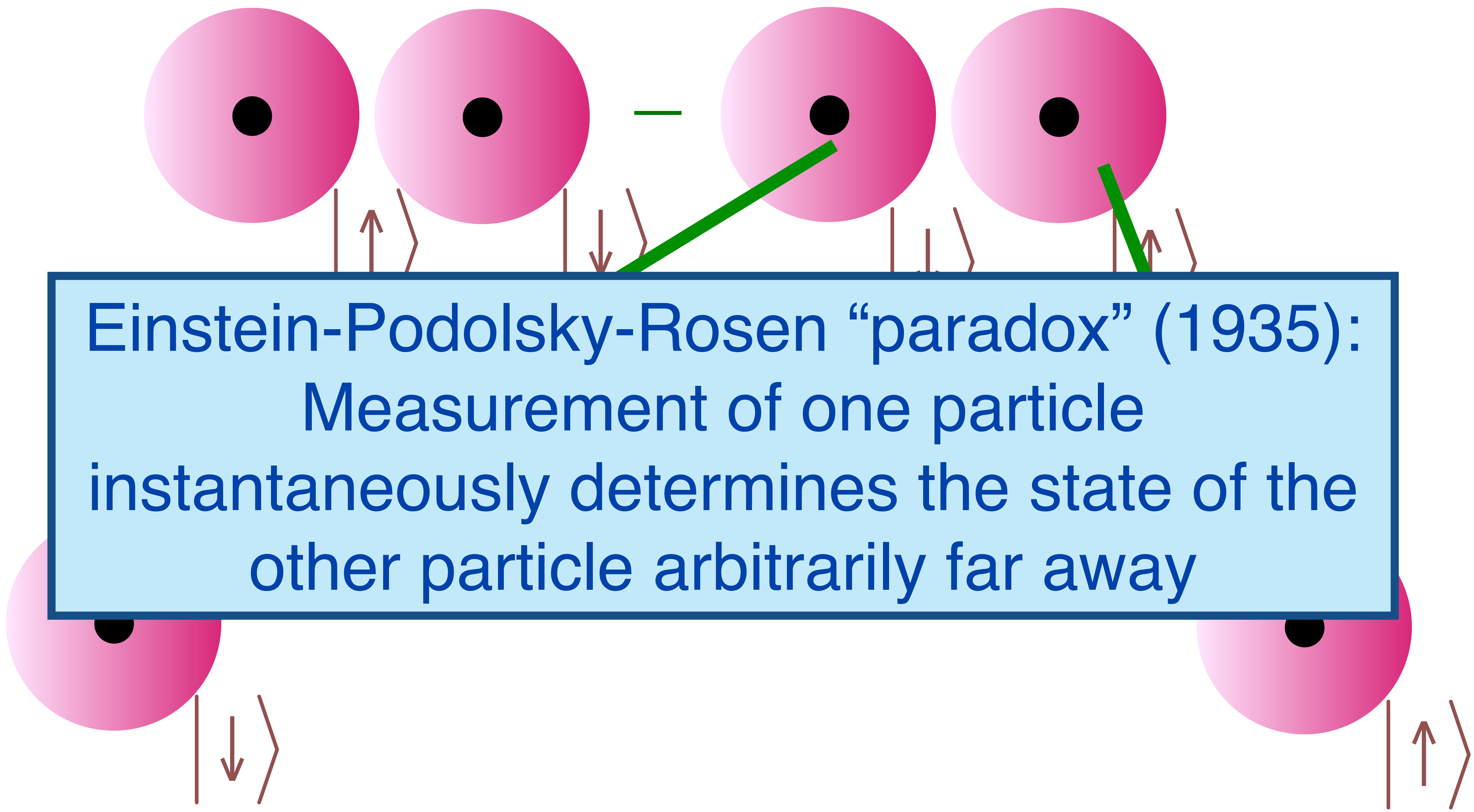
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen "paradox" (1935):
Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

Quantum entanglement

Quantum entanglement

A simple qubit model

A qubit: 2 states $|\uparrow\rangle$, $|\downarrow\rangle$.

Pauli gates:

$$X\,|\uparrow\rangle = |\downarrow\rangle \quad , \quad X\,|\downarrow\rangle = |\uparrow\rangle$$

$$Y\,|\uparrow\rangle = i\,|\downarrow\rangle \quad , \quad Y\,|\downarrow\rangle = -i\,|\uparrow\rangle$$

$$Z\,|\uparrow\rangle = |\uparrow\rangle \quad , \quad Z\,|\downarrow\rangle = -\,|\downarrow\rangle$$

A qubit: 2 states $|\uparrow\rangle$, $|\downarrow\rangle$.

Pauli gates:

$$X|\uparrow\rangle = |\downarrow\rangle \quad , \quad X|\downarrow\rangle = |\uparrow\rangle$$
$$Y|\uparrow\rangle = i|\downarrow\rangle \quad , \quad Y|\downarrow\rangle = -i|\uparrow\rangle$$
$$Z|\uparrow\rangle = |\uparrow\rangle \quad , \quad Z|\downarrow\rangle = -|\downarrow\rangle$$

A 2-qubit Hamiltonian: $\mathcal{H} = J\left(X_1X_2 + Y_1Y_2 + Z_1Z_2\right)$

Ground state: $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$

Energy $= -3J$

Excited states: $|\uparrow\rangle_1|\uparrow\rangle_2$, $|\downarrow\rangle_1|\downarrow\rangle_2$, $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\right)$

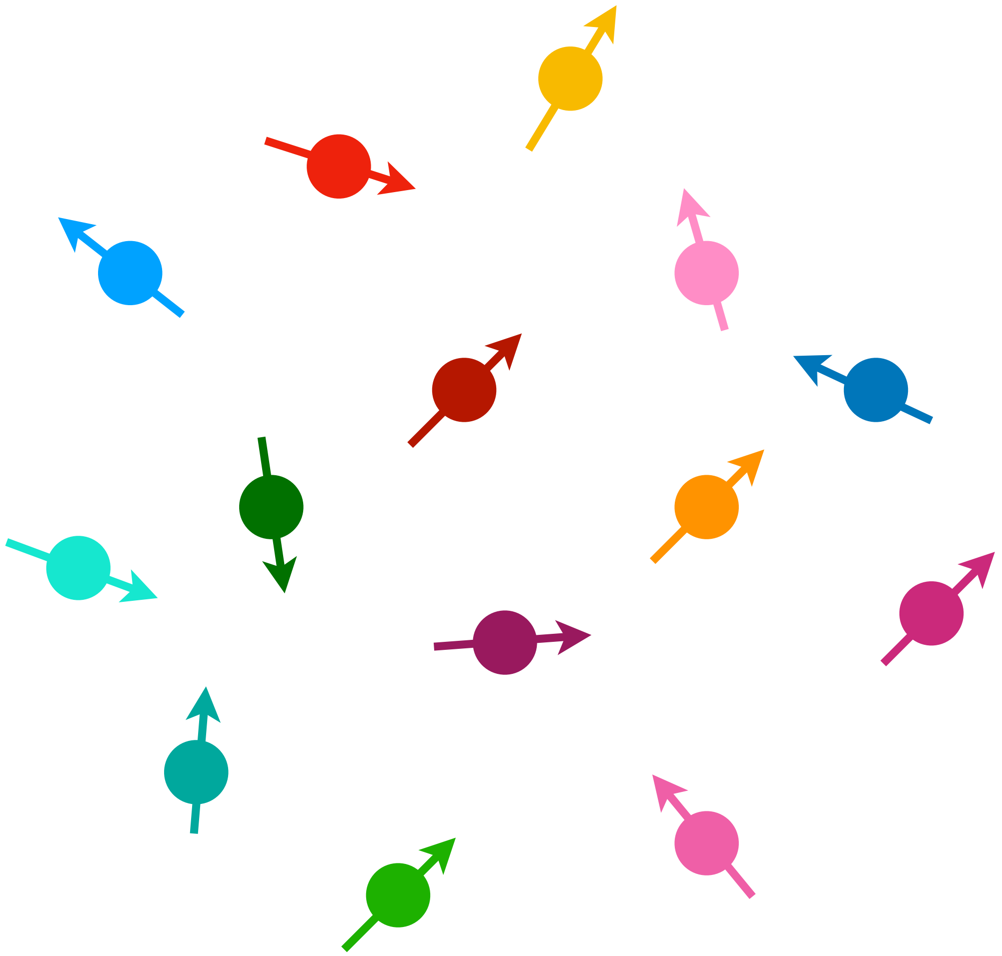
Energy $= J$

The simple model

$$\mathcal{H} = \sum_{i<j=1}^{N} J_{ij} \left(X_i X_j + Y_i Y_j + Z_i Z_j \right)$$

J_{ij} are random numbers with zero mean, and variance $\sim J^2/N$

(Technical comment: the solvable model has states in the self-conjugate representation of SU(M), and we take the limit $N \to \infty$ followed by the limit $M \to \infty$)



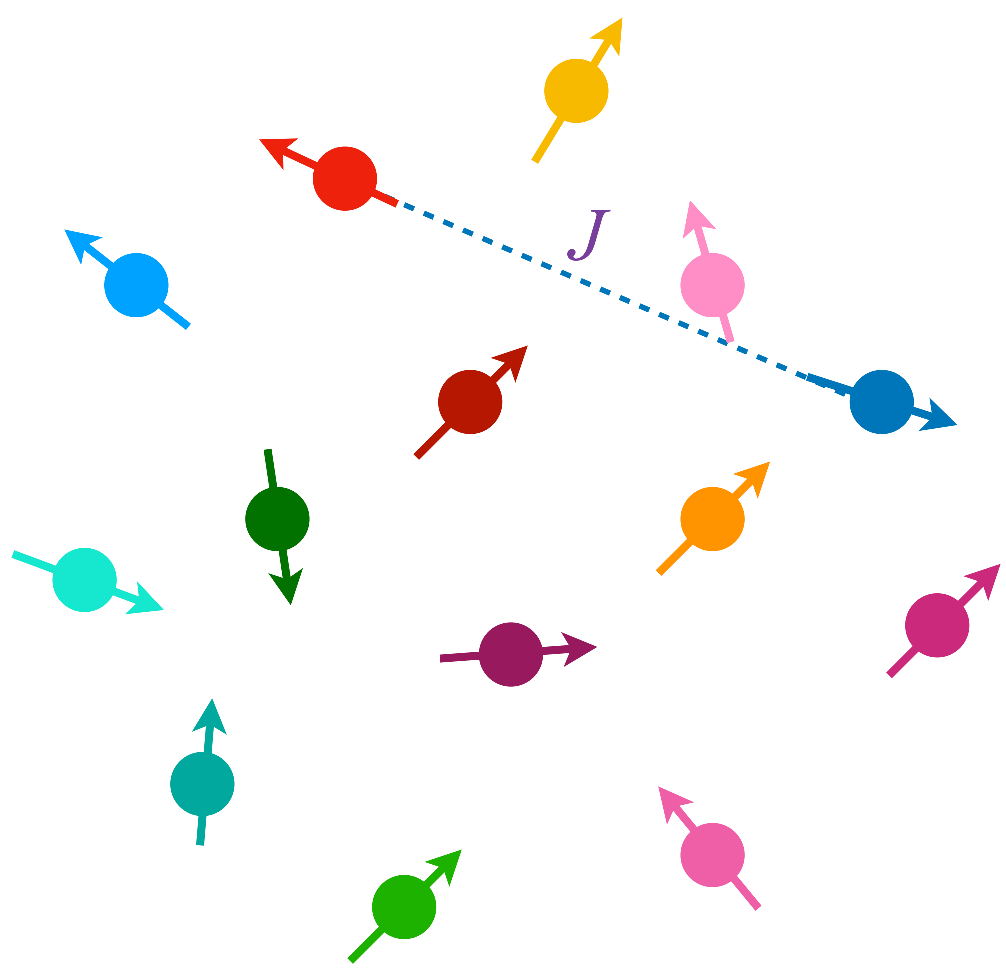
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The simple model

$$\mathcal{H} = \sum_{i<j=1}^{N} J_{ij} \left(X_i X_j + Y_i Y_j + Z_i Z_j \right)$$

J_{ij} are random numbers with zero mean, and variance $\sim J^2/N$

(Technical comment: the solvable model has states in the self-conjugate representation of SU(M), and we take the limit $N \to \infty$ followed by the limit $M \to \infty$)



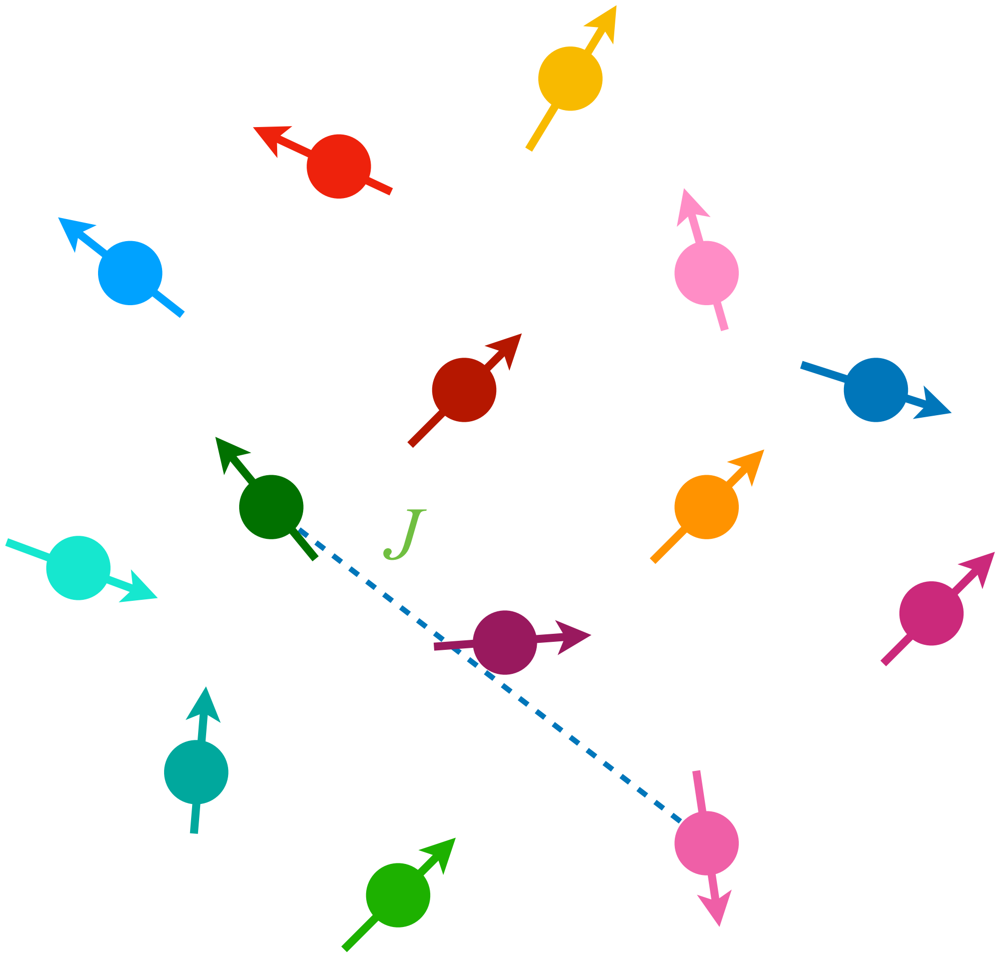
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The simple model

$$\mathcal{H} = \sum_{i<j=1}^{N} J_{ij} \left(X_i X_j + Y_i Y_j + Z_i Z_j \right)$$

J_{ij} are random numbers with zero mean, and variance $\sim J^2/N$

(Technical comment: the solvable model has states in the self-conjugate representation of SU(M), and we take the limit $N \to \infty$ followed by the limit $M \to \infty$)



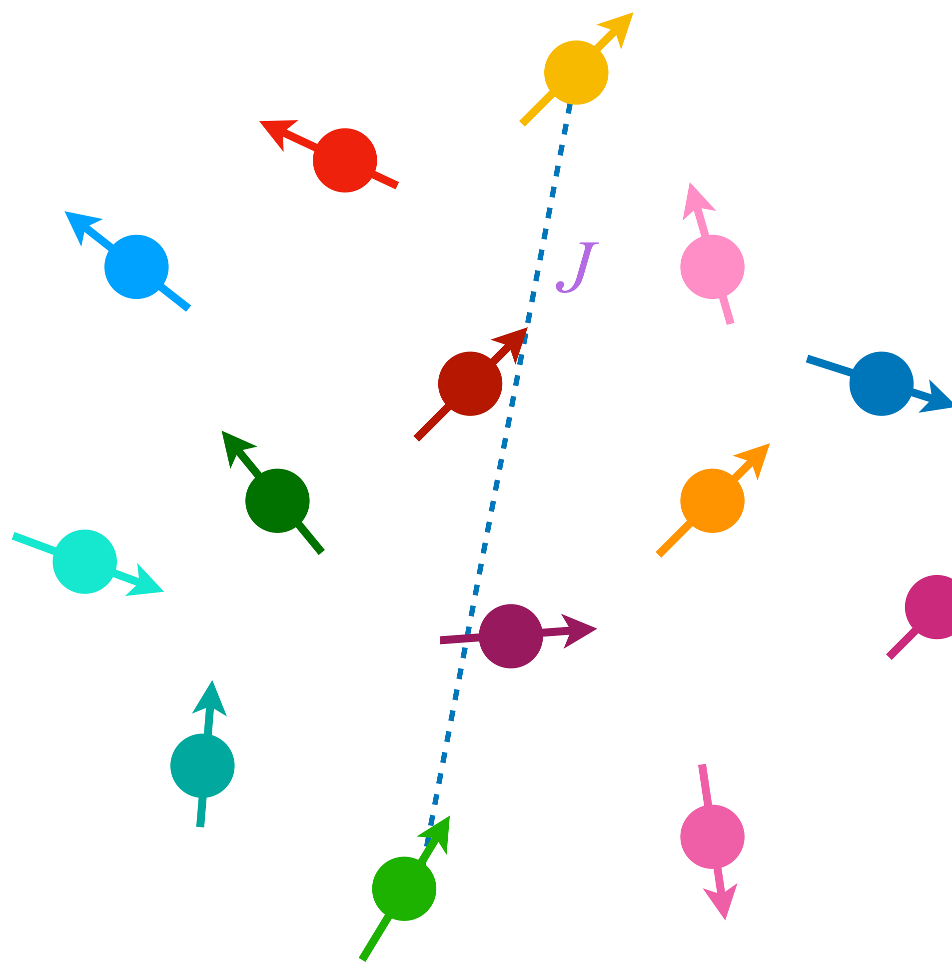
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The simple model

$$\mathcal{H} = \sum_{i<j=1}^{N} J_{ij} \left(X_i X_j + Y_i Y_j + Z_i Z_j \right)$$

J_{ij} are random numbers with zero mean, and variance $\sim J^2/N$

(Technical comment: the solvable model has states in the self-conjugate representation of SU(M), and we take the limit $N \to \infty$ followed by the limit $M \to \infty$)



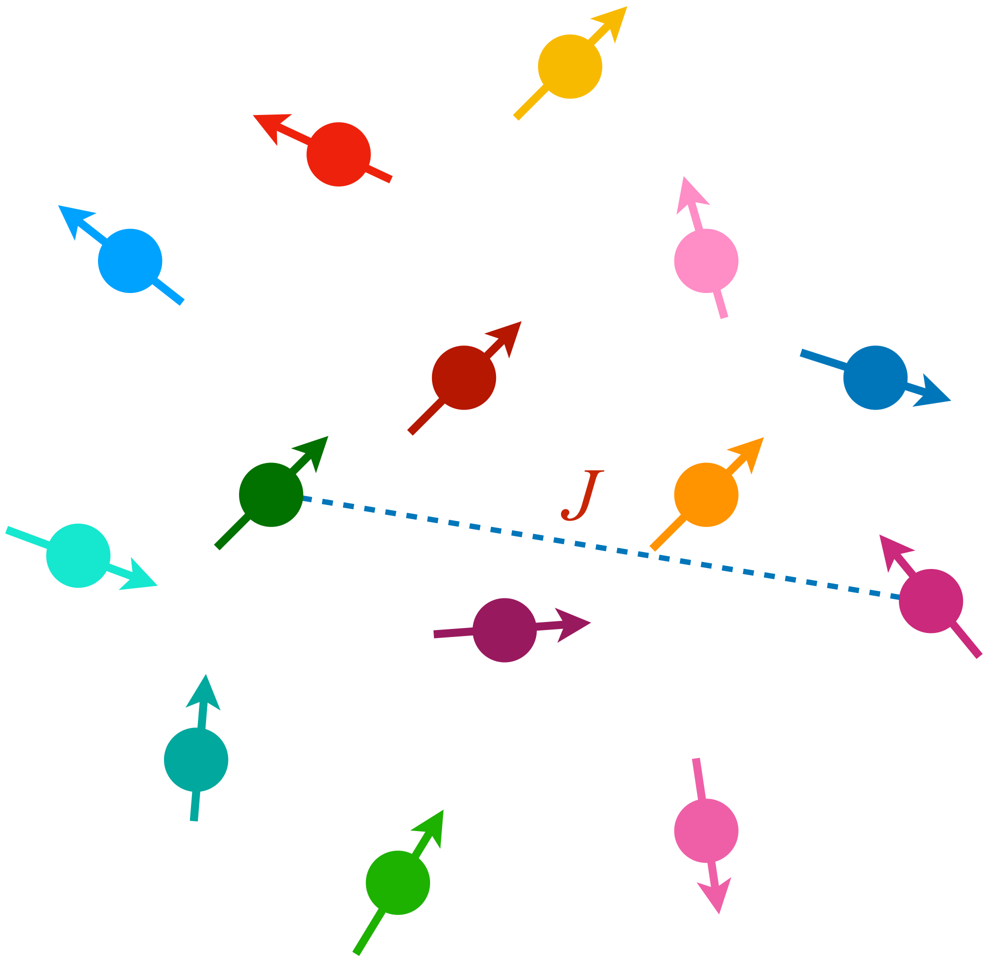
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The simple model

$$\mathcal{H} = \sum_{i<j=1}^{N} J_{ij}\, (X_i X_j + Y_i Y_j + Z_i Z_j)$$

J_{ij} are random numbers with zero mean, and variance $\sim J^2/N$

(Technical comment: the solvable model has states in the self-conjugate representation of SU(M), and we take the limit $N \to \infty$ followed by the limit $M \to \infty$)



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Main result I

For $k_B T \ll J$

$$
\begin{aligned}
\mathcal{Z} &= \mathrm{Tr}\exp\left(-\frac{\mathcal{H}}{k_B T}\right) \\
&= \exp\left(N\frac{S_0}{k_B}\right)\int \mathcal{D}f(\tau)\exp\left(-\frac{1}{\hbar}\mathcal{S}_{\rm 2D-gravity}\left[f(\tau)\right]\right)
\end{aligned}
$$

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

Main result I

For $k_B T \ll J$

$$\mathcal{Z} = \mathrm{Tr}\exp\left(-\frac{\mathcal{H}}{k_B T}\right)$$
$$= \exp\left(N\frac{S_0}{k_B}\right)\int \mathcal{D}f(\tau)\exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{2D}-\text{gravity}}\left[f(\tau)\right]\right)$$

S_0 is the $T \to 0$ entropy of the qubit model.
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

It maps on to the Bekenstein-Hawking
entropy of charged black holes

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

Main result I

For $k_B T \ll J$

$$\begin{aligned}\mathcal{Z} &= \mathrm{Tr} \exp\left(-\frac{\mathcal{H}}{k_B T}\right) \\ &= \exp\left(N\frac{S_0}{k_B}\right)\int \mathcal{D}f(\tau)\exp\left(-\frac{1}{\hbar}\mathcal{S}_{\rm 2D-gravity}\left[f(\tau)\right]\right)\end{aligned}$$

- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: τ on a circle of circumference $\hbar/(k_B T)$.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

Main result I

For $k_B T \ll J$

$$\begin{aligned}\mathcal{Z} &= \mathrm{Tr}\exp\left(-\frac{\mathcal{H}}{k_B T}\right)\\ &= \exp\left(N\frac{S_0}{k_B}\right)\int \mathcal{D}f(\tau)\exp\left(-\frac{1}{\hbar}\mathcal{S}_{\mathrm{2D-gravity}}\left[f(\tau)\right]\right)\end{aligned}$$

- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: τ on a circle of circumference $\hbar/(k_B T)$.
- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a 'boundary graviton'.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

Main result I

For $k_B T \ll J$

$$\begin{aligned} \mathcal{Z} &= \mathrm{Tr}\exp\left(-\frac{\mathcal{H}}{k_B T}\right) \\ &= \exp\left(N\frac{S_0}{k_B}\right)\int \mathcal{D}f(\tau)\exp\left(-\frac{1}{\hbar}\mathcal{S}_{\mathrm{2D-gravity}}\left[f(\tau)\right]\right) \end{aligned}$$

- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: τ on a circle of circumference $\hbar/(k_B T)$.
- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a 'boundary graviton'.
- The action of 2D-gravity, $\mathcal{S}_{\mathrm{2D-gravity}}$, is constrained by an emergent time reparameterization symmetry which is broken down to a conformal symmetry (SL(2,R)).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

Derivation of main result I

We write the spin operator $\vec{S}_i = (X_i, Y_i, Z_i)$ in terms of spin-1/2 fermions $\vec{S}_i = (1/2) c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$, where $\vec{\sigma}$ are the Pauli matrices. After introducing replicas $a = 1 \ldots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D} c_{ia}(\tau) \exp \Bigg[- \sum_{ia} \int_0^\beta d\tau \, c^\dagger_{ia} \left(\frac{\partial}{\partial \tau} - \mu \right) c_{ia} \\ - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c^\dagger_{ia}(\tau) c_{ib}(\tau') \right|^4 \Bigg] .$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D} G(\tau_1, \tau_2) \mathcal{D} \Sigma(\tau_1, \tau_2) \exp \Bigg[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \Bigg(G(\tau_2, \tau_1) \\ + \frac{1}{N} \sum_i c_i(\tau_2) c^\dagger_i(\tau_1) \Bigg) \Bigg] .$$

Derivation of main result I

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2)\exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2)\left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrizations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ is an arbitrary function.

Derivation of main result I

Reparametrization mode

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization symmetry by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4}G_s(f(\tau_1) - f(\tau_2))$$

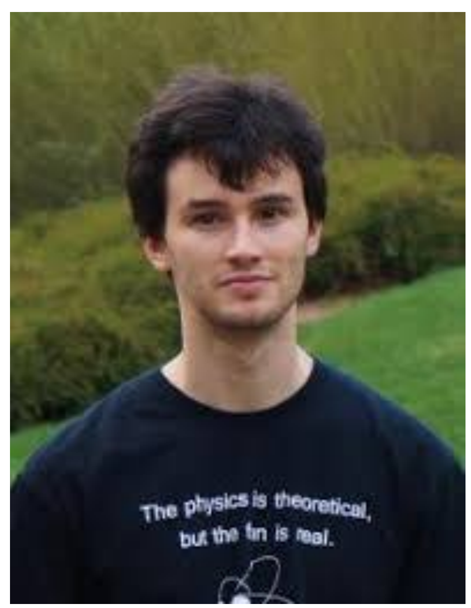
(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)e^{-N\mathcal{S}_{\rm 2D-gravity}[f(\tau)]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Maria Tikhanovskaya

Haoyu Guo



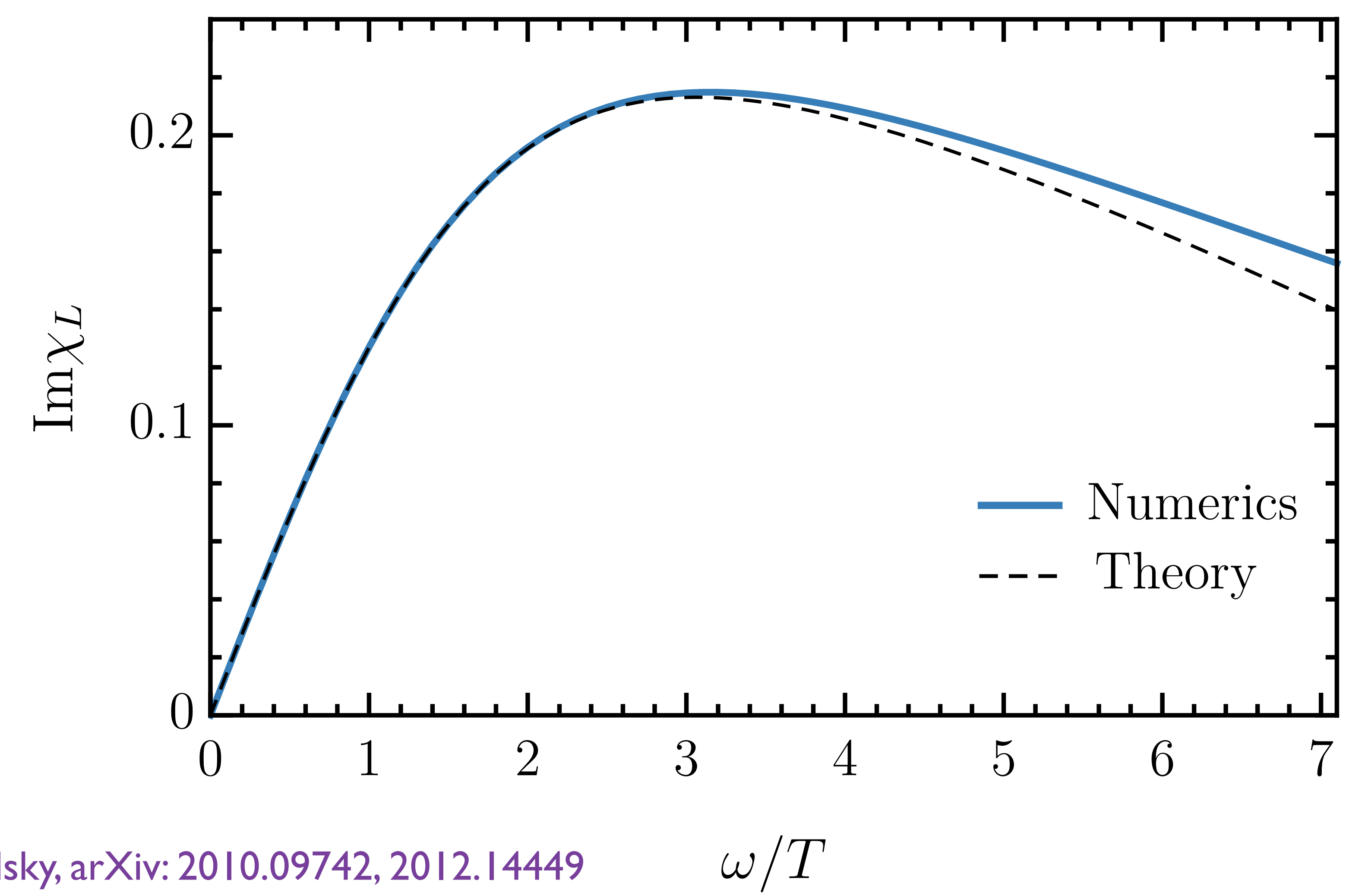
Grigory Tarnopolsky

arXiv:2010.09742
arXiv:2012.14449

Consequences of 2D-gravity for the dynamic spin susceptibility

$$\chi_L(\omega) = \sum_n \left|\langle 0| X_i |n\rangle\right|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right)\left[1 - \mathcal{C}\gamma\,\omega\tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \ldots\right]$$



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

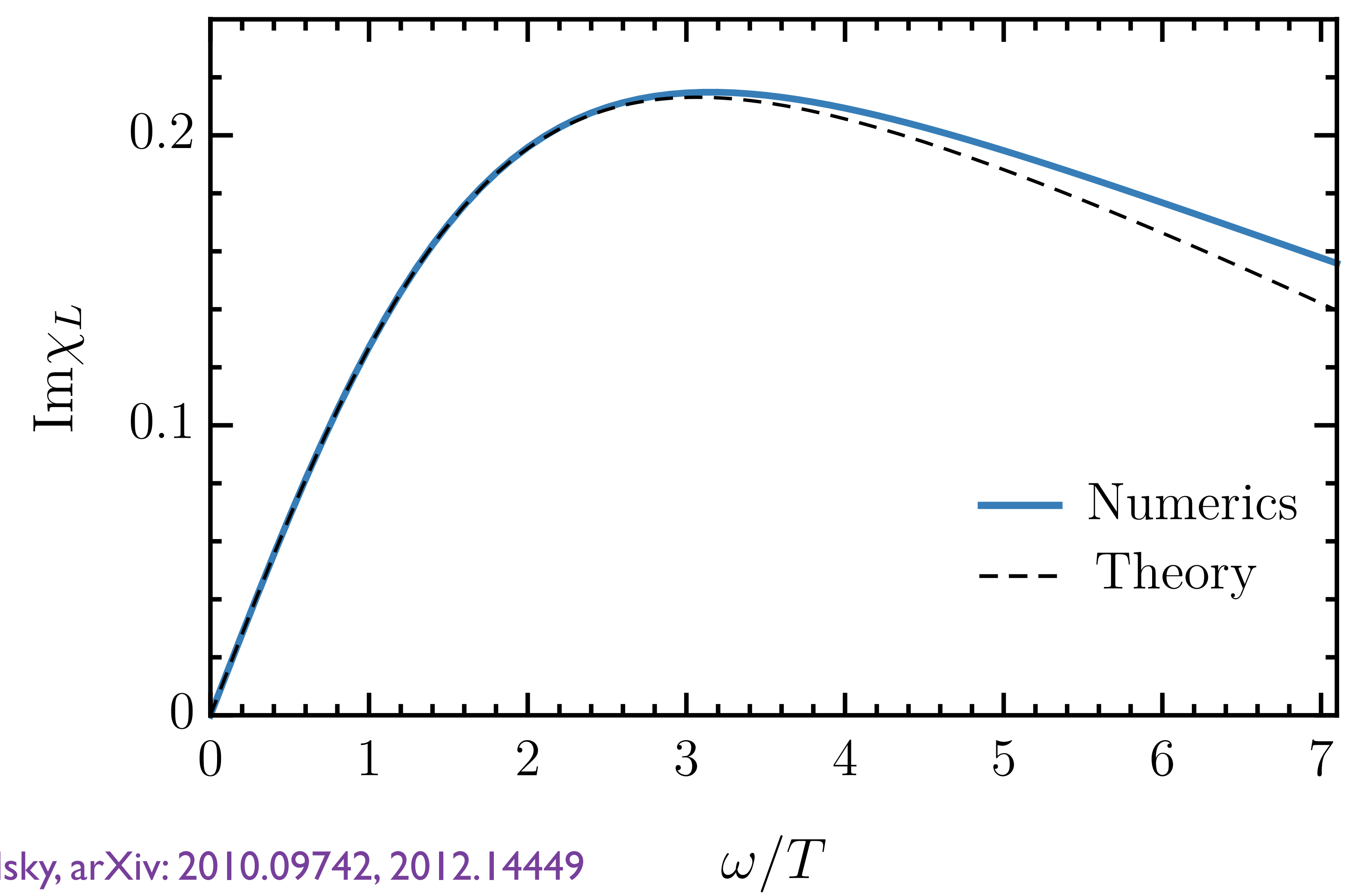
Consequences of 2D-gravity for the dynamic spin susceptibility

$$\chi_L(\omega) = \sum_n |\langle 0| X_i |n\rangle|^2 \, \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right)\left[1 - \mathcal{C}\gamma\,\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \ldots\right]$$

Conformally (SL(2,R)) invariant result with characteristic dissipative time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)



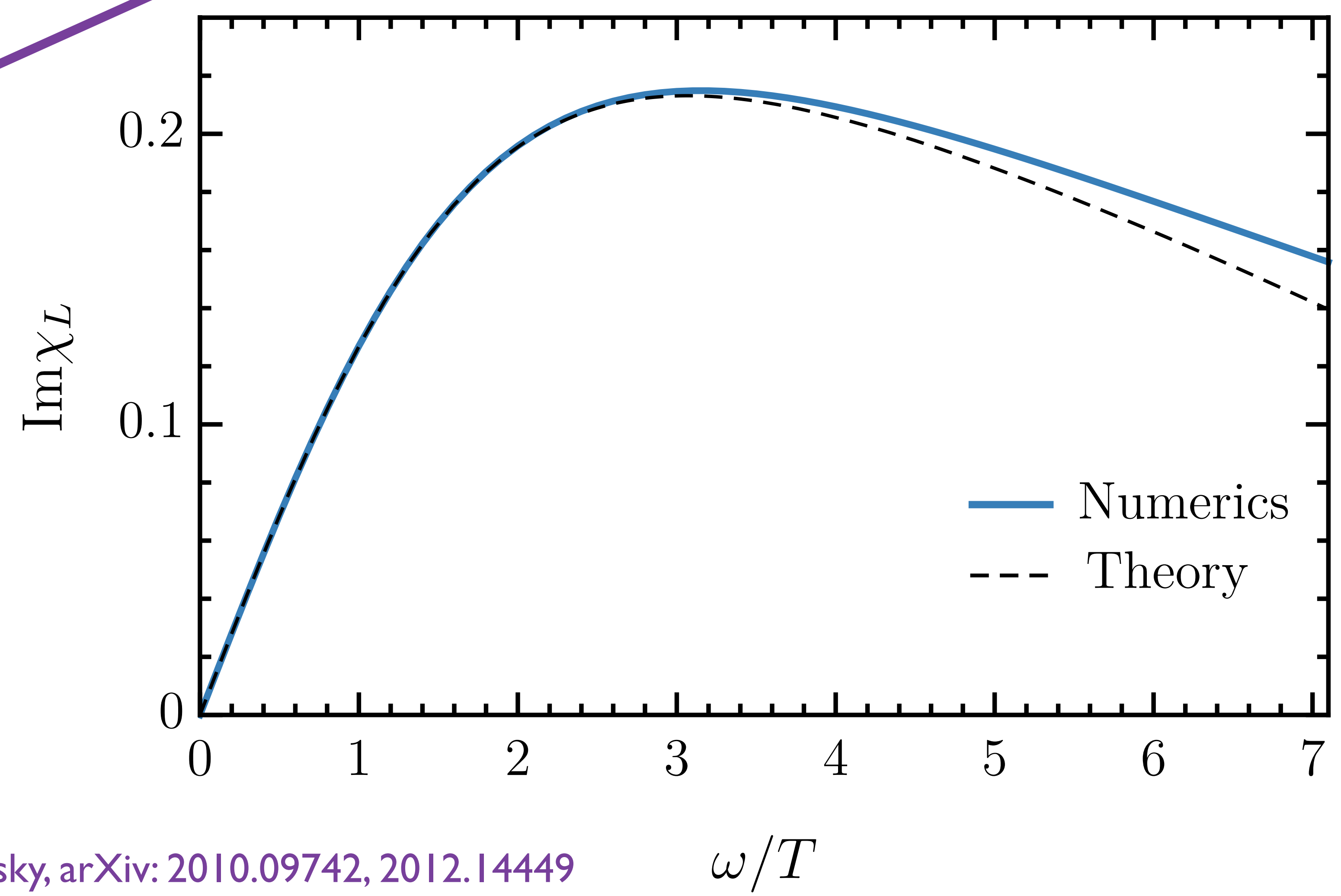
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Consequences of 2D-gravity for the dynamic spin susceptibility

$$\chi_L(\omega) = \sum_n |\langle 0| X_i |n\rangle|^2 \, \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right) \left[1 - \mathcal{C}\gamma\,\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \dots\right]$$

Correction from the boundary graviton



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Quantum entanglement

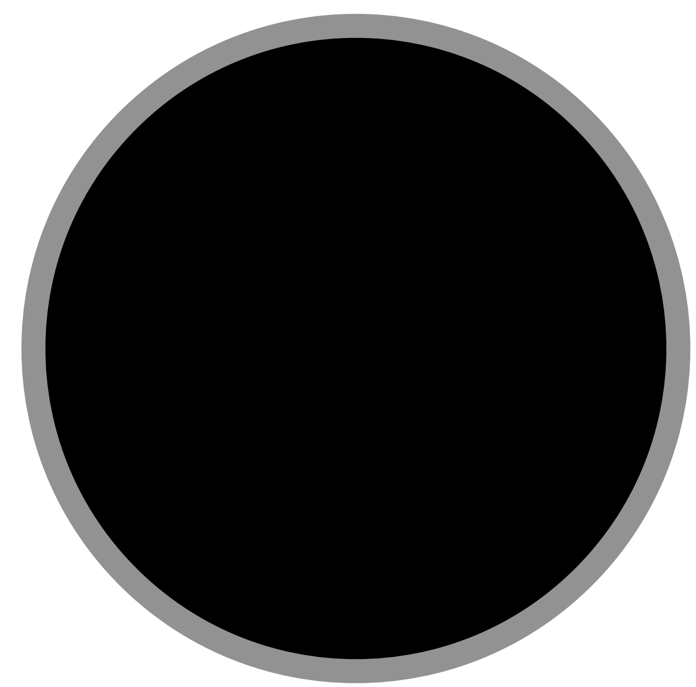
A simple qubit model

Black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

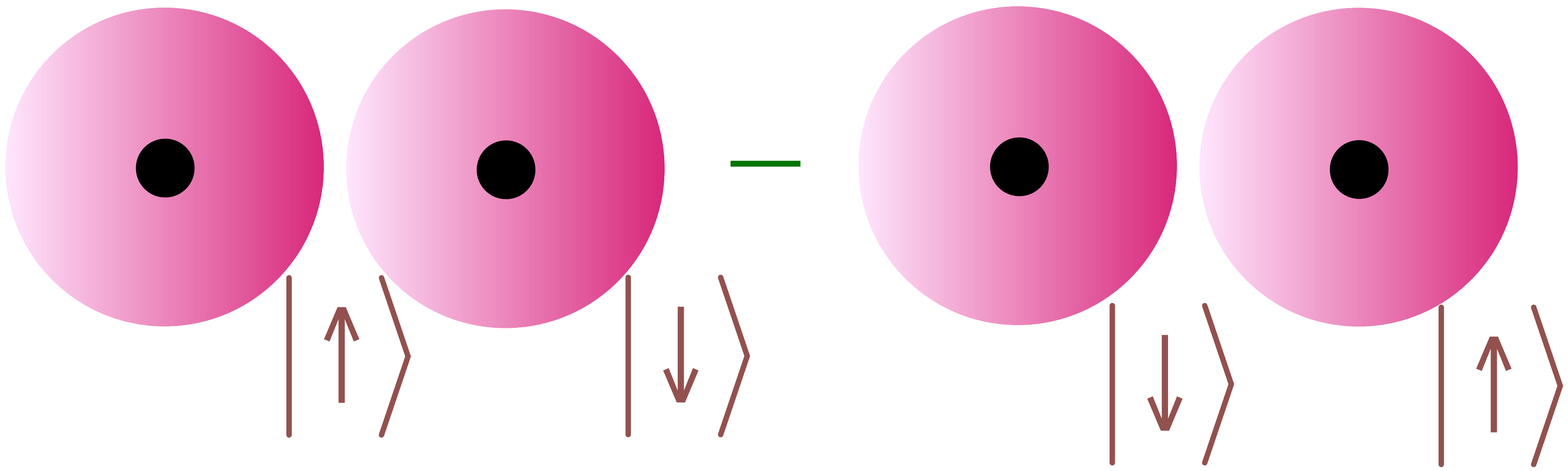
In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.



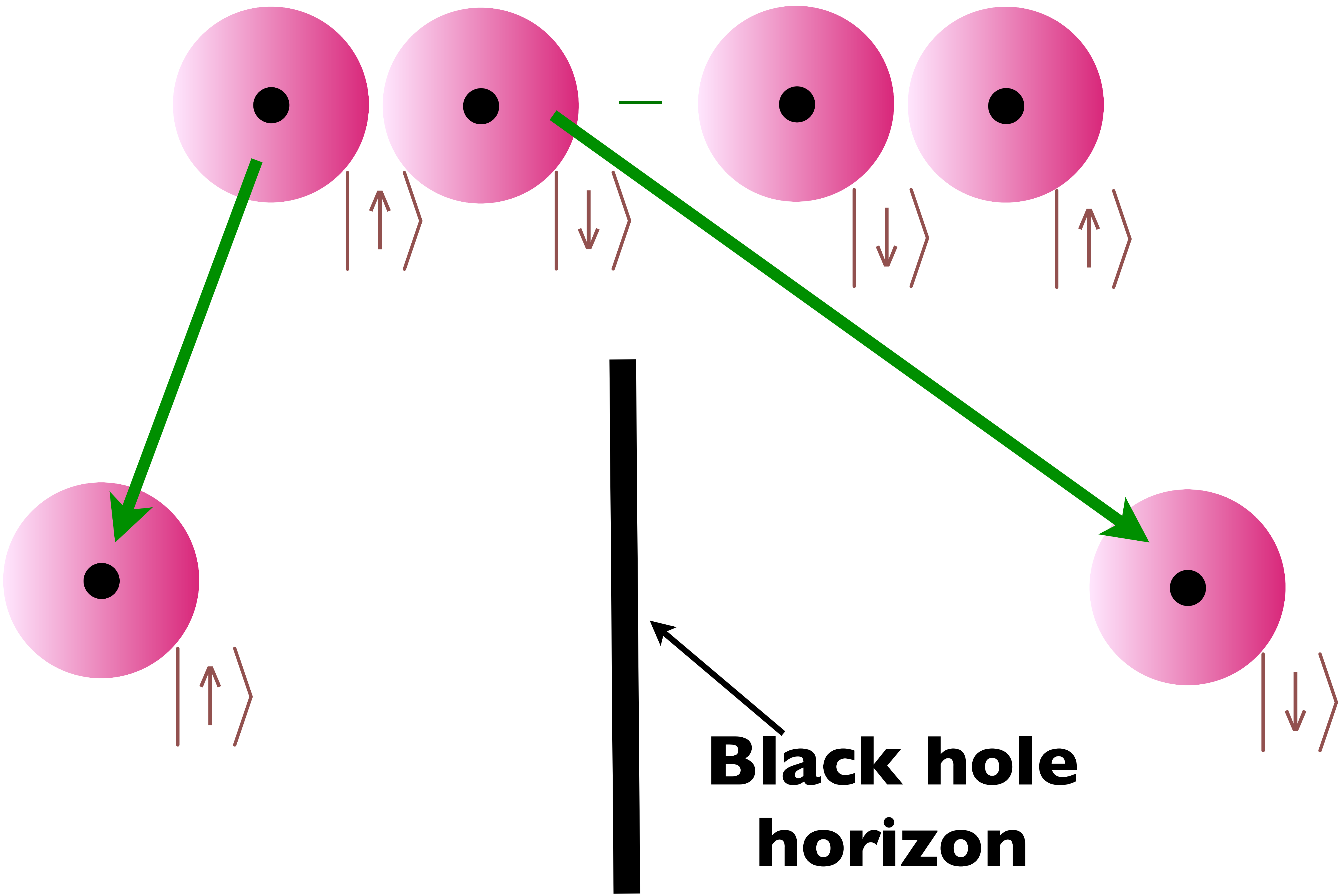
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

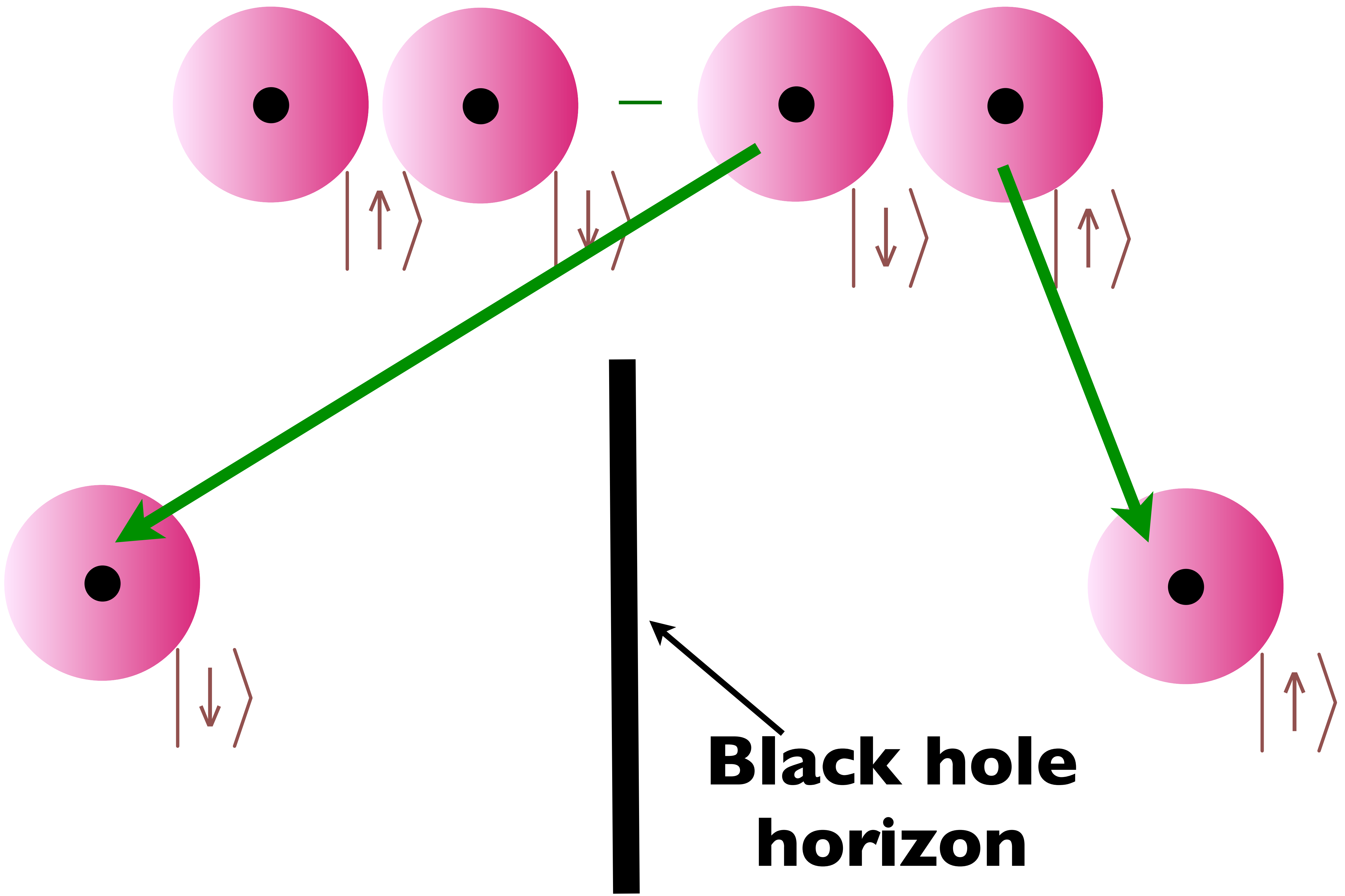
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

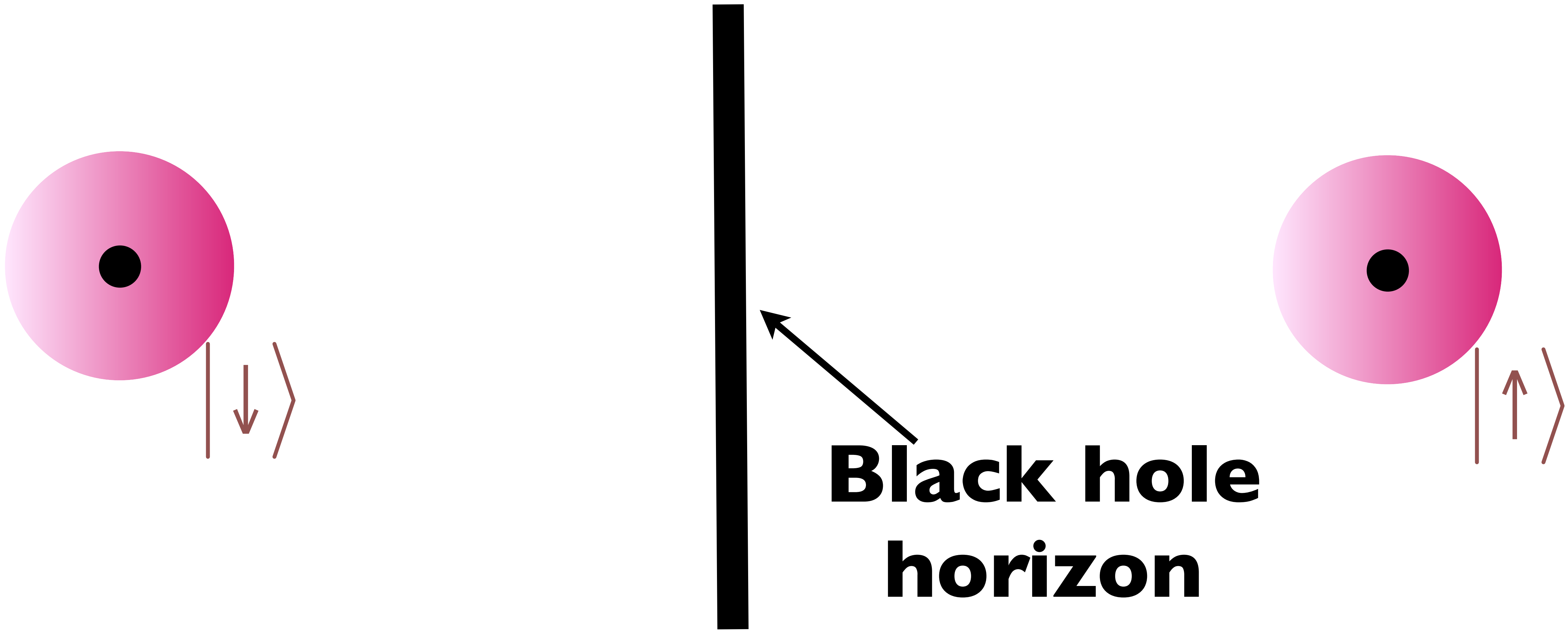


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

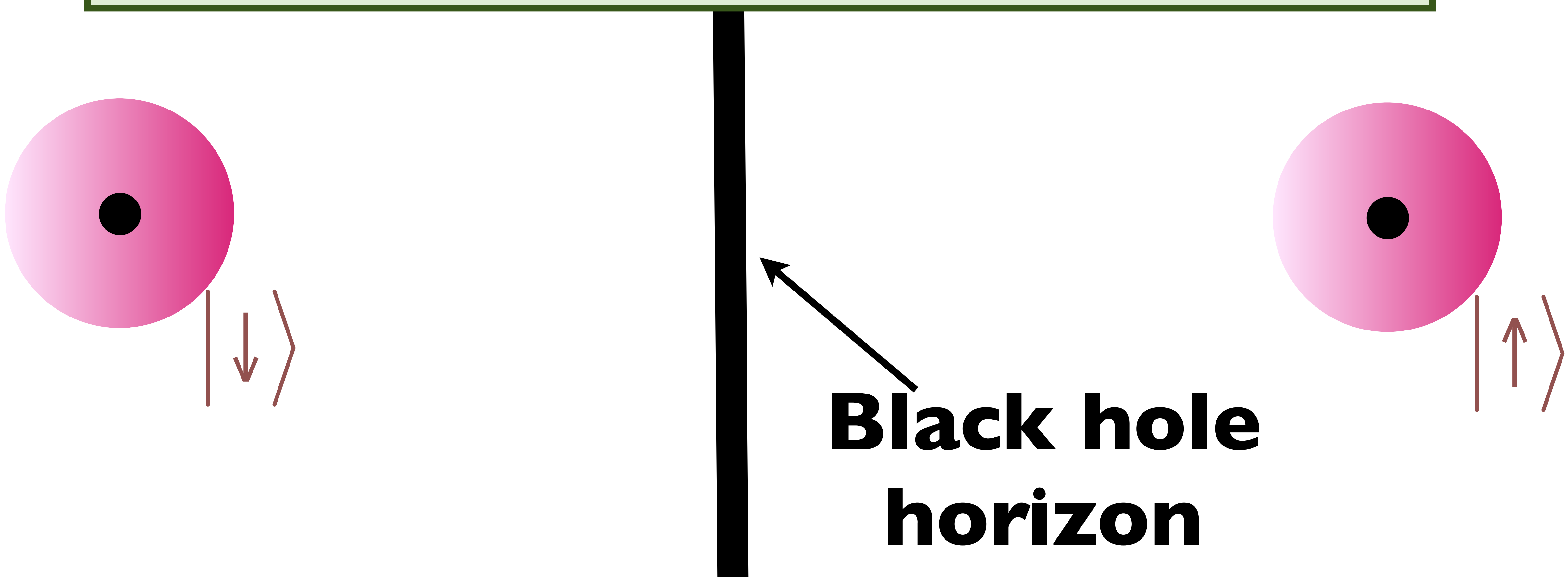
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

(because to an outside observer, the state of the electron inside the black hole is an unknown)



Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy, S_{BH} is proportional to their surface area.

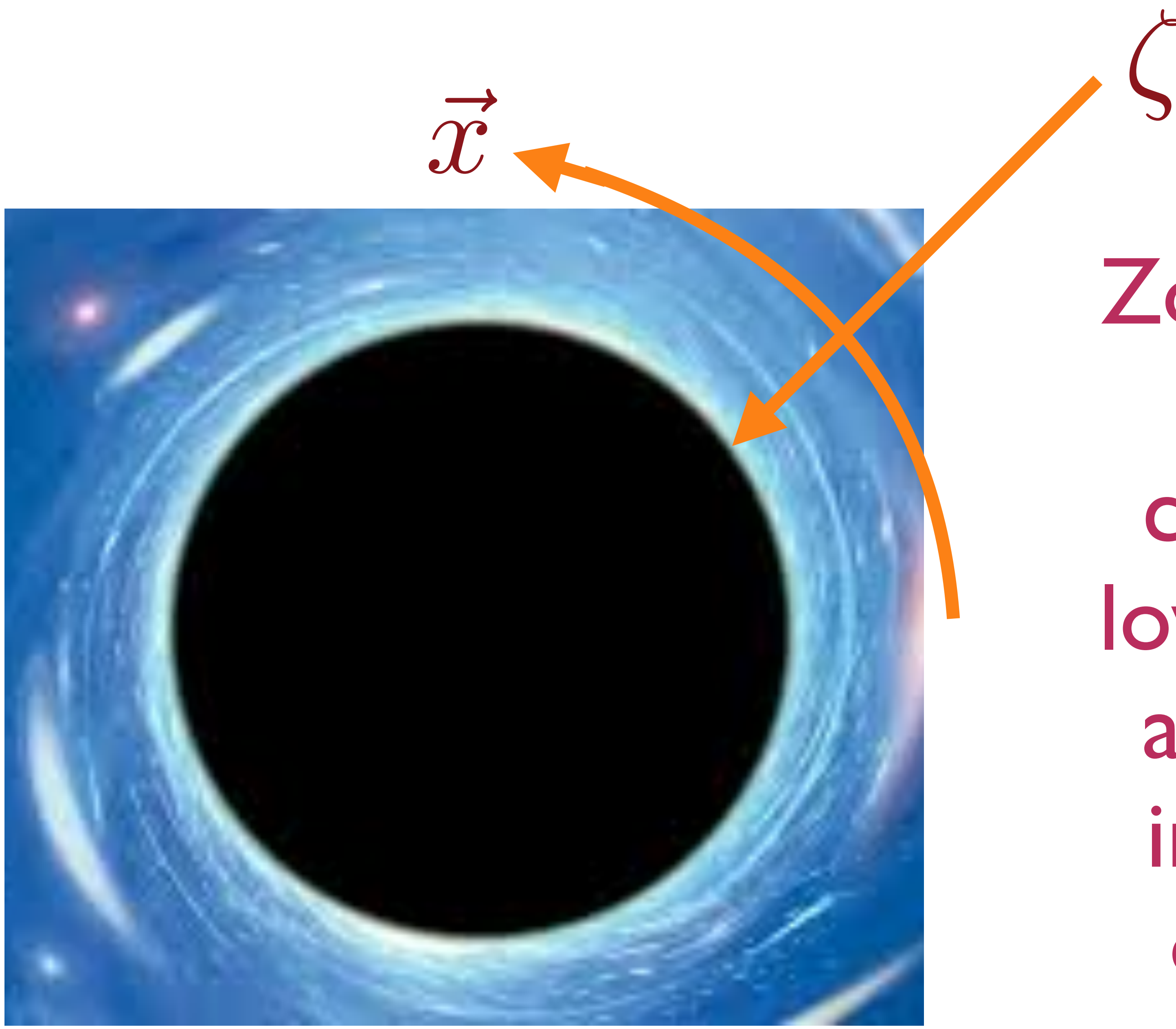
J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)

Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy, S_{BH} is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

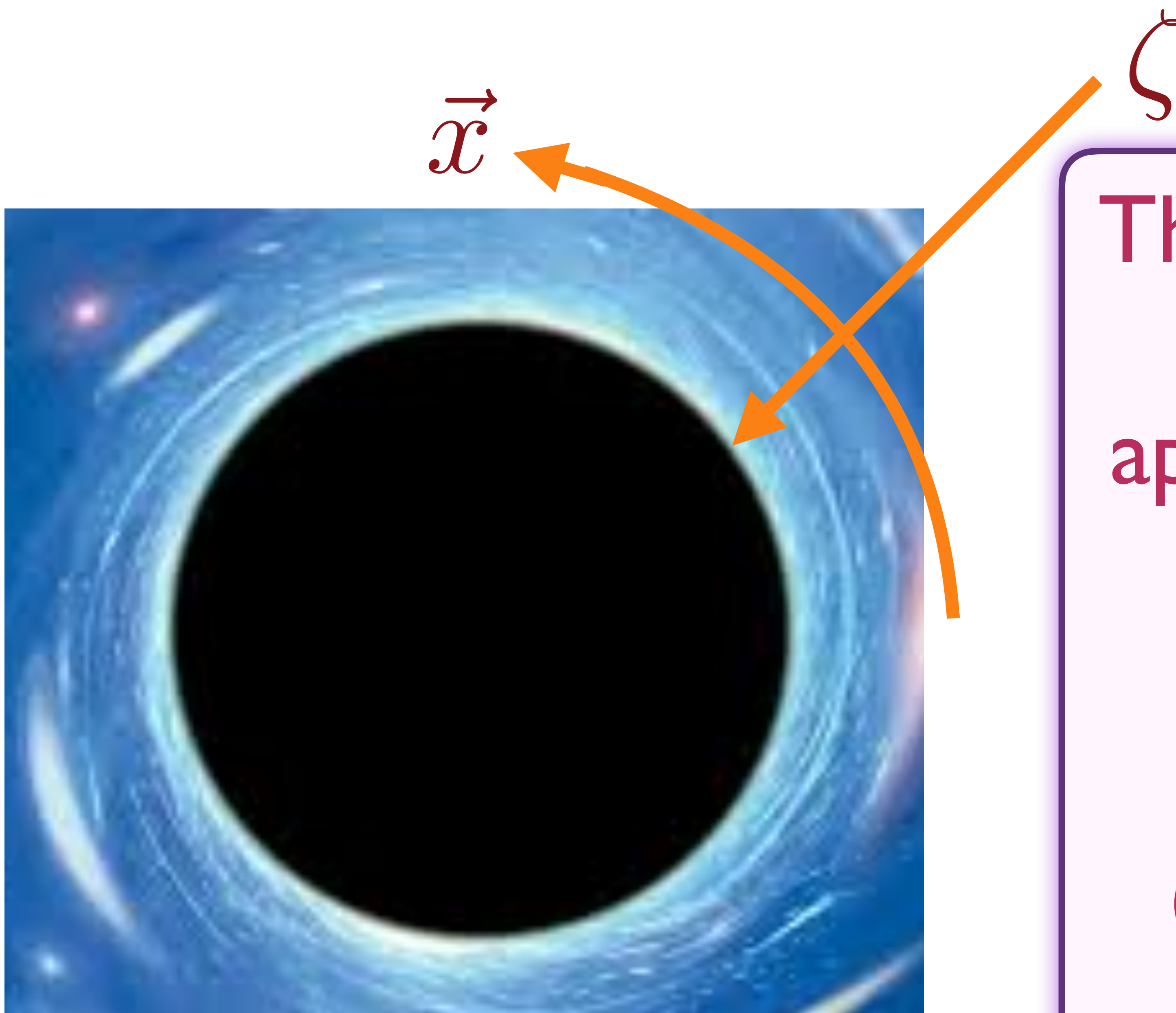
Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



This 2D-gravity theory is precisely that appearing in the low T limit of the Sachdev-Ye-Kitaev (SYK) models (including the qubit model)!

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

$$I_{EM} = \int d^{d+2}x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2} \right) + \frac{1}{4g_F^2} F^2 \right]$$

Metric $g_{\mu\nu}$

Ricci scalar in $d+2$ dimensions, $\mathcal{R}_{d+2}$

Cosmological constant $\Lambda = -d(d+1)/L^2$

U(1) gauge field A_μ

Electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Boundary conditions at spatial infinity:

Metric $\rightarrow$ AdS$_{d+2}$

Electric field $\rightarrow Q/(4\pi r^2)$

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

$$I_{EM} = \int d^{d+2}x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2}\right) + \frac{1}{4g_F^2}F^2\right]$$

Quantum gravity is 'defined' by the path integral

$$\mathcal{Z}_{\rm gravity} = \int \mathcal{D}g\mathcal{D}A \exp(-I_{EM}/\hbar)$$

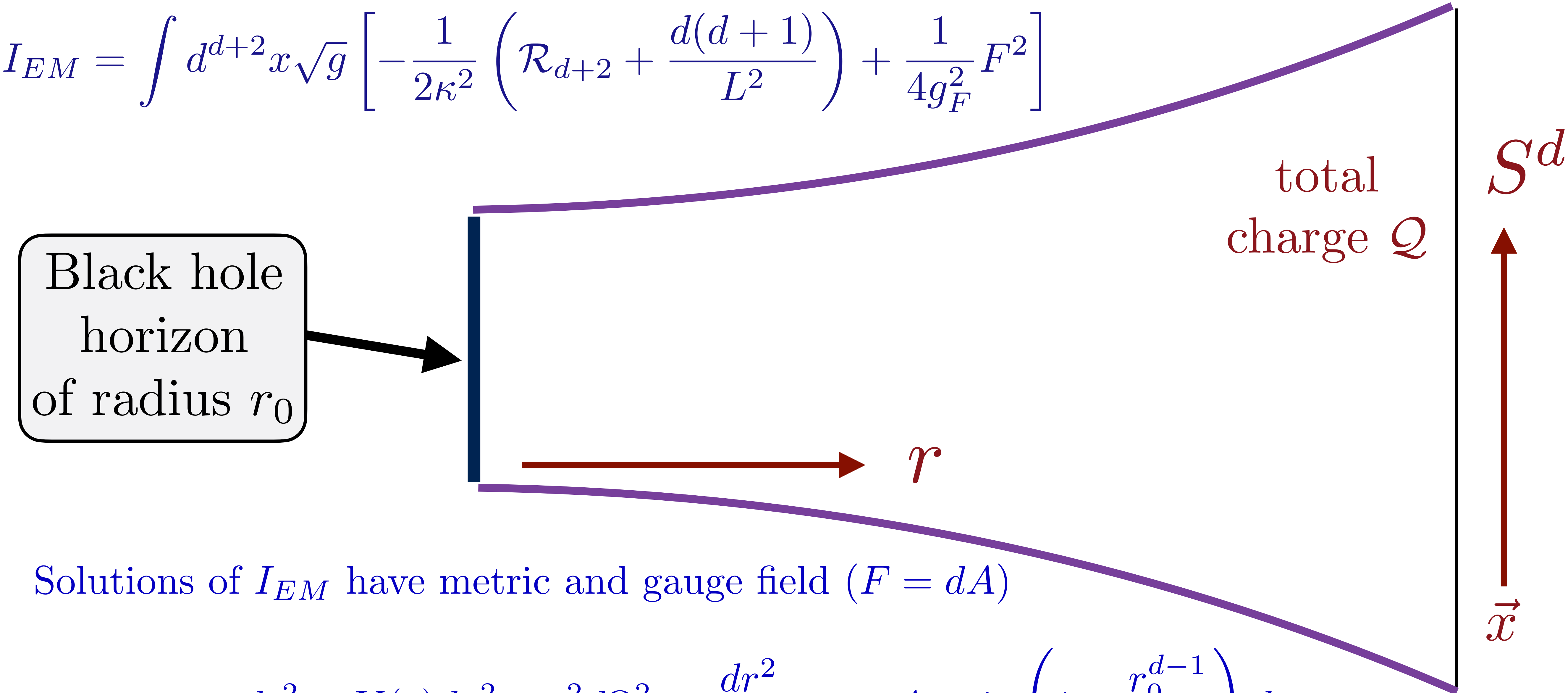
We will evaluate this integral exactly in a certain low temperature limit for charged black holes.

Charged black holes

$$I_{EM} = \int d^{d+2}x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2} \right) + \frac{1}{4g_F^2} F^2 \right]$$

Charged black holes

$$I_{EM} = \int d^{d+2}x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2}\right) + \frac{1}{4g_F^2}F^2\right]$$



Solutions of I_{EM} have metric and gauge field $(F = dA)$

$$ds^2 = V(r)d\tau^2 + r^2 d\Omega_d^2 + \frac{dr^2}{V(r)} \quad , \quad A = i\mu\left(1 - \frac{r_0^{d-1}}{r^{d-1}}\right)d\tau$$

$$V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}.$$

where $d\Omega_d^2$ is the metric of the d-sphere. All parameters of the solution are determined in terms of the chemical potential μ (related to the charge Q), and the Hawking temperature of horizon, T_H (related to the mass M).

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

Charged black holes

In the $T \to 0$ limit, at fixed μ, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of R_h

- In the near-horizon region, we change co-ordinates from r to ζ so that

$$r - R_h = \frac{R_2^2}{\zeta} \quad , \quad R_2 = \frac{L R_h}{\sqrt{d(d+1)R_h^2 + (d-1)^2 L^2}}.$$

Then the near-horizon metric becomes AdS$_2\times$ S$_d$, with

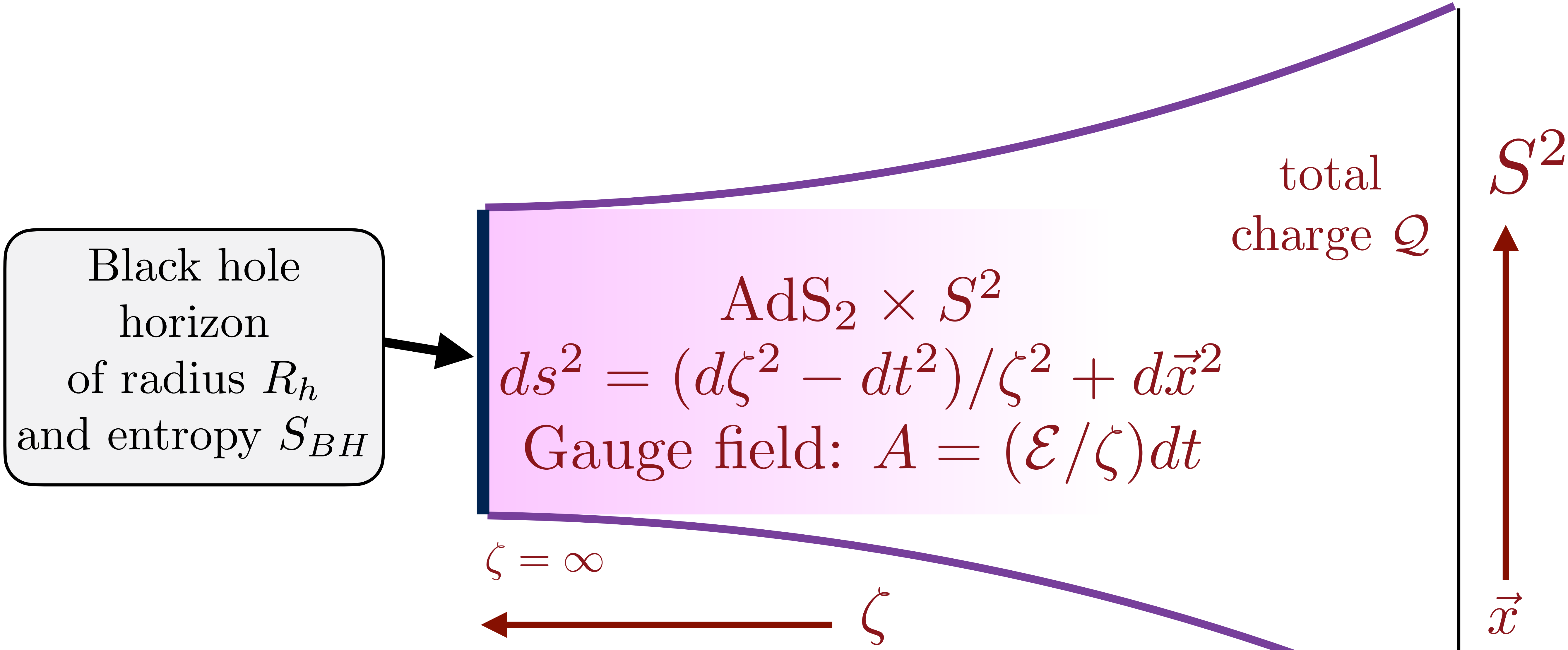
$$ds^2 = R_2^2 \left[\frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_d^2 \quad , \quad A = \frac{\mathcal{E}}{\zeta} dt \,.$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{2d\left[(d+1)R_h^2 + (d-1)L^2\right]}}{2\left[d(d+1)R_h^2 + (d-1)^2 L^2\right]}.$$

S. Sachdev, Journal of Mathematical Physics **60**, 052303 (2019)

Charged black holes



- The entropy S_{BH}, the charge $\mathcal{Q}$, and the dimensionless electric field $\mathcal{E}$ obey the same thermodynamic relation as the SYK model

$$\frac{dS_{BH}}{d\mathcal{Q}} = 2\pi\mathcal{E}$$

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

2D gravity and black holes

- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

Euclidean AdS_2

2D gravity and black holes

- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)
- The time reparameterization invariance of general relativity is broken down to SL(2,R) by the AdS_2 saddle point.

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

Euclidean AdS_2

A. Kitaev, 2015
J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

2D gravity and black holes

- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

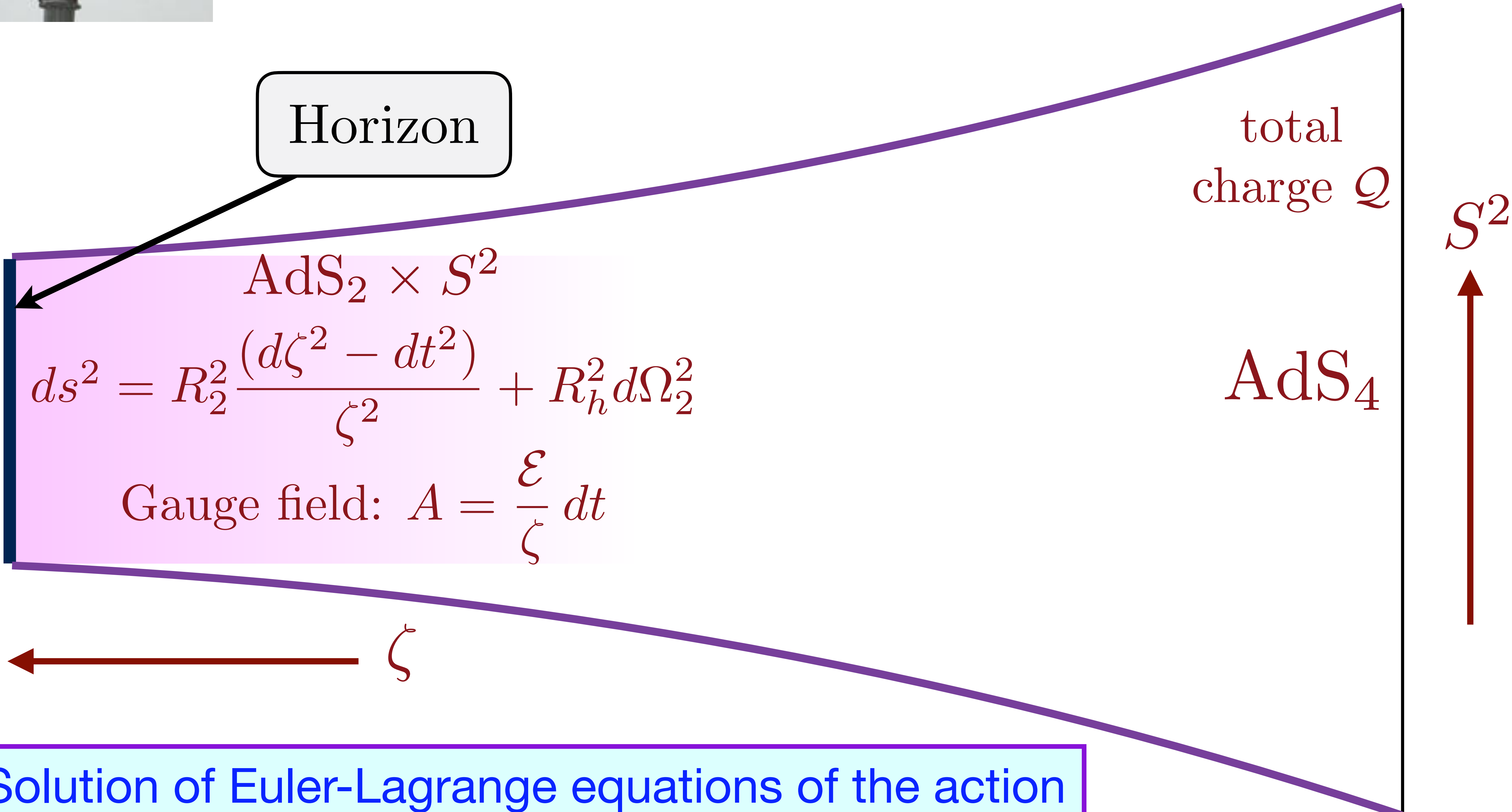
- The time reparameterization invariance of general relativity is broken down to SL(2,R) by the AdS_2 saddle point.

Low T quantum fluctuations about the Einstein-Maxwell theory of charged black holes in $d \geq 2$ spatial dimensions leads to the same 2D gravity theory as the SYK models.

Euclidean AdS_2

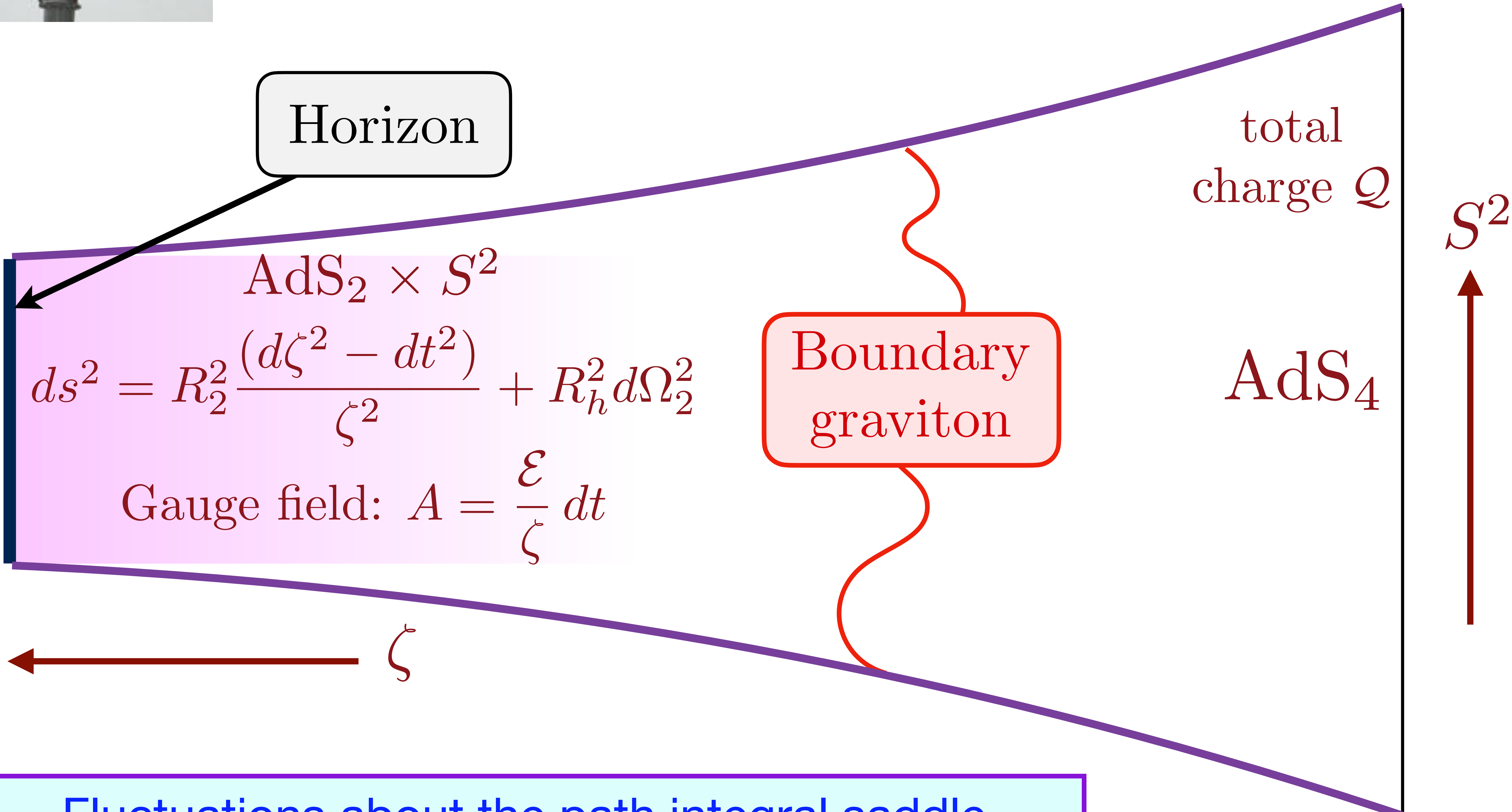
P. Nayak, A. Shukla, R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547
U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239
P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746
S. Sachdev, Journal of Mathematical Physics **60**, 052303 (2019)

SYK model and charged black holes



Solution of Euler-Lagrange equations of the action of Einstein gravity and Maxwell electromagnetism

SYK model and charged black holes



Fluctuations about the path integral saddle

Main result II

For $T \ll 1/R_h$

$$\mathcal{Z}_{\text{charged black hole in EM theory}} = \exp\left(\frac{S_{BH}}{k_B}\right)\int \mathcal{D}f(\tau)\exp\left(-\frac{1}{\hbar}\mathcal{S}_{2\text{D}-\text{gravity}}\left[f(\tau)\right]\right)$$

Main result II

For $T \ll 1/R_h$

$$\mathcal{Z}_{\text{charged black hole in EM theory}} = \exp\left(\frac{S_{BH}}{k_B}\right) \int \mathcal{D}f(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{2D-gravity}}\left[f(\tau)\right]\right)$$

$$\mathcal{S}_{\text{2D-gravity}}\left[f(\tau)\right] = -\frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, and we have used the *Schwarzian*:

$$\{g, \tau\} \equiv \frac{d^3g/d\tau^3}{dg/d\tau} - \frac{3}{2}\left(\frac{d^2g/d\tau^2}{dg/d\tau}\right)^2.$$

The defining property of the Schwarzian is its invariance under SL(2,R) transformations

$$\left\{\frac{ag(\tau)+b}{cg(\tau)+d}, \tau\right\} = \{g(\tau), \tau\}$$

Remarkably, this path integral can be evaluated exactly, using the Duistermaat–Heckman formula (Stanford, Witten, arXiv:1703.04612).

Derivation of main result II

We write the $(d+2)$-dimensional metric g of I_{EM} in terms of a two-dimensional metric h and a scalar field Φ:

$$ds^2 = \frac{ds_2^2}{\Phi^{d-1}} + \Phi^2 \, d\Omega_d^2 \,.$$

The Einstein-Maxwell and Gibbons-Hawking actions reduce to and extension of Jackiw-Tietelbaum gravity $(x \equiv (\tau, \zeta))$

$$\begin{aligned} I_{EM} &= \int d^2 x \sqrt{h} \left[-\frac{s_d}{2\kappa^2} \, \Phi^d \, \mathcal{R}_2 + U(\Phi) + \frac{Z(\Phi)}{4 g_F^2} F^2 \right] \\ I_{GH} &= -\frac{s_d}{\kappa^2} \int_{\partial} dx \sqrt{h_b} \Phi^d \, \mathcal{K}_1 \end{aligned}$$

The explicit forms of the potentials $U(\Phi)$ and $Z(\Phi)$ are,

$$U(\Phi) = -\frac{s_d}{2\kappa^2} \left(\frac{d(d-1)}{\Phi} + \frac{d(d+1)\Phi}{L^2} \right) \quad , \quad Z(\Phi) = s_d \Phi^{2d-1} \,.$$

S. Sachdev, Journal of Mathematical Physics **60**, 052303 (2019)

Derivation of main result II

The exact saddle point of Φ relates to R_h the horizon radius at $T = 0$

$$\Phi(\zeta) = R_h + \frac{R_2^2}{\zeta} \quad , \quad R_h \equiv \frac{L}{g_F} \left[\frac{(d-1)(\mu_0^2 \kappa^2 (d-1) - d g_F^2)}{d(d+1)} \right]^{1/2} ,$$

while the near-horizon, low $T \ll 1/R_h$ metric is AdS_2

$$ds_2^2 = \frac{R_2^2 R_h^{d-1}}{\zeta^2} \left[(1 - 4\pi^2 T^2 \zeta^2) d\tau^2 + \frac{d\zeta^2}{1 - 4\pi^2 T^2 \zeta^2} \right] ,$$

where

$$R_2 = \frac{L R_h}{\sqrt{d(d+1) R_h^2 + (d-1)^2 L^2}}$$

The field coupling to $\mathcal{R}_2$ is Φ^d

$$[\Phi(\zeta)]^d = R_h^d + \frac{\Phi_1}{\zeta} + \ldots \quad , \quad \Phi_1 = d R_h^{d-1} R_2^2,$$

S. Sachdev, Journal of Mathematical Physics **60**, 052303 (2019)

Derivation of main result II

The field coupling to $\mathcal{R}_2$ is Φ^d

$$[\Phi(\zeta)]^d = R_h^d + \frac{\Phi_1}{\zeta} + \ldots \quad , \quad \Phi_1 = d R_h^{d-1} R_2^2,$$

We choose the boundary of the AdS_2 region at bulk co-ordinates $(f(\tau), \zeta(\tau))$ with the induced boundary metric fixed at $(R_2^2 R_h^{d-1}/\zeta_b^2) d\tau^2$ by choosing

$$\zeta(\tau) = \zeta_b f'(\tau) + \zeta_b^3 \left(\frac{[f''(\tau)]^2}{2 f'(\tau)} - 2\pi^2 T^2 [f'(\tau)]^3 \right) + \ldots .$$

Finally, we evaluate I_{GH} along this boundary curve

$$I_1[f] = -\frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \, \{\tan(\pi T f(\tau)), \tau\} \, ,$$

where

$$\gamma = \frac{4\pi^2 s_d \Phi_1}{\kappa^2} \, ,$$

matches the linear-in-T co-efficient of the specific heat of the full Reissner-Nördstorm solution in $d+2$ dimensions.

S. Sachdev, Journal of Mathematical Physics **60**, 052303 (2019)

Main result II

For $T \ll 1/R_h$

$$\mathcal{Z}_{\text{charged black hole in EM theory}} = \exp\left(\frac{S_{BH}}{k_B}\right) \int \mathcal{D}f(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{2D}-\text{gravity}}\left[f(\tau)\right]\right)$$

$$\mathcal{S}_{\text{2D}-\text{gravity}}\left[f(\tau)\right] = -\frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, and we have used the *Schwarzian*:

$$\{g, \tau\} \equiv \frac{d^3 g/d\tau^3}{dg/d\tau} - \frac{3}{2}\left(\frac{d^2 g/d\tau^2}{dg/d\tau}\right)^2.$$

The defining property of the Schwarzian is its invariance under SL(2,R) transformations

$$\left\{\frac{ag(\tau)+b}{cg(\tau)+d}, \tau\right\} = \{g(\tau), \tau\}$$

Remarkably, this path integral can be evaluated exactly, using the Duistermaat–Heckman formula (Stanford, Witten, arXiv:1703.04612).

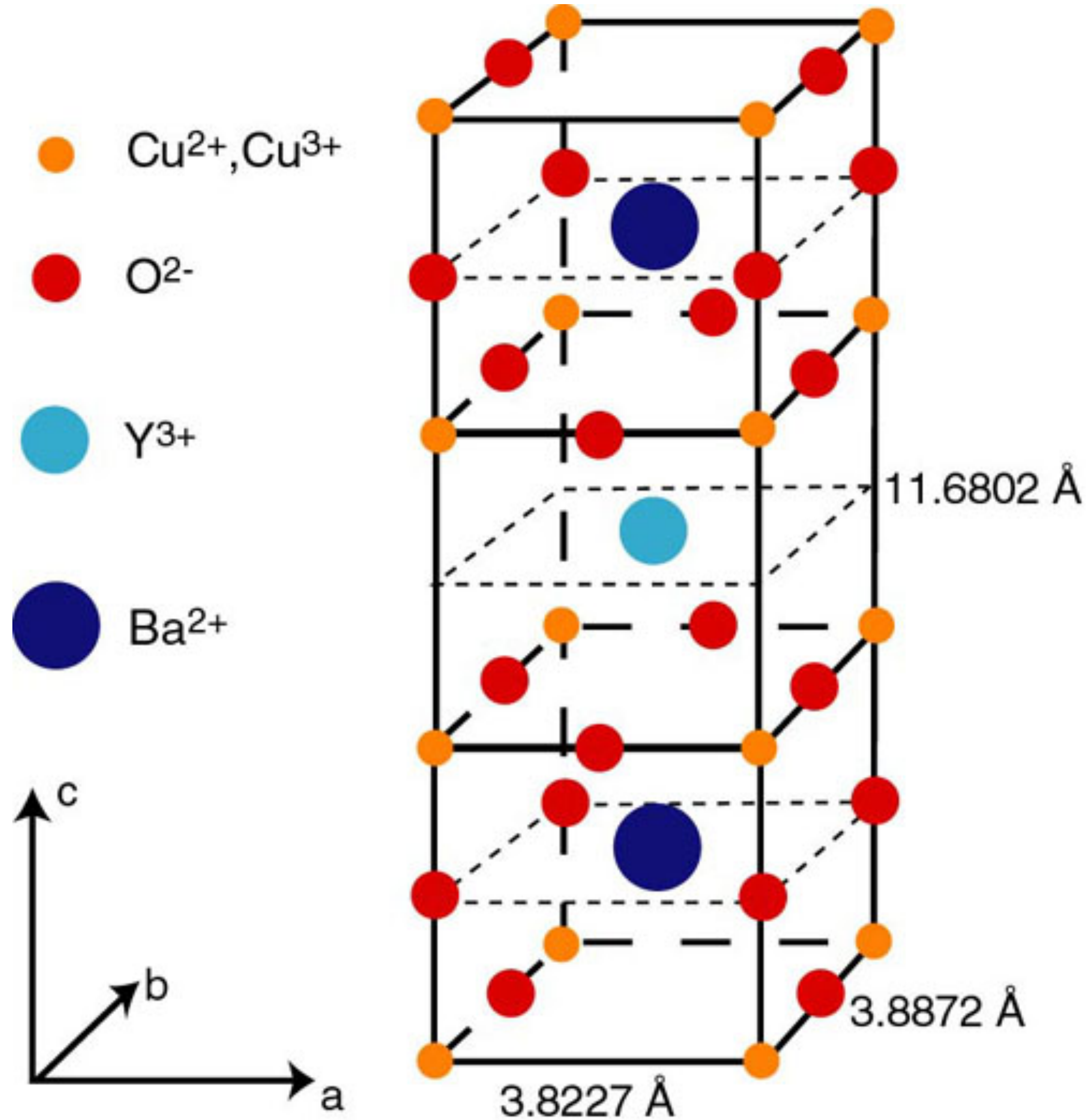
Quantum entanglement

A simple qubit model

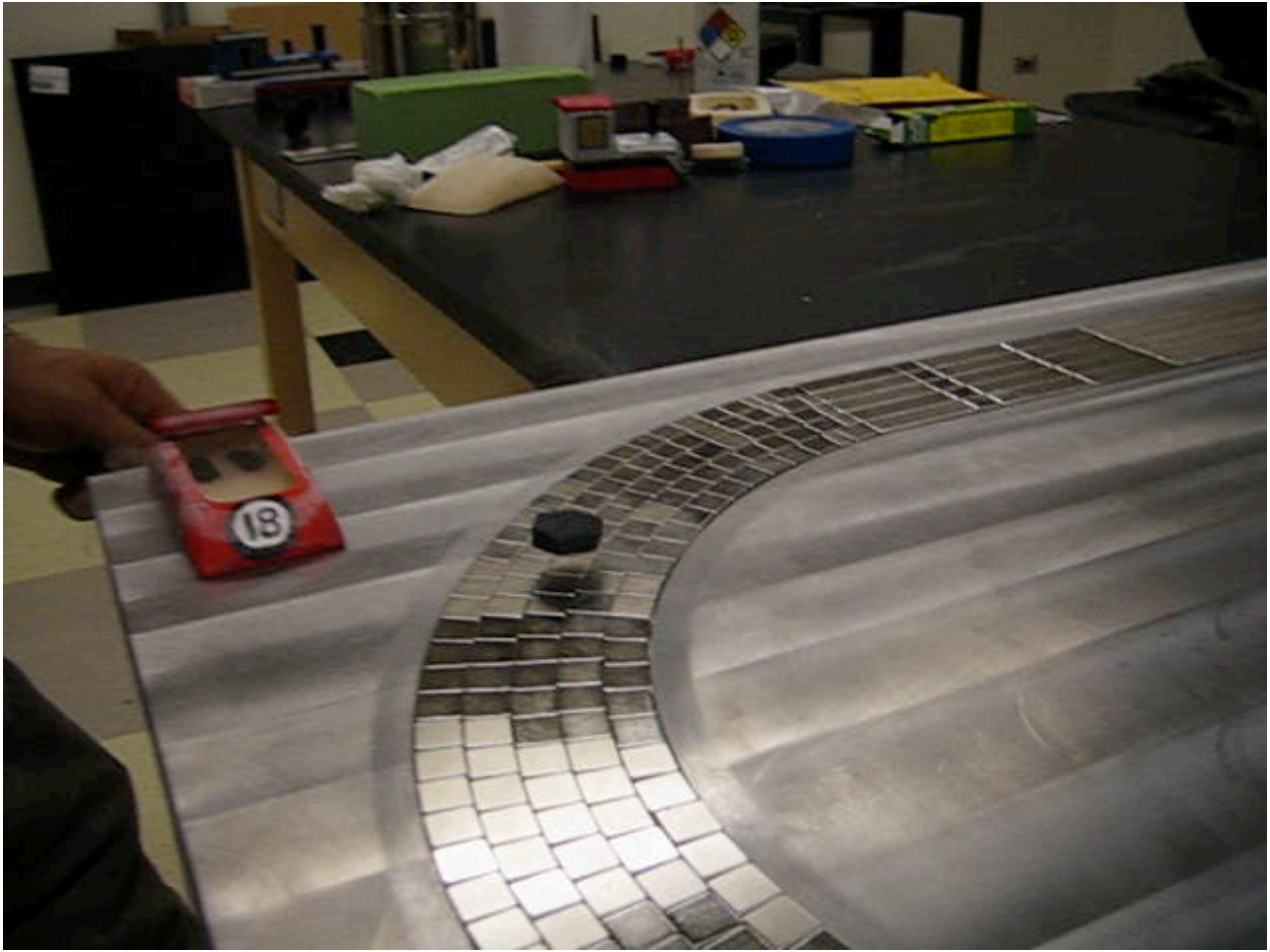
Black holes

Copper-based superconductors

Cuprate superconductors



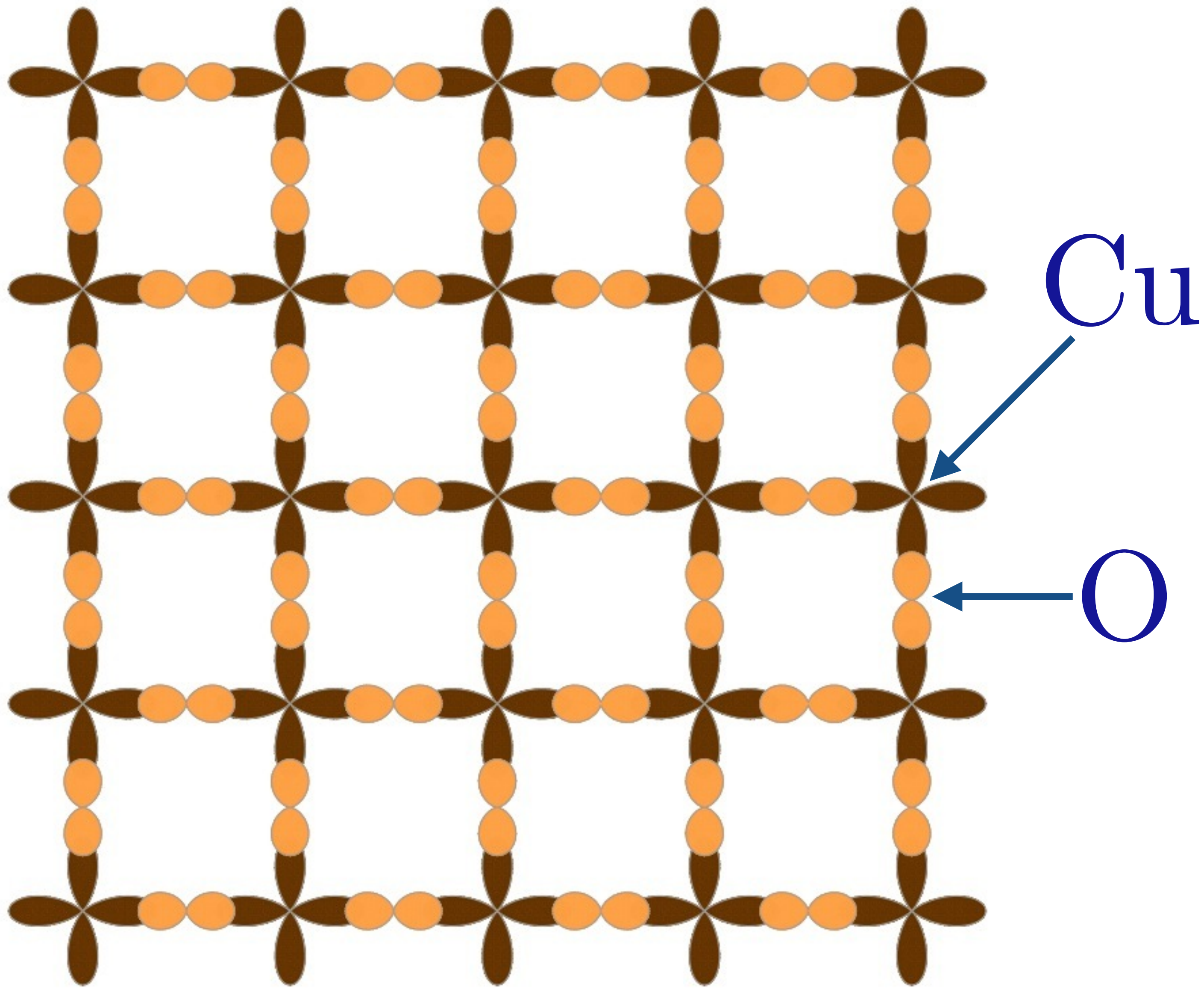
$YBa_2Cu_3O_{6+x}$



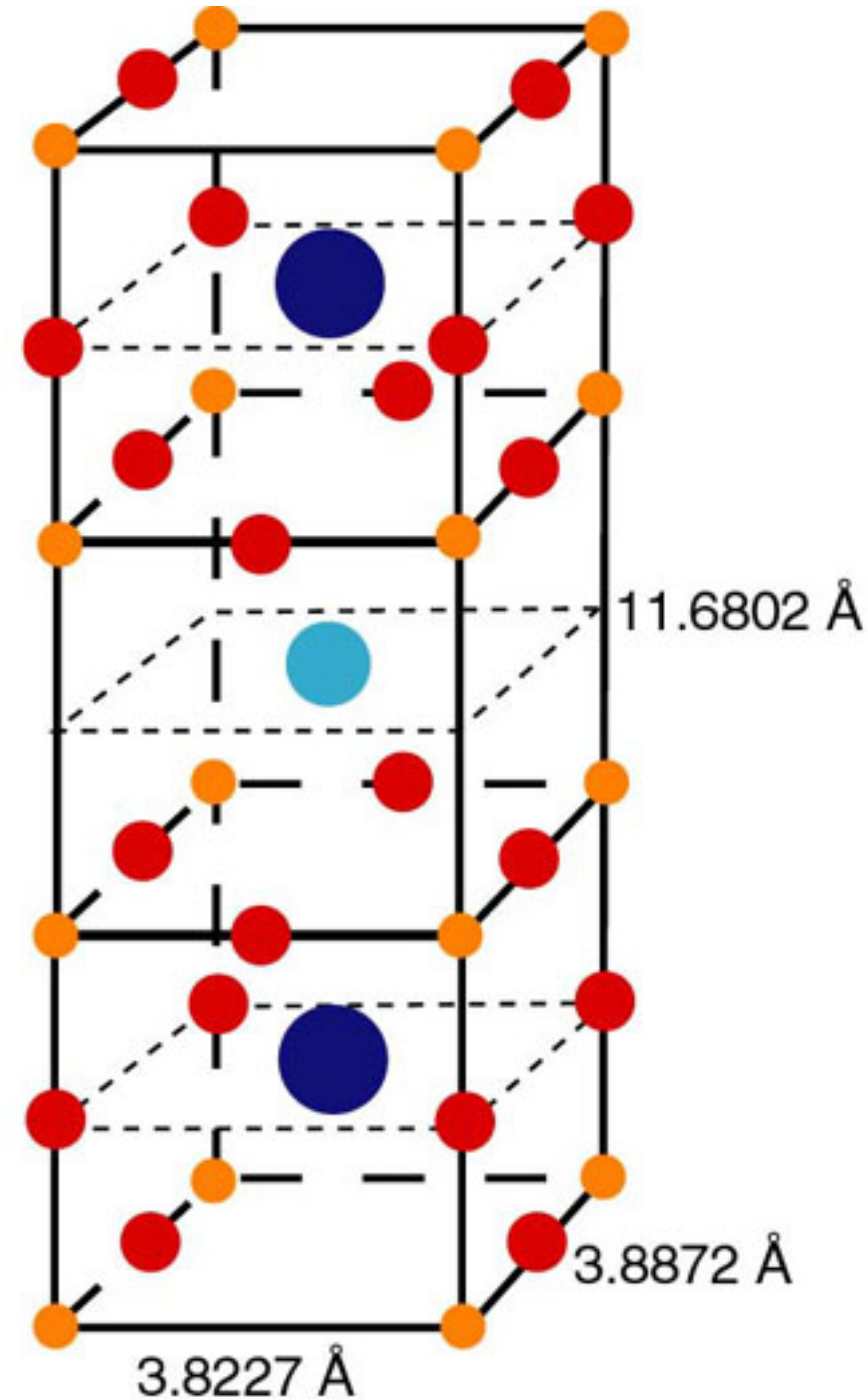
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

High temperature superconductors

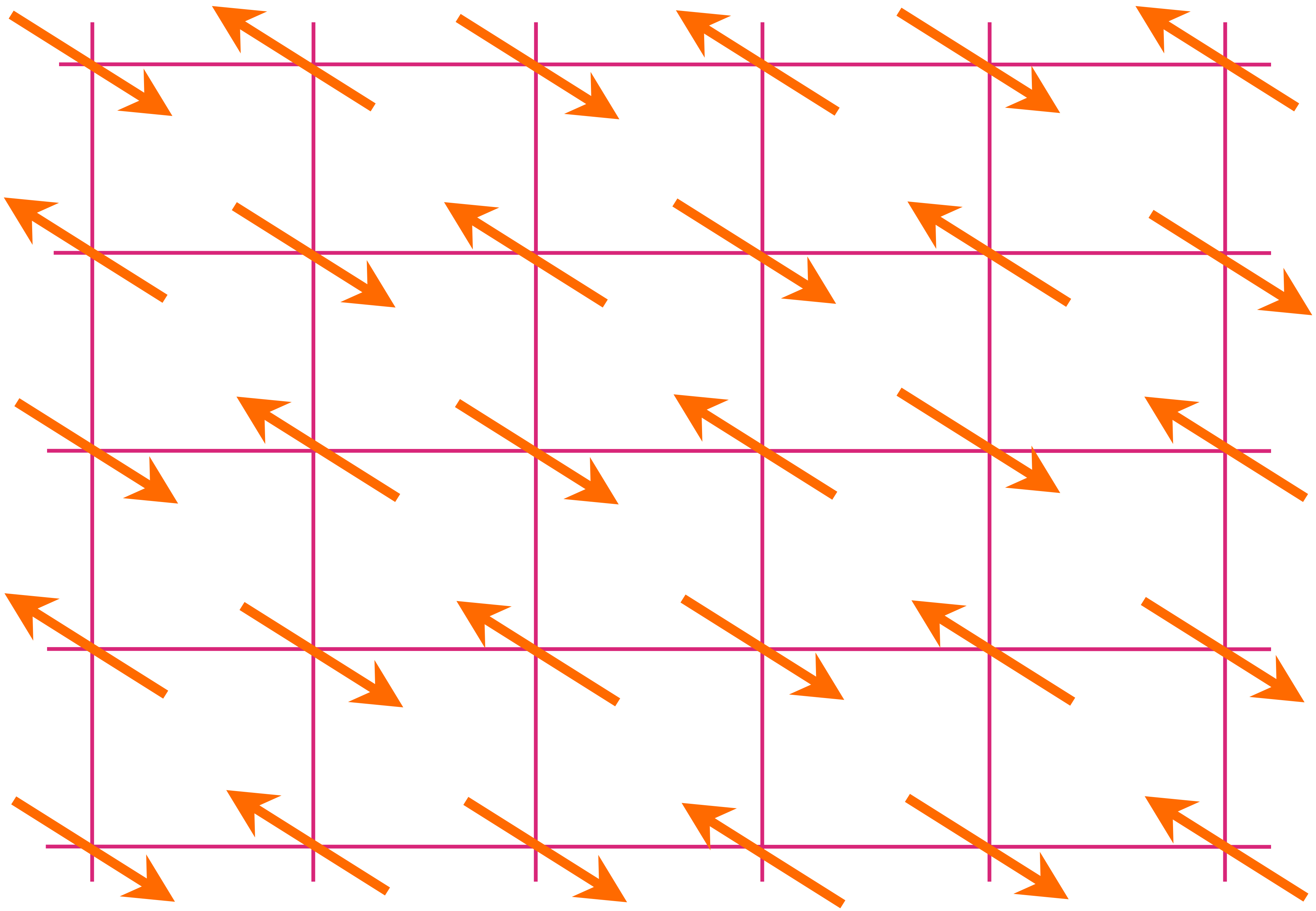


CuO_2 plane

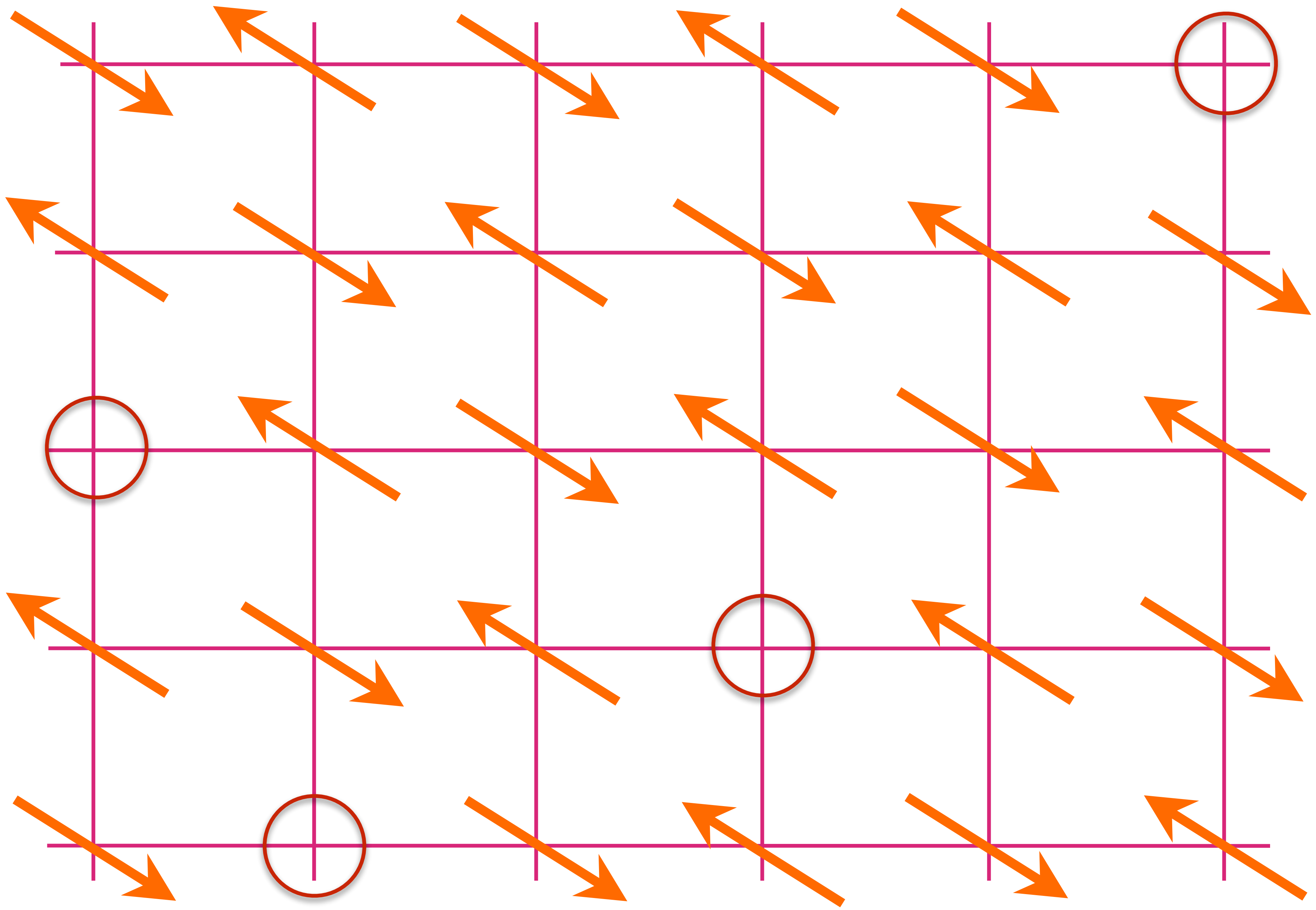


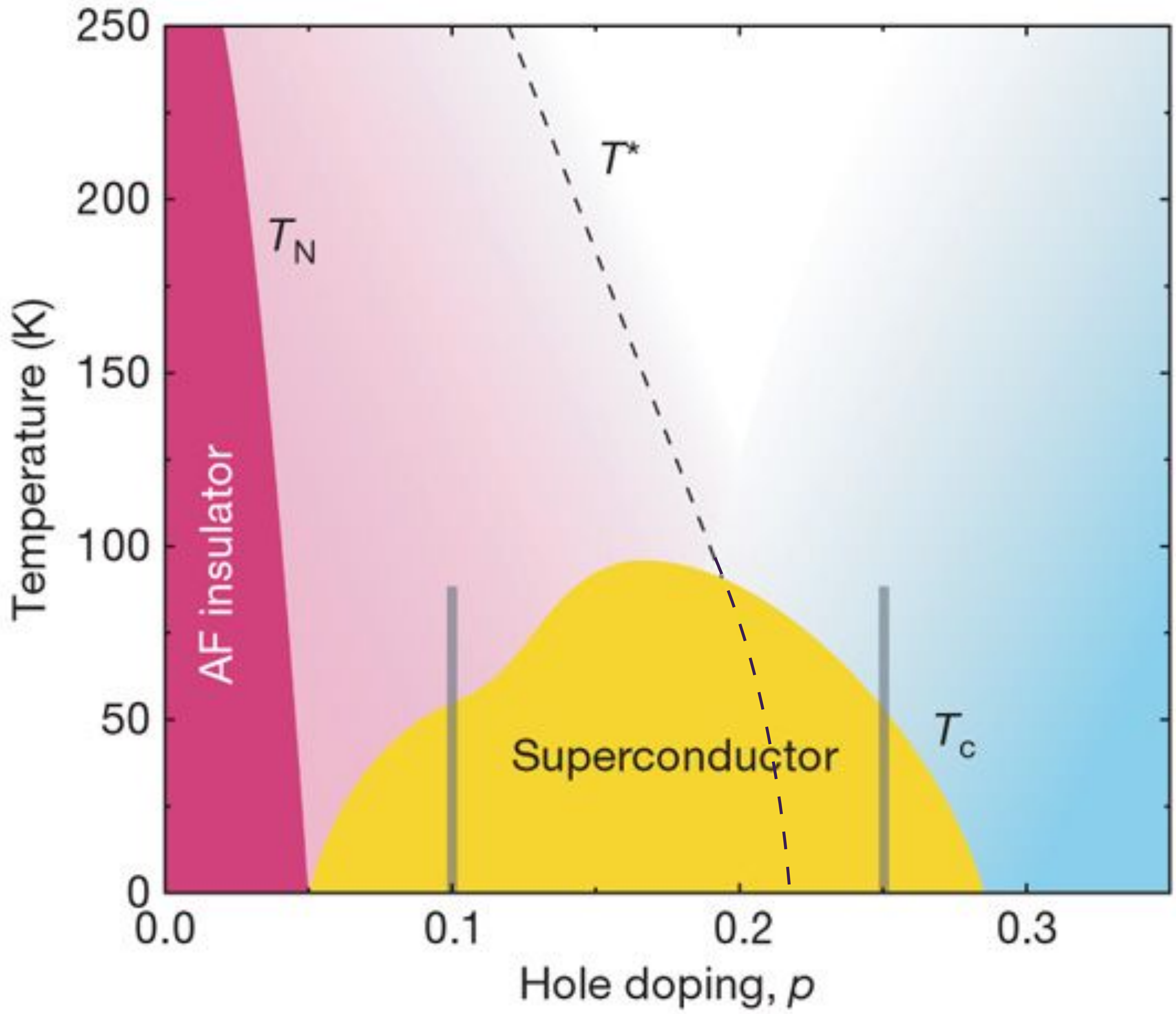
$YBa_2Cu_3O_{6+x}$

Insulating antiferromagnet

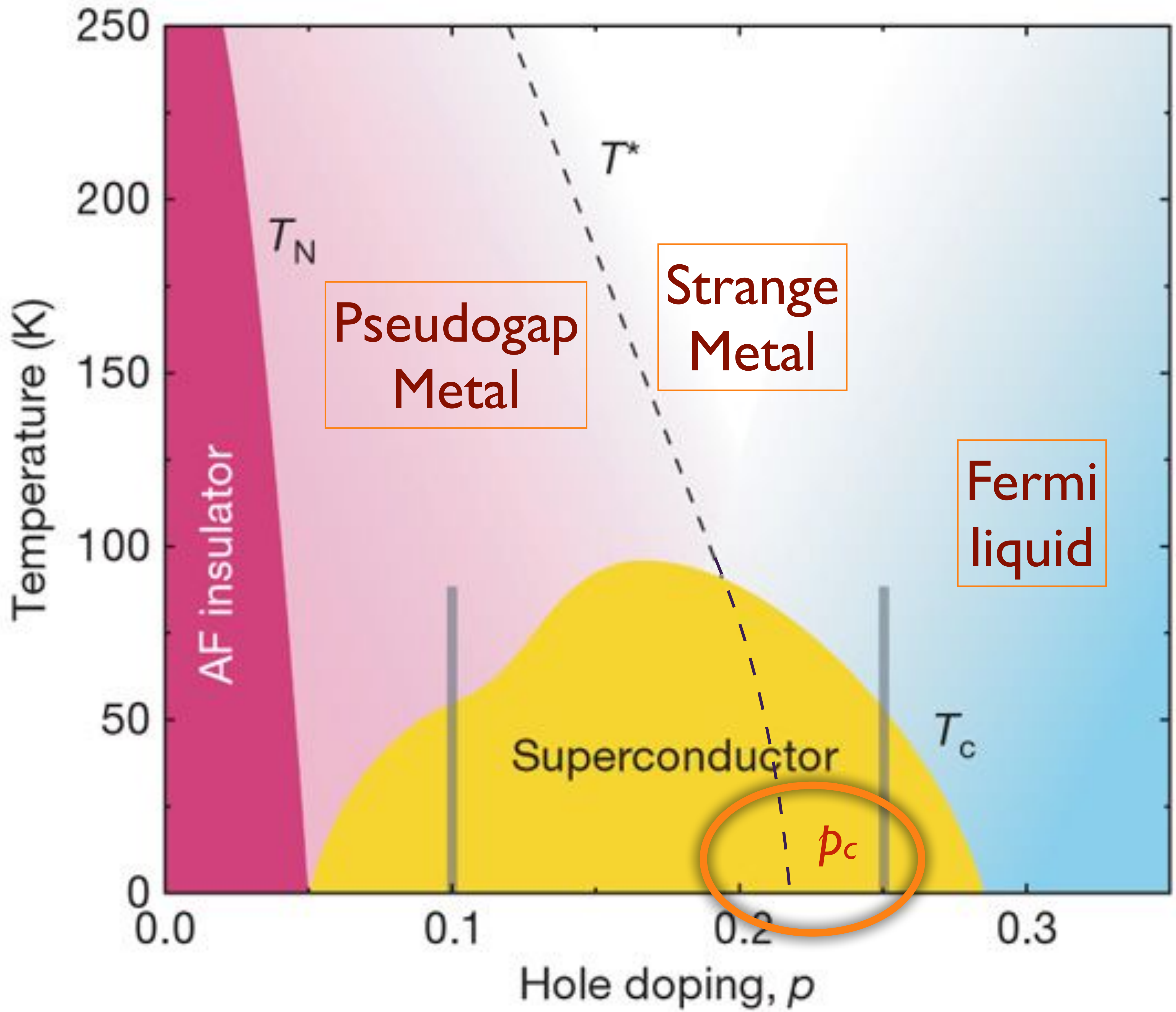


Antiferromagnet doped with hole density p





250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c

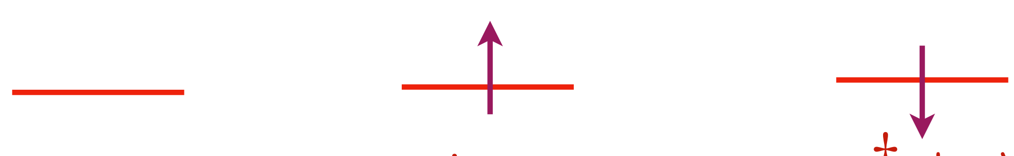
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



$$|0\rangle \qquad c_{\uparrow}^{\dagger}|0\rangle \qquad c_{\downarrow}^{\dagger}|0\rangle$$

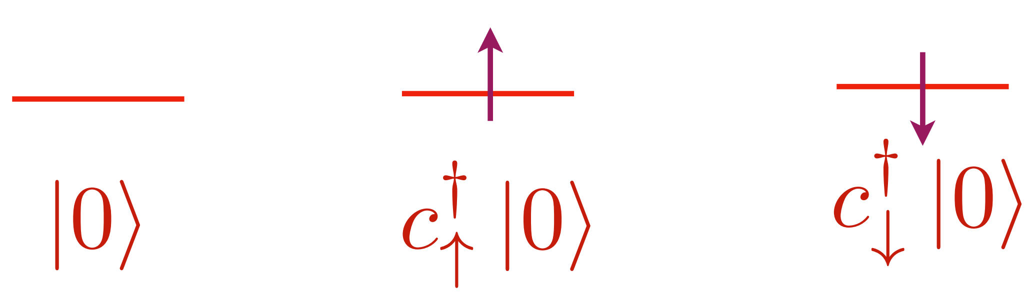
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

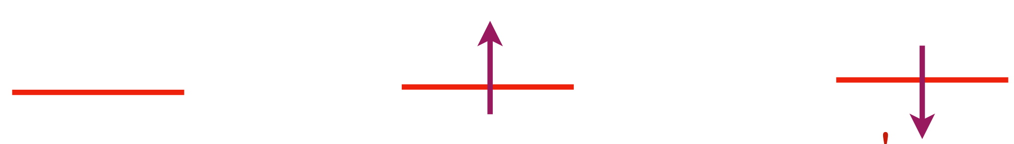
We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \; \overline{J_{ij}} = 0, \; \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random}, \; \overline{t_{ij}} = 0, \; \overline{t_{ij}^2} = t^2$$

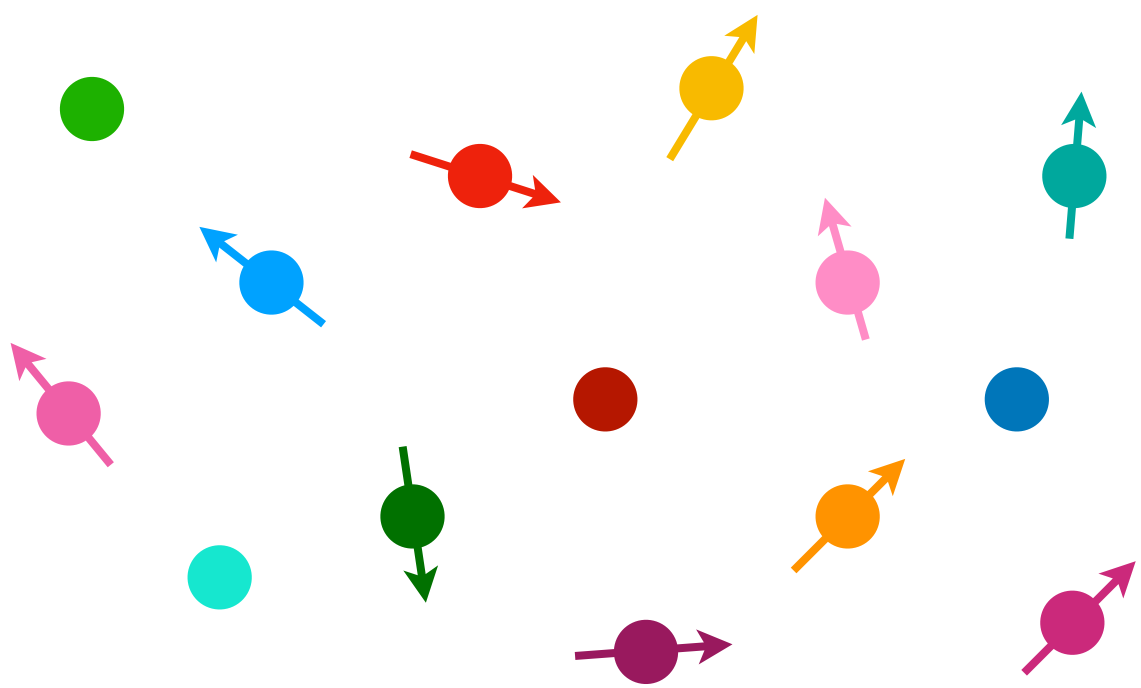


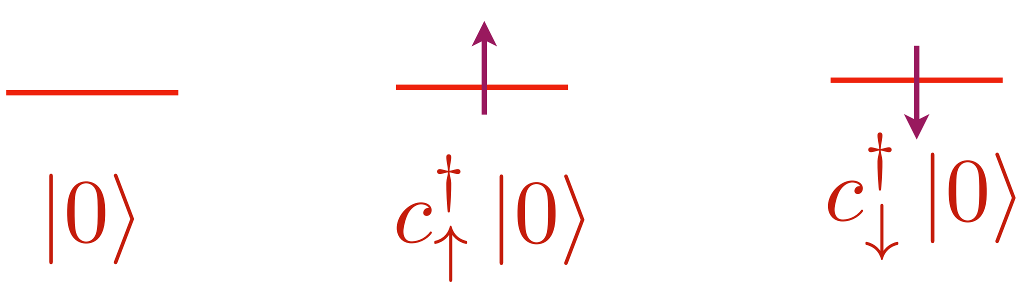
$|0\rangle$ $\quad c_{\uparrow}^{\dagger}|0\rangle$ $\quad c_{\downarrow}^{\dagger}|0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

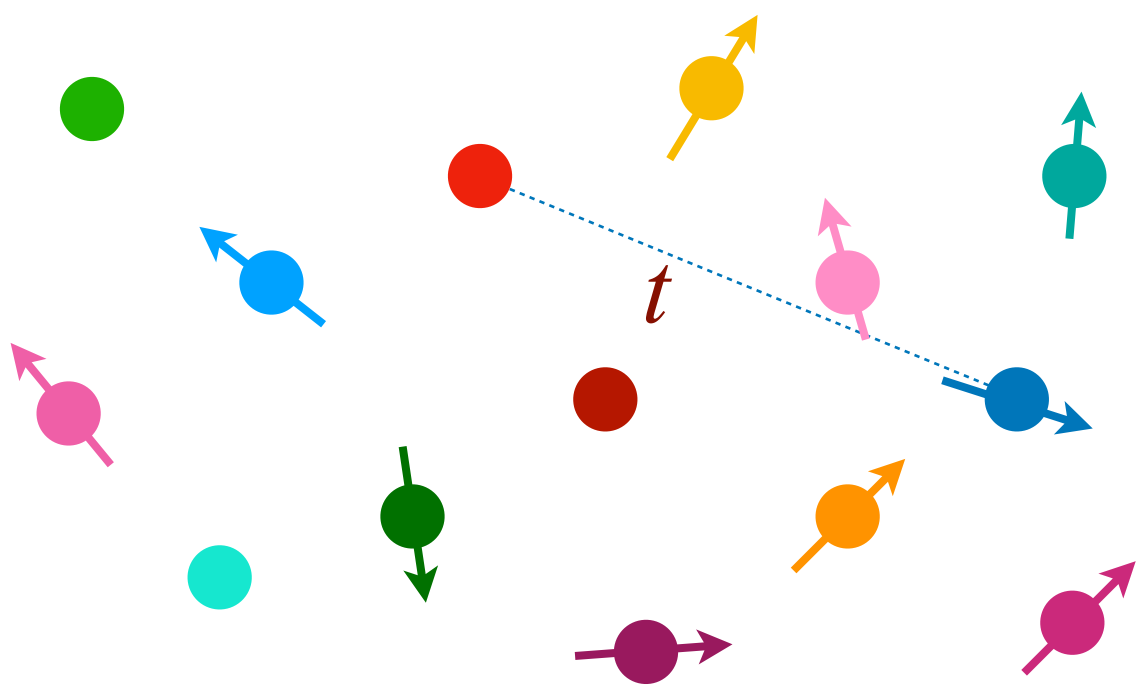


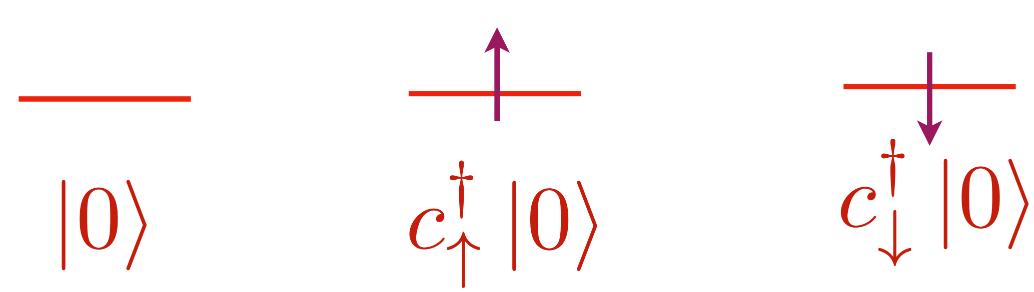


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

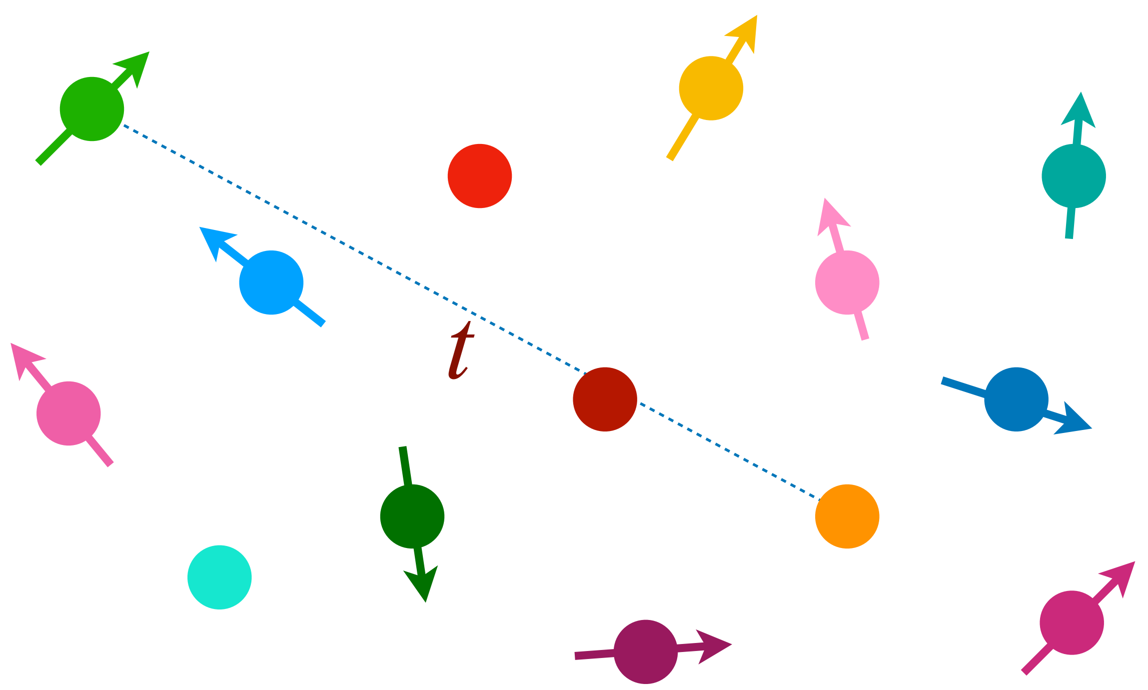


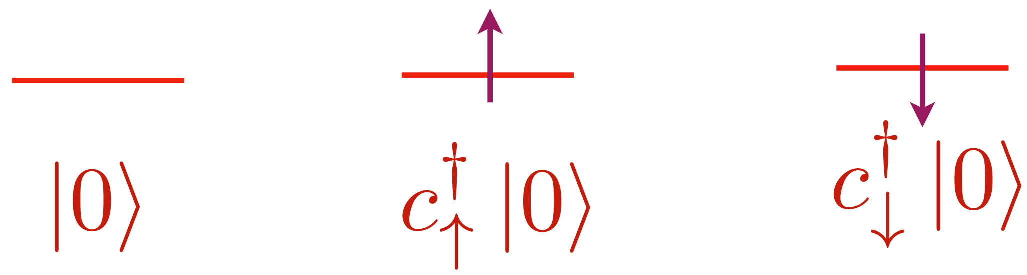


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

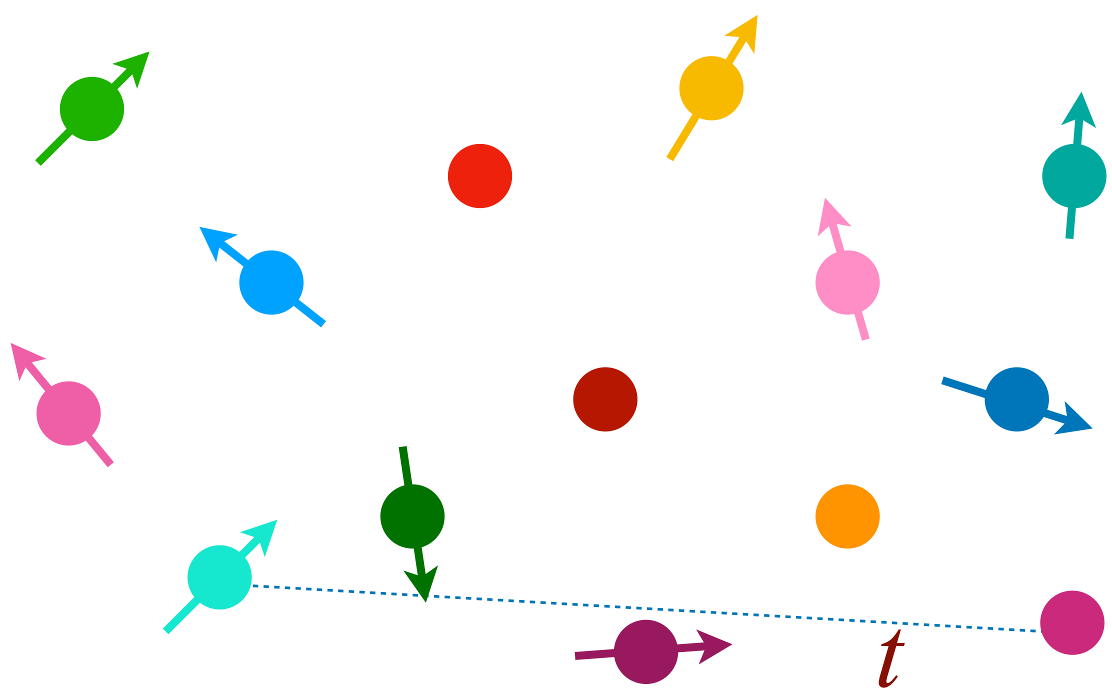


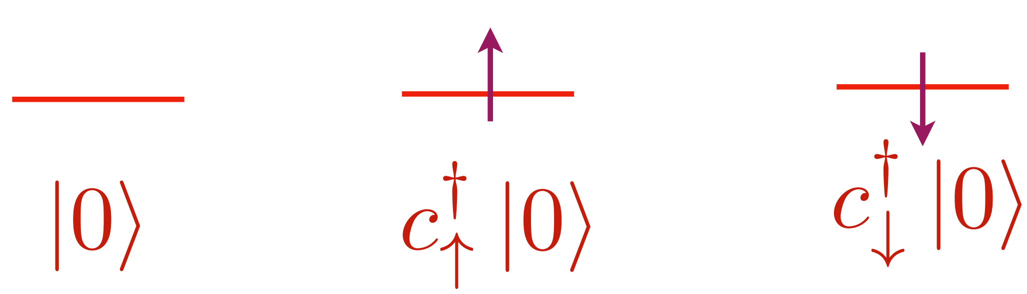


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

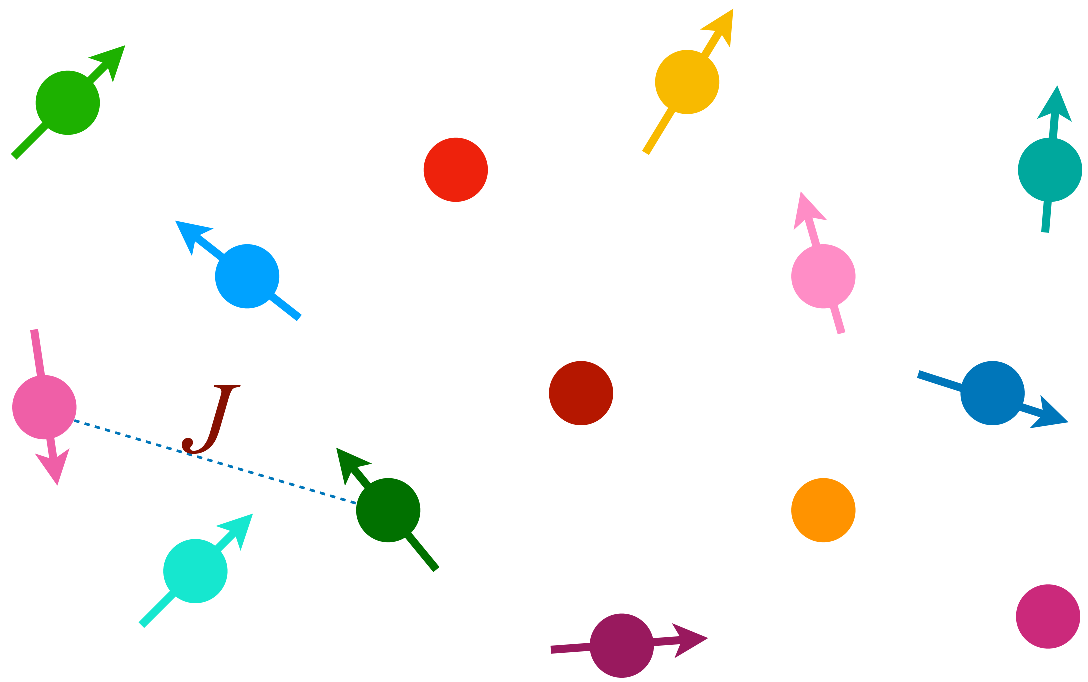


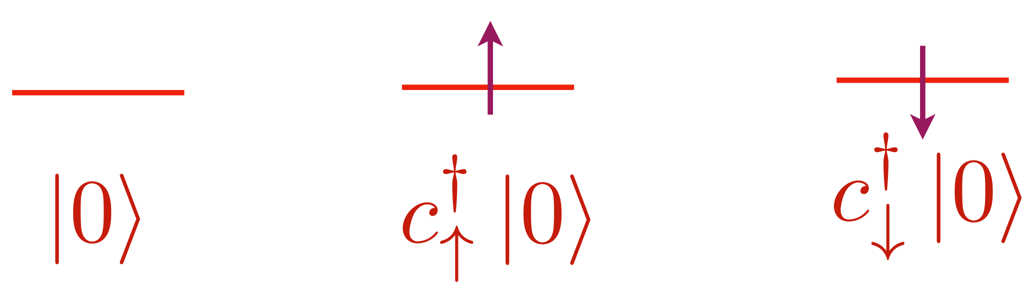


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

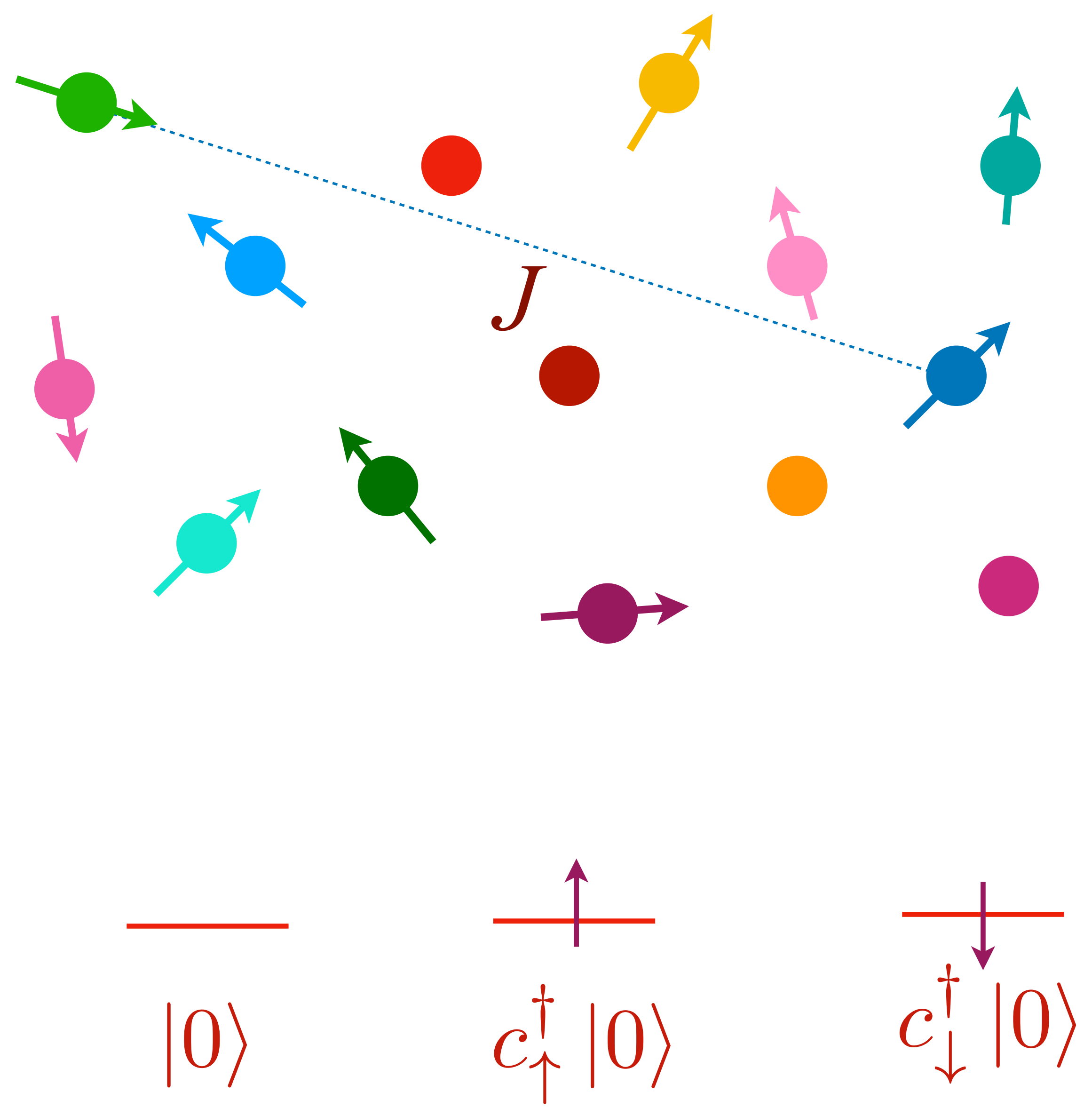




Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

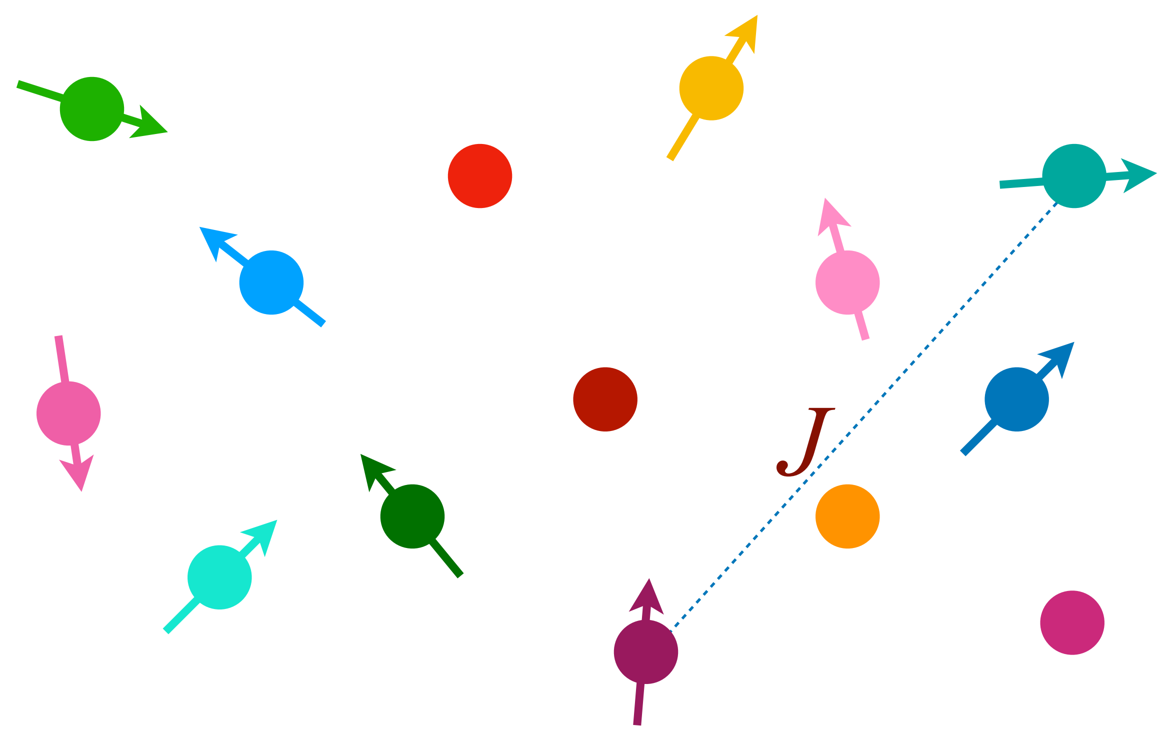
We consider the hole-doped case, with no double occupancy.

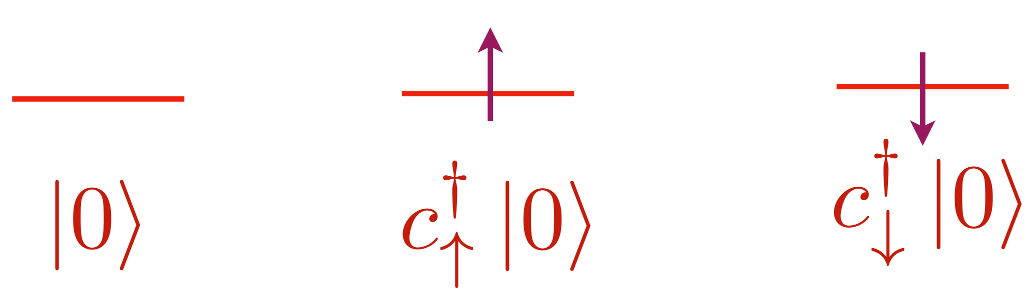


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





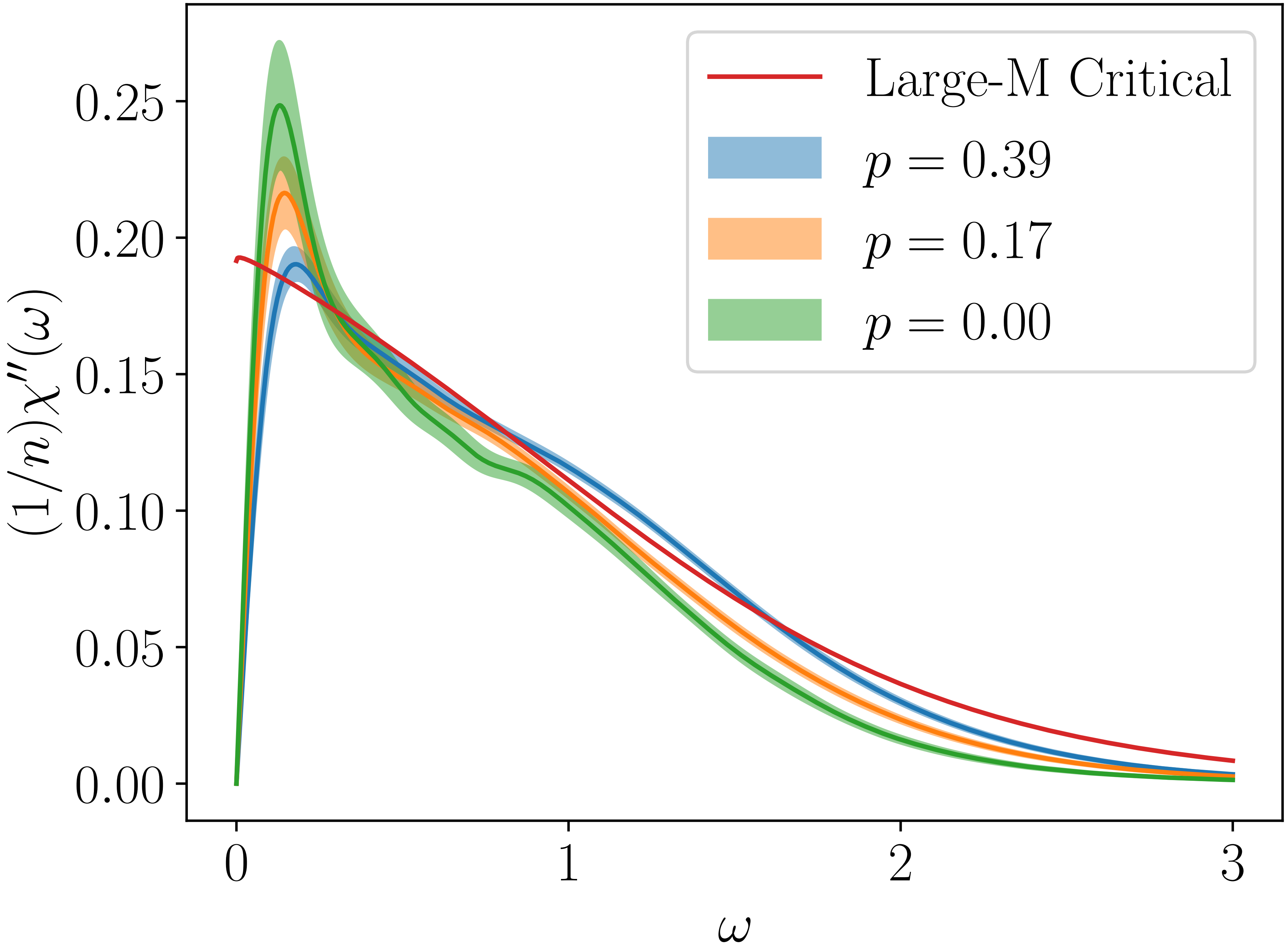
Henry Shackleton

Alexander Wietek

Antoine Georges

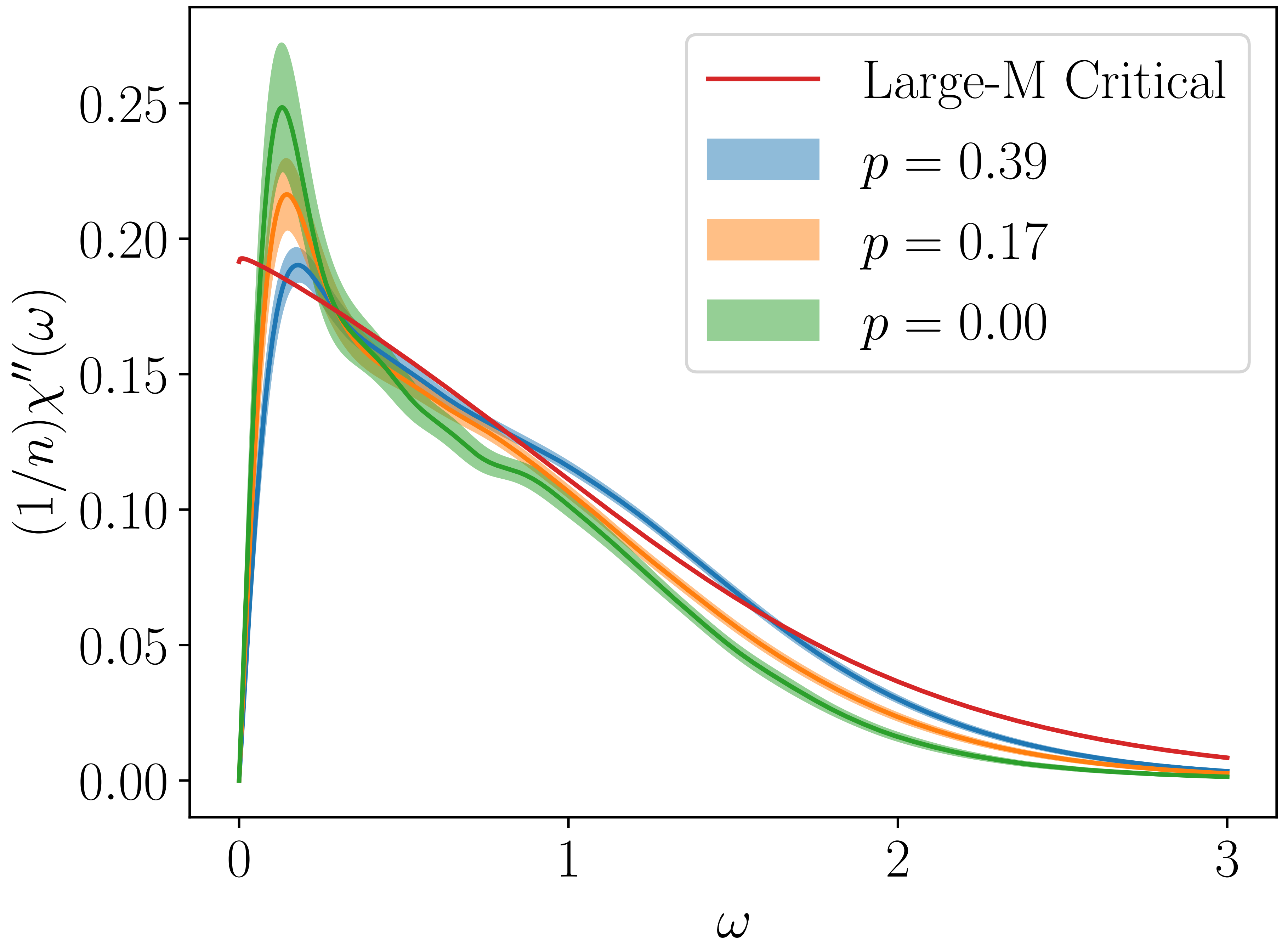
arXiv:2012.06589

Numerical exact diagonalization and SYK theory



Evidence for a quantum critical point at $p = p_c \approx 0.3$
with SYK criticality.
Spin glass order for $p < p_c$

Numerical exact diagonalization and SYK theory



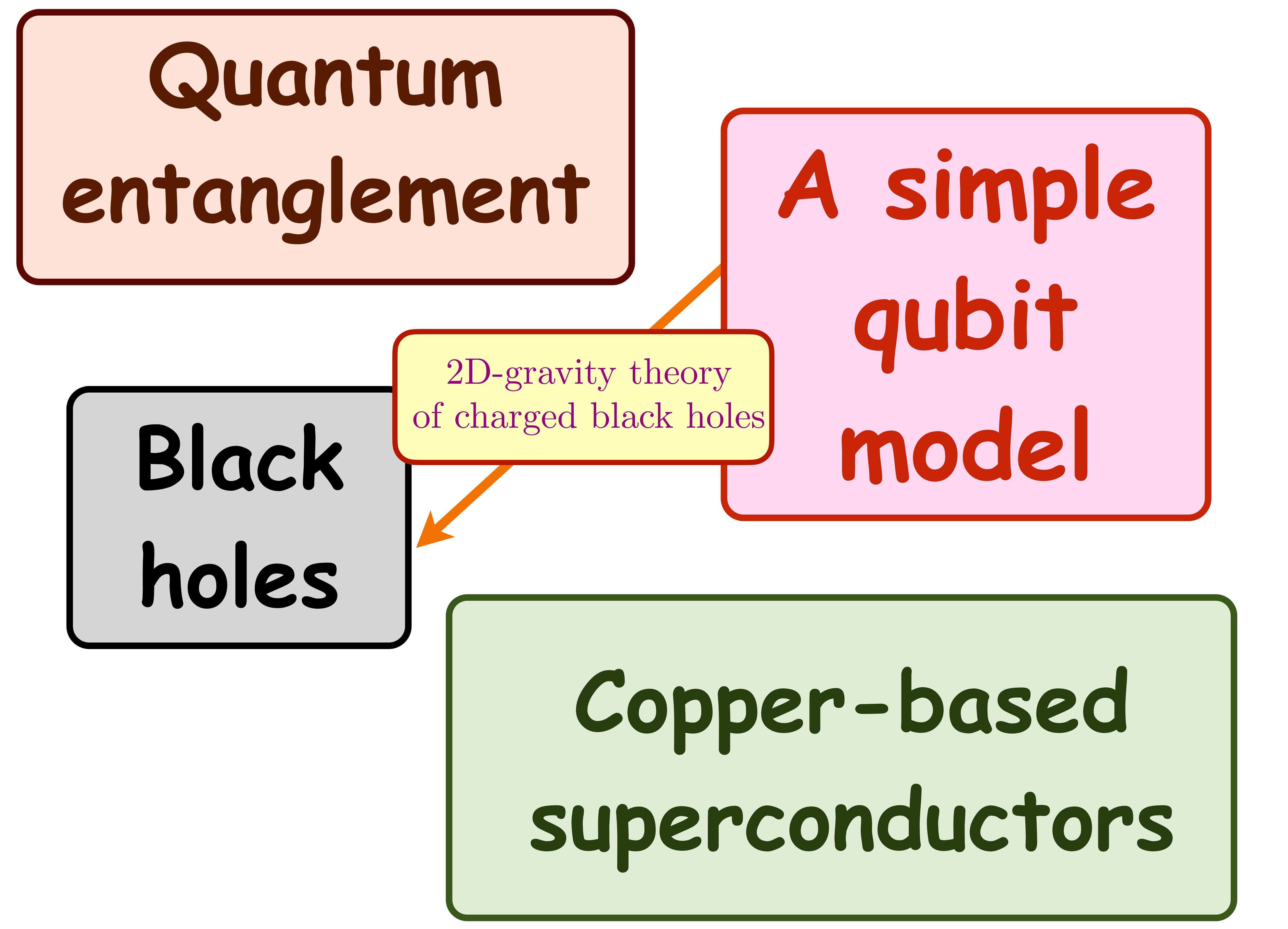
Numerics matches many other observations, including the breakdown of the Luttinger-volume Fermi surface for $p < p_c$, and Planckian dissipation at scale $\hbar/(k_B T)$.

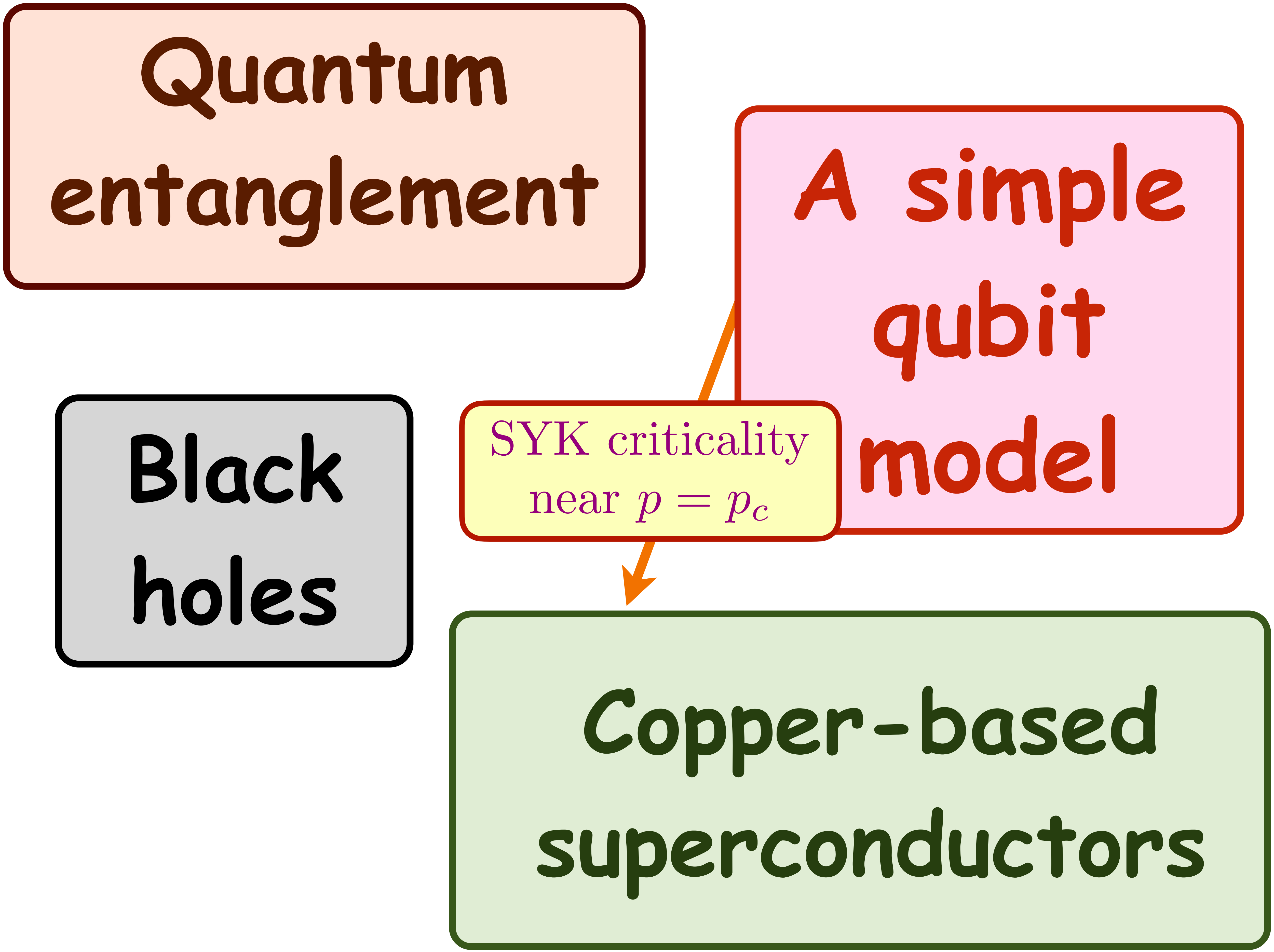
Quantum entanglement

A simple qubit model

Black holes

Copper-based superconductors


Quantum entanglement
A simple qubit model
2D-gravity theory of charged black holes
Black holes
Copper-based superconductors


Quantum entanglement
A simple qubit model
Black holes
SYK criticality near $p = p_c$
Copper-based superconductors