

Emergent “light” and the high temperature superconductors

Pennsylvania State University
State College, January 21, 2016

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

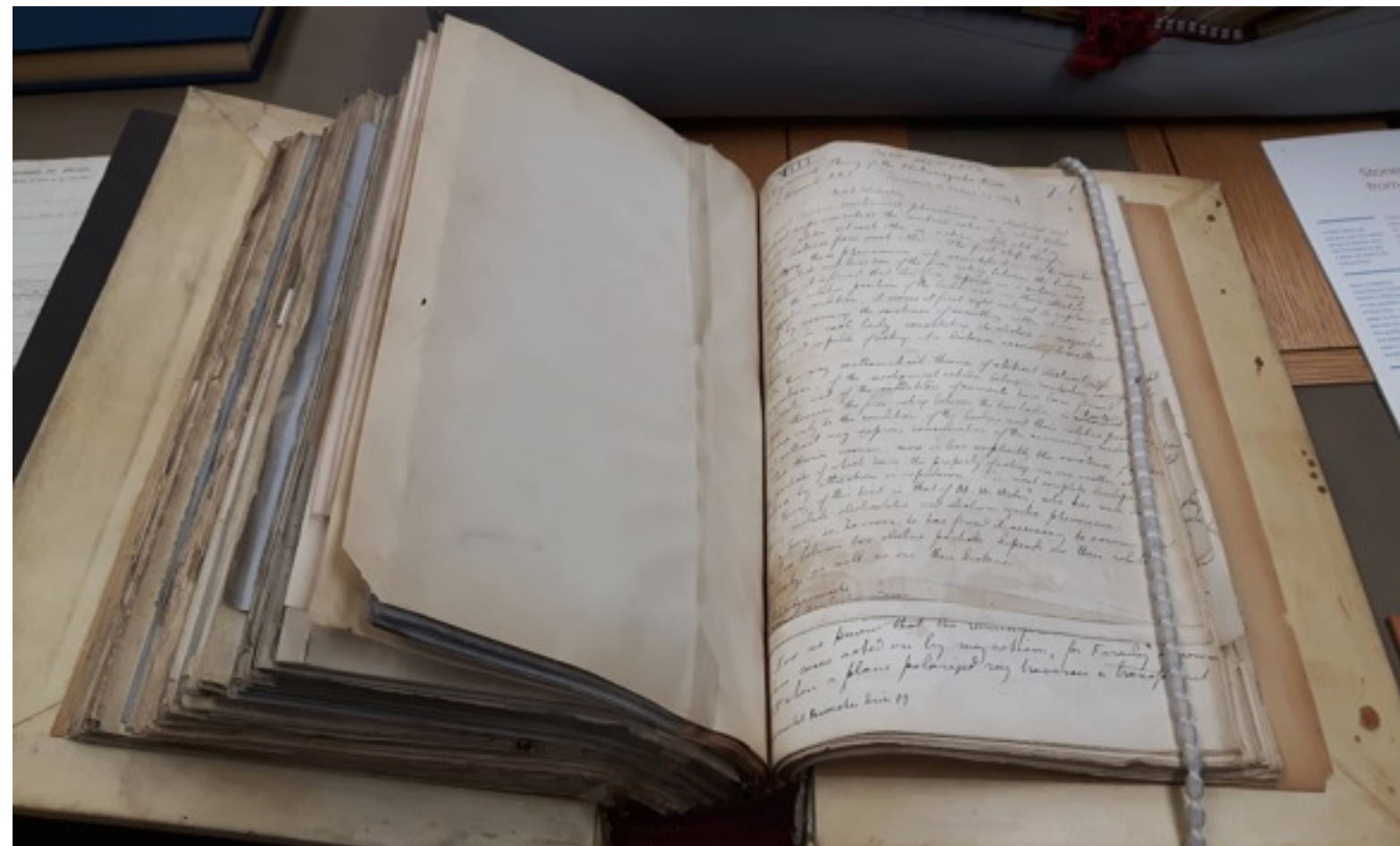


Maxwell's equations: 150 years of light

A century and a half ago, James Clerk Maxwell submitted a long paper to the Royal Society containing his famous equations. Inspired by Michael Faraday's experiments and insights, the equations unified electricity, magnetism and optics. Their far-reaching consequences for our civilisation, and our universe, are still being explored

Jon Butterworth

Sunday 22 November 2015 08.38 GMT, The Guardian



VIII
The Theory of the Electrodynamic Force
Revised Edition 1871-1872

... mechanical phenomena in electrical and
... at each other in action while at a
... from each other. The first step, therefore
... these phenomena into accounts for...
... and direction of the force acting between the bodies
... I found that this force depends on a certain way
... the relative position of the bodies and on their distance
... condition, it seems at first sight natural to explain the
... assuming the existence of something either at rest or in
... in each body, constituting its electric or magnetic
... and repulsive forces - in distance according to mathematical
... the very mathematical theory of static electricity, of
... of the mechanical action between conductors carrying
... of the interaction of currents has been found
... the force acting between the two bodies is
... theories to the condition of the bodies and their relative position
... only to the condition of the bodies and their relative position
... without any express consideration of the surrounding medium
... theories assume, more or less explicitly, the existence of a
... which have the property of acting on one another by
... attraction or repulsion. The most complete development
... of this kind is that of M. W. Weber, who has made
... include electrostatic and dynamo-quantic phenomena
... so, however, he has found it necessary to assume
... between two electric particles depends on their relative
... as well as on their distance.

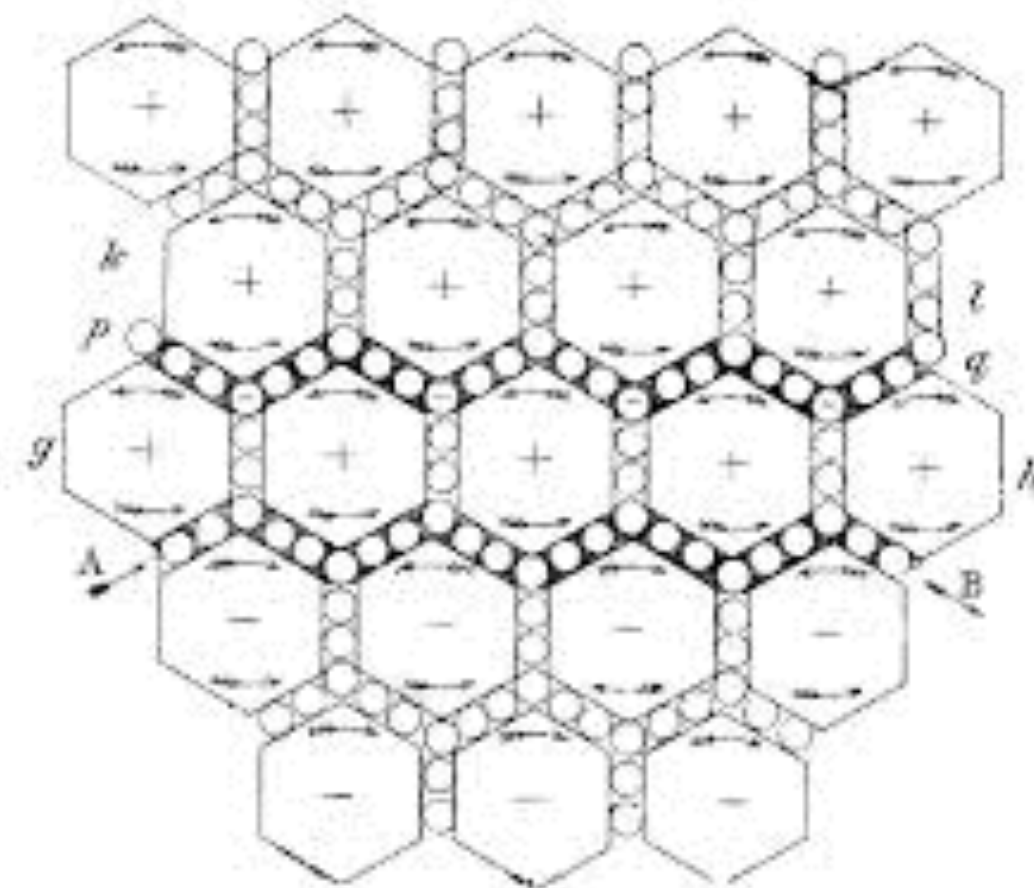
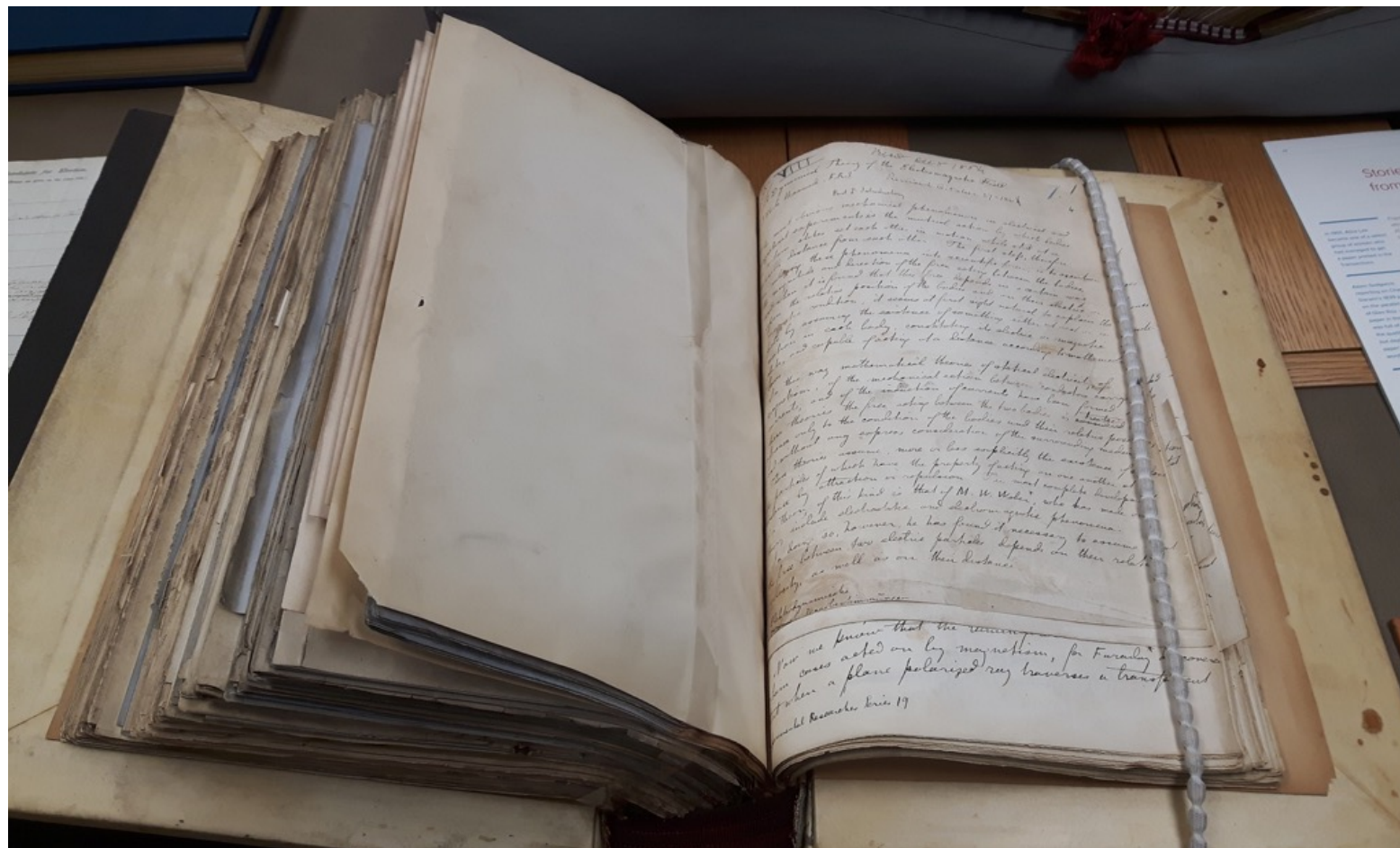
... we know that the velocity
... cases acted on by magnetism, for Faraday's
... when a plane polarized ray traverses a transparent
... Remark. Eric 19

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Modern point-of-view:

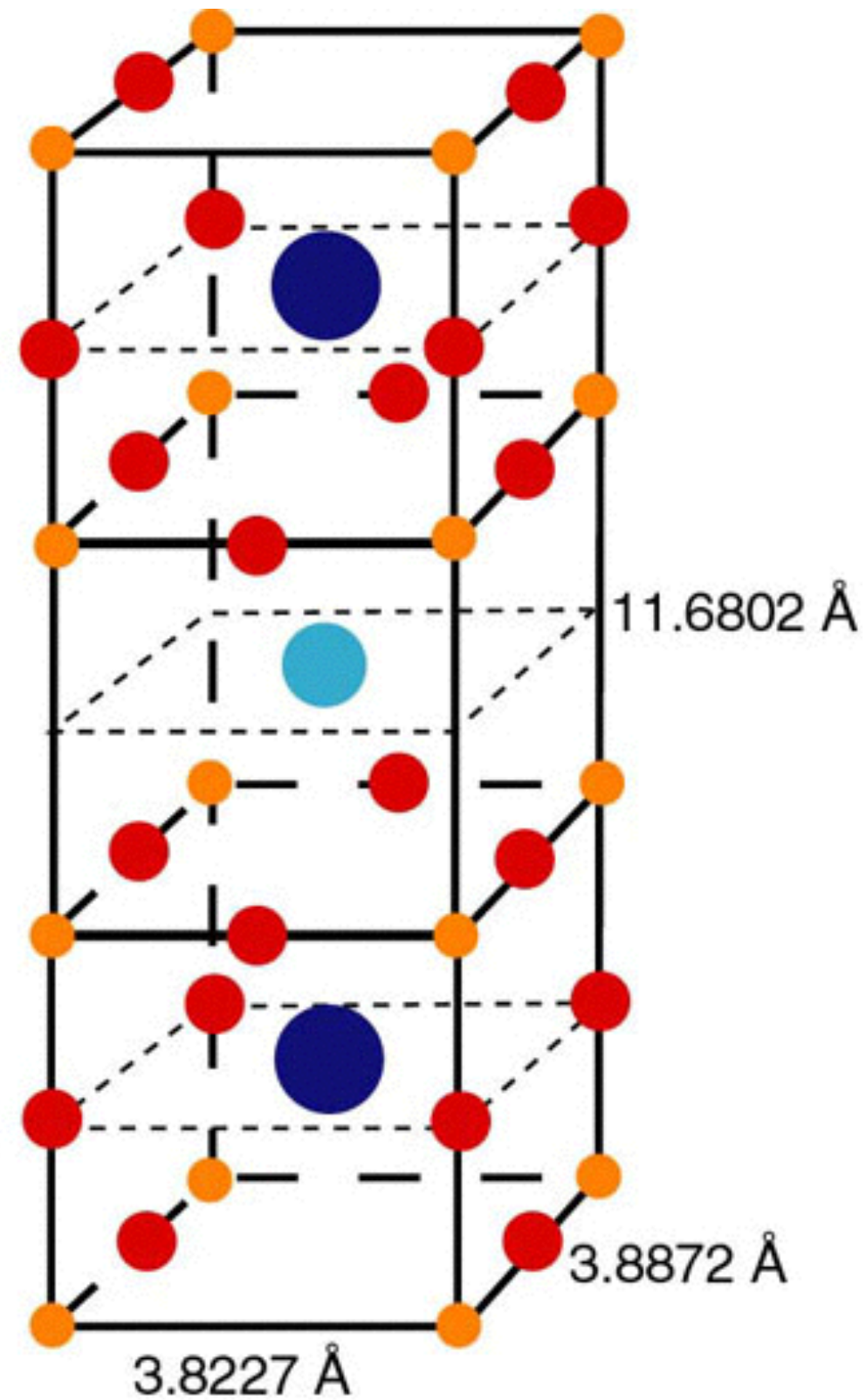
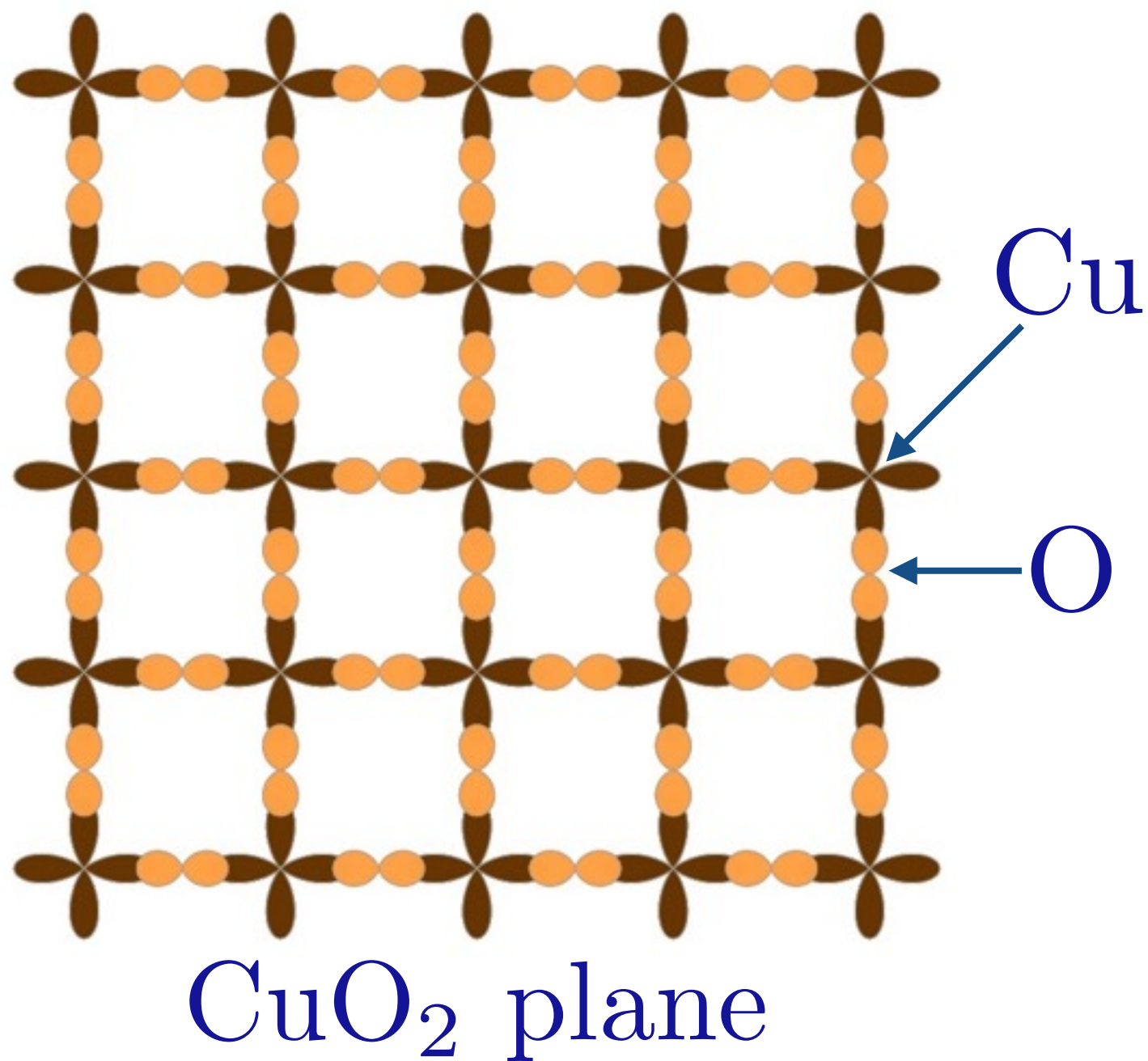
Electromagnetic gauge fields are needed to describe the long-range quantum entanglement of the “vacuum”.

Modern point-of-view:

Electromagnetic gauge fields are needed to describe the long-range quantum entanglement of the “vacuum”.

Electrons in crystals provide a new “vacuum”, and their interactions can naturally lead to quantum states which have long-range quantum entanglement, and require “emergent” gauge fields.

High temperature superconductors



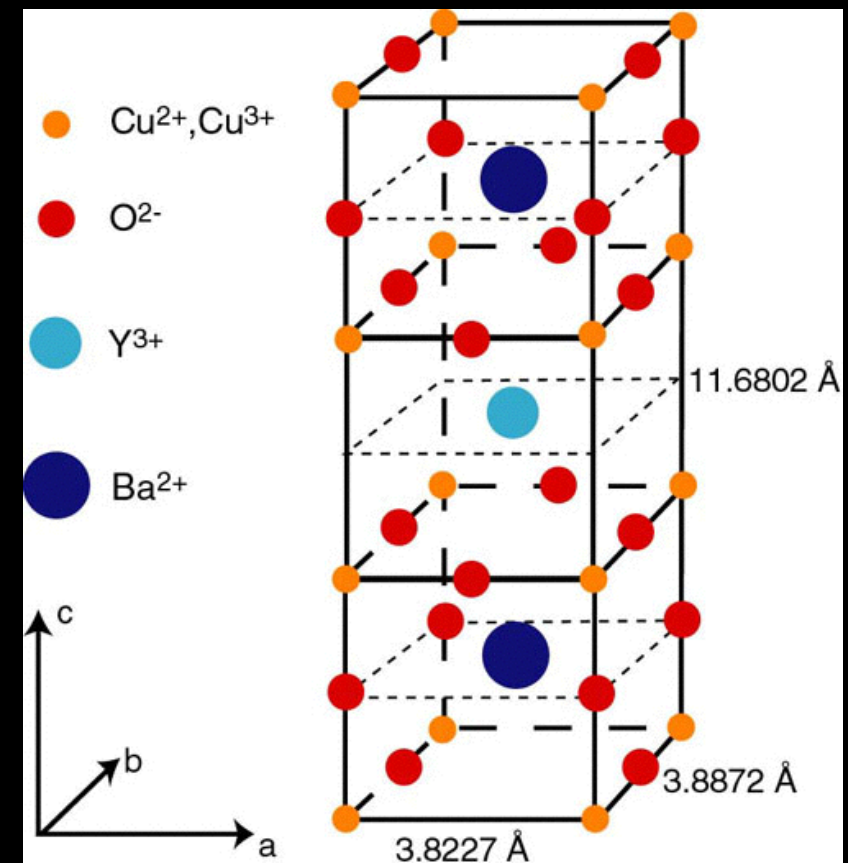
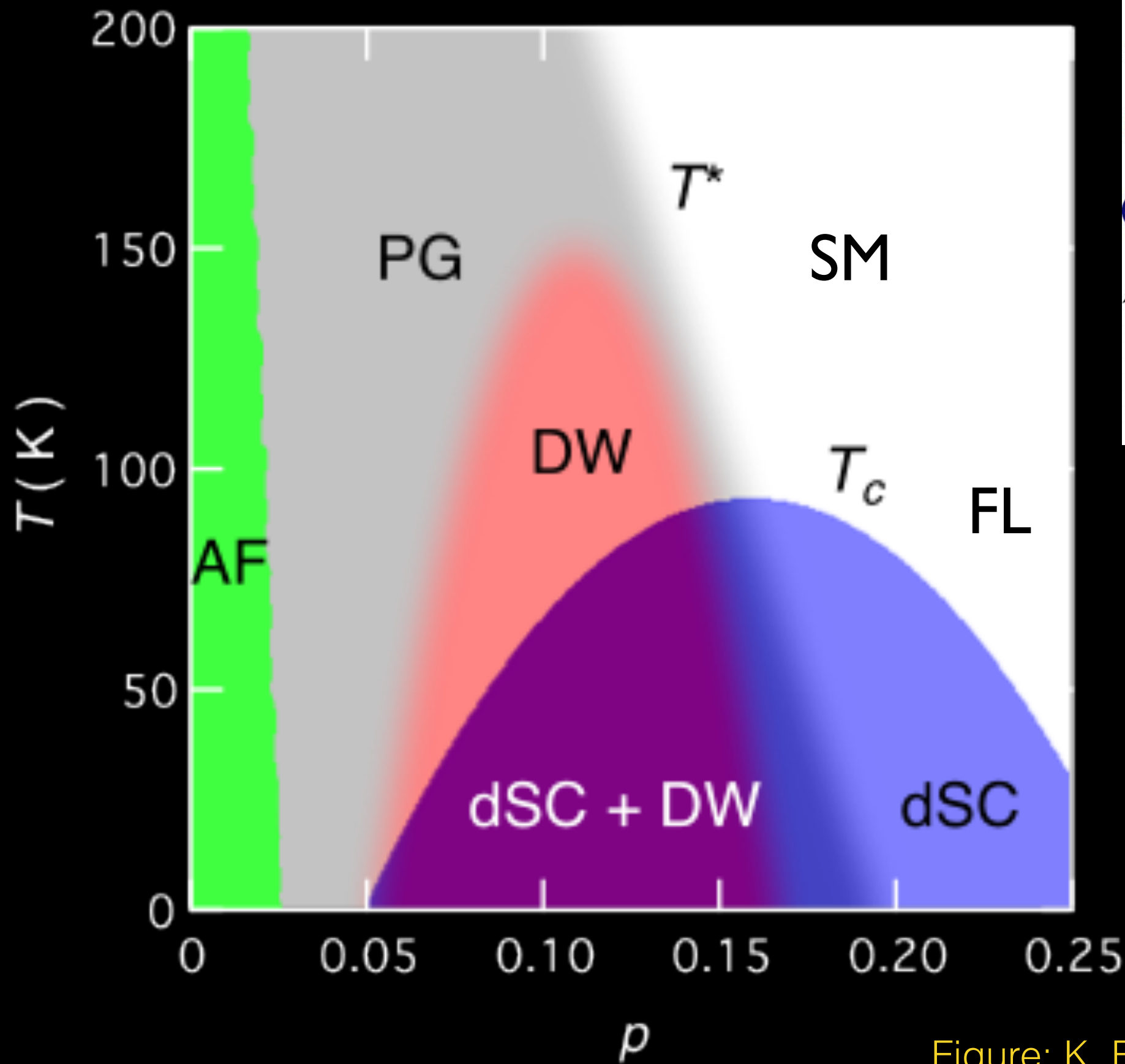
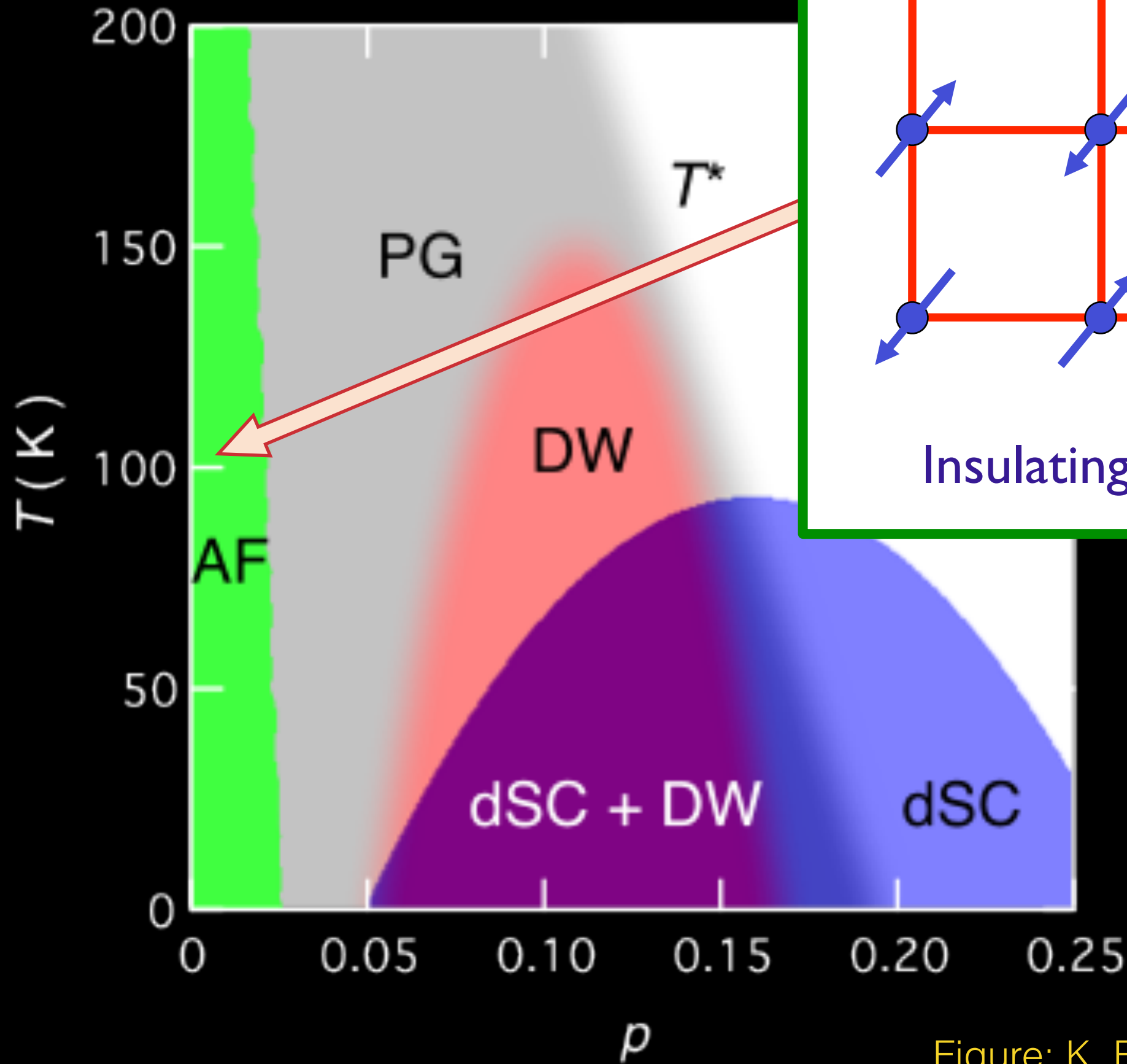


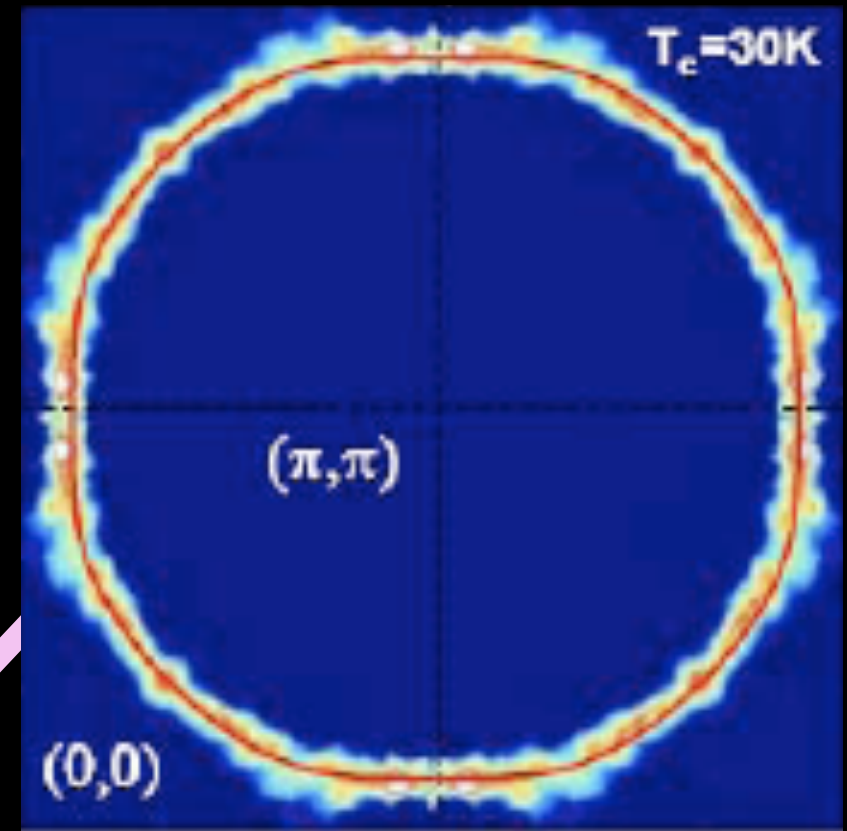
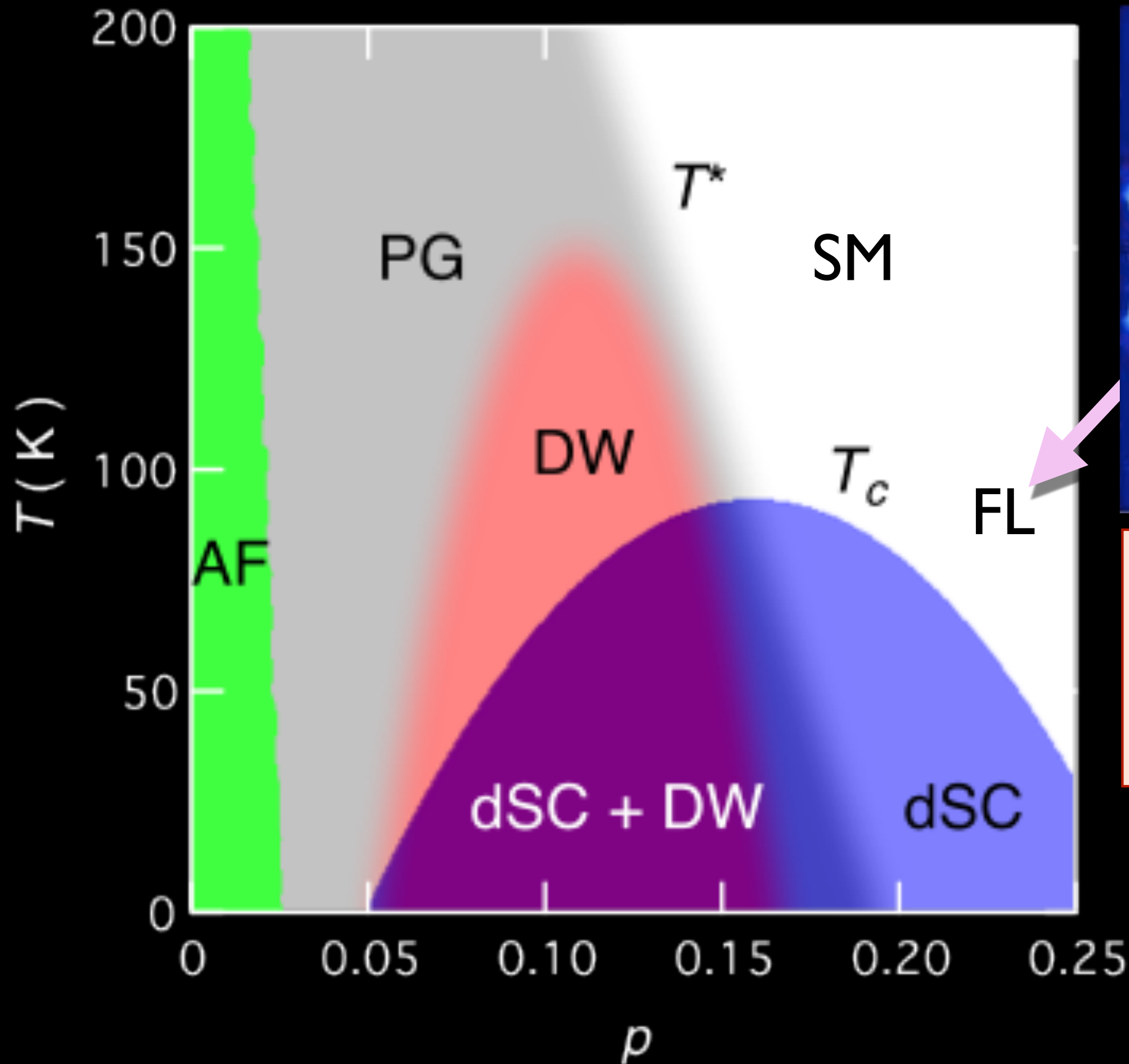
Figure: K. Fujita and J. C. Seamus Davis



$$T = Da^2 \cup a_3 \cup 6 + x$$

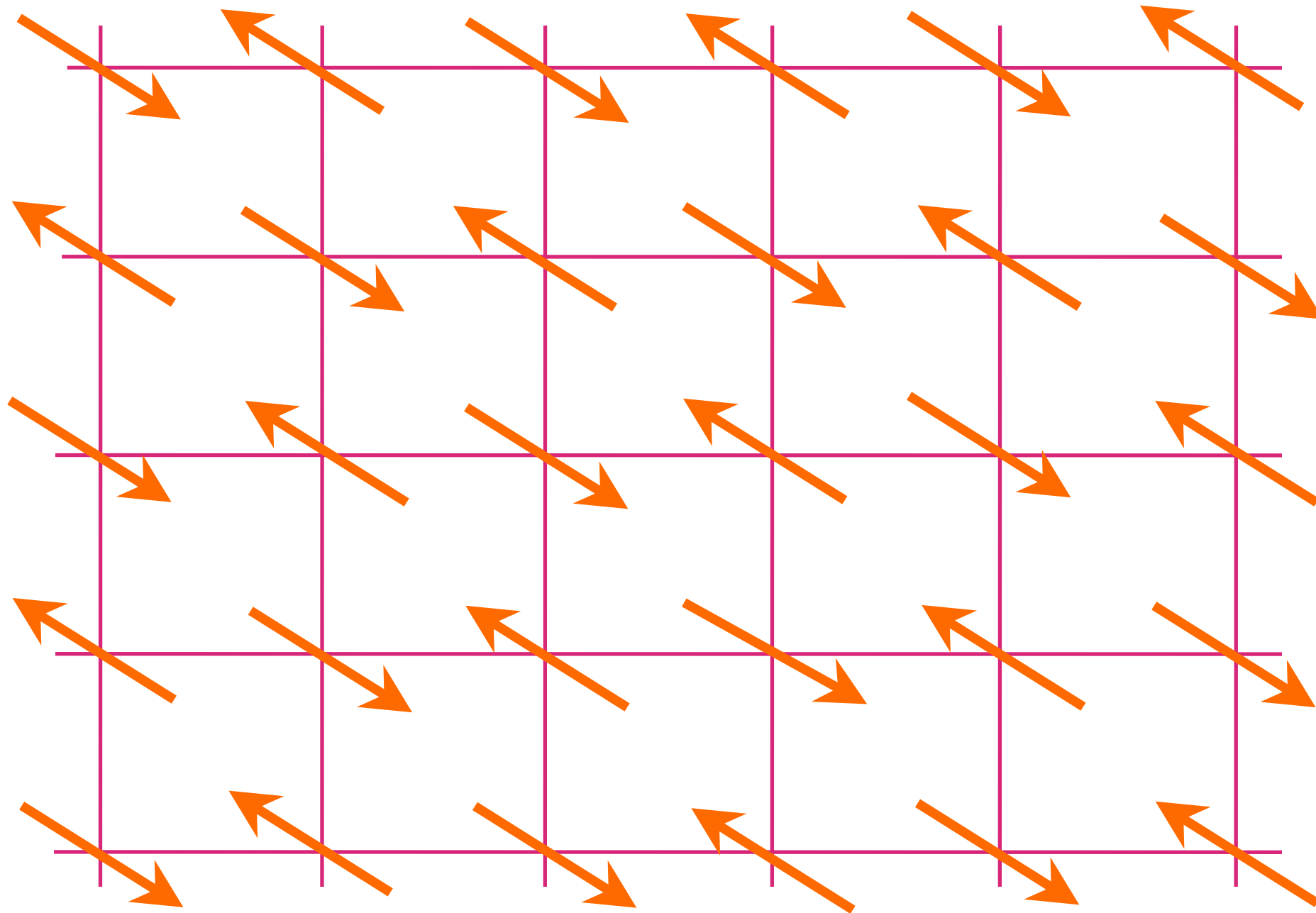
Figure: K. Fujita and J. C. Seamus Davis

M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)




A conventional metal:
the Fermi liquid

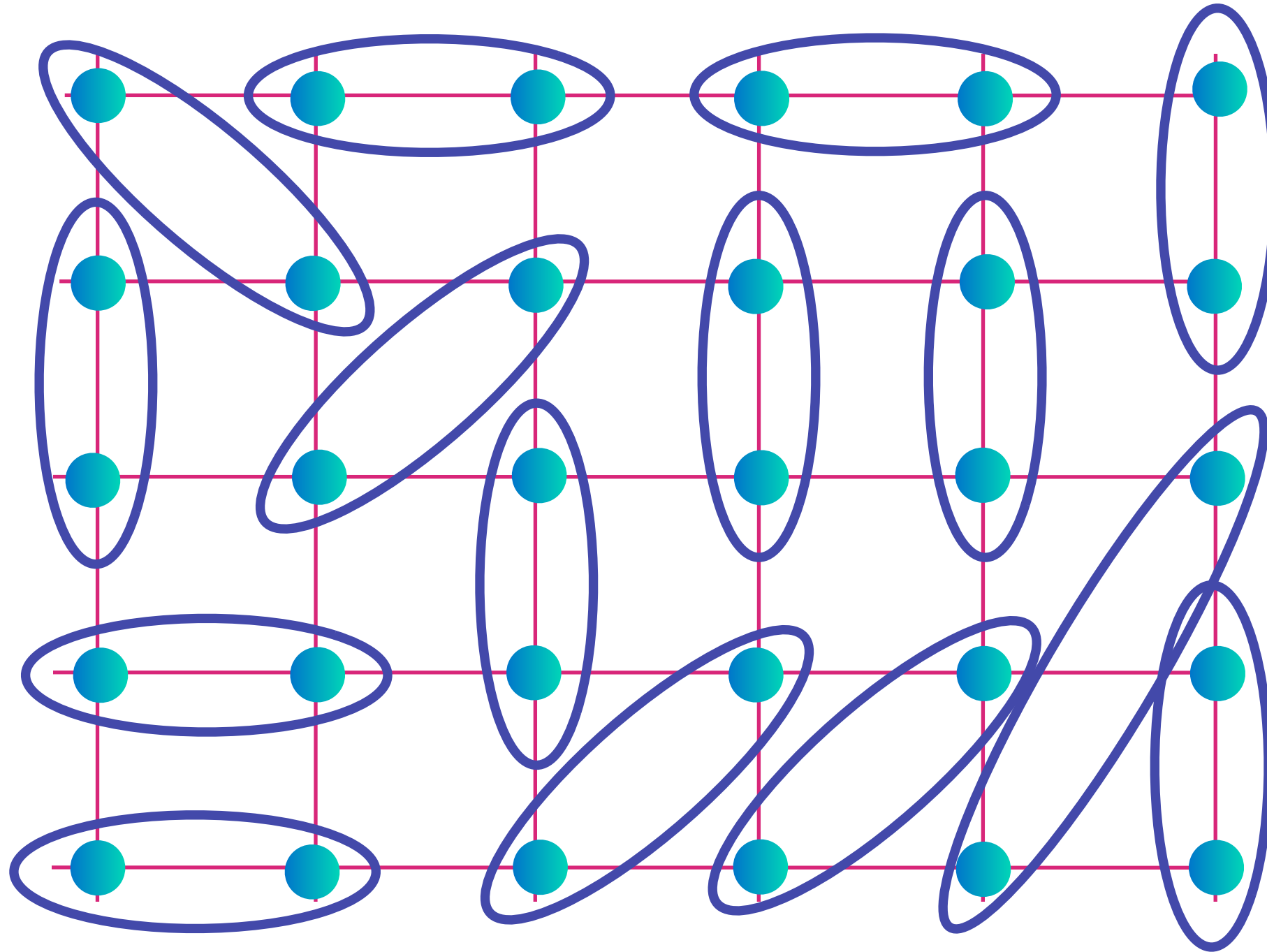
I. Emergent gauge fields and long-range entanglement in insulators



“Undoped”
Anti-
ferromagnet

Insulating spin liquid


$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$




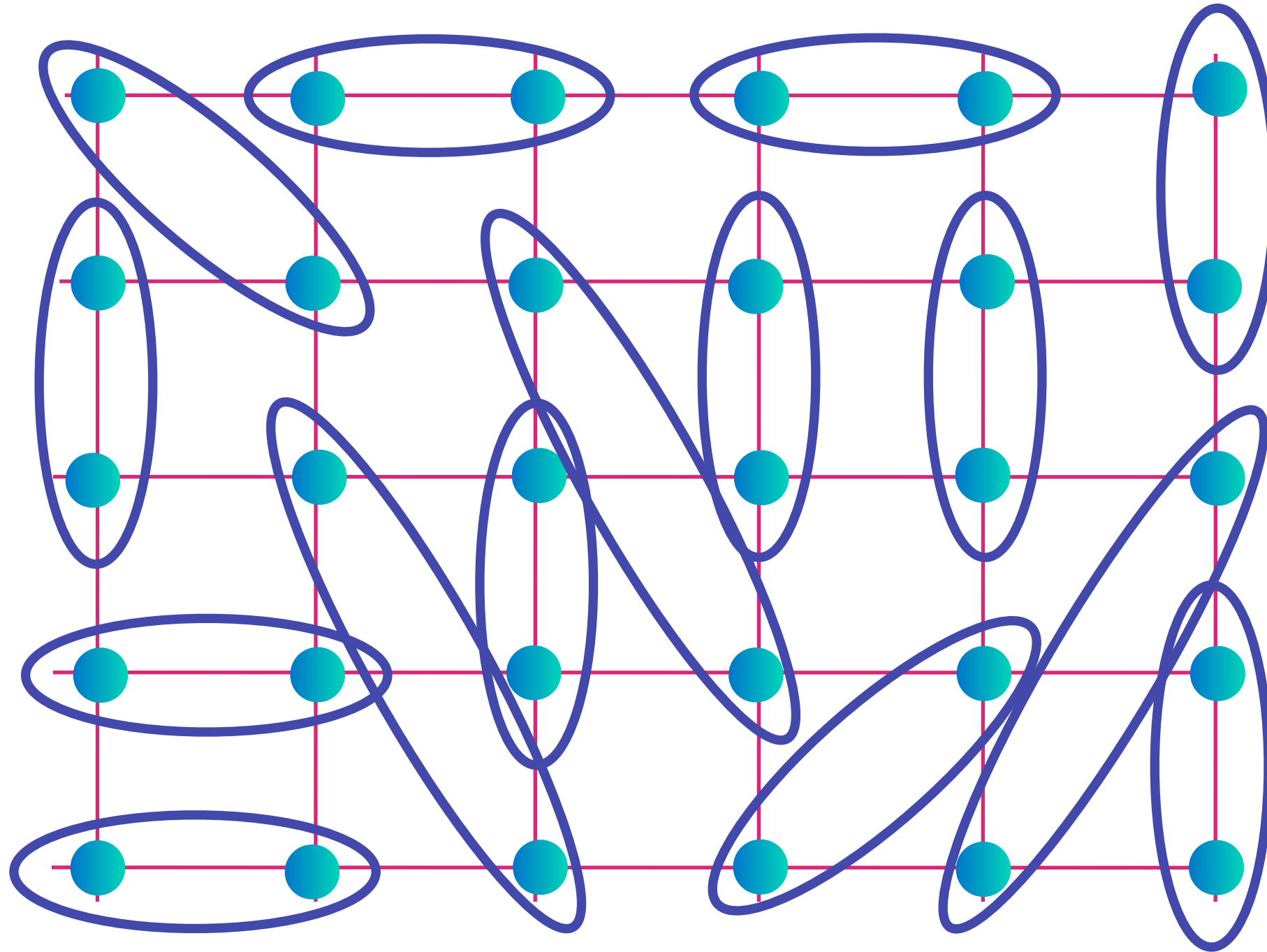
An insulator with emergent gauge fields: the first proposal of a quantum state with long-range entanglement

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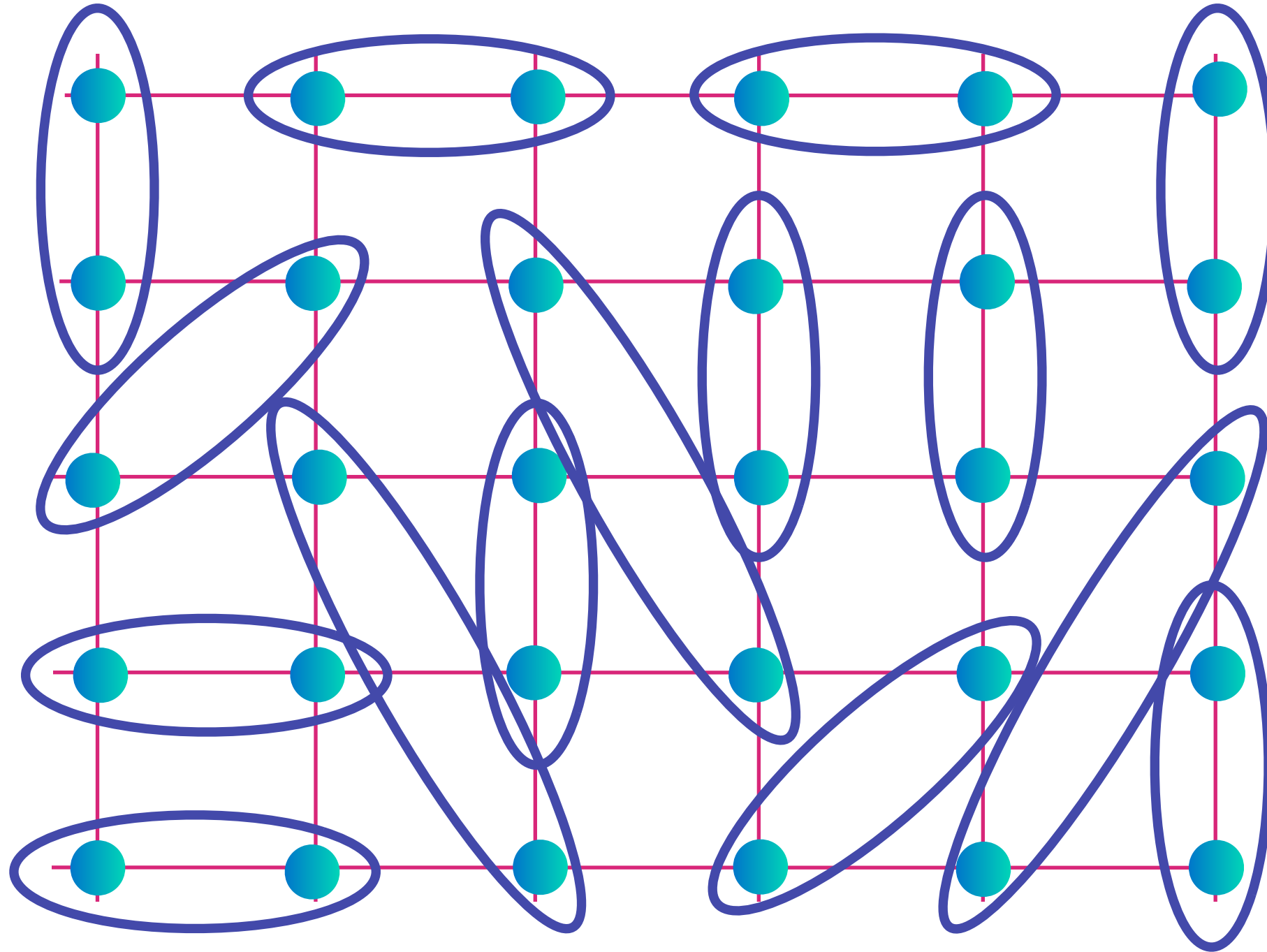
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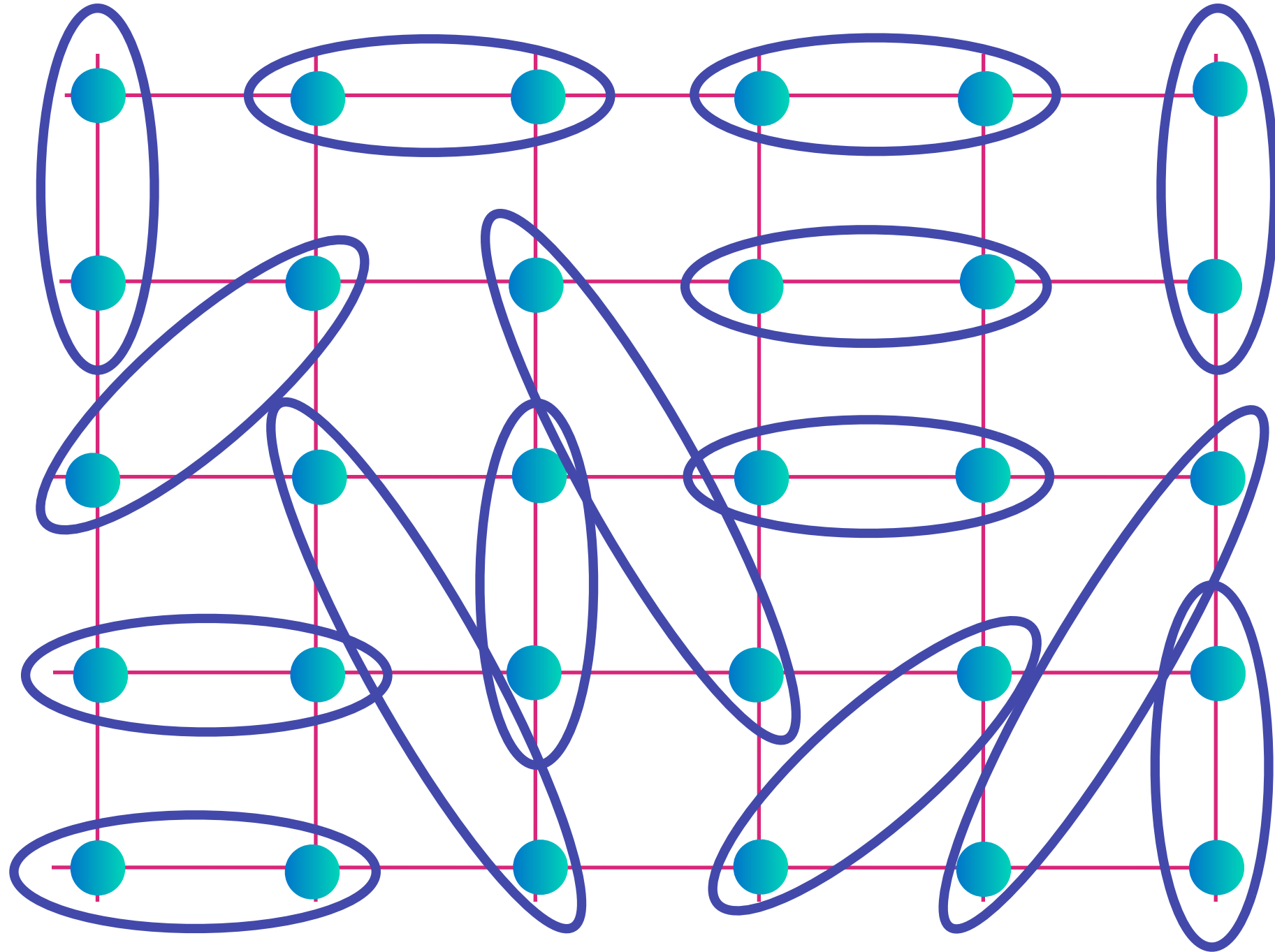
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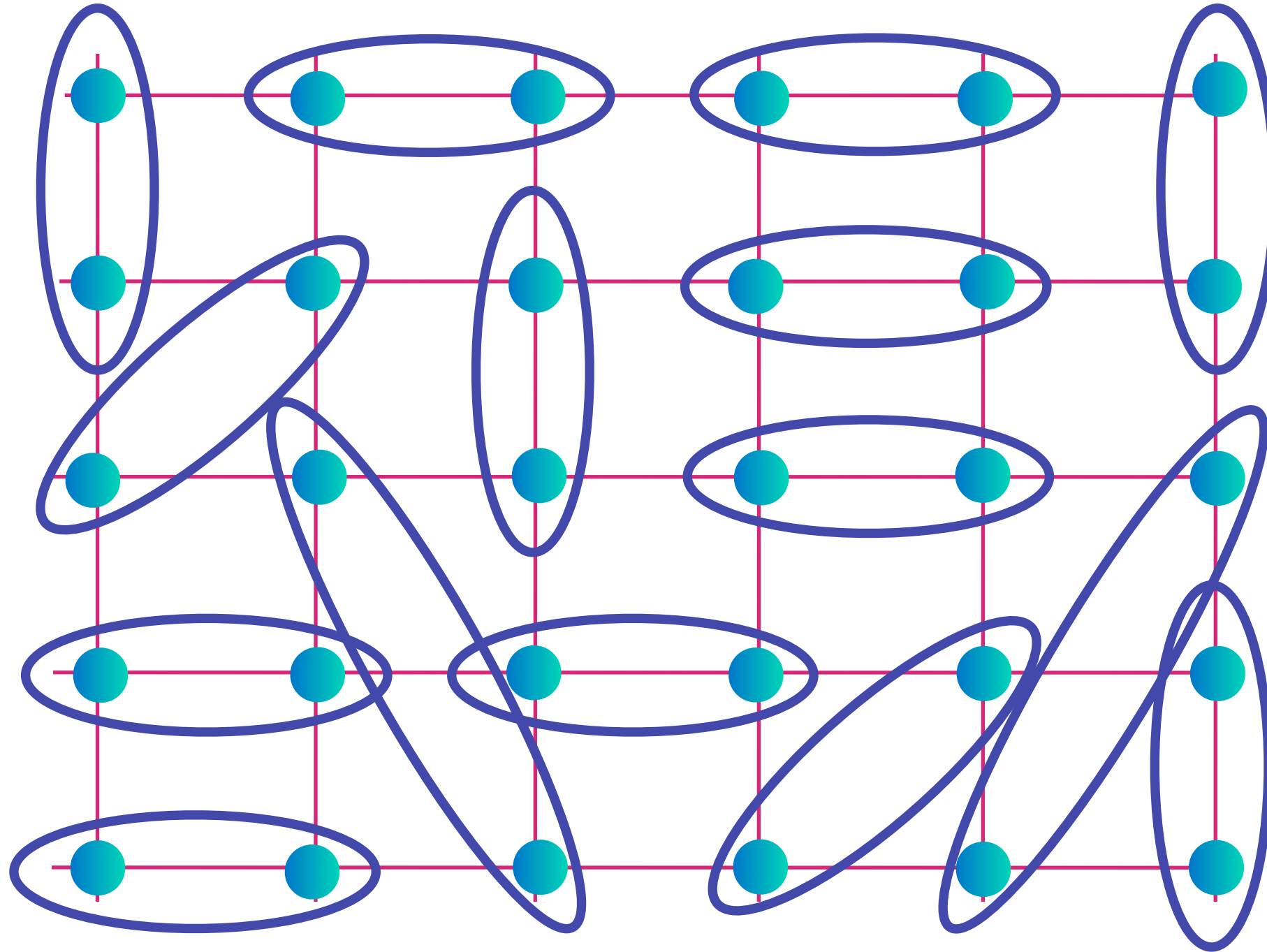
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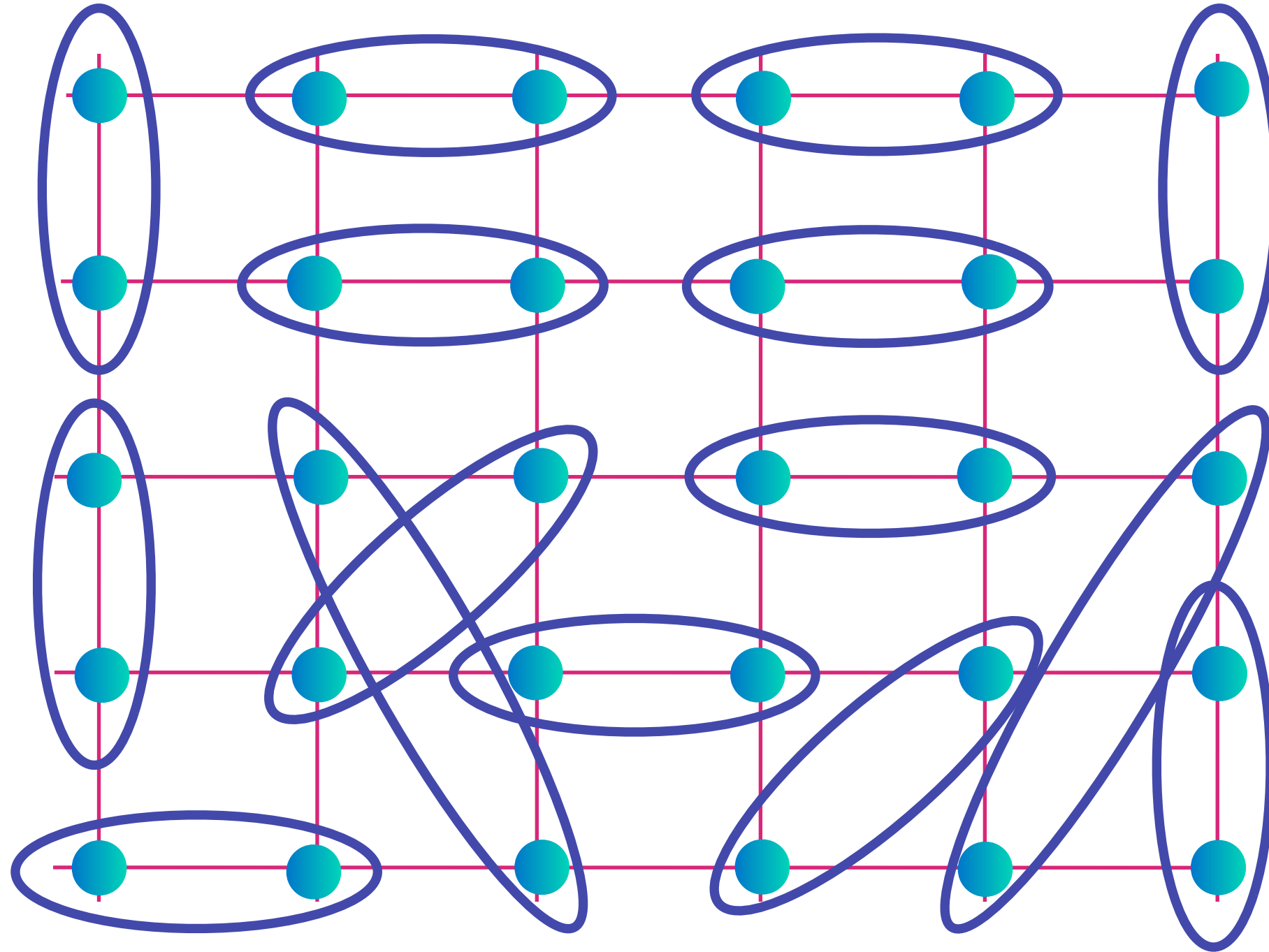
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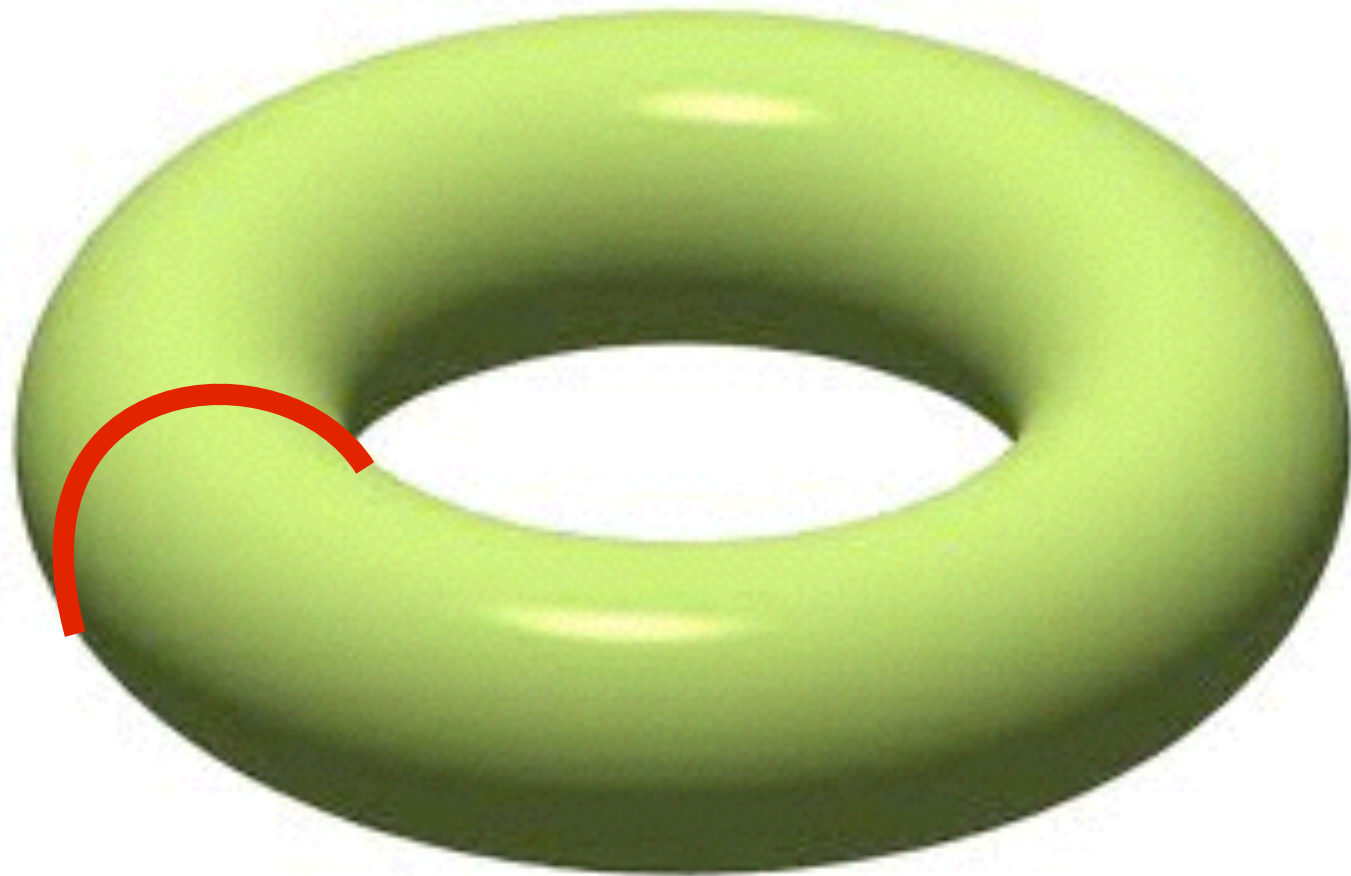
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Ground state degeneracy

Place
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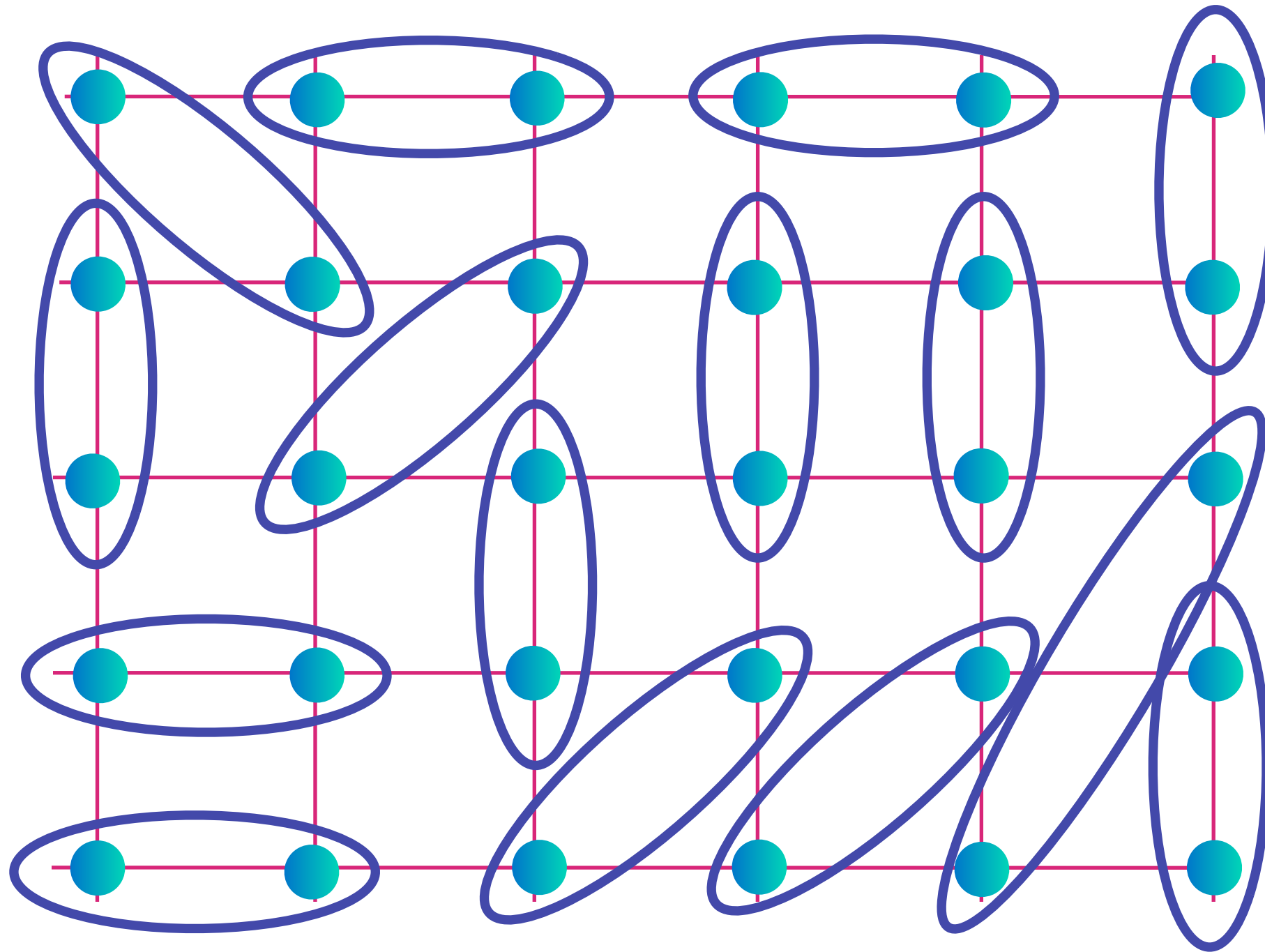
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D.J. Thouless, PRB 36, 7187 (1987)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. 6, 353 (1988)

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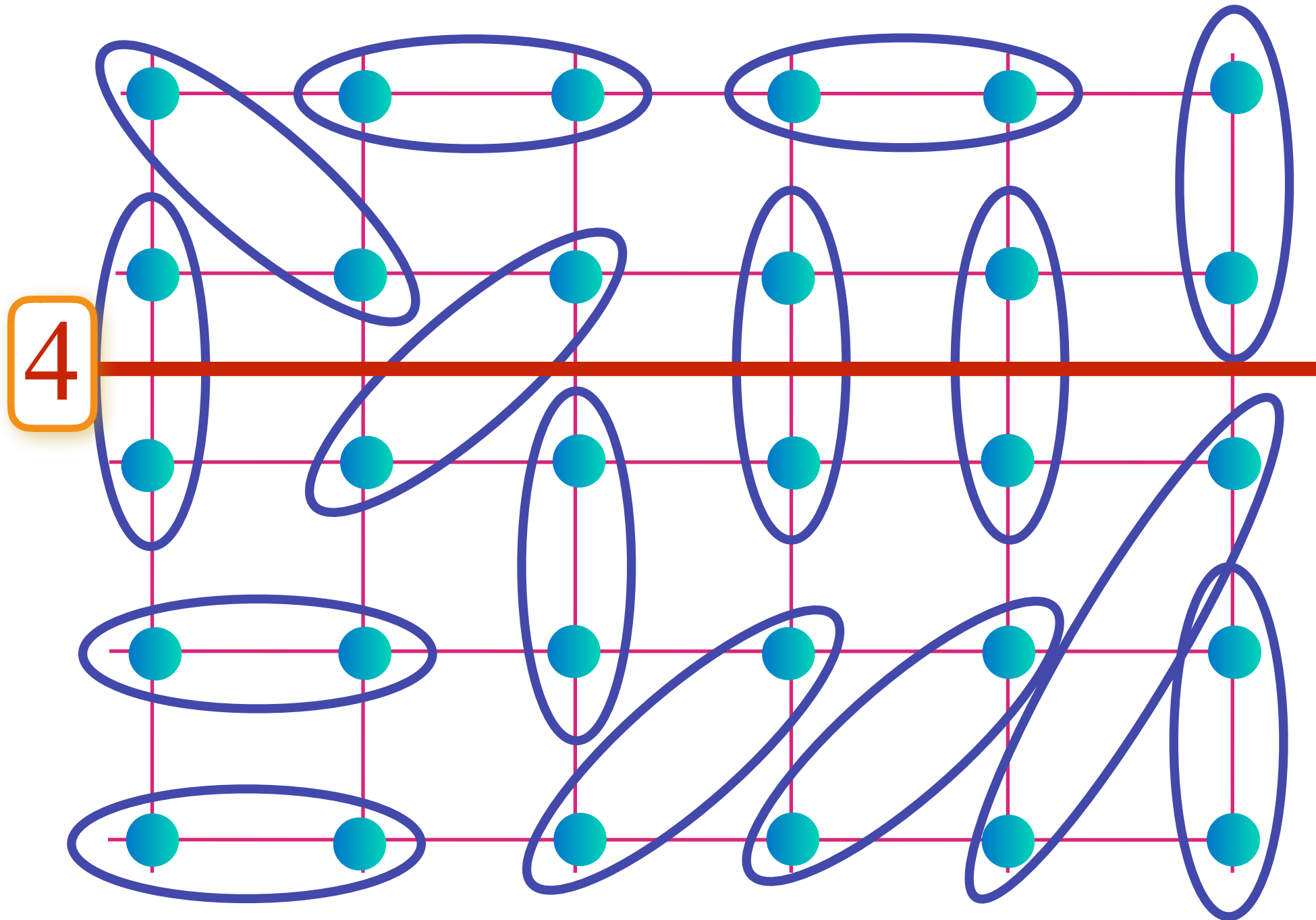
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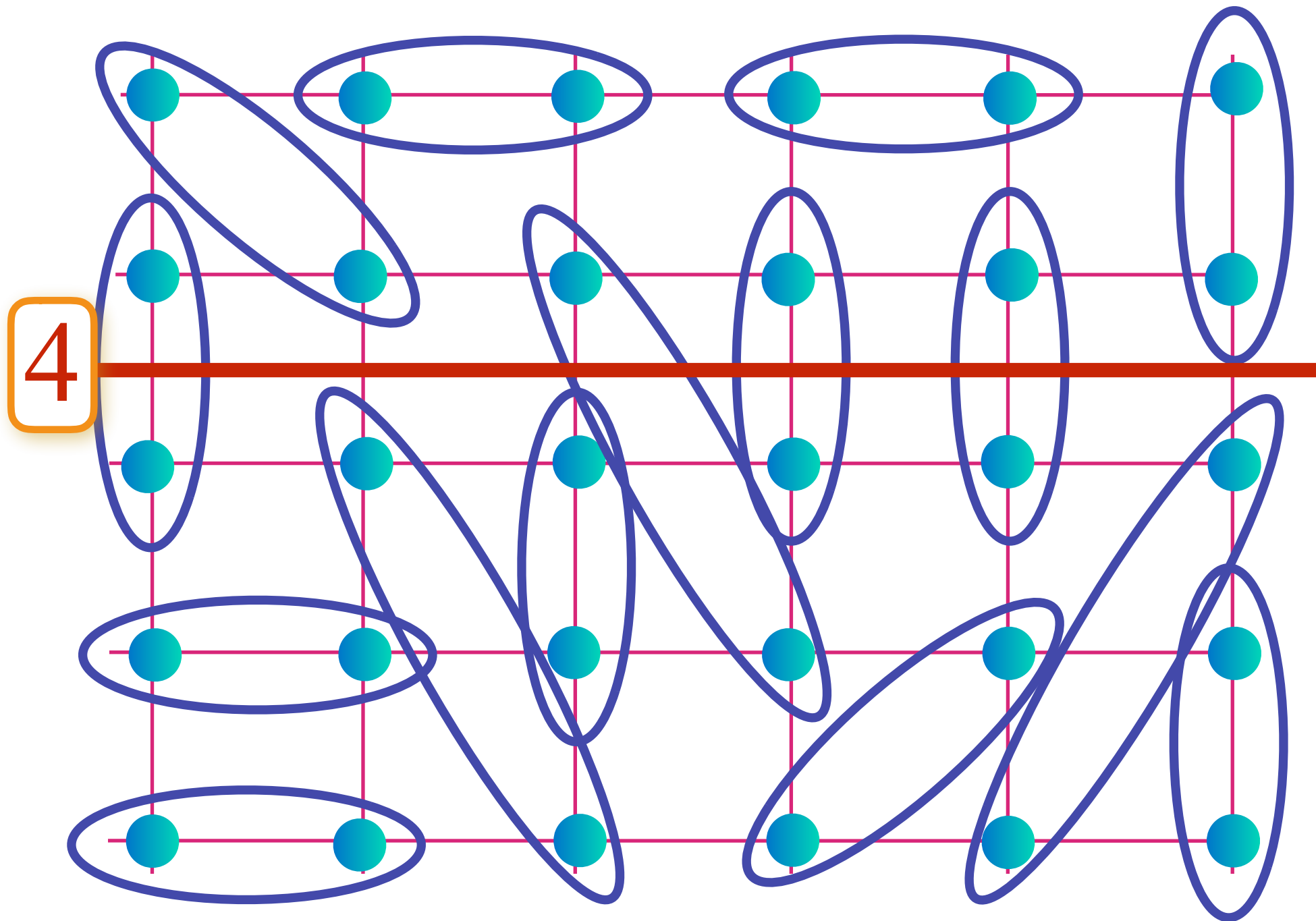
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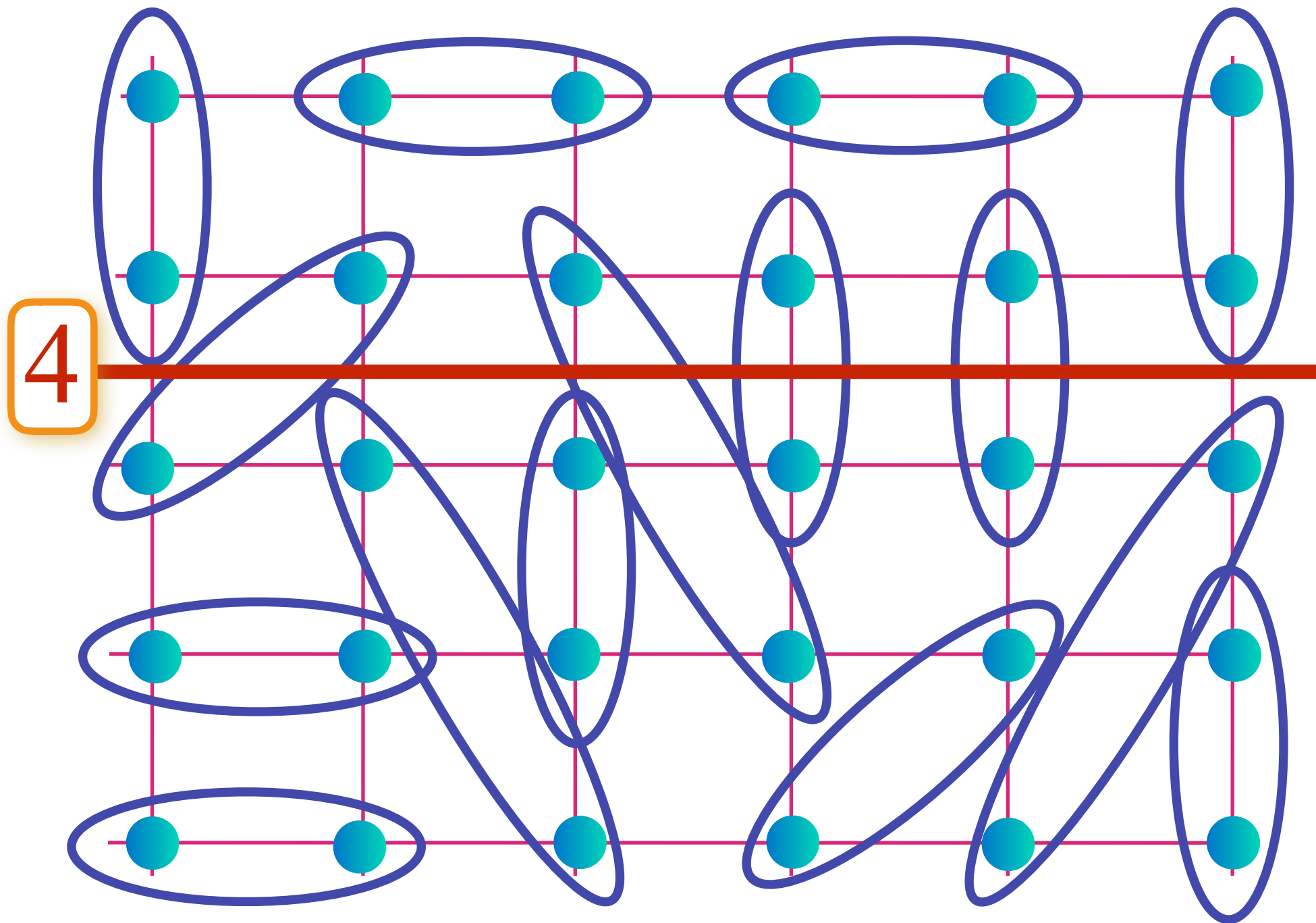
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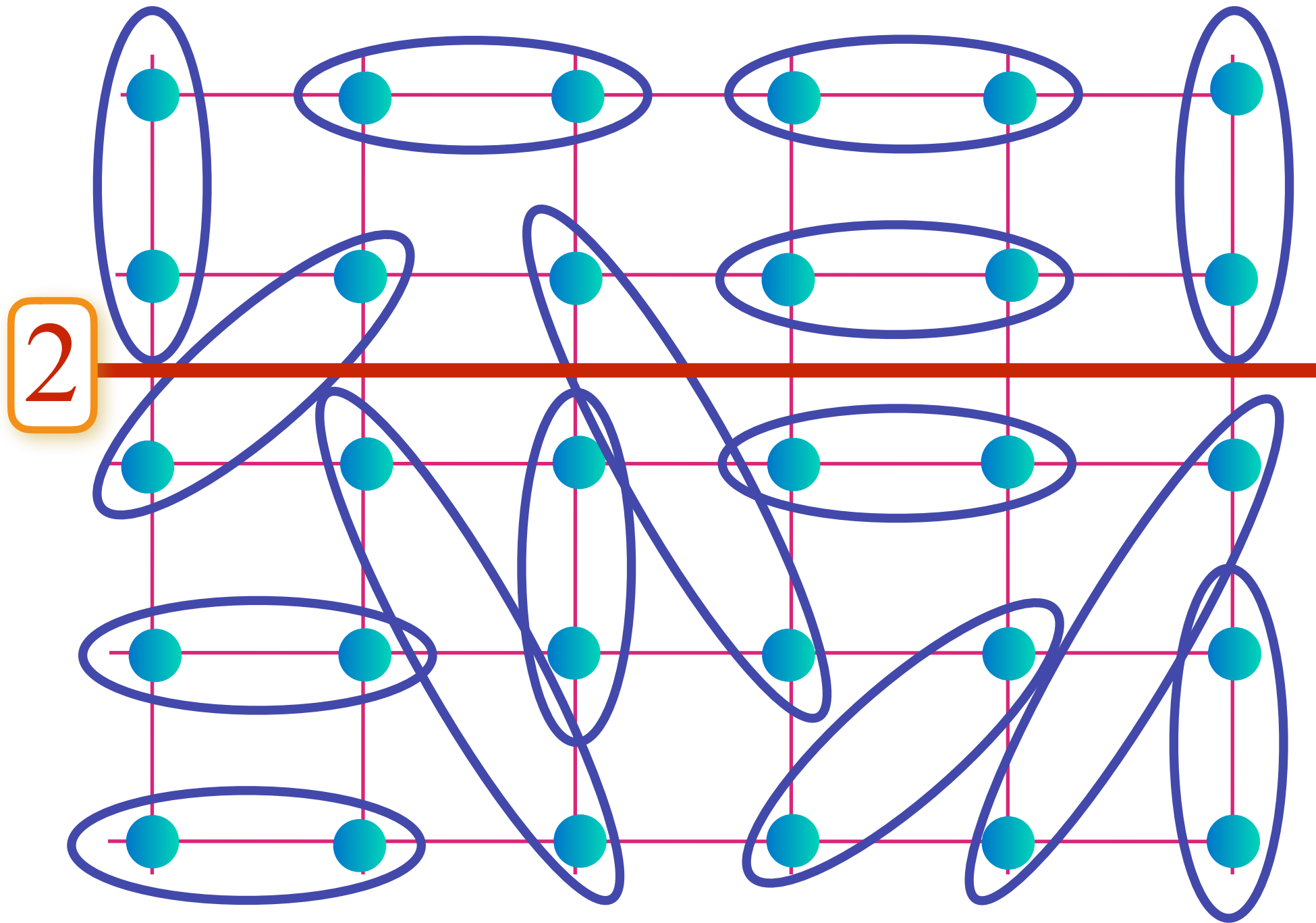
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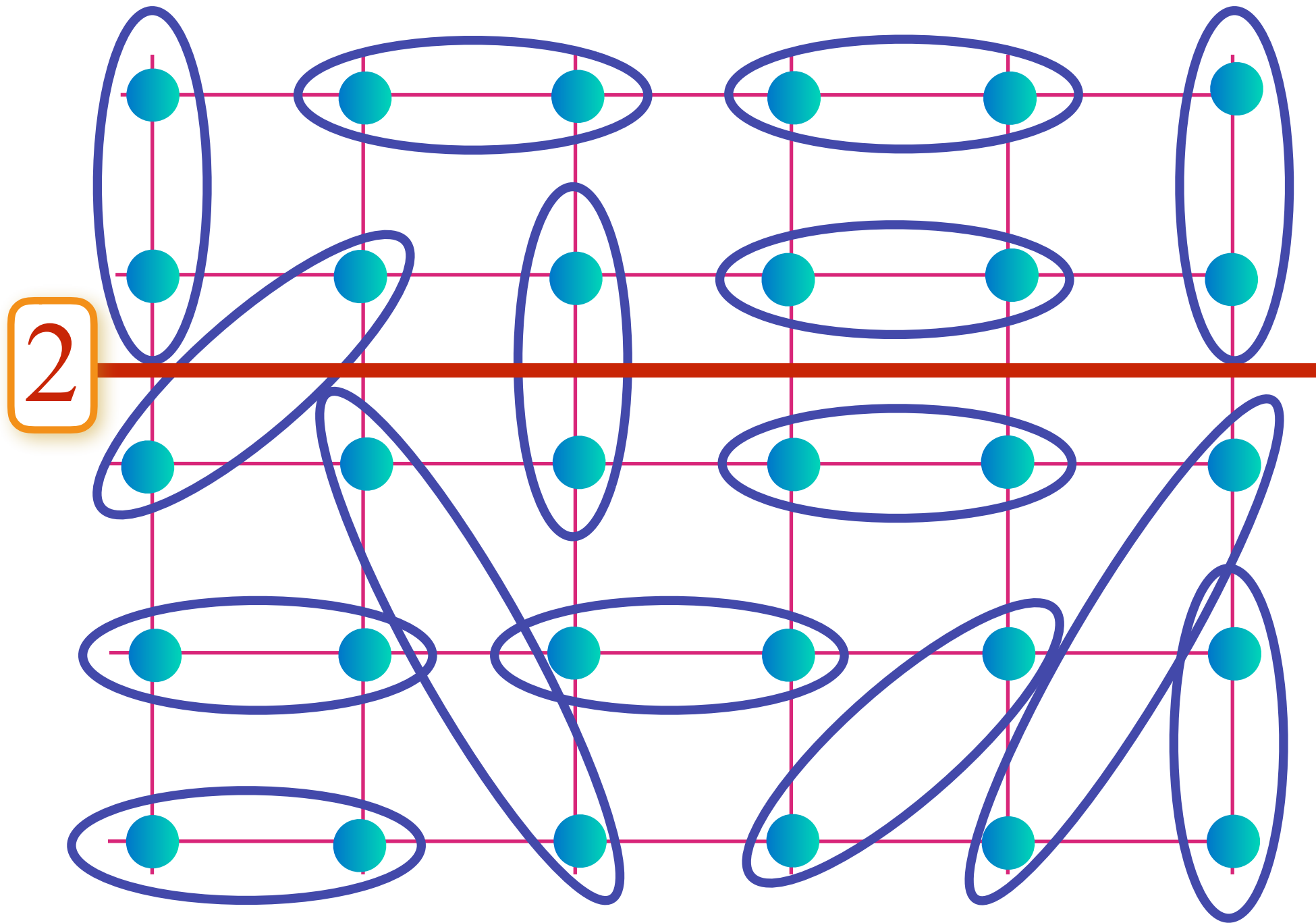
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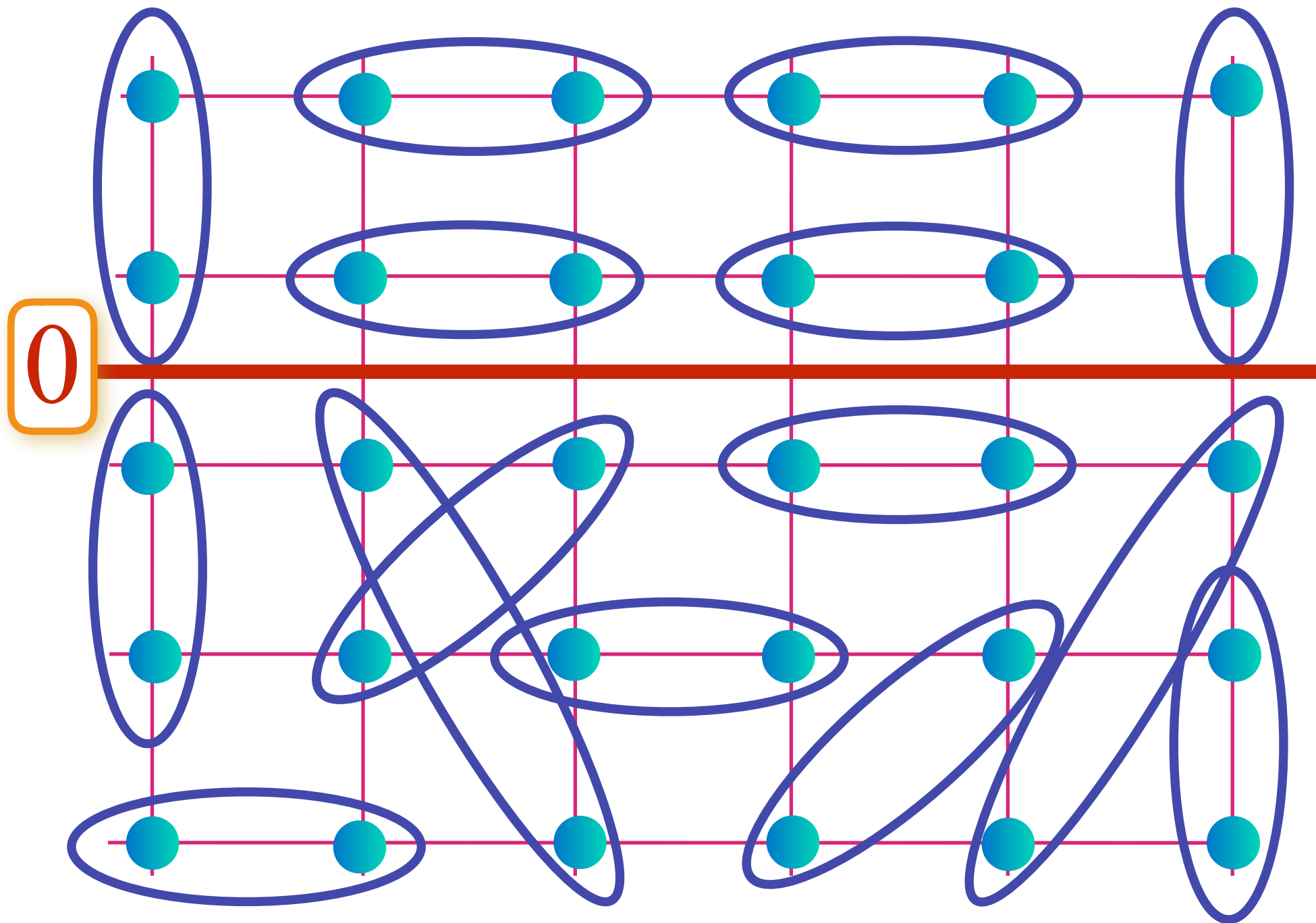
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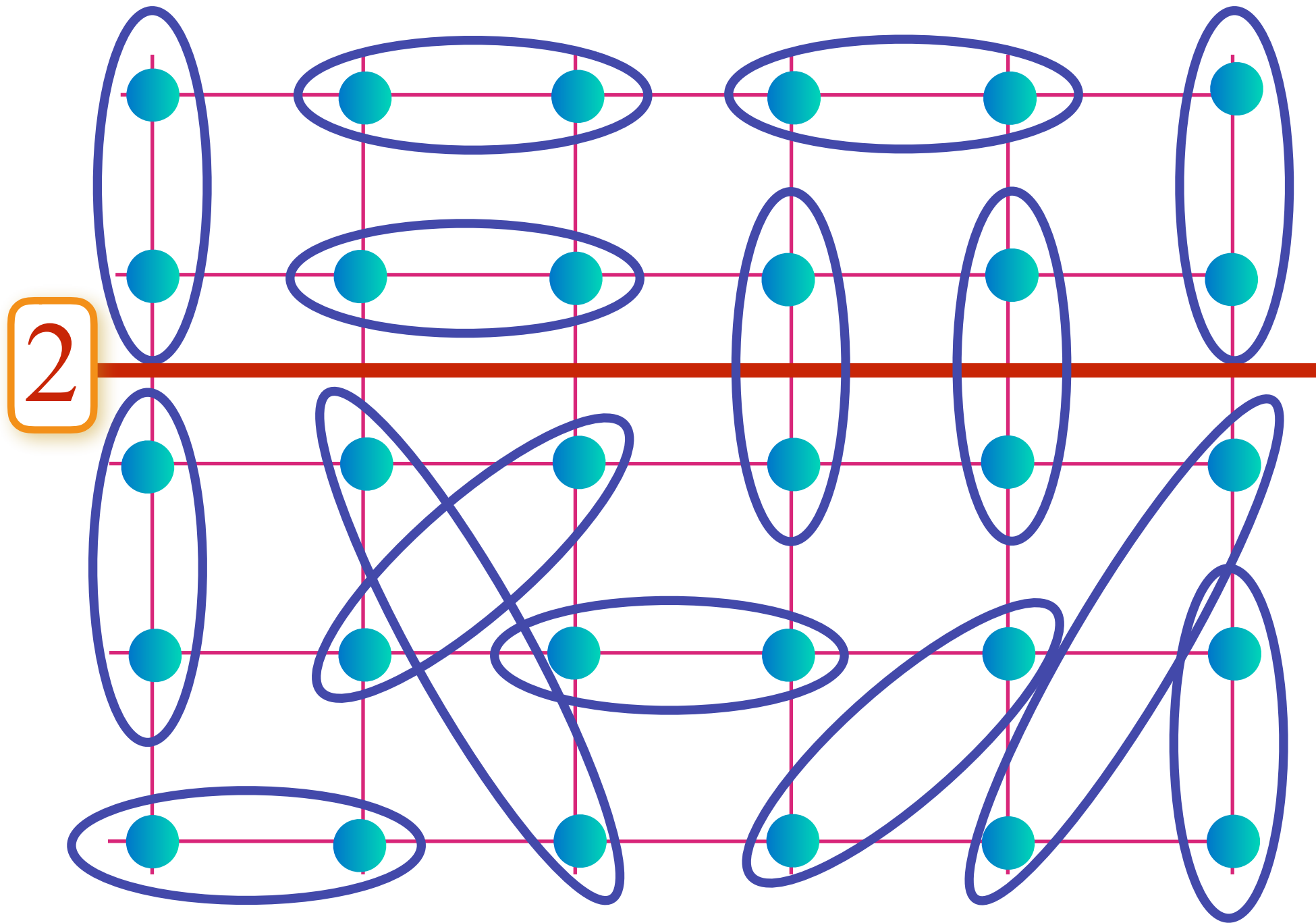
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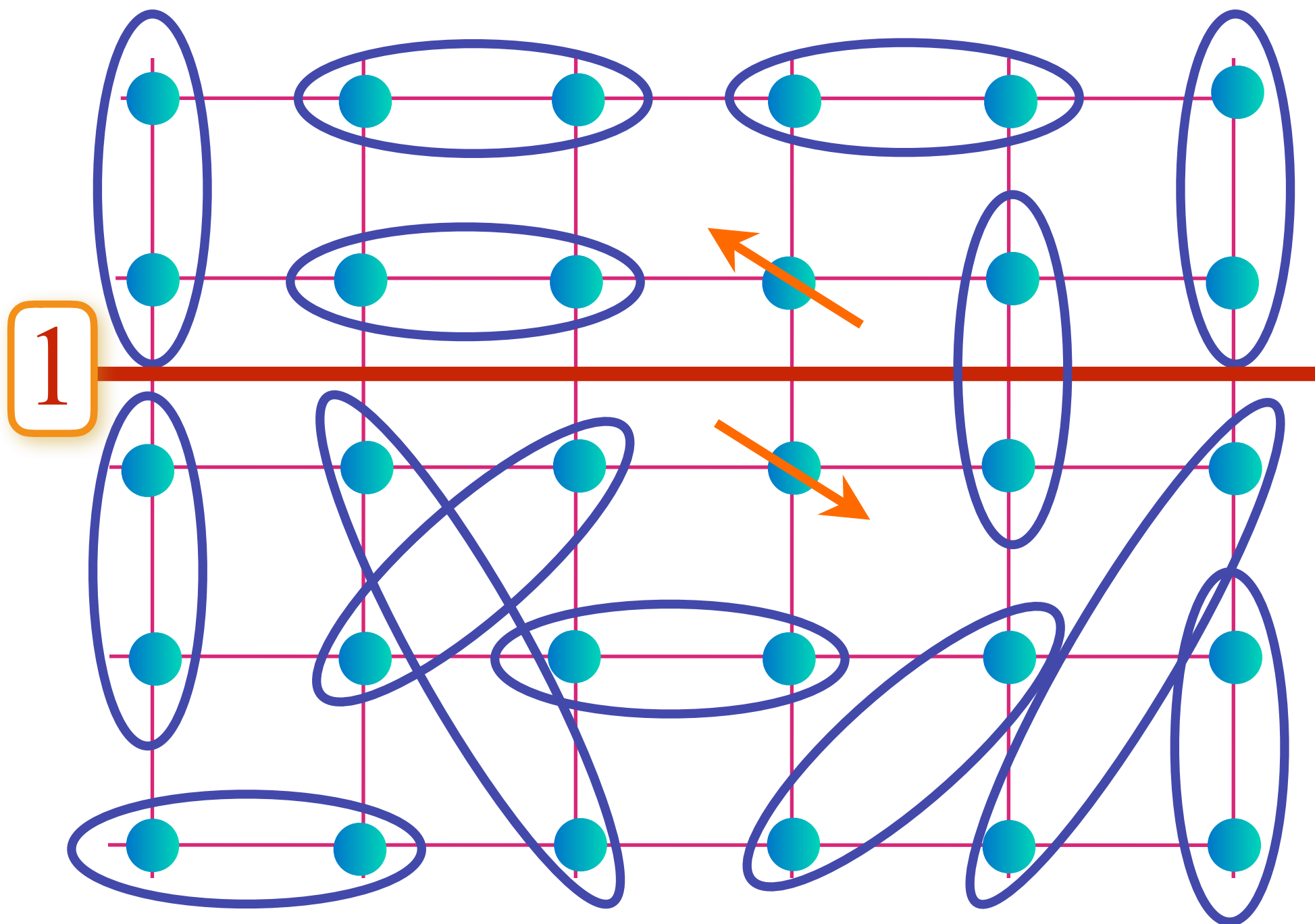
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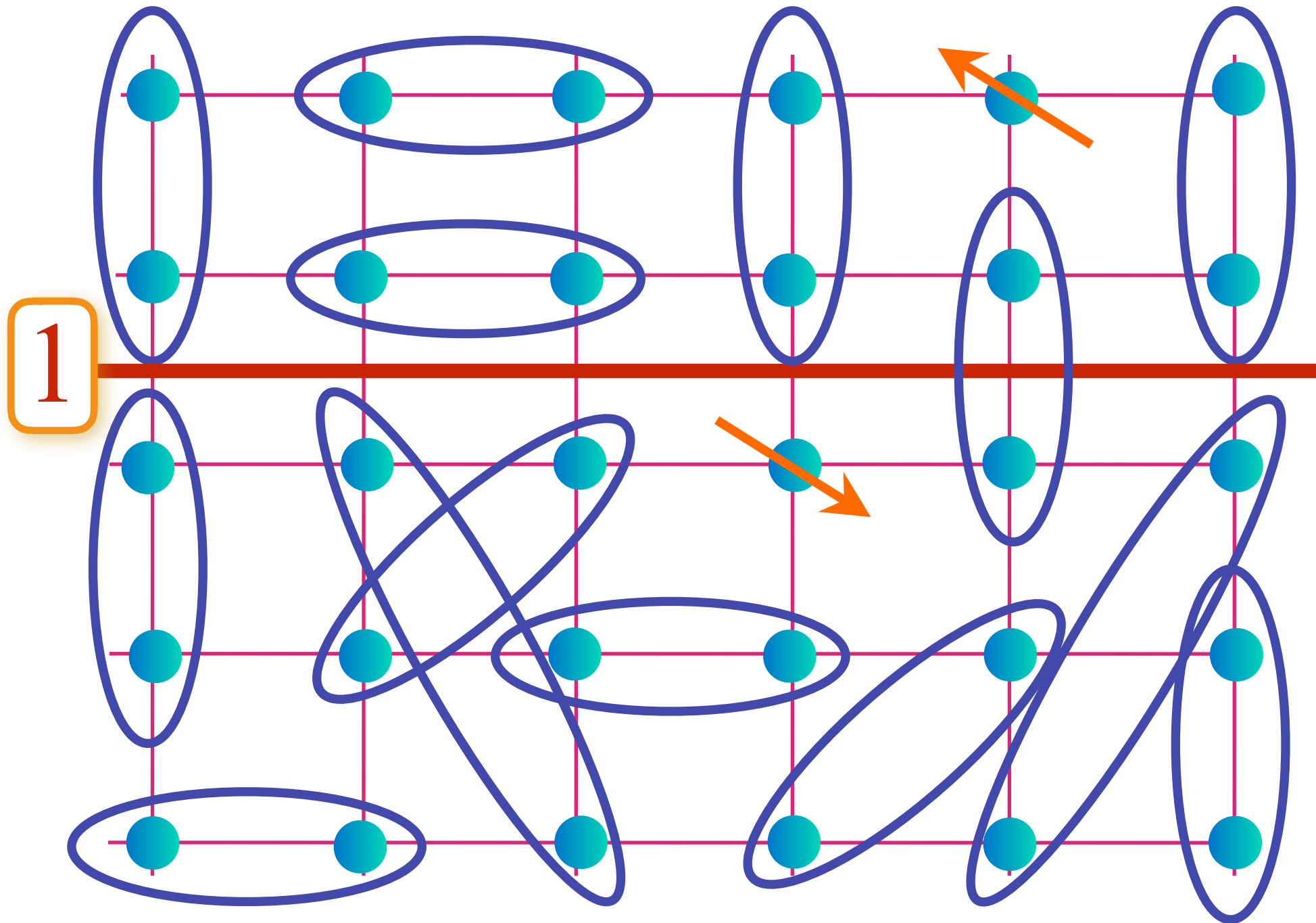
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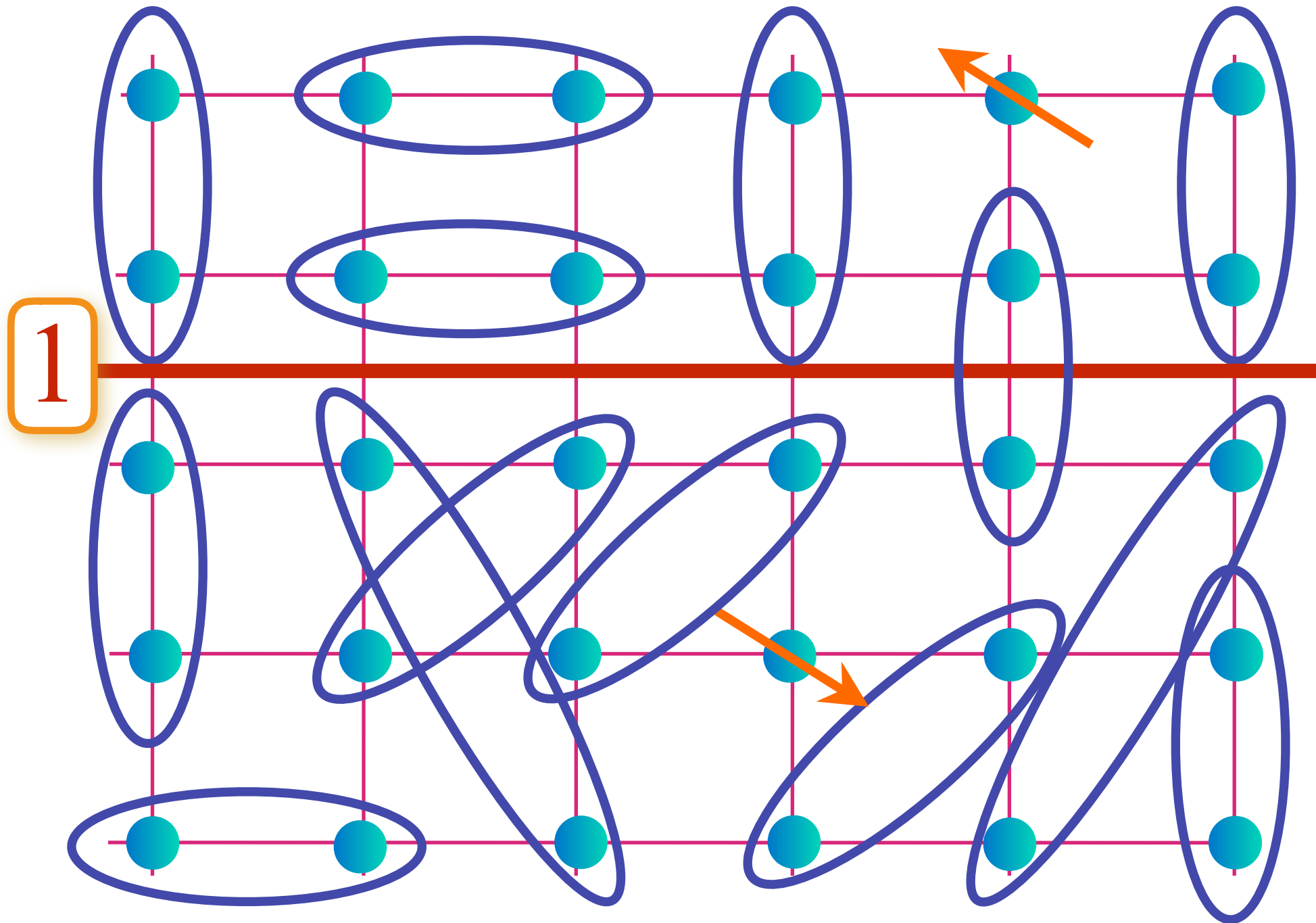
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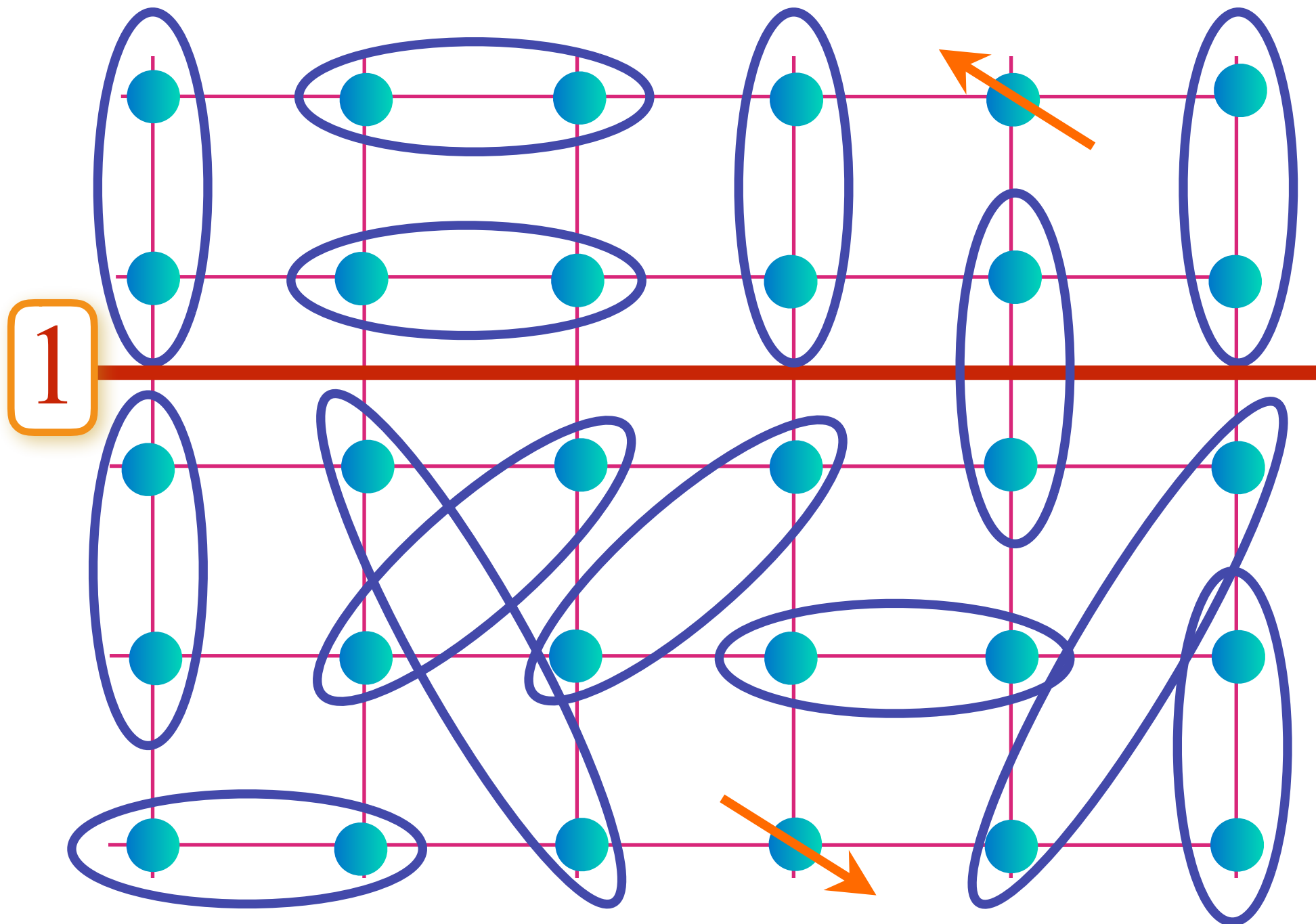
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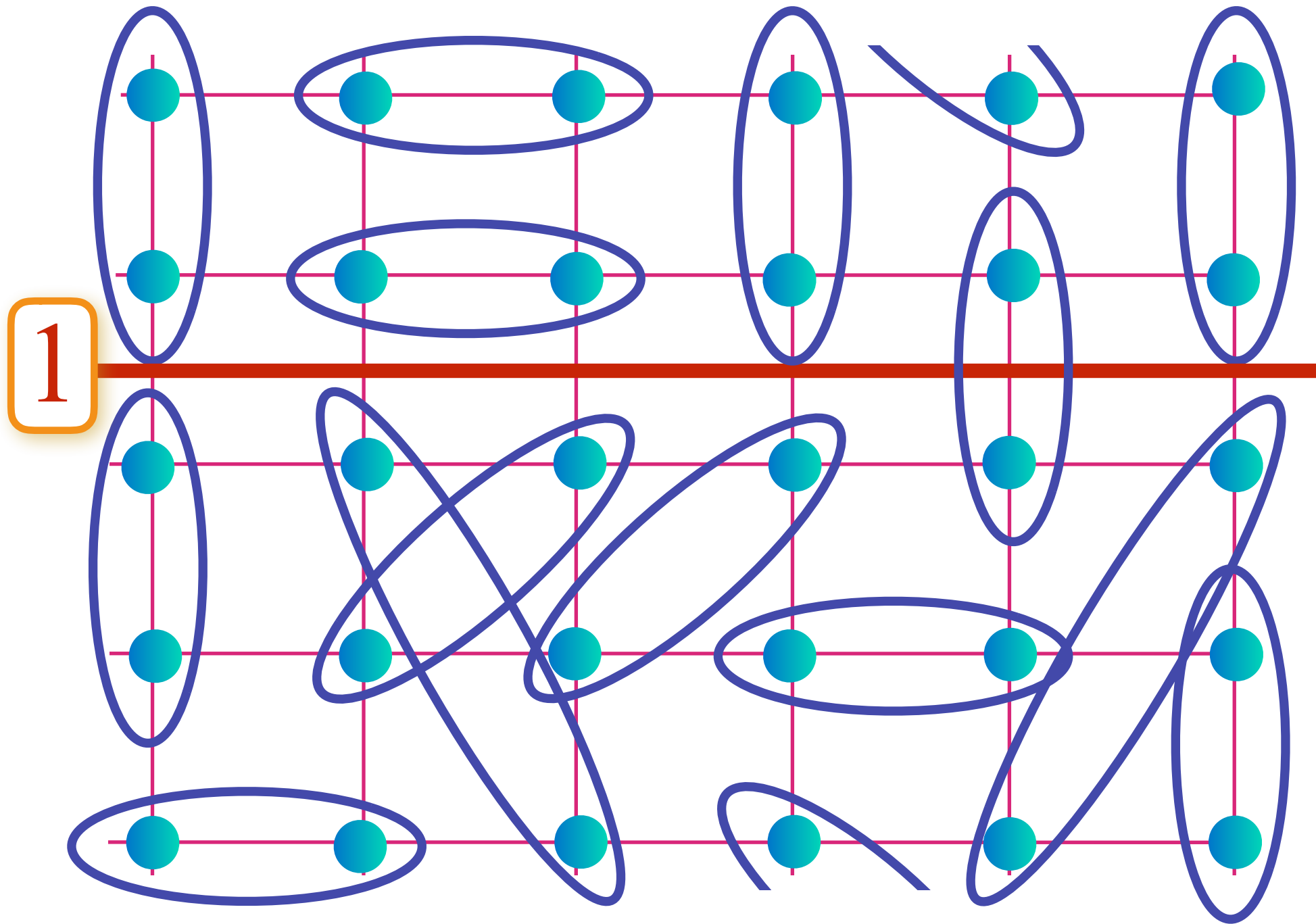
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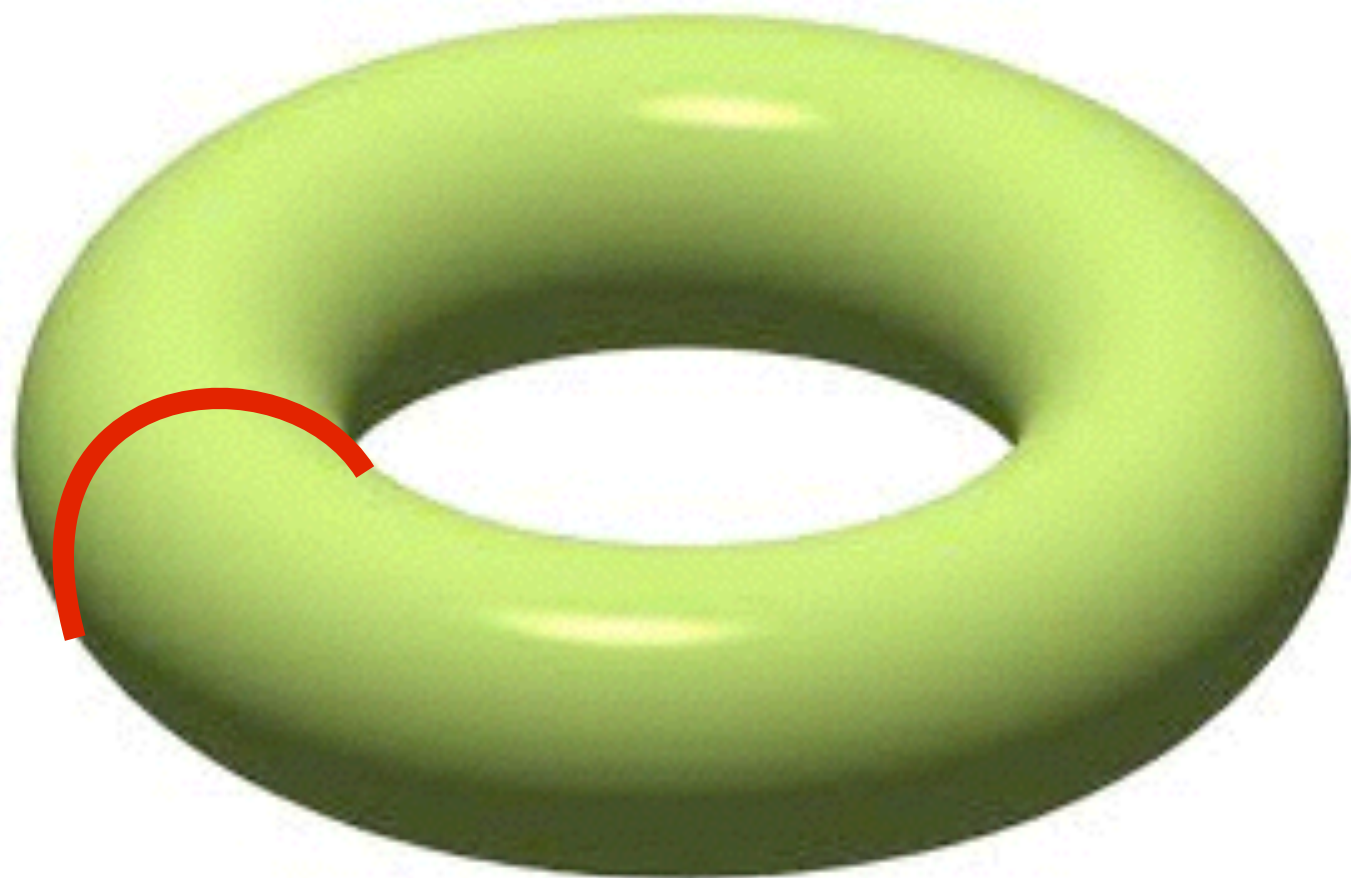
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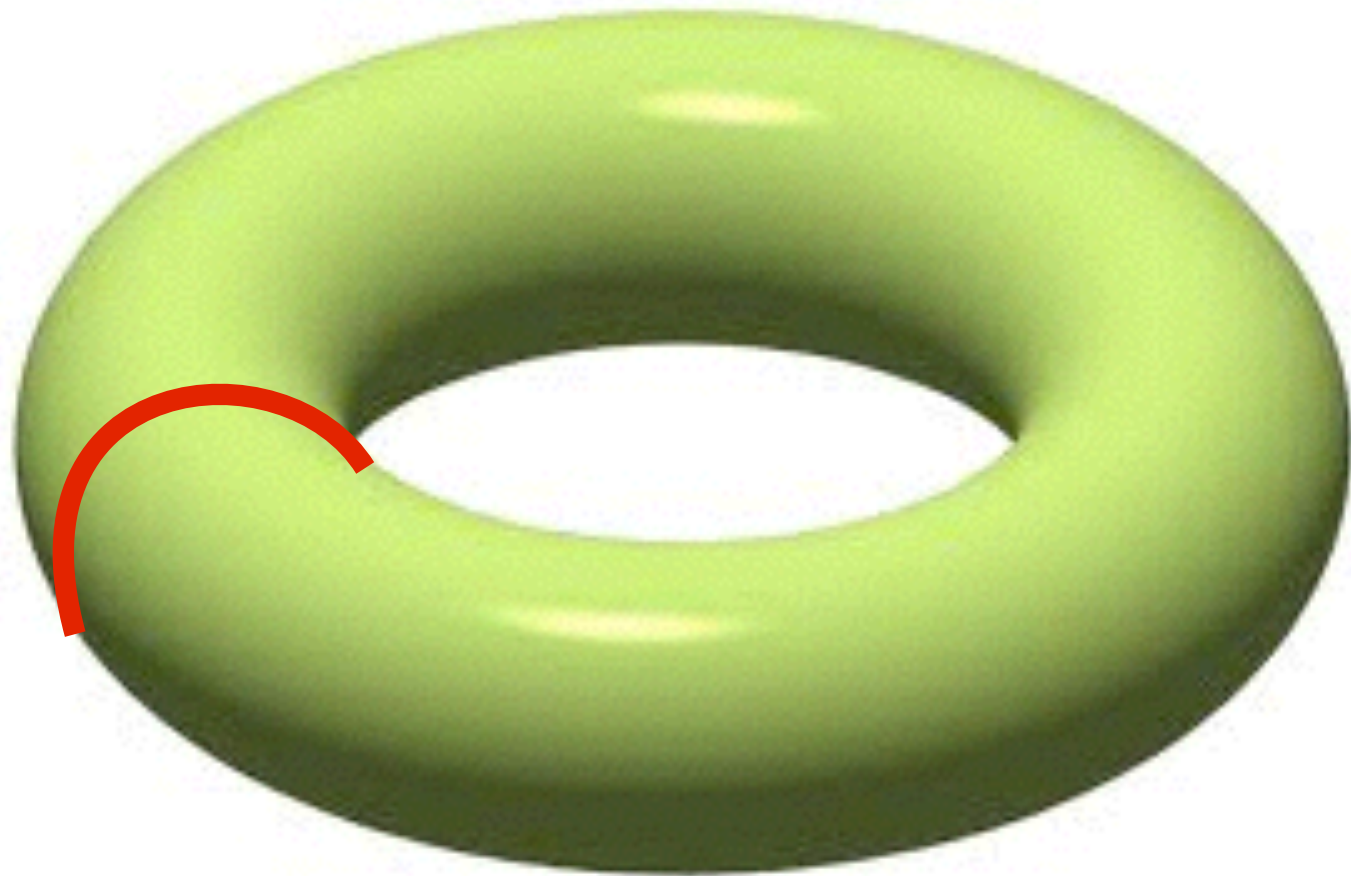
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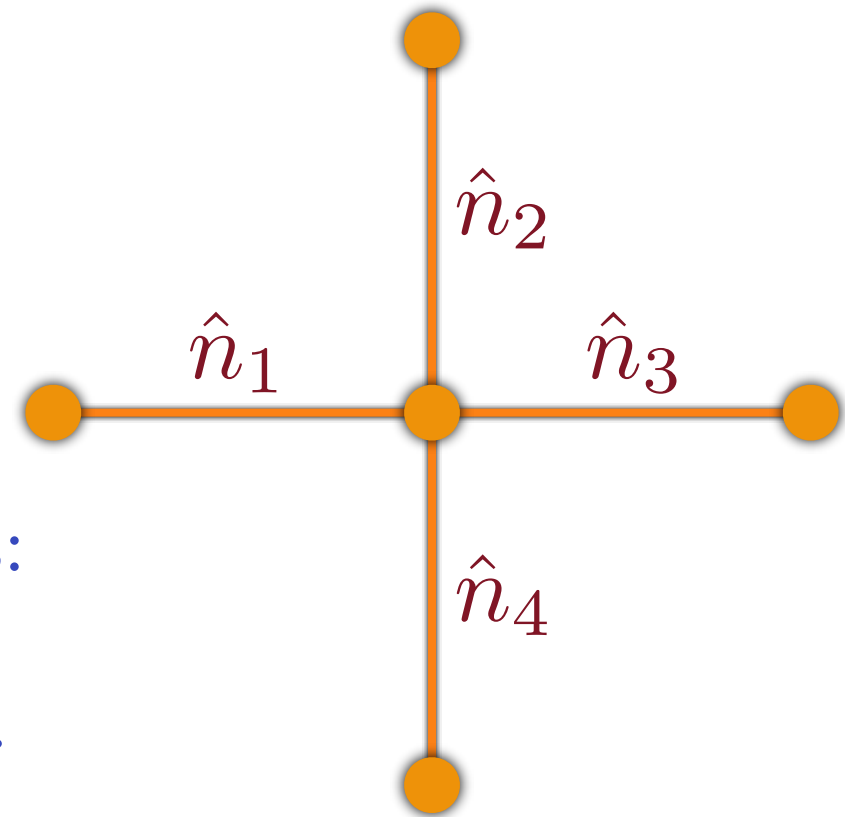


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Emergent gauge fields



Local constraint on dimer number operators:

$$\hat{n}_1 + \hat{n}_2 + \hat{n}_3 + \hat{n}_4 = 1.$$

Identify dimer number with an ‘electric’ field, $\hat{E}_{i\alpha} = (-1)^{i_x+i_y} \hat{n}_{i\alpha}$, ($\alpha = x, y$); the constraint becomes ‘Gauss’s Law’:

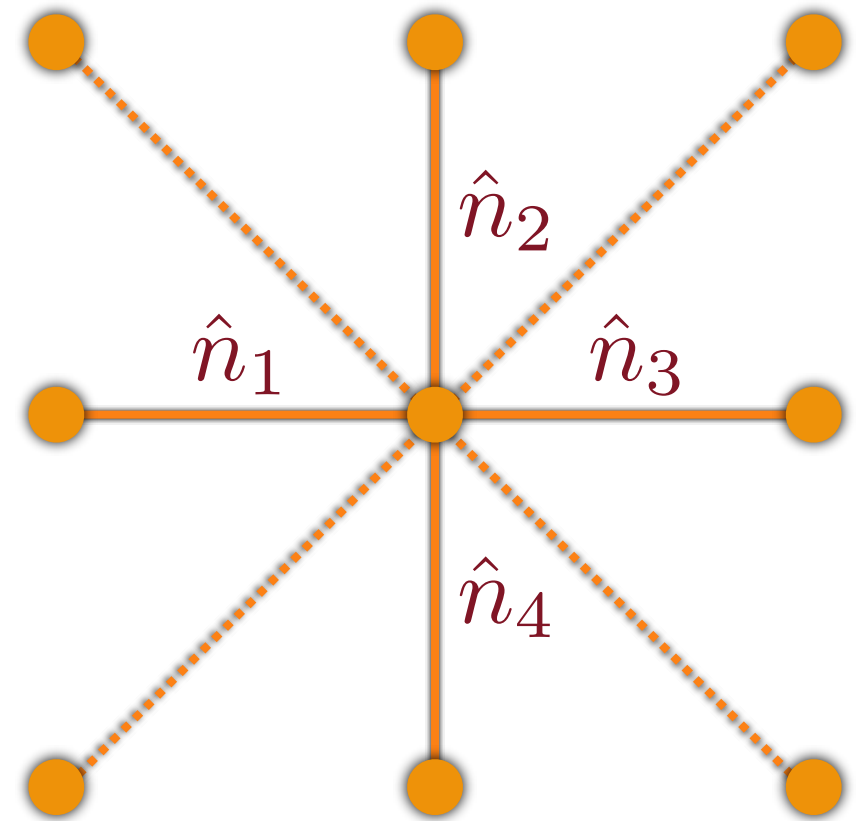
$$\Delta_\alpha \hat{E}_{i\alpha} = (-1)^{i_x+i_y}.$$

The theory of the dimers is *compact* U(1) quantum electrodynamics in the presence of static background charges. The *compact* theory allows the analog of Dirac’s magnetic monopoles as tunneling events/excitations.

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

E. Fradkin and S. A. Kivelson, Mod. Phys. Lett. B **4**, 225 (1990)

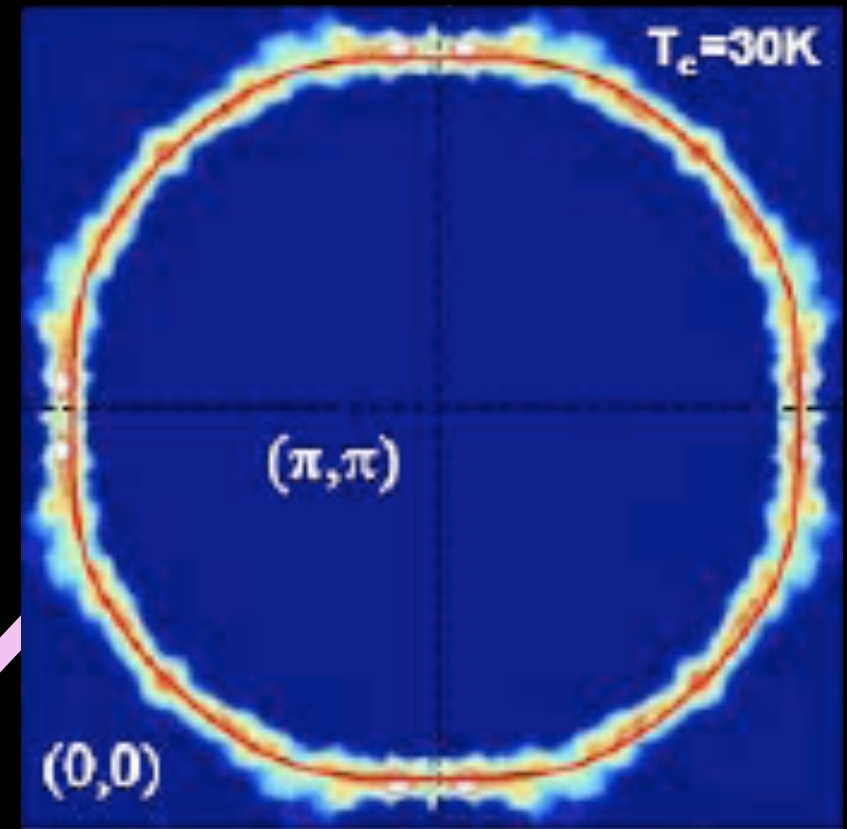
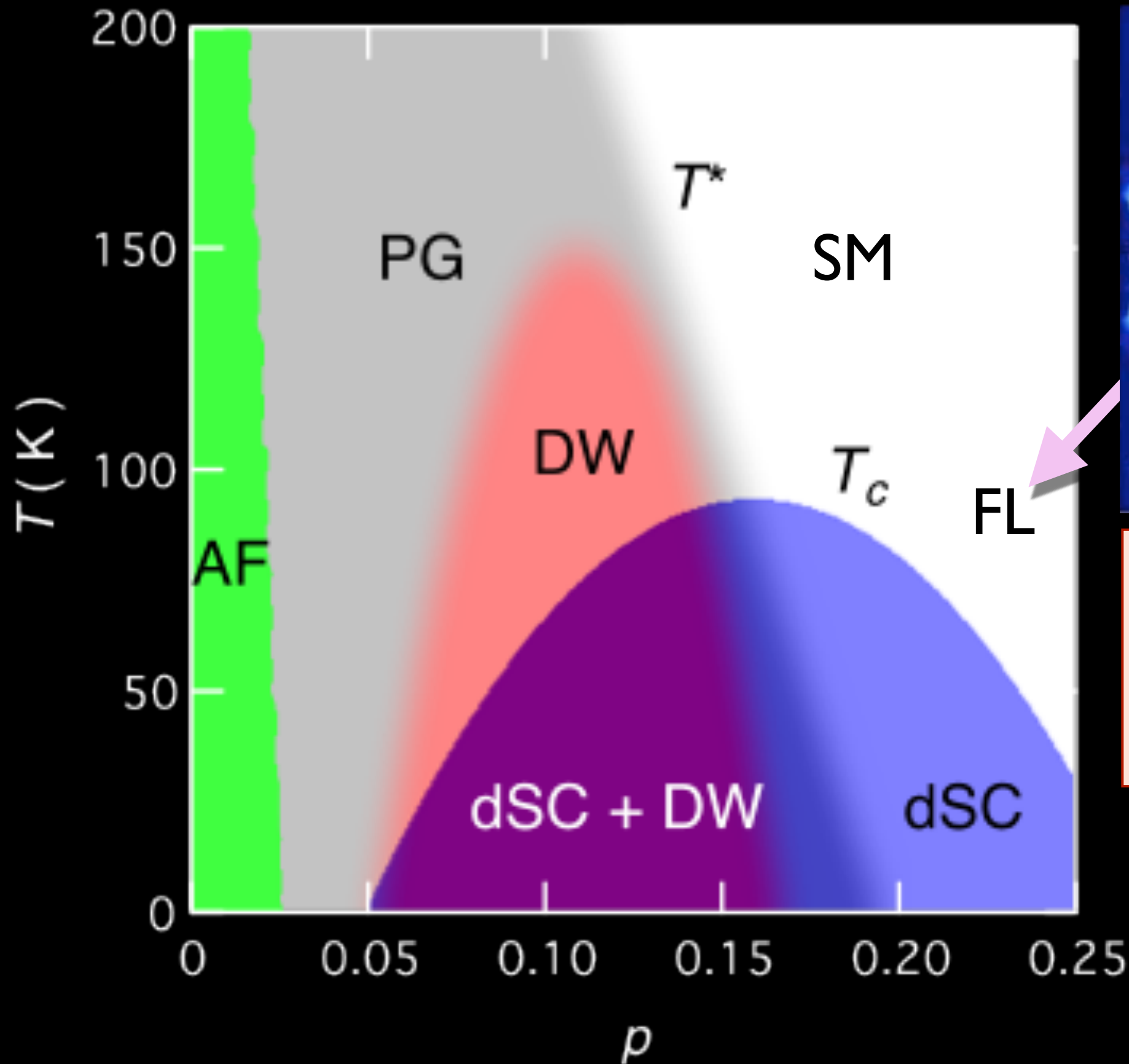
Emergent gauge fields



Including dimers connecting the same sublattice leads to a \mathbb{Z}_2 gauge theory in the presence of Berry phases of static background charges. This has a stable deconfined phase in $2+1$ dimensions. By varying parameters it can undergoes a confinement transition to a valence bond solid, described by a frustrated Ising model.

I. Emergent gauge fields and long-range entanglement in insulators

M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



A conventional metal:
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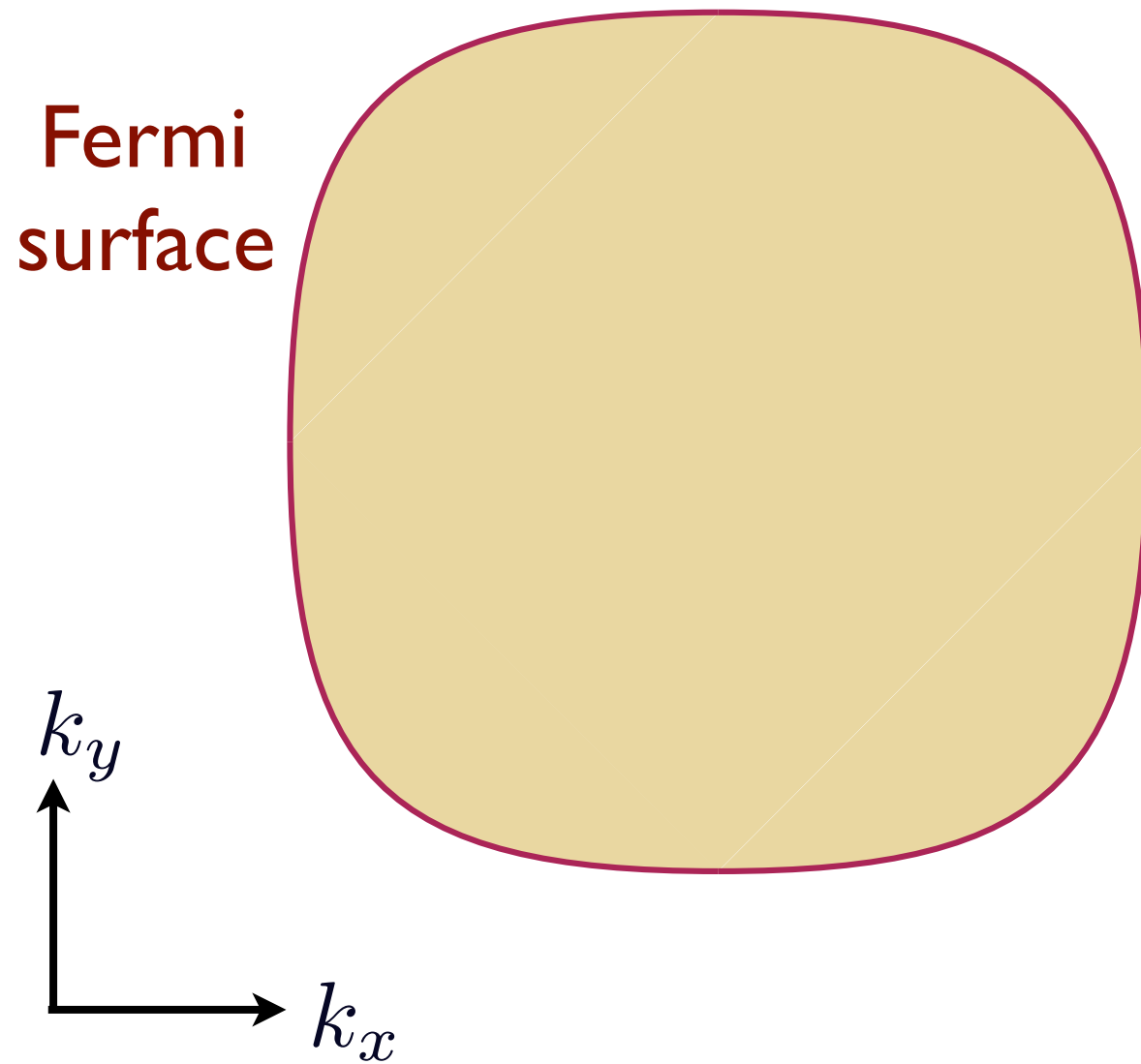
2. Theory of ordinary metals: Fermi liquids (FL)

(a) Quasiparticles

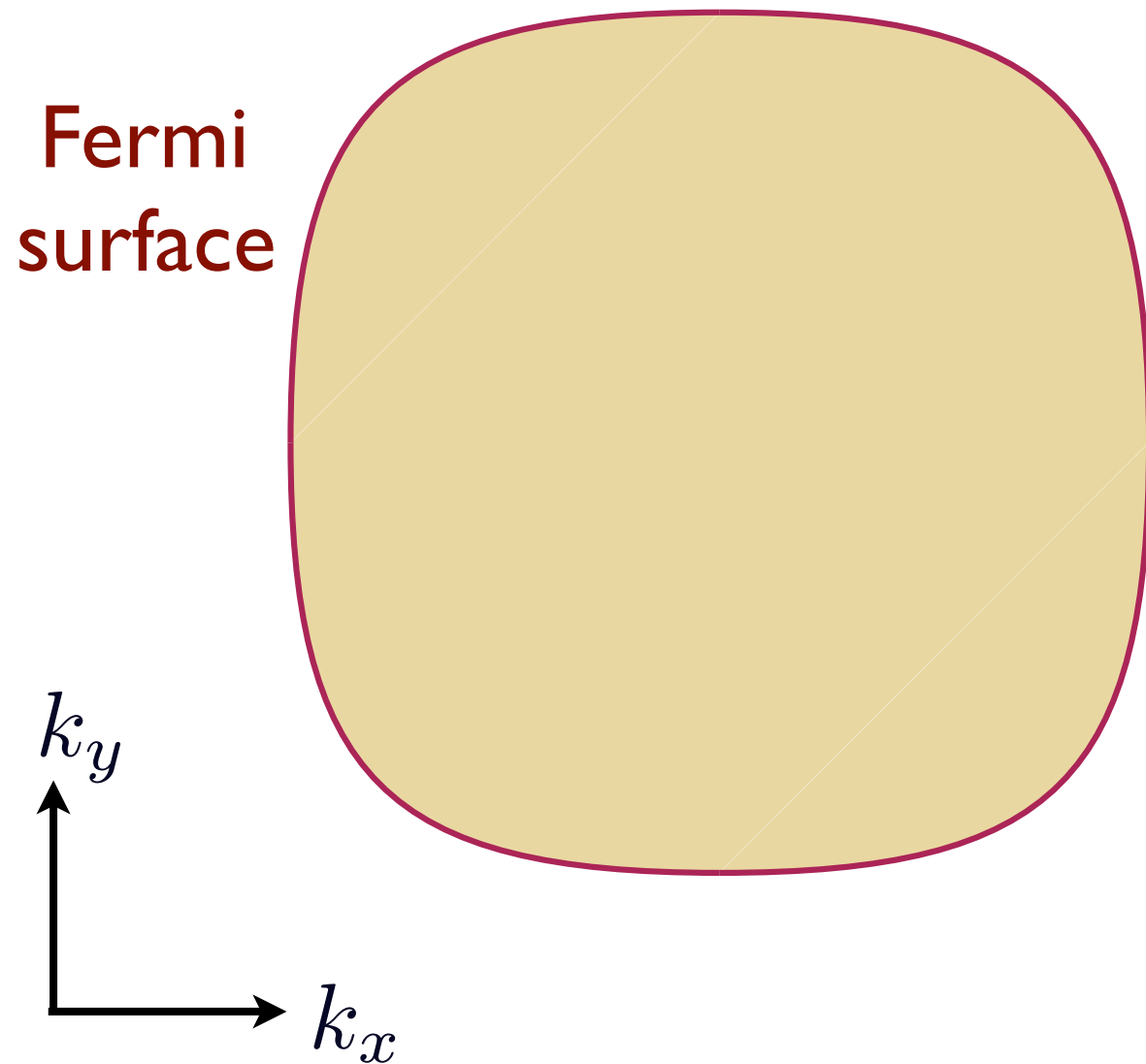
(b) Luttinger theorem for volume enclosed by Fermi surface

Ordinary metals: the Fermi liquid

- Fermi surface separates empty and occupied states in momentum space.



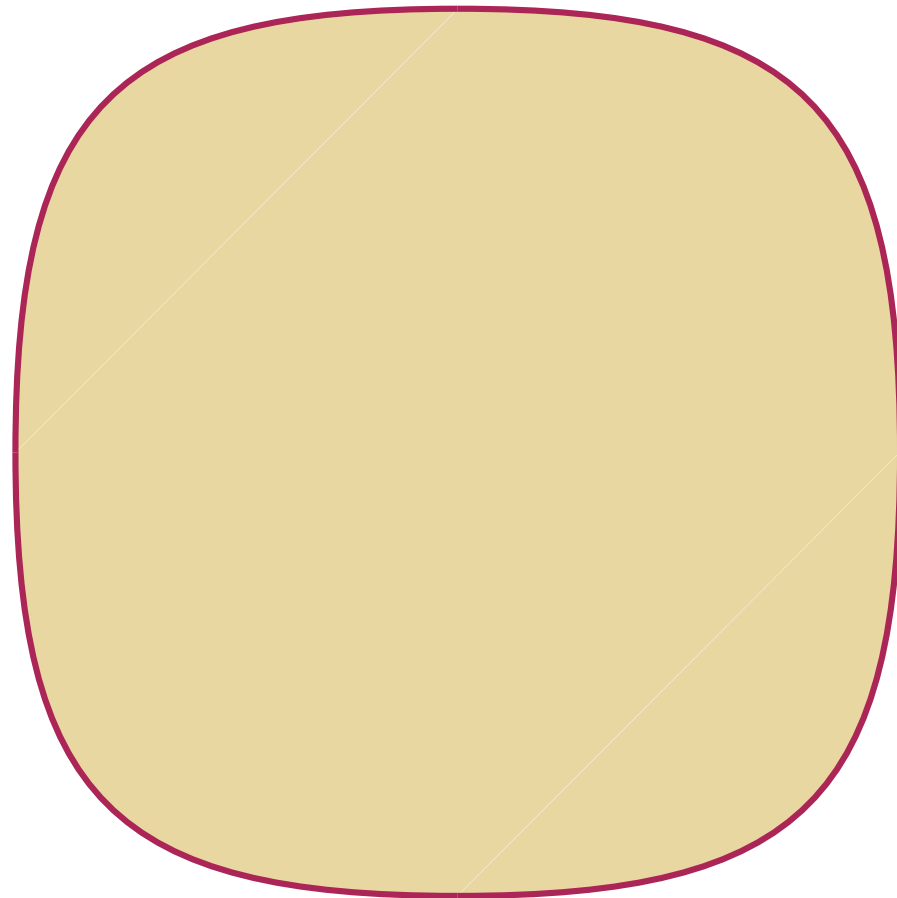
Ordinary metals: the Fermi liquid



- Fermi surface separates empty and occupied states in momentum space.
- *Luttinger Theorem*: volume (area) enclosed by Fermi surface = the electron density.

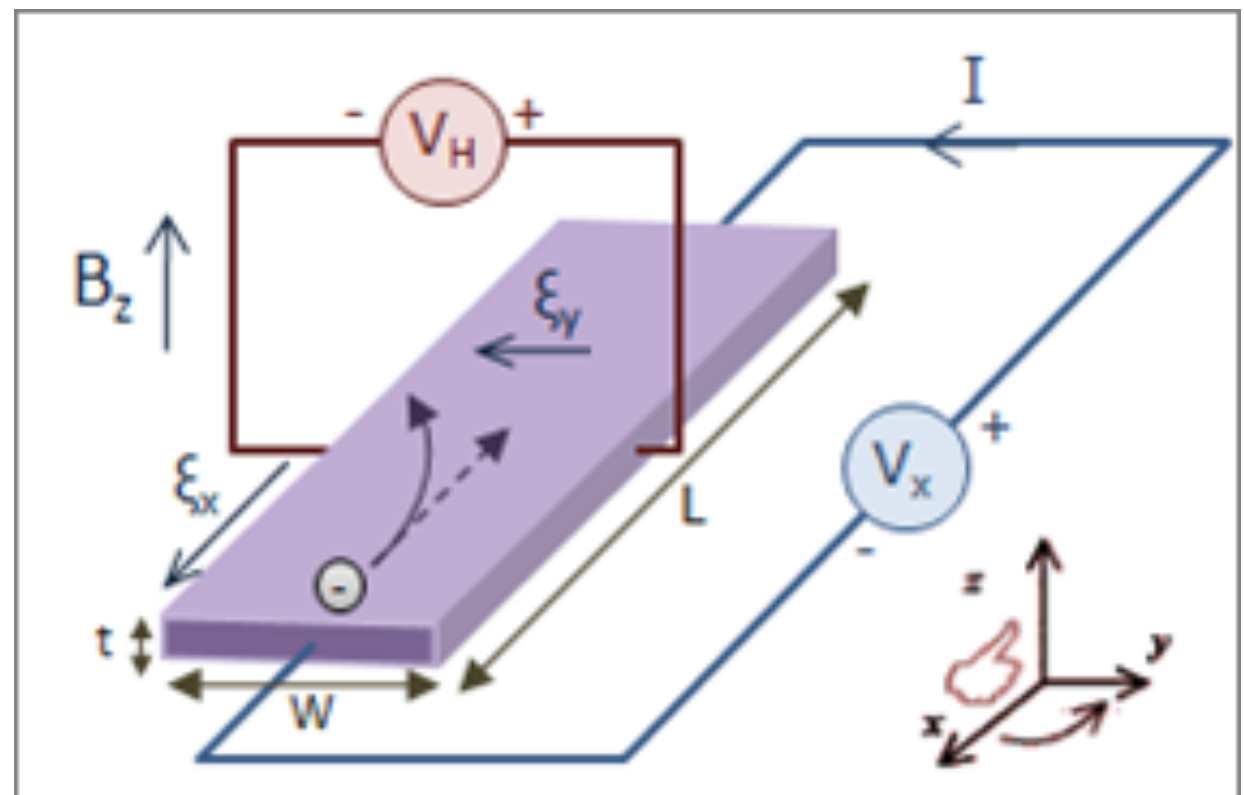
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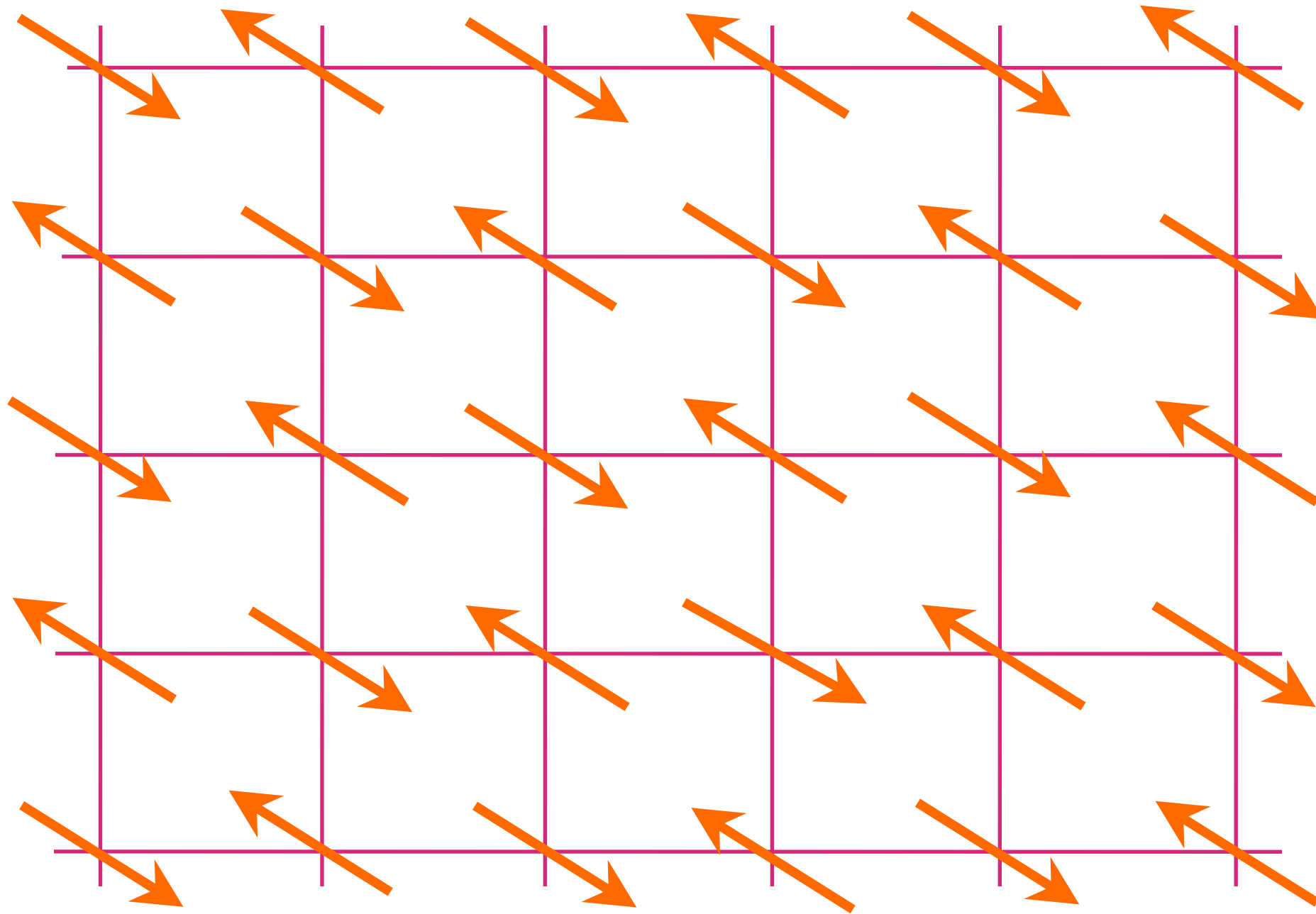
Fermi surface



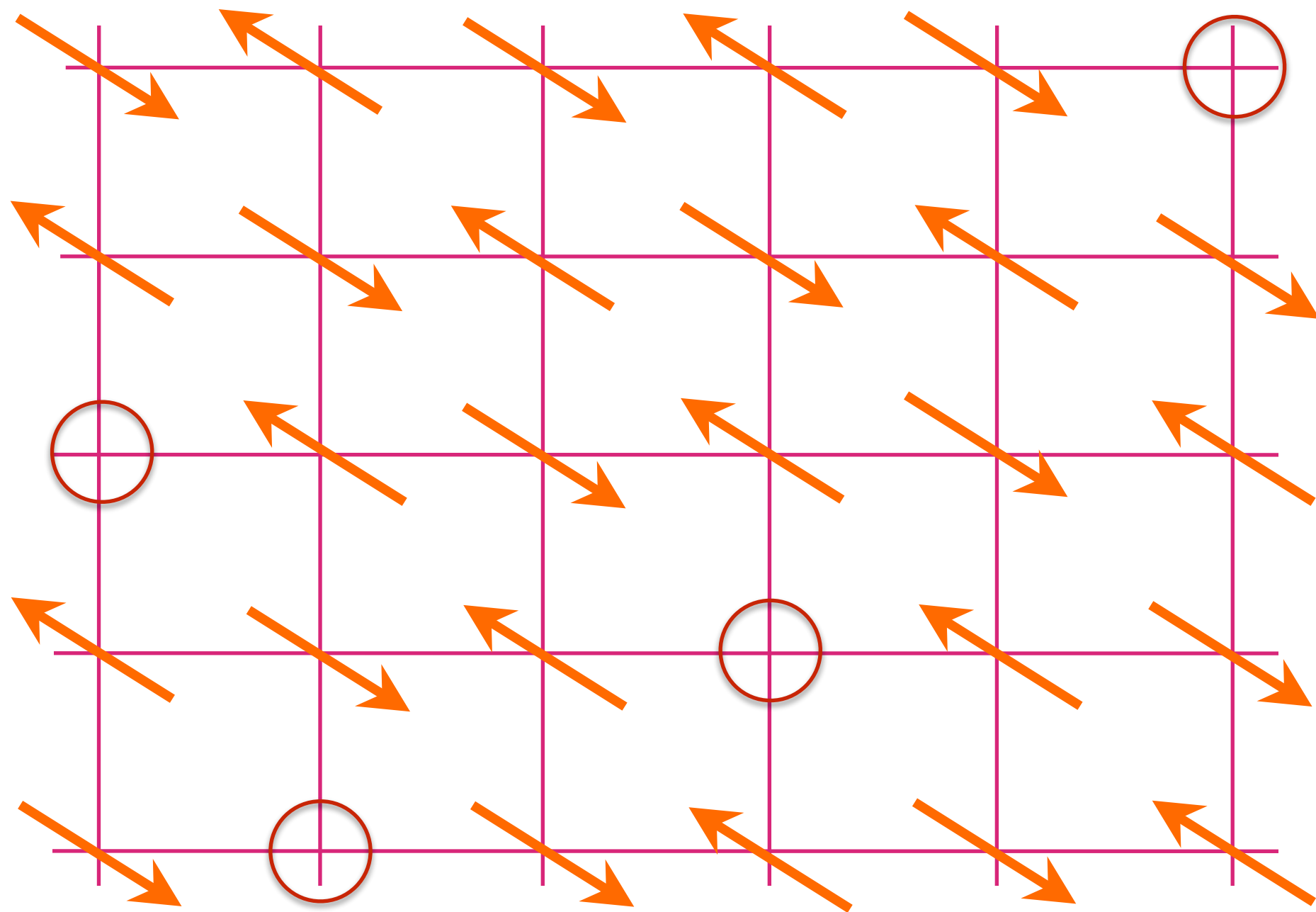
k_y
 k_x

- Fermi surface separates empty and occupied states in momentum space.
- *Luttinger Theorem*: volume (area) enclosed by Fermi surface = the electron density.
- Hall coefficient
 $R_H = -1/((\text{Fermi volume}) \times e)$.

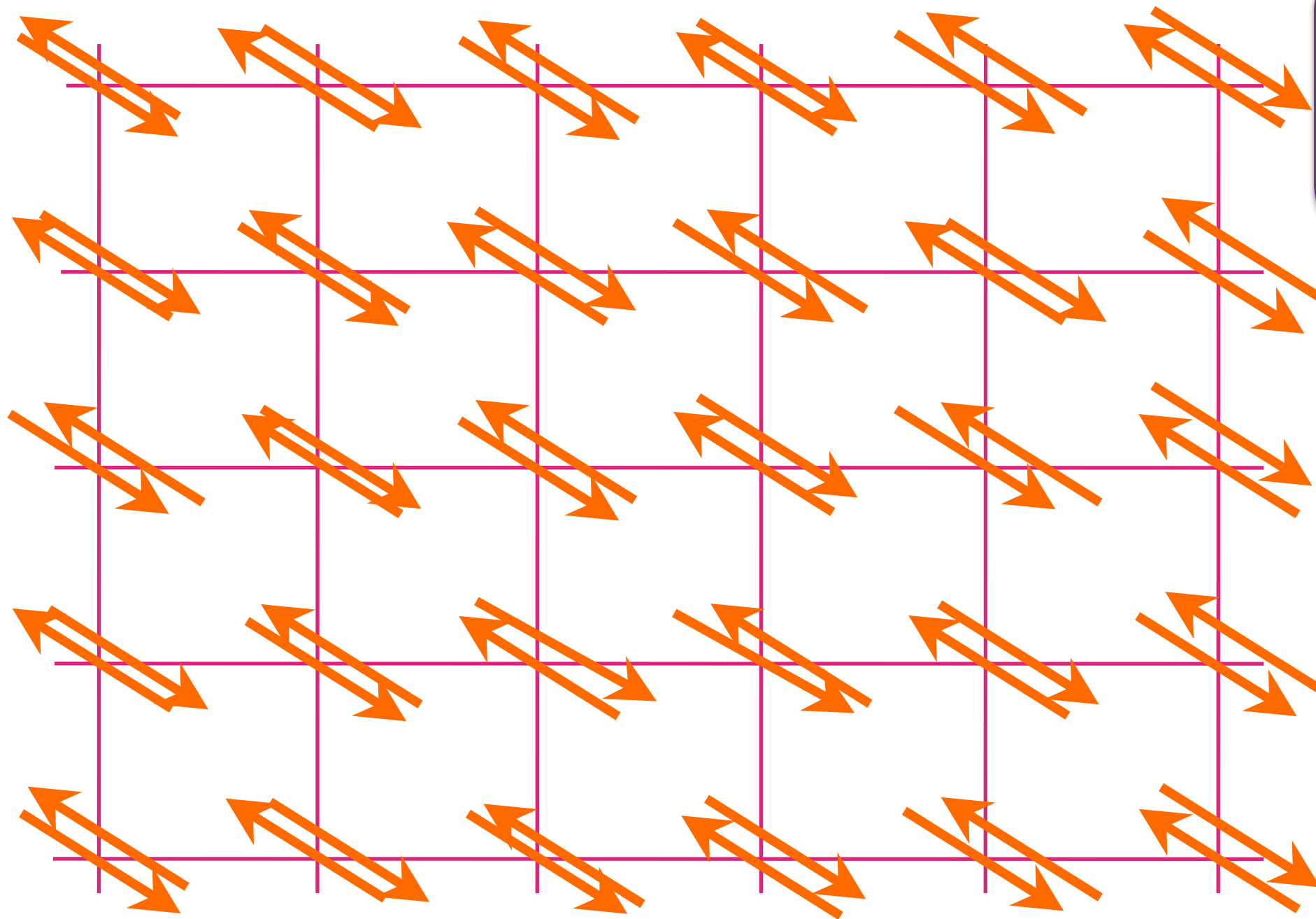




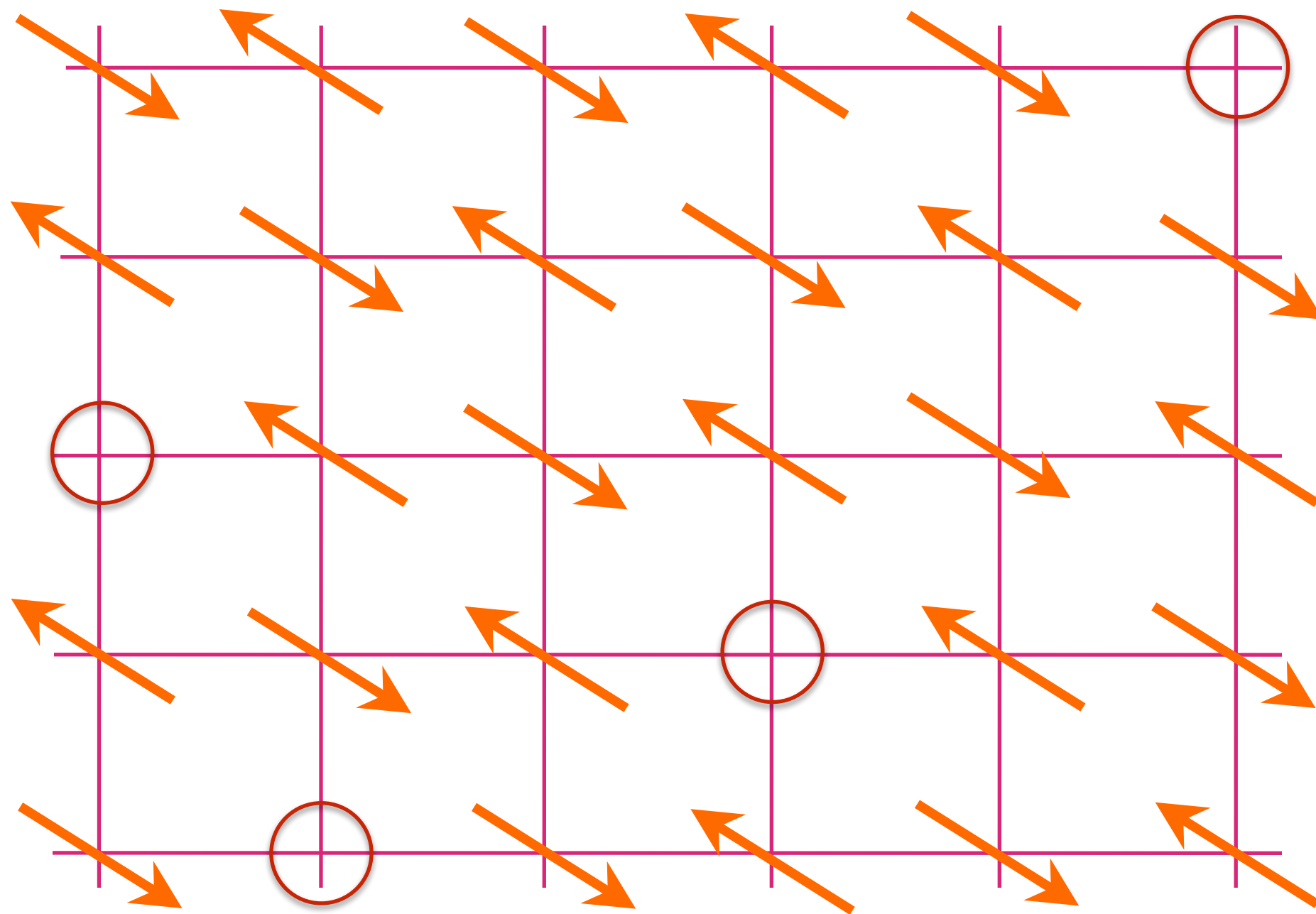
“Undoped”
Anti-
ferromagnet



Anti-ferromagnet
with p holes
per square



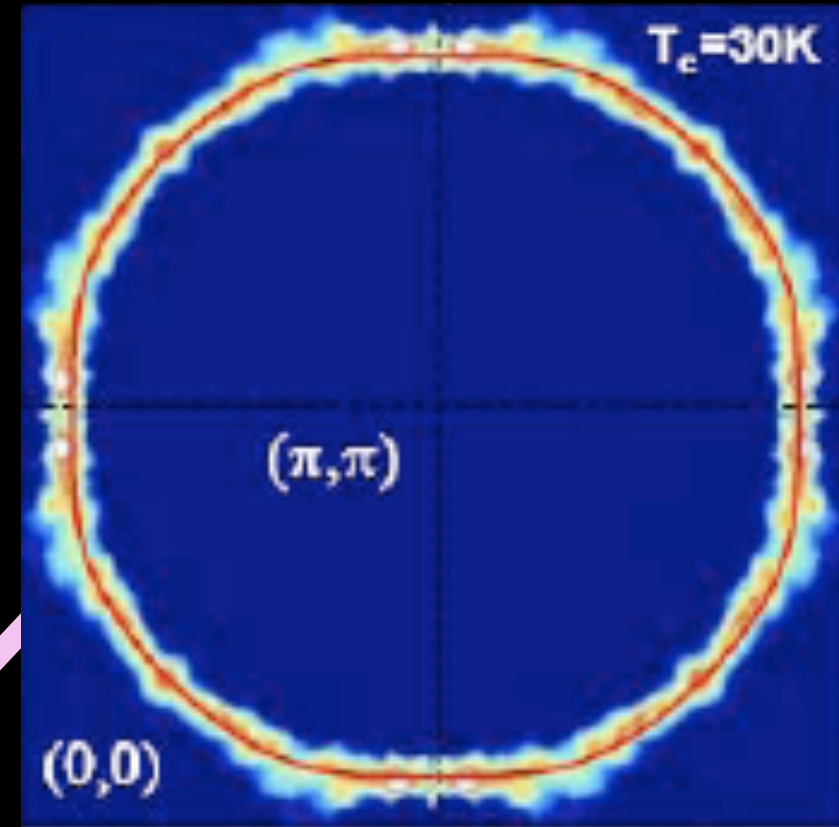
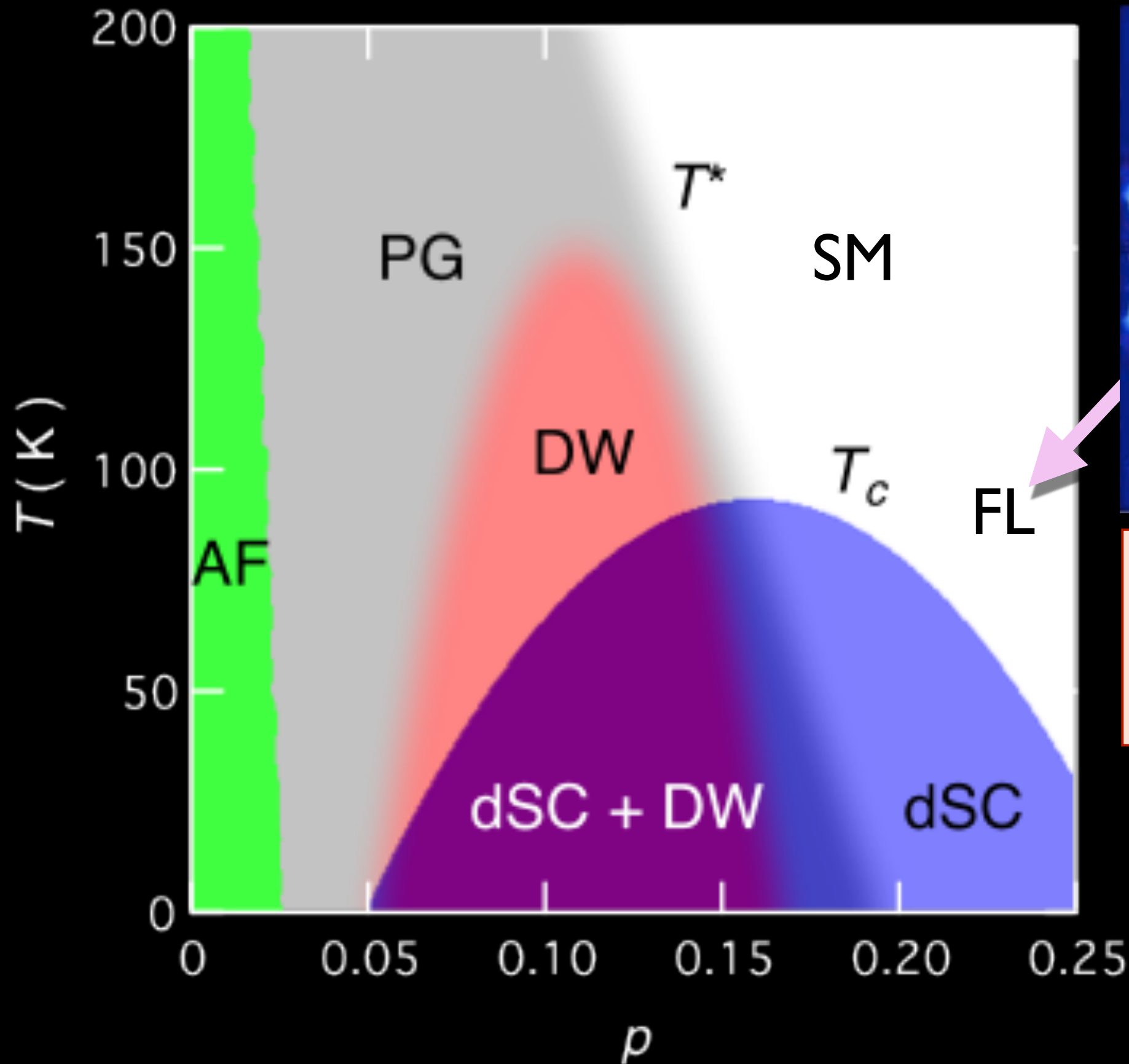
Filled
Band



Anti-ferromagnet with p holes per square

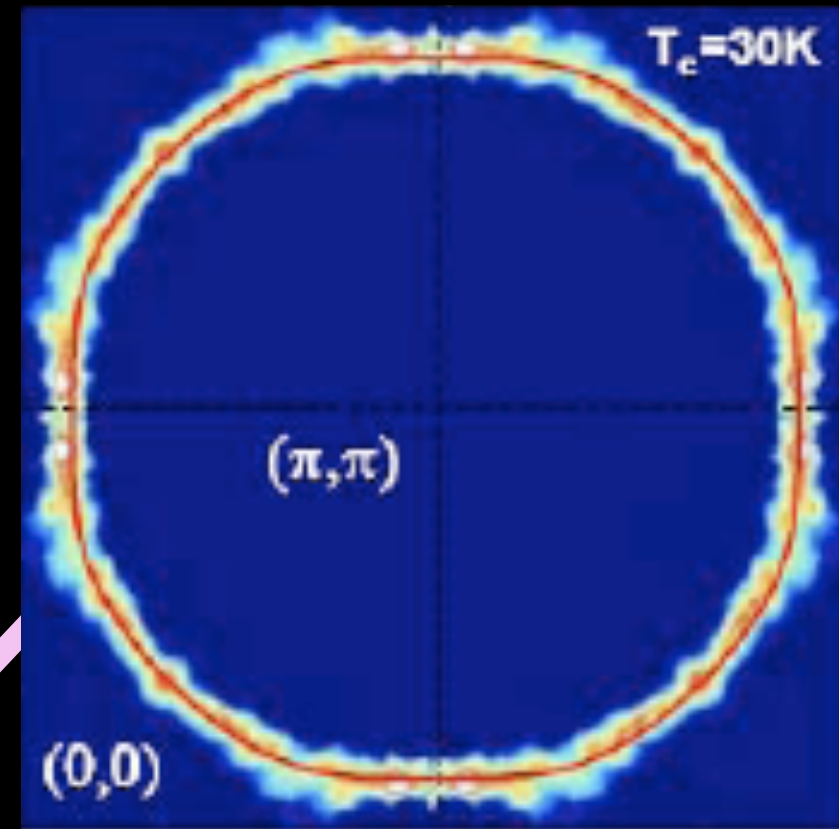
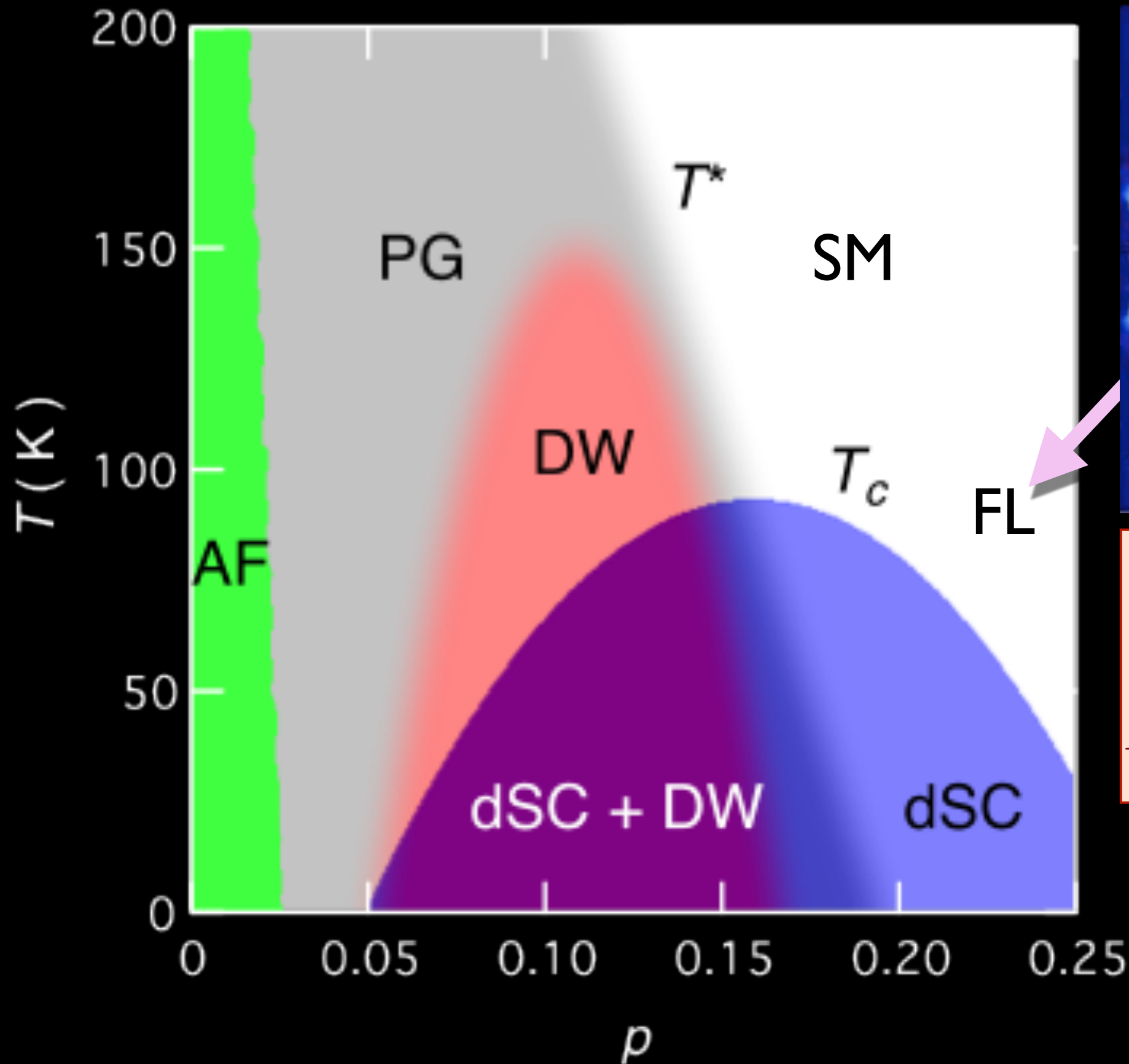
But relative to the band insulator, there are $1 + p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1 + p$

M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



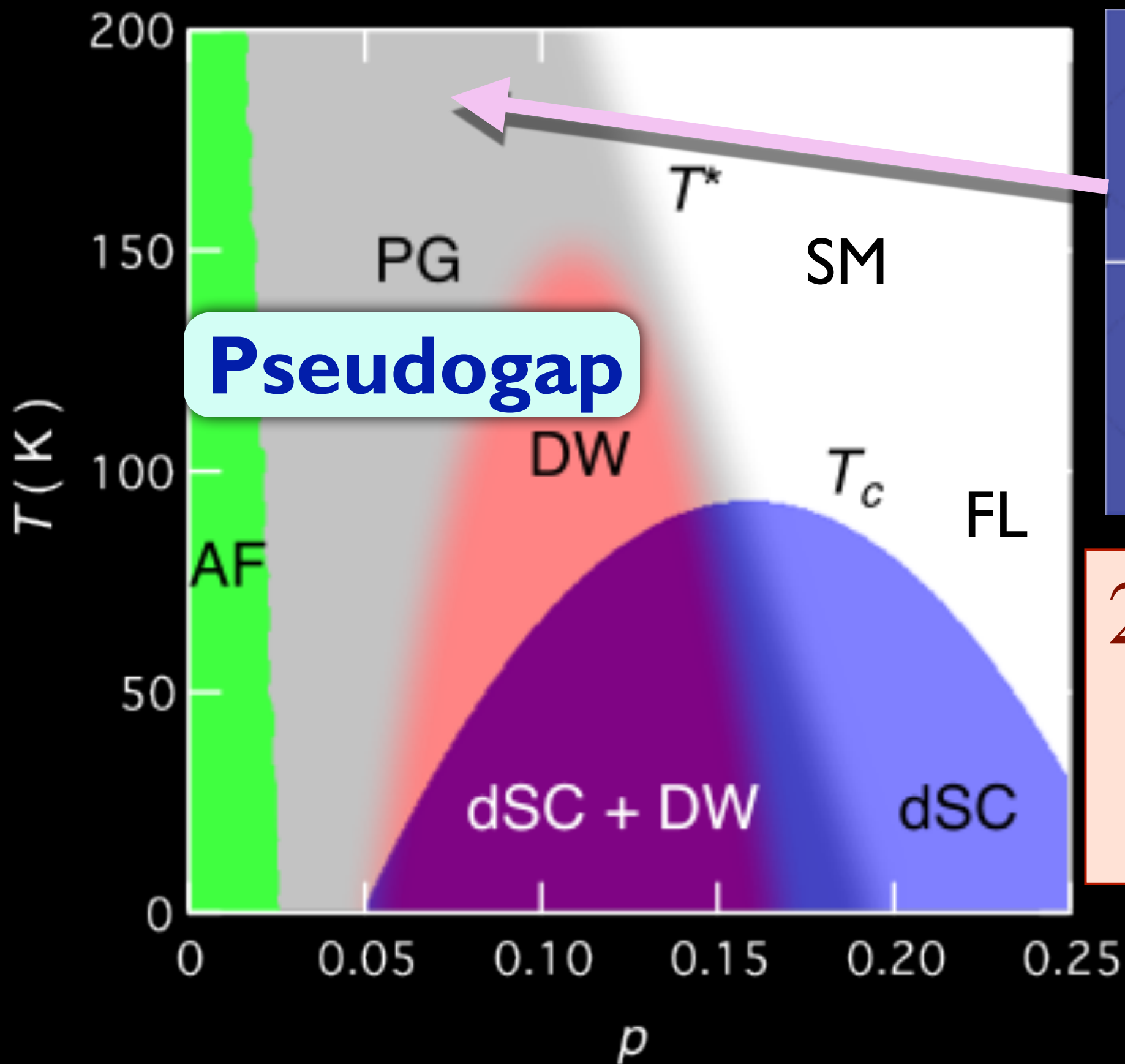
Fermi liquid
Area enclosed by
Fermi surface = $1+p$

M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

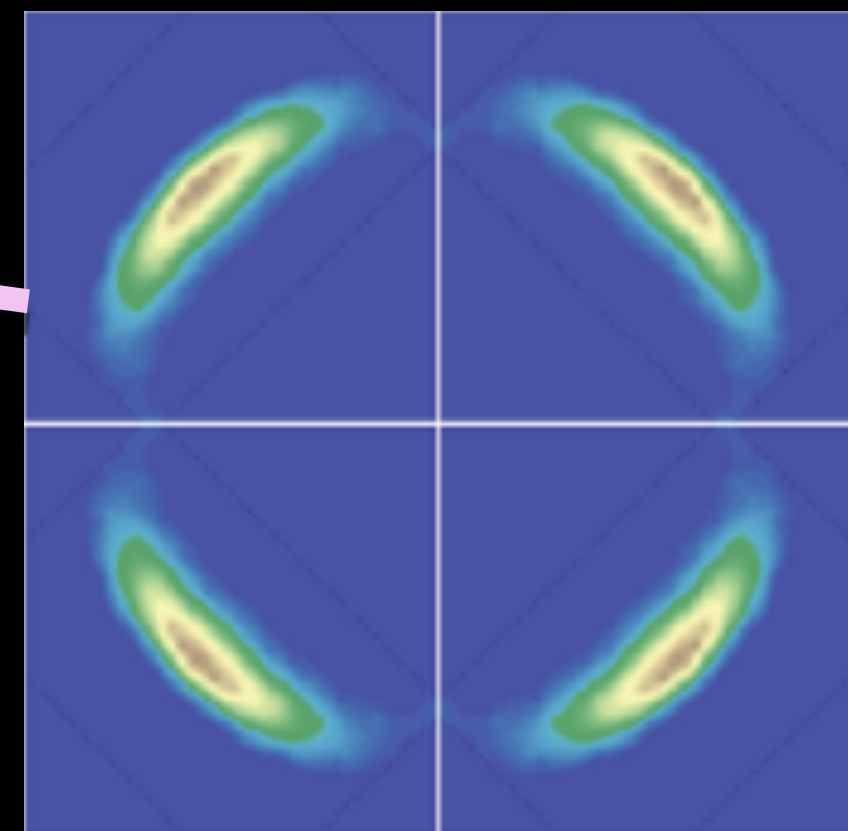


Fermi liquid
Hall co-efficient
 $R_H = +1/((1 + p)e)$

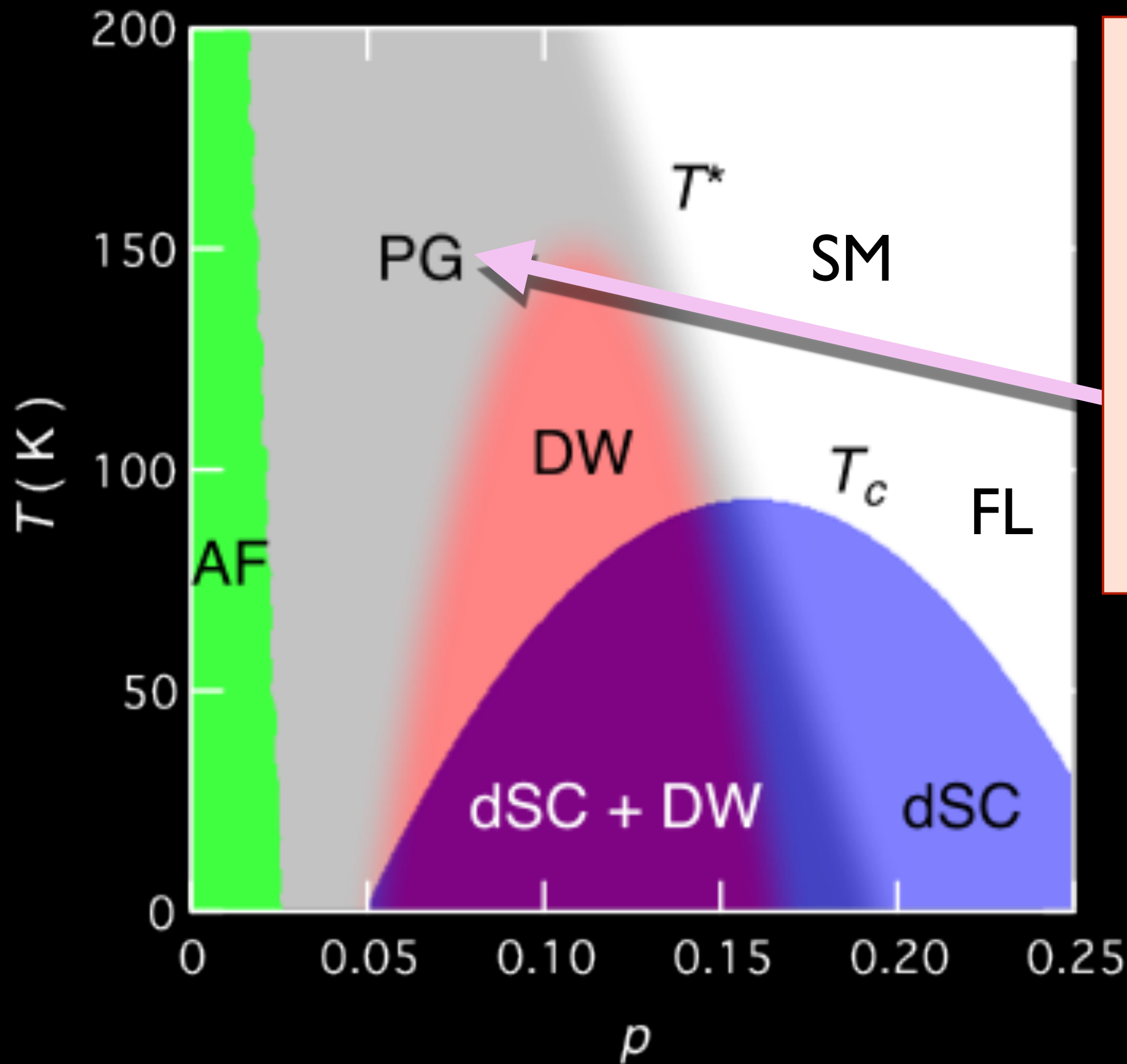
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



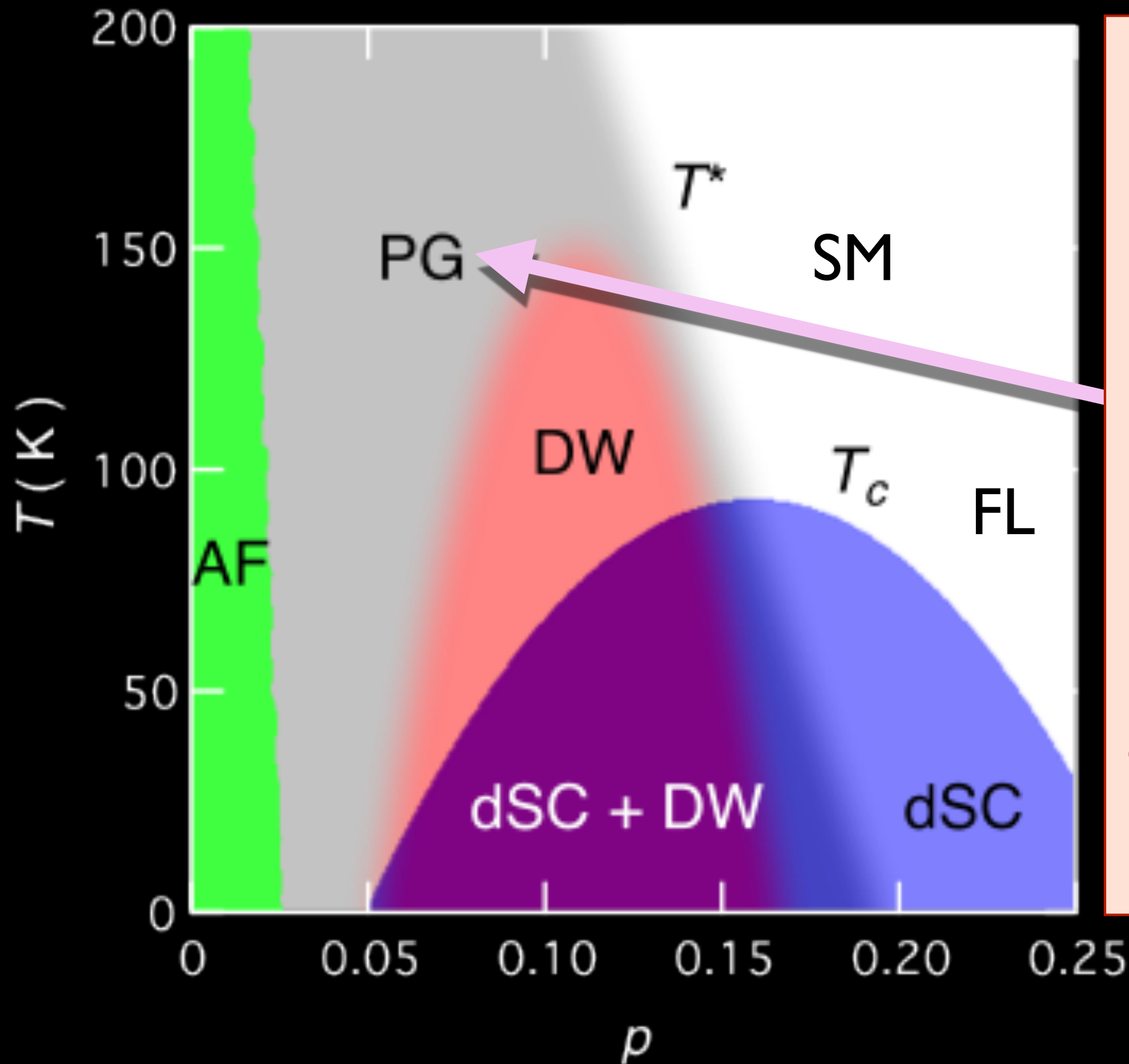
Pseudogap



2. Pseudogap metal
at low p



The PG regime behaves in many respects like a Fermi liquid, but with a Fermi surface size of p and not $1+p$



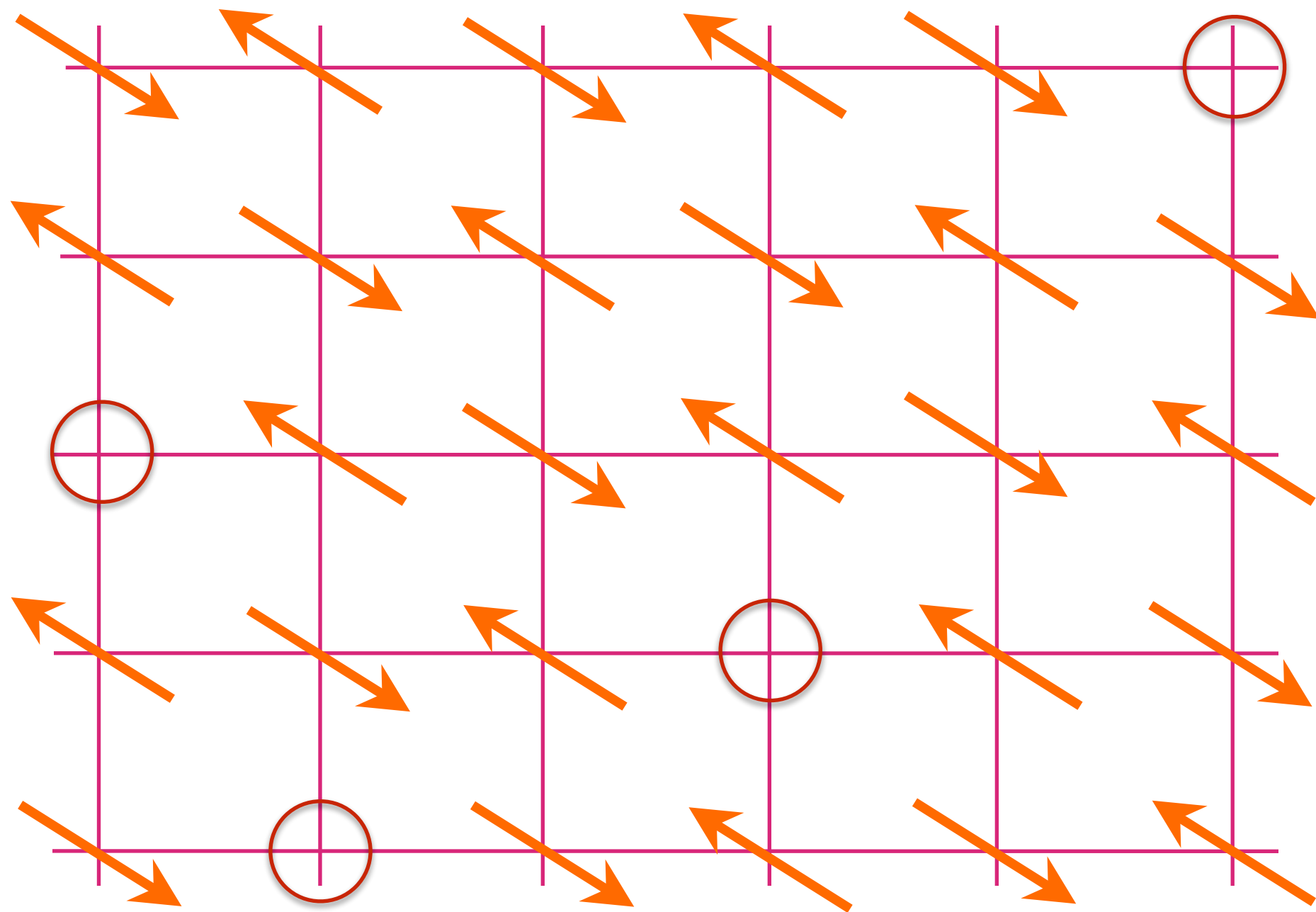
The PG regime behaves in many respects like a Fermi liquid, but with a Fermi surface size of p and not $1+p$

.e.g. Hall co-efficient $R_H = +1/(pe)$.

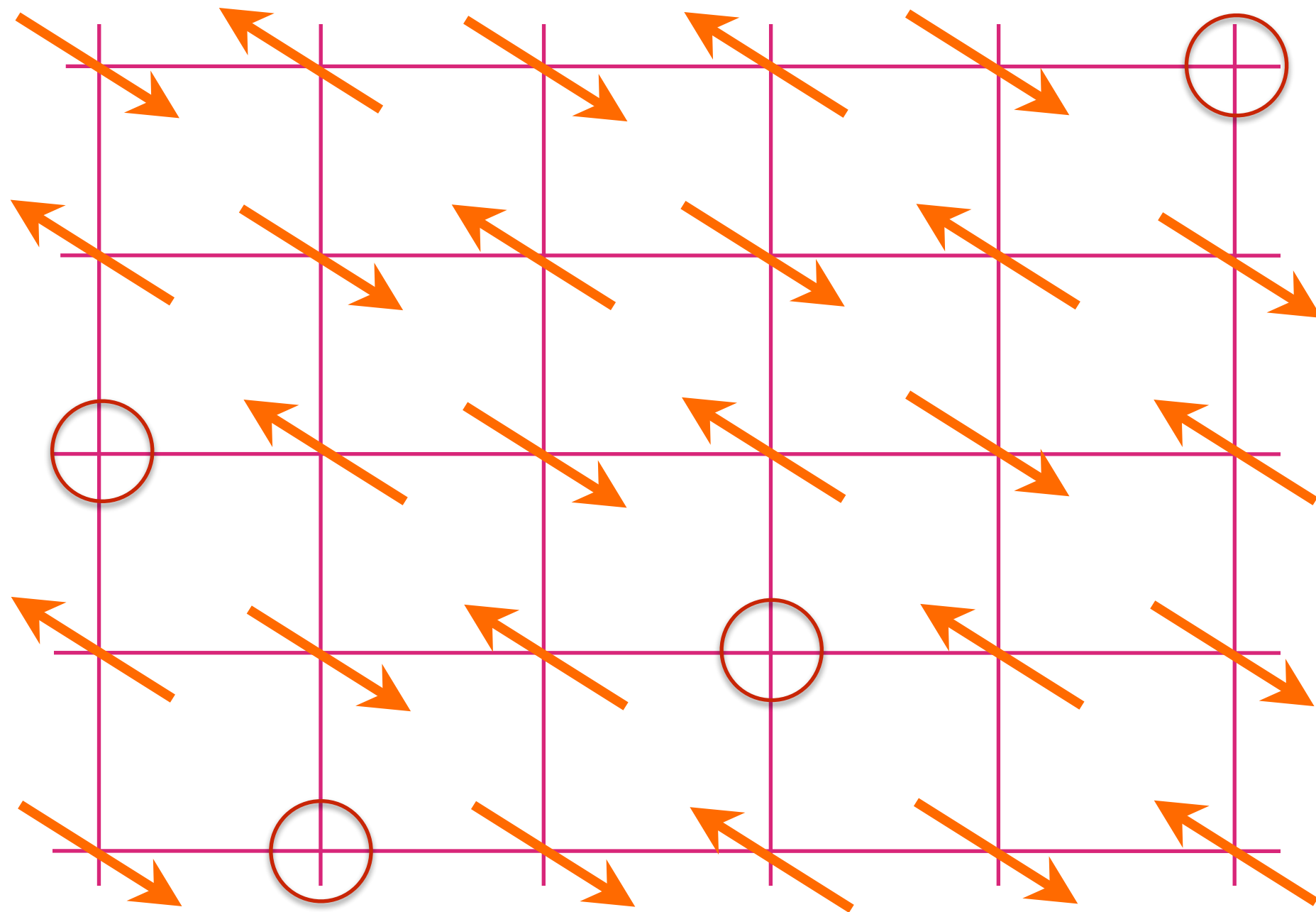
1. Emergent gauge fields and long-range entanglement in insulators

2. Theory of ordinary metals: Fermi liquids (FL)
 - (a) *Quasiparticles*
 - (b) *Luttinger theorem for volume enclosed by Fermi surface*

3. The FL* phase:
*Quasiparticles with a non-Luttinger volume,
and emergent gauge fields*



Anti-ferromagnet
with p holes
per square

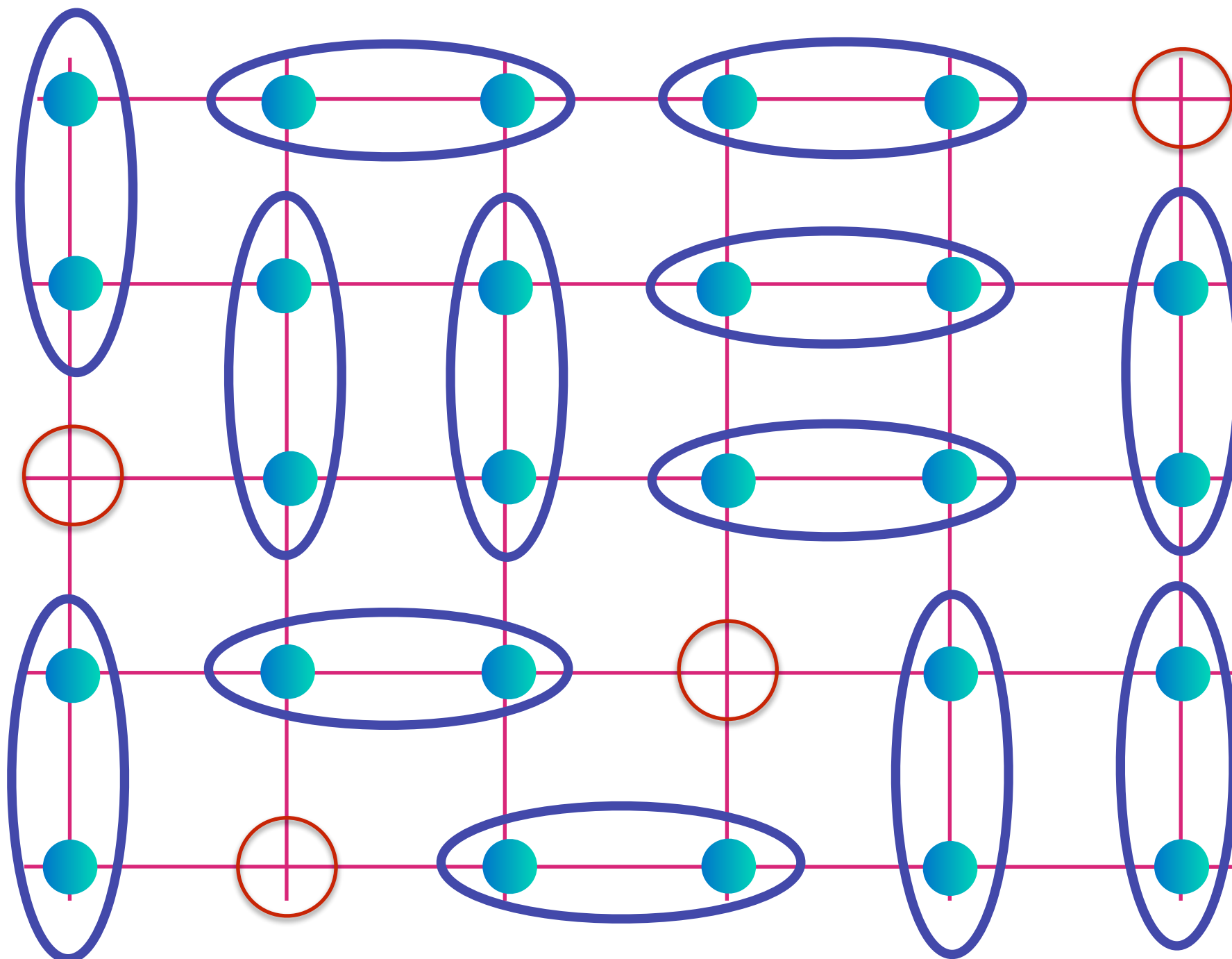


Anti-ferromagnet with p holes per square

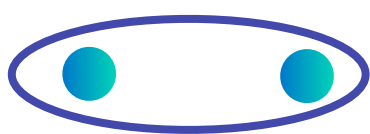
Can we get a Fermi surface of size p ?
(and full square lattice symmetry)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB 35, 8865 (1987)

N. Read and B. Chakraborty, PRB 40, 7133 (1989)

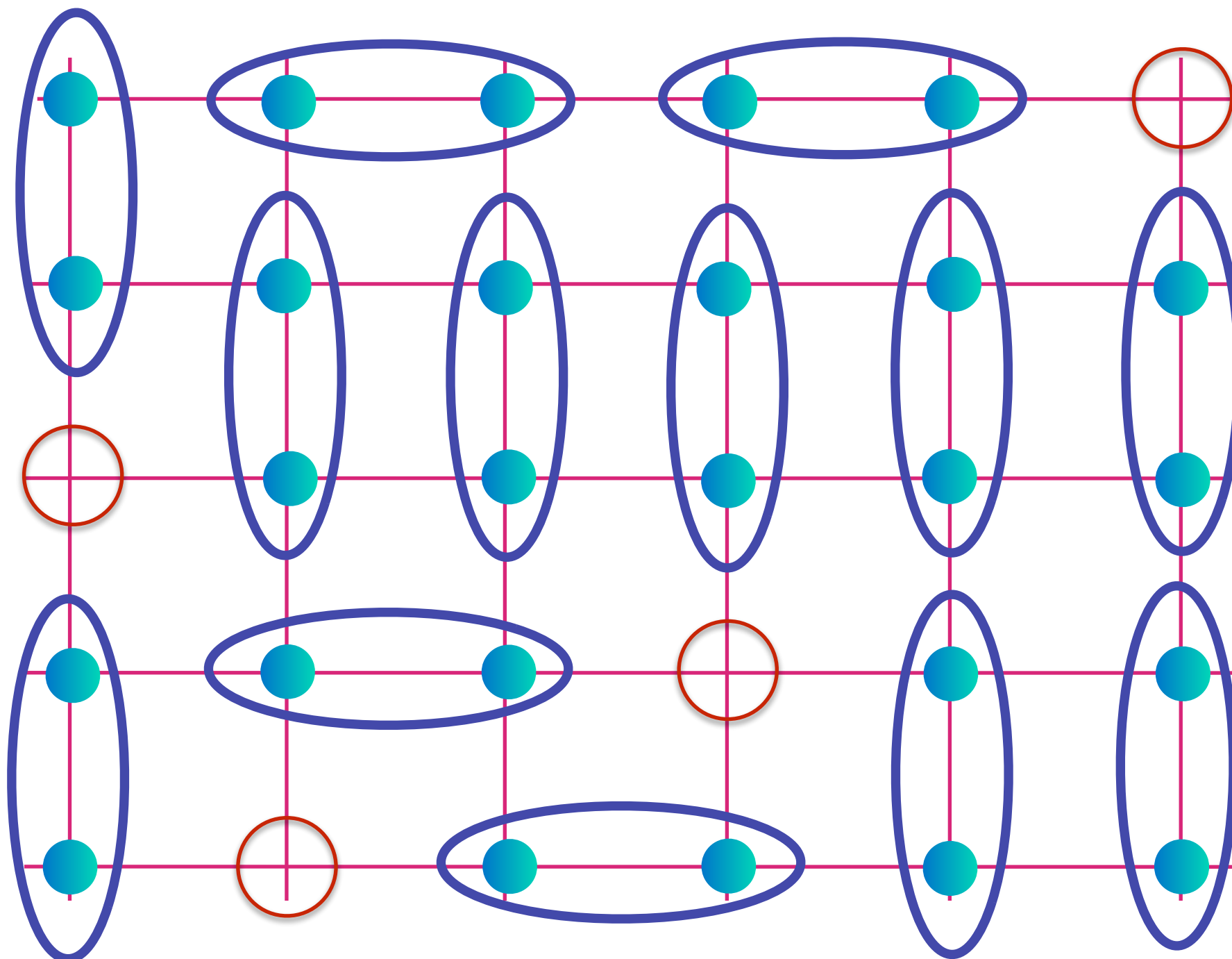


Spin liquid with density ρ of spinless, charge $+e$ "holons". These can form a Fermi surface of size ρ , but this is not visible in electron photo-emission

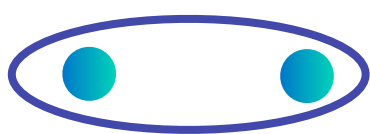
 = $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

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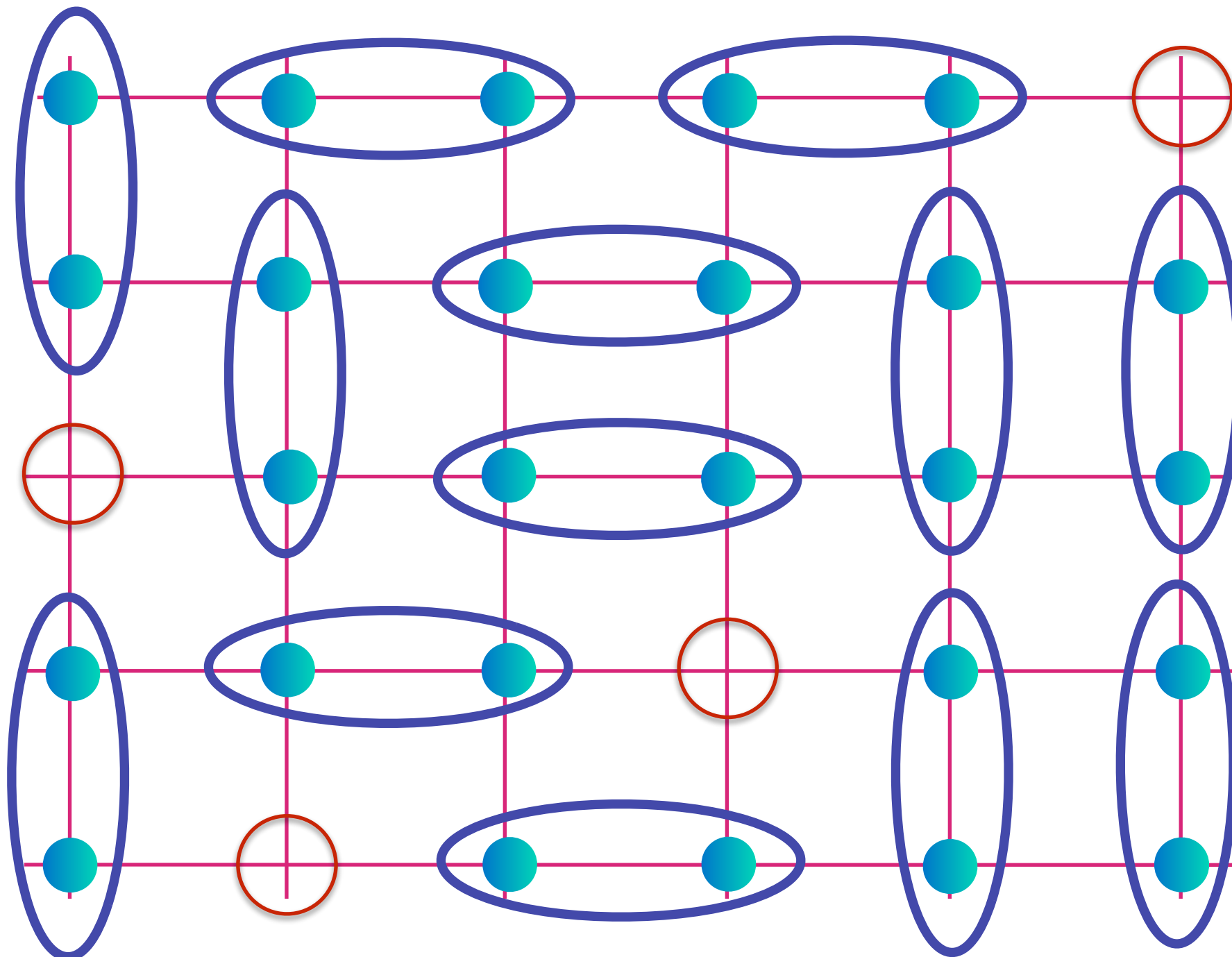


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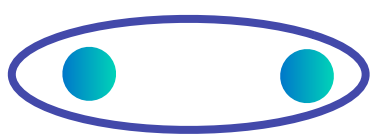
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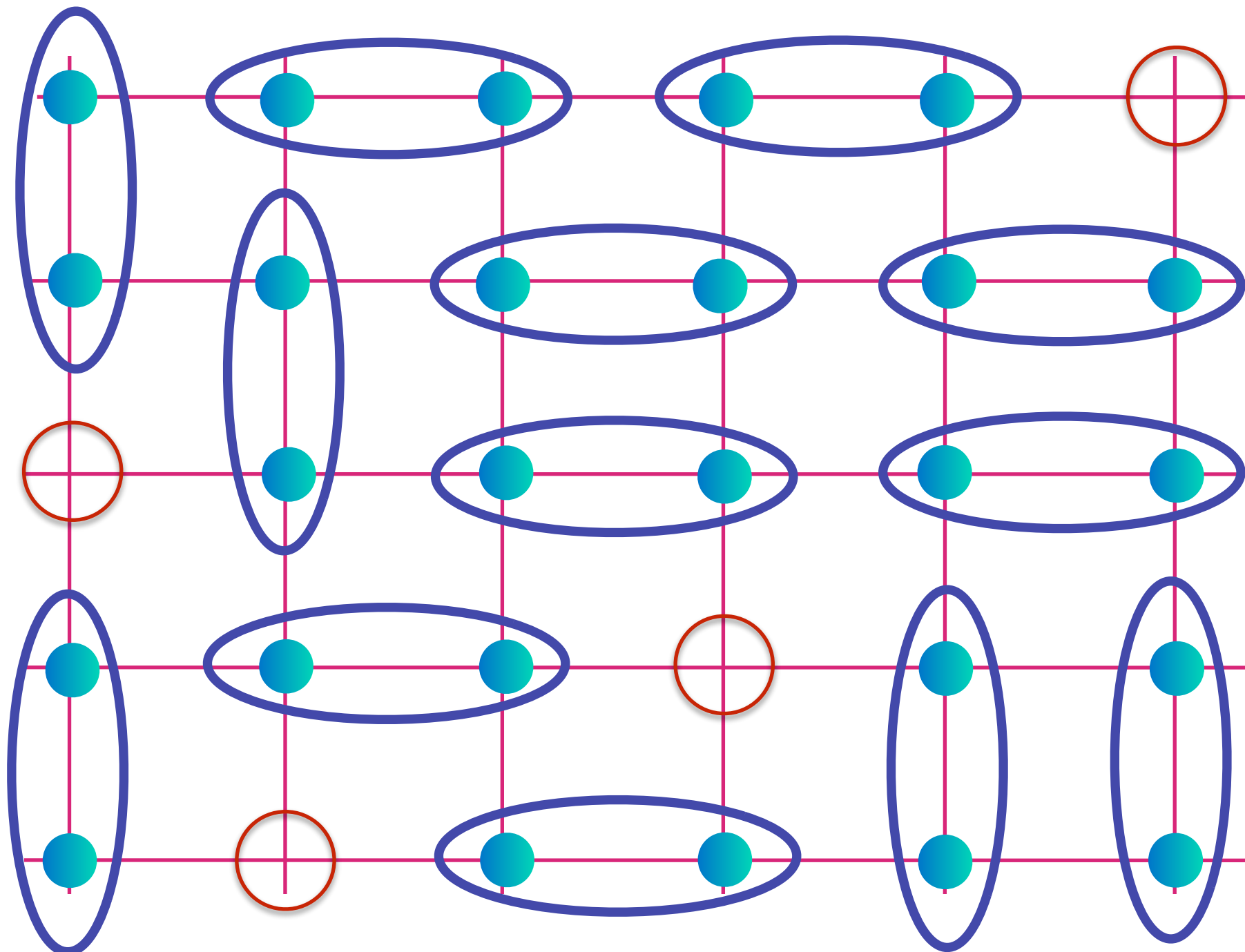


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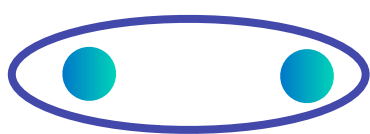
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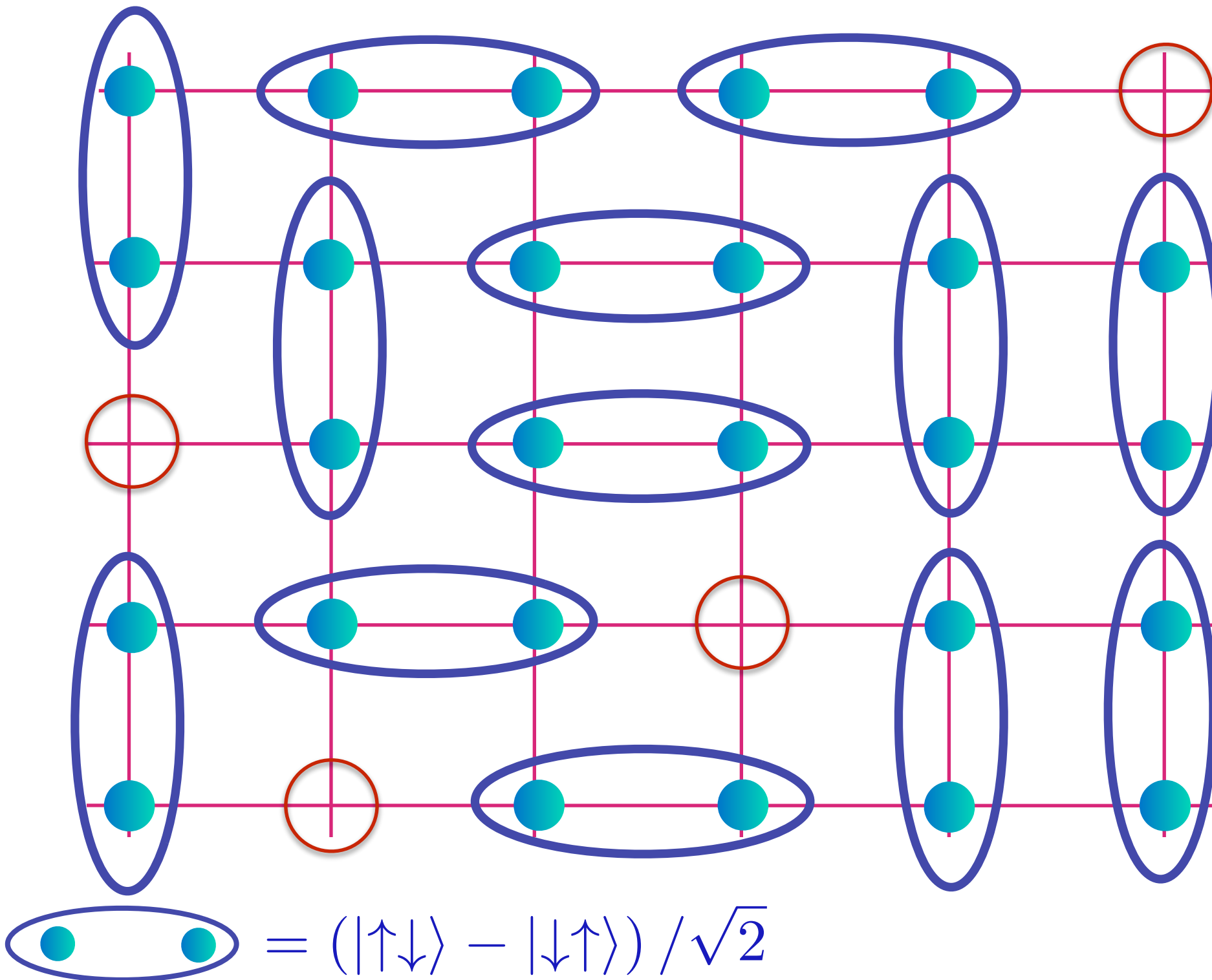


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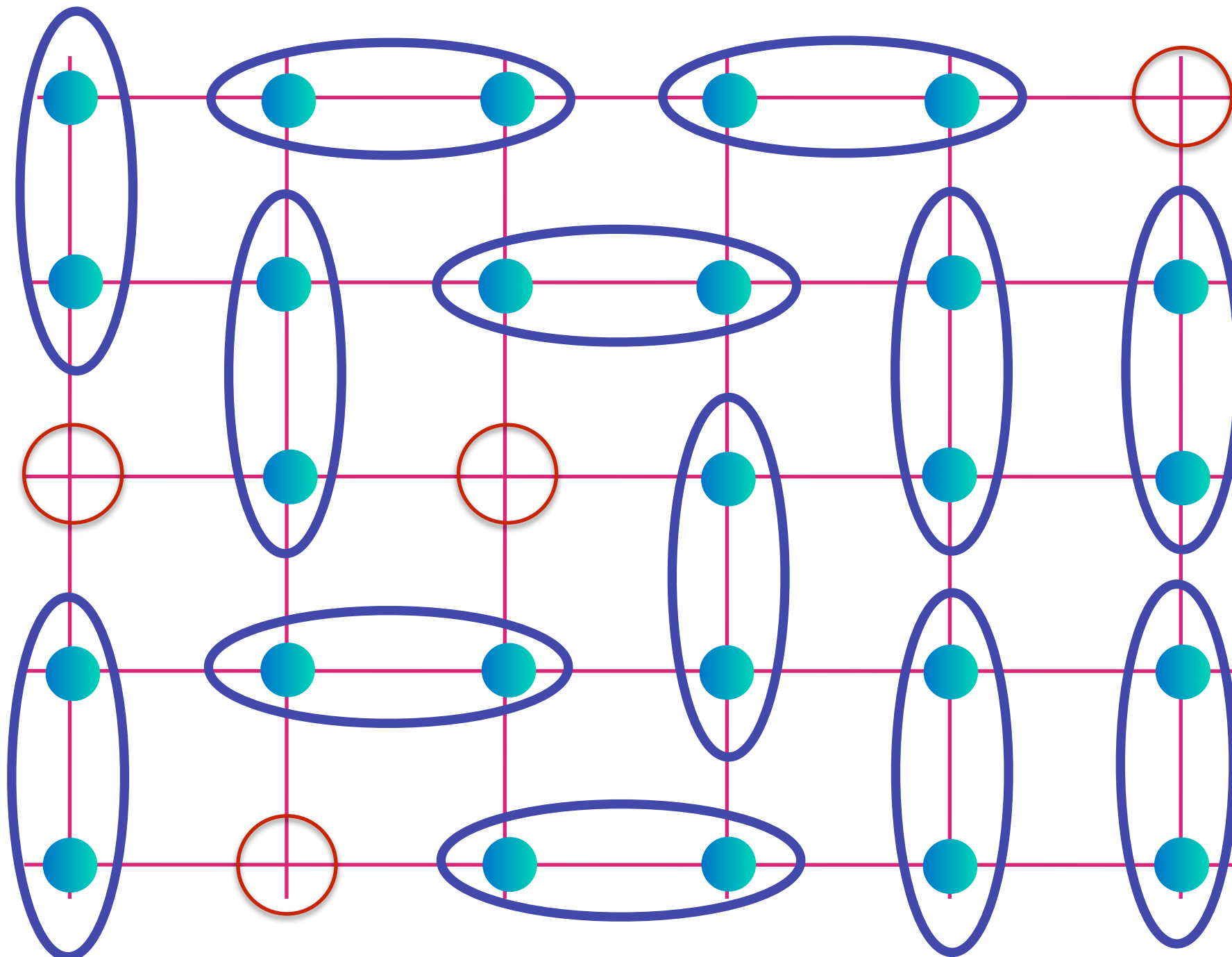
N. Read and B. Chakraborty, PRB 40, 7133 (1989)



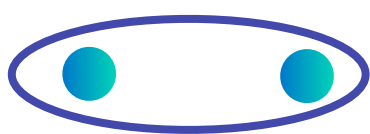
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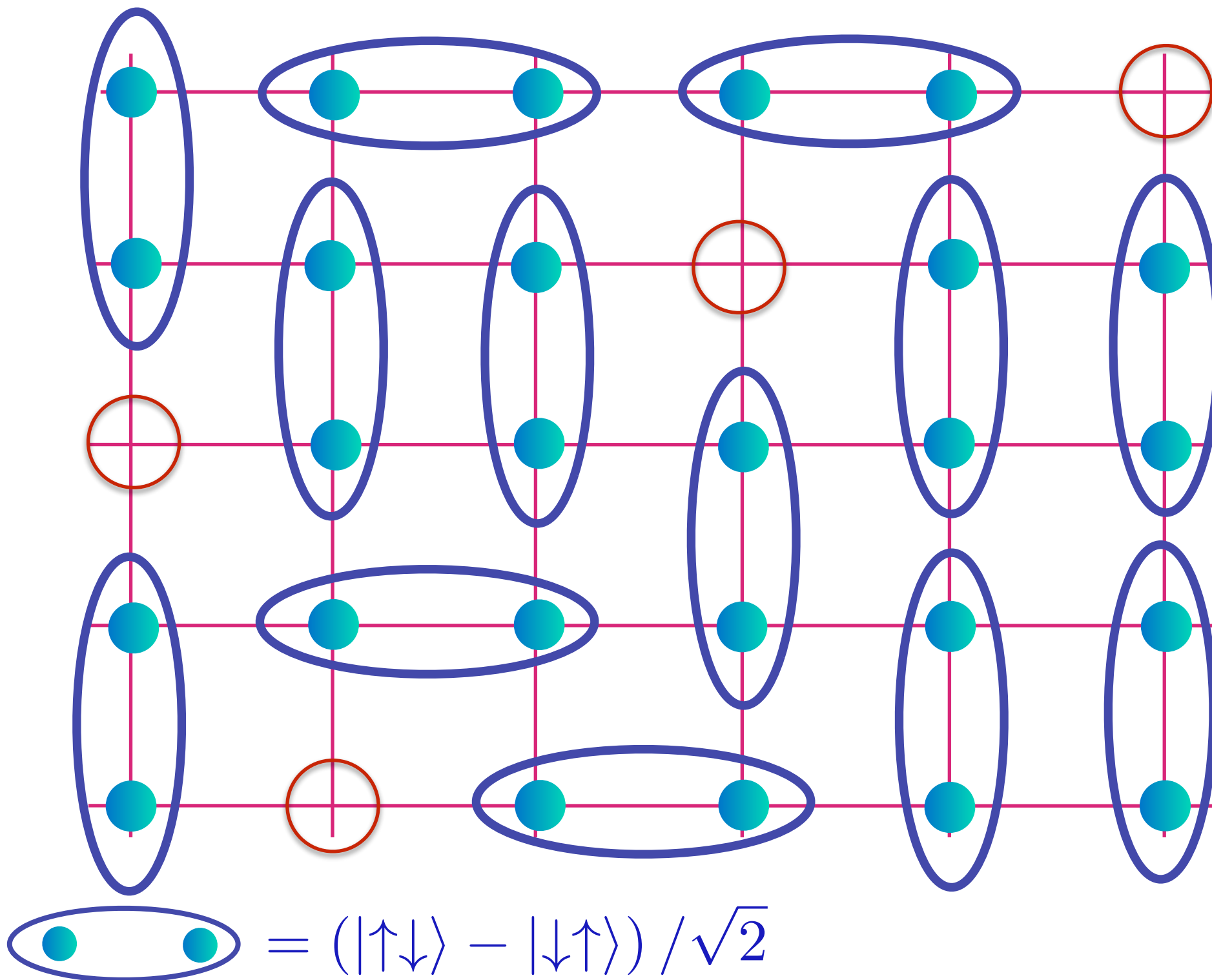


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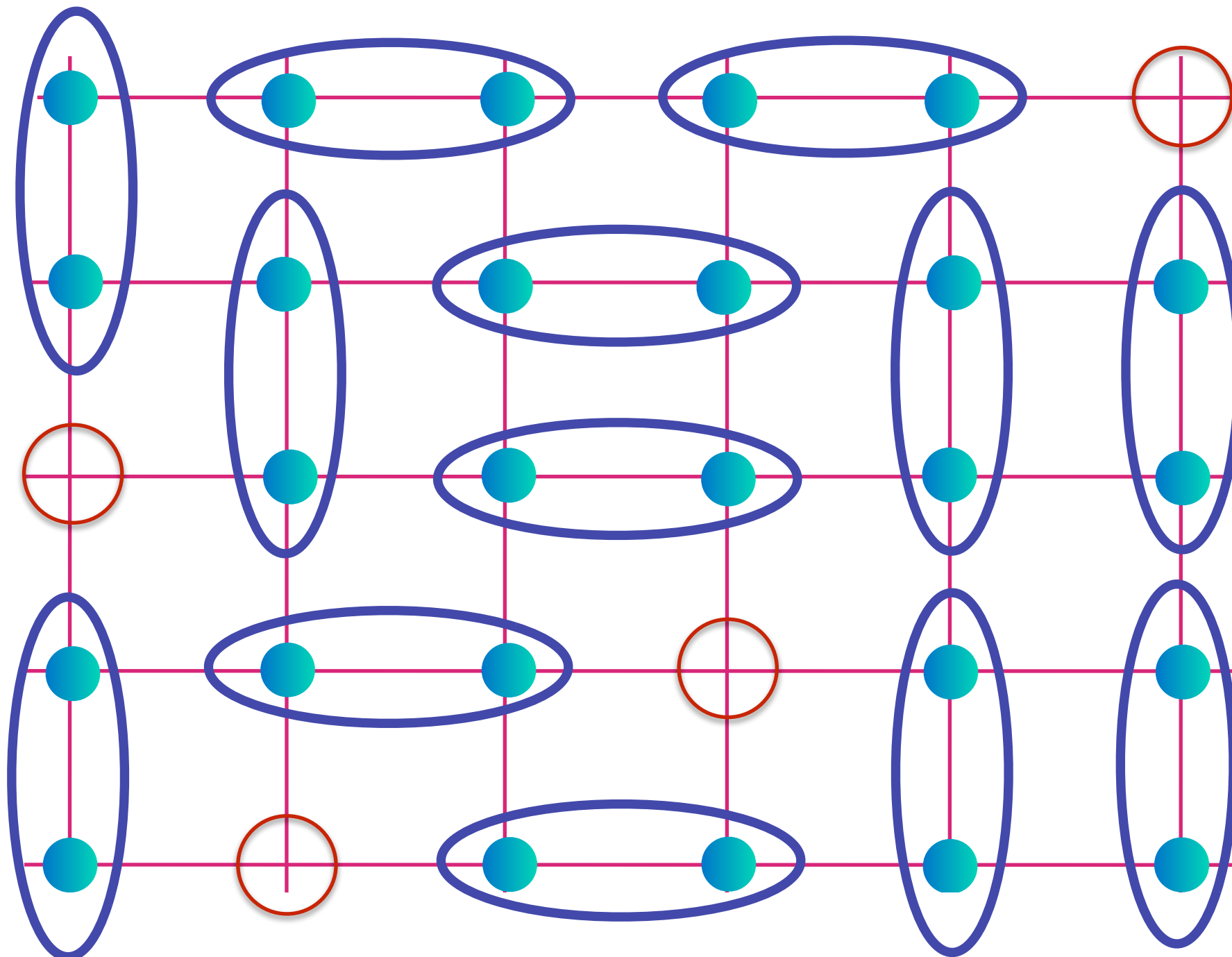
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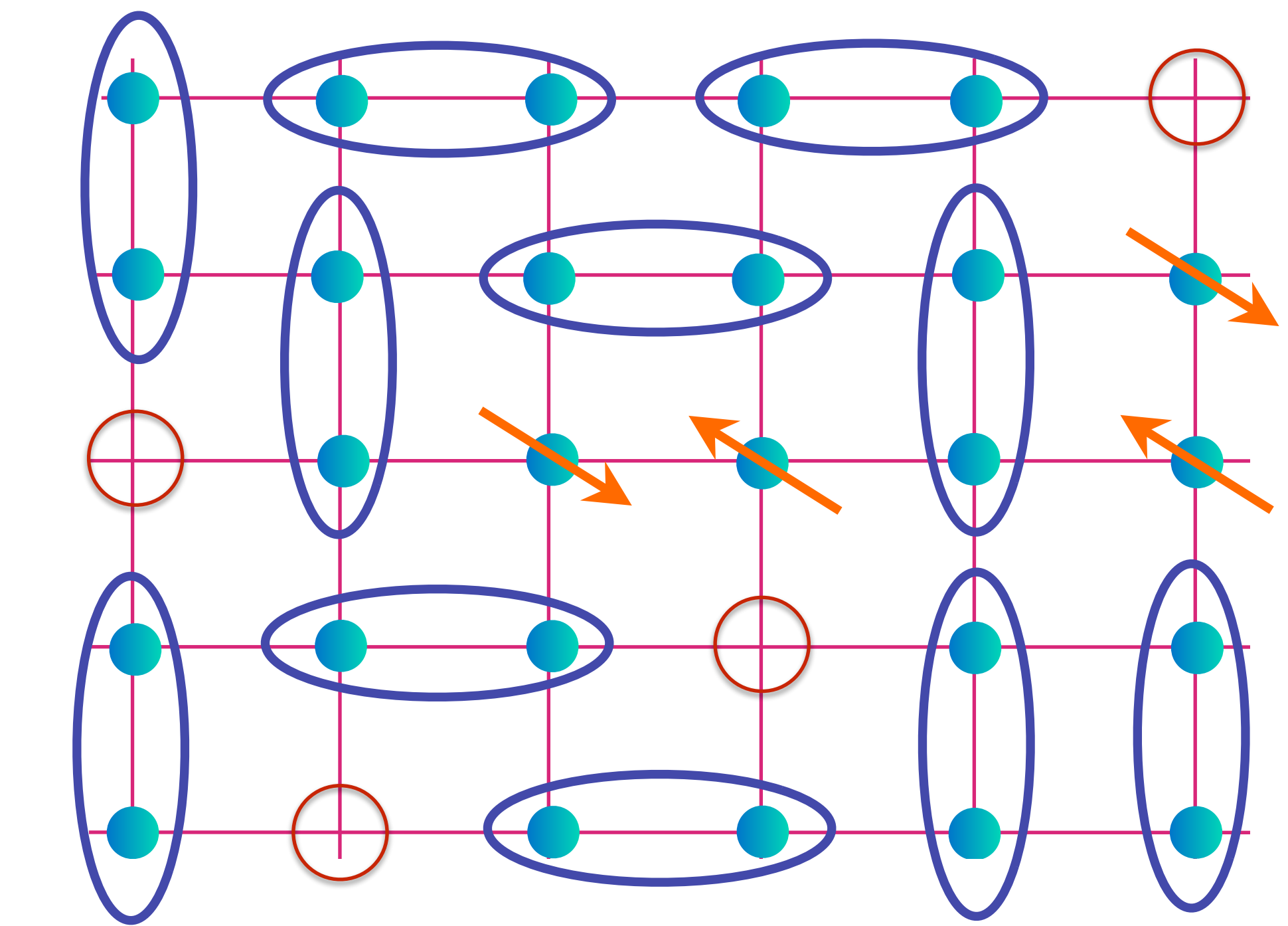
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB 35, 8865 (1987)

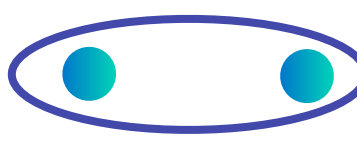
N. Read and B. Chakraborty, PRB 40, 7133 (1989)

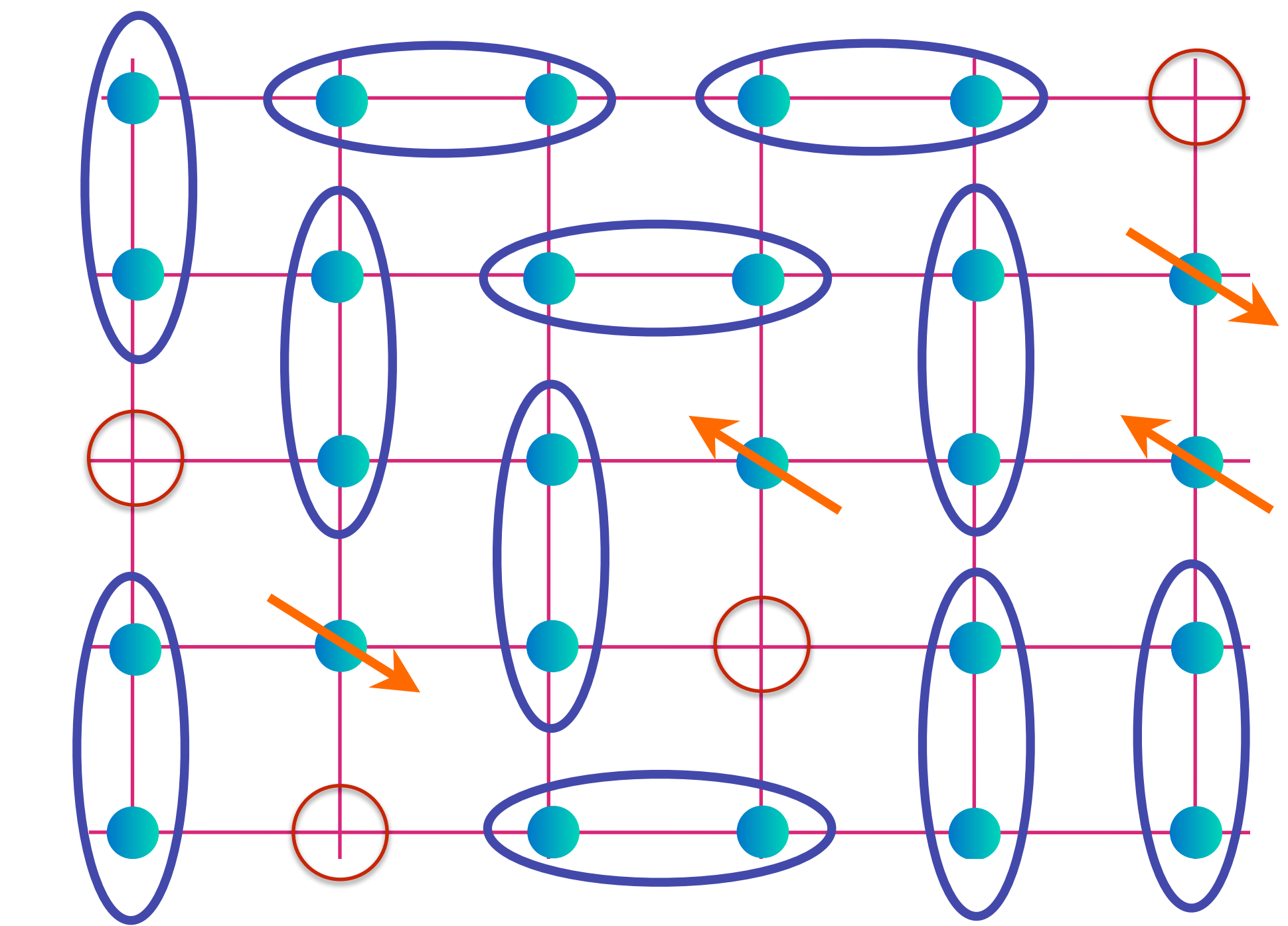


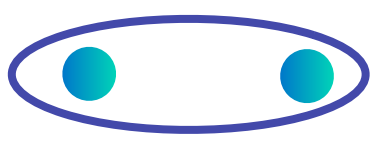
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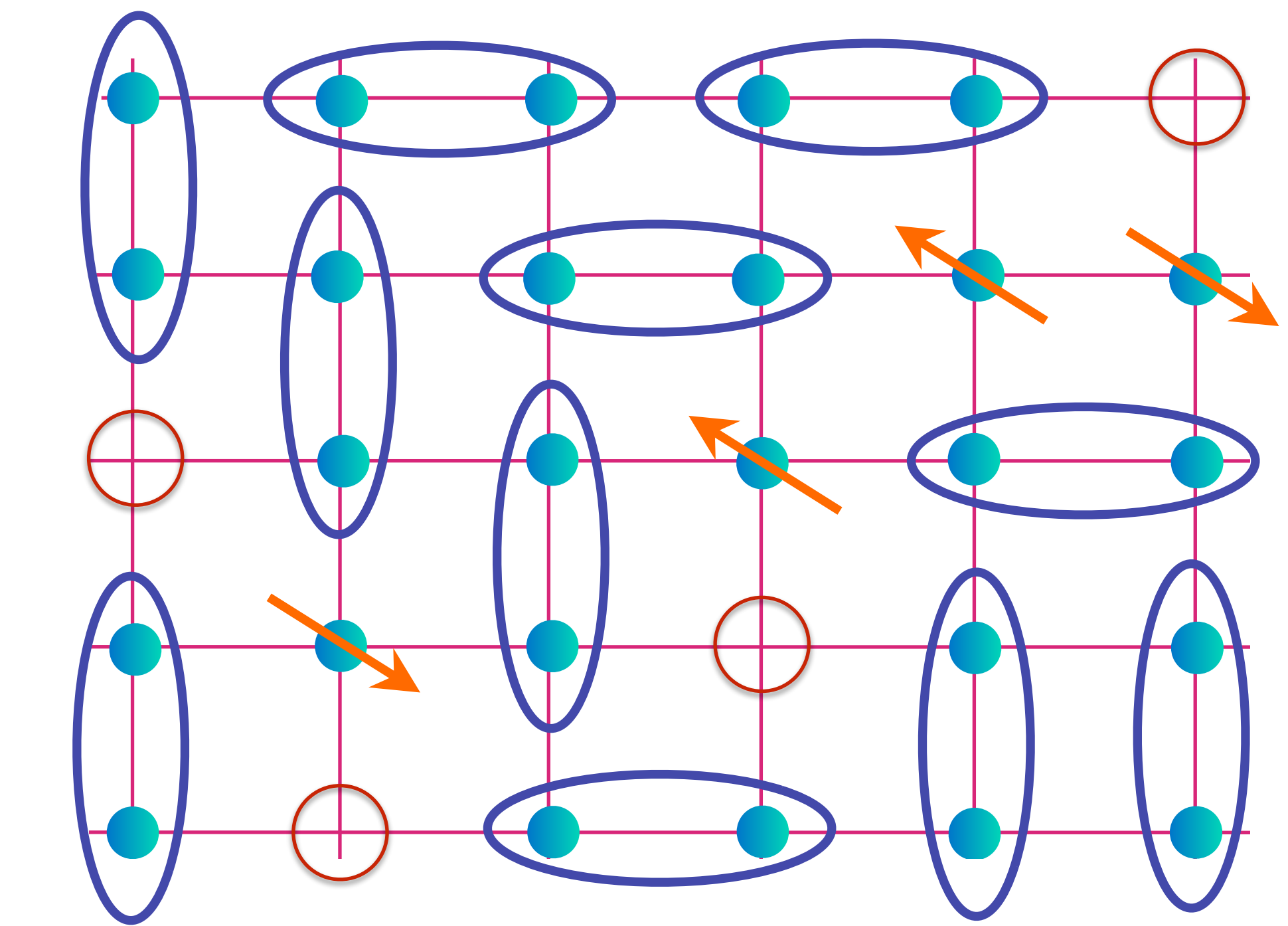
$$\text{[Diagram of two red dots in a blue oval]} = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

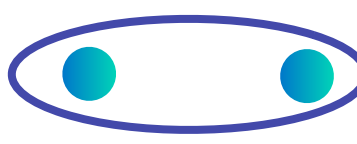


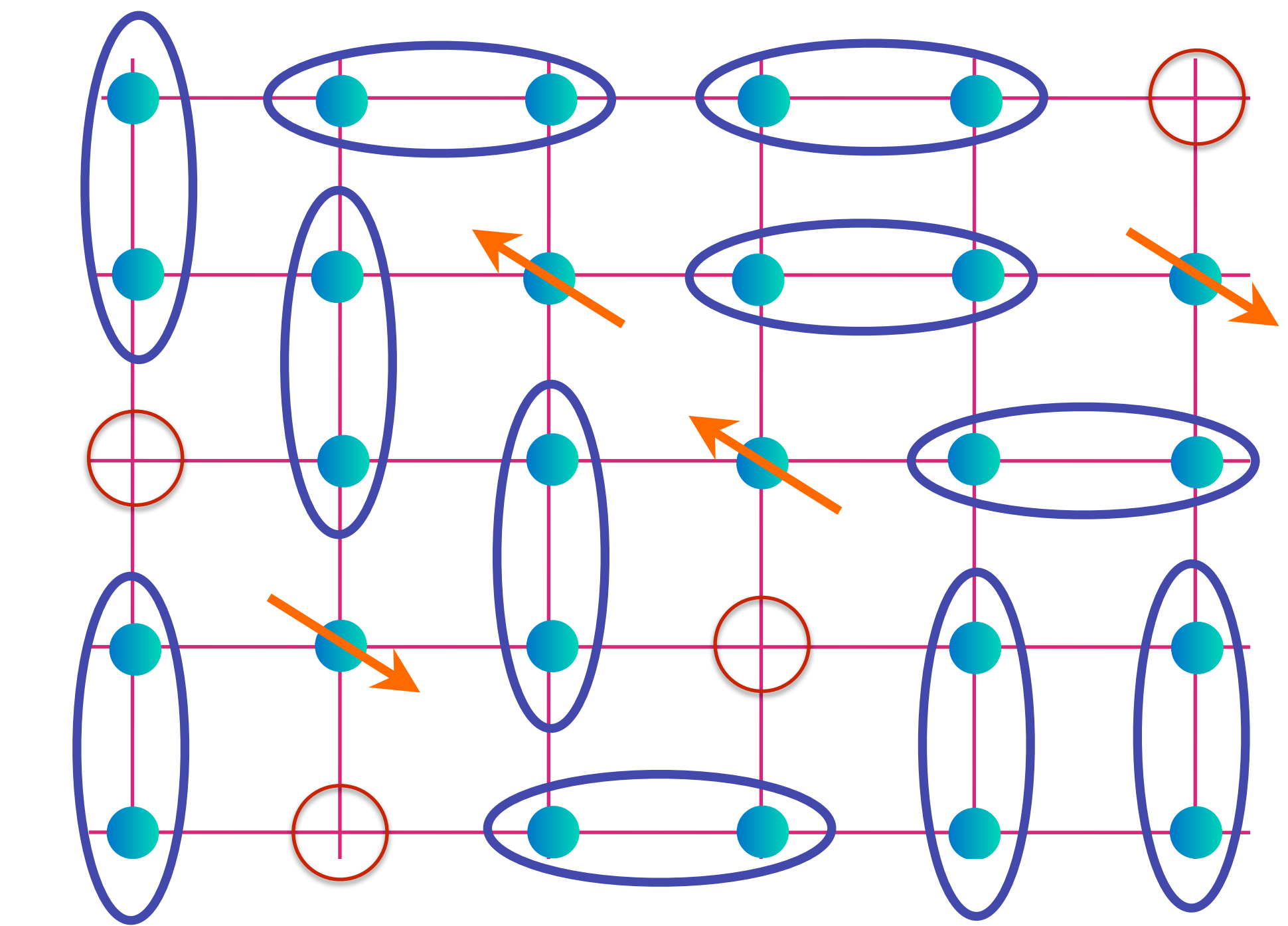

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

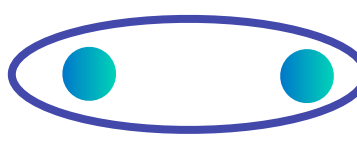


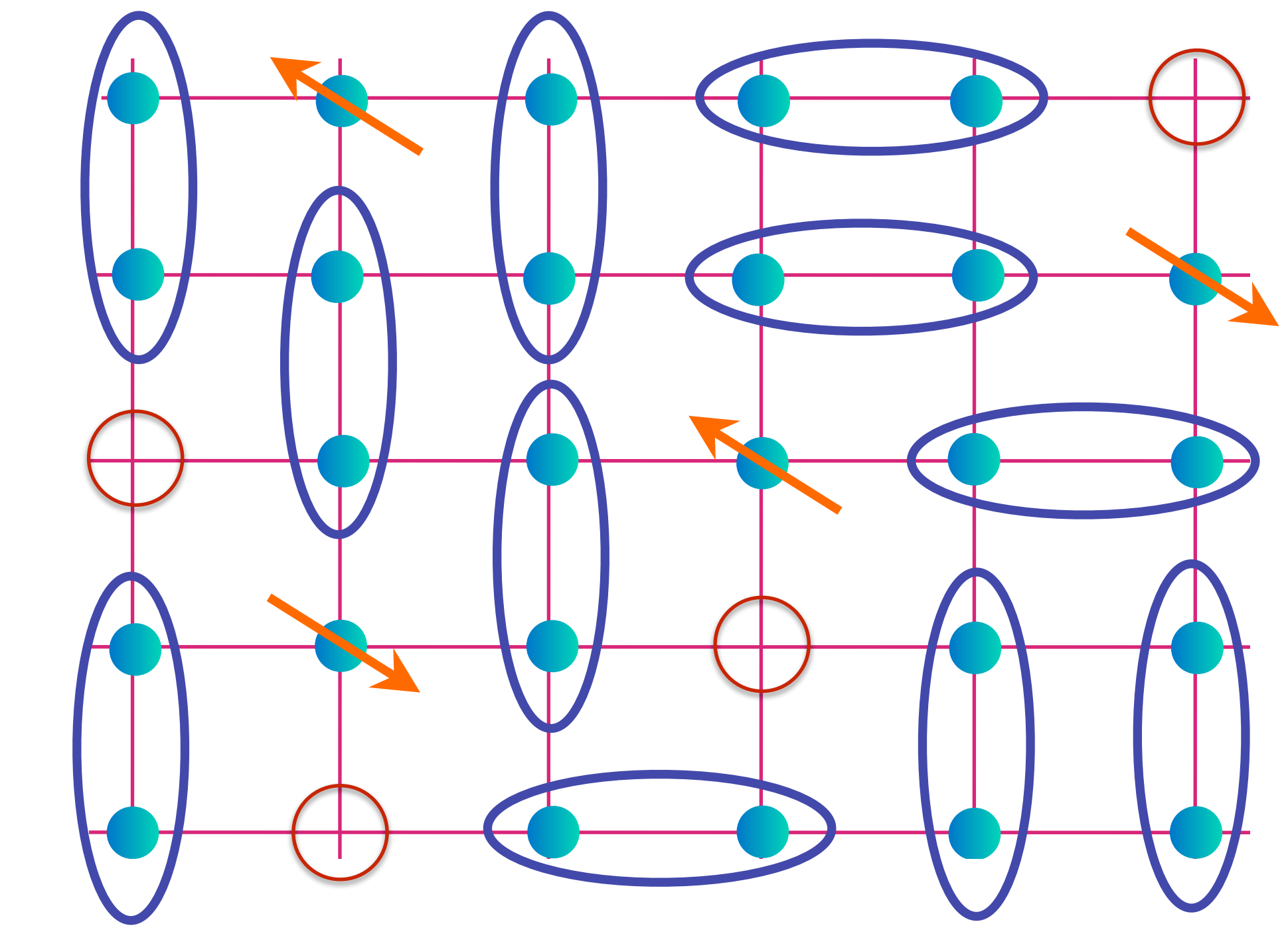

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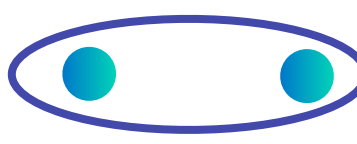


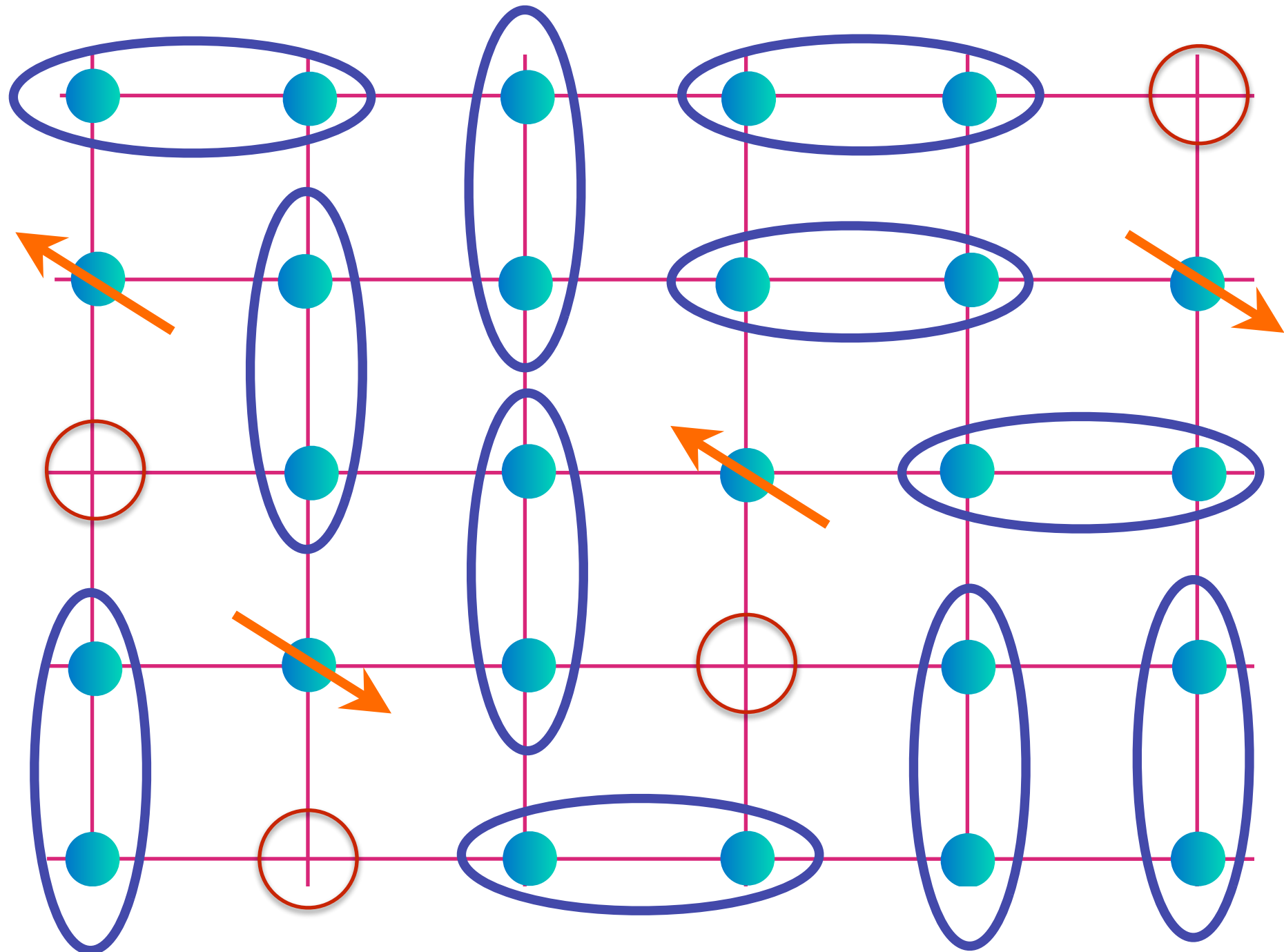

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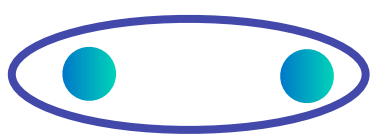


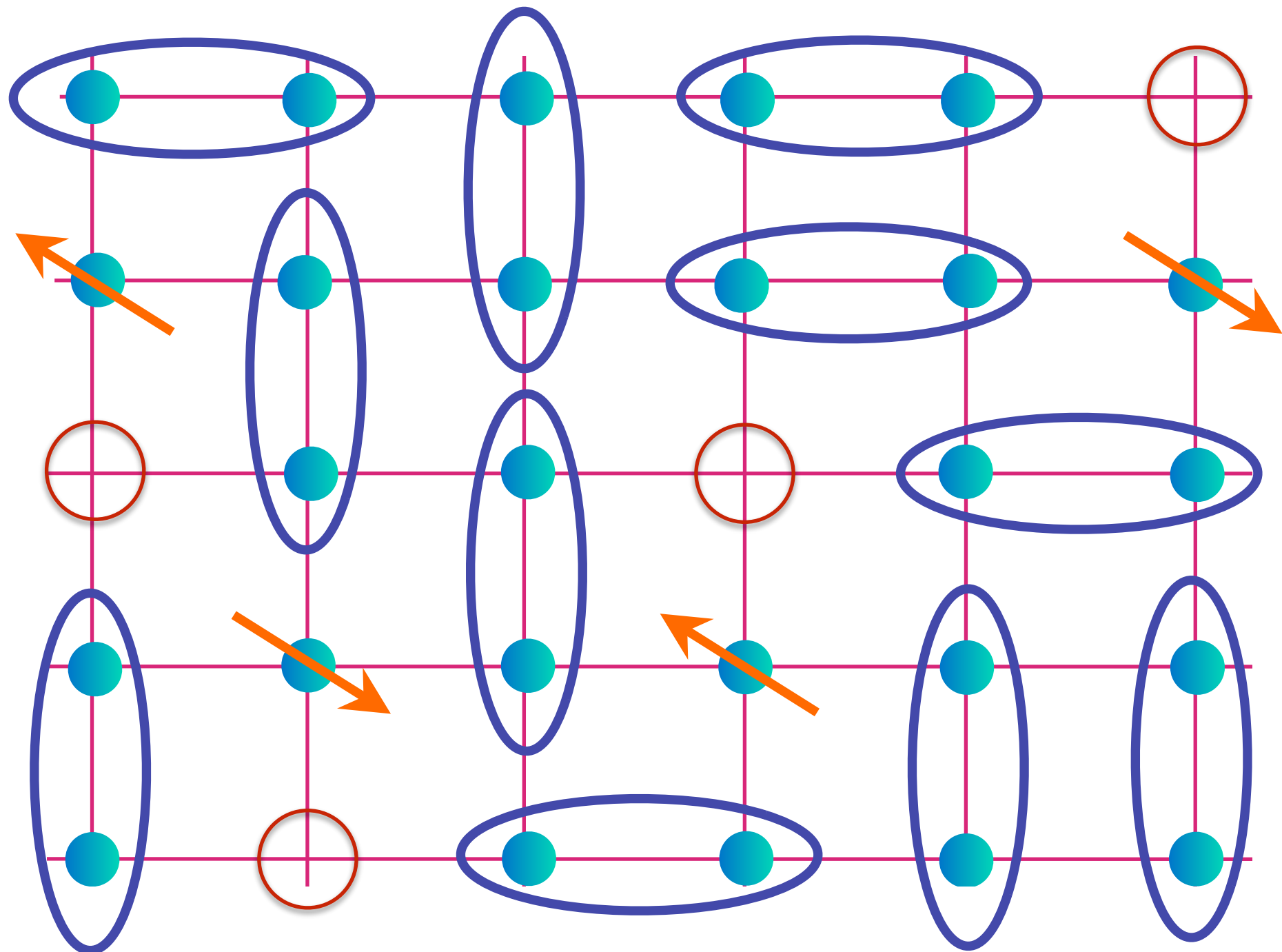

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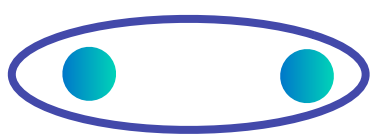


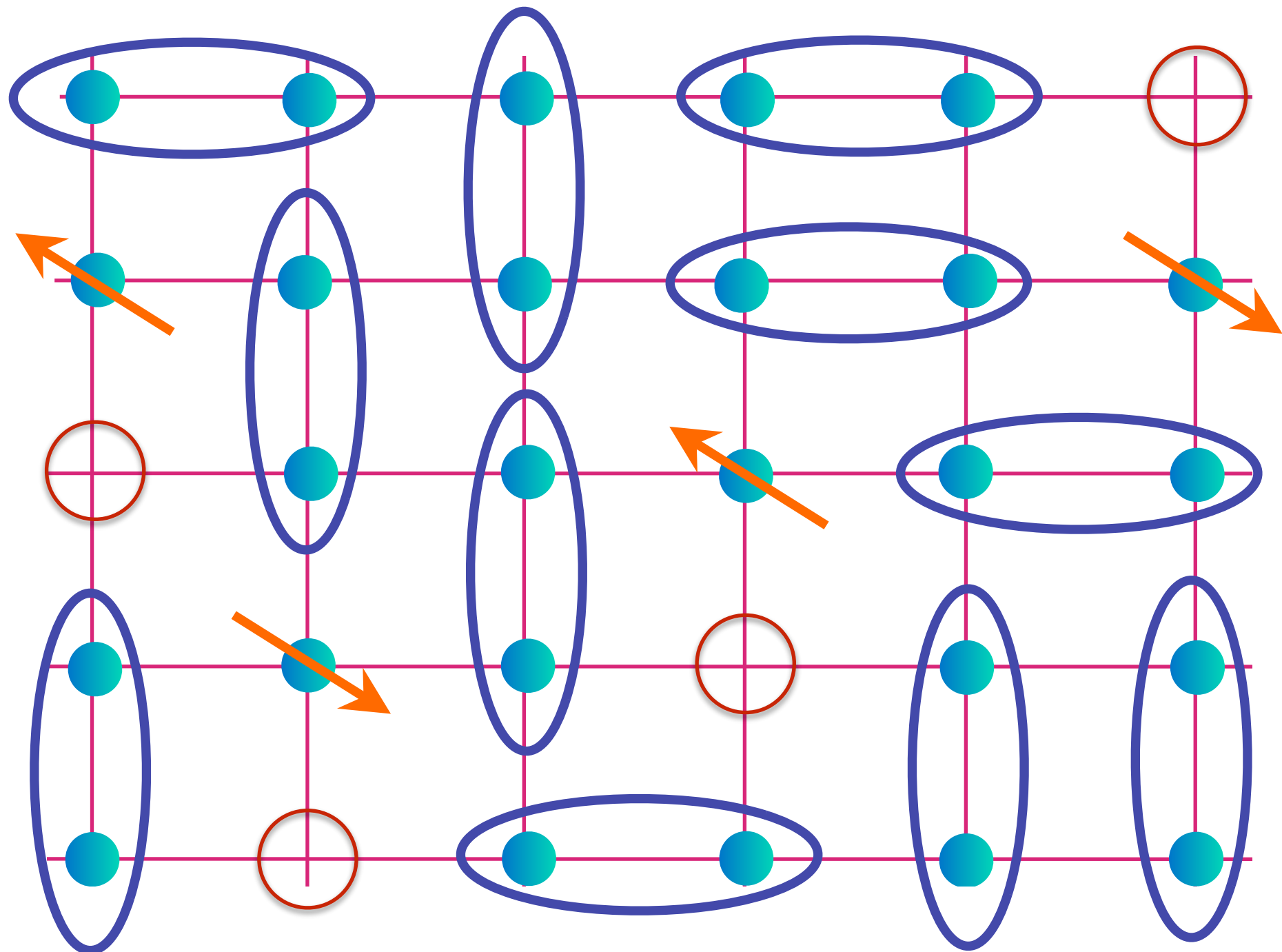

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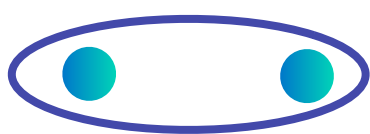



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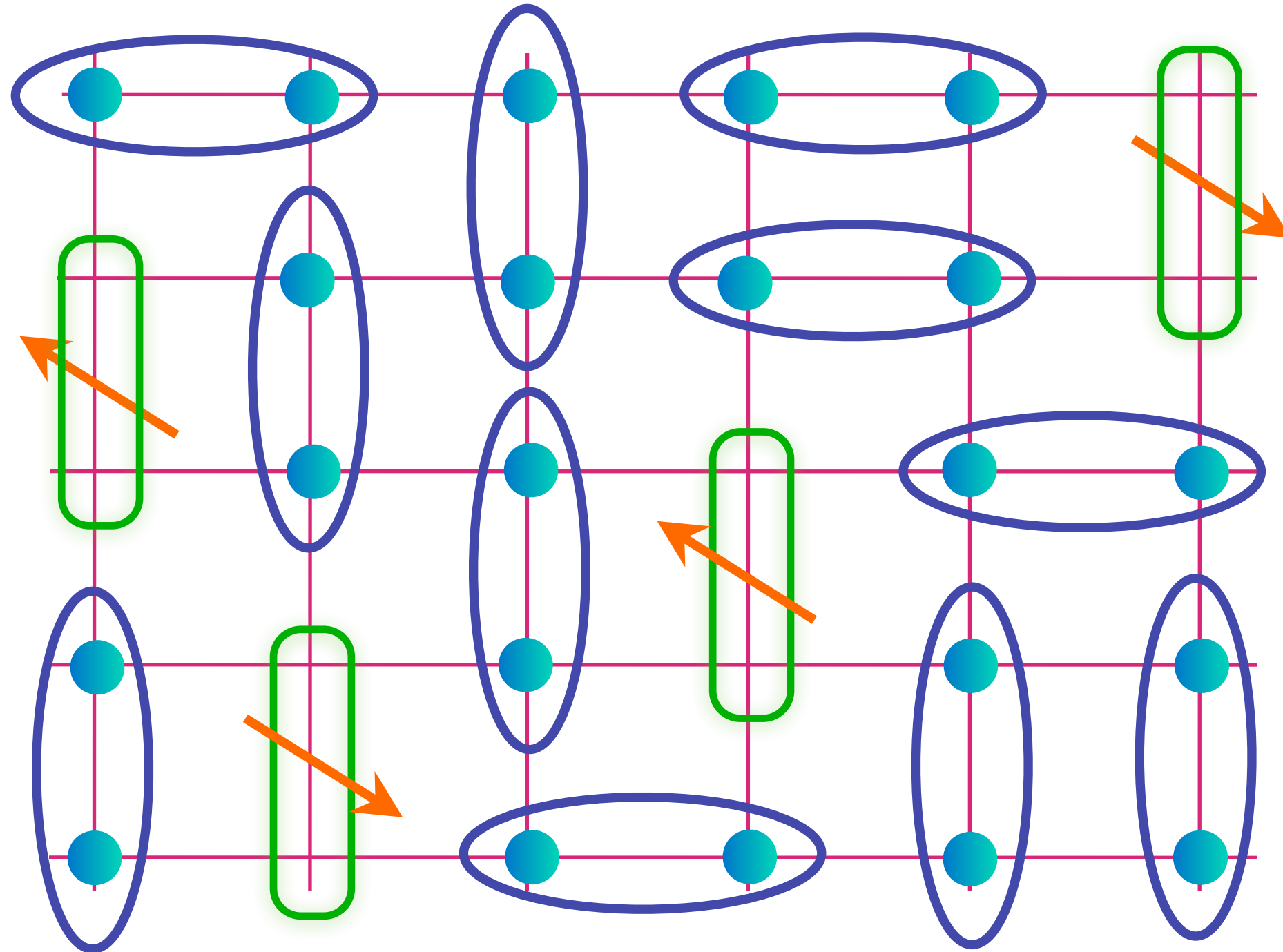



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FL*

S. Sachdev PRB 49, 6770 (1994); X.-G. Wen and P.A. Lee PRL 76, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB 75, 235122 (2007)



Mobile
 $S=1/2$, charge
 $+e$ fermionic
dimers: form
a Fermi
surface of
size p visible
in electron
photo-
emission

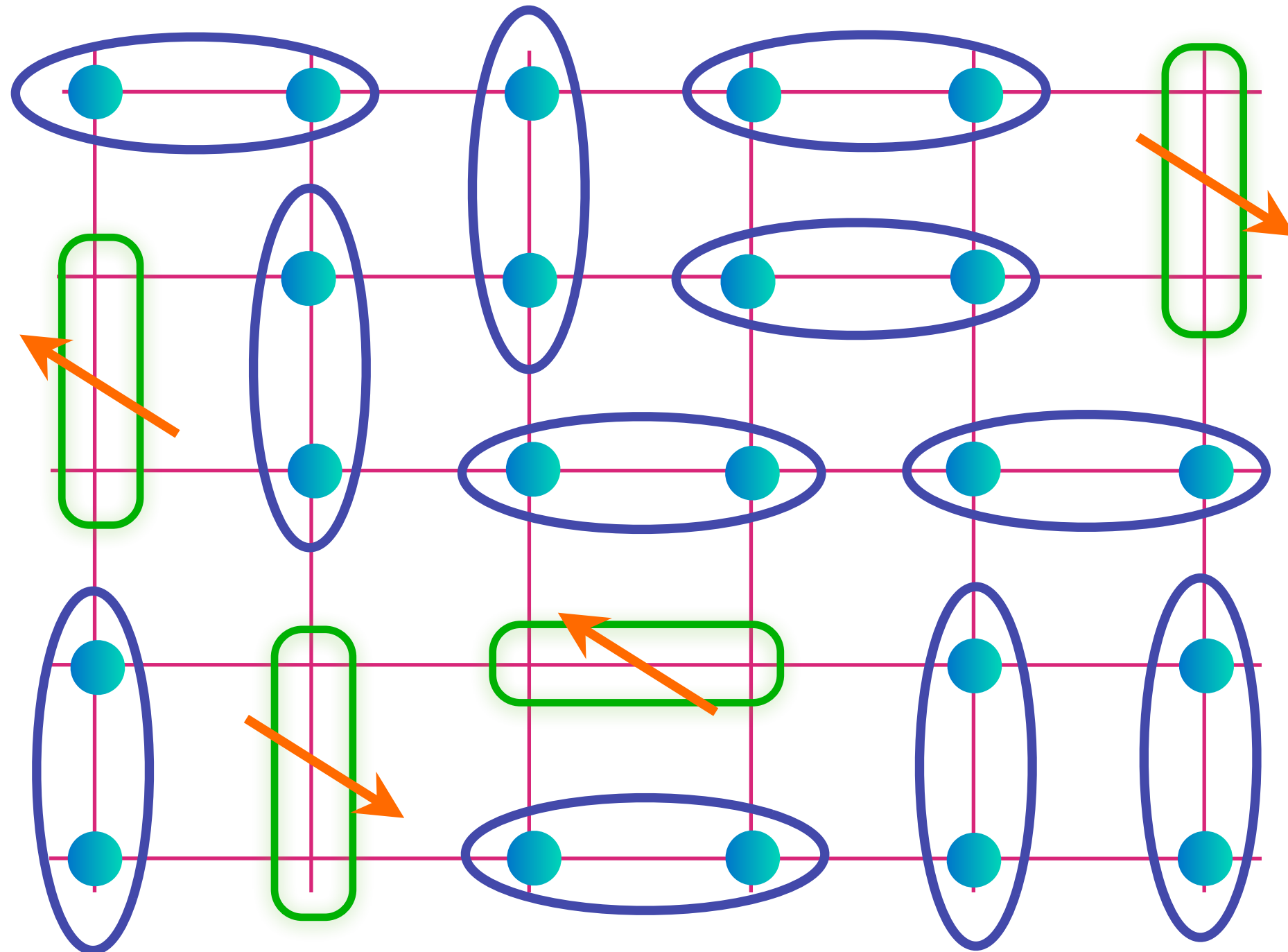
$$\text{Blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{Green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

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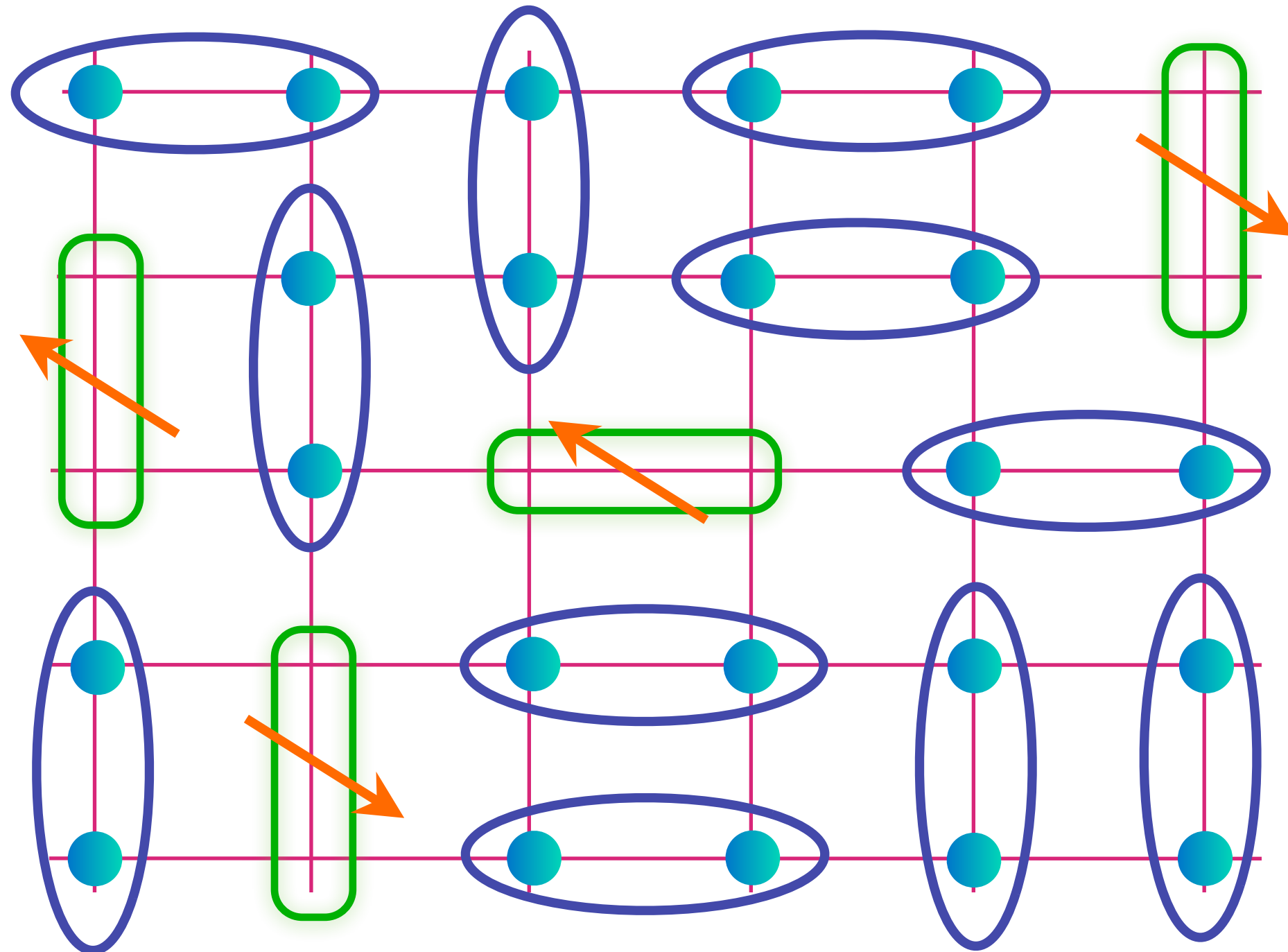
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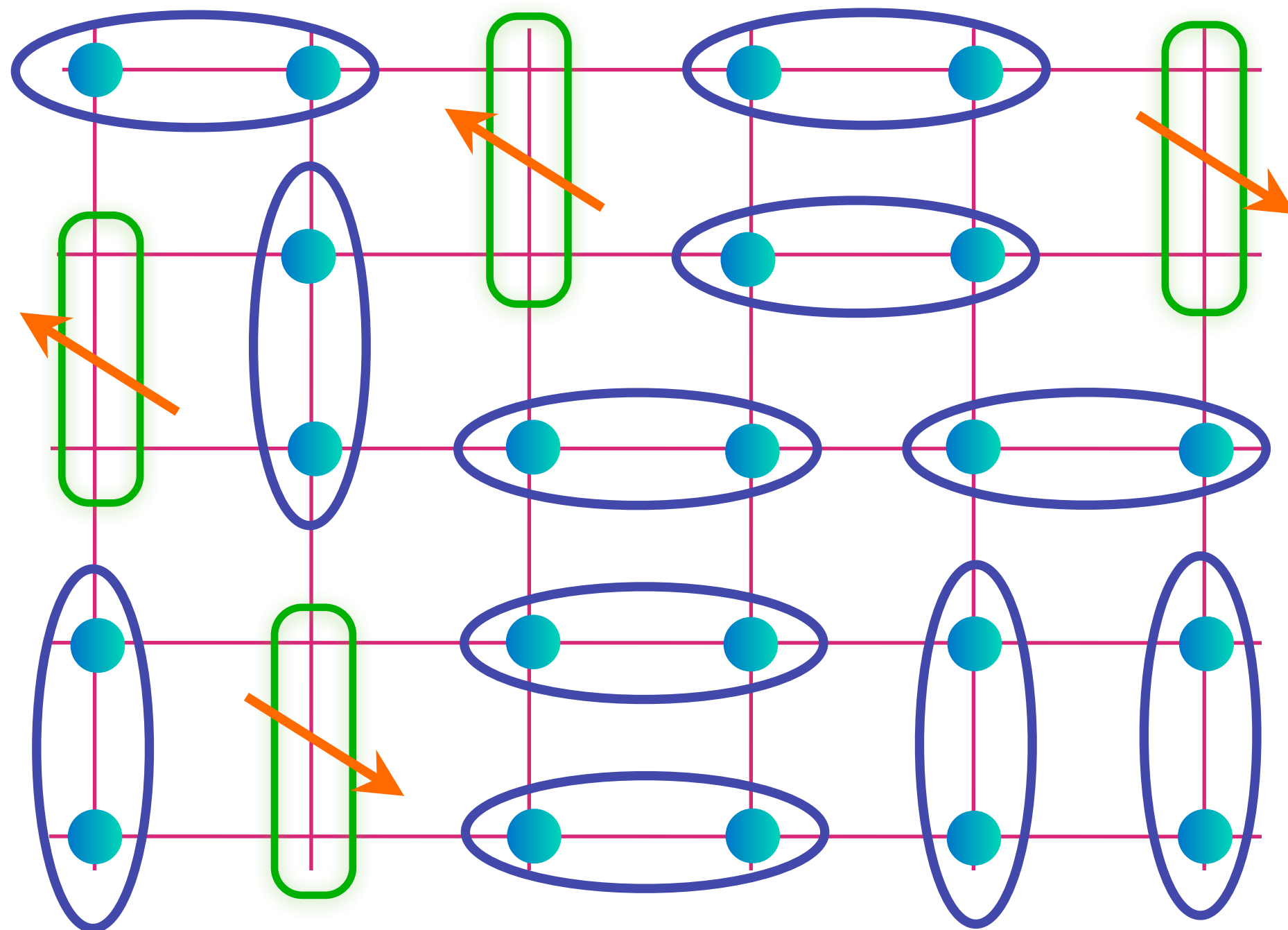
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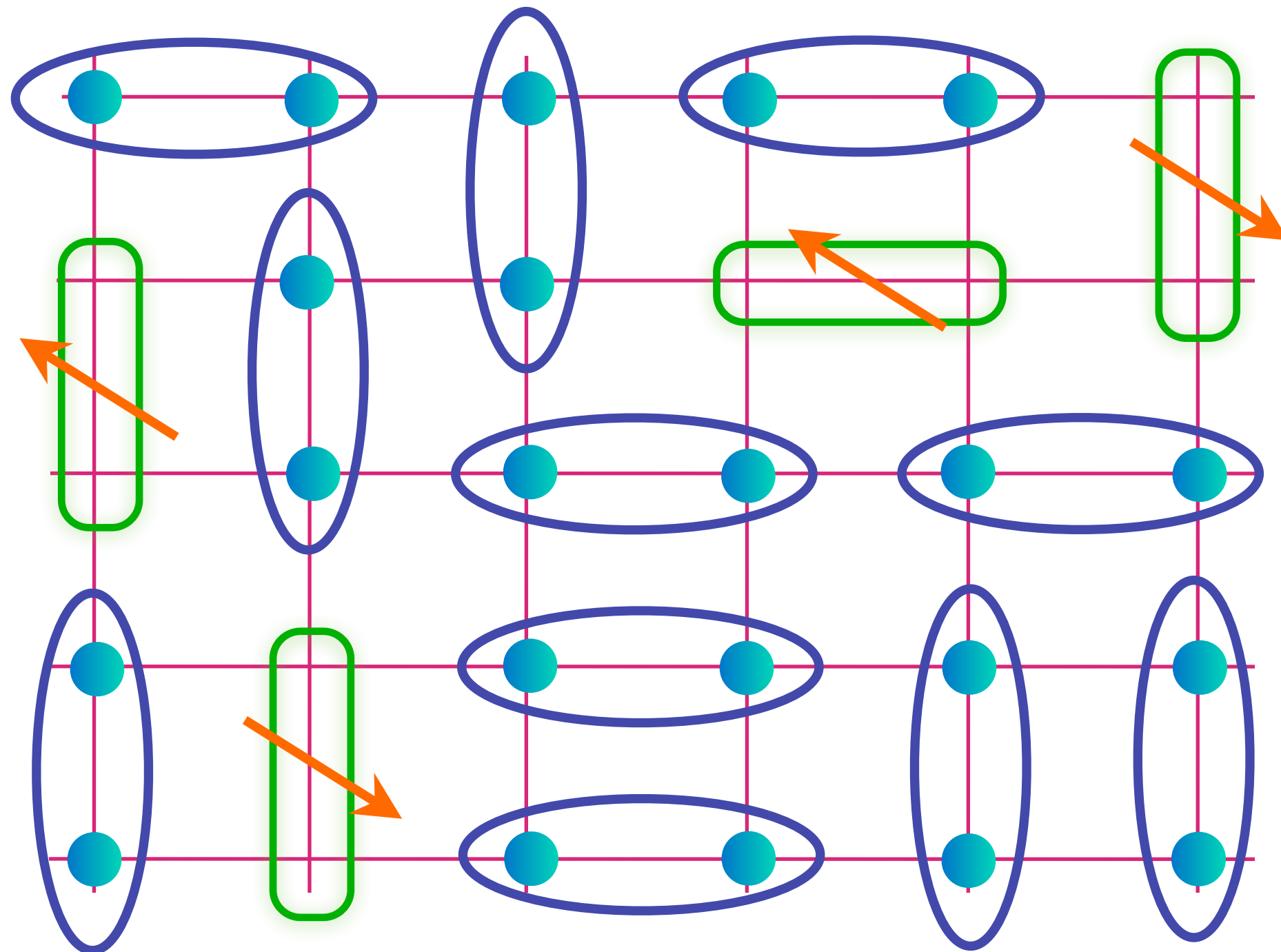
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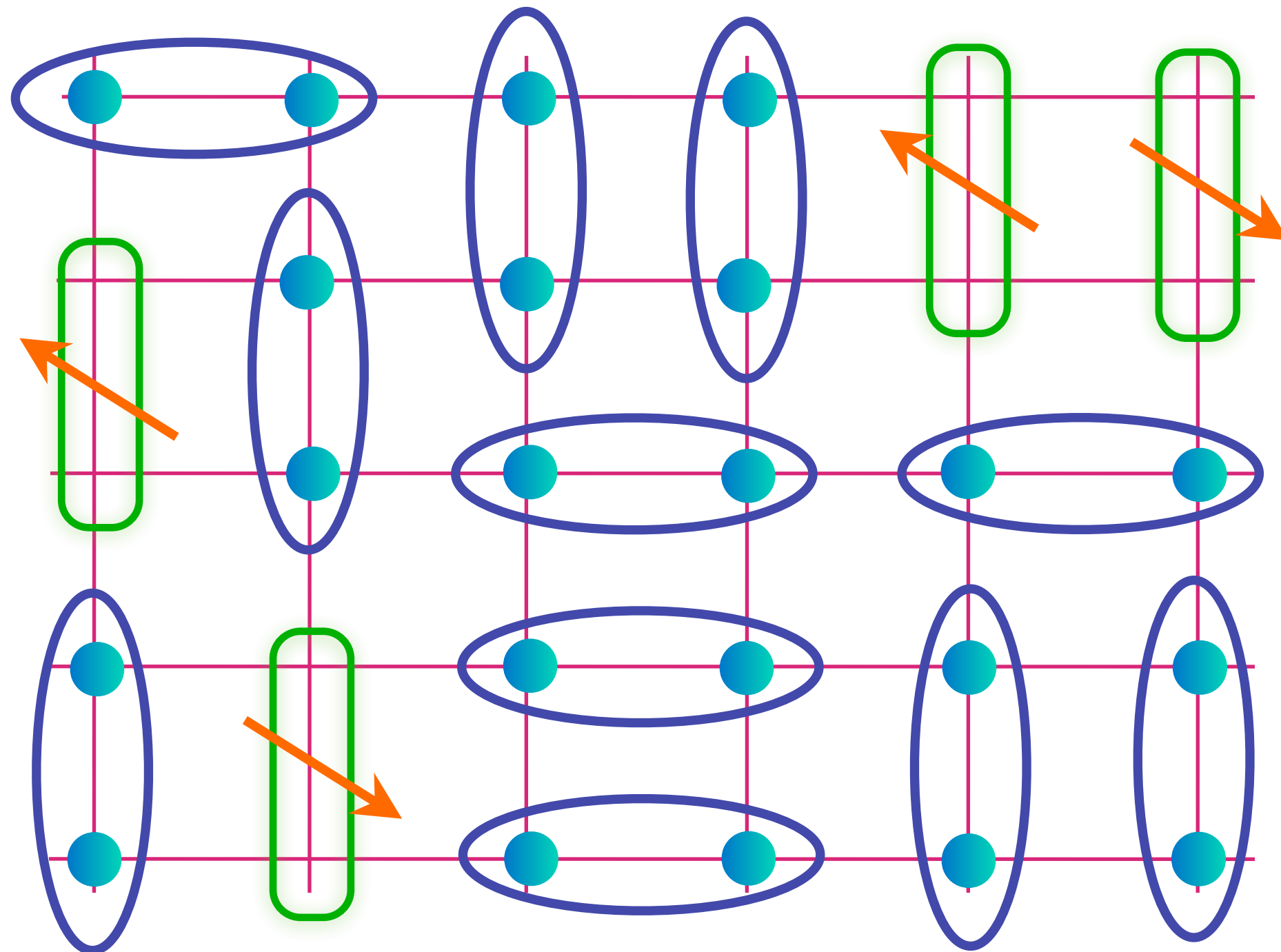
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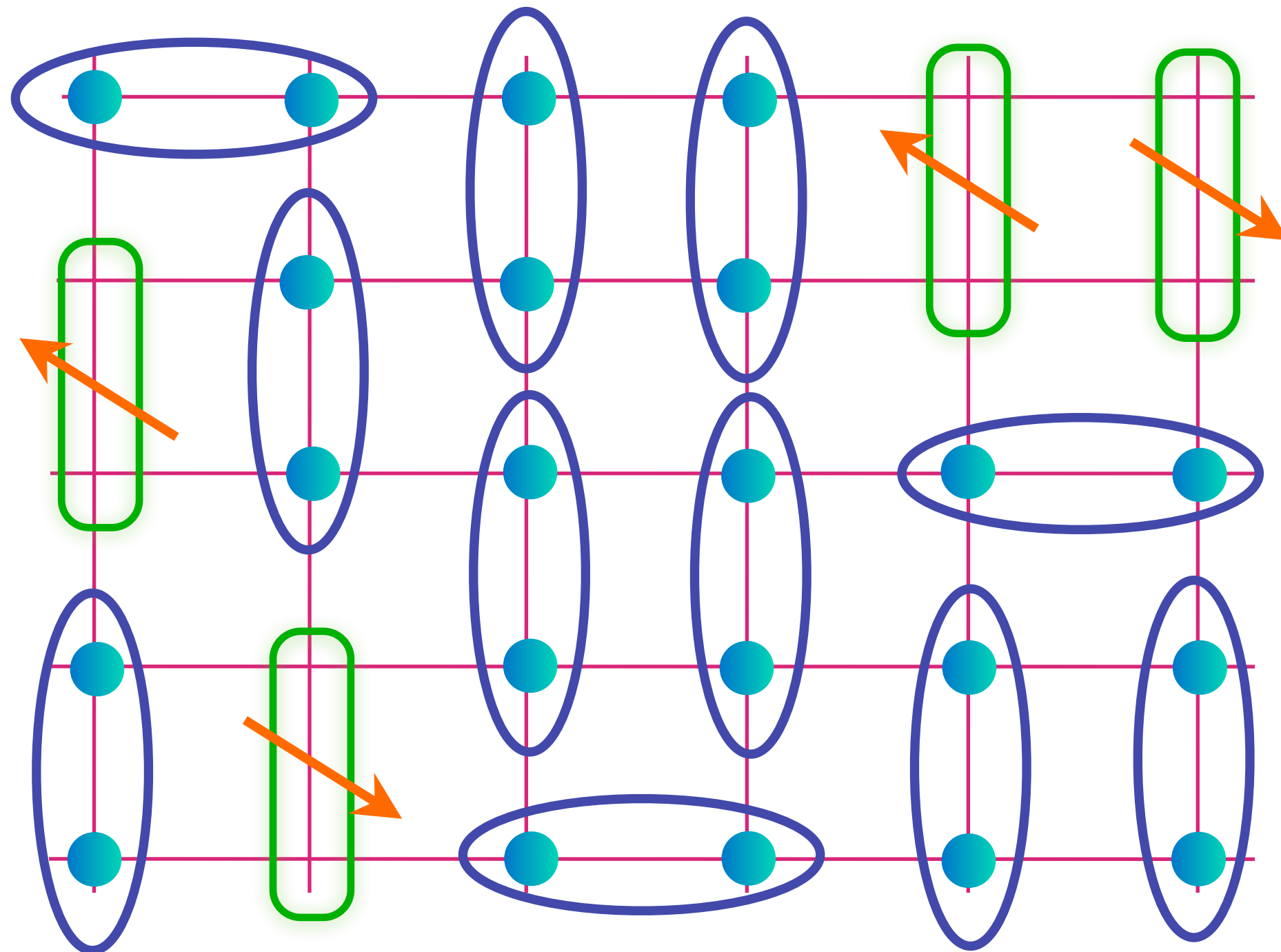
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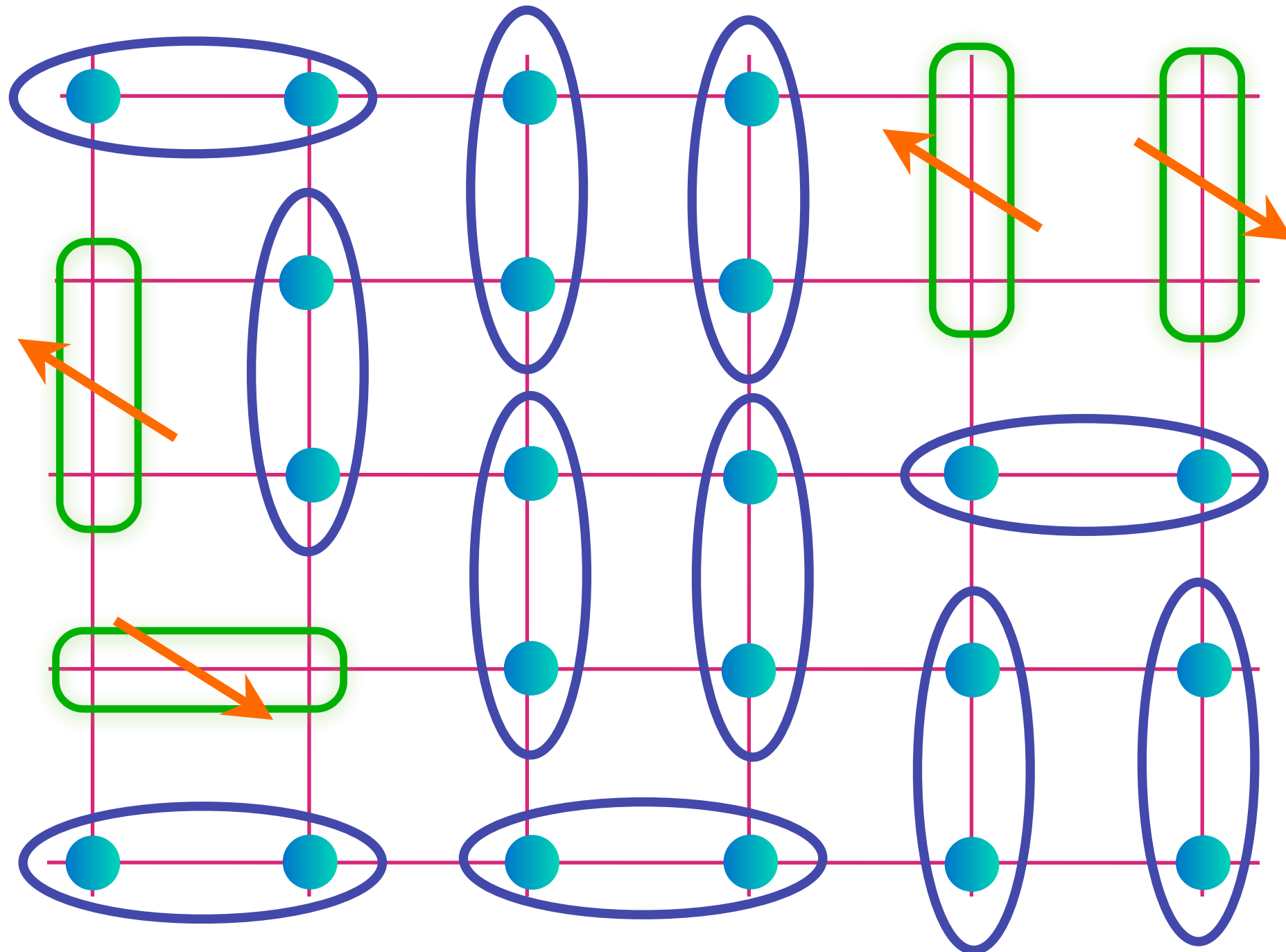
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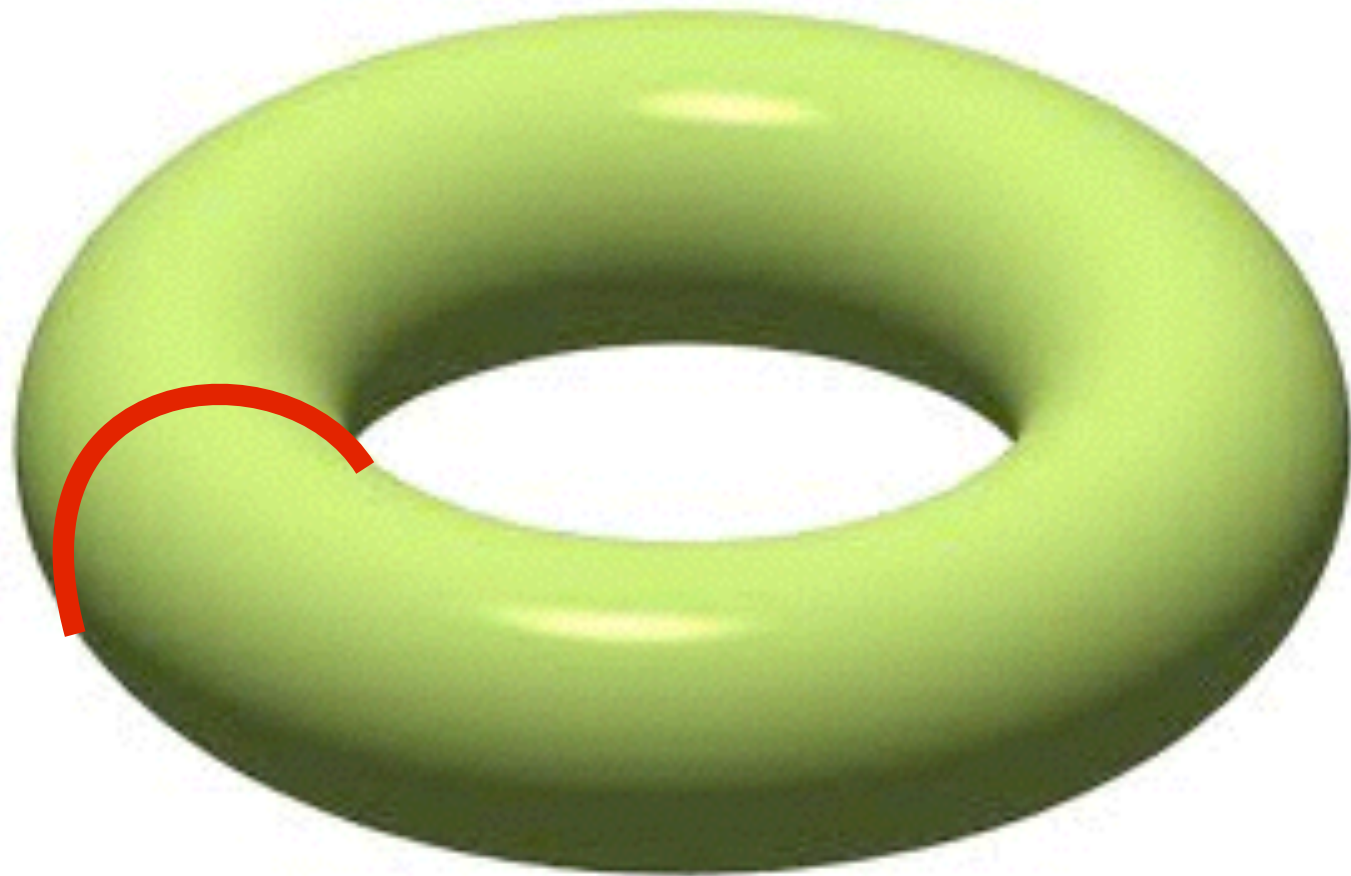
$$\text{Green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Ground state degeneracy

Place FL^*
on a torus:

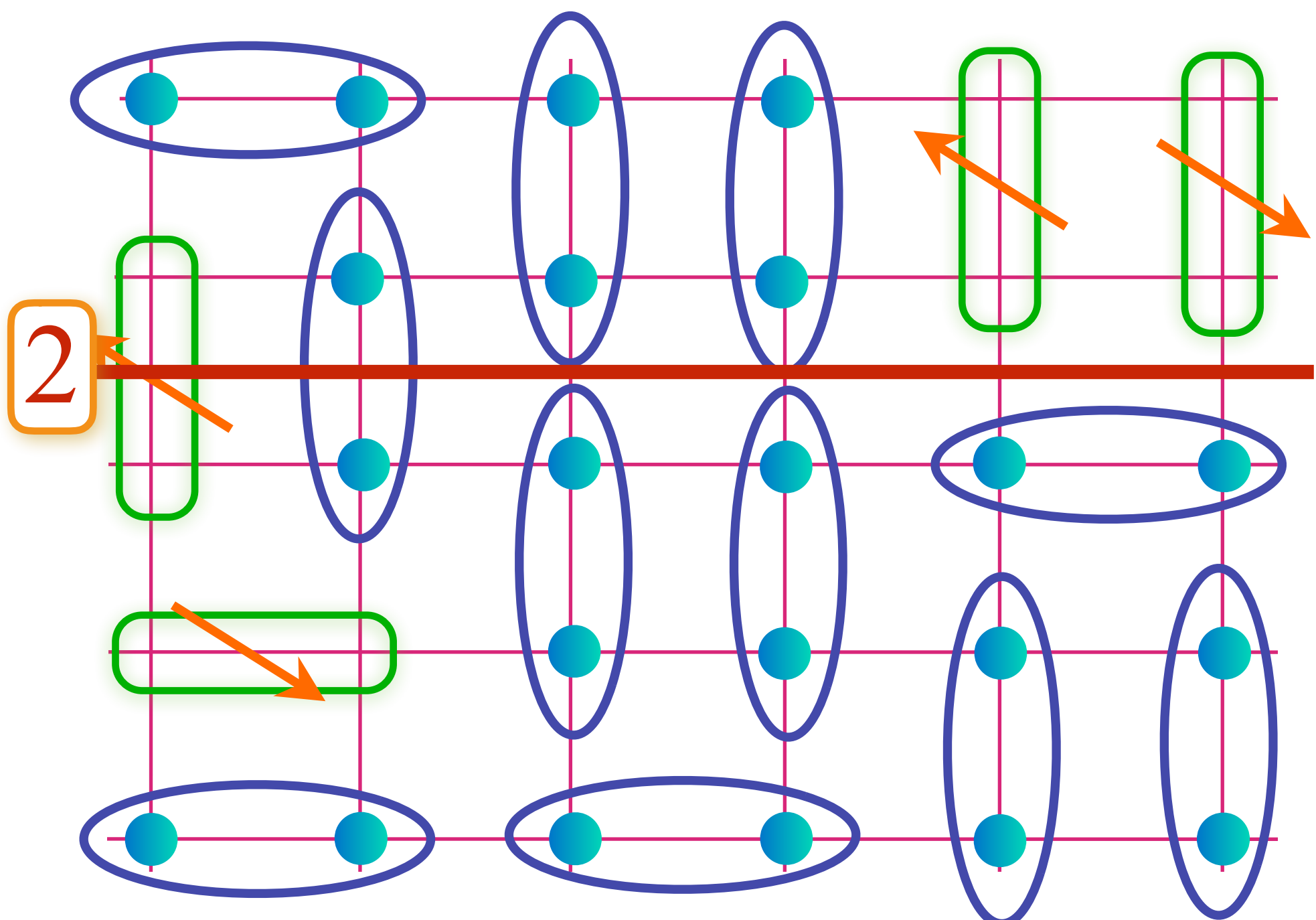


Ground state degeneracy

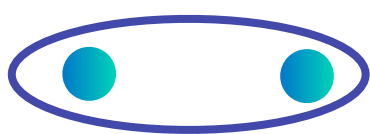


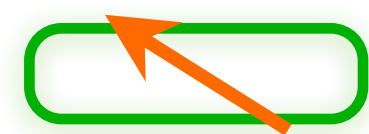
Place FL*
on a torus:
obtain
“topological”
states nearly
degenerate with
quasiparticle
states: number
of dimers
crossing red line
is conserved
modulo 2

FL*

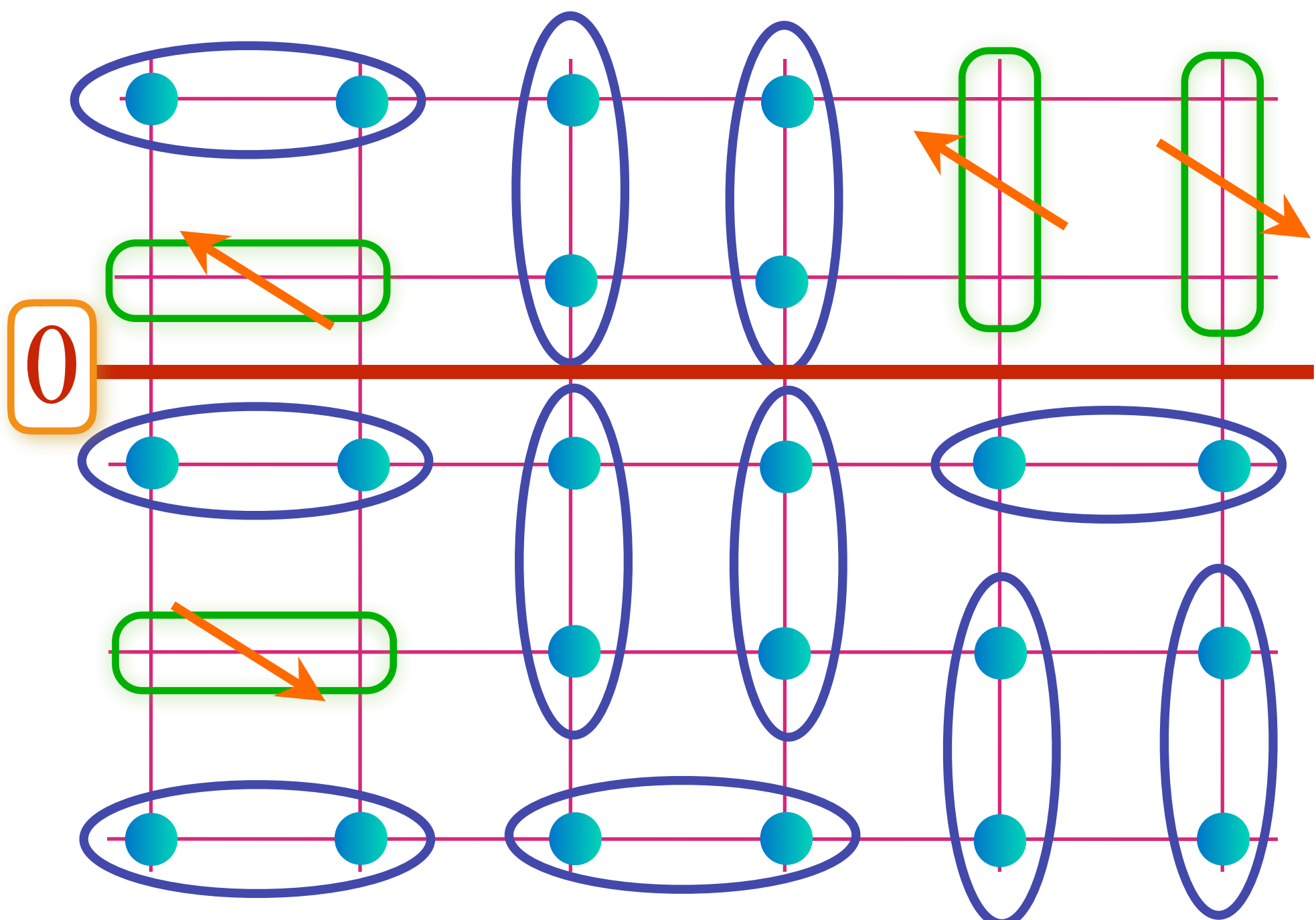


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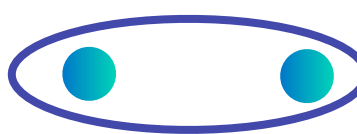
 = $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

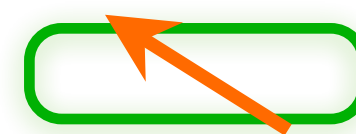
 = $(|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$

FL*

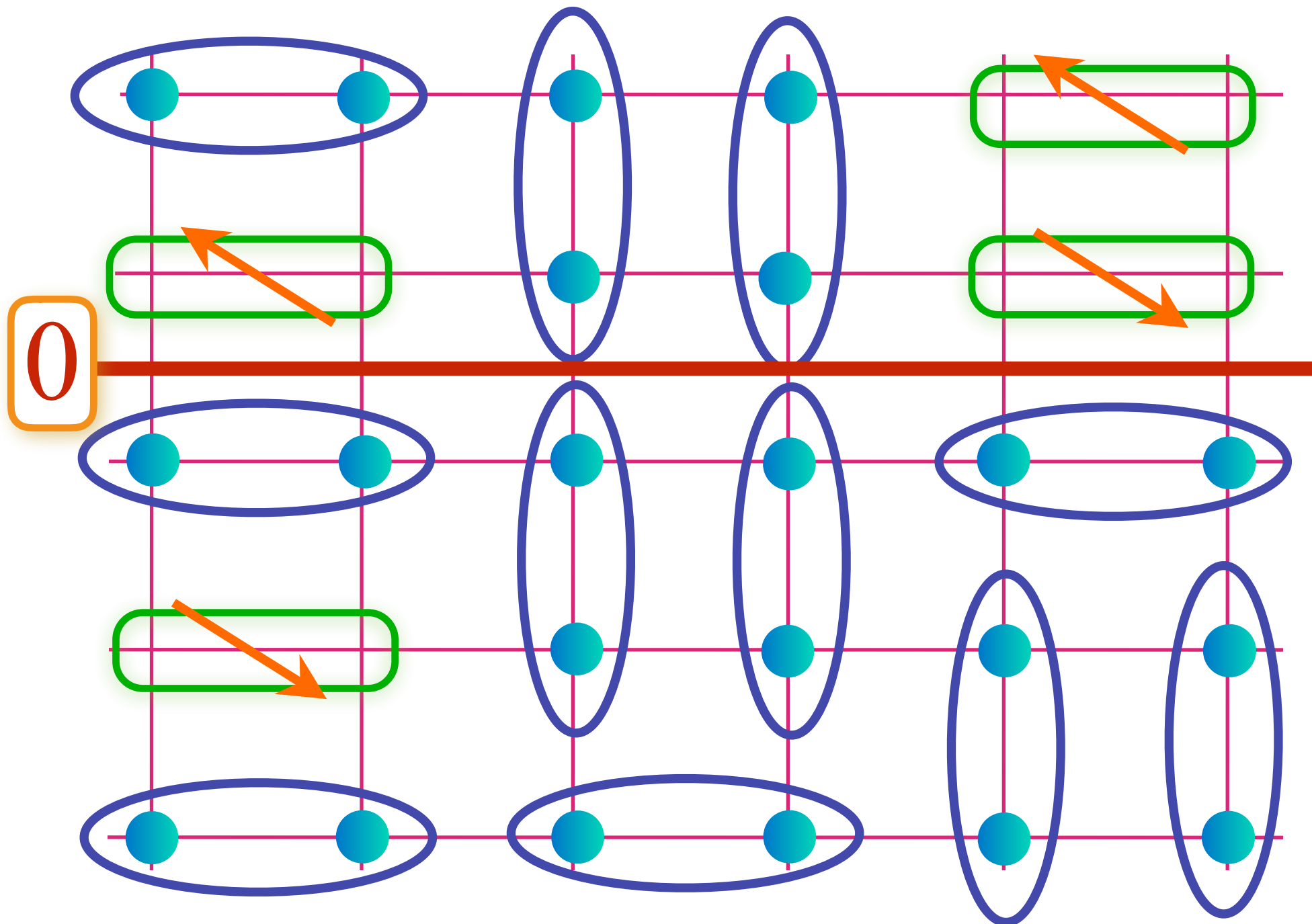


Place FL*
on a torus:
obtain
“topological”
states nearly
degenerate with
quasiparticle
states: number
of dimers
crossing red line
is conserved
modulo 2

 = $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

 = $(|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$

FL*



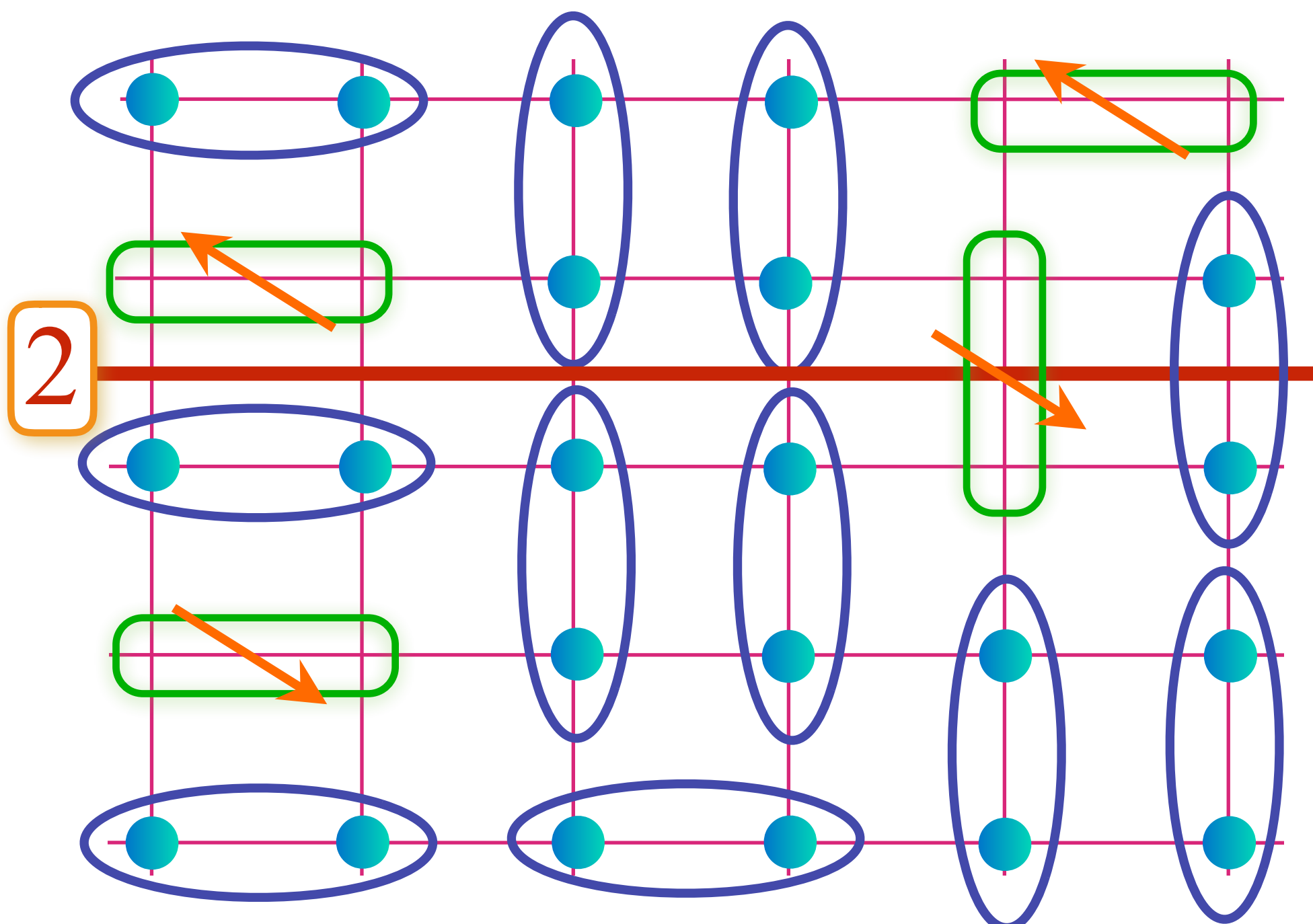
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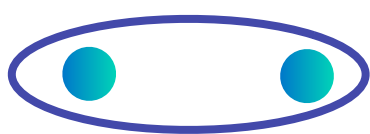
$$\text{Blue oval with two dots} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

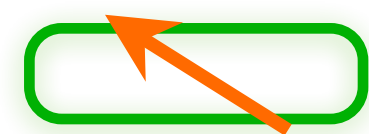
$$\text{Green box with arrow} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

FL*



Place FL*
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 = $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

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FL*

We have described a metal with:

- A Fermi surface of electrons enclosing volume p , and not the Luttinger volume of $l+p$
- Additional low energy quantum states on a torus not associated with quasiparticle excitations *i.e.* emergent gauge fields

FL*

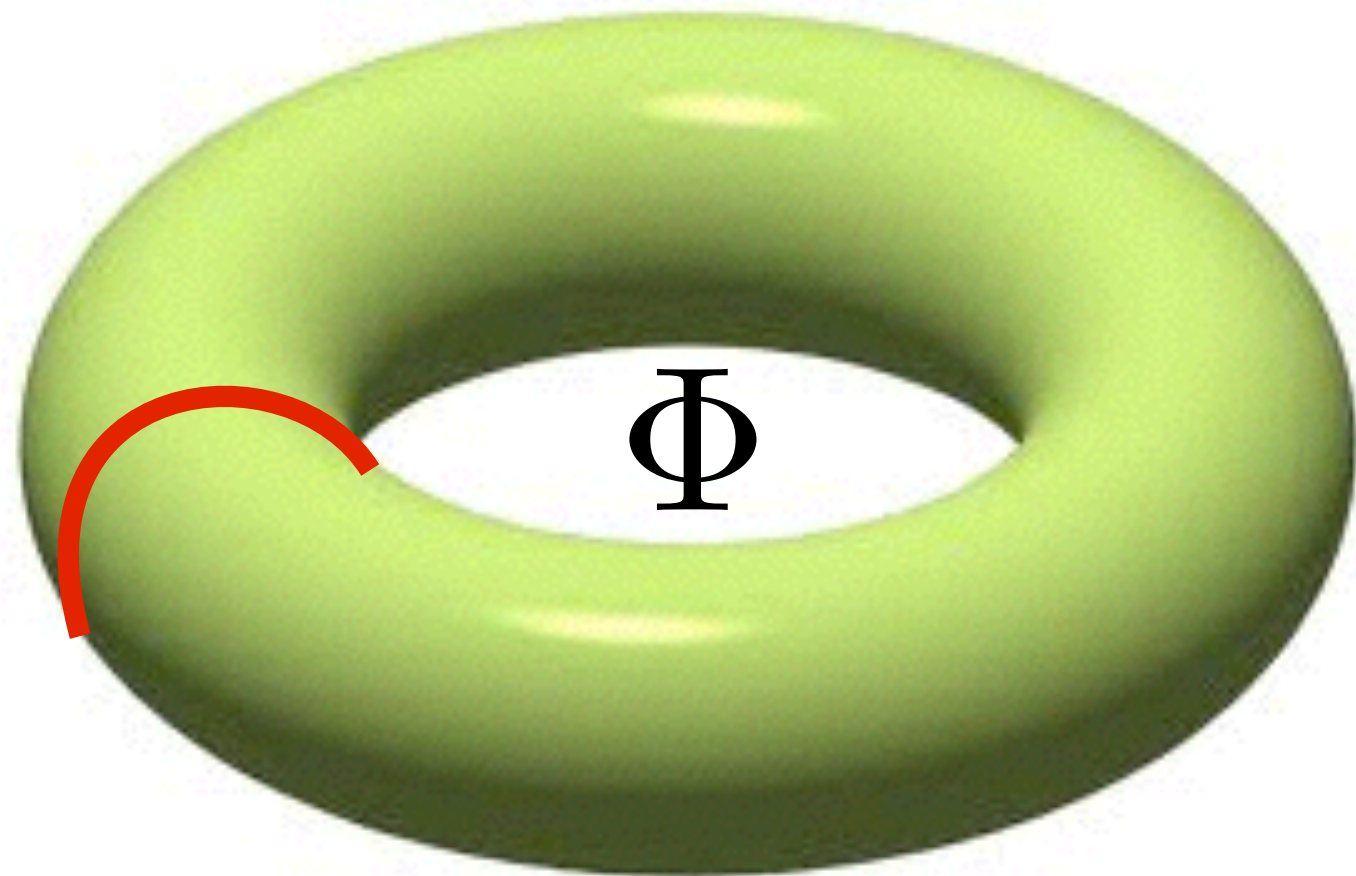
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There is a general and fundamental relationship between these two characteristics.

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)



Following the evolution of the quantum state under adiabatic insertion of a flux quantum leads to a non-perturbative argument for the volume enclosed by the Fermi surface

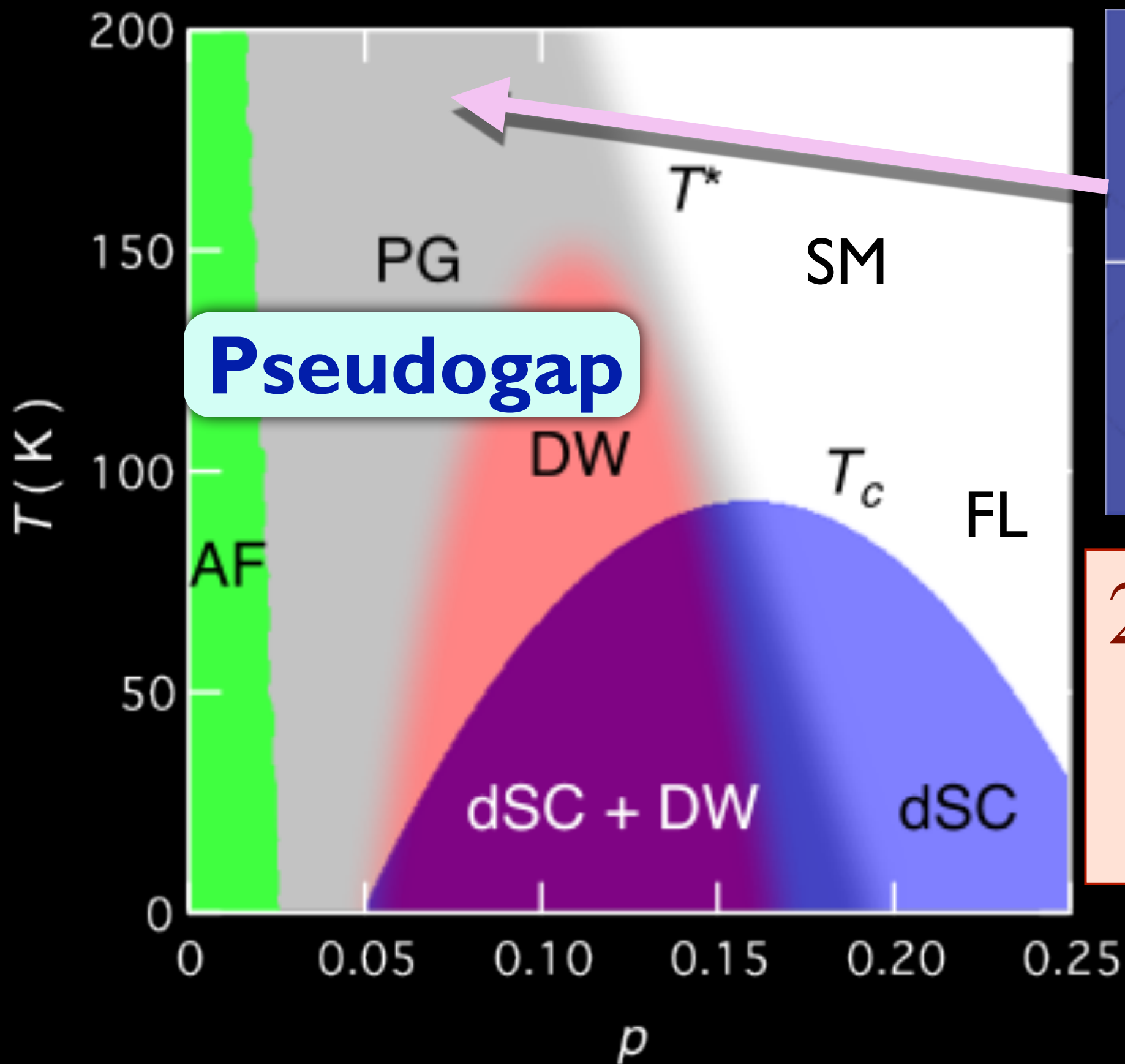
M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

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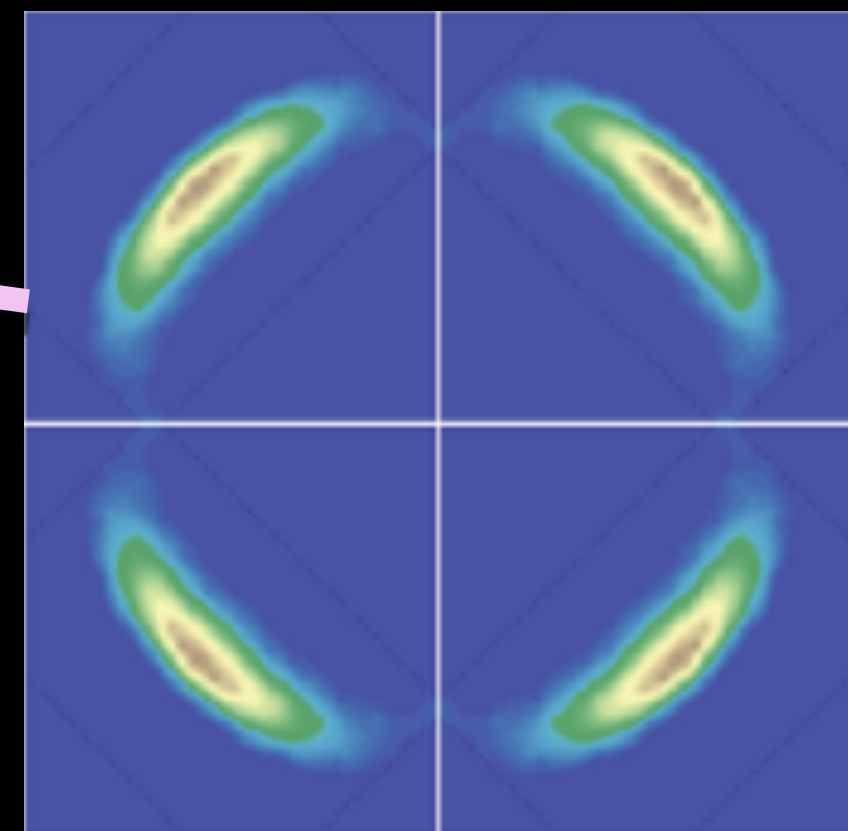
1. Emergent gauge fields and long-range entanglement in insulators
2. Theory of ordinary metals: Fermi liquids (FL)
 - (a) *Quasiparticles*
 - (b) *Luttinger theorem for volume enclosed by Fermi surface*
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4. The pseudogap metal of the cuprate superconductors

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

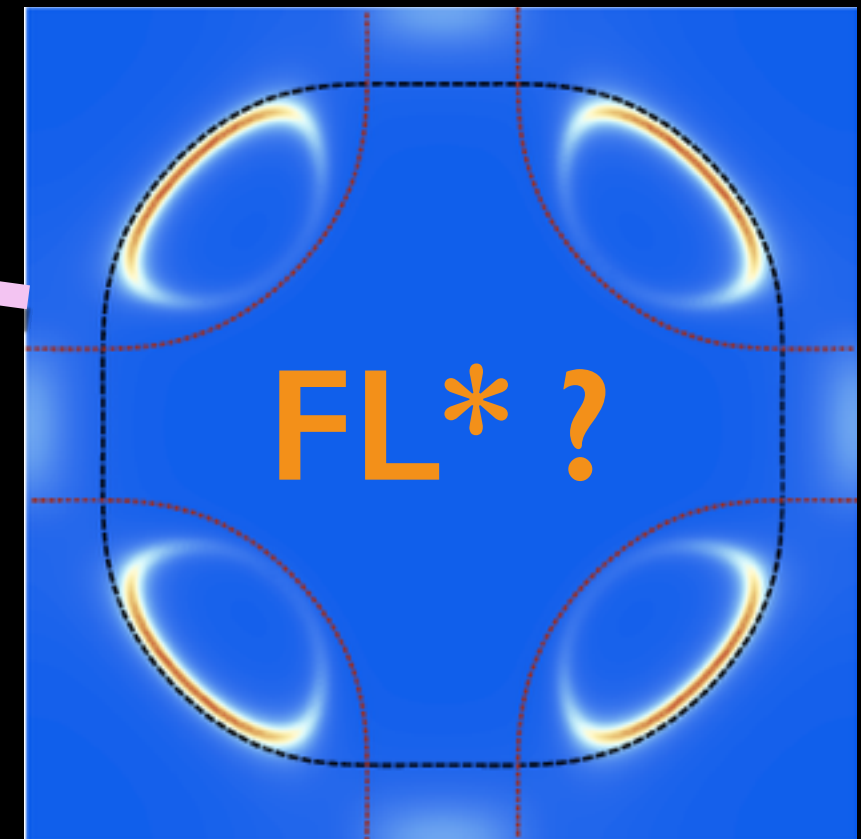
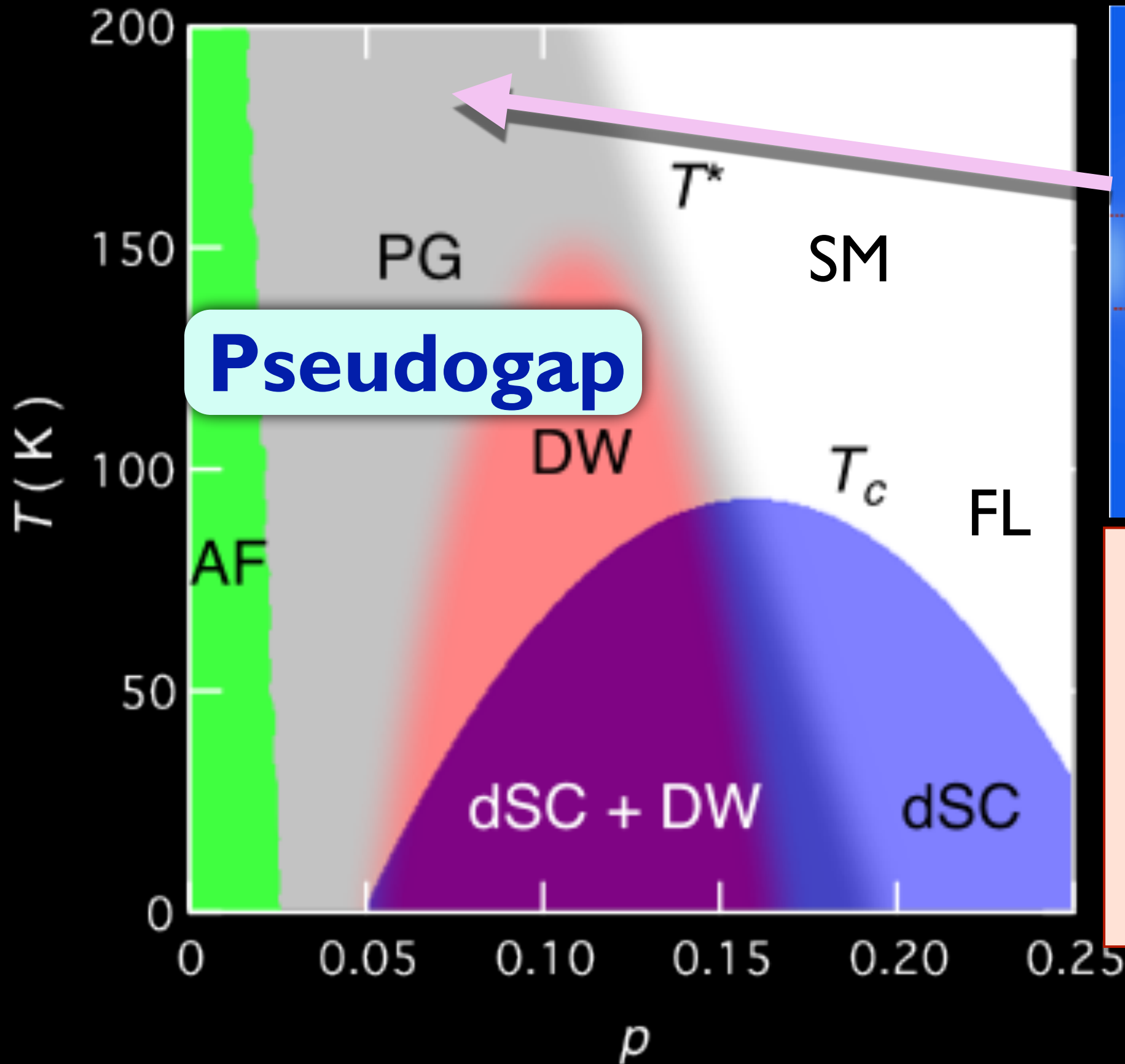


Pseudogap



2. Pseudogap
metal
at low p

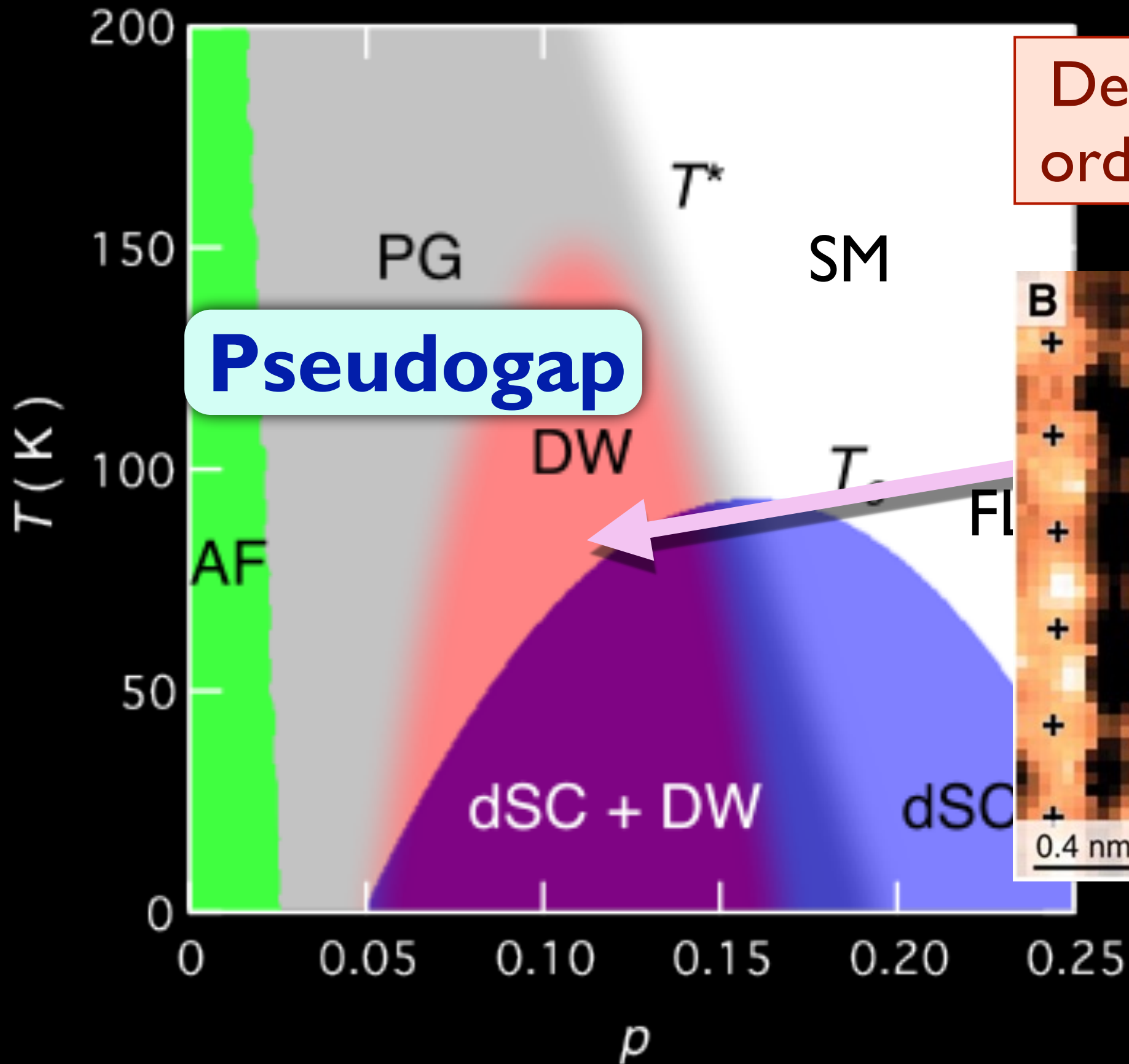
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)



A new metal - FL*:
with electron-like
quasiparticles on a
Fermi surface of
size p and emergent
gauge fields

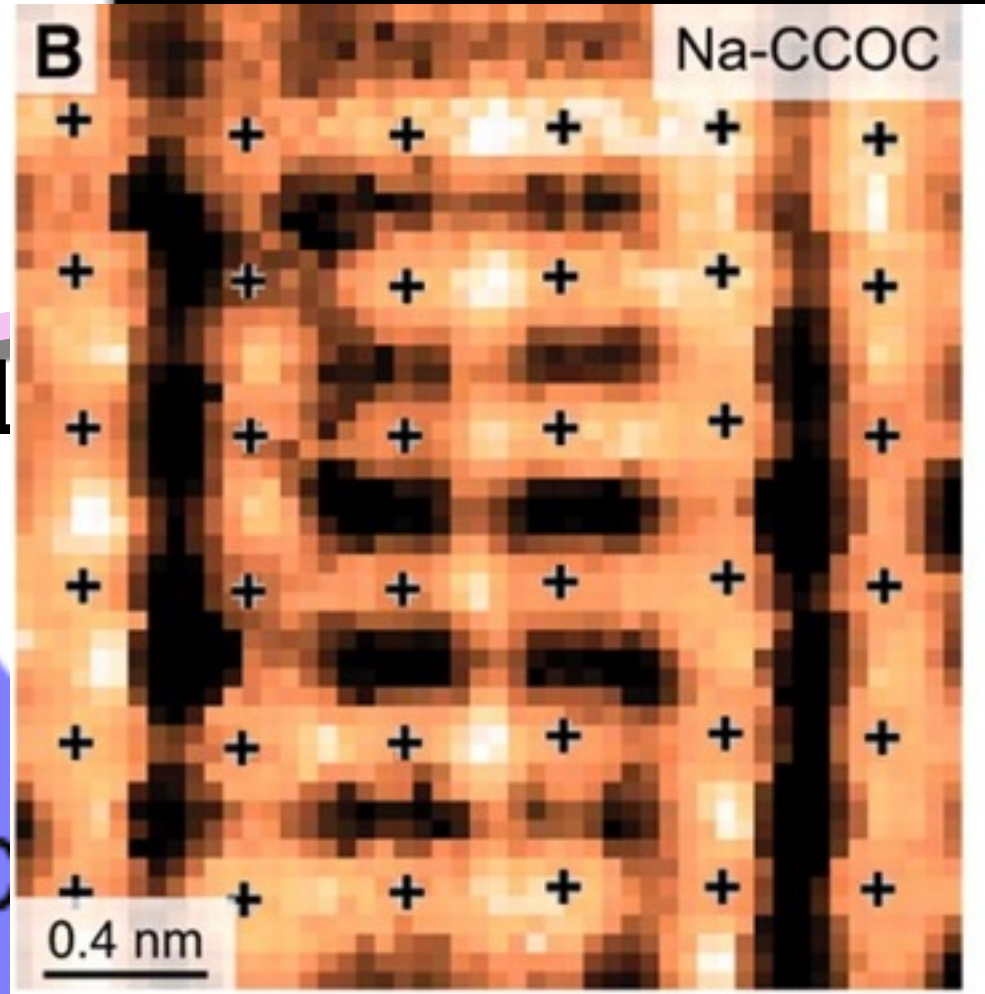
Recent evidence for pseudogap metal as FL*

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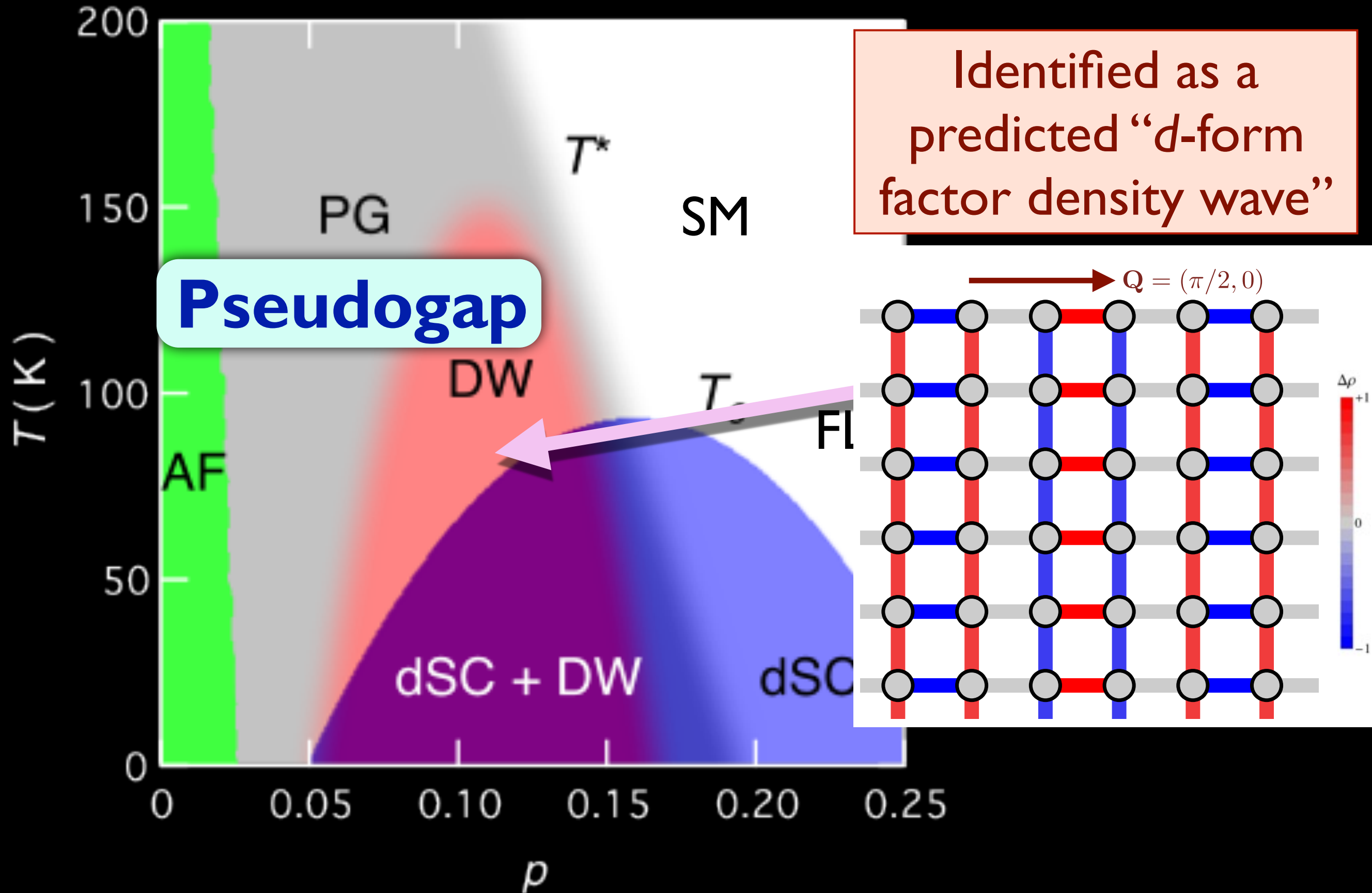
Pseudogap

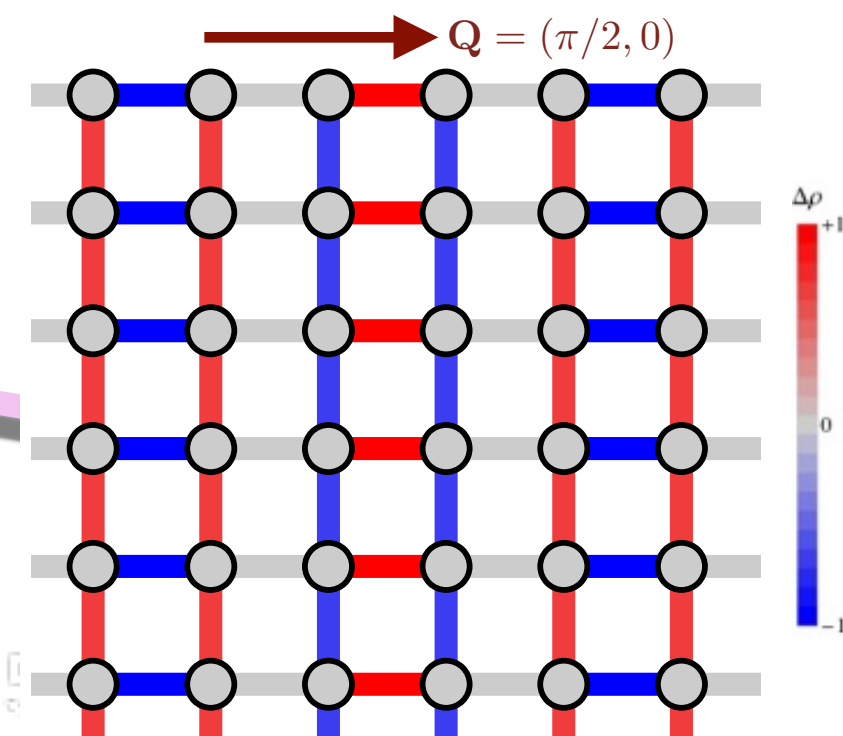
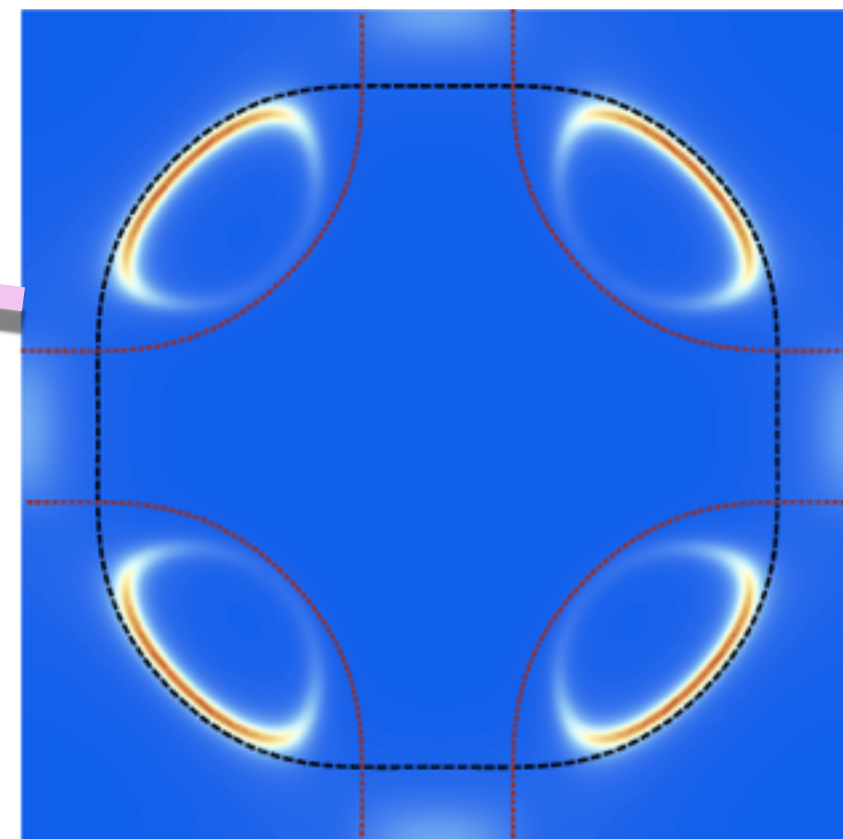
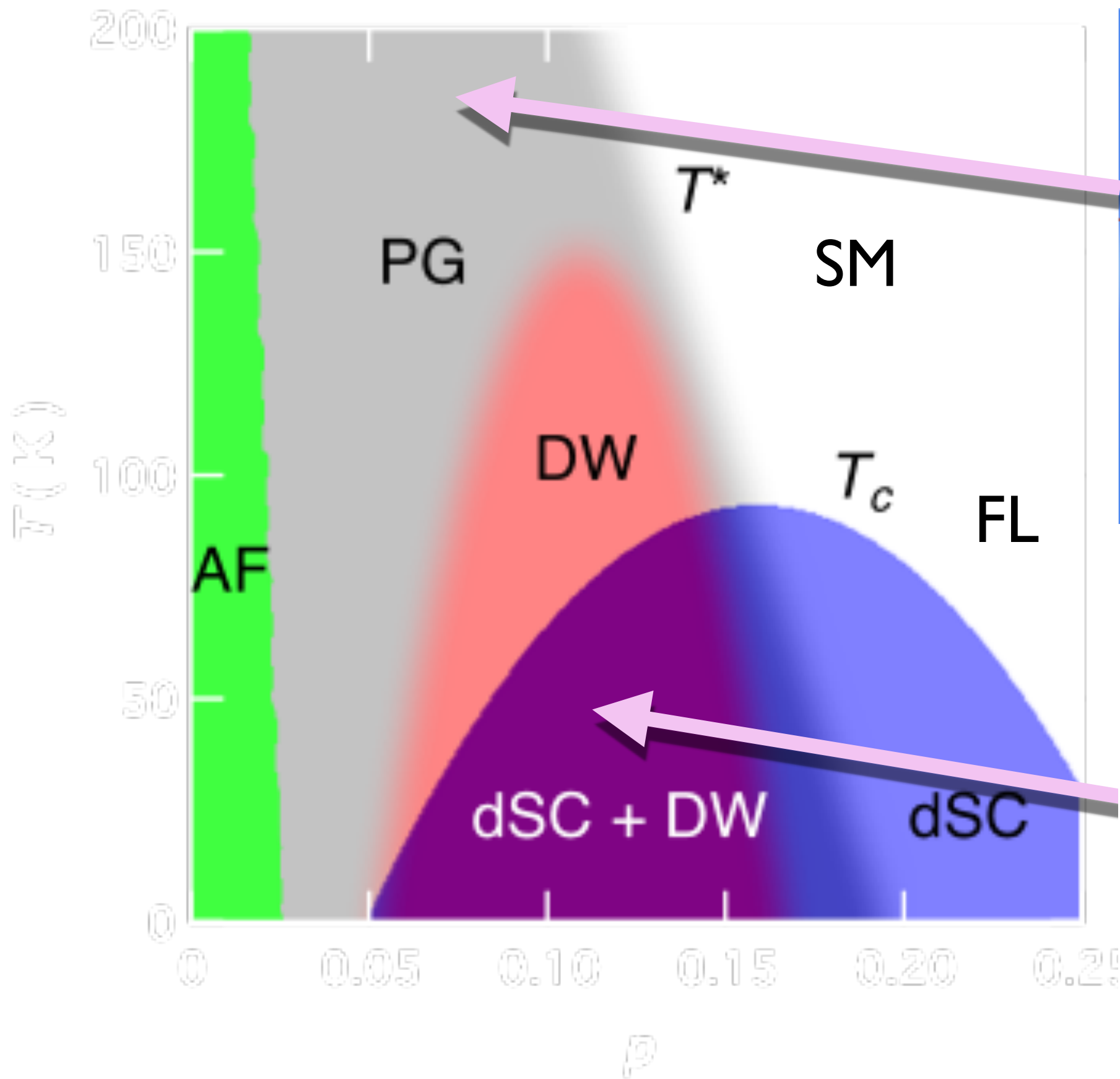
Density wave (DW) order at low T and p

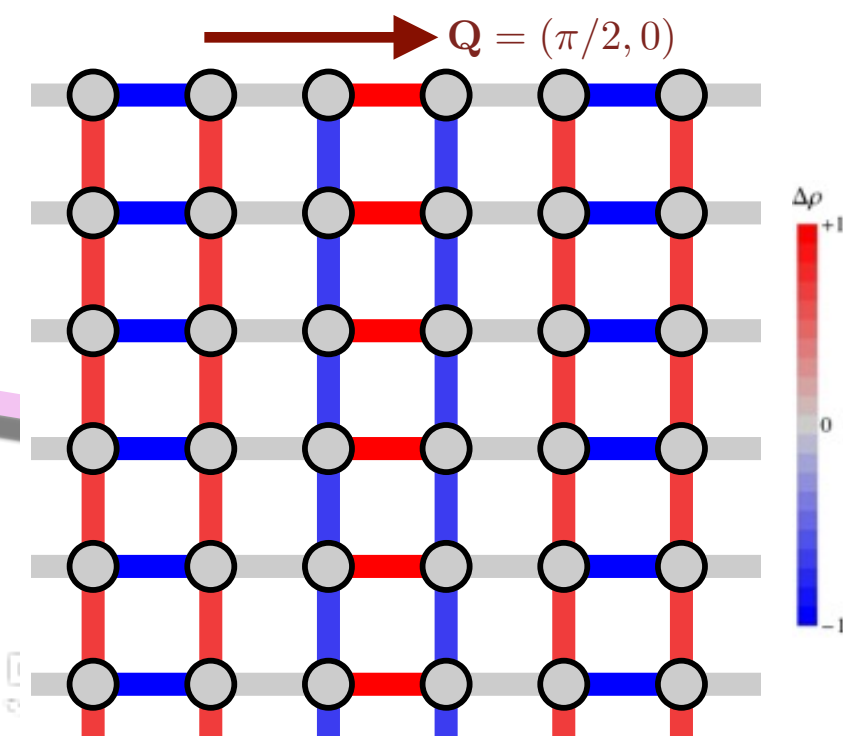
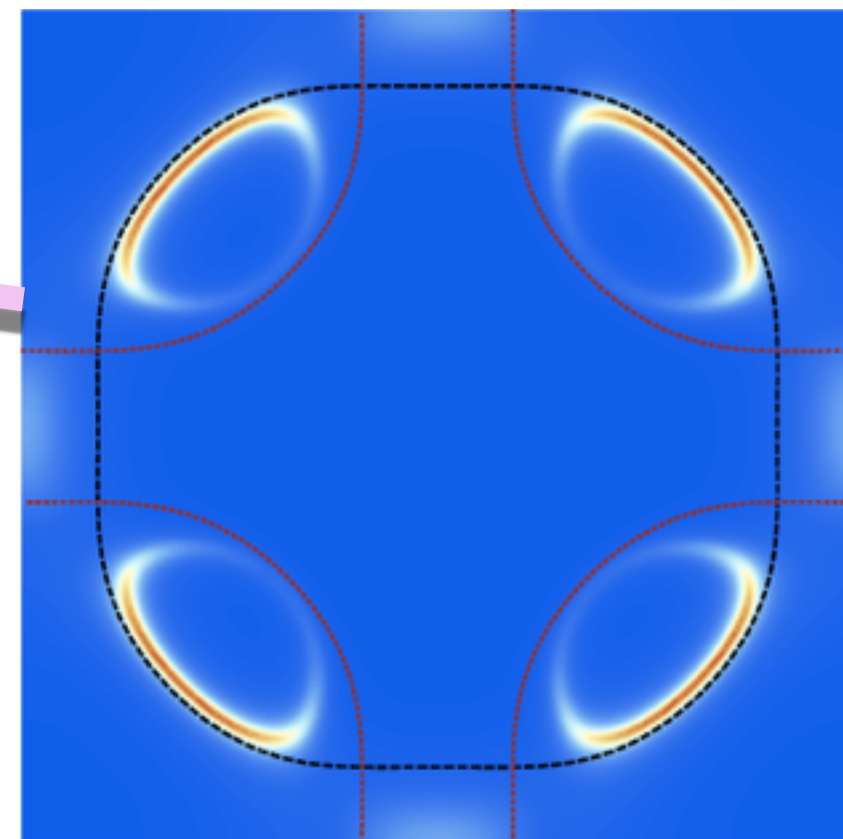
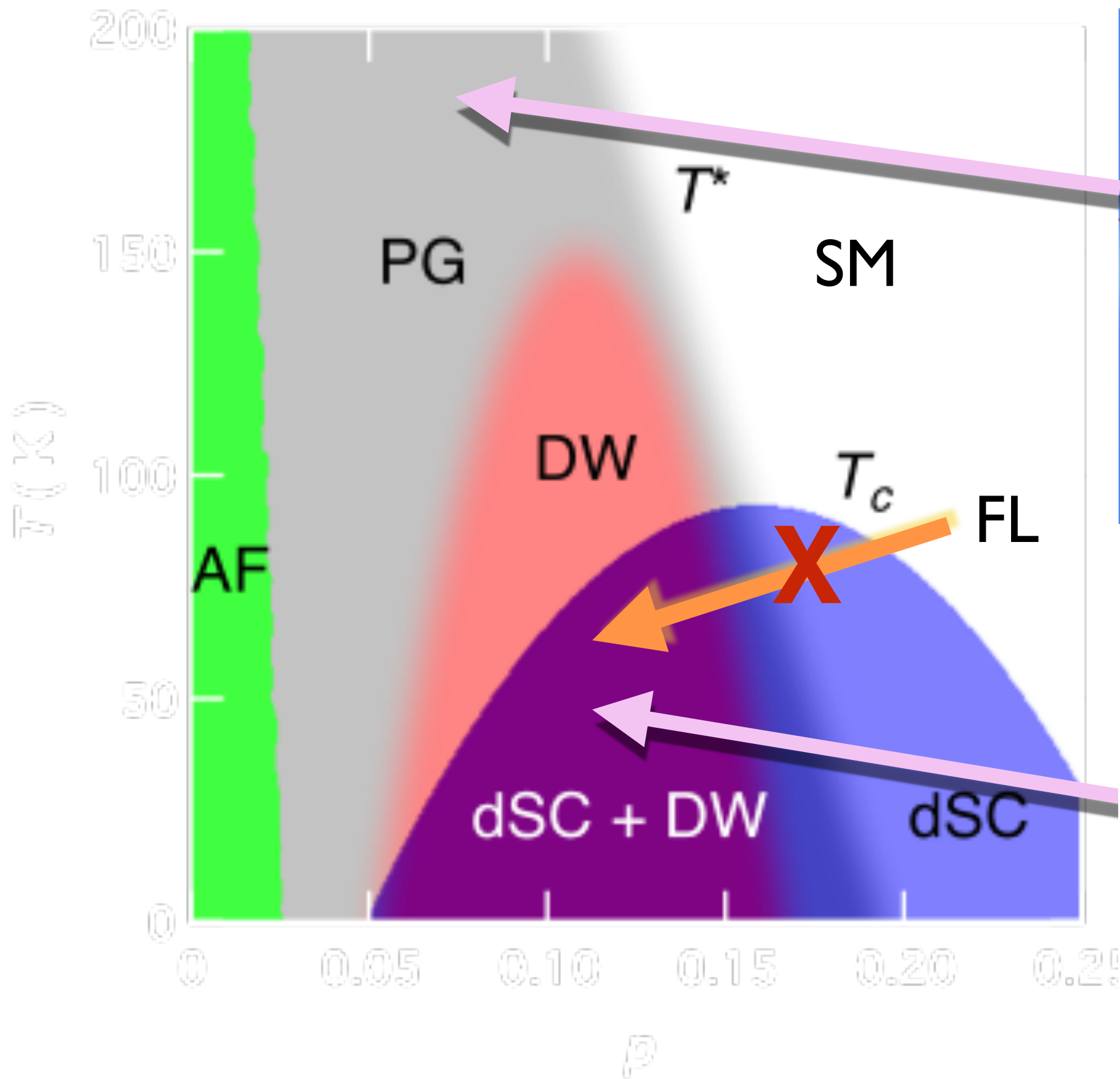


M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).

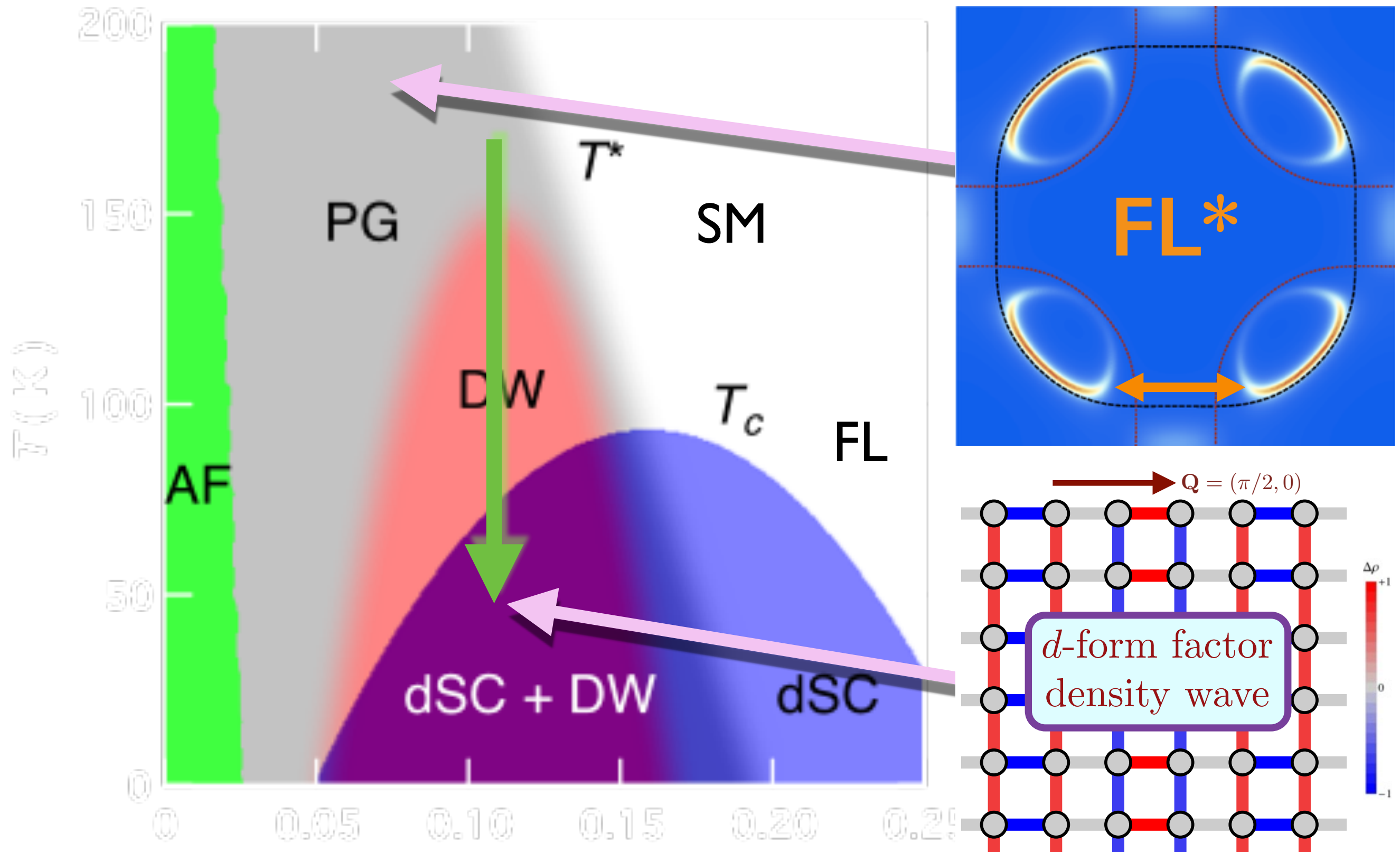
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)







The high T FL* can help explain the “d-form factor density wave” observed at low T



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- T -independent positive Hall coefficient, R_H , corresponding to carrier density p in the higher temperature pseudogap (Ando *et al.*, PRL **92**, 197001 (2004)) and in recent measurements at high fields, low T , and around $p \approx 0.16$ in YBCO (Proust-Taillefer-UBC collaboration, Badoux *et al.*, arXiv:1511.08162).

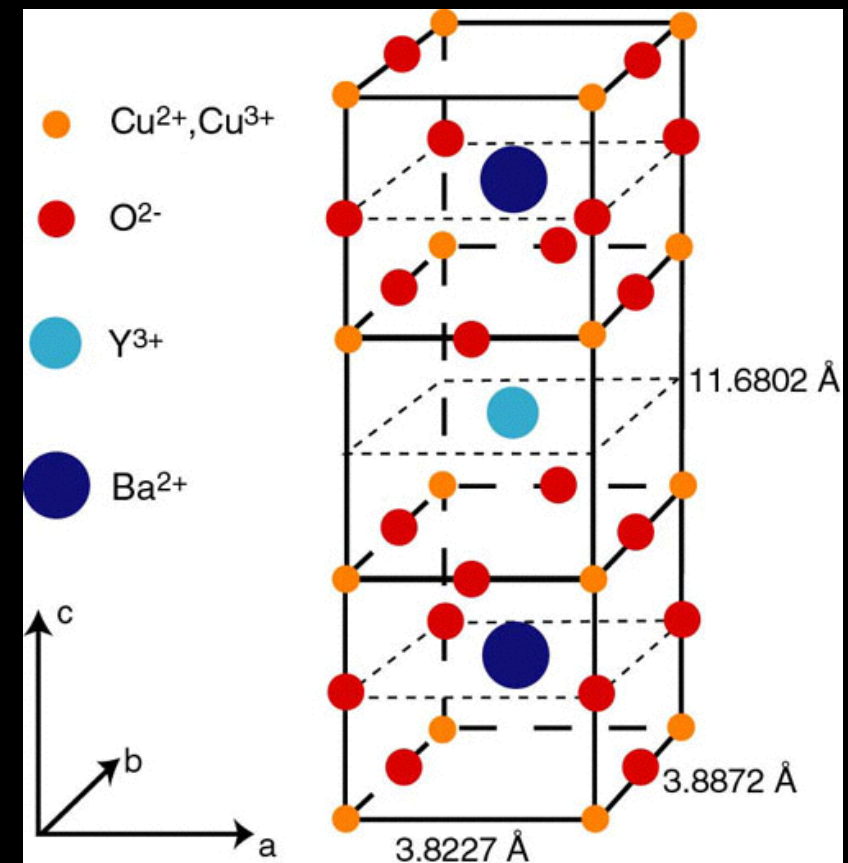
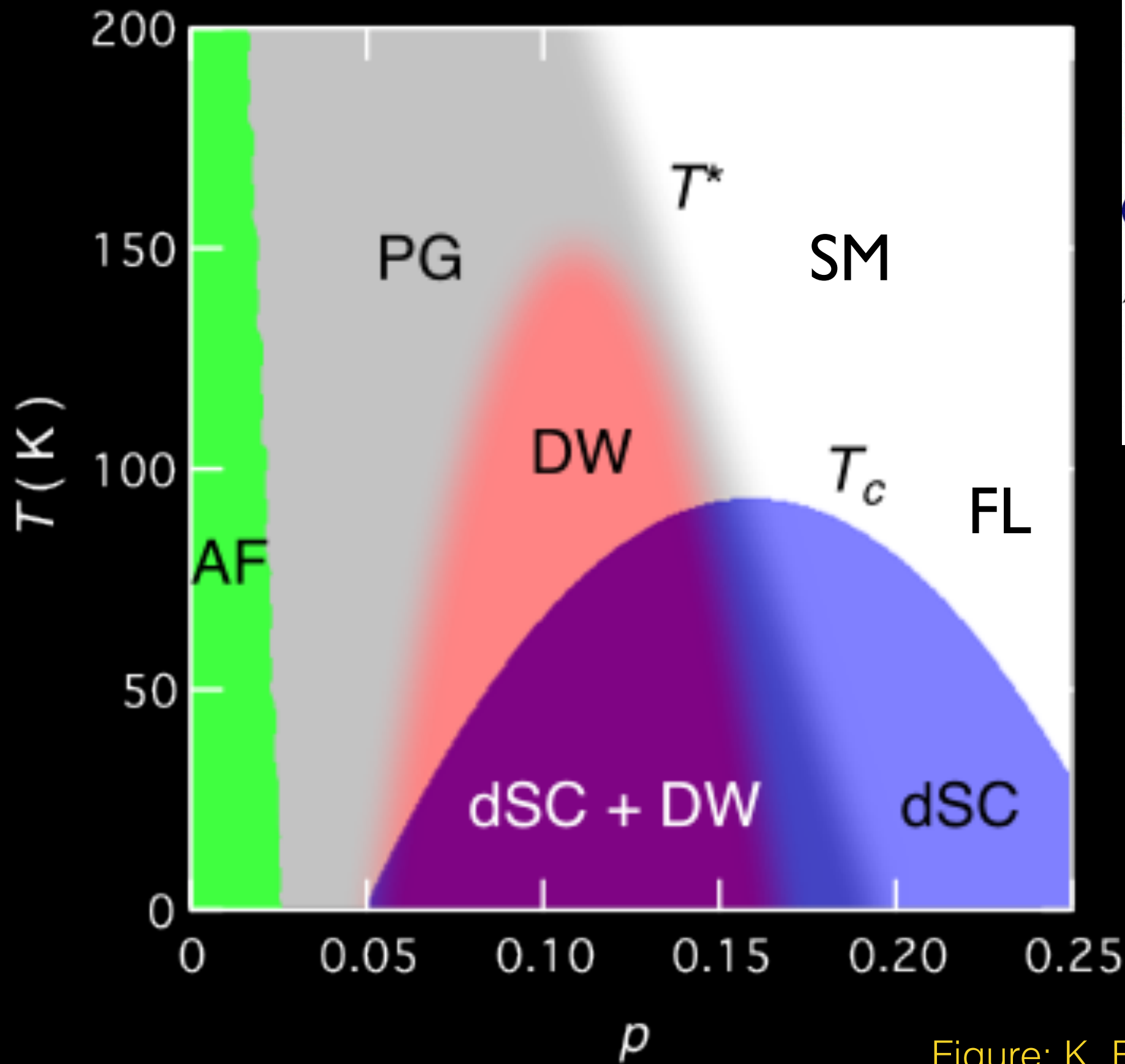
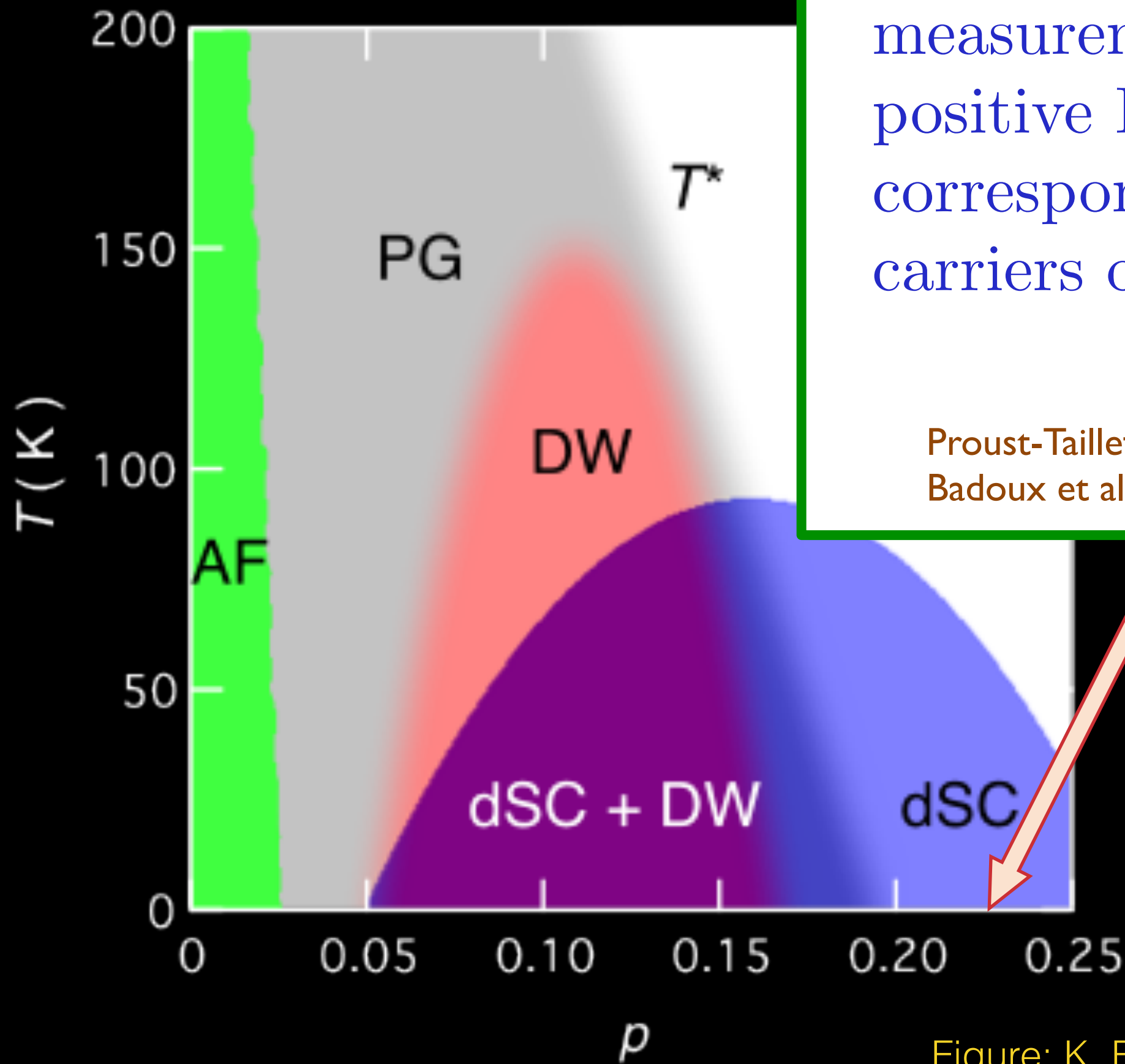


Figure: K. Fujita and J. C. Seamus Davis

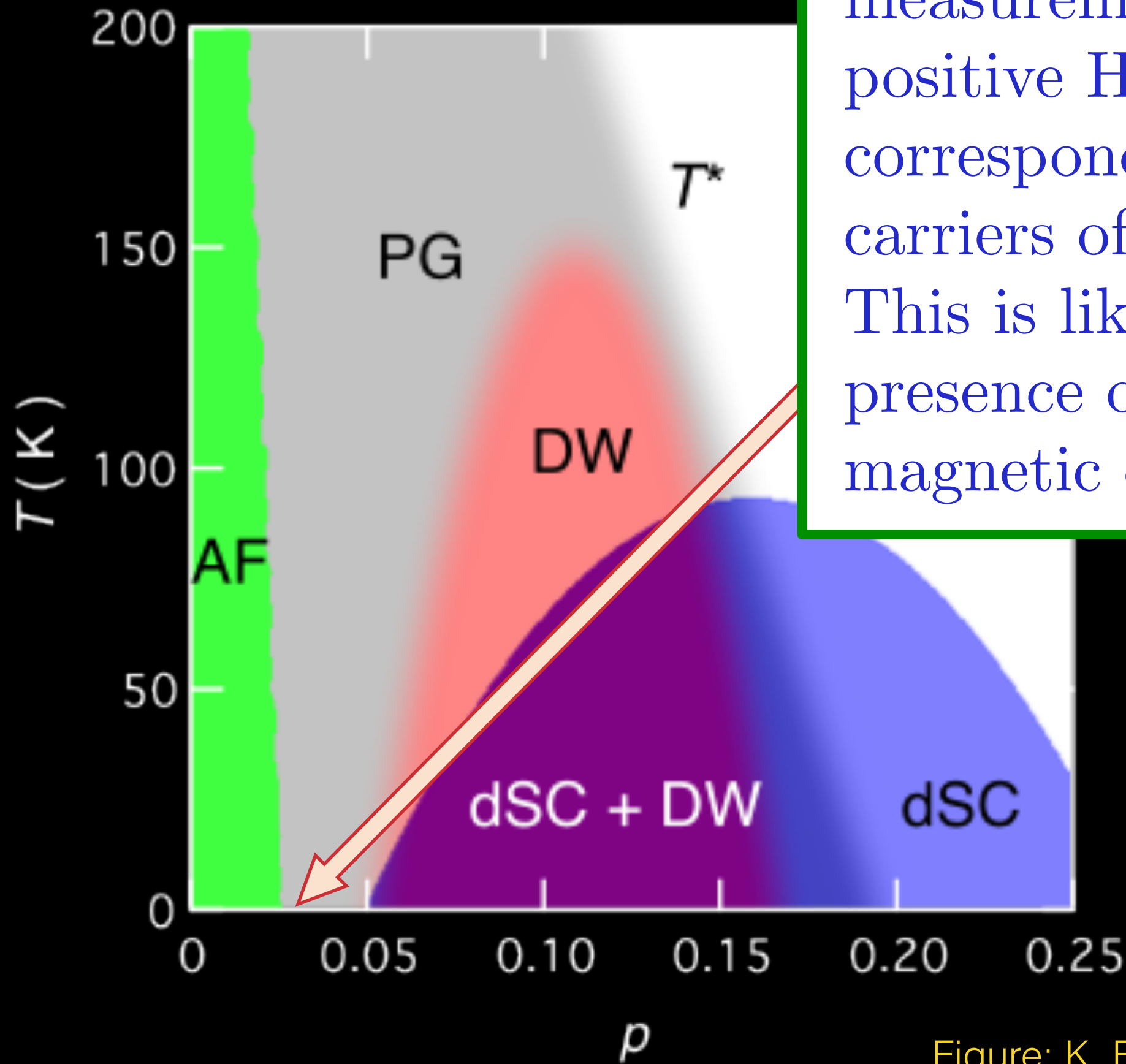


High field, low T measurements show a positive Hall co-efficient corresponding to carriers of density $1 + p$

Proust-Taillefer-UBC collaboration,
Badoux et al. arXiv:1511.08162

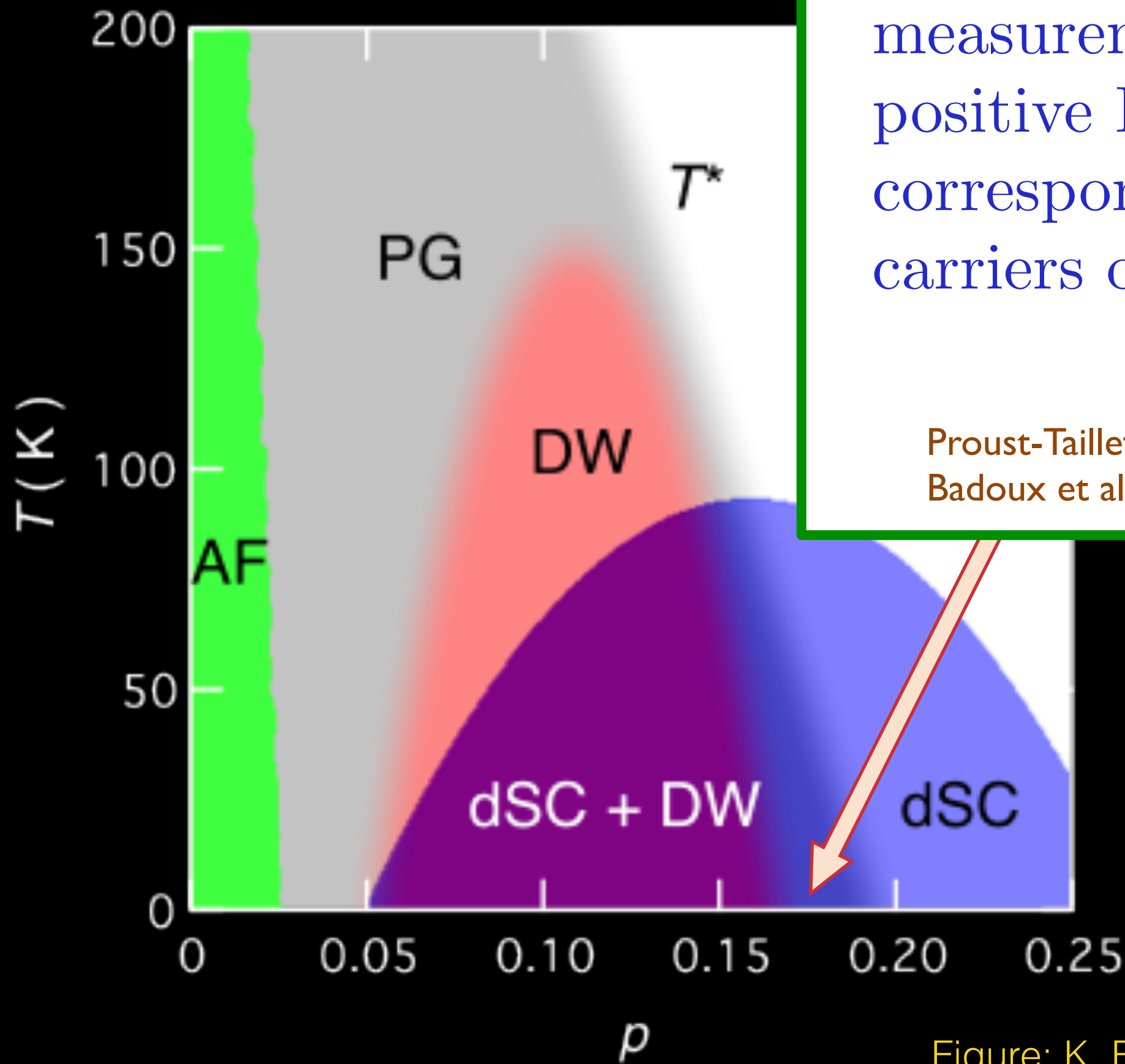
$D \propto \alpha_2 \cup \alpha_3 \cup 6+x$

Figure: K. Fujita and J. C. Seamus Davis



High field, low T measurements show a positive Hall co-efficient corresponding to carriers of density p . This is likely due to the presence of antiferromagnetic order.

Figure: K. Fujita and J. C. Seamus Davis

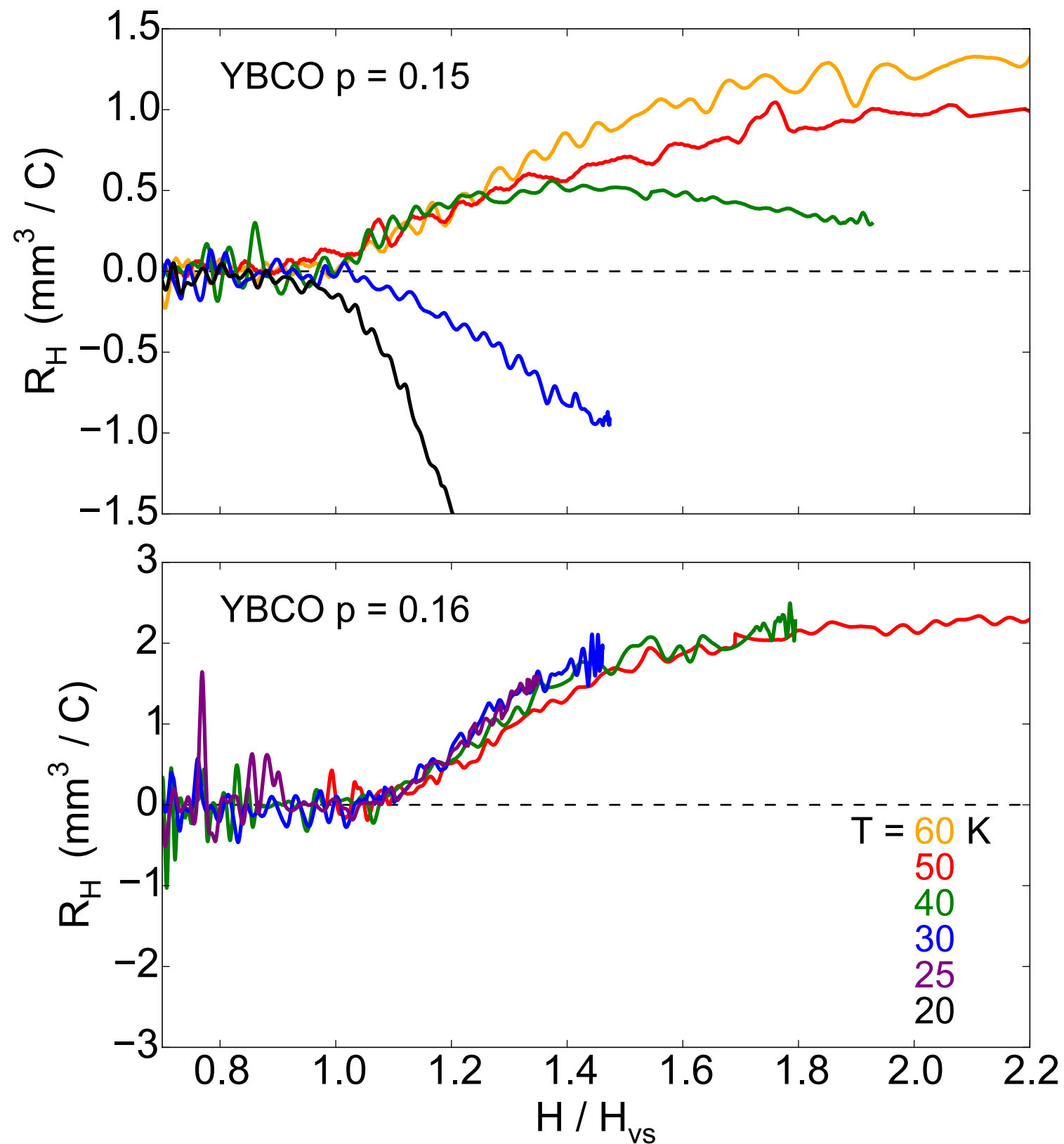


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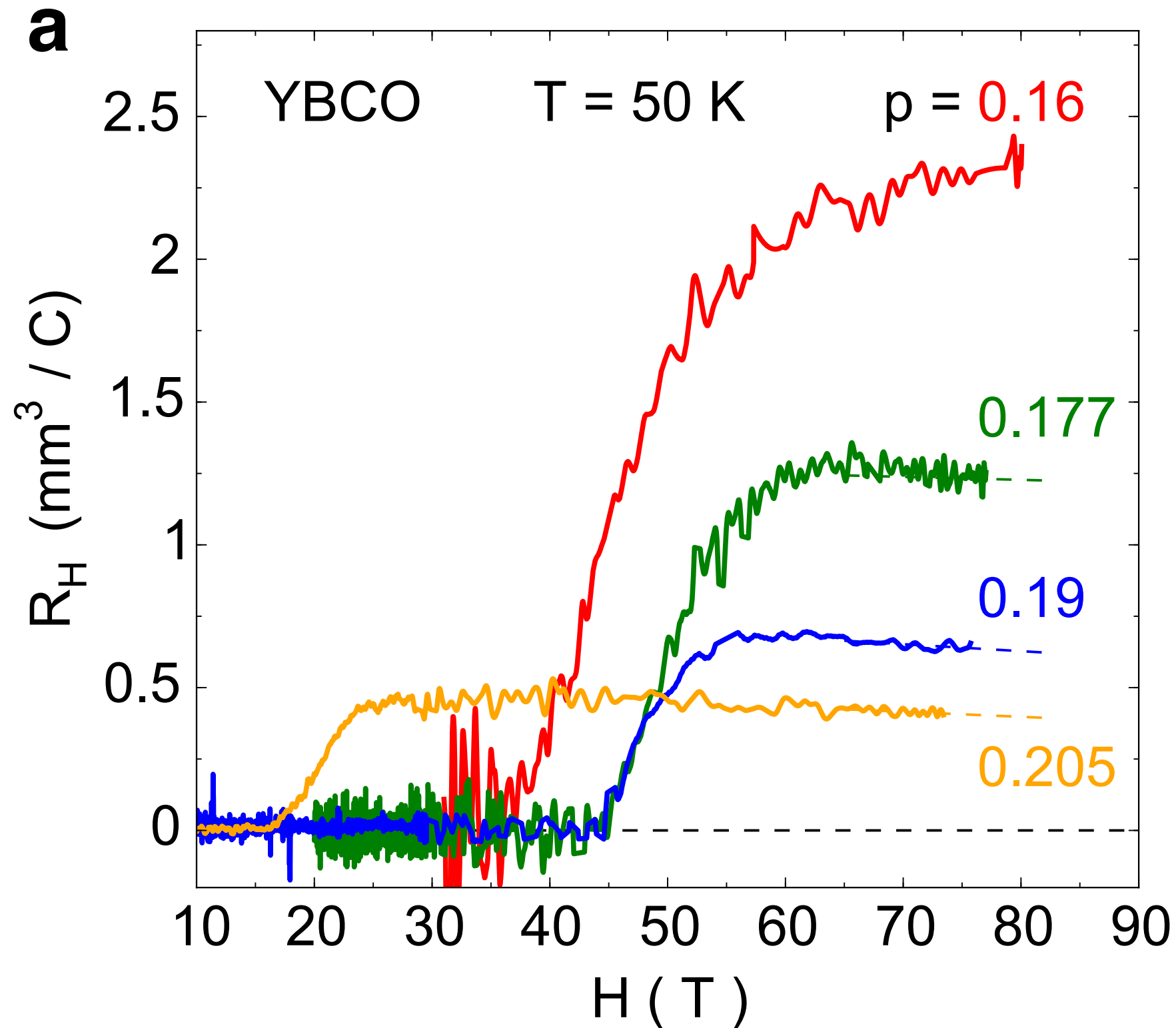
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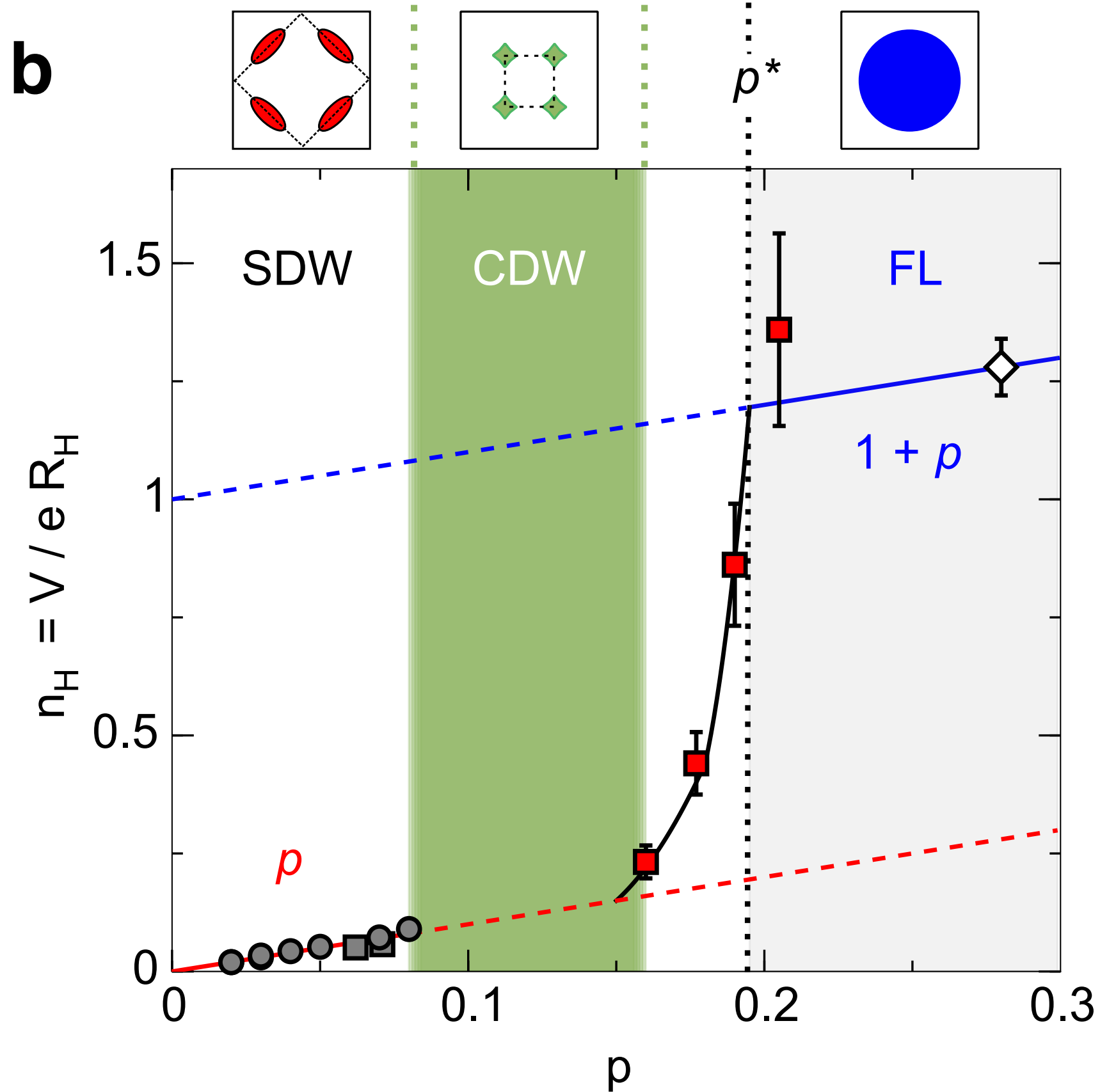
Recent evidence for pseudogap metal as FL*



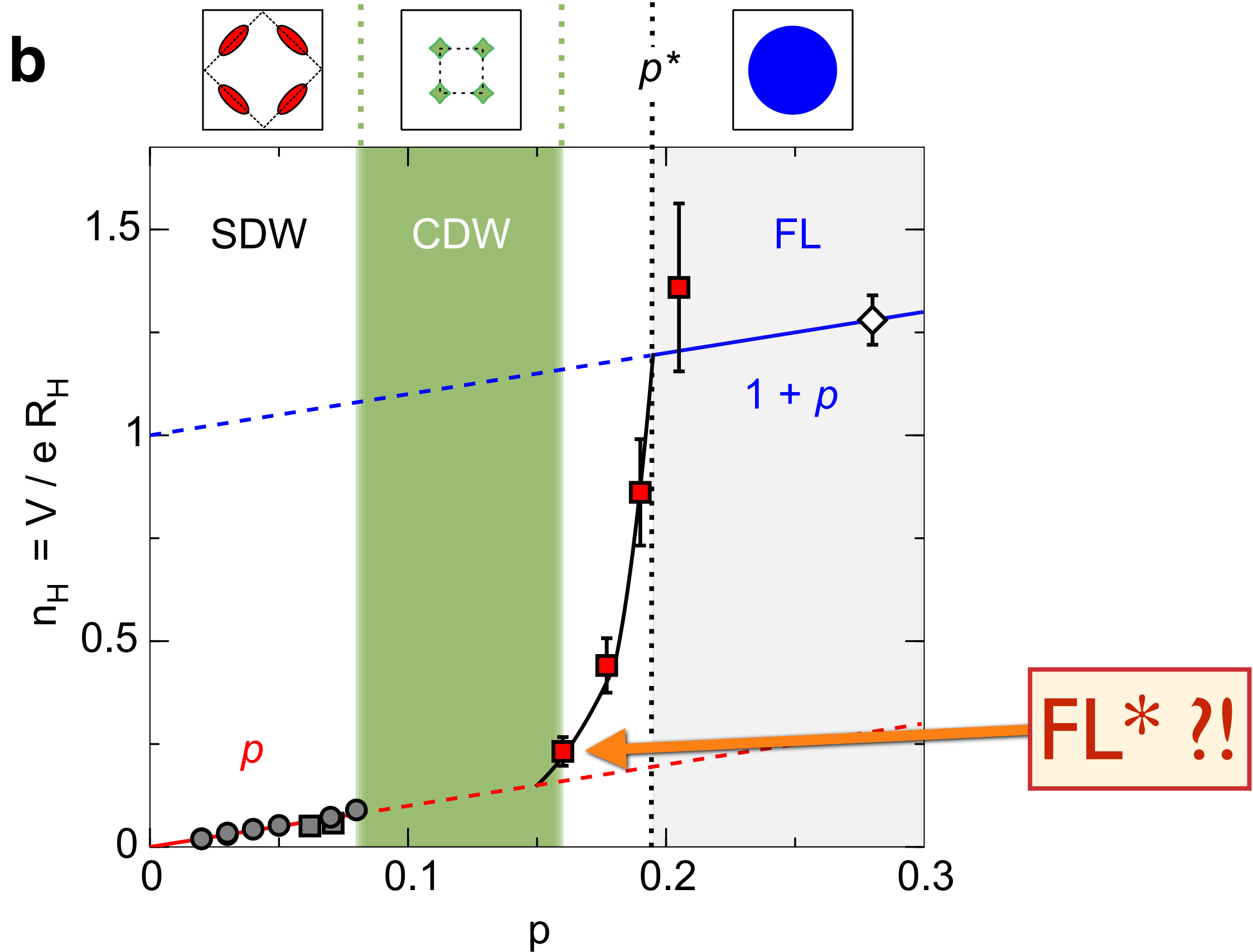
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There is a general and fundamental relationship between these two characteristics. Promising indications that such a metal describes the pseudogap of the cuprate superconductors