Competing orders in the cuprate superconductors

Eugene Demler (Harvard)
Kwon Park
Anatoli Polkovnikov
Subir Sachdev
Matthias Vojta (Augsburg)
Ying Zhang

Talk online at
http://pantheon.yale.edu/~subir

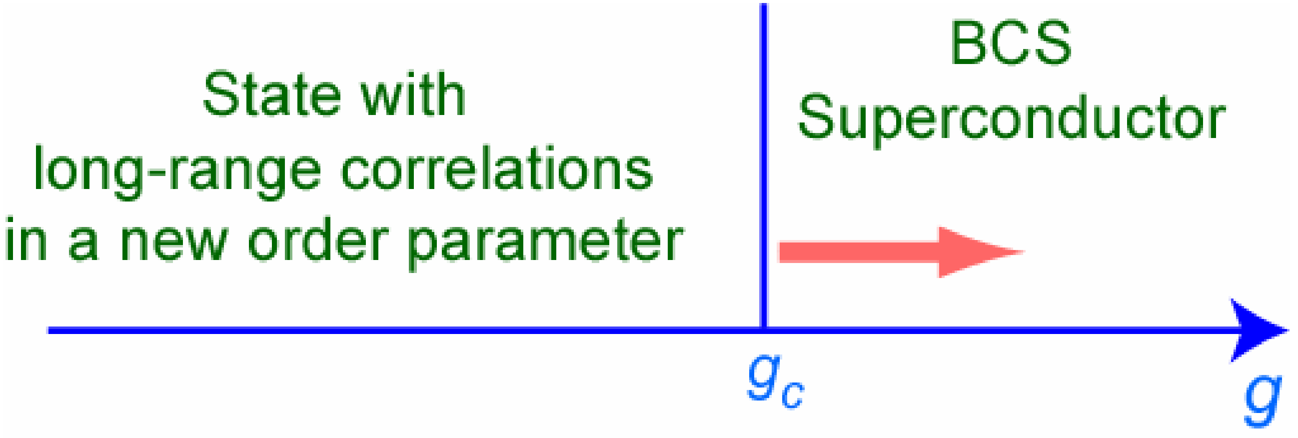
Superconductivity in a doped Mott insulator

Review: S. Sachdev, *Science* **286**, 2479 (1999).

Hypothesis: cuprate superconductors are characterized by additional order parameters, associated with the proximate Mott insulator, along with the familiar order associated with the Bose condensation of Cooper pairs in BCS theory. These orders lead to new low energy excitations

Superconductivity in a doped Mott insulator

Review: S. Sachdev, *Science* **286**, 2479 (1999).



Study physics in a generalized phase diagram which includes new phases (which need not be experimentally accessible) with long-range correlations in the additional order parameters. Expansion away from qauntum critical points provides a systematic and controlled theory of the low energy excitations (including their behavior near imperfections such as impurities and vortices and their response to applied fields) and of crossovers into “incoherent” regimes at finite temperature.

Outline

I. **Order in Mott insulators**

Magnetic order

A. Collinear spins

B. Non-collinear spins

Paramagnetic states

A. Bond order and confined spinons

B. Topological order and deconfined spinons

II. Doping Mott insulators with collinear spins and bond order

Global phase diagram

III. Spin density waves (SDW) in LSCO

Tuning order and transitions by a magnetic field.

IV. Connection with LDOS modulations

STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

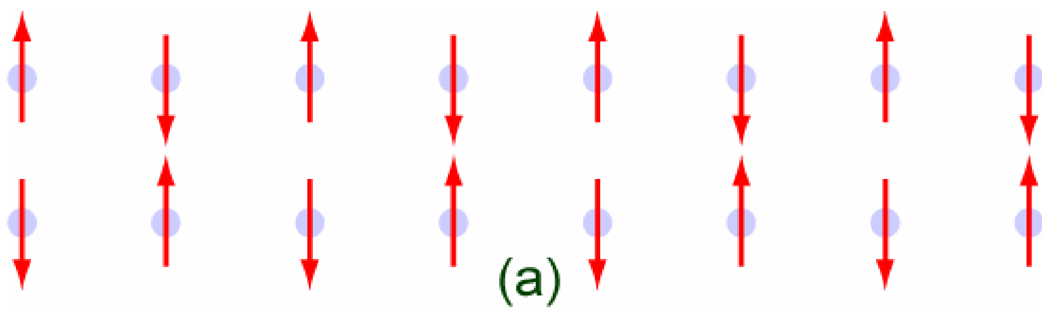
V. Conclusions

I. Order in Mott insulators

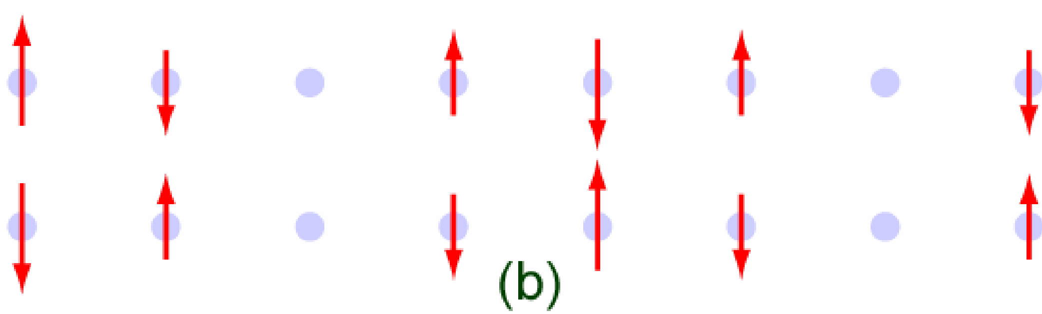
Magnetic order

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \bullet \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \bullet \vec{r}_j\right)$$

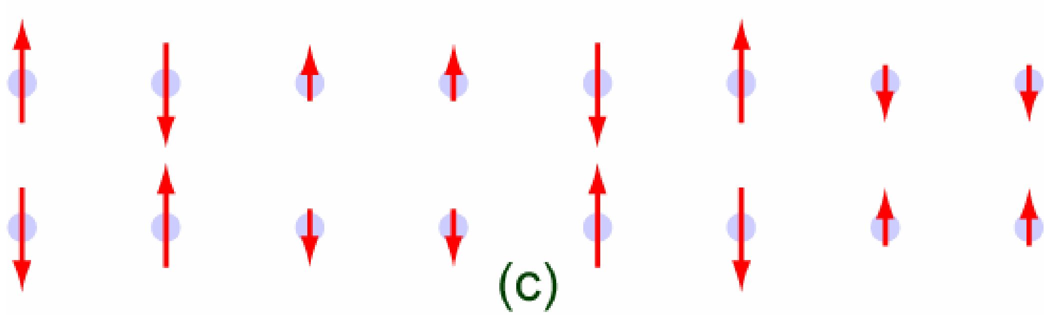
A. Collinear spins



$$\vec{K} = (\pi, \pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$



$$\vec{K} = (3\pi/4, \pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$



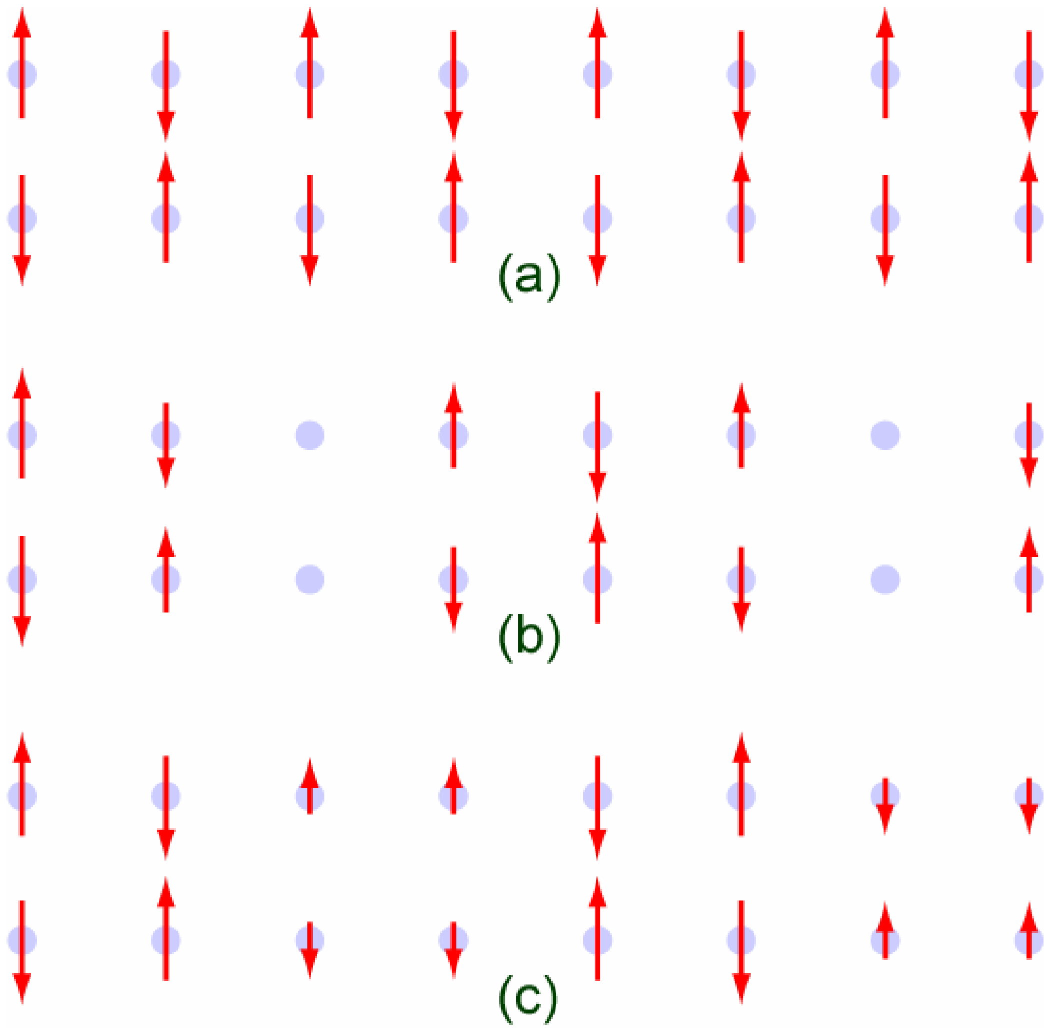
$$\vec{K} = (3\pi/4, \pi) \;\; ;$$

$$\boldsymbol{N}_2 = \left(\sqrt{2} - 1\right) \boldsymbol{N}_1$$

I. Order in Mott insulators

Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K}\bullet\vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K}\bullet\vec{r}_j\right)$

A. Collinear spins



Key properties

States can also have ***bond order***.

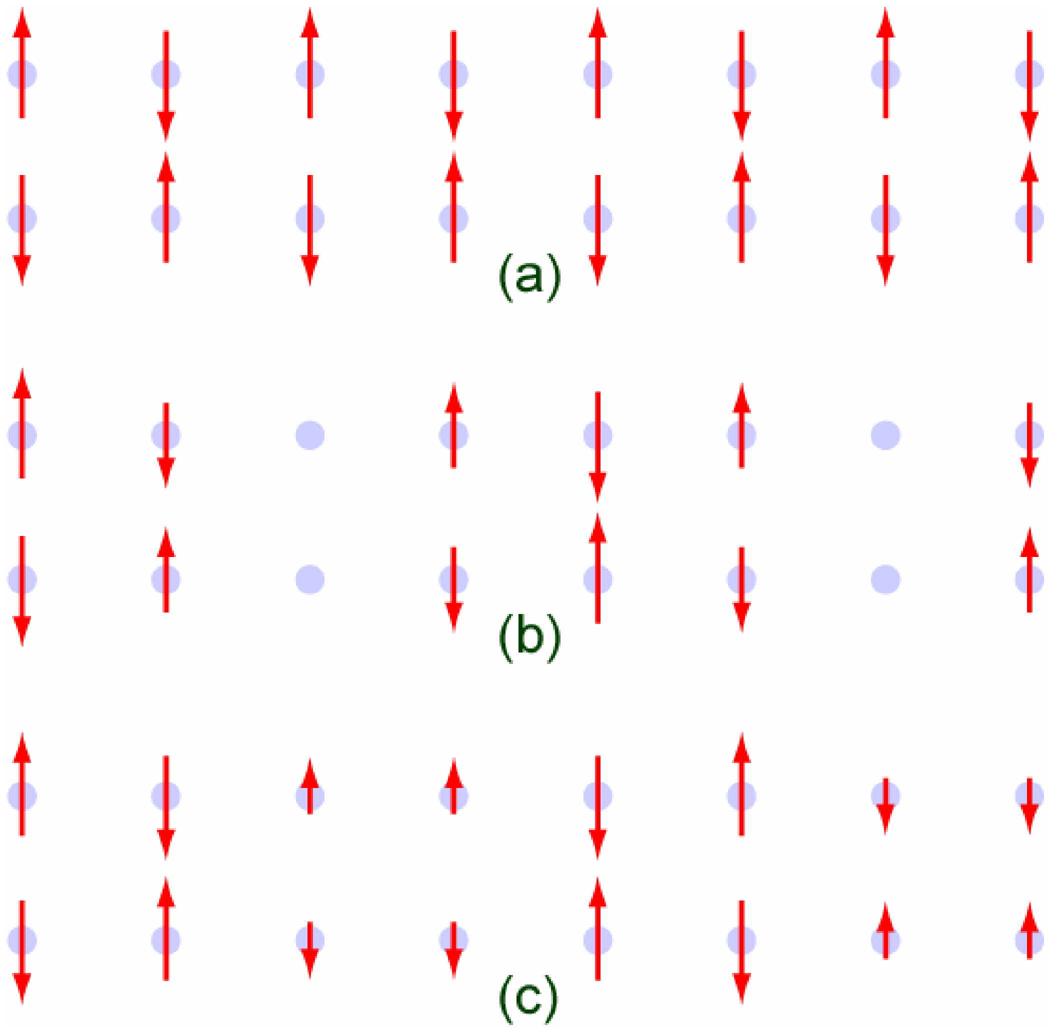
$$Q_a\left(\boldsymbol{r}_i\right) \equiv \boldsymbol{S}_i \bullet \boldsymbol{S}_{i+a}$$

Values of $\left\langle Q_a\left(\boldsymbol{r}_i\right)\right\rangle$ modulate with wavevector $2\vec{K}$;
$\left\langle Q_a\left(\boldsymbol{0}\right)\right\rangle$ is a measure of site charge density

I. Order in Mott insulators

Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \bullet \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \bullet \vec{r}_j\right)$

A. Collinear spins



Key properties

Order specified by a single vector $\boldsymbol{N}$.

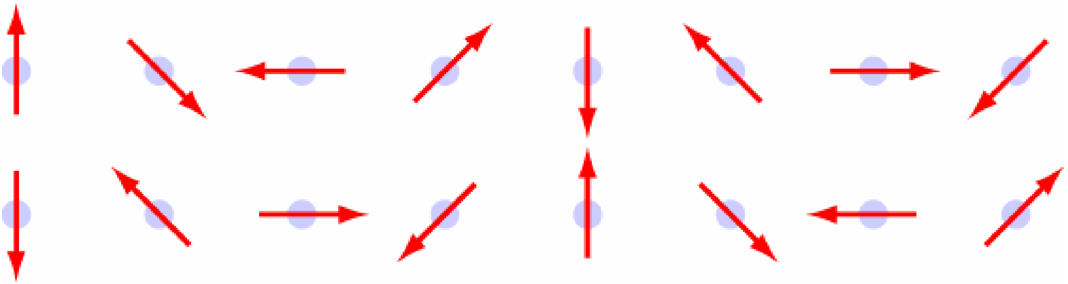
Quantum fluctuations leading to loss of magnetic order should produce a paramagnetic state with a vector (S=1) quasiparticle excitation.

I. Order in Mott insulators

Magnetic order $\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K}\cdot\vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K}\cdot\vec{r}_j\right)$

B. Noncollinear spins

(B.I. Shraiman and E.D. Siggia, *Phys. Rev. Lett.* **61**, 467 (1988))



$\vec{K} = (3\pi/4, \pi)$;

$\mathbf{N}_2^2 = \mathbf{N}_1^2 \ , \ \mathbf{N}_1 \cdot \mathbf{N}_2 = 0$

Solve constraints by expressing $\mathbf{N}_{1,2}$ in terms of two complex numbers $z_\uparrow$, $z_\downarrow$

$$\mathbf{N}_1 + i\mathbf{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $\left(z_\uparrow, z_\downarrow\right)$ (modulo an overall sign)

Order parameter space: S_3/Z_2

Physical observables are invariant under the Z_2 gauge transformation $z_a \to \pm z_a$

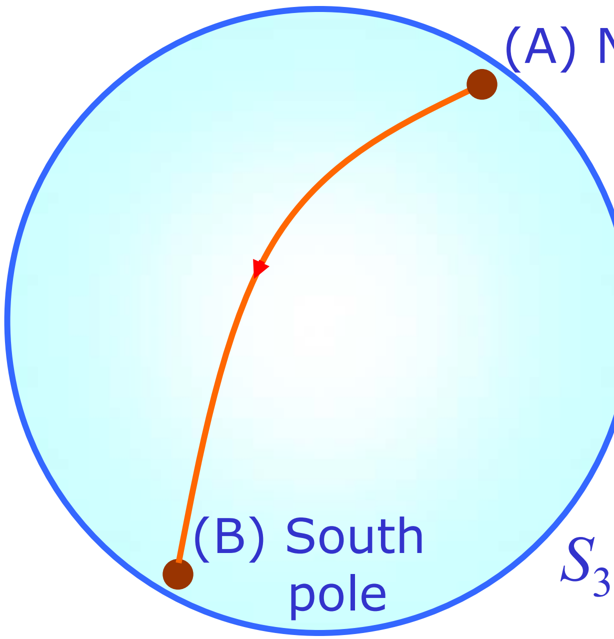
I. Order in Mott insulators

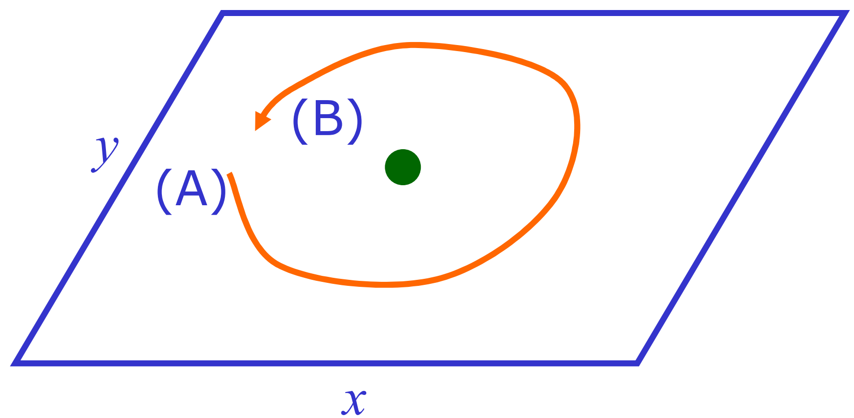
Magnetic order $\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K} \bullet \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \bullet \vec{r}_j\right)$

B. Noncollinear spins

$$\mathbf{N}_1 + i\mathbf{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2z_\uparrow z_\downarrow \end{pmatrix}$$

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$ (*visons*)





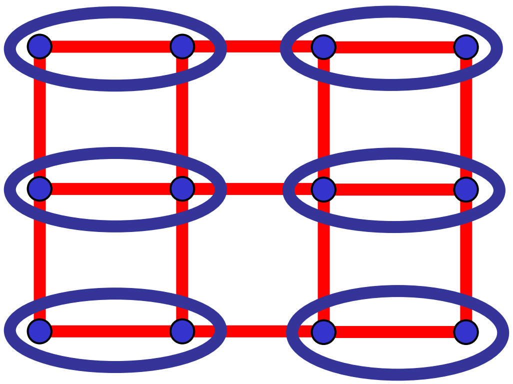
Quantum fluctuations leading to loss of magnetic order produce a paramagnetic state with a spinor (S=1/2) quasiparticle excitation, $\left(z_\uparrow, z_\downarrow\right)$, with a Z_2 gauge charge, a vison vortex gap, and topological order associated with vison suppression in the ground state.

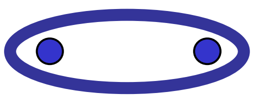
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

I. Order in Mott insulators

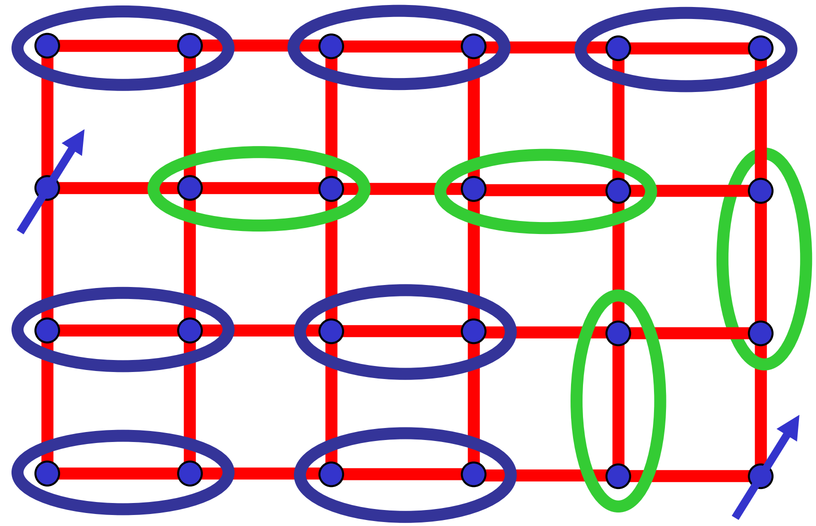
Paramagnetic states $\left\langle \mathbf{S}_j \right\rangle = 0$

A. Bond order and spin excitons





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$



S=1/2 spinons are ***confined*** by a linear potential into a S=1 spin exciton

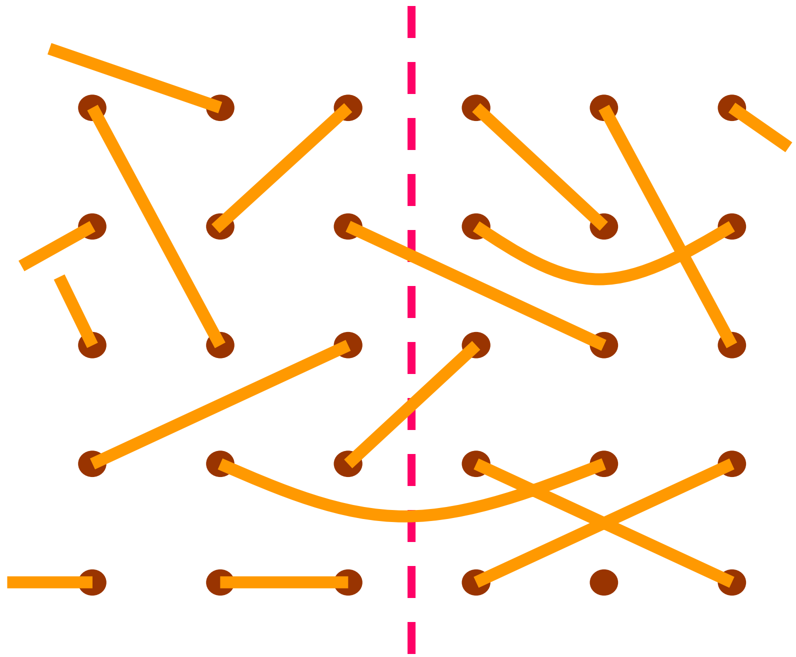
Such a state is obtained by quantum-``disordering" collinear state with $\vec{K} = (\pi, \pi)$: fluctuating $\boldsymbol{N}$ becomes the S=1 spin exciton and Berry phases induce bond order

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

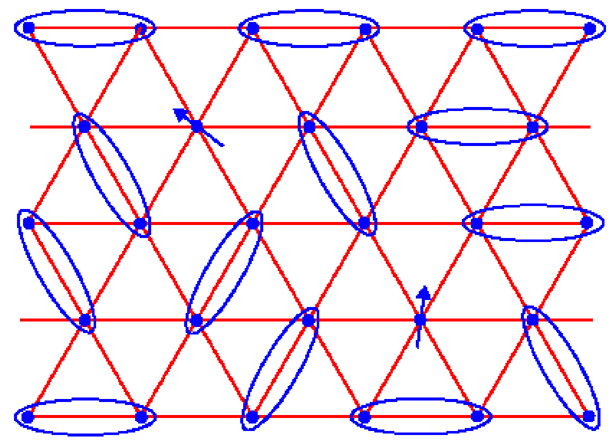
I. Order in Mott insulators

Paramagnetic states $\left\langle \boldsymbol{S}_j \right\rangle = 0$

B. Topological order and deconfined spinons



Number of valence bonds cutting line is conserved modulo 2 – this is described by the same Z_2 gauge theory as non-collinear spins



RVB state with free spinons

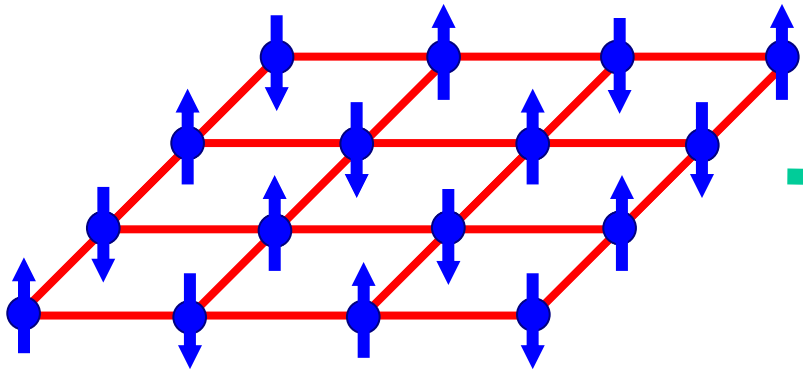
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

D.S. Rokhsar and S. Kivelson, *Phys. Rev. Lett*. **61**, 2376 (1988)
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991);
R. Jalabert and S. Sachdev, *Phys. Rev*. B **44**, 686 (1991);
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).

Orders of Mott insulators in two dimensions

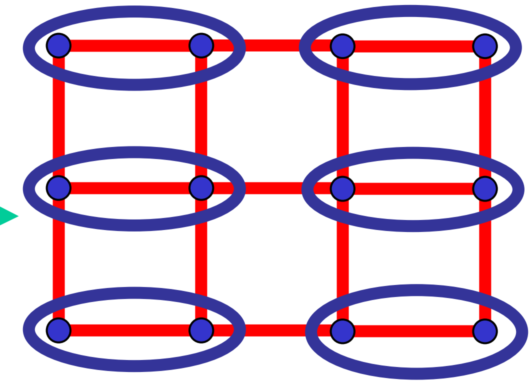
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991); S.S. and N.R. *Int. J. Mod. Phys.* B **5**, 219 (1991).

A. Collinear spins, Berry phases, and bond order



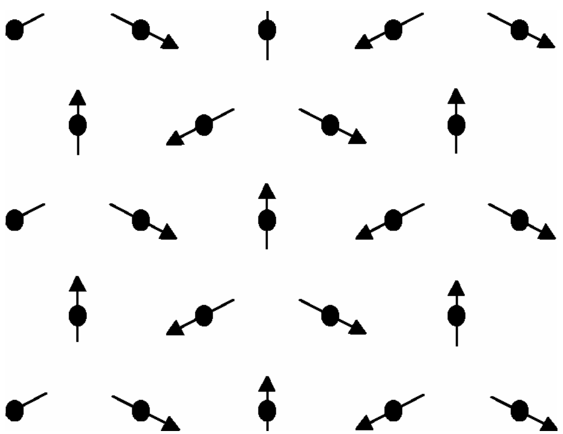
Néel ordered state

Quantum transition restoring spin rotation invariance



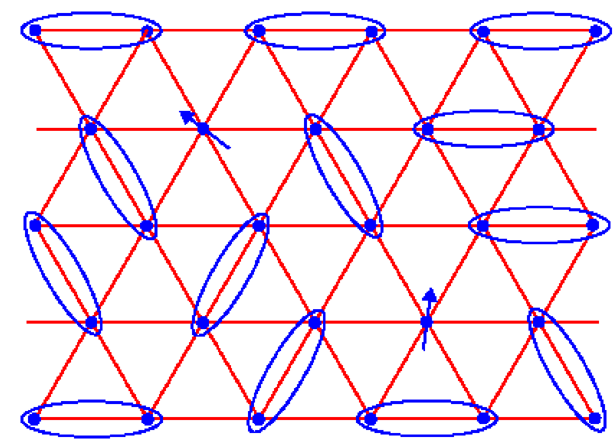
Bond order and *S=1* spin exciton

B. Non-collinear spins and deconfined spinons.



Non-collinear ordered antiferromagnet

Quantum transition restoring spin rotation invariance

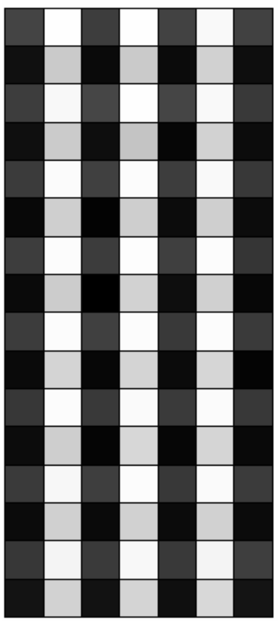


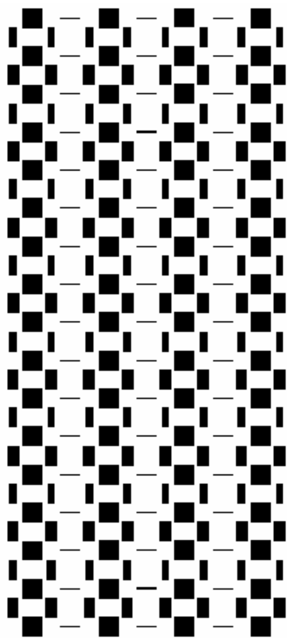
Topological order: RVB state with Z_2 gauge visons, *S=1/2* spinons

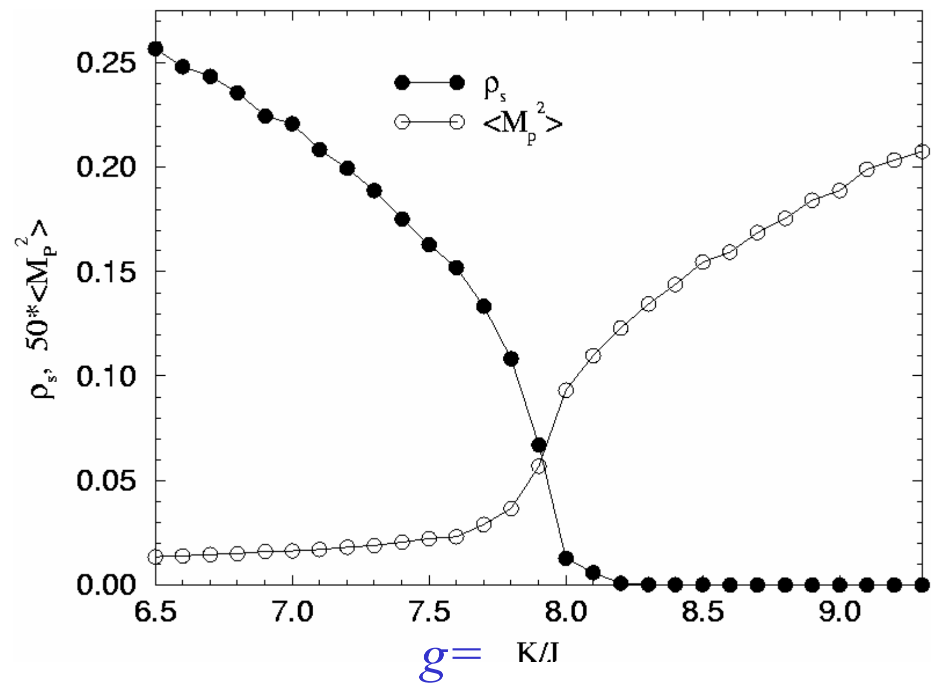
Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry







$$H = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle \subset \square} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

Outline

I. Order in Mott insulators

Magnetic order

A. Collinear spins

B. Non-collinear spins

Paramagnetic states

A. Bond order and confined spinons

B. Topological order and deconfined spinons

II. Doping Mott insulators with collinear spins and bond order

Global phase diagram

III. Spin density waves (SDW) in LSCO

Tuning order and transitions by a magnetic field.

IV. Connection with LDOS modulations

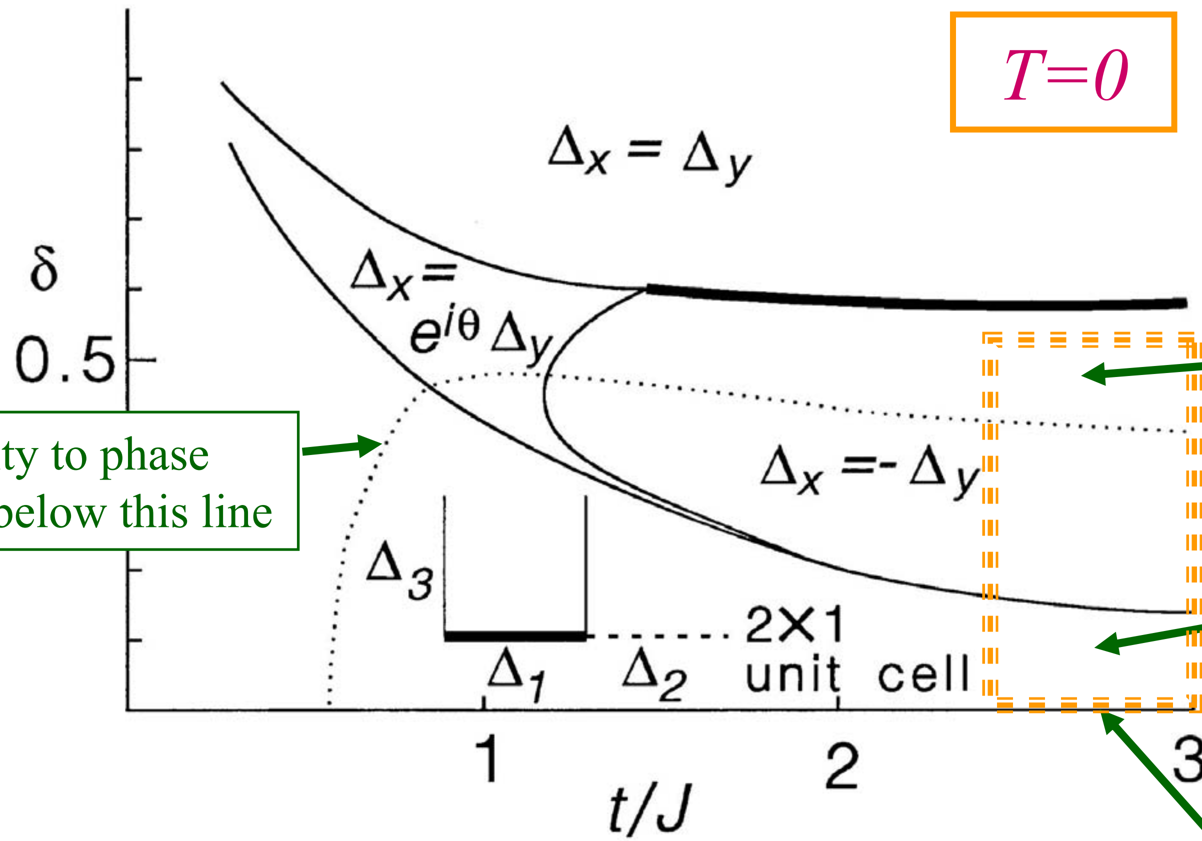
STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

V. Conclusions

II. Doping Mott insulators with collinear spins and bons order

Doping a paramagnetic bond-ordered Mott insulator

systematic Sp(N) theory of translational symmetry breaking, while preserving spin rotation invariance.



d-wave superconductor

Instability to phase separation below this line

Superconductor with co-existing bond-order

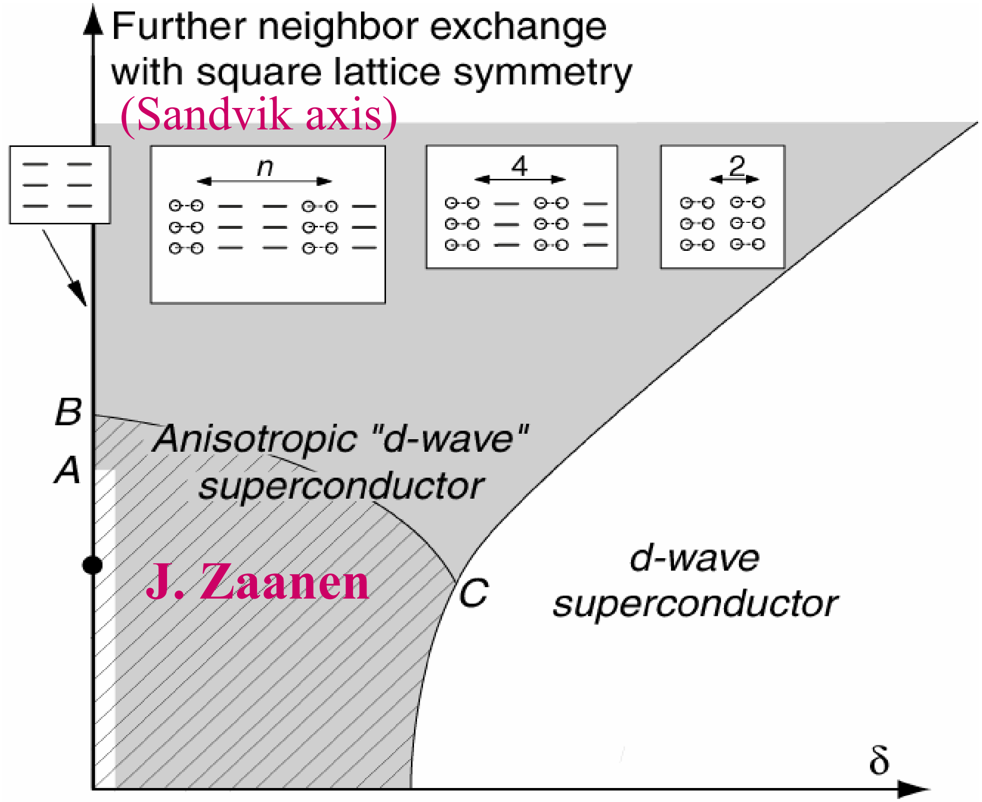
Mott insulator with bond-order

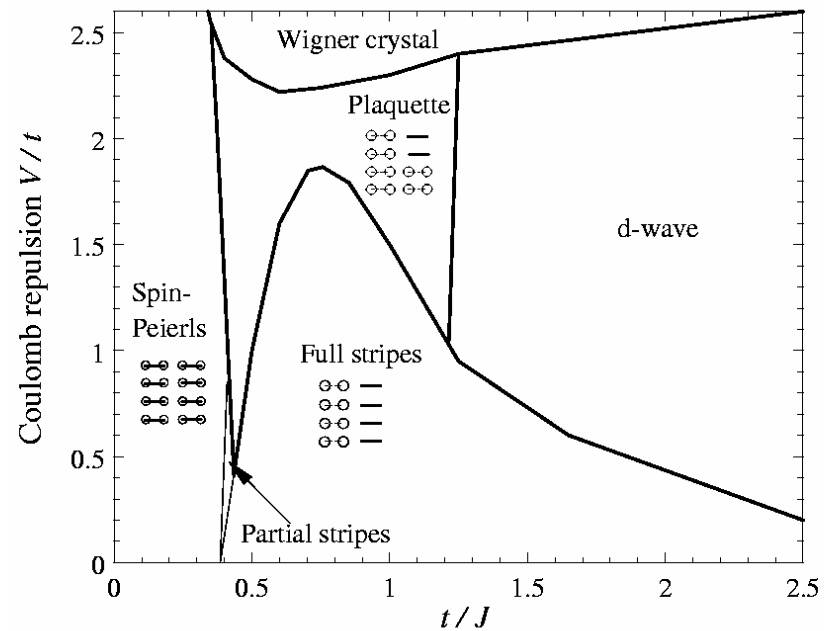
S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

II.Global phase diagram

Include long-range Coulomb interactions: frustrated phase separation

V.J. Emery, S.A. Kivelson, and H.Q. Lin, *Phys. Rev. Lett*. **64**, 475 (1990).





Hatched region --- static spin order
Shaded region ---- static bond/charge order

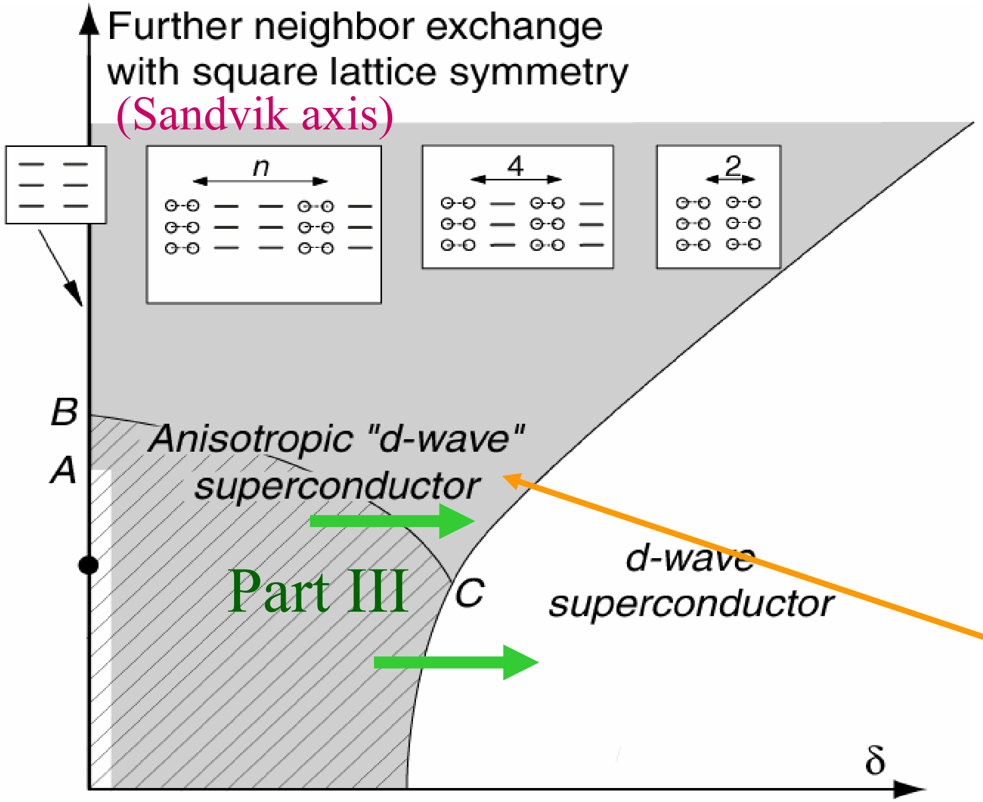
M. Vojta and S. Sachdev,
Phys. Rev. Lett. **83**, 3916 (1999)
M. Vojta, Y. Zhang, and S. Sachdev,
Phys. Rev. B **62**, 6721 (2000).
M. Vojta, cond-mat/0204284.

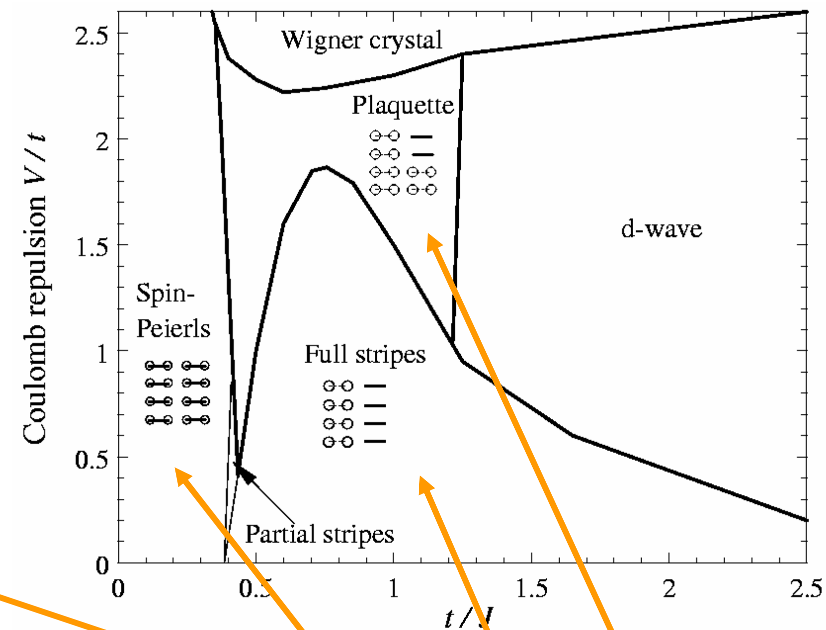
See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

II.Global phase diagram

Include long-range Coulomb interactions: frustrated phase separation

V.J. Emery, S.A. Kivelson, and H.Q. Lin, *Phys. Rev. Lett*. **64**, 475 (1990).





Hatched region --- static spin order
Shaded region ---- static bond/charge order

Non-magnetic “d-wave” superconductor with even period bond order.

M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999)
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).
M. Vojta, cond-mat/0204284.

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

Outline

I. Order in Mott insulators

Magnetic order

A. Collinear spins

B. Non-collinear spins

Paramagnetic states

A. Bond order and confined spinons

B. Topological order and deconfined spinons

II. Doping Mott insulators with collinear spins and bond order

Global phase diagram

III. Spin density waves (SDW) in LSCO

Tuning order and transitions by a magnetic field.

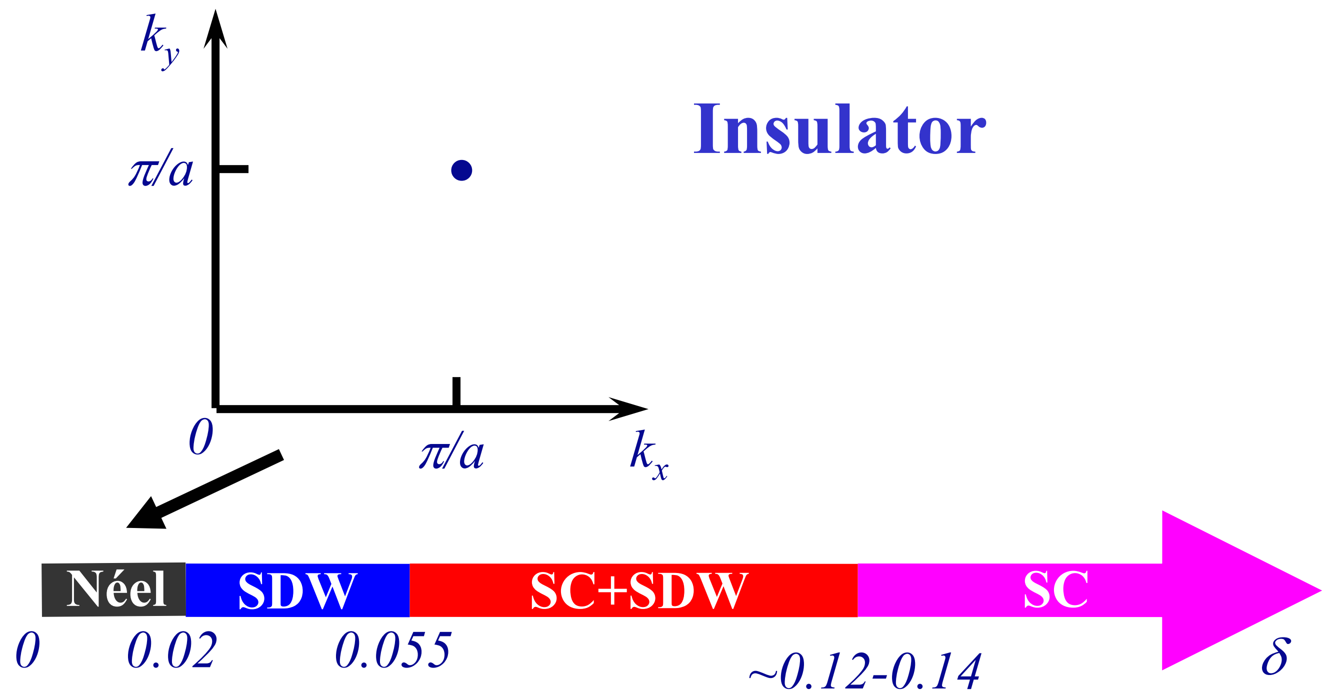
IV. Connection with LDOS modulations

STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

V. Conclusions

III. Tuning magnetic order in LSCO by a magnetic field

T=0 phases of LSCO

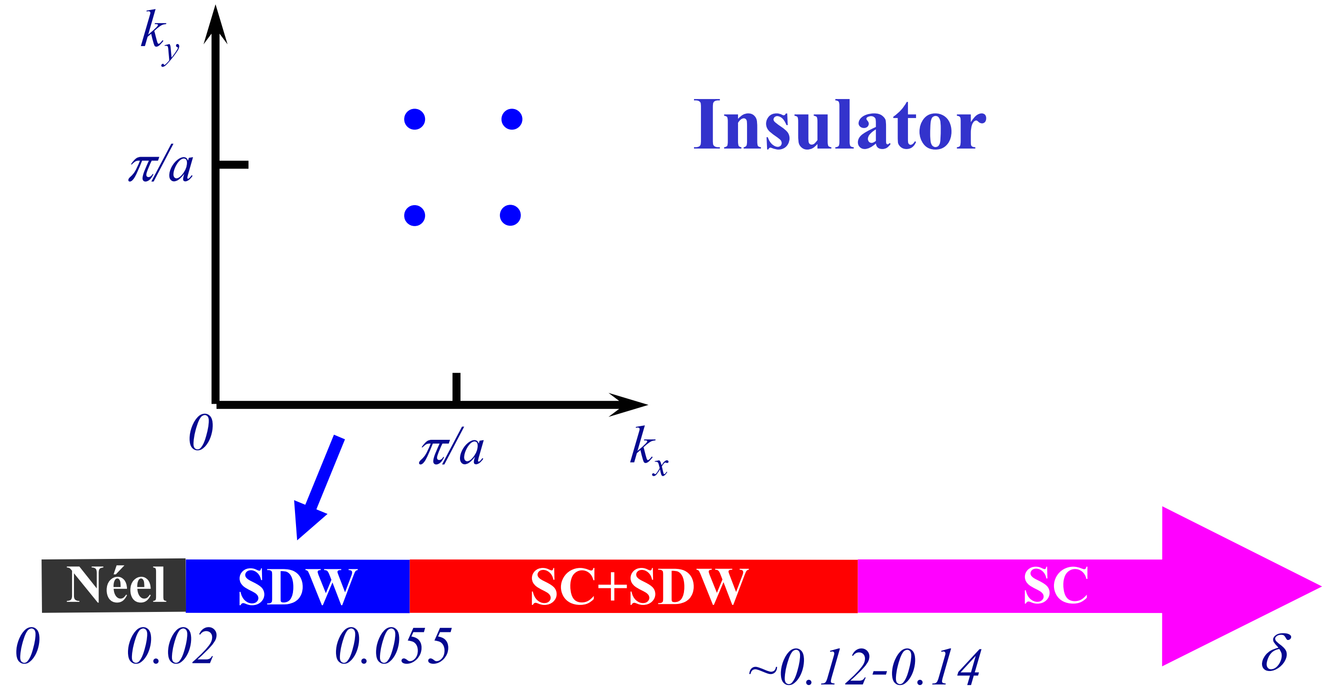


(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al., Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

III. Tuning magnetic order in LSCO by a magnetic field

T=0 phases of LSCO

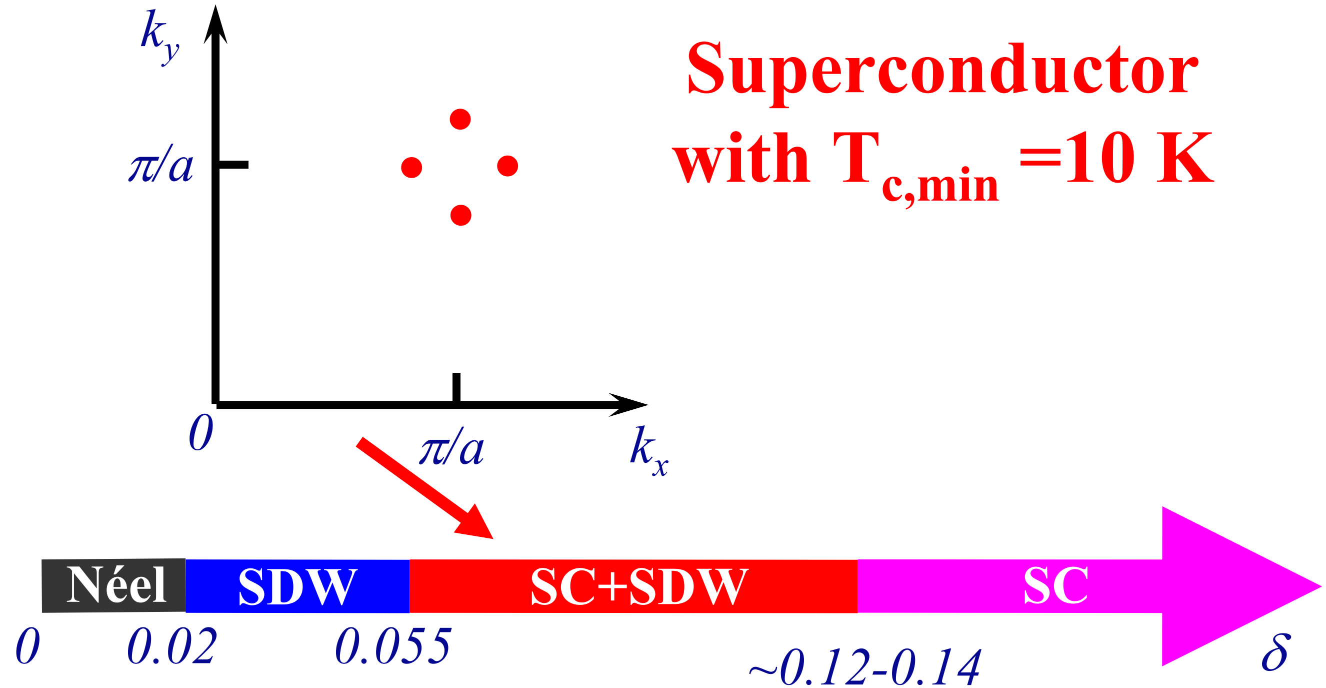


(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

III. Tuning magnetic order in LSCO by a magnetic field

T=0 phases of LSCO

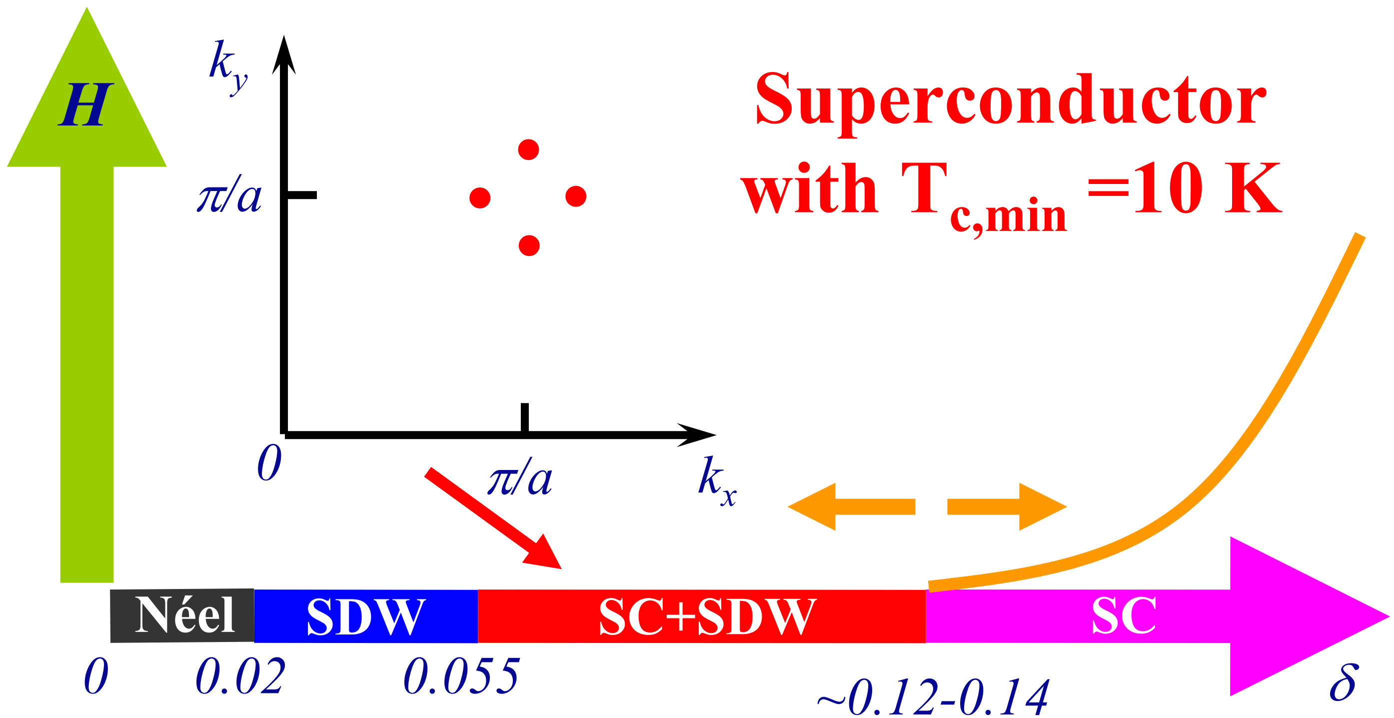


(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

III. Tuning magnetic order in LSCO by a magnetic field

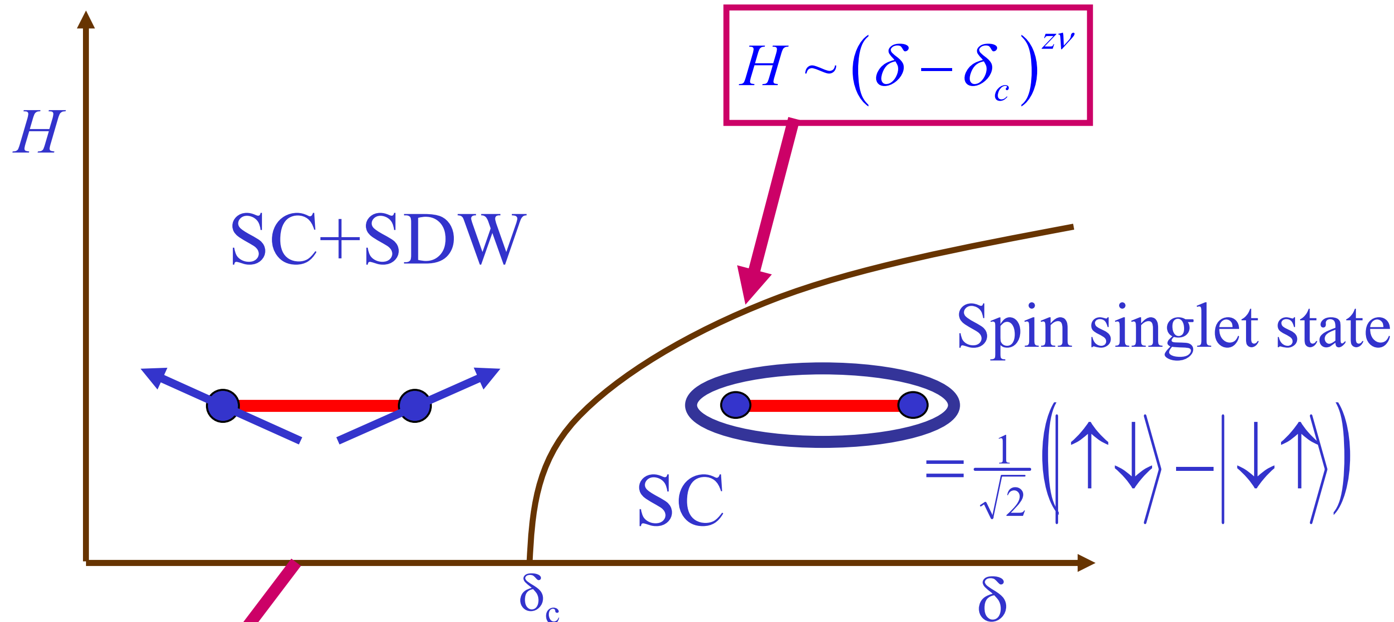
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field

Effect of the Zeeman term: precession of SDW order about the magnetic field



Elastic scattering intensity

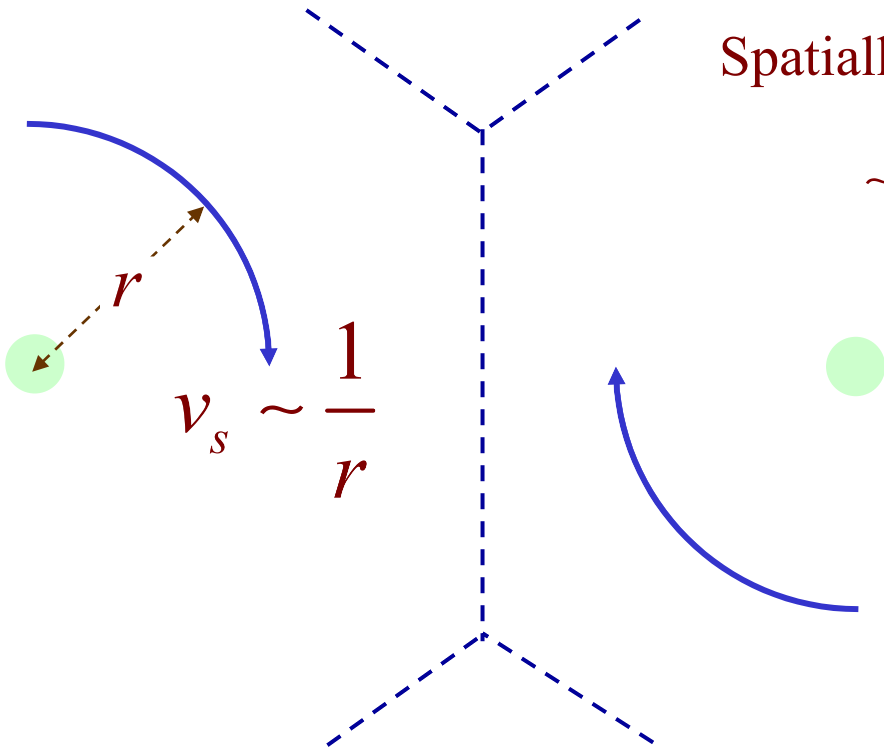
$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

Effect is negligible over experimental field scales

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

The suppression of SC order appears to the SDW order as an effective δ:

$$\delta_{\text{eff}}\left(H\right) = \delta - C\frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Main results

$T=0$

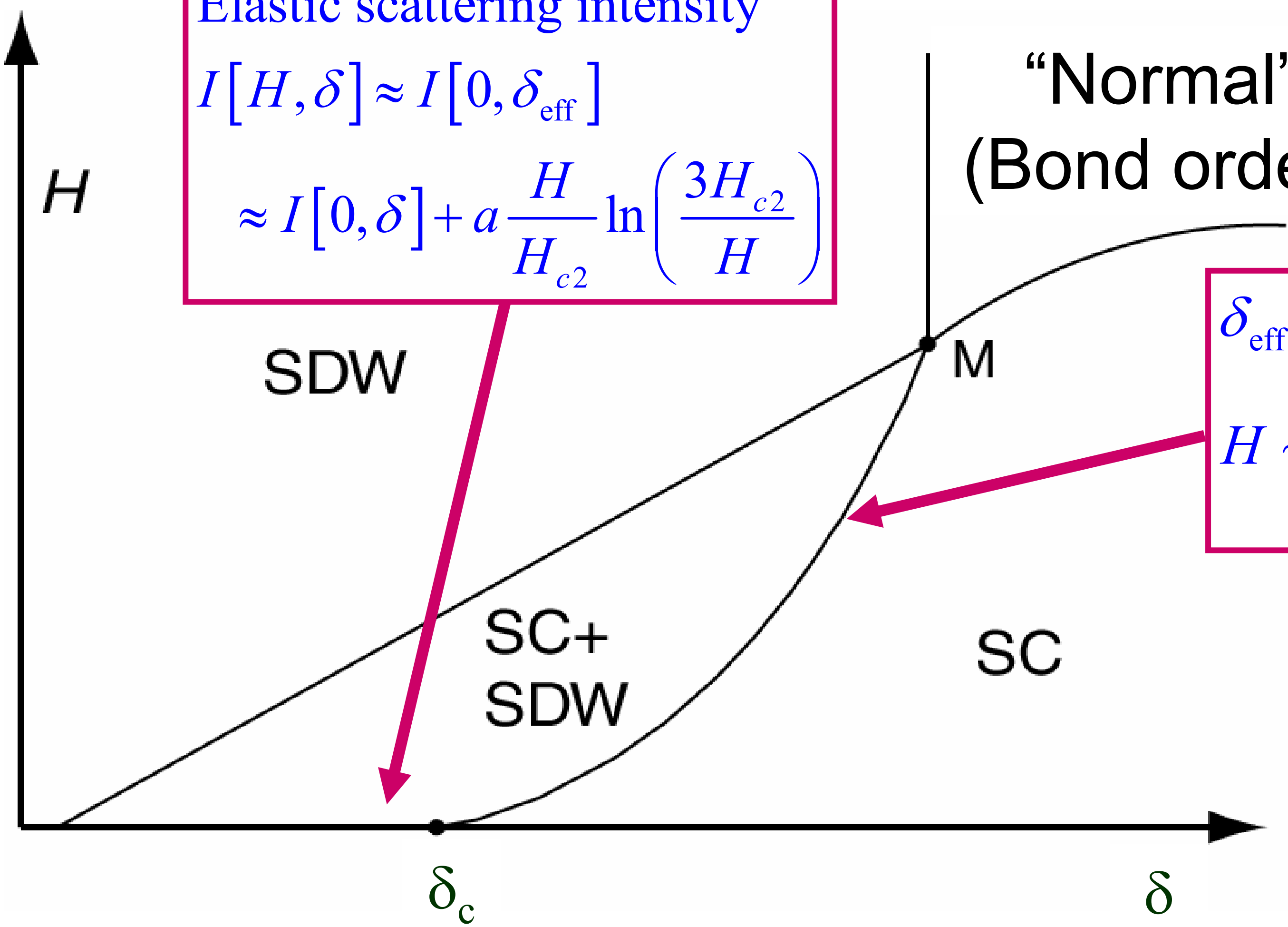
Elastic scattering intensity

$$I[H,\delta] \approx I[0,\delta_{\text{eff}}]$$

$$\approx I[0,\delta] + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$\delta_{\text{eff}}(H) = \delta_c \quad \Rightarrow$$

$$H \sim \frac{(\delta-\delta_c)}{\ln\left(1/(\delta-\delta_c)\right)}$$

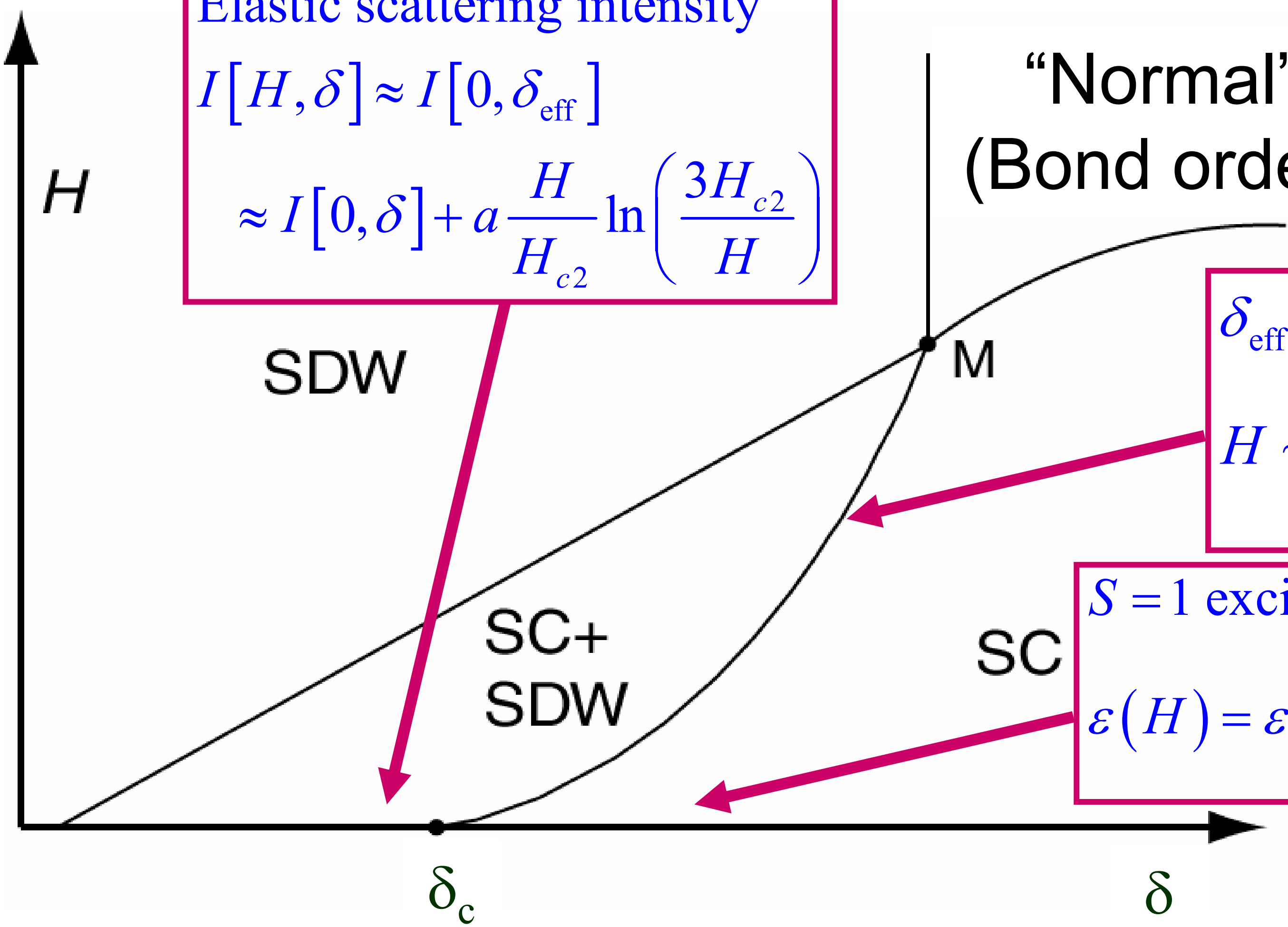


E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) proposed static antiferromagnetism in vortex cores in SC phase

Main results

$T=0$



Elastic scattering intensity

$$I[H,\delta] \approx I[0,\delta_{\rm eff}]$$

$$\approx I[0,\delta] + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$\delta_{\rm eff}(H) = \delta_c \quad \Rightarrow$$

$$H \sim \frac{(\delta-\delta_c)}{\ln\left(1/(\delta-\delta_c)\right)}$$

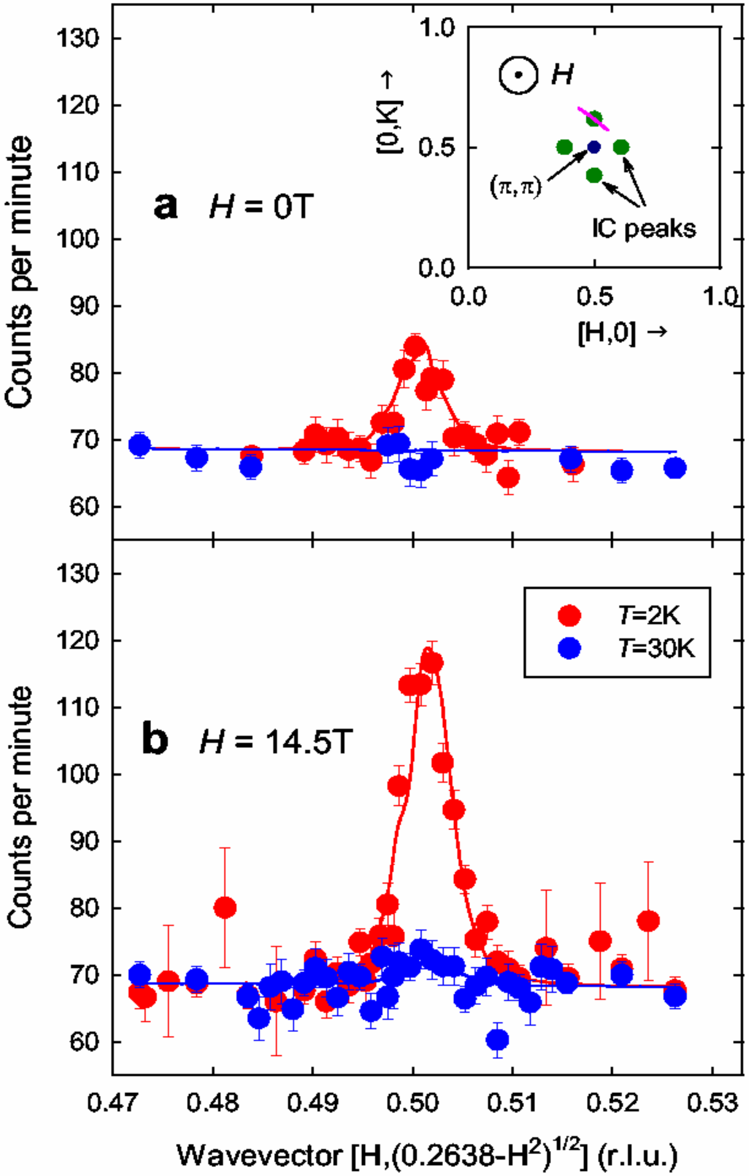
$S=1$ exciton energy

$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

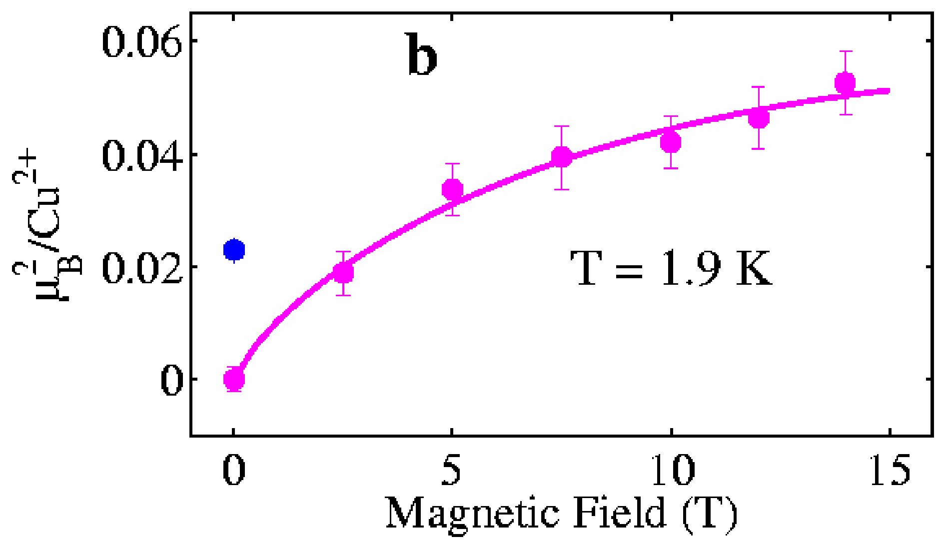
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Lowering of characteristic energy of dynamic spin fluctuations was measured earlier in LSCO by B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



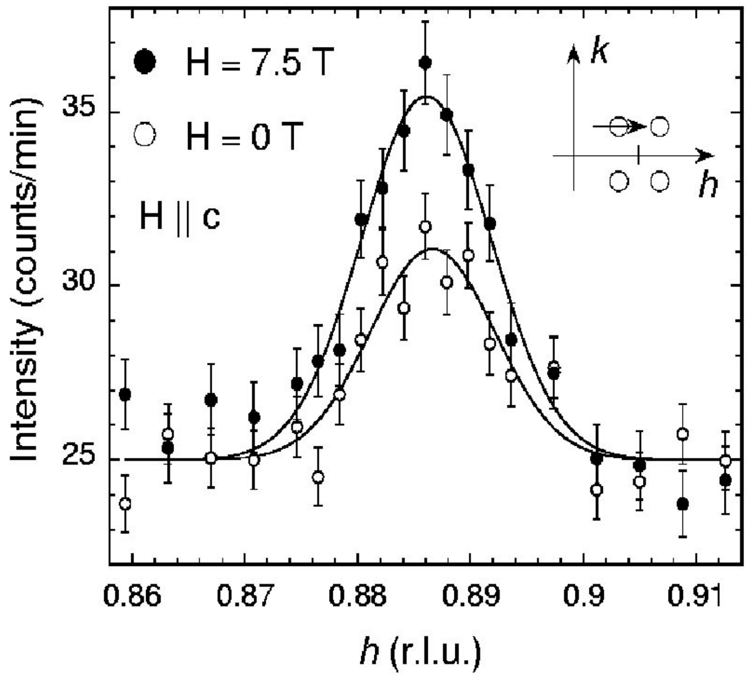
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

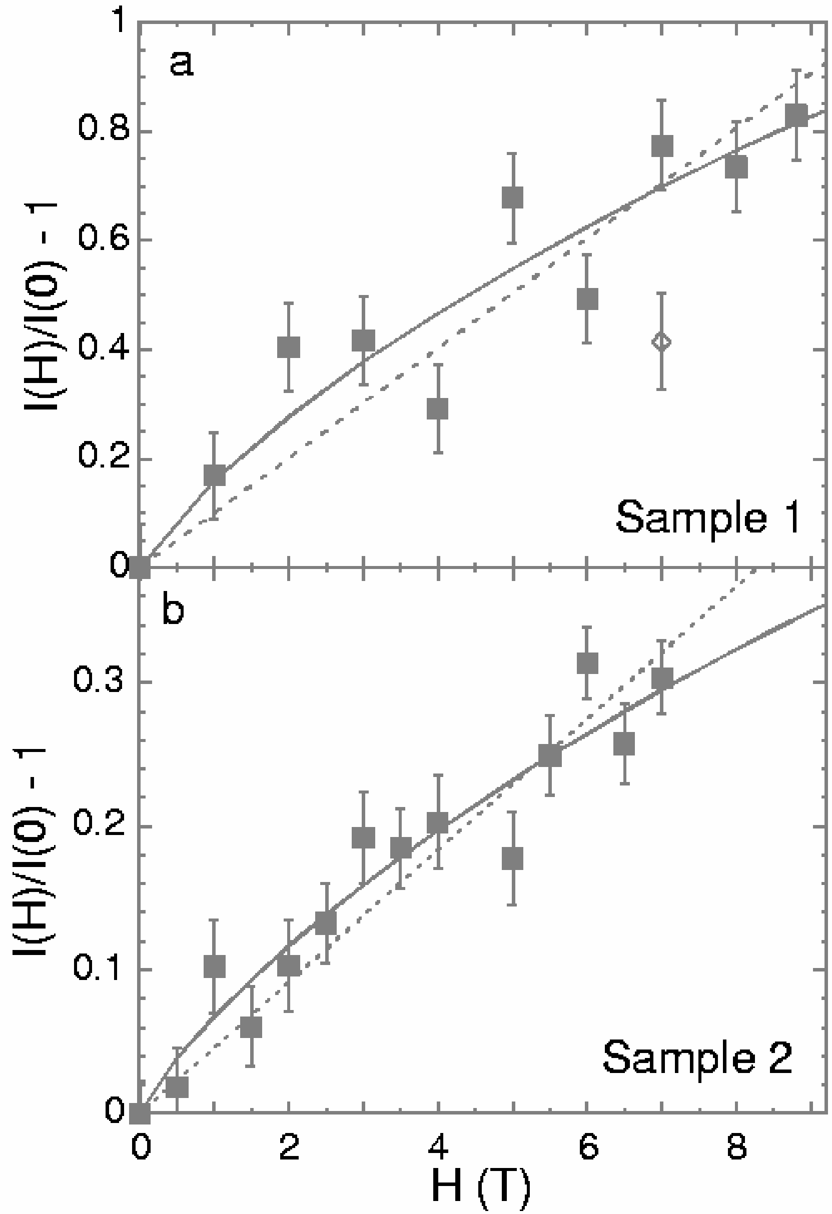
See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev*. B **62**, R14677 (2000).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, *Phys. Rev.* B **66**, 014528 (2002).





Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

Best fit value - $a = 2.4$ with $H_{c2} = 60$ T

Aside: Topological order with collinear spins (J. Zaanen)

SDW order: $S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$

Collinear spins: $\Rightarrow \Phi_\alpha = n_\alpha e^{i\theta}$ with n_α real

Z_2 gauge symmetry: $n_\alpha \to -n_\alpha$ and $\theta \to \theta + \pi$

Effective action

$$S = -J \sum_{\langle ij \rangle} \sigma_{ij} n_{\alpha i} n_{\alpha j} - J \sum_{\langle ij \rangle} \sigma_{ij} \cos(\theta_i - \theta_j) - K \sum_{\square} \prod_{\square} \sigma_{ij}$$

$\sigma_{ij} \to Z_2$ gauge field

Can obtain a topologically ordered state with

$$\langle n_\alpha \rangle = 0 \quad ; \quad \langle e^{i\theta} \rangle = 0$$

but Z_2 gauge flux suppressed

P. E. Lammert, D. S. Rokhsar, and J. Toner, *Phys. Rev. Lett*. **70**, 1650 (1993) ;
Phys. Rev. E **52**, 1778 (1995) (for nematic liquid crystals)
Y. Zhang, E. Demler and S. Sachdev, *Phys. Rev*. B **66,** 094501 (2002).

Outline

I. Order in Mott insulators

Magnetic order

A. Collinear spins

B. Non-collinear spins

Paramagnetic states

A. Bond order and confined spinons

B. Topological order and deconfined spinons

II. Doping Mott insulators with collinear spins and bond order

Global phase diagram

III. Spin density waves (SDW) in LSCO

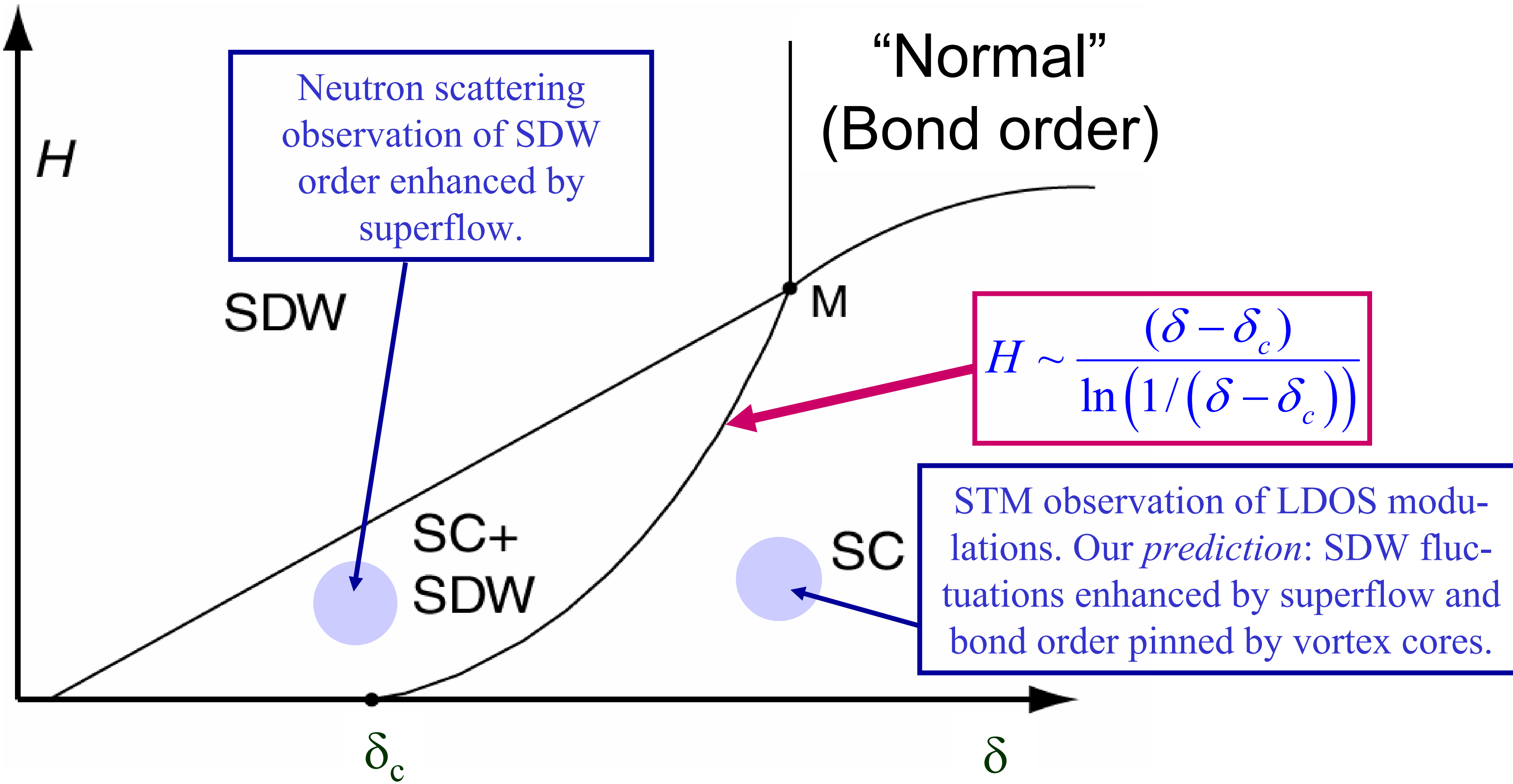
Tuning order and transitions by a magnetic field.

IV. Connection with LDOS modulations

STM experiments on $Bi_2Sr_2CaCu_2O_{8+\delta}$

V. Conclusions

IV. Connections with LDOS modulations



K. Park and S. Sachdev *Phys. Rev*. B **64**, 184510 (2001).
Y. Zhang, E. Demler and S. Sachdev, *Phys. Rev*. B **66,** 094501 (2002).

IV. Connections with LDOS modulations

SDW order: $S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$

Bond order: $Q_{\boldsymbol{a}}(\boldsymbol{r}) = \sum_\alpha S_\alpha(\boldsymbol{r}) S_\alpha(\boldsymbol{r}+\boldsymbol{a}) \approx \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{a}} e^{2i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$

Superflow reduces energy of dynamic spin exciton, but action so far does not lead to static charge order because all terms are invariant under the "sliding" symmetry:

$$\Phi_\alpha(\boldsymbol{r}) \rightarrow \Phi_\alpha(\boldsymbol{r}) e^{i\theta}$$

Small vortex cores break this sliding symmetry on the lattice scale, and lead to a pinning term, which picks particular phase of the local bond order

$$\mathcal{S}_{\text{pin}} = \zeta \sum_{\text{All } \boldsymbol{r}_v \text{ where } \psi(\boldsymbol{r}_v)=0} \int_0^{1/T} d\tau \left[\sum_\alpha \Phi_\alpha^2(\boldsymbol{r}_v) e^{i\vartheta} + \text{c.c.} \right]$$

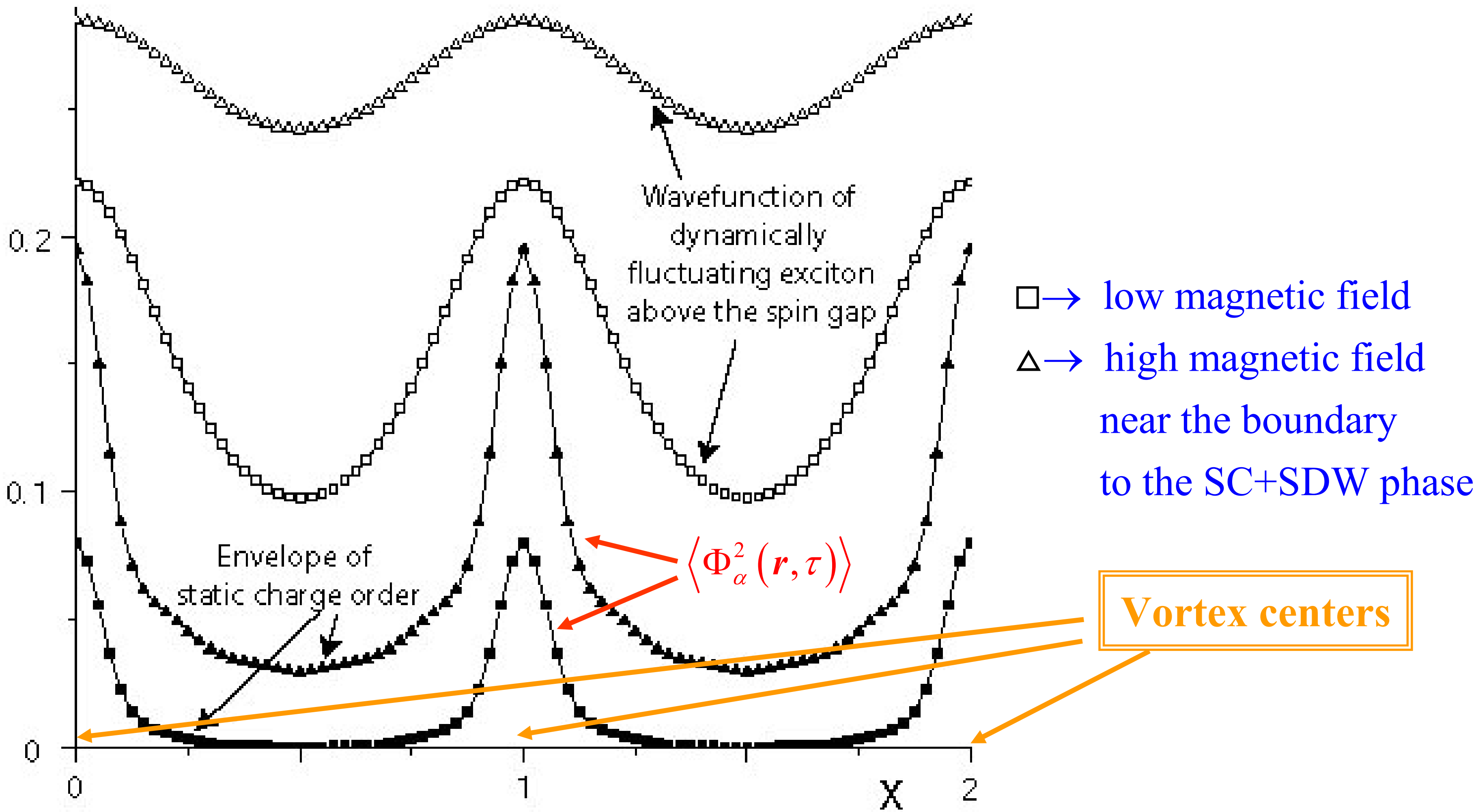
With this term, SC phase has bond order but dynamic SDW

i.e. **there is no static spin order (no "spins in vortices")**

$$\left\langle \Phi_\alpha^2(\boldsymbol{r}) \right\rangle \neq 0 \quad ; \quad \left\langle \Phi_\alpha(\boldsymbol{r}) \right\rangle = 0$$

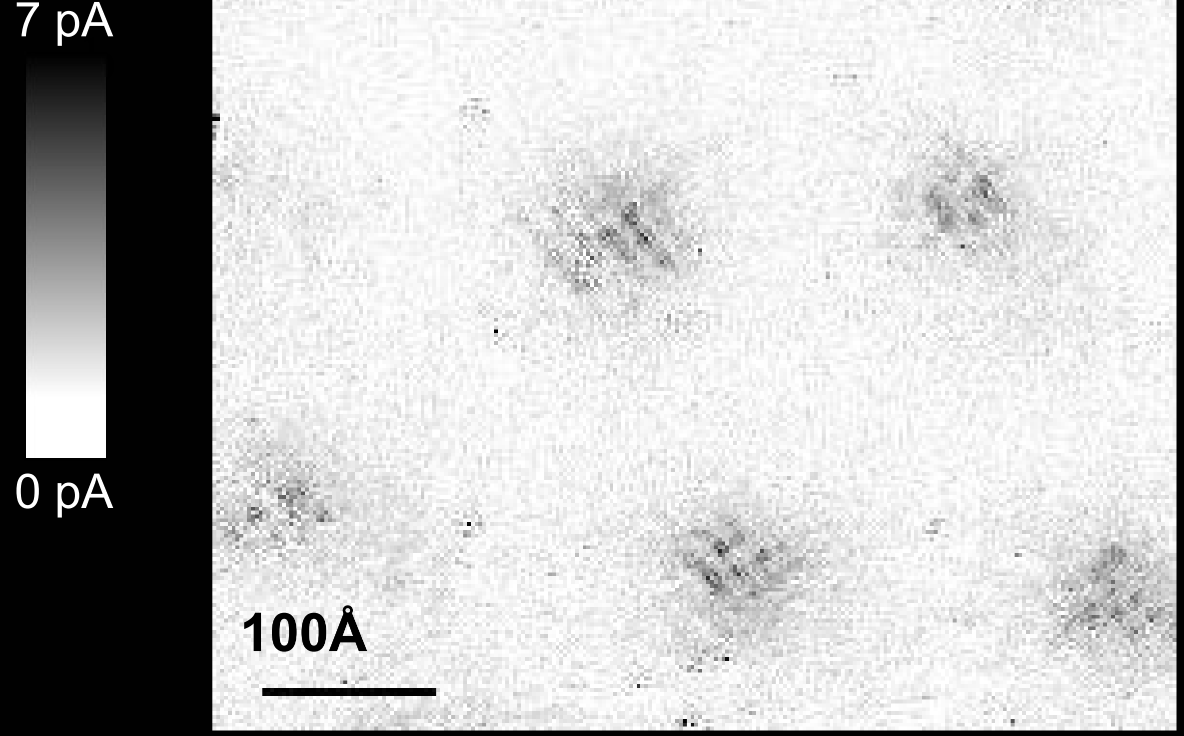
Pinning of static bond order by vortex cores in SC phase, with dynamic SDW correlations

$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\Phi_\alpha^*(\boldsymbol{r}_v,\tau_1)\right\rangle^2 \quad ; \quad \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\right\rangle = 0$$



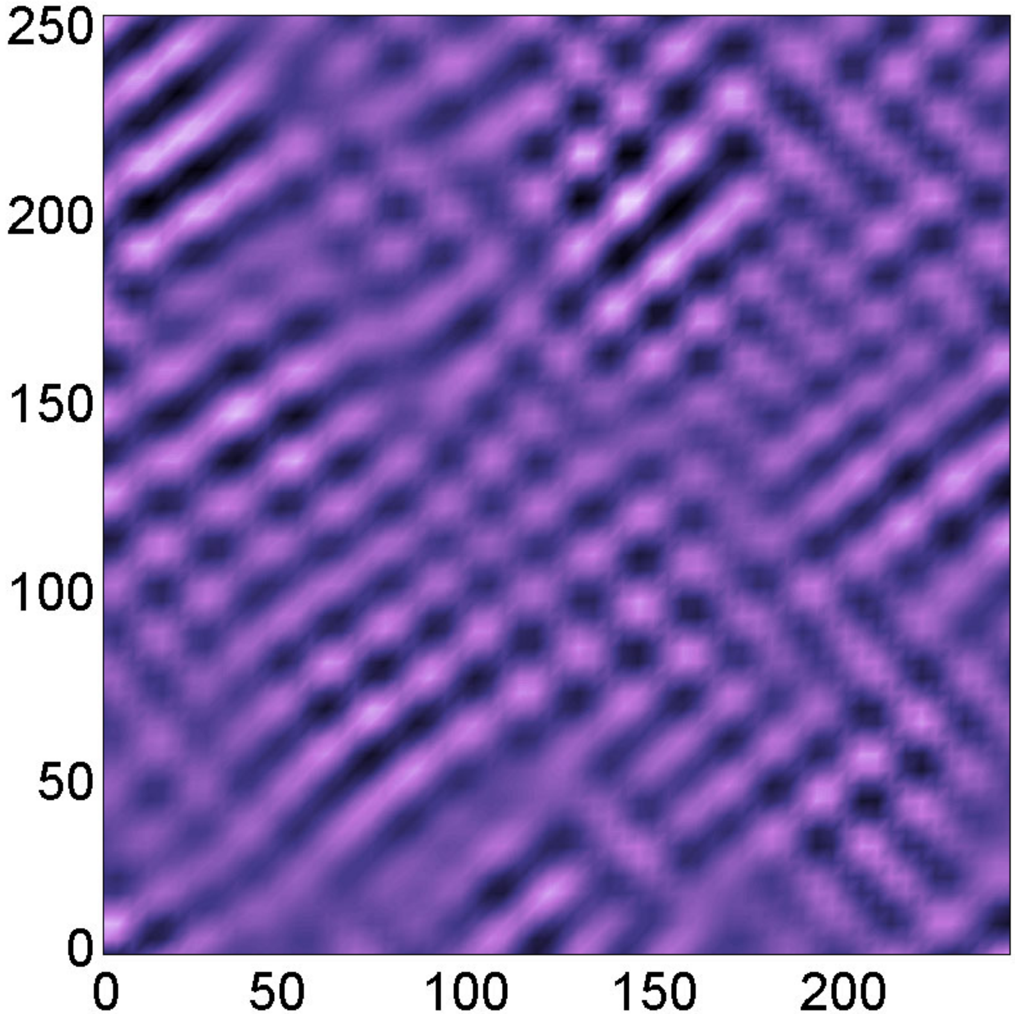
Ying Zhang, E. Demler, and S. Sachdev, *Phys. Rev.* B **66,** 094501 (2002).

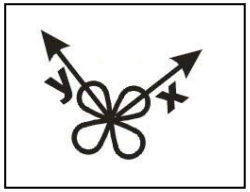
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

IV. STM image of LDOS modulations in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

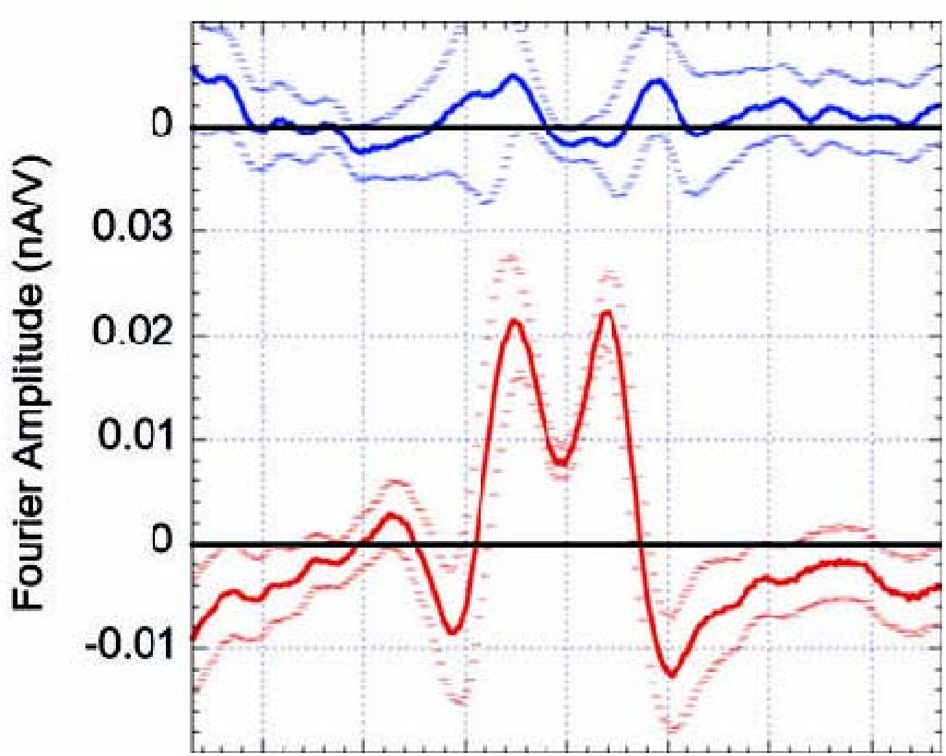




Period = 4 lattice spacings

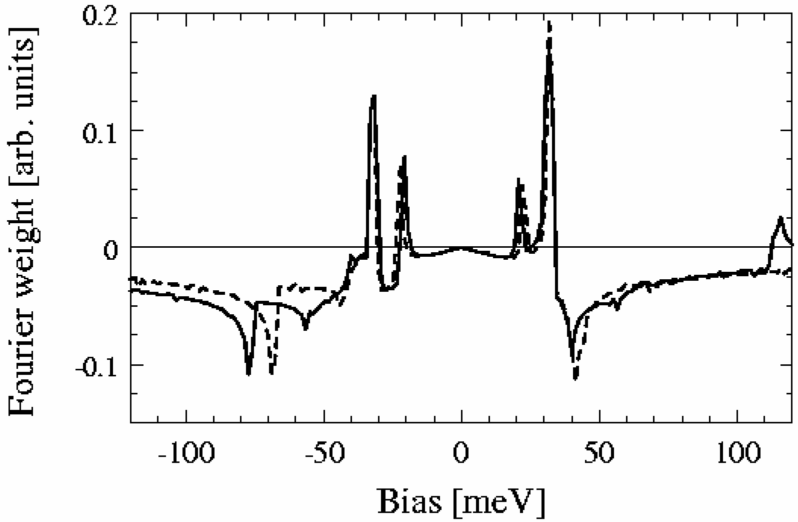
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

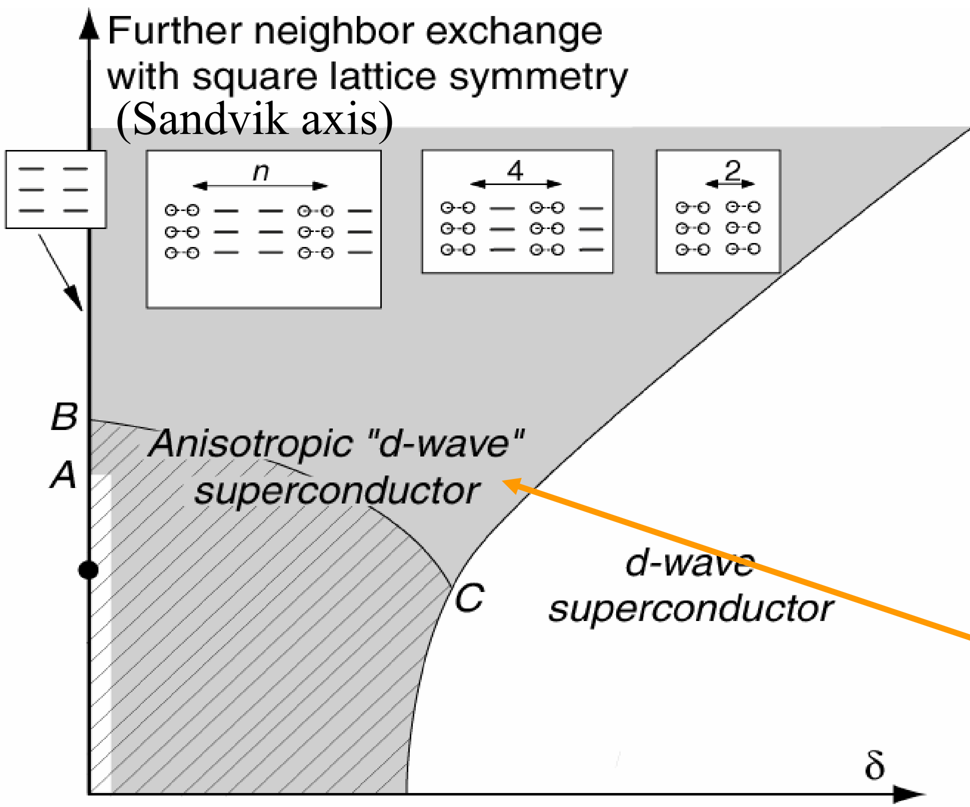
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

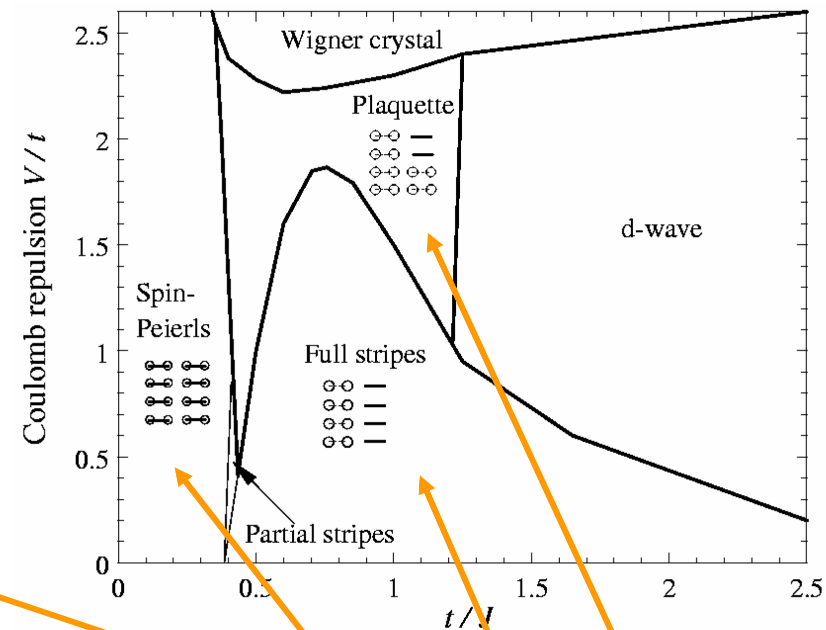


Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, Phys. Rev. B **66**, 104505 (2002); D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, cond-mat/0204011

Global phase diagram





Hatched region --- static spin order
Shaded region ---- static bond/charge order

Non-magnetic "d-wave" superconductor with even period bond order.

M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999)
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).
M. Vojta, cond-mat/0204284.

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).
S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers. The correct paramagnetic Mott insulator has bond-order and confinement of spinons (collinear spins in magnetically ordered state.

II. Theory of quantum phase transitions provides semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.

III. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

IV. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?