

Emergent “light” and the high temperature superconductors

Perimeter Institute
January 19, 2016

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

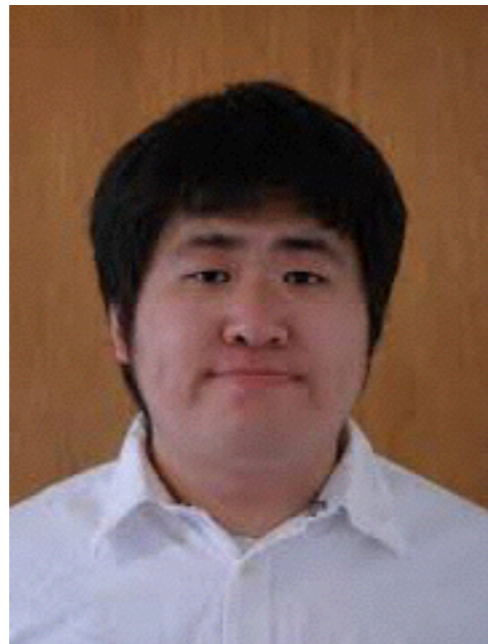




Debanjan
Chowdhury



Andrea Allais

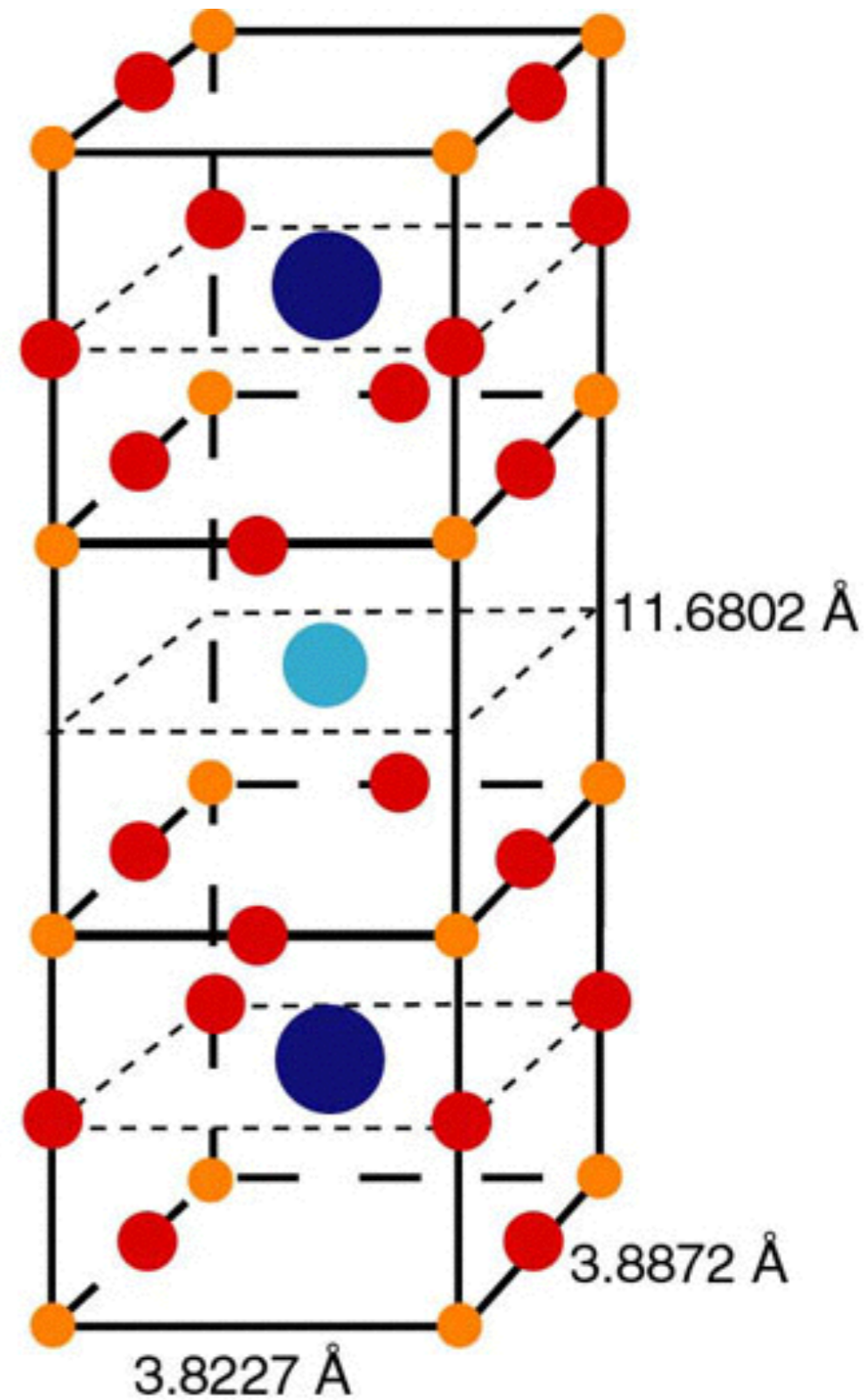
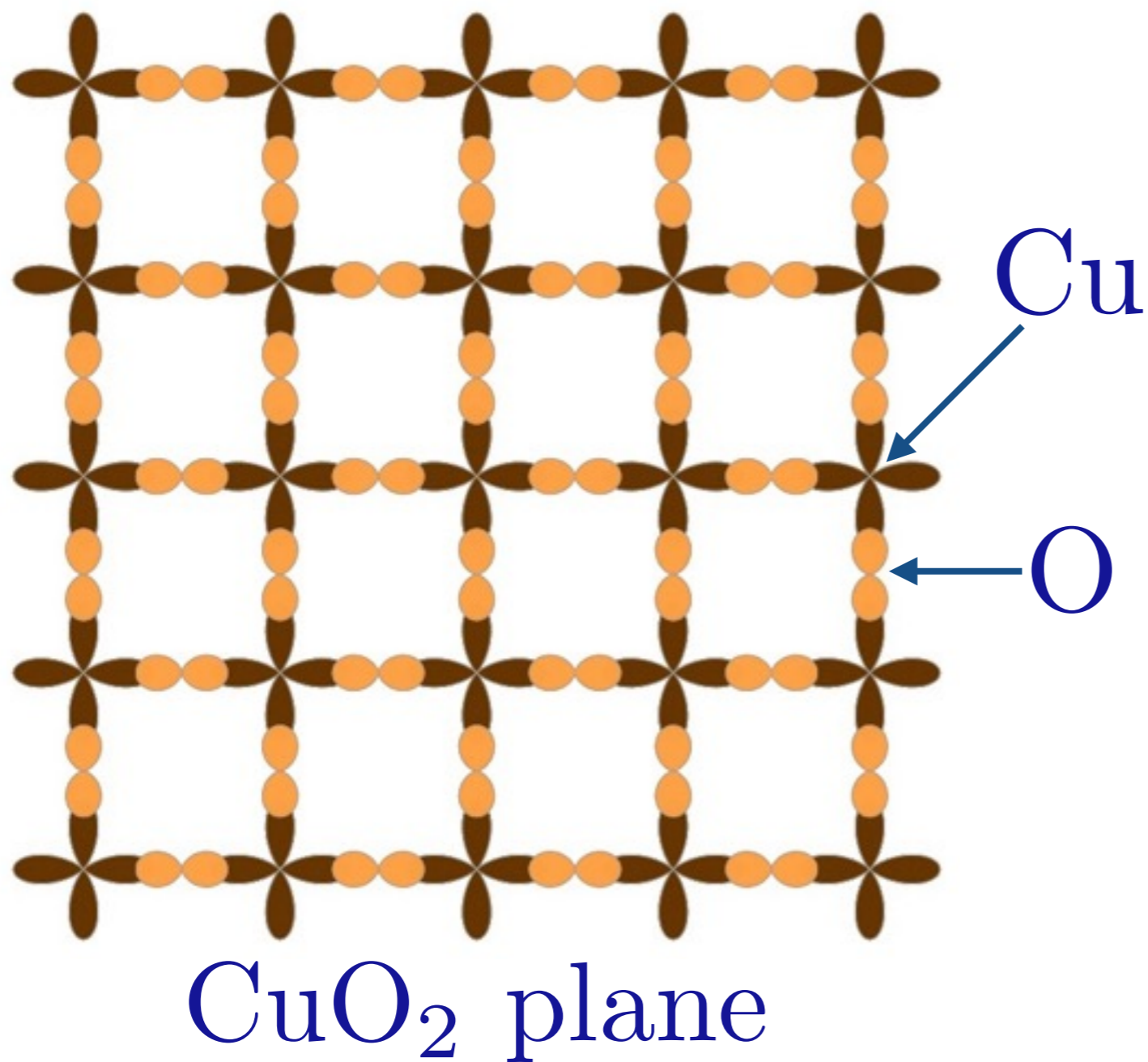


Yang Qi



Matthias Punk

High temperature superconductors



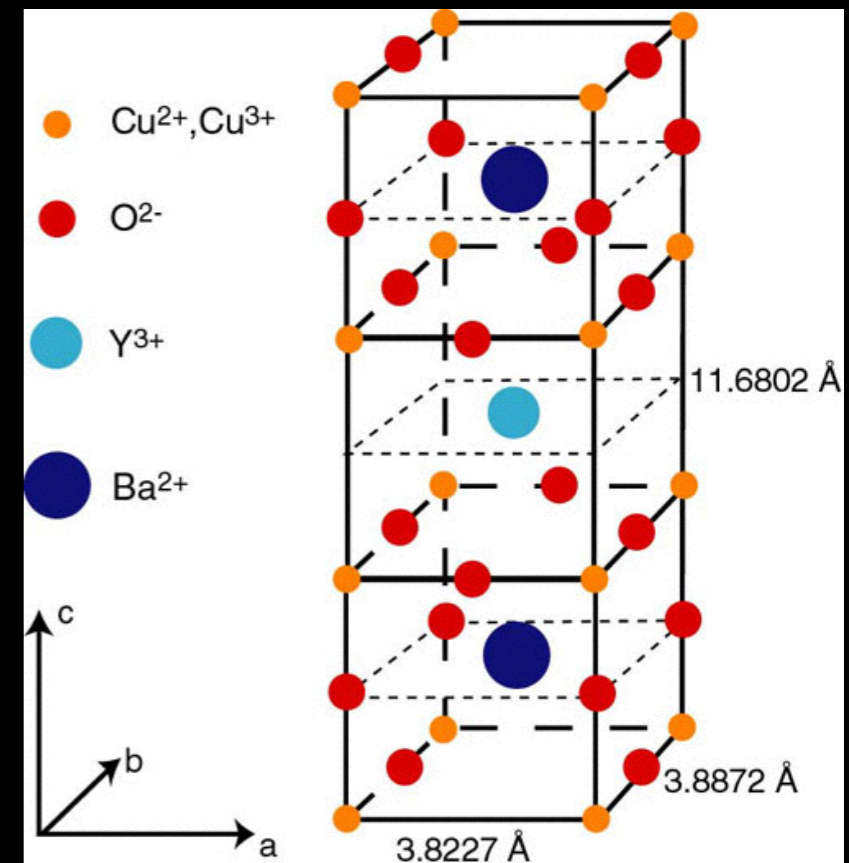
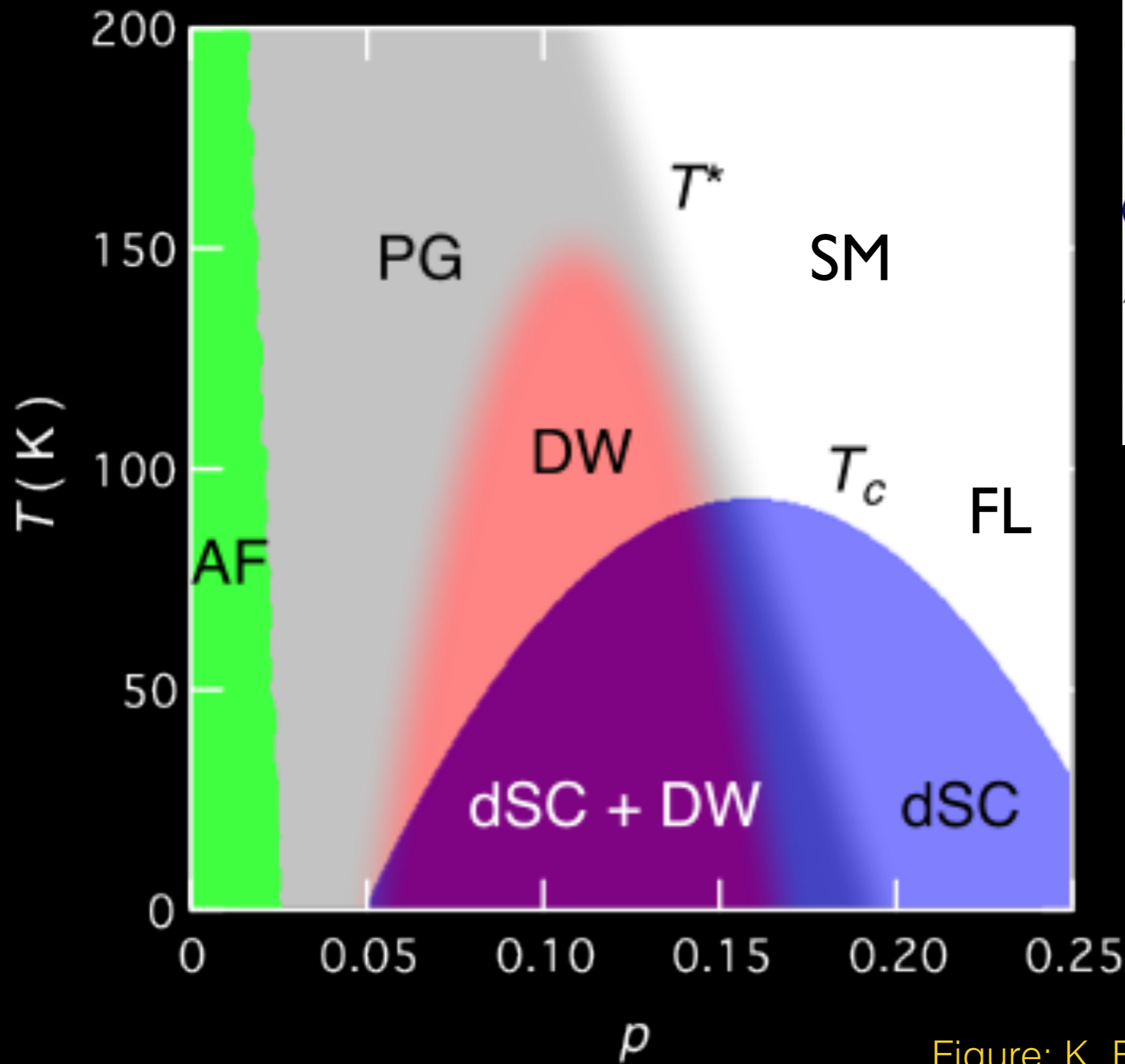
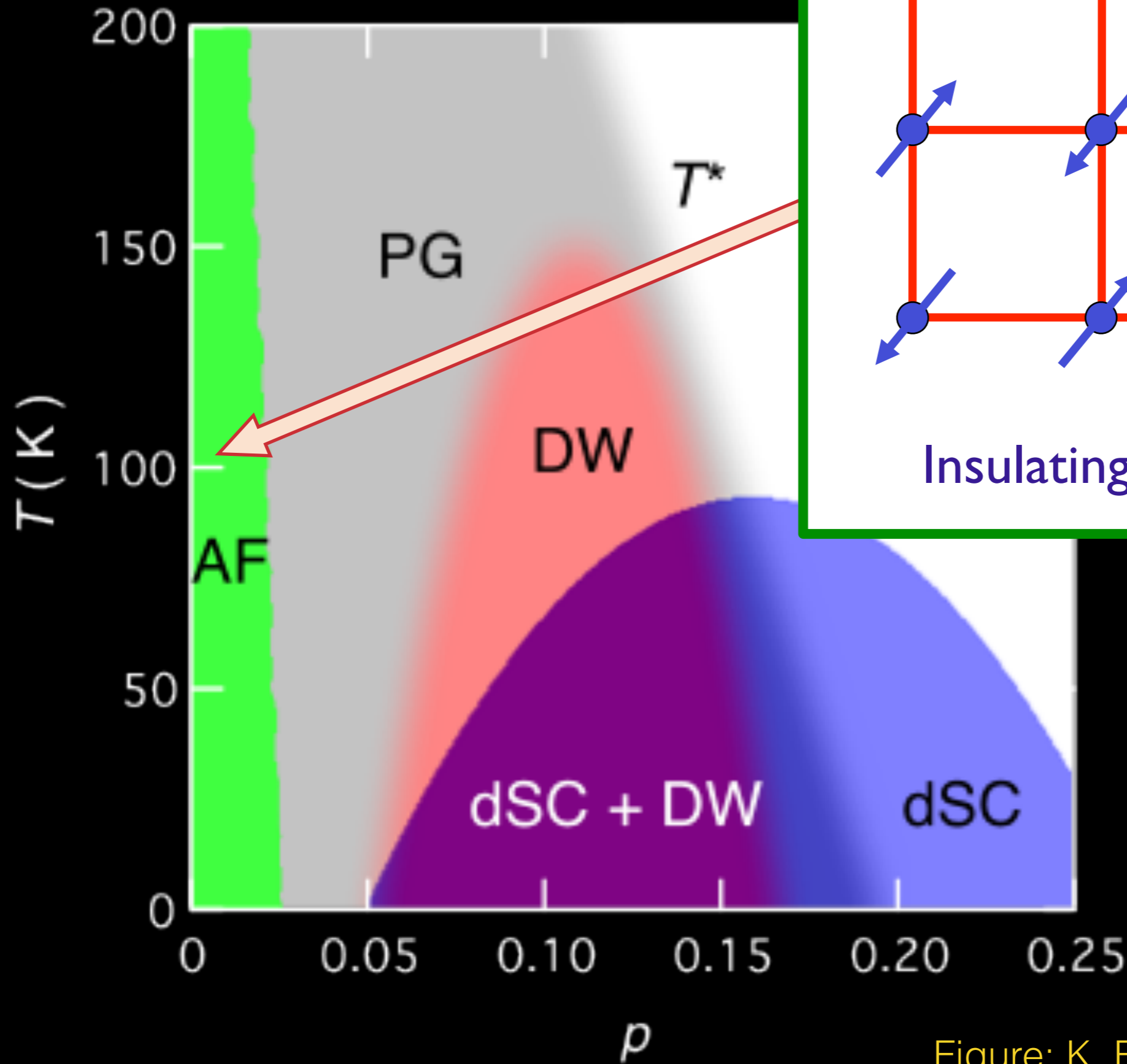


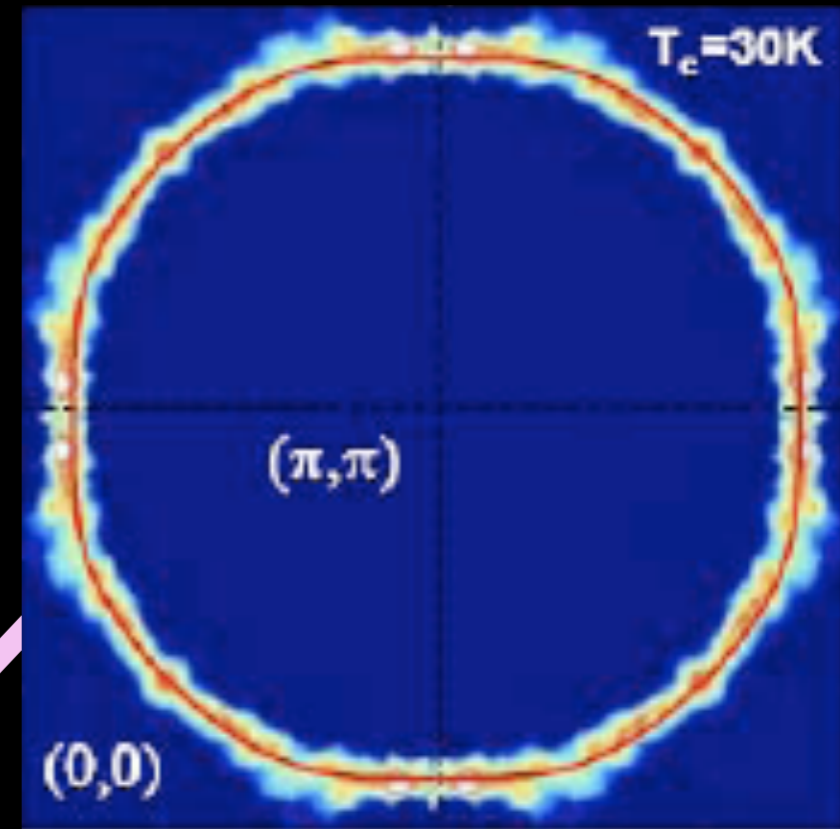
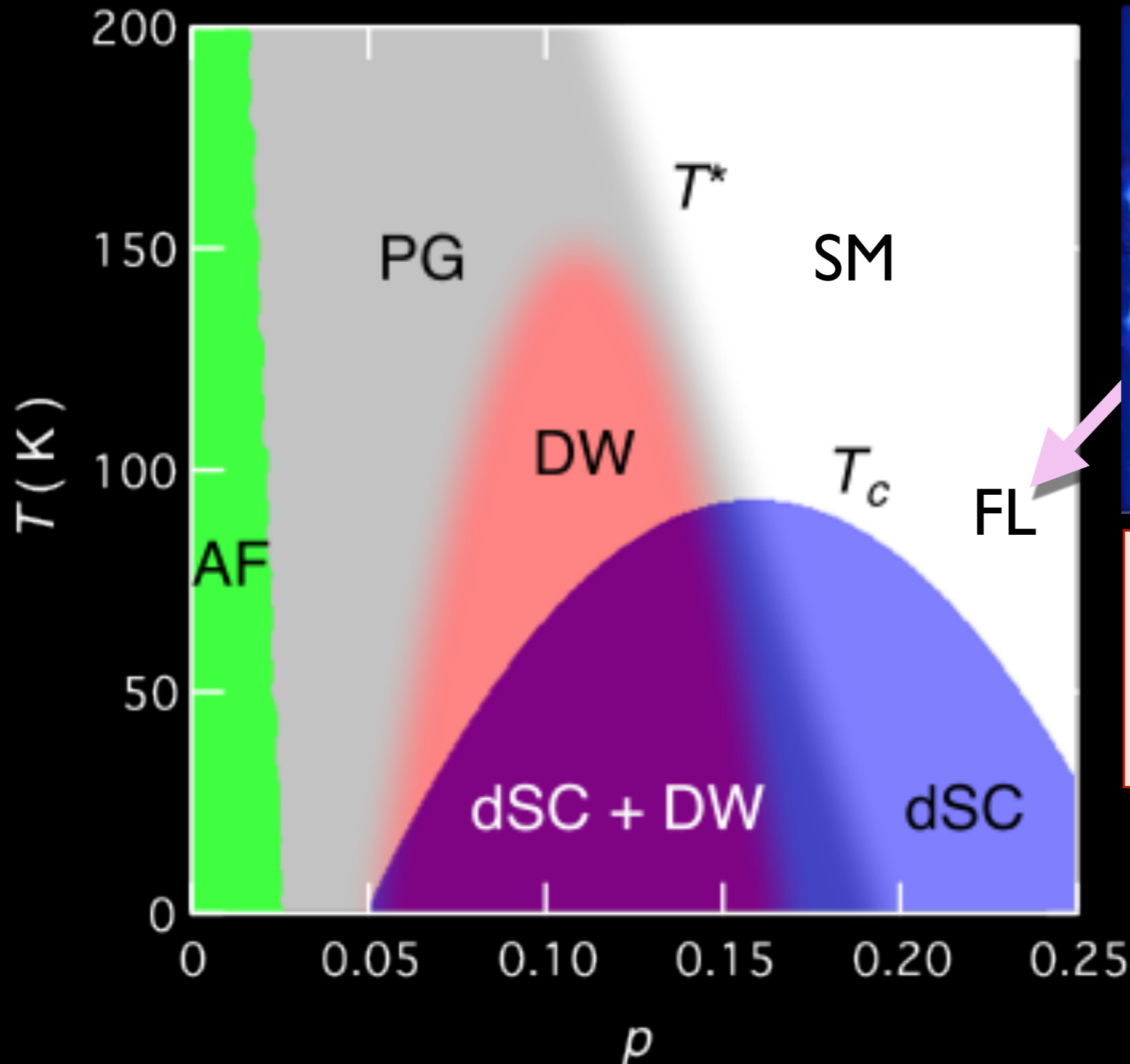
Figure: K. Fujita and J. C. Seamus Davis



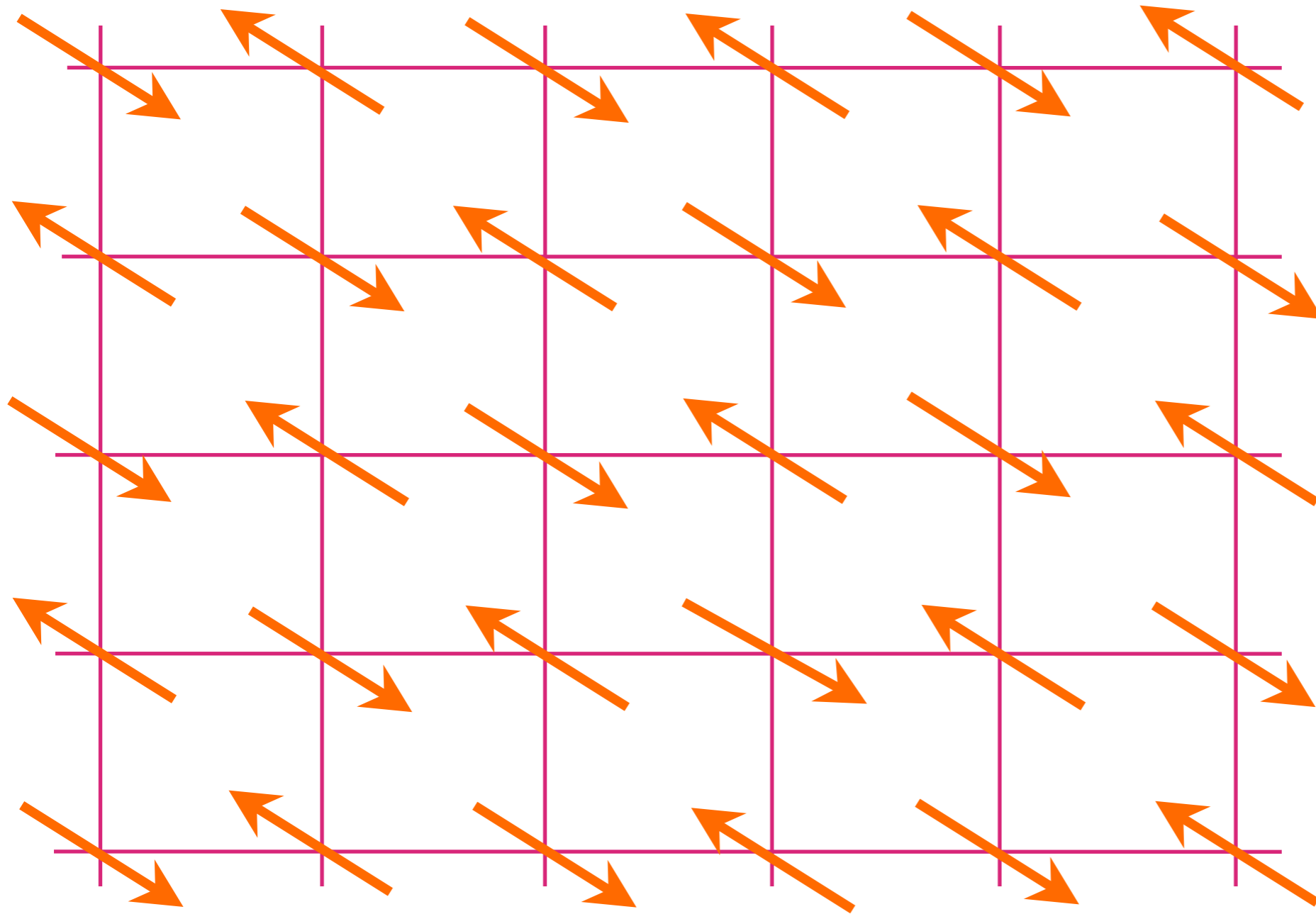
$$T = Da^2 \cup a_3 \cup 6 + x$$

Figure: K. Fujita and J. C. Seamus Davis

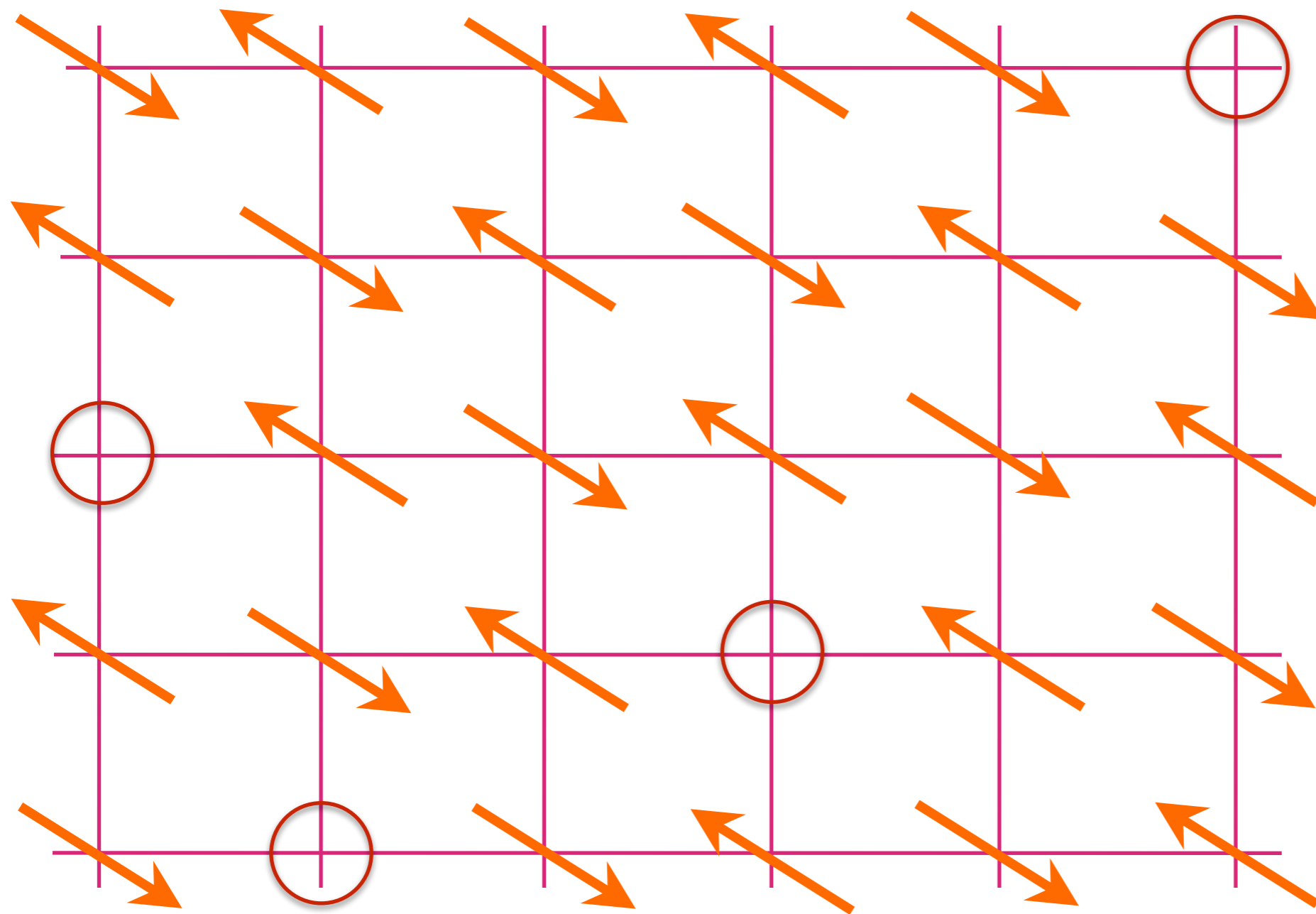
M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



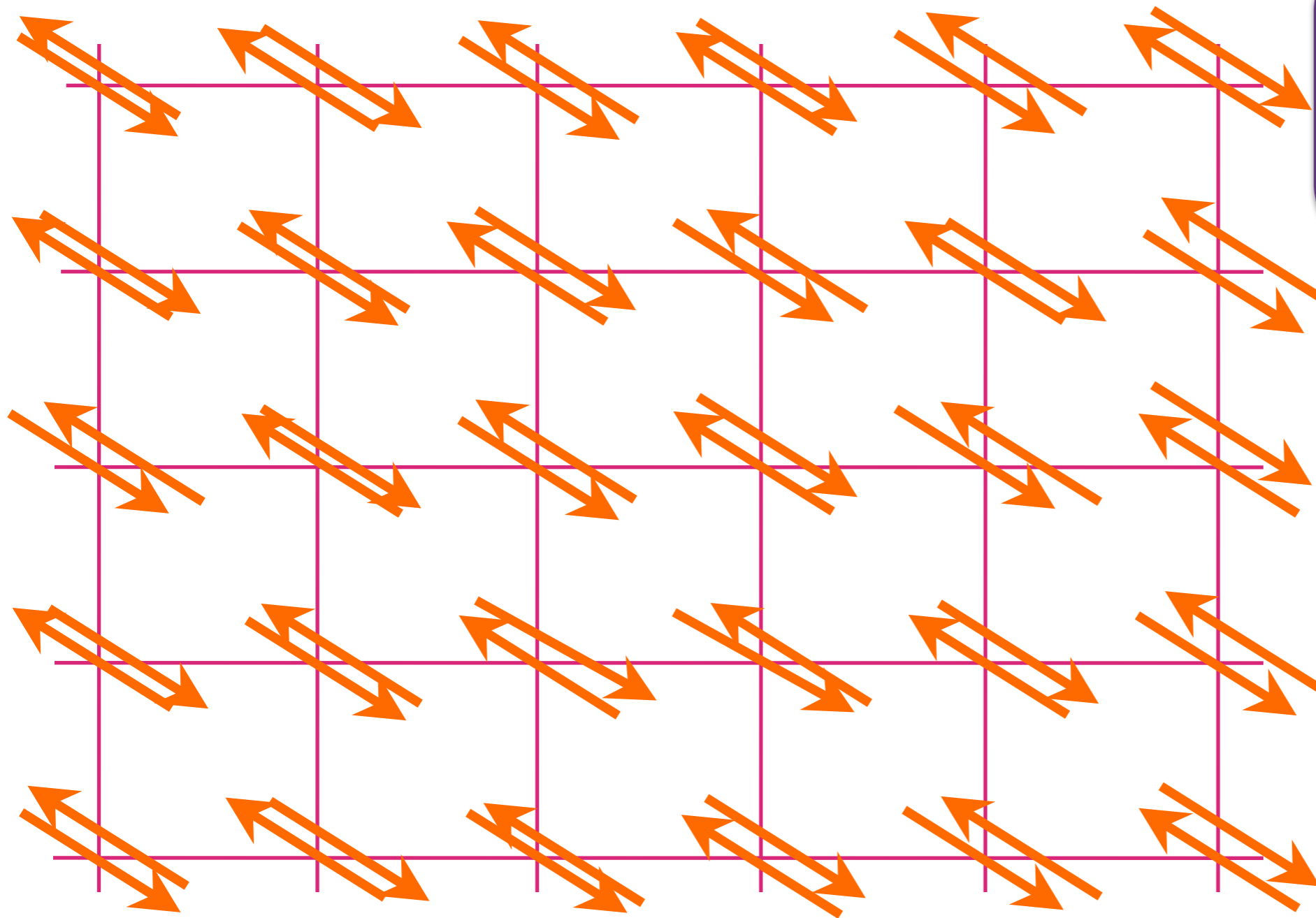
A conventional metal:
the Fermi liquid



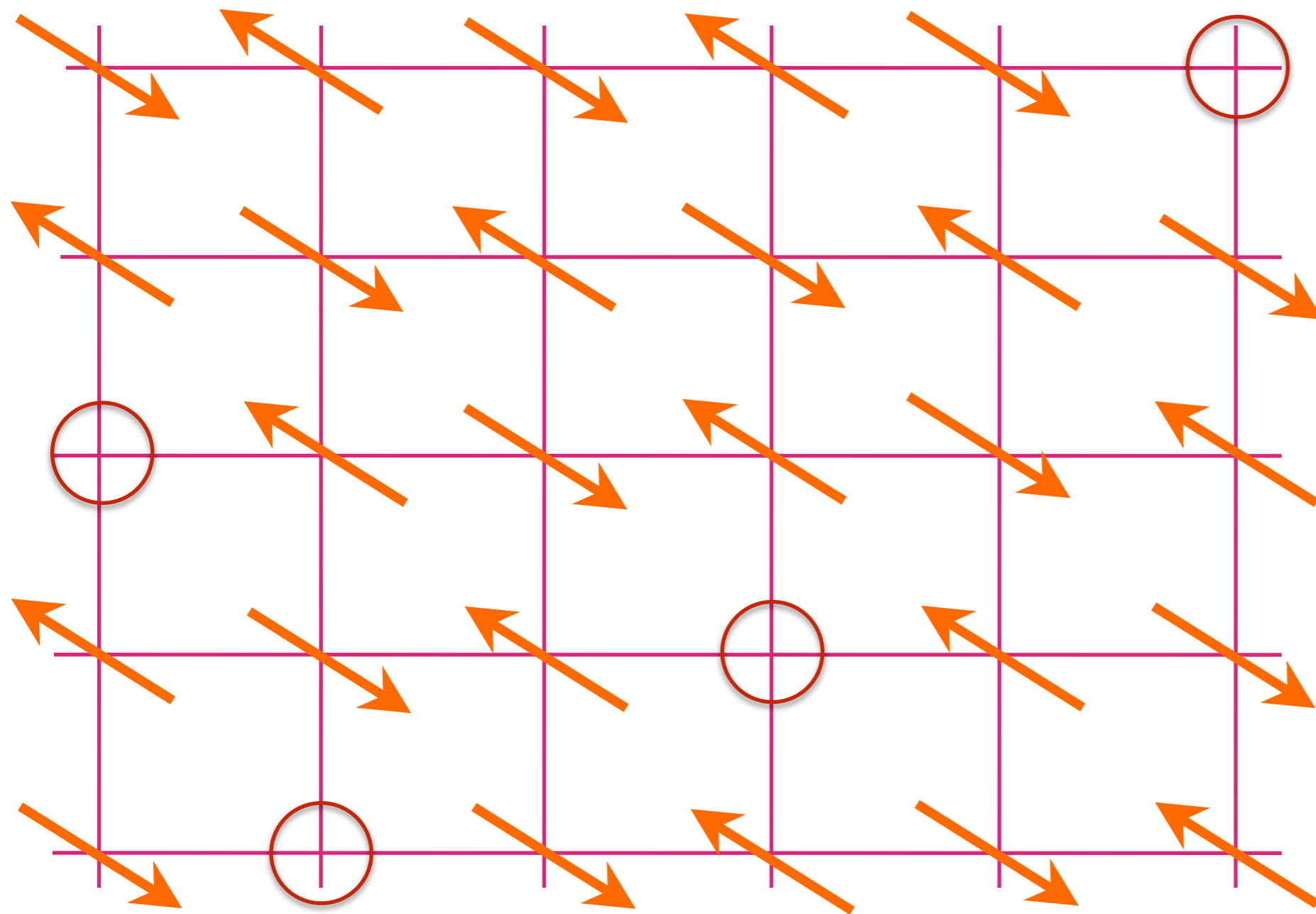
“Undoped”
Anti-
ferromagnet



Anti-ferromagnet
with p holes
per square



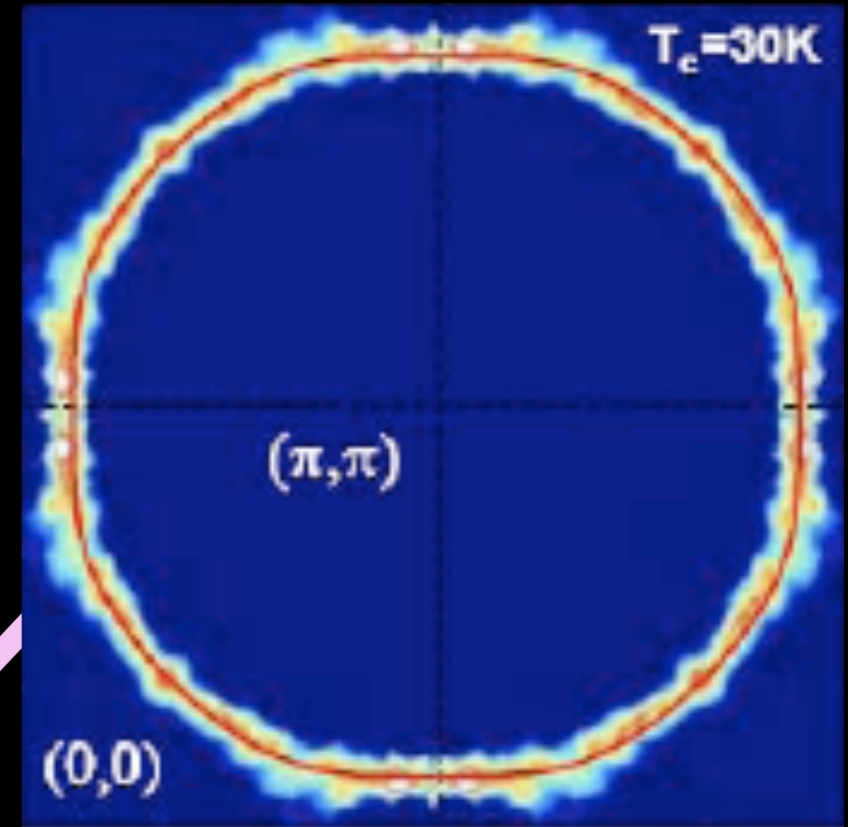
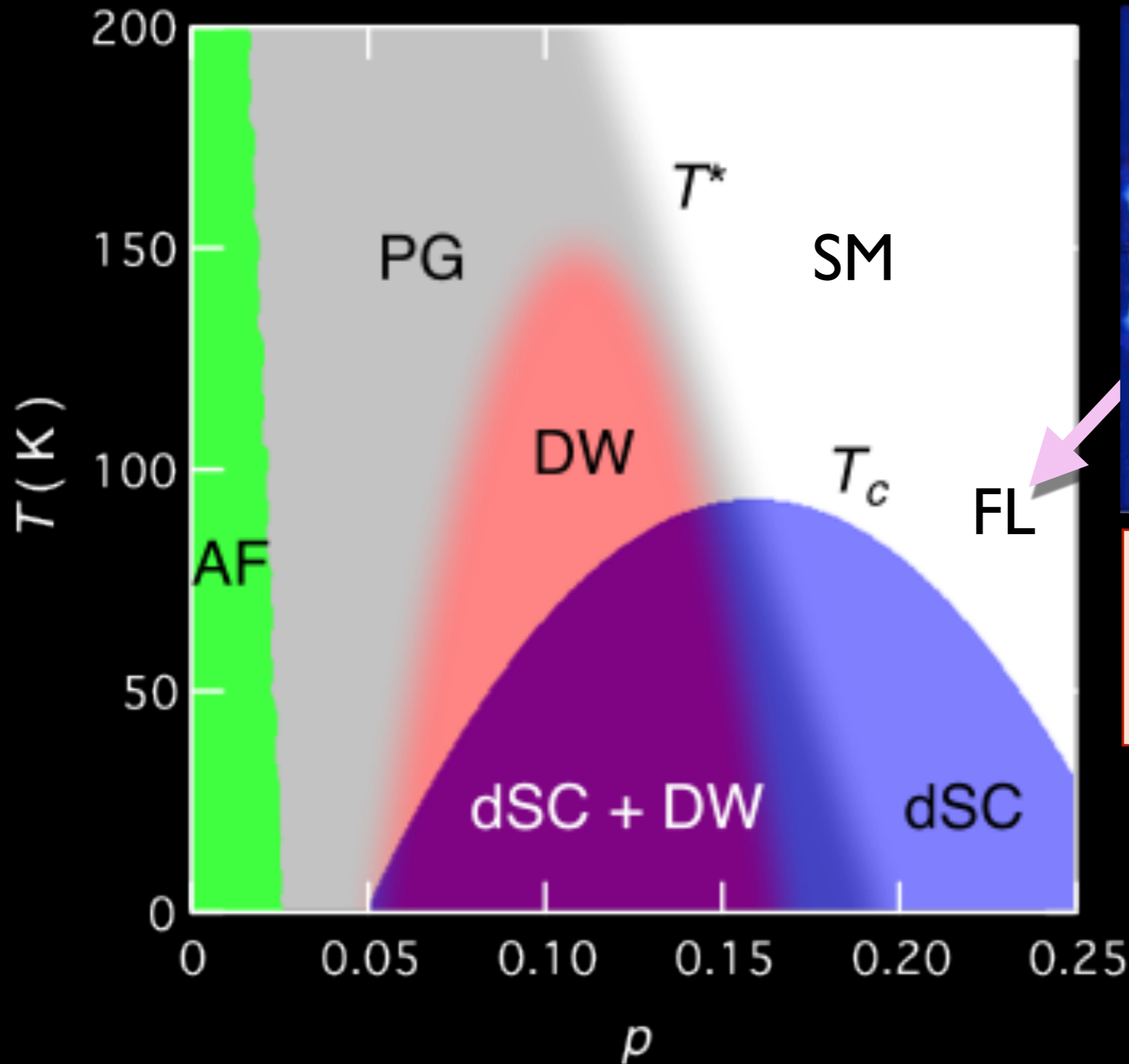
Filled
Band



Anti-ferromagnet with p holes per square

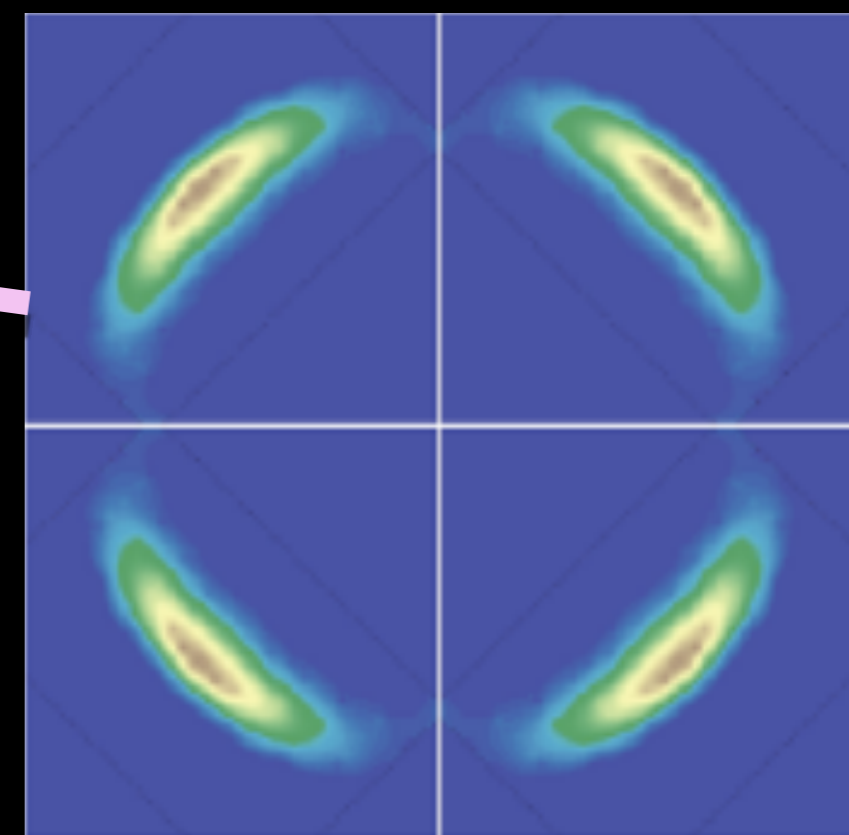
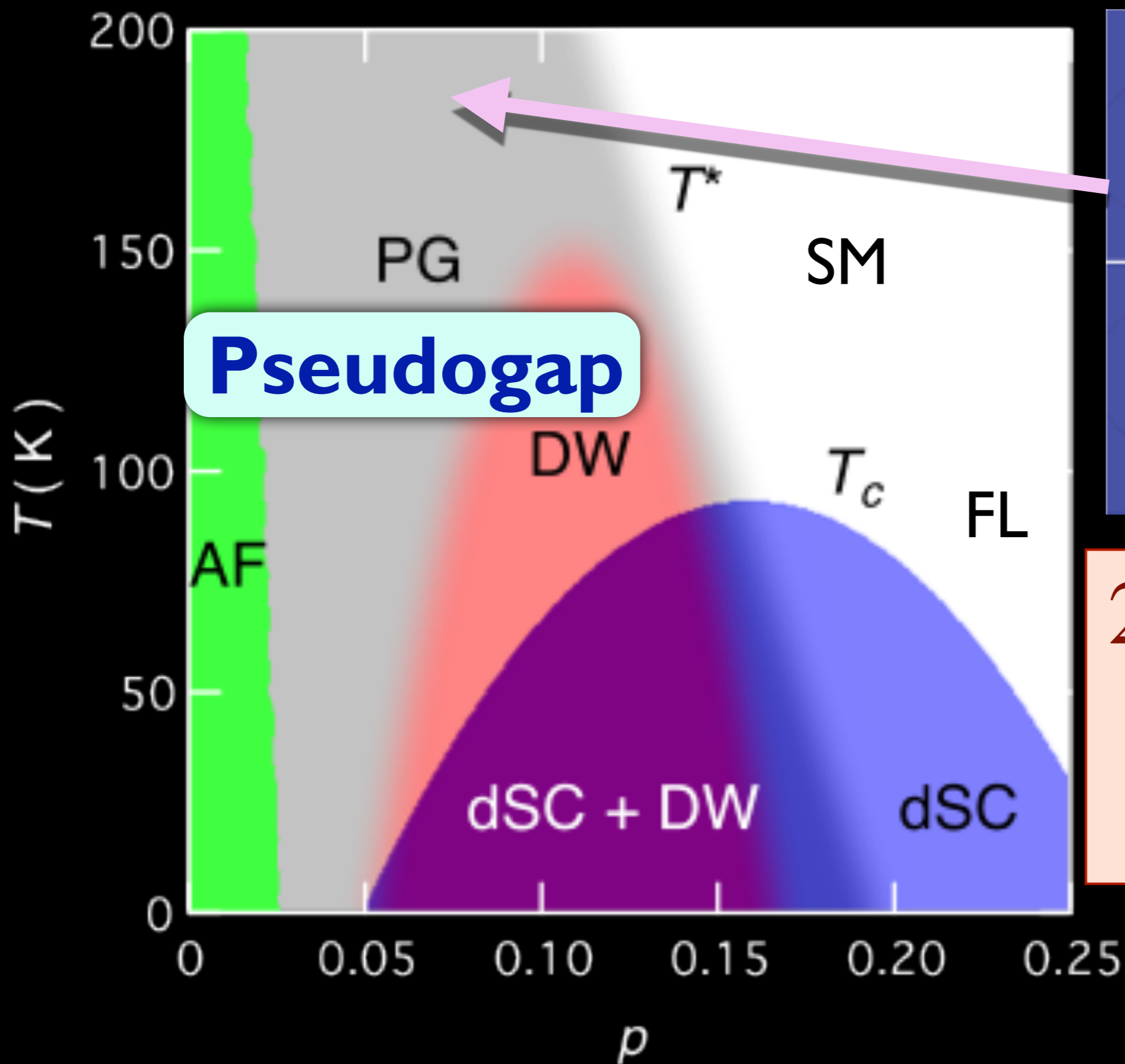
But relative to the band insulator, there are $1 + p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1 + p$

M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



Fermi liquid
Area enclosed by
Fermi surface = $1+p$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



2. Pseudogap
metal
at low p

Is the higher temperature pseudogap
(with ``Fermi arc'' spectra) described by

(A) Thermal fluctuations of the low
temperature orders (superconductivity,
density wave, antiferromagnetism...)

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(B) A new type of metal, which can be stable
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(C) The possibilities (A) and (B) are merely
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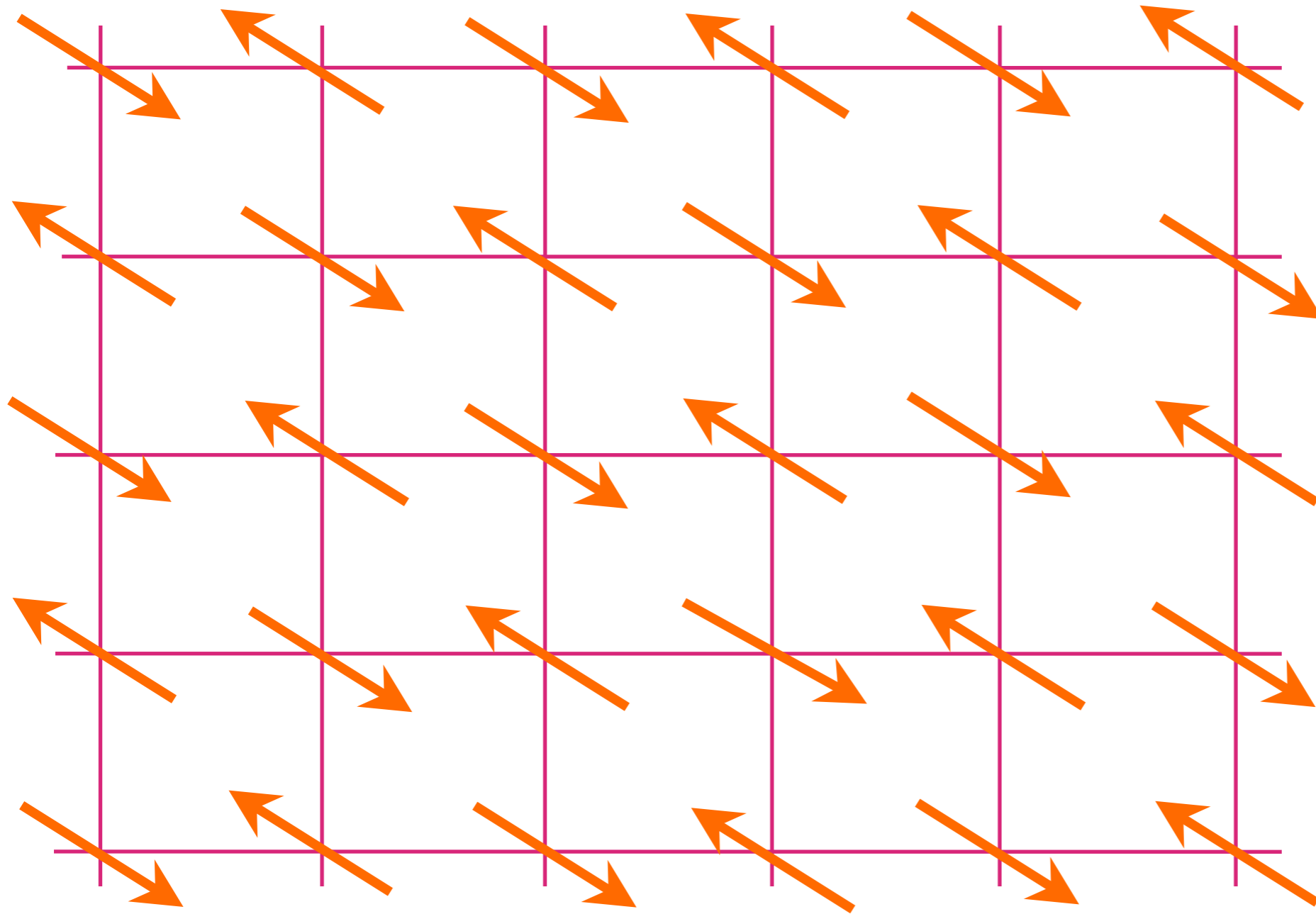
(A) Thermal fluctuations of the low

Answer (B) must have “emergent” gauge fields, and these are (in principle) detectable in low temperature experiments. There are also qualitative differences between (A) and (B) at higher temperatures.

OR


(C) The possibilities (A) and (B) are merely two limits of the same physics?

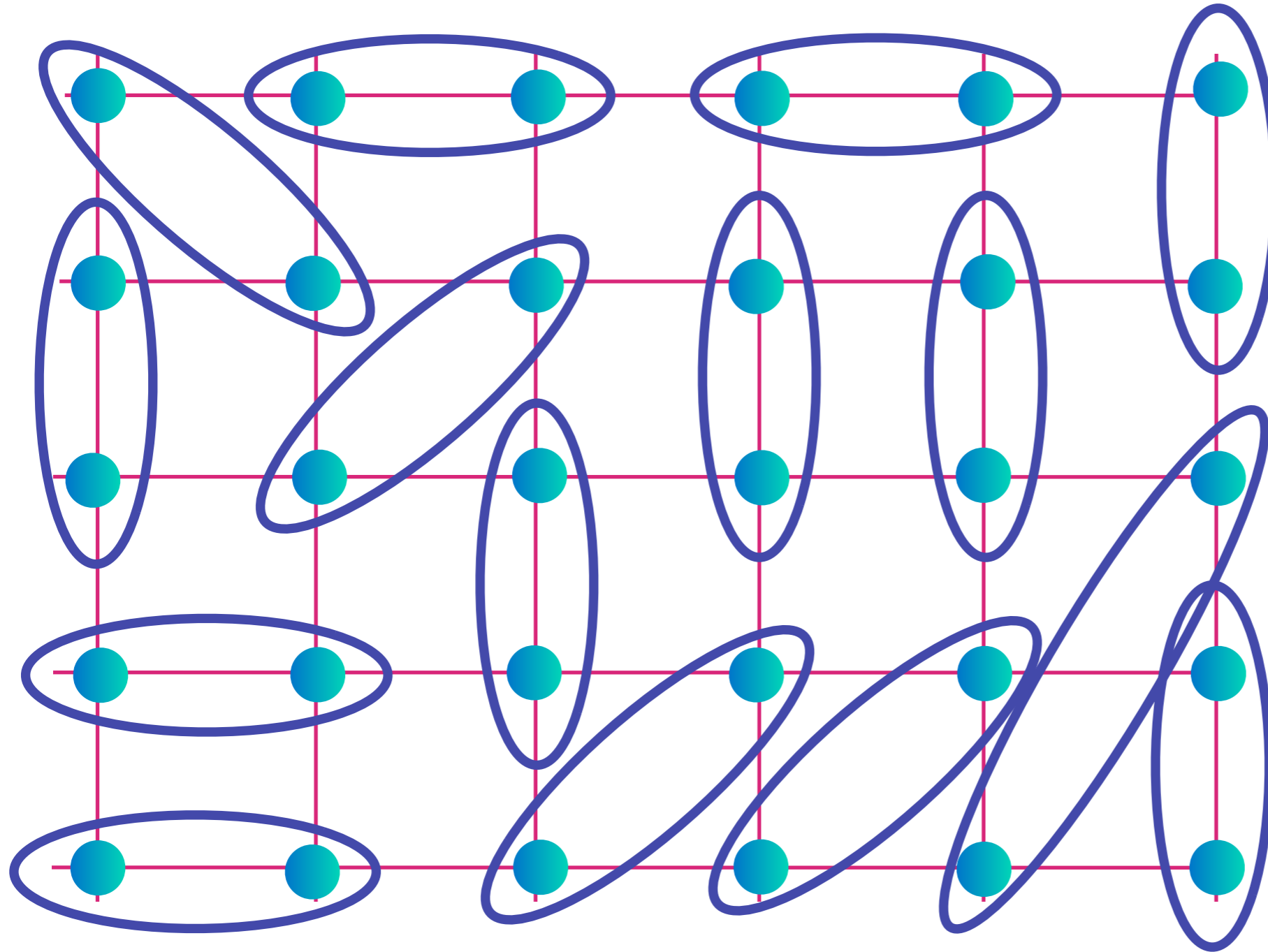
I. Emergent gauge fields and long-range entanglement in insulators



“Undoped”
Anti-
ferromagnet

Insulating spin liquid


$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$




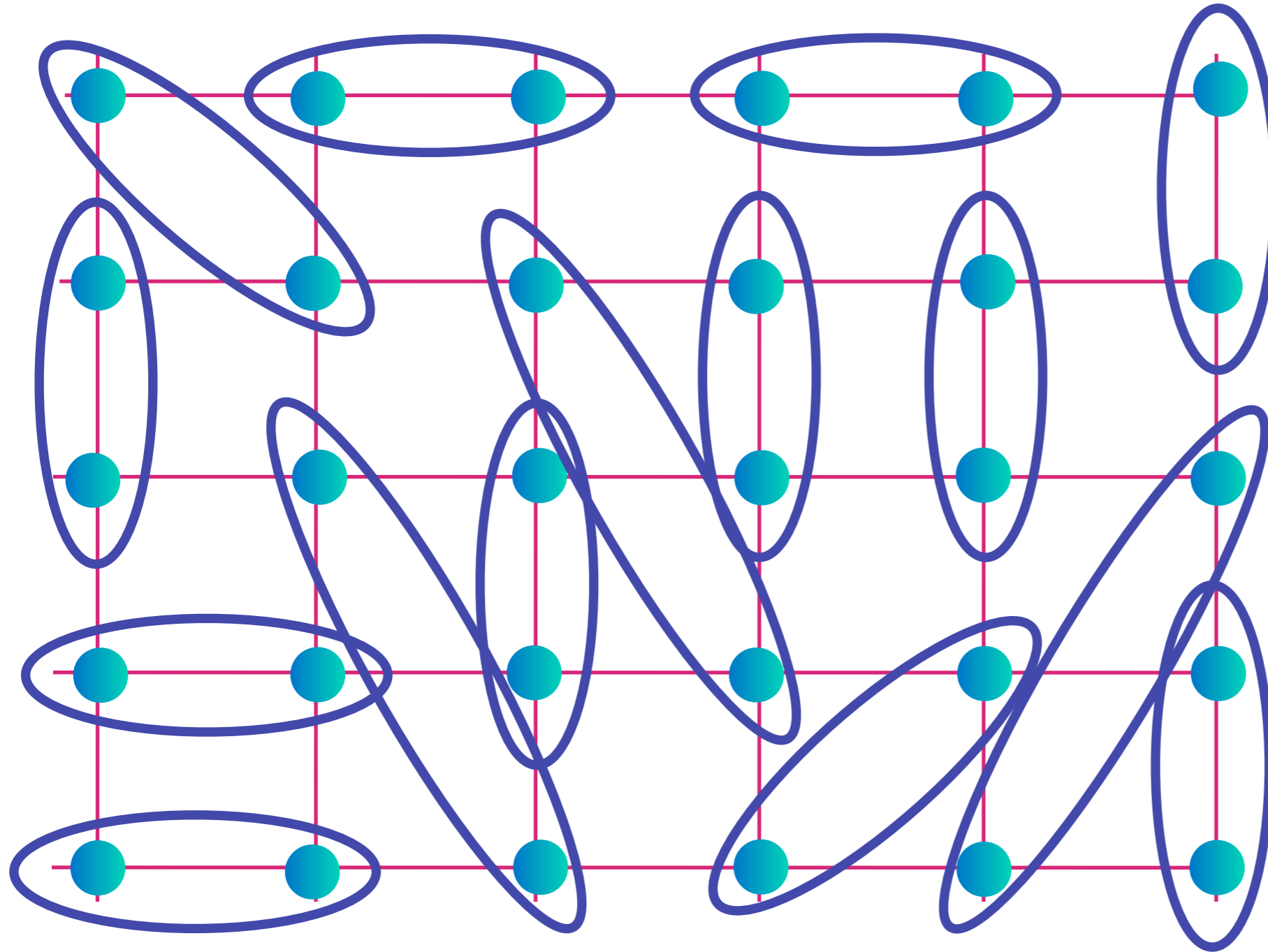
An insulator
with
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L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P.W.Anderson, Materials Research Bulletin **8**, 153 (1973)

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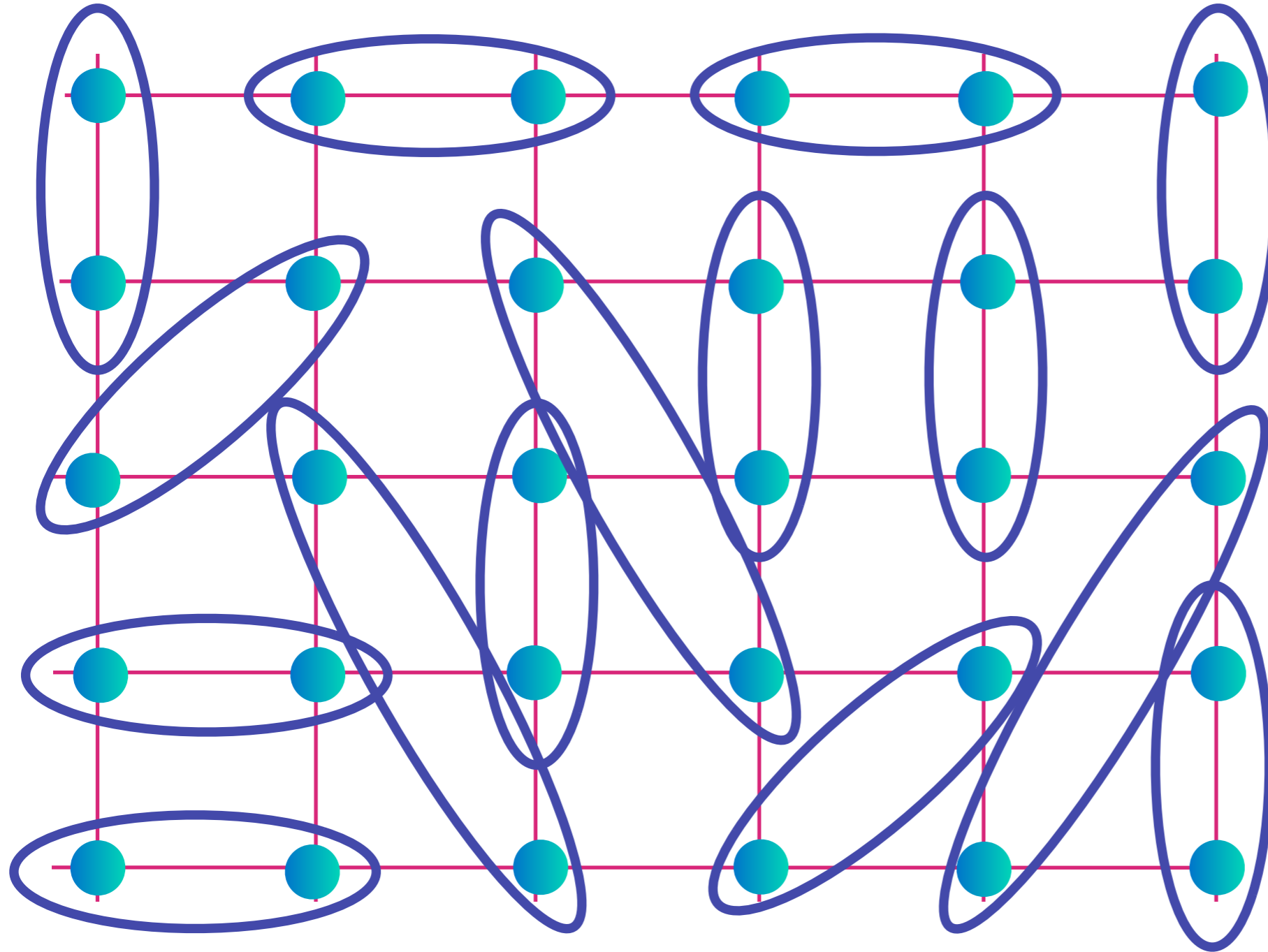
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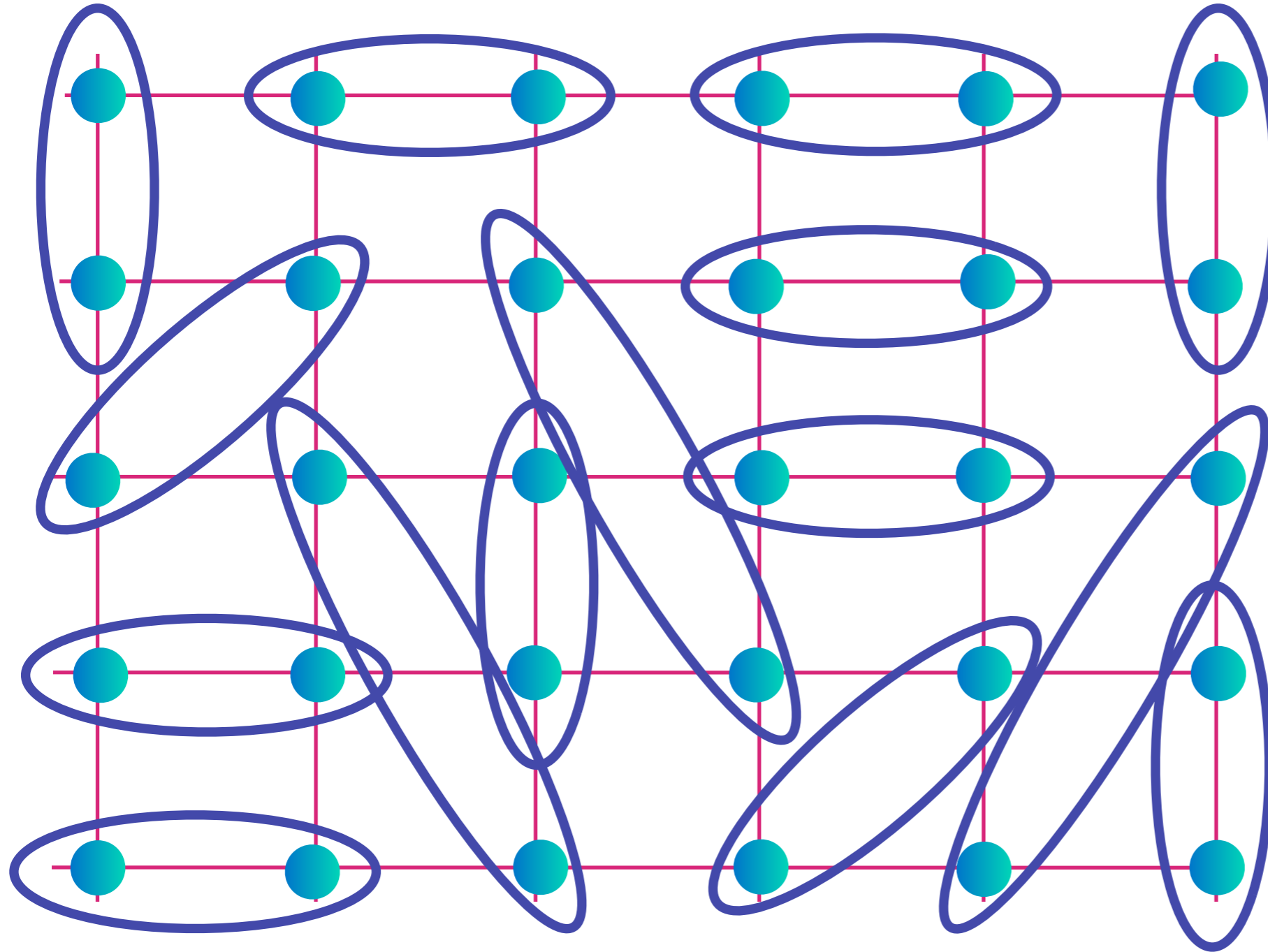
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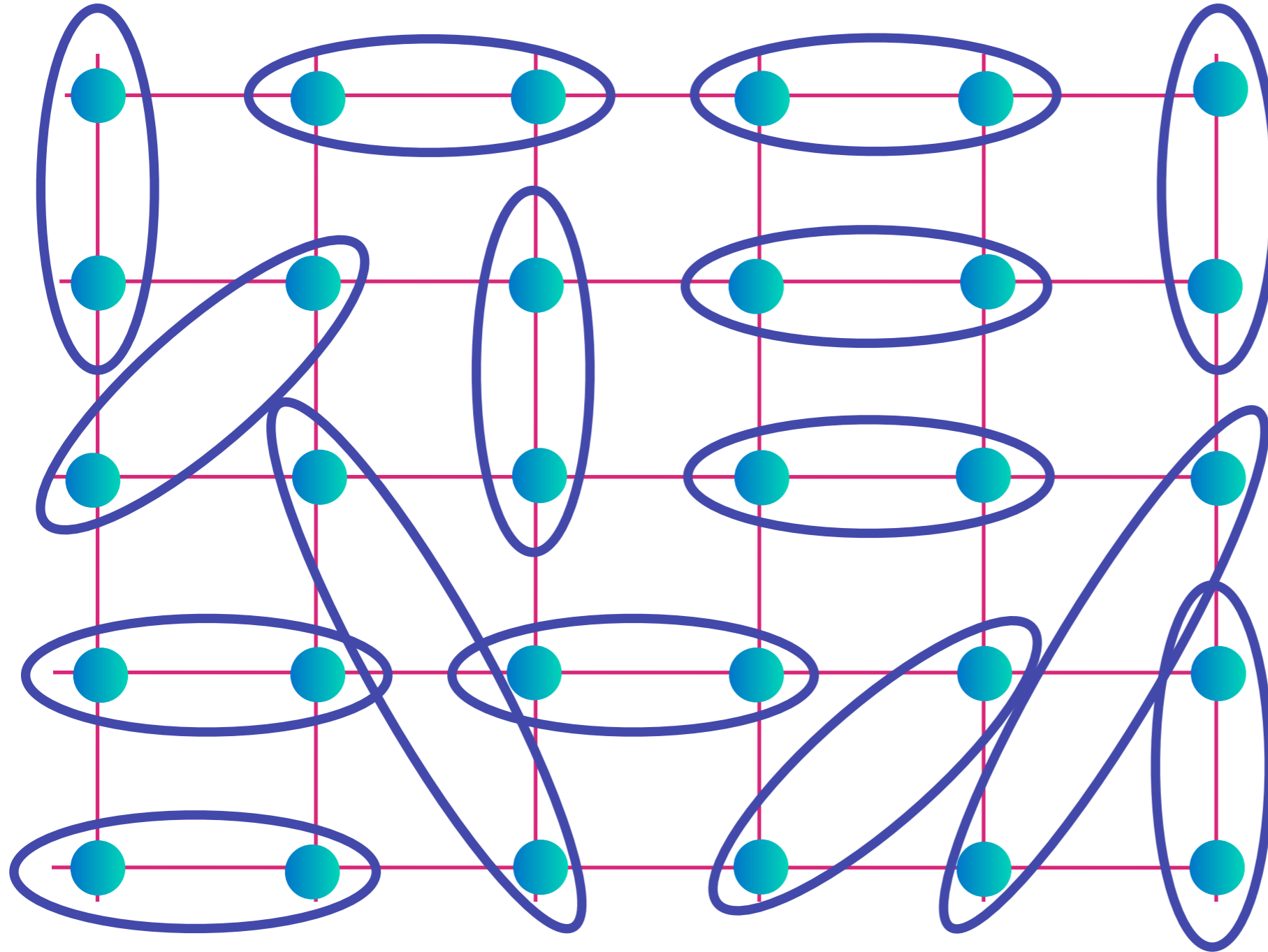
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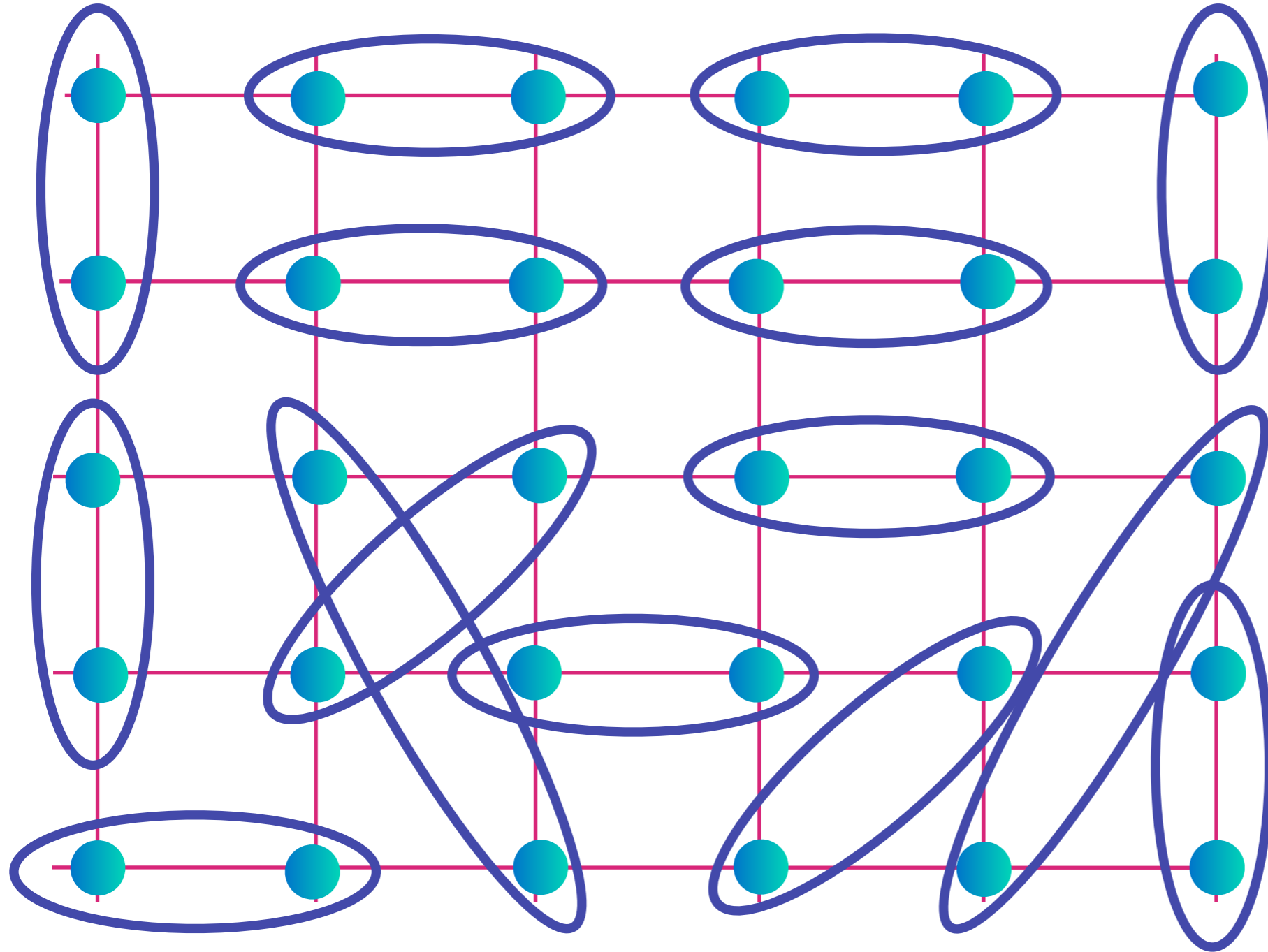
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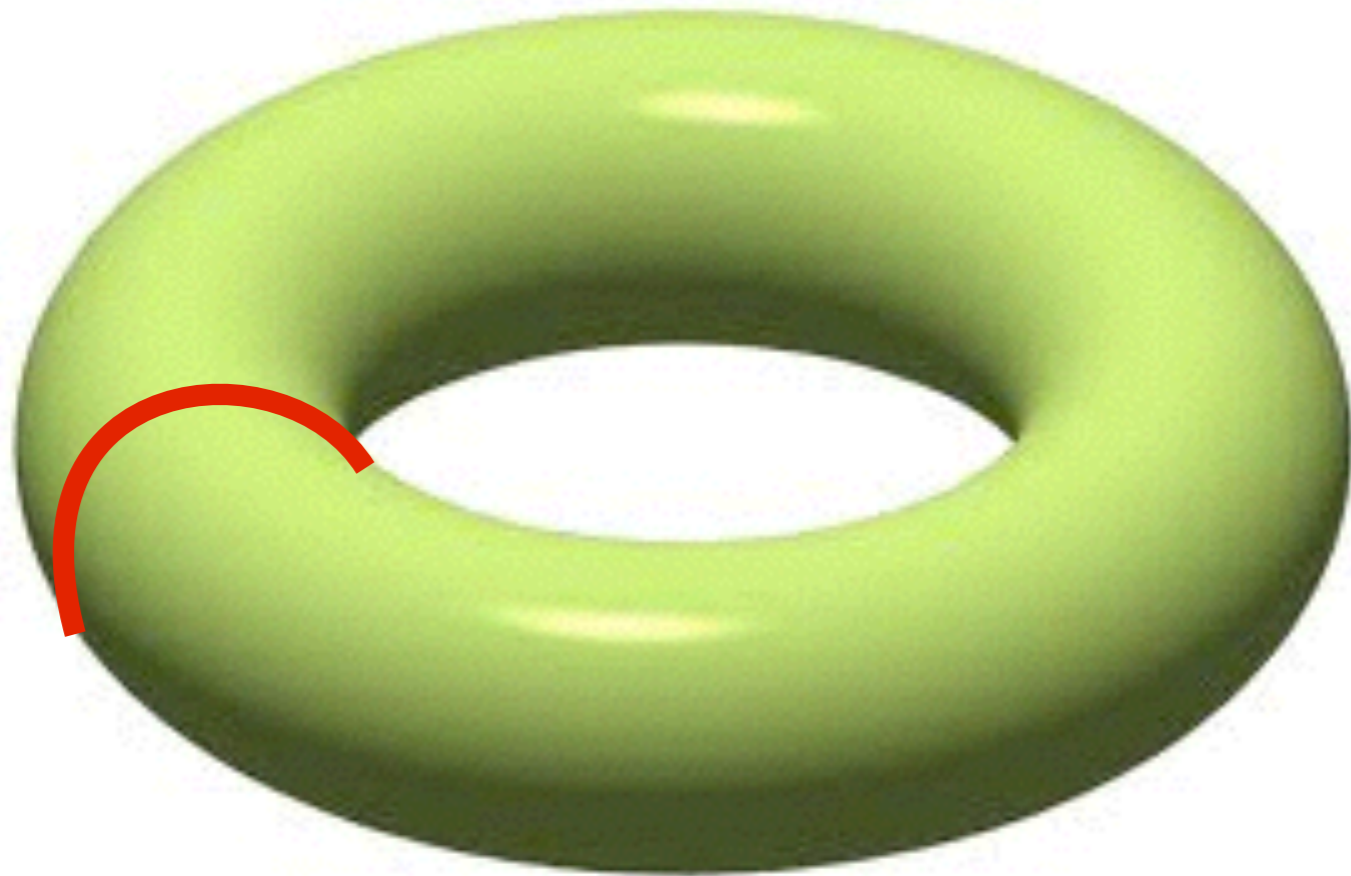
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Ground state degeneracy

Place
insulator
on a torus;



Ground state degeneracy




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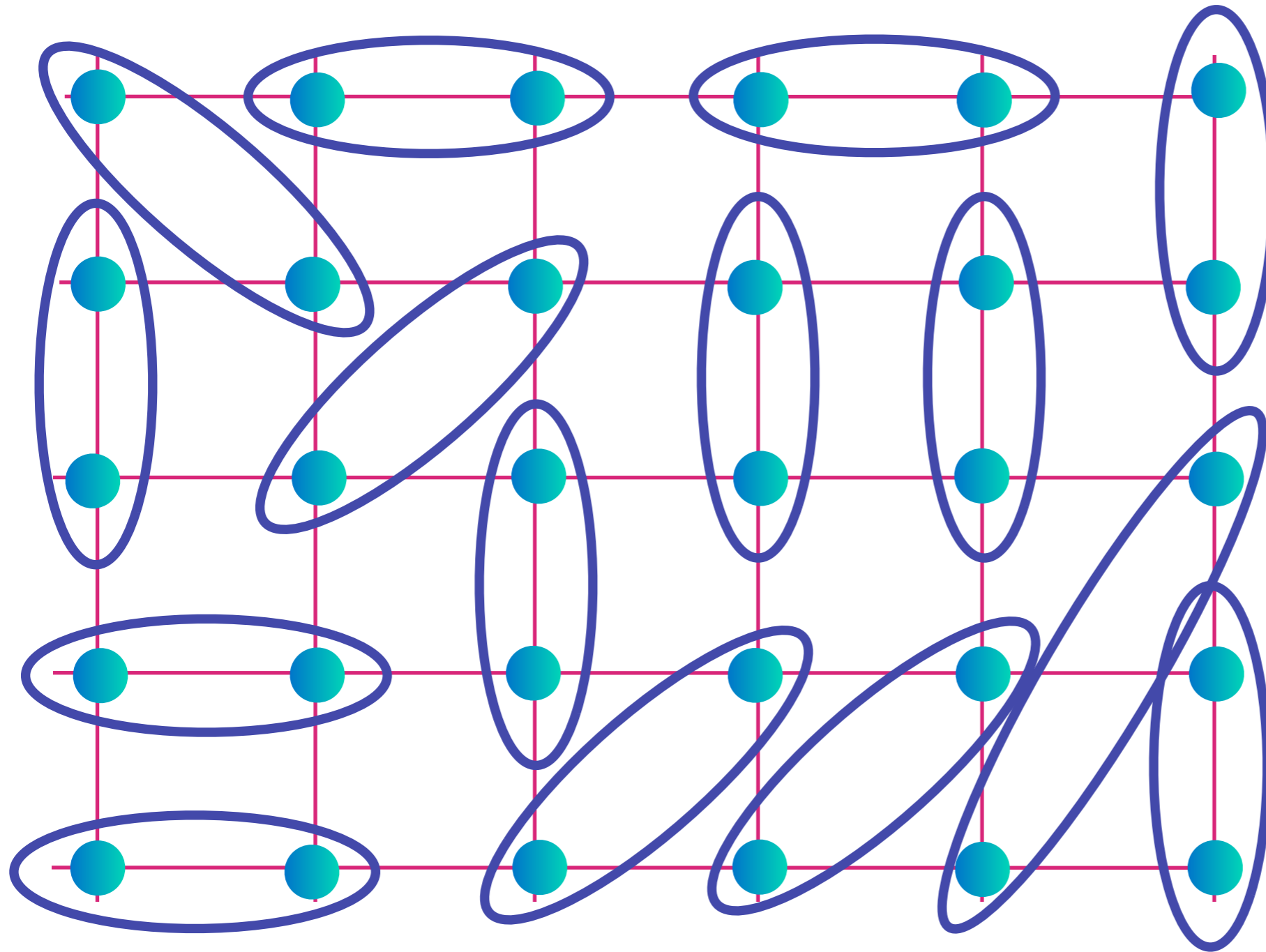
obtain
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states nearly
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number of
dimers crossing
red line is
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D.J. Thouless, PRB 36, 7187 (1987)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. 6, 353 (1988)

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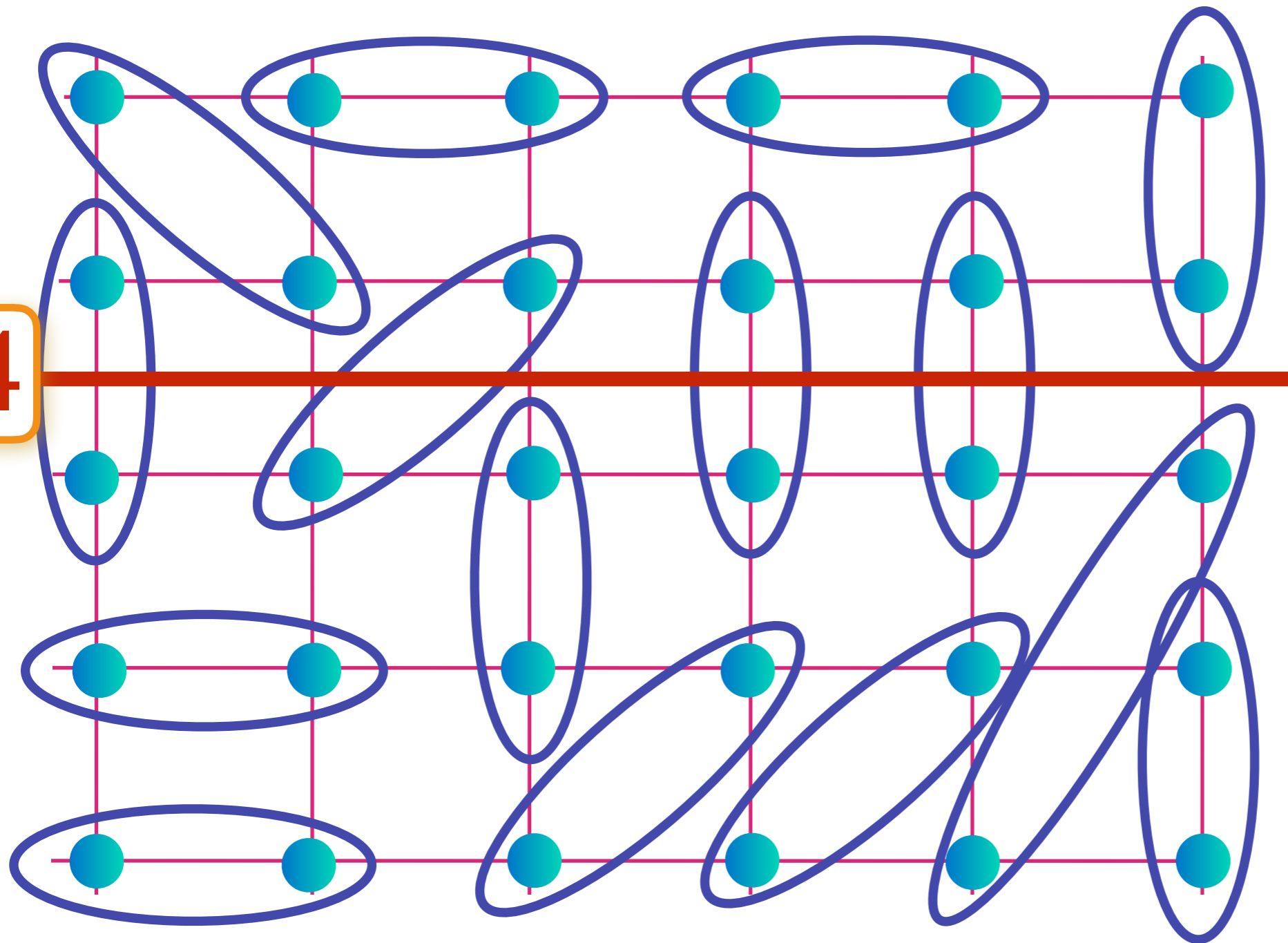
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$$\text{[Diagram of two teal dots in a blue oval]} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

4



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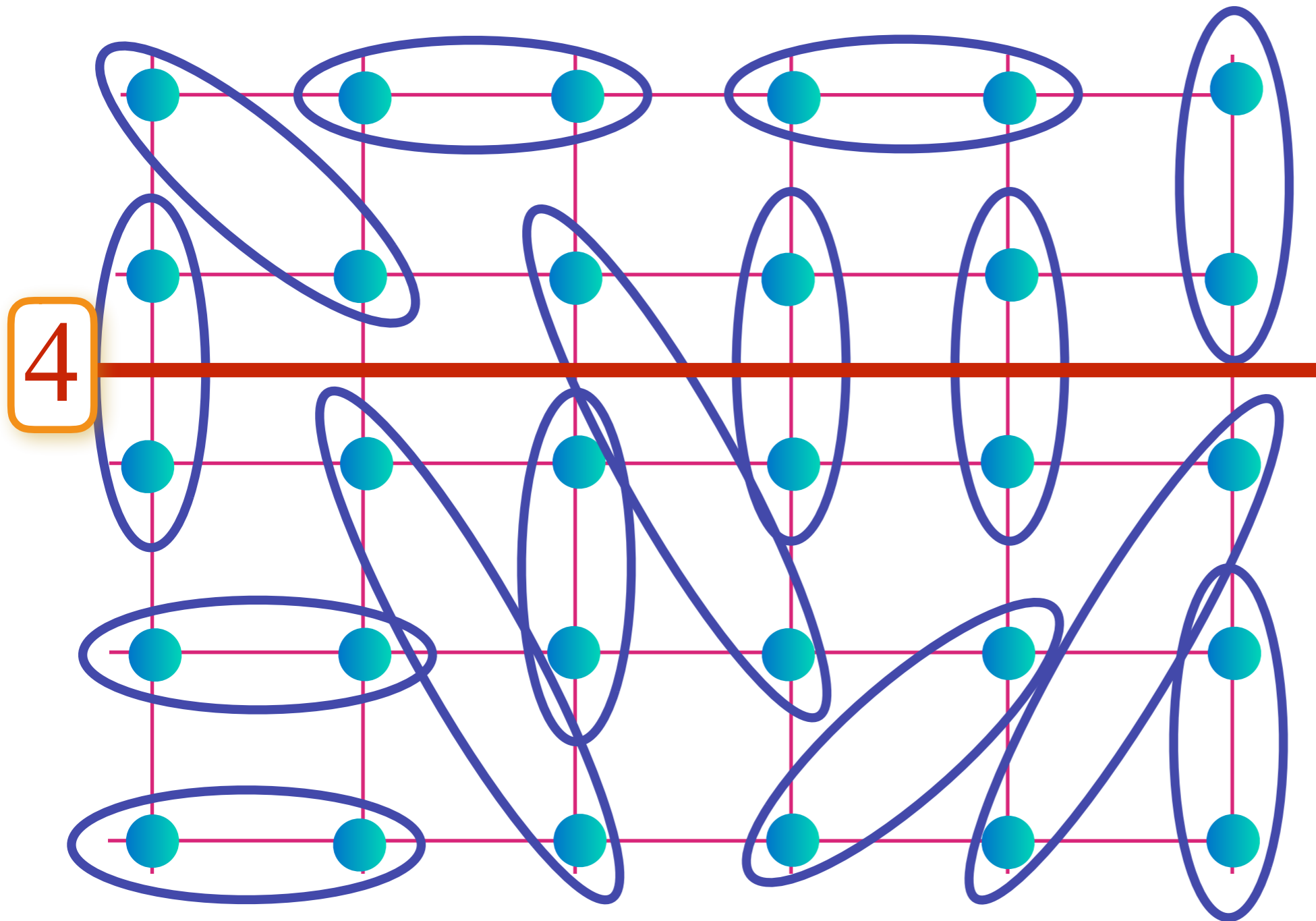
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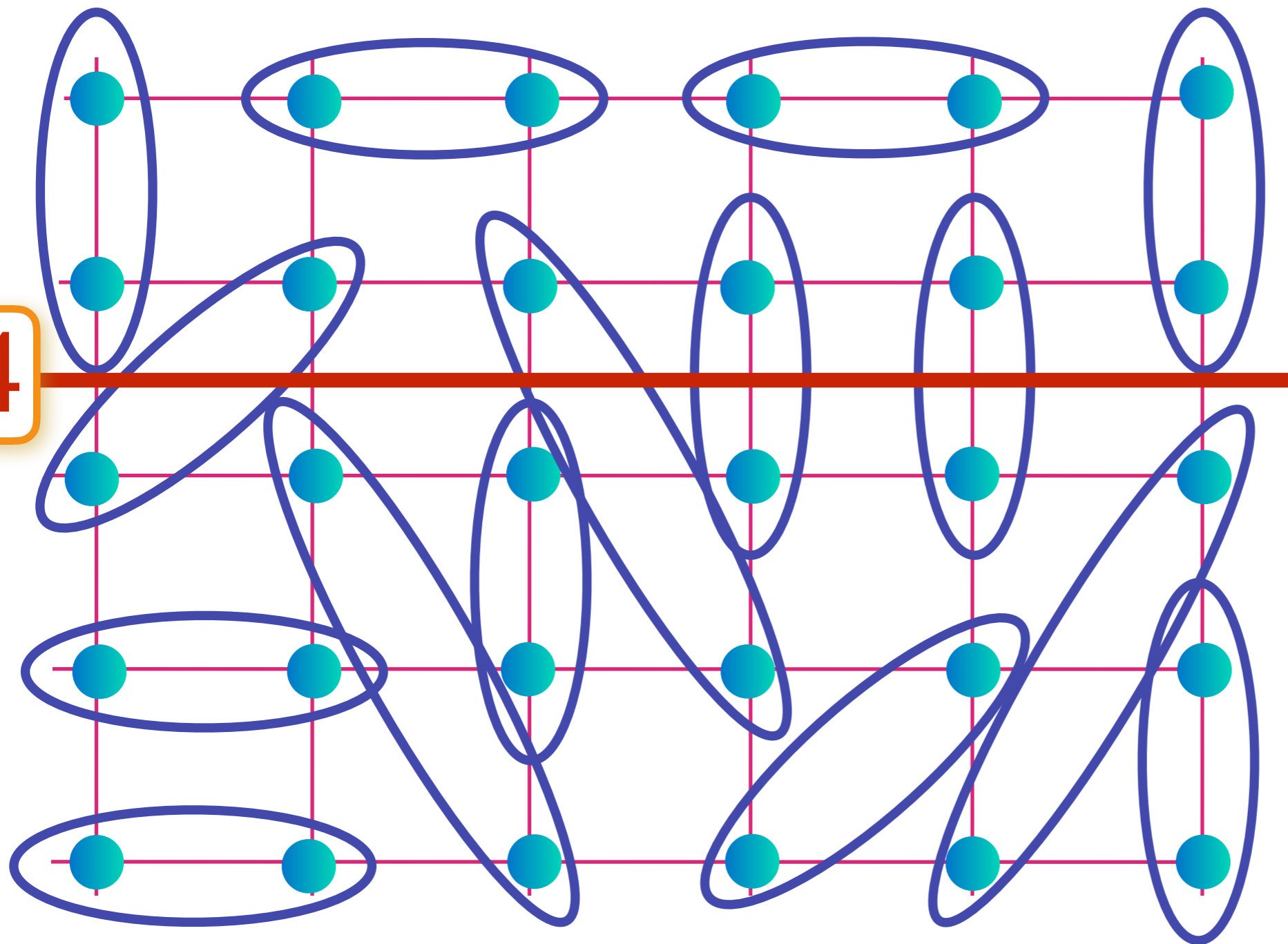
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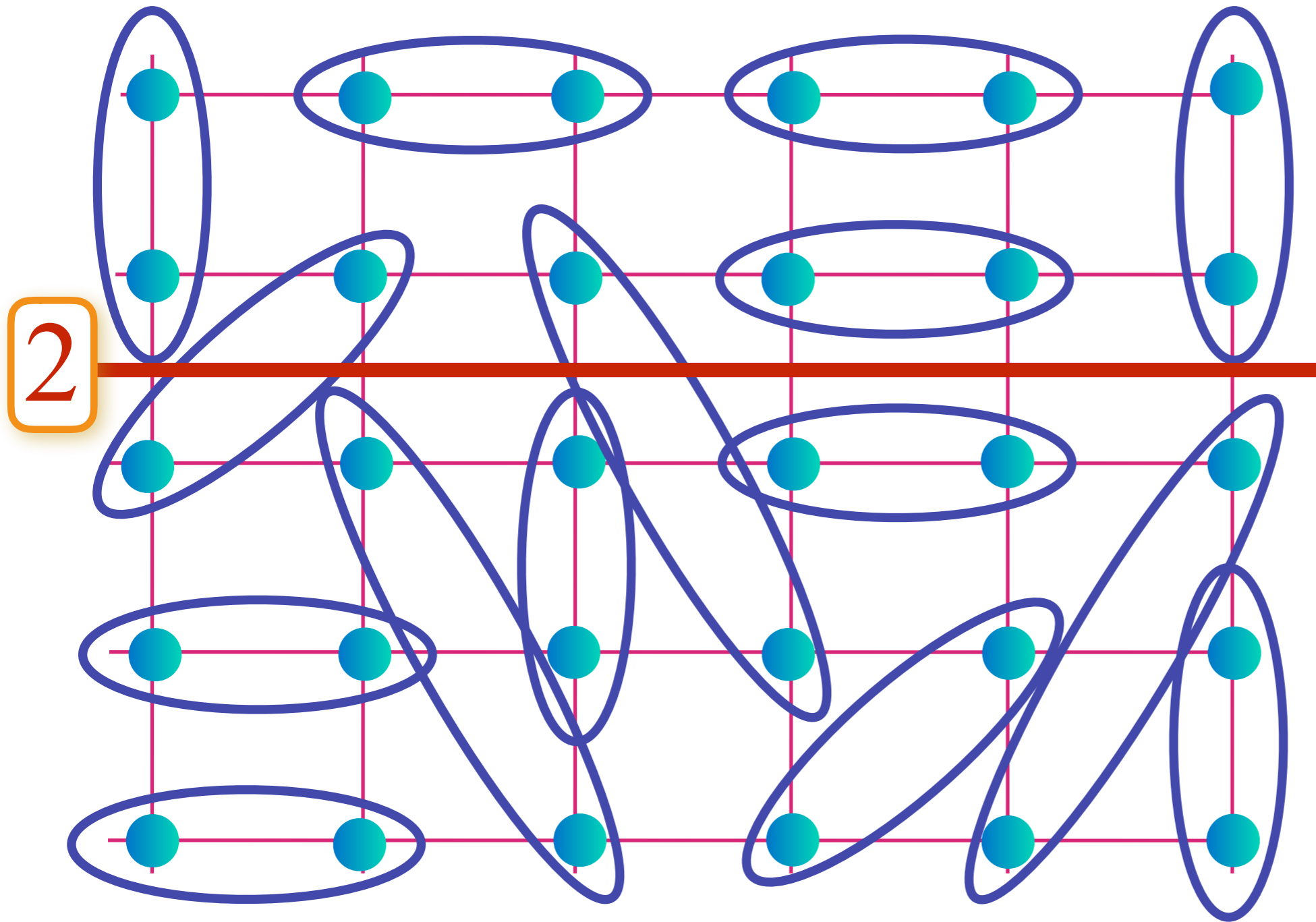
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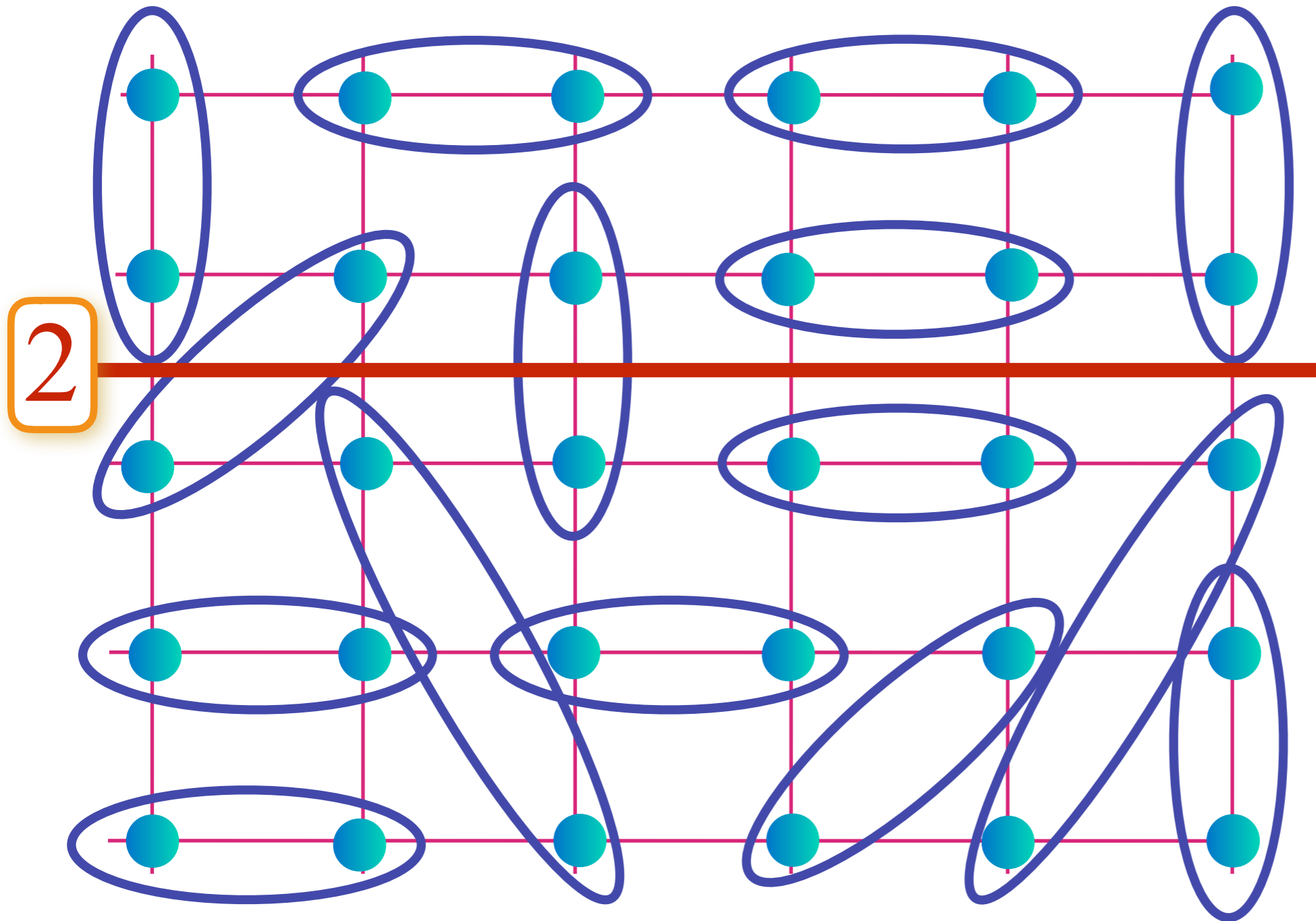
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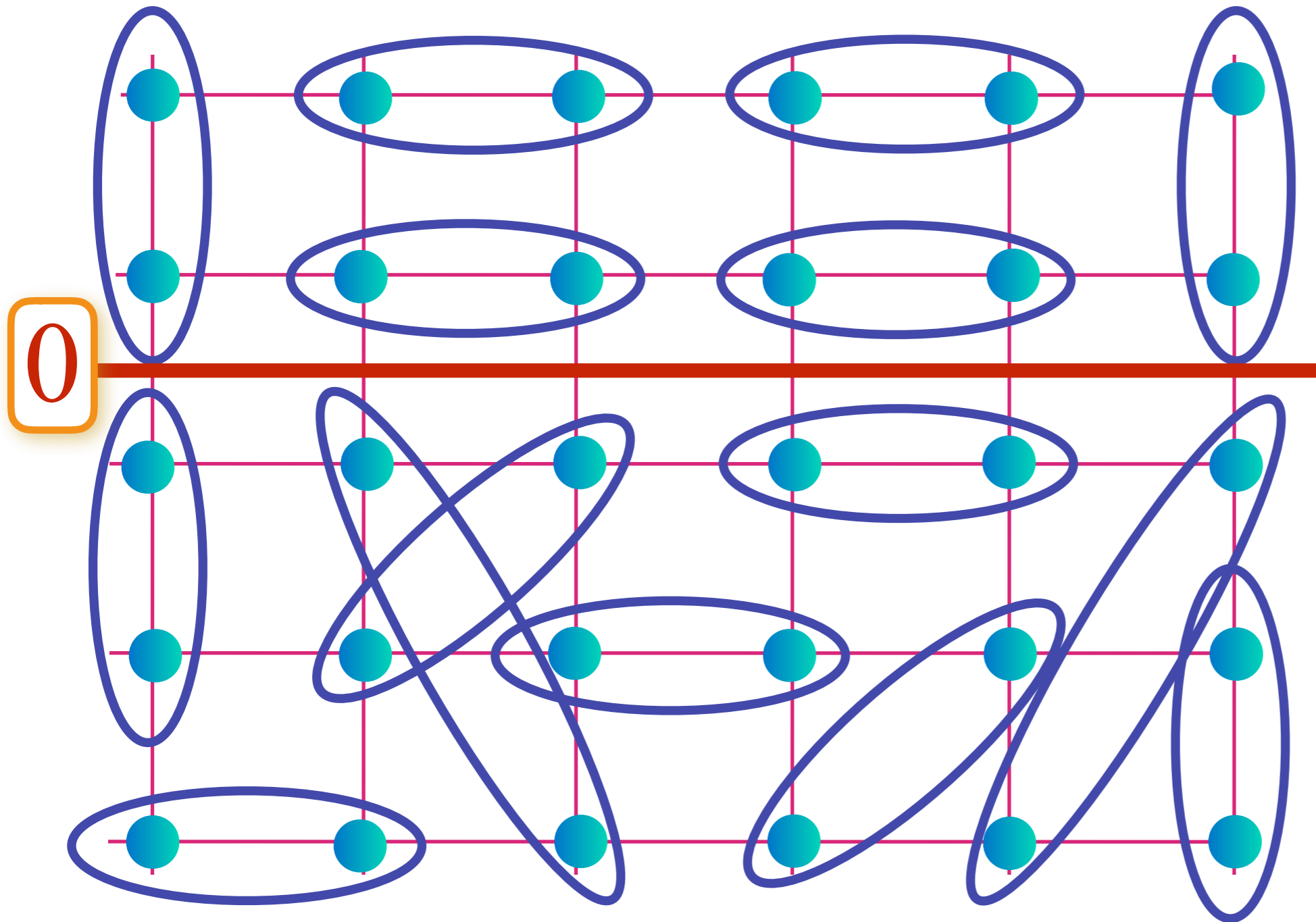
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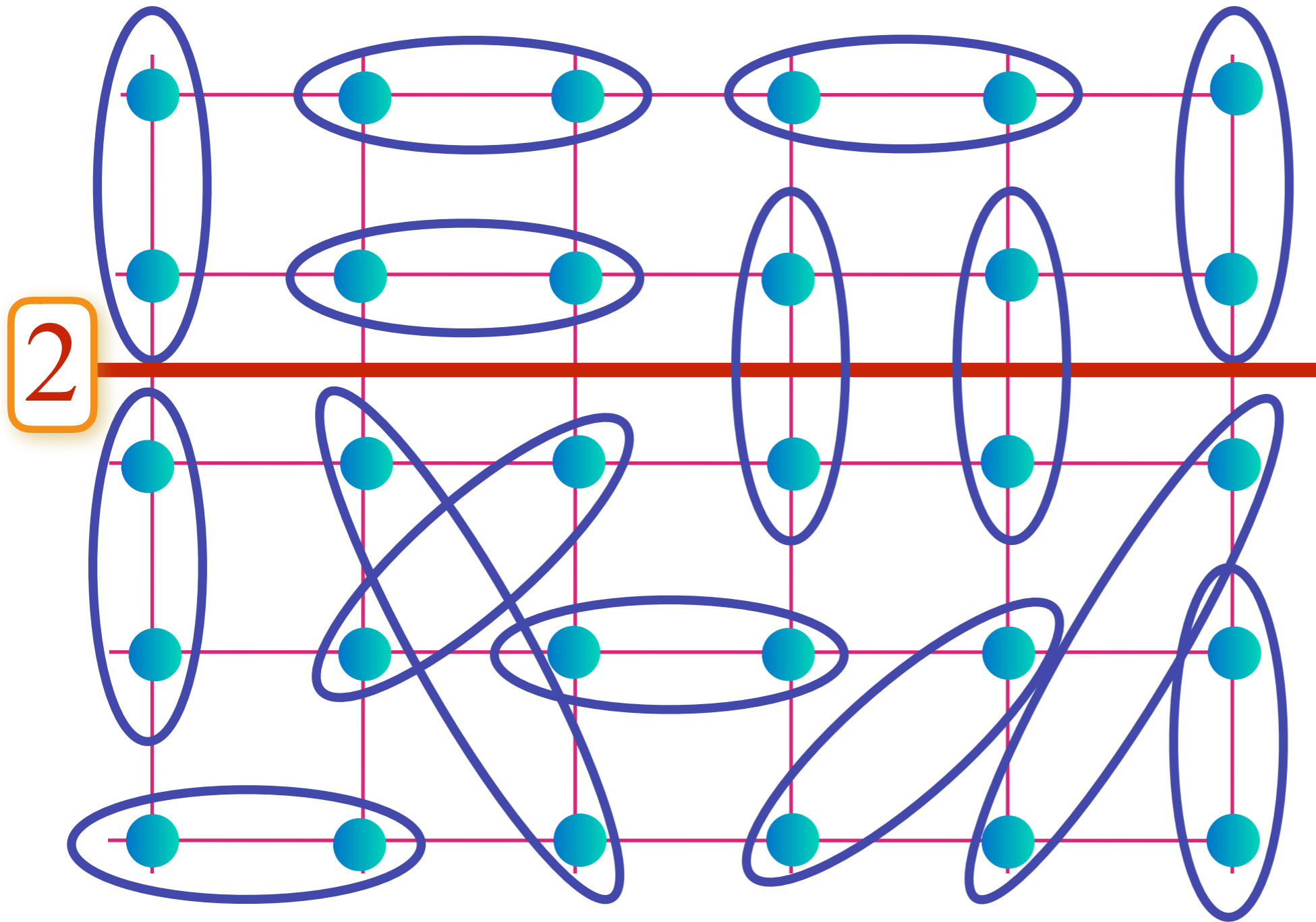
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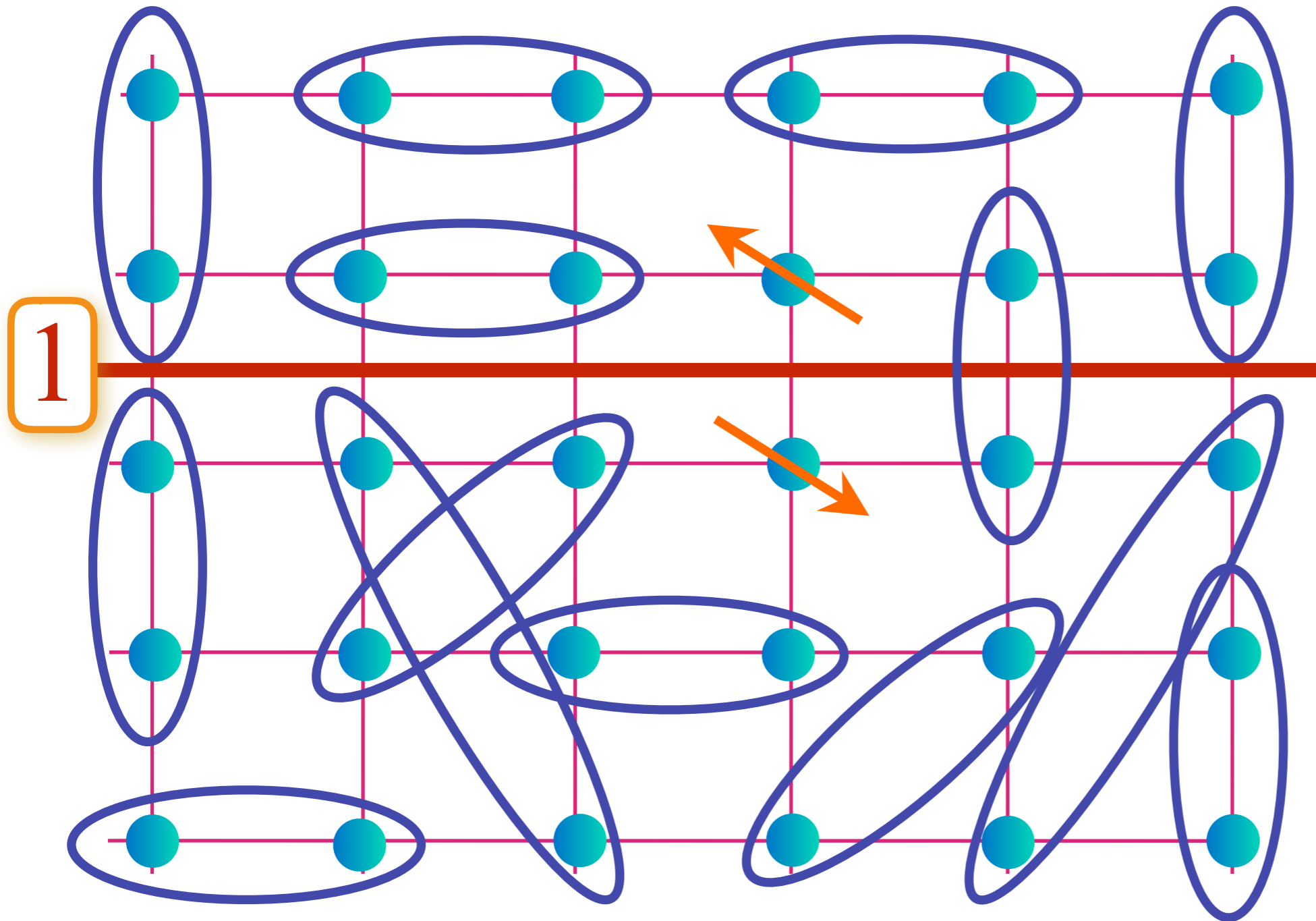
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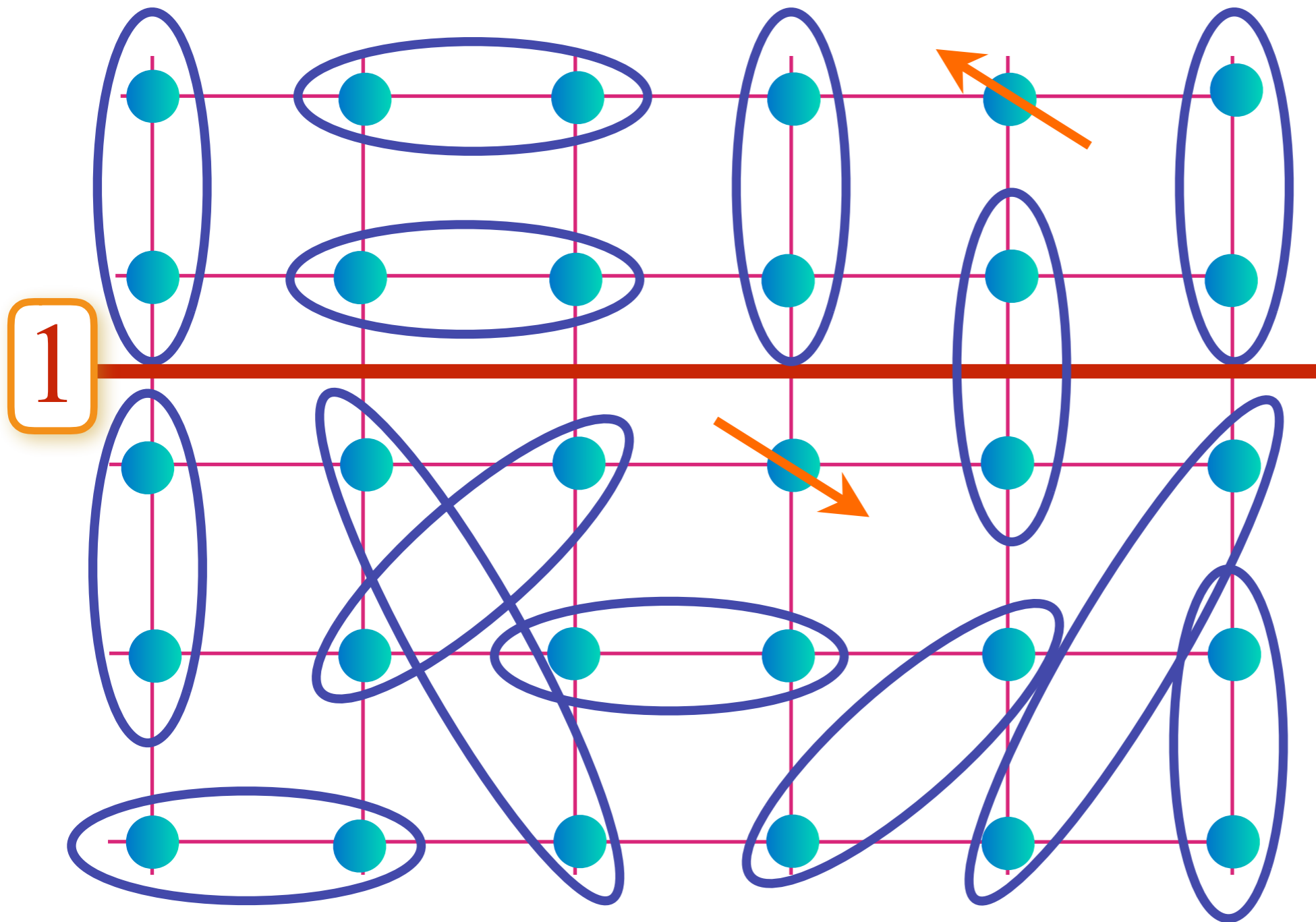
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number parity
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sample.

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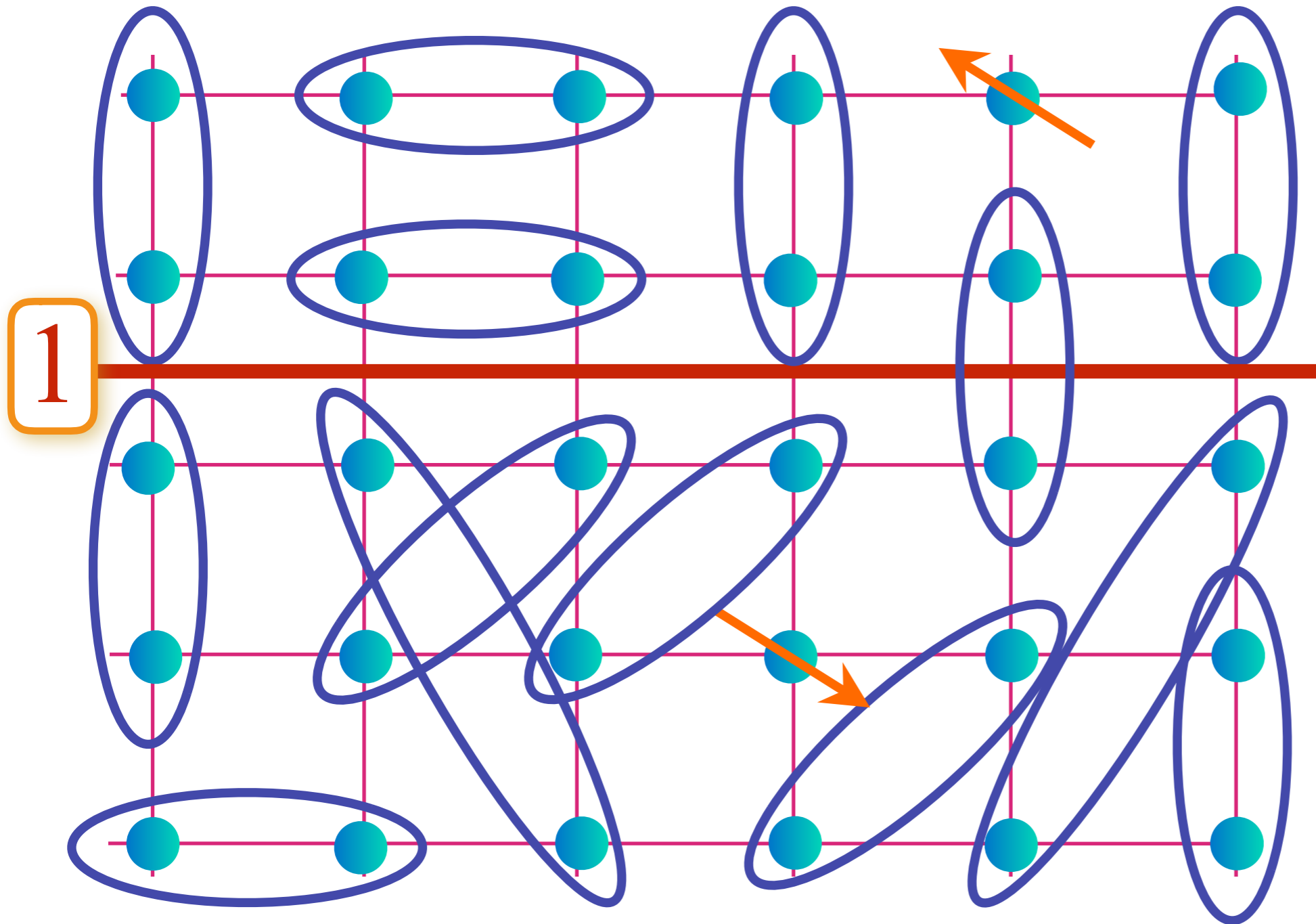
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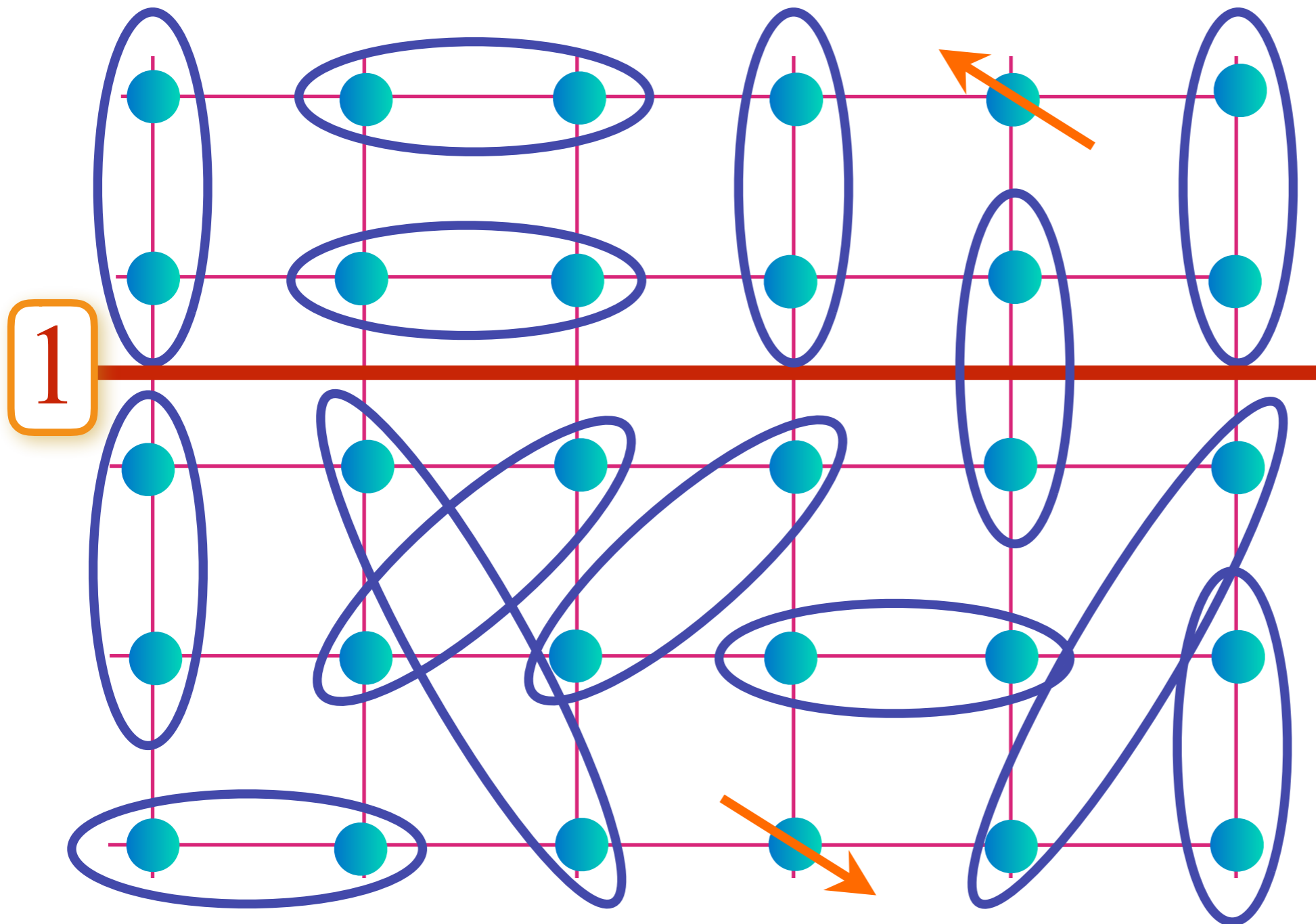
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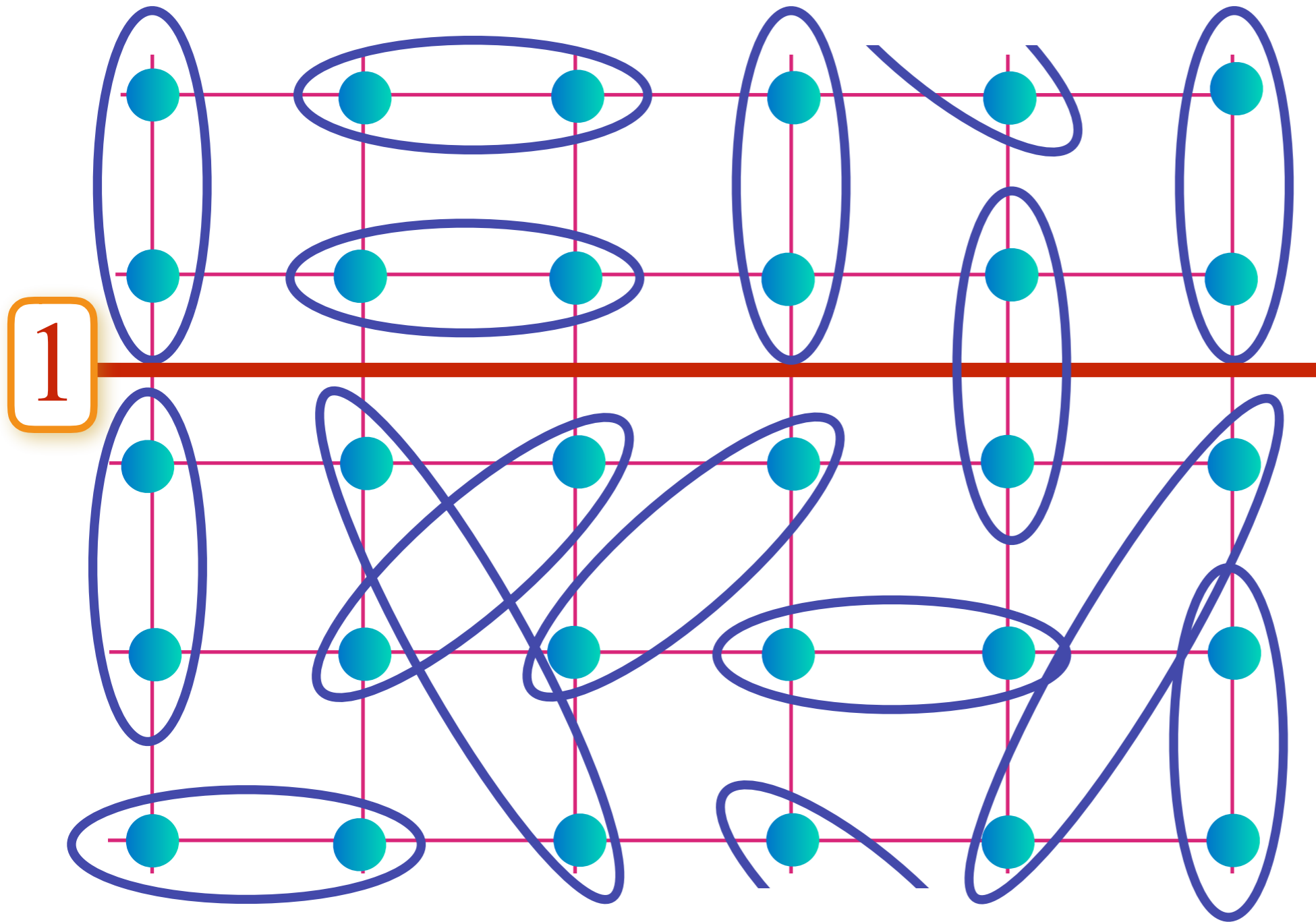
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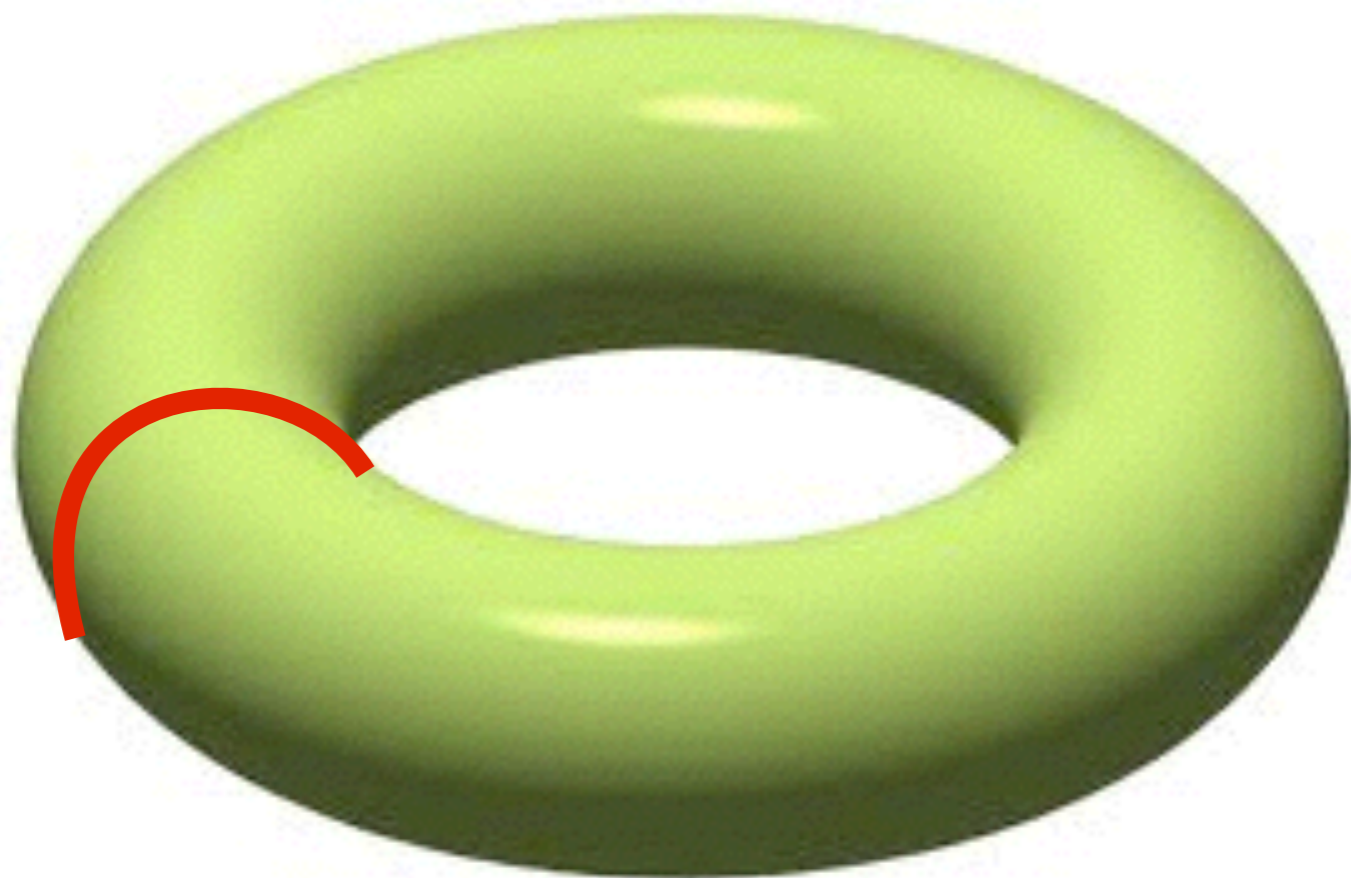
**Place
insulator
on a torus;**

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Ground state degeneracy



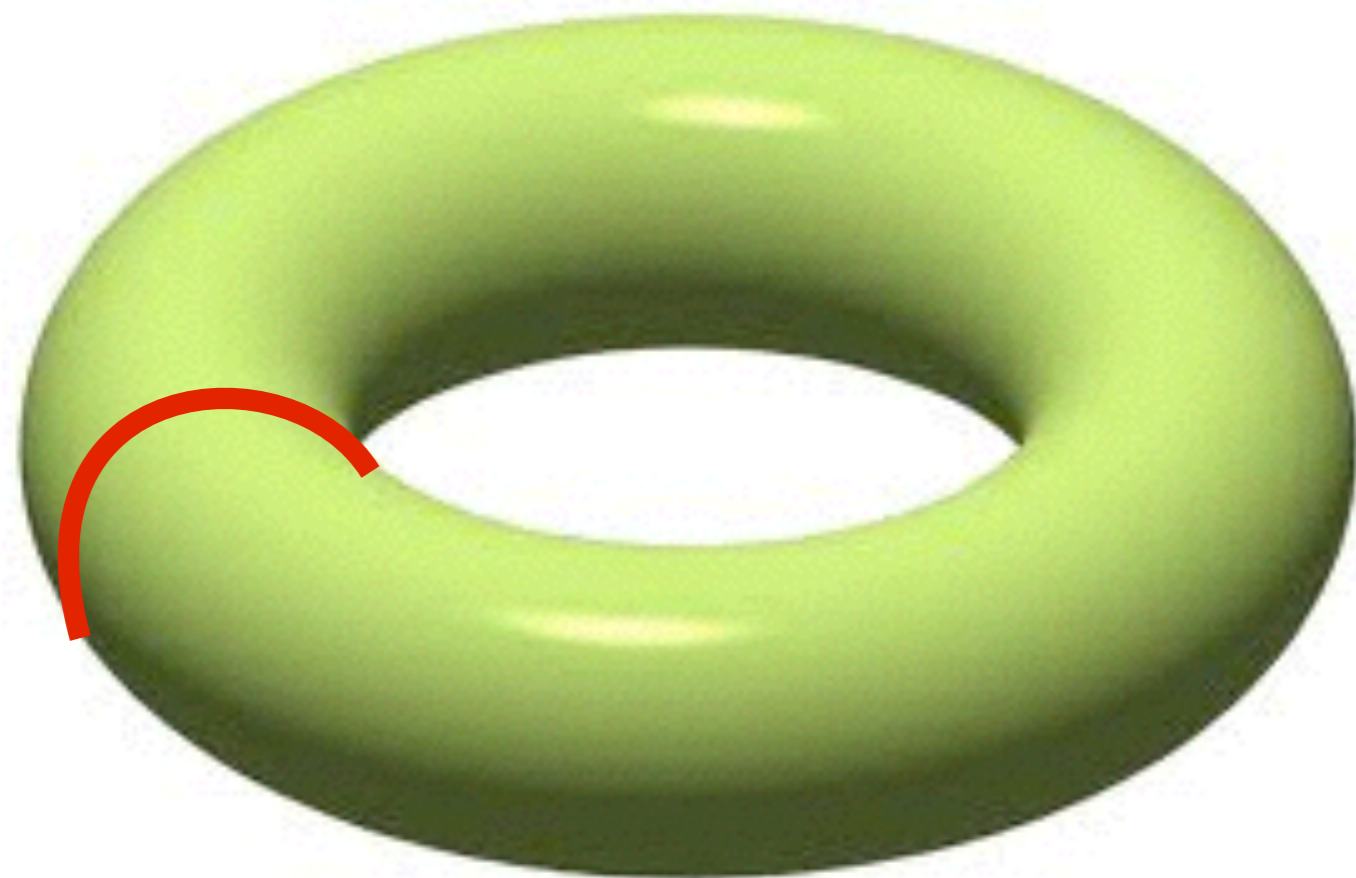
**Place
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The sensitivity
of the
degeneracy to
the global
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D.J. Thouless, PRB 36, 7187 (1987)

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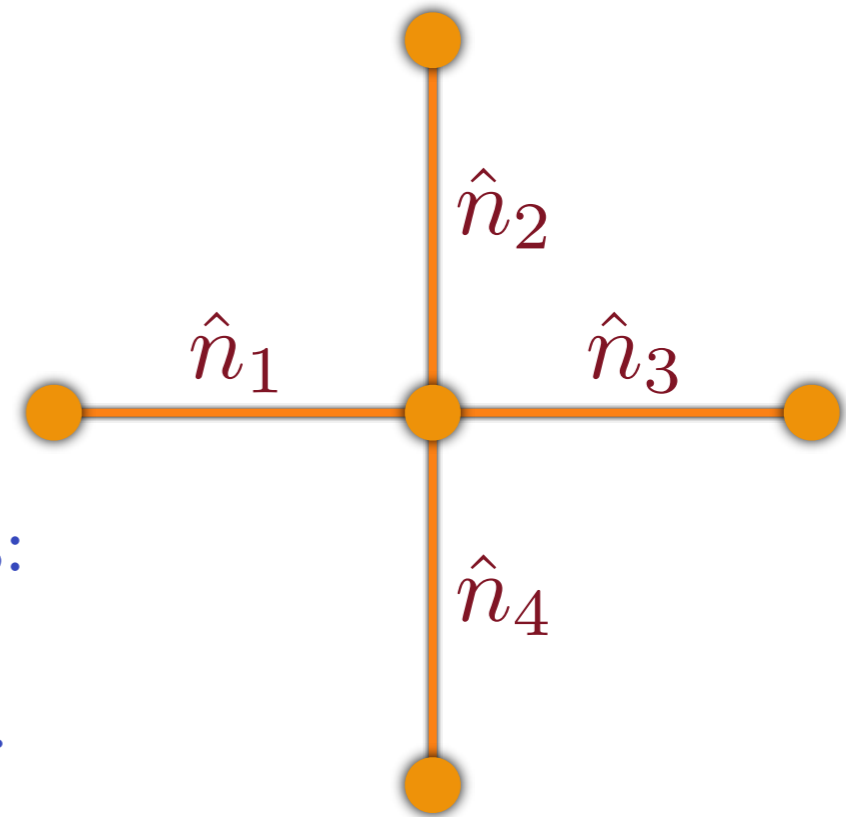


Place insulator on a torus;
The degenerate states are conjugate to the flux of an emergent gauge field piercing the cycles of the torus

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Emergent gauge fields



Local constraint on dimer number operators:

$$\hat{n}_1 + \hat{n}_2 + \hat{n}_3 + \hat{n}_4 = 1.$$

Identify dimer number with an ‘electric’ field, $\hat{E}_{i\alpha} = (-1)^{i_x+i_y} \hat{n}_{i\alpha}$, ($\alpha = x, y$); the constraint becomes ‘Gauss’s Law’:

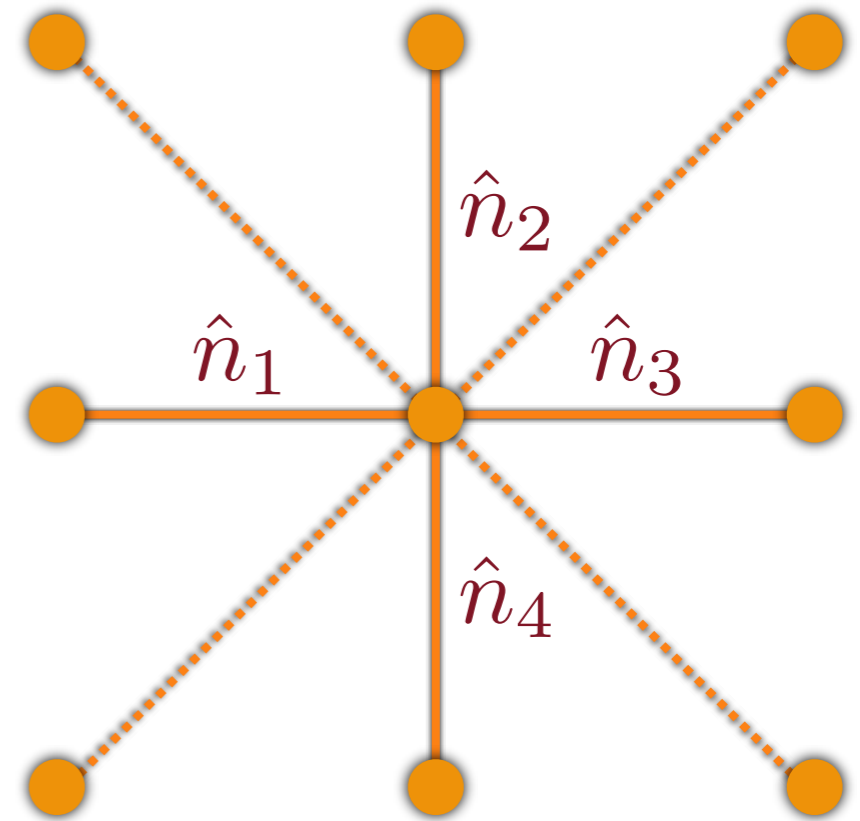
$$\Delta_\alpha \hat{E}_{i\alpha} = (-1)^{i_x+i_y}.$$

The theory of the dimers is *compact* U(1) quantum electrodynamics in the presence of static background charges. The *compact* theory allows the analog of Dirac’s magnetic monopoles as tunneling events/excitations.

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

E. Fradkin and S. A. Kivelson, Mod. Phys. Lett. B **4**, 225 (1990)

Emergent gauge fields



Including dimers connecting the same sublattice leads to a \mathbb{Z}_2 gauge theory in the presence of Berry phases of static background charges. This has a stable deconfined phase in 2+1 dimensions. By varying parameters it can undergoes a confinement transition to a valence bond solid, described by a frustrated Ising model.

R.A. Jalabert and S. Sachdev, Phys. Rev. B **44**, 686 (1991)

S. Sachdev and M.Vojta, J. Phys. Soc. Jpn **69**, Supp. B, 1 (1999)

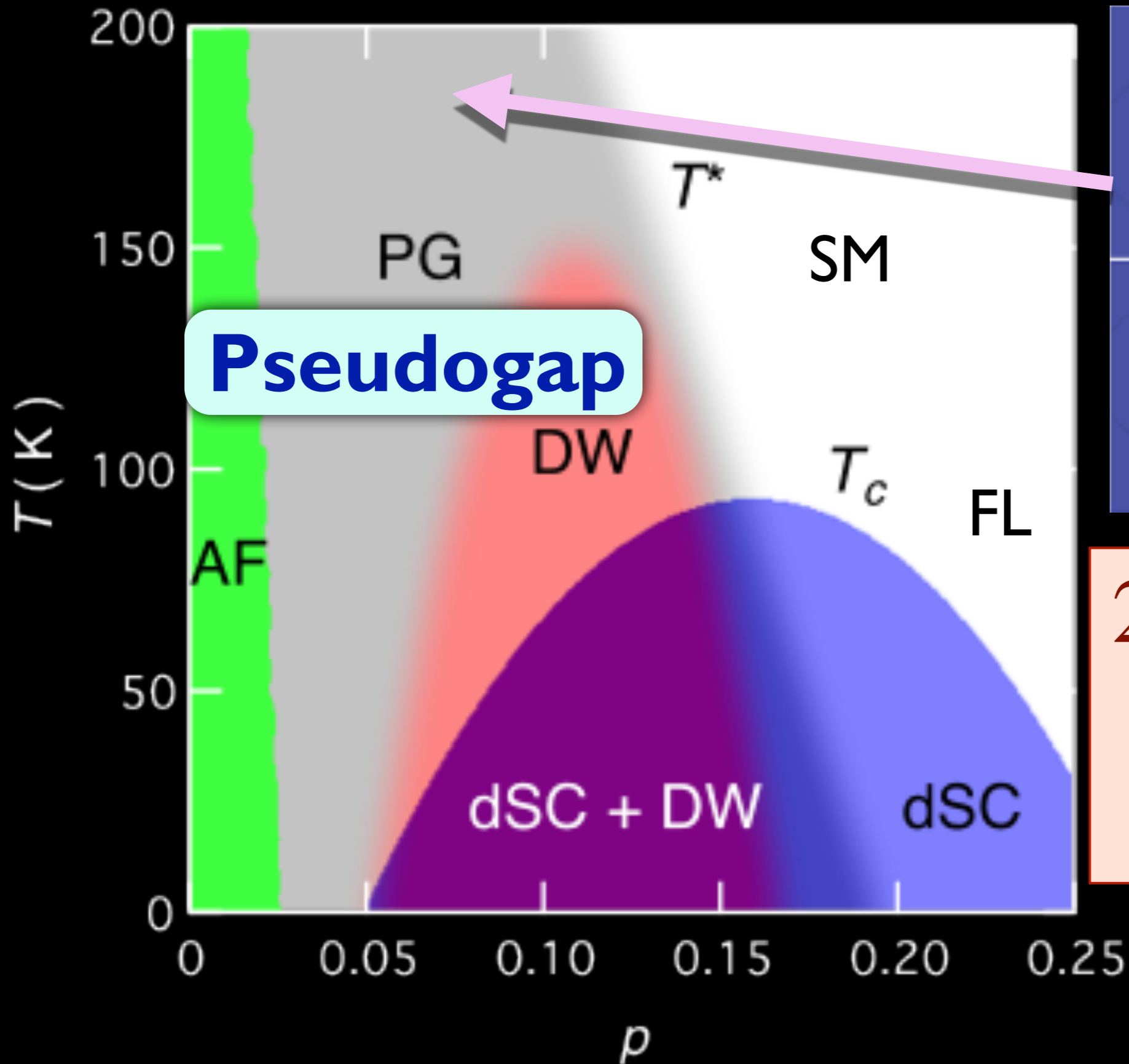
I. Emergent gauge fields and long-range entanglement in insulators

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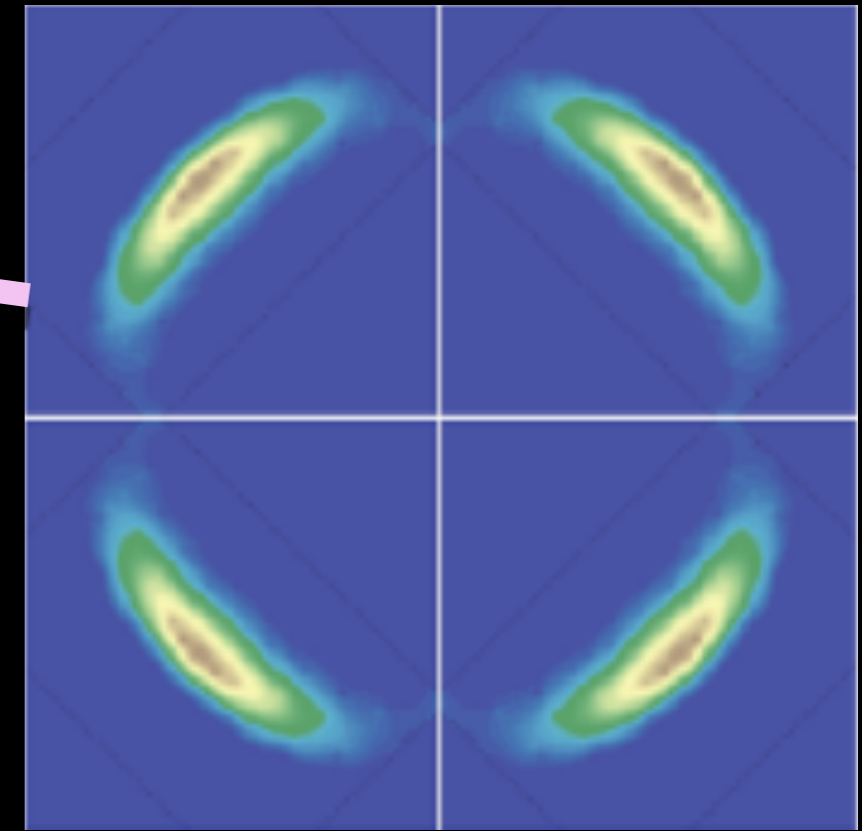
2. Fractionalized Fermi liquids (FL*)

*Quasiparticles with a non-Luttinger volume,
and emergent gauge fields*

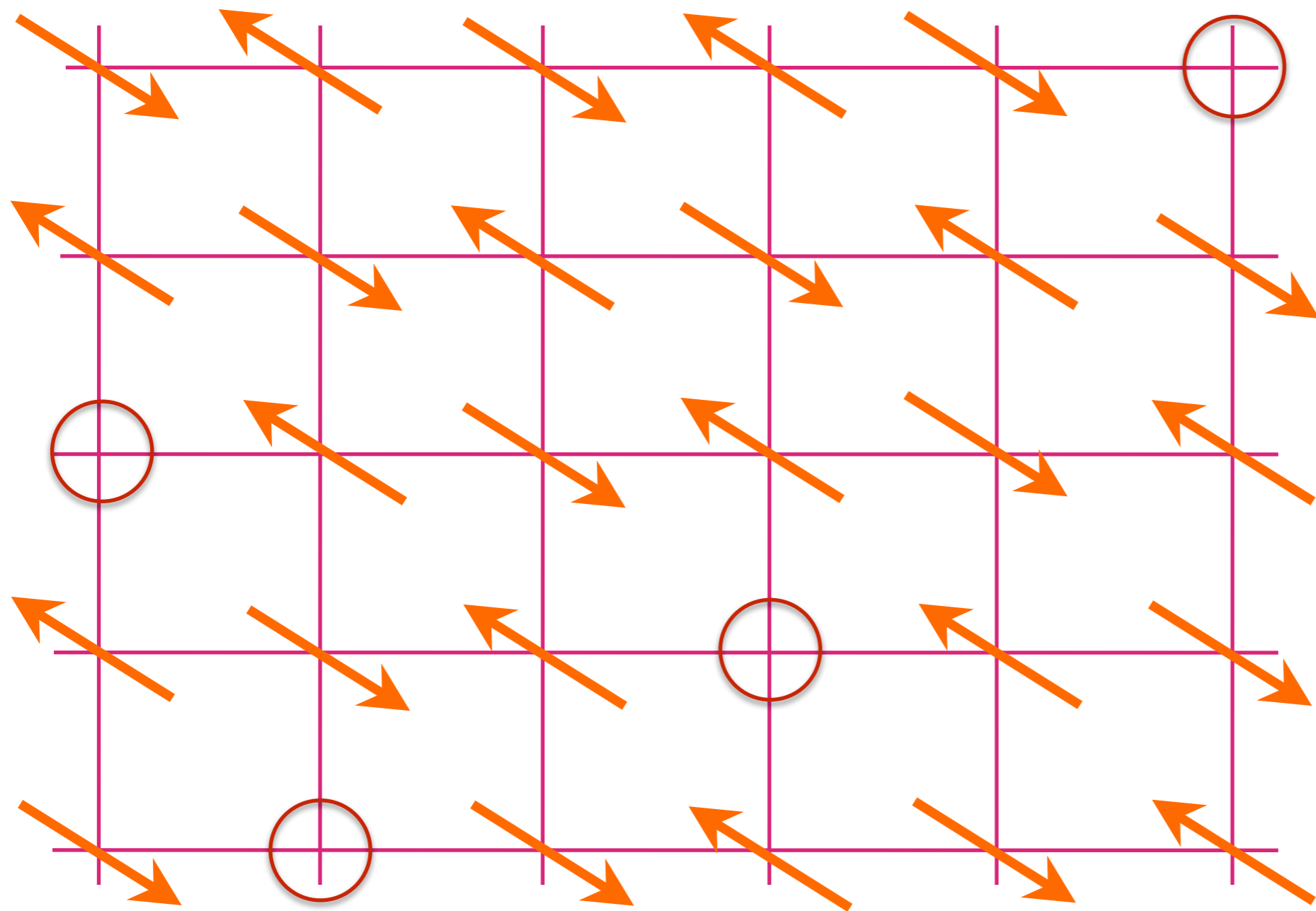
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



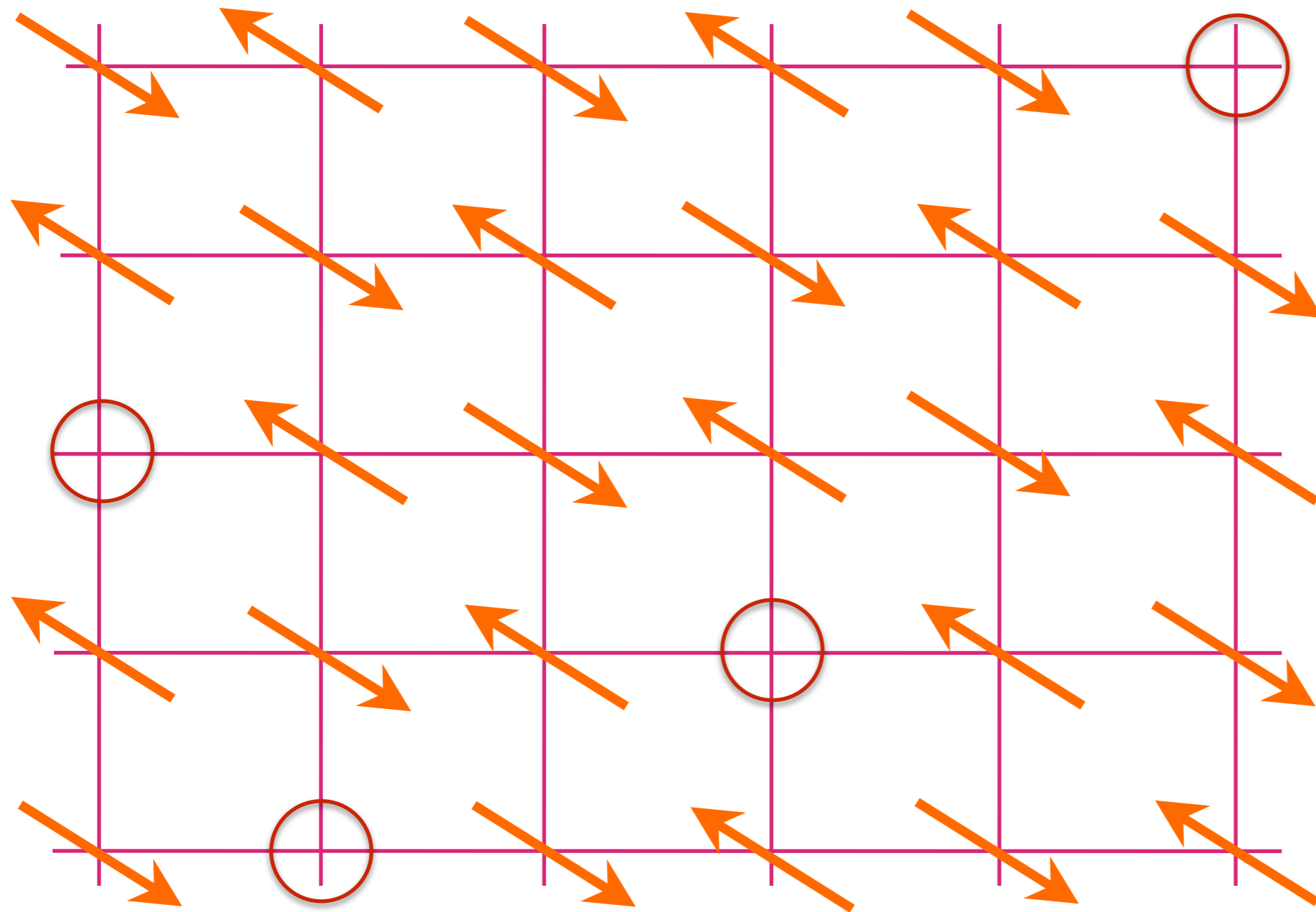
Pseudogap



2. Pseudogap
metal
at low p



Anti-ferromagnet
with p holes
per square

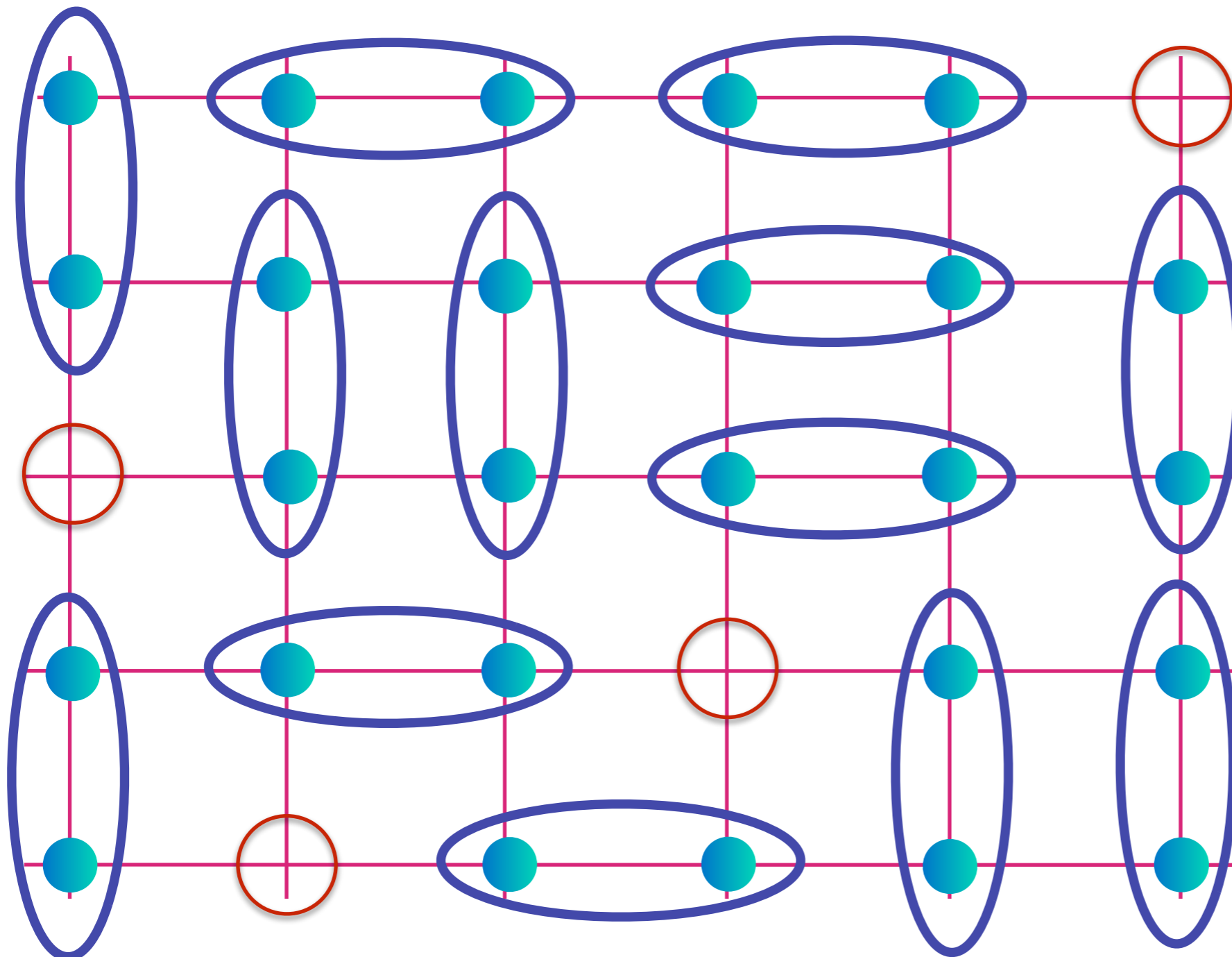


Anti-ferromagnet with p holes per square

Can we get a Fermi surface of size p ?
(and full square lattice symmetry)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB 35, 8865 (1987)

N. Read and B. Chakraborty, PRB 40, 7133 (1989)

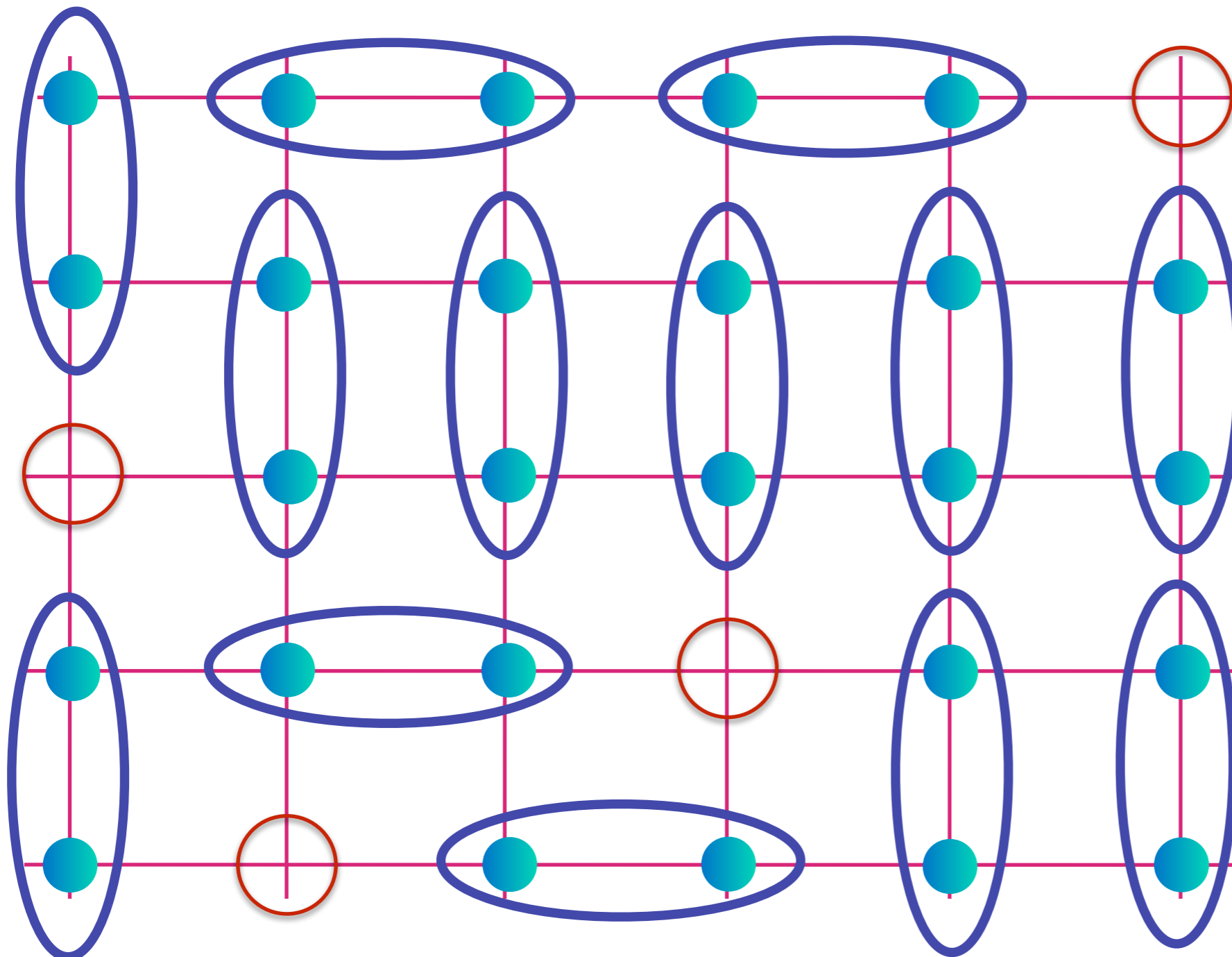


Spin liquid with density ρ of spinless, charge $+e$ "holons". These can form a Fermi surface of size ρ , but this is not visible in electron photo-emission

$$\text{[Diagram of two red dots in a blue oval]} = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB 35, 8865 (1987)

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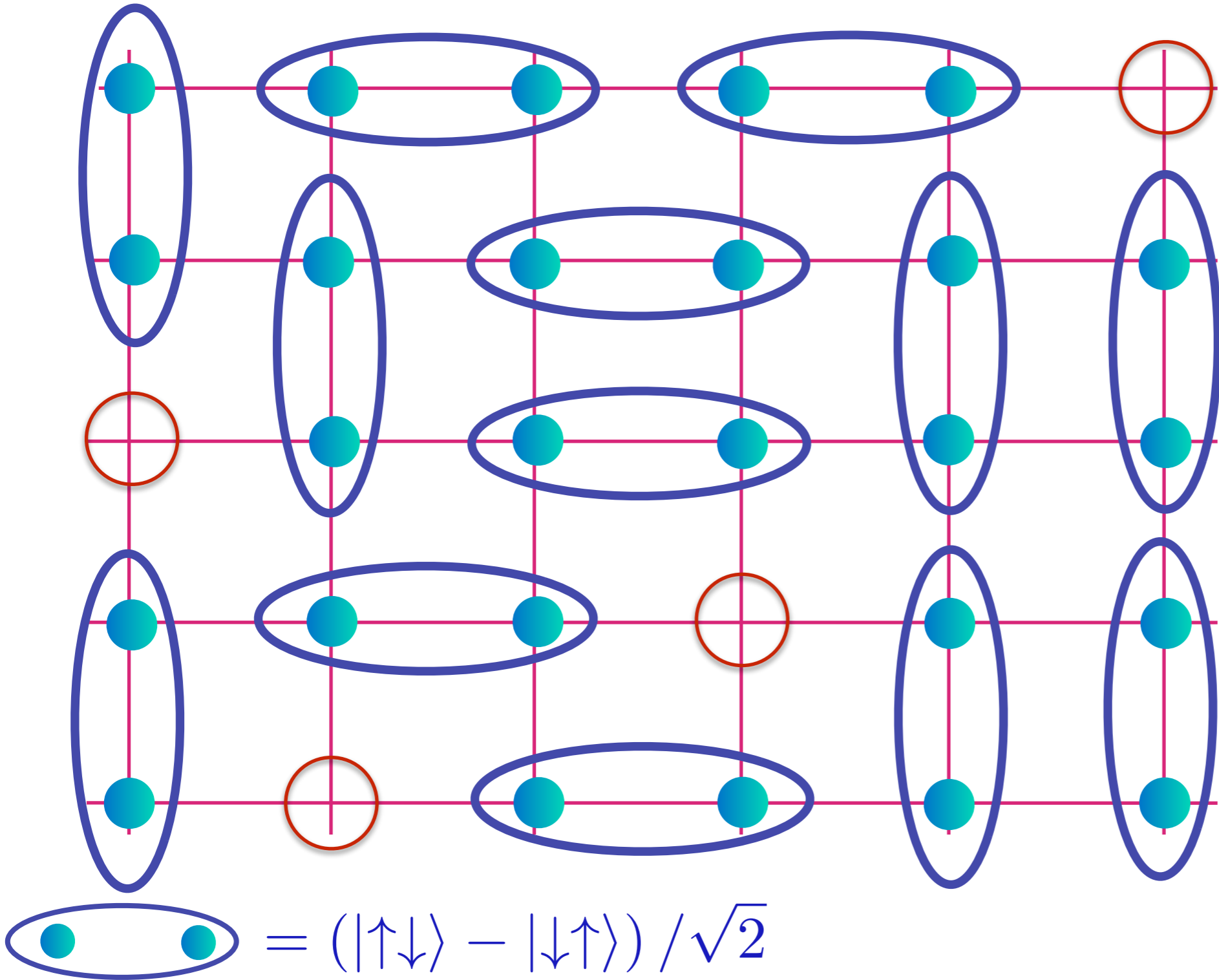


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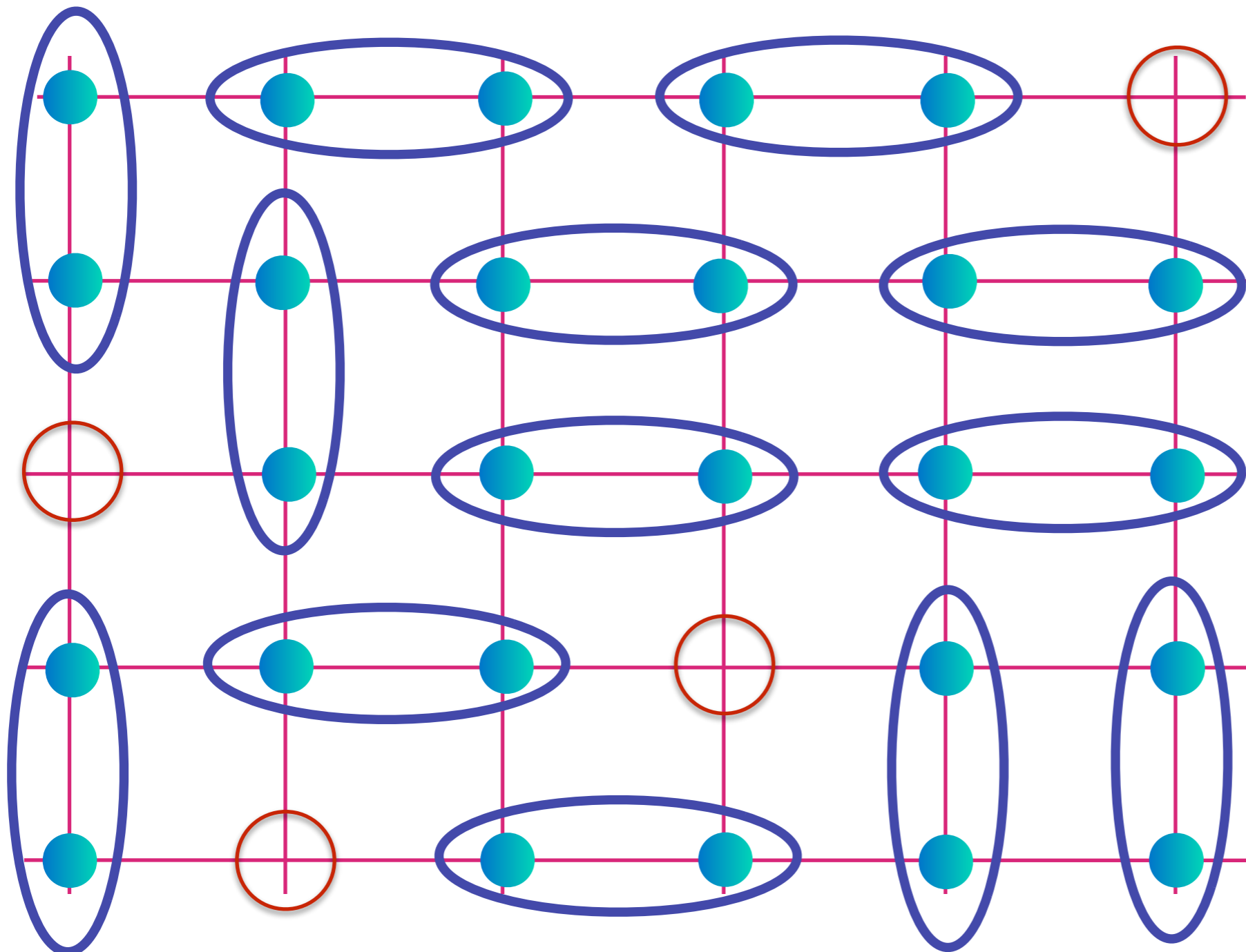
N. Read and B. Chakraborty, PRB 40, 7133 (1989)



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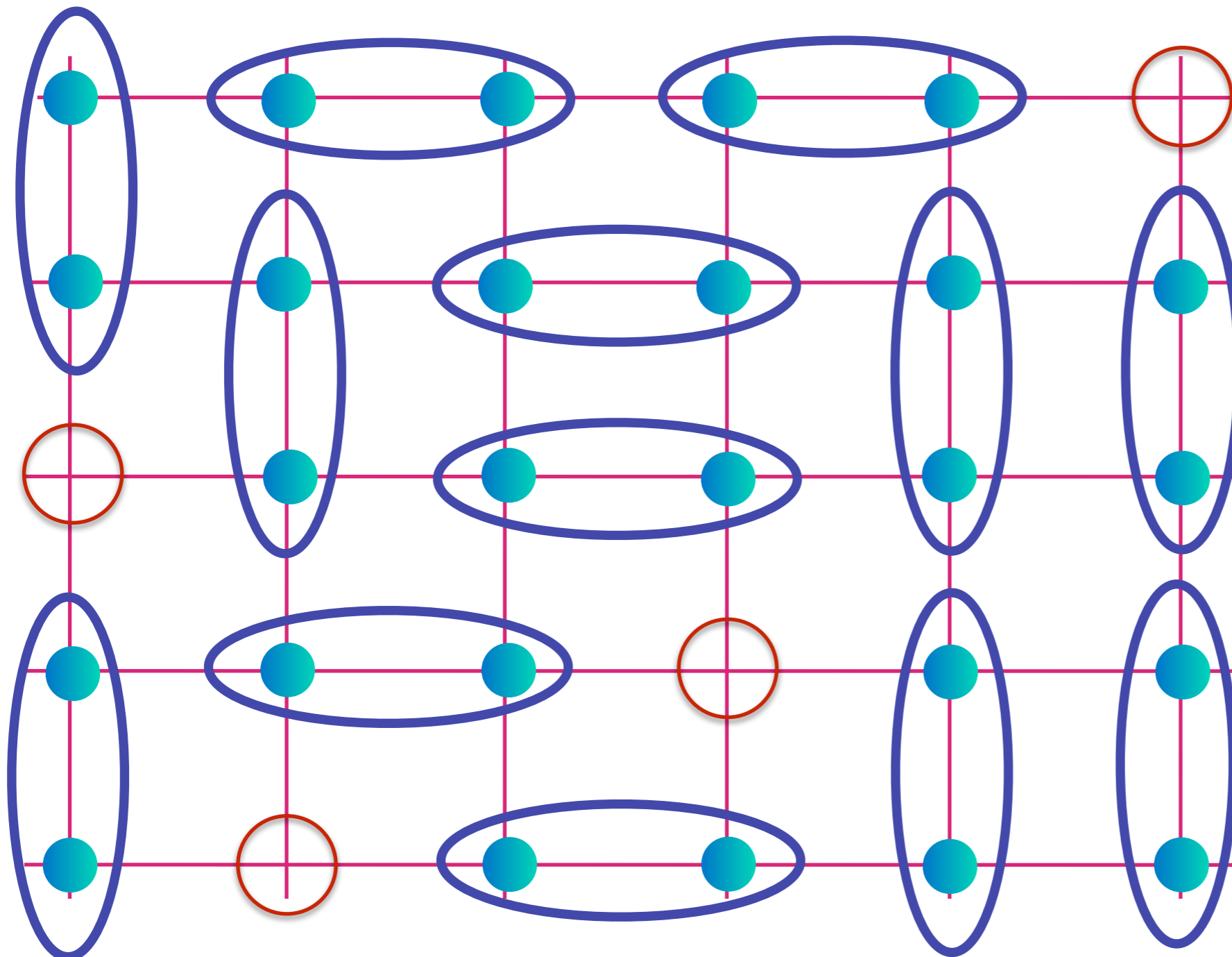


Spin liquid with density ρ of spinless, charge $+e$ "holons". These can form a Fermi surface of size ρ , but this is not visible in electron photo-emission

 = $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB 35, 8865 (1987)

N. Read and B. Chakraborty, PRB 40, 7133 (1989)

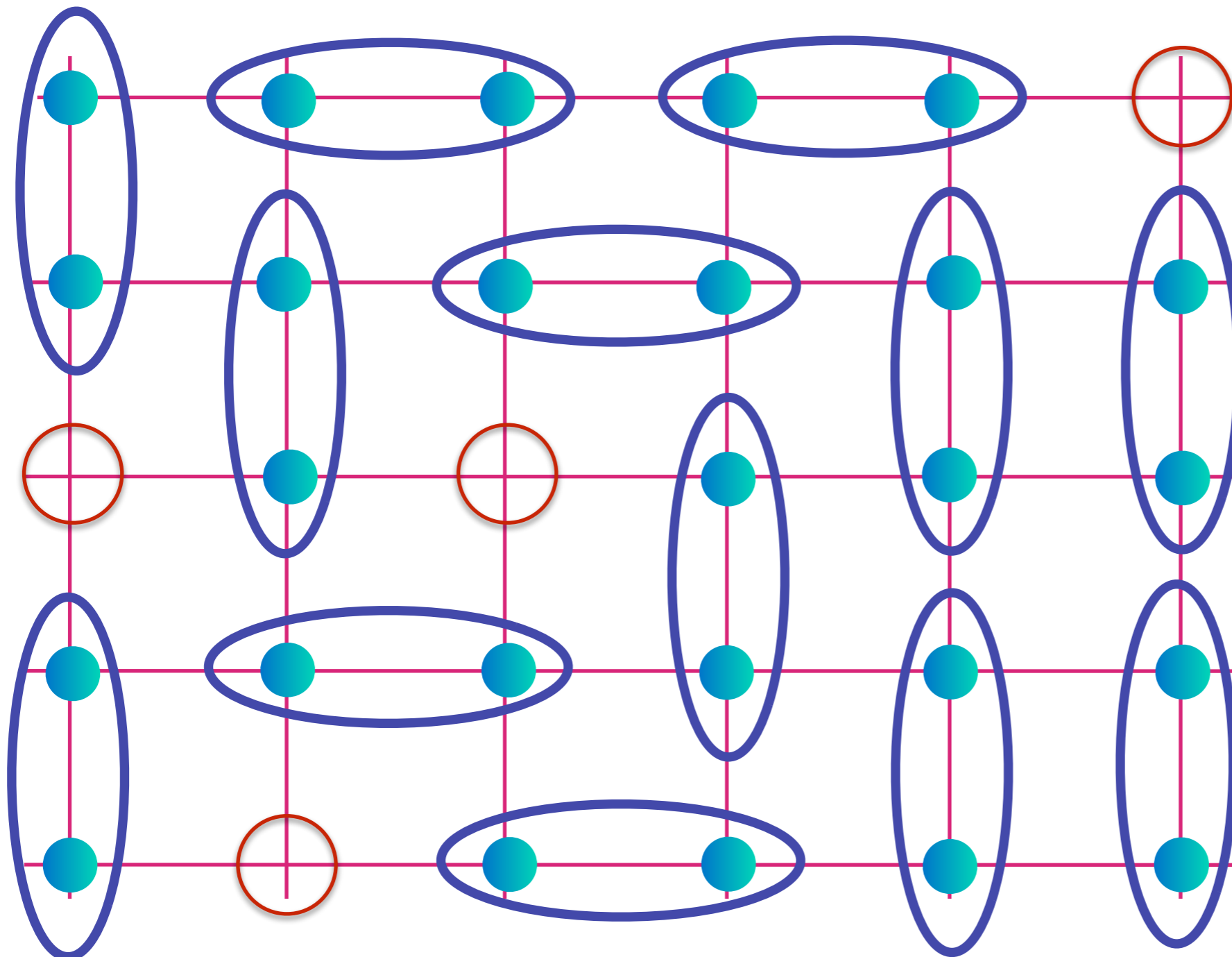


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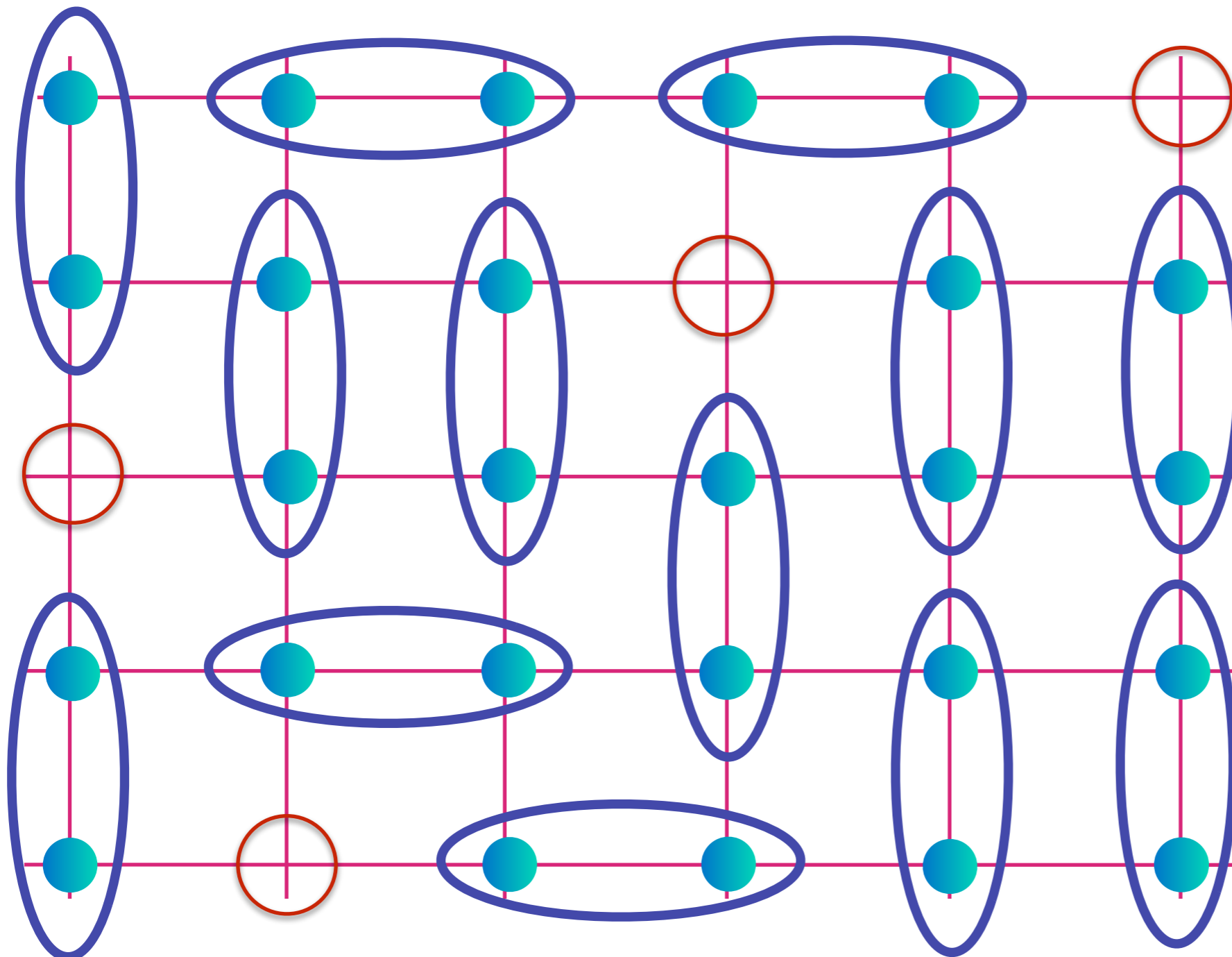


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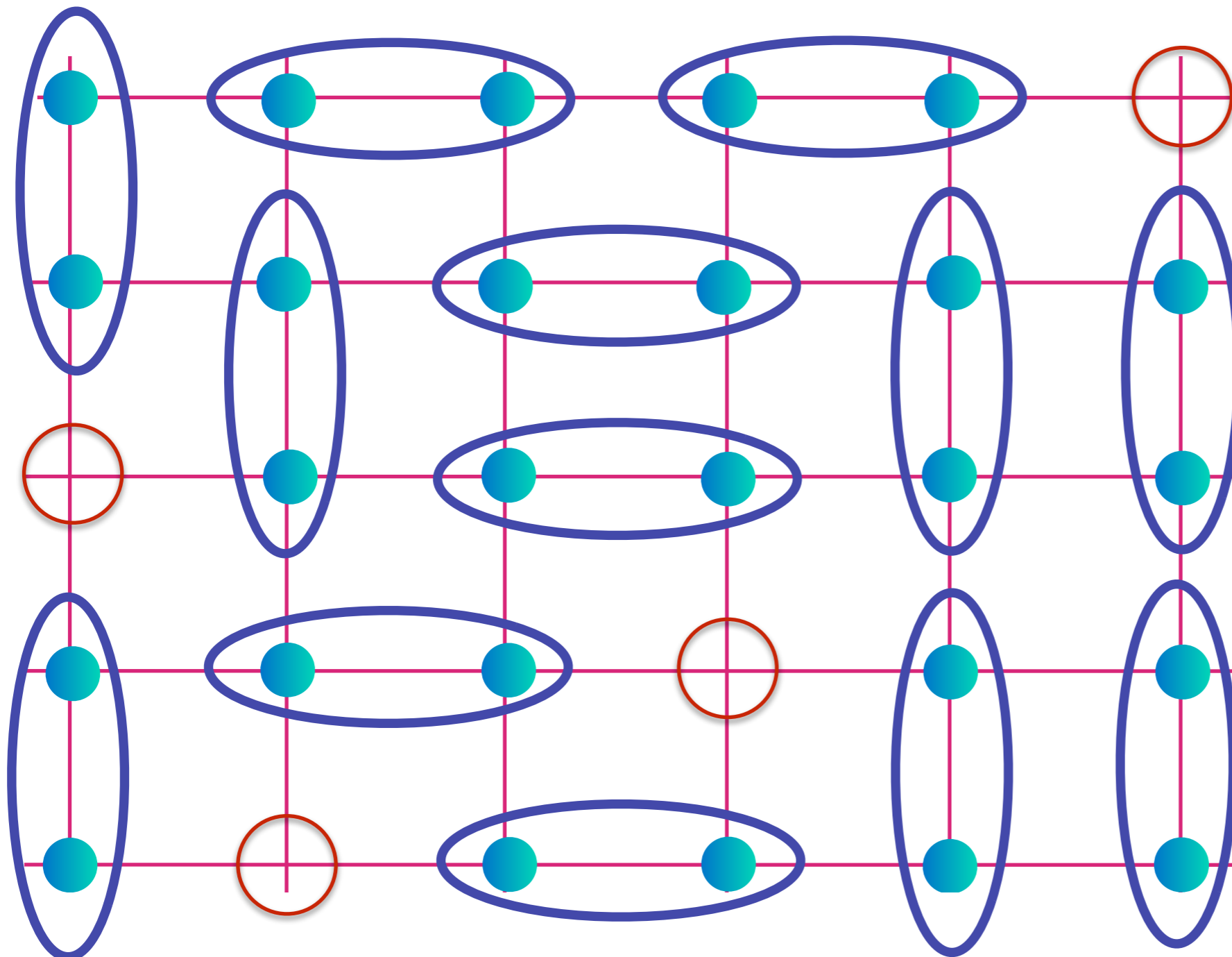


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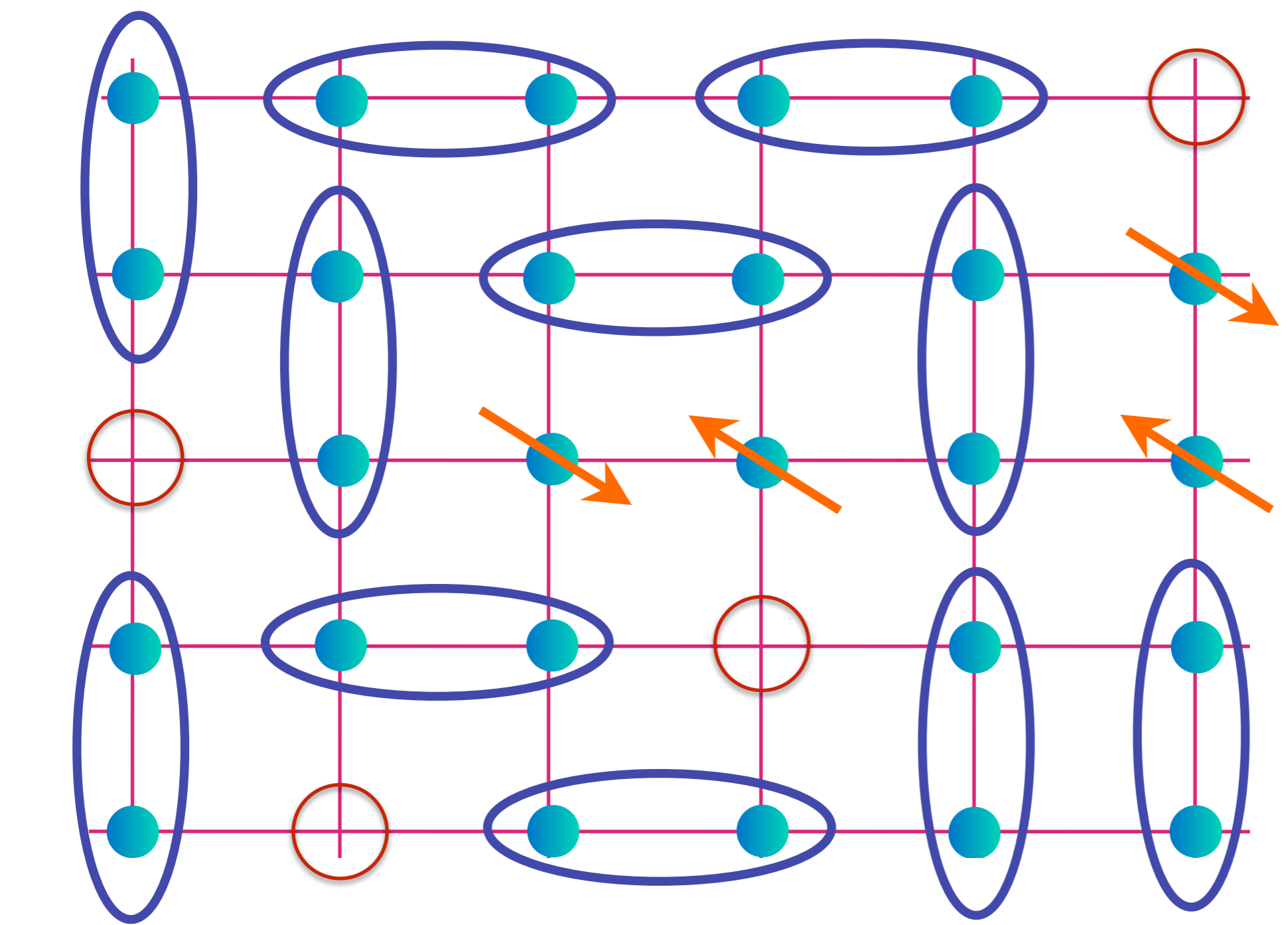
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB 35, 8865 (1987)

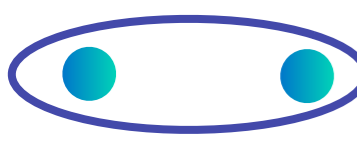
N. Read and B. Chakraborty, PRB 40, 7133 (1989)

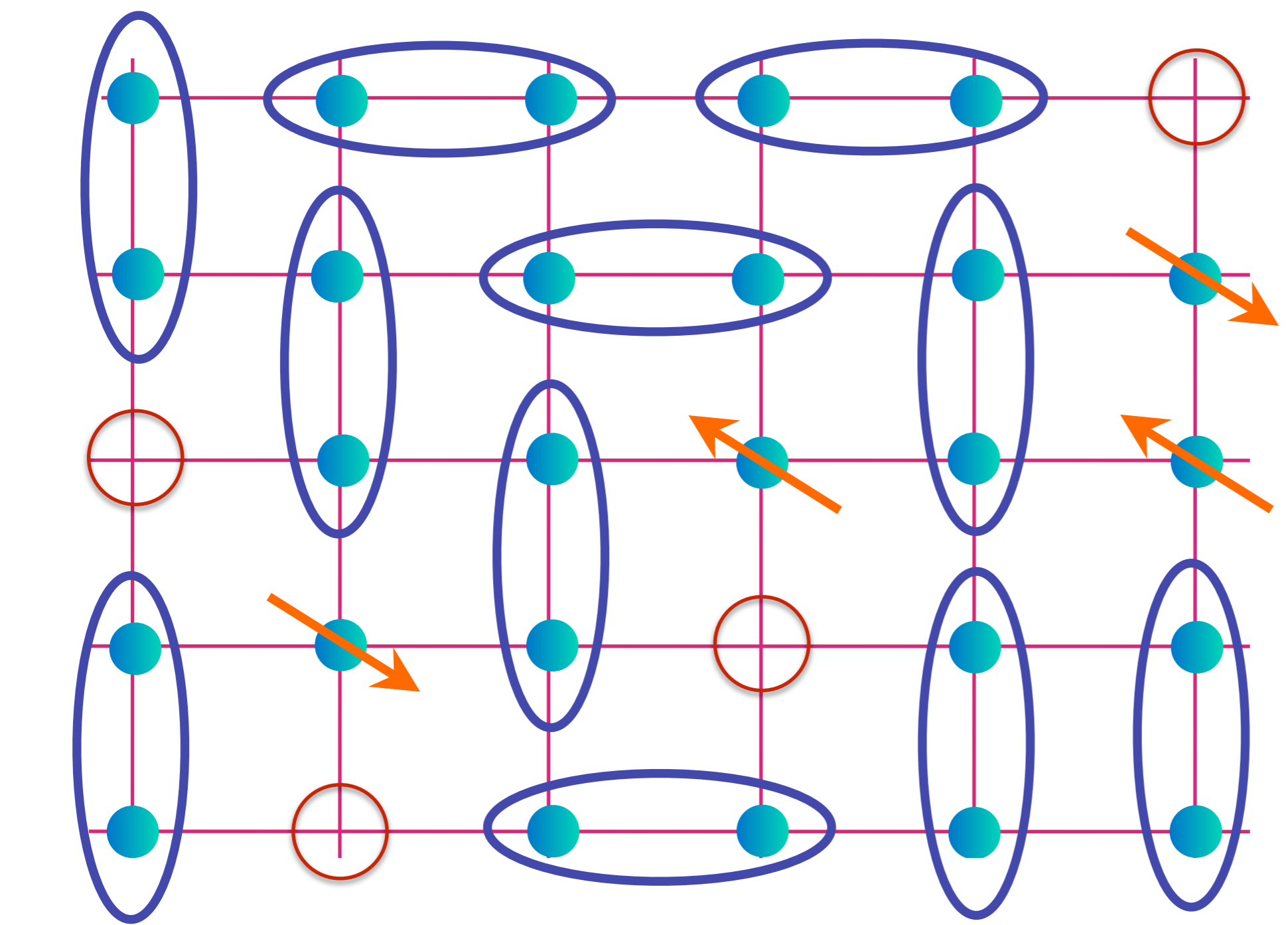


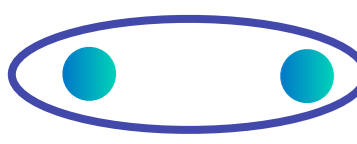
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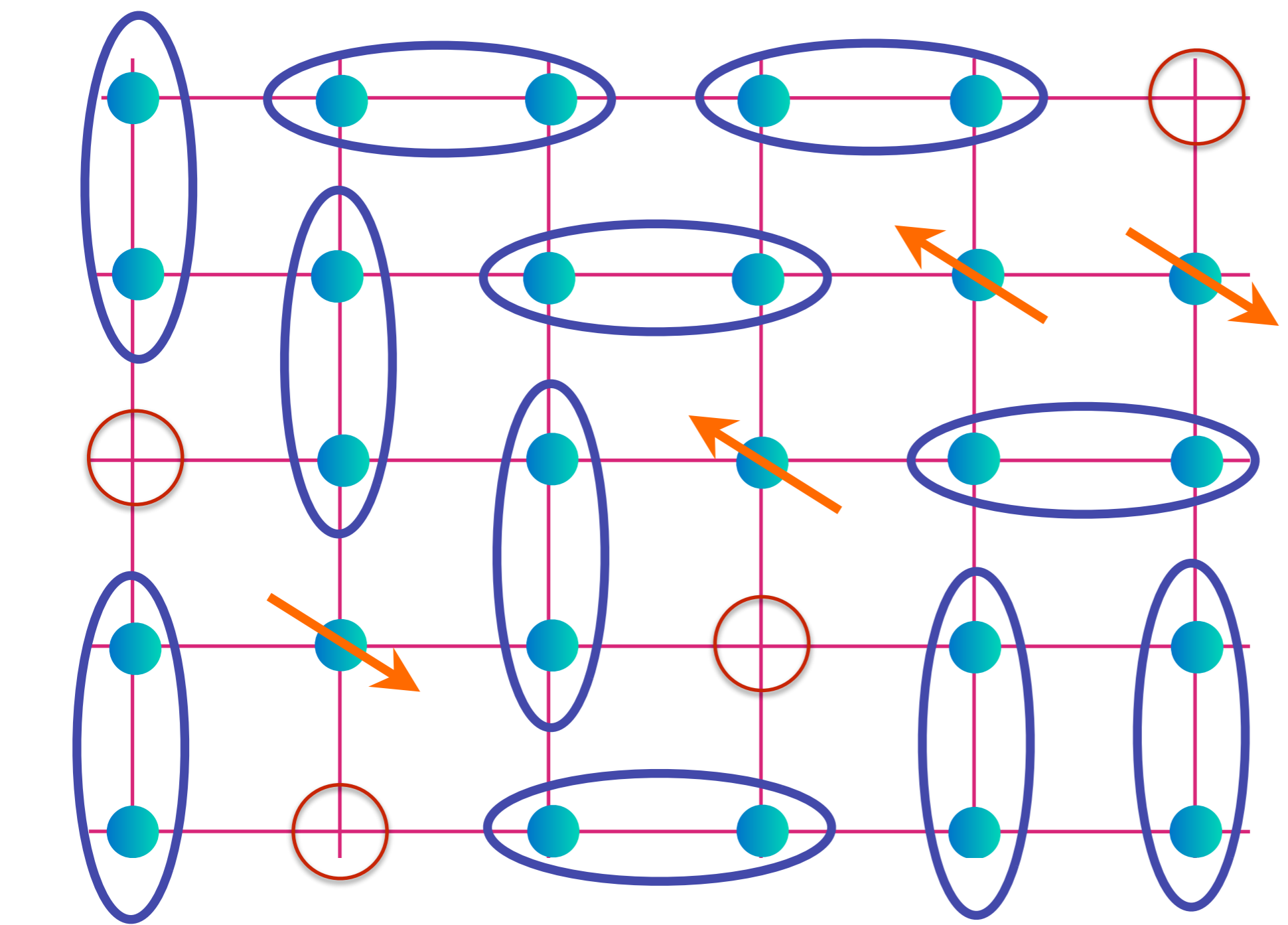
$$\text{[Diagram of two cyan circles in a blue oval]} = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

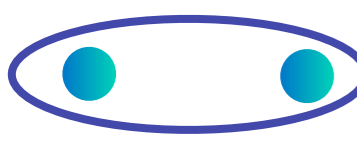


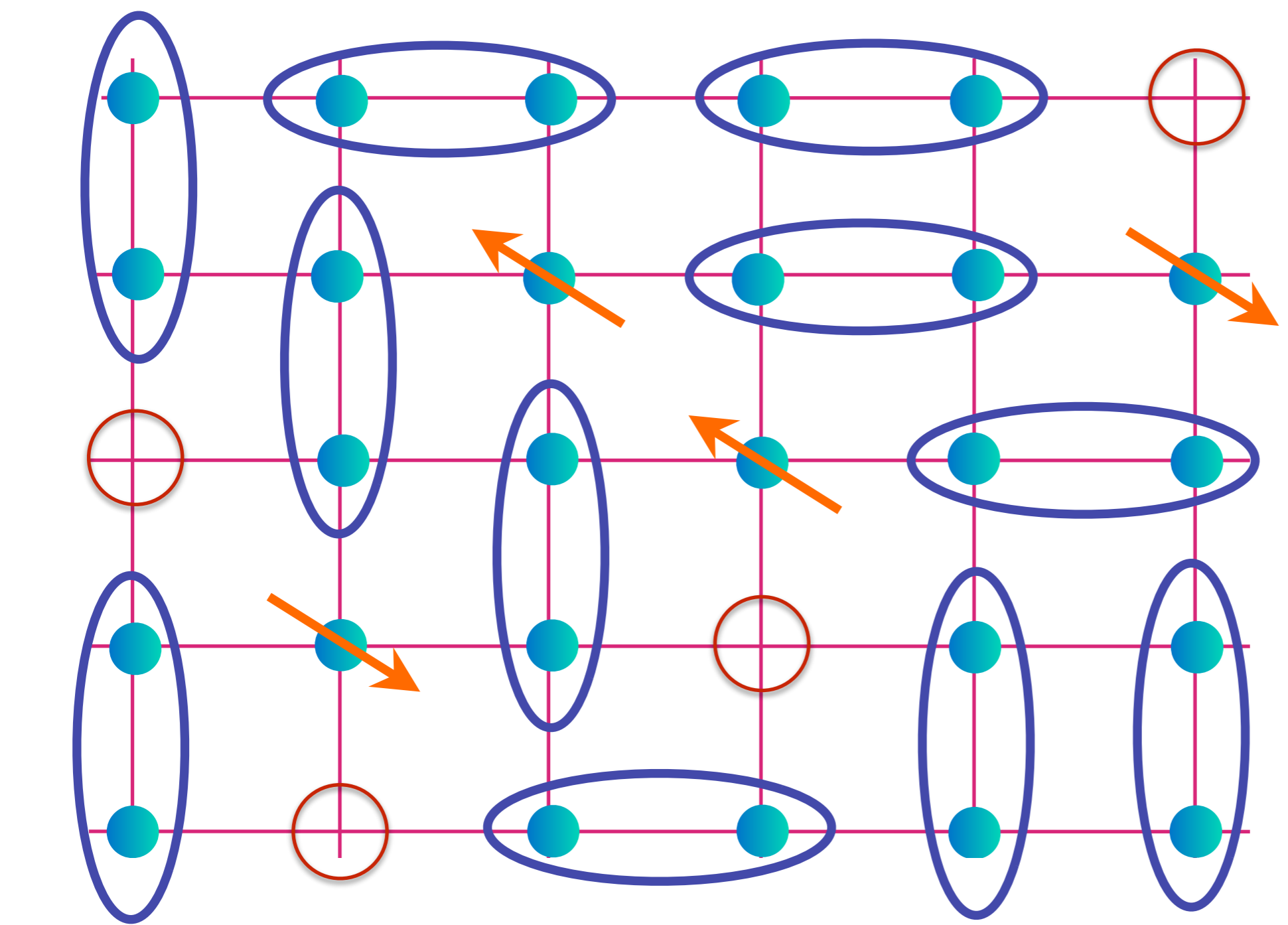

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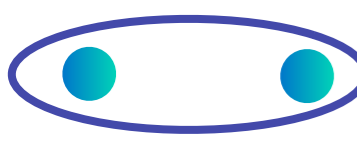


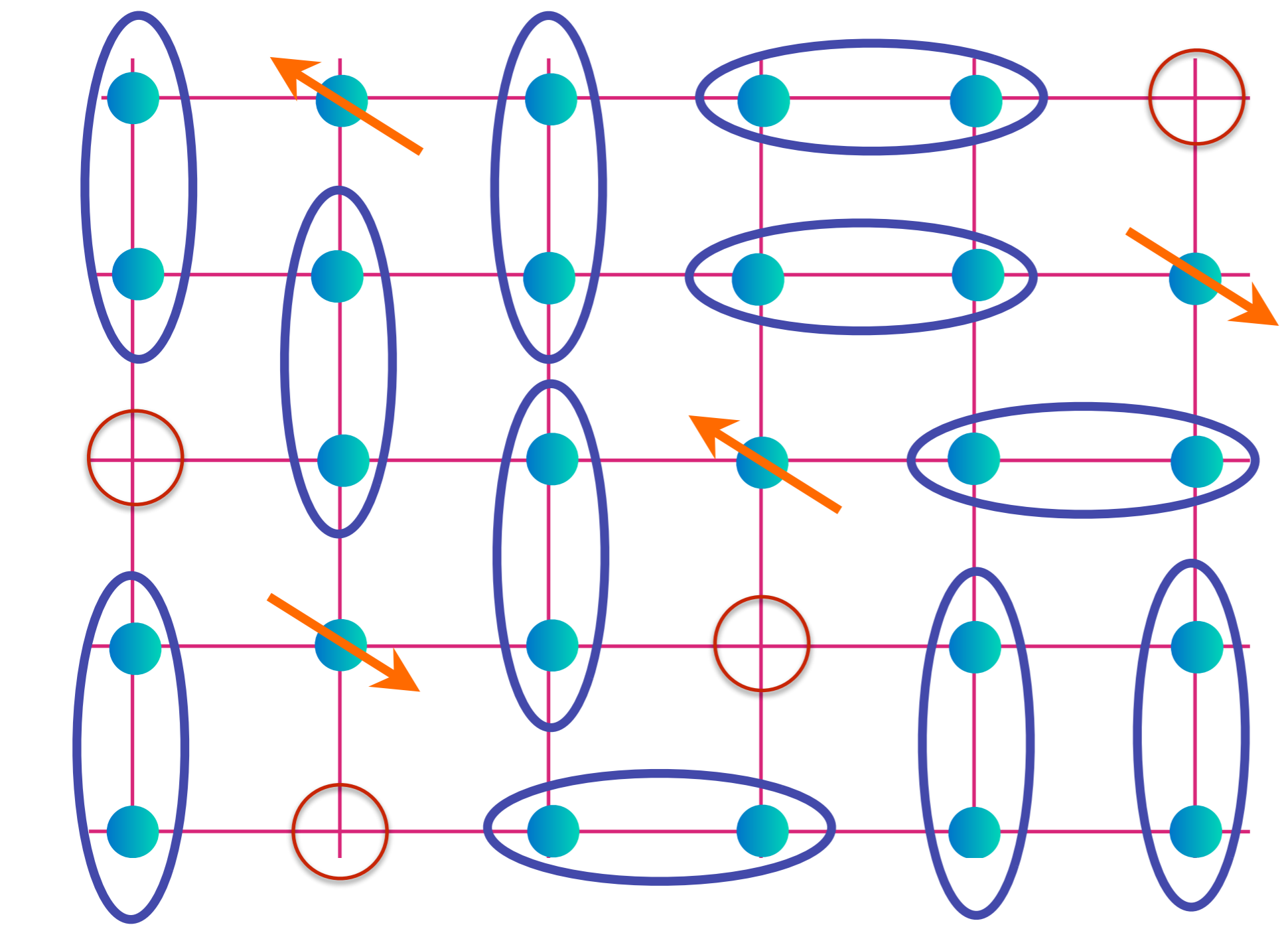

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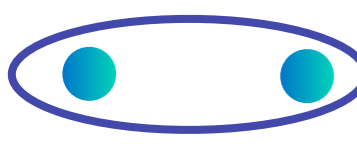


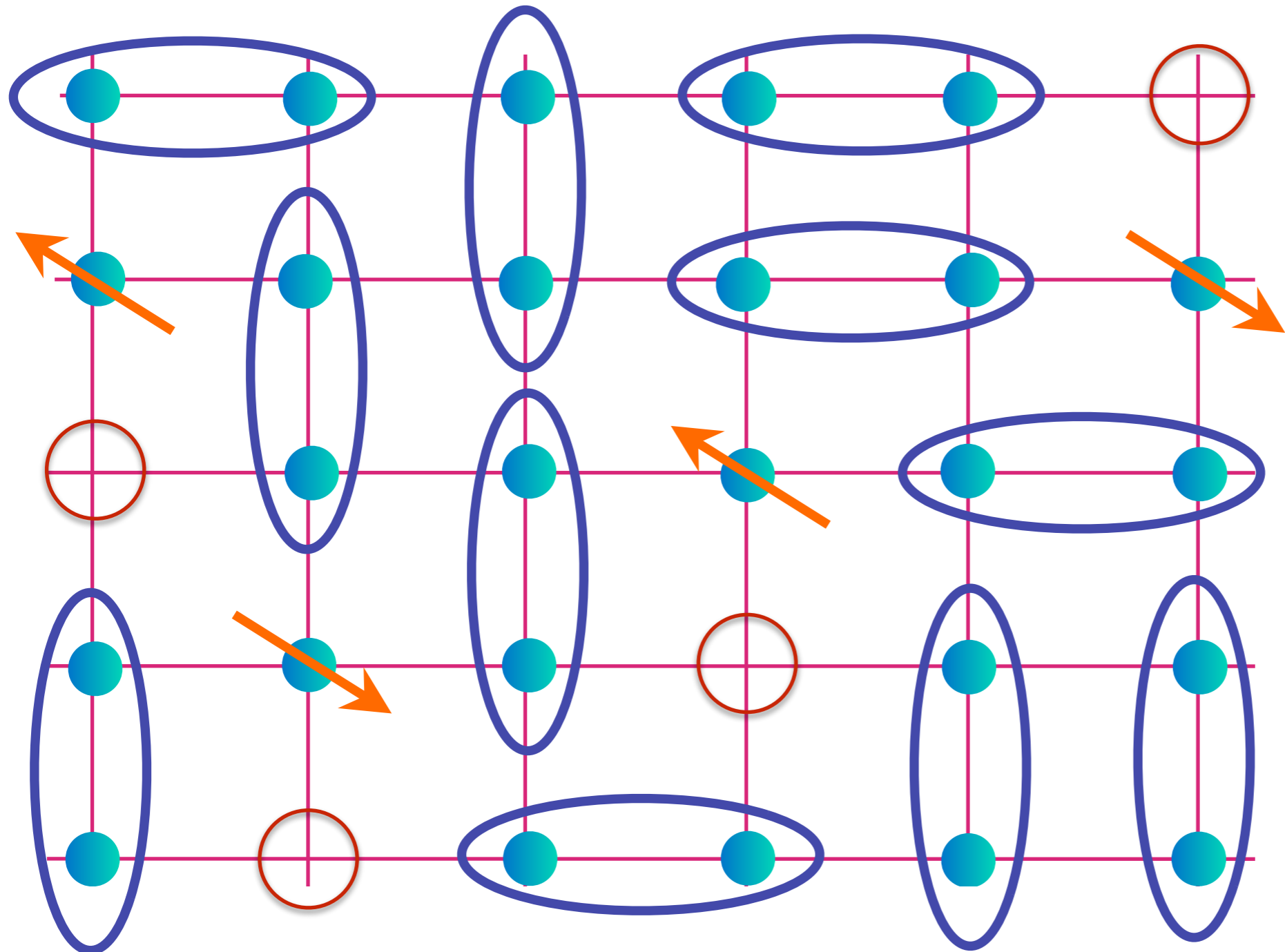

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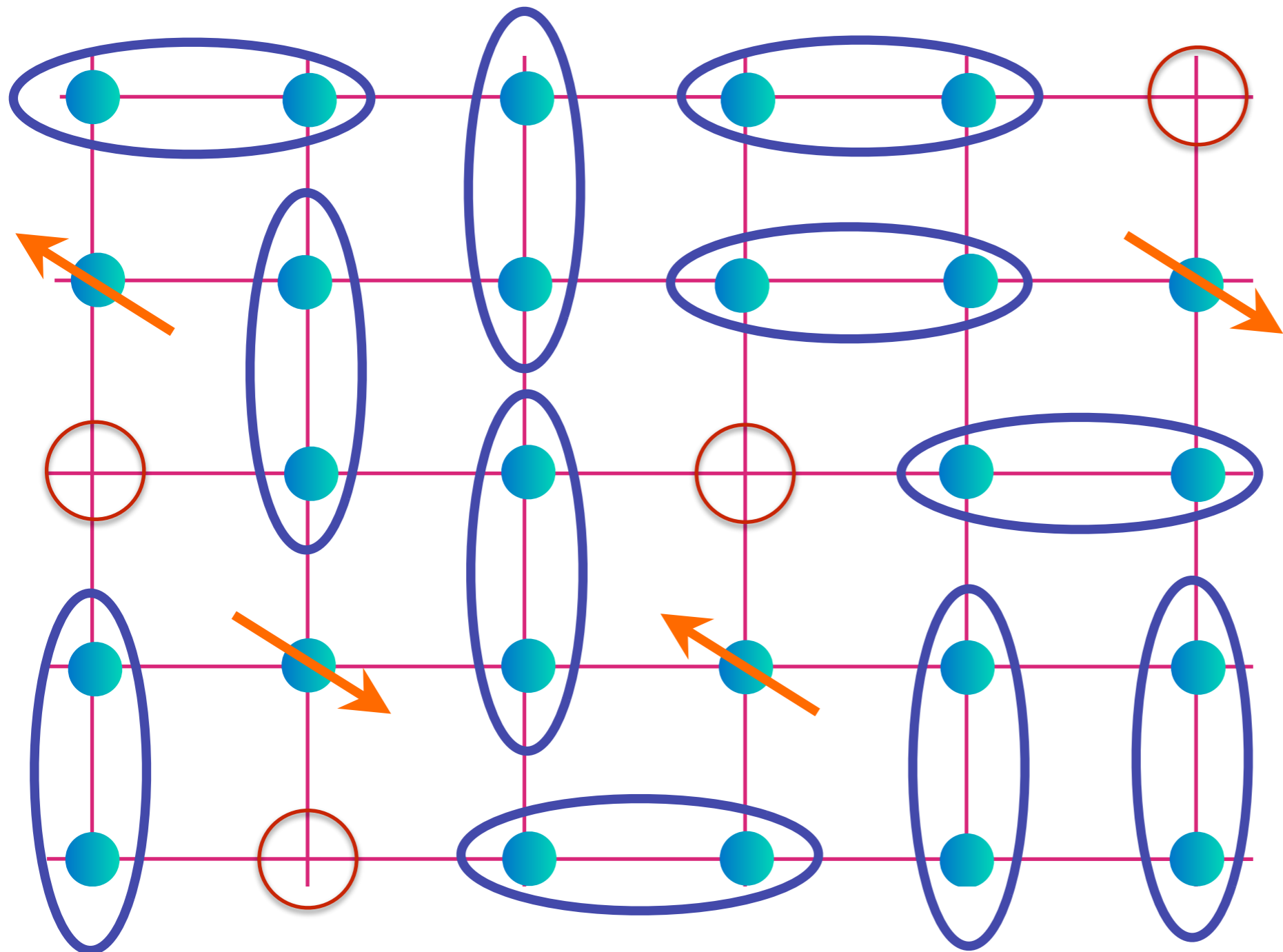

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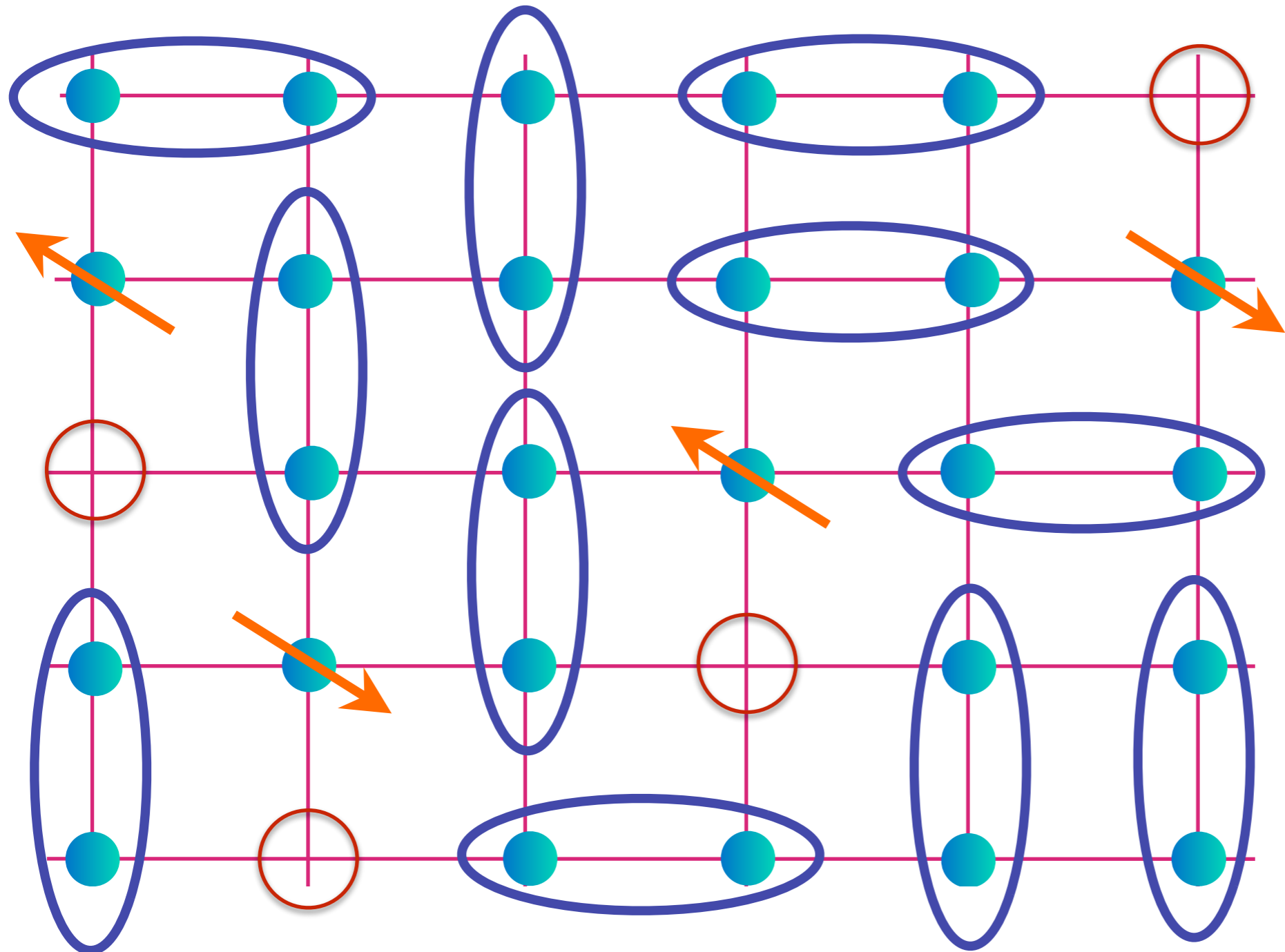

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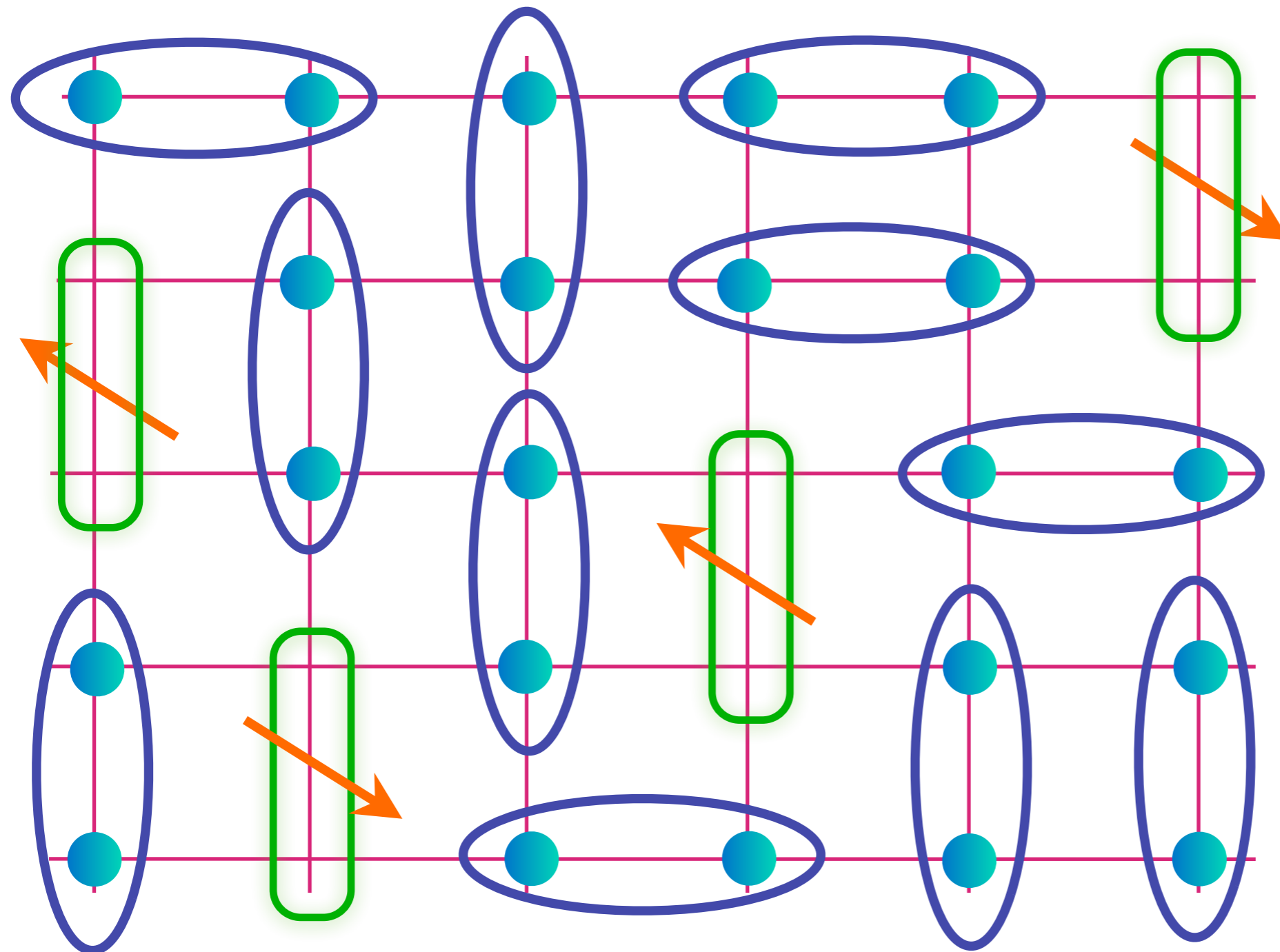



 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

Fractionalized Fermi liquid (FL*)

S. Sachdev PRB 49, 6770 (1994); X.-G. Wen and P.A. Lee PRL 76, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB 75, 235122 (2007)



Mobile
 $S=1/2$, charge
 $+e$ fermionic
 dimers: form
 a Fermi
 surface of
 size p visible
 in electron
 photo-
 emission

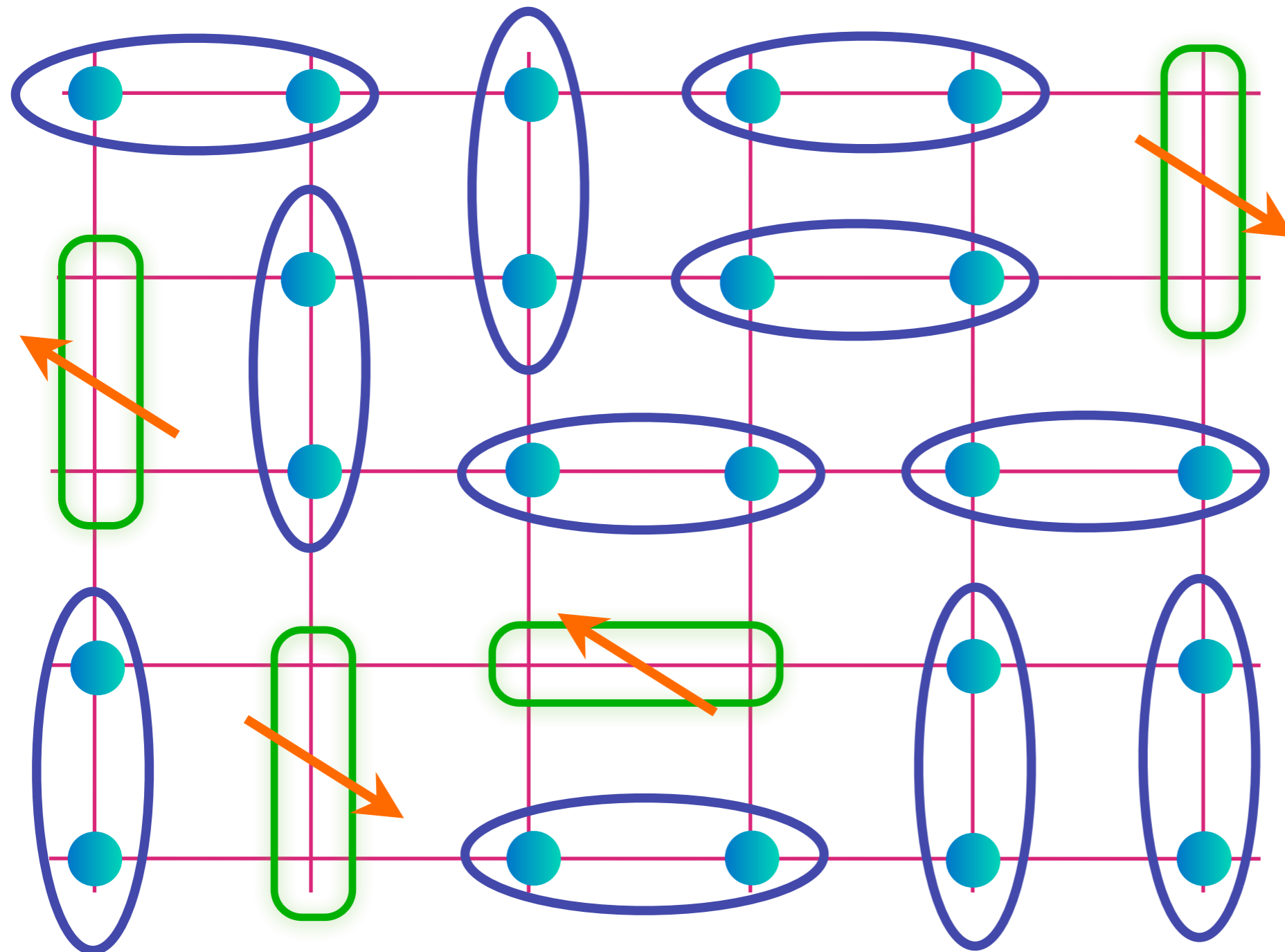
$$\text{Blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{Green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

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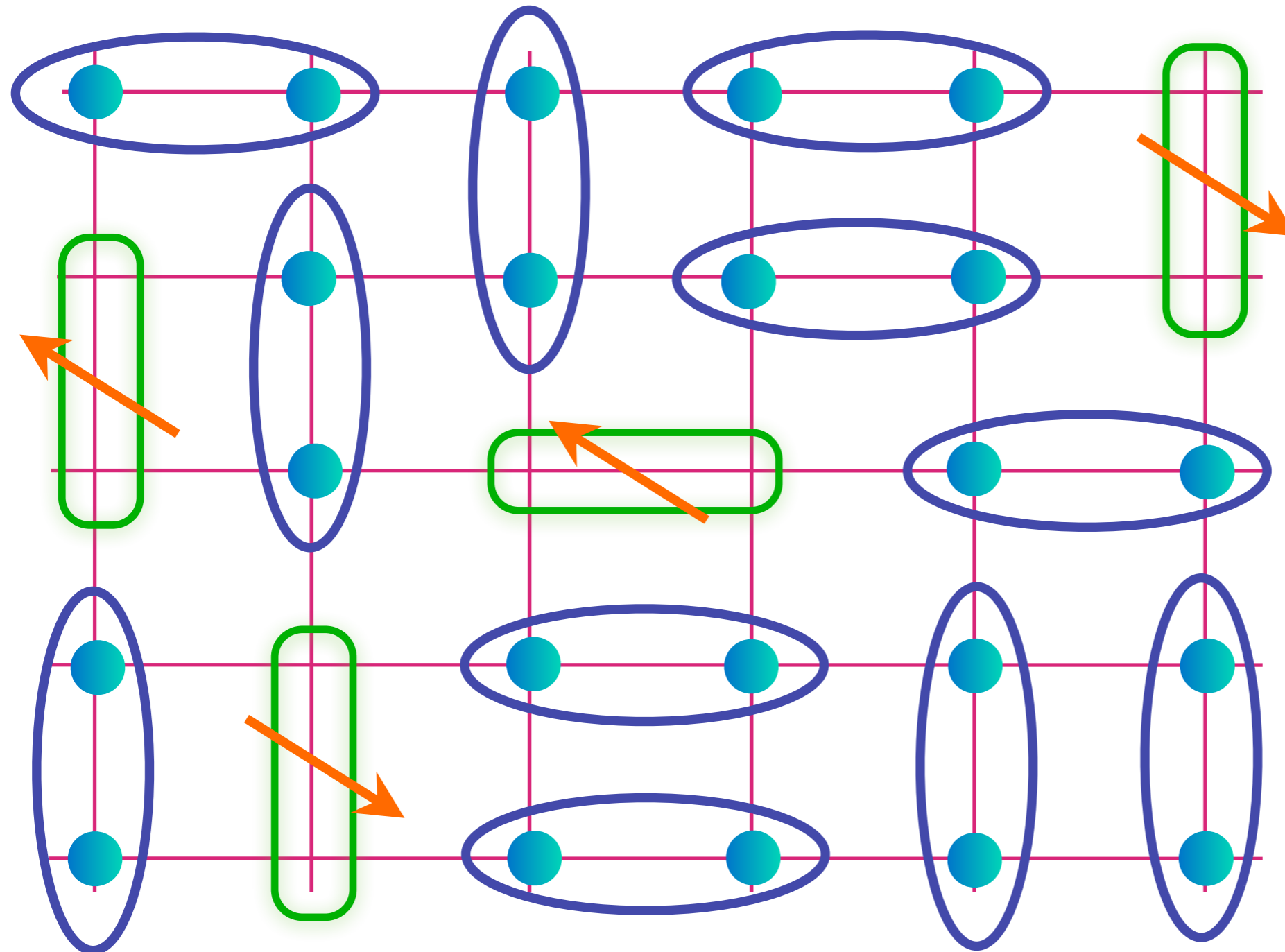
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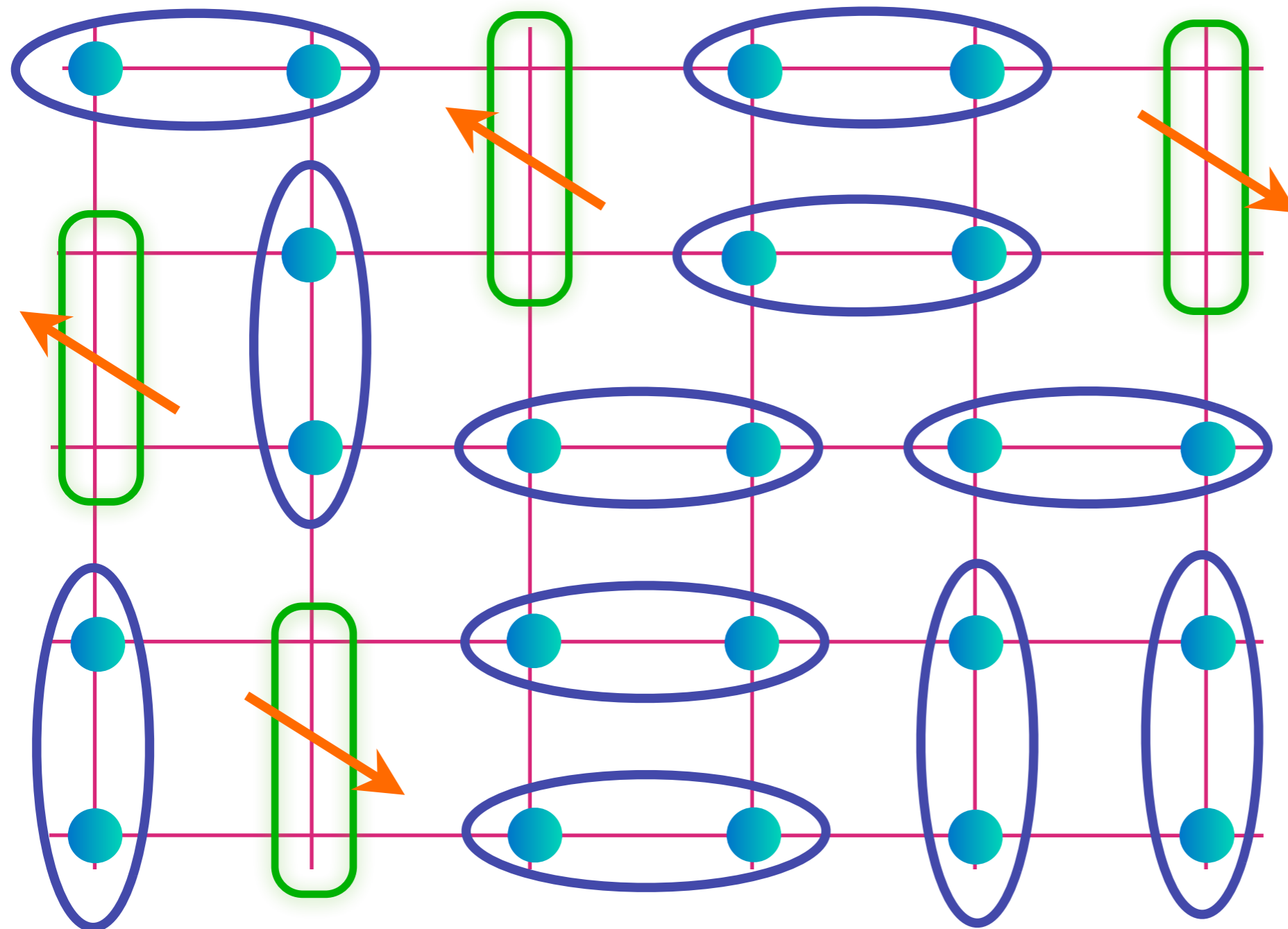
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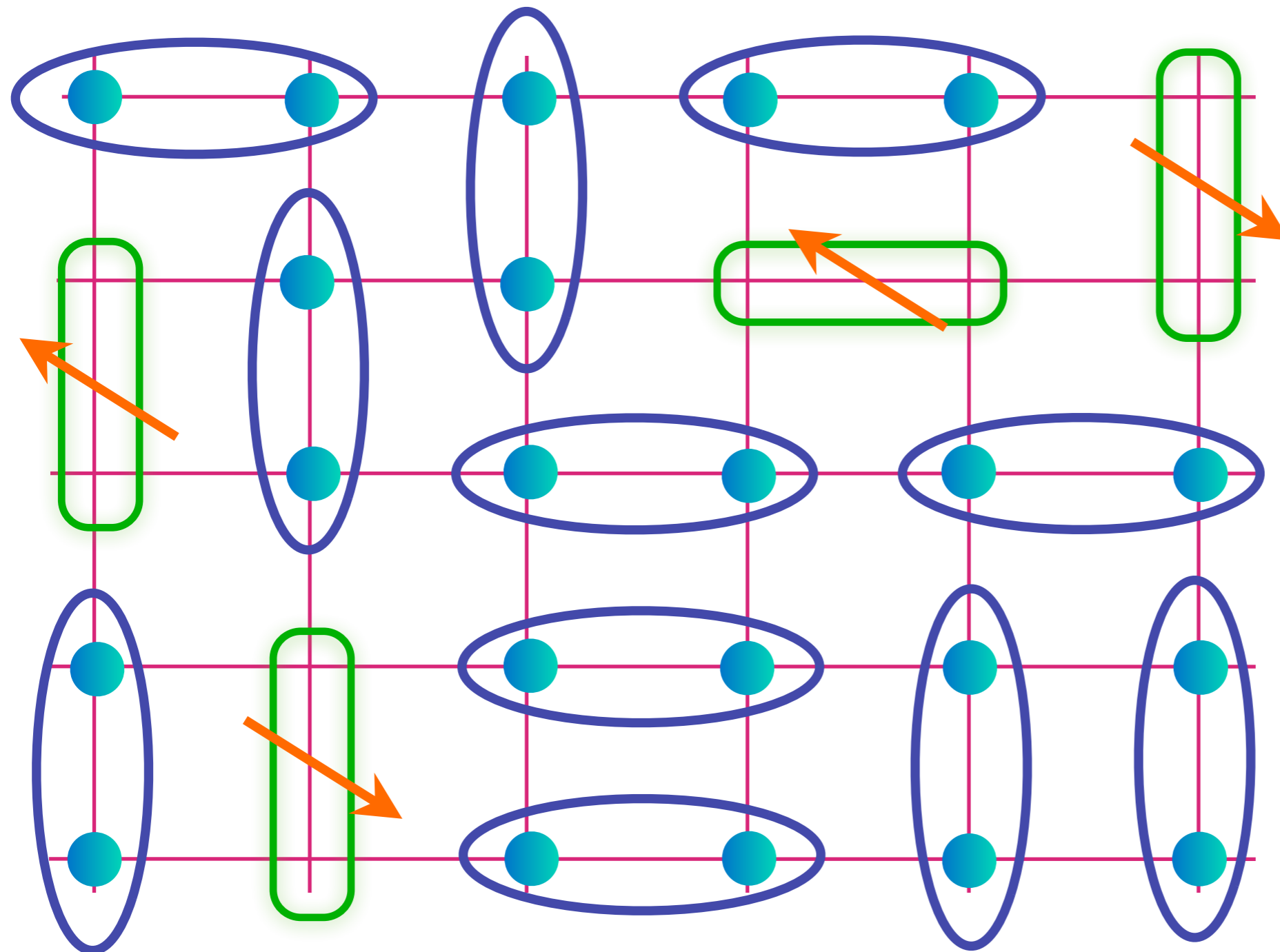
$$\text{Blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{Green rounded rectangle} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

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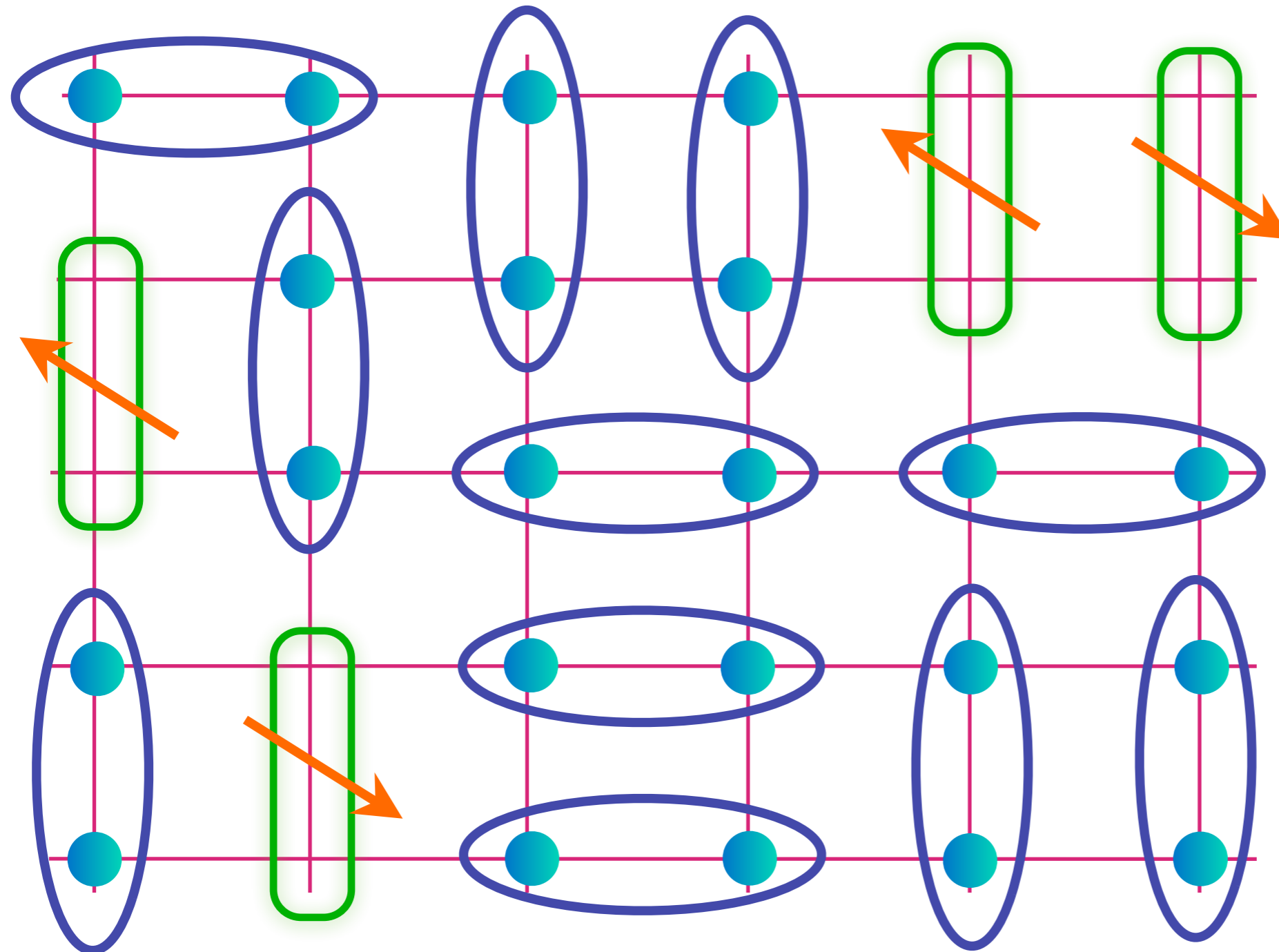
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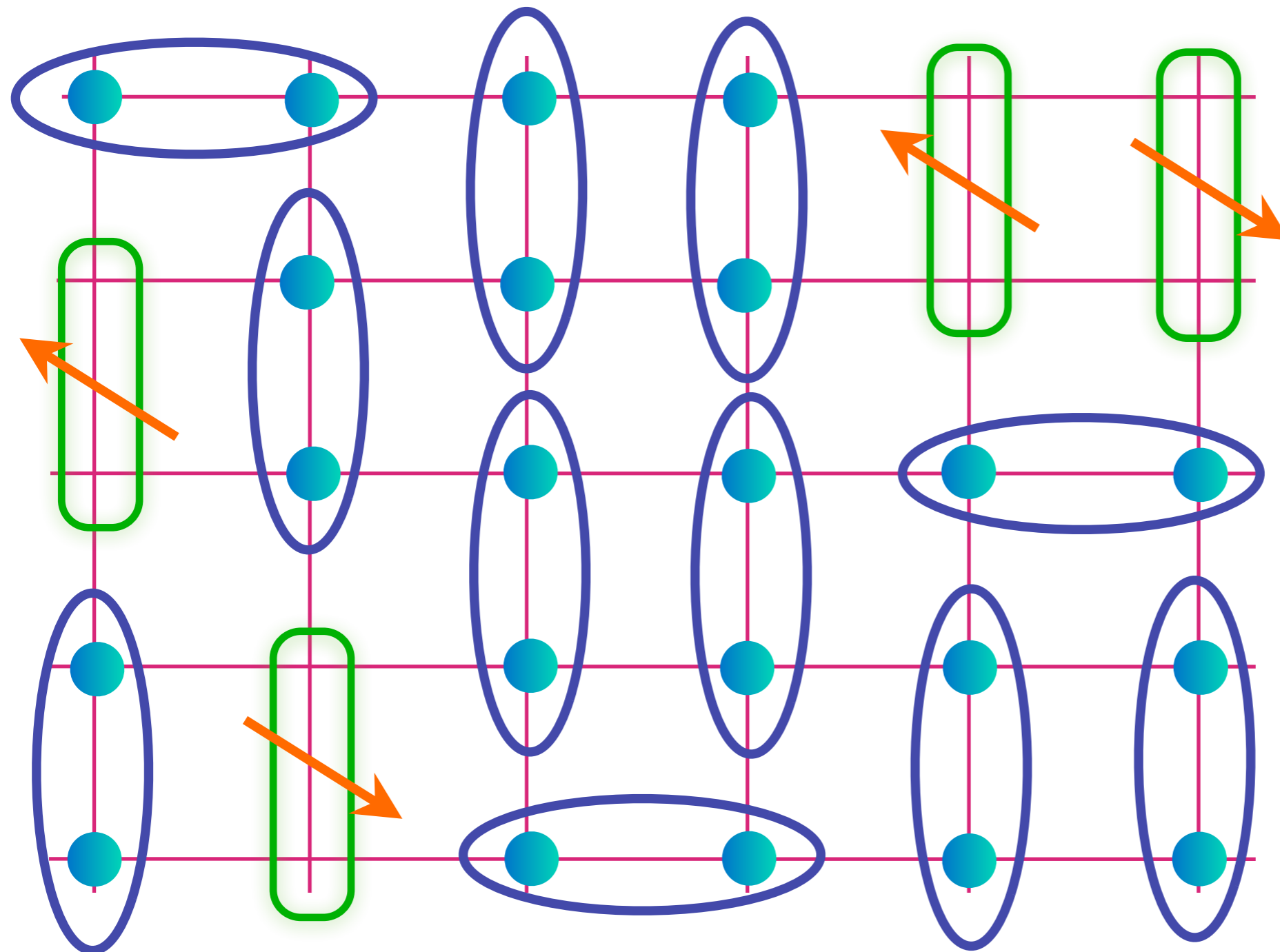
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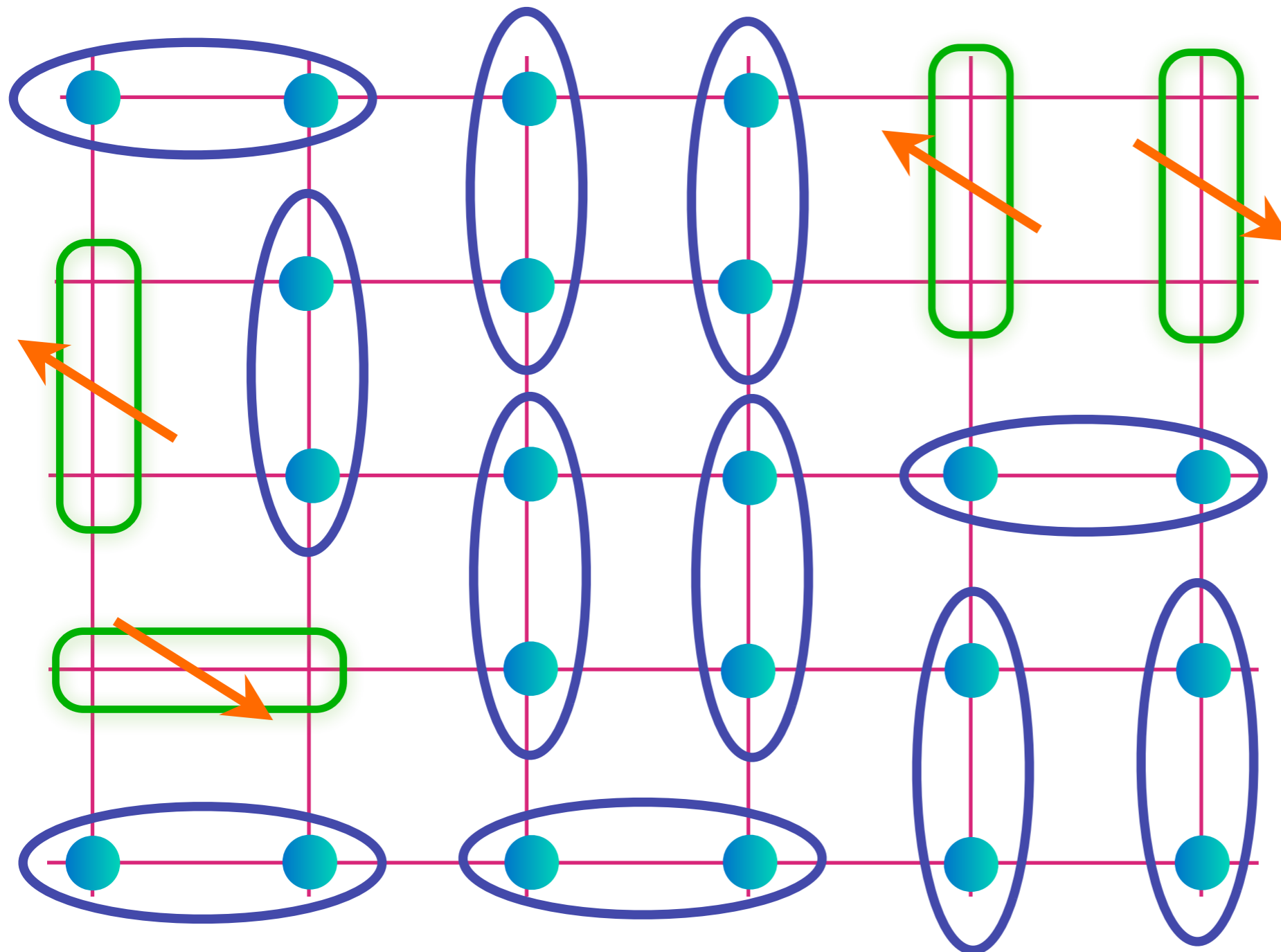
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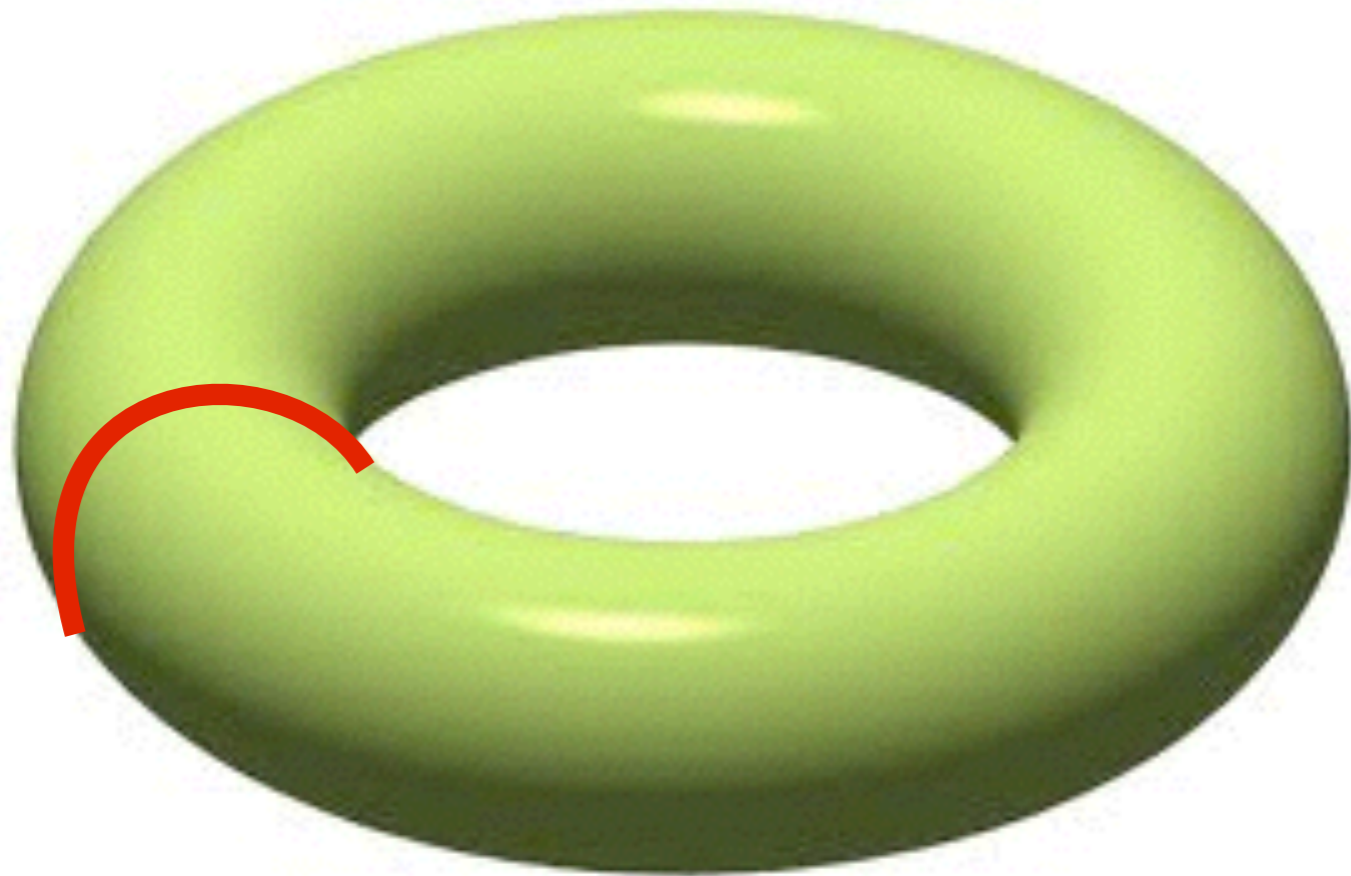
$$\text{Green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Ground state degeneracy

Place FL^*
on a torus:

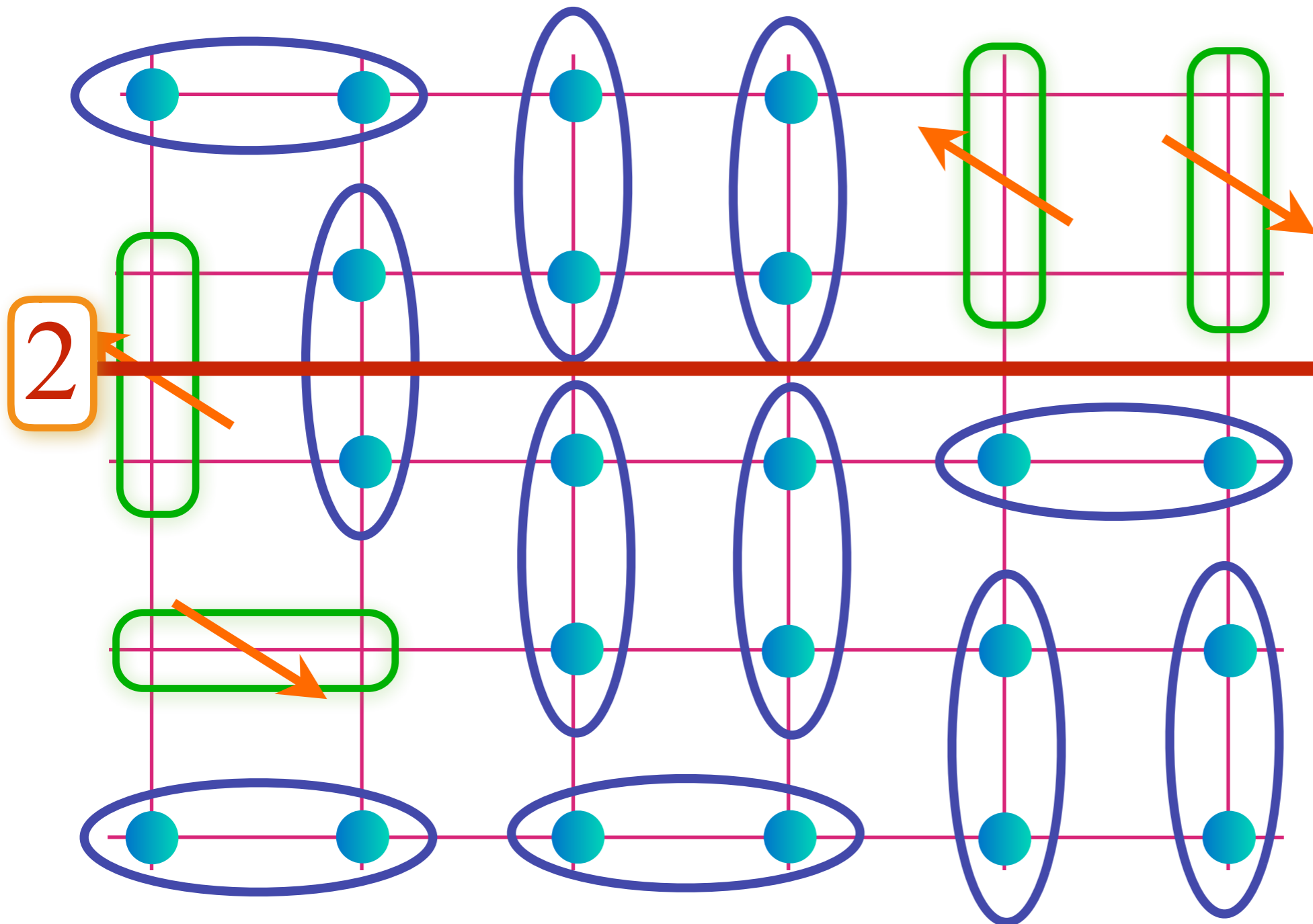


Ground state degeneracy



Place FL*
on a torus:
obtain
“topological”
states nearly
degenerate with
quasiparticle
states: number
of dimers
crossing red line
is conserved
modulo 2

Fractionalized Fermi liquid (FL*)

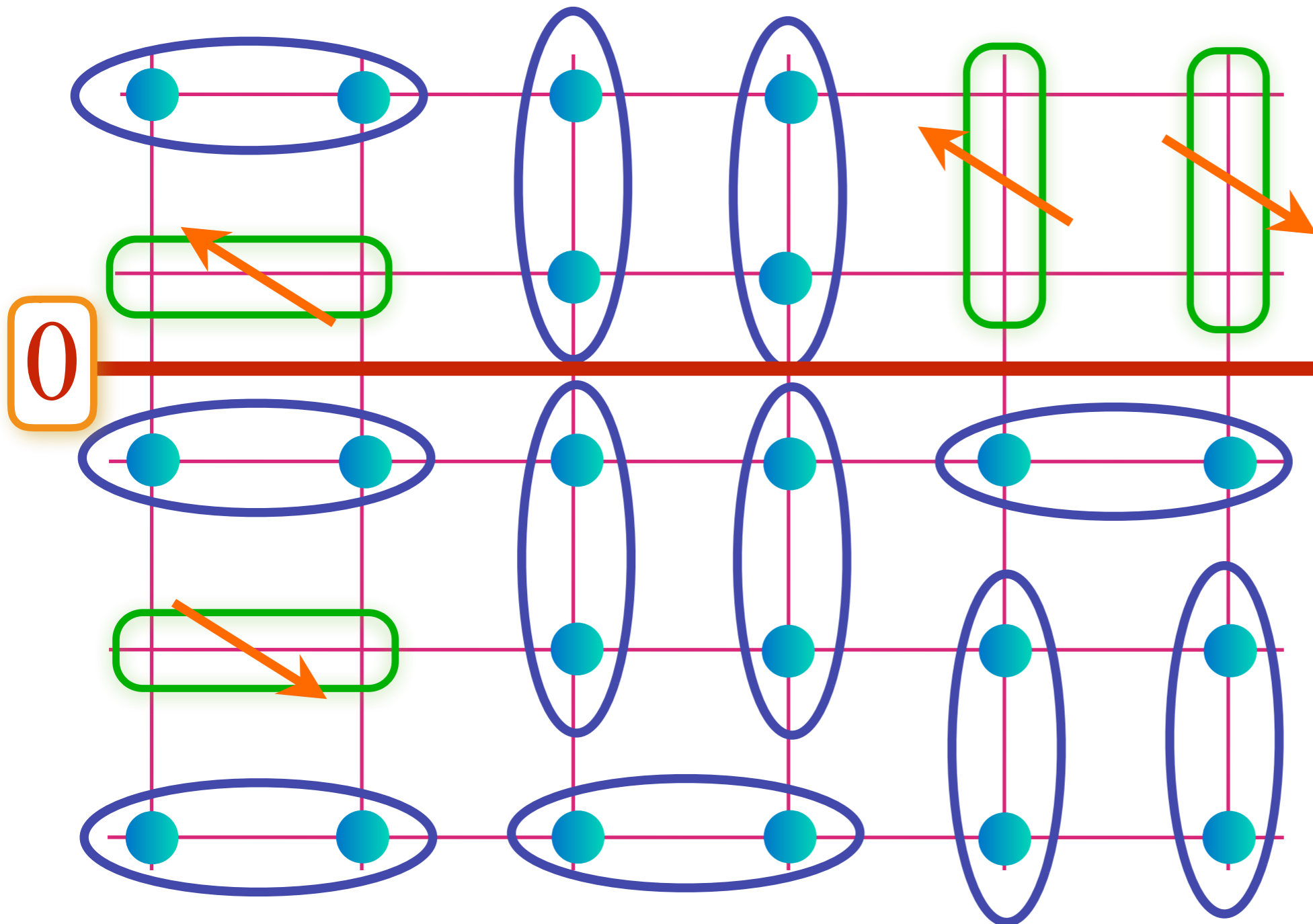


Place FL*
on a torus:
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Fractionalized Fermi liquid (FL*)

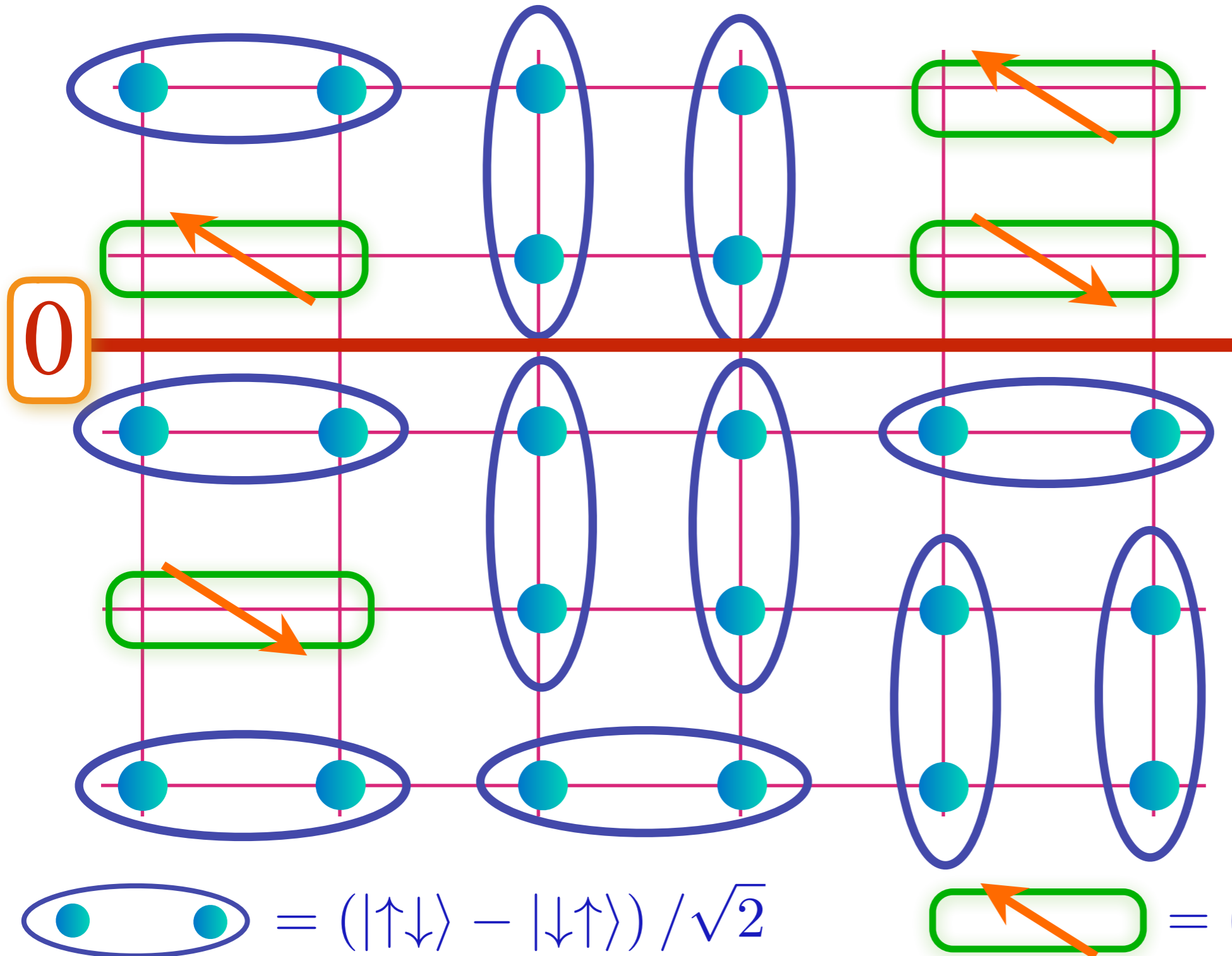


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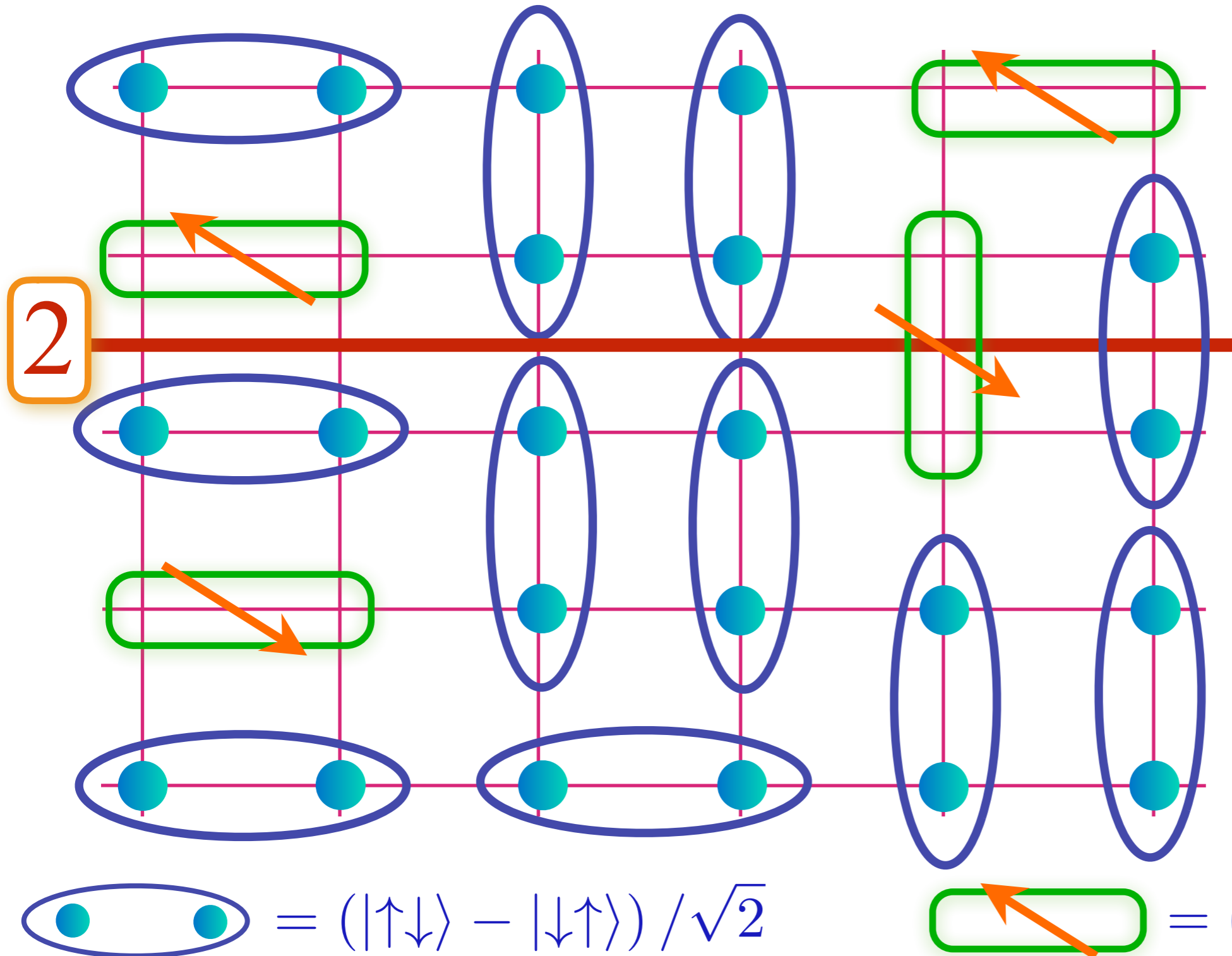
$$\text{Green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Fractionalized Fermi liquid (FL*)



Place FL*
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obtain
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Fractionalized Fermi liquid (FL*)



Place FL* on a torus: obtain “topological” states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

Fractionalized Fermi liquid (FL*)

We have described a metal with:

- A Fermi surface of electrons enclosing volume p , and not the Luttinger volume of $l+p$
- Additional low energy quantum states on a torus not associated with quasiparticle excitations *i.e.* emergent gauge fields

Fractionalized Fermi liquid (FL*)

We have described a metal with:

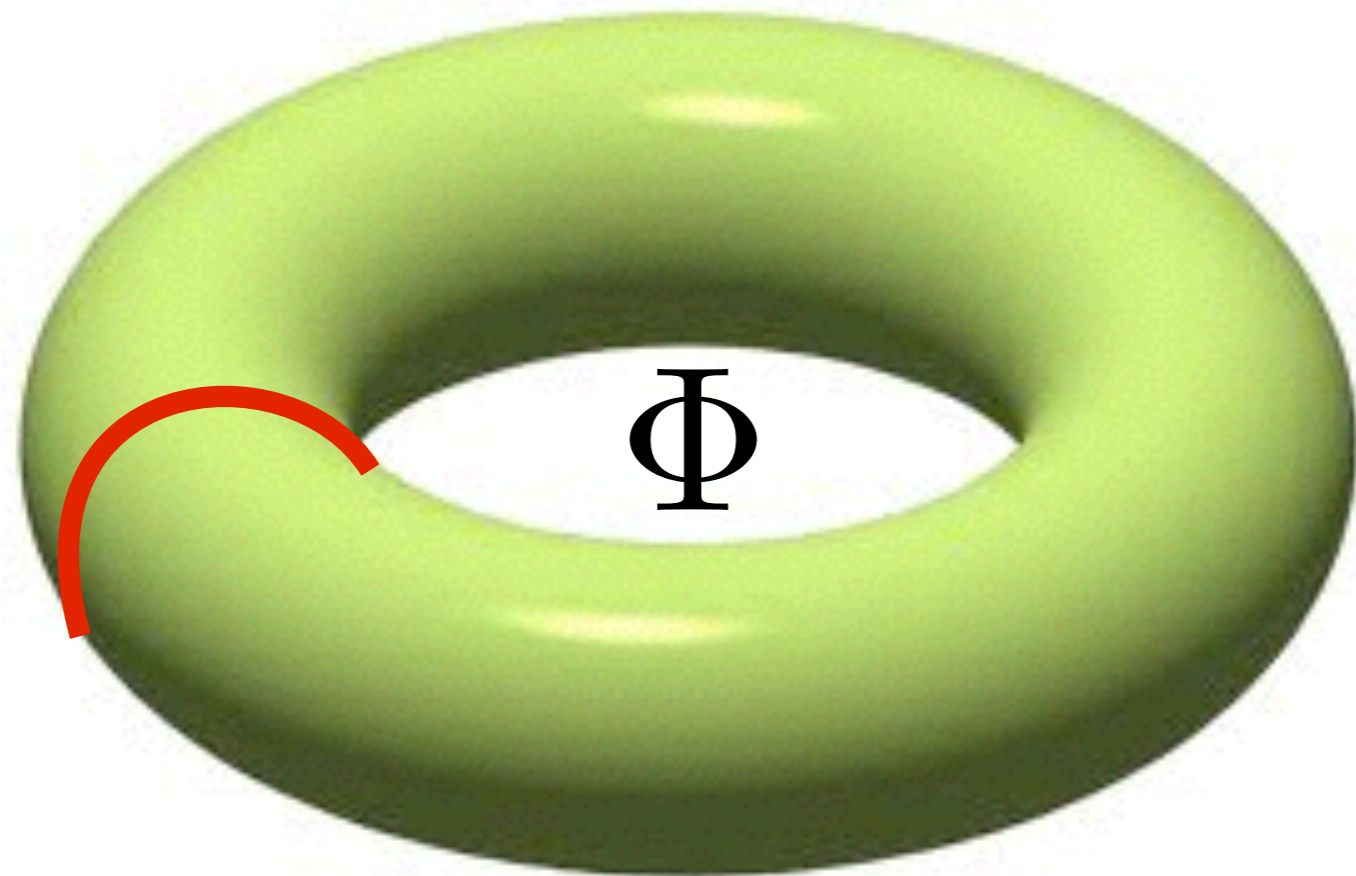
- A Fermi surface of electrons enclosing volume p , and not the Luttinger volume of $l+p$
- Additional low energy quantum states on a torus not associated with quasiparticle excitations *i.e.* emergent gauge fields

There is a general and fundamental relationship between these two characteristics.

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)



Following the evolution of the quantum state under adiabatic insertion of a flux quantum leads to a non-perturbative argument for the volume enclosed by the Fermi surface

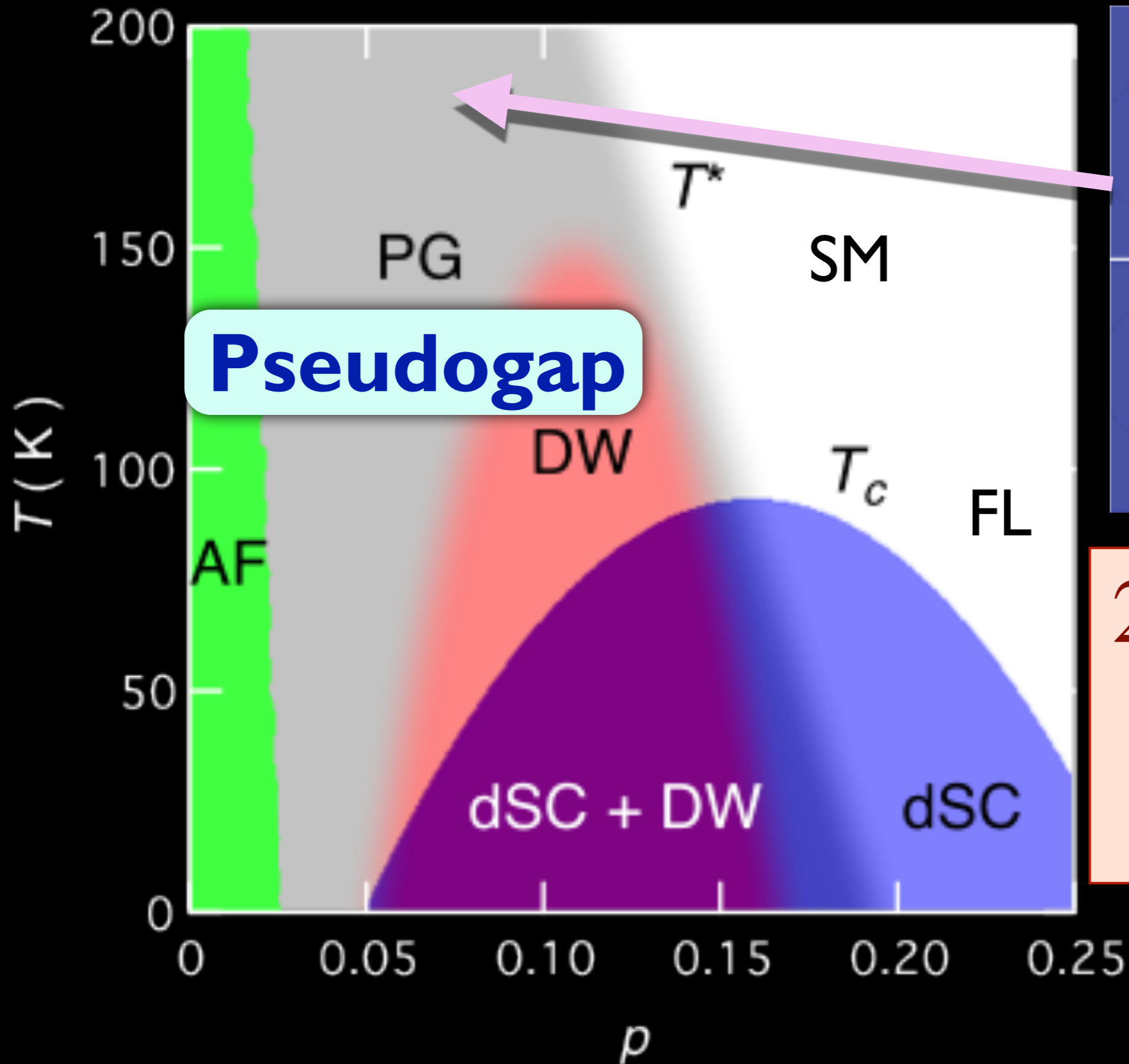
M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

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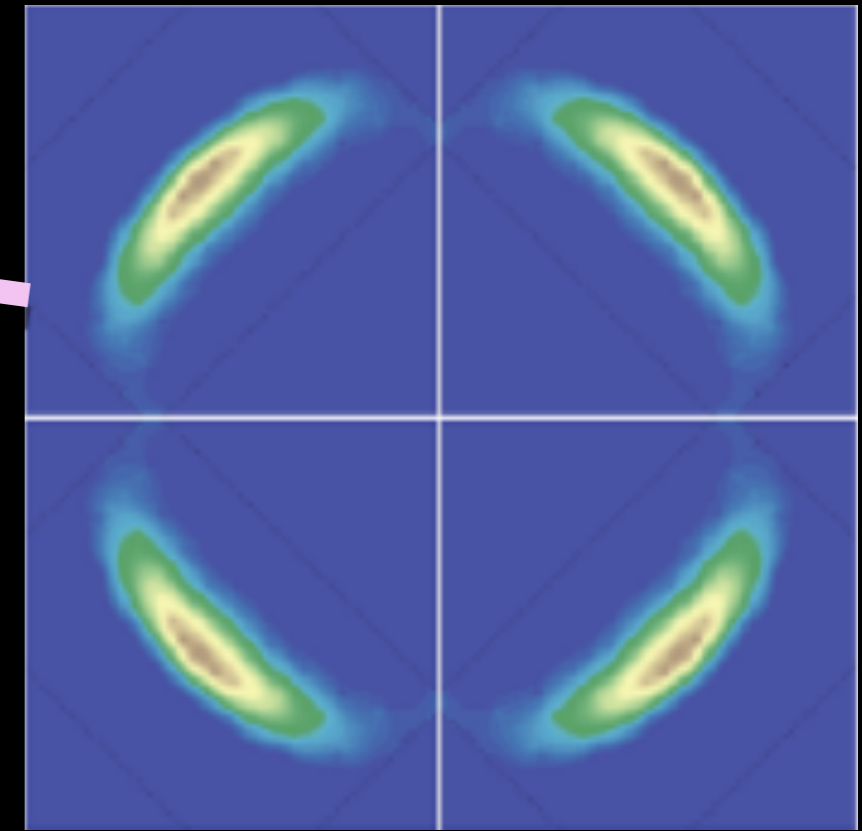
1. Emergent gauge fields and long-range entanglement in insulators
2. Fractionalized Fermi liquids (FL*)
*Quasiparticles with a non-Luttinger volume,
and emergent gauge fields*

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2. Fractionalized Fermi liquids (FL*)
*Quasiparticles with a non-Luttinger volume,
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3. The pseudogap metal of the cuprate superconductors

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

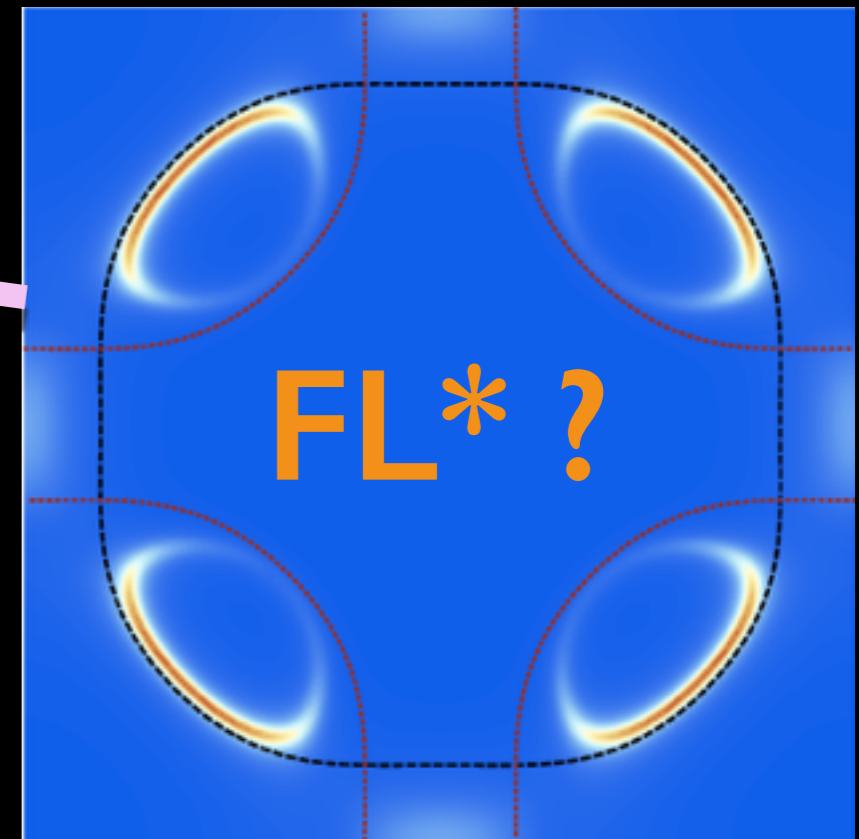
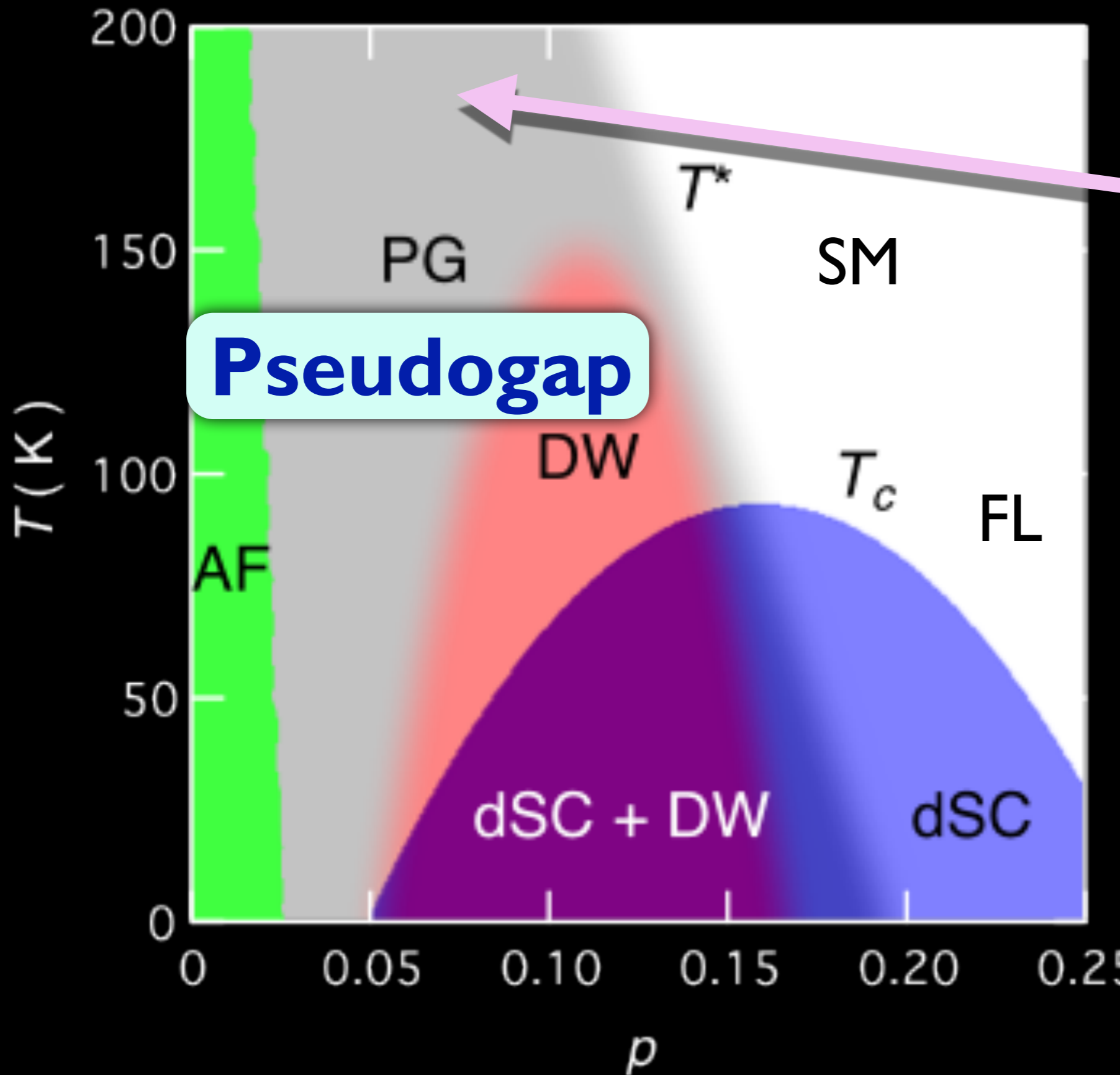


Pseudogap



2. Pseudogap
metal
at low p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)



A new metal —
a fractionalized
Fermi liquid (FL*)
— with electron-
like quasiparticles
on a Fermi surface
of size p

Recent evidence for pseudogap metal as FL*

Recent evidence for pseudogap metal as FL*

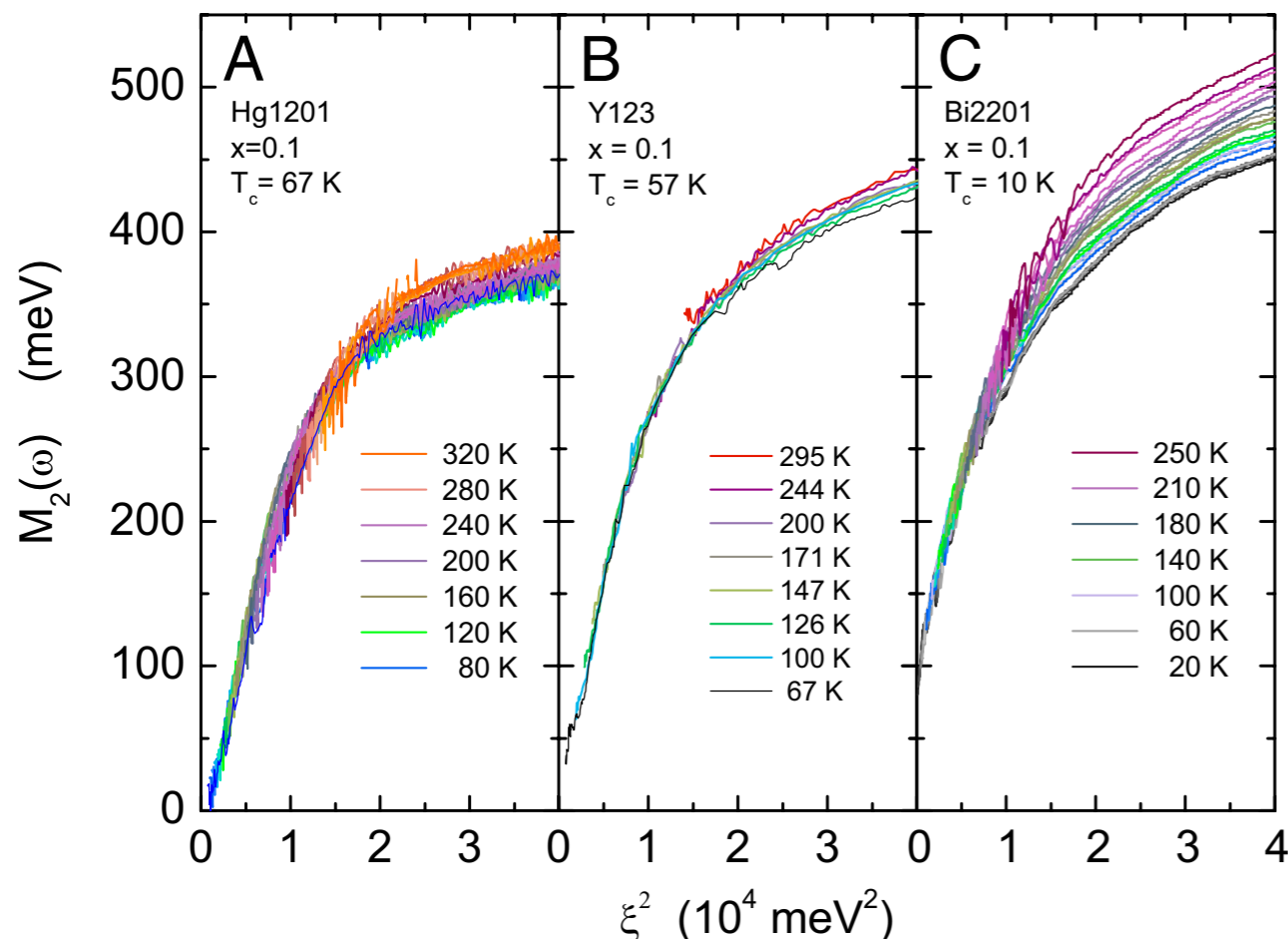
- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).

Optical evidence for Fermi surface of long-lived quasiparticles of density ρ

Spectroscopic evidence for Fermi liquid-like energy and temperature dependence of the relaxation rate in the pseudogap phase of the cuprates

Seyed Iman Mirzaei^a, Damien Stricker^a, Jason N. Hancock^{a,b}, Christophe Berthod^a, Antoine Georges^{a,c,d}, Erik van Heumen^{a,e}, Mun K. Chan^f, Xudong Zhao^{f,g}, Yuan Li^h, Martin Greven^f, Neven Barišić^{f,i,j}, and Dirk van der Marel^{a,1}

PNAS 110, 5774 (2013)



$$\sigma_{xx} \sim \frac{1}{(-i\omega + 1/\tau)}$$

with $\frac{1}{\tau} \sim \omega^2 + T^2$

Fig. 6. Collapse of the frequency and temperature dependence of the relaxation rate of underdoped cuprate materials. Normal state $M_2(\omega, T)$ as a function of $\xi^2 \equiv (\hbar\omega)^2 + (\rho\pi k_B T)^2$

Recent evidence for pseudogap metal as FL*

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).

Recent evidence for pseudogap metal as FL*

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).
- Magnetoresistance $\rho_{xx} \sim \tau^{-1} (1 + aH^2T^2)$ with $\tau \sim T^{-2}$ (Chan *et al.*, PRL **113**, 177005 (2014)).

Electrical evidence for Fermi surface of long-lived quasiparticles of density p

In-Plane Magnetoresistance Obeys Kohler's Rule in the Pseudogap Phase of Cuprate Superconductors

M. K. Chan,^{1,*} M. J. Veit,¹ C. J. Dorow,^{1,†} Y. Ge,¹ Y. Li,¹ W. Tabis,^{1,2} Y. Tang,¹ X. Zhao,^{1,3}
N. Barišić,^{1,4,5,‡} and M. Greven^{1,§}

PRL 113, 177005 (2014)

We report in-plane resistivity (ρ) and transverse magnetoresistance (MR) measurements for underdoped $\text{HgBa}_2\text{CuO}_{4+\delta}$ (Hg1201). Contrary to the long-standing view that Kohler's rule is strongly violated in underdoped cuprates, we find that it is in fact satisfied in the pseudogap phase of Hg1201. The transverse MR shows a quadratic field dependence, $\delta\rho/\rho_0 = aH^2$, with $a(T) \propto T^{-4}$. In combination with the observed $\rho \propto T^2$ dependence, this is consistent with a single Fermi-liquid quasiparticle scattering rate. We show that this behavior is typically masked in cuprates with lower structural symmetry or strong disorder effects.

$$\rho_{xx} \sim \frac{1}{\tau} (1 + aH^2\tau^2 + \dots)$$

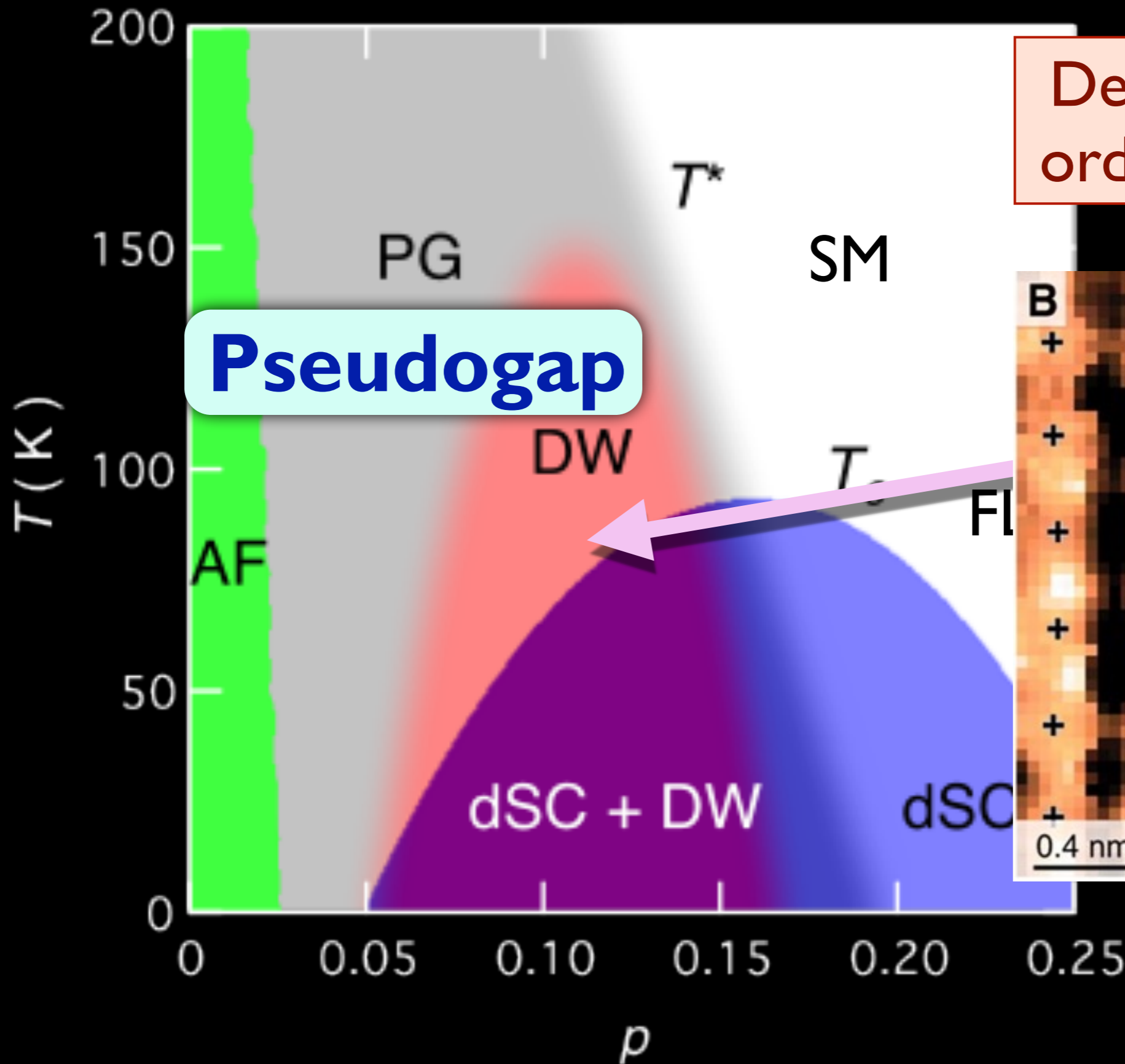
$$\text{with } \frac{1}{\tau} \sim T^2$$

Recent evidence for pseudogap metal as FL*

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).
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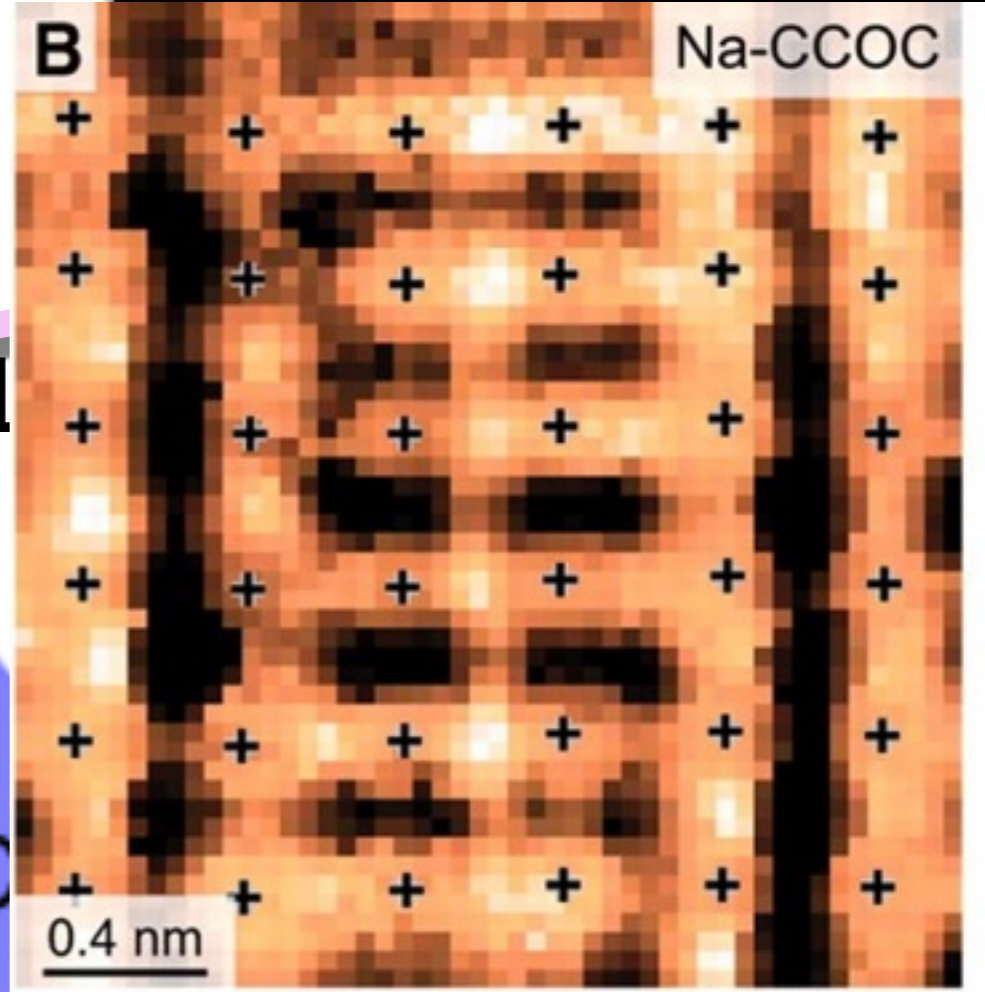
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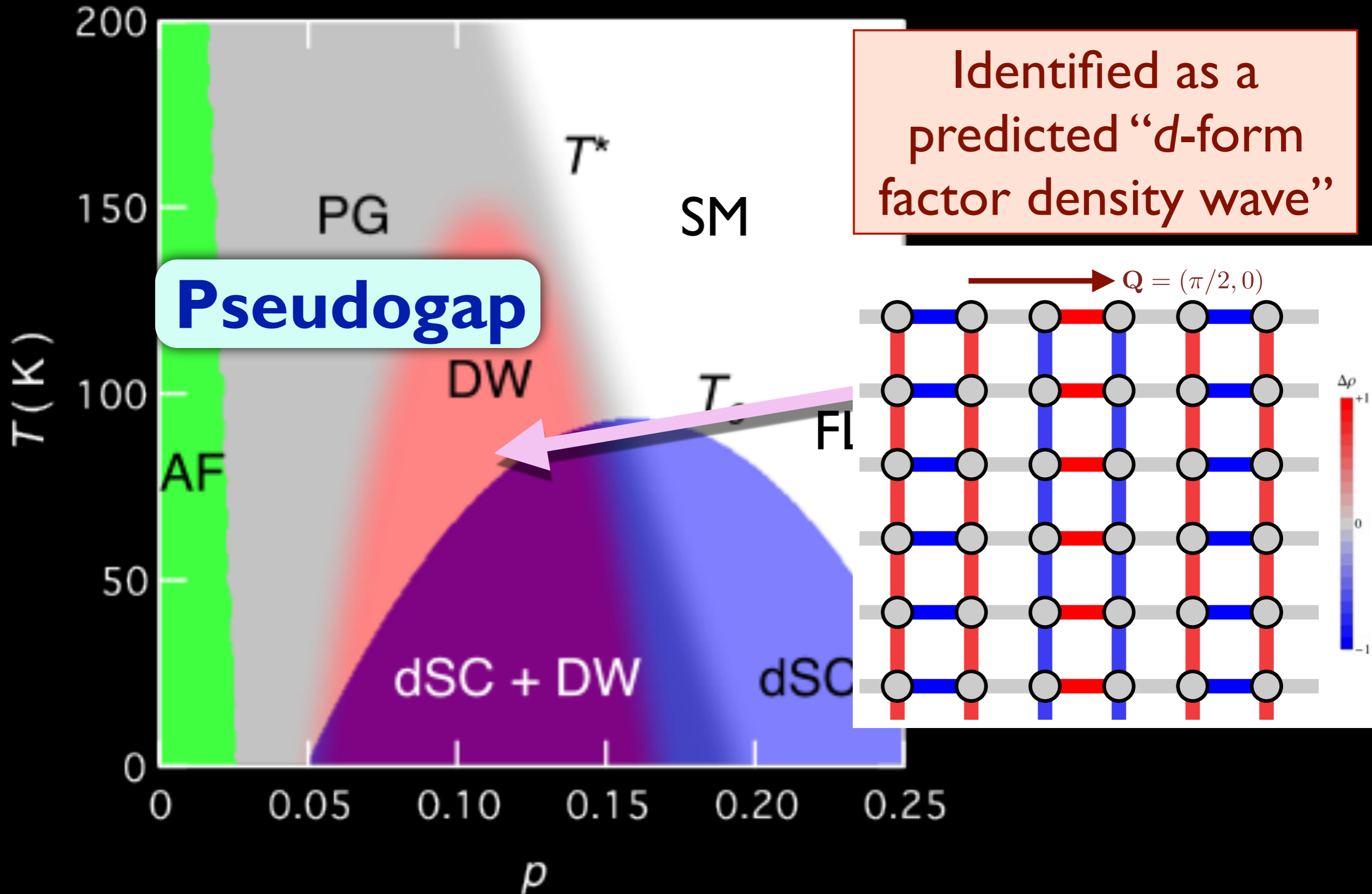
Pseudogap

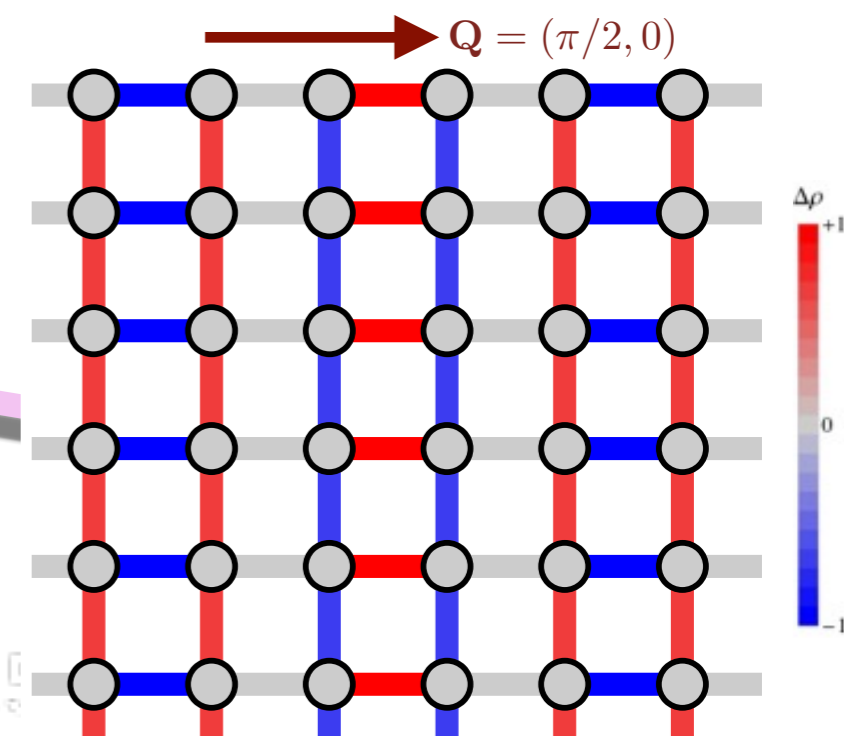
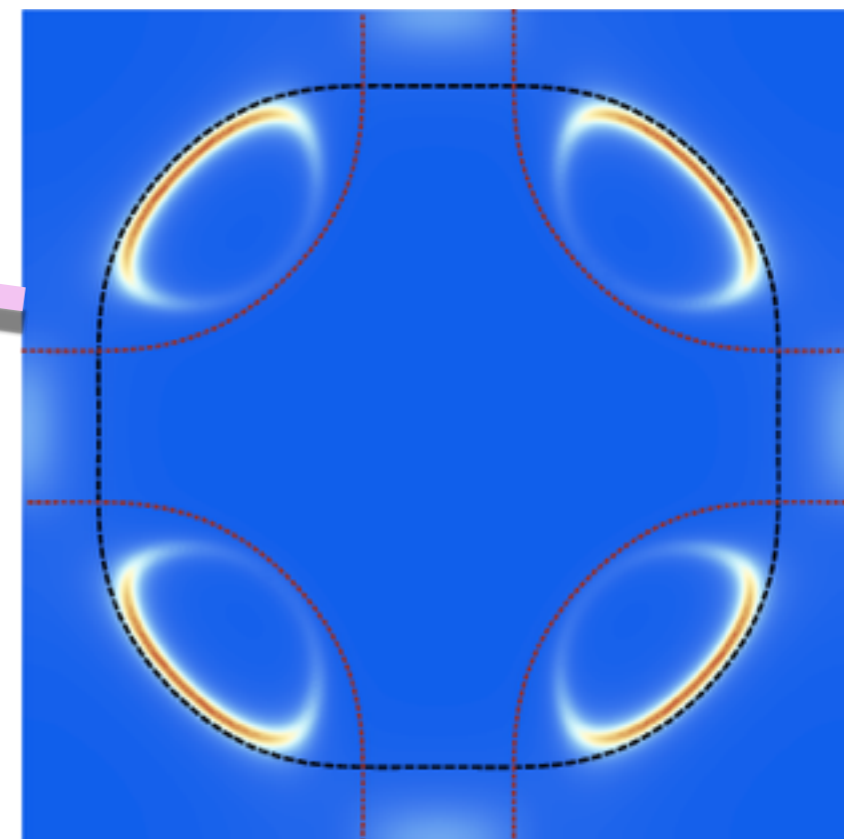
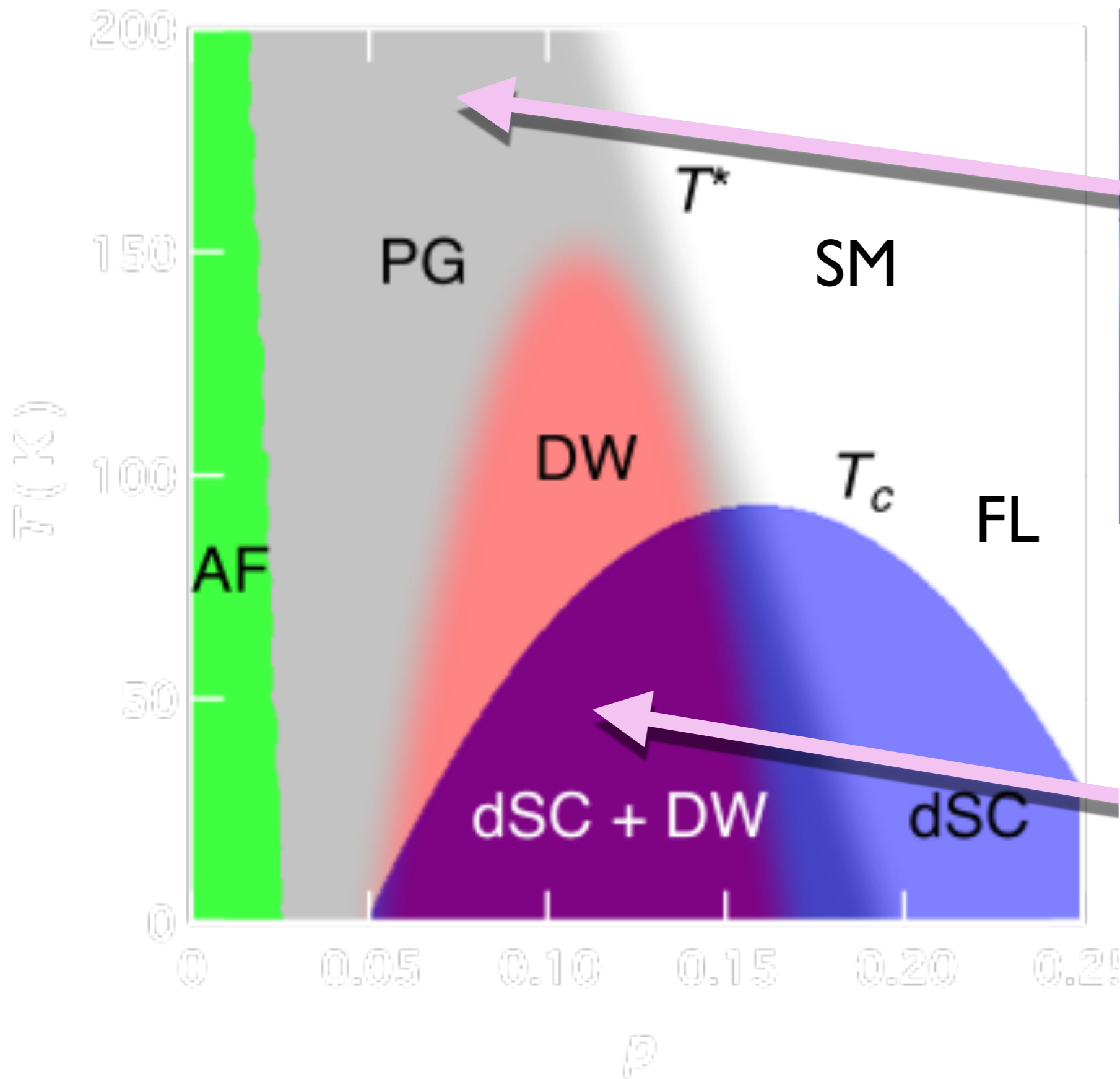
Density wave (DW) order at low T and p

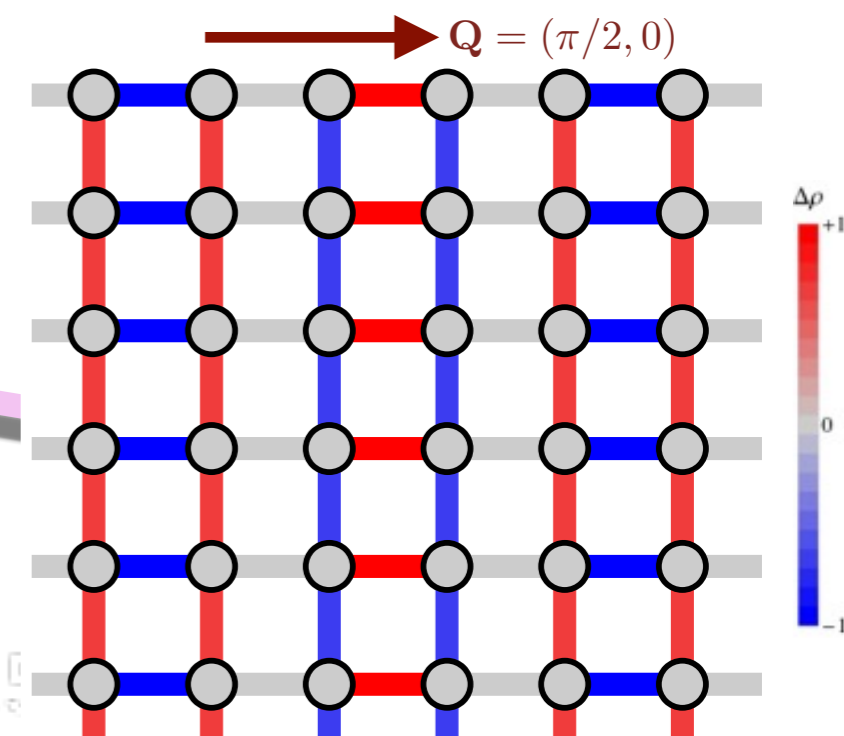
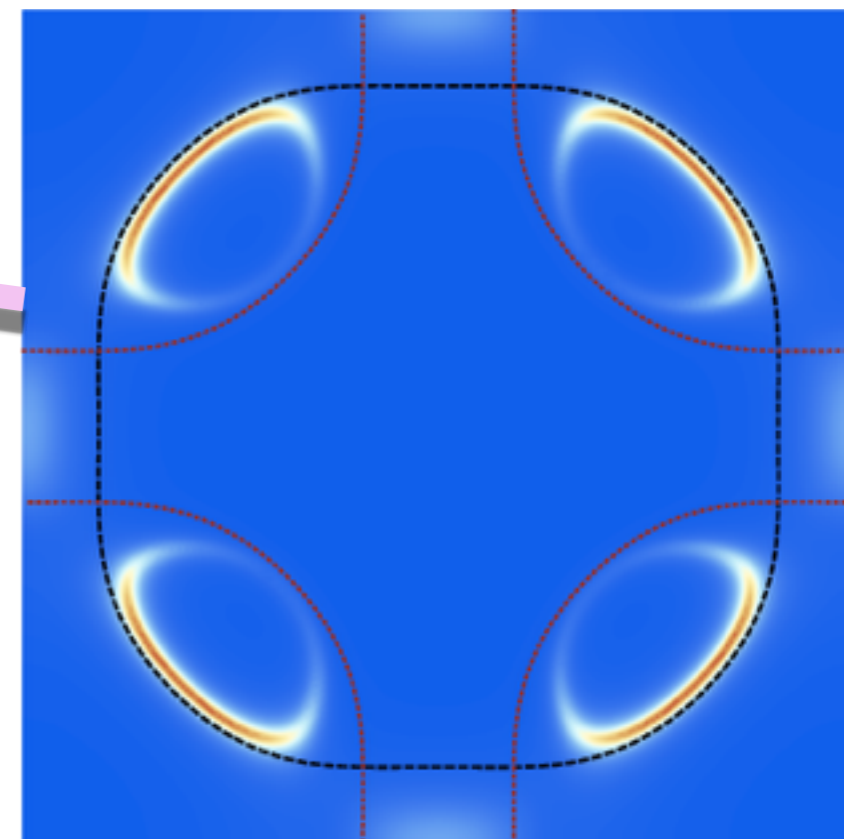
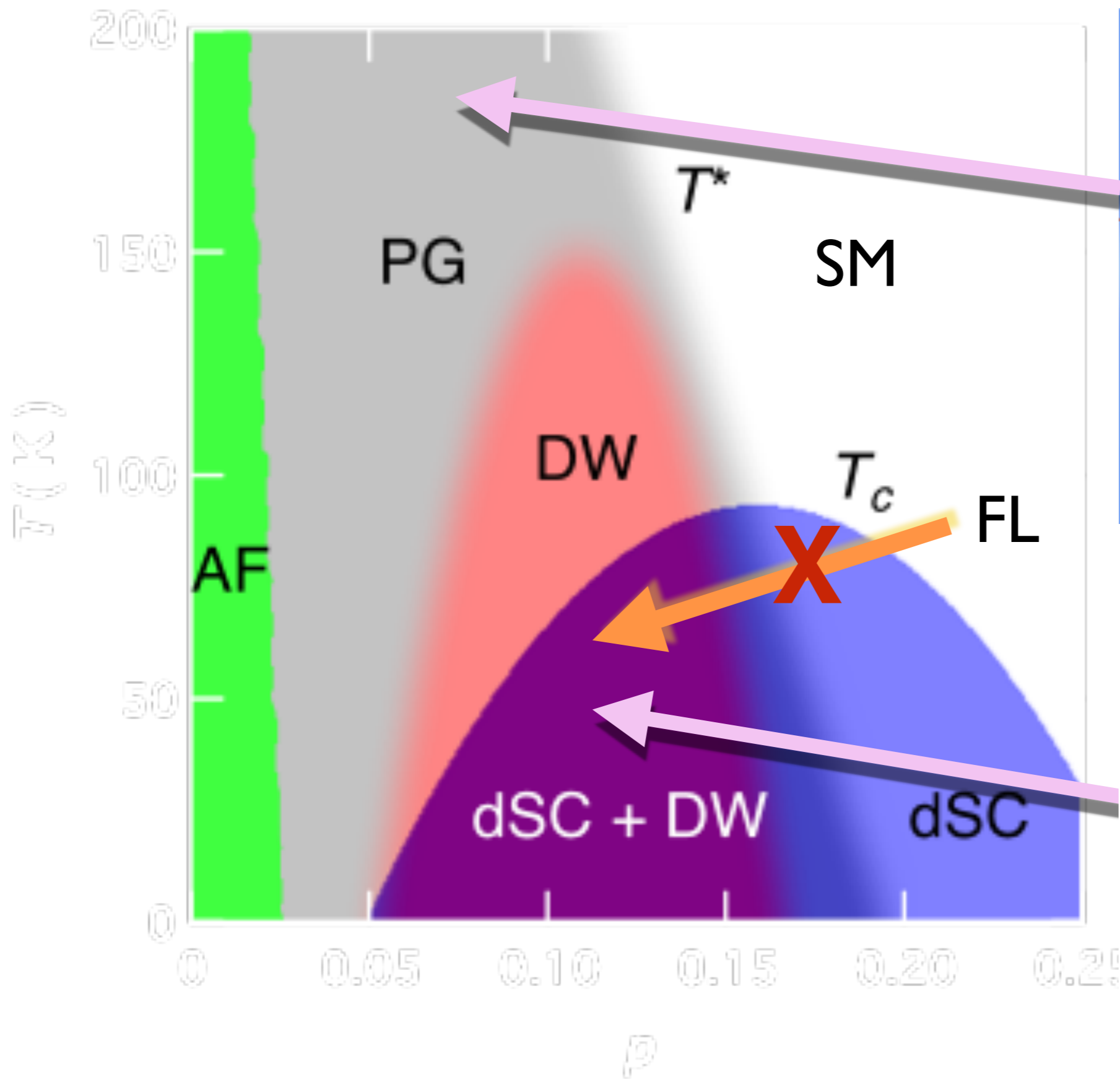


M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).

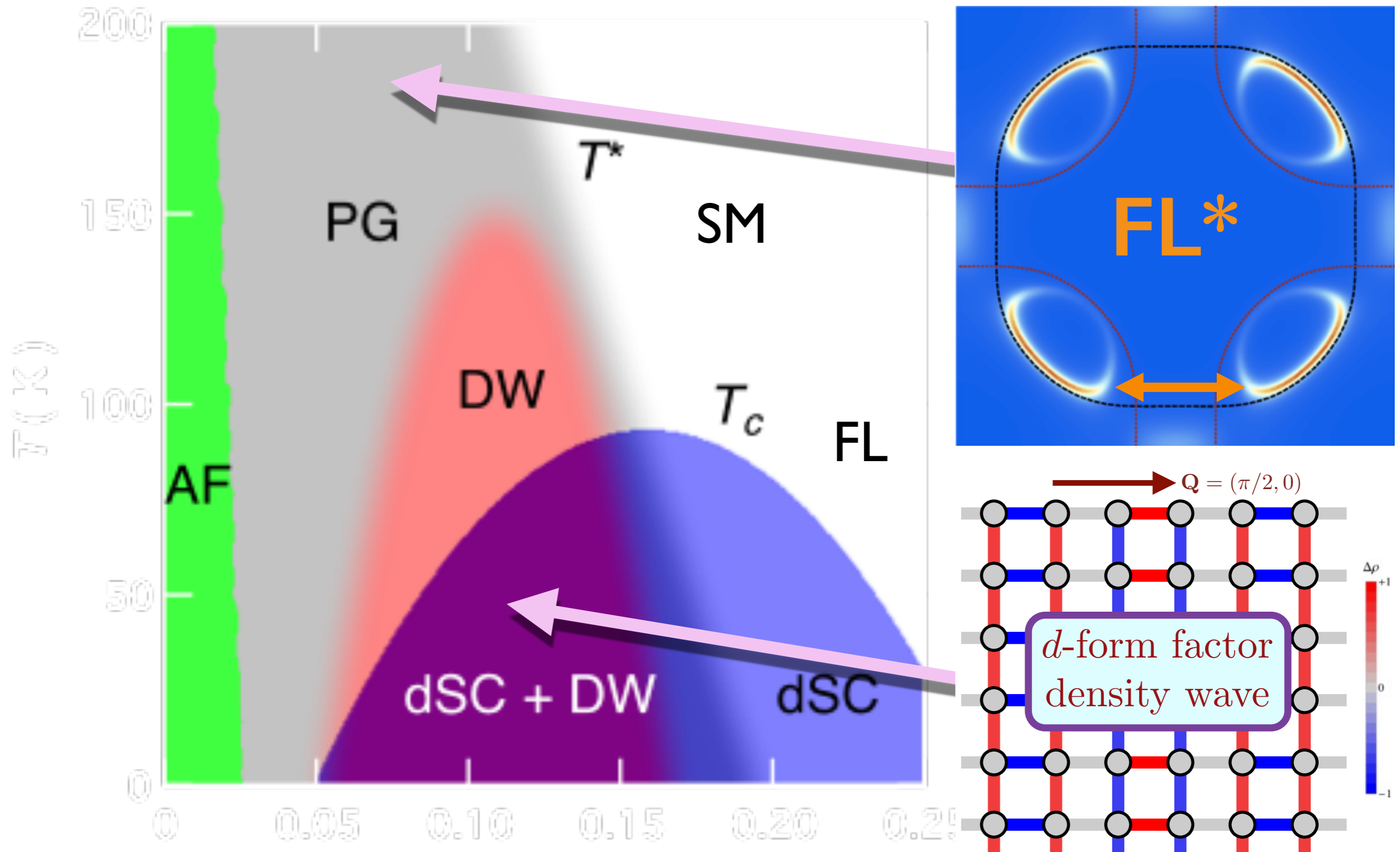
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



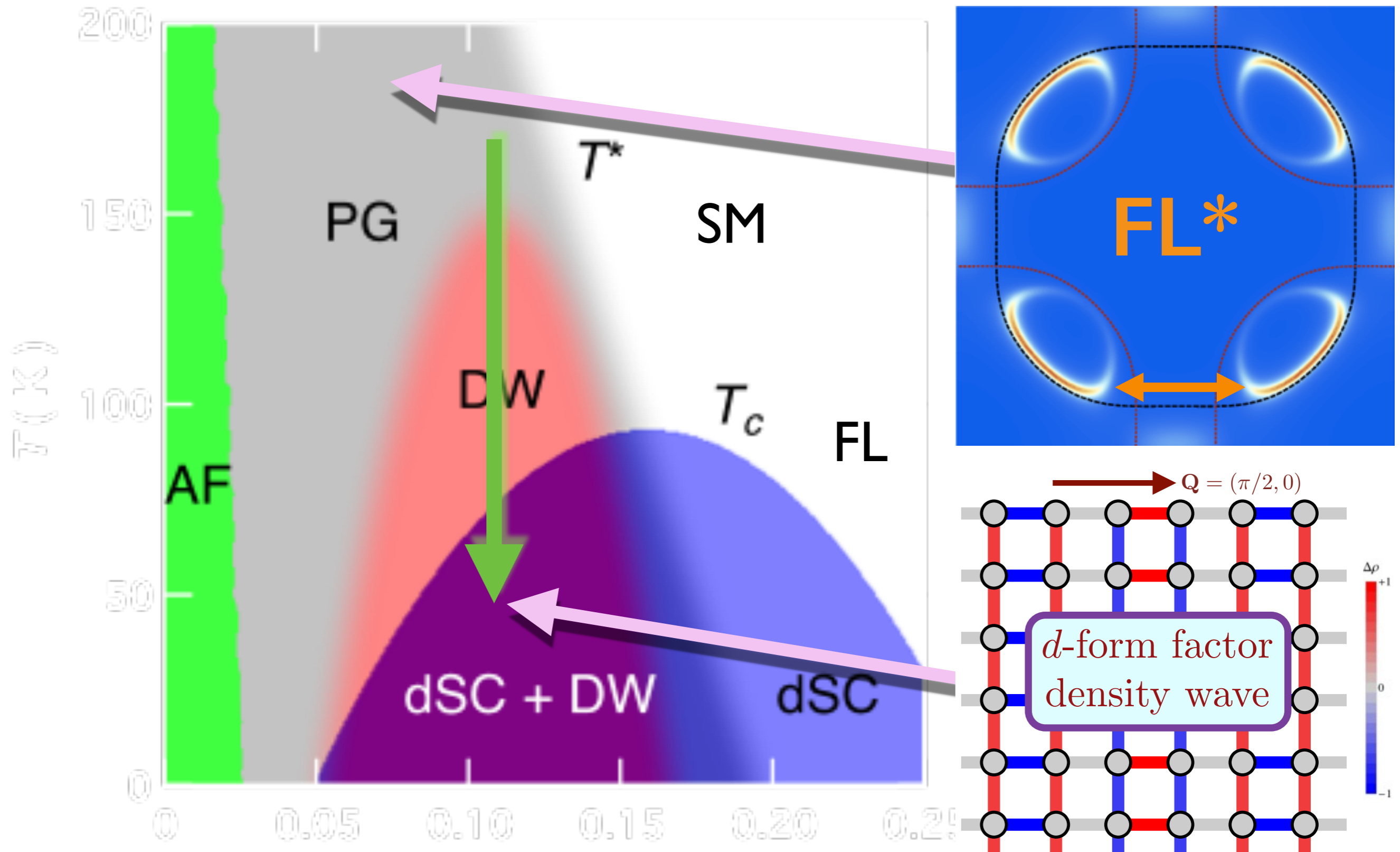




The high T FL* can help explain the “d-form factor density wave” observed at low T



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- T -independent positive Hall coefficient, R_H , corresponding to carrier density p in the higher temperature pseudogap (Ando *et al.*, PRL **92**, 197001 (2004)) and in recent measurements at high fields, low T , and around $p \approx 0.16$ in YBCO (Proust-Taillefer-UBC collaboration, Badoux *et al.*, arXiv:1511.08162).

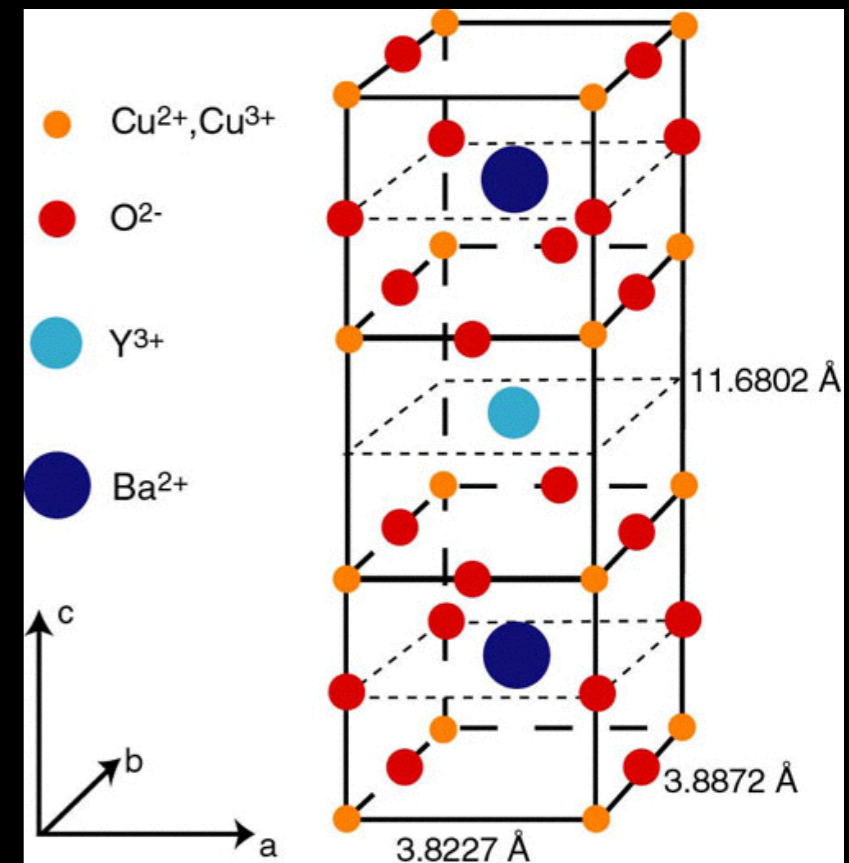
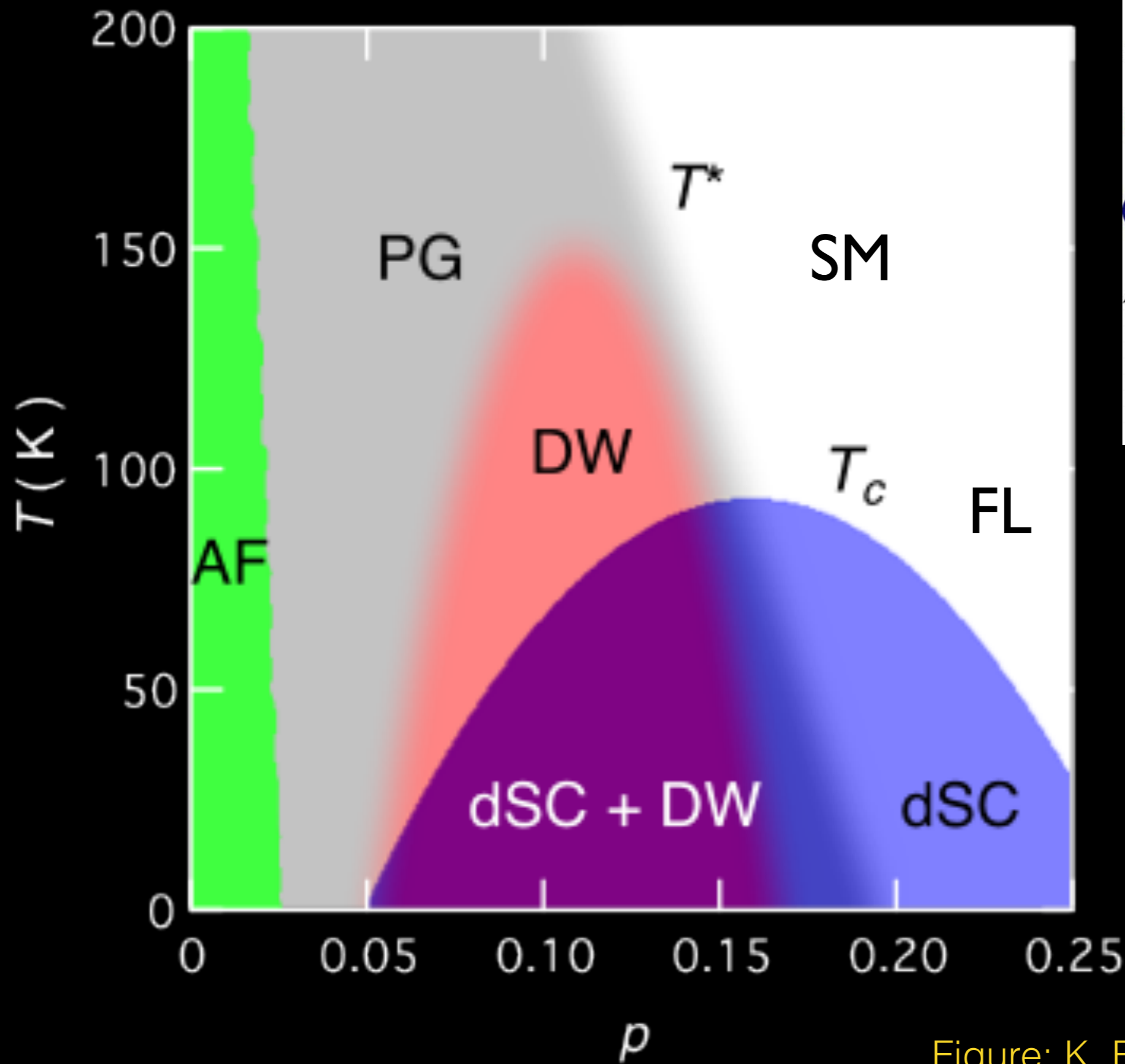
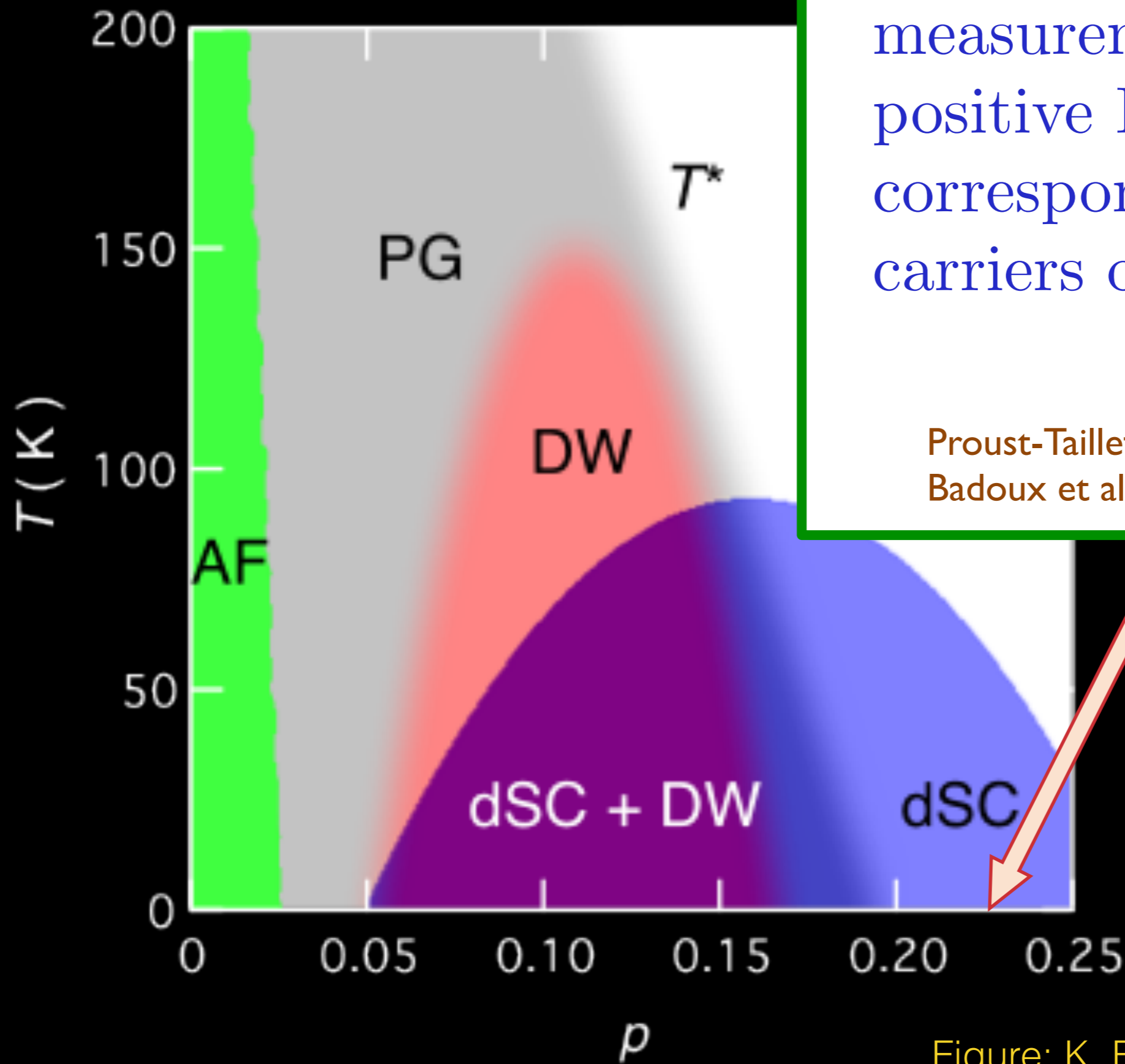


Figure: K. Fujita and J. C. Seamus Davis

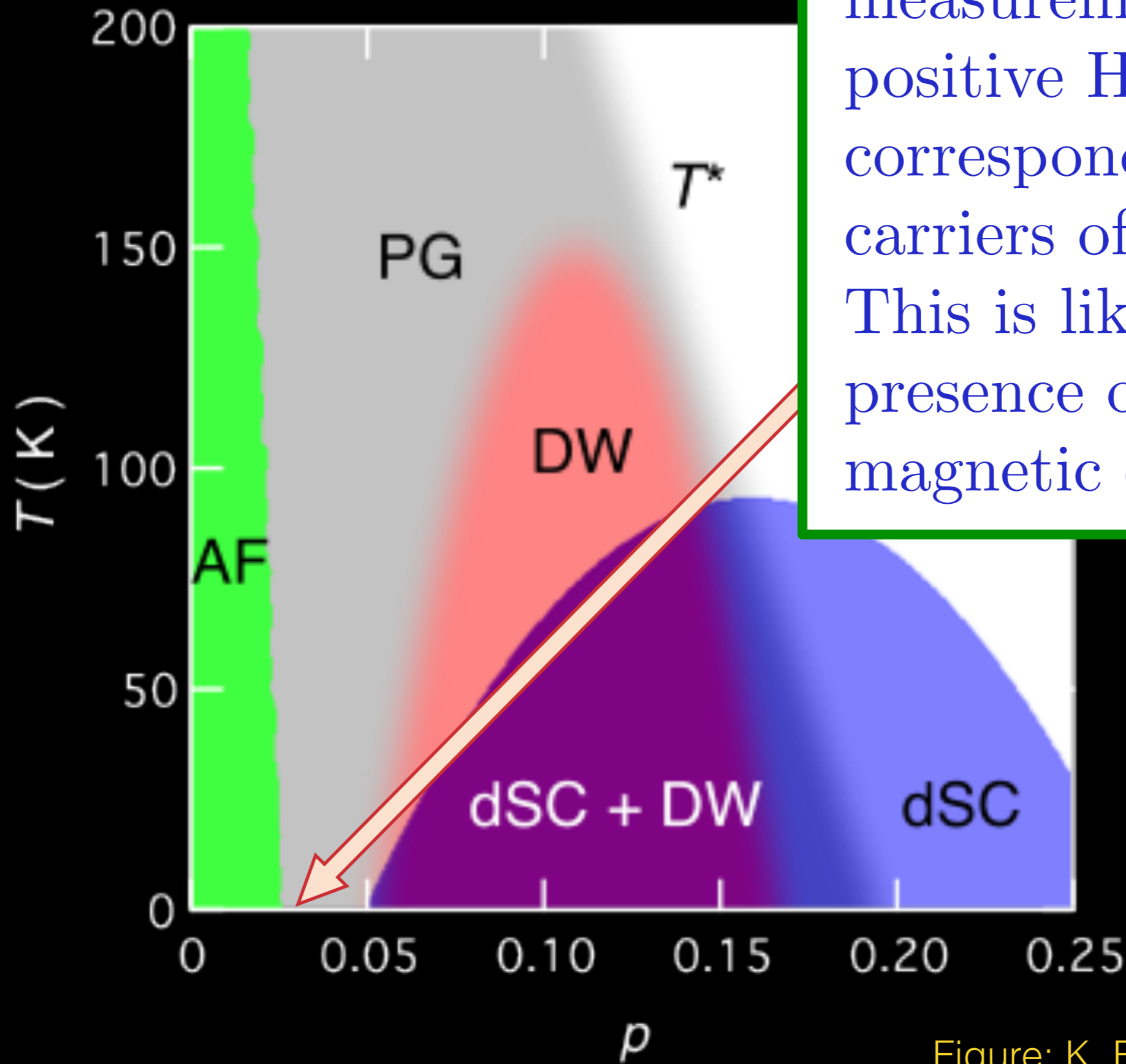


High field, low T measurements show a positive Hall co-efficient corresponding to carriers of density $1 + p$

Proust-Taillefer-UBC collaboration,
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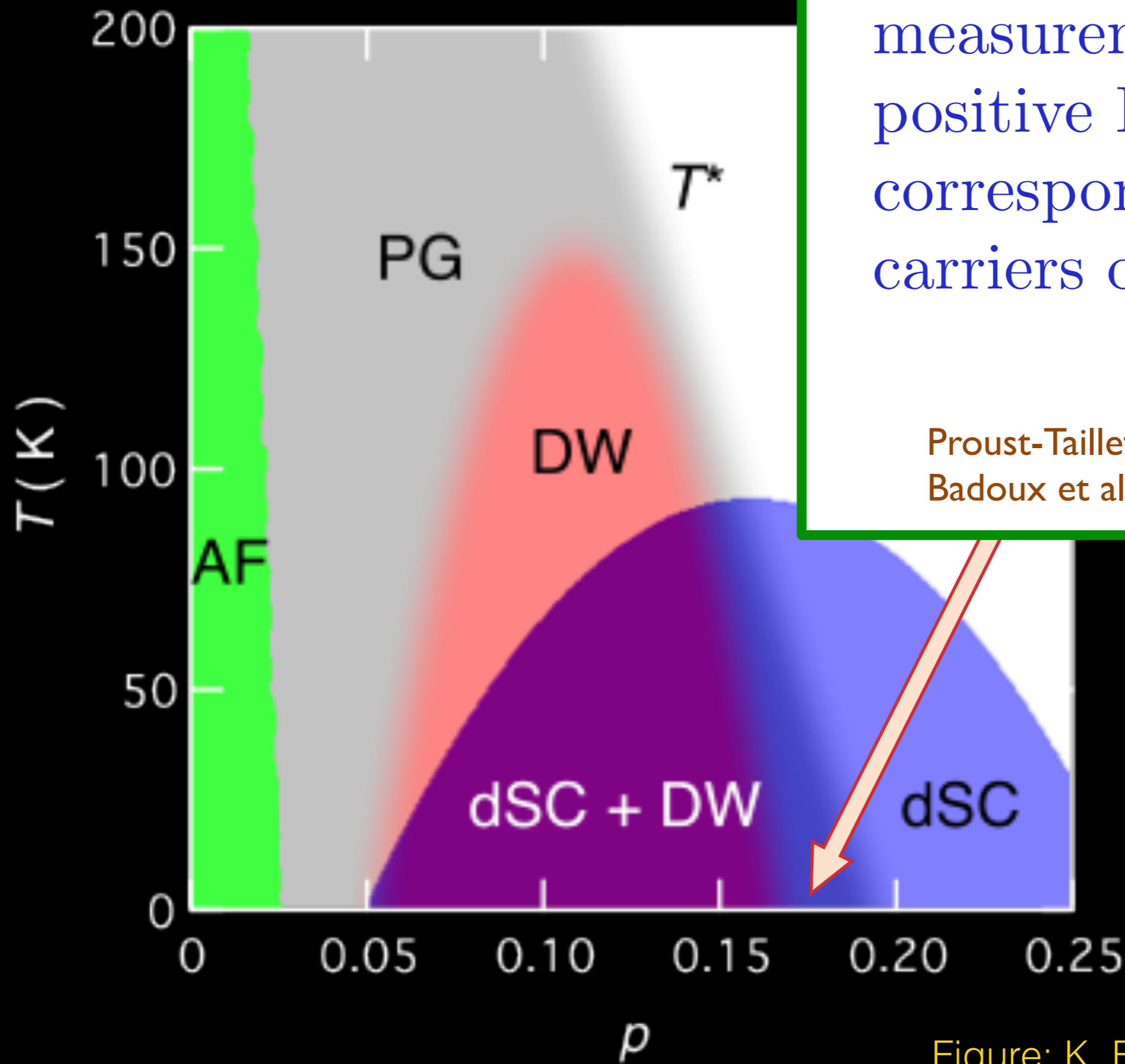
$D \propto \alpha_2 \cup \alpha_3 \cup \alpha_6 + x$

Figure: K. Fujita and J. C. Seamus Davis



High field, low T measurements show a positive Hall co-efficient corresponding to carriers of density p . This is likely due to the presence of antiferromagnetic order.

Figure: K. Fujita and J. C. Seamus Davis



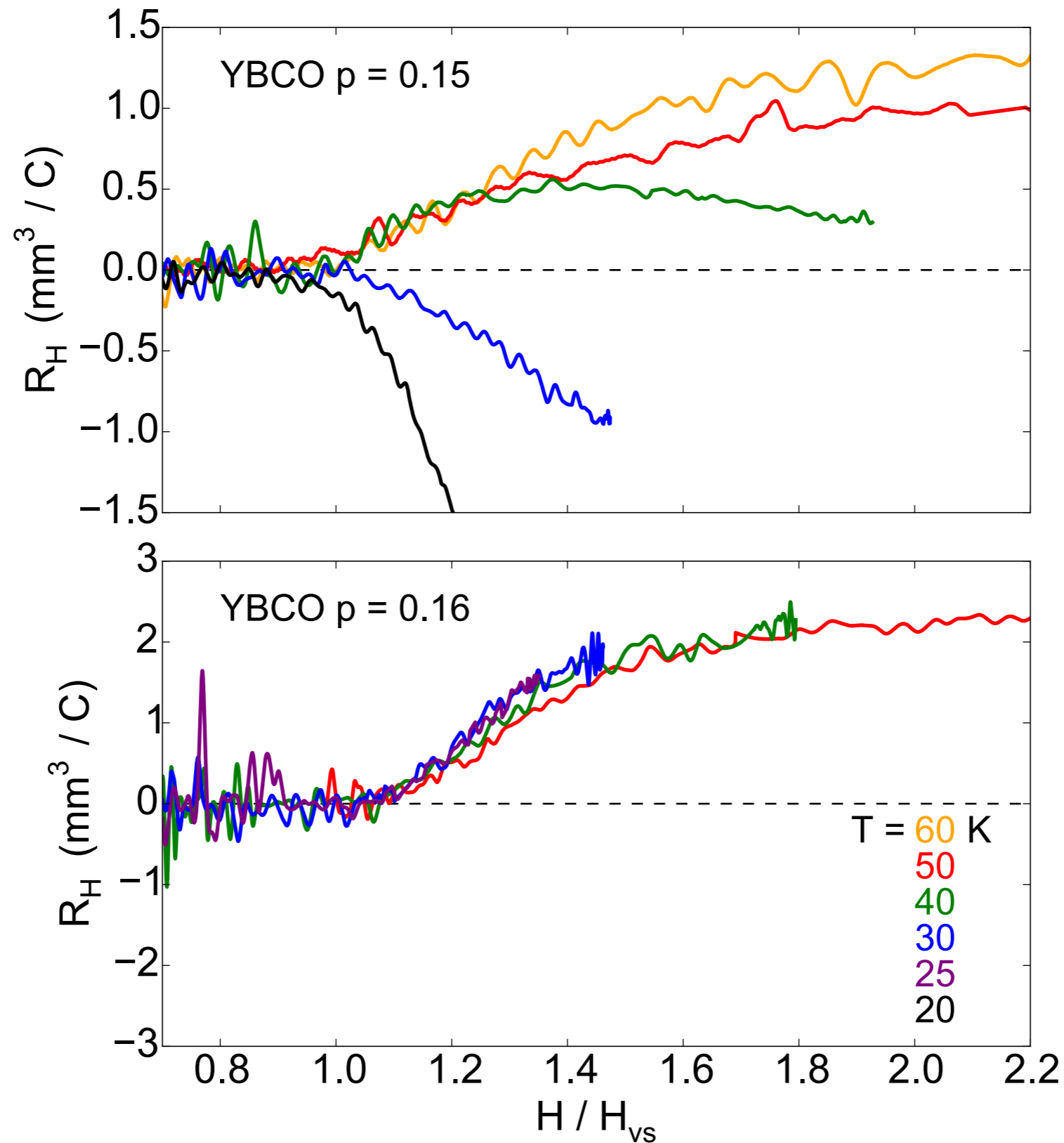
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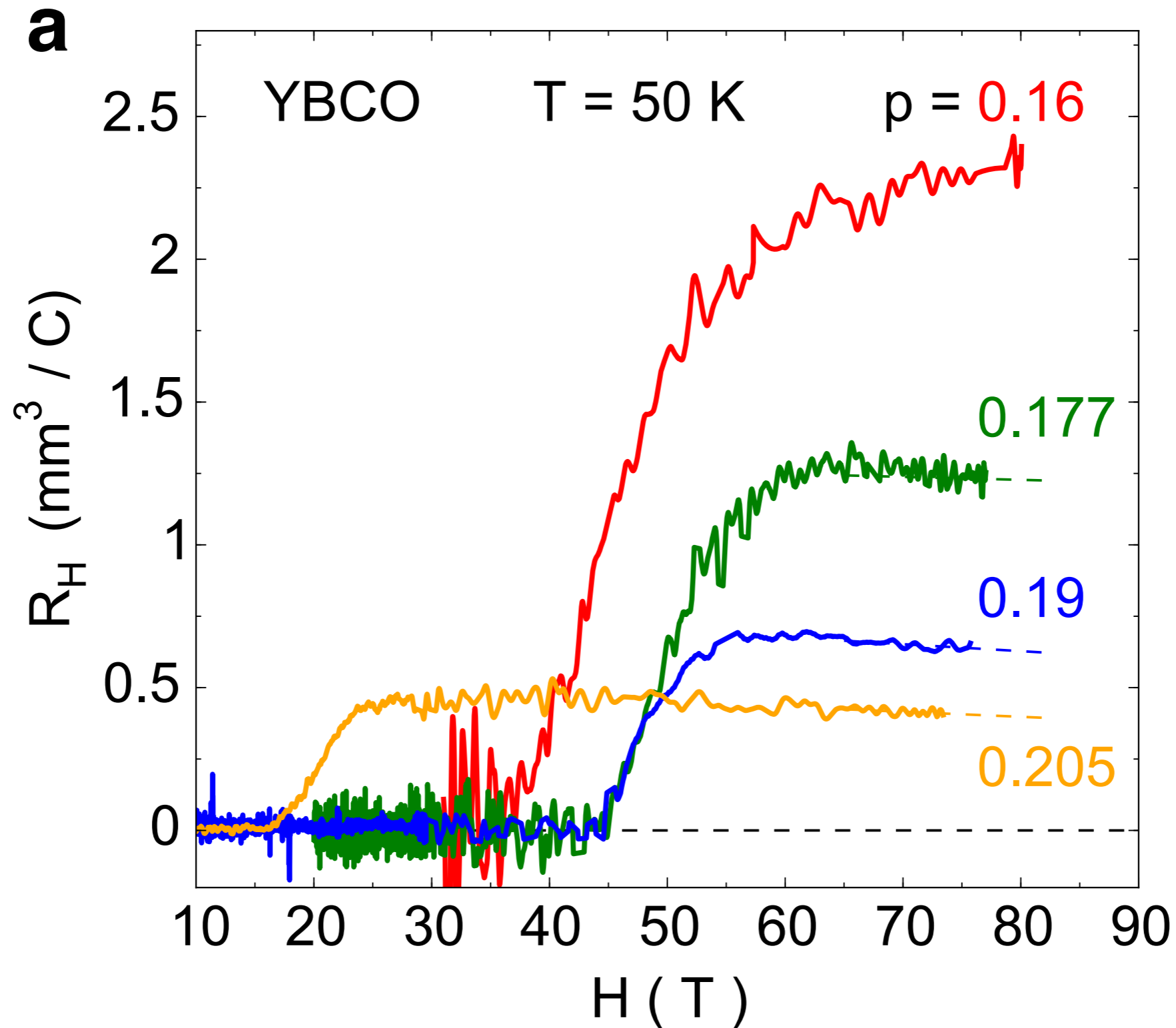
$T = Da^2 \cup a_3 \cup 6 + x$

Figure: K. Fujita and J. C. Seamus Davis

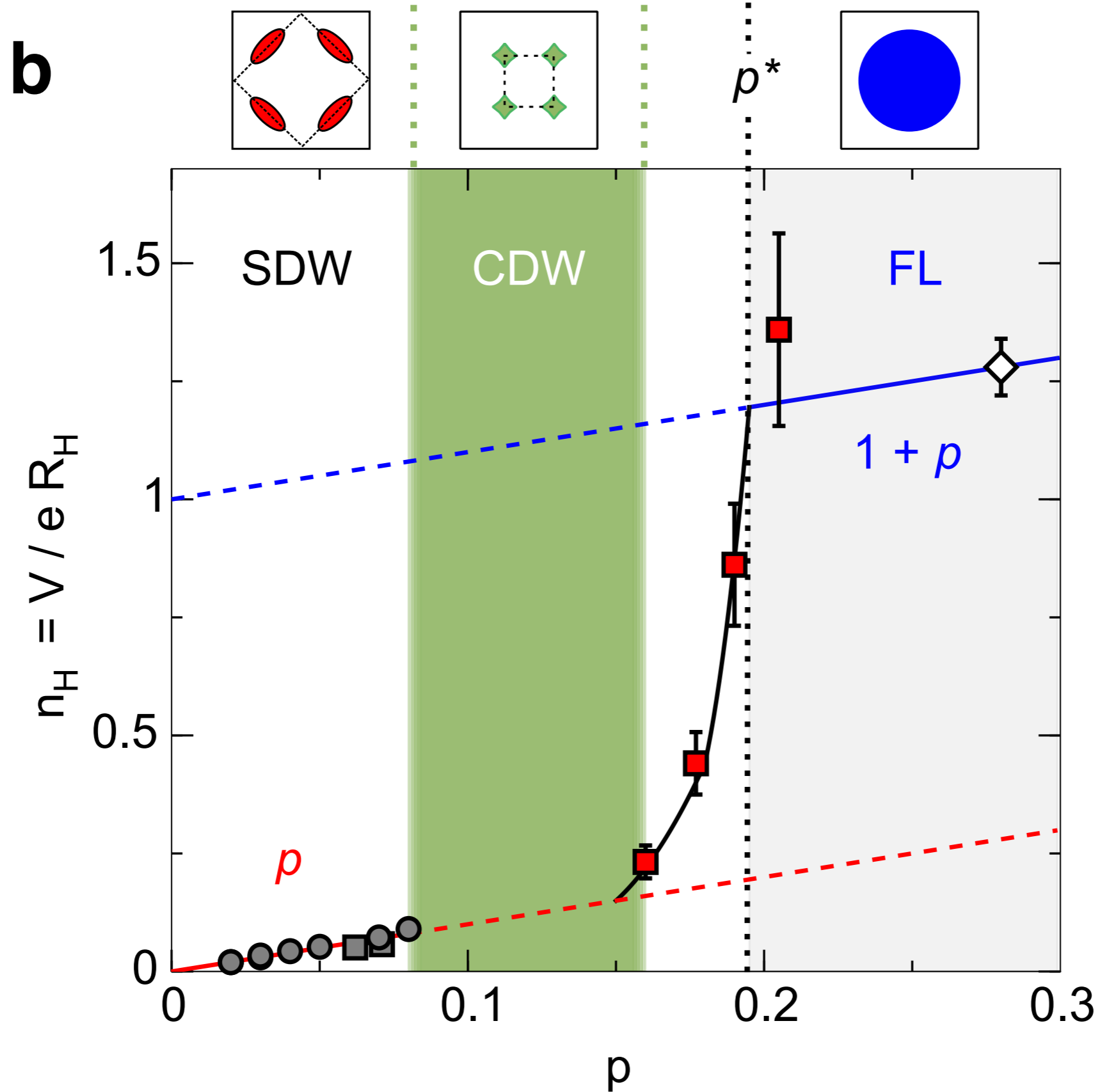
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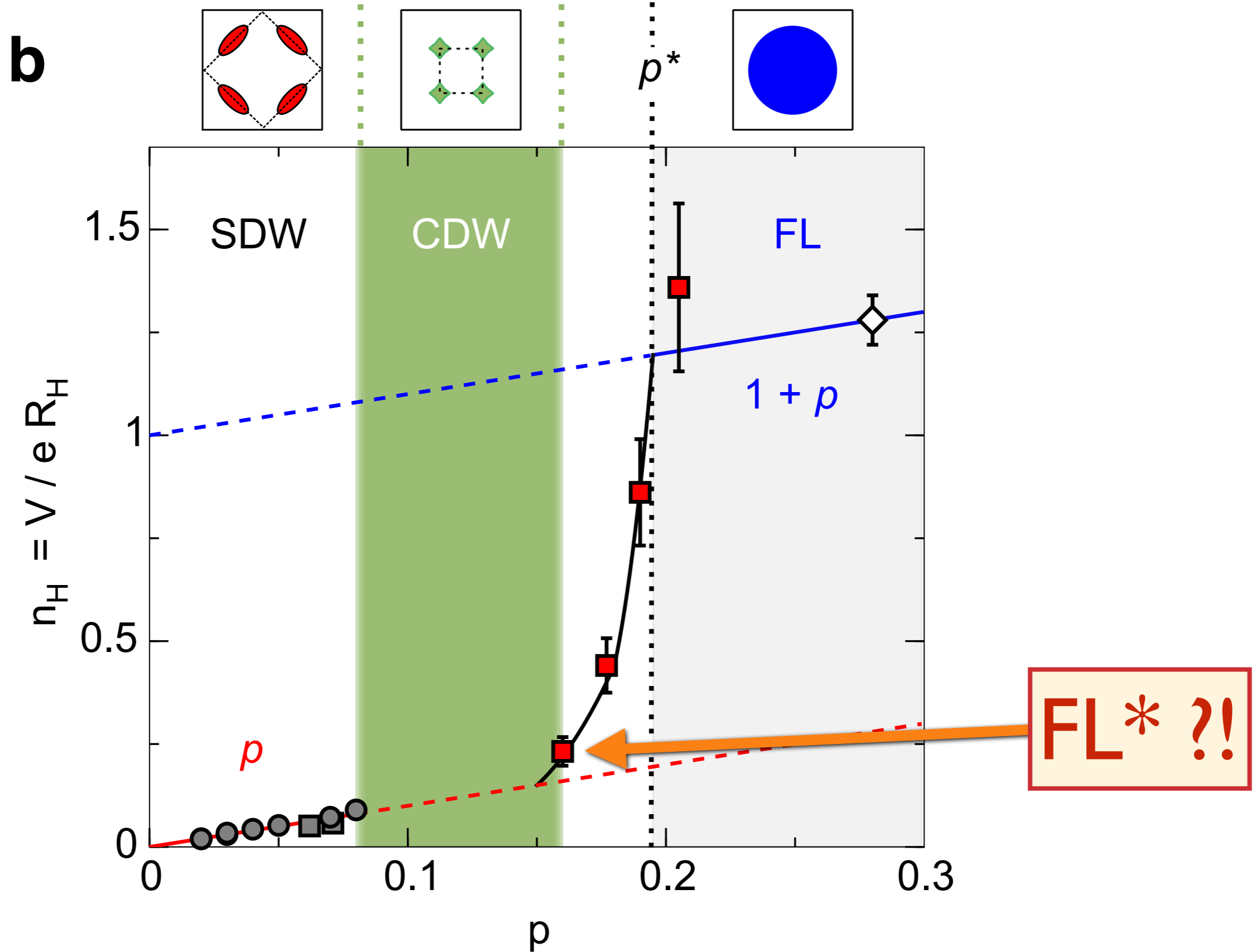
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Fractionalized Fermi liquid (FL*)

We have described a metal with:

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There is a general and fundamental relationship between these two characteristics. Promising indications that such a metal describes the pseudogap of the cuprate superconductors