Bekenstein-Hawking entropy from strange metals

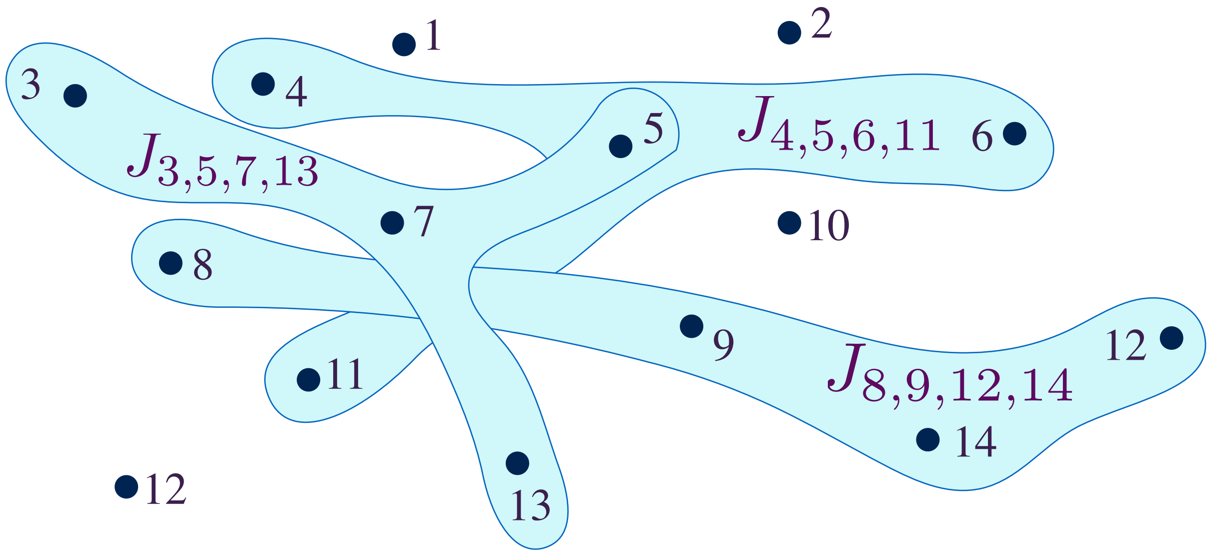
Perimeter Institute
June 16, 2015

Subir Sachdev

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

PHYSICS
VE RI TAS
HARVARD

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

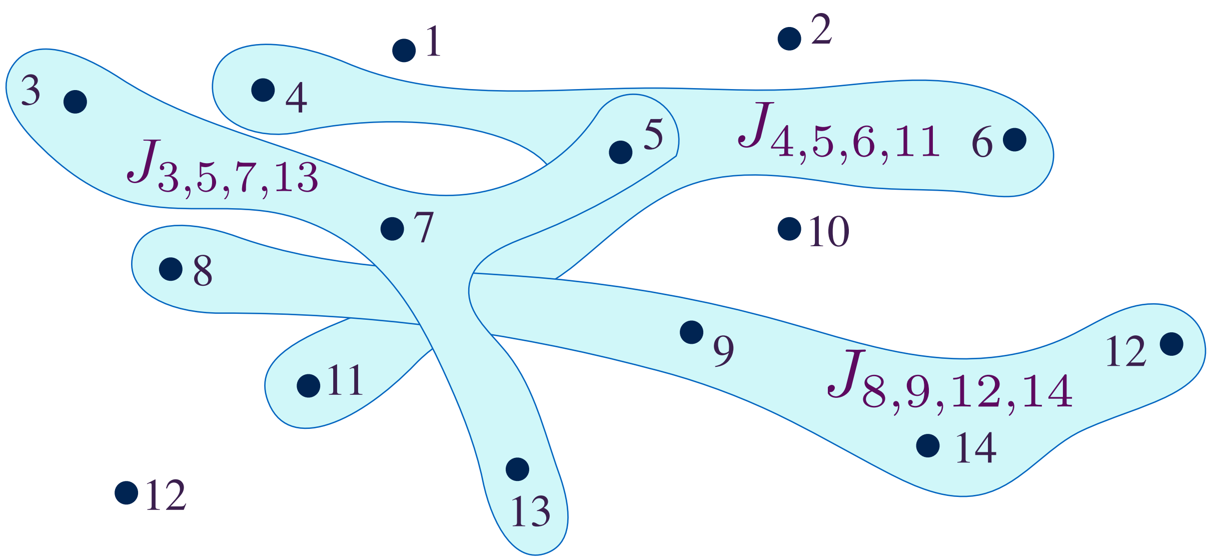


$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

A mean-field model of a strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



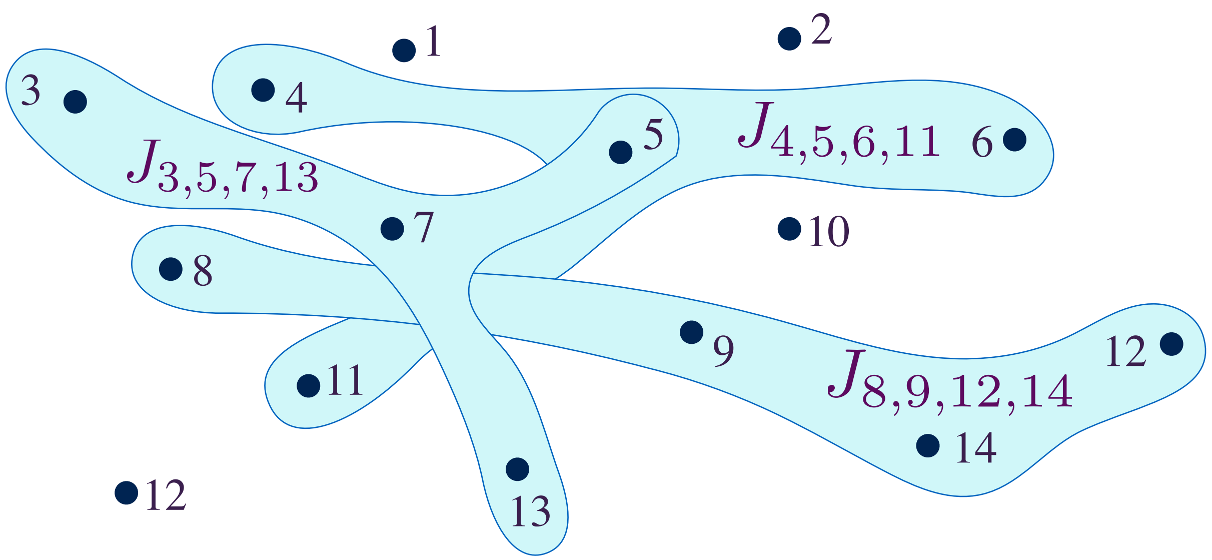
$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

$$-\text{Im}\left[G^R(\omega)\right] \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



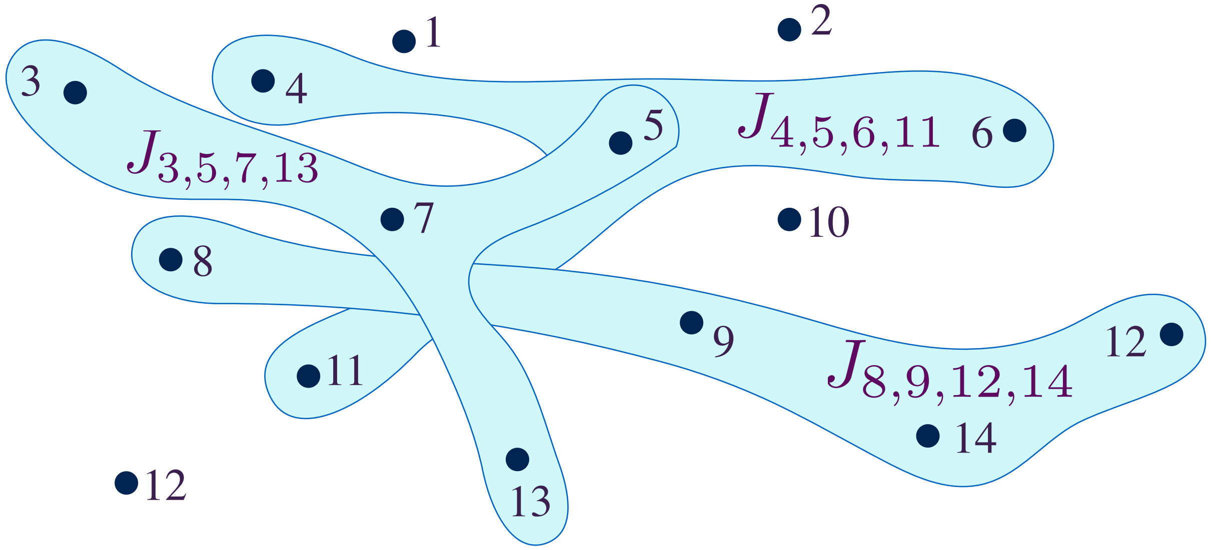
$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

A. Georges, O. Parcollet, and S. Sachdev
Phys. Rev. B **63**, 134406 (2001)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\,\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}}\, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

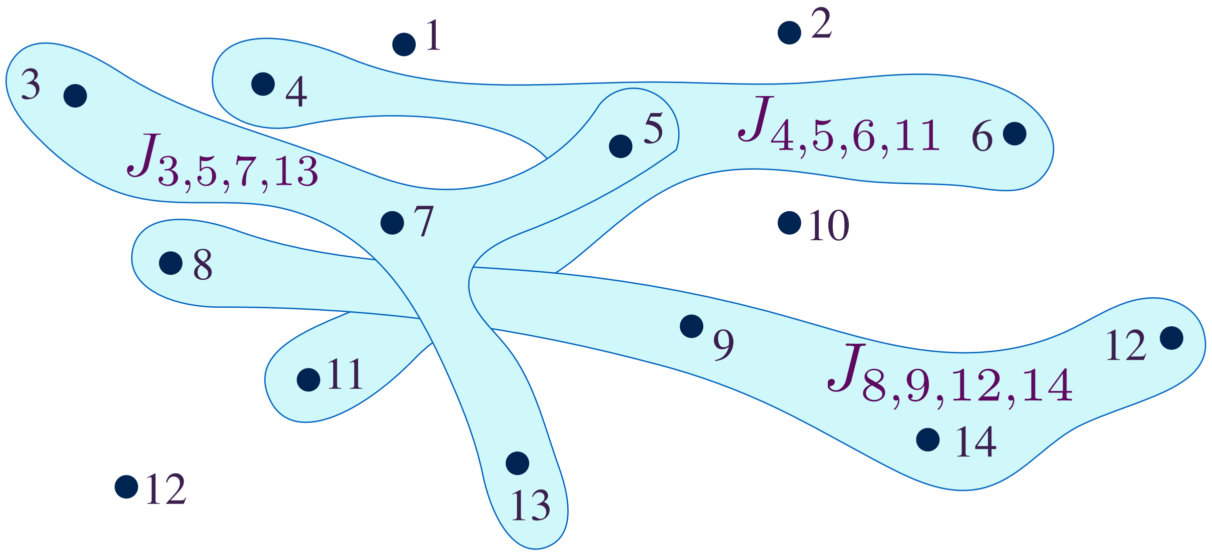
Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
Phys. Rev. B **58**, 3794 (1998)

A. Georges, O. Parcollet, and S. Sachdev
Phys. Rev. B **63**, 134406 (2001)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N}\sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}}\, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

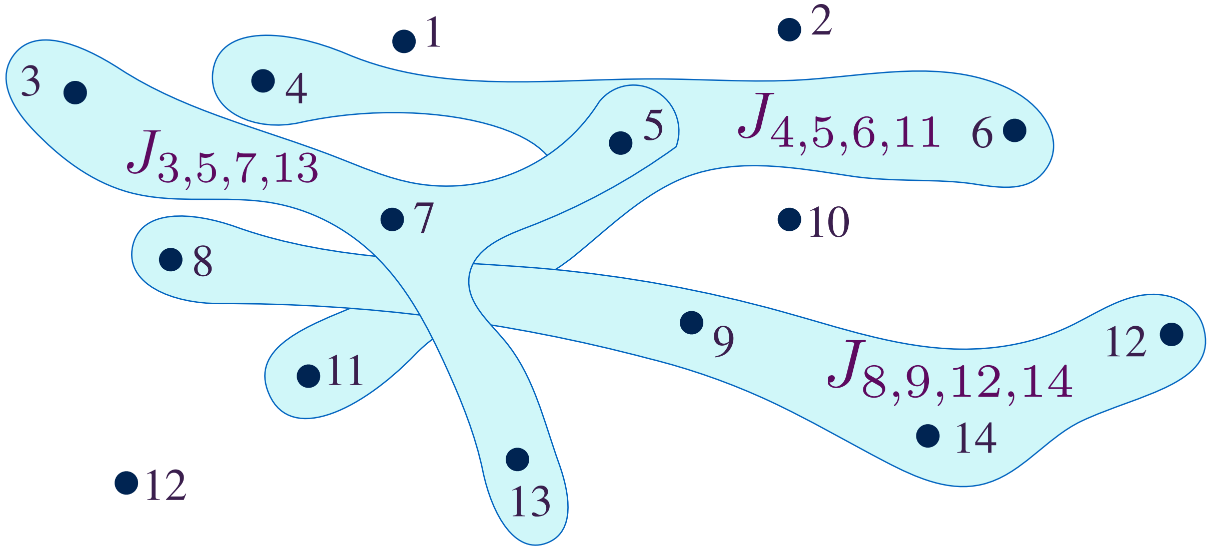
Known ‘equation of state’ determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

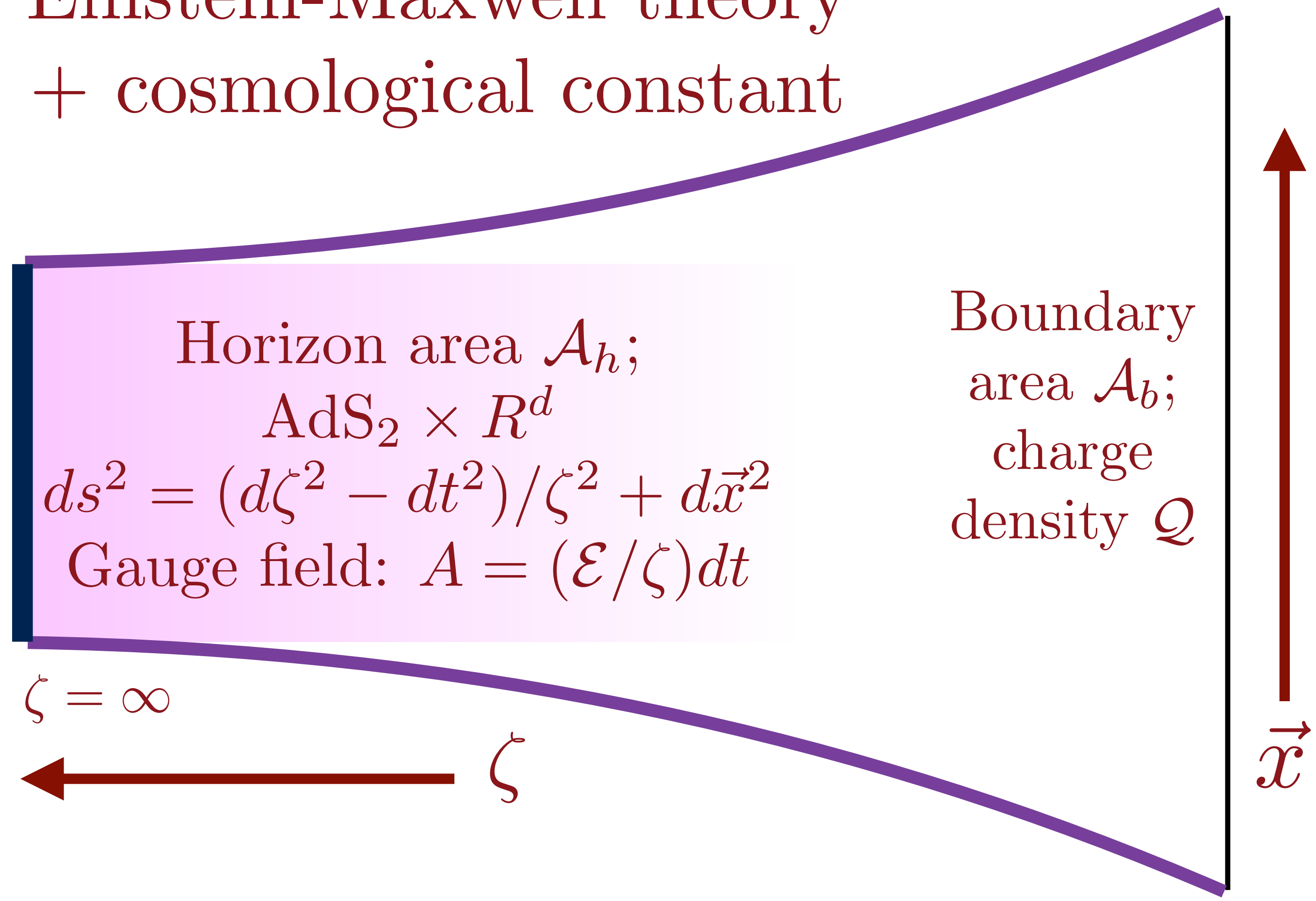
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

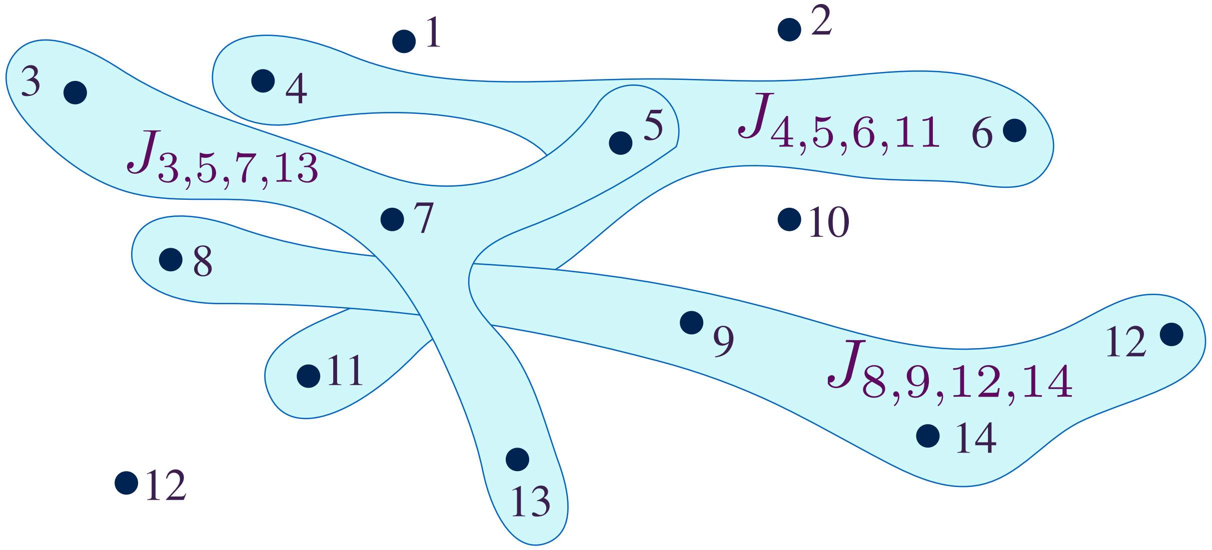
$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant



A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers
Phys. Rev. D **60**, 064018 (1999)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



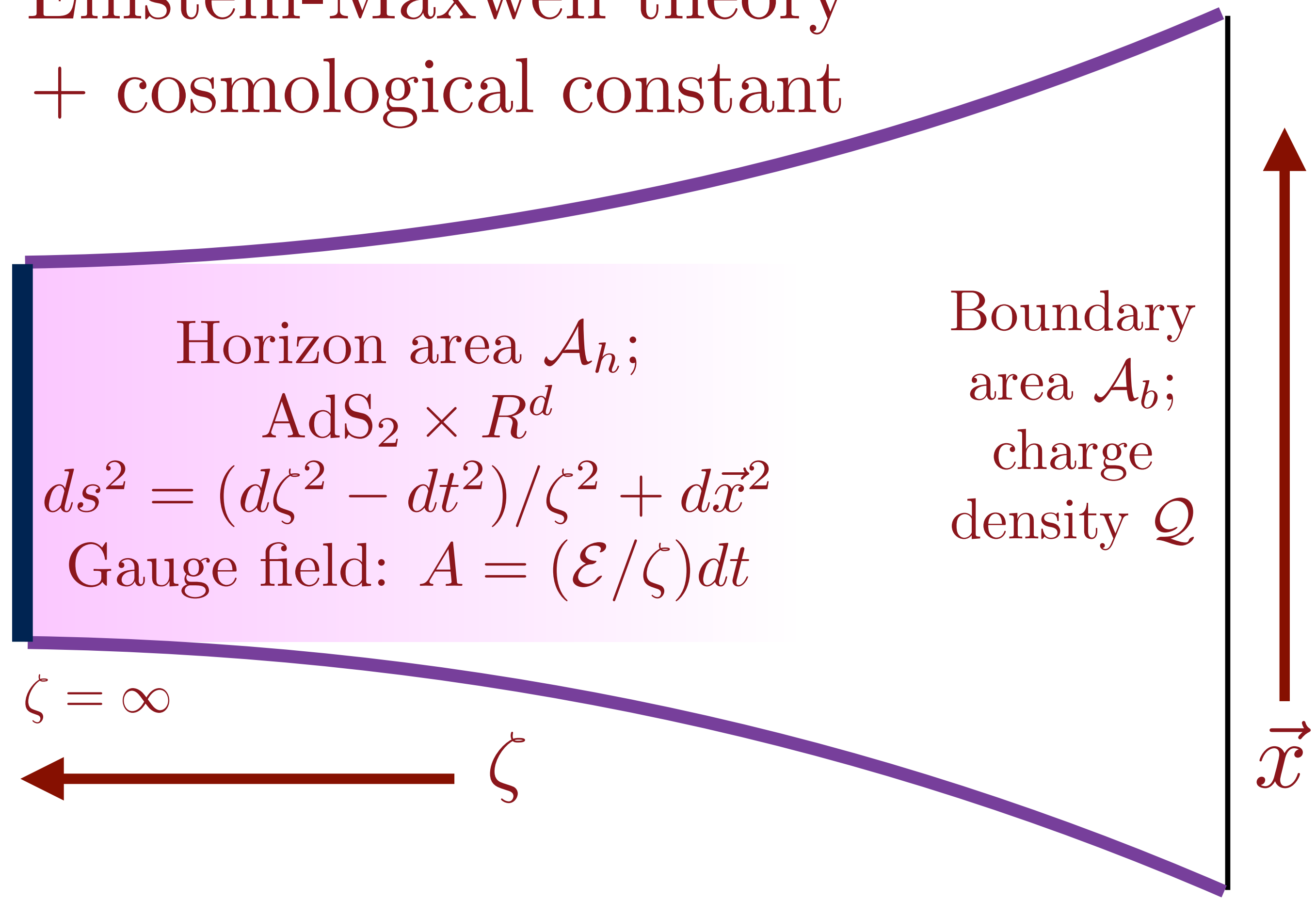
$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

> Microscopic zero temperature entropy density, $\mathcal{S}$, obeys
> $$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant

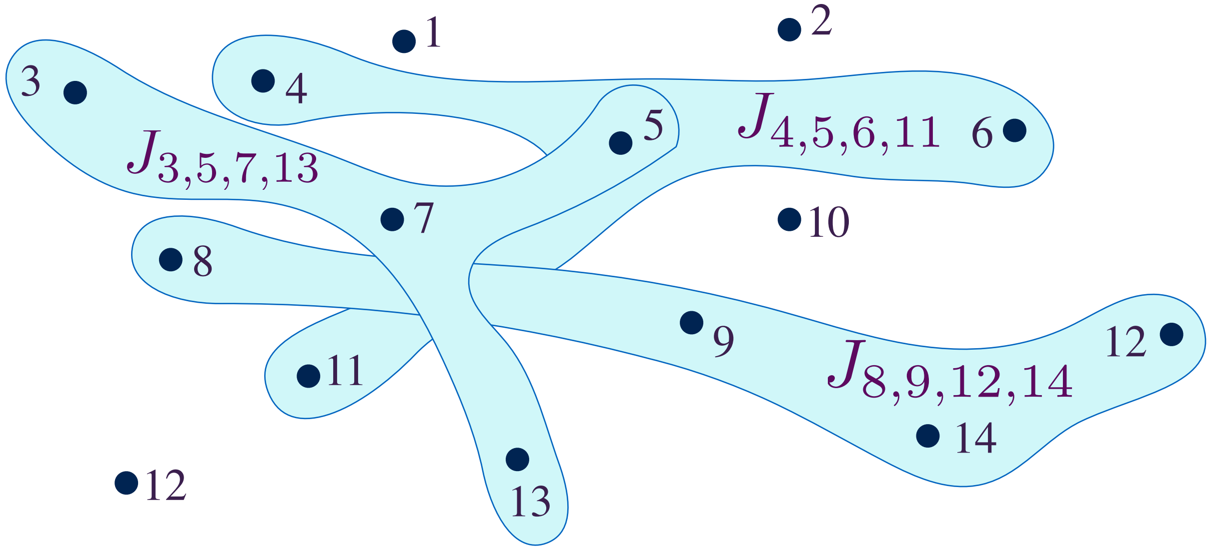


$$\mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi$$

$$-\left\langle \psi(\tau) \overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh
Phys. Rev. D **83**, 125002 (2011)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

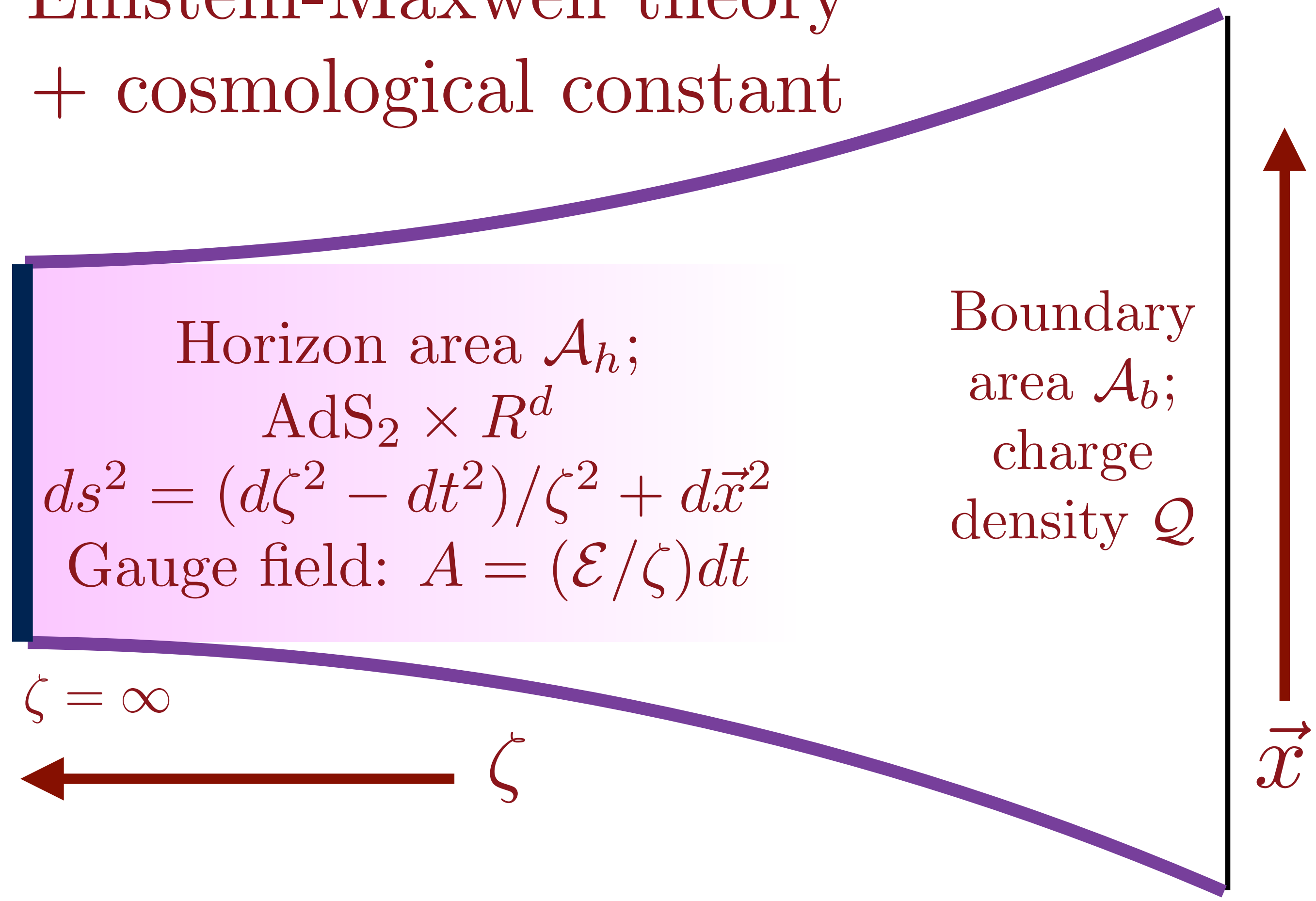
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant



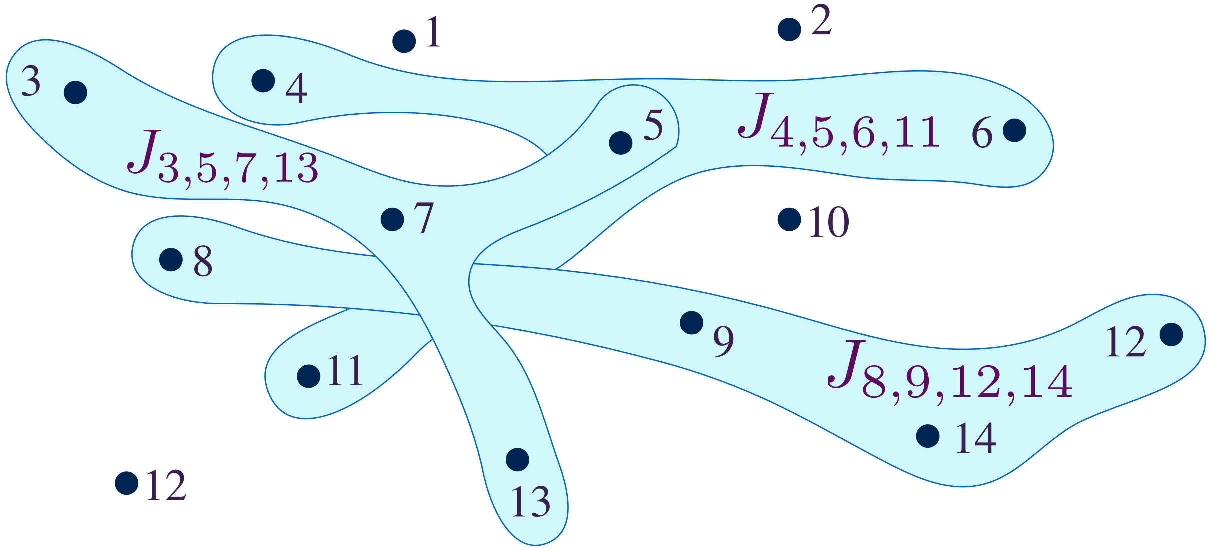
$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

$$-\left\langle \psi(\tau)\overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

S. Sachdev, arXiv:1506.05111

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

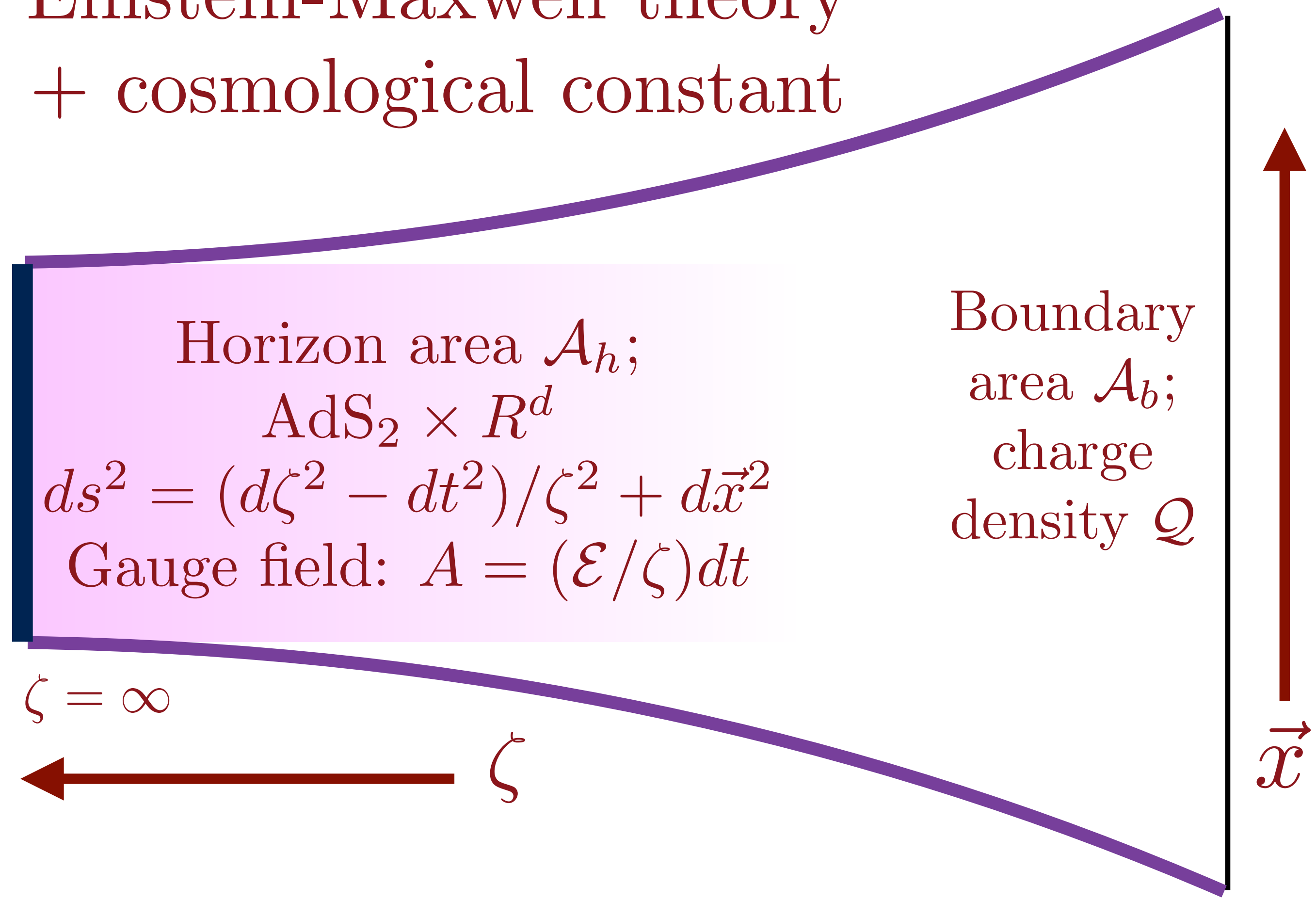
$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi$$

$$-\left\langle \psi(\tau) \overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \ \tau > 0 \\ e^{-2\pi\mathcal{E}} \, |\tau|^{-1/2} & , \ \tau < 0. \end{cases}$$

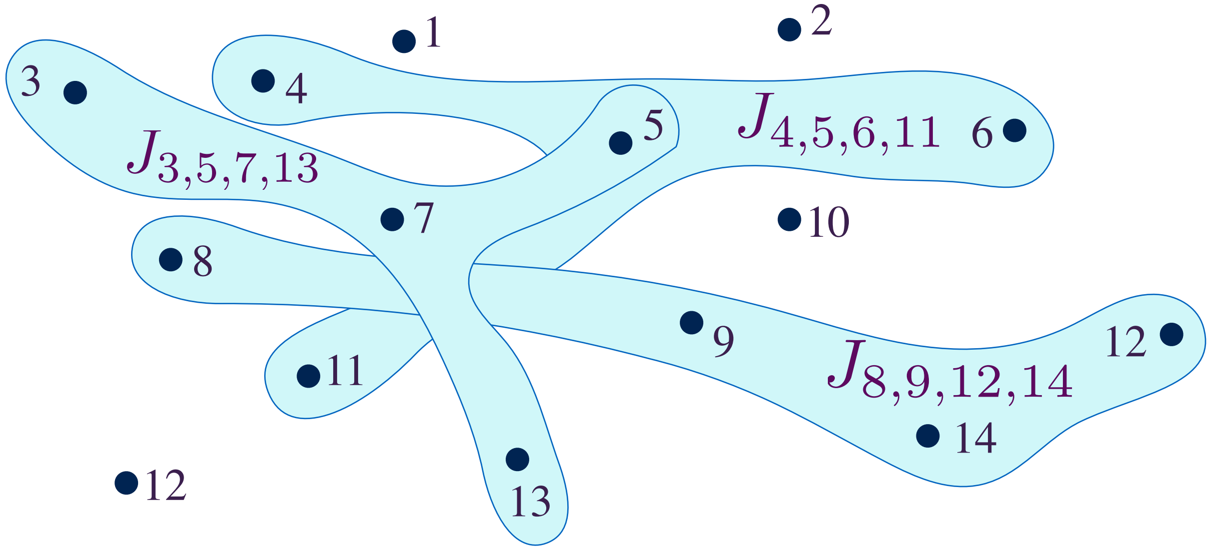
'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical GR) yields

$$\frac{1}{\mathcal{A}_b} \frac{\partial \mathcal{A}_h}{\partial \mathcal{Q}} = 8\pi G_N \mathcal{E}$$

S. Sachdev, arXiv:1506.05111

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

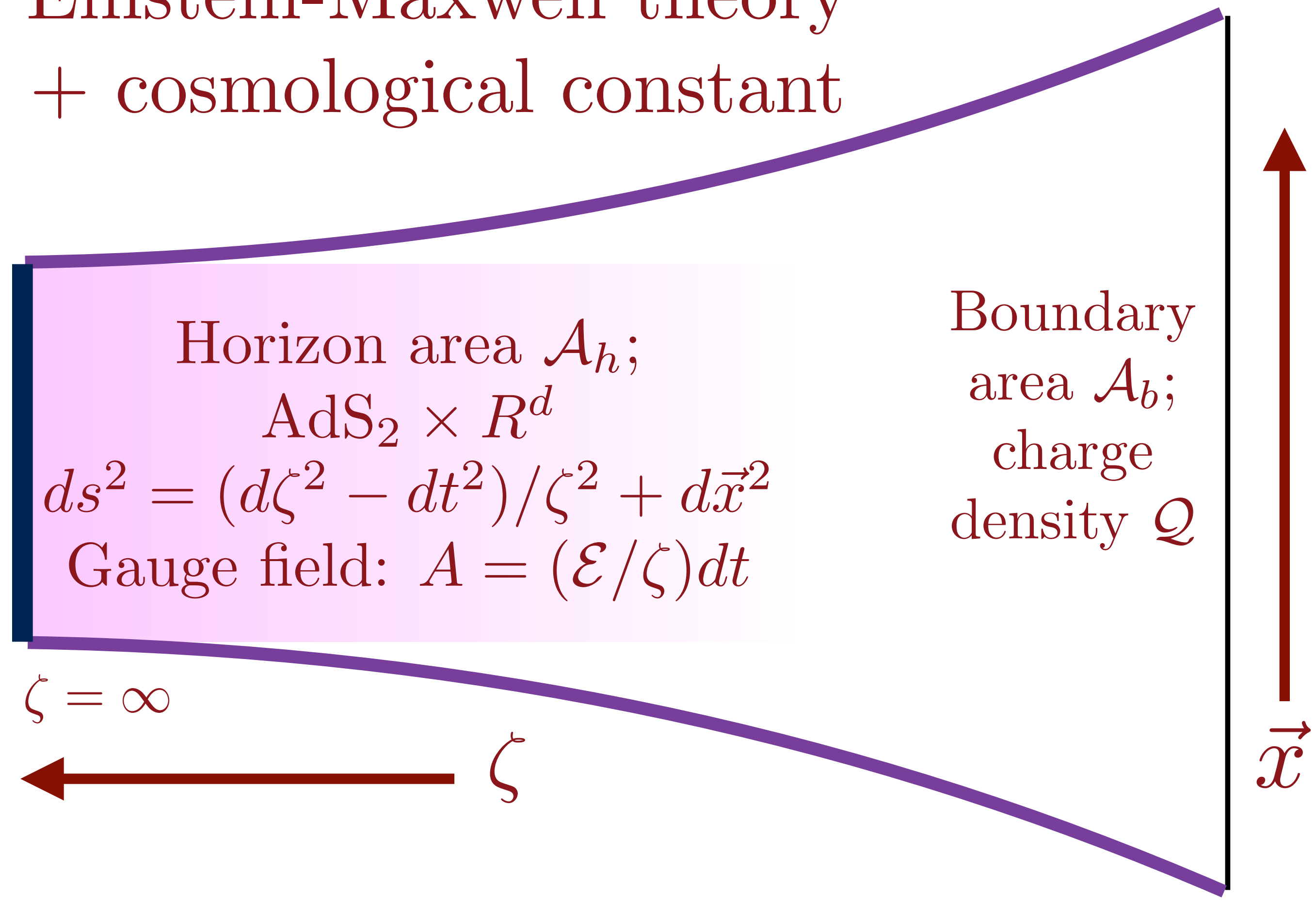


$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

$$-\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}} |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Einstein-Maxwell theory + cosmological constant



$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

$$-\left\langle \psi(\tau)\overline{\psi}(0) \right\rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi\mathcal{E}} |\tau|^{-1/2}, & \tau < 0. \end{cases}$$

'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Combination:

$$\mathcal{S} = \frac{\mathcal{A}_h}{4G_N \mathcal{A}_b}$$

Black hole thermodynamics (classical GR) yields

$$\frac{1}{\mathcal{A}_b}\frac{\partial \mathcal{A}_h}{\partial \mathcal{Q}} = 8\pi G_N \mathcal{E}$$

S. Sachdev, arXiv:1506.05111