

Quantum matter without quasiparticles

Low Energy Challenges for High Energy Physicists

Perimeter Institute, Waterloo, Canada

May 29, 2014

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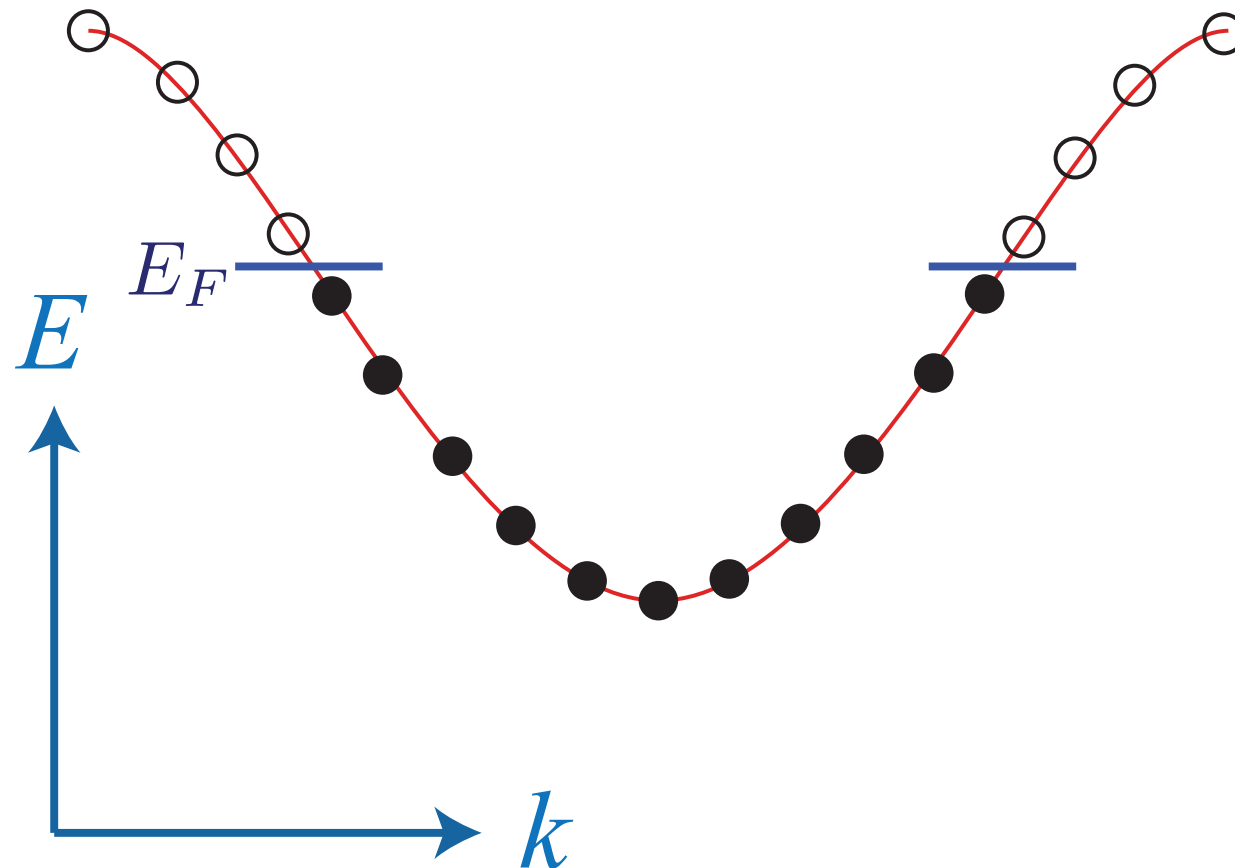
Talk online: sachdev.physics.harvard.edu



Foundations of quantum many body theory:

I. Ground states connected adiabatically to independent electron states

Metals

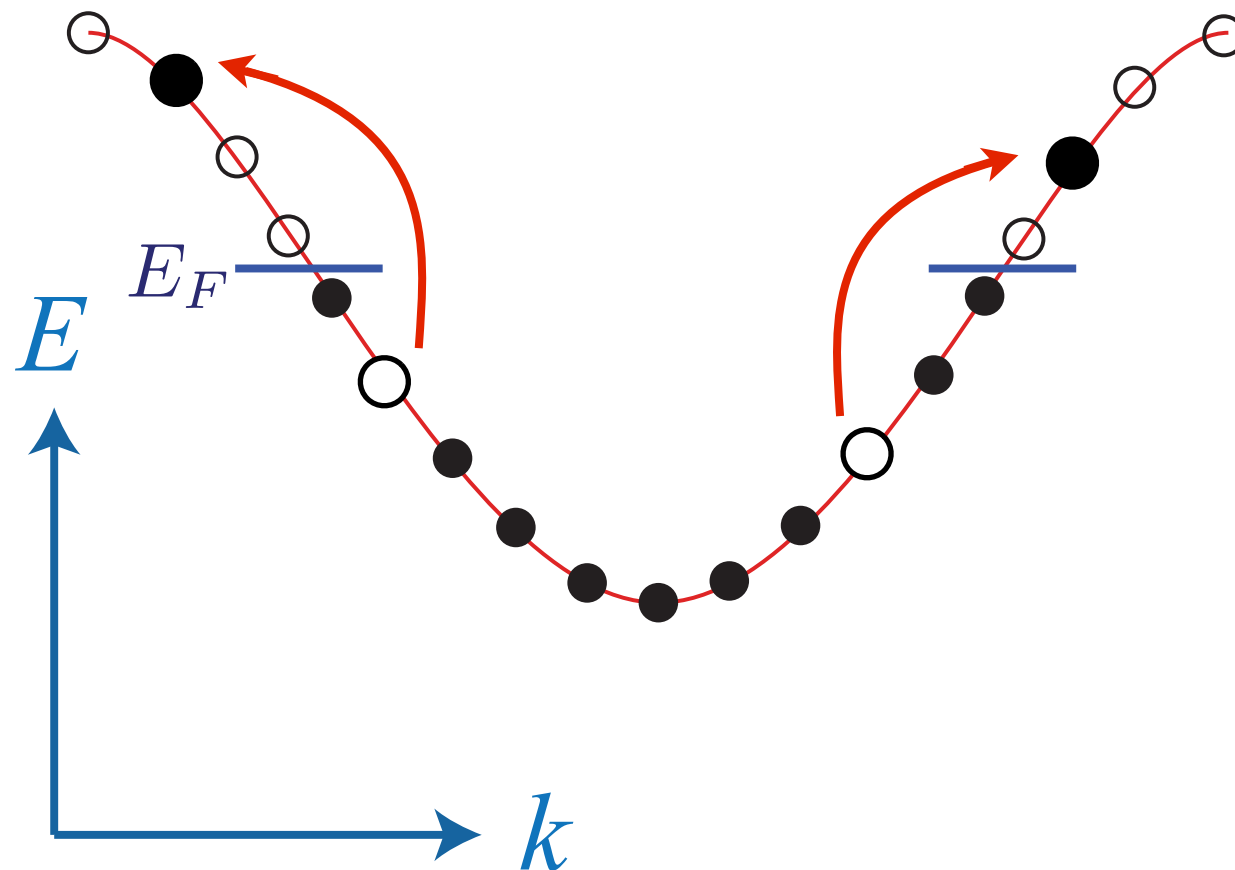


Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

Metals



Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. Boltzmann-Landau theory of quasiparticles*

Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. Boltzmann-Landau theory of quasiparticles*

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. Boltzmann-Landau theory of quasiparticles*

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations (“phonons”) are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. No quasiparticles**

Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. No quasiparticles**

Only 2 examples:

1. Conformal field theories in spatial dimension $d > 1$
2. Quantum critical metals in dimension $d=2$

Outline

1. Conformal field theories in $2+1$ dimensions

Superfluid-insulator transition

of ultracold bosonic atoms in an optical lattice

2. Theory of a non-Fermi liquid

Non-quasiparticle transport at the

Ising-nematic quantum critical point

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1. Conformal field theories in $2+1$ dimensions

*Superfluid-insulator transition
of ultracold bosonic atoms in an optical lattice*

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*Non-quasiparticle transport at the
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The dynamics of quantum criticality revealed by quantum Monte Carlo and holography

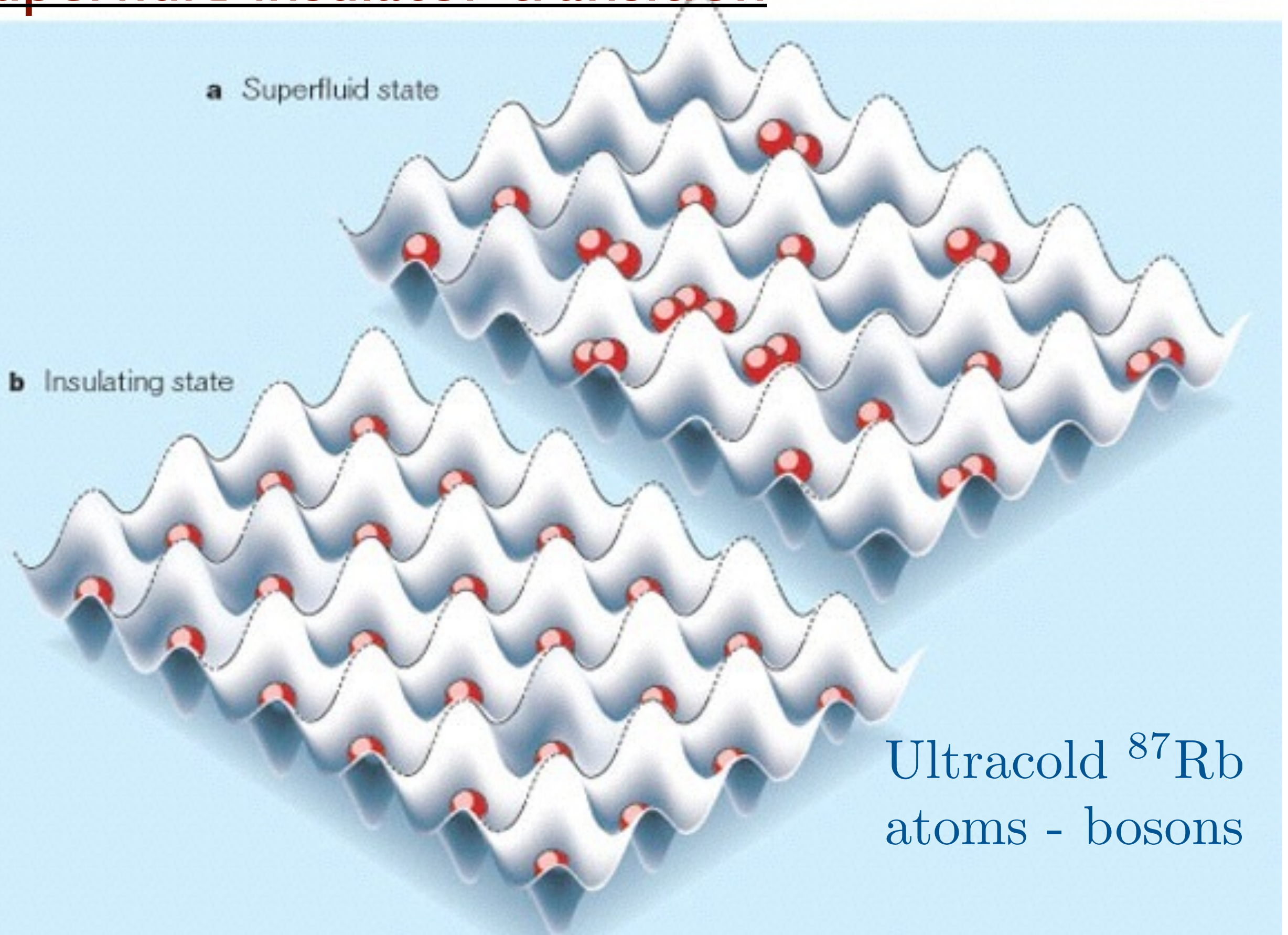


William Witczak-Krempa
Perimeter



Erik Sorensen
McMaster

Superfluid-insulator transition

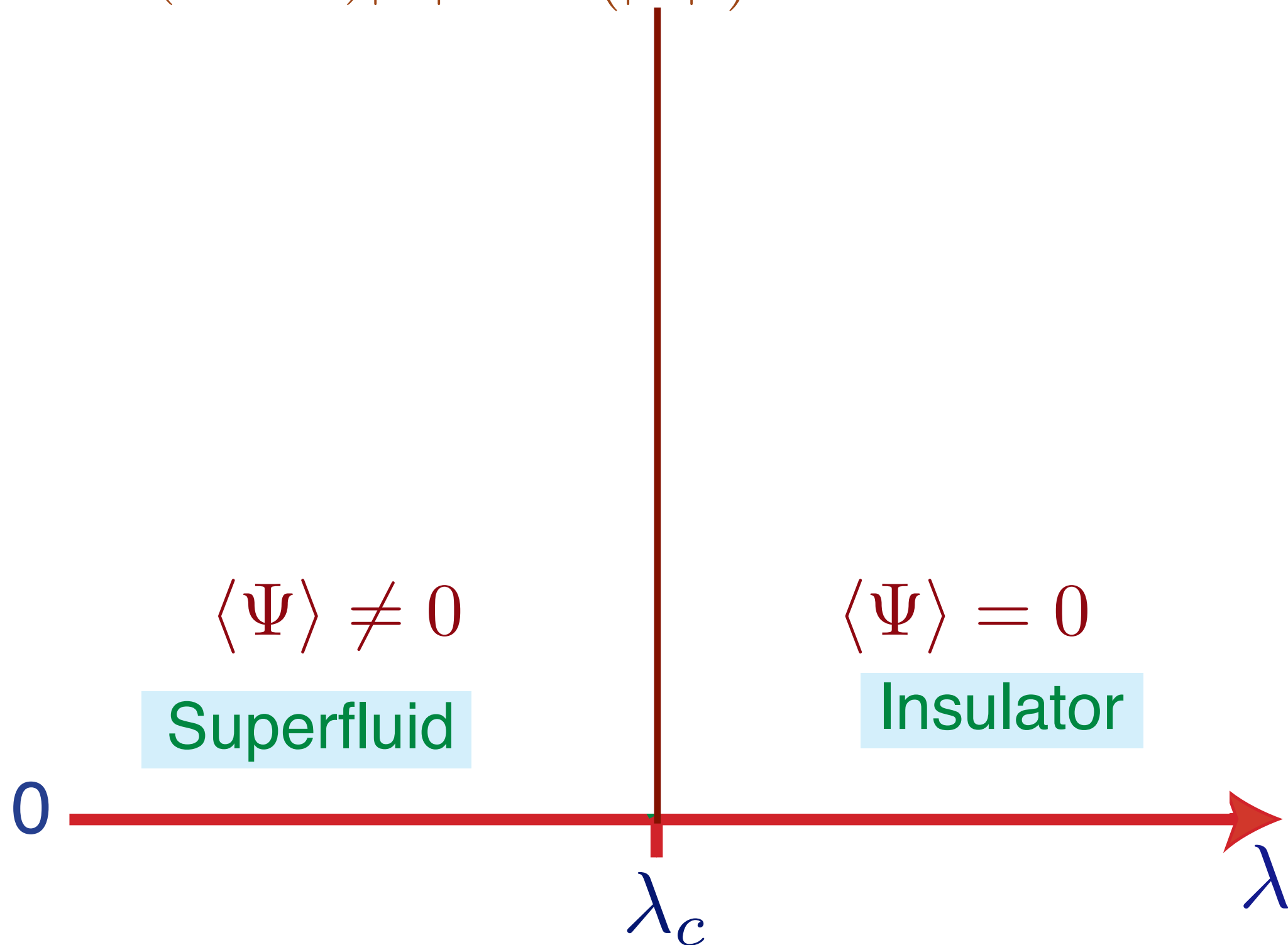


Ultracold ^{87}Rb
atoms - bosons

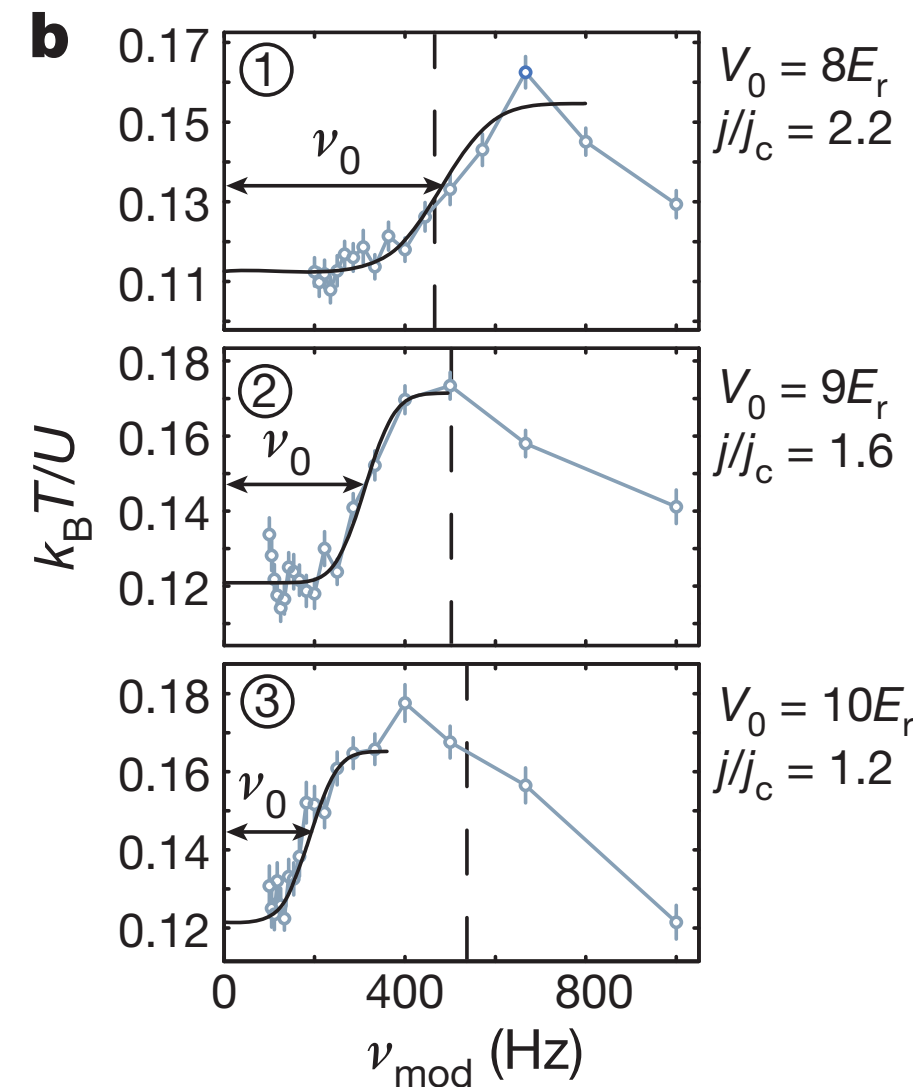
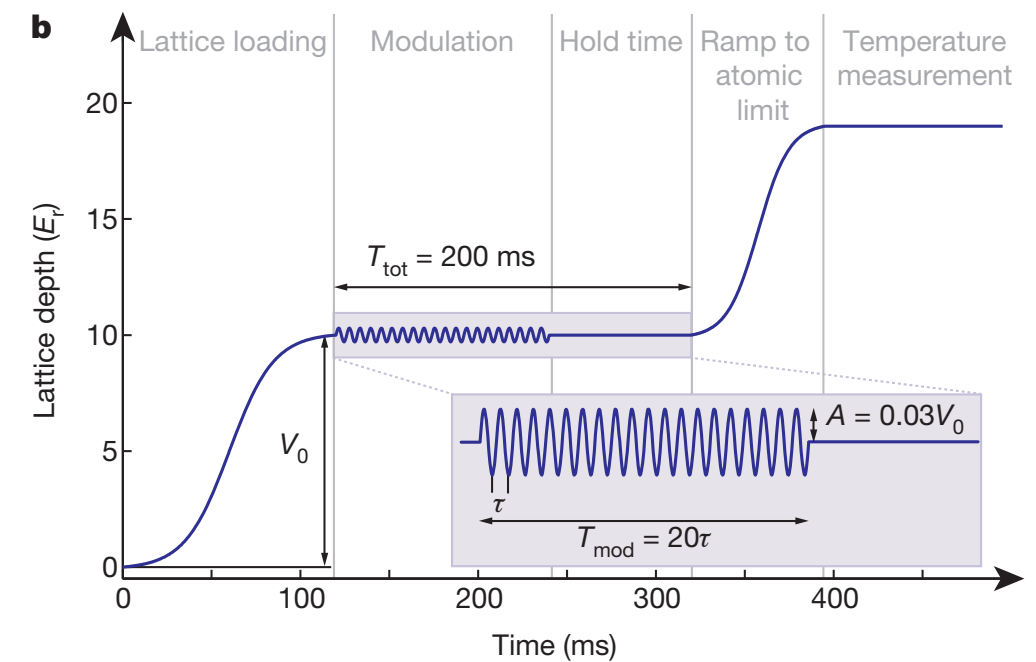
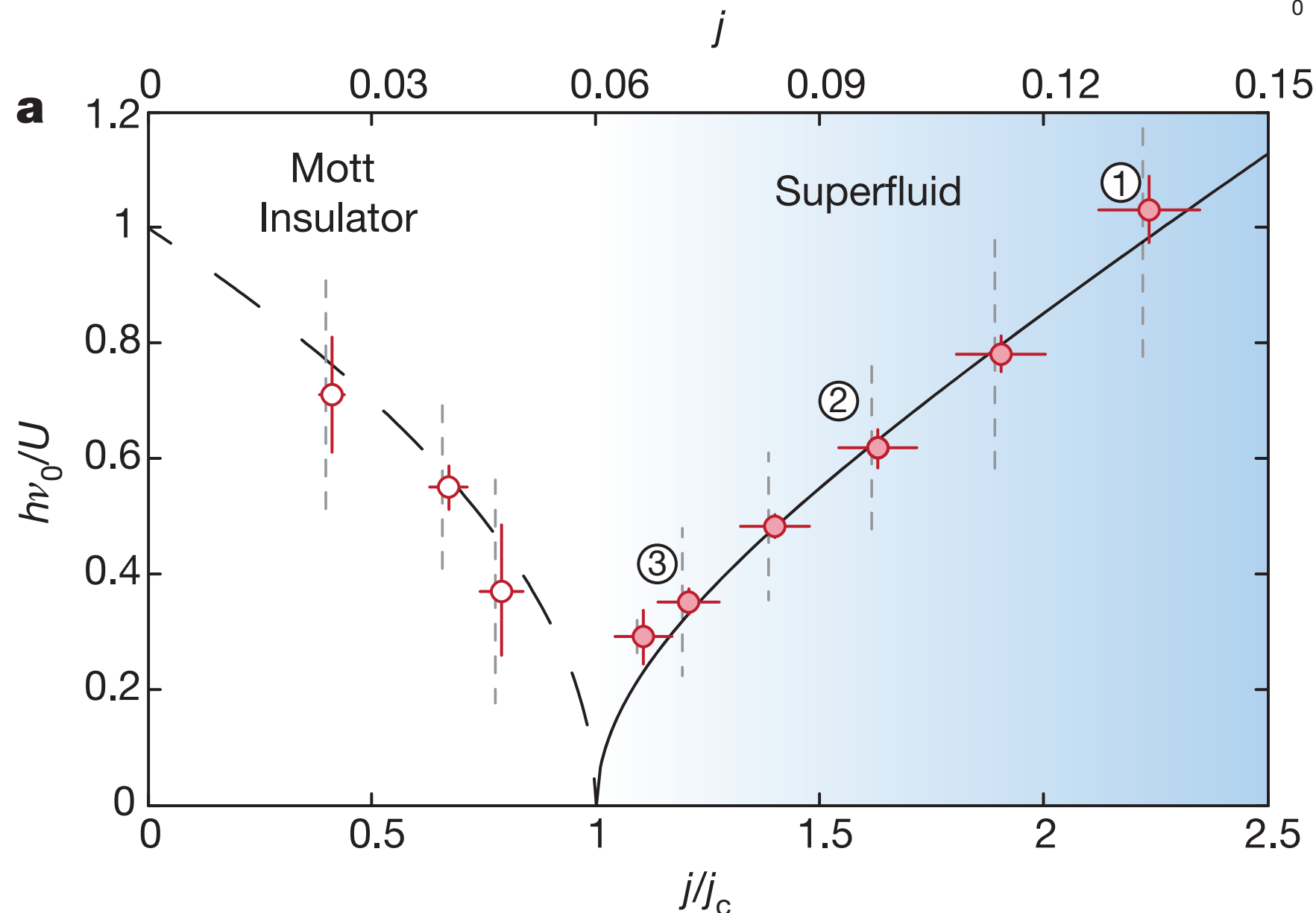
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

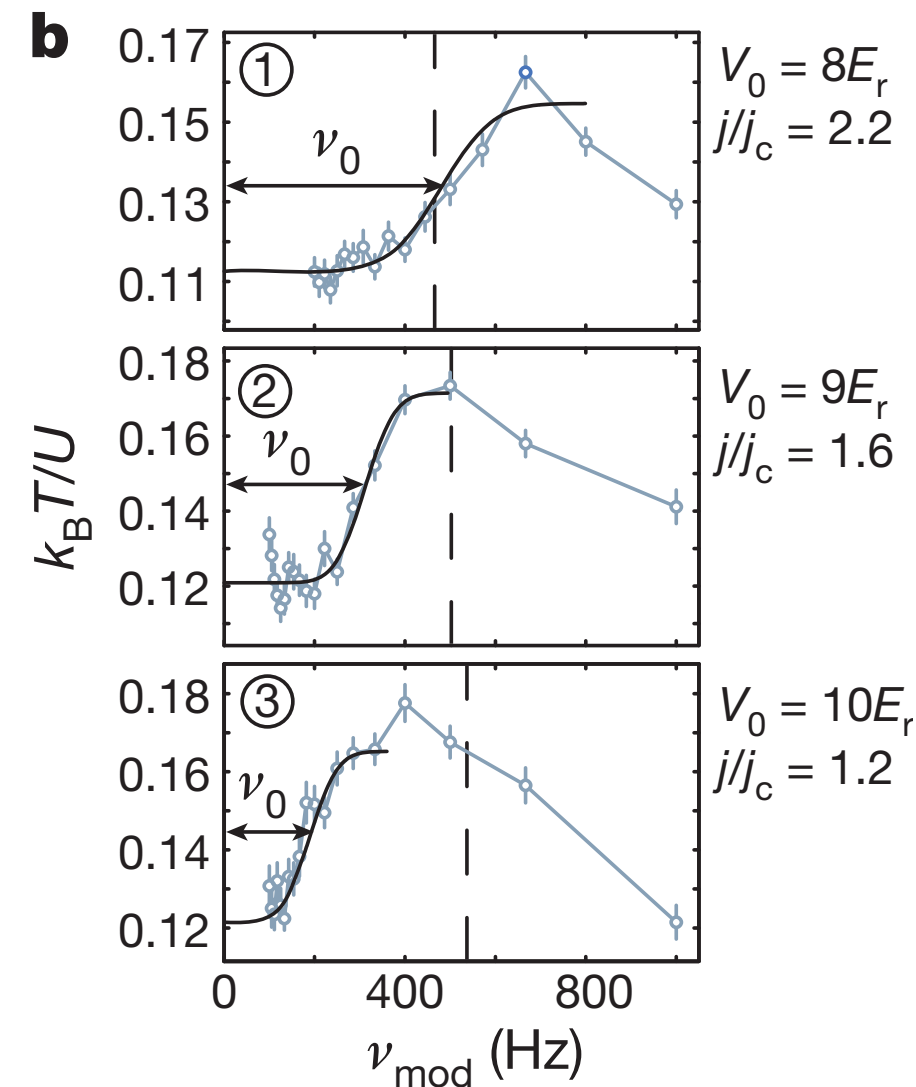
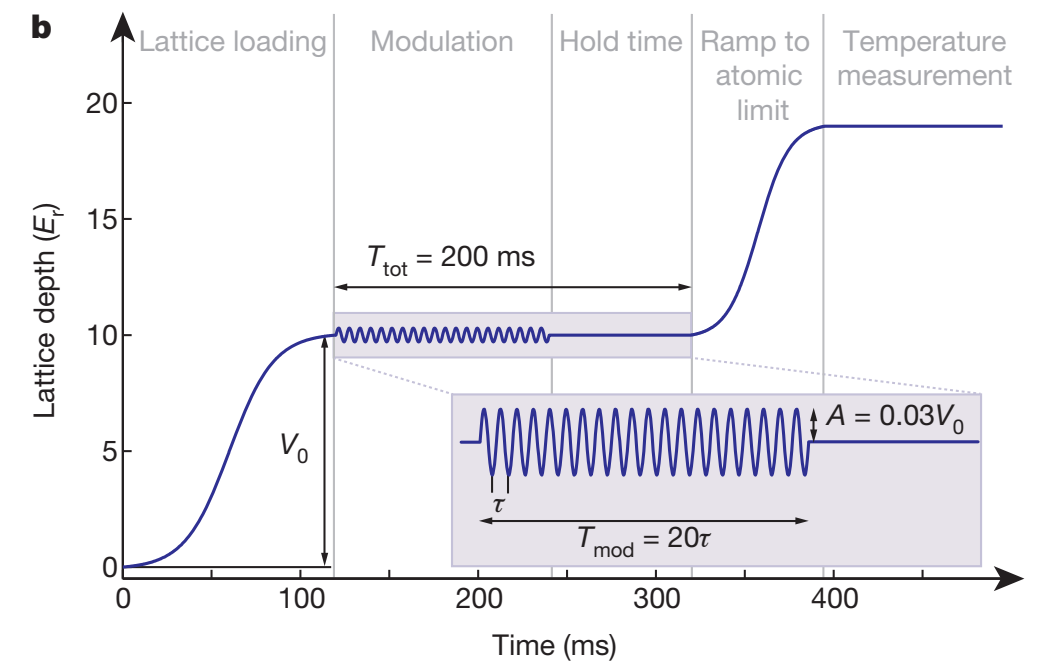
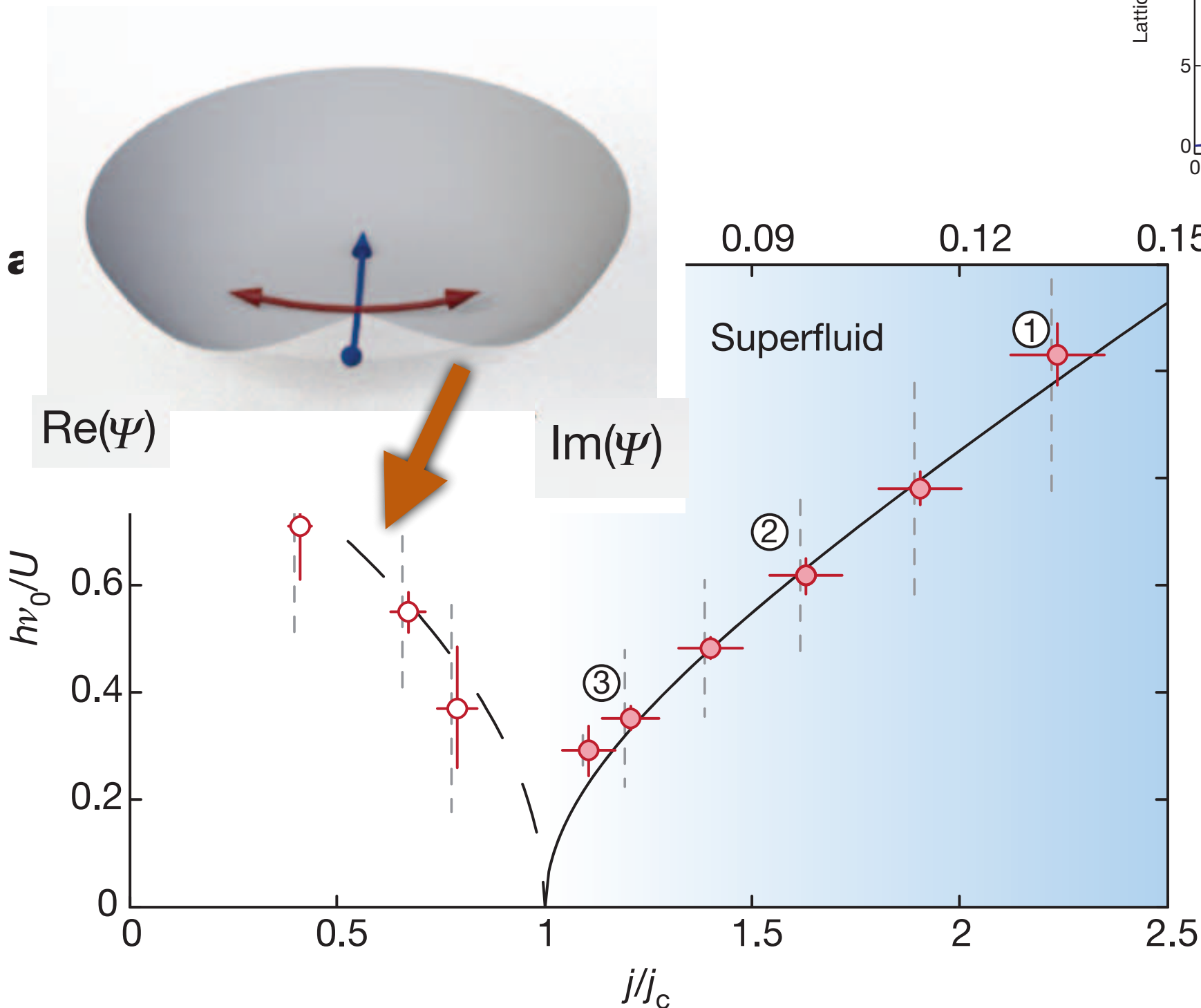


Excitations across the superfluid-insulator transition



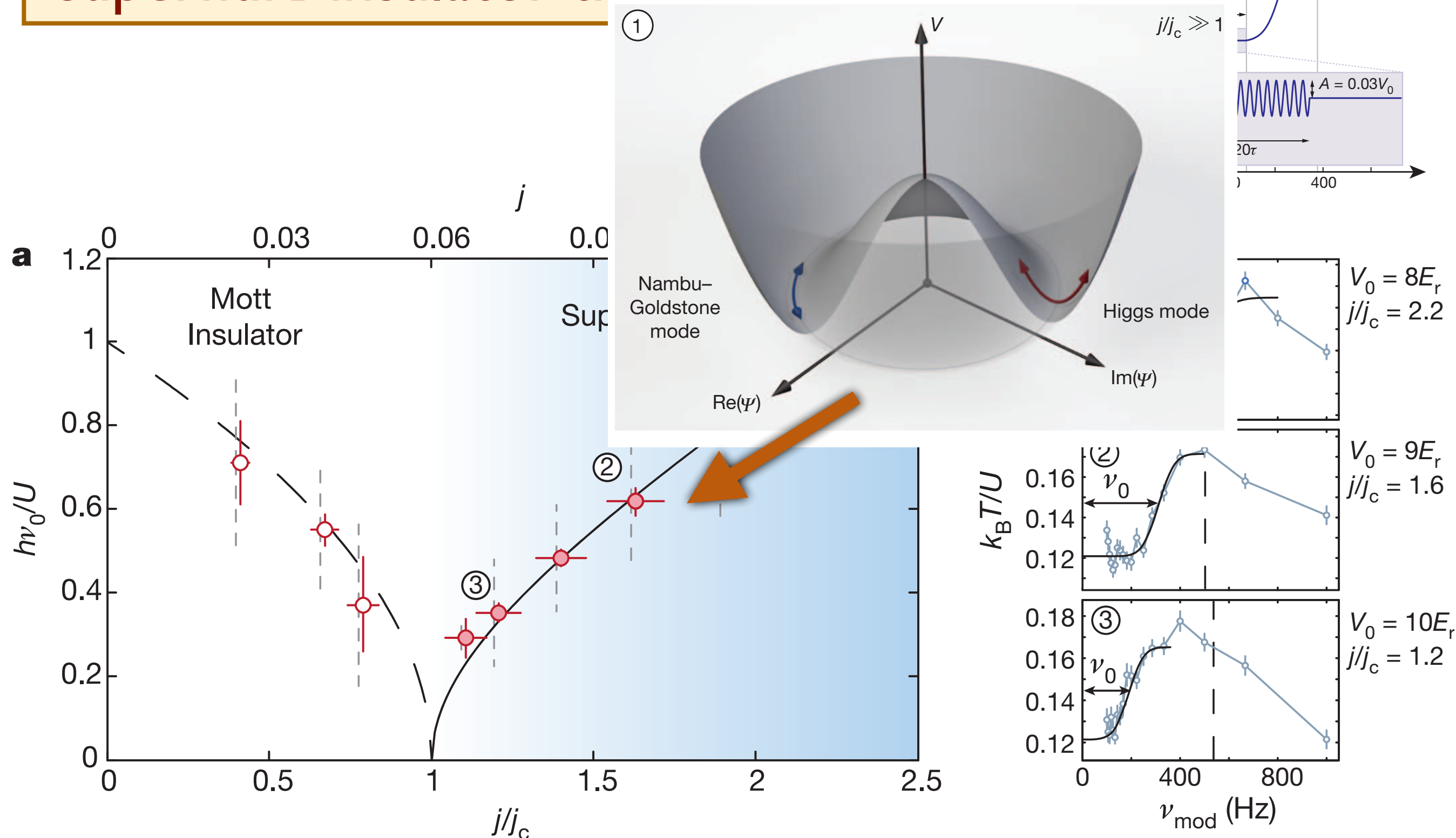
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Excitations across the Mott transition



Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Excitations across the superfluid-insulator transition

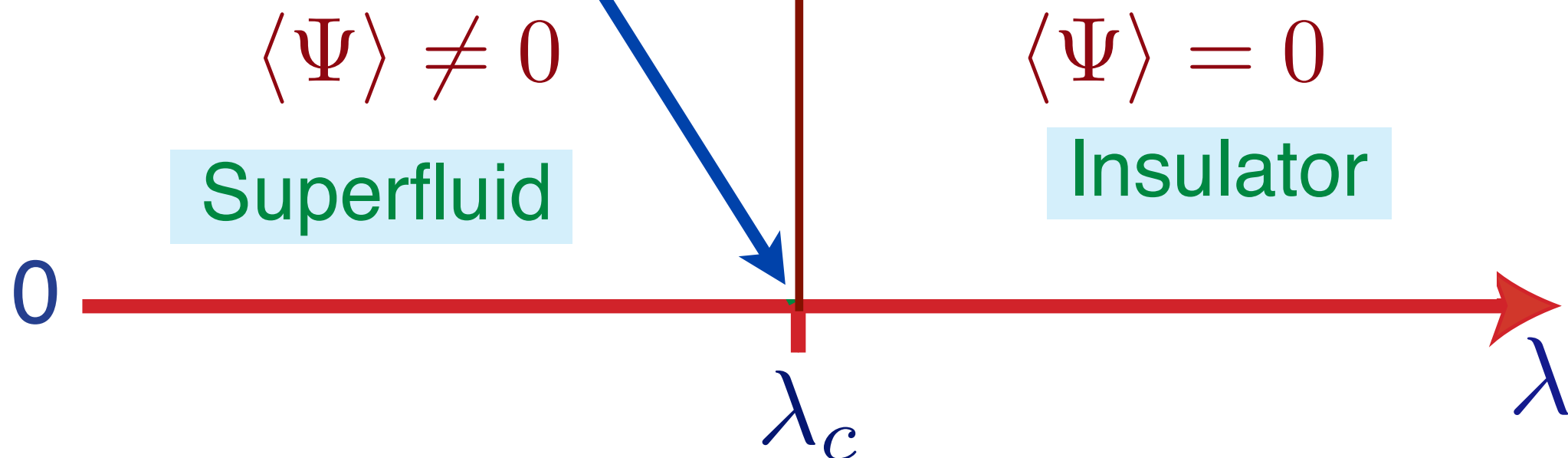


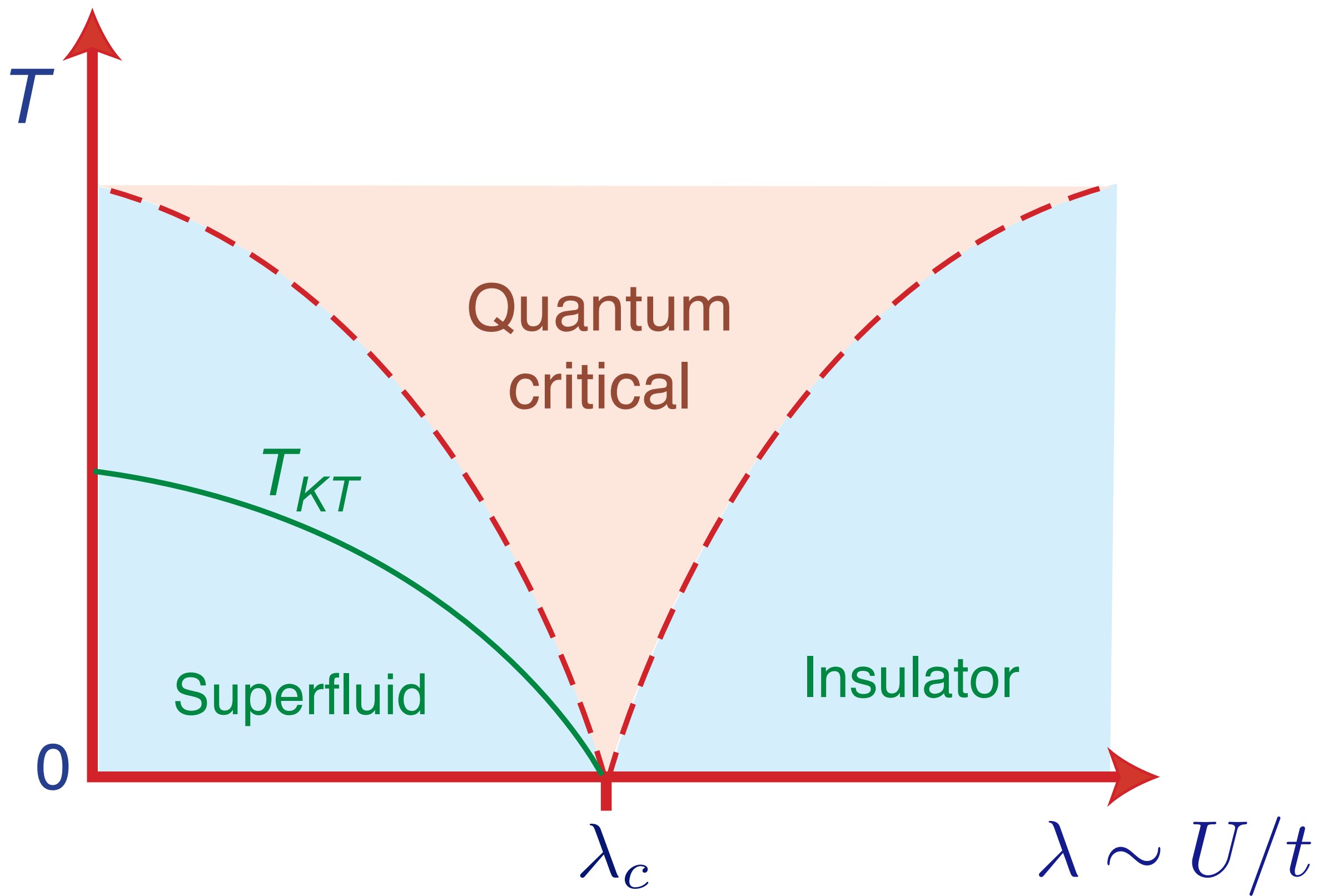
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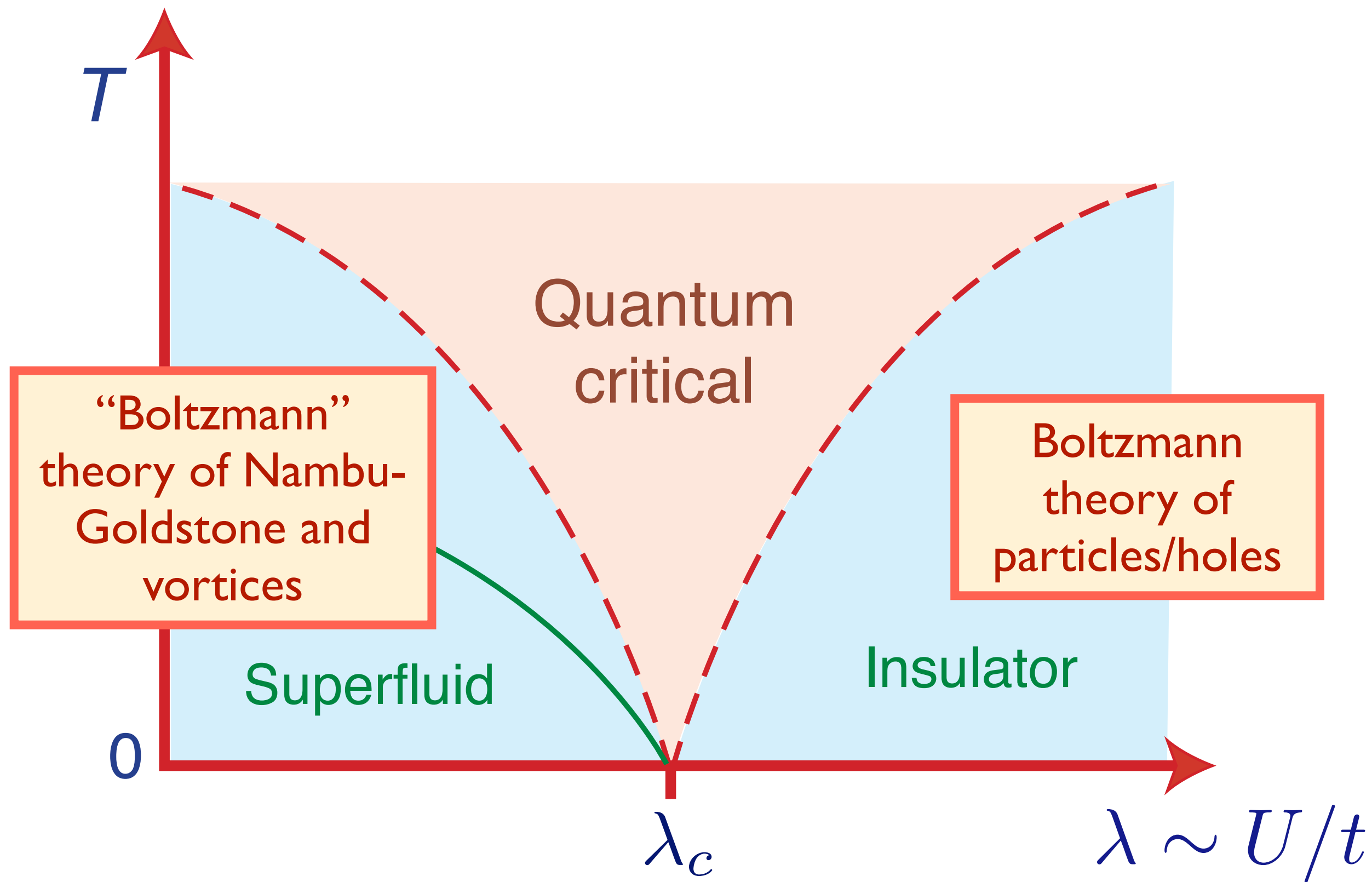
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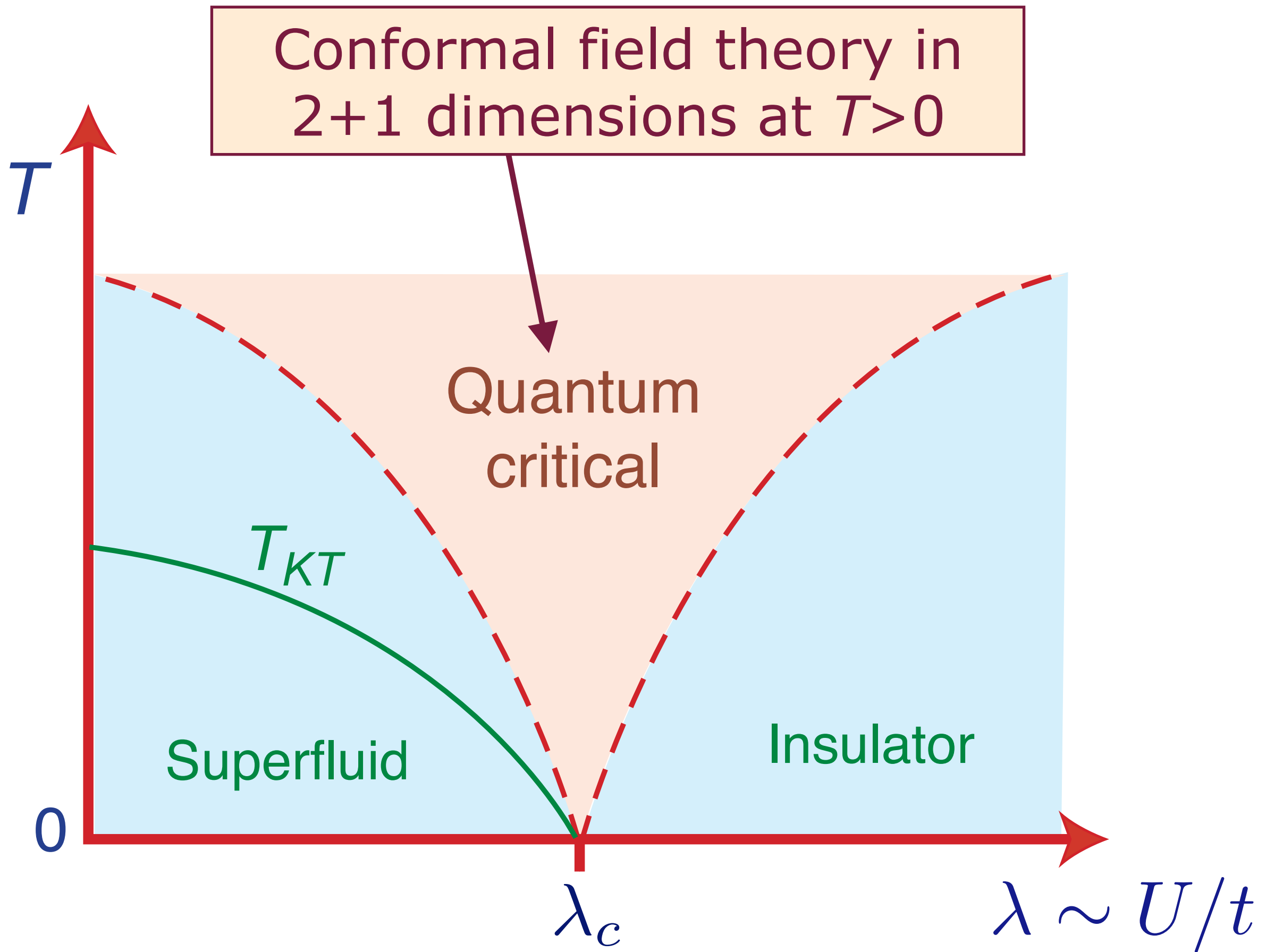
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

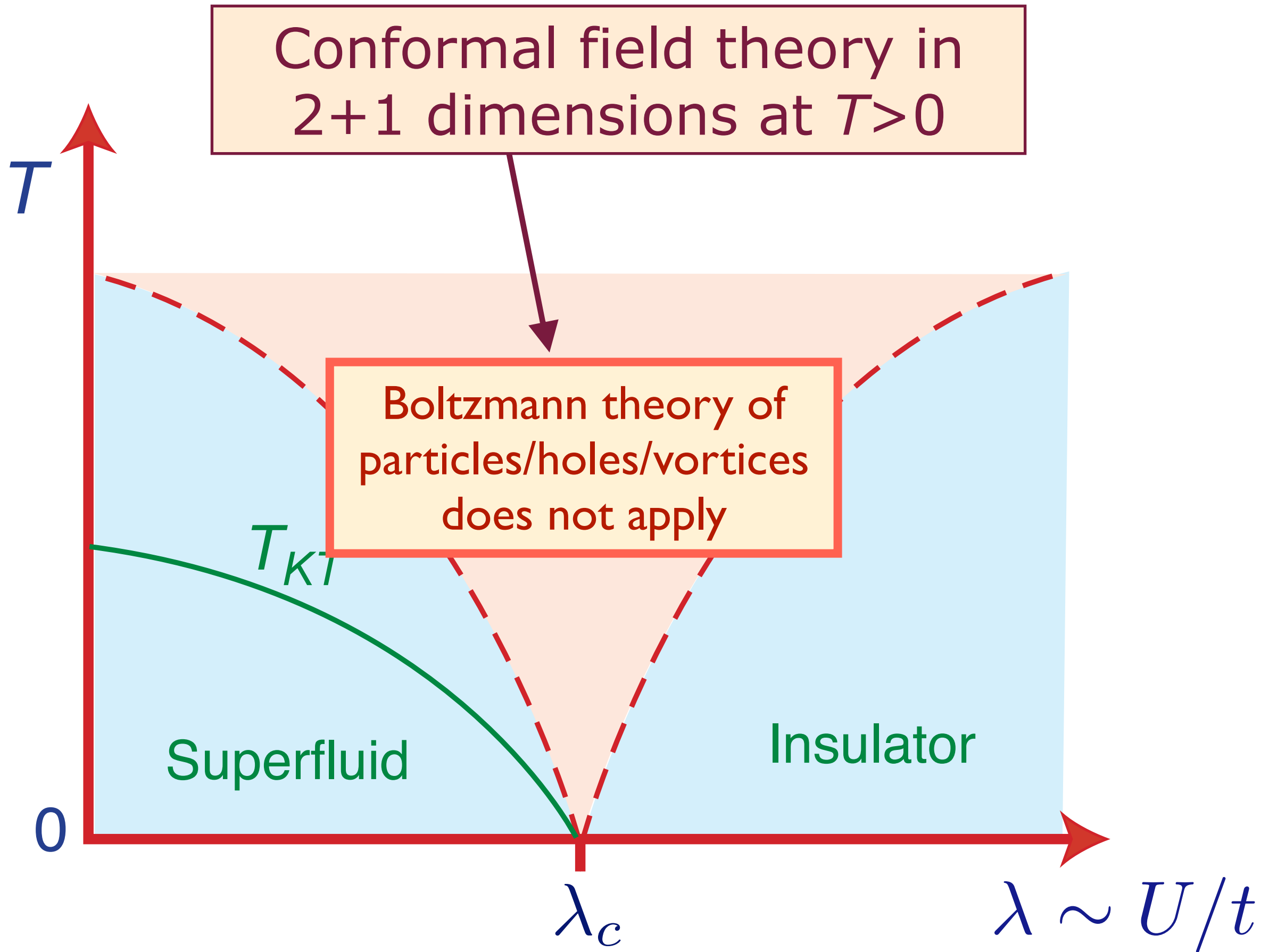
Quantum state with
 “long-range” quantum entanglement
and no quasiparticles.
 A 2+1 dim. conformal field theory (CFT3)

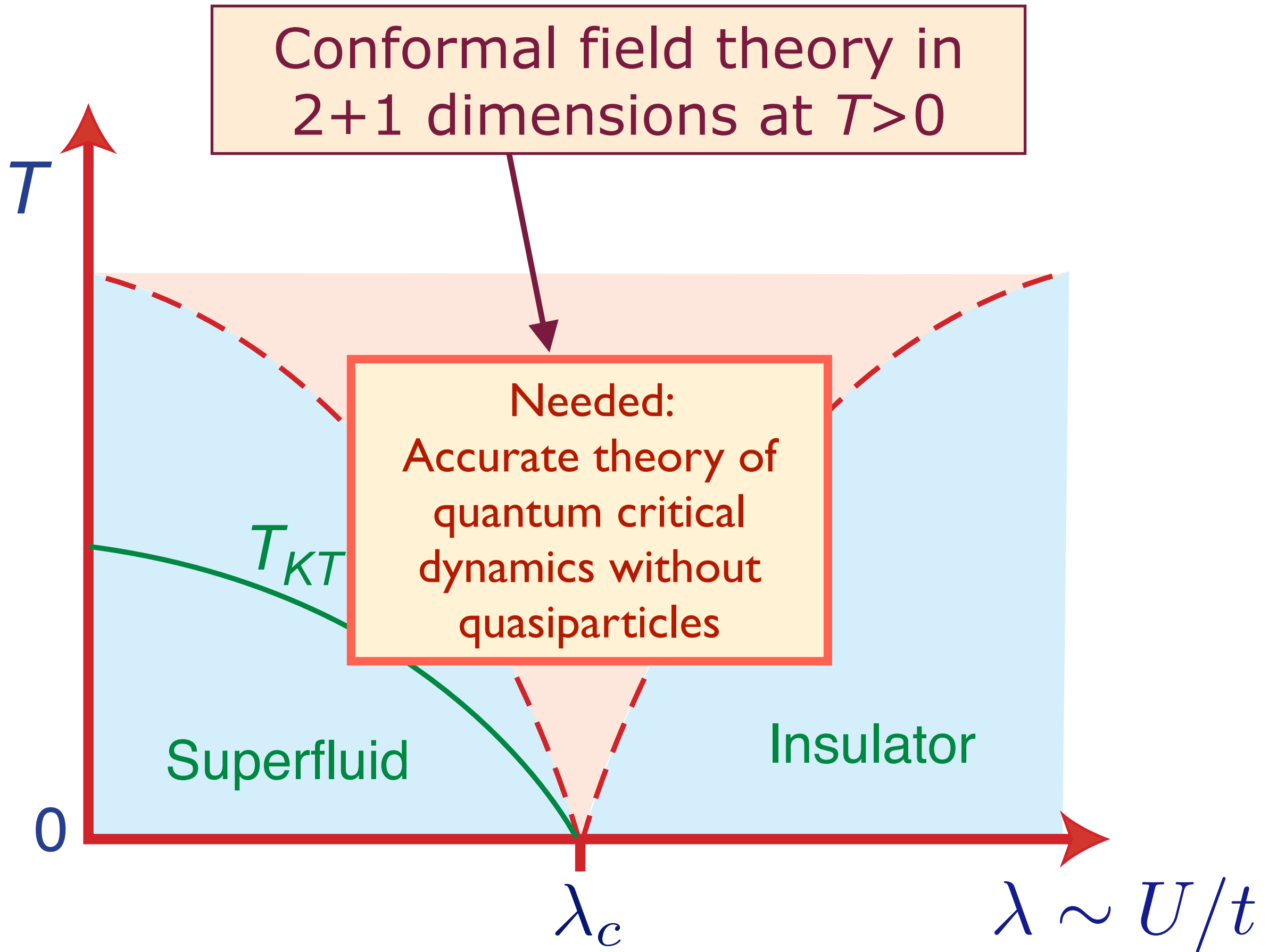












Traditional CMT

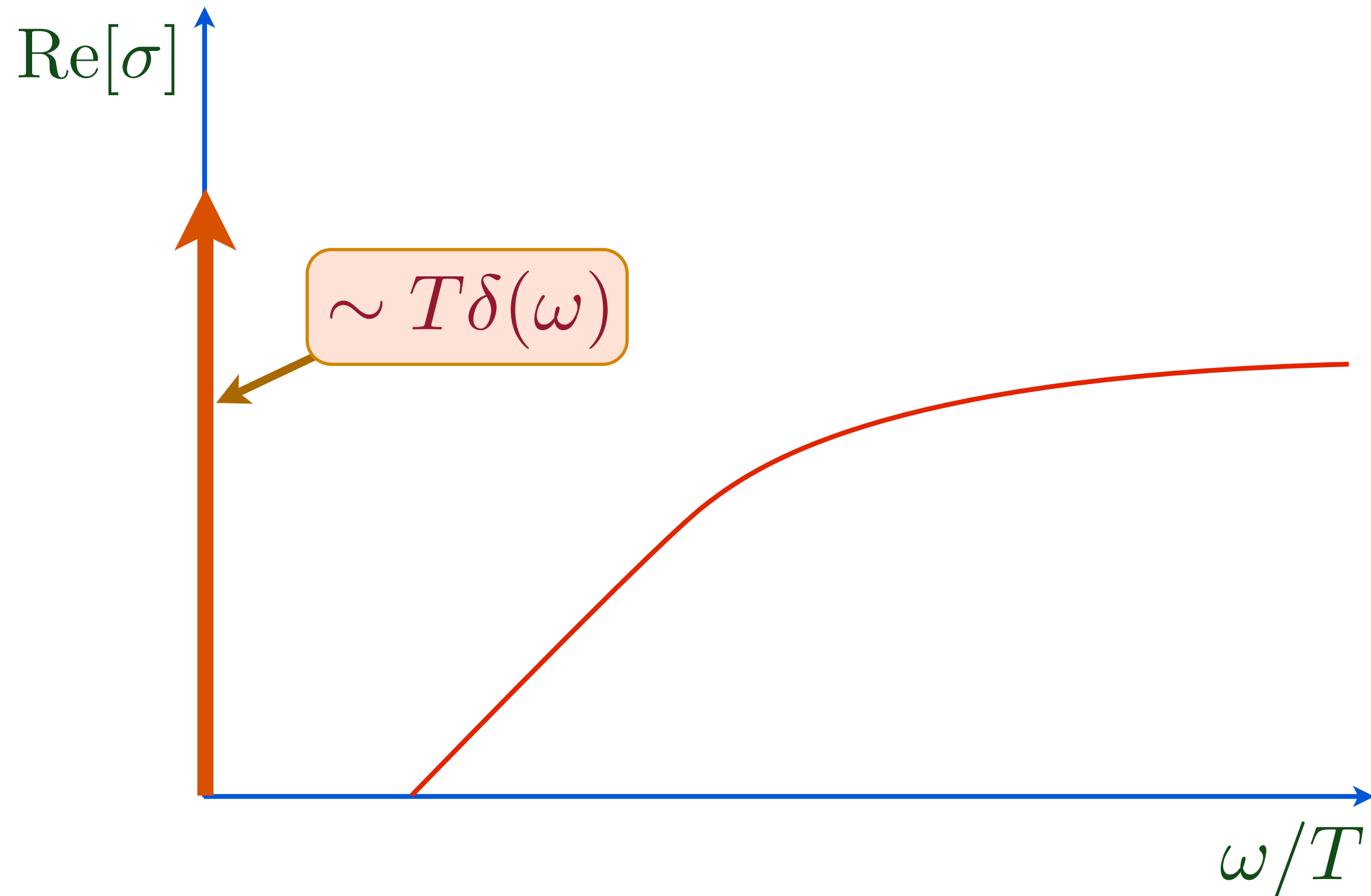
- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

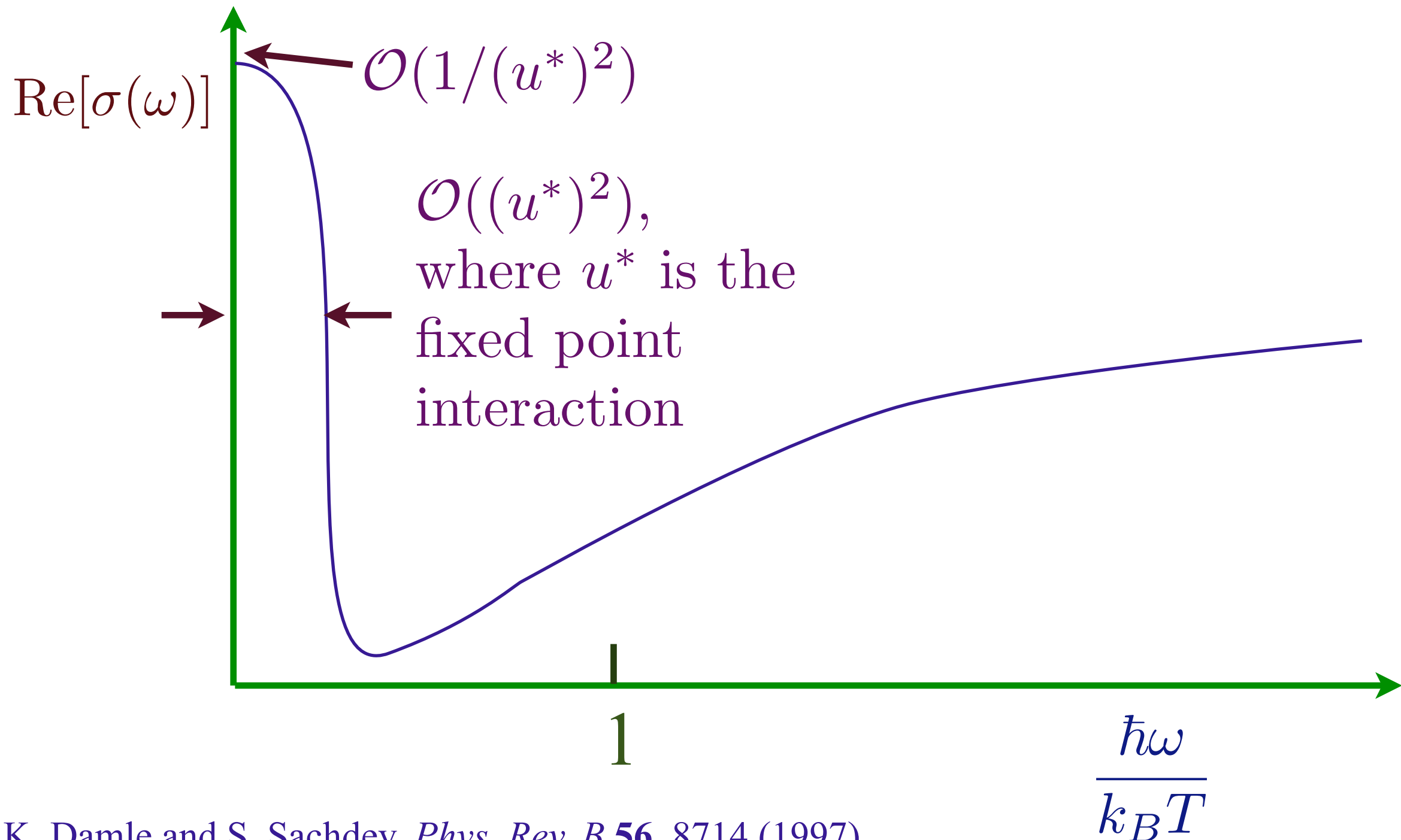
Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Quasiparticle view of quantum criticality (Boltzmann equation):
Electrical transport for a free CFT3



Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



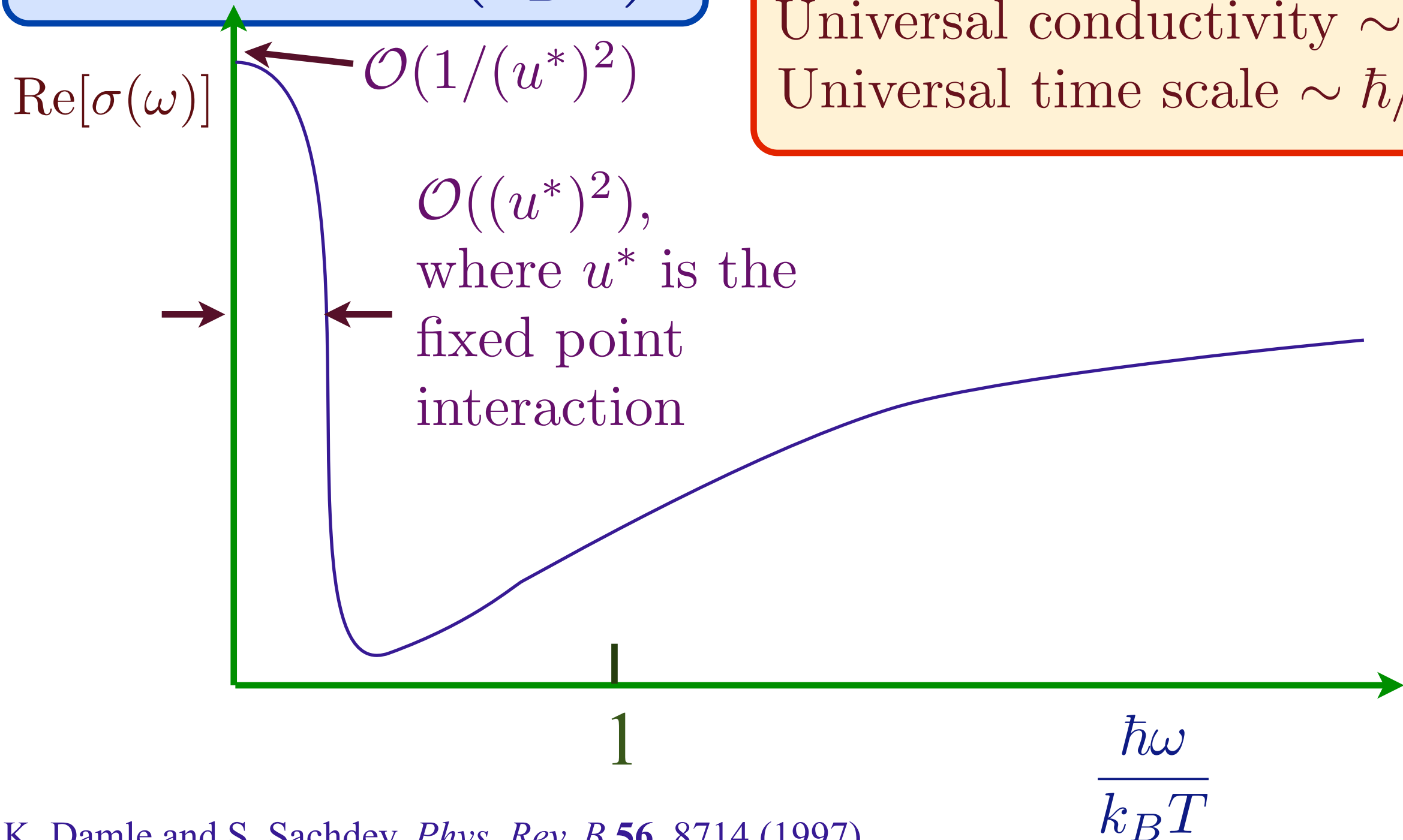
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right);$$

$\Sigma \rightarrow$ a universal function

Universal conductivity $\sim e^2/h$

Universal time scale $\sim \hbar/k_B T$



Quasiparticle view of quantum criticality (Boltzmann equation):
Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

$\text{Re}[\sigma(\omega)]$

$$\Sigma(\infty) = 2\pi \times (1/16) + \mathcal{O}(1/N)$$

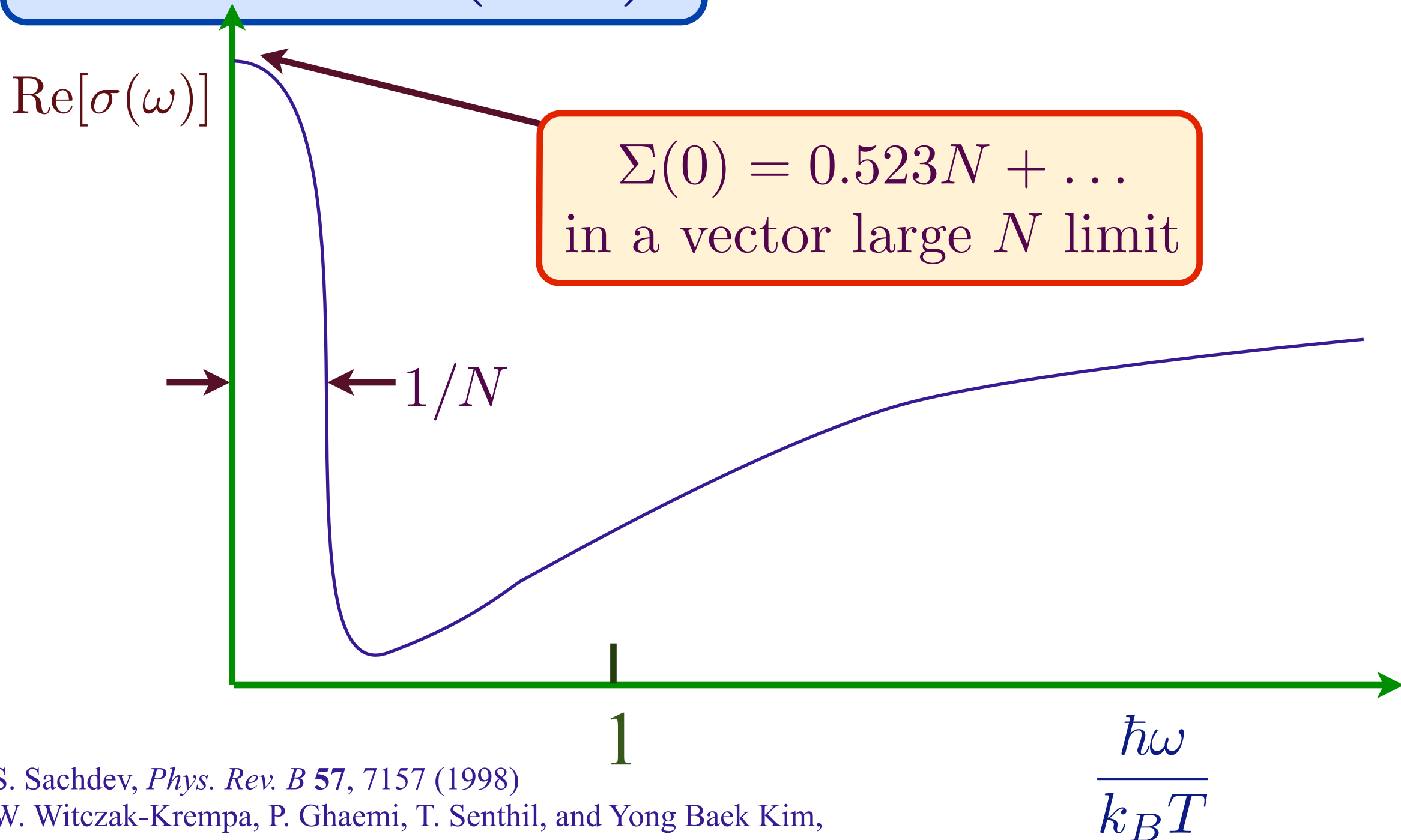
in a vector large N limit

1

$\frac{\hbar\omega}{k_B T}$

Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$



S. Sachdev, *Phys. Rev. B* **57**, 7157 (1998)

W. Witczak-Krempa, P. Ghaemi, T. Senthil, and Yong Baek Kim,
Phys. Rev. B **86**, 24102 (2012)

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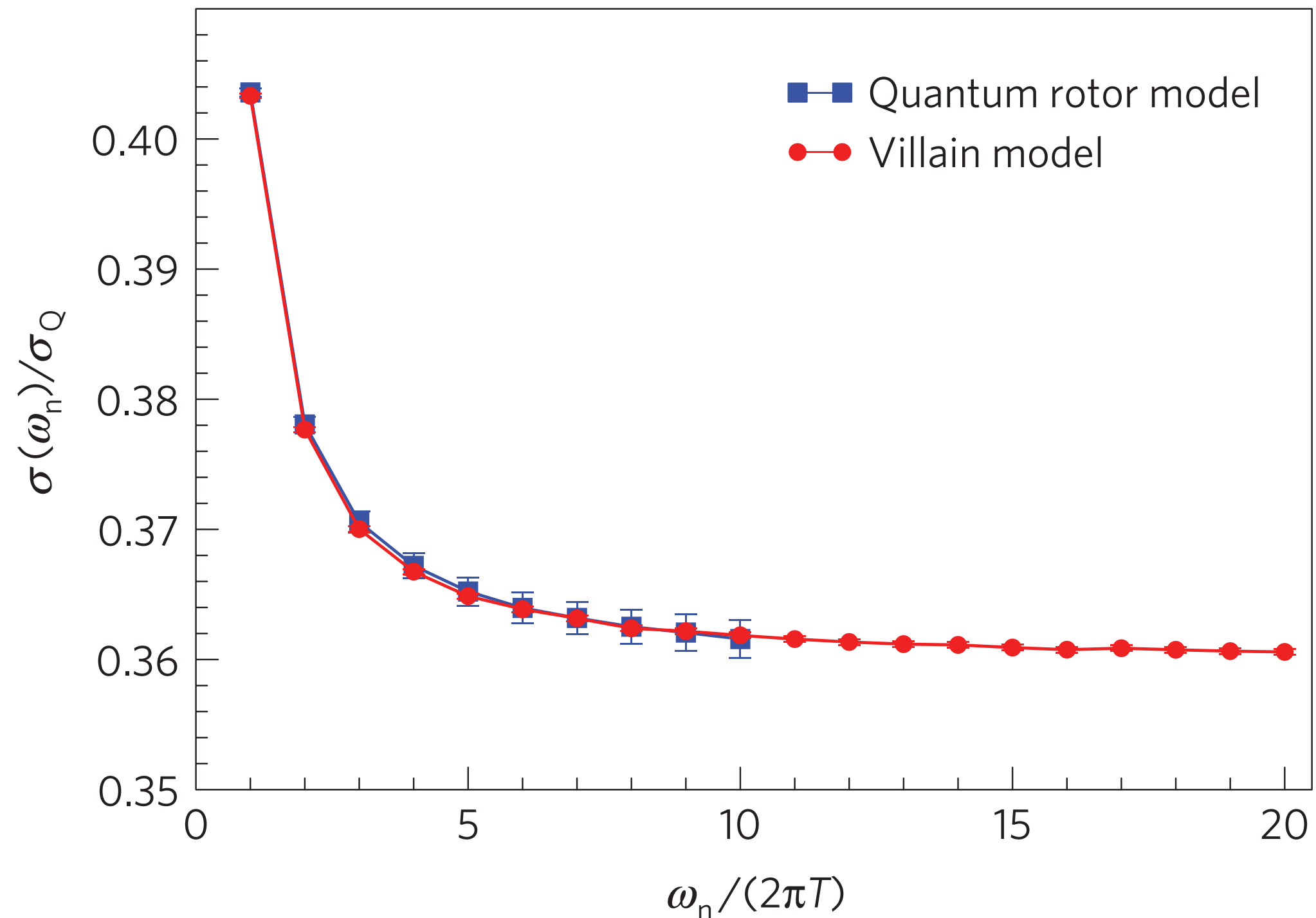
$$\frac{\sigma(0)}{\sigma(\infty)} = 1.332N + \dots$$

$\leftarrow 1/N$

1

$\frac{\hbar\omega}{k_B T}$

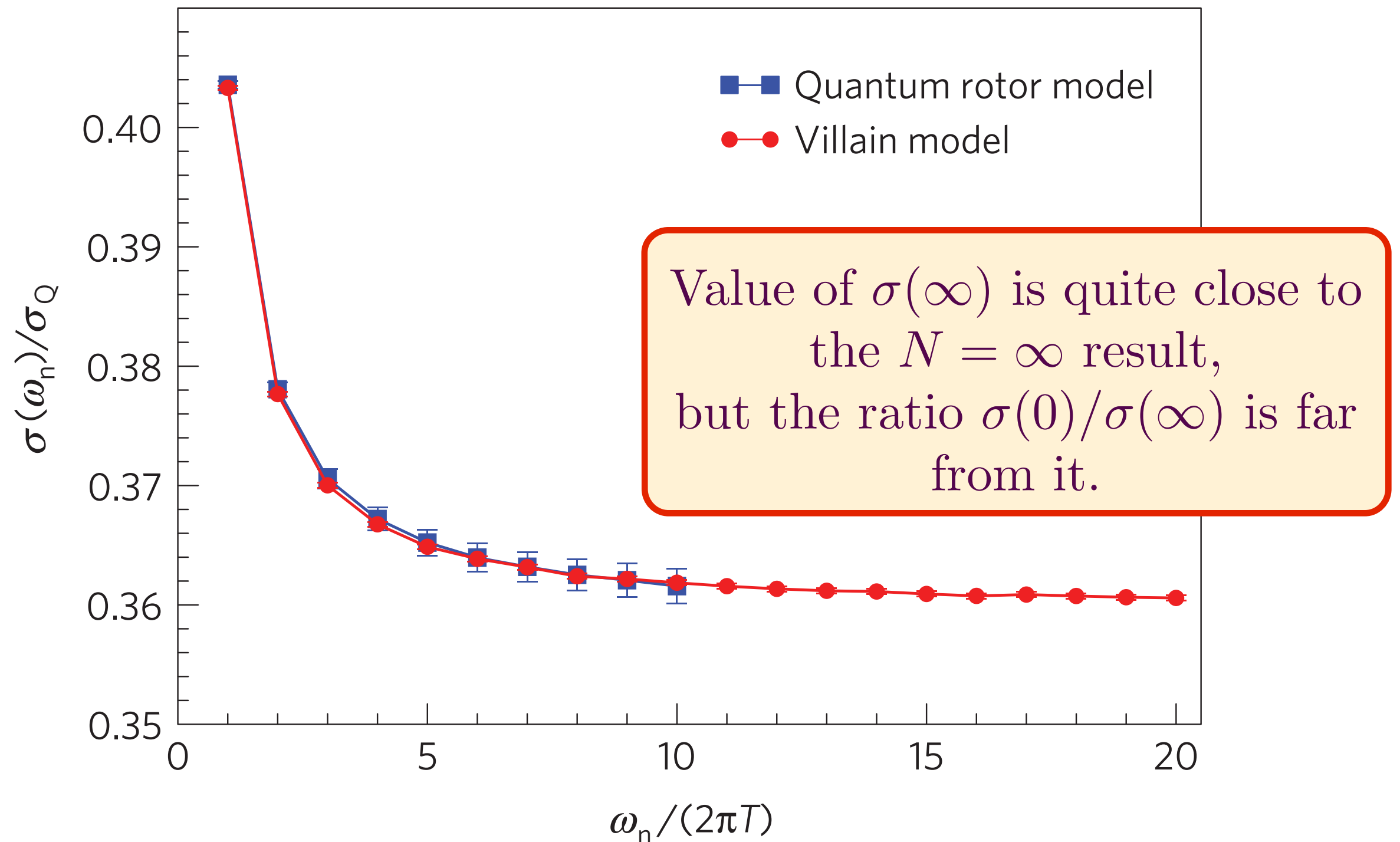
Quantum Monte Carlo for lattice bosons



W. Witczak-Krempa, E. Sorensen, and S. Sachdev, Nature Physics **10**, 361 (2014)

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, Phys. Rev. Lett. **112**, 030402 (2013)

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Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT

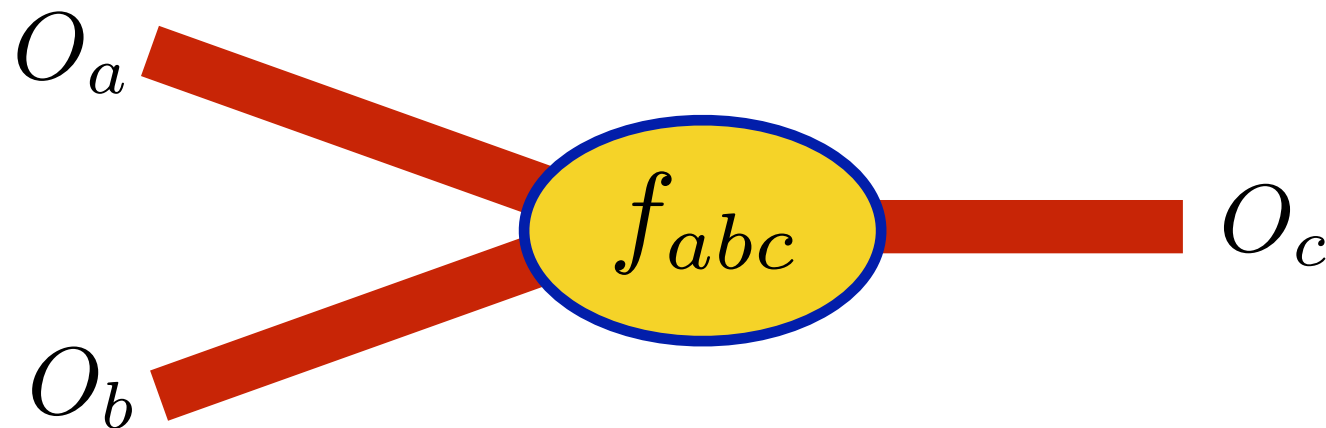
Basic characteristics of CFTs

Primary operators of CFT, $O_a(x)$, obey (at $T = 0$):

$$\langle O_a(x) O_b(0) \rangle = \frac{\delta_{ab}}{|x|^{2\Delta_a}}$$

where Δ_a is their scaling dimension. Their “interactions” are determined by the OPE (considering scalar operators only)

$$\lim_{x' \rightarrow x} \langle O_a(x') O_b(x) O_c(0) \rangle = \frac{f_{abc}}{|x|^{\Delta_a + \Delta_b + \Delta_c}}$$



The values of $\{\Delta_a, f_{abc}\}$ determine (in principle) all observable properties of the CFT, as constrained by conformal Ward identities. For the Wilson-Fisher CFT₃, systematic methods exist to compute (in principle) all the $\{\Delta_a, f_{abc}\}$, and we will assume this data is *known*. This knowledge will be taken as an *input* to the computation of the finite T dynamics

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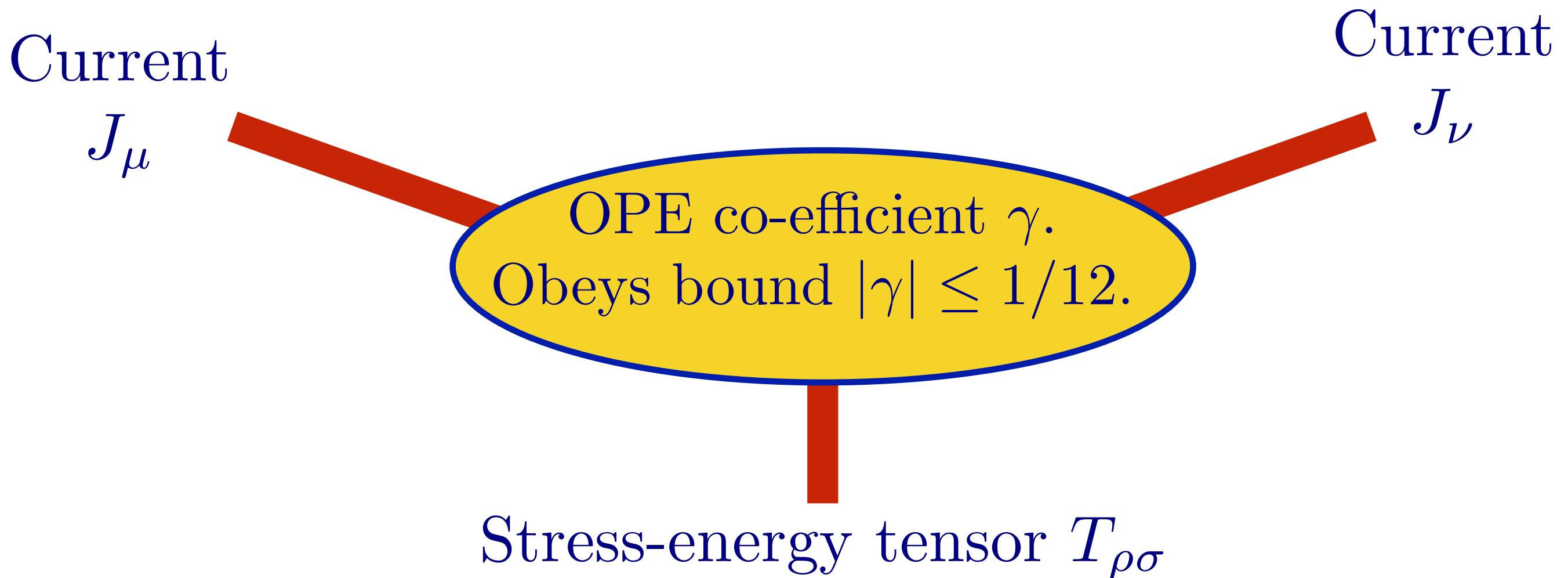
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Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT
- Relate OPE co-efficients to couplings of an effective gravitational theory on AdS
- Non-zero T dynamics of CFT maps to dynamics of a “horizon” in (Einstein’s) gravitational theory

Physical picture of electrical transport in a CFT3



Conductivity at $T > 0$ determined by
“scattering” of current by
thermal stress-energy tensor.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* **87**, 085138 (2013).

D. M. Hofman and J. Maldacena, *JHEP* **0805** (2008) 012.

AdS₄ theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \\ + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833..$

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS₄ theory of quantum criticality

- The AdS₄ solutions satisfy two sum rules which are expected to be satisfied by all CFT3s:

$$\int_0^\infty d\omega \left(\text{Re} [\Sigma(\omega)] - \Sigma(\infty) \right) = 0$$
$$\int_0^\infty d\omega \left(\text{Re} \left[\frac{1}{\Sigma(\omega)} \right] - \frac{1}{\Sigma(\infty)} \right) = 0$$

The second sum rule relies on the existence of a S-dual CFT3.

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- The poles *and* zeros of the complex function $\Sigma(\omega)$ are expected to be in the lower-half plane.

AdS₄ theory of quantum criticality

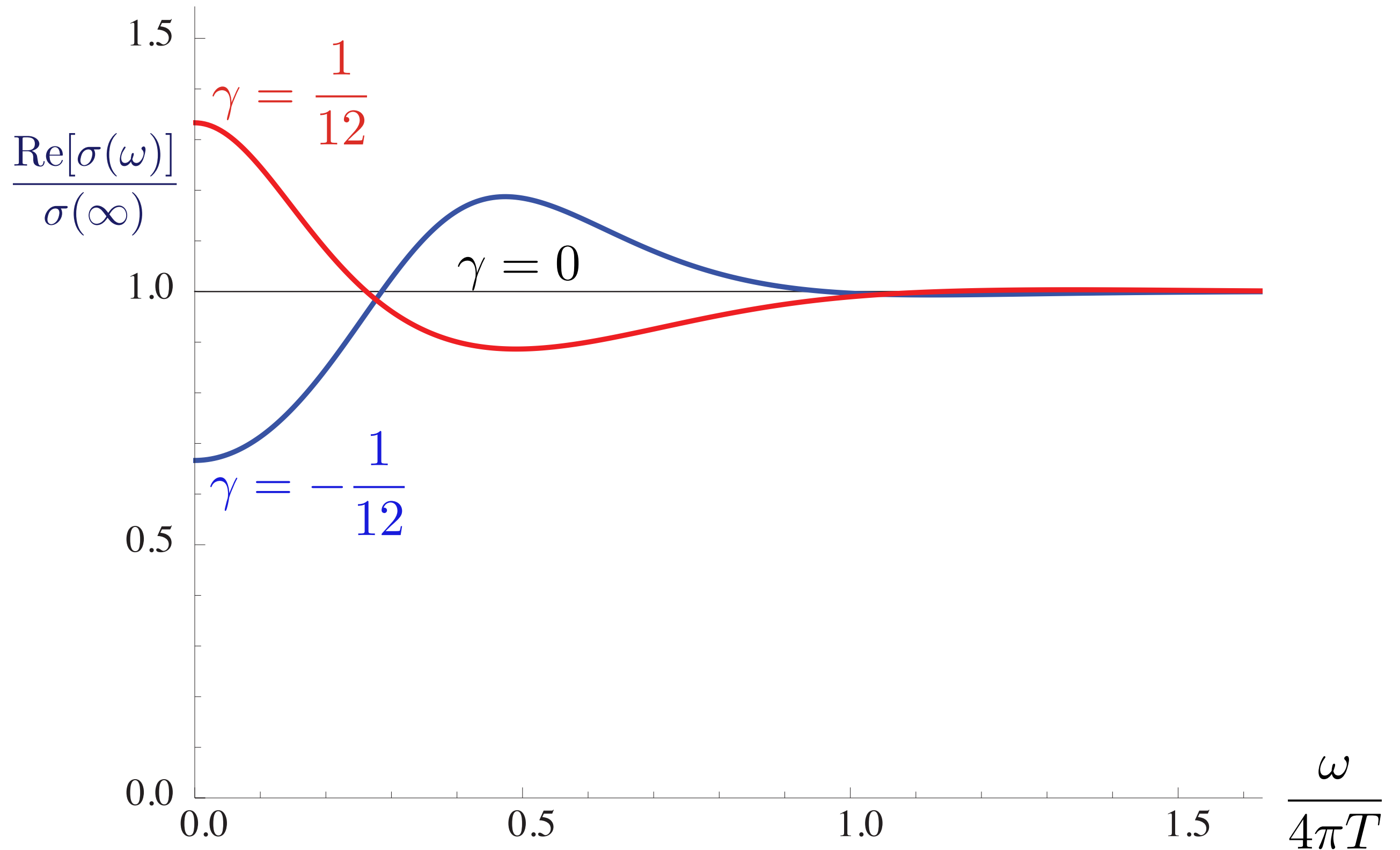
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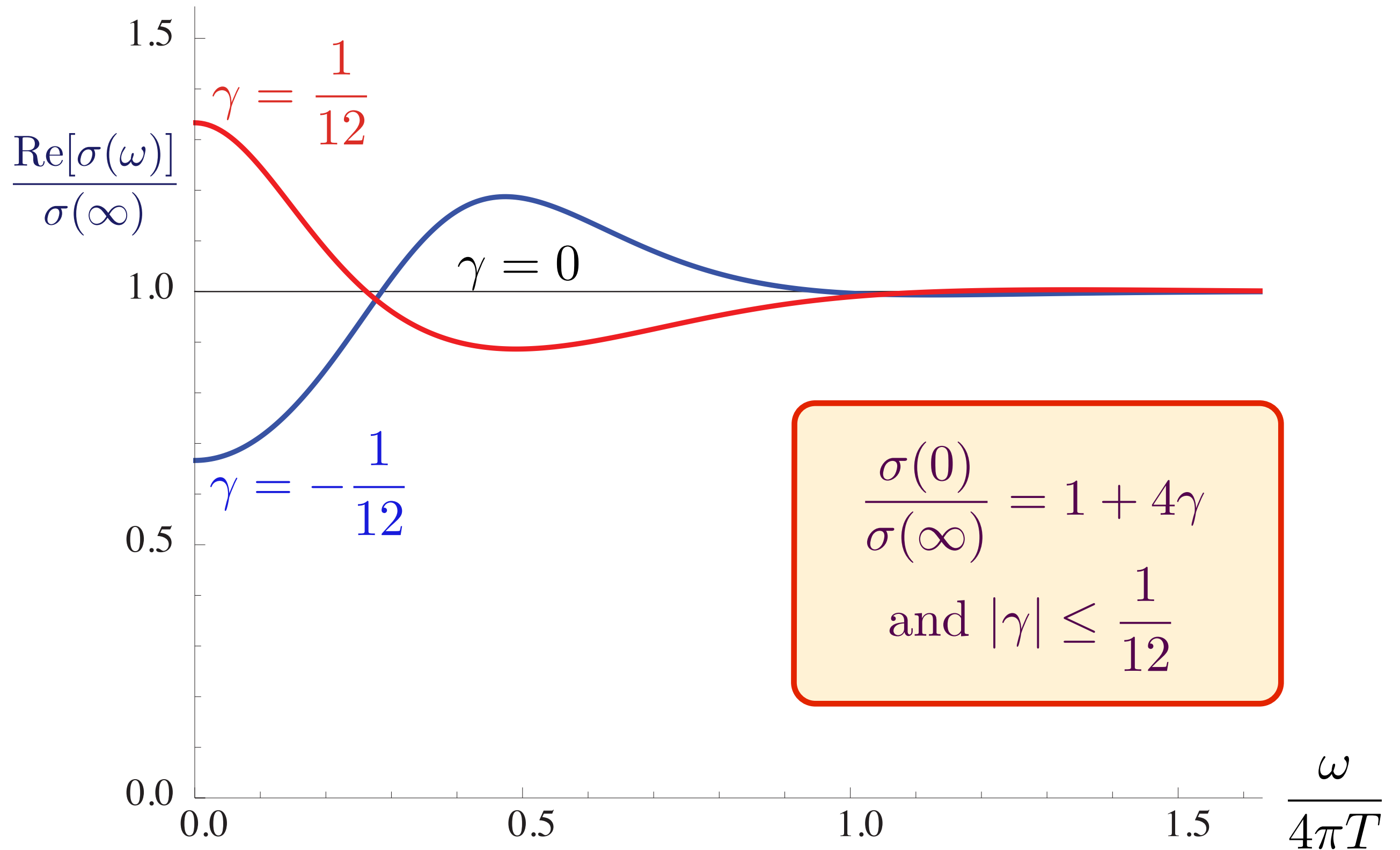
The second sum rule relies on the existence of a S-dual CFT3.

- The poles *and* zeros of the complex function $\Sigma(\omega)$ are expected to be in the lower-half plane.
- The Boltzmann theory does *not* obey the above exact properties.

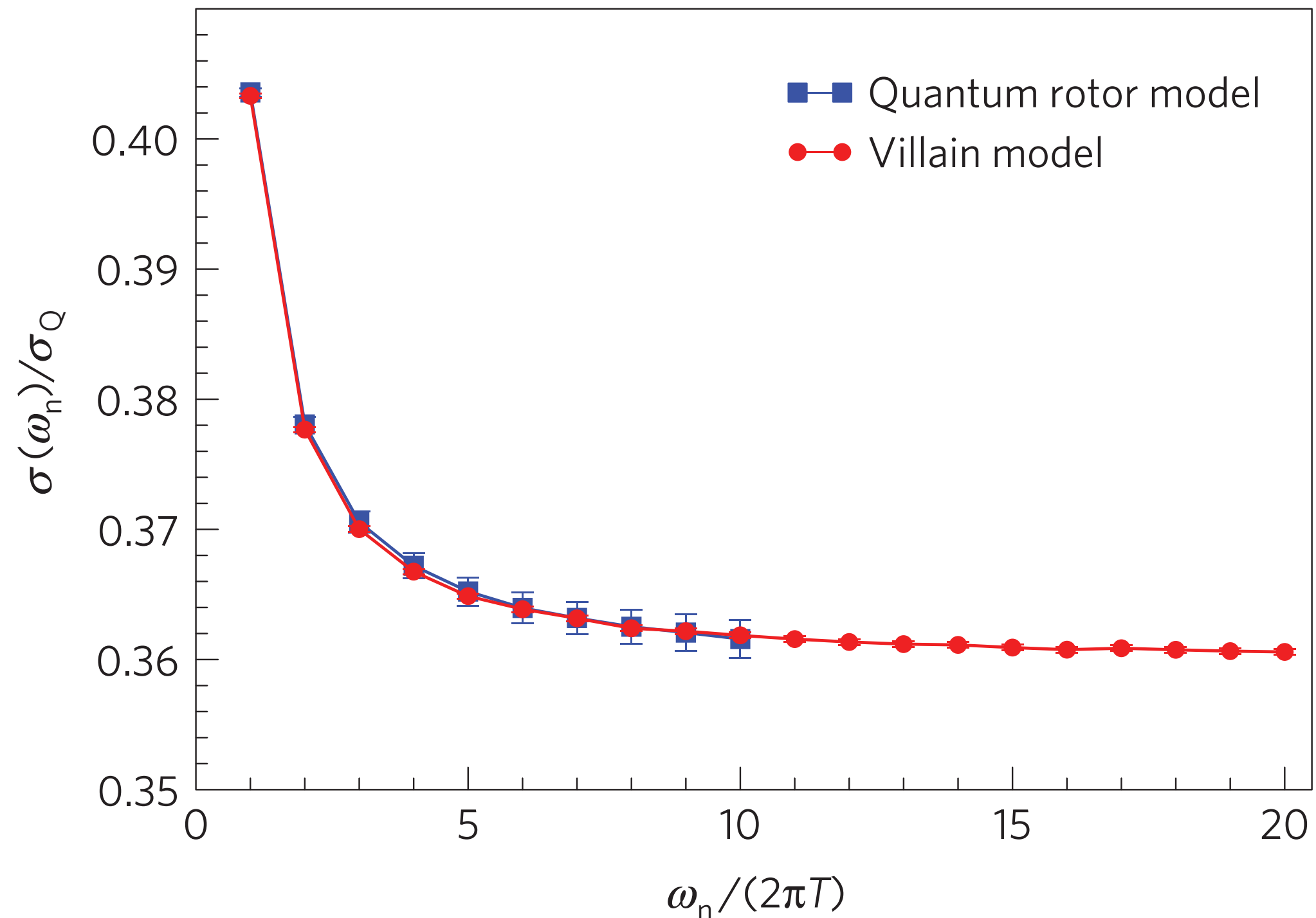
AdS₄ theory of quantum criticality



AdS₄ theory of quantum criticality



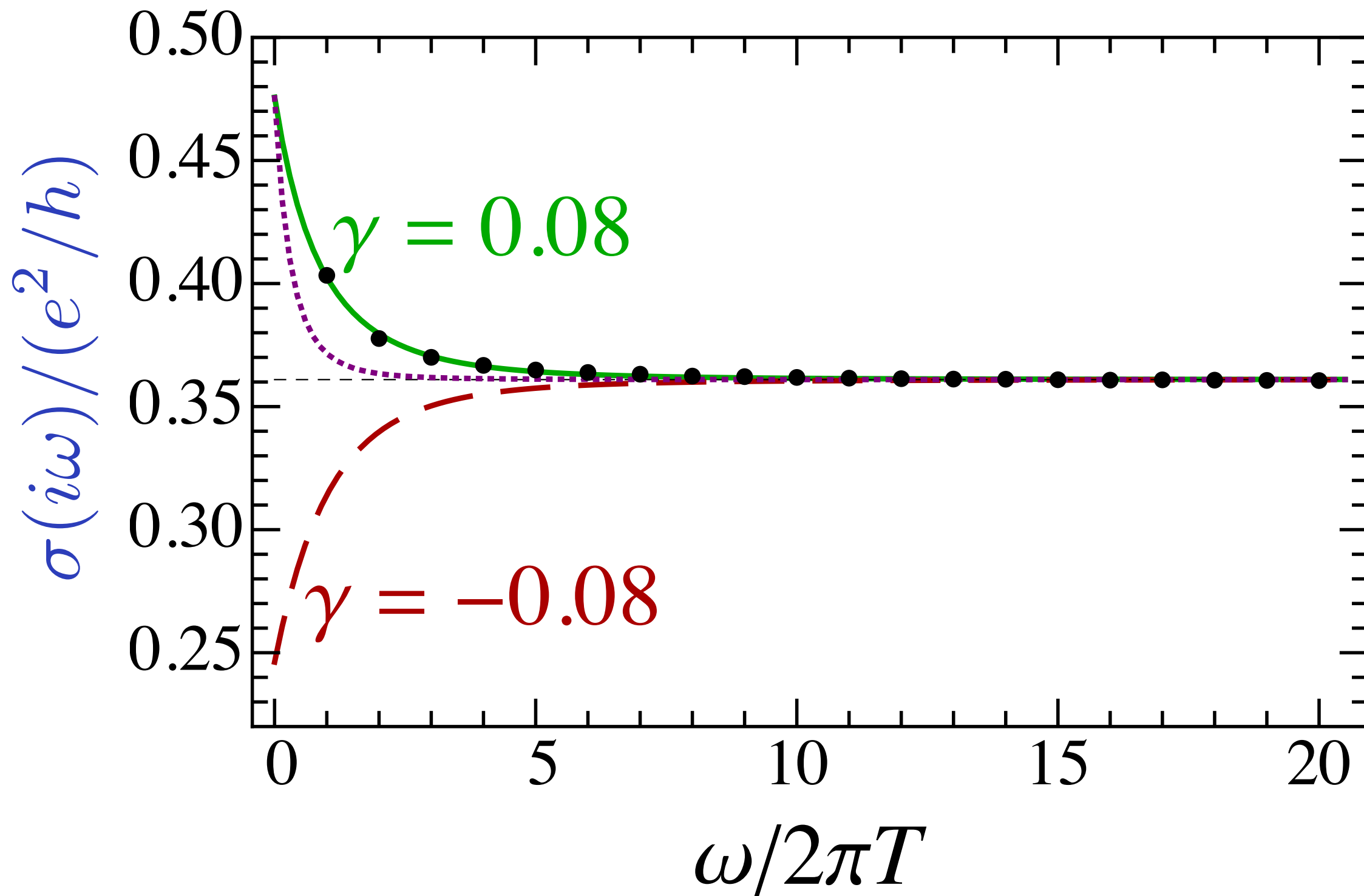
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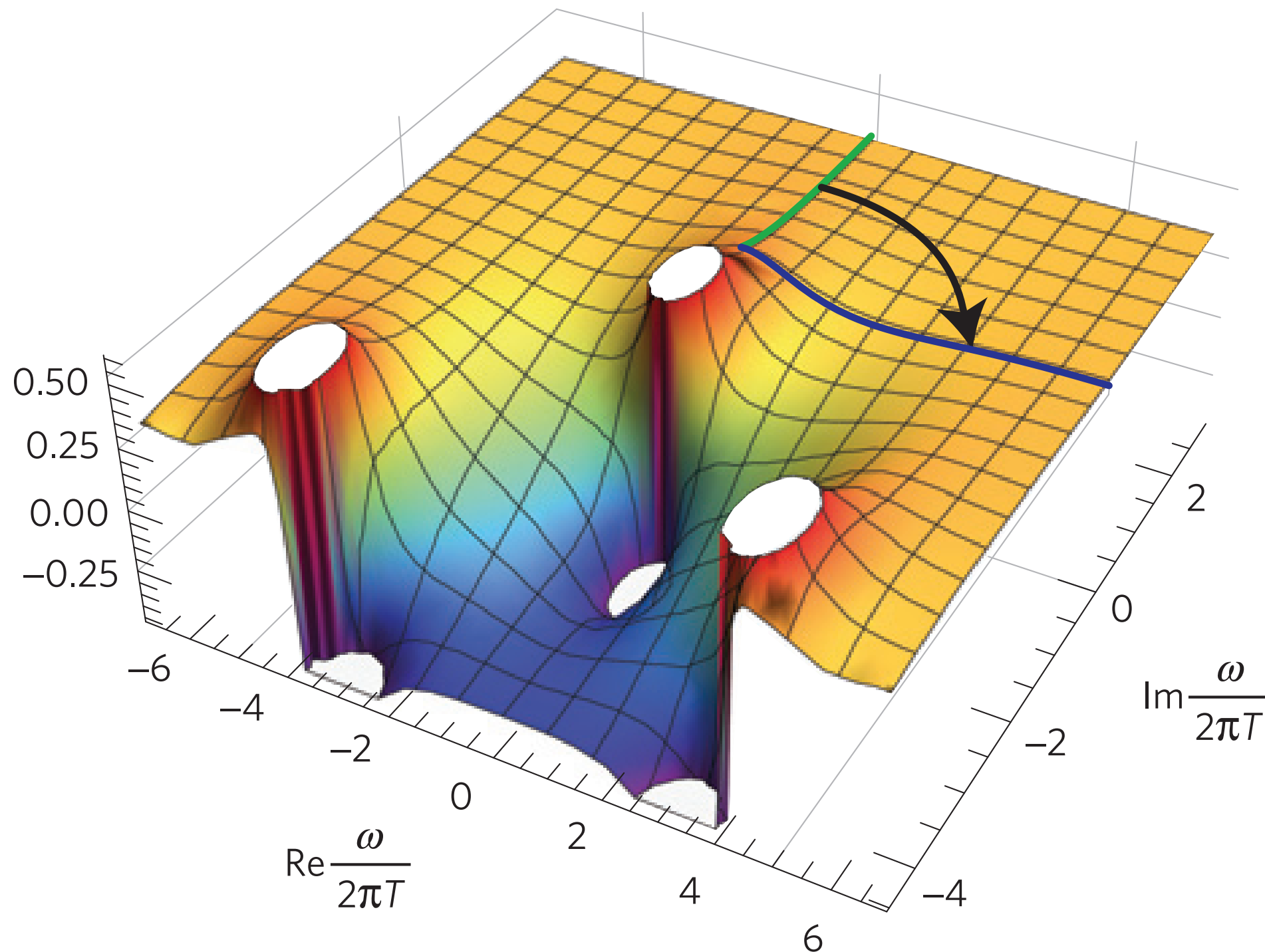
See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, Phys. Rev. Lett. **112**, 030402 (2013)

Transport revealed by QMC and holography



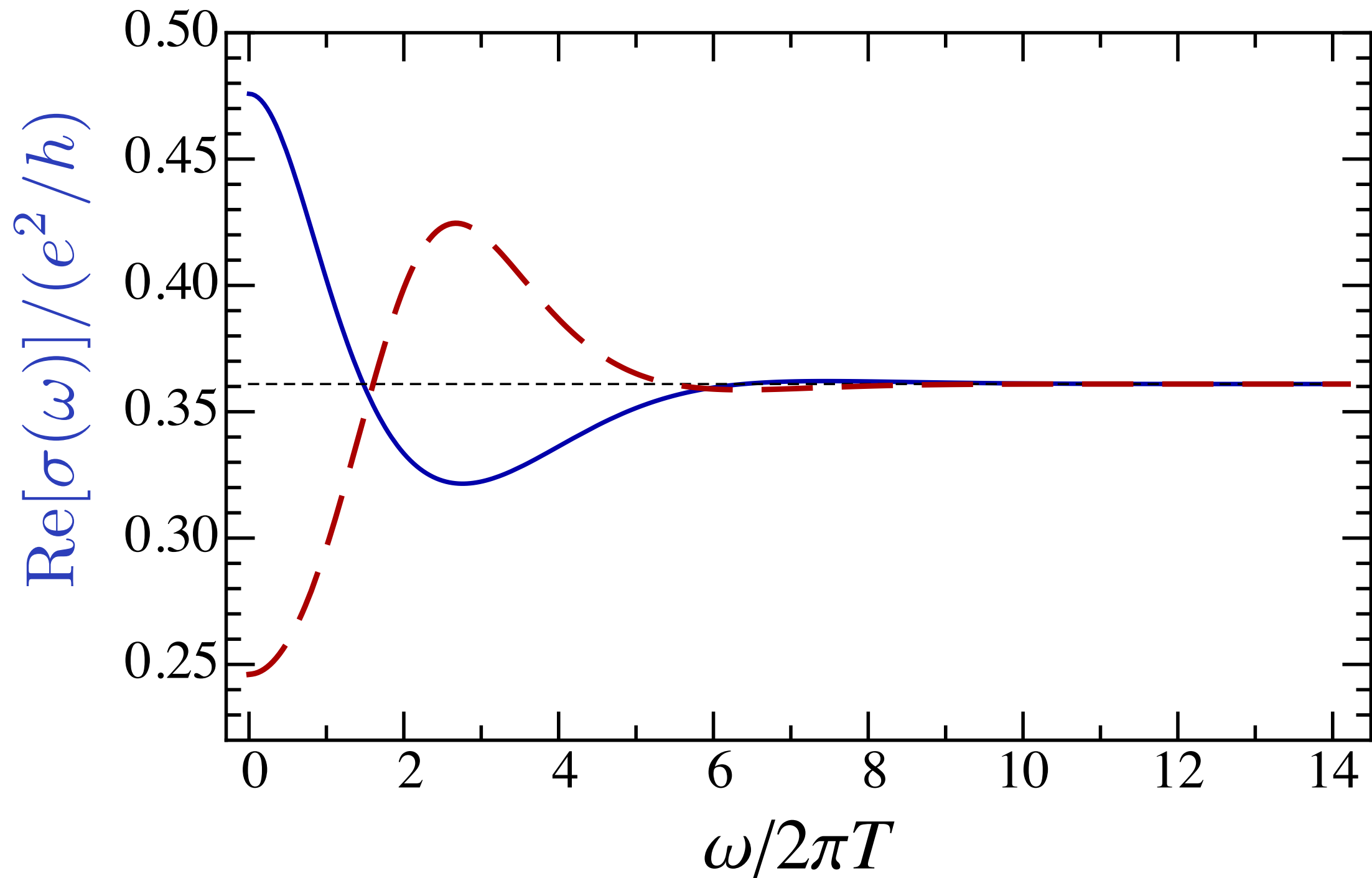
Fit of holographic theory to Monte Carlo,
after rescaling frequency axis

Analytic continuation by a holographic model



Obeys 2
non-trivial
sum rules,
and all poles
and zeros are
in the
lower-half
plane

Transport revealed by QMC and holography



Predictions of holographic theory,
after analytic continuation to real frequencies

Outline

1. Conformal field theories in $2+1$ dimensions

Superfluid-insulator transition

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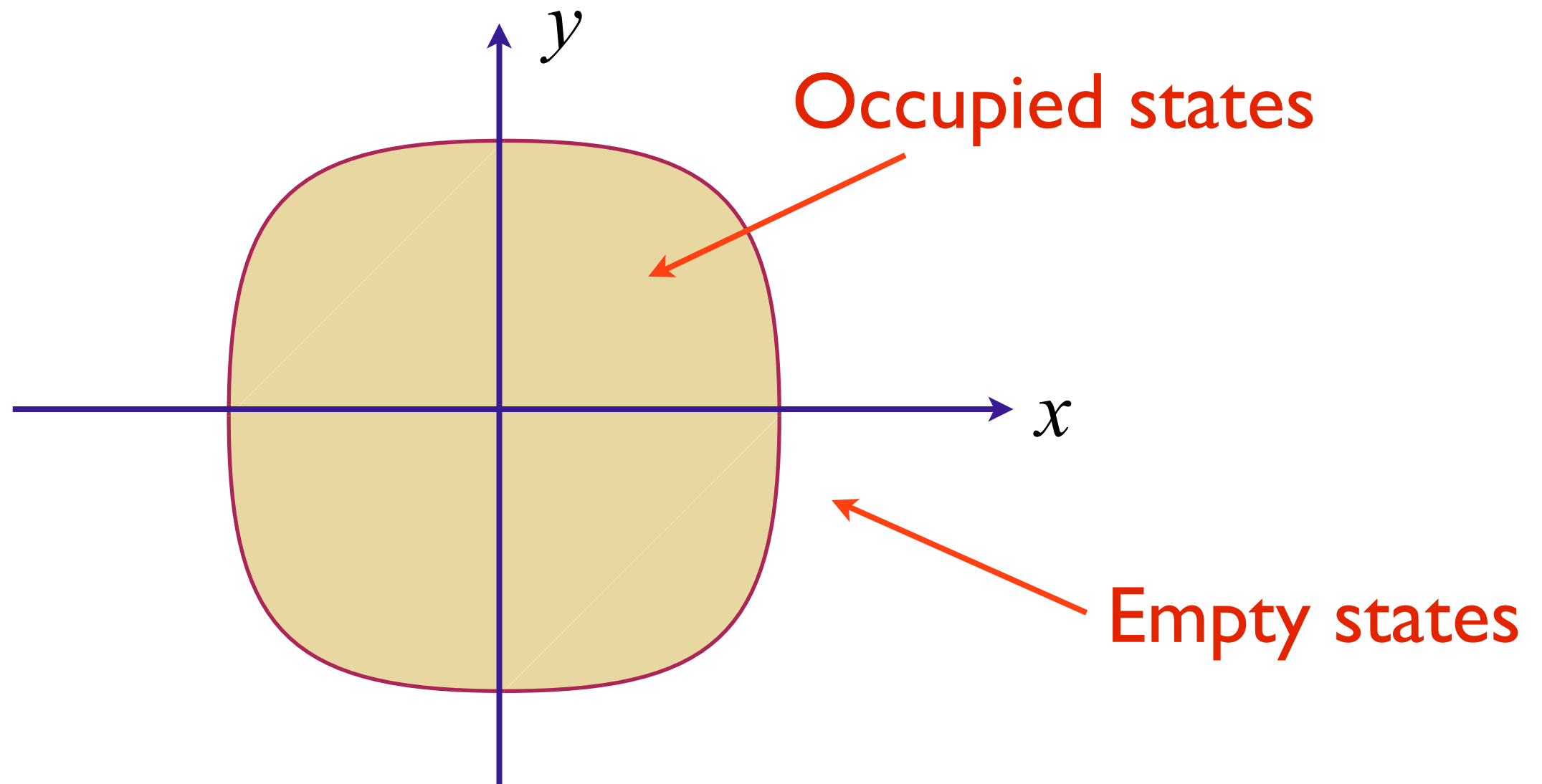
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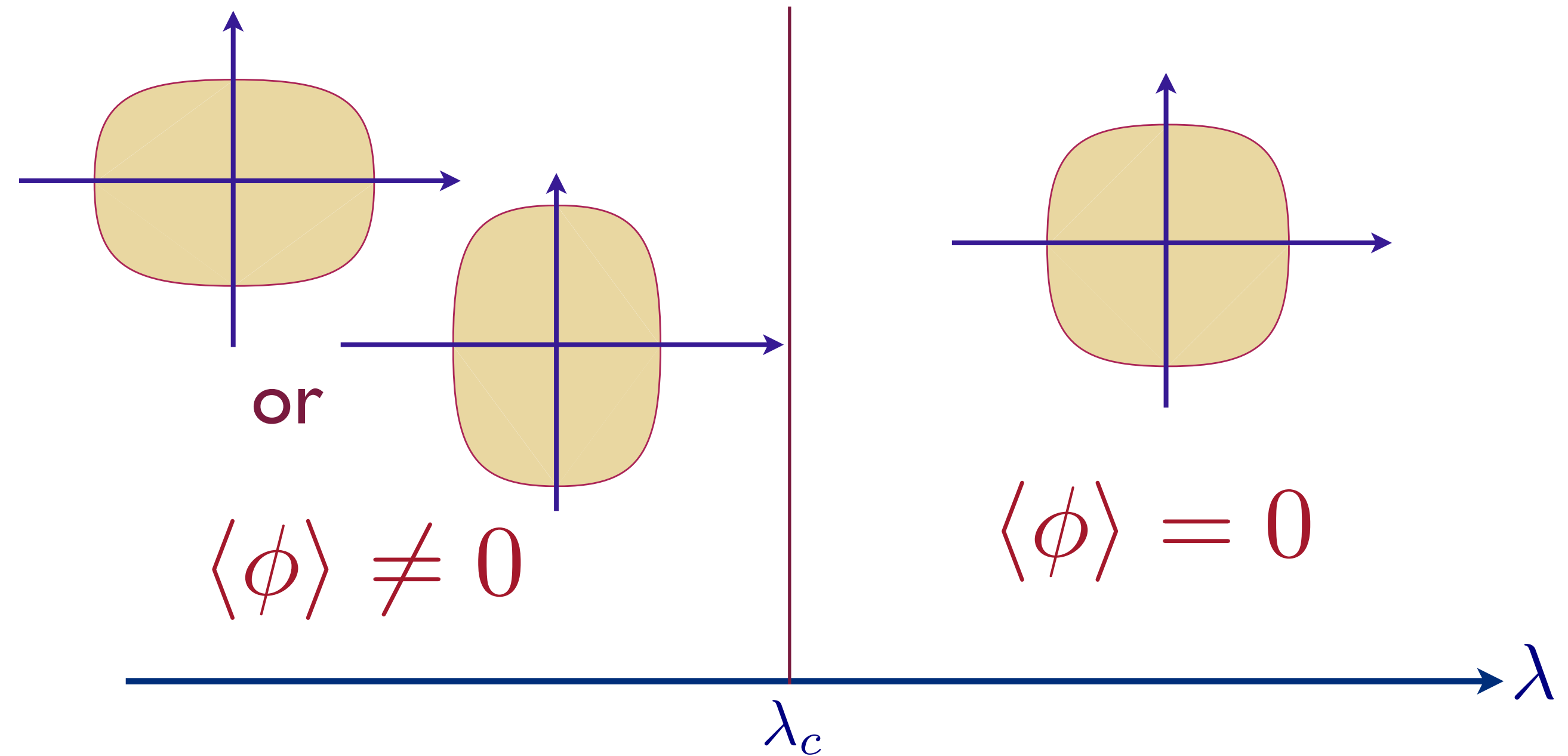
Max Metlitski

Quantum criticality of Ising-nematic ordering in a metal



A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

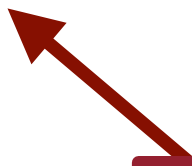
Quantum criticality of Ising-nematic ordering in a metal

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Field theory of
bosonic order
parameter

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Electrons with a
Fermi surface: $\varepsilon_{\mathbf{k}} =$
 $-2t(\cos k_x + \cos k_y) - \mu \dots$

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Quantum criticality of Ising-nematic ordering in a metal


The “standard model”:

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda -$$

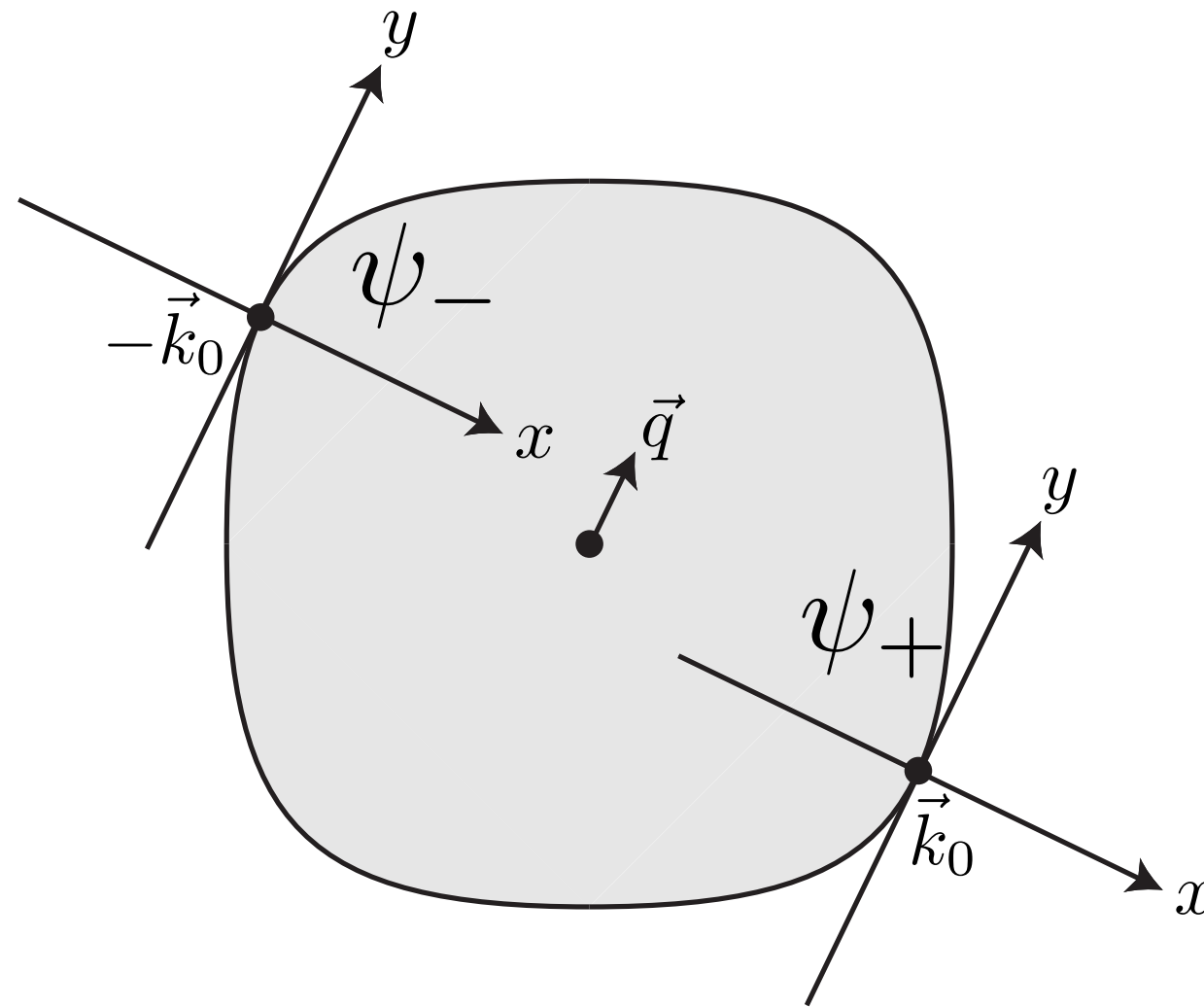
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**“Yukawa”
coupling
between bosons
and fermions**

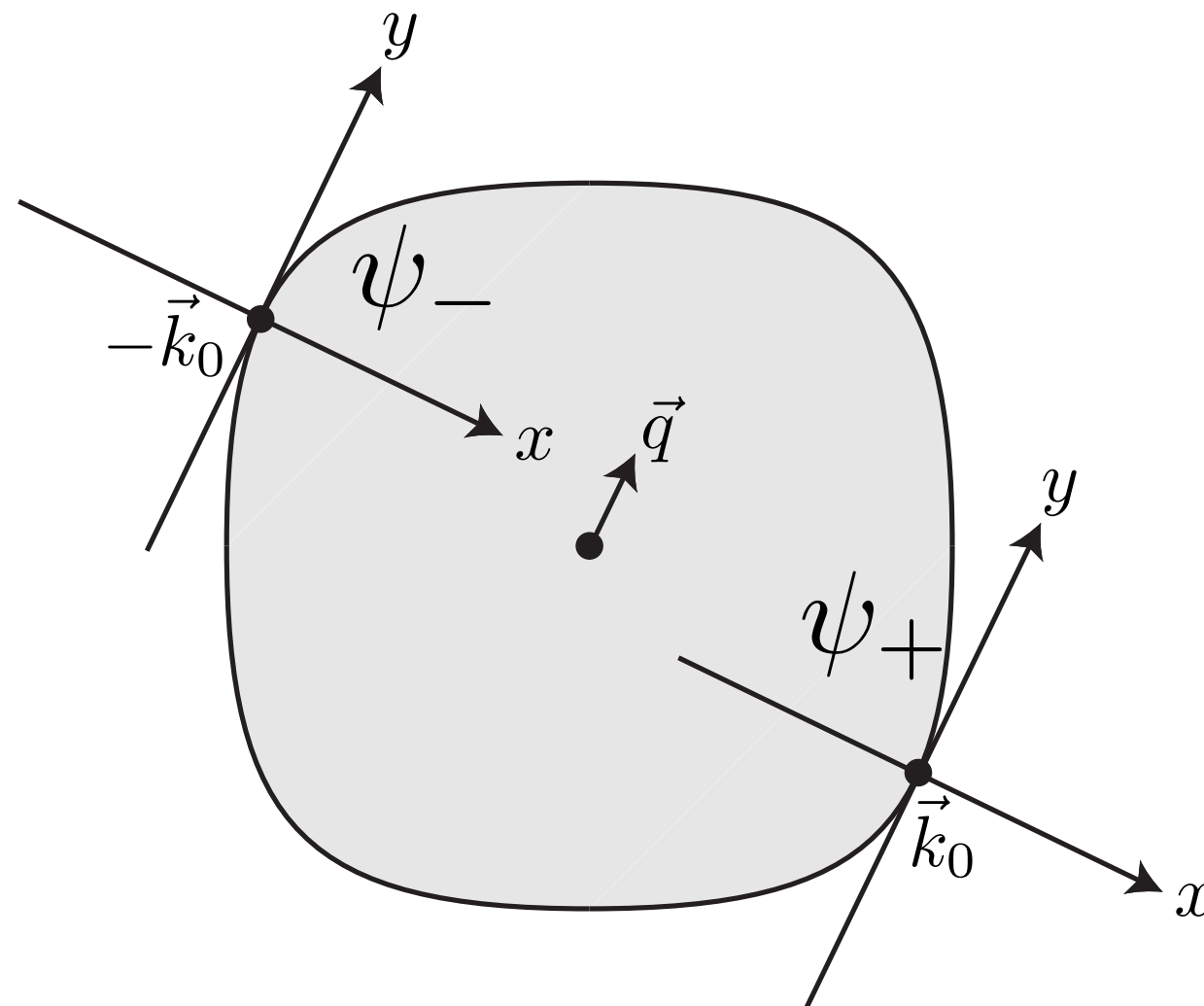


Quantum criticality of Ising-nematic ordering in a metal



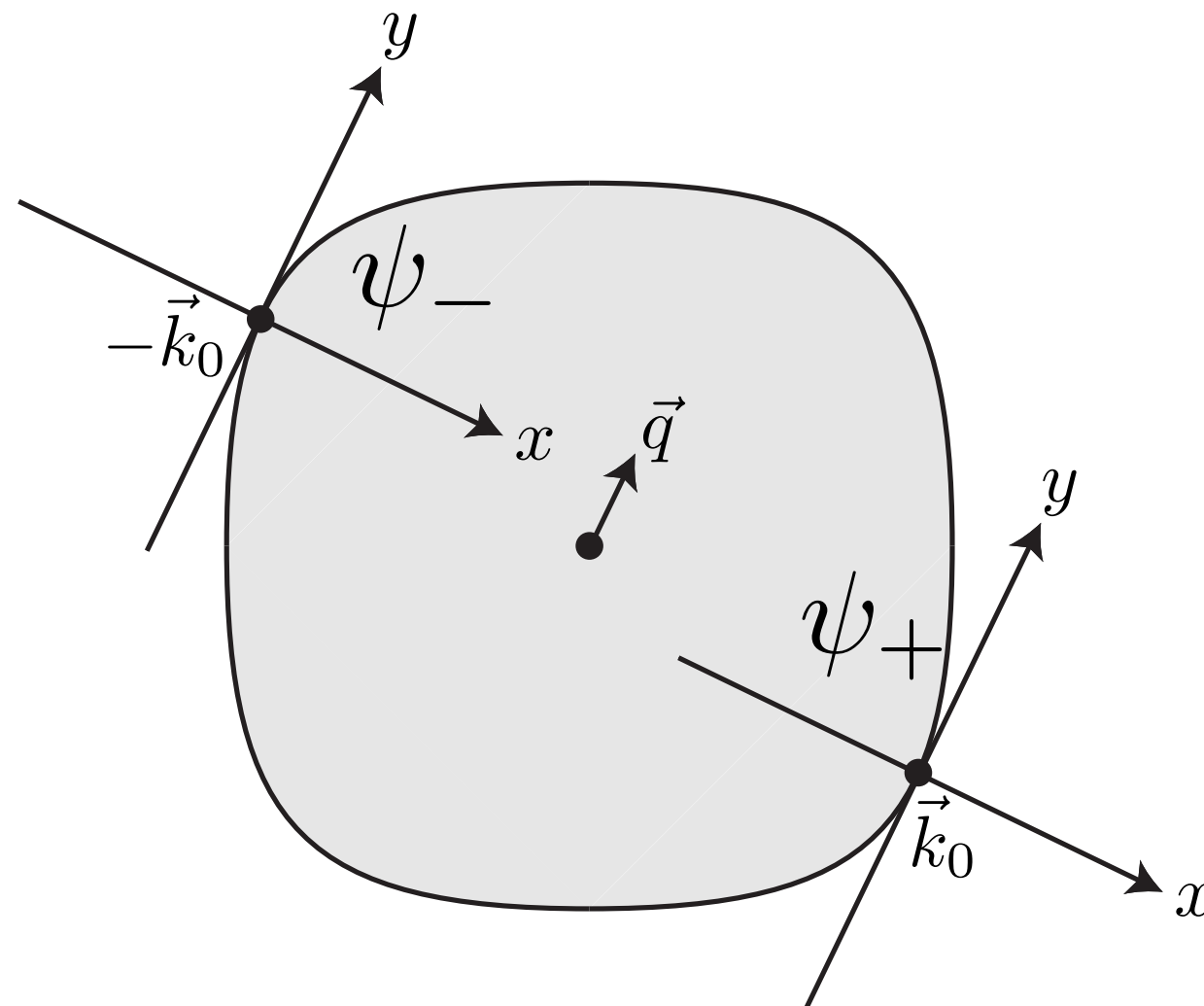
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

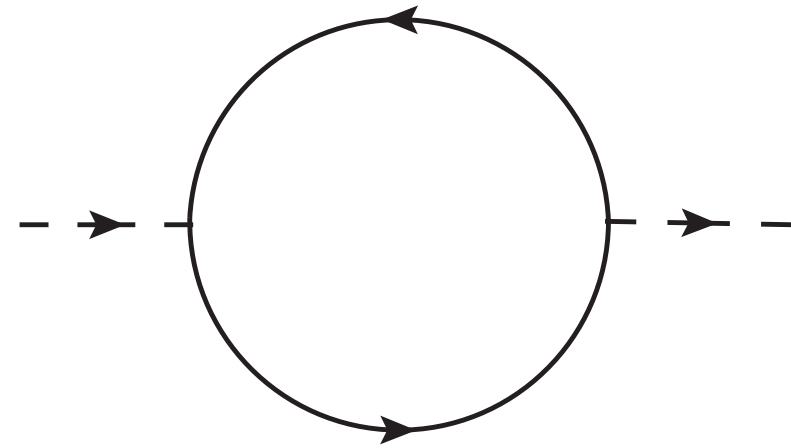
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$$\begin{aligned} \mathcal{L}[\psi_{\pm}, \phi] = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

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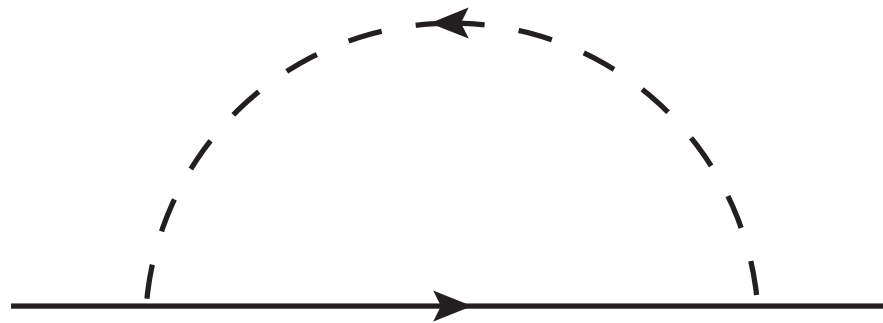
One loop ϕ self-energy with N_f fermion flavors:

$$\begin{aligned}\Sigma_\phi(\vec{q}, \omega) &= N_f \int \frac{d^2 k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{[-i(\Omega + \omega) + k_x + q_x + (k_y + q_y)^2] [-i\Omega - k_x + k_y^2]} \\ &= \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|}\end{aligned}$$

Landau-damping

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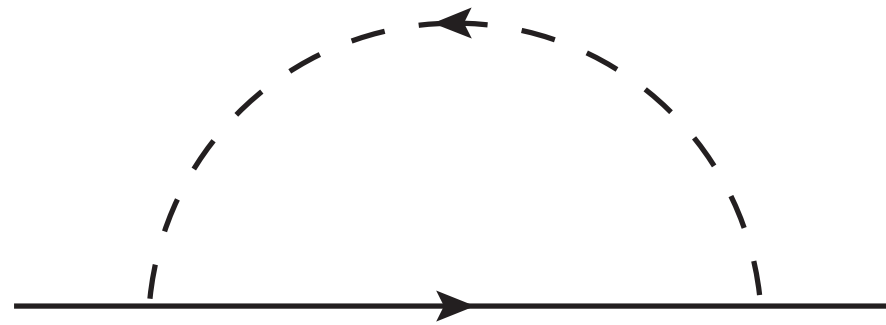


Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k}, \Omega) = -\frac{1}{N_f} \int \frac{d^2 q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2] \left[\frac{q_y^2}{g^2} + \frac{|\omega|}{|q_y|} \right]} \\ = -i \frac{2}{\sqrt{3} N_f} \left(\frac{g^2}{4\pi} \right)^{2/3} \text{sgn}(\Omega) |\Omega|^{2/3}$$

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Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_\perp^2 + \frac{|\omega|}{|q_\perp|}}, \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_\perp^2 - i \text{sgn}(\omega) |\omega|^{d/3} / N_f}$$

In the boson case, $q_\perp^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.

In the fermion case, $q_x \sim q_\perp^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2 \Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.

Strongly-coupled theory in $d = 2$.

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In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

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Simple scaling argument for $z = 3/2$.

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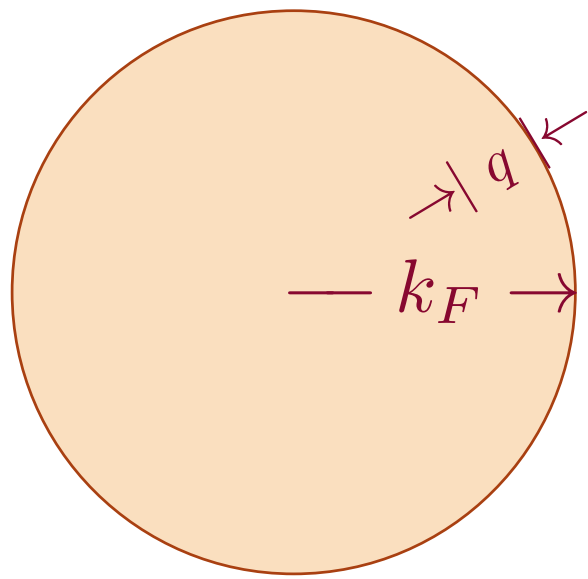
Simple scaling argument for $z = 3/2$.

Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\rightarrow \phi s \\ \psi &\rightarrow \psi s^{(2z+1)/4} \\ g &\rightarrow g s^{(3-2z)/4} \end{aligned}$$

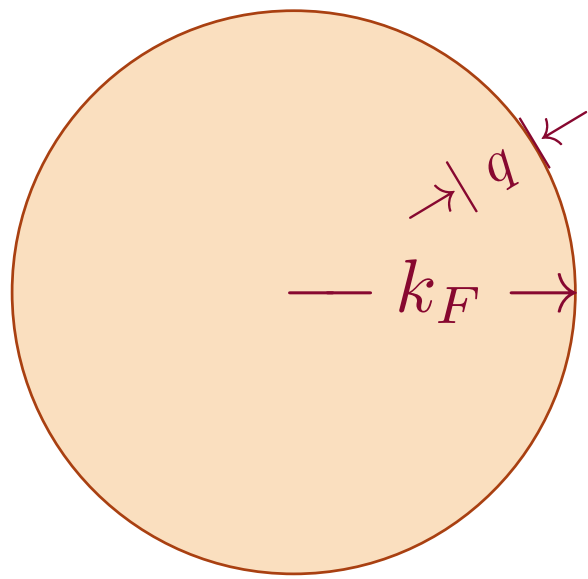
So the action is invariant provided $z = 3/2$.

FL Fermi liquid



- $k_F^d \sim Q$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

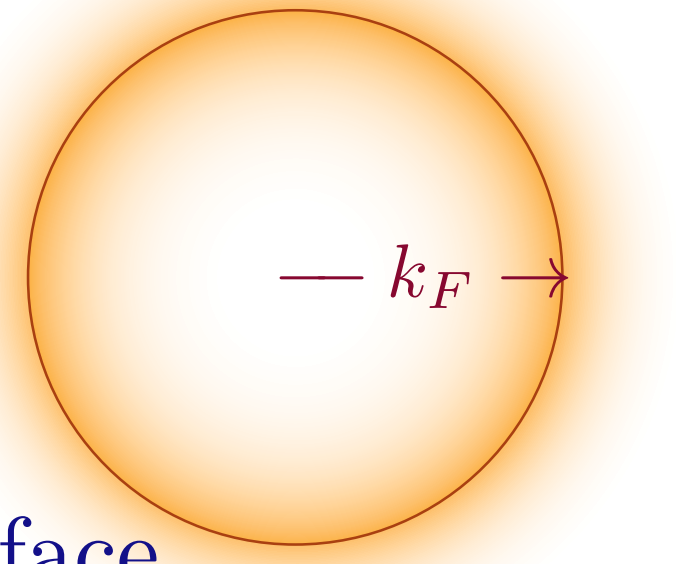
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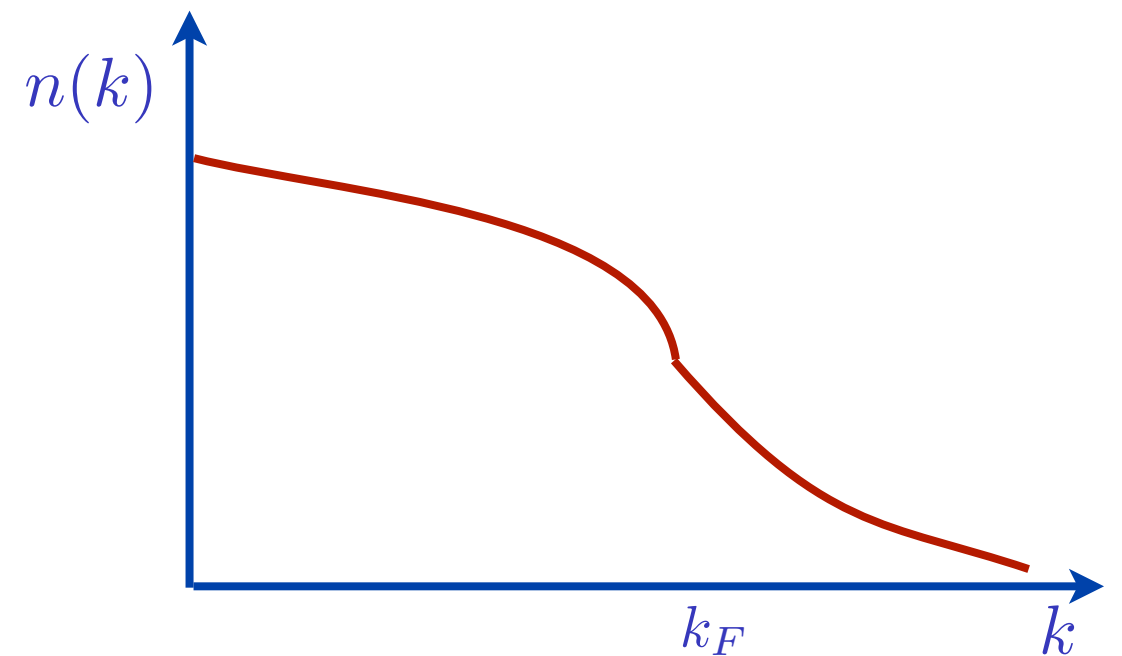
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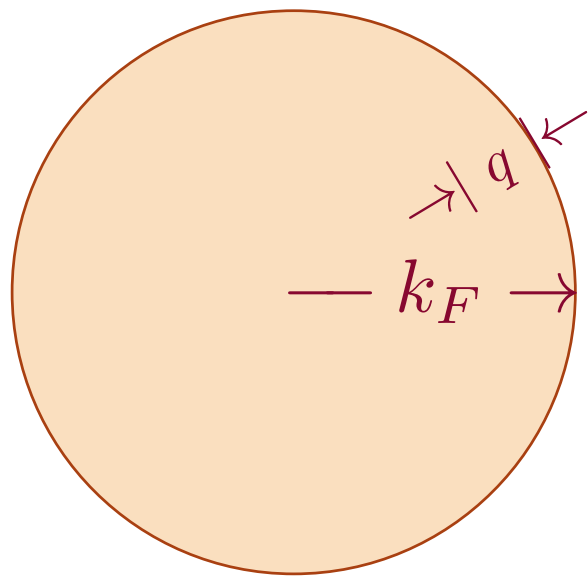
NFL Nematic QCP



- Fermi surface with $k_F^d \sim Q$.



FL Fermi liquid



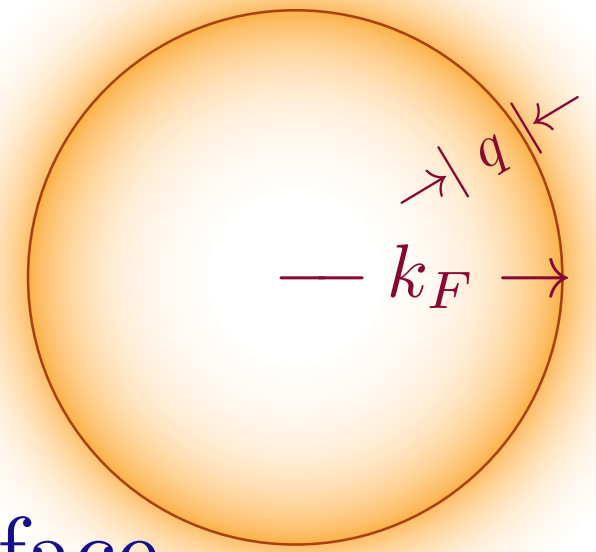
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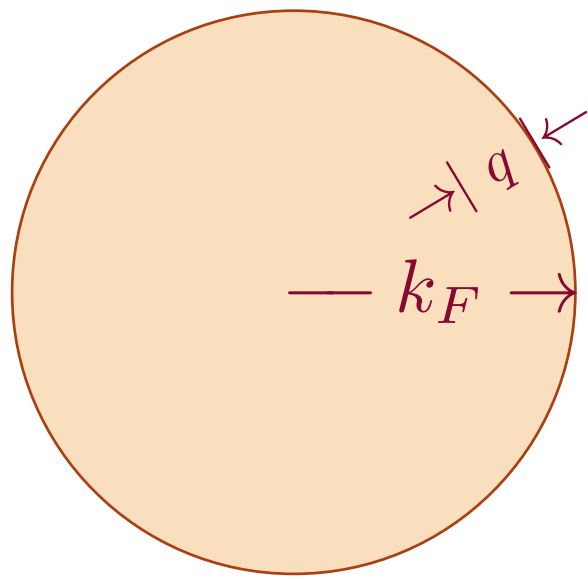
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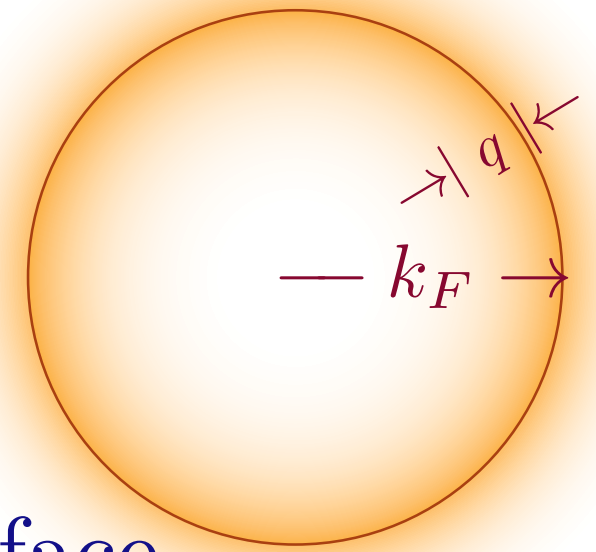
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

FL Fermi liquid



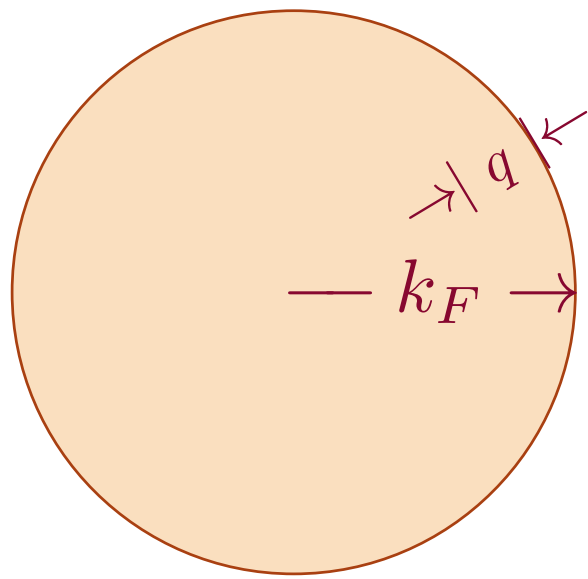
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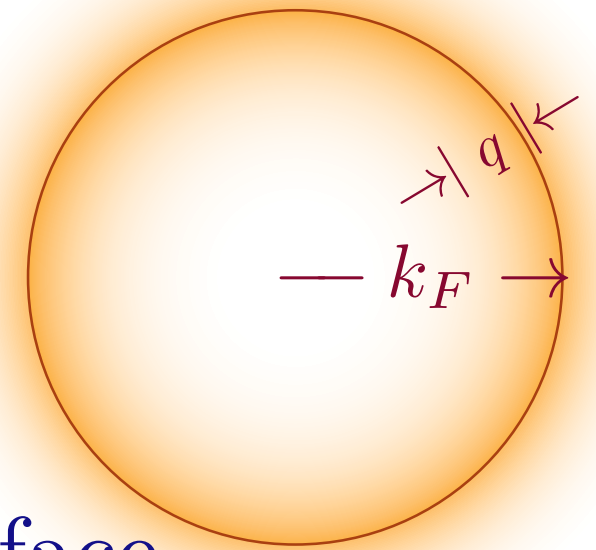
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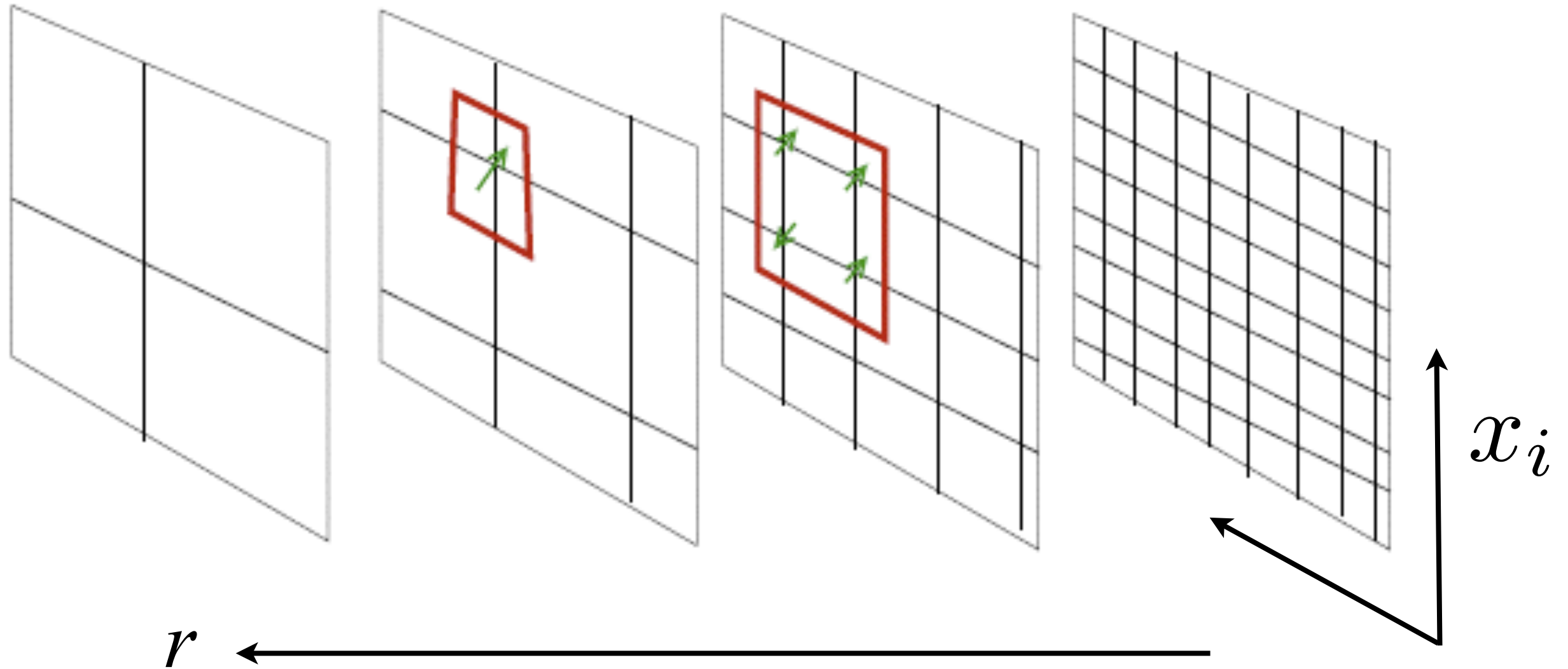


Liza Huijse

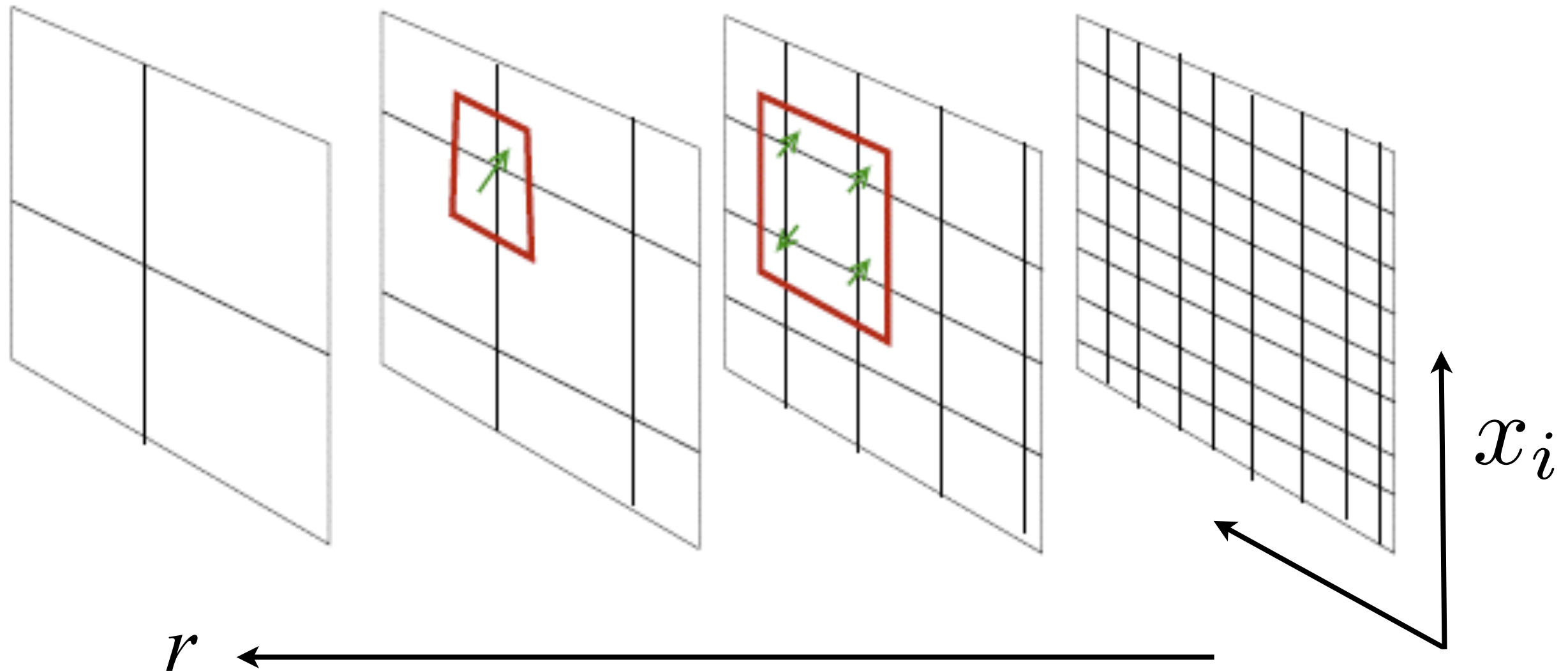


Brian Swingle

Generalized holography



Generalized holography

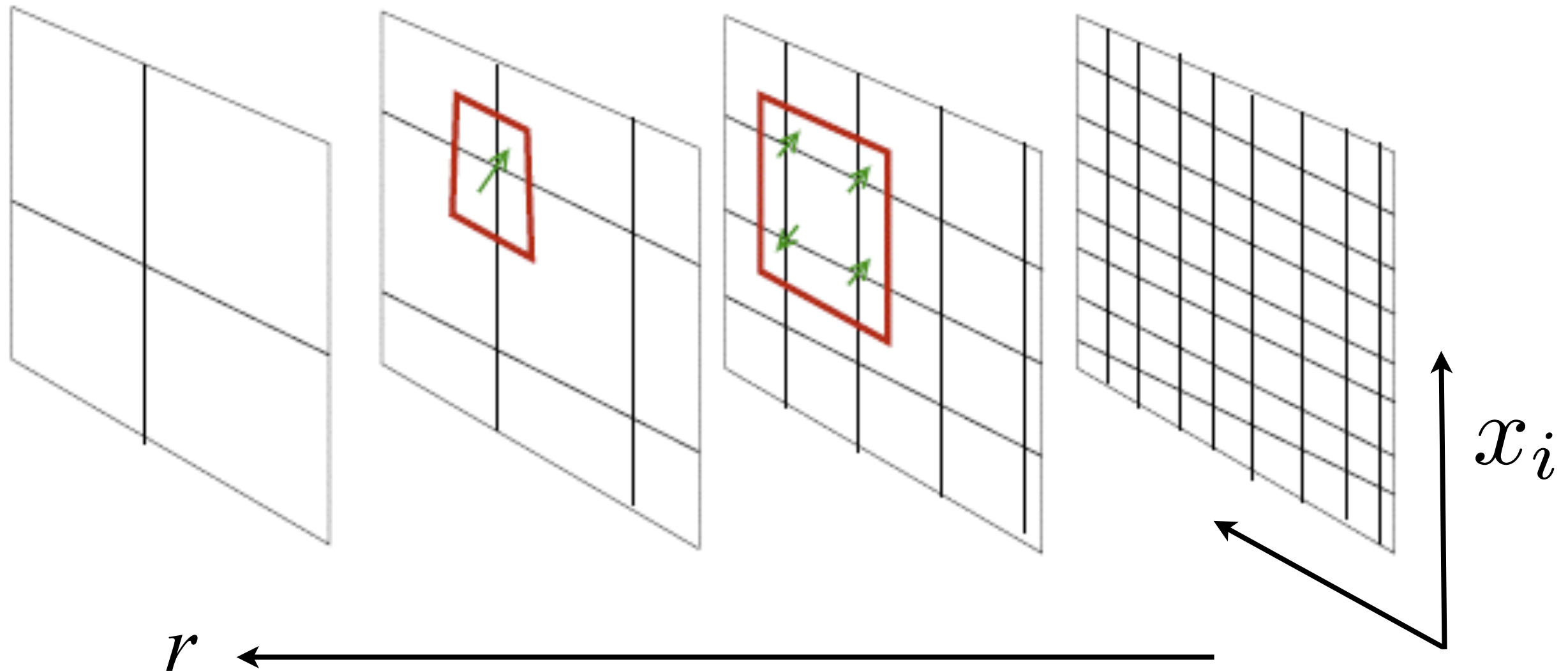


Consider a metric which transforms under rescaling as

$$x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Generalized holography



The most general such metric is

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

This is the most general metric which is invariant under the scale transformation

$$\begin{aligned} x_i &\rightarrow \zeta x_i \\ t &\rightarrow \zeta^z t \\ ds &\rightarrow \zeta^{\theta/d} ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points). We will see shortly that θ is the violation of hyperscaling exponent.

We have used reparametrization invariance in r to define it so that it scales as

$$r \rightarrow \zeta^{(d-\theta)/d} r.$$

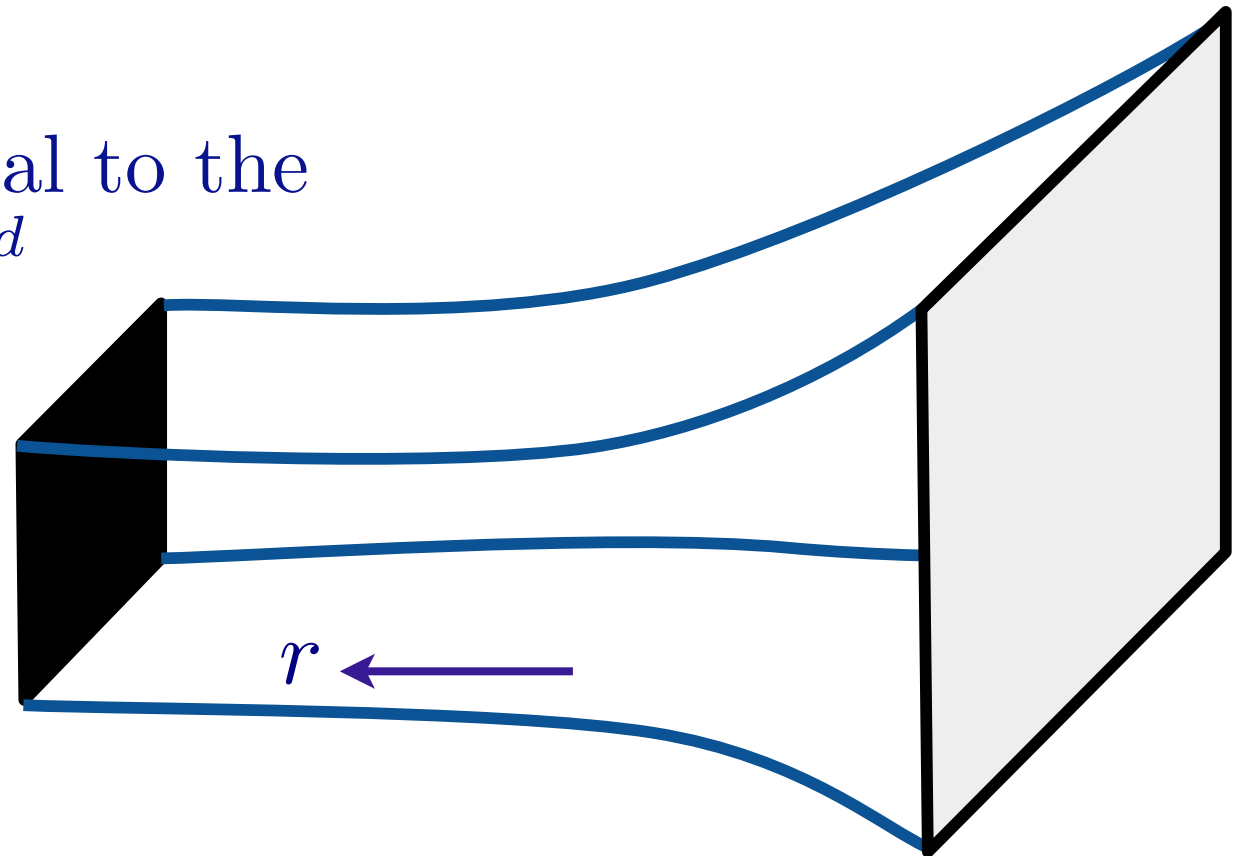
Generalized holography

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At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



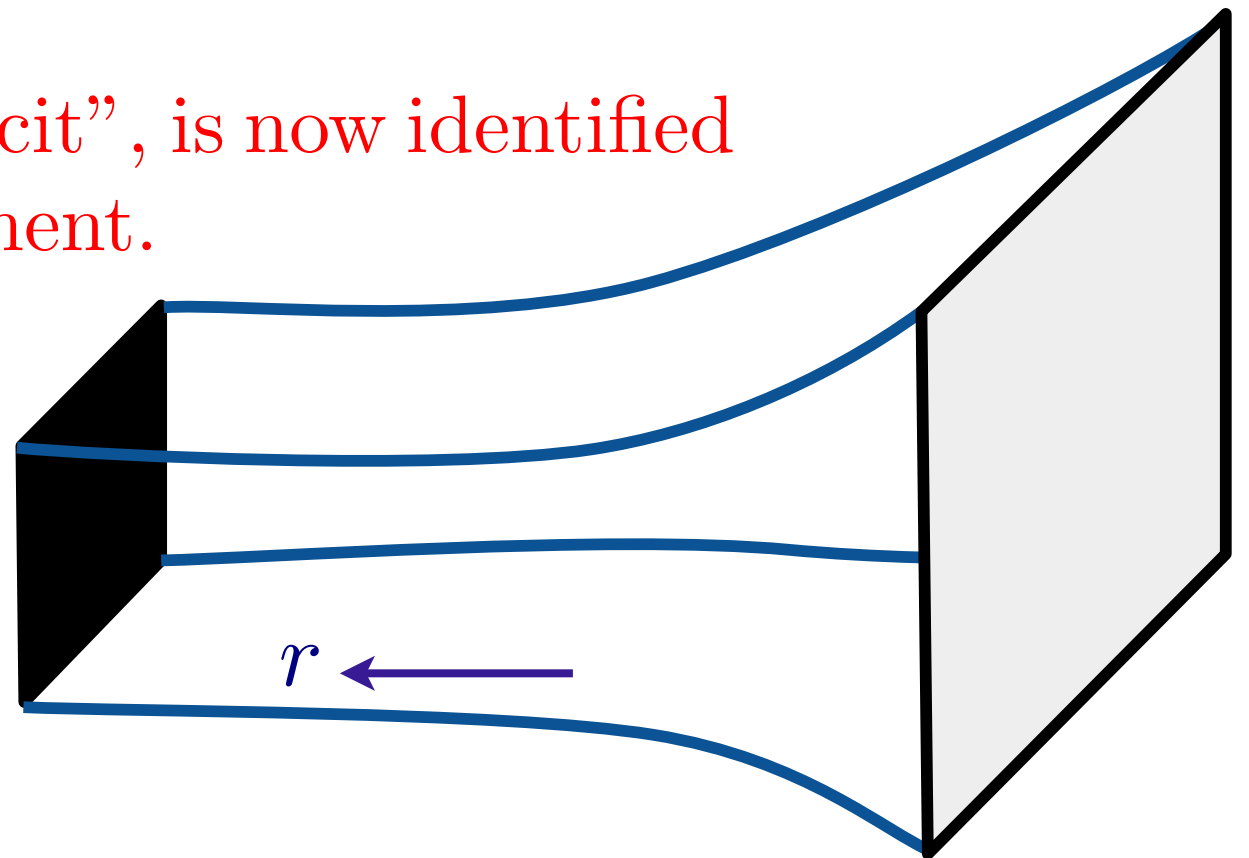
Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$, the “dimension deficit”, is now identified as the violation of hyperscaling exponent.



Generalized holography

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$$z \geq 1 + \frac{\theta}{d}$$

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The non-Fermi liquid in $d = 2$ has $\theta = d - 1$, and this implies $z \geq 3/2$. So the lower bound is precisely the value obtained for the non-Fermi liquid!

Generalized holography

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases} .$$

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The non-Fermi liquid has log-violation of “area law”, and this appears precisely at the correct value $\theta = d - 1$!

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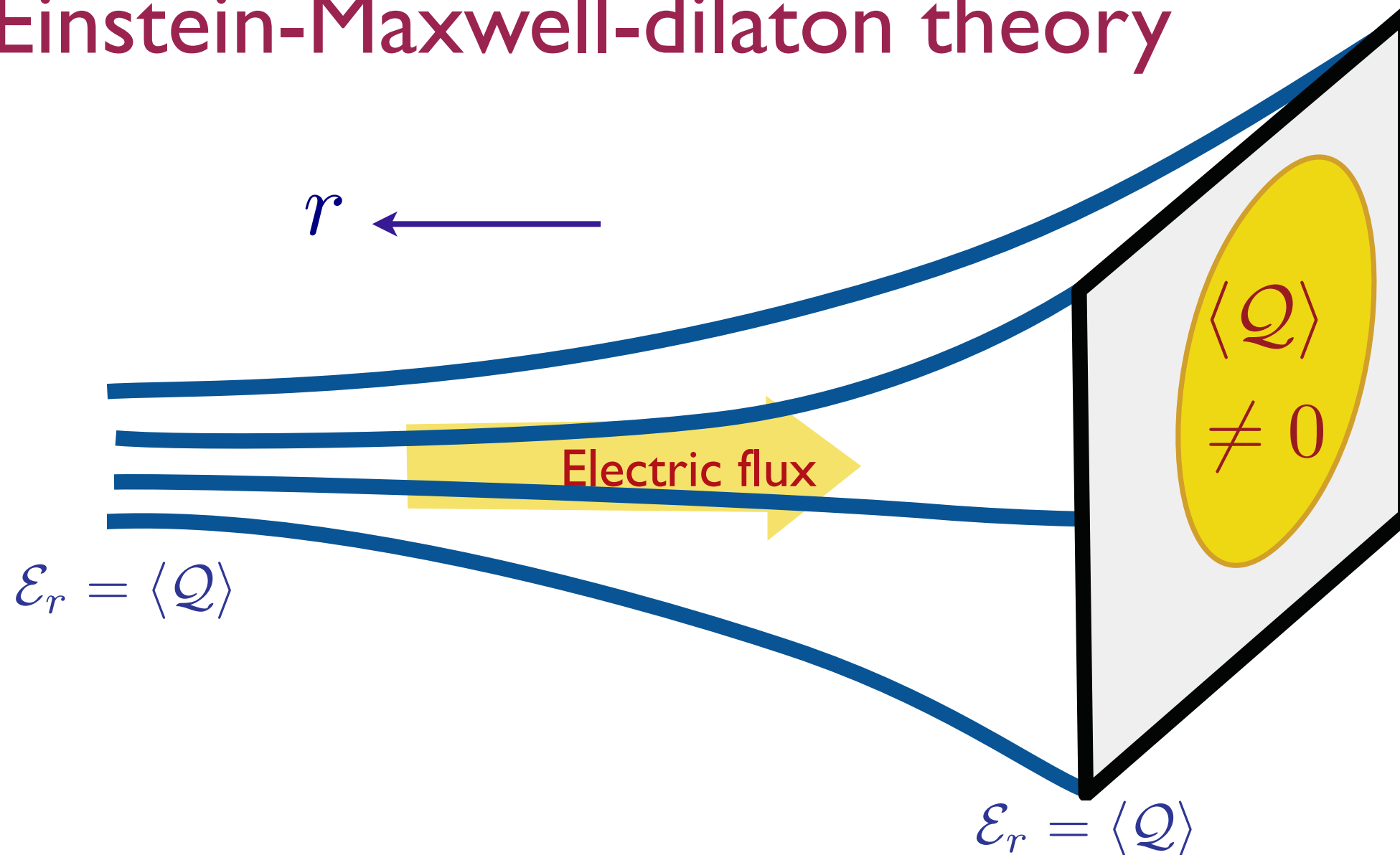
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The non-Fermi liquid has log-violation of “area law”, and this appears precisely at the correct value $\theta = d - 1$!

Moreover, the co-efficient of $P \ln P$ computed holographically is independent of the shape of the entangling region just as expected for a circular Fermi surface!!

Holography of a non-Fermi liquid

Einstein-Maxwell-dilaton theory



$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).

S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).

N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

The $r \rightarrow \infty$ limit of the metric of the Einstein-Maxwell-dilaton (EMD) theory has the most general form with

$$\begin{aligned} \theta &= \frac{d^2 \beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta) + \theta)^2}{d^2(d-\theta)\alpha^2}. \end{aligned}$$

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Computation of the entanglement entropy in the EMD theory via the Ryu-Takayanagi formula for $\theta = d - 1$ yields

$$\mathcal{S}_E = \mathcal{C}_E \mathcal{Q}^{(d-1)/d} P \ln P$$

where \mathcal{C}_E is independent of UV details.

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This is precisely as expected for a Fermi surface, when combined with the Luttinger theorem!

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Computation of the entanglement entropy in the EMD

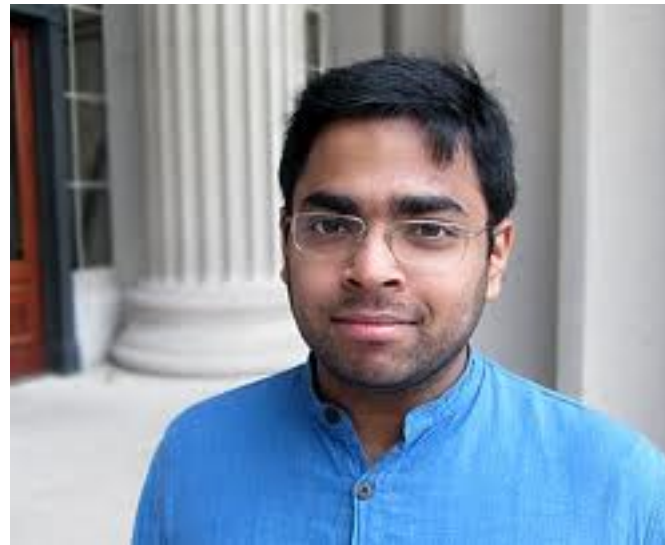
Corrections to the Ryu-Takayangi formula from the bulk entanglement entropy also correctly yield the entanglement entropy and Fermi surface volume of probe fermions.

L. Huijse, S. Sachdev, and B. Swingle, arXiv:1308.3234

combined with the Luttinger theorem!



Sean Hartnoll
Stanford



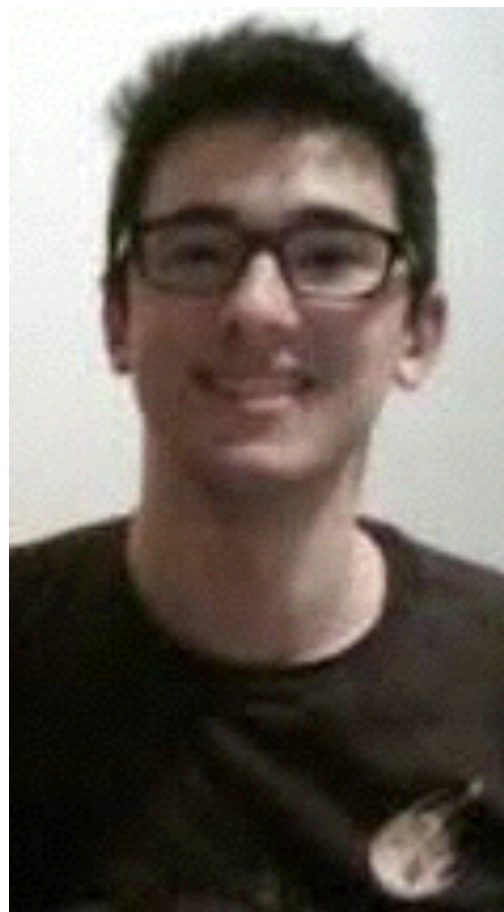
Raghu Mahajan
Stanford



Matthias Punk
Innsbruck



Koenraad Schalm
Leiden



Andrew Lucas
Harvard

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic ϕ fluctuations.

Quantum criticality of Ising-nematic ordering in a metal

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- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$.

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- Analogous to electron-phonon scattering in metals, where we have “Bloch’s law”: a resistivity $\rho(T) \sim T^5$.
- “Bloch’s law” for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.

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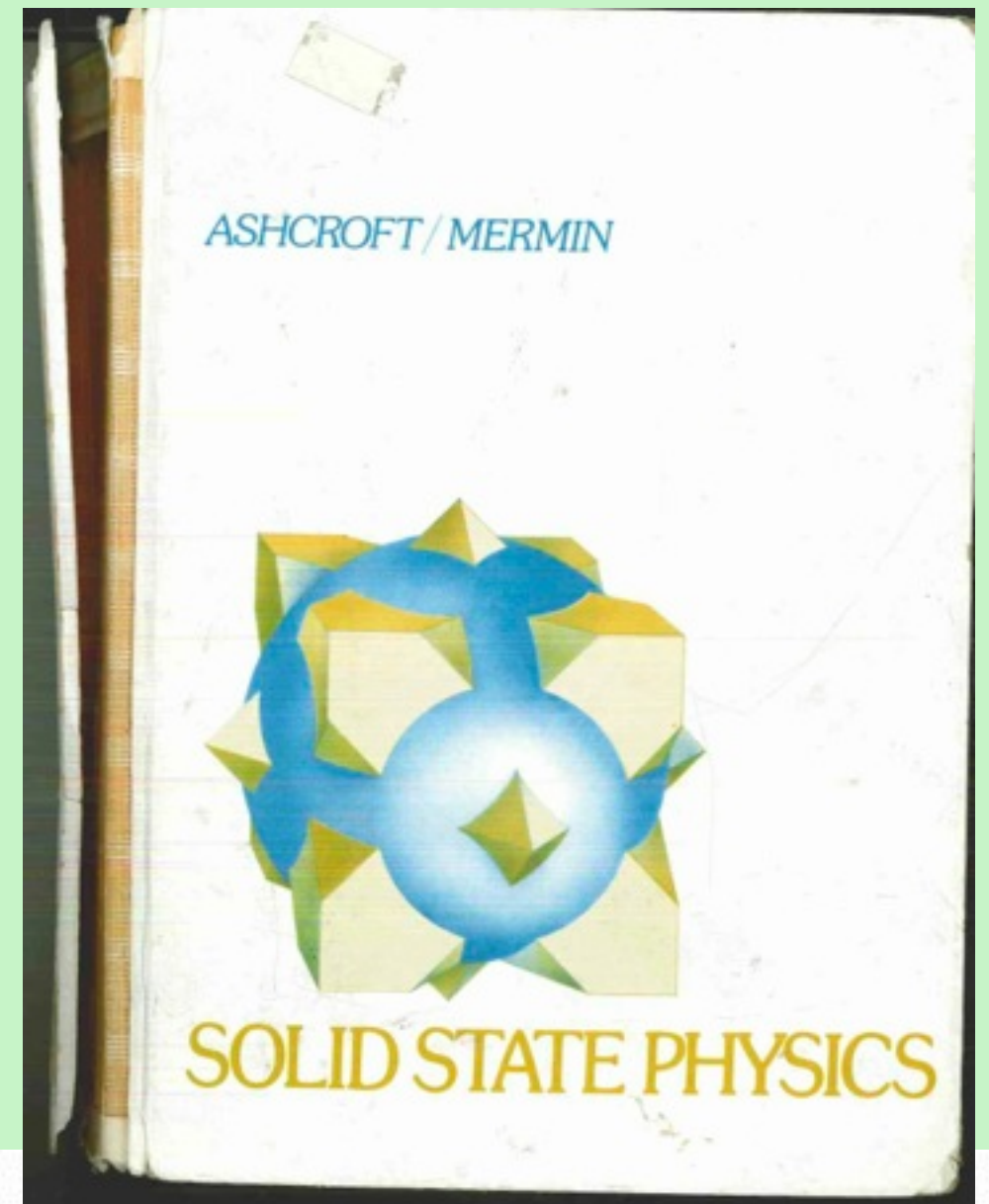
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**However, this ignores
“phonon drag”**

PHONON DRAG

Peierls²⁸ pointed out a way in which the low temperature resistivity might decline more rapidly than T^5 .

²⁸ R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).



Quantum criticality of Ising-nematic ordering in a metal

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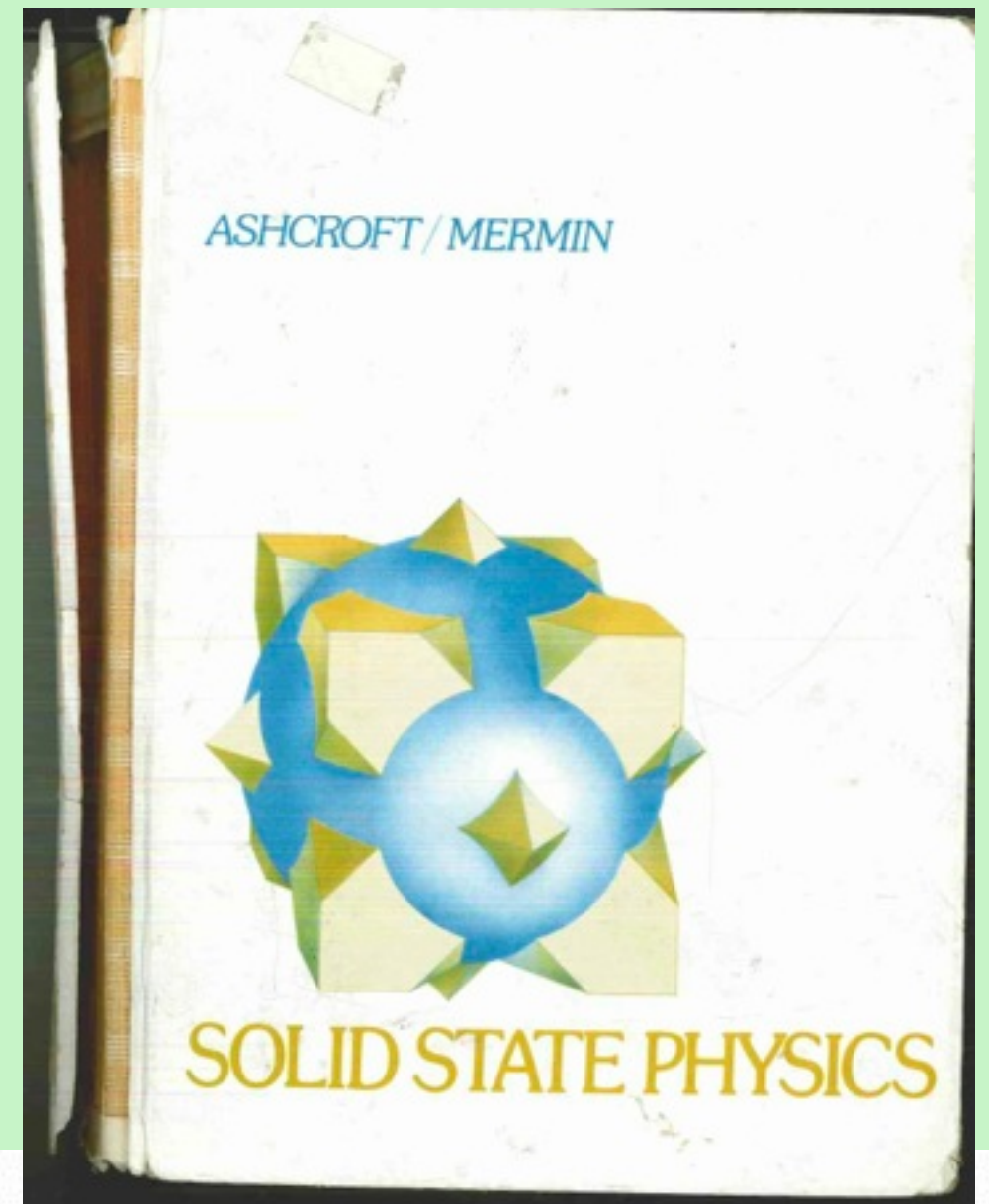
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**However, this ignores
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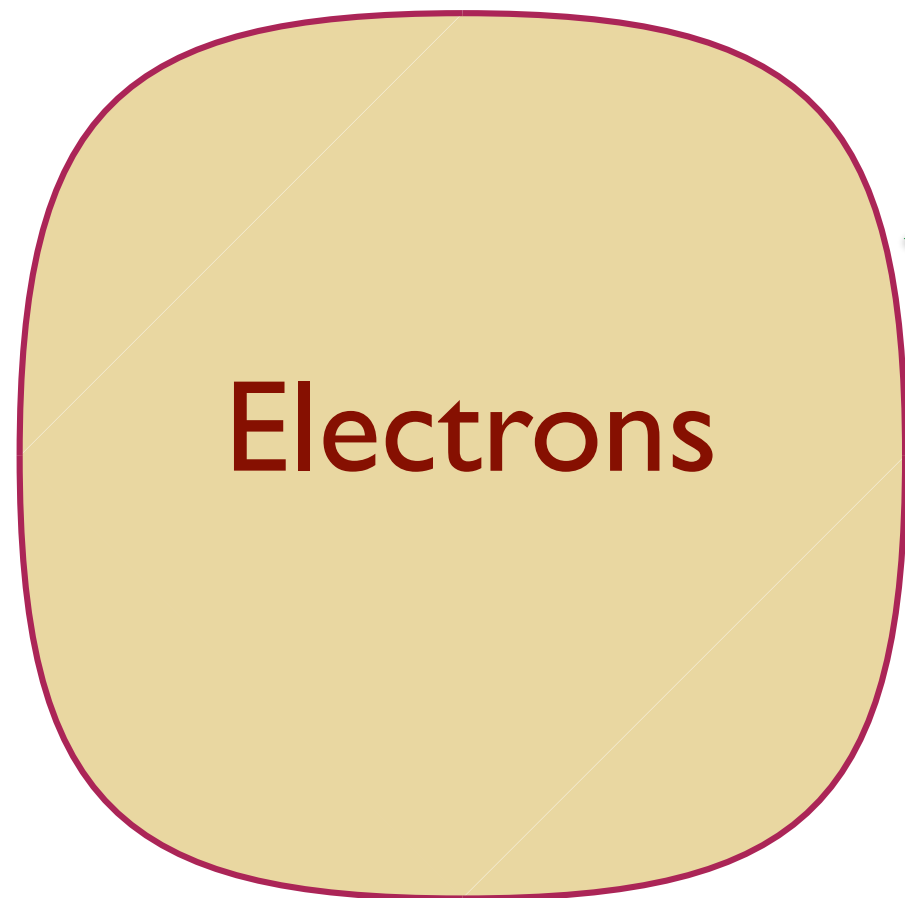
PHONON DRAG

Peierls²⁸ pointed out a way in which the low temperature resistivity might decline more rapidly than T^5 . This behavior has yet to be observed.

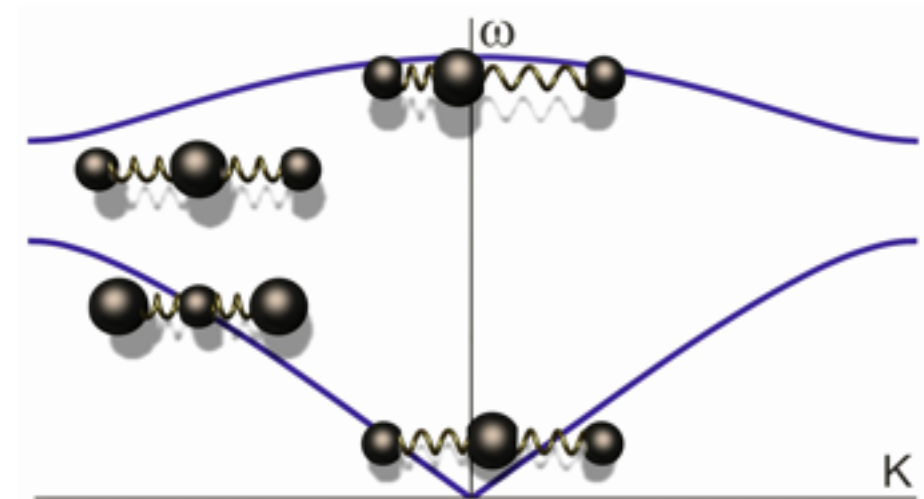
²⁸ R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).



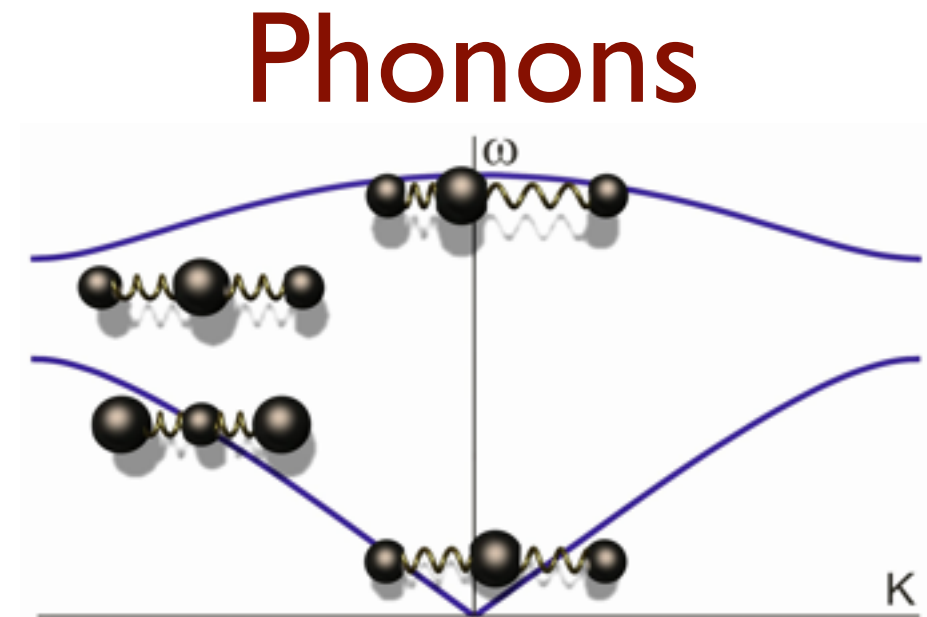
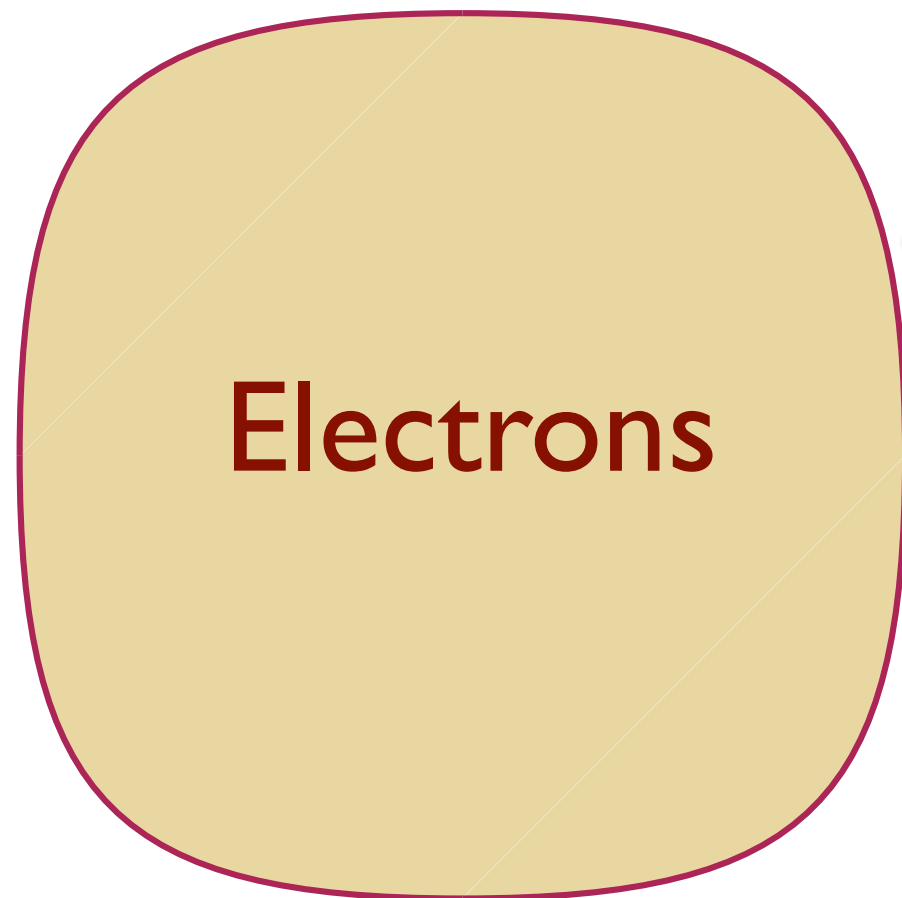
Rates of Momentum Flow



Phonons

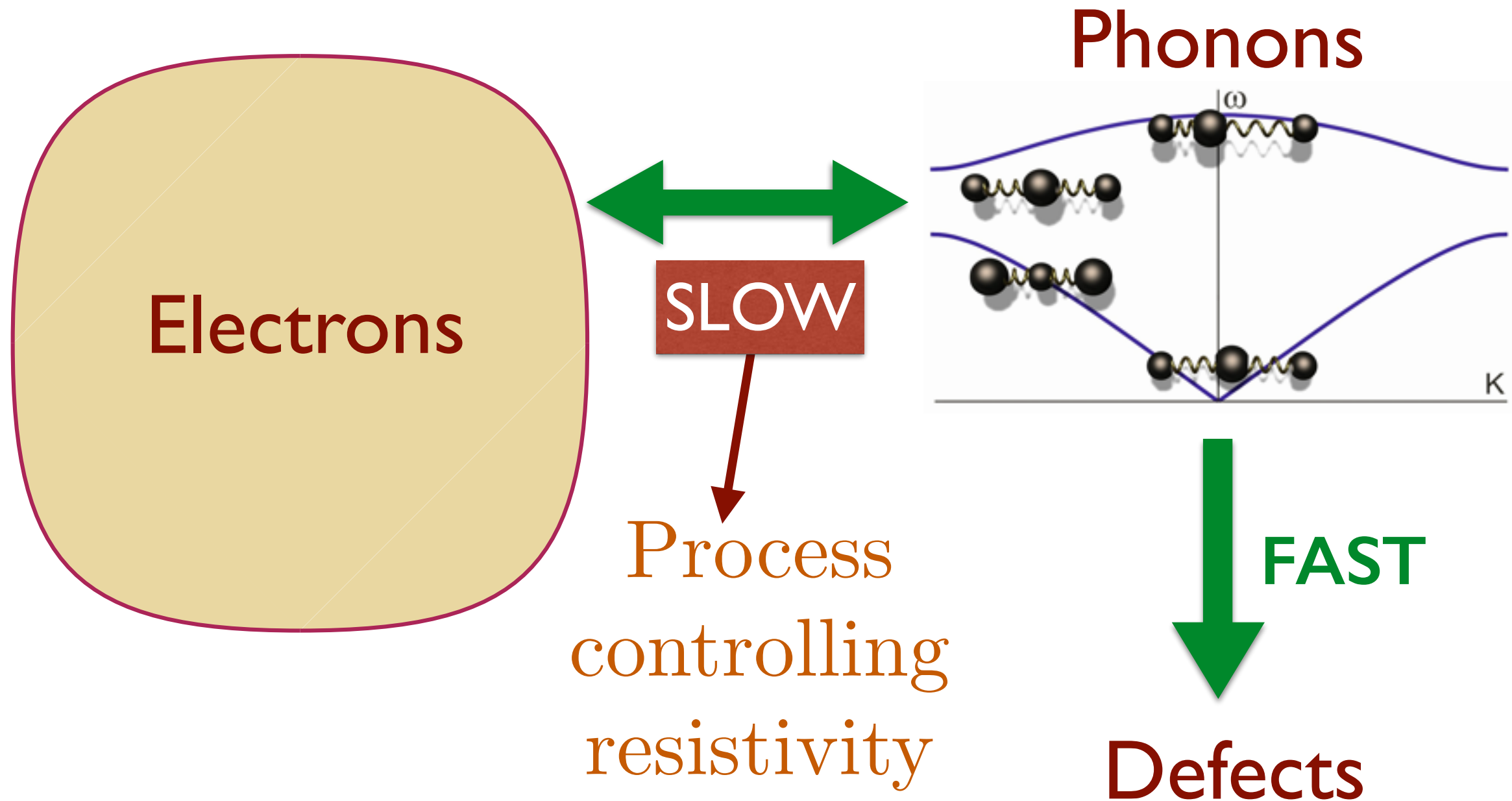


Rates of Momentum Flow

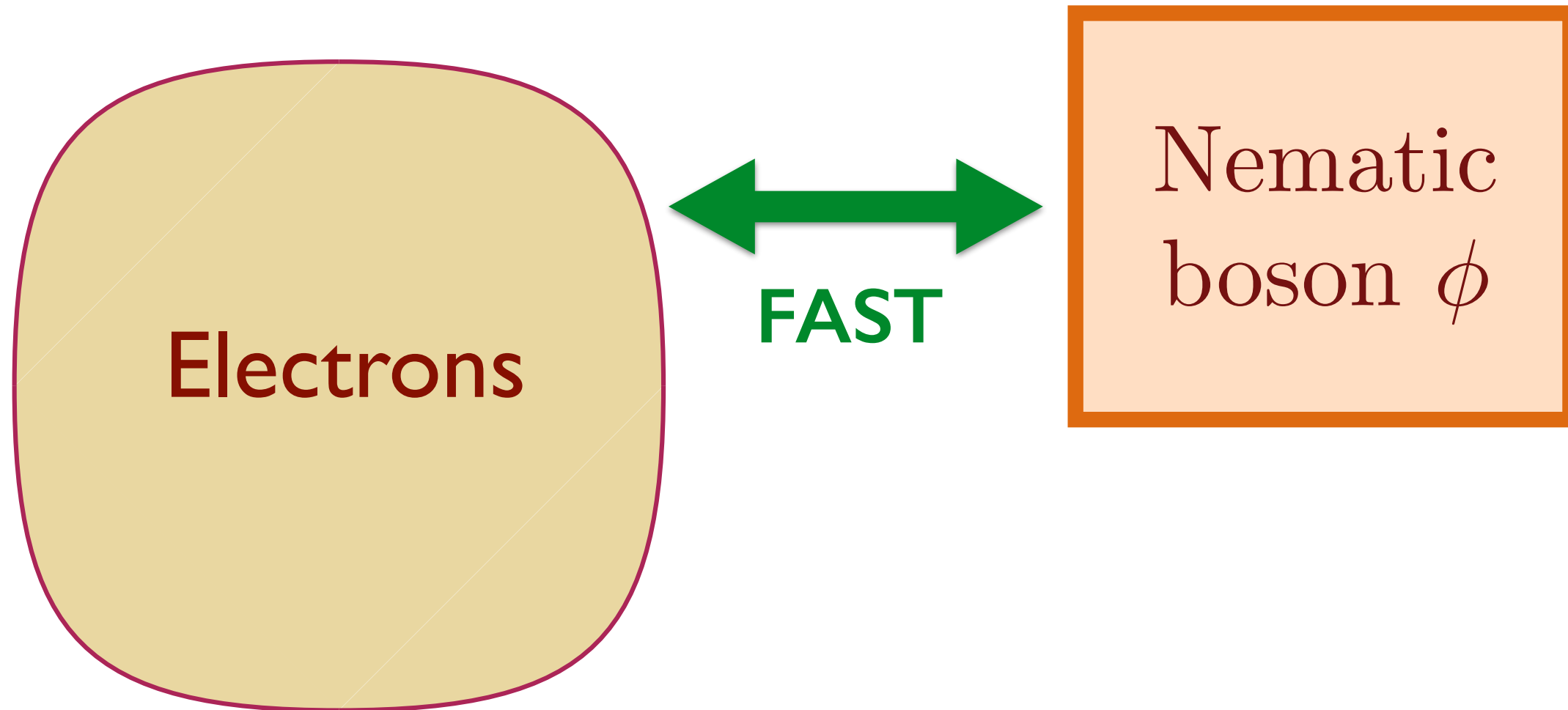


Defects

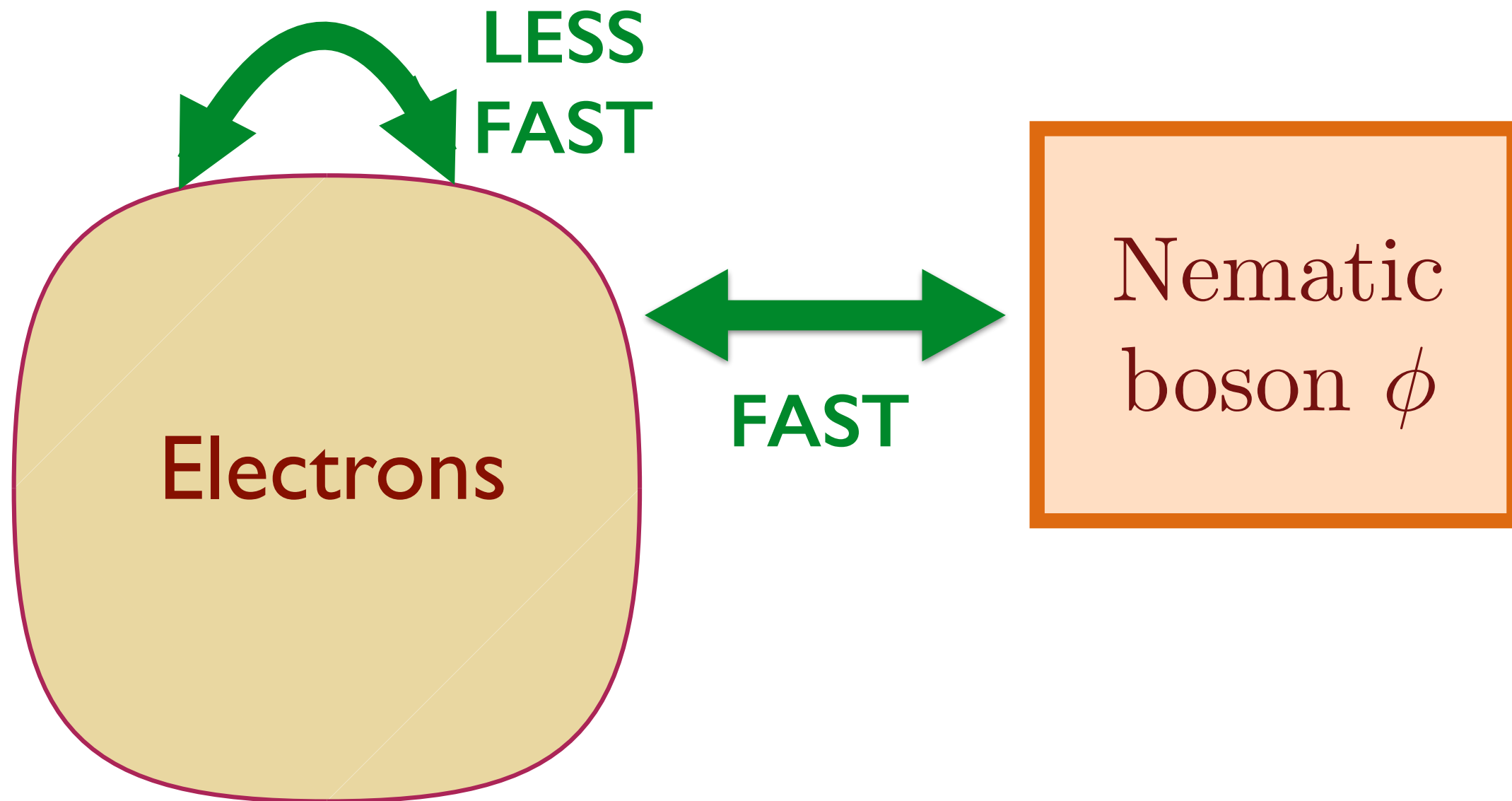
Rates of Momentum Flow



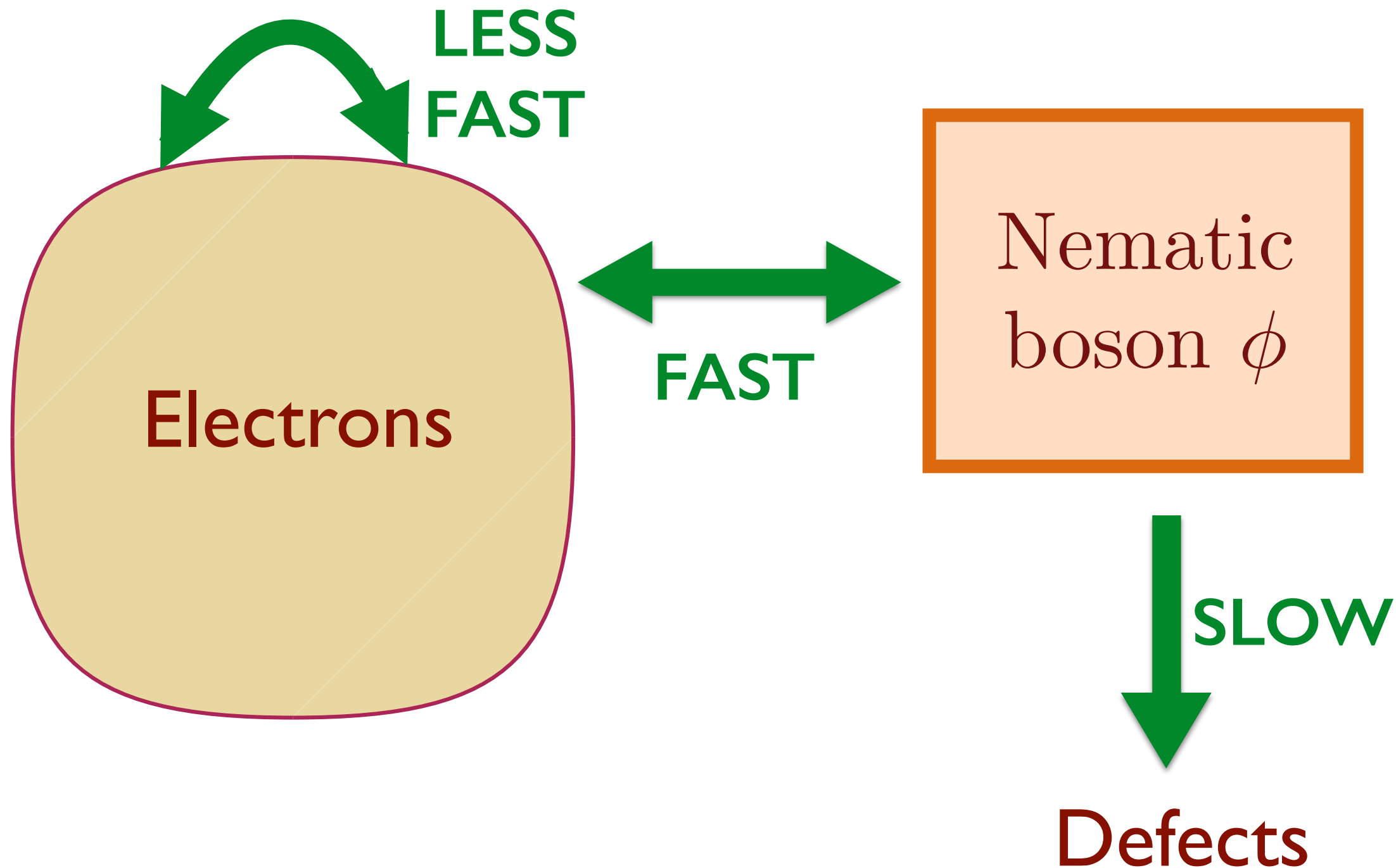
Rates of Momentum Flow



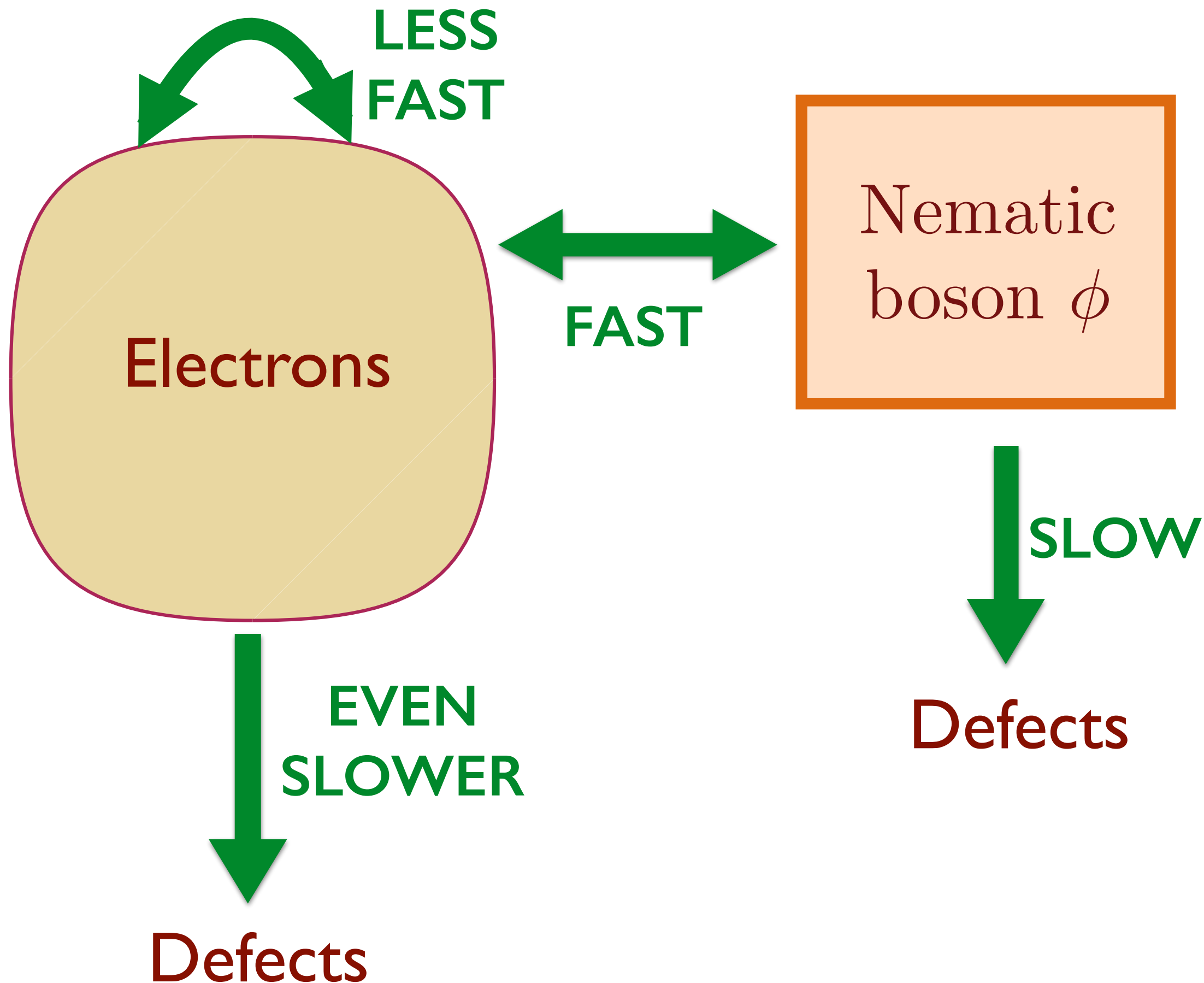
Rates of Momentum Flow



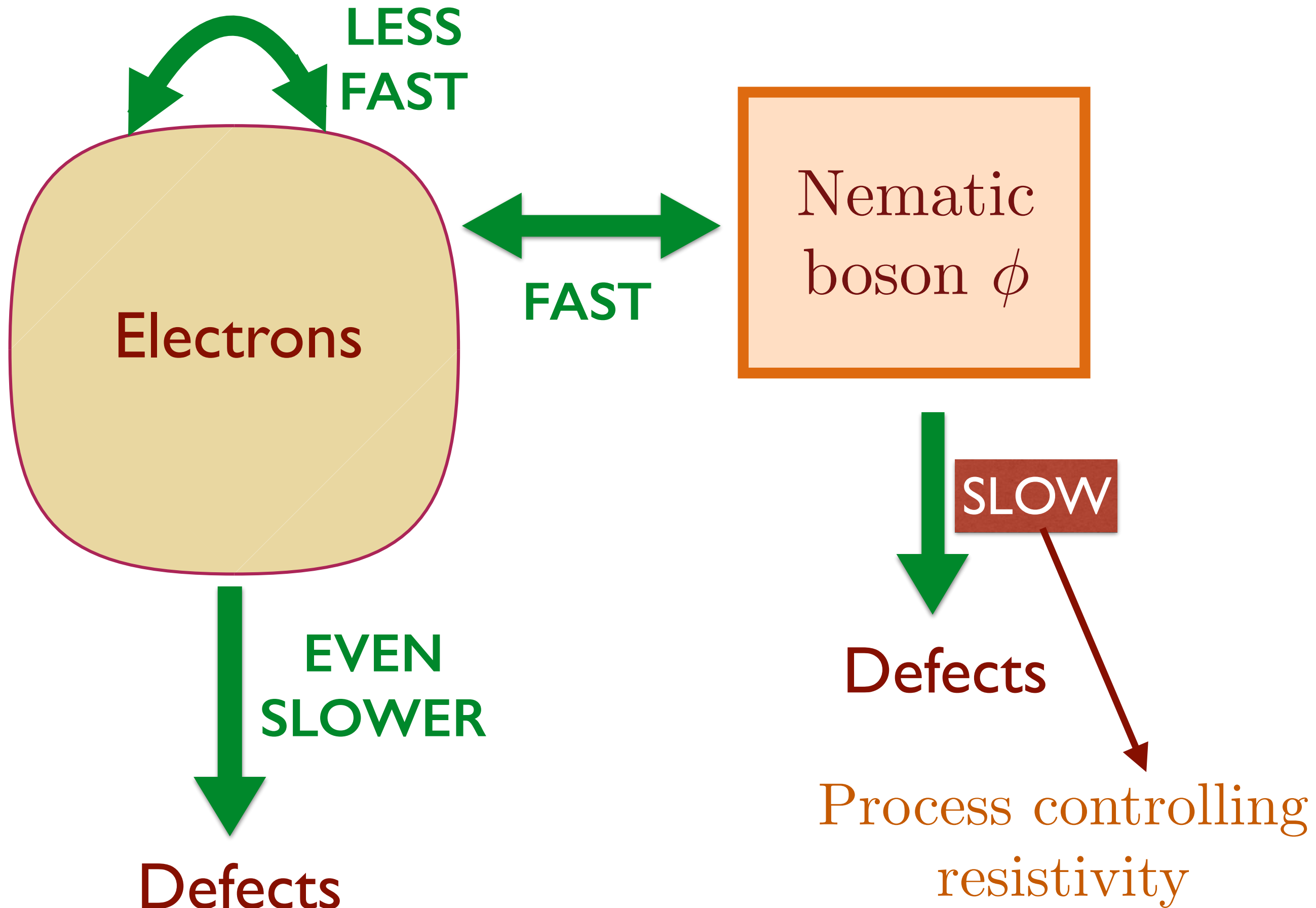
Rates of Momentum Flow



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Rates of Momentum Flow



Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

- Focus on the interplay between J_μ and $T_{\mu\nu}$!



The most-probable state with a non-zero current \mathbf{J} has a non-zero momentum \mathbf{P} (and vice versa).

At non-zero density, \mathbf{J} “drags” \mathbf{P} .

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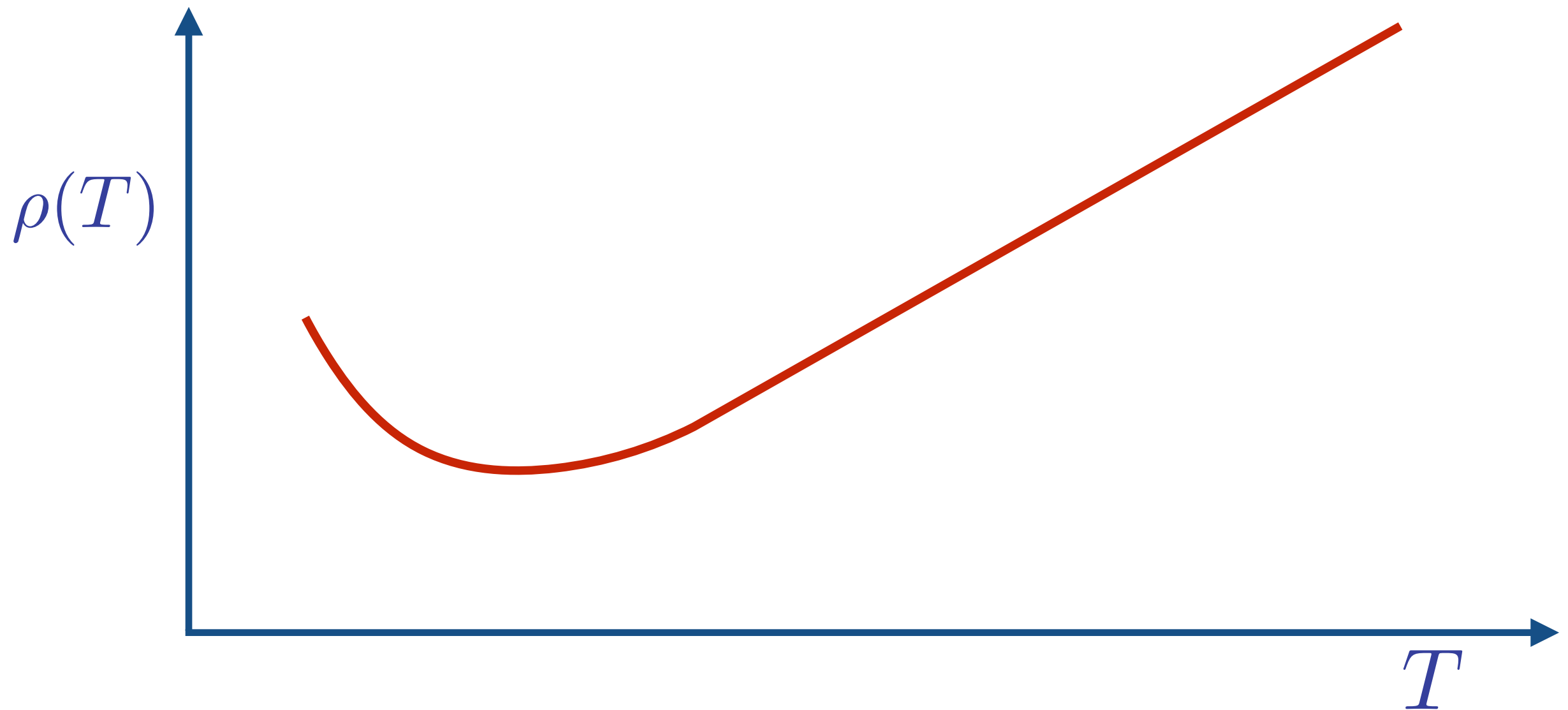
At non-zero density, \mathbf{J} “drags” \mathbf{P} .

The resistivity of this metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

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Transport without quasiparticles:

Resistivity from random-field disorder



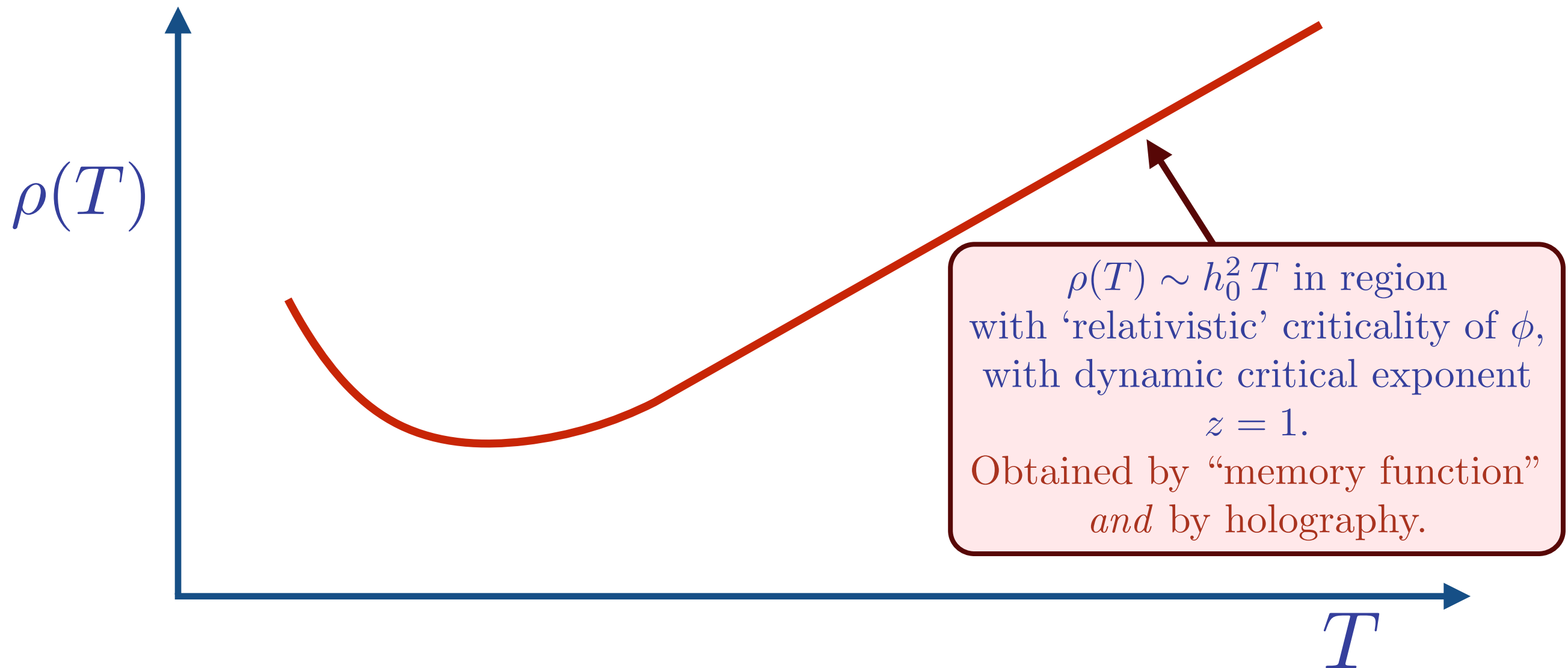
S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, Physical Review B **89**, 155130 (2014)

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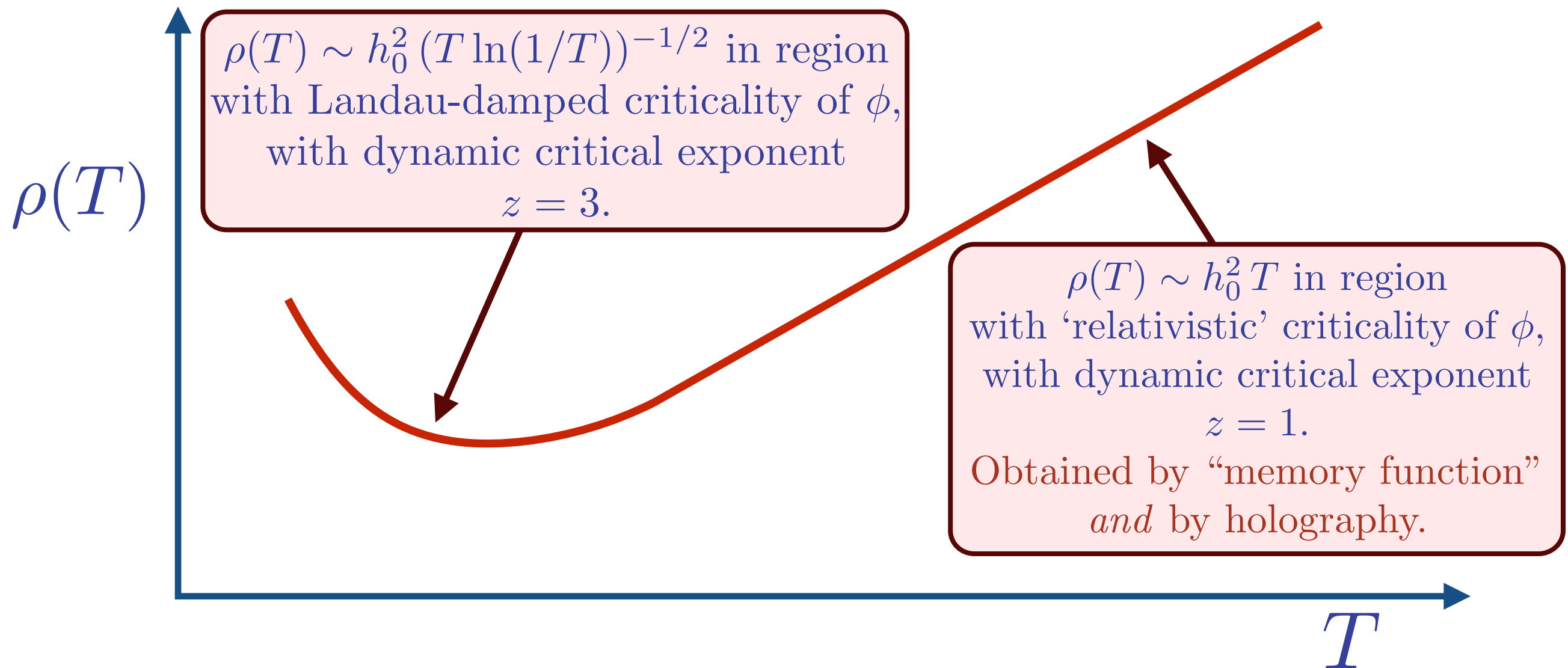
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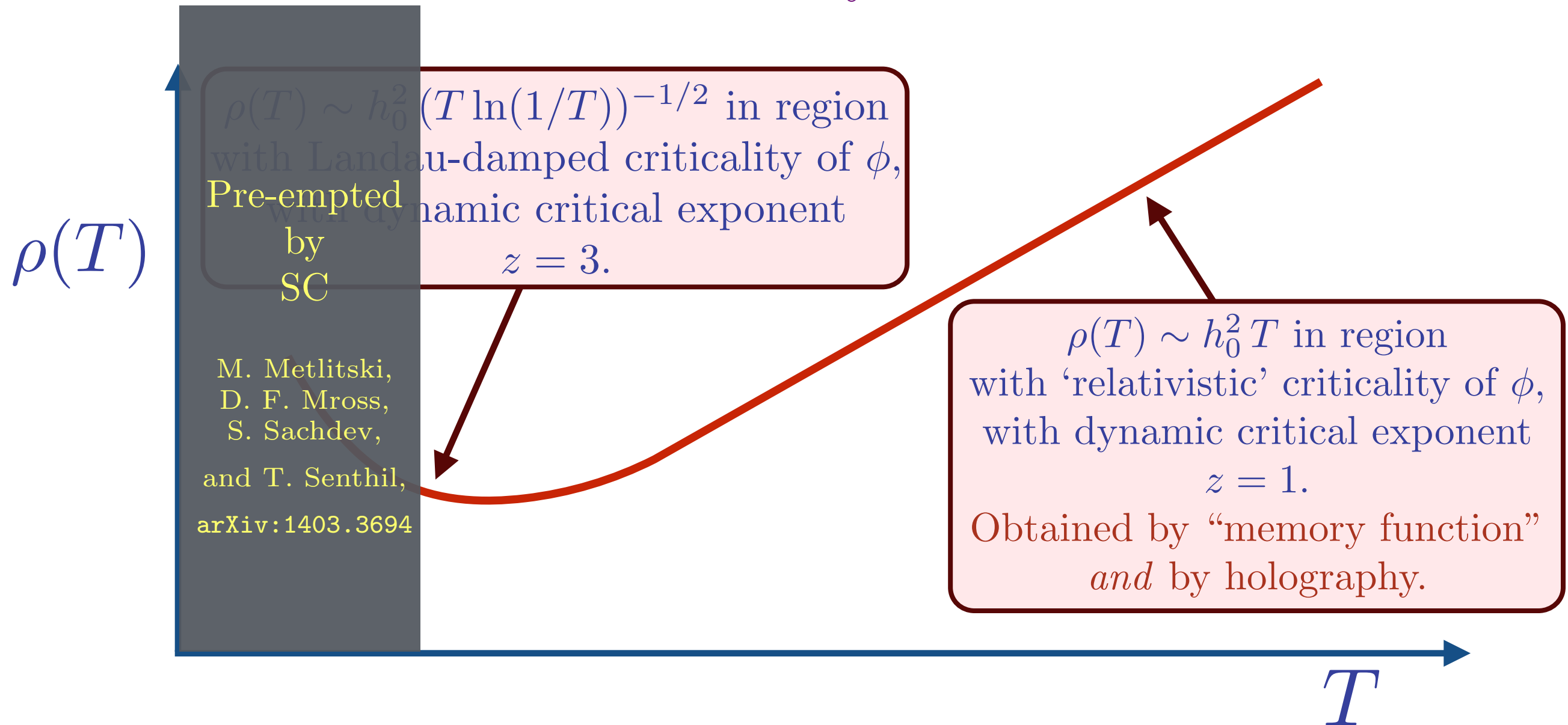
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- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography.