Entangling antiferromagnetism and superconductivity:

Quantum Monte Carlo without the sign problem

Perimeter Institute, July 11, 2012

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Max Metlitski

Erez Berg

Matthias Punk

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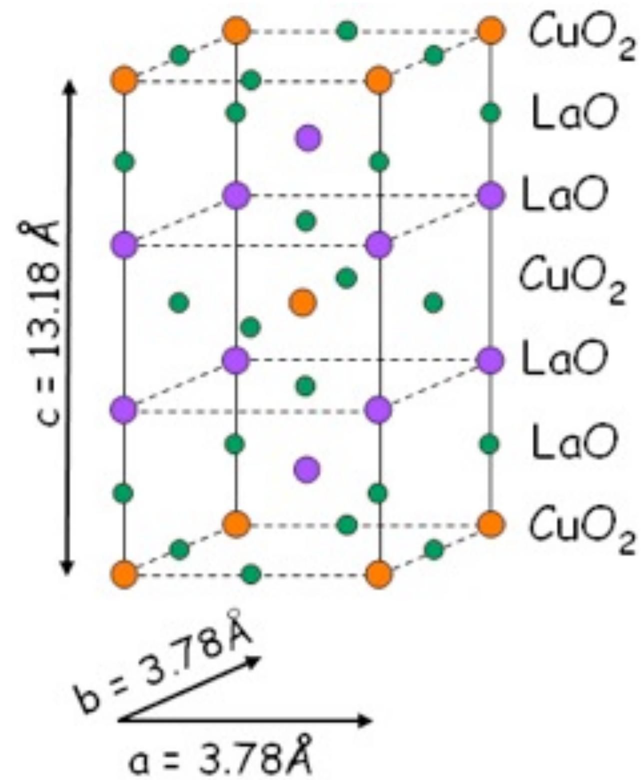
CuO2
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c = 13.18 Å
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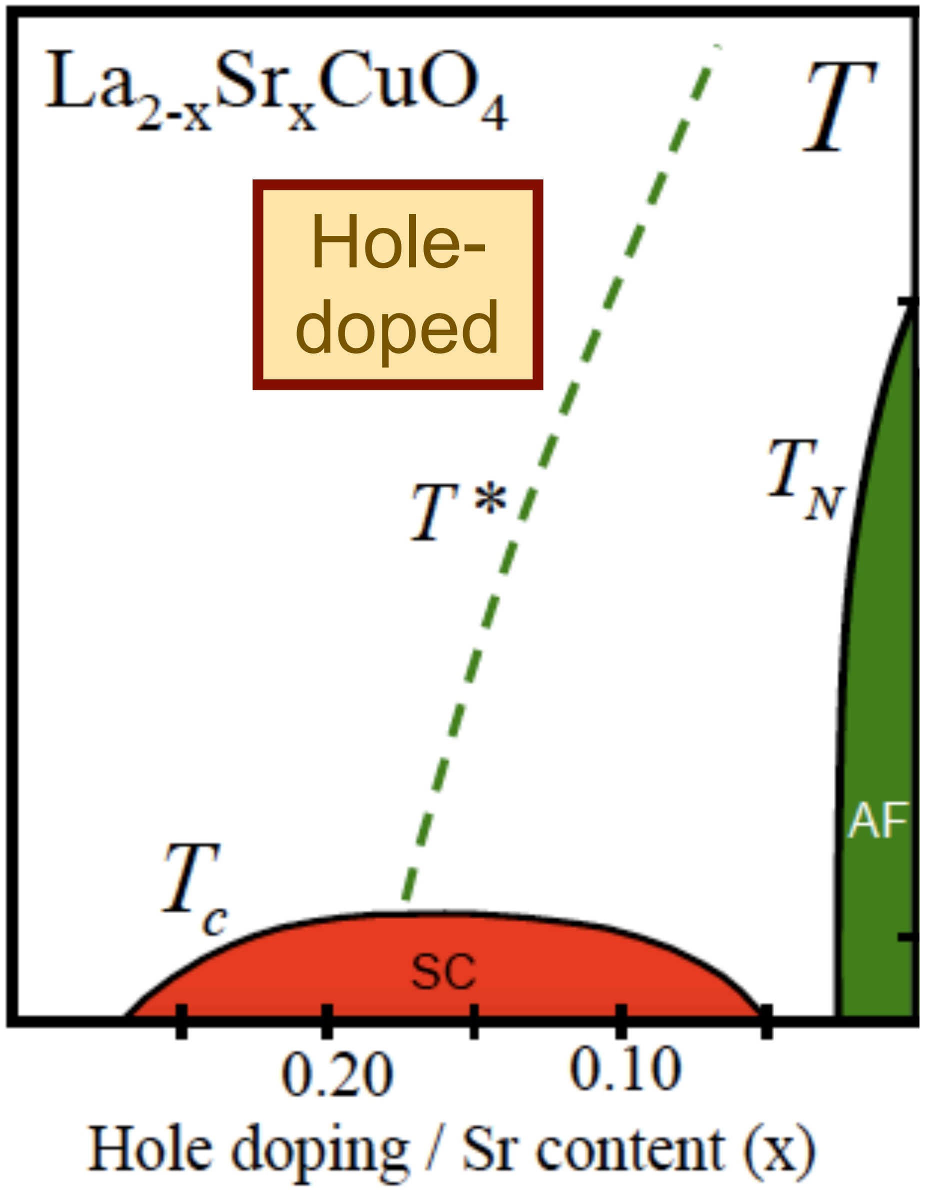
La2-xSrxCuO4
Hole-doped
T
T*
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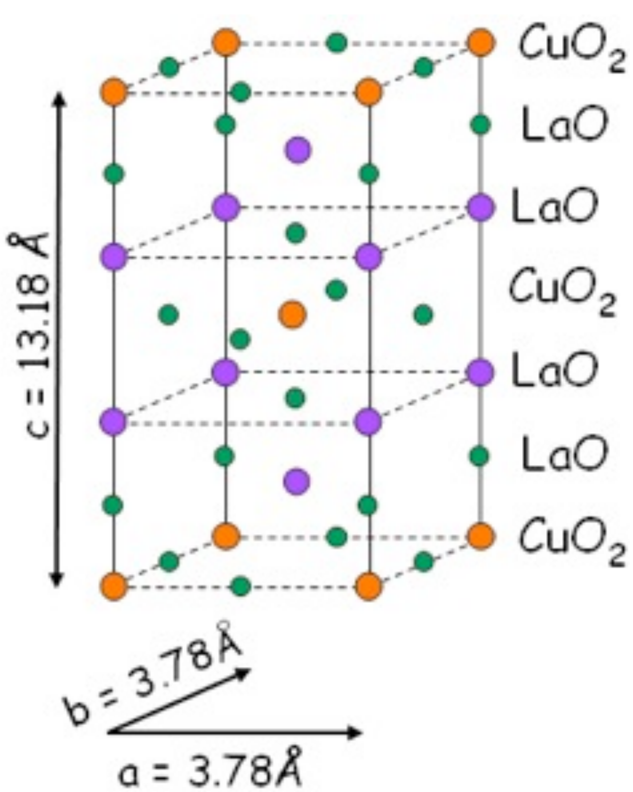
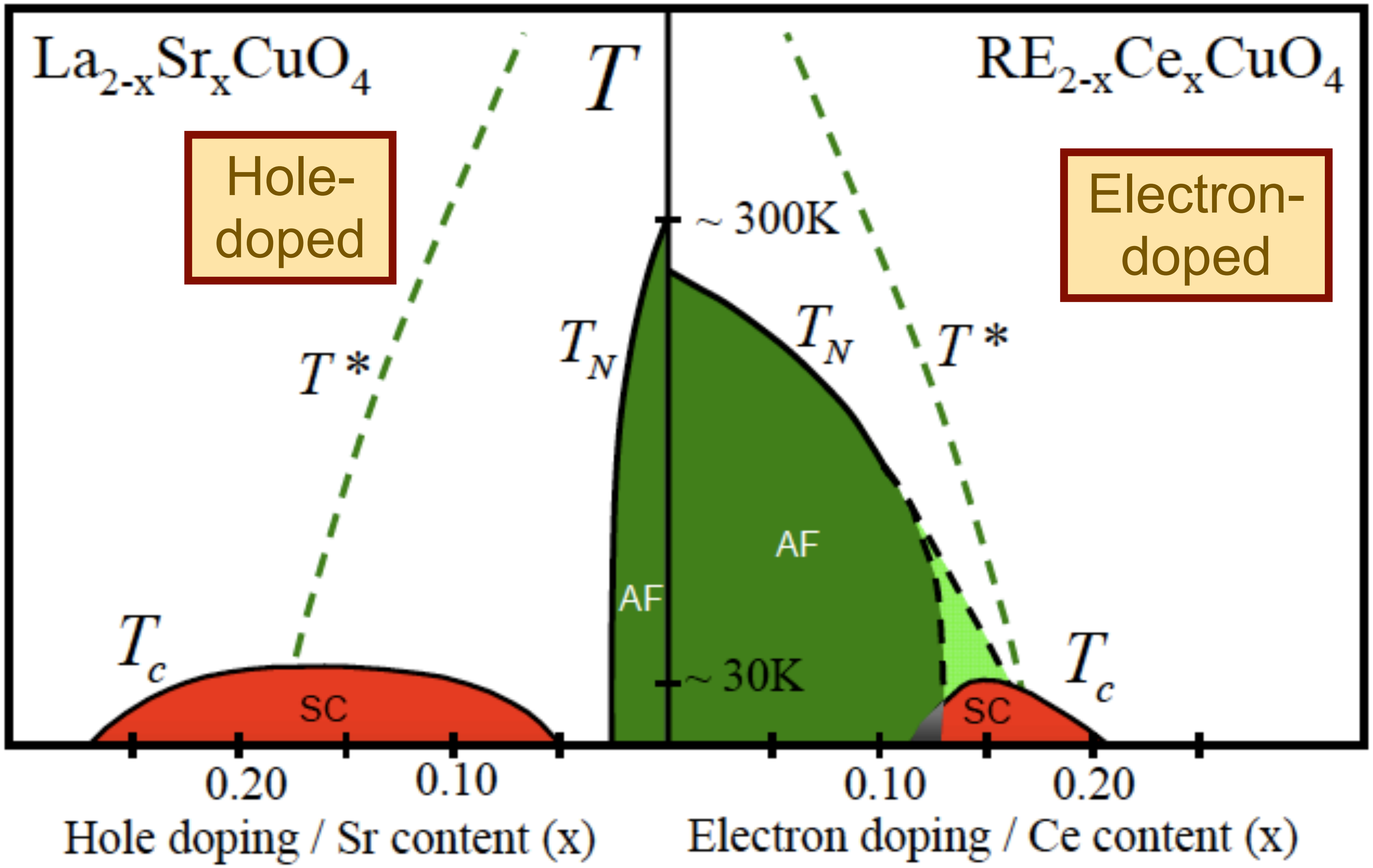
AF
Tc
SC
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors





Electron-doped cuprate superconductors

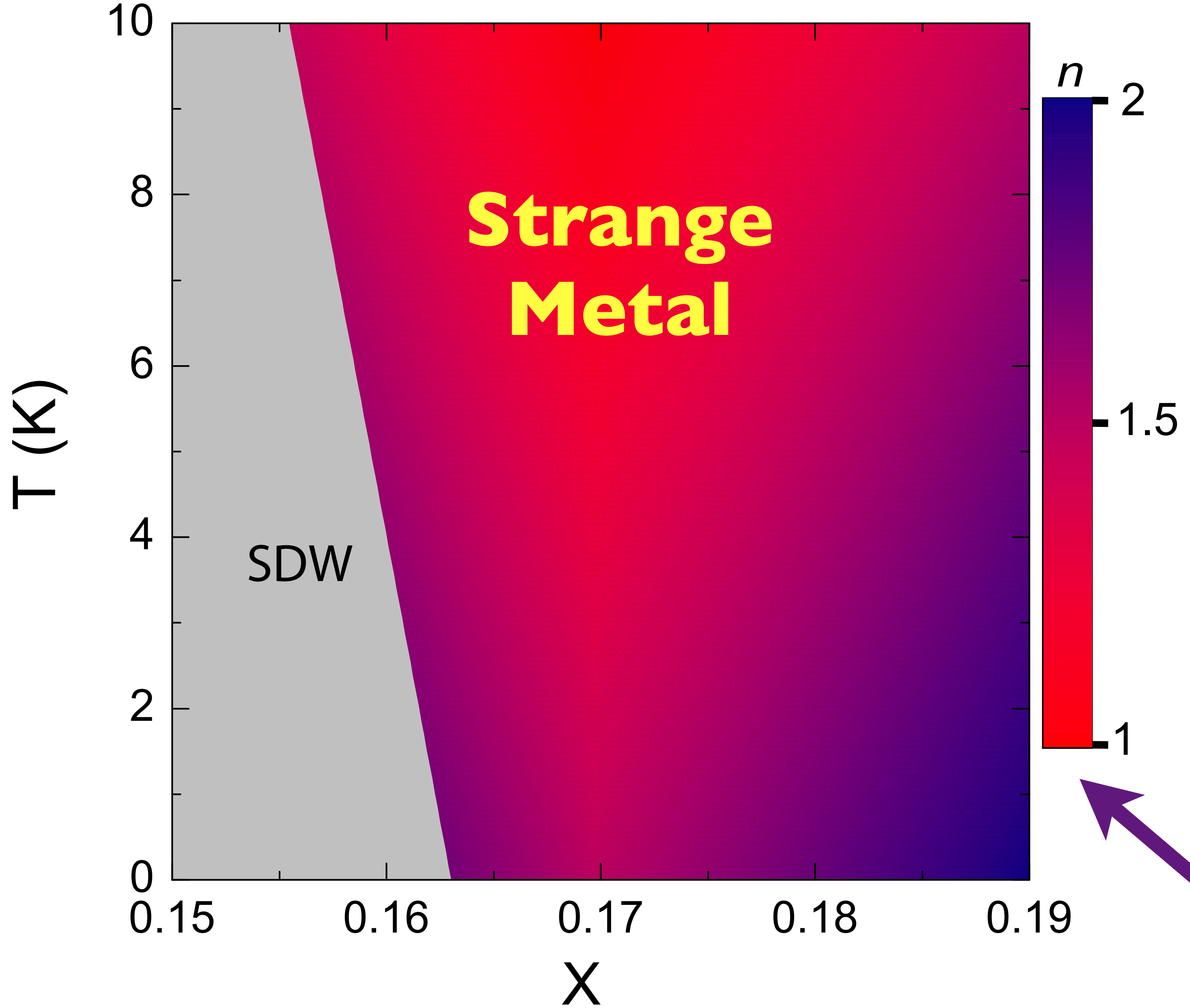
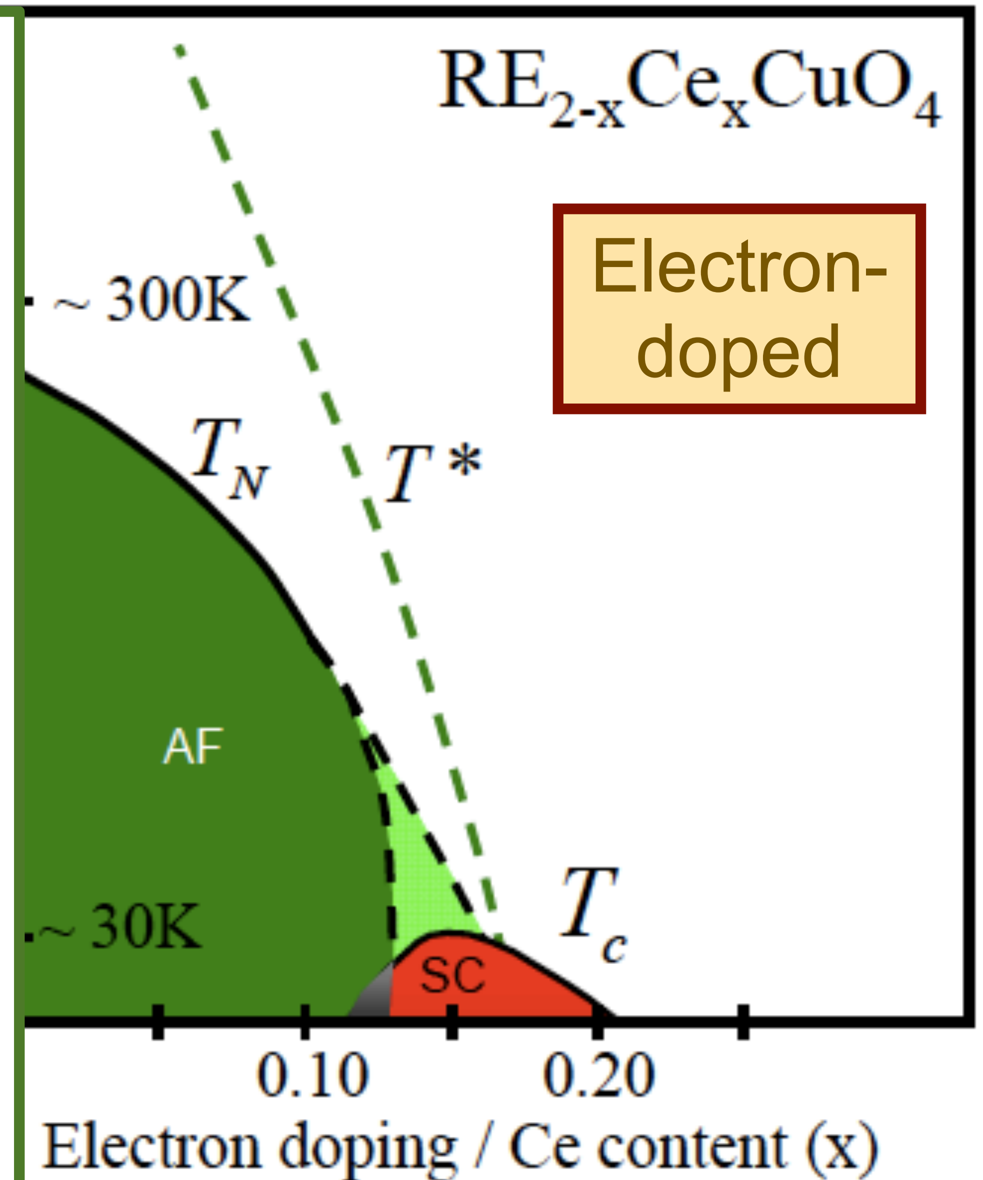
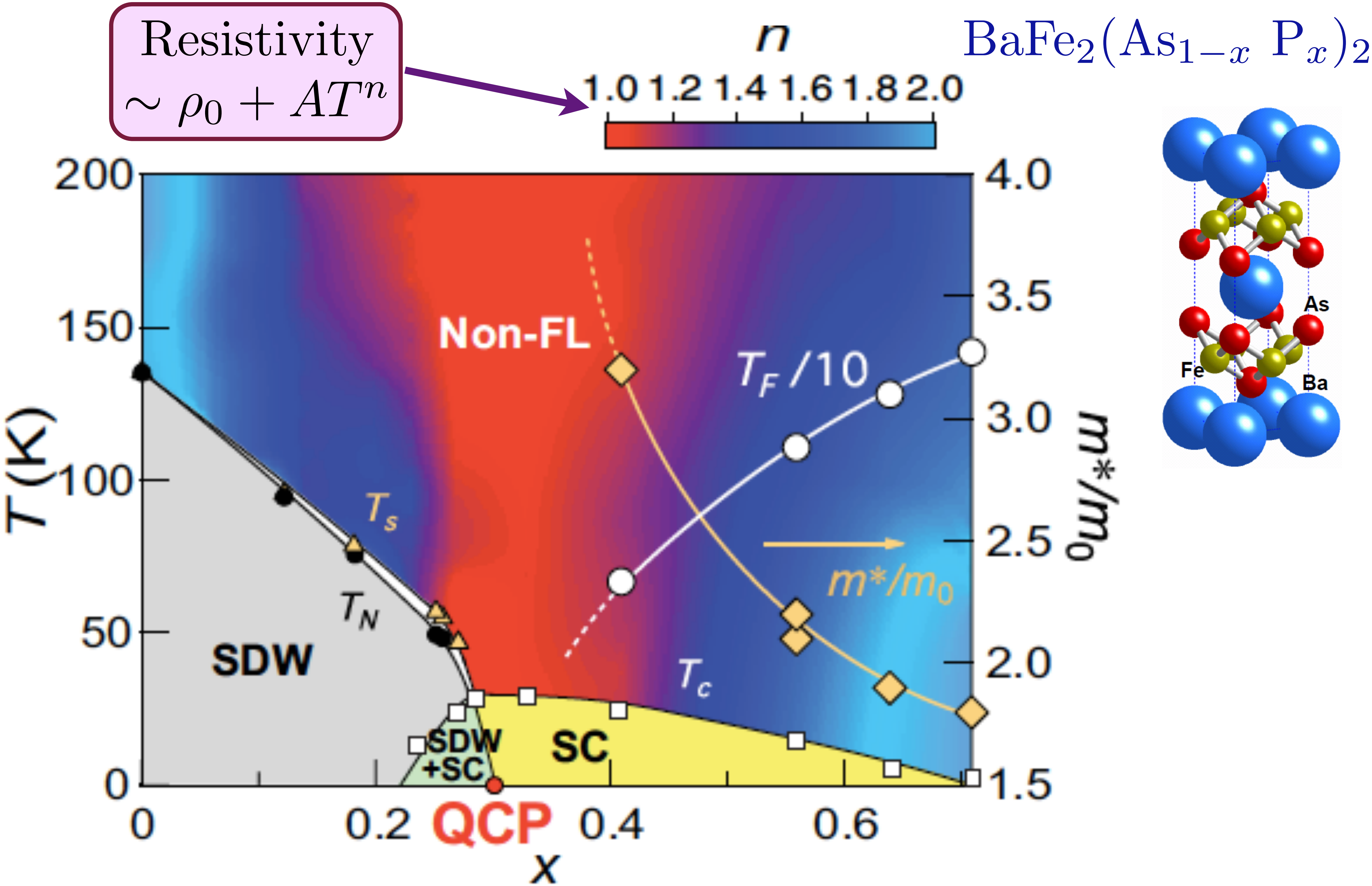


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



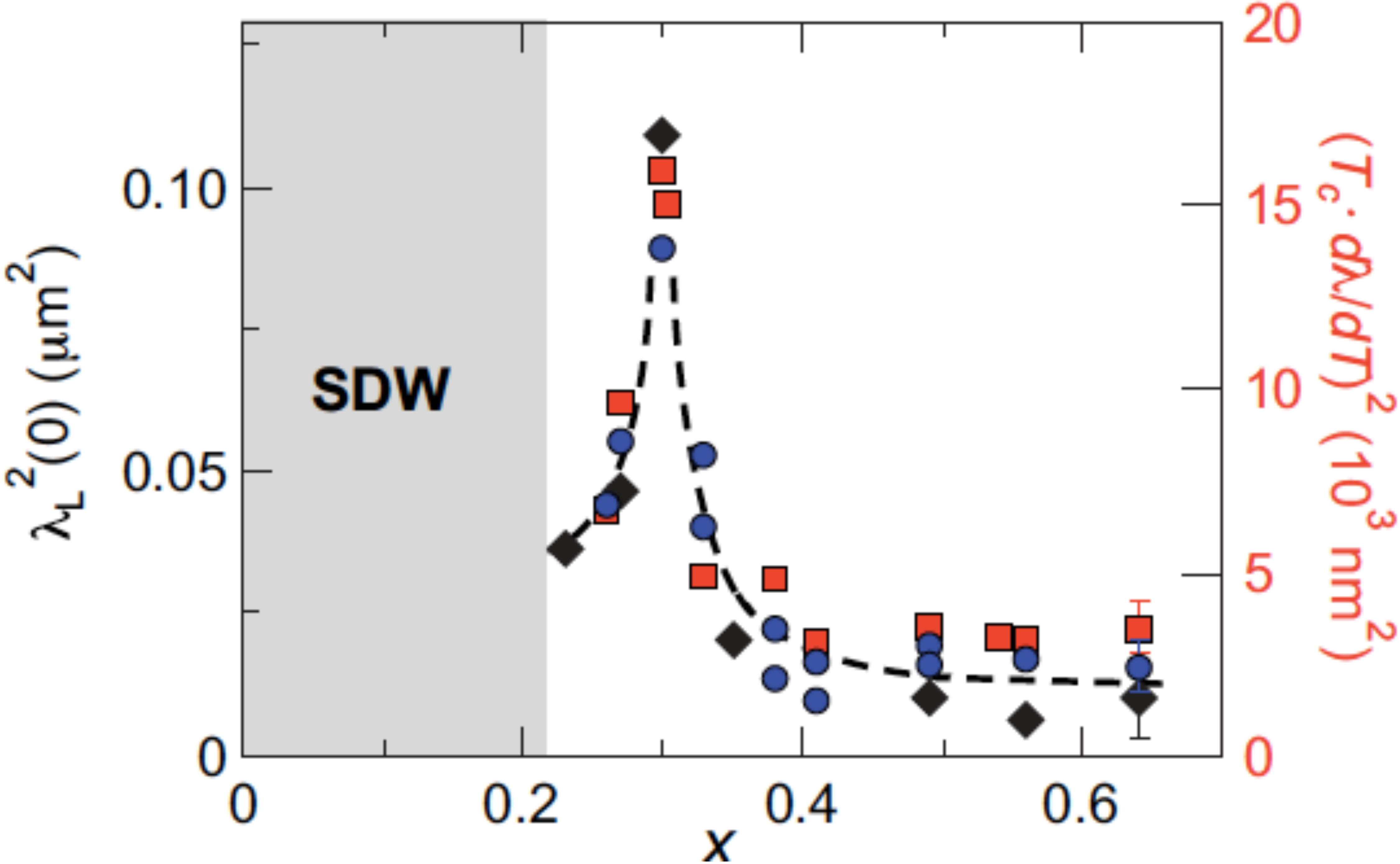
Resistivity

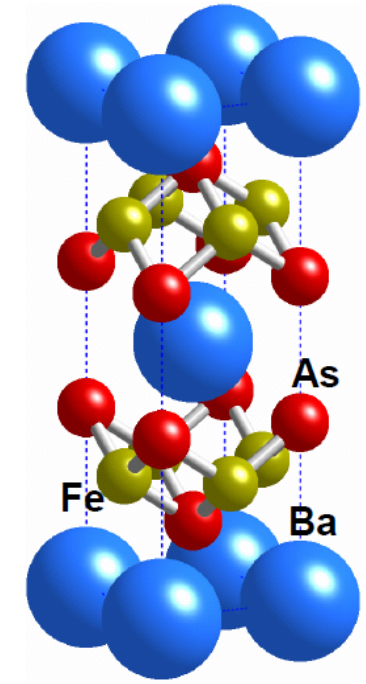
$\sim \rho_0 + AT^n$



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

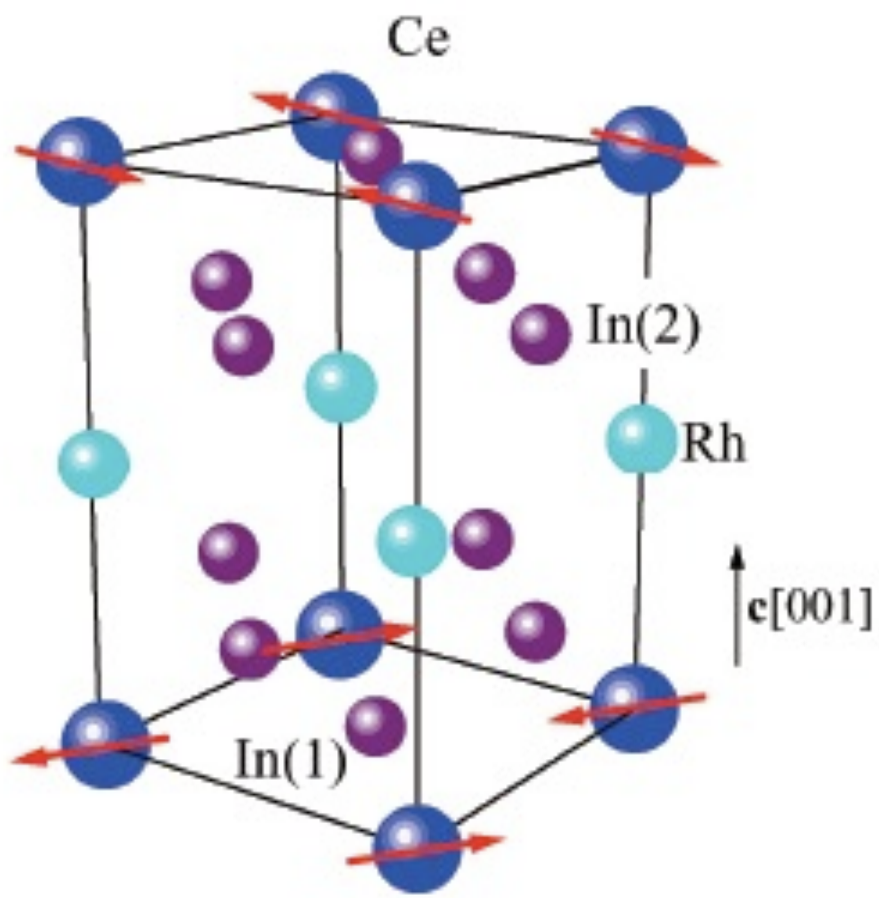
$BaFe_2(As_{1-x}\ P_x)_2$

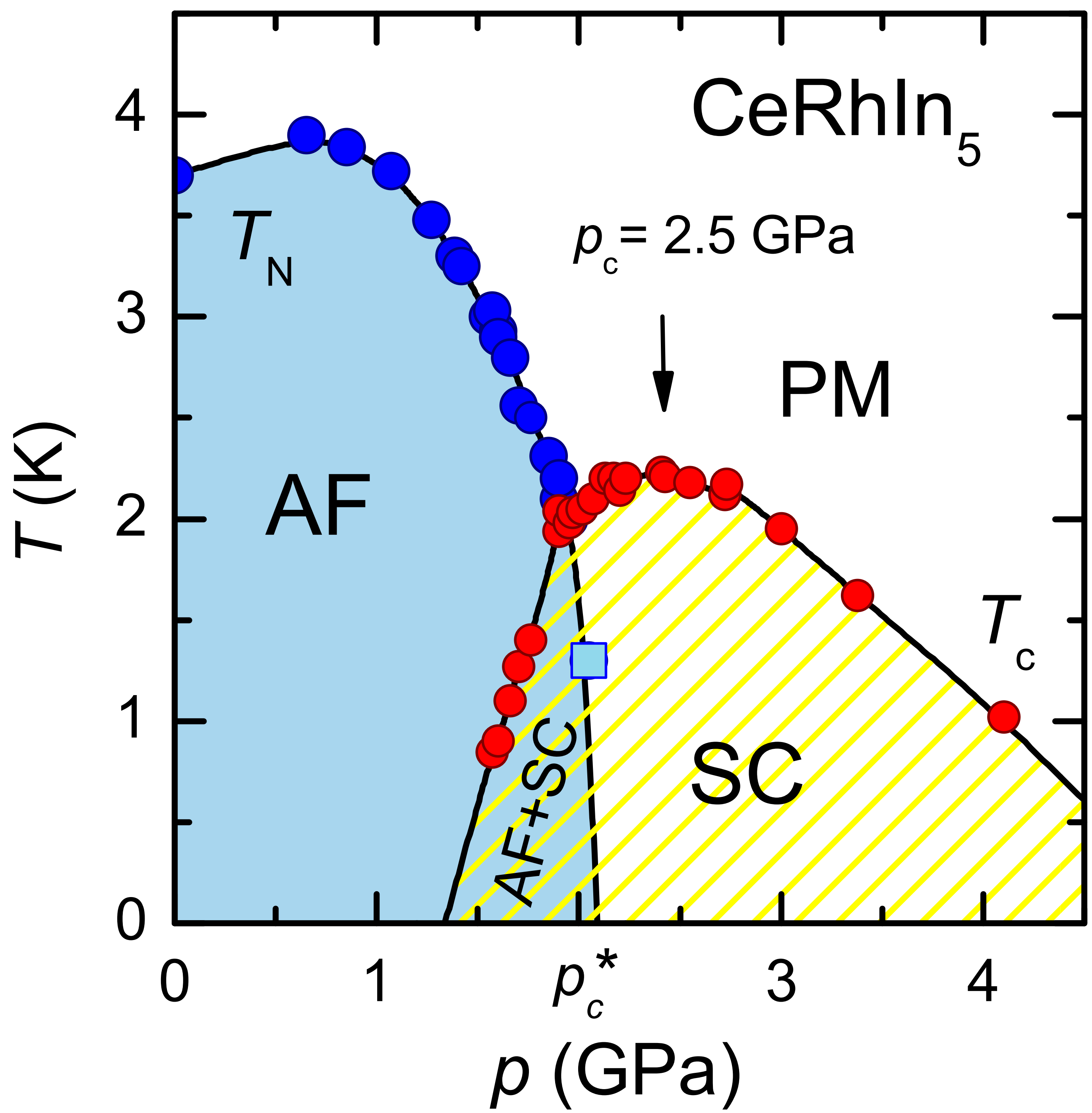




K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

Lower T_c superconductivity in the heavy fermion compounds





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

Outline

1. Weak-coupling theory for the onset of antiferromagnetism in a metal

2. Quantum field theory of the onset of antiferromagnetism in a metal

3. Quantum Monte Carlo without the sign problem

4. Fractionalization in metals, and the hole-doped cuprates

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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha}$$

$$c^\dagger_{i\alpha} c_{j\beta} + c_{j\beta} c^\dagger_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

The Hubbard Model

Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}\right]$$

$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

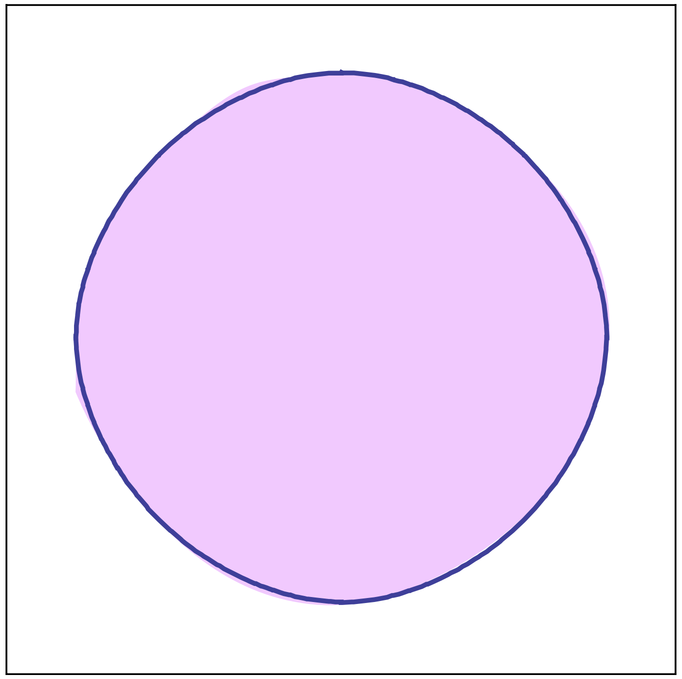
$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda\, \varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b .$$

"Yukawa" coupling between fermions and antiferromagnetic order:
$\lambda^2 \sim U$, the Hubbard repulsion

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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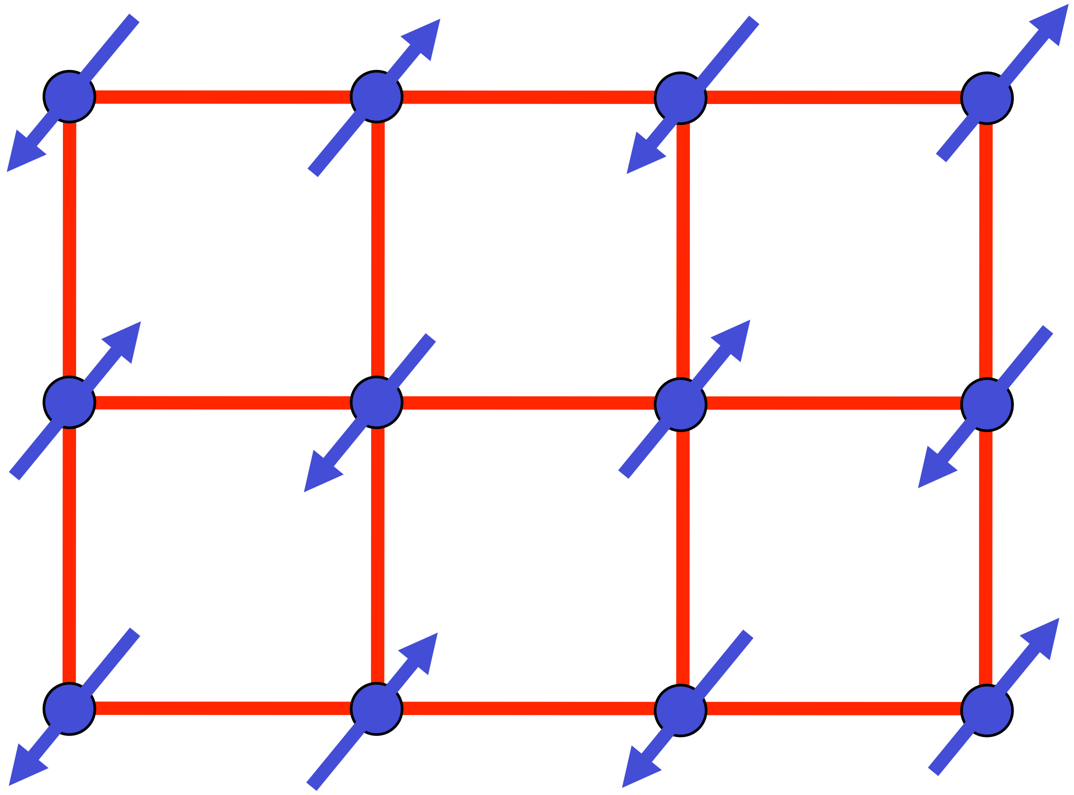


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism

Mean field theory

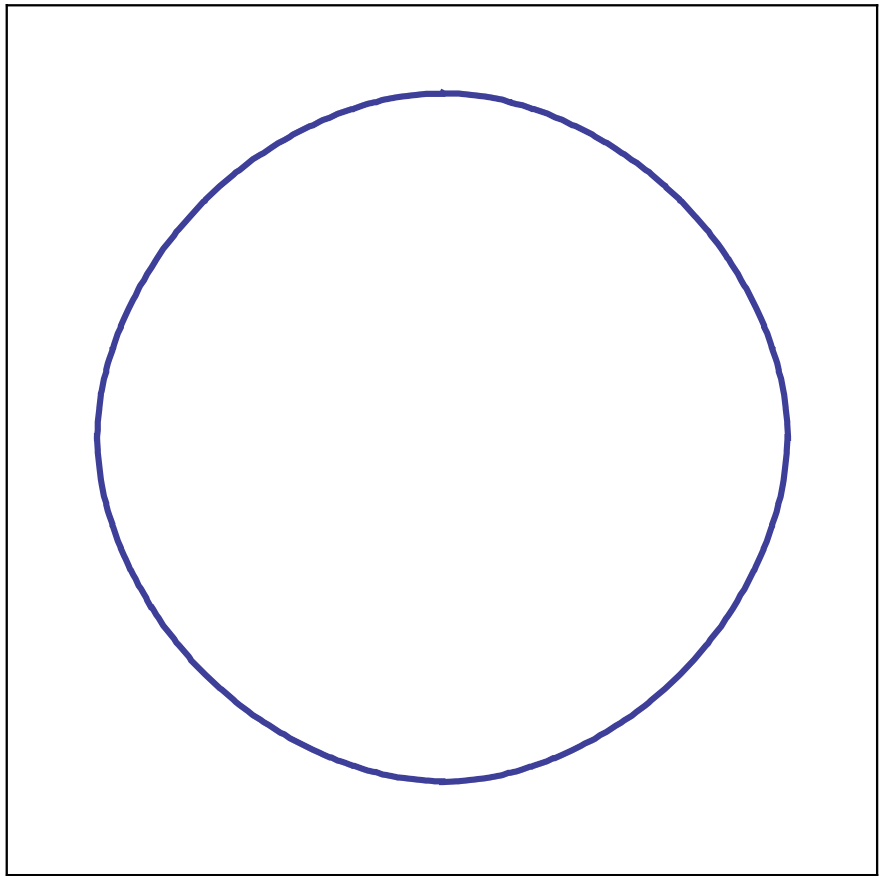
In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

$$H_{\mathrm{sdw}} = \lambda \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\mathrm{sdw}}$ for $\vec{\varphi} \propto (0,0,1)$ are

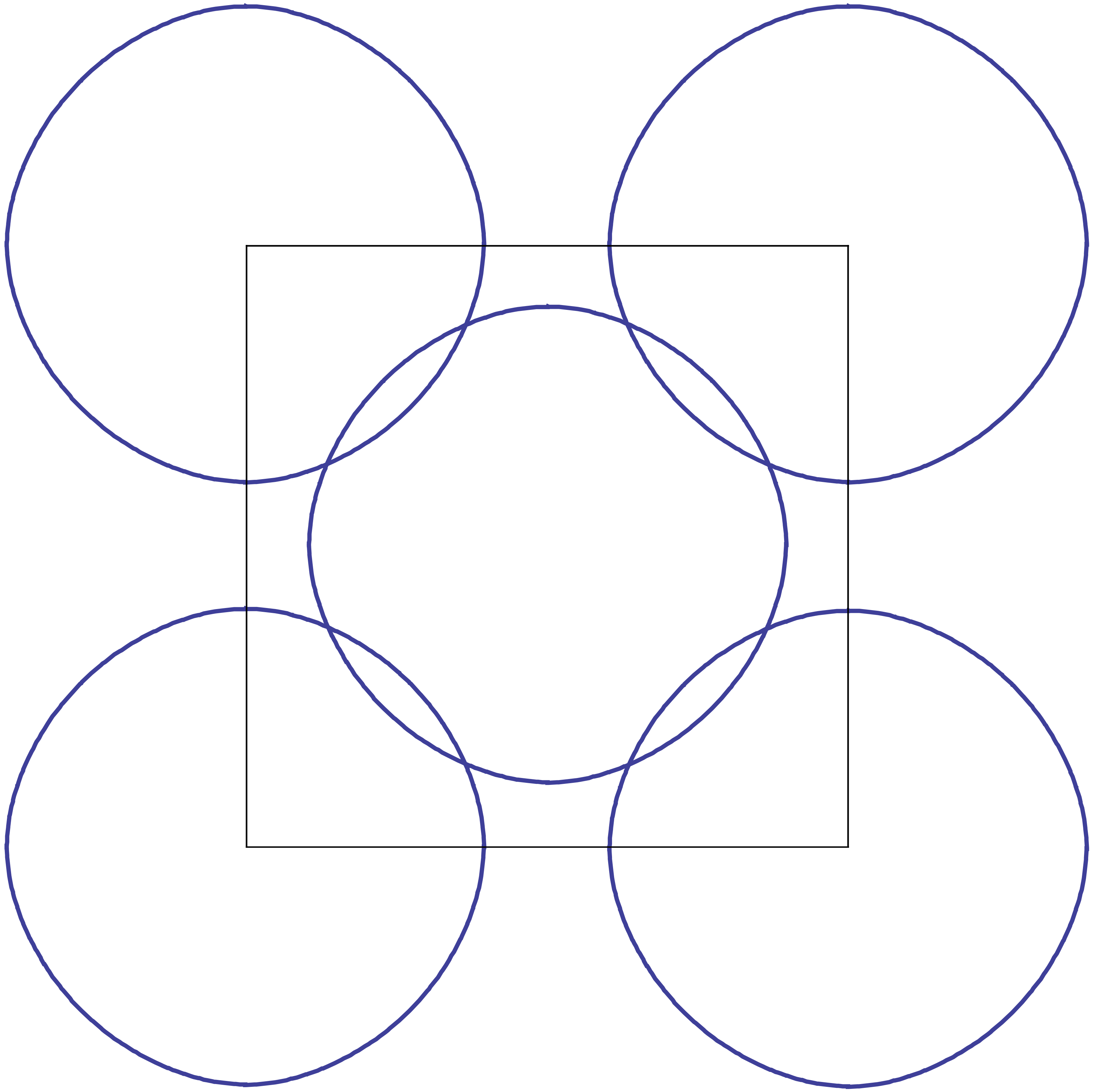
$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \lambda^2 \varphi^2}$$

Fermi surface+antiferromagnetism



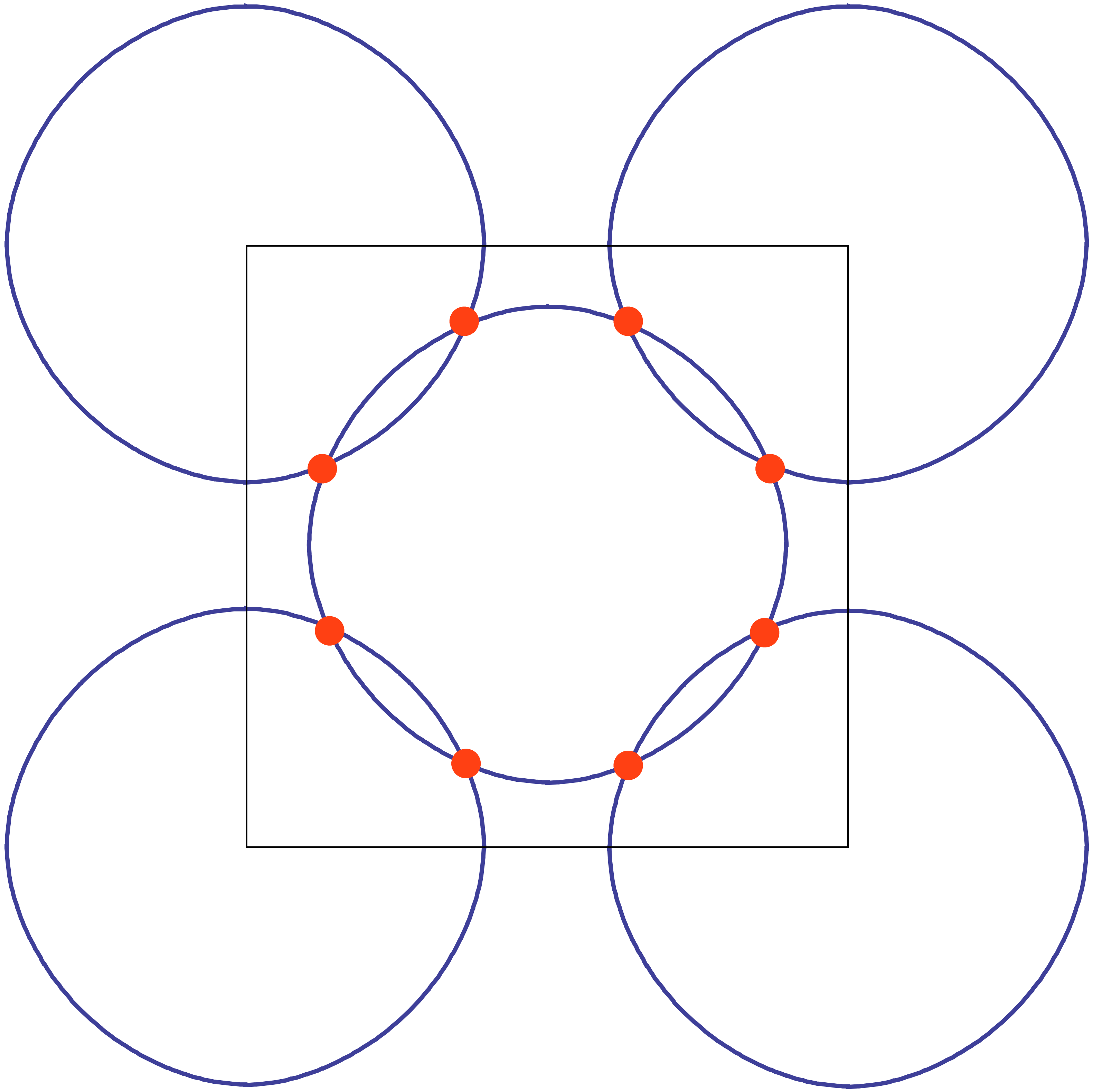
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



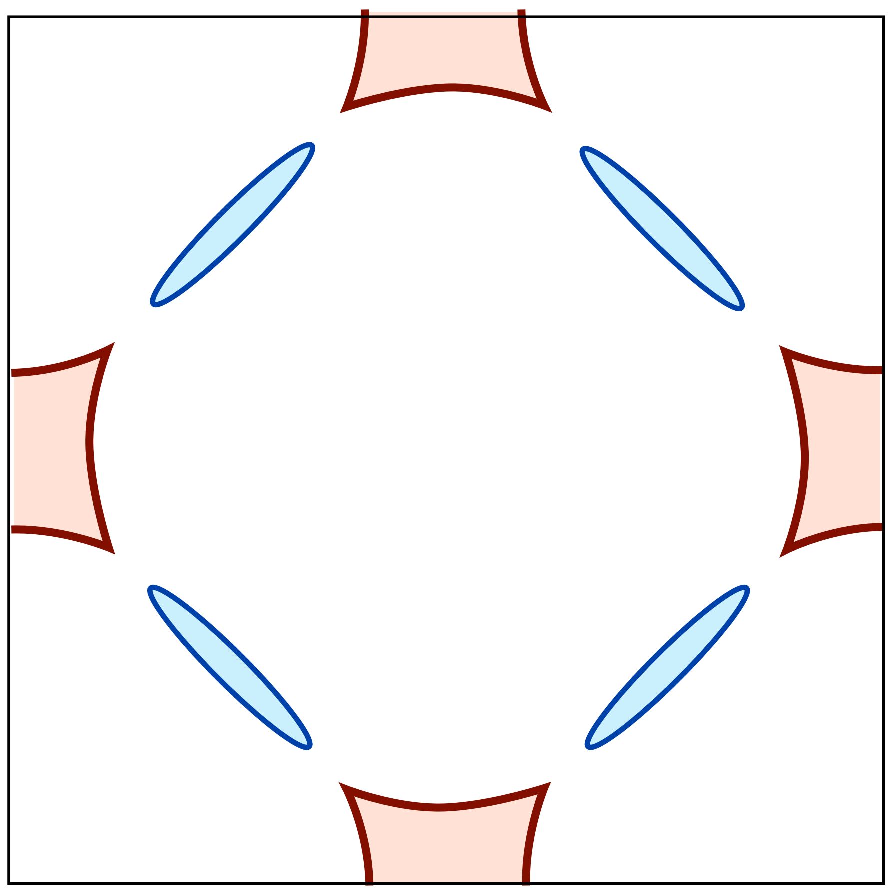
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



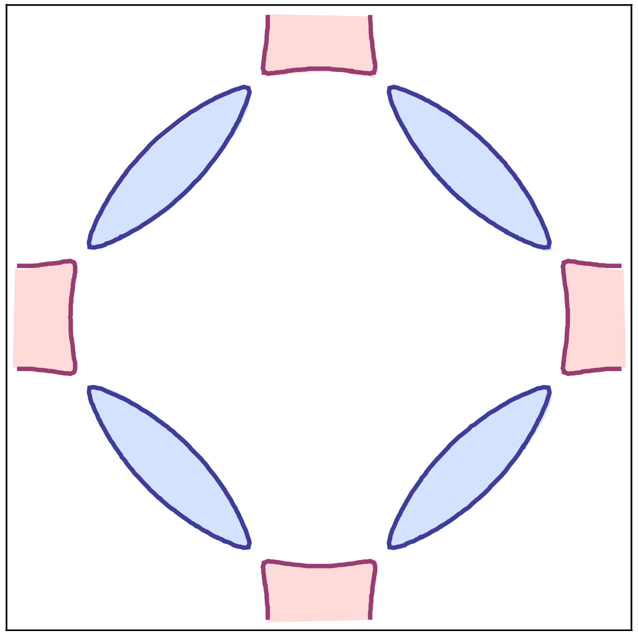
“Hot” spots

Fermi surface+antiferromagnetism



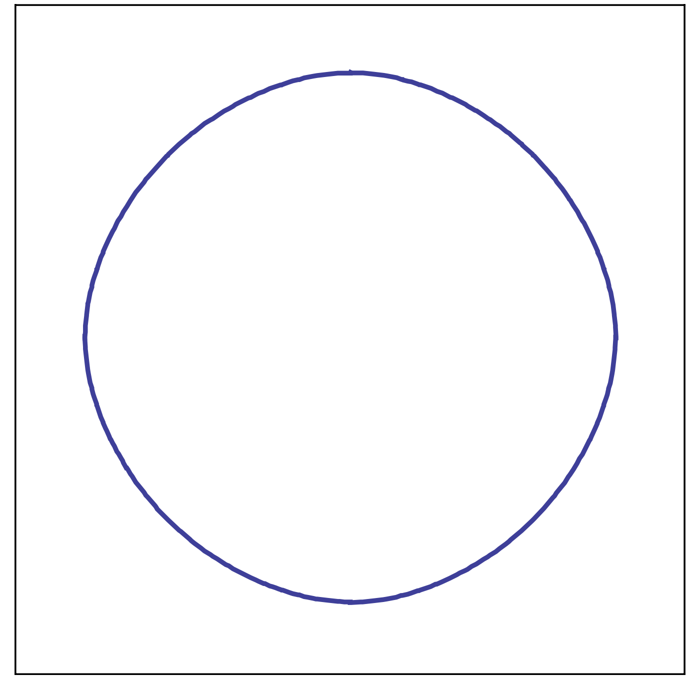
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



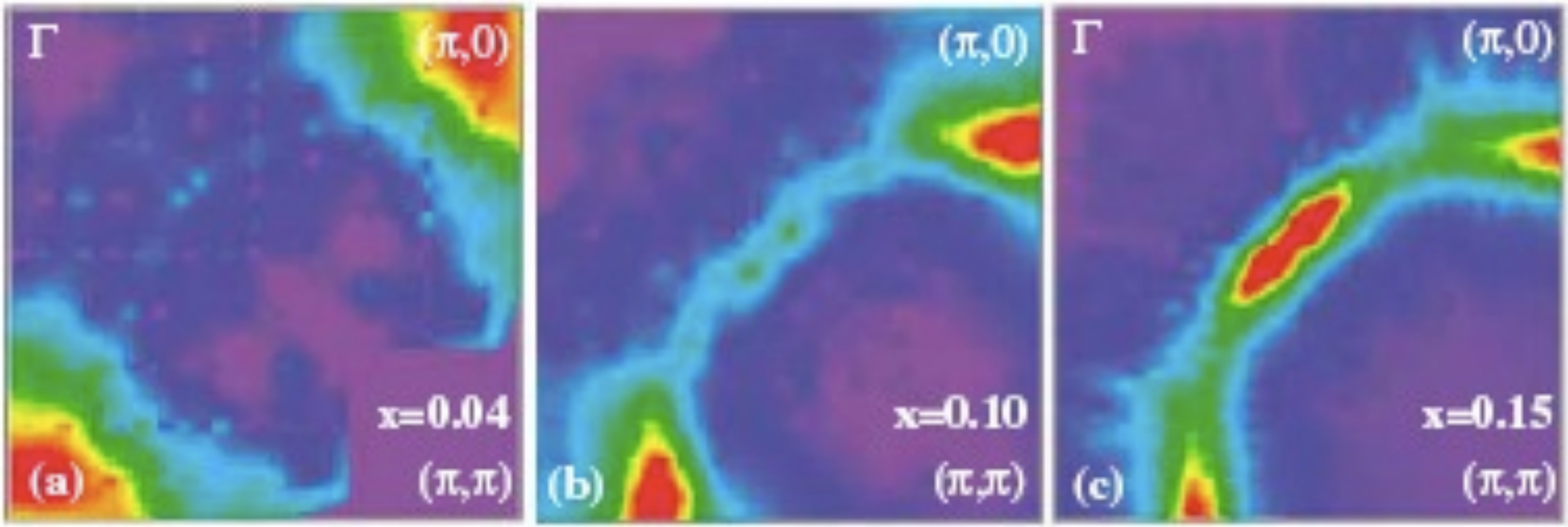
$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

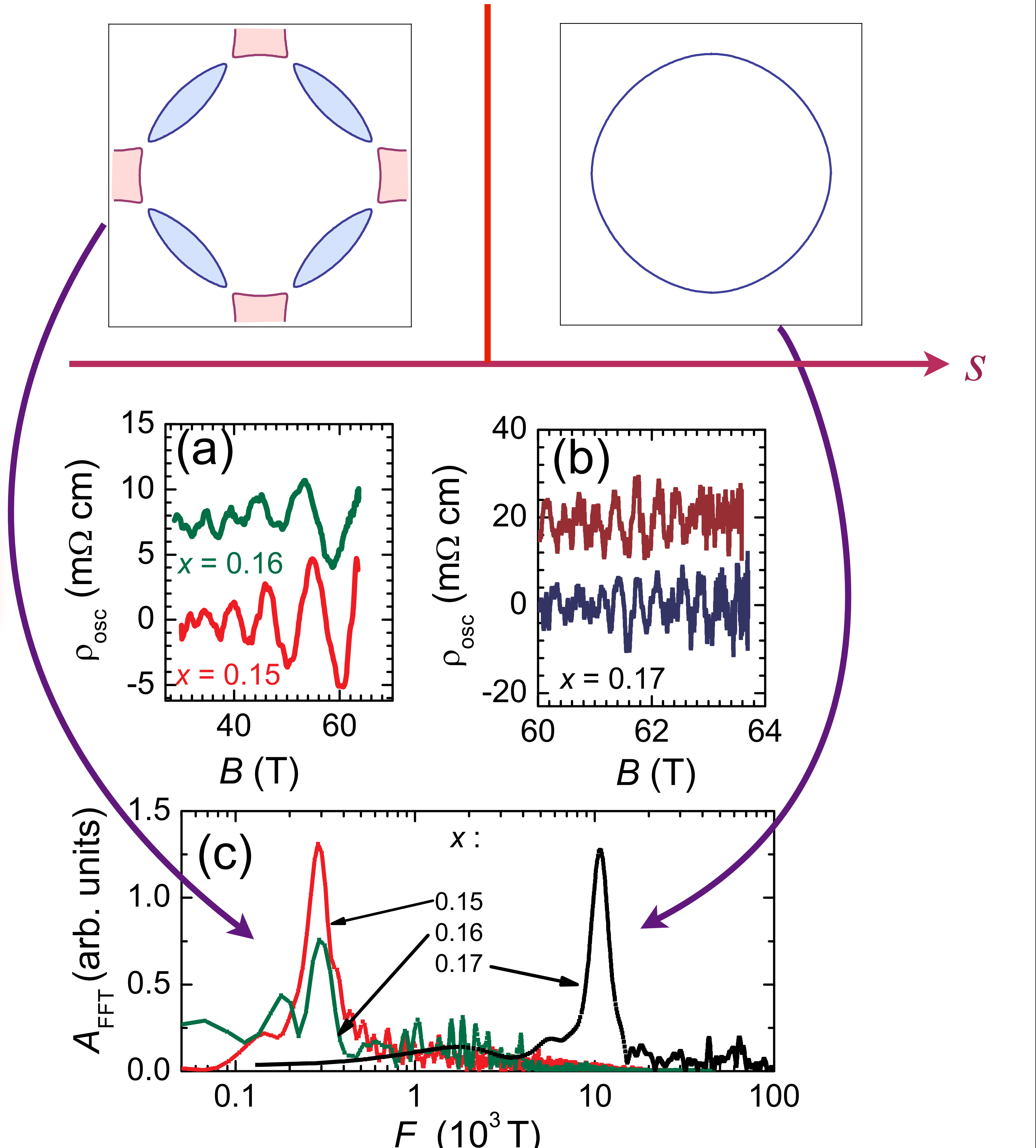
Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Photoemission in $Nd_{2-x}Ce_xCuO_4$



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).



Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

d-wave pairing near a spin-density-wave instability

D. J. Scalapino, E. Loh, Jr.,* and J. E. Hirsch†

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 23 June 1986)

We investigate the three-dimensional Hubbard model and show that paramagnon exchange near a spin-density-wave instability gives rise to a strong singlet d-wave pairing interaction. For a cubic band the singlet ($d_{x^2-y^2}$ and $d_{3z^2-r^2}$) channels are enhanced while the singlet (d_{xy}, d_{xz}, d_{yz}) and triplet p-wave channels are suppressed. A unique feature of this pairing mechanism is its sensitivity to band structure and band filling.

Physical Review B **34**, 8190 (1986)

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\lambda^2}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\lambda^2 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

Pairing by SDW fluctuation exchange

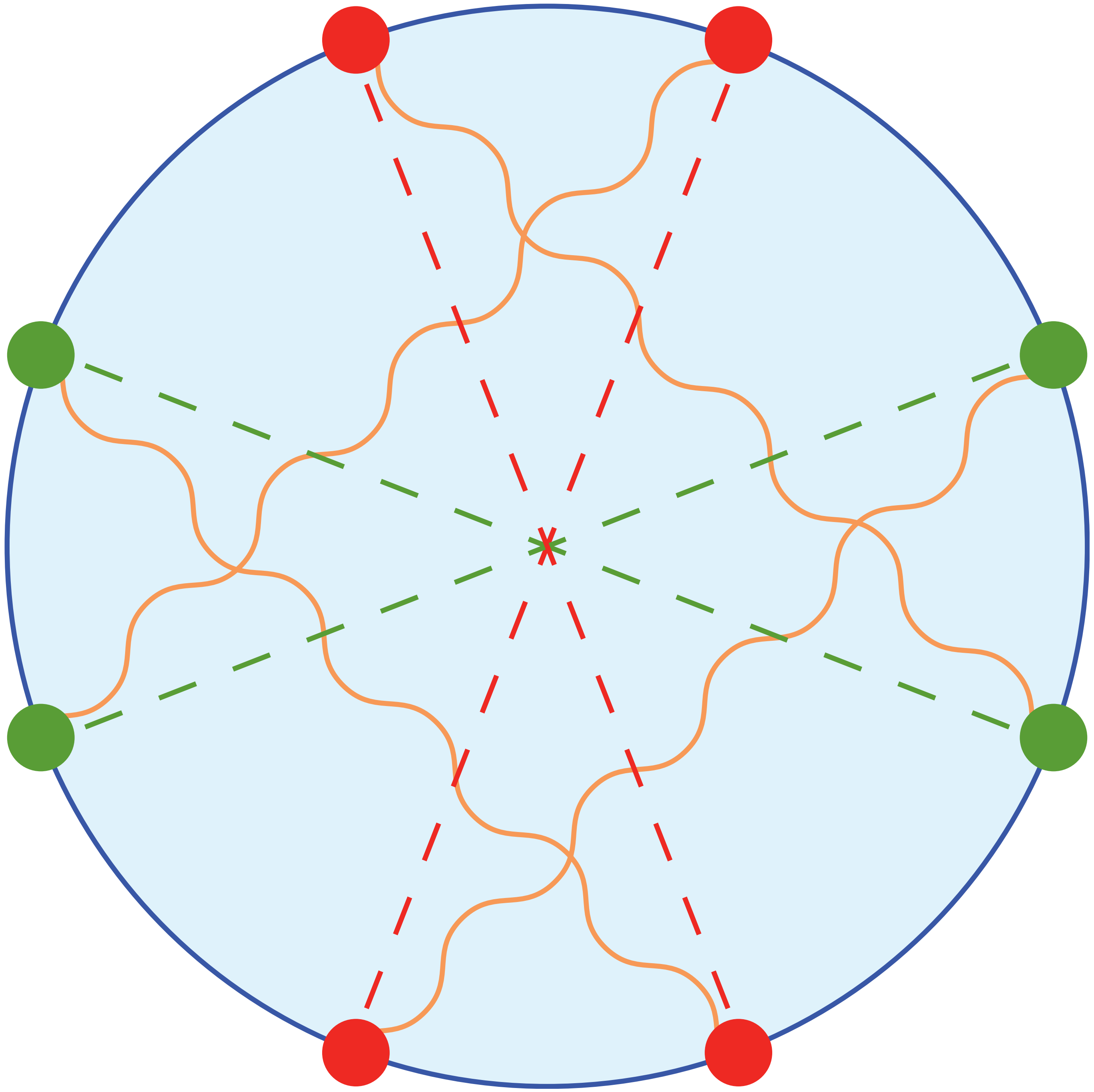
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

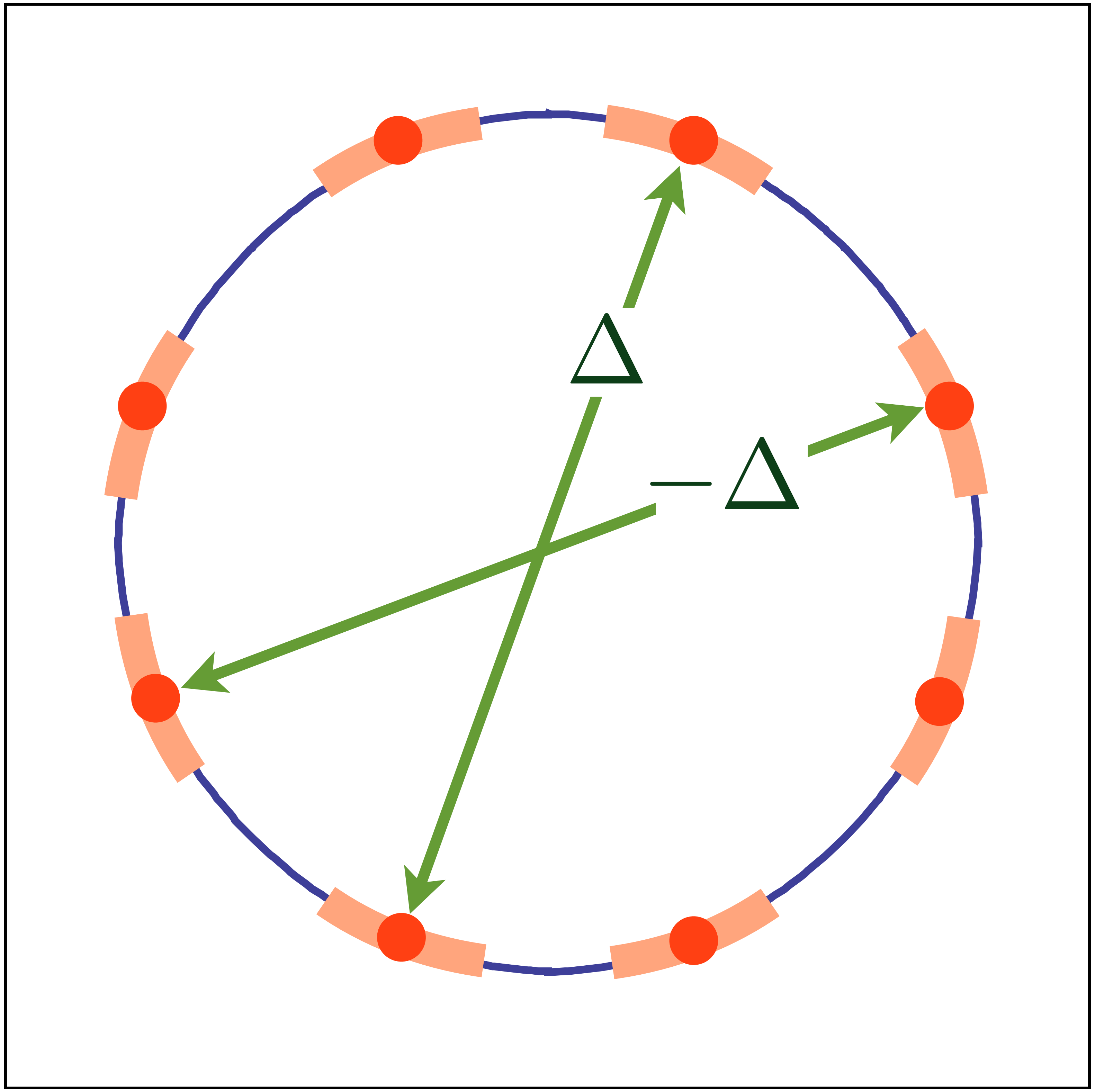
$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\lambda^2}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

Pairing “glue” from antiferromagnetic fluctuations

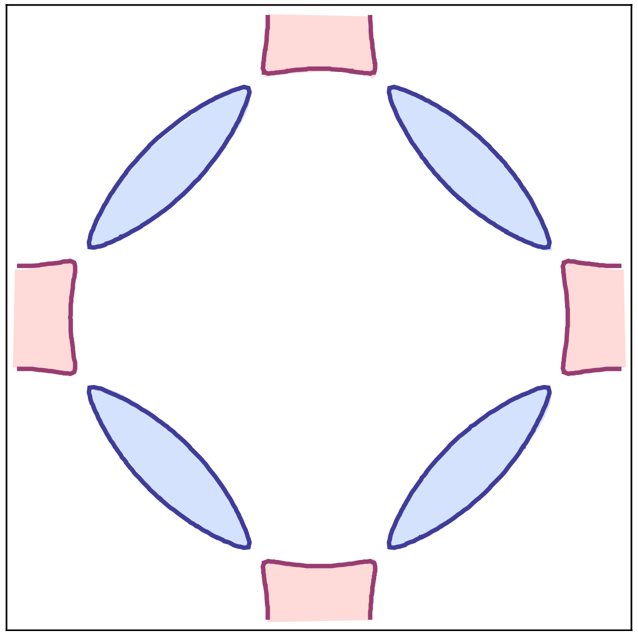


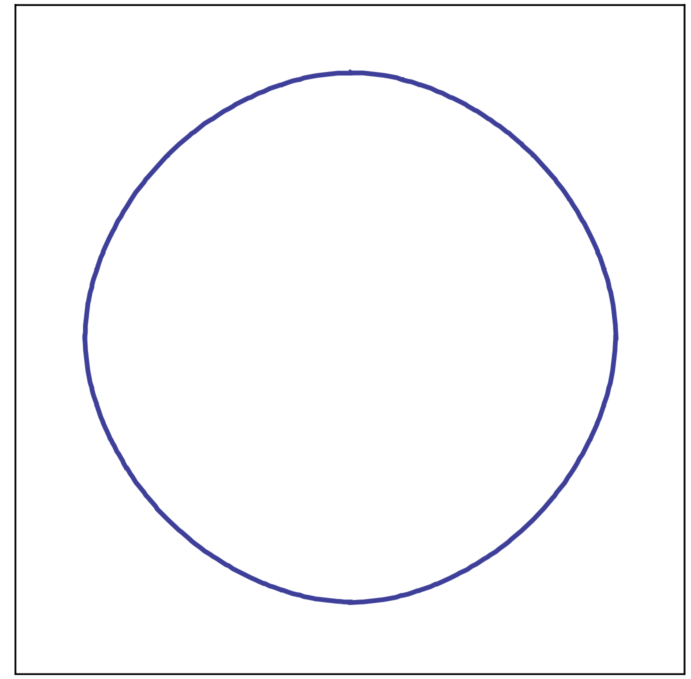
$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta(\cos k_x - \cos k_y)$$



Unconventional pairing at <u>*and near*</u> hot spots

Fermi surface+antiferromagnetism





$$\langle \vec{\varphi} \rangle \neq 0$$

$$\langle \vec{\varphi} \rangle = 0$$

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A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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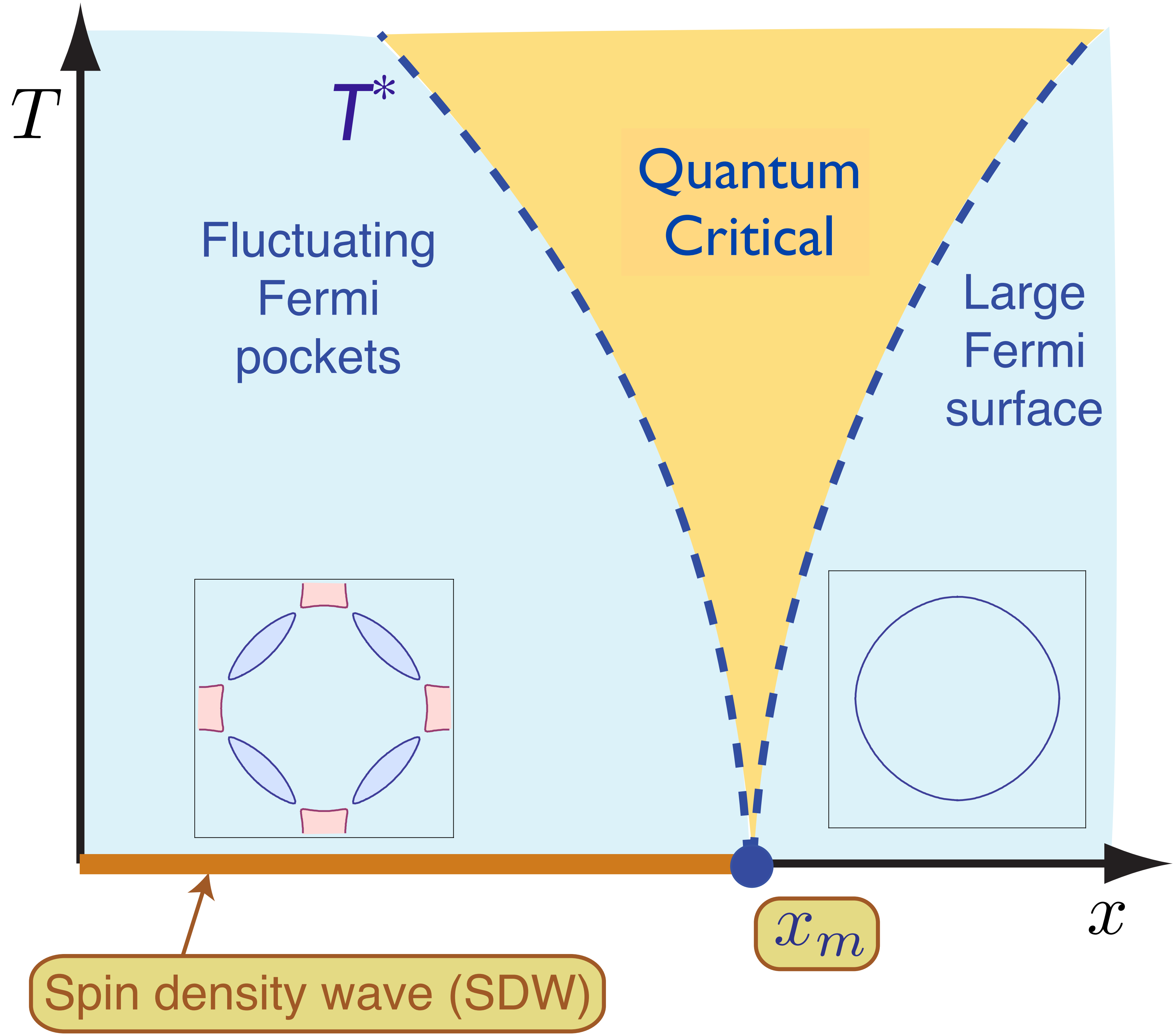
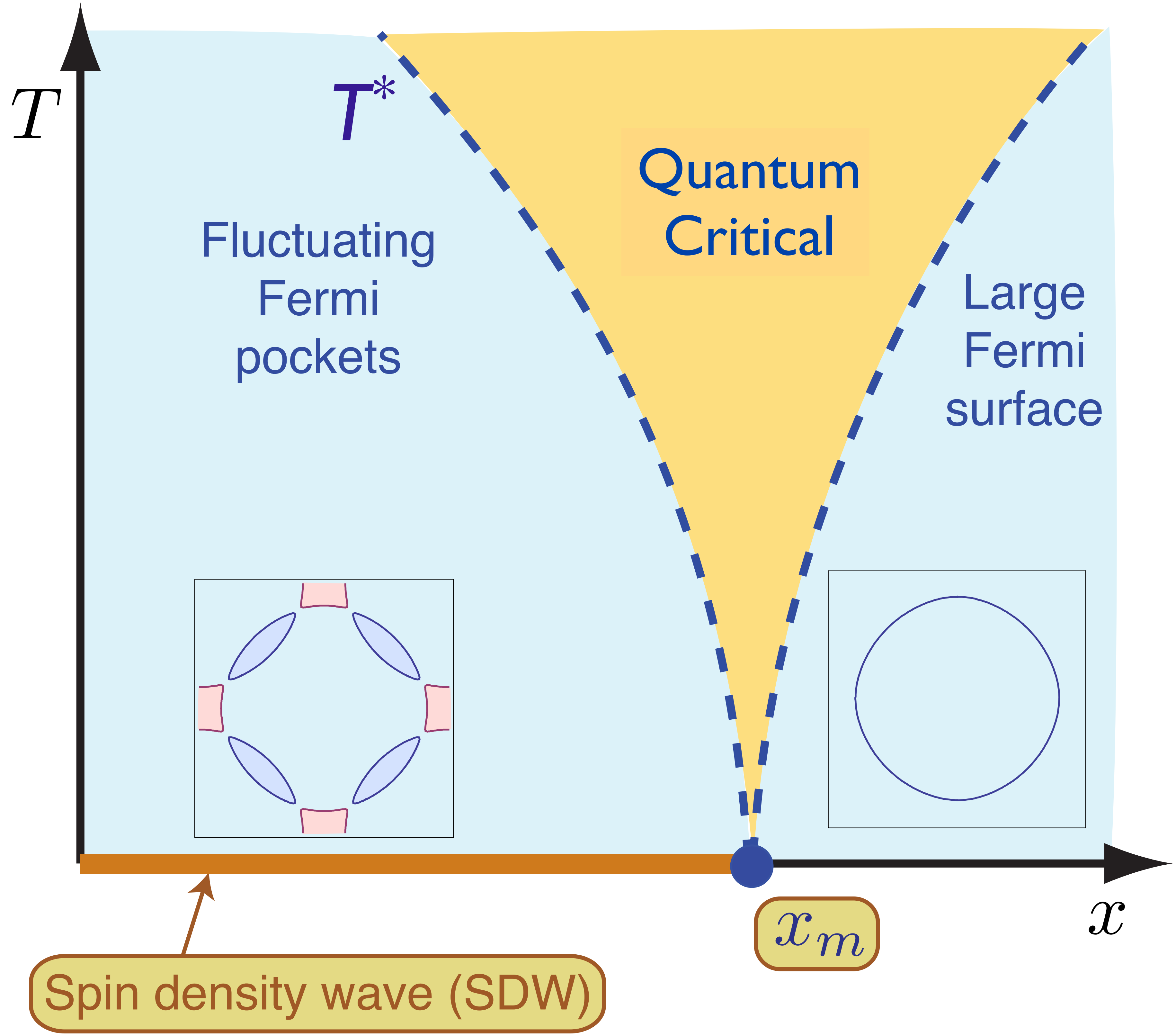


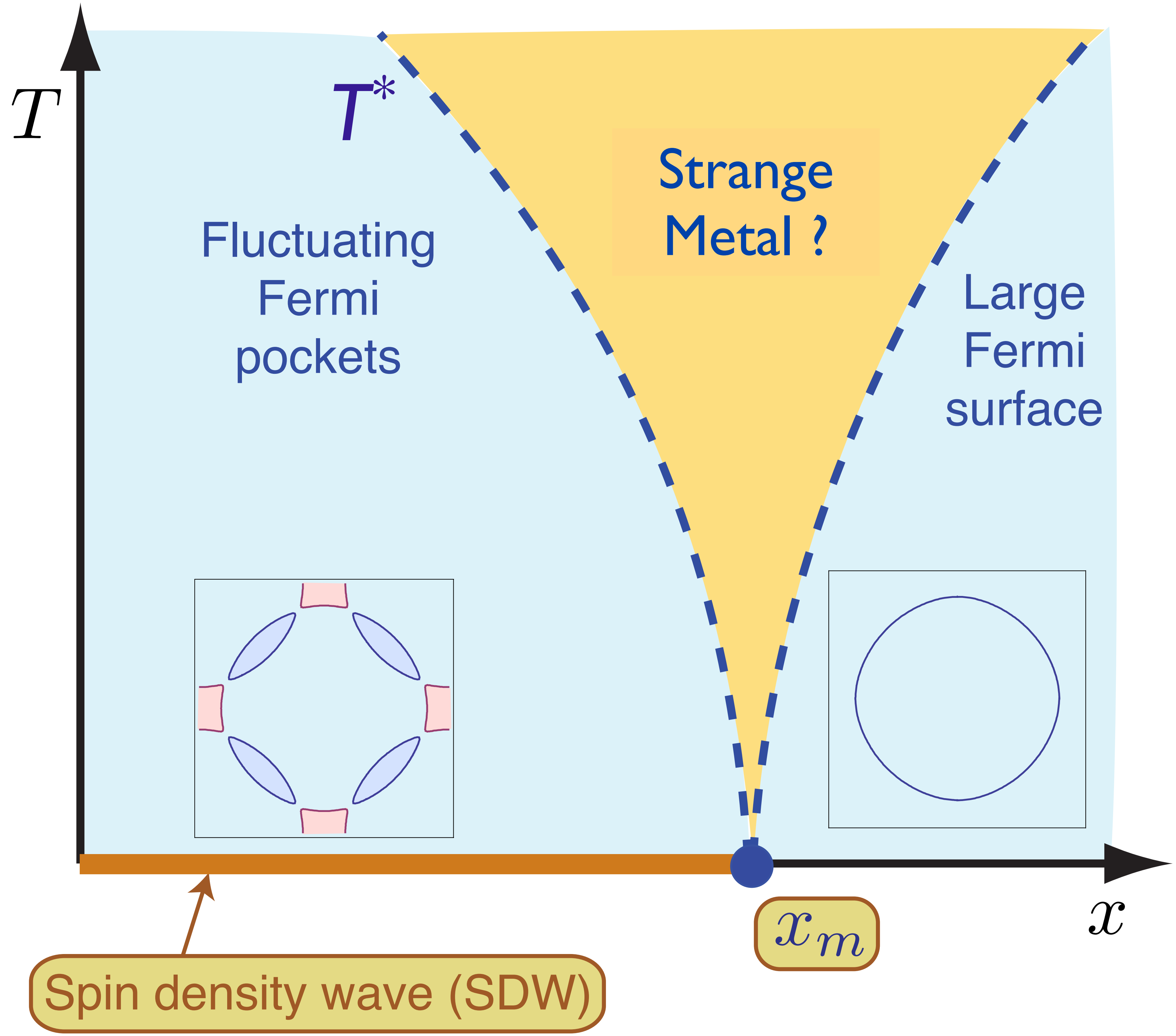
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surface+antiferromagnetism



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

Fermi surface+antiferromagnetism



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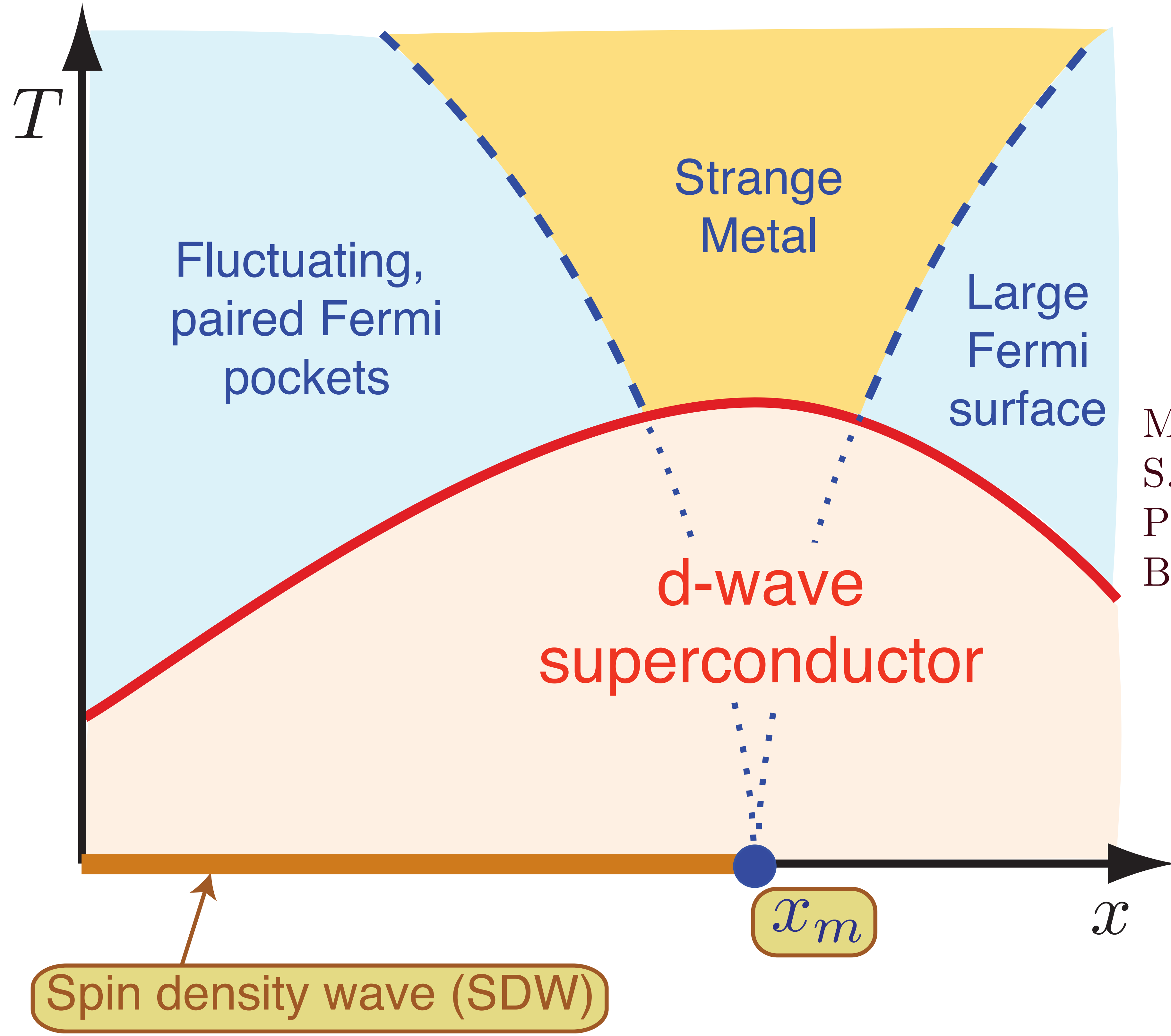
Fermi surface+antiferromagnetism



M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

Pairing "glue" from antiferromagnetic fluctuations

At stronger coupling,
different effects compete:

- Pairing glue becomes stronger.

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- Pairing glue becomes stronger.

- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

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- Pairing glue becomes stronger.

- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

- Other instabilities can appear *e.g.* to charge density waves/stripe order.

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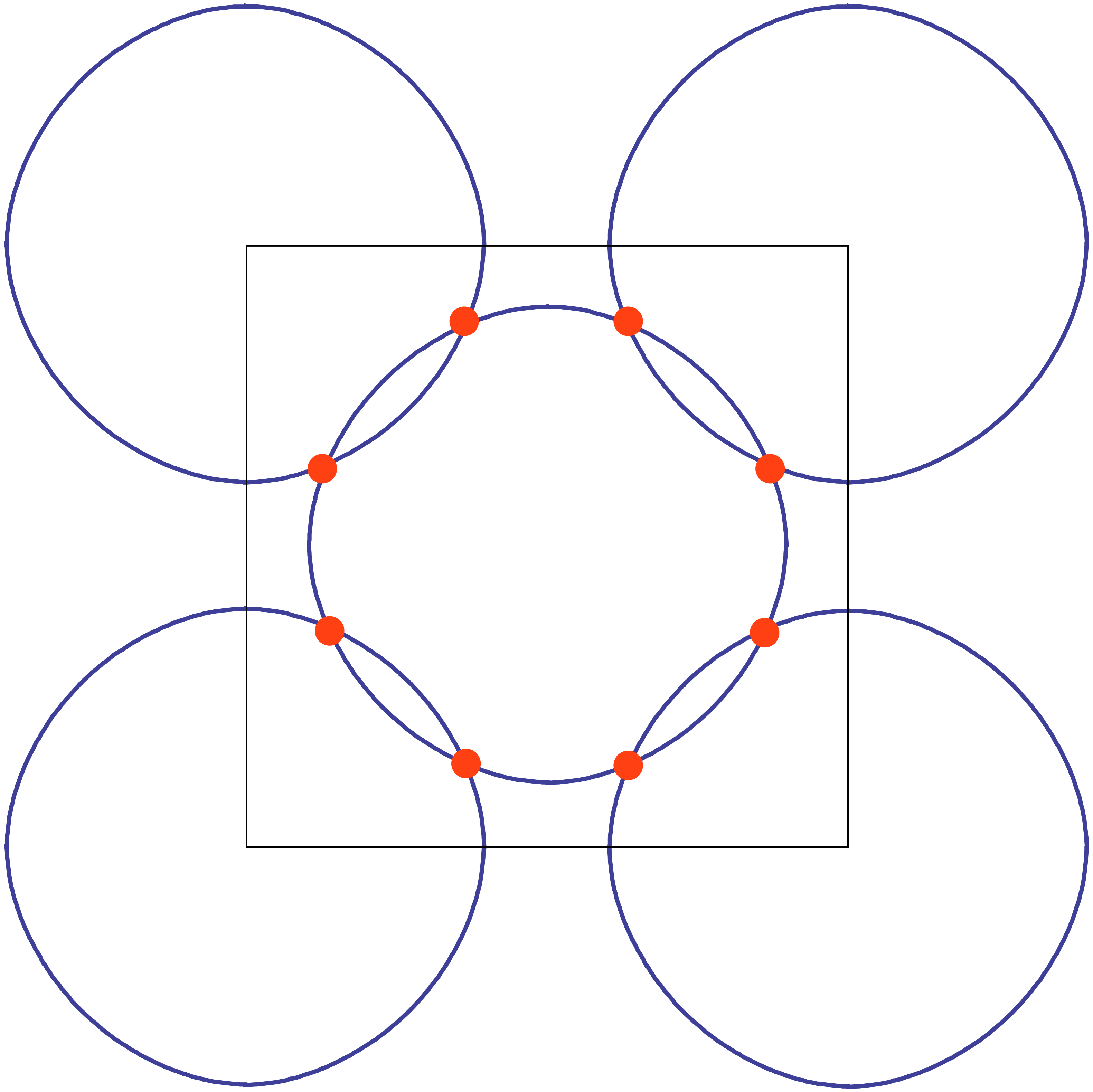
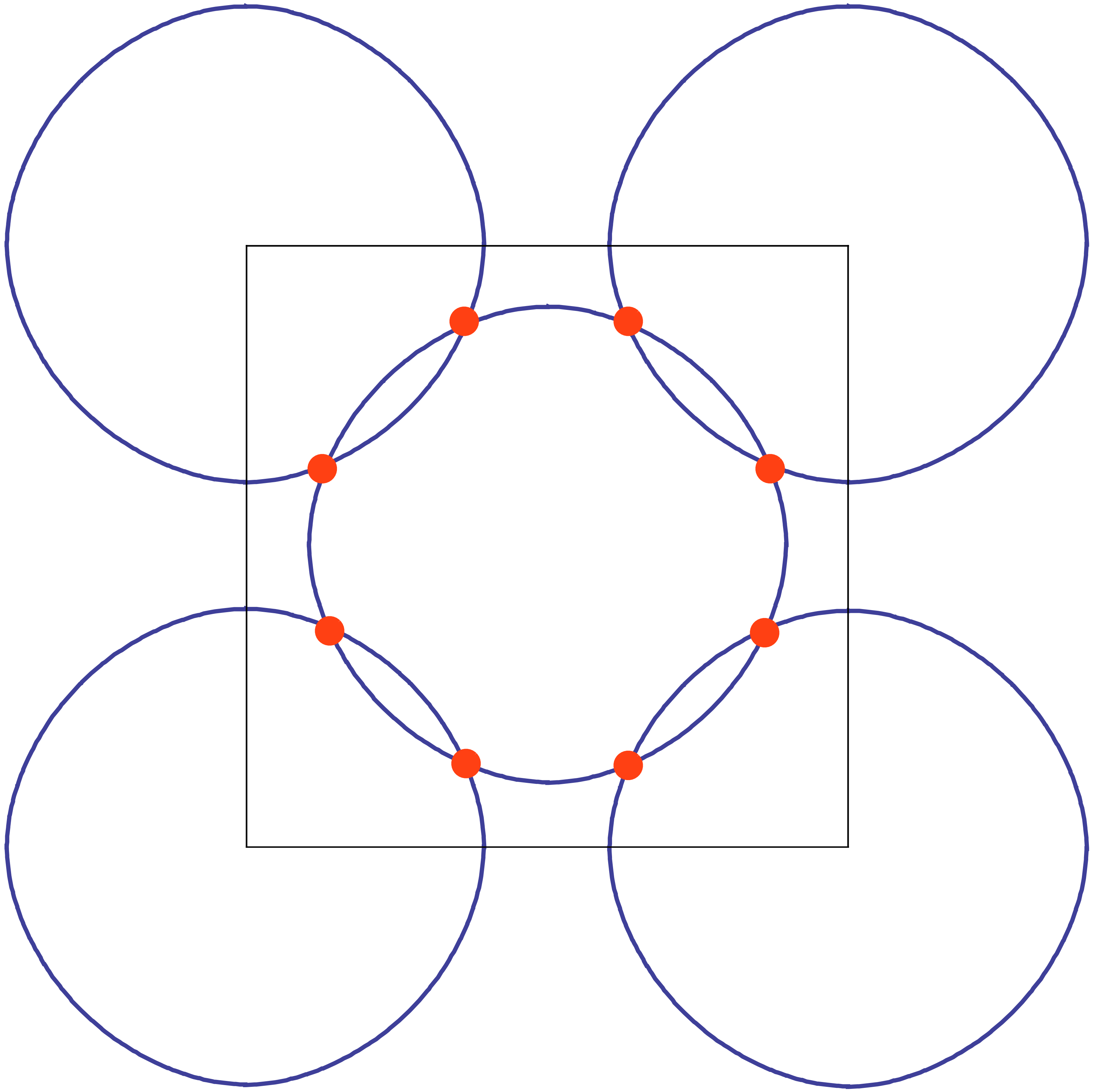
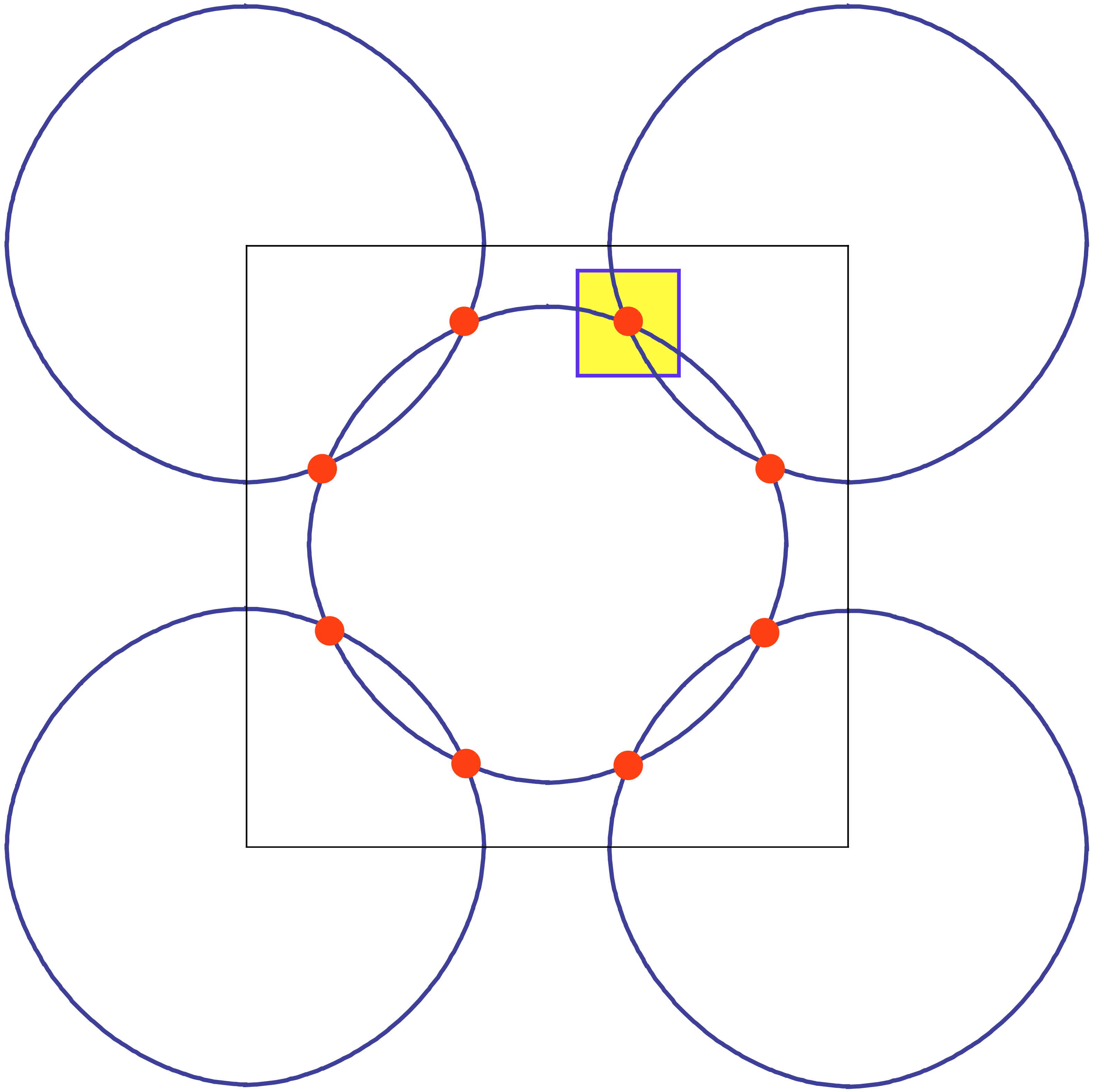


“Hot” spots

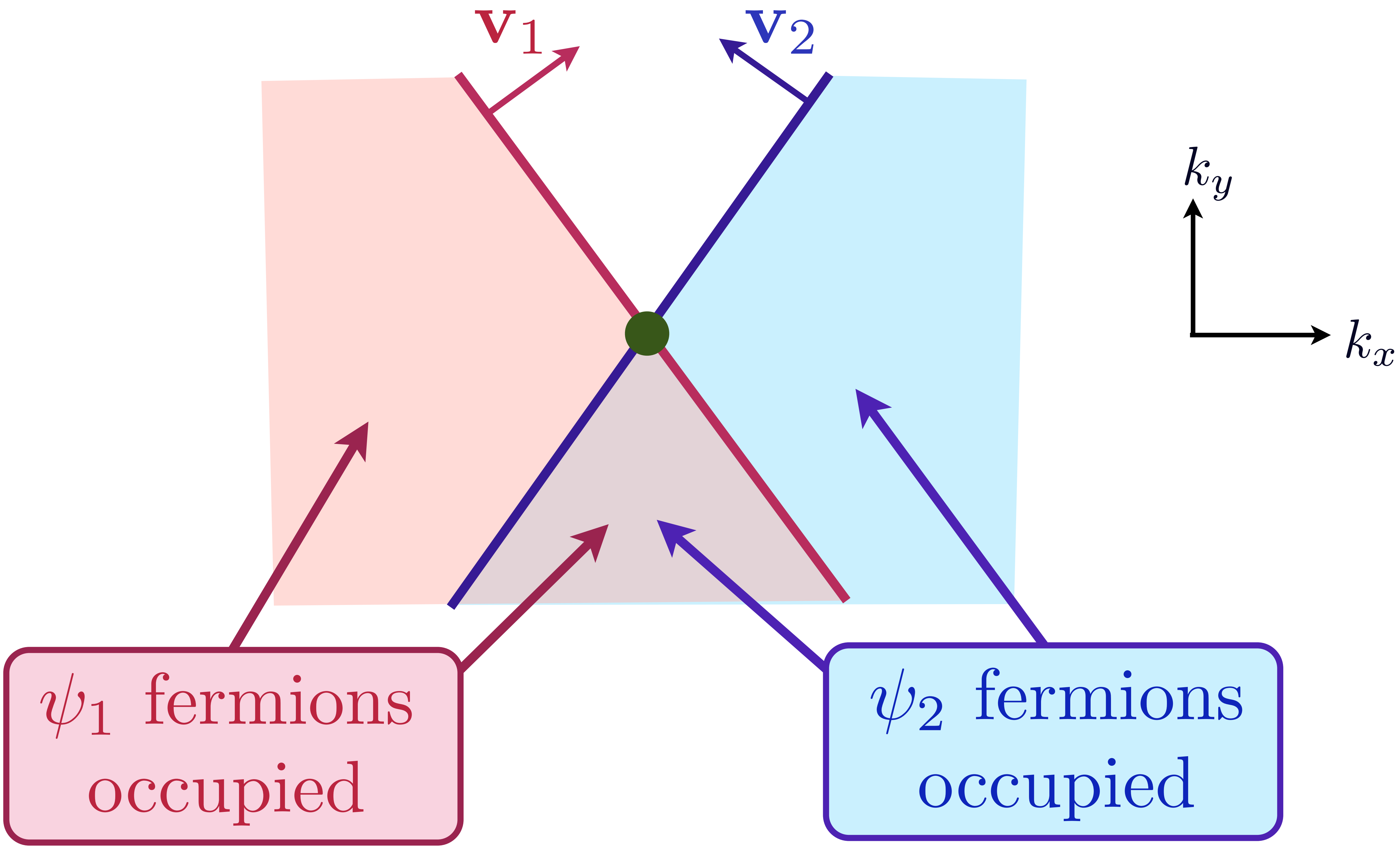


Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$)
and boson order parameter $\vec{\varphi}$,
interacting with coupling λ



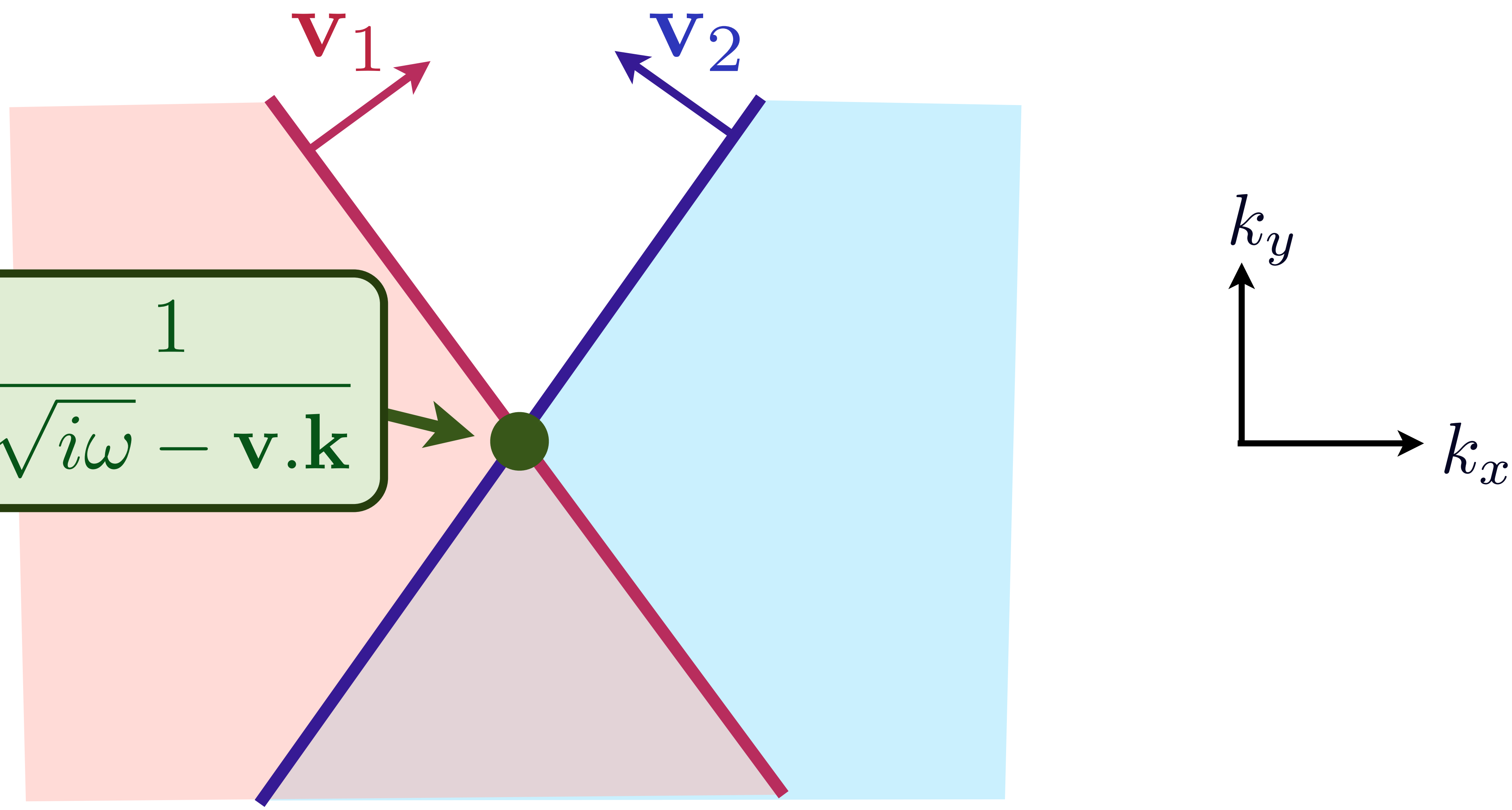
- In $d = 2$, we *must* work in local theores which keeps both the order parameter and the Fermi surface quasiparticles "alive".

- The theories can be organized in a $1/N$ expansion, where N is the number of fermion "flavors".

- At subleading order, resummation of all "planar" graphics is required (at least): this theory is even more complicated than QCD.

Sung-Sik Lee, *Phys. Rev. B* **80**, 165102 (2009)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075128 (2010)

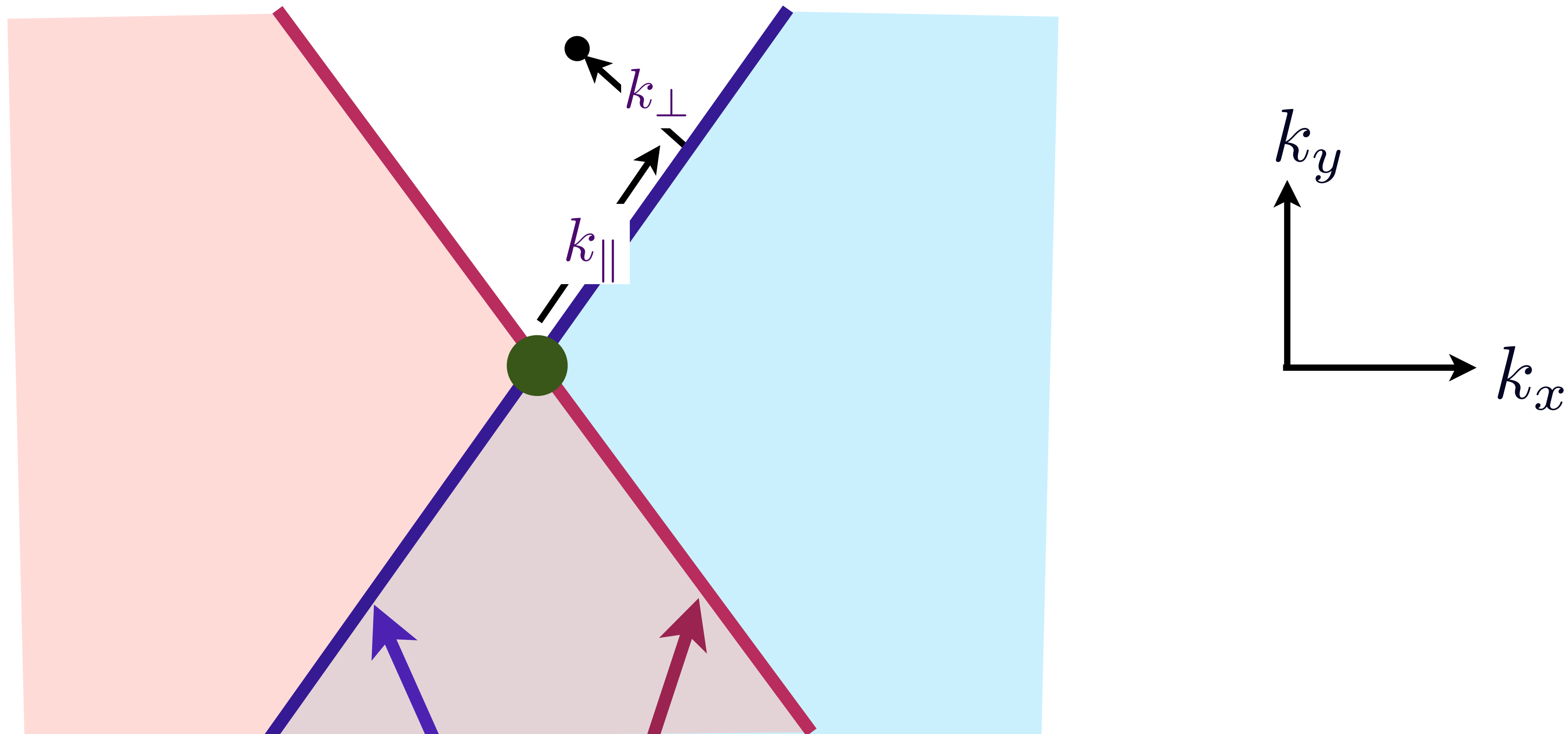
Two loop results: Non-Fermi liquid spectrum at hot spots



$$G_{\text{fermion}} \sim \frac{1}{\sqrt{i\omega} - \mathbf{v}.\mathbf{k}}$$

A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004)

Two loop results: Quasiparticle weight vanishes upon approaching hot spots



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{\omega - v_F(k_\parallel) k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

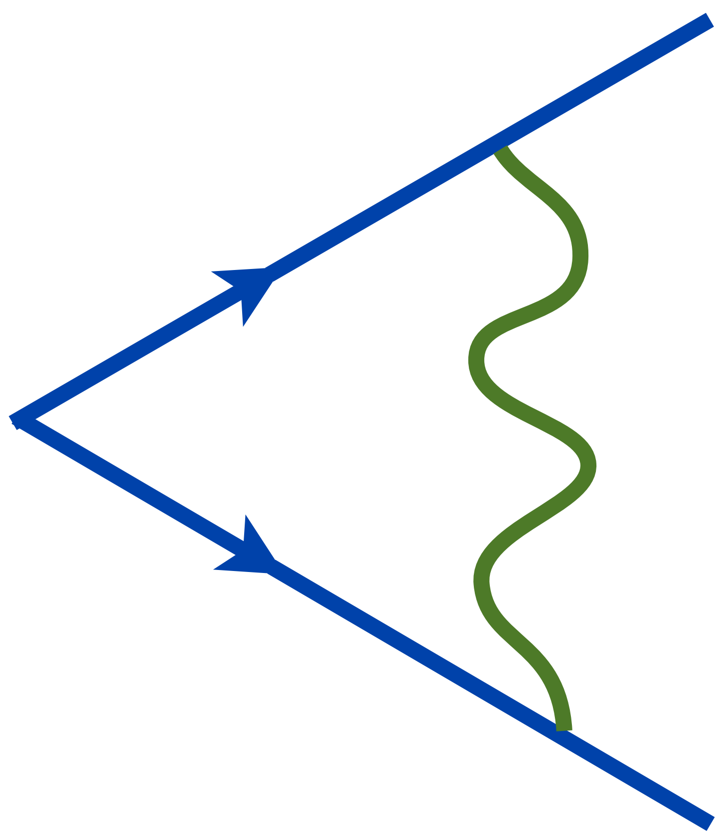
Pairing by SDW fluctuation exchange

Weak-coupling theory

Fermi energy

$$1 + \lambda^2 \rho(E_F) \log\left(\frac{E_F}{\omega}\right)$$

Density of states at Fermi energy



Pairing by SDW fluctuation exchange

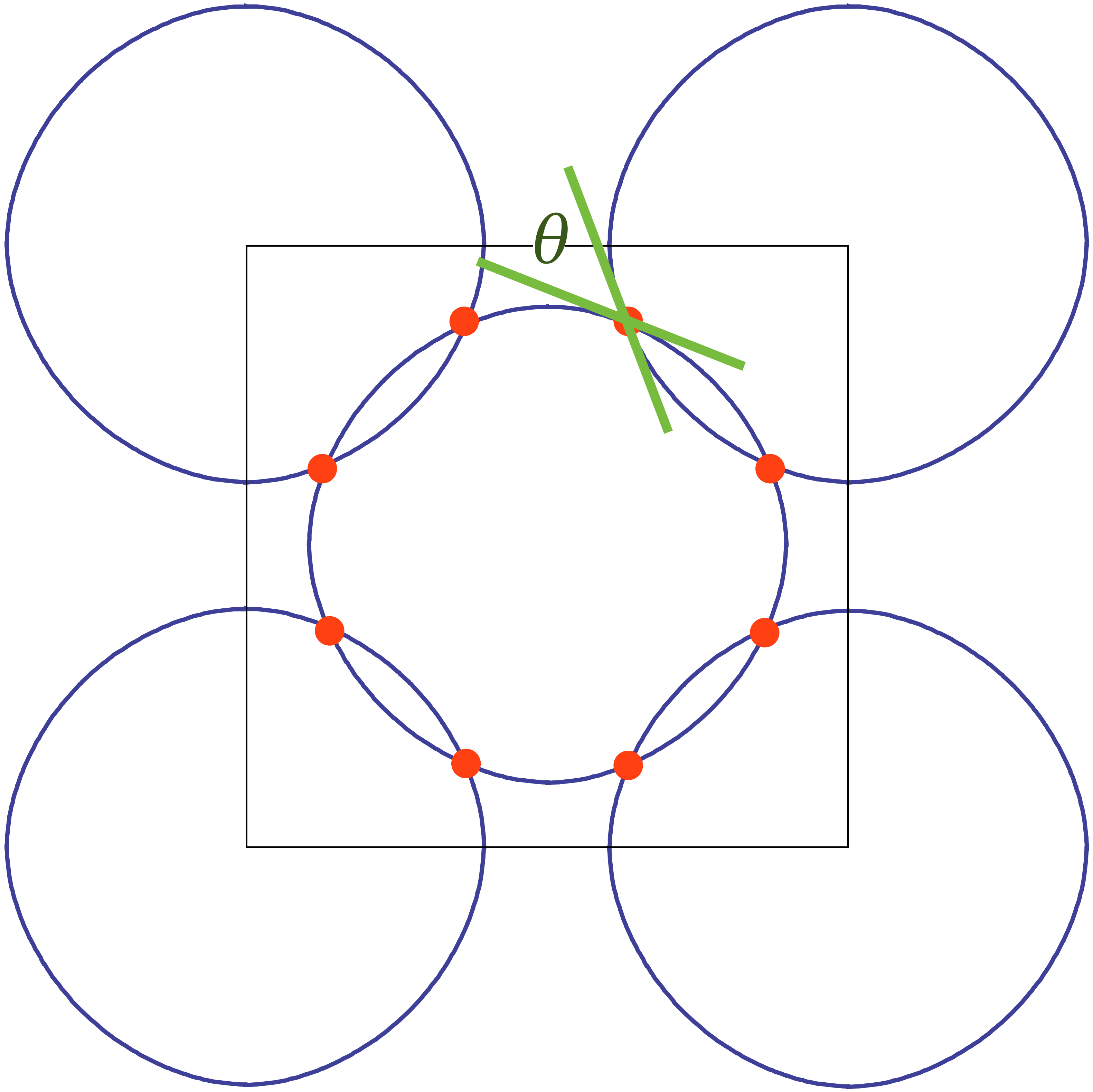
Antiferromagnetic critical point

Fermi energy

$$1 + \frac{\sin\theta}{2\pi} \log^2\left(\frac{E_F}{\omega}\right)$$

θ is the angle between Fermi lines. *Independent* of interaction strength U in 2 dimensions.

(see also Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)


θ

Pairing by SDW fluctuation exchange

Antiferromagnetic critical point

$$1 + \frac{\sin\theta}{2\pi} \log^2 \left(\frac{E_F}{\omega} \right)$$

- **Universal** $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Summary:

- Field theory/RG provide strong evidence that there is unconventional ("pairing-amplitude-sign-changing") spin-singlet superconductivity at the antiferromagentic quantum critical point in all two-dimensional metals.

- The flow to strong-coupling indicates that Feynman graph/field theory/RG methods have reached their limits, and we have reached an impasse........

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4. Fractionalization in metals, and the hole-doped cuprates

Outline

1. Weak-coupling theory for the onset of antiferromagnetism in a metal

2. Quantum field theory of the onset of antiferromagnetism in a metal

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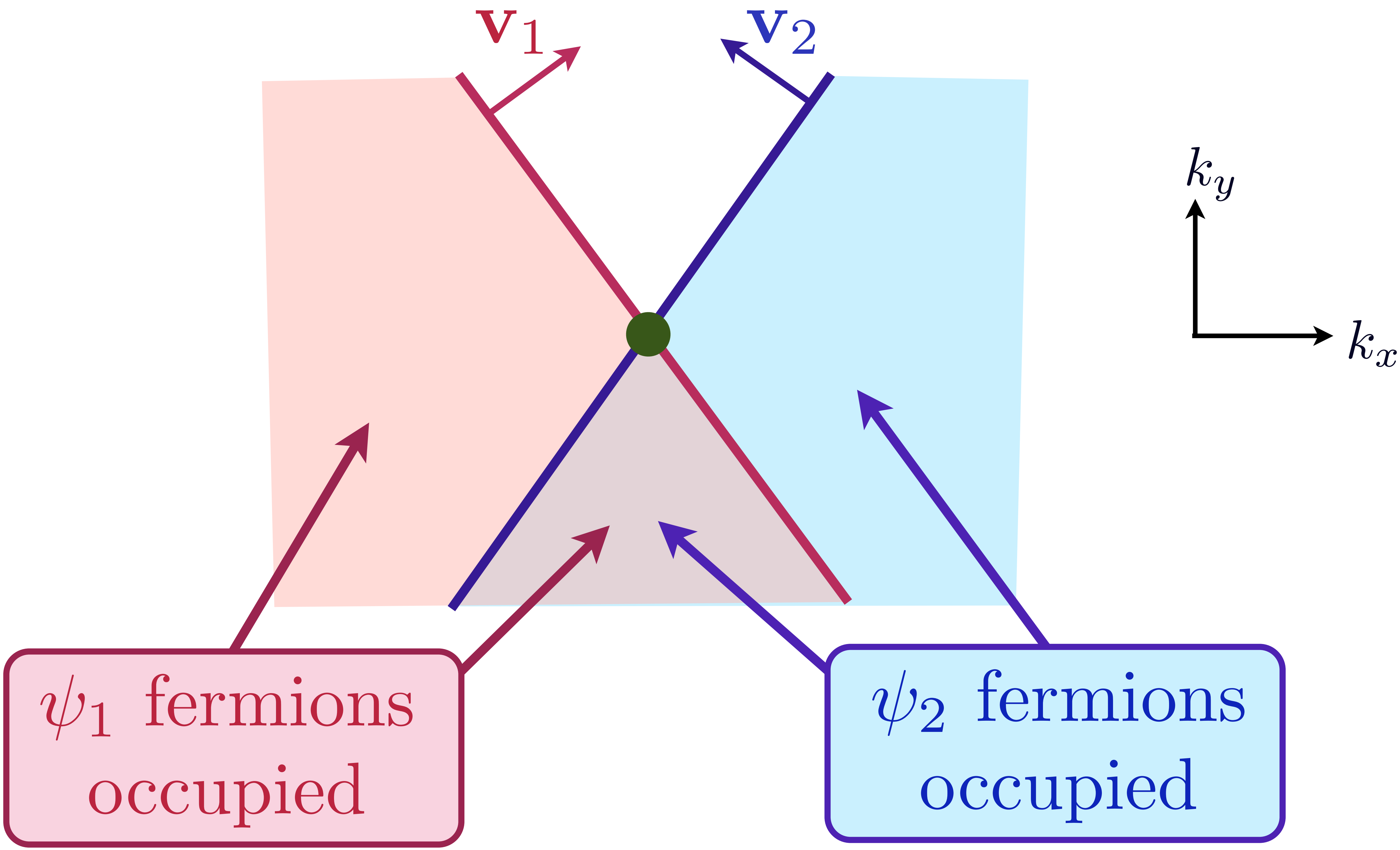
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$)
and boson order parameter $\vec{\varphi}$,
interacting with coupling λ



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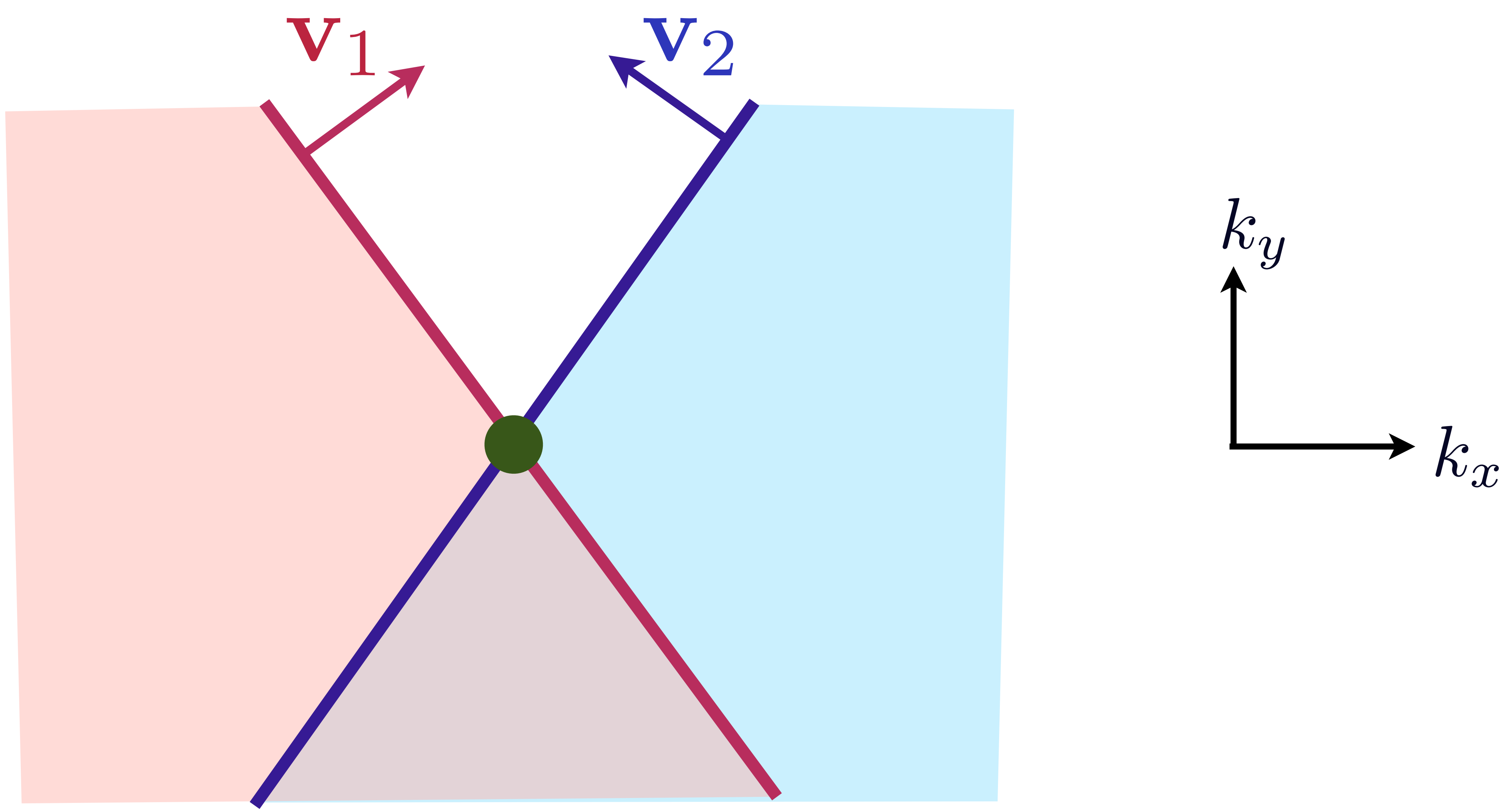
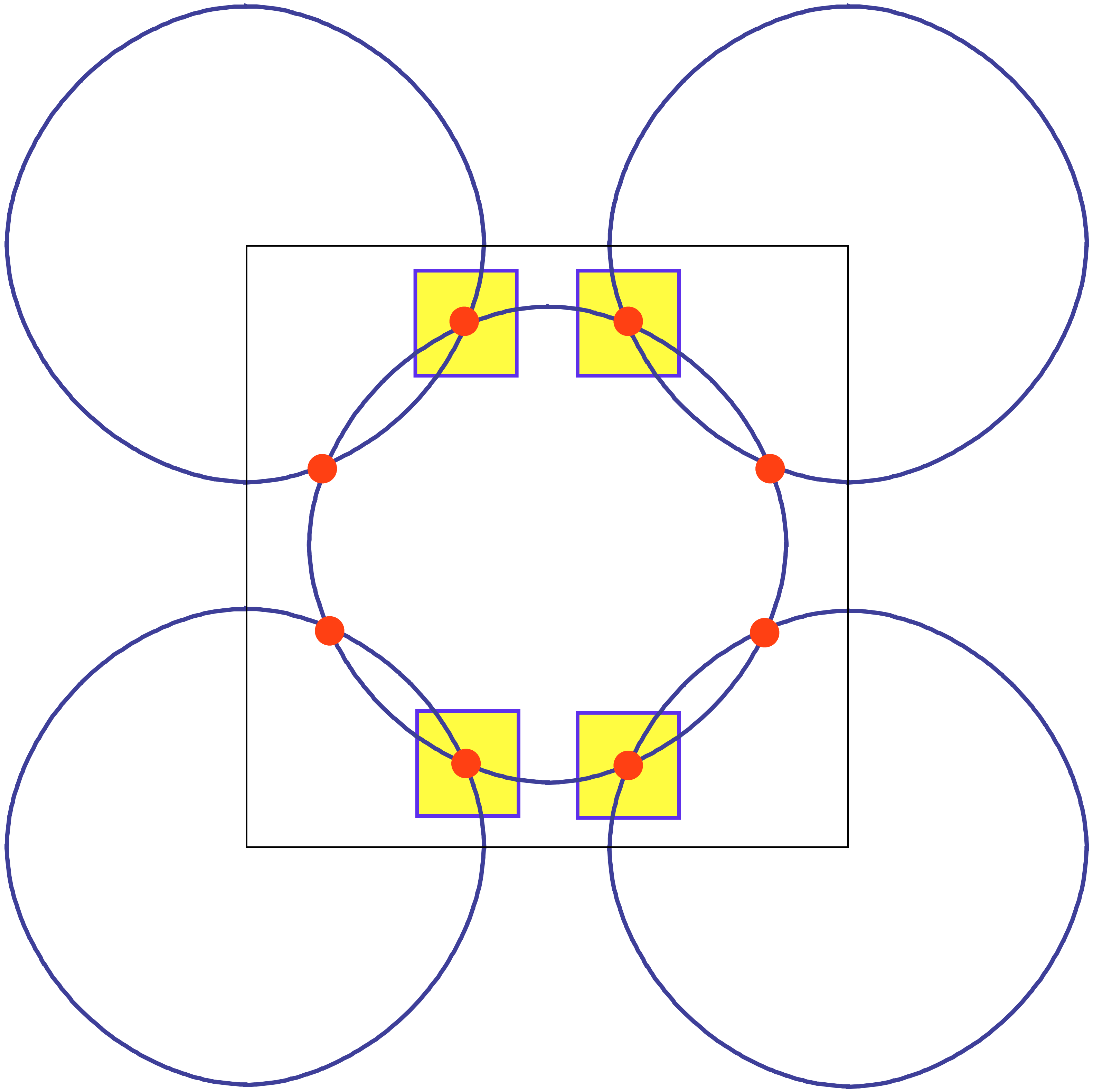
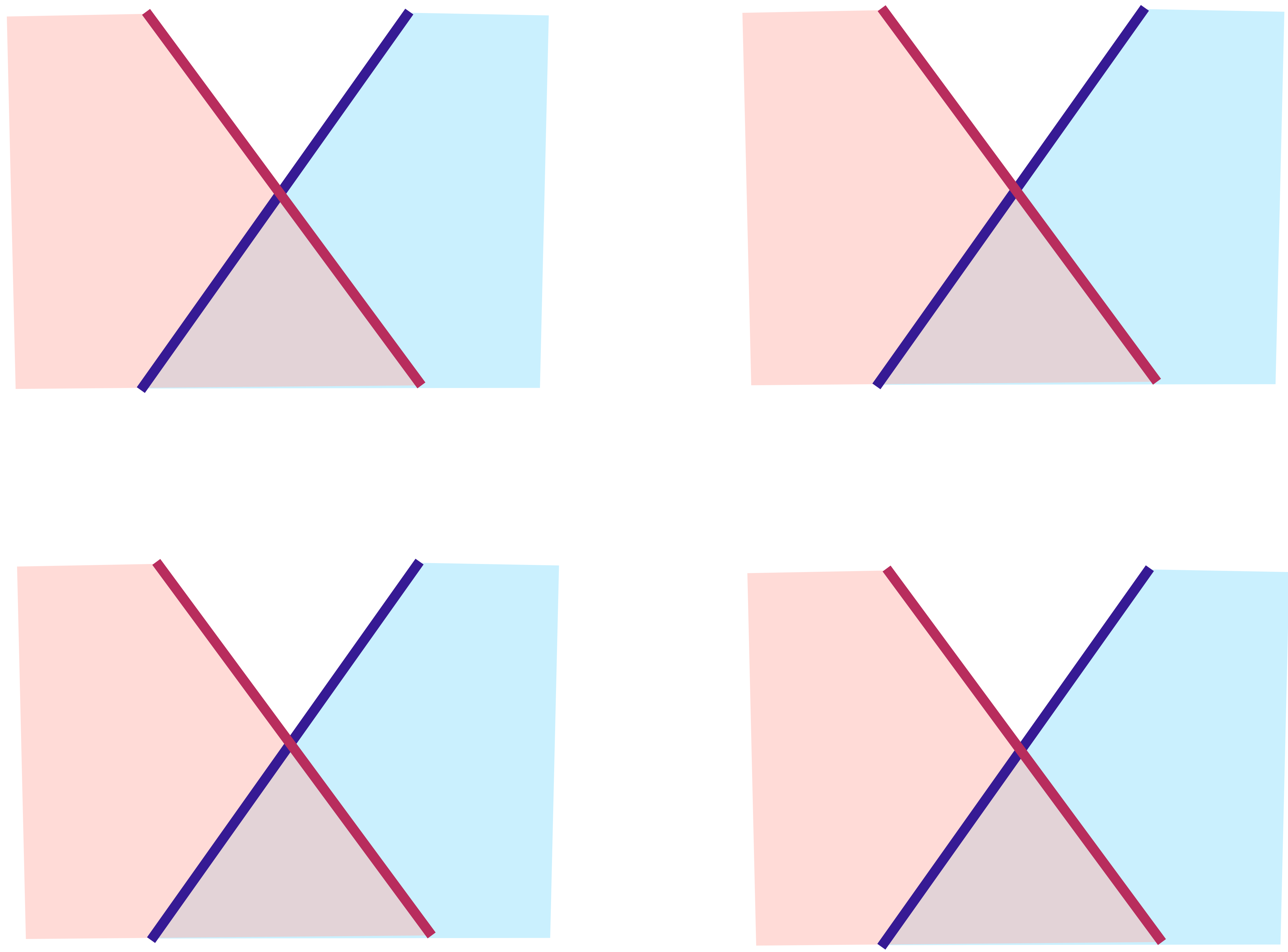


To faithfully realize low energy theory in quantum Monte Carlo, we need a UV completion in which Fermi lines don't end and all weights are positive.

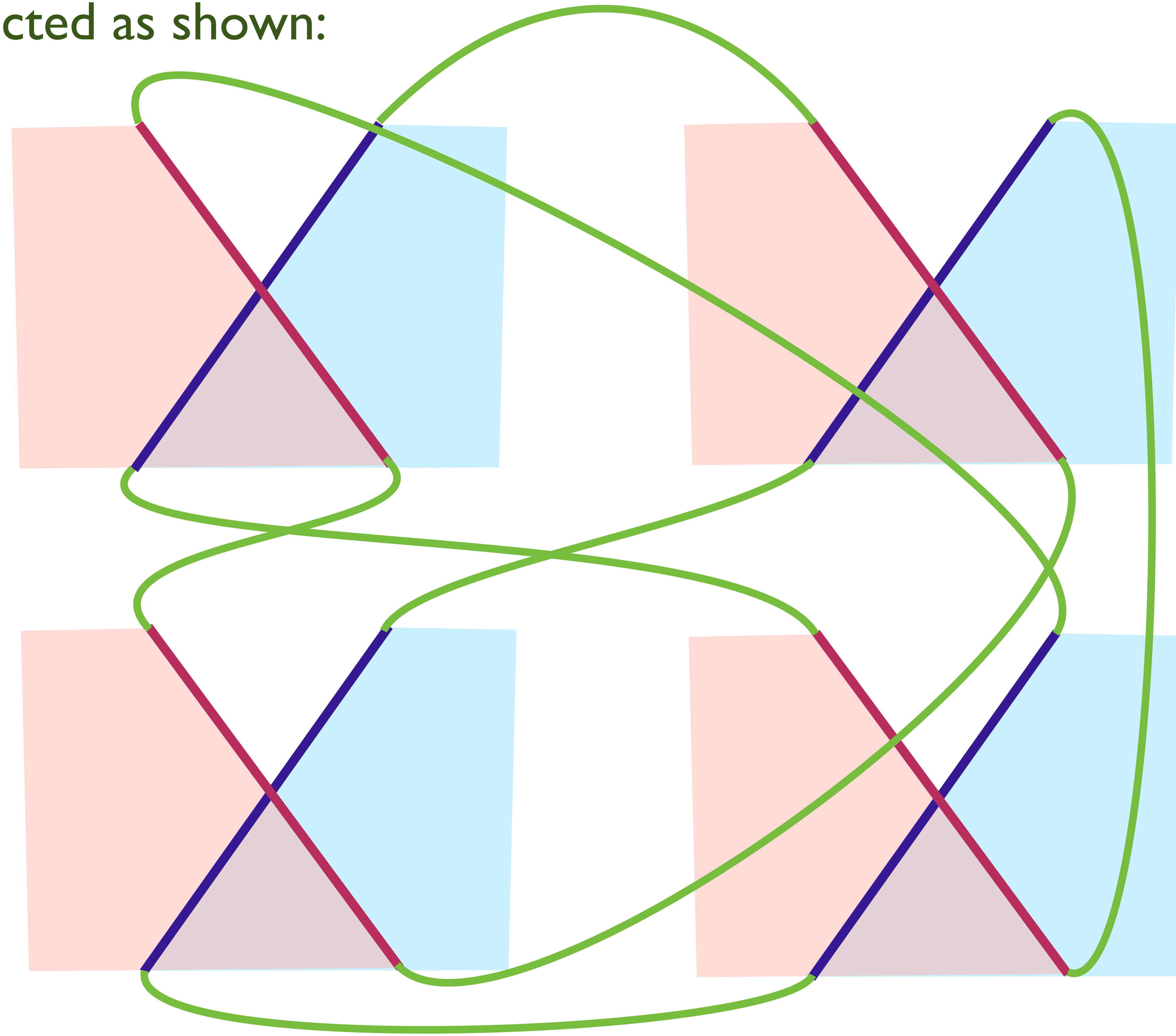


Low energy theory for critical point near hot spots

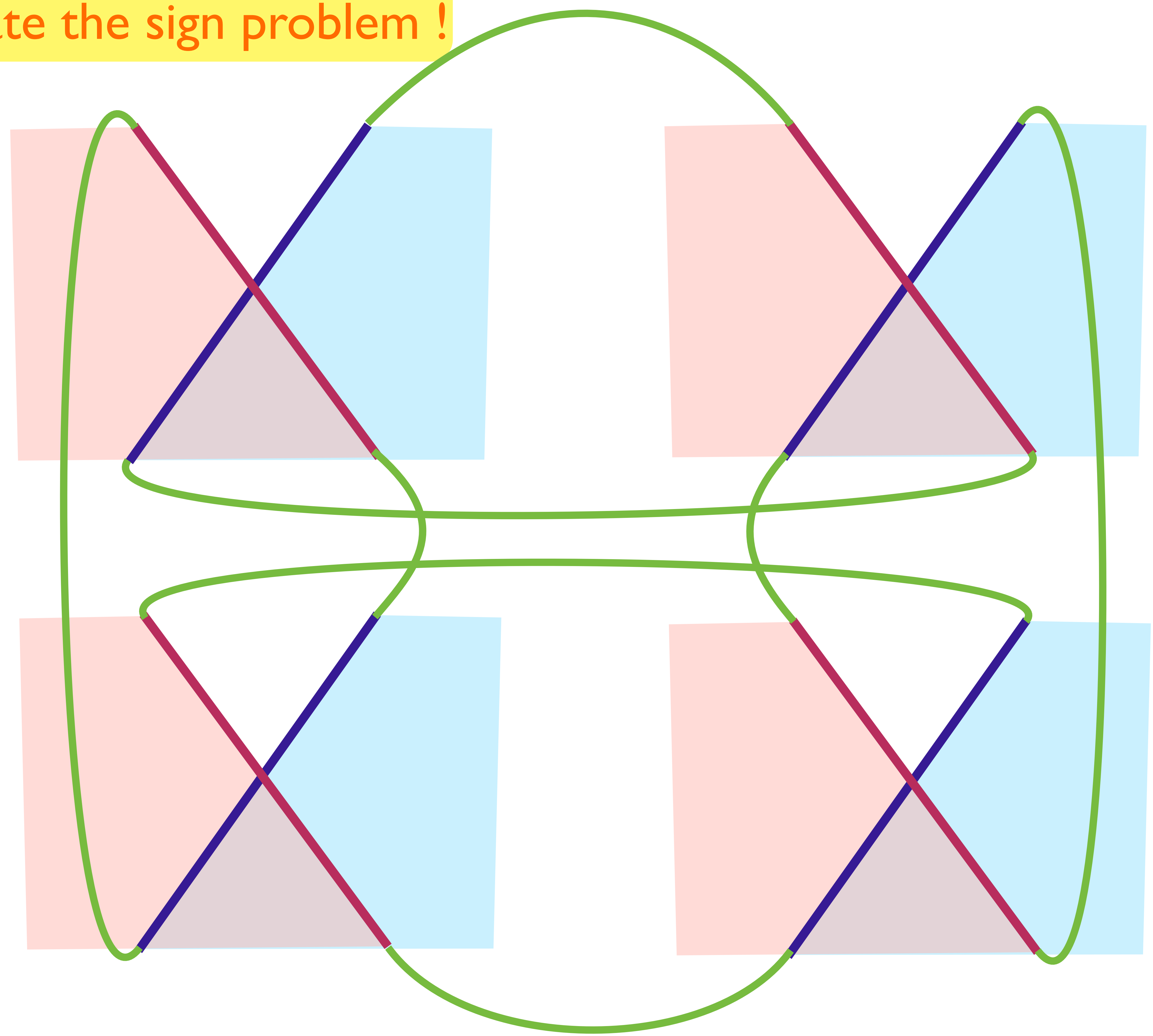
We have 4 copies of the hot spot theory......



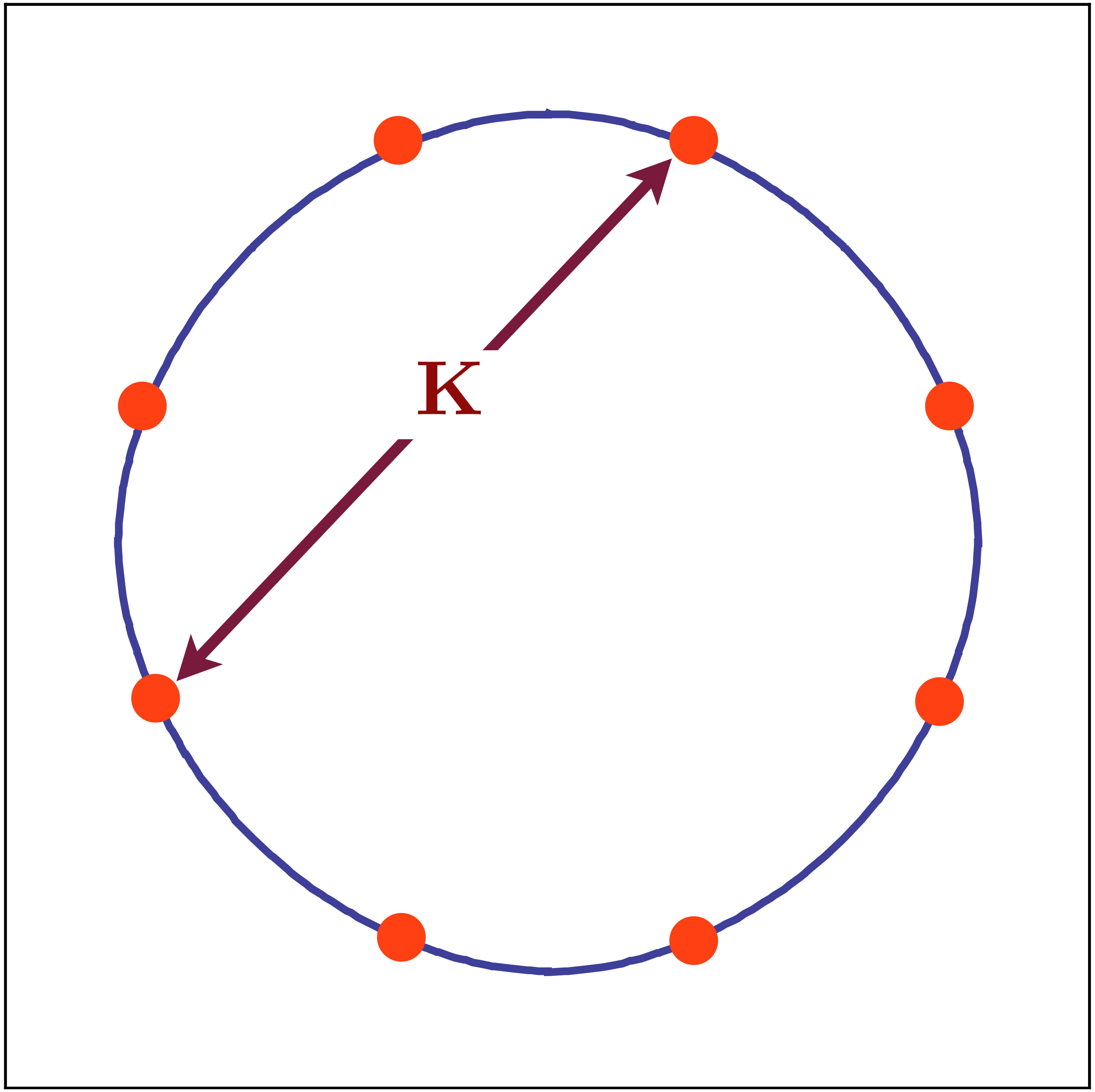
and their Fermi lines are connected as shown:



Reconnect Fermi lines and eliminate the sign problem !

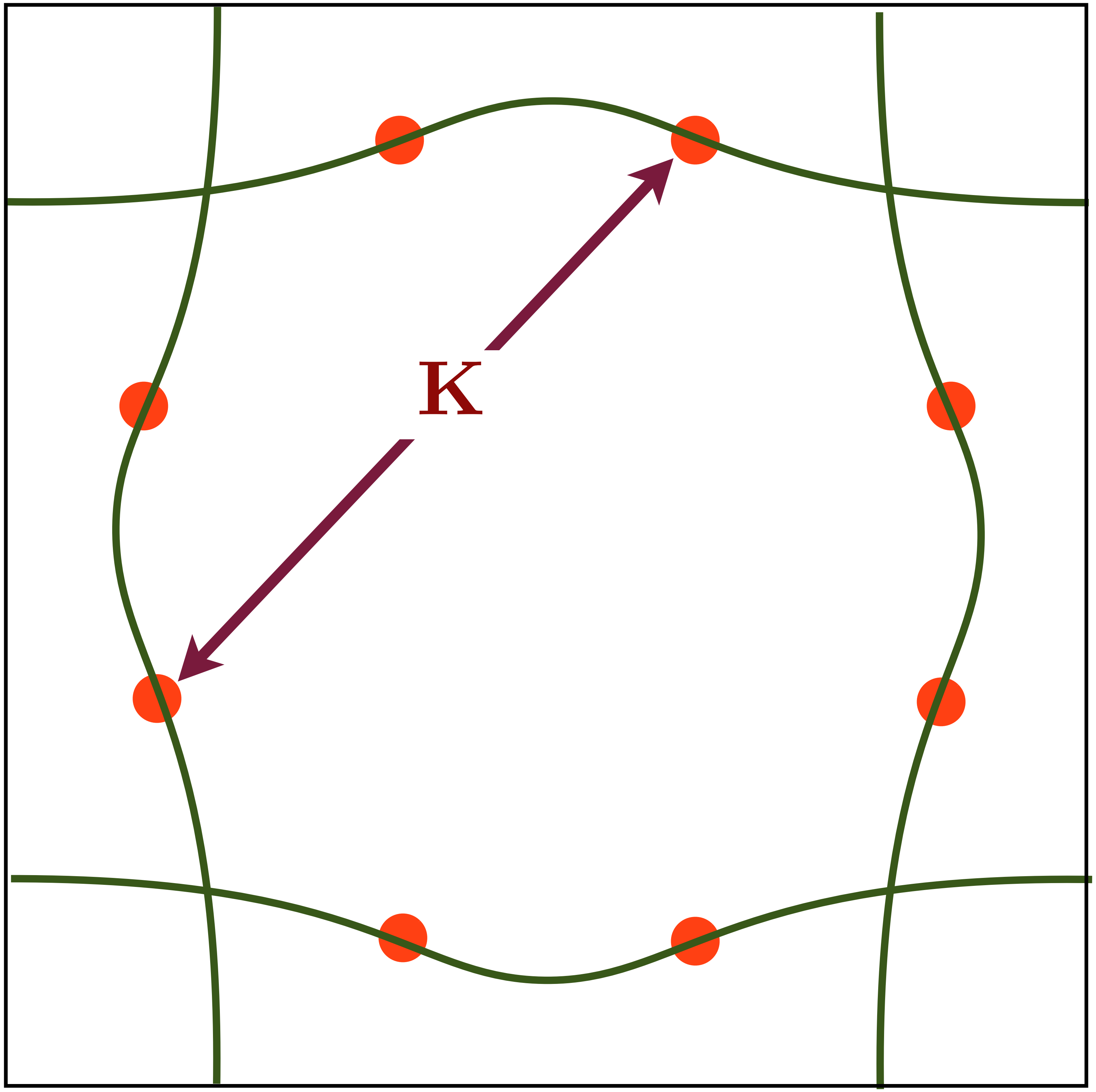


QMC for the onset of antiferromagnetism



Hot spots in a single band model

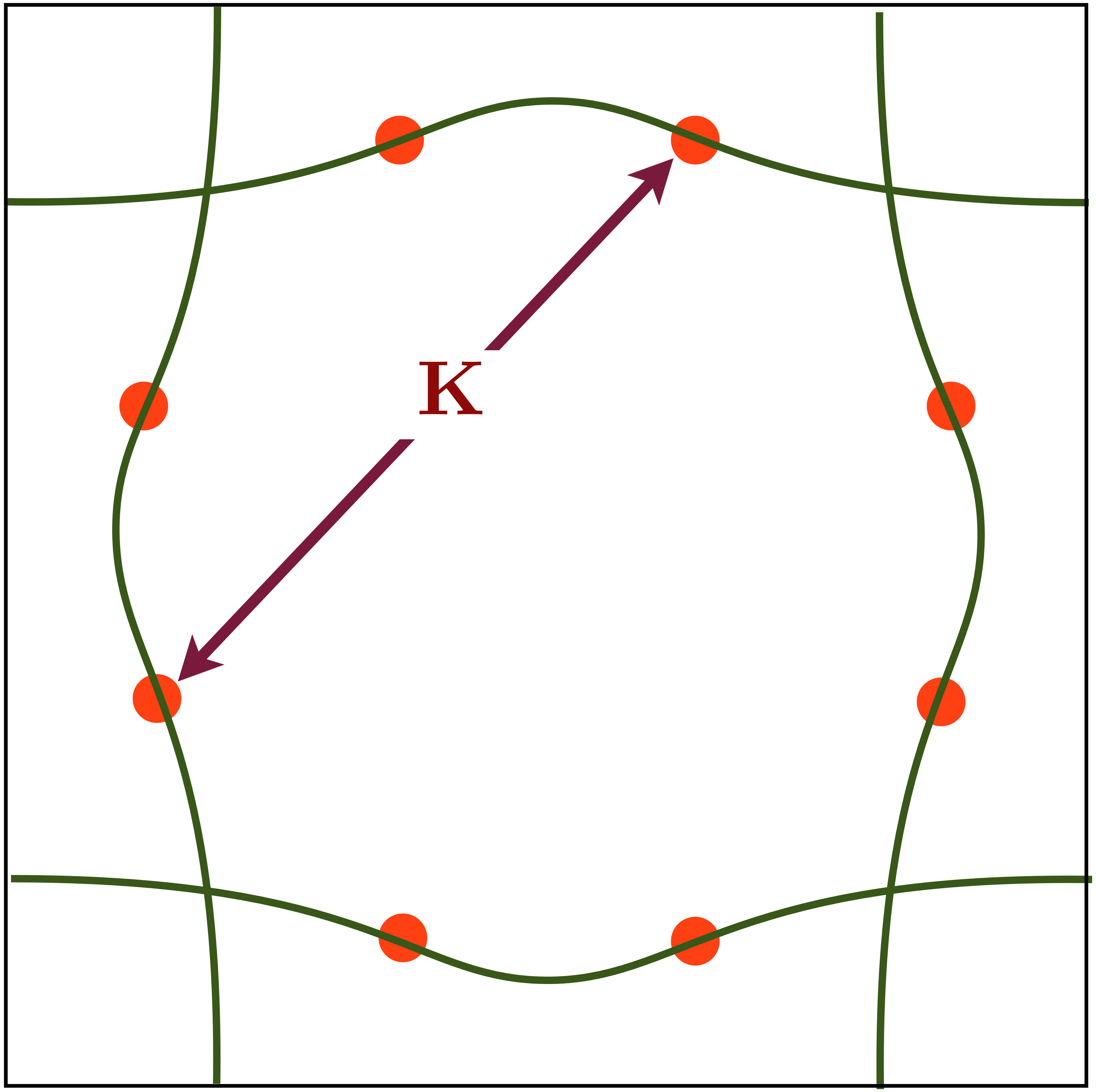
QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Hot spots in a two band model

QMC for the onset of antiferromagnetism



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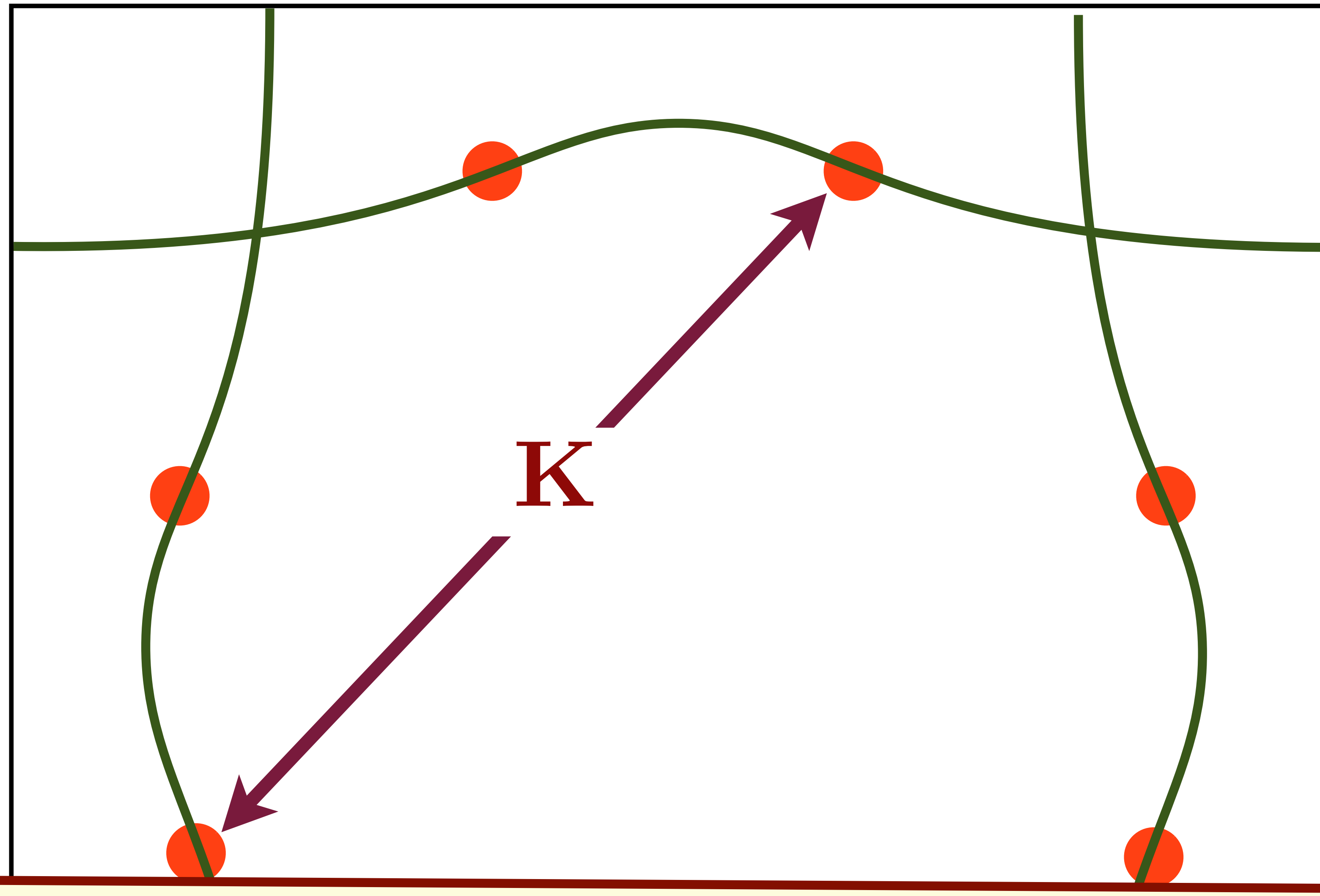

E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

No sign problem in
fermion determinant Monte Carlo !

Determinant is positive because of Kramer's degeneracy, and no additional symmetries are needed; holds for arbitrary band structure and band filling, provided ***K*** only connects hot spots in distinct bands

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

$$+ \int d\tau d^2x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

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E. Berg,
M. Metlitski, and
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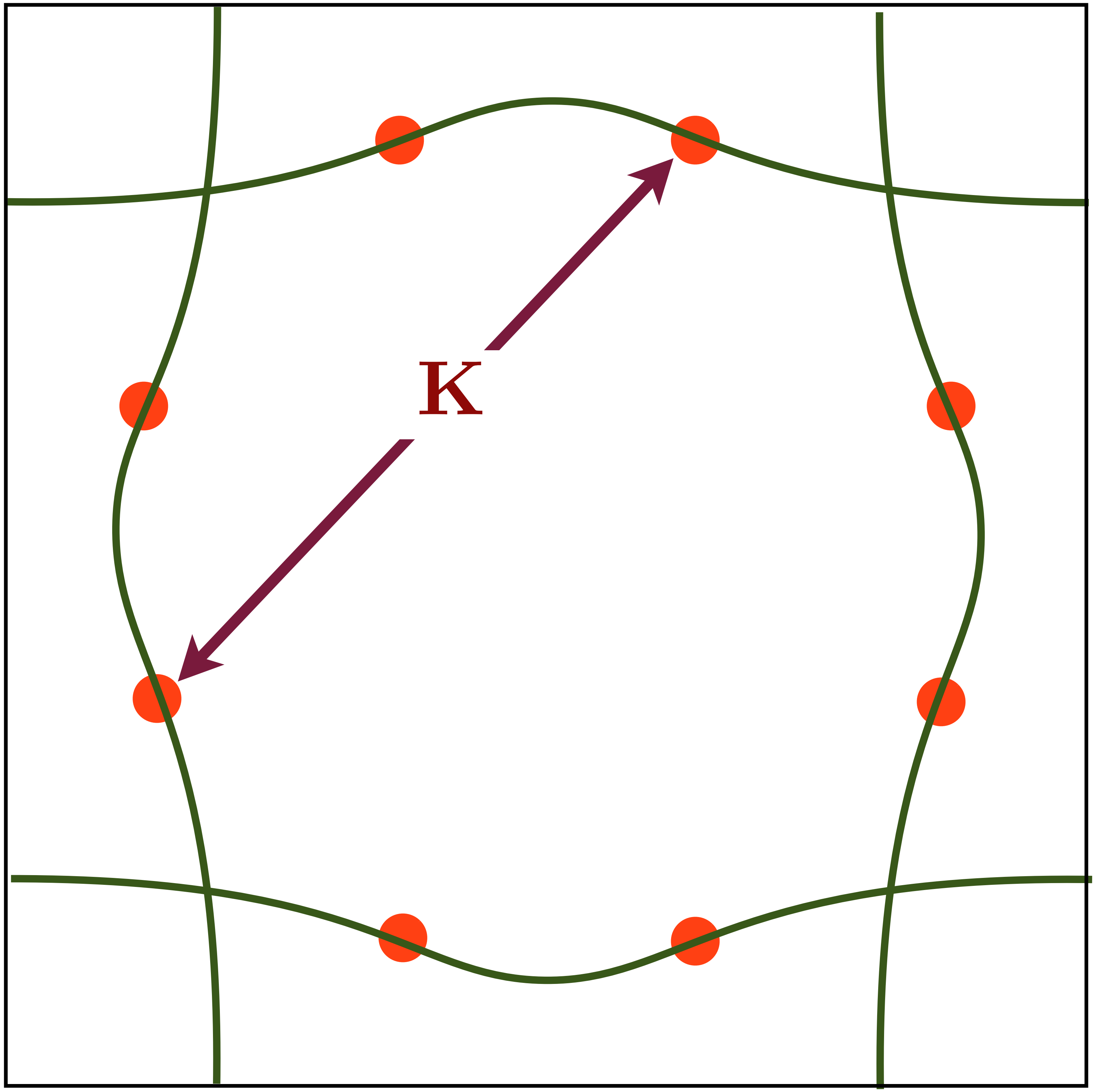
E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

No sign problem !

QMC for the onset of antiferromagnetism

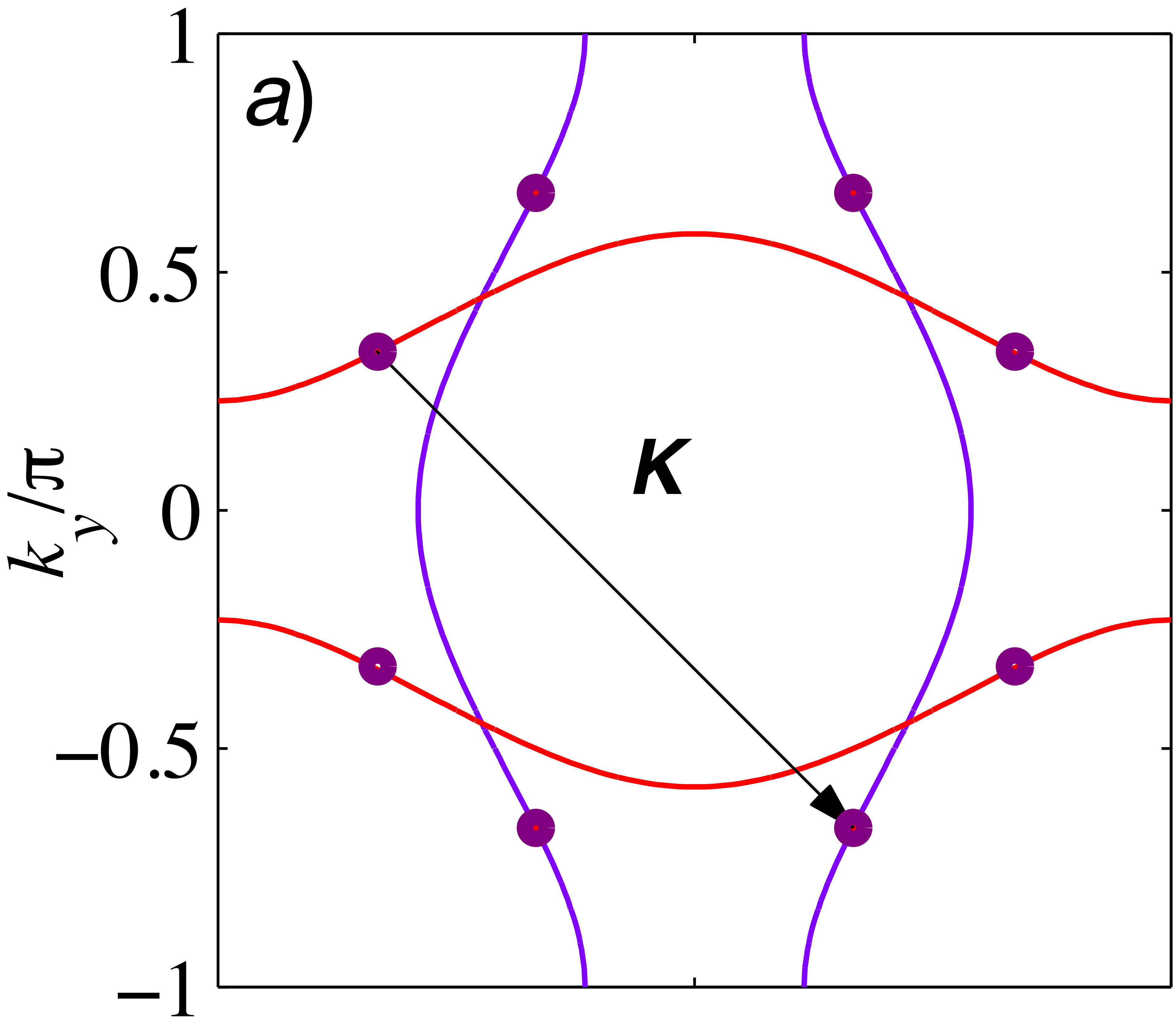


Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,

arXiv:1206.0742

QMC for the onset of antiferromagnetism

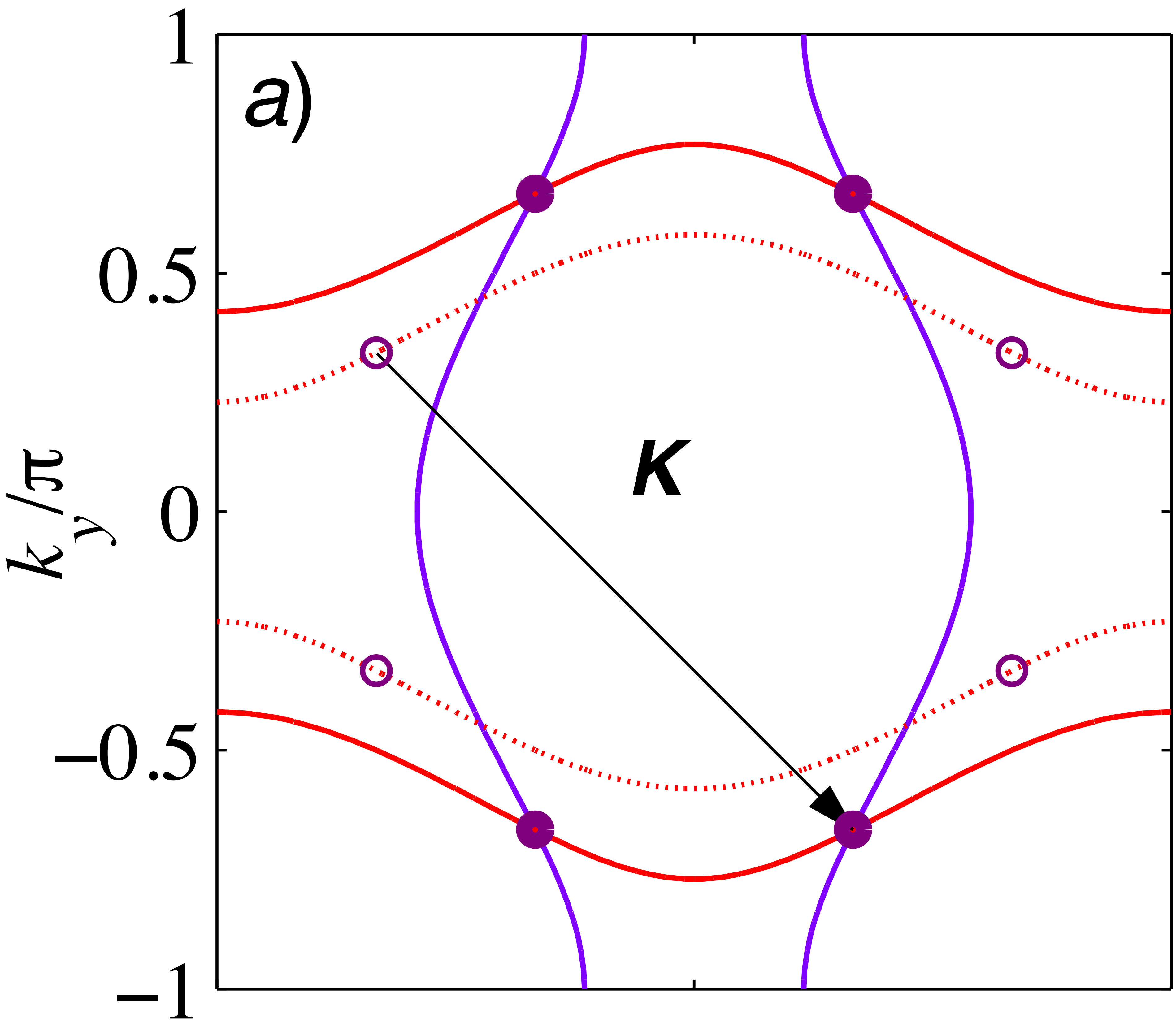


Center Brillouin zone at (π,π,)

E. Berg,
M. Metlitski, and
S. Sachdev,

arXiv:1206.0742

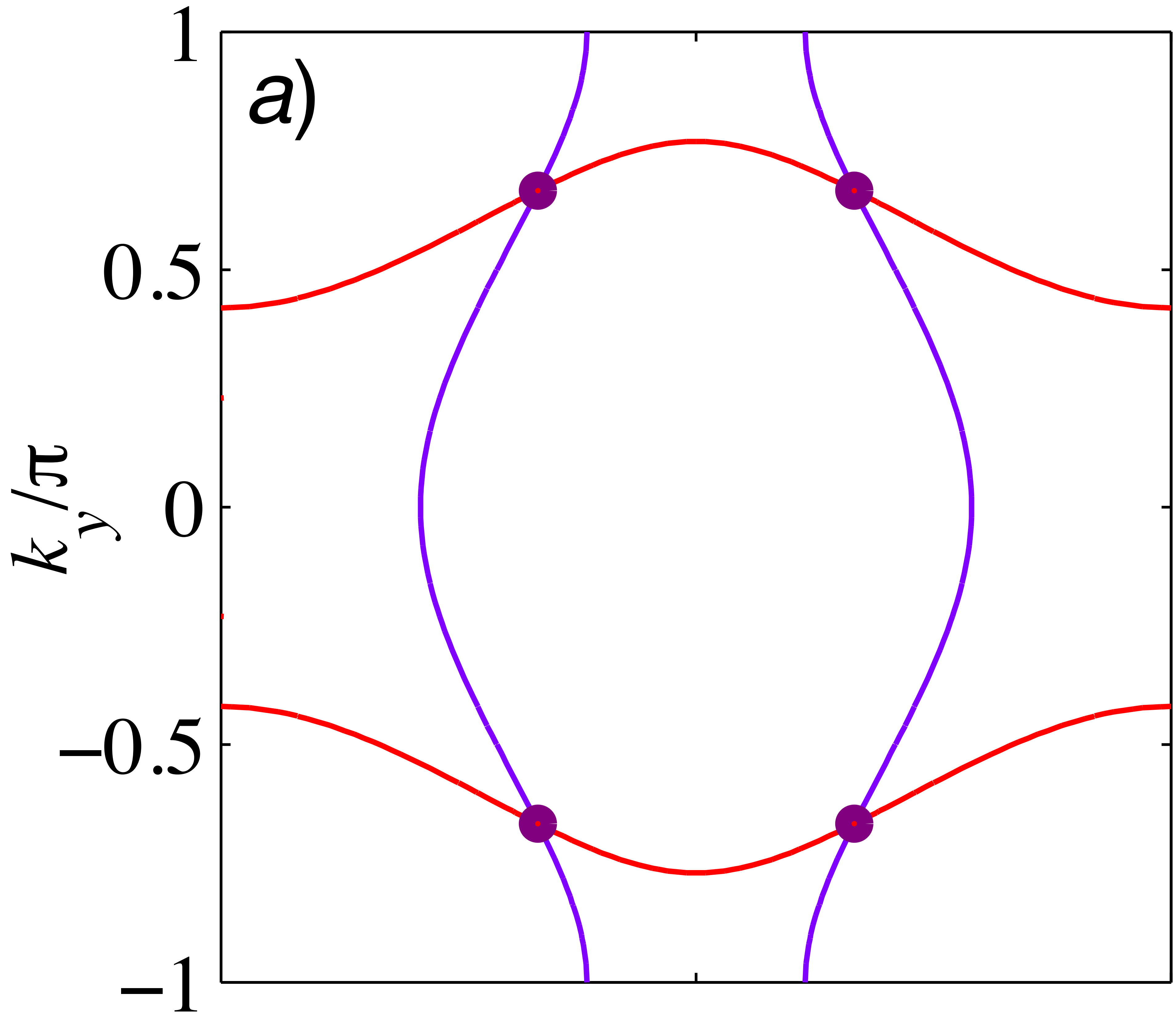
QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Move one of the Fermi surface by (π,π,)

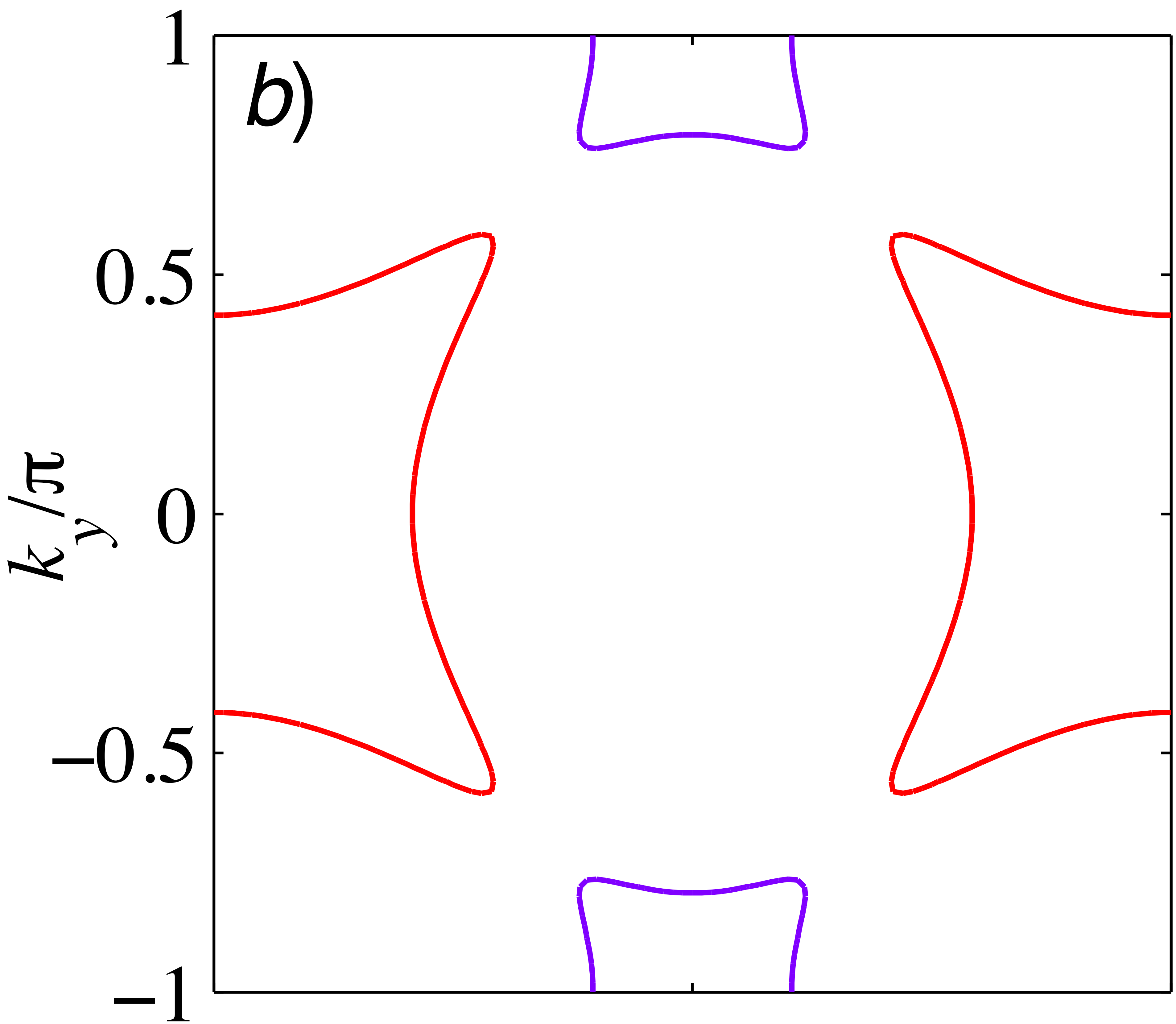
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

Now hot spots are at Fermi surface intersections

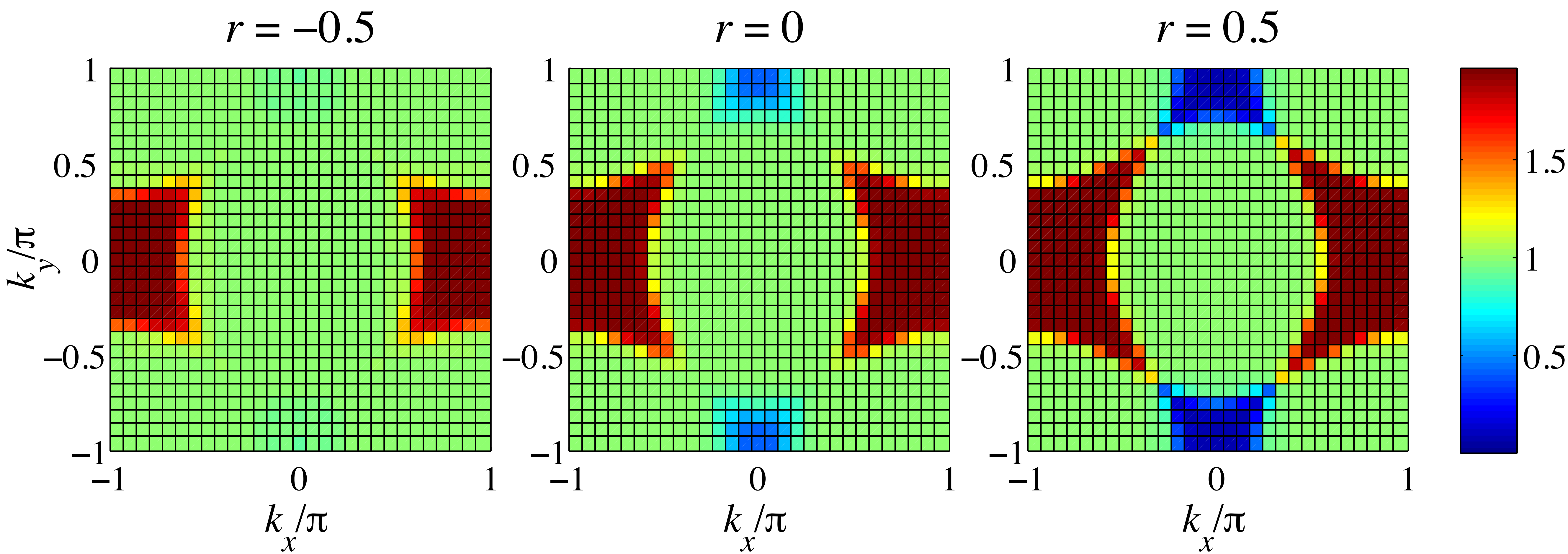
QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Expected Fermi surfaces in the AFM ordered phase

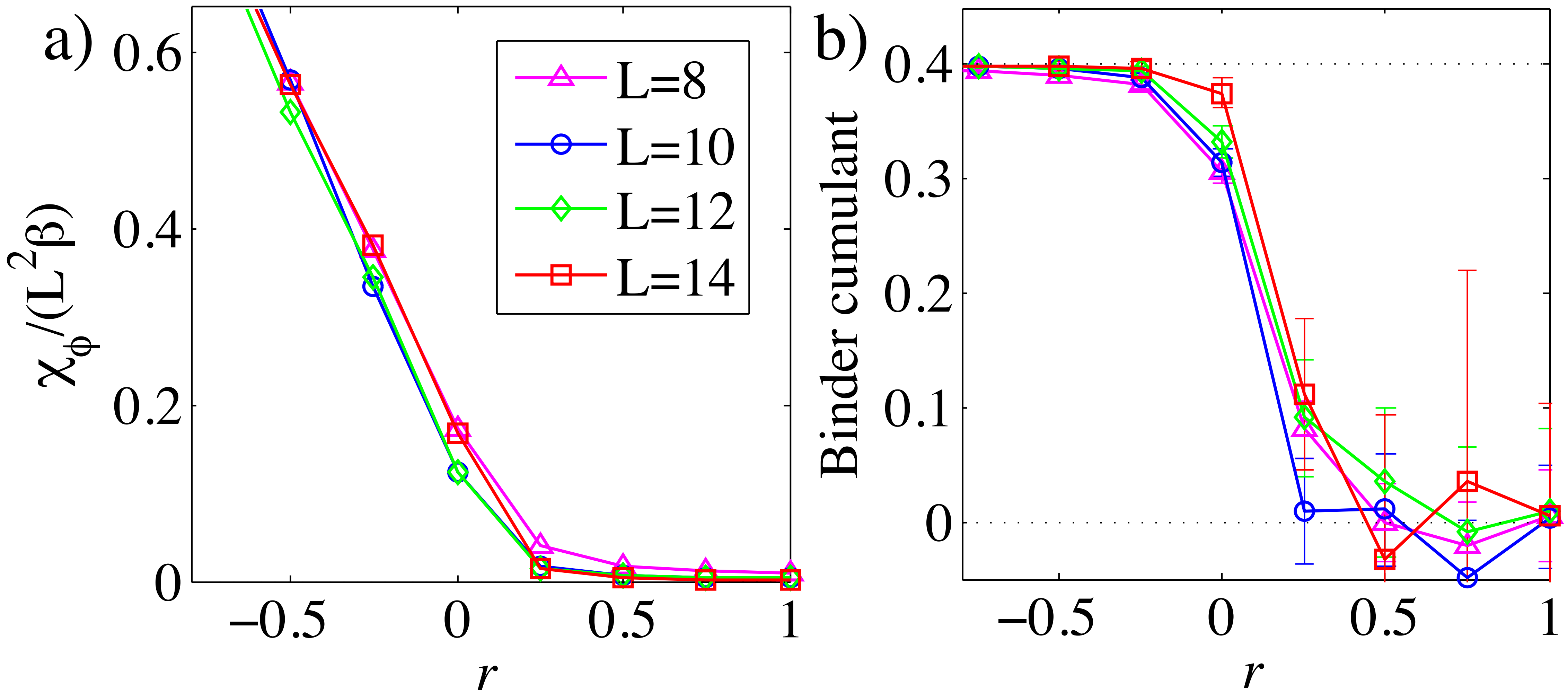
QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

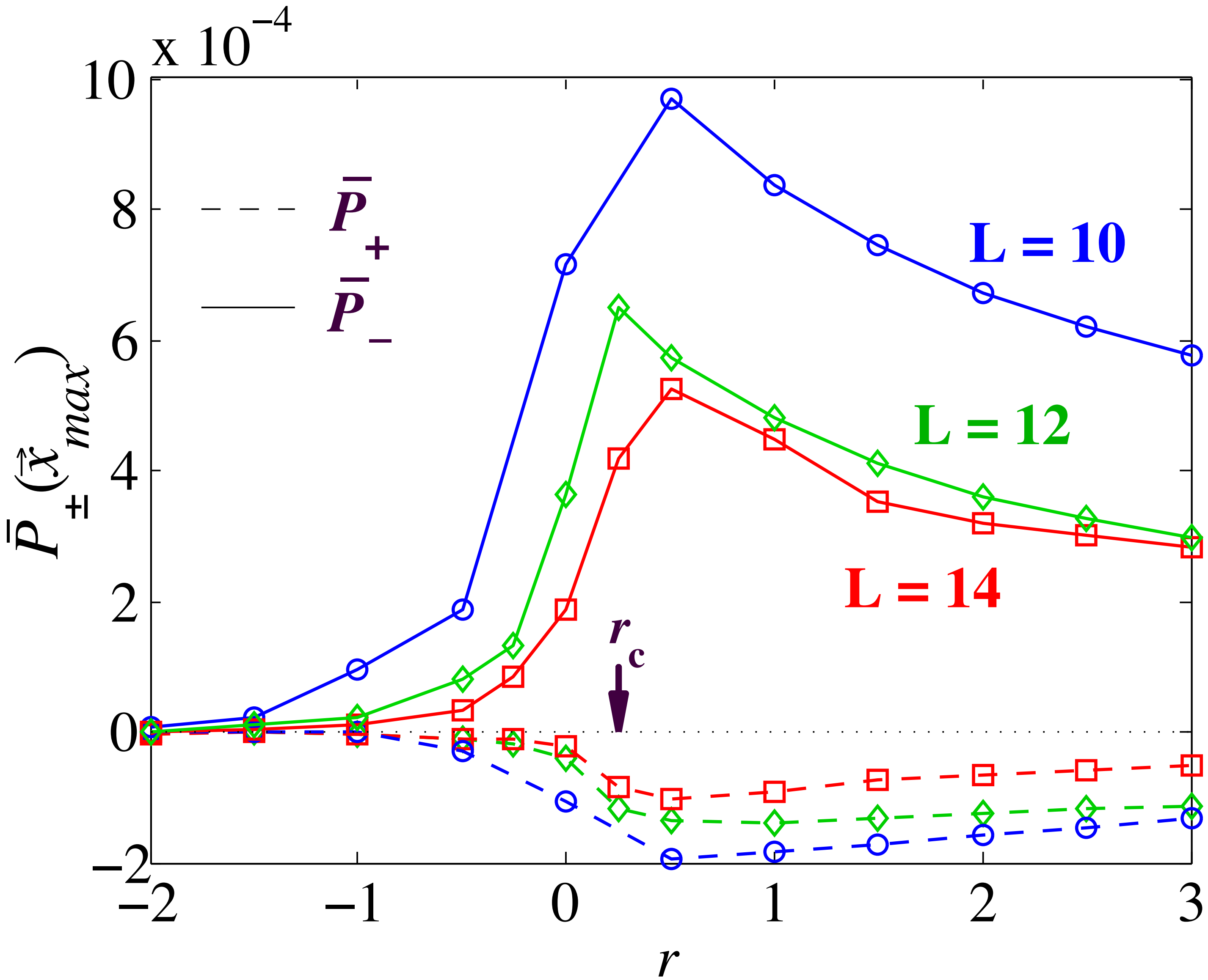
QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

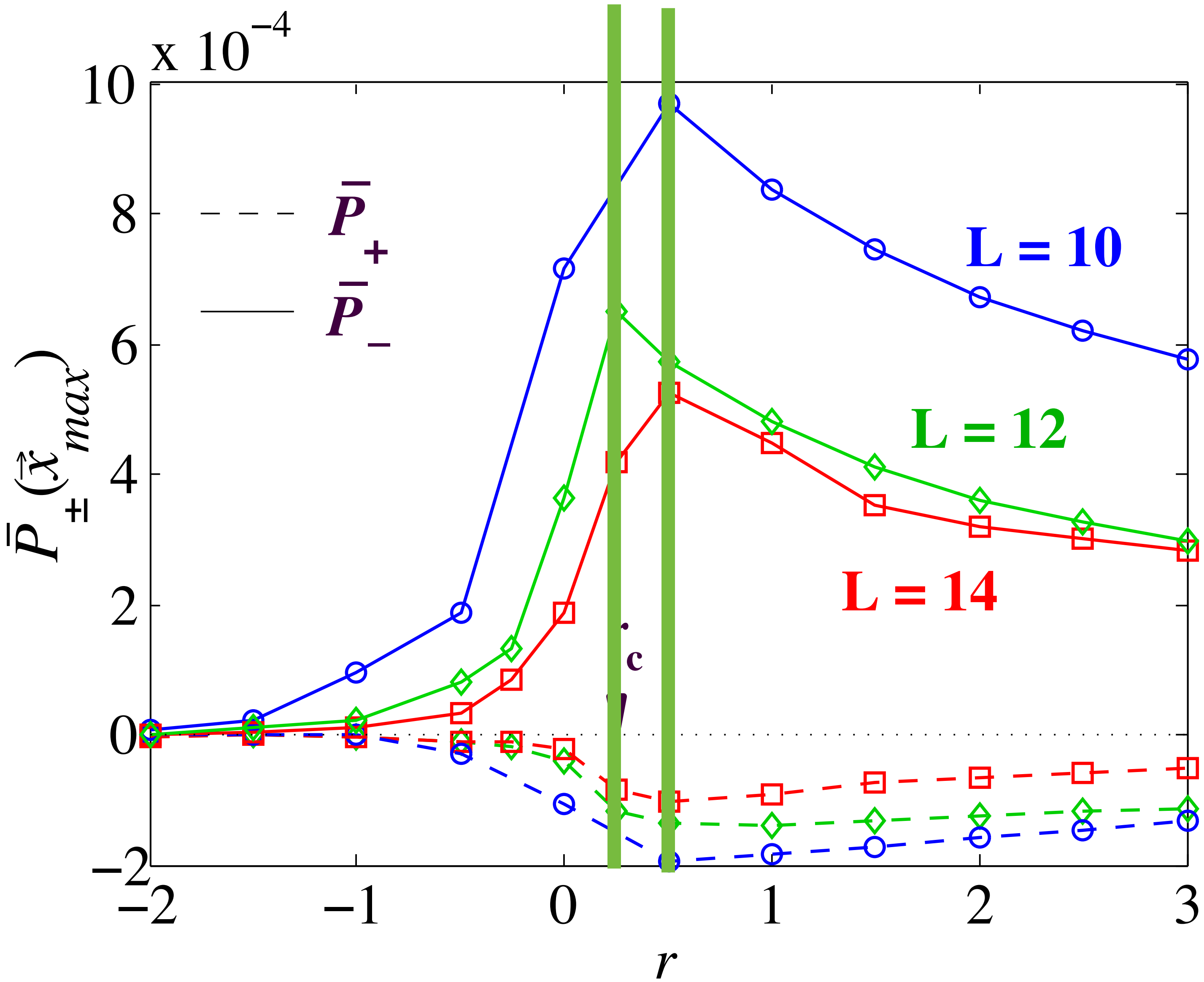
QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

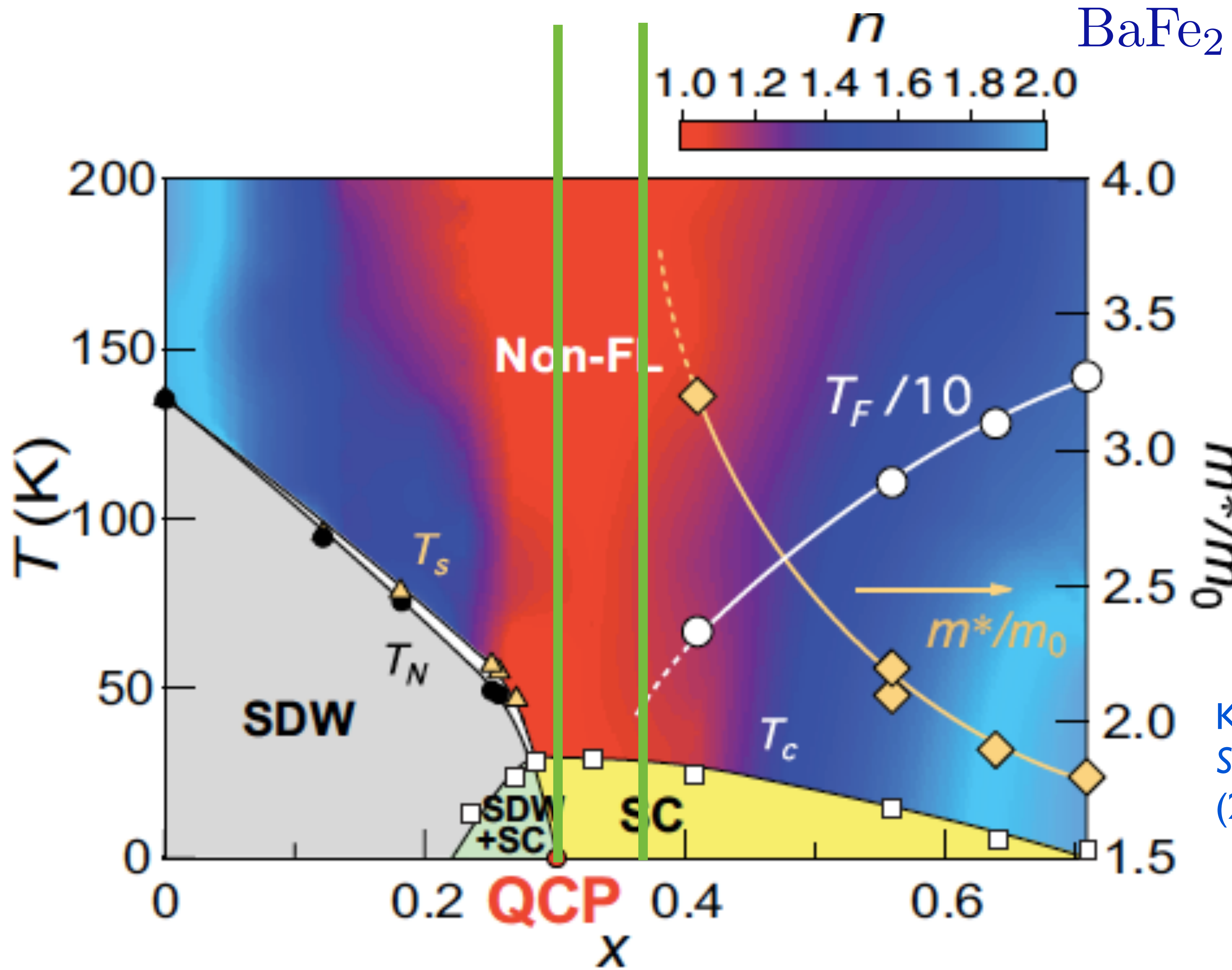
QMC for the onset of antiferromagnetism

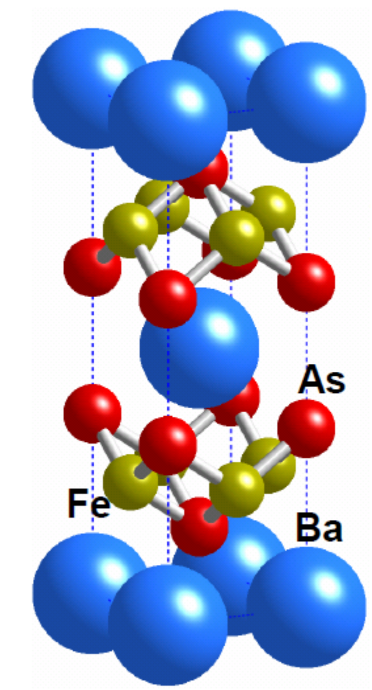


Notice shift between the position of the QCP in the superconductor, and the position of maximum pairing. This was predicted and is found in numerous experiments.

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

$BaFe_2(As_{1-x}\ P_x)_2$





K. Hashimoto *et al.*, *Science* **336**, 1554 (2012).

Notice shift between the position of the QCP in the superconductor, and the divergence in effective mass in the metal measured at high magnetic fields

Outline

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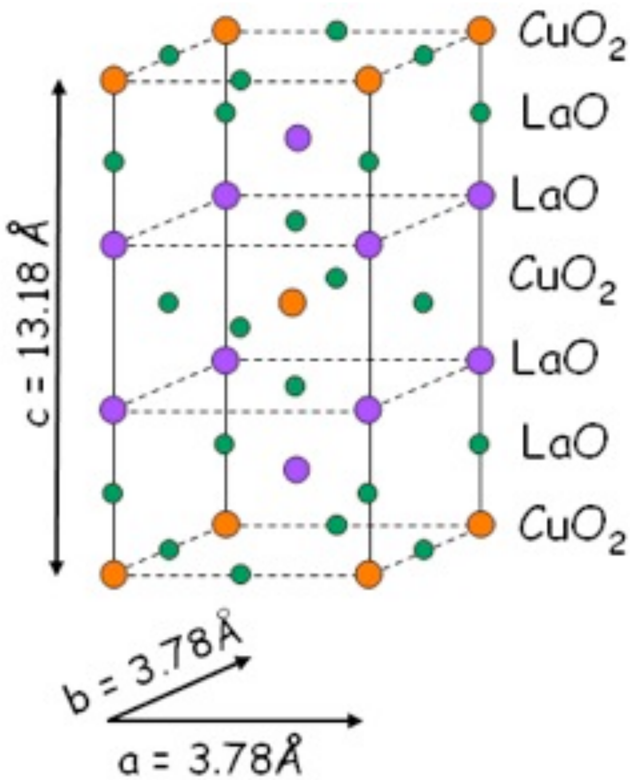
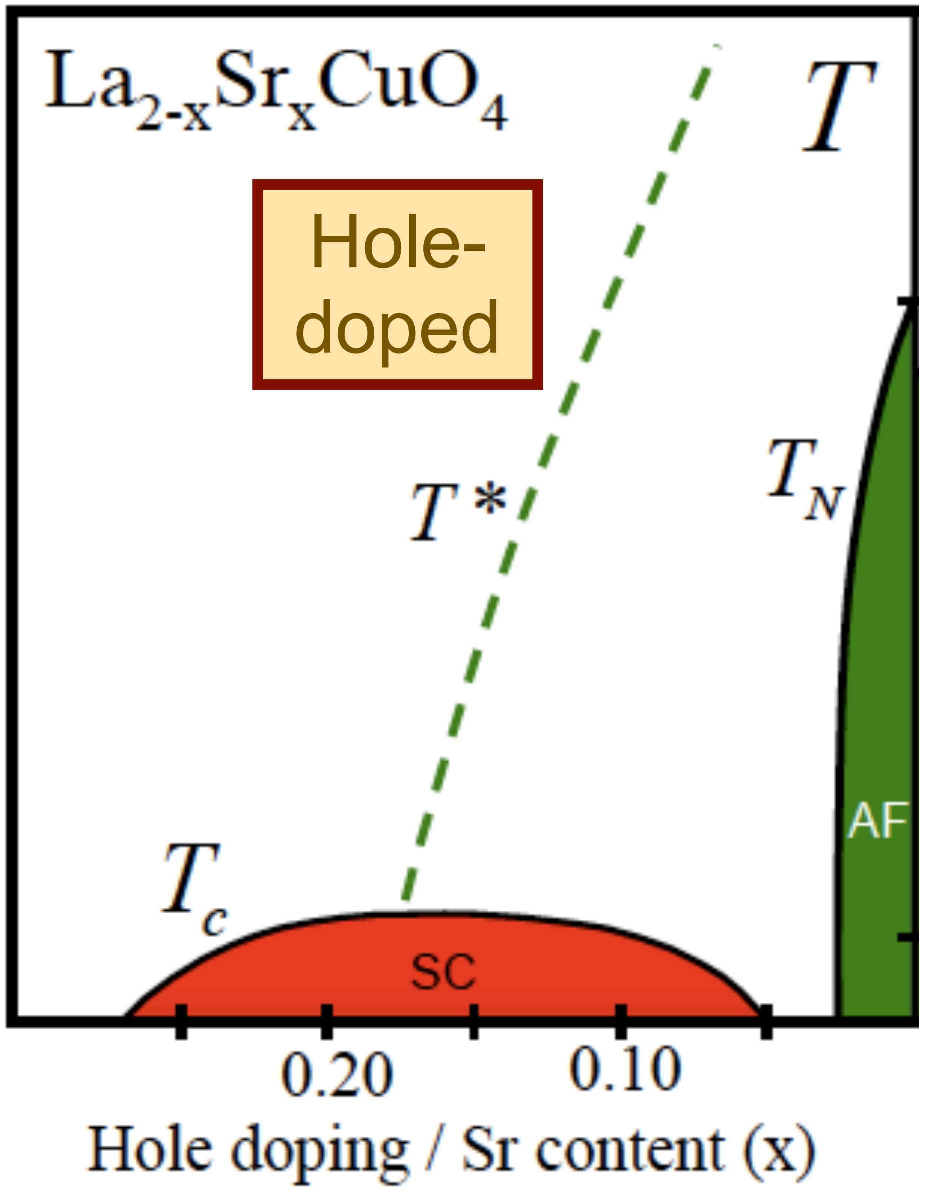
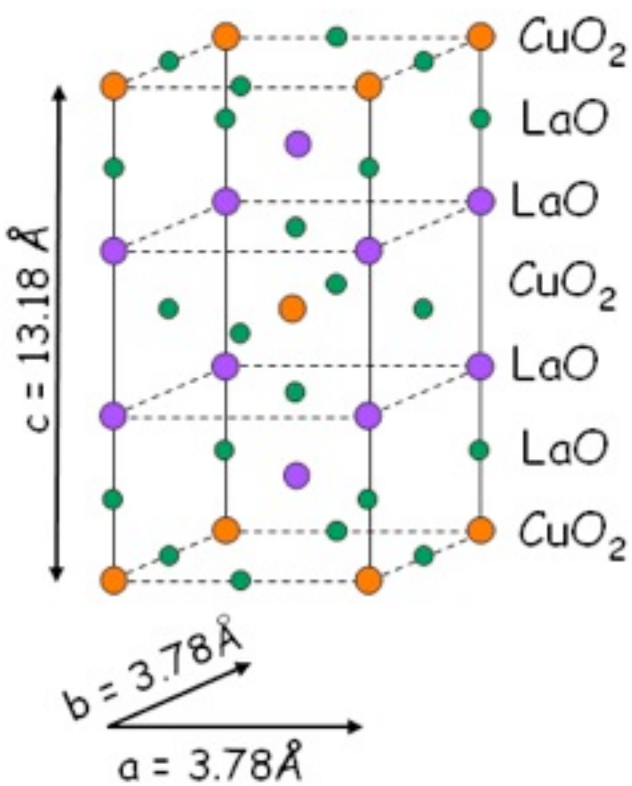

CuO2
LaO
LaO
CuO2
LaO
LaO
CuO2
c = 13.18 Å
b = 3.78Å
a = 3.78Å


La2-xSrxCuO4
Hole-doped
T
T*
TN
AF
Tc
SC
0.20
0.10
Hole doping / Sr content (x)



CuO2
LaO
LaO
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LaO
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c = 13.18 Å
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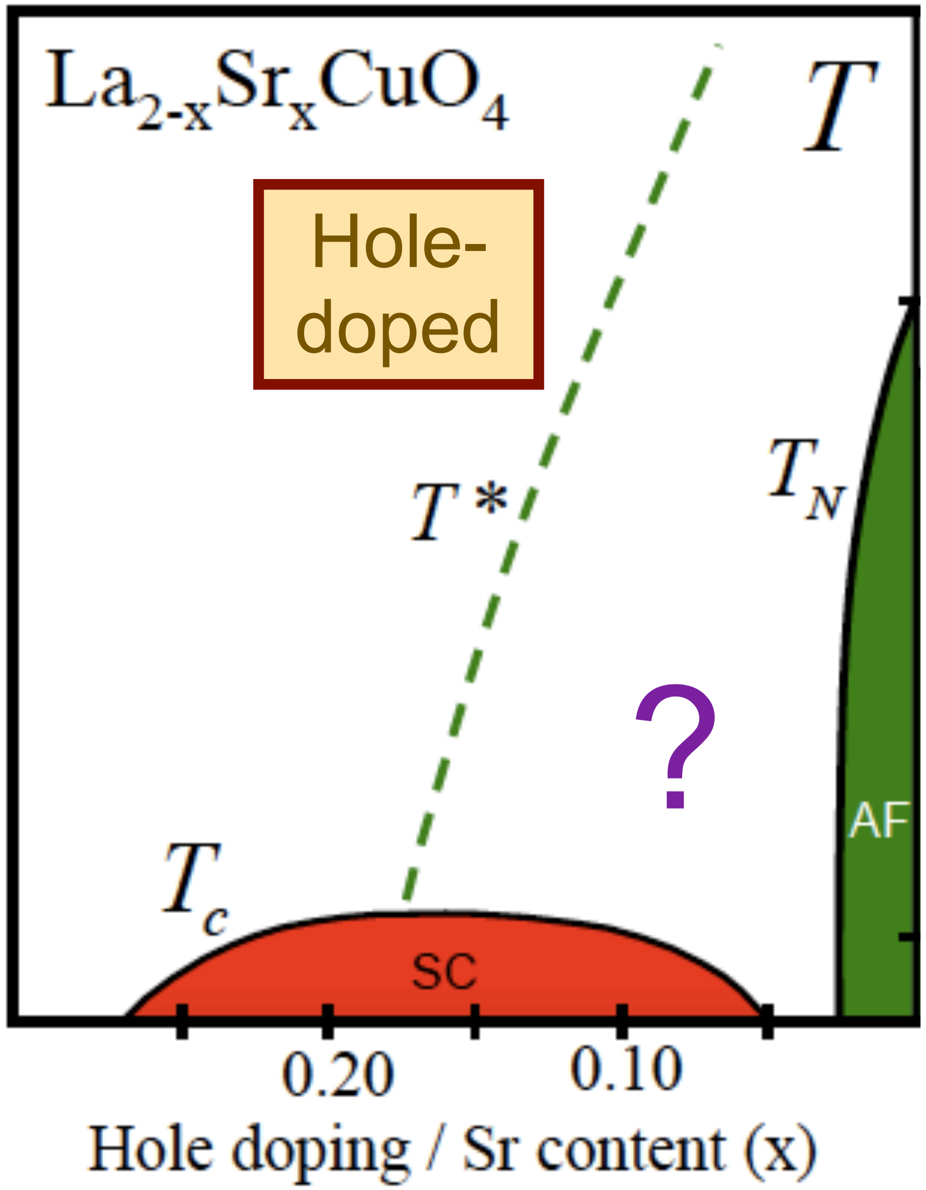
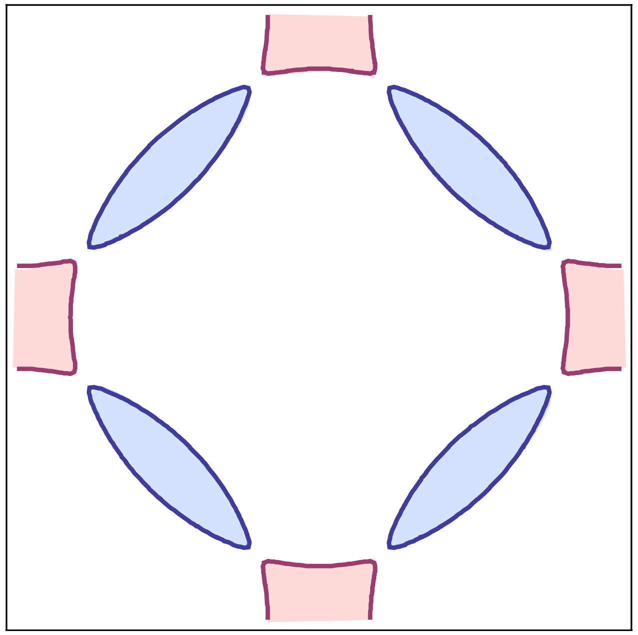


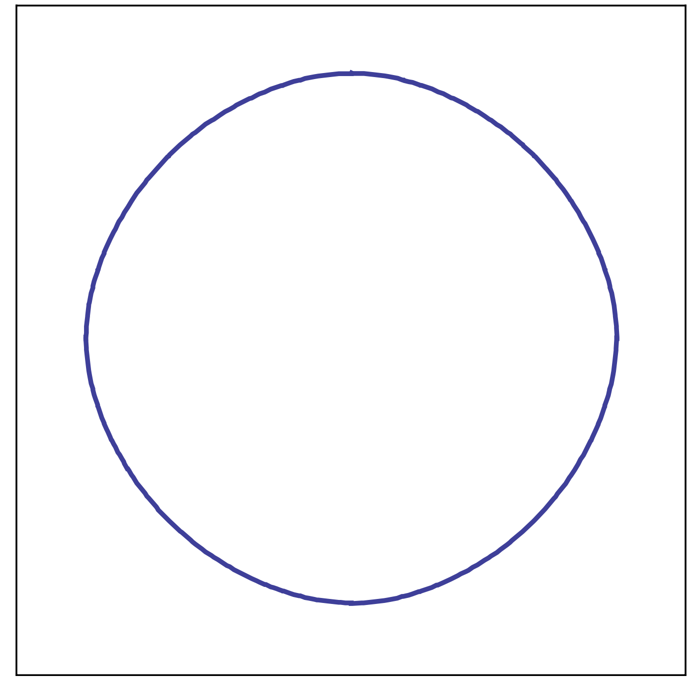
La2-xSrxCuO4
T
Hole-doped
T*
TN
?
AF
Tc
SC
0.20
0.10
Hole doping / Sr content (x)

Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

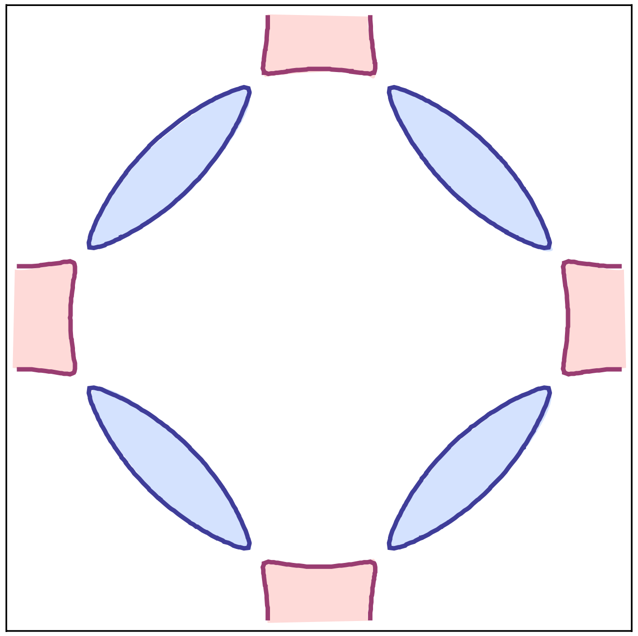
Metal with electron and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

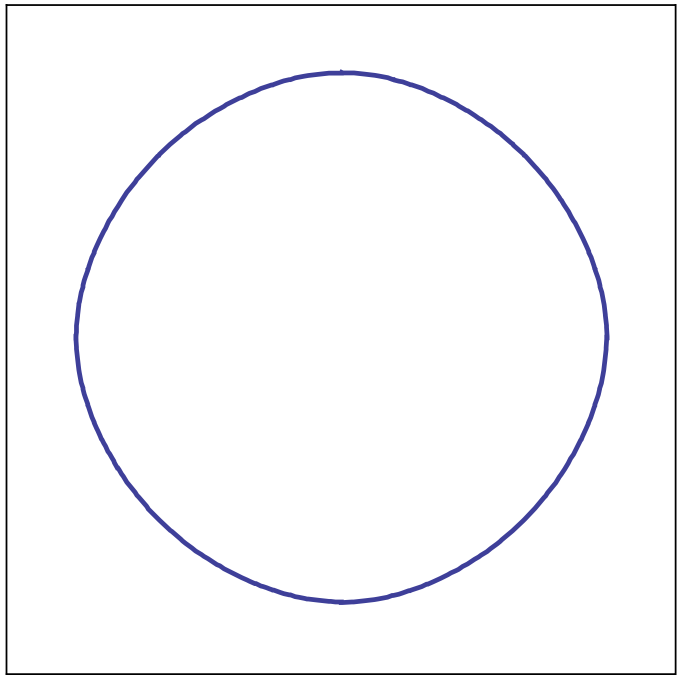
Metal with "large" Fermi surface

Separating onset of SDW order and Fermi surface reconstruction



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

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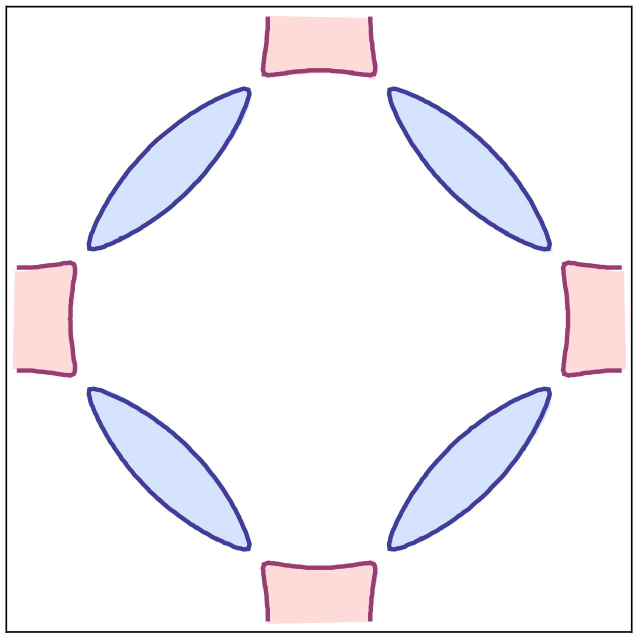
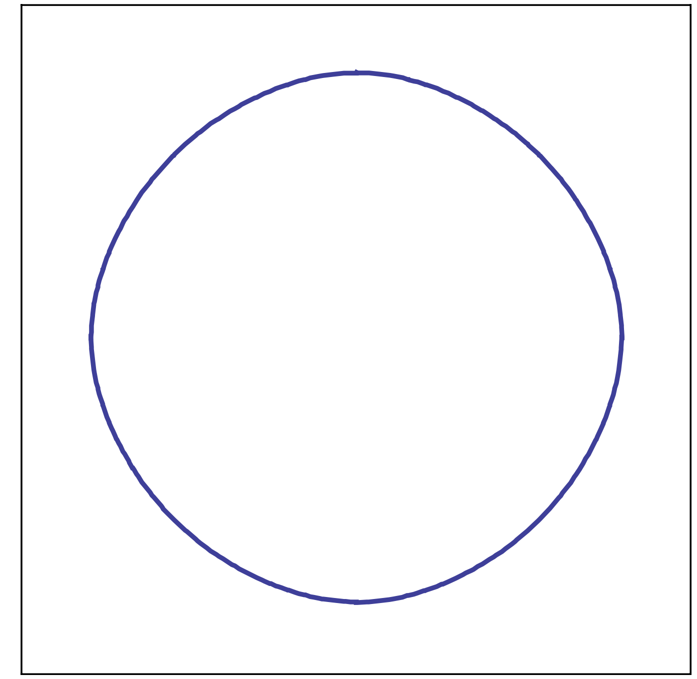


Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order



$$\langle \vec{\varphi} \rangle \neq 0$$

$$\langle \vec{\varphi} \rangle = 0$$

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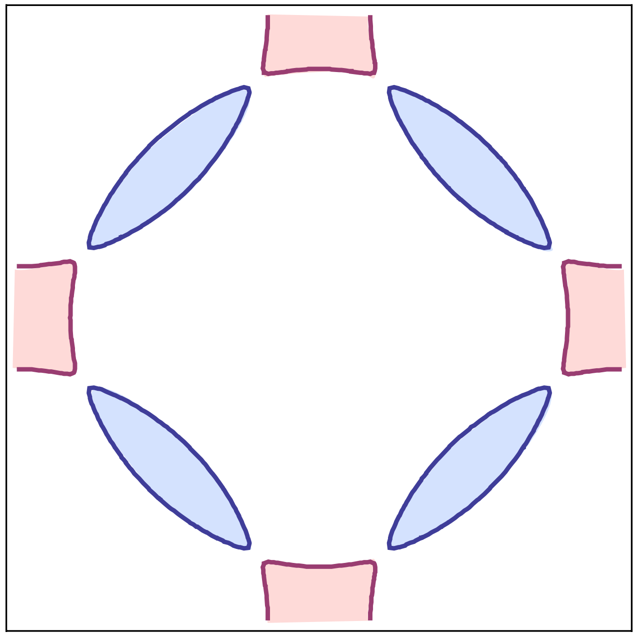
Metal with electron and hole pockets

Metal with "large" Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Separating onset of SDW order and Fermi surface reconstruction



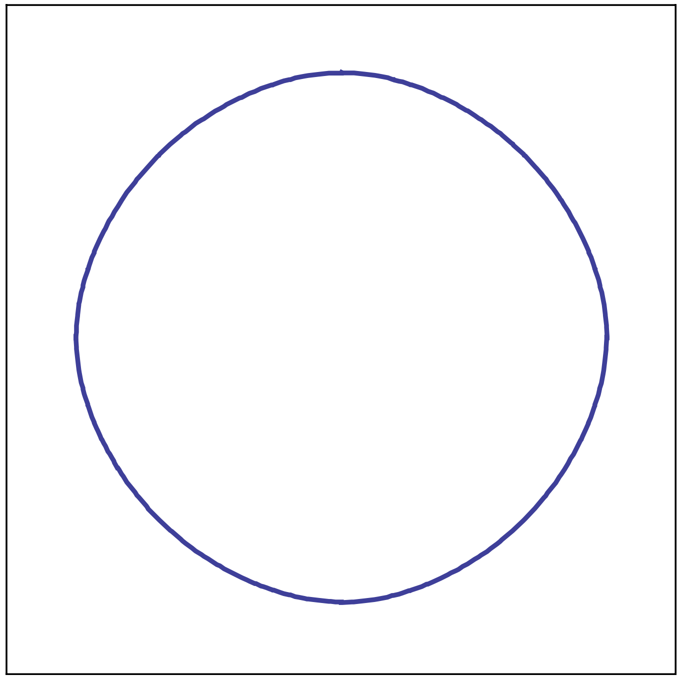
$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

$\langle \vec{\varphi} \rangle = 0$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface



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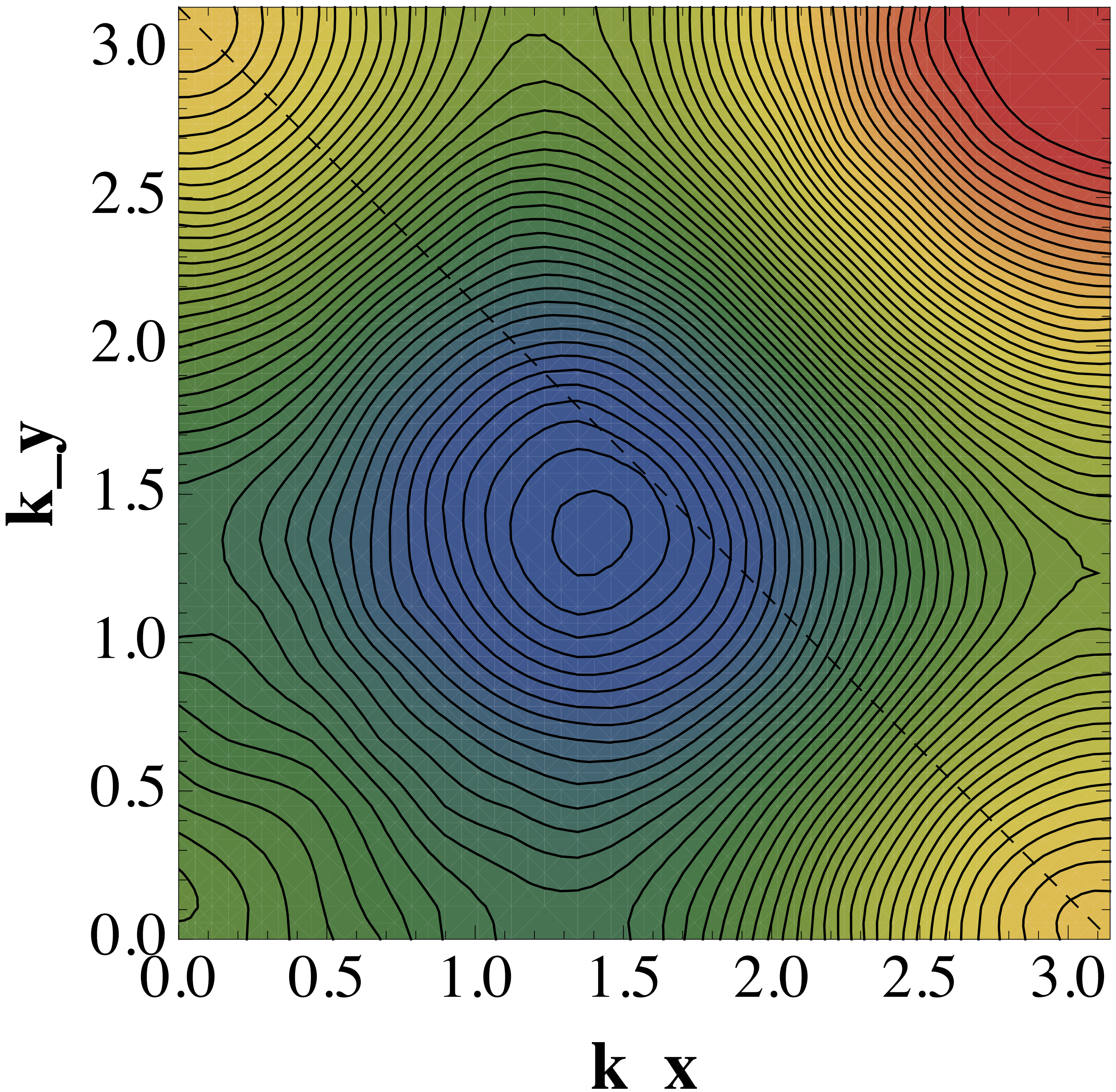
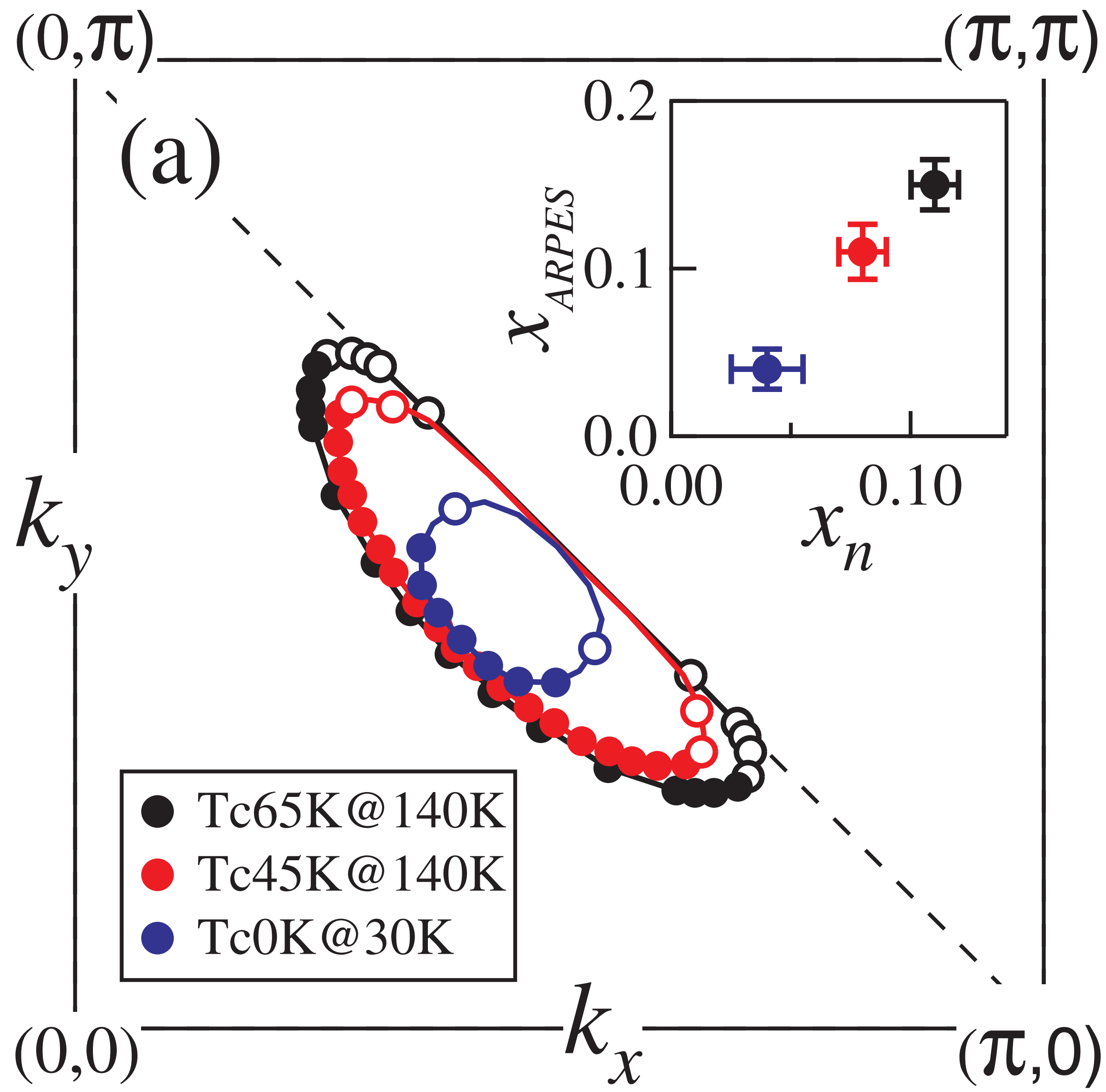
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)



Hole pocket of a $\mathbb{Z}_2$-FL* phase in a *single*-band t-J model

M. Punk and S. Sachdev, *Phys. Rev. B* **85**, 195123 (2012)



PRL **107,** 047003 (2011) PHYSICAL REVIEW LETTERS week ending 22 JULY 2011

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors

H.-B. Yang,[1] J. D. Rameau,[1] Z.-H. Pan,[1] G. D. Gu,[1] P. D. Johnson,[1] H. Claus,[2] D. G. Hinks,[2] and T. E. Kidd[3]

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.
- Such a phase *must* have neutral $S = 1/2$ excitations ("spinons"), and collective spinless gauge excitations ("topological" order).
- These topological excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa's proof of the Luttinger theorem.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Questions

- *Can quantum fluctuations near the onset of antiferromagnetism induce higher temperature superconductivity ?*

- *How should such a theory be extended to apply to the hole-doped cuprates ?*

- *What is the physics of the strange metal ?*

Questions and Answers

- *Can quantum fluctuations near the onset of antiferromagnetism induce higher temperature superconductivity ?*

  Yes; convincing evidence from field theory and sign-problem free quantum Monte Carlo

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  The QCP shift from the metal to the superconductor is large. New physics (charge order, fractionalization...) is likely present in the intermediate regime

- *What is the physics of the strange metal ?*

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- *What is the physics of the strange metal ?*

  Strongly-coupled quantum criticality of Fermi surface change in a metal