Quantum "disordering" magnetic order in insulators, metals, and superconductors

Perimeter Institute, Waterloo, May 29, 2010

Talk online: sachdev.physics.harvard.edu

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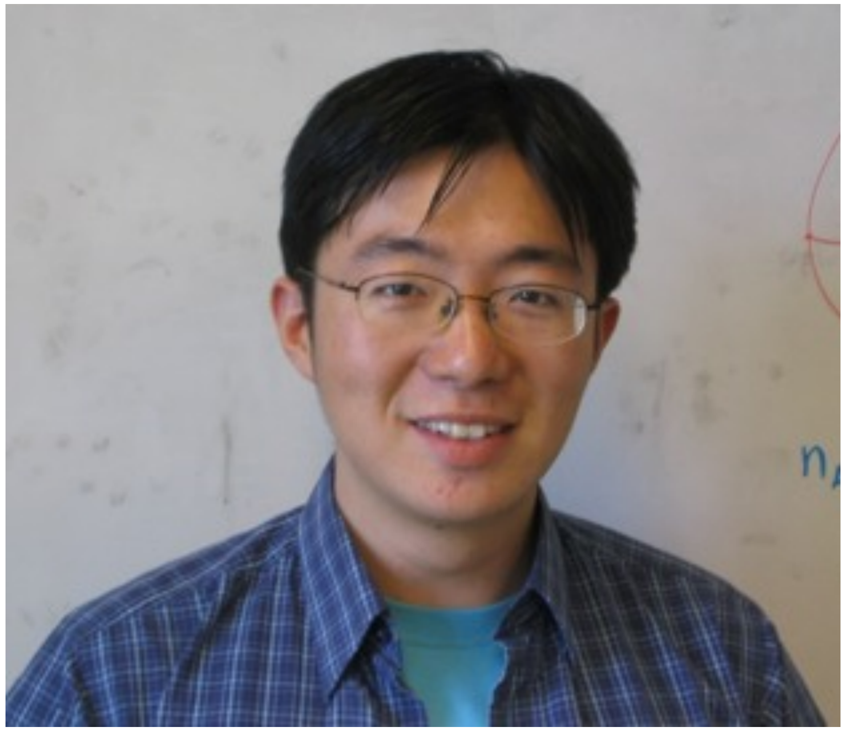
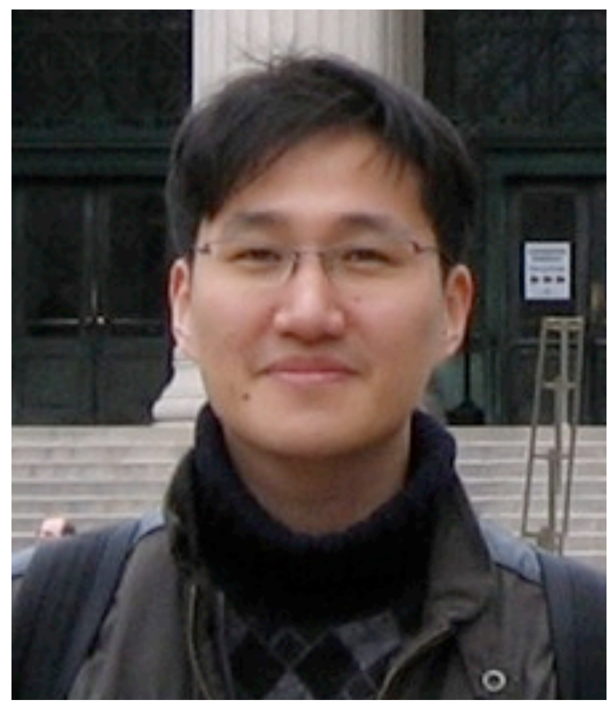


Cenke Xu, Harvard
arXiv:1004.5431

Max Metlitski, Harvard
arXiv:1005.1288



Eun Gook Moon, Harvard
arXiv:1005.3312

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Outline

1. Quantum “disordering” magnetic order in two-dimensional antiferromagnets

   Topological defects and their Berry phases

2. Unified theory of spin liquids

   Majorana liquids

3. Loss of magnetic order in a metal

   d-wave pairing and (modulated) Ising-nematic order

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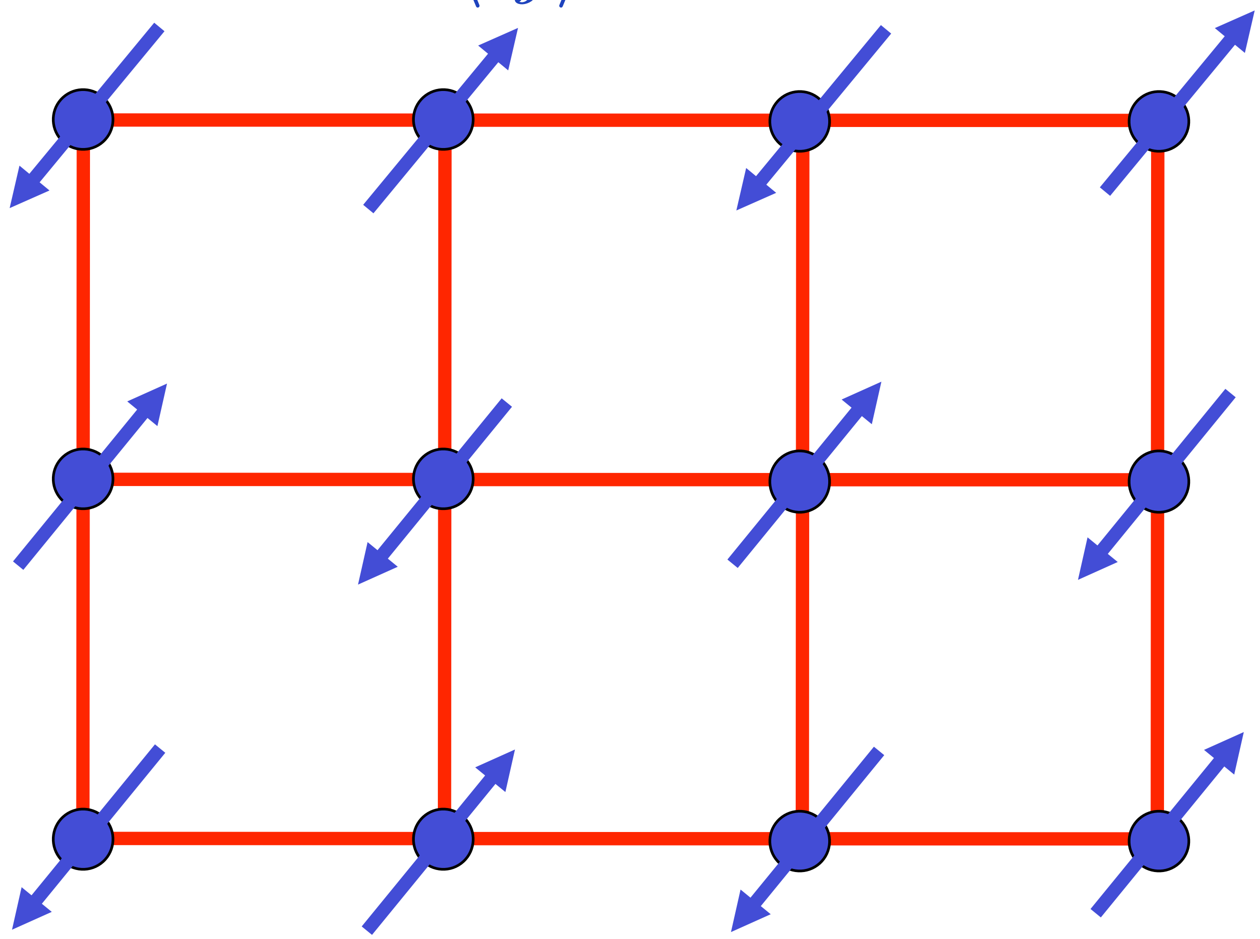
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \cdots + \cdots$$

Add perturbations so ground state no longer has long-range Néel order

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Add perturbations so ground state no longer has long-range Néel order

Describe the resulting state by an effective theory of fluctuations of the Néel order:

$$\mathcal{R}_z(x,\tau) \left| \text{Néel} \right\rangle$$

where R is a SU(2) spin rotation matrix related to the Néel order

$$\mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \quad ; \quad \vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

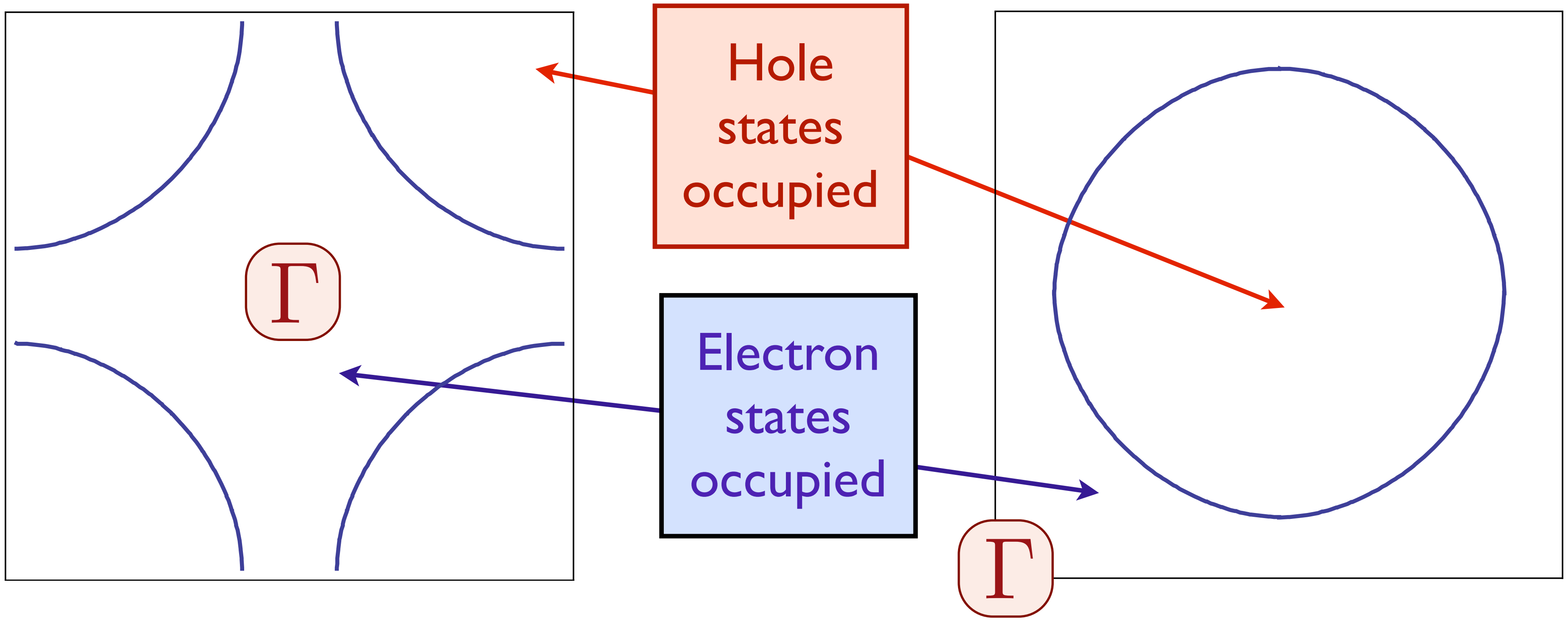
Ginzburg-Landau paradigm: Effective action for $\mathcal{R}_z(x,\tau)$, deduced by symmetries, describes quantum transitions and phases "near" the Néel state.

Order parameter description is incomplete

Underlying electrons cannot be ignored even though charged excitations are fully gapped.

They endow topological defects in the order parameter (hedgehogs, vortices...) with Berry phases: the defects acquire additional degeneracies and transform non-trivially under lattice space group *e.g.* with non-zero crystal momentum

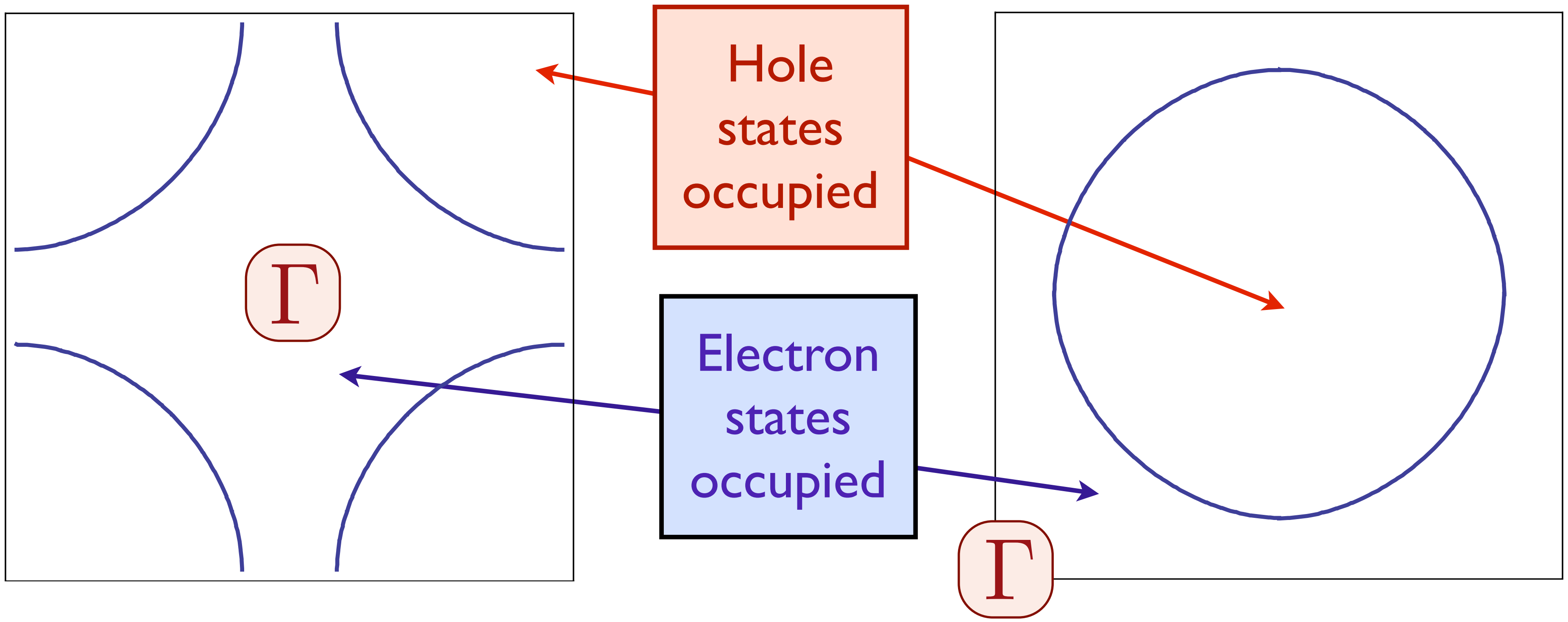
Metals (in the cuprates)

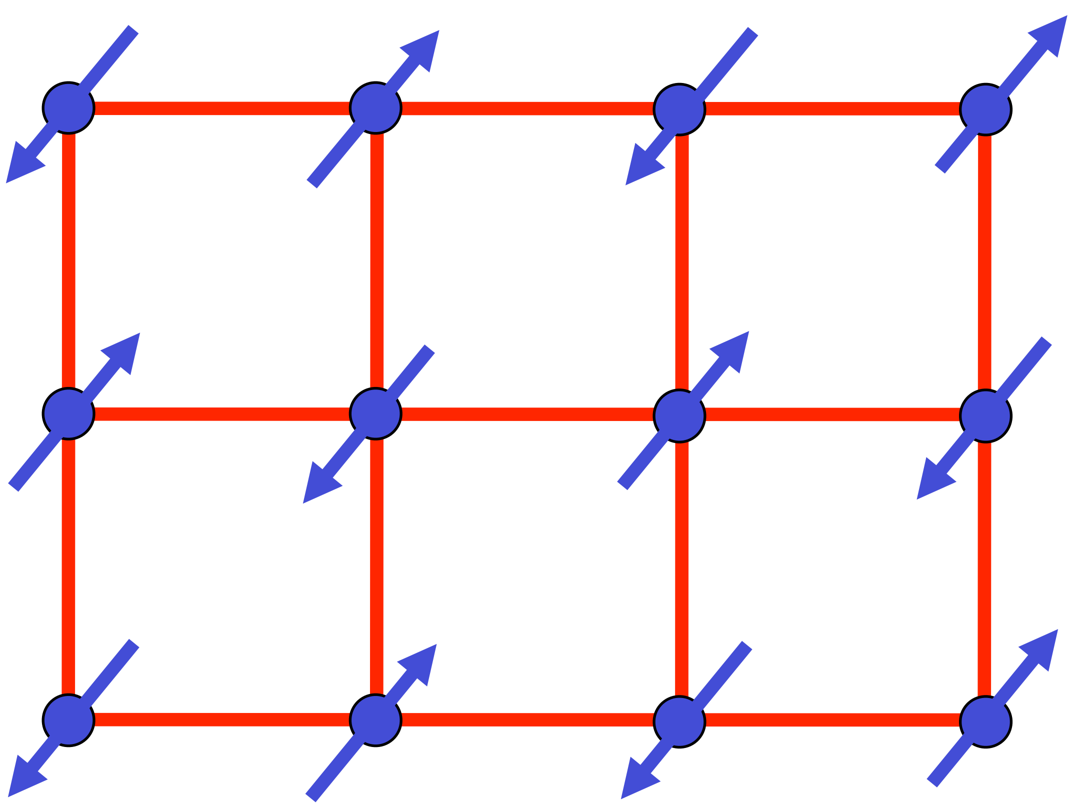


$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

- Begin with free electrons.

Fermi surface+antiferromagnetism





The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

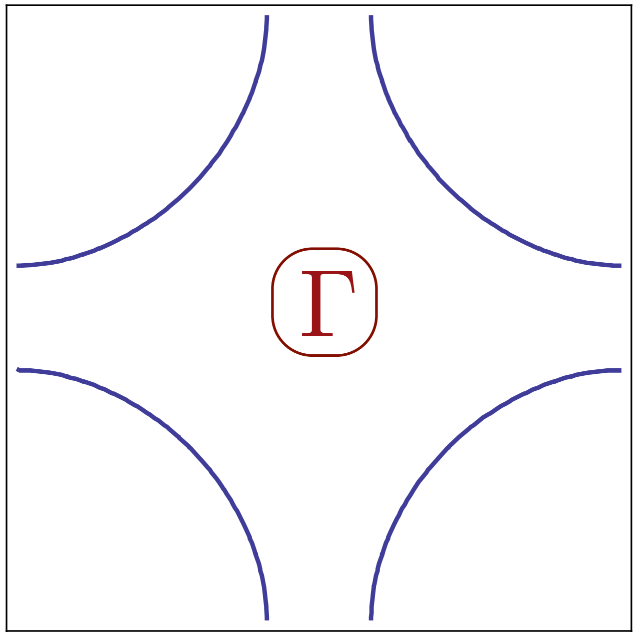
where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\rm sdw}$ for $\vec{\varphi} \propto (0, 0, 1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

This leads to the Fermi surfaces shown in the following slides for half-filling

Half-filled band

Increasing SDW order



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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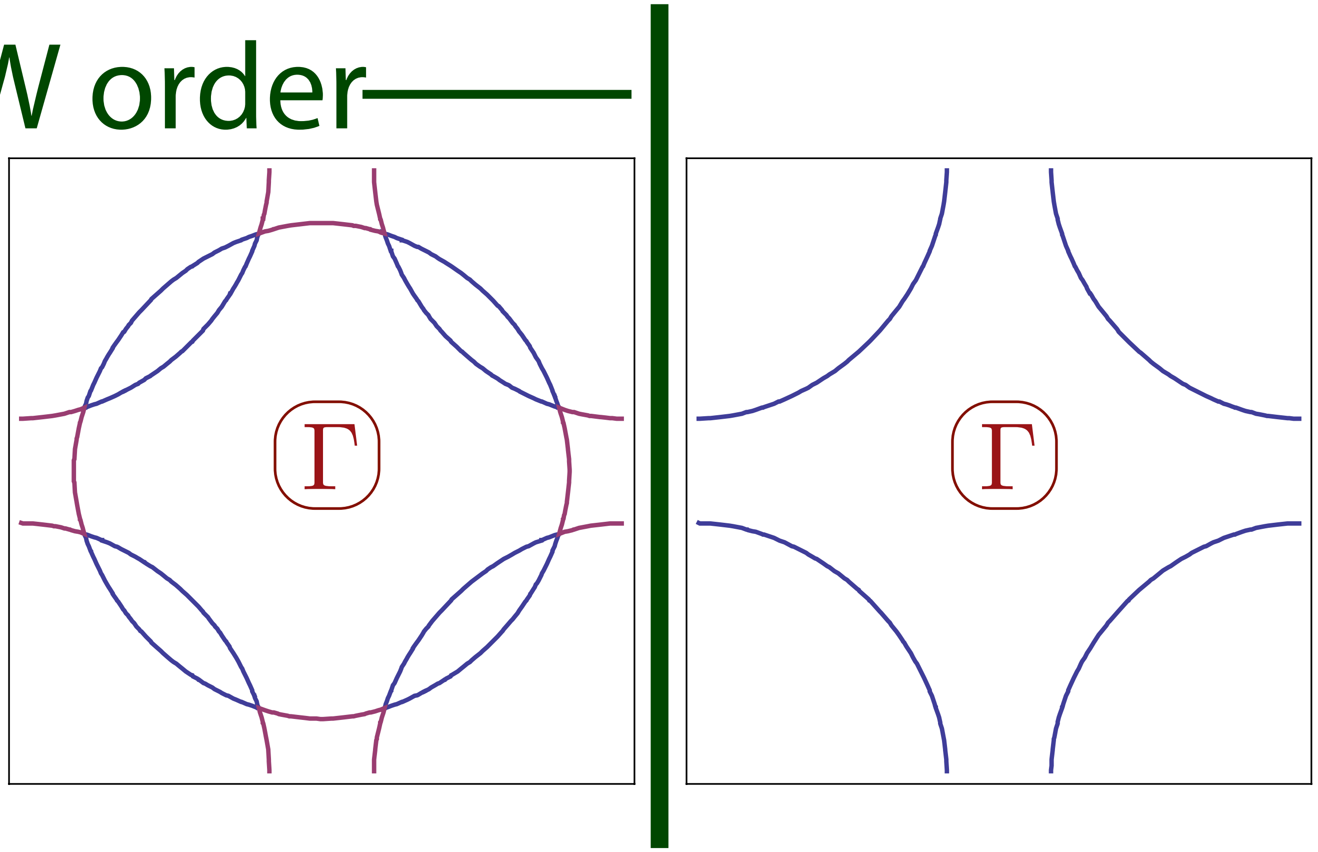

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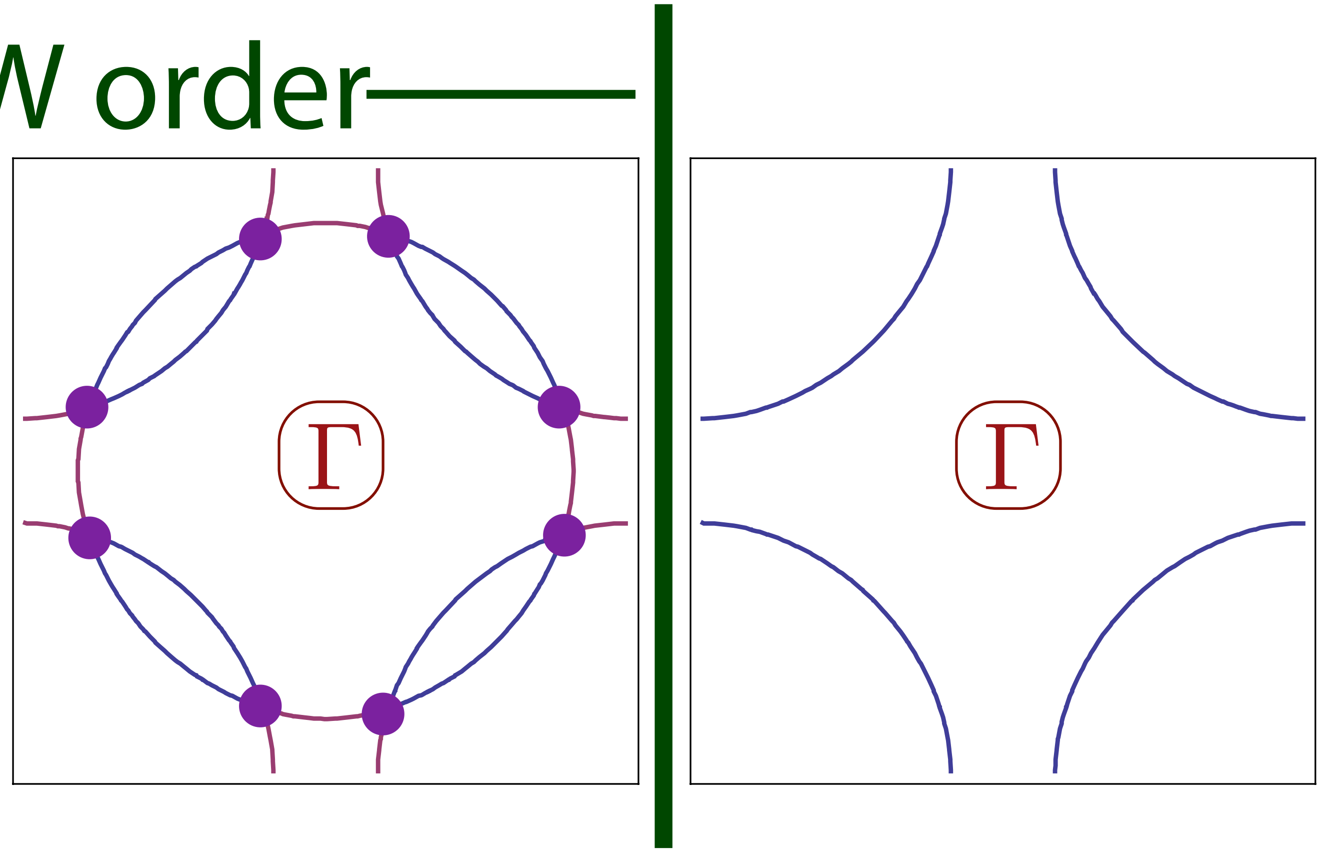

Hot spots

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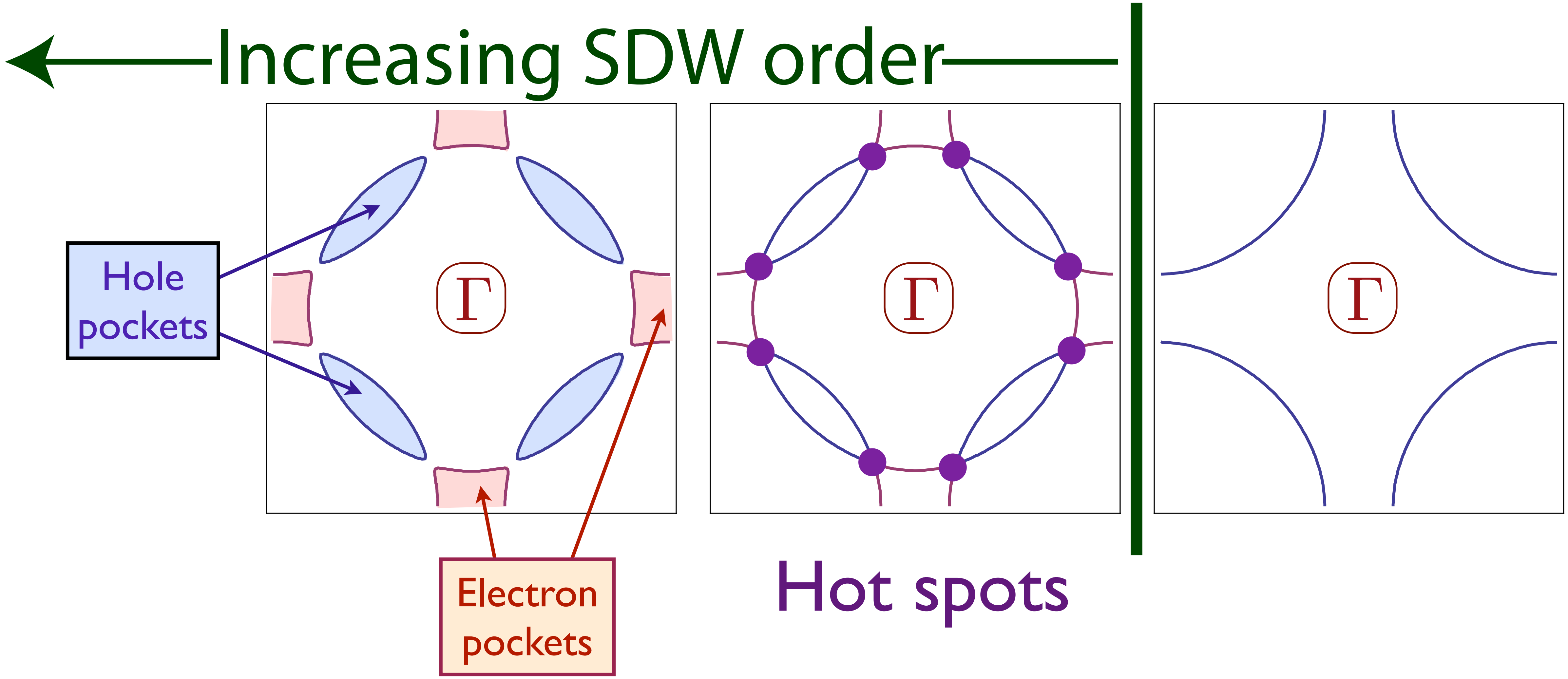

Fermi surface breaks up at hot spots
into electron and hole "pockets"

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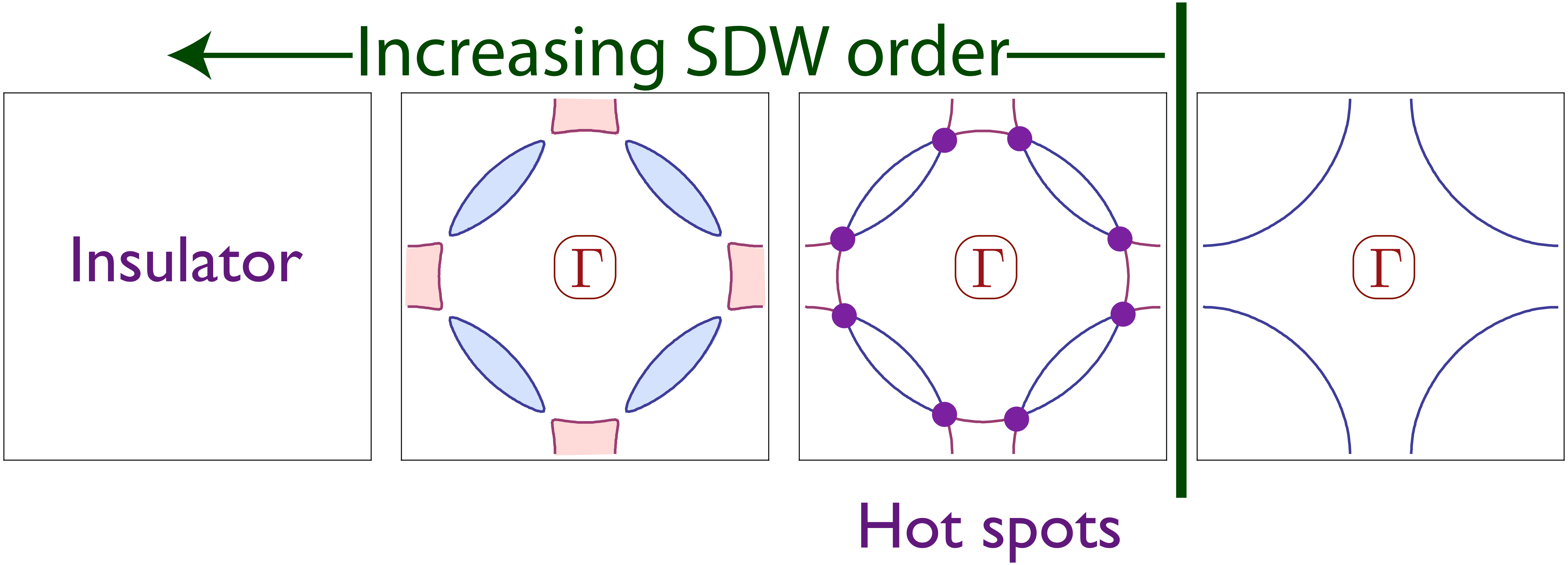

Insulator with Neel order has electrons filling a band, and no Fermi surface

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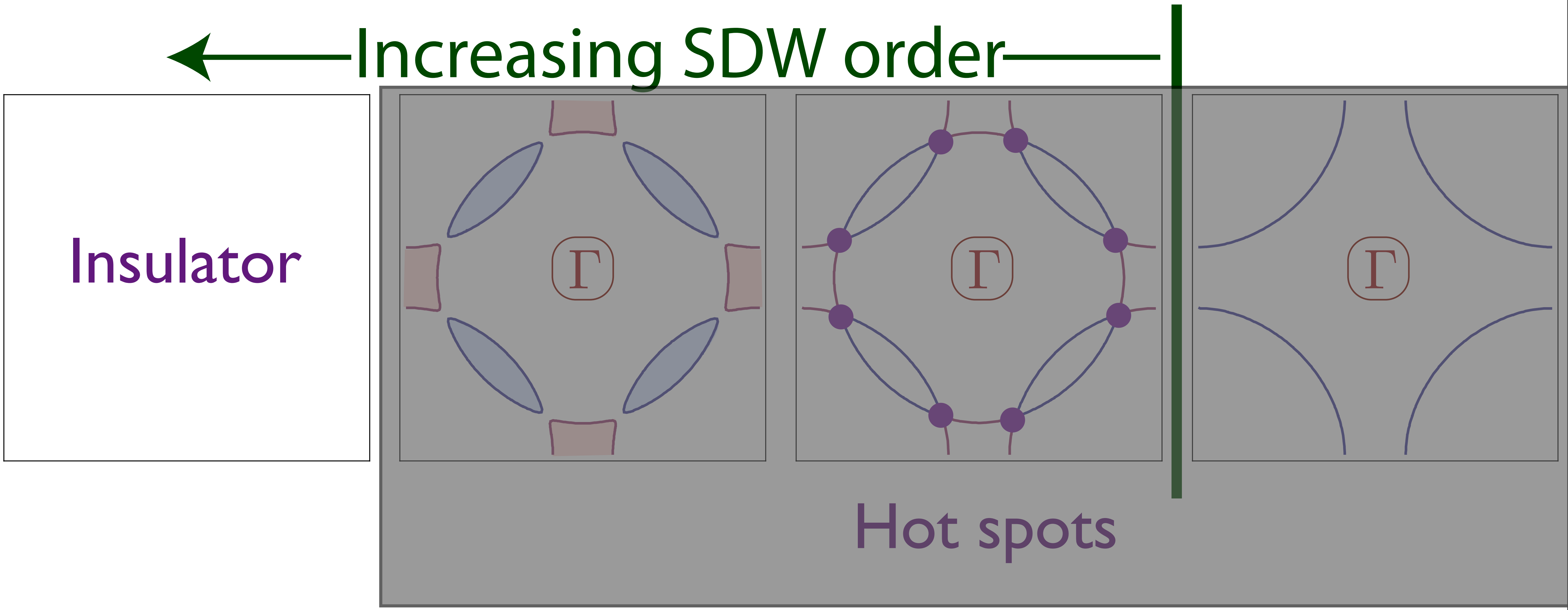

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Perform SU(2) rotation $\mathcal{R}_z$ on filled band of electrons:

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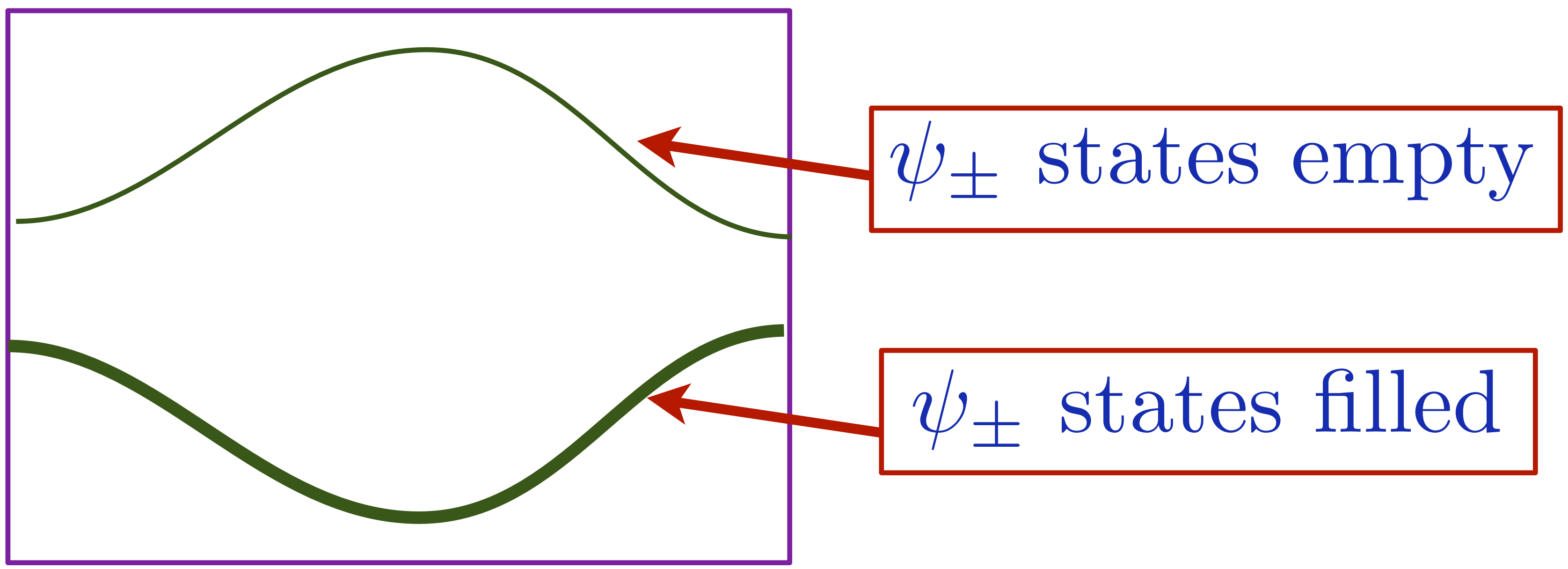

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This is invariant under

$$z_\alpha \to e^{i\theta} z_\alpha \;\; ; \;\; \psi_+ \to e^{-i\theta}\psi_+ \;\; ; \;\; \psi_- \to e^{i\theta}\psi._-$$

We obtain a U(1) gauge theory of

- bosonic neutral spinons z_α;
- spinless, charged fermions $\psi_\pm$ occupying filled bands;
- an emergent U(1) gauge field A_μ.

The Néel phase is the Higgs state with $\langle z_\alpha \rangle \neq 0$.

Nature of quantum "disordered" phase

The Néel phase is the Higgs state with $\langle z_\alpha \rangle \neq 0$.

In the quantum "disordered" phase, with $\langle z_\alpha \rangle = 0$ and z_α excitations gapped, let us examine the theory for the $\psi_\pm$ fermions. For simplicity, we focus on the honeycomb lattice, where this can be written in Dirac notation:

$$\mathcal{L}_\psi = i\overline{\psi}\gamma^\mu \left(\partial_\mu - iA_\mu \sigma^z\right)\psi + m\overline{\psi}\rho^y\sigma^z\psi$$

where $\vec{\sigma}/\vec{\rho}$ are Pauli matrices in spin/valley space.

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Introduce an external gauge field B_μ to probe the structure of the gapped $\psi_\pm$ phase

Nature of quantum "disordered" phase

After integrating out the fermions, the quantum spin Hall physics implies a mutual Chern-Simons term between A_μ and B_μ

$$\mathcal{L}_{\text{eff}} = \frac{i}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu B_\lambda$$

Changing the A_μ flux (analog of electric field in QSHE), induces a B_μ charge (analog of spin in QSHE).

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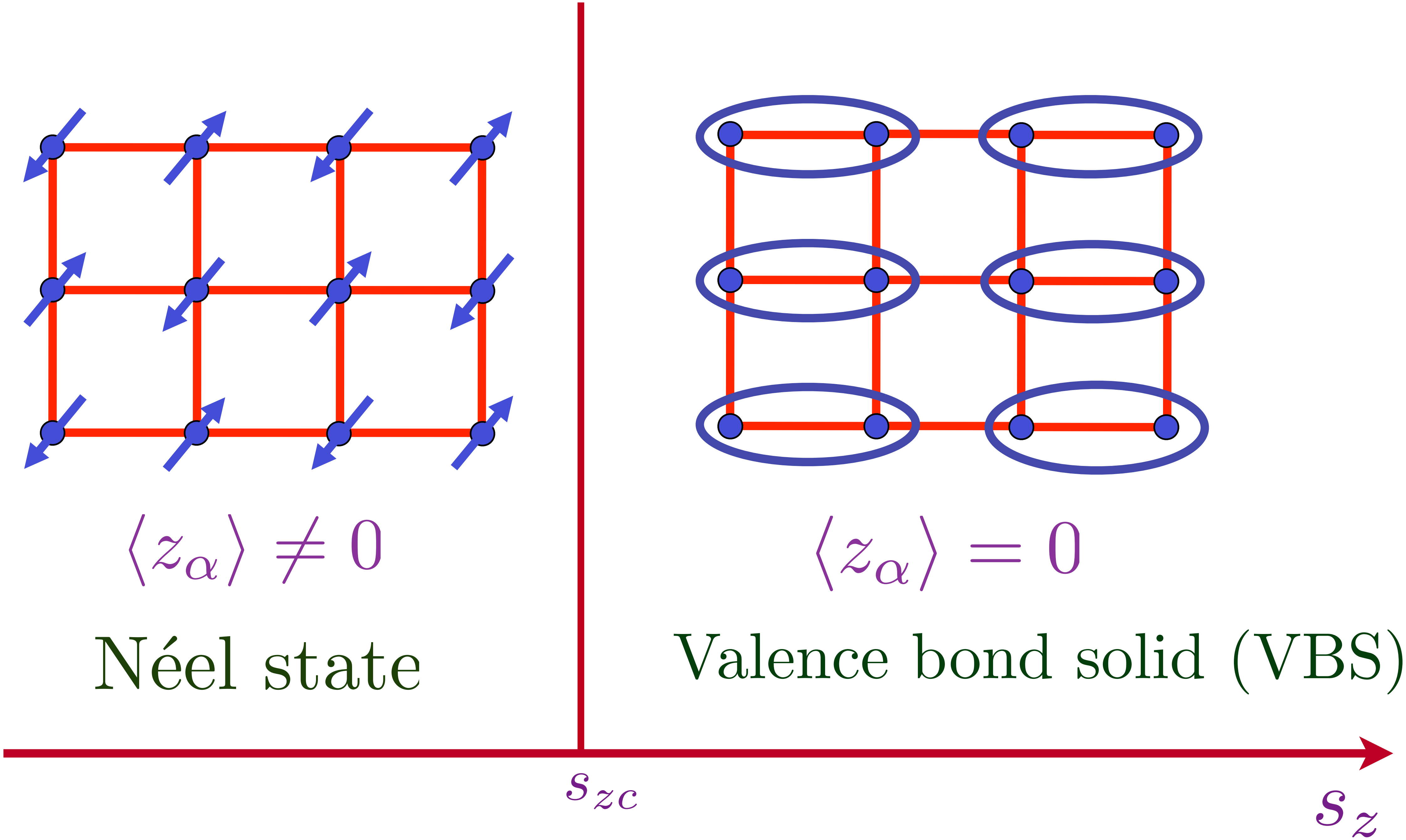
Monopoles in A_μ carry B_μ charge.

This endows A_μ monopoles with non-zero crystal momentum.

Nature of quantum "disordered" phase



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)

Phase diagram of frustrated antiferromagnets

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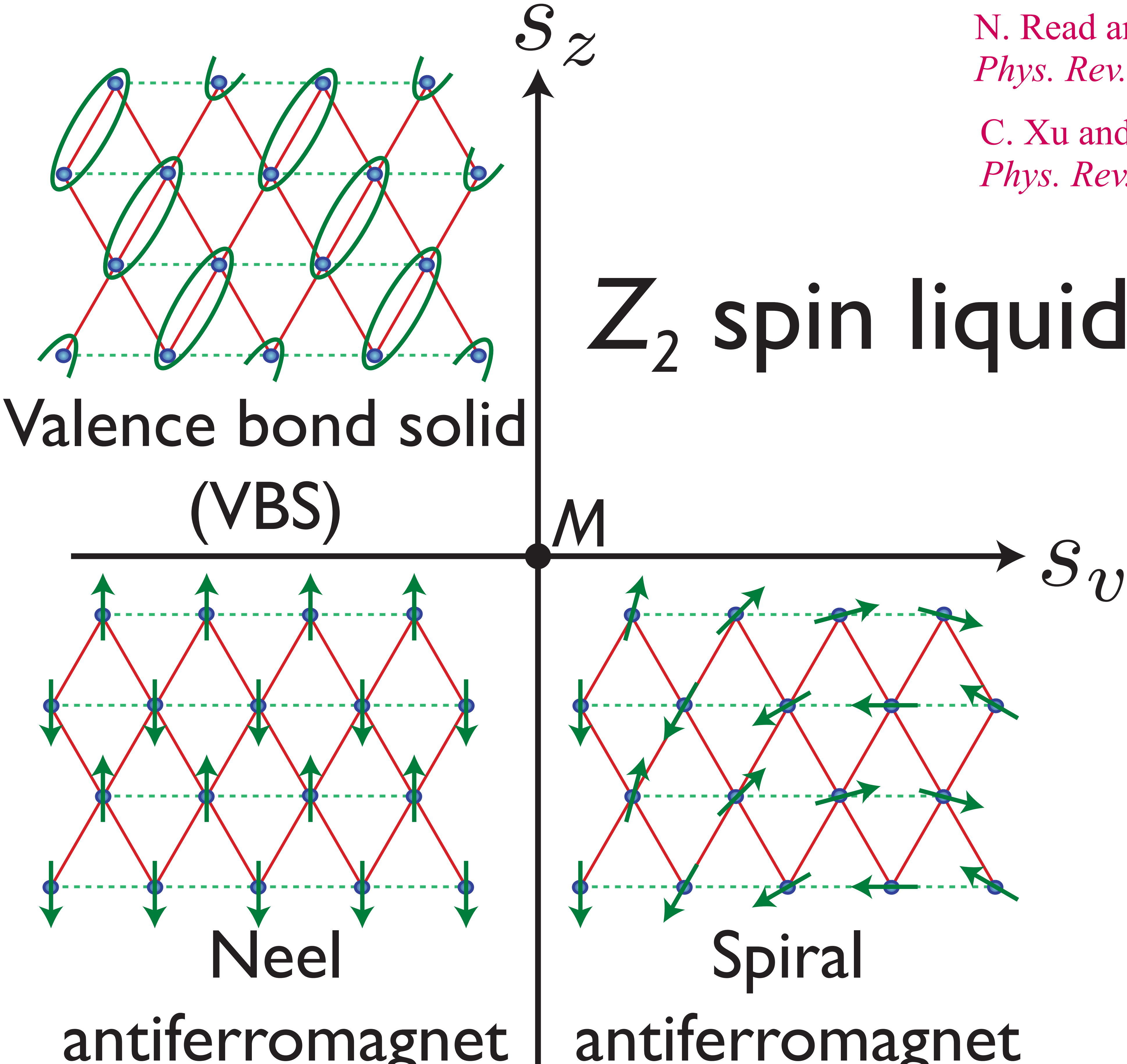
Phys. Rev. Lett. **63**, 1773 (1991)

C. Xu and S. Sachdev,
Phys. Rev. B **79**, 064405 (2009)



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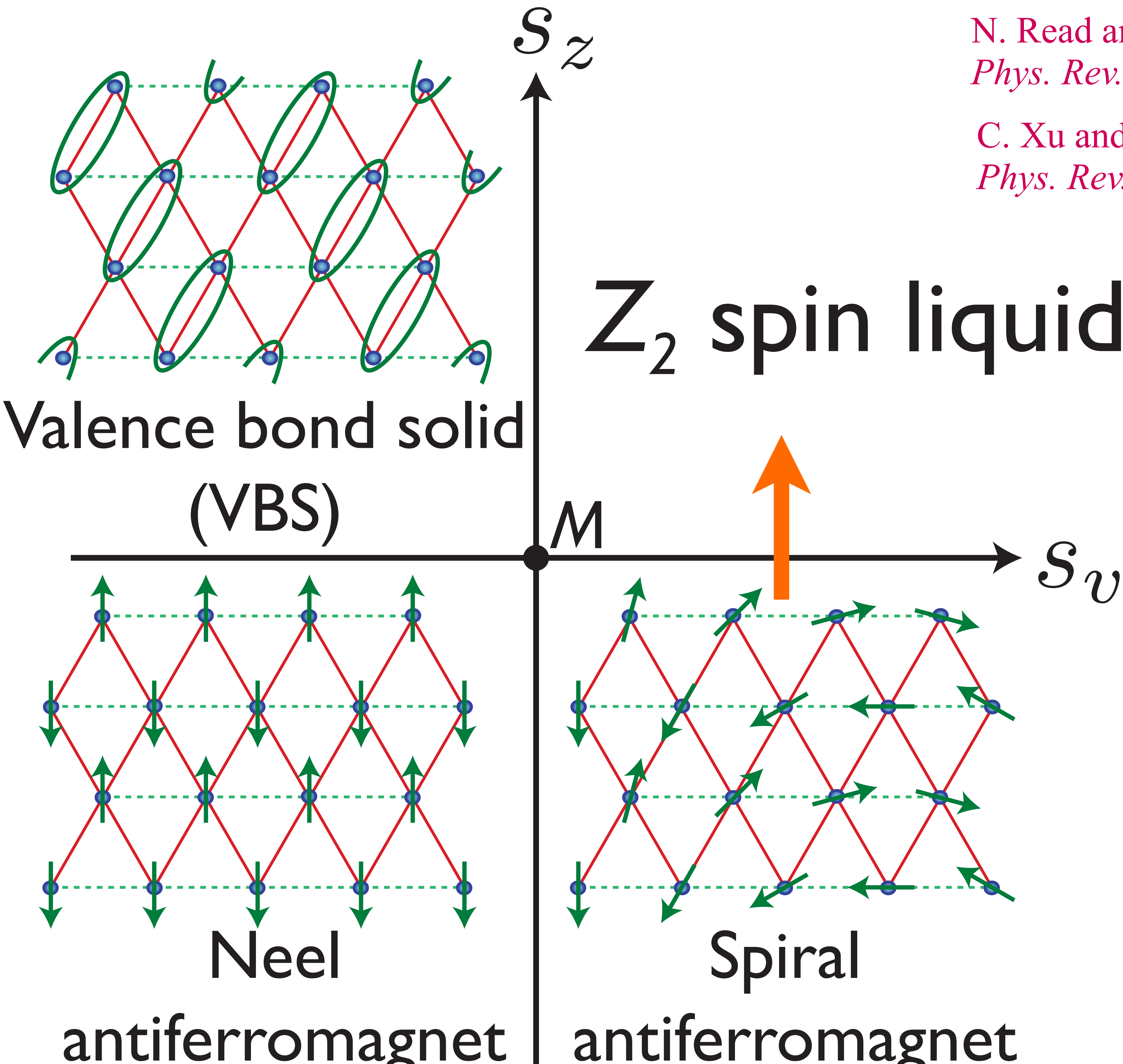

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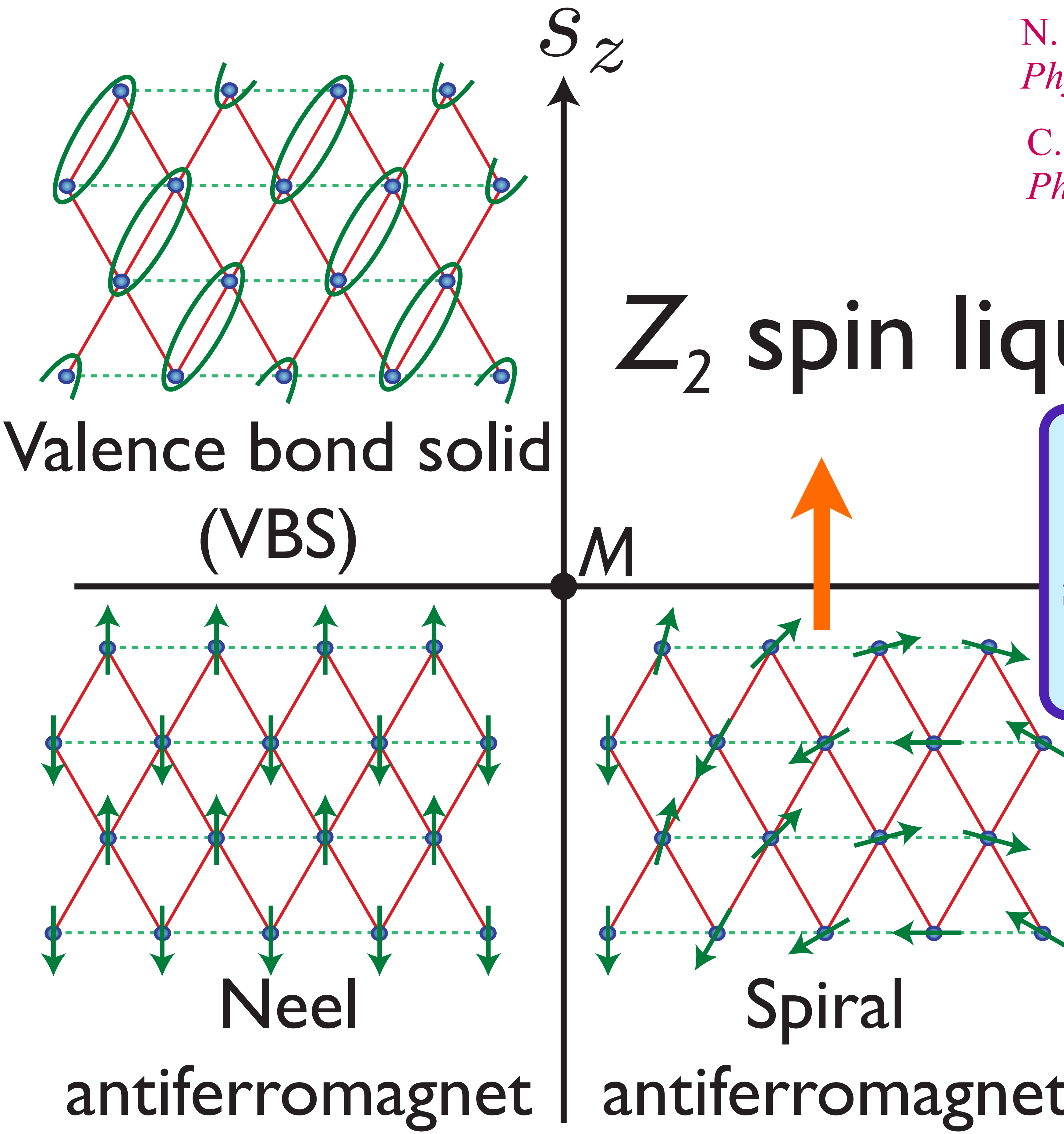

Quantum "disordering" spiral order leads to a Z_2 spin liquid

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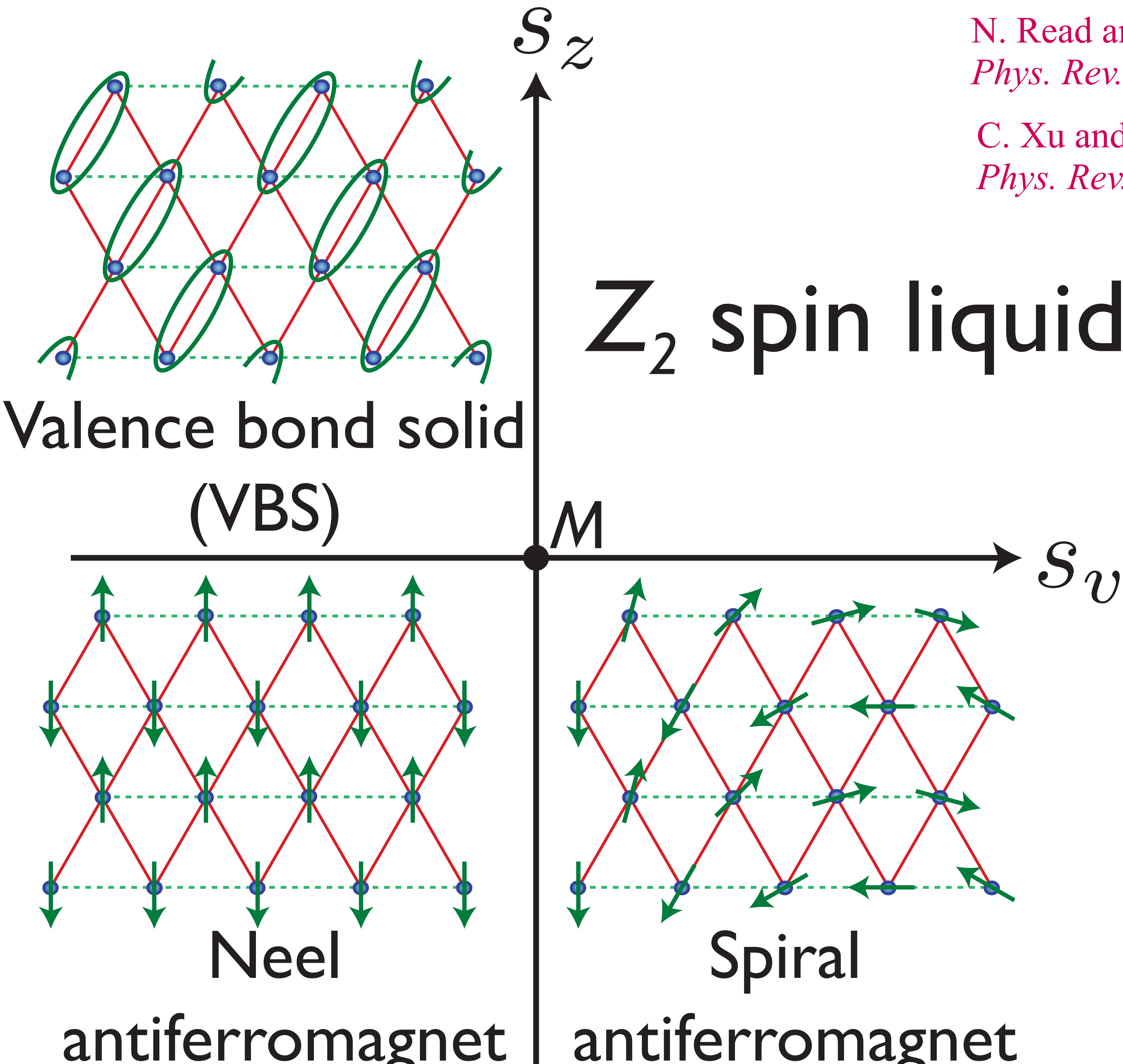

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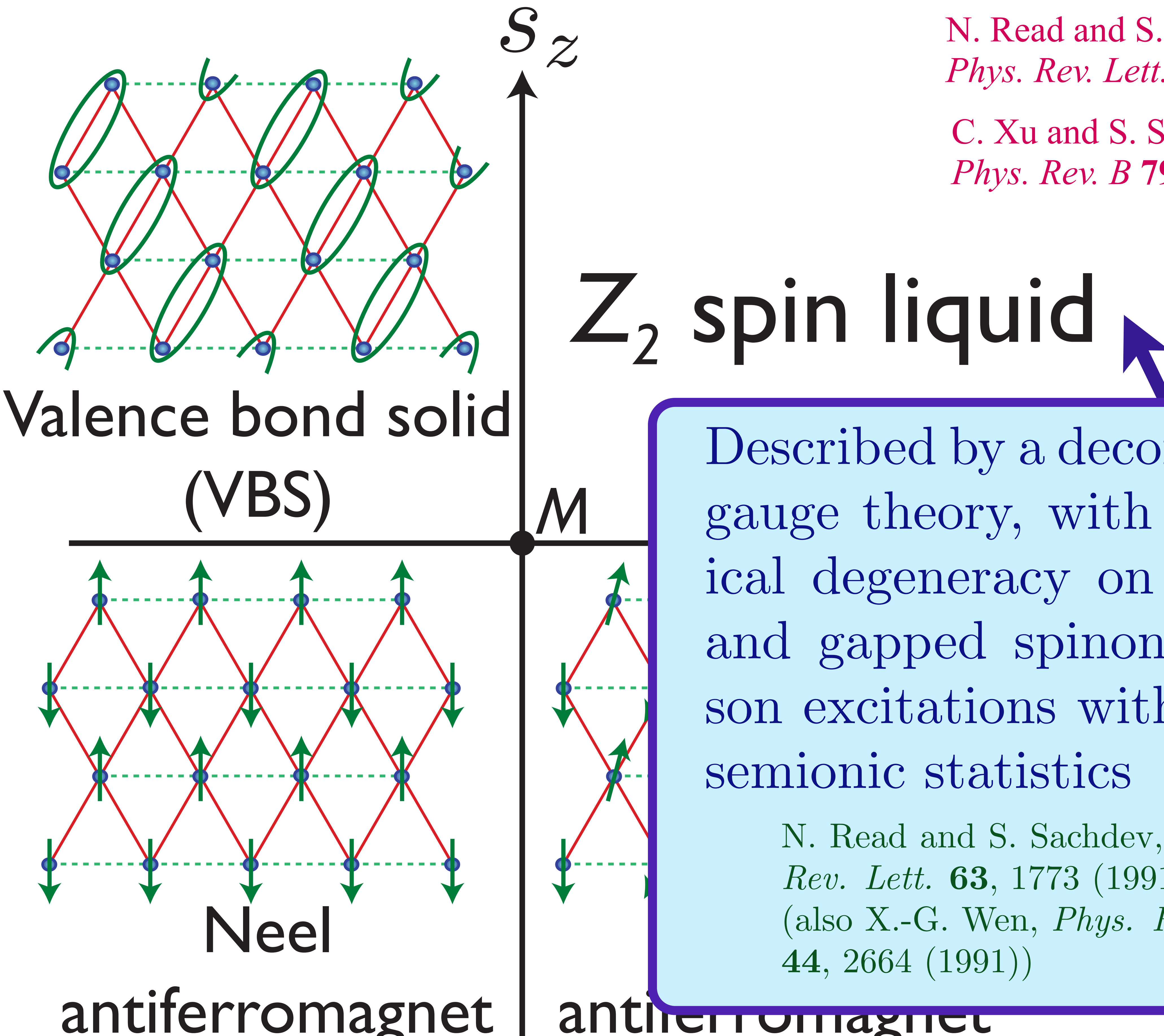

Described by a deconfined Z_2 gauge theory, with topological degeneracy on a torus, and gapped spinon and vison excitations with mutual semionic statistics

N. Read and S. Sachdev, *Phys. Rev. Lett.* **63**, 1773 (1991).
(also X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991))

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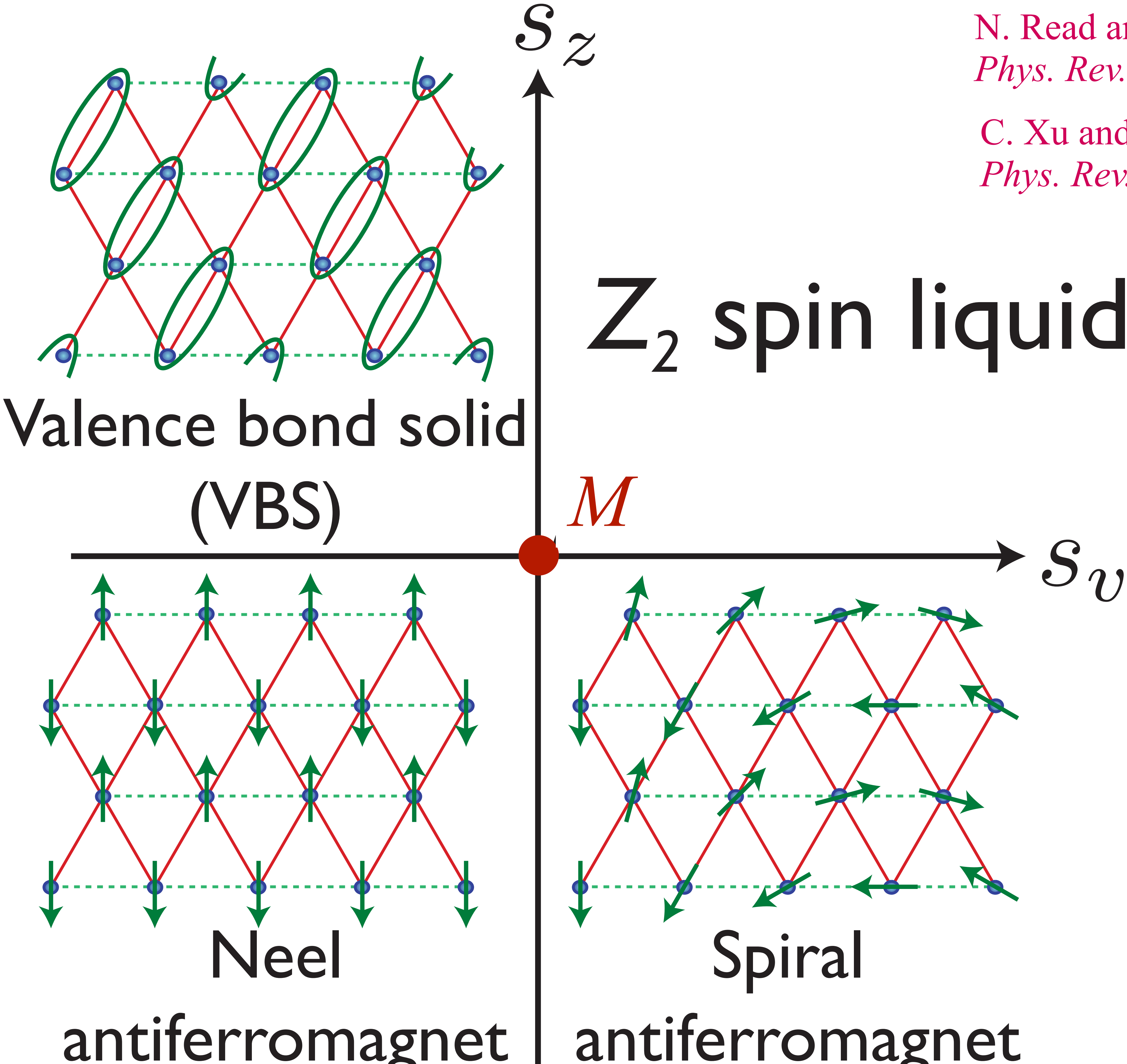

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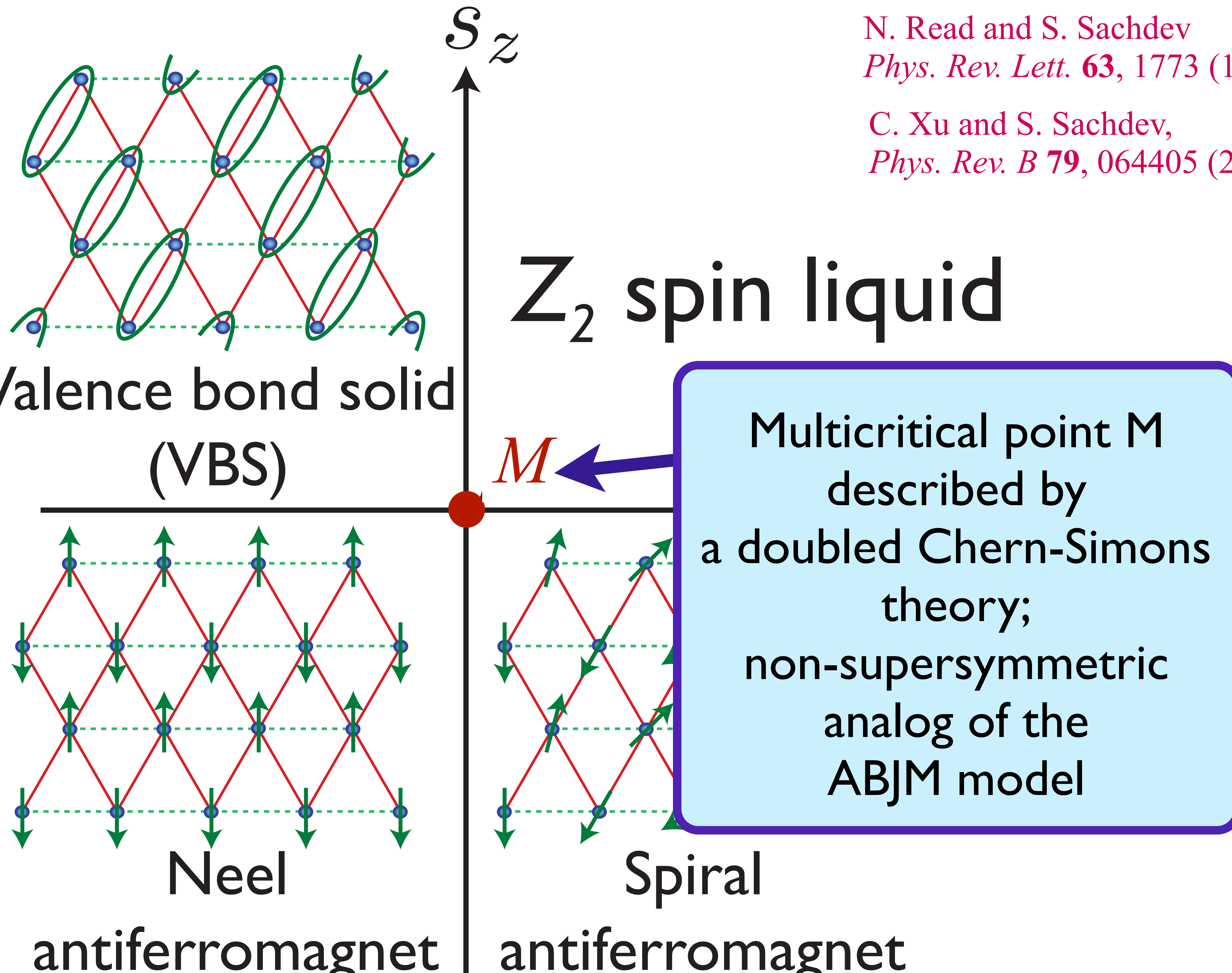
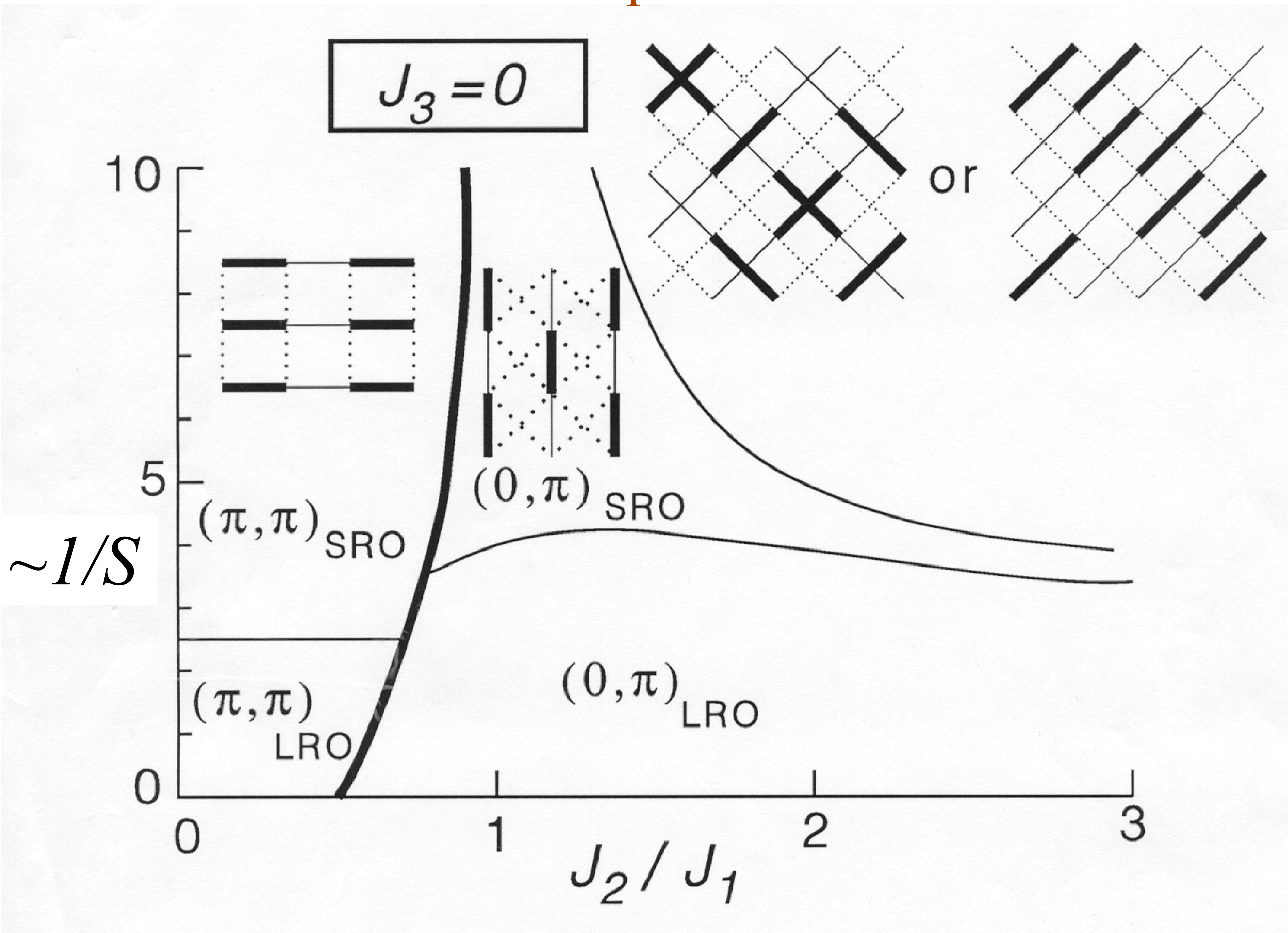


Phase diagram of J_1-J_2-J_3 antiferromagnet on the square lattice



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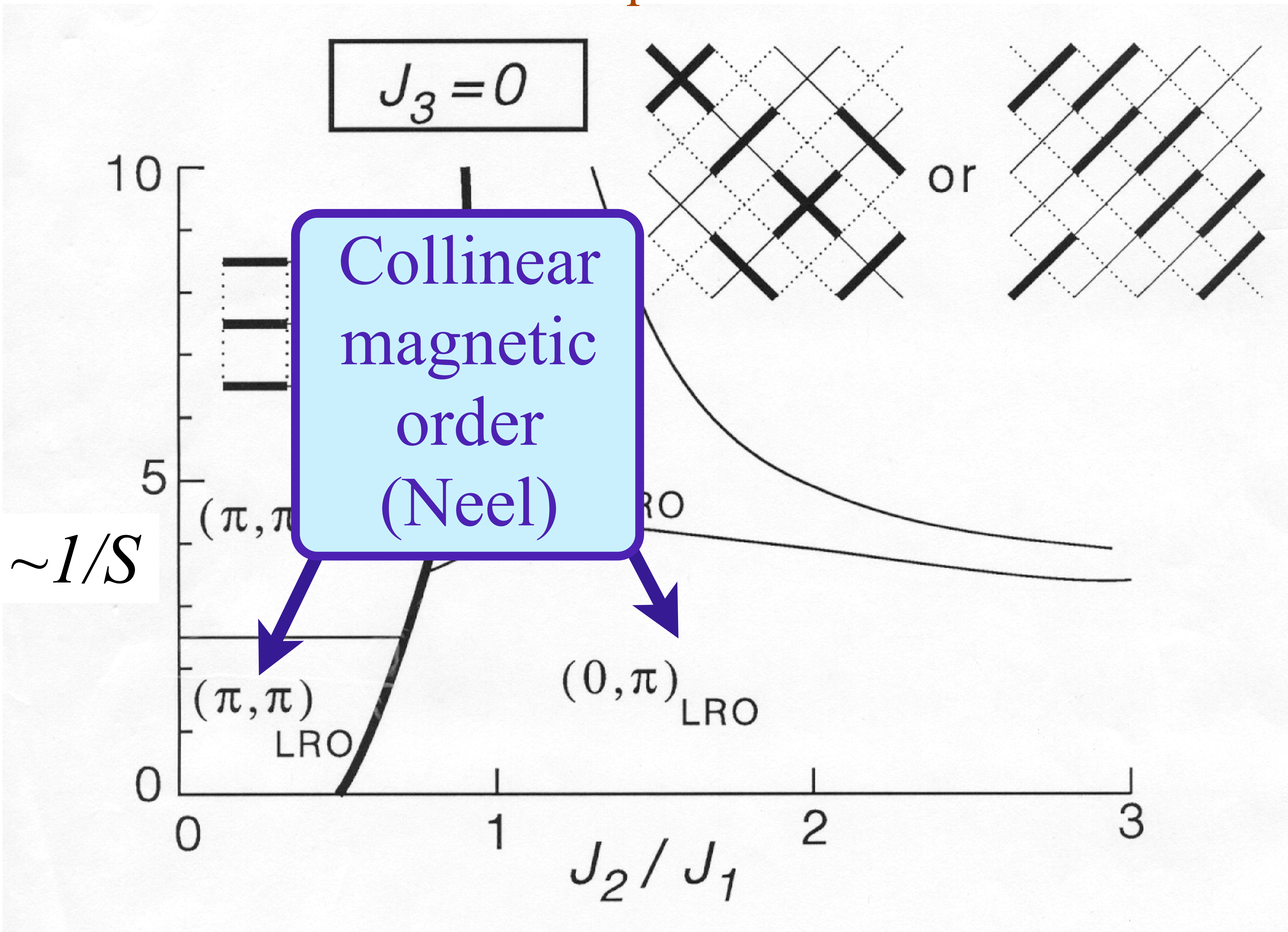

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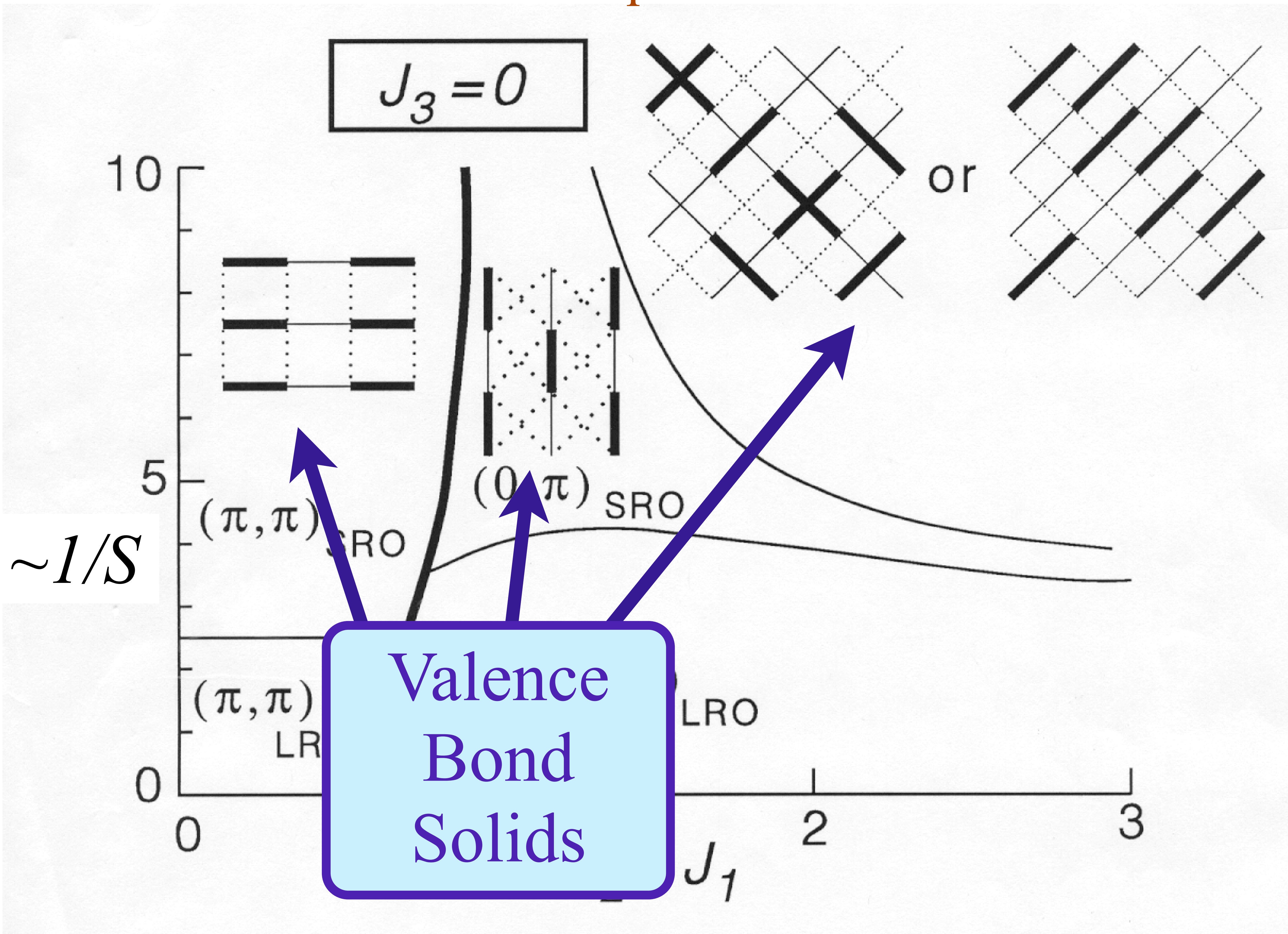

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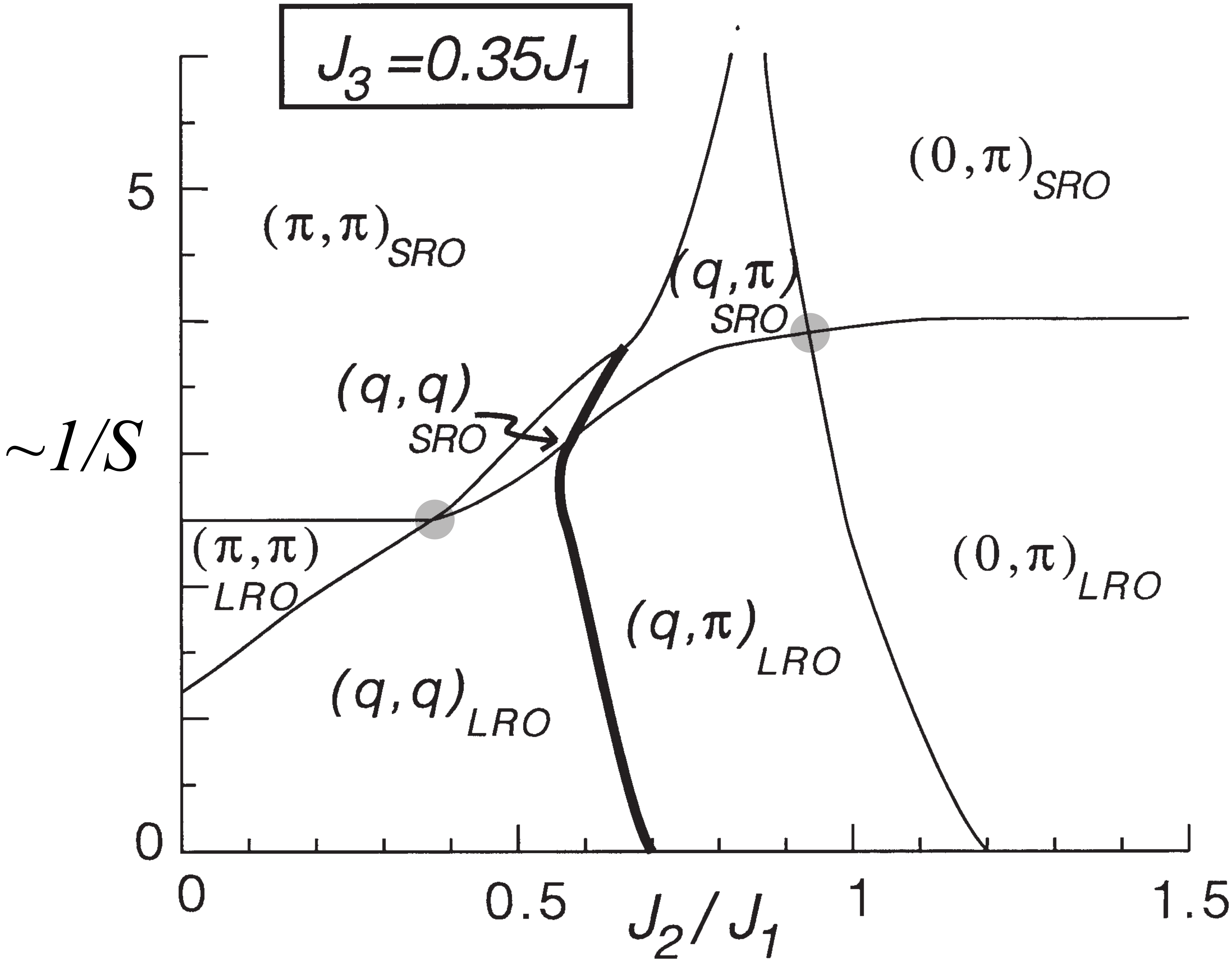

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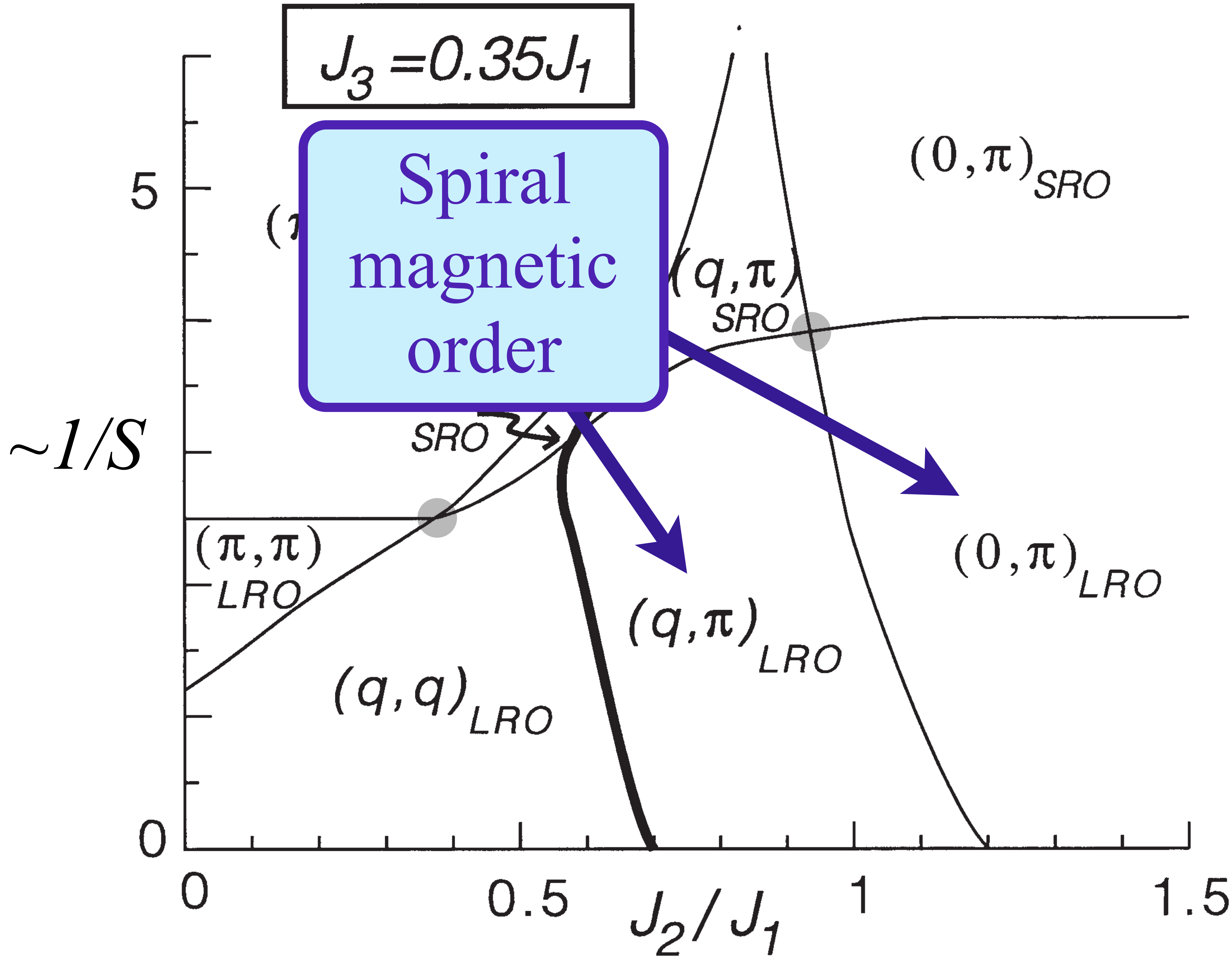

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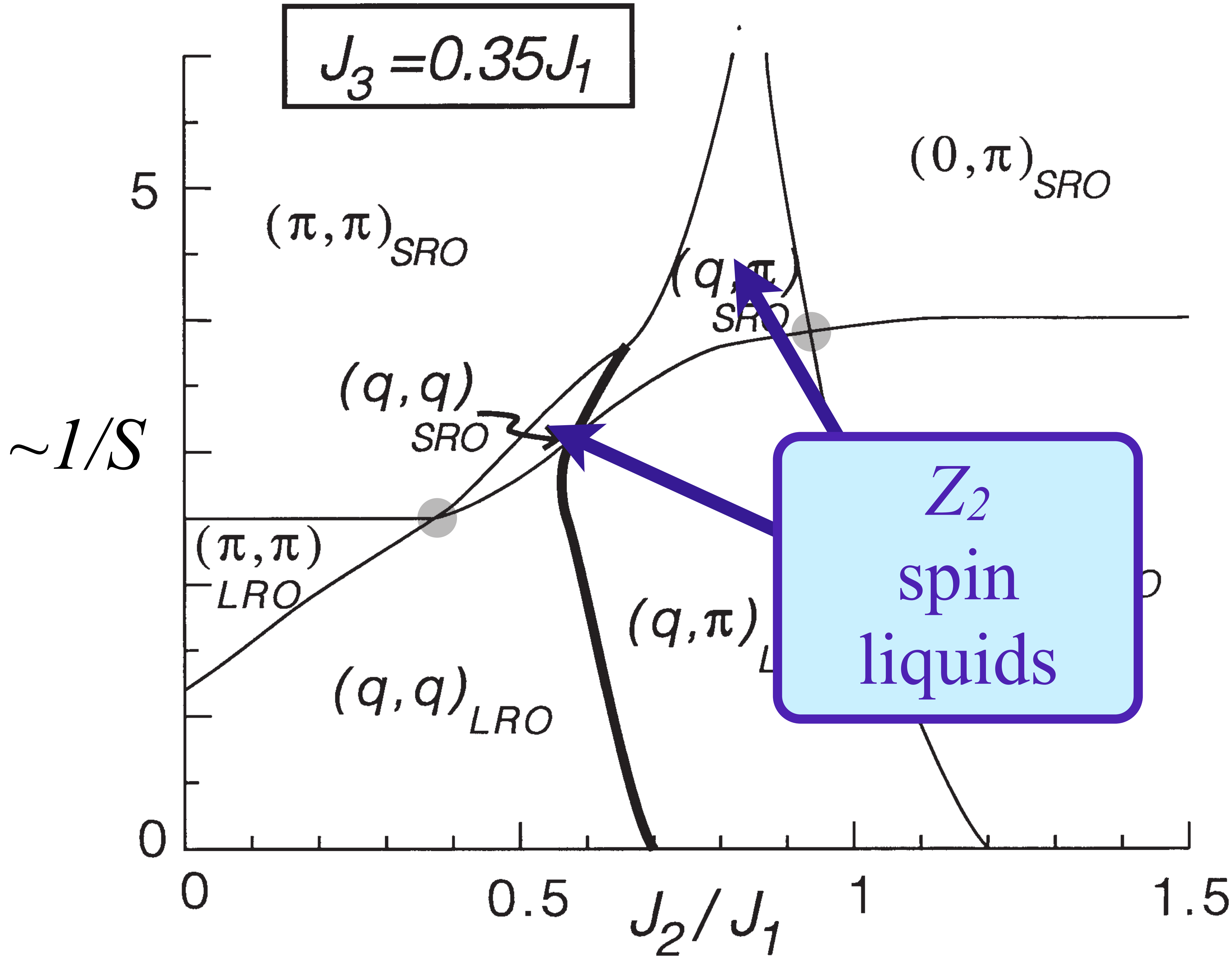

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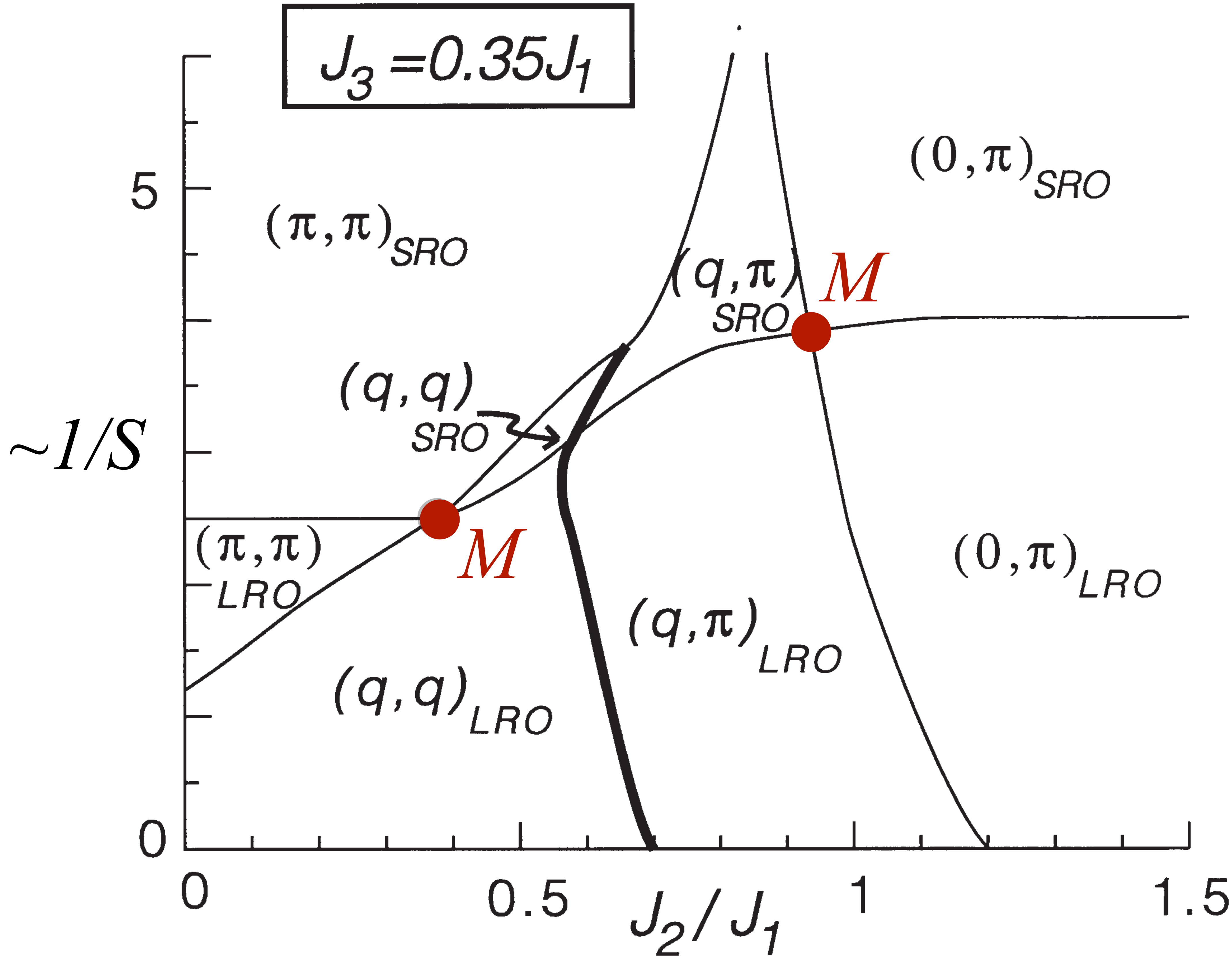

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- Spin rotations, $SU(2)_{\mathrm{spin}}$
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$$\boxed{\begin{array}{c} U \times U^{-1} \\ SU(2)_{s;gauge} \end{array}}$$

S. Sachdev, M. A. Metlitski, Y. Qi, and S. Sachdev *Phys. Rev. B* **80**, 155129 (2009)

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neutral fermionic spinons

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charged slave boson/rotor

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Saturday, May 29, 2010

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$$\begin{pmatrix} c_\uparrow \\ c_\downarrow^\dagger \end{pmatrix} = \begin{pmatrix} b_1^* & b_2^* \\ -b_2 & b_1 \end{pmatrix} \bullet \begin{pmatrix} f_1 \\ f_2^\dagger \end{pmatrix}$$

$SU(2)_{\text{spin}}$

Global symmetry operations:

- Spin rotations, $SU(2)_{\text{spin}}$
- Combine electromagnetic charge (electron number) $U(1)_{\text{charge}}$ with particle-hole transformations to obtain $SU(2)_{\text{pseudospin}}$.

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$$\boxed{\begin{array}{c} U \times U^{-1} \\ SU(2)_{\text{p;gauge}} \end{array}}$$

Unified spin liquid theory

Decompose electron operator into real fermions, χ:

$$c_\uparrow = \chi_1 + i\chi_2 \quad ; \quad c_\downarrow = \chi_3 + i\chi_4$$

Introduce a 4-component Majorana fermion ζ_i, $i = 1 \ldots 4$ and a SO(4) matrix $\mathcal{R}$, and decompose:

$$\chi = \quad \mathcal{R} \quad \zeta$$

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$$O \times O^T$$

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By breaking SO(4)$_{\text{gauge}}$ with different Higgs fields, we can reproduce essentially all earlier theories of spin liquids.

We also find many new spin liquid phases, some with Majorana fermion excitations which carry neither spin nor charge

Outline

1. Quantum "disordering" magnetic order in two-dimensional antiferromagnets
   Topological defects and their Berry phases

2. Unified theory of spin liquids
   Majorana liquids

3. Loss of magnetic order in a metal
   d-wave pairing and (modulated) Ising-nematic order

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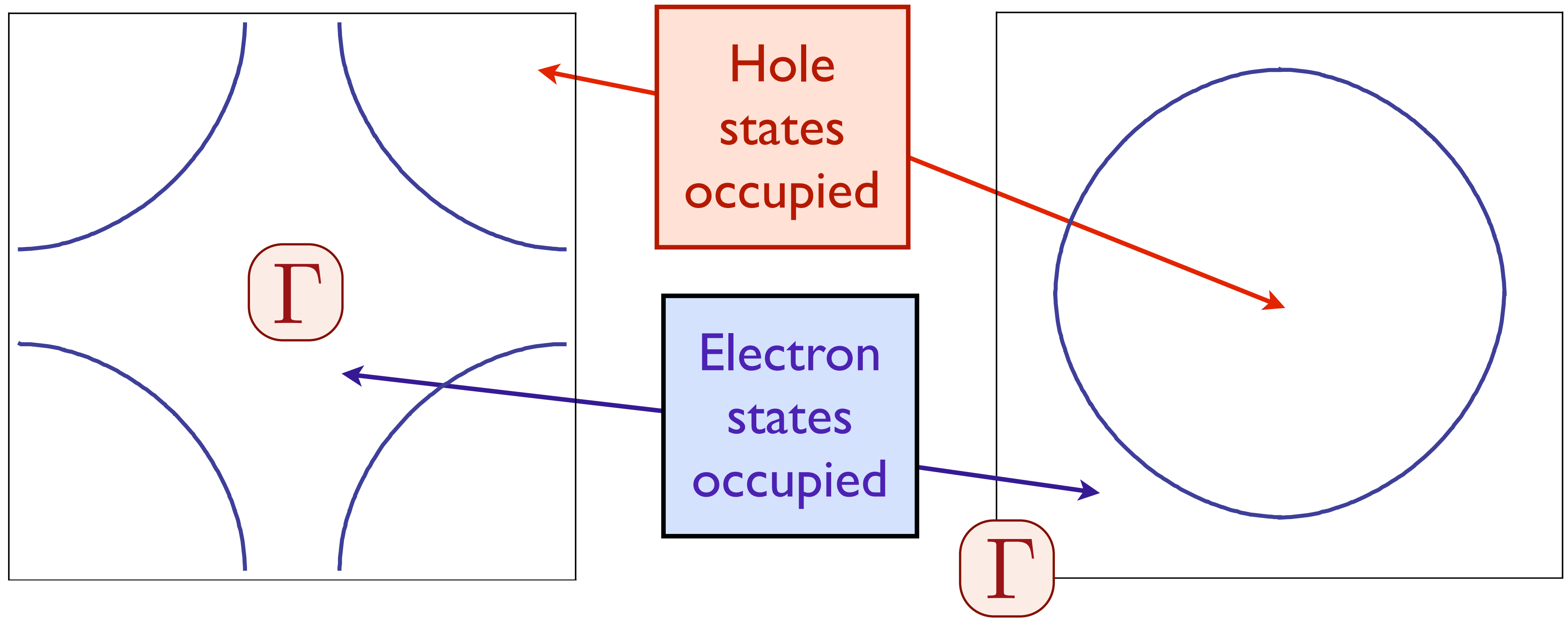
Fermi surface+antiferromagnetism



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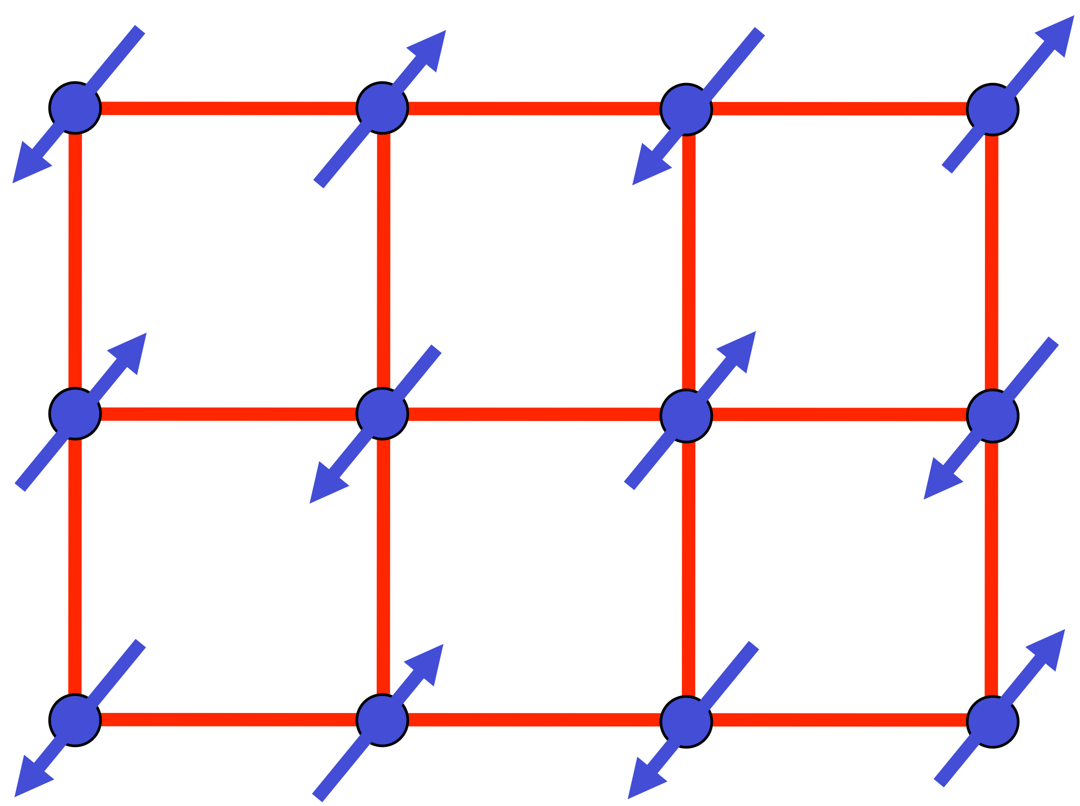
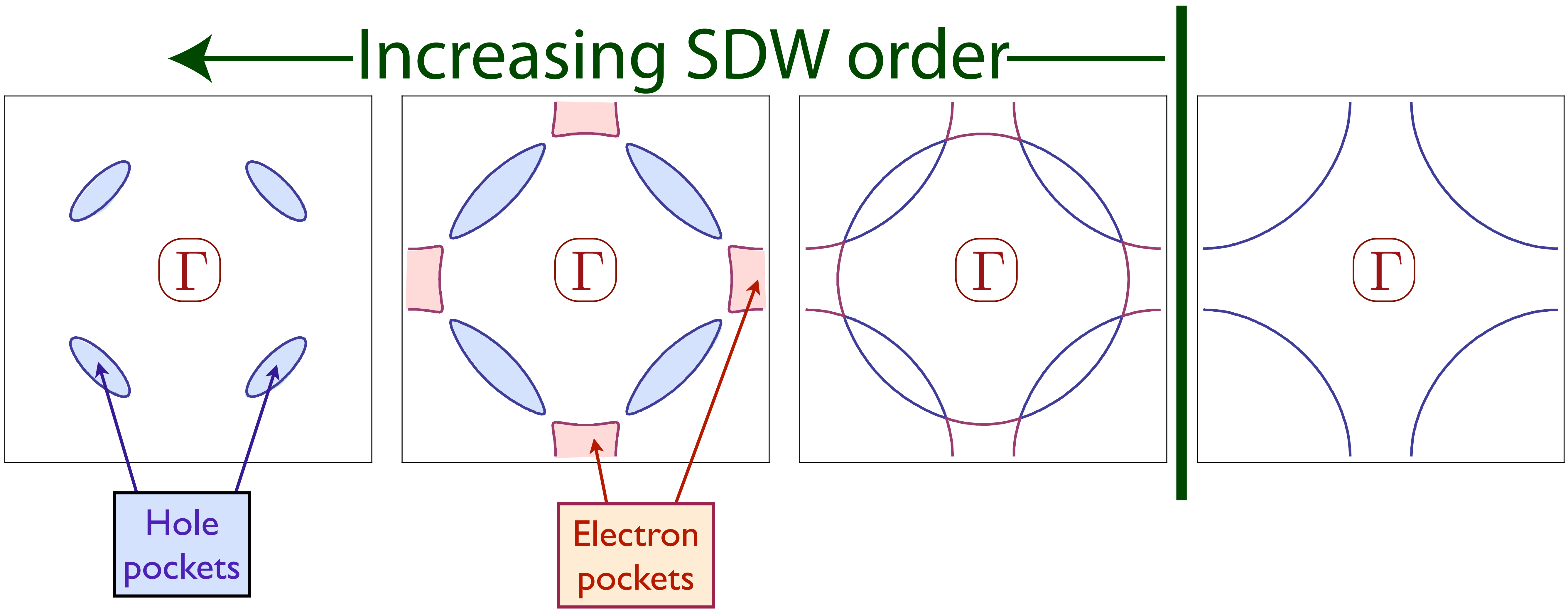


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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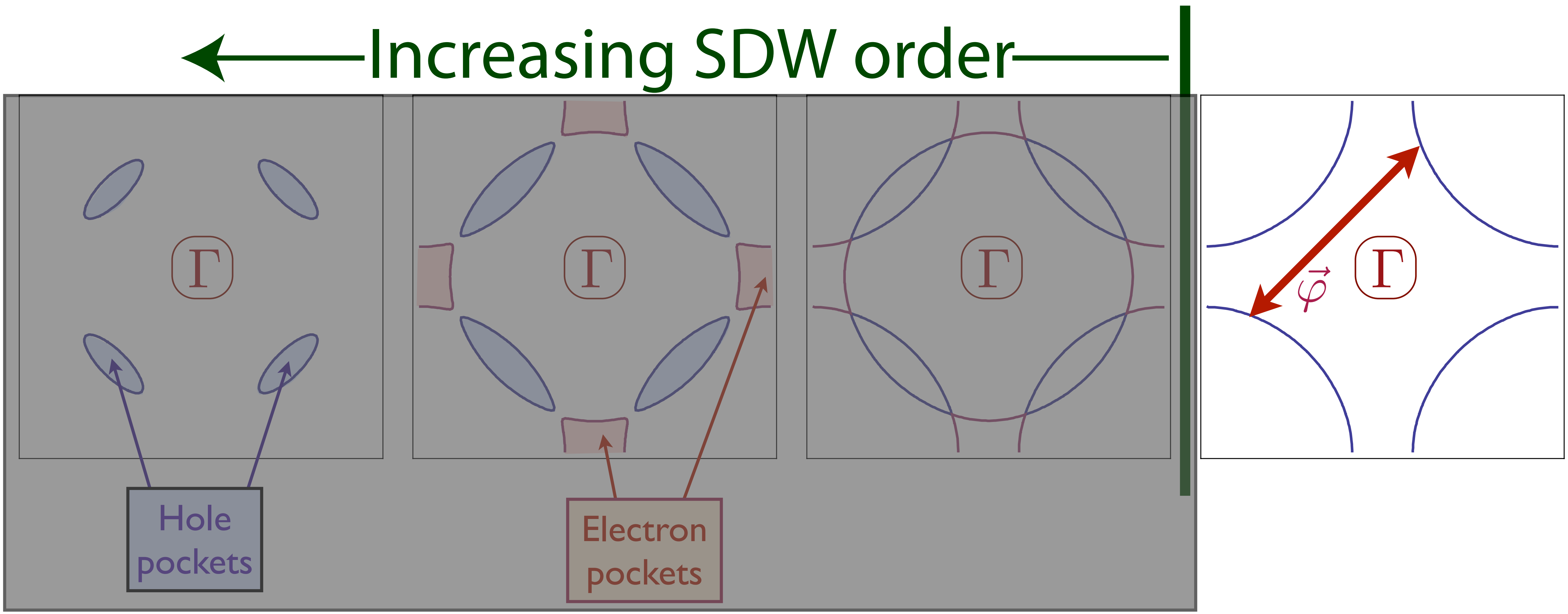


$\vec{\varphi}$ fluctuations act on the large Fermi surface

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

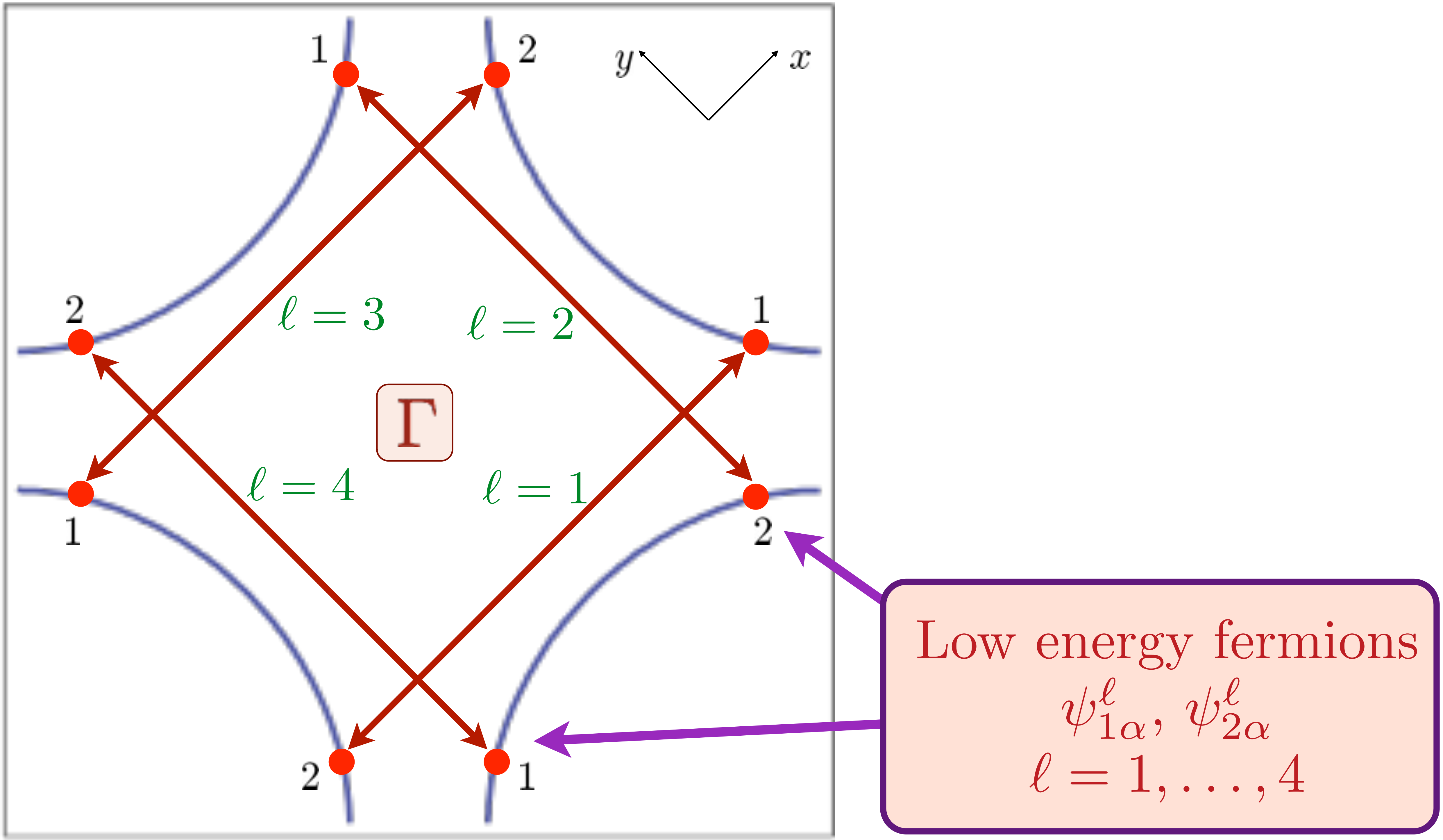
Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}\right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

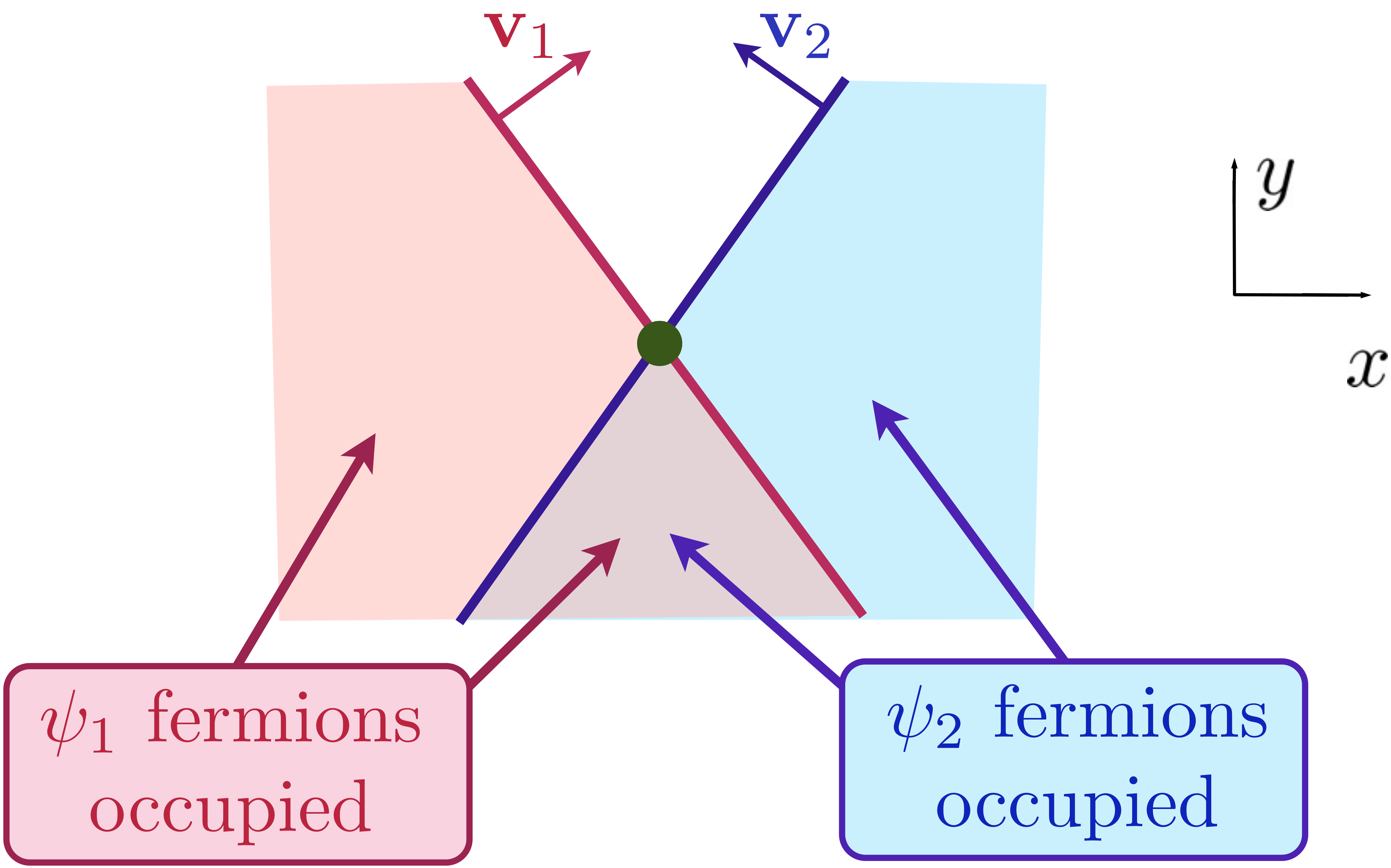
$$+ \int d\tau d^2 r \left[\frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4\right]$$



$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha} \left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_1 \cdot \boldsymbol{\nabla}_r\right) \psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha} \left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_2 \cdot \boldsymbol{\nabla}_r\right) \psi^{\ell}_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y), \ \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



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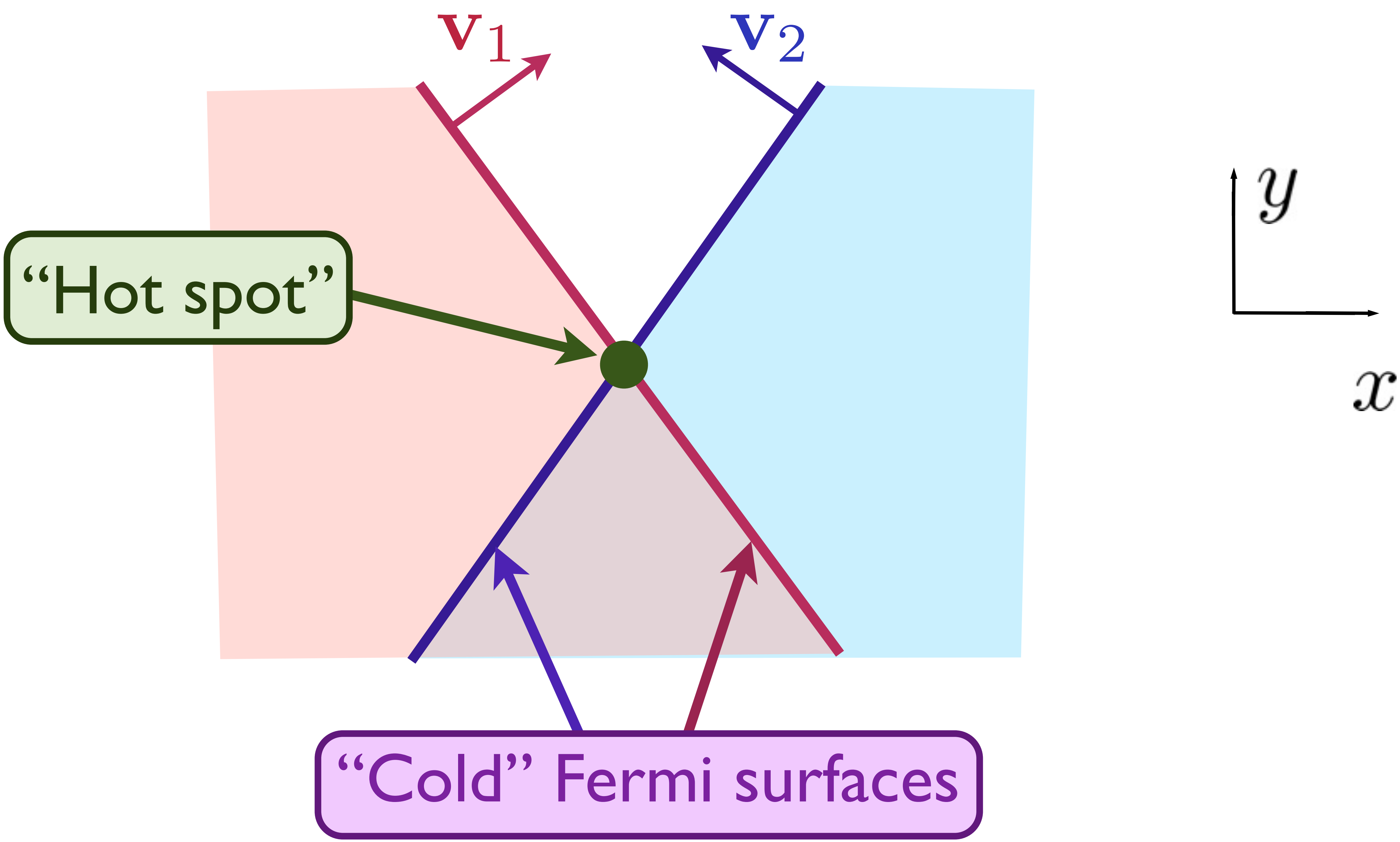

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Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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"Yukawa" coupling: $\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$

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Hertz theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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OK in $d = 3$, but higher order terms contain an infinite number of marginal couplings in $d = 2$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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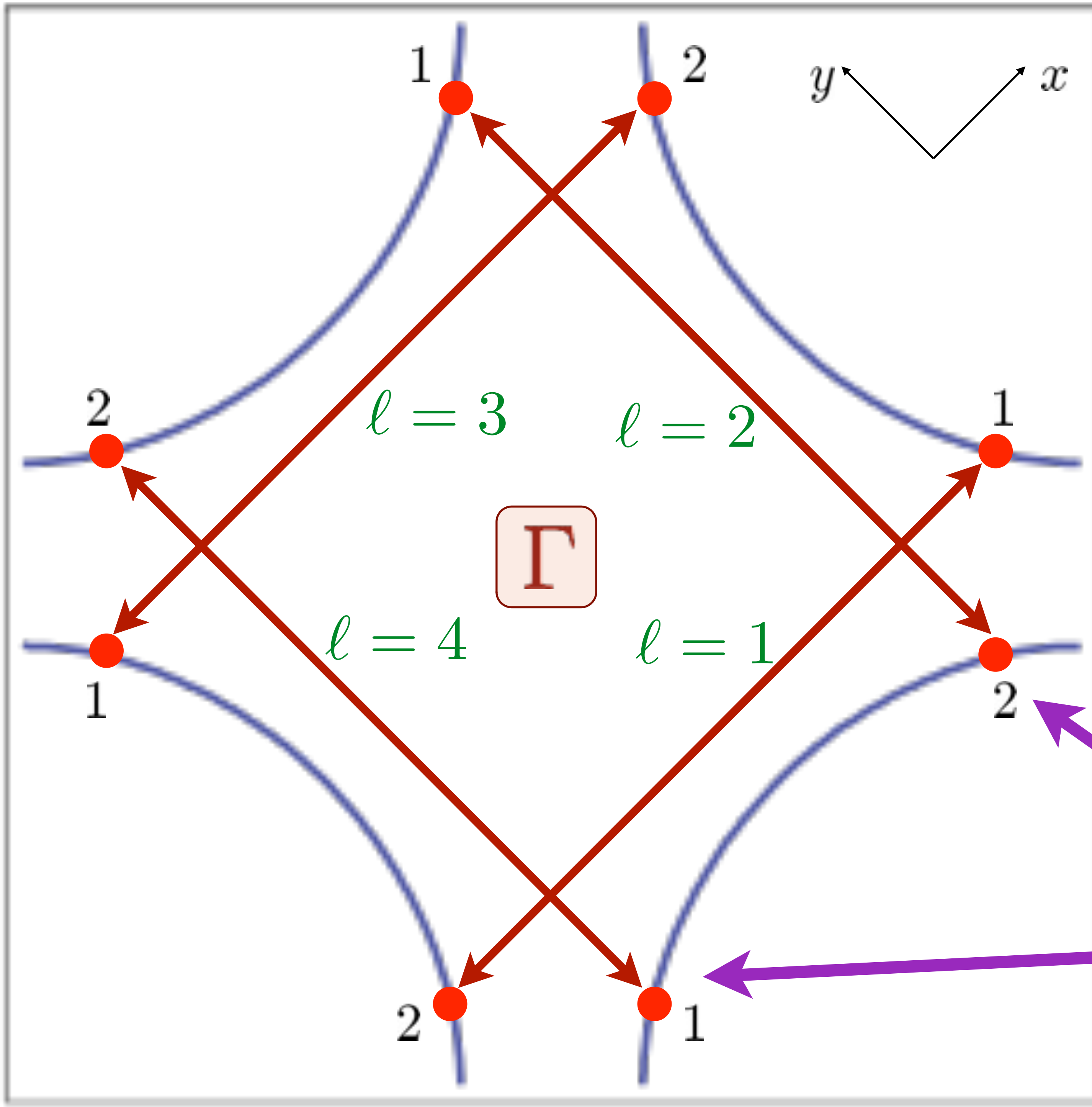
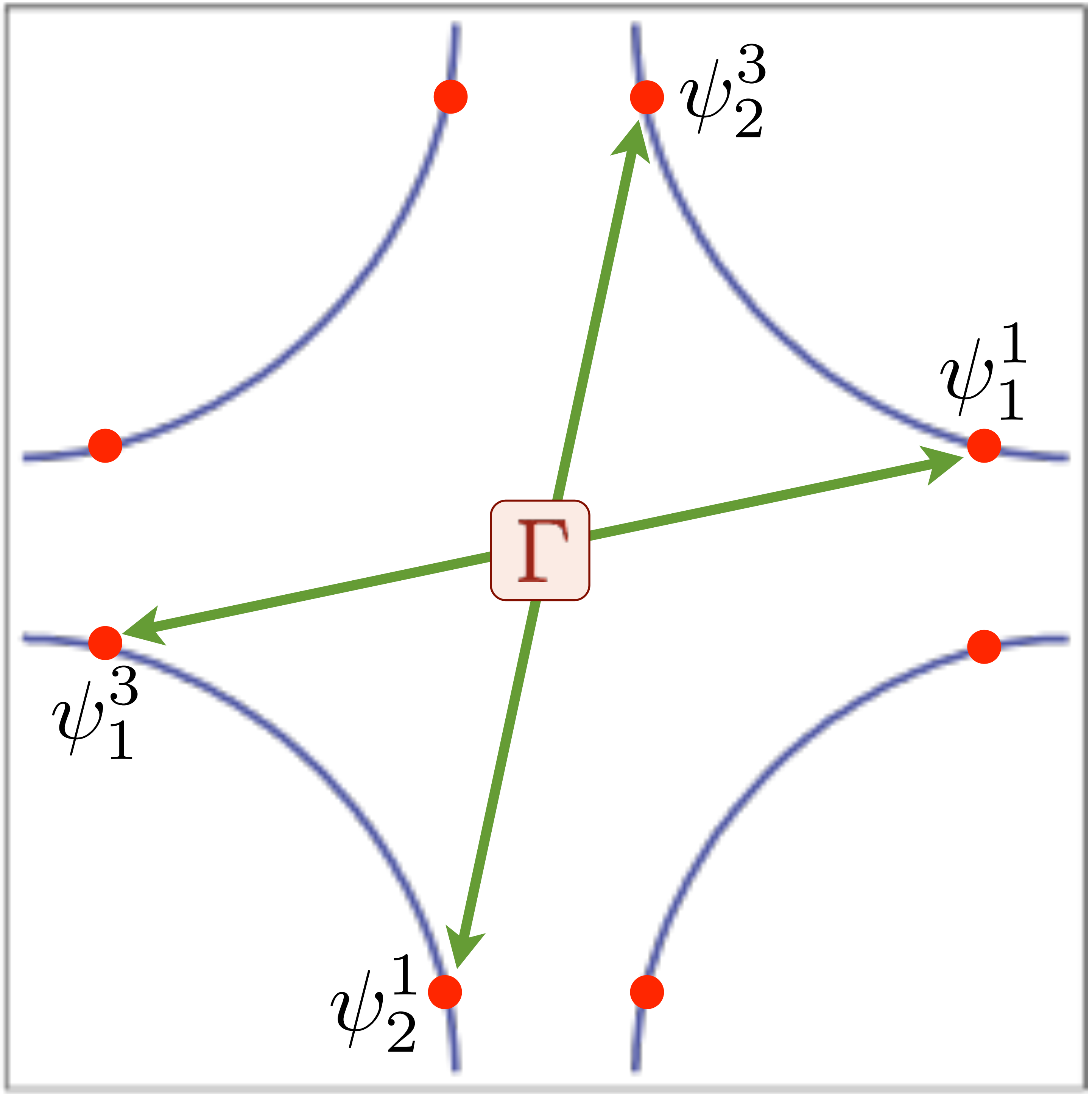
Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.



Low energy fermions

$\psi^{\ell}_{1\alpha}$, $\psi^{\ell}_{2\alpha}$

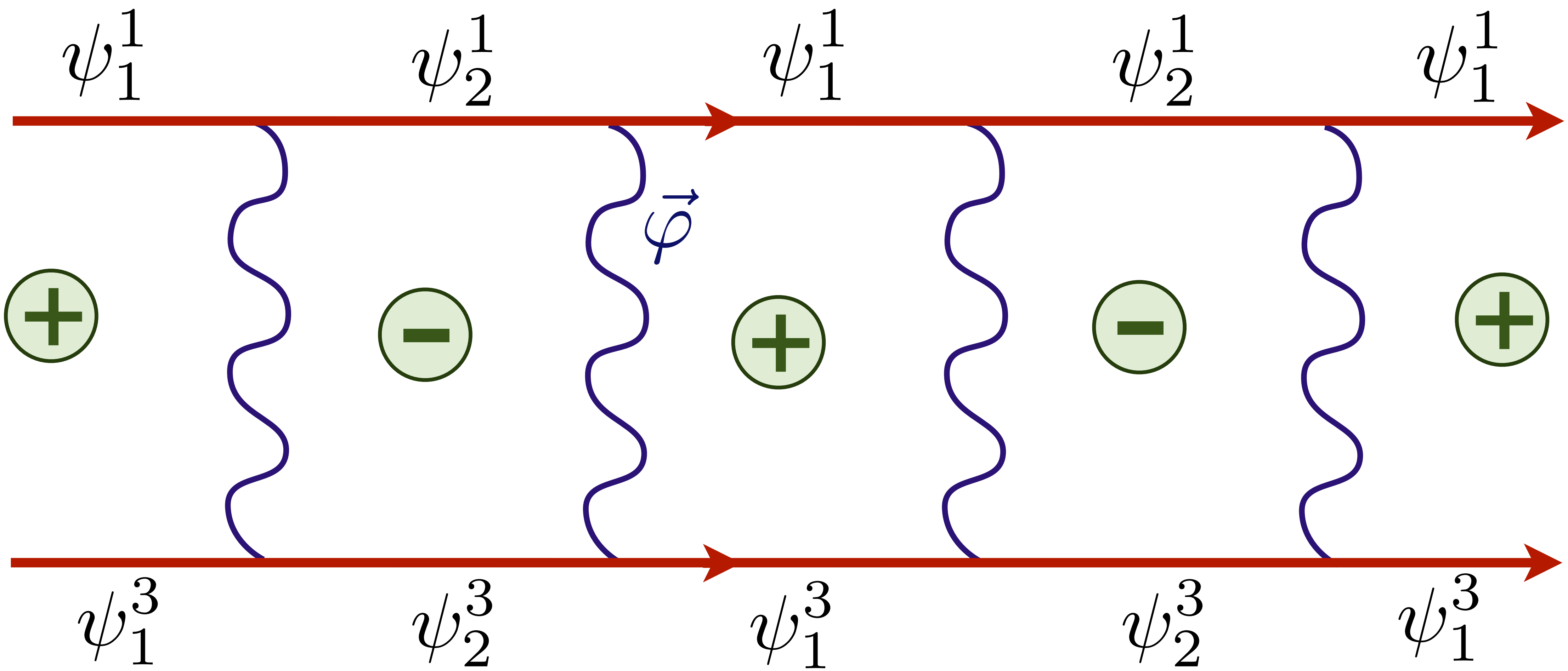
$\ell = 1, \ldots, 4$



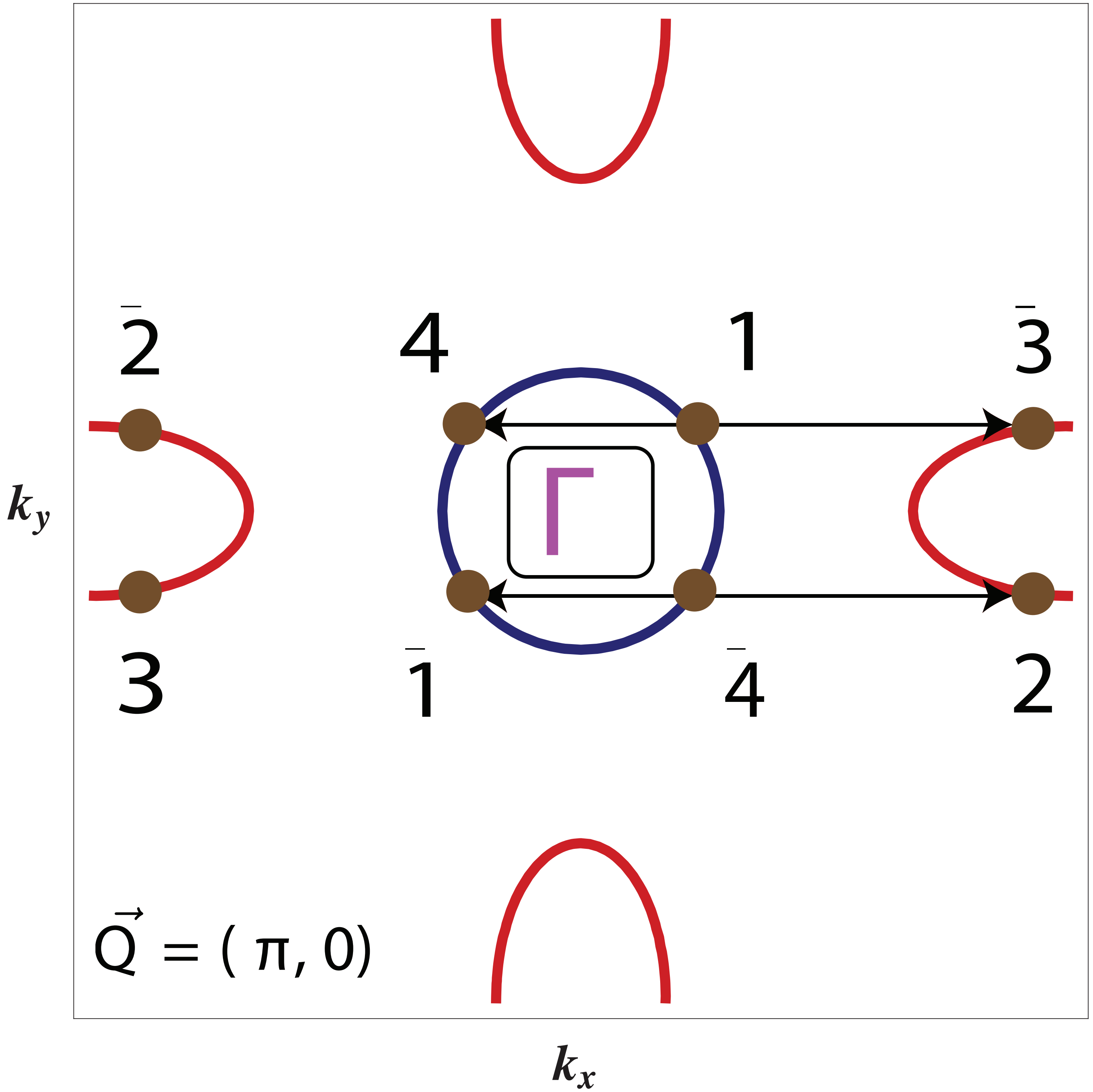
Hot spots have strong instability to d-wave pairing near SDW critical point. This instability is stronger than the BCS instability of a Fermi liquid.

Pairing order parameter:

$$\varepsilon^{\alpha\beta}\left(\psi^3_{1\alpha}\psi^1_{1\beta} - \psi^3_{2\alpha}\psi^1_{2\beta}\right)$$



d-wave Cooper pairing instability in particle-particle channel



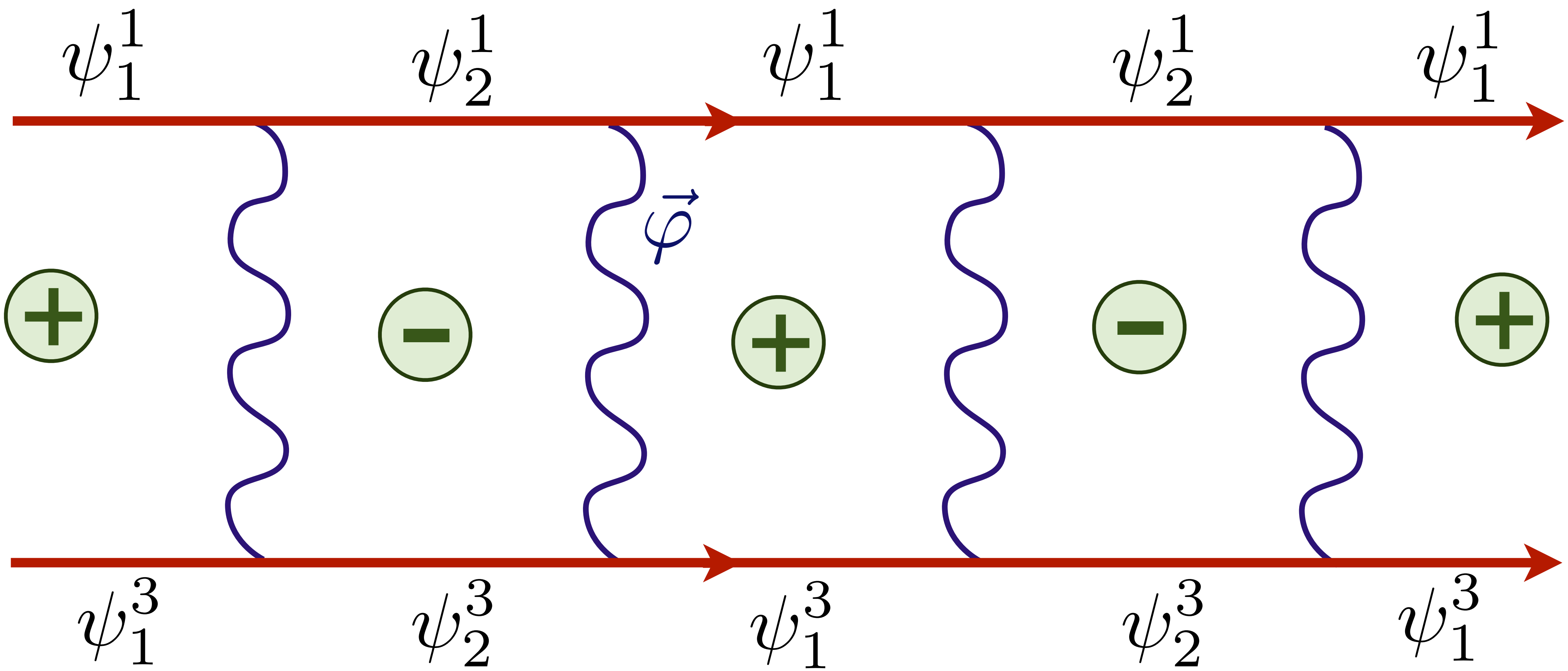
Similar theory applies to the pnictides, and leads to $s_{\pm}$ pairing.

Emergent Pseudospin symmetry

Continuum theory of hotspots in invariant under:

$$\begin{pmatrix} \psi_{\uparrow}^{\ell} \\ \psi_{\downarrow}^{\ell\dagger} \end{pmatrix} \rightarrow U^{\ell} \begin{pmatrix} \psi_{\uparrow}^{\ell} \\ \psi_{\downarrow}^{\ell\dagger} \end{pmatrix}$$

where U^{ℓ} are arbitrary SU(2) matrices which can be *different* on different hotspots ℓ.



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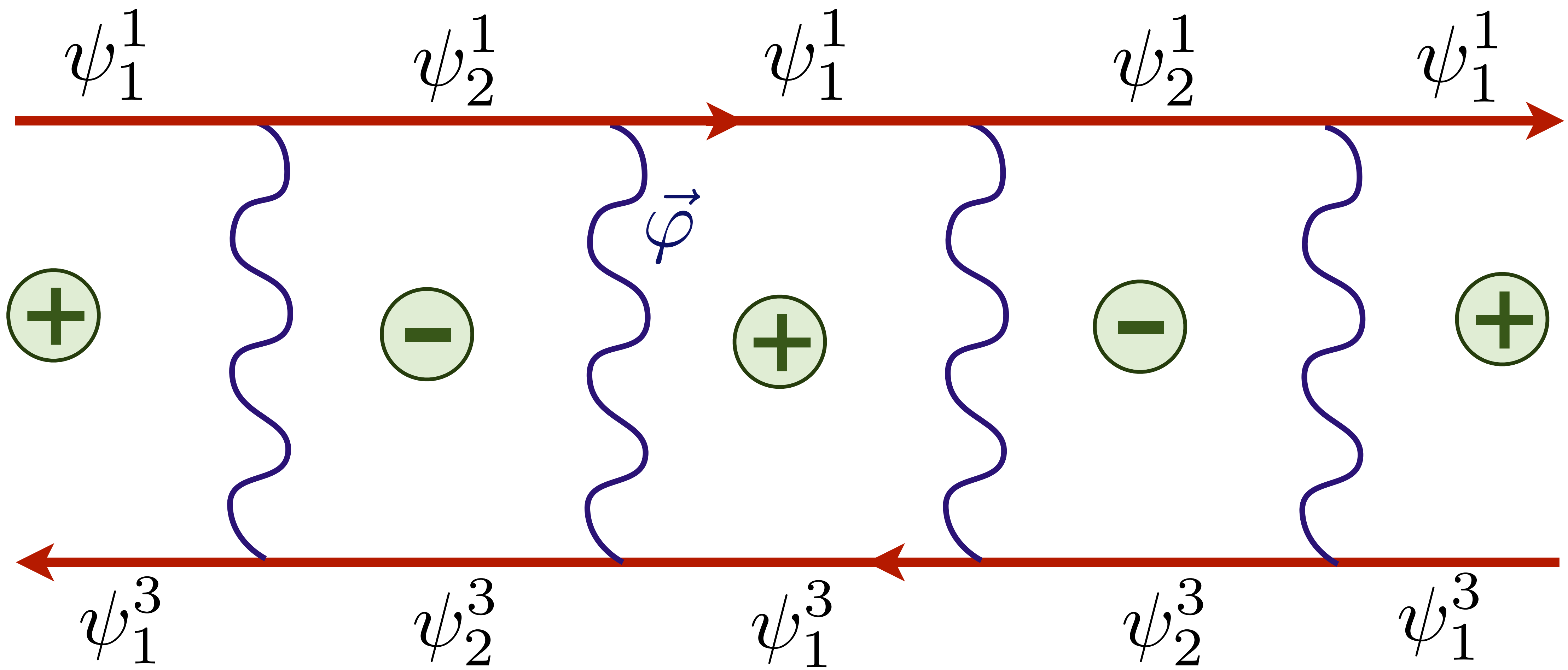
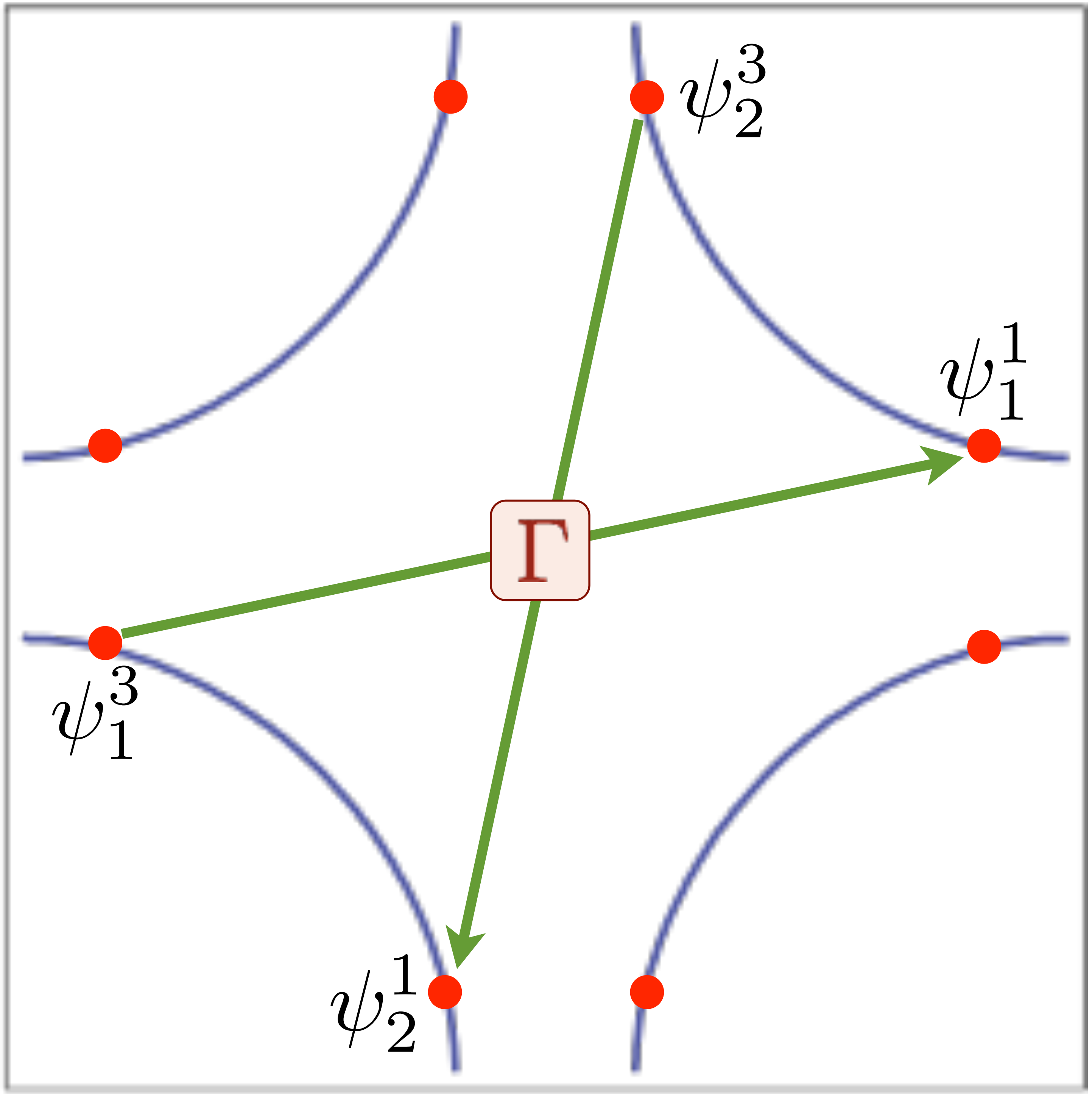


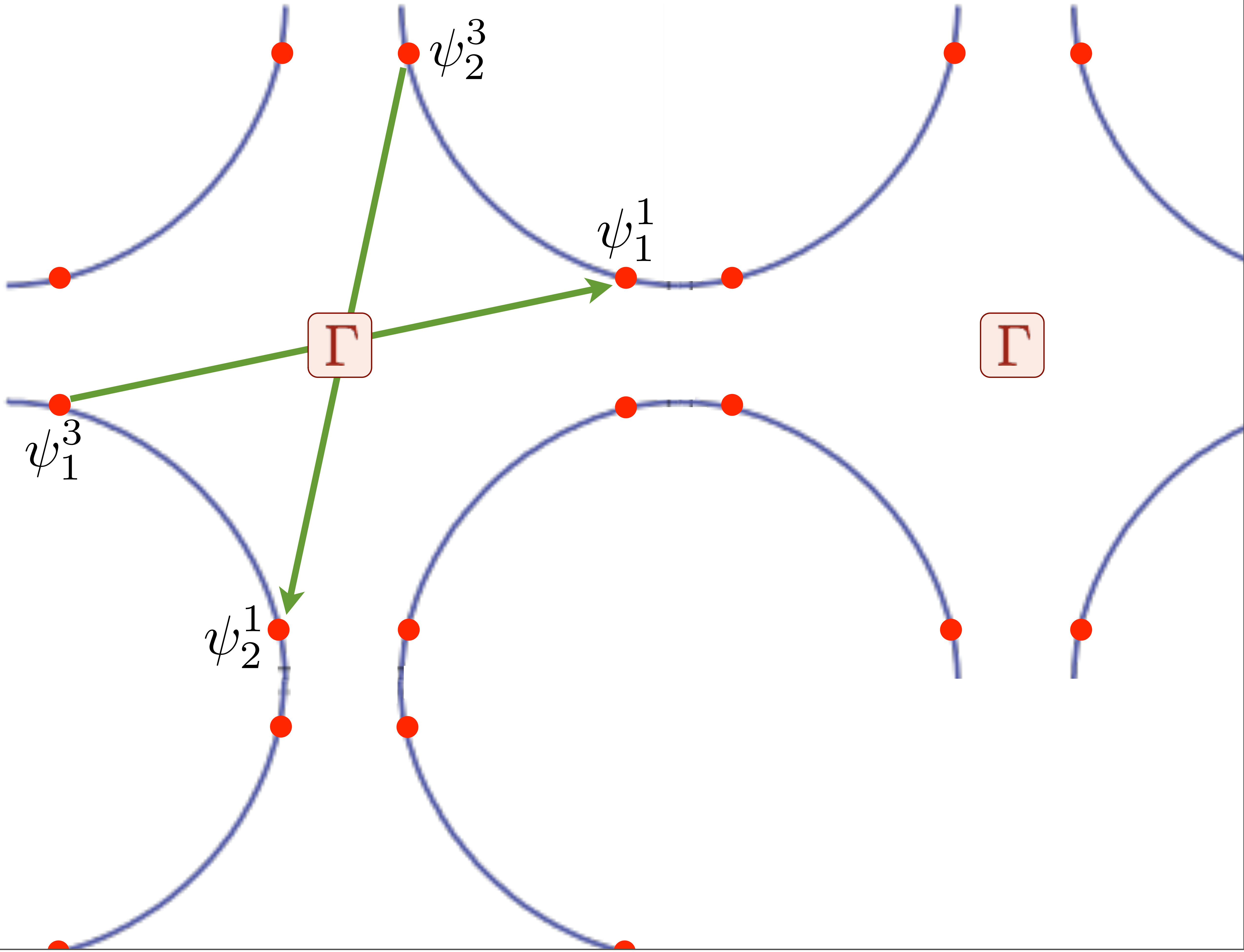
Bond density wave (with local Ising-nematic order) instability in particle-hole channel

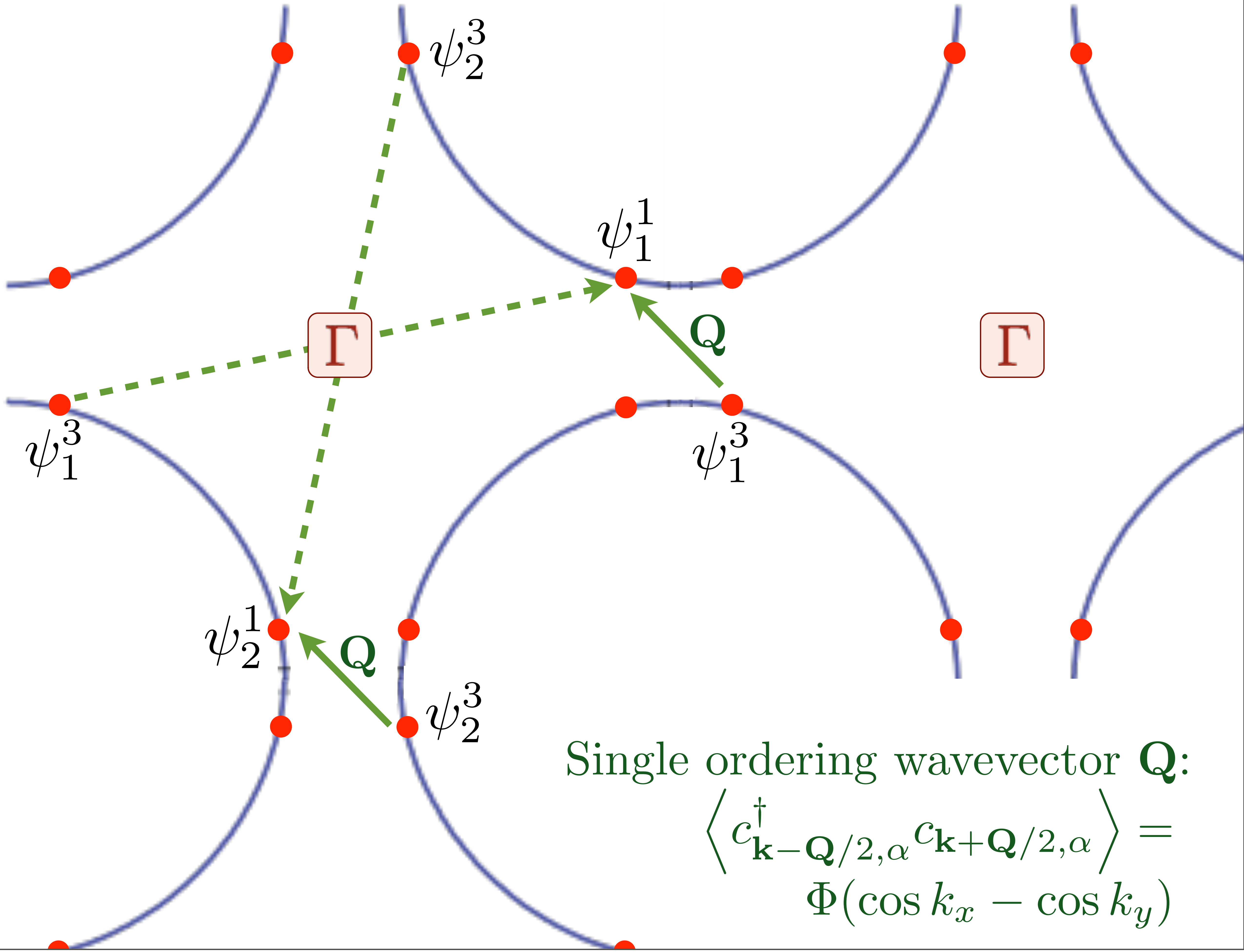


d-wave pairing has a partner instability in the particle-hole channel

Density-wave order parameter:

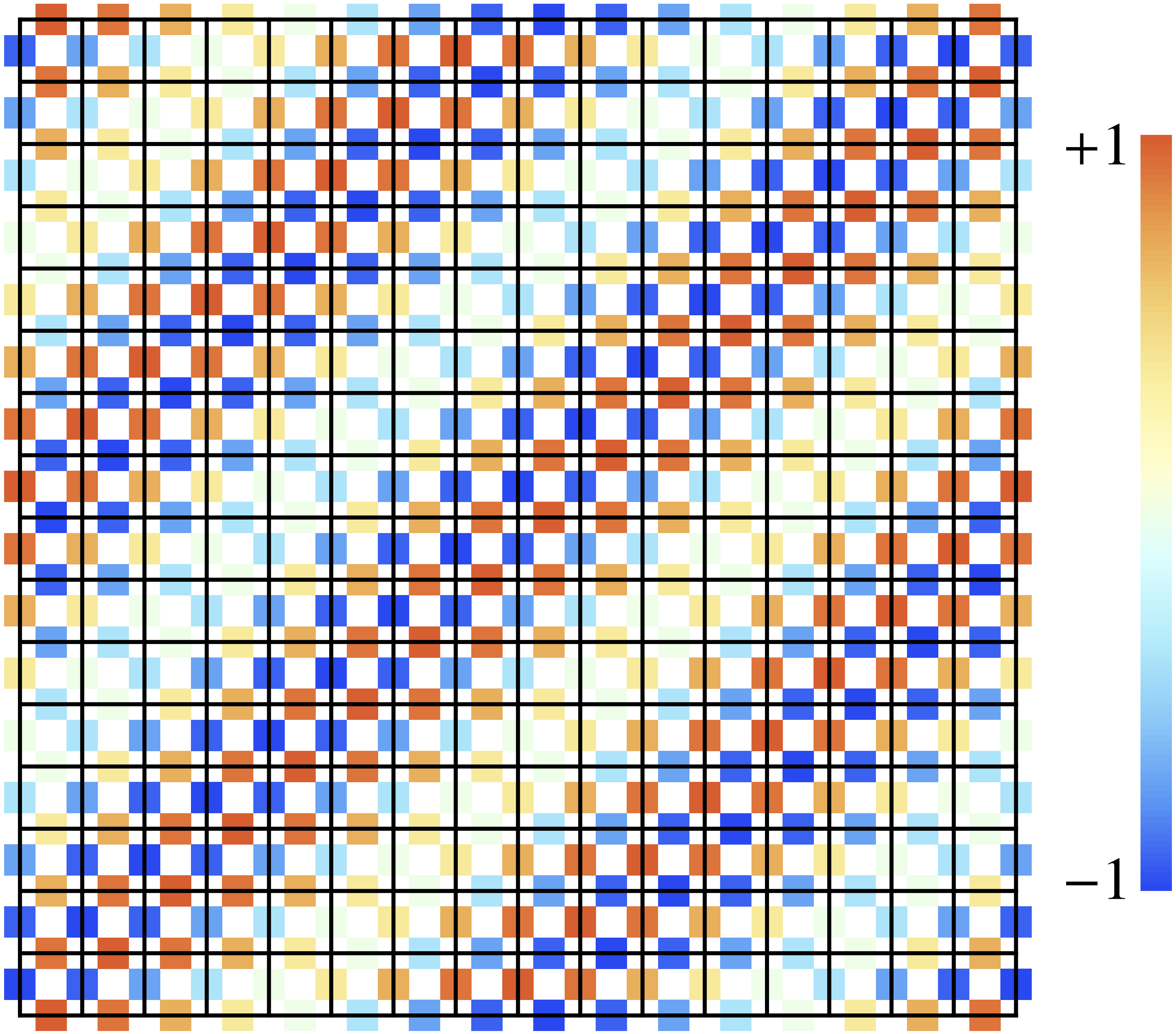
$$\left(\psi_{1\alpha}^{3\dagger}\psi_{1\alpha}^{1} - \psi_{2\alpha}^{3\dagger}\psi_{2\alpha}^{1}\right)$$


ψ_2^3
ψ_1^1
Γ
Γ
ψ_1^3
ψ_2^1



Single ordering wavevector $\mathbf{Q}$:

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond: VBS order

No modulations on sites. Modulated bond-density wave with local Ising-nematic ordering:

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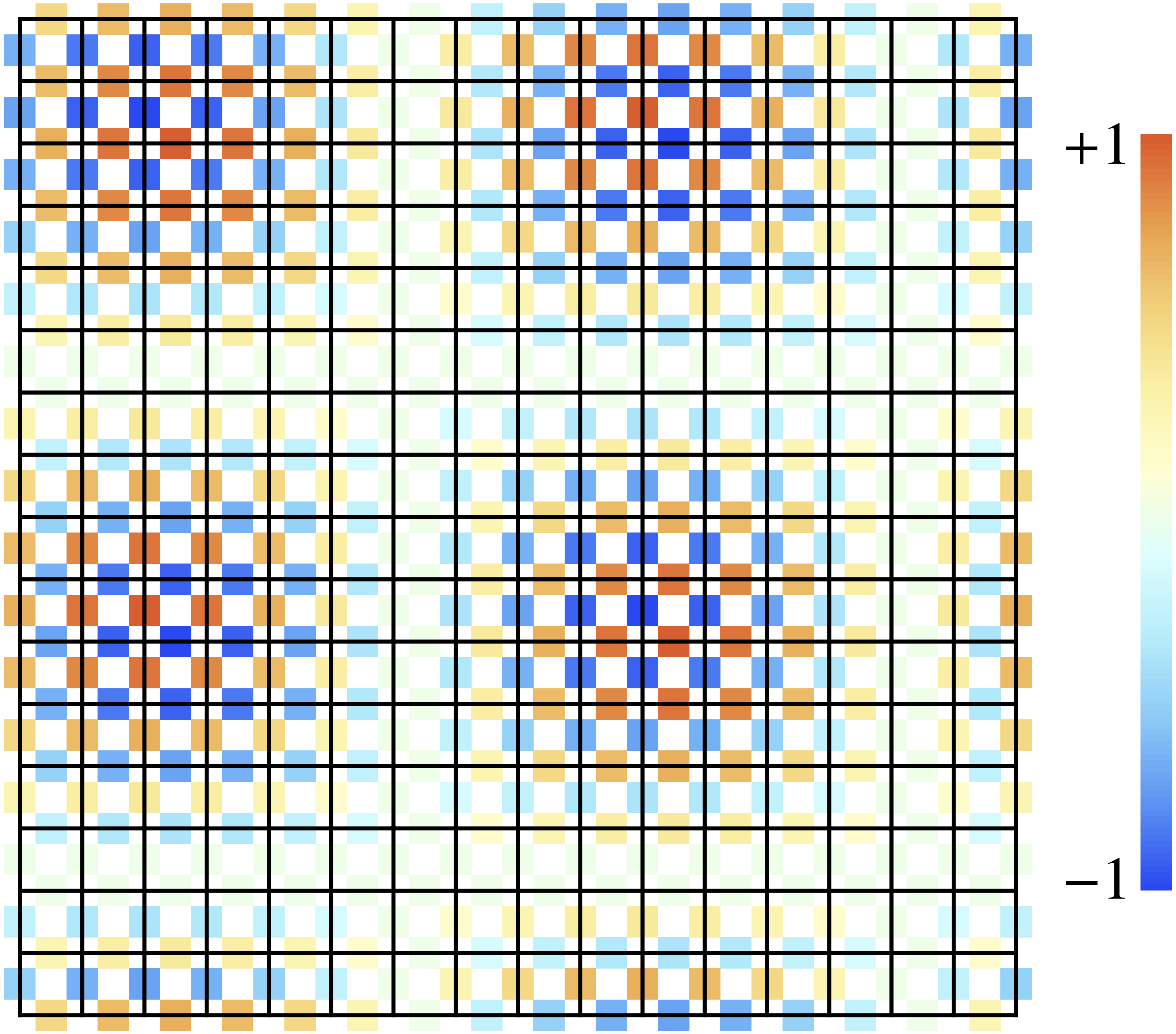

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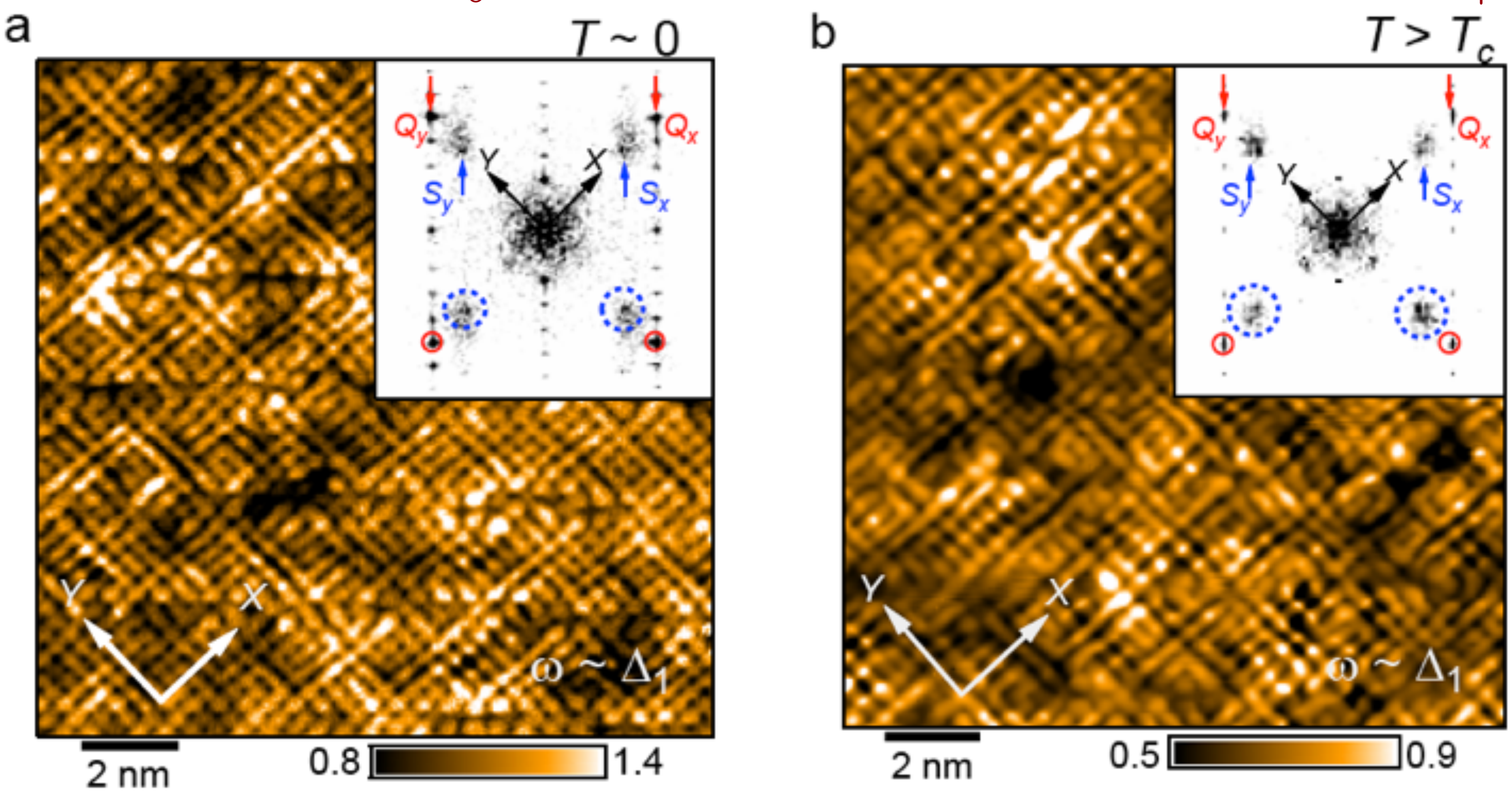
STM measurements of $Z(r)$, the energy asymmetry in density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$.



M. J. Lawler, K. Fujita, Jhinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, preprint

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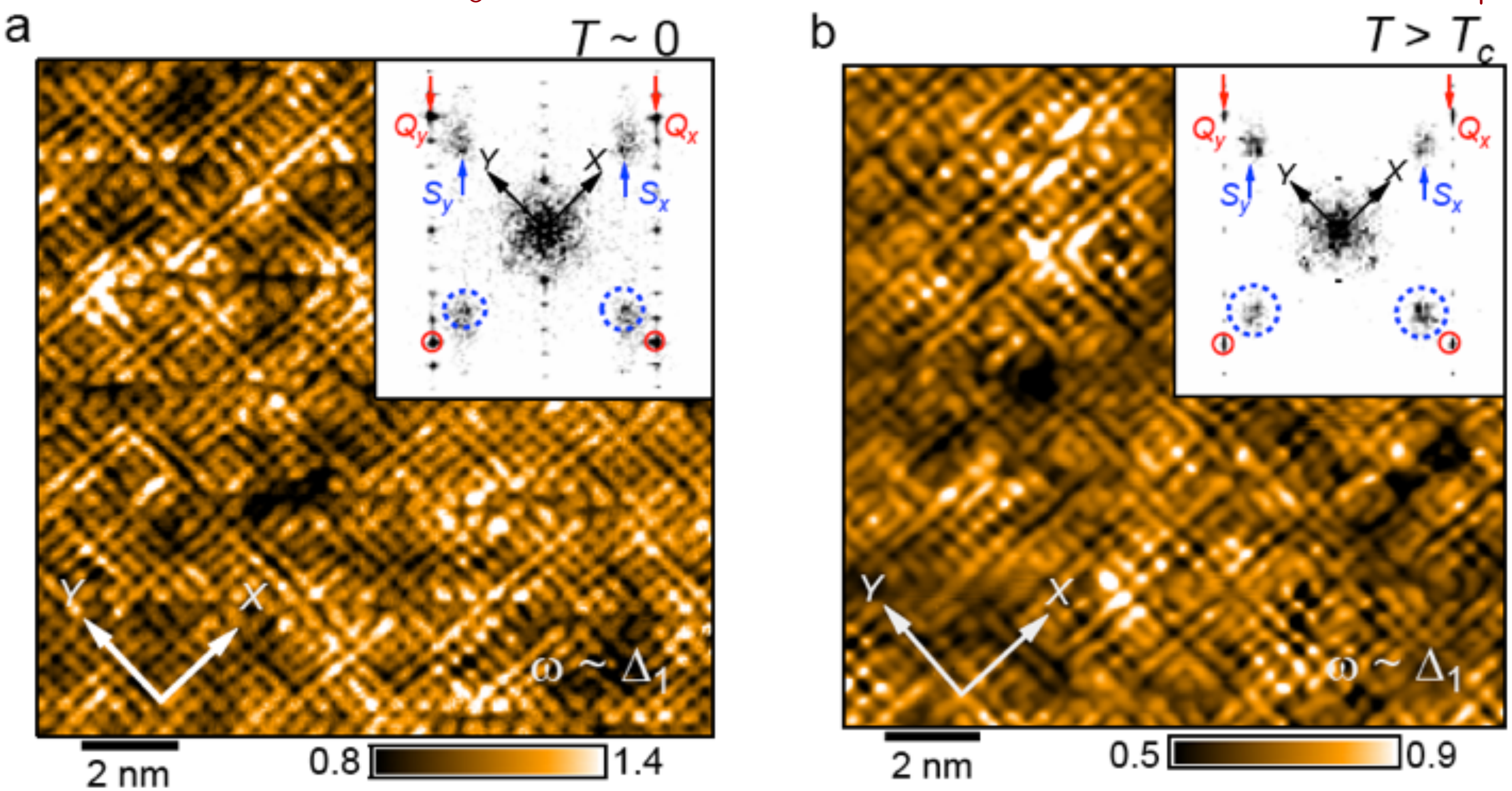

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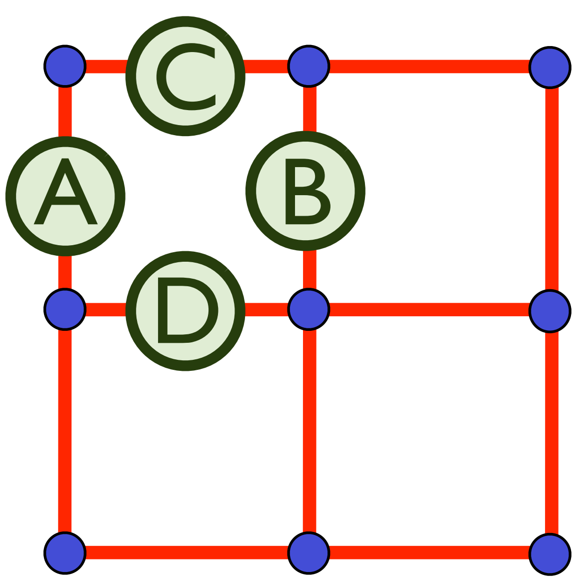

$$O_N = Z_A + Z_B - Z_C - Z_D$$

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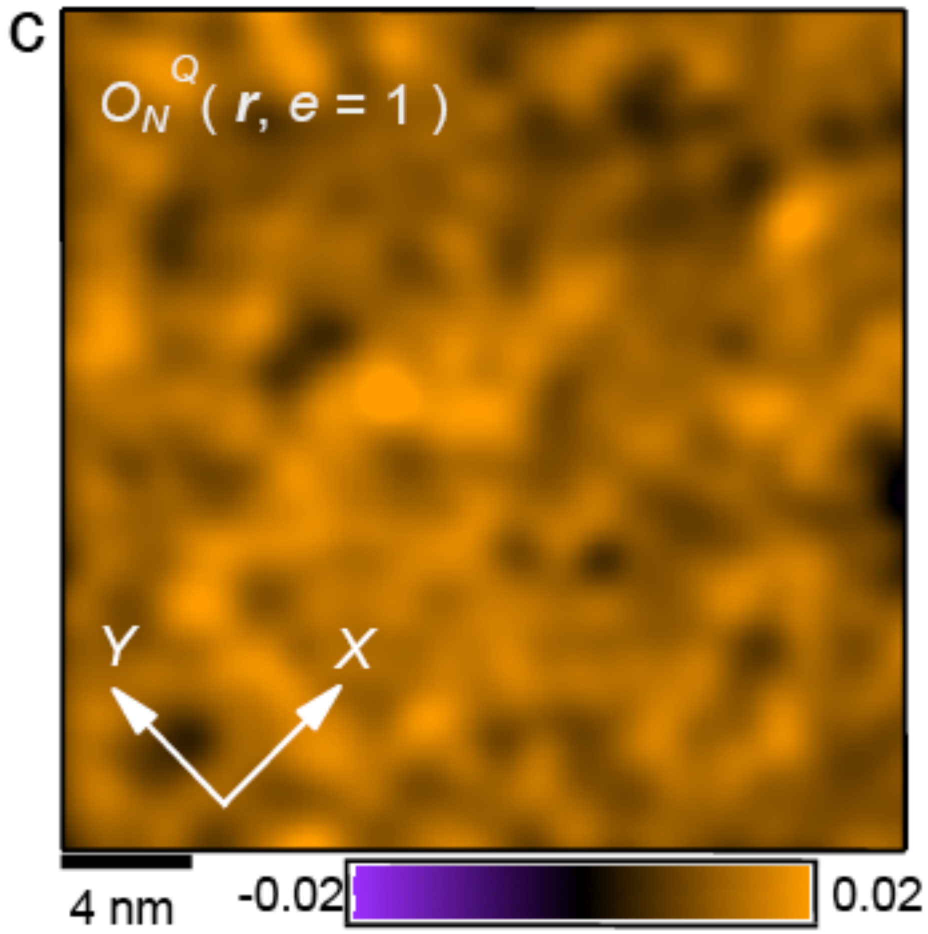

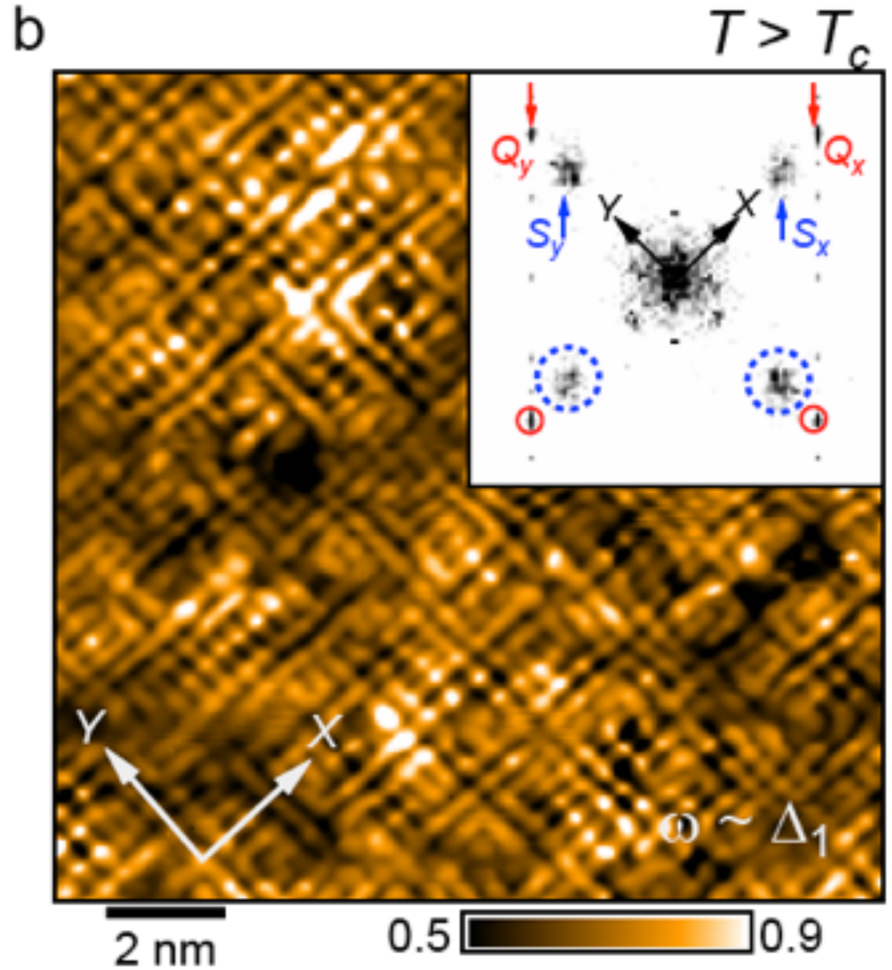

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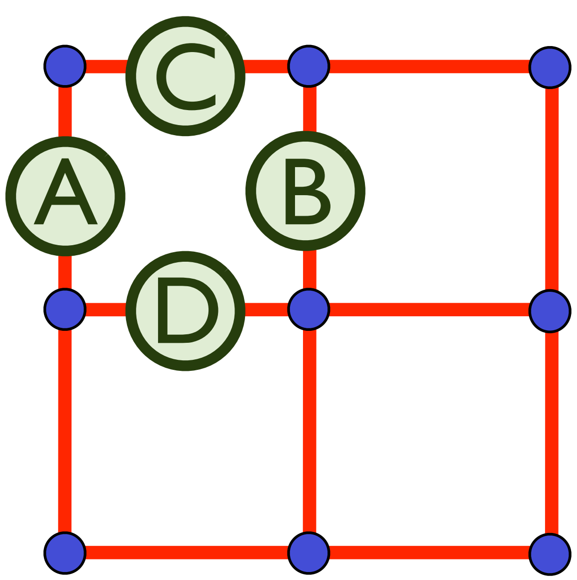

$$O_N = Z_A + Z_B - Z_C - Z_D$$

Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

Conclusions

Theory for the onset of spin density wave in metals is *strongly* coupled in two dimensions

For the cuprate Fermi surface, there are strong instabilities near the quantum critical point to *d*-wave pairing
and
bond density waves with local Ising-nematic ordering

Conclusions

Quantum "disordering" magnetic order leads to valence bond solids and Z_2 spin liquids

Unified theory of spin liquids using Majorana fermions: also includes states obtained by projecting free fermion determinants