Strong coupling problems in condensed matter and the AdS/CFT correspondence

Reviews:

arXiv:0910.1139
arXiv:0901.4103

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD

Yejin Huh, Harvard

Max Metlitski, Harvard

Frederik Denef, Harvard
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

PHYSICS
VE RI TAS
HARVARD

1. Quantum-critical transport

   Collisionless-to-hydrodynamic crossover of CFT3s

2. Quantum criticality of Dirac fermions

   "Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

   The genus expansion

1. Quantum-critical transport

Collisionless-to-hydrodynamic crossover of CFT3s

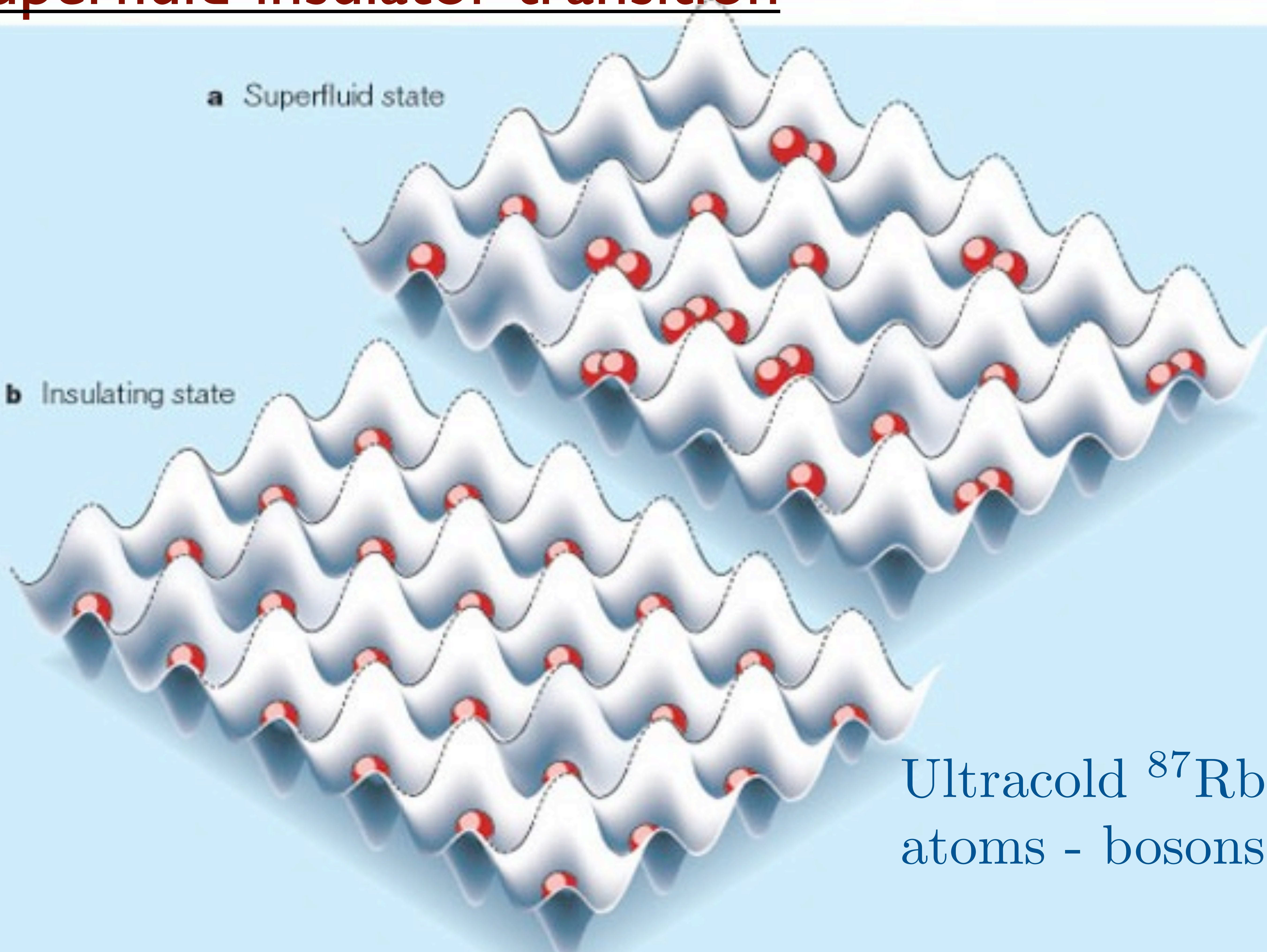
2. Quantum criticality of Dirac fermions

"Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

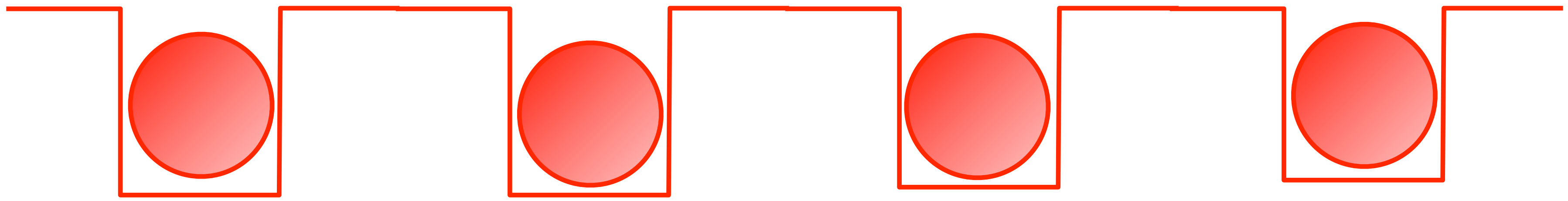
The genus expansion

Superfluid-insulator transition



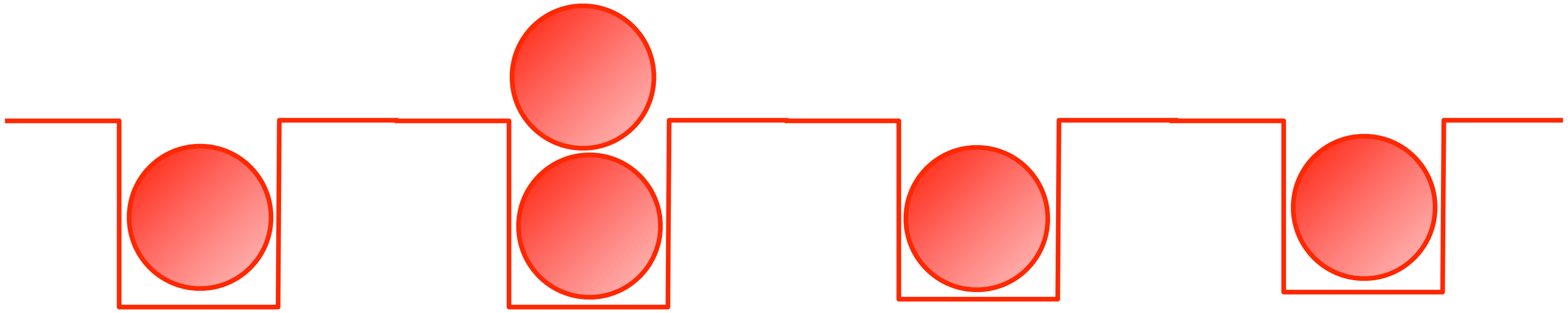
Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



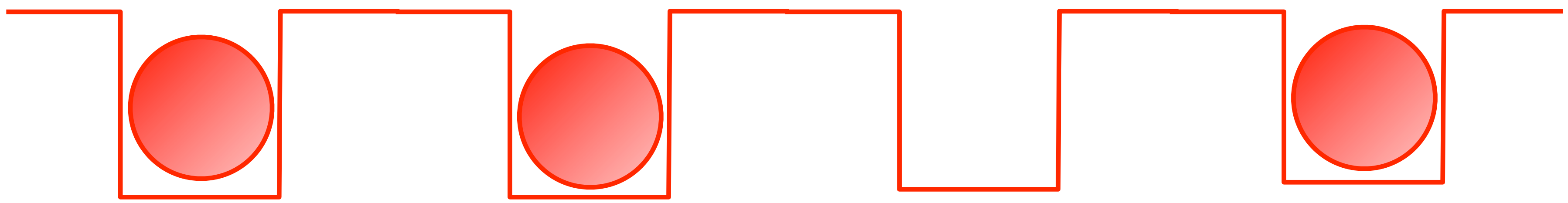
Insulator (the vacuum) at large U

Excitations:



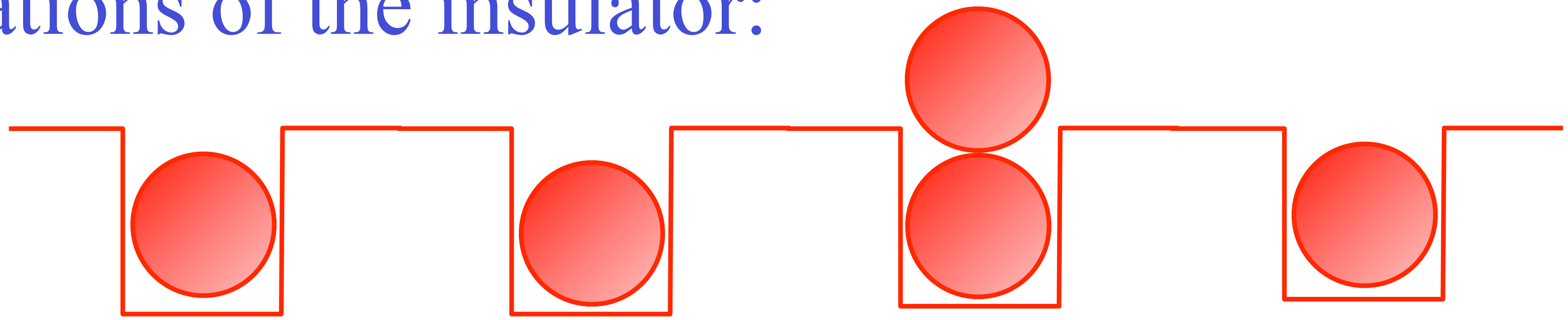
Particles ~ $\psi^\dagger$

Excitations:



Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$

Holes ~ ψ

Density of particles = density of holes ⇒
"relativistic" field theory for ψ:

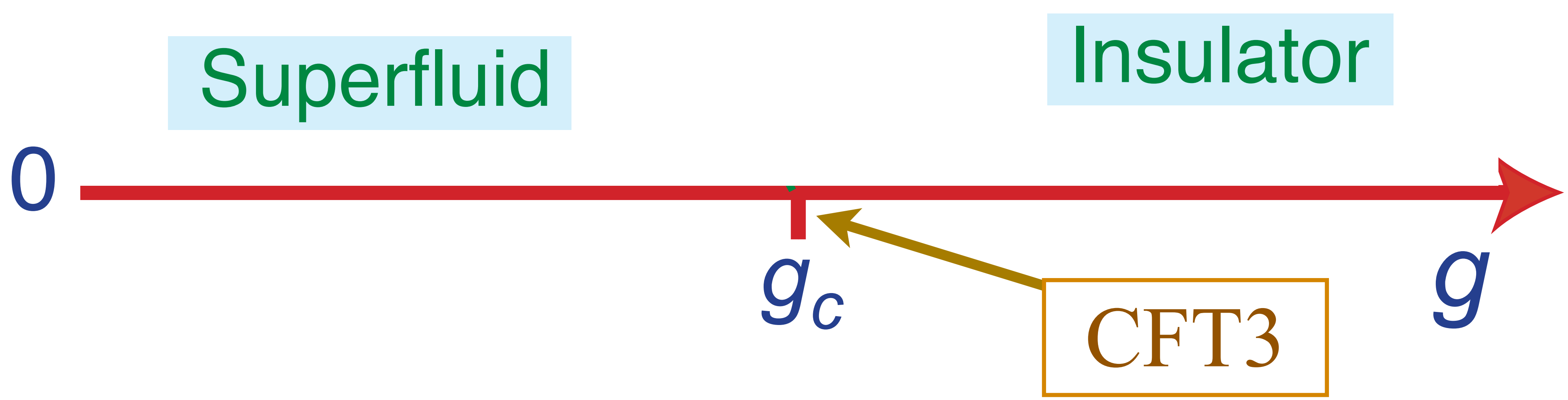
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

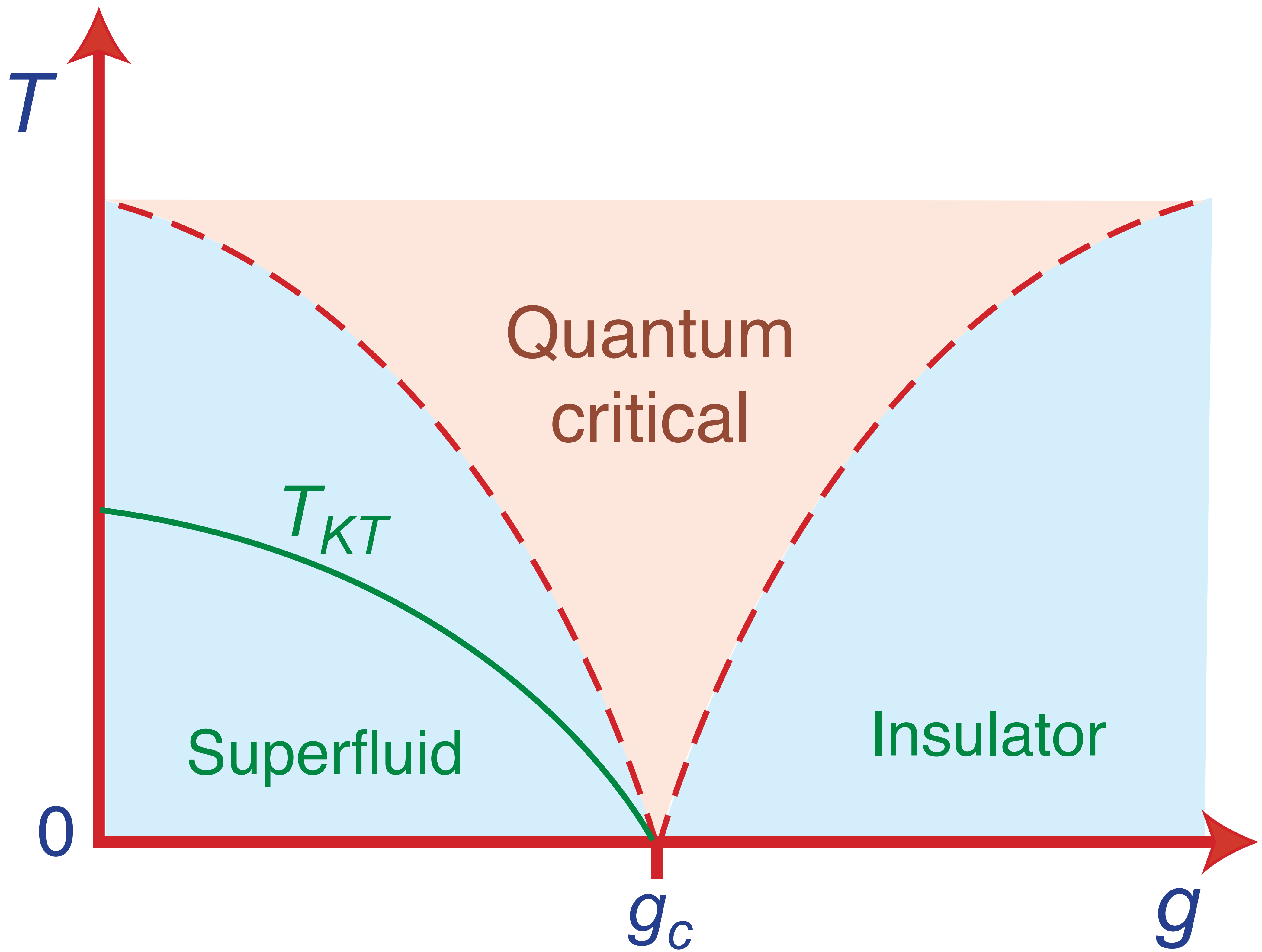
Insulator ⇔ $\langle \psi \rangle = 0$

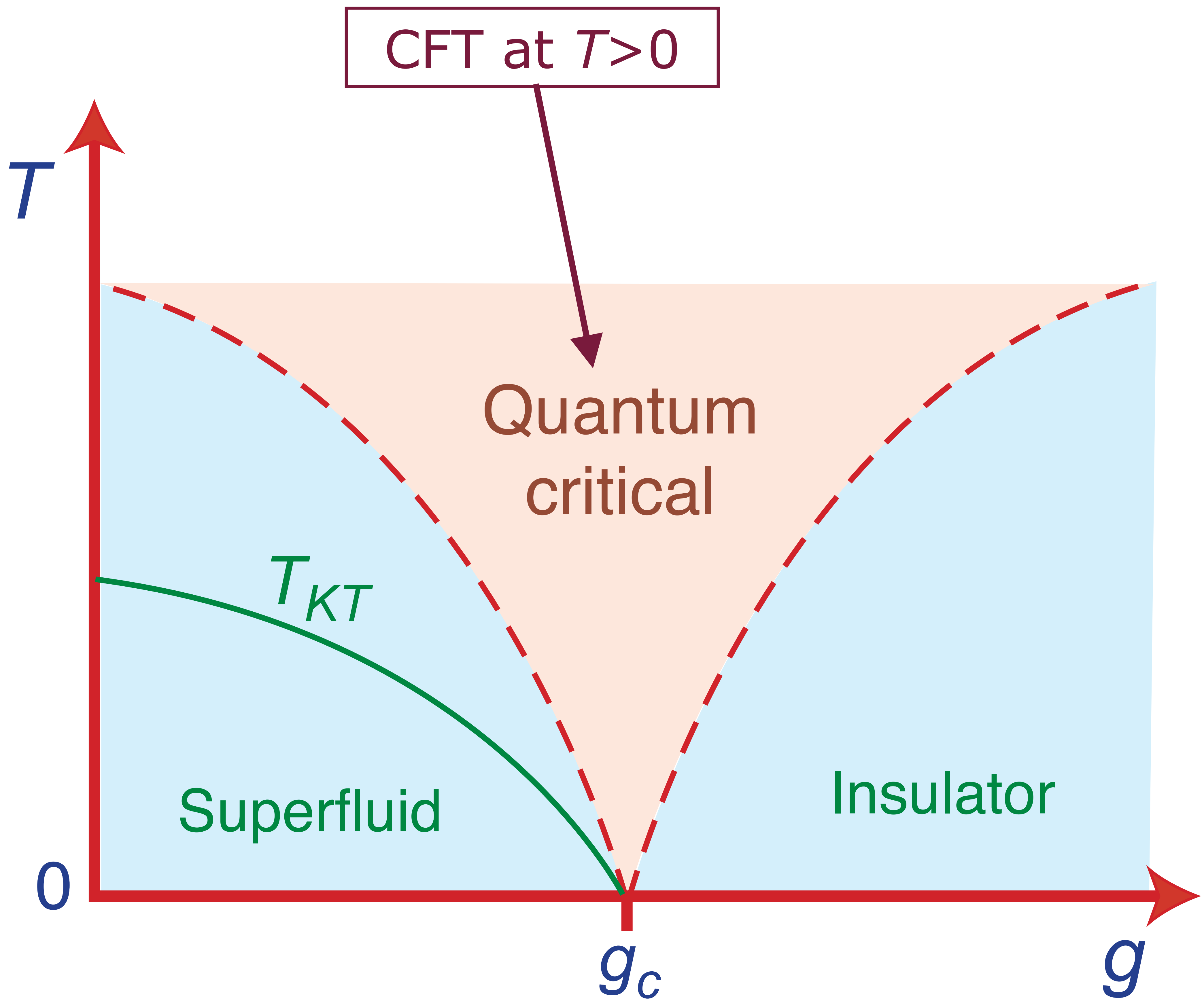
Superfluid ⇔ $\langle \psi \rangle \neq 0$

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$\langle \psi \rangle \neq 0$ $\langle \psi \rangle = 0$




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"perfect fluid"* with shortest possible relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{4e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

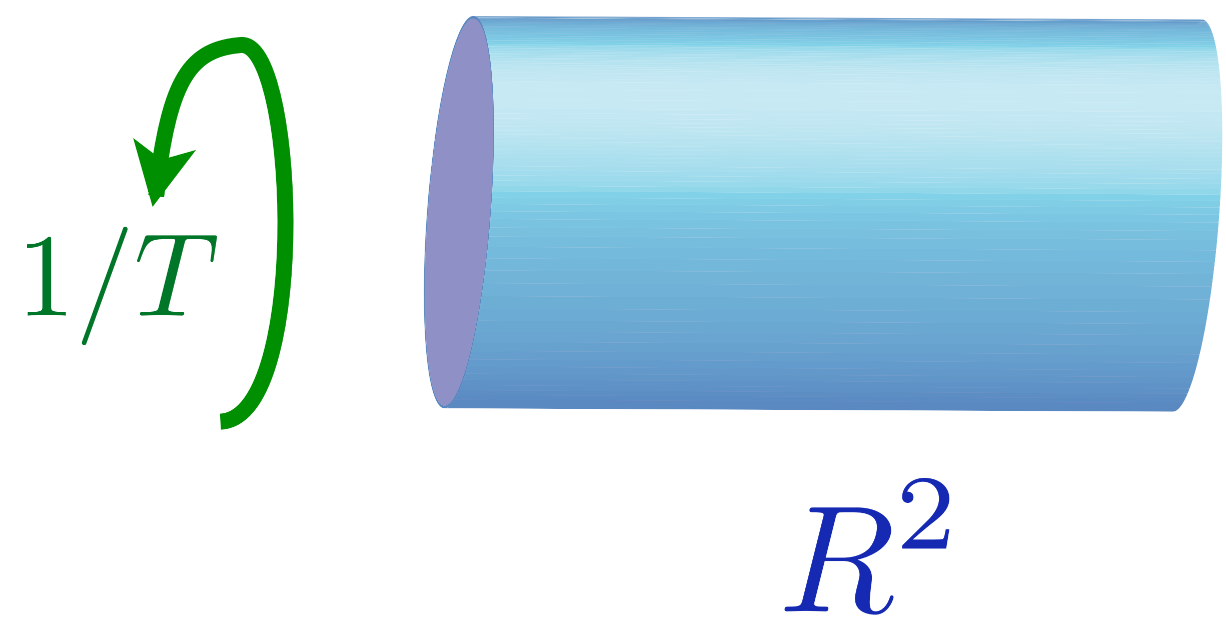
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

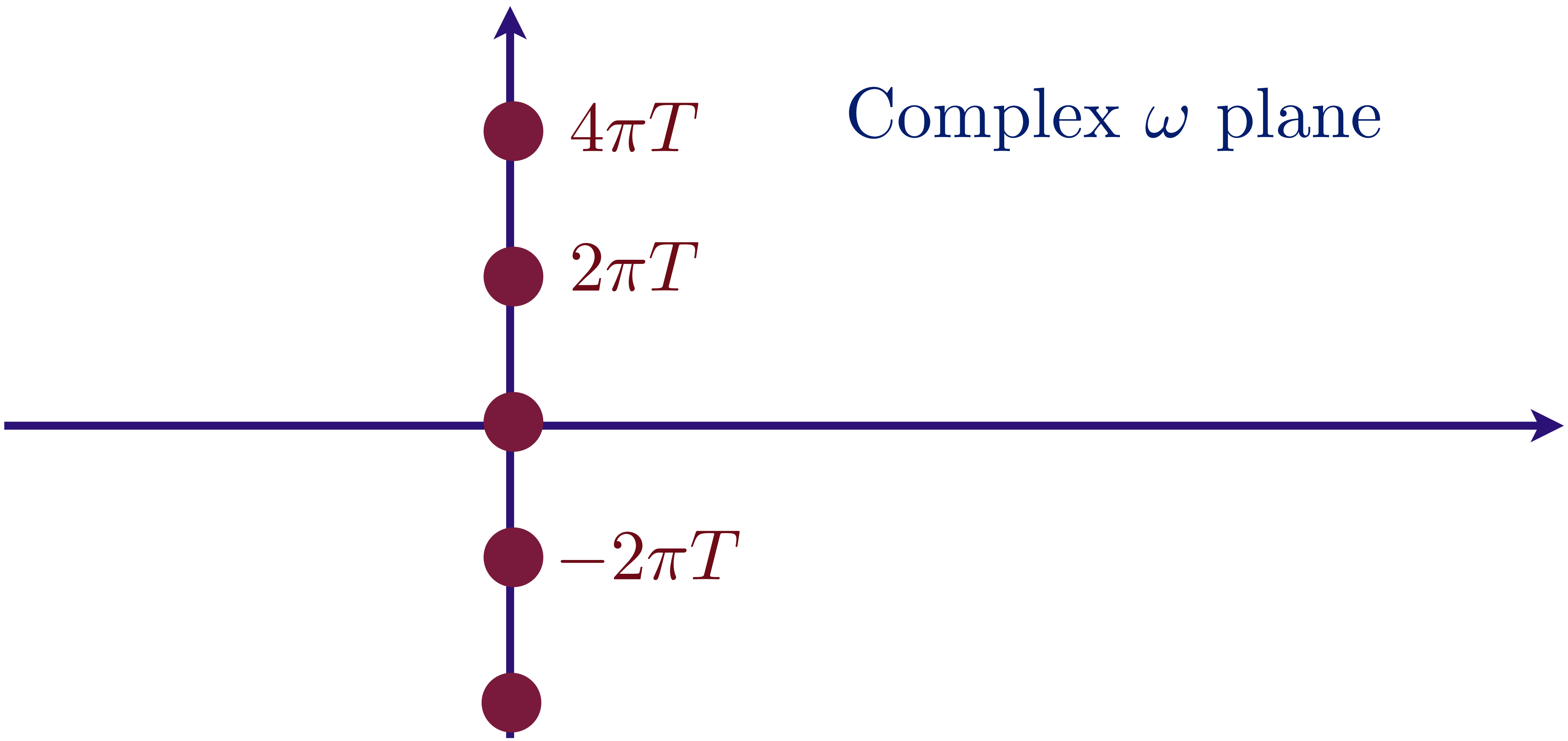
Euclidean field theory:

Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Quantum critical transport

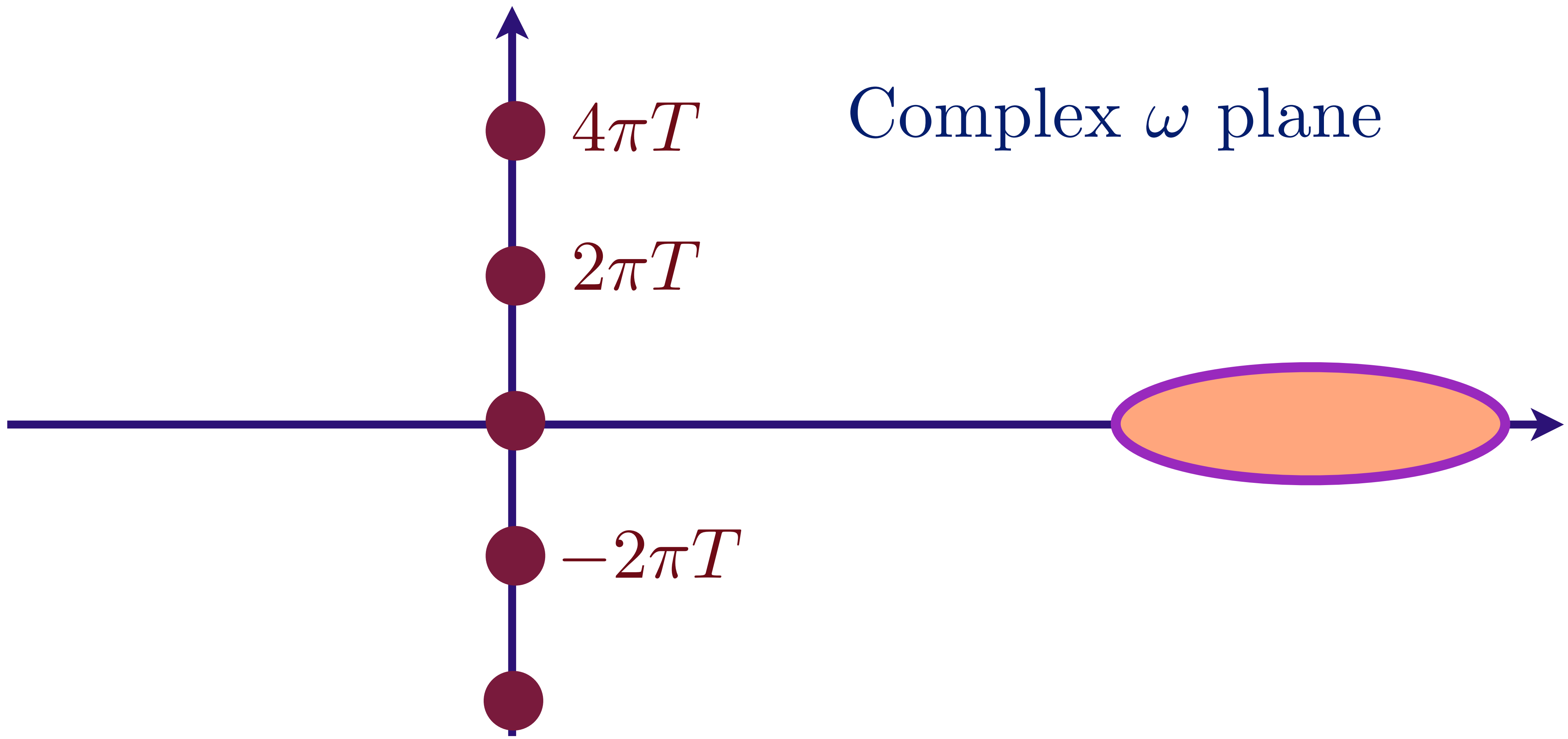
Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer) or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer)
or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Density correlations in CFTs at $T>0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

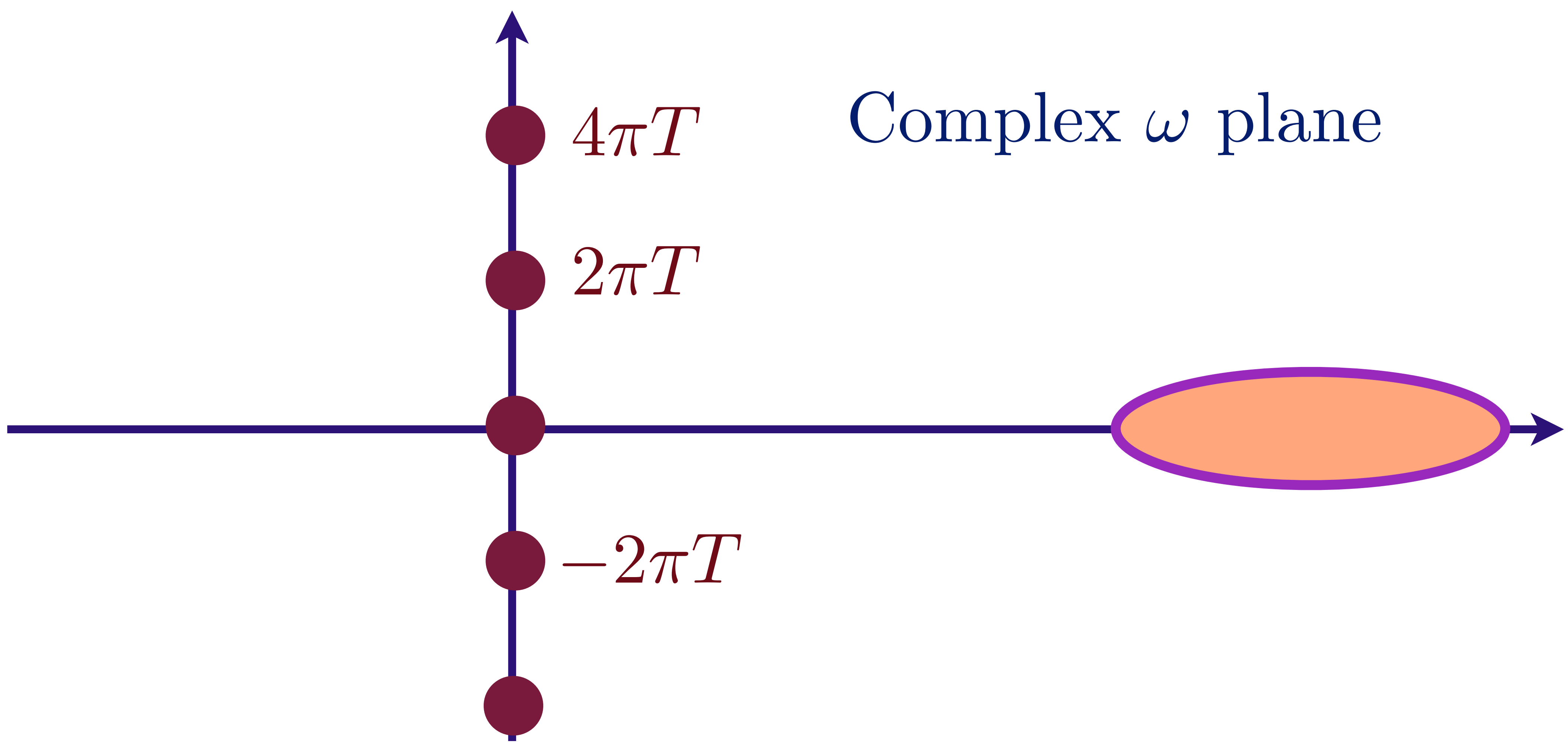
For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2k^2-\omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Quantum critical transport

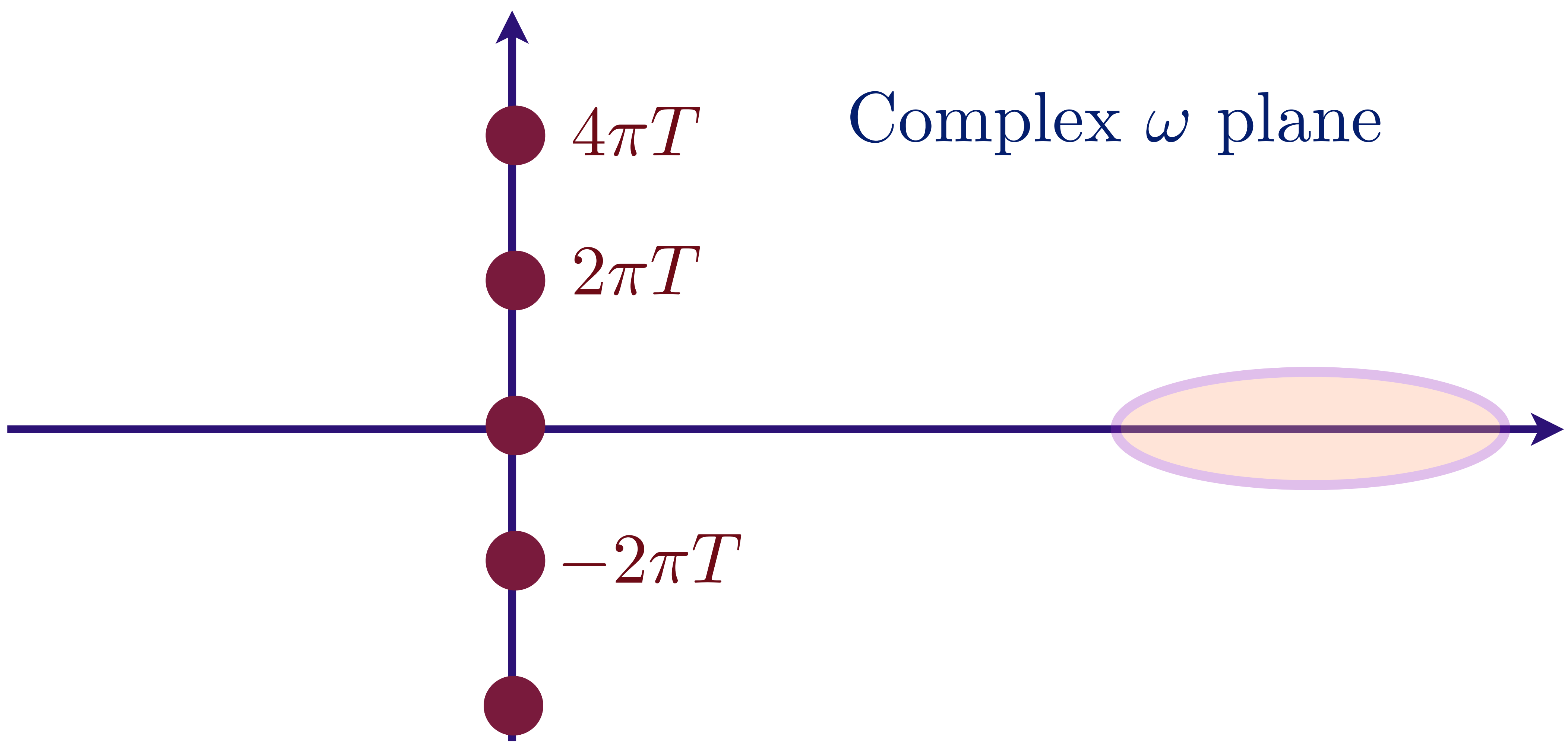
Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer)
or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$

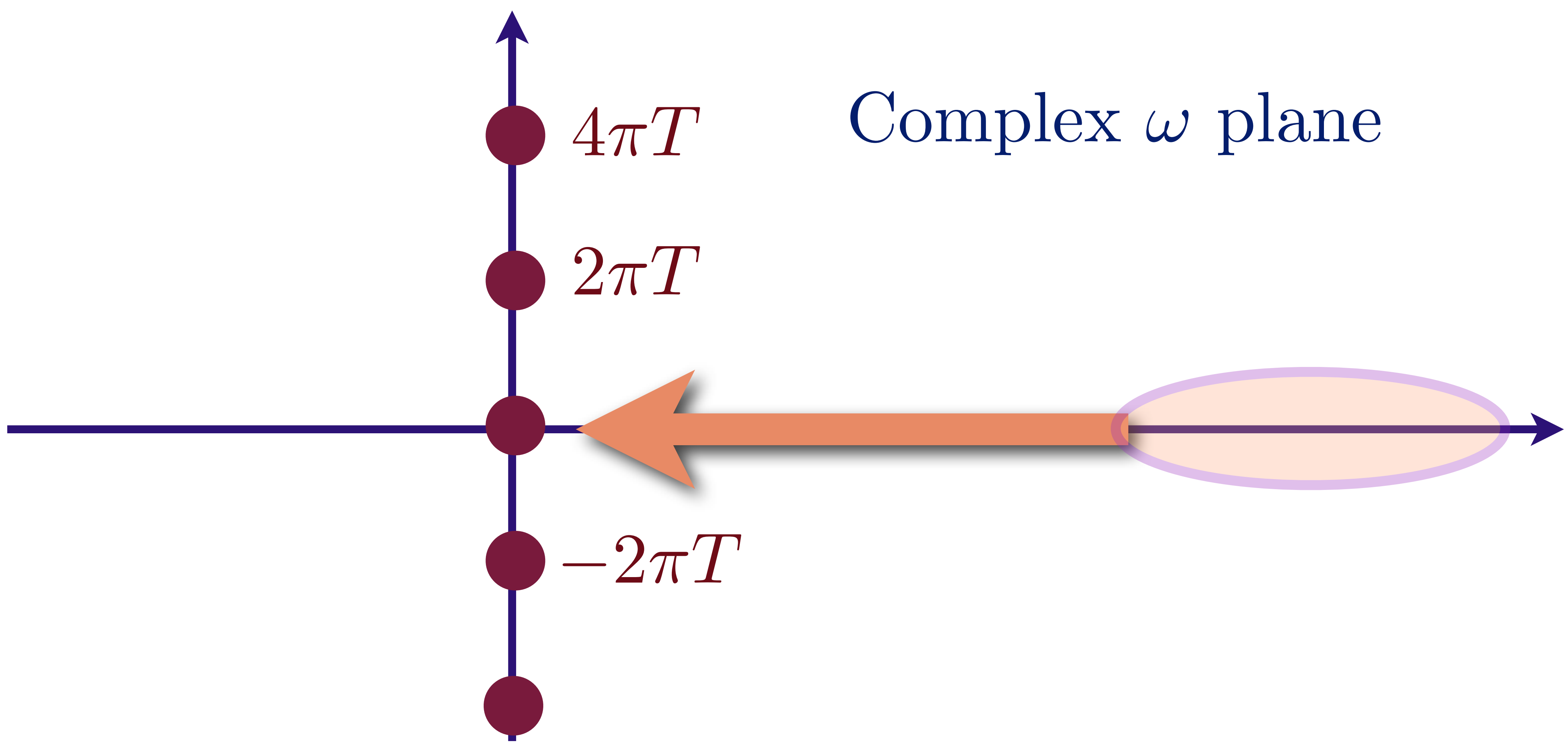


Strong coupling problem:

Correlators at $\hbar\omega \ll k_B T$, along the real axis, in the collision-dominated hydrodynamic regime.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Strong coupling problem:
Correlators at $\hbar\omega \ll k_B T$, along the real axis, in the collision-dominated hydrodynamic regime.

Density correlations in CFTs at $T>0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k,\omega) = 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

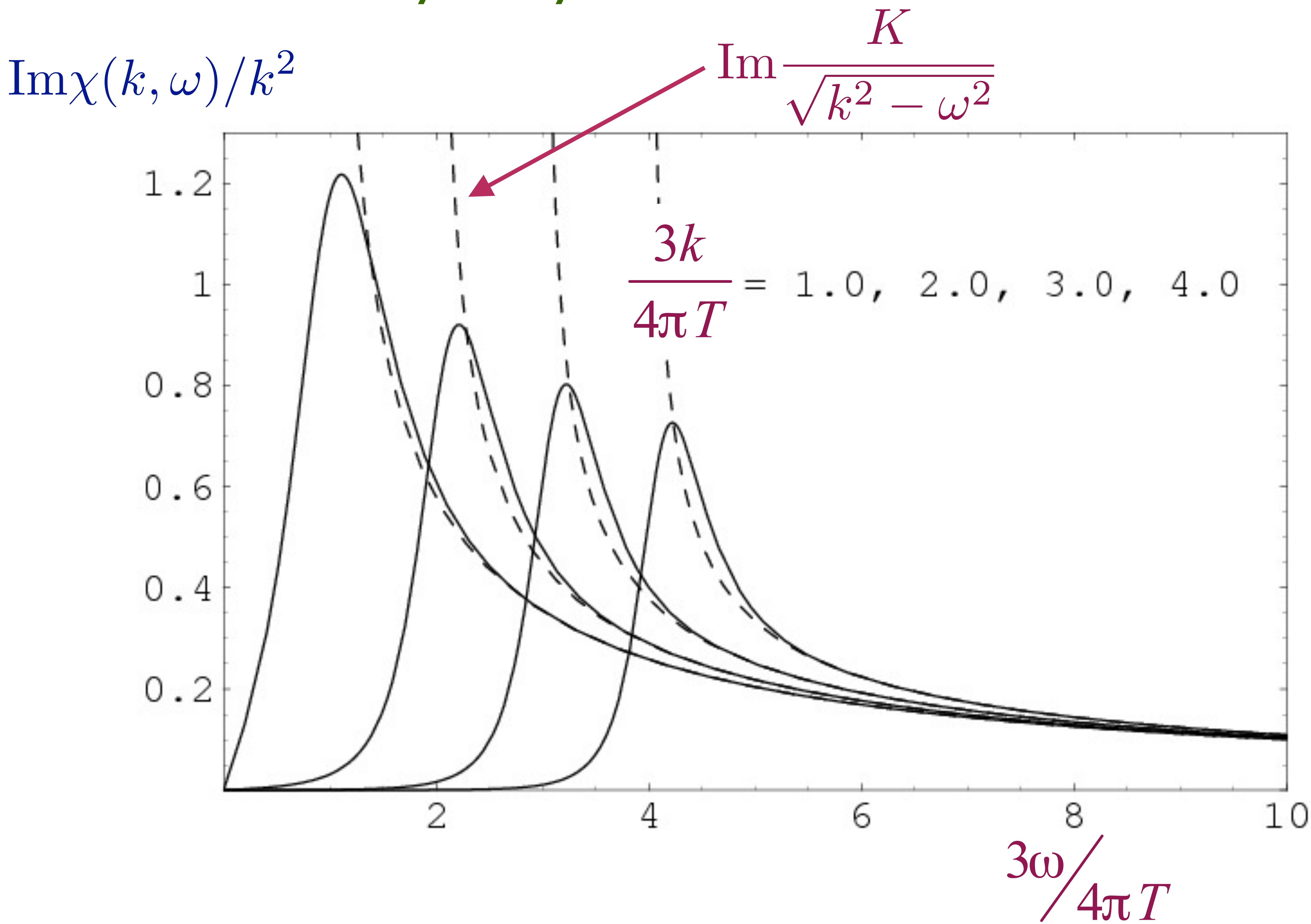
where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

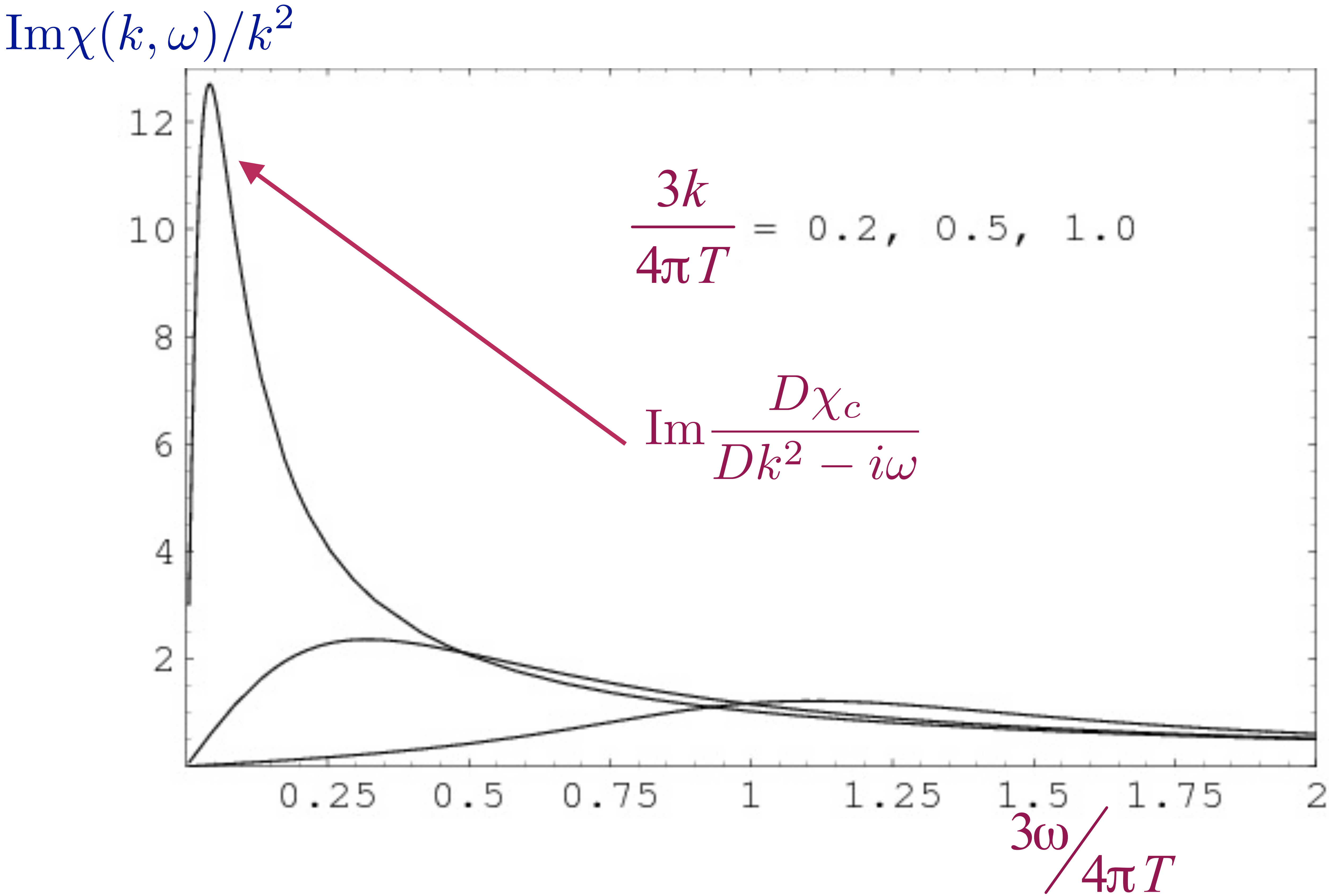
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

1. Quantum-critical transport

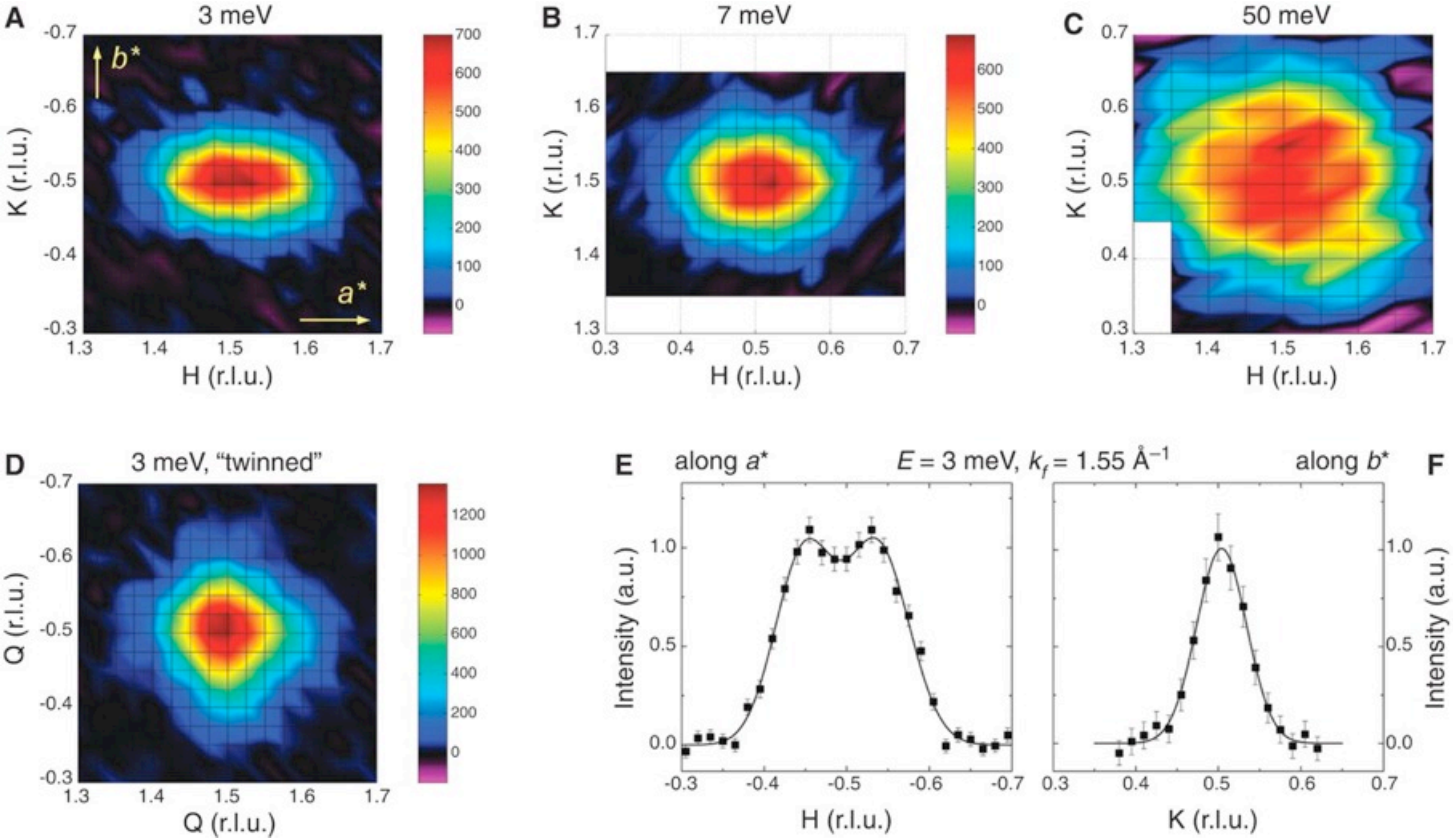
   Collisionless-to-hydrodynamic crossover of CFT3s

2. Quantum criticality of Dirac fermions

   "Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

   The genus expansion

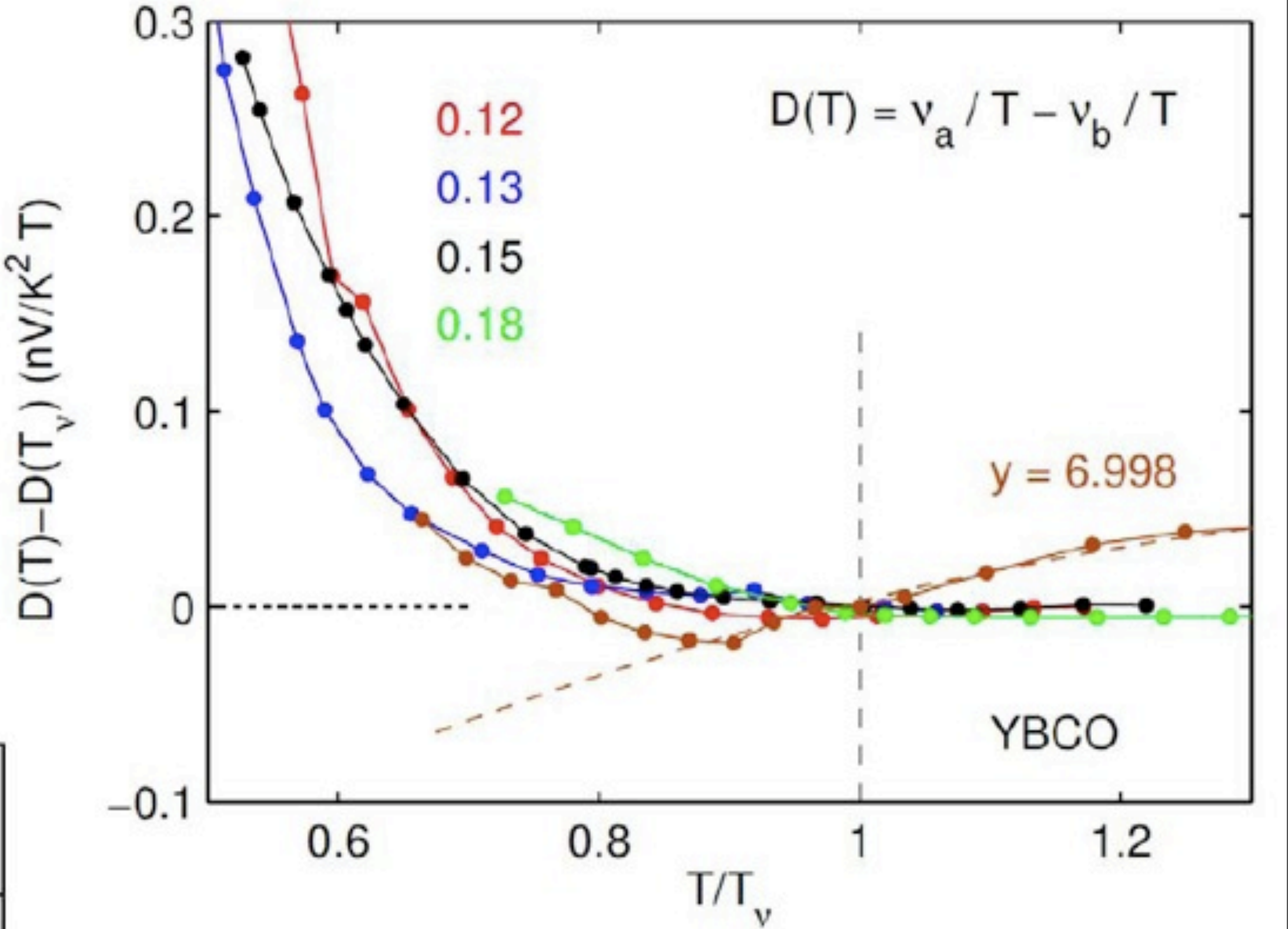


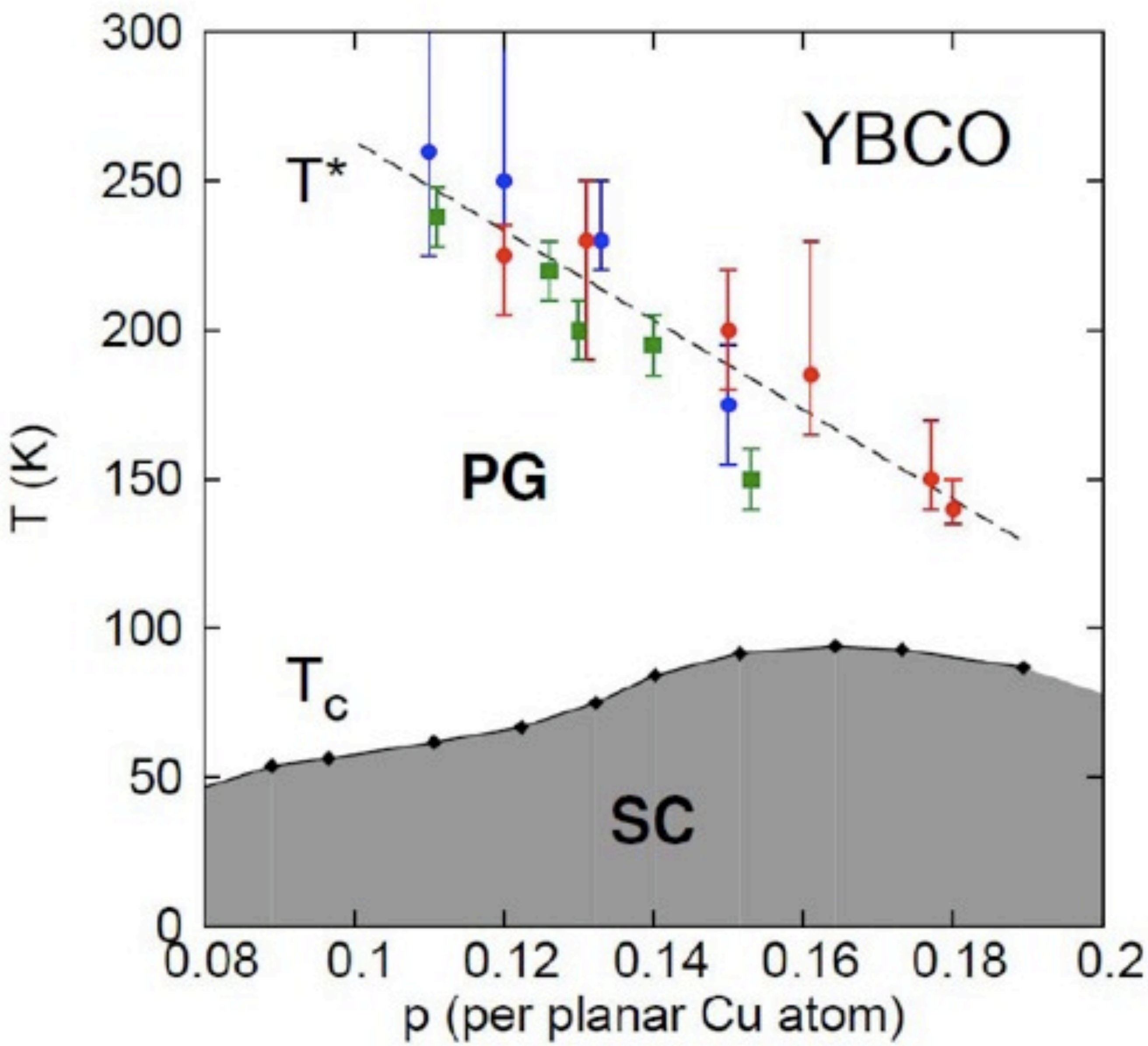
Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D. A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430





S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).

1. Quantum-critical transport

Collisionless-to-hydrodynamic crossover of CFT3s

2. Quantum criticality of Dirac fermions

"Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

The genus expansion

1. Quantum-critical transport

Collisionless-to-hydrodynamic crossover of CFT3s

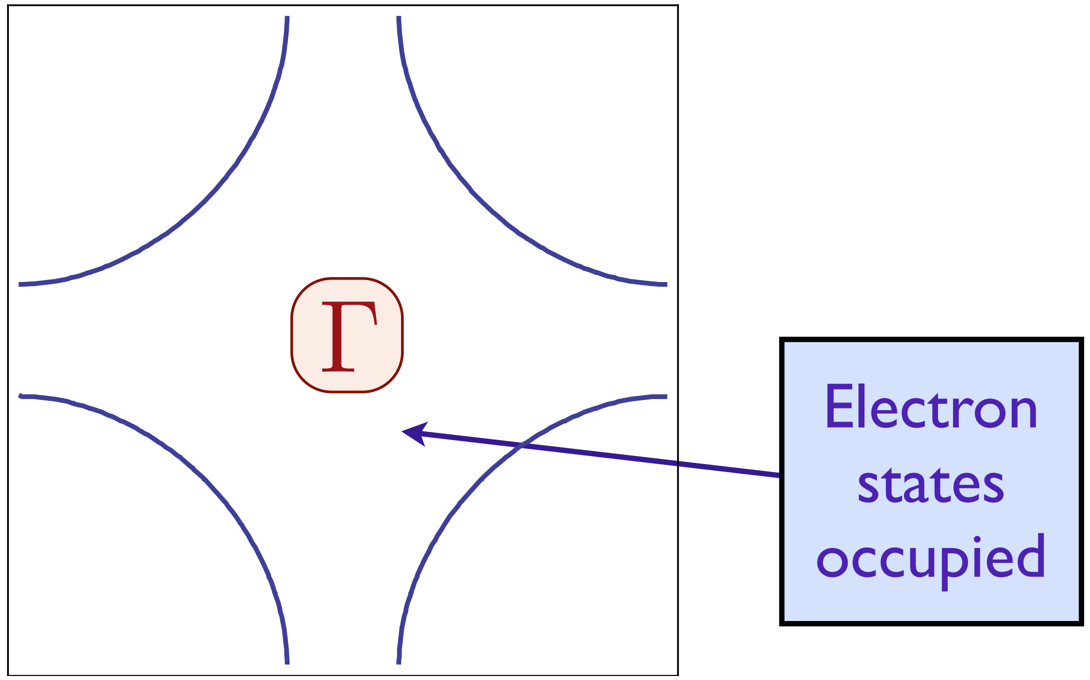
2. Quantum criticality of Dirac fermions

"Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

The genus expansion

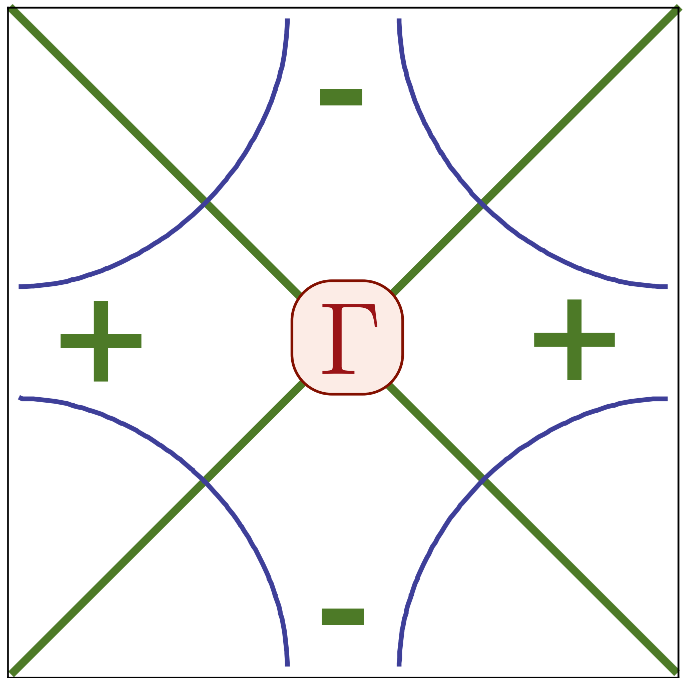
d-wave superconductivity in cuprates



$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

- Begin with free electrons.

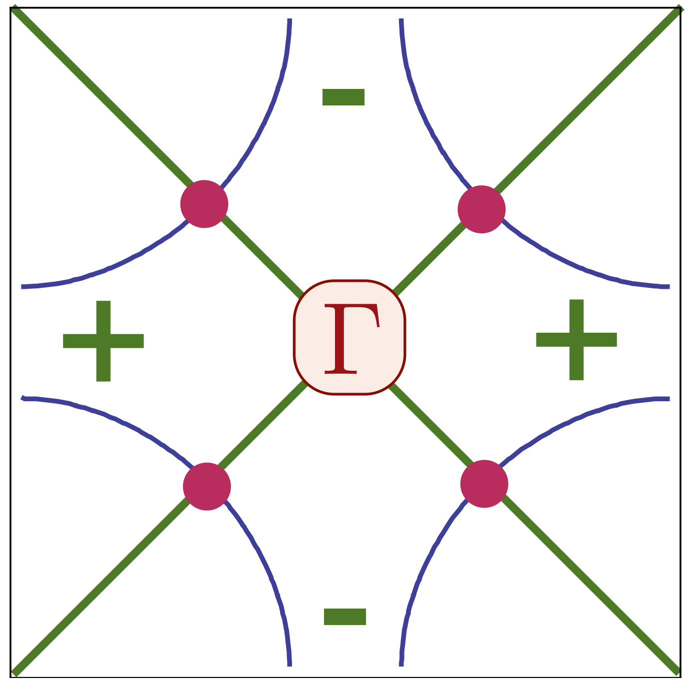
d-wave superconductivity in cuprates



$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

- Begin with free electrons.
- Add d-wave pairing interaction $\Delta_k \sim \cos k_x - \cos k_y$ which vanishes along diagonals

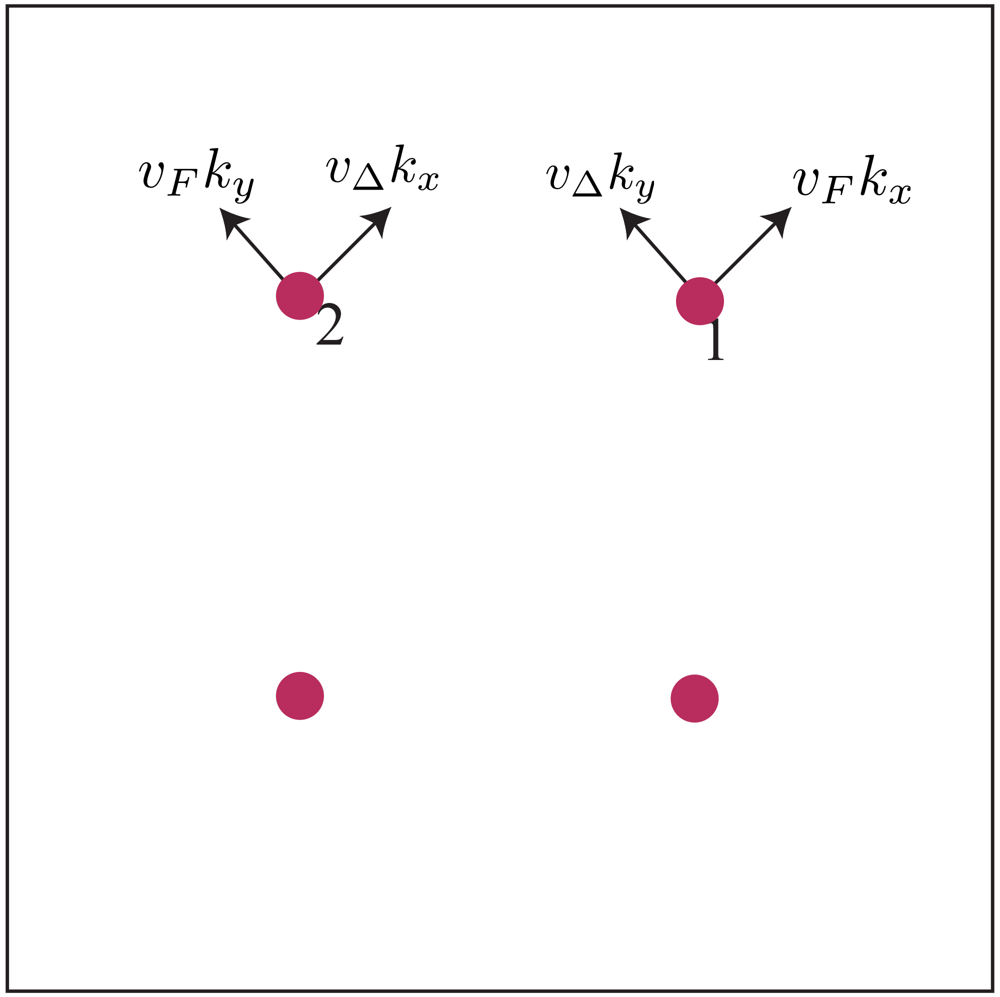
d-wave superconductivity in cuprates



$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

- Begin with free electrons.
- Add d-wave pairing interaction Δ_k which vanishes along diagonals
- Obtain Bogoliubov quasiparticles with dispersion $\sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$

d-wave superconductivity in cuprates



4 two-component Dirac fermions

$$S_\Psi = \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{1a} \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a}$$
$$+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{2a} \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}.$$

d-wave superconductivity in cuprates

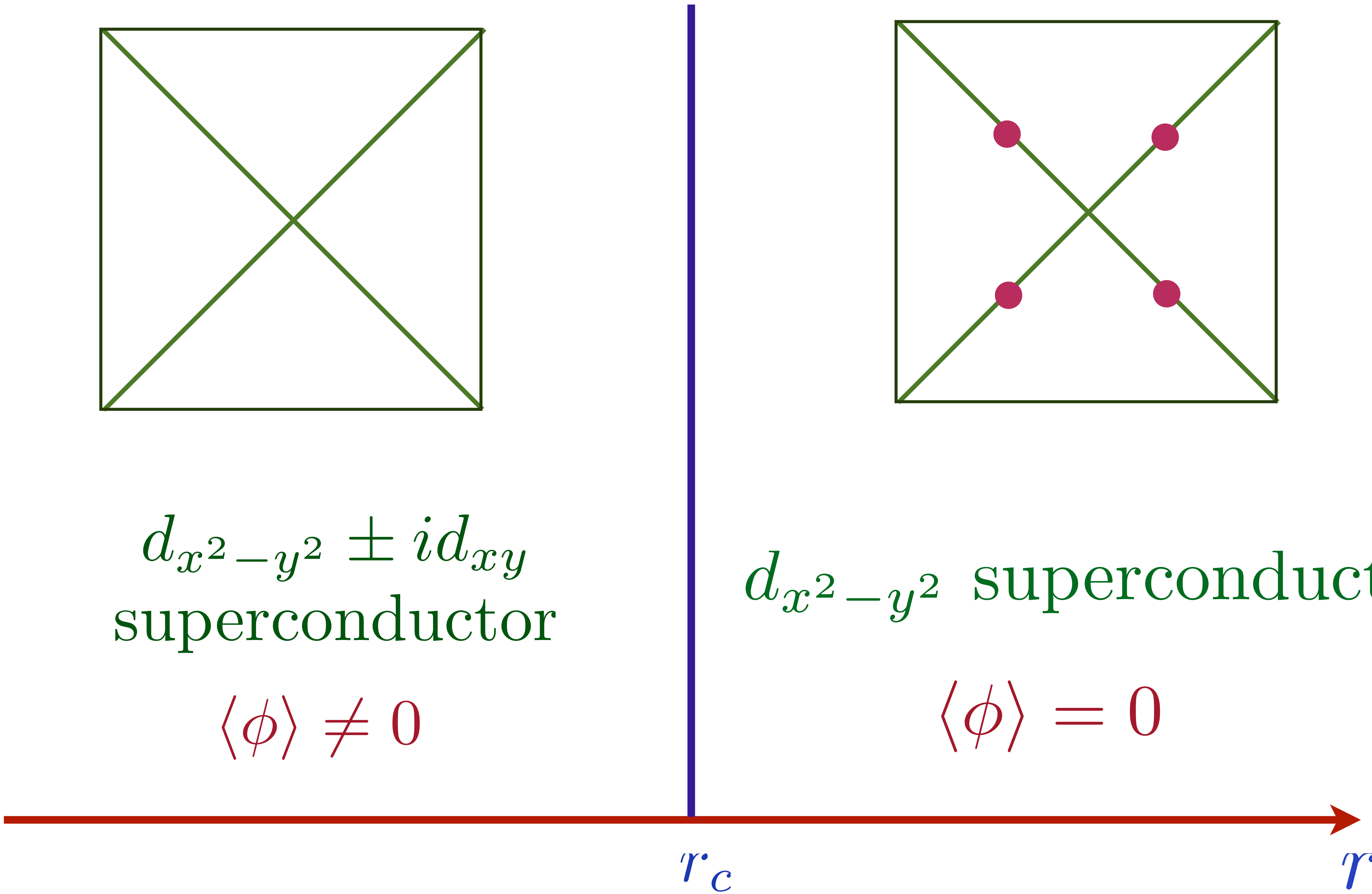
Now consider a discrete spontaneous symmetry breaking, with Ising symmetry, described by a real scalar field ϕ.
Two cases of experimental interest are:

- Time-reversal symmetry breaking: leads to a $d_{x^2-y^2} + id_{xy}$ superconductor, in which the Dirac fermions are massive
- Break 4-fold lattice rotation symmetry to 2-fold lattice rotations: leads to a superconductor with **nematic** order.

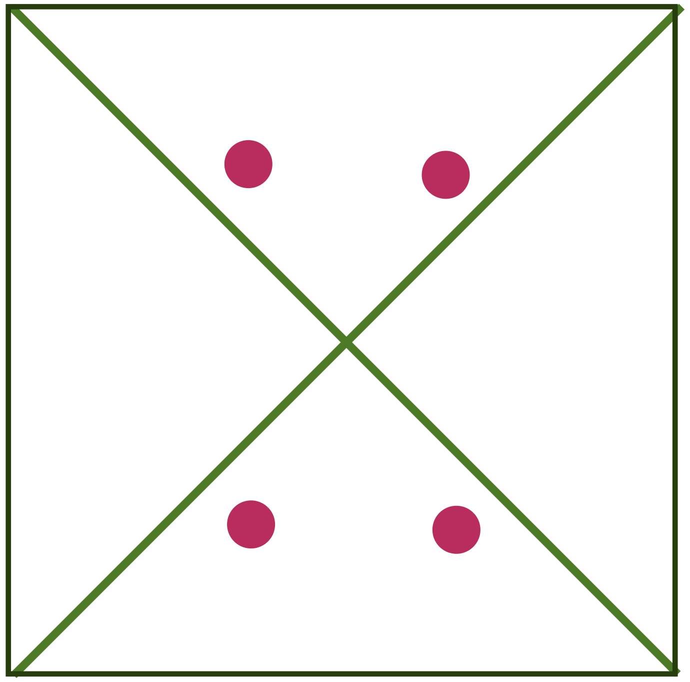
We can write down the usual ϕ^4 theory for the scalar field:

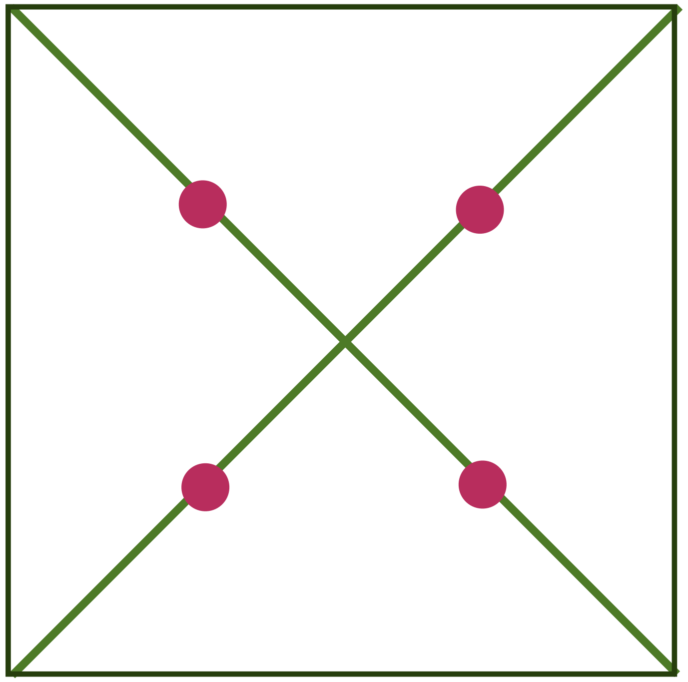
$$S_\phi^0 = \int d^2x d\tau \left[\frac{1}{2} (\partial_\tau \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 + \frac{r}{2} \phi^2 + \frac{u_0}{24} \phi^4 \right]$$

Time-reversal symmetry breaking



Lattice rotation symmetry breaking





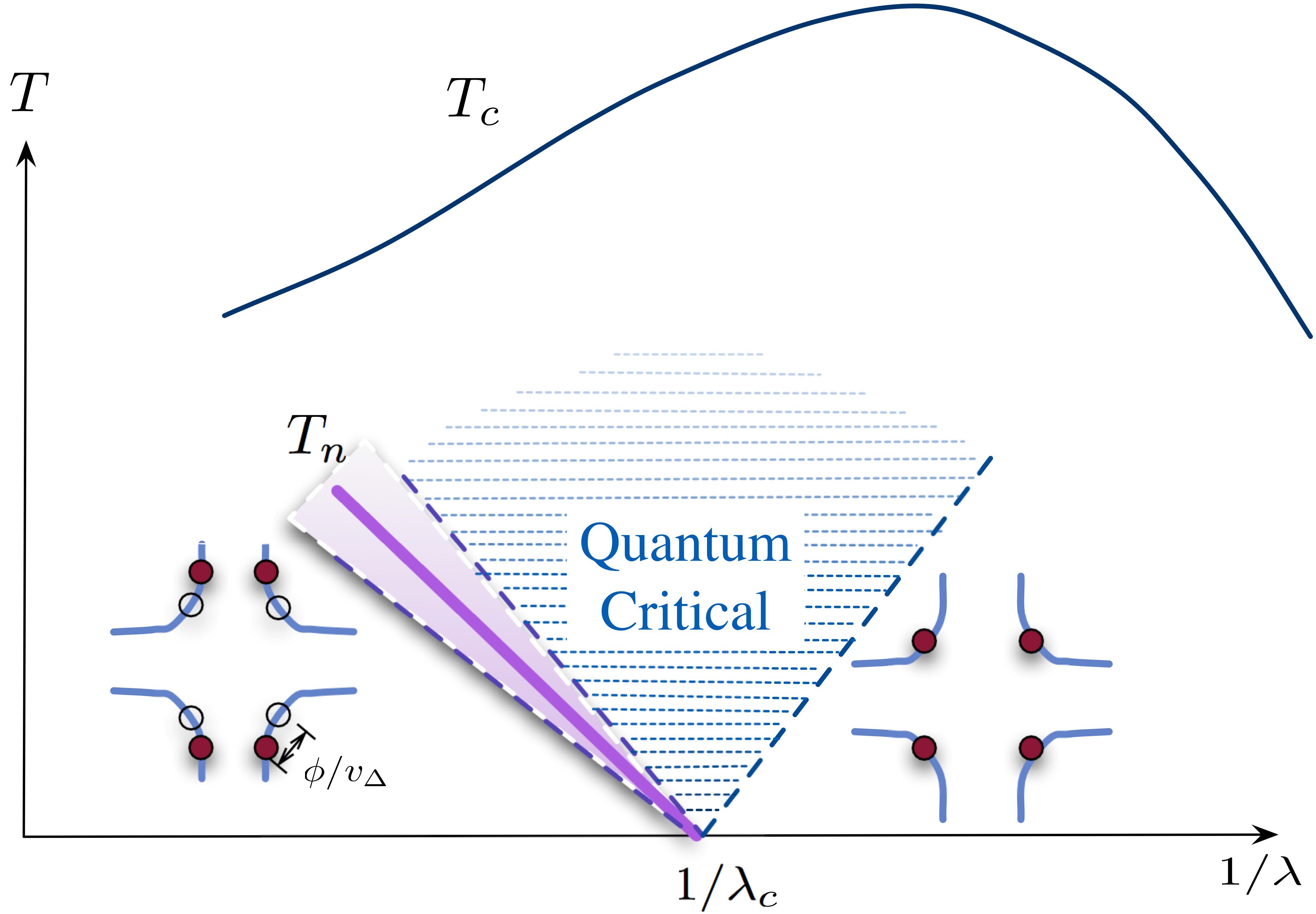
$d_{x^2-y^2}$ superconductor + nematic order

$\langle \phi \rangle \neq 0$

$d_{x^2-y^2}$ superconductor

$\langle \phi \rangle = 0$

r_c r



M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000)
E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, Phys. Rev. B **77**, 184514 (2008).

Ising order and Dirac fermions couple via a "Yukawa" term.

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^x \Psi_{1a} + \Psi_{2a}^\dagger \tau^x \Psi_{2a} \right) \right],$$

Nematic ordering

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^y \Psi_{1a} + \Psi_{2a}^\dagger \tau^y \Psi_{2a} \right) \right]$$

Time reversal symmetry breaking

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Ising order and Dirac fermions couple via a "Yukawa" term.

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^{\dagger} \tau^x \Psi_{1a} + \Psi_{2a}^{\dagger} \tau^x \Psi_{2a} \right) \right],$$

Nematic ordering

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^{\dagger} \tau^y \Psi_{1a} + \Psi_{2a}^{\dagger} \tau^y \Psi_{2a} \right) \right]$$

Time reversal symmetry breaking

For the latter case *only*, with $v_F = v_\Delta = c$, theory reduces to relativistic Gross-Neveu model

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the scalar order parameter

$$
\begin{aligned}
S_\phi &= \frac{N_f}{v_\Delta v_F}\Gamma\left[\lambda_0\phi(x,\tau);\frac{v_\Delta}{v_F}\right] \\
&\quad + \frac{N_f}{2}\int d^2x d\tau \Big(r\phi^2(x,\tau)\Big) \\
&\quad + \text{irrelevant terms}
\end{aligned}
$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ/v_F.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the scalar order parameter

$$
\begin{aligned}
S_\phi &= \frac{N_f}{v_\Delta v_F}\Gamma\left[\lambda_0\phi(x,\tau);\frac{v_\Delta}{v_F}\right] \\
&\quad + \frac{N_f}{2}\int d^2x d\tau \left(r\phi^2(x,\tau)\right) \\
&\quad + \text{irrelevant terms}
\end{aligned}
$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ/v_F.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the scalar order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F}\Gamma\left[\lambda_0\phi(x,\tau);\frac{v_\Delta}{v_F}\right] + \frac{N_f}{2}\int d^2x d\tau\left(r\phi^2(x,\tau)\right) + \text{irrelevant terms}$$

where Γ is a non-local and non-analytic functional of ϕ.

There is a systematic expansion in powers of $1/N_f$ for renormalization group equations and all critical properties.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

1. Quantum-critical transport

   Collisionless-to-hydrodynamic crossover of CFT3s

2. Quantum criticality of Dirac fermions

   "Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

   The genus expansion

1. Quantum-critical transport

   Collisionless-to-hydrodynamic crossover of CFT3s

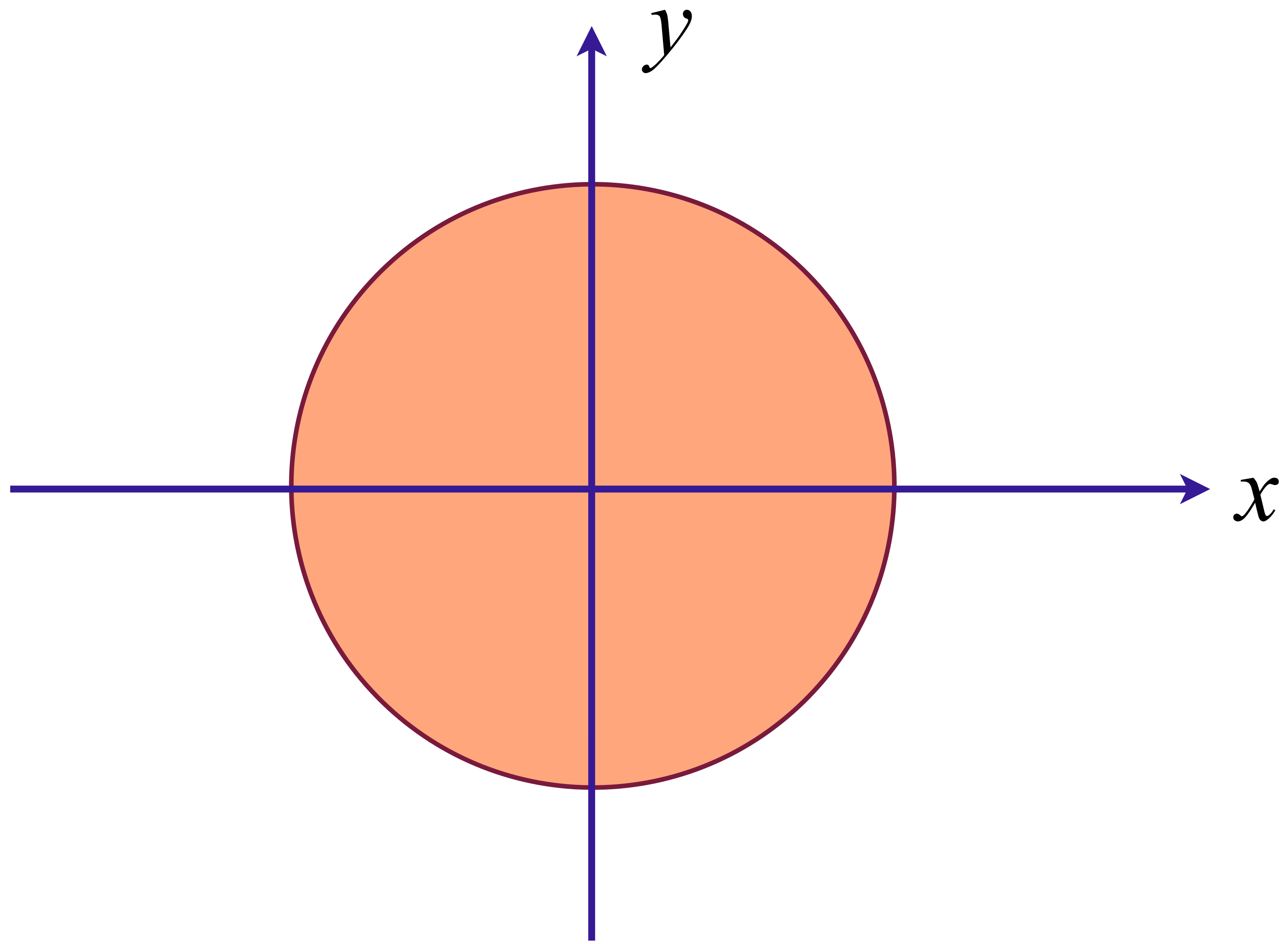
2. Quantum criticality of Dirac fermions

   "Vector" 1/N expansion

3. Quantum criticality of Fermi surfaces

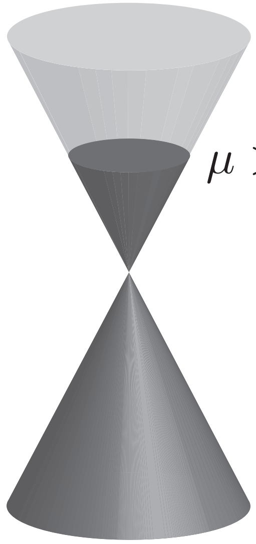
   The genus expansion

Quantum criticality of Pomeranchuk instability



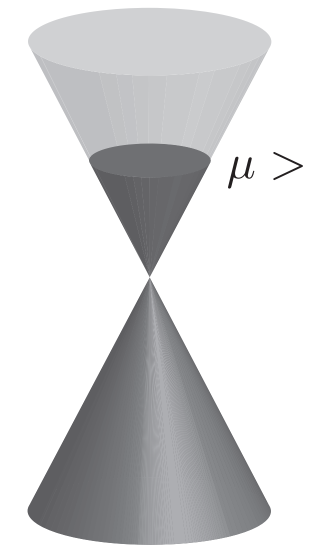
Fermi surface with full square lattice symmetry

Electron Green's function in Fermi liquid (T=0)



$$G(k,\omega) = \frac{Z}{\omega - v_F(k-k_F) - i\omega^2 \mathcal{F}\left(\frac{k-k_F}{\omega}\right)} + \ldots$$

Electron Green's function in Fermi liquid (T=0)

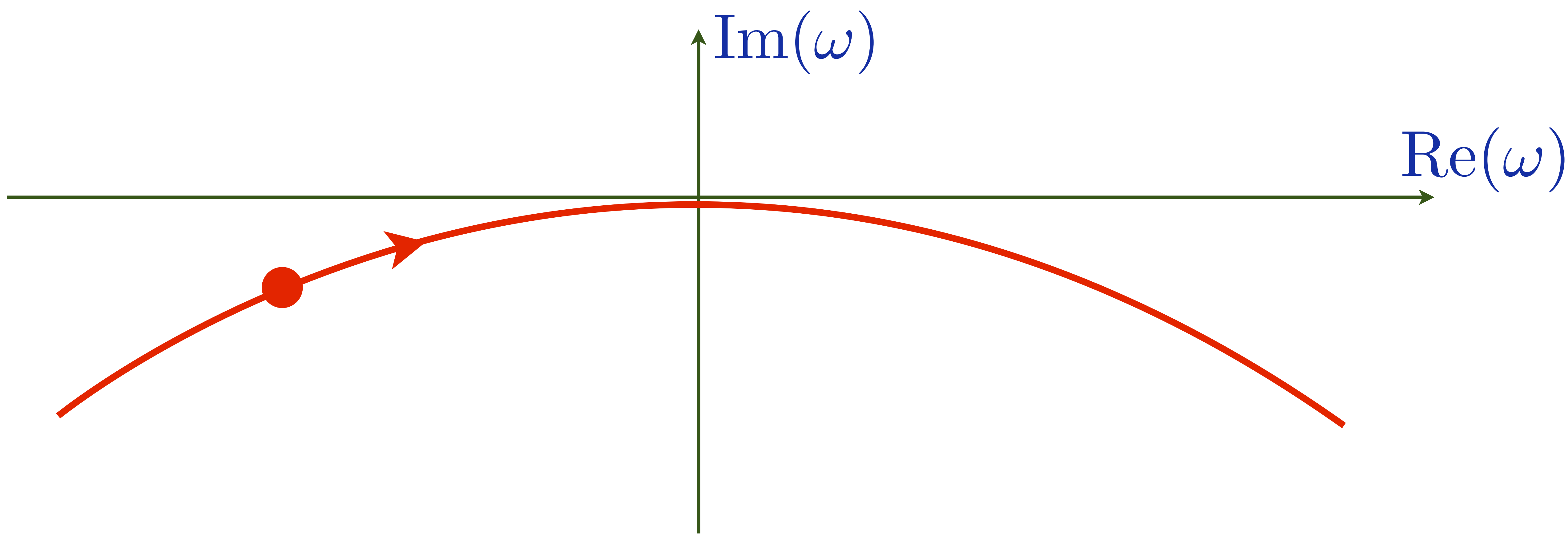


$$G(k,\omega) = \frac{Z}{\omega - v_F(k-k_F) - i\omega^2\mathcal{F}\left(\frac{k-k_F}{\omega}\right)} + \ldots$$

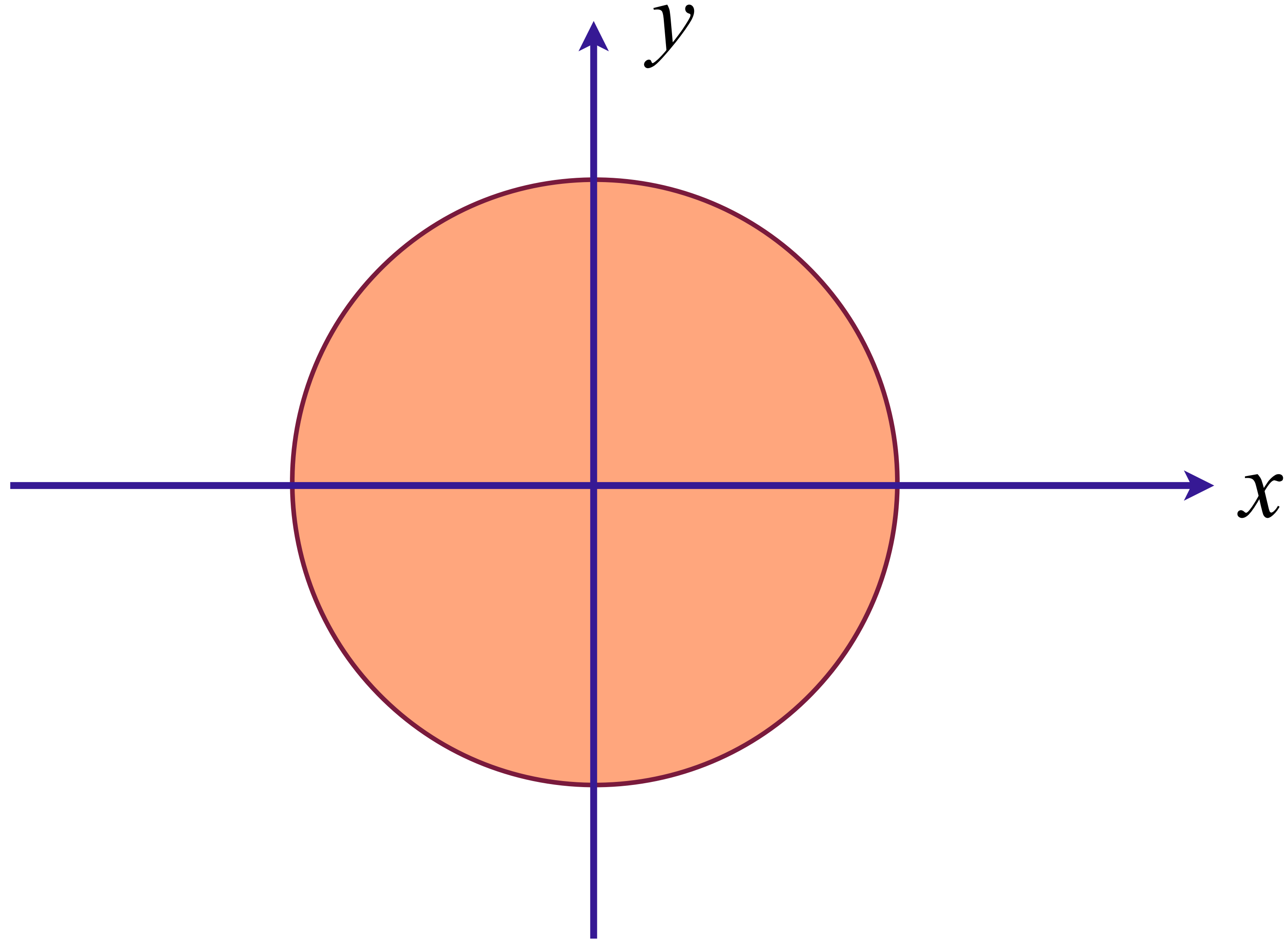
Green's function has a pole in the LHP at

$$\omega = v_F(k-k_F) - i\alpha(k-k_F)^2 + \ldots$$

Pole is at $\omega = 0$ precisely at $k = k_F$ *i.e.* on a sphere of radius k_F in momentum space. This is the *Fermi surface.*

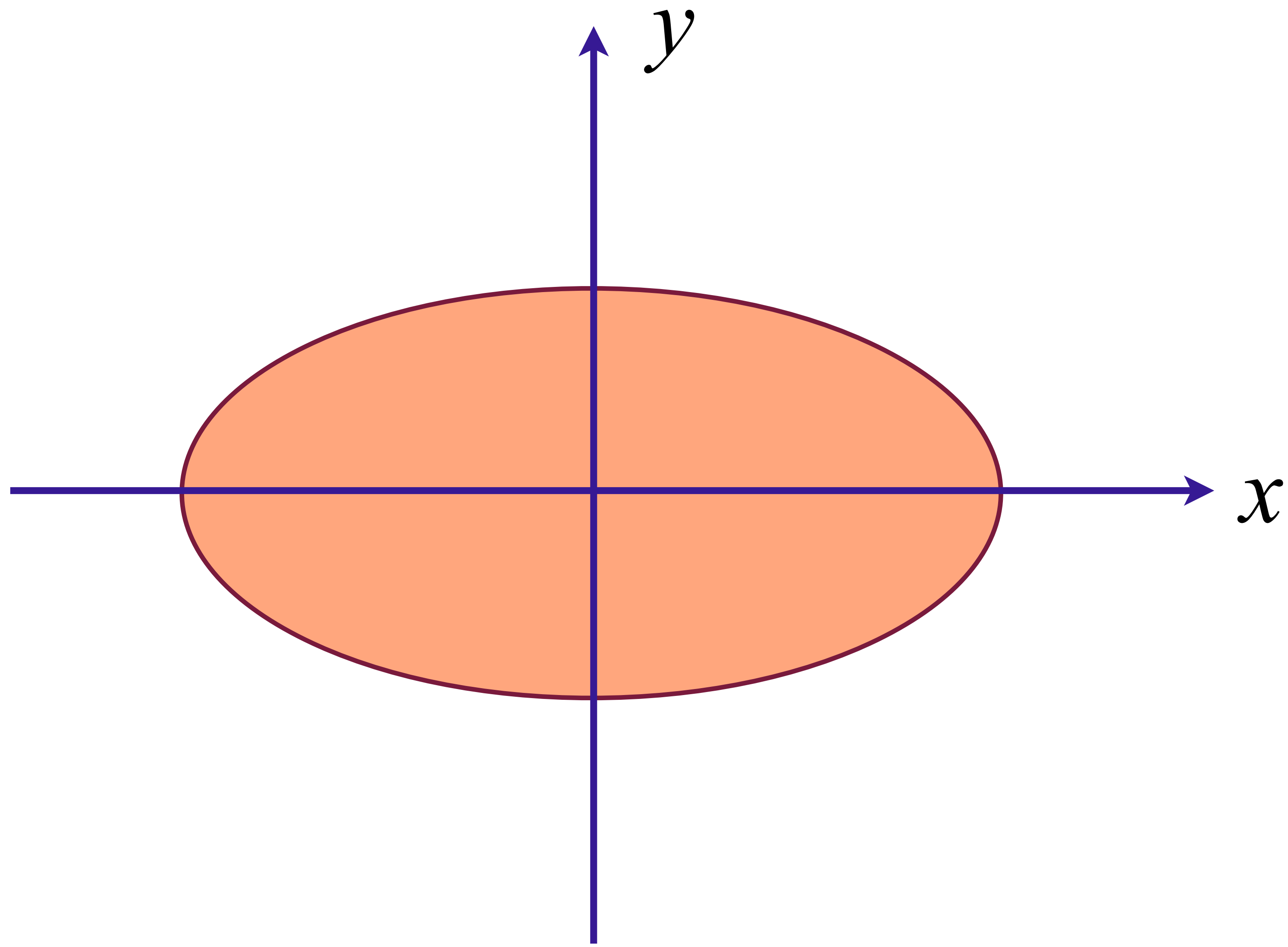


Quantum criticality of Pomeranchuk instability



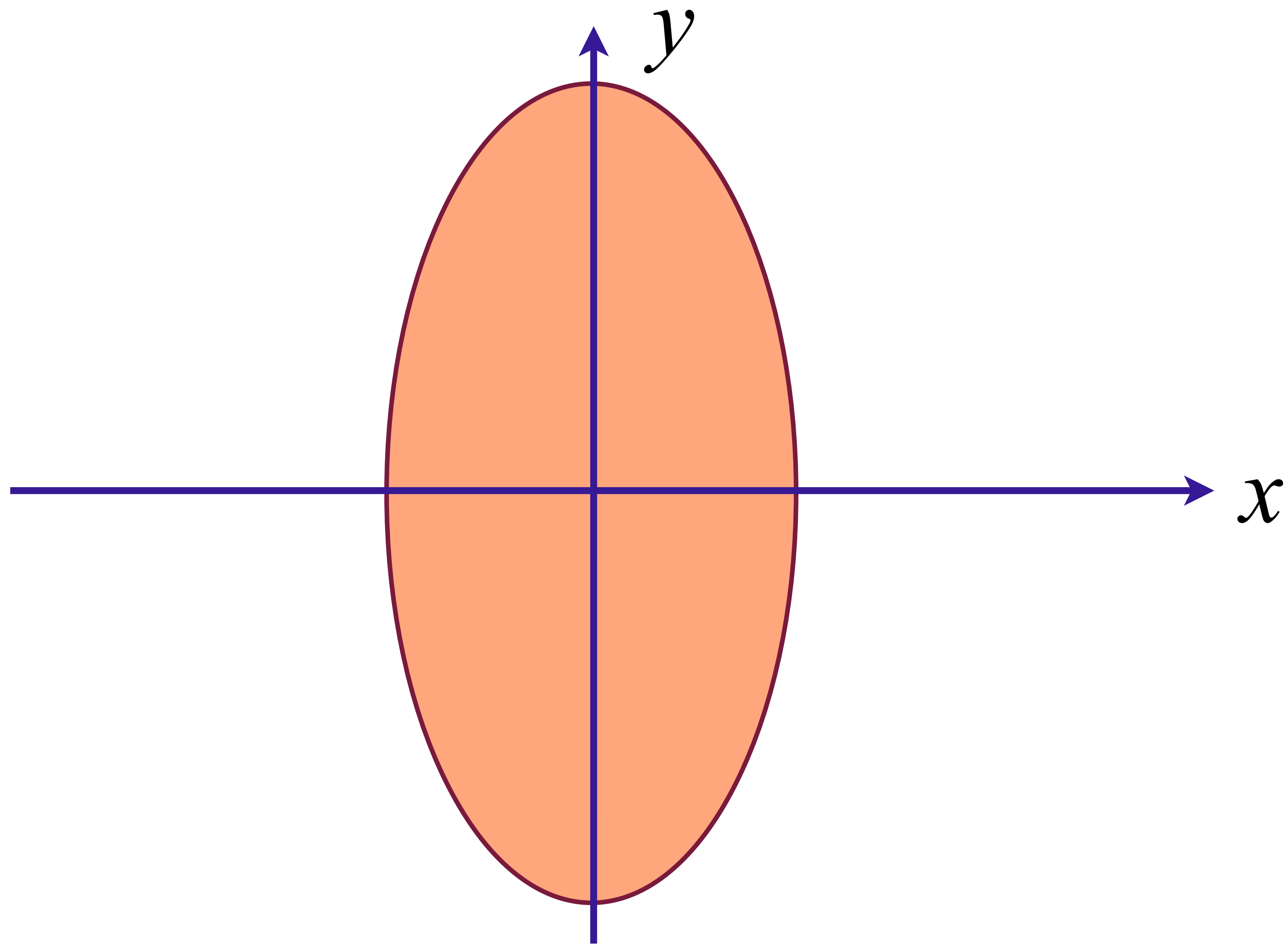
Fermi surface with full square lattice symmetry

Quantum criticality of Pomeranchuk instability



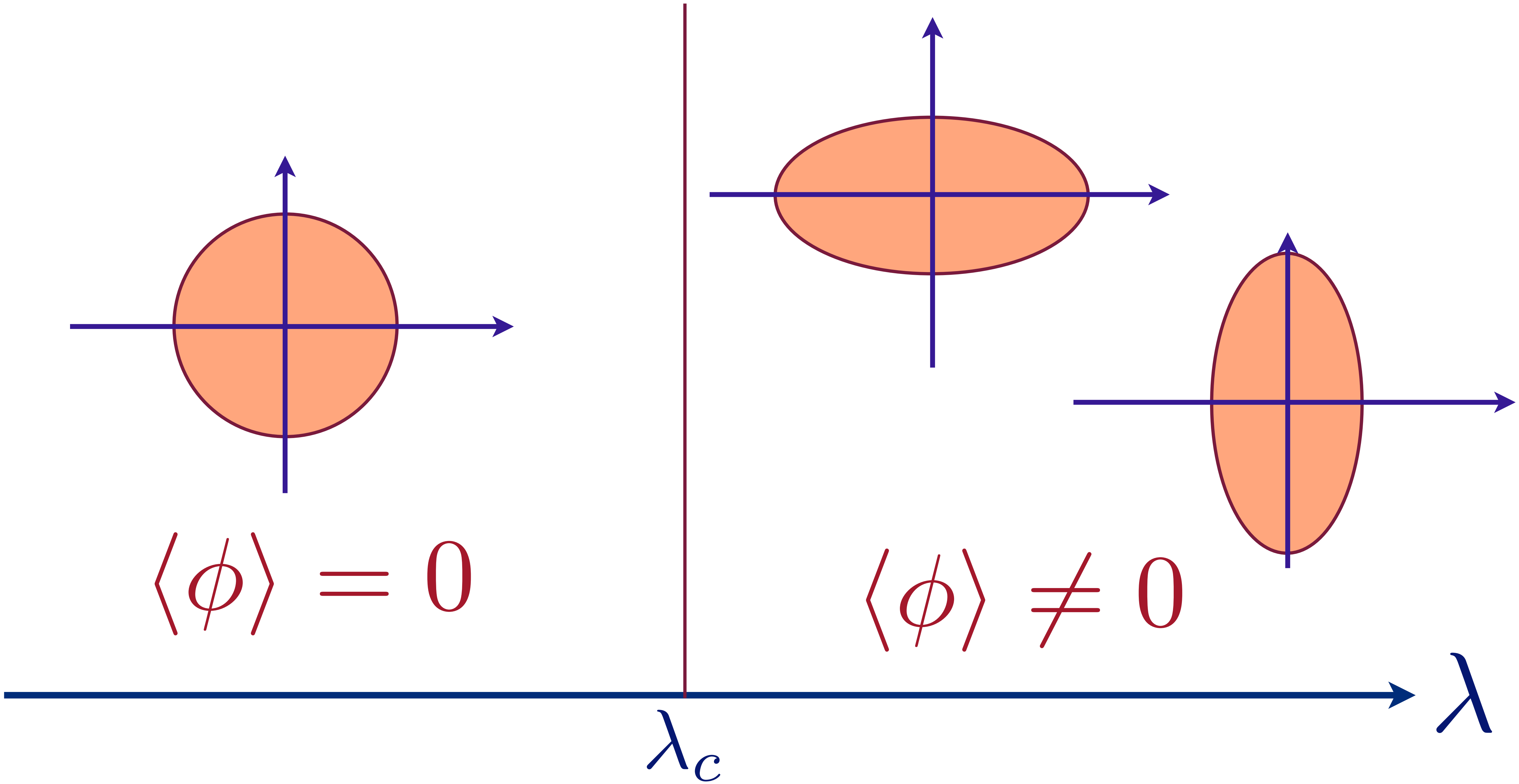
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability



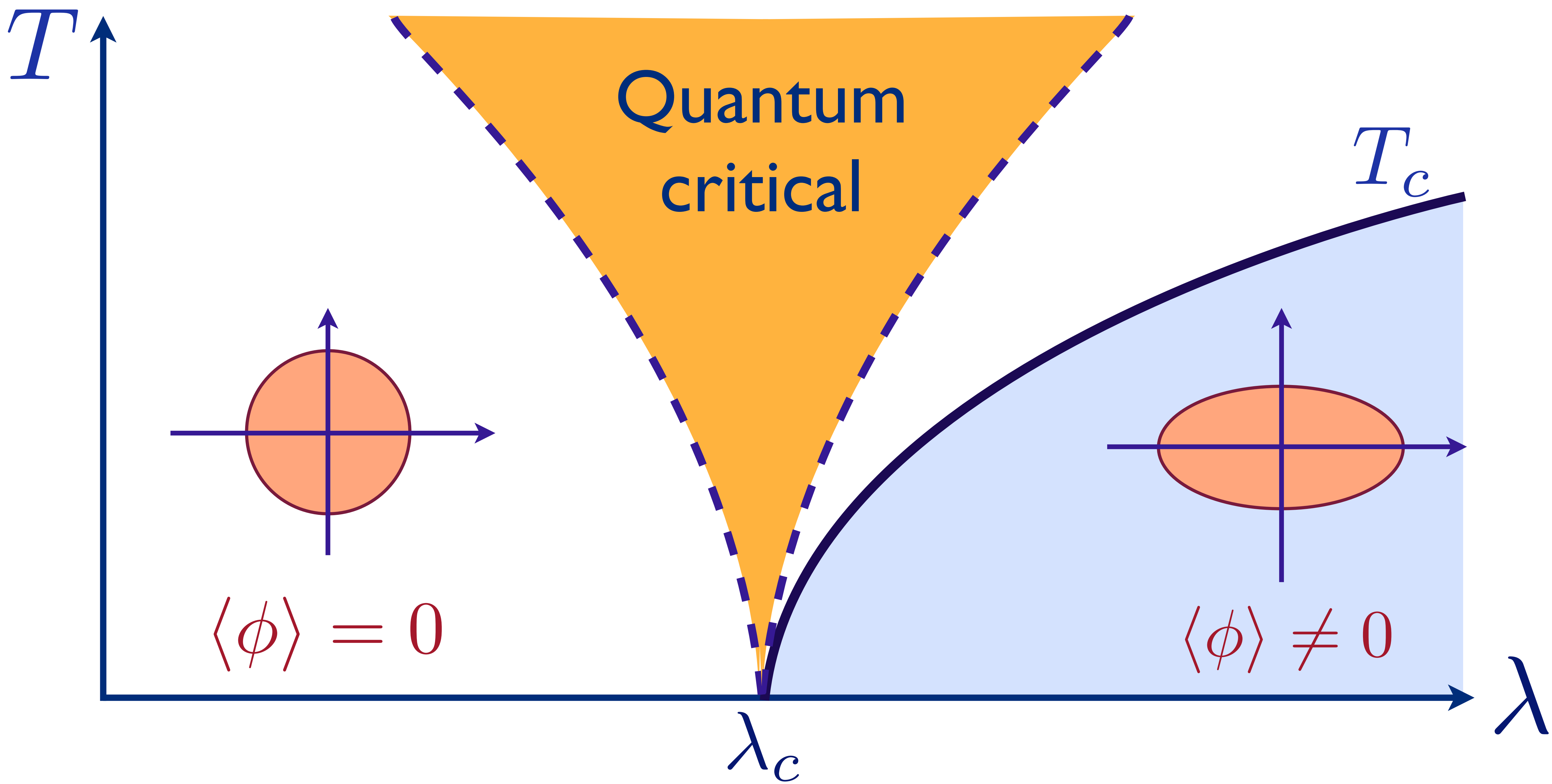
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Pomeranchuk instability



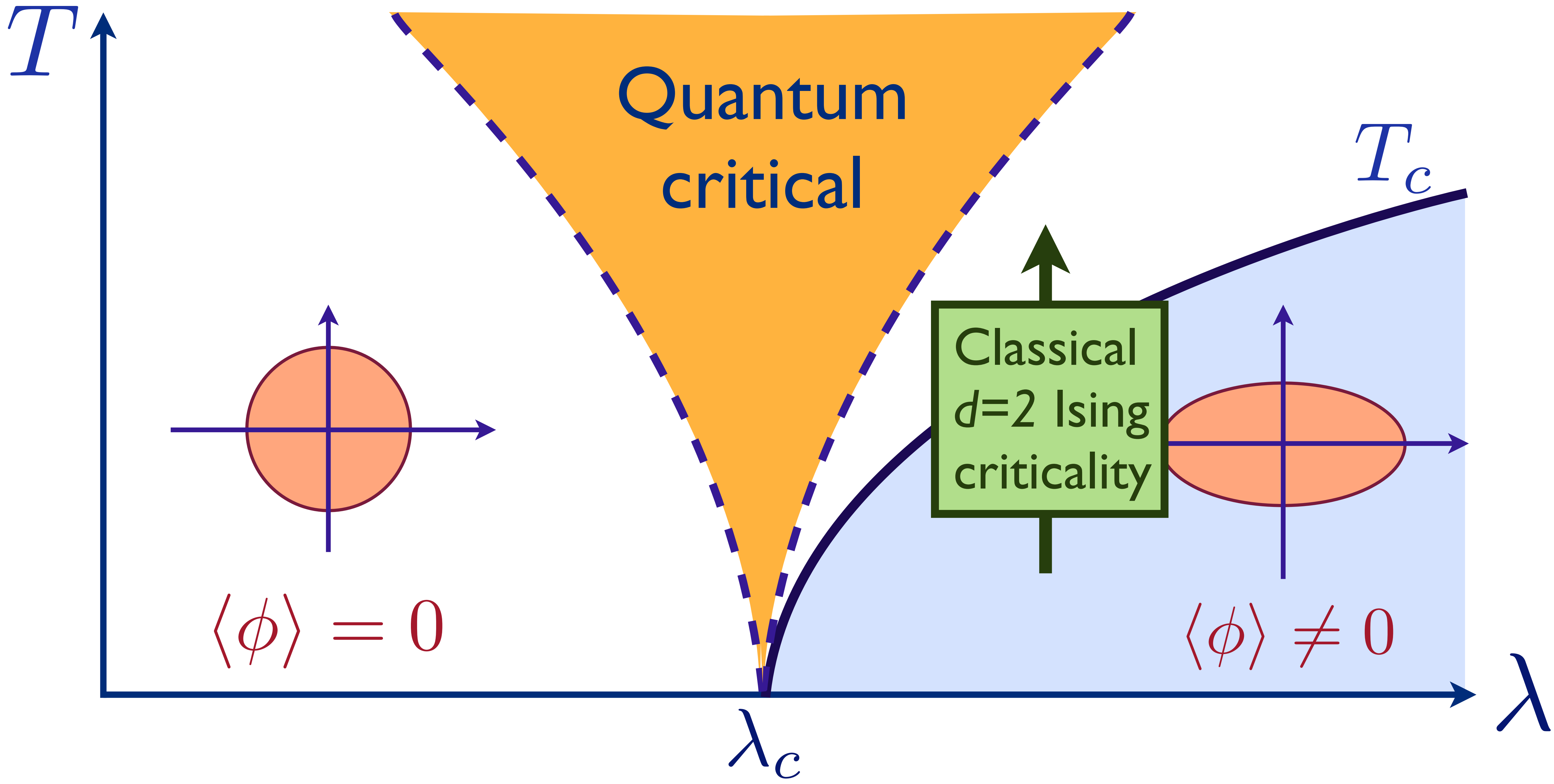
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Pomeranchuk instability



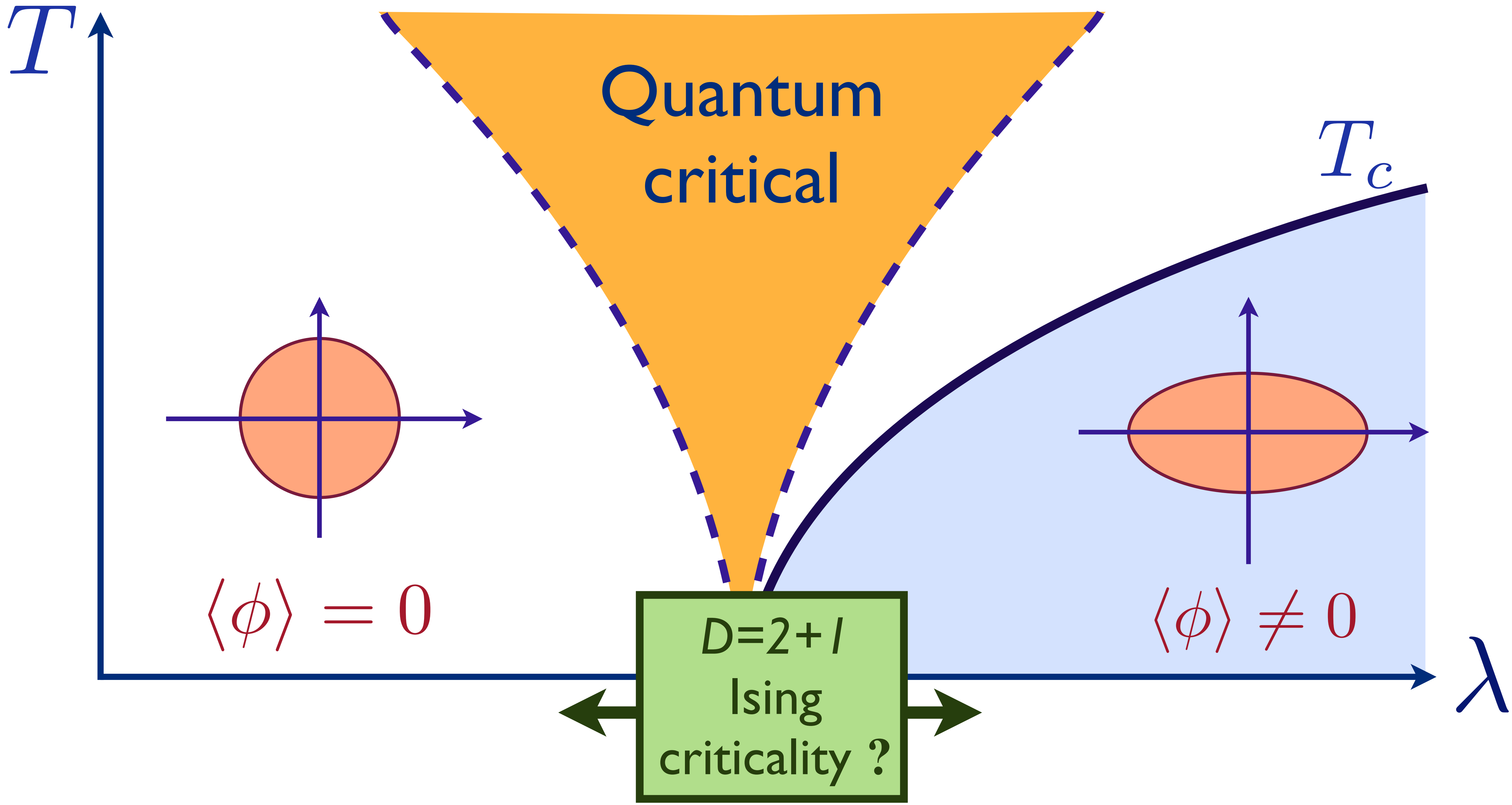
Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

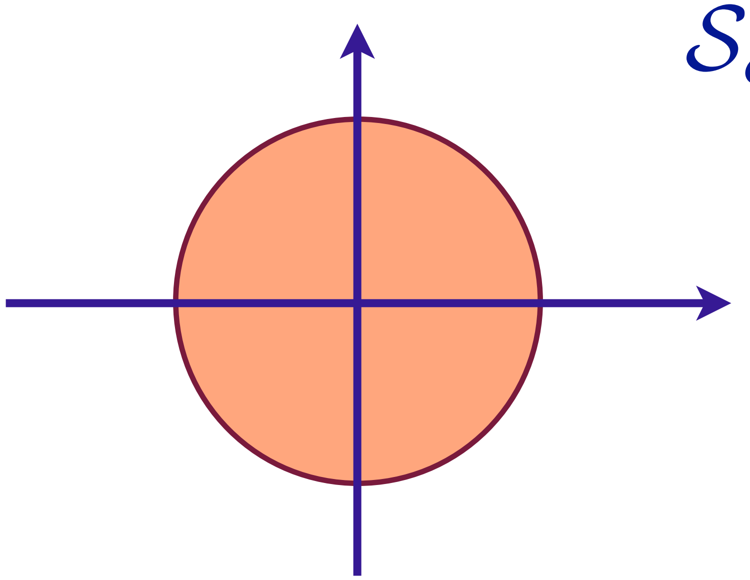
$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

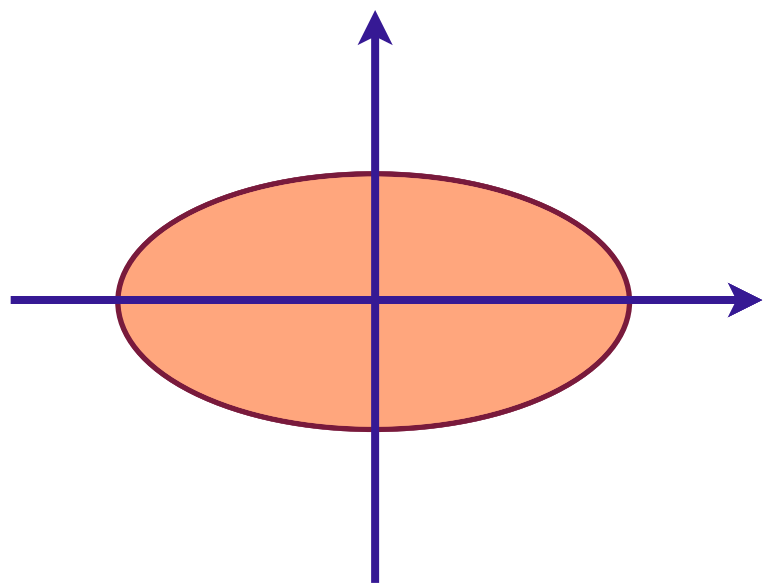
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Pomeranchuk instability

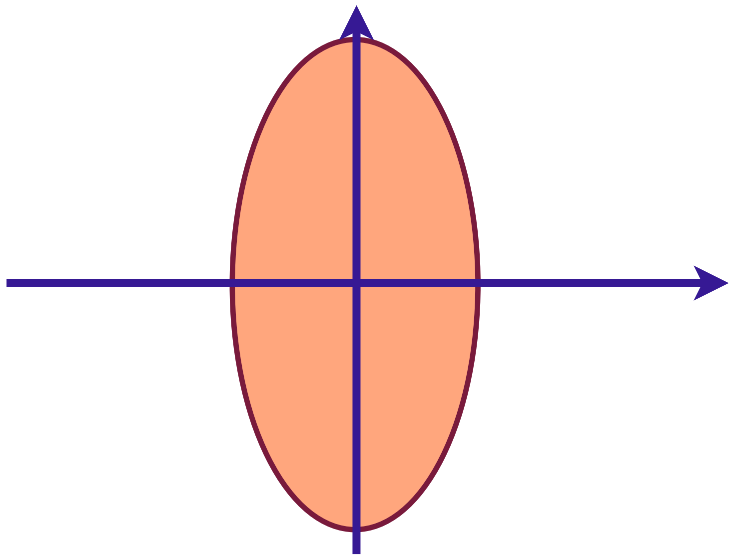
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$$\langle \phi \rangle > 0$$



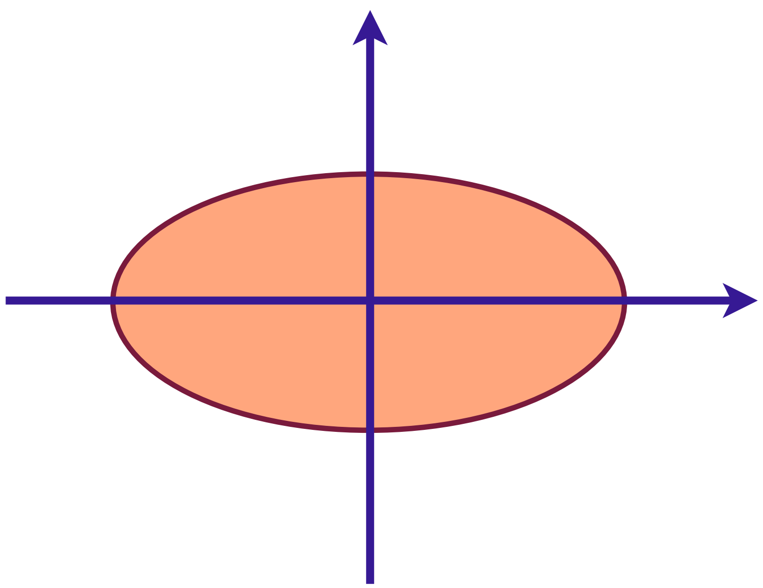
$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

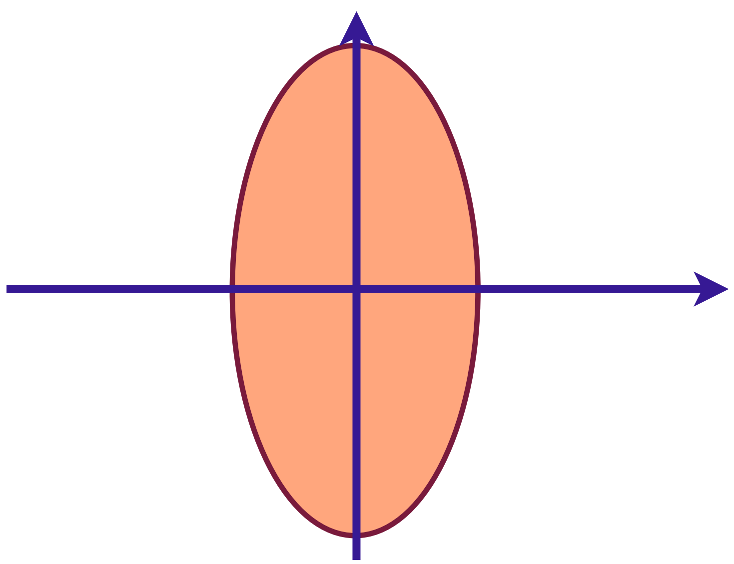
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



$\langle \phi \rangle < 0$

Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla\phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4\right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}}\right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum critical field theory

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}c_{i\alpha} \exp\left(-\mathcal{S}_\phi - \mathcal{S}_c - \mathcal{S}_{\phi c}\right)$$

Quantum criticality of Pomeranchuk instability

Hertz theory

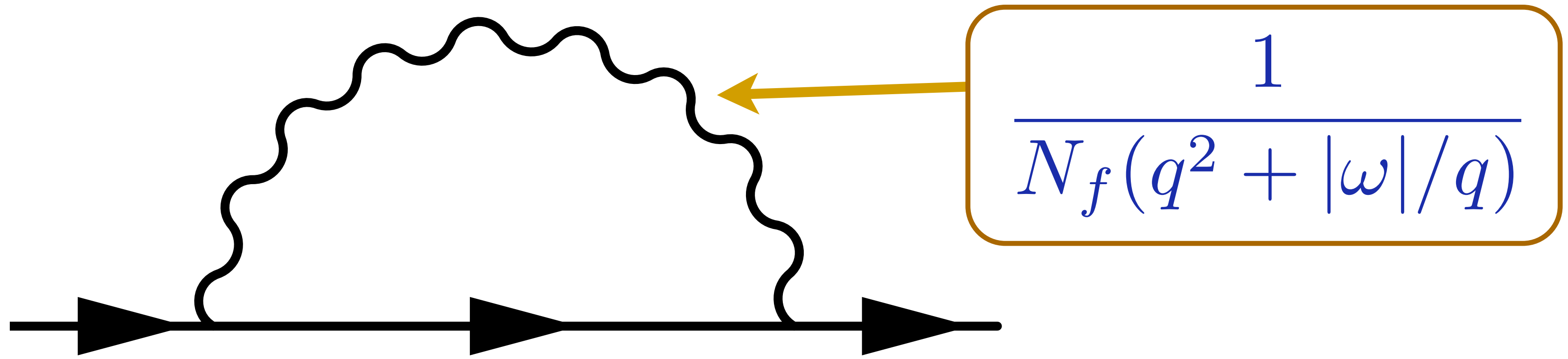
Integrate out c_α fermions and obtain non-local corrections to ϕ action

$$\delta \mathcal{S}_\phi \sim N_f \gamma^2 \int \frac{d^2 q}{4\pi^2} \int \frac{d\omega}{2\pi} |\phi(\mathbf{q},\omega)|^2 \left[\frac{|\omega|}{q} + q^2 \right] + \ldots$$

This leads to a critical point with dynamic critical exponent $z = 3$ and quantum criticality controlled by the Gaussian fixed point.

Quantum criticality of Pomeranchuk instability

Hertz theory



Self energy of c_α fermions to order $1/N_f$

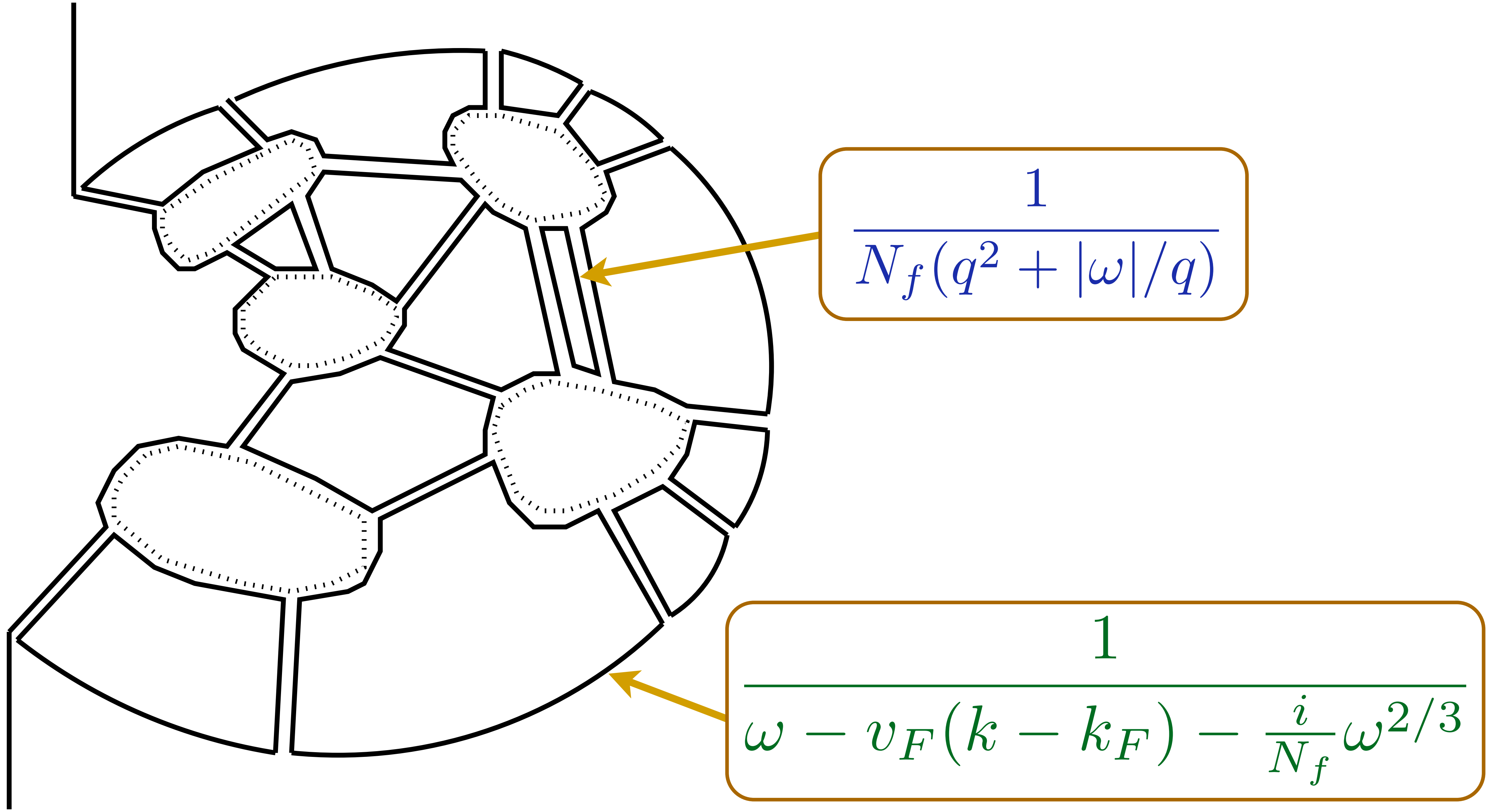
$$\Sigma_c(k,\omega) \sim \frac{i}{N_f}\omega^{2/3}$$

This leads to the Green's function

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - \frac{i}{N_f}\omega^{2/3}}$$

Note that the order $1/N_f$ term is more singular in the infrared than the bare term; this leads to problems in the bare $1/N_f$ expansion in terms that are dominated by low frequency fermions.

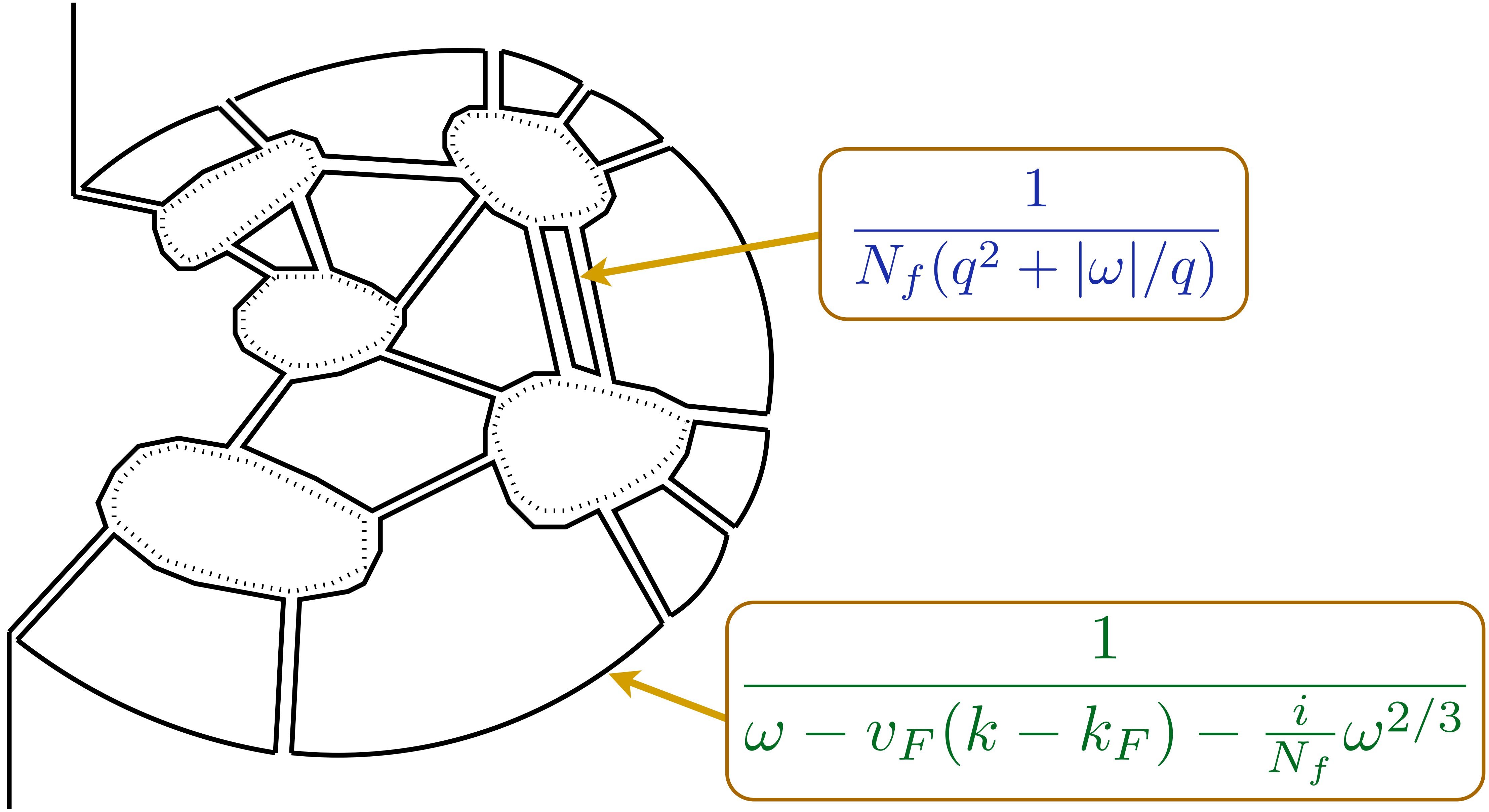
Quantum criticality of Pomeranchuk instability



The infrared singularities of fermion particle-hole pairs are most severe on planar graphs: these all contribute at leading order in $1/N_f$.

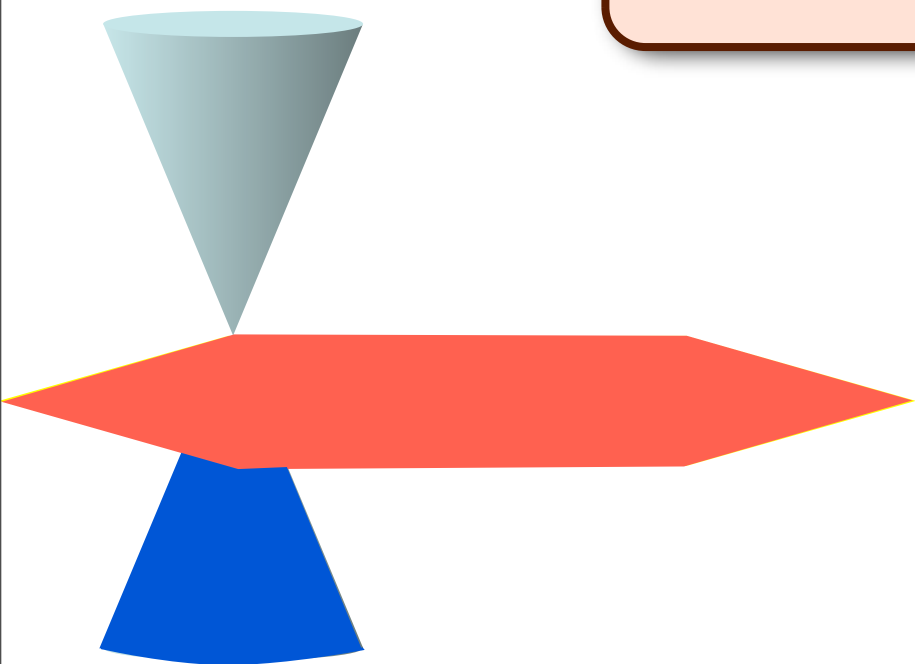
Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Quantum criticality of Pomeranchuk instability



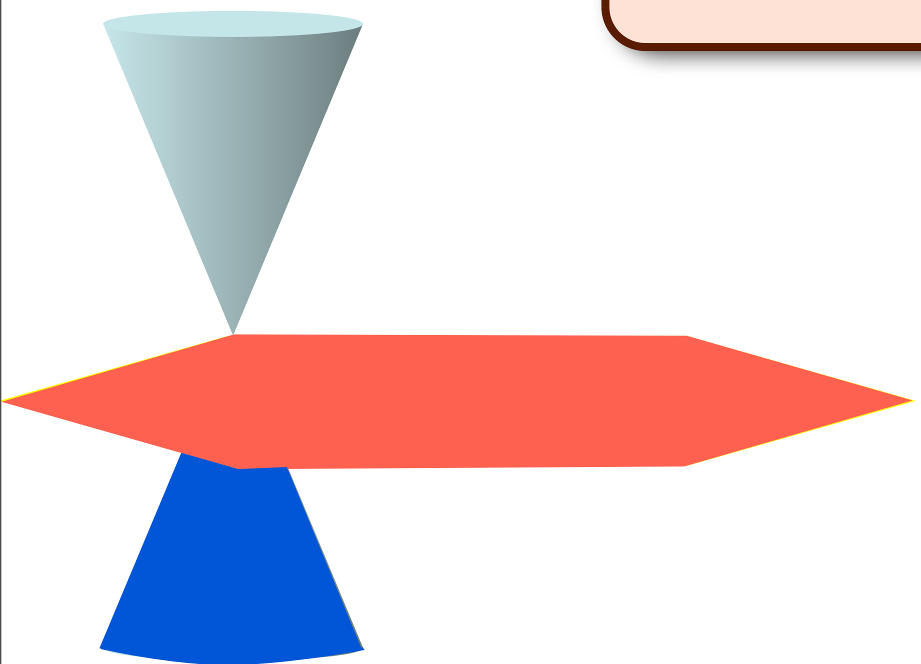
A string theory for the Fermi surface ?

Conformal field theory
in 2+1 dimensions at $T = 0$



Einstein gravity
on AdS_4

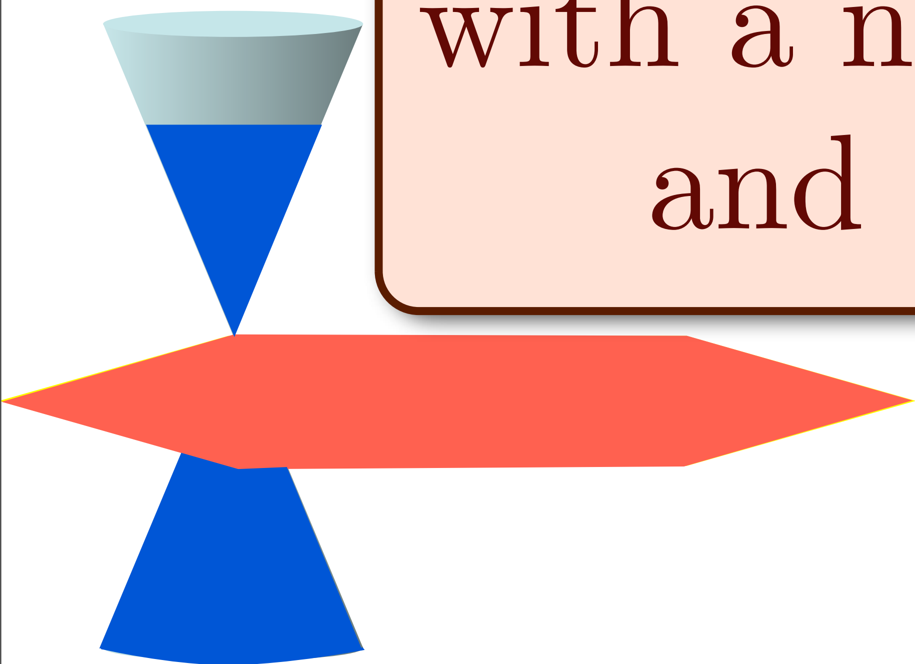
Conformal field theory
in 2+1 dimensions at $T > 0$



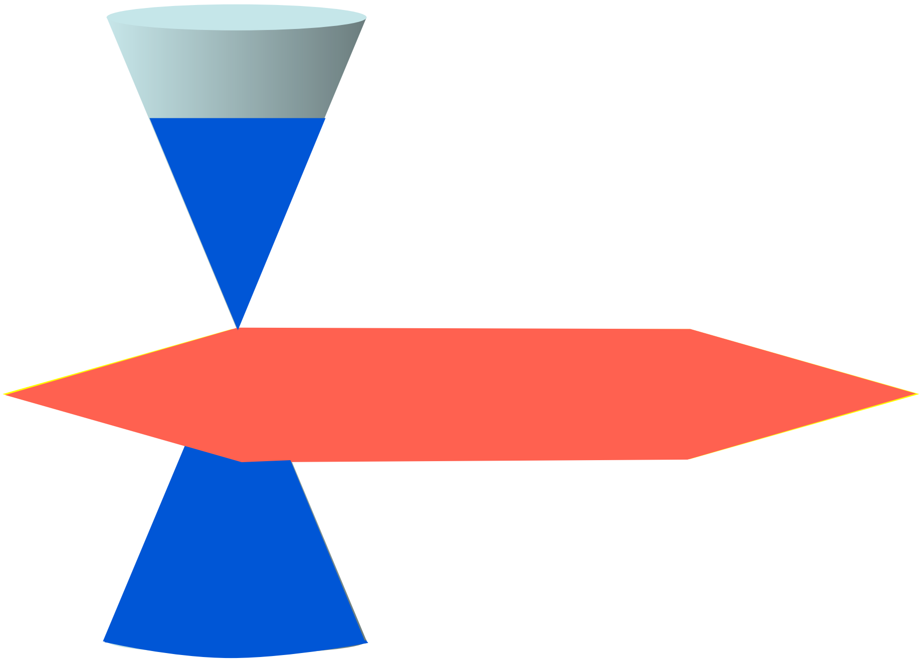
$\Updownarrow$

Einstein gravity on AdS_4
with a Schwarzschild
black hole

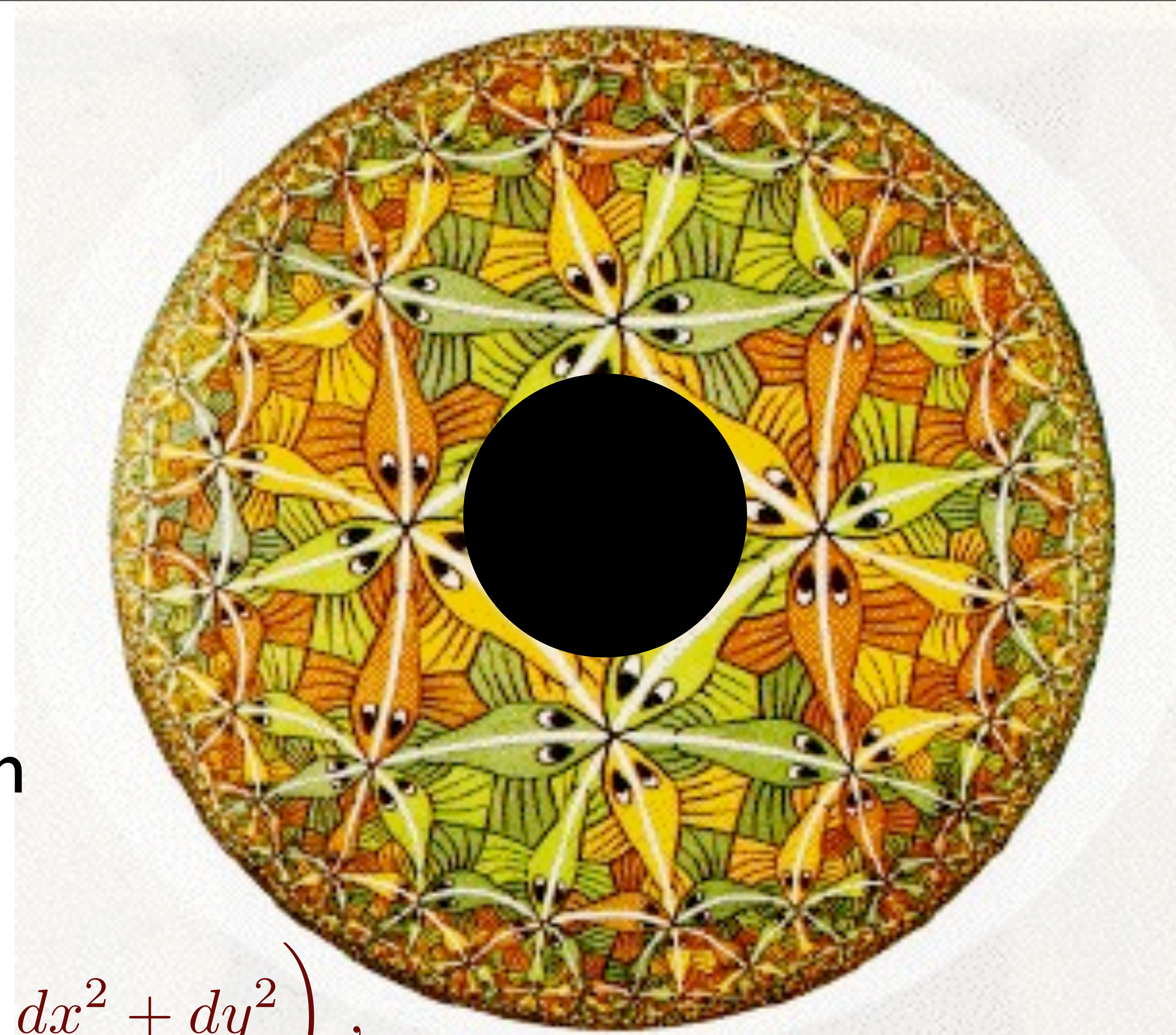
Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges



AdS4-Reissner-Nordstrom black hole

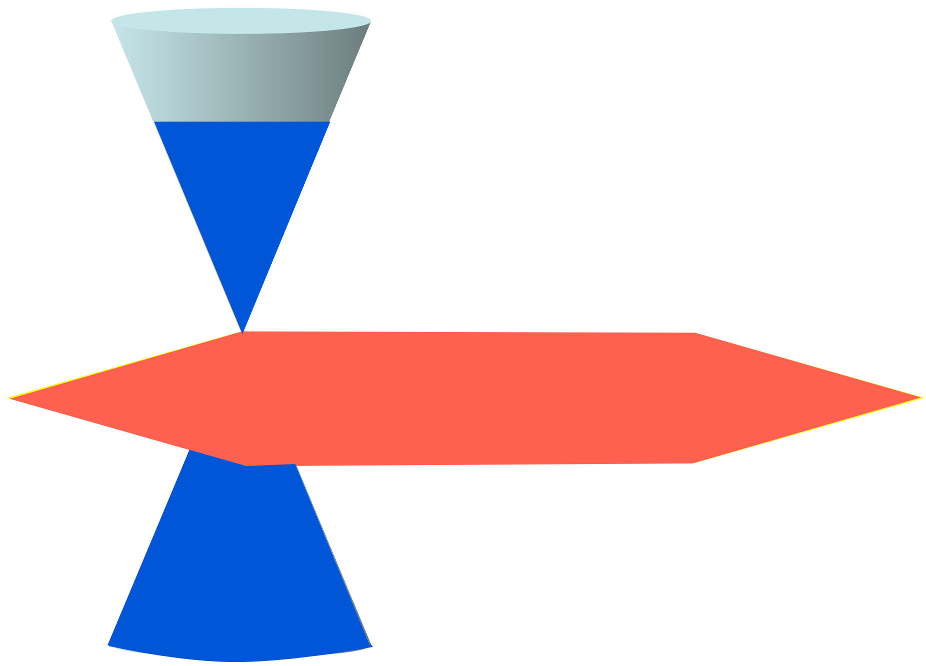


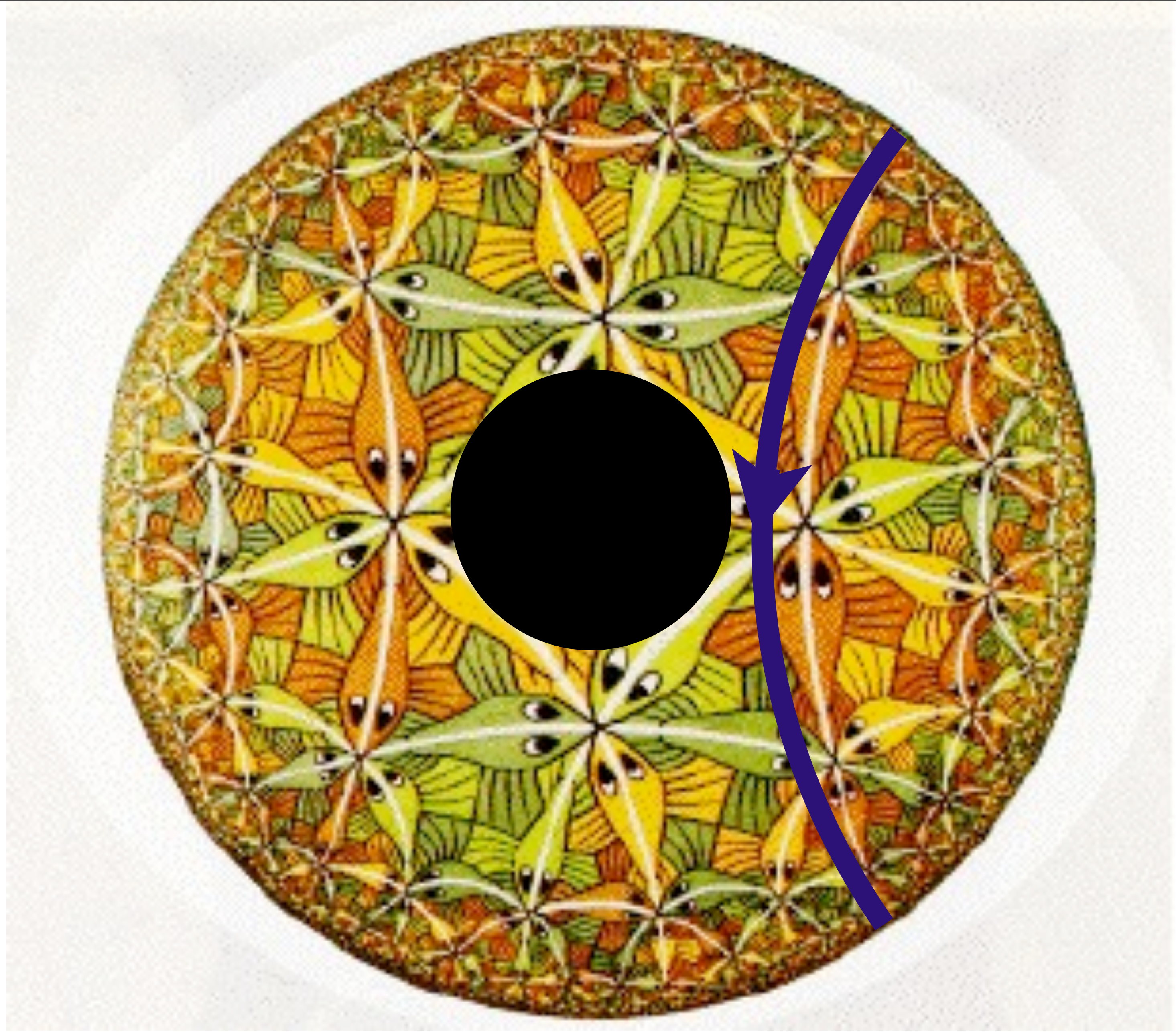
$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right),$$

$$f(r) = 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4,$$

$$A = i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy\,.$$

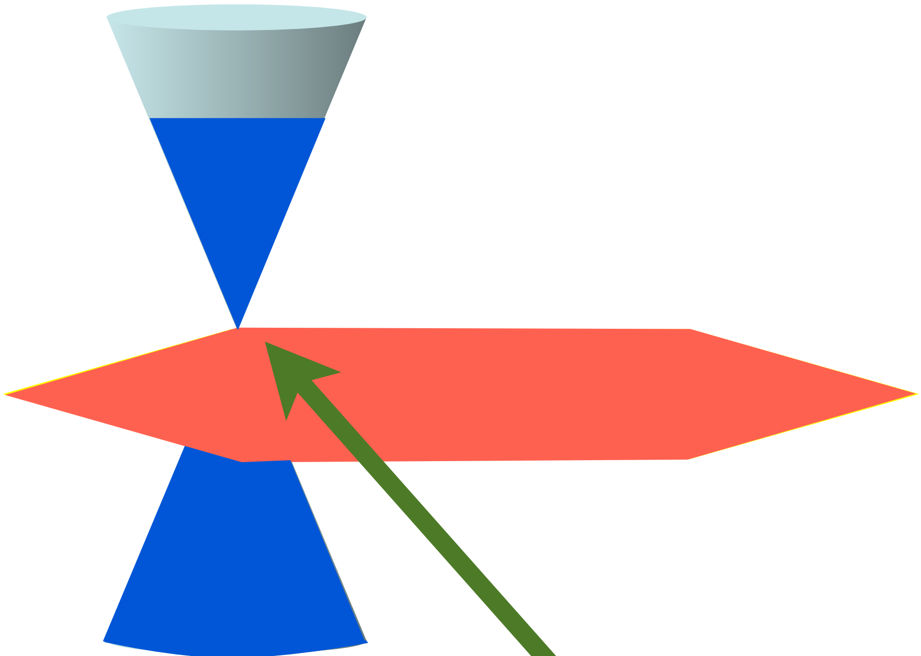
$$T = \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).$$

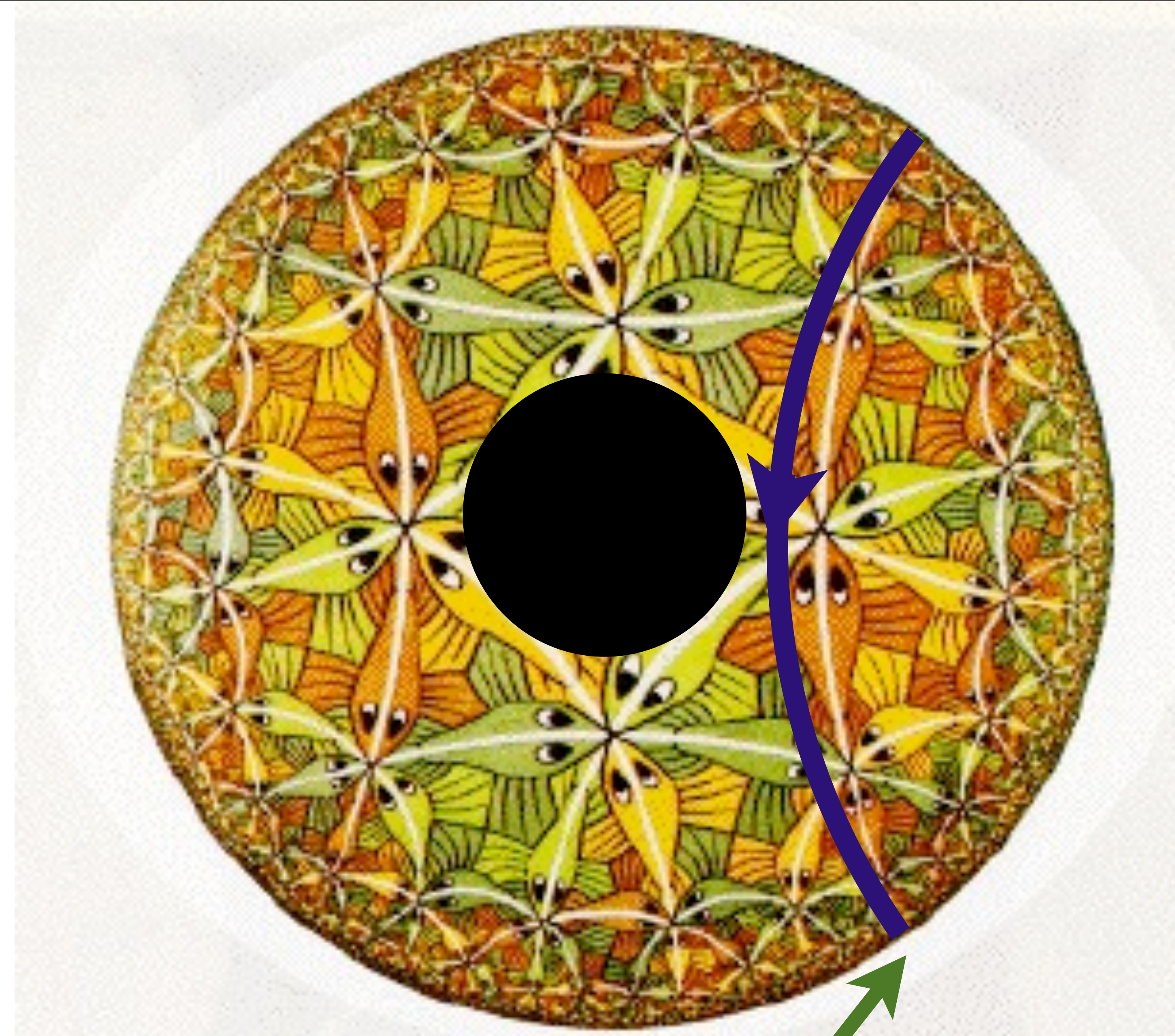




Examine free energy and Green's function of a probe particle

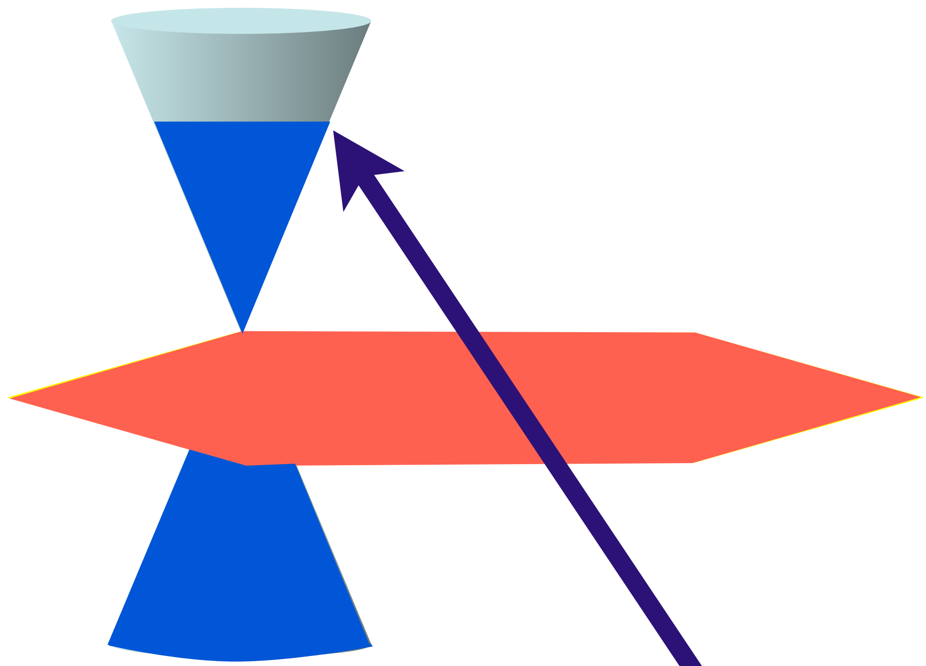
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

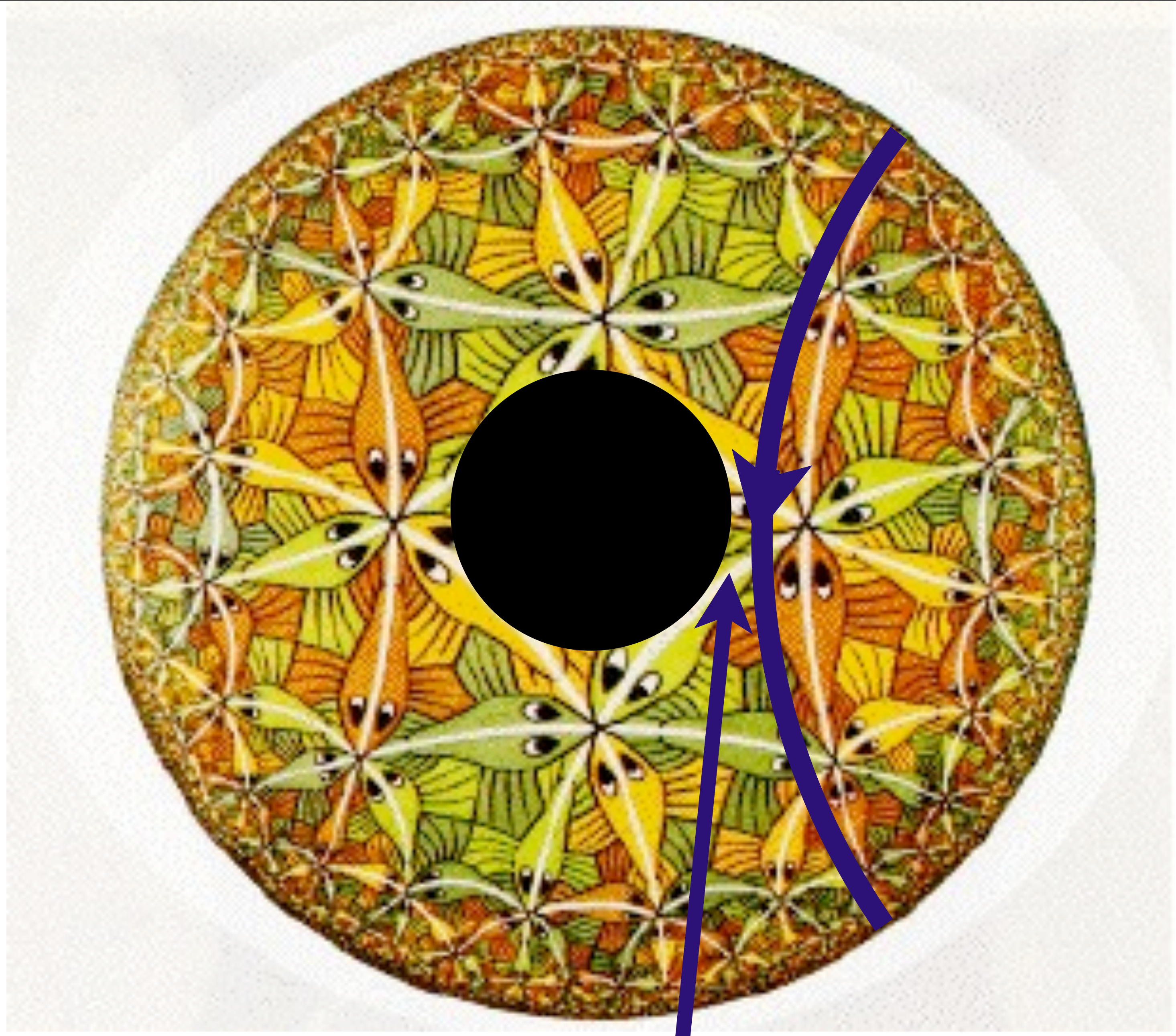




Short time behavior depends upon conformal AdS_4 geometry near boundary

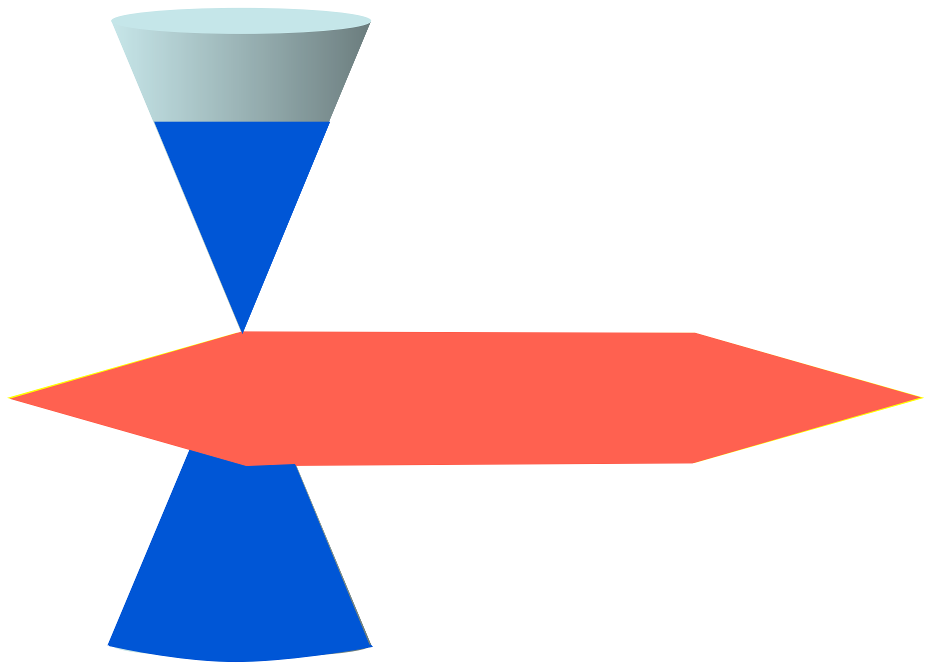
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

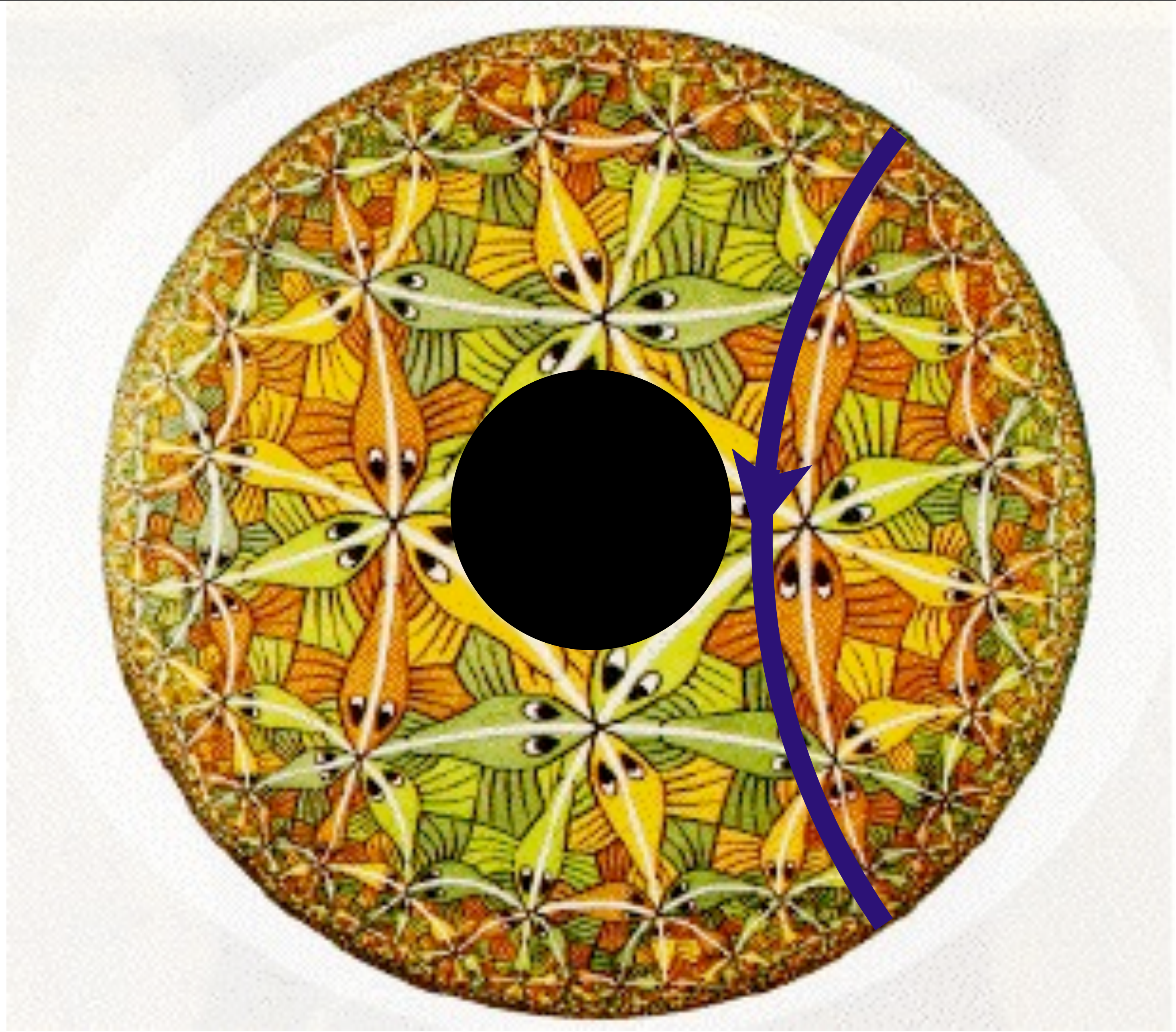




Long time behavior depends upon near-horizon geometry of black hole

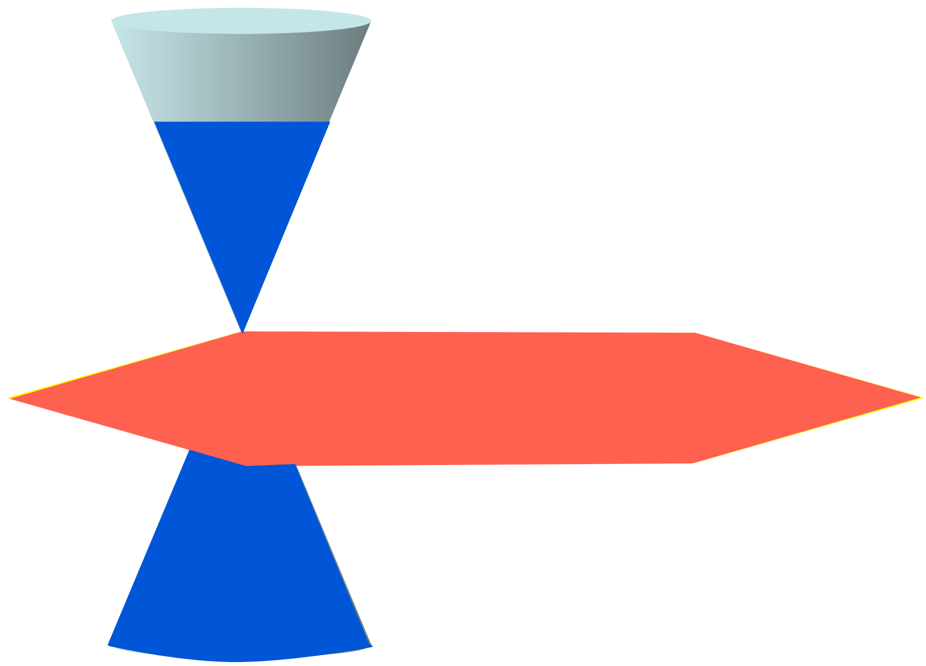
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

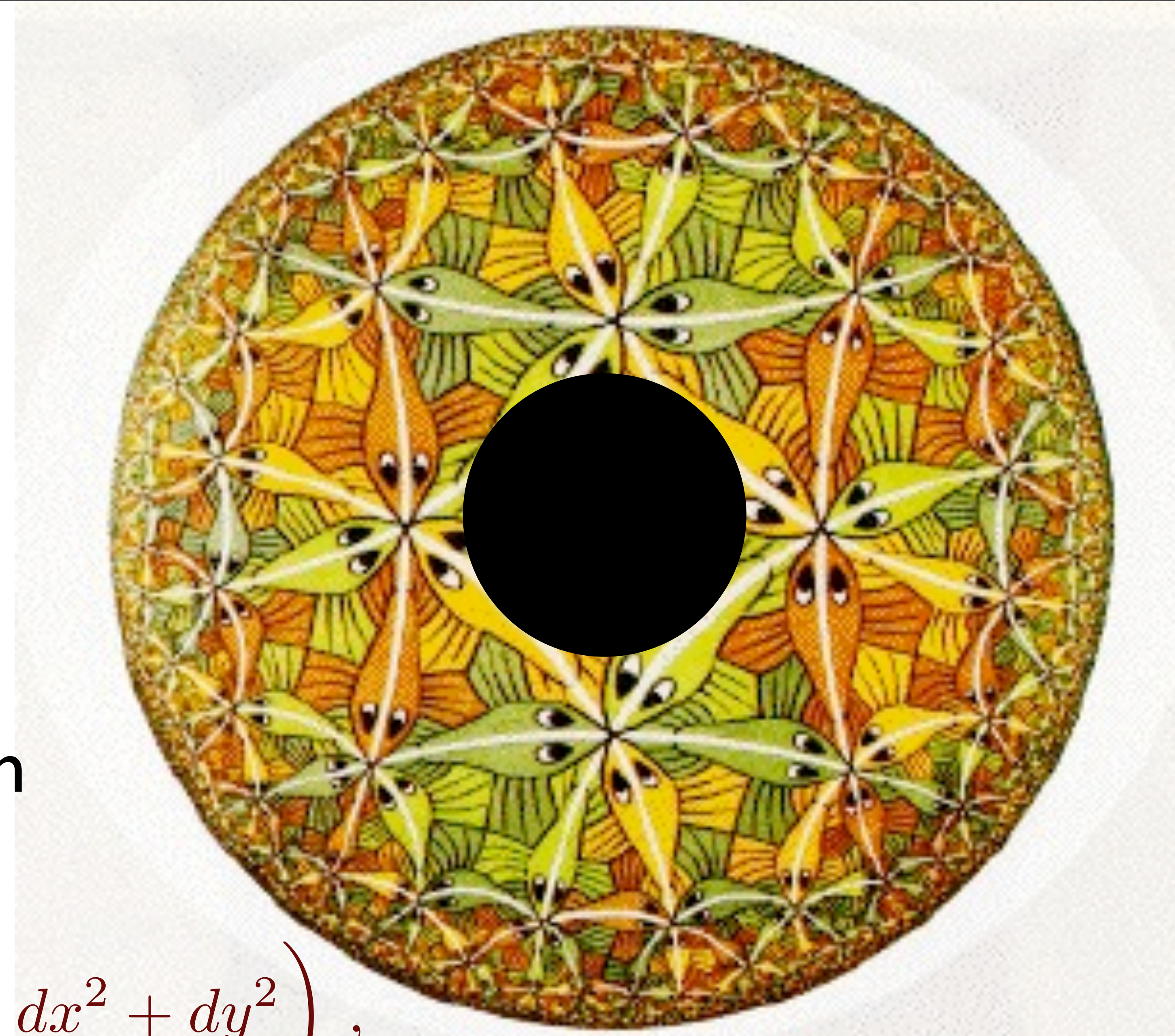




Radial direction of gravity theory is measure of energy scale in CFT

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788





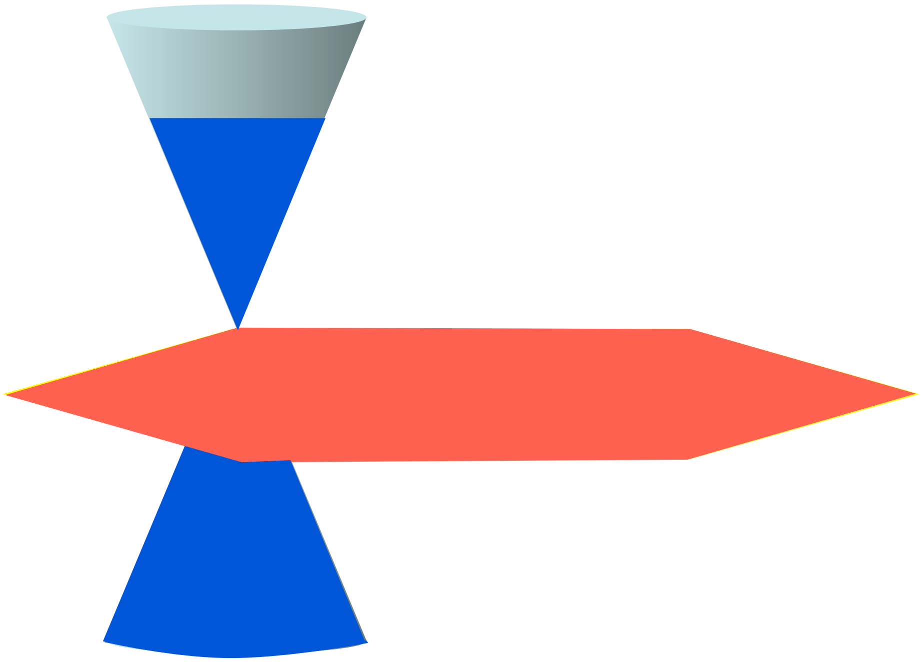
AdS_4-Reissner-Nordstrom black hole

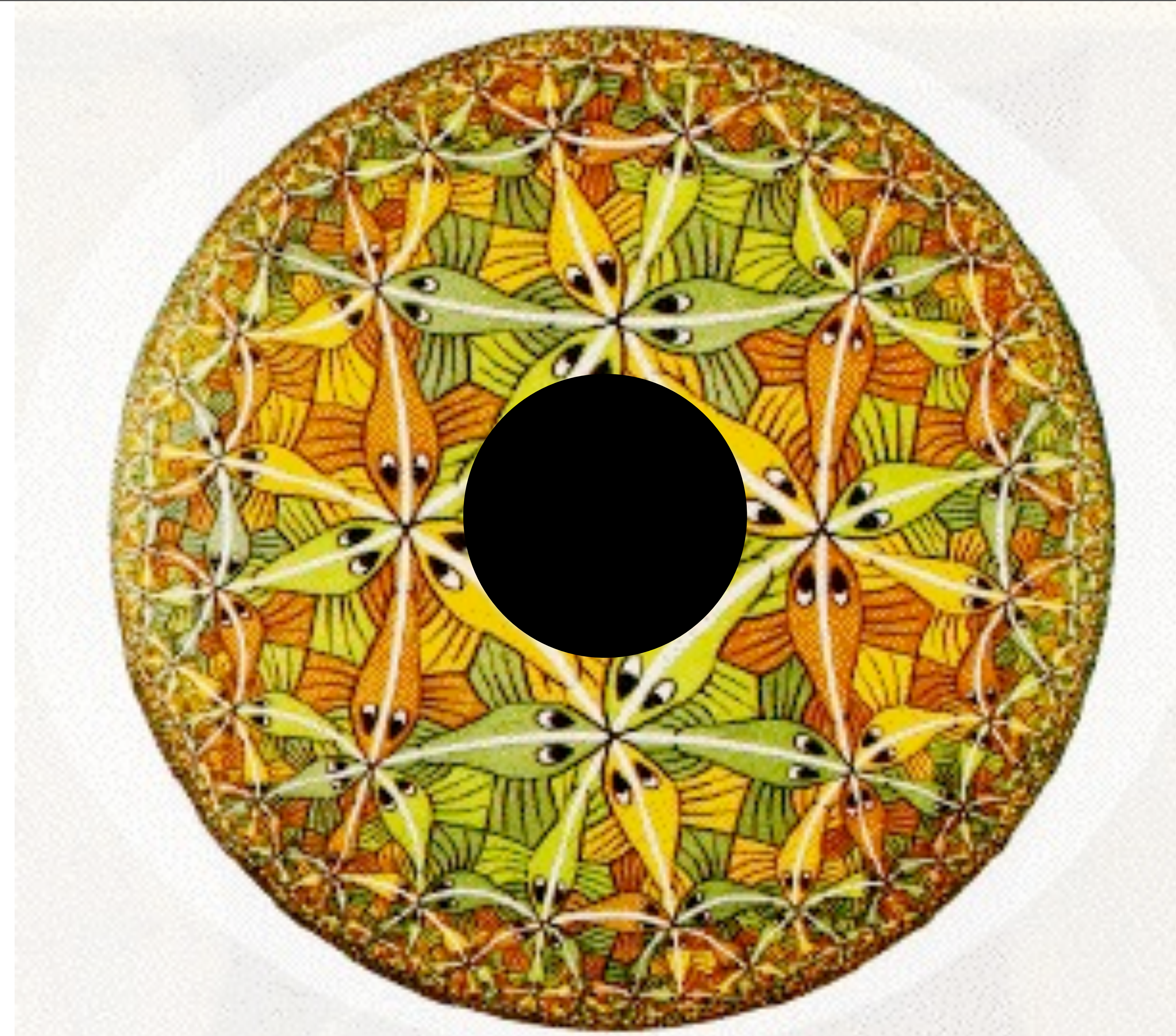
$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right),$$

$$f(r) = 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4,$$

$$A = i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy\,.$$

$$T = \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).$$

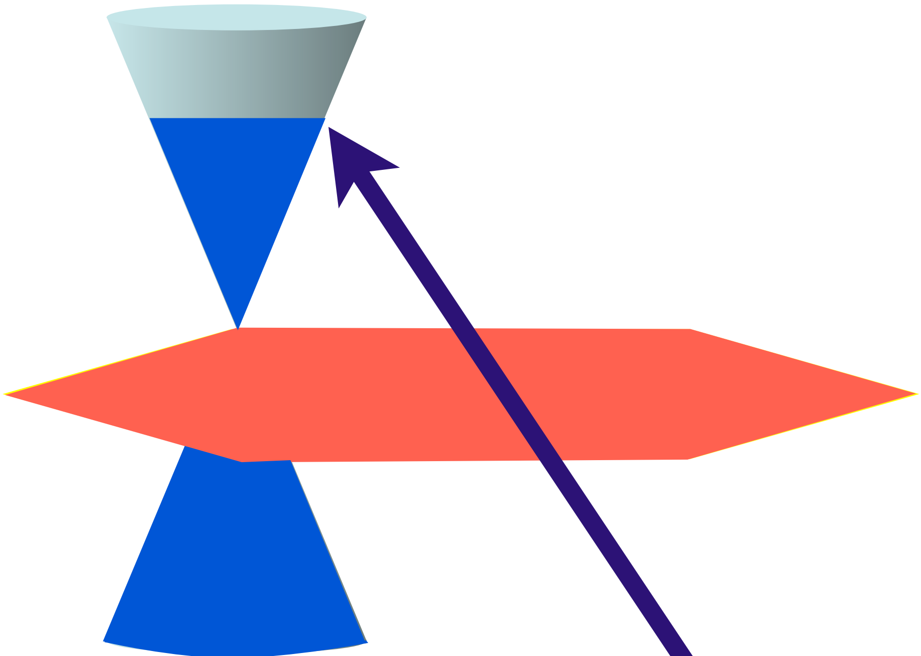


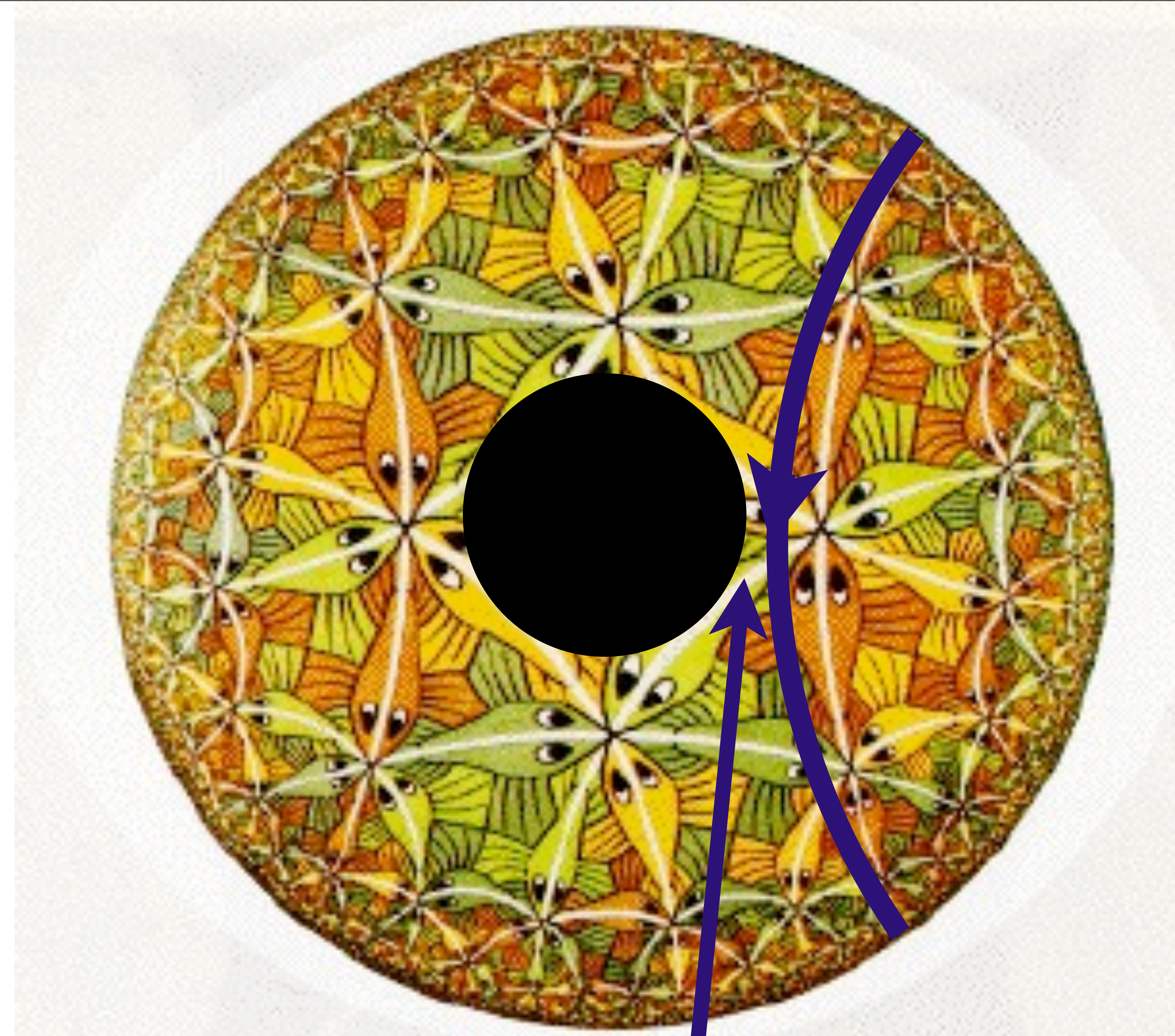


$AdS_2 \times R^2$ near-horizon geometry

$$r - r_+ \quad \sim \quad \frac{1}{\zeta}$$

$$ds^2 \quad = \quad \frac{R^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_+^2}{R^2}\left(dx^2 + dy^2\right)$$

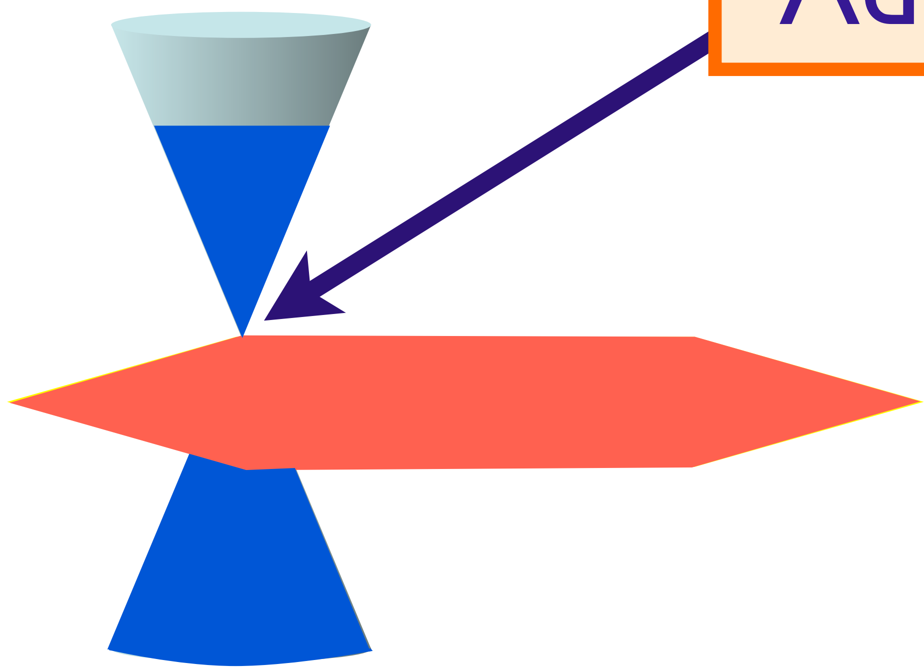


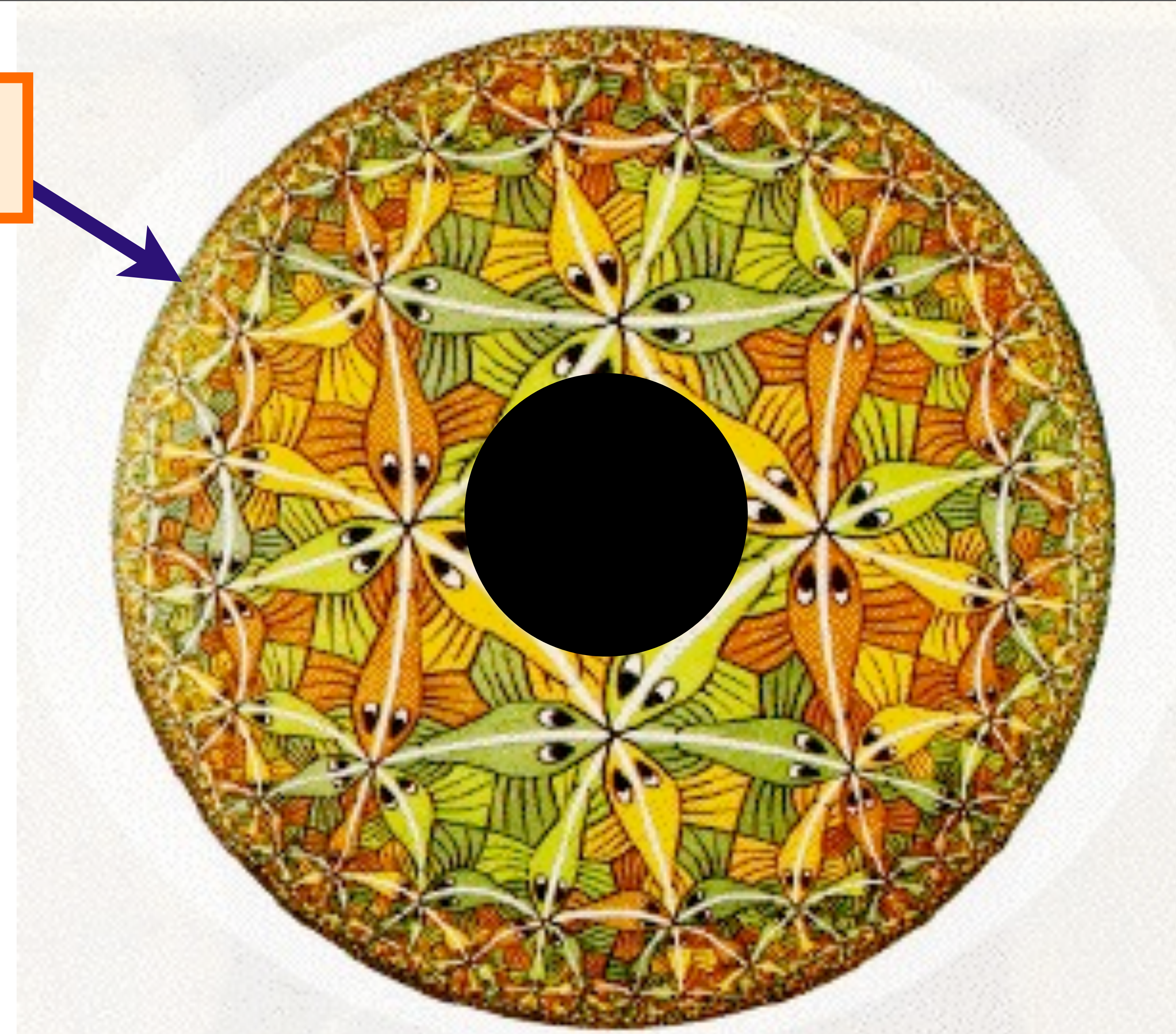


Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

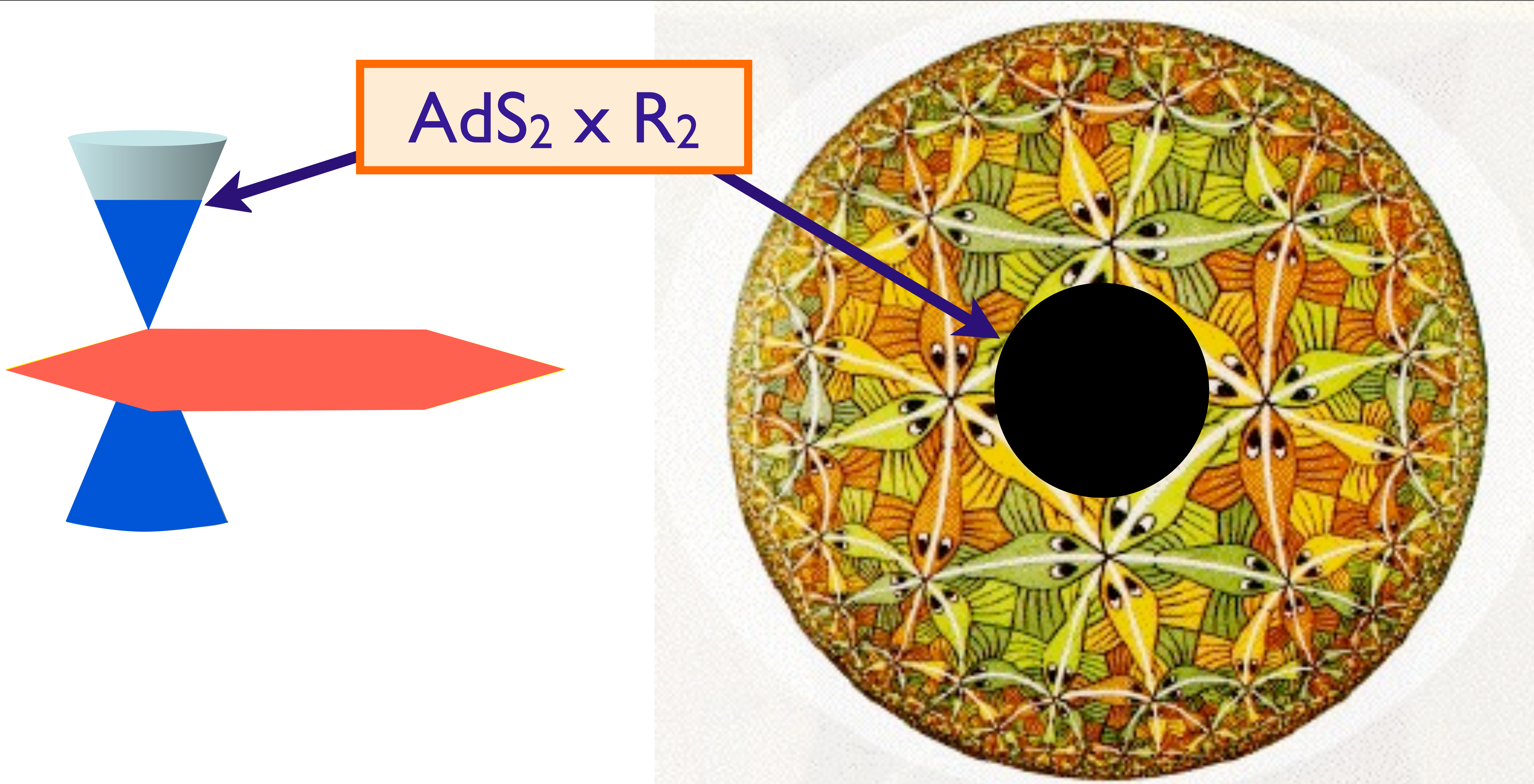
AdS$_4$





Geometric interpretation of RG flow

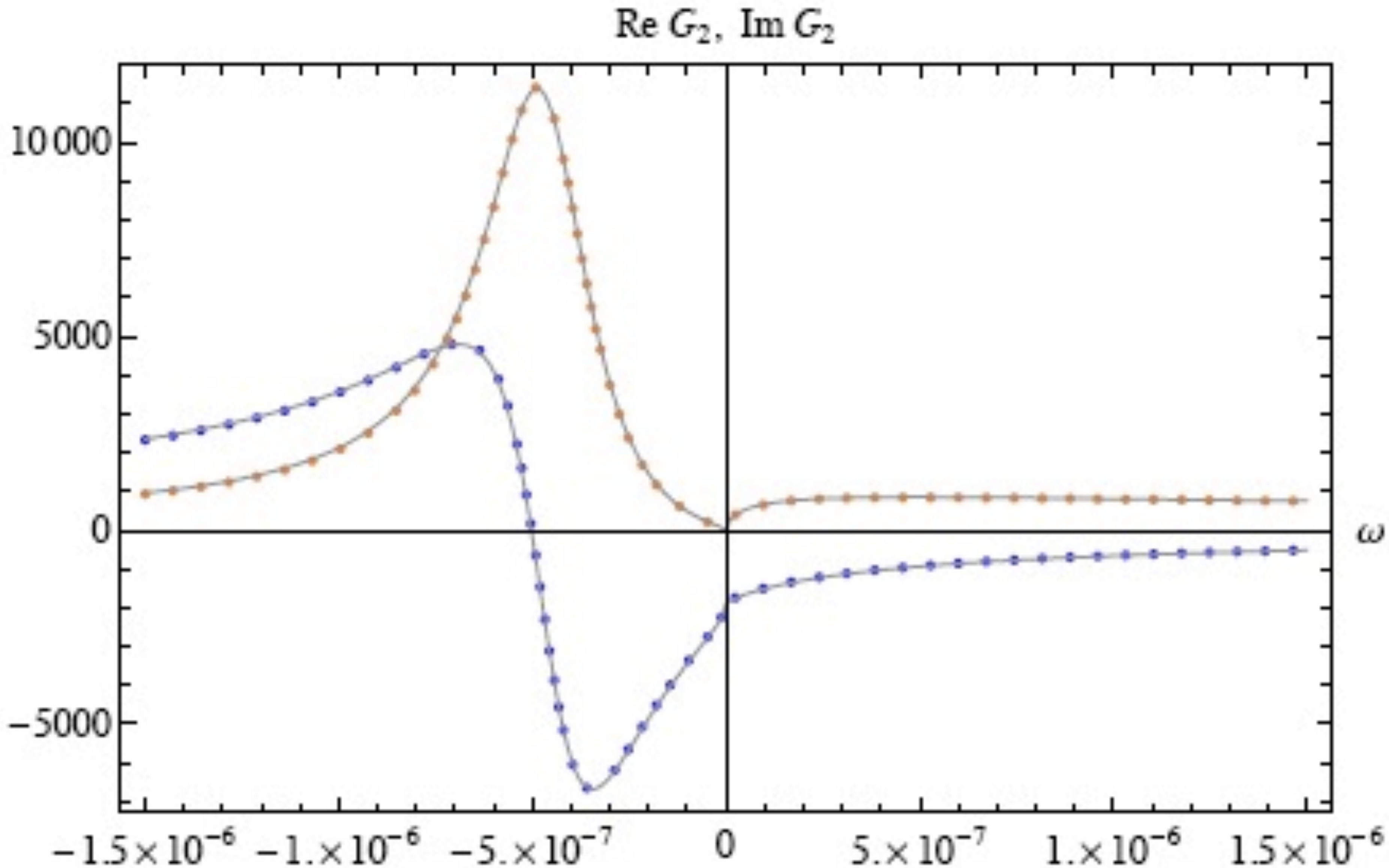
T. Faulkner, H. Liu, J. McGreevy, and D.Vegh, arXiv:0907.2694



Geometric interpretation of RG flow

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Green's function of a fermion

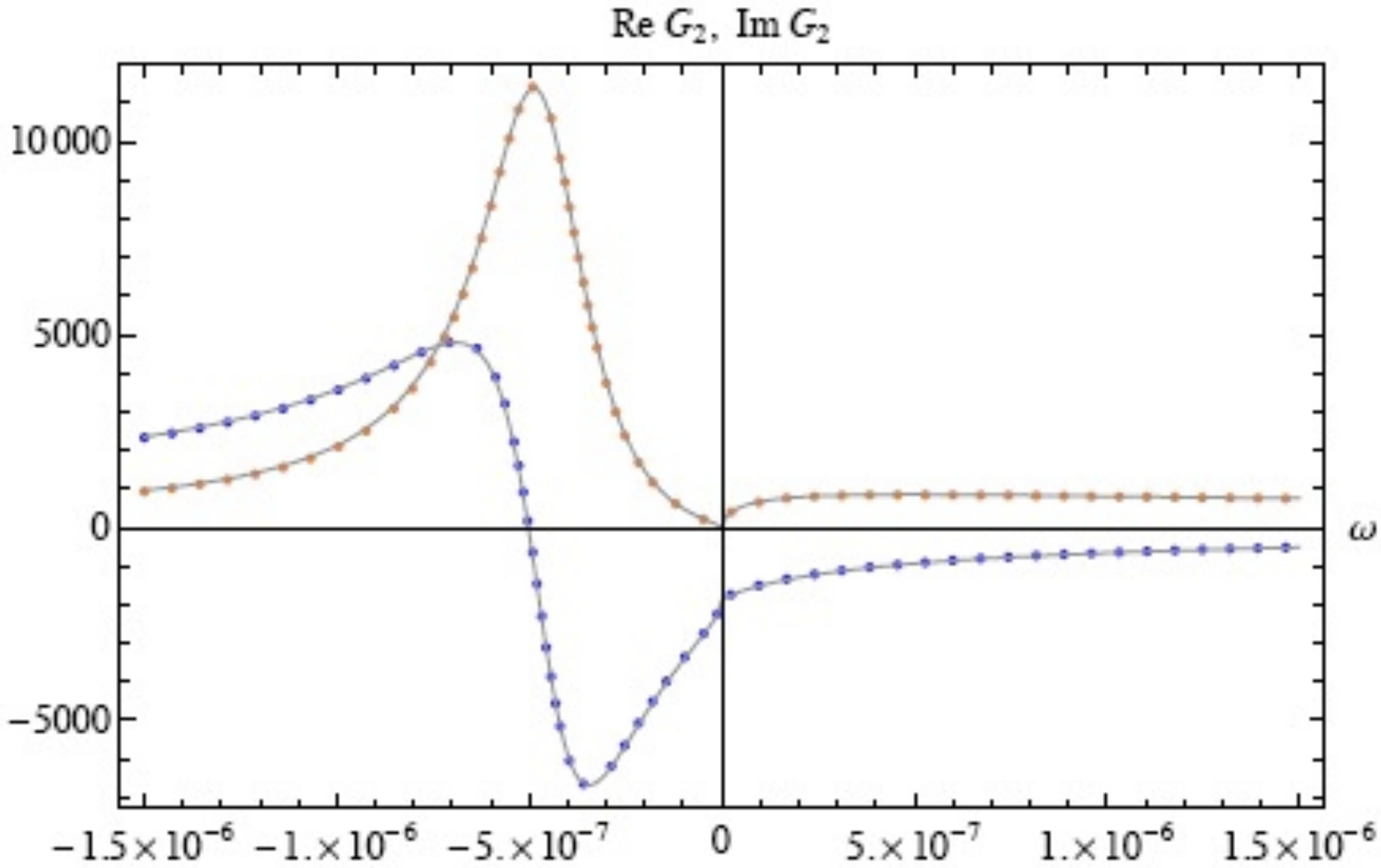


T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also M. Cubrovic, J Zaanen, and K. Schalm, arXiv:0904.1993

Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the "quasinormal frequencies", z_ℓ, of the black hole. Here ℓ represents any set of quantum numbers:

$$\mathcal{F}_{\text{boson}} = -T \sum_\ell \ln \left(\frac{|z_\ell|}{2\pi T} \left| \Gamma \left(\frac{i z_\ell}{2\pi T} \right) \right|^2 \right)$$

$$\mathcal{F}_{\text{fermion}} = T \sum_\ell \ln \left(\left| \Gamma \left(\frac{i z_\ell}{2\pi T} + \frac{1}{2} \right) \right|^2 \right)$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period (2π/(Fermi surface ares)) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density