Entanglement, holography, and strange metals

PCTS, Princeton, October 26, 2012

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Liza Huijse

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"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Z_2 Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

S. Sachdev, arXiv:1203.4565

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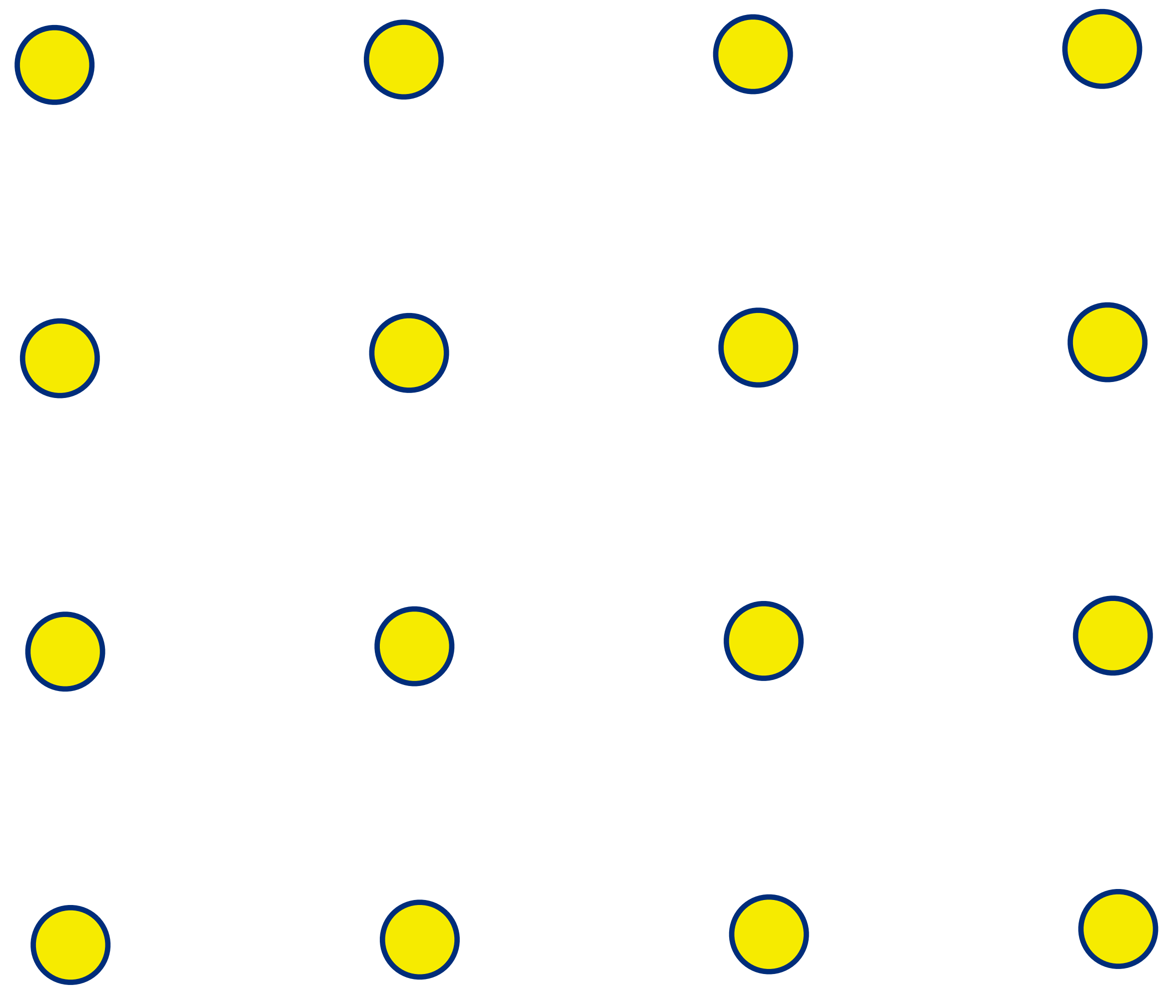
- Conformal systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

Compressible quantum matter

The only compressible phase of traditional condensed matter physics which does not break the translational or $U(1)$ symmetries is the Landau Fermi liquid

Compressible quantum matter

I. Field theory

A. Fermi liquids

B. Non-Fermi liquids

II. Gauge-gravity duality

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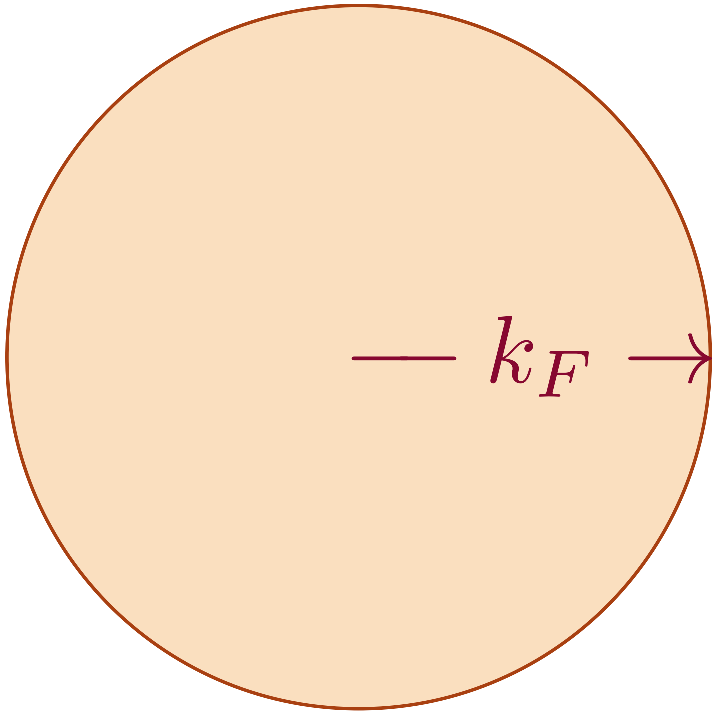
The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$



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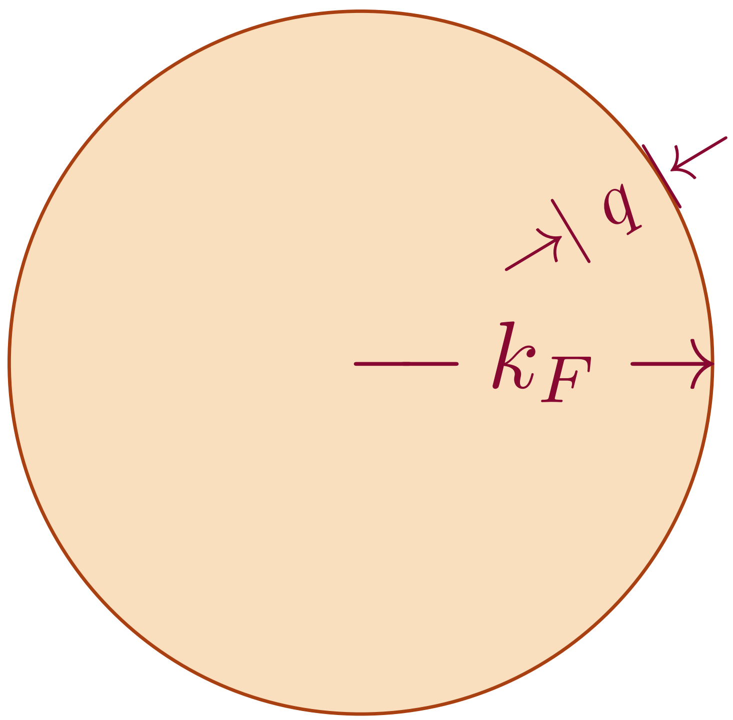

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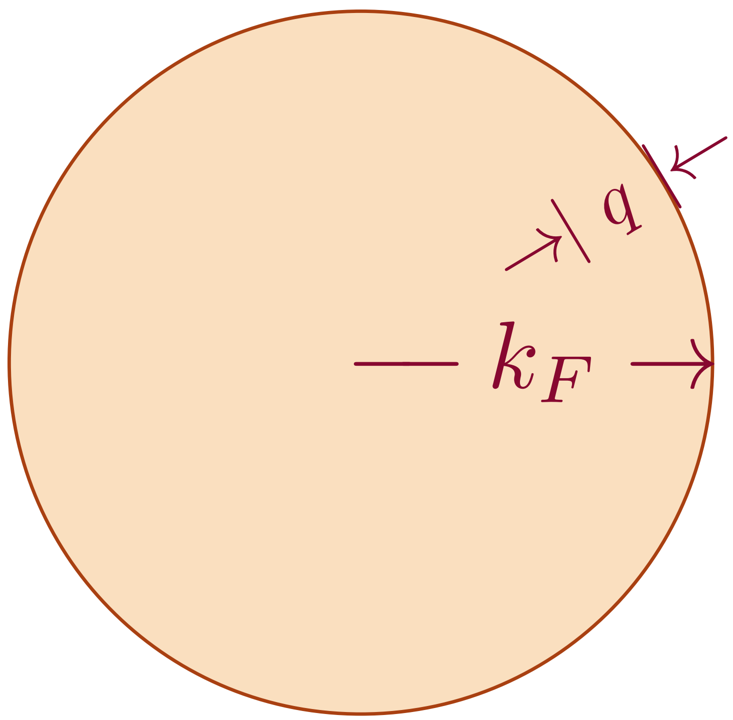

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- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this is as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Hyperscaling

Hyperscaling is the property that the free energy density $F = -(1/V)\log Z$ has the canonical scaling dimension. For a *classical* thermal system at its critical temperature in D spatial dimensions placed in a finite box of size L, we have (Casimir; Fisher, de Gennes 1978):

$$F = F_\infty - \widetilde{c}\, L^{-D},$$

where $\widetilde{c}$ is a universal constant. Similarly, moving away from the critical point to a system with a finite correlation length ξ, we have in an infinite system

$$F = F_\infty - \bar{c}\, \xi^{-D}.$$

For systems which violate hyperscaling, we write

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For a <u>*quantum*</u> system at a quantum critical point in d spatial dimensions, T is the analog of $1/L^z$, and so its free energy density obeys

$$F(T) = F(0) - \bar{c}\, T^{(d+z-\theta)/z}.$$

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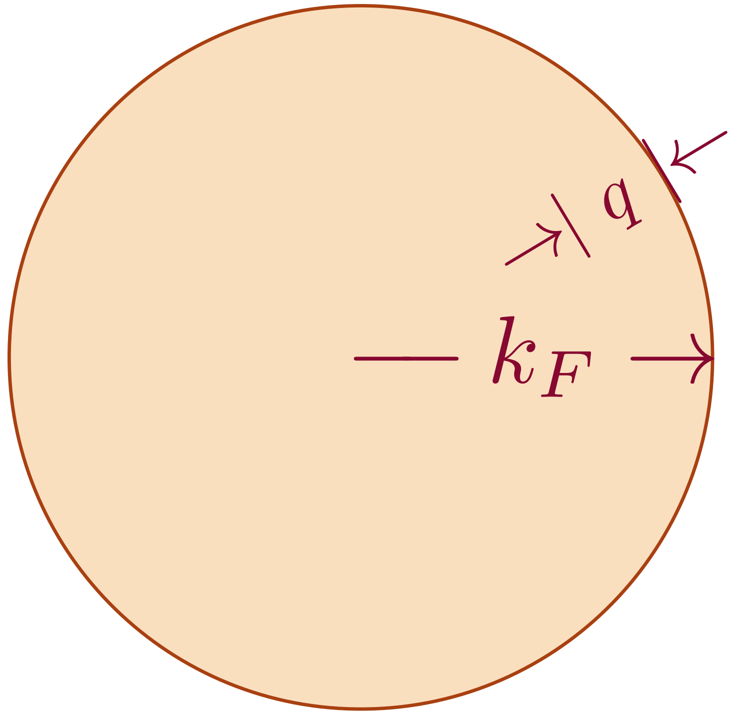

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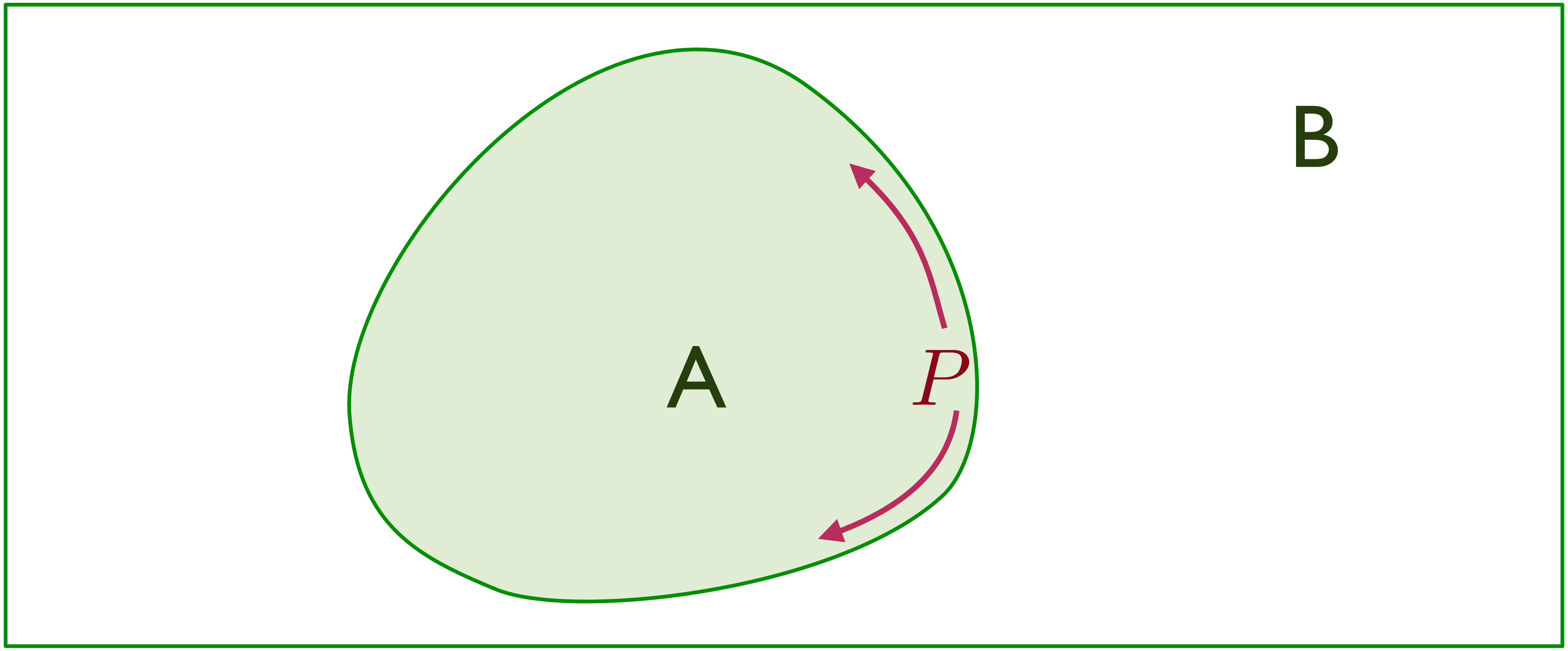
Entanglement entropy of the Fermi liquid



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
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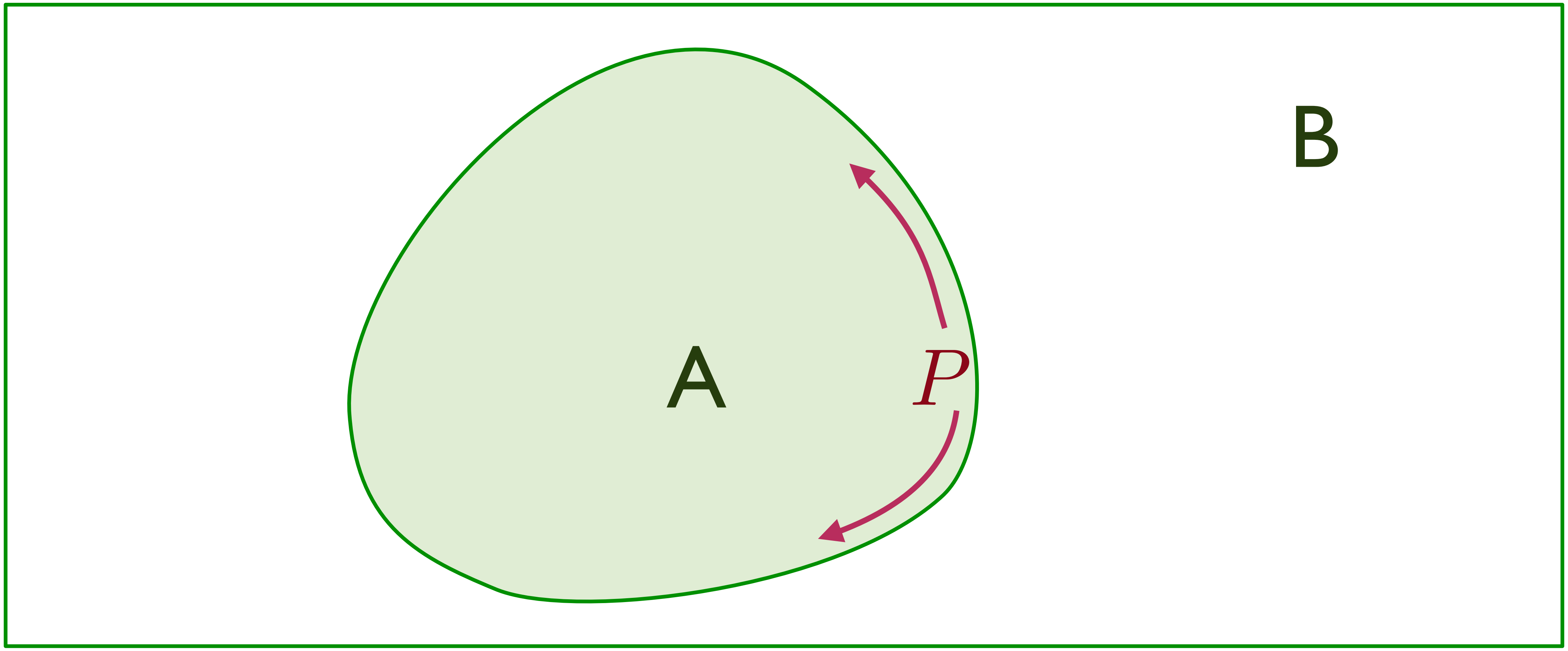

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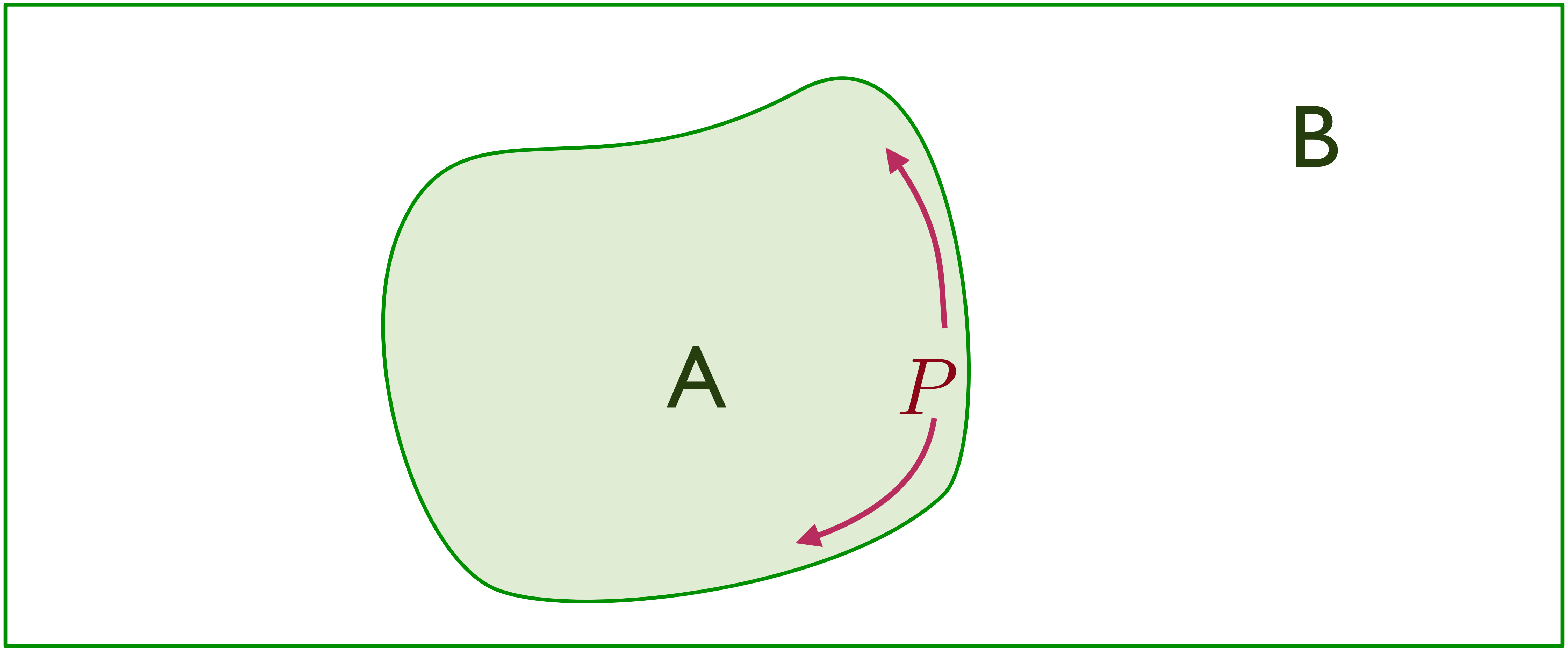

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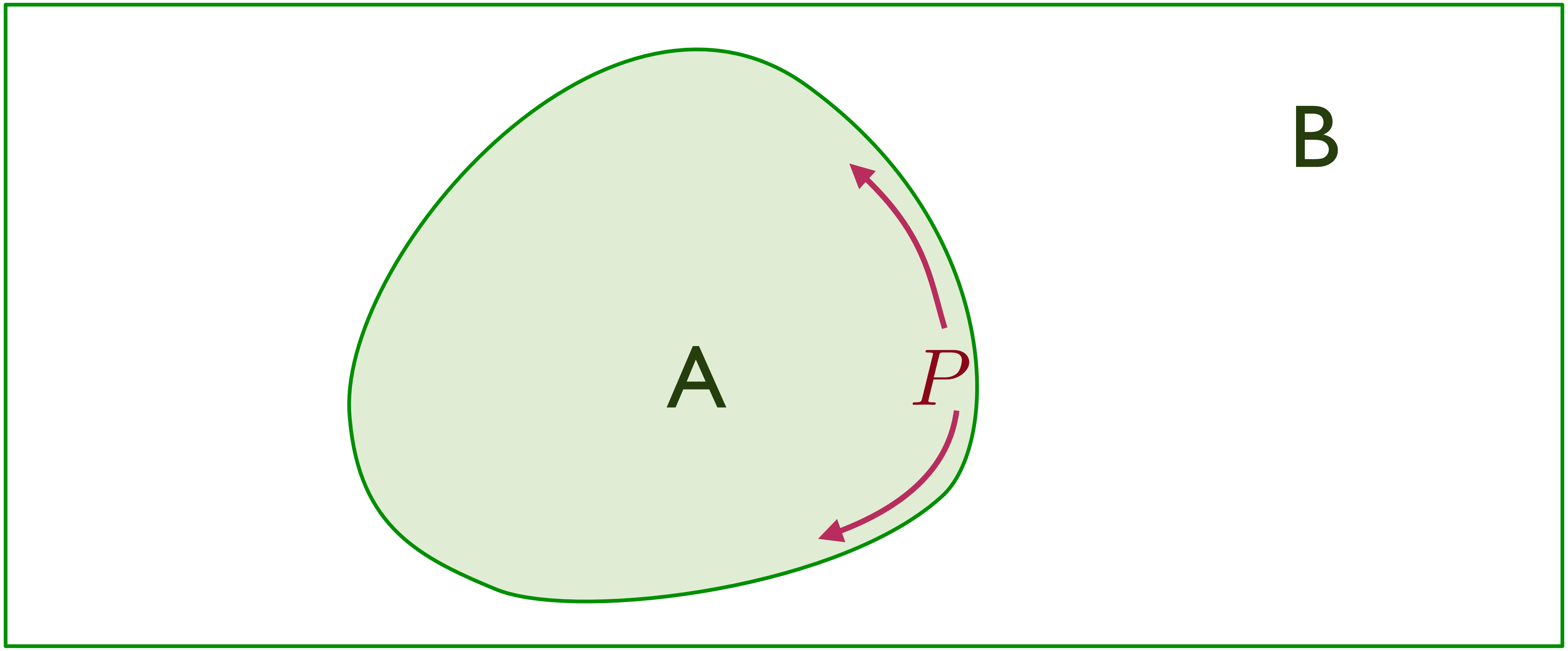

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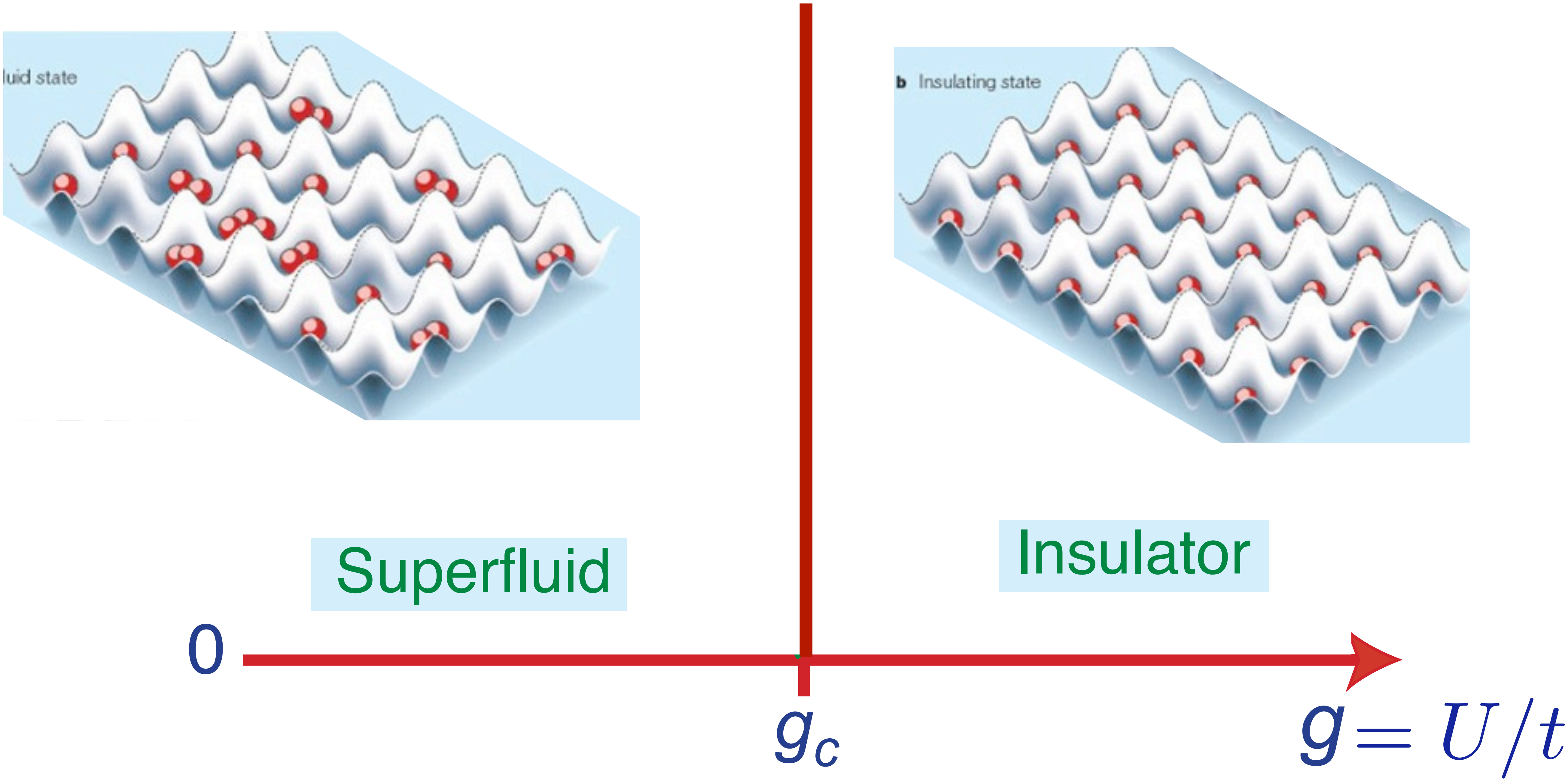
Bose-Hubbard model at integer filling

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) \quad ; \quad n_i \equiv b_i^\dagger b_i$$



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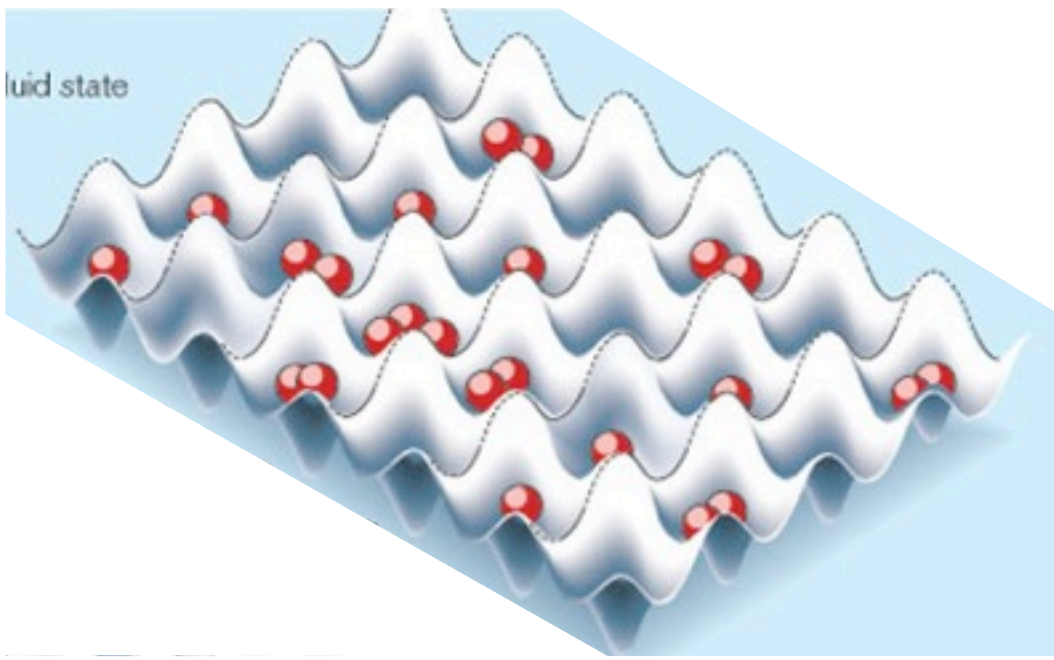
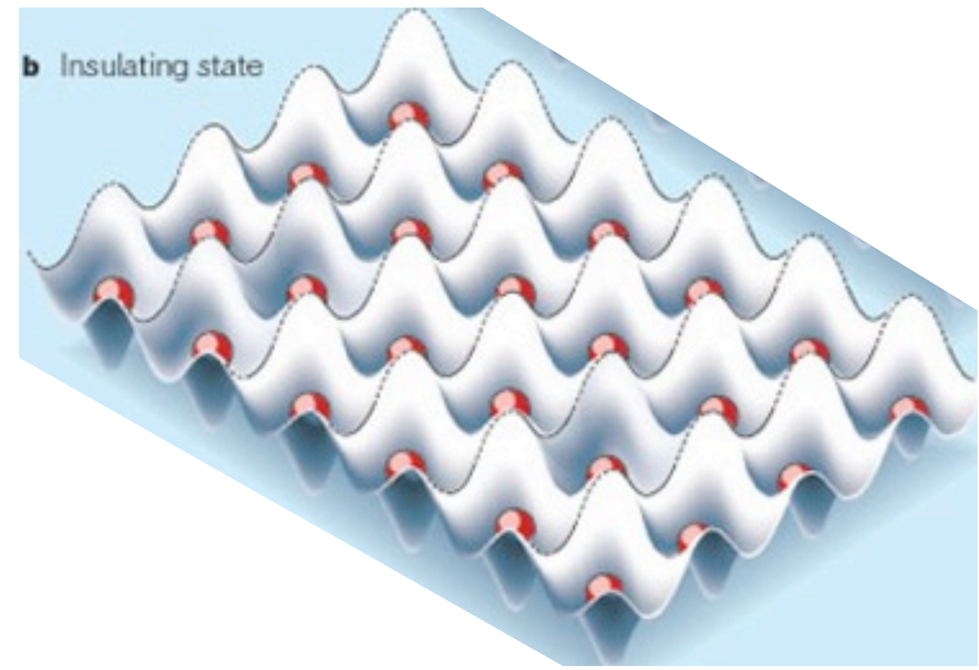




Superfluid

Insulator

0

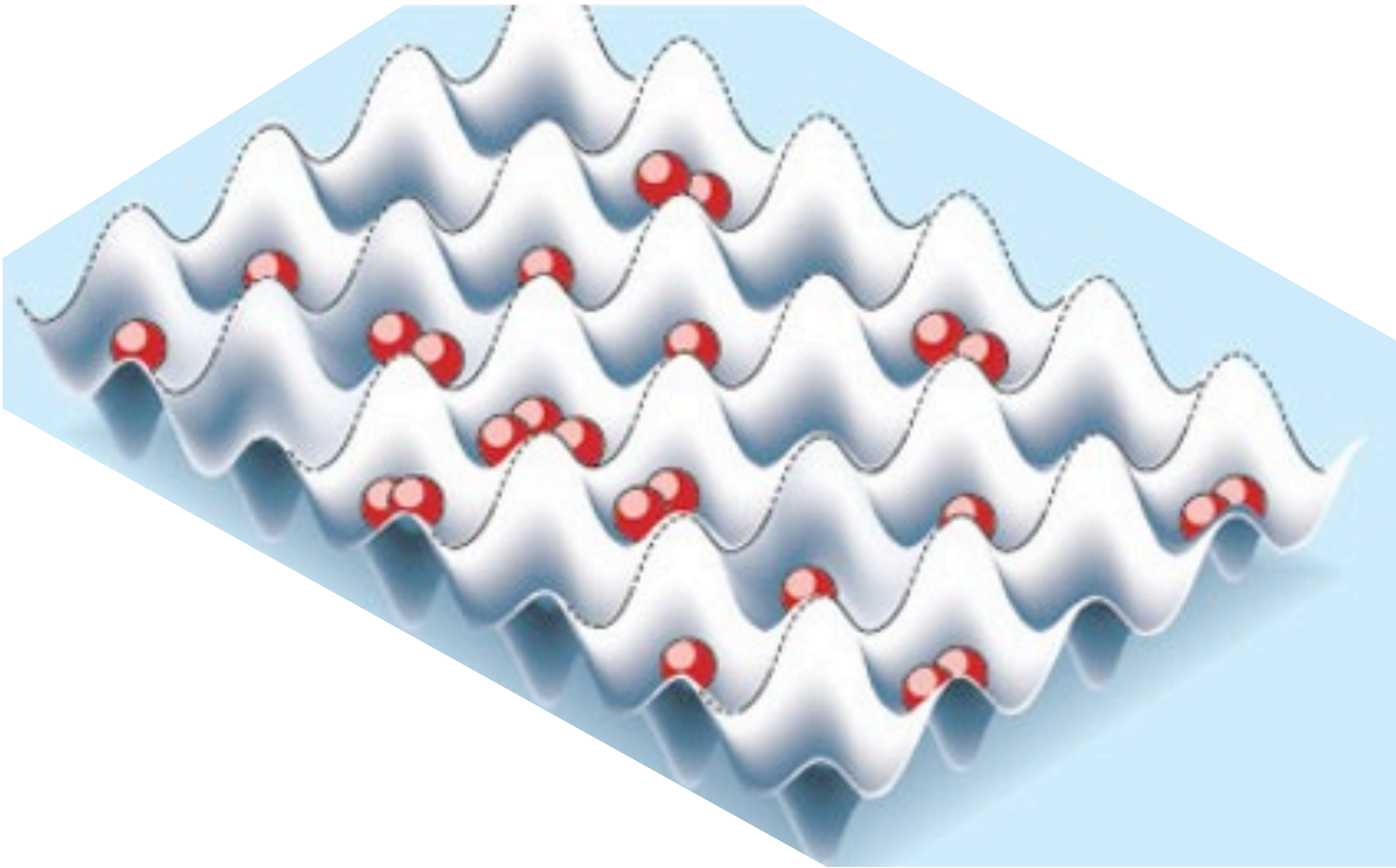
g_c

$g = U/t$

CFT3 of the XY model:
$\mathcal{L} = |\partial \psi|^2 + s|\psi|^2 + u|\psi|^4$

Bosons with correlated hopping

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$



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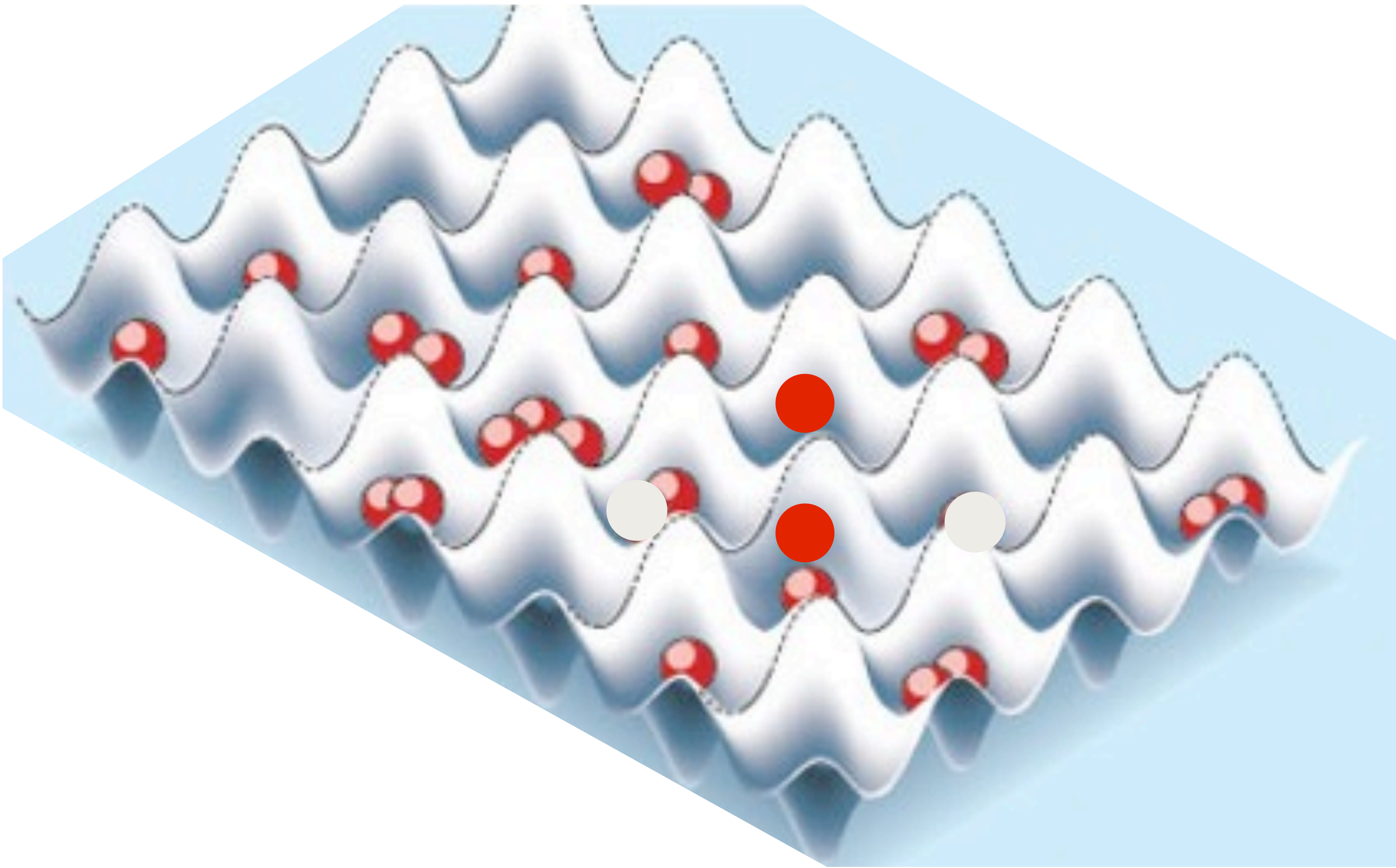

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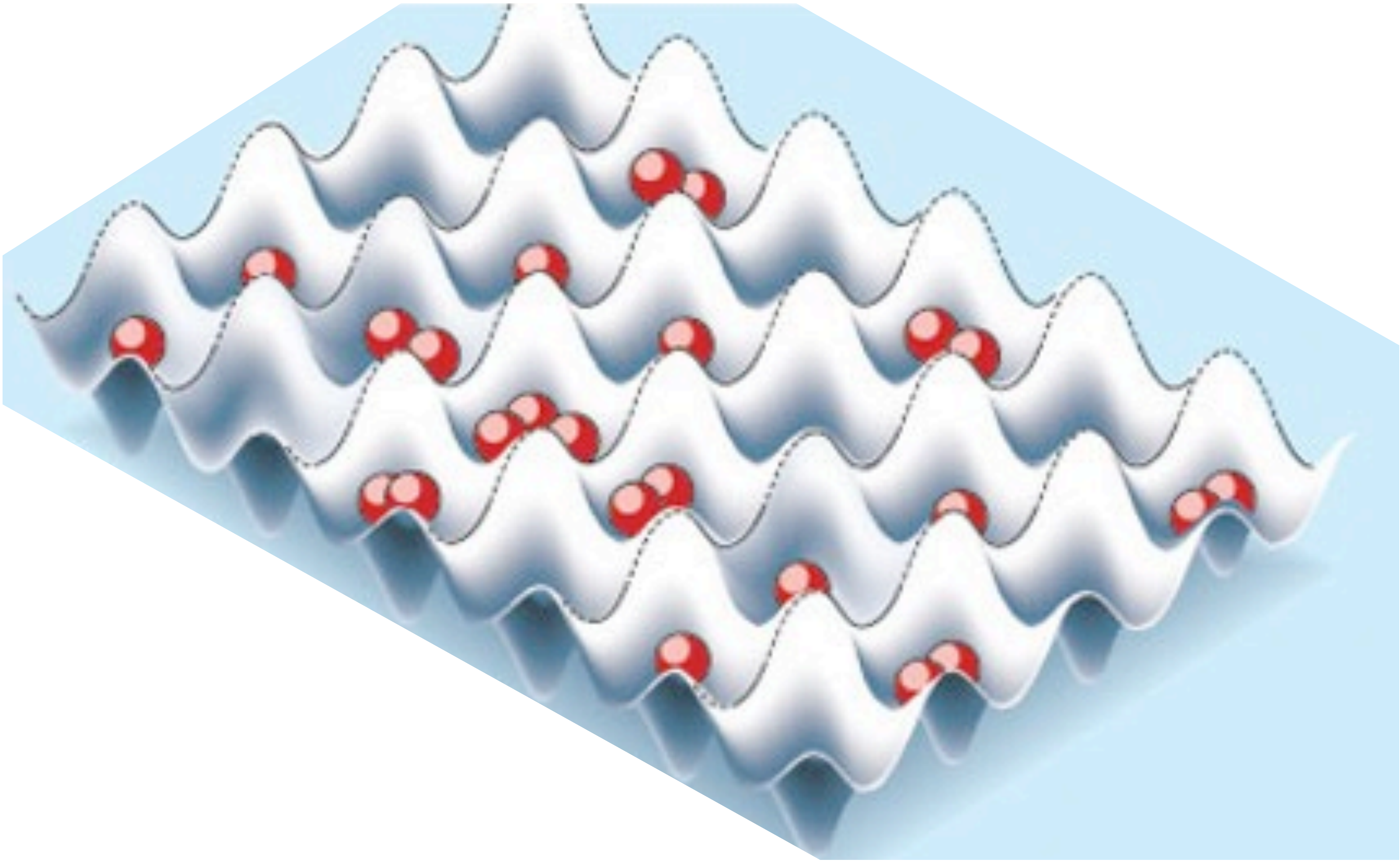

Bosons with correlated hopping at half-filling

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$

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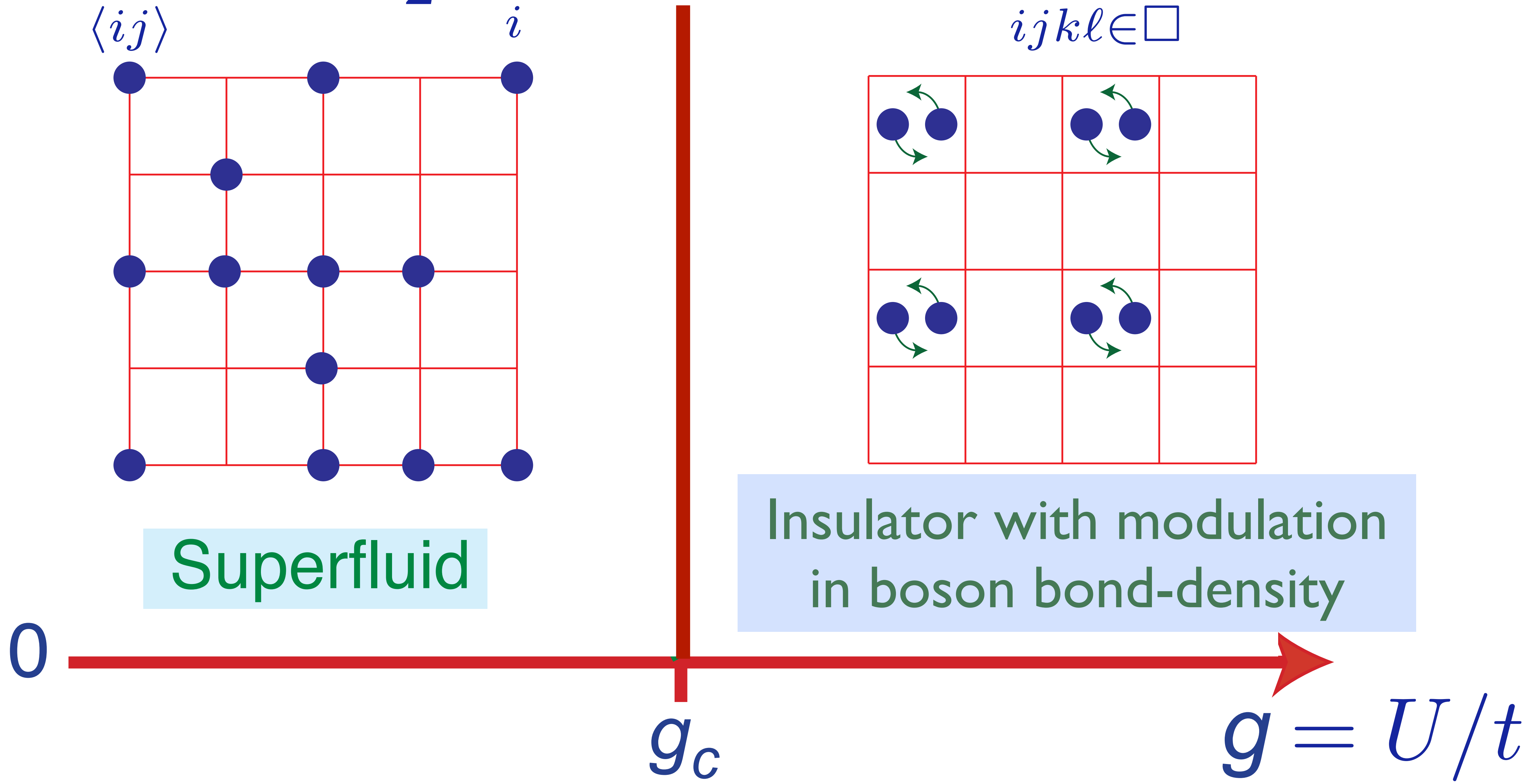

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

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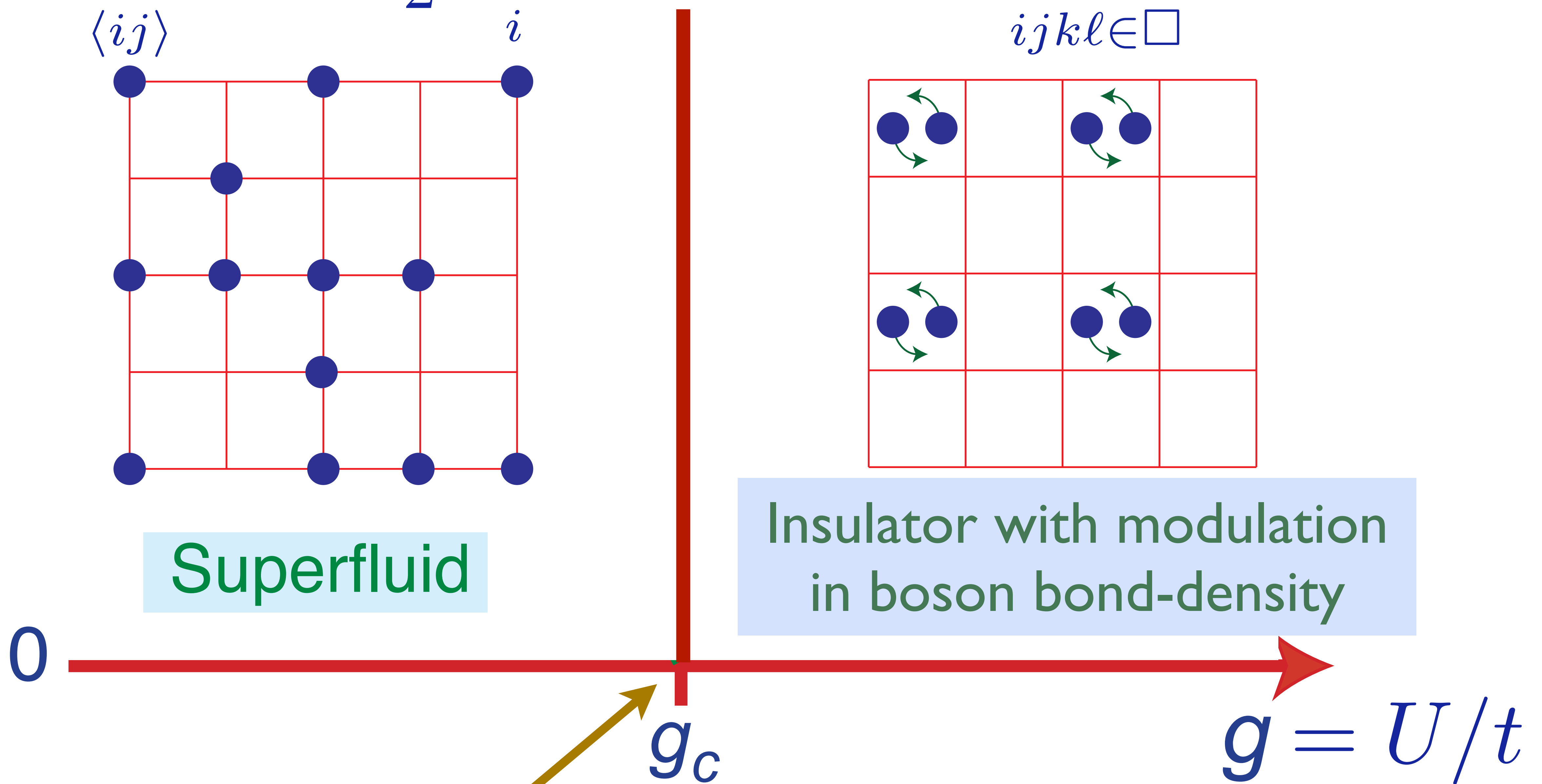

'Deconfined' critical point: boson fractionalizes $b \sim z_1 z_2$, and the fractionalized bosons are coupled to an emergent U(1) gauge field

$$\mathcal{L} = |(\partial_\mu - iA_\mu)z_1|^2 + |(\partial_\mu + iA_\mu)z_2|^2 + s(|z_1|^2 + |z_2|^2) + u(|z_1|^2 + |z_2|^2)^2 - v|z_1|^2|z_2|^2$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Bosons with correlated hopping close to half-filling

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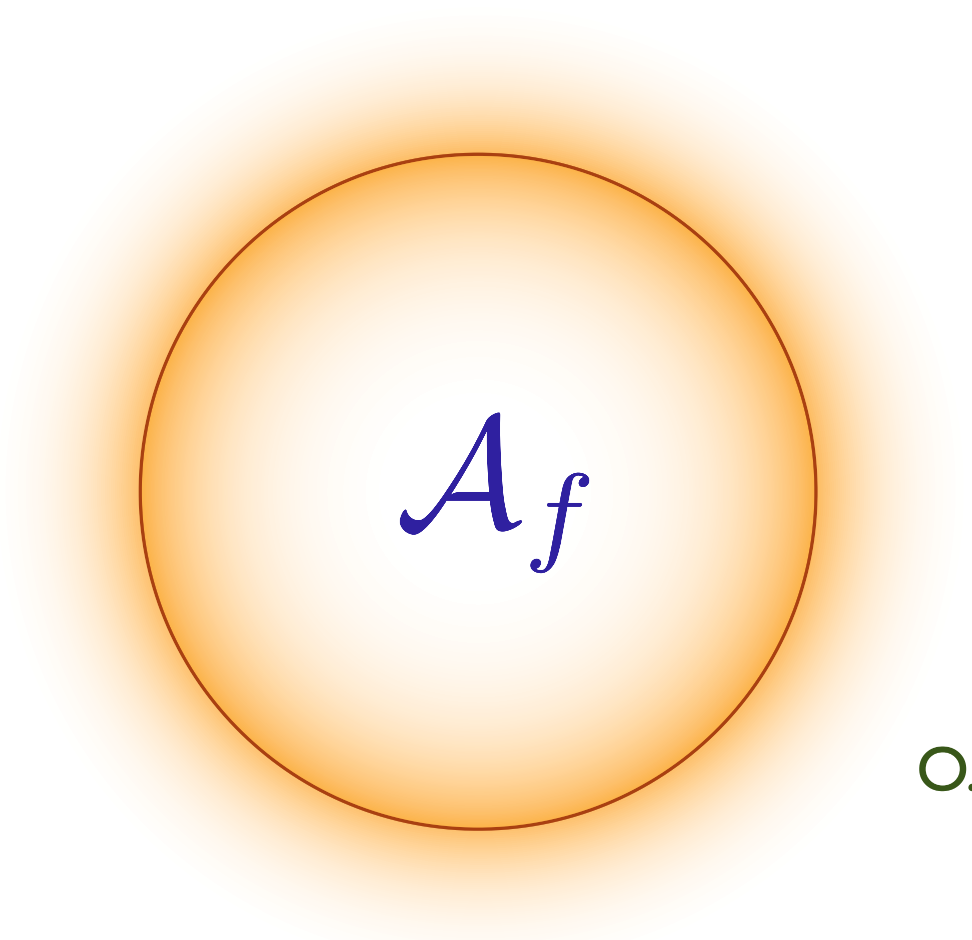
- **NFL**, the non-Fermi liquid *Bose metal*. The z_1, z_2 quanta fermionize into f_1, f_2, each of which forms a Fermi surface. Both fermions are gauge-charged, and so the Fermi surfaces are partially "hidden".



$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Phys. Rev.* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Phys. Rev.* D **84**, 026001 (2011)
S. Sachdev, arXiv:1209.1637

Non-Fermi liquid Bose Metal

For suitable interactions, we can have the boson, b, *fractionalize* into two fermions $f_{1,2}$:

$$b \to f_1 f_2$$

This implies the effective theory for $f_{1,2}$ is invariant under the U(1) gauge transformation

$$f_1 \to f_1 e^{i\theta(\boldsymbol{x},\tau)} \quad , \quad f_2 \to f_2 e^{-i\theta(\boldsymbol{x},\tau)}$$

Consequently, the effective theory of the Bose metal has an emergent gauge field A_μ and has the structure

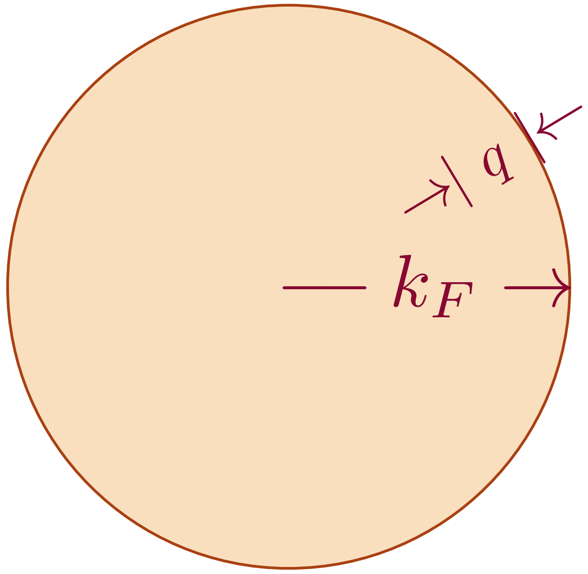
$$\mathcal{L} = f_1^\dagger \left(\partial_\tau - iA_\tau - \frac{(\boldsymbol{\nabla} - i\boldsymbol{A})^2}{2m} - \mu \right) f_1 + f_2^\dagger \left(\partial_\tau + iA_\tau - \frac{(\boldsymbol{\nabla} + i\boldsymbol{A})^2}{2m} - \mu \right) f_2$$

The gauge-dependent $f_{1,2}$ Green's functions have Fermi surfaces obeying $\mathcal{A}_f = \langle \mathcal{Q} \rangle$. However, these Fermi surfaces are not directly observable because it is gauge-dependent. Nevertheless, gauge-independent operators, such as b or $b^\dagger b$, will exhibit *Friedel oscillations* associated with fermions scattering across these <u>hidden</u> Fermi surfaces.

FL

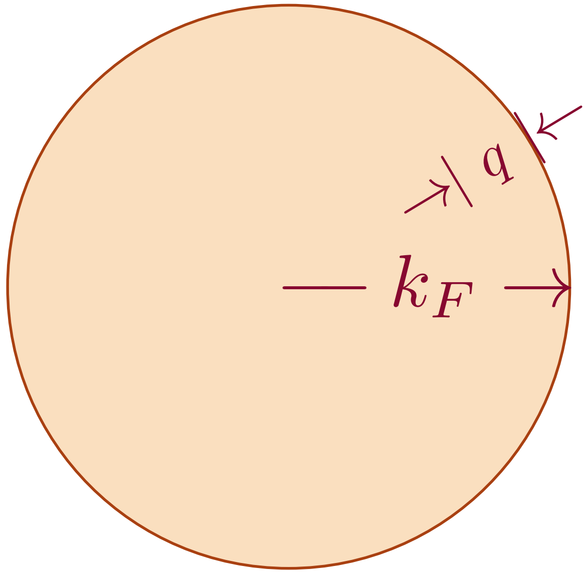
Fermi

liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL
Fermi liquid



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NFL
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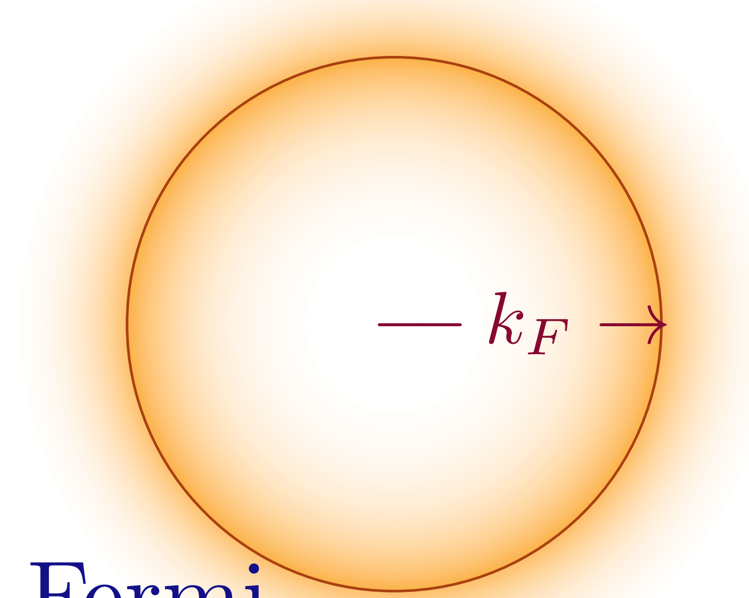
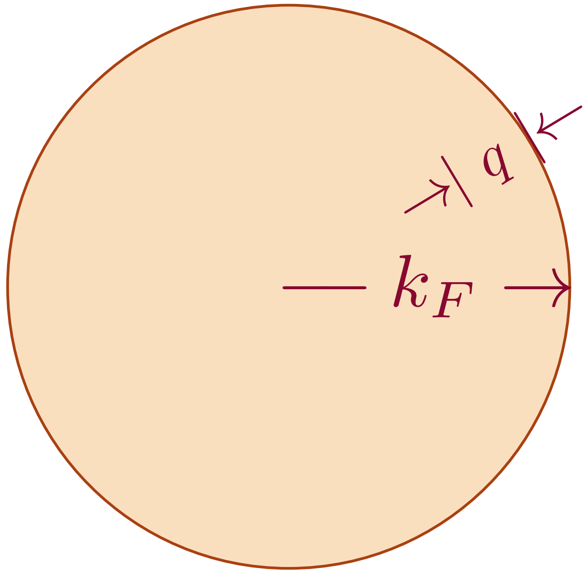
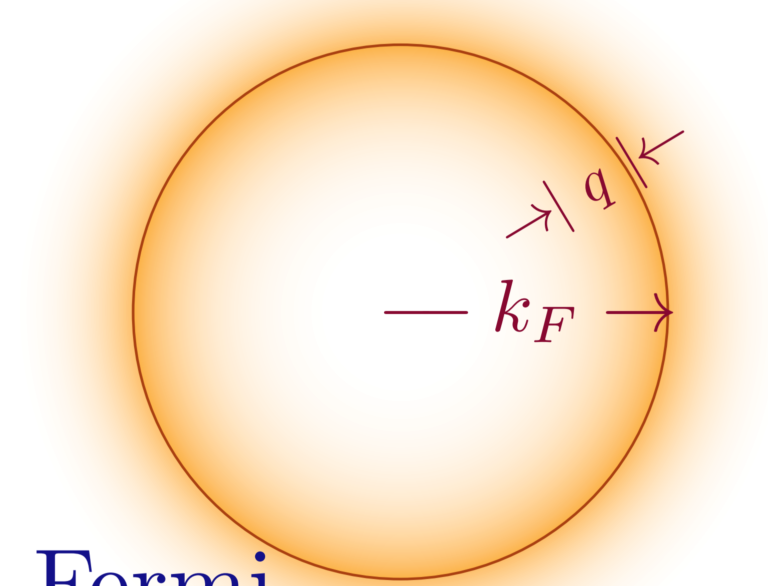
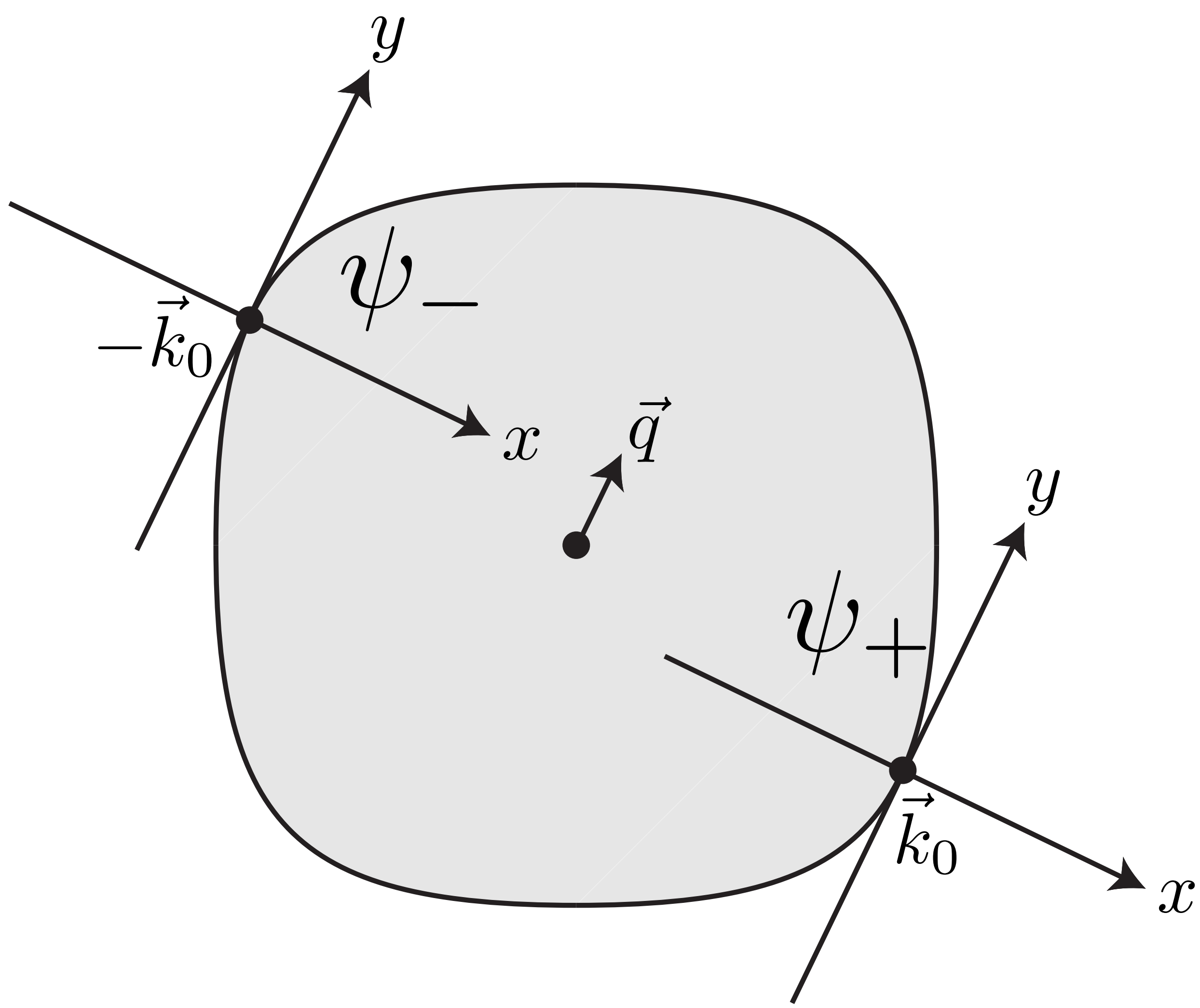


- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

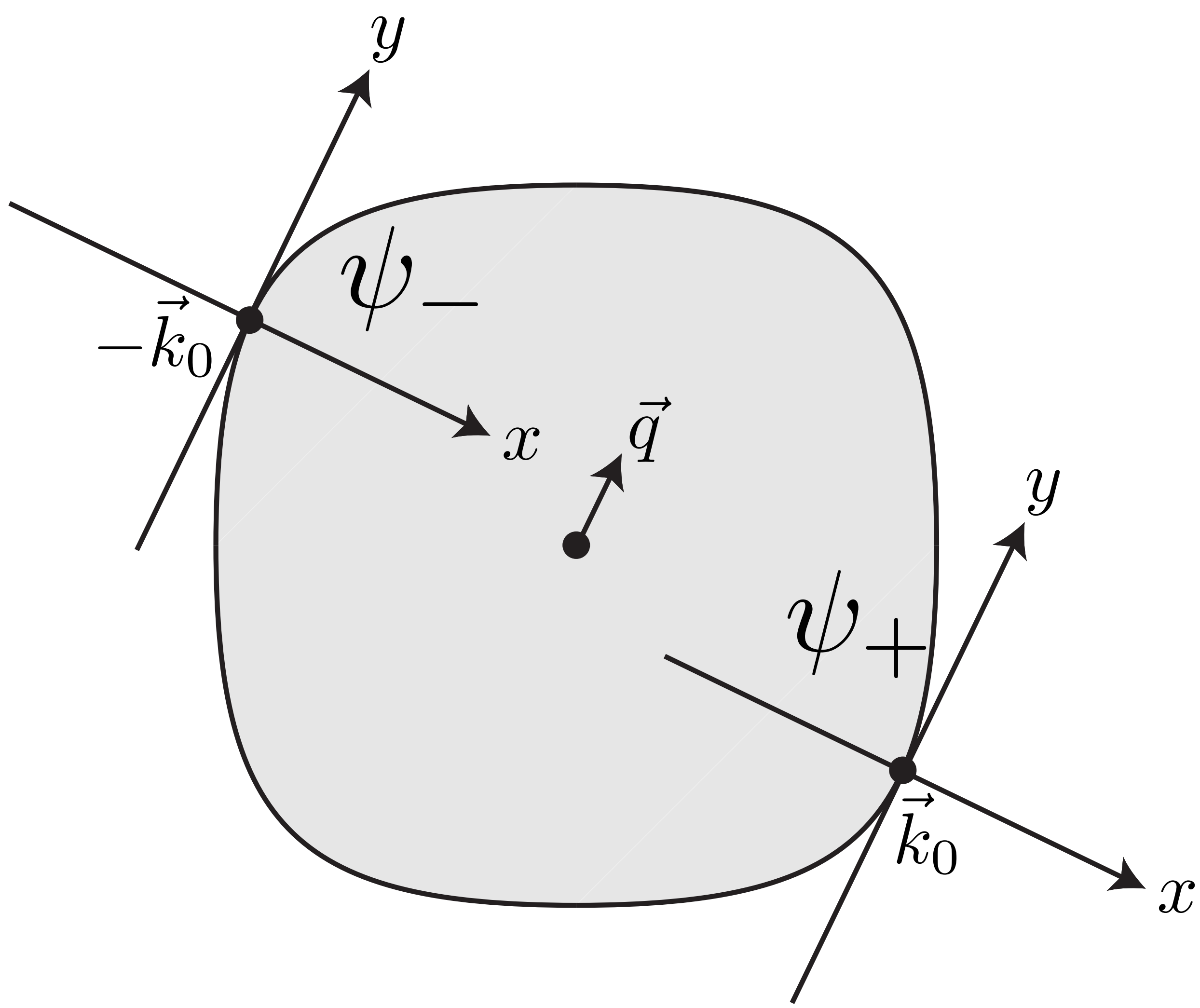
FL Fermi liquid 	**NFL Bose metal** 
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$. • Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989) M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid



- $\vec{A}$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Field theory of non-Fermi liquid



- In Landau gauge, write $A_i = (\epsilon_{ij} q_j / |q|) a(\vec{q})$. Then the correlators of $a(\vec{q})$ obey a matrix-model-like clustering property w.r.t. different orientations of $\vec{q}$:

$$\langle a(\vec{q}_1) a(\vec{q}_2) a(\vec{q}_3) \ldots a(\vec{q}_n) \rangle = \prod_\alpha \langle a(\vec{q}_{\alpha 1}) a(\vec{q}_{\alpha 2}) \ldots \rangle$$

where all momenta for a given α are collinear to each other, and those with $\alpha \neq \beta$ are non-collinear.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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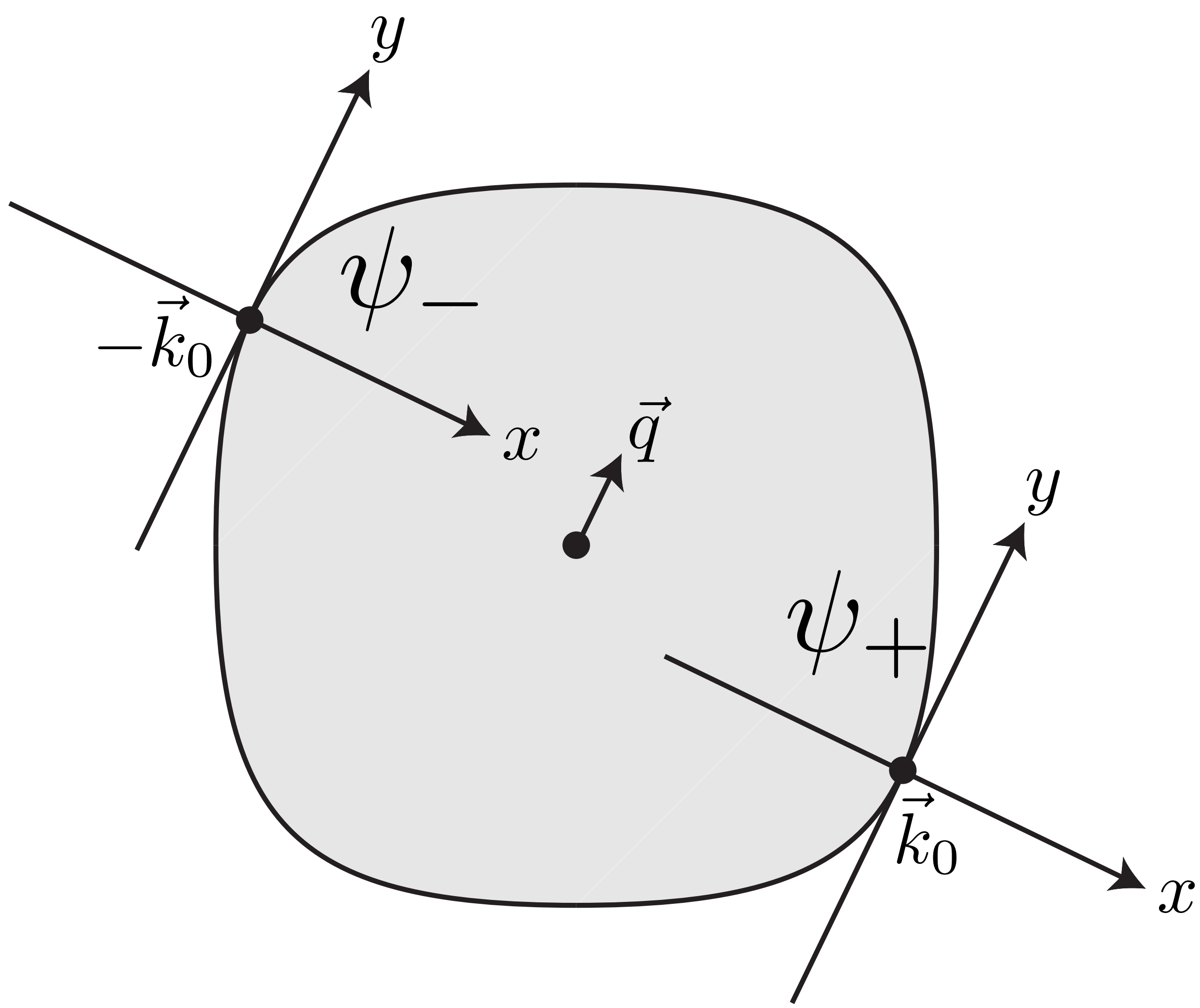

$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

Field theory of non-Fermi liquid

$$\mathcal{L} = \psi_+^\dagger\left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger\left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - a\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

Simple scaling argument for $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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Simple scaling argument for $z = 3/2$.

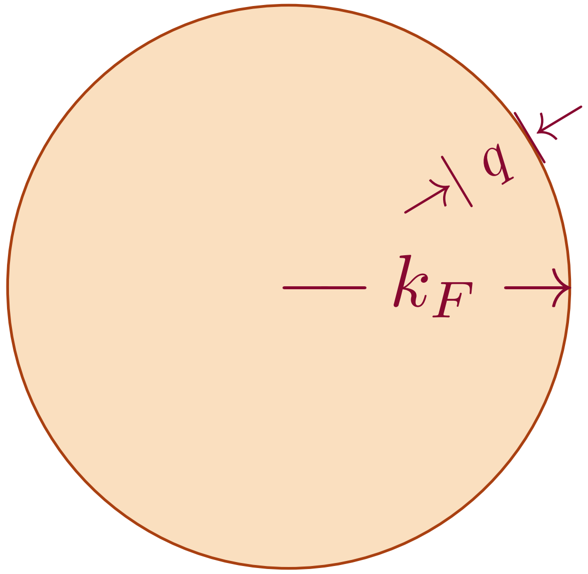
Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} a &\to a\, s \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

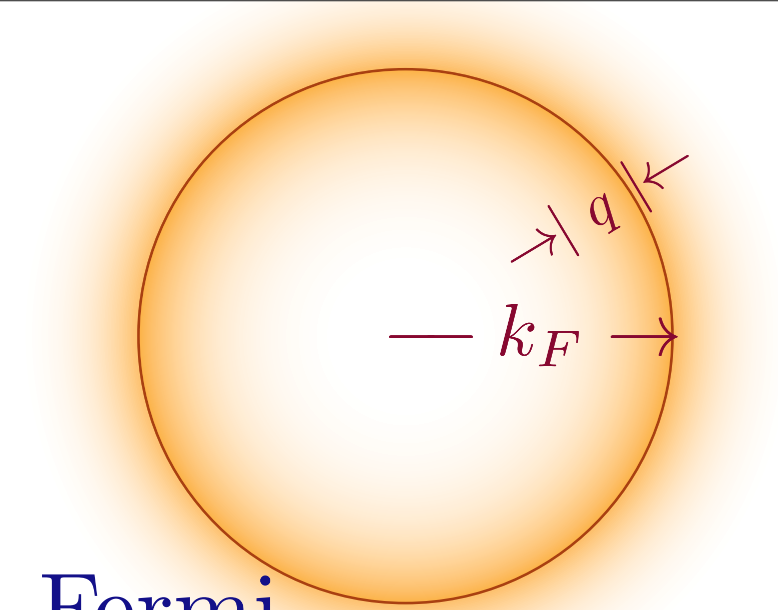
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

FL Fermi liquid



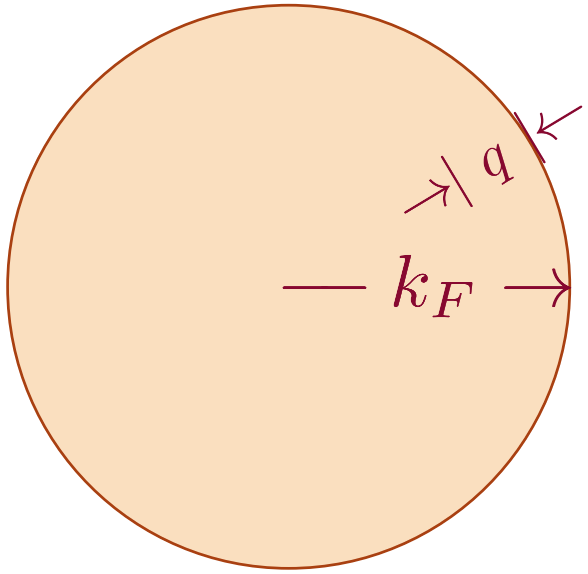
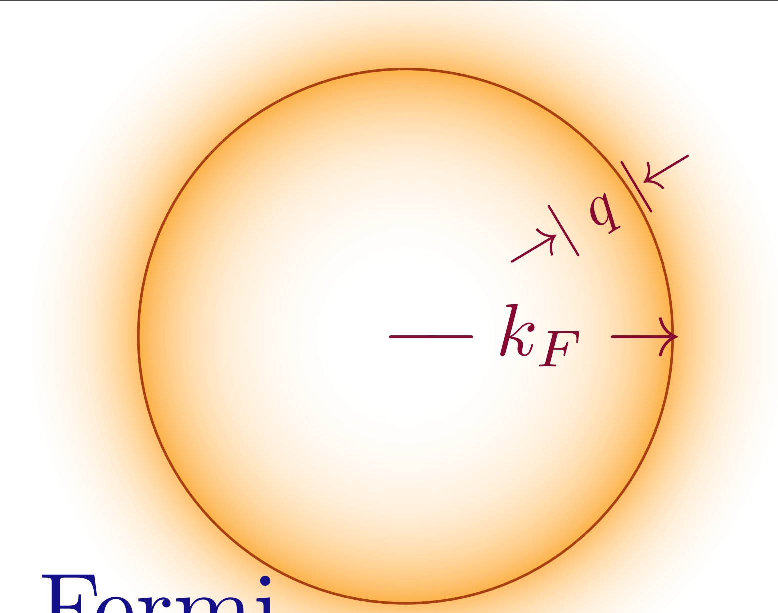
- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

NFL Bose metal



- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

FL Fermi liquid 	NFL Bose metal 
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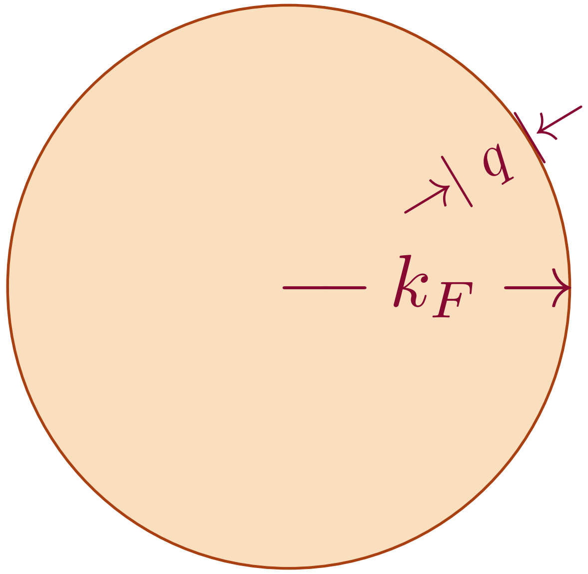

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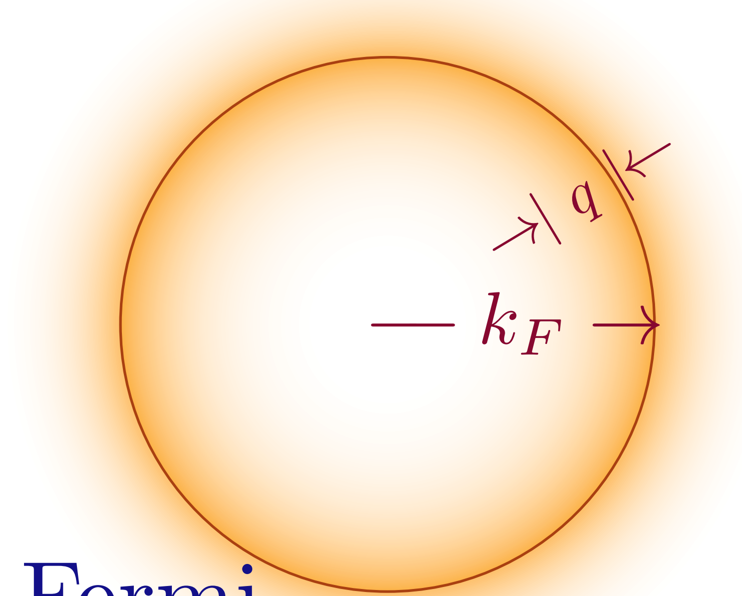

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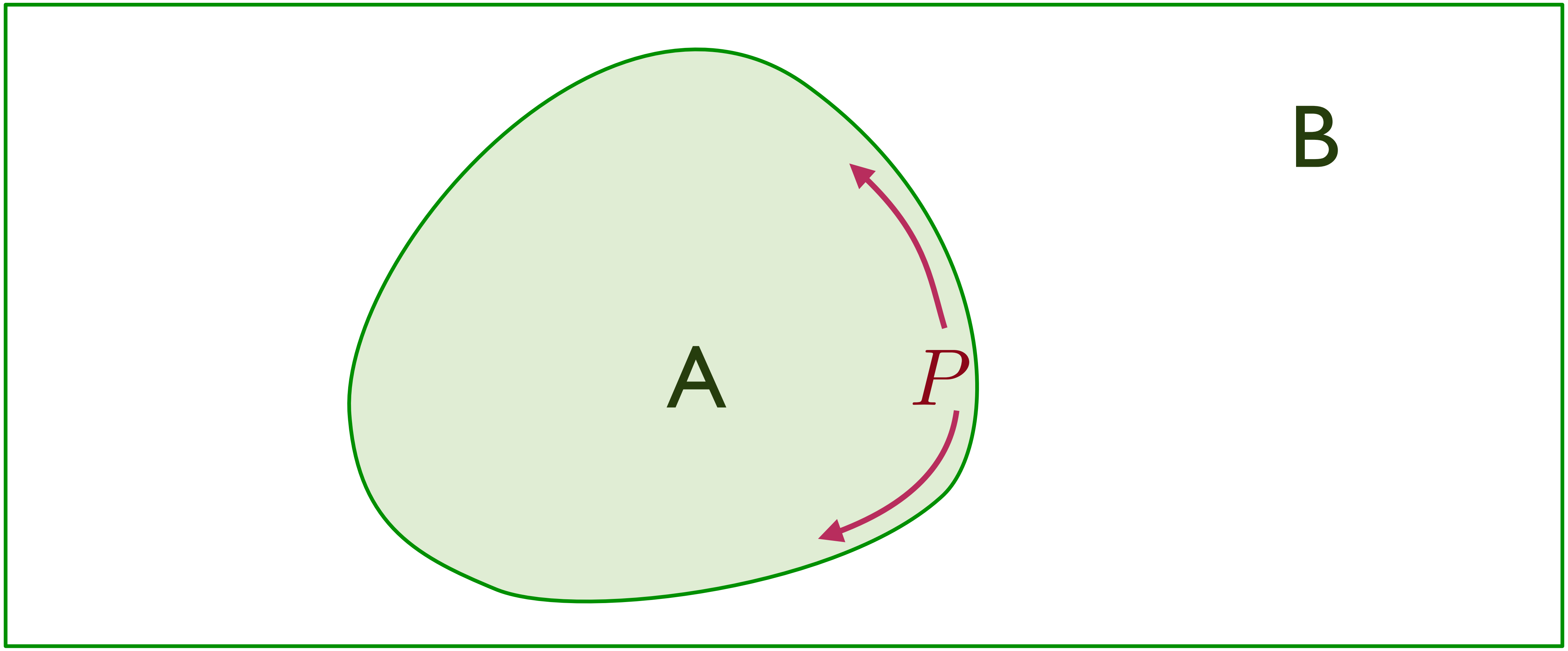
Entanglement entropy of the non-Fermi liquid



Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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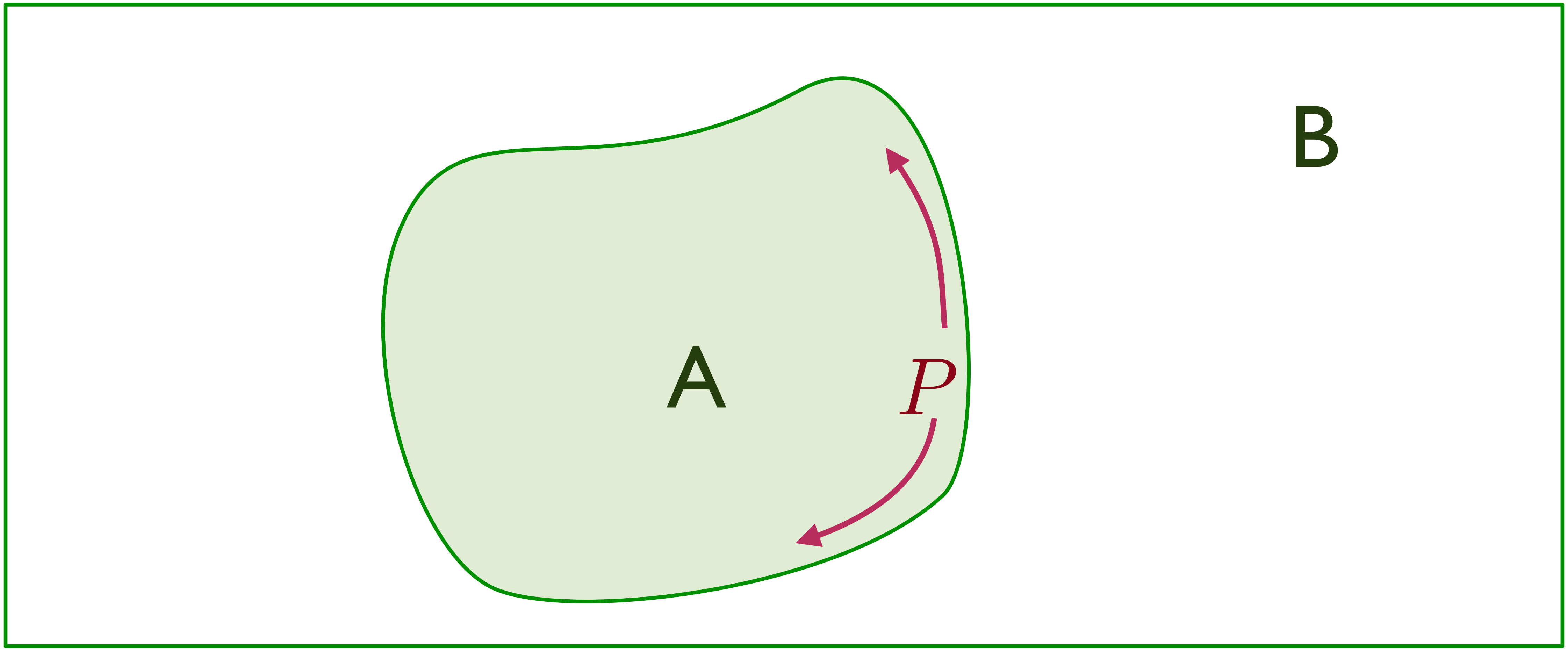

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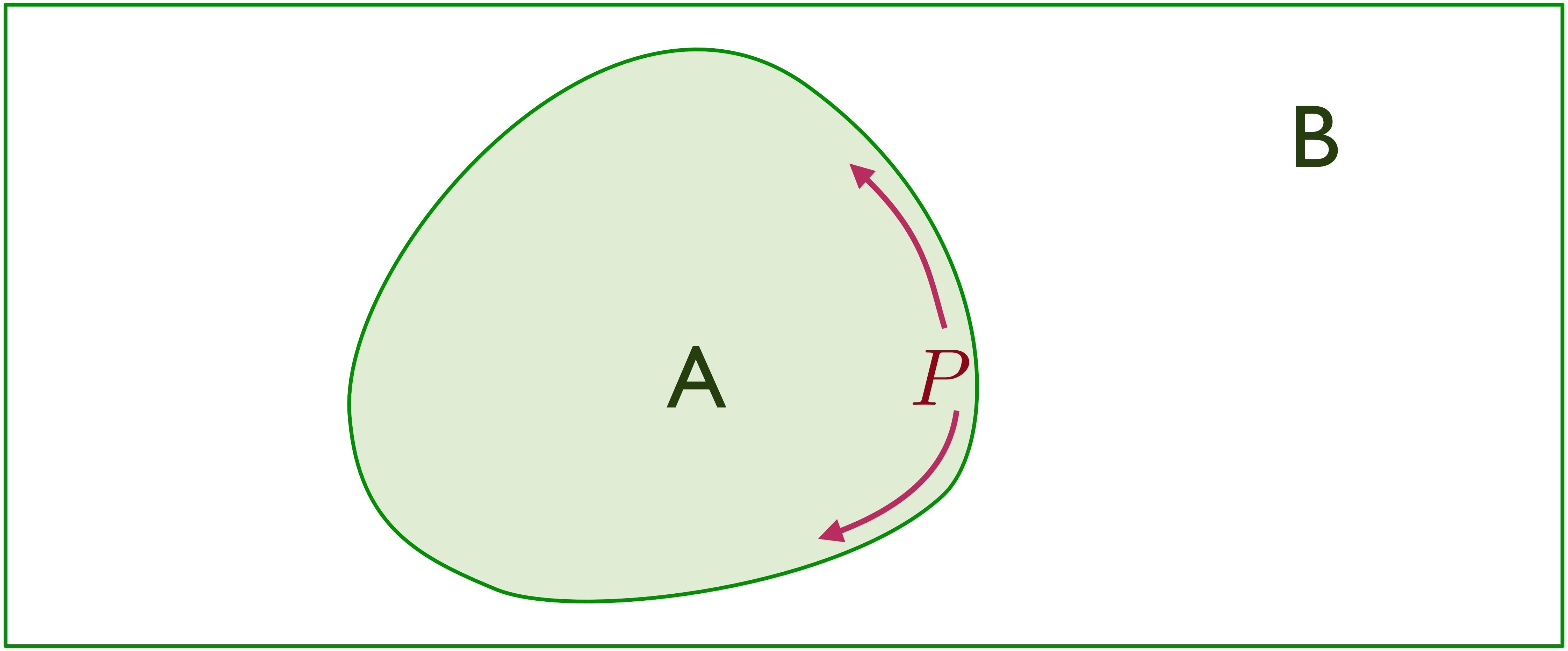

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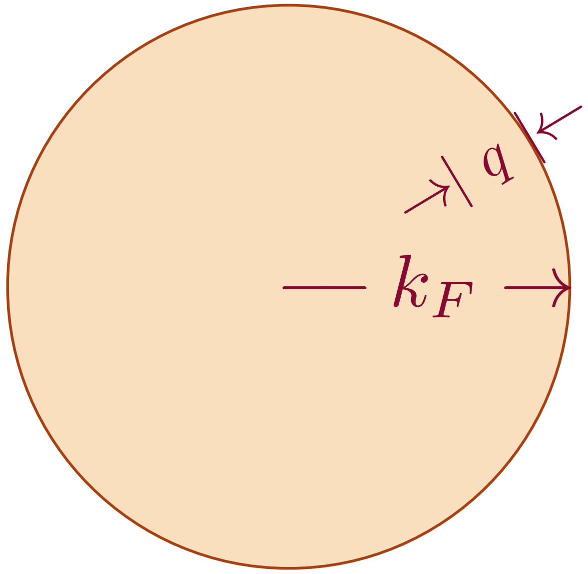	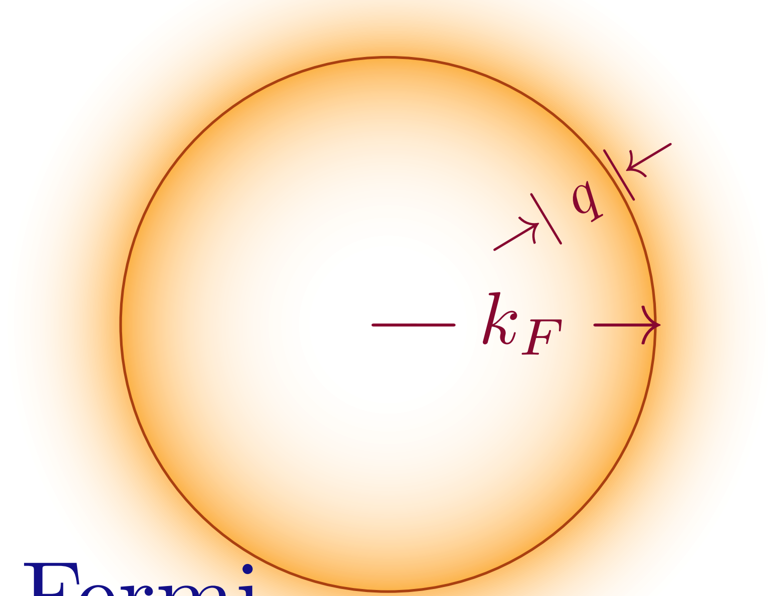
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Compressible quantum matter

I. Field theory

A. Fermi liquids

B. Non-Fermi liquids

II. Gauge-gravity duality

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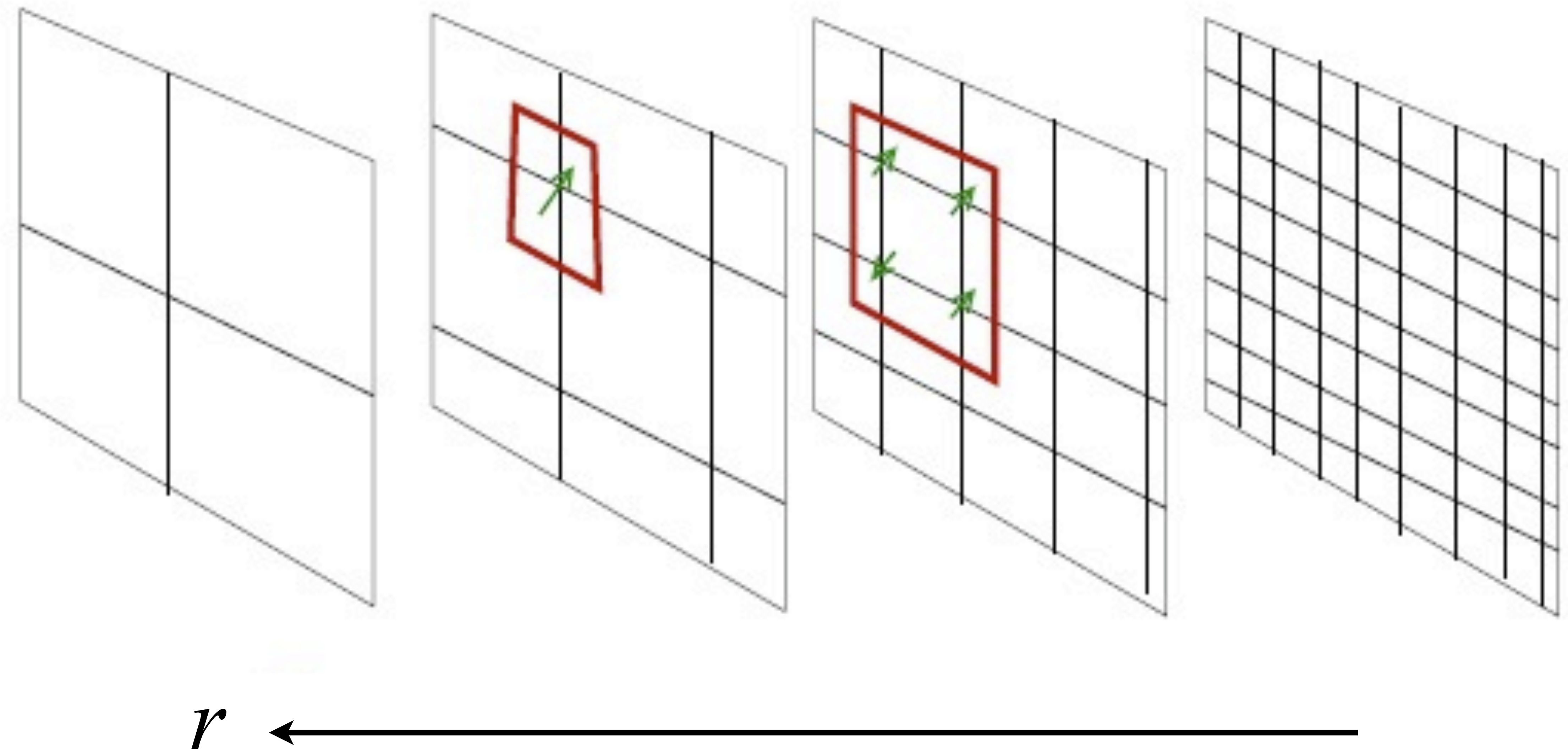
Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

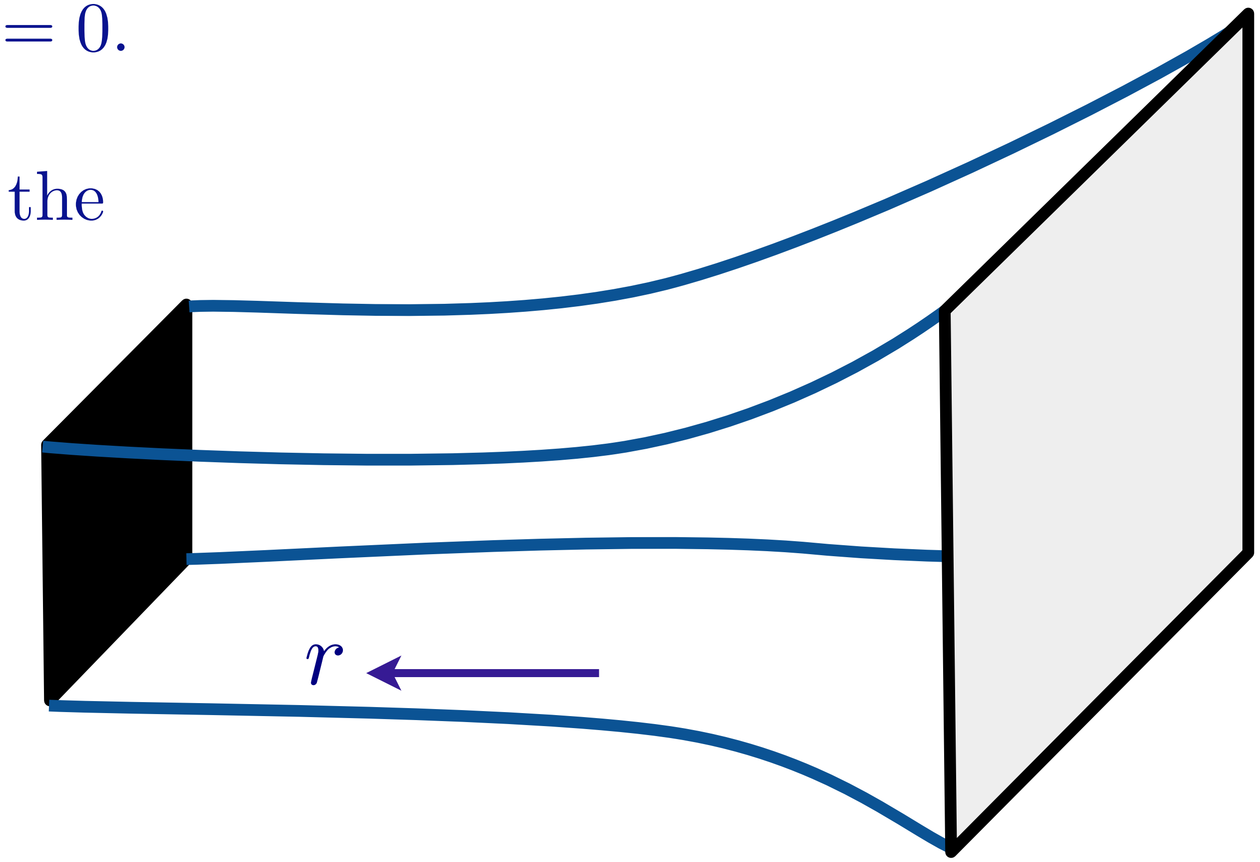
We have used reparametrization invariance in r to choose so that it scales as $r \to \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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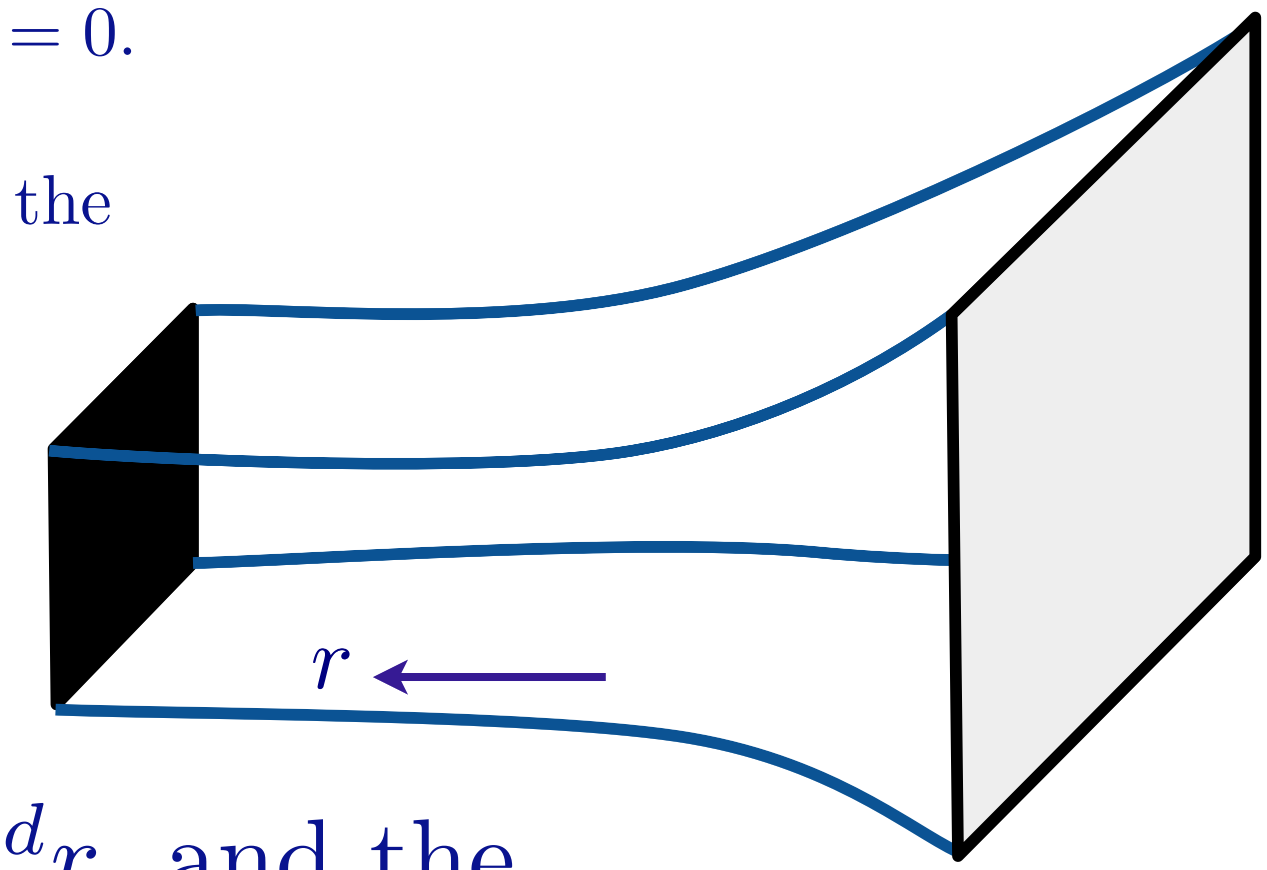

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So θ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.
No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$\theta = d - 1$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim P \ln P$$

with a co-efficient *independent* of the shape of the entangling region. These properties are just as expected for a circular Fermi surface.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
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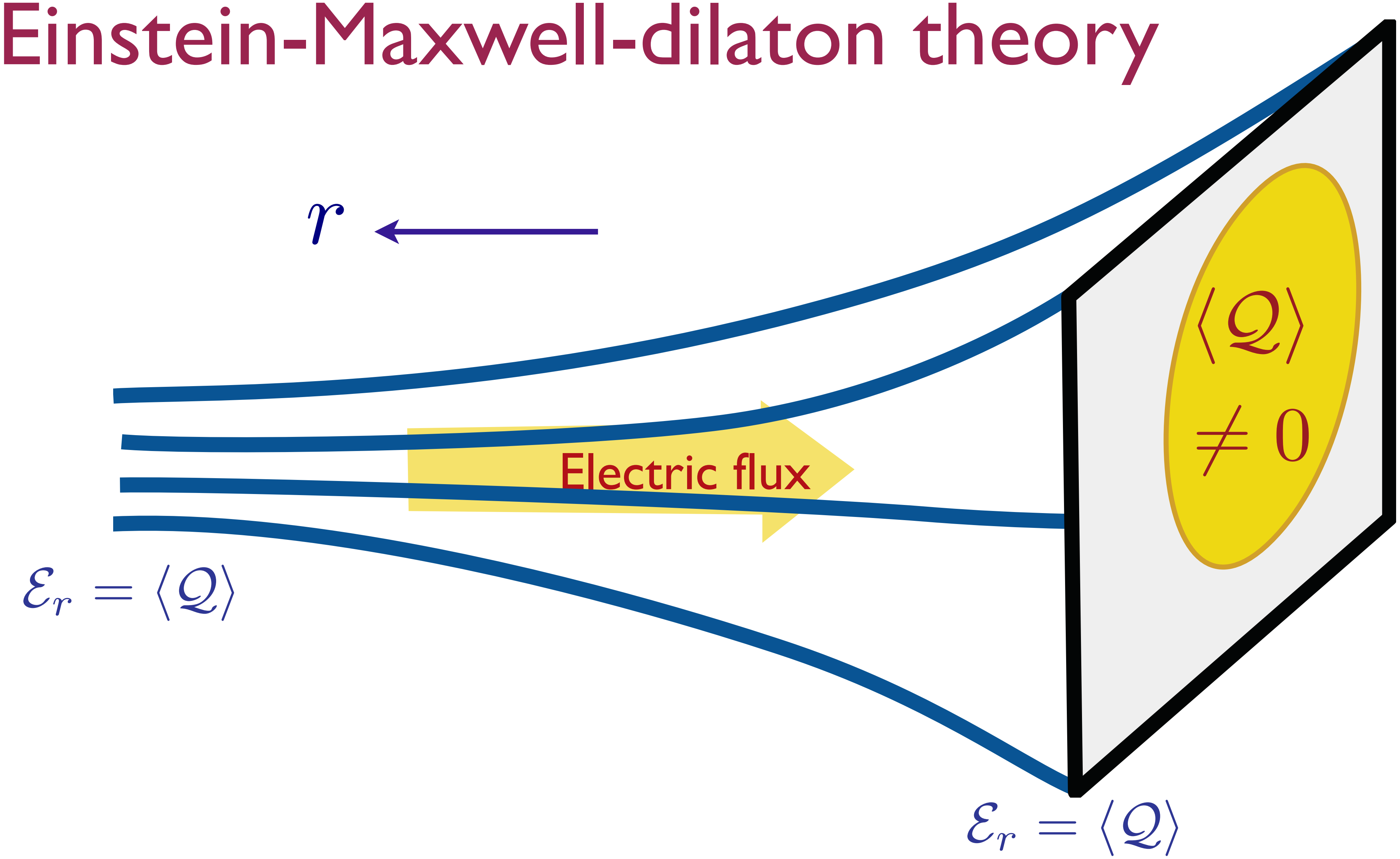
Einstein-Maxwell-dilaton theory

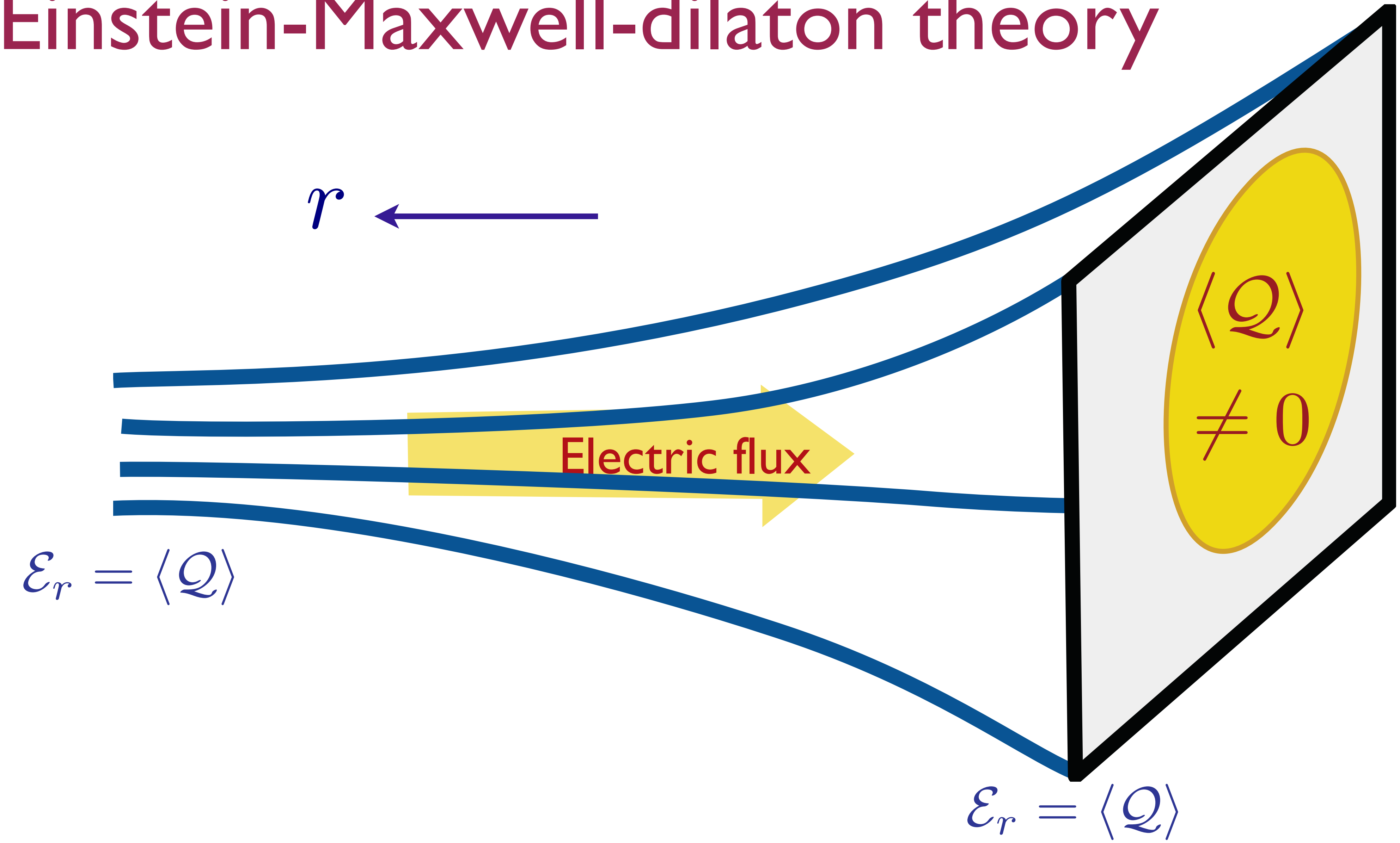


$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab} F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Einstein-Maxwell-dilaton theory



Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \dfrac{dx^2 + dy^2}{r^2} \right)$

with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^{\delta}$, $\Phi(r) \sim \ln(r)$ as $r \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
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Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The $r \to \infty$ metric has the above form with

$$\begin{aligned}\theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}.\end{aligned}$$

Note $z \geq 1 + \theta/d$.

In the present theory, we have to choose α or β so that $\theta = d - 1$.
Needed: a dynamical quantum analysis which automatically selects this value of θ.

Holography of a non-Fermi liquid

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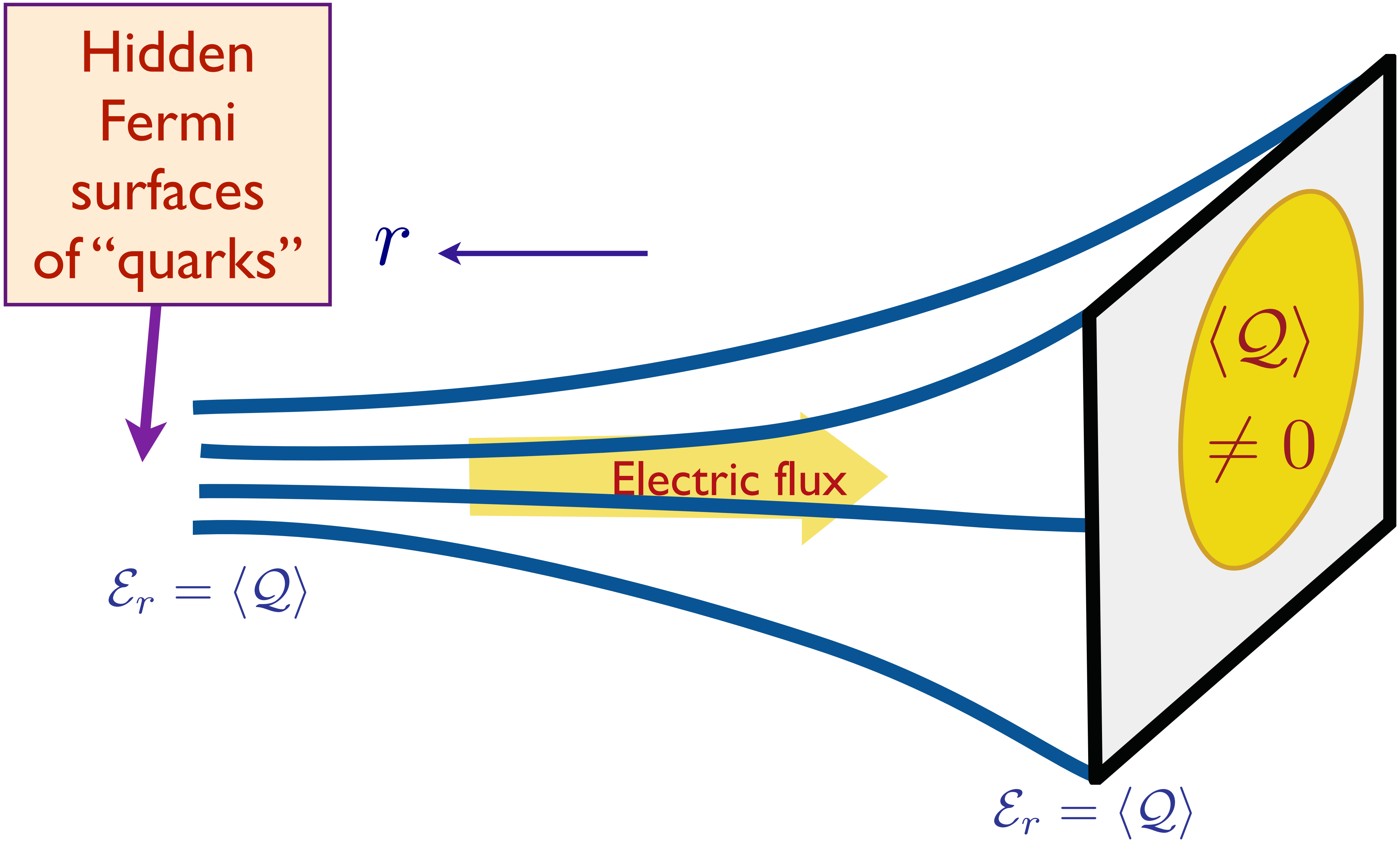
Using the Einstein-Maxwell-dilaton theory we obtain a more precise result for the entanglement entropy

$$S_E = \mathcal{C}_E\, \mathcal{Q}^{(d-1)/d} P \ln(\mathcal{Q}^{(d-1)/d} P)$$

where the co-efficient $\mathcal{C}_E$ is *independent* of all UV details (*e.g.* boundary conditions on the dilaton), but depends on z and other IR characteristics. These properties are just as expected for a circular Fermi surface with a Fermi wavevector obeying $\mathcal{Q} \sim k_F^d$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of a non-Fermi liquid



This is a "bosonization" of the *hidden* Fermi surface

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

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<u>Spatial dimension $d=1$</u>

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Spatial dimension *d=1*

Monopoles in the 2+1 dimensional bulk U(1) gauge field acquire a Berry phase determined by the boundary U(1) charge density $\mathcal{Q}$, and a dilute gas theory of monopoles leads to Friedel oscillations with

$$\langle \rho(x)\rho(0)\rangle \sim \frac{\cos(2k_F x)}{|x|^{2\Delta_F}}$$

T. Faulkner and N. Iqbal, arXiv:1207.4208

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T. Faulkner and N. Iqbal, arXiv:1207.4208

Exact solution of adjoint Dirac fermions at non-zero density coupled to a SU(N_c) gauge field: low energy theory has an emergent $\mathcal{N} = (2,2)$ supersymmetry, the global U(1) symmetry becomes the R-symmetry, and there are Friedel oscillations with

$$\Delta_F = 1/3 \quad \text{for all } N_c \geq 2$$

R. Gopakumar, A. Hashimoto, I.R. Klebanov, S. Sachdev, and K. Schoutens, arXiv:1206.4719

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S. Sachdev, arXiv:1209.1637

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Spatial dimension *d=2*

- For every CFT in 2+1 dimensions with a globally conserved U(1), we can define a monopole operator which transforms as a scalar under conformal transformations.
  e.g. for the XY model, we insert a monopole at x_m by including a *fixed* background gauge flux α_μ so that

$$\mathcal{L} = |(\partial_\mu - i\alpha_\mu)\psi|^2 + s|\psi|^2 + u|\psi|^4$$

where the flux $\beta_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu\alpha_\lambda$ obeys

$$\partial_\mu\beta_\mu = 2\pi\delta(x - x_m) \quad , \quad \epsilon_{\mu\nu\lambda}\partial_\nu(\Omega\beta_\nu) = 0$$

where the CFT lives on the conformally flat space with is $ds^2 = \Omega^{-2}dx_\mu^2$.

S. Sachdev, arXiv:1209.1637

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension d=2

- In the holographic theory, we have a bulk scalar field Φ_m (conjugate to the monopole operator of the CFT) which carries the charge of the S-dual of the 4-dimensional bulk U(1) gauge field:

$$\mathcal{S}_m = \int d^4x \sqrt{-g} \left[|(\nabla - 2\pi i \widetilde{A})\Phi_m|^2 + \ldots \right]$$

where $\widetilde{F} = d\widetilde{A} = *F = *dA$.

S. Sachdev, arXiv:1209.1637

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- When a chemical potential is applied to the boundary CFT, Φ_m experiences a magnetic flux. Consequently condensation of Φ_m leads to a vortex-lattice-like state, which corresponds to the formation of a *crystal* in the CFT. *The crystal has unit $\mathcal{Q}$ charge per unit cell.*

- We expect that a vortex-liquid-like state of the Φ_m will yield the Friedel oscillations of the Fermi surface, with the correct Fermi wavevector. We are working on the theory of such a state ...

S. Sachdev, arXiv:1209.1637

Conclusions

Compressible quantum matter

Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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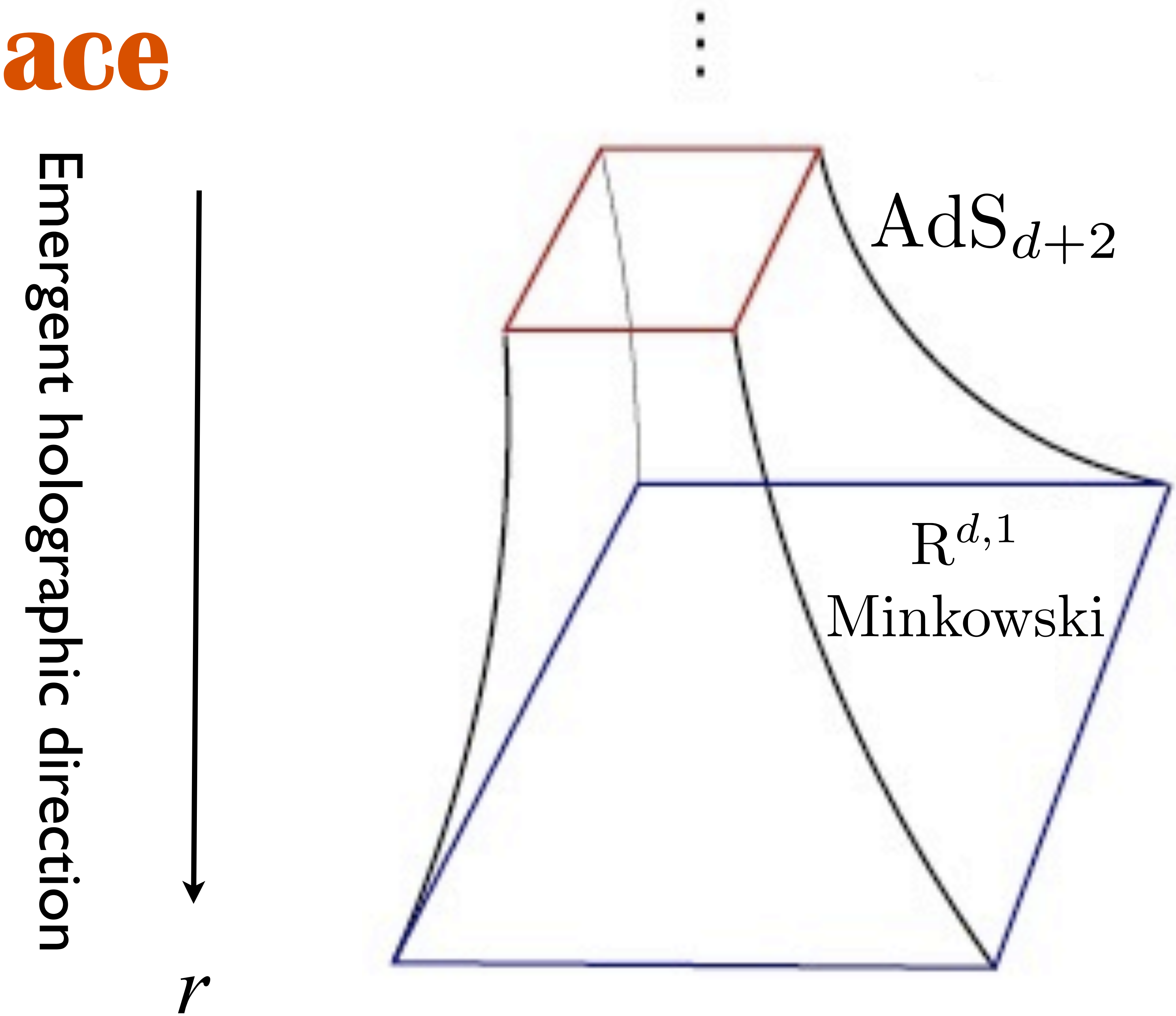
- Evidence for Luttinger theorem in prefactor of S_E.
- Monopole operators lead to crystalline state, and have the correct features to yield Friedel oscillations of a Fermi surface.

anti-de Sitter space



anti-de Sitter space