The uses of gauge-gravity duality in condensed matter physics

PCTS, September 13, 2011

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Outline

1. Conformal quantum matter

2. Compressible quantum matter

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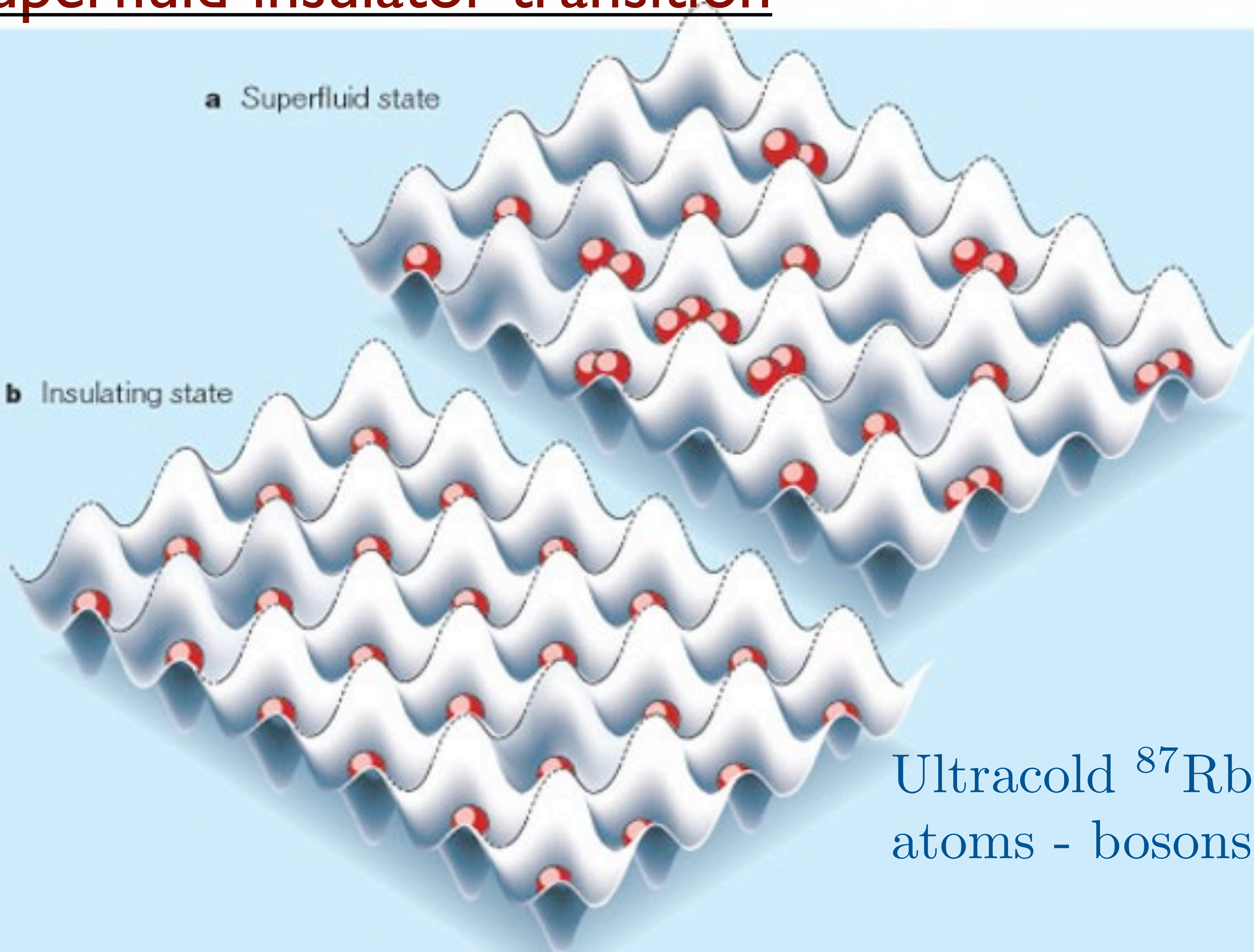
2. Compressible quantum matter

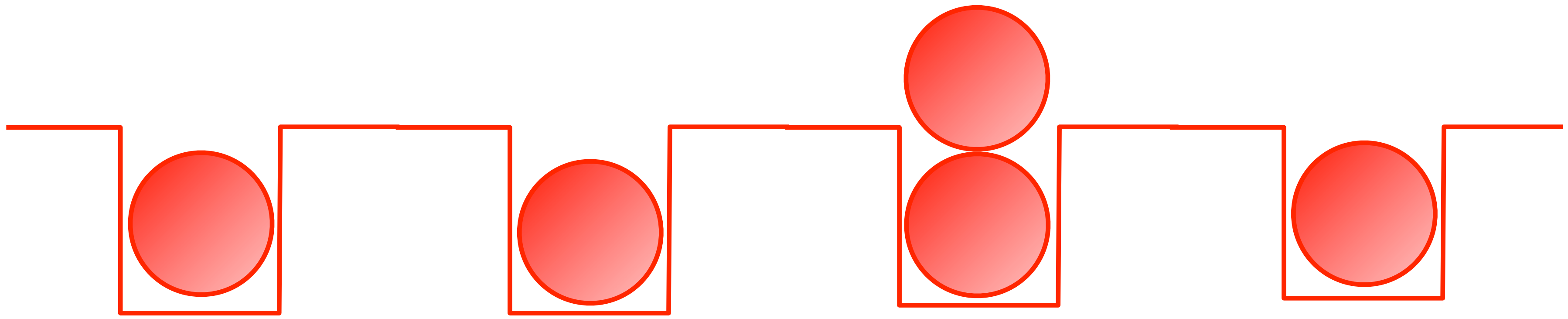
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Excitations of the insulator:



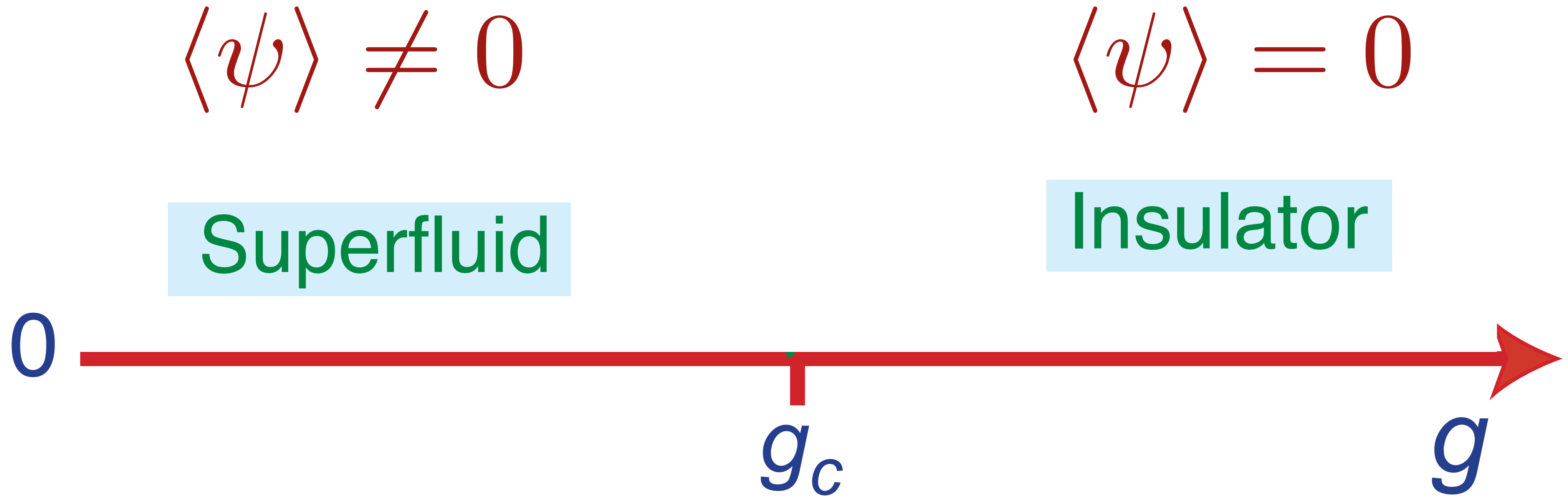
Particles ~ $\psi^{\dagger}$

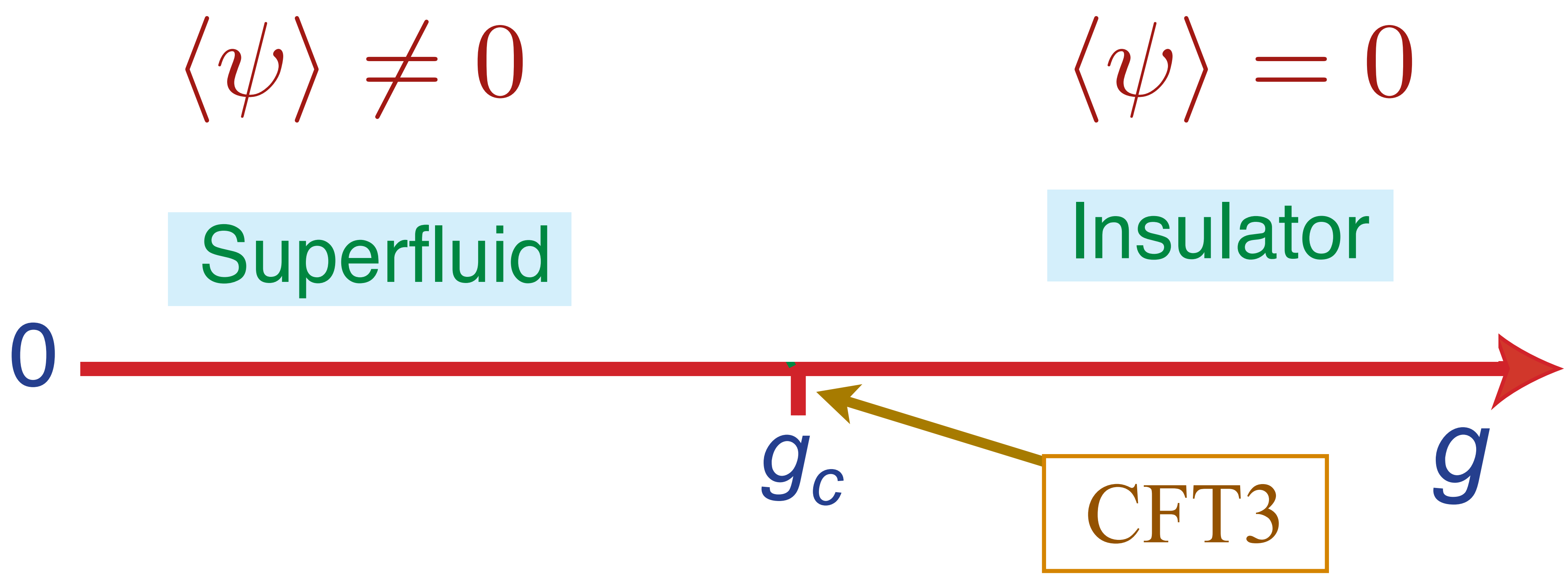
Holes ~ ψ

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

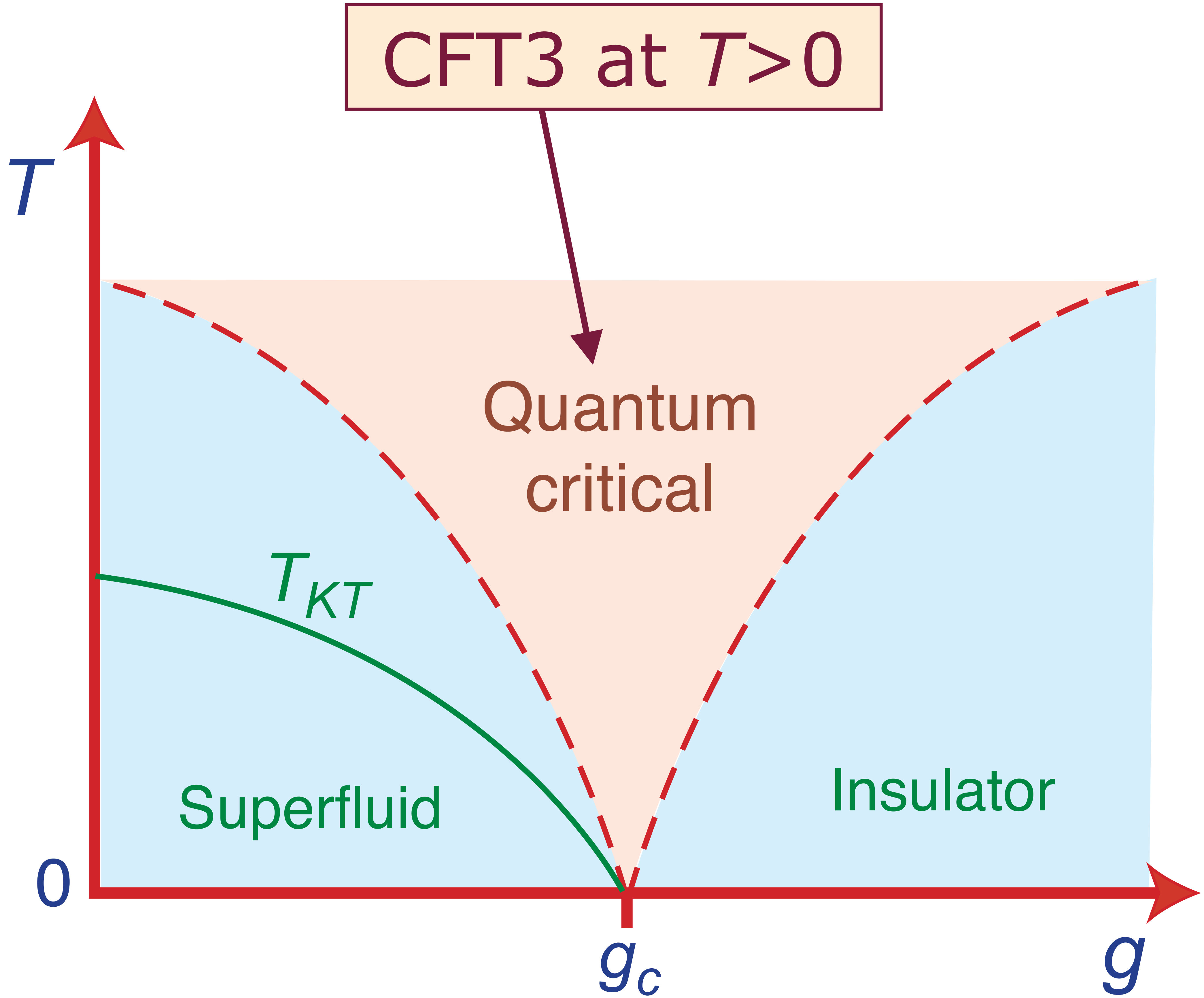
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g



$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
CFT3
g


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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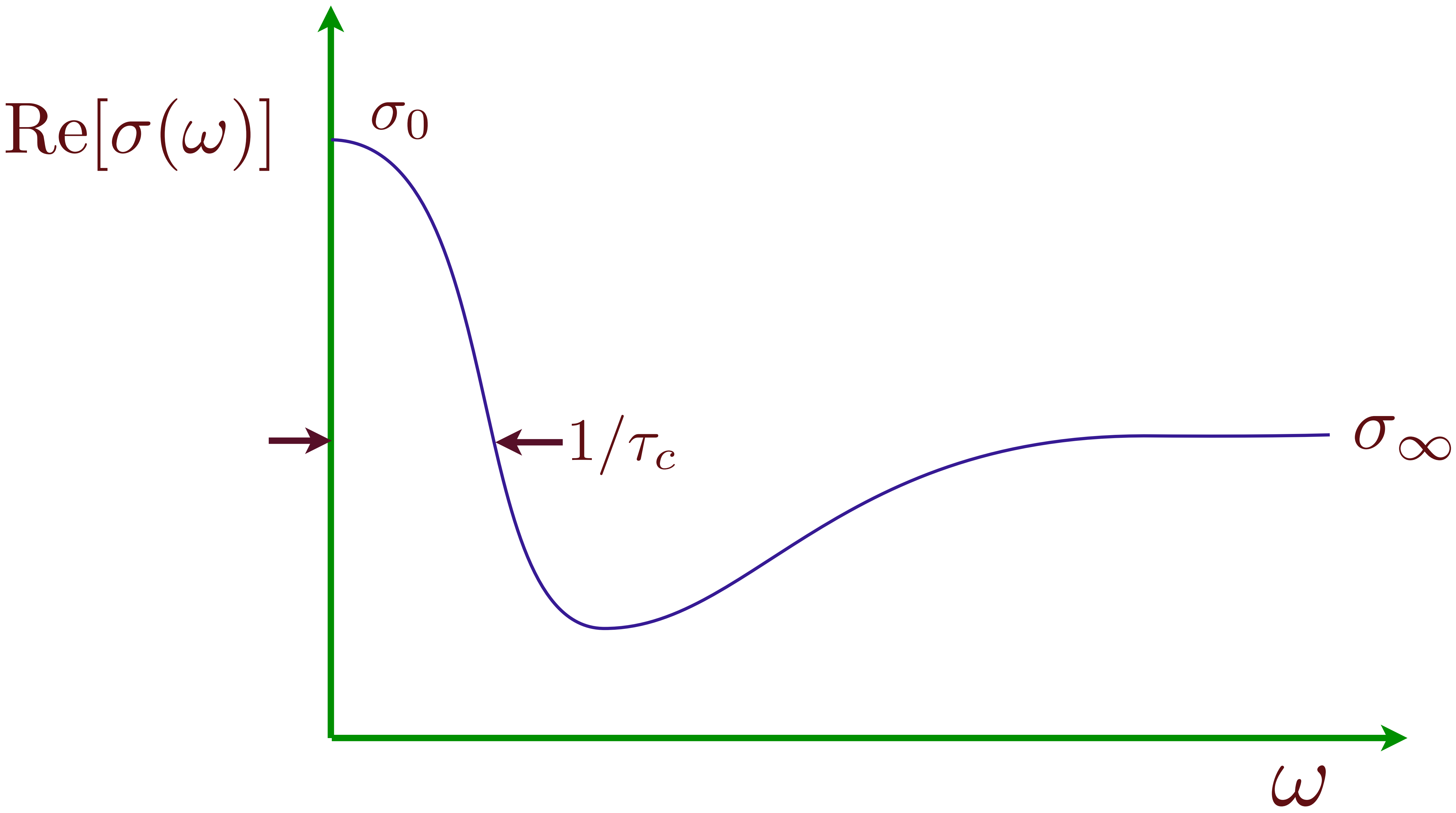
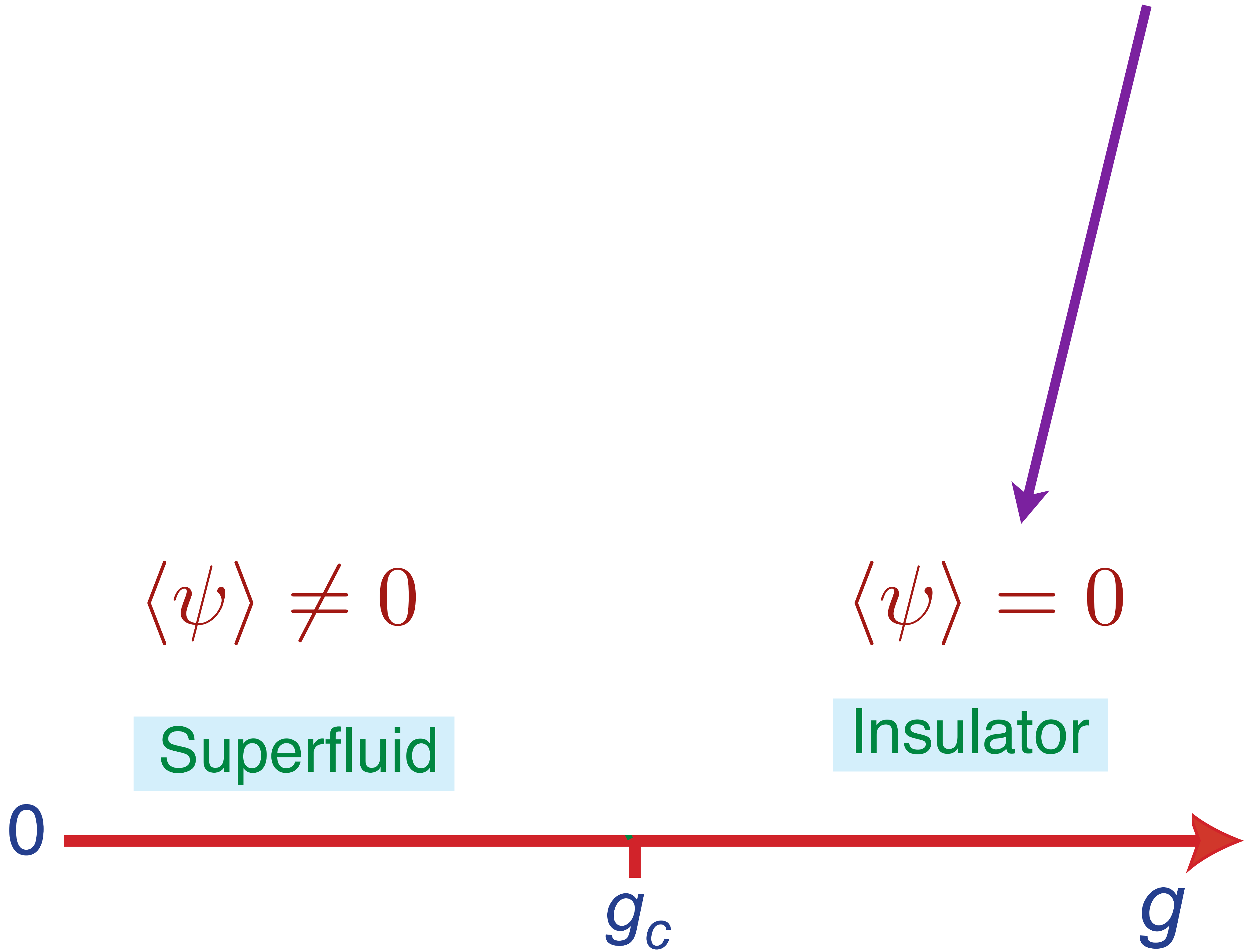
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

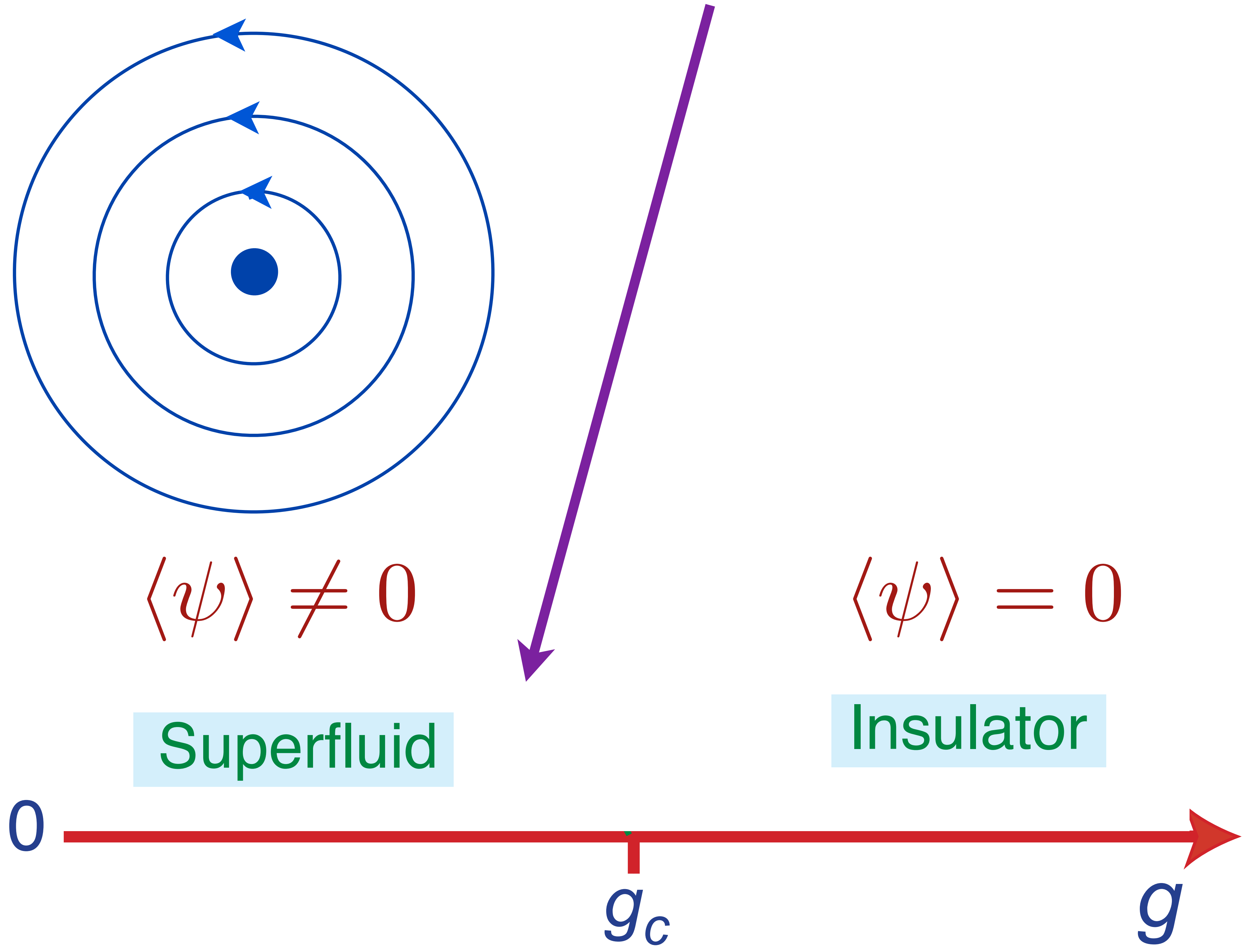
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



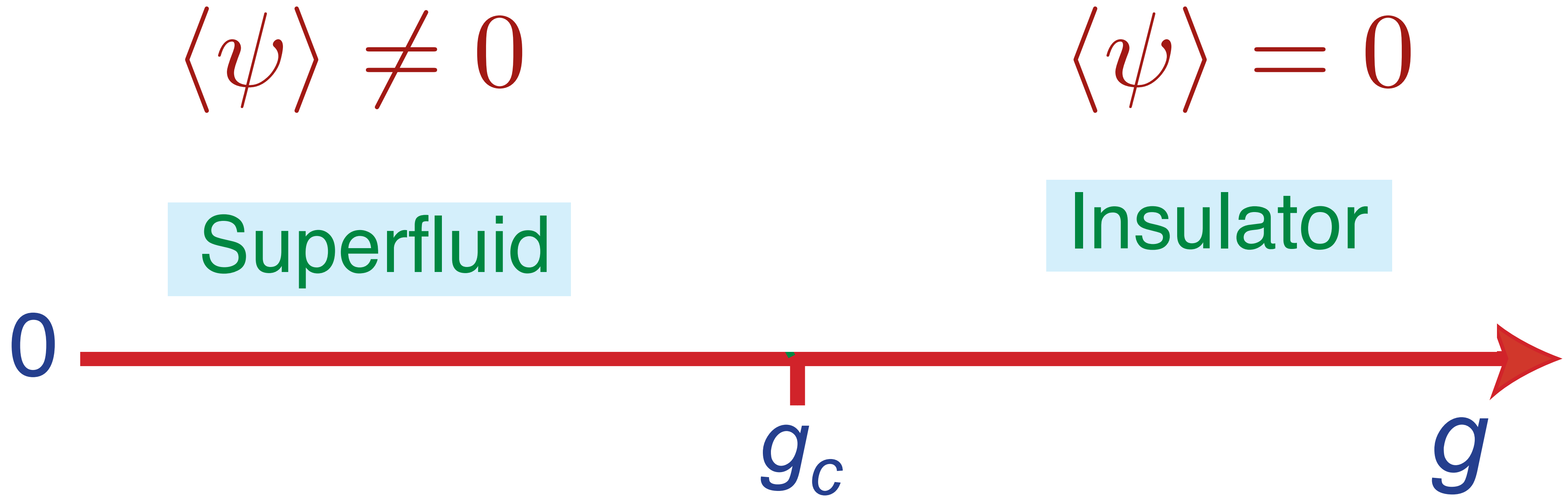
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

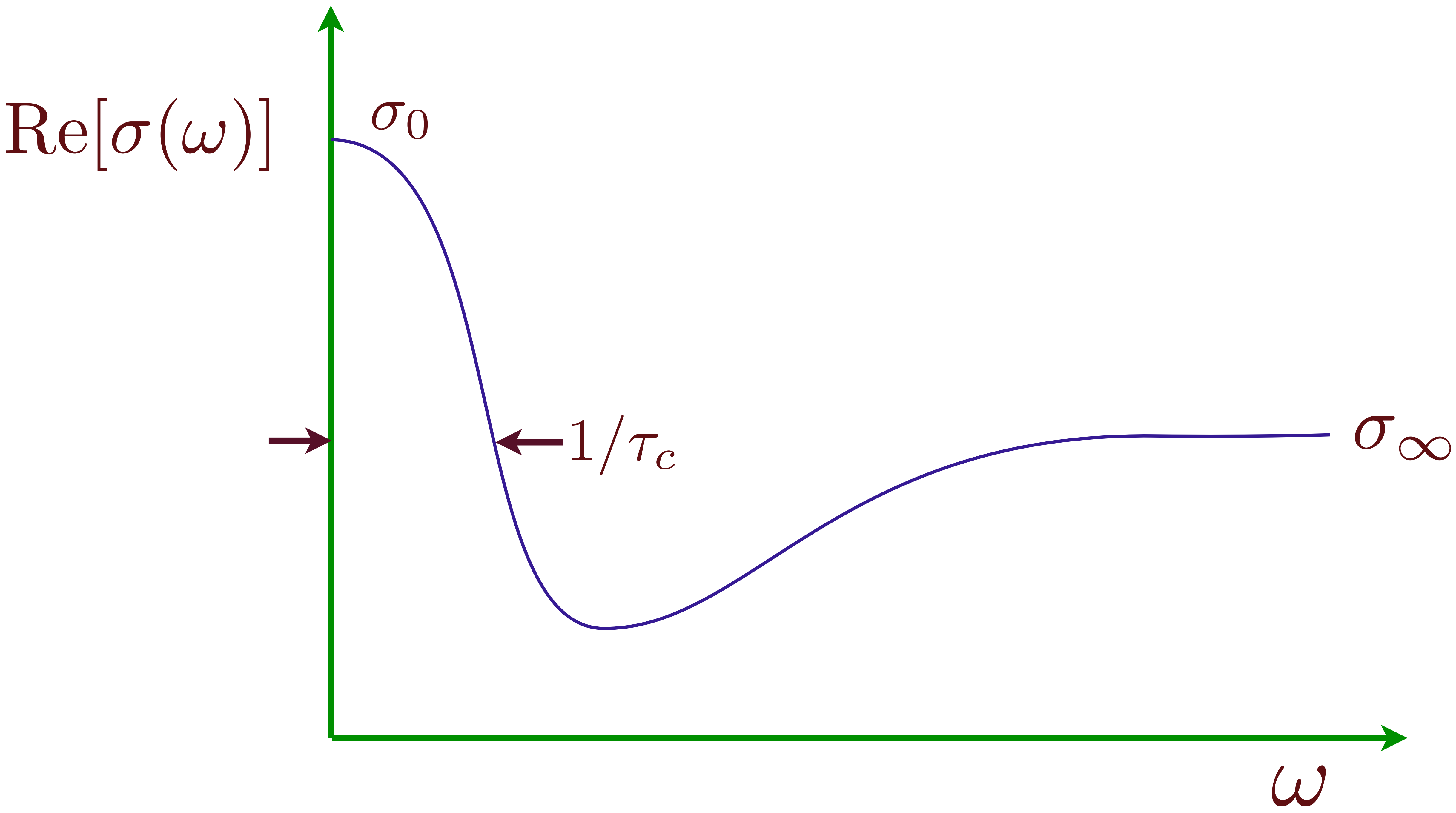
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) "dual" CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

$$\text{Conductivity} = \text{Resistivity of vortices}$$

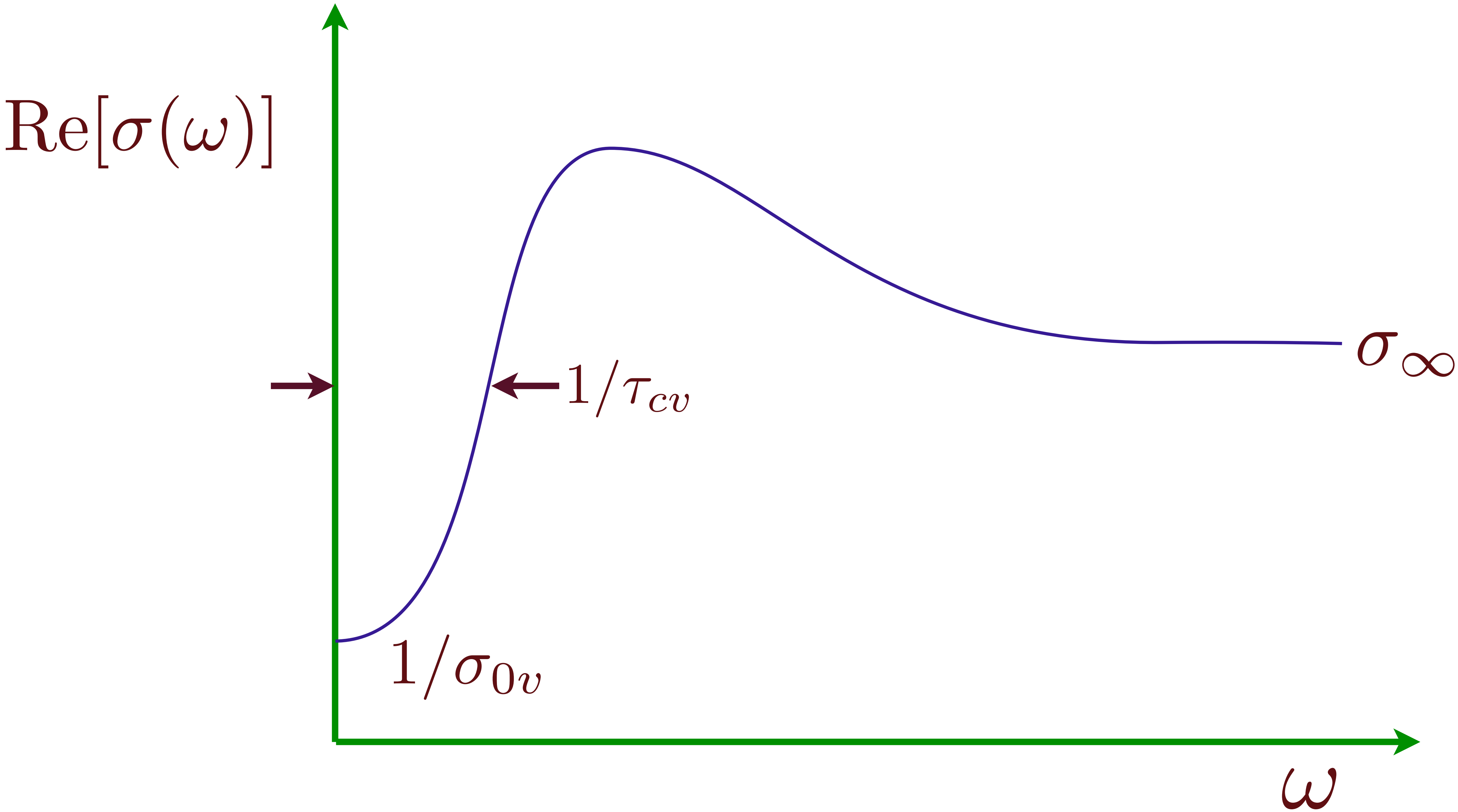


M.P.A. Fisher, *Physical Review Letters* **65**, 923 (1990)

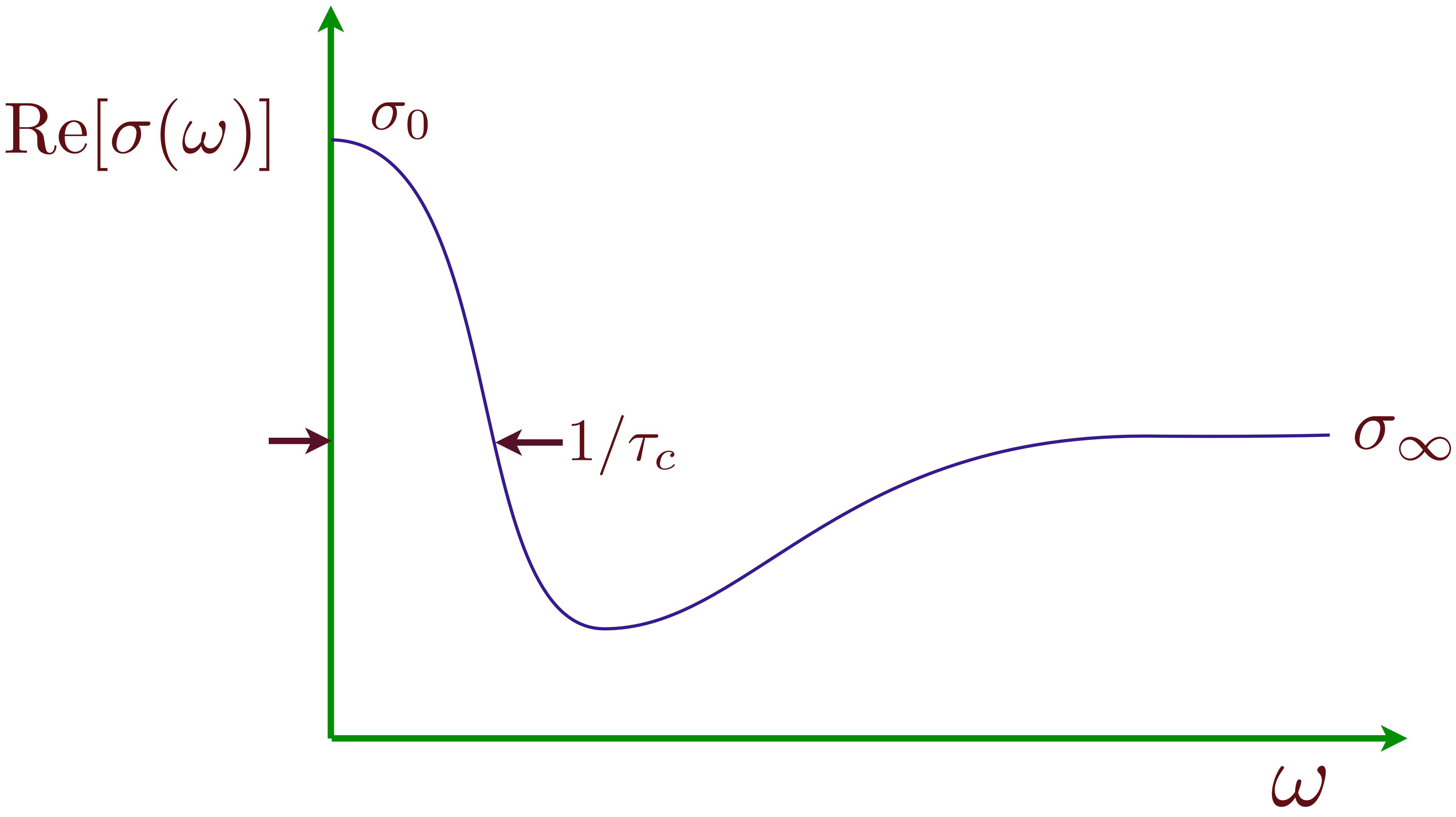
Boltzmann theory of bosons



Boltzmann theory of vortices

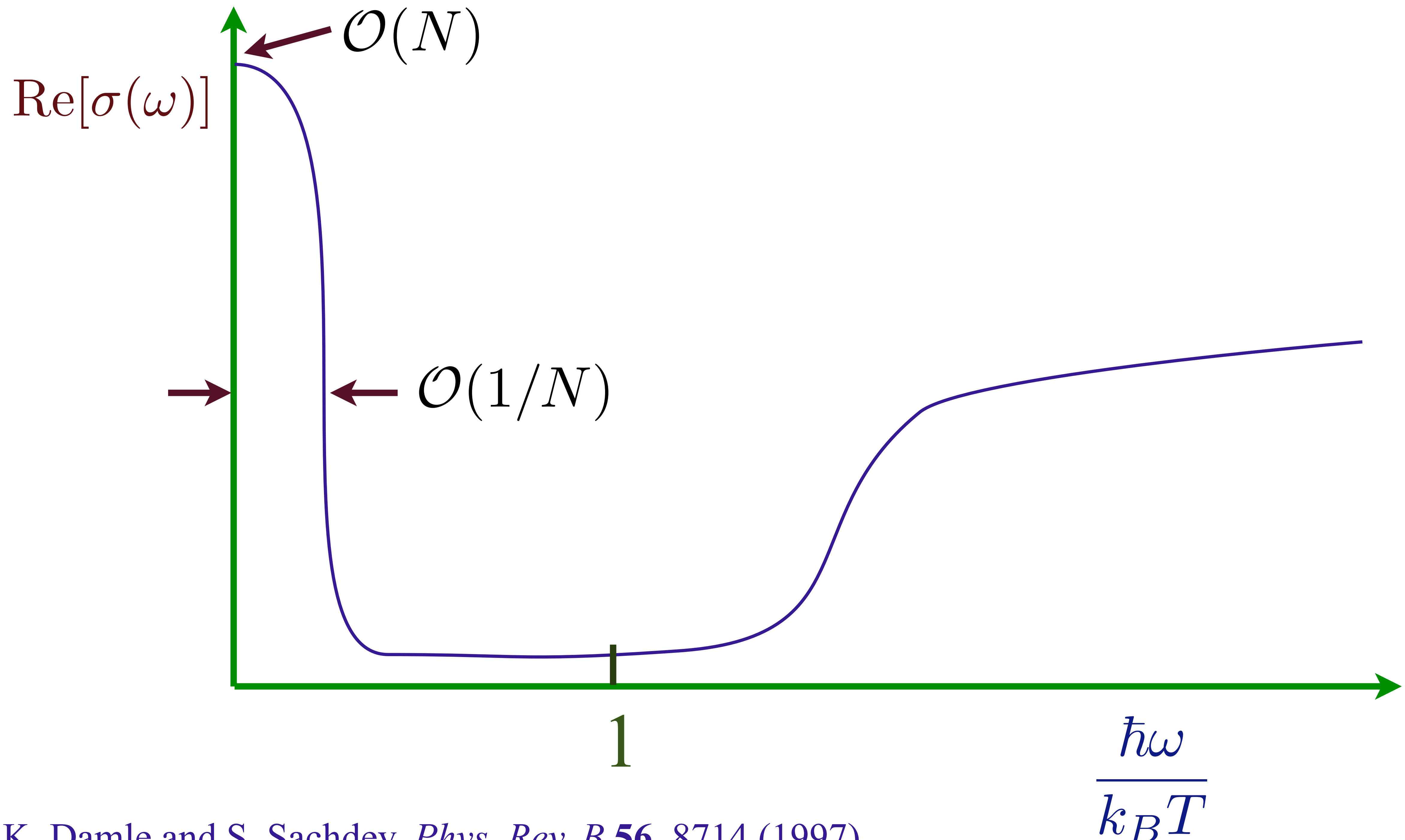


Boltzmann theory of bosons



Vector large *N* expansion for CFT3

$$\sigma = \frac{Q^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right); \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Outline

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The AdS_4 - Schwarzschild black brane

2. Compressible quantum matter

Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

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Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \rightarrow x^\mu / b \quad , \quad u \rightarrow b\, u$$

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This is an invariance of the *metric* of the theory in $D+1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+1}, and L is the AdS radius.

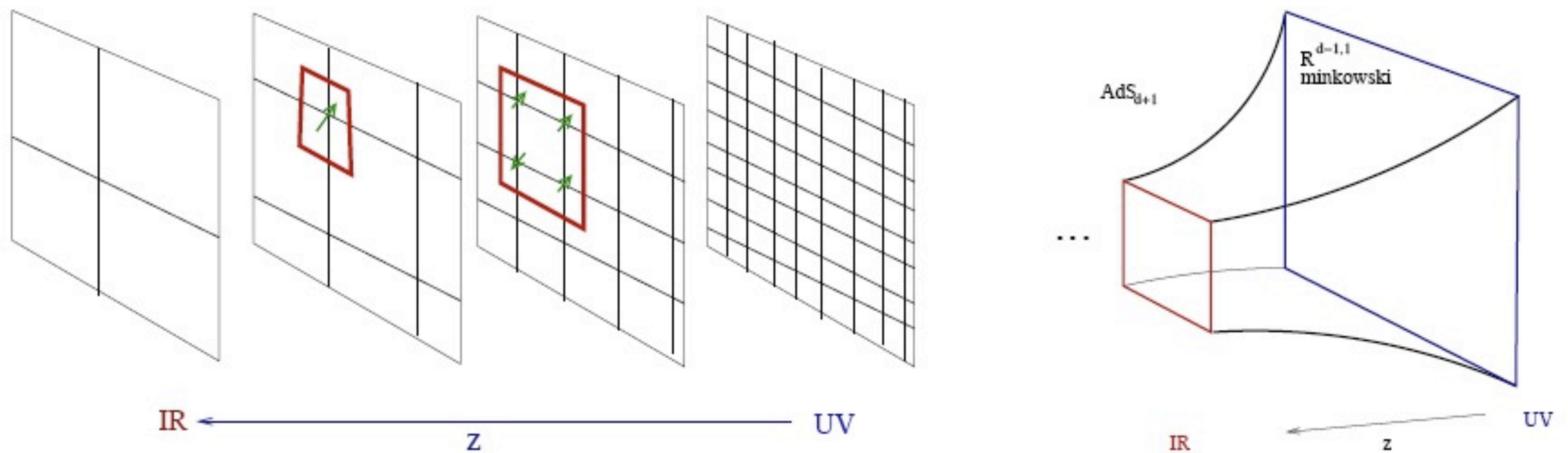
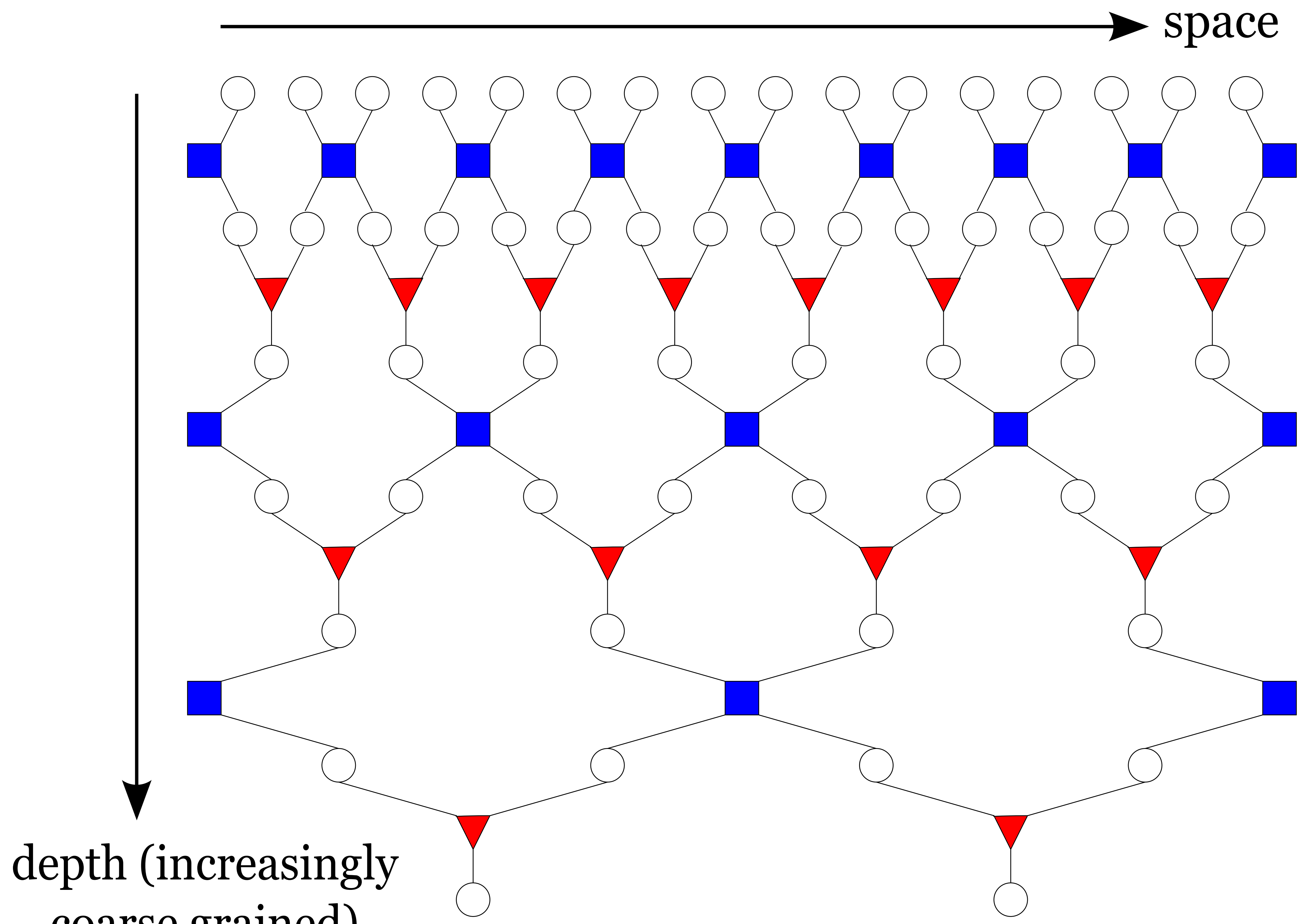


Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

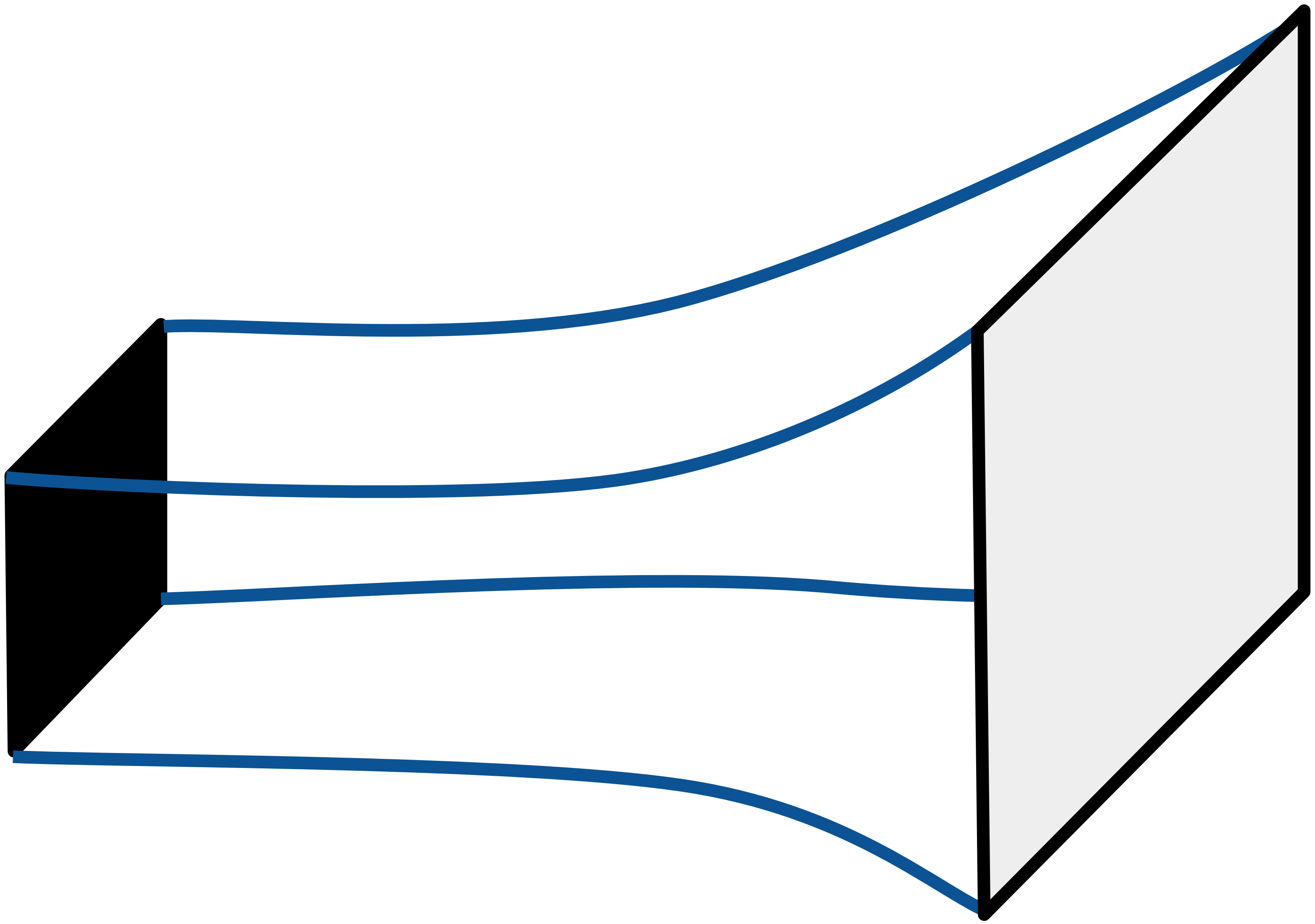
J. McGreevy, arXiv0909.0518

Emergent direction as a representation of an entanglement tensor network



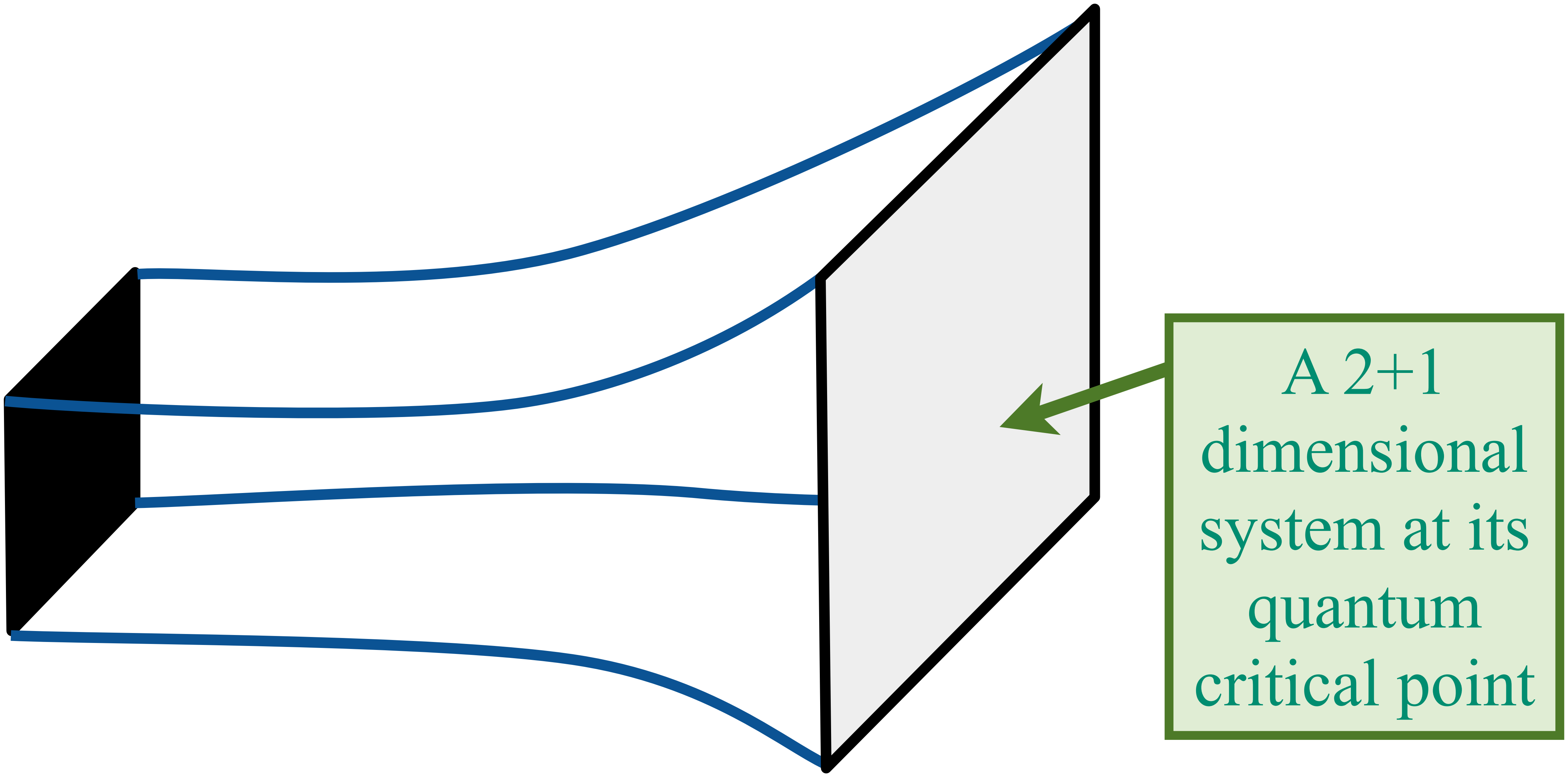
B. Swingle, arXiv:0905.1317

AdS_4-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS$_4$-Schwarzschild black-brane



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AdS/CFT correspondence

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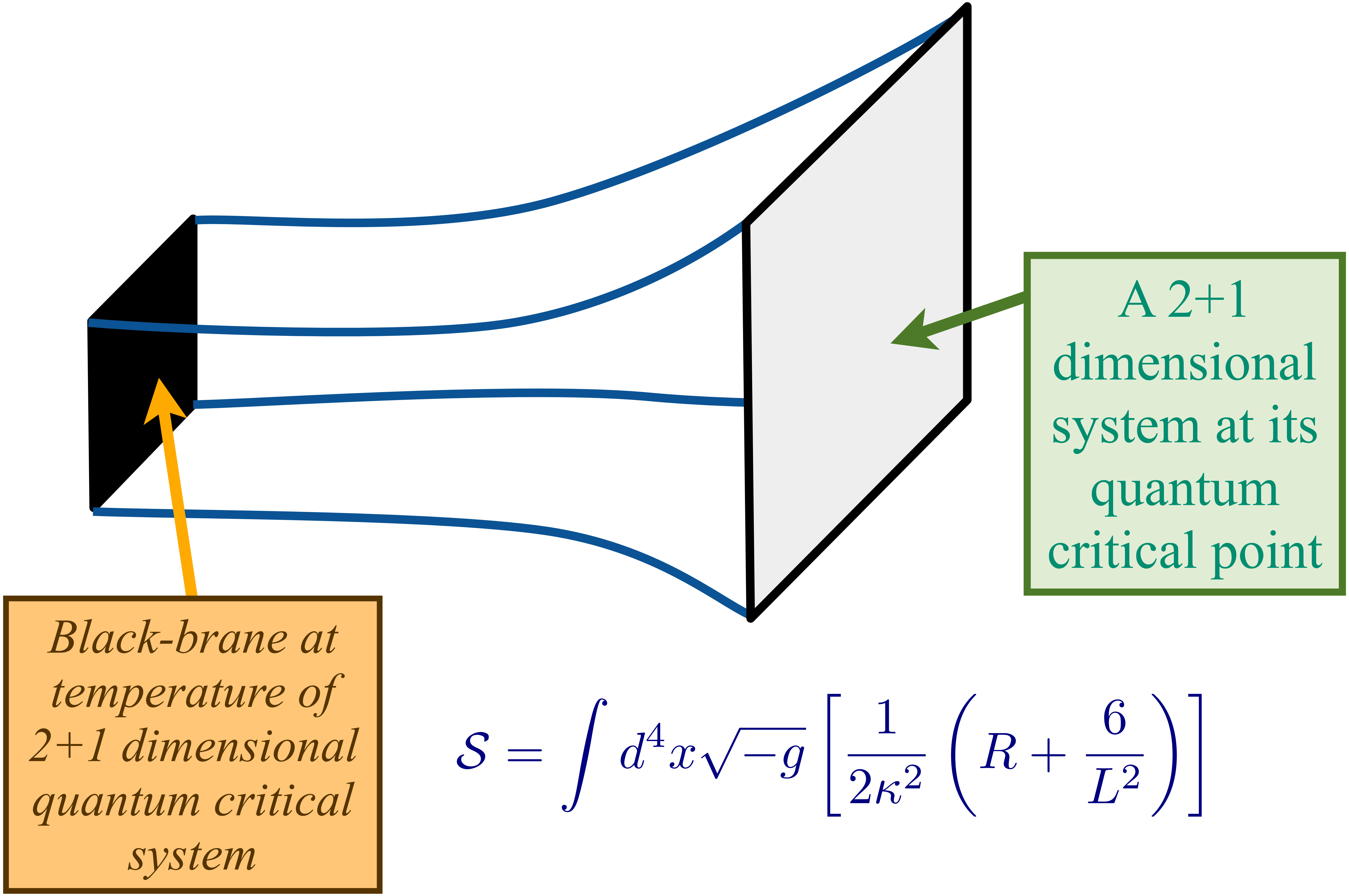

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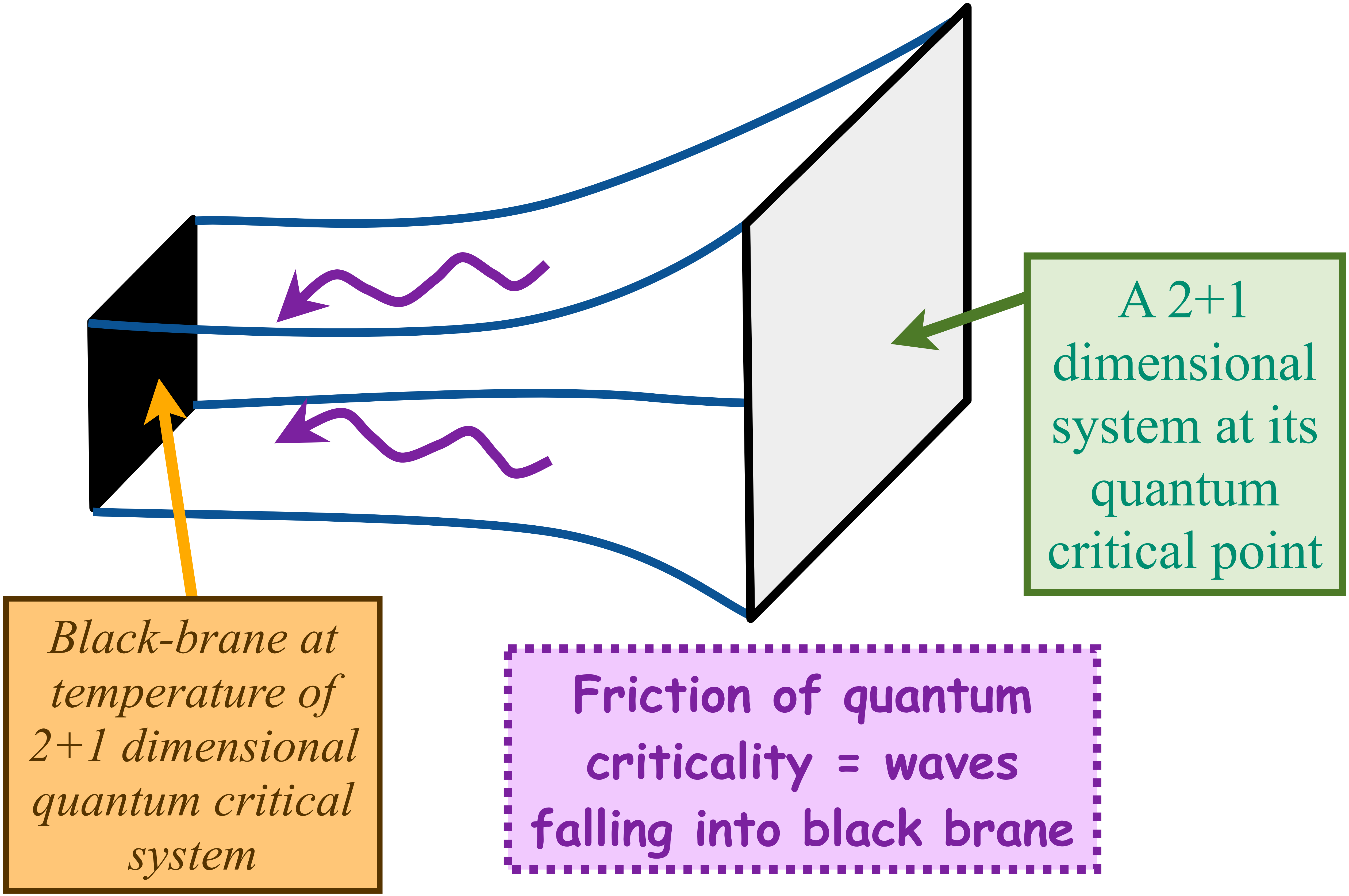

AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS$_4$ theory of "nearly perfect fluids"

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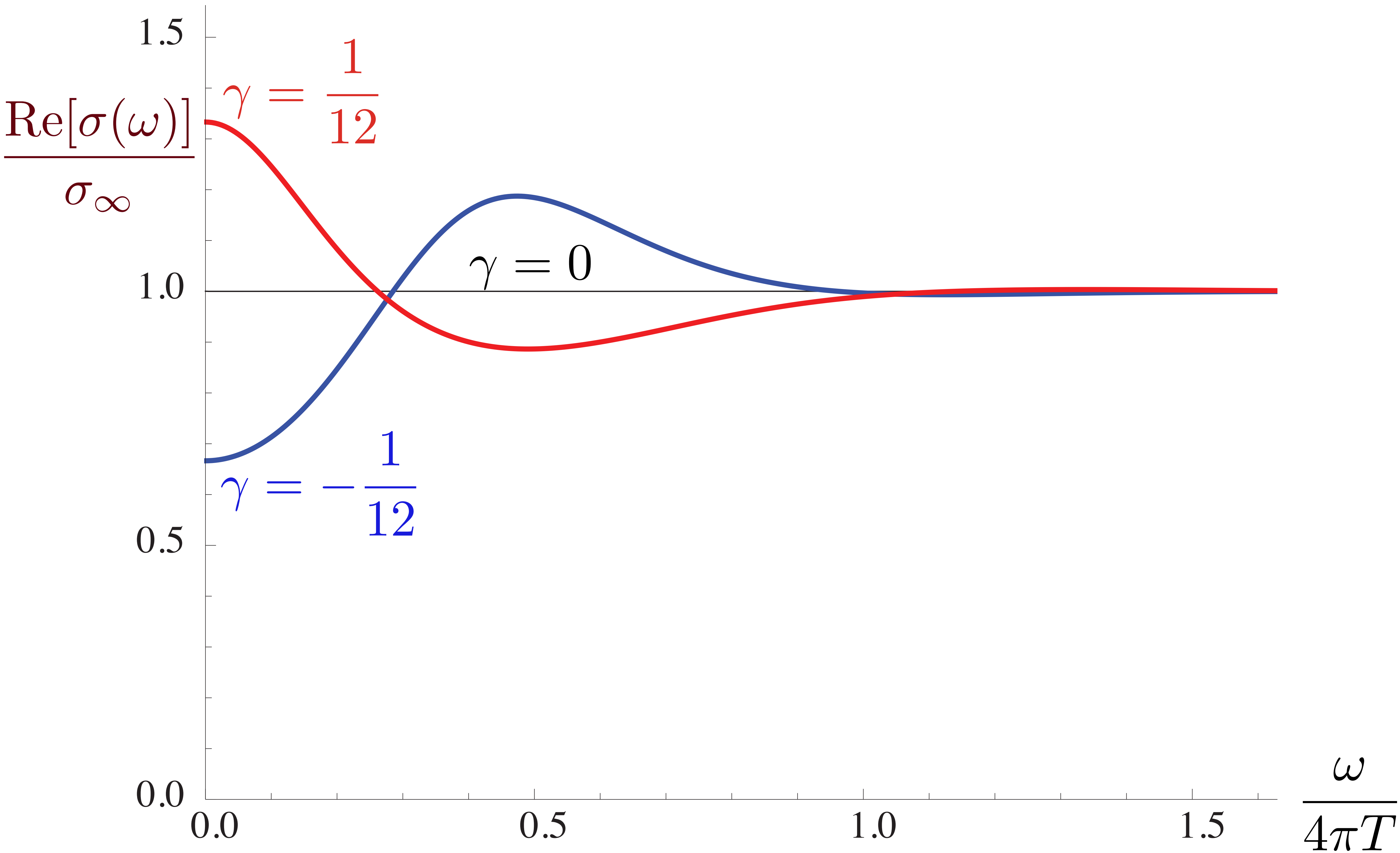
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS$_4$):

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of strongly interacting "perfect fluids"



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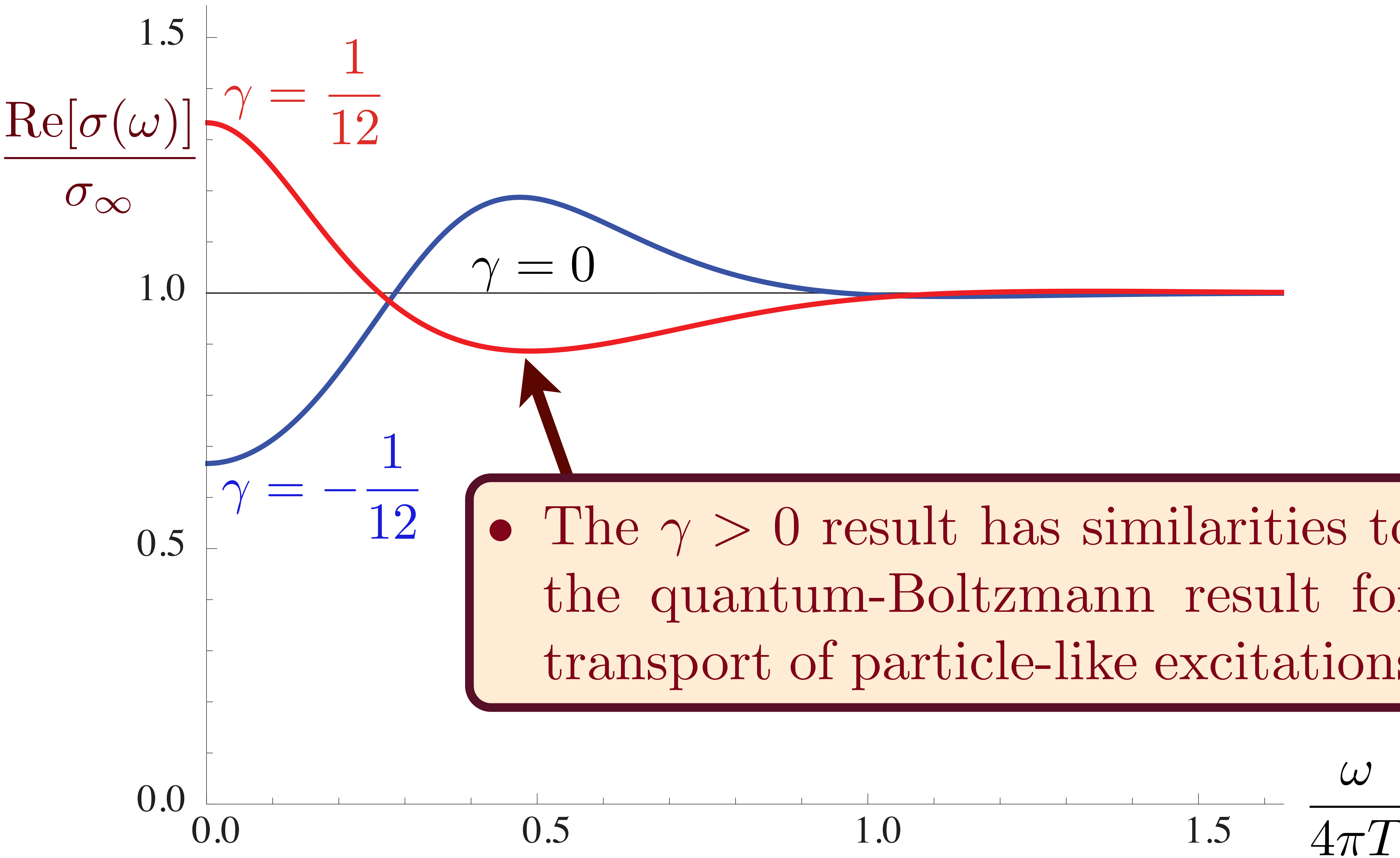

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

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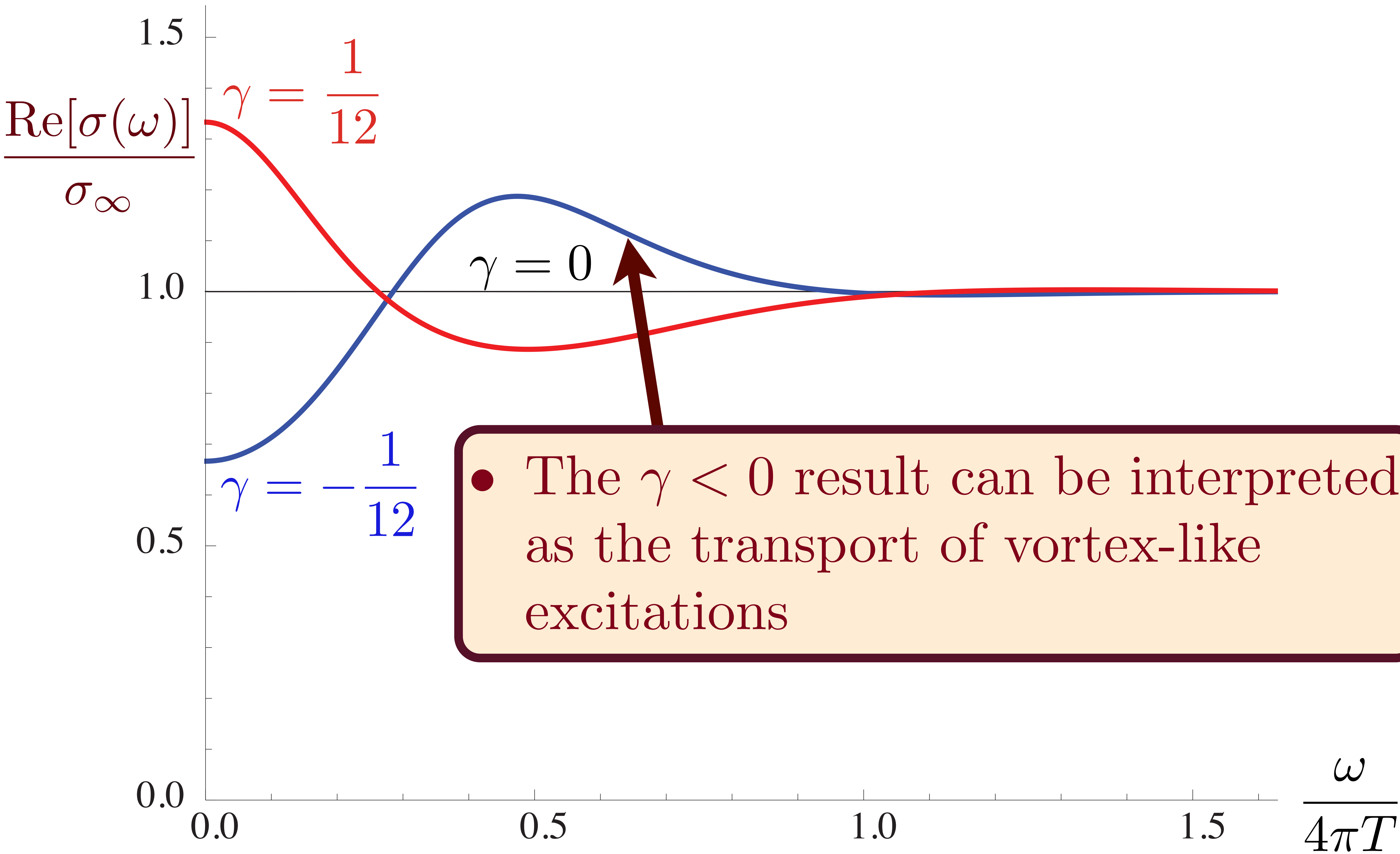

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

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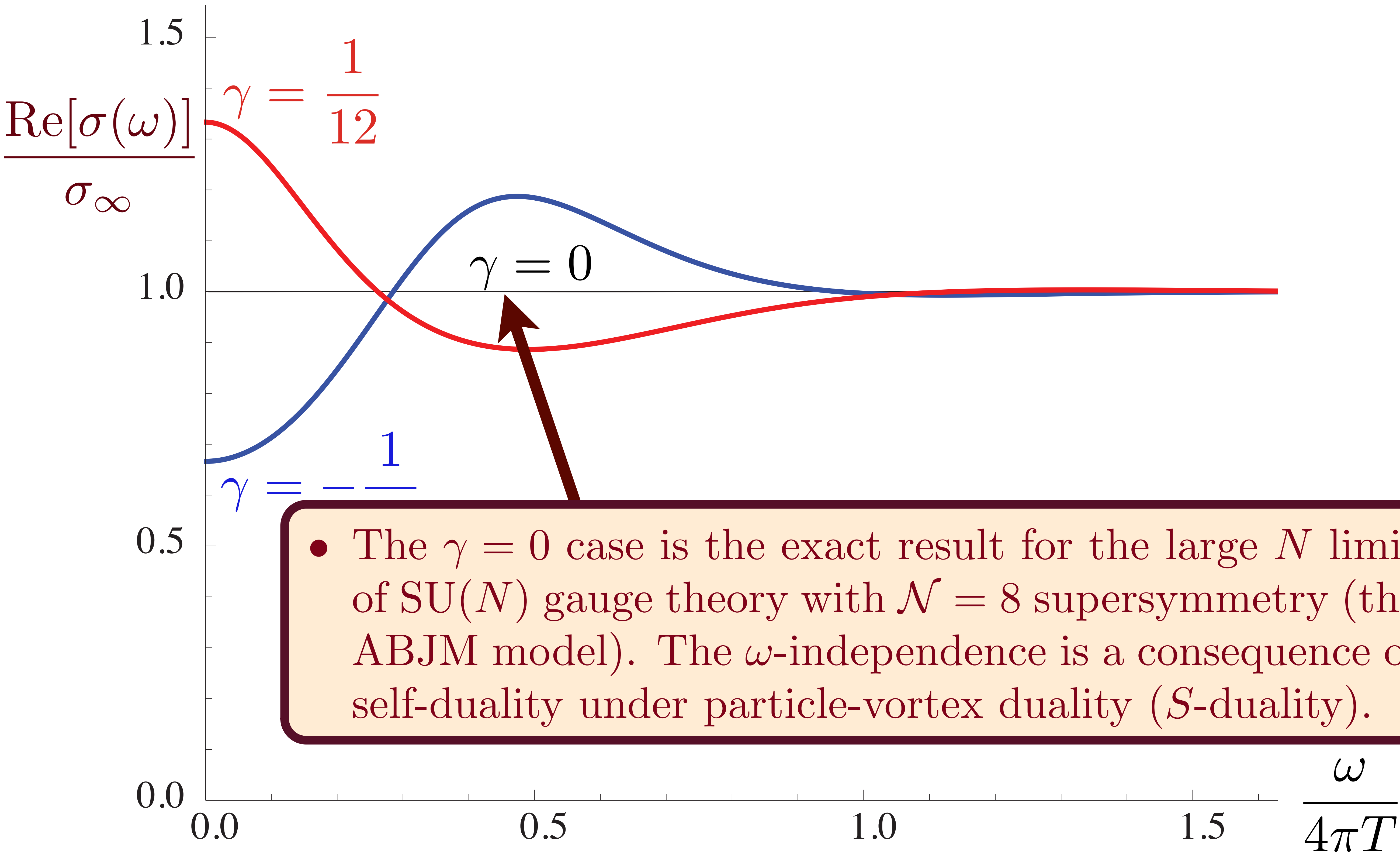

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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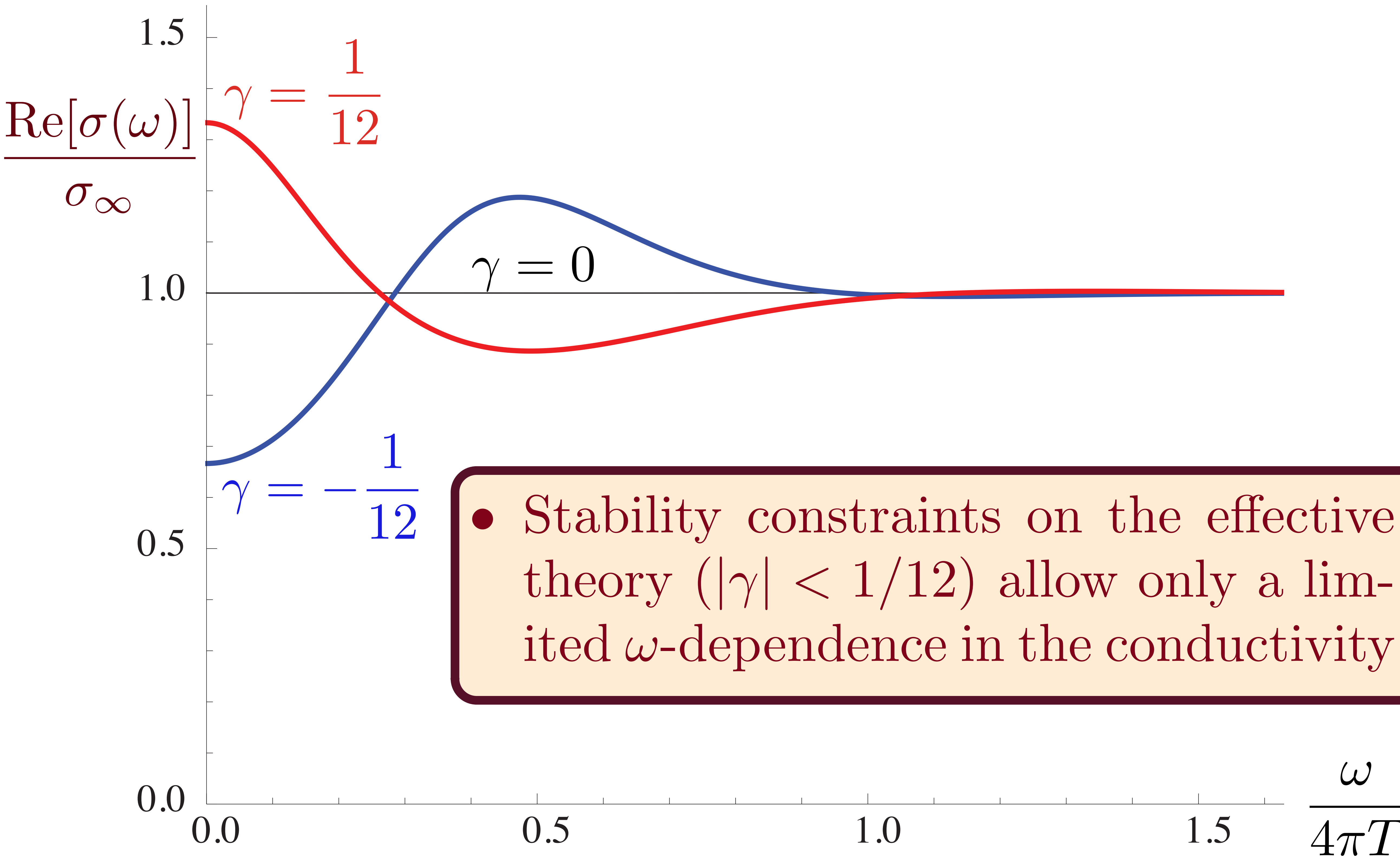

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

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where C_{abcd} is the Weyl curvature tensor.

General approach:

- Theory has 2 free dimensionless parameters: e^2 and γ. We match these to correlators of the CFT3 of interest at $\omega \gg T$: e^2 is determines the current correlator $\langle J_\mu J_\nu \rangle$, while γ determines the 3-point function $\langle T_{\mu\nu} J_\rho J_\sigma \rangle$, where $T_{\mu\nu}$ is the stress-energy tensor.

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- We use $\mathcal{S}_{EM}$ to extrapolate to transport properties for $\omega \ll T$. This step is traditionally carried out by descendants of the Boltzmann equation.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value

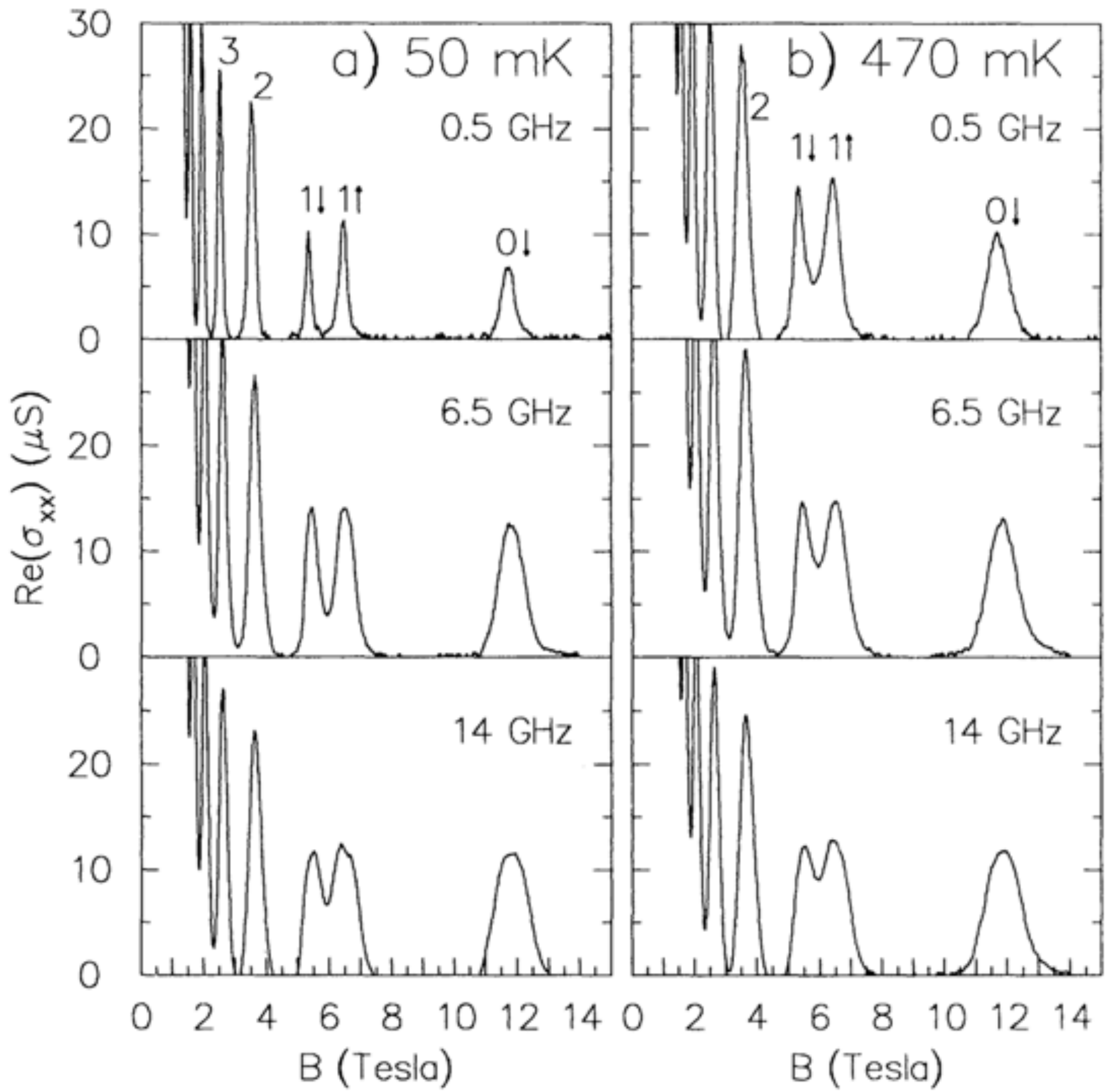

FIG. 3. Re(σ_{xx}) vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, *Physical Review Letters* **71**, 2638 (1993).

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A. Condensed matter vs. continuum QFTs

Compressible quantum matter

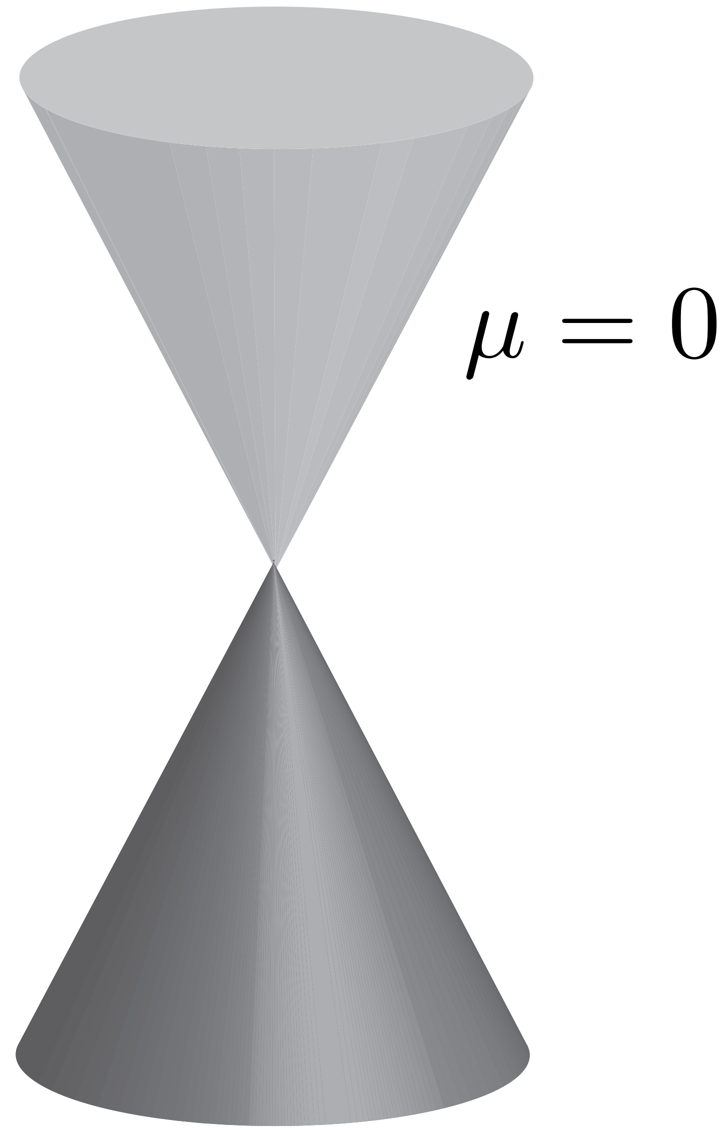
- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $\langle \mathcal{Q} \rangle$ varies smoothly as a function of μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.

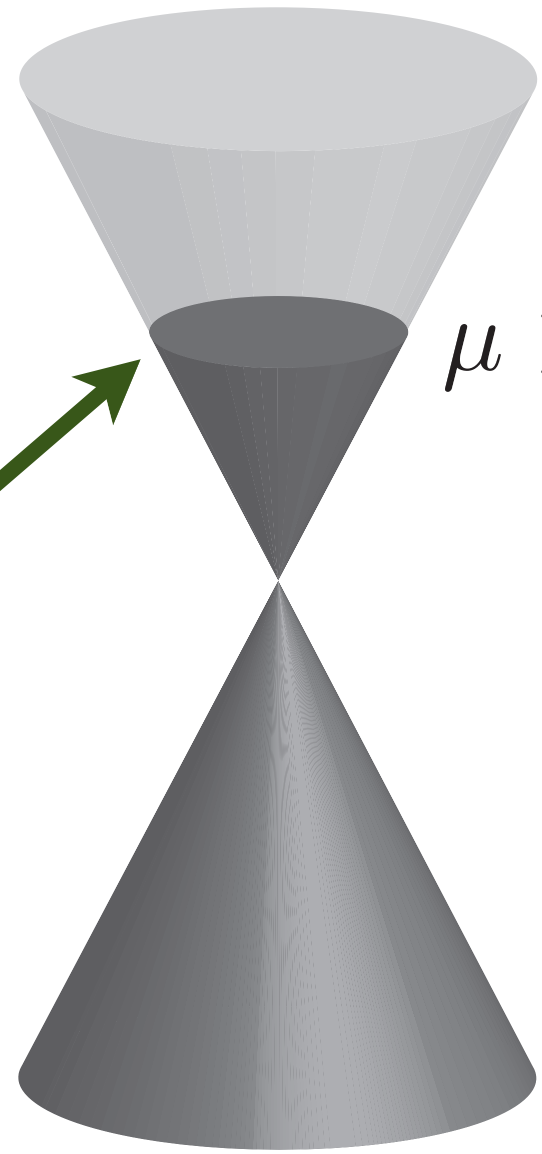
Turning on a chemical potential on a CFT



Massless Dirac fermions
(e.g. graphene)

Turning on a chemical potential on a CFT

Compressible phase is a **Fermi Liquid** with a **Fermi surface**



Massless Dirac fermions
(e.g. graphene)

The Fermi surface

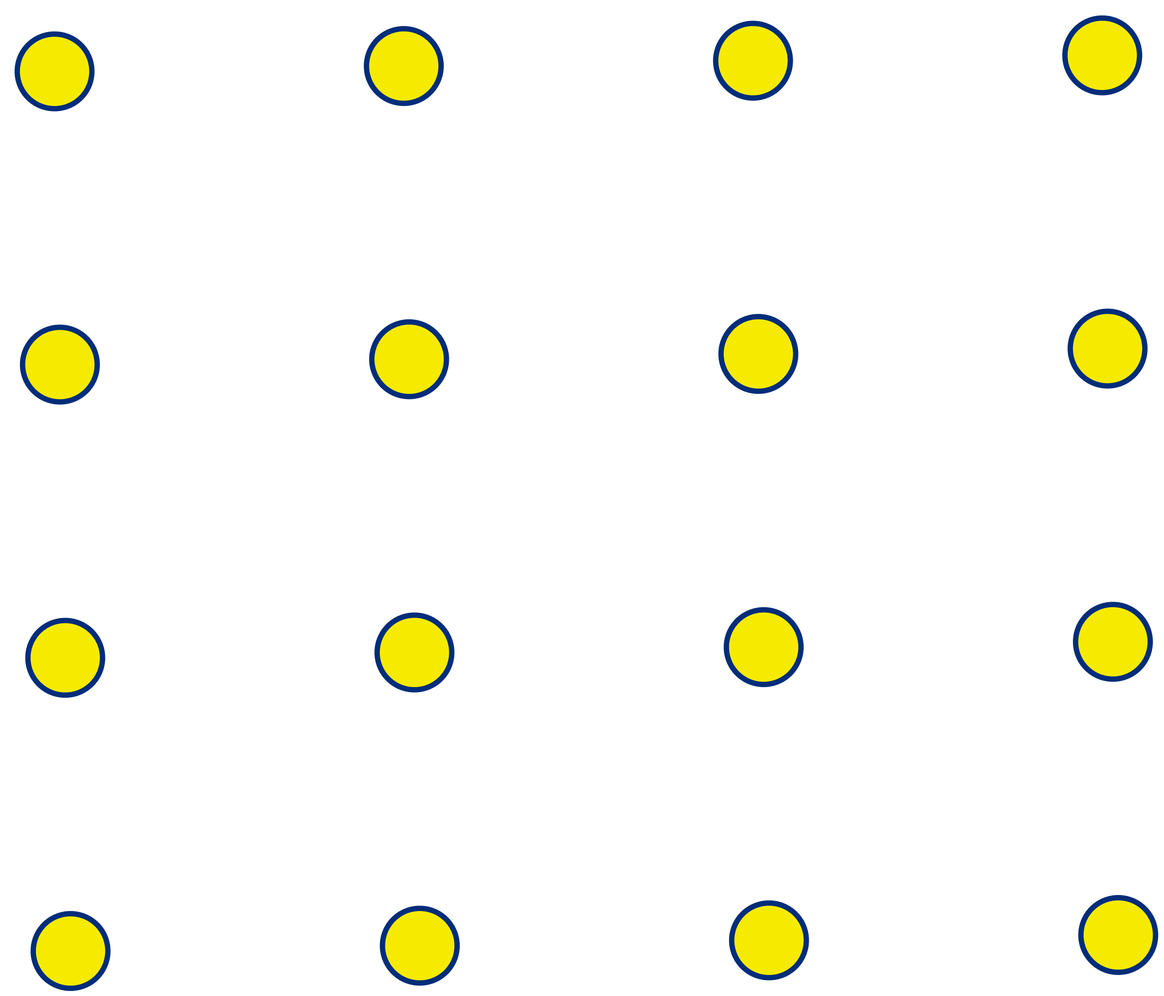
This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge $\mathcal{Q}$.

$$G^{-1}_{\text{fermion}}(k = k_F, \omega = 0) = 0.$$

Luttinger relation: The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling.

Compressible quantum matter

Another compressible state is the **solid**
(or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

The only other familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

Compressible quantum matter

Conjecture: All compressible states which preserve translational and global $U(1)$ symmetries must have Fermi surfaces, *but they are not necessarily Fermi liquids.*

Compressible quantum matter

Conjecture: All compressible states which preserve translational and global U(1) symmetries must have FERMI SURFACES, *but they are not necessarily Fermi liquids.*

- Such states obey the Luttinger relation

$$\sum_{\ell} q_\ell \mathcal{A}_\ell = \langle \mathcal{Q} \rangle,$$

where the ℓ'th Fermi surface has fermionic quasiparticles with global U(1) charge q_ℓ and encloses area $\mathcal{A}_\ell$.

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- Non-Fermi liquids have quasiparticles coupled to deconfined gauge fields (or gapless bosonic modes at quantum critical points).

Consider mixture of fermions f and bosons b.

$$
\begin{aligned}
\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\
&+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots
\end{aligned}
$$

Consider mixture of fermions f and bosons b.
There is a $\mathrm{U}(1)\times\mathrm{U}_b(1)$ symmetry
and 2 conserved charges:

$$\mathcal{Q} = f^\dagger f$$
$$\mathcal{Q}_b = b^\dagger b$$

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Consider mixture of fermions f and bosons b.
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and 2 conserved charges:

The 2 symmetries imply 2 Luttinger constraints. However, bosons at non-zero density invariably Bose condense at $T = 0$, and so $\mathrm{U}_b(1)$ is broken. So there is only the single constraint on the f Fermi surface. This describes mixtures of ^{3}He and ^{4}He.

$$\mathcal{Q} = f^\dagger f$$
$$\mathcal{Q}_b = b^\dagger b$$

$$\begin{aligned}\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu\right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b\right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots\end{aligned}$$

Consider mixture of fermions f and bosons b.
There is a $\mathrm{U}(1)\times\mathrm{U}_b(1)$ symmetry
and 2 conserved charges:

$$\mathcal{A} = \langle \mathcal{Q} \rangle$$

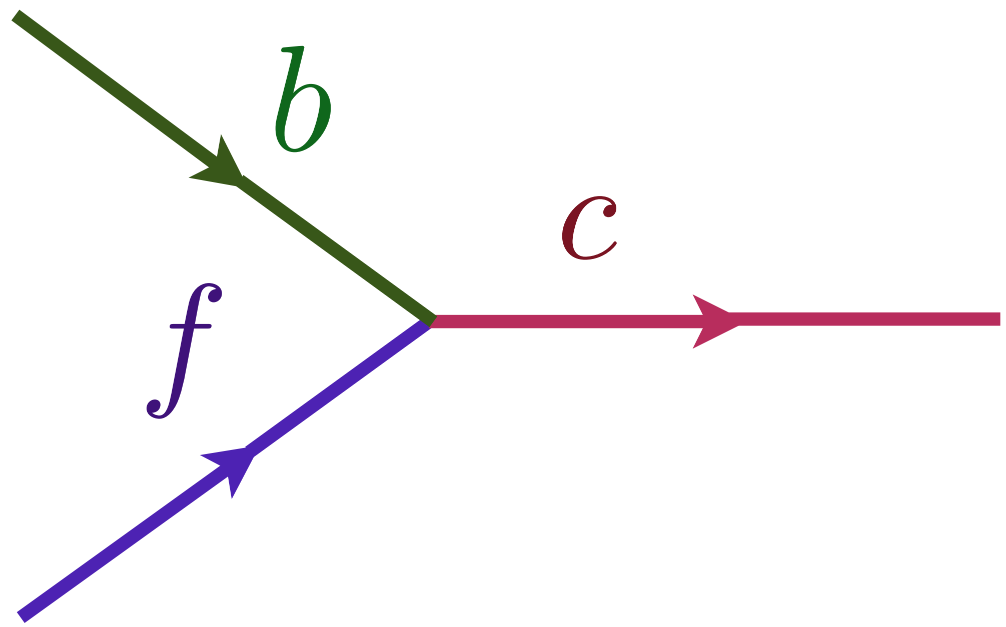
$$\mathcal{Q} = f^\dagger f$$
$$\mathcal{Q}_b = b^\dagger b$$

Superfluid: $\langle b \rangle \neq 0$

$\mathrm{U}_b(1)$ broken; $\mathrm{U}(1)$ unbroken

$$\begin{aligned} \mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots \end{aligned}$$

Increase the coupling g until the boson, b, and fermion, f, can bind into a 'molecule', the fermion c.



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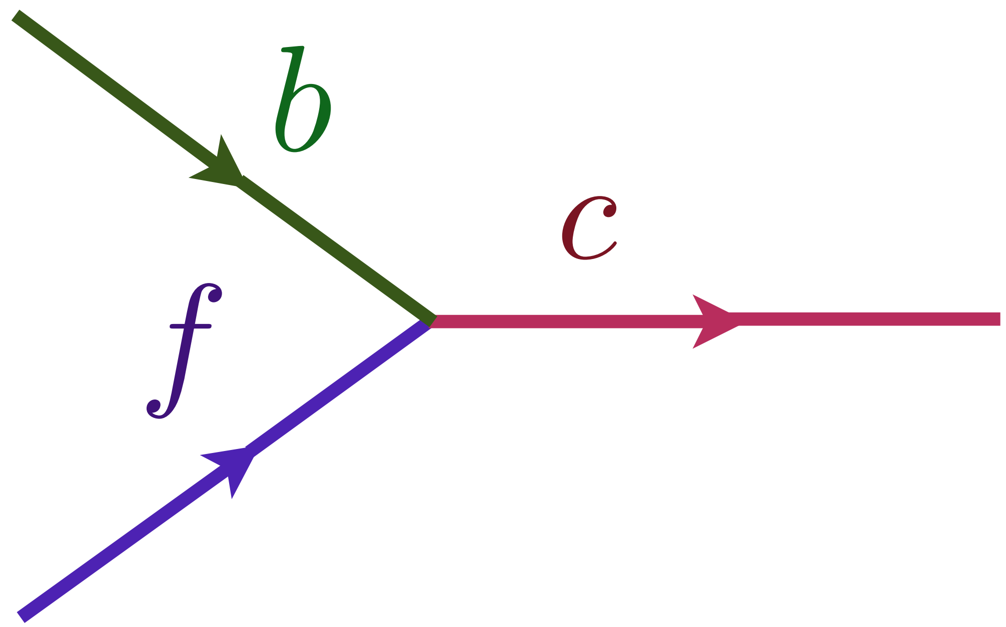
Increase the coupling g until the boson, b, and fermion, f, can bind into a 'molecule', the fermion c.

Decouple the interaction between b and f by a fermion c

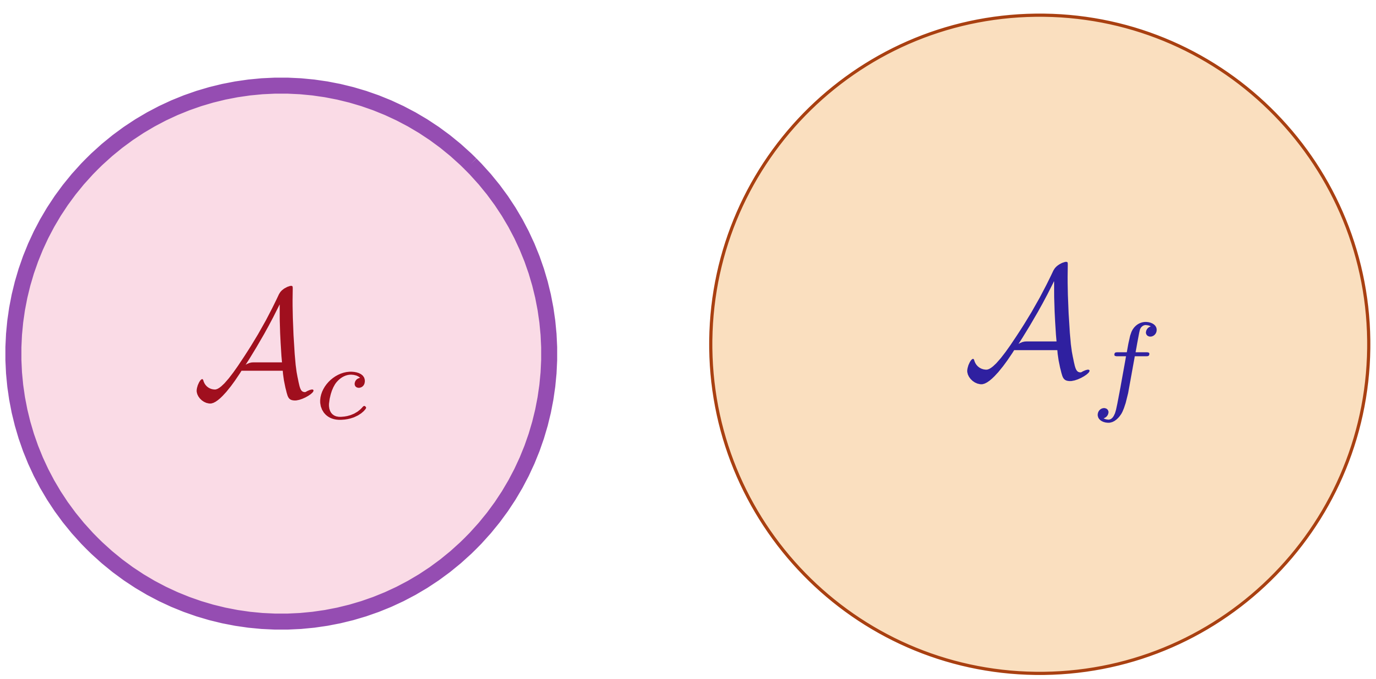


$$\mathcal{Q} = f^\dagger f$$
$$\mathcal{Q}_b = b^\dagger b$$

$$\begin{aligned} \mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + \frac{1}{g} c^\dagger c - c^\dagger f b - c b^\dagger f^\dagger + \ldots \end{aligned}$$

In a phase with $U_b(1)$ unbroken, there is a Luttinger relation for each conserved U(1) charge. However, the boson, b cannot have a Fermi surface in its Green's function, and so there is no area associated with it, although the boson density is *included* in the Luttinger relation

$$\begin{aligned} \mathcal{A}_c + \mathcal{A}_f &= \left\langle f^\dagger f \right\rangle = \langle \mathcal{Q} \rangle \\ \mathcal{A}_c &= \left\langle b^\dagger b \right\rangle = \langle \mathcal{Q}_b \rangle \end{aligned}$$



The b bosons have bound with f fermions to form c "molecules"

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

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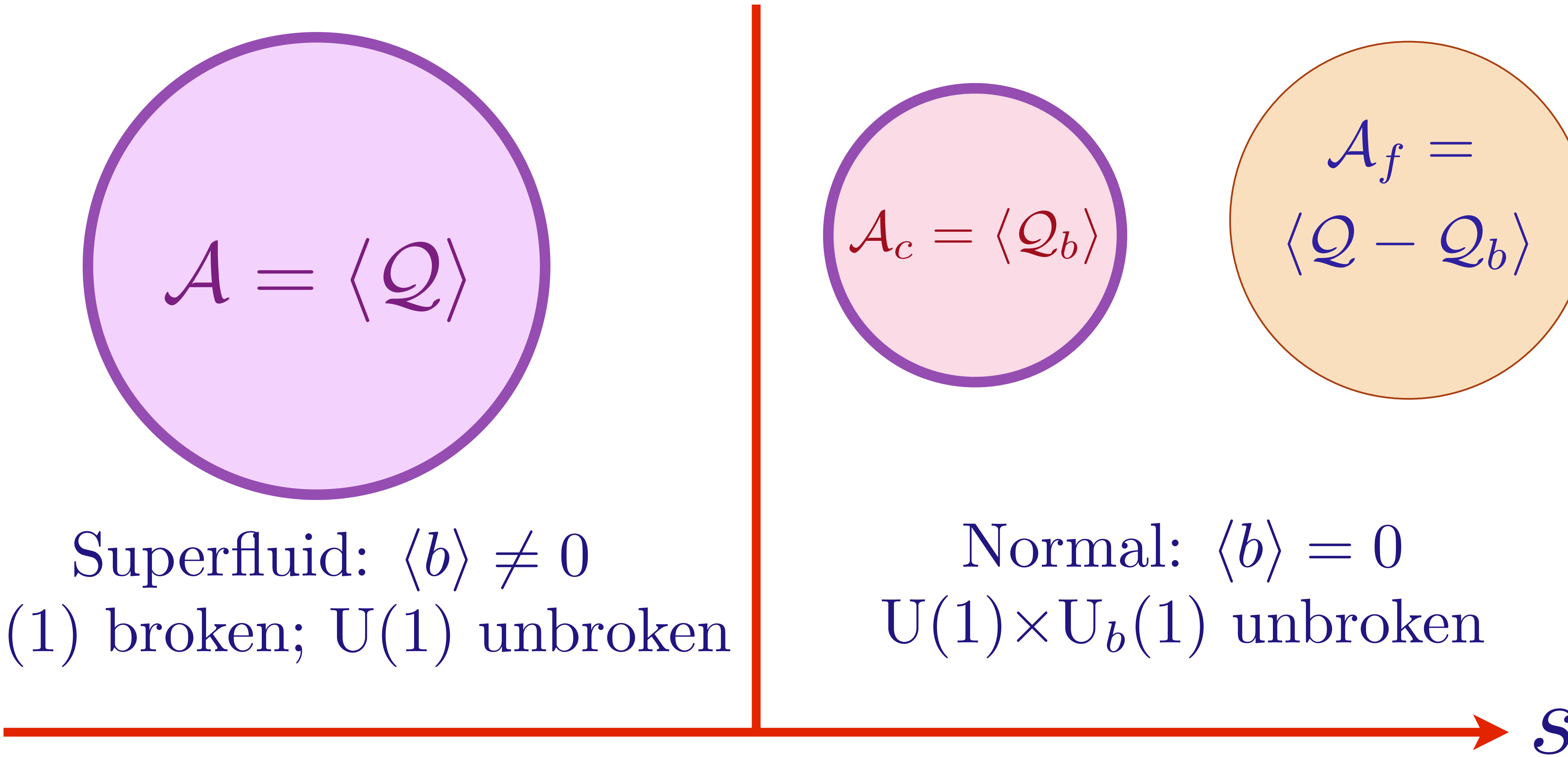
Phase diagram of boson-fermion mixture



$$\begin{aligned}\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots\end{aligned}$$

- Now gauge $\mathcal{Q} - \mathcal{Q}_b$ by a dynamic gauge field A_a. This leaves fermion c gauge-invariant

$$\begin{aligned}
\mathcal{L} &= f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f \\
&+ b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots
\end{aligned}$$

- Now gauge $\mathcal{Q} - \mathcal{Q}_b$ by a dynamic gauge field A_a. This leaves fermion c gauge-invariant

(Need a background neutralizing charge)

$$\begin{aligned}\mathcal{L} &= f^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau + iA_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m_b} - \mu_b \right) b + s|b|^2 + -g\, b^\dagger f^\dagger f b + \ldots\end{aligned}$$

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)
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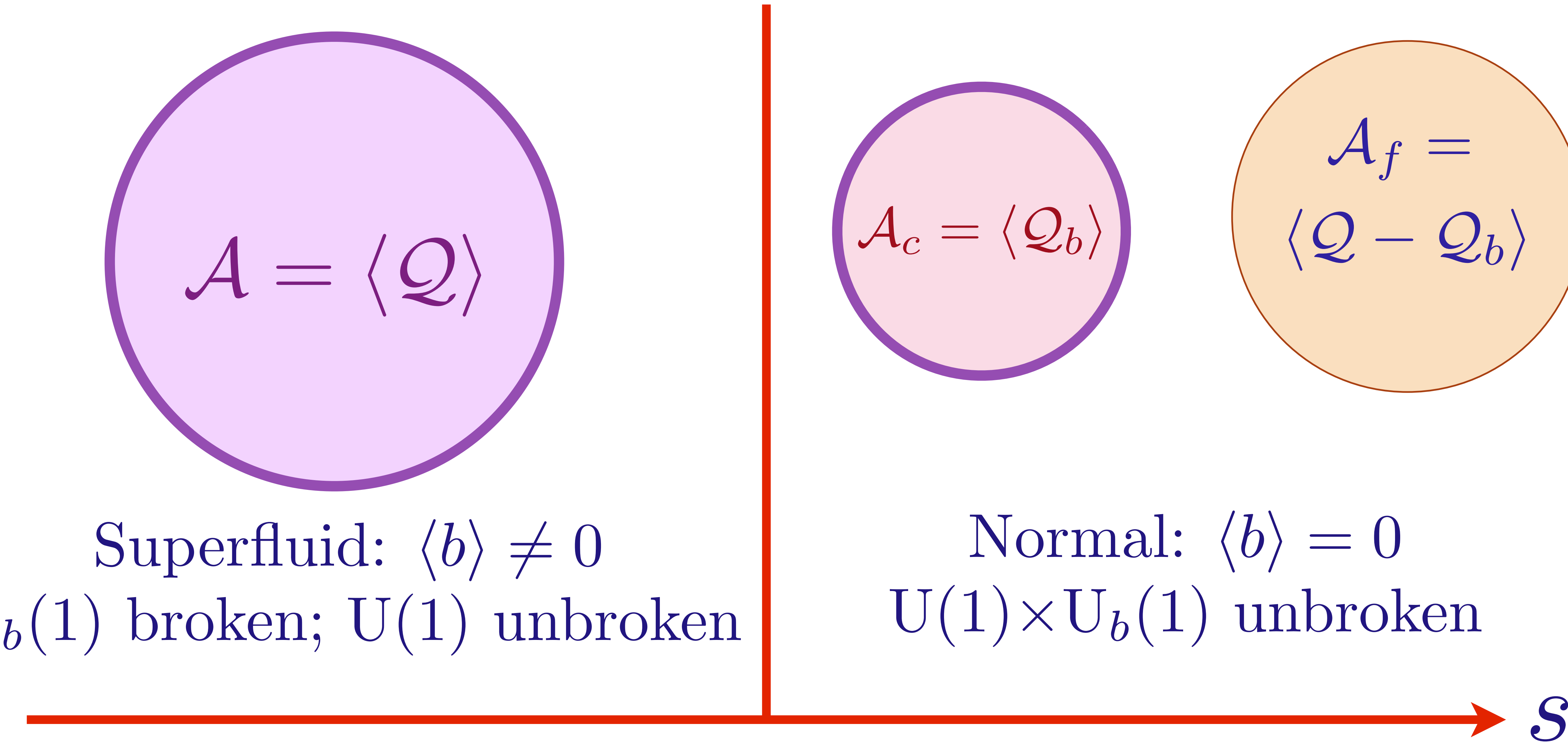

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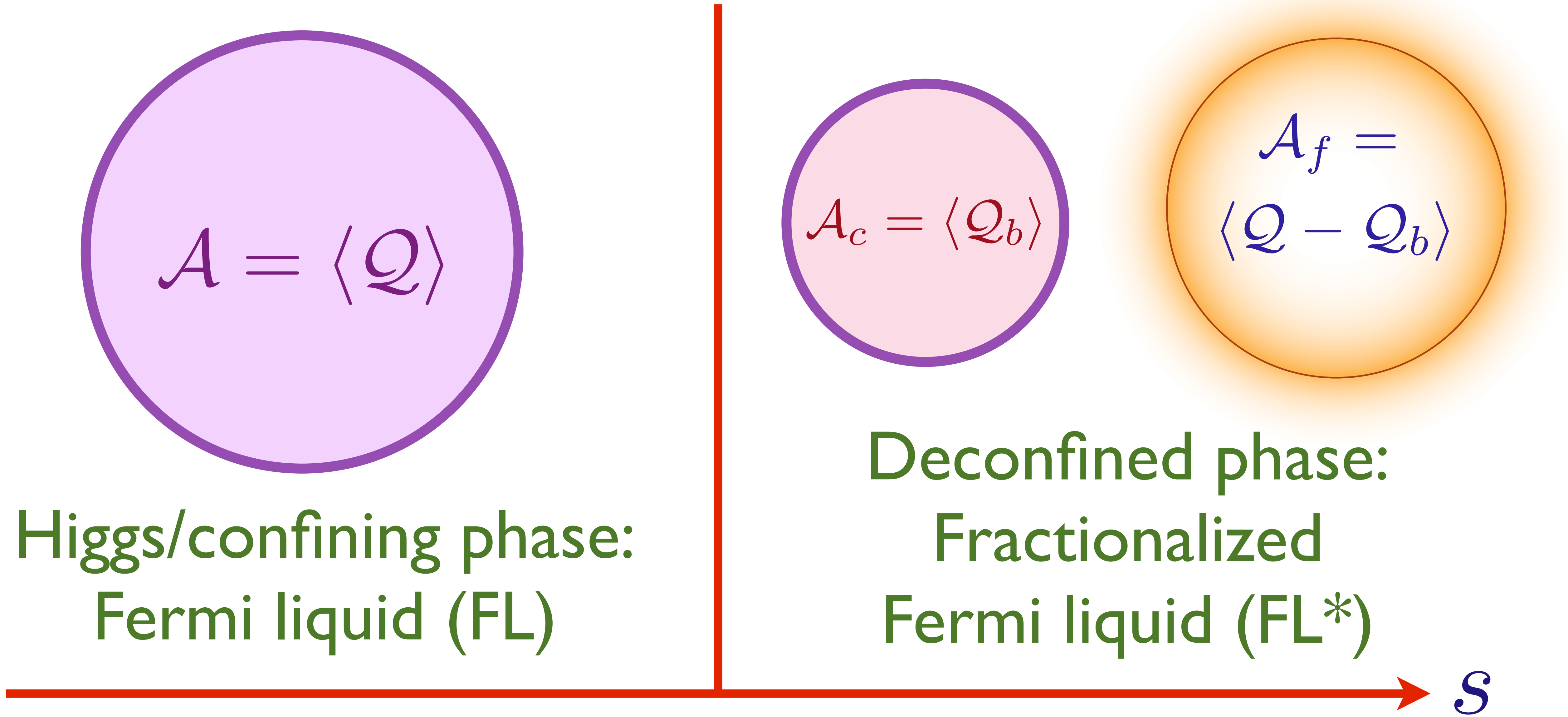
T. Senthil, M. Vojta, and S. Sachdev, *Physical Review* B **69**, 035111 (2004)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

Phase diagram of U(1) gauge theory



$$\mathcal{L} = f^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f$$
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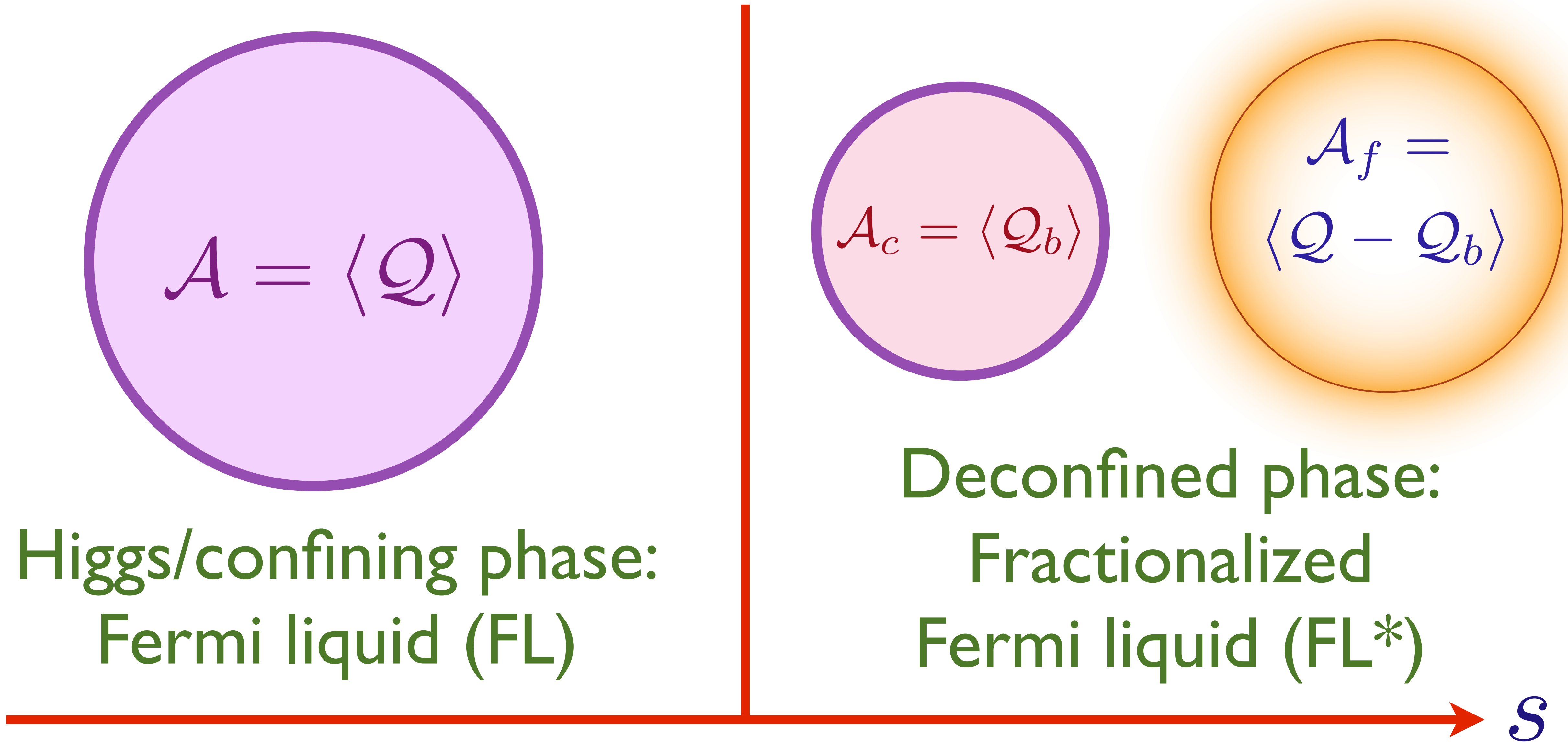

$$
\begin{aligned}
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P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

Phase diagram of U(1) gauge theory

- FL phase: Fermi surface of gauge-neutral fermions encloses total global charge $\mathcal{Q}$

- FL* phase: Fermi surface of gauge neutral fermions encloses only part of the global charge $\mathcal{Q}$

Higgs/confining phase: Fermi liquid (FL)

Fractionalized Fermi liquid (FL*)

s

$$\mathcal{L} = f^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f$$

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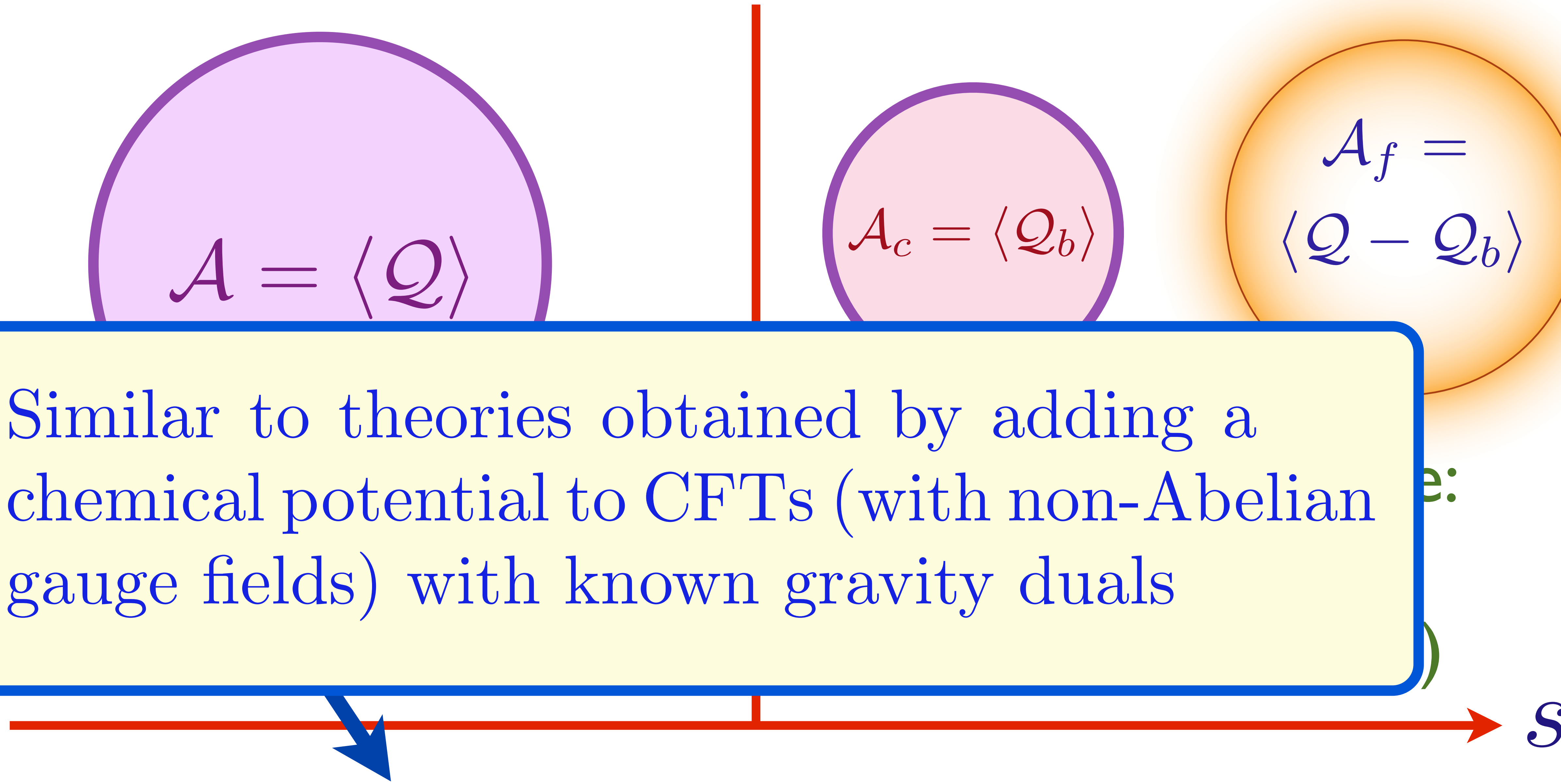

Similar to theories obtained by adding a chemical potential to CFTs (with non-Abelian gauge fields) with known gravity duals

$$\mathcal{L} = f^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f$$
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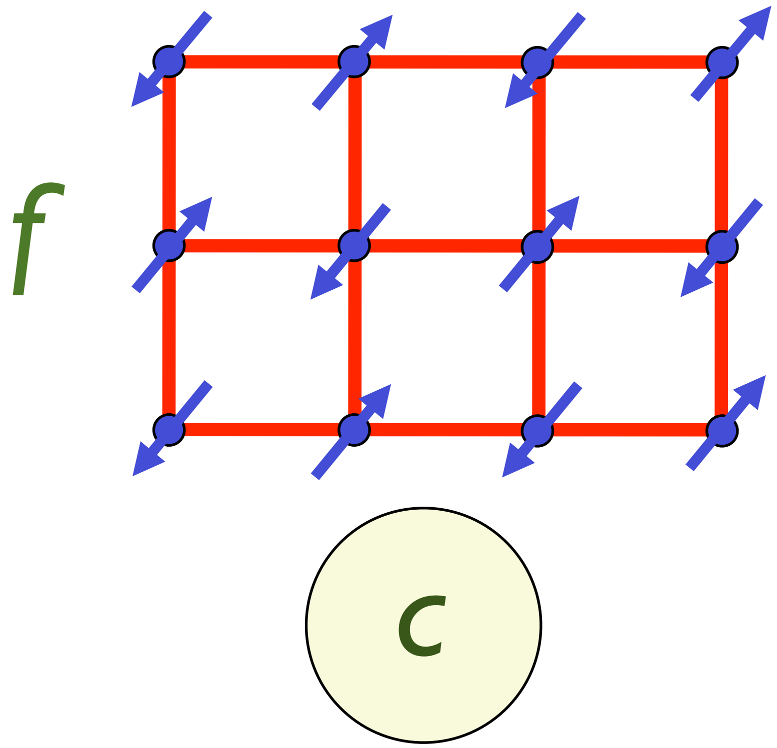
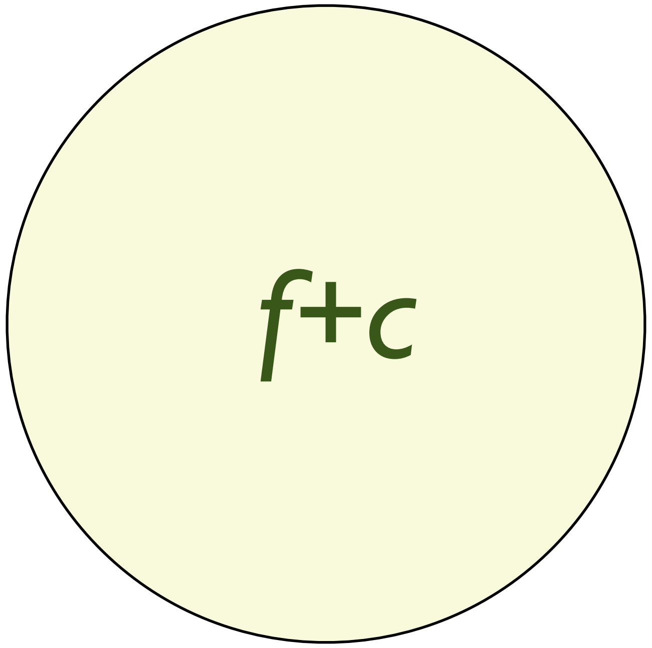
Magnetic order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

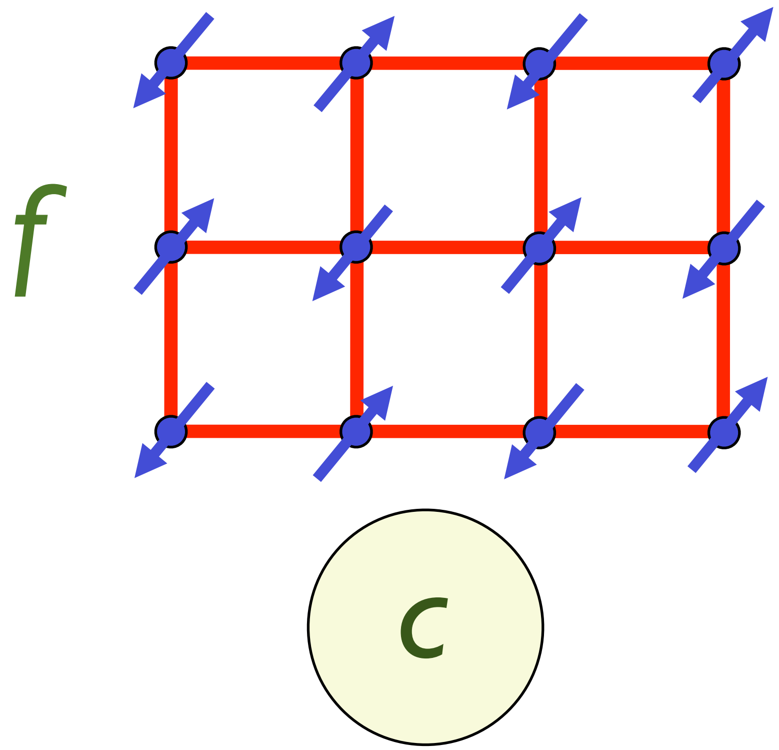
Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid
with “large” Fermi
surface of
hydridized *f* and
c-conduction
electrons

Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



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Magnetic Metal:
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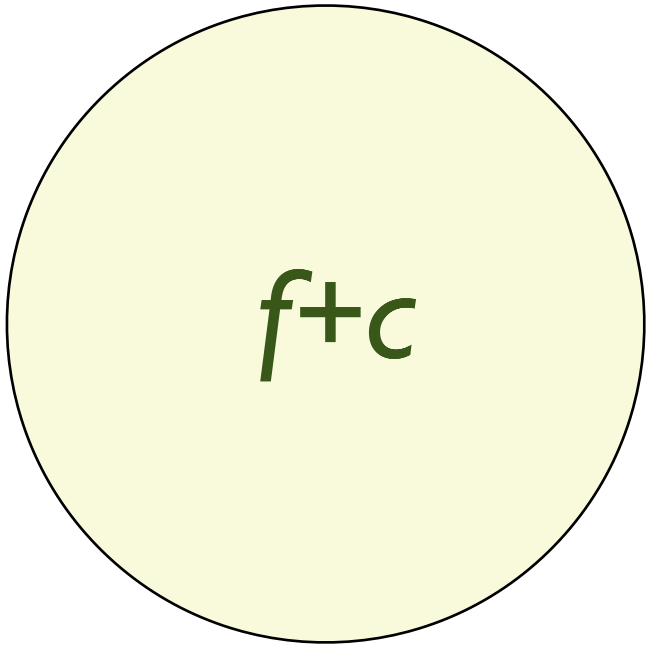

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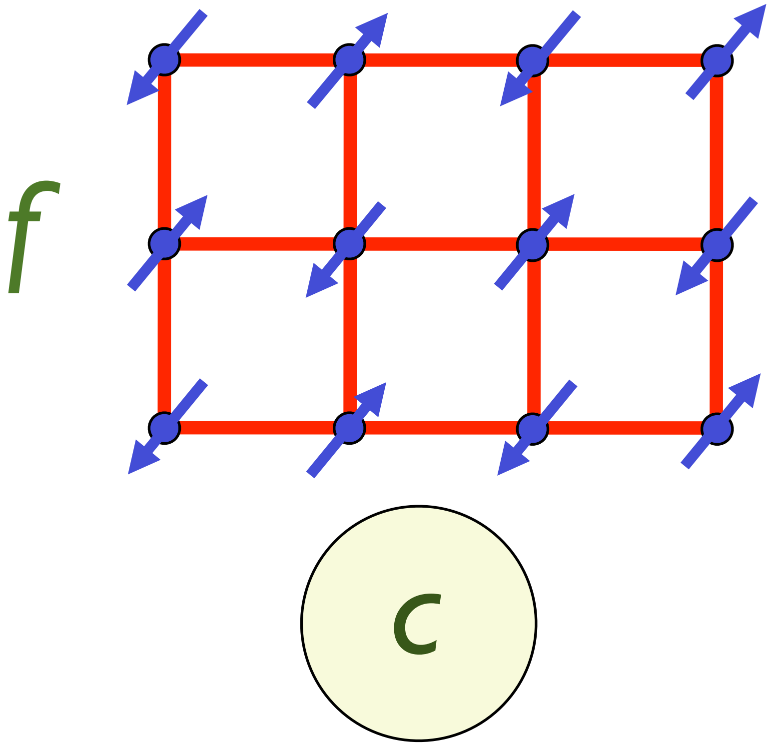
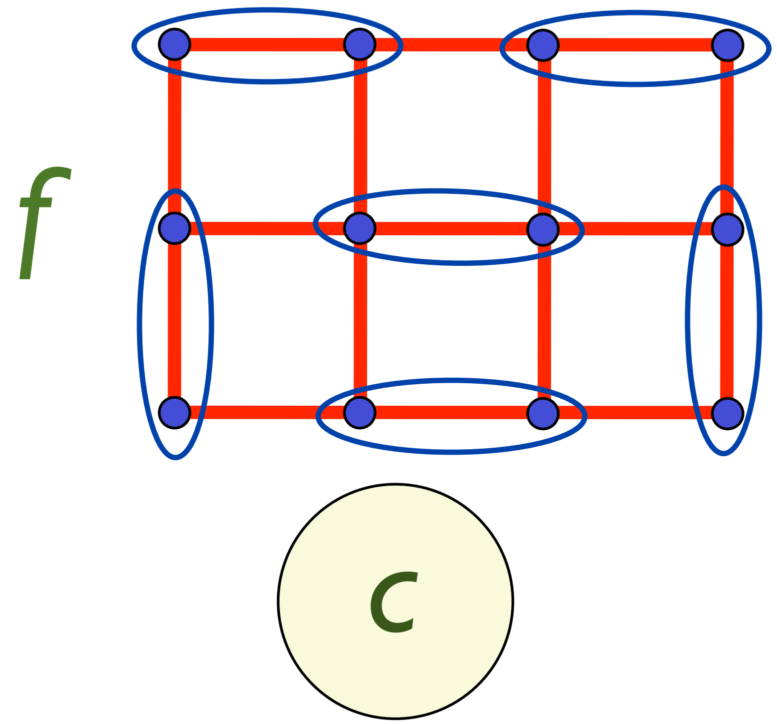


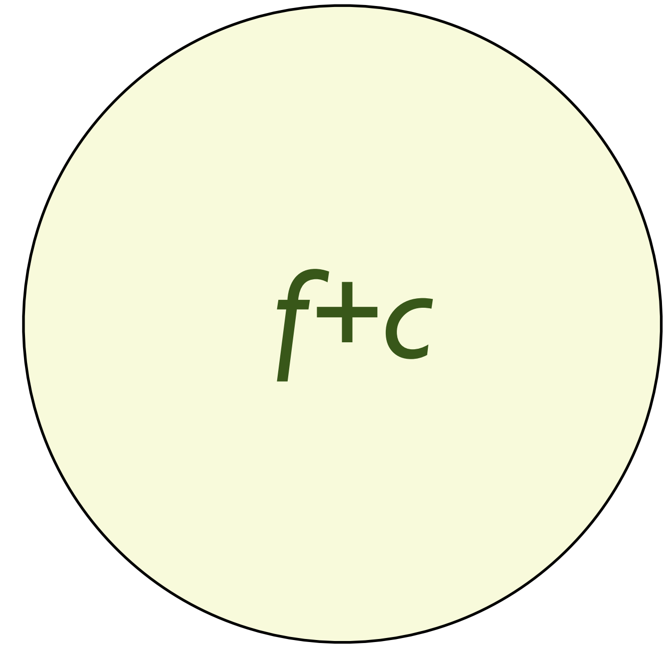
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Conduction electron
Fermi surface
and
spin-liquid of
f-electrons



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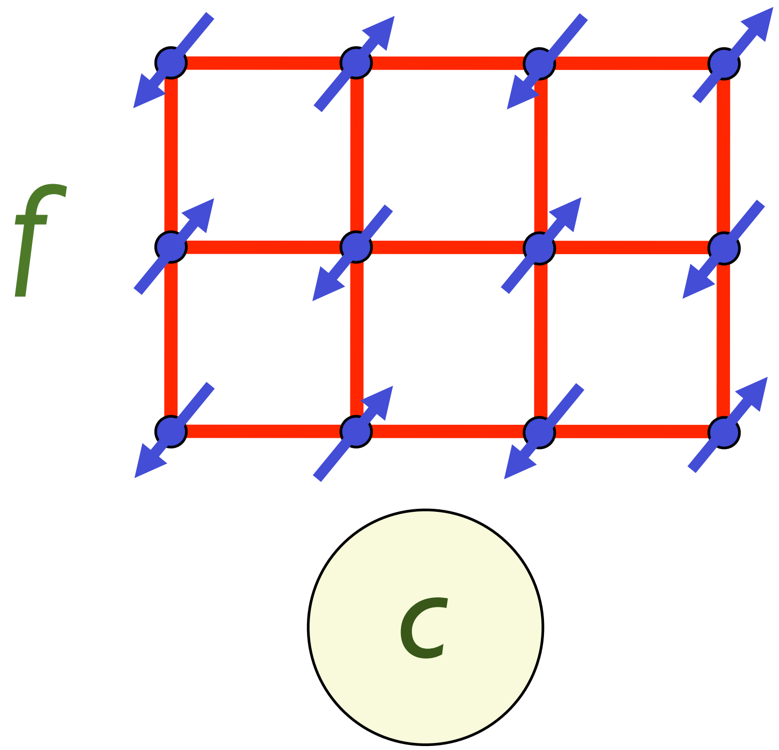
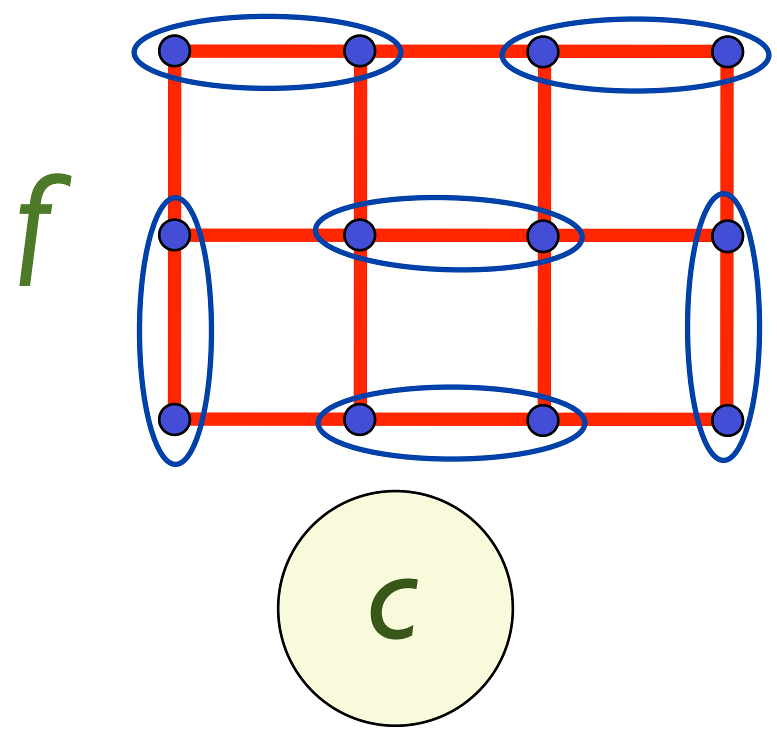


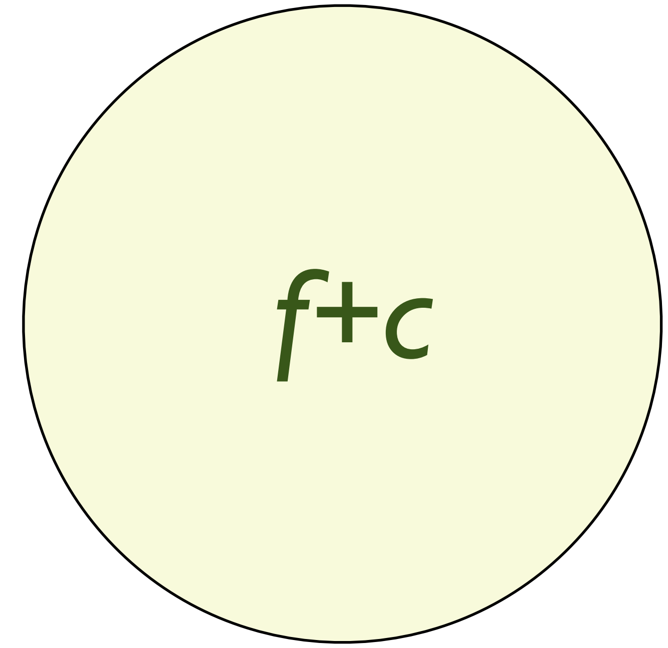
$\langle \vec{\varphi} \rangle \neq 0$

Magnetic Metal: *f*-electron moments and *c*-conduction electron Fermi surface



$\langle \vec{\varphi} \rangle = 0$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface

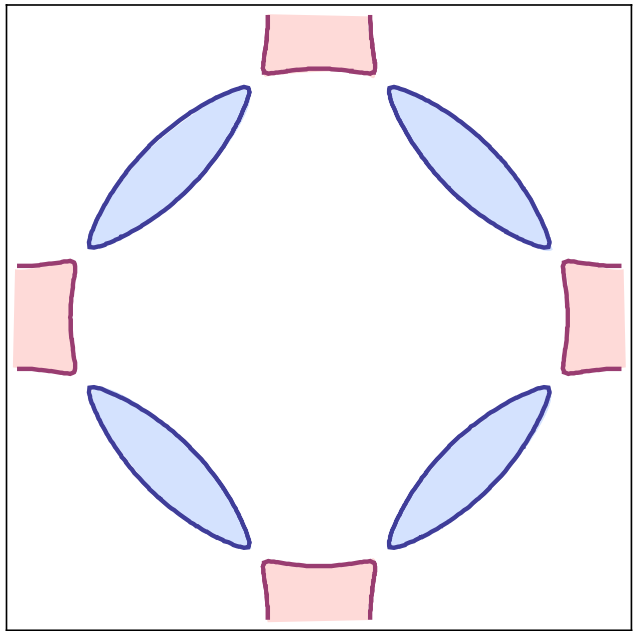


$\langle \vec{\varphi} \rangle = 0$

Heavy Fermi liquid with "large" Fermi surface of hydridized *f* and *c*-conduction electrons

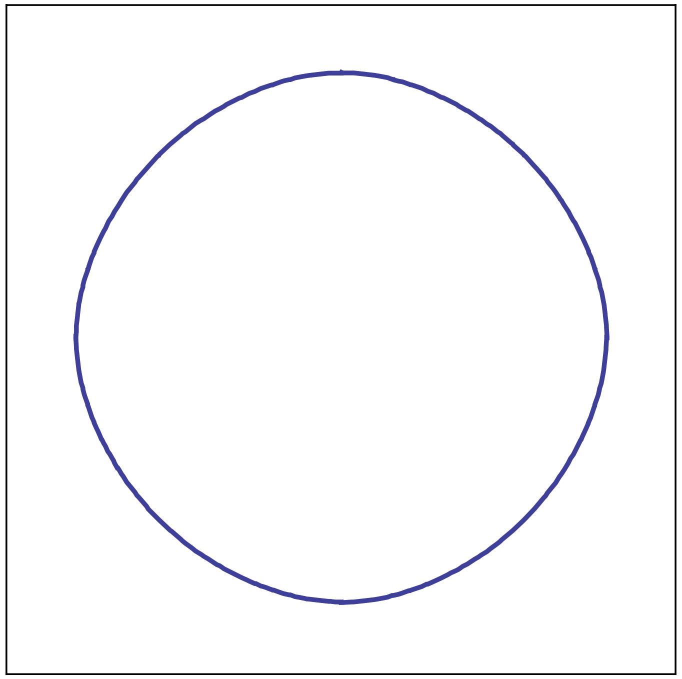
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Fermi surface reconstruction in a single band model



$$\langle \vec{\varphi} \rangle \neq 0$$

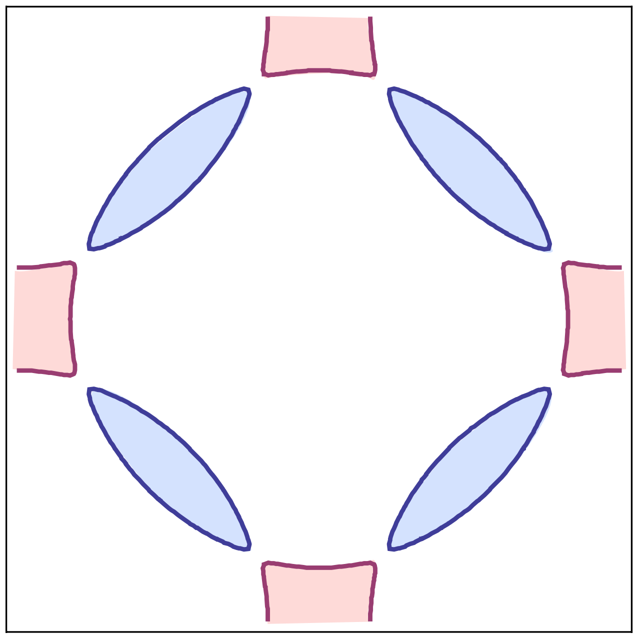
Metal with electron and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

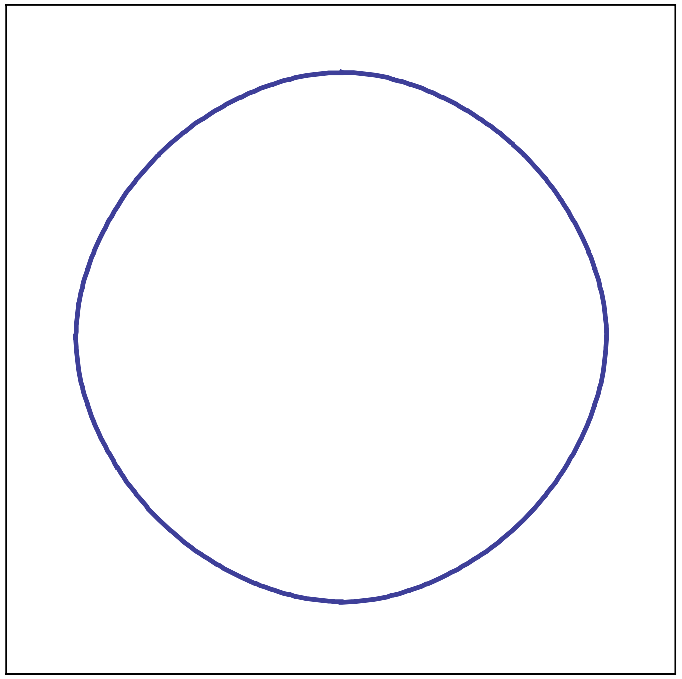
Metal with "large" Fermi surface

Separating onset of SDW order and Fermi surface reconstruction



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



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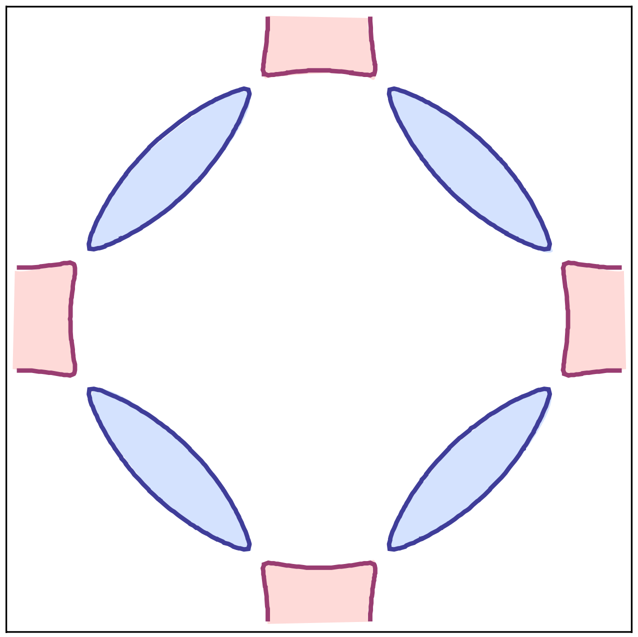
Separating onset of SDW order and Fermi surface reconstruction



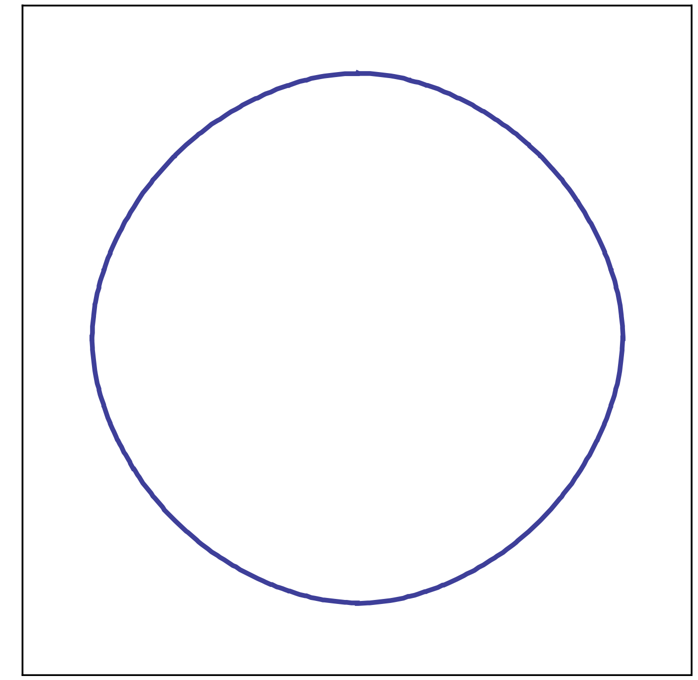
$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

$\langle \vec{\varphi} \rangle = 0$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

Y. Qi and S. Sachdev, *Physical Review B* **81**, 115129 (2010); M. Punk and S. Sachdev, to appear; see also T. C. Ribeiro and X.-G. Wen, *Physical Review B* **74**, 155113 (2006)

Outline

1. Conformal quantum matter

The AdS_4 - Schwarzschild black brane

2. Compressible quantum matter

A. Condensed matter vs. continuum QFTs

Outline

1. Conformal quantum matter

The AdS_4 - Schwarzschild black brane

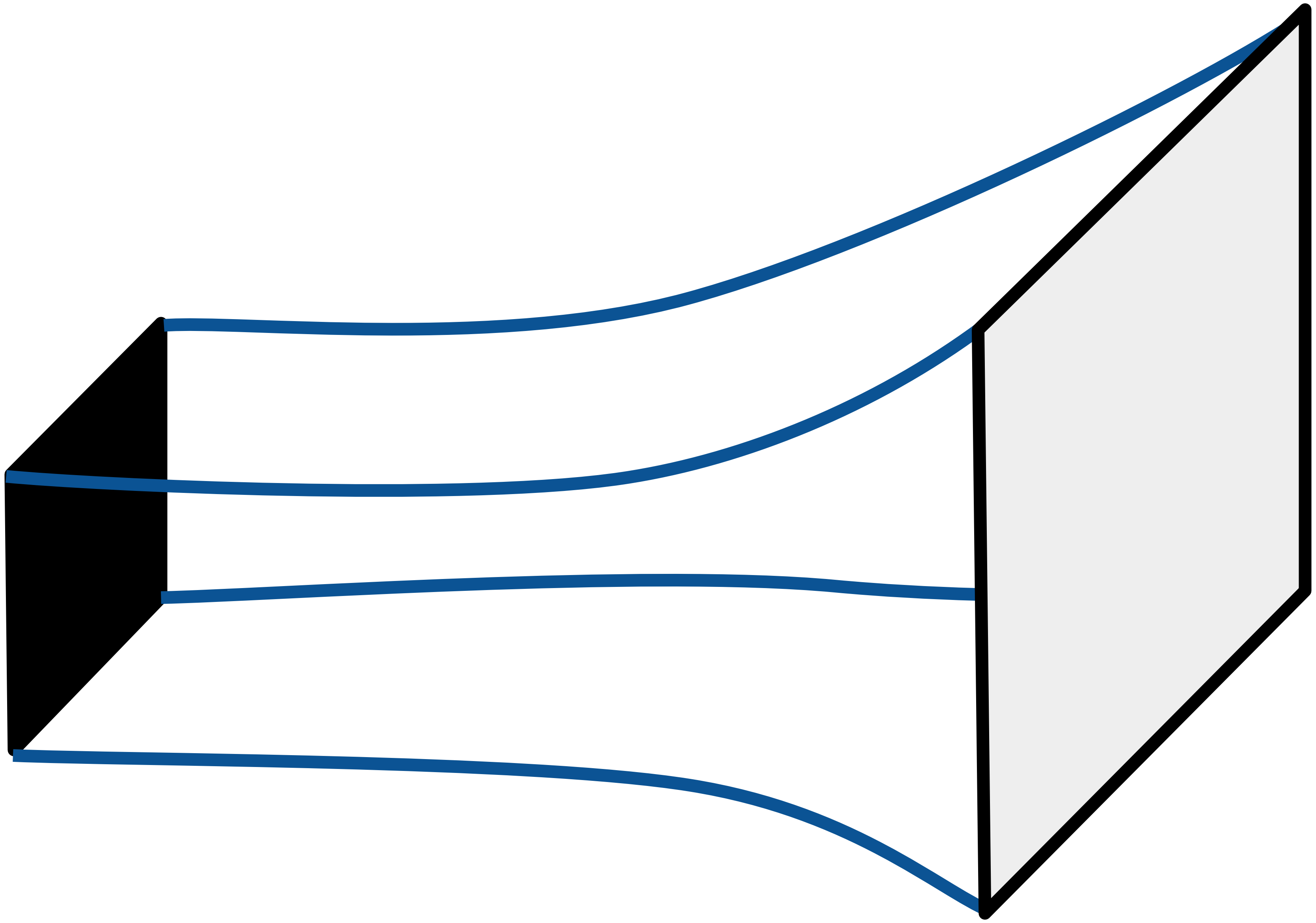
2. Compressible quantum matter

A. Condensed matter vs. continuum QFTs

B. The AdS_4 - Reissner-Nordström black-brane and $AdS_2 \times R^2$

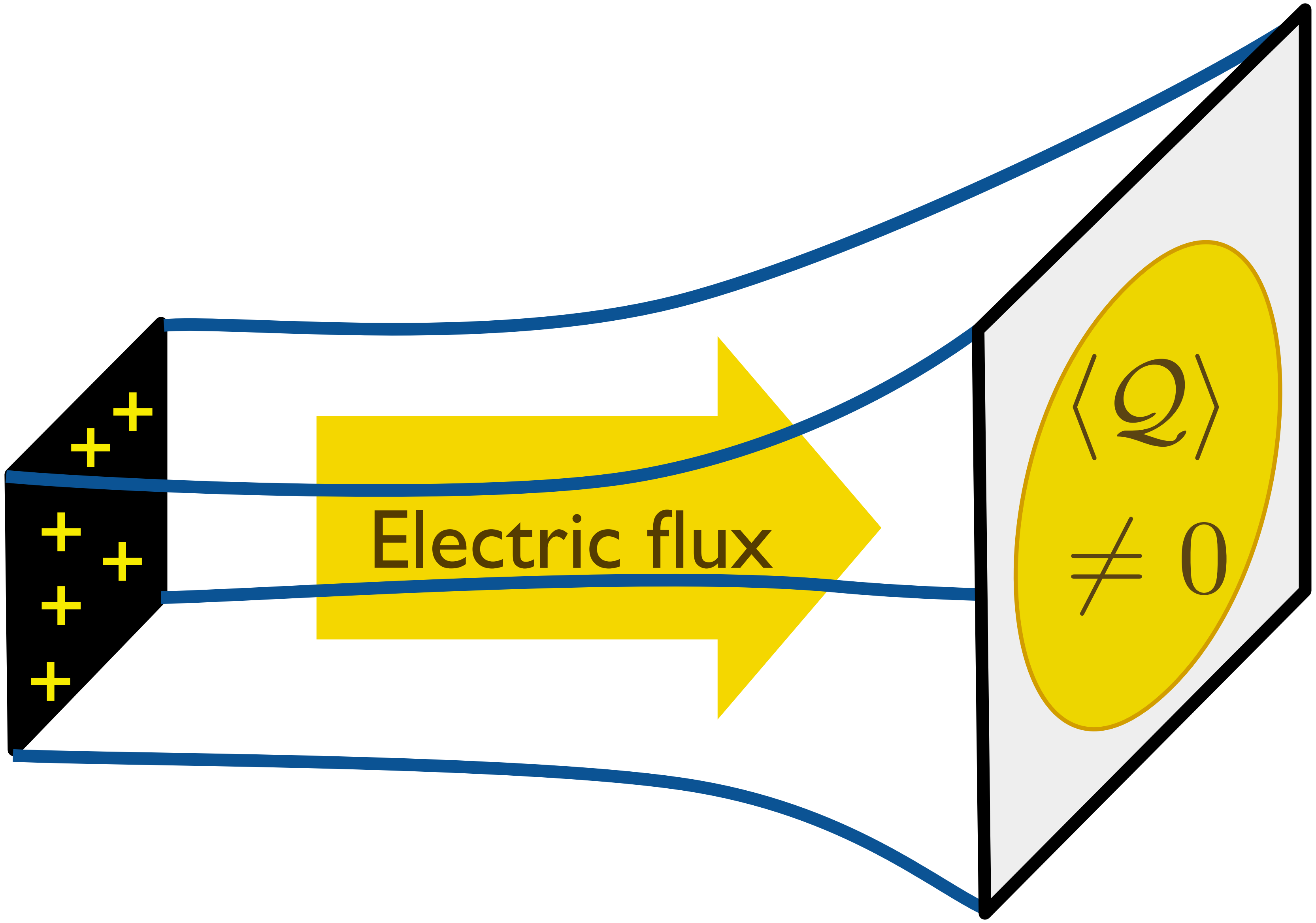
C. Beyond $AdS_2 \times R^2$

AdS$_4$-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

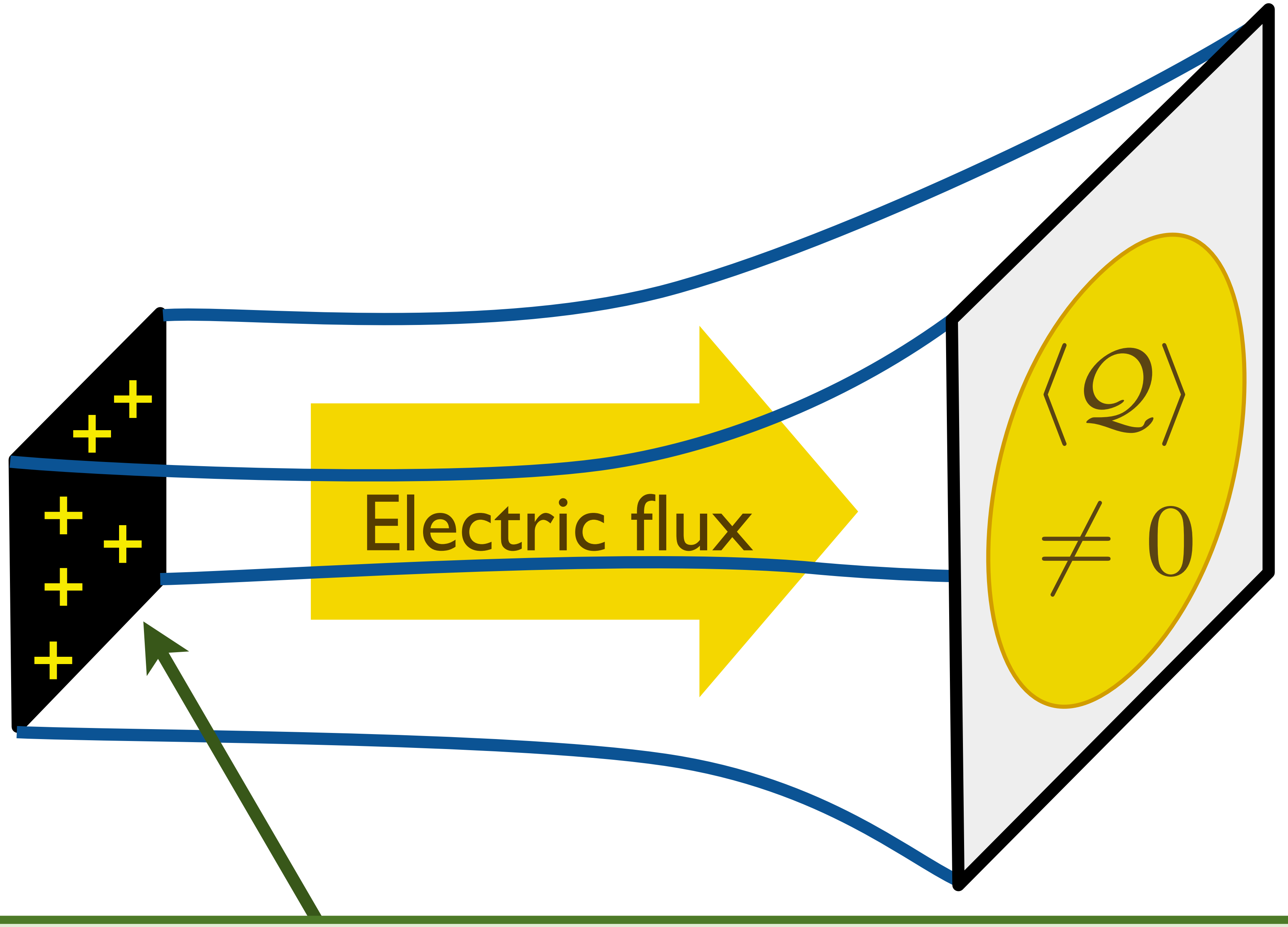
AdS$_4$-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

AdS_4-Reissner-Nordtröm black-brane



At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\mathrm{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

Properties of $AdS_2 \times R^2$

This state appears stable in the presence of matter fields (with large enough bulk mass). The single-particle Green's function of the boundary theory has the IR (small ω) limit

$$G^{-1}(k,\omega) = A(k) + B(k)\omega^{\nu_k}$$

where $A(k)$, $B(k)$, and ν_k are smooth functions of k.

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

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For fermions, if $A(k)$ changes sign at a $k = k_F$, we have a *Fermi surface* at $k = k_F$. This Fermi surface is non-Fermi liquid like.

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Lee; Denef, Hartnoll, Sachdev; Cubrovic, Zaanen, Schalm; Faulkner, Polchinski

Properties of AdS_2 X R^2

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Properties of AdS$_2$ X R^2

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There is a deficit in the Luttinger count. This suggests there are ``hidden Fermi surfaces" of gauge-charged particles as in a FL* phase.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).
L. Huijse and S. Sachdev, Phys. Rev. D 84, 026001 (2011)

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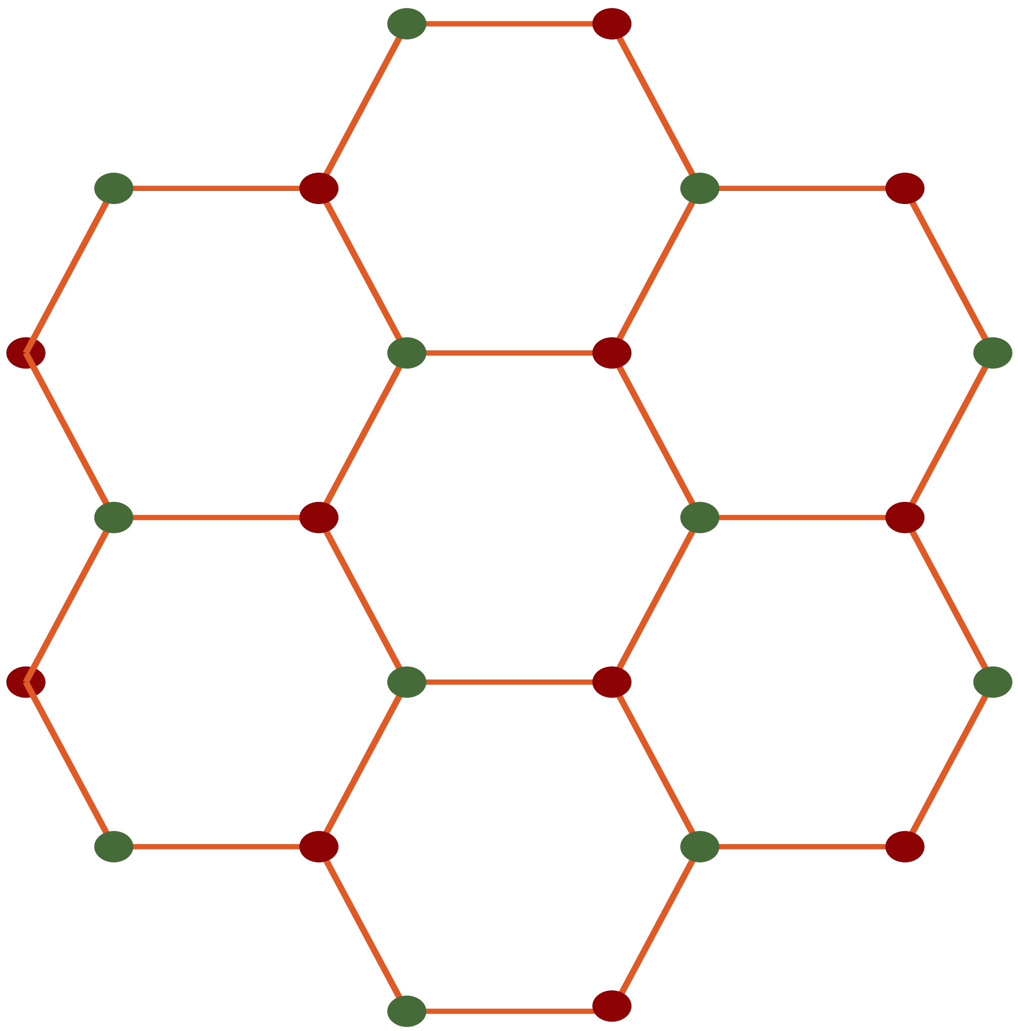
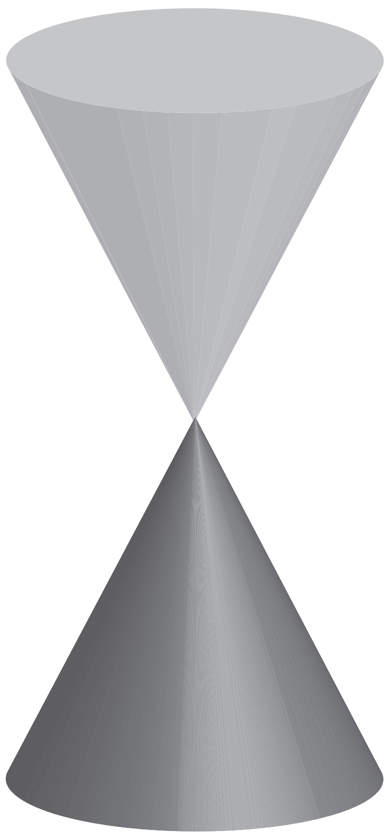
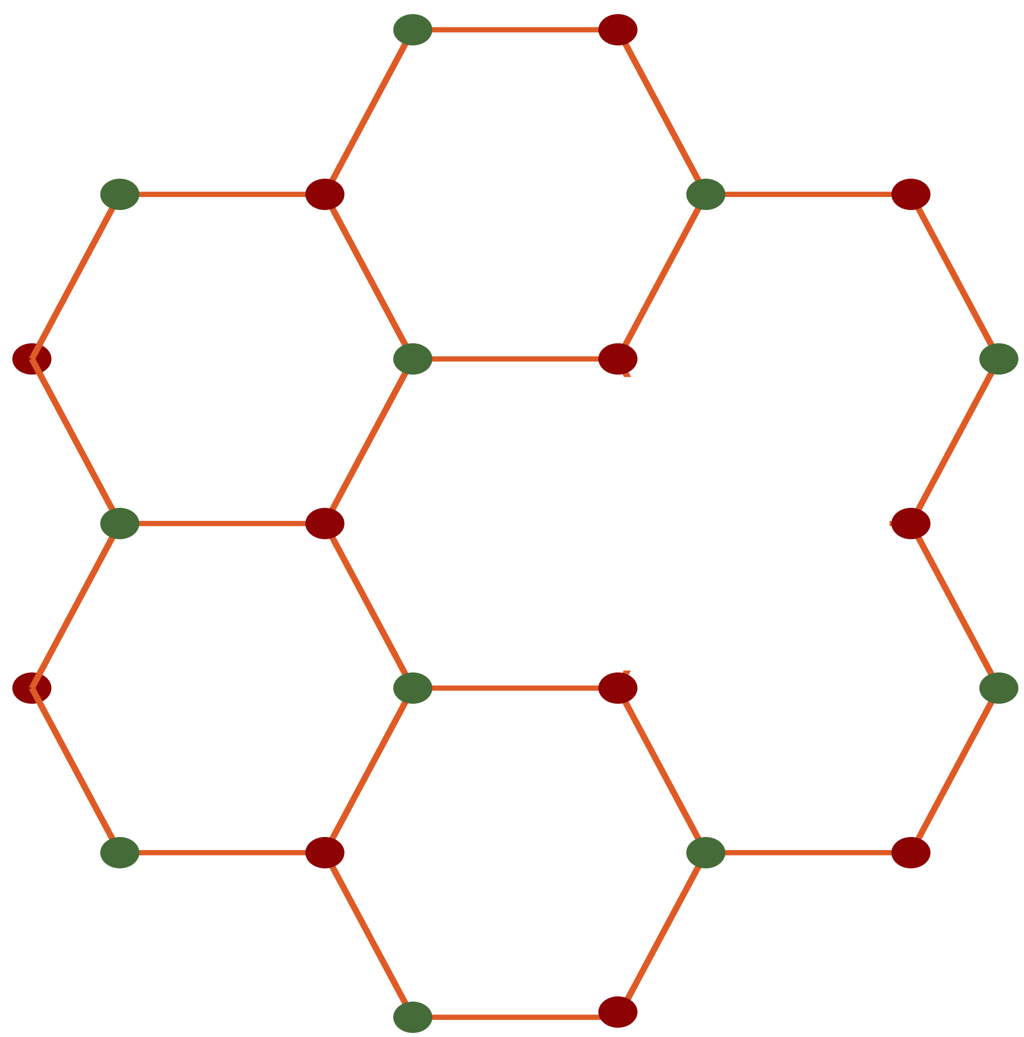
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
Lee; Denef, Hartnoll, Sachdev; Cubrovic, Zaanen, Schalm; Faulkner, Polchinski

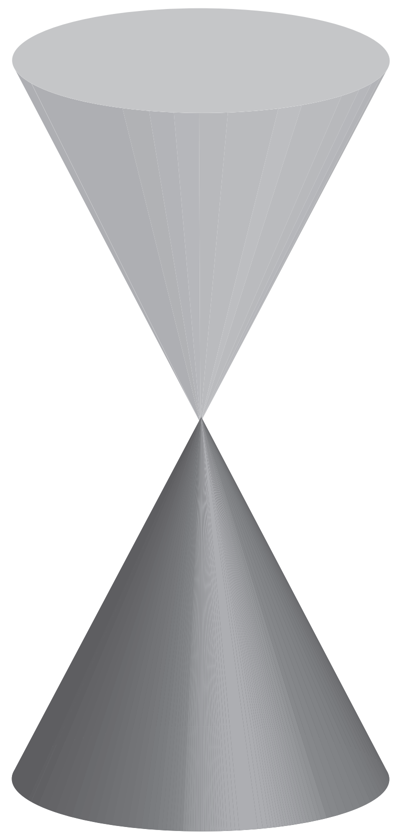
Interpretation of AdS_2

CFT on graphene

Interpretation of AdS_2

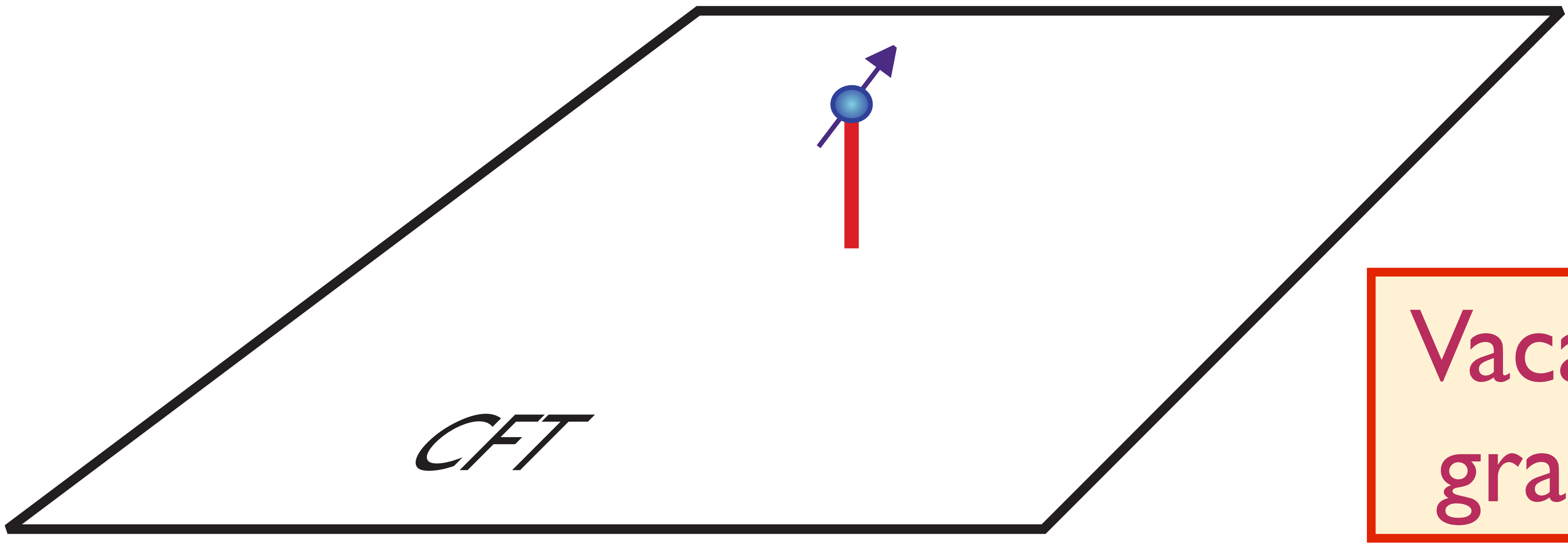




Add "matter" one-at-a-time: honeycomb lattice with a vacancy.

There is a zero energy quasi-bound state with $|\psi(r)| \sim 1/r$.
We represent this by a localized fermion field $\chi_\alpha(\tau)$.

Interpretation of AdS_2



Vacancy in graphene

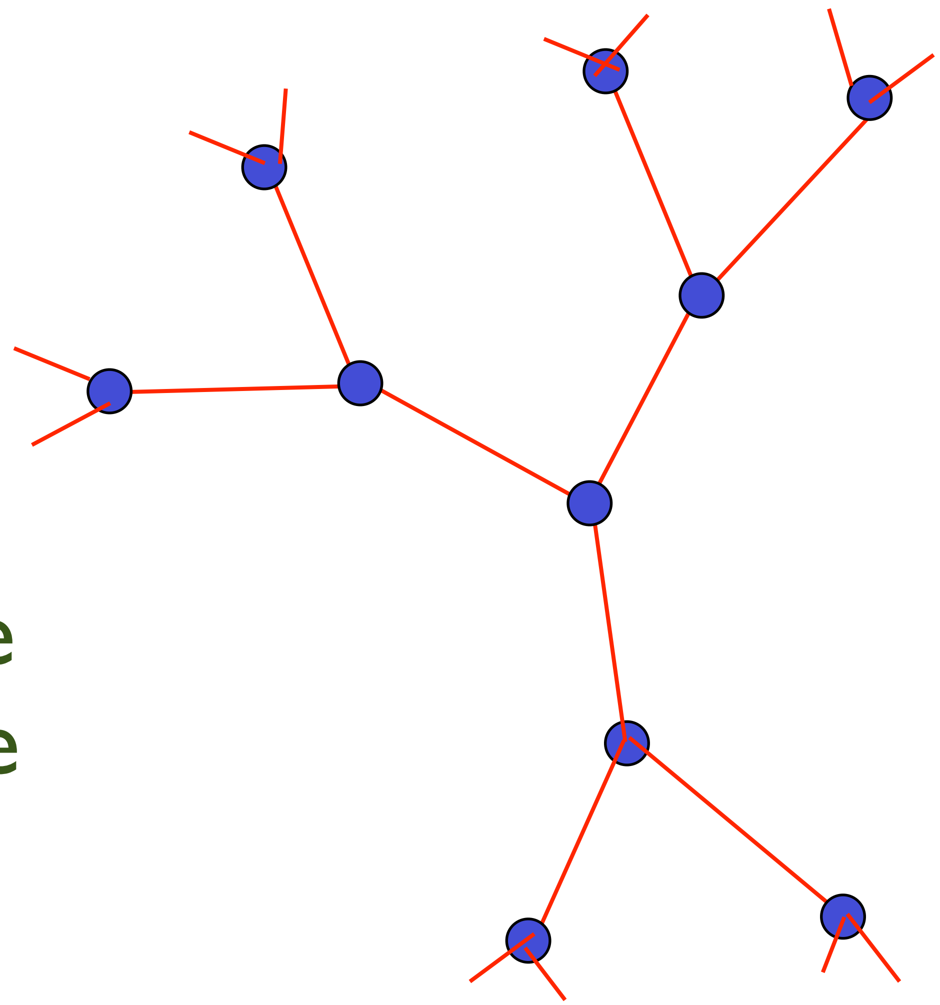
$$\mathcal{S} = \int d^3x \, \mathcal{L}_{\rm CFT} - \int d\tau \mathcal{L}_{\rm imp}$$

$$\mathcal{L}_{\rm imp} = \chi_\alpha^\dagger \frac{\partial \chi_\alpha}{\partial \tau} - \kappa \, \chi_\alpha^\dagger \sigma^a_{\alpha\beta} \chi_\beta \, \varphi^a(\mathbf{r} = 0, \tau)$$

AdS_2: "Boundary" conformal field theory obtained when κ flows to a fixed point $\kappa \to \kappa^*$.

S. Sachdev, C. Buragohain, and M. Vojta, *Science* **286**, 2479 (1999)

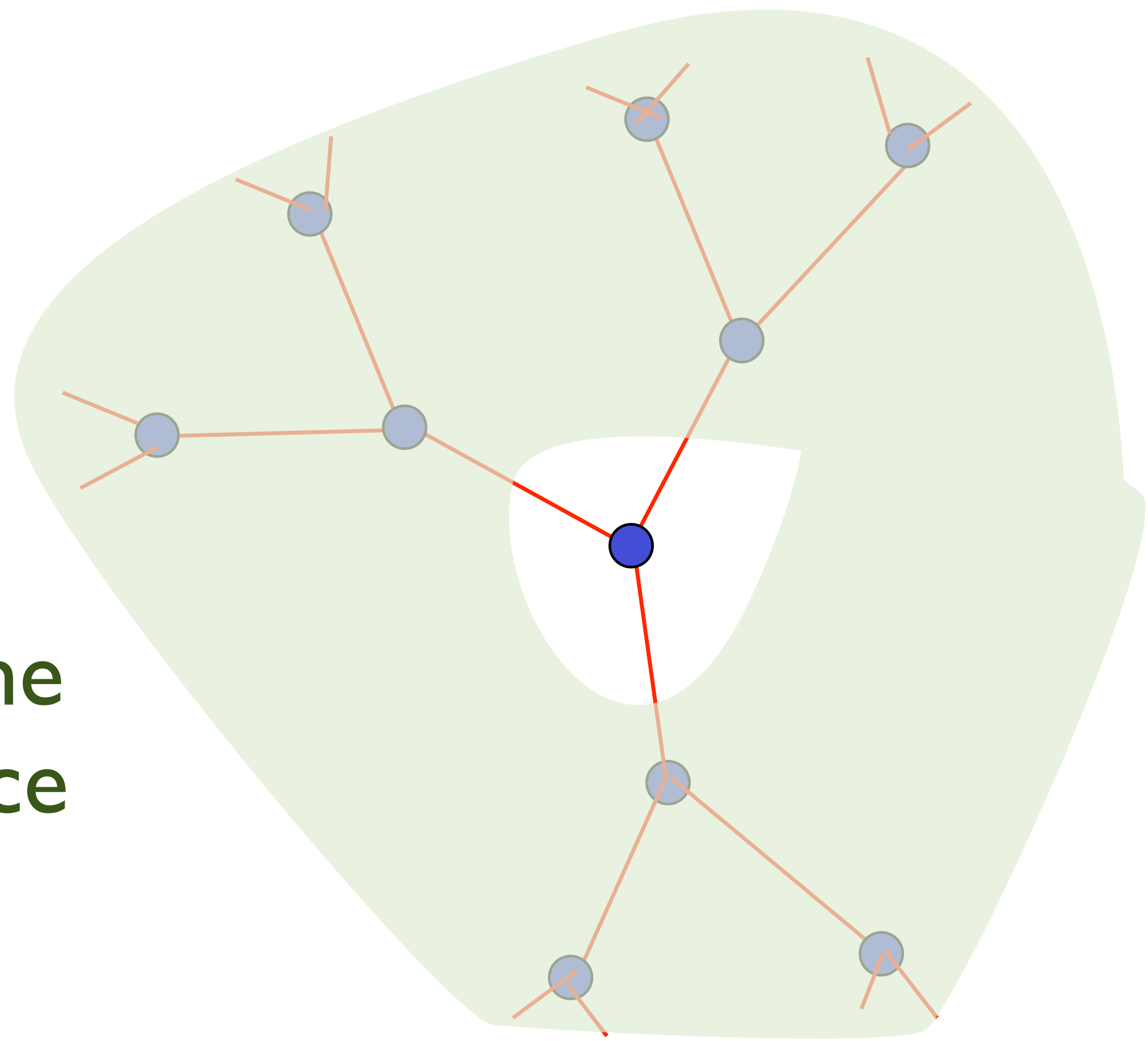
Interpretation of AdS_2 X R^2



Bethe
lattice

Solve electronic models in the limit of large number of nearest-neighbors

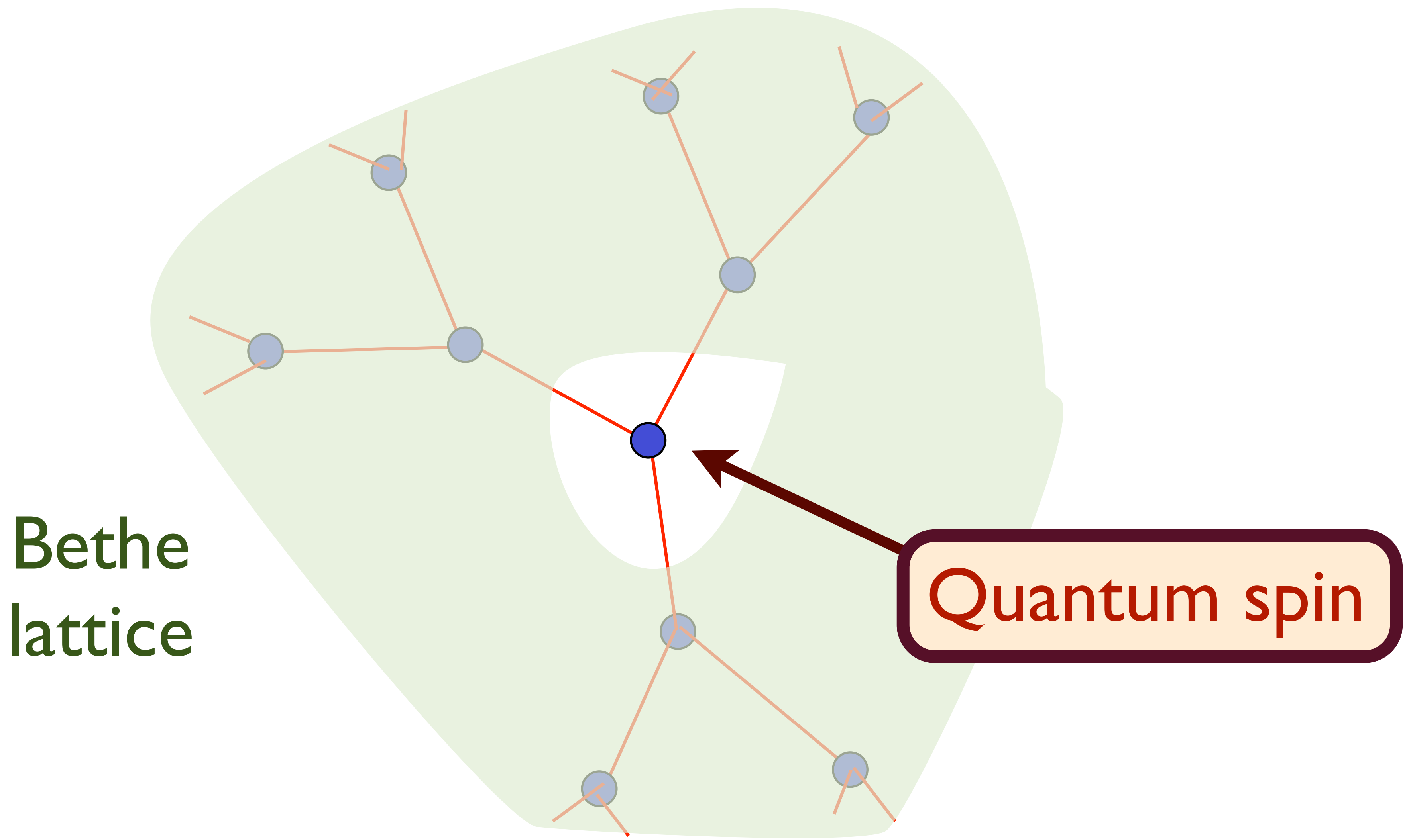
Interpretation of $AdS_2 \times R^2$



Bethe lattice

Theory is expressed as a "quantum spin" coupled to an "environment": solution is often a boundary CFT in 0+1 dimension

Interpretation of $AdS_2 \times R^2$



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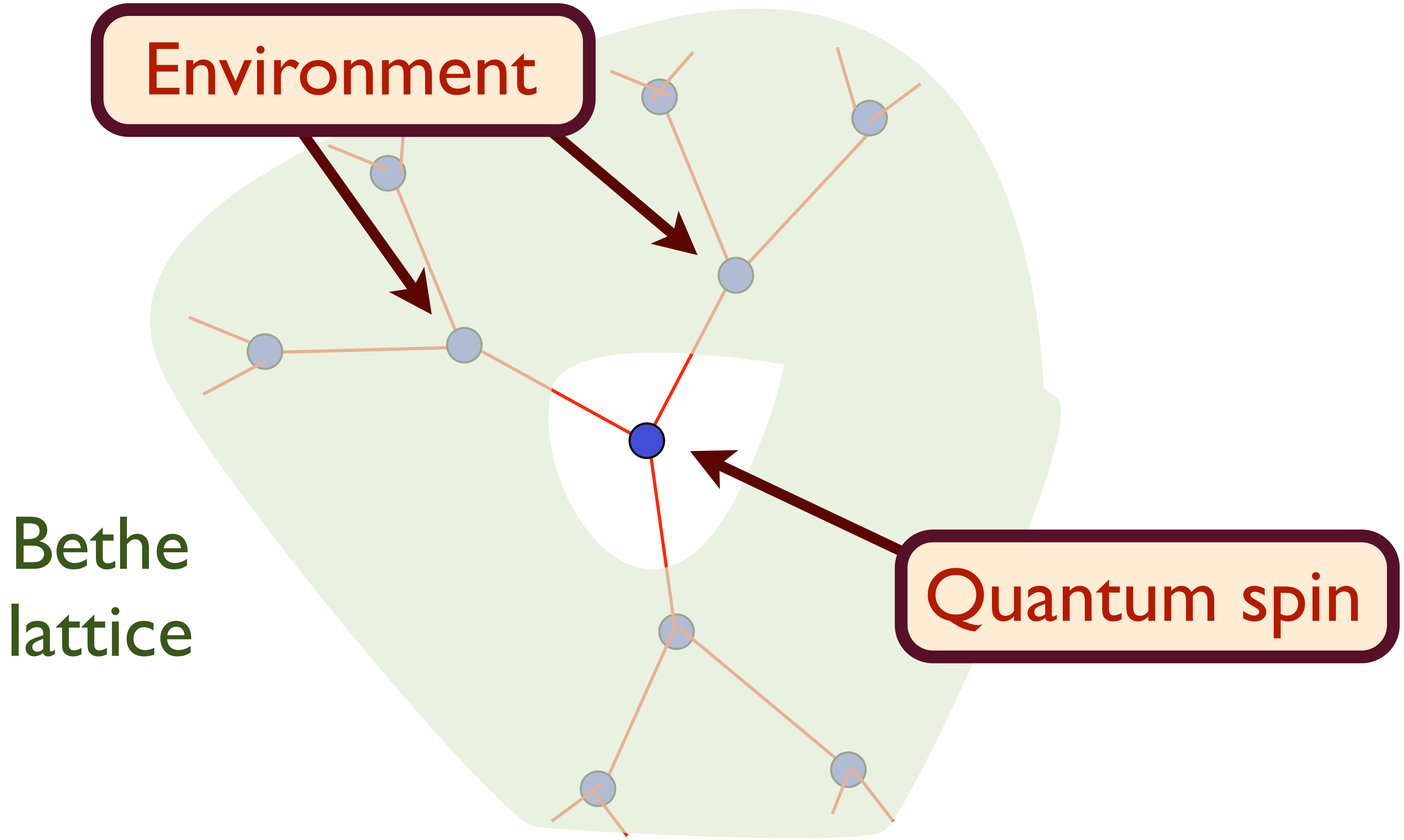

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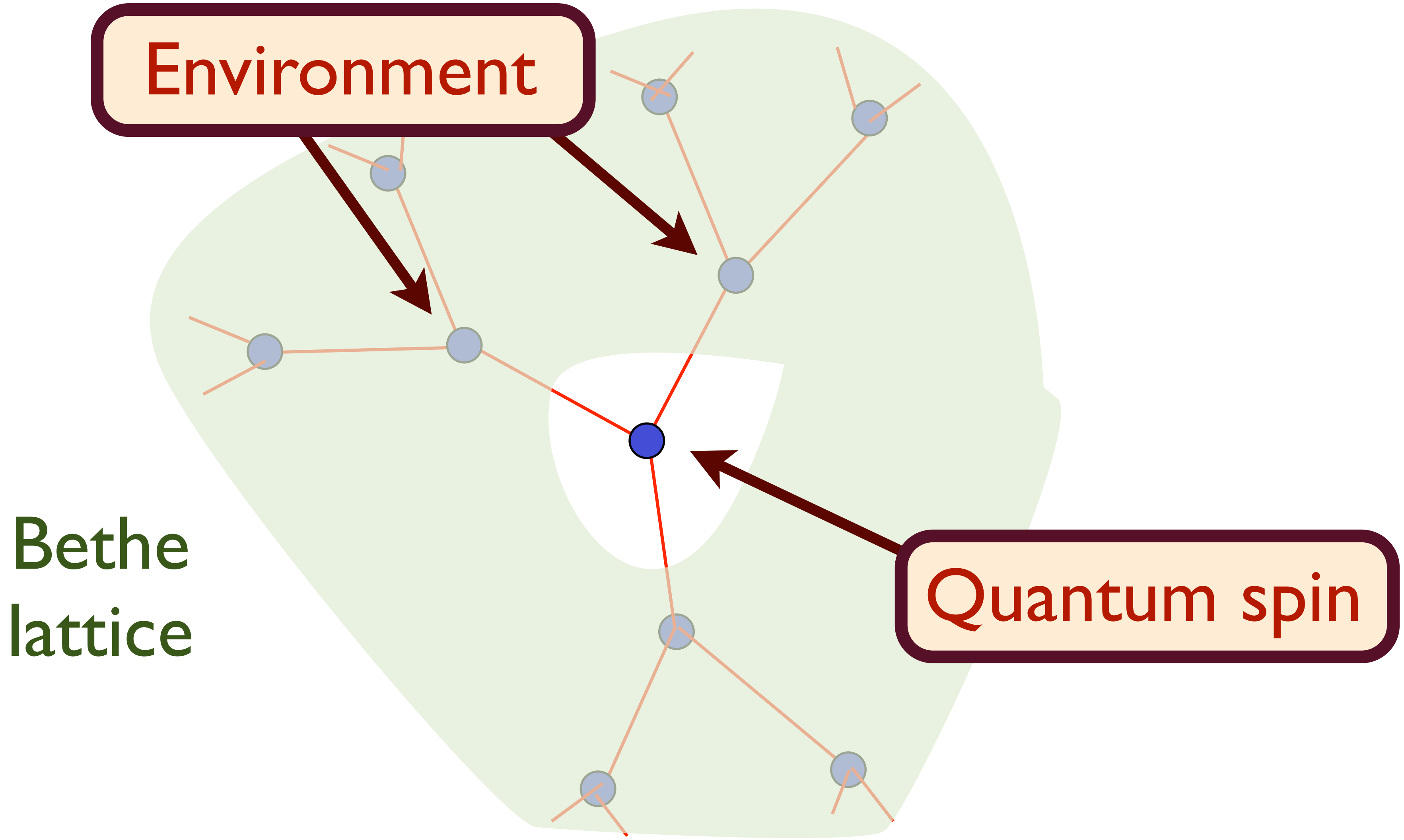
Interpretation of $AdS_2 \times R^2$



Exponents are determined by self-consistency condition between "spin" and "environment".

Artifacts of AdS_2 X R^2

- The large-neighbor-limit solution matches with those of the $\mathrm{AdS}_2 \times R^2$ holographic solutions:
  - A non-zero ground state entropy.
  - Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
  - A marginal Fermi liquid spectrum for fermionic quasi-particles (for the holographic solution, this requires tuning a free parameter).
  - The low energy sector has conformally invariant correlations.

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Outline

1. Conformal quantum matter

The AdS_4 - Schwarzschild black brane

2. Compressible quantum matter

A. Condensed matter vs. continuum QFTs

B. The AdS_4 - Reissner-Nordström black-brane and $AdS_2 \times R^2$

C. Beyond $AdS_2 \times R^2$

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1. Conformal quantum matter

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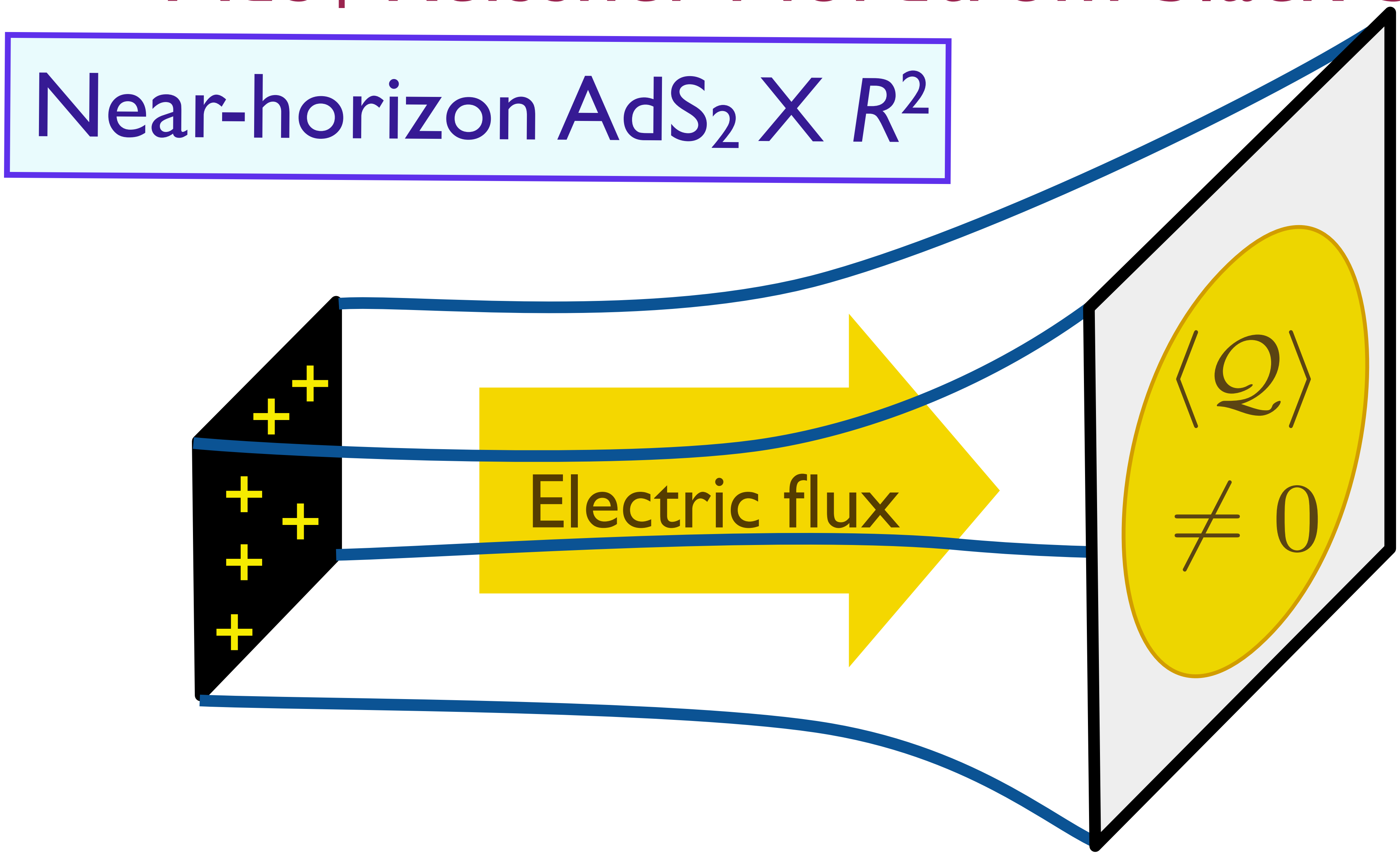
AdS$_4$-Reissner-Nordtröm black-brane

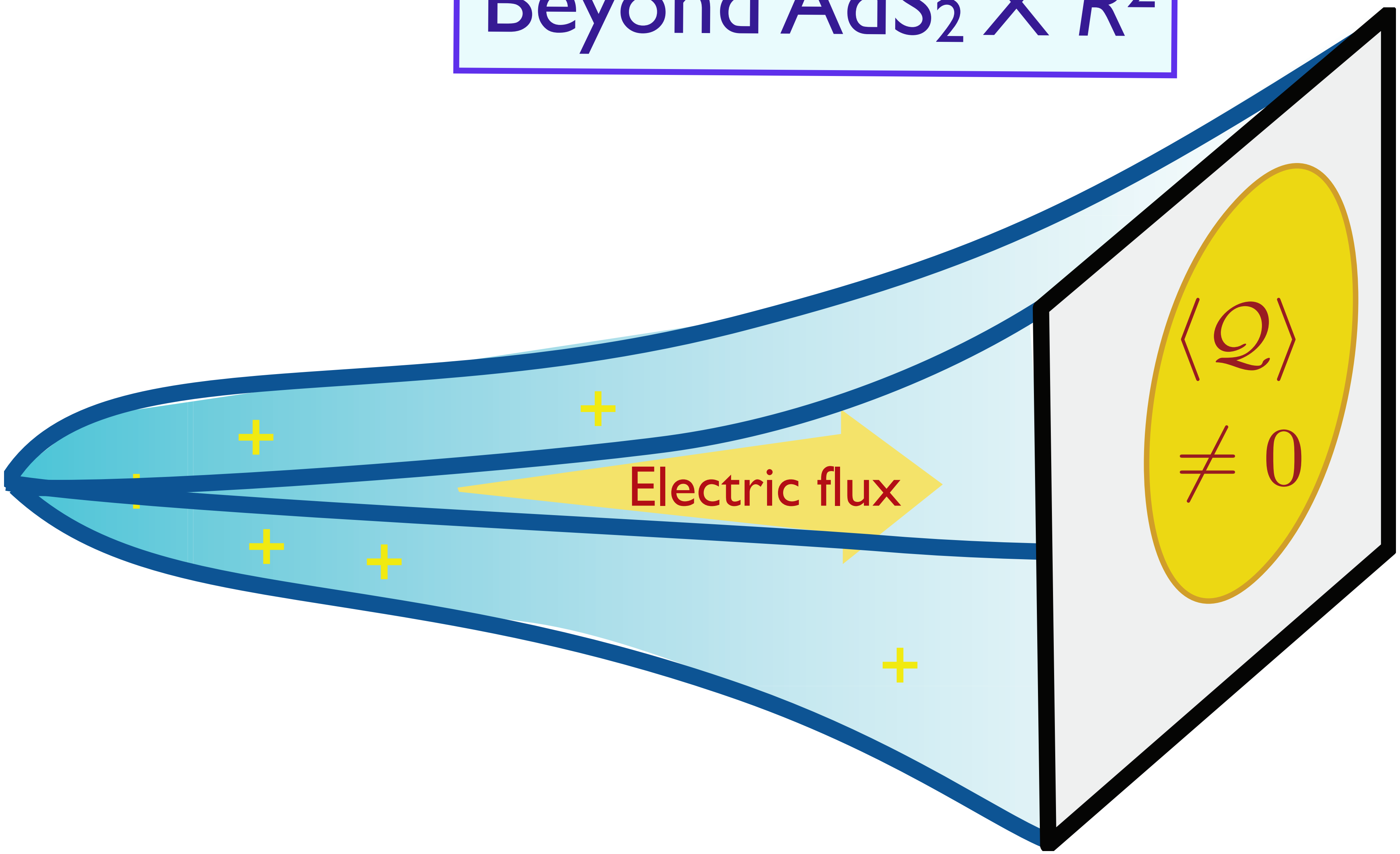


$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

Beyond $AdS_2 \times R^2$

S. Sachdev
arXiv:1107.5321

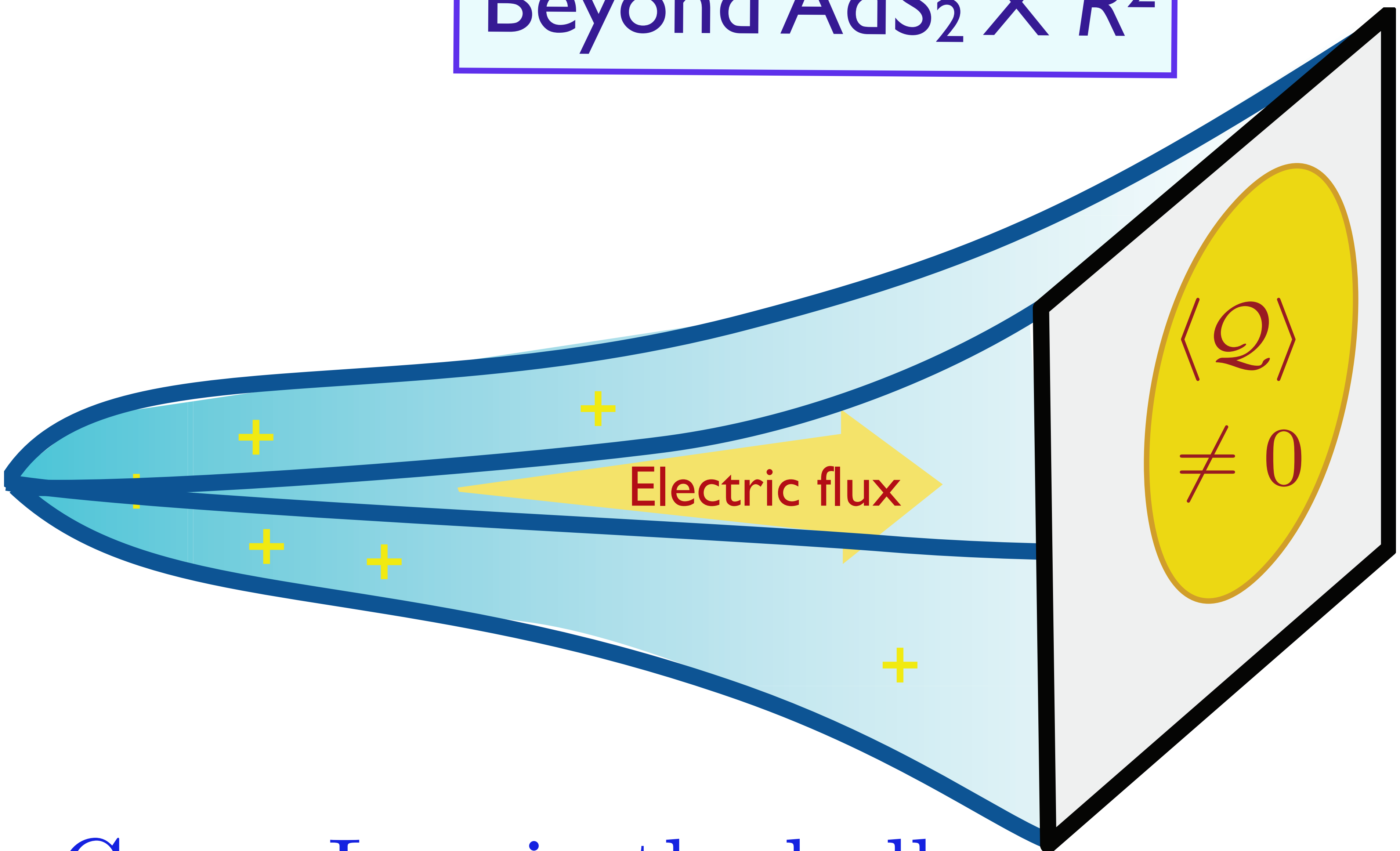


$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} + \mathcal{L}_{\text{matter}} \right]$$

In a confining phase, the horizon disappears,
the charge density is delocalized in the bulk spacetime,
and a Fermi liquid phase is obtained on the boundary

Beyond $AdS_2 \times R^2$

S. Sachdev
arXiv:1107.5321



Gauss Law in the bulk
$\Leftrightarrow$ Luttinger theorem on the boundary

In a confining phase, the horizon disappears, the charge density is delocalized in the bulk spacetime, and a Fermi liquid phase is obtained on the boundary

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Conclusions

Compressible quantum matter

- The Reissner-Nordström solution provides the simplest holographic theory of a compressible state. The solution is similar to those of (extended) DMFT.

- Much current work on realizing Fermi liquid (FL), fractionalized Fermi liquid (FL*), and non-Fermi liquid (nFL) phases