

Entanglement, holography, and strange metals

Rencontre theoricienes, Paris, June 7, 2012

Subir Sachdev

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World, chair D.J. Gross.
arXiv:1203.4565

sachdev.physics.harvard.edu





Liza Huijse



Max Metlitski



Brian Swingle

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement

“Complex entangled” states of quantum matter in d spatial dimensions

Useful classification is provided by nature of excitations with vanishing energy:

1. Gapped systems without zero energy excitations
2. “Relativistic” systems with zero energy excitations at isolated points in momentum space
3. “Compressible” systems with zero energy excitations on $d-1$ dimensional surfaces in momentum space.

“Complex entangled” states of quantum matter in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

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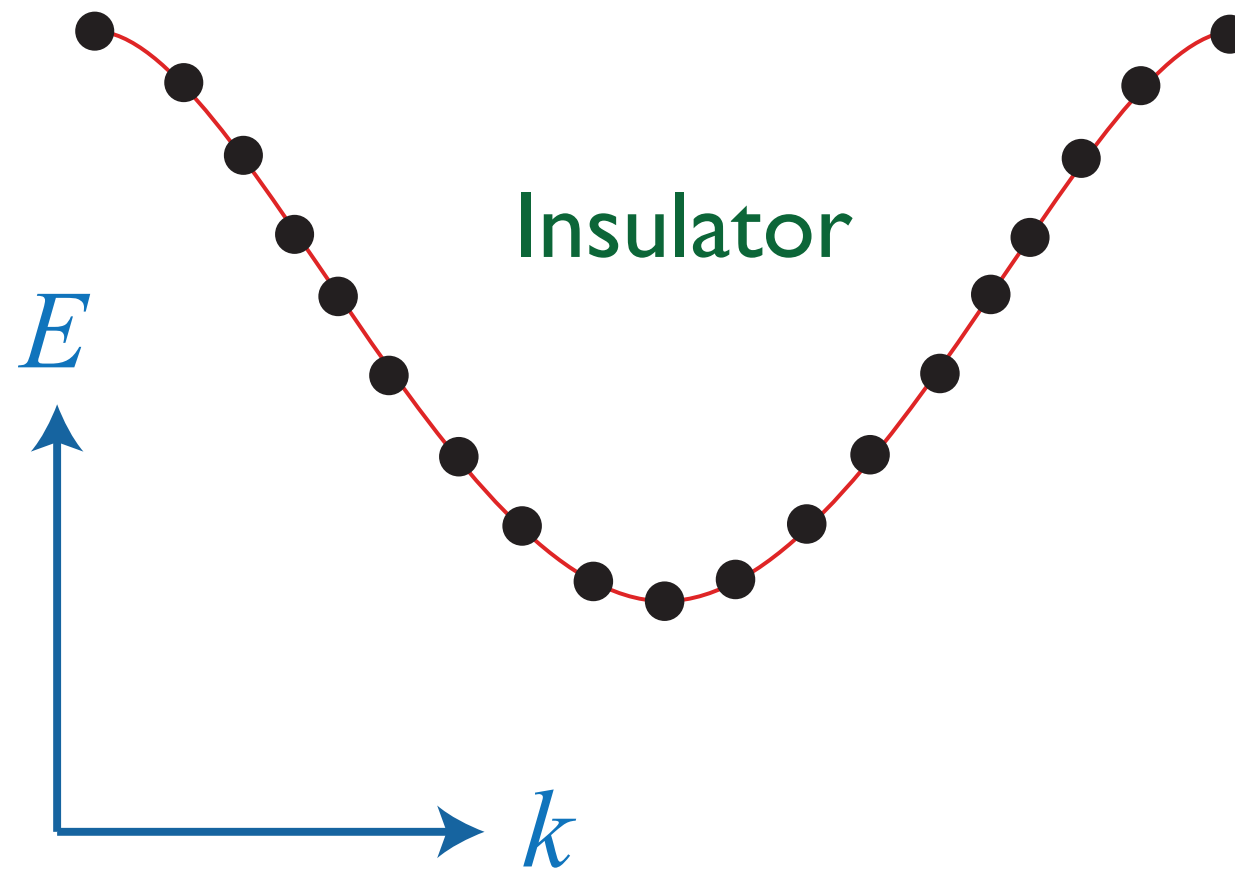
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Band insulators



An even number of electrons per unit cell

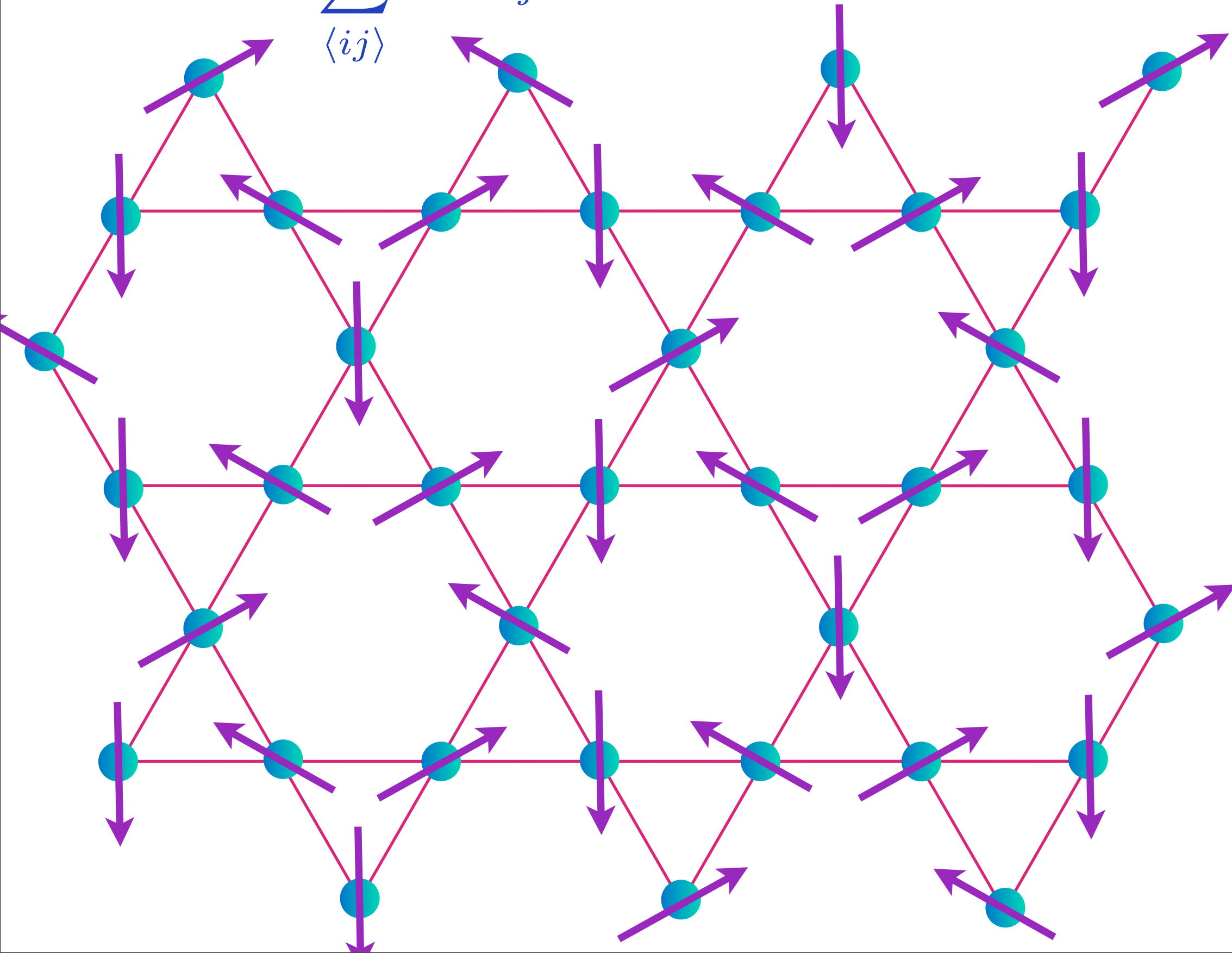
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Kagome antiferromagnet

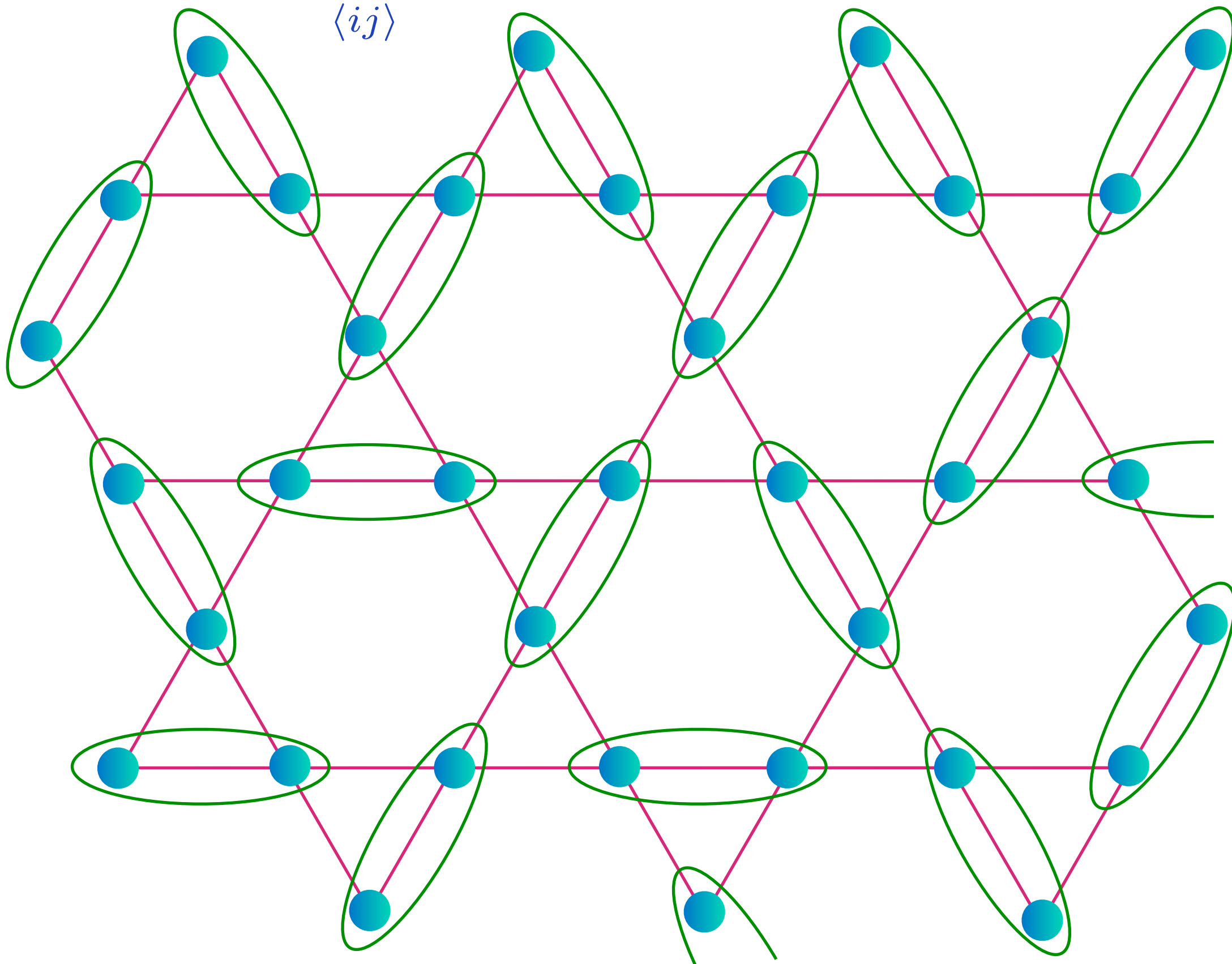
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Mott insulator: Kagome antiferromagnet

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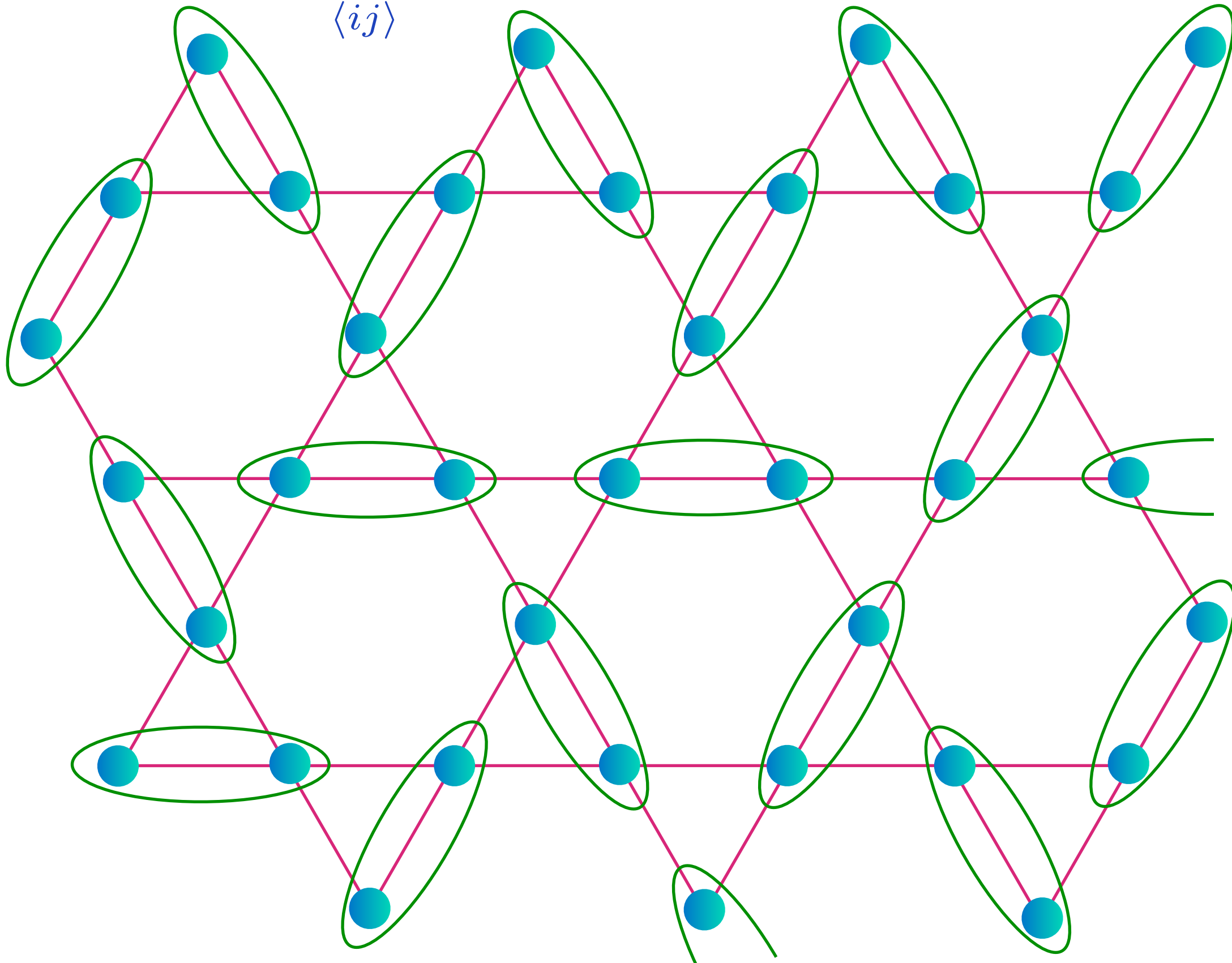


P. Fazekas and
P. W. Anderson,
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30, 23 (1974).

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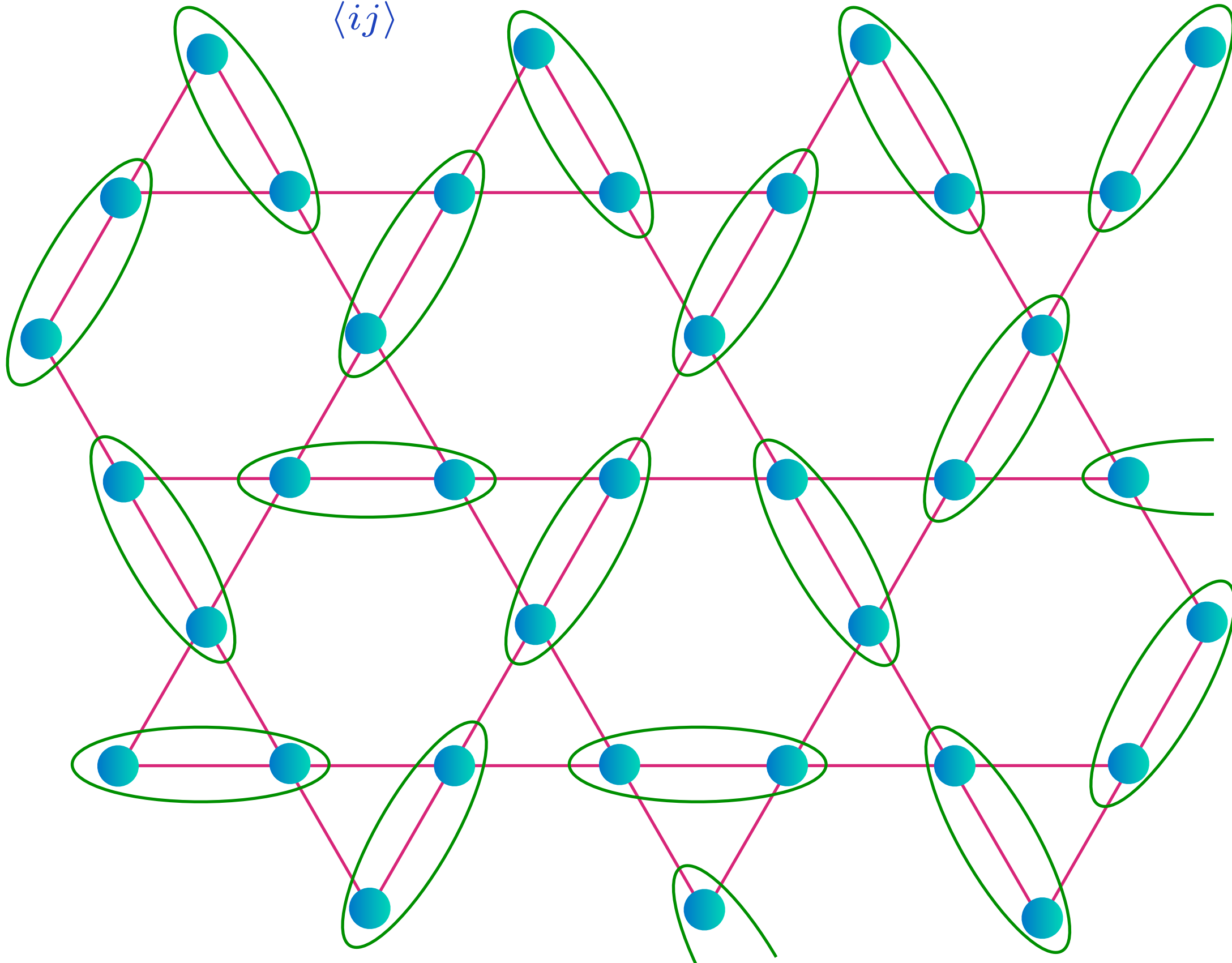


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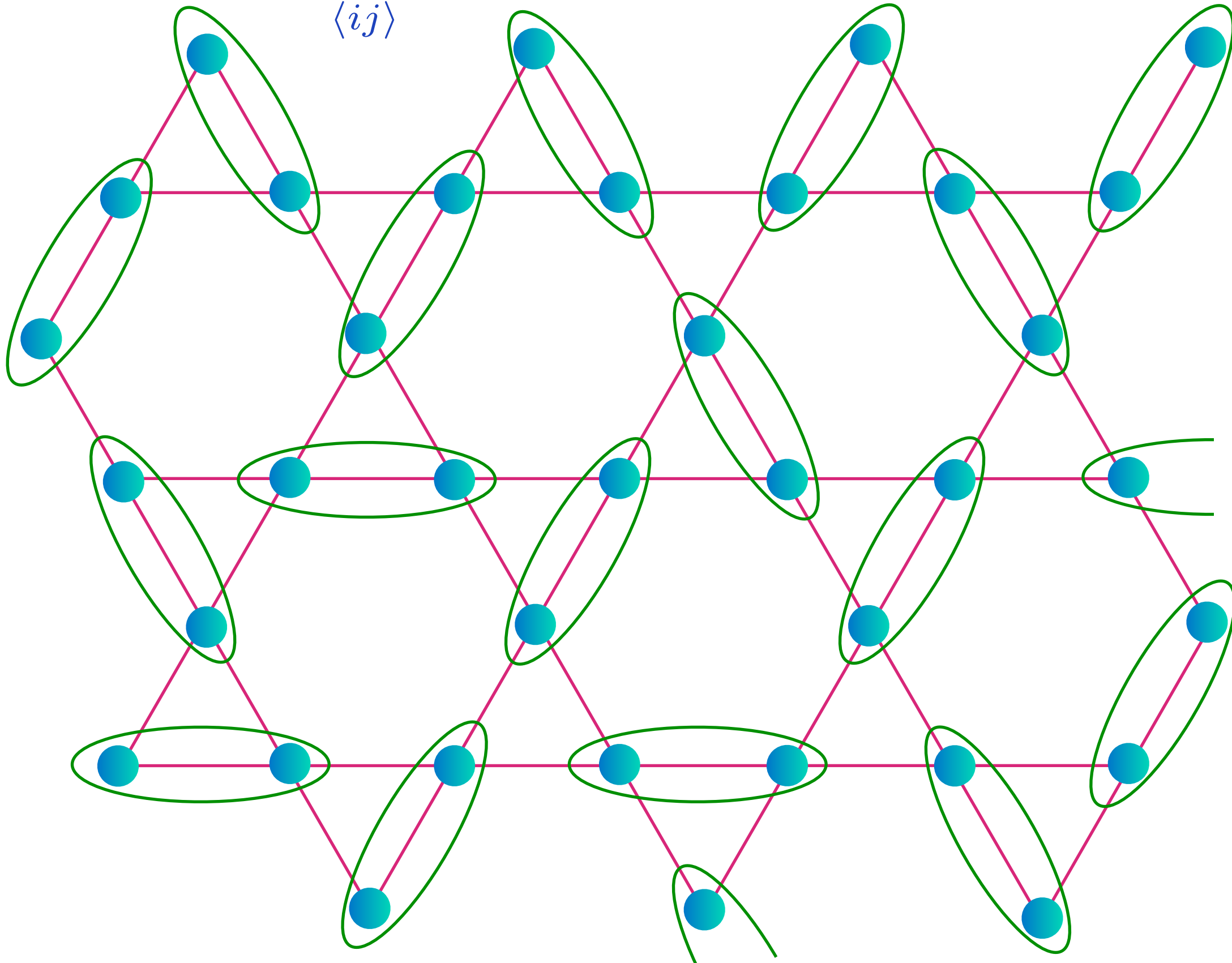


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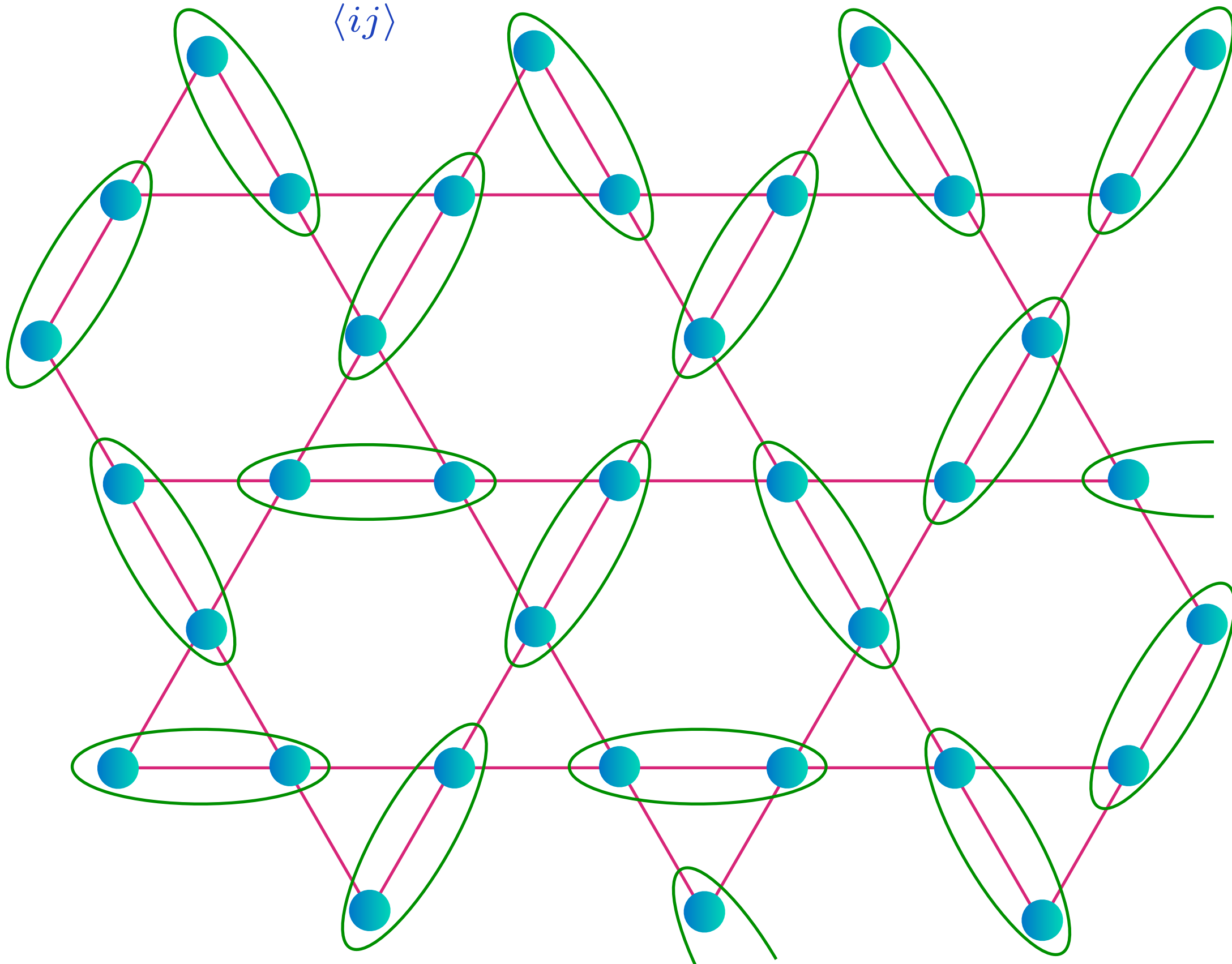


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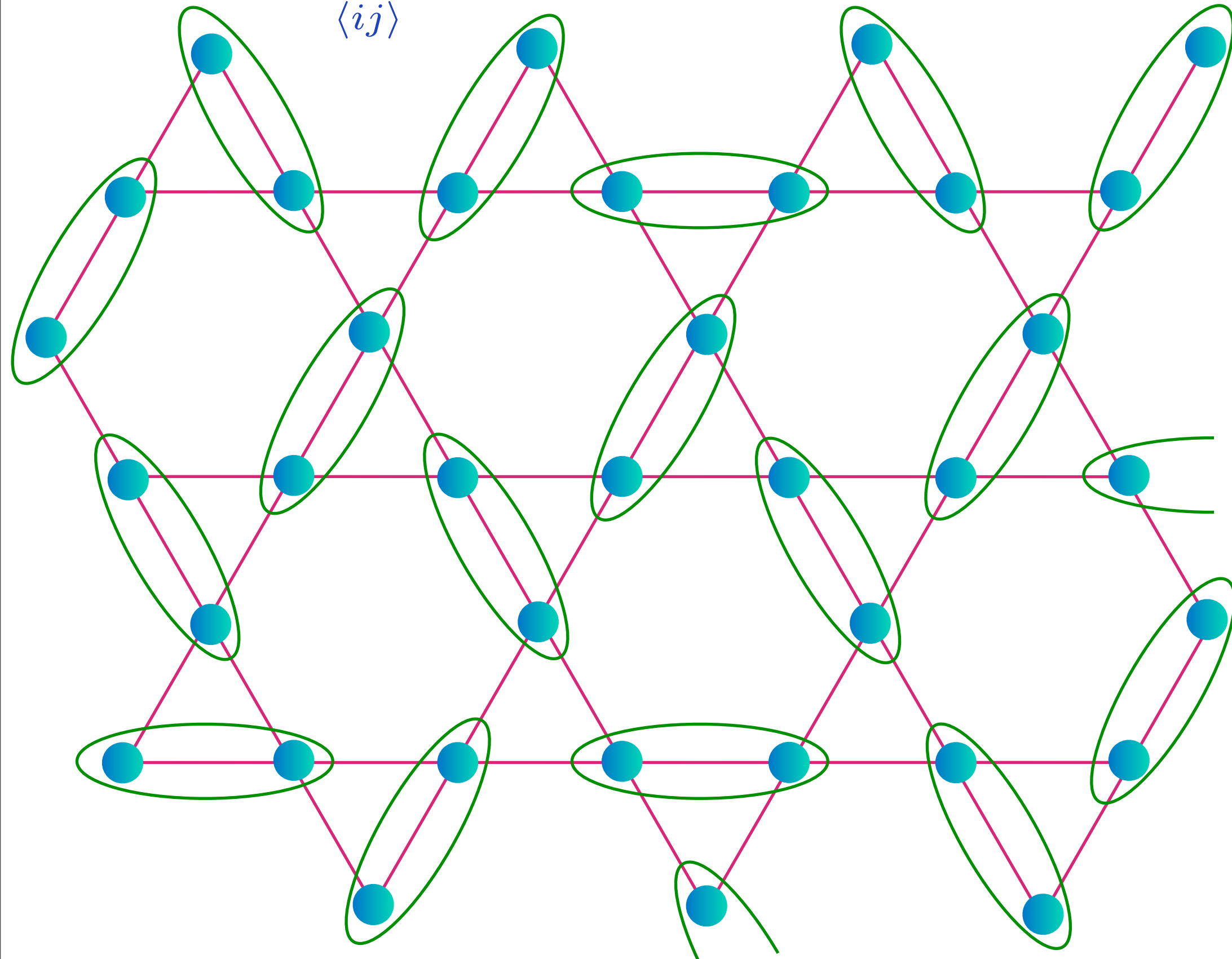


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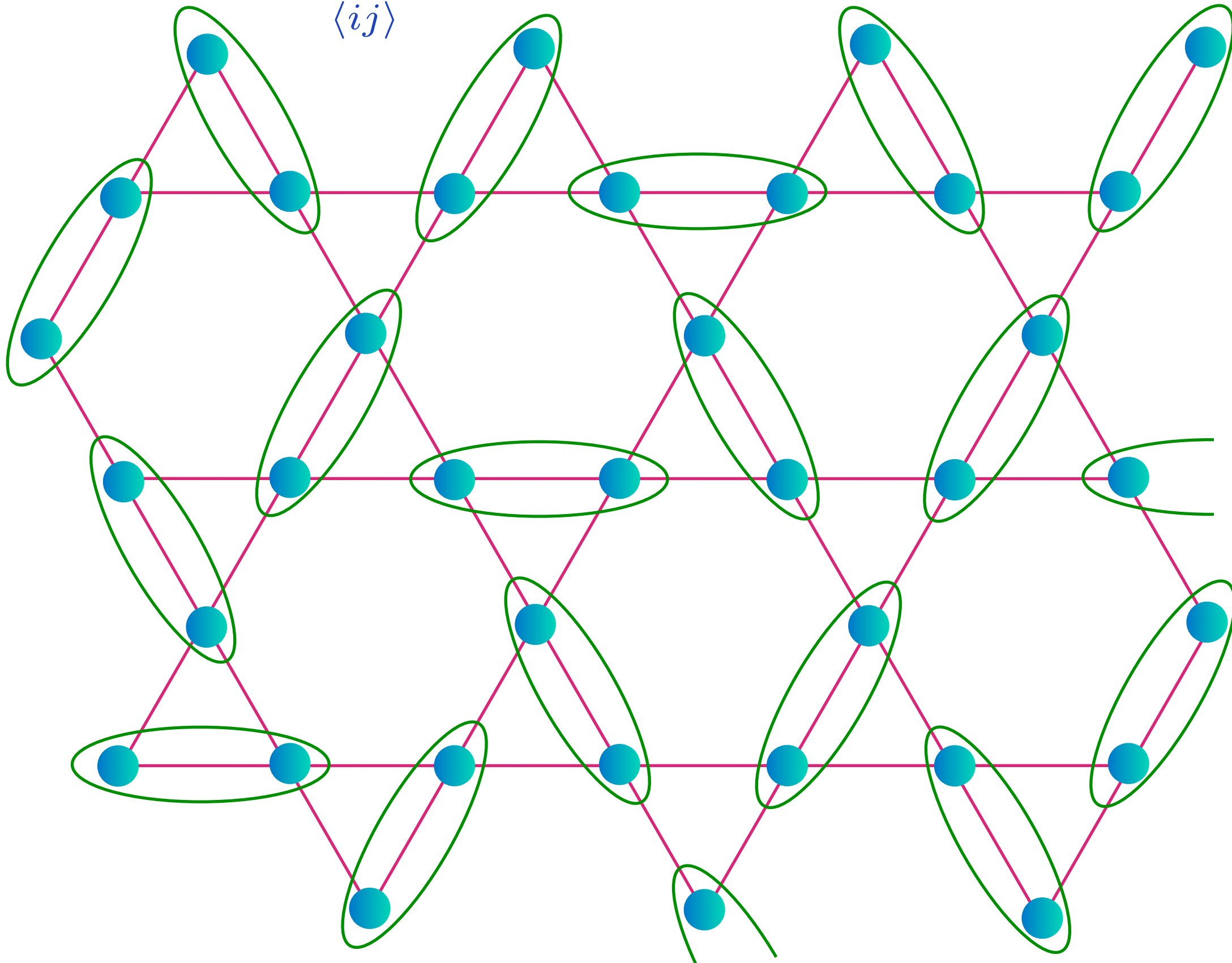


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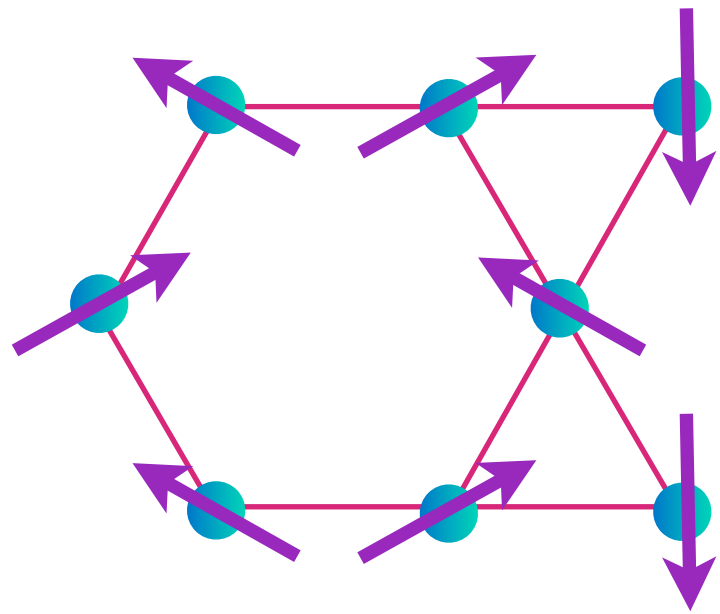
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Mott insulator: kagome antiferromagnet



non-collinear Néel state

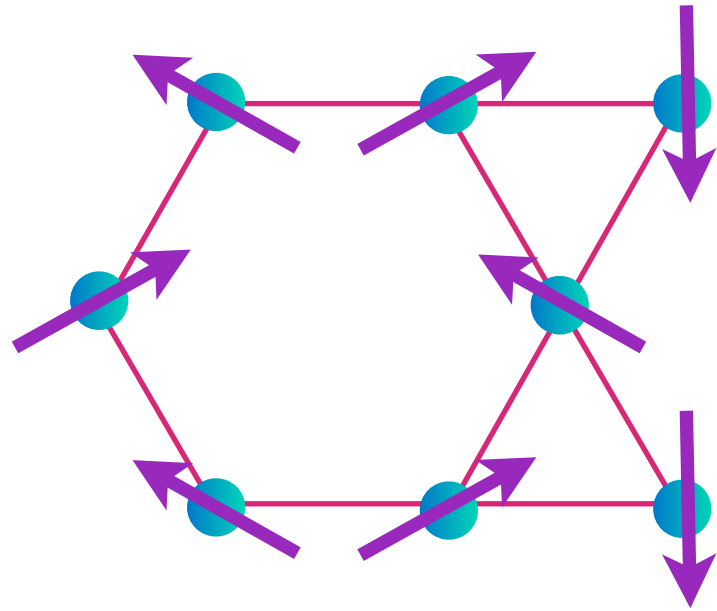
Quantum “disordered” state with exponentially decaying spin correlations.

S_c

S

S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)

Mott insulator: kagome antiferromagnet



non-collinear Néel state

Quantum “disordered” state with exponentially decaying spin correlations.

Spin liquid with topological features described by a \mathbb{Z}_2 gauge theory, or (equivalently) a doubled Chern-Simons field theory.

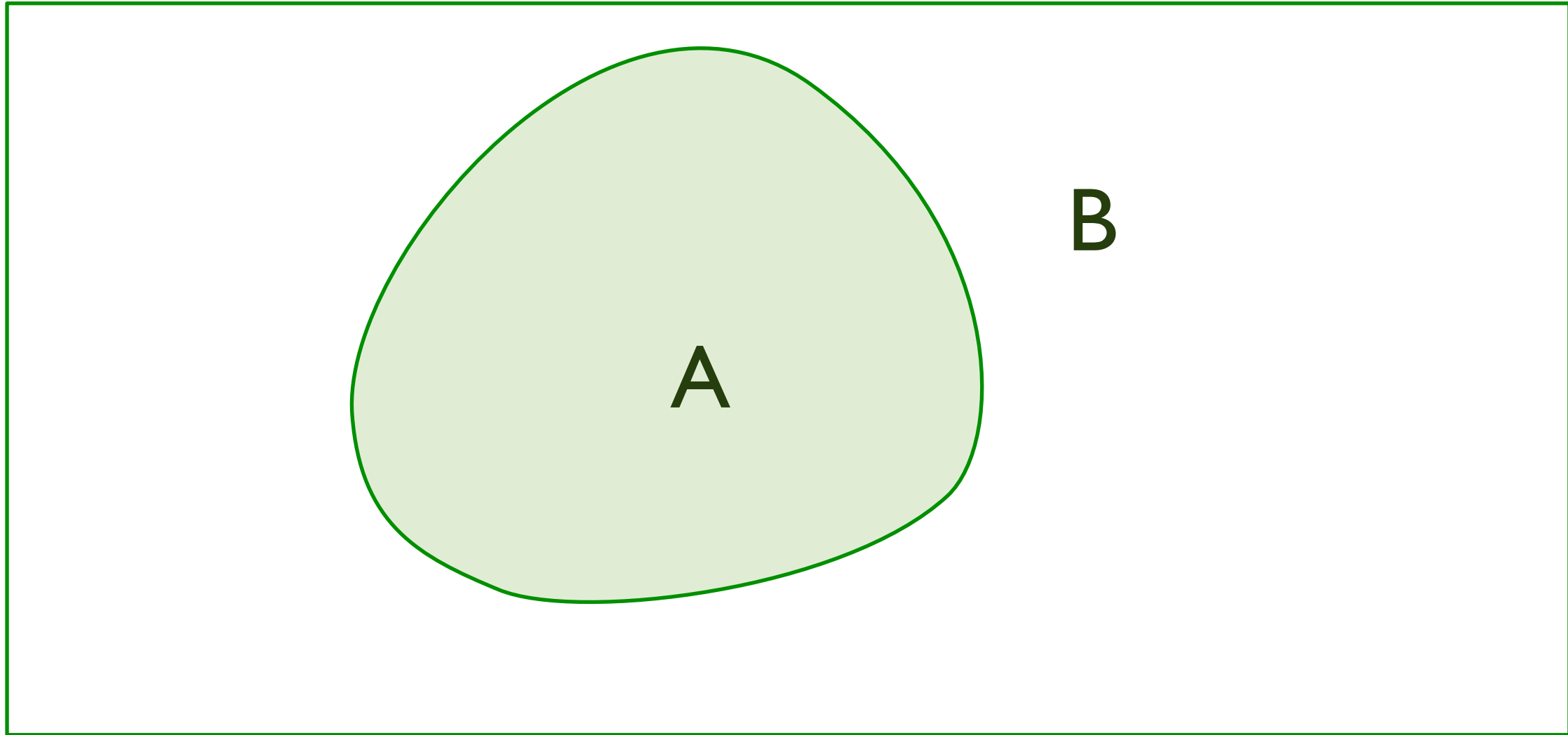
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S

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, *Annals of Physics* **310**, 428 (2004).

Entanglement in the Z_2 spin liquid ground state

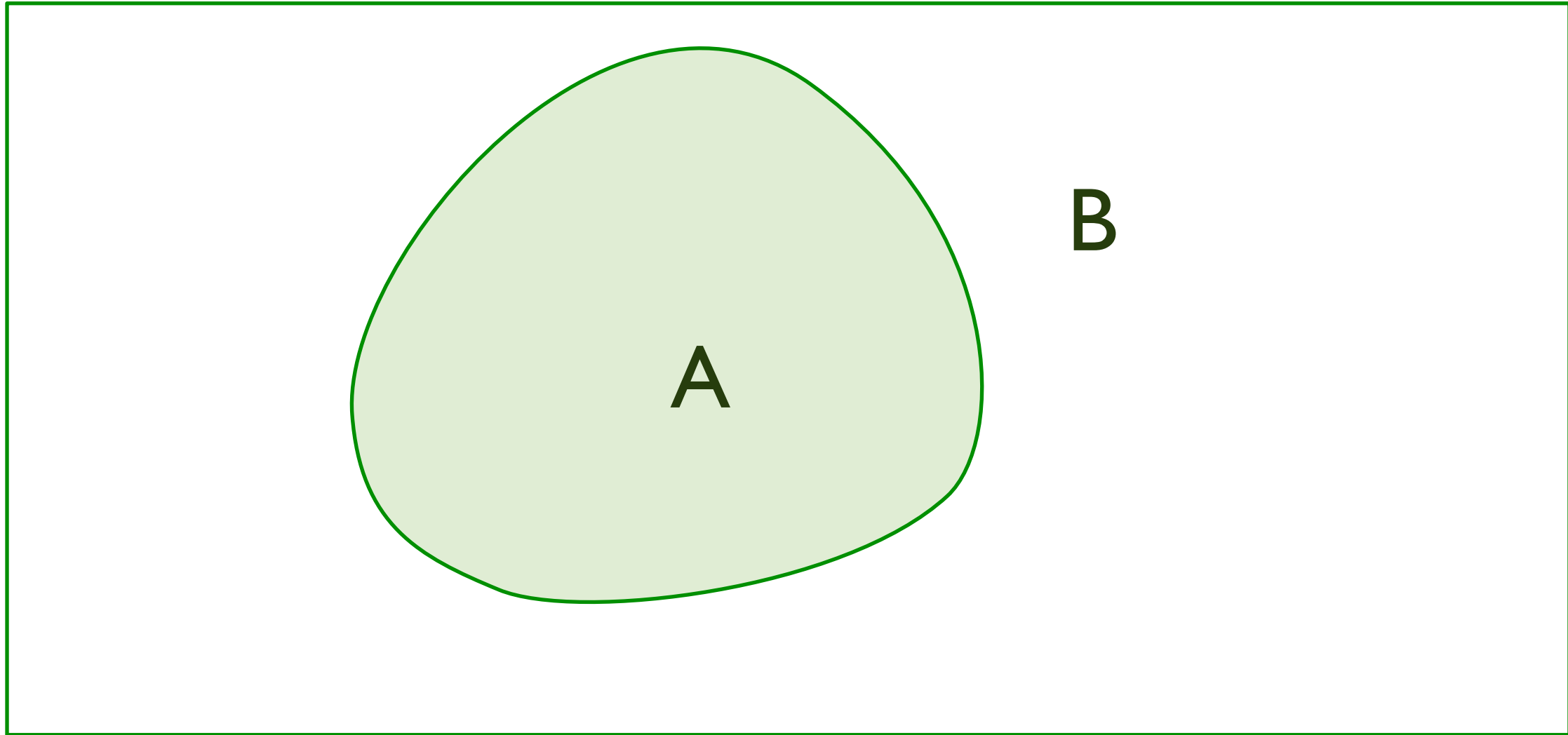


$|\Psi\rangle \Rightarrow$ Ground state of entire system,
 $\rho = |\Psi\rangle\langle\Psi|$

$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_{EE} = -\text{Tr}(\rho_A \ln \rho_A)$

Entanglement in the Z_2 spin liquid ground state

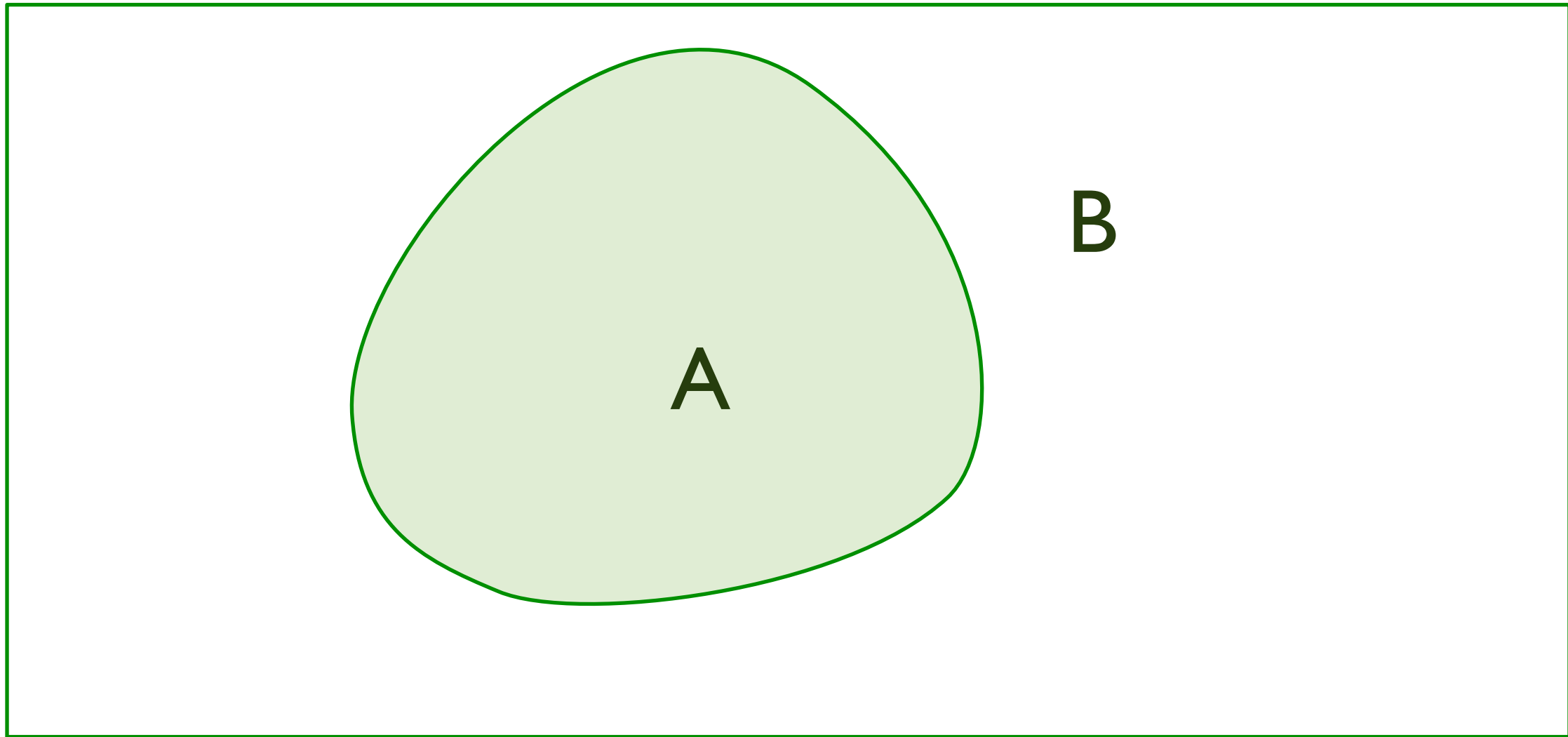


Entanglement entropy of a band insulator:

$$S_E = aP - b \exp(-cP)$$

where P is the surface area (perimeter)
of the boundary between A and B.

Entanglement in the \mathbb{Z}_2 spin liquid ground state

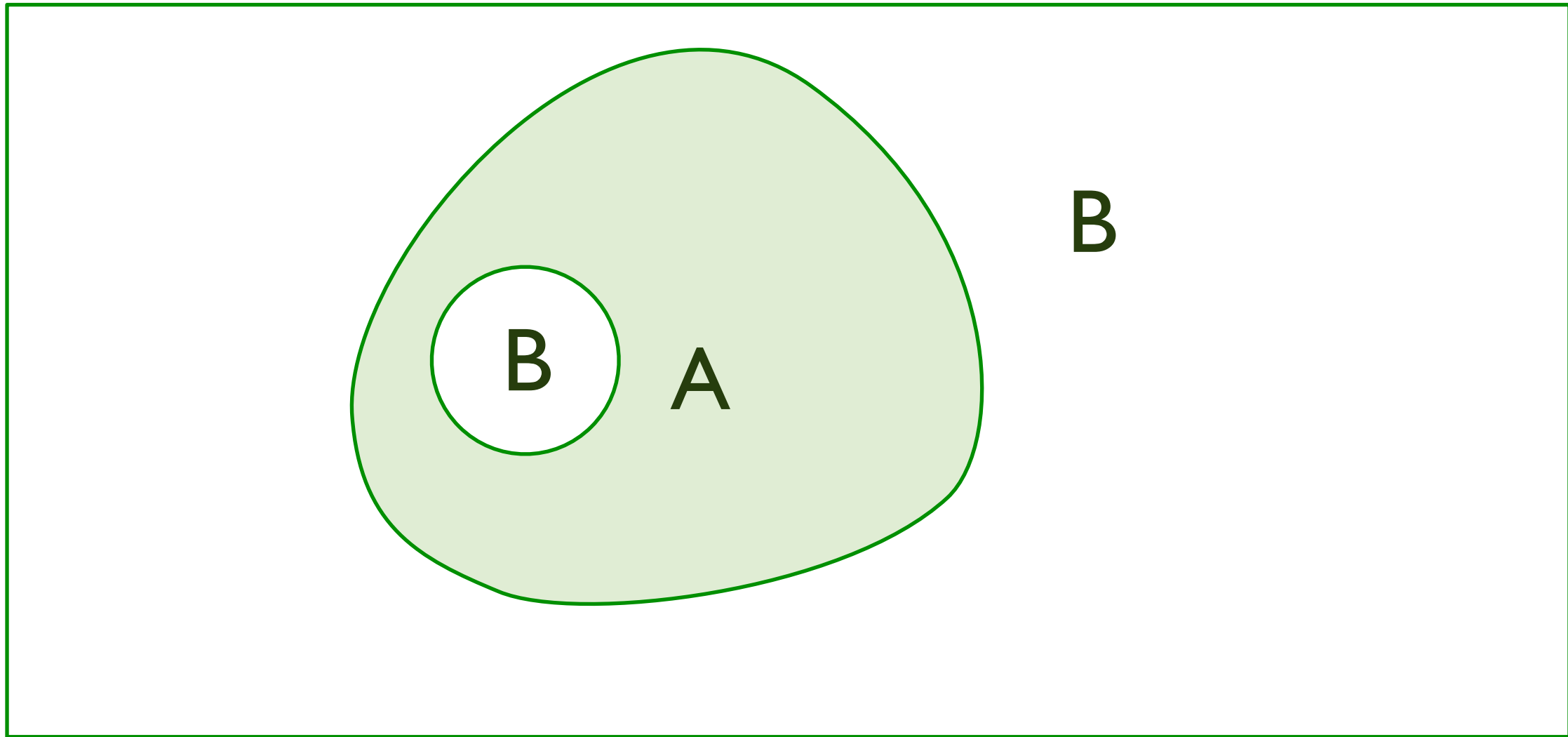


Entanglement entropy of a \mathbb{Z}_2 spin liquid:

$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter)
of the boundary between A and B.

Entanglement in the \mathbb{Z}_2 spin liquid ground state

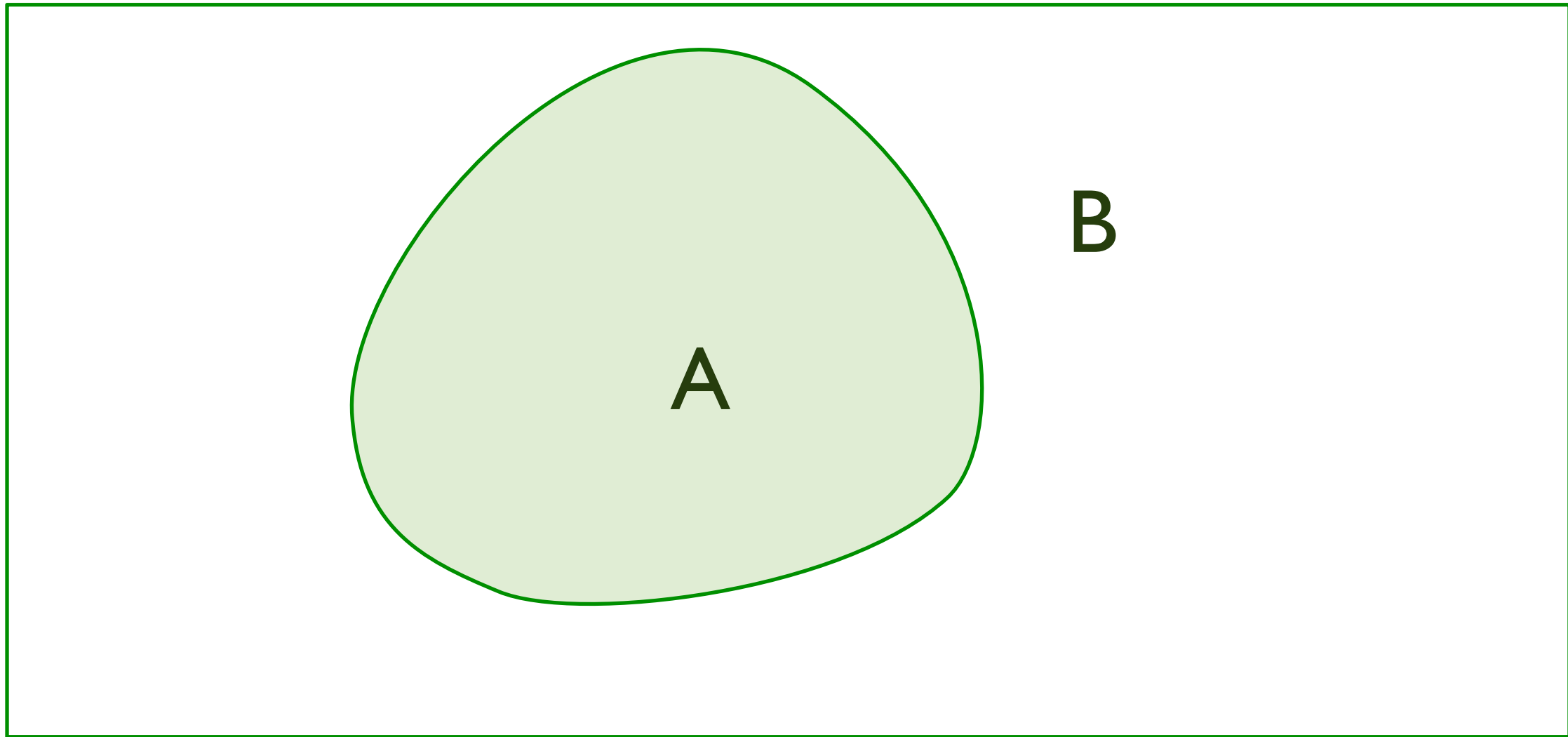


Entanglement entropy of a \mathbb{Z}_2 spin liquid:

$$S_E = aP - \ln(4)$$

where P is the surface area (perimeter) of the boundary between A and B.

Entanglement in the \mathbb{Z}_2 spin liquid ground state

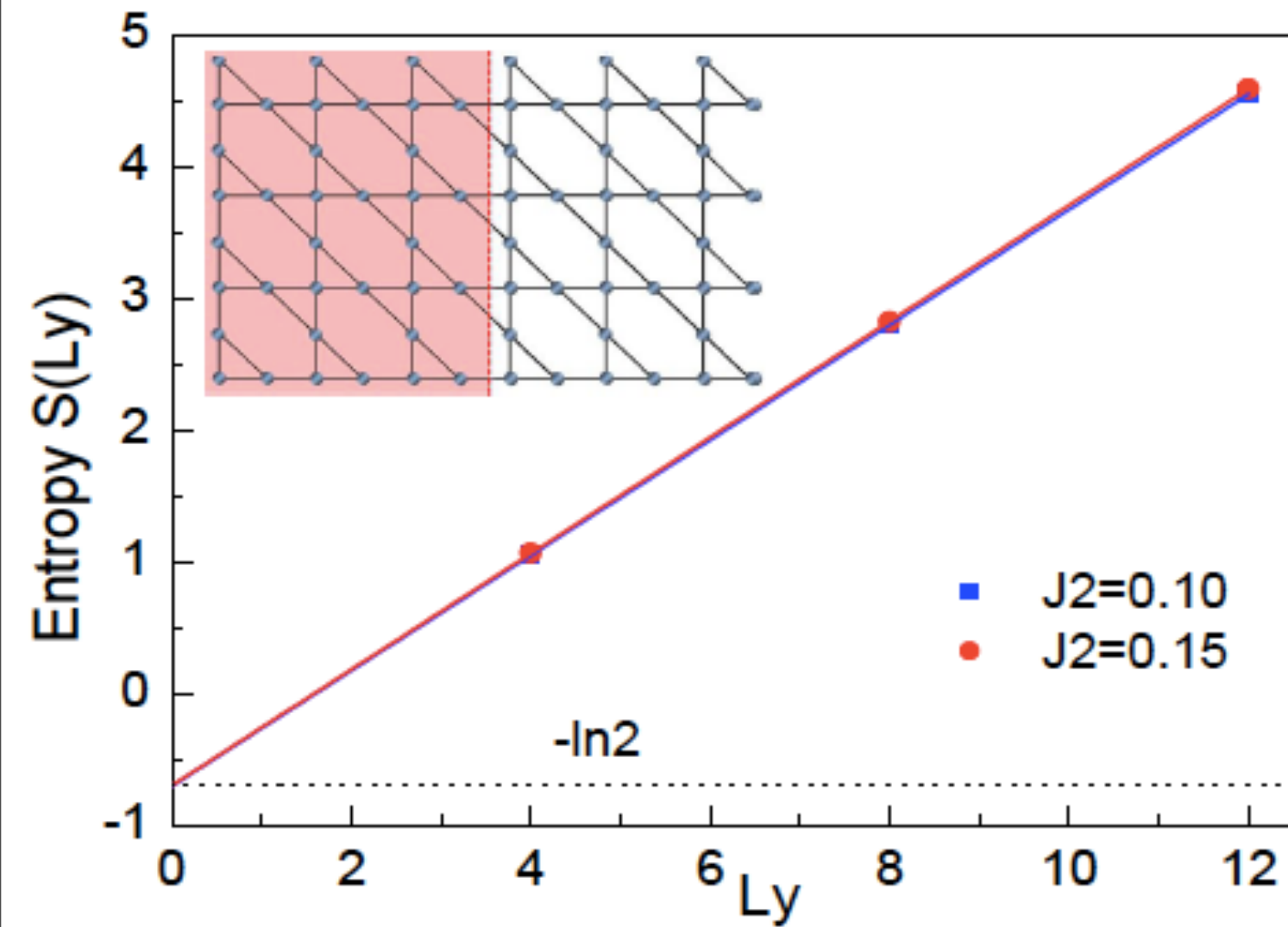


Entanglement entropy of a \mathbb{Z}_2 spin liquid:

$$S_E = aP - \ln(2)$$

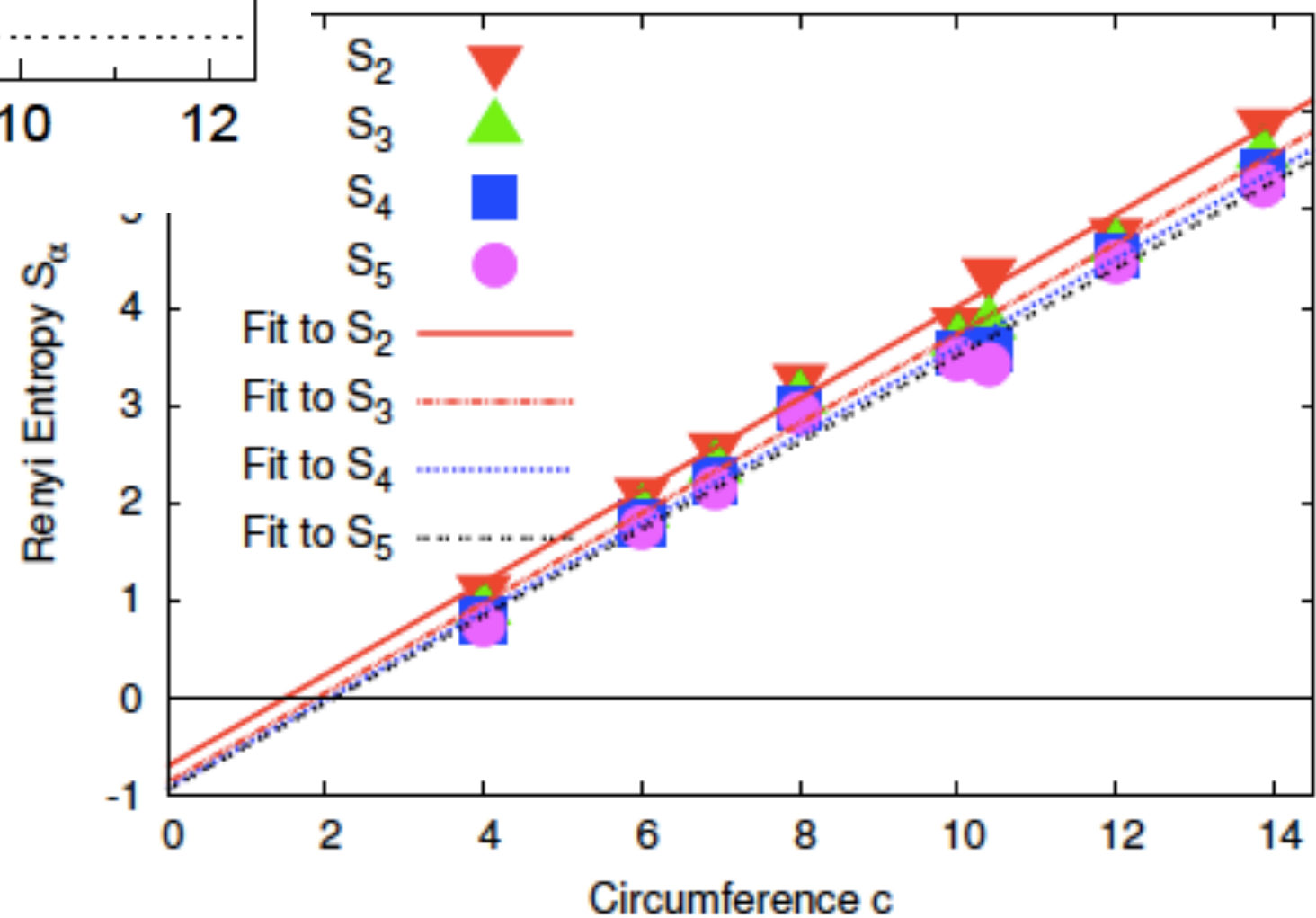
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of the boundary between A and B.

Kagome antiferromagnet



S. Depenbrock,
I. P. McCulloch,
and
U. Schollwoeck,
arXiv:1205.4858

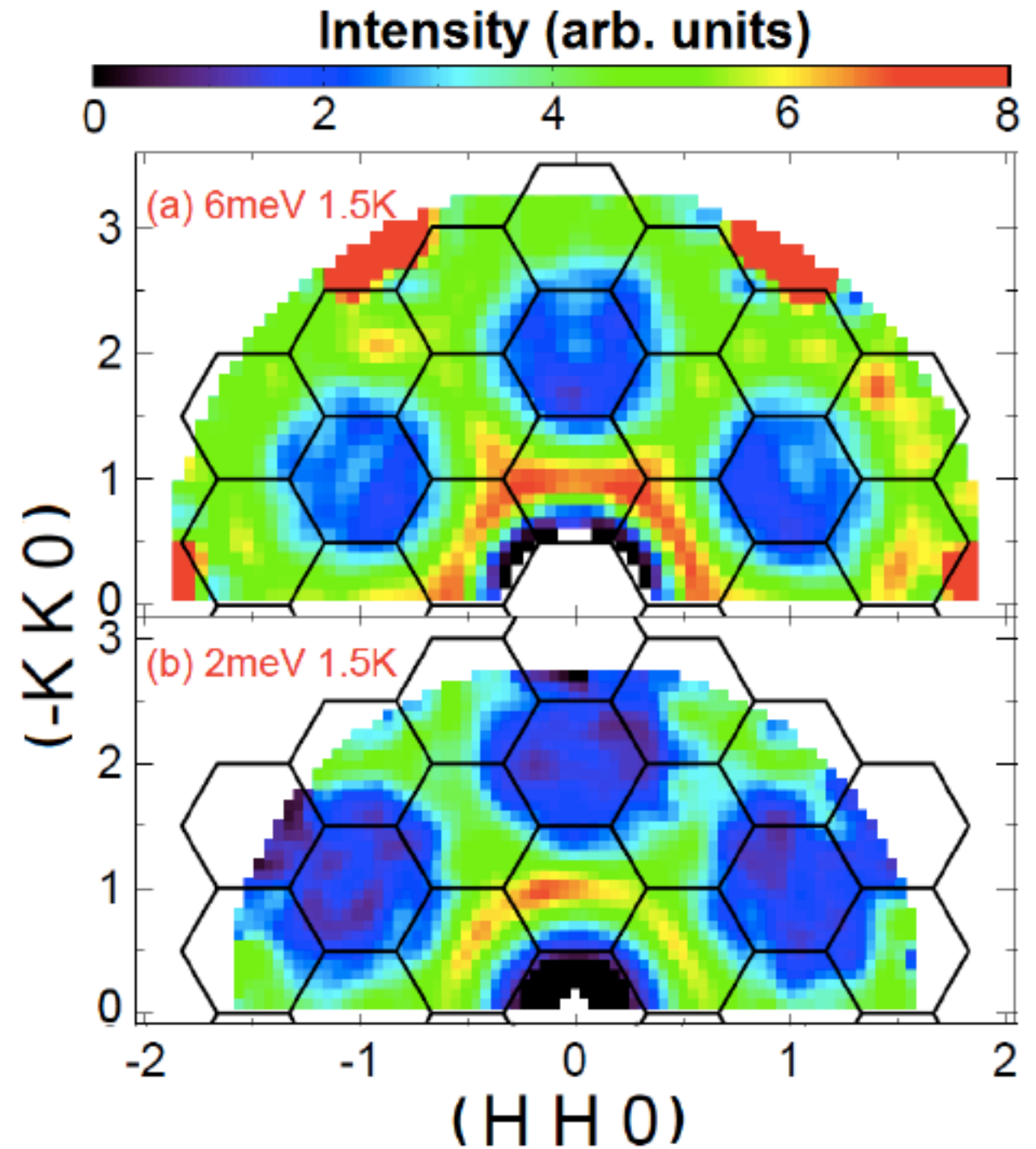
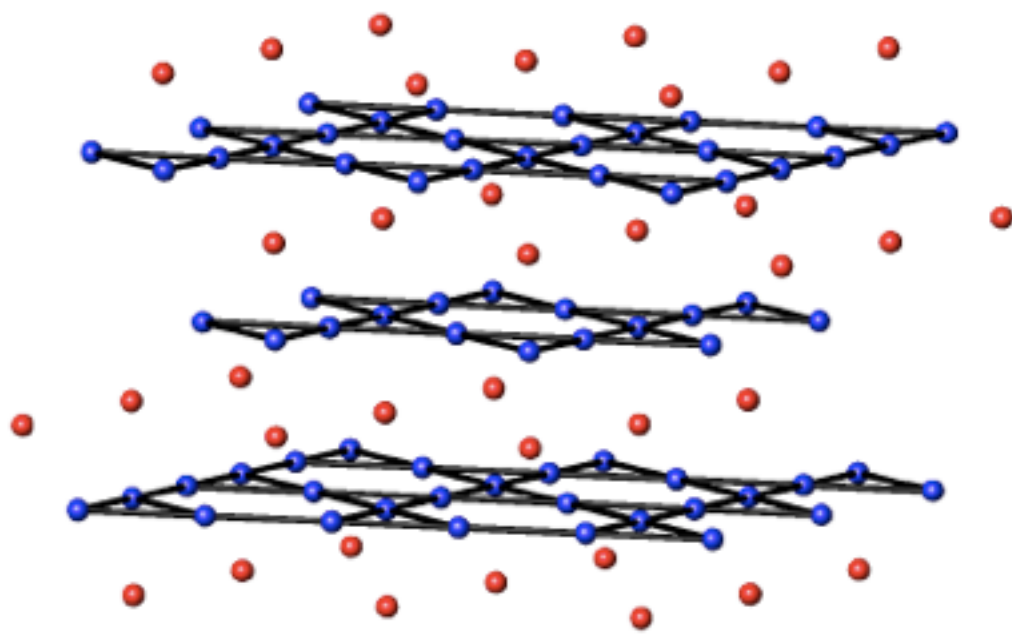
Hong-Chen Jiang,
Z. Wang,
and L. Balents,
arXiv:1205.4289



Kagome antiferromagnet: evidence for spinons

Young Lee,
APS meeting, March 2012

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$ (also called Herbertsmithite)



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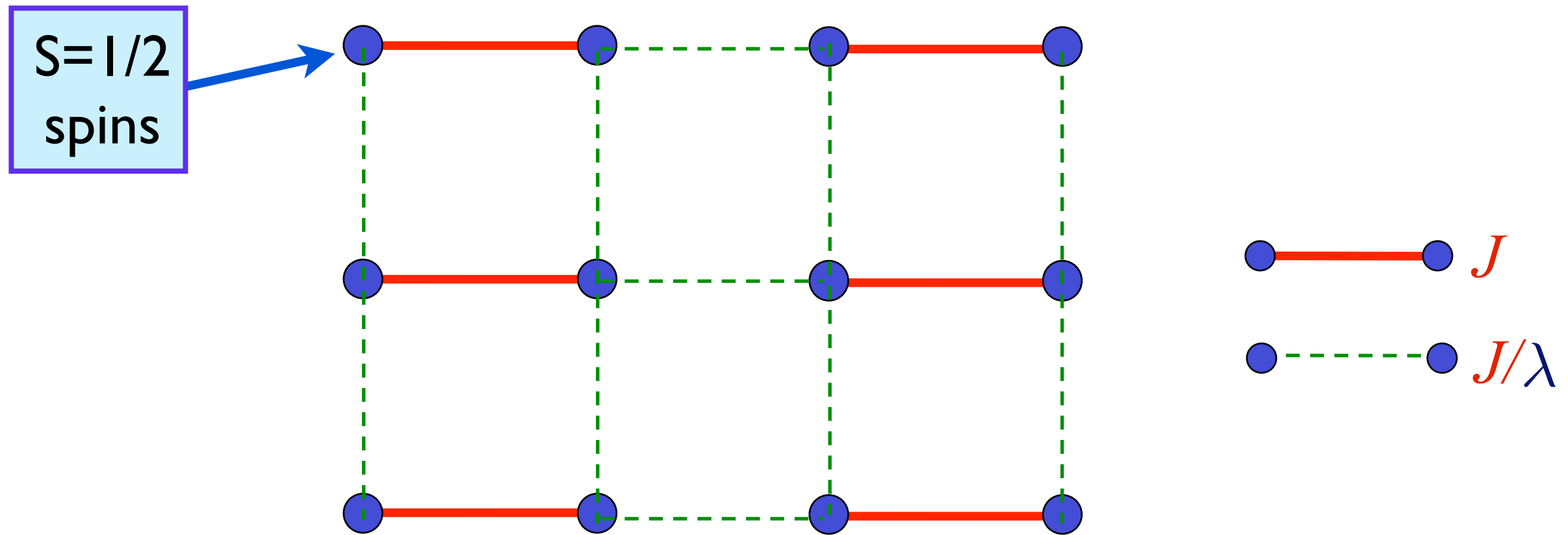
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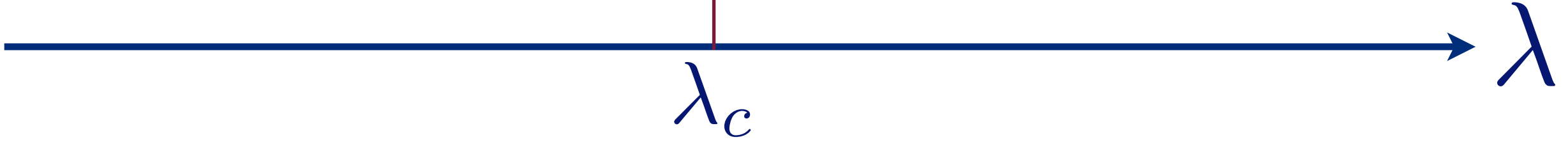
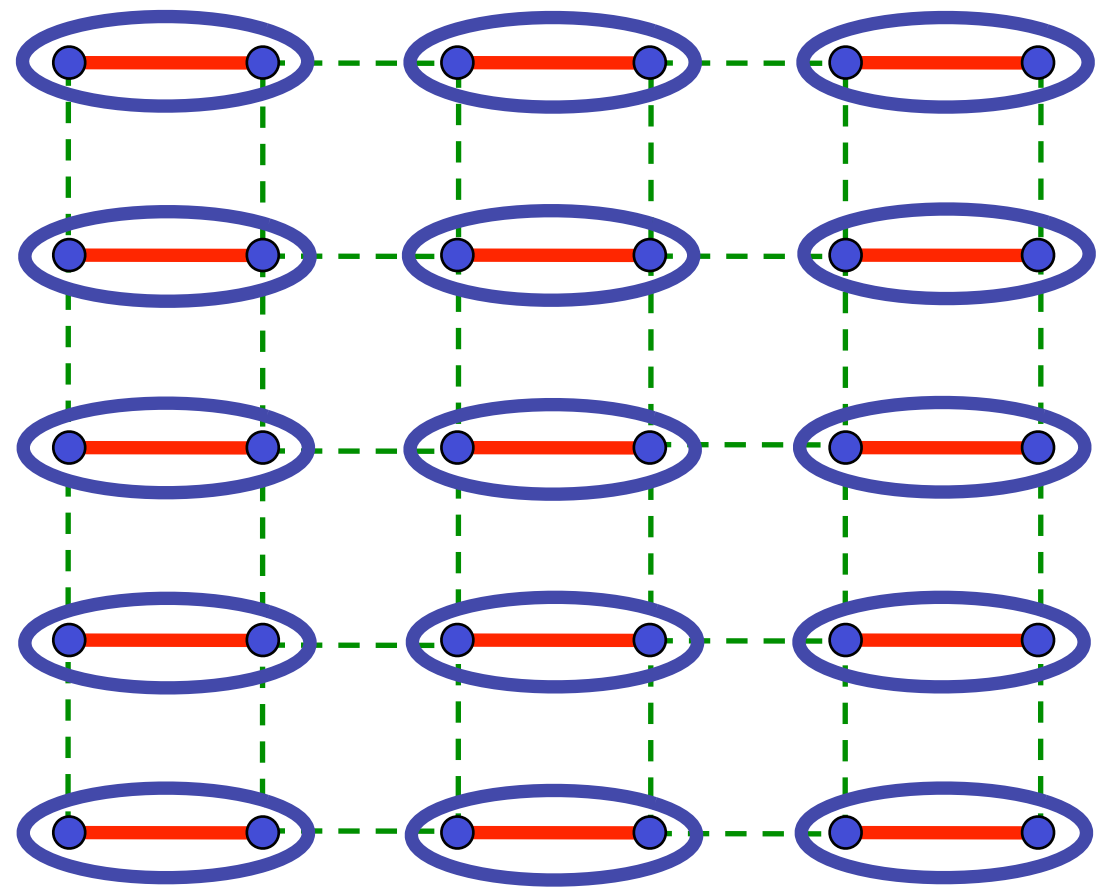
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

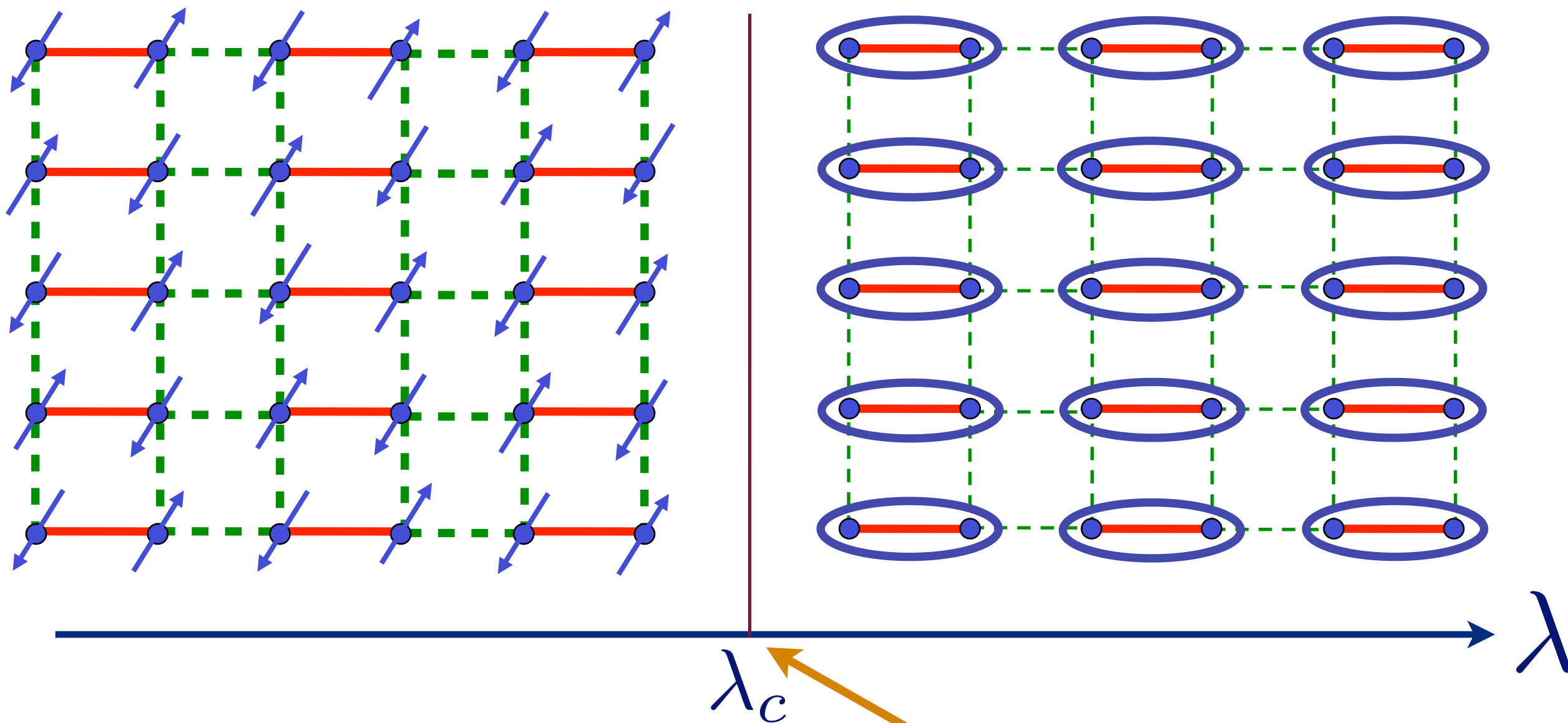


Examine ground state as a function of λ

$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

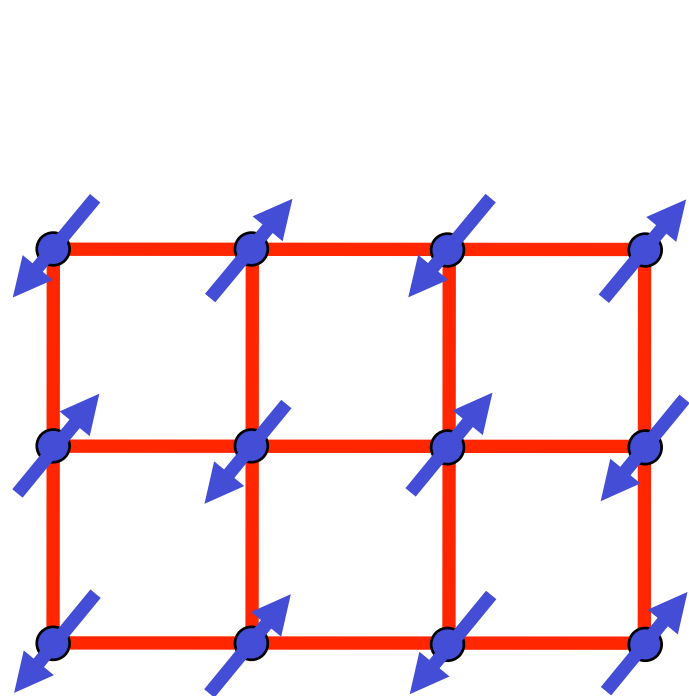


$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

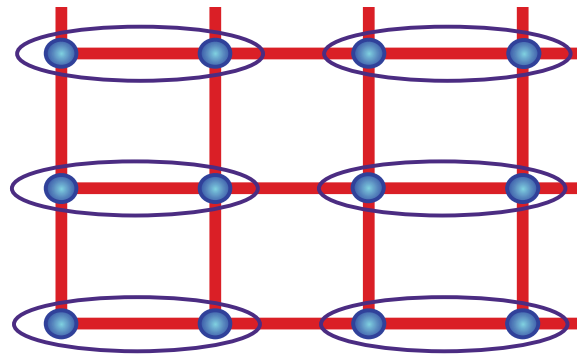


Quantum critical point described by
a CFT3 (O(3) Wilson-Fisher)

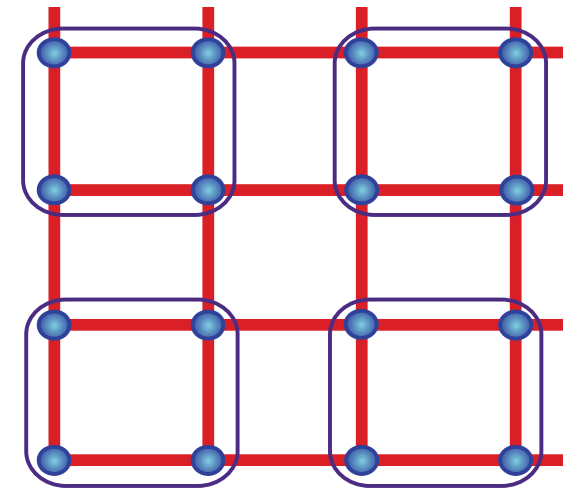
Quantum critical point in a frustrated square lattice antiferromagnet



Néel state



or



Valence bond solid (VBS) state
with a nearly gapless, emergent “photon”

s_c

s

Critical theory for photons and deconfined spinons:

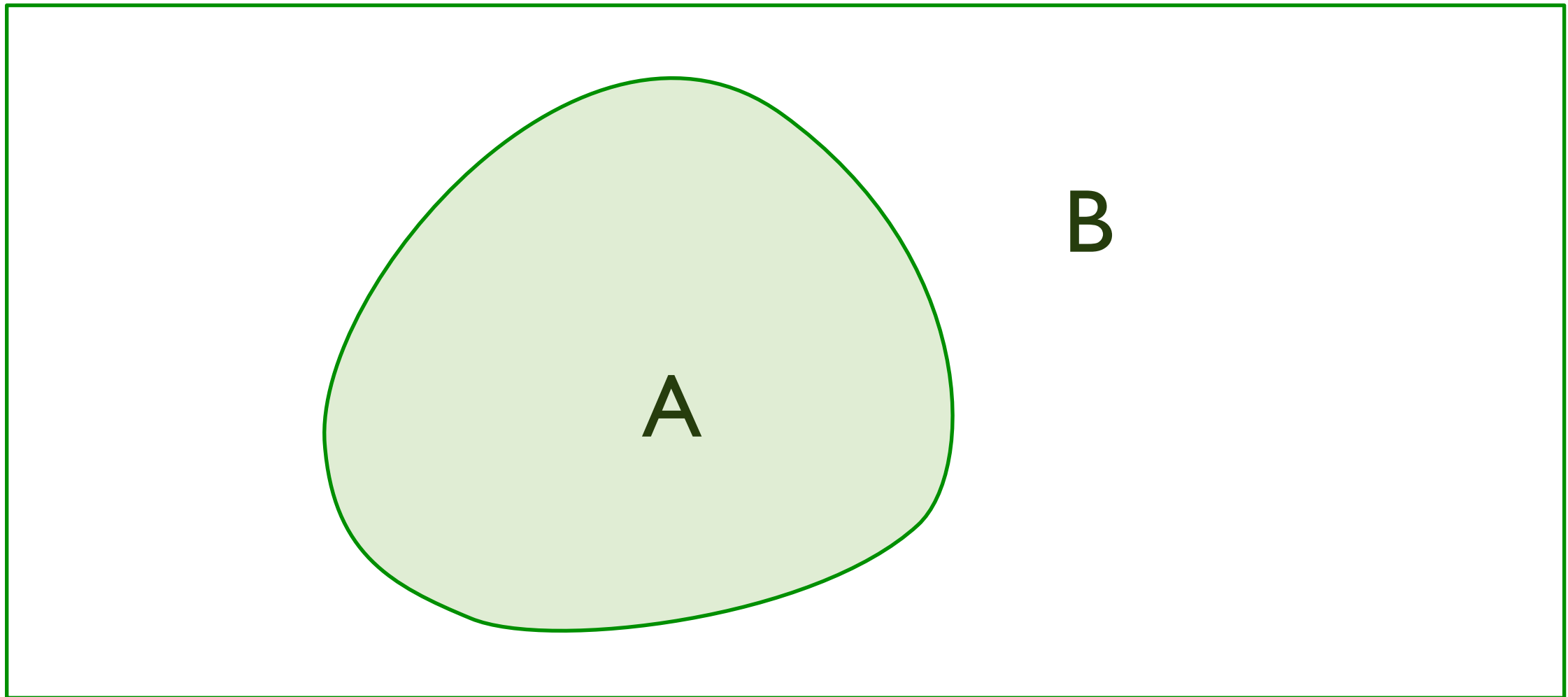
$$\mathcal{S}_z = \int d^2r d\tau \left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aL - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.

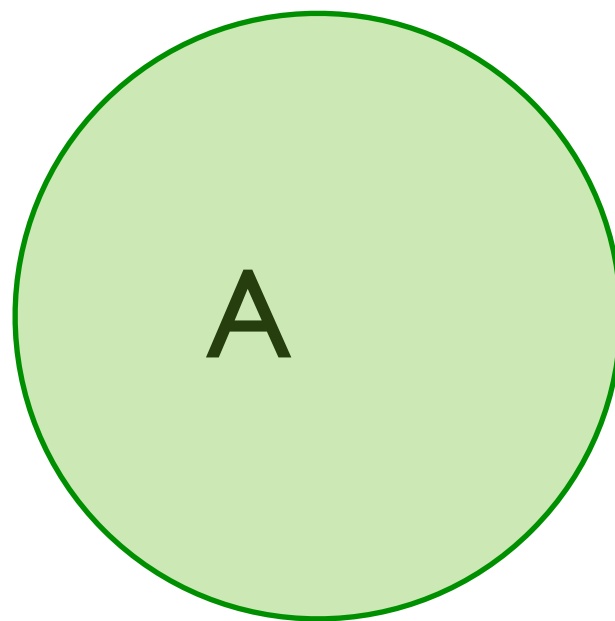


M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

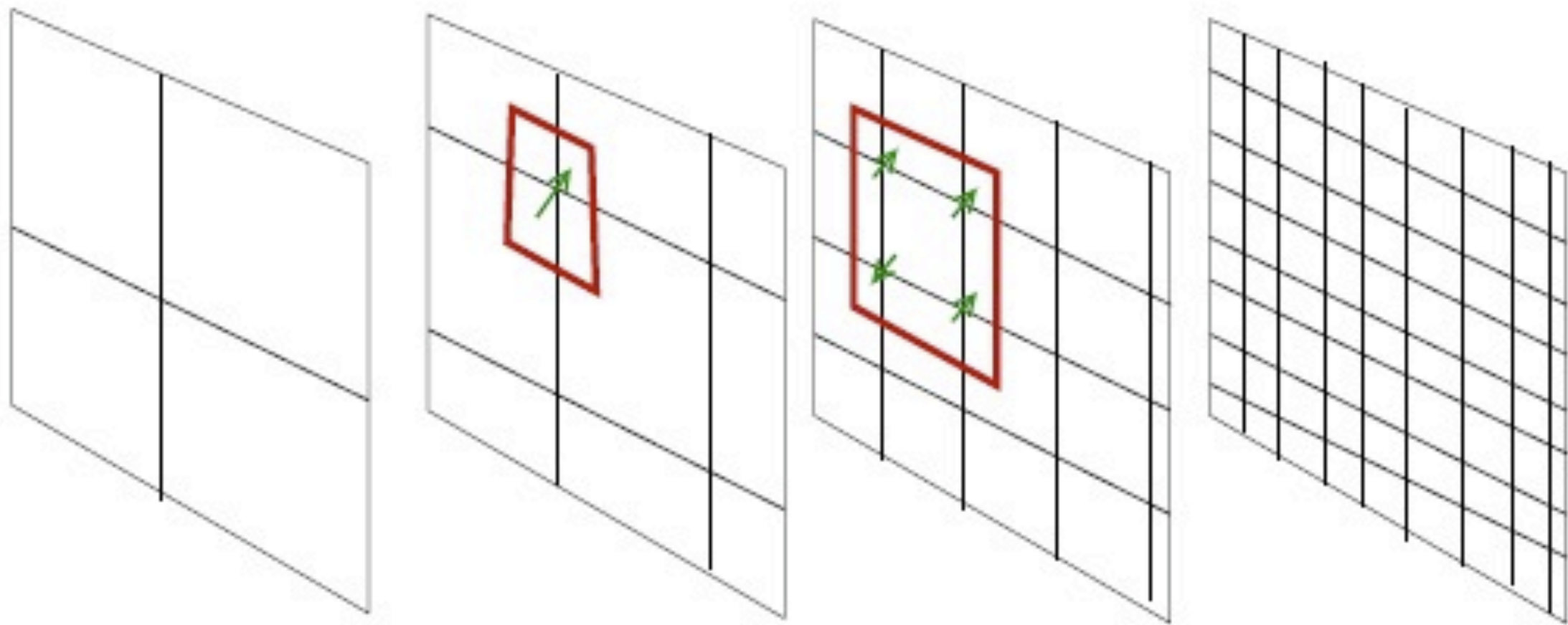
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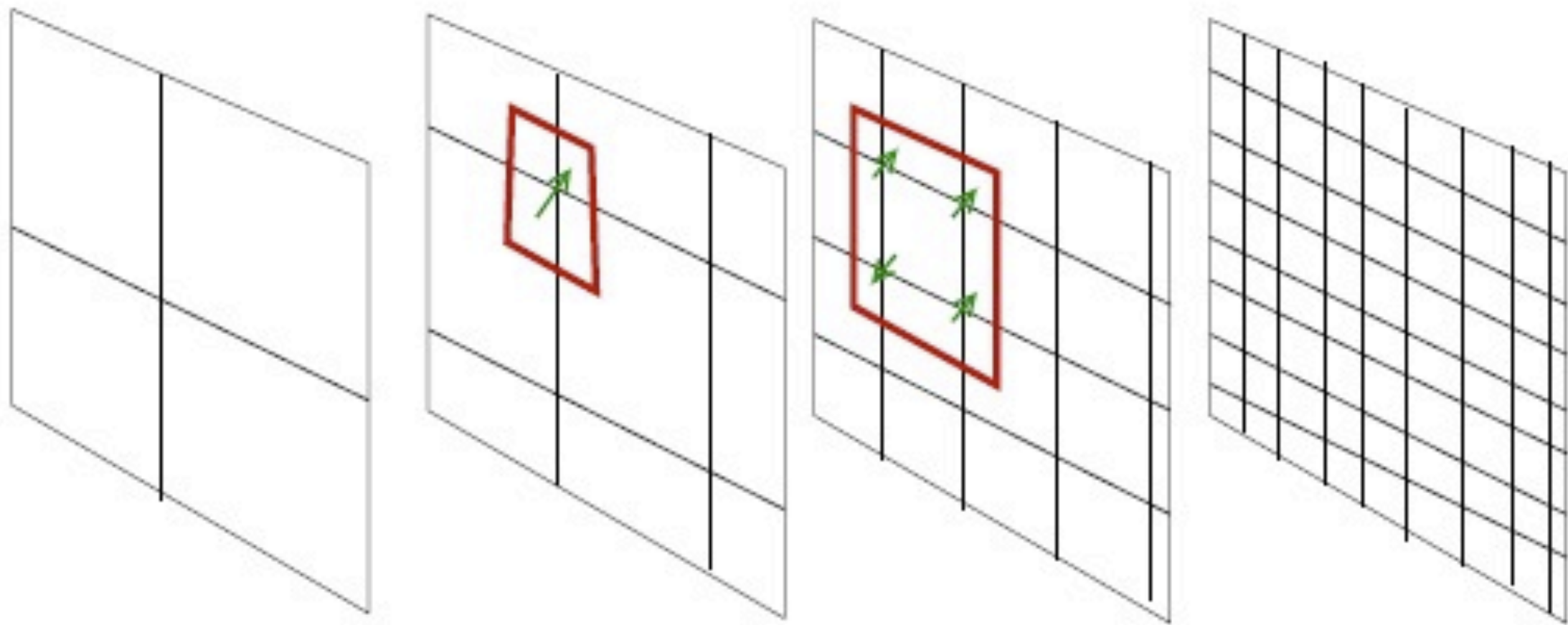
- When A is a circle, $e^{-\gamma} =$ partition function of CFT3 on S^3 .



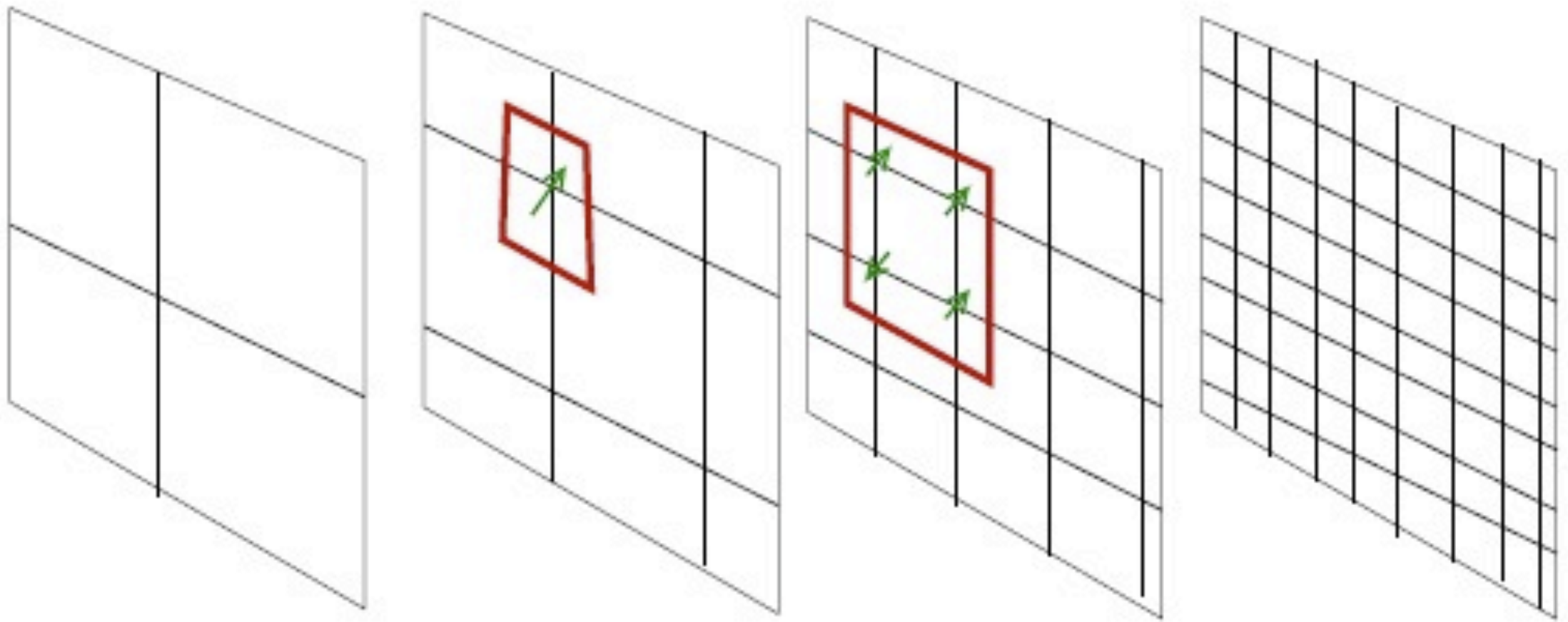
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r ←



r ←



r ←

Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

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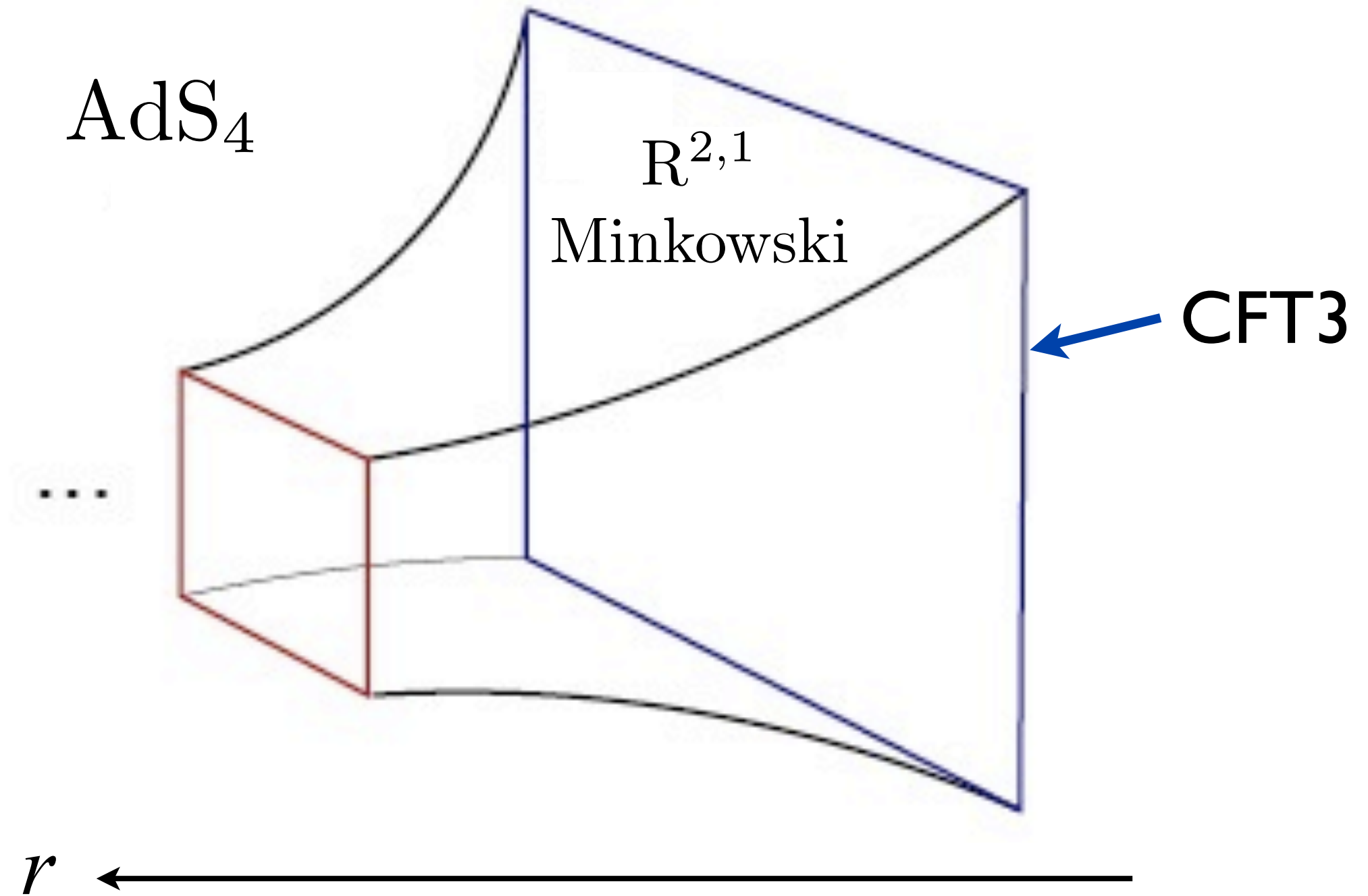
$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

This gives the unique metric

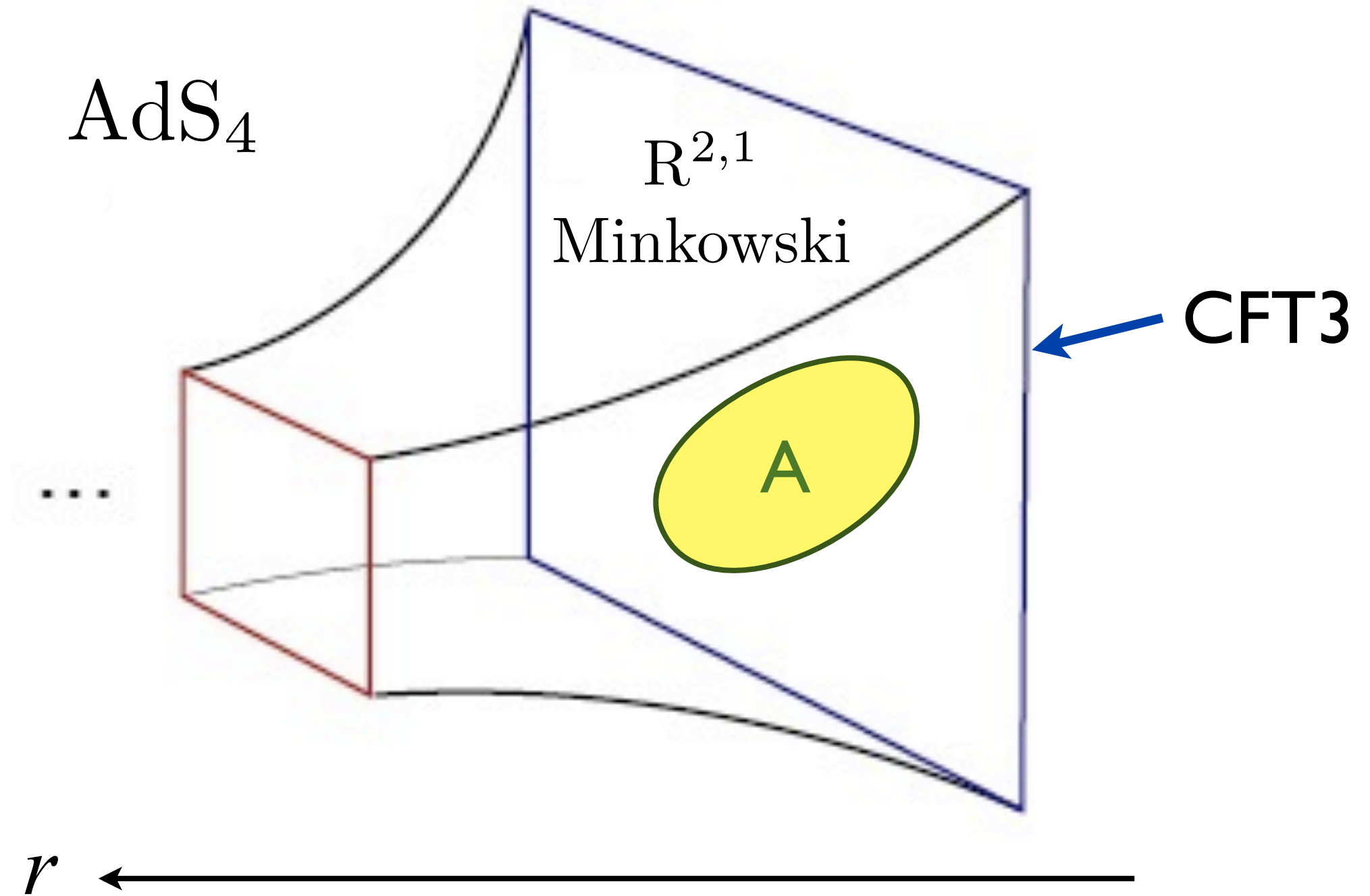
$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2} .

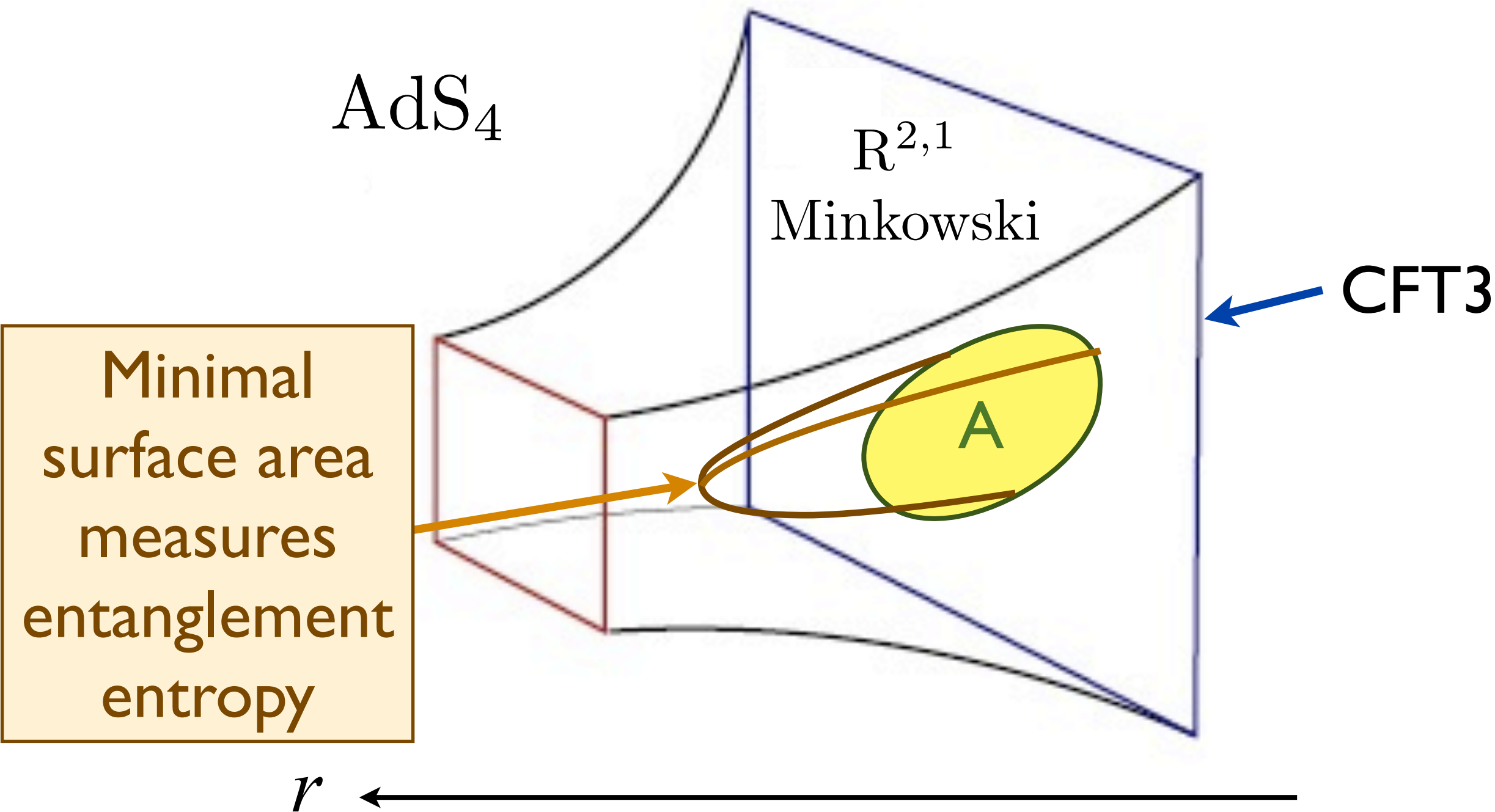
AdS/CFT correspondence



AdS/CFT correspondence



AdS/CFT correspondence



- Minimal surface area yields $S_E = aL - \gamma$, where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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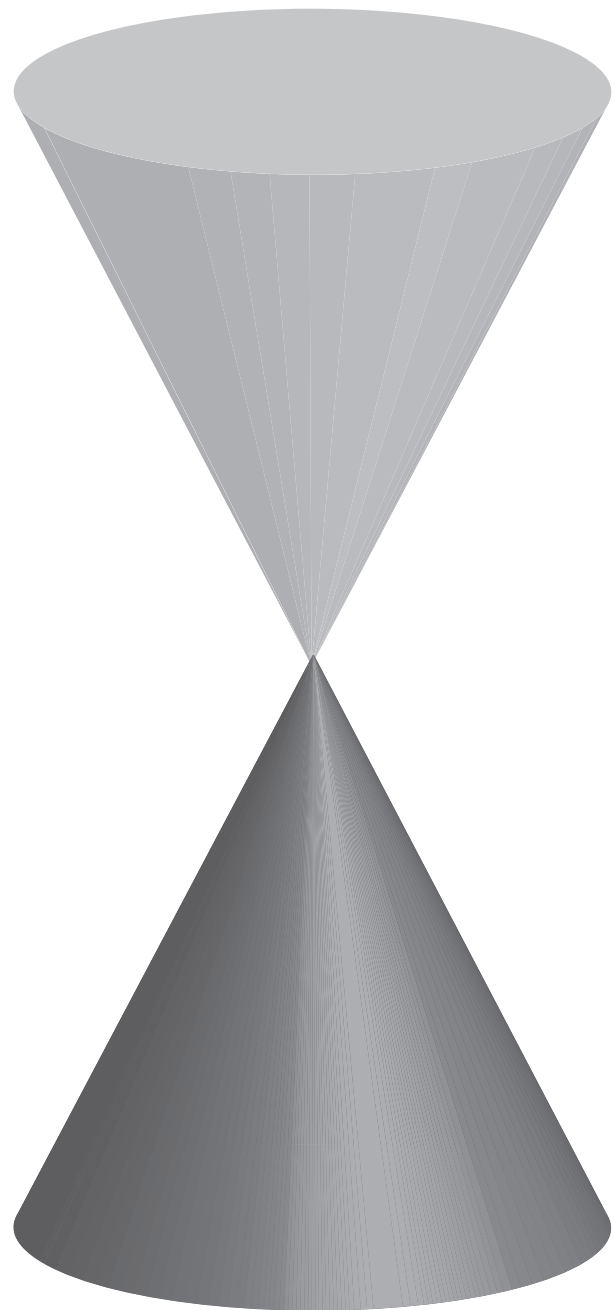
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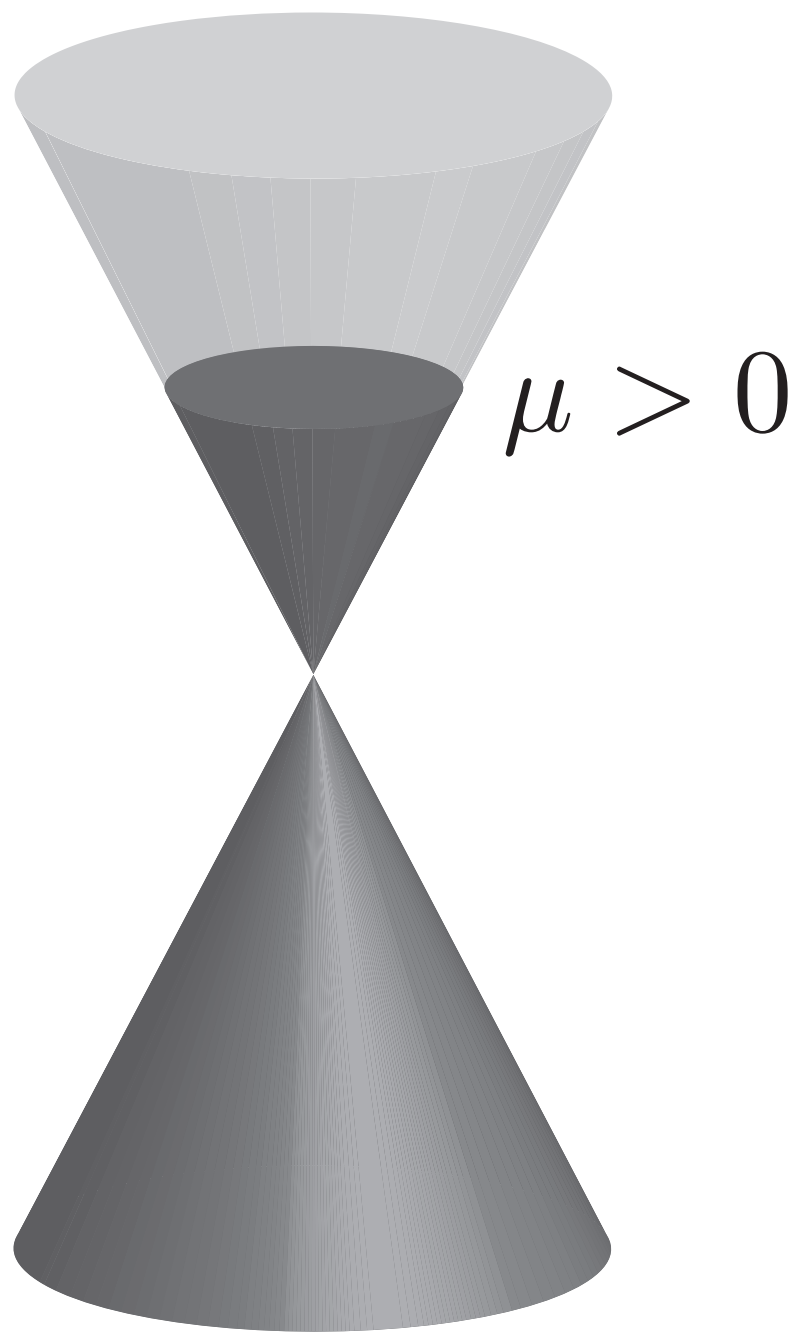
Graphene, strange metals in high temperature superconductors, spin liquids

Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge Q (the “electron density”) in spatial dimension $d > 1$.
- Describe zero temperature phases where $d\langle Q \rangle / d\mu \neq 0$, where μ (the “chemical potential”) which changes the Hamiltonian, H , to $H - \mu Q$.

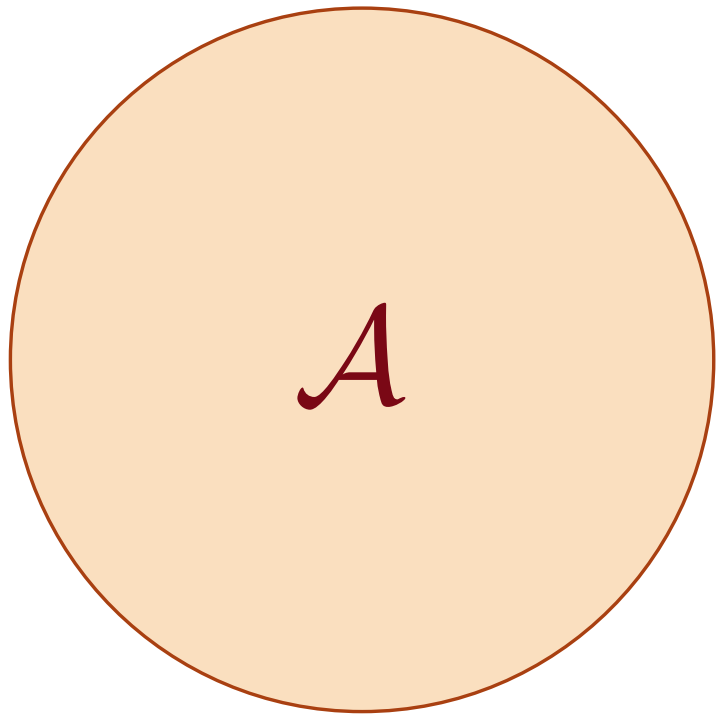


Conformal quantum matter



Compressible quantum matter

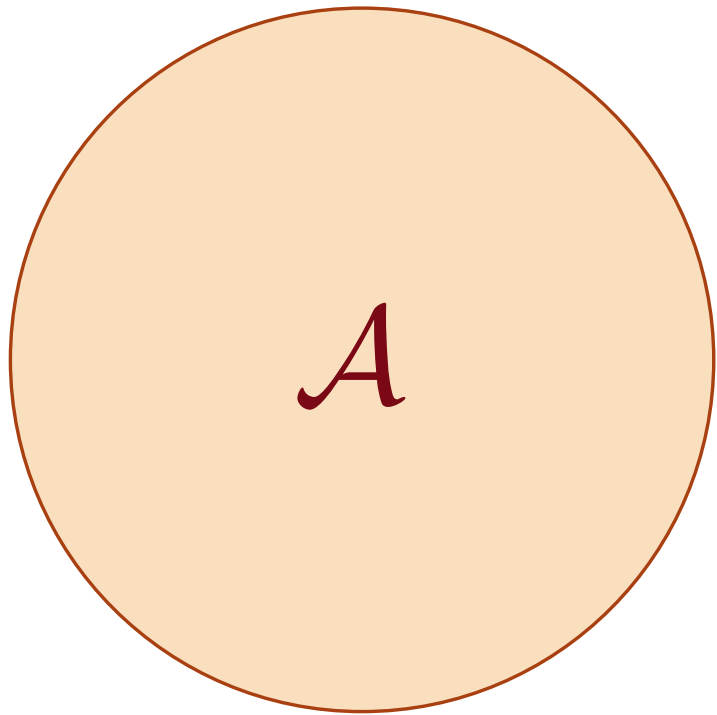
The Fermi liquid



$$\mathcal{L} = f_{\sigma}^{\dagger} \left(\partial_{\tau} - \frac{\nabla^2}{2m} - \mu \right) f_{\sigma}$$

+ short-range 4-Fermi terms

The Fermi liquid

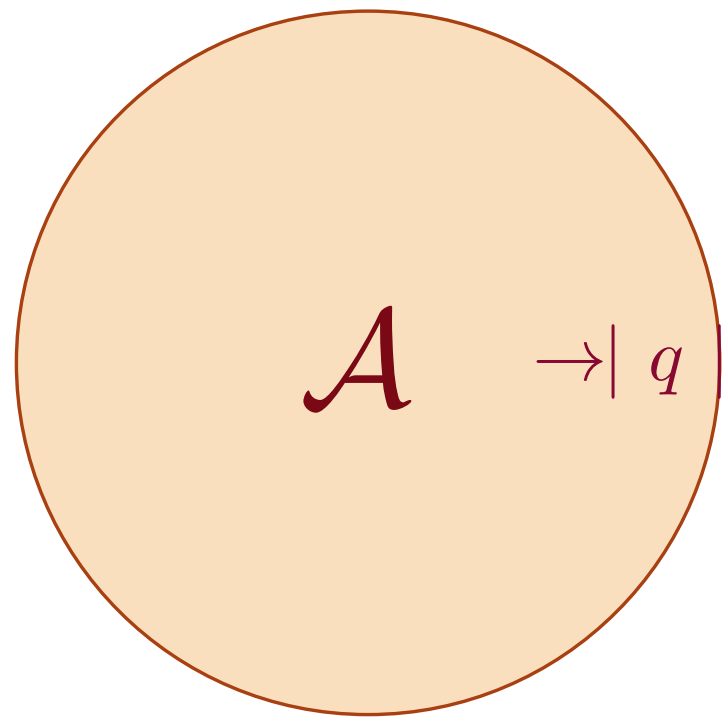


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+ short-range 4-Fermi terms

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

The Fermi liquid

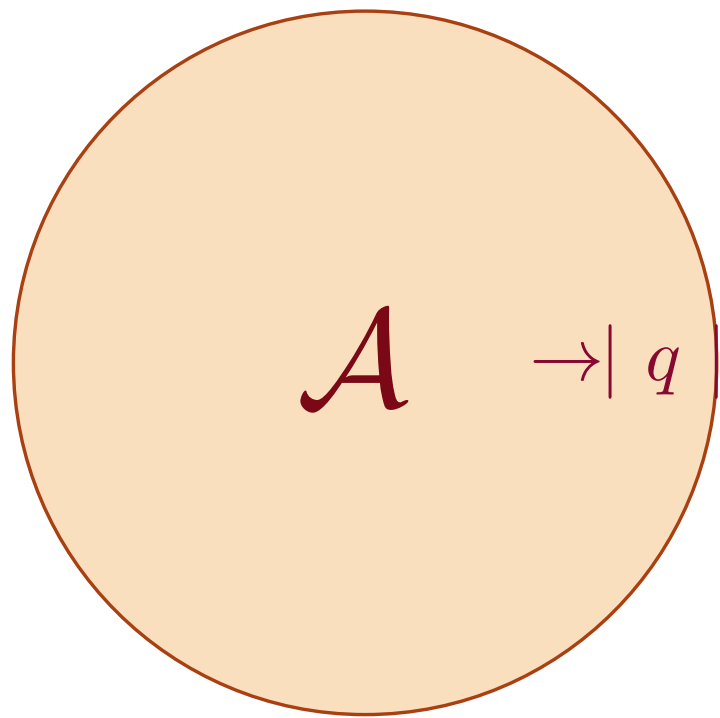


$$\mathcal{L} = f_{\sigma}^{\dagger} \left(\partial_{\tau} - \frac{\nabla^2}{2m} - \mu \right) f_{\sigma}$$

+ short-range 4-Fermi terms

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|$.

The Fermi liquid

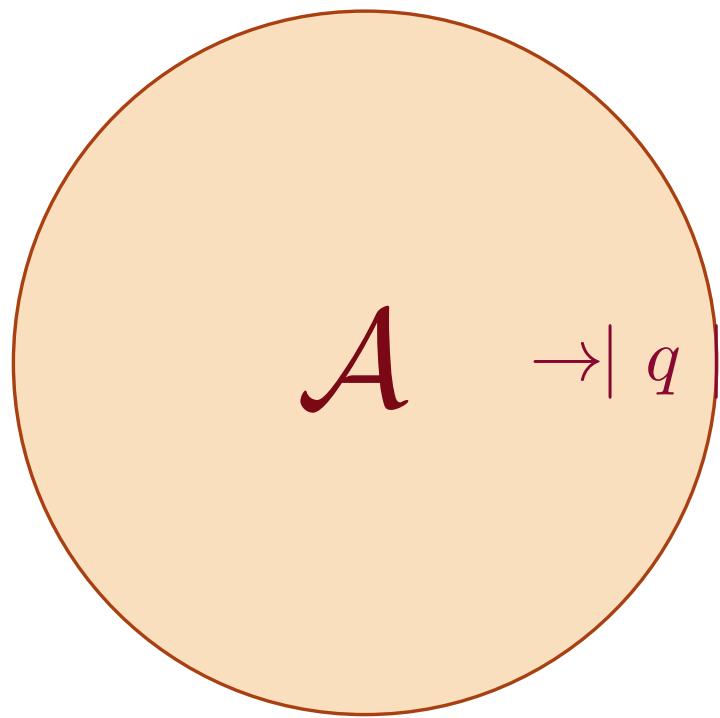


$$\mathcal{L} = f_{\sigma}^{\dagger} \left(\partial_{\tau} - \frac{\nabla^2}{2m} - \mu \right) f_{\sigma}$$

+ short-range 4-Fermi terms

- Fermion Green's function $G_f^{-1} = \omega - v_F q + i\mathcal{O}(\omega^2, q^2)$.

The Fermi liquid

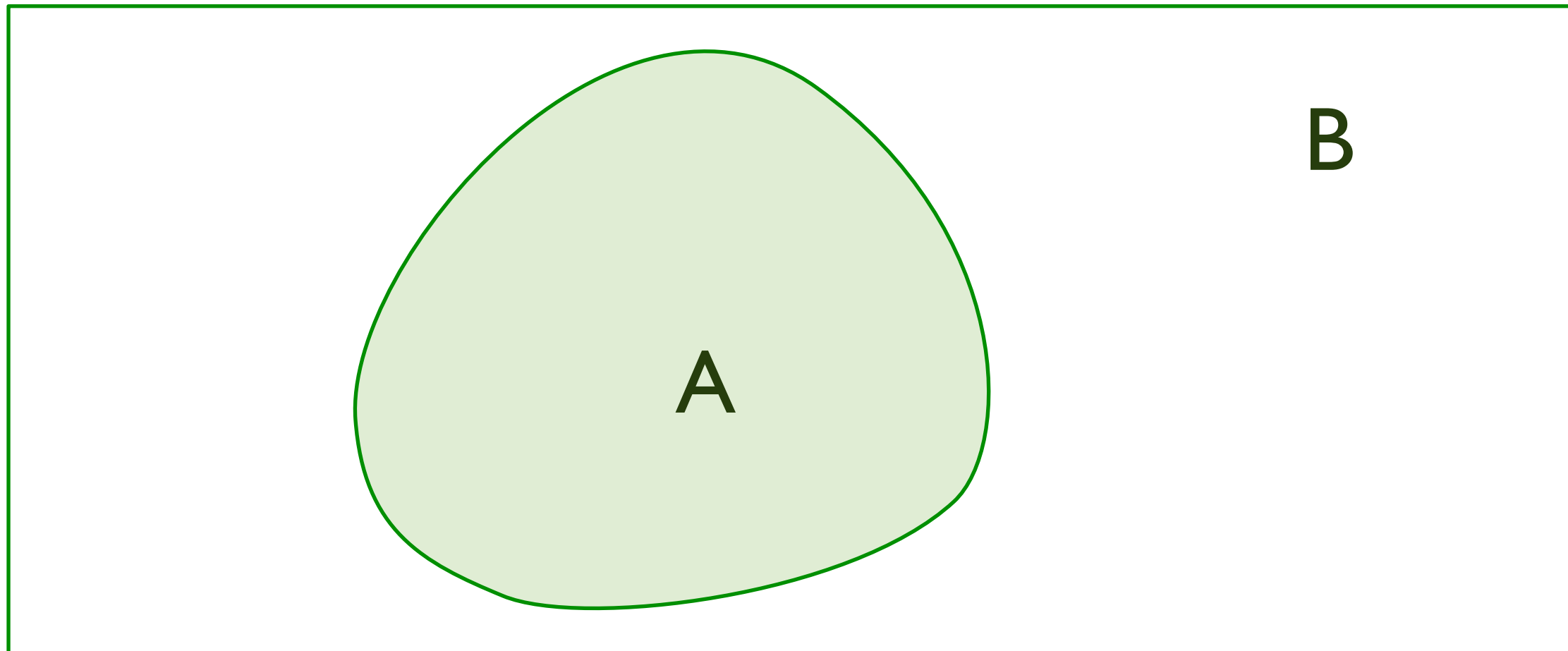


$$\mathcal{L} = f_{\sigma}^{\dagger} \left(\partial_{\tau} - \frac{\nabla^2}{2m} - \mu \right) f_{\sigma}$$

+ short-range 4-Fermi terms

- Fermion Green's function $G_f^{-1} = \omega - v_F q + i\mathcal{O}(\omega^2, q^2)$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}}$ with $d_{\text{eff}} = 1$.

Entanglement entropy of Fermi surfaces



Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F ,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Non-Fermi liquids

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless scalar field, ϕ , which has low energy excitations near $\mathbf{q} = 0$.

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- ferromagnetic order

Non-Fermi liquids

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- ferromagnetic order
- breaking of point-group symmetry (Ising-nematic order)

Non-Fermi liquids

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless scalar field, ϕ , which has low energy excitations near $\mathbf{q} = 0$. The field ϕ could represent

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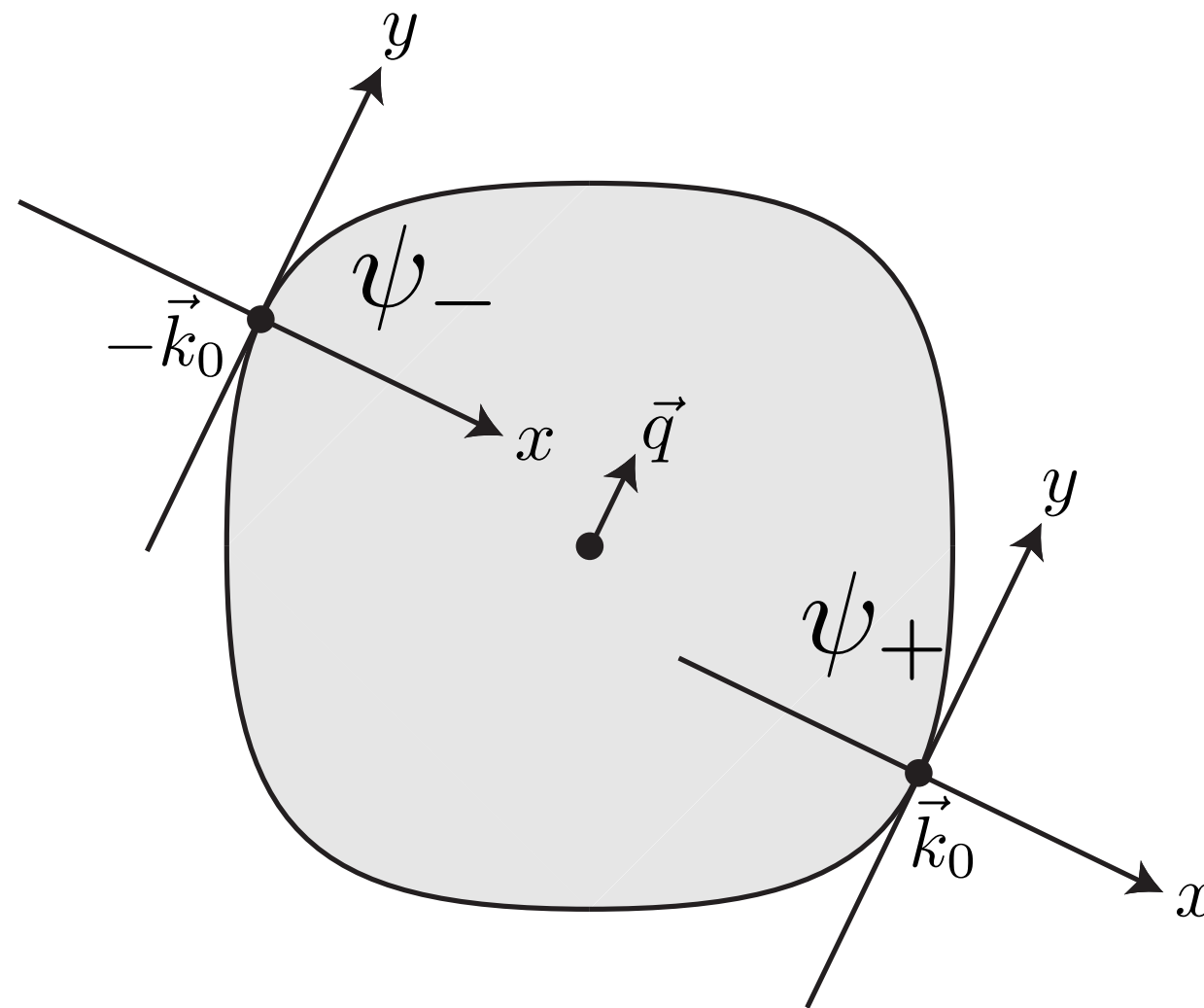
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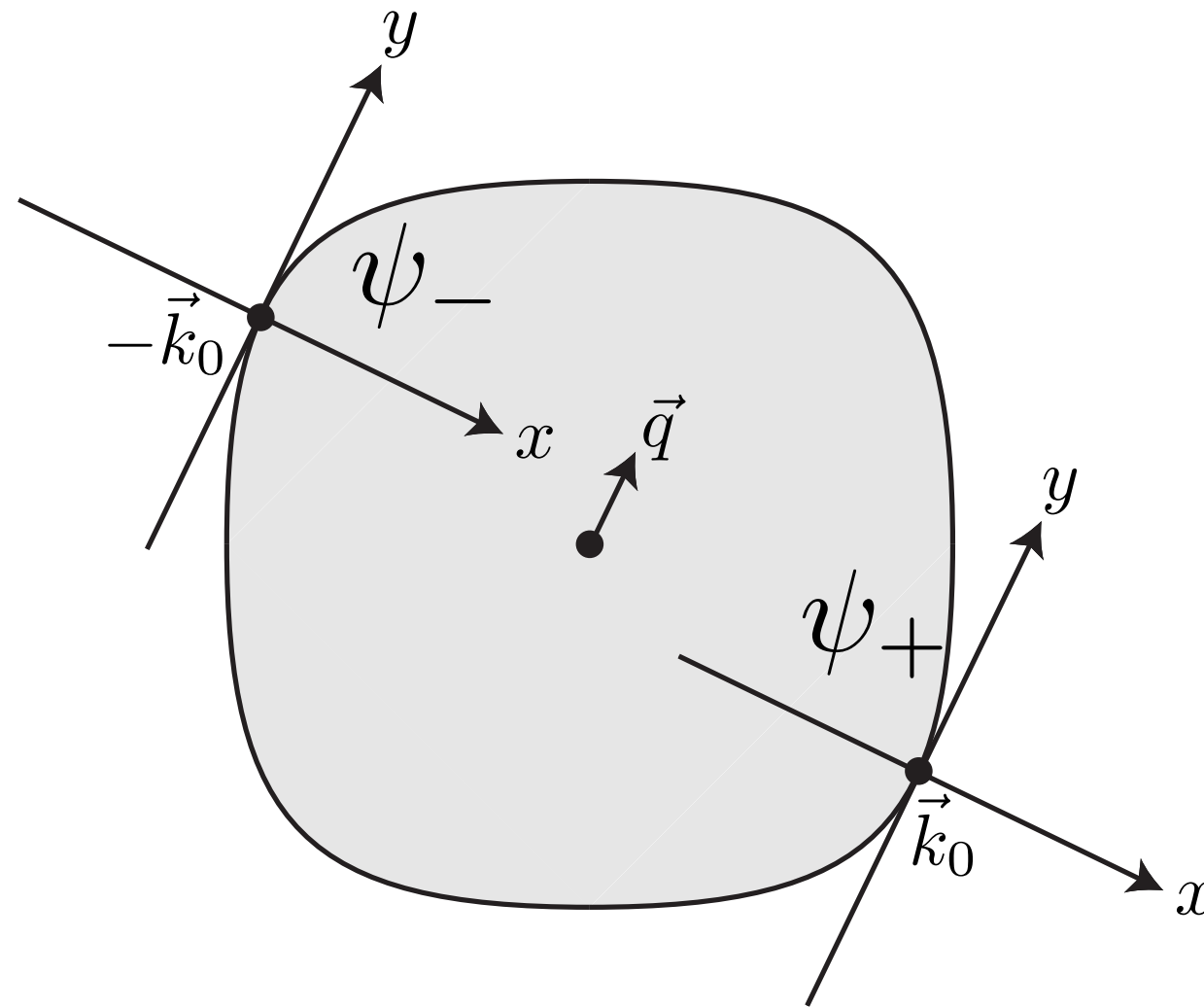
- ferromagnetic order
- breaking of point-group symmetry (Ising-nematic order)
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- circulating currents
- transverse component of an Abelian or non-Abelian gauge field.
- ...

Non-Fermi liquids



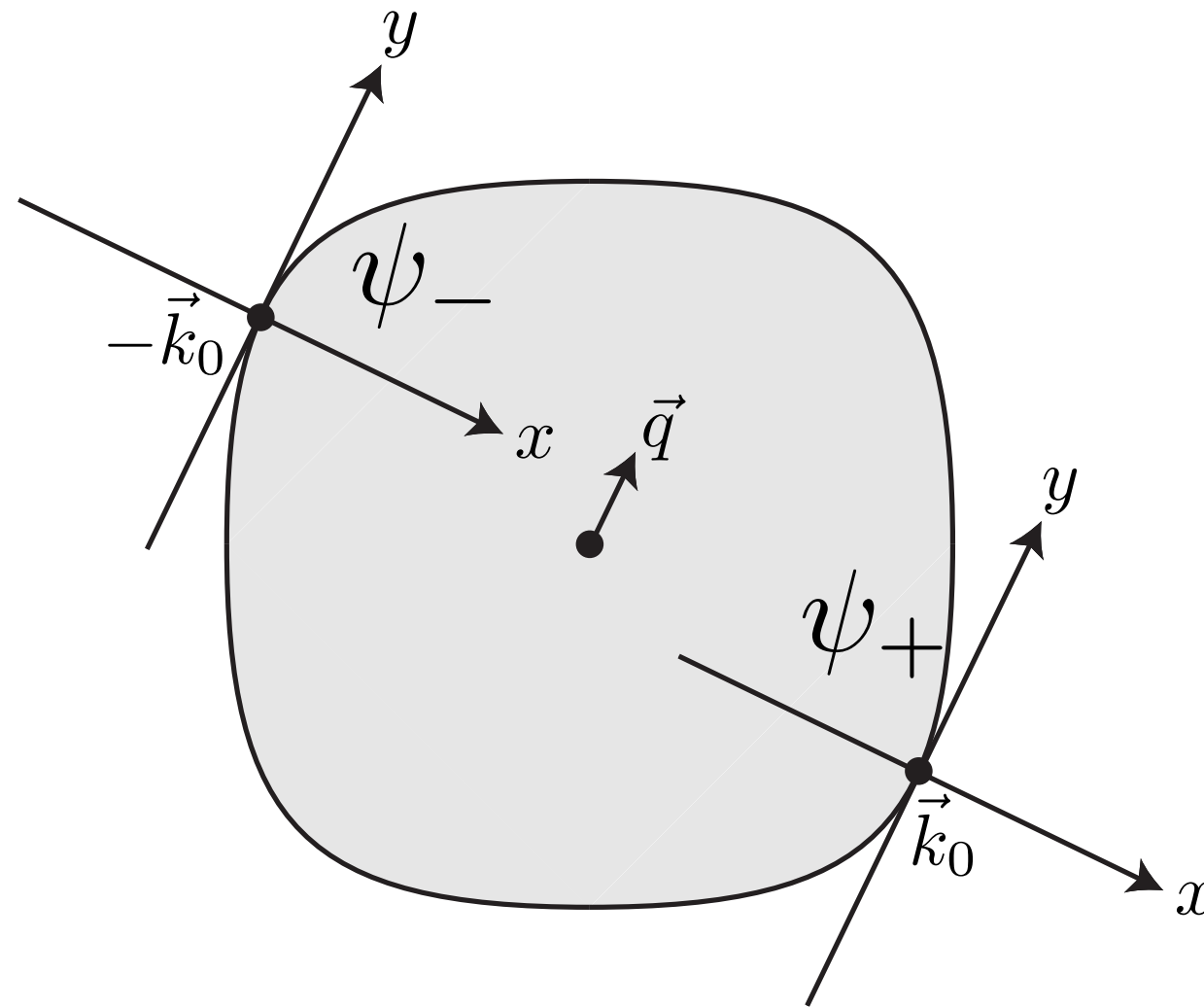
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Non-Fermi liquids



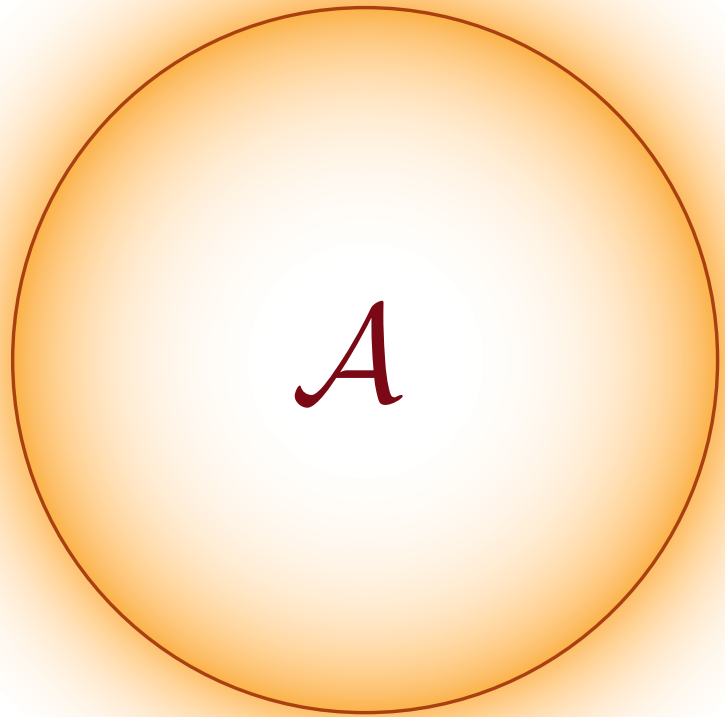
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Non-Fermi liquids



$$\mathcal{L}[\psi_{\pm}, \phi] = \psi_{+}^{\dagger} (\partial_{\tau} - i\partial_x - \partial_y^2) \psi_{+} + \psi_{-}^{\dagger} (\partial_{\tau} + i\partial_x - \partial_y^2) \psi_{-} - g\phi \left(\psi_{+}^{\dagger} \psi_{+} \pm \psi_{-}^{\dagger} \psi_{-} \right) + (\partial_y \phi)^2$$

Non-Fermi liquids



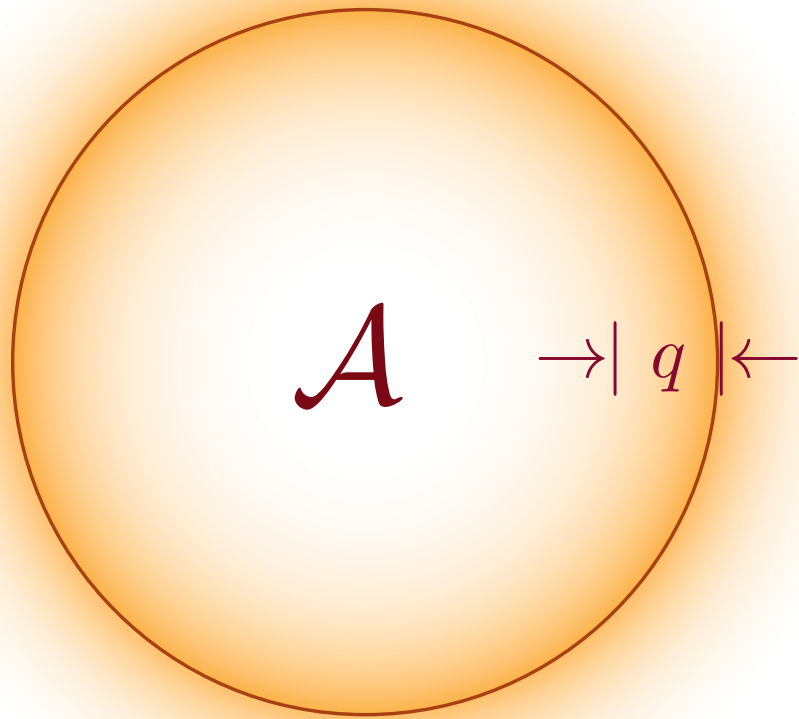
- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

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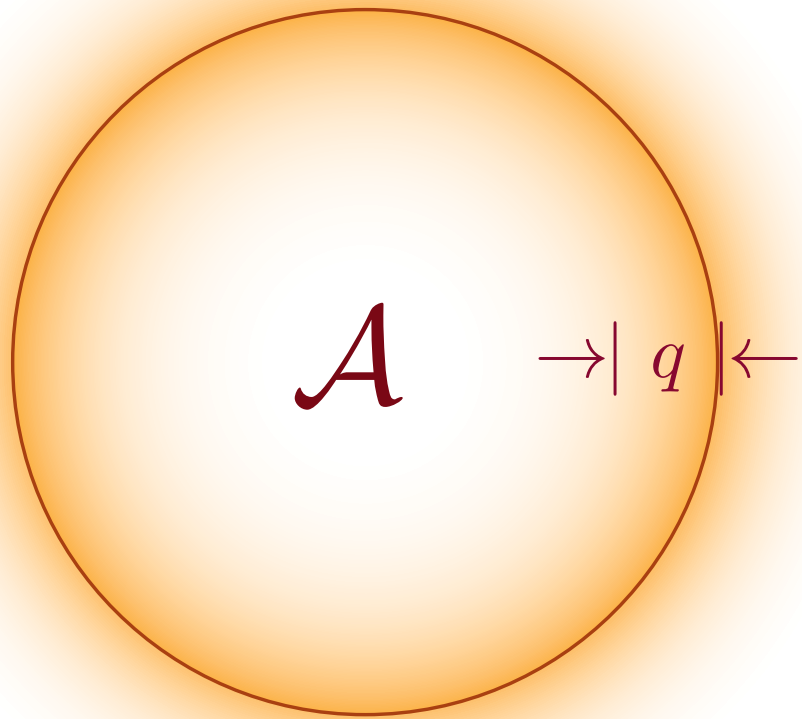
- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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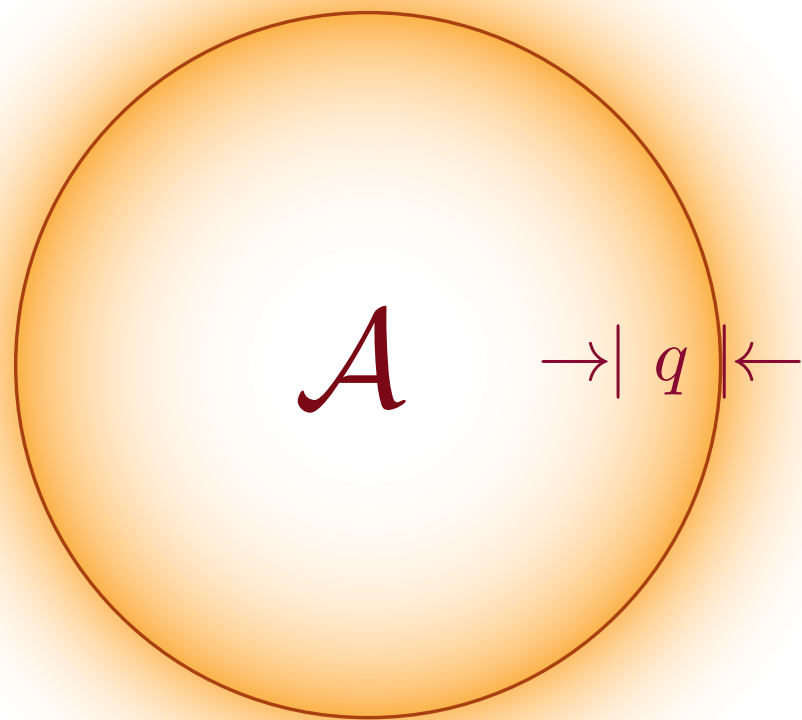
- Gauge-dependent Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$.
Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

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- Gauge-dependent Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.
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S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

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Non-Fermi liquids

Simple scaling argument for $z = 3/2$.

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ - g\phi \left(\psi_+^\dagger \psi_+ \pm \psi_-^\dagger \psi_- \right) + (\partial_y \phi)^2$$

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Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

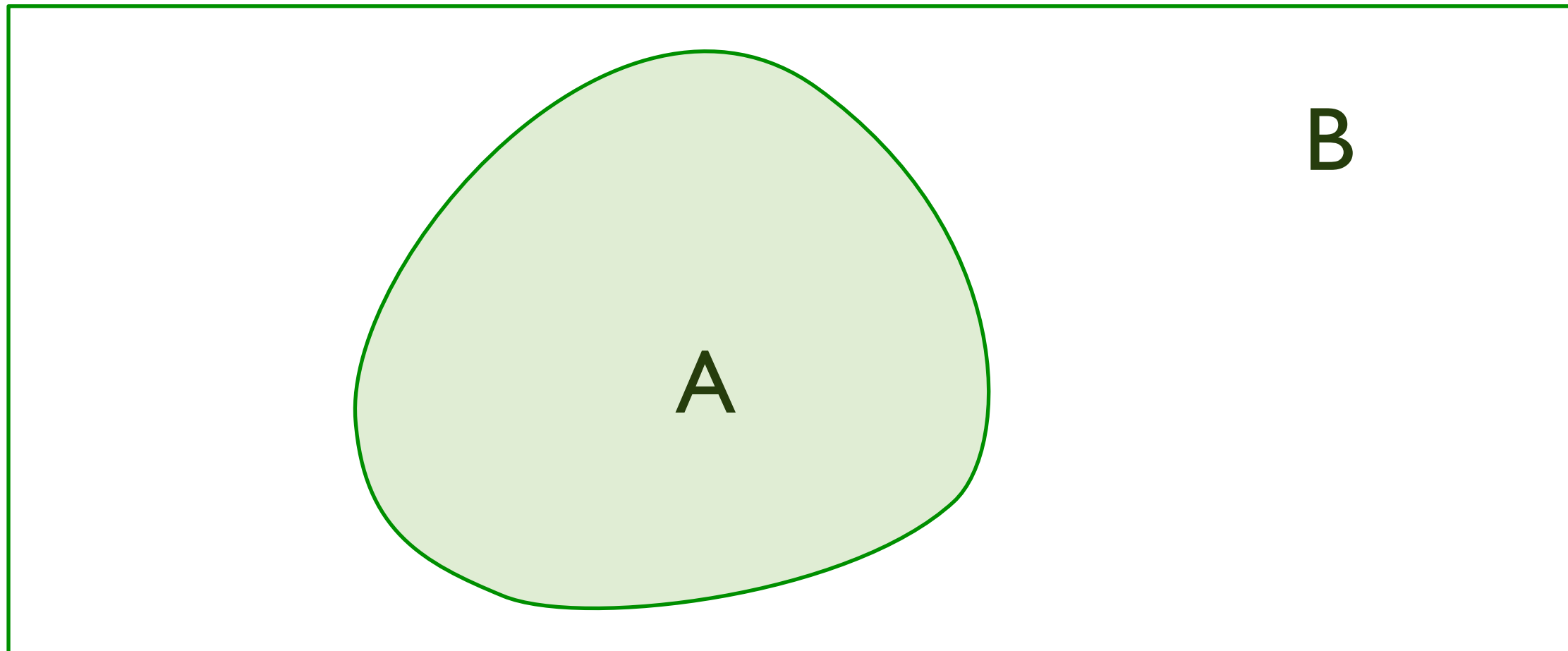
$$\phi \rightarrow \phi s^{(2z+1)/4}$$

$$\psi \rightarrow \psi s^{(2z+1)/4}$$

$$g \rightarrow g s^{(3-2z)/4}$$

So the action is invariant provided $z = 3/2$.

Entanglement entropy of Fermi surfaces



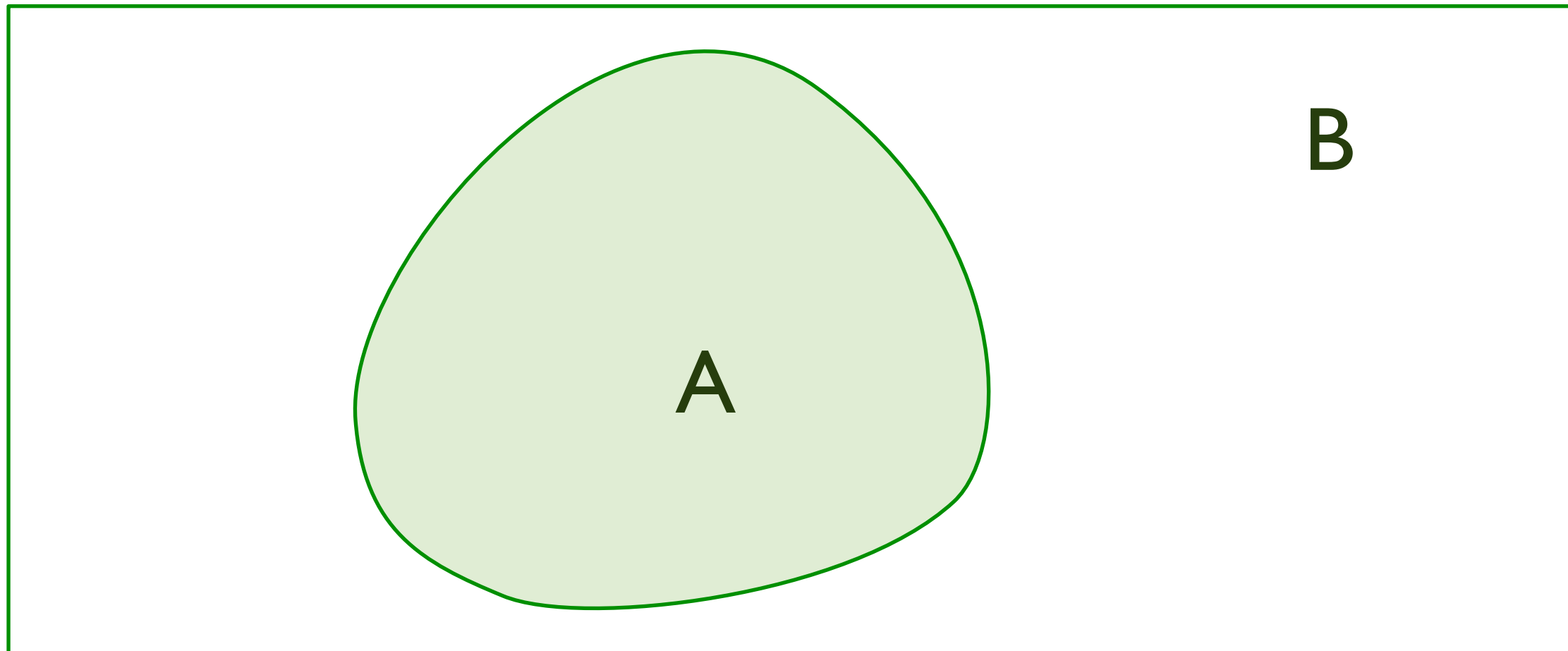
Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the “1/12” prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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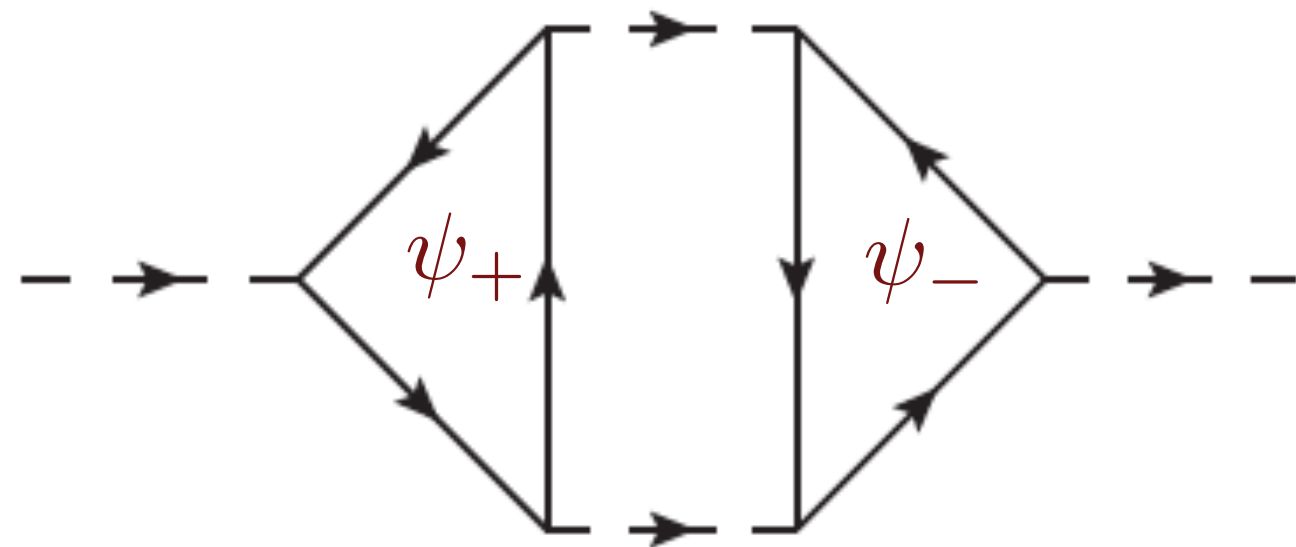
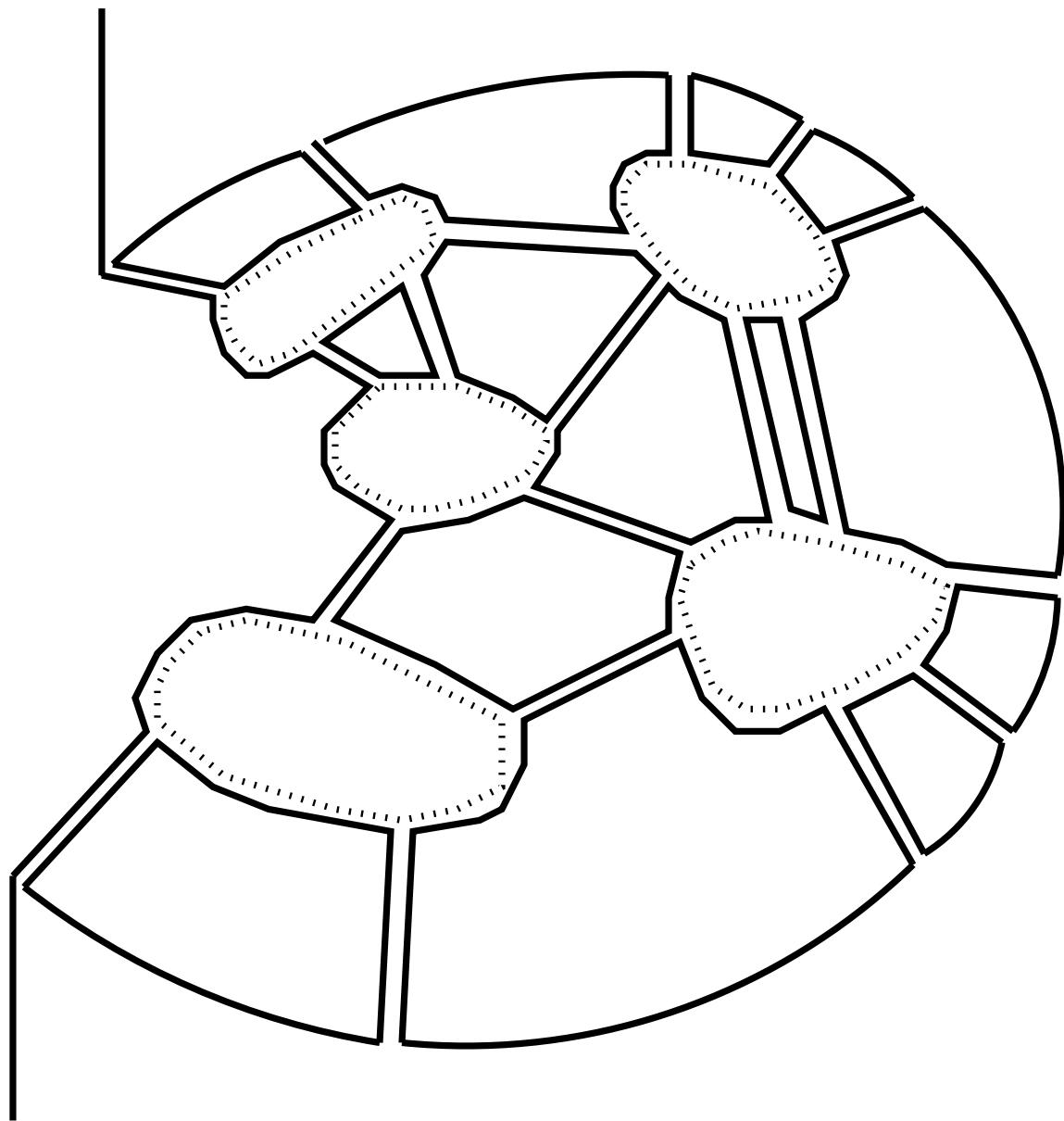
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Computations in the $1/N$ expansion

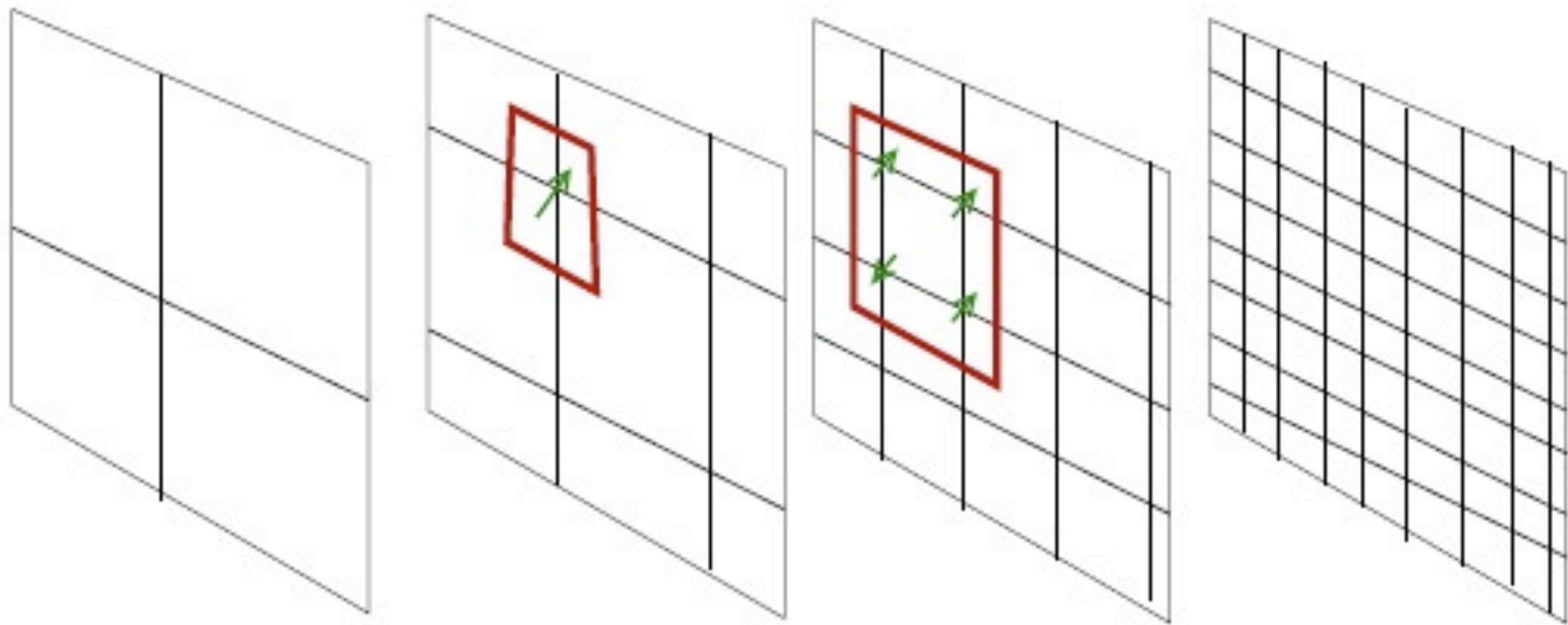


Graph mixing ψ_+ and ψ_- is $\mathcal{O}(N^{3/2})$ (instead of $\mathcal{O}(N)$), violating genus expansion

All planar graphs of ψ_+ alone are as important as the leading term

M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review B* **80**, 165102 (2009)



r ←

Consider the metric which transforms under rescaling as

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\t &\rightarrow \zeta^z t \\ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

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The most general choice of such a metric is

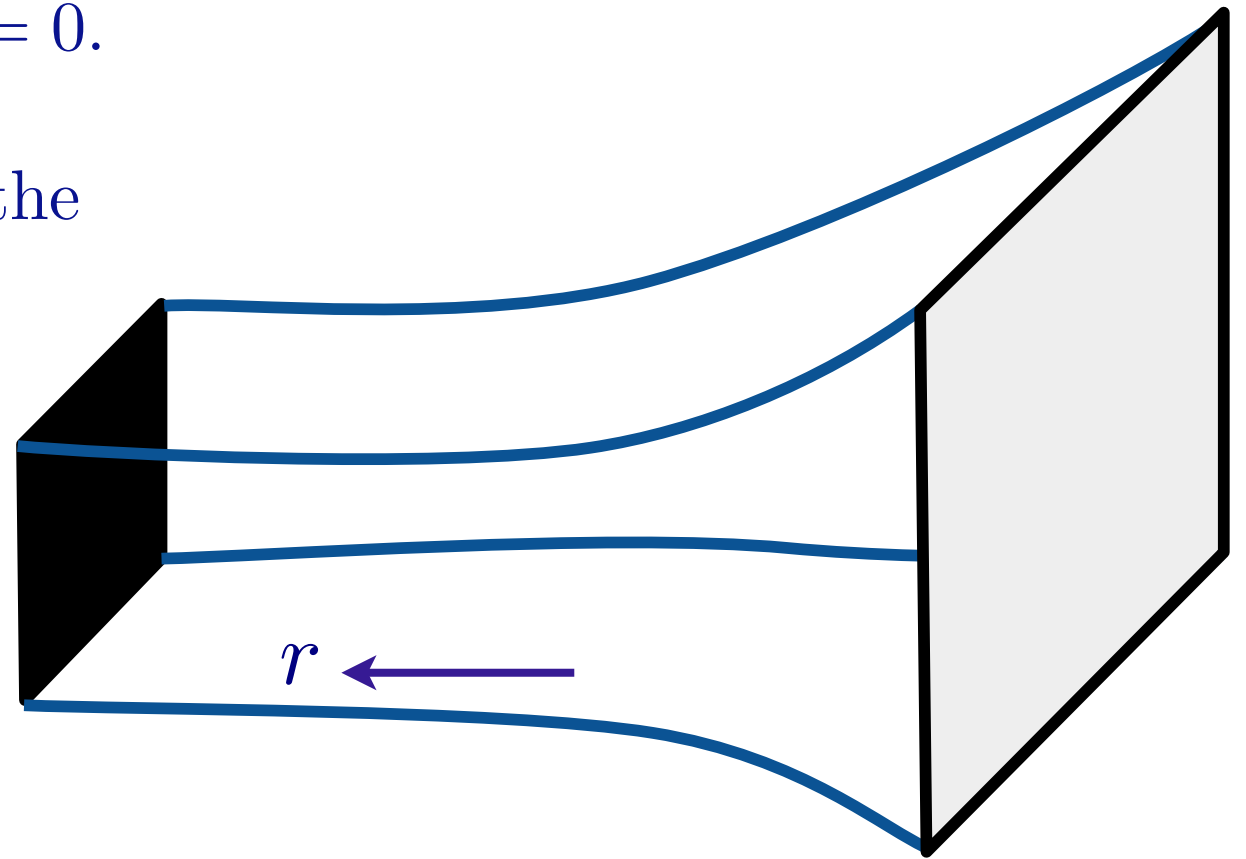
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Bekenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

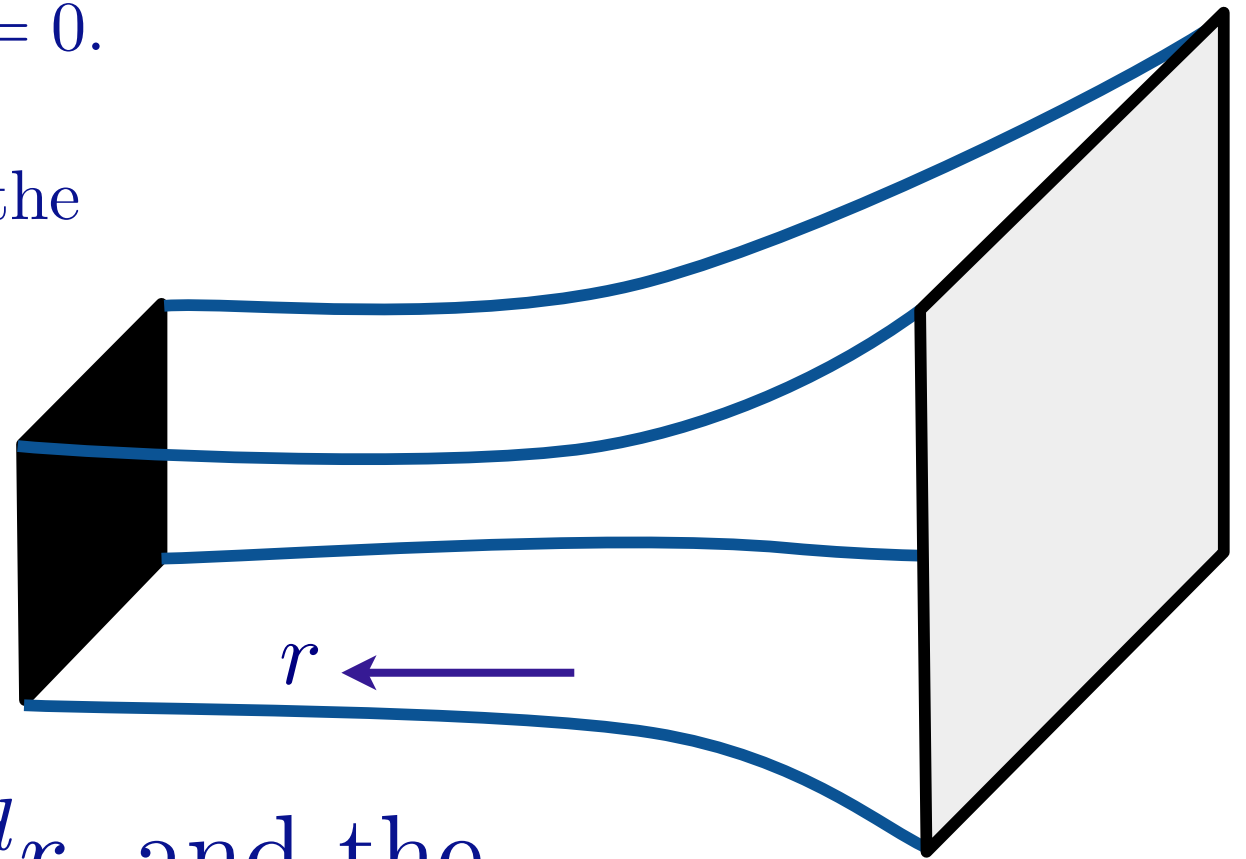
The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



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Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.

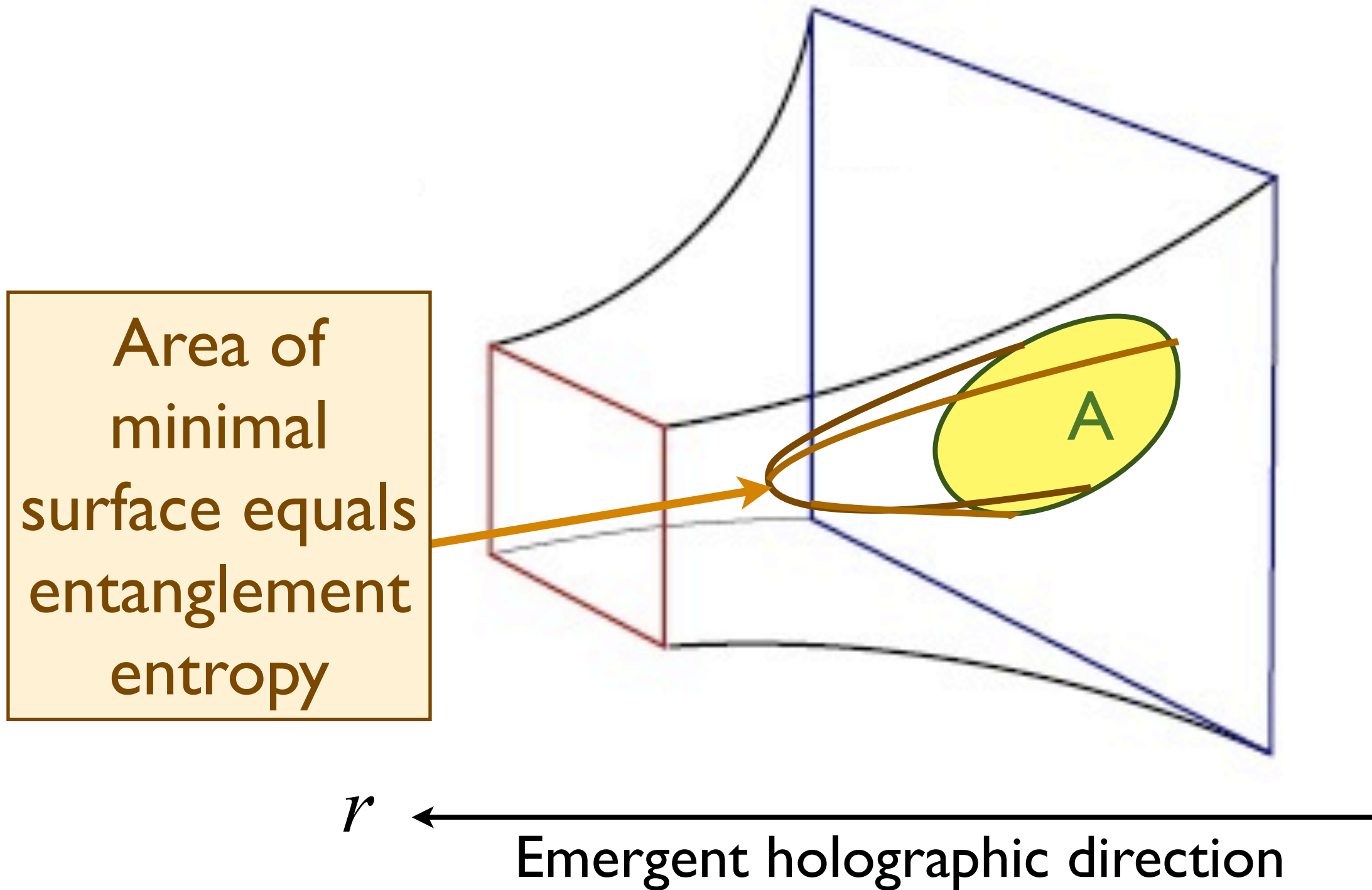
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$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

Holographic entanglement entropy



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases}$$

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Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

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- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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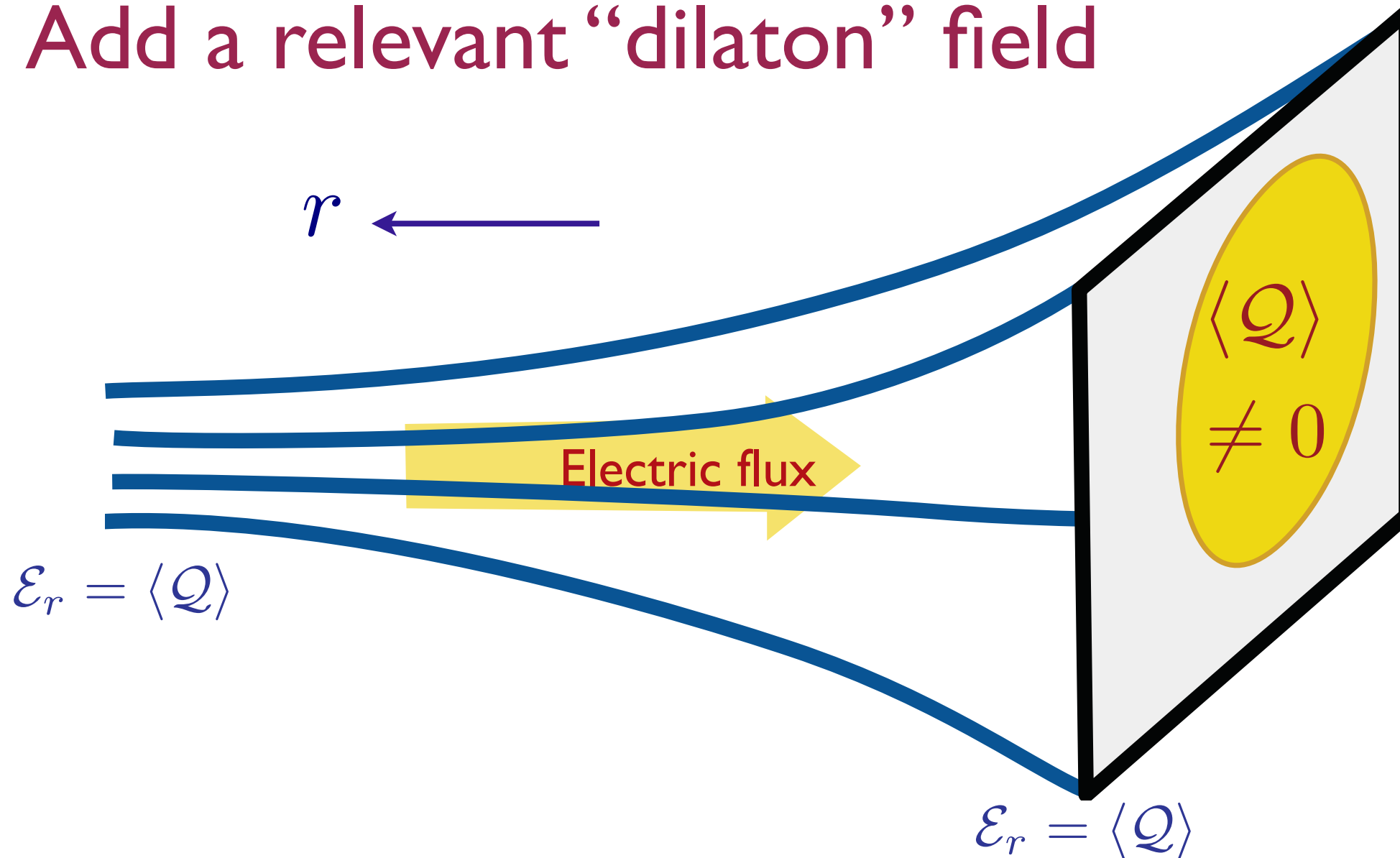
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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



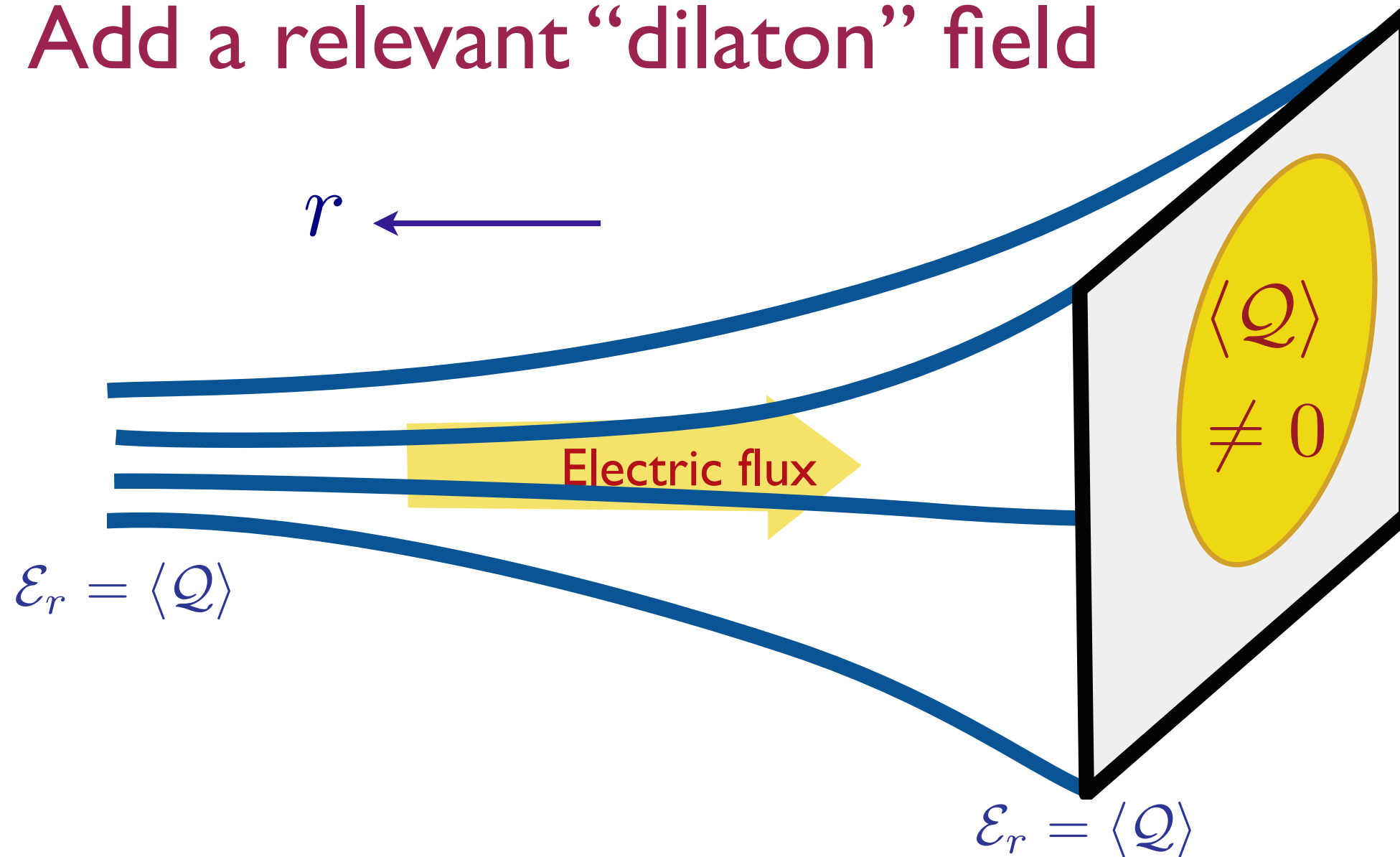
$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

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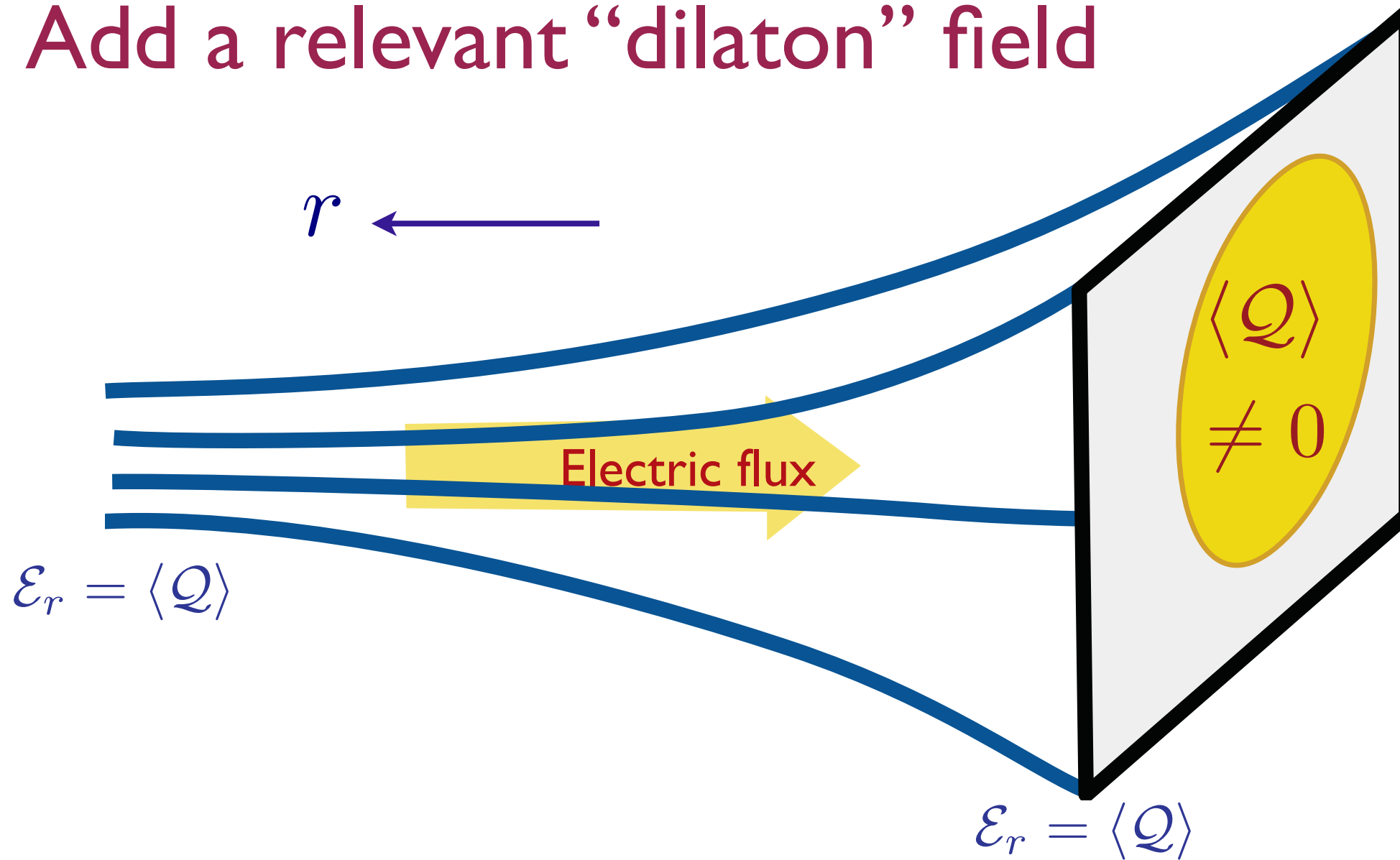
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This is a “bosonization” of the Fermi surface

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$
with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^\delta$, $\Phi(r) \sim \ln(r)$ as $r \rightarrow \infty$.

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$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

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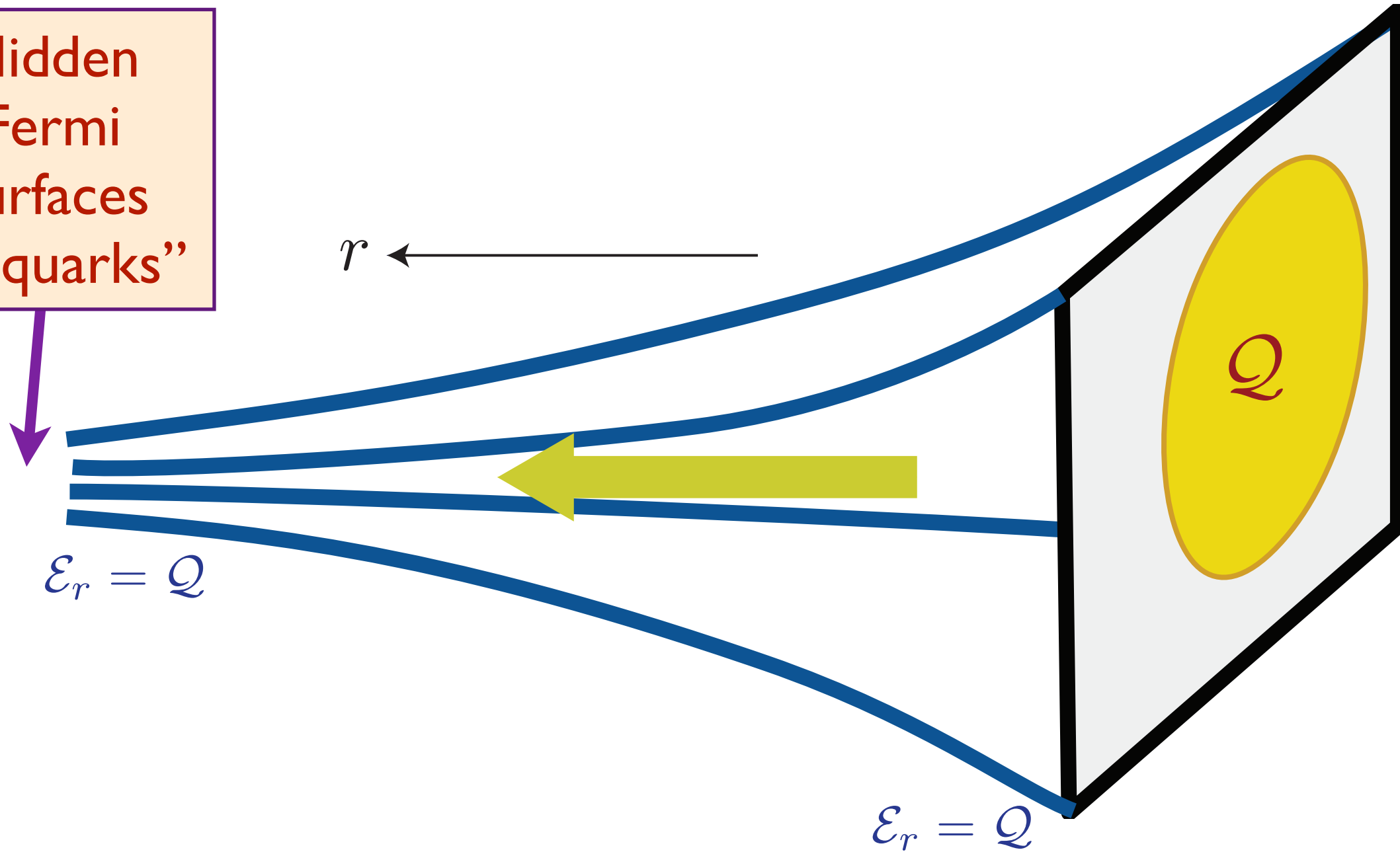
- The entanglement entropy has log-violation of the area law

$$S_E = \Xi Q^{(d-1)/d} P \ln P.$$

where P is surface area of the entangling region, and Ξ is a dimensionless constant which is **independent of all UV details**, of Q , and of any property of the entangling region. Note $Q^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

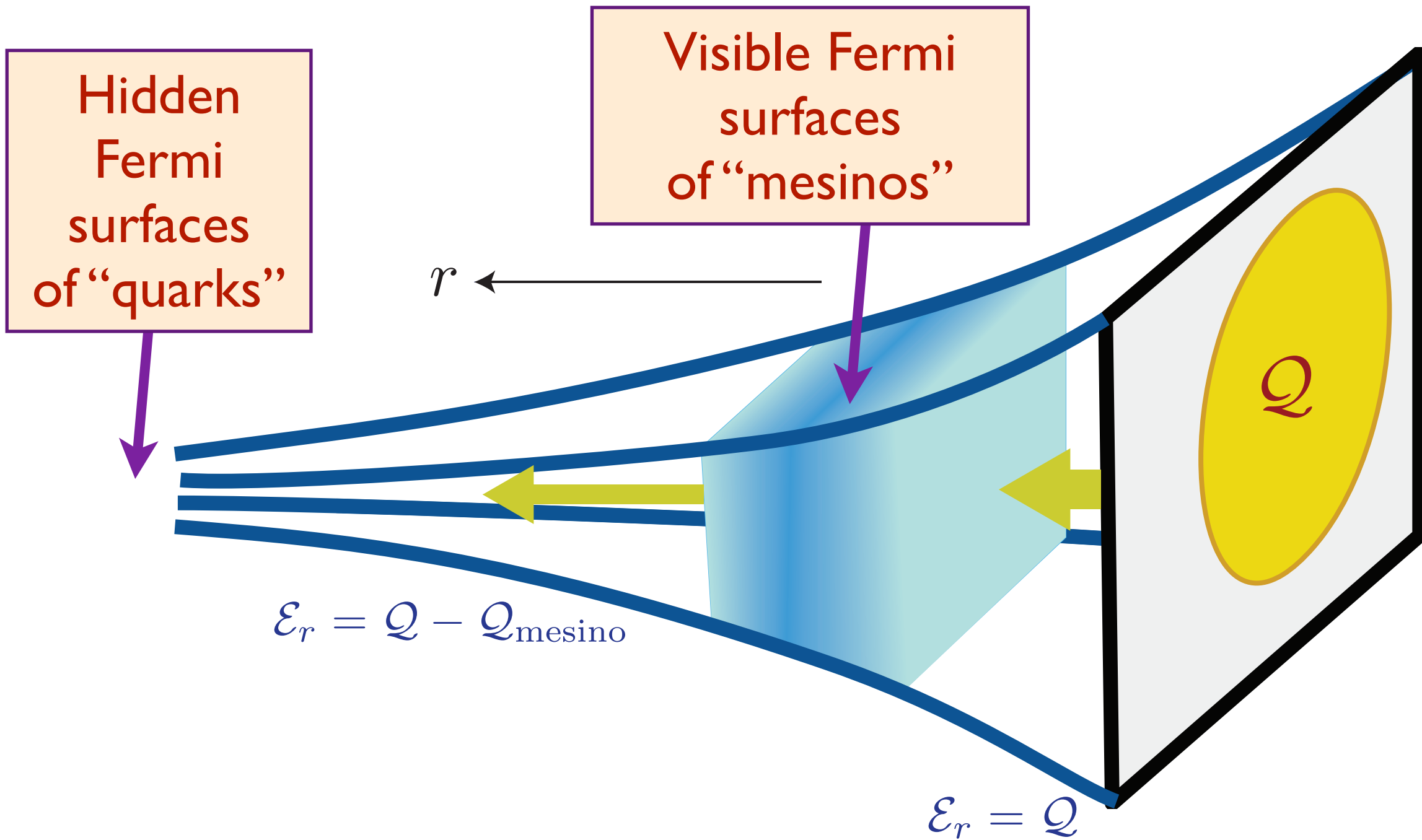
Holographic theory of a non-Fermi liquid (NFL)

Hidden Fermi surfaces of “quarks”



Gauss Law and the “attractor” mechanism
 \Leftrightarrow Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)

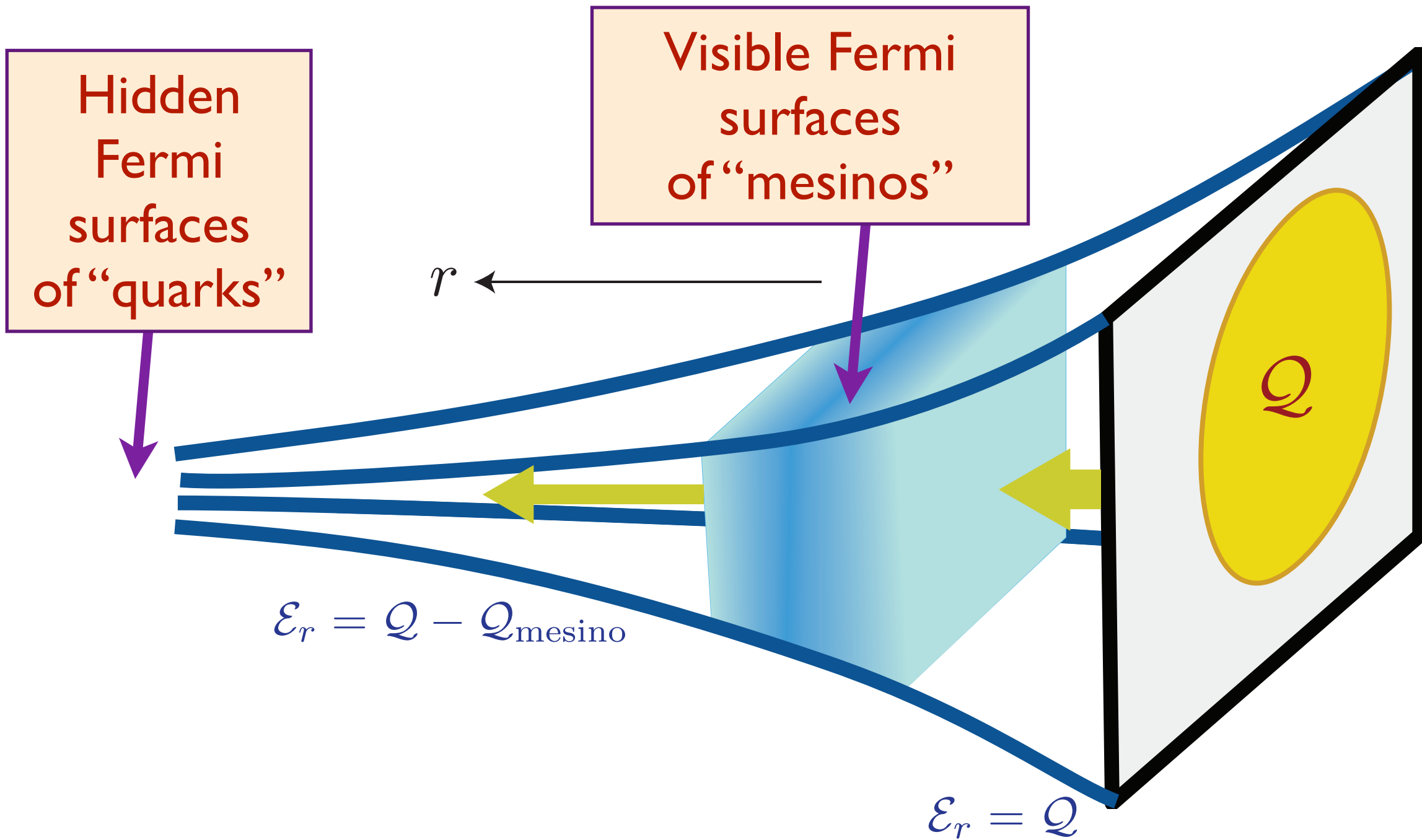


A state with *partial* confinement

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

S. Sachdev, *Physical Review D* **84**, 066009 (2011)

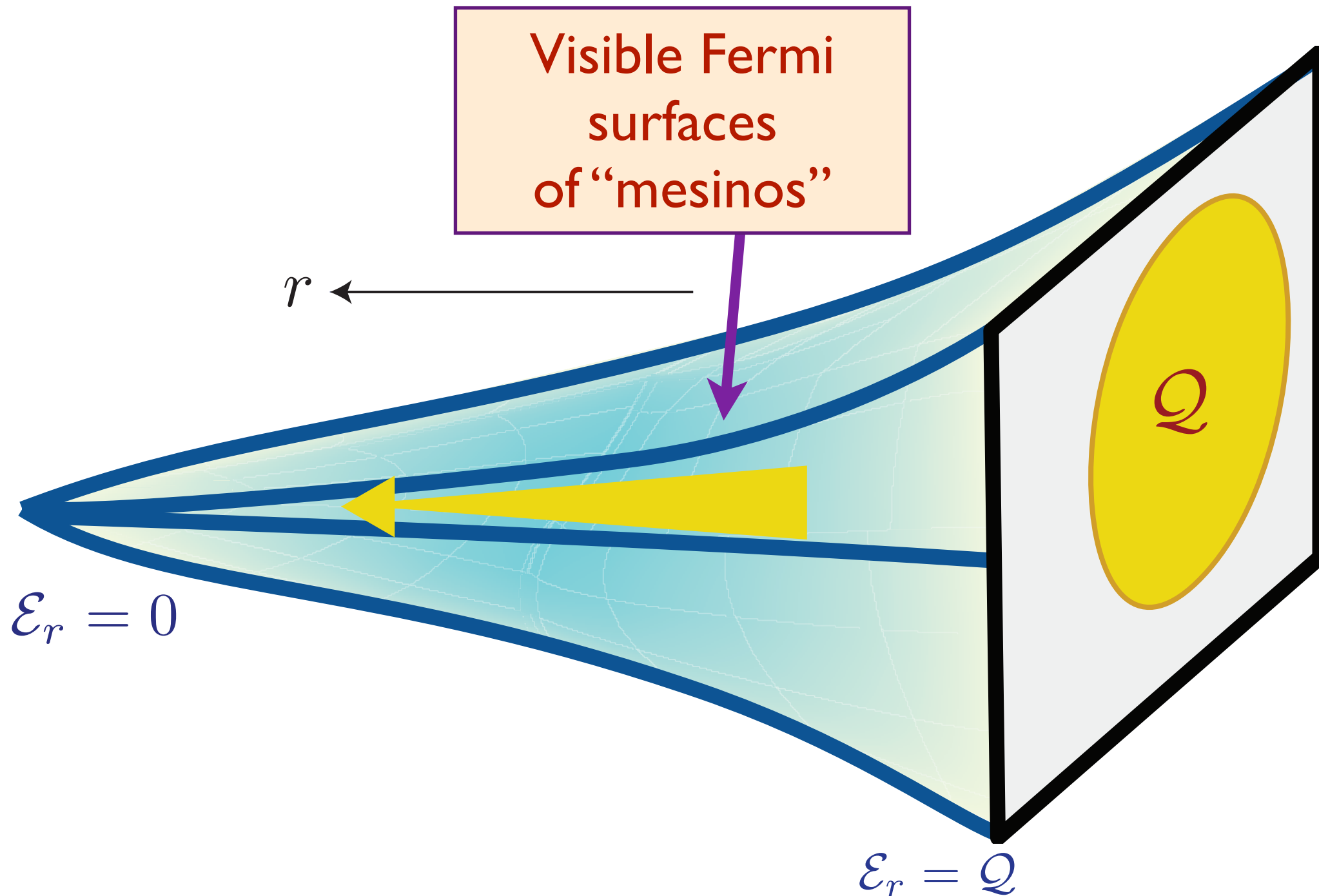
Holographic theory of a fractionalized-Fermi liquid (FL*)



The “mesinos” corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

These are spectators, and are expected to have well-defined quasiparticle excitations.

Holographic theory of a Fermi liquid (FL)




- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)


Conclusions

Compressible quantum matter

 Field theory of a non-Fermi liquid obtained by coupling a Fermi surface to a gapless scalar field with low energy excitations near zero wavevector

Conclusions

Compressible quantum matter

 Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

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