Quantum criticality and the phase diagram of the cuprates

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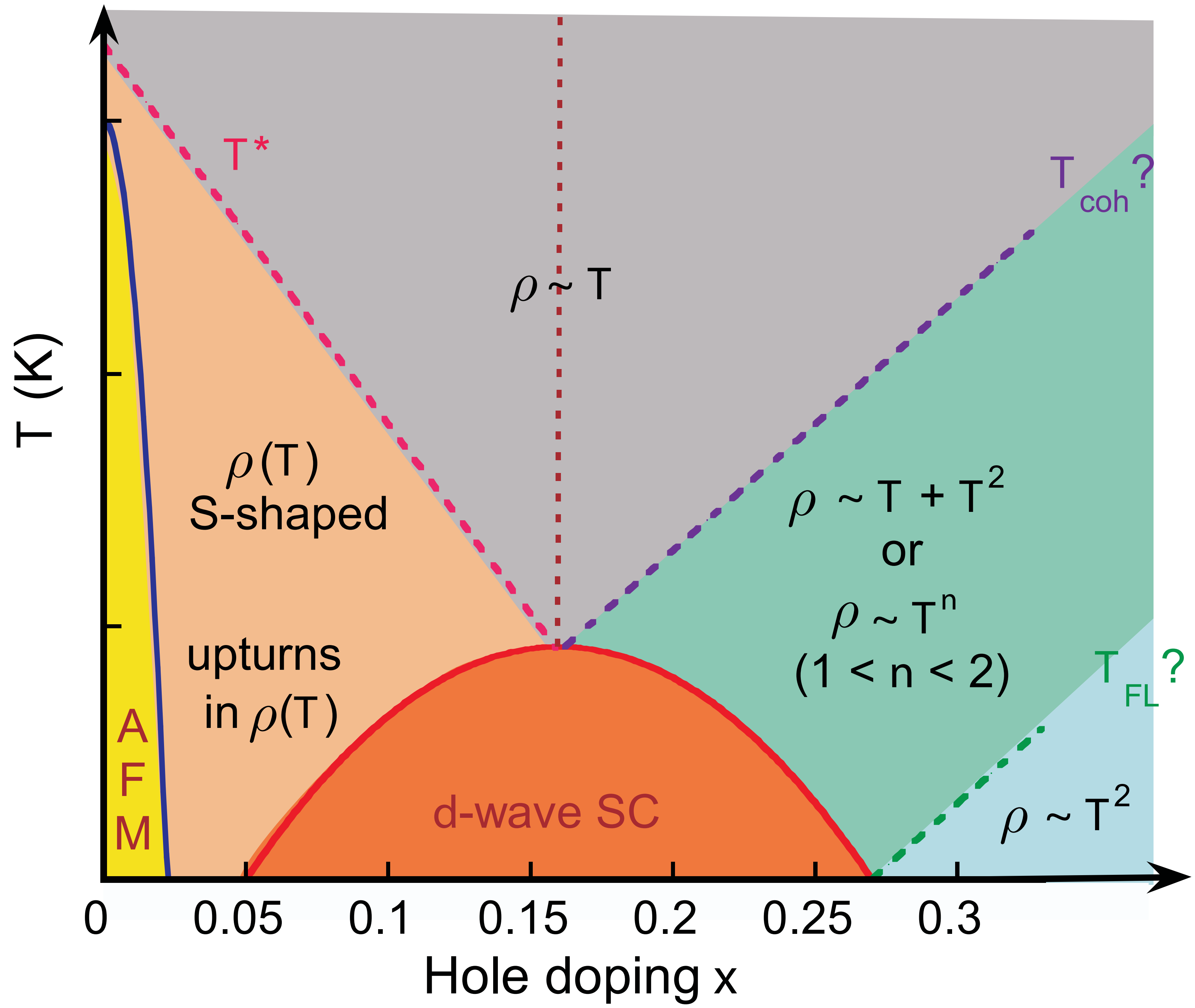
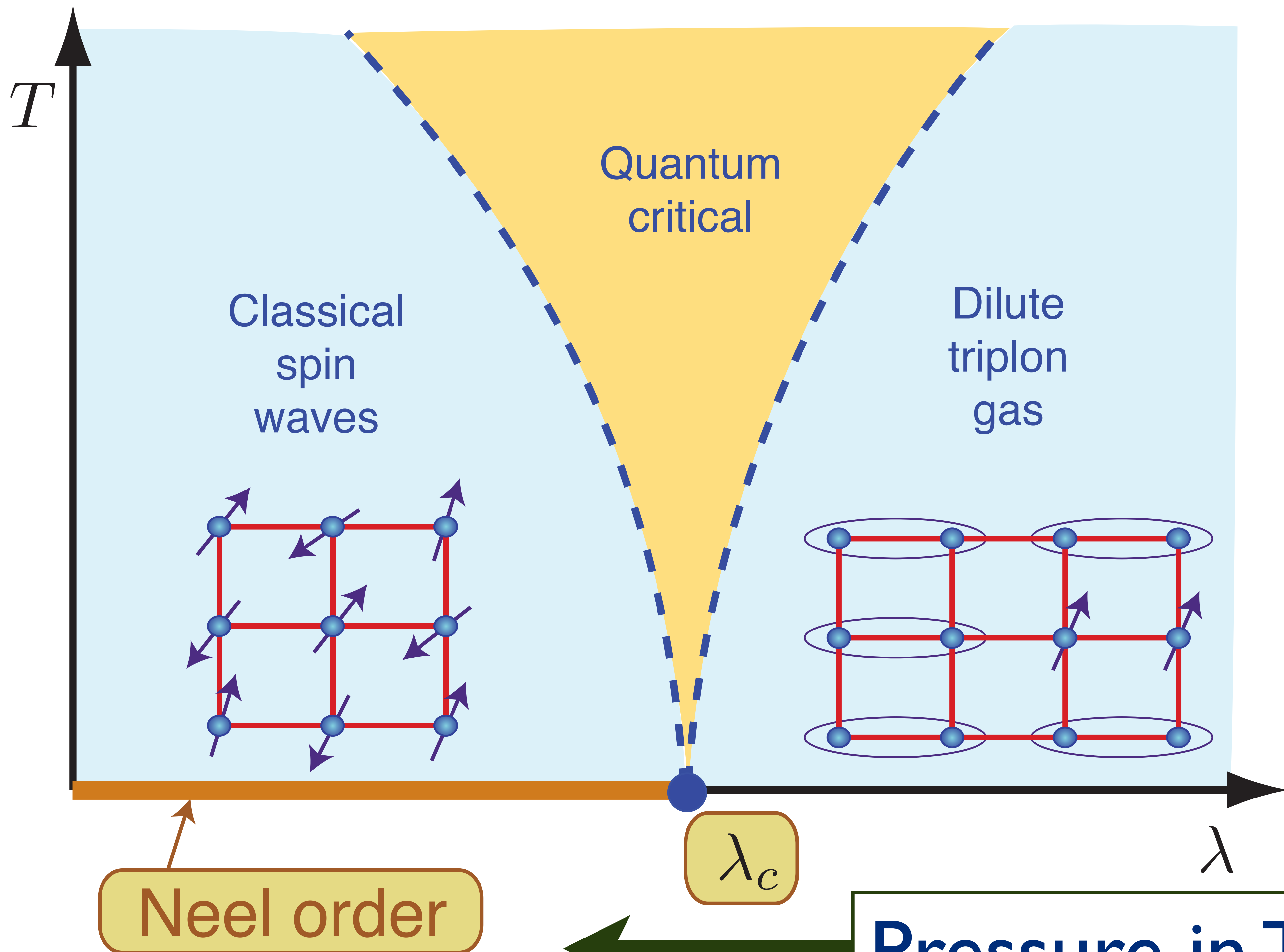
Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Canonical quantum critical phase diagram of coupled-dimer antiferromagnet

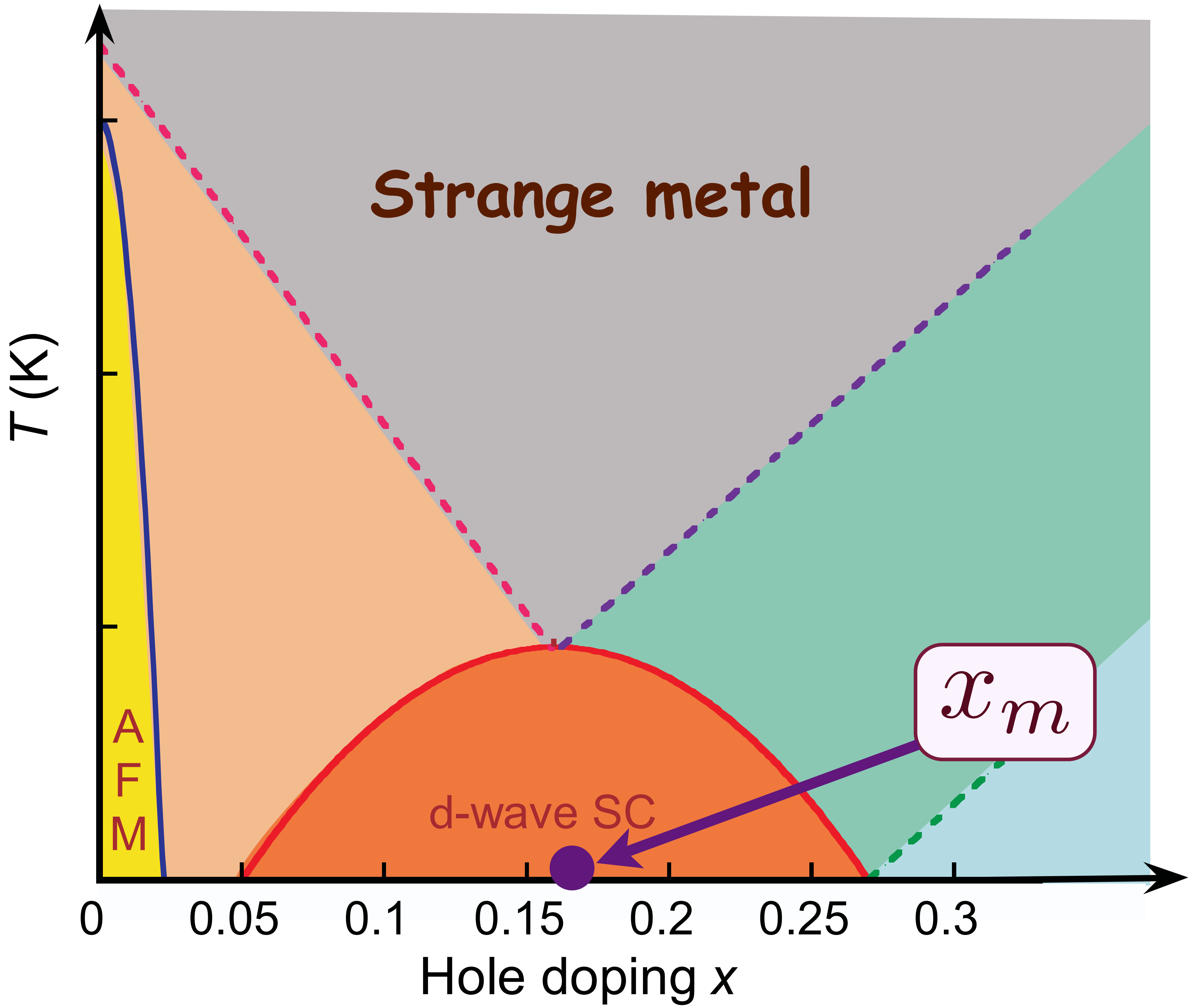


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Pressure in $TlCuCl_3$

Christian Ruegg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

Crossovers in transport properties of hole-doped cuprates



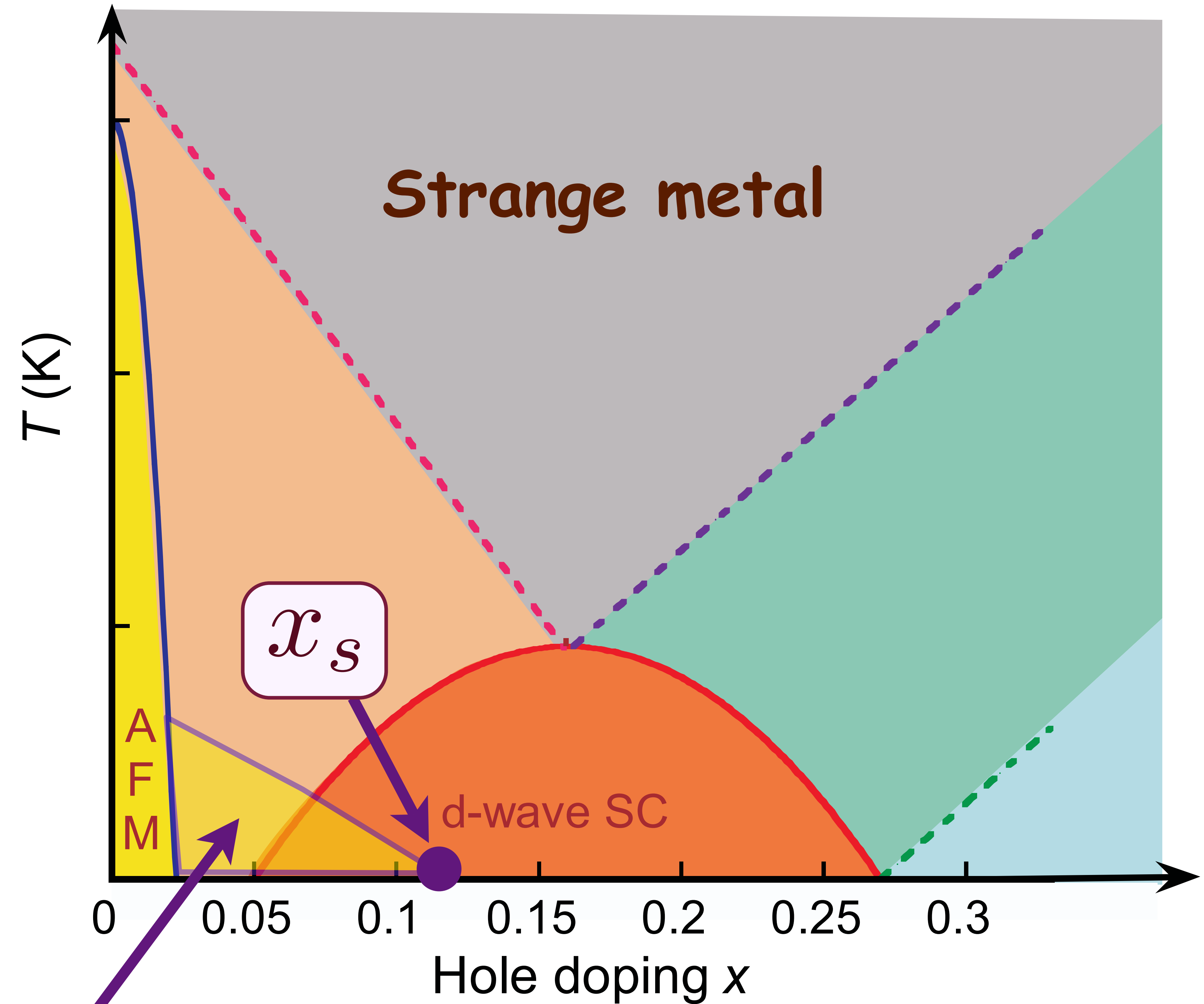
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

A. J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

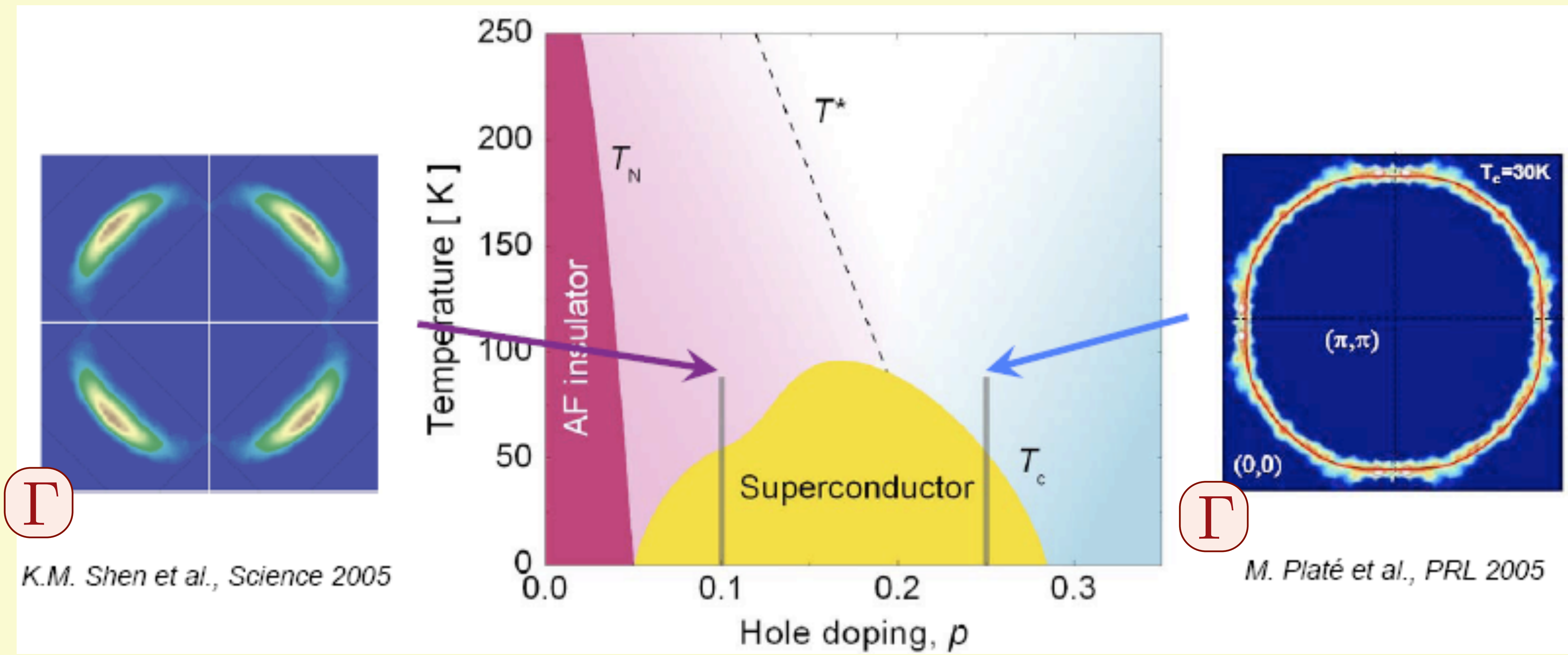
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

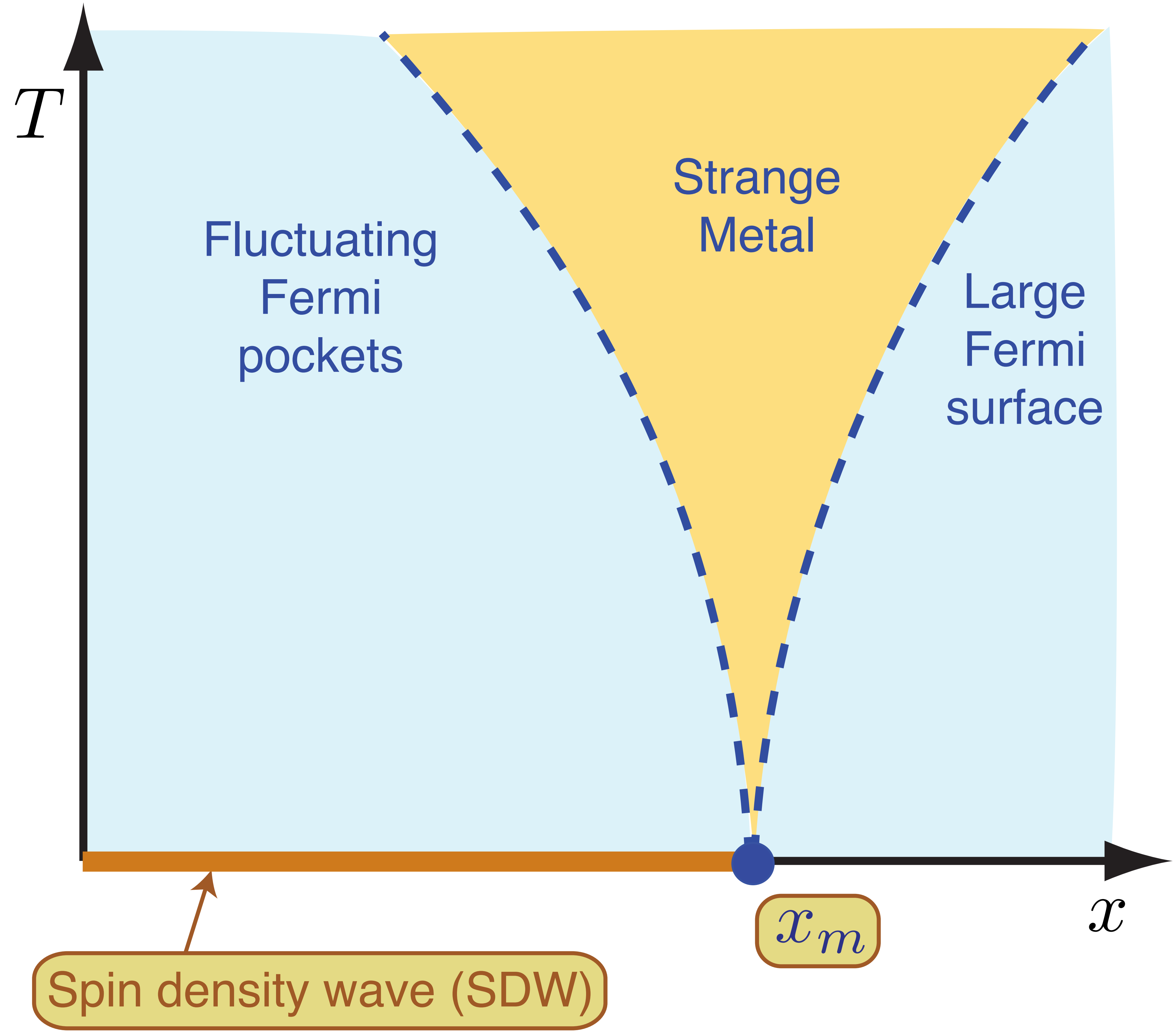
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

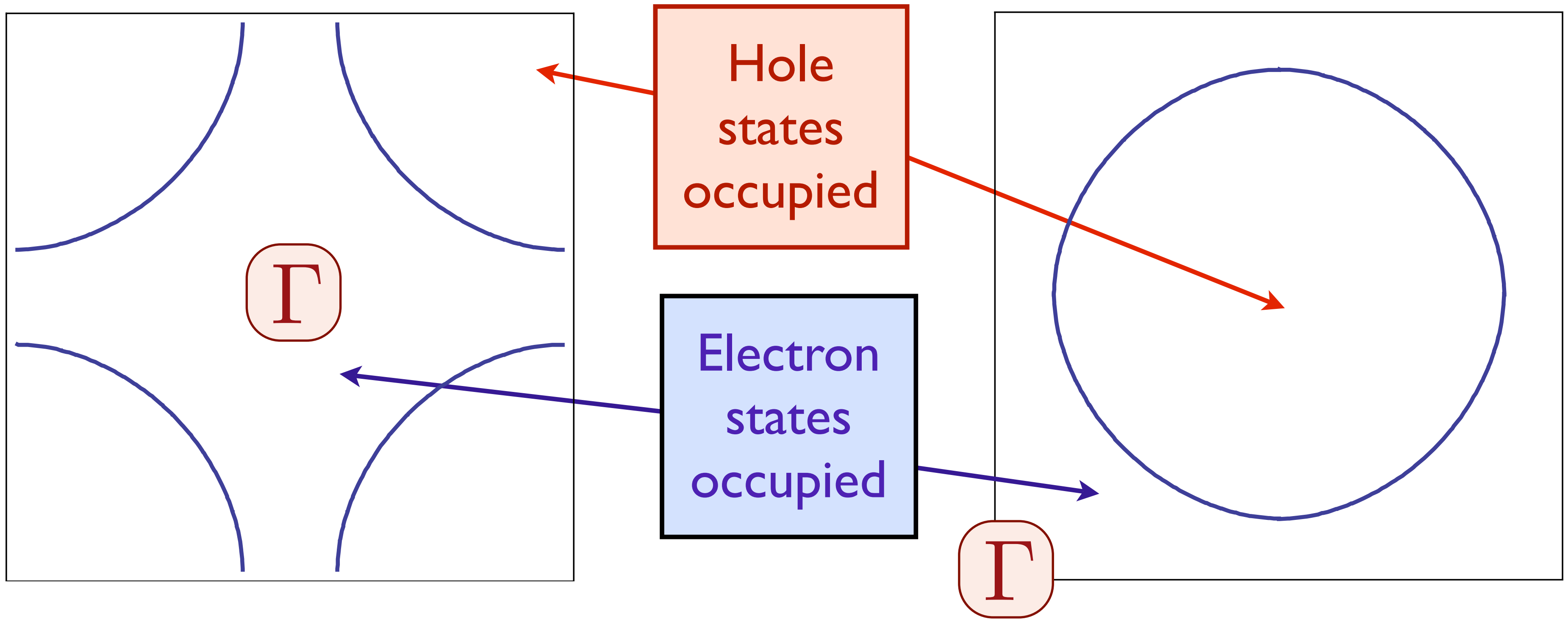
Large hole Fermi surface

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in **metal** at $x = x_m$

"Large" Fermi surfaces in cuprates



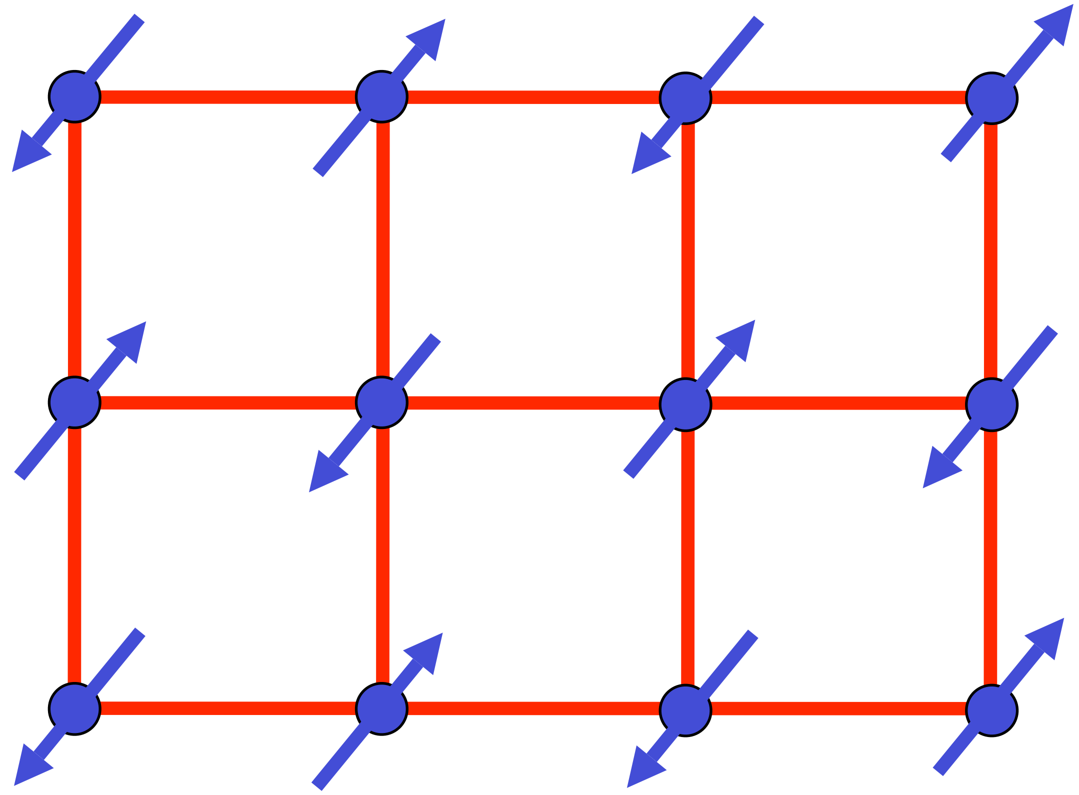
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$$

Spin density wave theory

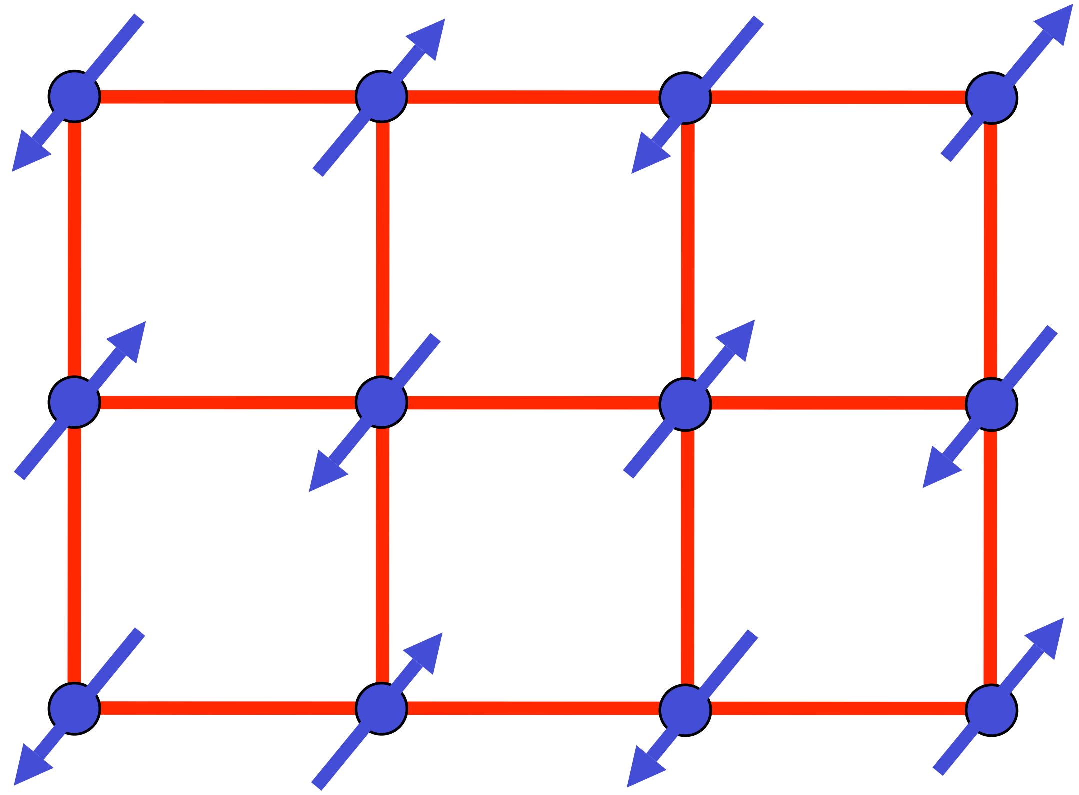


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and $\mathbf{K}$ is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



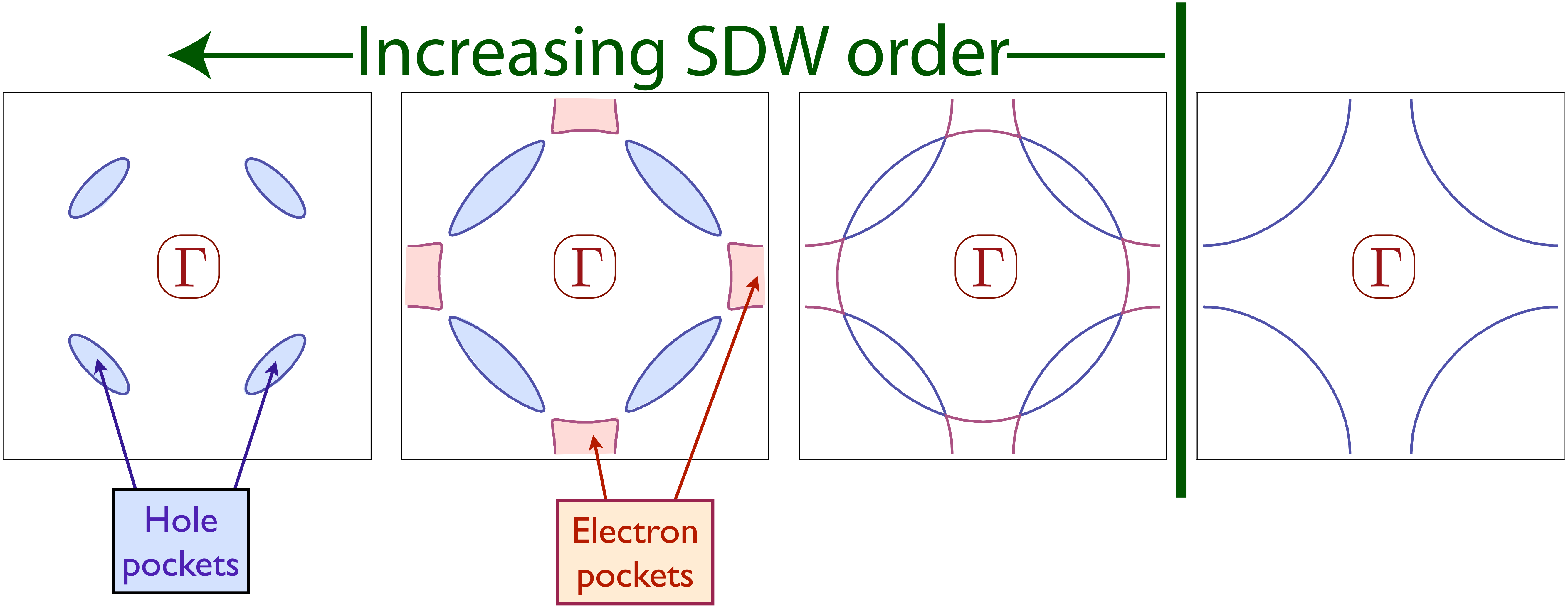
Spin density wave Hamiltonian

$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

Diagonalize $H_0 + H_{\rm sdw}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

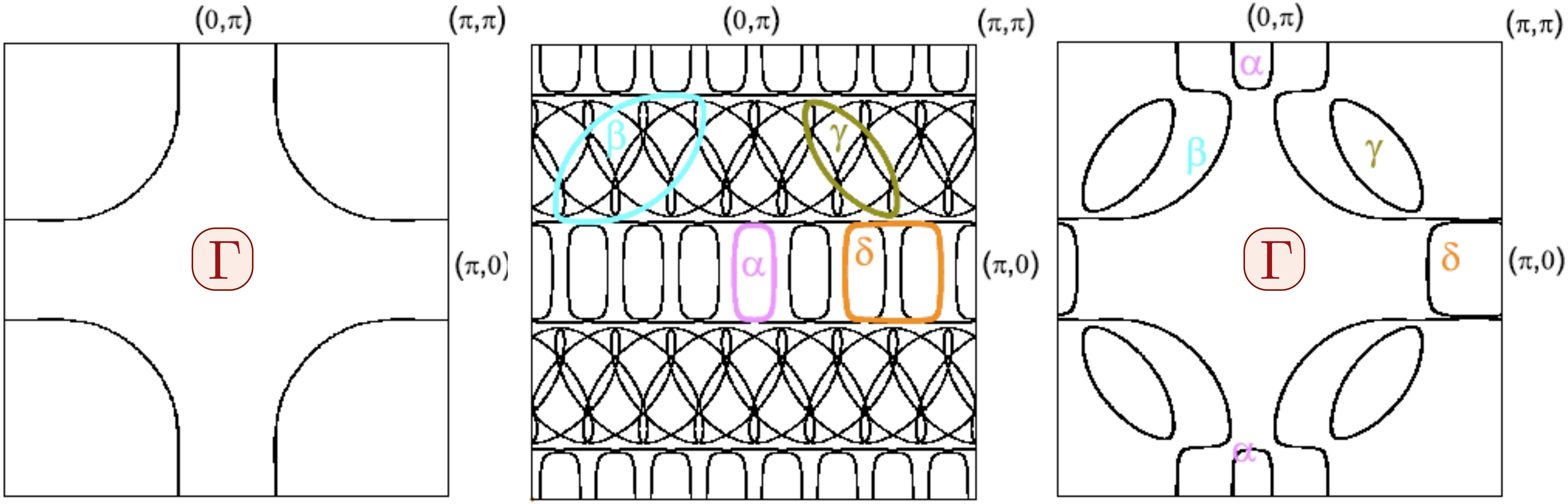
Spin density wave theory



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

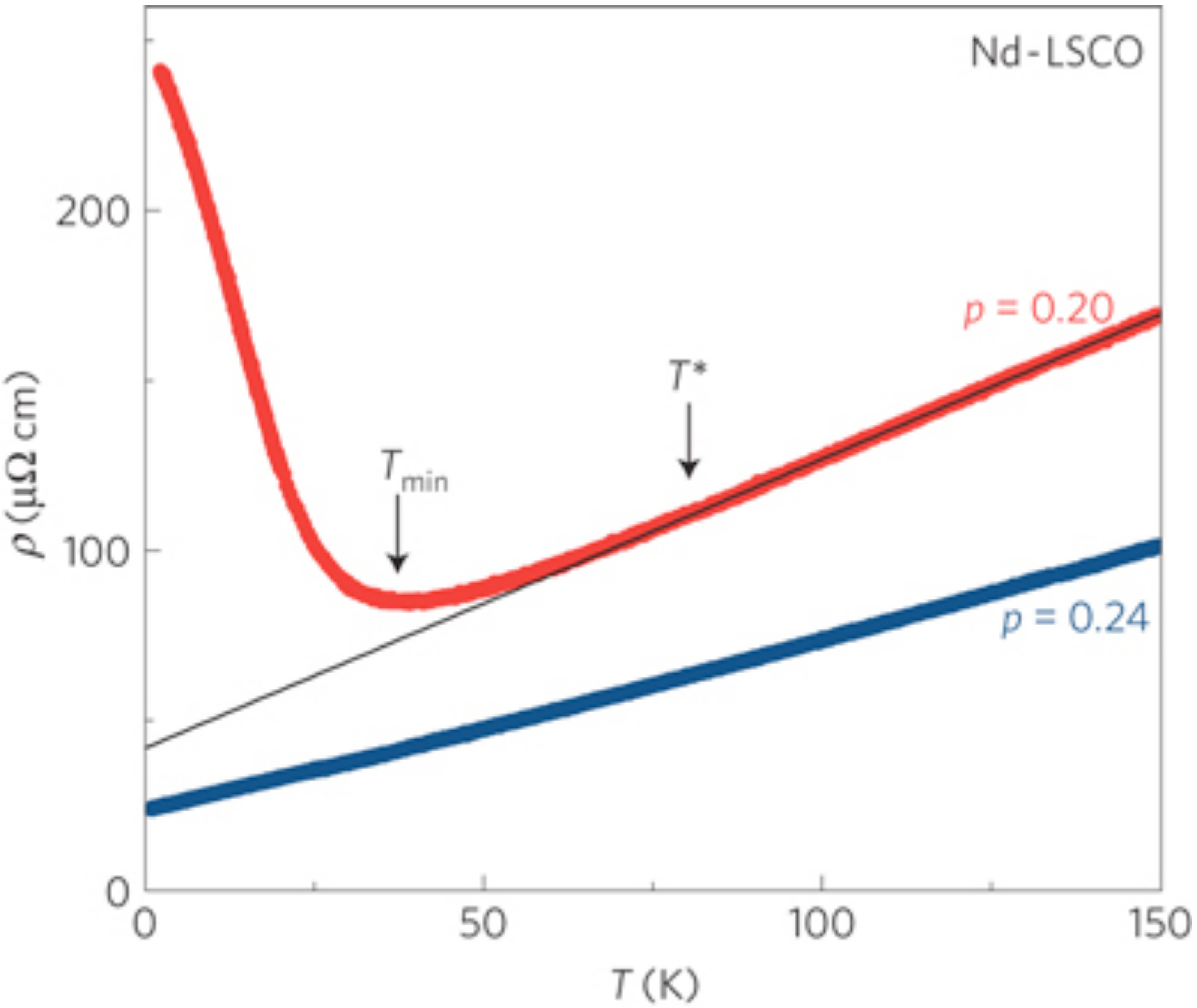
Spin density wave theory in hole-doped cuprates



Incommensurate order in $YBa_2Cu_3O_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

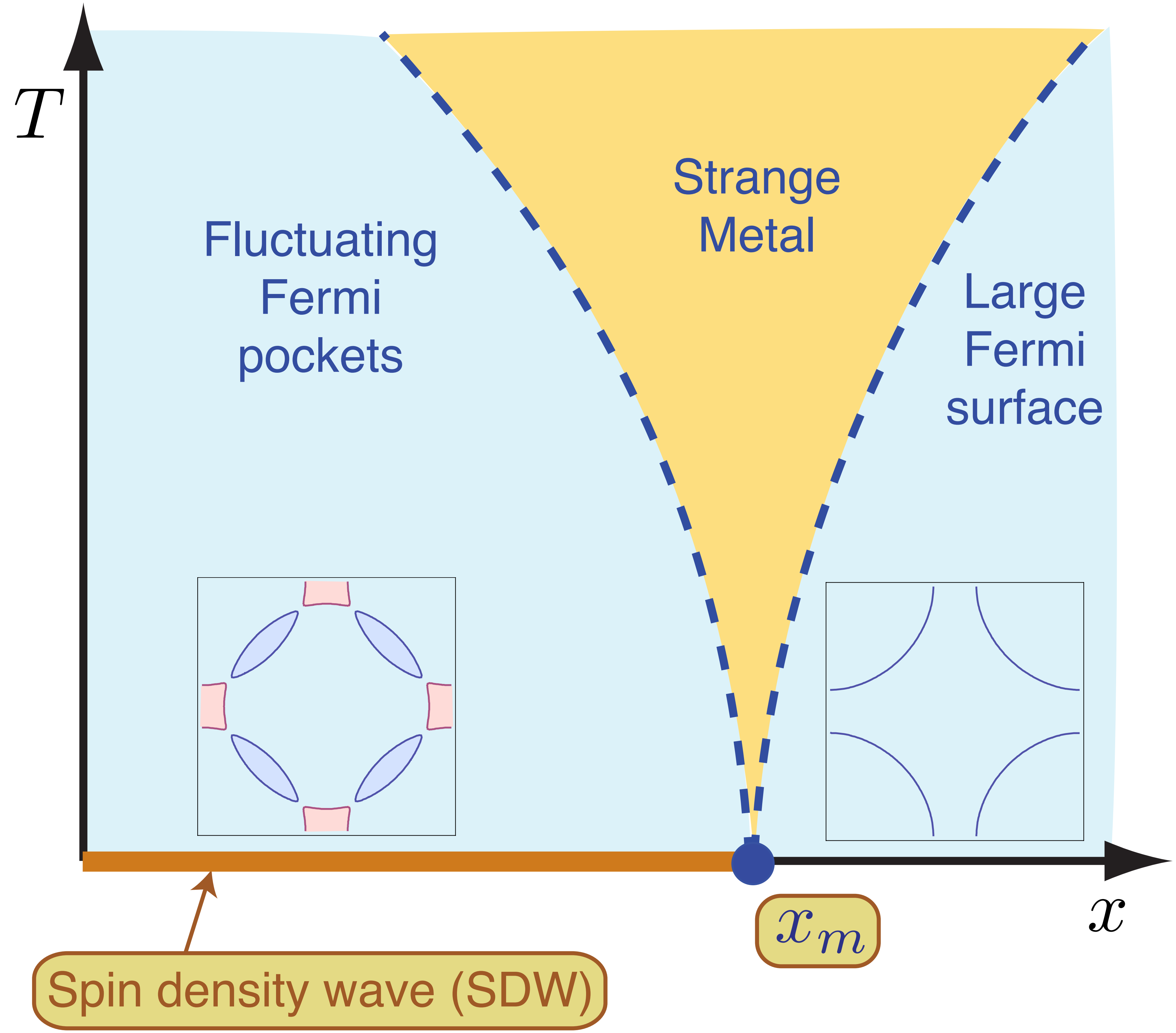
Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c



Magnetic field of upto 35 T used to suppress superconductivity

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor
R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

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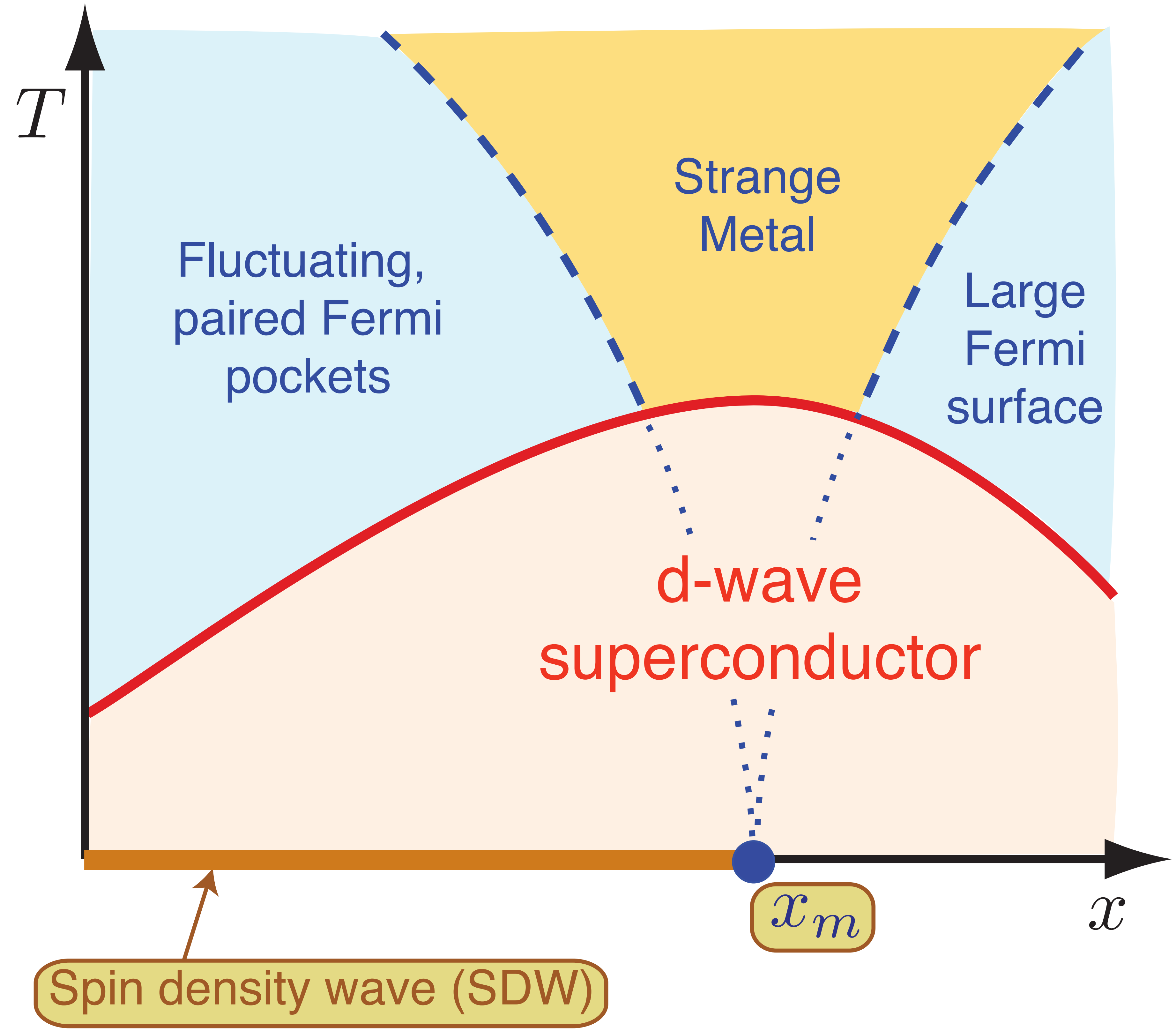
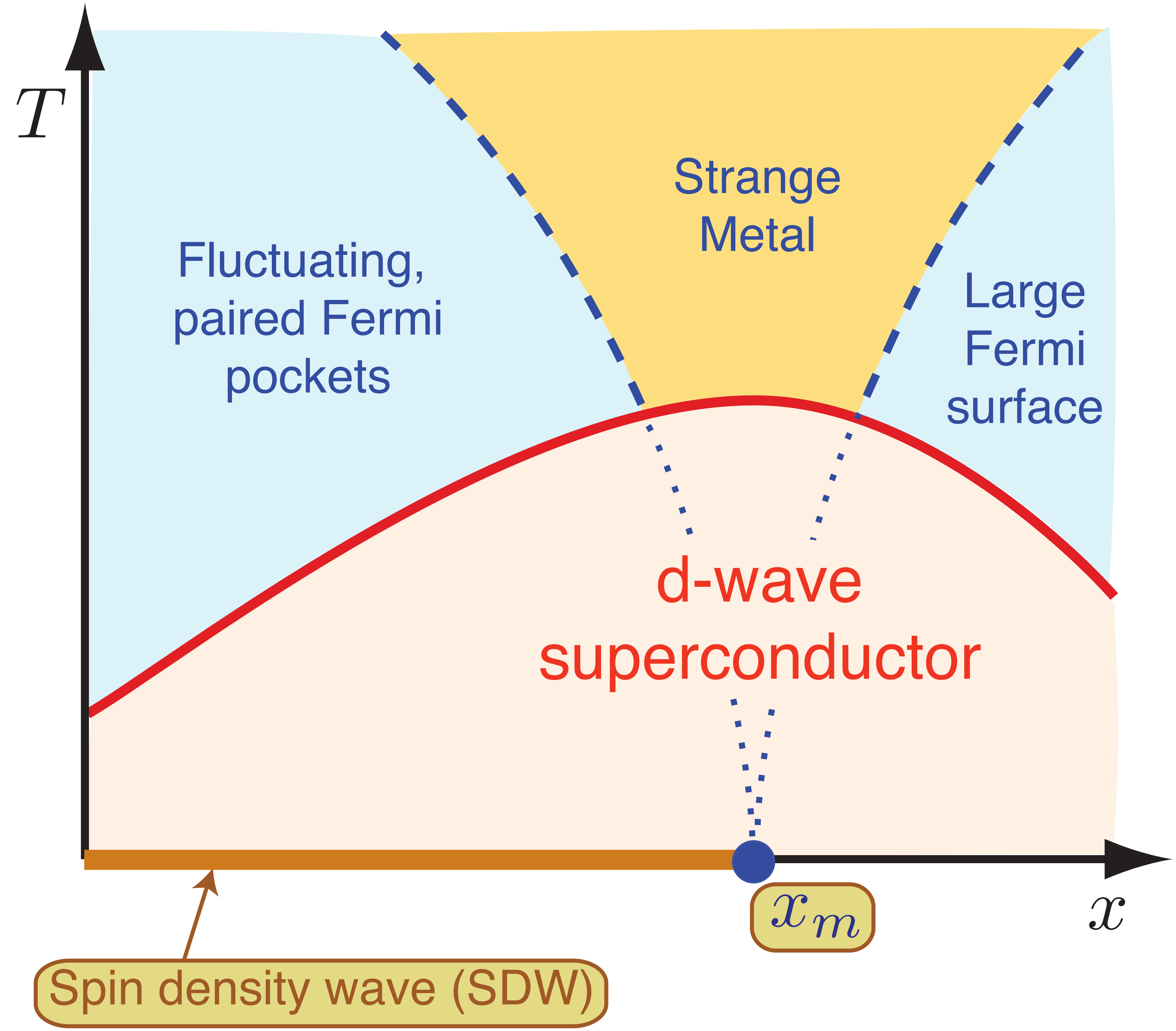


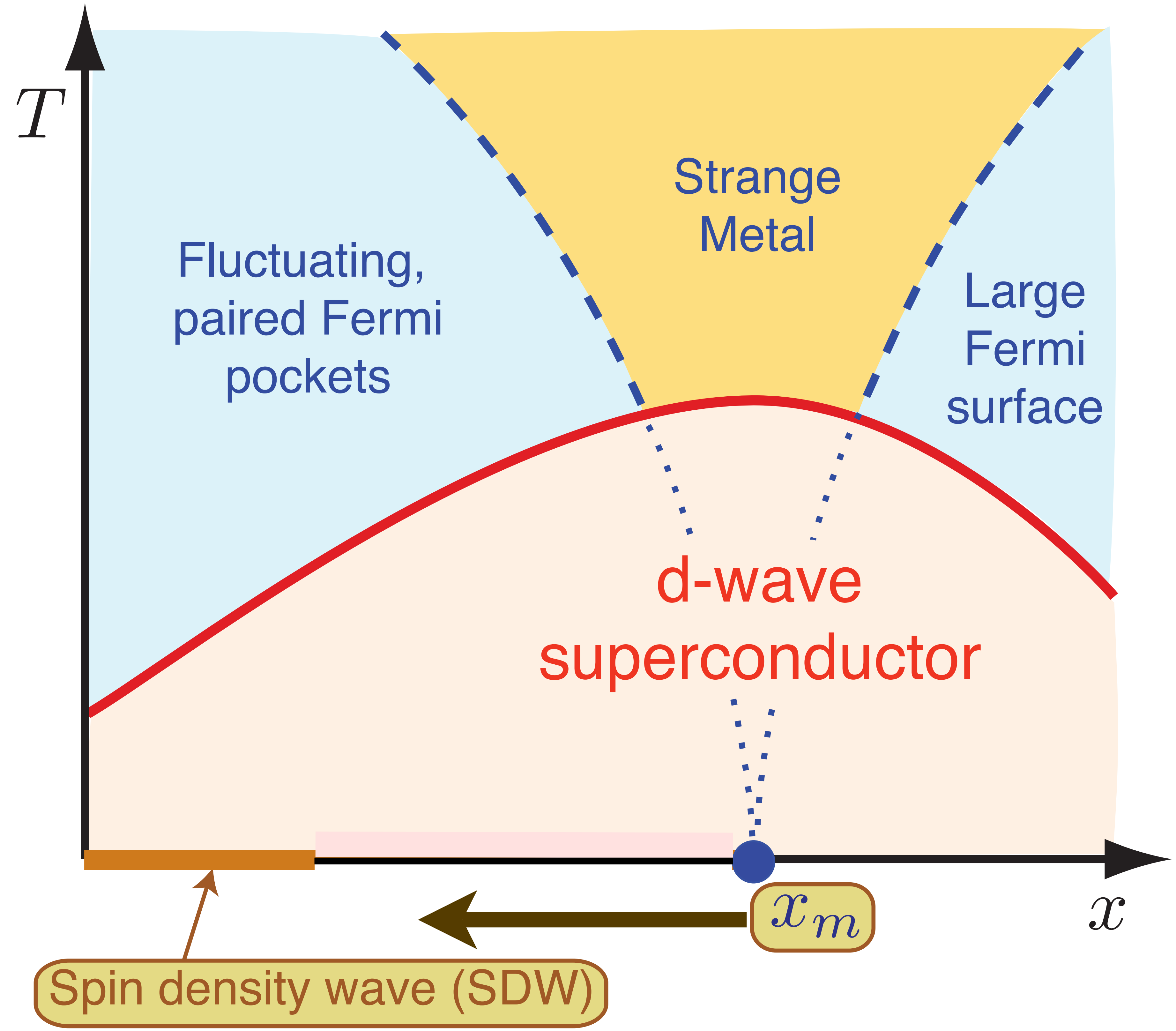
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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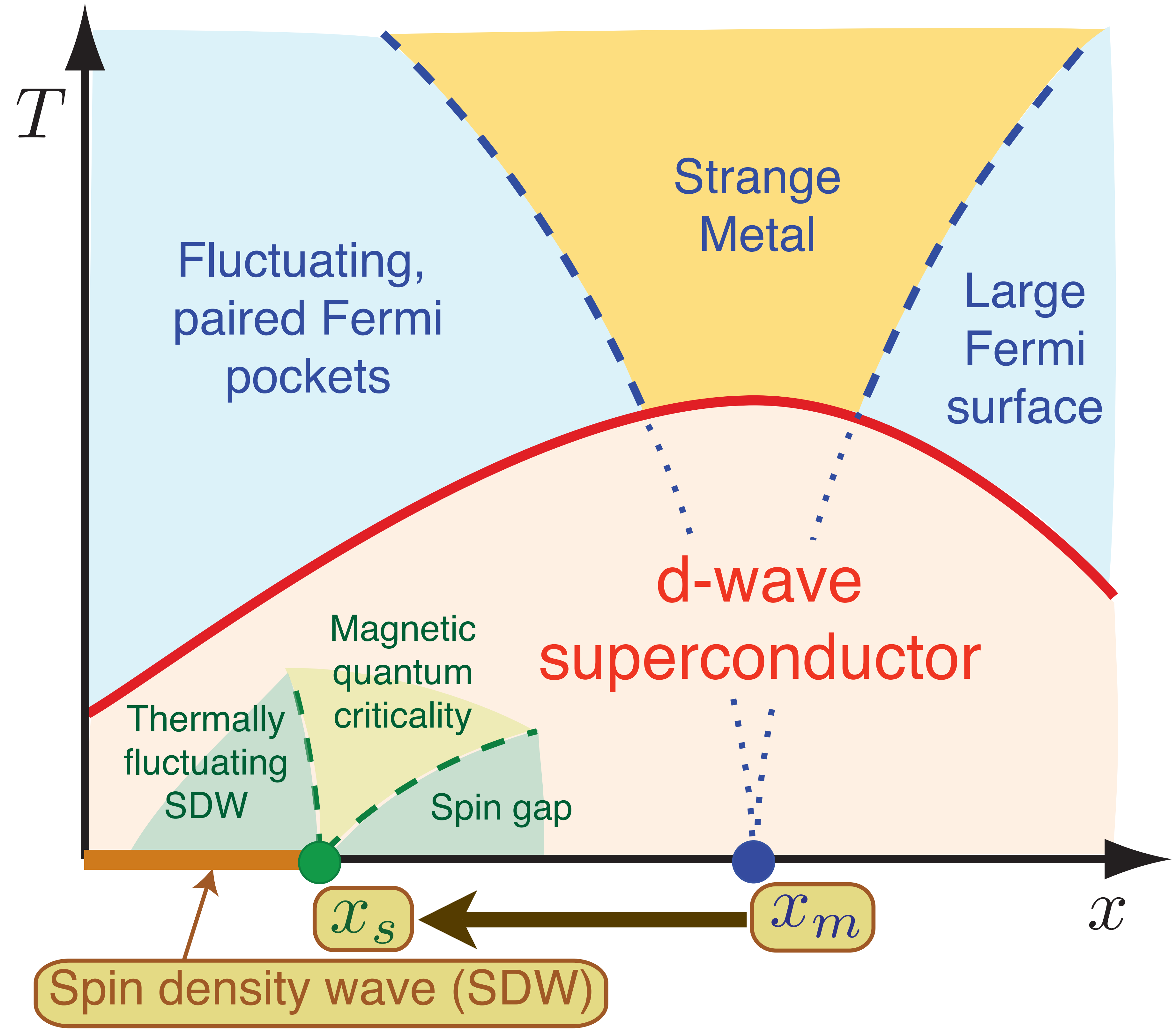

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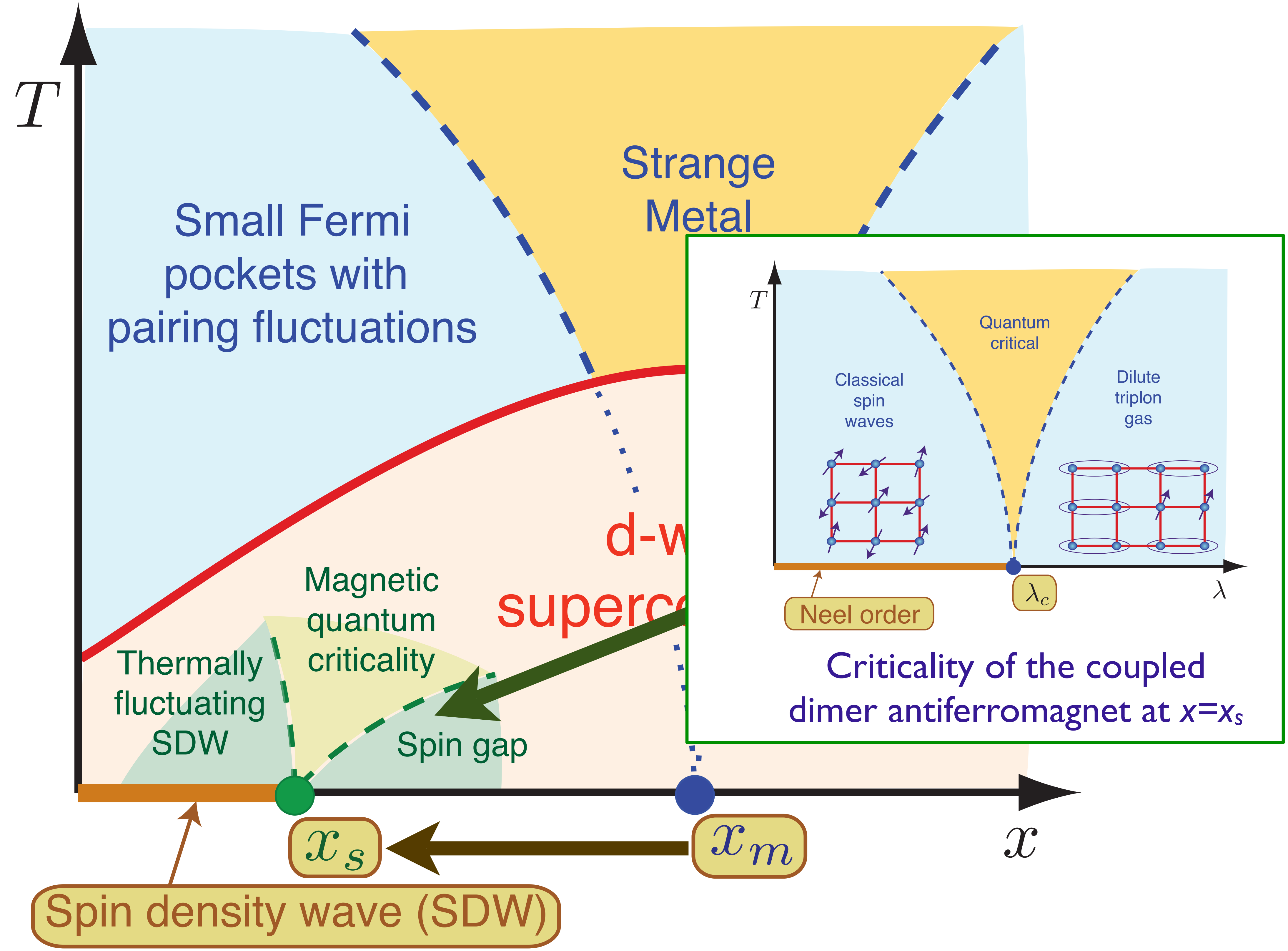

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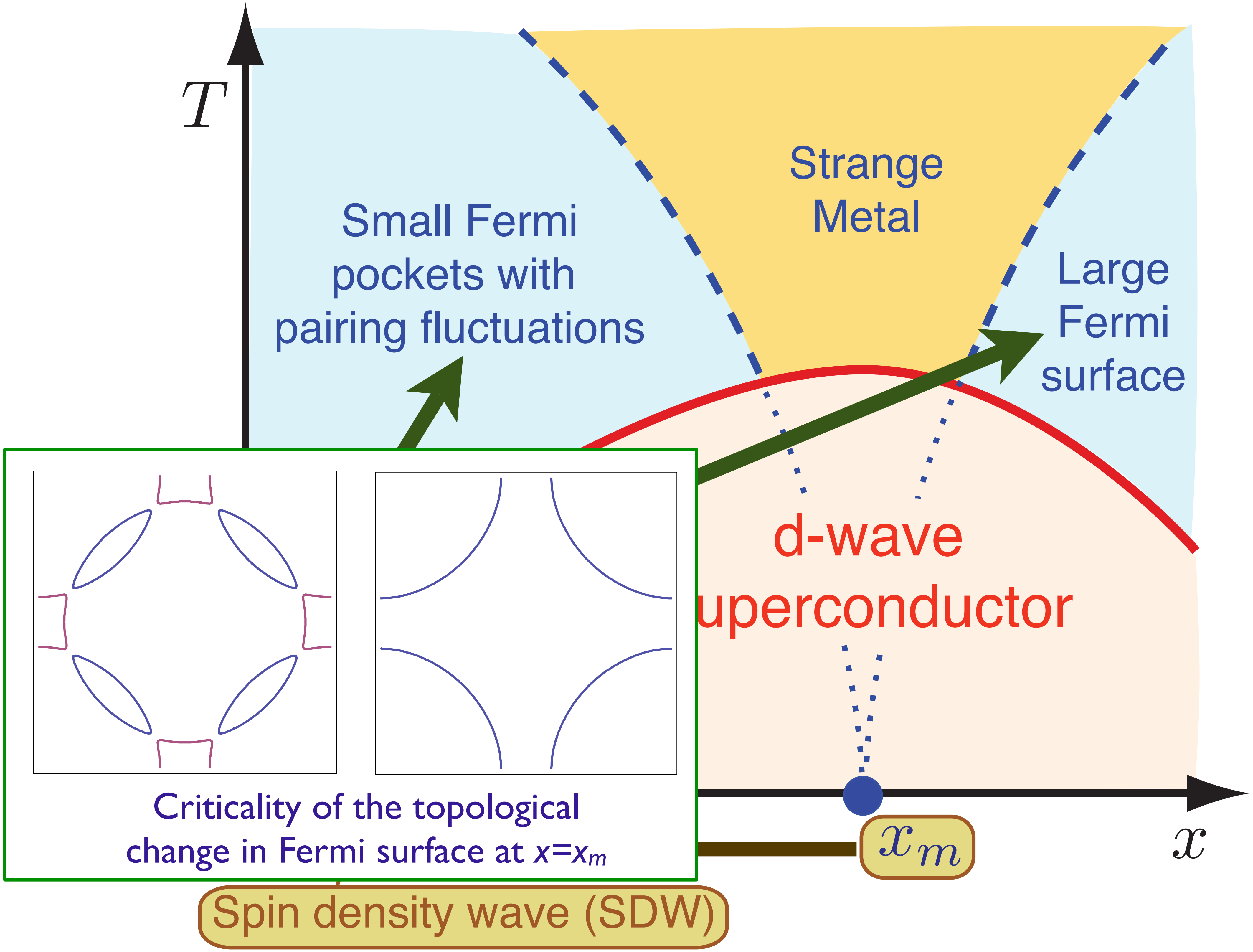

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Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order

   Survey of recent experiments

2. Overdoped vs. underdoped pairing

   Electronic theory of competing orders

3. Theory of SDW quantum critical point

   Dominance of planar graphs

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   Dominance of planar graphs

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 \right.$$

$$\left. + \kappa\, \vec{\varphi}^2 \left|\psi\right|^2 \right]$$

$$+ \int d^2 r \left[\left|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi\right|^2 - \left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516 (2002).

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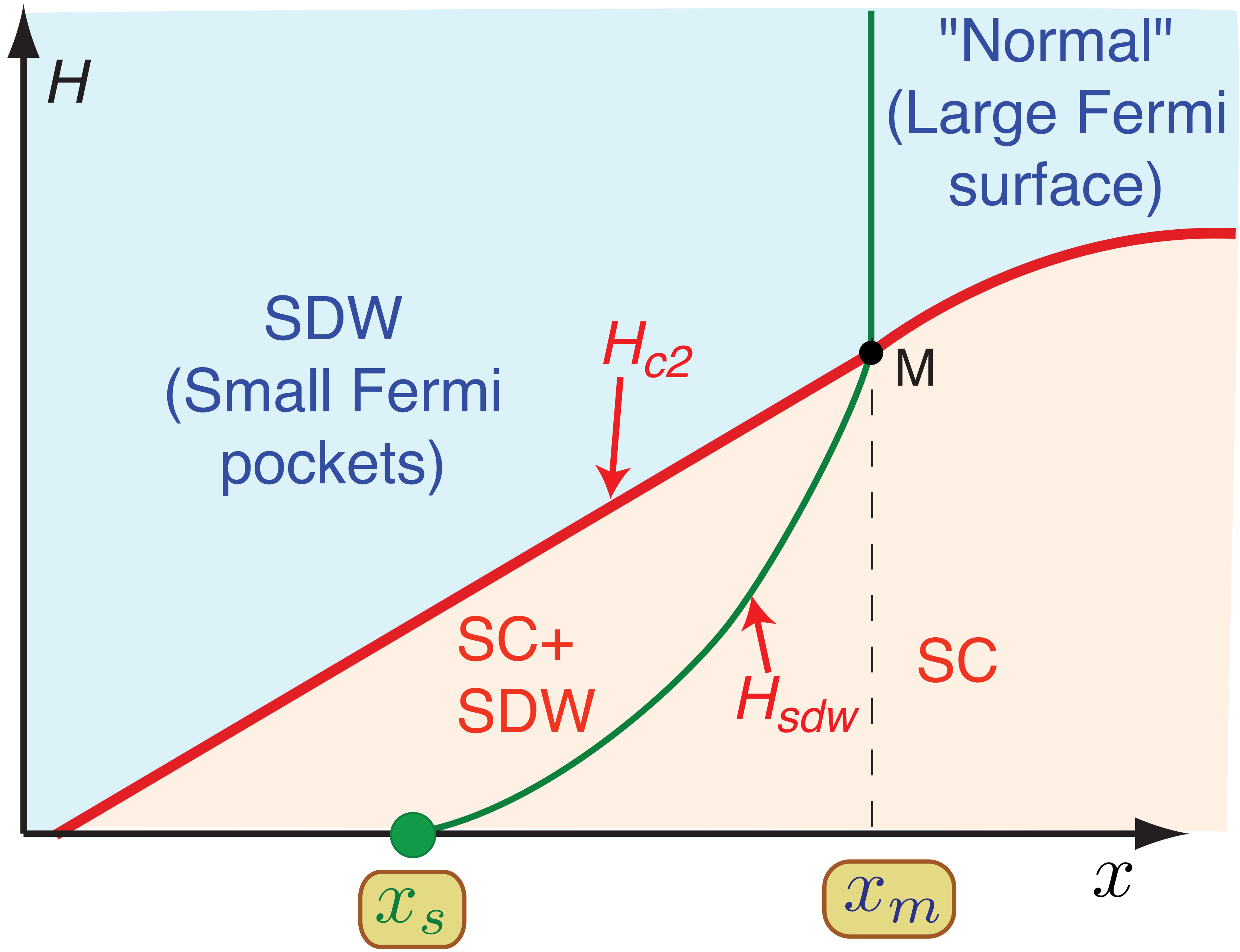

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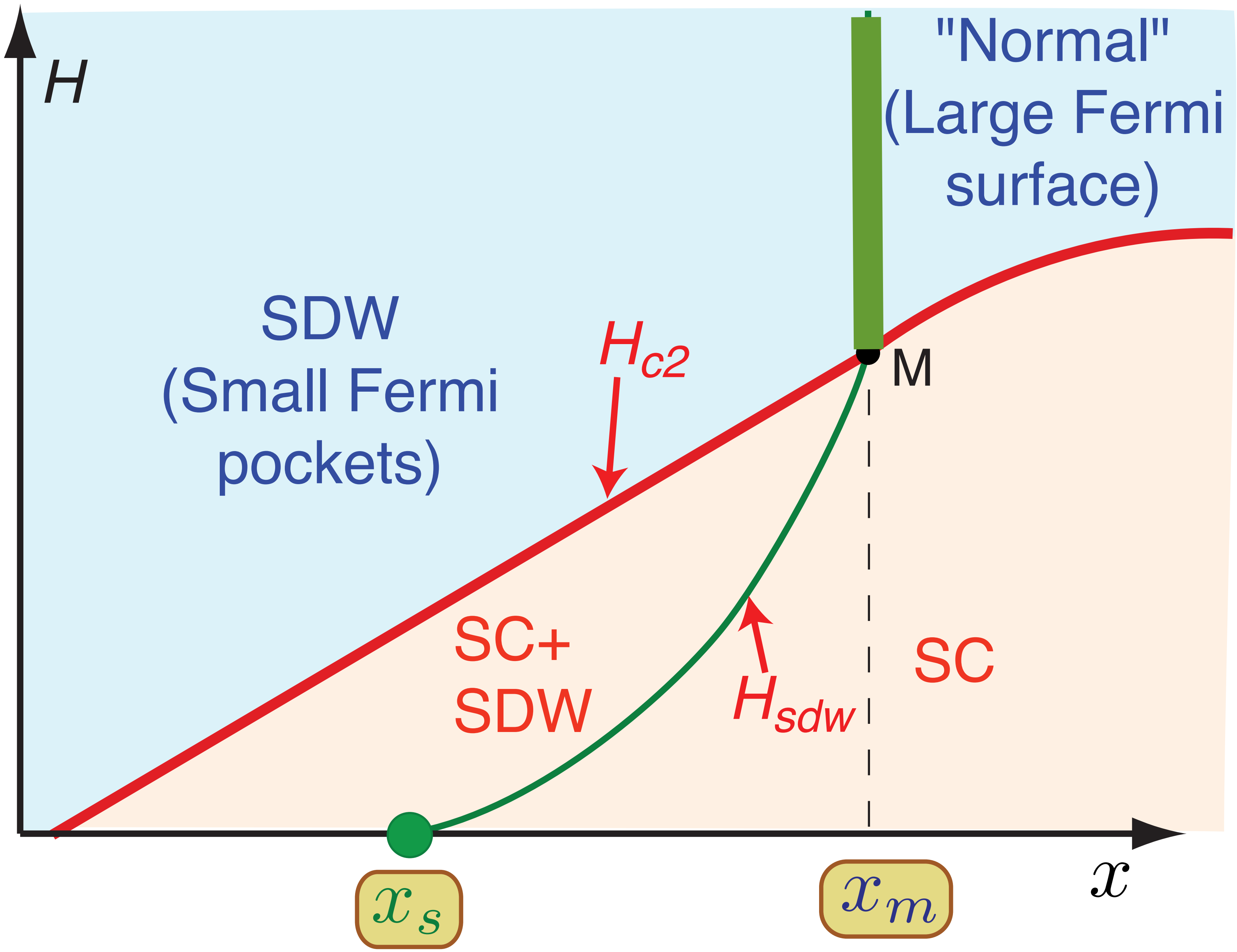

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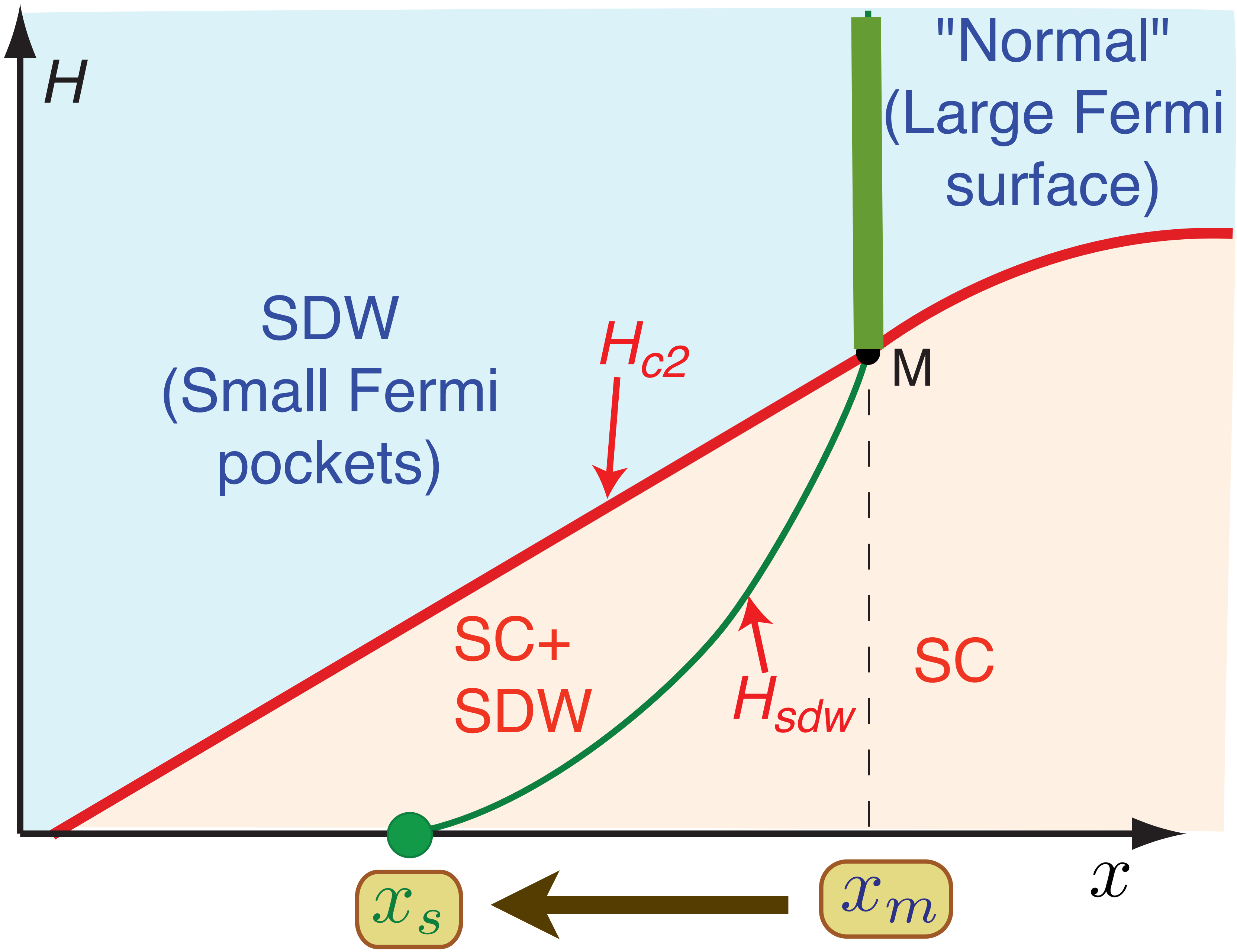

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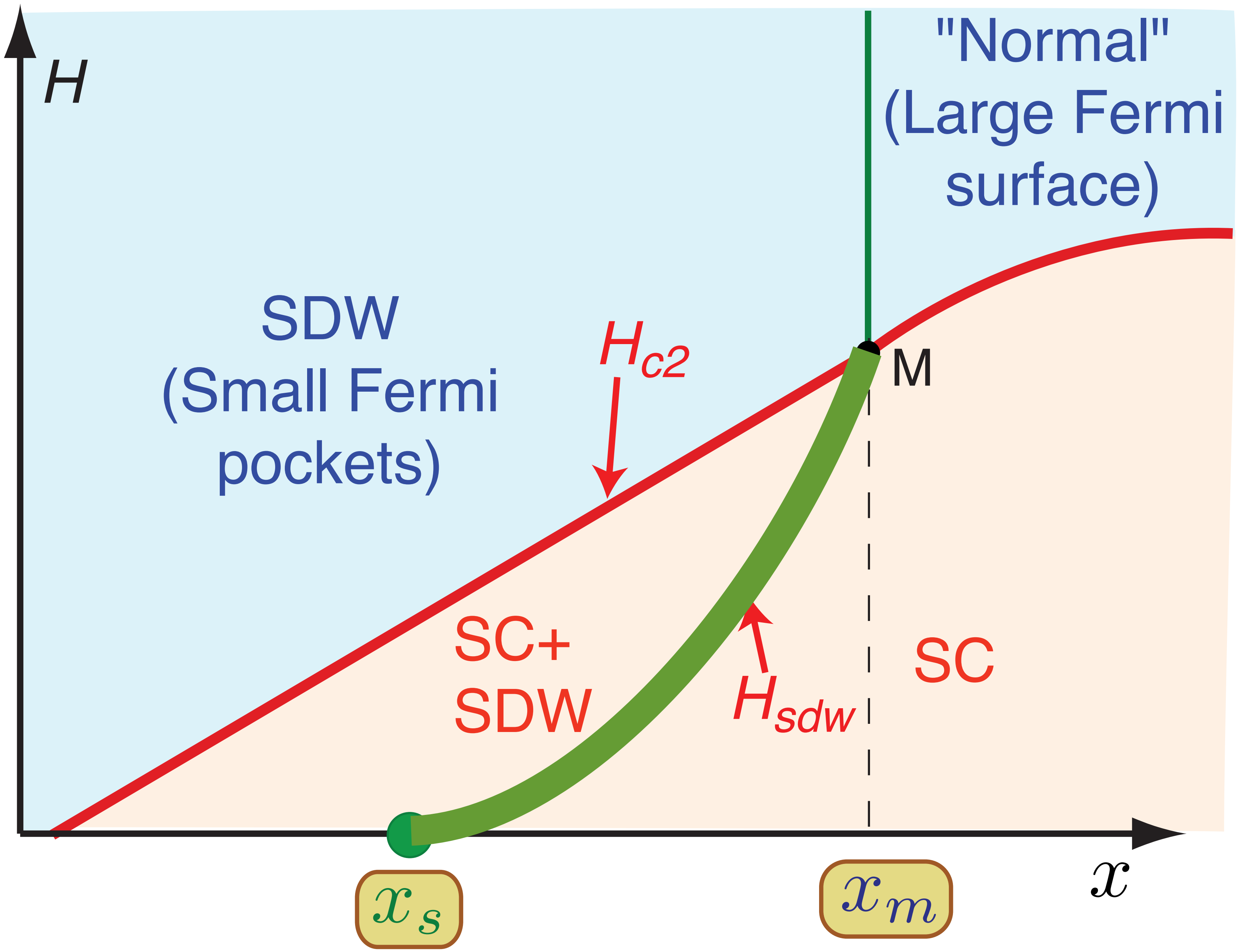

- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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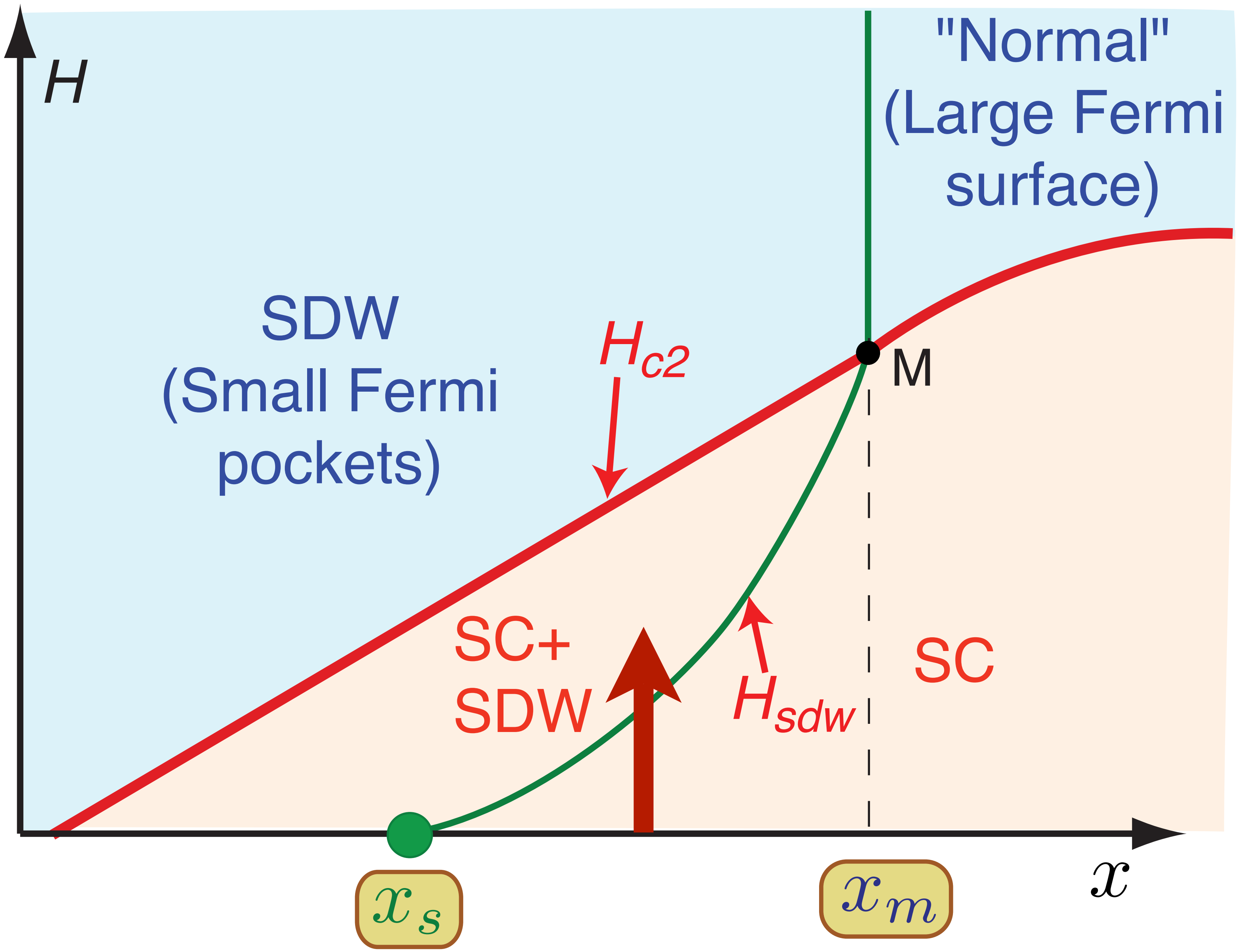

Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
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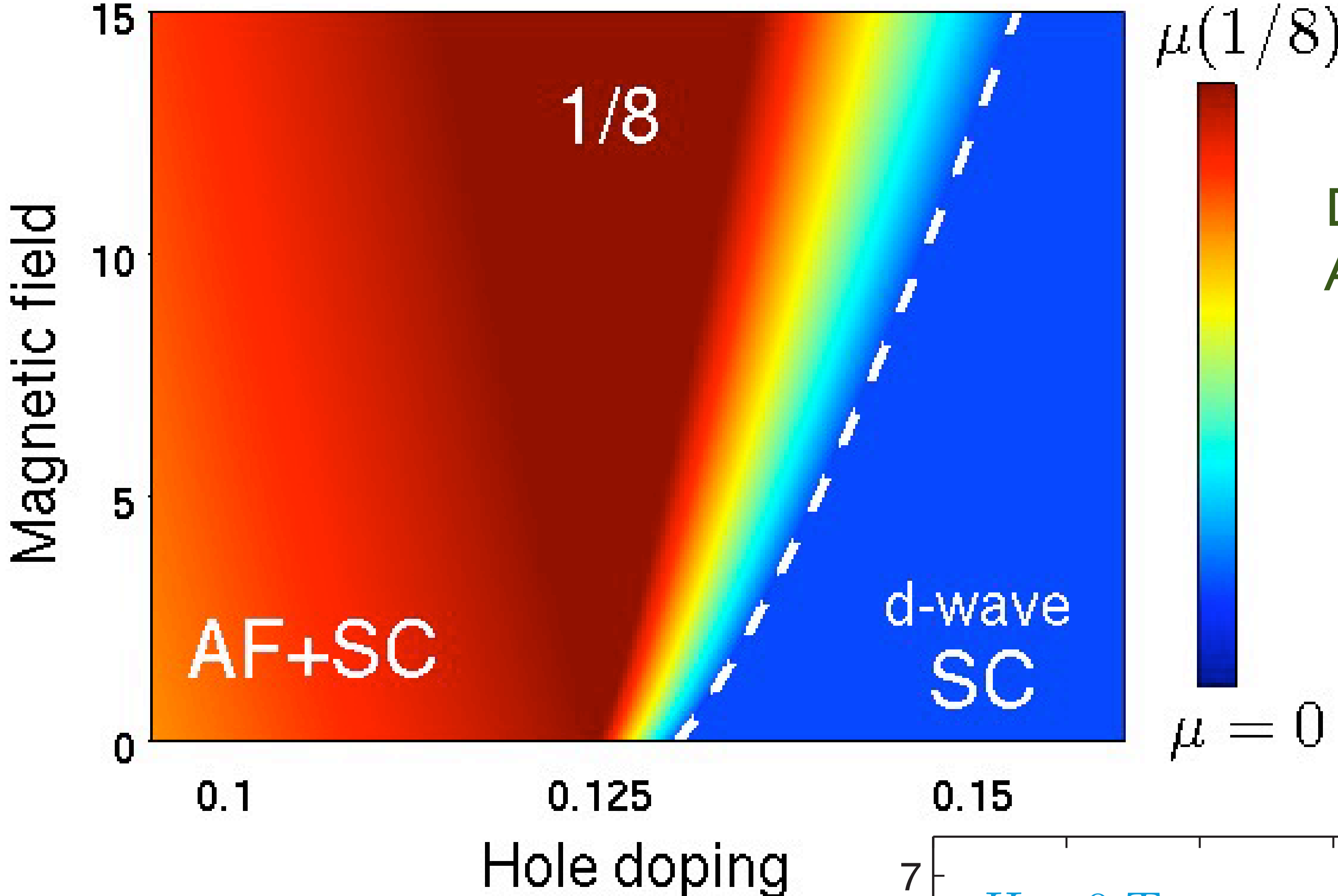
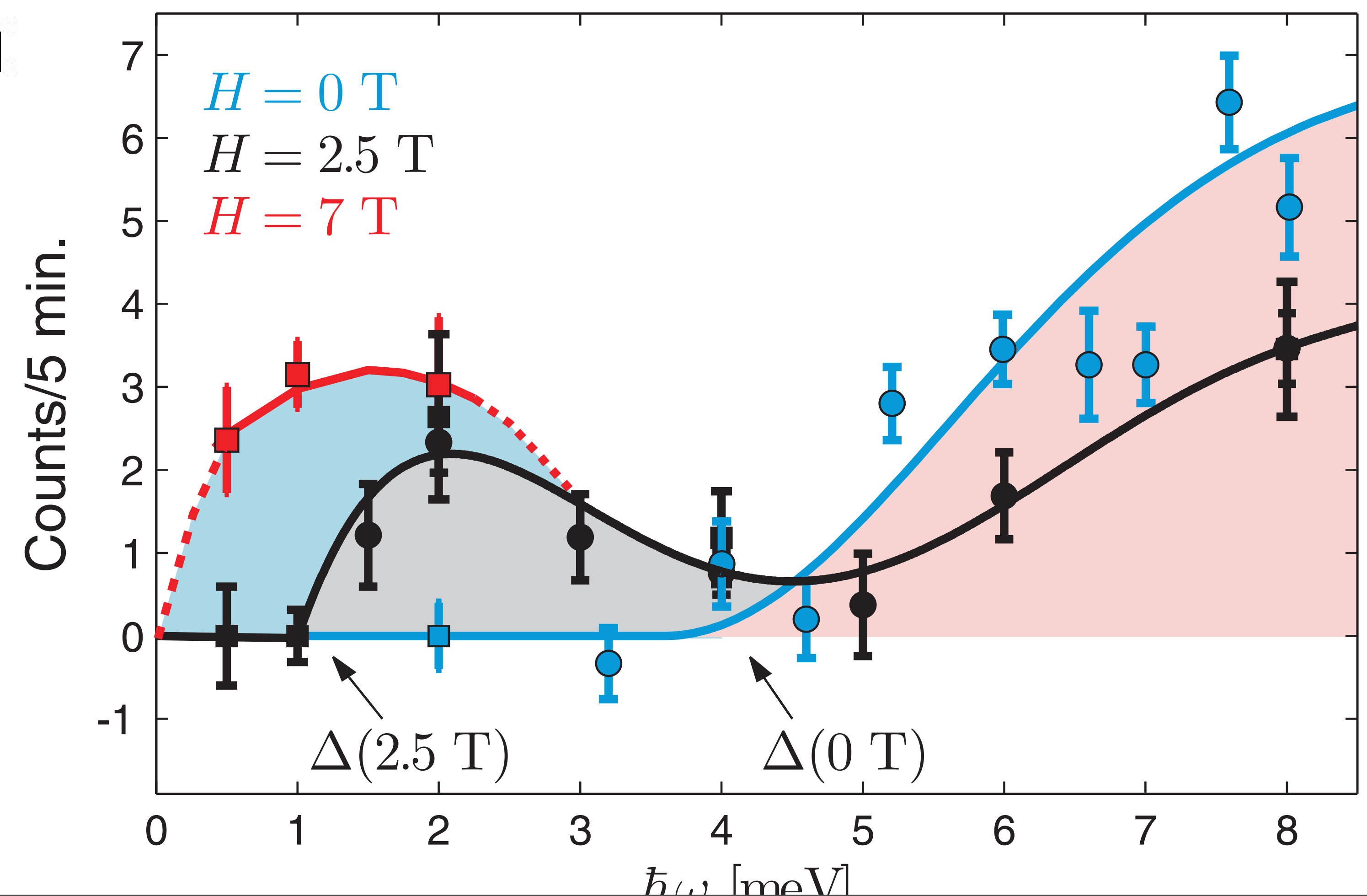


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



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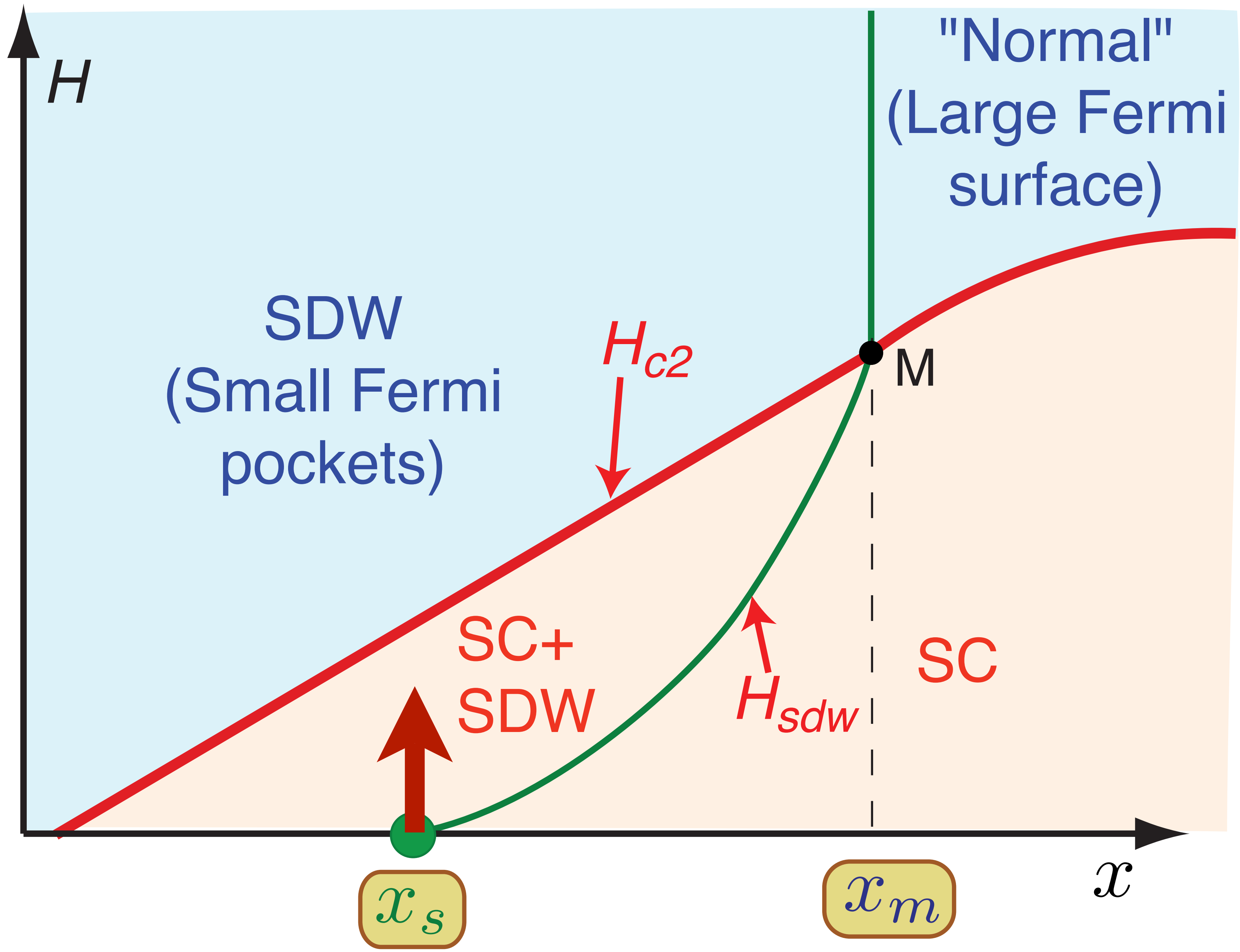

Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, *Phys. Rev. Lett.* **103**, 017001 (2009).

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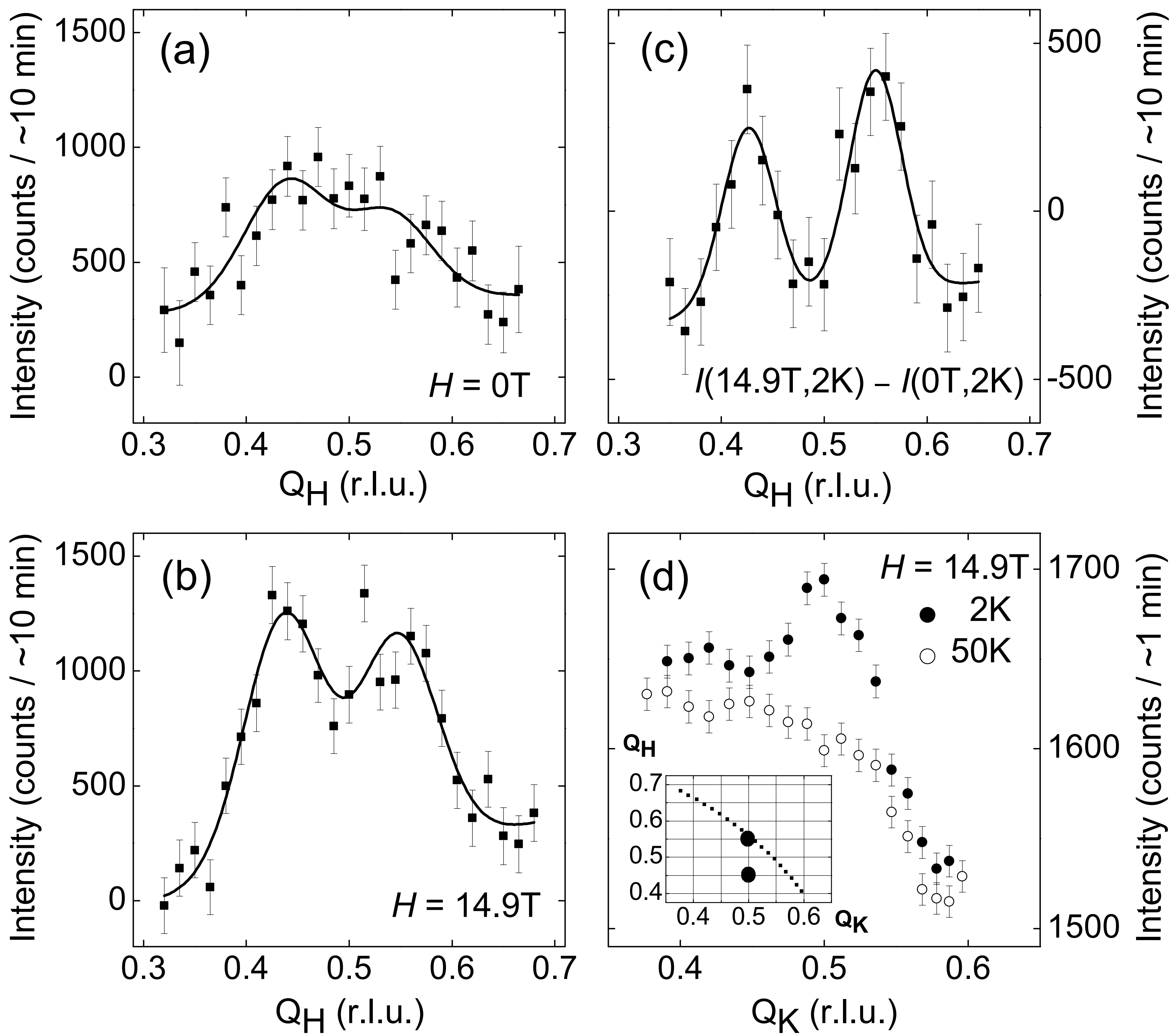

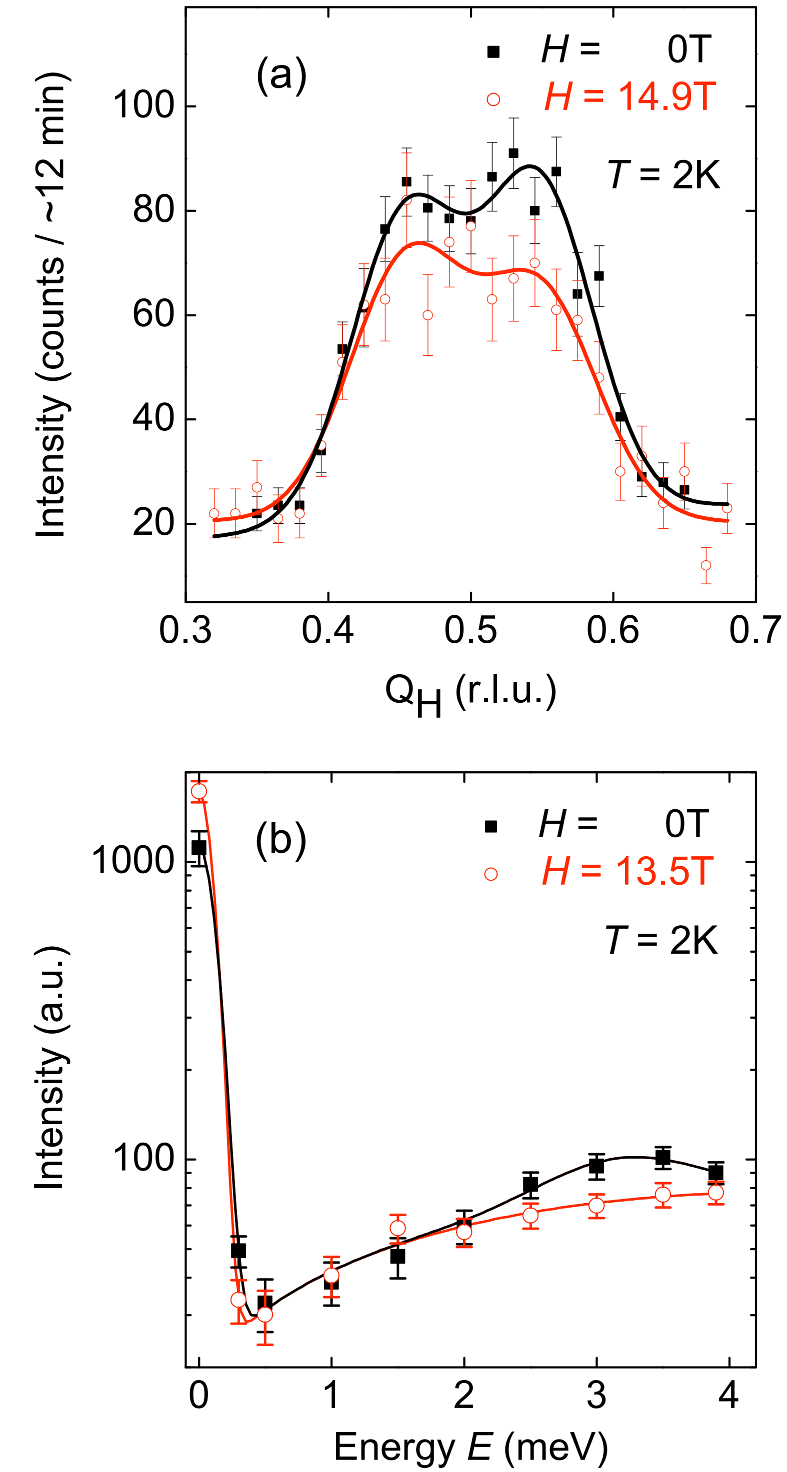

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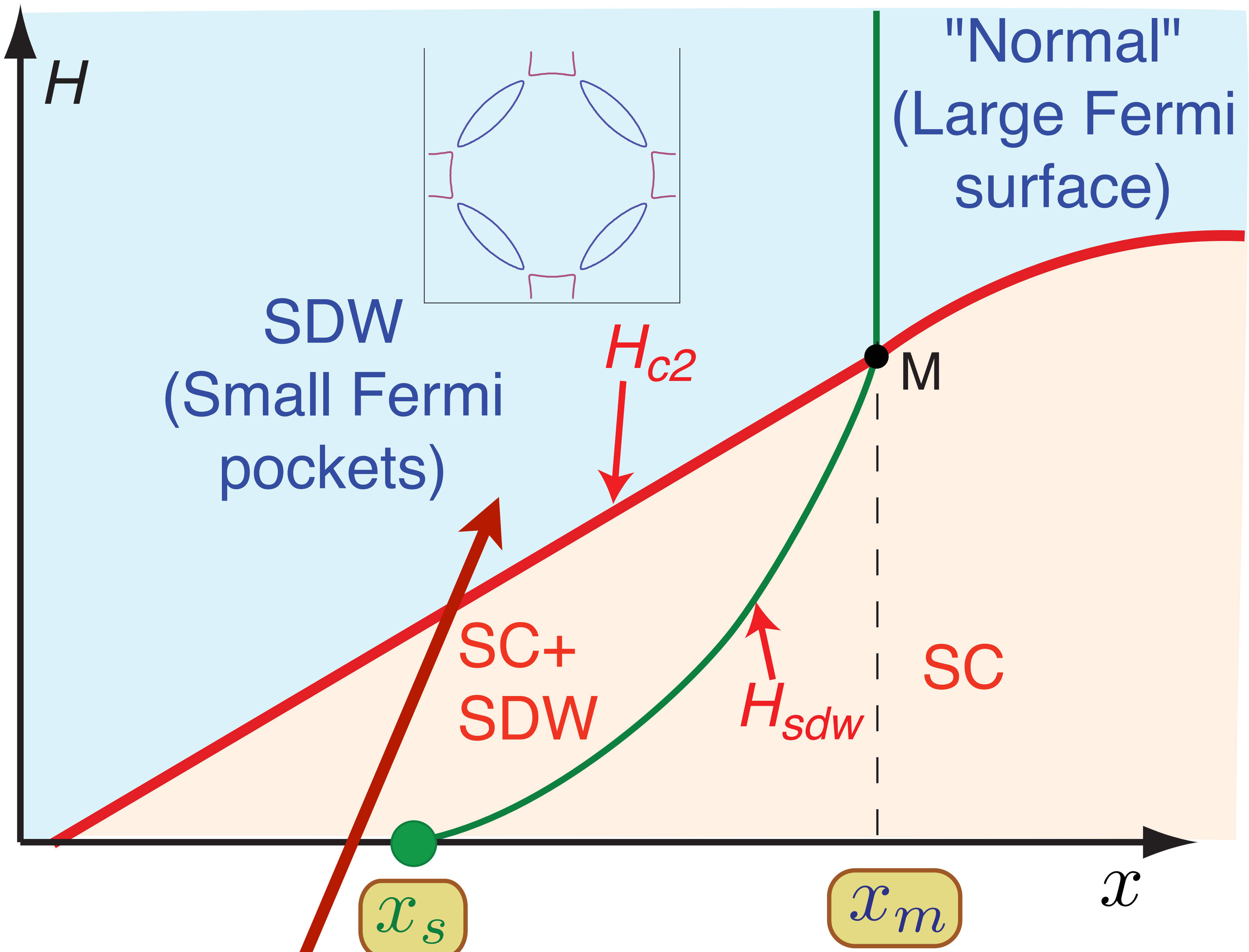


Quantum oscillations without Zeeman splitting

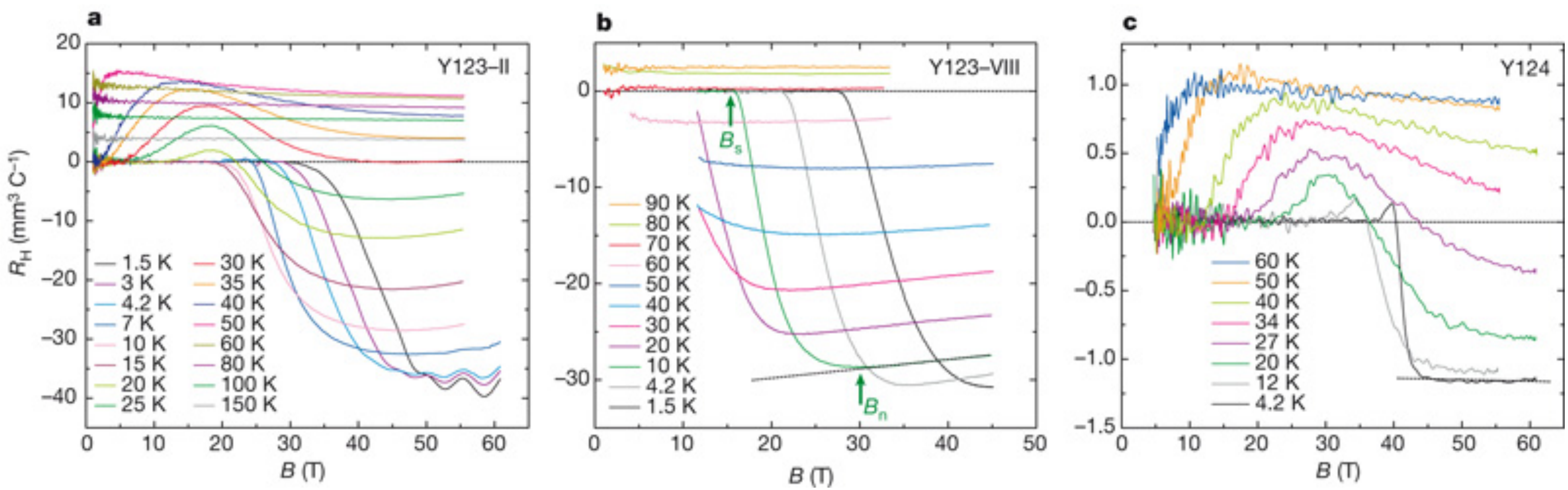
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007).
S. E. Sebastian, N. Harrison, C. H. Mielke, Ruixing Liang, D. A. Bonn, W.~N.~Hardy, and G. G. Lonzarich, arXiv:0907.2958

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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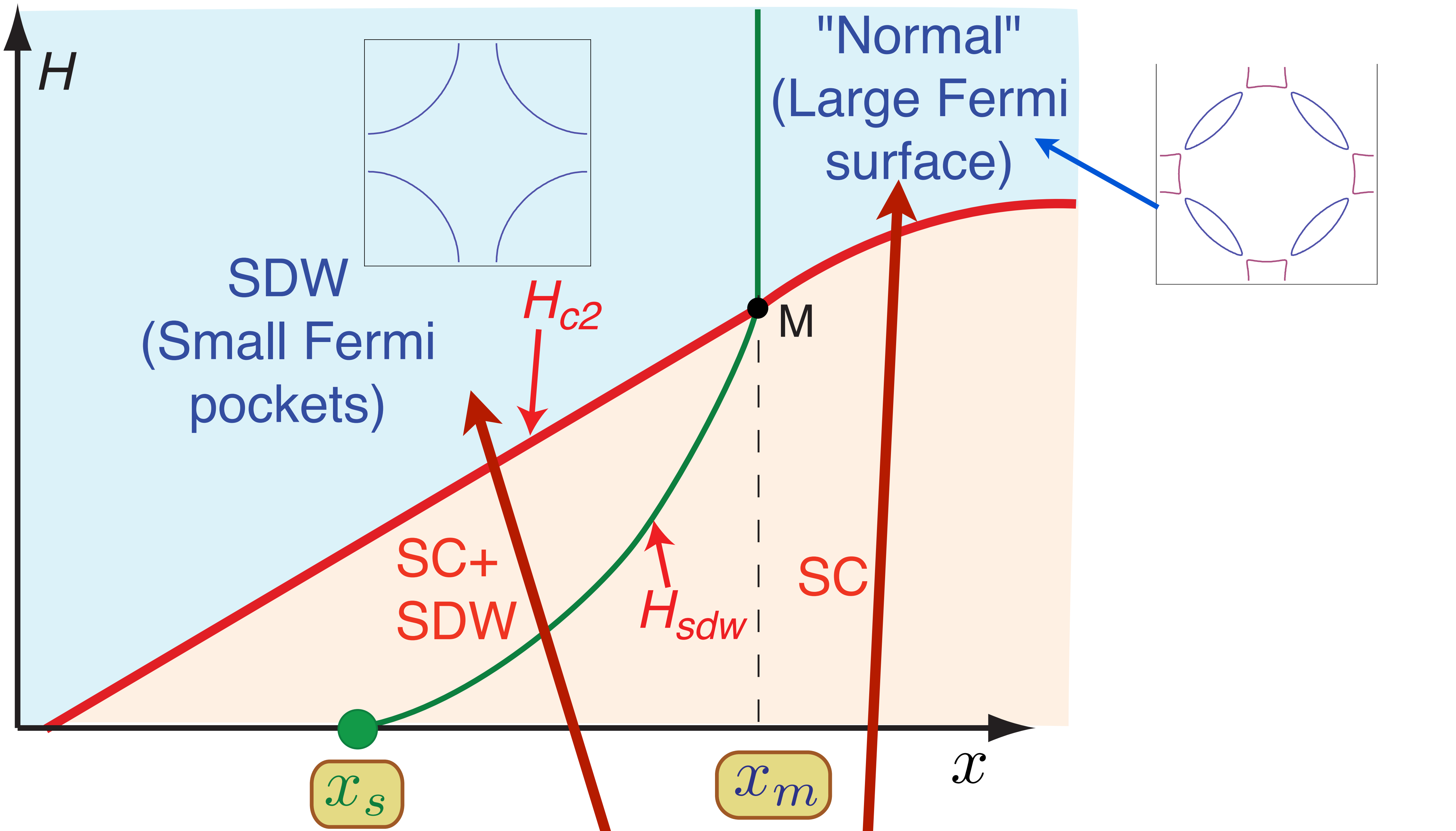
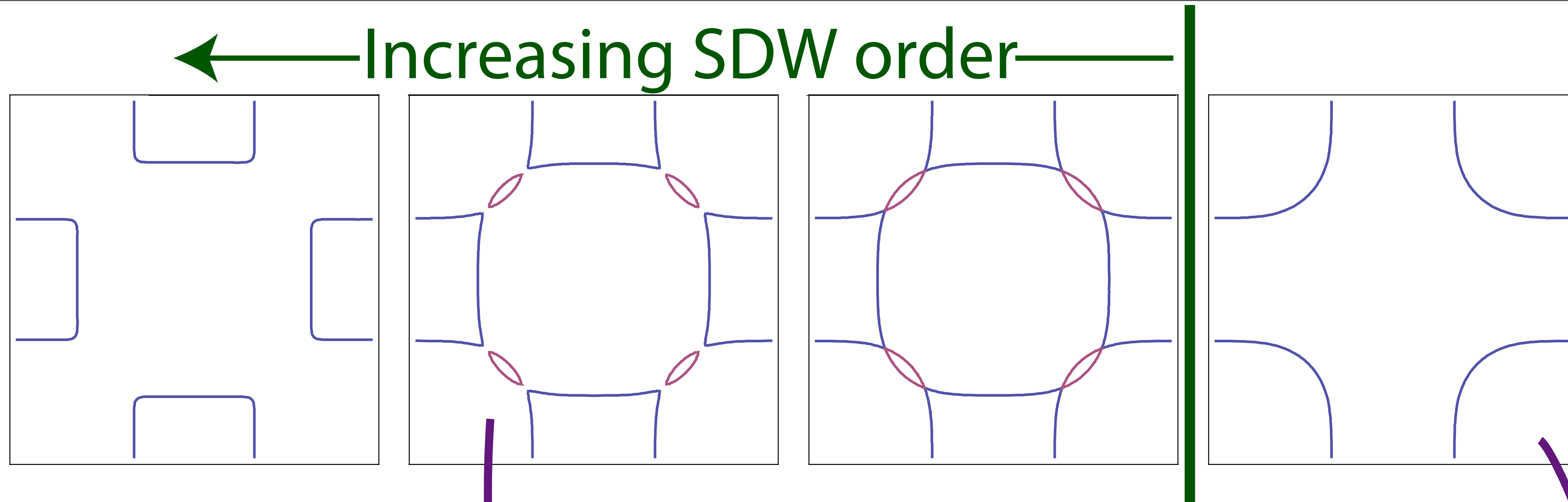
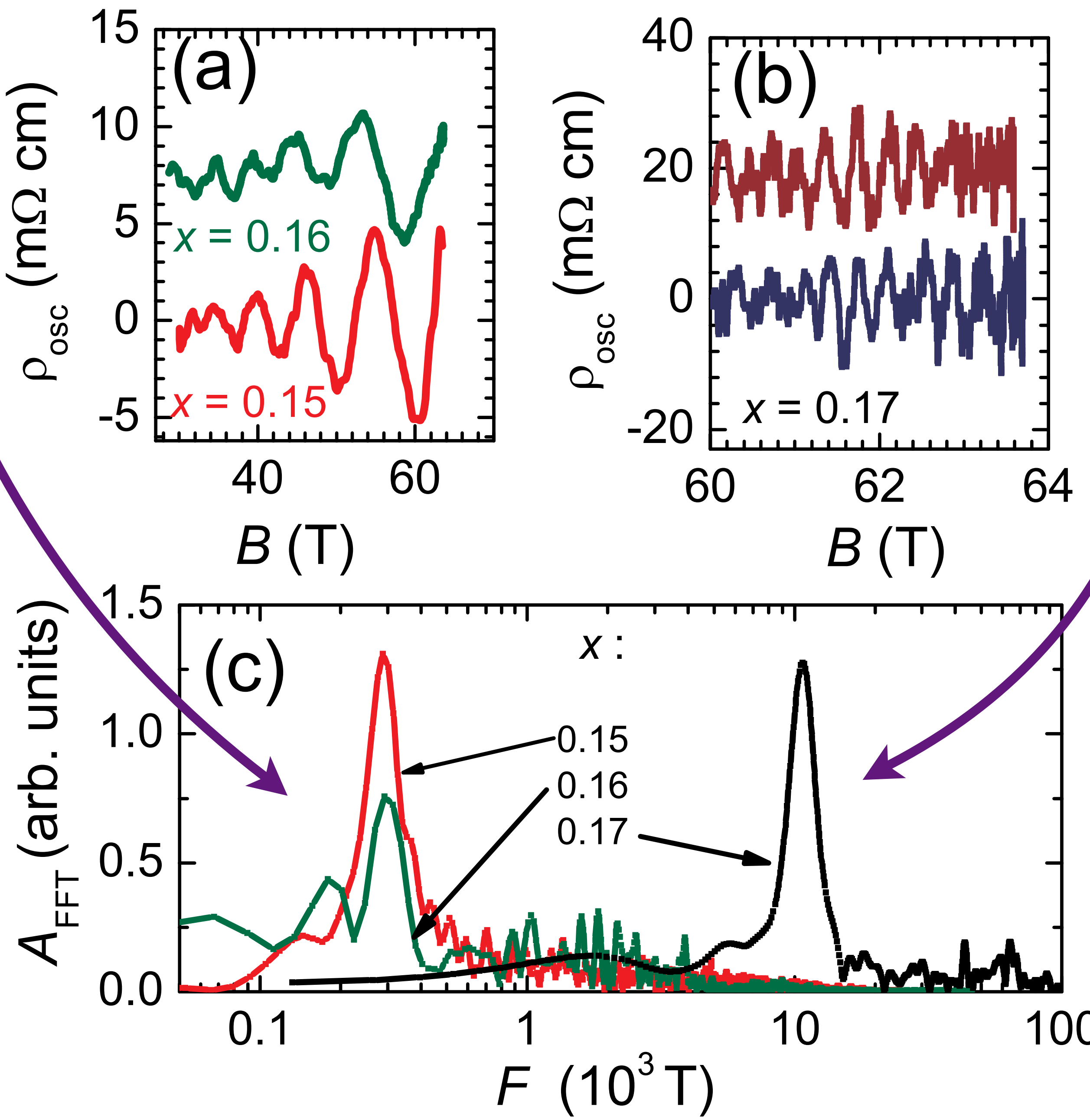


Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$

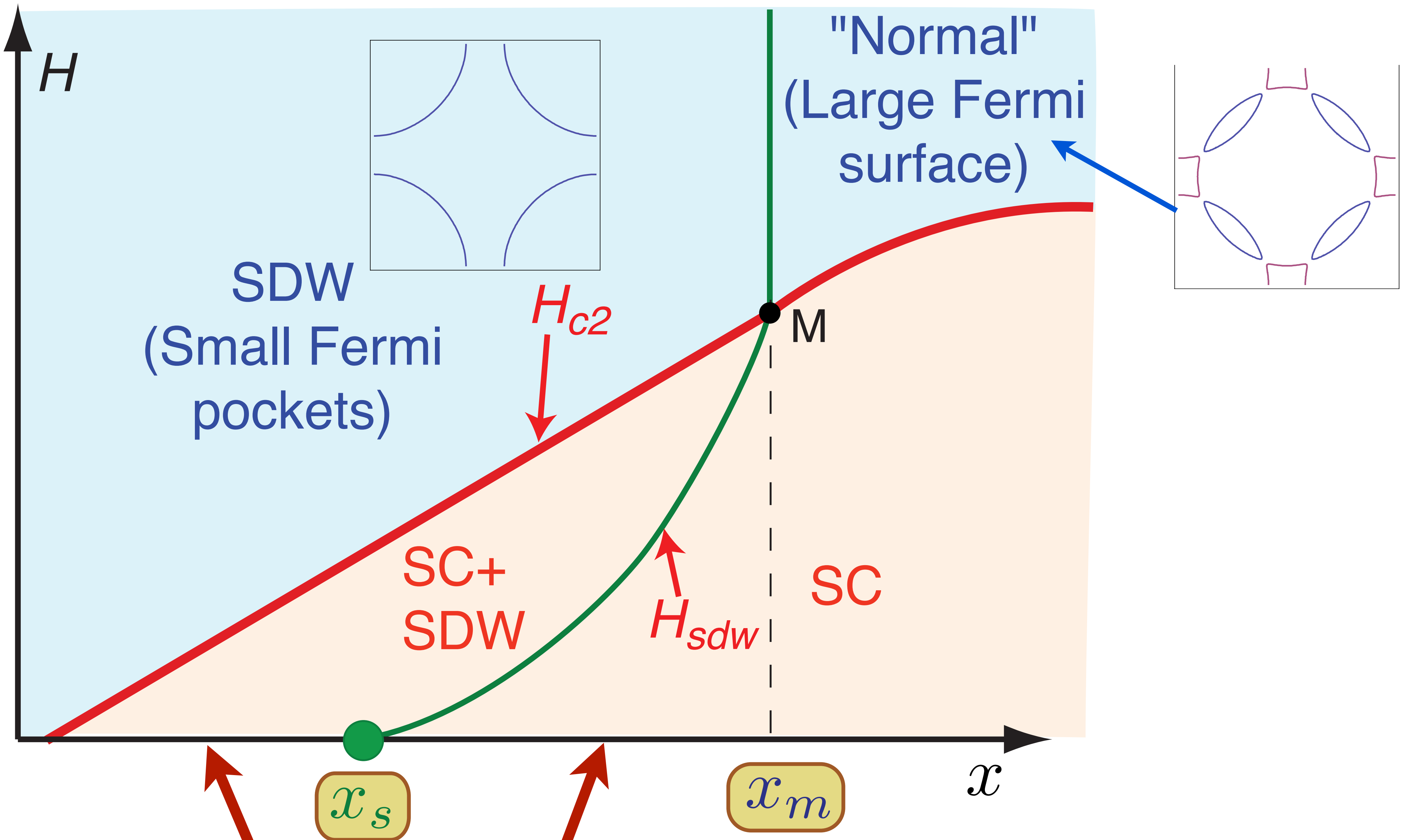




$\mathrm{Nd}_{2-x}\mathrm{Ce}_x\mathrm{CuO}_4$

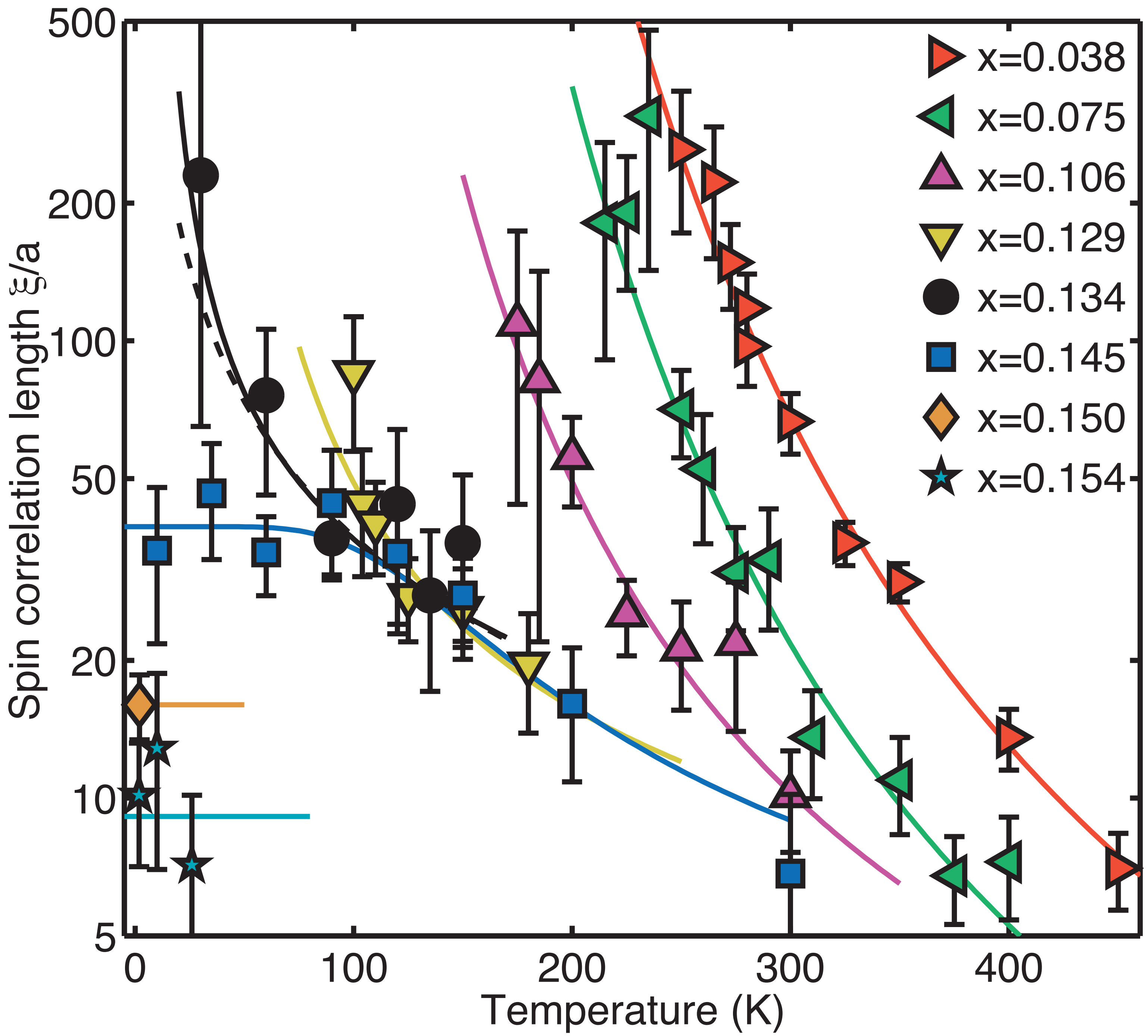
T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



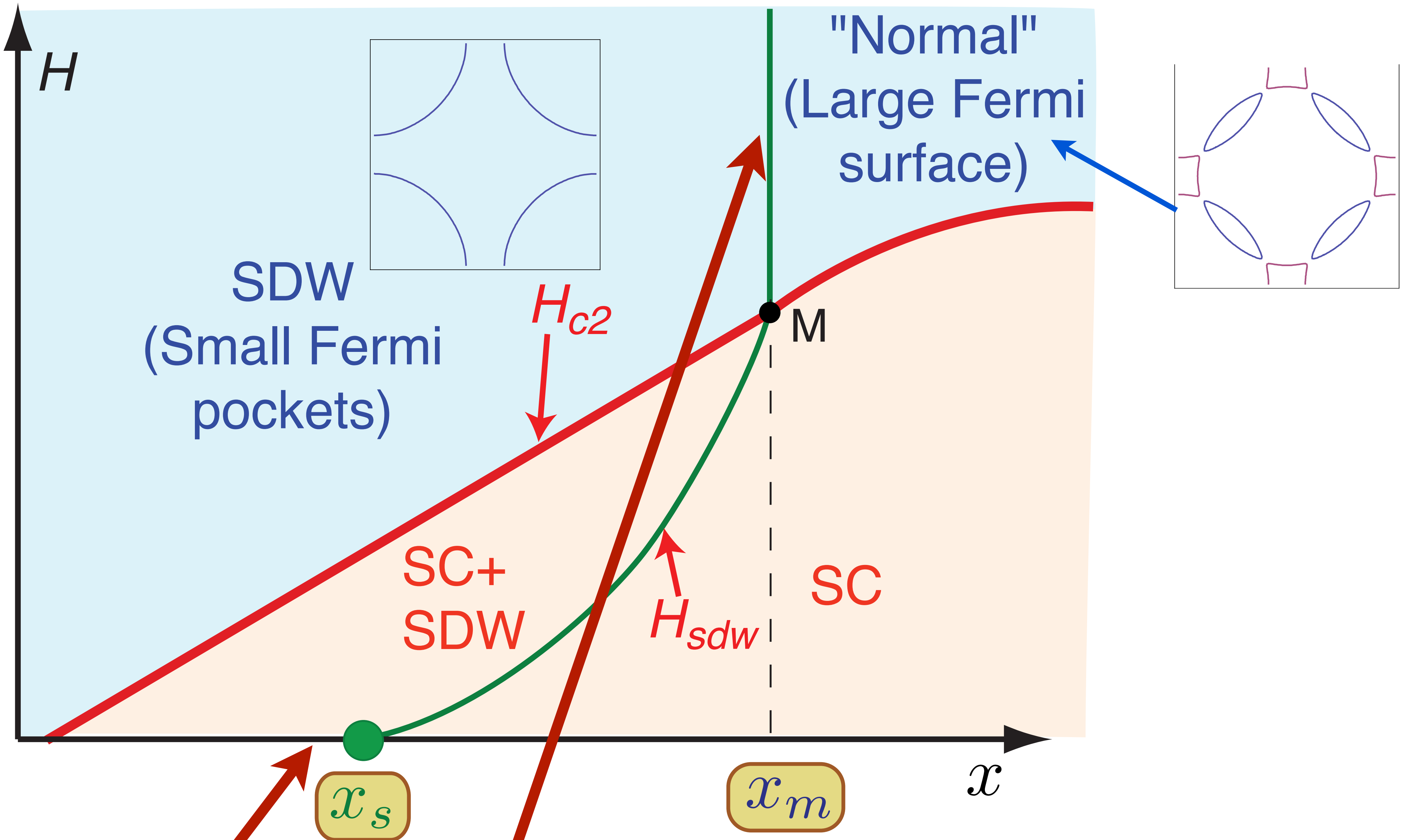
Neutron scattering at *H=0* in **same** material identifies $x_s = 0.14 < x_m$

$Nd_{2-x}Ce_xCuO_4$

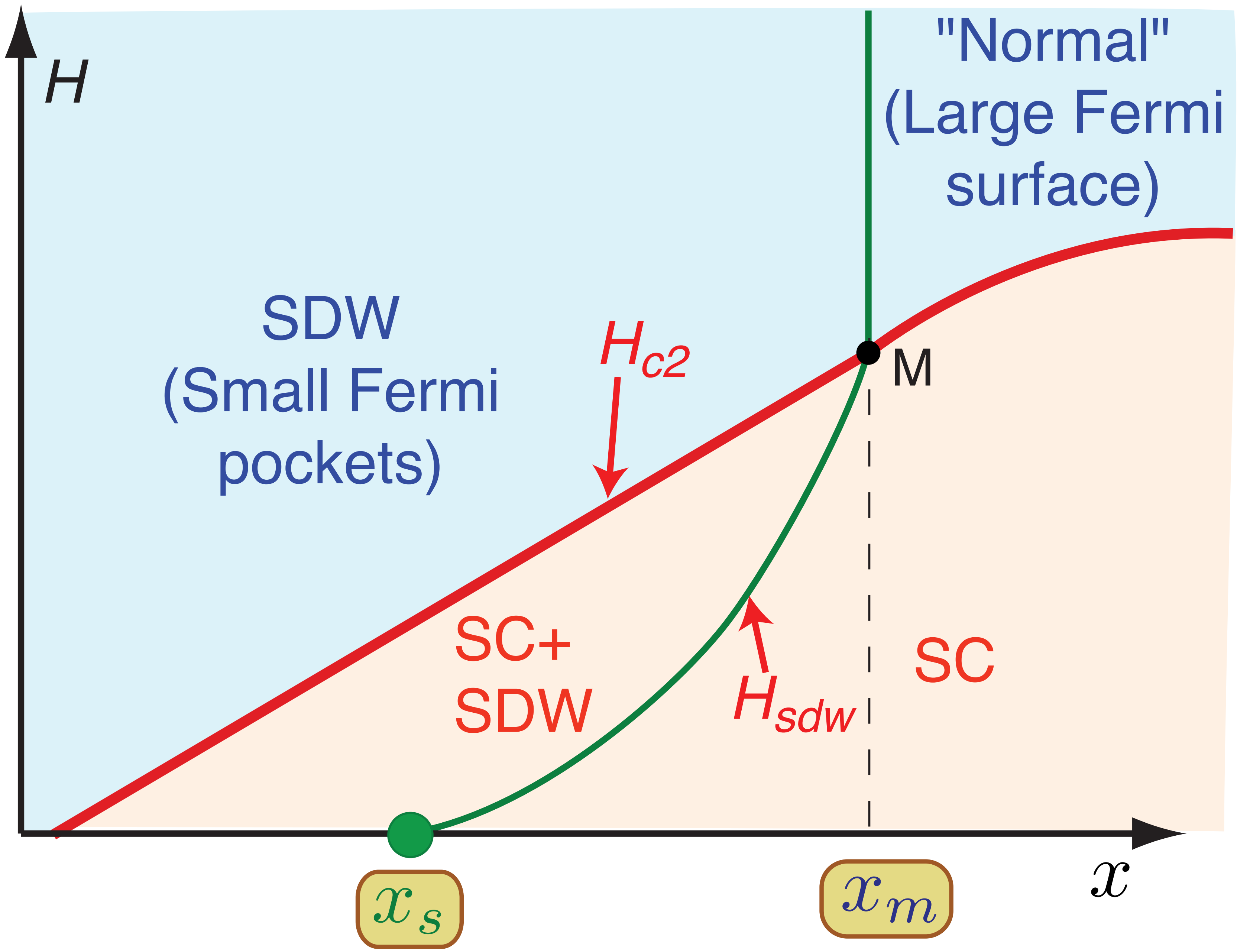


E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).

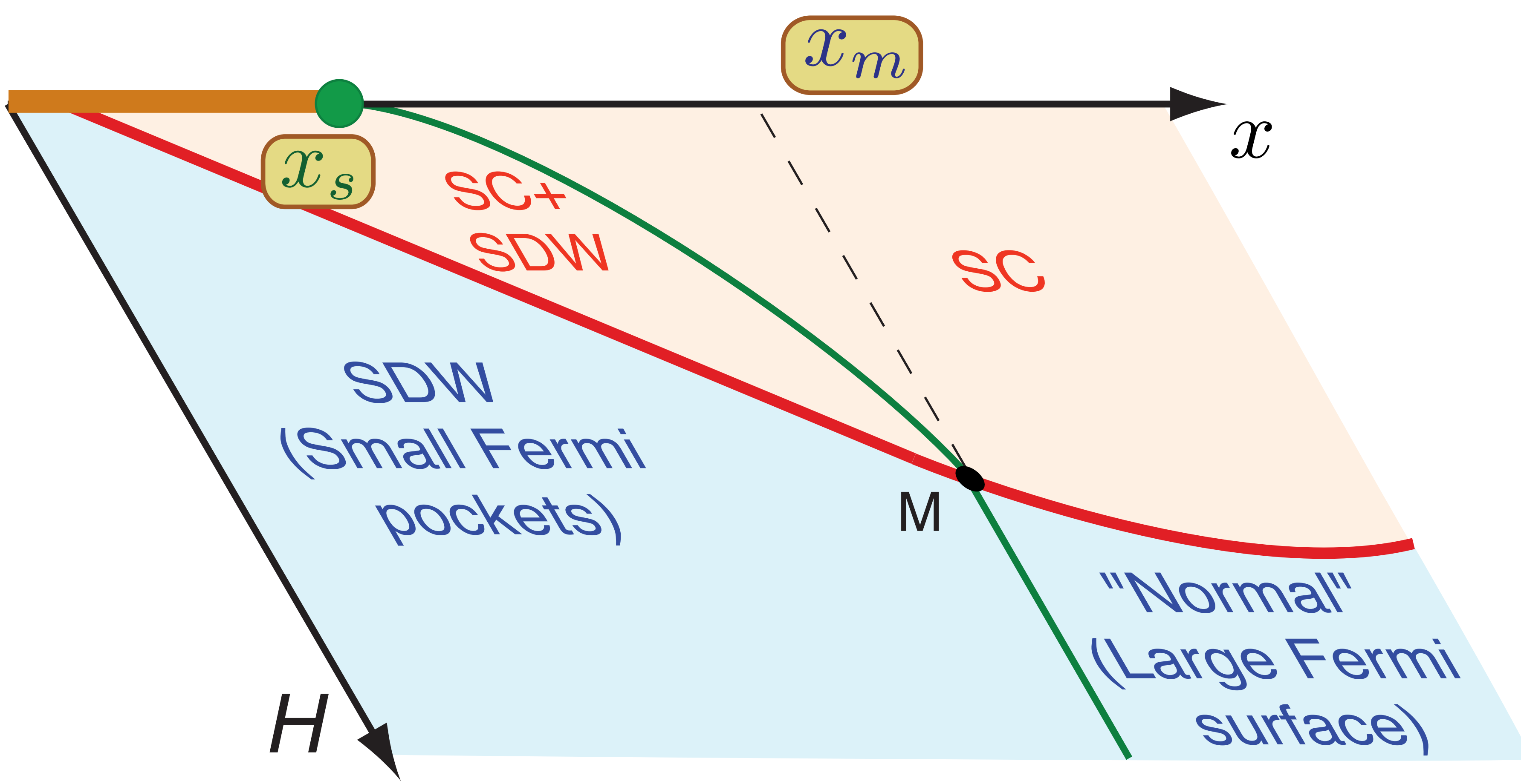
Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



Experiments on $\mathrm{Nd}_{2-x}\mathrm{Ce}_x\mathrm{CuO}_4$ show that at low fields $x_s = 0.14$, while at high fields $x_m = 0.165$.



H
"Normal"
(Large Fermi
surface)
SDW
(Small Fermi
pockets)
H_{c2}
M
SC+
SDW
H_{sdw}
SC
x
x_s
x_m



x_m
x
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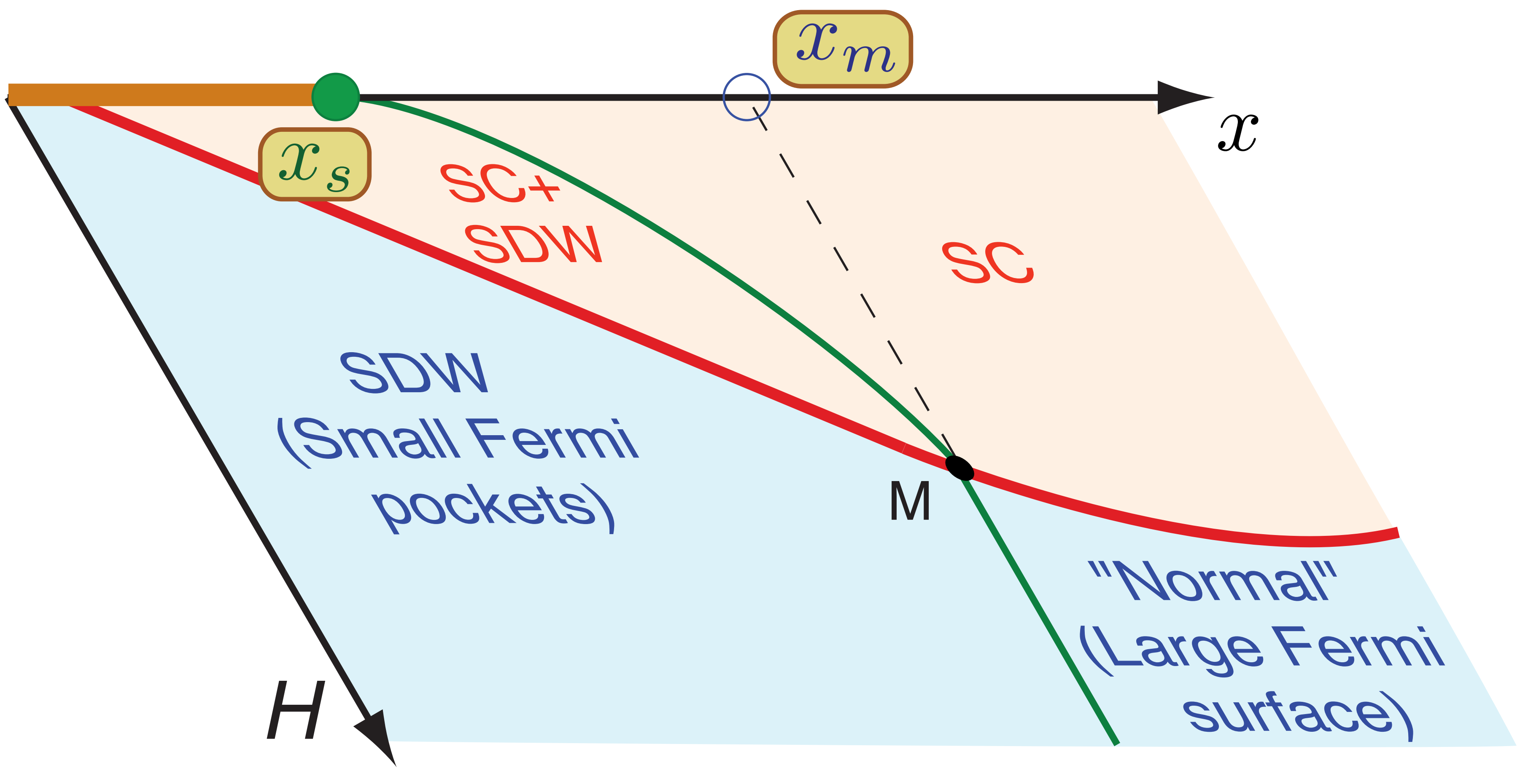
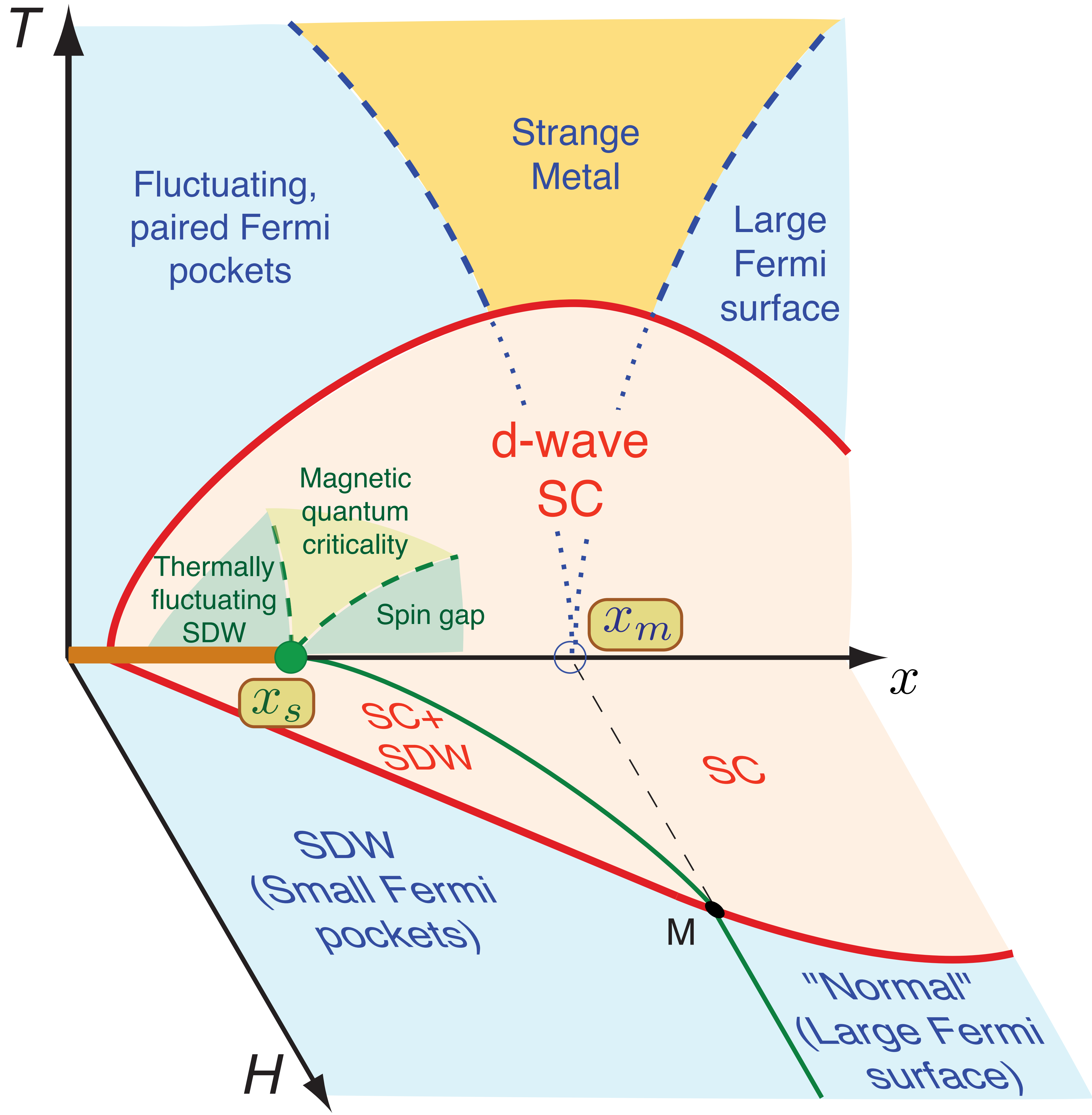
Fluctuating, paired Fermi pockets





T
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
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x_s
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Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order

   Survey of recent experiments

2. Overdoped vs. underdoped pairing

   Electronic theory of competing orders

3. Theory of SDW quantum critical point

   Dominance of planar graphs

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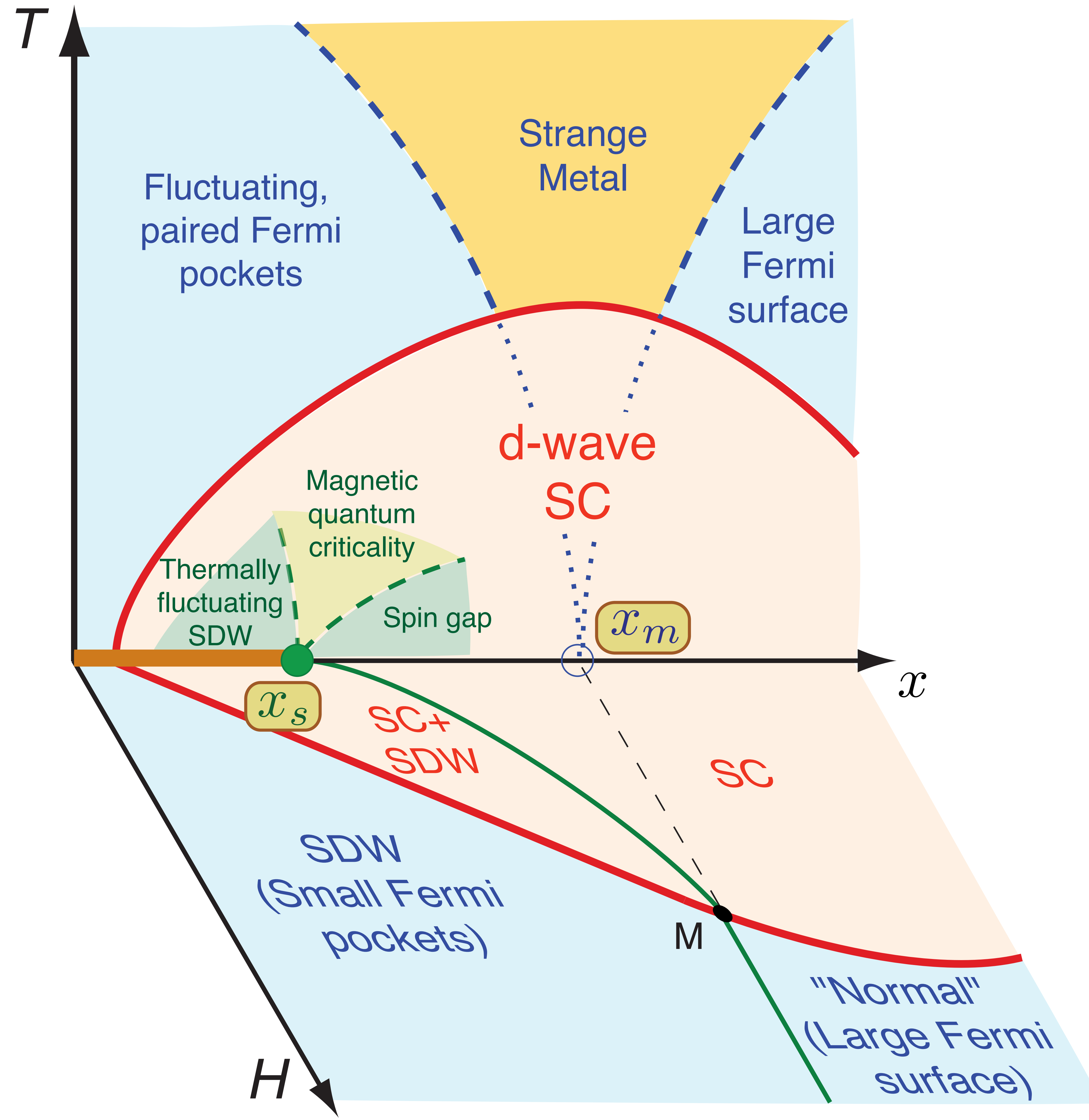

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Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x
x_s
SC+ SDW
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SDW (Small Fermi pockets)
M
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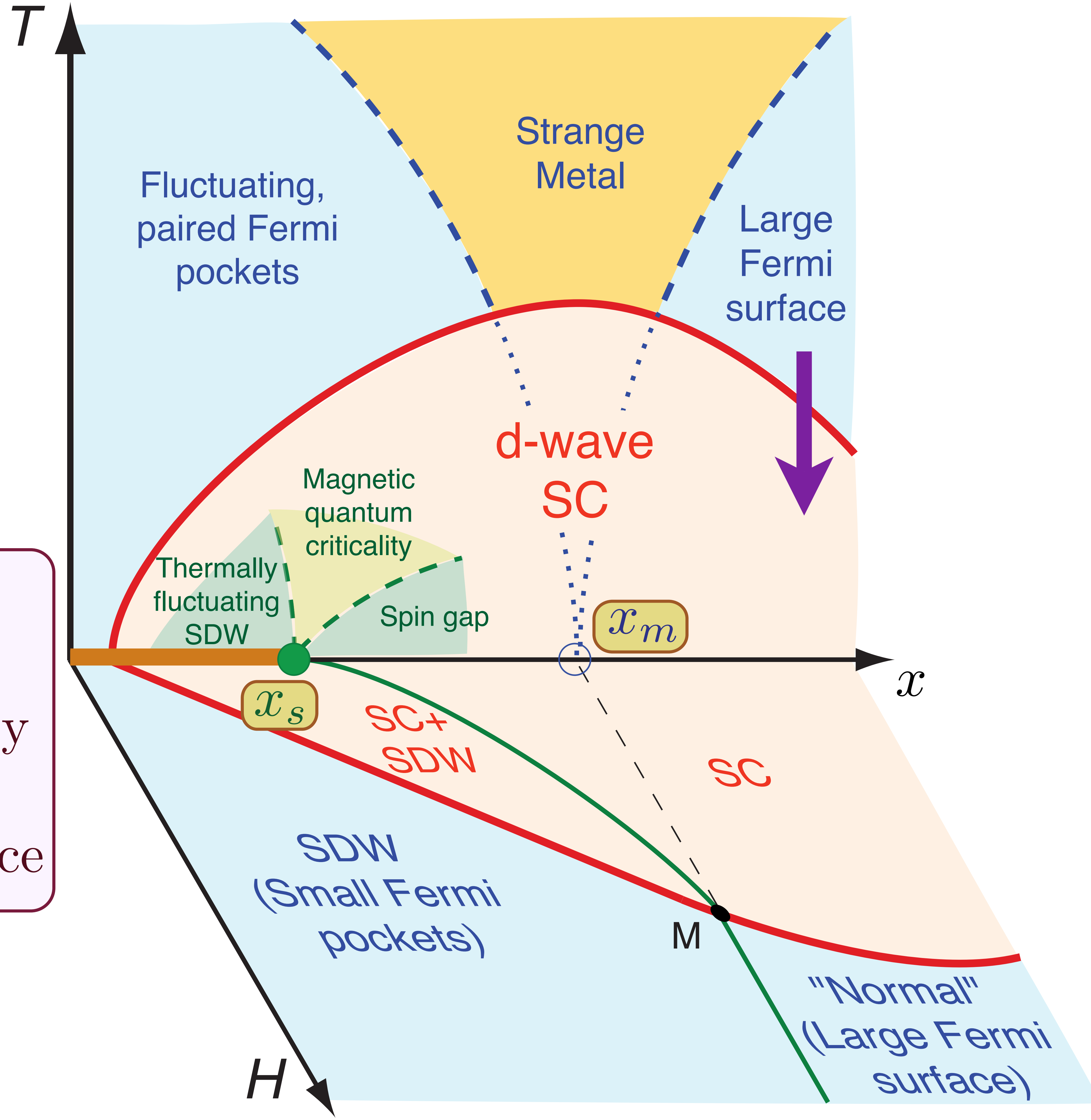
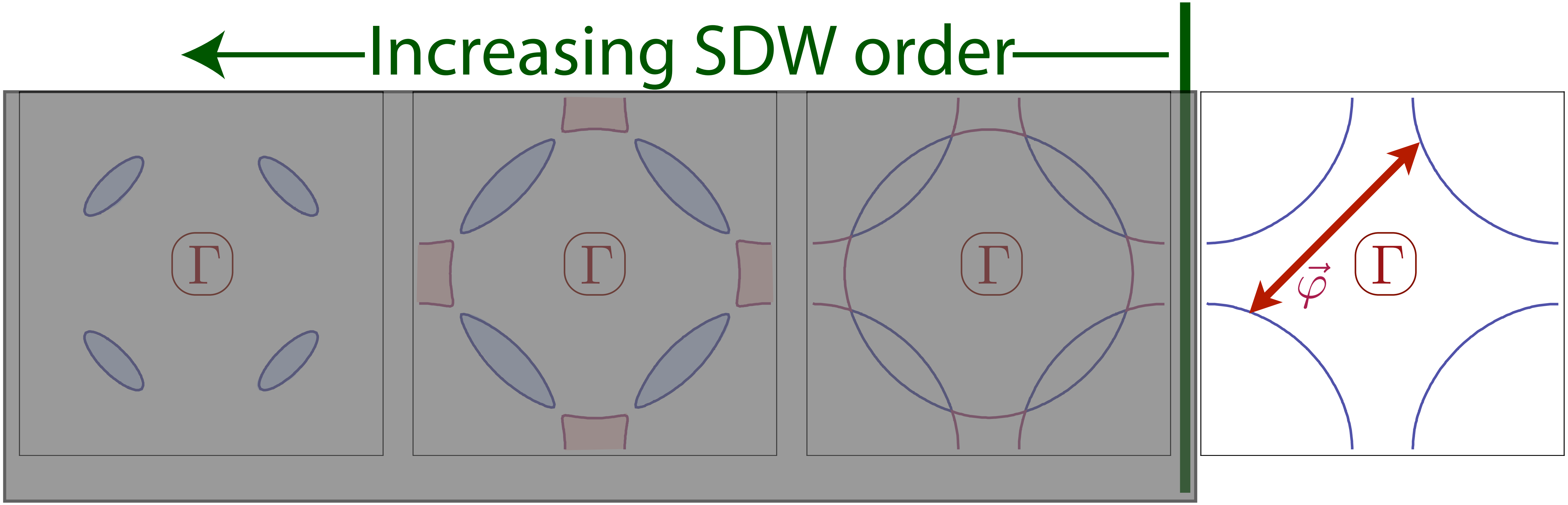
Theory of the onset of d-wave superconductivity from a large Fermi surface



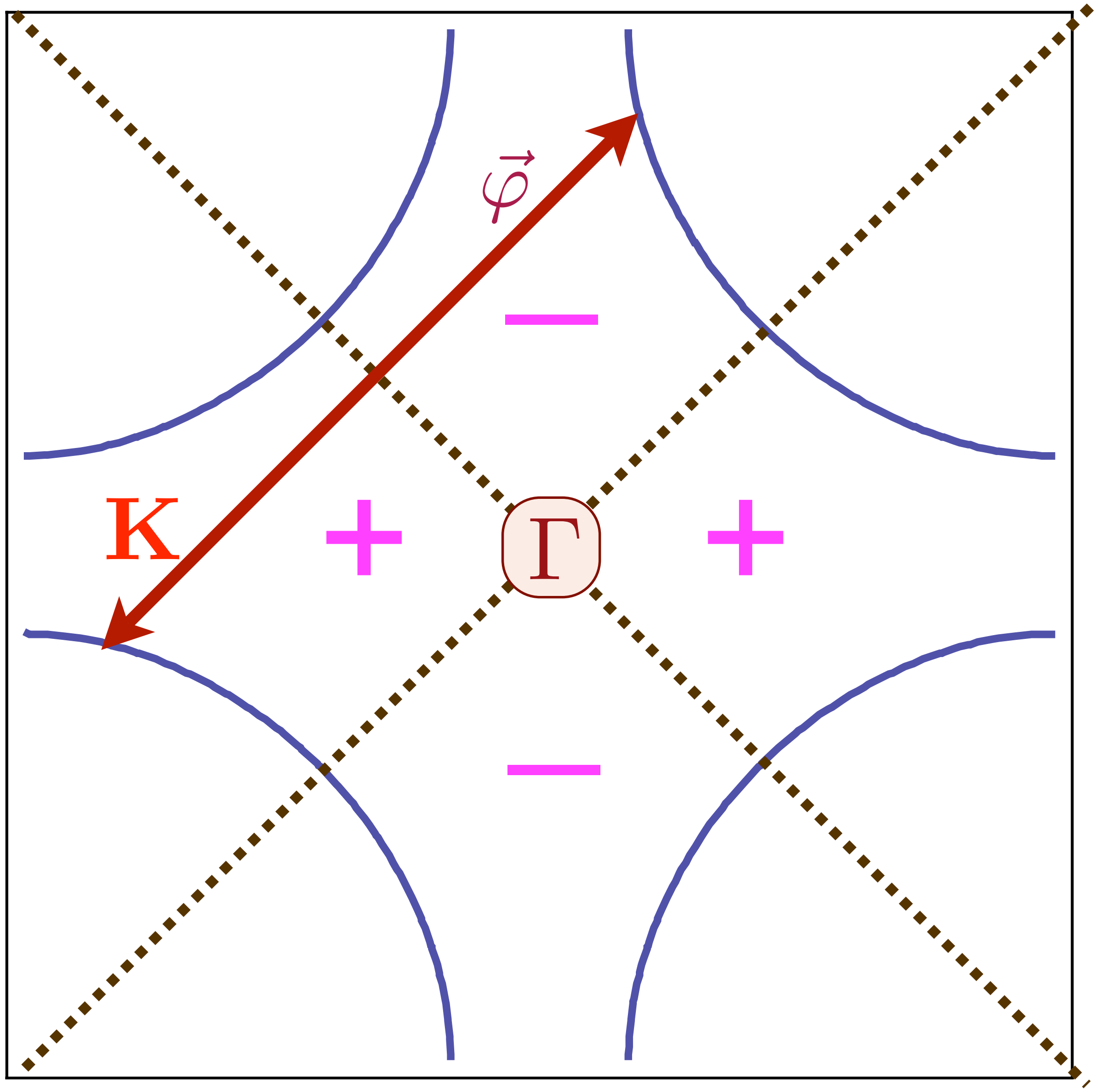
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

d-wave pairing of the large Fermi surface



$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
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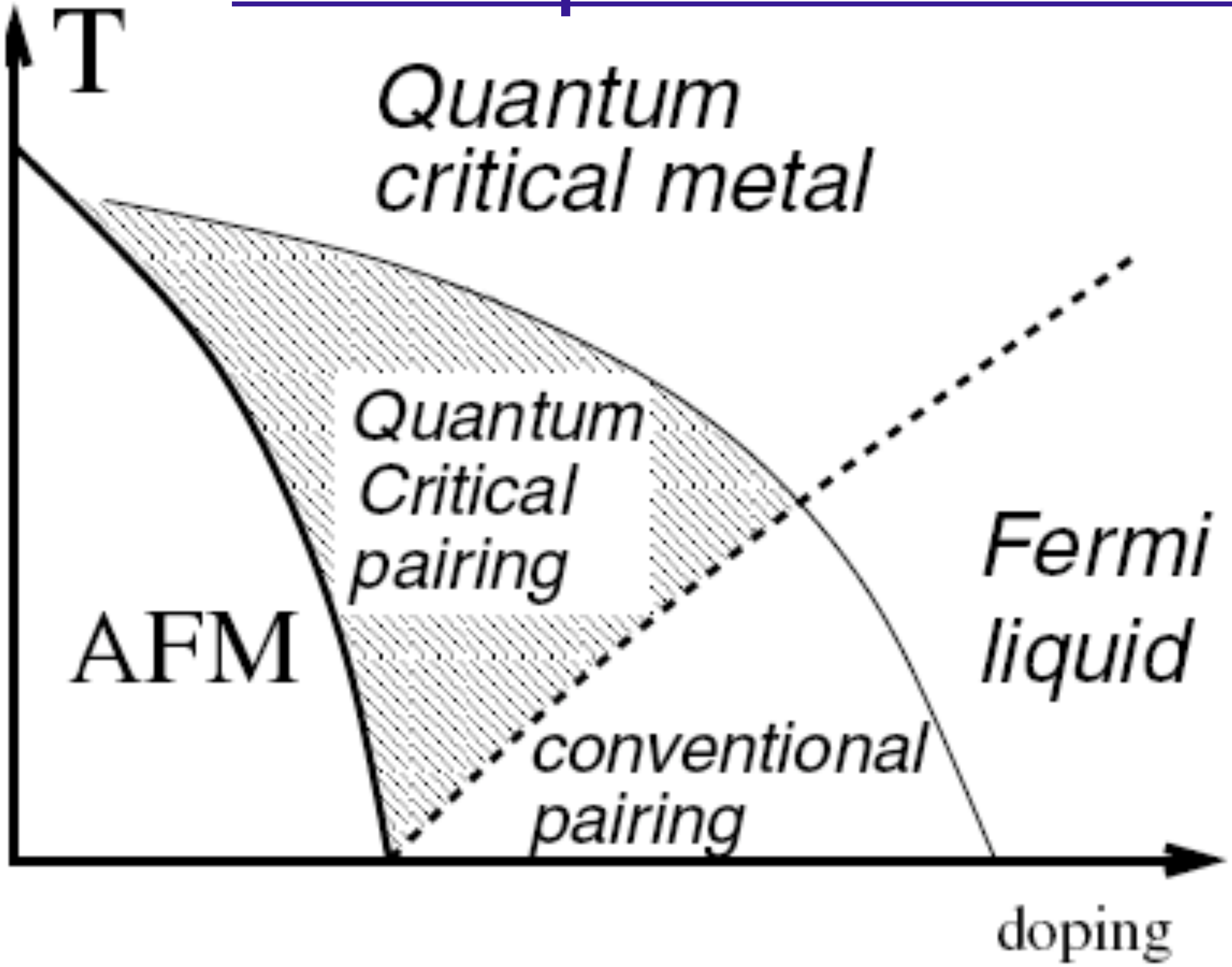
Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

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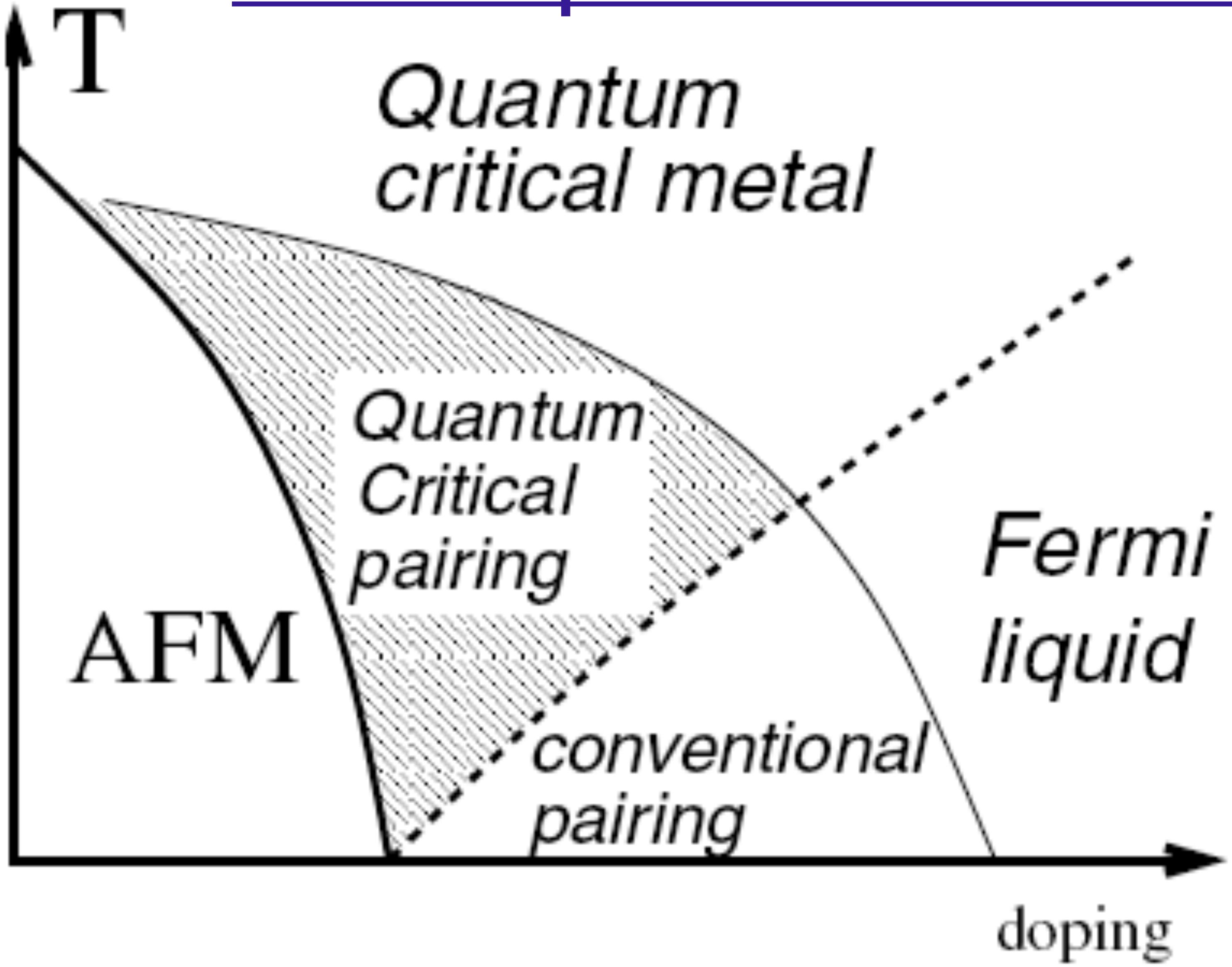
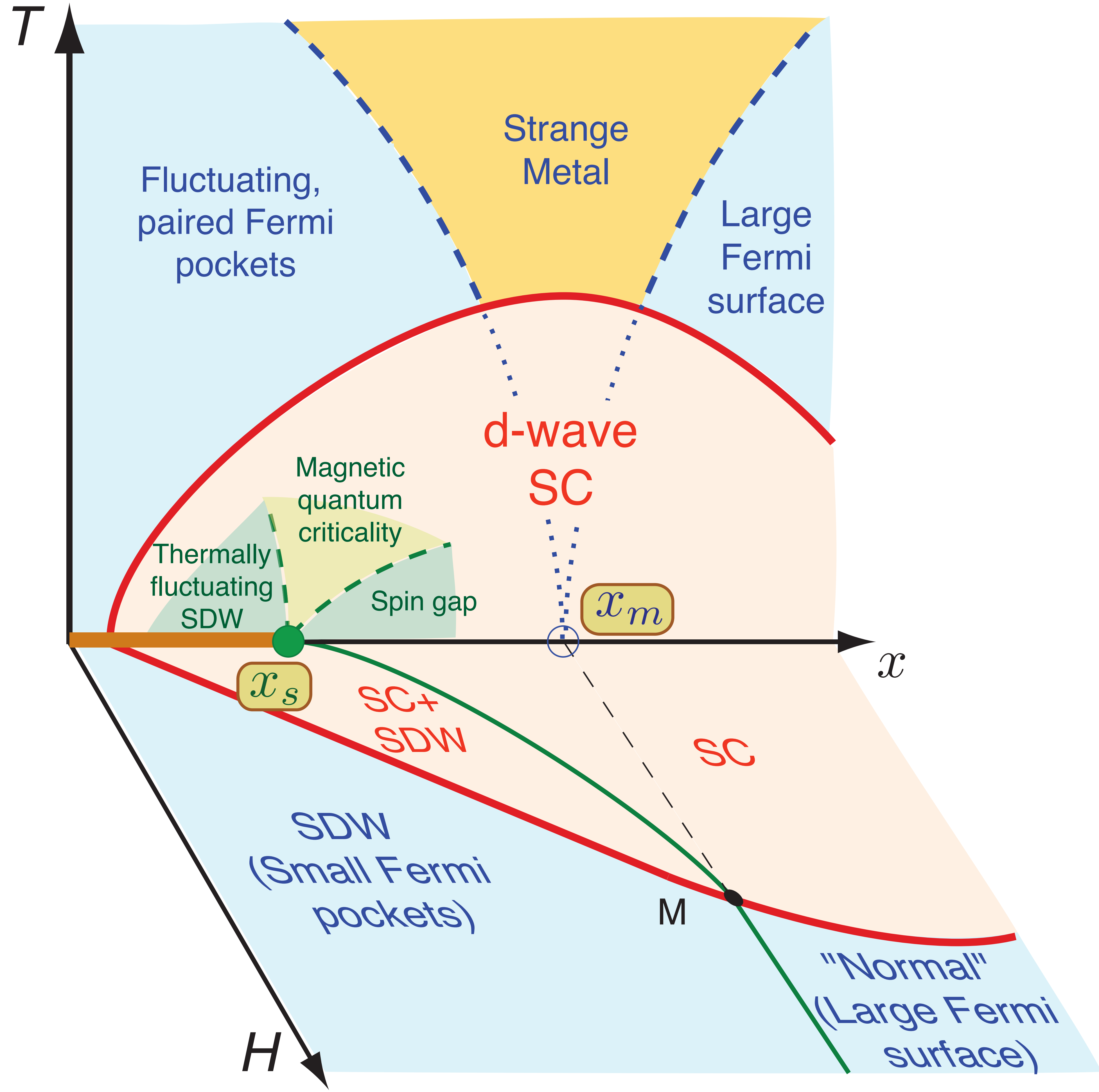


- T_c *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.

Ar. Abanov, A. V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).



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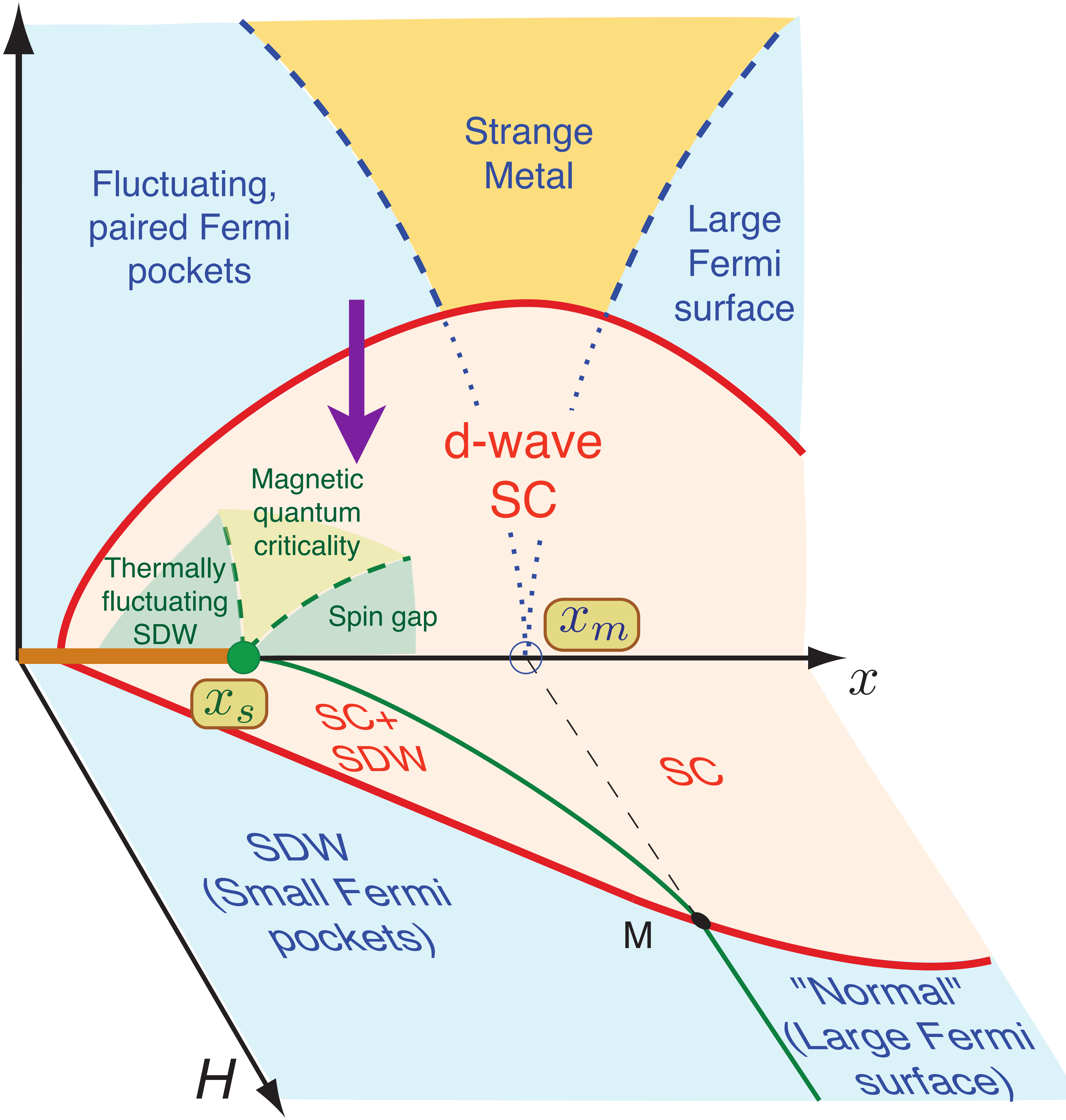
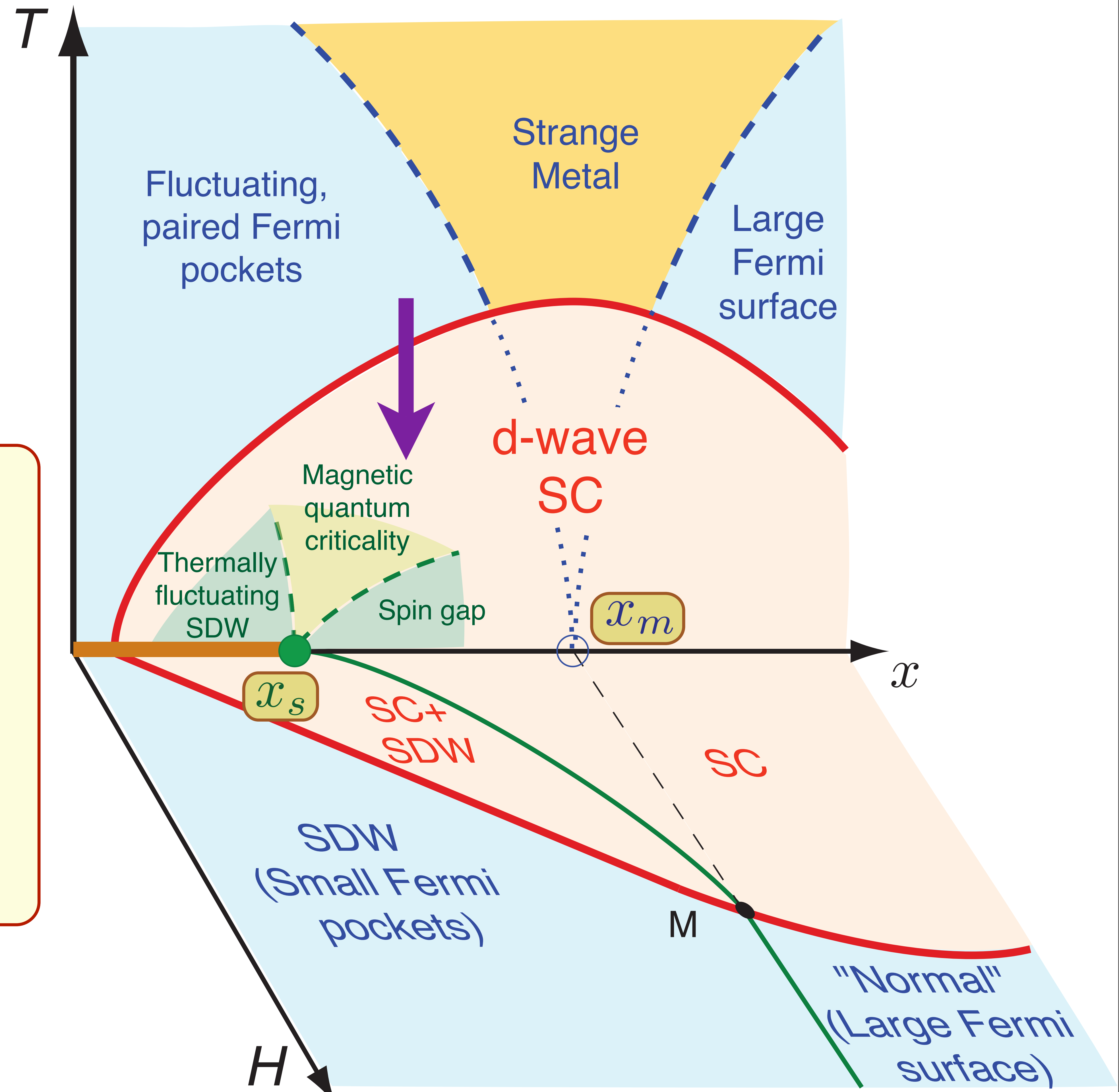
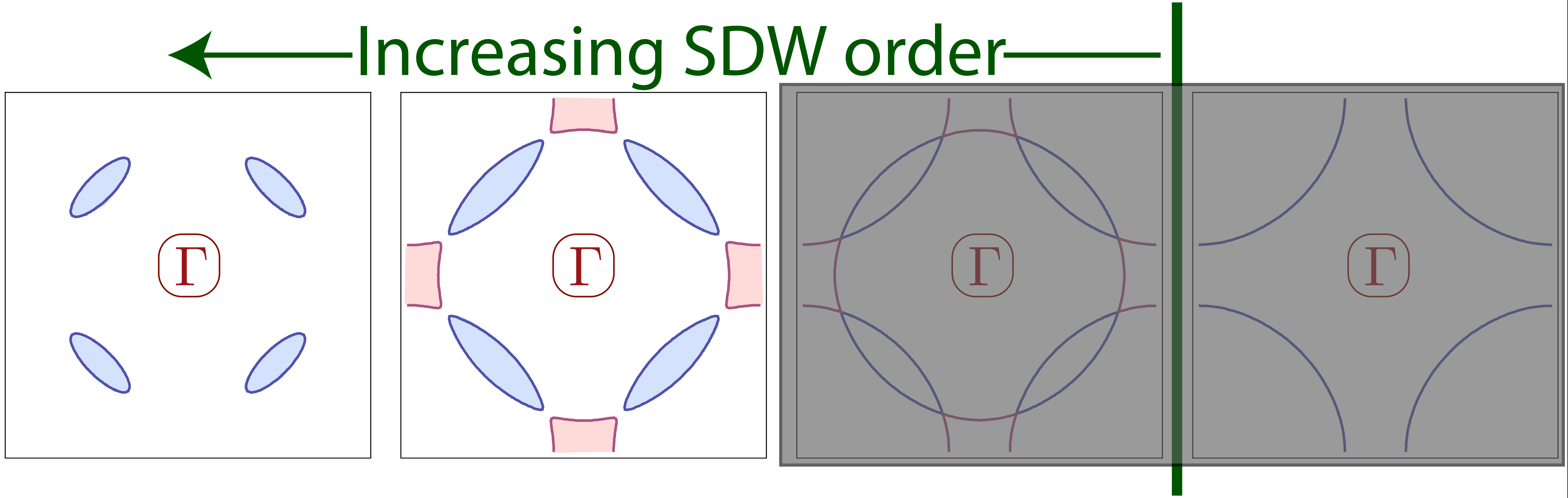
Theory of the onset of d-wave superconductivity from small Fermi pockets



Physics of competition: d-wave SC and SDW “eat up’ same pieces of the large Fermi surface.



Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\left(\begin{array}{c} c_\uparrow \\ c_\downarrow \end{array} \right) = R \left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \quad ; \quad \psi_+ \rightarrow e^{-i\theta} \psi_+ \quad ; \quad \psi_- \rightarrow e^{i\theta} \psi_-$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

Theory of underdoped cuprates

Starting from the "SDW-fermion" model with Lagrangian

$$\mathcal{L} = \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} - E_{sdw} \sum_{i} c_{i\alpha}^{\dagger} \hat{\vec{\varphi}}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} + \frac{1}{2t} \left(\partial_\mu \hat{\vec{\varphi}} \right)^2$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

$$\mathcal{L}_\psi = \sum_{\mathbf{k},p=\pm} \Bigg[\psi^\dagger_{\mathbf{k}p} \left(\frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} \\ - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \Bigg]$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order

$$\mathcal{L}_z = \frac{1}{t}\left[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - i\mathbf{A})z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1)\right]$$

Features of superconductivity

- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.

V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Features of superconductivity

- d-wave superconductivity.
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- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.

Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

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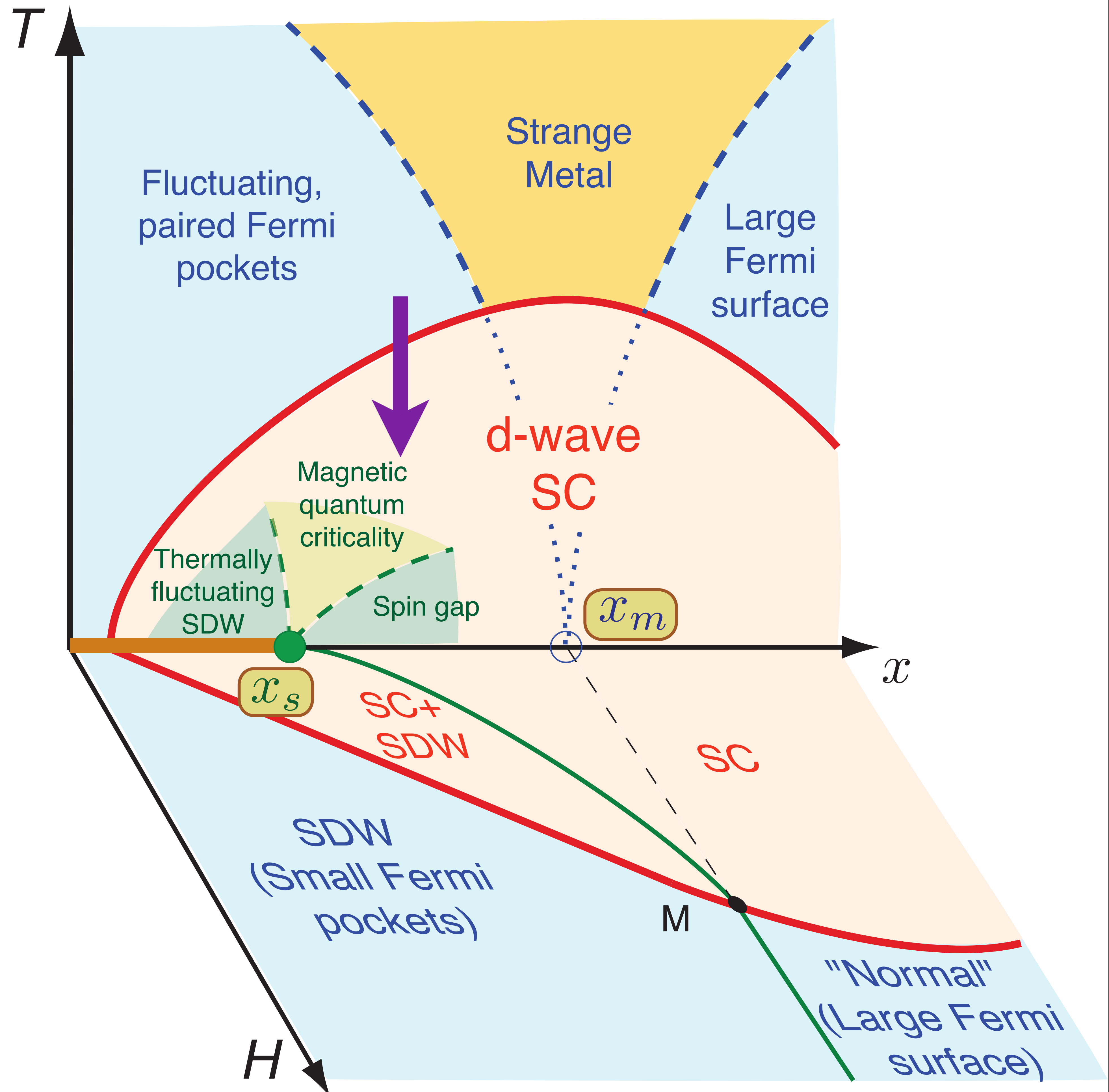
- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.
- After onset of superconductivity, monopoles condense and lead to confinement and **nematic** and/or **valence bond solid** (VBS) order.

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).



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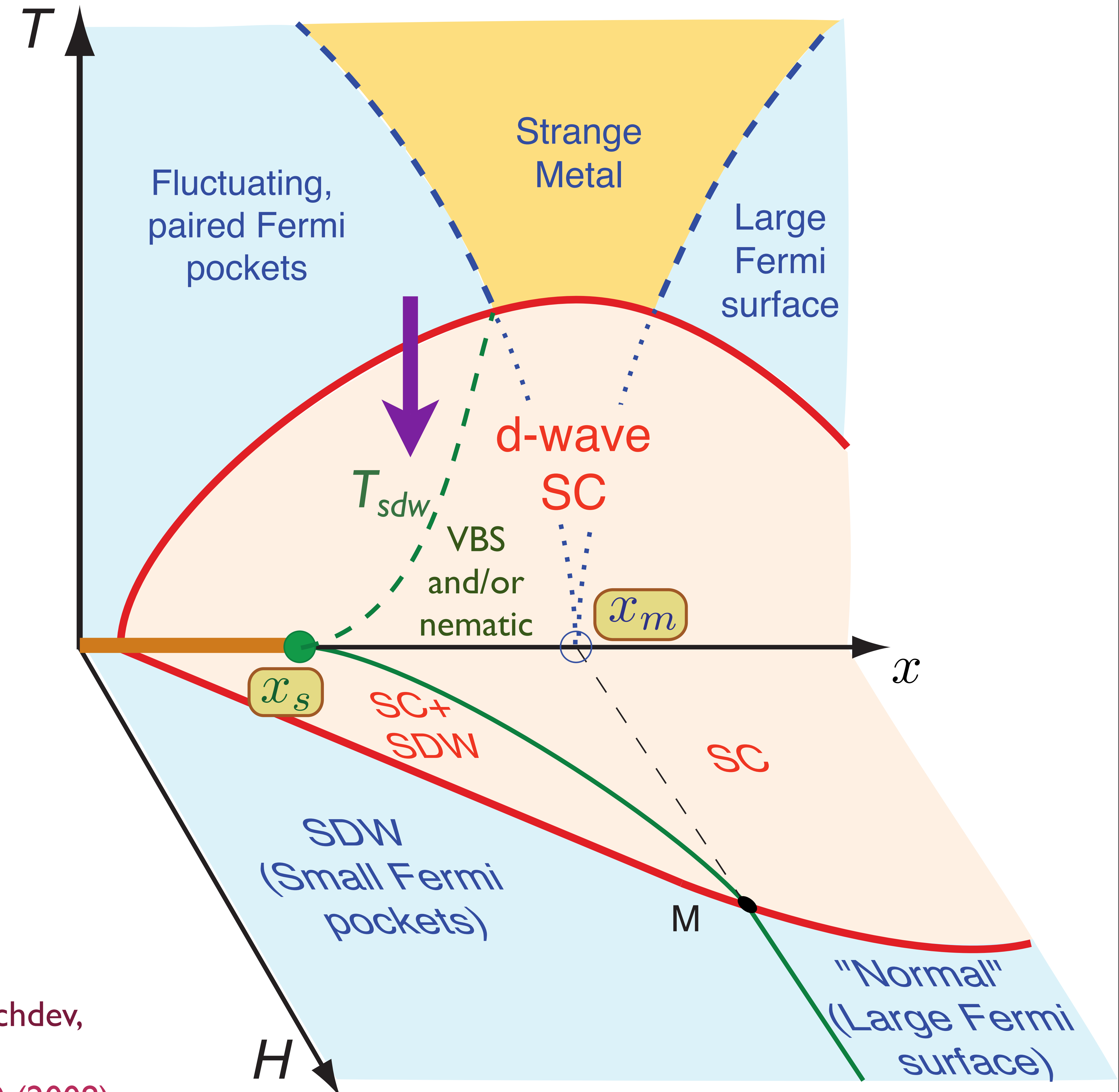

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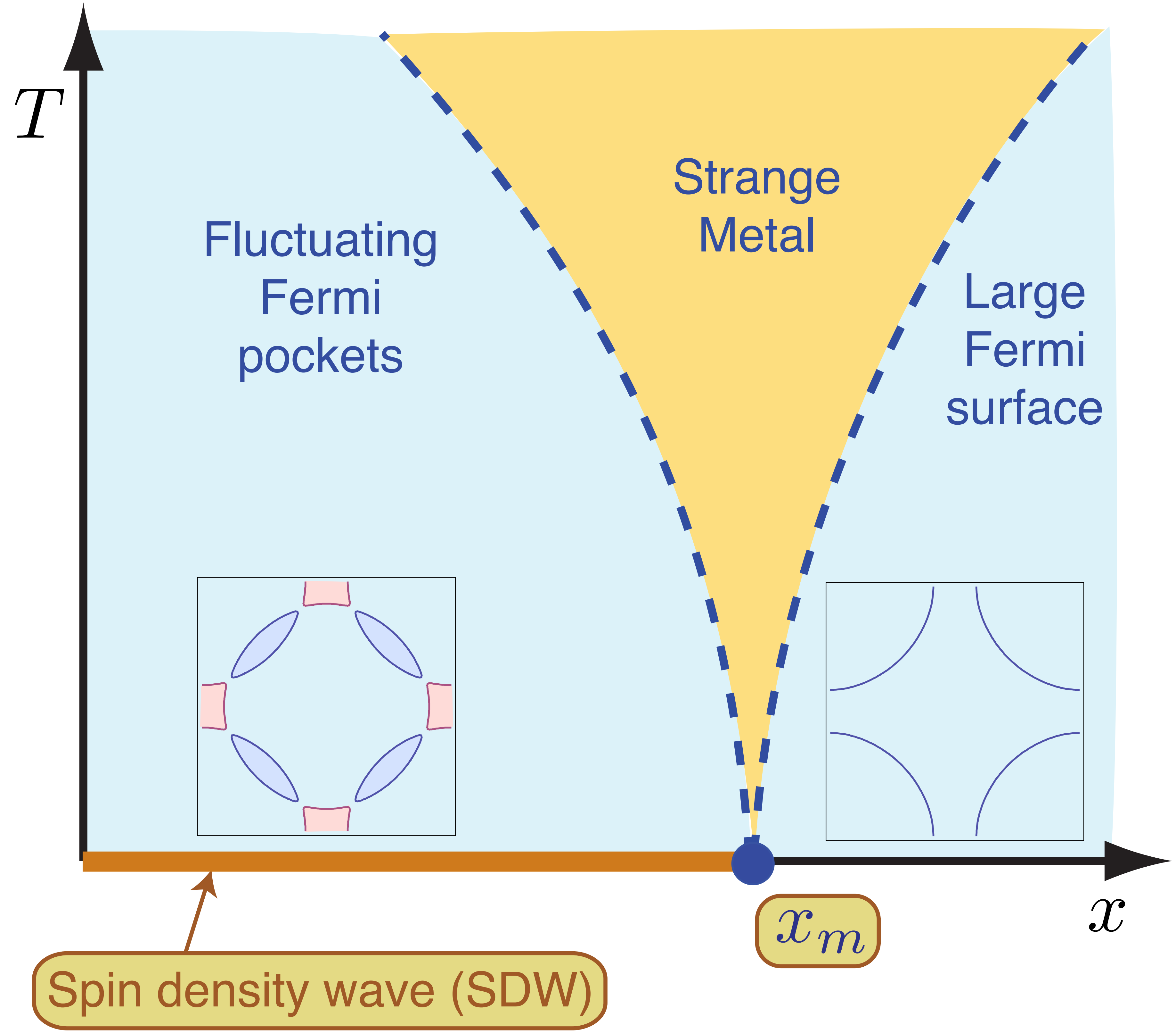
Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Max Metlitski

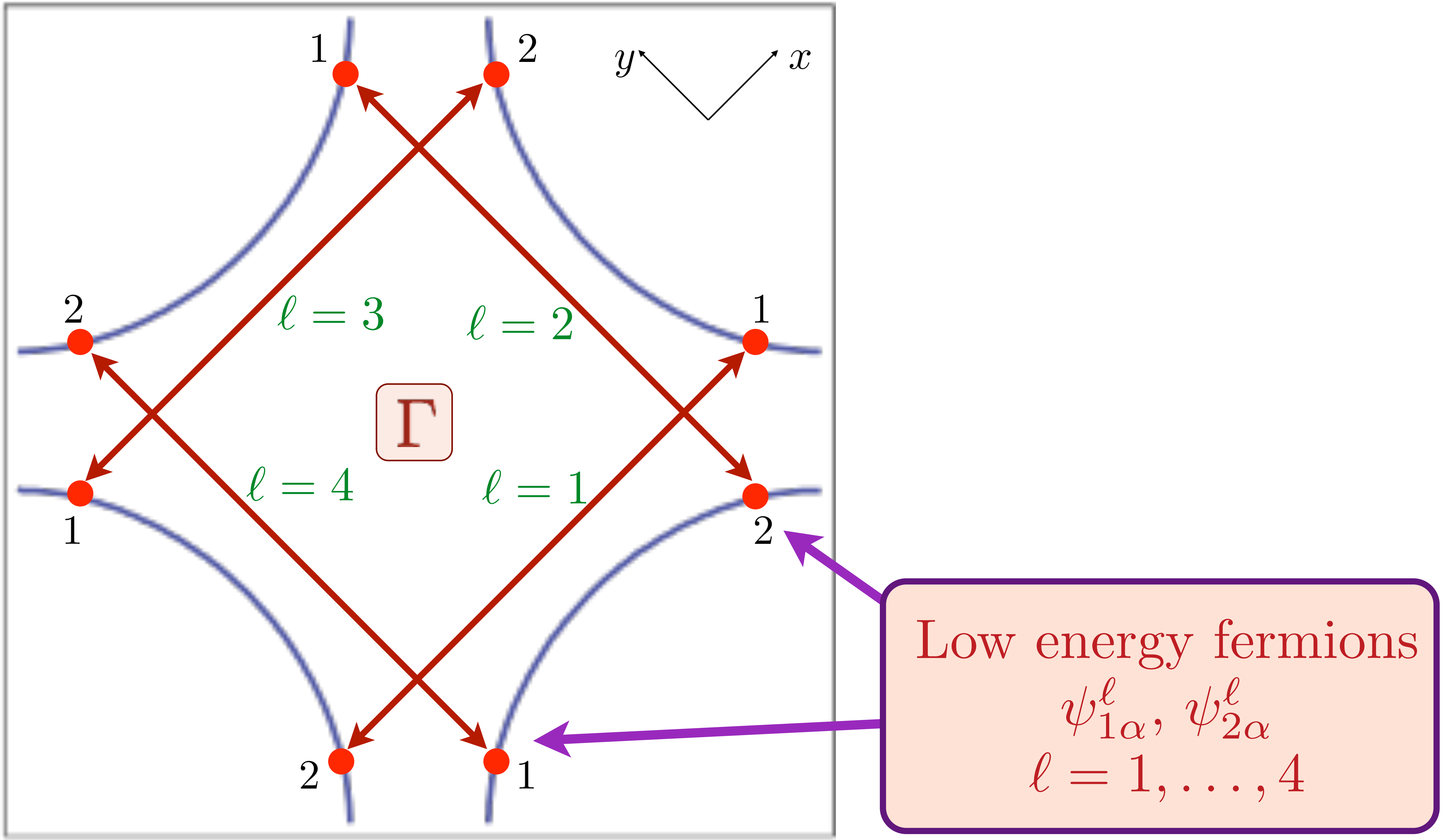
M. Metlitski and S. Sachdev, *to appear*

Ar. Abanov, A.V. Chubukov, and J. Schmalian, *Advances in Physics* **52**, 119 (2003)

Sung-Sik Lee, arXiv:0905.4532.

Start from the "spin-fermion" model

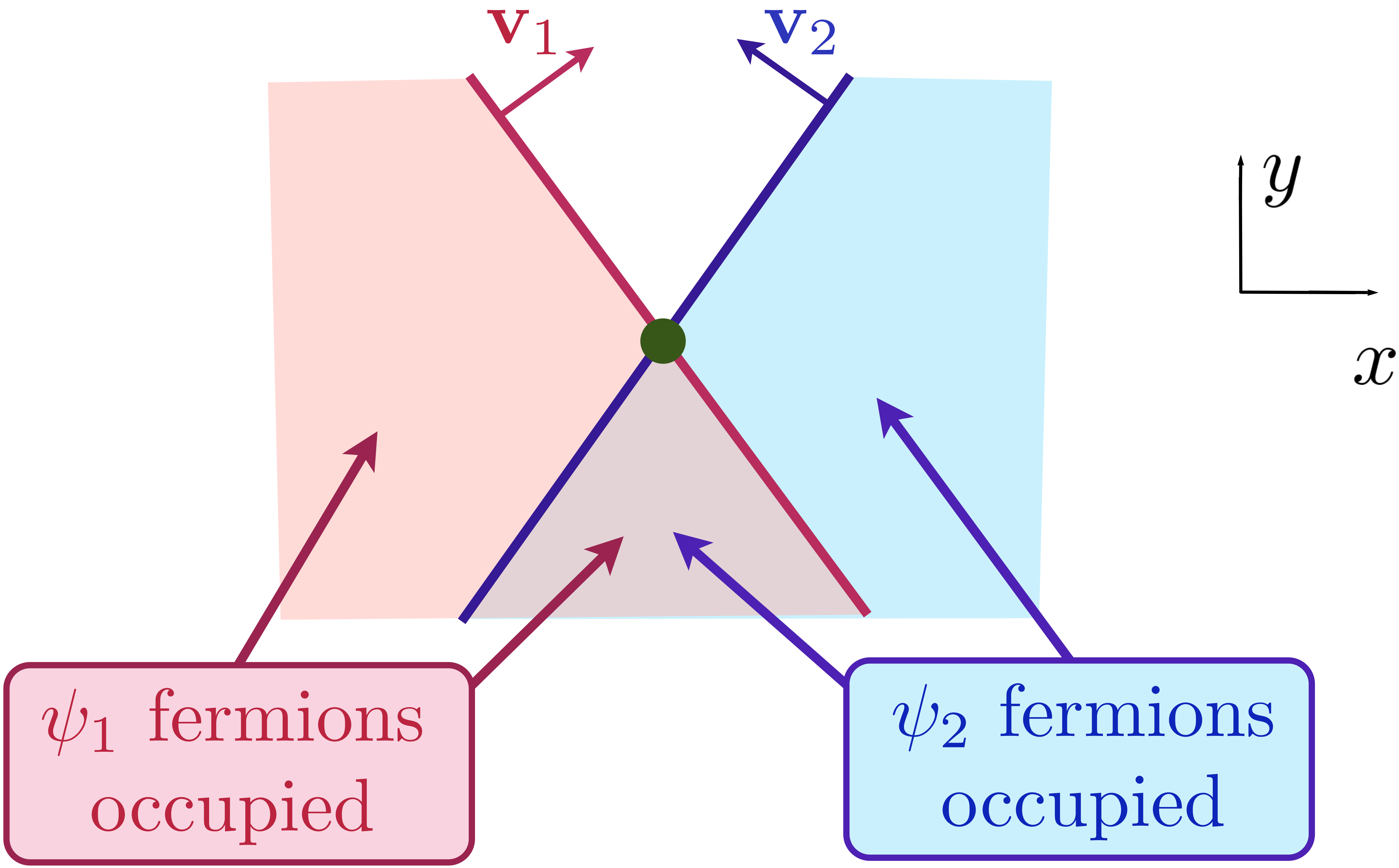
$$\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right) \\
\mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\
&\quad - \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\
&\quad + \int d\tau d^2 r \left[\left(\partial_r \vec{\varphi}\right)^2 + \frac{1}{c^2}\left(\partial_\tau \vec{\varphi}\right)^2 \right]
\end{aligned}$$



$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_1 \cdot \boldsymbol{\nabla}_r\right)\psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_2 \cdot \boldsymbol{\nabla}_r\right)\psi^{\ell}_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y),\ \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f \;=\; \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$



$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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"Yukawa" coupling: $\mathcal{L}_c = -\vec{\varphi}\cdot\left(\psi^{\ell\dagger}_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{2\beta} + \psi^{\ell\dagger}_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{1\beta}\right)$

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Hertz-Millis theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings . . . . . .

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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With $z = 2$ scaling, ζ is irrelevant.
So we take $\zeta \to 0$
(⚠ watch for dangerous irrelevancy).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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Set $\vec{\varphi}$ wavefunction renormalization by keeping co-efficient of $(\boldsymbol{\nabla}_r\vec{\varphi})^2$ fixed (as usual).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Set fermion wavefunction renormalization by keeping Yukawa coupling fixed.

Y. Huh and S. Sachdev, *Phys. Rev. B* **78**, 064512 (2008).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_1\cdot\boldsymbol{\nabla}_r\right)\psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_2\cdot\boldsymbol{\nabla}_r\right)\psi^{\ell}_{2\alpha}$$

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We find consistent two-loop RG factors, as $\zeta \to 0$, for the velocities v_x, v_y, and the wavefunction renormalizations.

Consistency check: the expression for the boson damping constant, $\gamma = \dfrac{2}{\pi v_x v_y}$, is preserved under RG.

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\boxed{\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}}$$

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$$\boxed{\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}}$$

The velocities flow as

$$\frac{1}{v_x}\frac{dv_x}{d\ell} = \frac{\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2} \quad ; \quad \frac{1}{v_y}\frac{dv_y}{d\ell} = \frac{-\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2}$$

$$\mathcal{A}(\alpha) \equiv \frac{3}{\pi N}\frac{\alpha}{1+\alpha^2}$$

$$\mathcal{B}(\alpha) \equiv \frac{1}{2\pi N}\left(\frac{1}{\alpha}-\alpha\right)\left(1+\left(\frac{1}{\alpha}-\alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\boxed{\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}}$$

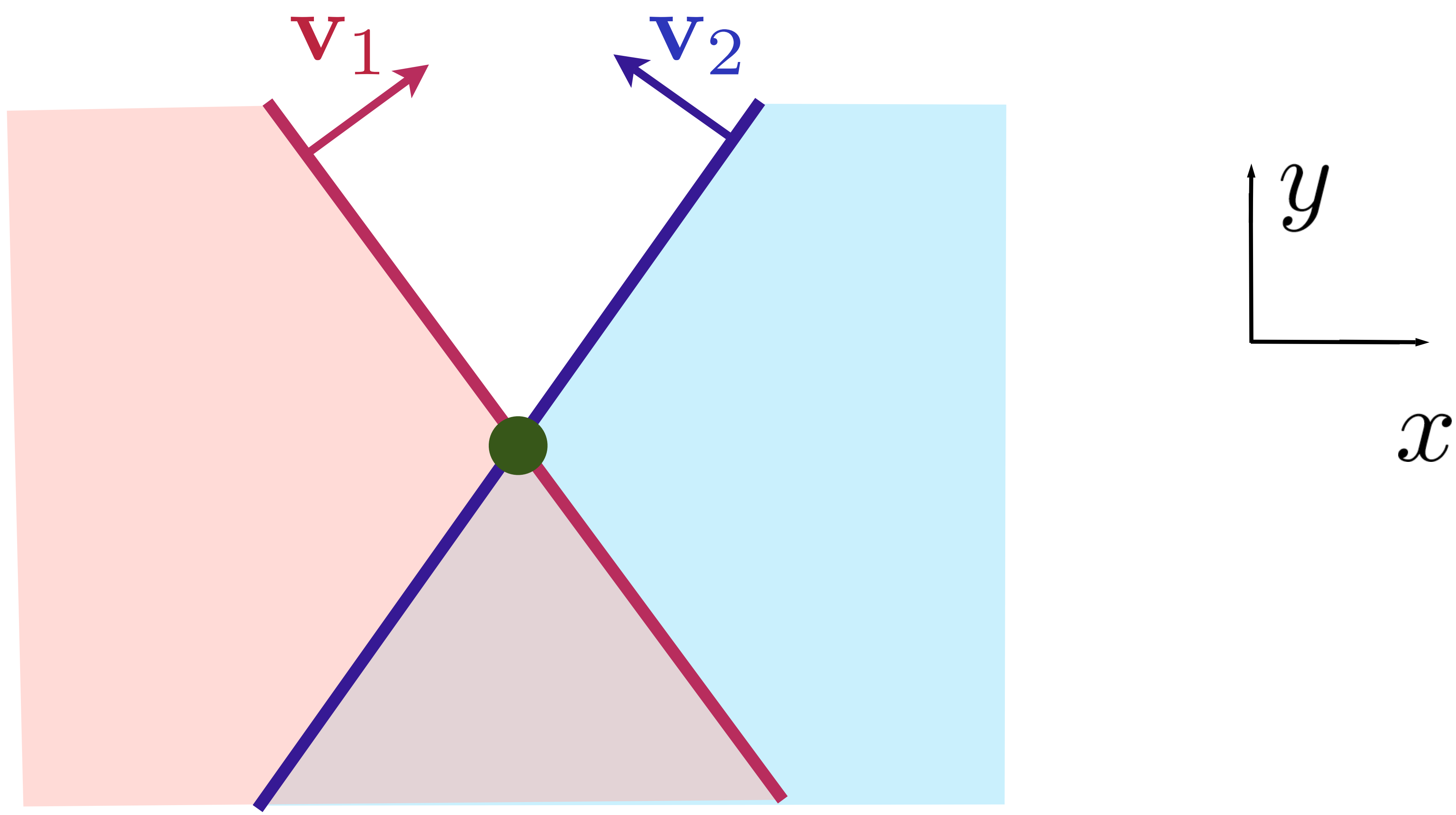
The anomalous dimensions of $\vec{\varphi}$ and ψ are

$$\begin{aligned}
\eta_\varphi &= \frac{1}{2\pi N}\left(\frac{1}{\alpha} - \alpha + \left(\frac{1}{\alpha^2} + \alpha^2\right)\tan^{-1}\frac{1}{\alpha}\right) \\
\eta_\psi &= -\frac{1}{4\pi N}\left(\frac{1}{\alpha} - \alpha\right)\left(1 + \left(\frac{1}{\alpha} - \alpha\right)\tan^{-1}\frac{1}{\alpha}\right)
\end{aligned}$$

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting

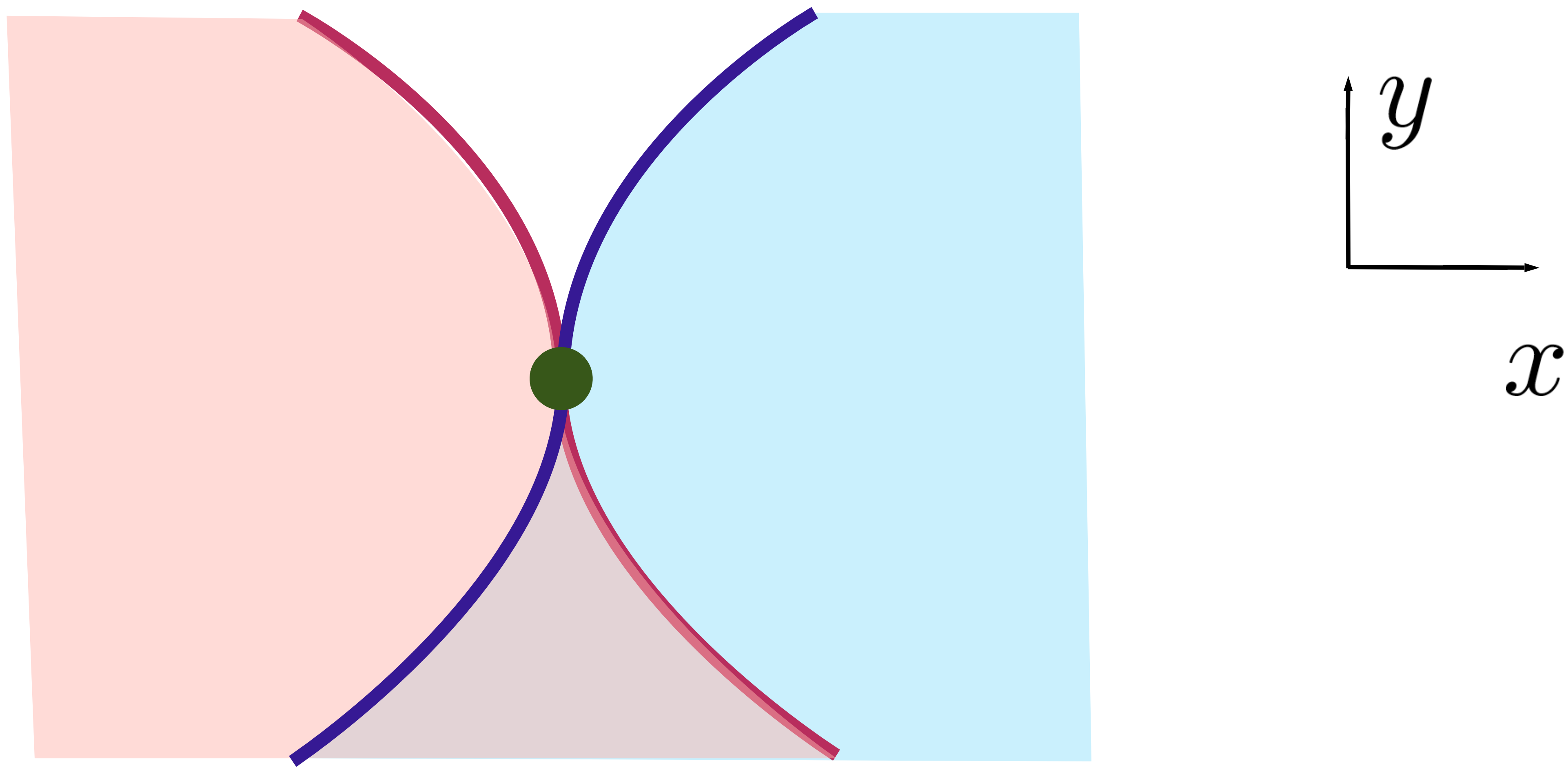


Bare Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y / v_x \to 0$ logarithmically in the infrared.

<u>Dynamical Nesting</u>

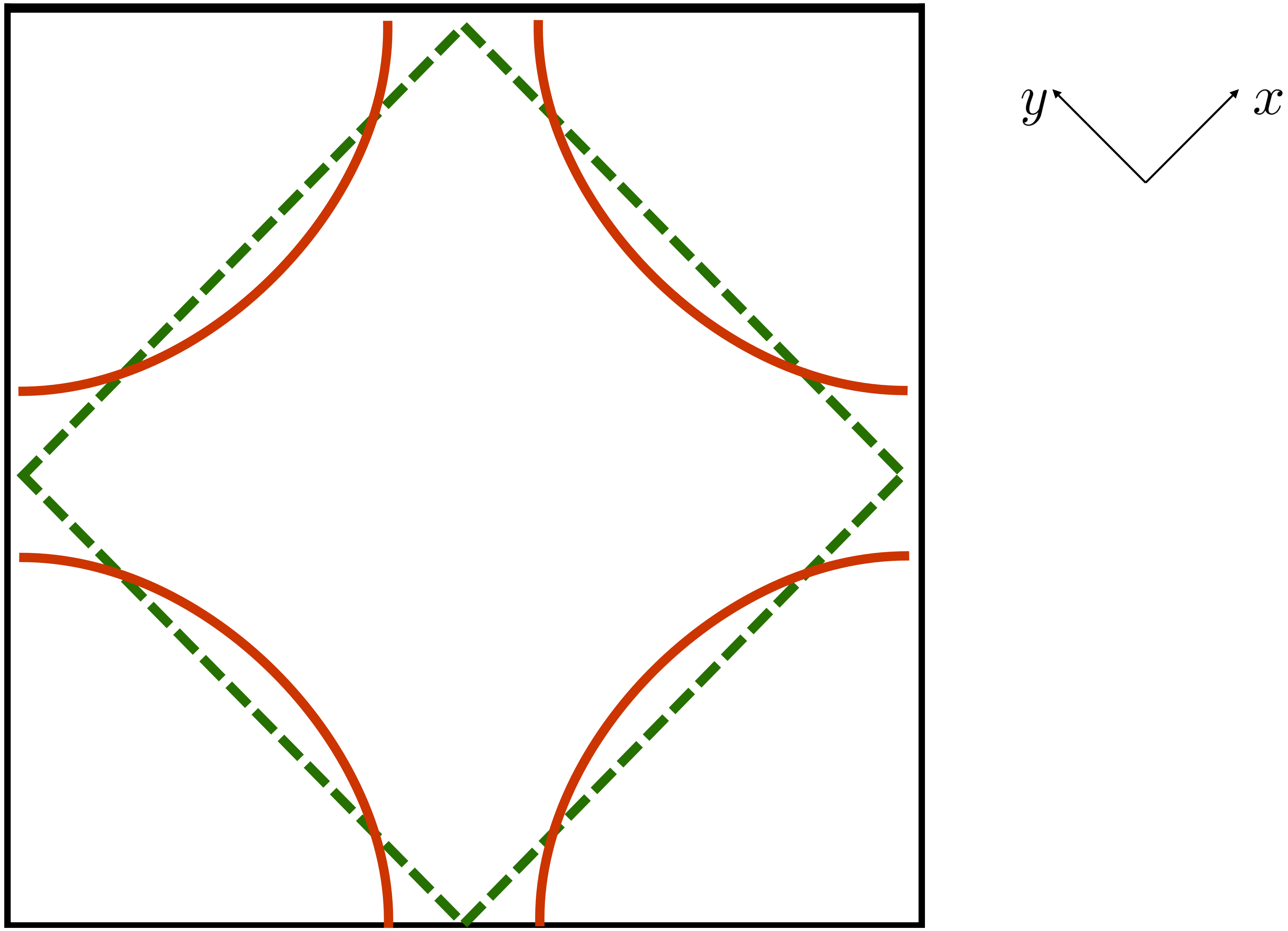


Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting

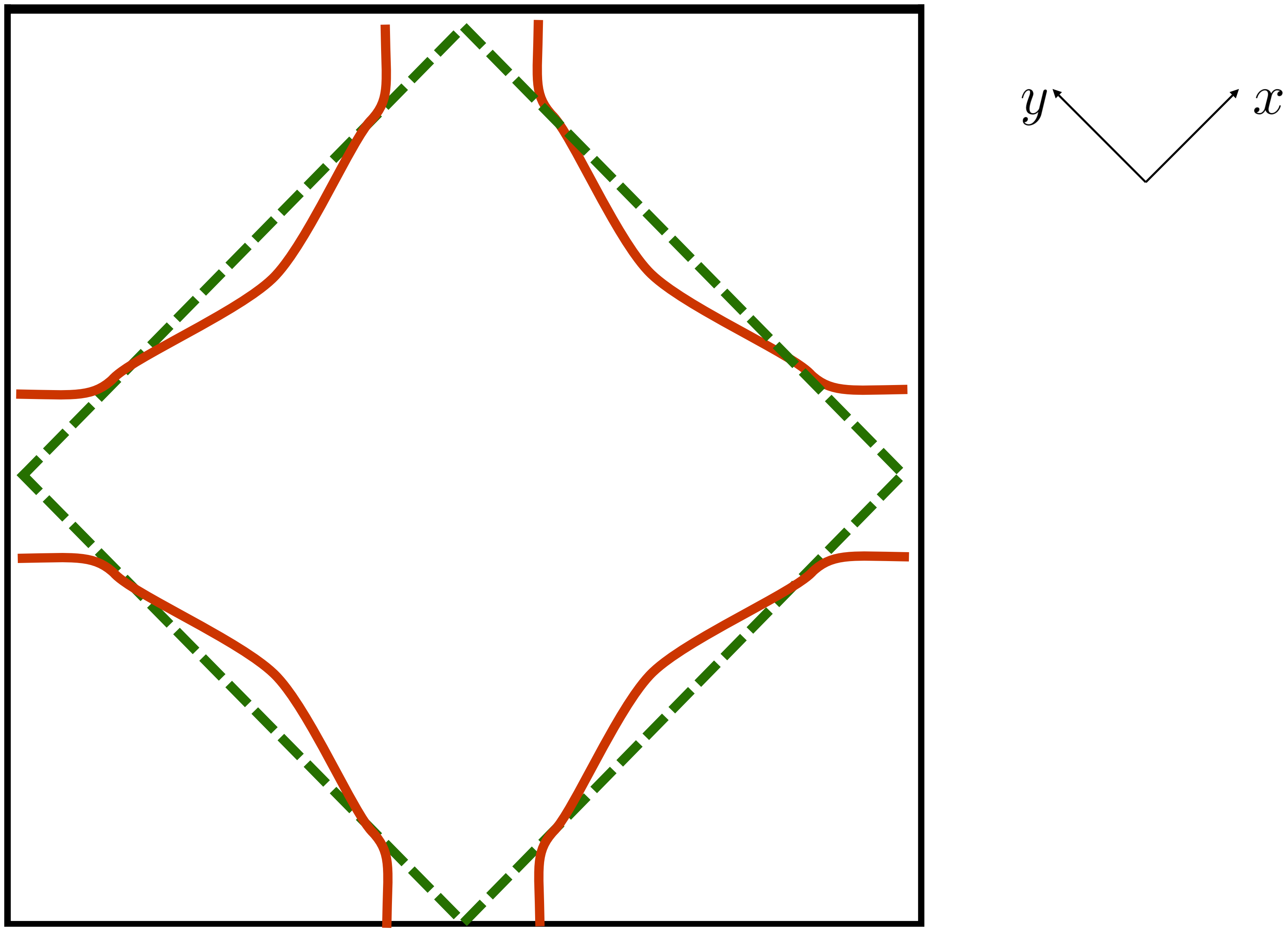


Bare Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting



Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

In $\vec{\varphi}$ SDW fluctuations, characteristic q and ω scale as

$$q \sim \omega^{1/2} \exp\left(-\frac{3}{64\pi^2}\left(\frac{\ln(1/\omega)}{N}\right)^3\right).$$

However, $1/N$ expansion cannot be trusted in the asymptotic regime.

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N}\frac{1}{(q^2+\gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v}\cdot\mathbf{q}+i\zeta\omega+i\frac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\frac{v^2q^2}{\omega}\right)}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

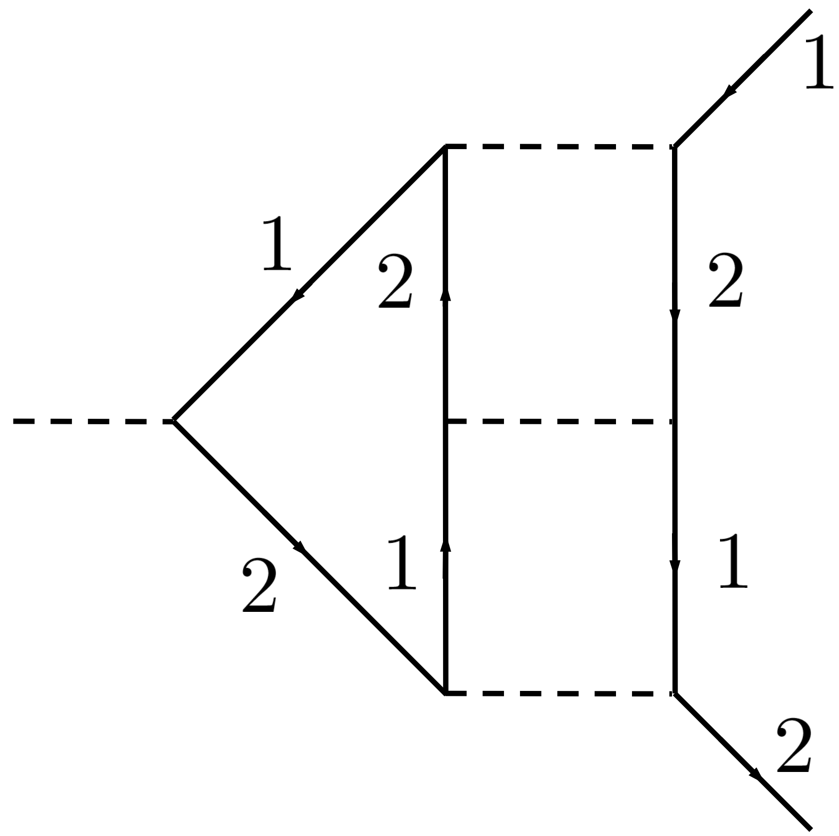
$$\frac{1}{N}\frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v}\cdot\mathbf{q} + i\zeta\omega + i\frac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\frac{v^2q^2}{\omega}\right)}$$

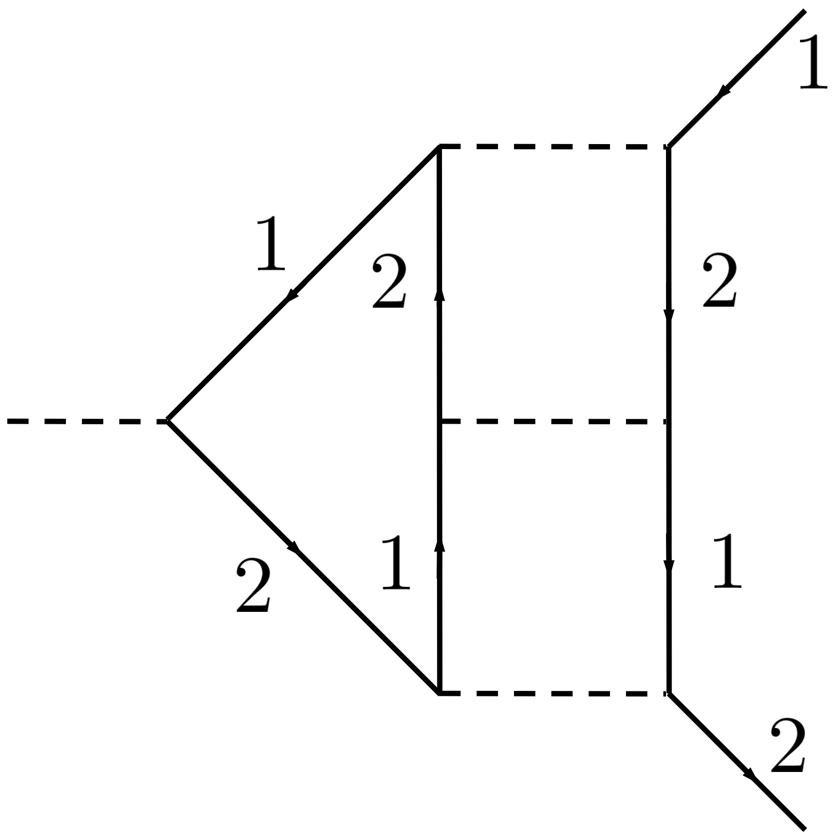
⚠ Dangerous

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \frac{1}{N^2} \times \frac{1}{\zeta^2} \times \frac{1}{\omega}$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

Actual order $\sim \dfrac{1}{N^0}$

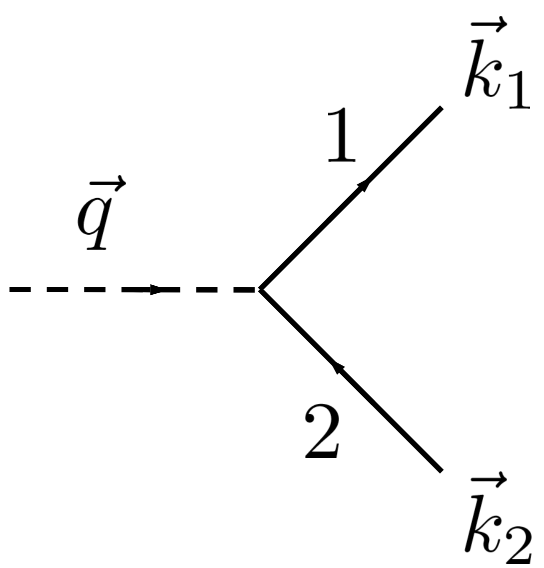
Double line representation

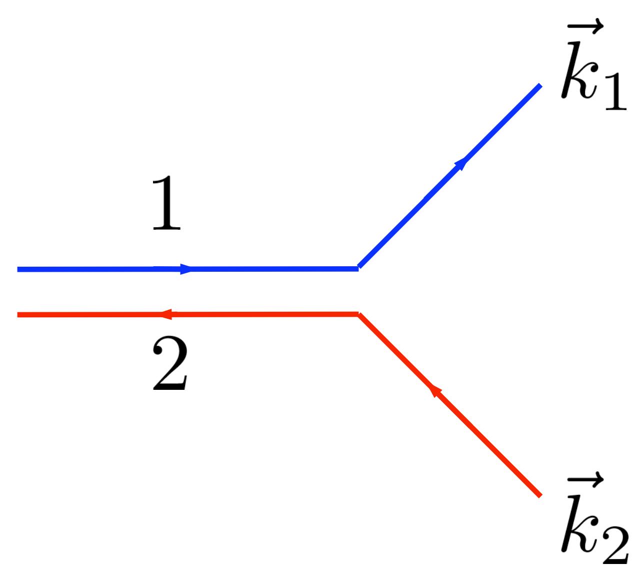
- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega, \vec{k}) = \frac{1}{-\Sigma_1(\omega, \vec{k}) - \vec{v} \cdot \vec{k}} \qquad \Sigma_1 \sim \frac{1}{N}$$

- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

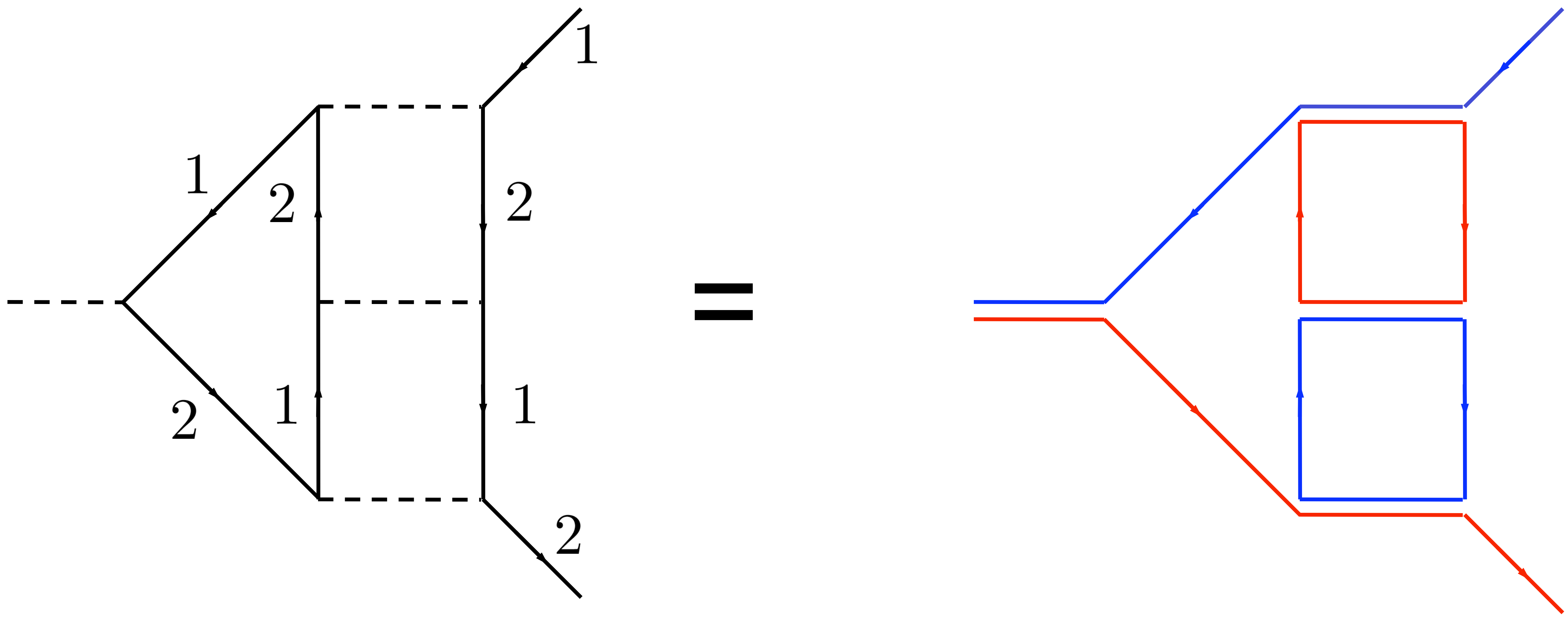
$$\vec{q} = \vec{k}_1 - \vec{k}_2 \qquad \vec{k}_1 \in \text{FS of } \psi_1 \qquad \vec{k}_2 \in \text{FS of } \psi_2$$





R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
S. W. Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

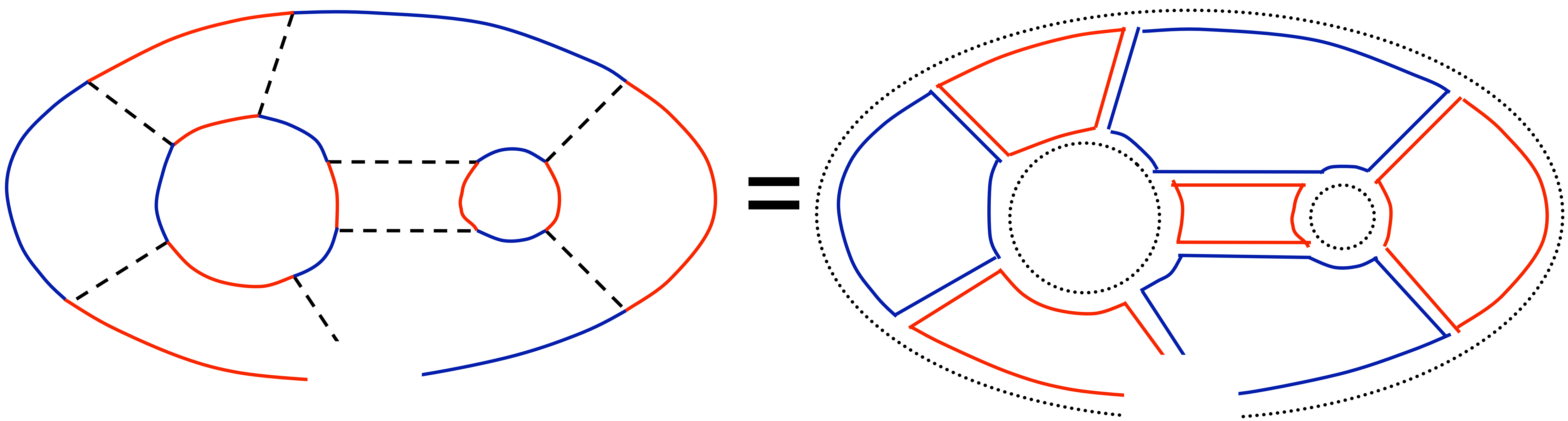
New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Singularities as $\zeta \to 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

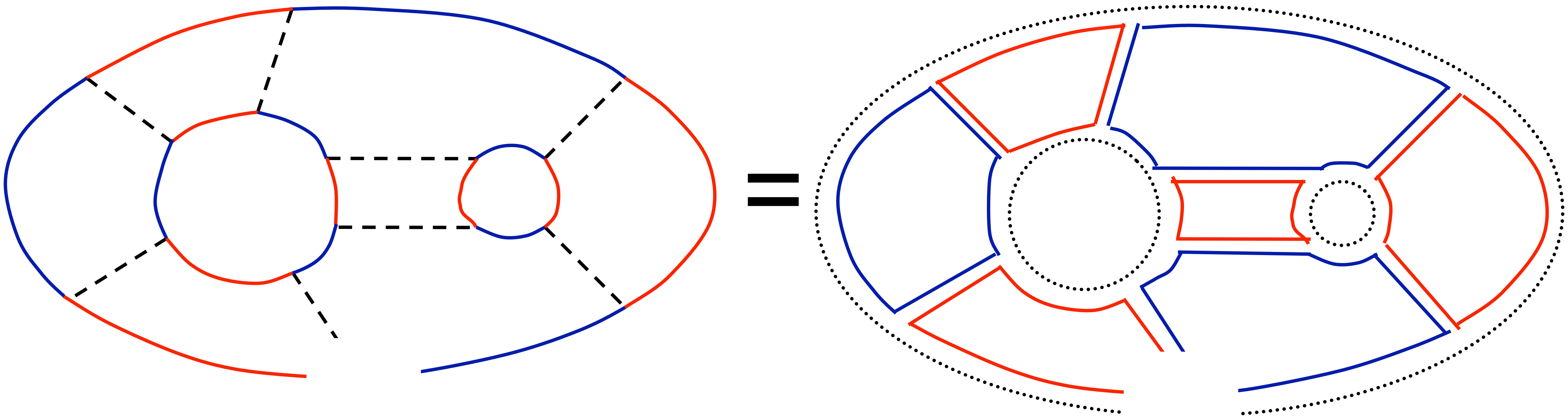


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines by drawing additional dotted single line loops for each fermion loop

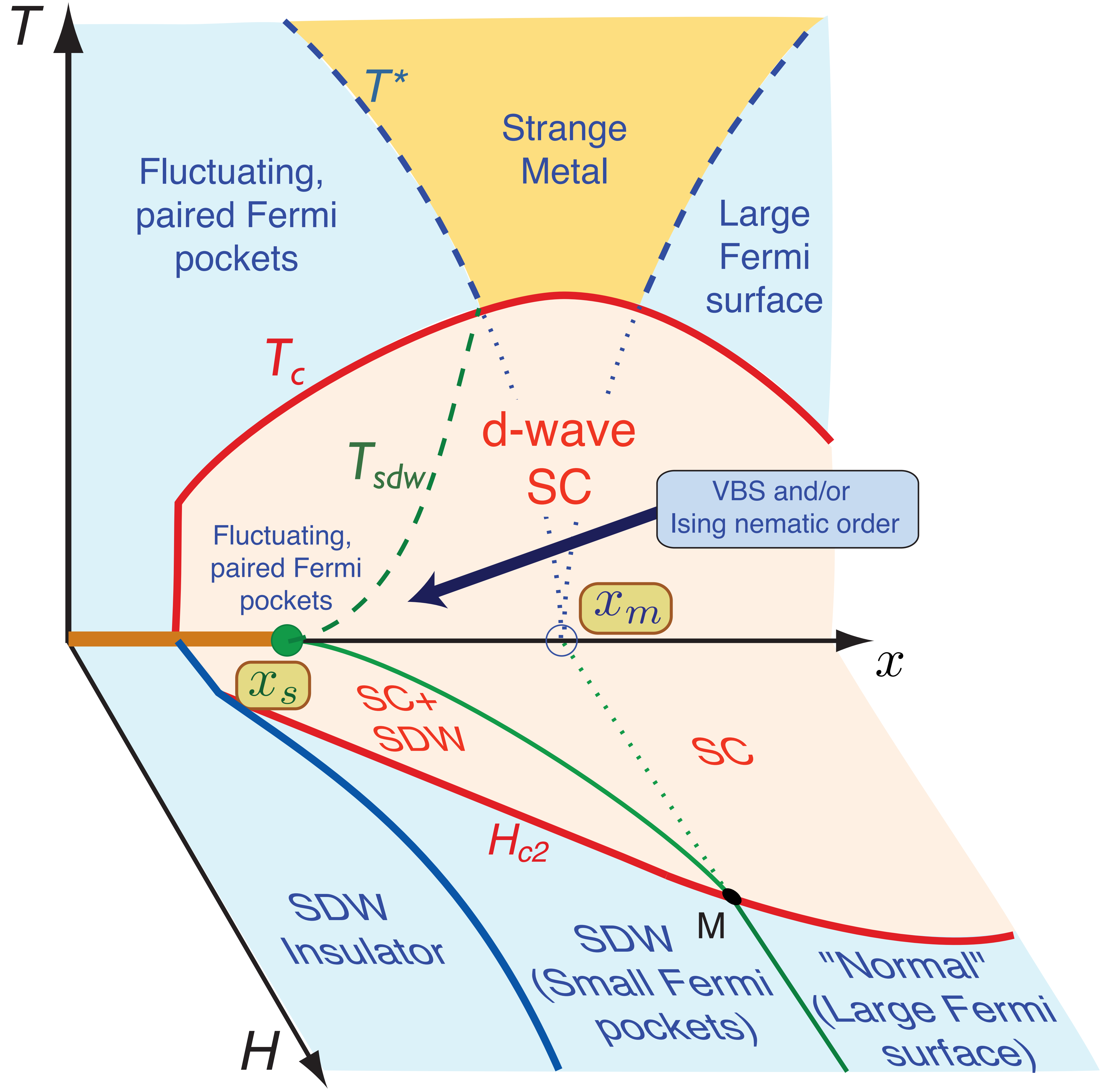
Sung-Sik Lee, arXiv:0905.4532

New infra-red singularities as $\zeta \to 0$ at higher loops
(Breakdown of Migdal-Eliashberg)



$$\text{Actual order} \sim \frac{1}{N^0}$$

A consistent analysis requires resummation of all planar graphs


T
T*
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
T_c
T_{sdw}
d-wave SC
VBS and/or Ising nematic order
Fluctuating, paired Fermi pockets
x_m
x_s
x
SC+ SDW
SC
H_{c2}
SDW Insulator
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity