Topology and the phases of quantum matter in two dimensions

Albanova and Nordita Colloquium,
2016 Nobel prize in physics, impact on physics research and future directions
December 8, 2016

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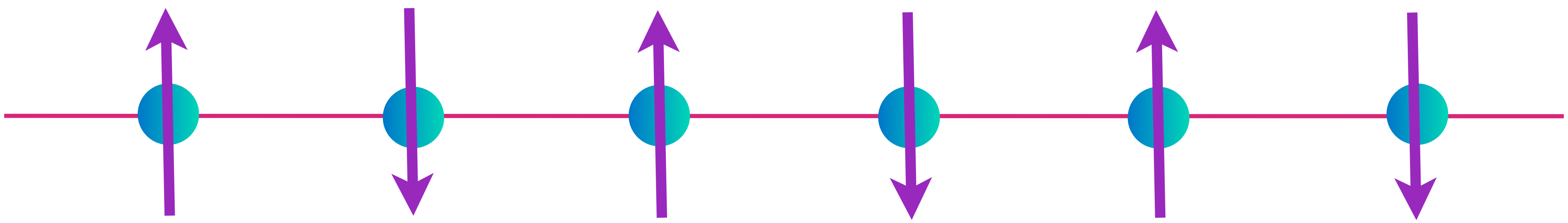
PHYSICS
HARVARD

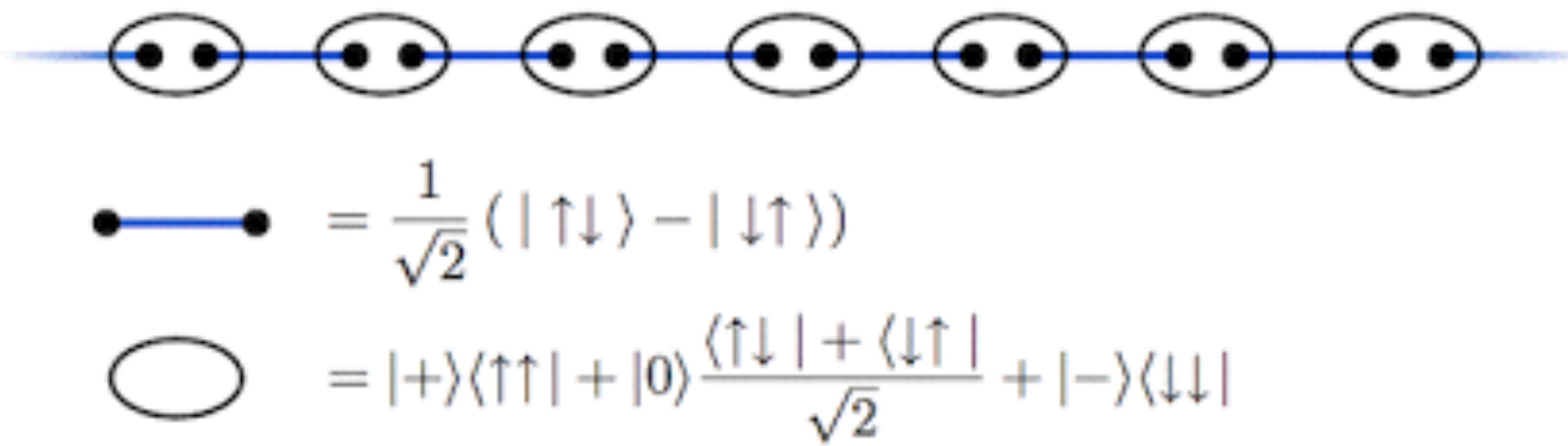
Quantum antiferromagnets in one dimension

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



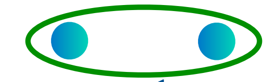
Haldane: Semiclassical theory of quantum fluctuations

Berry phases of spacetime Skyrmions textures lead to sensitivity to 2S (mod 2). For S=1, we obtain the Haldane gap SPT state.



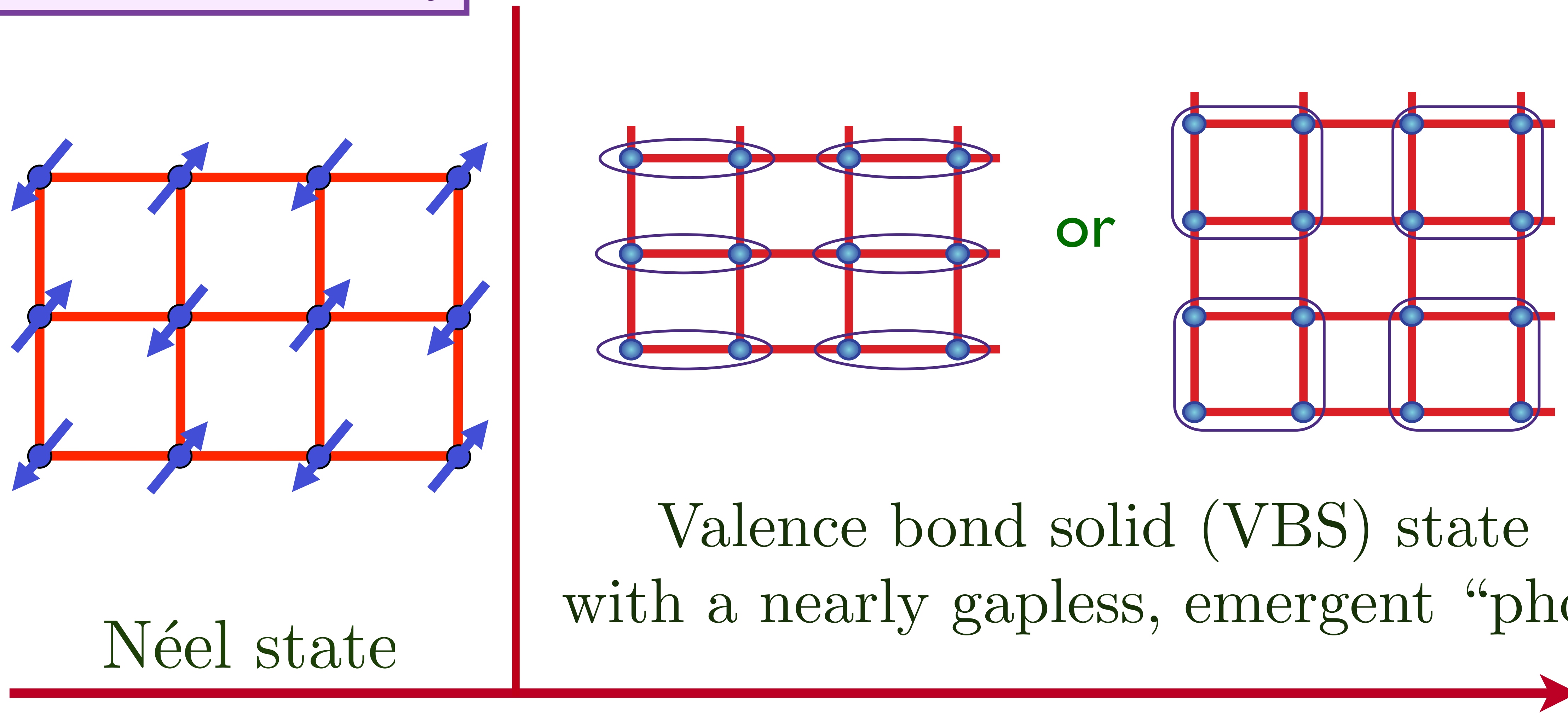
Quantum antiferromagnets on the square lattice

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Semiclassical theory

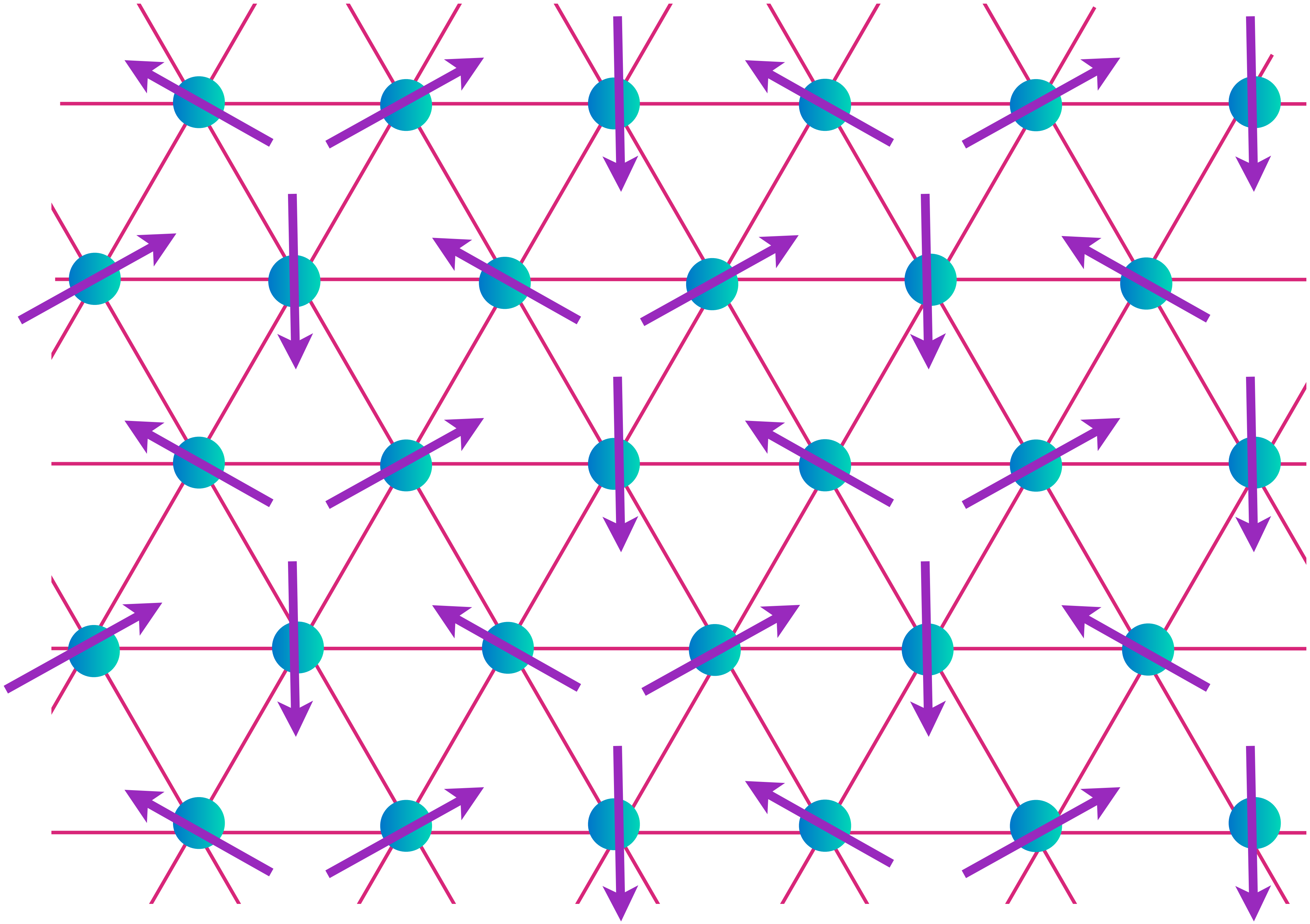


Haldane: Berry phases of spacetime instantons ("hedgehogs") lead to sensitivity to 2S (mod 4).

Read and S.S. : Valence bond solids break lattice translational symmetry except for 2S (mod 4) = 0.

Quantum antiferromagnets on the triangular lattice

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



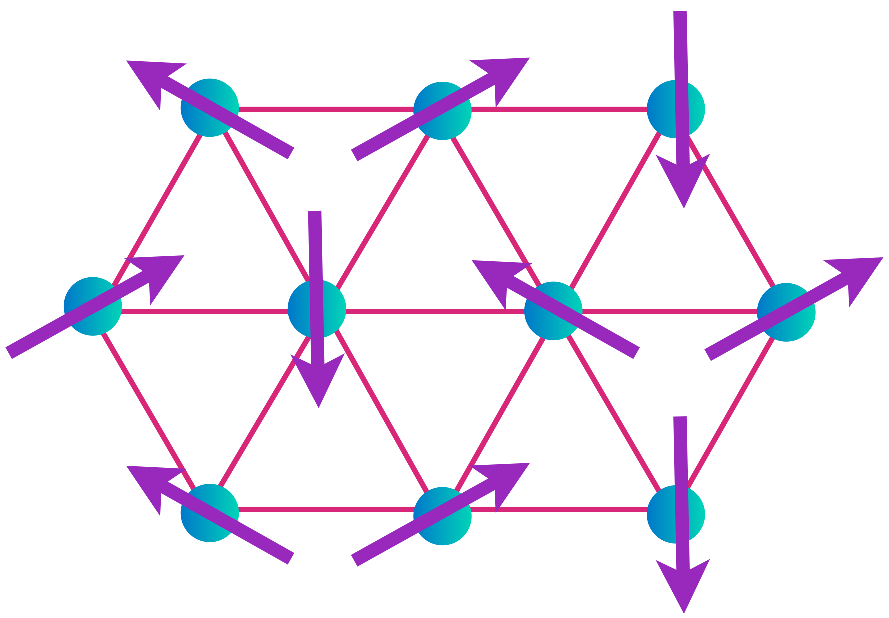
Nearest-neighbor model has non-collinear Neel order

Quantum antiferromagnets on the triangular lattice

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Semiclassical theory

(Read and S.S. (1991))



non-collinear Néel state

Semiclassical theory leads to a spin liquid state with Z_2 topological order.

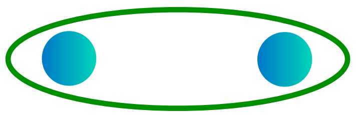
The topological order is associated with suppressed vortex defects in the non-collinear Neel order

s_c s

Vortex suppression is similar to low temperature phase of the Kosterlitz-Thouless transition. (However spin correlations decay exponentially, not as a power-law)

Quantum antiferromagnets on the triangular lattice

Suppressed-vortex-state is the resonating valence bond state with Z_2 topological order.



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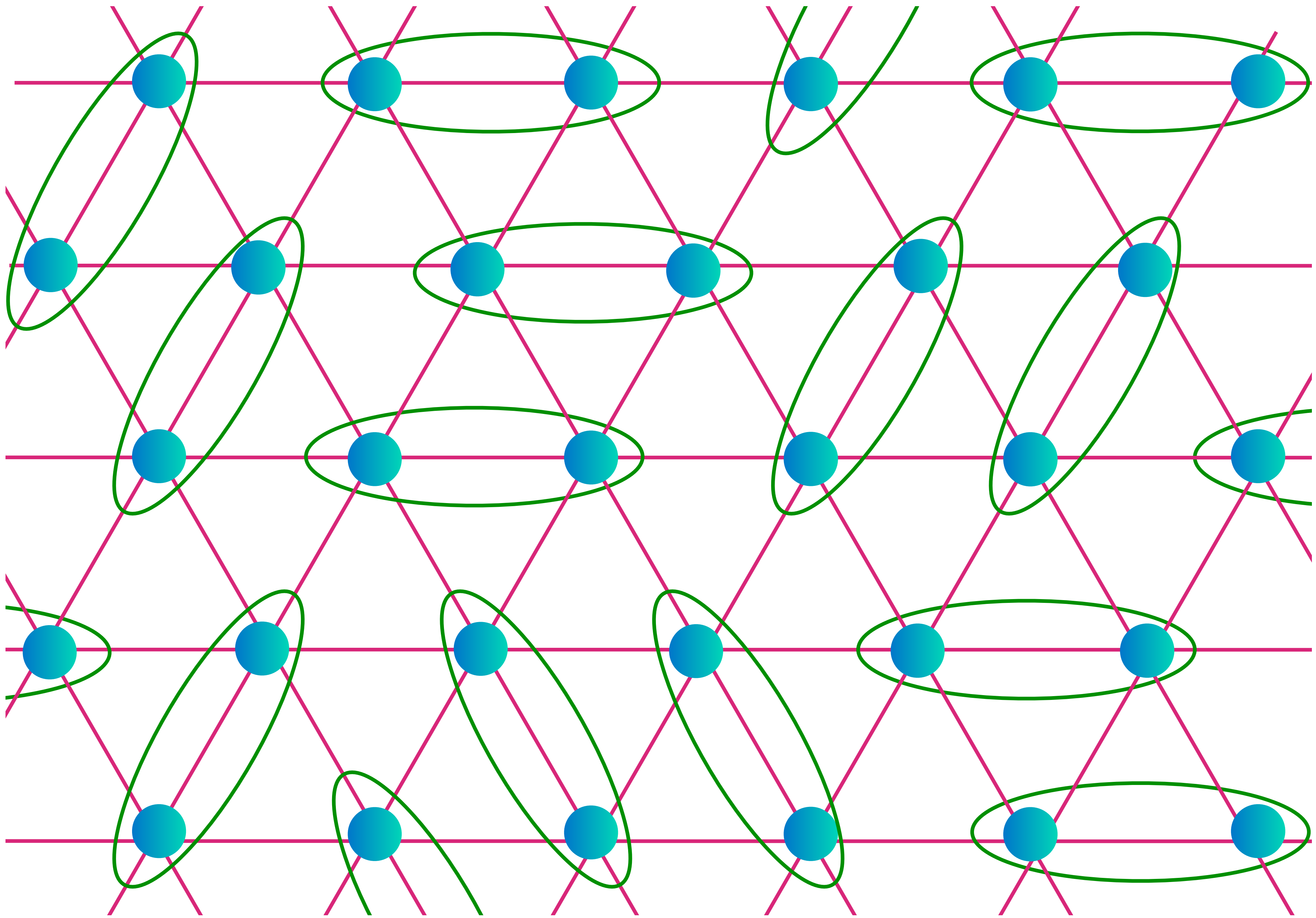

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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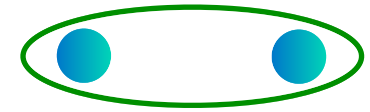

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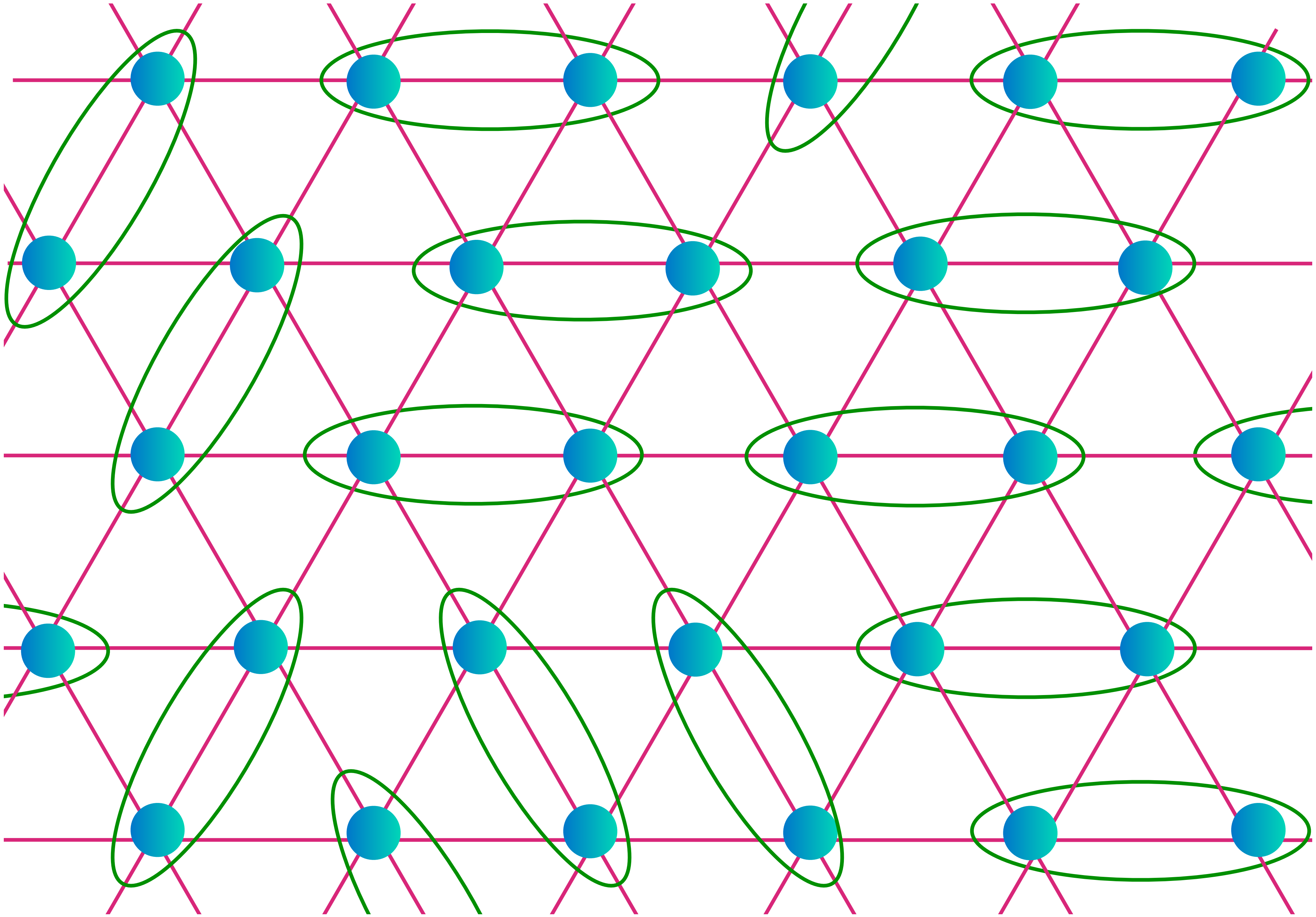

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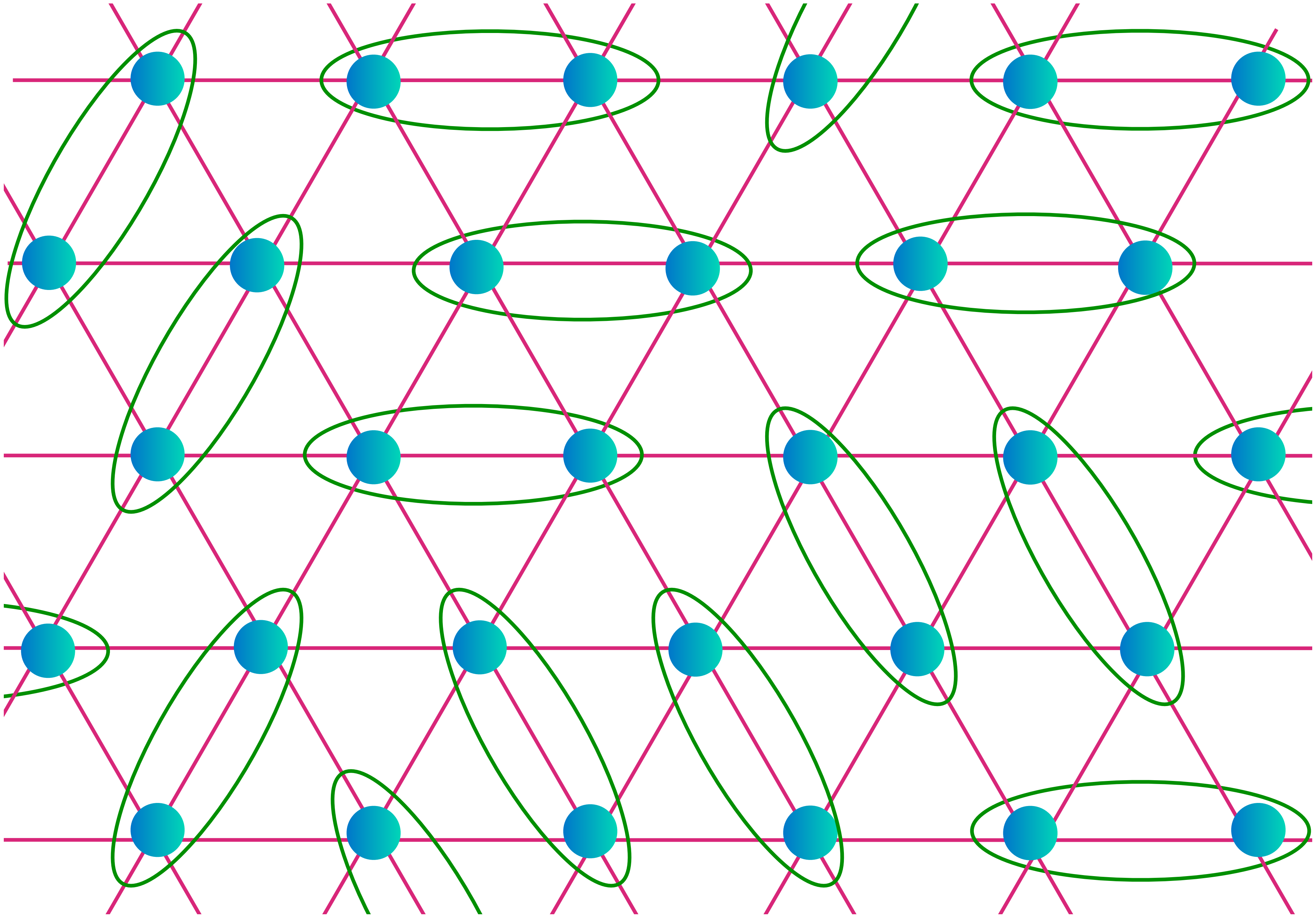

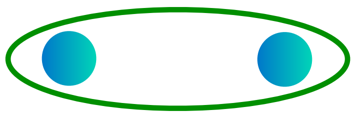

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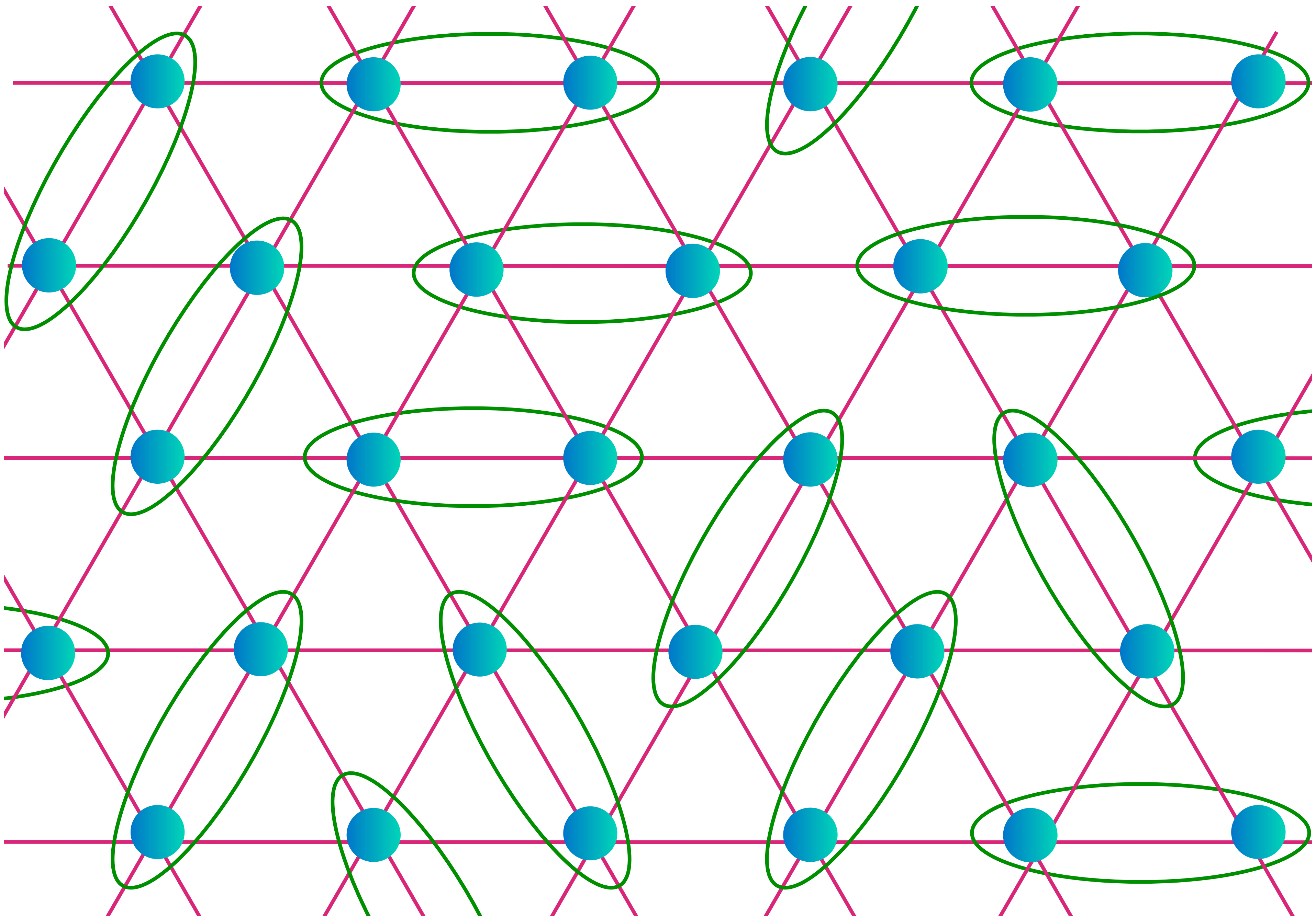

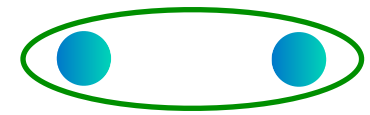

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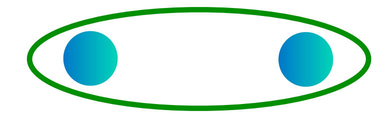

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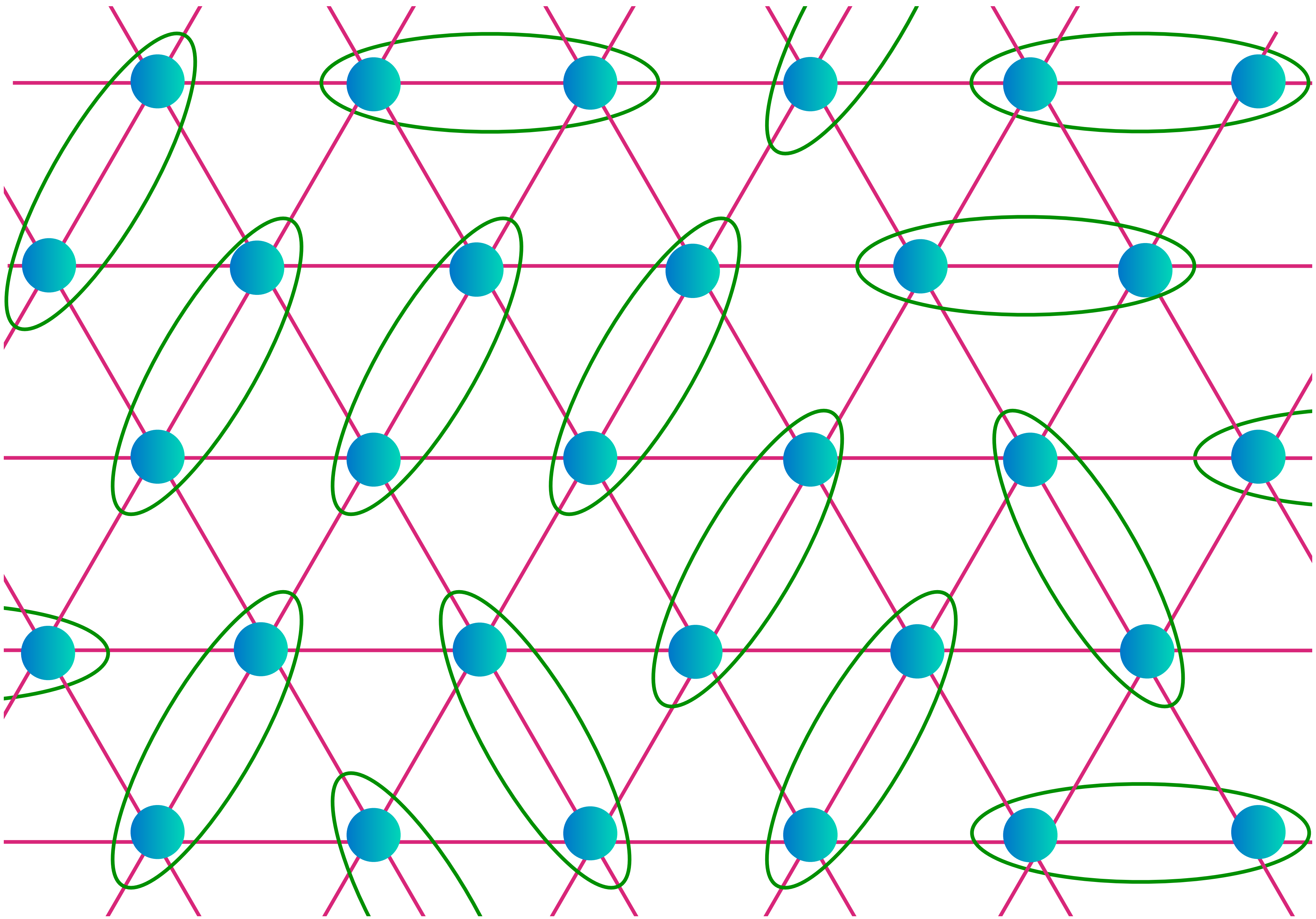

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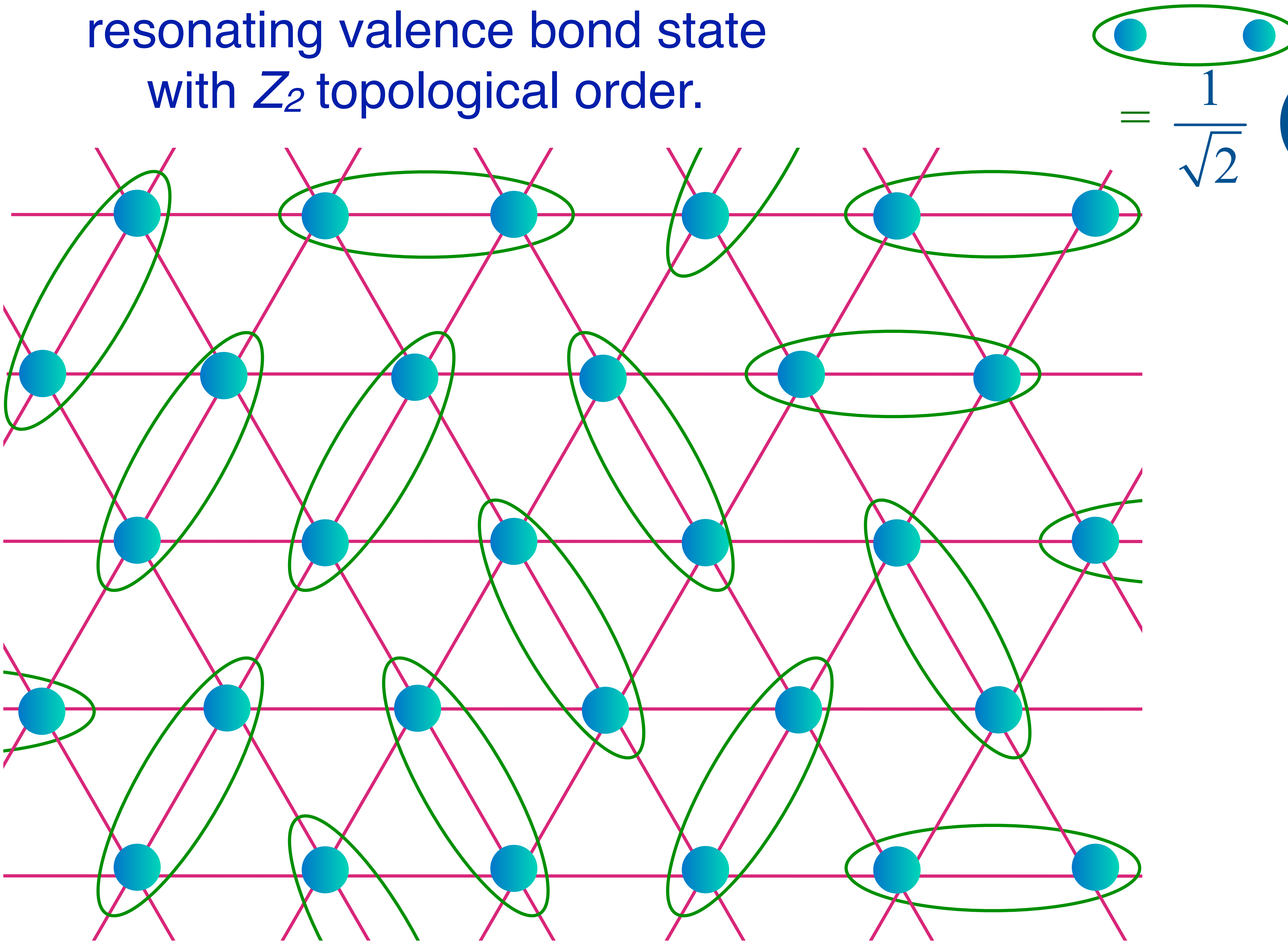

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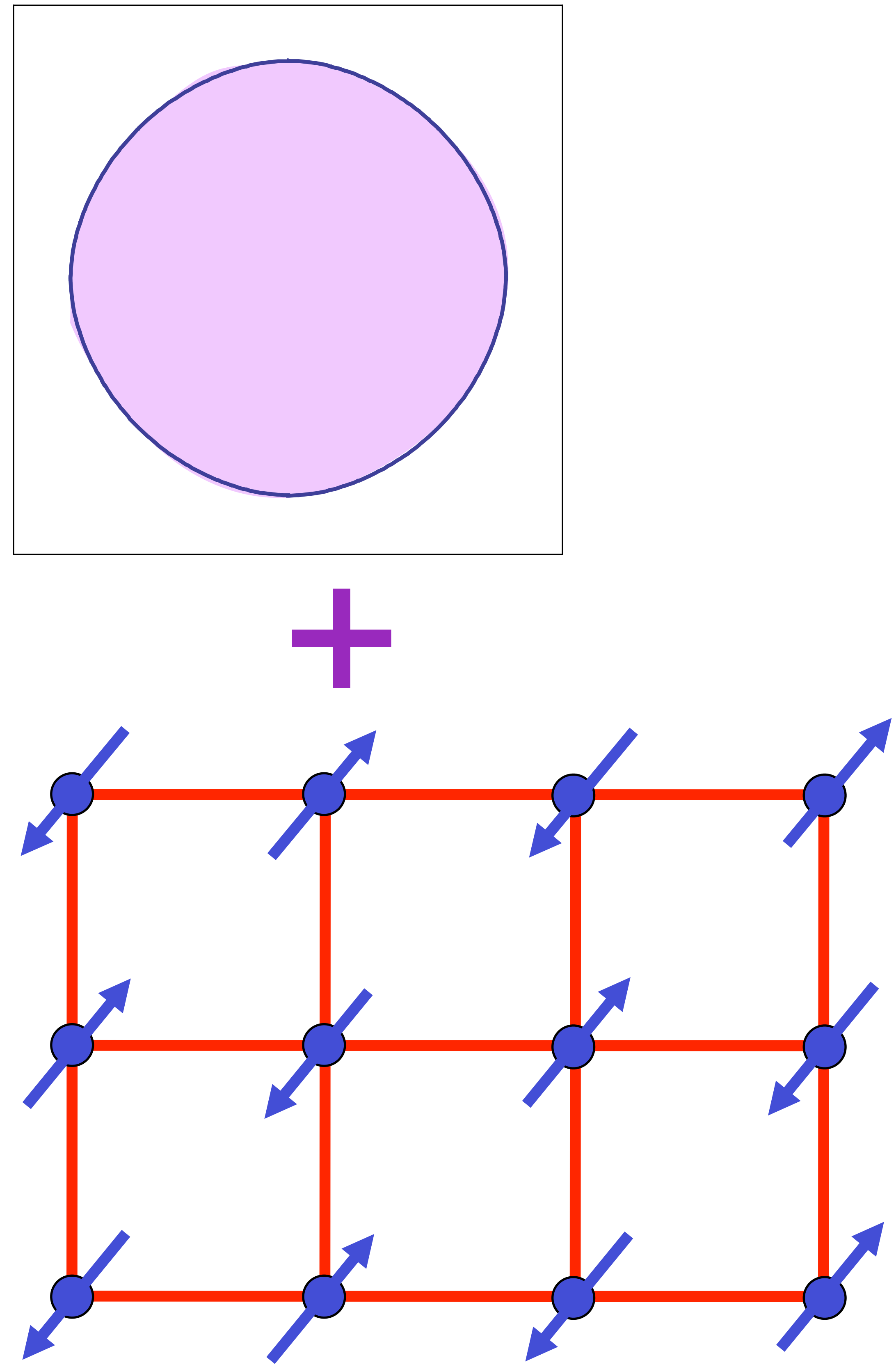
Quantum antiferromagnetism in metals

Metal with “large”
Fermi surface

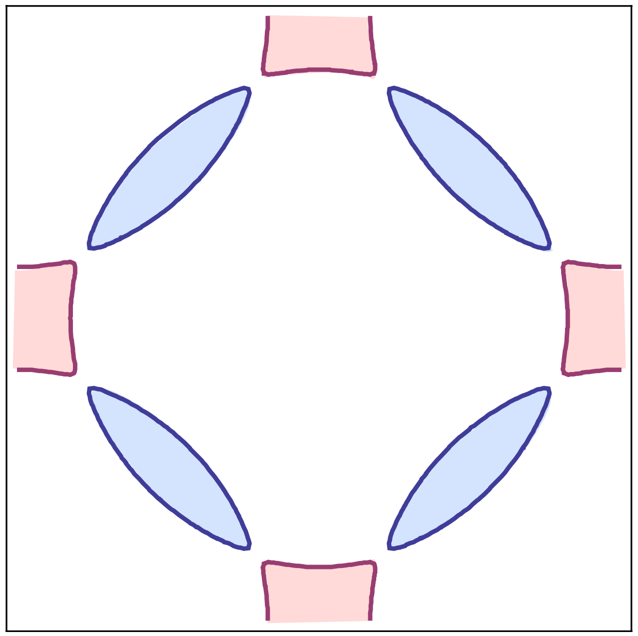


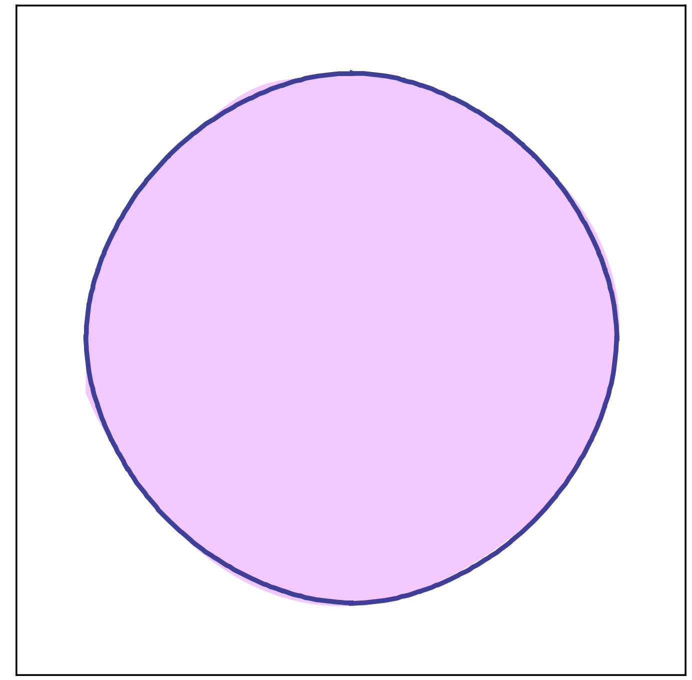
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Quantum antiferromagnetism in metals





$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with antiferromagnetic order and electron and hole pockets

$$\langle \vec{\varphi} \rangle = 0$$

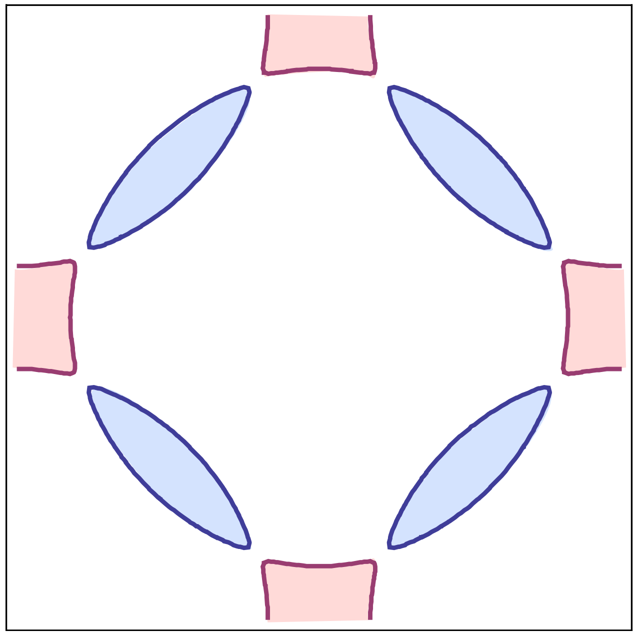
Metal with "large" Fermi surface

Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

Topological order in metals

(Senthil, S.S., Vojta (2003))



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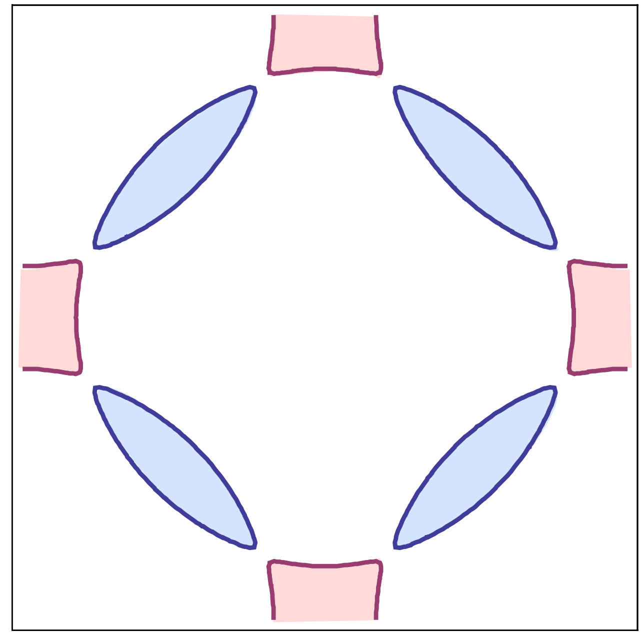
Metal with *antiferromagnetic* order and electron and hole pockets



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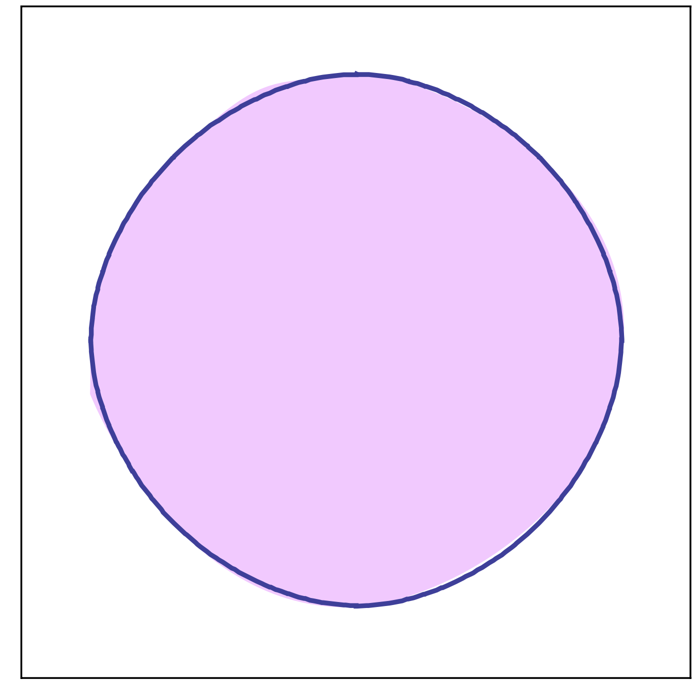

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Metal with "large" Fermi surface

Vortex suppression similar to low temperature phase of the Kosterlitz-Thouless transition.

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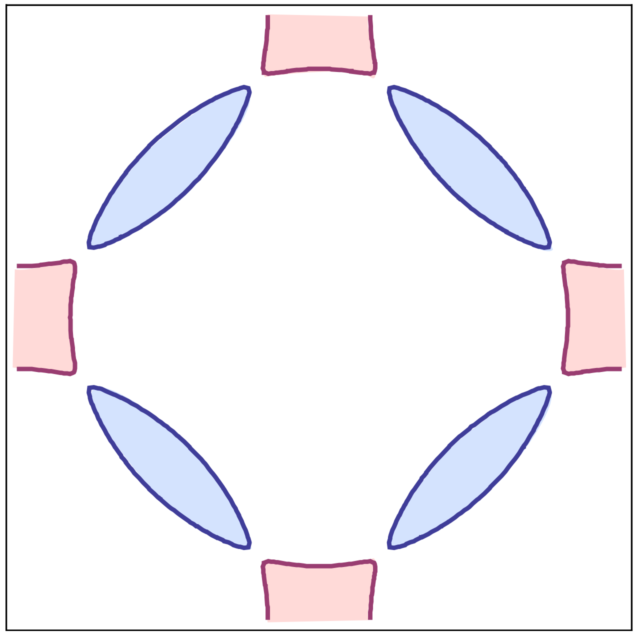

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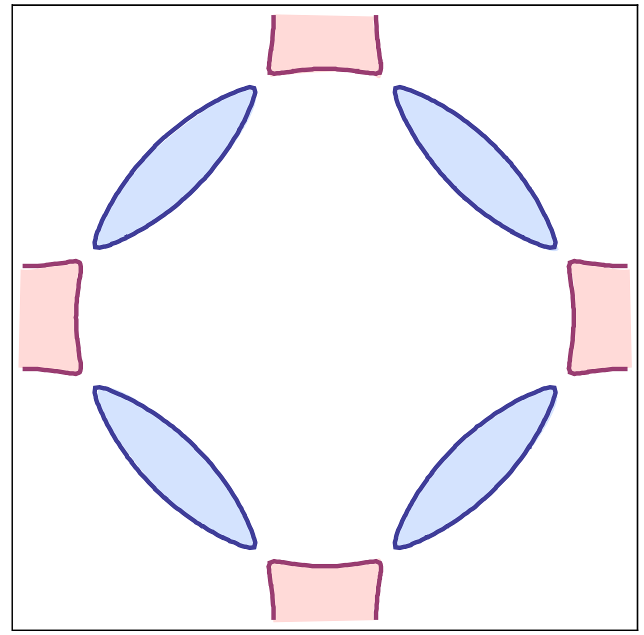

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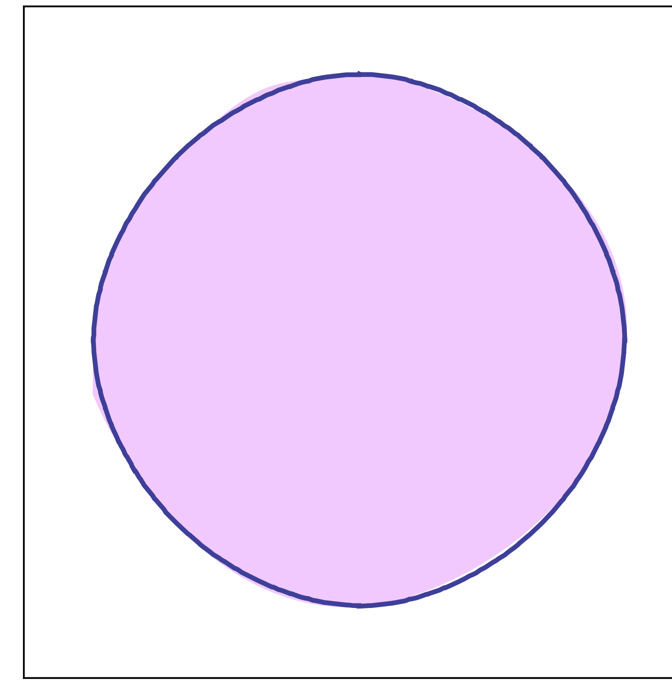

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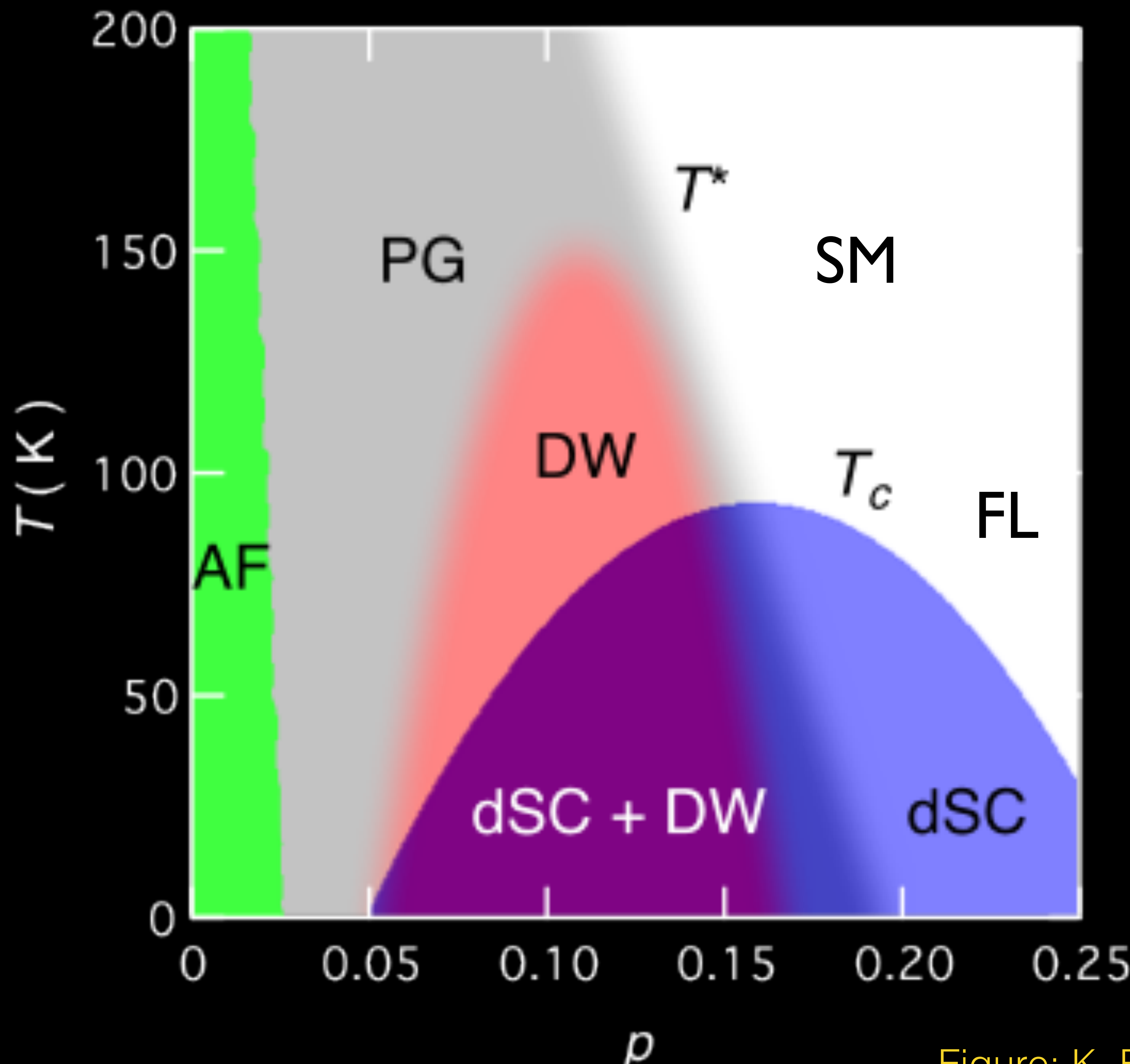
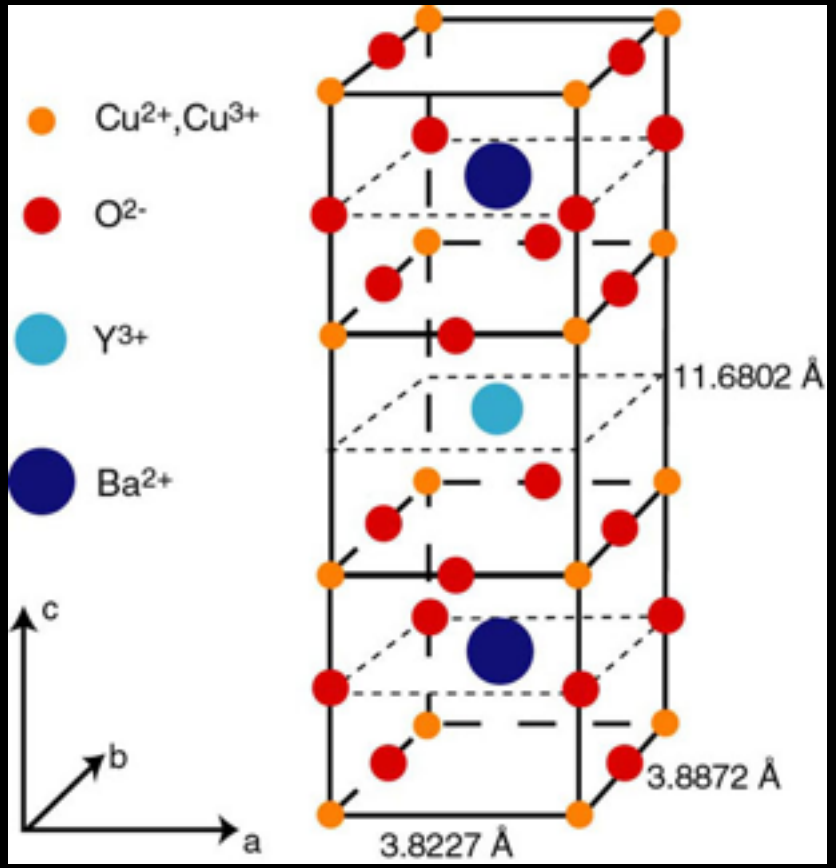
Metal with "large" Fermi surface

Metal with topological order does not obey the Luttinger theorem for the volume enclosed by the Fermi surface





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

Topological order in metals

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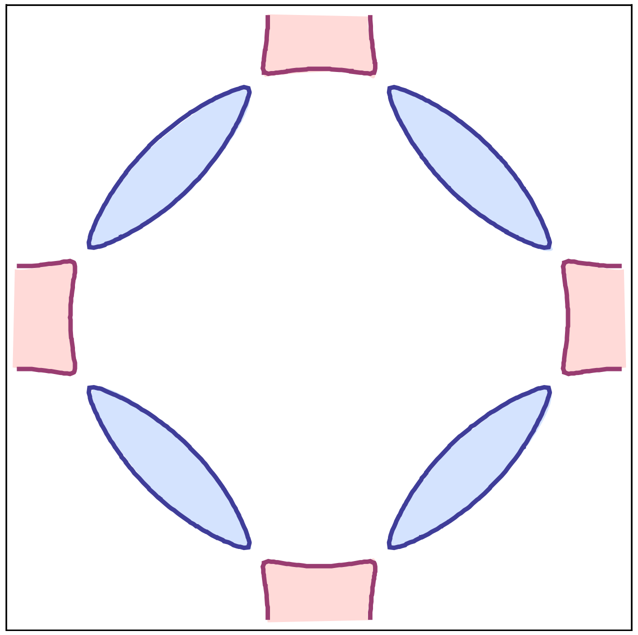

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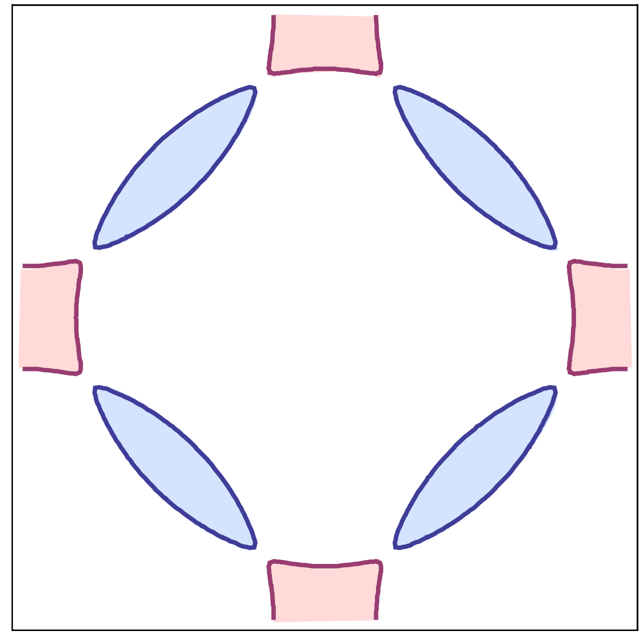

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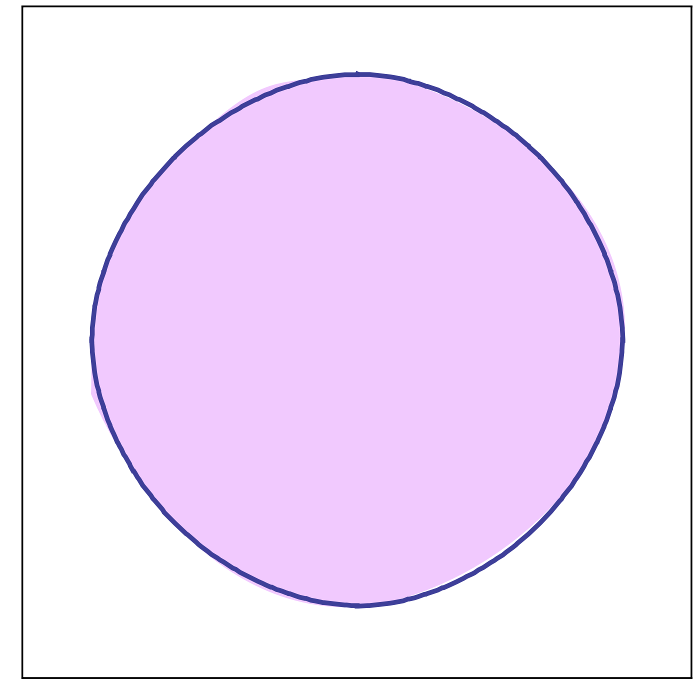

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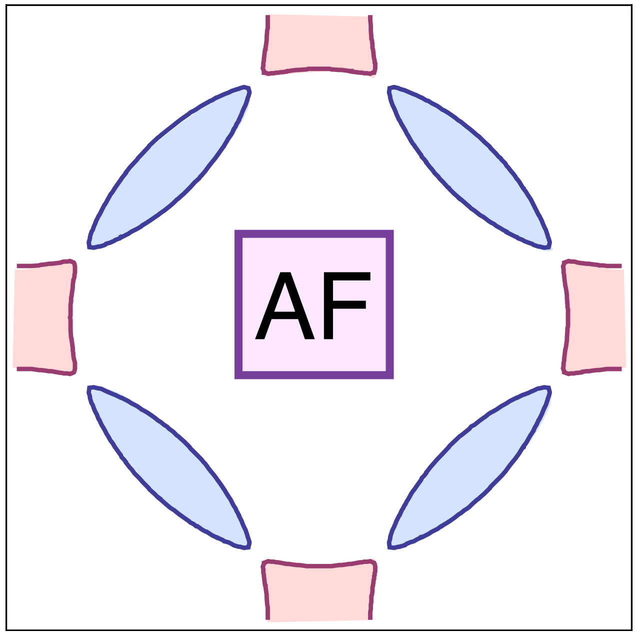

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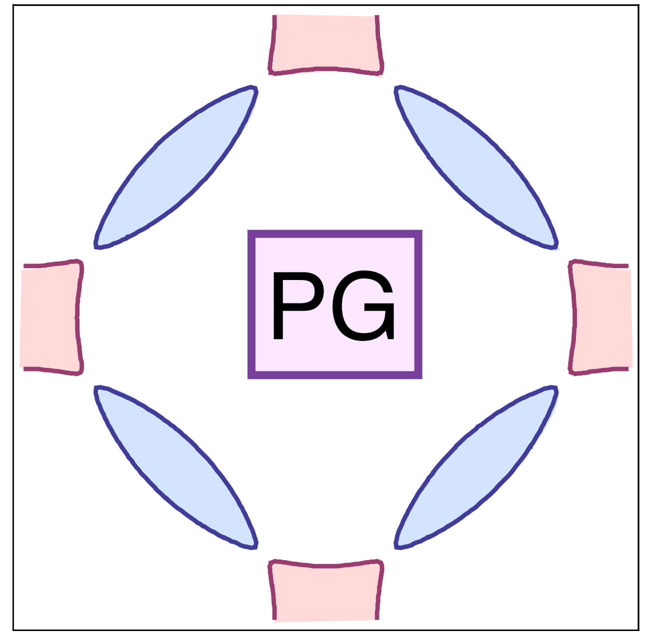

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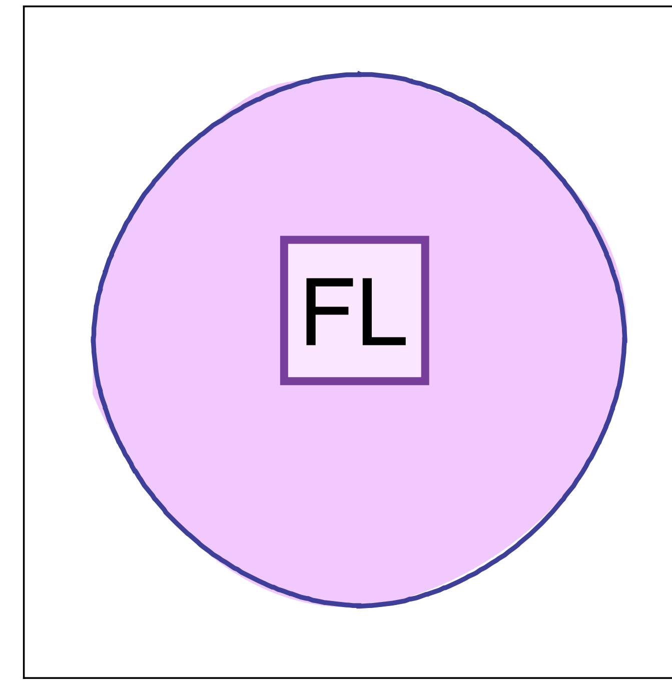

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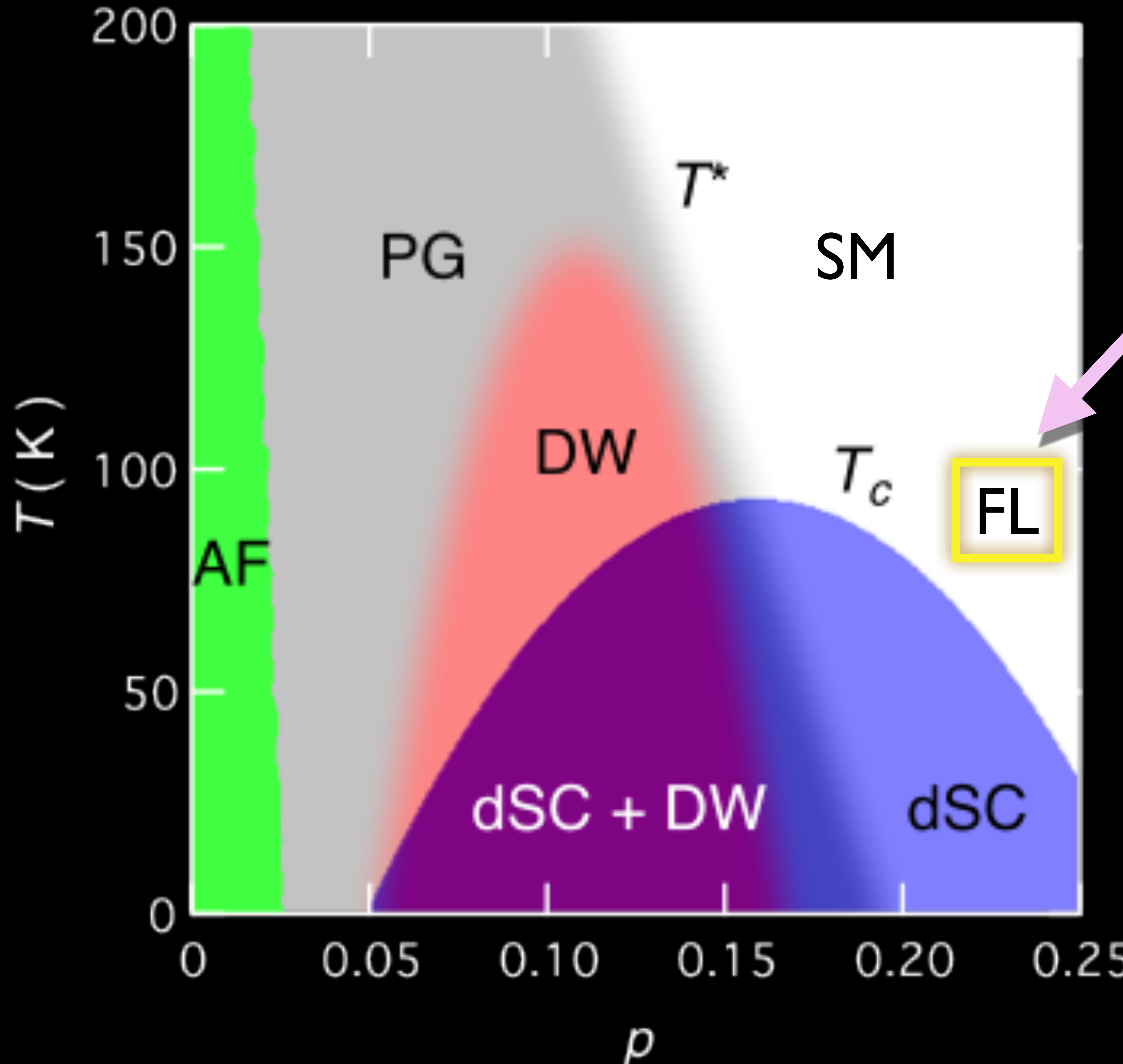
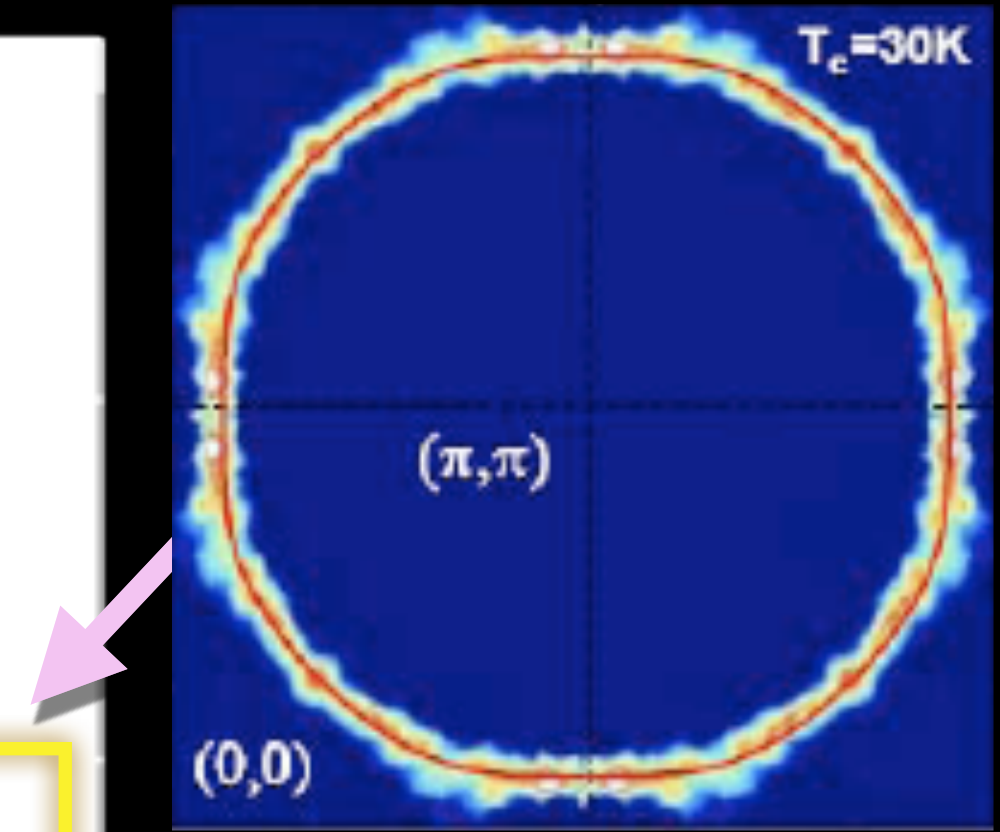
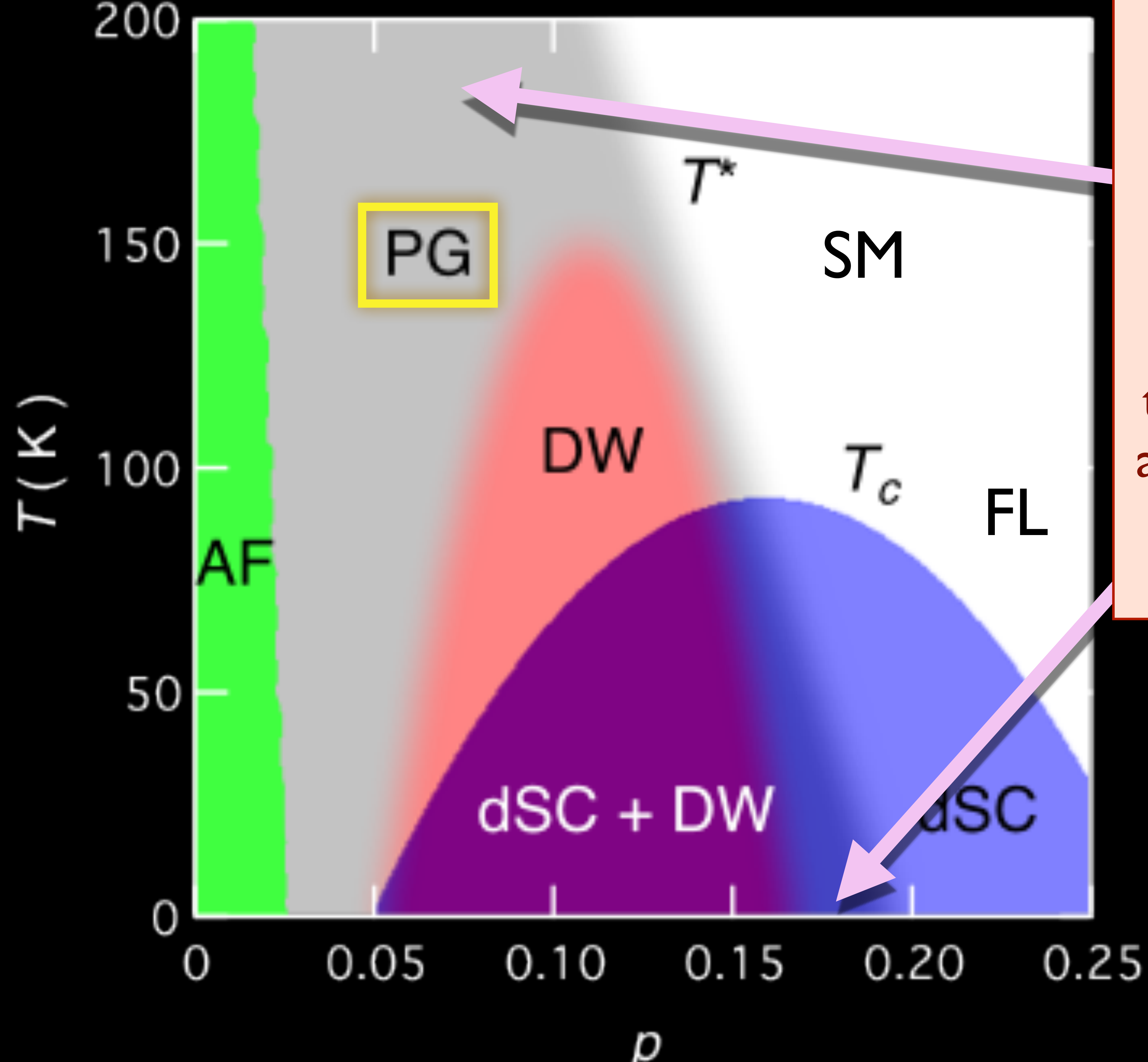
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





A conventional metal:
the Fermi liquid with Fermi surface of size $1+p$

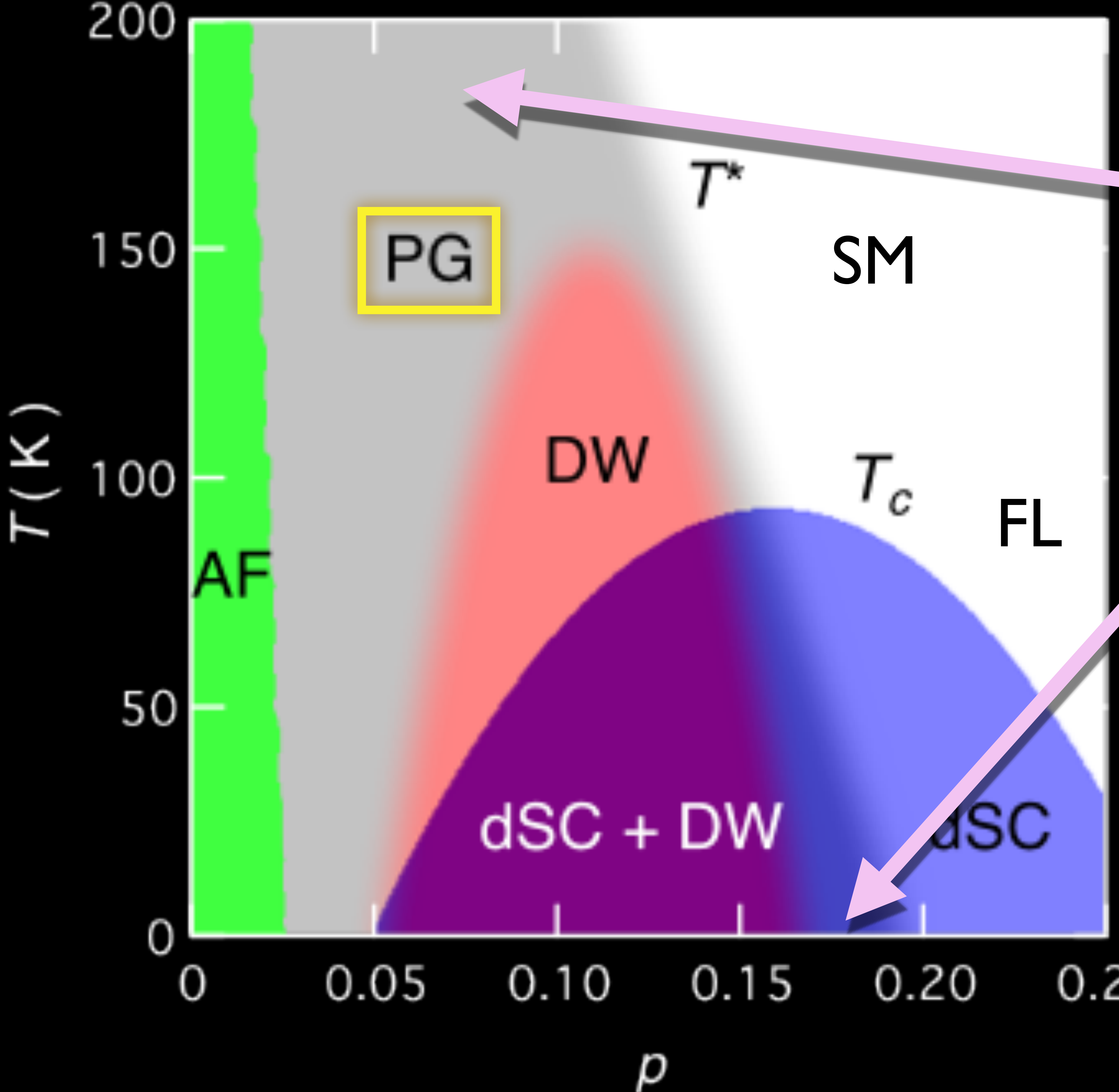
S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D.A. Bonn, W.N. Hardy, R. Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Nature **531**, 210 (2016).



Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)



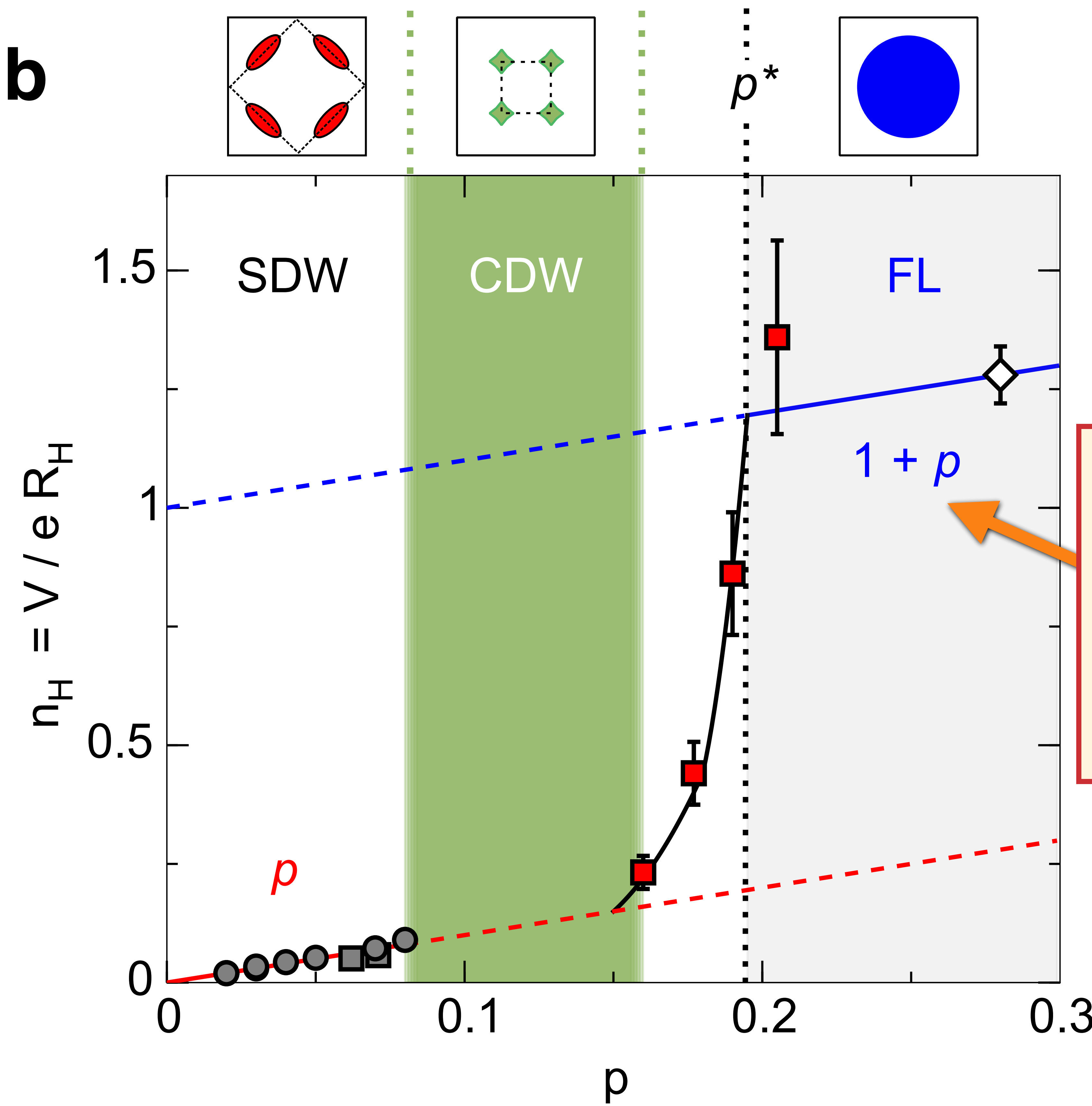
<u>Pseudogap metal</u>

at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

If present at $T=0$, a metal with a size p Fermi surface (and translational symmetry preserved) *<u>must</u>* have *<u>topological order</u>*

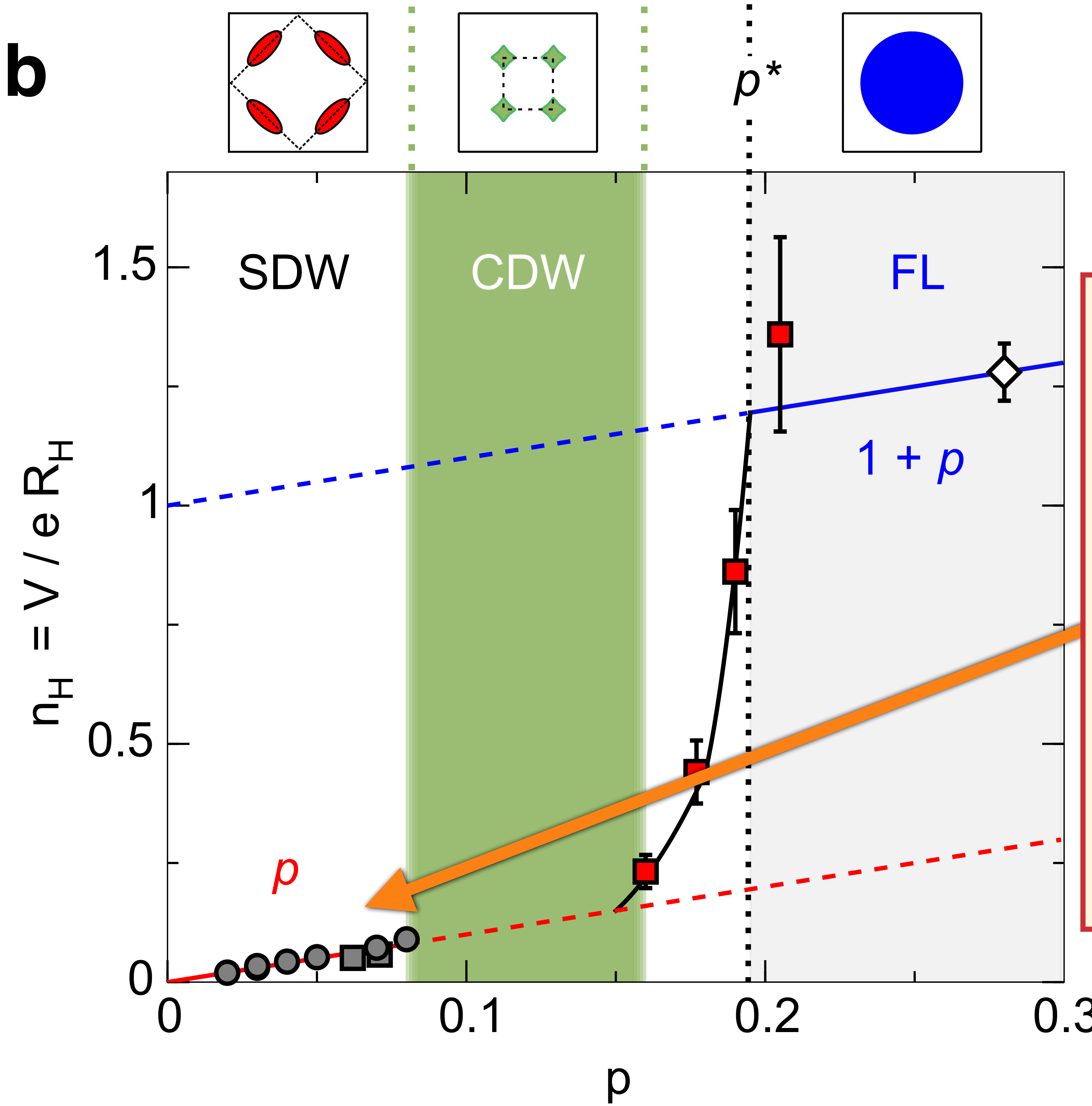
Hall effect measurements in YBCO



Fermi liquid (FL) with carrier density 1+*p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

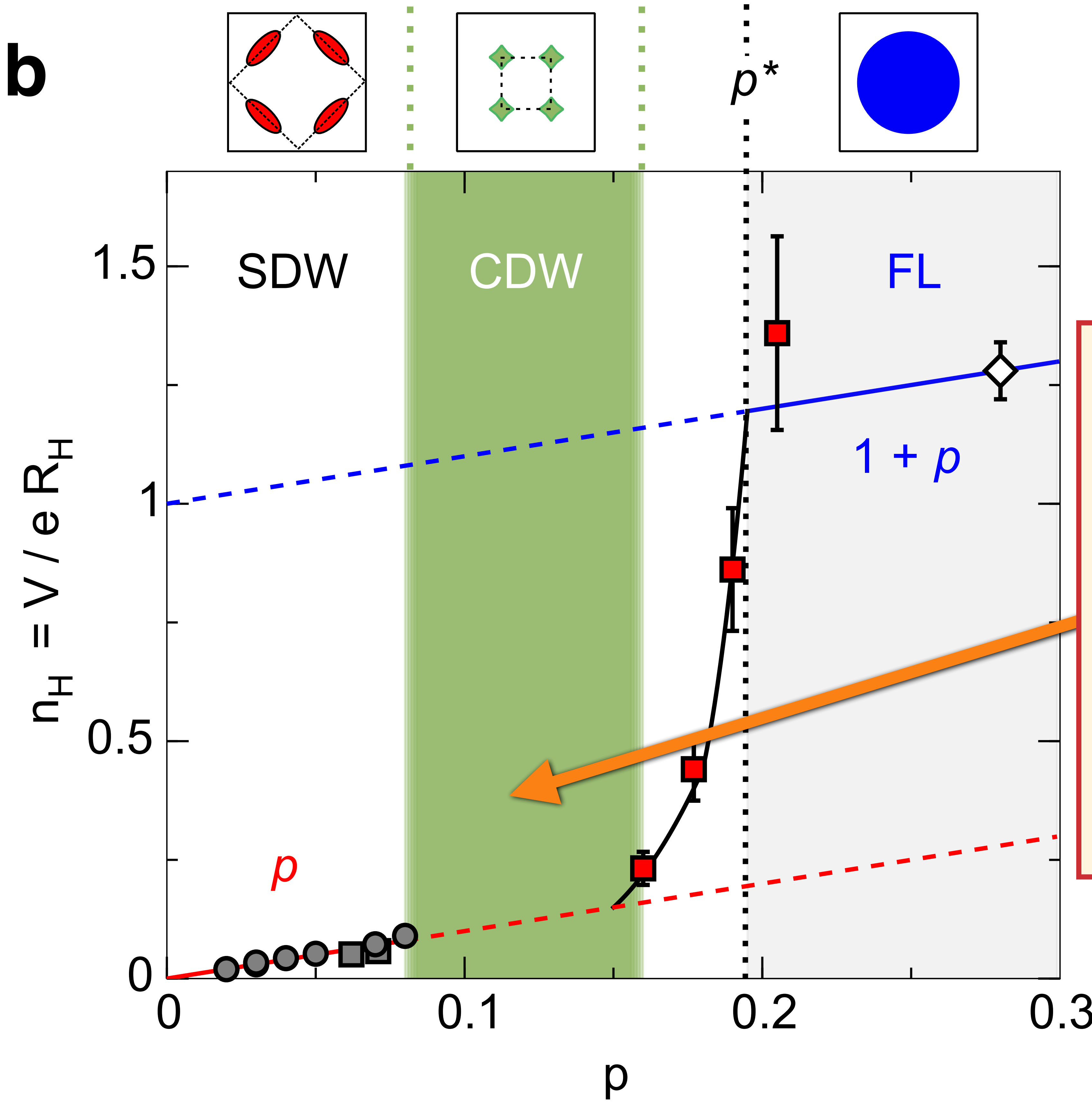
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density *p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

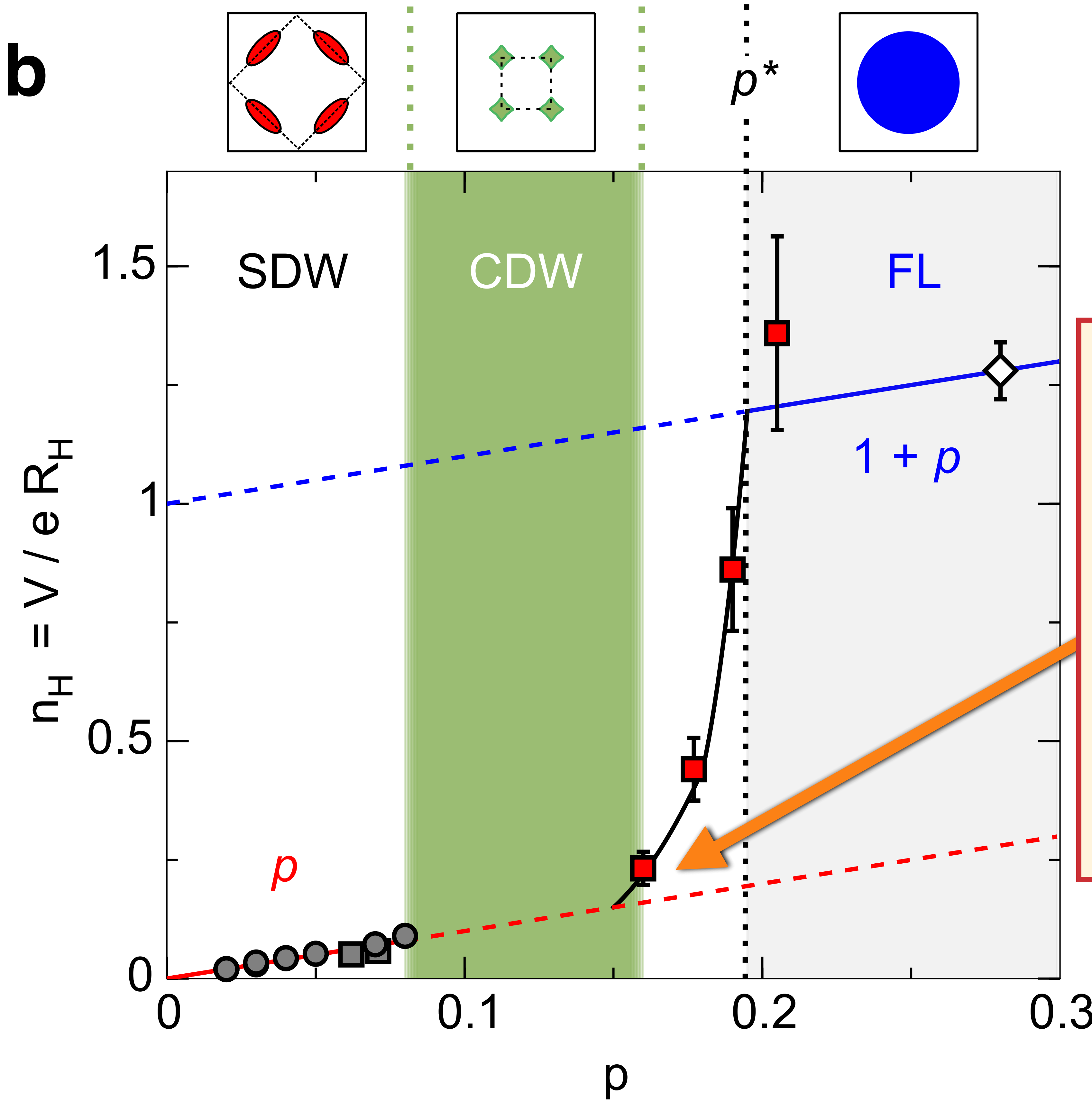
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

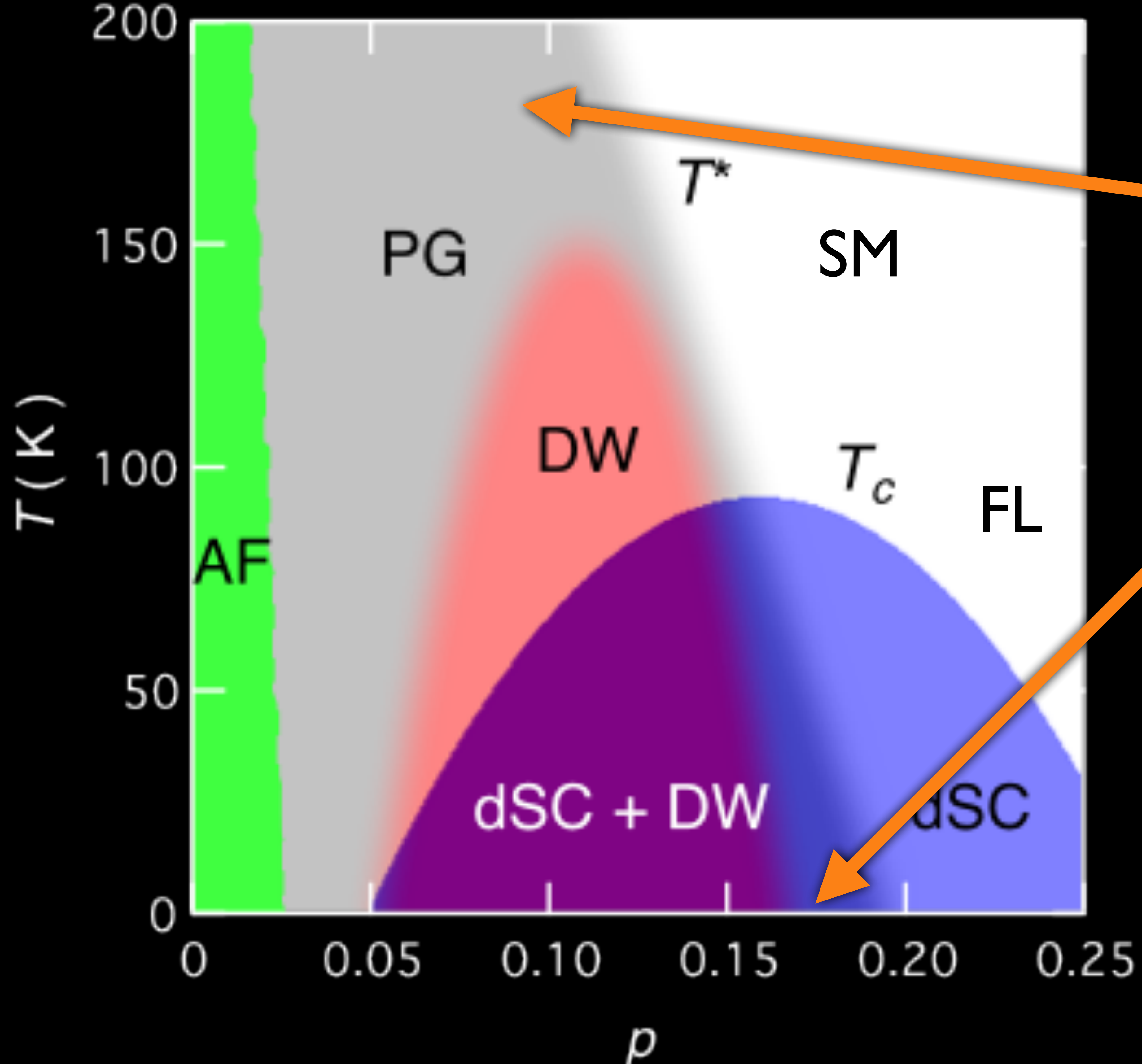
Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

Hall effect measurements in YBCO



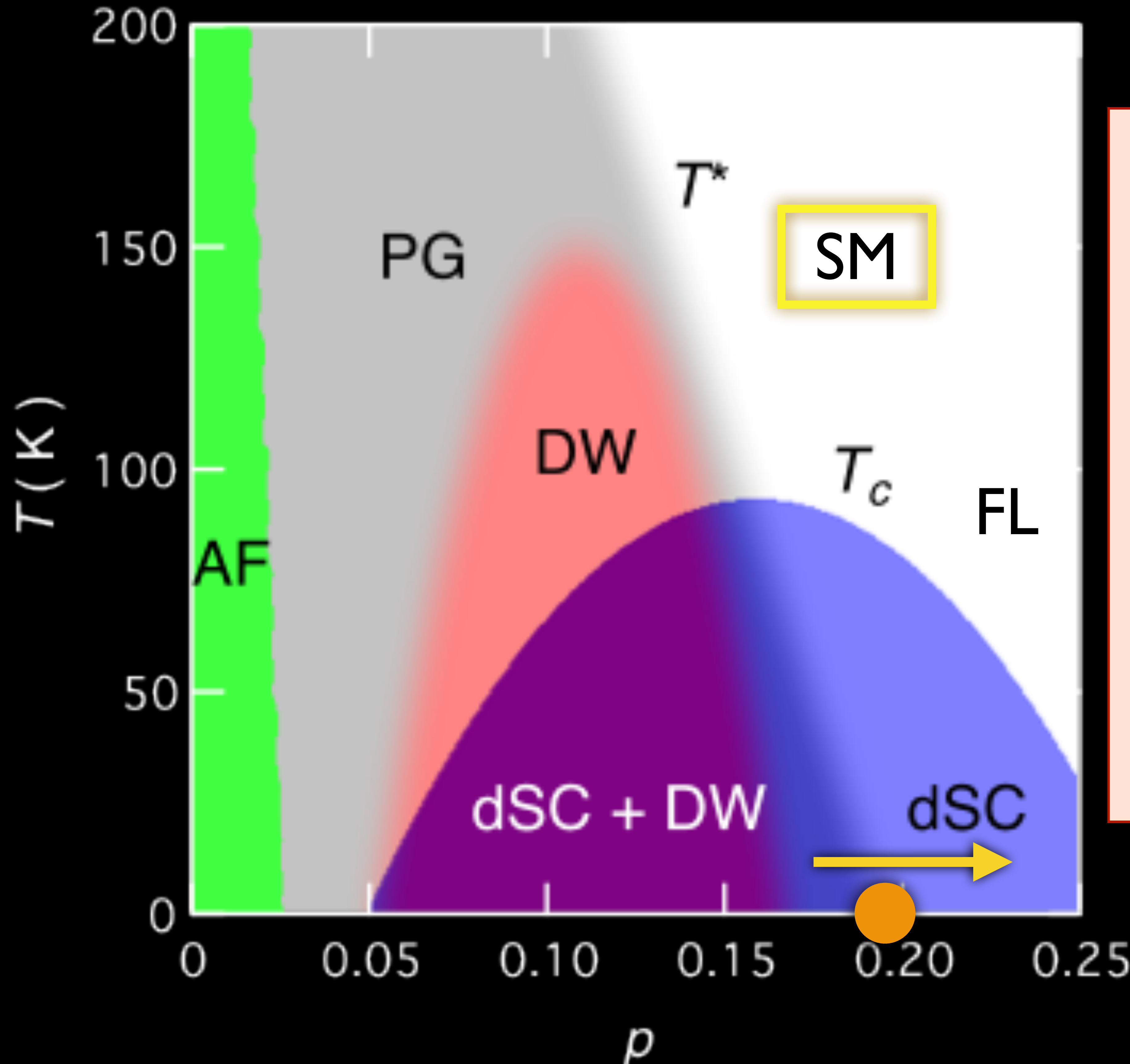
Evidence for a metal with topological order: Fermi surface of size *p* !

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)



Metal with topological order ?

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579.



Gauge theory for a topological phase transition, and for the strange metal (SM)