A theory of the underdoped cuprates

Talk online: sachdev.physics.harvard.edu

A theory of the underdoped cuprates

Hole dynamics in an antiferromagnet across a deconfined quantum critical point, R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Physical Review* B **75** , 235122 (2007).

Algebraic charge liquids and the underdoped cuprates, R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008).

Destruction of Neel order in the cuprates by electron doping, R. K. Kaul, M. Metlitksi, S. Sachdev, and C. Xu, *Physical Review B* **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates, V. Galitski and S. Sachdev, arXiv:0901.0005

Outline

1. Nodal-anti-nodal dichotomy in the cuprates

    Survey of recent experiments

2. Spin density wave theory of normal metal

    From a "large" Fermi surface to electron and hole pockets

3. Loss of Neel order in insulating square lattice antiferromagnets

    Landau-Ginzburg theory vs. gauge theory for spinons

4. Algebraic charge liquids

    Pairing by gauge forces, d-wave superconductivity, and the nodal-anti-nodal dichotomy

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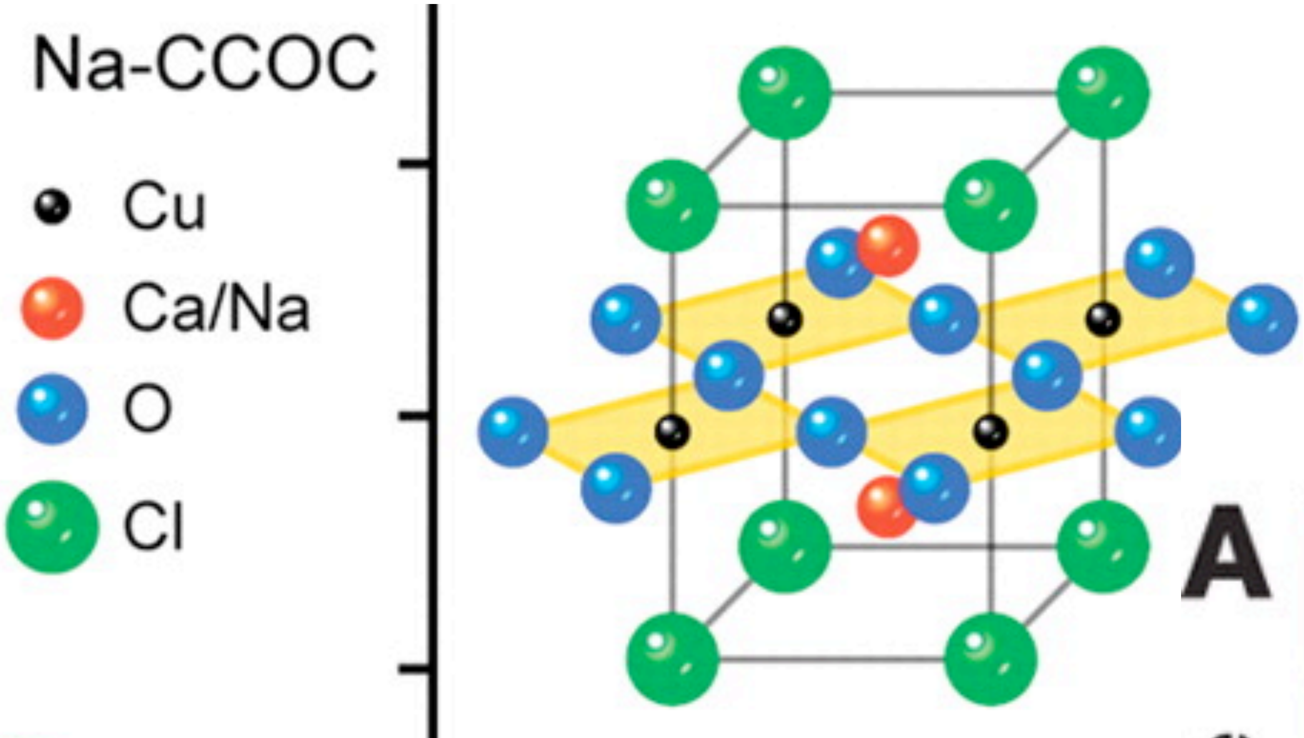
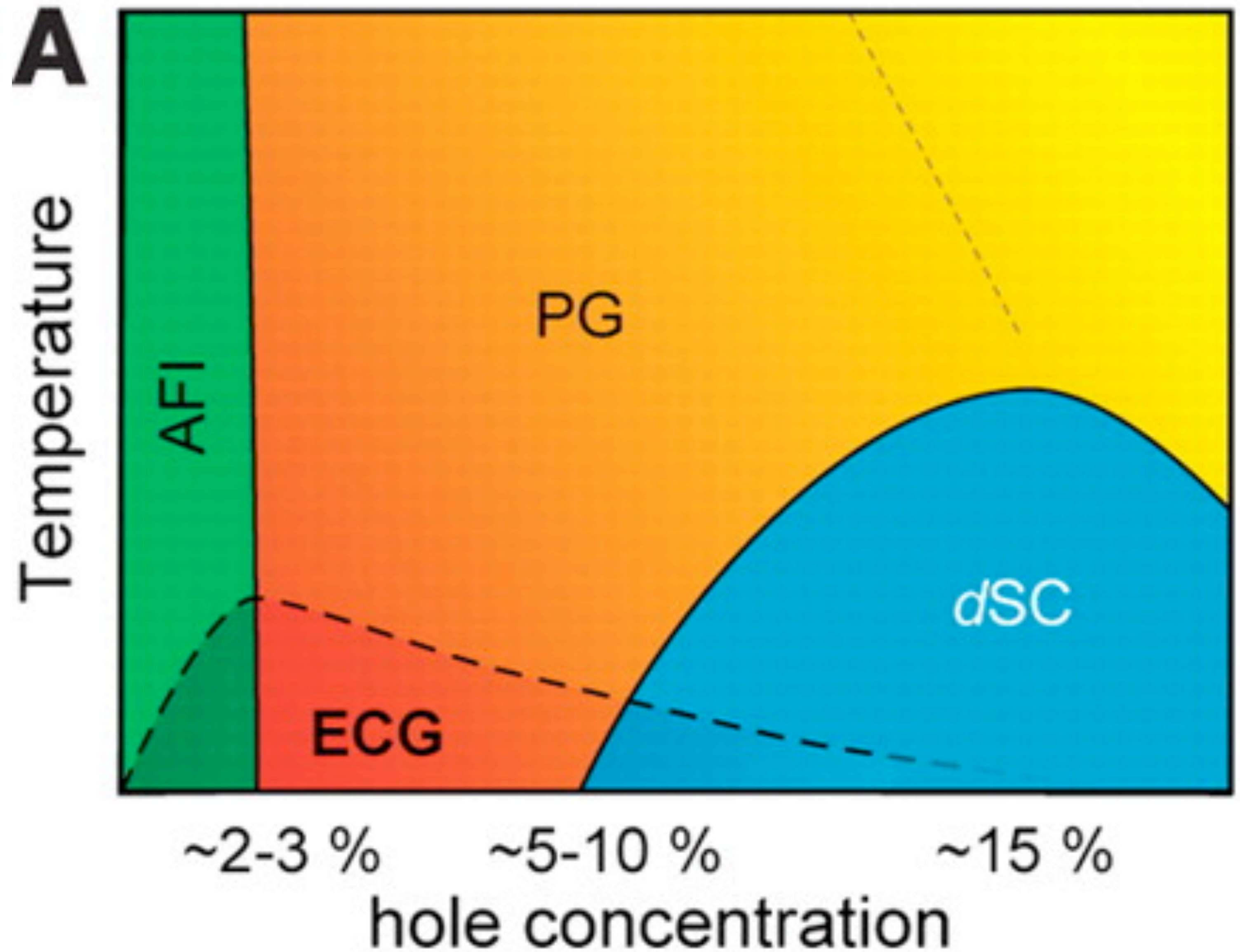
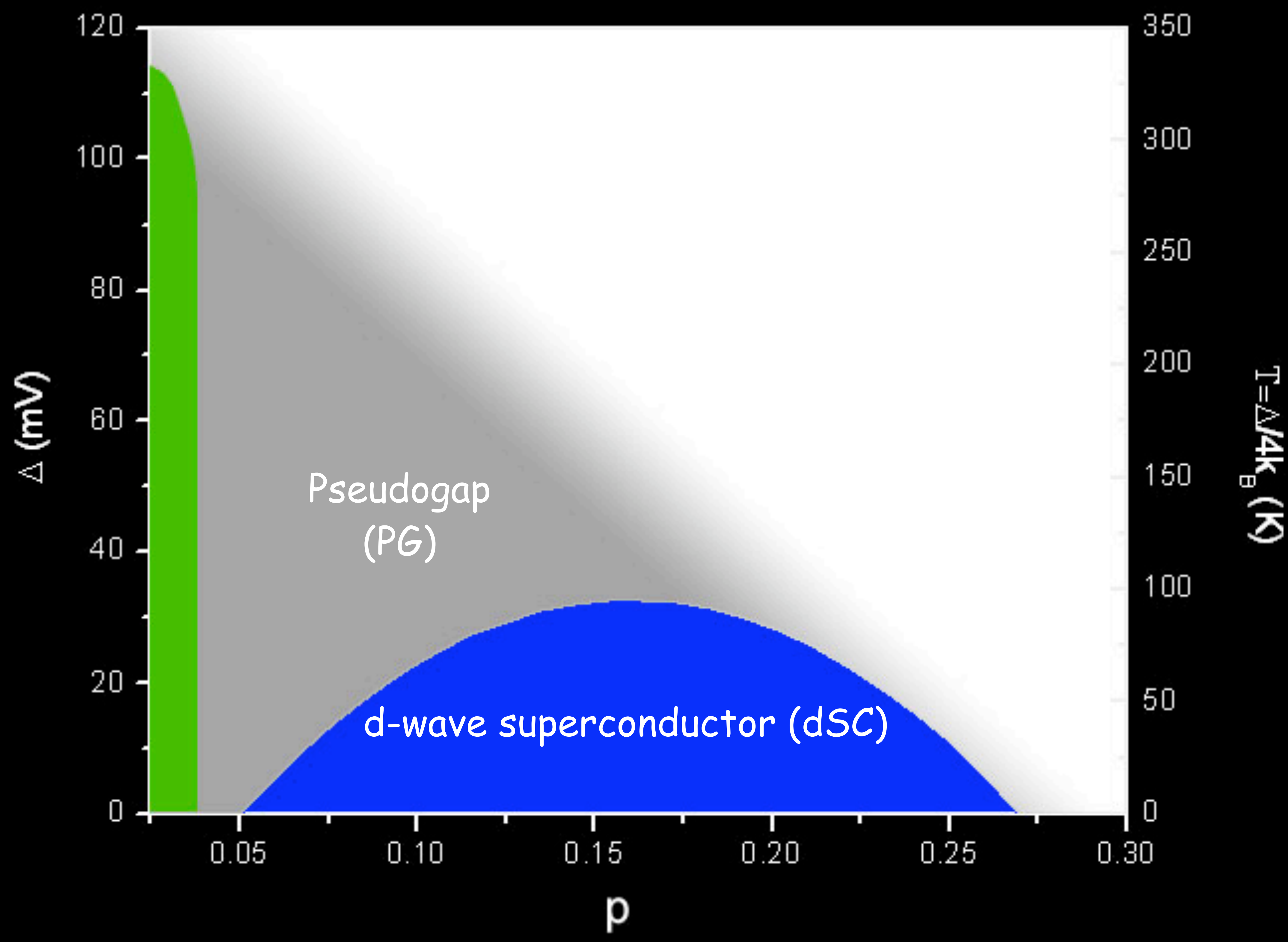
The cuprate superconductors





Near p~ 20% -- electronic structure consistent with d-BCS



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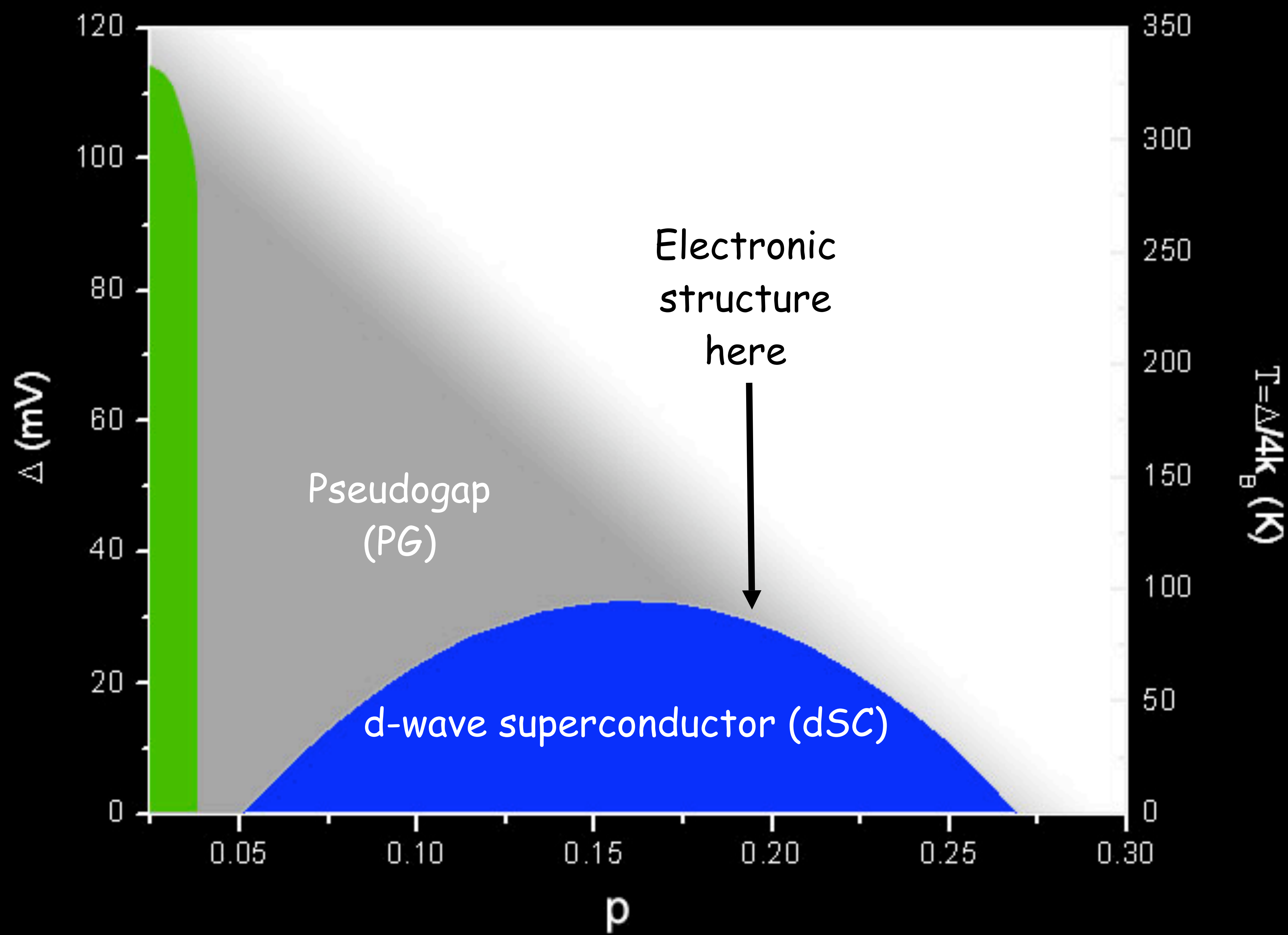
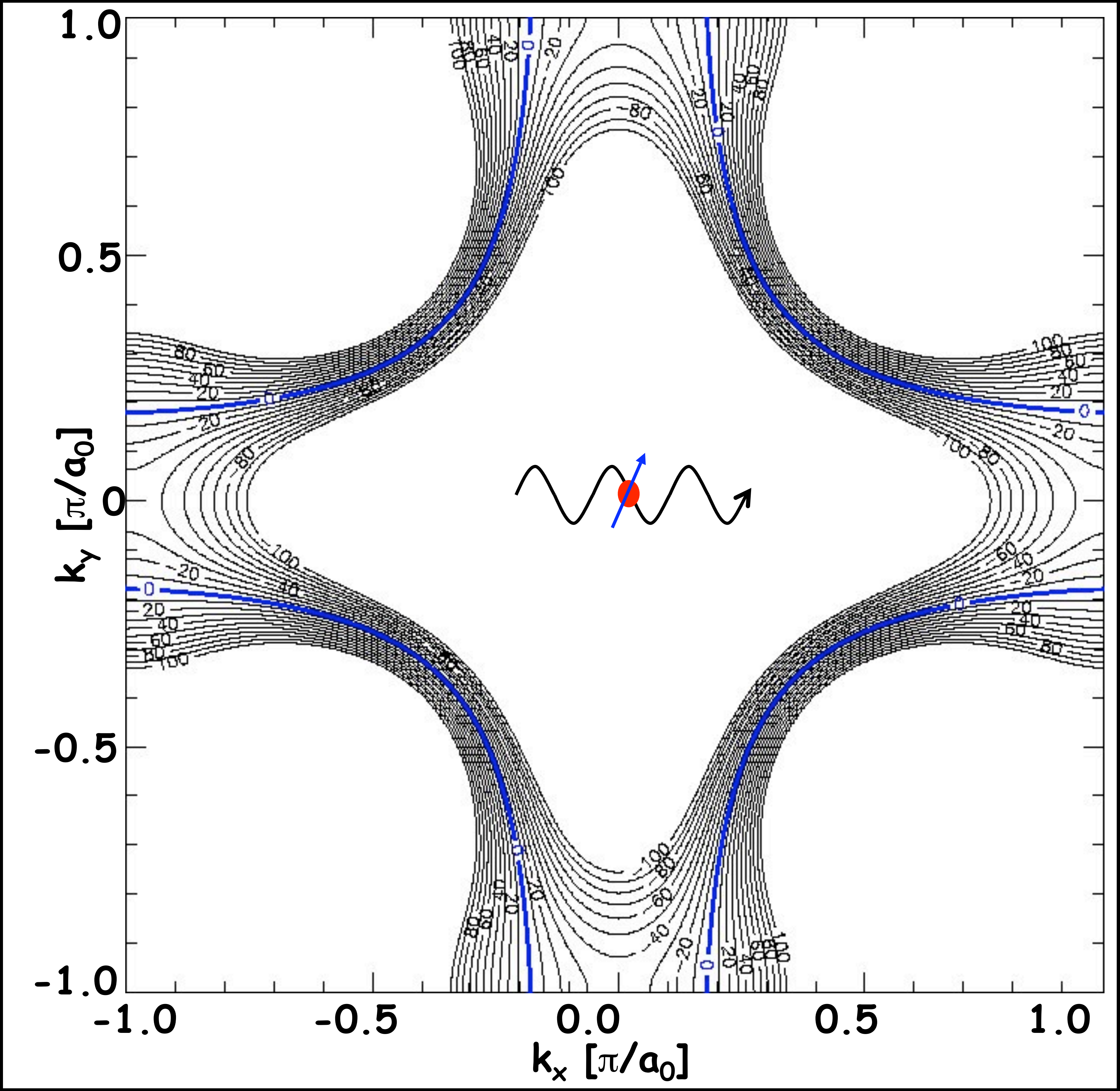


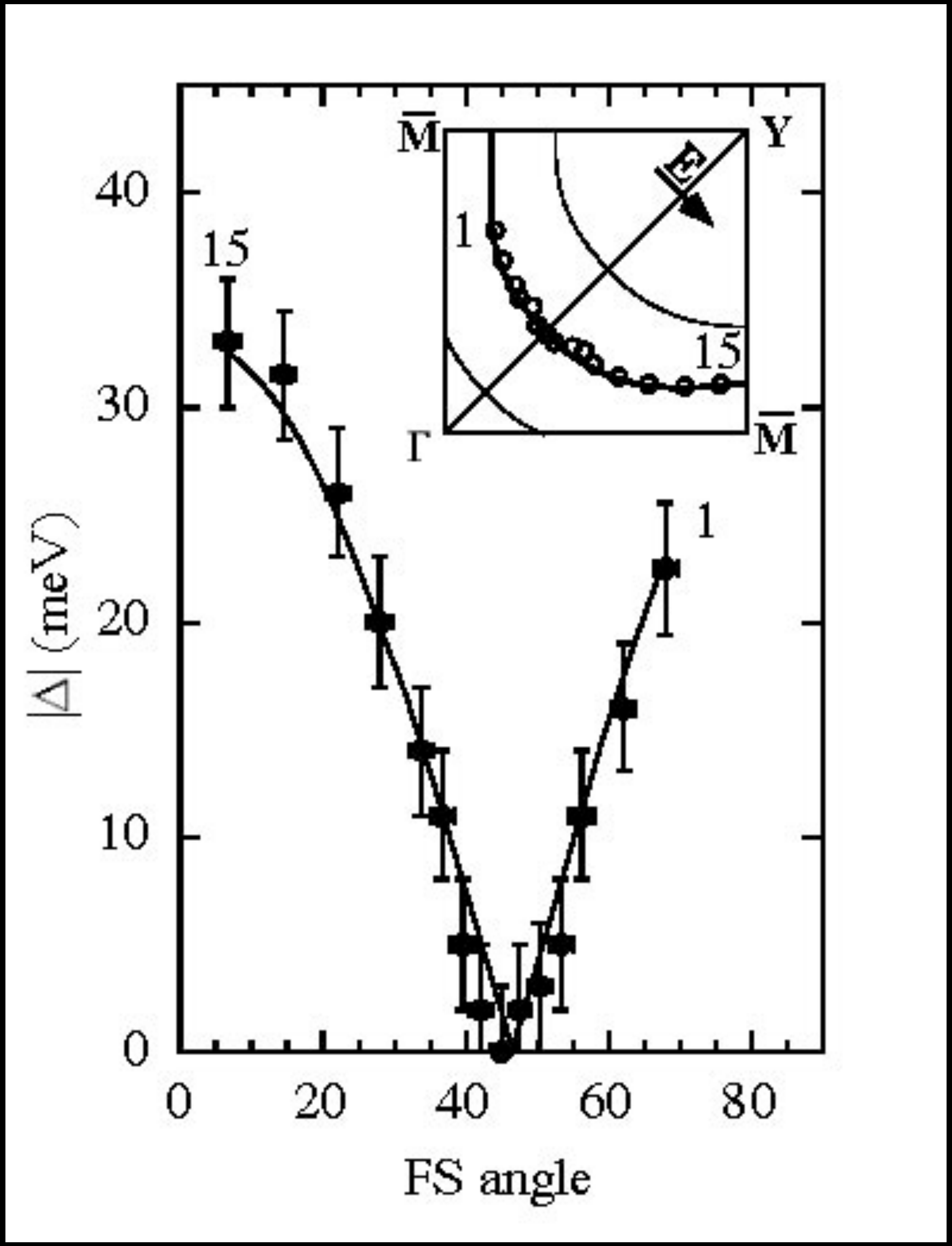
Normal State k-Space Electronic Structure



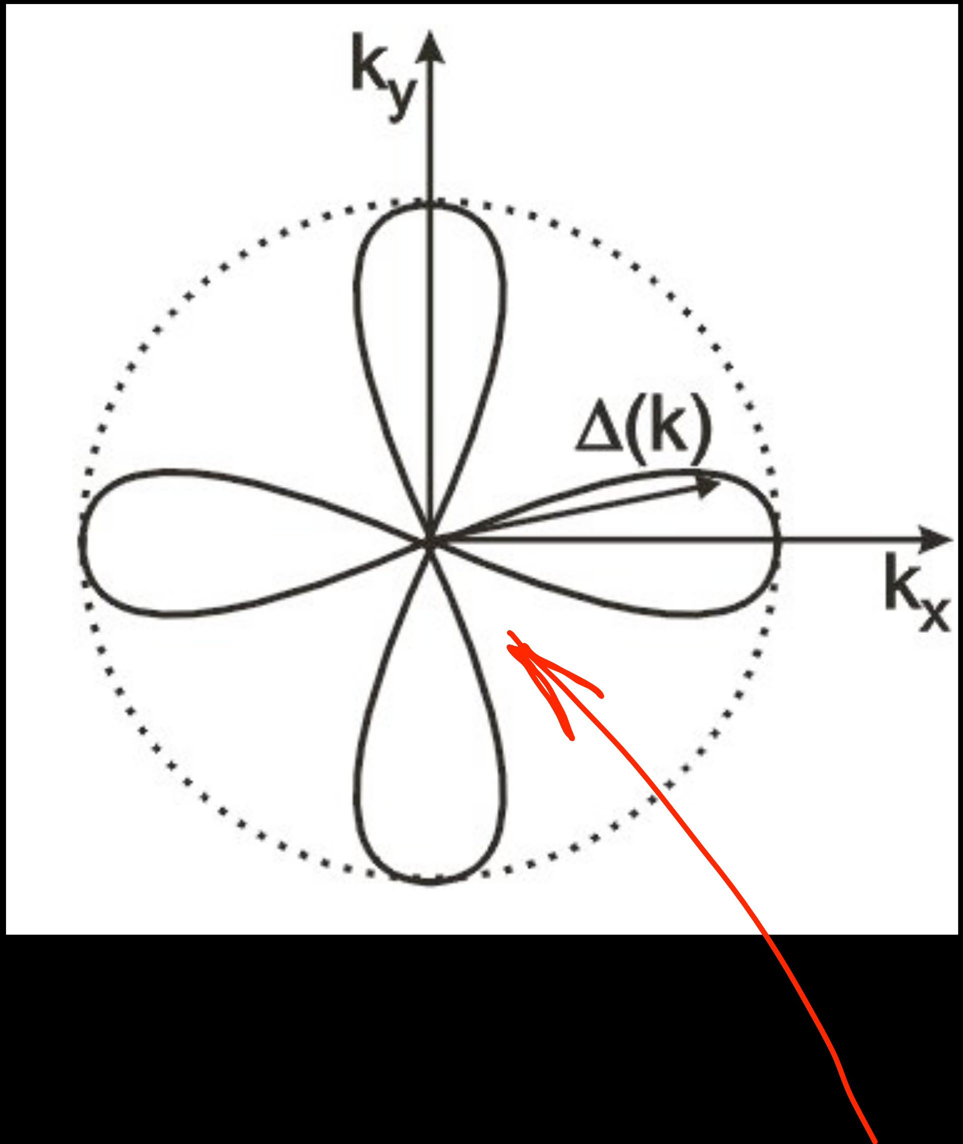
Parameterization:
M. Norman
PRB 52, 615 (1995).

Based on data:
Ding et al.,
PRL 74, 2784 (1995).

SC State: Momentum-dependent Pair Energy Gap $\Delta(\vec{k})$

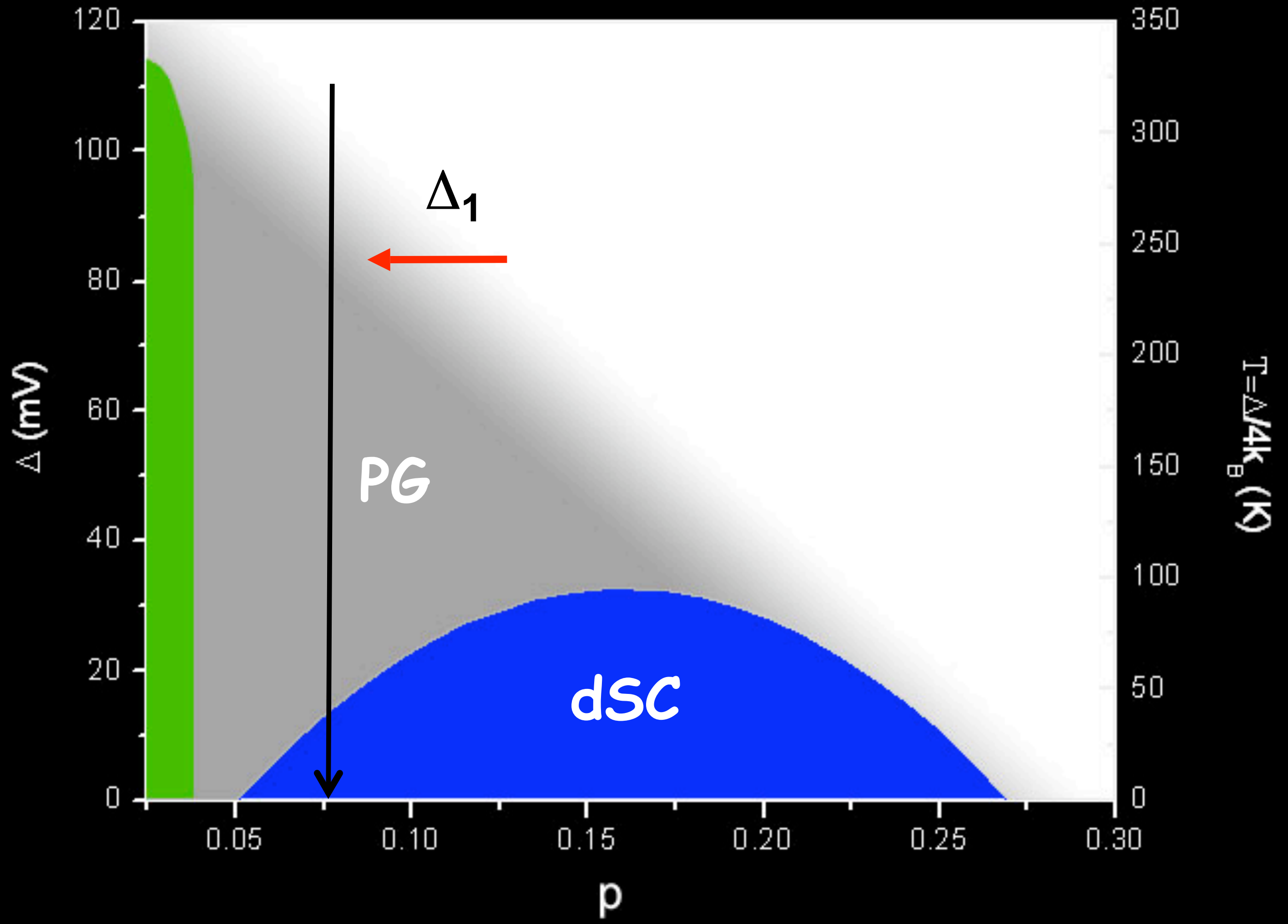


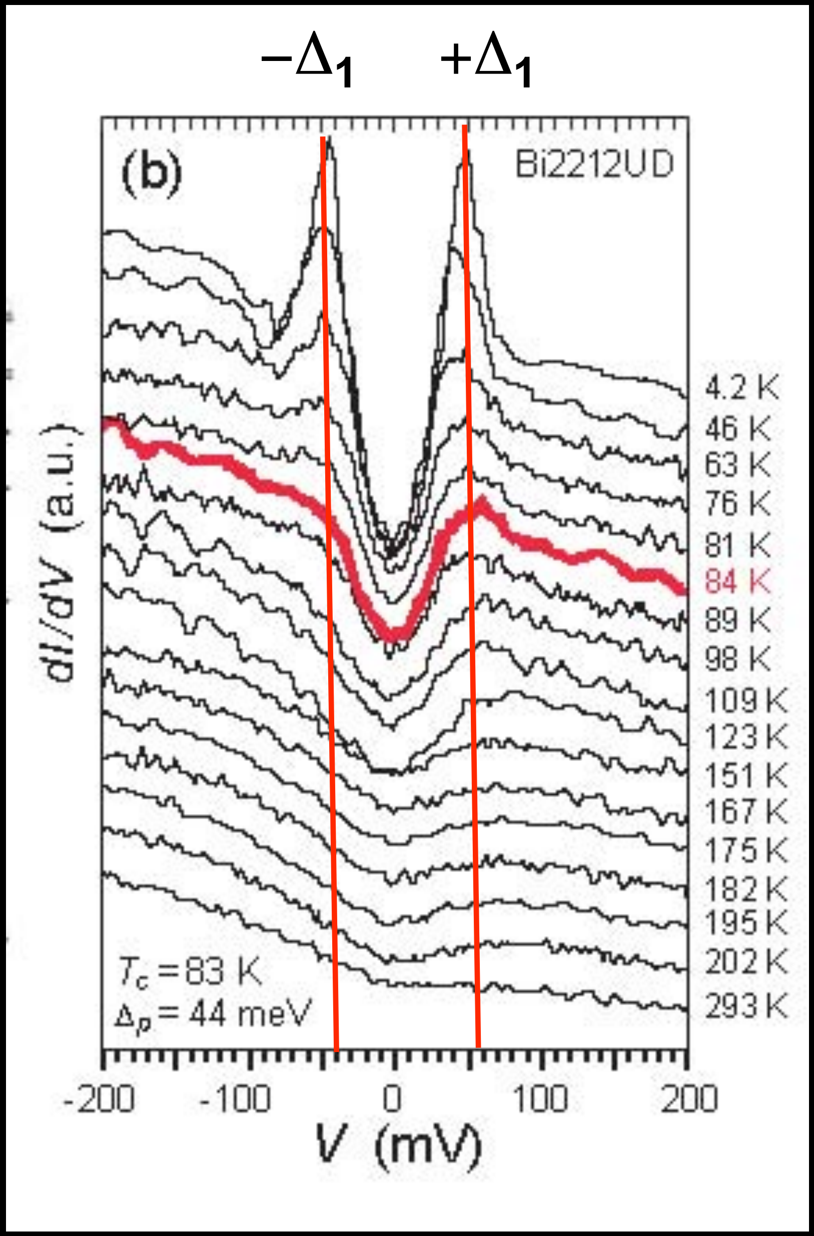
Shen et al PRL **70**, 3999 (1993)
Ding et al PRB **54** 9678 (1996)
Mesot et al PRL **83** 840 (1999)



The SC energy gap $\Delta(\vec{k})$ has four nodes.

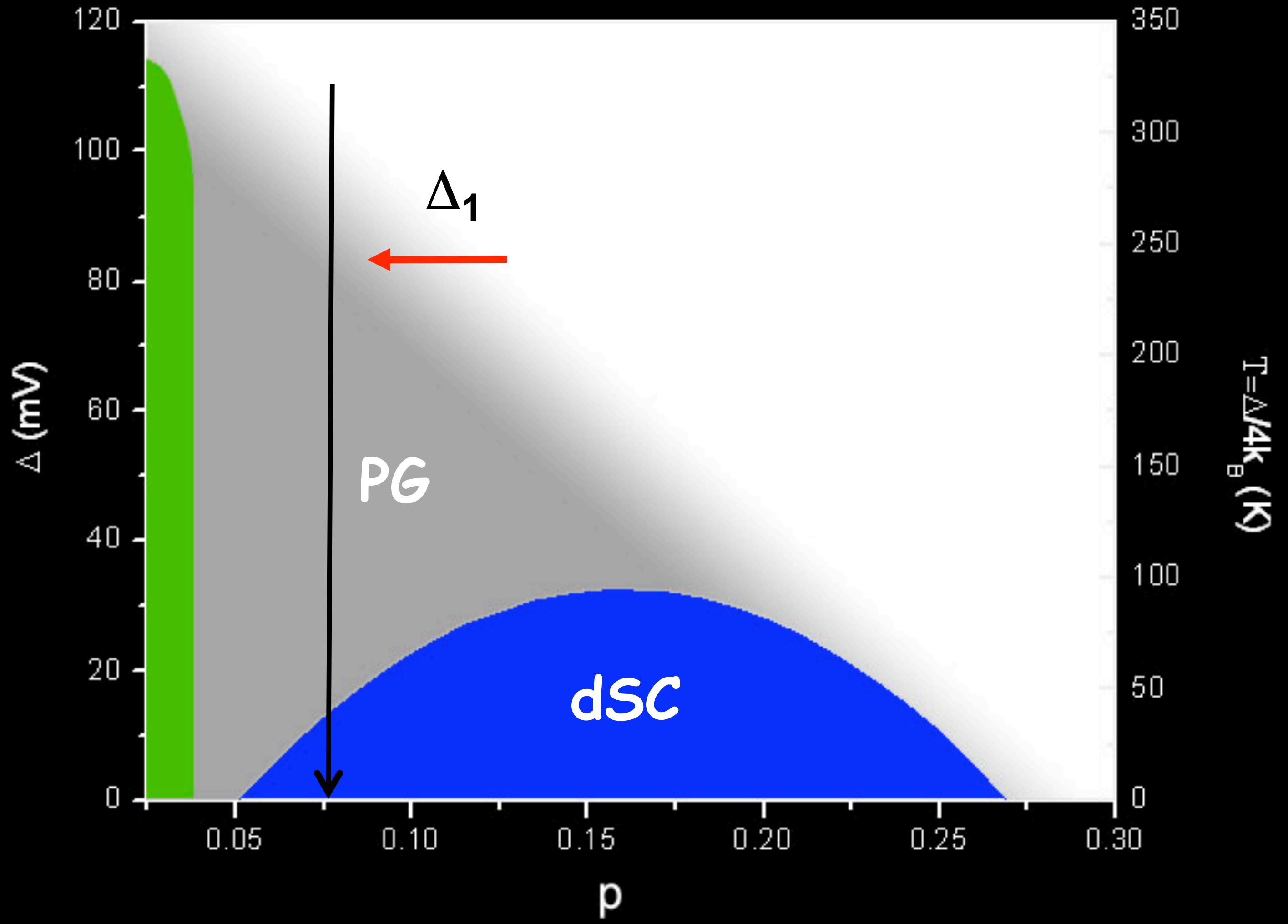
Pseudogap: Temperature-independent energy gap exists $T \gg T_c$

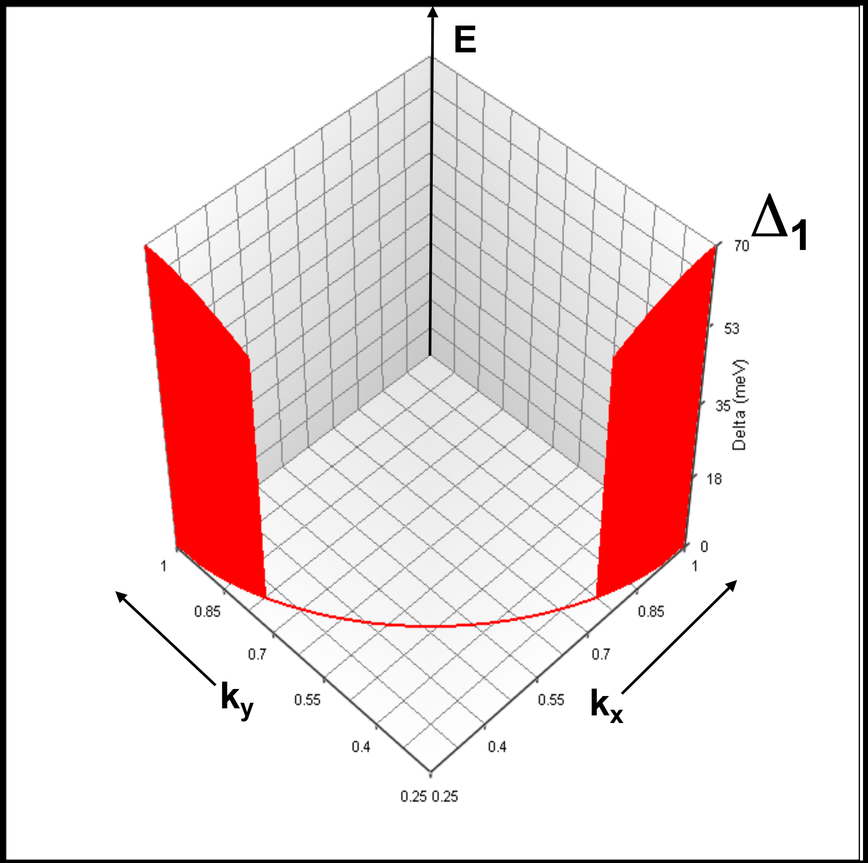


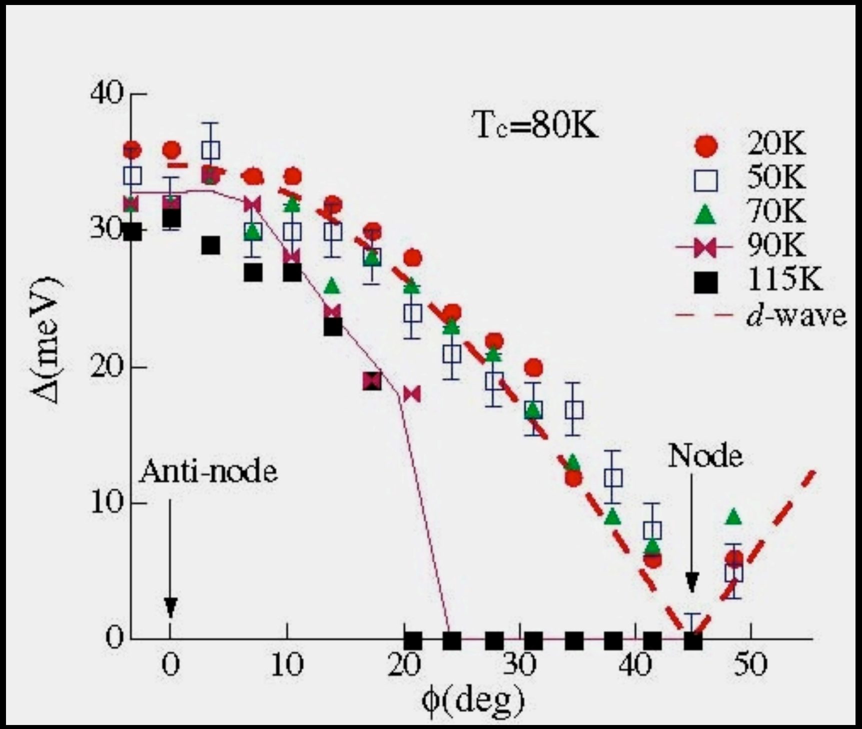


Ch. Renner et al, PRL 80, 149 (1998)
Ø. Fischer et al, RMP 79, 353 (2007)

Pseudogap: Temperature-independent energy gap near k~(π,0)

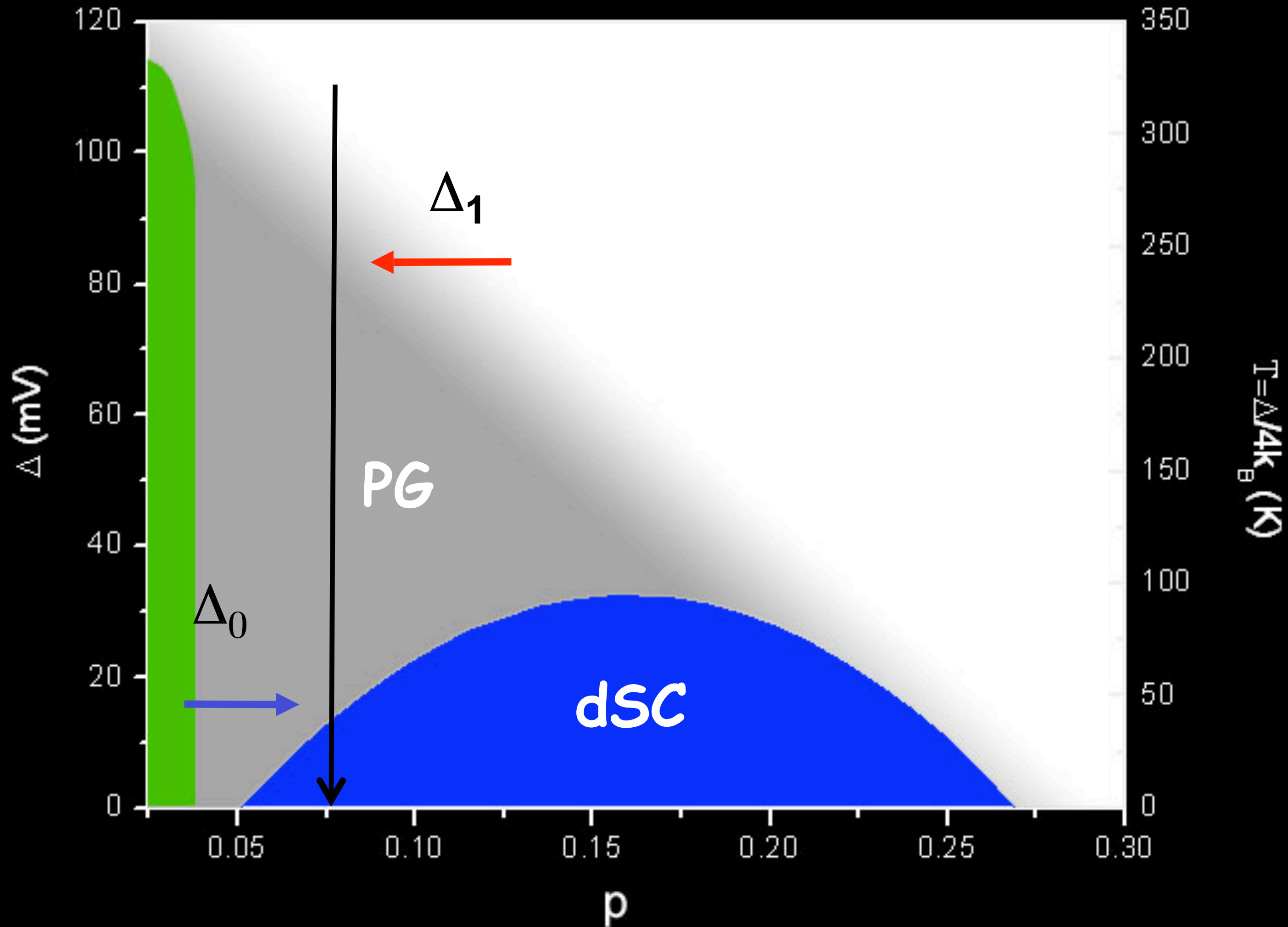


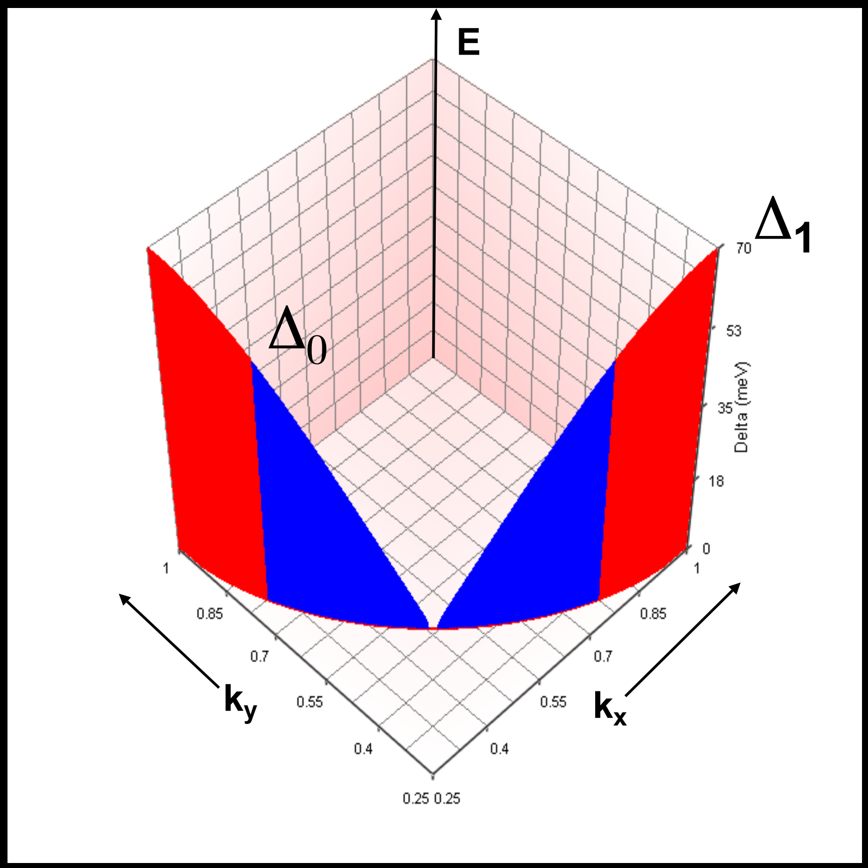


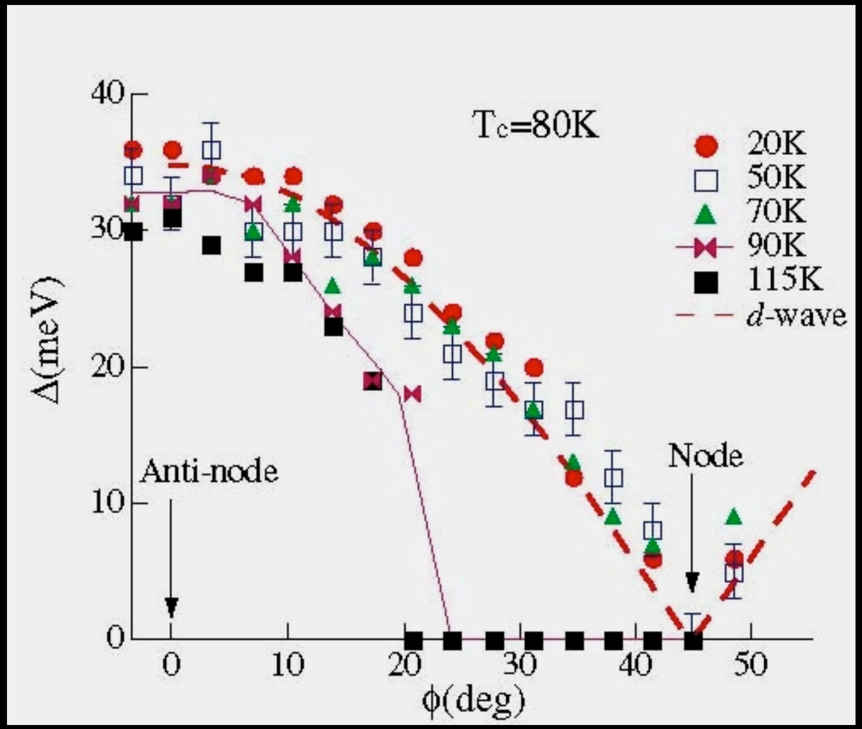


Loeser et al, Science 273 325 (1996)
Ding et al, Nature 382 51, (1996)
Norman et al, Nature 392 , 157 (1998)
Shen et al Science 307, 902 (2005)
Kanigel et al, Nature Physics 2,447 (2006)
Tanaka et al, Science 314, 1912 (2006)

Pseudogap: Temperature-dependent energy gap near node







Loeser et al, Science 273 325 (1996)
Ding et al, Nature 382 51, (1996)
Norman et al, Nature 392 , 157 (1998)
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Nodal-anti-nodal dichotomy in the underdoped cuprates

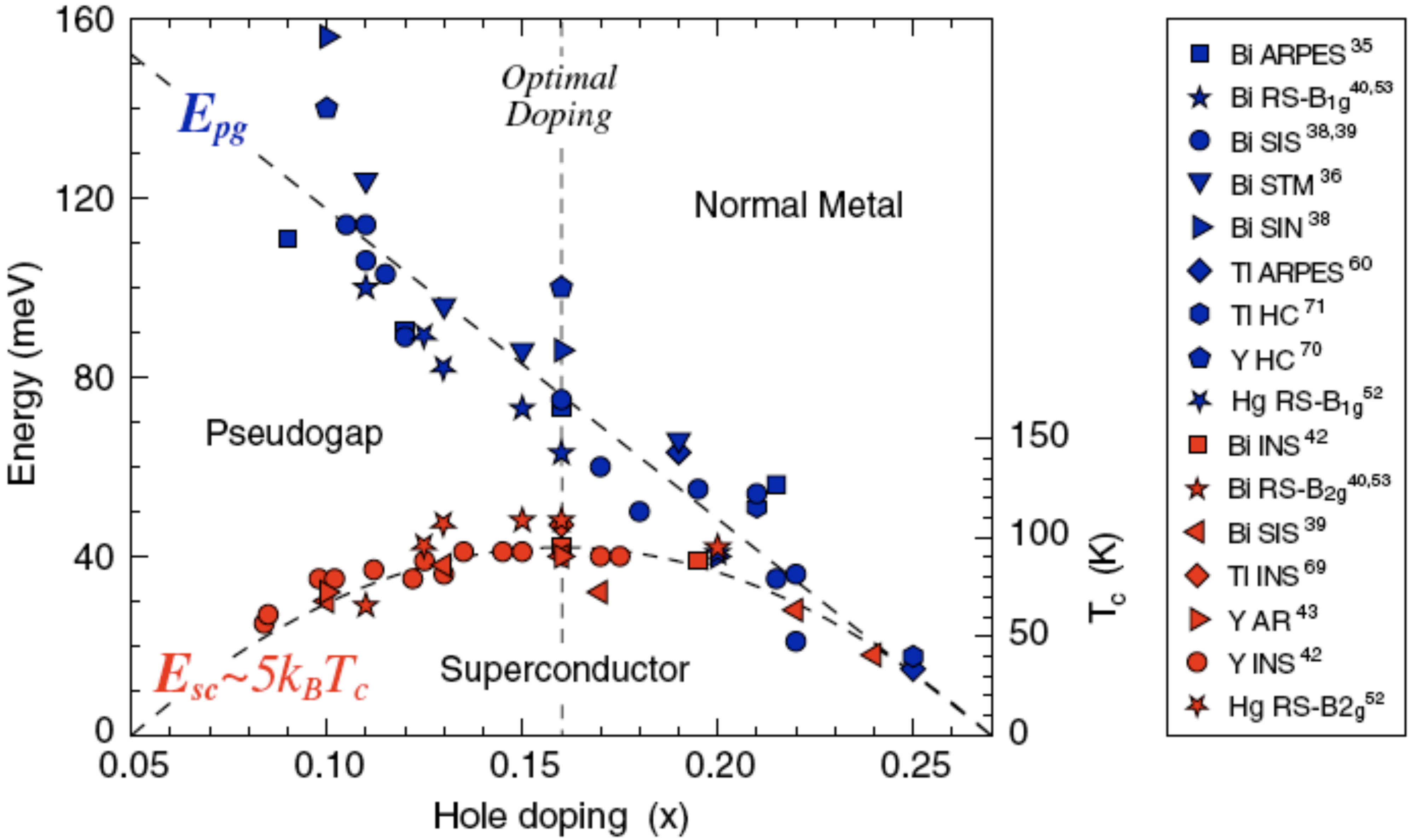

Figure 2. Pseudogap ($E_{pg} = 2\Delta_{pg}$) and superconducting ($E_{sc} \sim 5k_BT_c$) energy scales for a number of HTSCs with $T_c^{max} \sim 95$ K (Bi2212, Y123, Tl2201 and Hg1201). The datapoints were obtained, as a function of hole doping x, by angle-resolved photoemission spectroscopy (ARPES), tunneling (STM, SIN, SIS), Andreev reflection (AR), Raman scattering (RS) and heat conductivity (HC). On the same plot we are also including the energy Ω_r of the magnetic resonance mode measured by inelastic neutron scattering (INS), which we identify with E_{sc} because of the striking quantitative correspondence as a function of T_c. The data fall on two universal curves given by $E_{pg} = E_{pg}^{max}(0.27 - x)/0.22$ and $E_{sc} = E_{sc}^{max}[1 - 82.6(0.16 - x)^2]$, with $E_{pg}^{max} = E_{pg}(x = 0.05) = 152 \pm 8$ meV and $E_{sc}^{max} = E_{sc}(x = 0.16) = 42 \pm 2$ meV (the statistical errors refer to the fit of the selected datapoints; however, the spread of all available data would be more appropriately described by ± 20 and ± 10 meV, respectively).

S. Hufner, M.A. Hossain, A. Damascelli, and G.A. Sawatzky, *Rep. Prog. Phys.* **71**, 062501 (2008)

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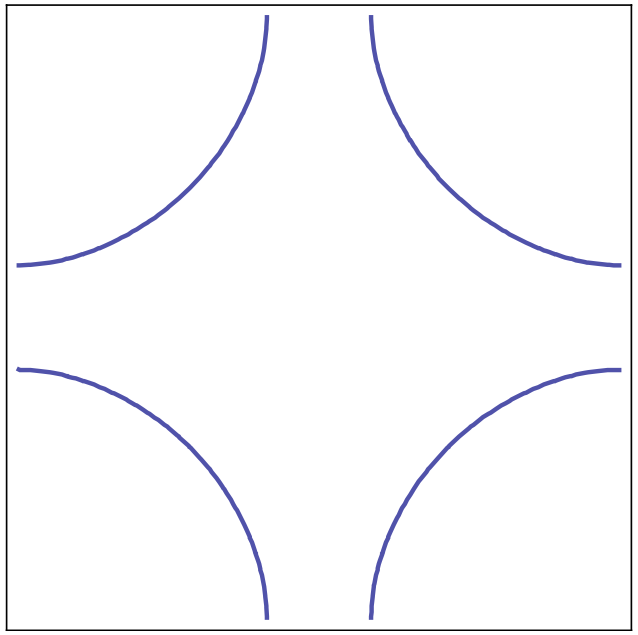
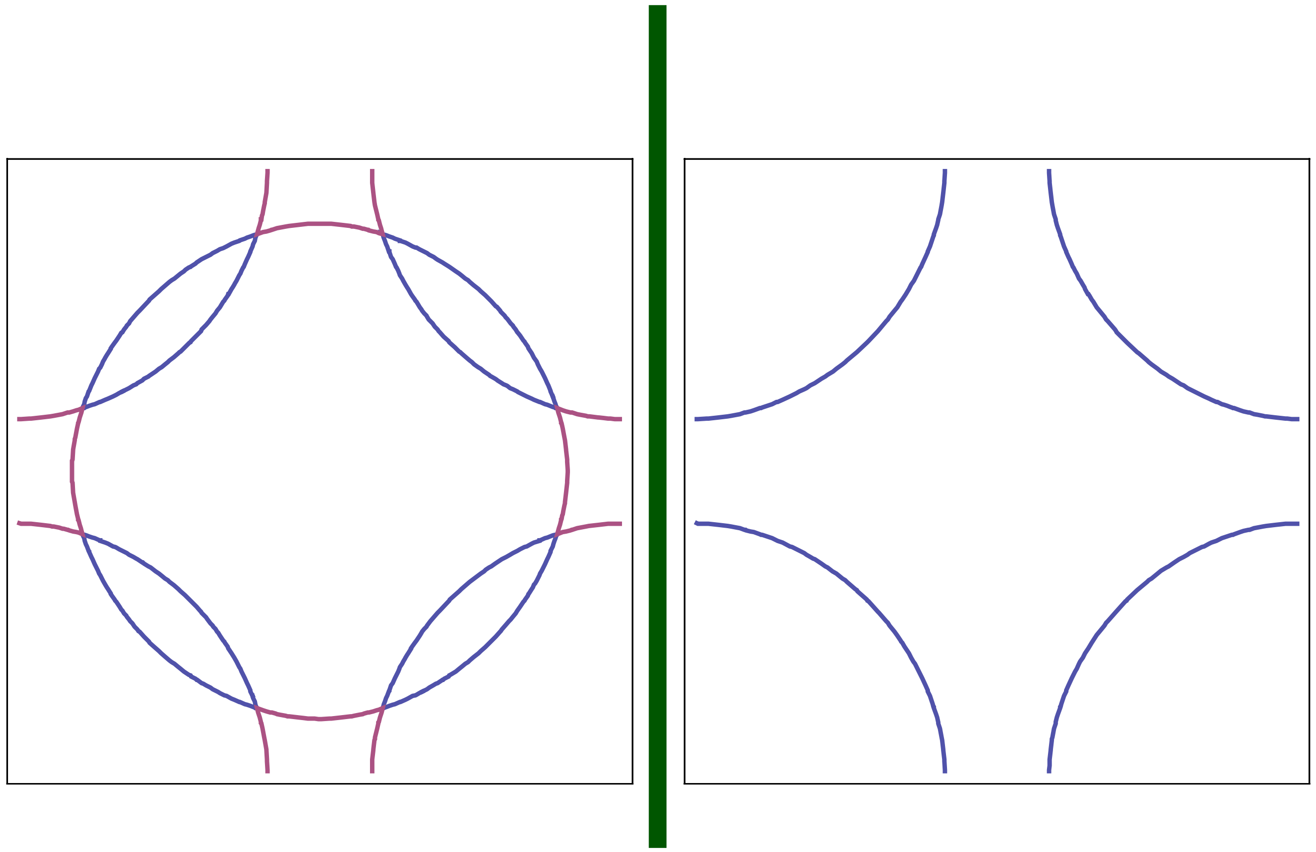
Spin density wave theory in hole-doped cuprates



A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



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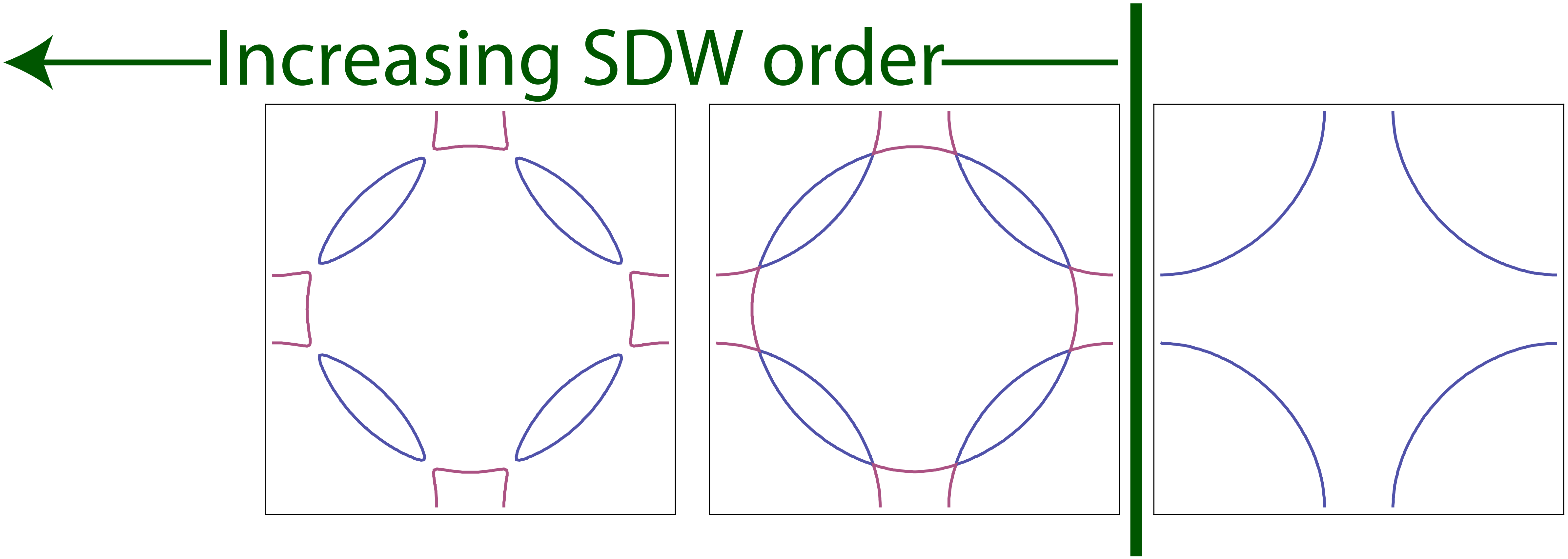

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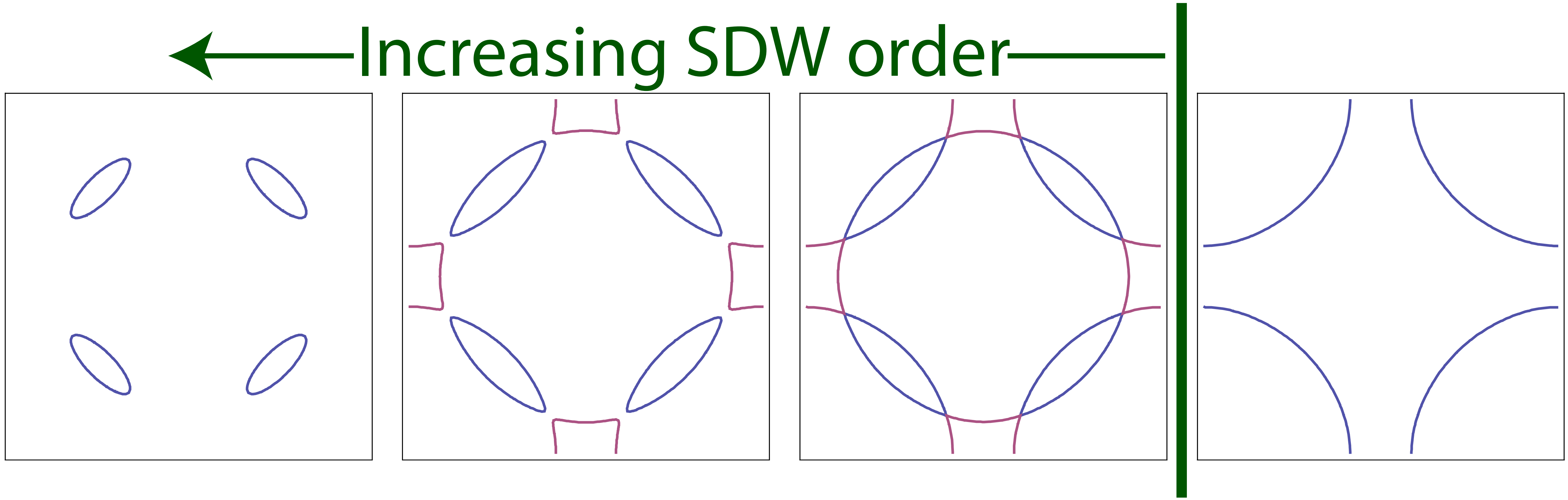

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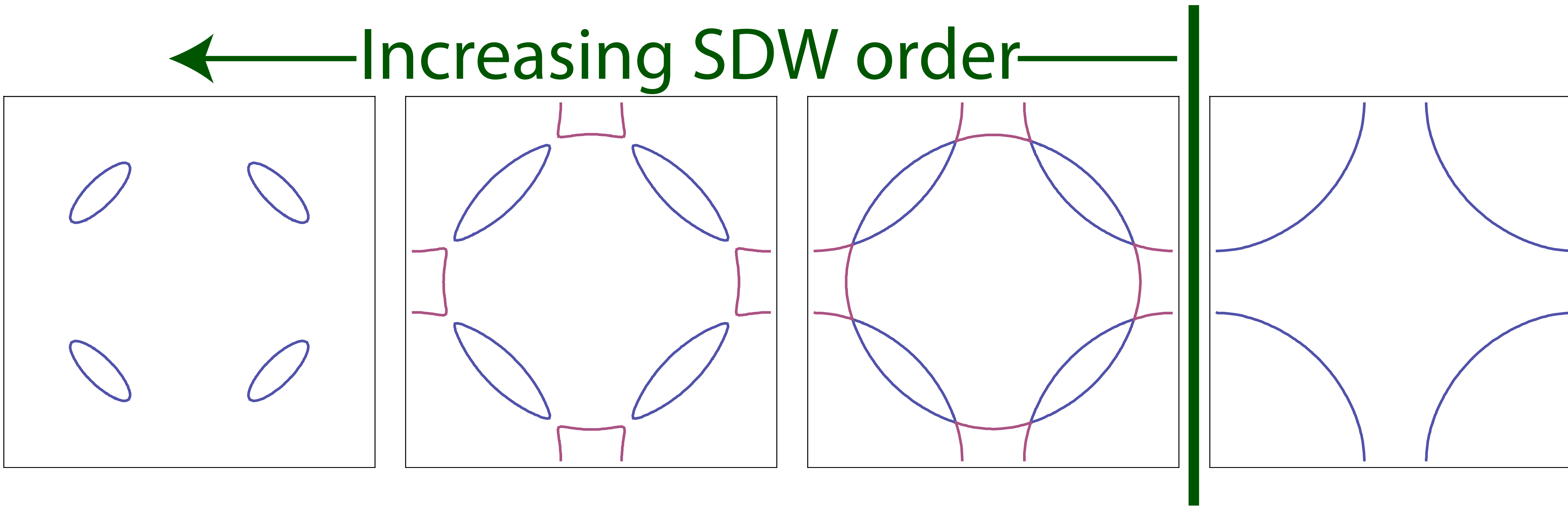

$$\mathrm{O}(3) \text{ vector order parameter } \vec{\varphi}$$

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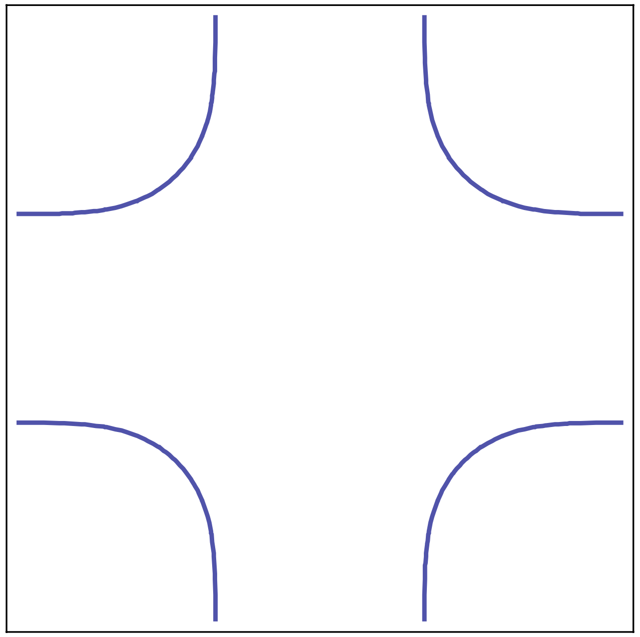
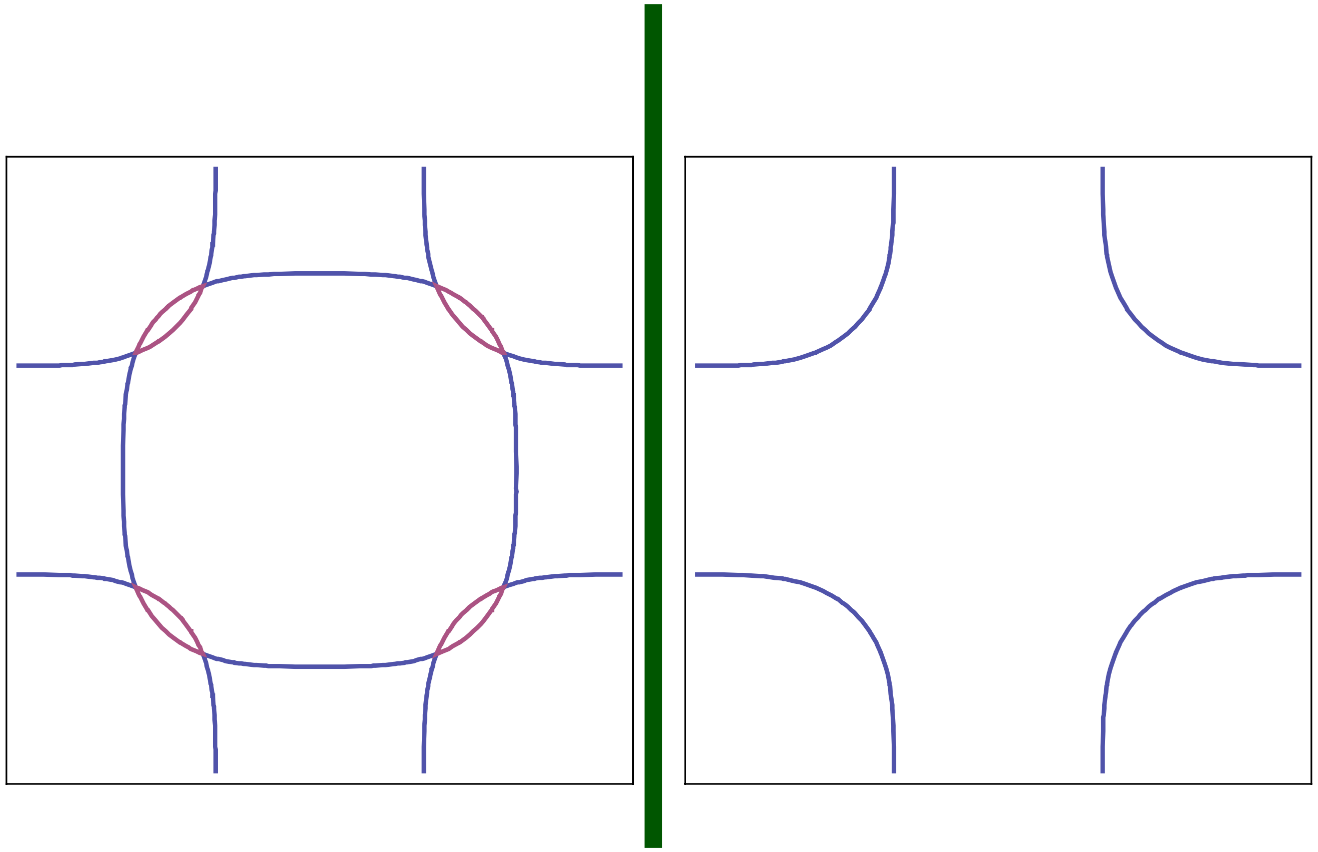
Spin density wave theory in electron-doped cuprates



A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in electron-doped cuprates



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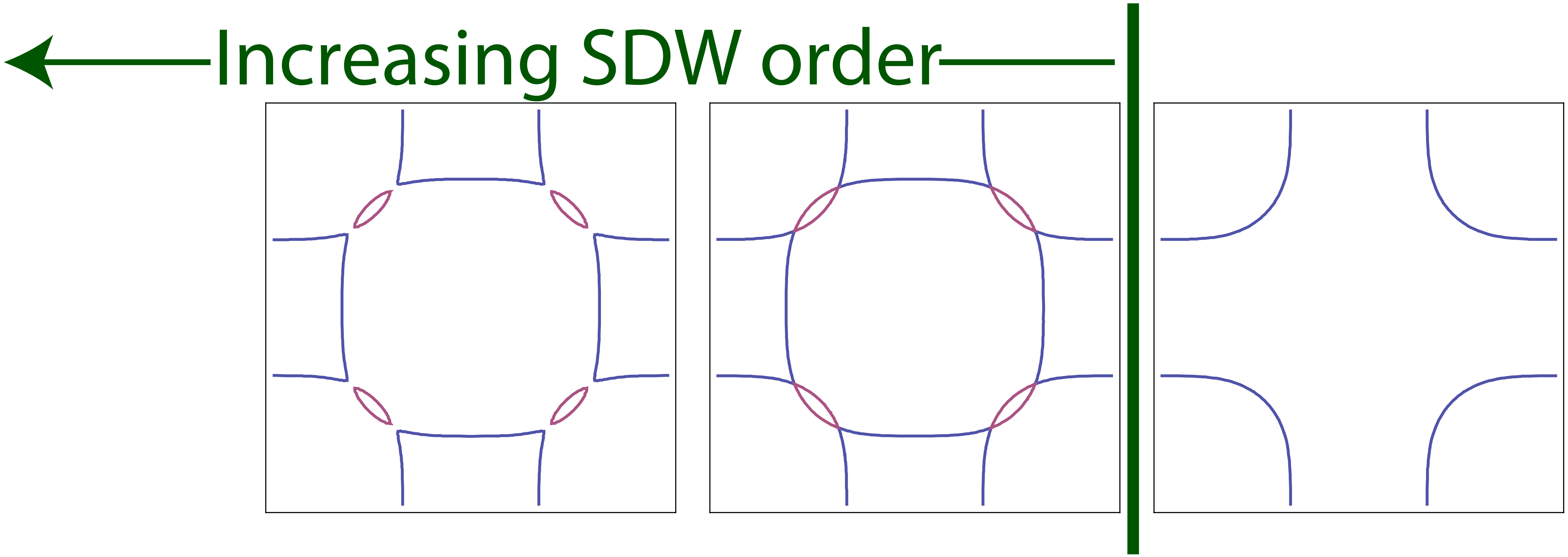

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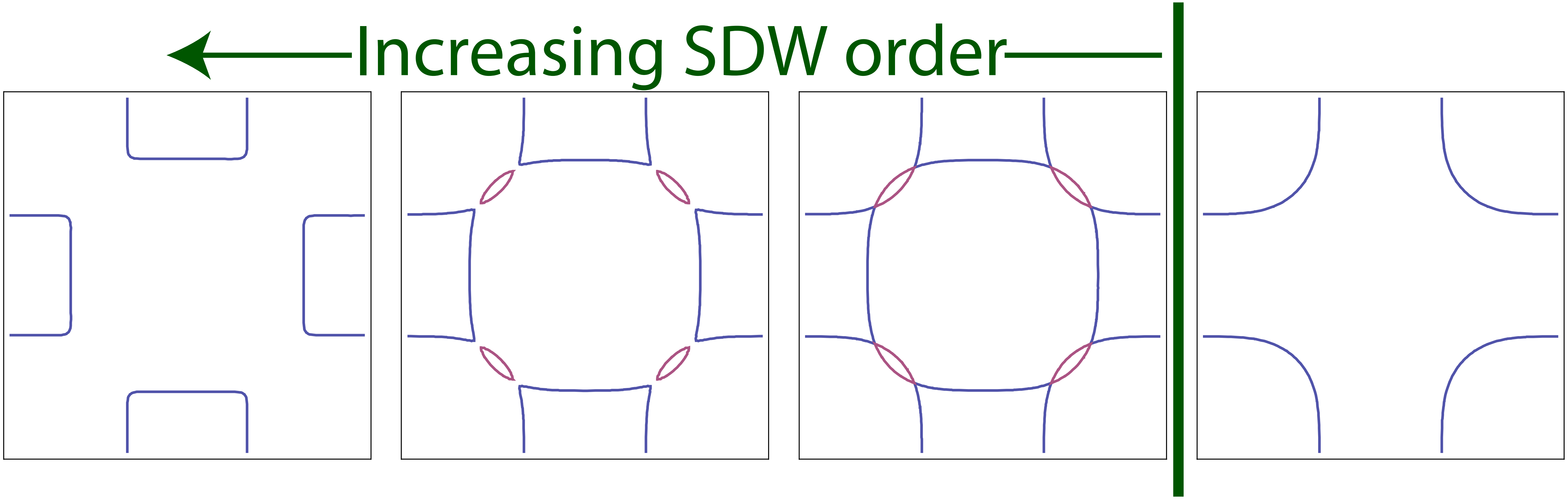
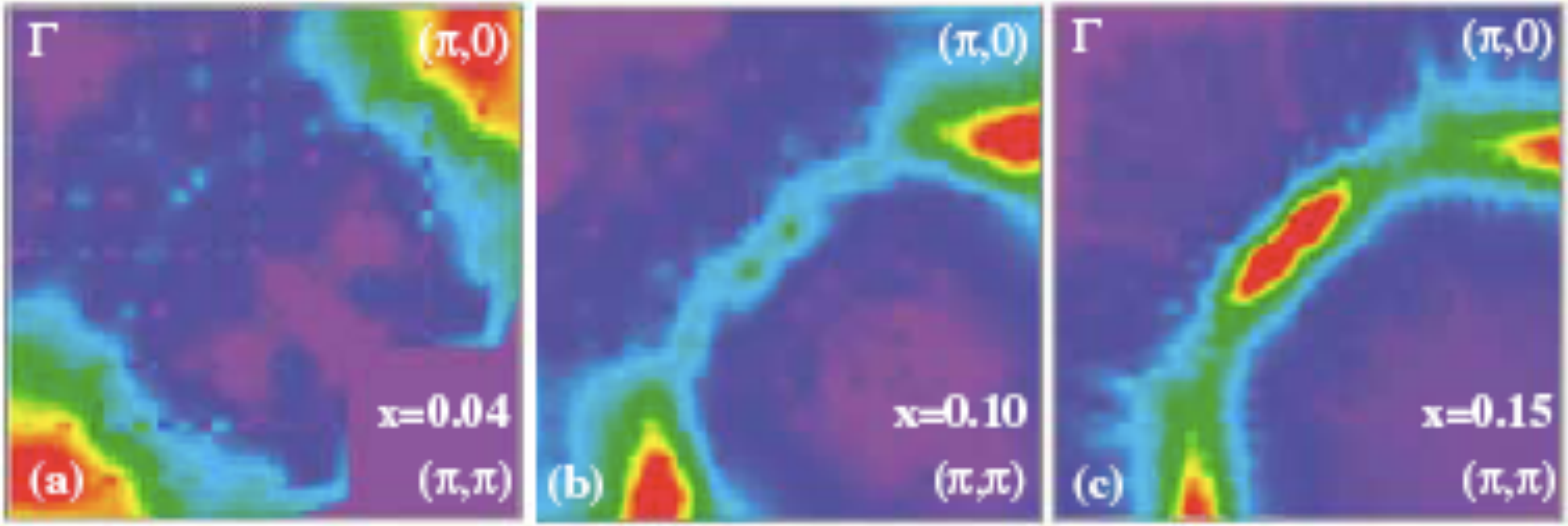


O(3) vector order parameter $\vec{\varphi}$

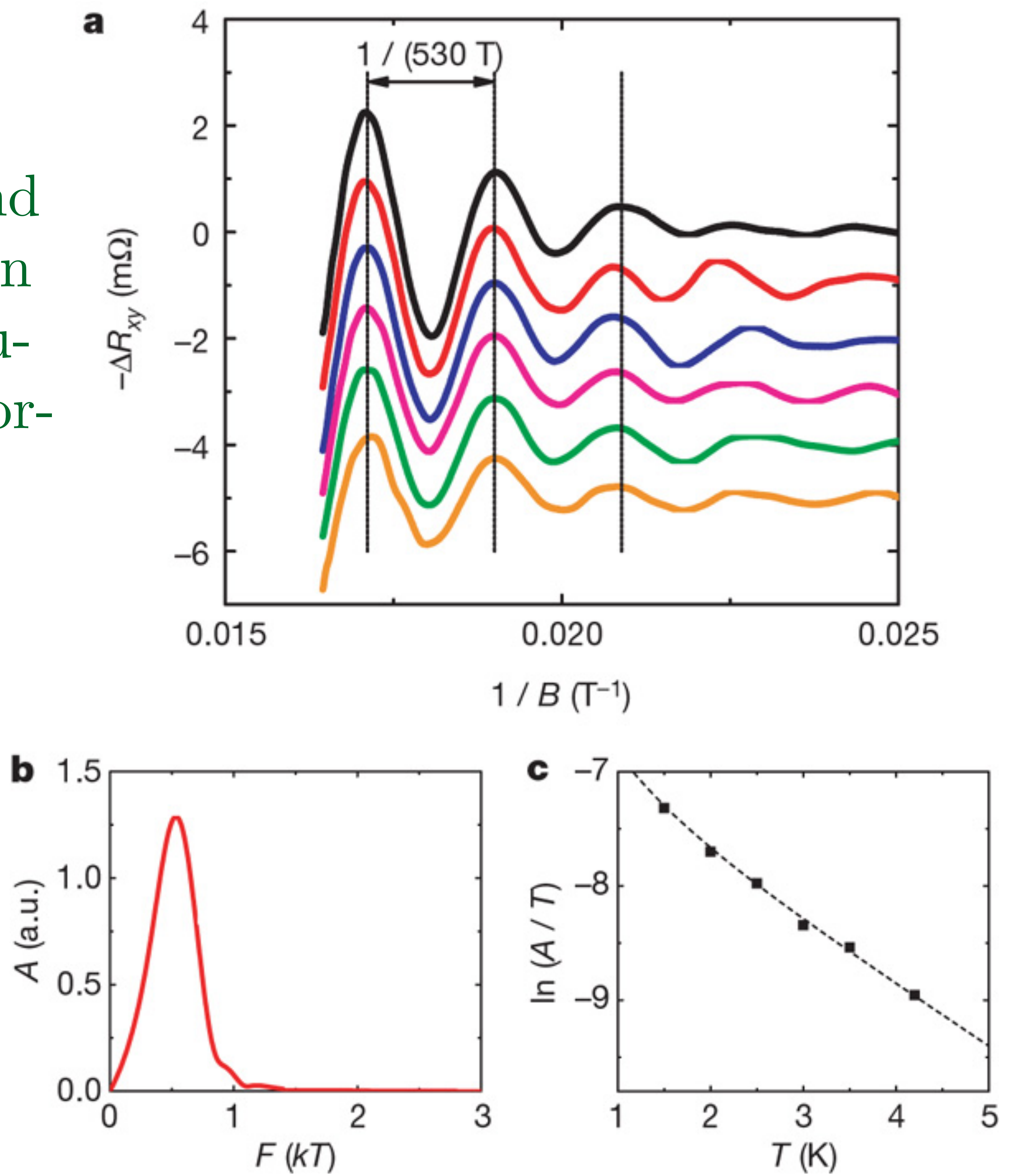
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Photoemission in NCCO (electron-doped)



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $YBa_2Cu_3O_{6.5}$).

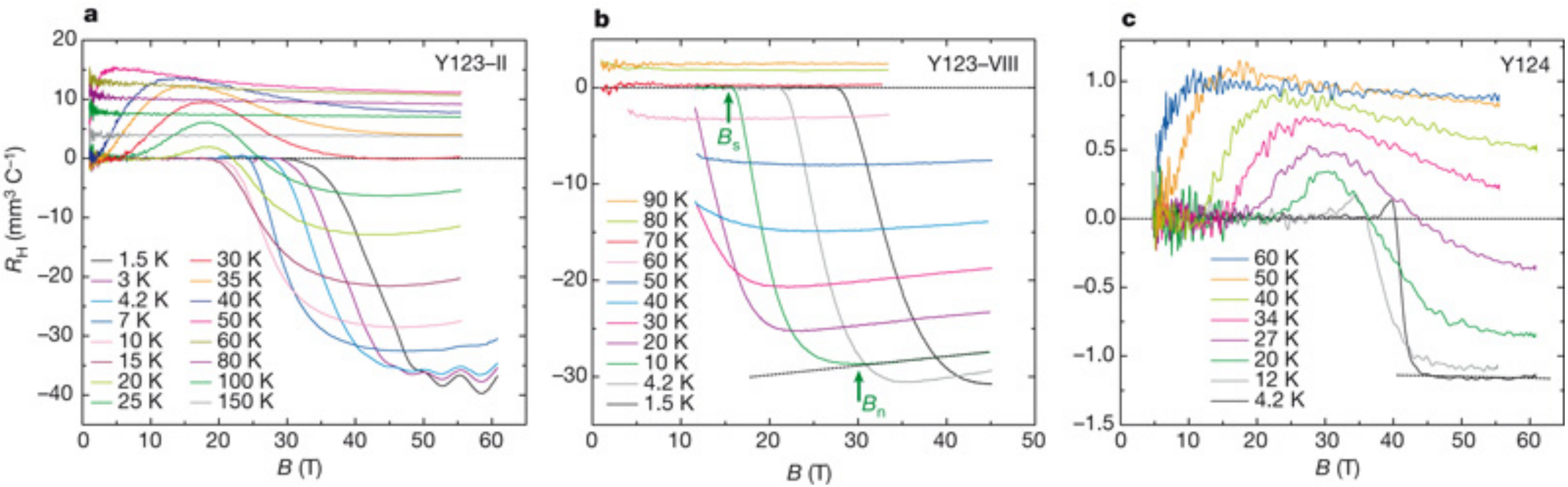


N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

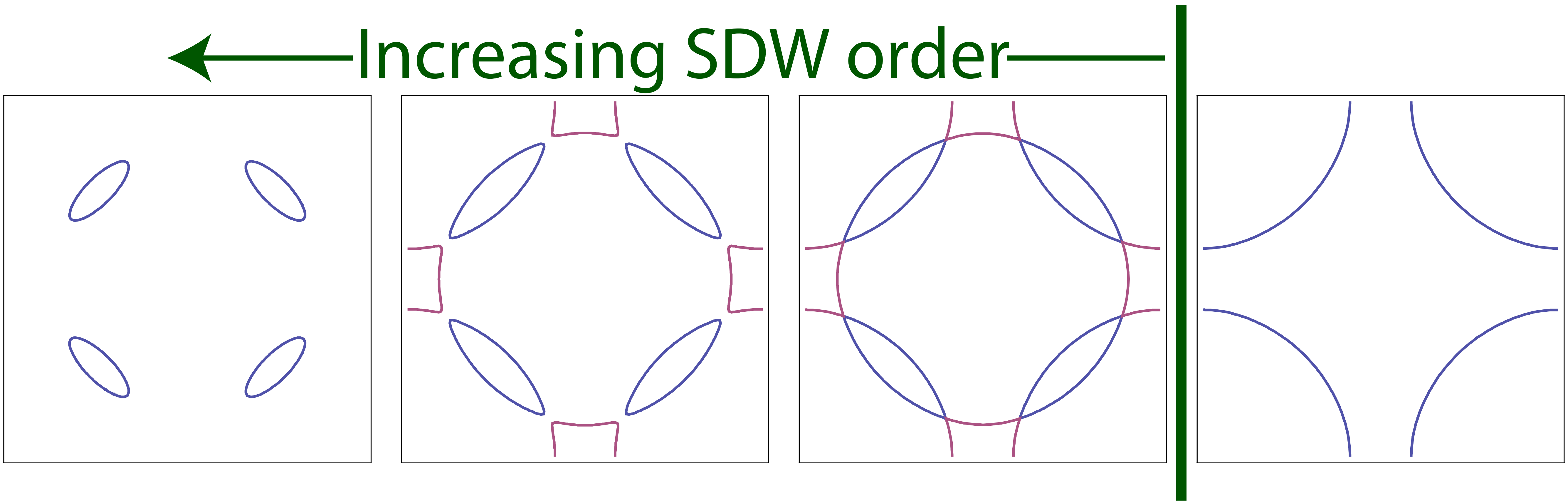
Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



Spin density wave theory in hole-doped cuprates



$$\mathrm{O}(3) \text{ vector order parameter } \vec{\varphi}$$

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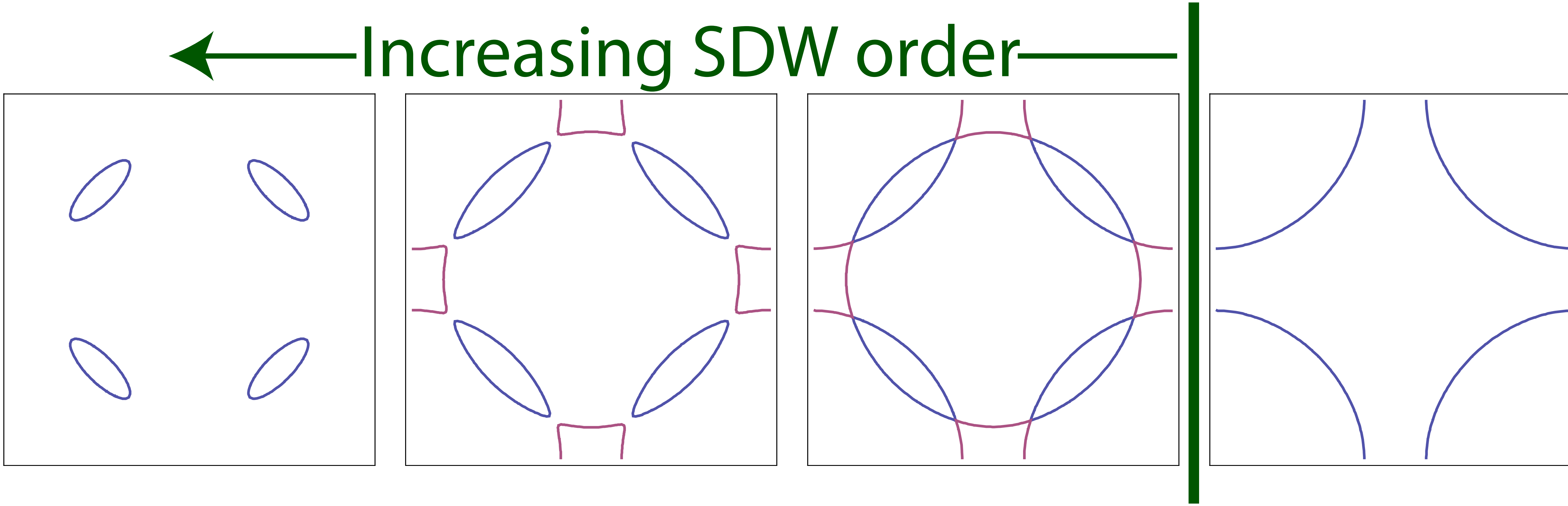

- Loss of SDW order co-incides with large Fermi surface to electron/hole pocket transition.

- Landau-Ginzburg-Hertz theory for SDW ordering:

$$\mathcal{S}_H = \int d^2r d\tau \left\{ \frac{1}{2}(\nabla\vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{2}\left(\vec{\varphi}^2\right)^2 \right\} + \int \frac{d^2k d\omega}{8\pi^3} |\omega||\vec{\varphi}(k,\omega)|^2$$

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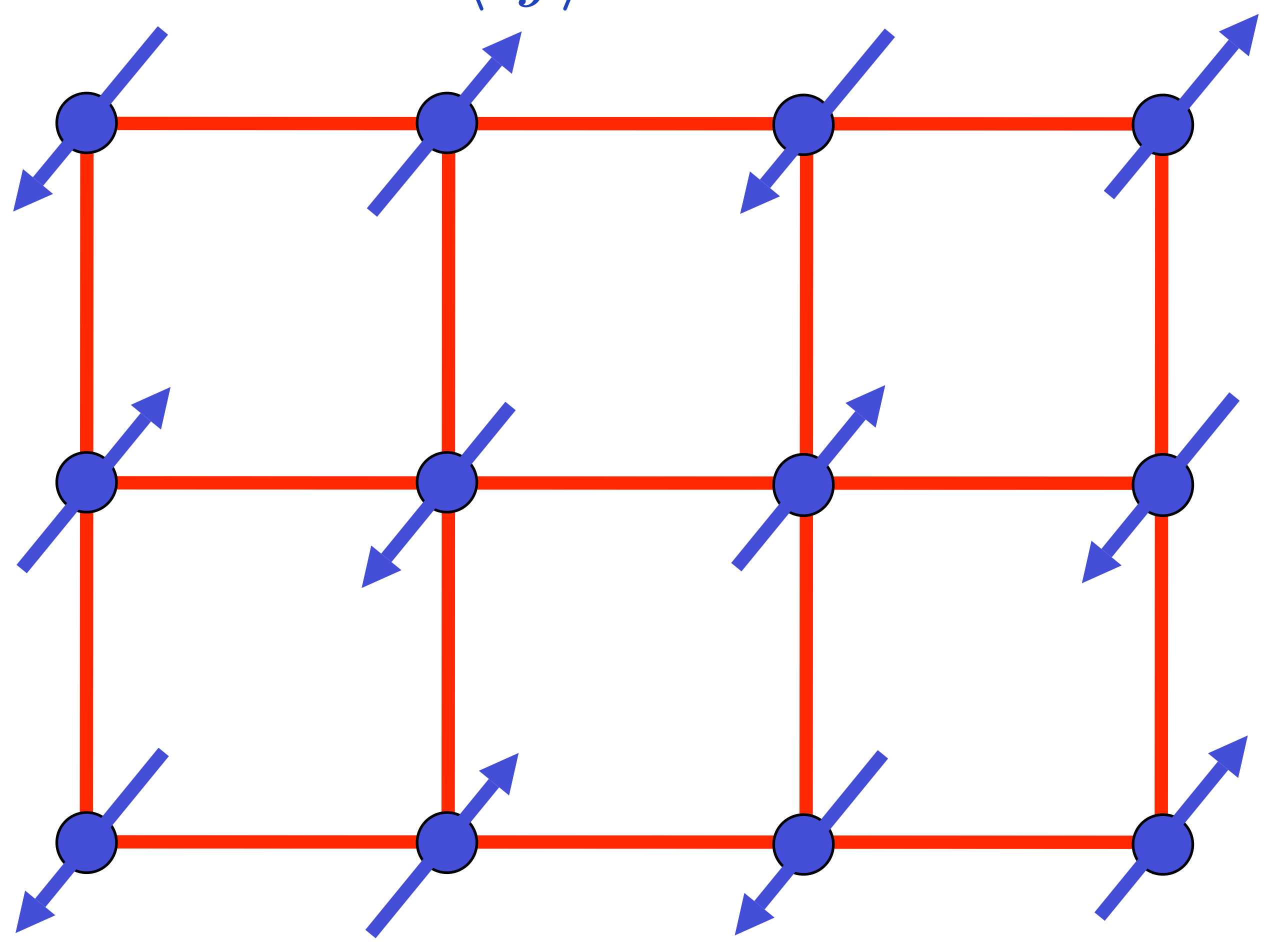
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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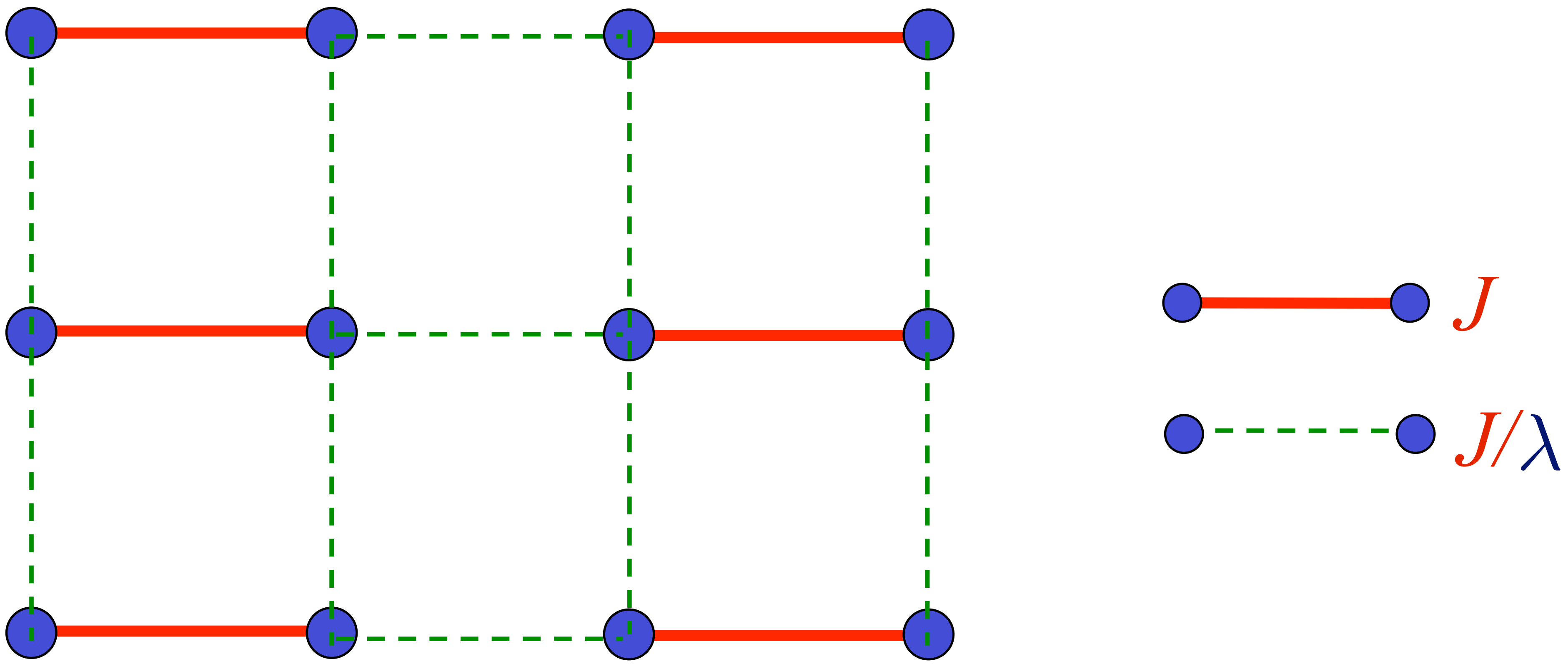
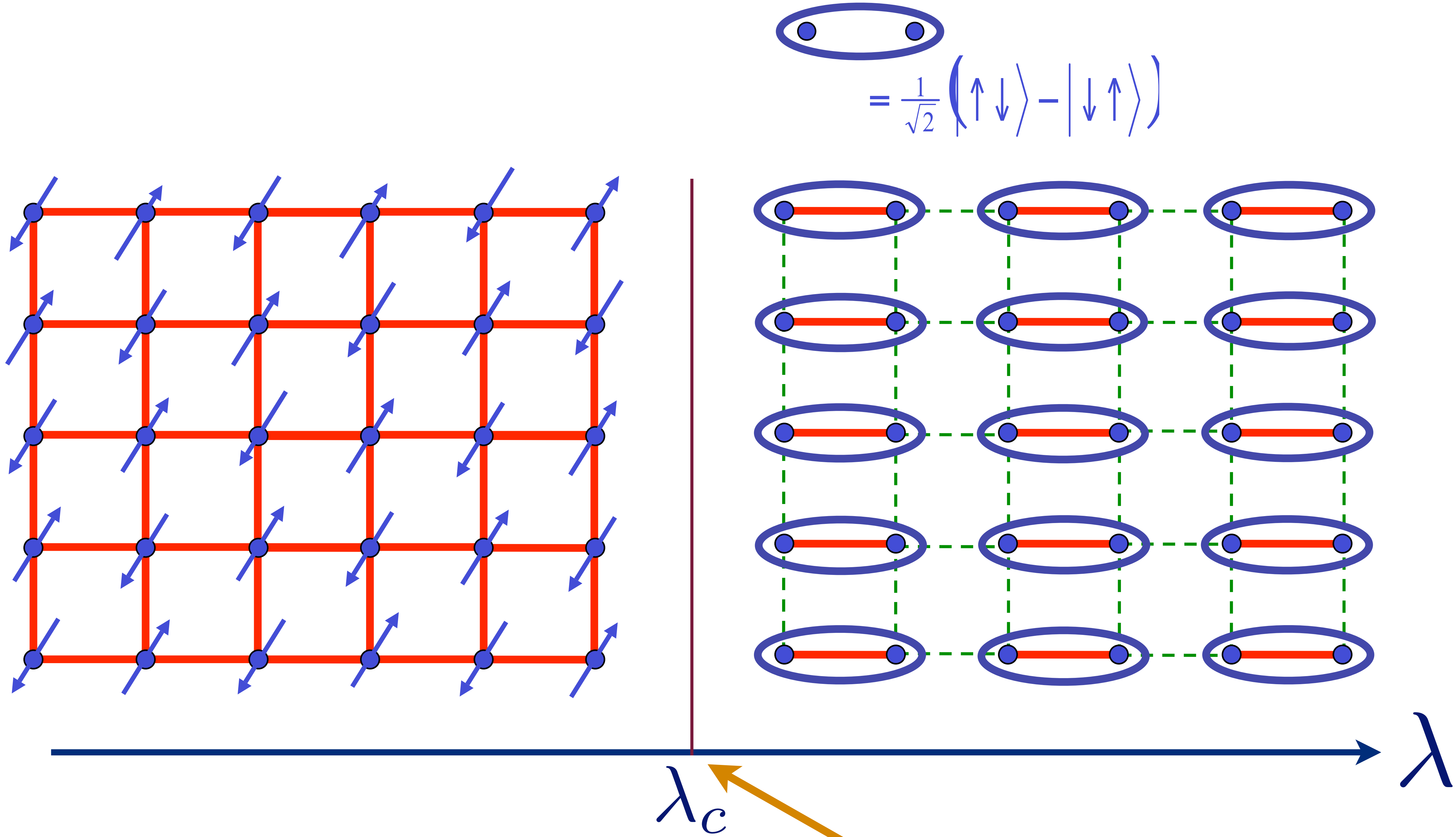
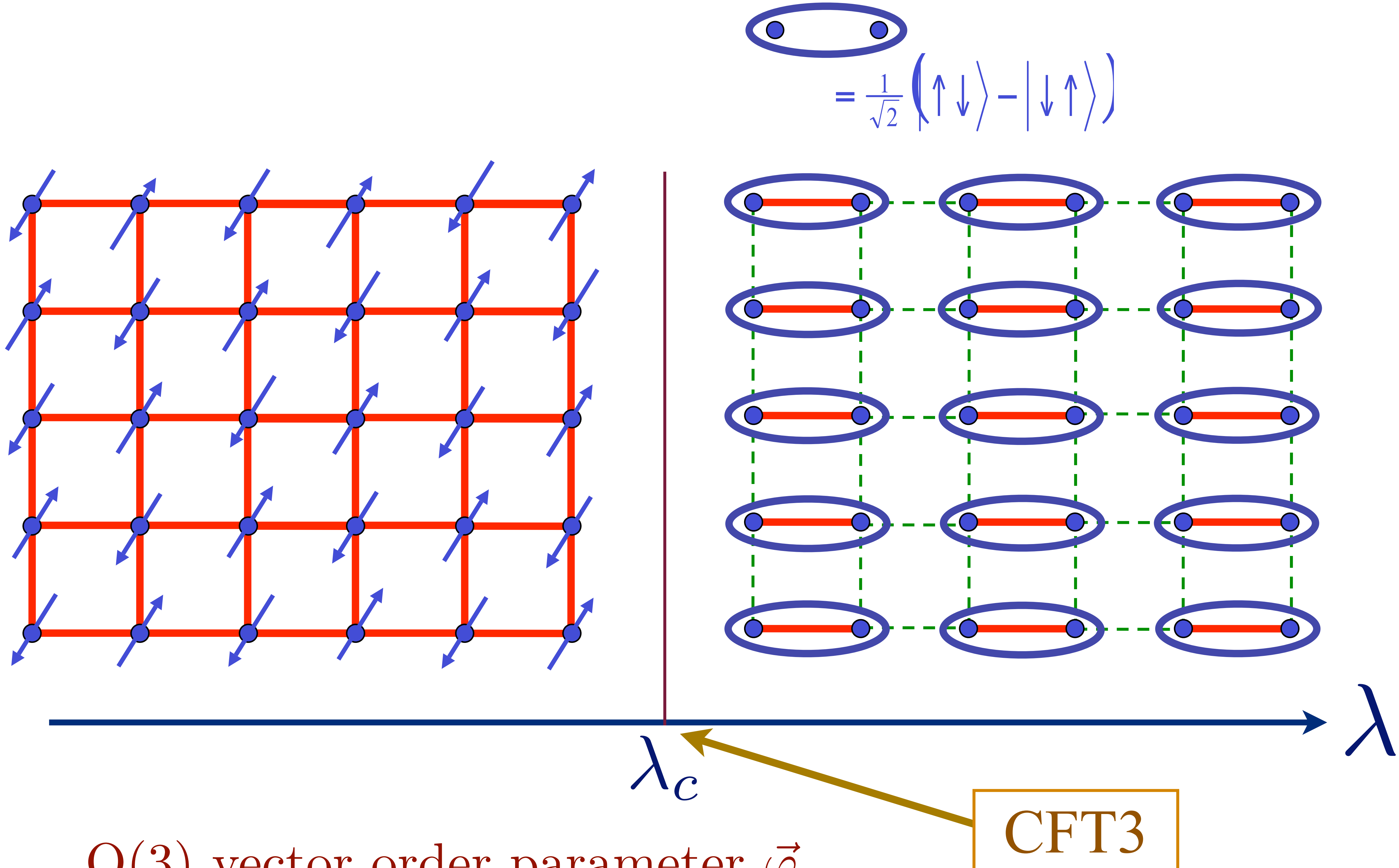


Weaken some bonds to induce spin entanglement in a new quantum phase



Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).



O(3) vector order parameter $\vec{\varphi}$

$$\mathcal{S}_{LG} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + s\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$

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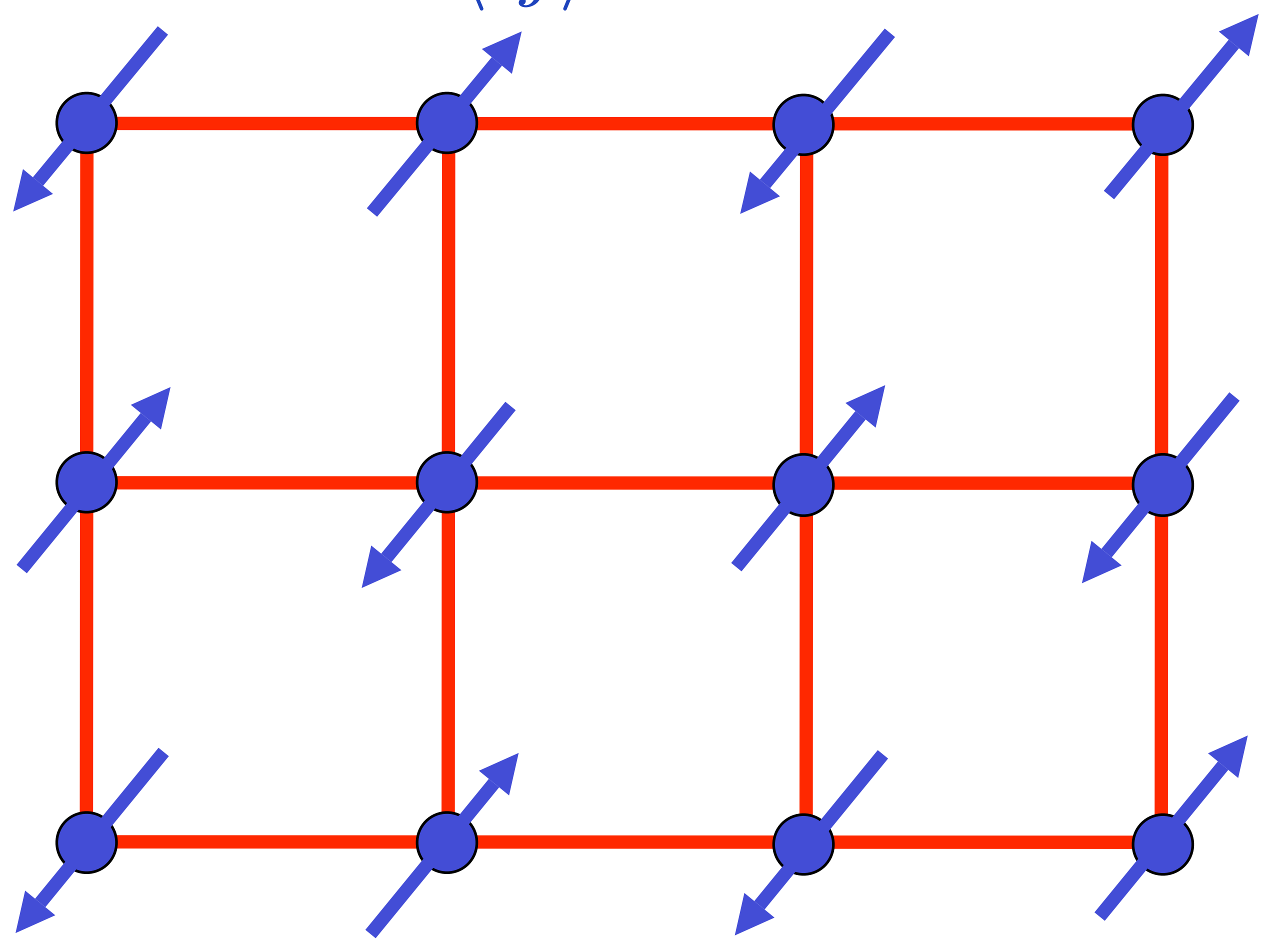

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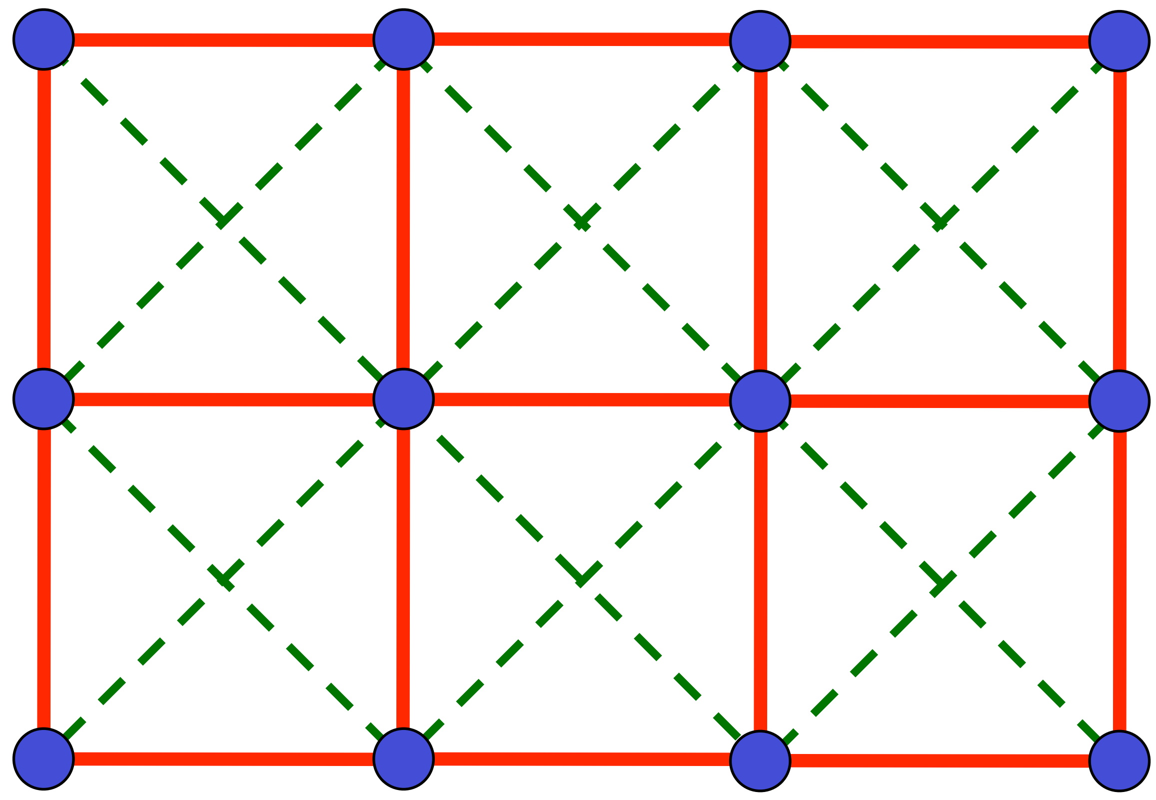

Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What are possible states with $\langle \vec{\varphi} \rangle = 0$?

Square lattice antiferromagnet

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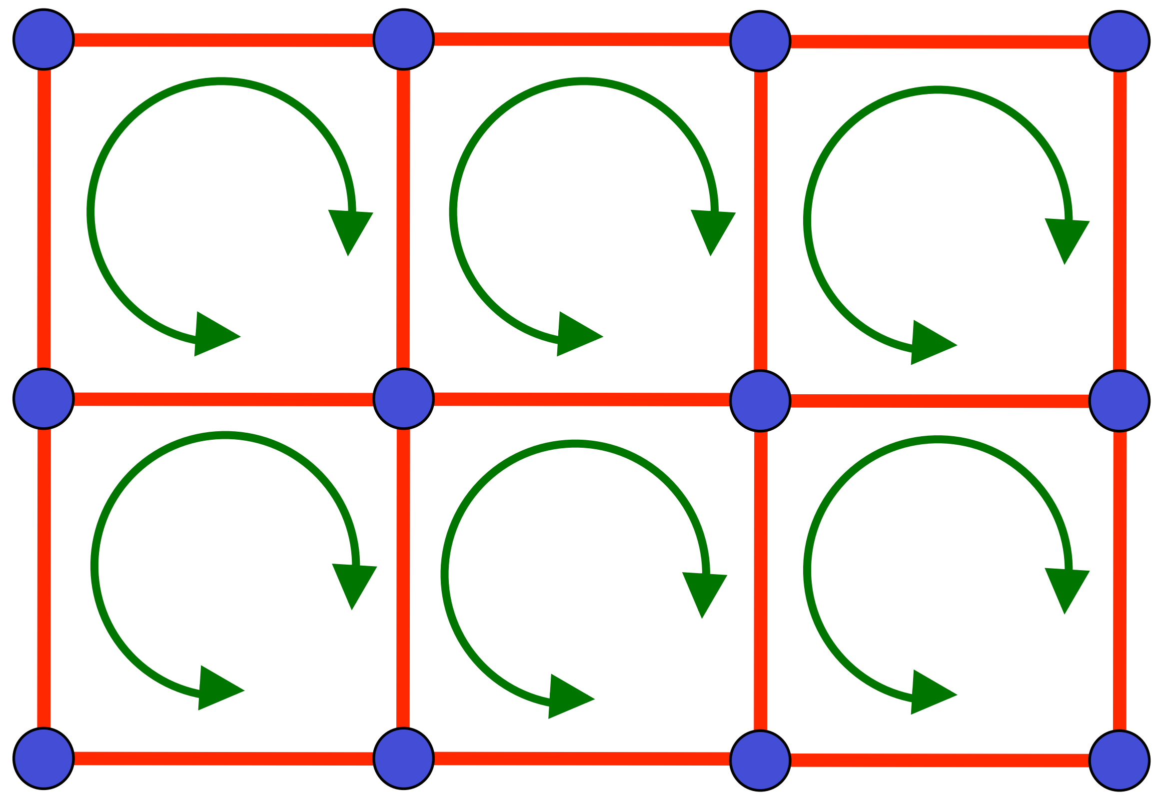

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Theory for loss of Neel order

Write the Néel order in terms of
Schwinger bosons (spinons) $z_{i\alpha}$, $\alpha = \uparrow, \downarrow$:

$$\vec{\varphi}_i = z_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} z_{i\beta}$$

where $\vec{\sigma}$ are Pauli matrices, and the bosons obey the local constraint

$$\sum_\alpha z_{i\alpha}^\dagger z_{i\alpha} = 2S$$

Effective theory for spinons must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \rightarrow e^{i\theta} z_{i\alpha}$$

Perturbation theory

Low energy spinon theory for "quantum disordering" the Néel state is the CP^1 model

$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x)z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau)z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + u\left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

where A_μ is an emergent U(1) gauge field which describes low-lying spin-singlet excitations.

Phases:

$\langle z_\alpha \rangle \neq 0$	$\Rightarrow$	Néel (Higgs) state
$\langle z_\alpha \rangle = 0$	$\Rightarrow$	Spin liquid (Coulomb) state

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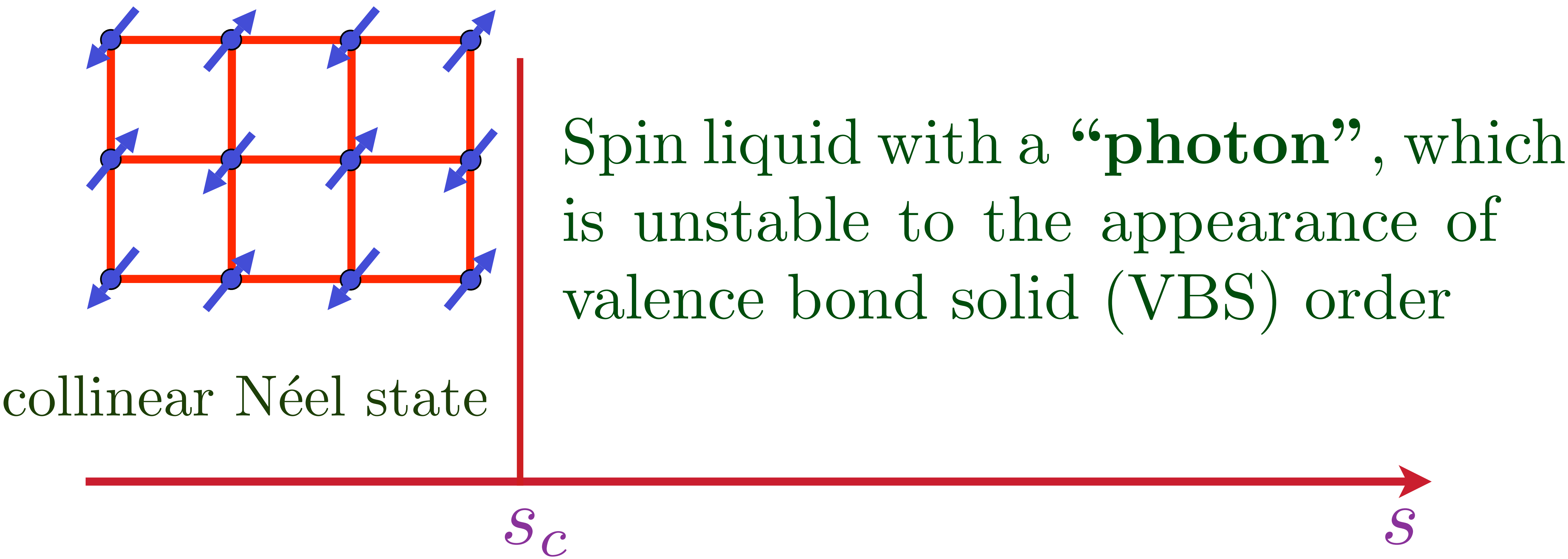
Distinct universality from O(3) model

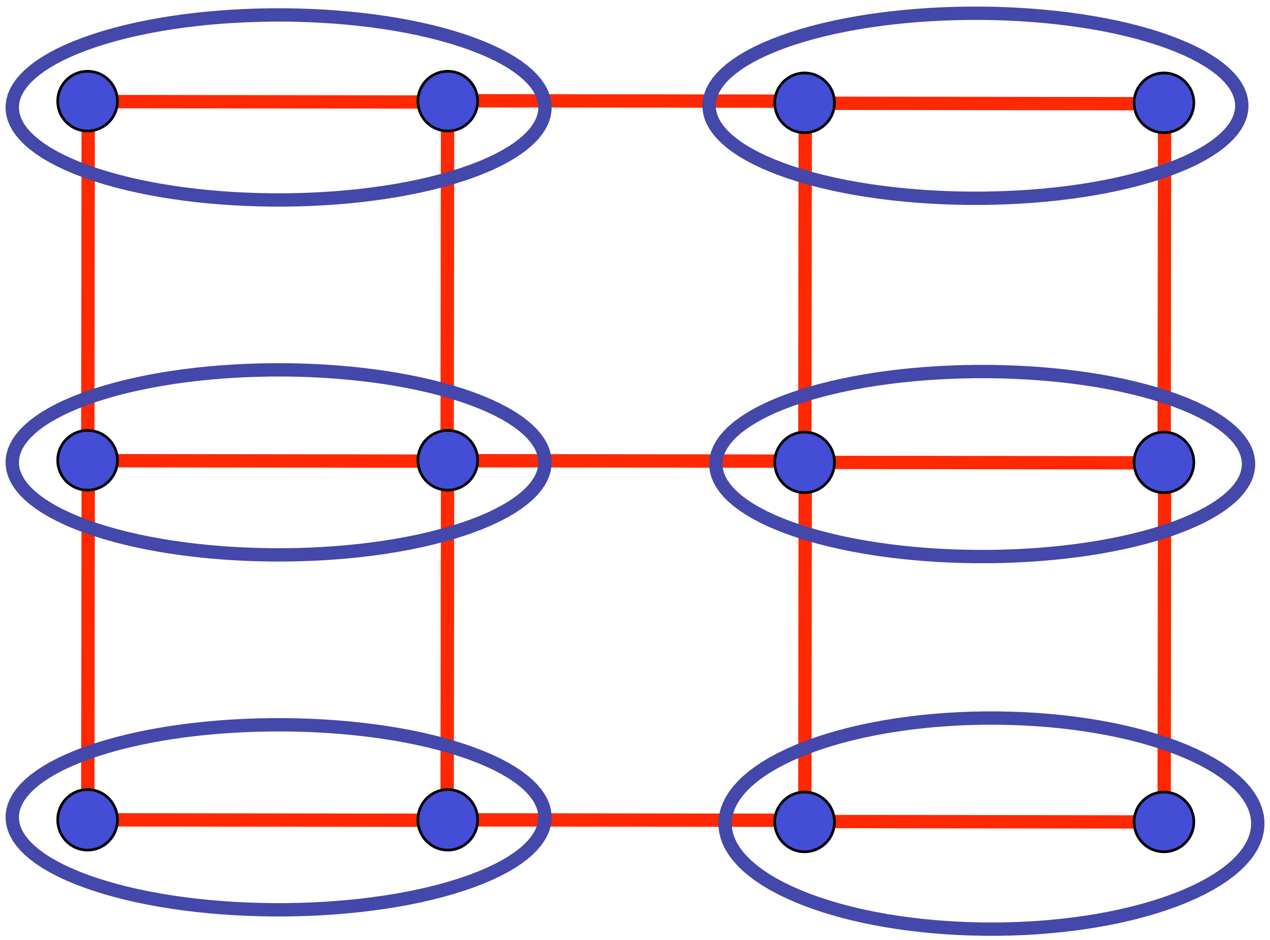
O. I. Motrunich and A. Vishwanath, *Phys. Rev.* B **70**, 075104 (2004)

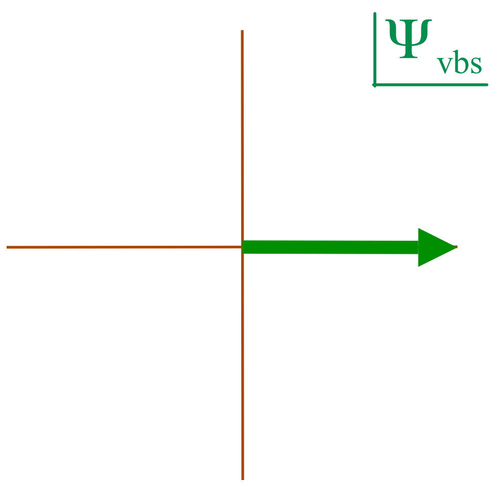
Quantum "disordering" magnetic order



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

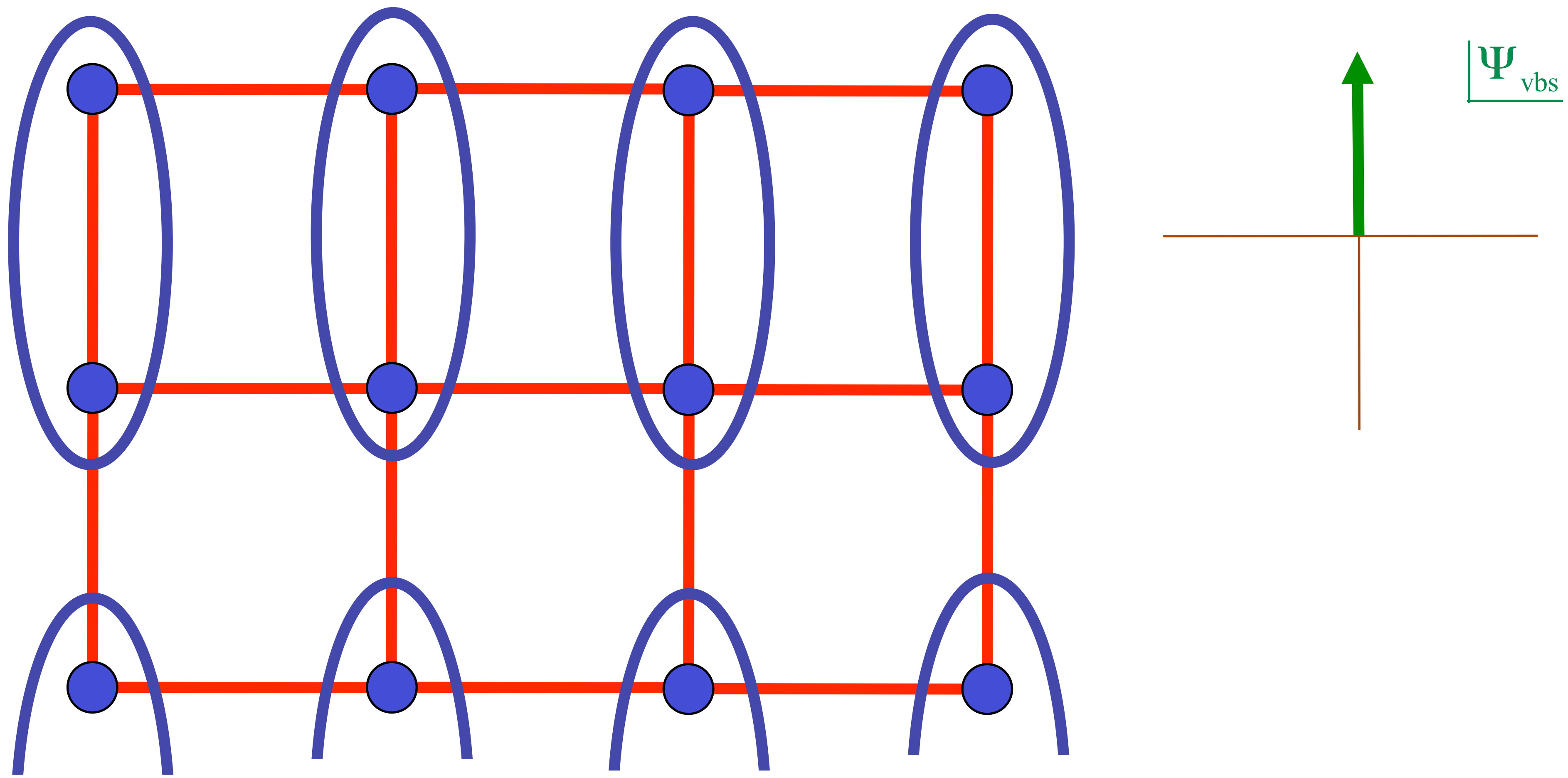
Order parameter of VBS state





$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Order parameter of VBS state



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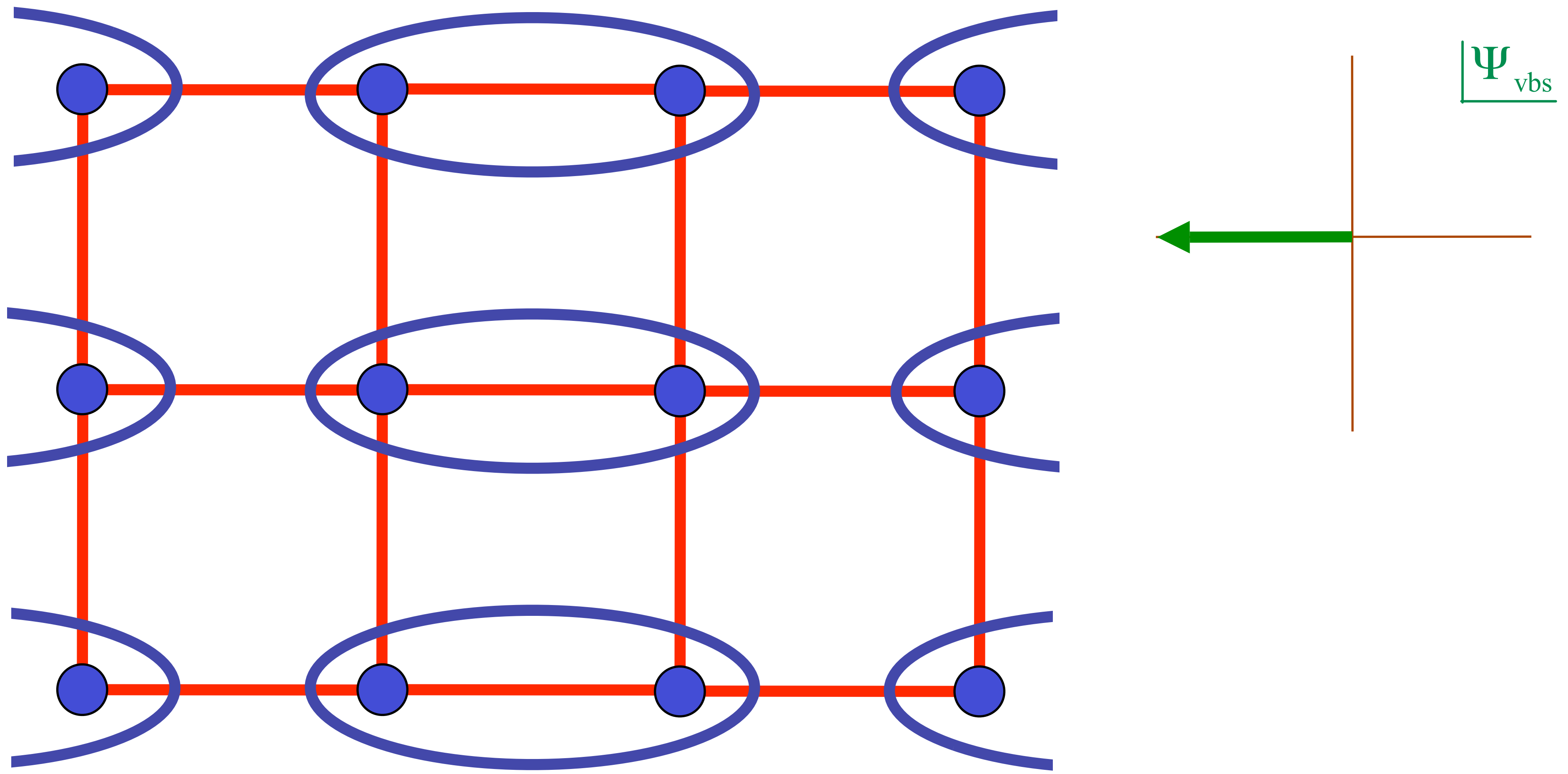

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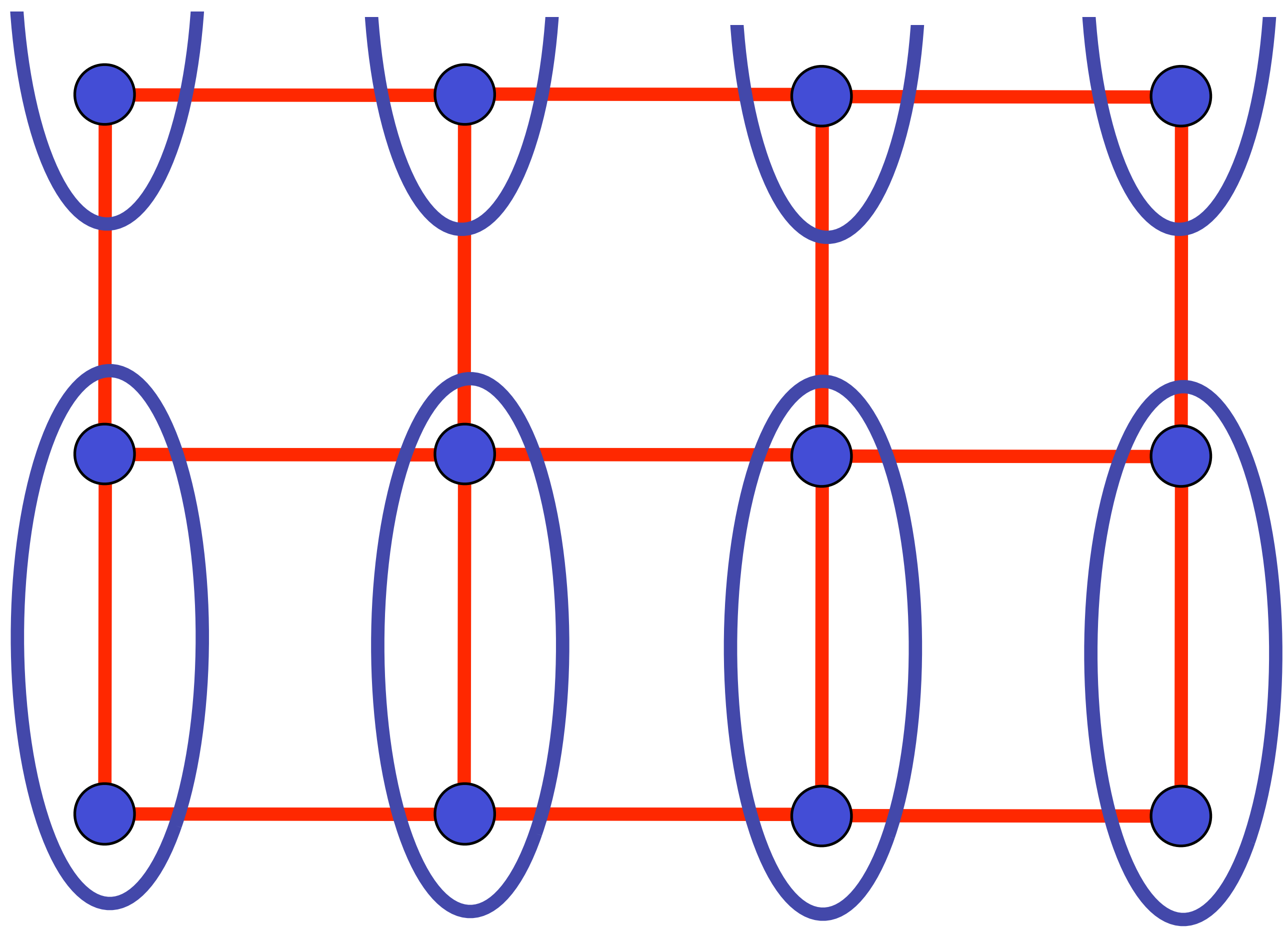

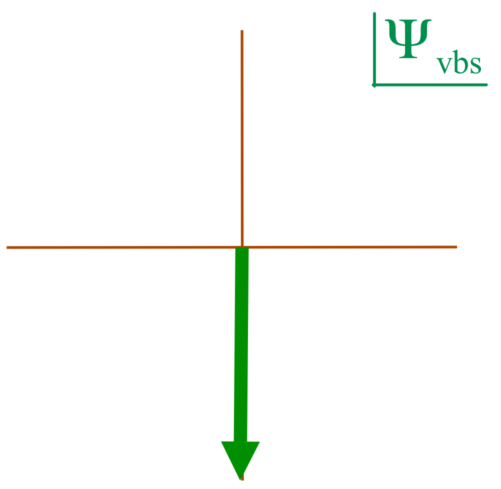

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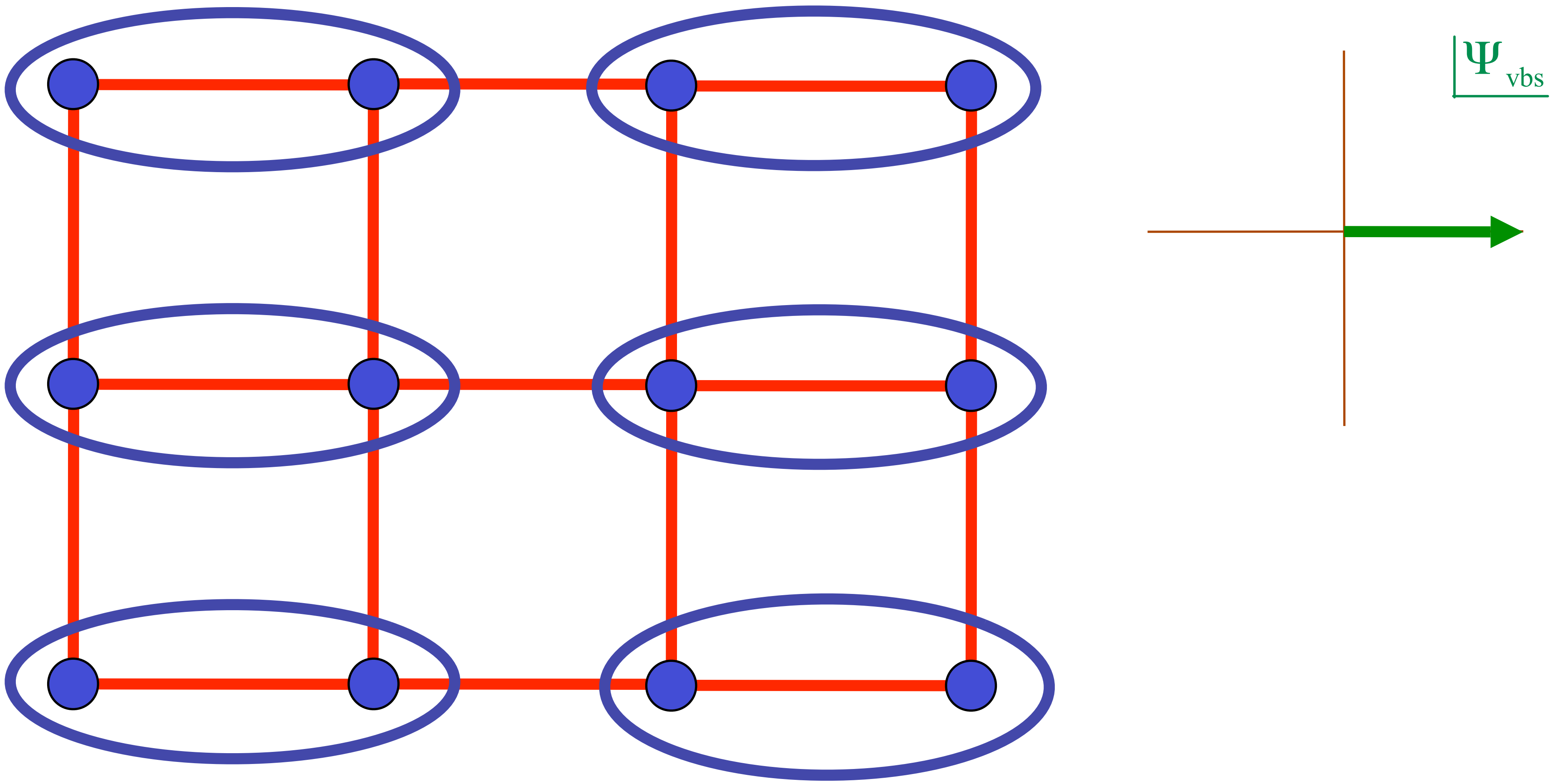

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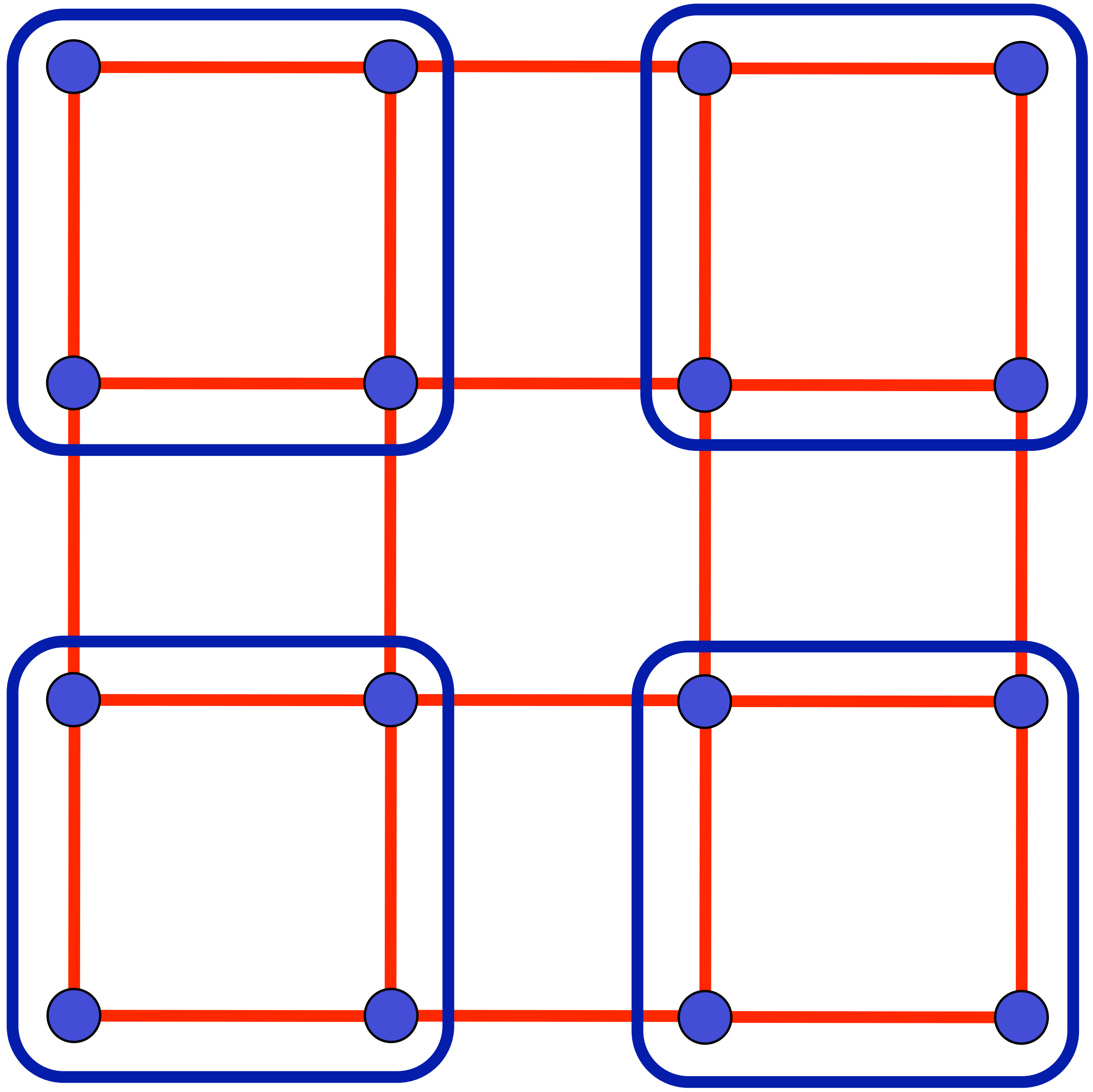

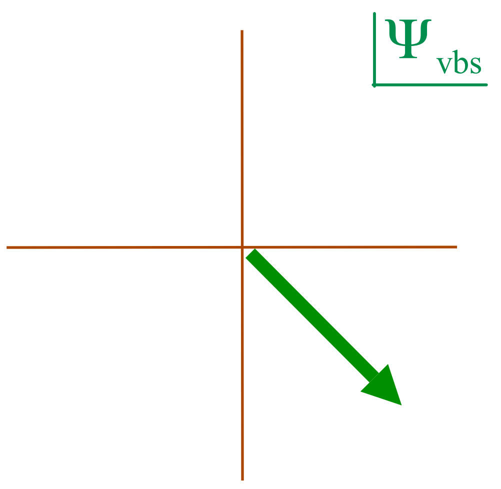

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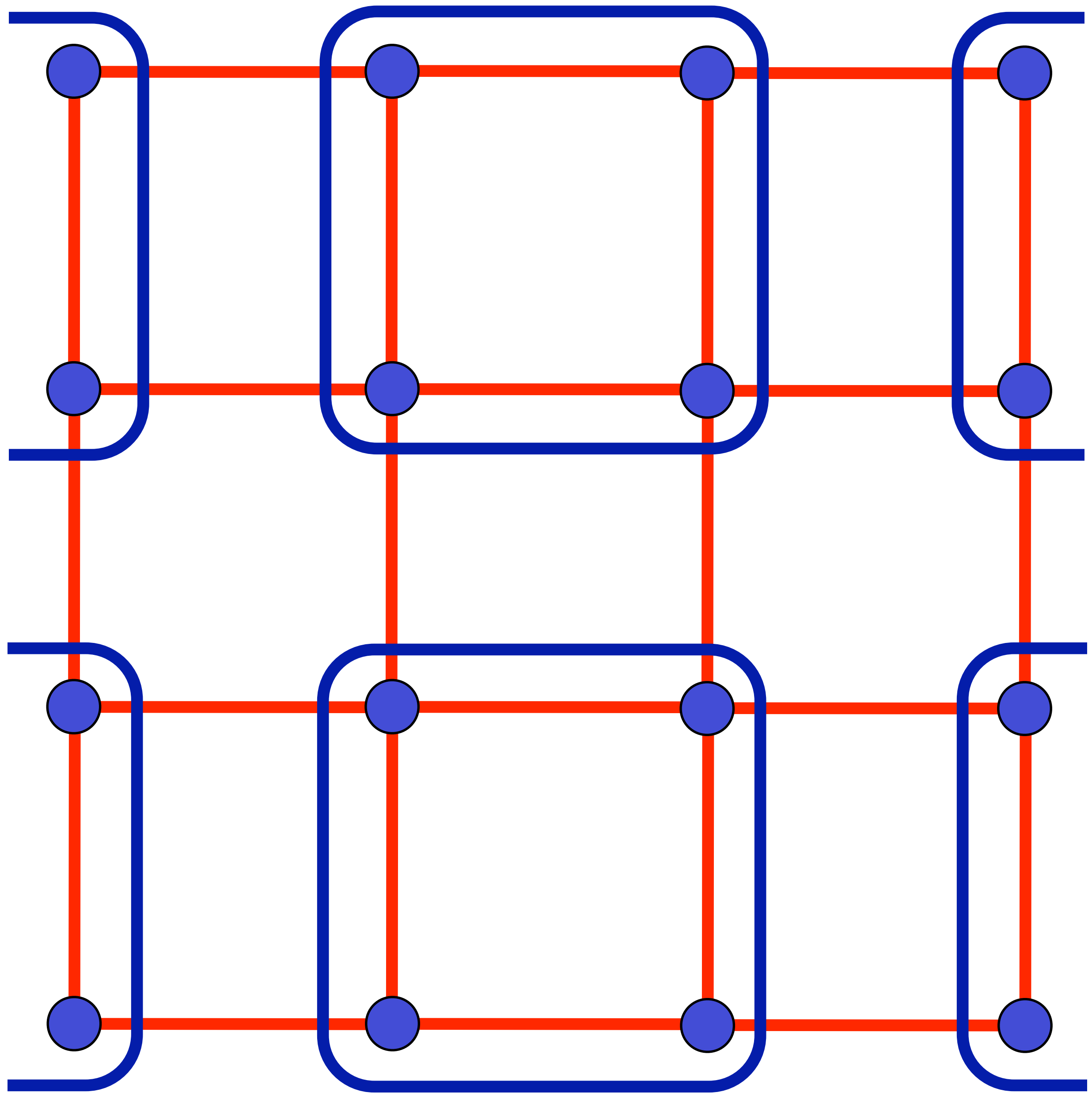

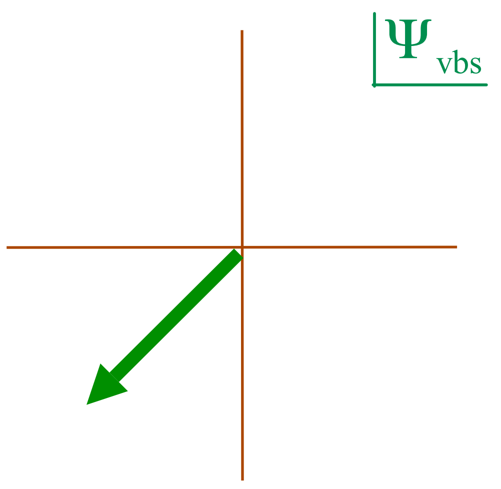

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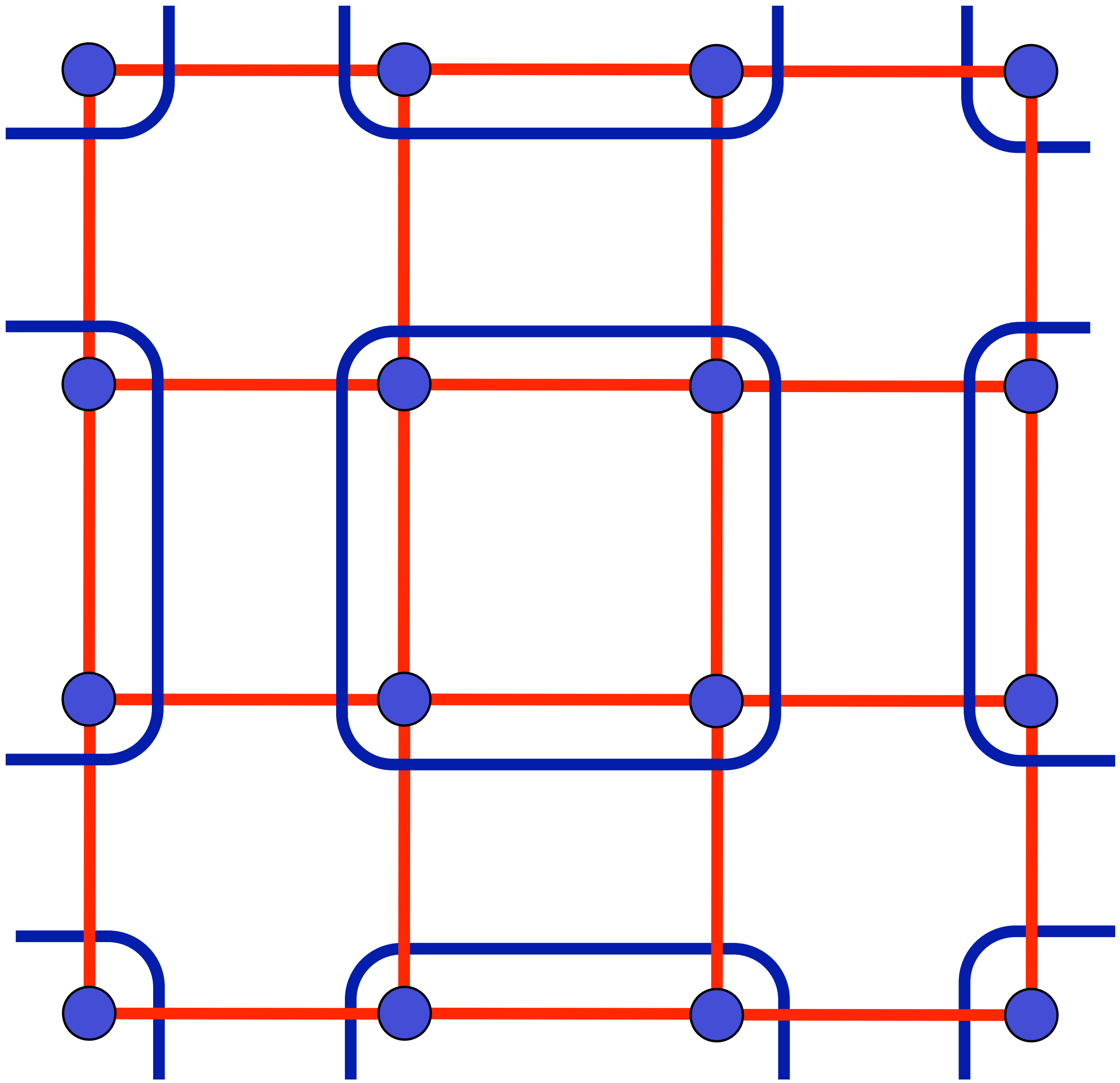

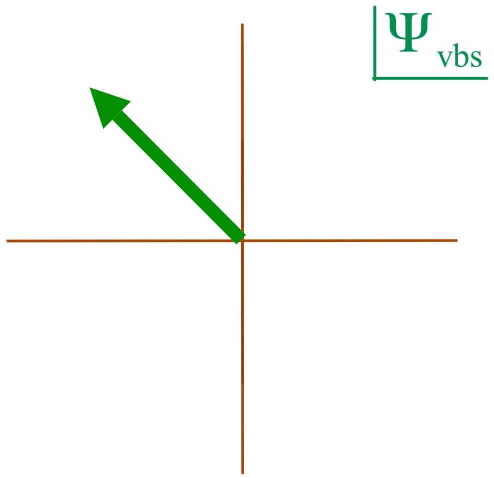

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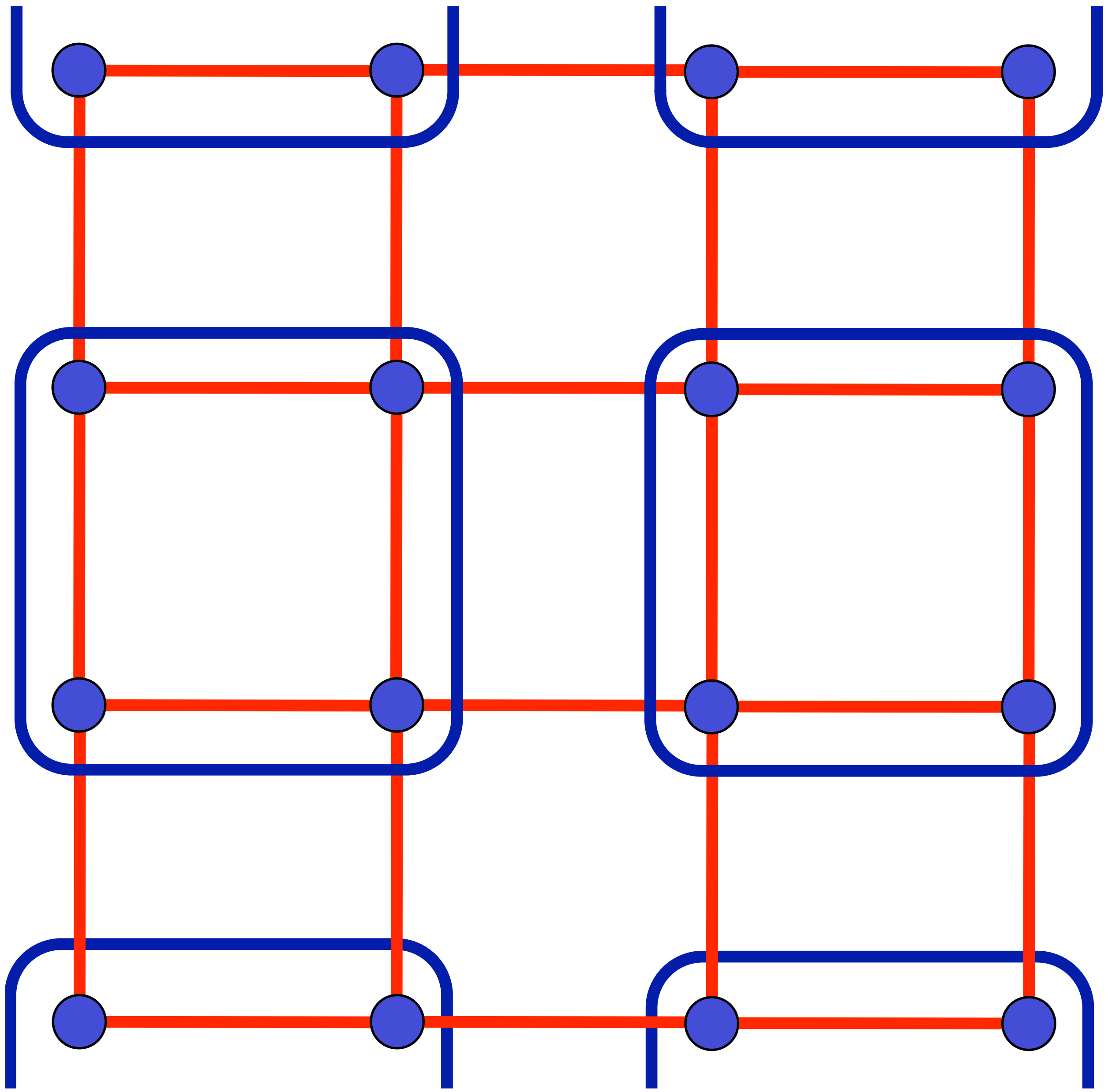

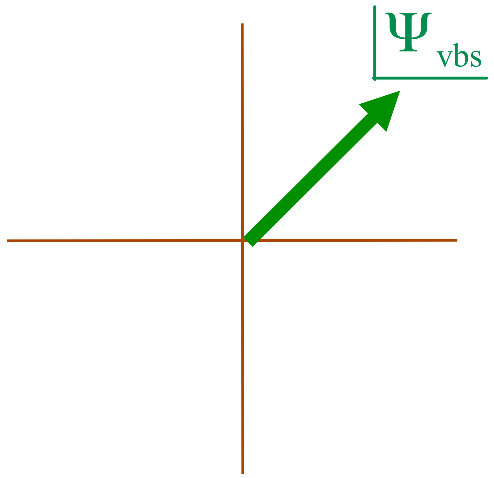

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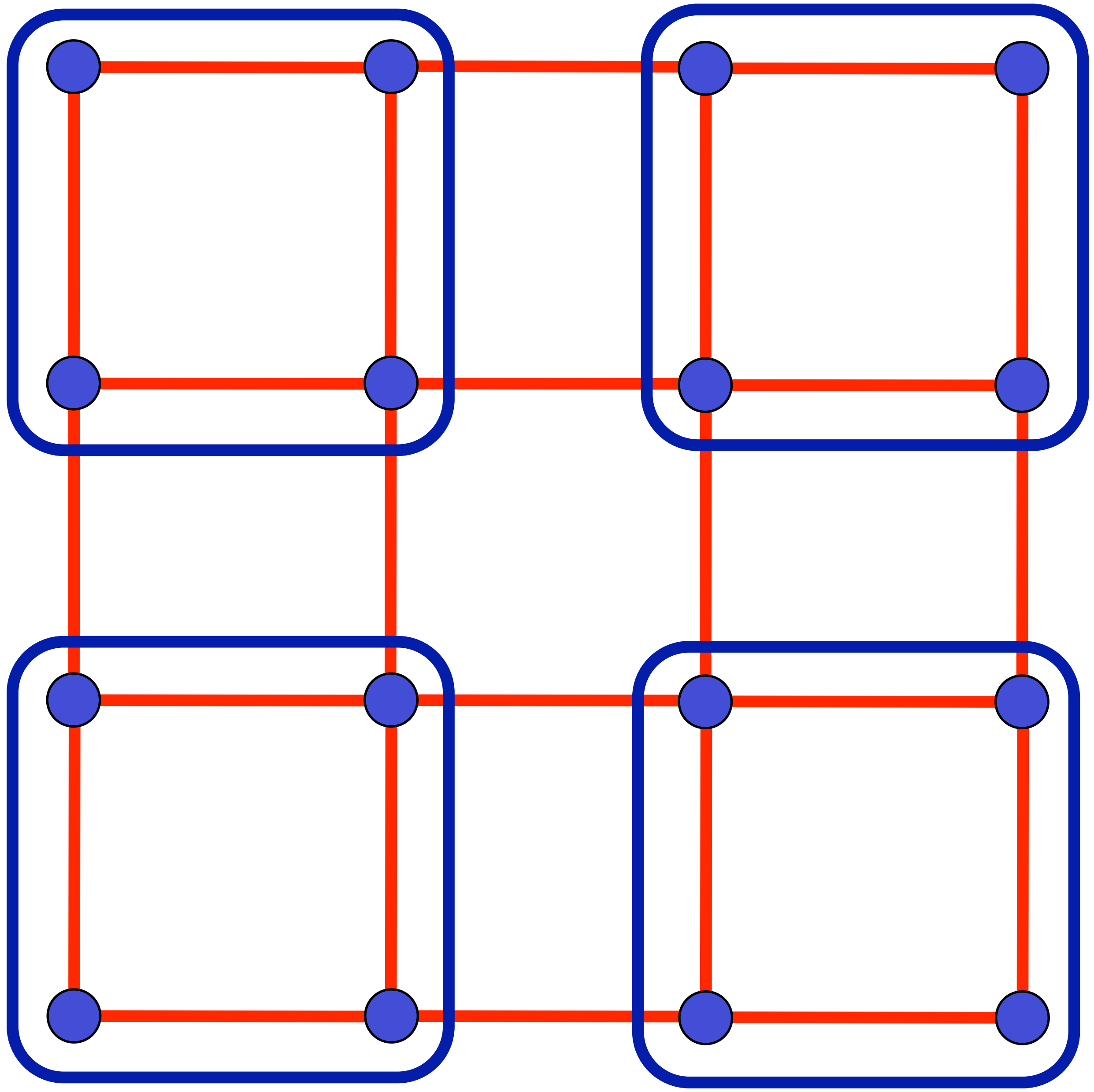
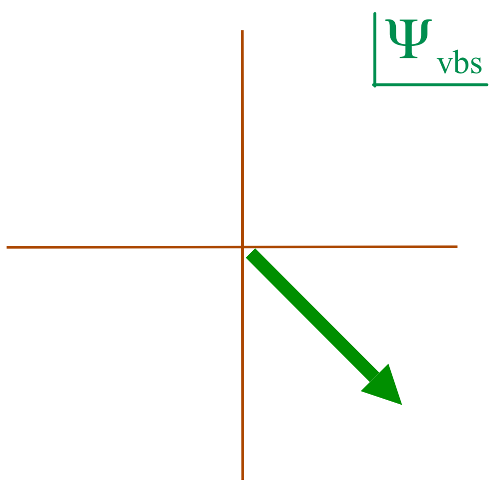
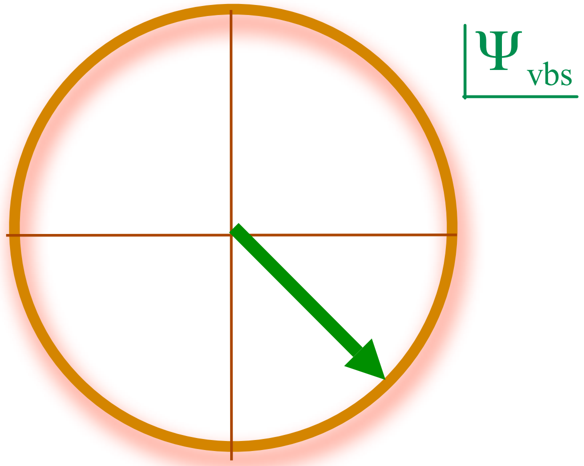




$$\Psi_{\rm vbs}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

- Near the Néel-VBS transition, the (nearly) gapless photon can be identified with the Goldstone mode associated with an emergent circular symmetry



$$\Psi_{\mathrm{vbs}} \rightarrow \Psi_{\mathrm{vbs}} e^{i\theta}.$$

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)
O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

Quantum Monte Carlo simulations display convincing evidence for a transition from a

Neel state at small Q
to a
VBS state at large Q

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).
F.-J. Jiang, M. Nyfeler, S. Chandrasekharan, and U.-J. Wiese, arXiv:0710.3926

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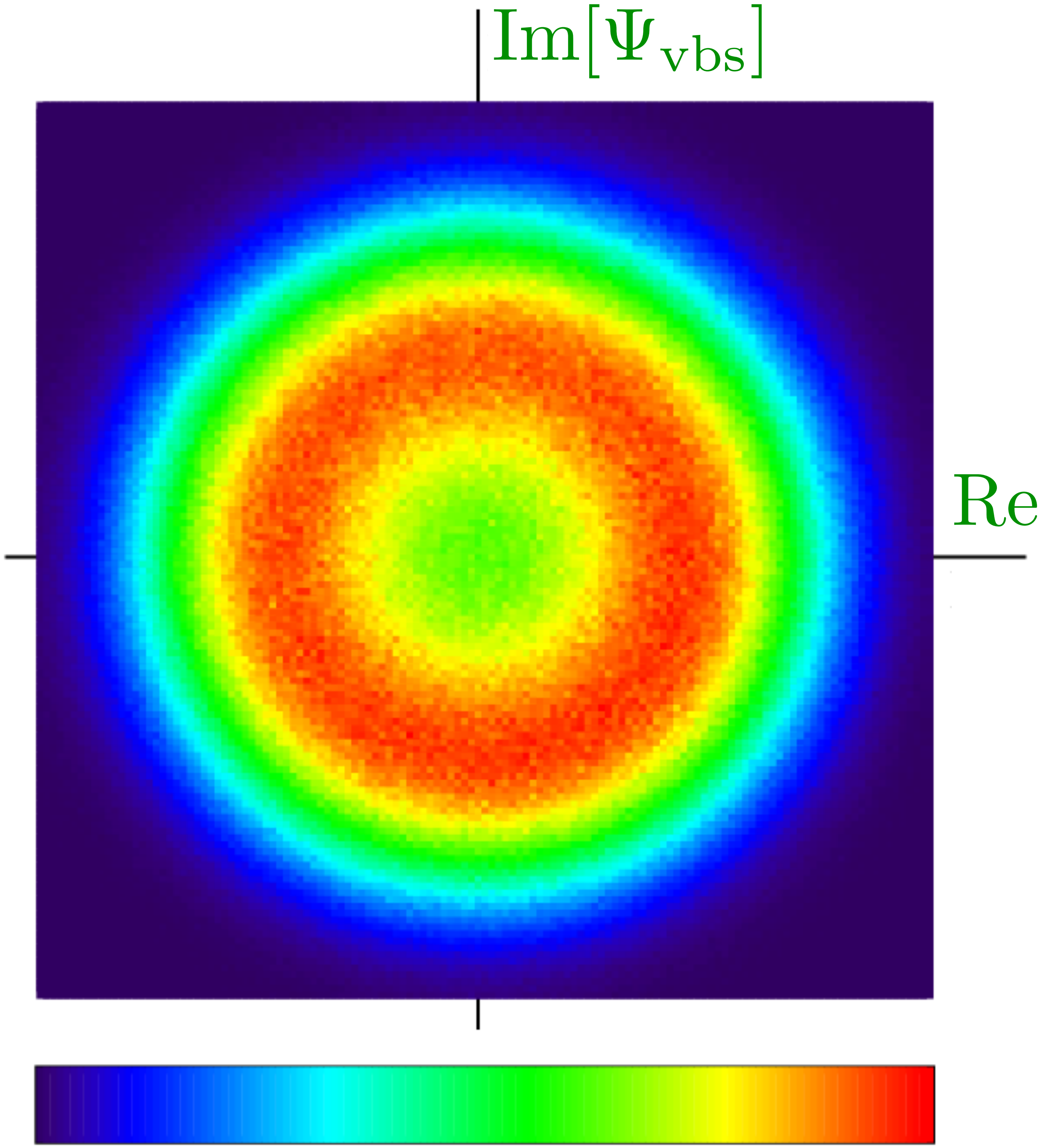

Distribution of VBS order Ψ_{vbs} at large Q

Emergent circular symmetry is evidence for U(1) photon and topological order

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Outline

1. Nodal-anti-nodal dichotomy in the cuprates

   Survey of recent experiments

2. Spin density wave theory of normal metal

   From a "large" Fermi surface to electron and hole pockets

3. Loss of Neel order in insulating square lattice antiferromagnets

   Landau-Ginzburg theory vs. gauge theory for spinons

4. Algebraic charge liquids

   Pairing by gauge forces, d-wave superconductivity, and the nodal-anti-nodal dichotomy

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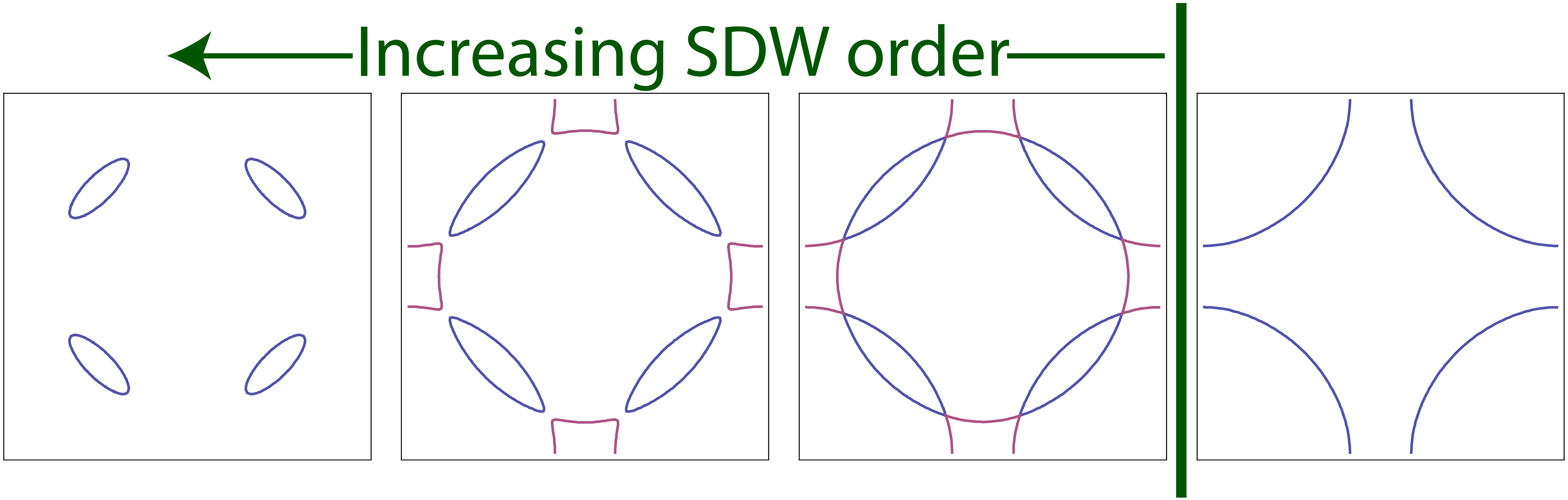
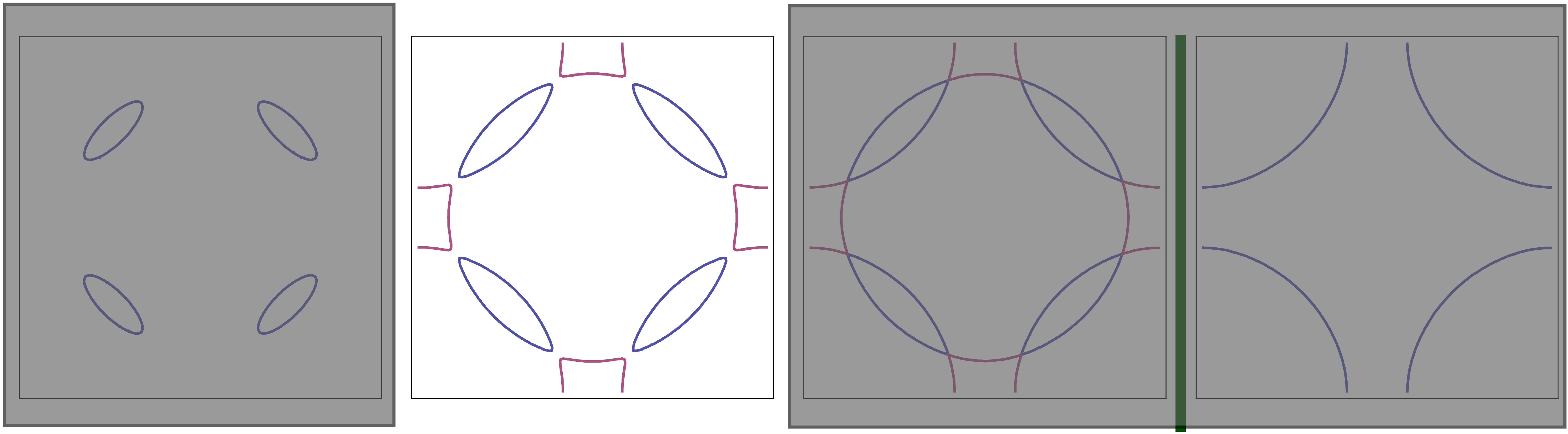
Spin density wave theory in hole-doped cuprates



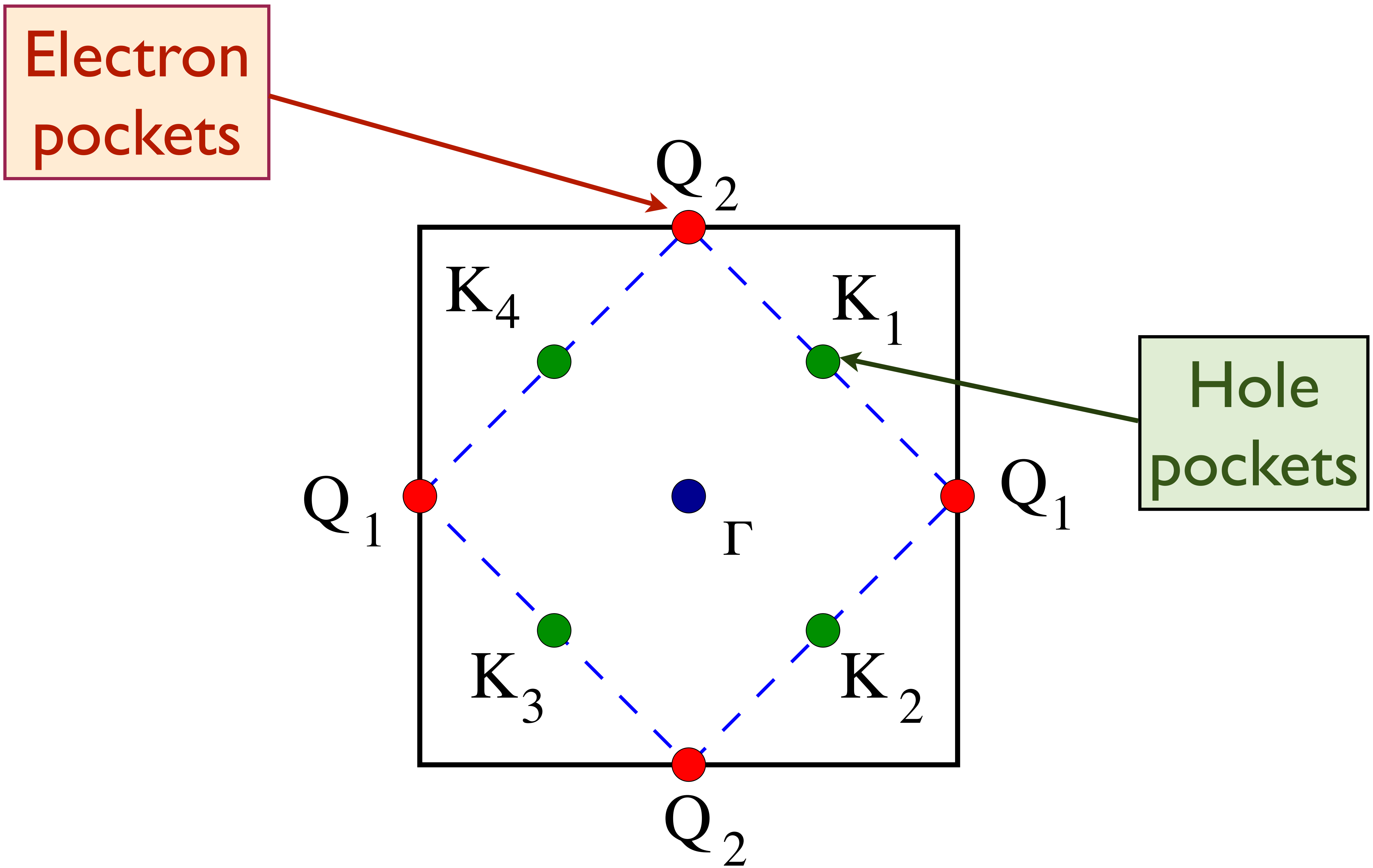
$$\mathrm{O}(3) \text{ vector order parameter } \vec{\varphi}$$

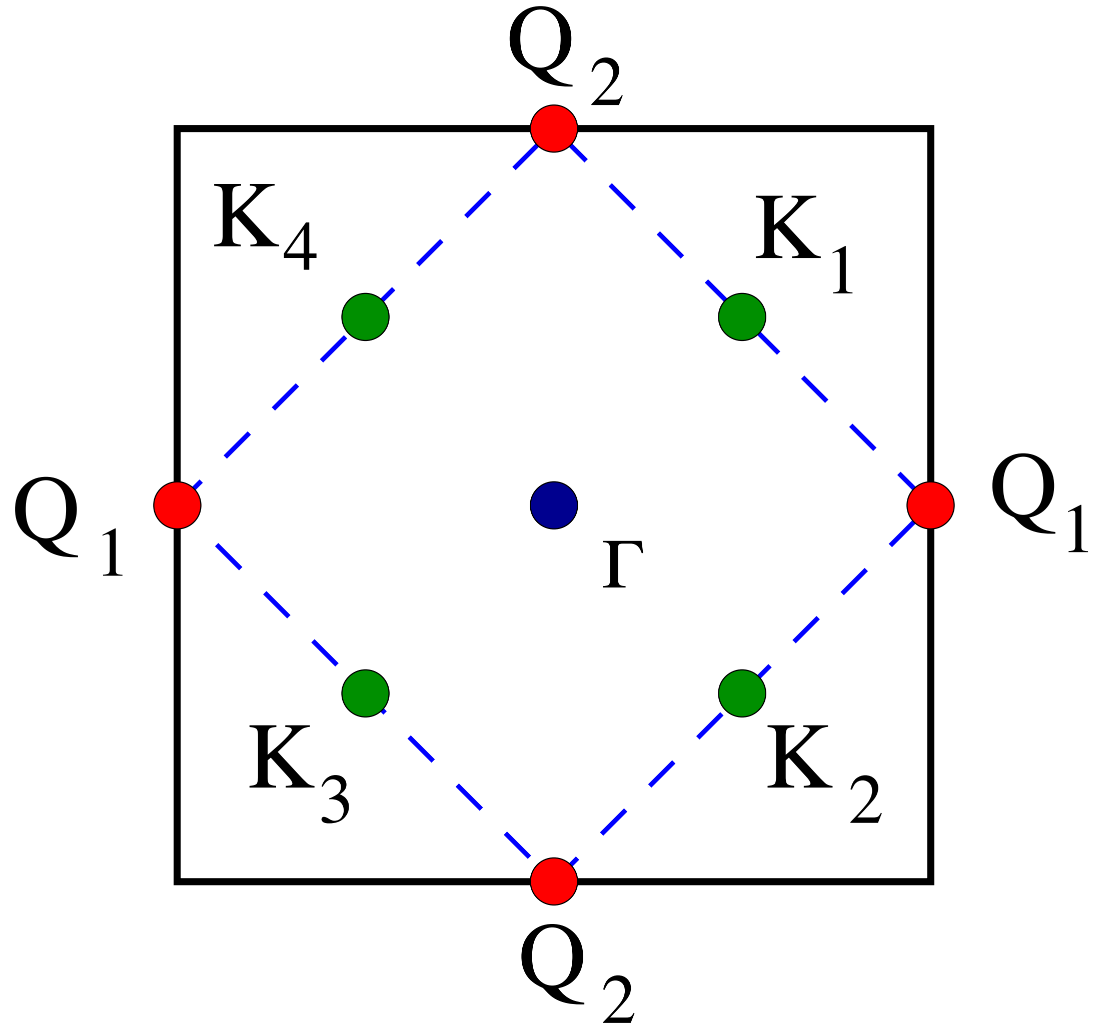
Spinon theory in hole-doped cuprates



SU(2) spinor order parameter z_α

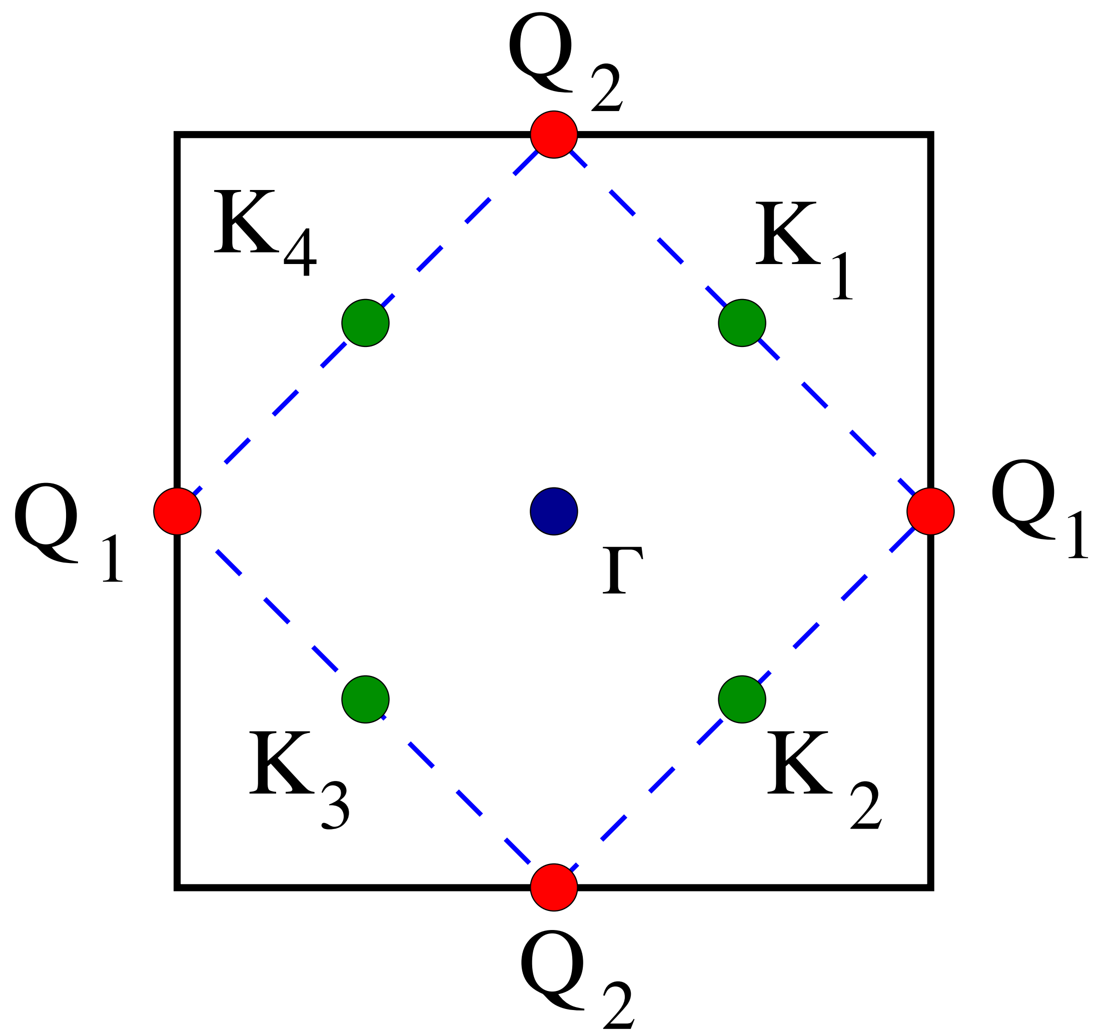
Charge carriers in the lightly-doped cuprates with Neel order





• Begin with the representation of the antiferromagnet as a CP^1 model (where $z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$ is the Néel order parameter, and A_μ is an emergent gauge field):

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + \frac{u}{2}\left(|z_\alpha|^2\right)^2 .$$



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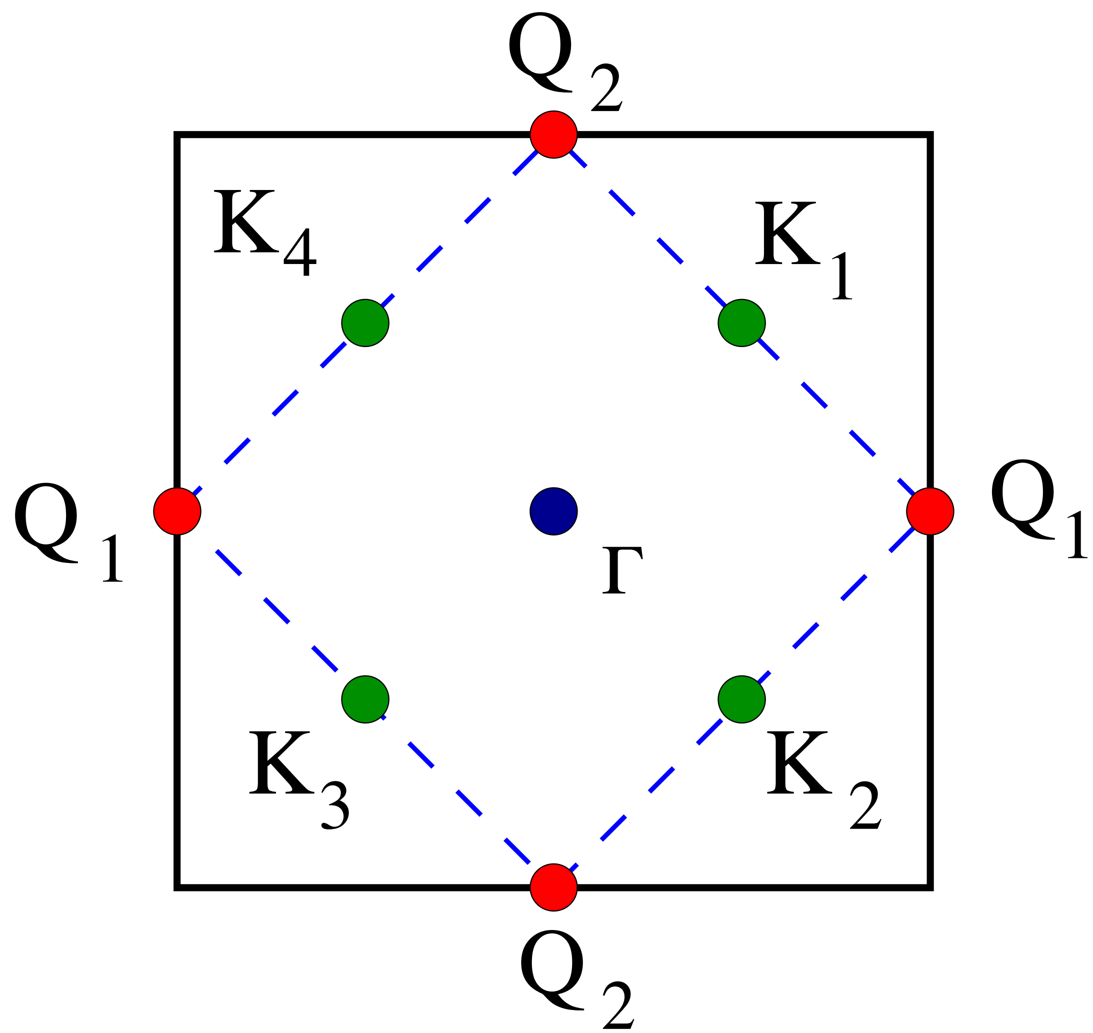
$$\mathcal{L}_z = |(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + \frac{u}{2}\left(|z_\alpha|^2\right)^2.$$

We have the conventional SDW metal for $s < 0$ where z_α condense



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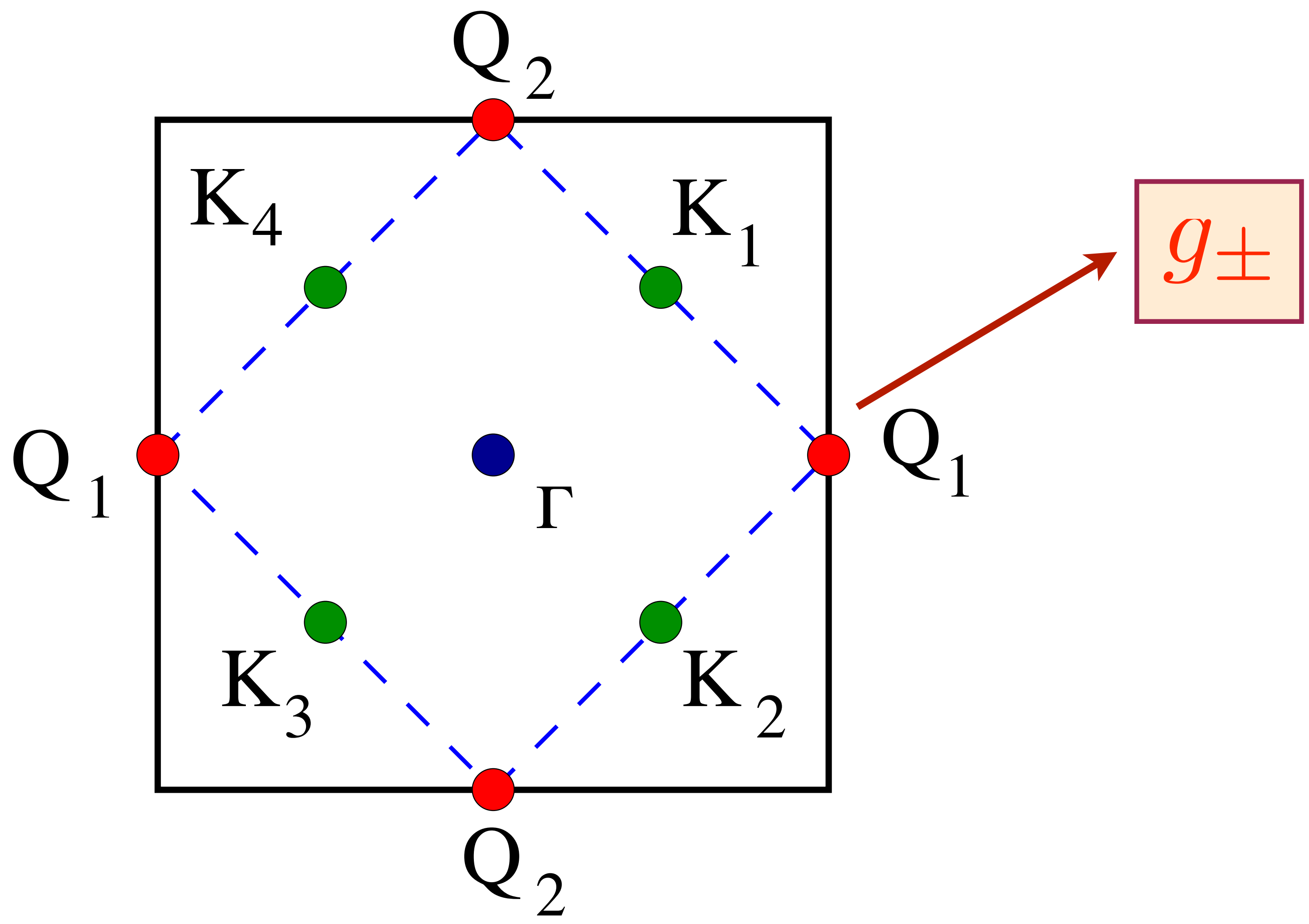
$$\mathcal{L}_z = |(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + \frac{u}{2}\left(|z_\alpha|^2\right)^2 .$$

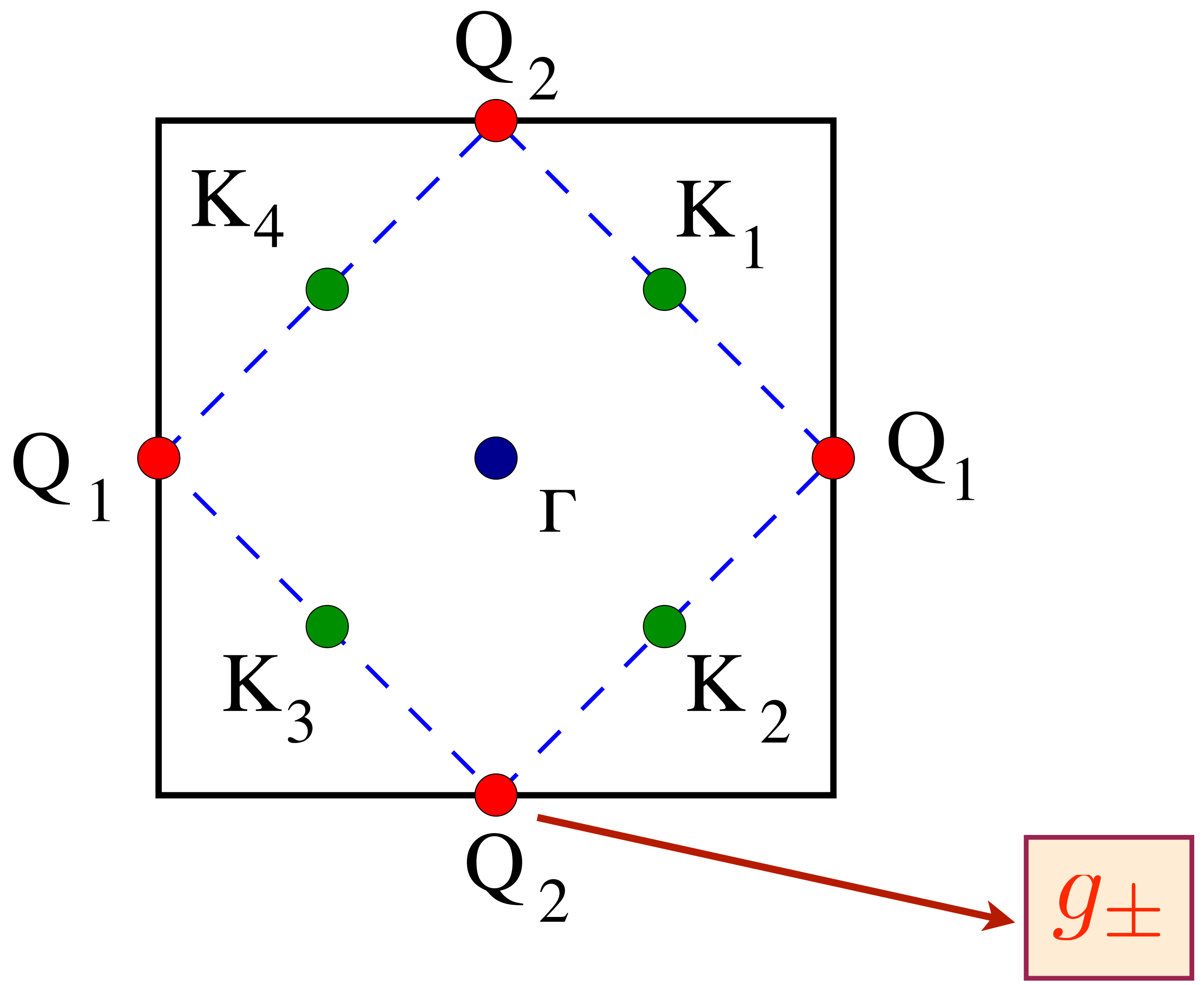
For $s > 0$ there is no SDW order,
but "ghosts" of electron/hole
pockets survive in an *algebraic charge liquid*



• Write the electron operator at wavevector Q_1 in terms of fermions $g_\pm$ polarized along the local direction of the SDW order:

$$\begin{pmatrix} c_\uparrow(Q_1) \\ c_\downarrow(Q_1) \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} g_+ \\ g_- \end{pmatrix}.$$

Electron pocket is polarized in a *rotating reference frame* defined by the local SDW order.



• This is linked to the electron operator at the pocket at Q_2 separated by the SDW ordering wavevector:

$$\begin{pmatrix} c_\uparrow(Q_2) \\ c_\downarrow(Q_2) \end{pmatrix} = \begin{pmatrix} z_\uparrow & z_\downarrow^* \\ z_\downarrow & -z_\uparrow^* \end{pmatrix} \begin{pmatrix} g_+ \\ g_- \end{pmatrix}.$$

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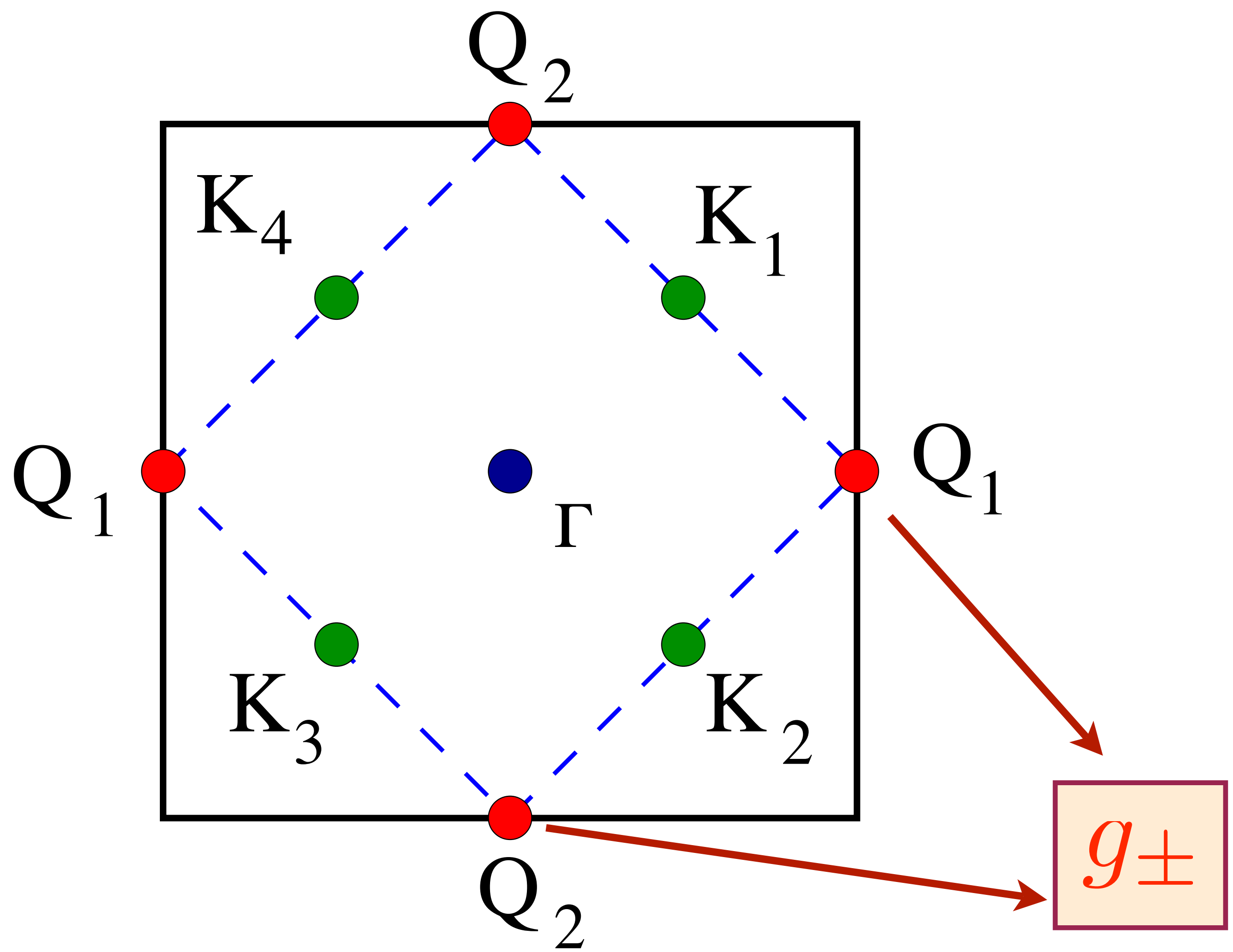

Low energy theory for spinless, charge $-e$ fermions $g_\pm$:

$$\mathcal{L}_g = g_+^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] g_+ + g_-^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] g_-$$

Two Fermi surfaces coupled to
a fluctuating gauge field with opposite charges.

Strong pairing of the $g_\pm$ electron pockets

- Problem is similar to double layer quantum Hall systems at total filling fraction $\nu = 1$. At large layer spacing we have 2 composite fermion Fermi surfaces each at filling fraction $\nu = 1/2$. At small layer spacing, there is a paired state formed by attractive interaction mediated by antisymmetric gauge field.

N. E. Bonesteel, I. A. McDonald, and C. Nayak, *Phys. Rev. Lett.* **77**, 3009 (1996).
I. Ussishkin and A. Stern, *Phys. Rev. Lett.* **81**, 3932 (1998).

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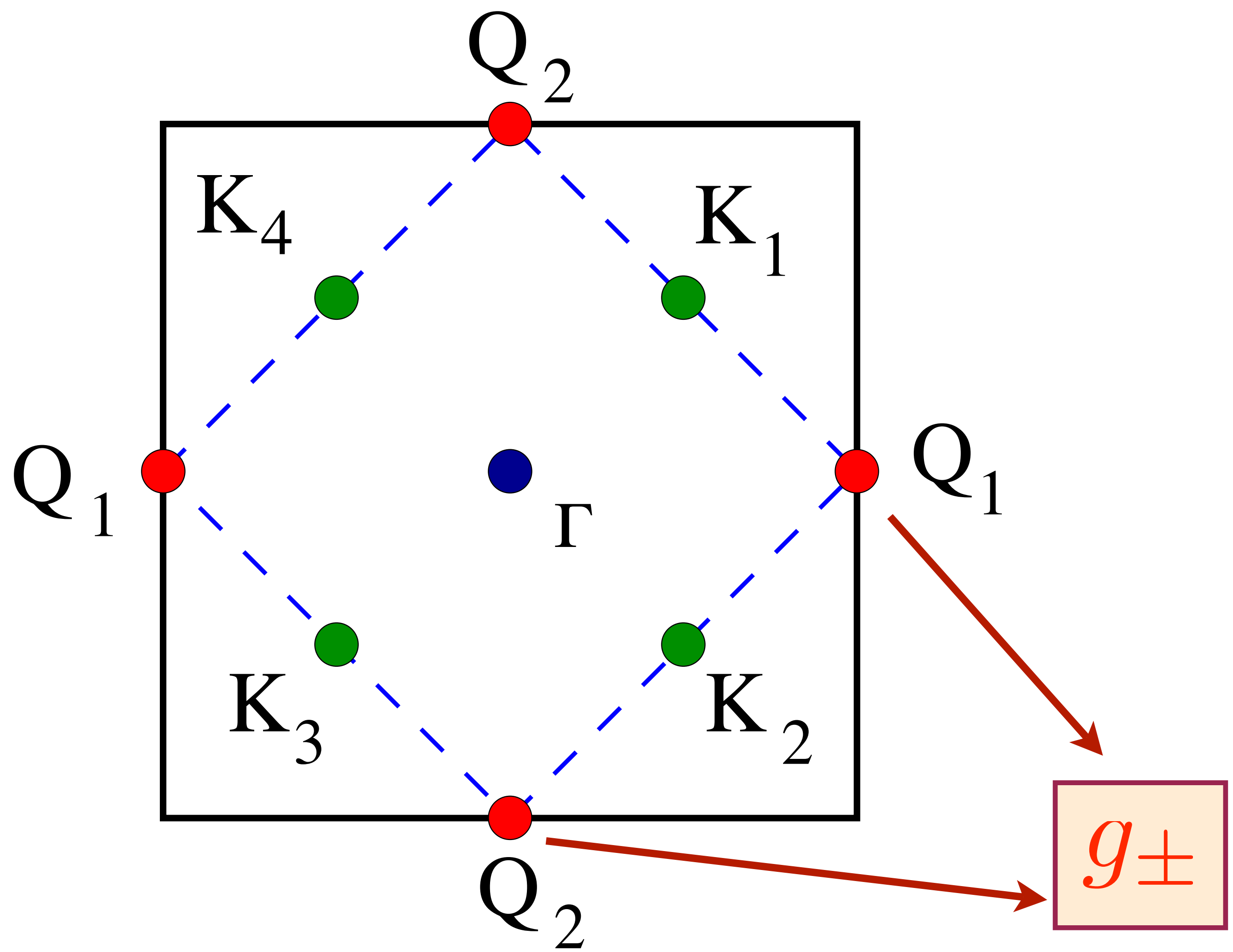
- Gauge forces lead to a s-wave paired state with a T_c of order the Fermi energy of the pockets. Inelastic scattering from low energy gauge modes lead to very singular $g_\pm$ self energy, but is *not* pair-breaking.

$$\langle g_+ g_- \rangle = \Delta$$

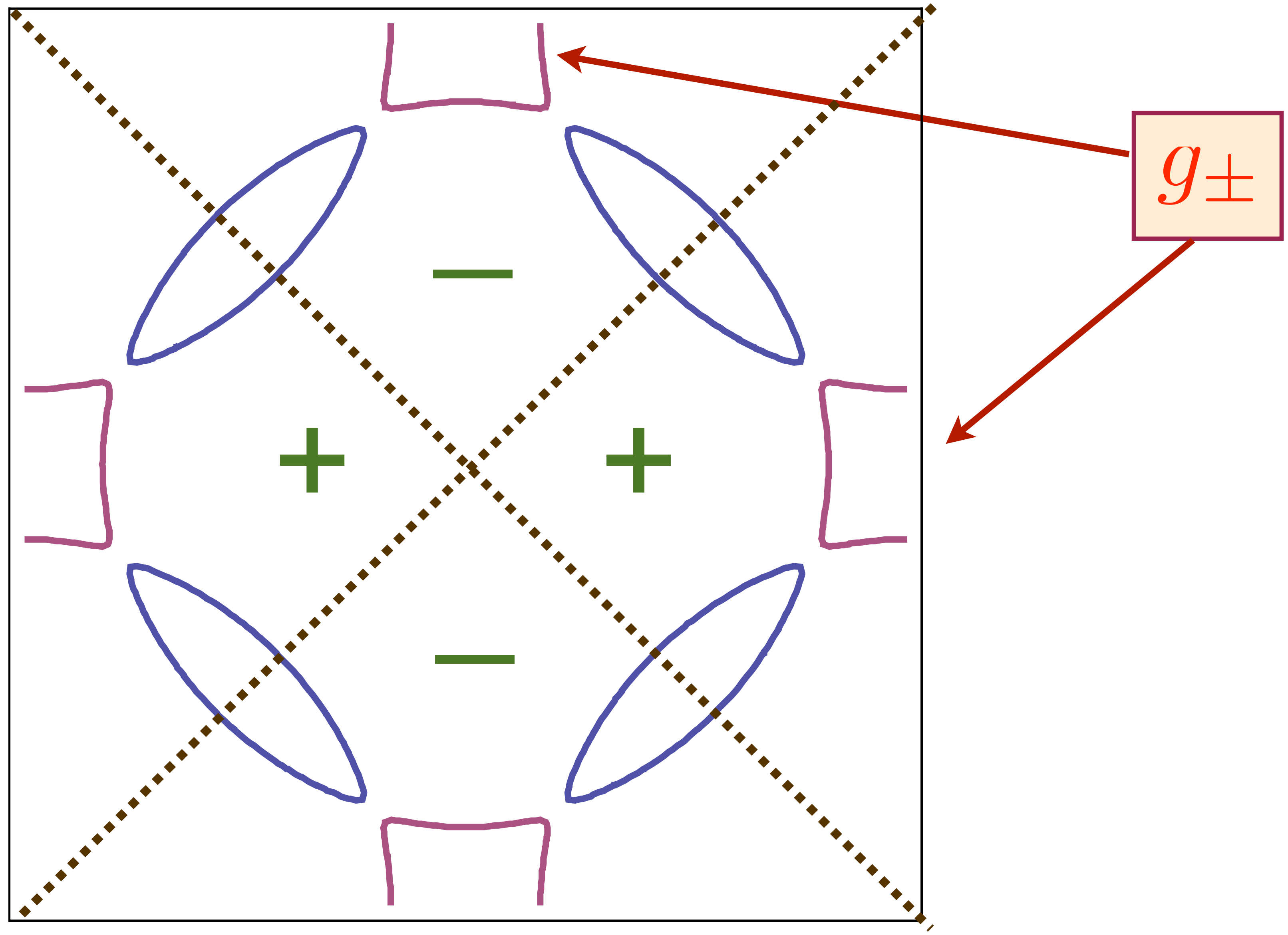


Strong pairing of the $g_\pm$ electron pockets

- Transforming back to the physical fermions we find

$$\langle c_\uparrow(Q_1) c_\downarrow(Q_1)\rangle = -\langle c_\uparrow(Q_2) c_\downarrow(Q_2)\rangle \sim \Delta$$

i.e. the pairing signature for the electrons is d-wave.



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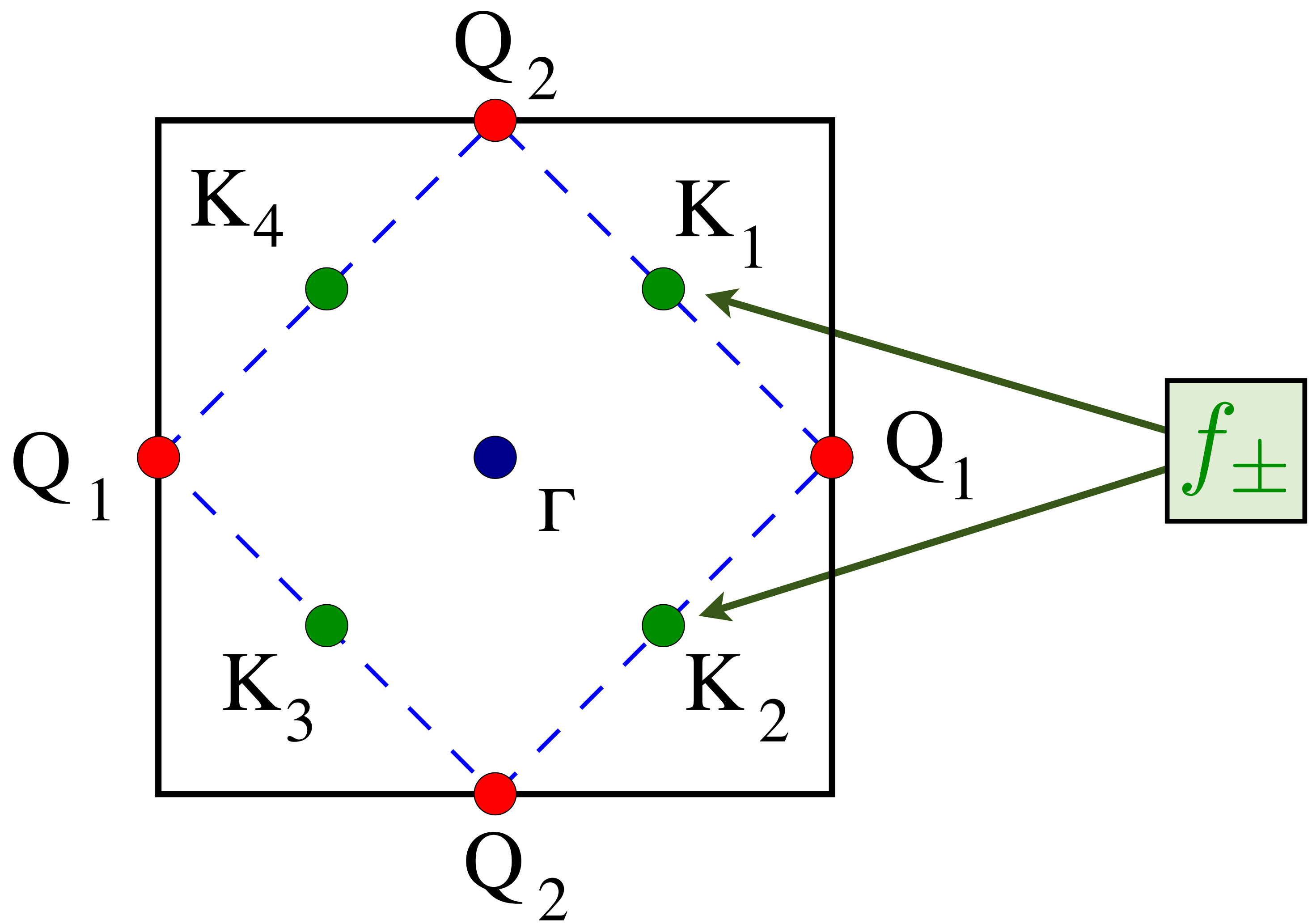


Low energy theory for spinless, charge $+e$ fermions $f_{\pm v}$:

$$\mathcal{L}_f = \sum_{v=1,2} \Big\{ f_{+v}^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] f_{+v}$$

$$+ f_{-v}^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] f_{-v} \Big\}$$

Two pairs of Fermi surfaces coupled to
a fluctuating gauge field with opposite charges

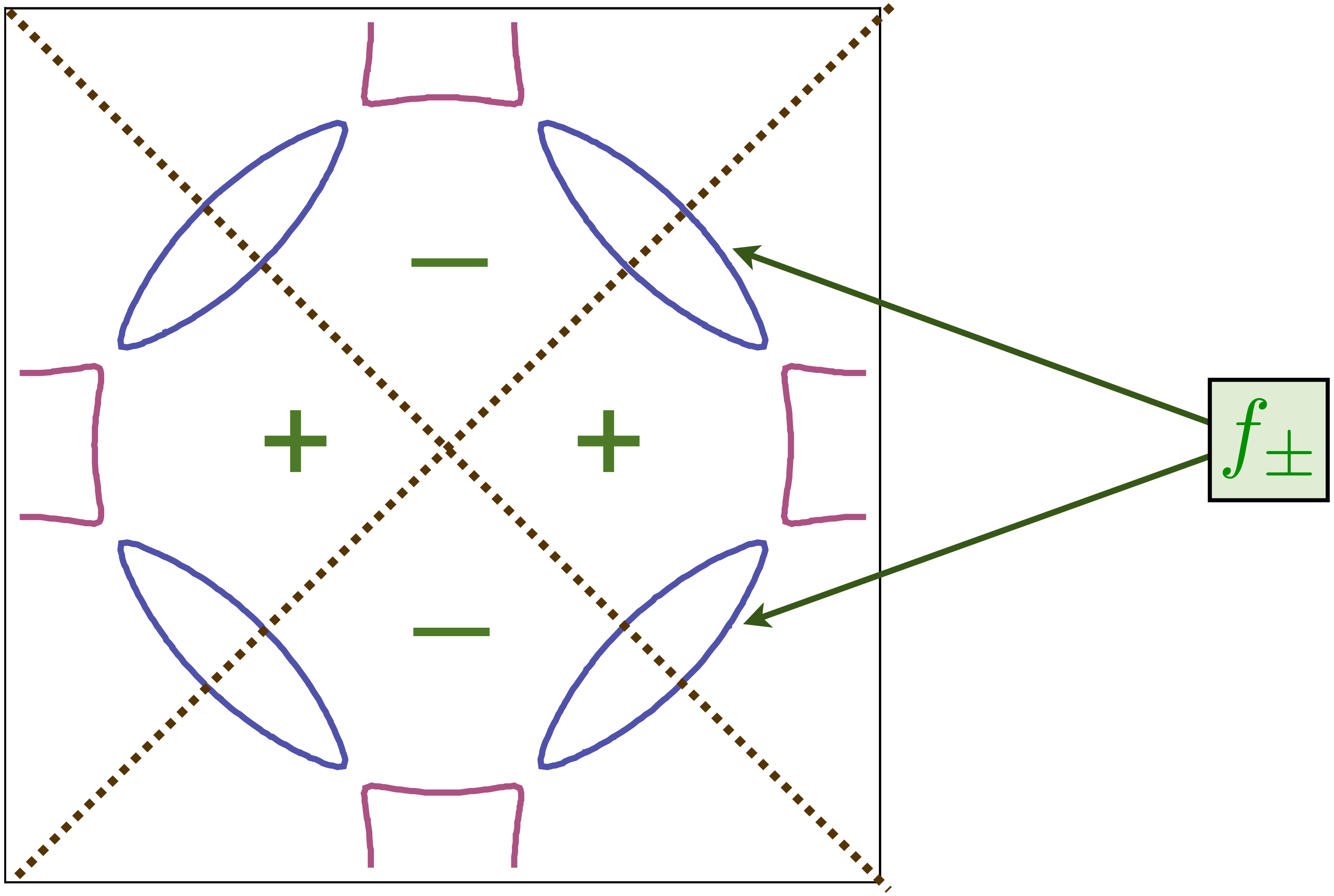
Weak pairing of the $f_\pm$ hole pockets

$$\begin{aligned}\mathcal{L}_{\rm Josephson} &= iJ_{fg}\left[g_+ g_-\right]\Big[f_{+1}\overleftrightarrow{\partial}_x f_{-1} - f_{+1}\overleftrightarrow{\partial}_y f_{-1} \\ &\qquad + f_{+2}\overleftrightarrow{\partial}_x f_{-2} + f_{+2}\overleftrightarrow{\partial}_y f_{-2}\Big] + \text{H.c.}\end{aligned}$$

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

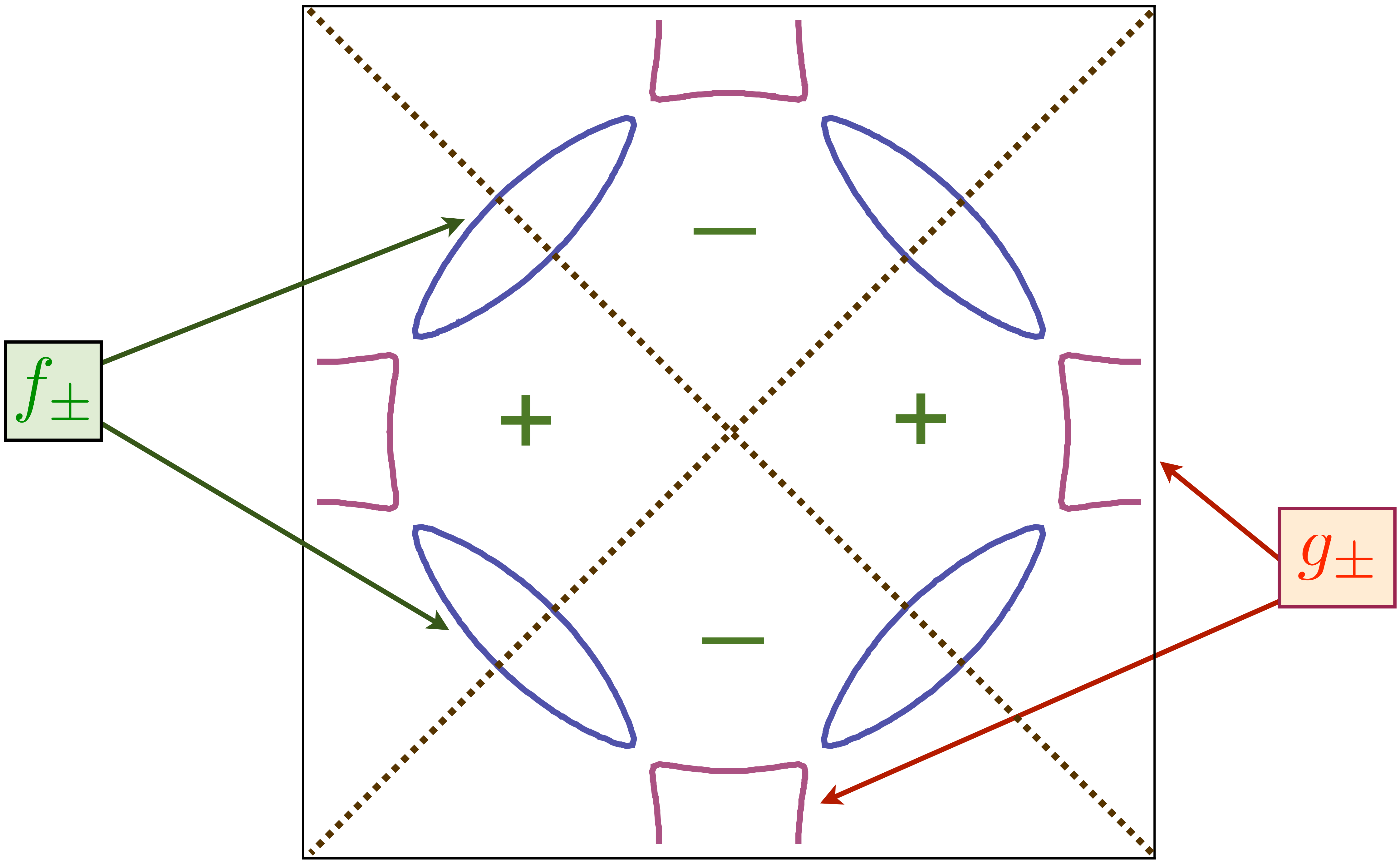
Proximity Josephson coupling to $g_\pm$ fermions leads to p-wave pairing of the $f_{\pm v}$ fermions. Gauge forces are strongly pair-breaking, and so the pairing is very weak.

$$\begin{aligned}\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J_{fg}\Delta; \\ \langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J_{fg}\Delta; \\ \langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,\end{aligned}$$



Weak pairing of the $f_\pm$ hole pockets

$$
\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle &\sim (k_x - k_y) J_{fg} \Delta; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle &\sim (k_x + k_y) J_{fg} \Delta; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle &= 0,
\end{aligned}
$$



d-wave pairing of the electrons is associated with

- Strong s-wave pairing of $g_\pm$
- Weak p-wave pairing of $f_{\pm v}$.

Conclusions

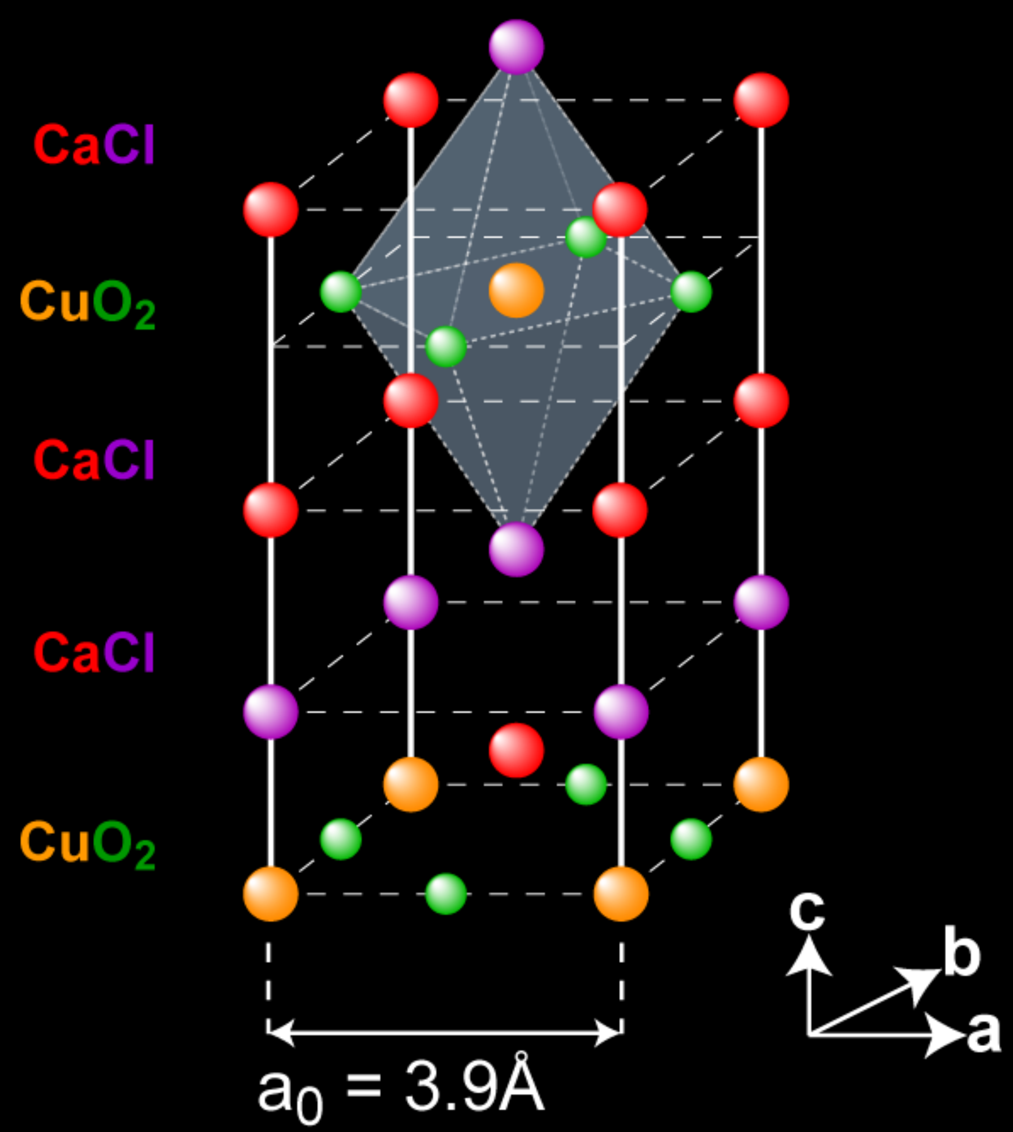
- ★ Non-Landau-Ginzburg theory for loss of antiferromagnetic order in a metal
- ★ New metallic state has "ghost" electron and hole pockets
- ★ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes

Conclusions

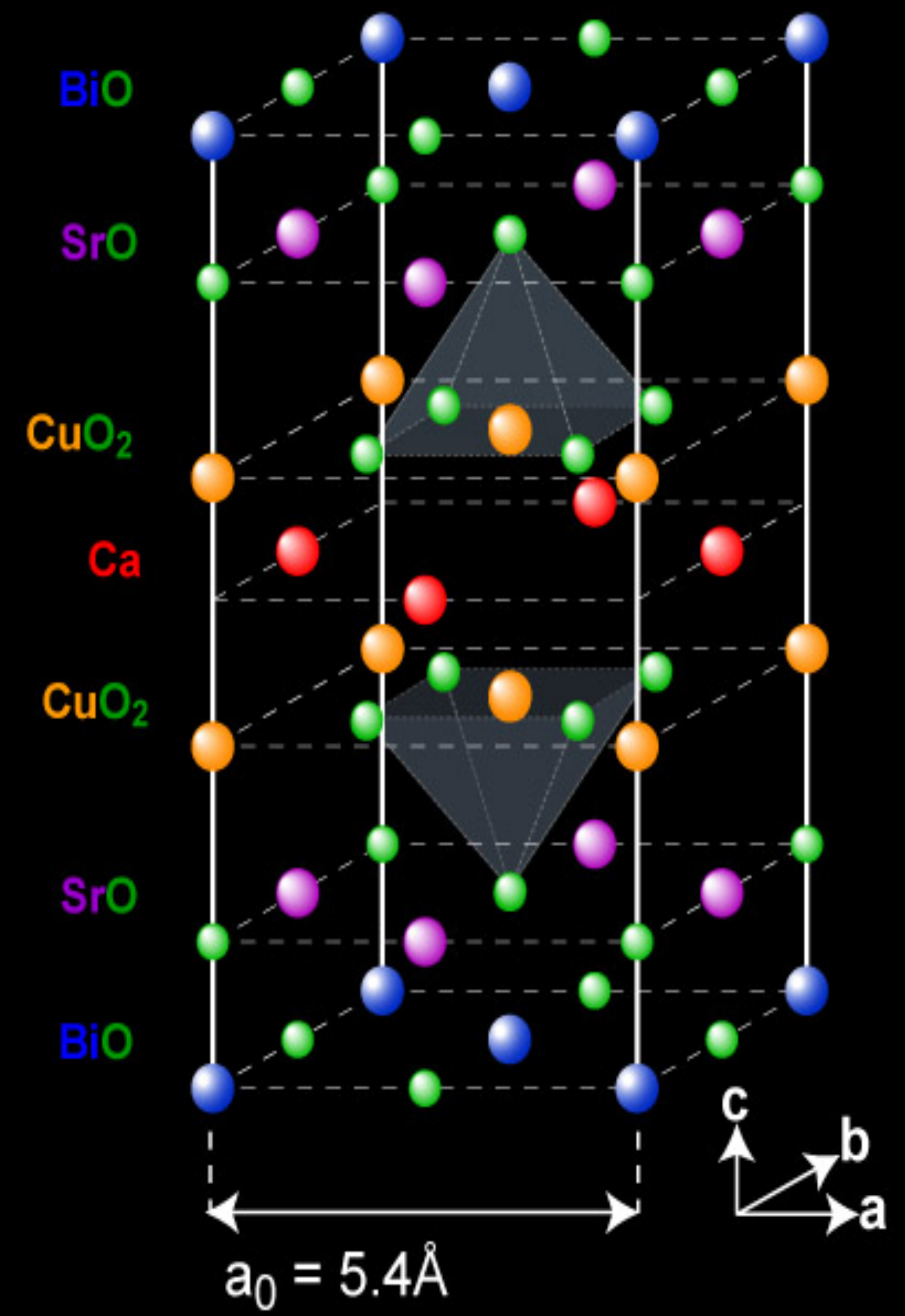
- ⋆ Non-Landau-Ginzburg theory for loss of antiferromagnetic order in a metal
- ⋆ New metallic state has "ghost" electron and hole pockets
- ⋆ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes
- ⋆ Paired electron pockets are expected to lead to valence-bond-solid modulations at low temperature

STM studies of the underdoped superconductor

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

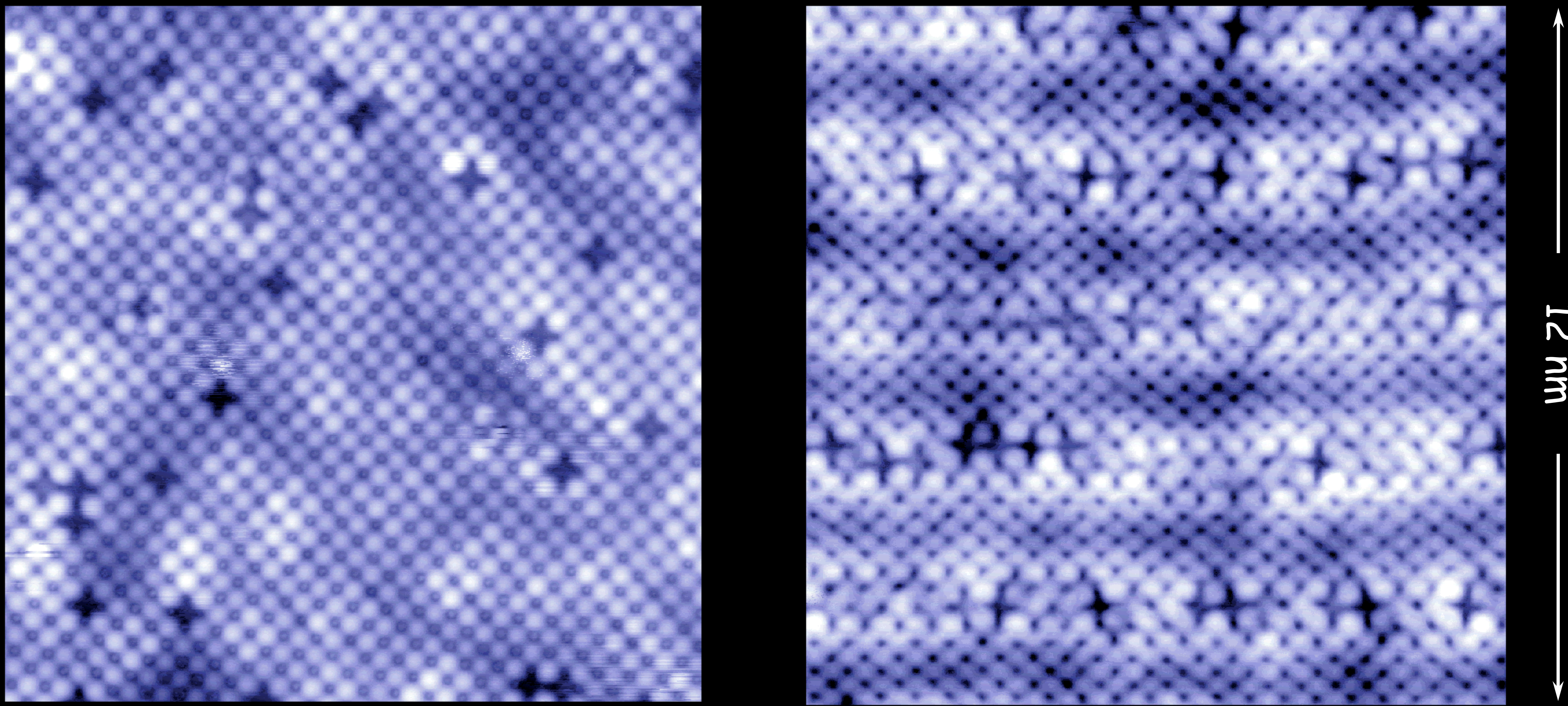


$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Topograph

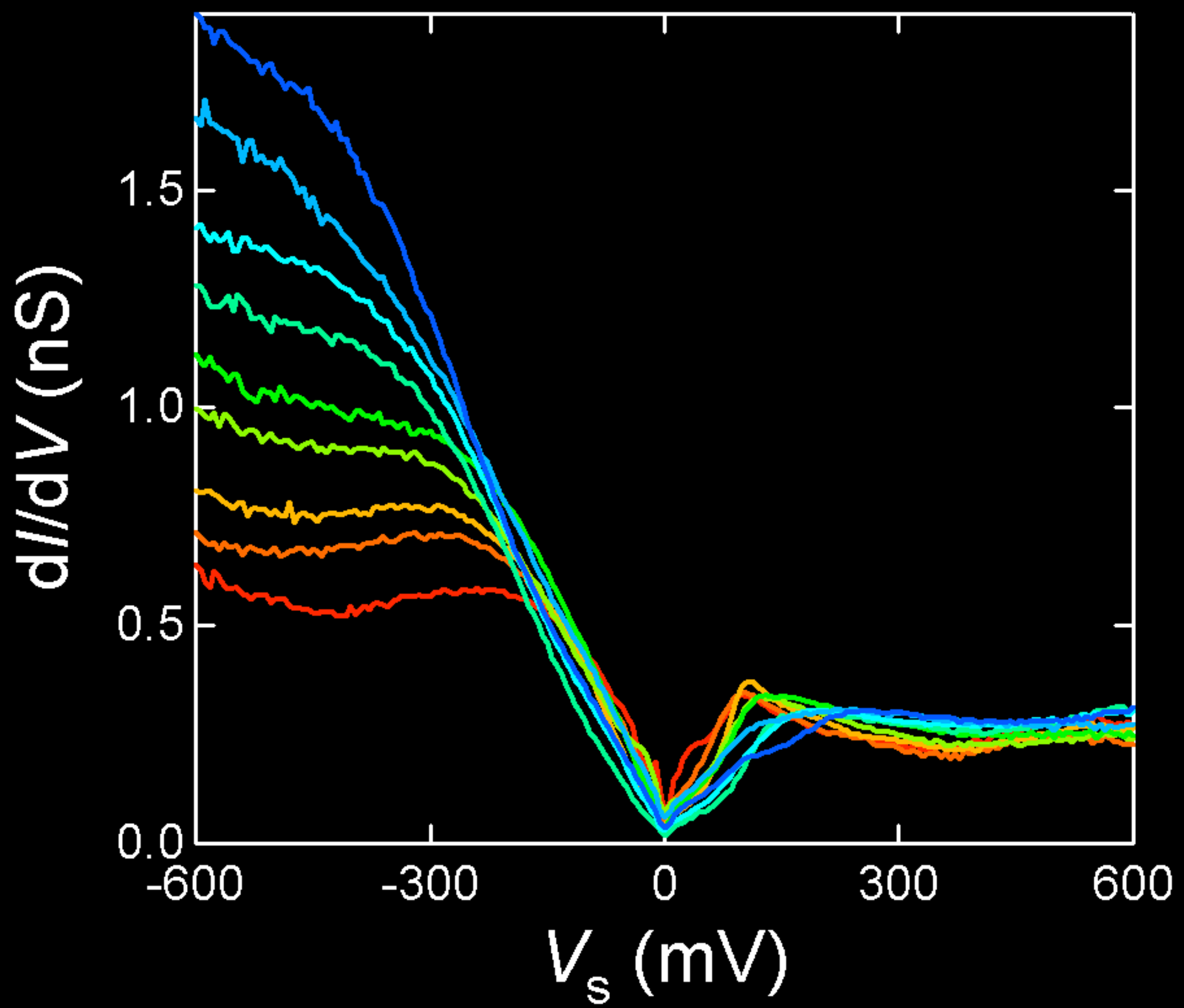
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

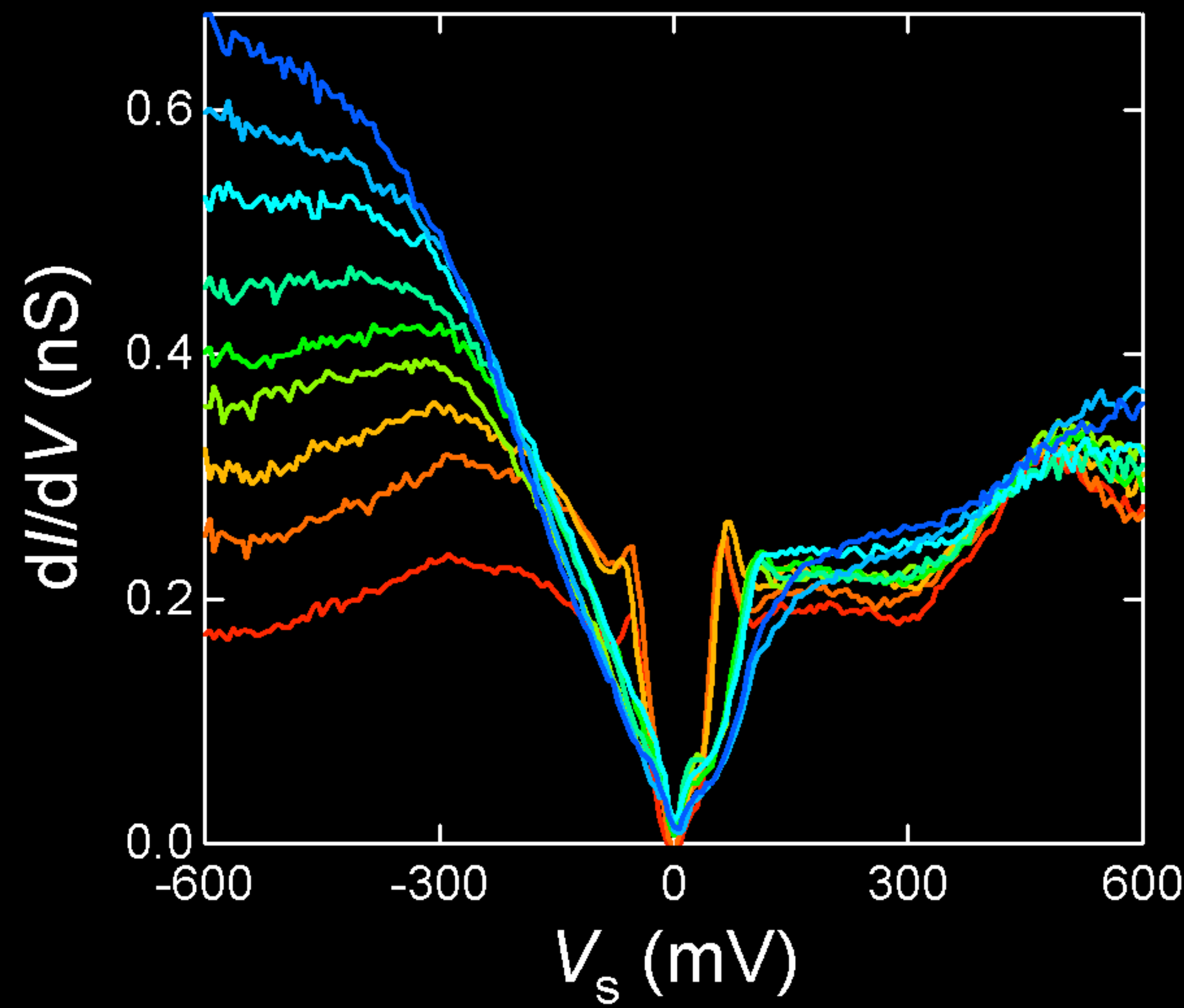


Y. Kohsaka et al. Science 315, 1380 (2007)

dI/dV Spectra

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$





Intense Tunneling-Asymmetry (TA)
variation are highly similar

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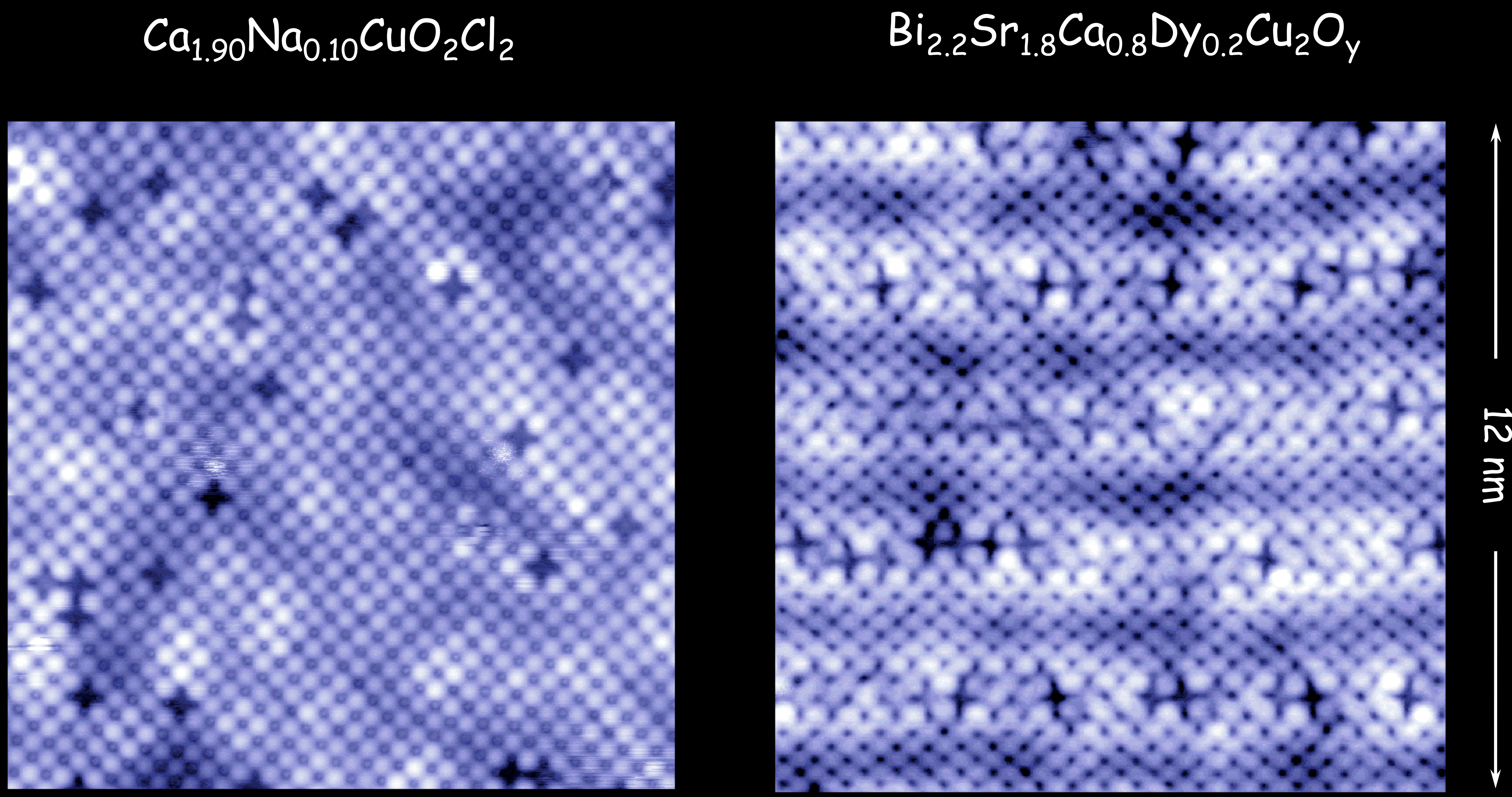
Topograph



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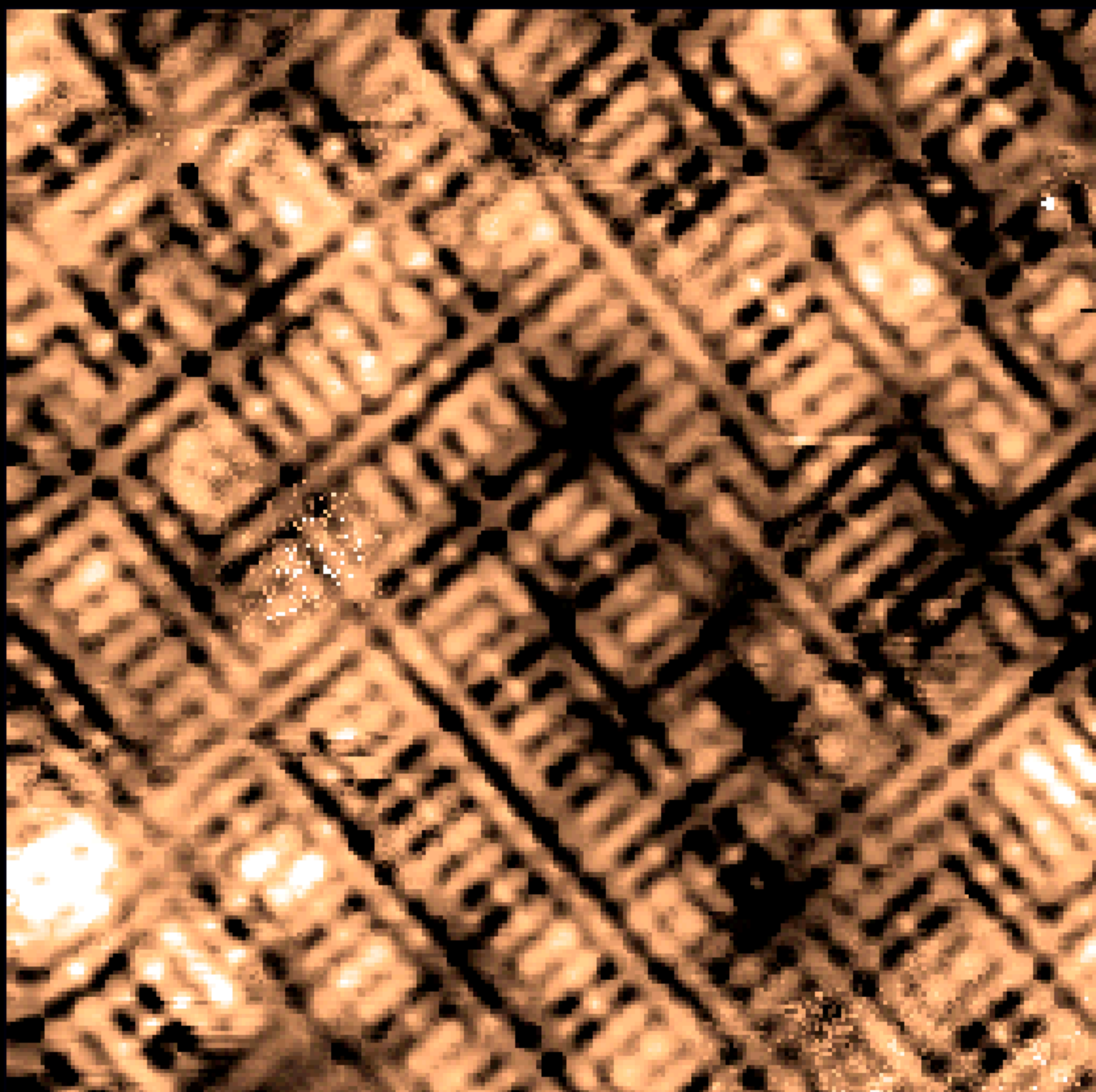
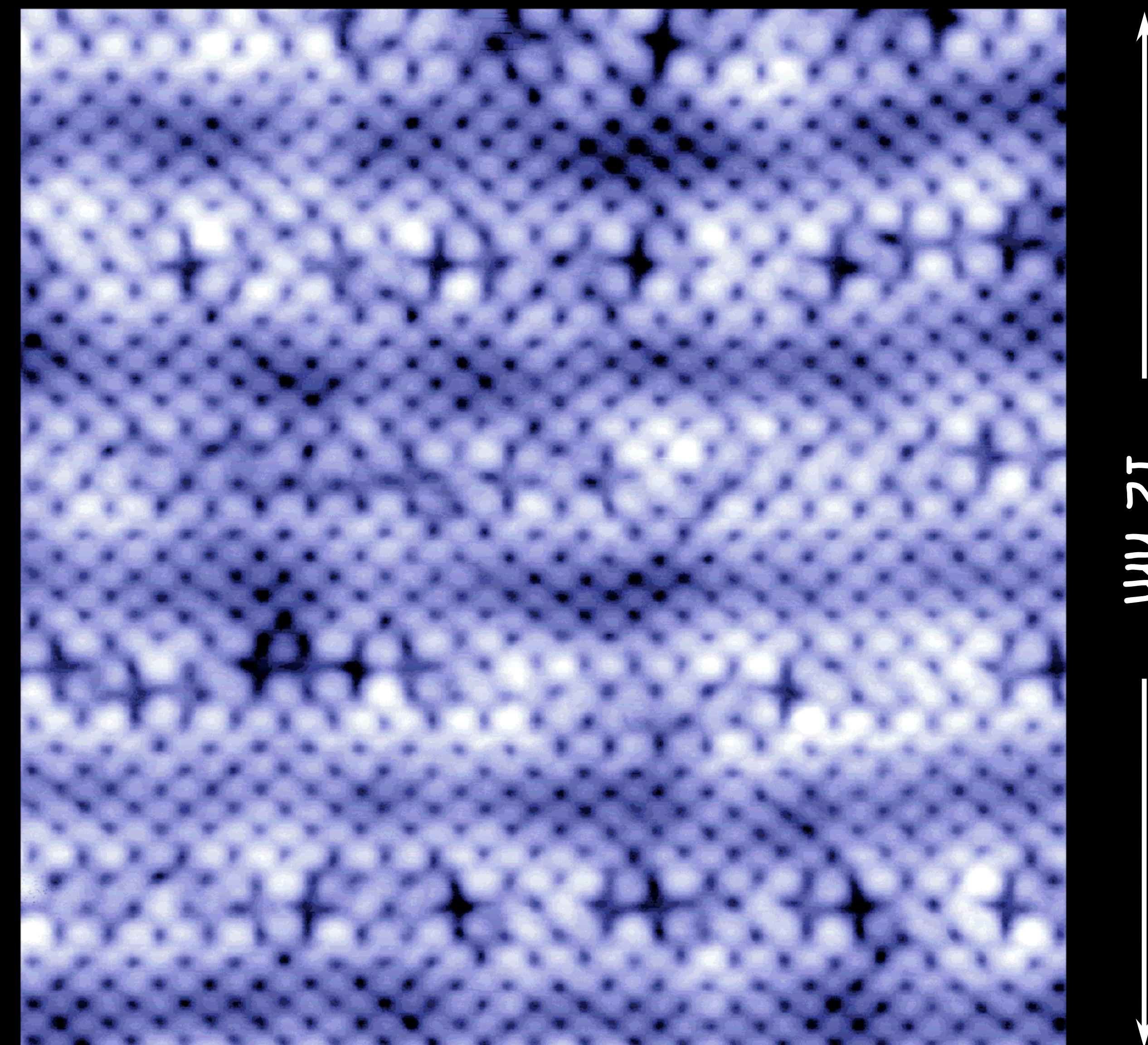
Tunneling Asymmetry (TA)-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



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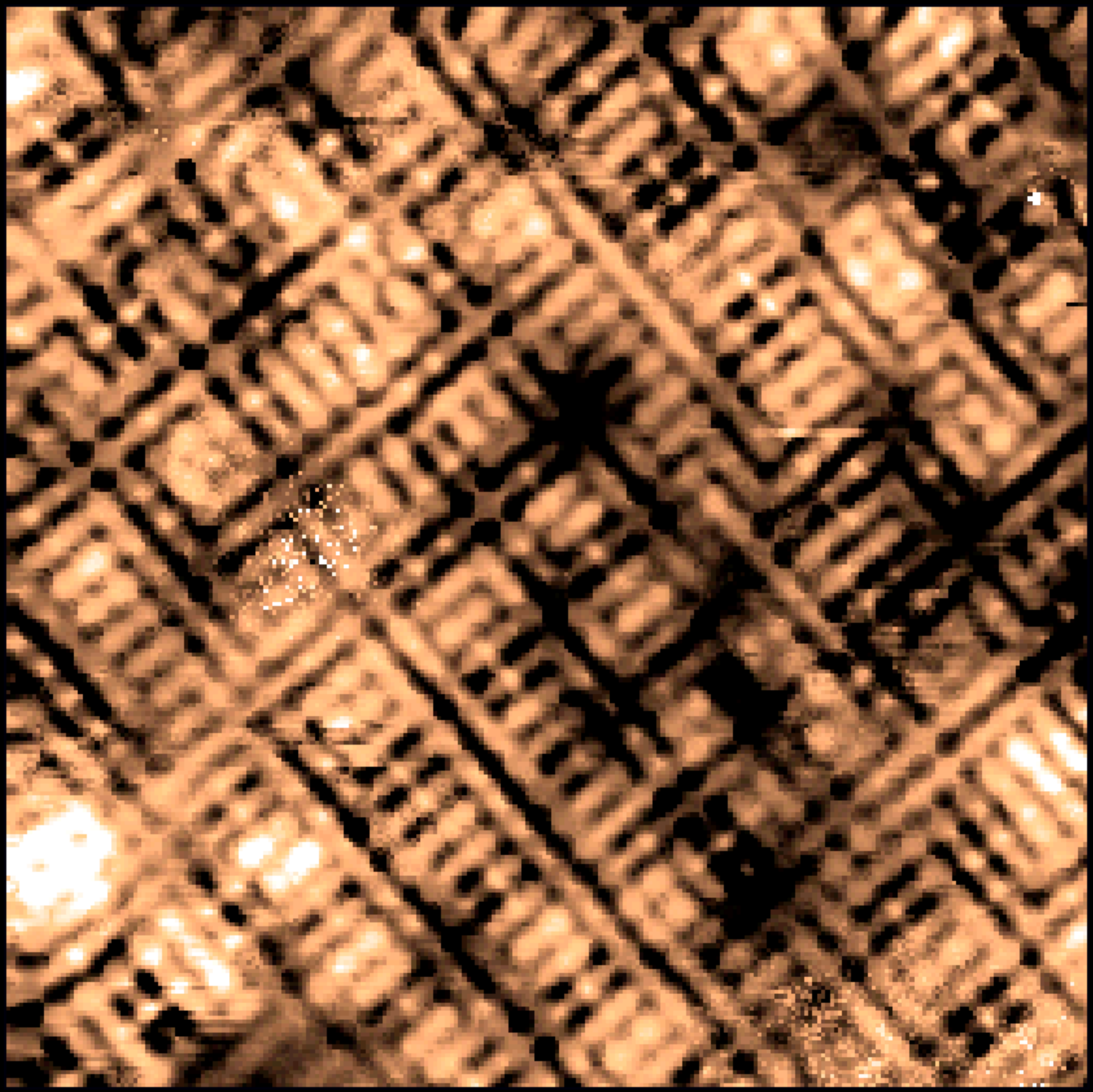
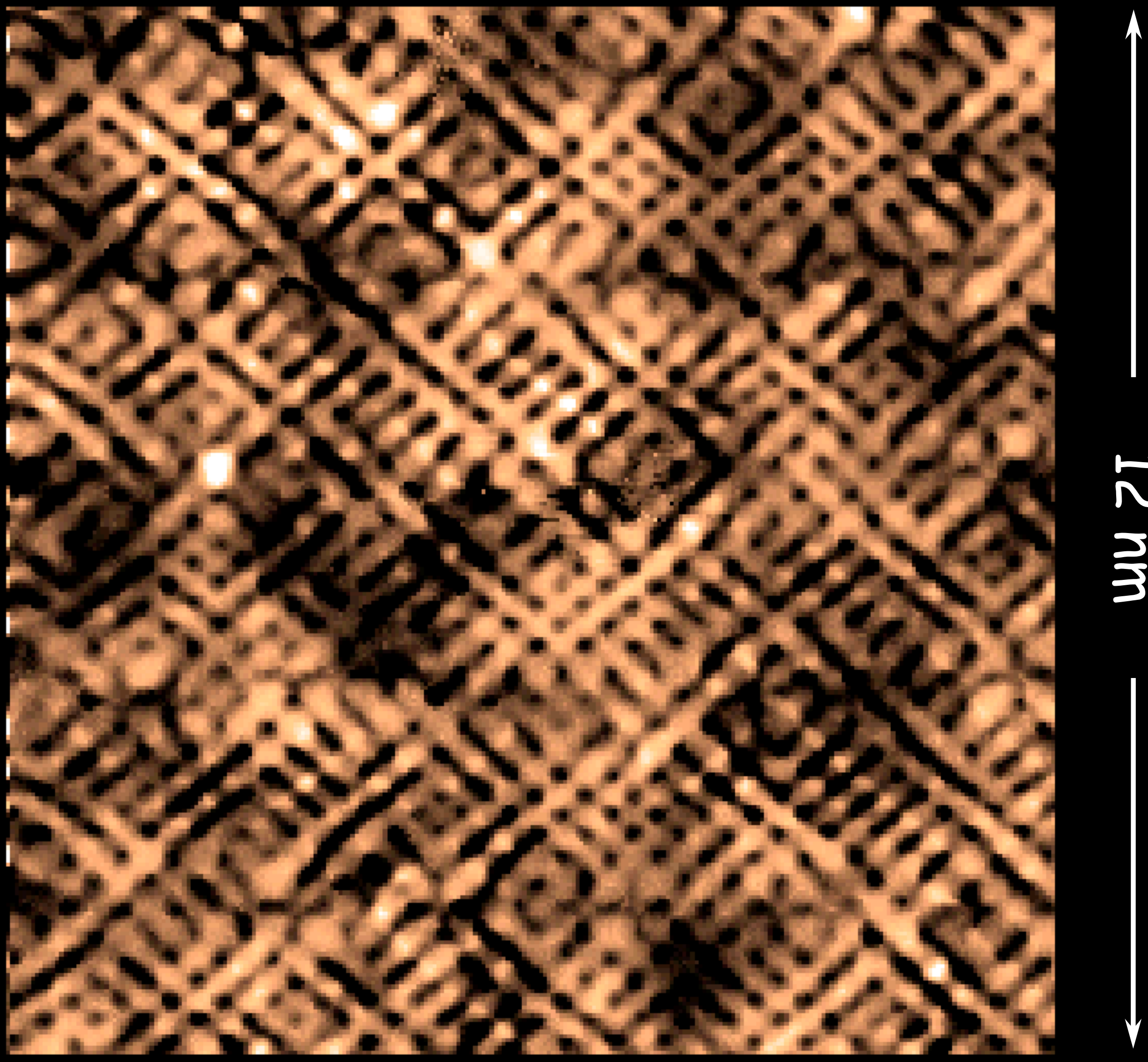
Tunneling Asymmetry (TA)-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



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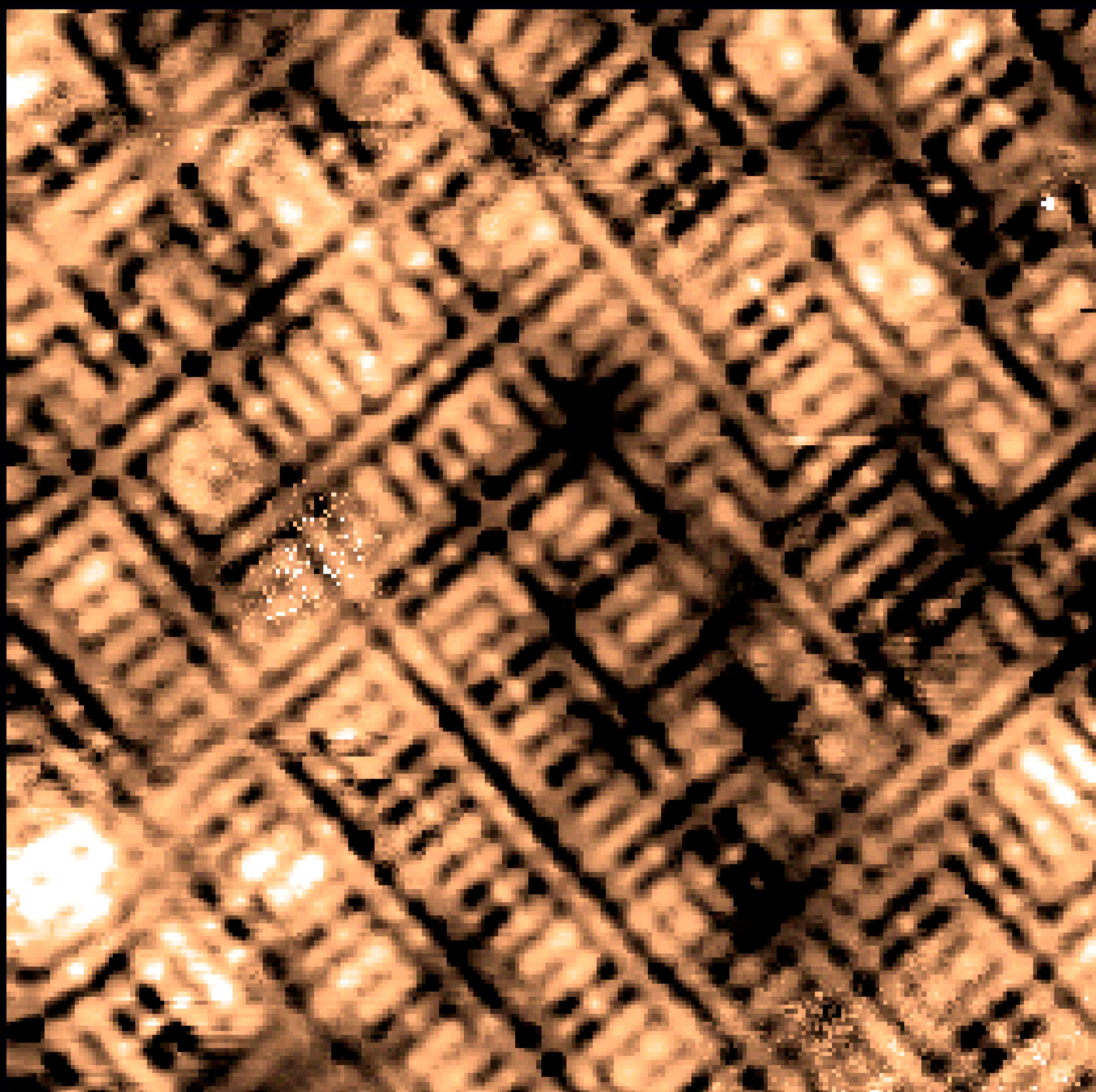

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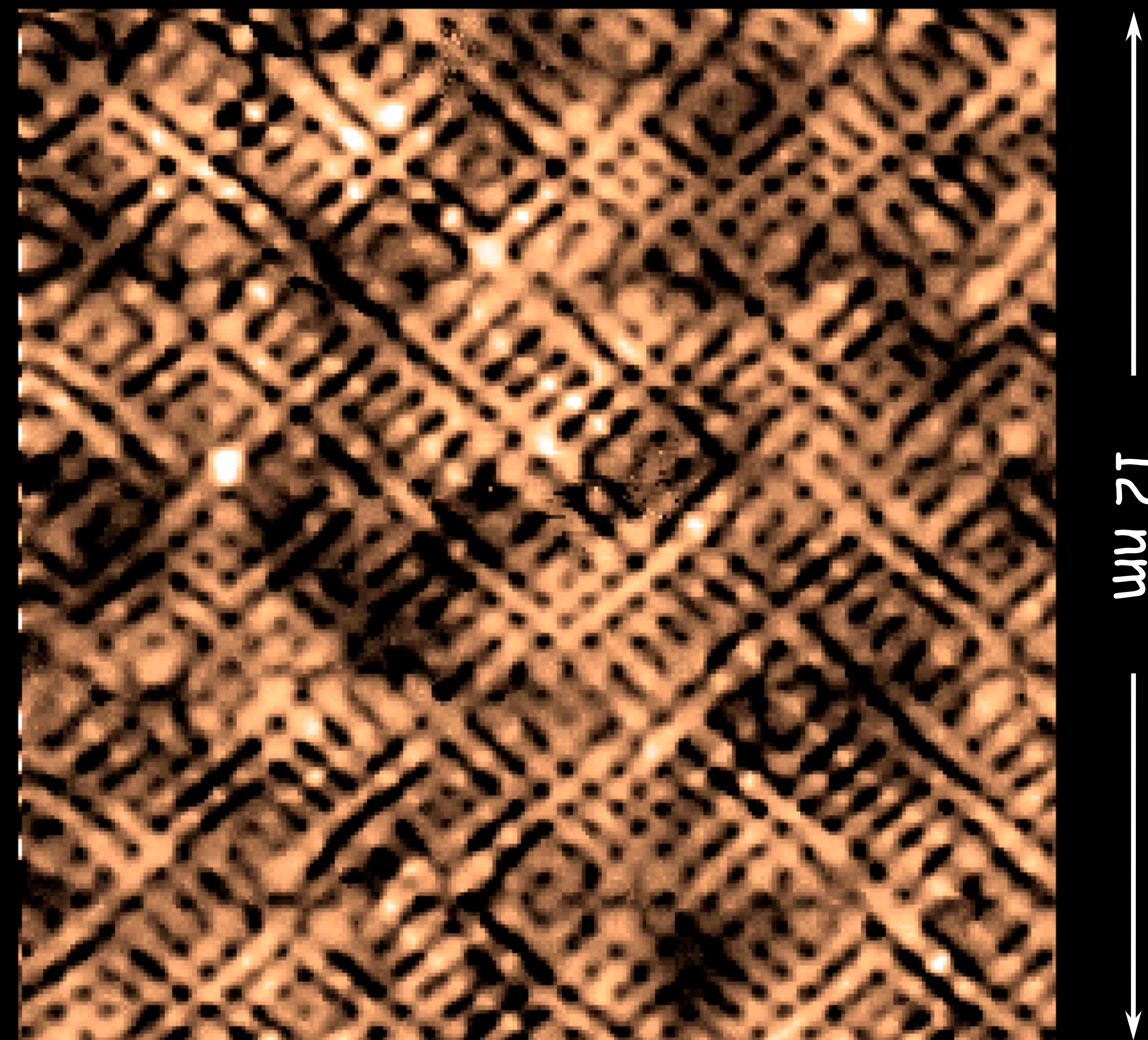

Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

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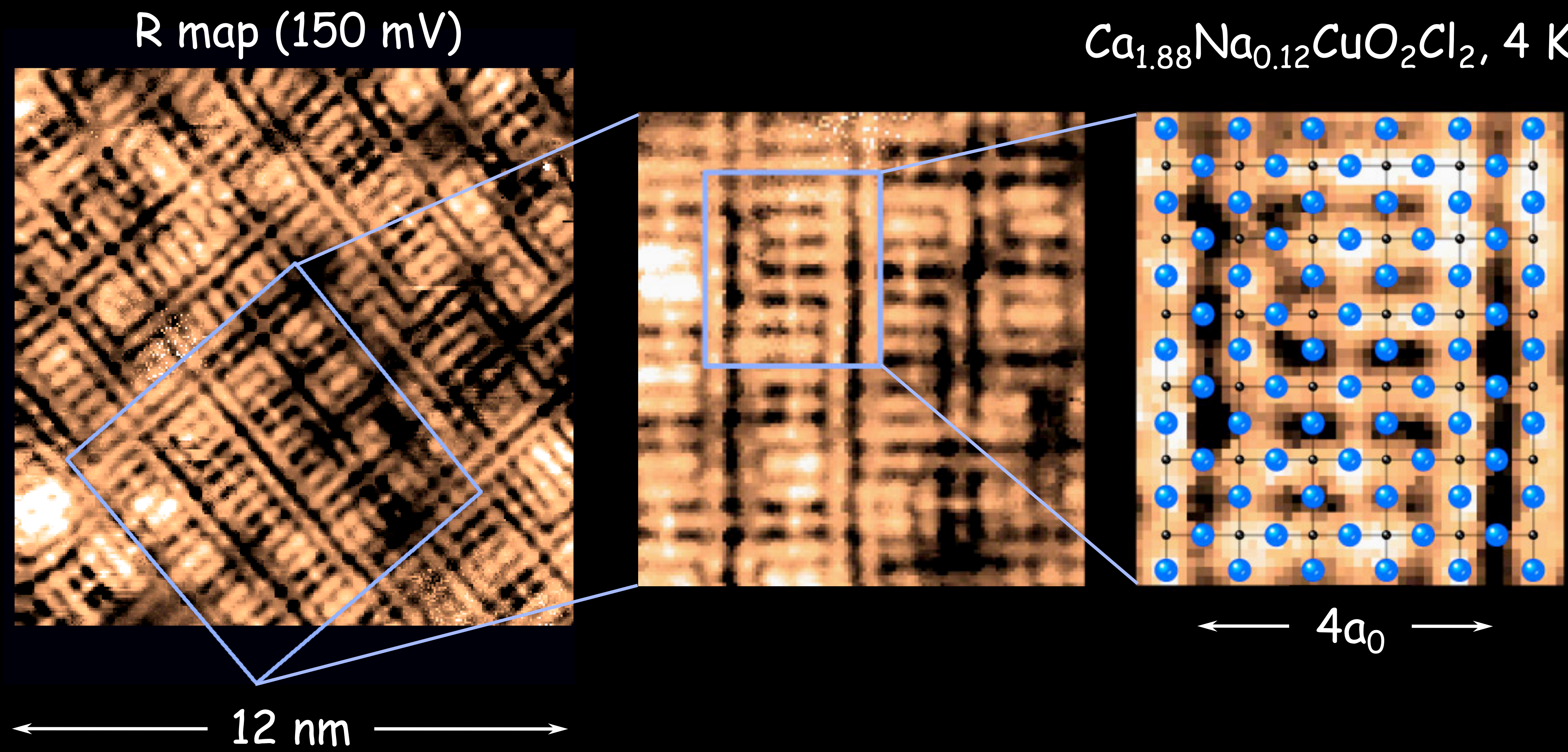
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



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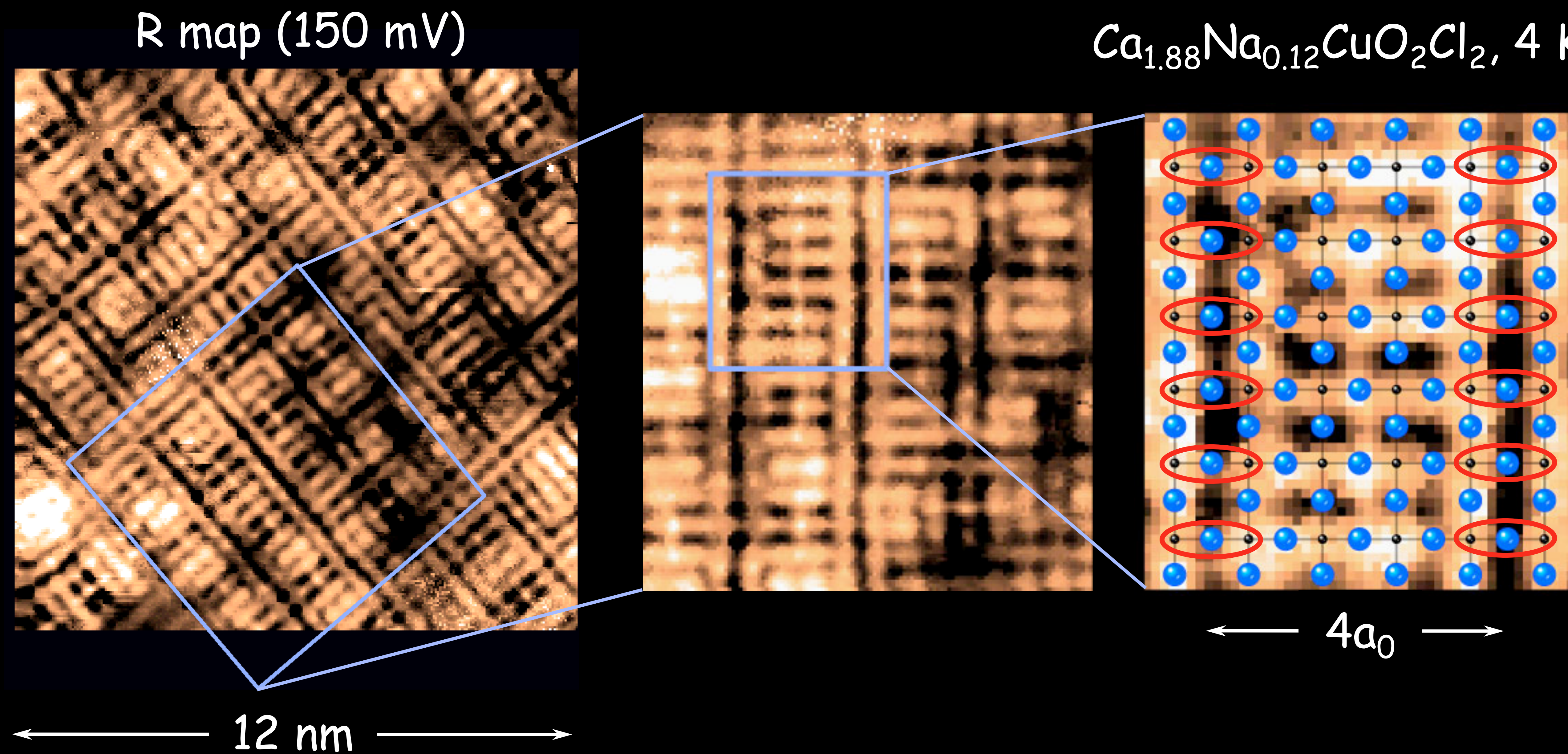
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



Evidence for a predicted valence bond supersolid

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).

Conclusions

- ⋆ Non-Landau-Ginzburg theory for loss of antiferromagnetic order in a metal
- ⋆ New metallic state has "ghost" electron and hole pockets
- ⋆ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes
- ⋆ Paired electron pockets are expected to lead to valence-bond-solid modulations at low temperature

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- ⋆ Paired electron pockets are expected to lead to valence-bond-solid modulations at low temperature
- ⋆ Needed: theory for transition to "large" Fermi surface at higher doping