Conformal field theories in a periodic potential: holography vs. field theory

Newton Institute, Sep 16, 2013

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Talk online at sachdev.physics.harvard.edu

Andy Lucas

Paul Chesler

arXiv:1308.0329

A. Electrical transport in CFT3s

B. Electrical transport in CFT3s in a constant chemical potential

C. CFT3s in a periodic chemical potential

D. Monopoles

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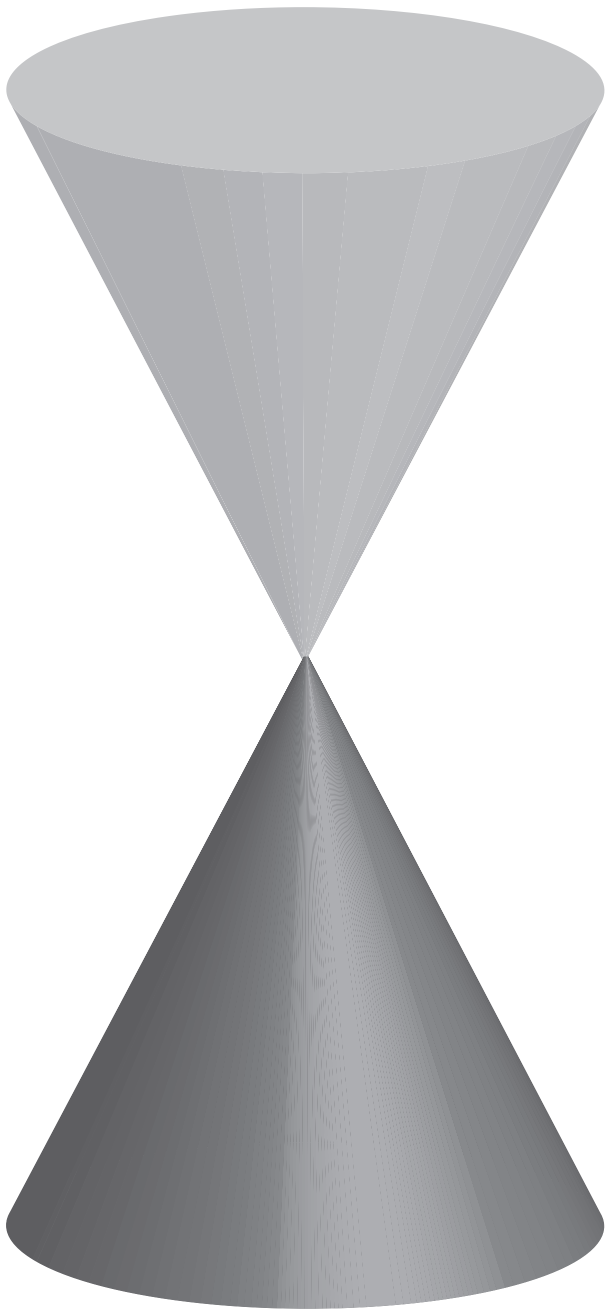
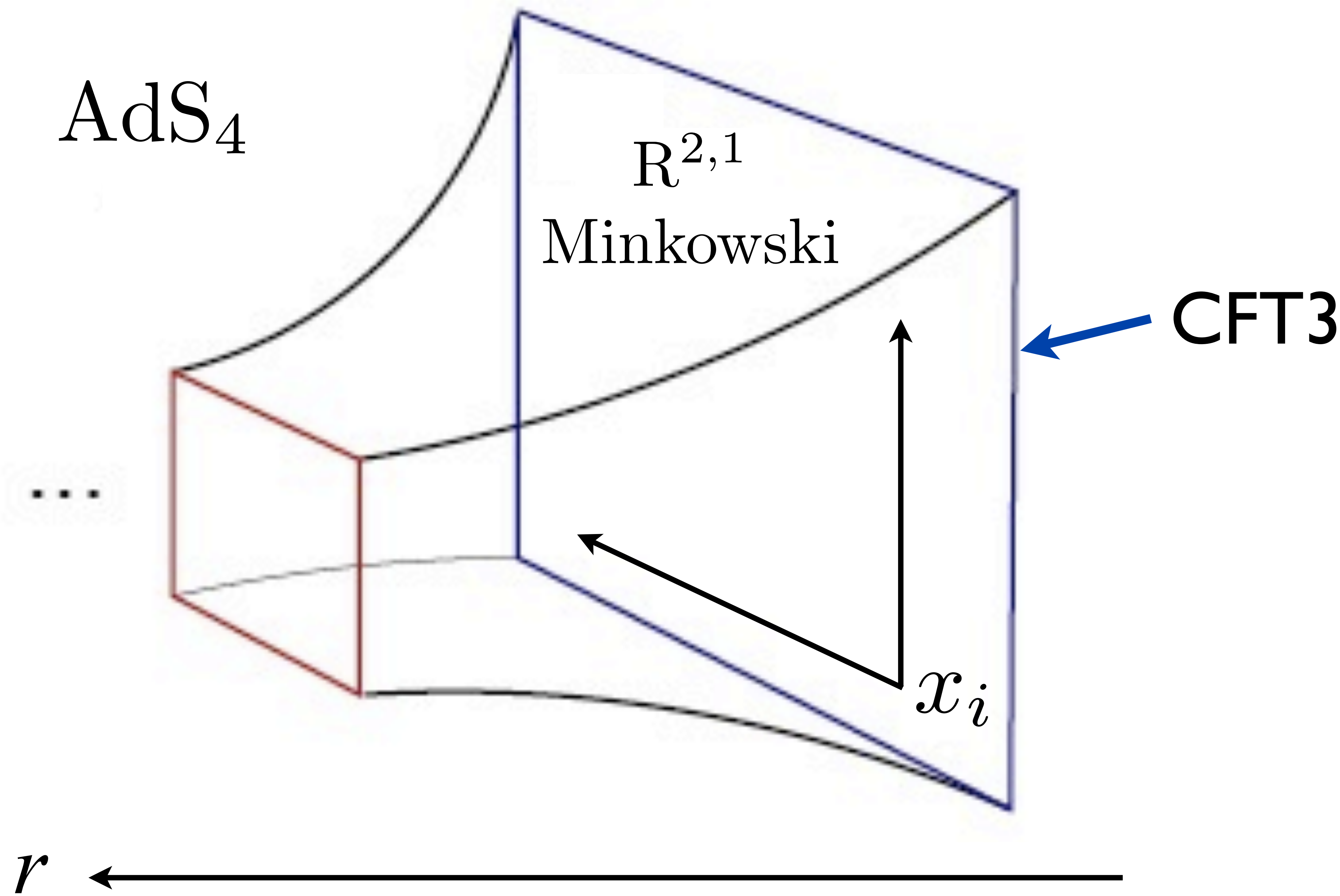
A conformal field theory (CFT3): Dirac fermions coupled to a gauge field



$$\begin{aligned} \mathcal{L} &= \overline{\psi}\,\gamma_\mu(\partial_\mu - i a_\mu)\psi \\ \psi &\rightarrow N_f \text{ flavors} \\ a_\mu &\rightarrow N_c \text{ colors} \end{aligned}$$

CFT3 is obtained in the $1/N_f$ expansion.

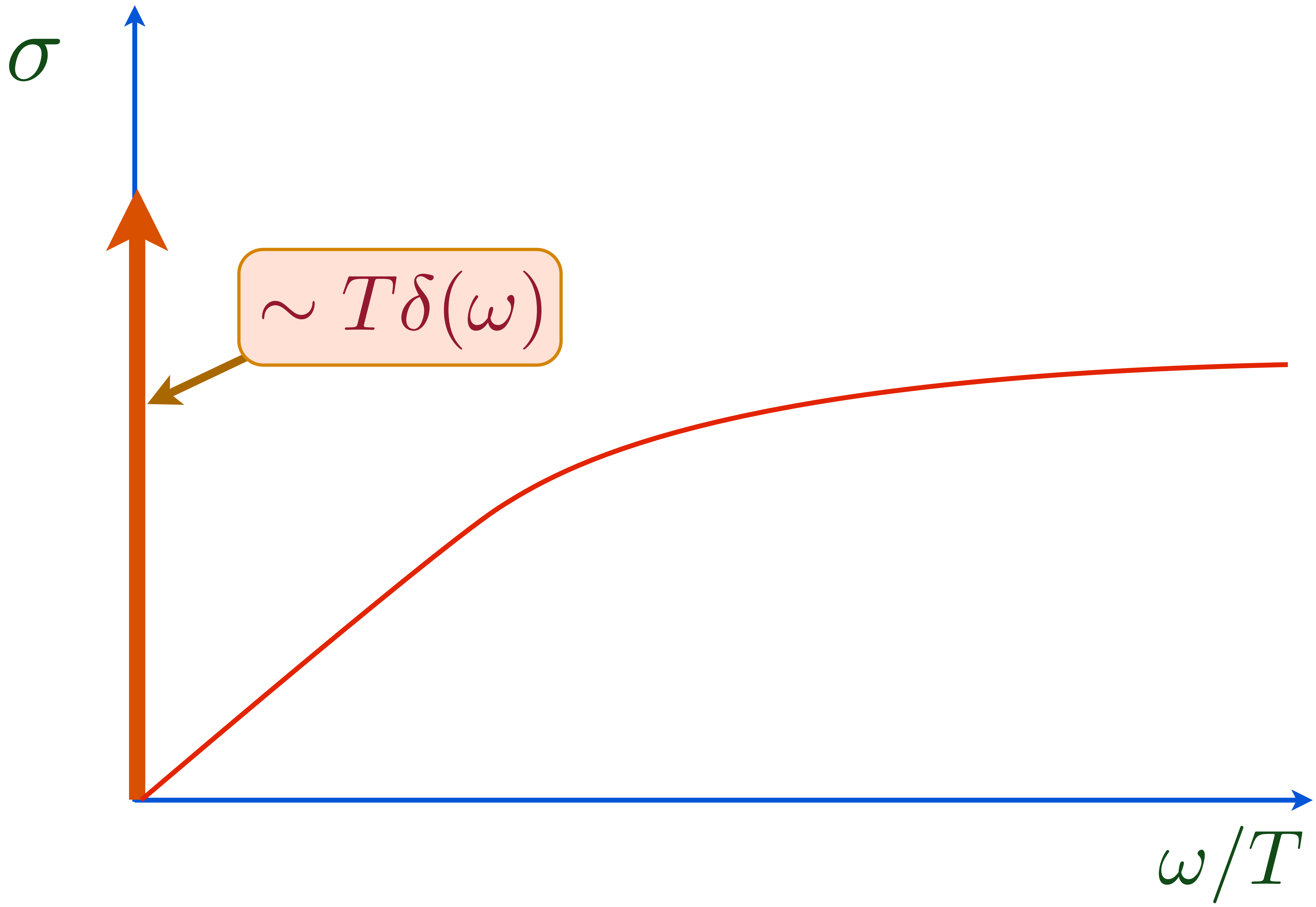
AdS/CFT correspondence



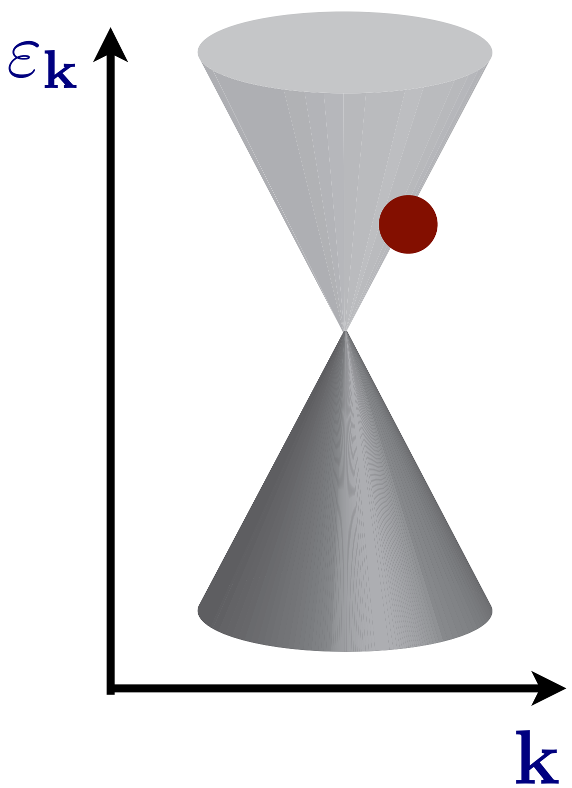
This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

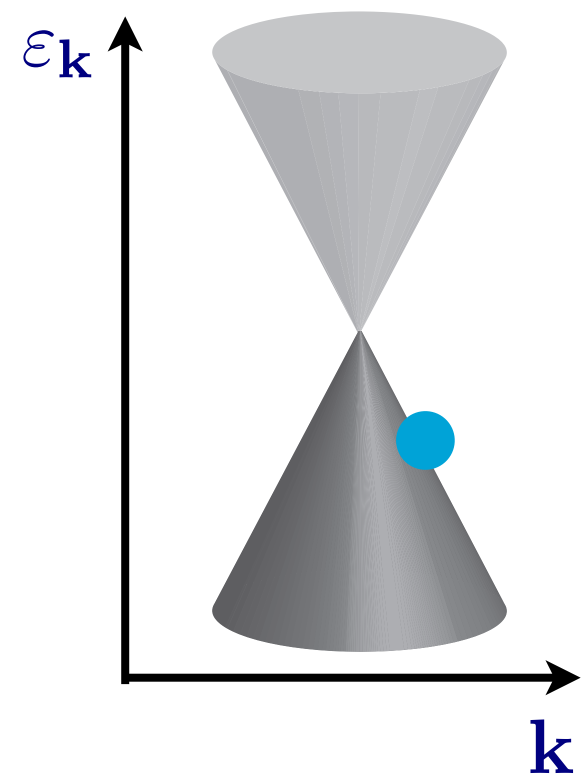
$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Electrical transport in the CFT3 at $N_f = \infty$.



Particles	Holes
Momentum →	Momentum ←
Current →	Current →

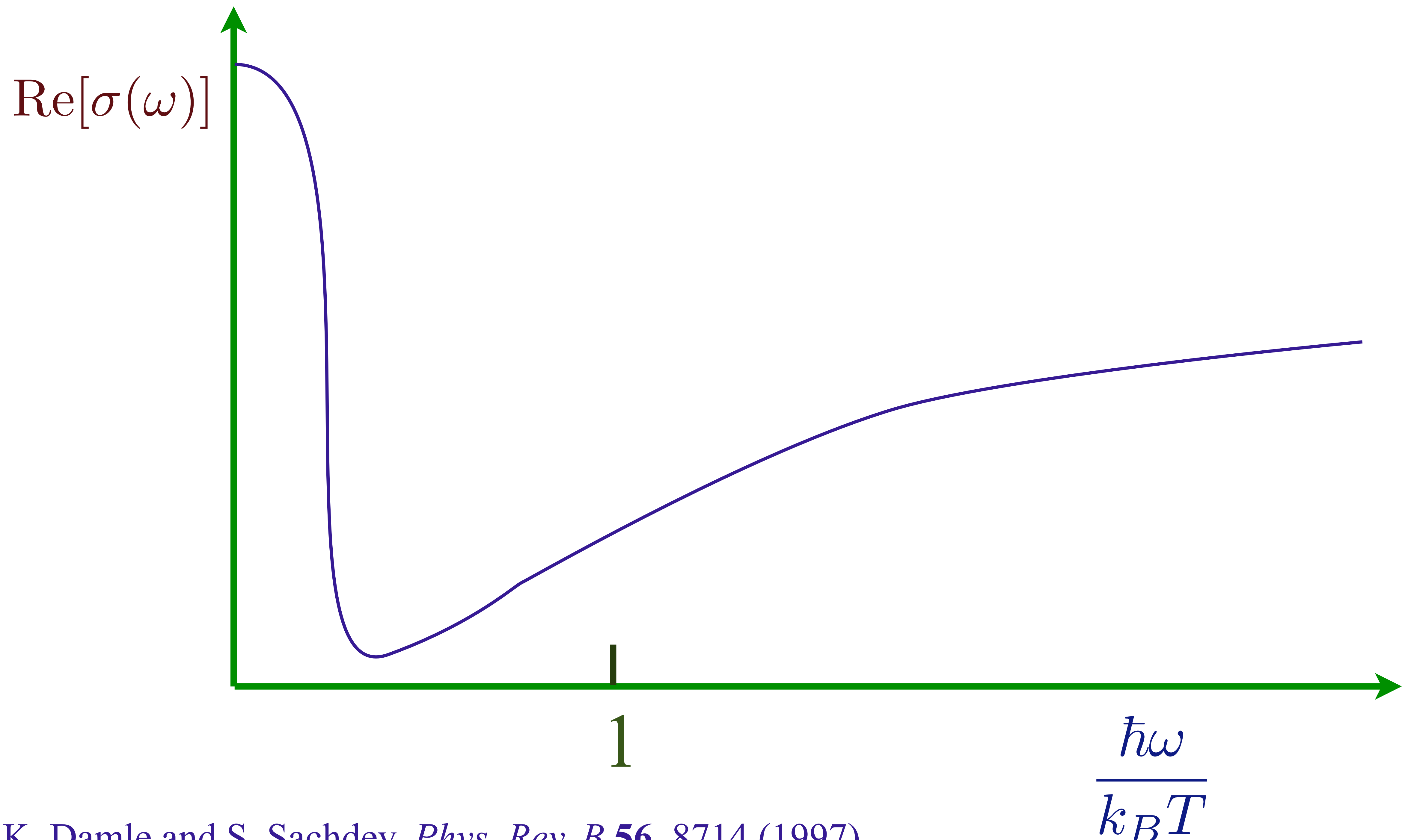




Particle hole symmetry: current carrying state has zero momentum, and collisions can relax current to zero

Electrical transport in the CFT3 at small $1/N_f$

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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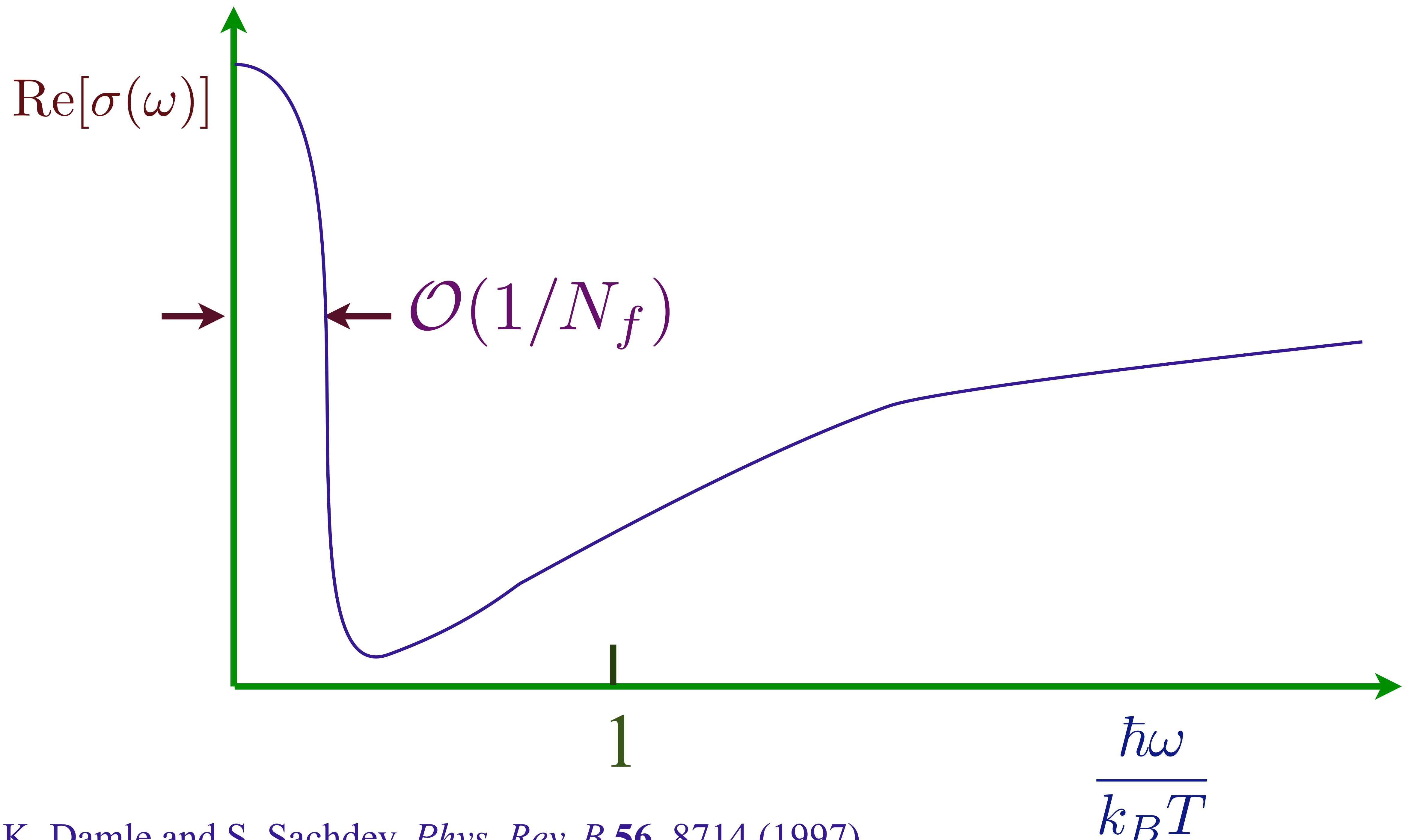

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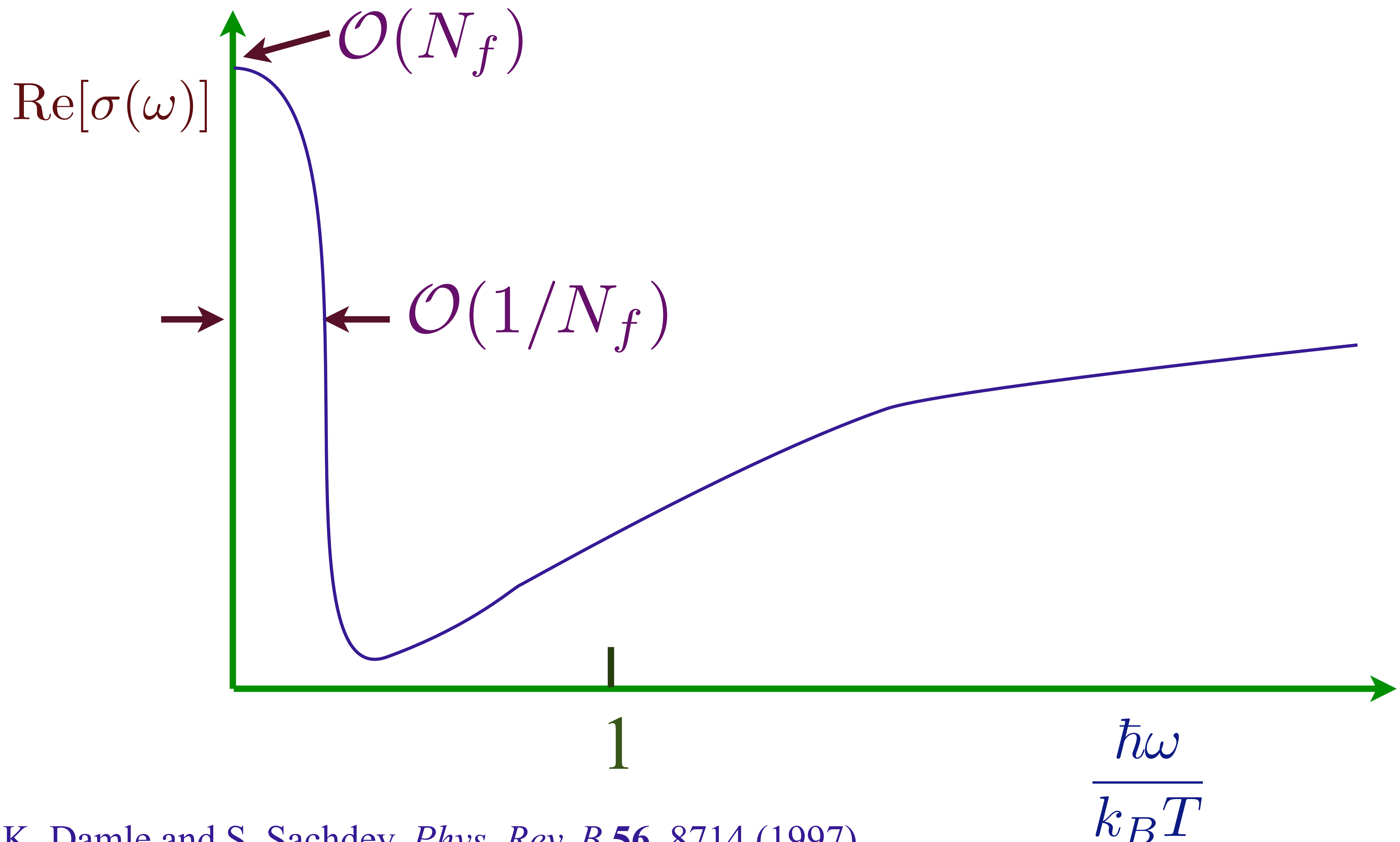

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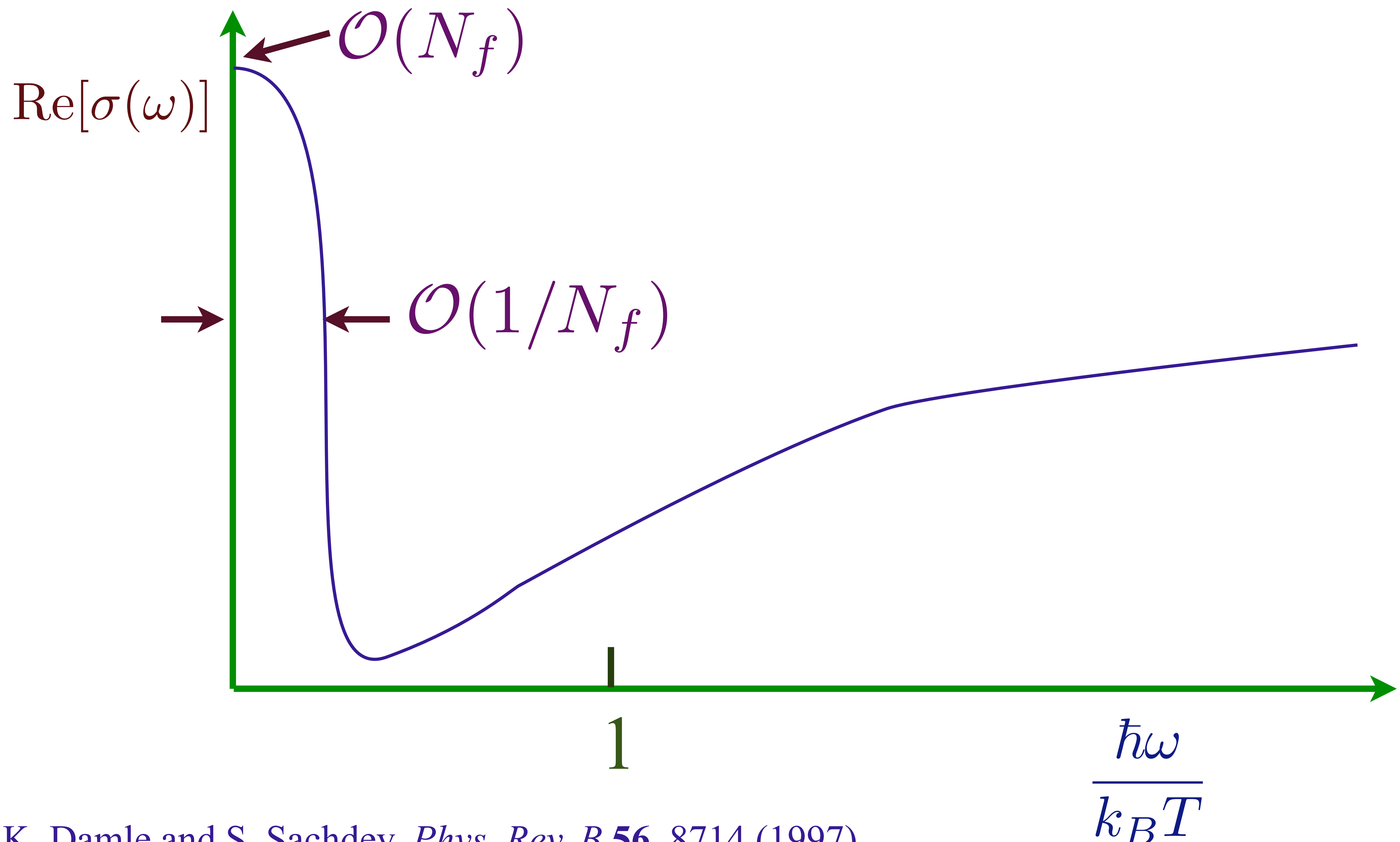

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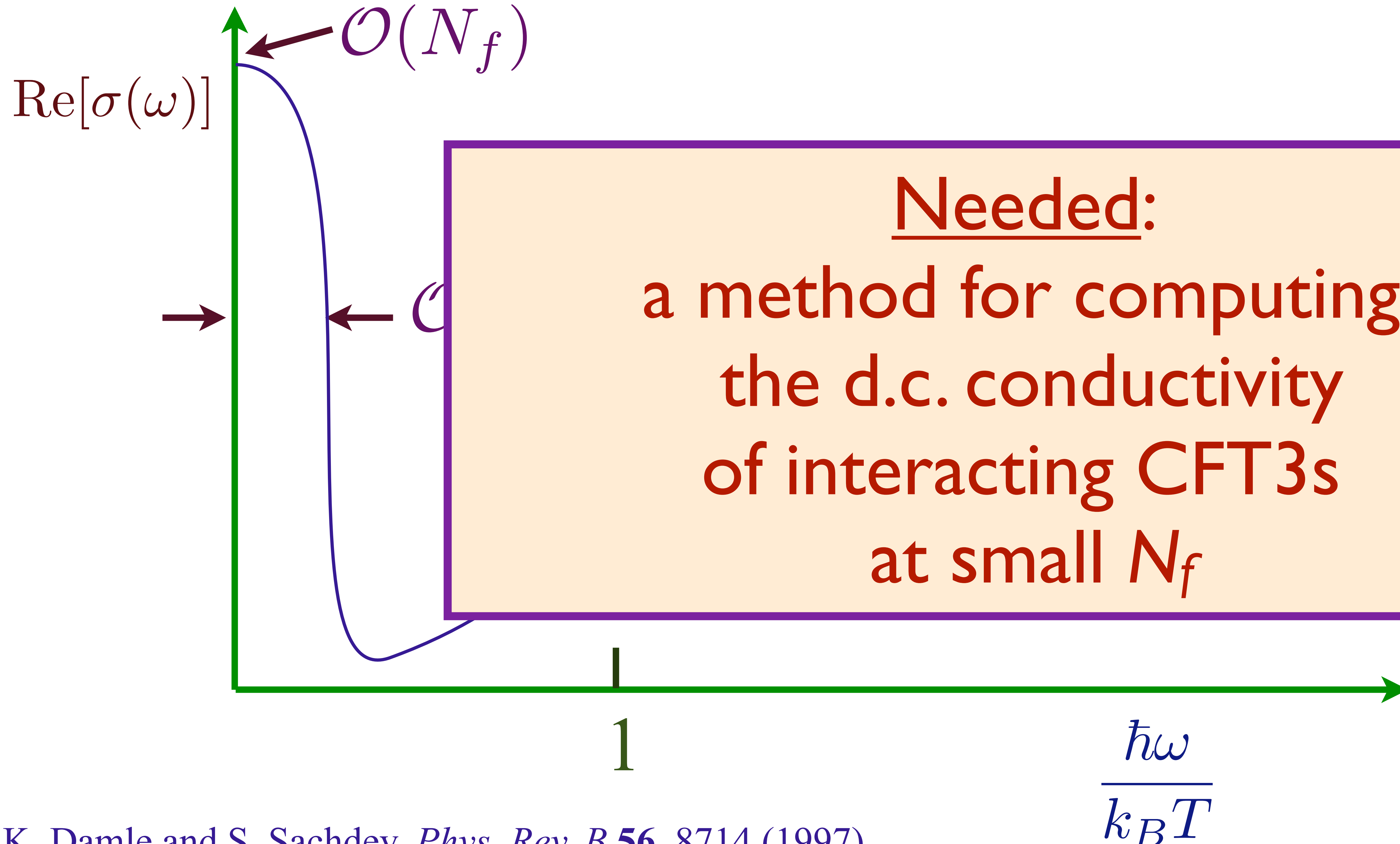
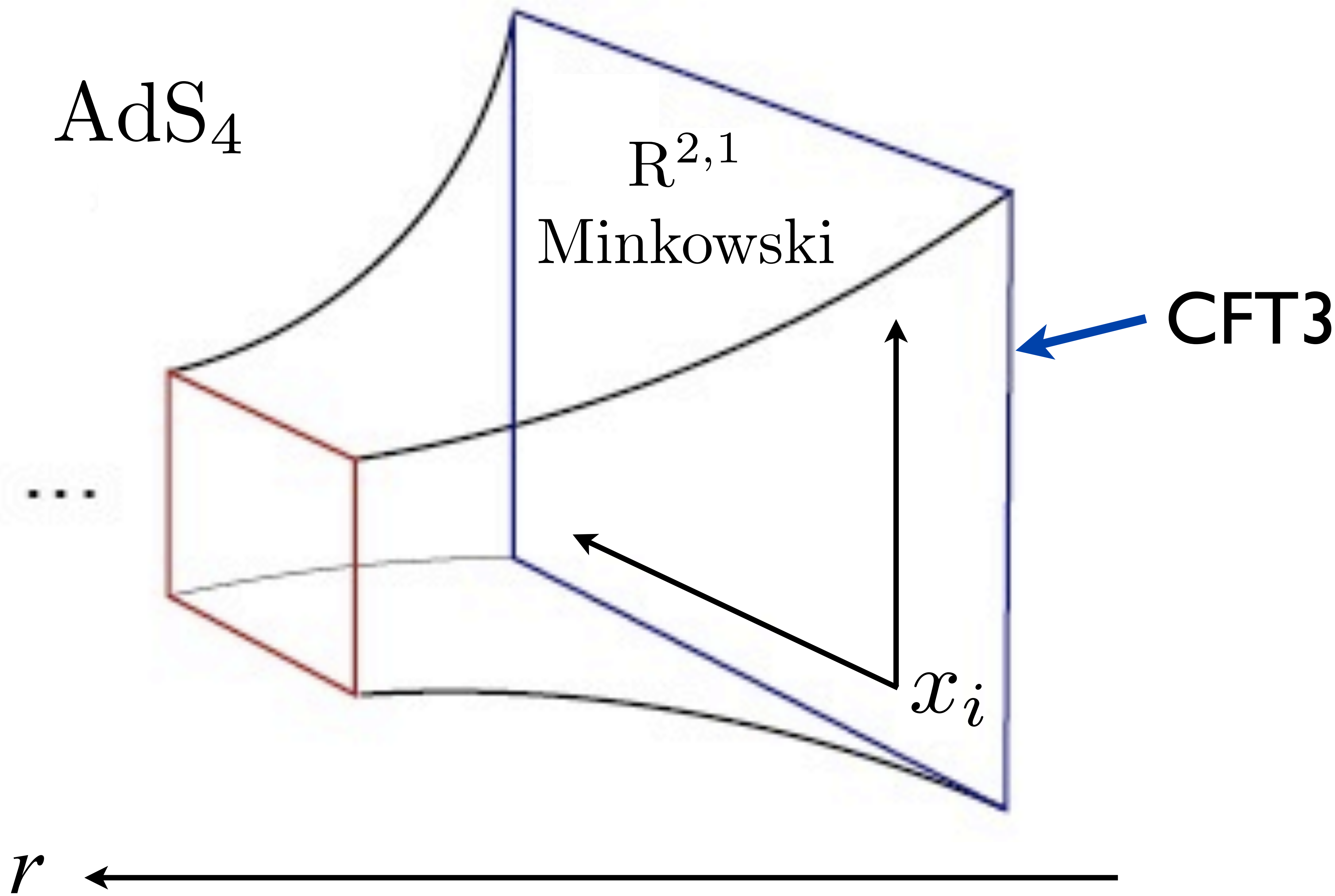


<u>Needed</u>:
a method for computing
the d.c. conductivity
of interacting CFT3s
at small N_f

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence

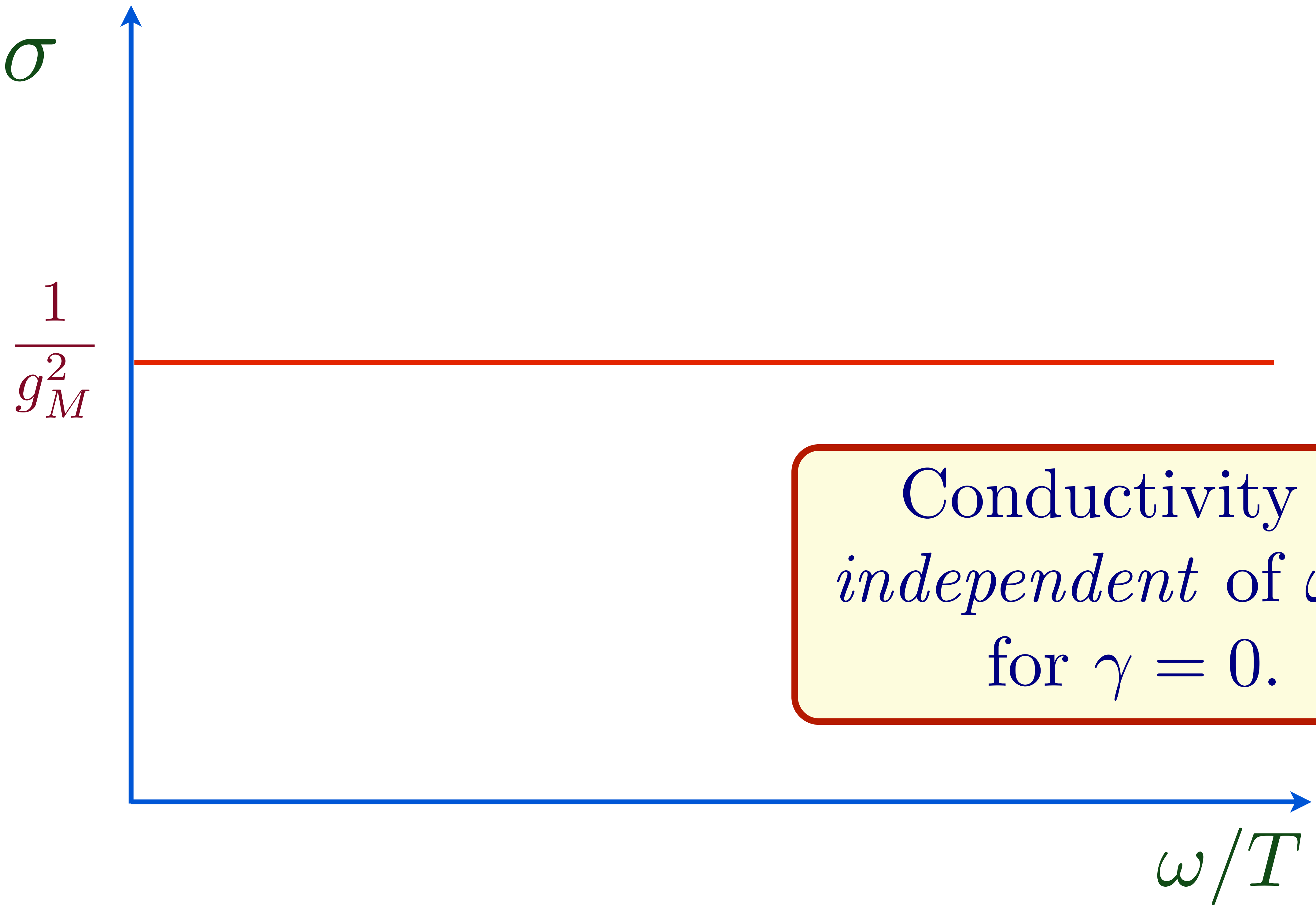
This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

where C_{abcd} is the Weyl tensor. The parameter γ can be related to 3-point correlators of J_μ and $T_{\mu\nu}$. Both boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

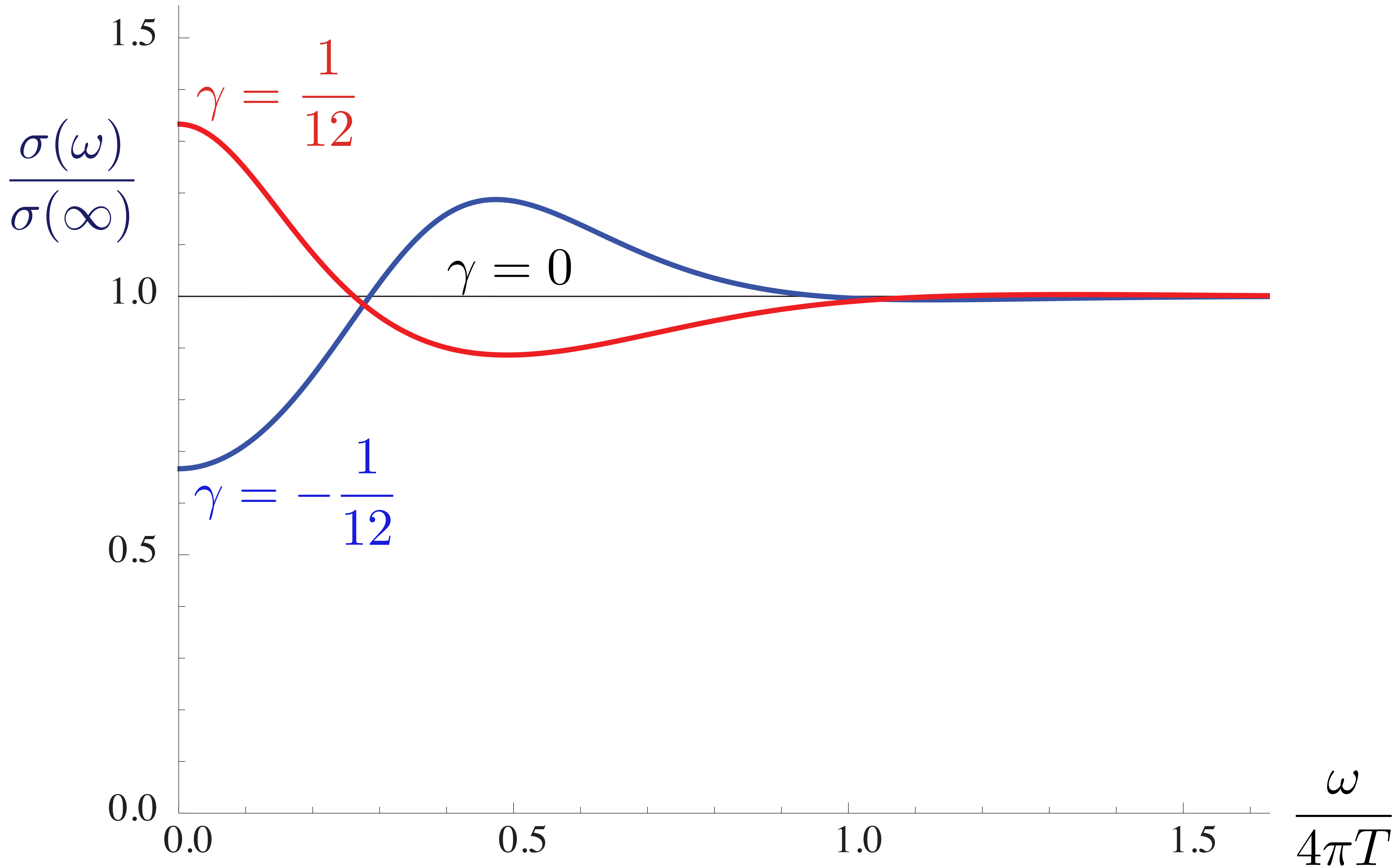
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*



Conductivity is *independent* of ω/T for $\gamma = 0$.

AdS_4 theory of "nearly perfect fluids"



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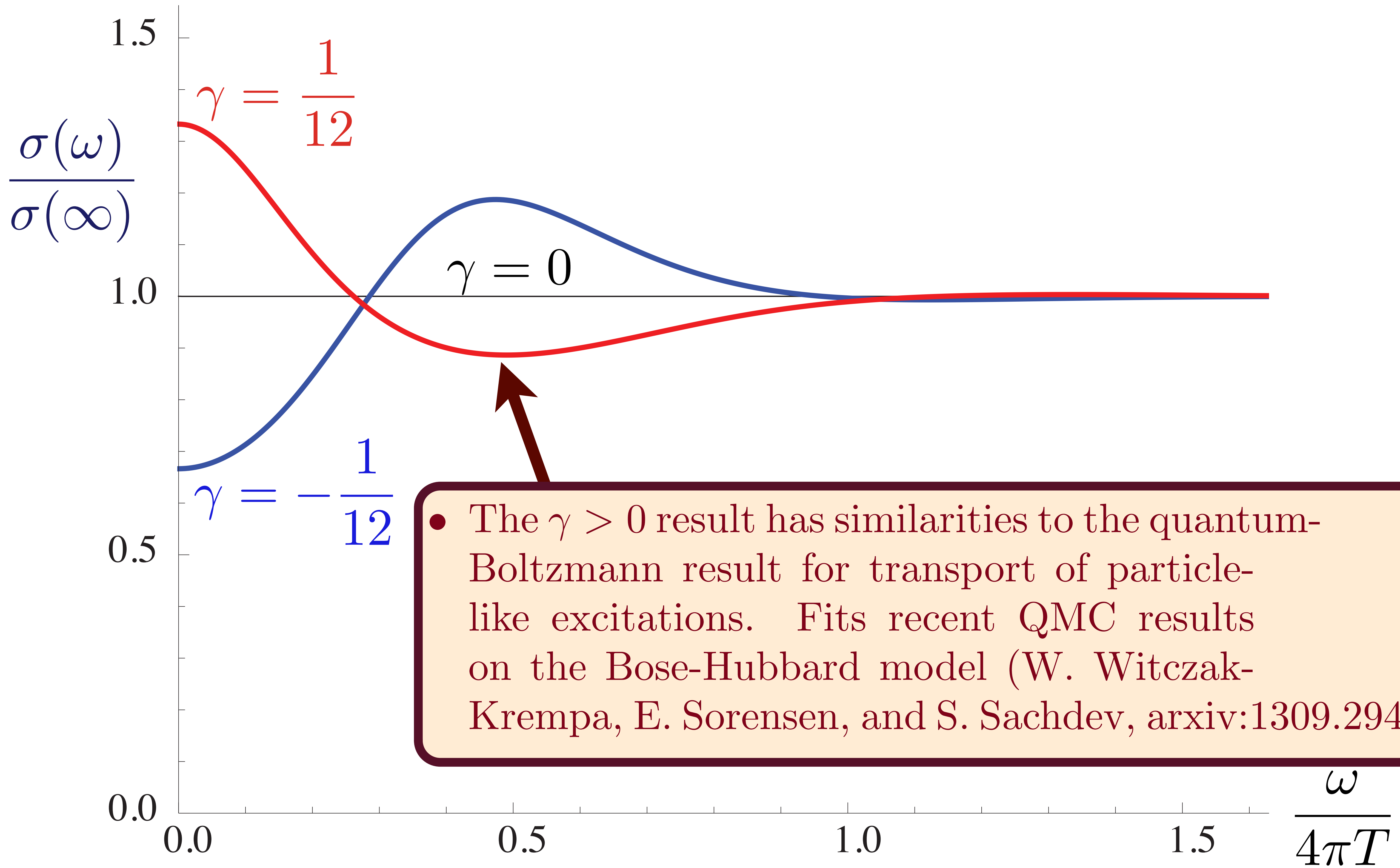
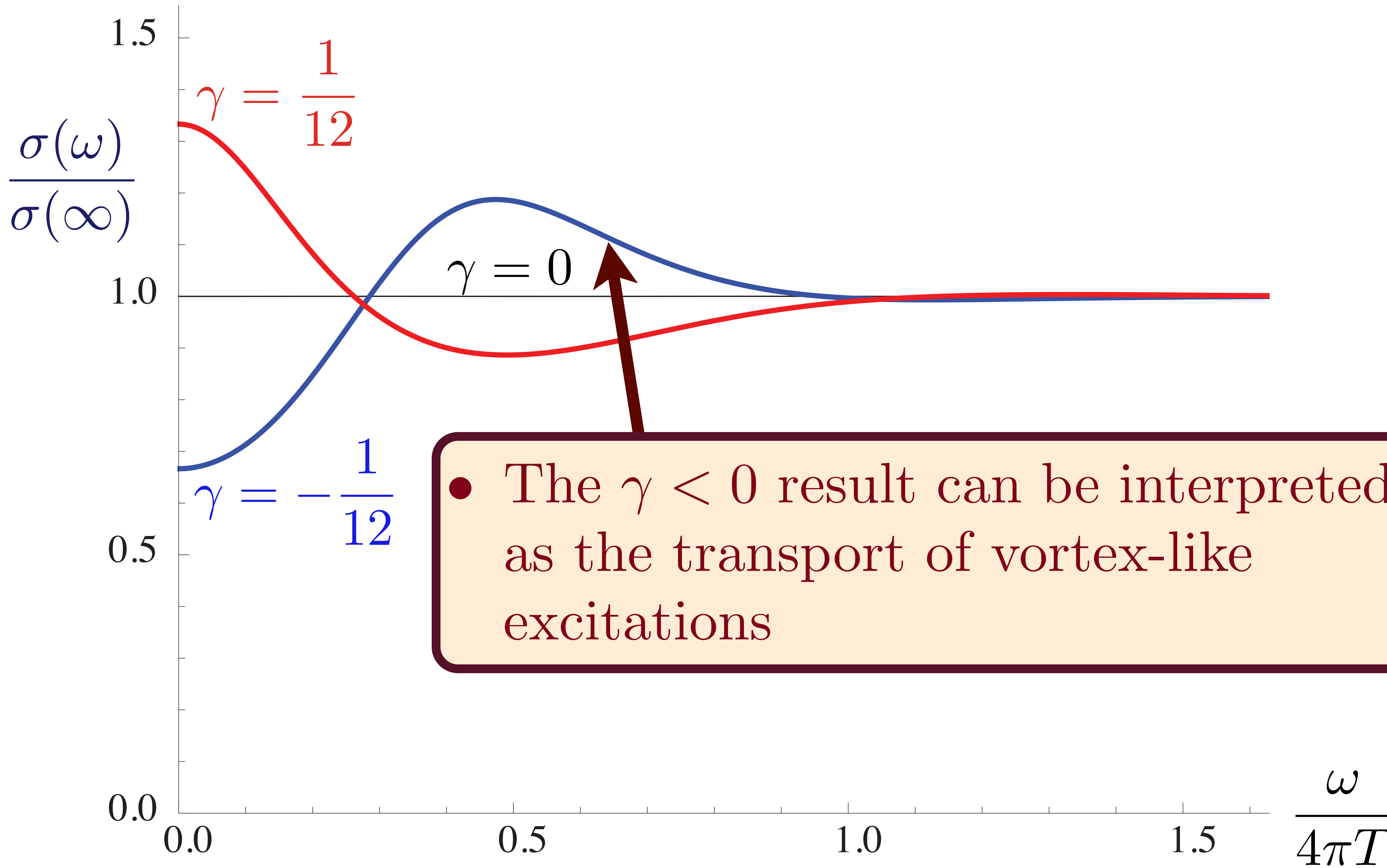


- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations. Fits recent QMC results on the Bose-Hubbard model (W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arxiv:1309.2941)

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

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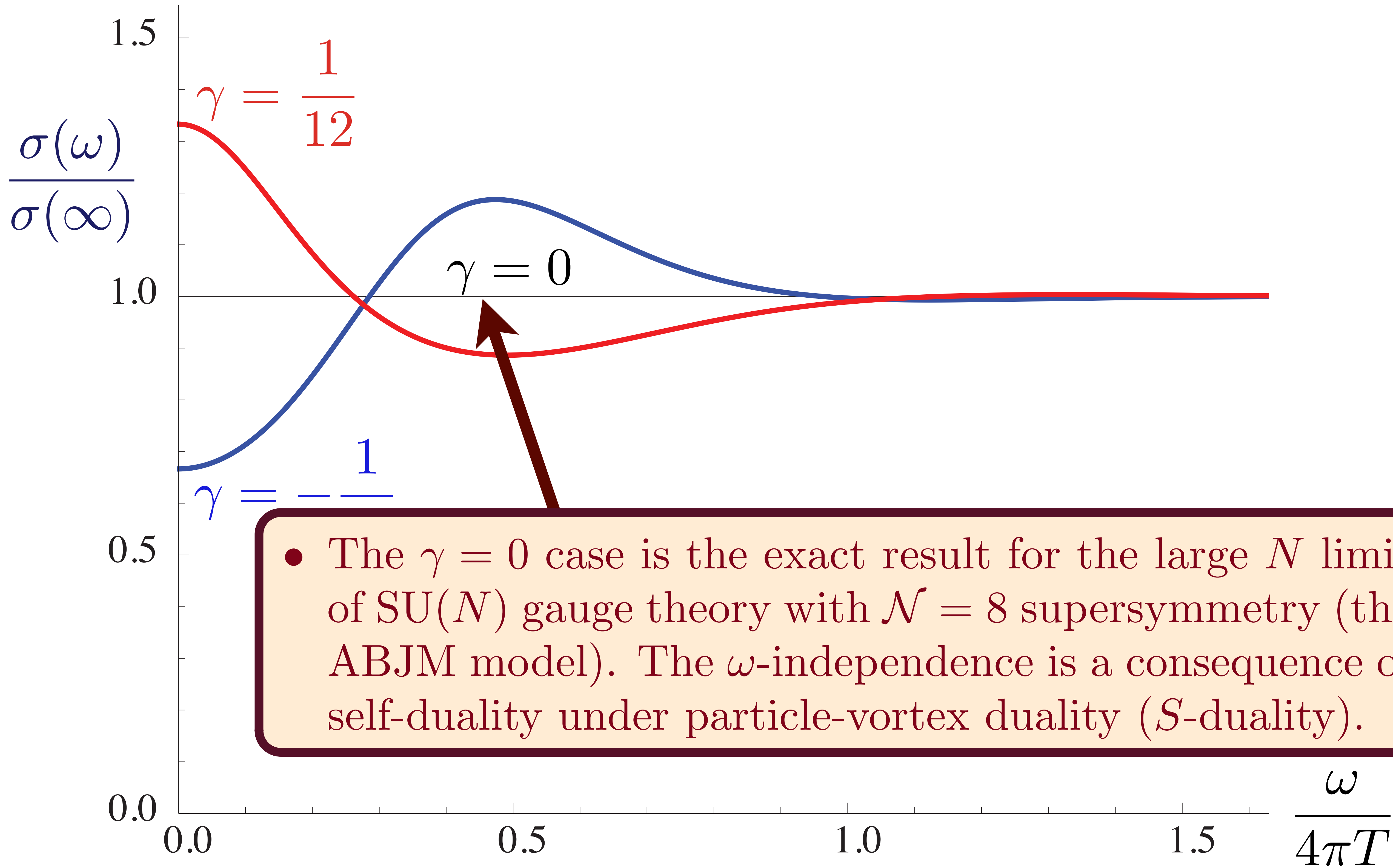

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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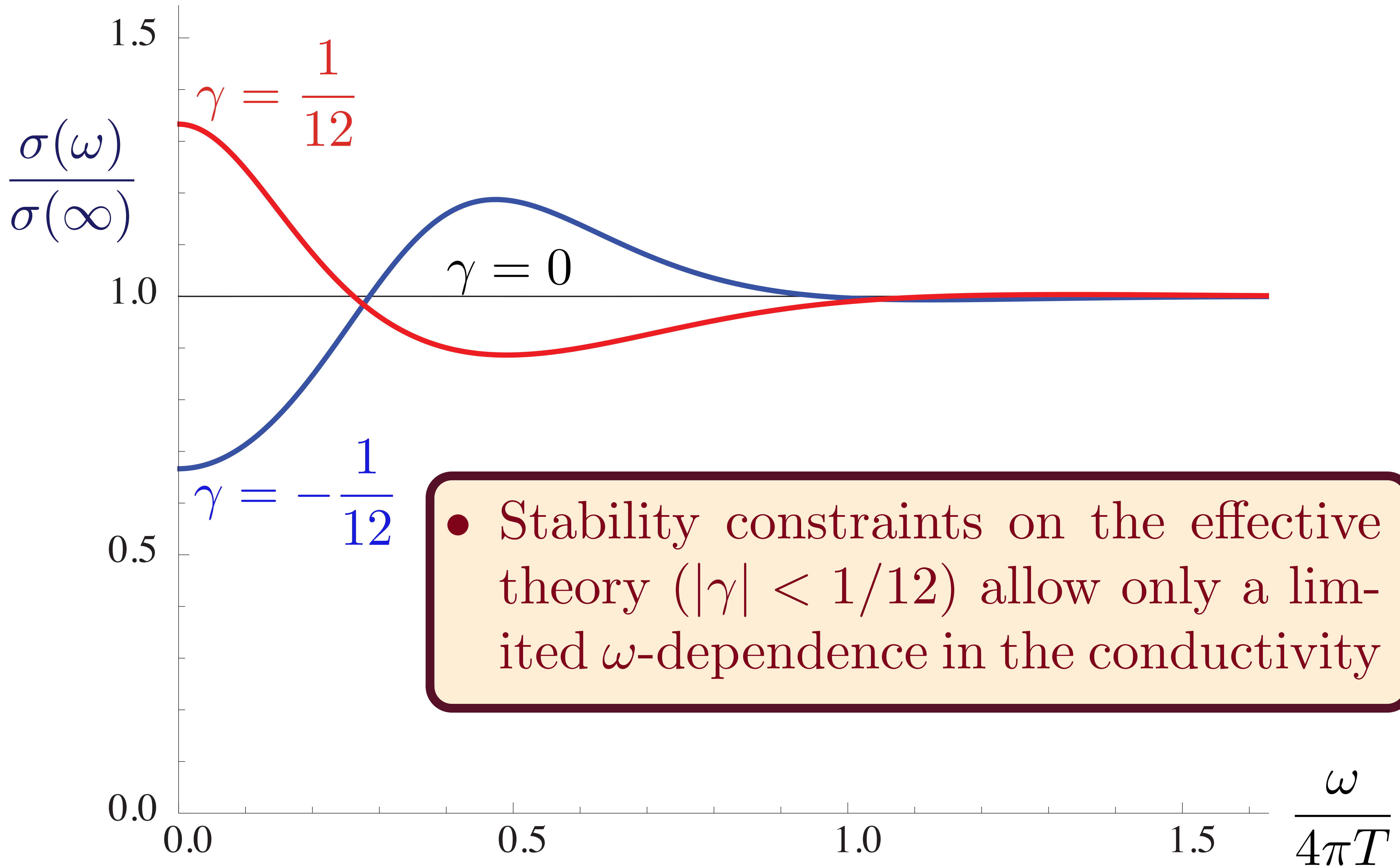

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

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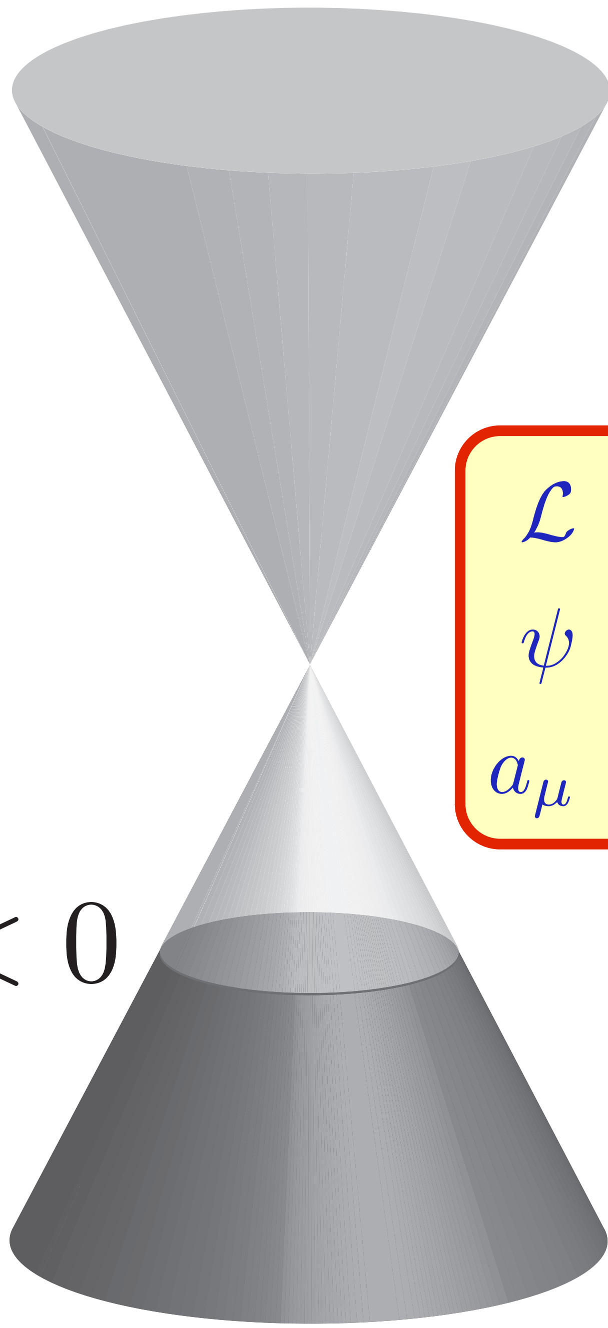
CFT3 in a chemical potential



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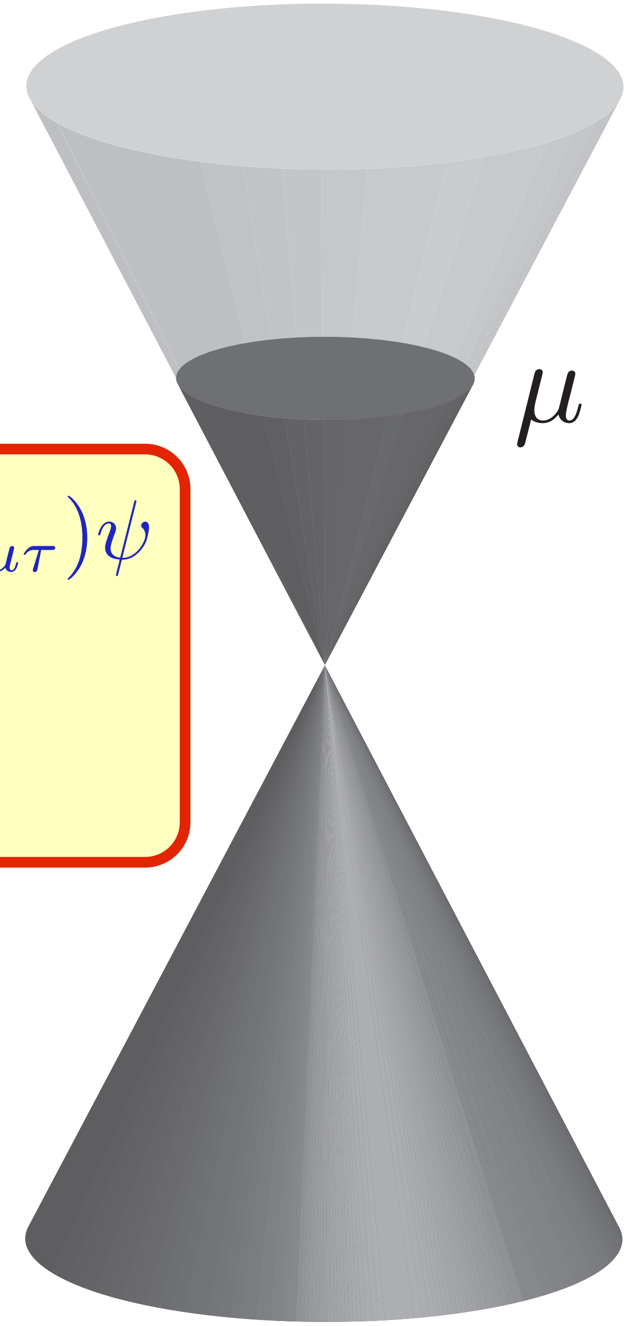
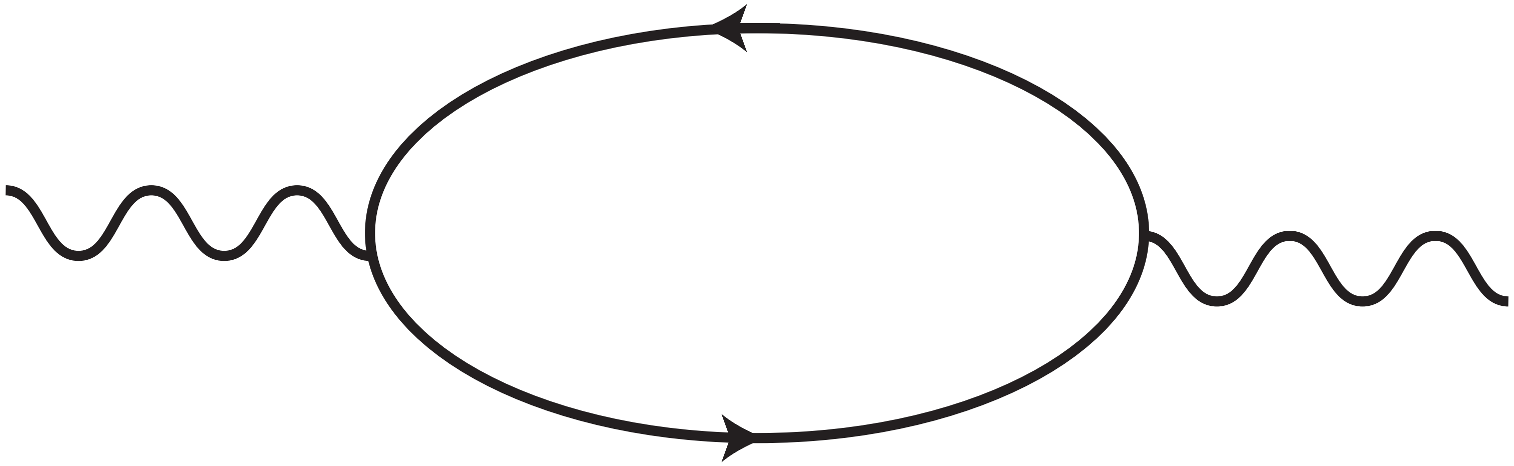
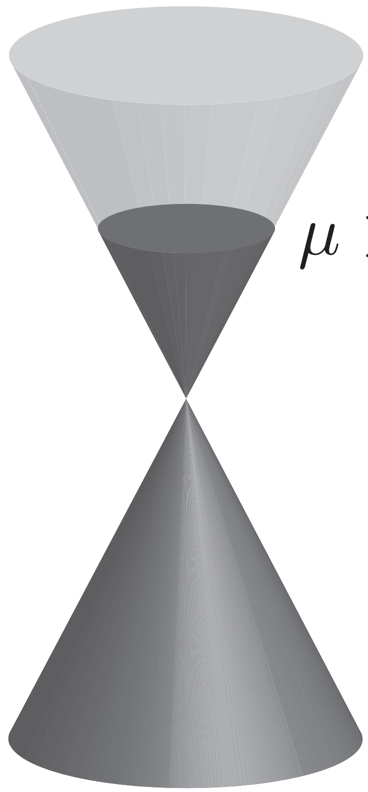


Hole
Fermi surface

Electron
Fermi surface

Transport at non-zero μ and $N_f = \infty$





From the Kubo formula

$$\sigma(\omega) = 2\,(ev_F)^2\,\frac{\hbar}{i}\sum_{ss'}\int \frac{d^2k}{4\pi^2}\,\frac{f(\varepsilon_s(\mathbf{k})) - f(\varepsilon_{s'}(\mathbf{k}))}{(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}))(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}) + \hbar\omega + i\eta)}$$

where $\varepsilon_s(\mathbf{k}) = s\hbar v_F|\mathbf{k}|$ and $s, s' = \pm 1$ for the valence and conduction bands.

T. Ando, Y. Zheng and H. Suzuura, J. Phys. Soc. Jpn. **71** (2002) pp. 1318-1324

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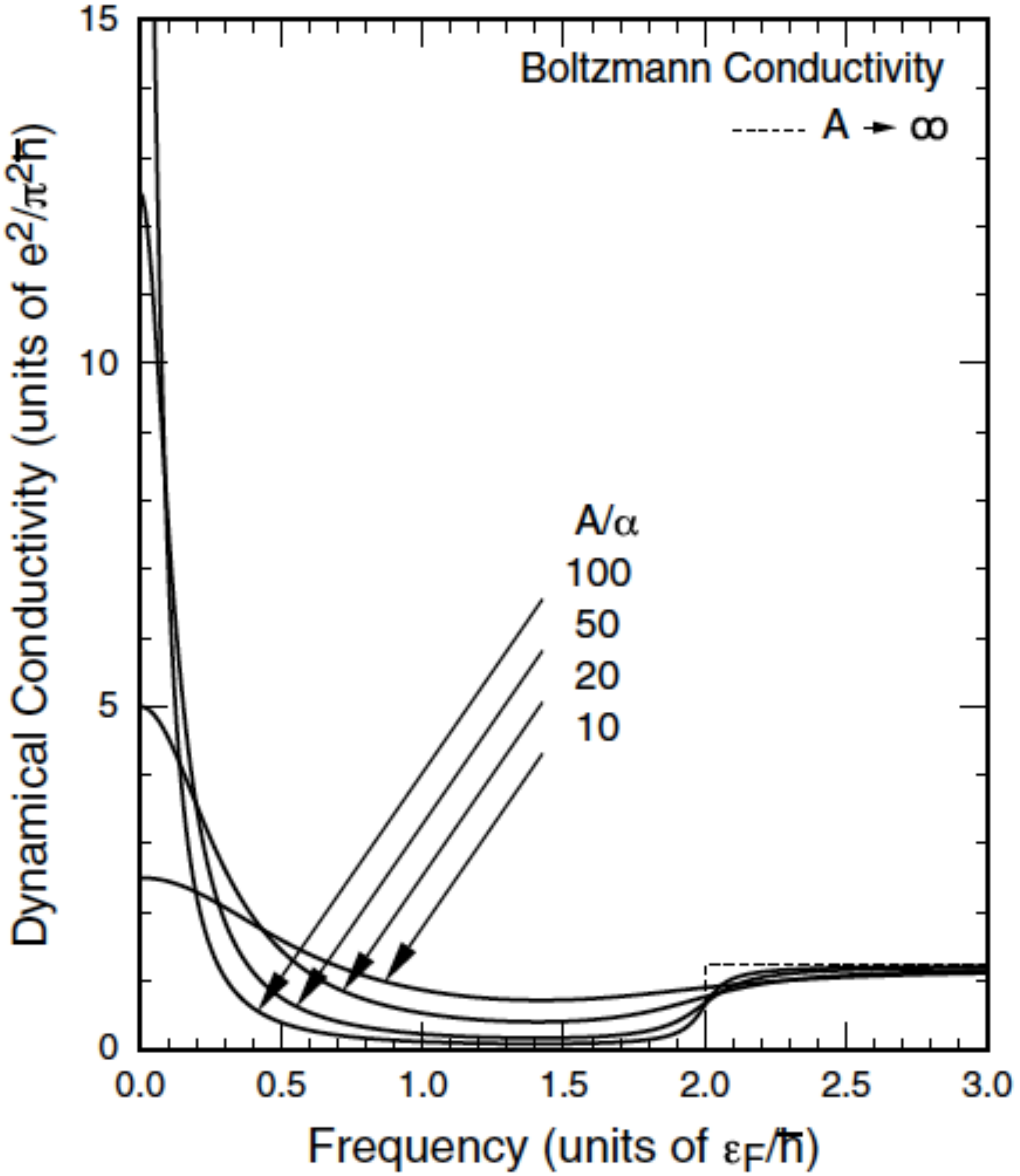

A is inversely proportional to disorder. In the clean limit $A \to \infty$, at $T = 0$

$$\mathrm{Re}[\sigma(\omega)] = \frac{e^2}{\hbar}\left[\frac{\varepsilon_F}{\hbar}\delta(\omega) + \frac{1}{4}\theta(|\omega| - 2\varepsilon_F)\right]$$

Notice delta function is present even at $T = 0$ at non-zero density: this is a generic consequence of the conservation of momentum in any clean interacting Fermi liquid. Only "umklapp" scattering can broaden this delta function.

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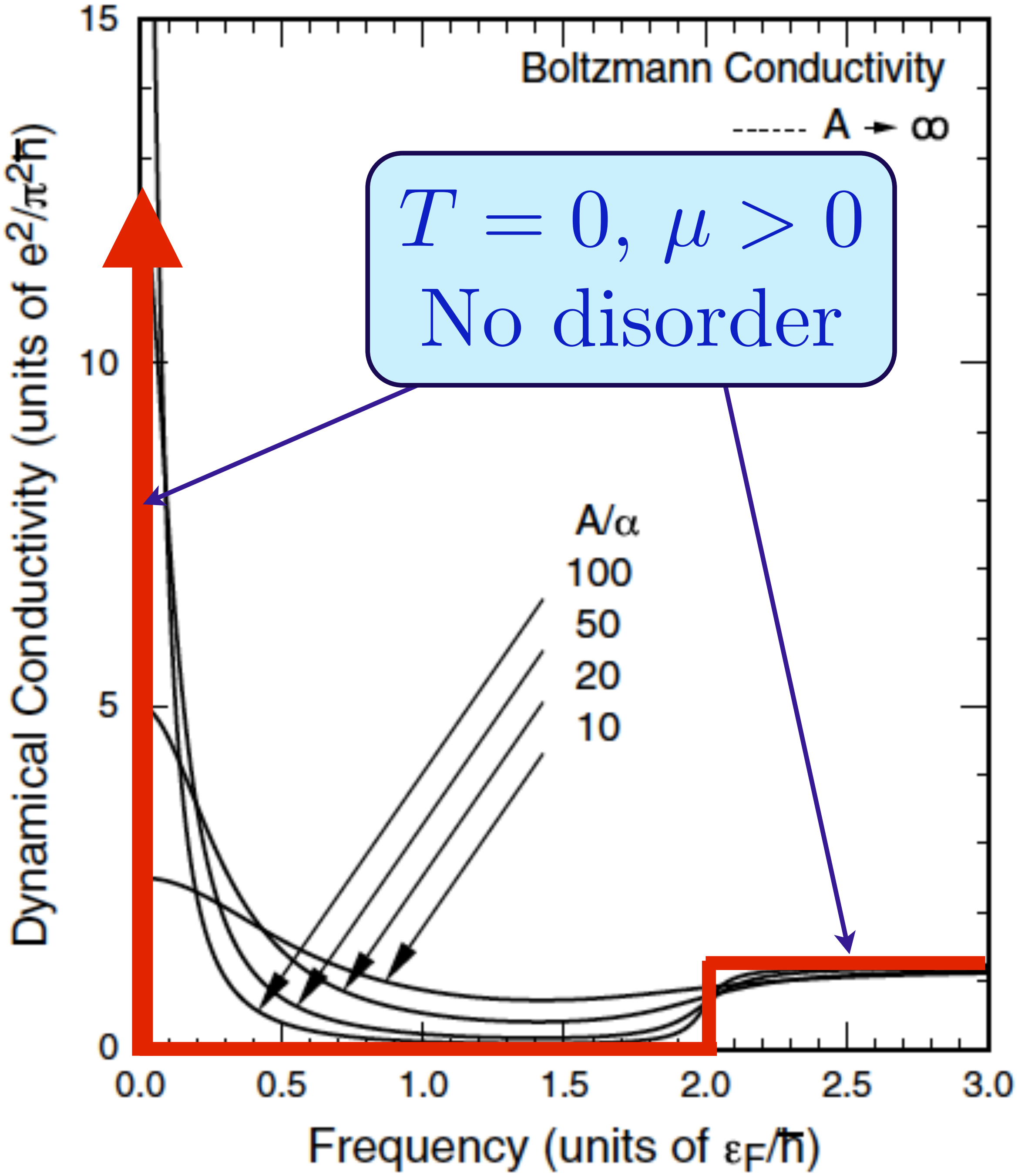

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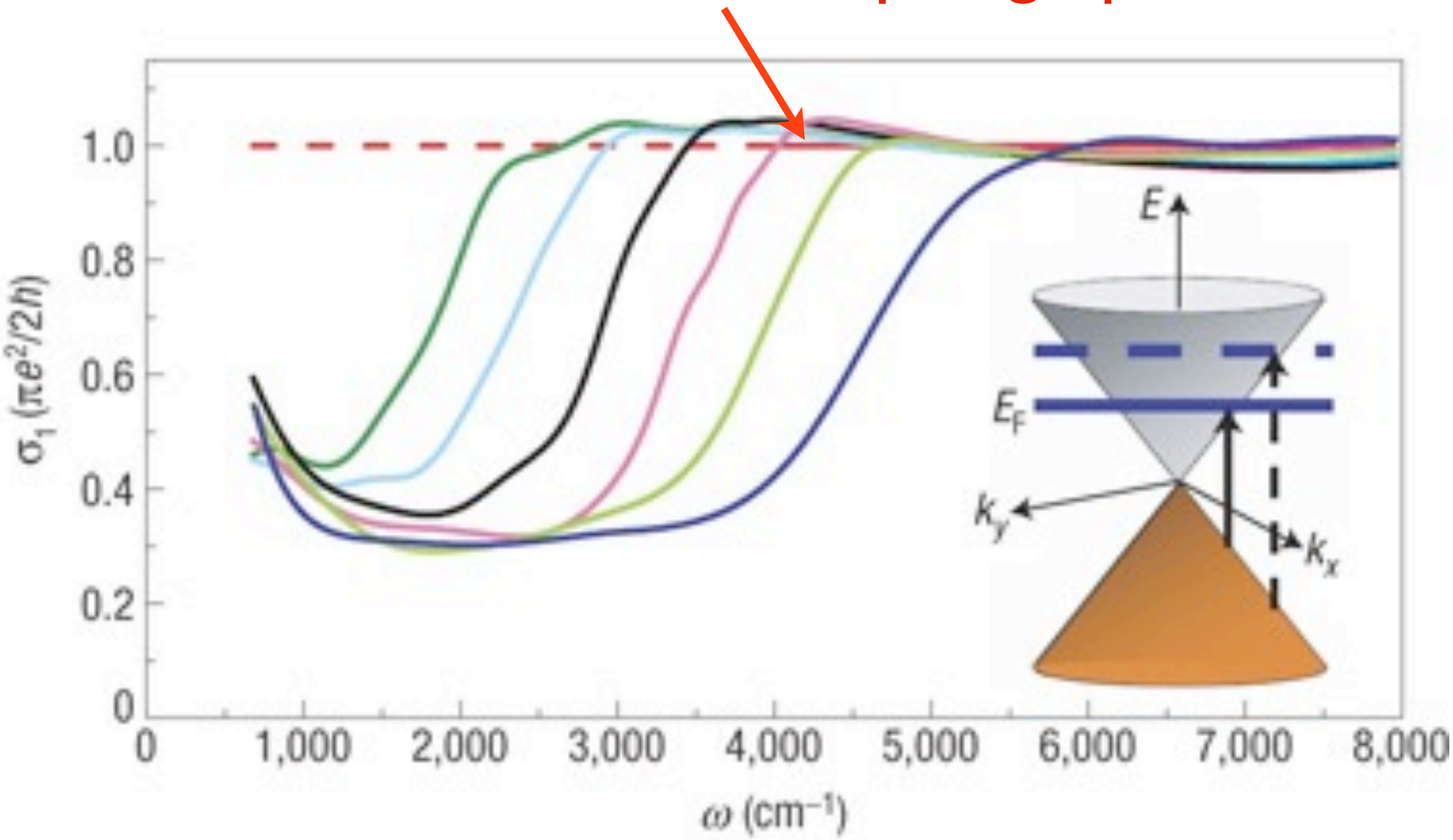
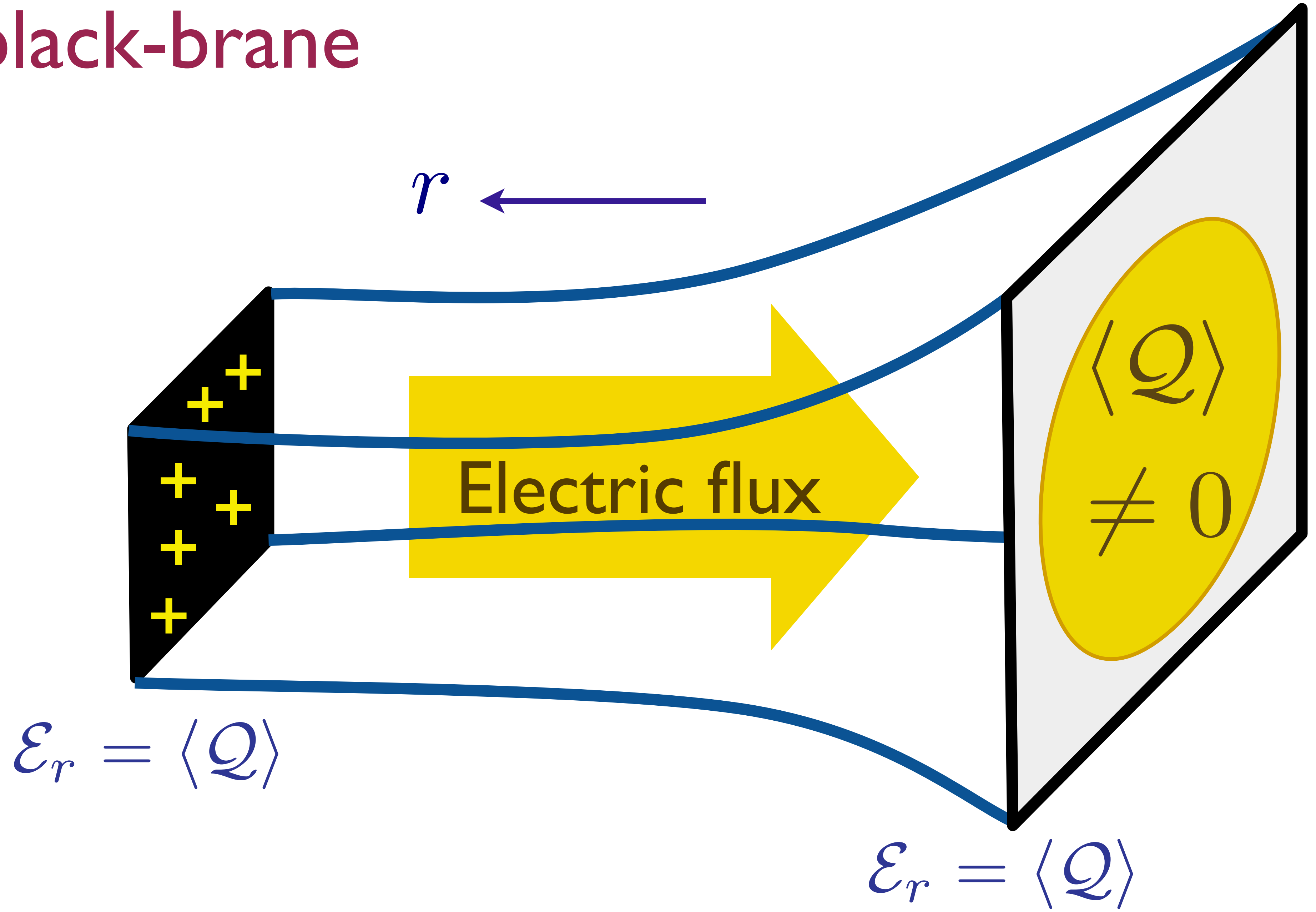
Optical conductivity of graphene

Undoped graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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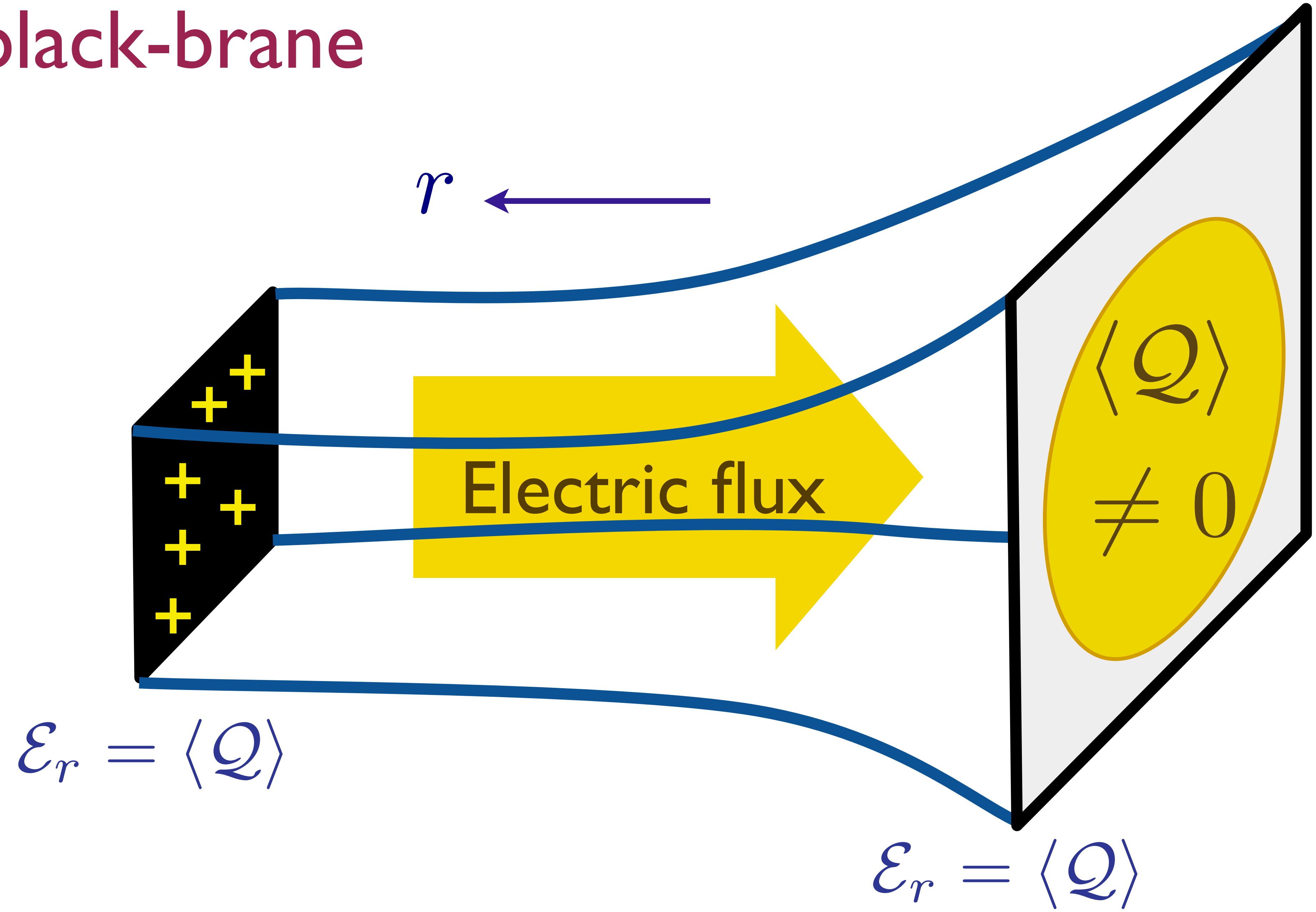

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right)$ and $R = \frac{\sqrt{6}Lg_4}{\kappa\mu}$, and $A_\tau = \mu\left(1 - \frac{r}{R}\right)$

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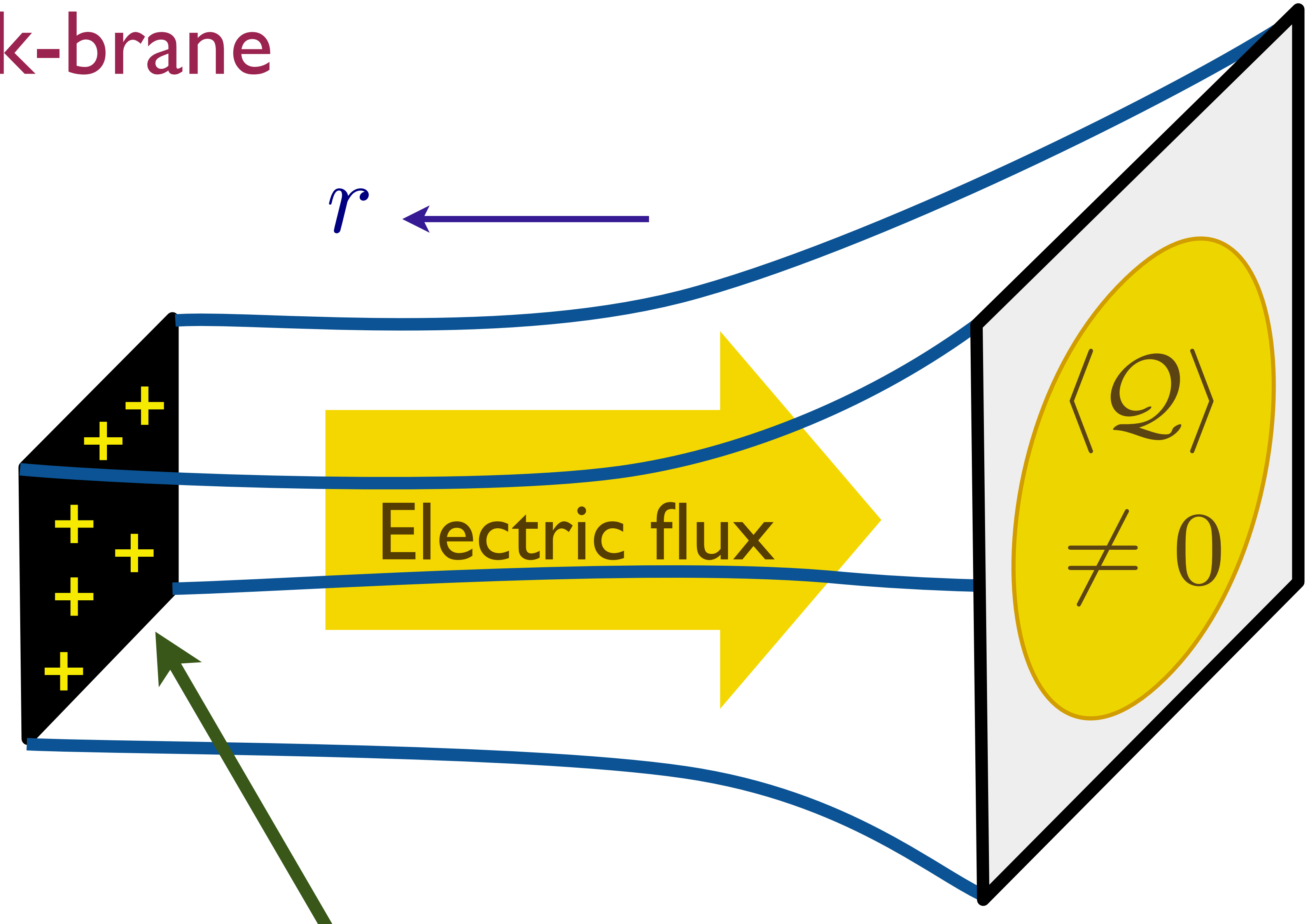
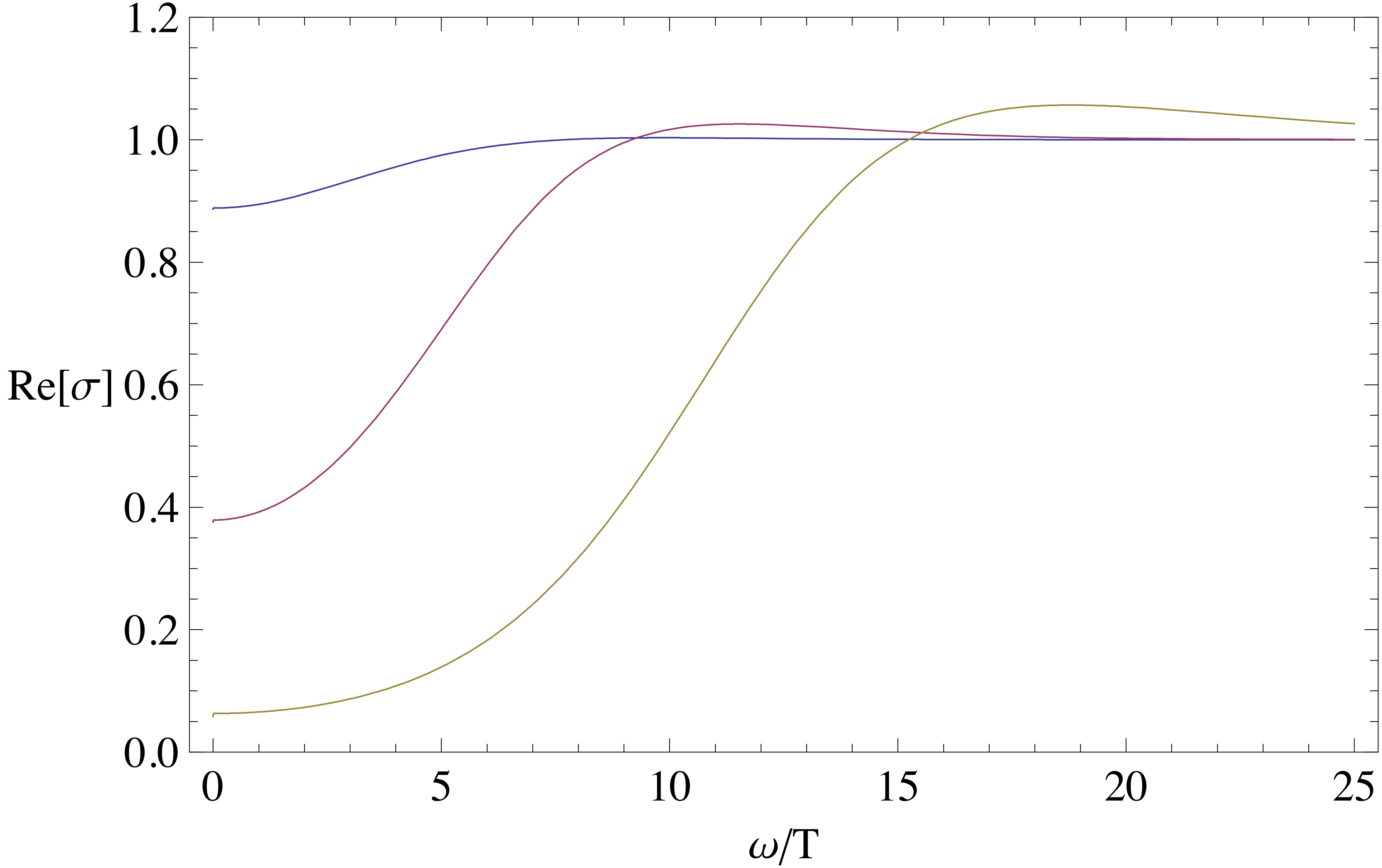


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $AdS_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Compute conductivity using response to a time-dependent vector potential as a function of ω/T and μ/T



S.A. Hartnoll, arXiv:0903.3246

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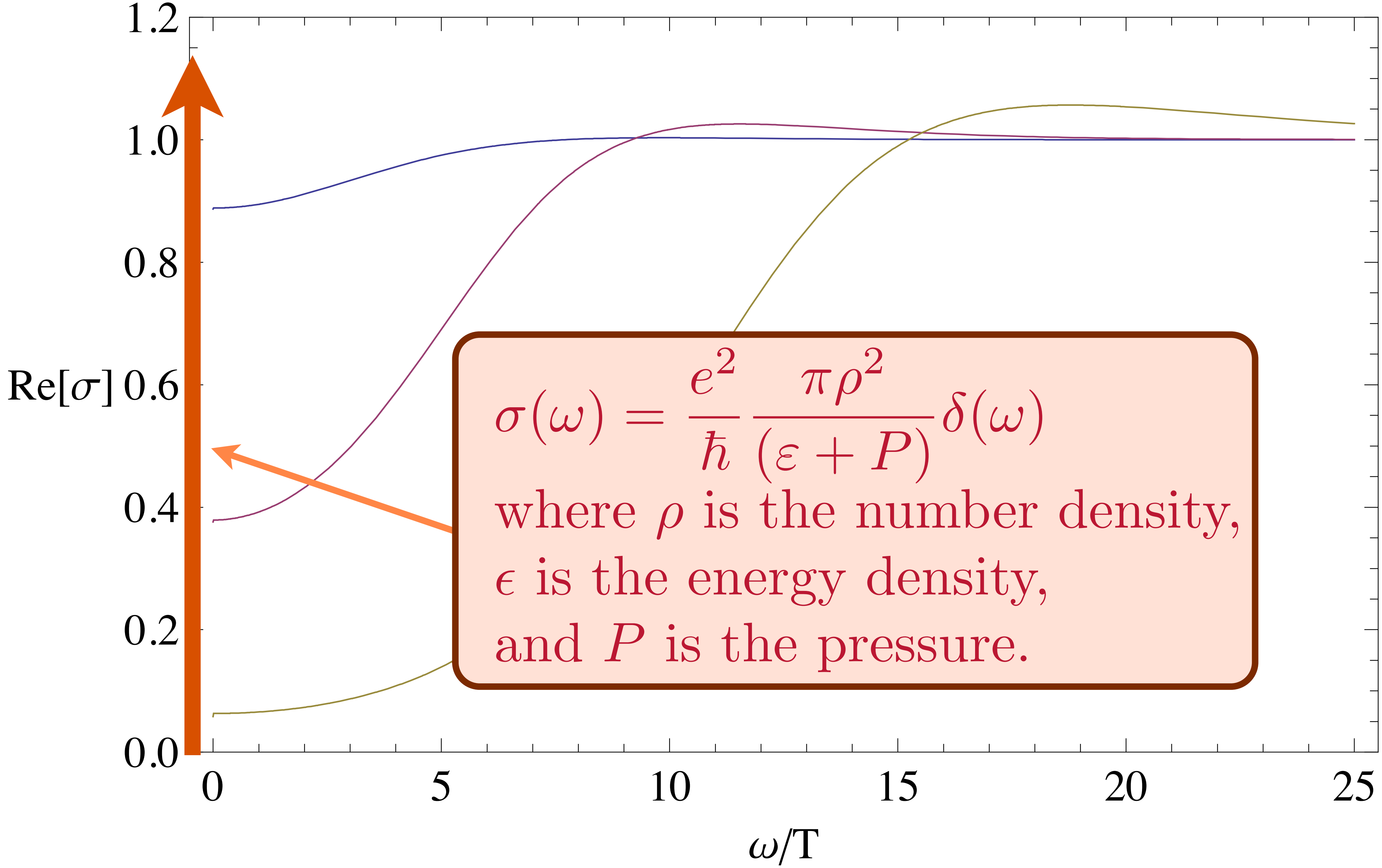

$$\sigma(\omega) = \frac{e^2}{\hbar}\frac{\pi\rho^2}{(\varepsilon + P)}\delta(\omega)$$
where ρ is the number density, ϵ is the energy density, and P is the pressure.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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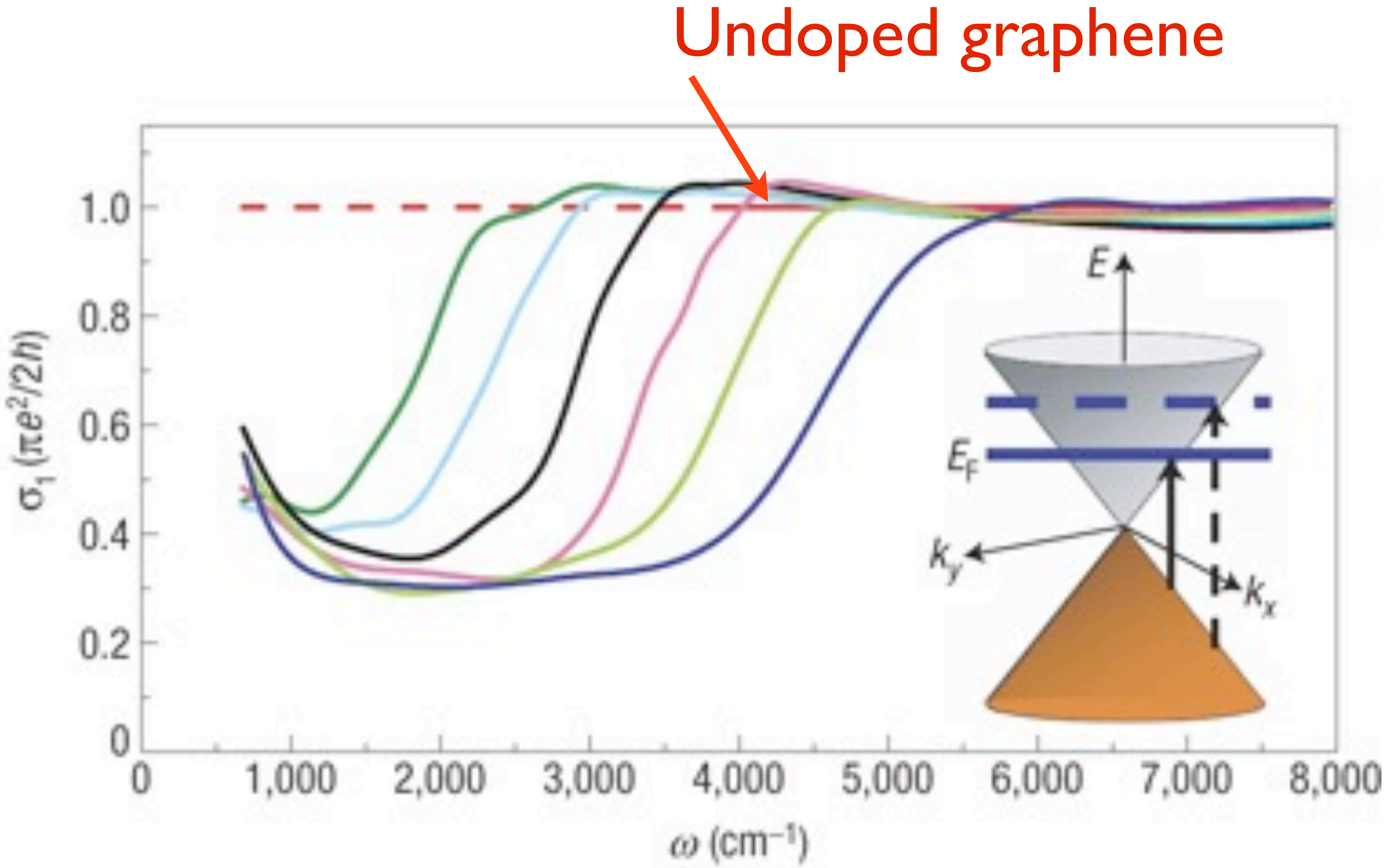

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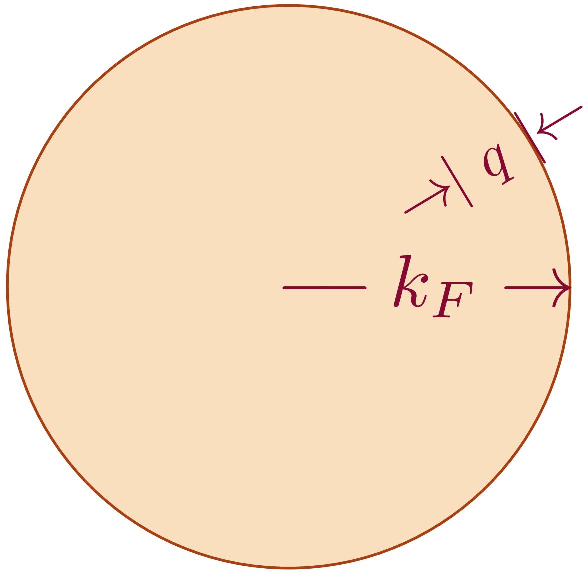
CFT3 in a chemical potential

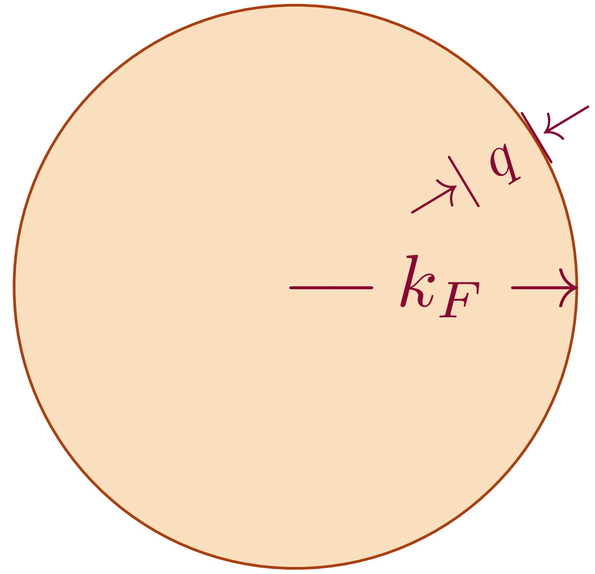
At non-zero $1/N_f$, we have at low energies the theory of a Fermi surface coupled to a gauge field. This described a "non-Fermi liquid". Its Fermi surface is *hidden*, the single fermion Green's function is not gauge-invariant. Nevertheless, the Fermi wavevector, k_F, does control the oscillations of gauge-invariant correlation functions, such as that of the density operator (Friedel oscillations). Also, the entanglement entropy of the non-Fermi liquid is best understood as arising from this "hidden" Fermi surface.

FL Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL Fermi liquid



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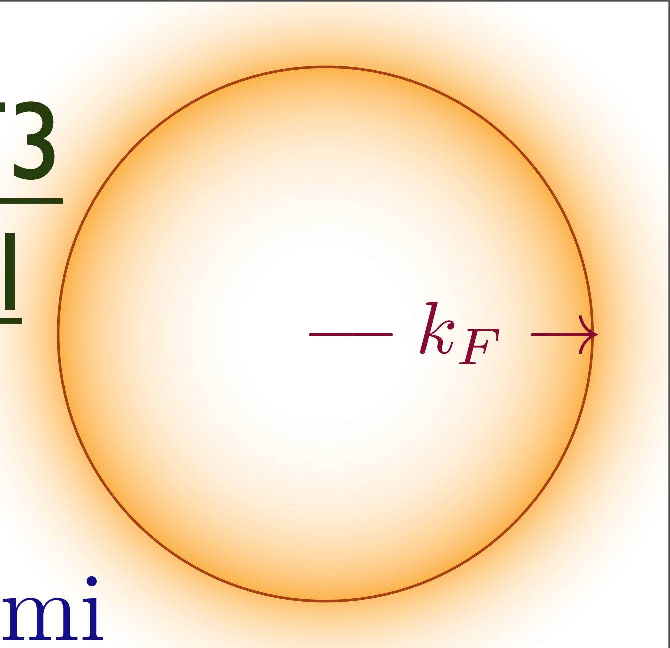
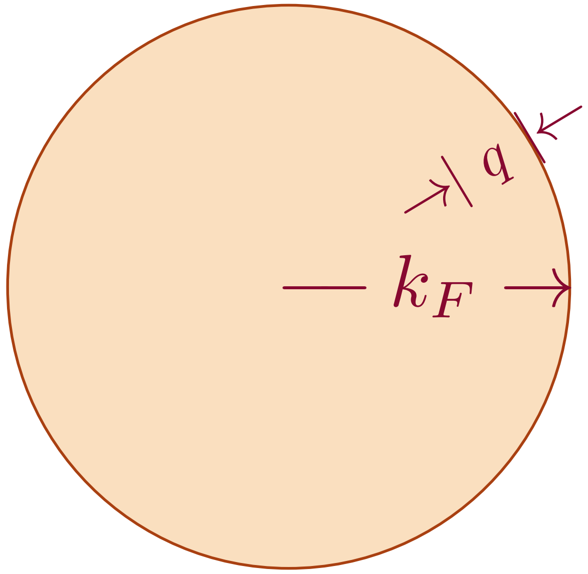
Large N_f CFT3 in a chemical potential



- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

FL Fermi liquid



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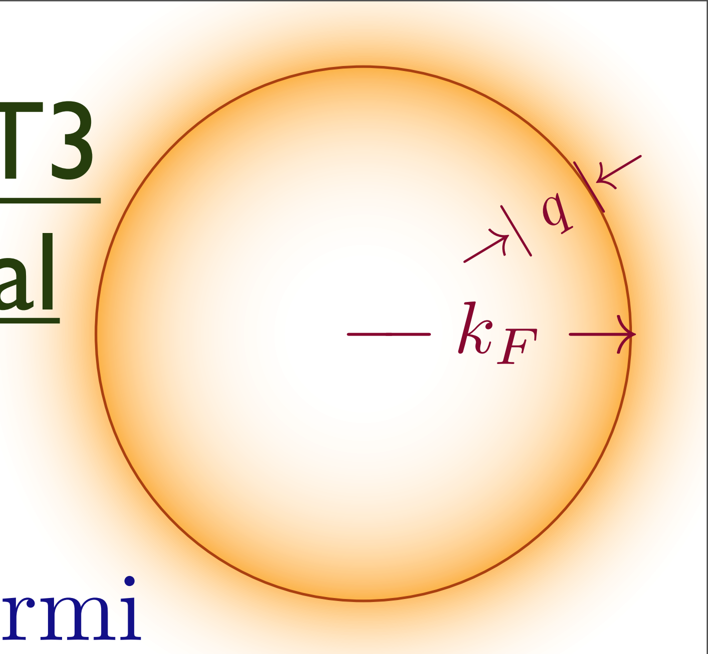
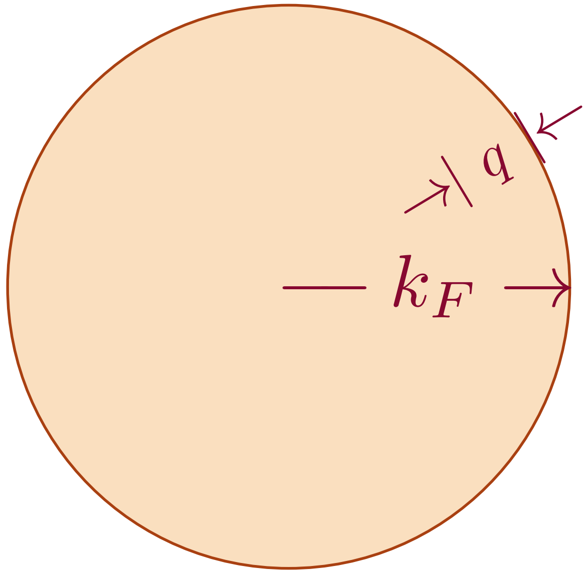


- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

FL Fermi liquid



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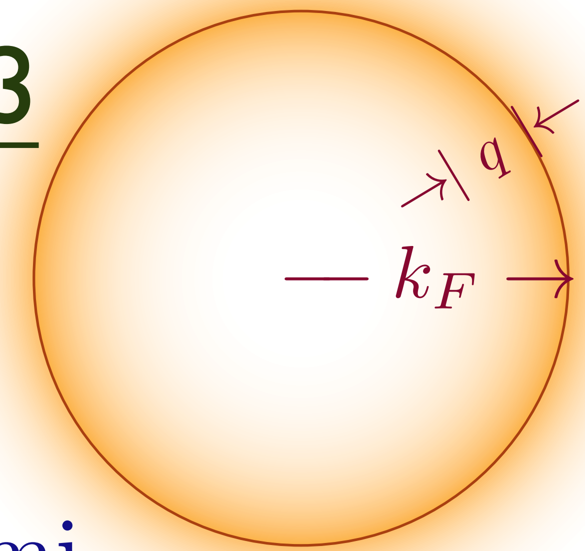

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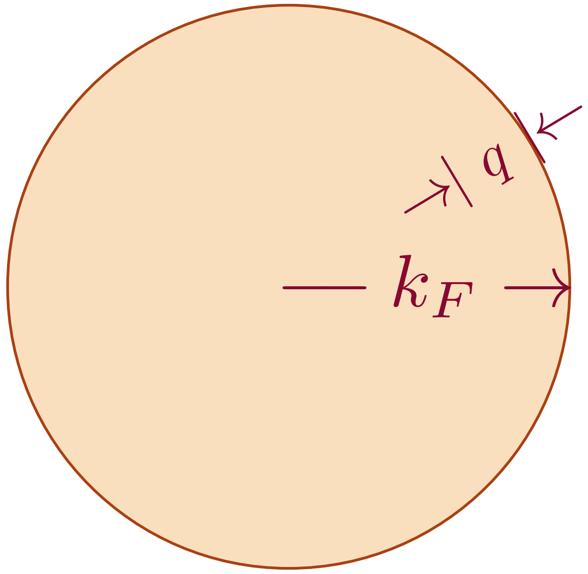
- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

FL Fermi liquid



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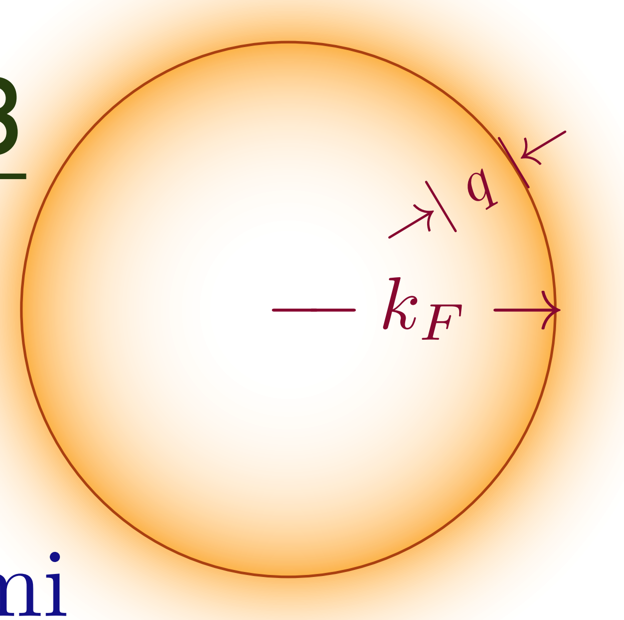
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

Large N_f CFT3 in a chemical potential



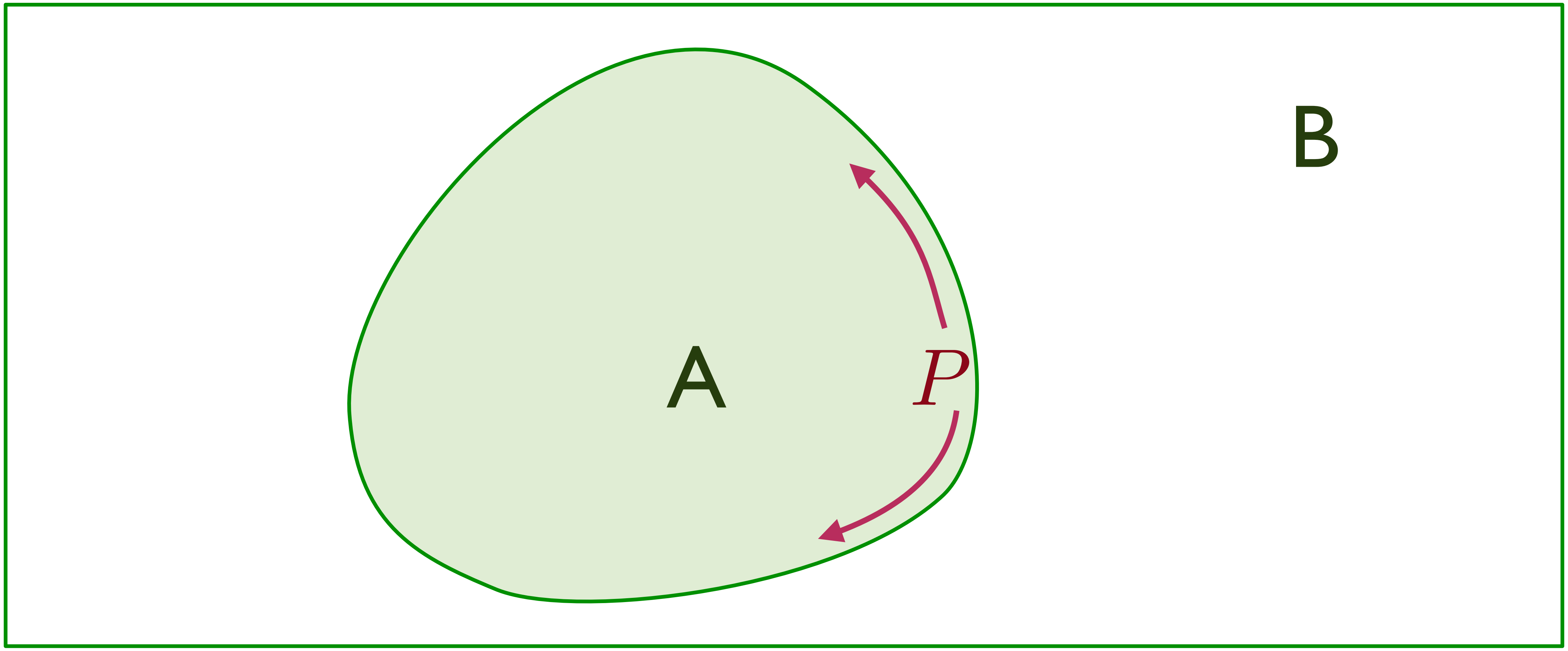
- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

- $S_E \sim k_F^{d-1} P \ln P$.

Entanglement entropy of the non-Fermi liquid



Logarithmic violation of "area law": $S_E = \mathcal{C}_E \, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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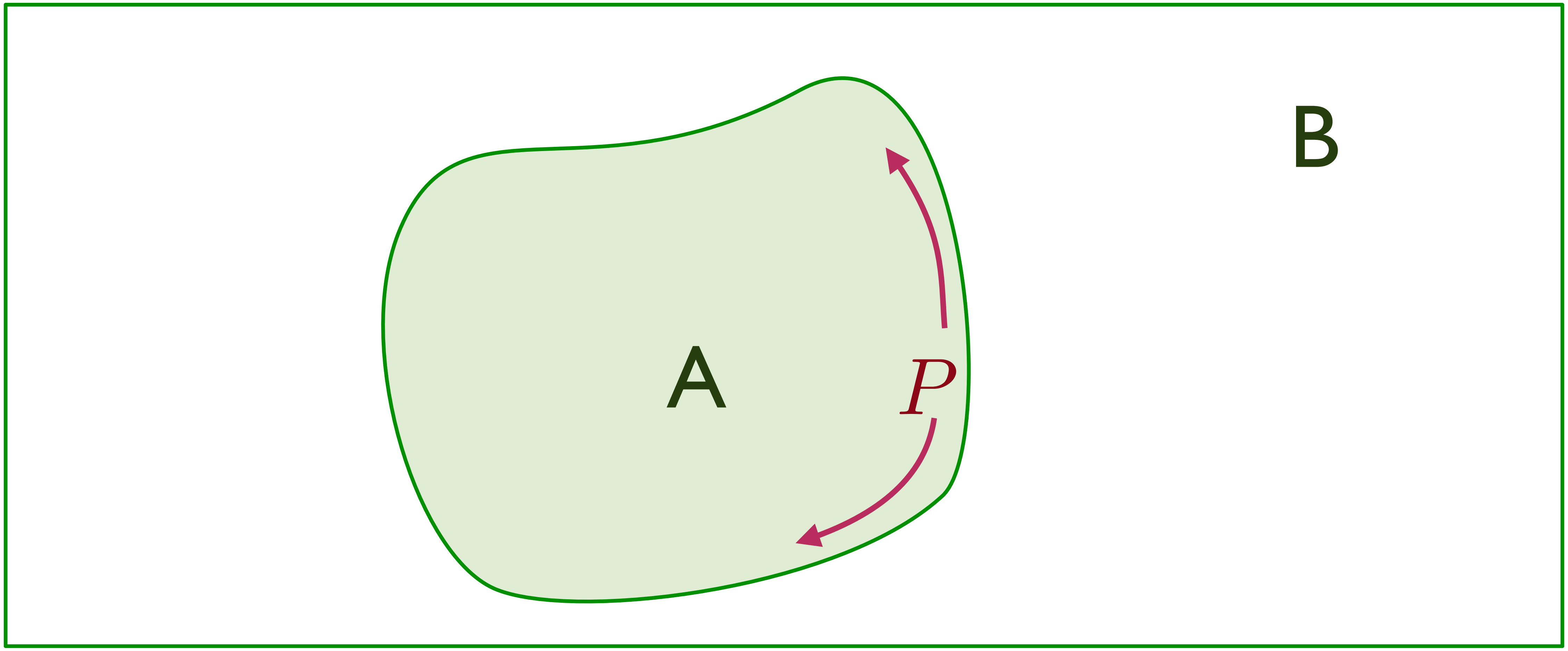

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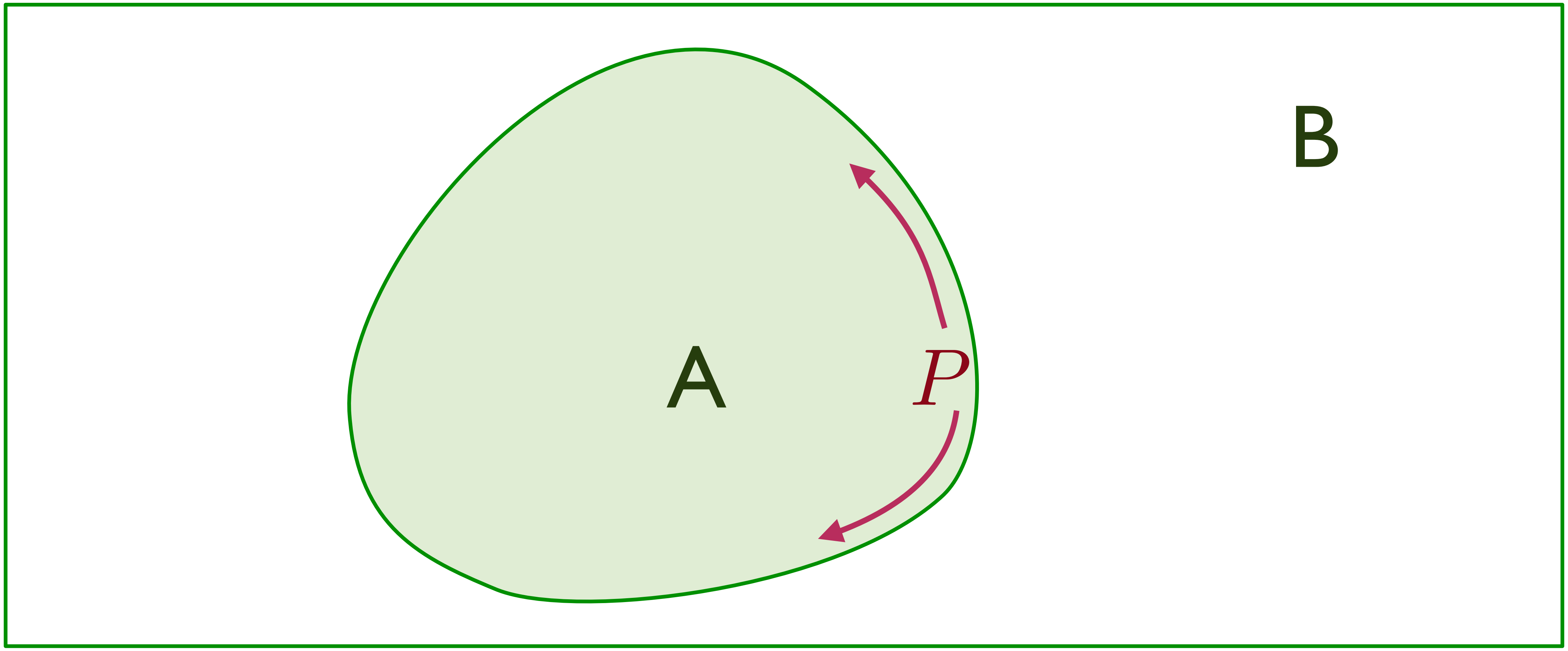

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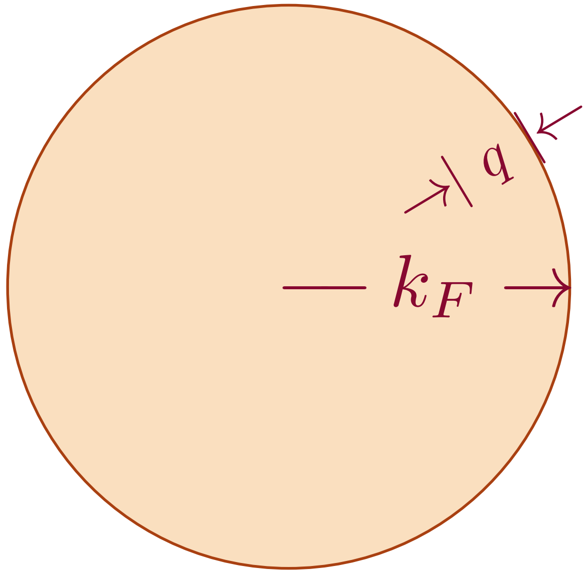
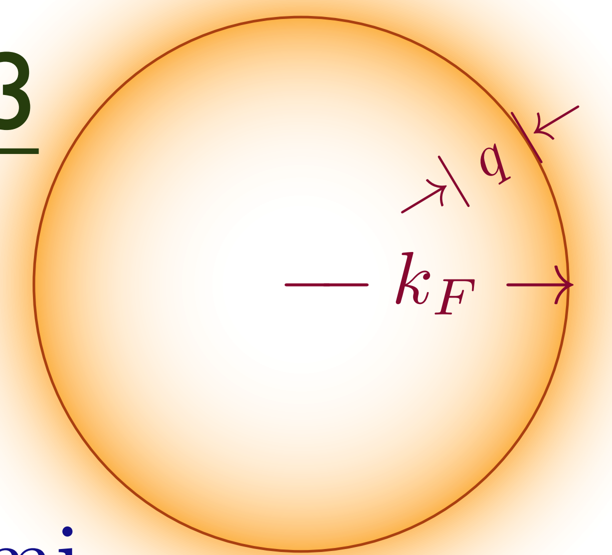
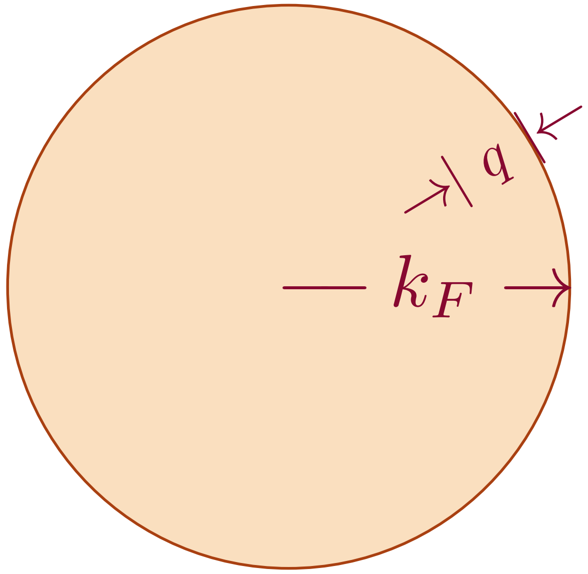
FL Fermi liquid	Large N_f CFT3 in a chemical potential
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \|q\|^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	• $S_E \sim k_F^{d-1} P \ln P$.

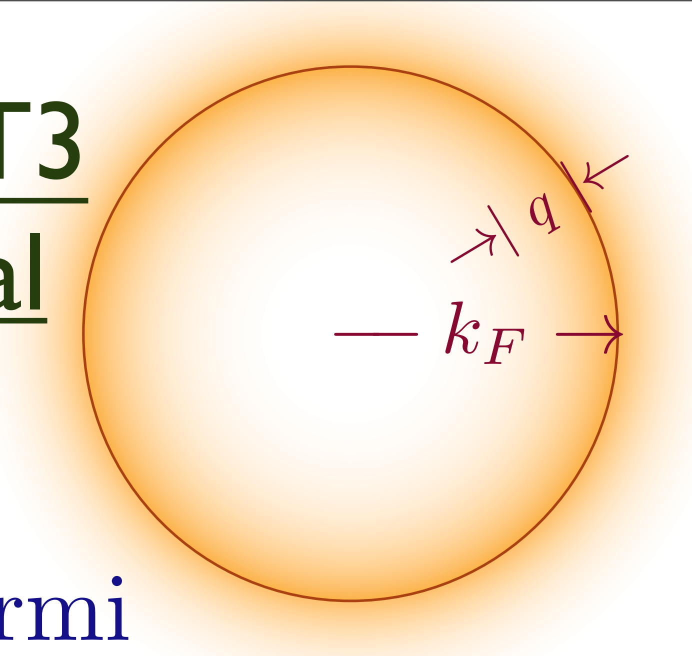
<u>FL Fermi liquid</u>



- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

<u>Large N_f CFT3 in a chemical potential</u>



- <u>Hidden</u> Fermi surface with $k_F^d \sim \mathcal{Q}$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

All characteristics of the entropy and entanglement entropy are reproduced in a holographic theory with a IR metric

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012).

A. Electrical transport in CFT3s

B. Electrical transport in CFT3s in a constant chemical potential

C. CFT3s in a periodic chemical potential

D. Monopoles

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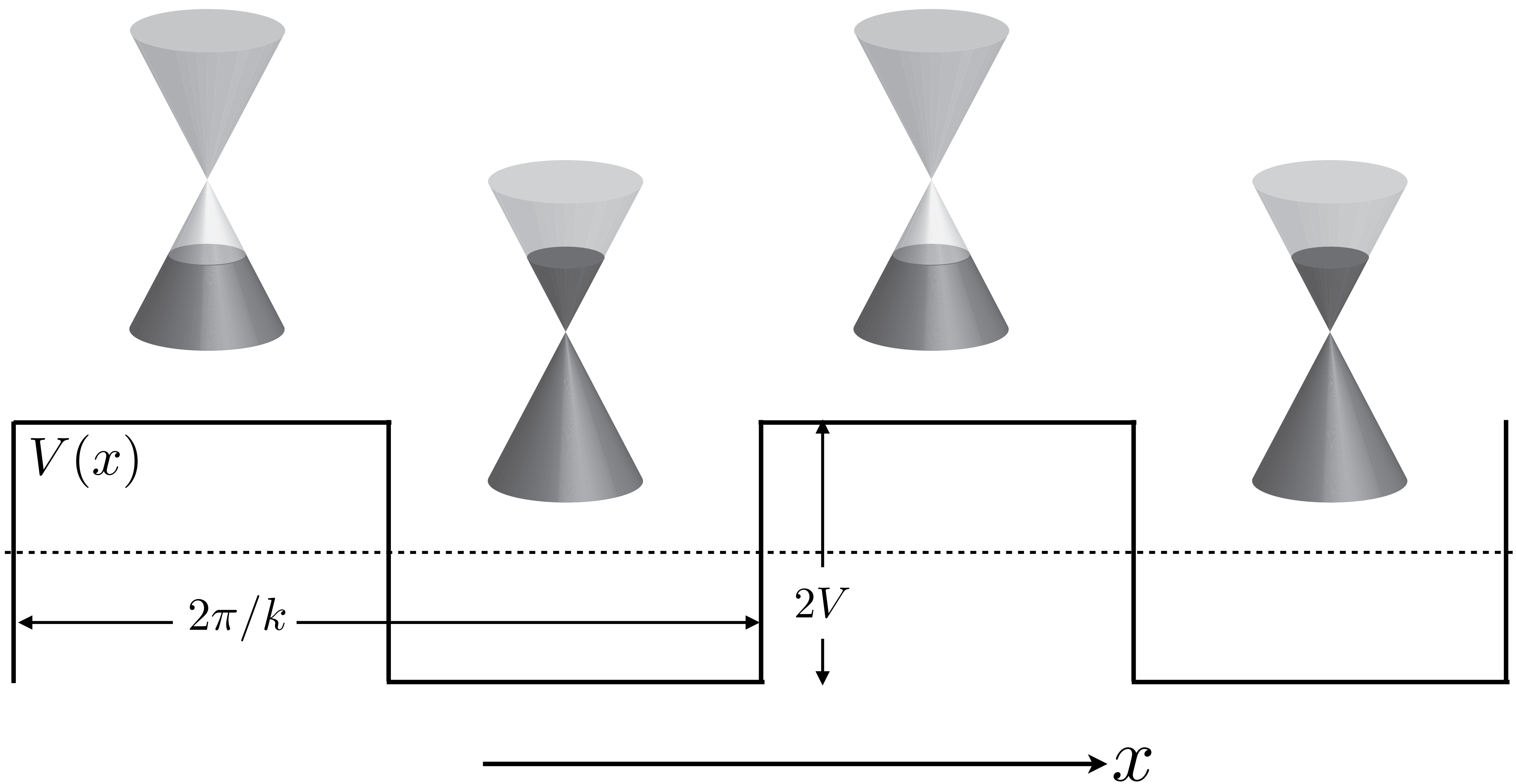
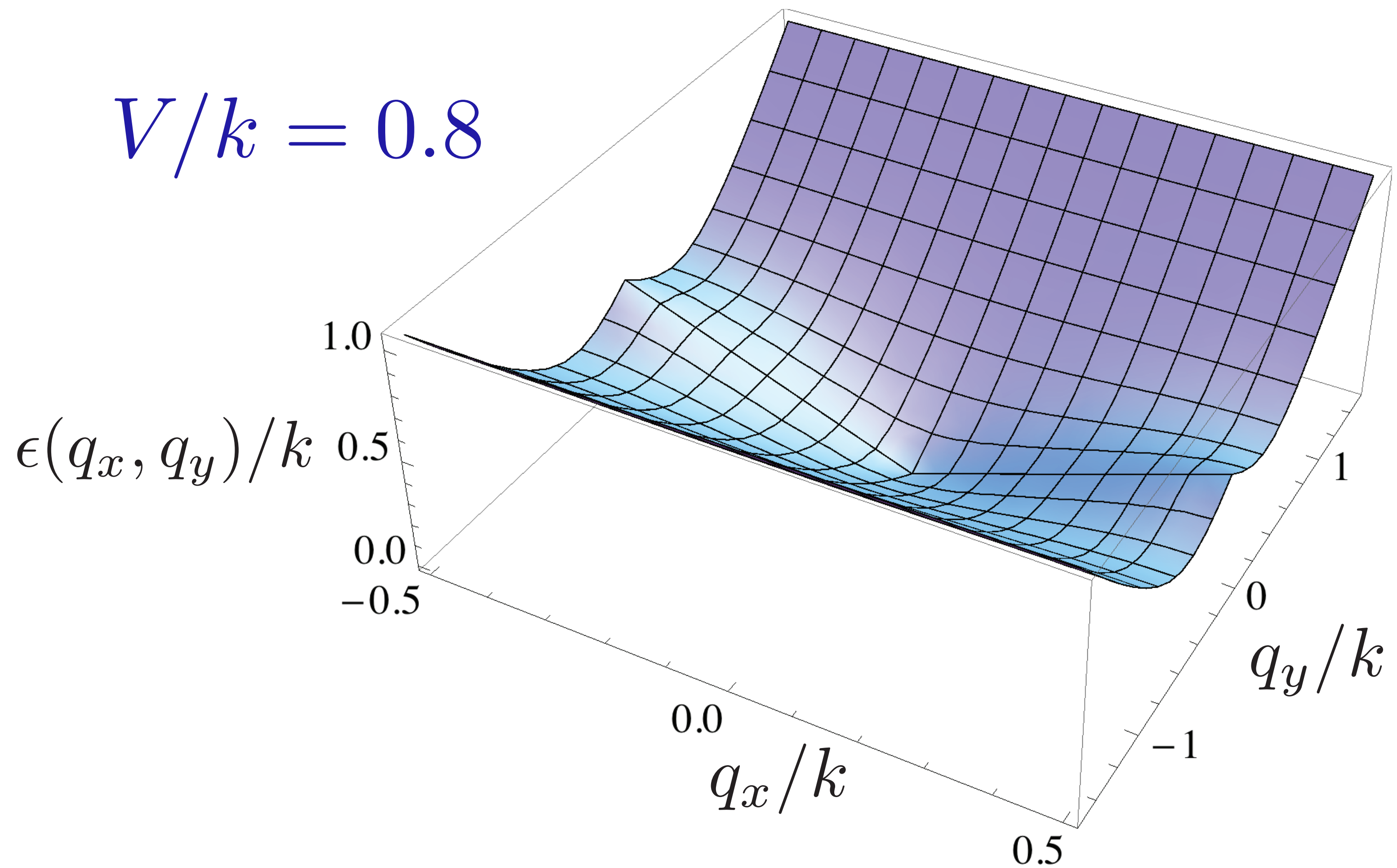
D. Monopoles


$V(x)$
$2\pi/k$
$2V$
x

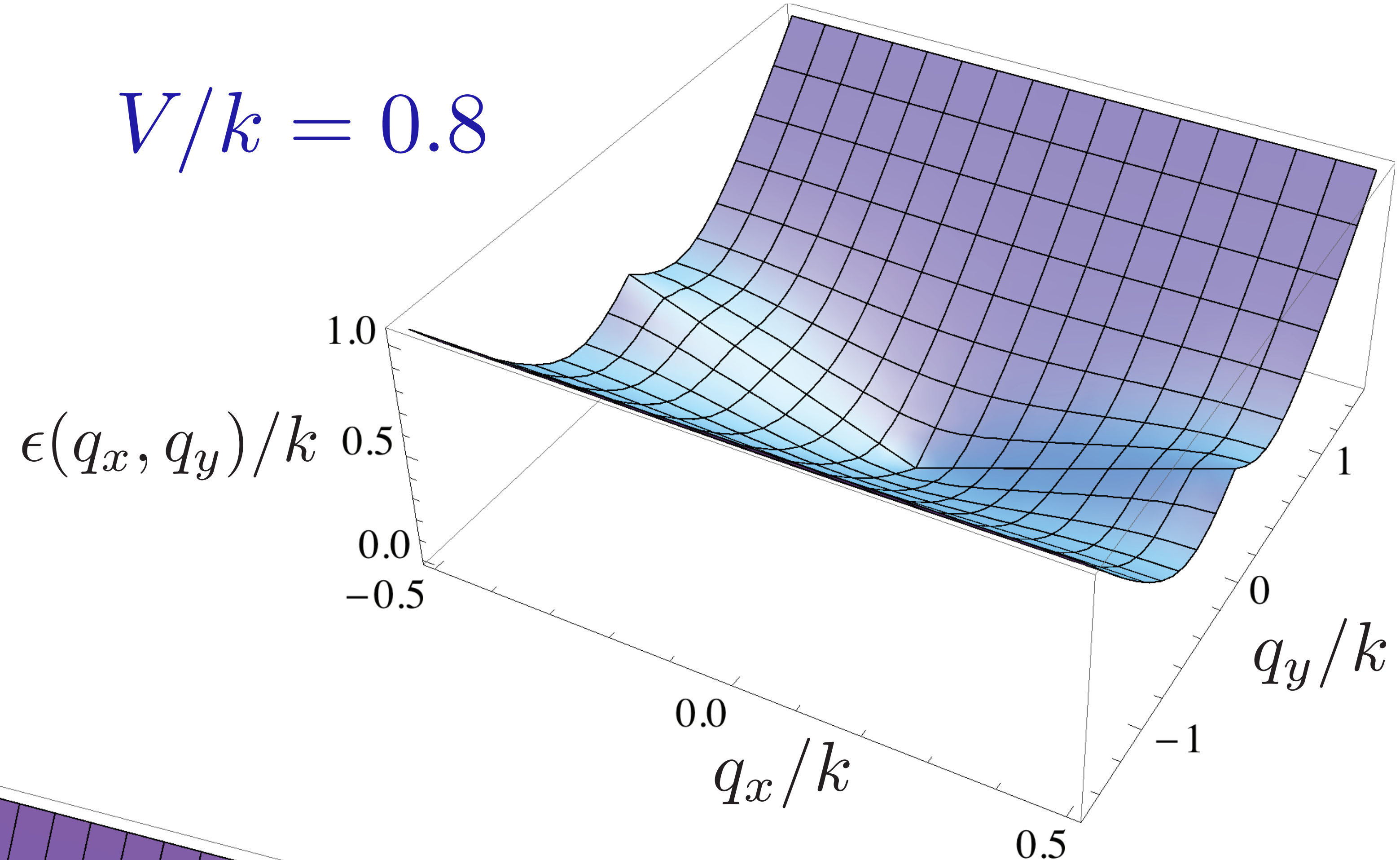
Fermion spectrum at $N_f = \infty$

$V/k = 0.8$

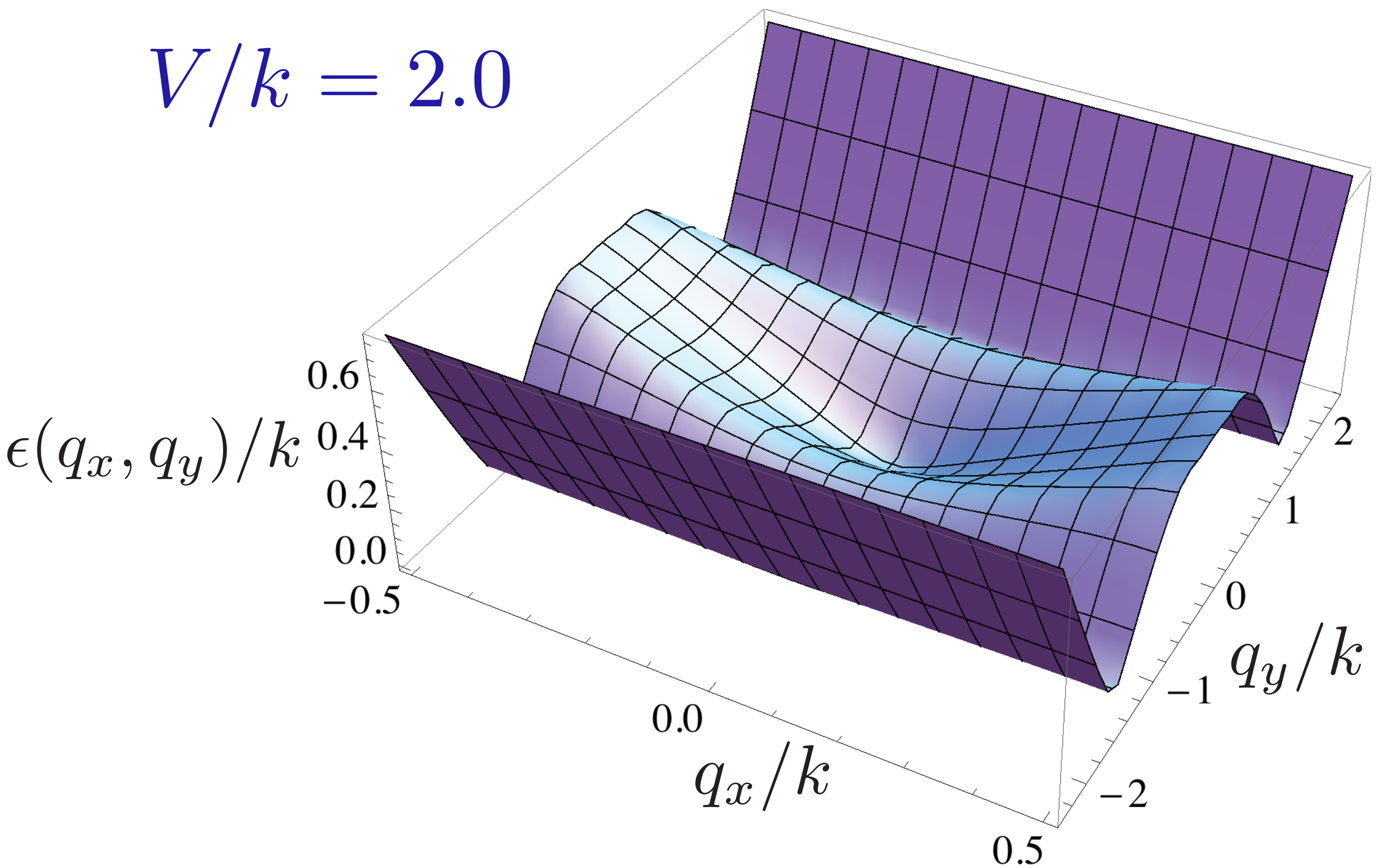


Fermion spectrum at $N_f = \infty$

$V/k = 0.8$



$V/k = 2.0$



Fermion spectrum at $N_f = \infty$

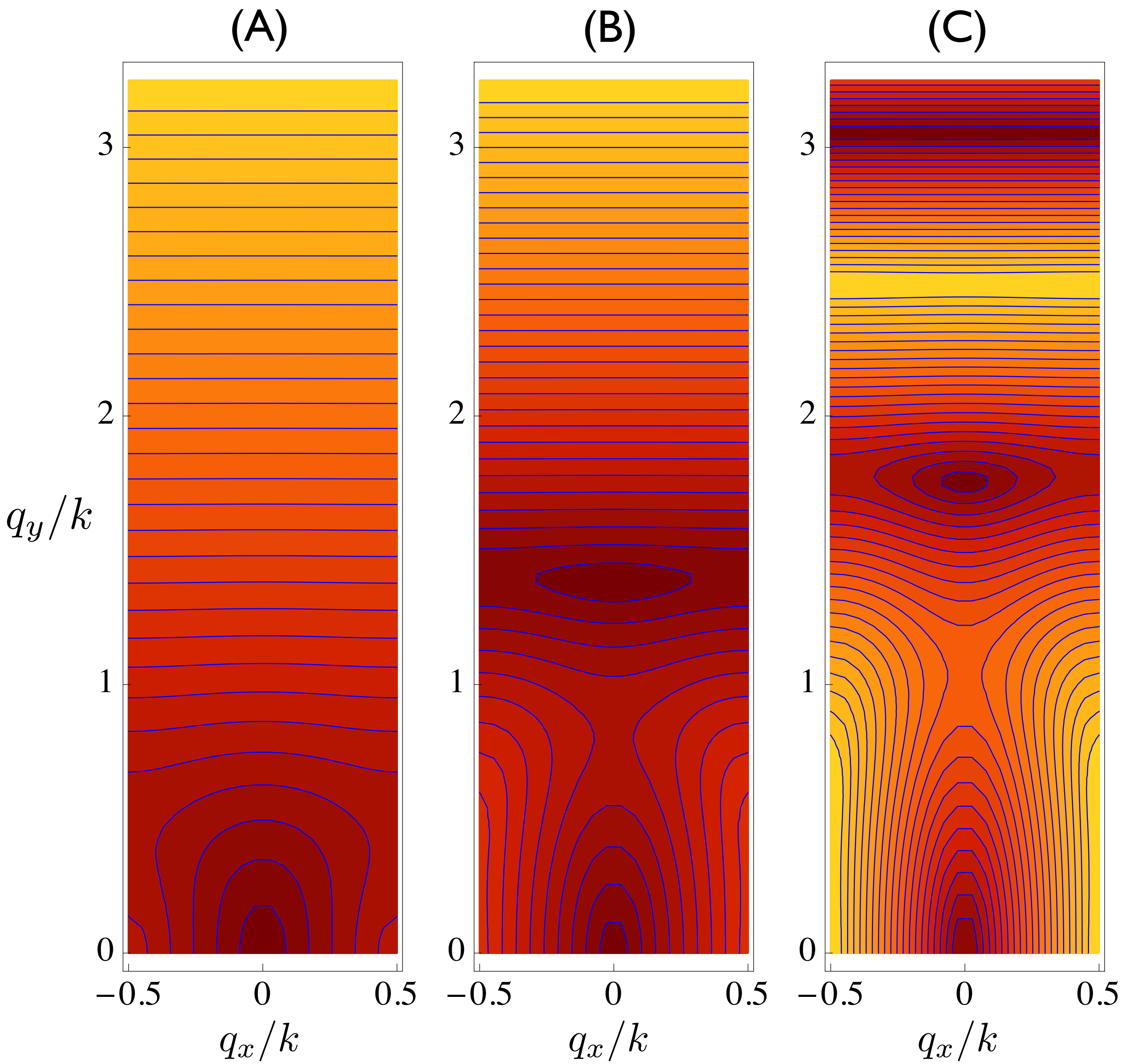


FIGURE 16: Contour plot of the lowest positive energy eigenvalues $\epsilon(q_x, q_y)$ for (A) $V/k = 0.8$, (B) $V/k = 2.0$, and (C) $V/k = 3.6$. All three plots show Dirac nodes at $(0, 0)$. However for larger V/k, additional Dirac nodes appear at (B) $(q_x/k = 0, q_y/k = \pm 1.38)$, and (C) $(q_x/k = 0, q_y/k = \pm 1.75)$, $(q_x/k = 0, q_y/k = \pm 3.06)$

Fermion spectrum at $N_f = \infty$

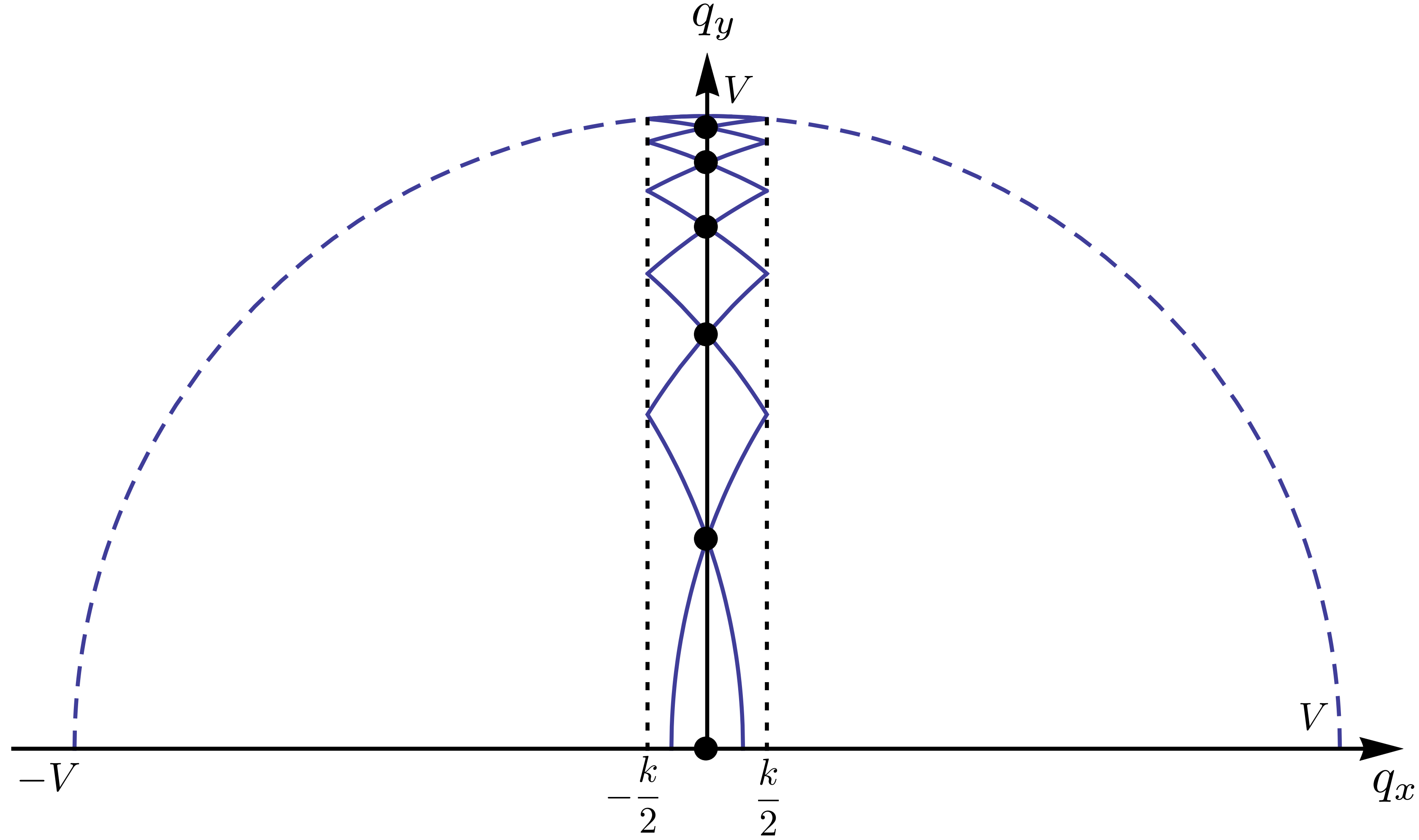
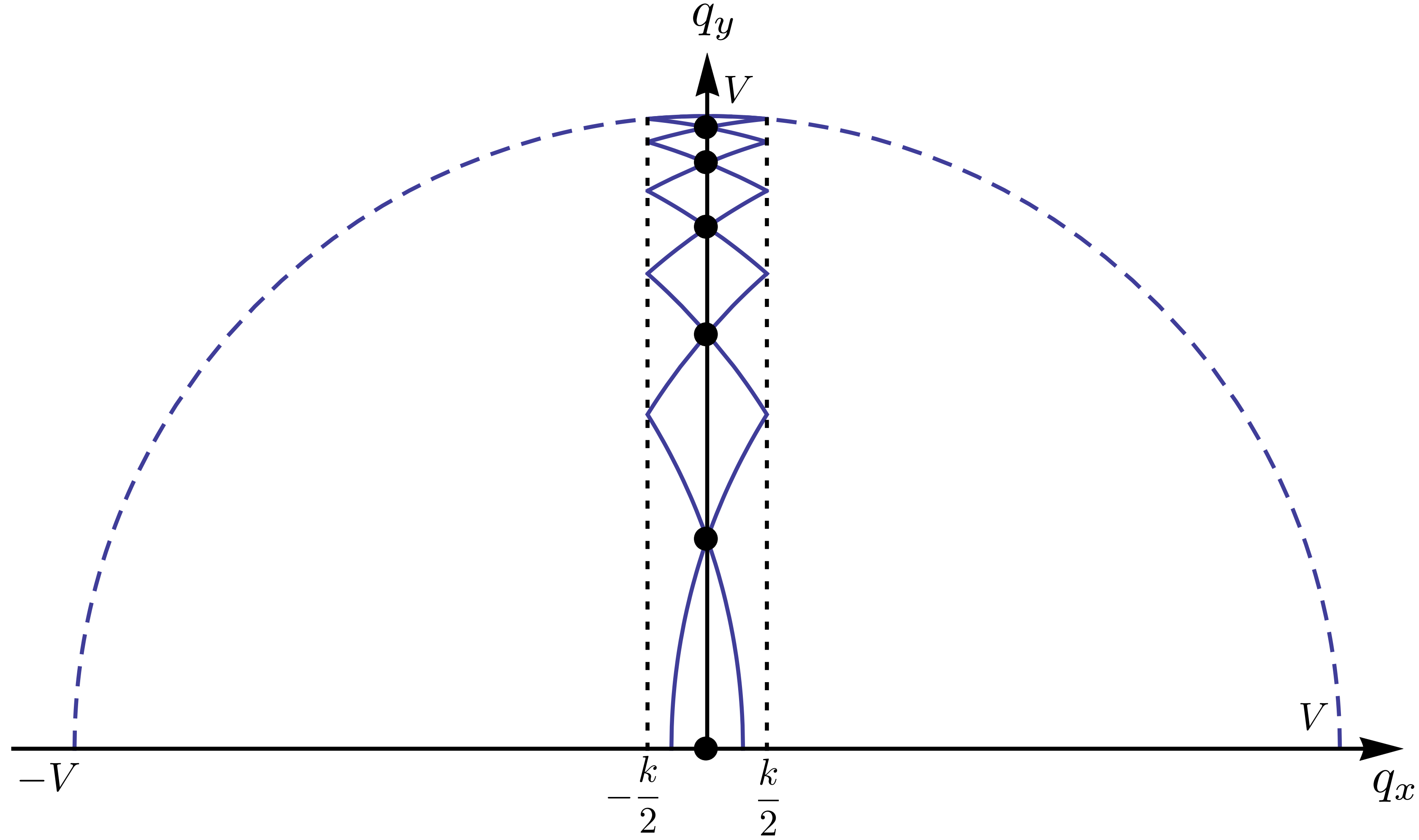


FIGURE 19: Illustration of the positions of the Dirac points with positive q_D for $V/k = 5.3$. The dashed line is the location of the electron and hole Fermi surfaces of Fig. 17. These are folded back into the first Brillouin zone $-k/2 < q_x < k/2$ and shown as the full lines. The Dirac points are the filled circles at the positions in Eq. (69), and these appear precisely at the intersection points of the folded Fermi surfaces in the first Brillouin zone.

Fermion spectrum at $N_f = \infty$



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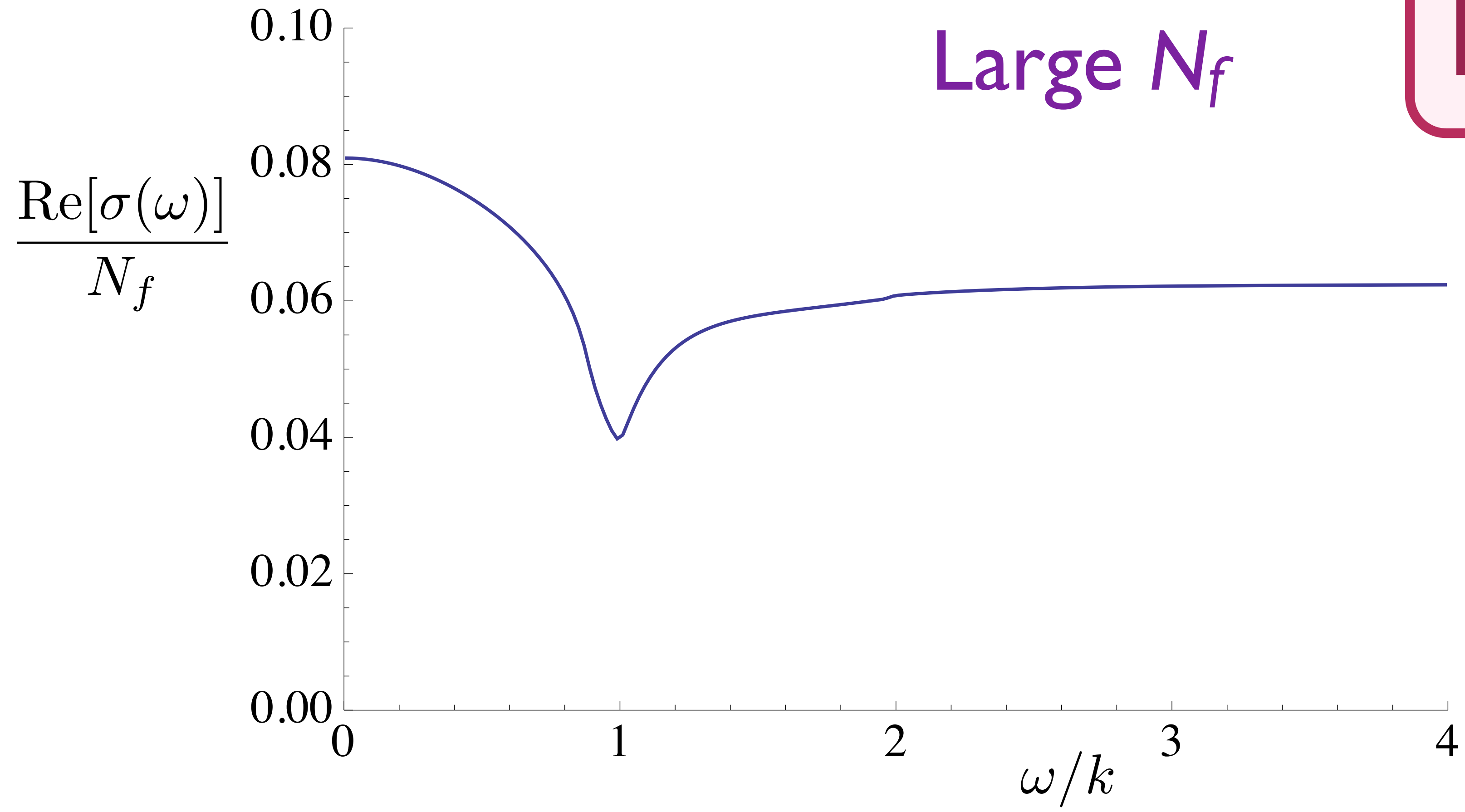
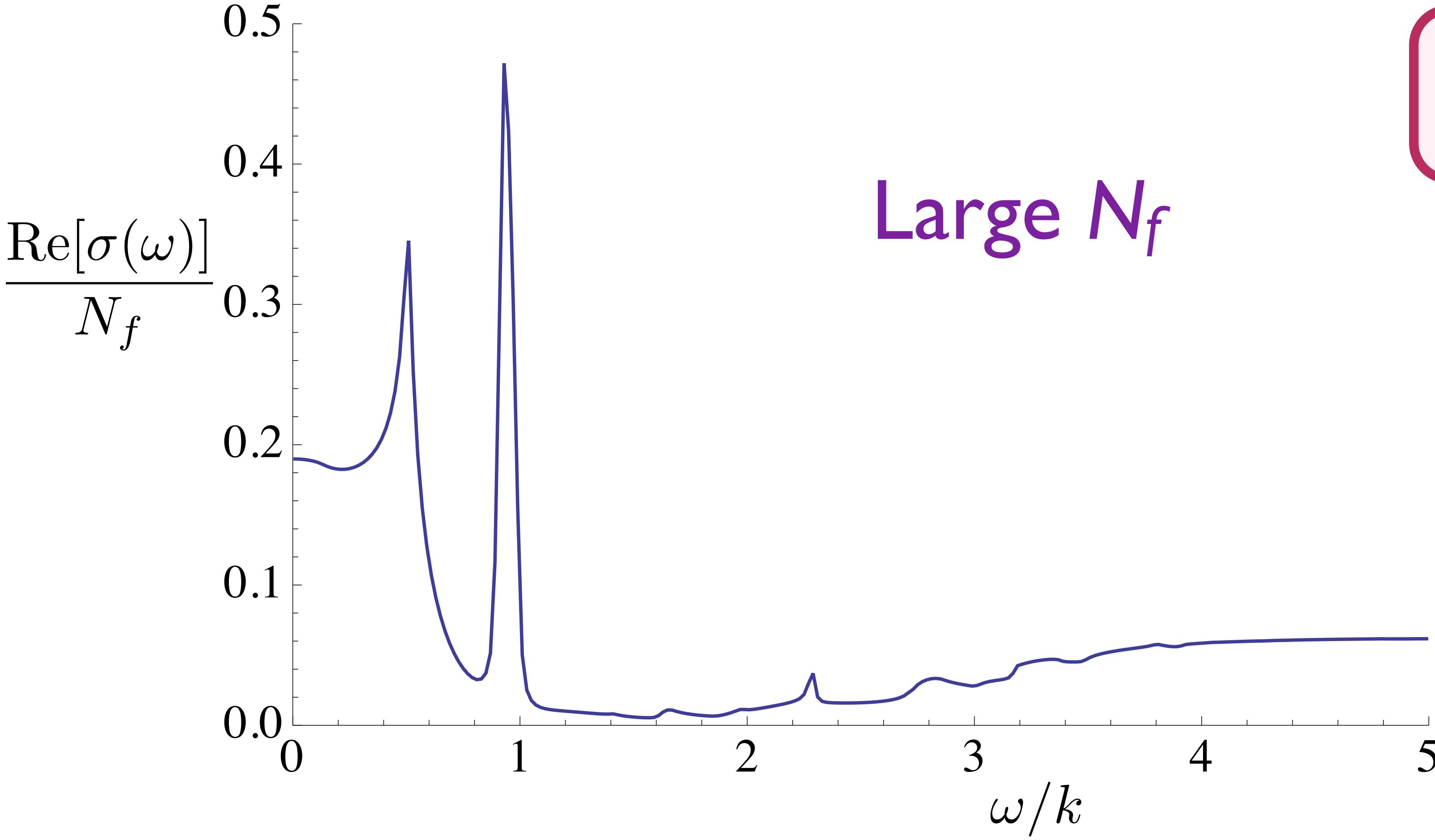
At small non-zero $1/N_f$, the IR is described by a CFT of $N_D N_f$ Dirac fermions coupled to a SU(N_c) gauge field. The value of N_D is a monotonically increasing function of V/k, and jumps in unit steps at an infinite set of critical values of V/k.

Electrical transport

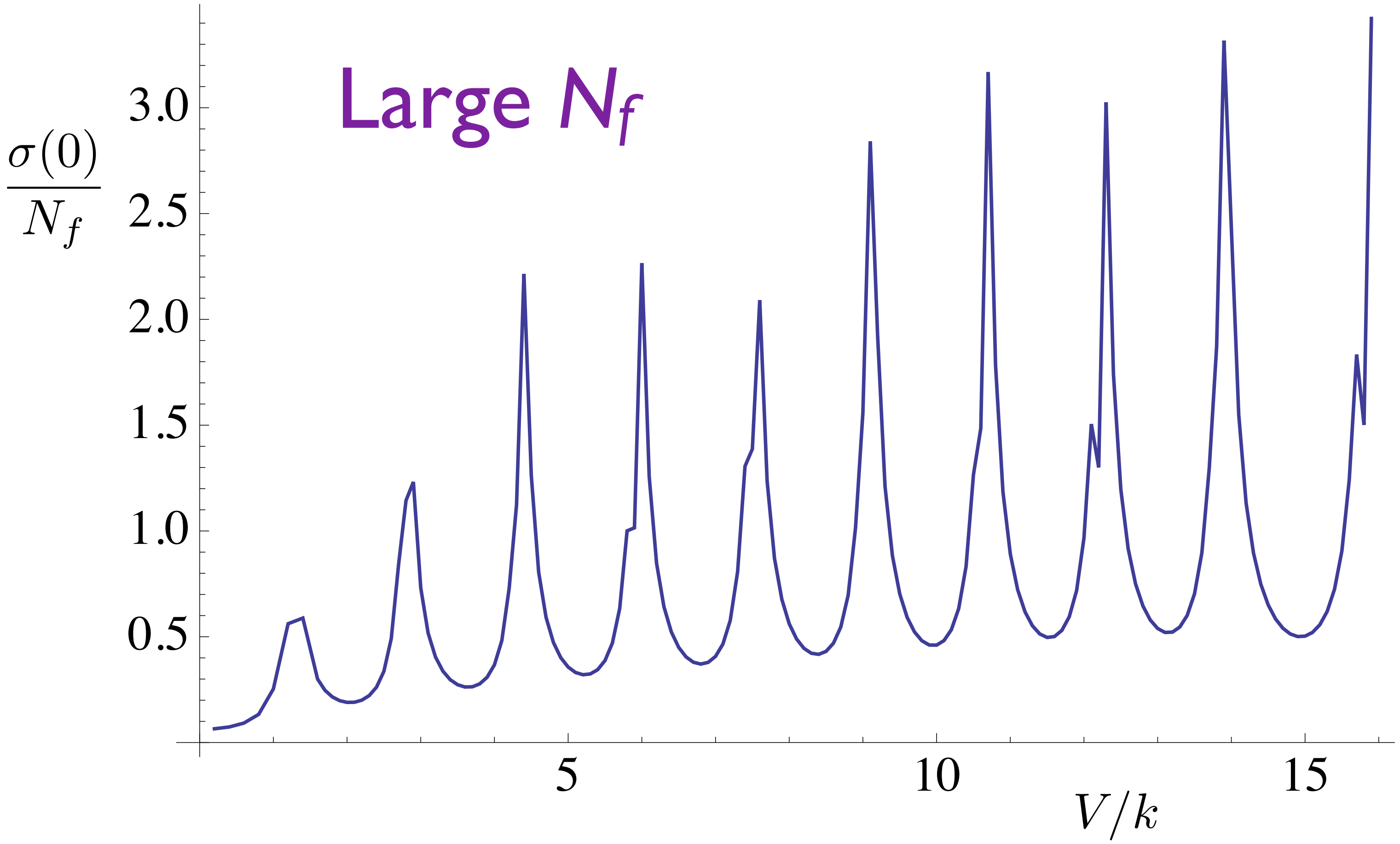


Small V/k

Electrical transport

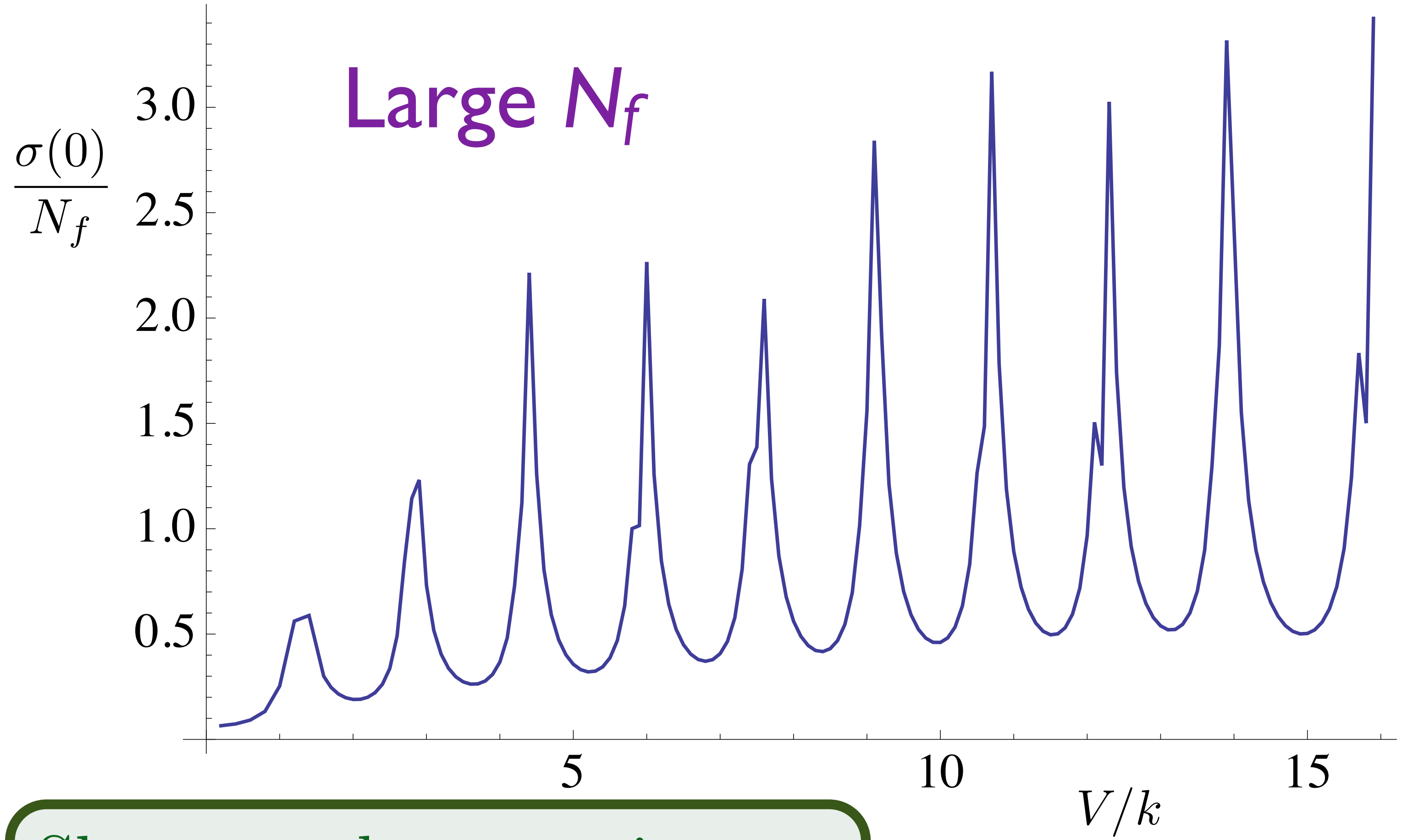


Large V/k



Electrical transport

d.c. conductivity



Electrical transport

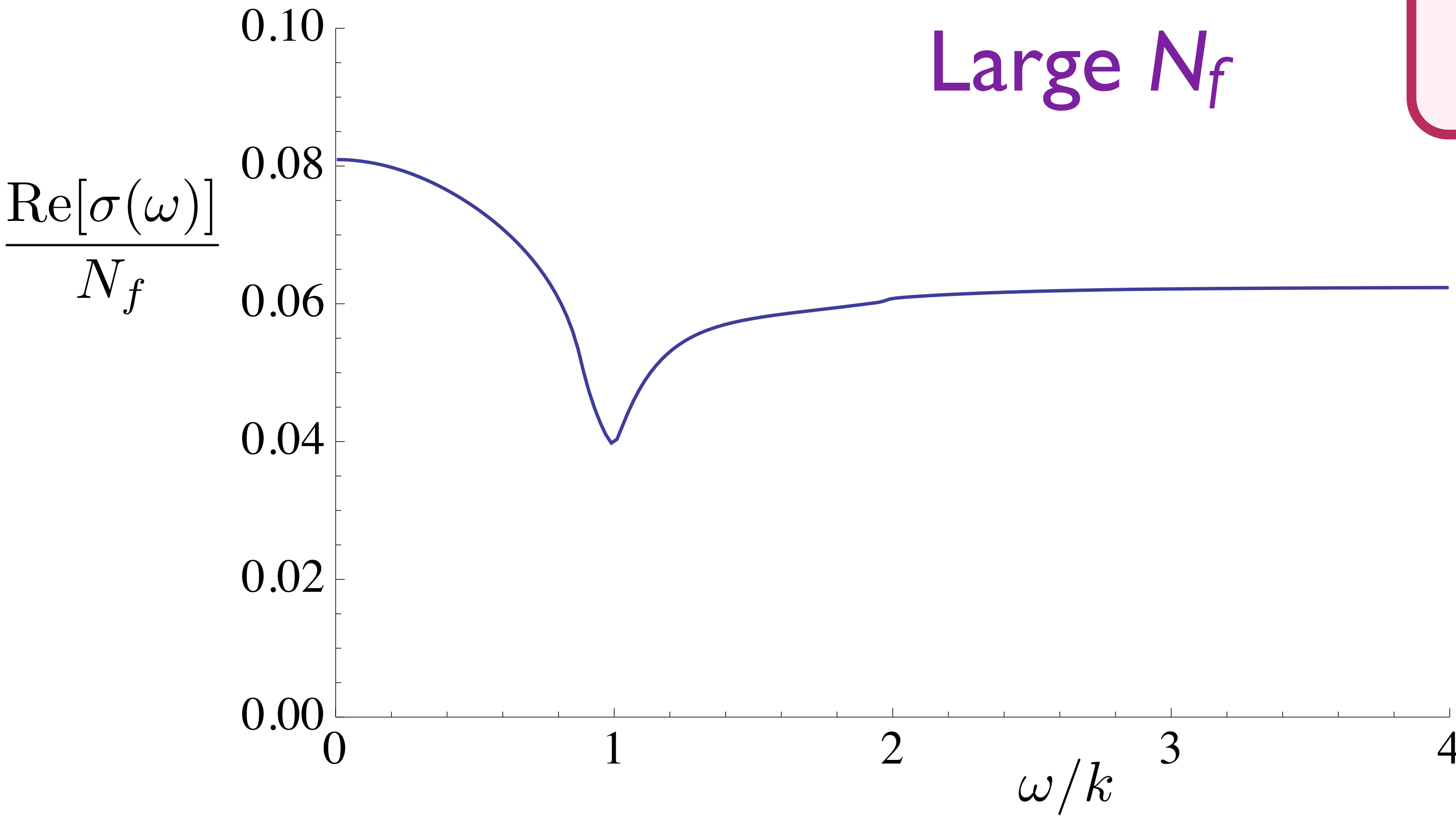
d.c. conductivity

Sharp peaks at points where IR CFT changes, and the value of N_D jumps. Signals of "hidden" Fermi surfaces in the local chemical potential.

CFT3 in a periodic chemical potential

In holography, we solved the Einstein-Maxwell equations in the presence of a periodic potential. This was done numerically, and also in a perturbation theory in V/k using Witten diagrams. Consistent results were obtained. For all values of V/k, the deep IR has a AdS_4 metric, with couplings that change smoothly as a function of V/k.

Electrical transport



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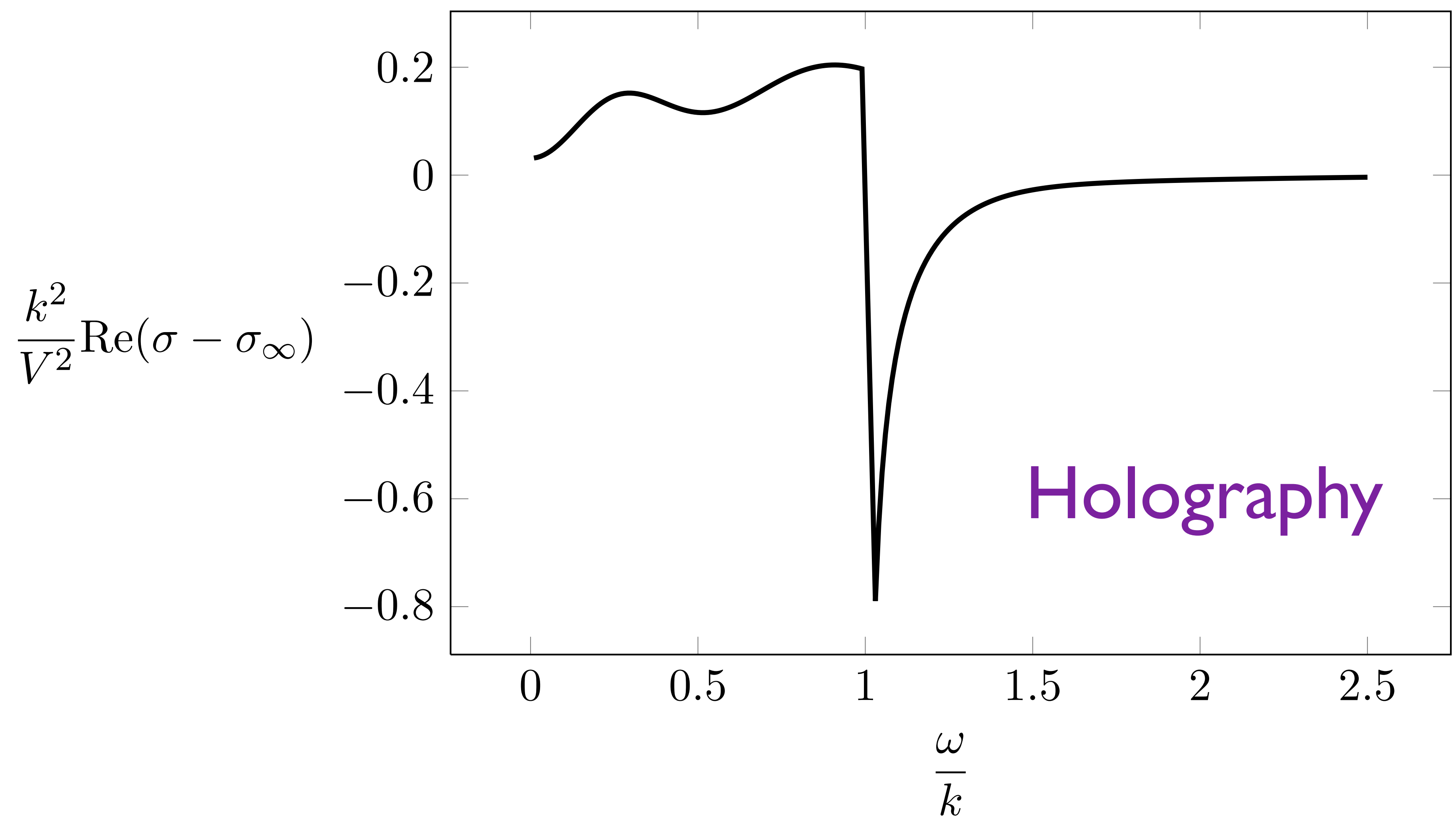
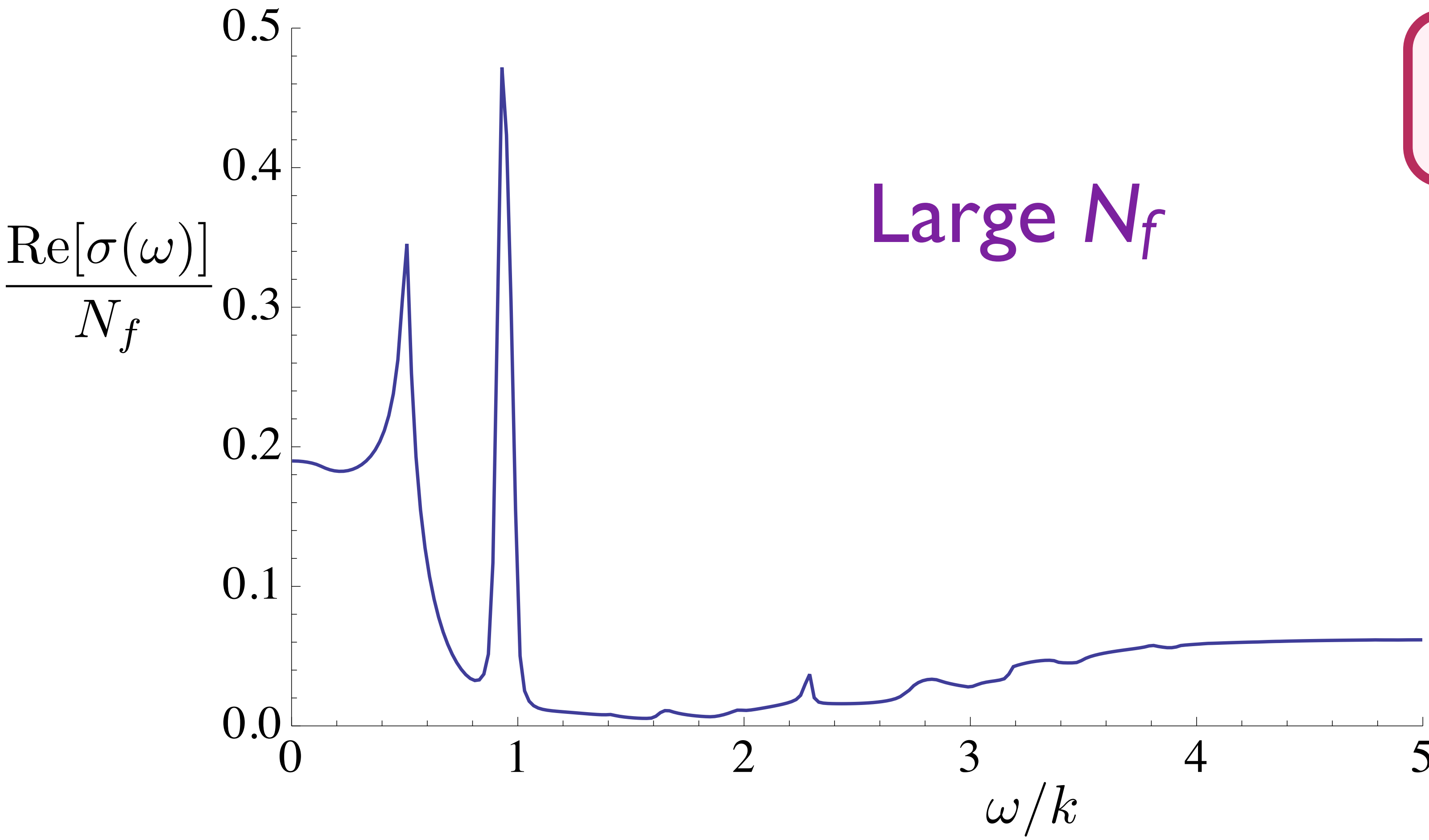


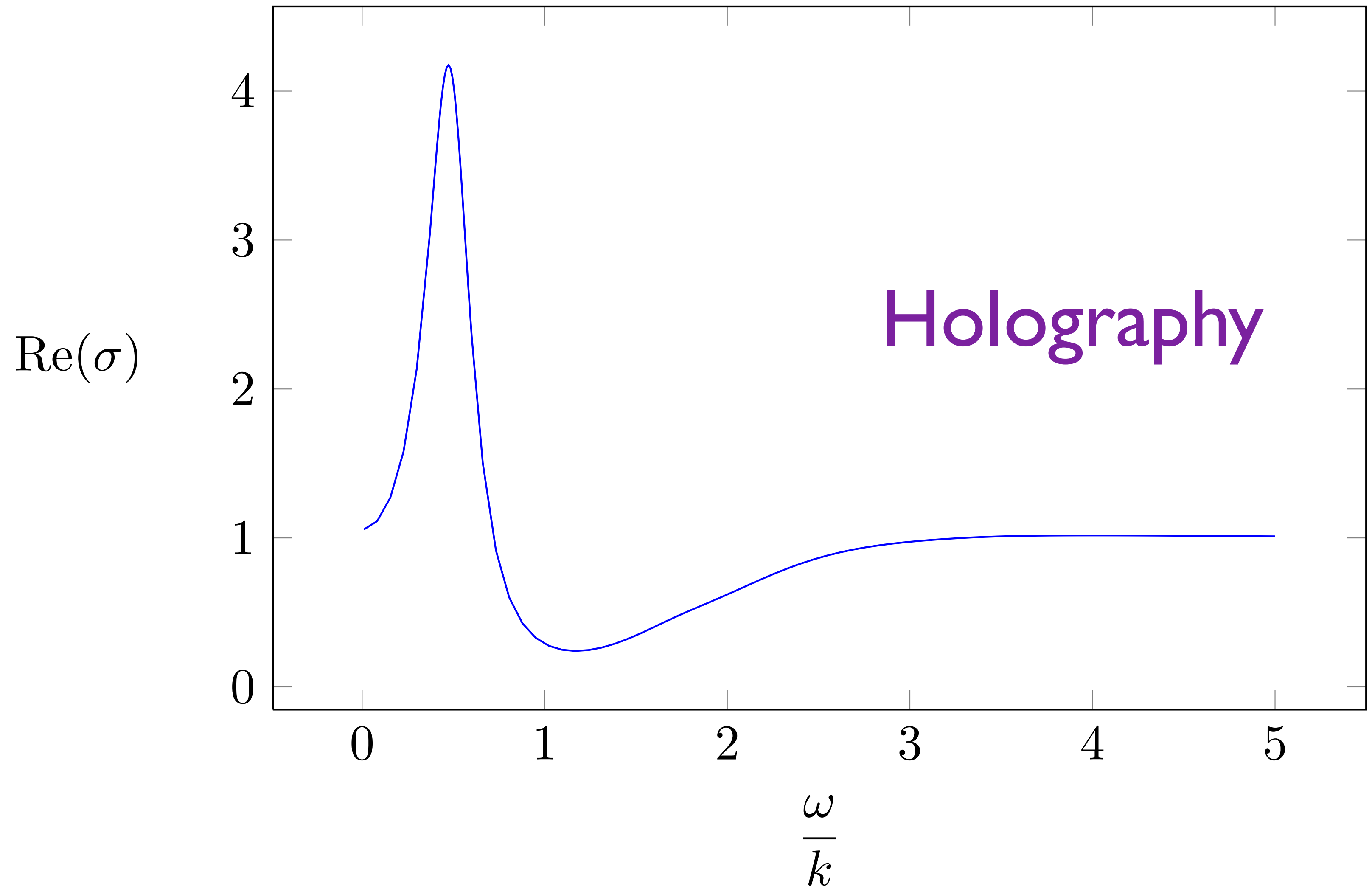
Electrical transport

Large N_f



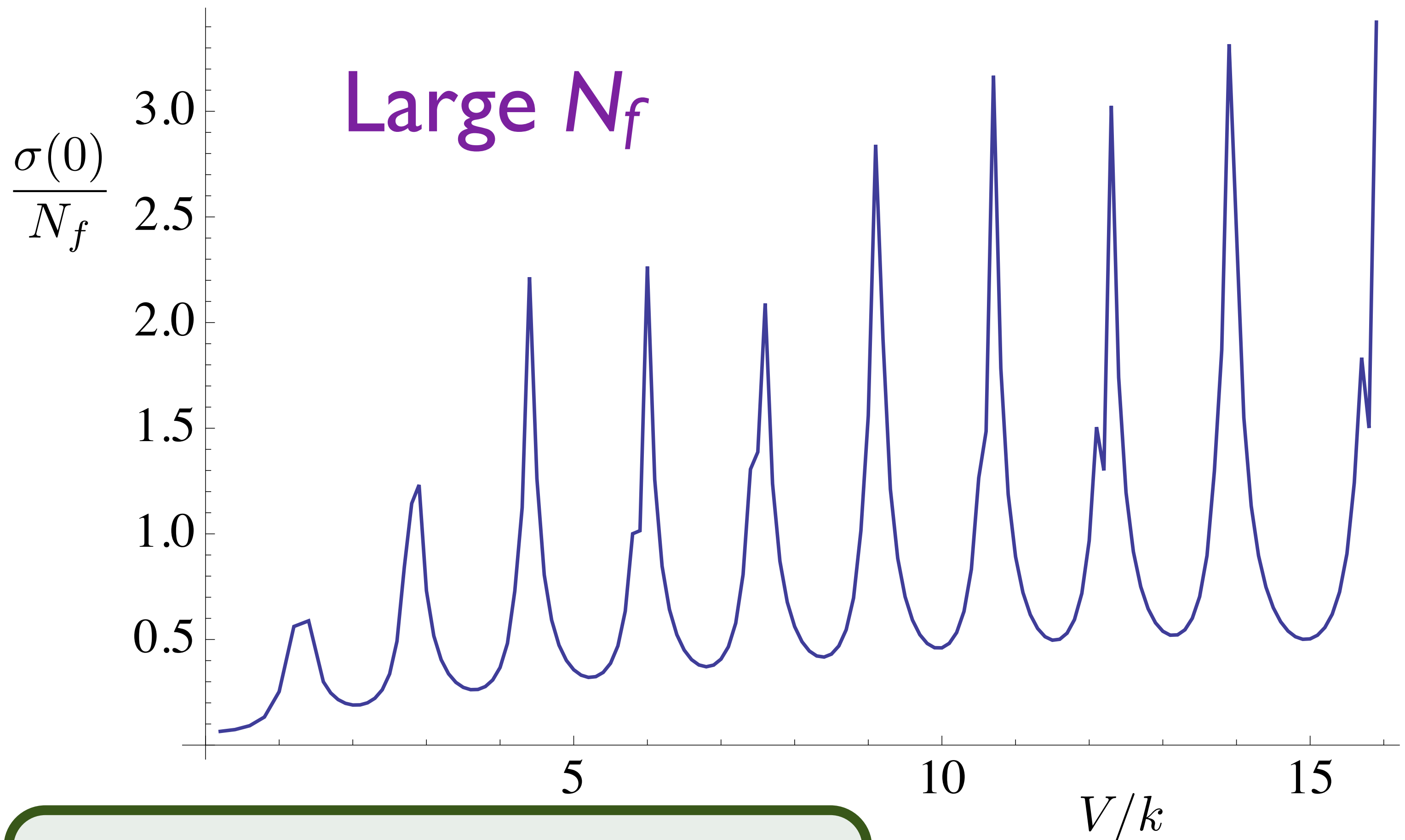
Large V/k

Holography



Electrical transport

d.c. conductivity



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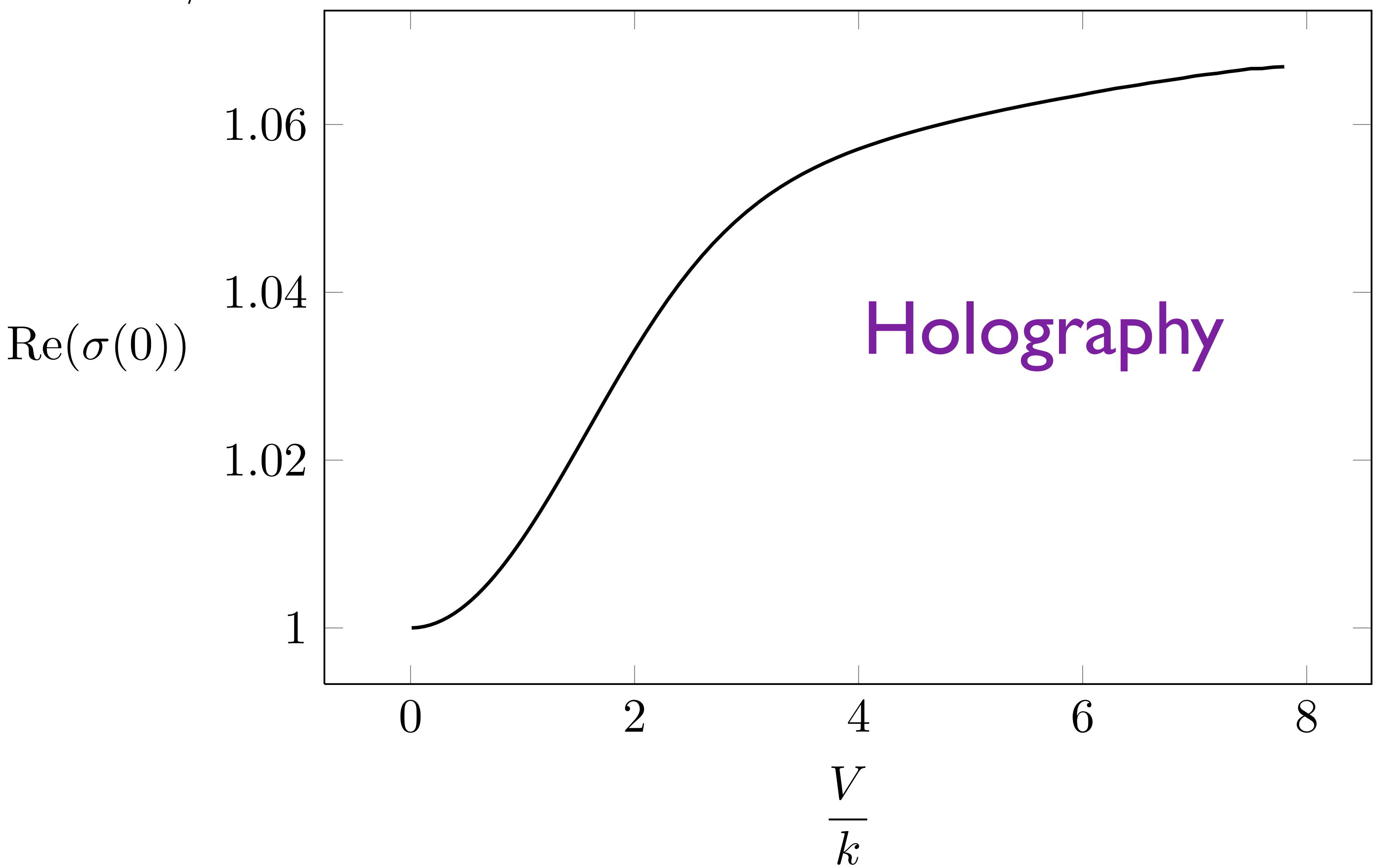

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Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension $d=1$

Monopoles in the 2+1 dimensional bulk U(1) gauge field acquire a Berry phase determined by the boundary U(1) charge density $\mathcal{Q}$, and a dilute gas theory of monopoles leads to Friedel oscillations with

$$\langle \rho(x) \rho(0) \rangle \sim \frac{\cos(2k_F x)}{|x|^{2\Delta_F}}$$

T. Faulkner and N. Iqbal, arXiv:1207.4208

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015

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T. Faulkner and N. Iqbal, arXiv:1207.4208

Exact solution of adjoint Dirac fermions at non-zero density coupled to a SU(N_c) gauge field: low energy theory has an emergent $\mathcal{N} = (2,2)$ supersymmetry, the global U(1) symmetry becomes the R-symmetry, and there are Friedel oscillations with

$$\Delta_F = 1/3 \quad \text{for all } N_c \geq 2$$

R. Gopakumar, A. Hashimoto, I.R. Klebanov, S. Sachdev, and K. Schoutens, arXiv:1206.4719

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension d=2

- For every CFT in 2+1 dimensions with a globally conserved U(1), we can define a monopole operator which transforms as a scalar under conformal transformations.
  e.g. for the XY model, we insert a monopole at x_m by including a *fixed* background gauge flux α_μ so that

$$\mathcal{L} = |(\partial_\mu - i\alpha_\mu)\psi|^2 + s|\psi|^2 + u|\psi|^4$$

  where the flux $\beta_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu \alpha_\lambda$ obeys

$$\partial_\mu \beta_\mu = 2\pi\delta(x - x_m) \quad , \quad \epsilon_{\mu\nu\lambda}\partial_\nu(\Omega\beta_\nu) = 0$$

  where the CFT lives on the conformally flat space with is $ds^2 = \Omega^{-2} dx_\mu^2$.

S. Sachdev, Phys. Rev. D **86**, 126003 (2012)

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension $d=2$

- In the holographic theory, we have a bulk scalar field Φ_m (conjugate to the monopole operator of the CFT) which carries the charge of the S-dual of the 4-dimensional bulk U(1) gauge field:

$$\mathcal{S}_m = \int d^4x\sqrt{-g}\left[|(\nabla - 2\pi i\widetilde{A})\Phi_m|^2 + \ldots\right]$$

where $\widetilde{F} = d\widetilde{A} = *F = *dA$.

S. Sachdev, Phys. Rev. D **86**, 126003 (2012)

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension $d=2$

- When a chemical potential is applied to the boundary CFT, Φ_m experiences a magnetic flux. Consequently condensation of Φ_m leads to a vortex-lattice-like state, which corresponds to the formation of a *crystal* in the CFT. *The crystal has unit $\mathcal{Q}$ charge per unit cell.*

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- The back-reaction of the vortex lattice on the metric *diverges* in the IR, suggesting a "rupturing" of spacetime and a transition into a confined, insulating solid.

N. Bao, S. Harrison, S. Kachru and S. Sachdev, Physical Review D **88**, 026002 (2013)

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Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

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- Does a vortex-liquid-like state of the Φ_m will yield the Friedel oscillations of the Fermi surface, with the correct Fermi wavevector ?

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension d=2

- Does a vortex-liquid-like state of the Φ_m will yield the Friedel oscillations of the Fermi surface, with the correct Fermi wavevector ?

- Can corrections from Φ_m fluctuations lead to the oscillations in the d.c. conductivity of a CFT3 in a periodic potential ?

A. Electrical transport in CFT3s

B. Electrical transport in CFT3s in a constant chemical potential

C. CFT3s in a periodic chemical potential

D. Monopoles