Quasiparticle damping and quantum phase transitions in d-wave superconductors

•M. Vojta
•Y. Zhang

Subir Sachdev

Transparencies on-line at
http://pantheon.yale.edu/~subir/newton.pdf

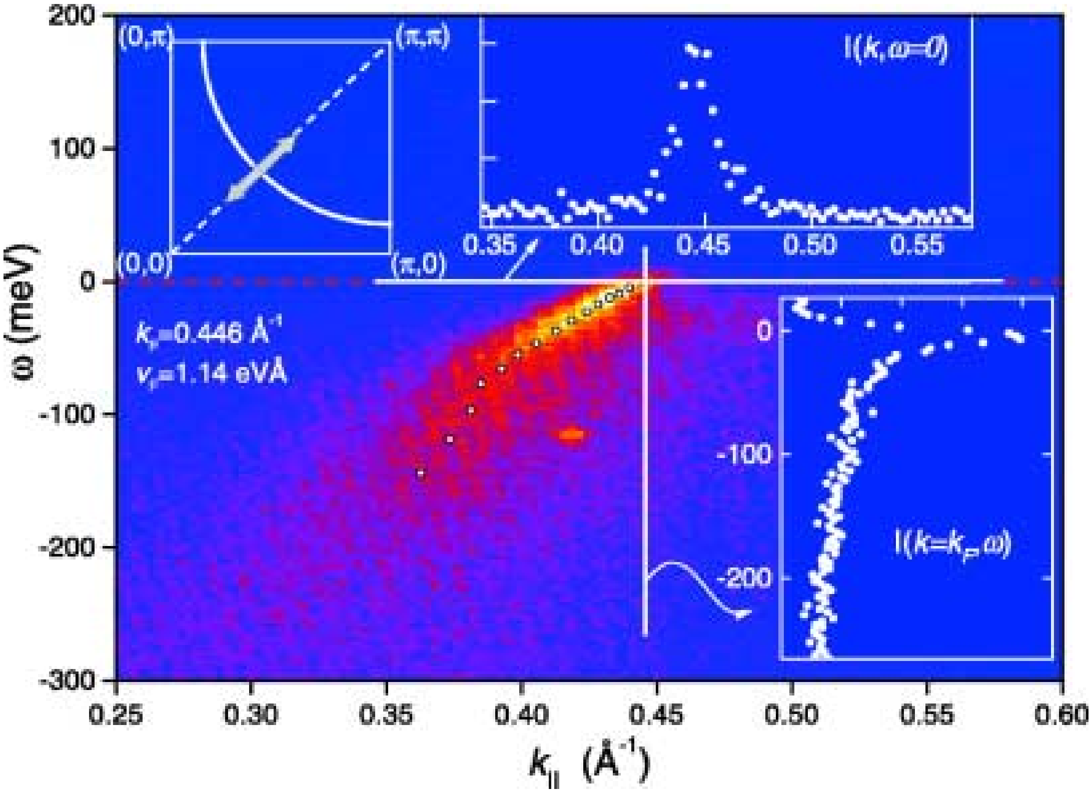
Phys. Rev. Lett. **83**, 3916 (1999).
cond-mat/0003163

Quantum Phase Transitions,
Cambridge University Press

Yale University

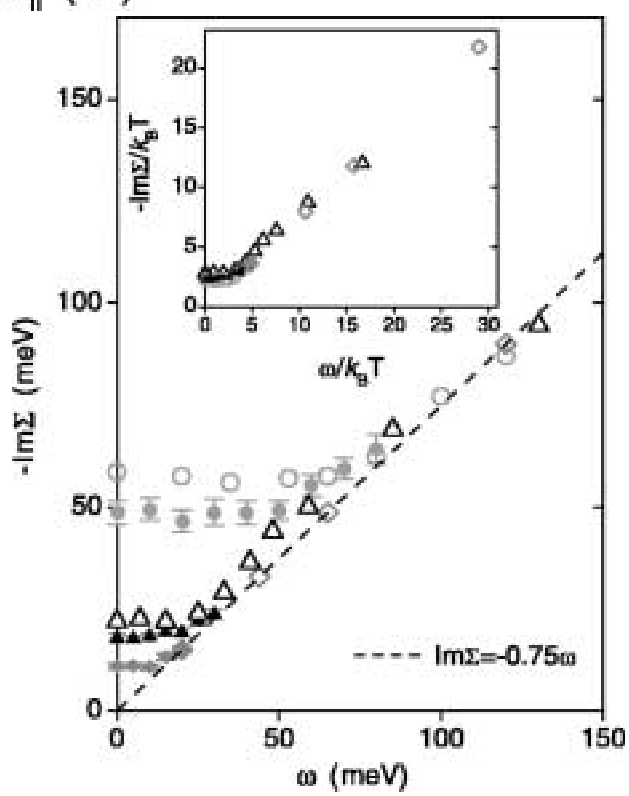
Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))



Quantum-critical damping of quasi-particles along (1,1)

Quasi-particles sharp along (1,0)



$$\mathrm{Im}\Sigma \sim k_B T \;\text{ for }\; \hbar\omega < k_B T$$

$$\mathrm{Im}\Sigma \sim \hbar\omega \;\text{ for }\; \hbar\omega > k_B T$$

"Marginal Fermi liquid" (Varma *et al* 1989) but only for nodal quasi-particles – strong ***k*** dependence at low temperatures

Origin of inelastic scattering ?

In a Fermi liquid

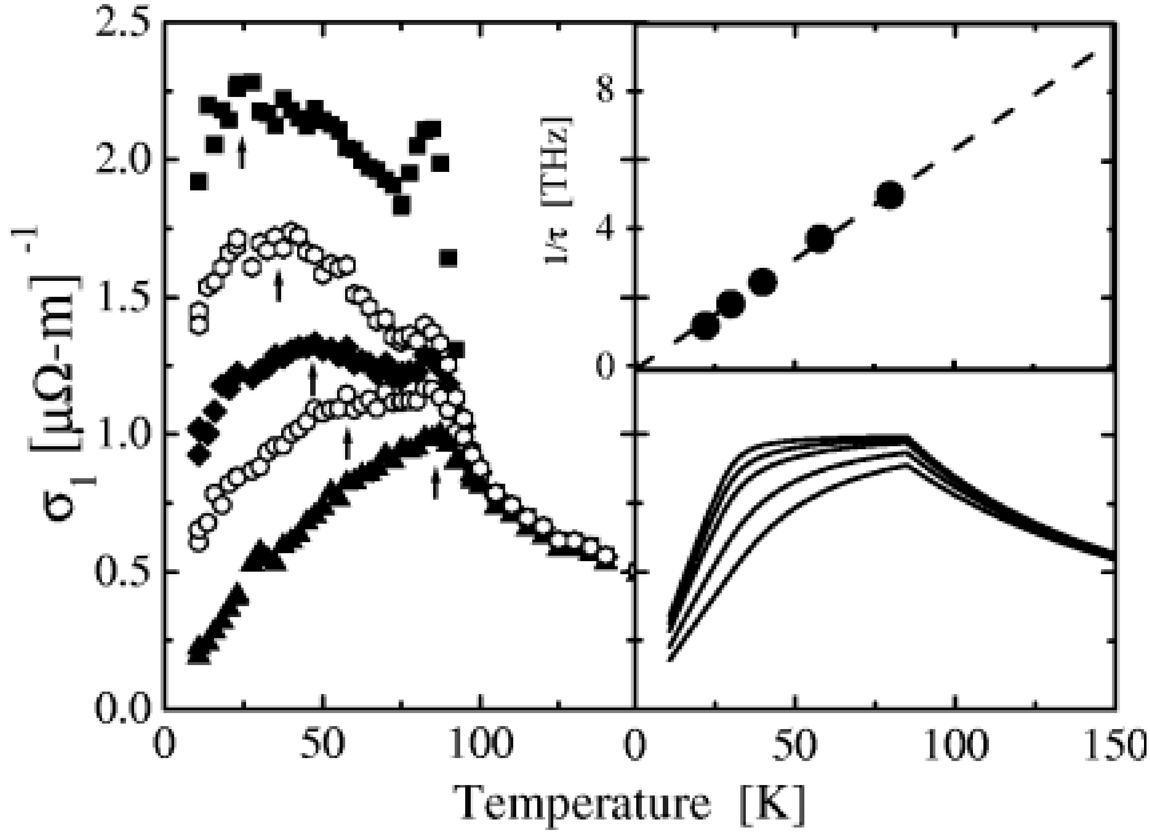
$$\mathrm{Im}\Sigma \sim T^2$$

In a BCS d-wave superconductor

$$\mathrm{Im}\Sigma \sim T^3$$

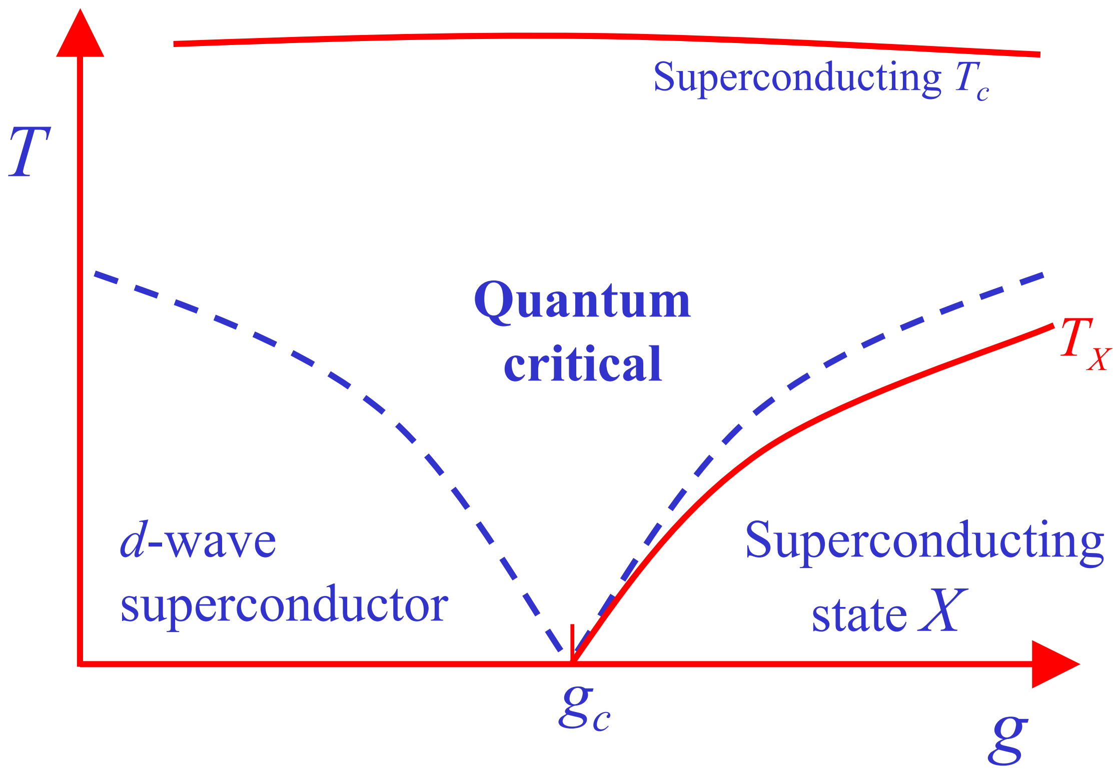
THz conductivity of BSCCO

(Corson et al cond-mat/0003243)



Quantum-critical damping of quasi-particles

Proximity to a quantum-critical point



(Crossovers analogous to those near quantum phase transitions in boson models
Weichmann *et al* 1986, Chakravarty *et al* 1989)

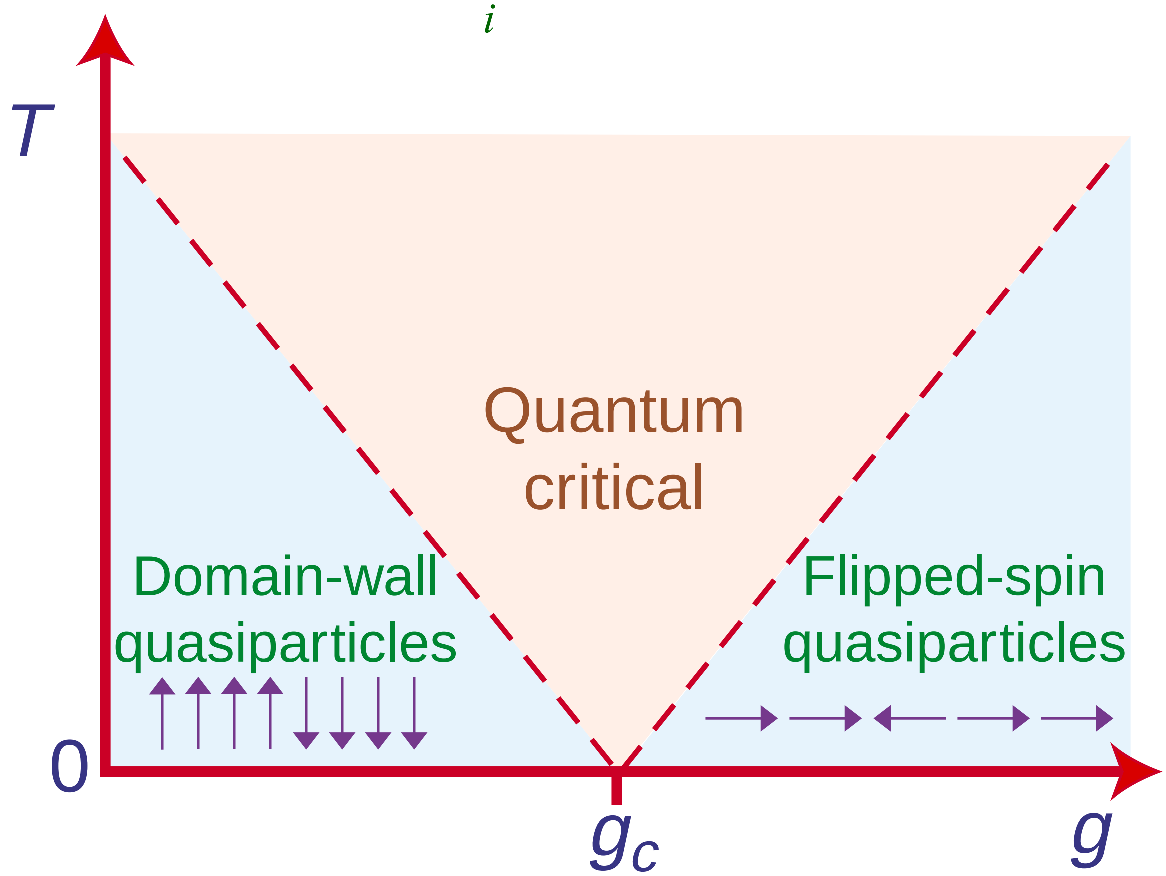
Relaxational dynamics in quantum critical region
(Sachdev+Ye, 1992)

$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \Phi\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$

Nodal quasiparticle Green's function
k → wavevector separation from node

Example: quantum Ising chain

$$H_I = -J\sum_i \left(g\sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$



In the quantum - critical regiom

$$\left\langle \sigma_i^z \sigma_j^z \right\rangle \sim |i-j|^{-1/4} \quad \text{at } T=0$$

$$\chi(\omega) \sim T^{-7/4} \Phi\left(\hbar\omega / k_B T\right)$$

$$\sim T^{-7/4}\left(1 - i\omega/\Gamma_R\right)^{-1}$$

$$\Gamma_R = \left(2\tan\frac{\pi}{16}\right)\frac{k_B T}{\hbar}$$

Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Critical field theory should not be free – required to obtain damping in the scaling limit. Combined with (1) this implies that characteristic relaxation times ~ $\hbar / k_B T$
3. Nodal quasi-particles should be part of the critical-field theory.
4. Quasi-particles along (1,0), (0,1) should not couple to critical degrees of freedom.

Outline

1. *d*-wave superconductors

2. Candidates for *X:*

   a) Superconductivity + charge density order (charge stripes)

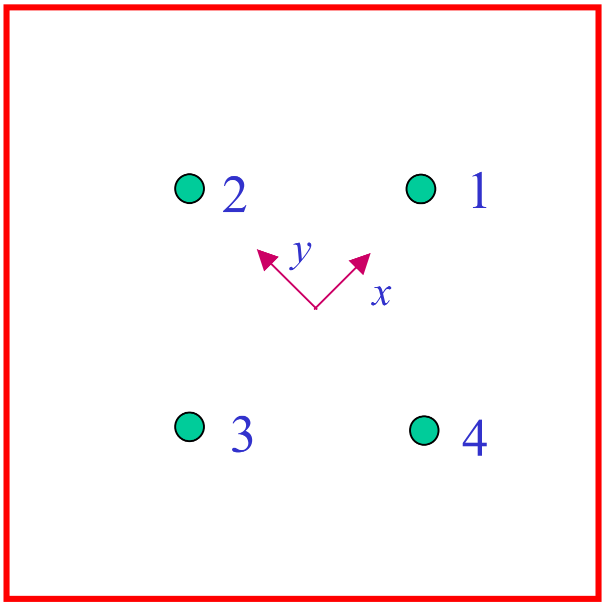
   b) Staggered-flux (or *orbital antiferromagnet*) order + *d*-wave superconductivity (breaks $\mathcal{T}$ – time-reversal symmetry).

   c) (*d*+*is*)-wave superconductivity (breaks $\mathcal{T}$)

   d) $d_{x^2-y^2} + id_{xy}$ wave superconductivity (breaks $\mathcal{T}$)

3. Phase diagram of *t*-*J* model with Sp(*N*) symmetry, *N* large.

1. *d*-wave superconductors



Gapless Fermi Points in a *d*-wave superconductor at wavevectors $(\pm K, \pm K)$

$K = 0.391\pi$

$$\Psi_1 = \begin{pmatrix} f_{1\uparrow} \\ f_{3\downarrow}^* \\ f_{1\downarrow} \\ -f_{3\uparrow}^* \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_{2\uparrow} \\ f_{4\downarrow}^* \\ f_{2\downarrow} \\ -f_{4\uparrow}^* \end{pmatrix}$$

$$S_\Psi = \int \frac{d^d k}{(2\pi)^d} T \sum_{\omega_n} \Psi_1^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_1 + \int \frac{d^d k}{(2\pi)^d} T \sum_{\omega_n} \Psi_2^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_2.$$

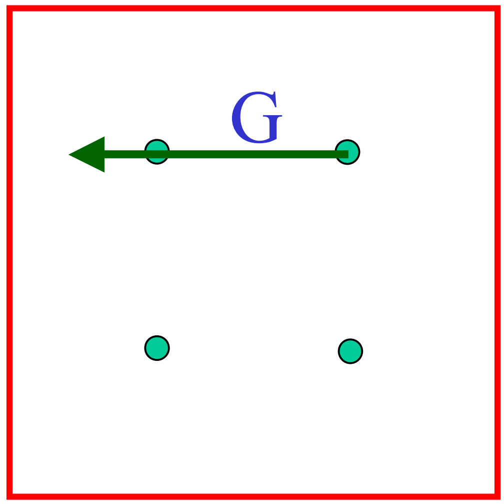
τ^x, τ^z are Pauli matrices in Nambu space

2a. Charge stripe order

Charge density

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$

If $G \neq 2K$ fermions do not couple efficiently to the order parameter and are not part of the critical theory



Action for quantum fluctuations of order parameter

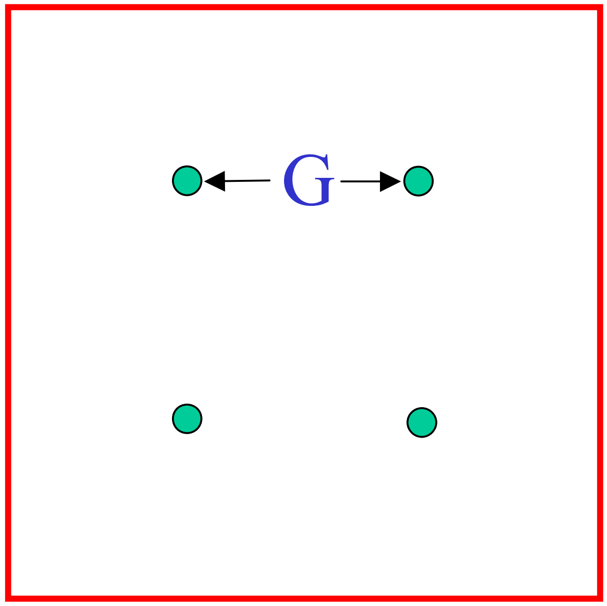
$$S_\Phi = \int d^d x d\tau \Big[|\partial_\tau \Phi_x|^2 + |\partial_\tau \Phi_y|^2 + |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + s_0 \left(|\Phi_x|^2 + |\Phi_y|^2 \right) + \frac{u_0}{2} \left(|\Phi_x|^4 + |\Phi_y|^4 \right) + v_0 |\Phi_x|^2 |\Phi_y|^2 \Big]$$

Coupling to fermions $\sim \lambda \int d^d x d\tau |\Phi_a|^2 \Psi\Psi$
and λ is irrelevant at the critical point

$$\mathrm{Im}\Sigma \sim T^{2d+1-2/\nu}$$

$$\sim T^{(\text{between 2 and 3})} \text{ for } 2/3 < \nu < 1$$

If $G=2K$:



Coupling between Φ_x, Φ_y, Ψ_1, Ψ_2 in critical field theory

$$S_{\Psi\Phi} = \int d^d x d\tau \Big[(\lambda_0 + \zeta_0) \left(\Phi_x \Psi_2^\dagger \tau^z \Psi_1 + \Phi_y \varepsilon_{ab} \Psi_{2a} \tau^x \Psi_{1b} \right) - (\lambda_0 - \zeta_0) \left(\Phi_x \Psi_2^\dagger \tau^x \Psi_1 + \Phi_y \varepsilon_{ab} \Psi_{2a} \tau^z \Psi_{1b} \right) + \text{H.c.} \Big]$$

(related work with spin-density wave order parameter: Balents, Fisher and Nayak 1998)

Central point: couplings λ_0 and ζ_0 take non-zero fixed-point values in the critical field theory

Strong inelastic scattering of nodal-quasiparticles in the scaling limit

Nodal quasiparticle lifetime $\sim \hbar / k_B T$

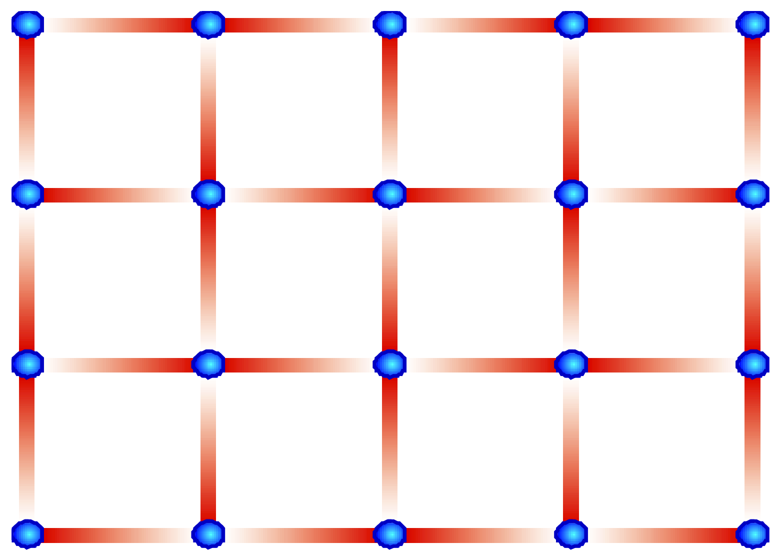
Momentum conservation inhibits scattering of quasi-particles along (1,0), (0,1) directions

However: Commensurability condition $G=2K$ appears to require fine-tuning and is not supported by experiments.

2b. Orbital antiferromagnet

Checkerboard pattern of spontaneous currents:

(Affleck+Marston 1988, Schulz 1989,
Wang, Kotliar, Wang, 1990, Wen+Lee, 1996)



$\mathcal{T}$-breaking Ising order parameter ϕ

$$\langle c^\dagger_{k+G,a} c_{k,a} \rangle = i\phi(\cos k_x - \cos k_y) \quad ; \quad G = (\pi, \pi)$$

(Nayak, 2000)

$$S_\phi = \int d^d x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{s_0}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

For K≠π/2, only coupling to nodal quasiparticles is $\sim \phi^2 \Psi\Psi$; this leads to

$$\mathrm{Im}\,\Sigma \sim T^{2d+1-2/\nu_{\mathrm{Ising}}} \sim T^{1.83}$$

<u>For completeness: assume K=π/2.</u>

Additional coupling:

$$\phi \varepsilon_{ab} \left[i\Psi_{1a}\tau^x \partial_y \Psi_{1b} + i\Psi_{2a}\tau^x \partial_x \Psi_{2b} + \mathrm{H.c.} \right].$$

The presence of the spatial derivative makes this coupling *irrelevant*.

Damping of nodal quasiparticles arises from irrelevant corrections to critical field theory

$$\mathrm{Im}\Sigma \sim T^{2+\eta_{\mathrm{Ising}}}$$

2c. (*d*+*is*)-wave superconductivity

(Kotliar, 1989)

$\mathcal{T}$-breaking Ising order parameter ϕ

$$\langle c_{k\uparrow} c_{-k\downarrow} \rangle = \Delta_0 (\cos k_x - \cos k_y) + i\phi (\cos k_x + \cos k_y).$$

Effective action:

$$S_\phi = \int d^d x d\tau \left[\frac{1}{2} (\partial_\tau \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 + \frac{s_0}{2} \phi^2 + \frac{u_0}{24} \phi^4 \right]$$

Efficient coupling to nodal quasi-particles (*generically*)

$$S_{\Psi\phi} = \int d^d x d\tau \left[\lambda_0 \phi \left(\Psi_1^\dagger \tau^y \Psi_1 + \Psi_2^\dagger \tau^y \Psi_2 \right) \right].$$

Coupling λ_0 takes a non-zero fixed-point value in the critical field theory

Strong inelastic scattering of nodal-quasiparticles in the scaling limit

Nodal quasiparticle lifetime ~ $\hbar / k_B T$

However: strong scattering of quasi-particles also along (1,0), (0,1) directions

2d. $d_{x^2-y^2} + id_{xy}$ -wave superconductivity

(Rokhsar 1993, Laughlin 1994)

$\mathcal{T}$-breaking Ising order parameter ϕ

$$\langle c_{k\uparrow} c_{-k\downarrow} \rangle = \Delta_0 (\cos k_x - \cos k_y) + i\phi \sin k_x \sin k_y .$$

Effective action:

$$S_\phi = \int d^d x d\tau \left[\frac{1}{2} (\partial_\tau \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 + \frac{s_0}{2} \phi^2 + \frac{u_0}{24} \phi^4 \right]$$

Efficient coupling to nodal quasi-particles (*generically*)

$$S_{\Psi\phi} = \int d^d x d\tau \left[\lambda_0 \phi \left(\Psi_1^\dagger \tau^y \Psi_1 - \Psi_2^\dagger \tau^y \Psi_2 \right) \right] .$$

Coupling λ_0 takes a non-zero fixed-point value in the critical field theory

Strong inelastic scattering of nodal-quasiparticles in the scaling limit

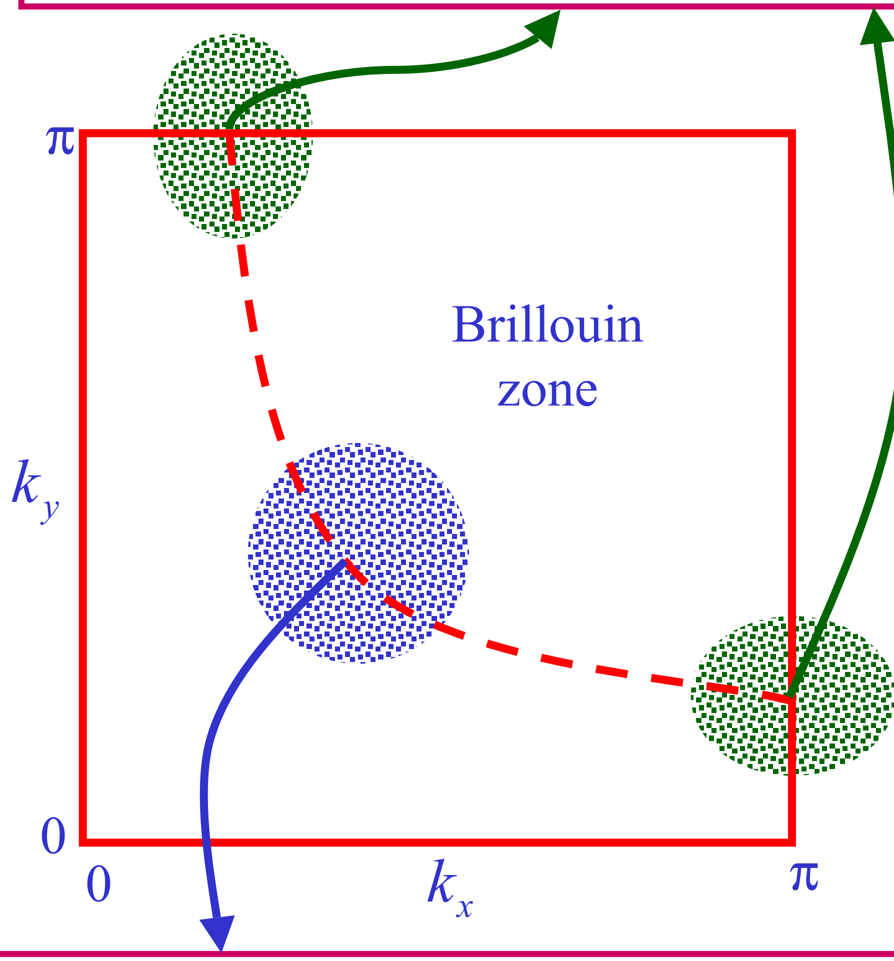
Nodal quasiparticle lifetime ~ $\hbar / k_B T$

Moreover: no scattering of quasi-particles along (1,0), (0,1) directions !

Gapped quasiparticles:

Below T_c : negligible damping

Above T_c: damping from strong coupling to superconducting phase and SDW fluctuations.



Nodal quasiparticles:

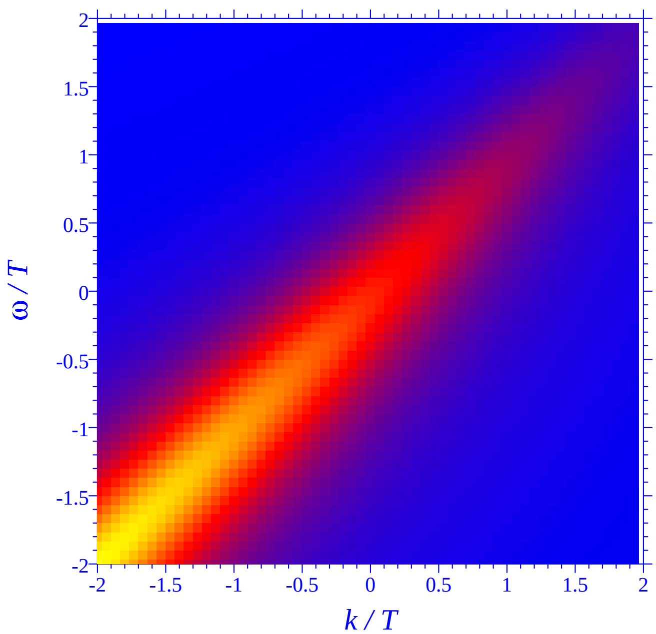
Below T_c : damping from fluctuations to $d_{x^2-y^2}+id_{xy}$ order

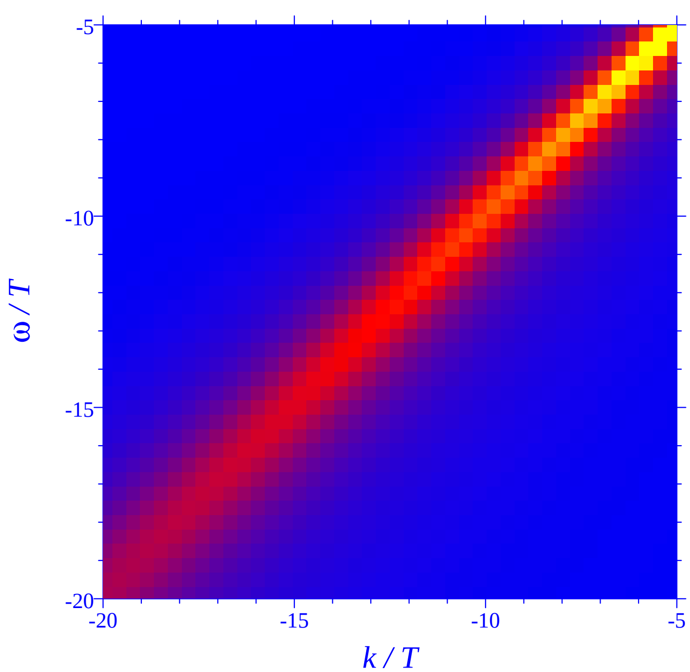
Above T_c: same mechanism applies as long as quantum-critical length < superconducting phase coherence length. Quasiparticles do not couple to phase or SDW fluctuations.

Results from expansion in ε=3-*d* for transition from *d* to $d_{x^2-y^2} + i d_{xy}$

T>0 fermion Green's function:

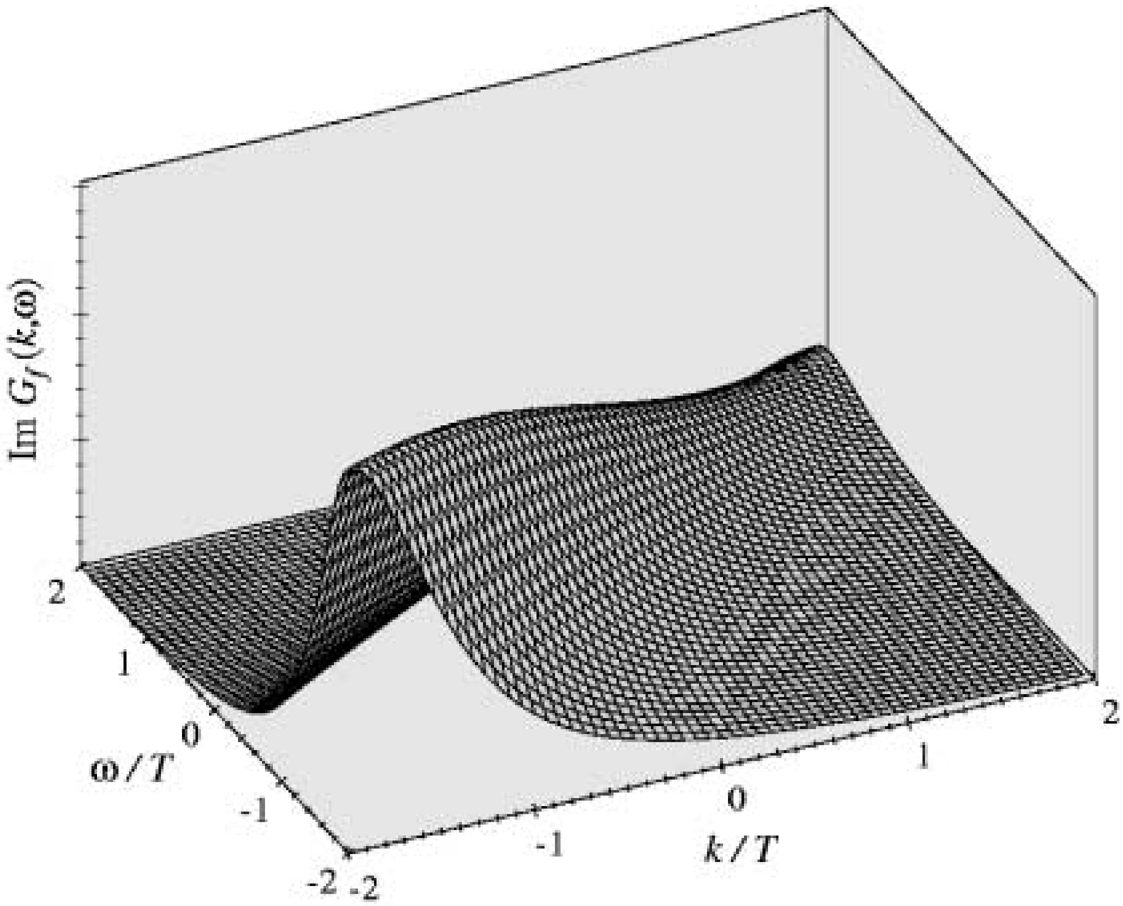
$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} X_1\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$





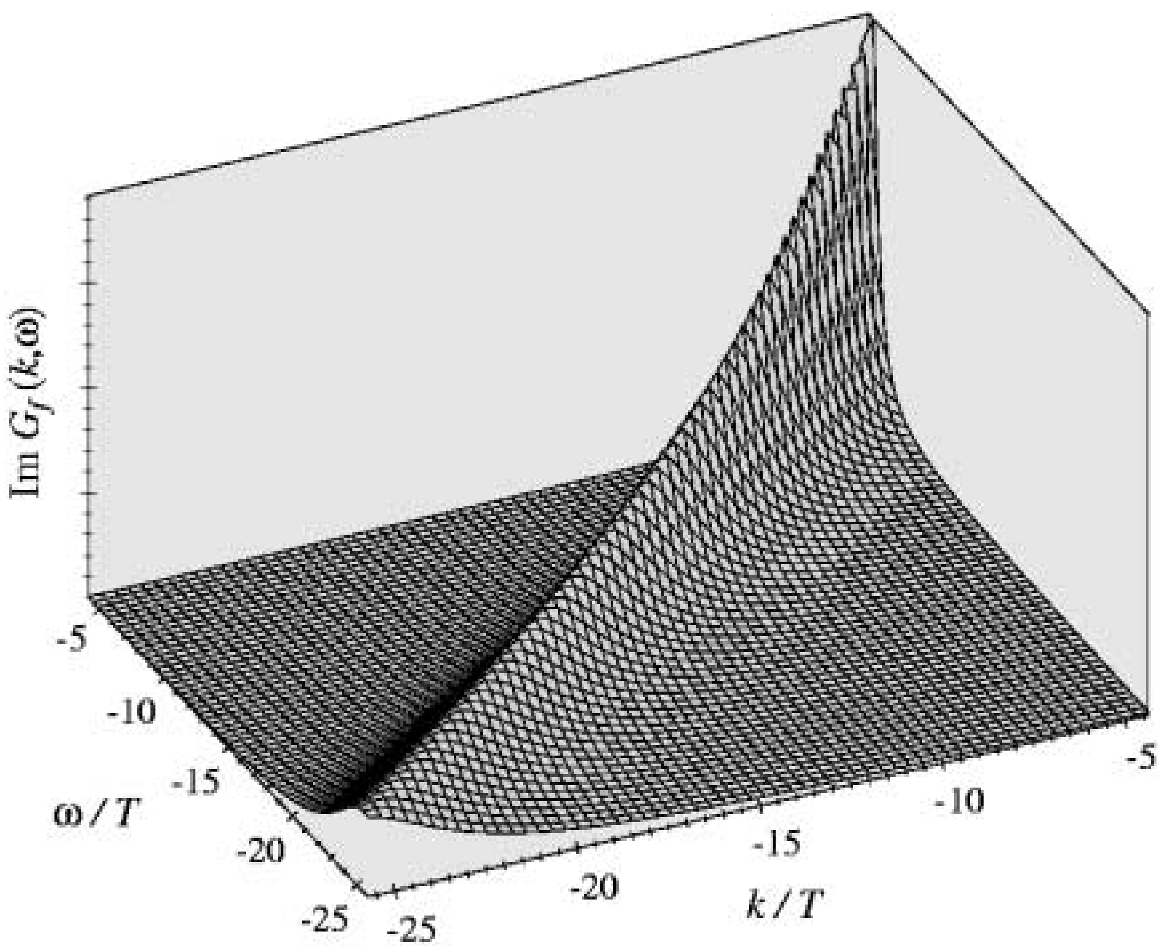
T>0 fermion Green's function:

$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \mathrm{X}_1\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$



T>0 fermion Green's function:

$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \mathrm{X}_1\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$



3. Phase diagram of an extended t-J model

Extended t-J model on the square lattice

$$H = \sum_{i>j} \left[-t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + J_{ij} \vec{S}_i \cdot \vec{S}_j + V_{ij} n_i n_j \right]$$

Plot phase diagram of stable ground states as a function of:

(1) Doping δ

(2) Frustration $\frac{J_2 \text{ (second neighbor)}}{J_1 \text{ (first neighbor)}}$

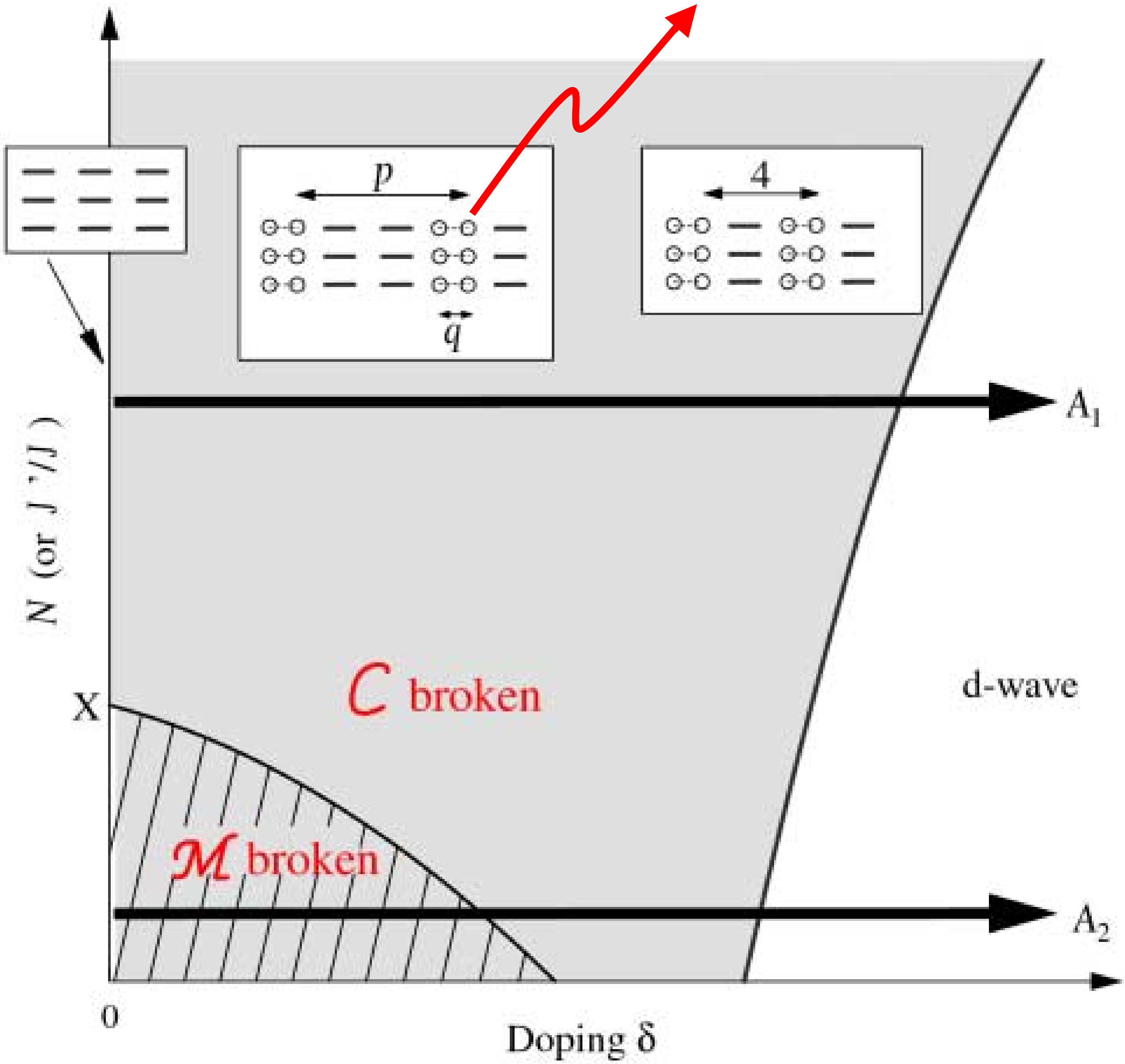
OR

N, where spin symmetry

$$SU(2) \rightarrow Sp(N)$$

Schematic phase diagram

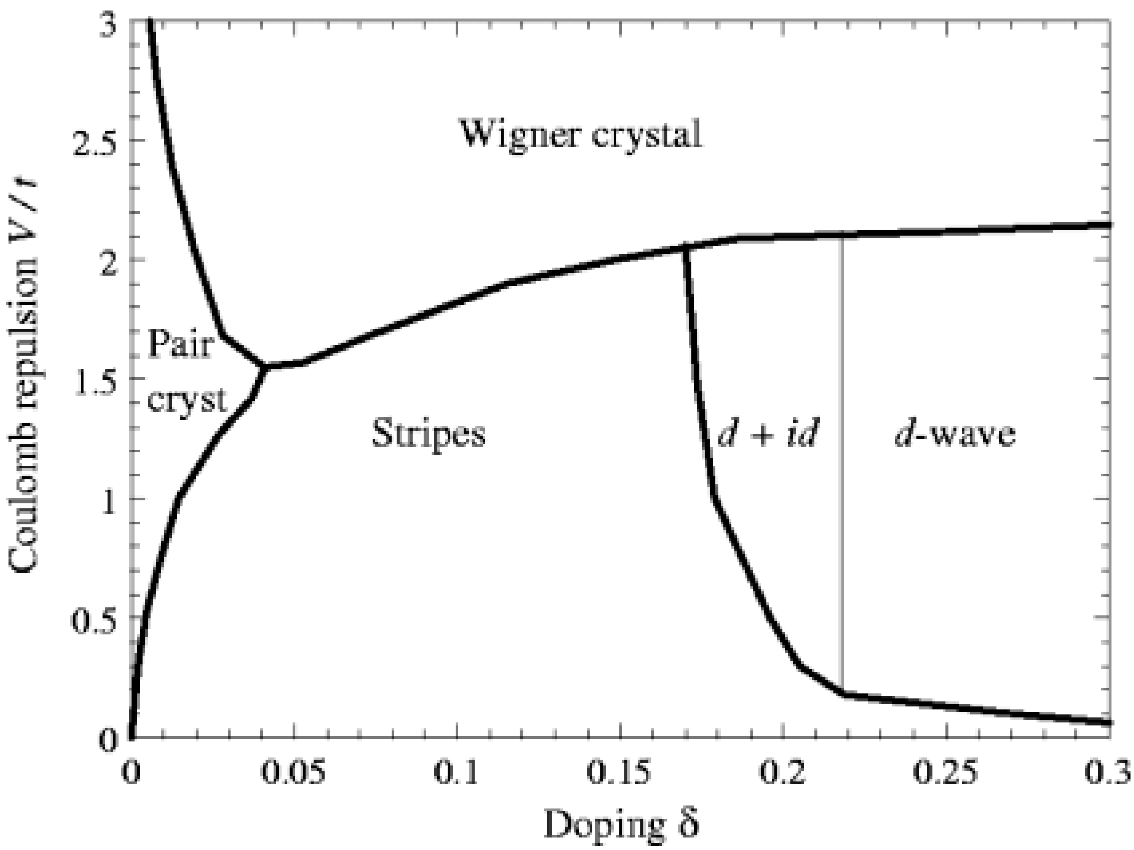
hole density ≈ 0.5/unit length



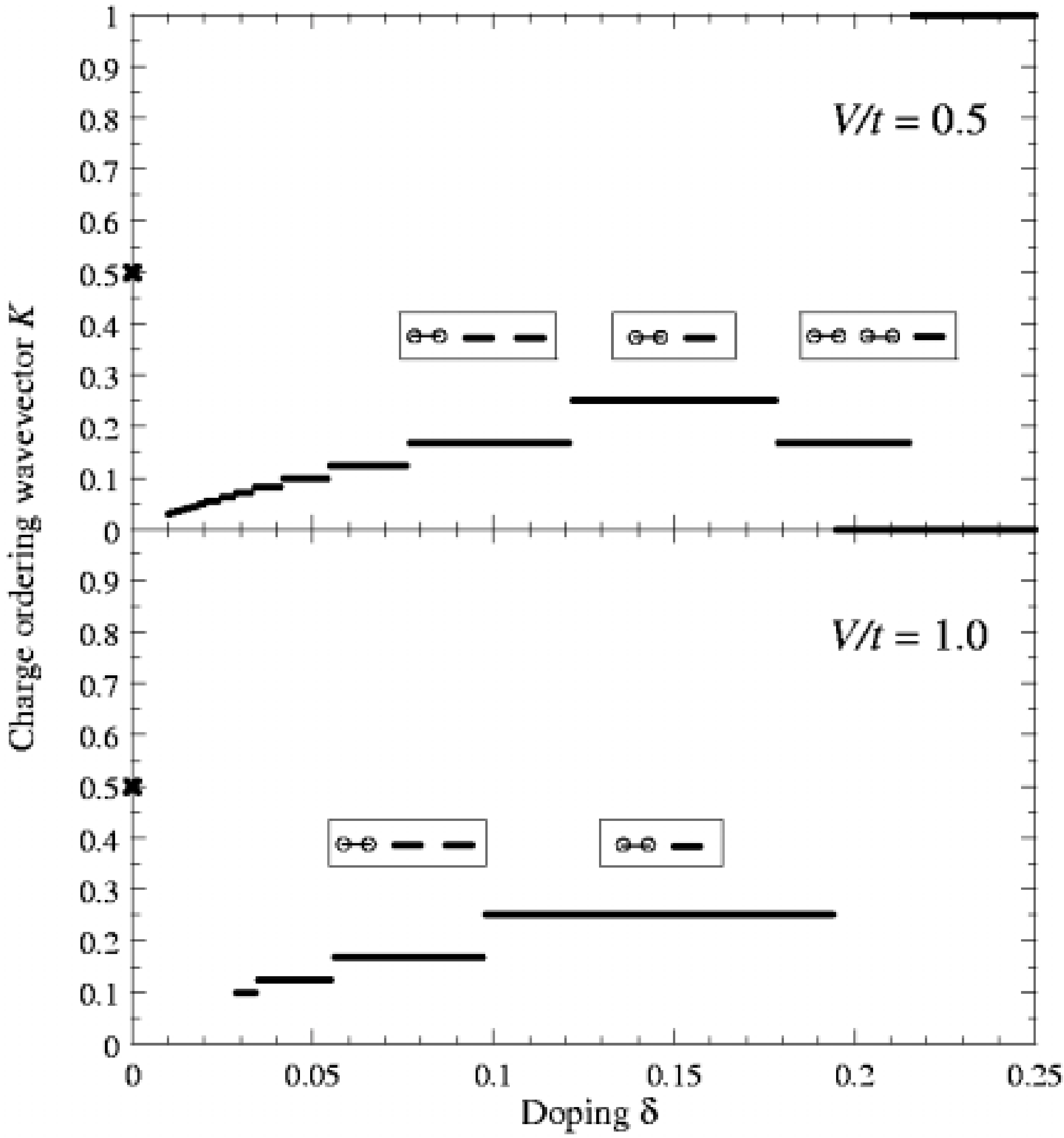
$\mathcal{S}$ broken for all δ>0 , large N

Time-reversal symmetry breaking and charge-ordering in a d-wave superconductor.

Sample phase diagram along A_1



Ordering wavevector of bond-centered stripes



Conclusions

Classification of quantum-critical points leading to critical damping of quasiparticles in superconductor

Most attractive possibility: $\mathcal{T}$ breaking transition from a $d_{x^2-y^2}$ superconductor to a $d_{x^2-y^2} + id_{xy}$ superconductor

Leads to quantum-critical damping along (1,1), and no damping along (1,0), with no unnatural fine-tuning.

Note: stable ground state of cuprates can always be a $d_{x^2-y^2}$ superconductor; only need thermal/quantum fluctuations to $d_{x^2-y^2} + id_{xy}$ order in quantum-critical region.

Experimental update: Tafuri+Kirtley (cond-mat/0003106) claim signals of $\mathcal{T}$ breaking near non-magnetic impurities in YBCO films

Quantum impurities in 2+1 dimensional conformal field theories: application to Zn impurities in YBCO

•M. Vojta
•C. Buragohain

Subir Sachdev

Transparencies on-line at
http://pantheon.yale.edu/~subir/newton.pdf

Science **286**, 2479 (1999).
cond-mat/9912020 (Phys. Rev. B in press)

Quantum Phase Transitions,
Cambridge University Press

Yale University

Deformation of quantum coherence by a dilute concentration of impurities n_{imp}

Magnetic impurities in a Fermi liquid

Quasiparticle scattering rate

$$\Gamma_{\text{imp}}(\varepsilon) \sim \begin{cases} n_{\text{imp}} J^2 a^{2d} \rho(E_F) & \varepsilon >> T_K \\ \dfrac{n_{\text{imp}}}{\rho(\mathrm{E_F})} & \varepsilon << \mathrm{T_K} \end{cases}$$

Pair-breaking in a non *s*-wave superconductor

Abrikosov-Gorkov pair-breaking parameter

$$\eta = \frac{\Gamma_{\text{imp}}(\Delta_{\text{sc}})}{\Delta_{\text{sc}}}$$

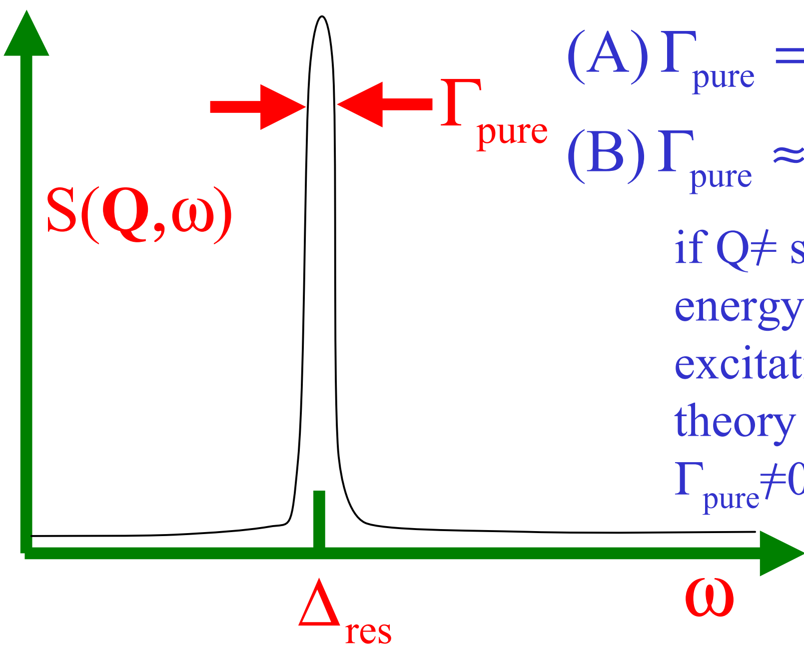
$\Delta_{\text{sc}} \rightarrow$ superconducting pairing energy

Magnetic quantum phase transitions in two dimensions

(A) Insulating Neel state (or collinear SDW at wavevector **Q**) ⇔ insulating quantum paramagnet

(B) *d*-wave superconductor with collinear SDW at wavevector **Q** ⇔ *d*-wave superconductor (paramagnet)

<u>Resonant *S*=1 collective mode in paramagnet</u>



(A) $\Gamma_{pure} = 0$ at $T = 0$

(B) $\Gamma_{pure} \approx 0$ at $T = 0$

if Q≠ separation between low energy nodal quasi-particle excitations. However, our theory applies <u>*also*</u> for $\Gamma_{pure} \neq 0$

$\Delta_{res} \rightarrow 0$ as paramagnet approaches quantum phase transition to magnetically ordered state

Effect of *arbitrary localized deformations* ("impurities") of density n_{imp}

Each impurity is characterized by an integer/half-odd-integer S

As $\Delta_{\text{res}} \to 0$

$$\frac{\Gamma_{\text{imp}}}{\Delta_{\text{res}}} = n_{\text{imp}} \left(\frac{\hbar c}{\Delta_{\text{res}}} \right)^2 \left[C_S + O\left(\frac{\Delta_{\text{res}}}{J} \right) \right]$$

Correlation length ξ

$C_S \to$ Universal numbers dependent only on S

$$C_0 = 0 \; ; \; C_{1/2} \approx 1$$

Zn impurities in YBCO have S=1/2

"Swiss-cheese" model of quantum impurities (Uemura):

Inverse Q of resonance ~ fractional volume of holes in Swiss cheese.

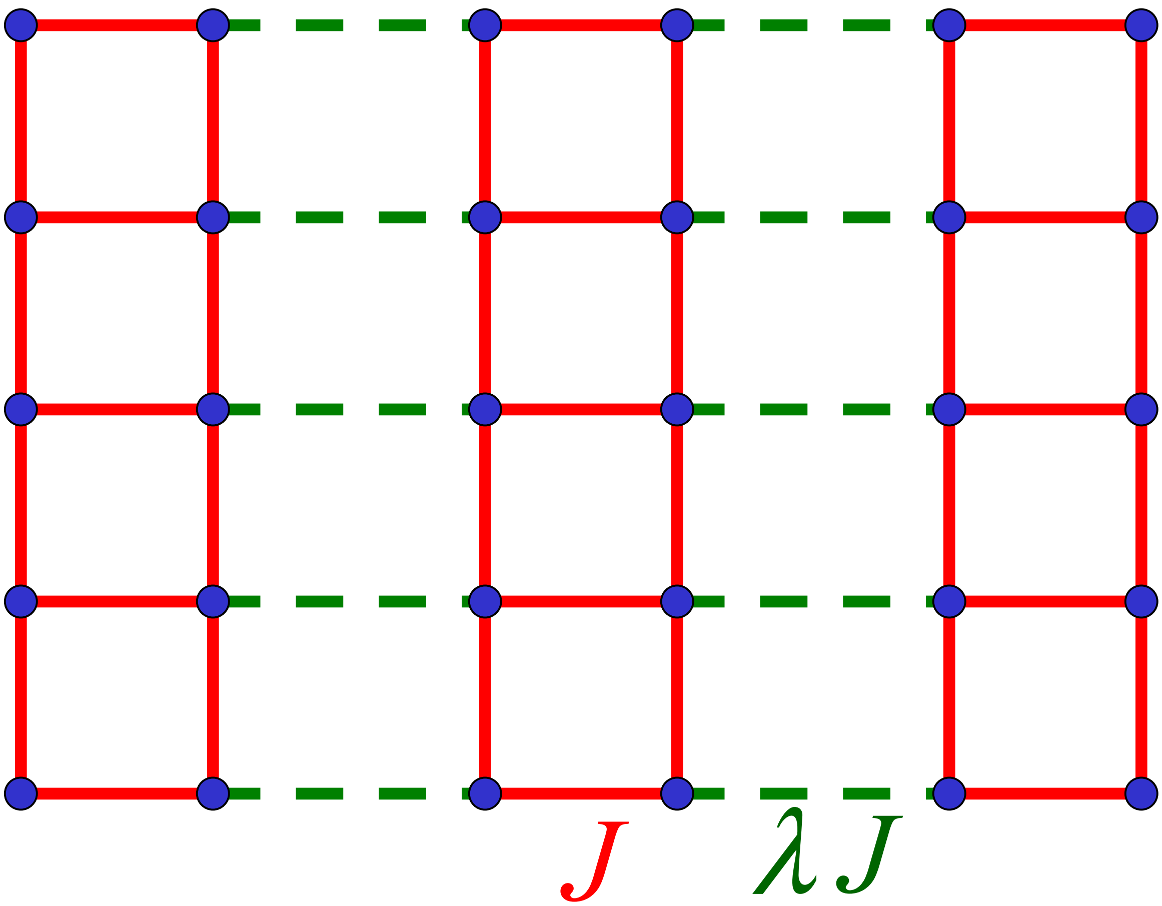
1. (A) Paramagnetic and Neel ground states in two dimensions --- coupled-ladder antiferromagnet.
   Field theory of quantum phase transition.

2. Non-magnetic impurities (Zn or Li) in two-dimensional paramagnets. Theory of host quantum-critical point provides a controlled expansion in Δ_{res}/J and accounts for strongly relevant self-interactions of the resonant mode.

3. Application to (B) d-wave superconductors.
   Comparison with, and predictions for, expts

1. Paramagnetic and Neel states in insulators

S=1/2 spins on coupled 2-leg ladders



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

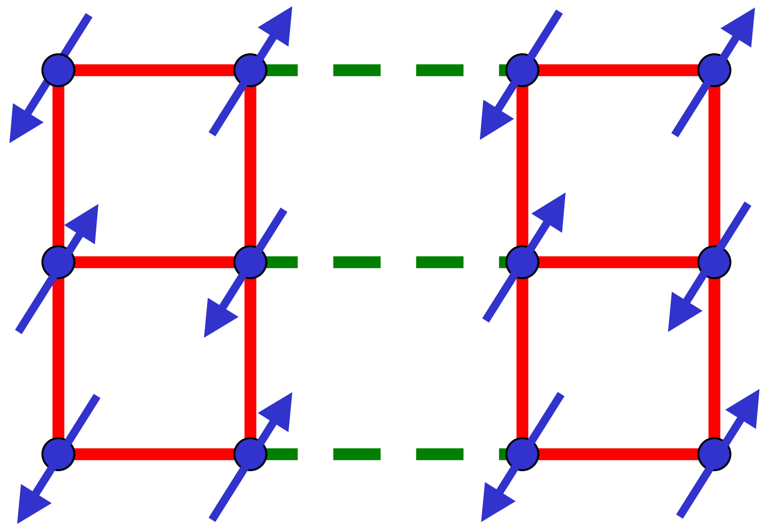
Follow ground state as a function of λ

$$0 \leq \lambda \leq 1$$

$\lambda = 1$

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



Ground state has long-range magnetic (Neel) order

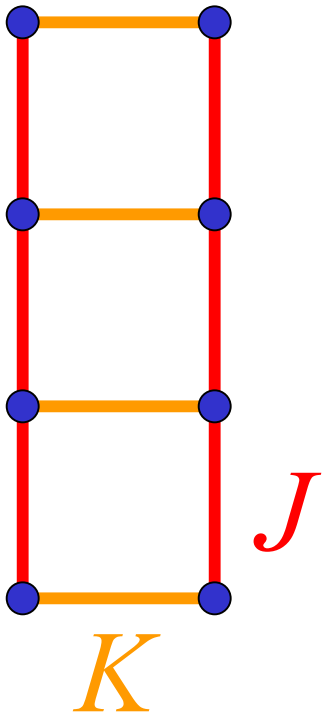
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves

Quasiclassical wave dynamics at low T

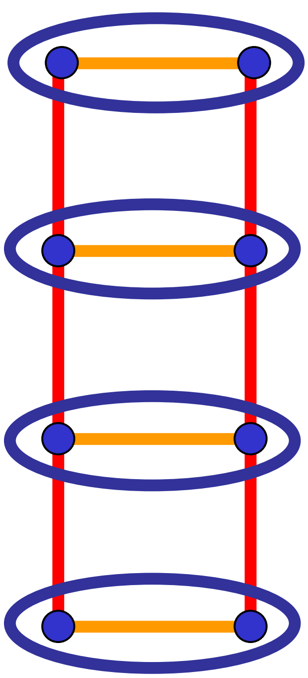
(Chakravarty et al, 1989;
Tyc et al, 1989)

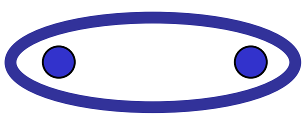
$\lambda = 0$



Decoupled 2-leg ladders

Allow $J \neq K$

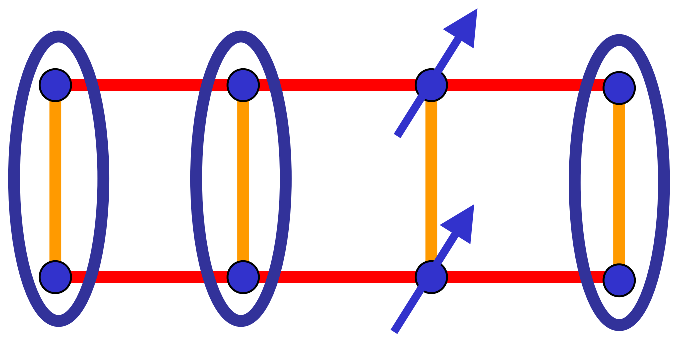


 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Quantum paramagnet ground state for $J \ll K$

Qualitatively similar ground state for all J/K

Excited states

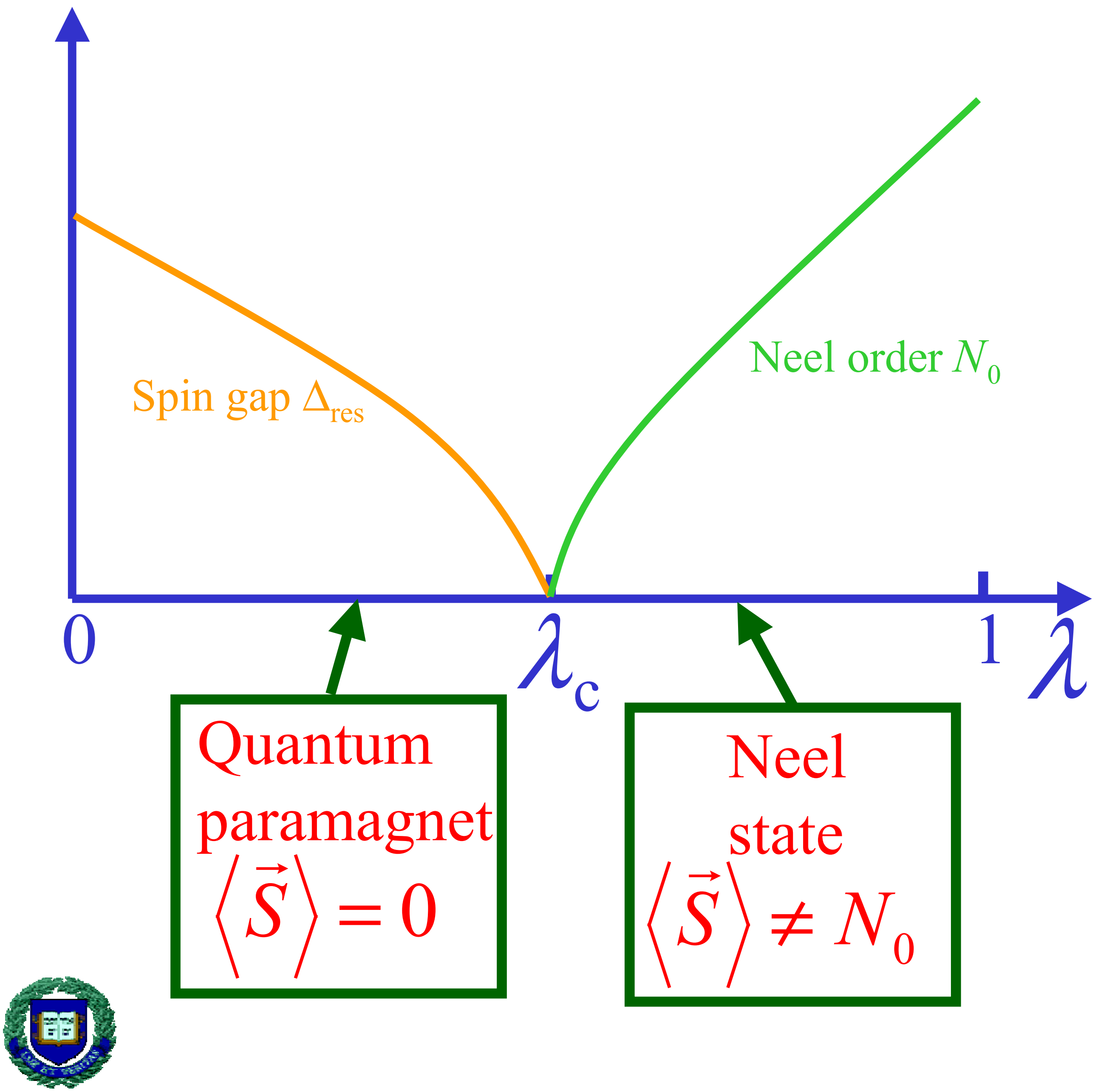


Triplet (S=1) particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta_{res} + \frac{c^2 k^2}{2\Delta_{res}}$$

$\Delta_{res} \rightarrow$ Spin gap



Spin gap Δ_{res}
Neel order N_0
0
λ_c
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

Nearly-critical paramagnets

λ is close to λ_c

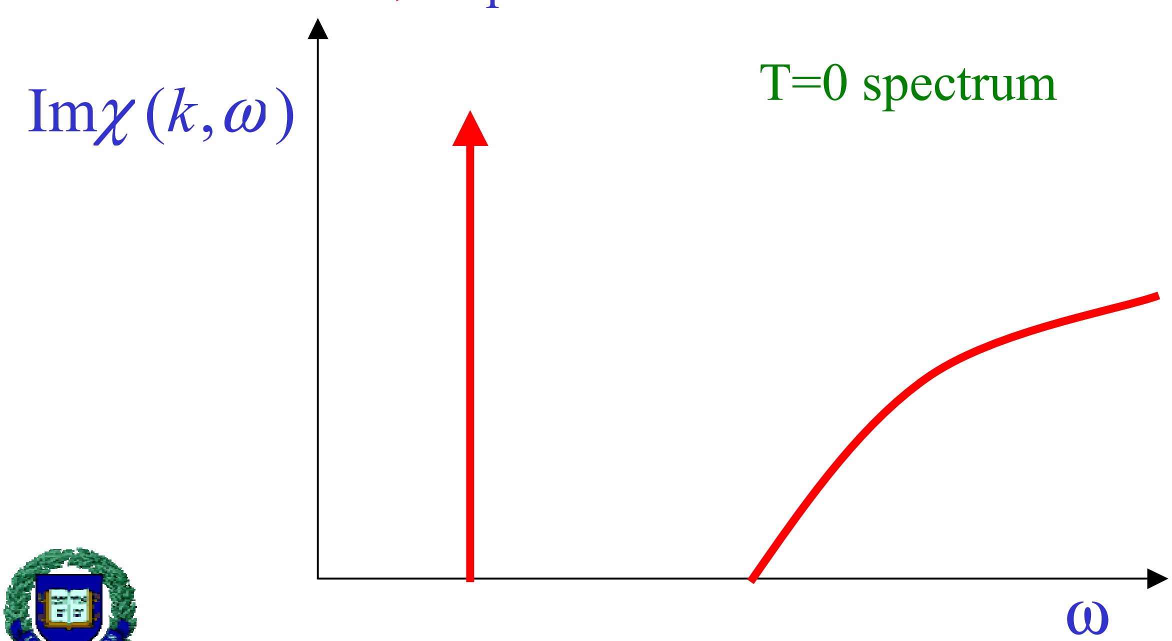
Quantum field theory:

$$S_b = \int d^d x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r \phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$	→	$\lambda < \lambda_c$
$r < 0$	→	$\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)
→ spin-1 collective mode



Coupling ***g*** approaches fixed-point value under renormalization group flow: beta function ($\varepsilon = 3$-d) :

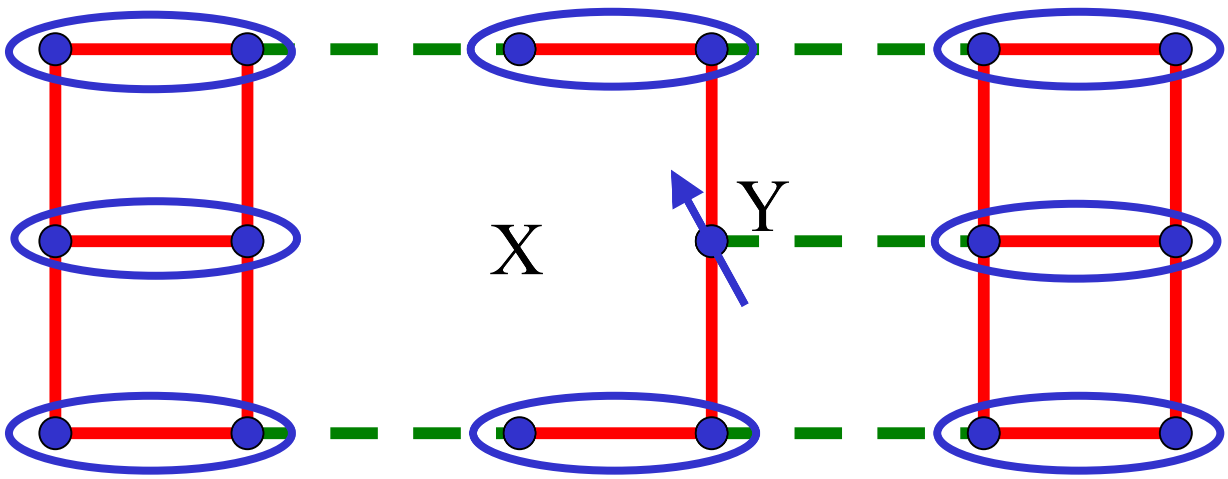
$$\beta(g) = -\epsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + \mathcal{O}(g^4)$$

Only relevant perturbation – ***r***
strength is measured by the spin gap Δ

Δ_{res} and c completely determine entire spectrum of quasi-particle peak and multiparticle continua, the S matrices for scattering between the excitations, and $T > 0$ modifications.

2. Quantum impurities in nearly-critical paramagnets

Make *any* localized deformation of antiferromagnet; e.g. remove a spin



Susceptibility $\chi = A\chi_b + \chi_{imp}$

(A = area of system)

In paramagnetic phase as $T \to 0$

$$\chi_b = \left(\frac{\Delta_{\text{res}}}{\hbar^2 c^2 \pi}\right) e^{-\Delta_{\text{res}}/k_B T} \; ; \; \chi_{imp} = \frac{S(S+1)}{3k_B T}$$

For a general impurity χ_{imp} *defines* the value of S

$$\lim_{\tau \to \infty} \left\langle \vec{S}_Y(\tau) \cdot \vec{S}_Y(0) \right\rangle = m^2 \neq 0$$

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$S_{\rm imp} = \int d\tau \left[i S A_\alpha(n) \frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau) \phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $S_b + S_{\rm imp}$

Recall -

$$S_b = \int d^d x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r \phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

Coupling γ approaches *also* approaches a fixed-point value under the renormalization group flow

(Sengupta, 97
Sachdev+Ye, 93
Smith+Si 99)

Beta function:

$$\beta(\gamma) = -\frac{\epsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \frac{\pi^2}{3}\left(S(S+1) - \frac{1}{3}\right) g\gamma^3 + \mathcal{O}\left((\gamma, \sqrt{g})^7\right)$$

No new relevant perturbations on the boundary;
All other boundary perturbations are irrelevant –
e.g.

$$\lambda \int d\tau \phi_\alpha^2(x=0,\tau)$$

(This is the simplest allowed boundary perturbation for S=0 – its irrelevance implies $C_0 = 0$)

Δ_{res} and c completely determine spin dynamics near an impurity –

No new parameters are necessary !

Universal properties at the critical point $\lambda = \lambda_c$

$$\left\langle \vec{S}_Y(\tau)\cdot \vec{S}_Y(0)\right\rangle = \frac{1}{\tau^{\eta'}}$$

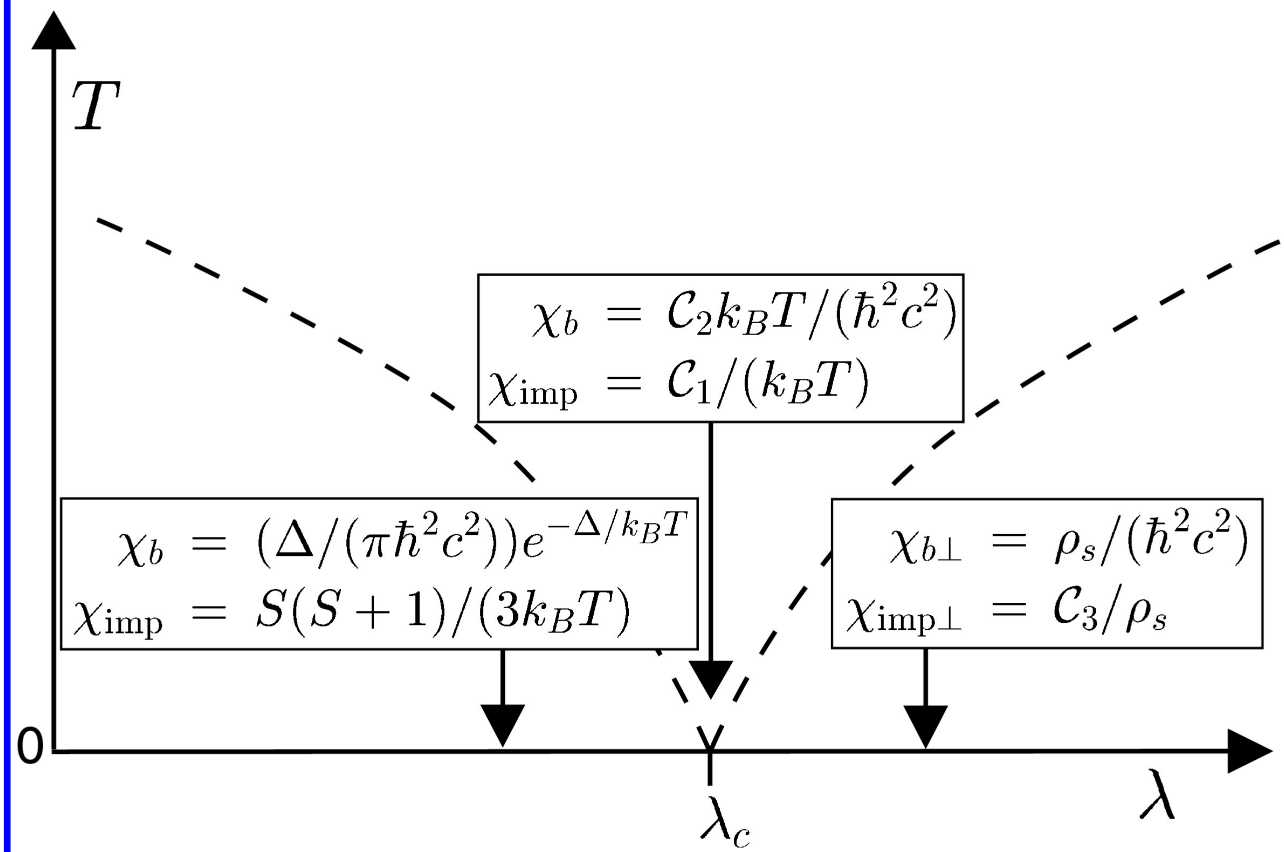
(and $m = |\lambda - \lambda_c|^{\eta'\nu}$)

However $$\chi_{imp} \neq \frac{1}{T^{1-\eta'}}$$

This last relationship holds in the multi-channel Kondo problem because the magnetic response of the screening cloud is negligible due to an exact "compensation" property. There is no such property here, and naïve scaling applies. This leads to

$$\chi_{imp} = \frac{\text{Universal number}}{k_B T}$$

Curie response of an irrational spin



In the Neel phase

$$\chi_{imp\perp} = \frac{\text{Universal number}}{\text{spin stiffness}}$$

spin stiffness $\rho_s = \left(\rho_{sx}\rho_{sy}\right)^{1/2}$

Bulk susceptibility vanishes while impurity susceptibility diverges as $\rho_s \to 0$

At $T > 0$, thermal averaging leads to

$$\chi_{\rm imp} = \frac{S^2}{3k_B T} + \frac{2}{3}\chi_{\rm imp\perp}$$

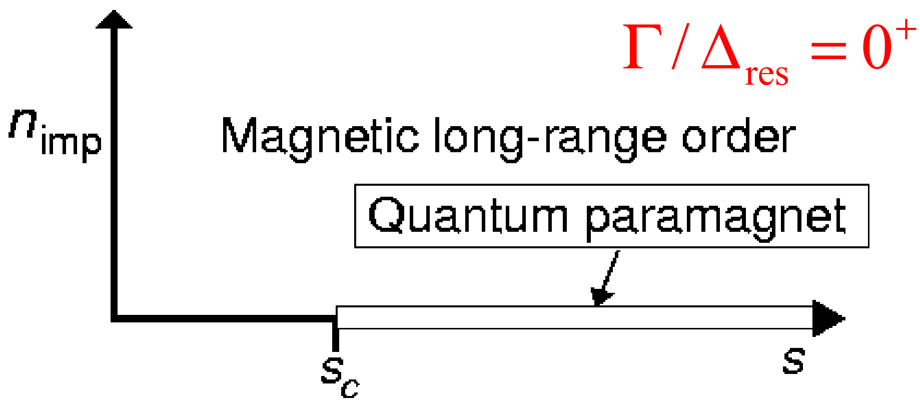
Finite density of impurities n_{imp}

Relevant perturbation – strength determined by only energy scale that is linear in n_{imp} and contains only bulk parameters

$$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta_{\text{res}}}$$

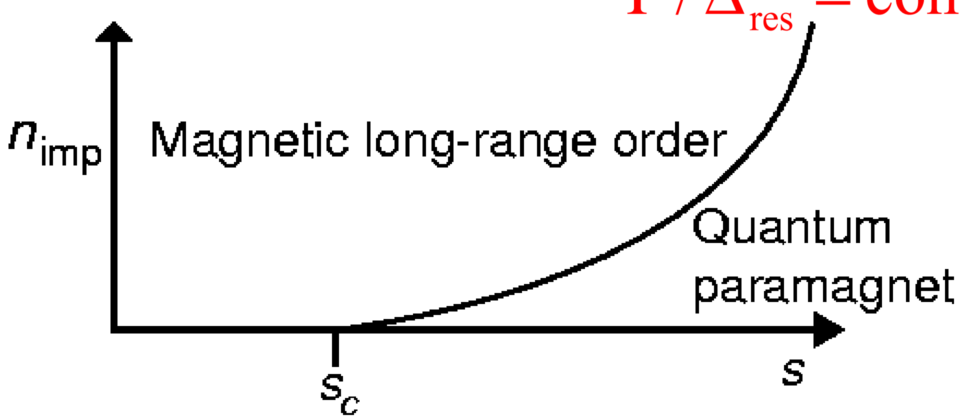
Two possible phase diagrams

$\Gamma / \Delta_{\text{res}} = 0^+$



(a)

$\Gamma / \Delta_{\text{res}} = \text{constant}$



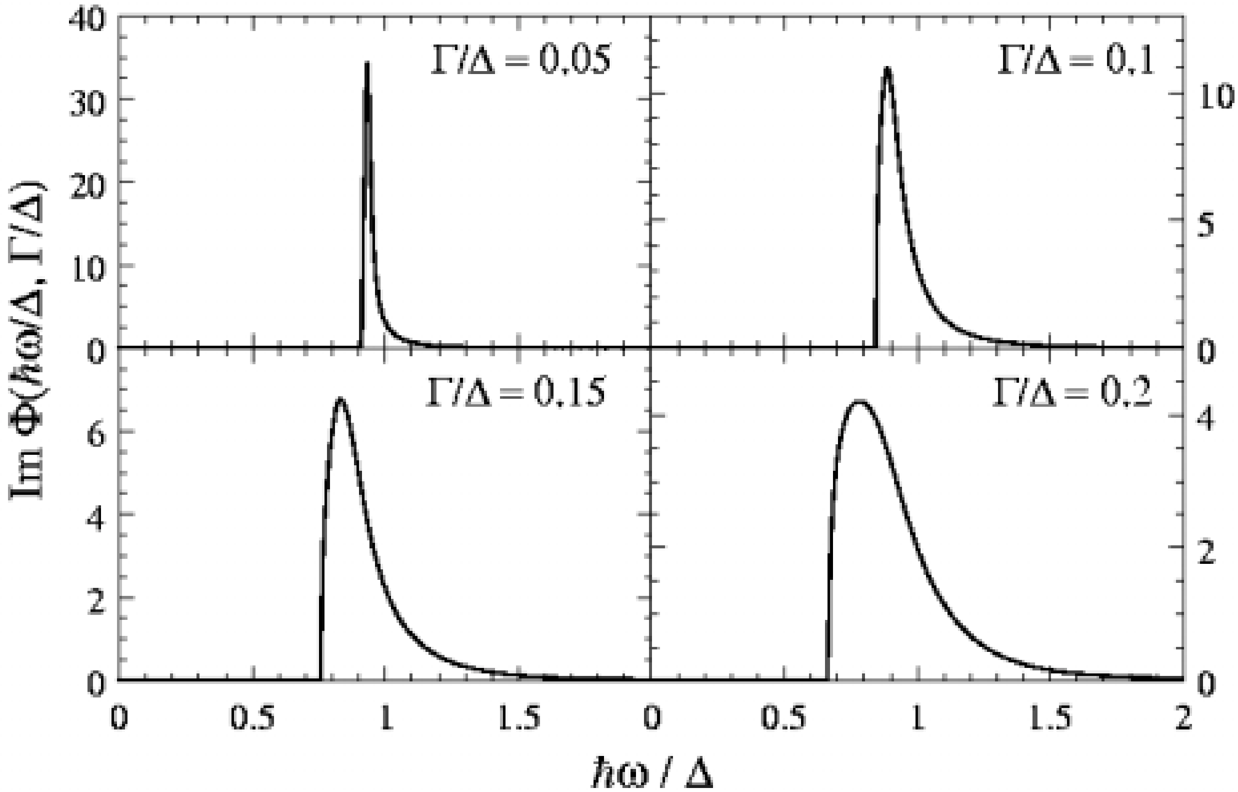
(b)

Fate of collective mode peak

Without impurities $\chi(G,\omega) = \dfrac{A}{\Delta_{\mathrm{res}}^2 - \omega^2}$

With impurities $\chi(G,\omega) = \dfrac{A}{\Delta_{\mathrm{res}}^2}\Phi\left(\dfrac{\hbar\omega}{\Delta_{\mathrm{res}}}, \dfrac{\Gamma}{\Delta_{\mathrm{res}}}\right)$

Φ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation

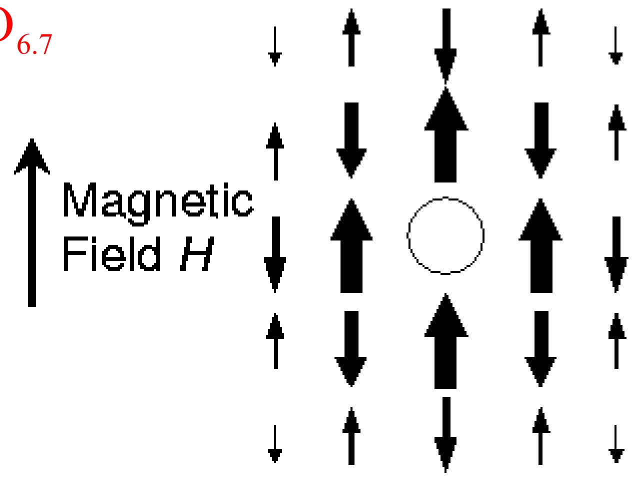


Predictions: Half-width of line $\approx \Gamma$

Universal asymmetric lineshape

3. Application to d-wave superconductors (YBCO)

Zn impurity in $YBa_2Cu_3O_{6.7}$



Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul

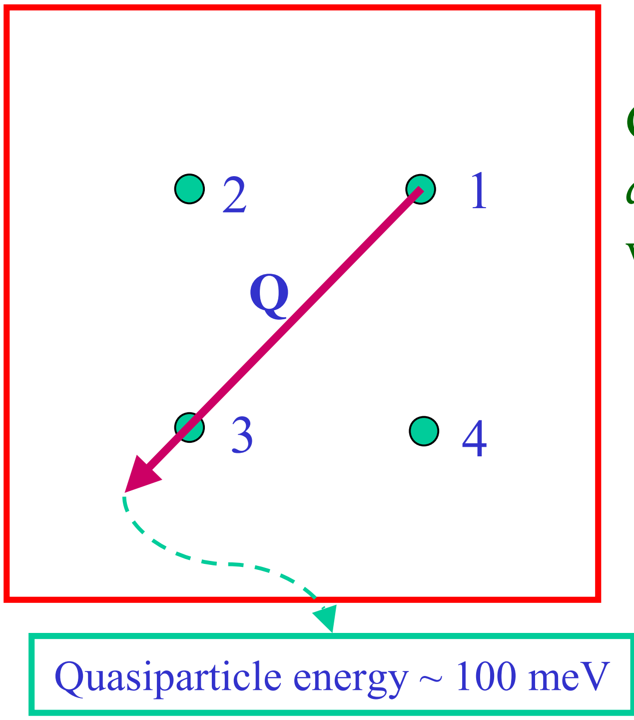
Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

Pepin and Lee: Modeled Zn impurity as a potential scatterer in the unitarity limit, and obtained quasi-bound states at the Fermi level.
Our approach: Each bound state captures only one electron and this yields a Berry phase of S=1/2; residual potential scattering of quasiparticles is not in the unitarity limit.

Additional low-energy spin fluctuations in a *d*-wave superconductor

Nodal quasiparticles Ψ

• Bulk coupling between Ψ and ϕ is forbidden by momentum conservation.



Gapless Fermi Points in a *d*-wave superconductor at wavevectors $(\pm K, \pm K)$

$K=0.391\pi$

(We also have a theory for the case **Q**=(2*K*,2*K*): scaling forms are the same, but scaling functions and exponents change)

• There is a Kondo coupling between moment around impurity and Ψ: $J_K S n_\alpha \Psi^* \sigma^\alpha \Psi$

However, because density of states vanishes linearly at the Fermi level, there is no Kondo screening for any finite J_K (below a finite J_K) with (without) particle-hole symmetry

(Withoff+Fradkin, Chen+Jayaprakash, Buxton+Ingersent)

H. F. Fong,
B. Keimer,
D. Reznik,
D. L. Milius,
and
I. A. Aksay,
Phys. Rev.
B **54**, 6708
(1996)

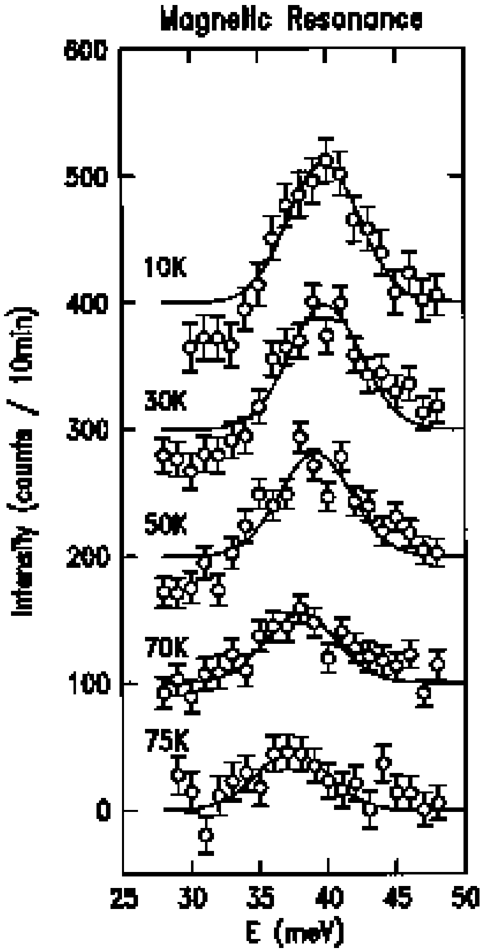
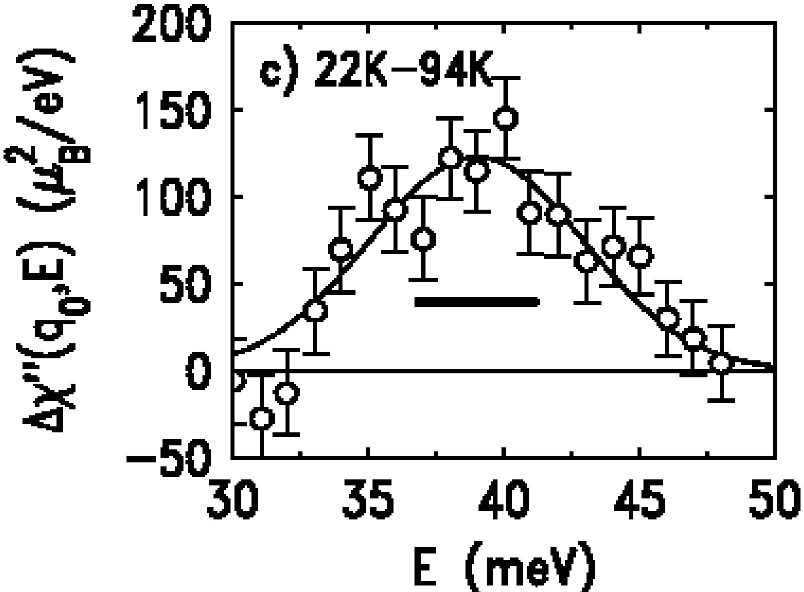


FIG. 8. Unpolarized beam, constant-Q data [Q=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Spin-1 collective mode in $YBa_2Cu_3O_7$ - little observable damping at low T → coupling to superconducting quasiparticles unimportant, and spin correlations in some regions of phase space are like those of a paramagnet. Nodal quasiparticles are at momenta (±*K*,±*K*) and do not couple to spin-1 collective mode at (π,π) unless *K*=π/2

$YBa_2Cu_3O_7 + 0.5\%\ Zn$

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



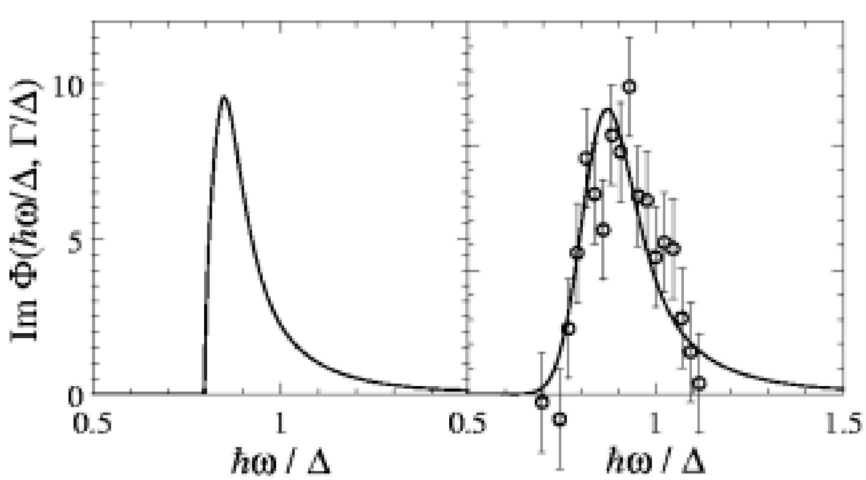
$$n_{\text{imp}} = 0.005$$

$$\Delta_{\text{res}} = 40\ \text{meV}$$

$$\hbar c = 0.2\ \text{eV}$$

$$\Rightarrow \quad \Gamma = 5\ \text{meV},\ \Gamma/\Delta_{\text{res}} = 0.125$$

Quoted half-width = 4.25 meV



Applicability to d-wave superconductor

Extended t-J model on the square lattice

$$H = \sum_{i>j} \left[- t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + J_{ij} \vec{S}_i \cdot \vec{S}_j + V_{ij} n_i n_j \right]$$

Plot phase diagram of stable ground states as a function of:

(1) Doping δ

(2) Frustration $\dfrac{J_2 \text{ (second neighbor)}}{J_1 \text{(first neighbor)}}$

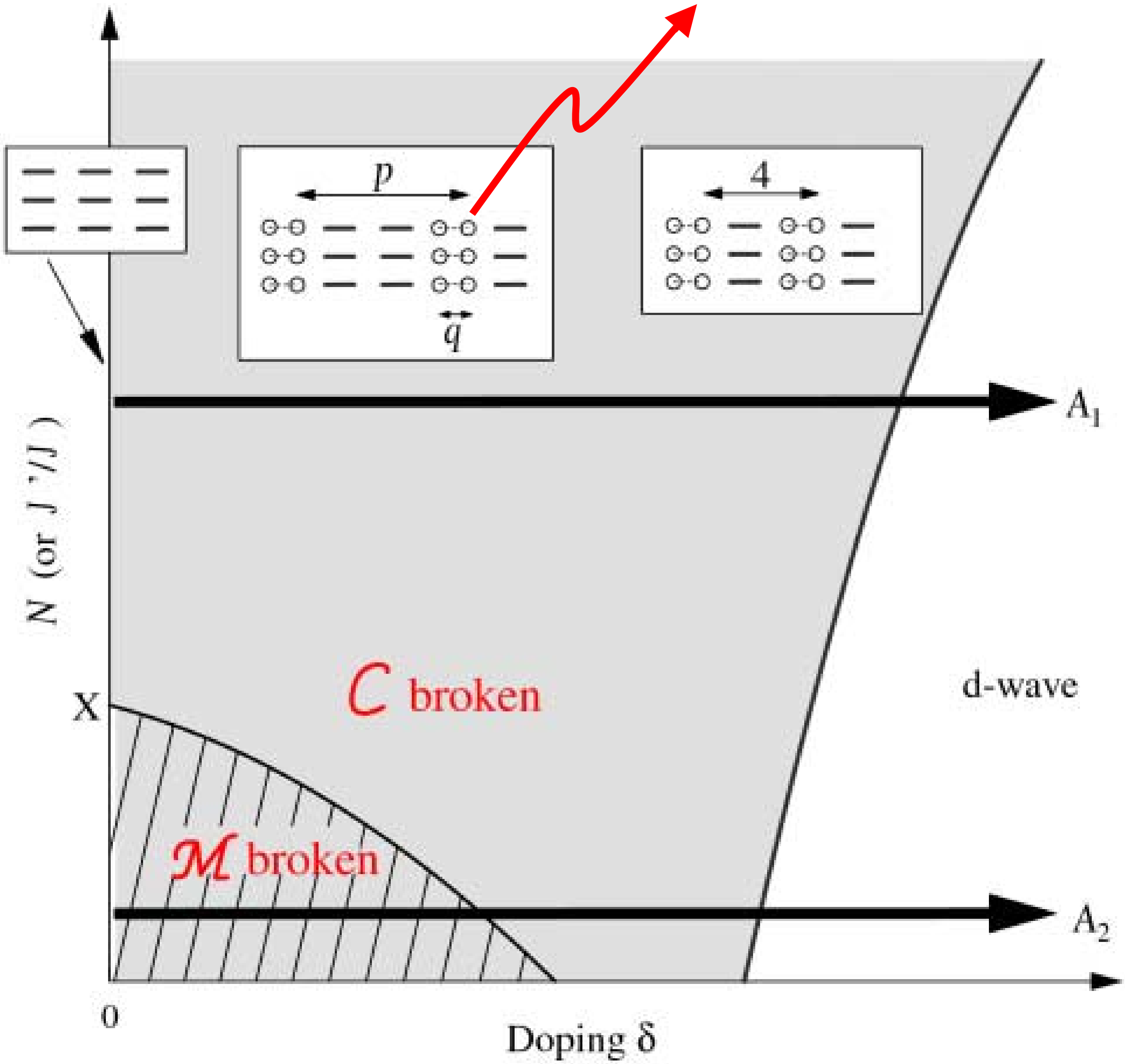
OR

N, where spin symmetry

$$SU(2) \rightarrow Sp(N)$$

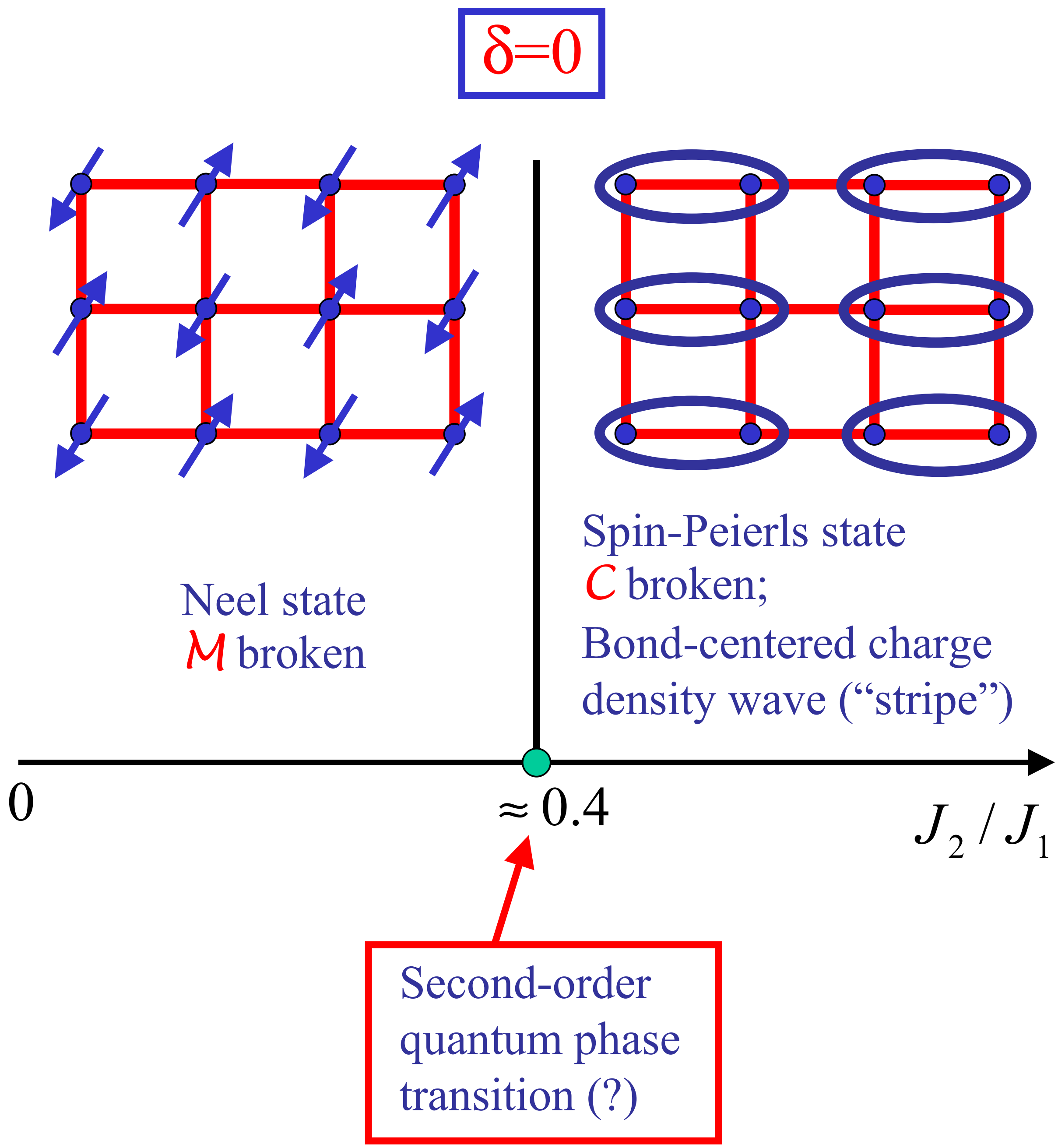
Schematic phase diagram

hole density ≈ 0.5/unit length



$\mathcal{S}$ broken for all δ>0 , large N

The magnetic ordering transition



(Read and Sachdev, PRL **62**, 1694 (1989)
Kotov et al, PRB **60**, 14613 (1999)
Singh et al, PRB **60**, 7278 (1999))

Conclusions

1. Universal *T=0* damping of S=1 collective mode by non-magnetic impurities.

   Linewidth: $$\Gamma \equiv \frac{n_{\text{imp}} (\hbar c)^2}{\Delta_{\text{res}}}$$

   independent of impurity parameters.

2. New interacting boundary conformal field theory in 2+1 dimensions

3. Universal irrational spin near the impurity at the critical point.