A simple model of many-particle entanglement: how it describes black holes and superconductors

University of New Mexico
February 5, 2021

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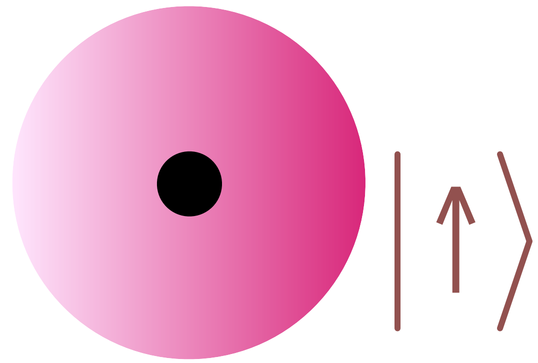
Talk online: sachdev.physics.harvard.edu

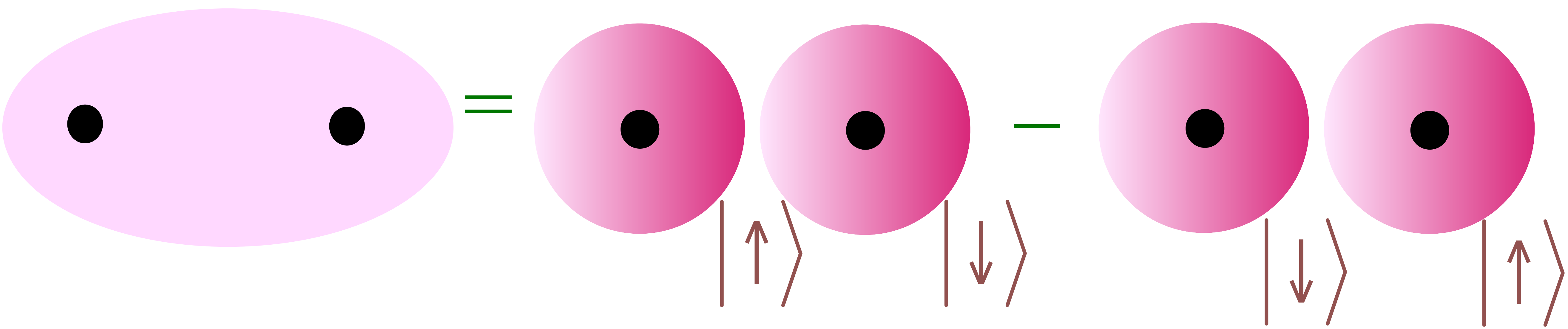
Quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

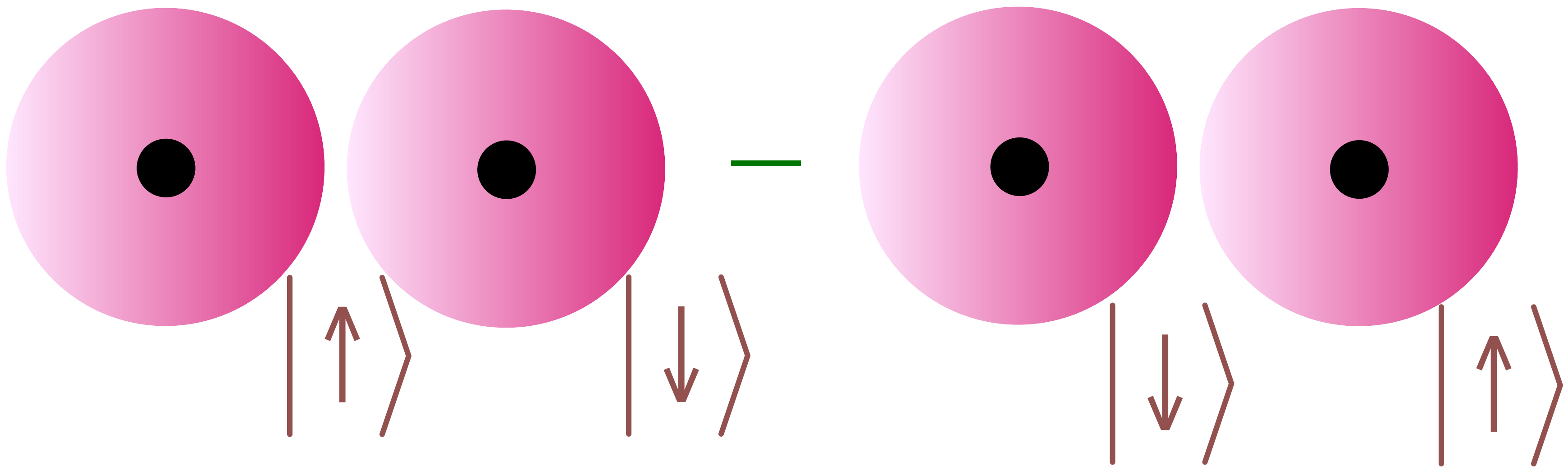


Hydrogen molecule:



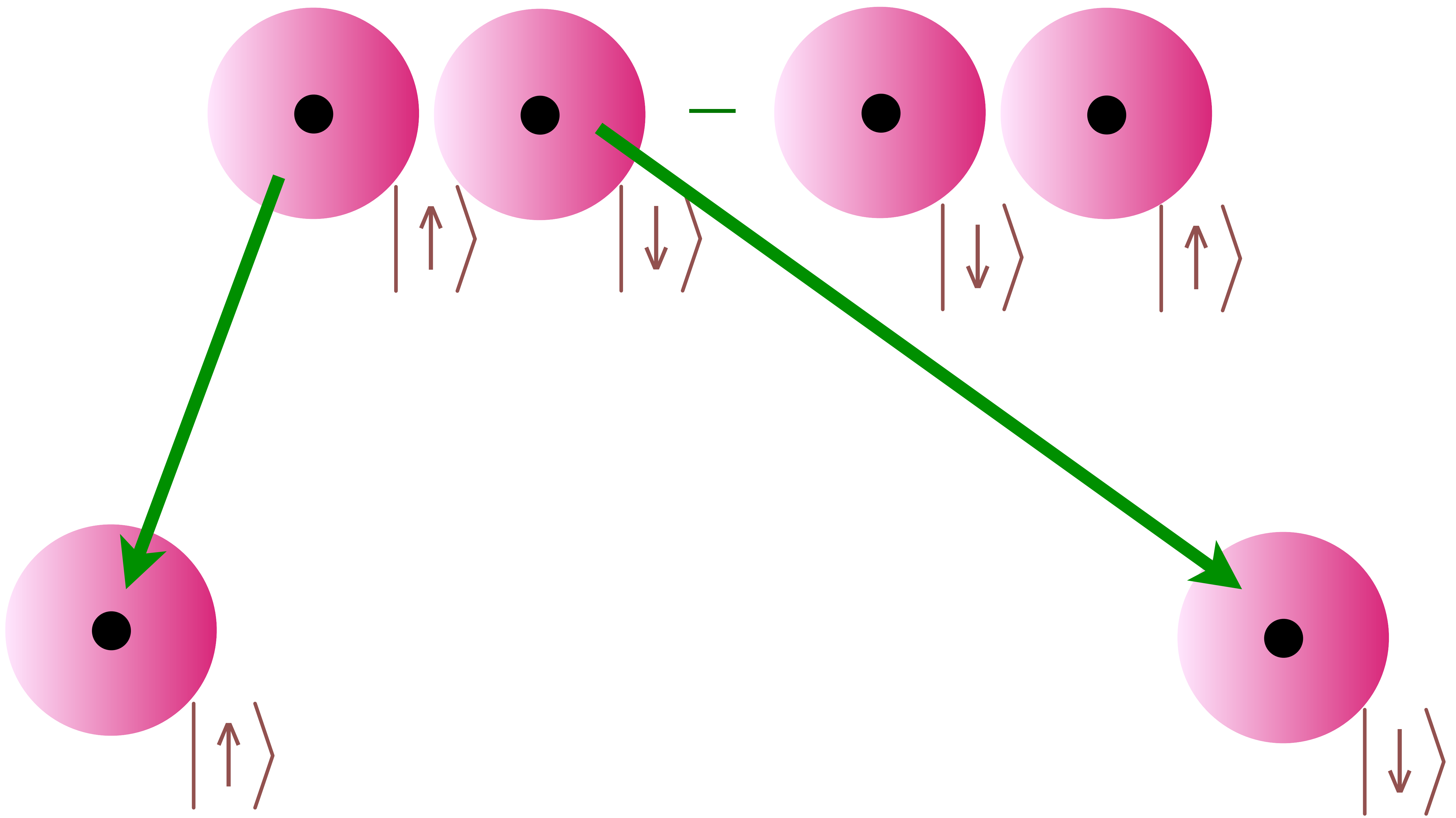
$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



$$|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle$$

Quantum Entanglement: quantum superposition with more than one particle



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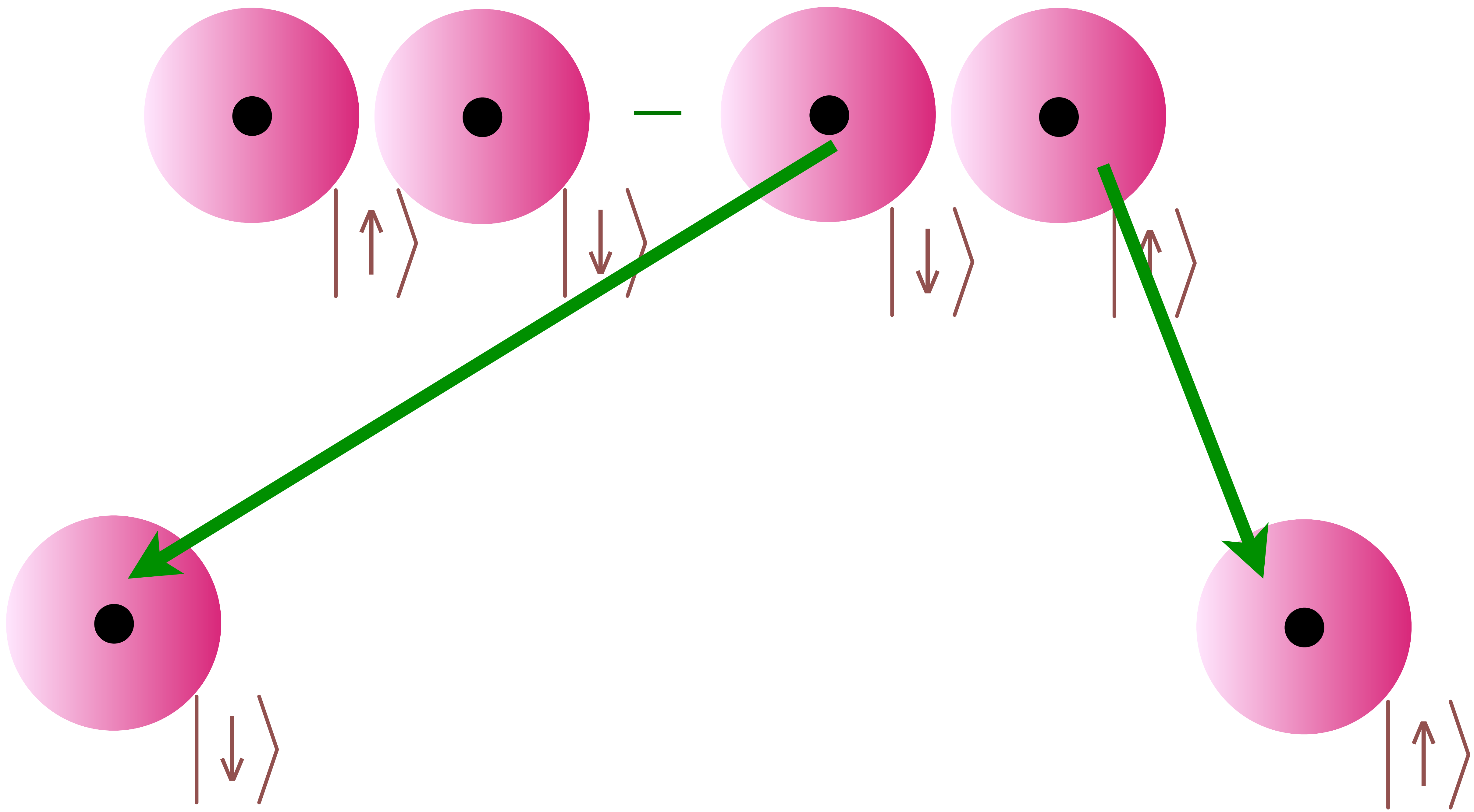
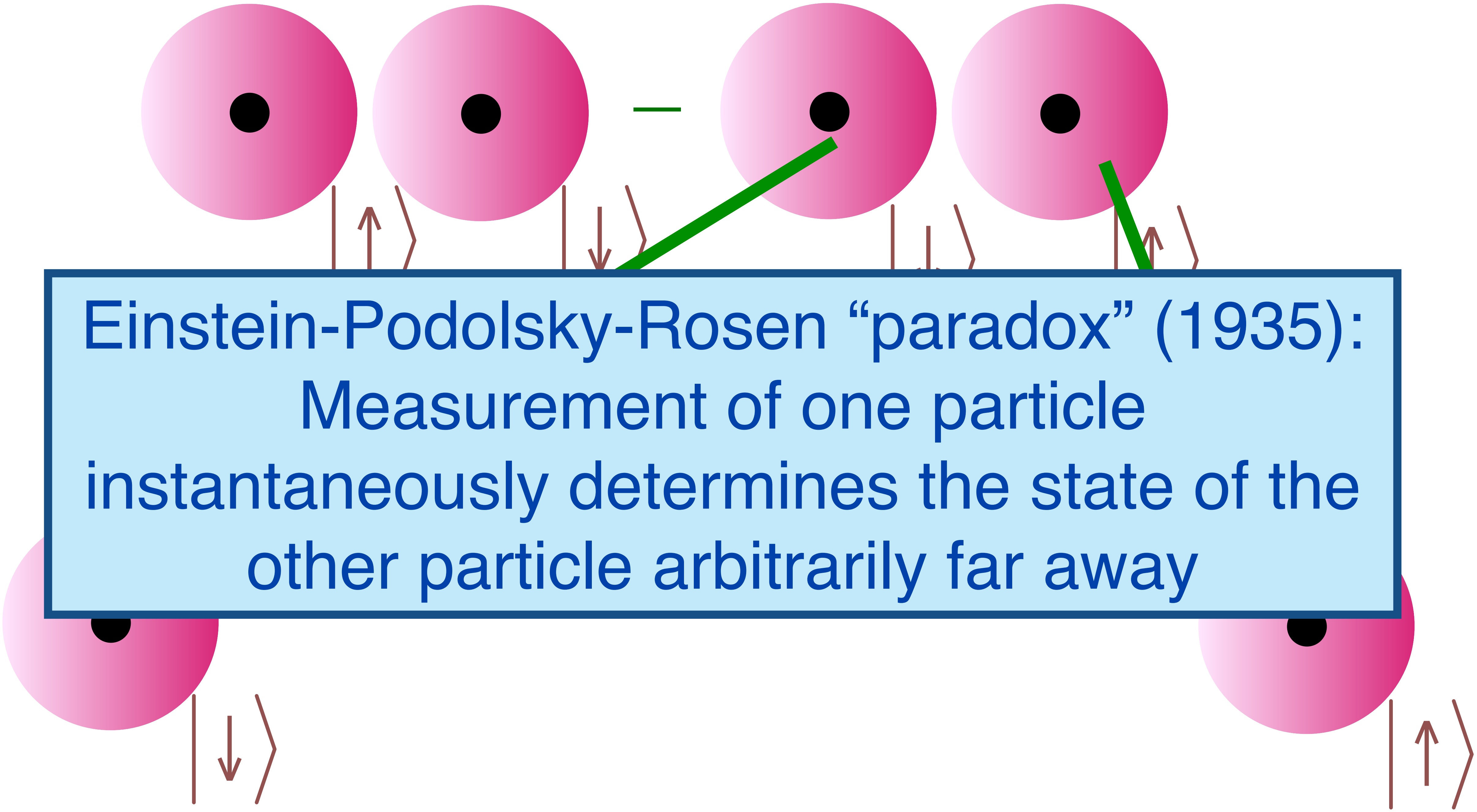


Quantum Entanglement: quantum superposition with more than one particle



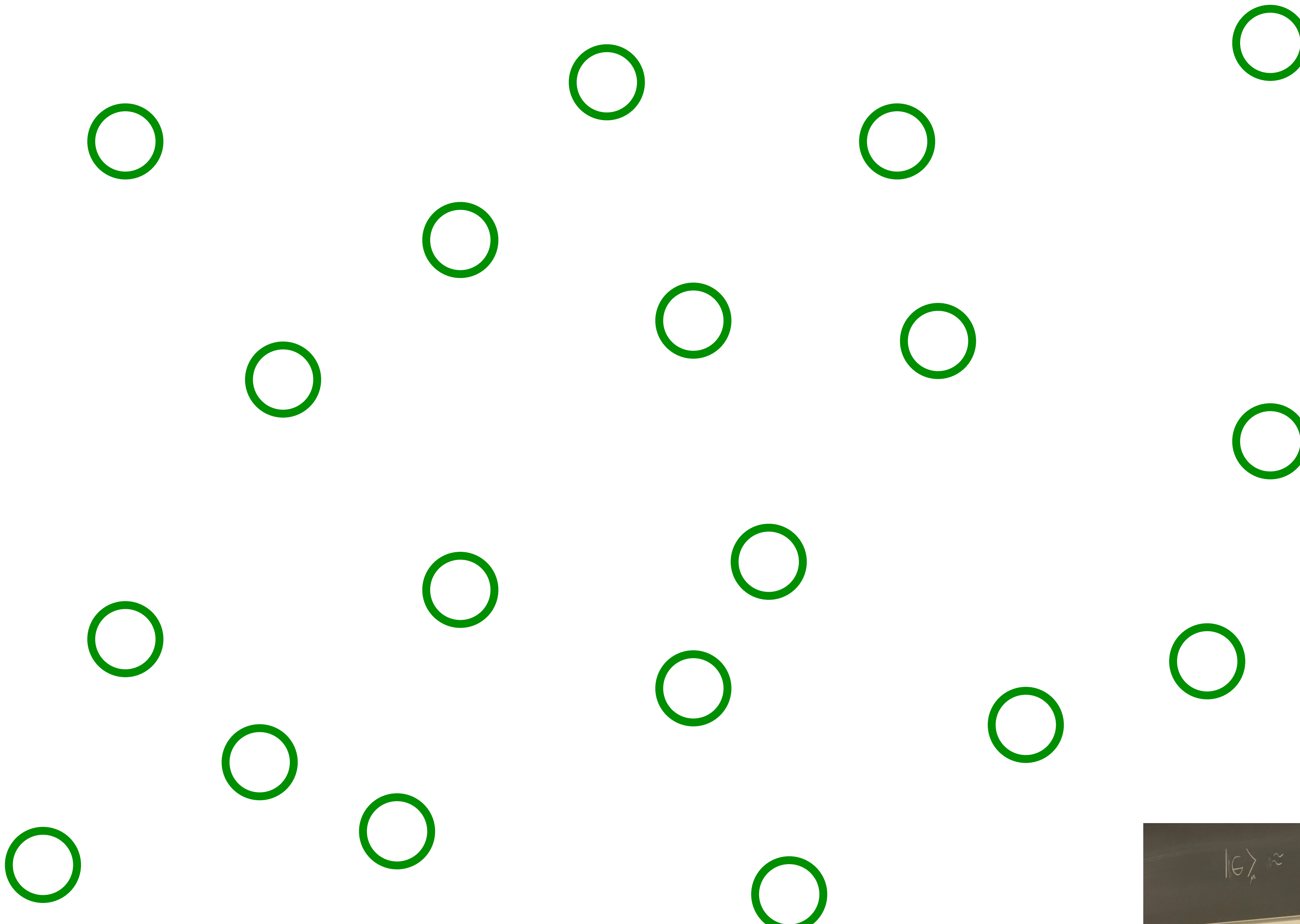
Einstein-Podolsky-Rosen "paradox" (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

Quantum entanglement

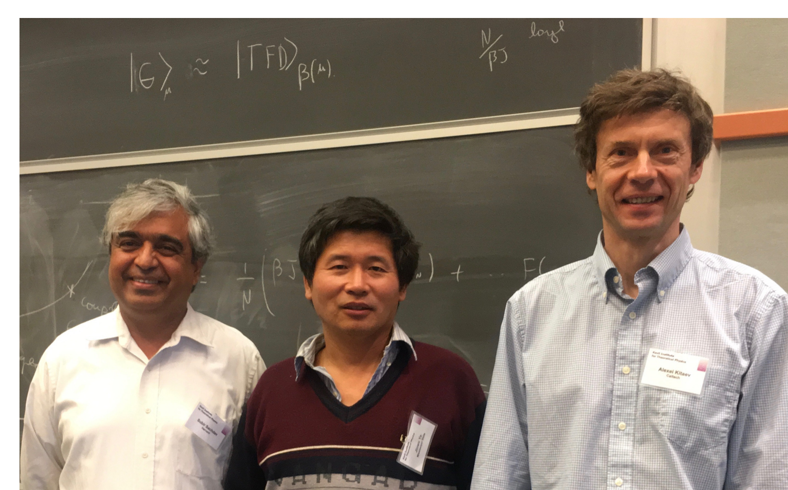
Quantum entanglement

A simple many-particle (SYK) model

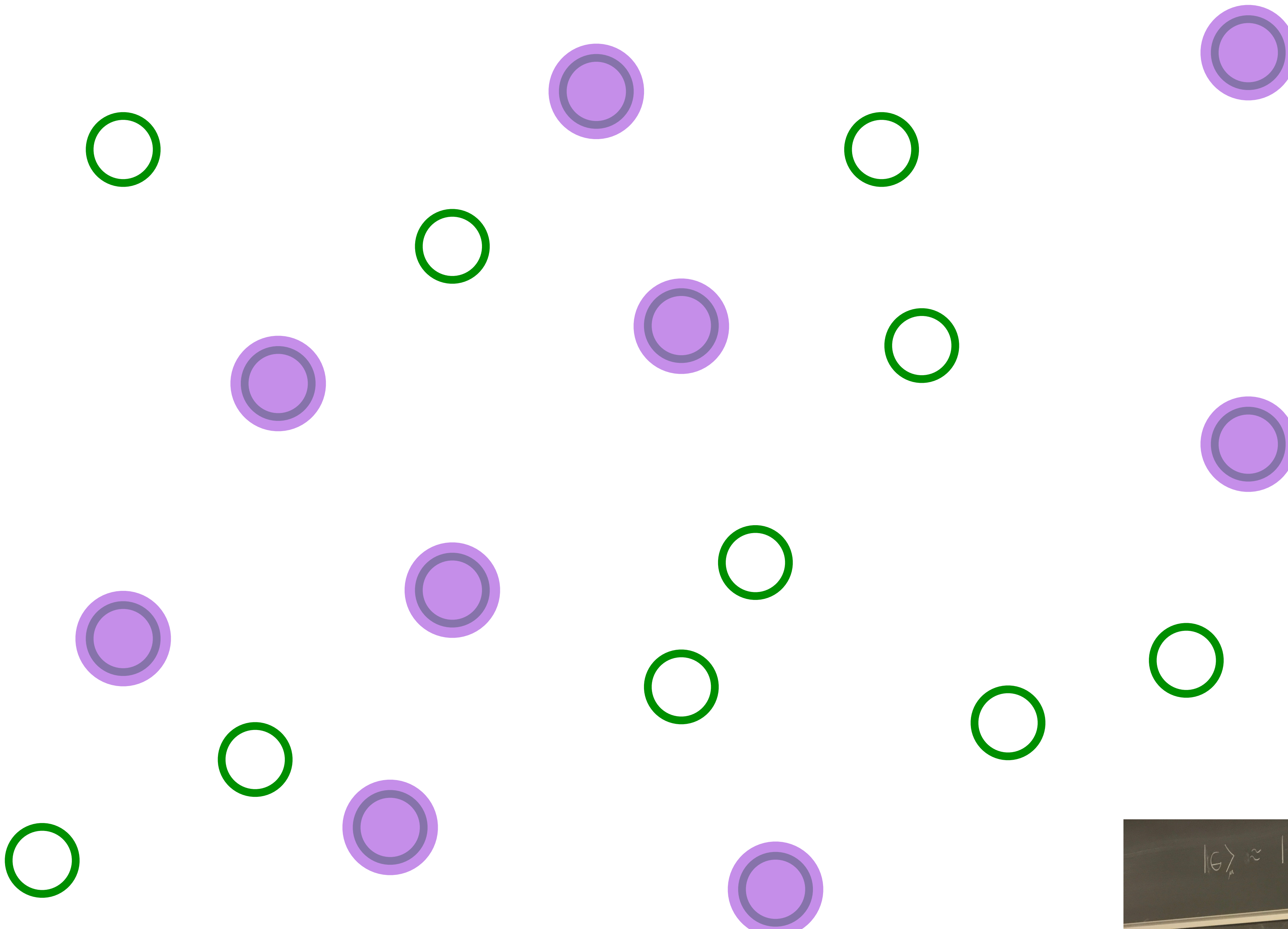
The Sachdev-Ye-Kitaev (SYK) model



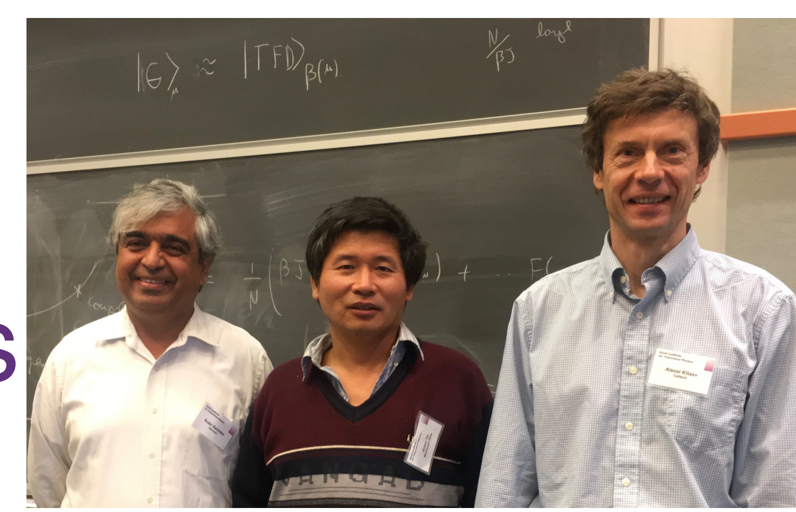
Pick a set of random positions



The SYK model



Place electrons randomly on some sites



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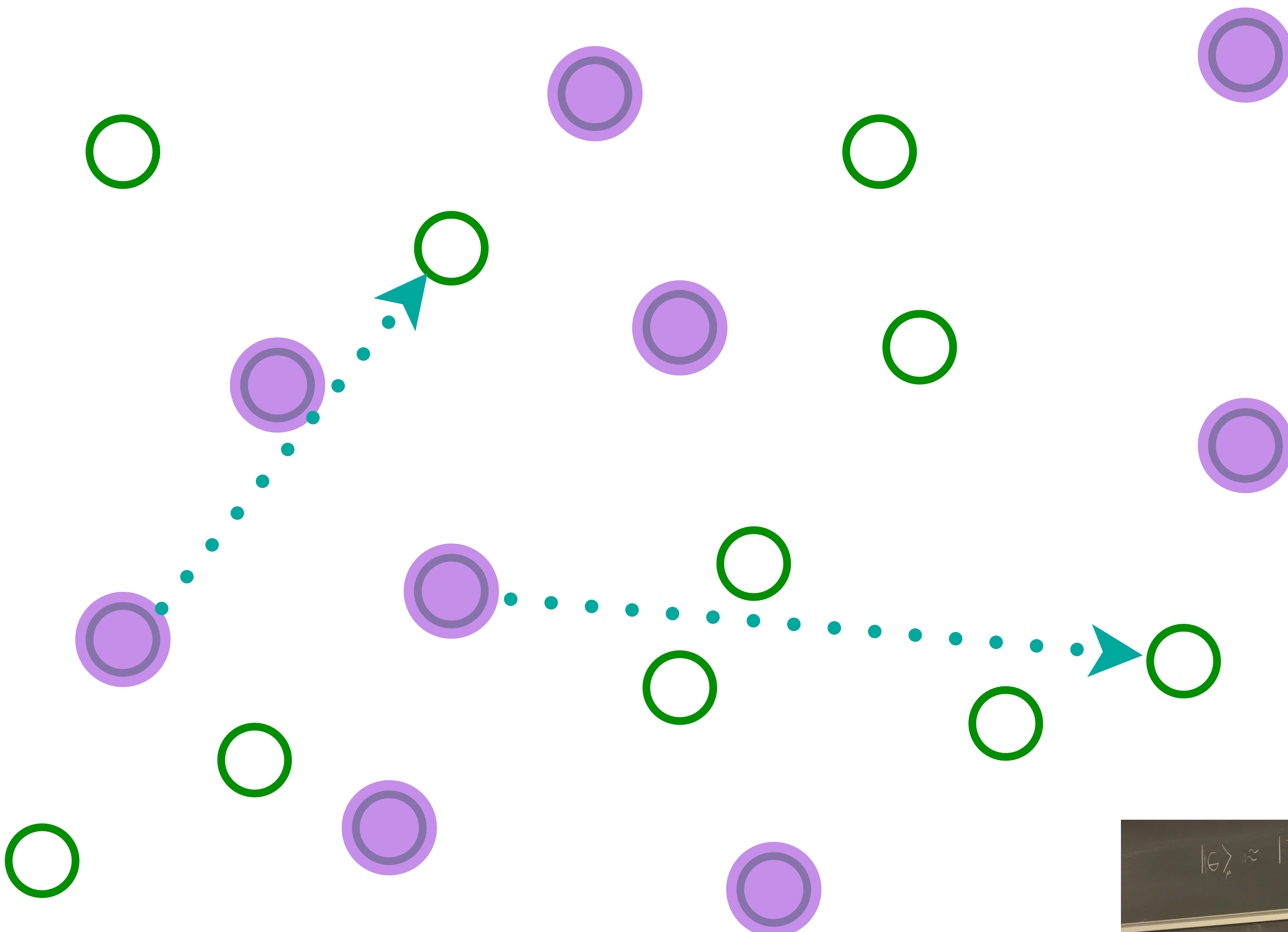

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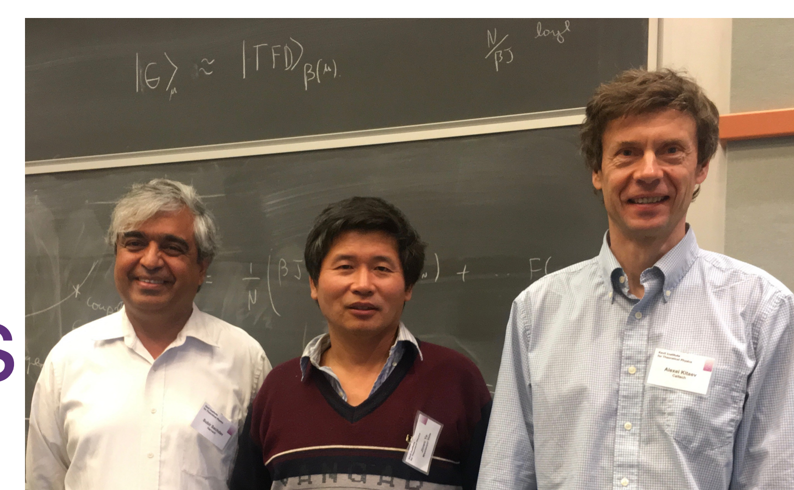

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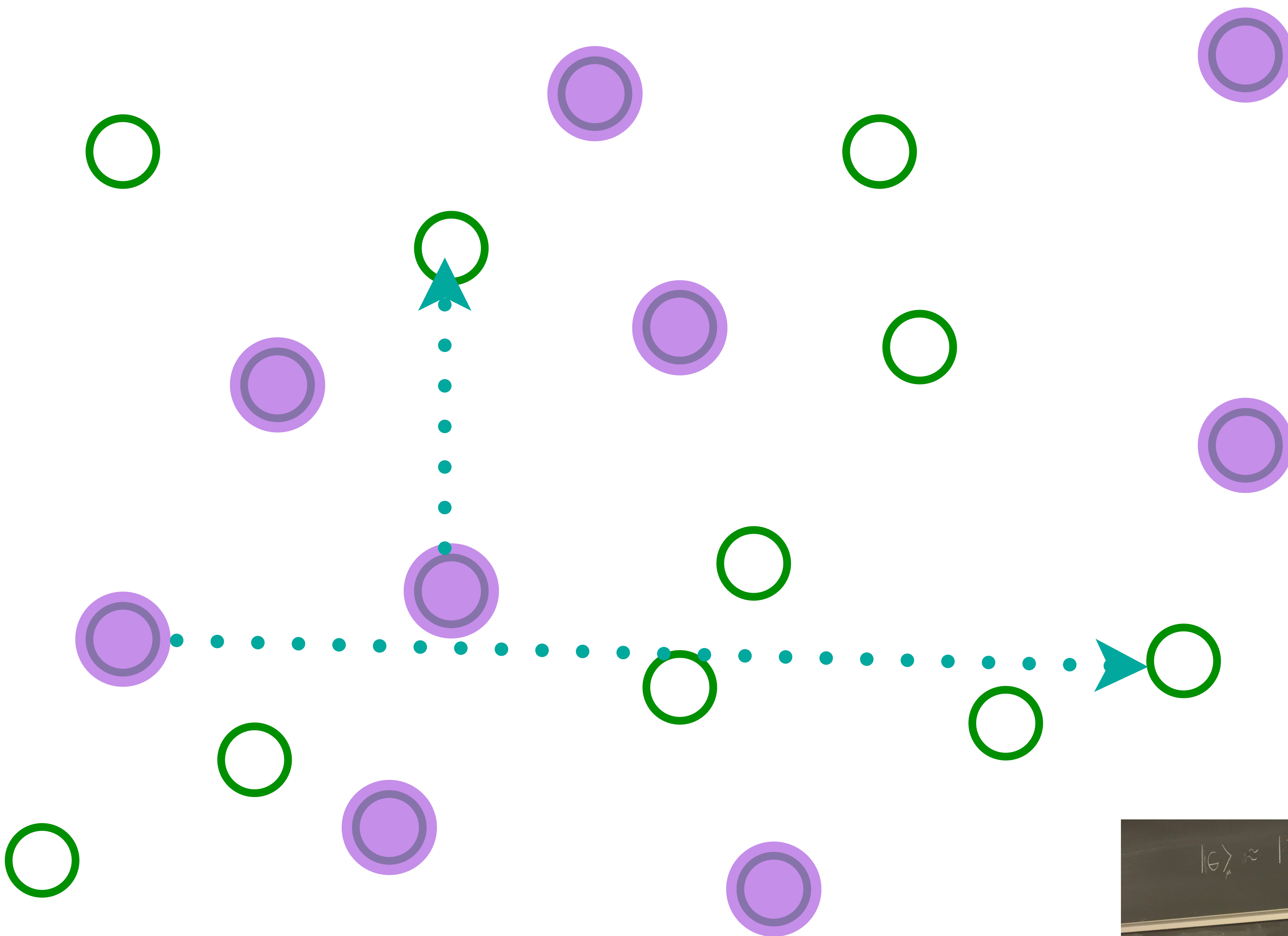

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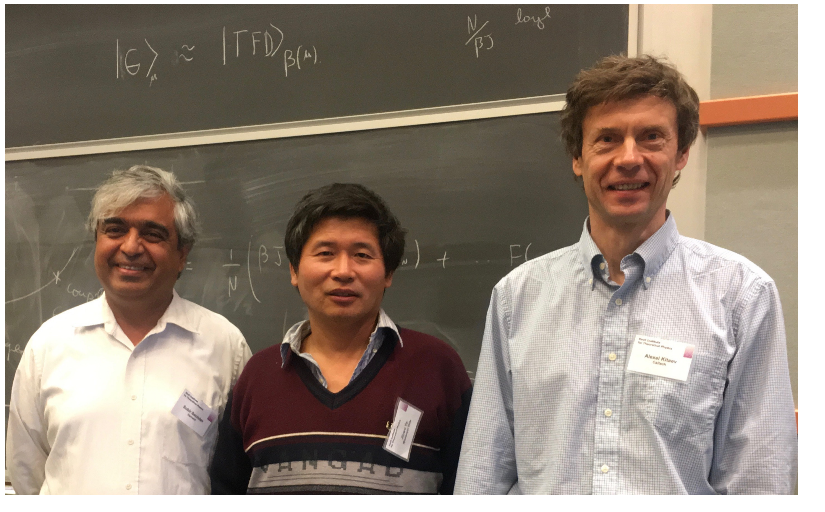

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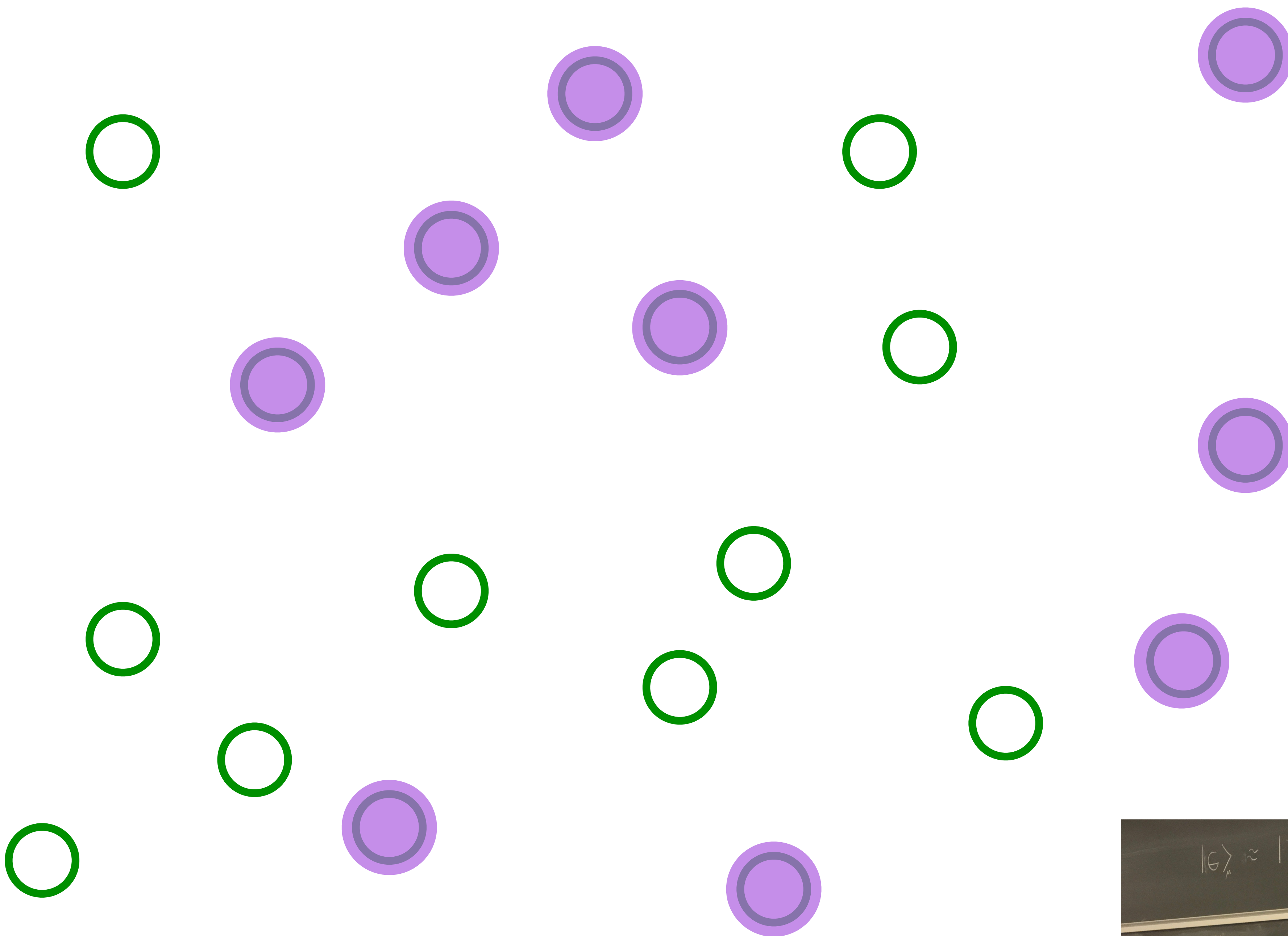
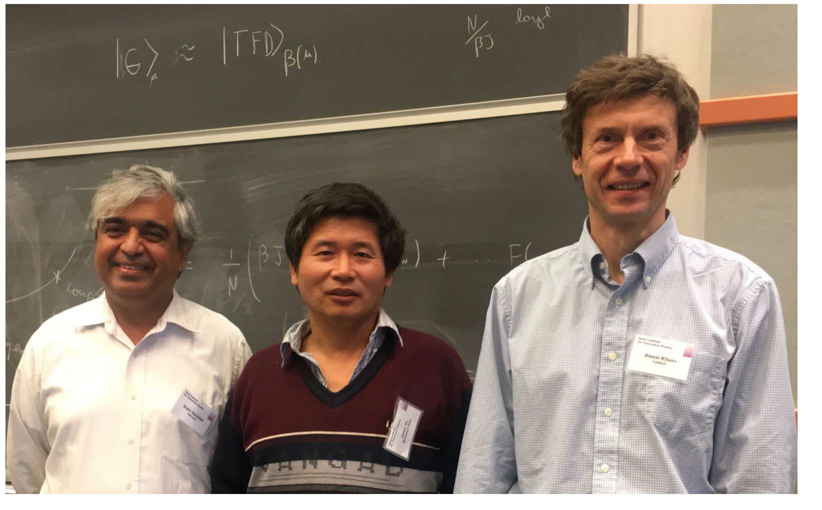


Entangle electrons pairwise randomly



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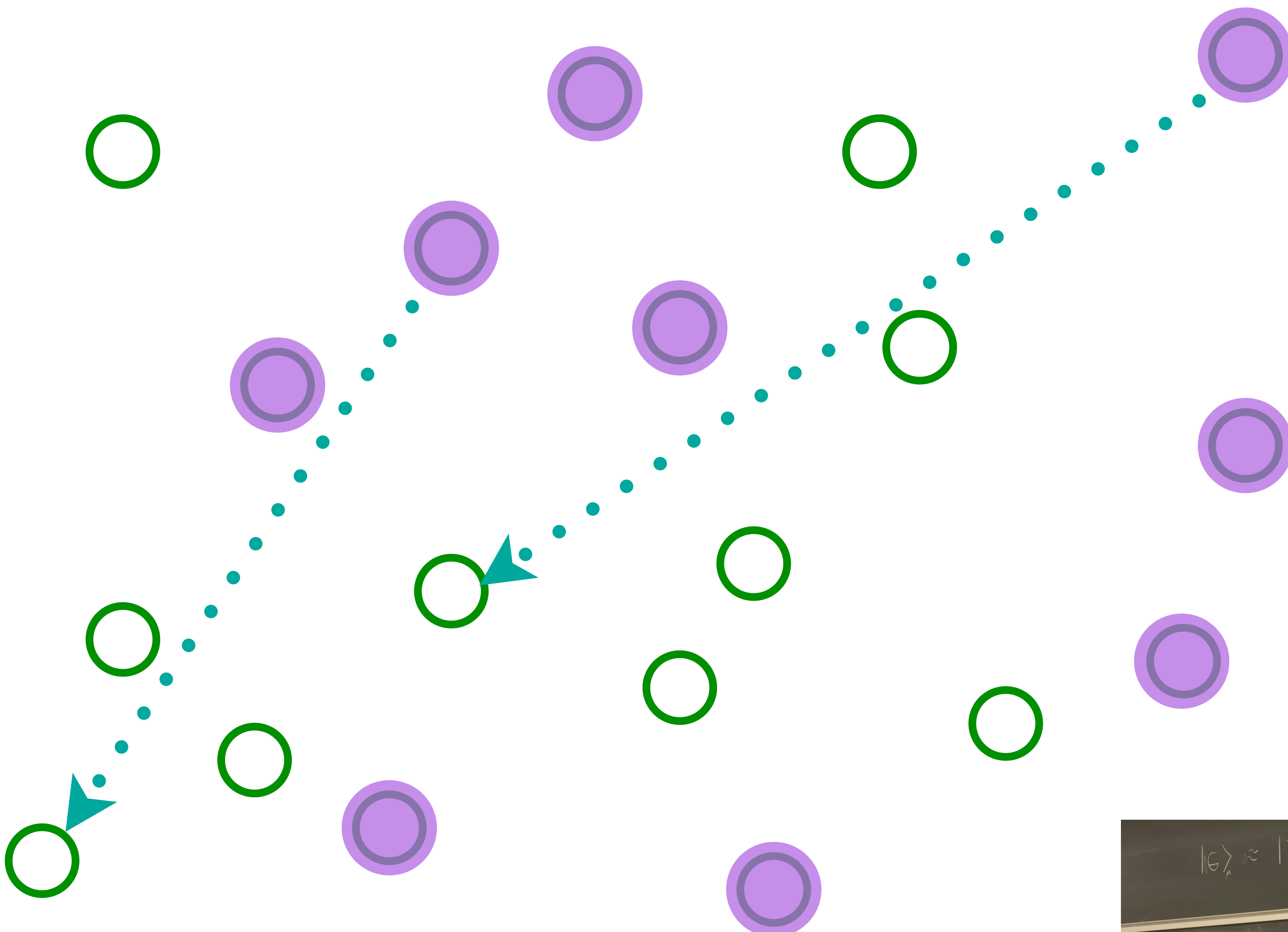

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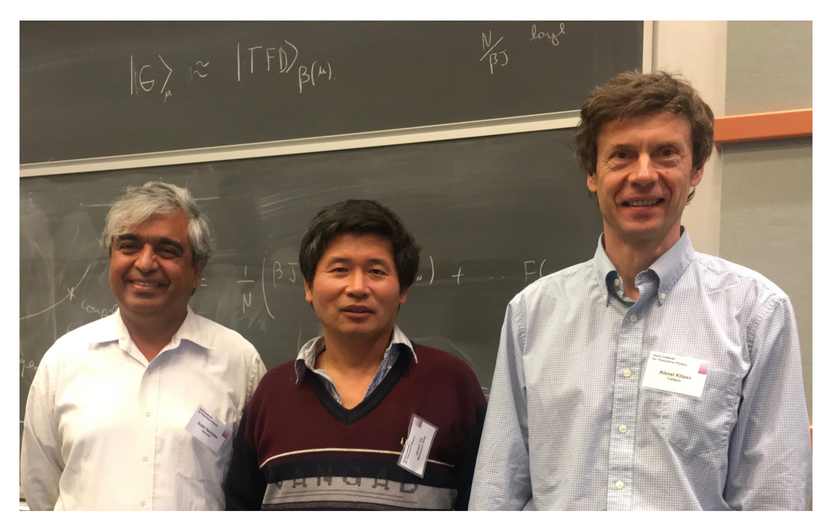

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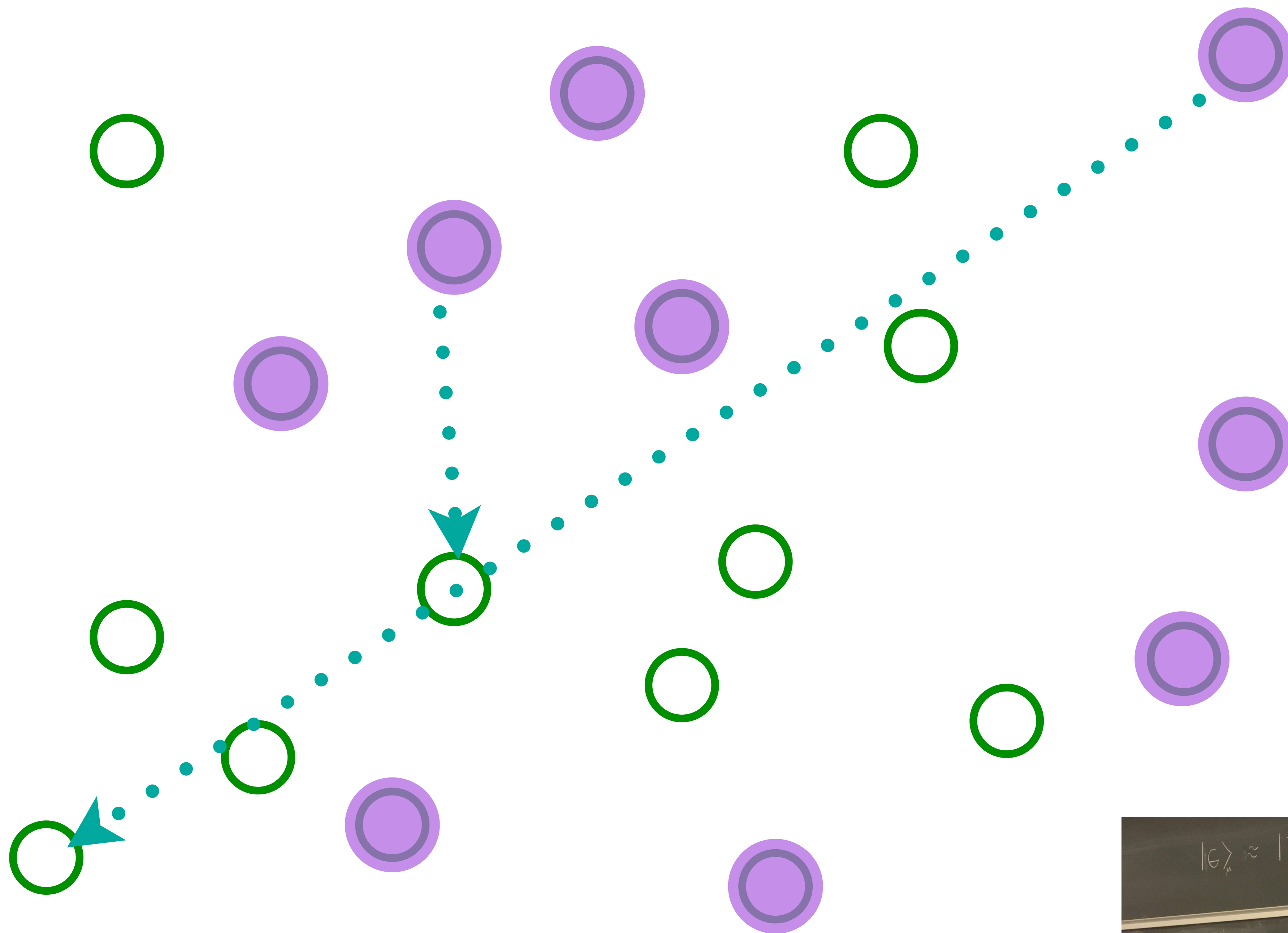

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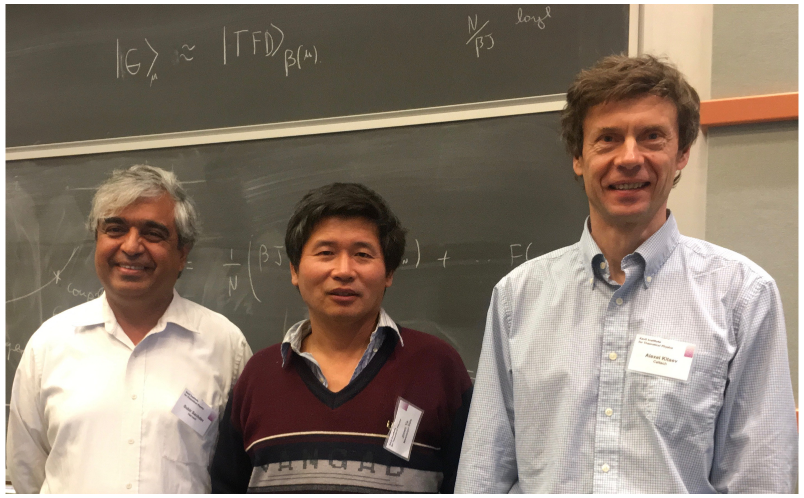

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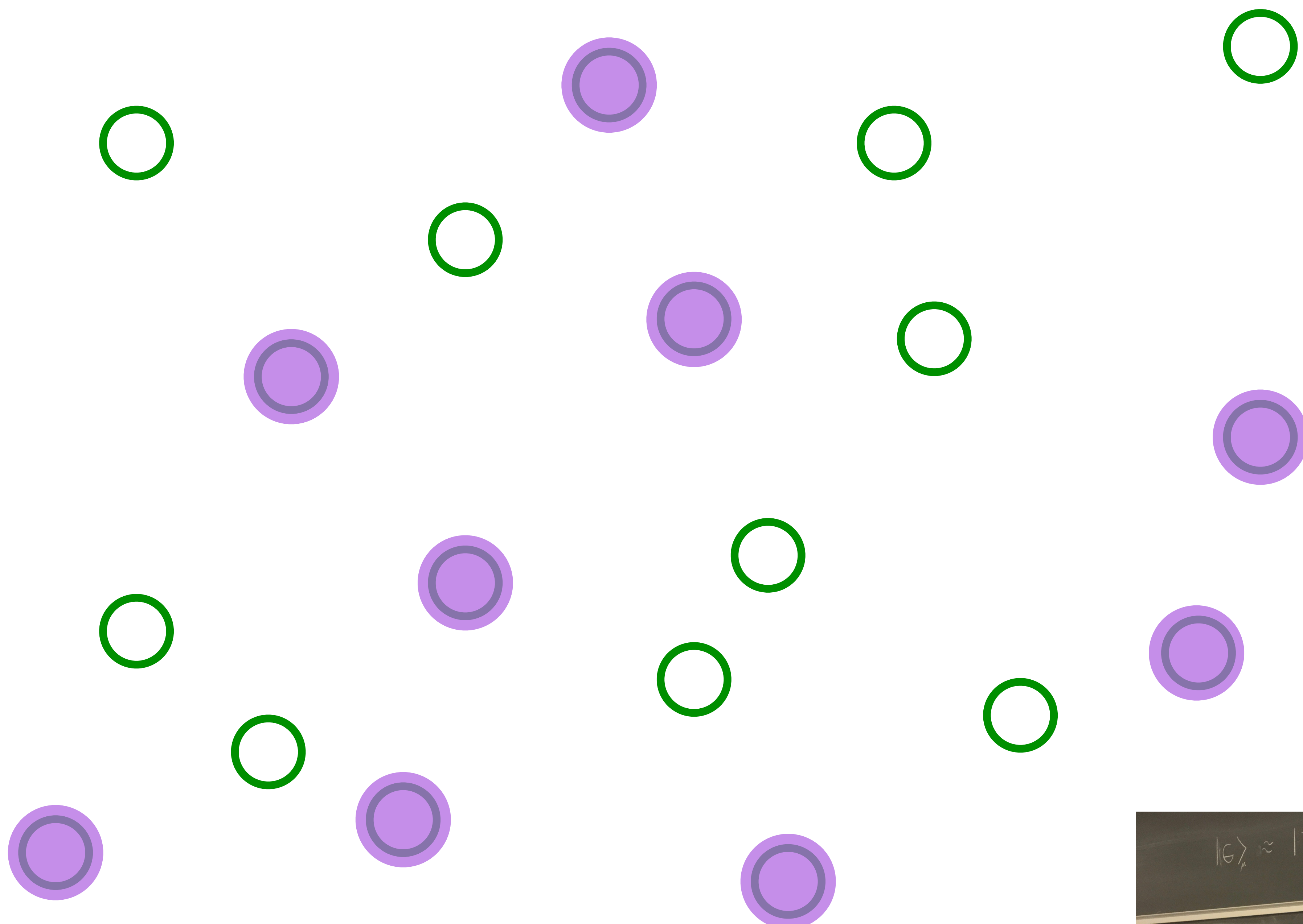

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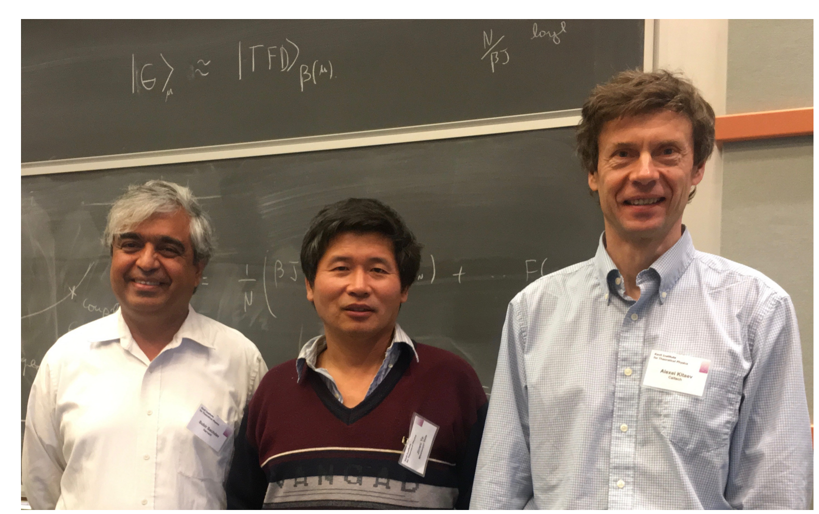

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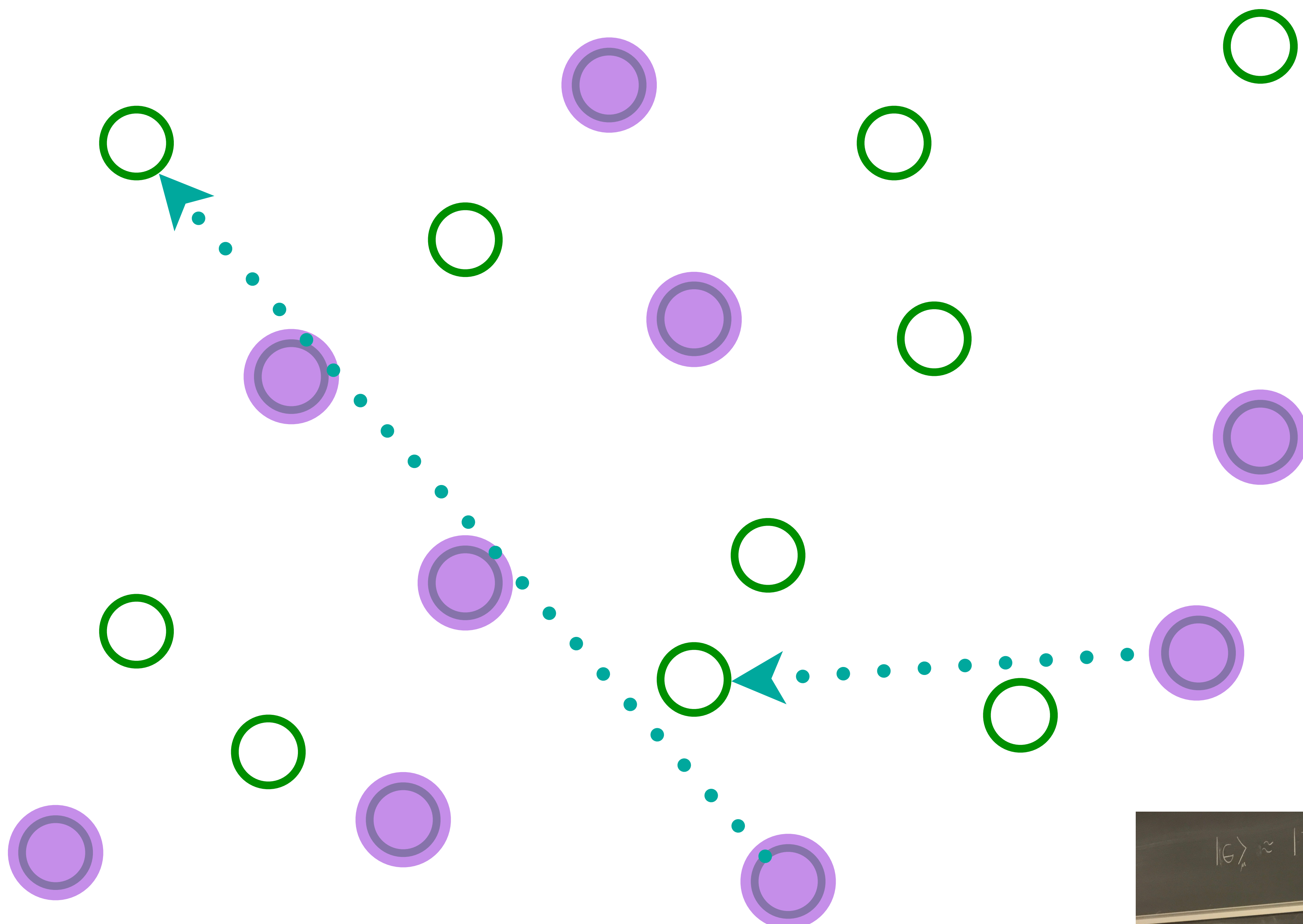

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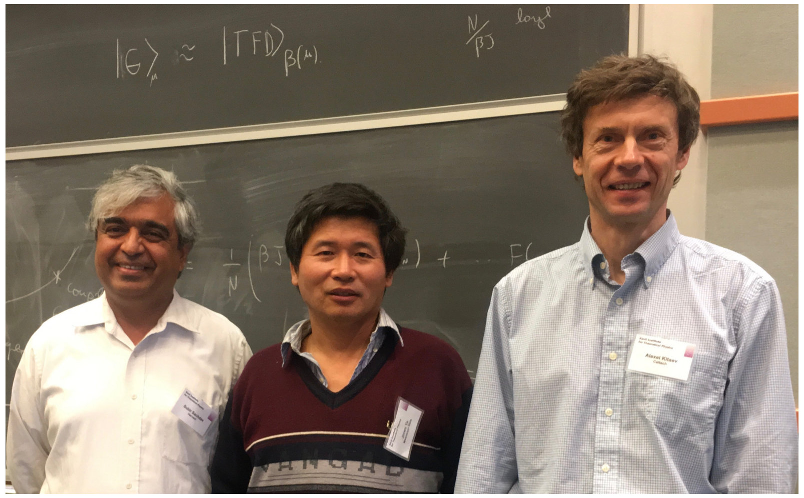

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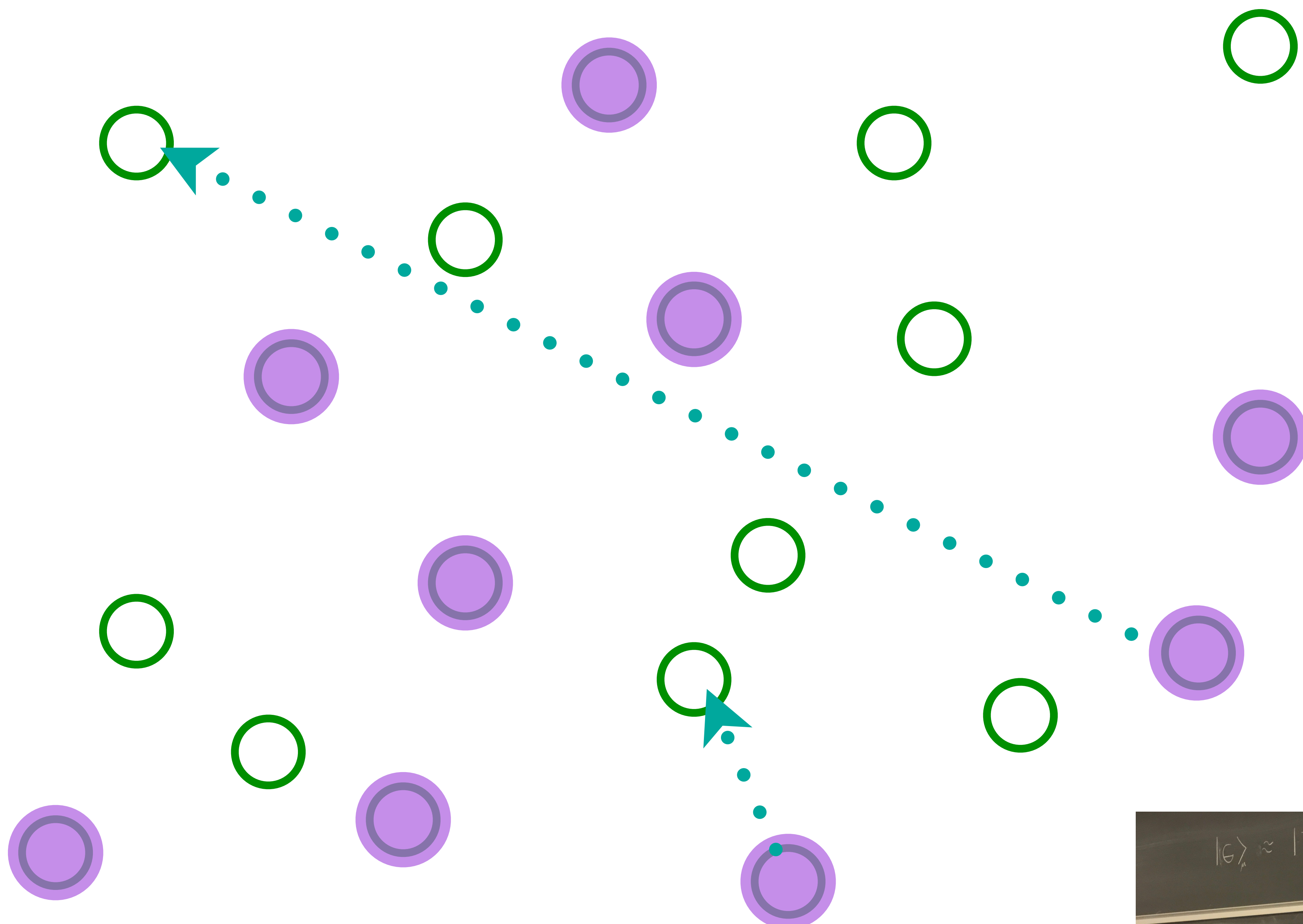

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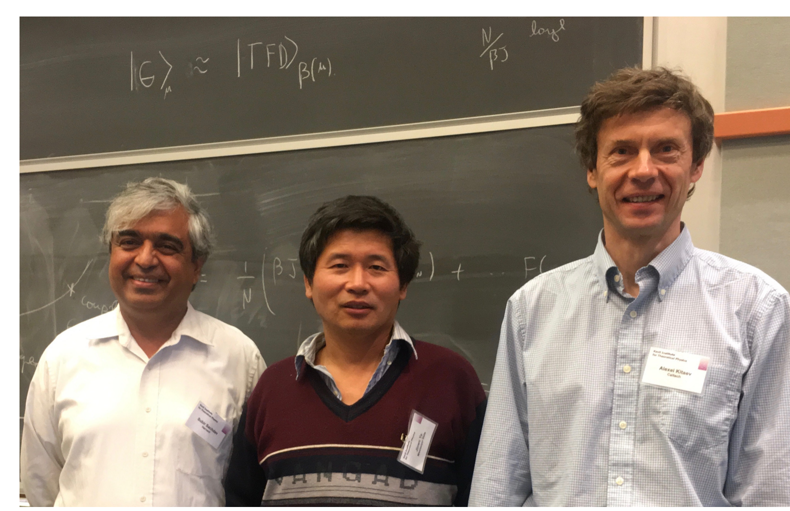

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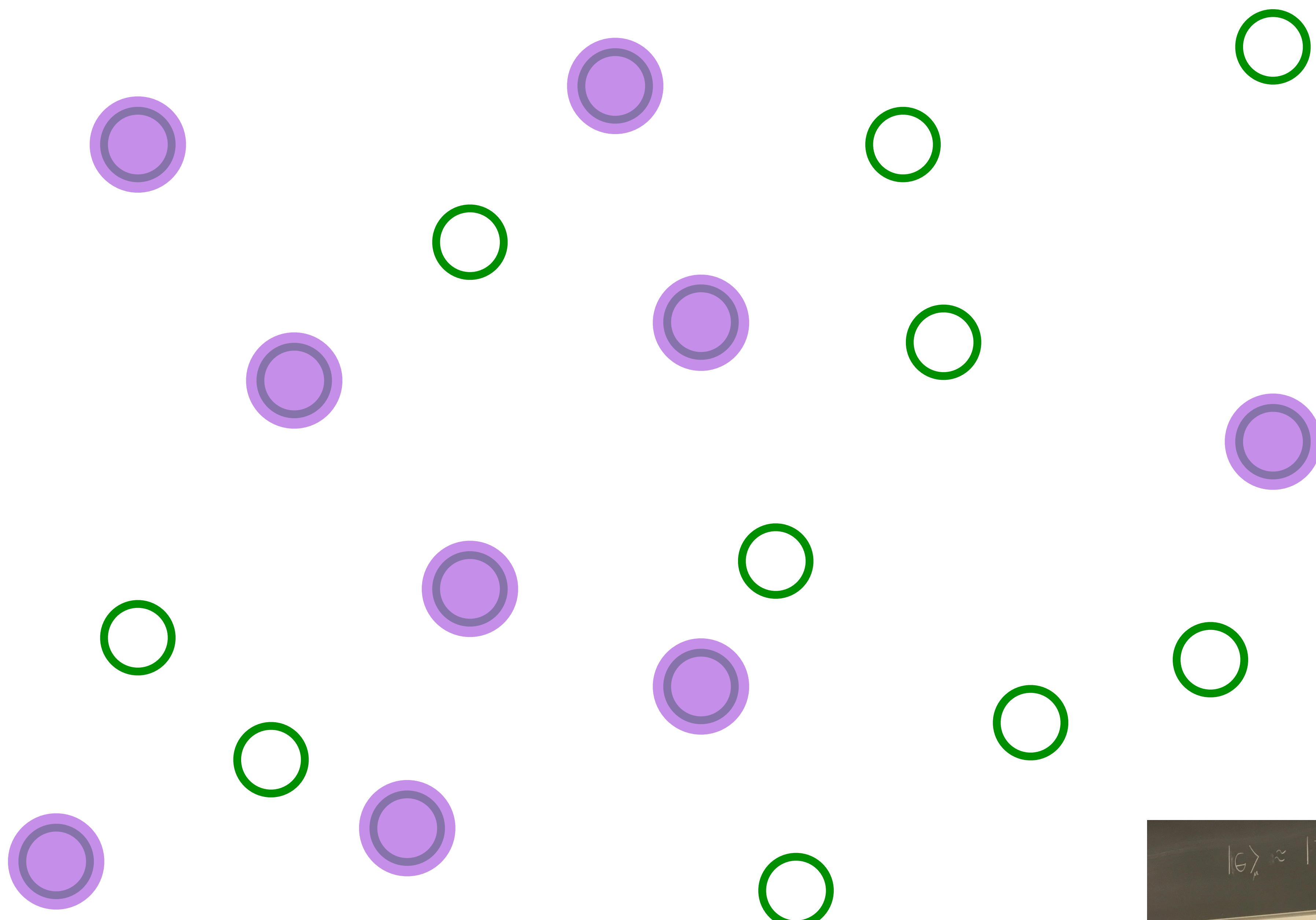

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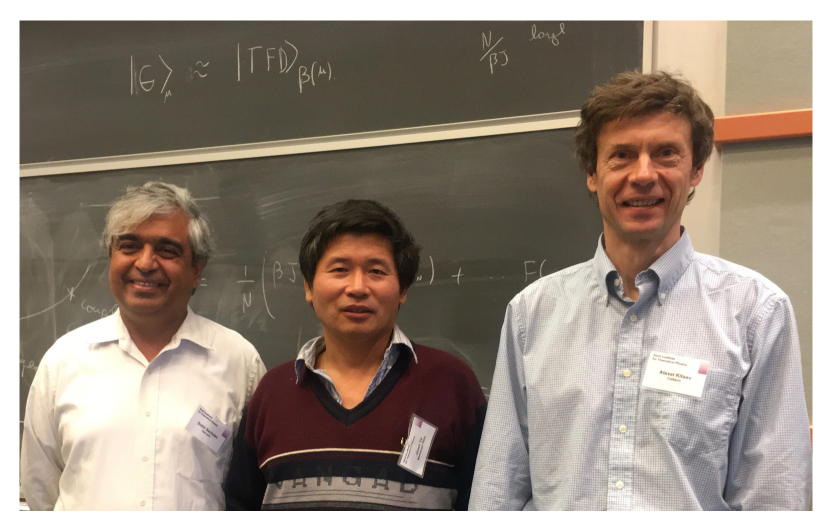

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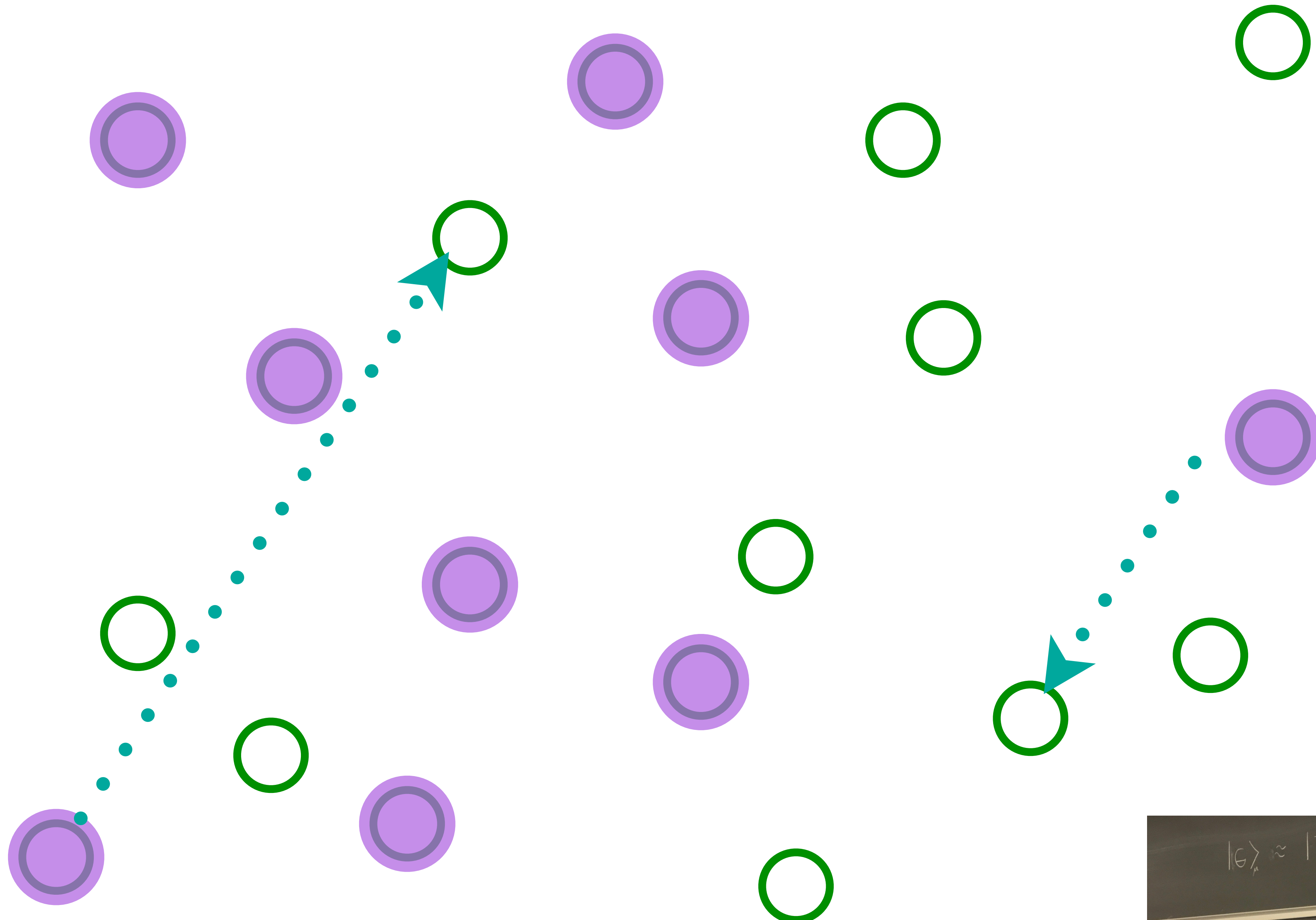

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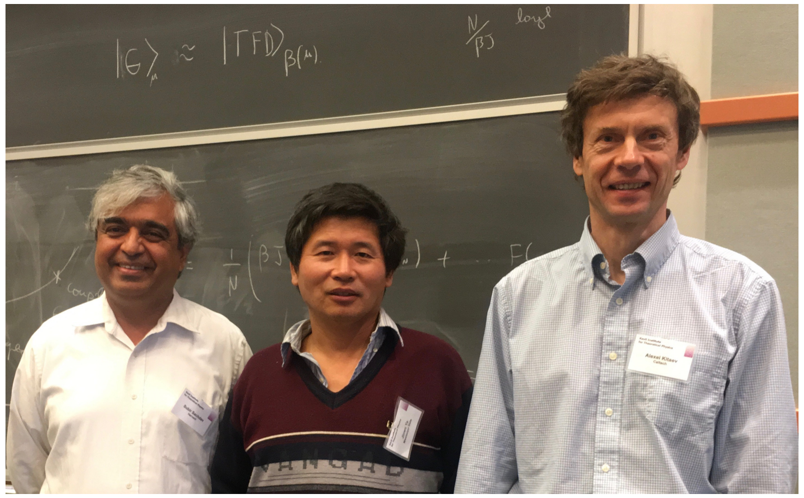

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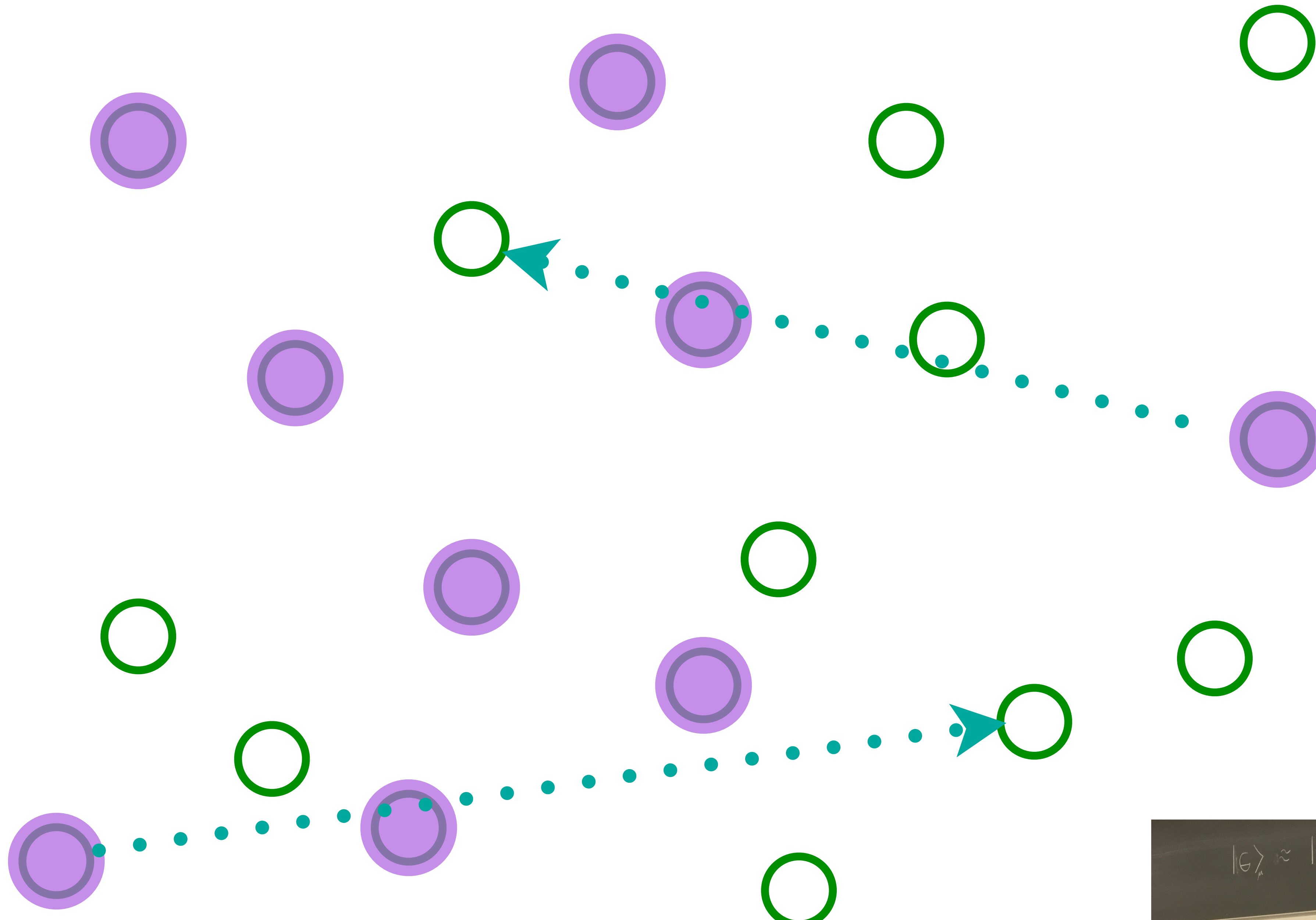

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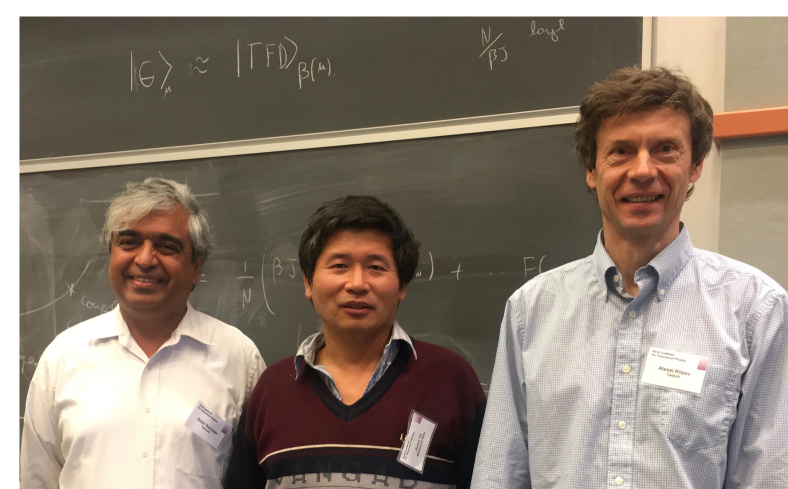

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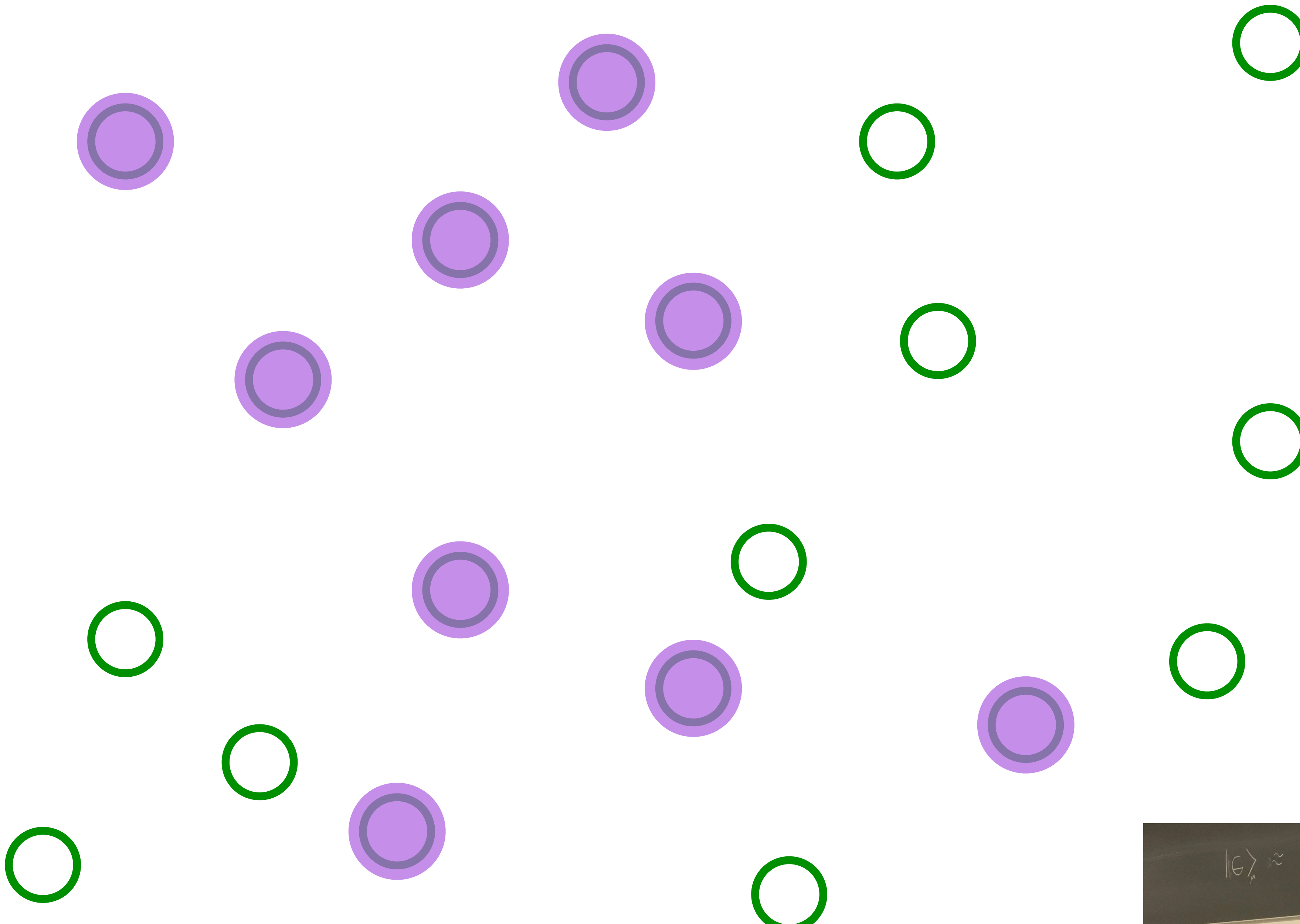

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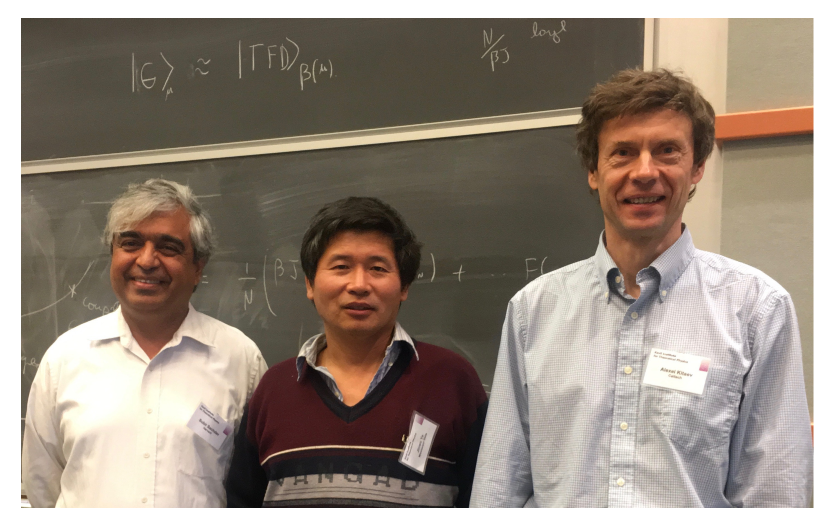

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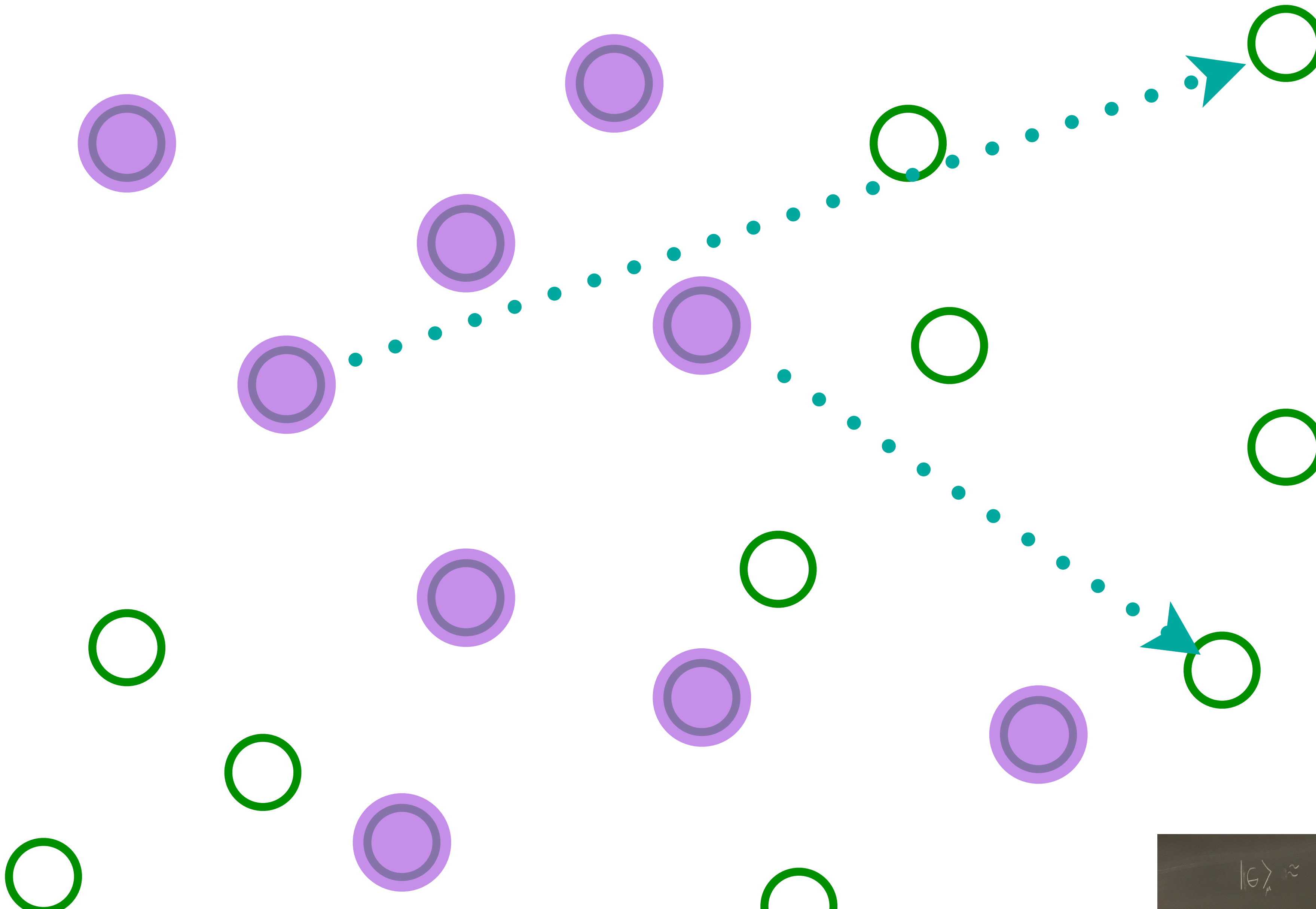

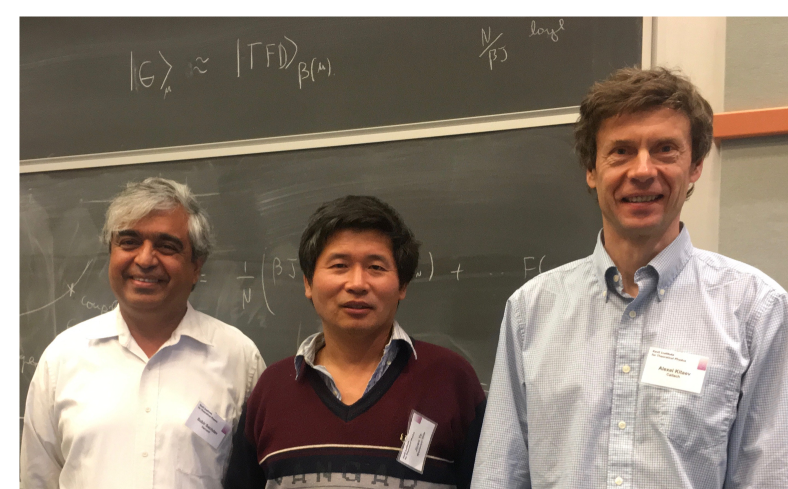

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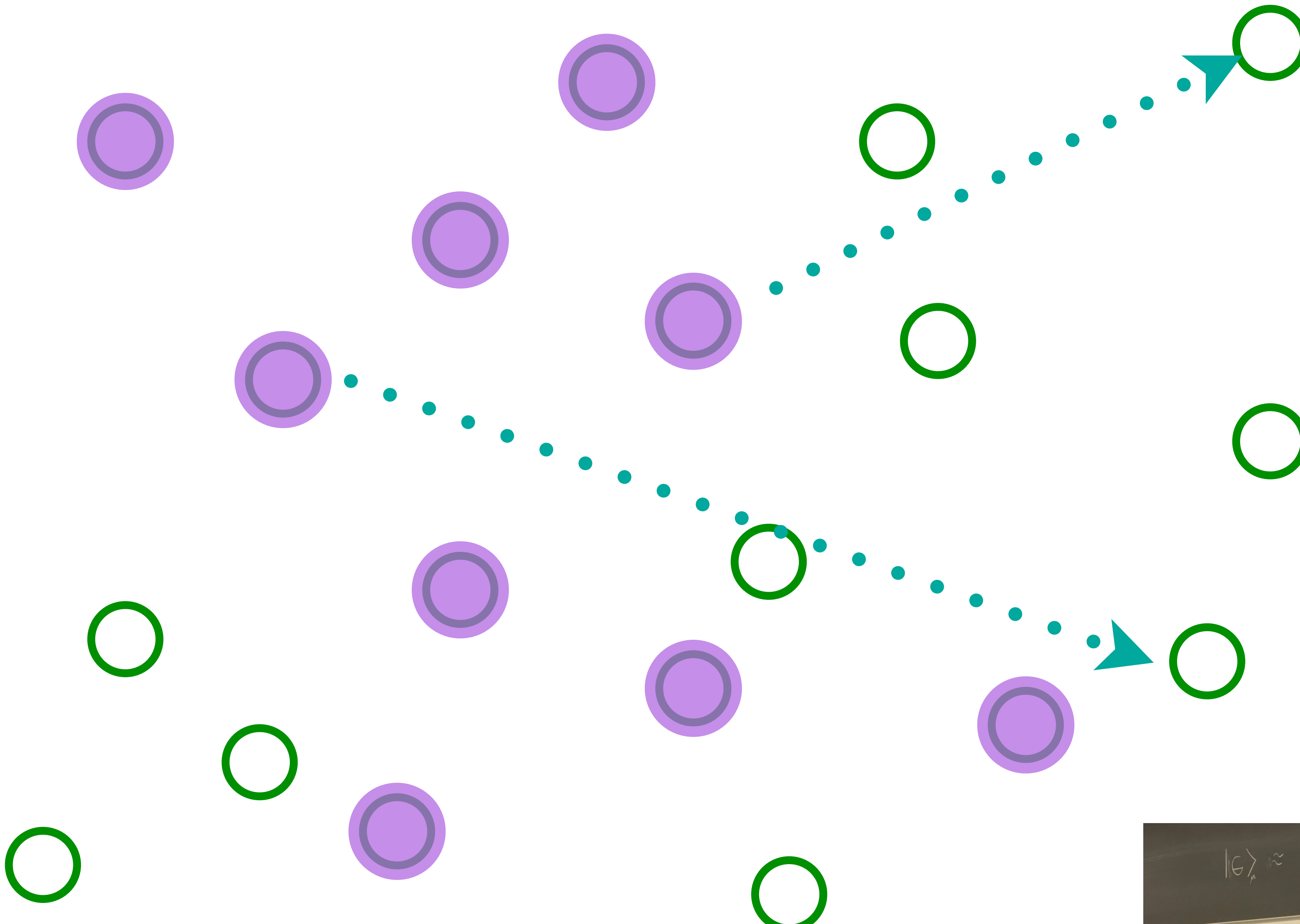

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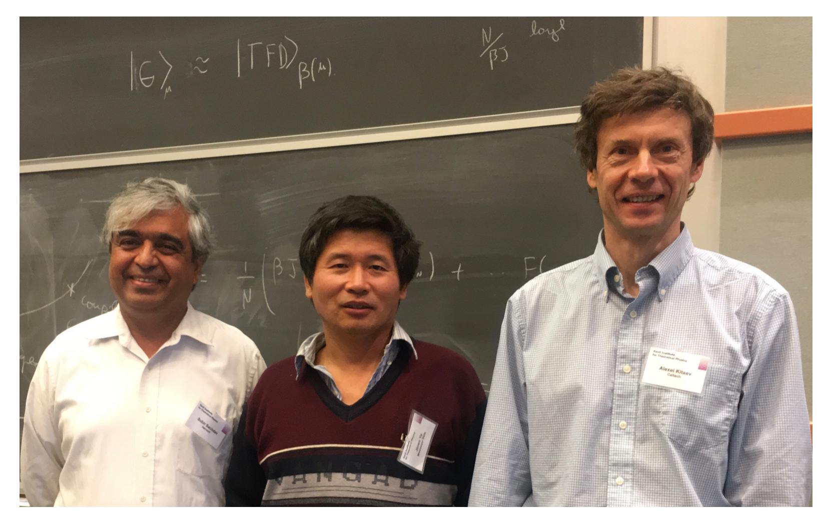

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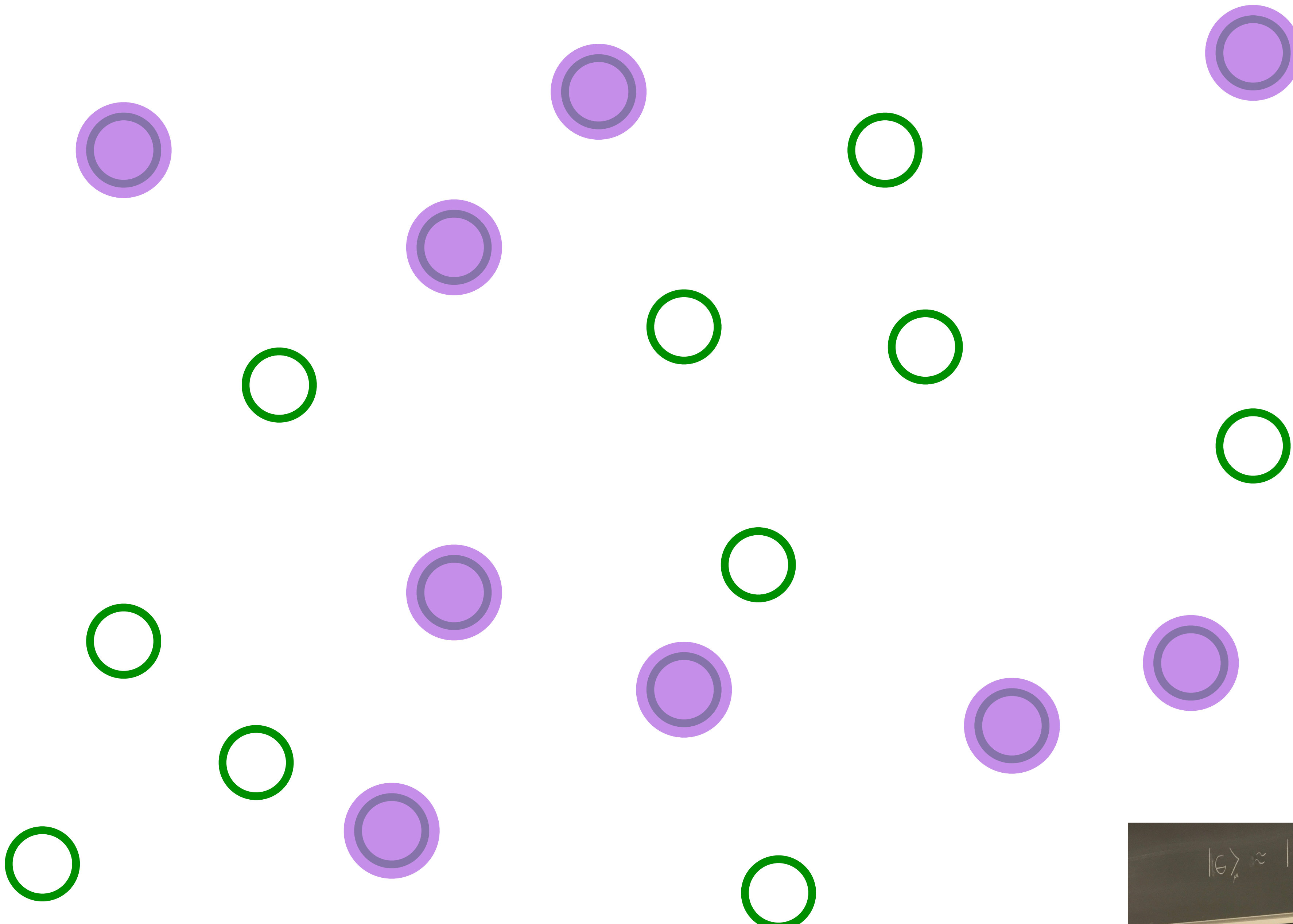

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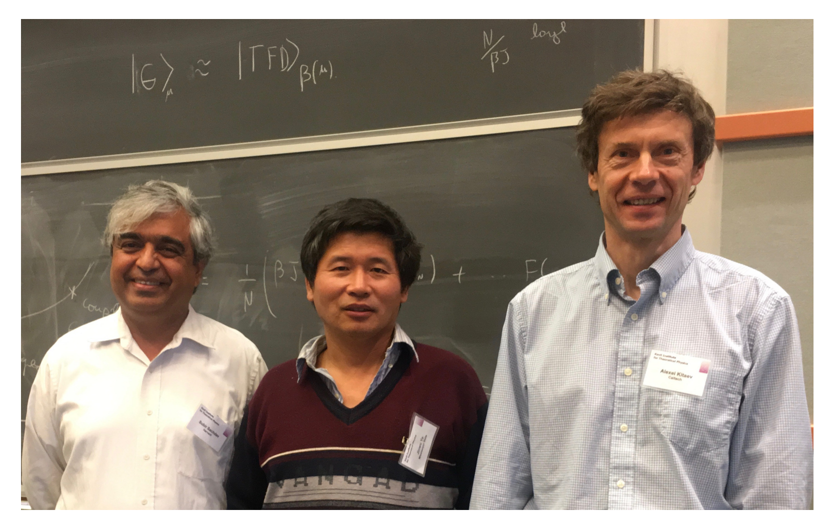

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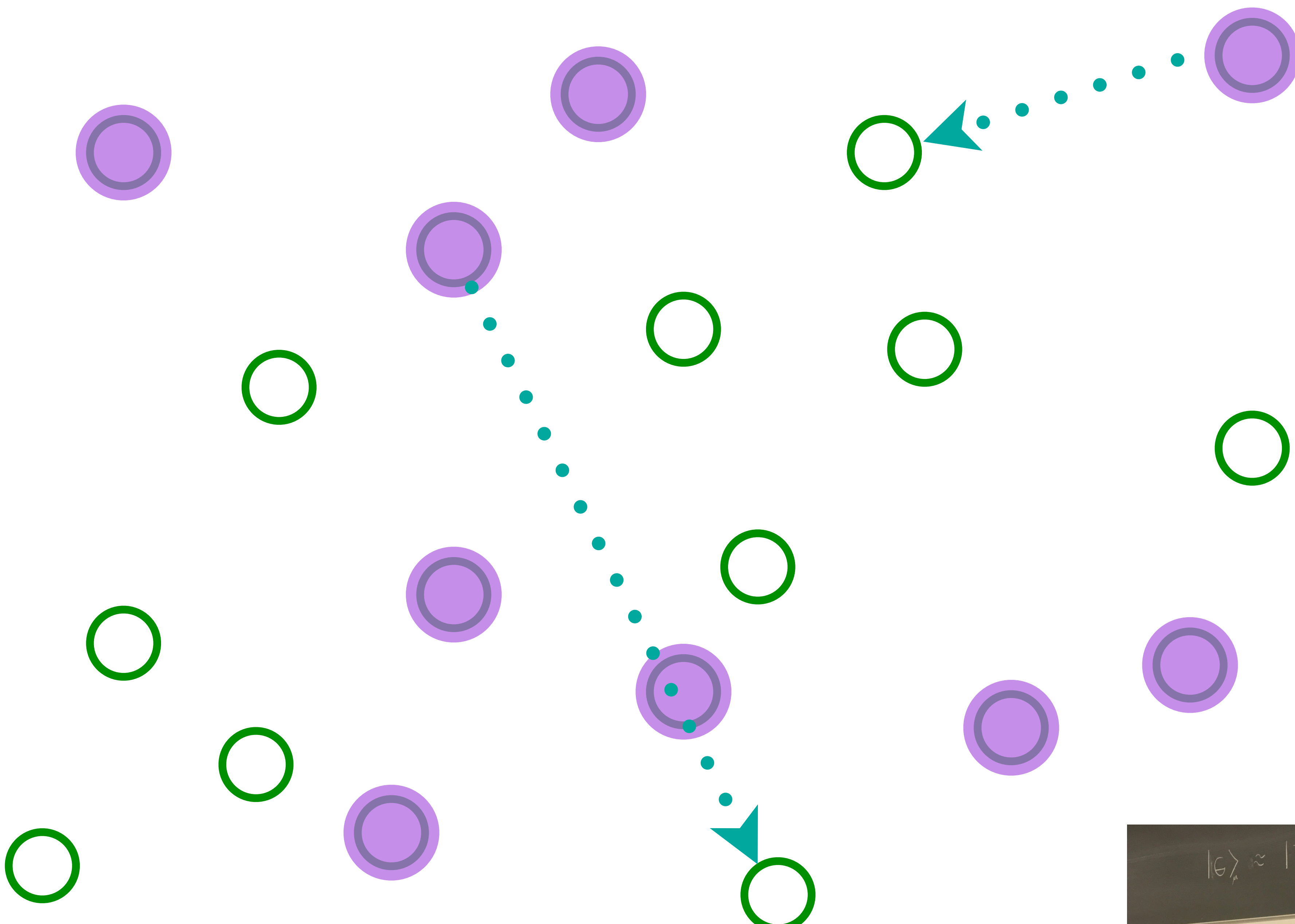

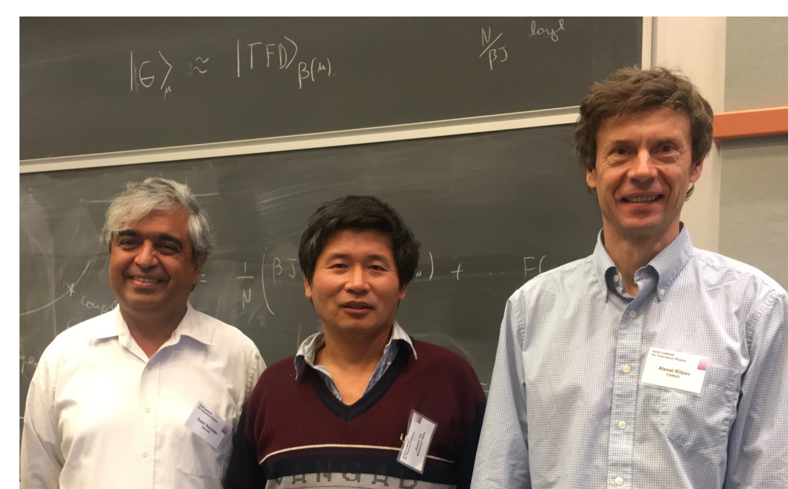

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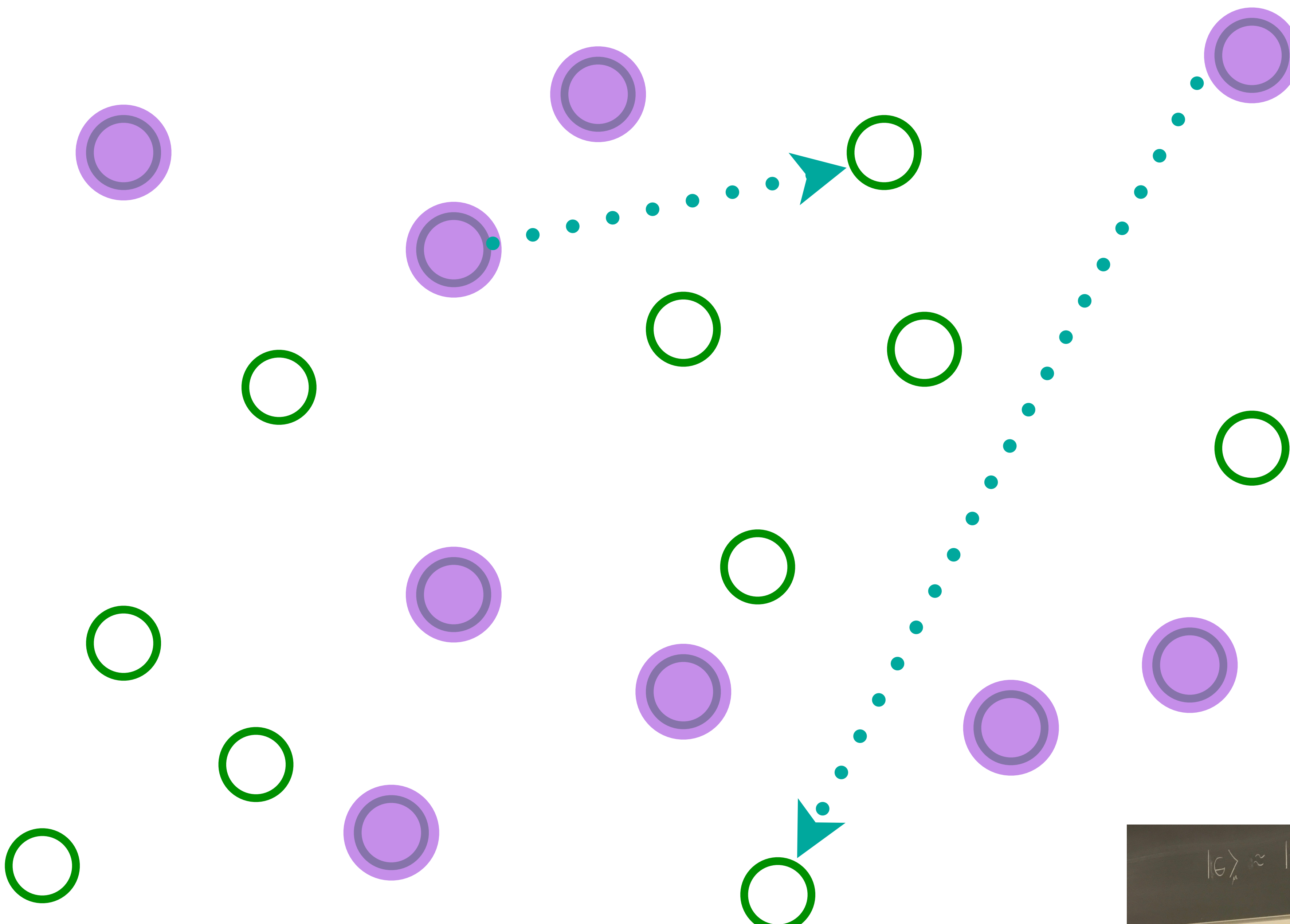

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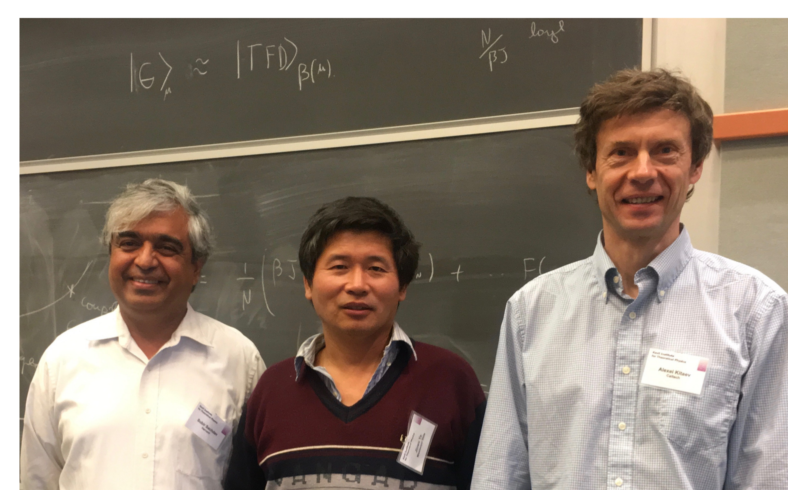

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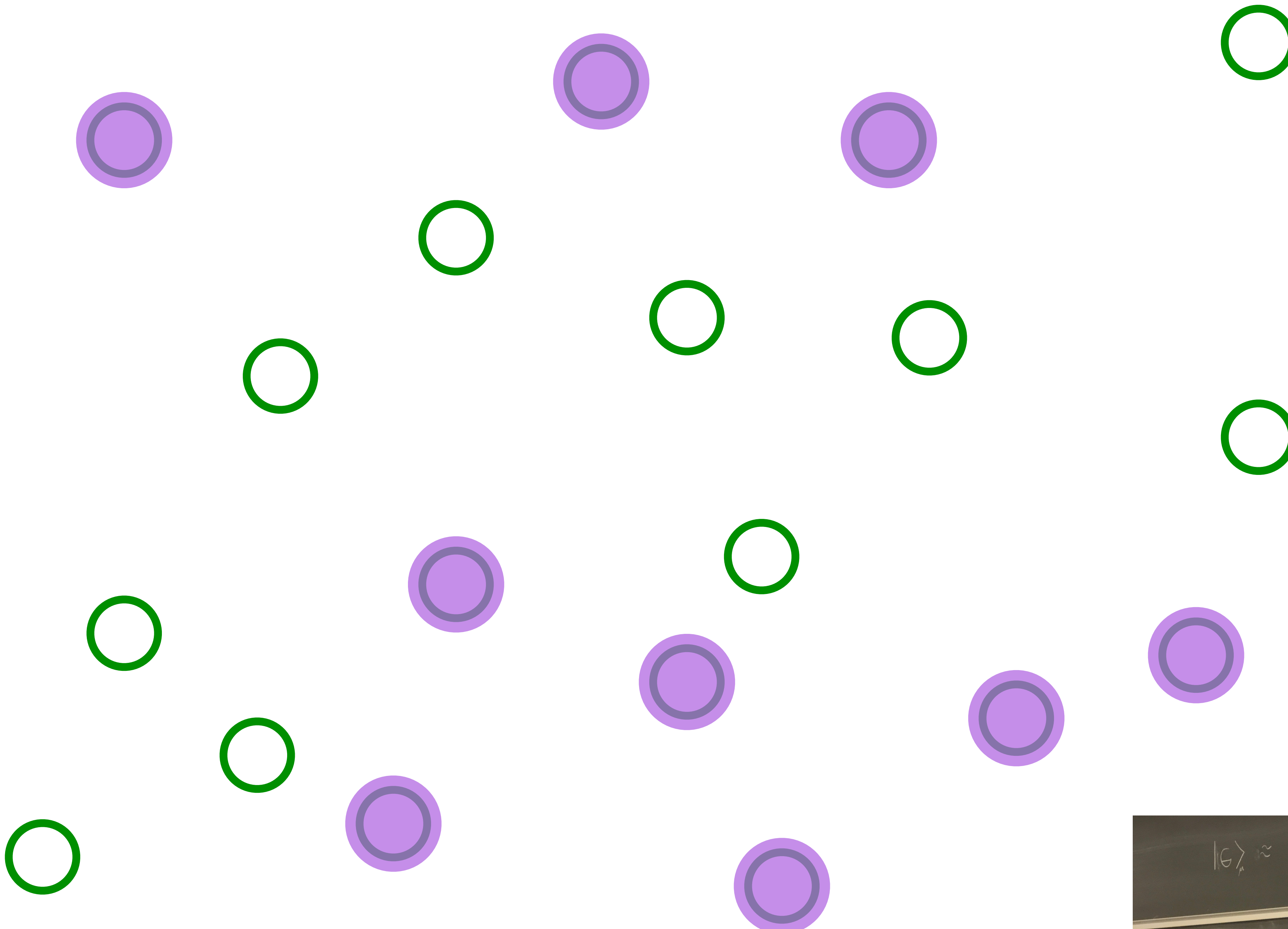

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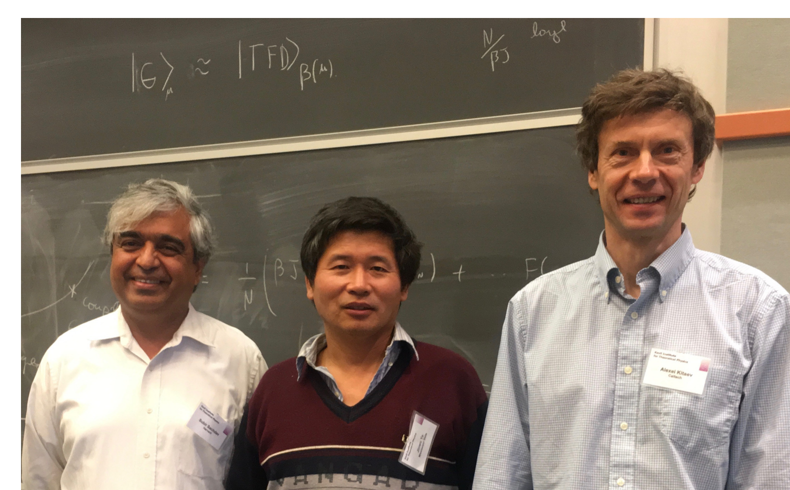


The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

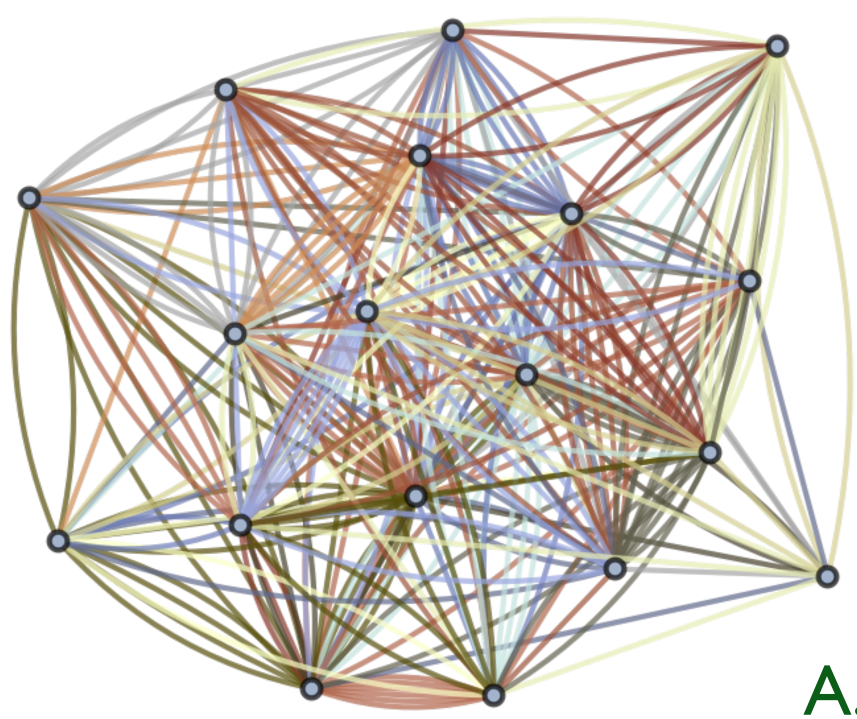
$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$

$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Complex multi-particle entanglement in the SYK model leads to a state without 'quasiparticle' excitations; *i.e.* multiple excitations cannot be built by composing an elementary set of 'quasiparticle' excitations.

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Many-body chaos and thermal equilibration in the shortest possible Planckian time $\sim \dfrac{\hbar}{k_B T}$.

Main result I

For $k_B T \ll U$

$$\mathcal{Z} = \mathrm{Tr} \exp\left(-\frac{\mathcal{H}}{k_B T}\right)$$

$$= \exp\left(N \frac{S_0}{k_B}\right) \int \mathcal{D} f(\tau) \exp\left(-\frac{1}{\hbar} \mathcal{S}_{2\mathrm{D-gravity}} \left[f(\tau)\right]\right)$$

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

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S_0 is the $T \to 0$ entropy of the SYK model.

$\dfrac{\partial S_0}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$, where $\mathcal{E}$ characterizes the particle-hole asymmetry of the spectrum.

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 133406 (2001)

$S(T) = S_0 + \ldots$ will map on to the Bekenstein-Hawking entropy of charged black holes

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
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- $f(\tau)$ is the reparameterization of the imaginary time of the SYK model: τ on a circle of circumference $\hbar/(k_B T)$.

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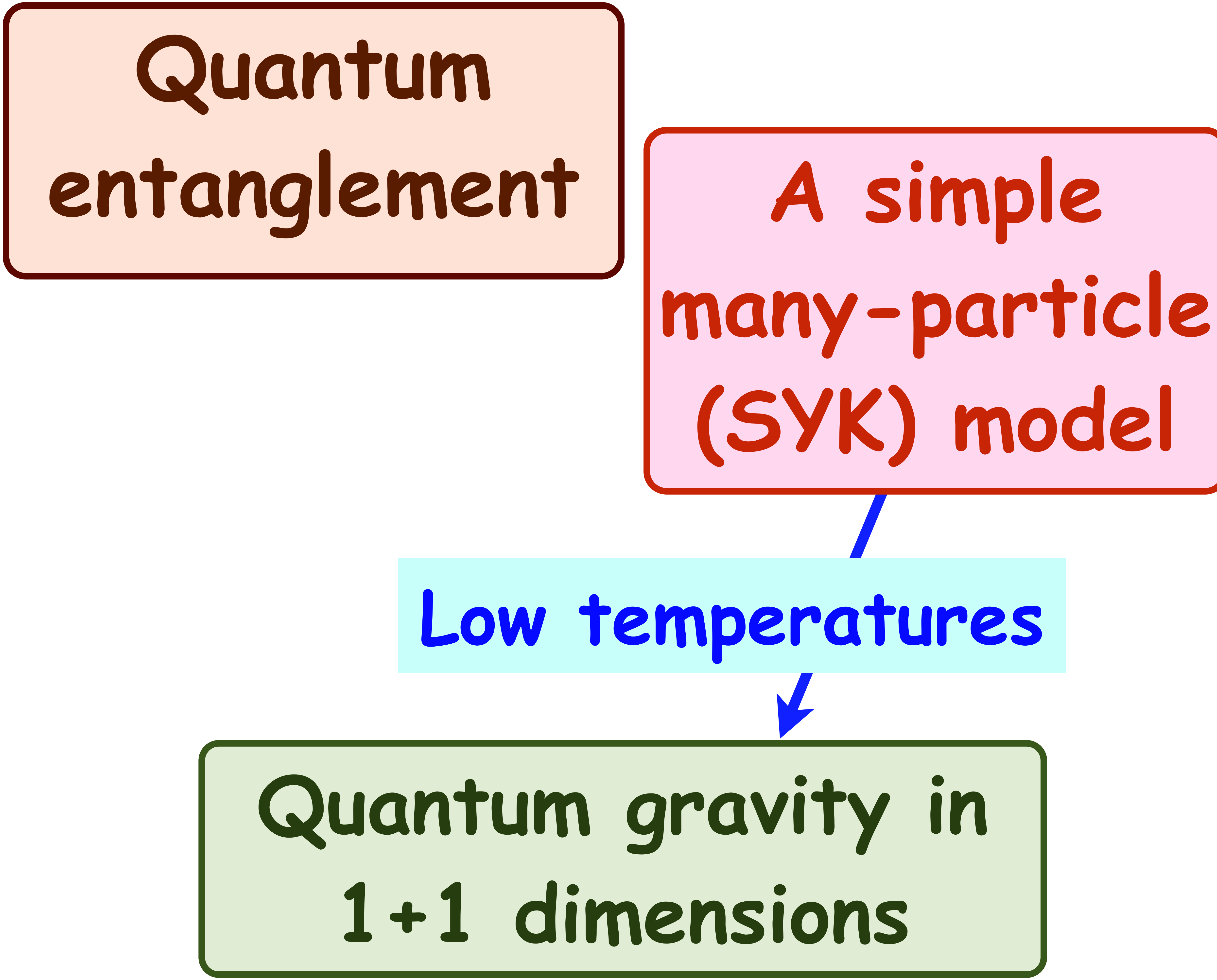
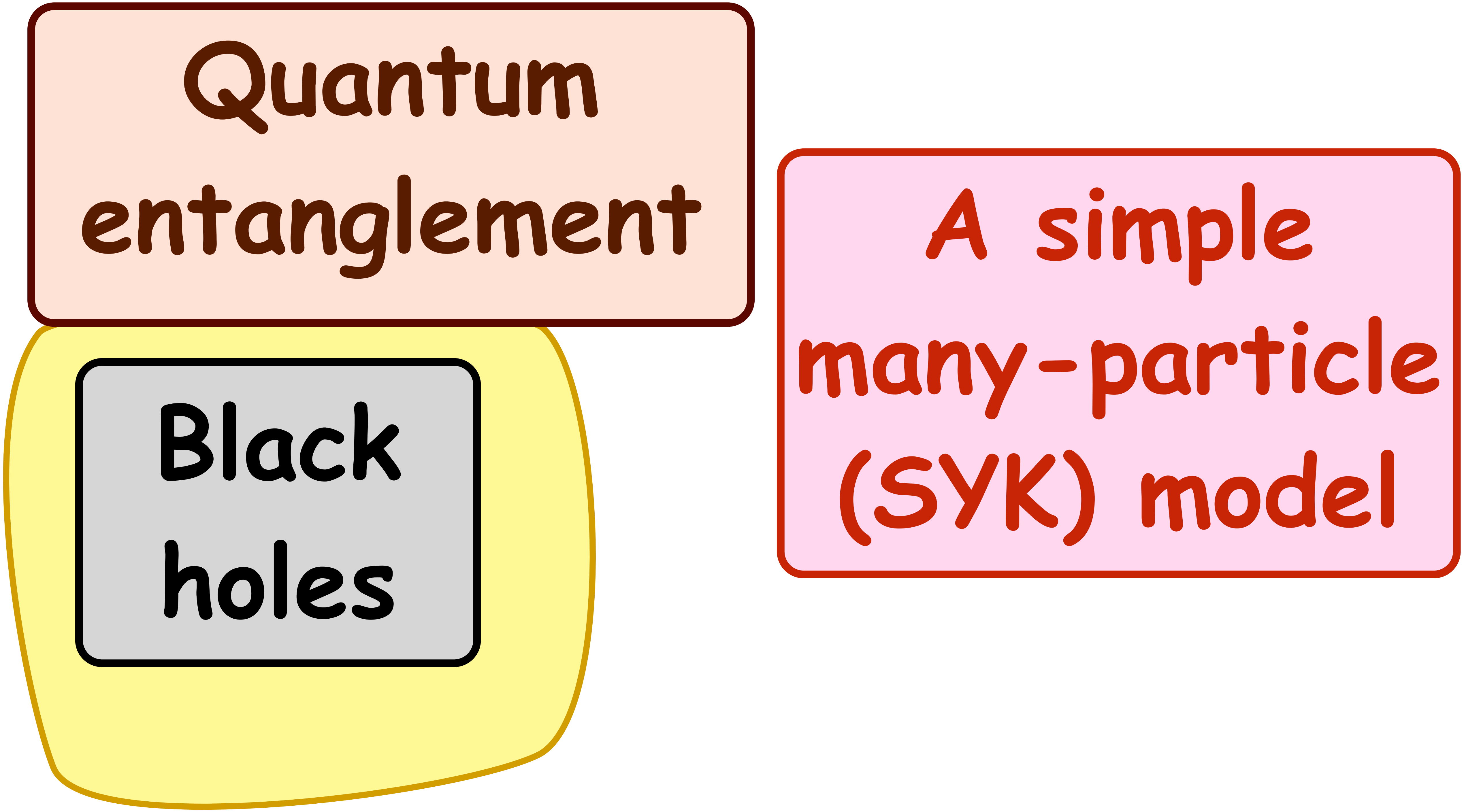
- $f(\tau)$ is the reparameterization of the imaginary time of the SYK model: τ on a circle of circumference $\hbar/(k_B T)$.
- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a 'boundary graviton'.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

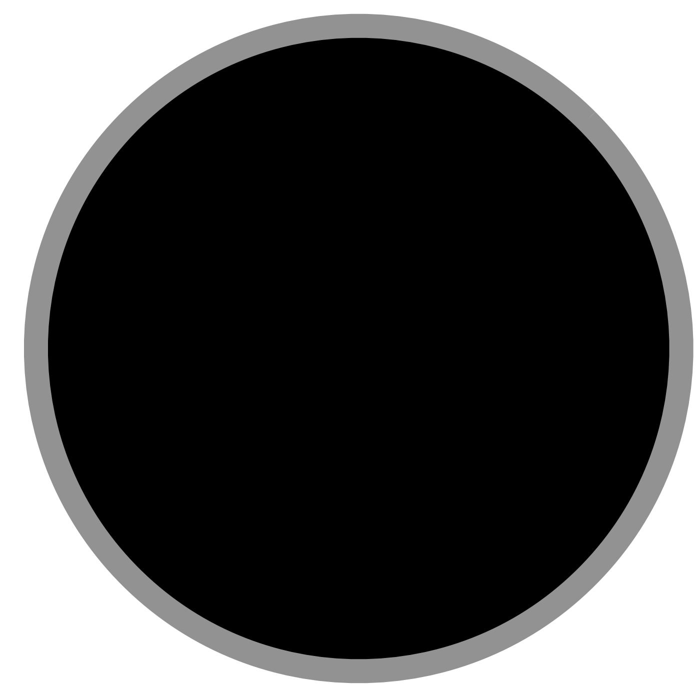

Quantum entanglement
A simple many-particle (SYK) model
Low temperatures
Quantum gravity in 1+1 dimensions


Quantum entanglement
Black holes
A simple many-particle (SYK) model

Black Holes

Objects so dense that light is gravitationally bound to them.

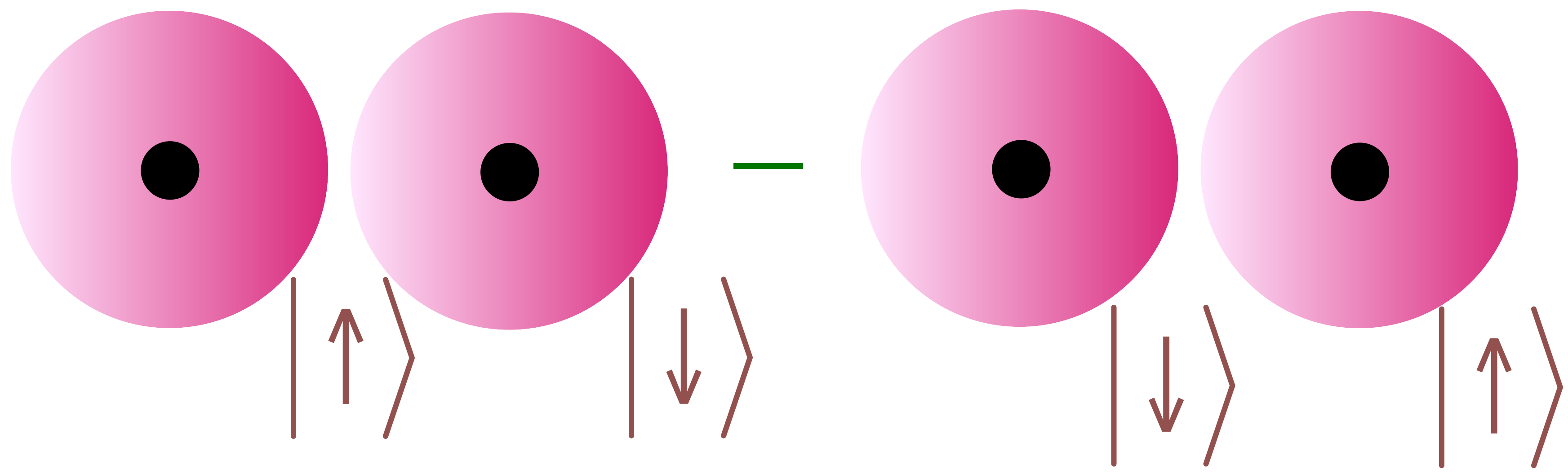
In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.



$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

Quantum Entanglement across a black hole horizon



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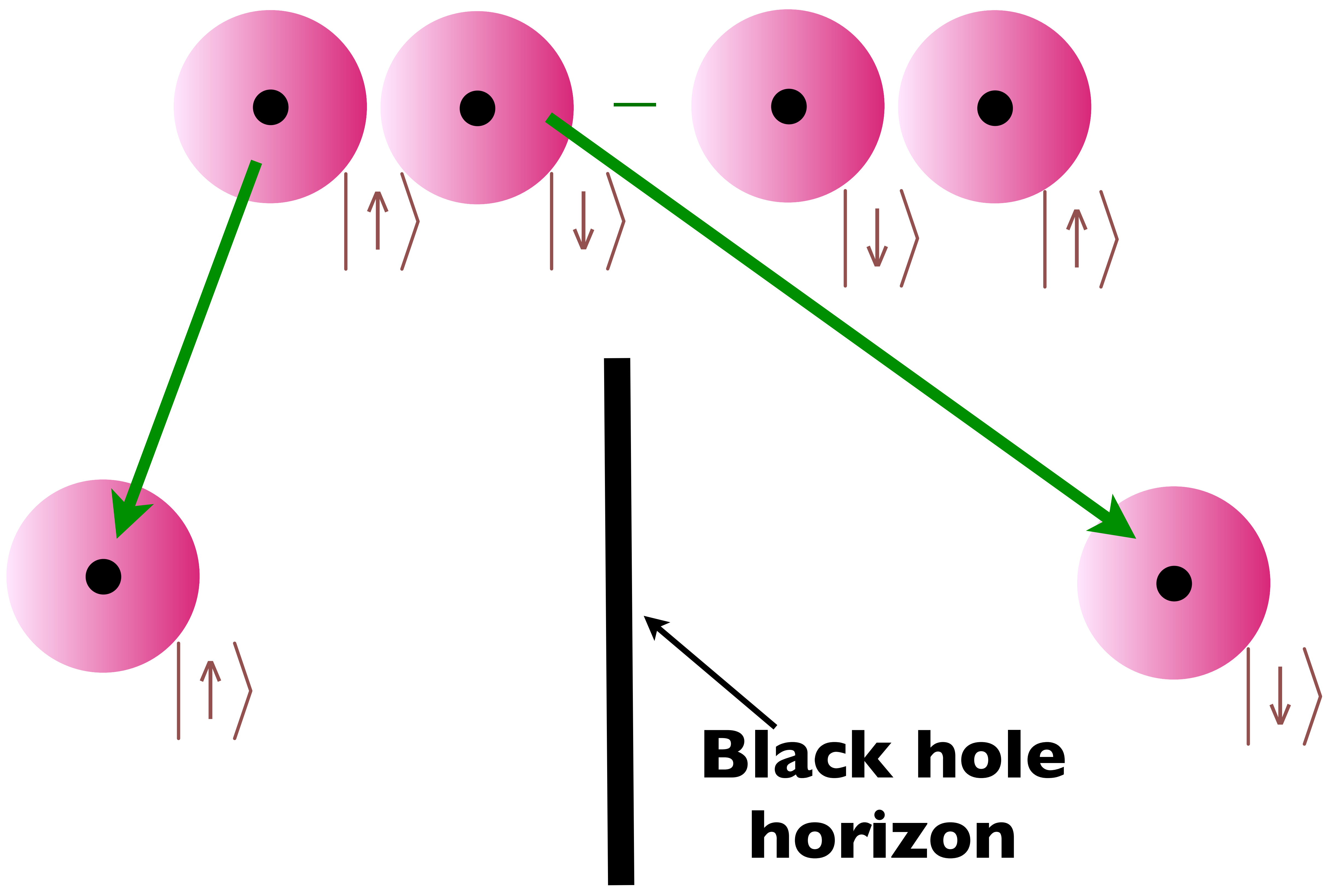

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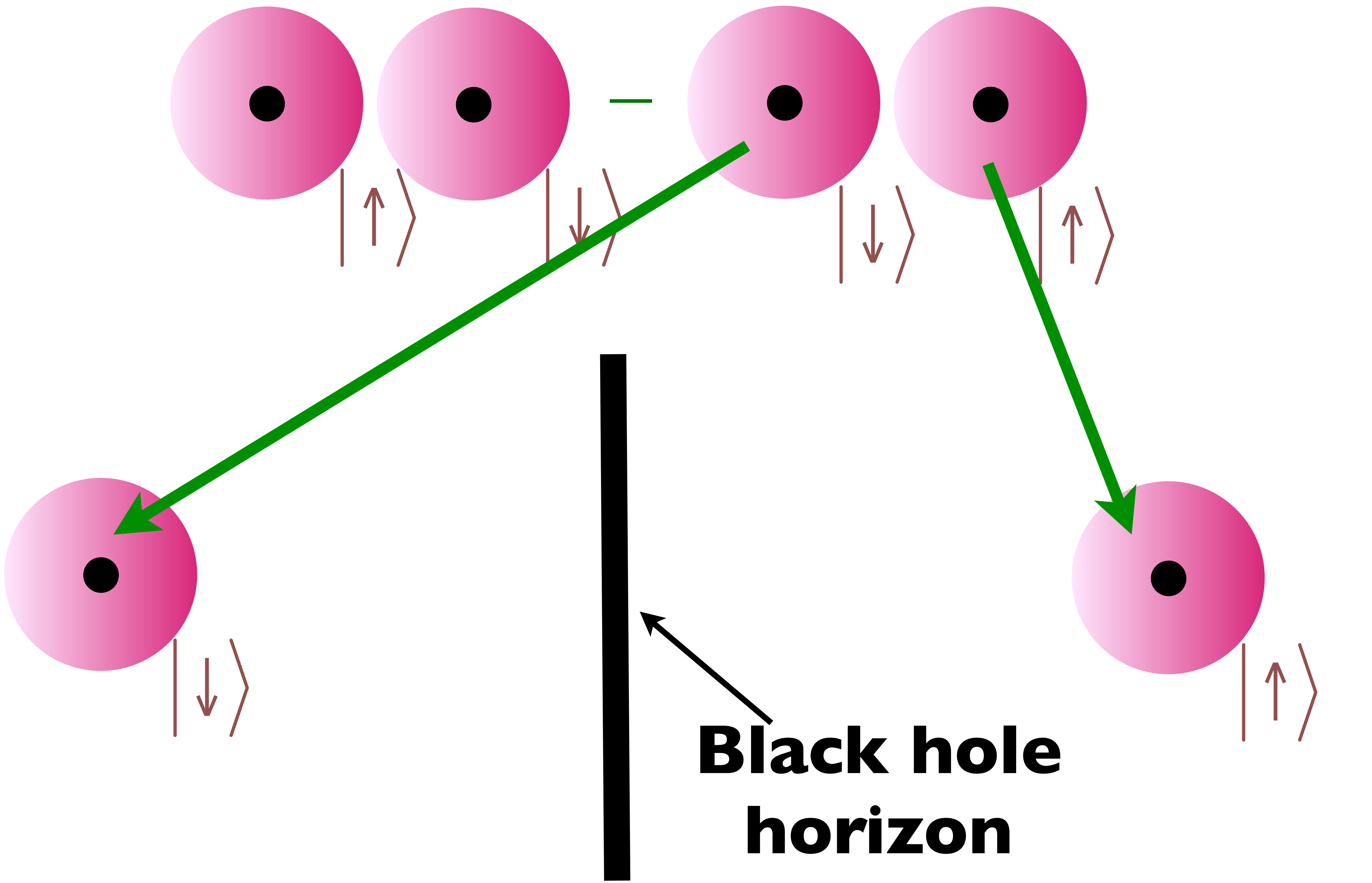
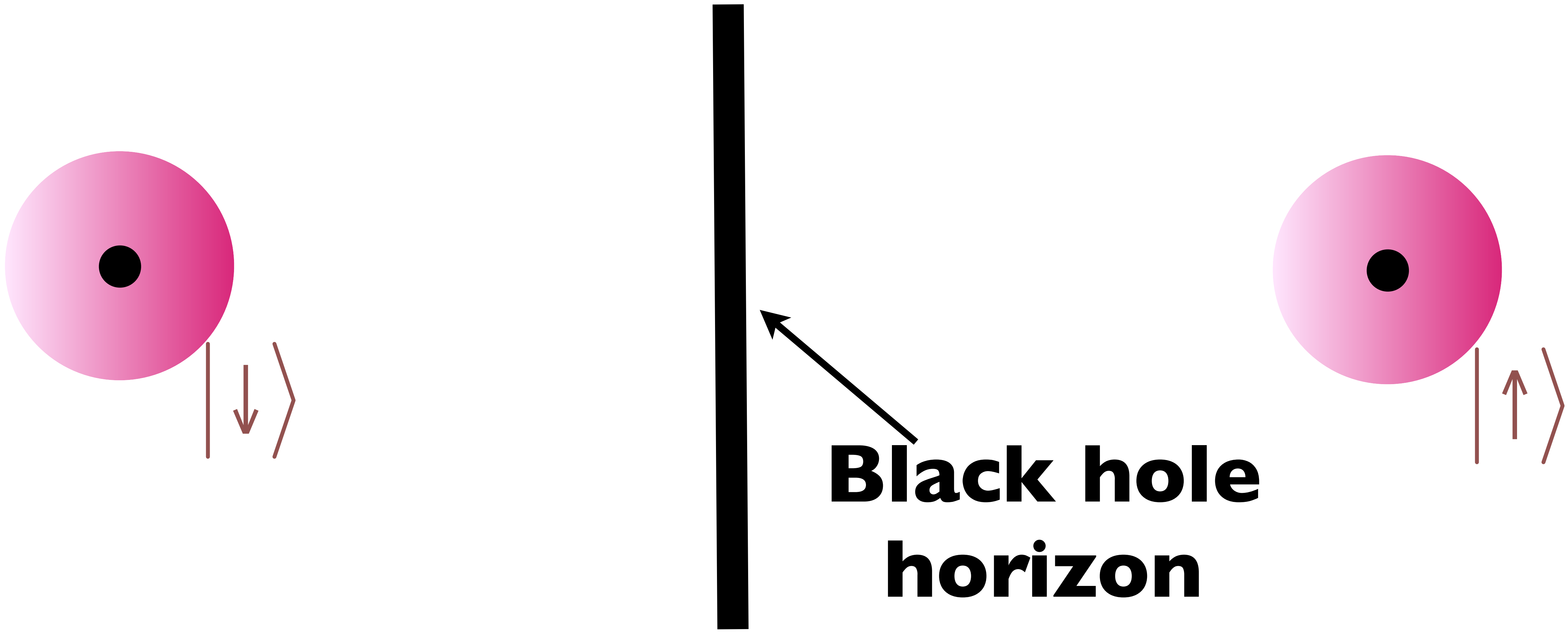


Quantum Entanglement across a black hole horizon

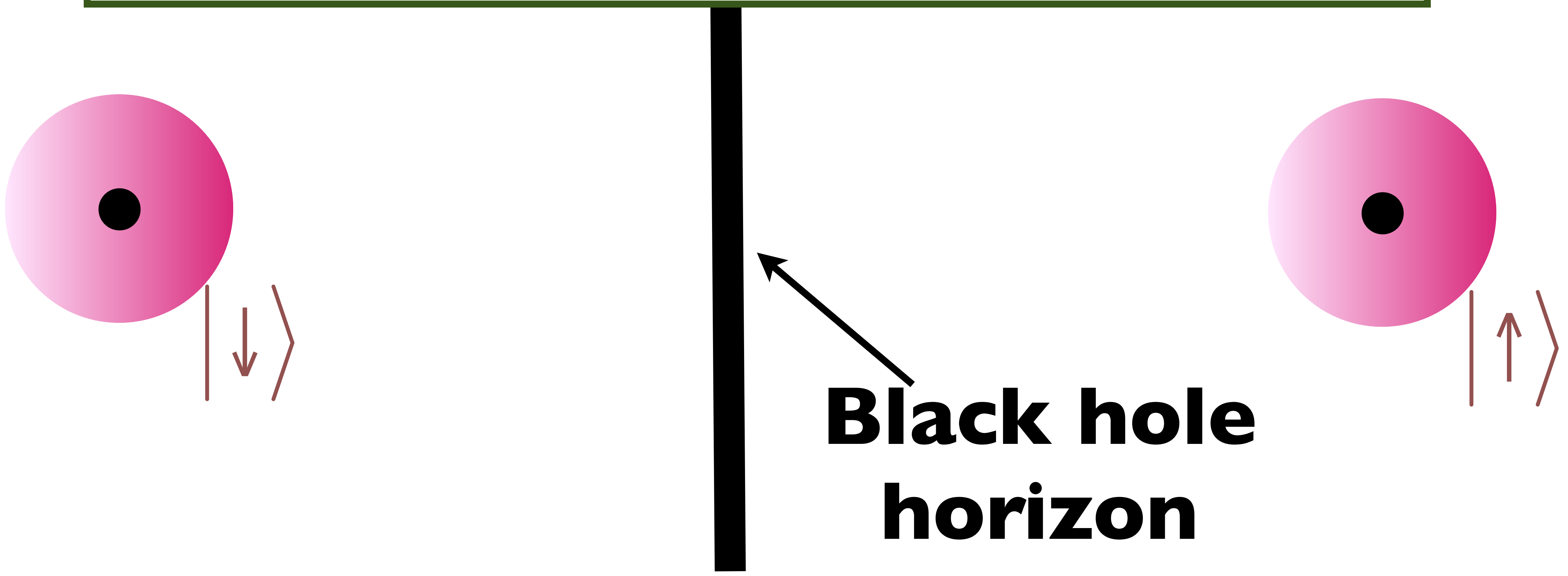
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

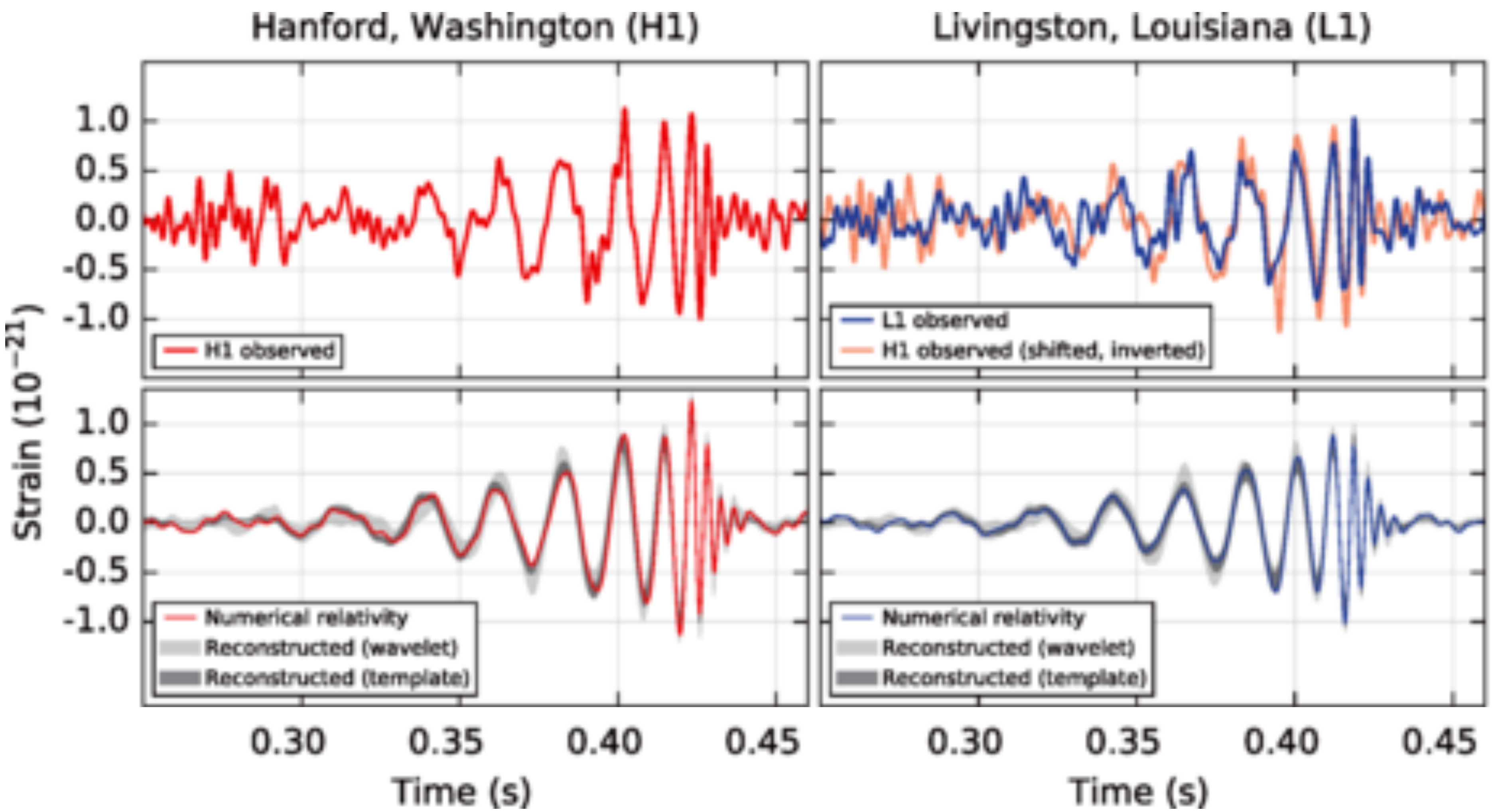
(because to an outside observer, the state of the electron inside the black hole is an unknown)

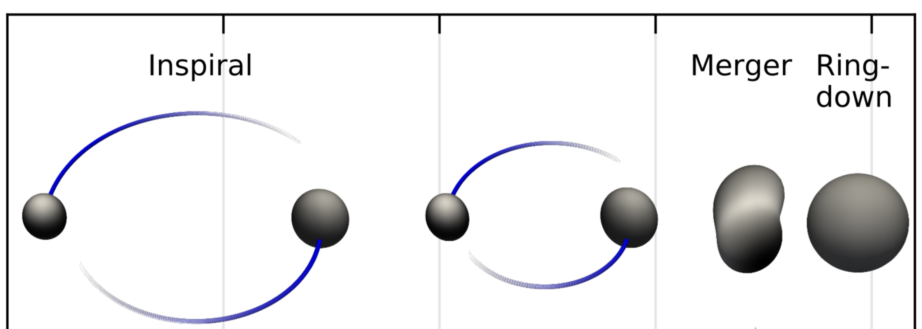


Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy, S_{BH} is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)





LIGO
September 14, 2015

- The ring-down time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals

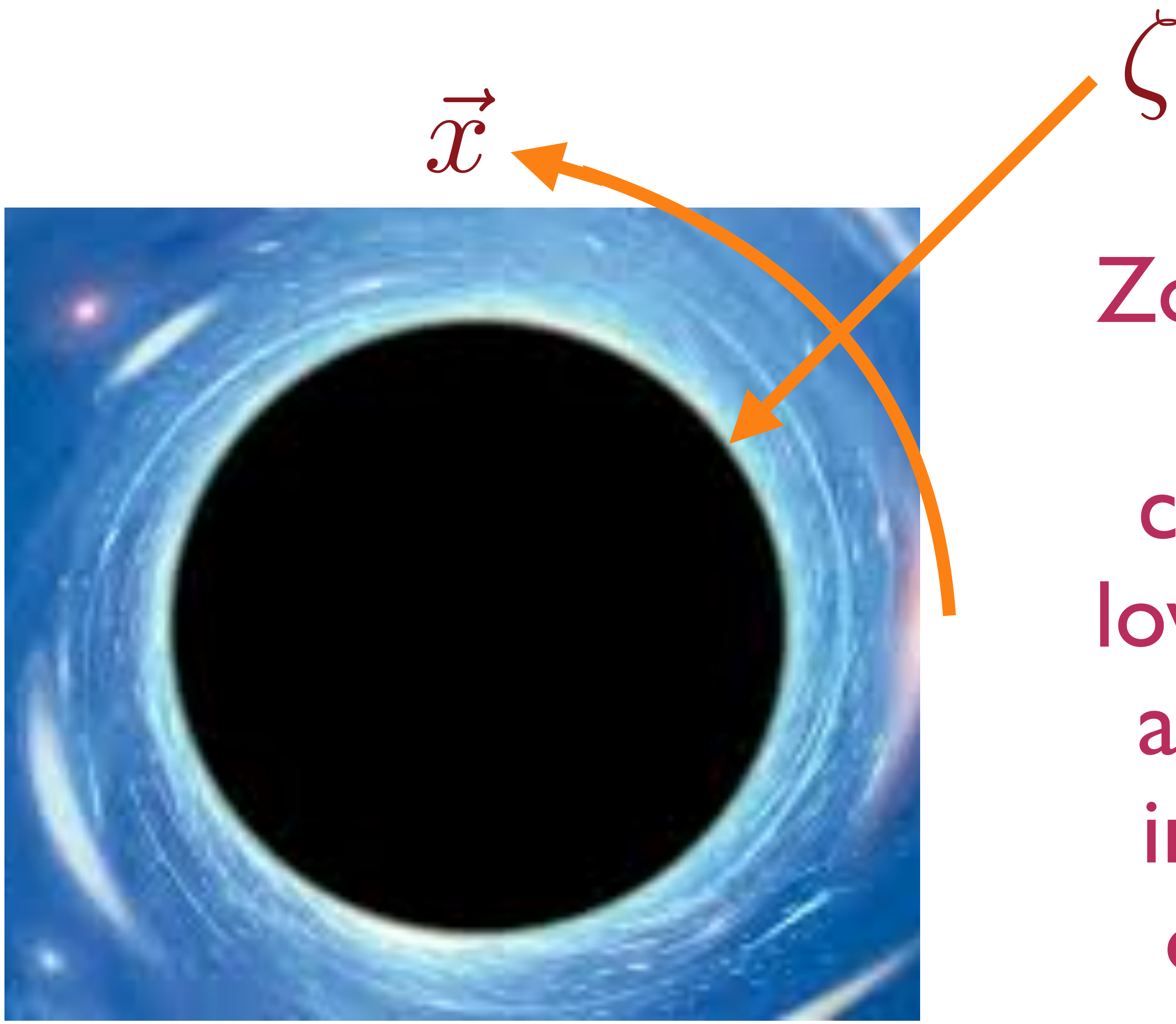
$$\boxed{\frac{\hbar}{k_B T_H},} \qquad \hbar \text{ Planck's constant}, \qquad k_B \text{ Boltzmann's constant}$$

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

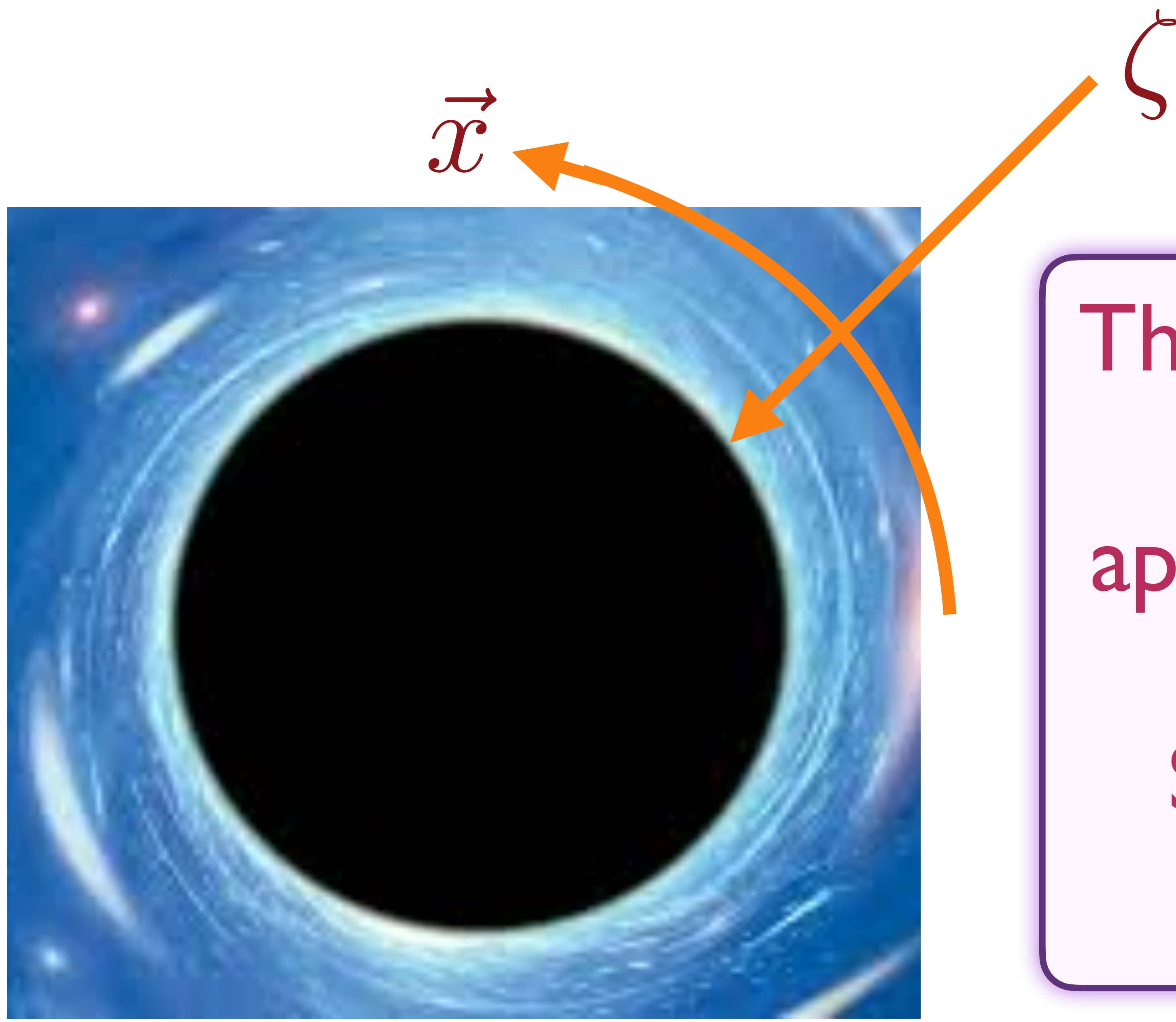
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



This 2D-gravity theory is precisely that appearing in the low T limit of the Sachdev-Ye-Kitaev (SYK) models

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

$$I_{EM} = \int d^{d+2}x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2} \right) + \frac{1}{4g_F^2} F^2 \right]$$

Metric $g_{\mu\nu}$
Ricci scalar in $d+2$ dimensions, $\mathcal{R}_{d+2}$
Cosmological constant $\Lambda = -d(d+1)/L^2$
U(1) gauge field A_μ
Electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Boundary conditions at spatial infinity:
Metric $\rightarrow$ AdS_{d+2}
Electric field $\rightarrow \mathcal{Q}/(4\pi r^2)$

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

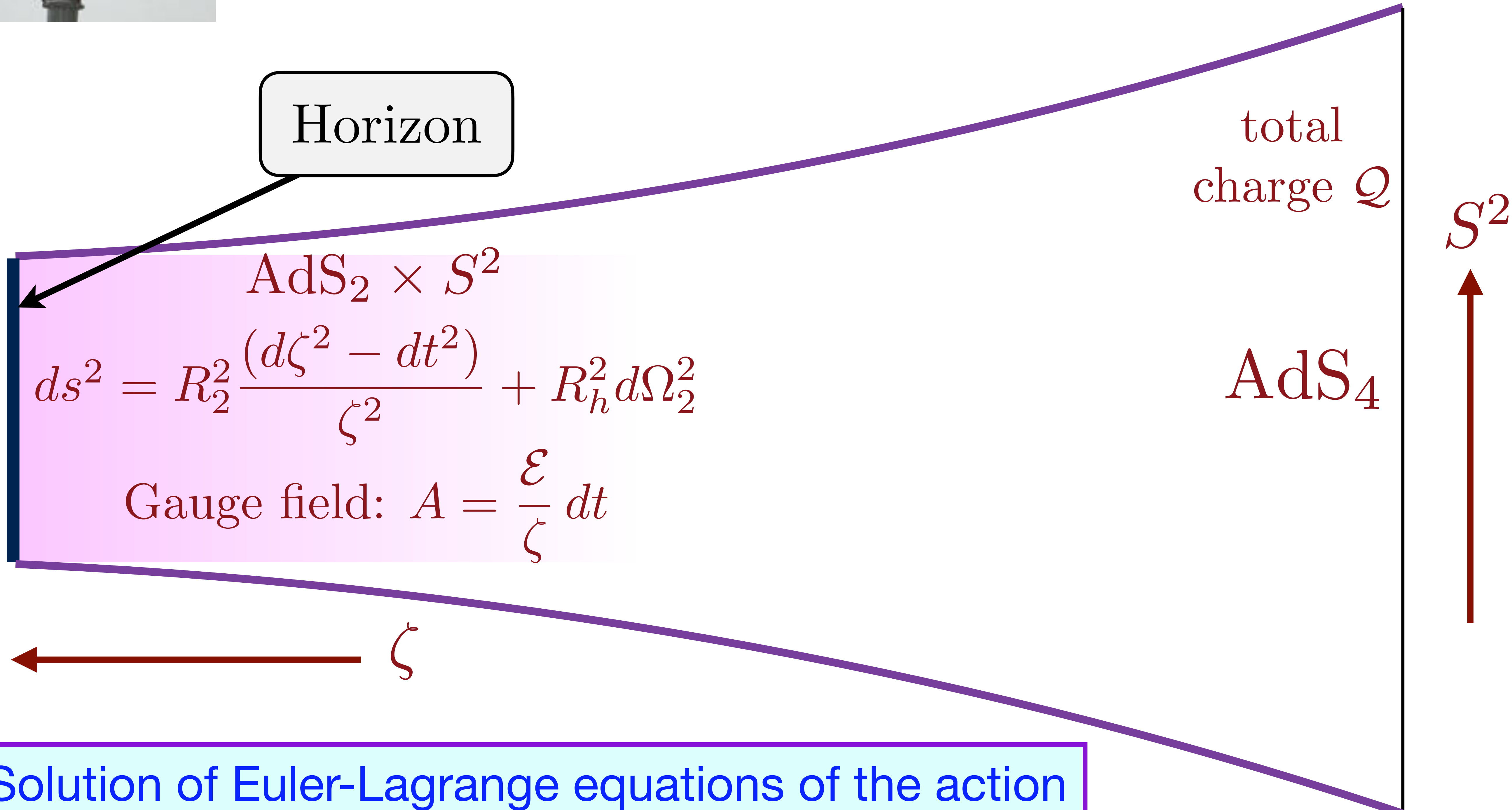
$$I_{EM} = \int d^{d+2}x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2} \right) + \frac{1}{4g_F^2} F^2 \right]$$

Quantum gravity is 'defined' by the path integral

$$\mathcal{Z}_{\text{gravity}} = \int \mathcal{D}g \mathcal{D}A \exp(-I_{EM}/\hbar)$$

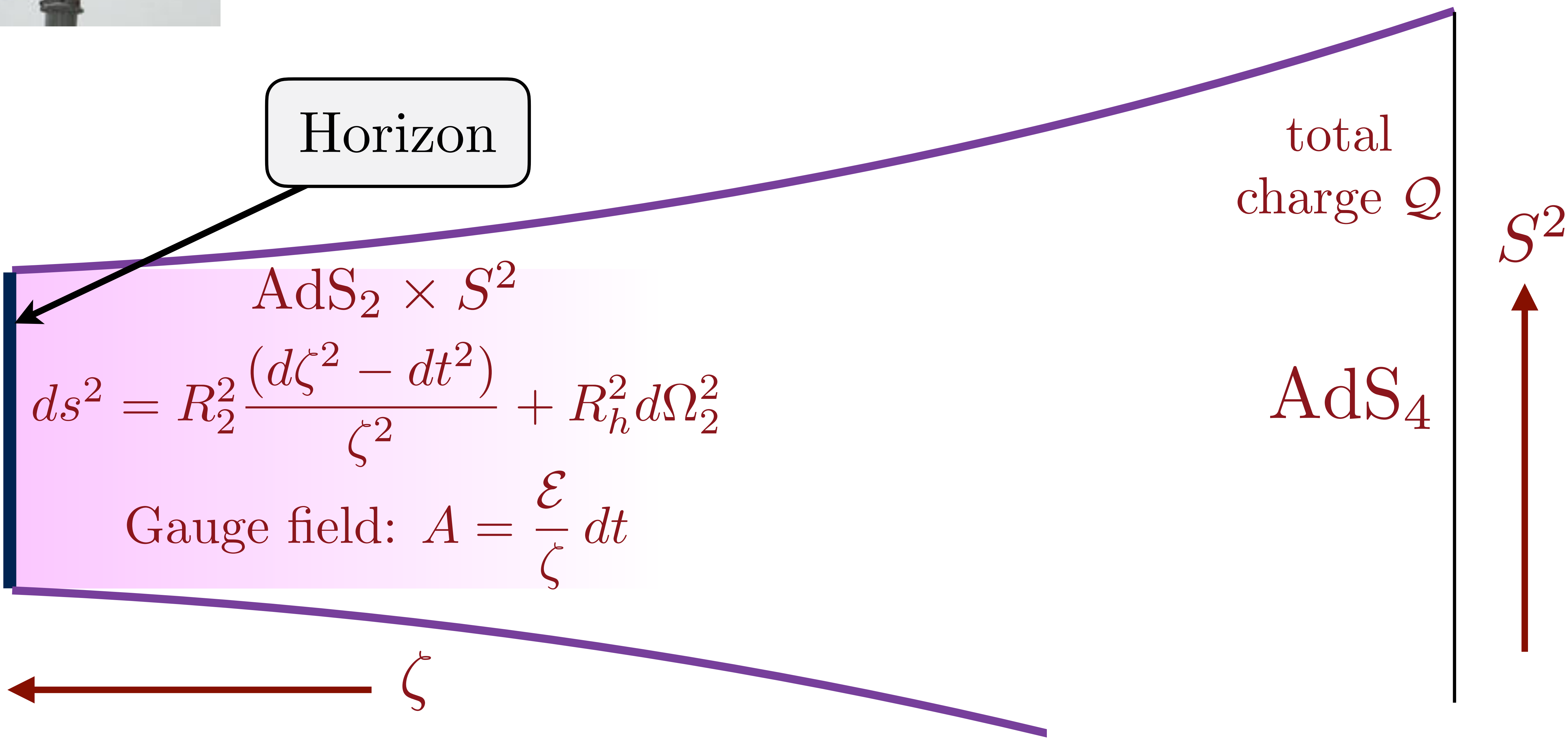
This integral is evaluated exactly in a certain low temperature limit for charged black holes.

SYK model and charged black holes



Solution of Euler-Lagrange equations of the action of Einstein gravity and Maxwell electromagnetism

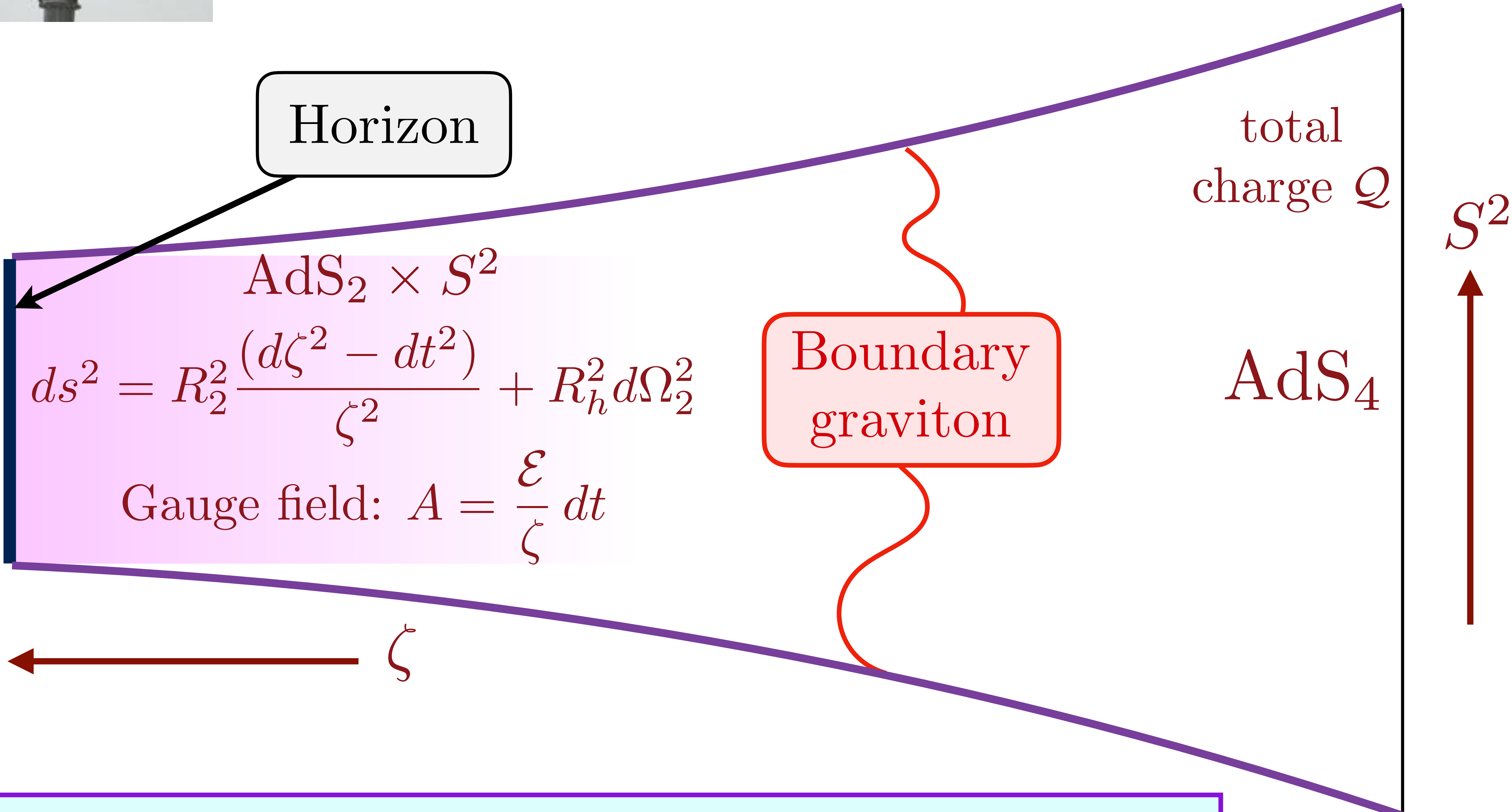
SYK model and charged black holes



- The entropy S_{BH}, the charge $\mathcal{Q}$, and the dimensionless electric field $\mathcal{E}$ obey the same thermodynamic relation as the SYK model

$$\frac{dS_{BH}}{d\mathcal{Q}} = 2\pi\mathcal{E}$$

SYK model and charged black holes



Fluctuations about the path integral saddle

Main result II

For $T \ll 1/R_h$

$$\mathcal{Z}_{\text{charged black hole in EM theory}} = \exp\left(\frac{S_{BH}}{k_B}\right) \int \mathcal{D}f(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{2D}-\text{gravity}}\left[f(\tau)\right]\right)$$

Main result II

For $T \ll 1/R_h$

$$\mathcal{Z}_{\text{charged black hole in EM theory}} =$$

$$\exp\left(\frac{S_{BH}}{k_B}\right) \int \mathcal{D}f(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{2D}-\text{gravity}}\left[f(\tau)\right]\right)$$

$$\mathcal{S}_{\text{2D}-\text{gravity}}\left[f(\tau)\right] = -\frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, and we have used the *Schwarzian*:

$$\{g, \tau\} \equiv \frac{d^3g/d\tau^3}{dg/d\tau} - \frac{3}{2}\left(\frac{d^2g/d\tau^2}{dg/d\tau}\right)^2 .$$

The defining property of the Schwarzian is its invariance under SL(2,R) transformations

$$\left\{\frac{ag(\tau)+b}{cg(\tau)+d}, \tau\right\} = \{g(\tau), \tau\}$$

Remarkably, this path integral can be evaluated exactly, using the Duistermaat–Heckman formula (Stanford, Witten, arXiv:1703.04612).

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
Phys. Rev. B **95**, 155131 (2017)

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746 P. Nayak, A. Shukla,

R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

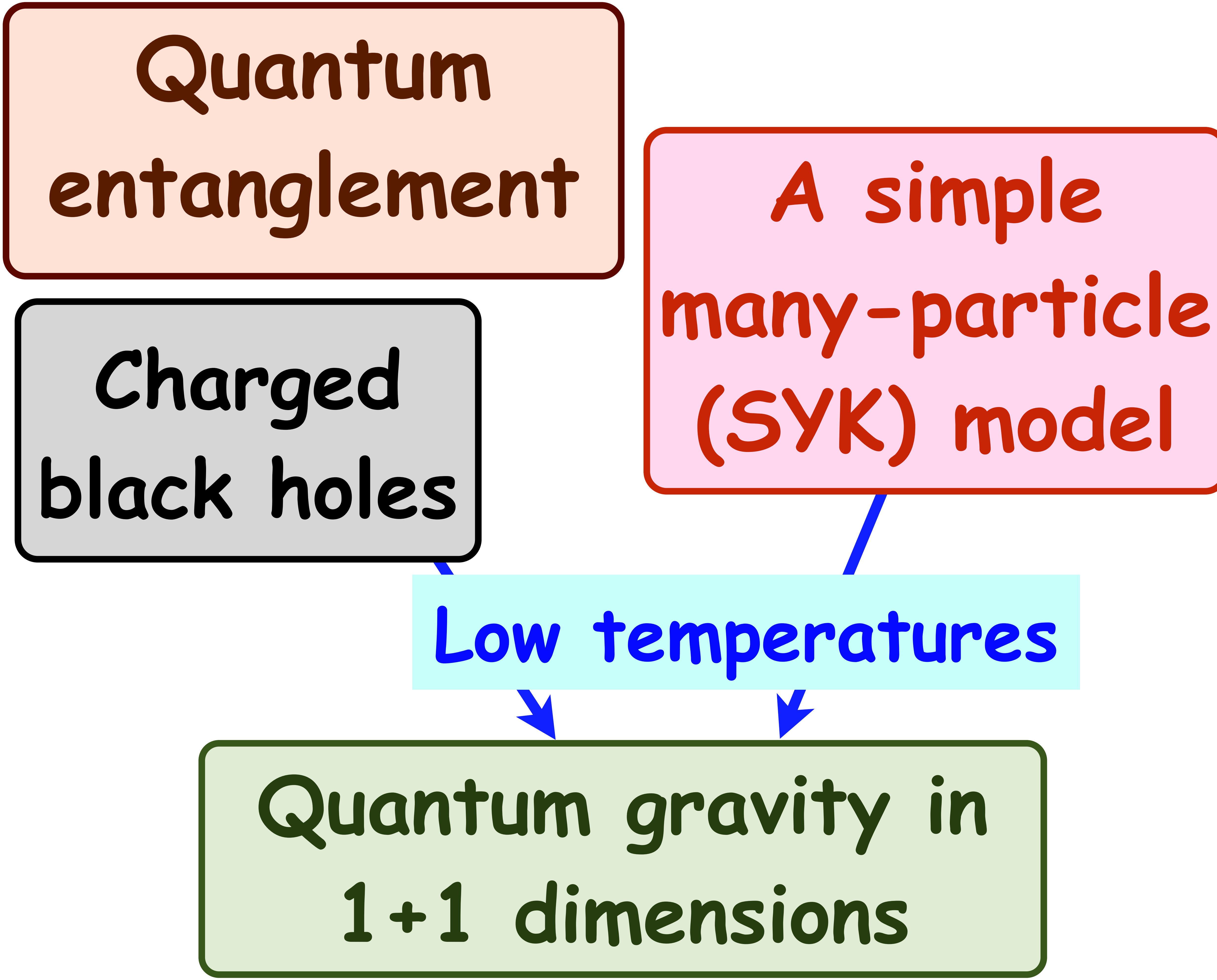
Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

Low temperatures

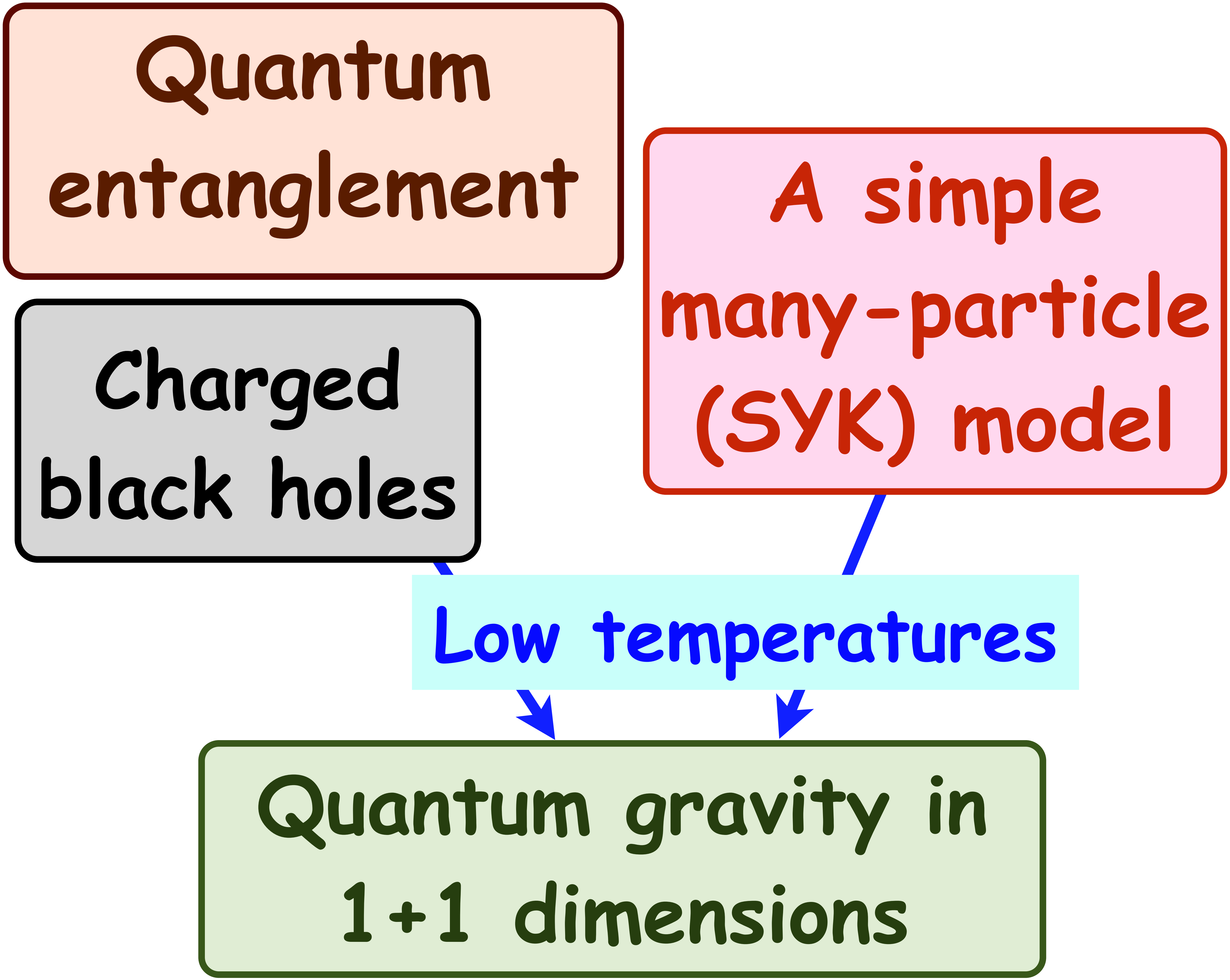
Quantum gravity in 1+1 dimensions


Quantum entanglement
Charged black holes
A simple many-particle (SYK) model
Low temperatures
Quantum gravity in 1+1 dimensions

Complex multi-particle entanglement leads to quantum systems without quasiparticle excitations.

Many-body chaos and thermal equilibration in the shortest possible

$$\text{Planckian time} \sim \frac{\hbar}{k_B T}.$$


Quantum entanglement
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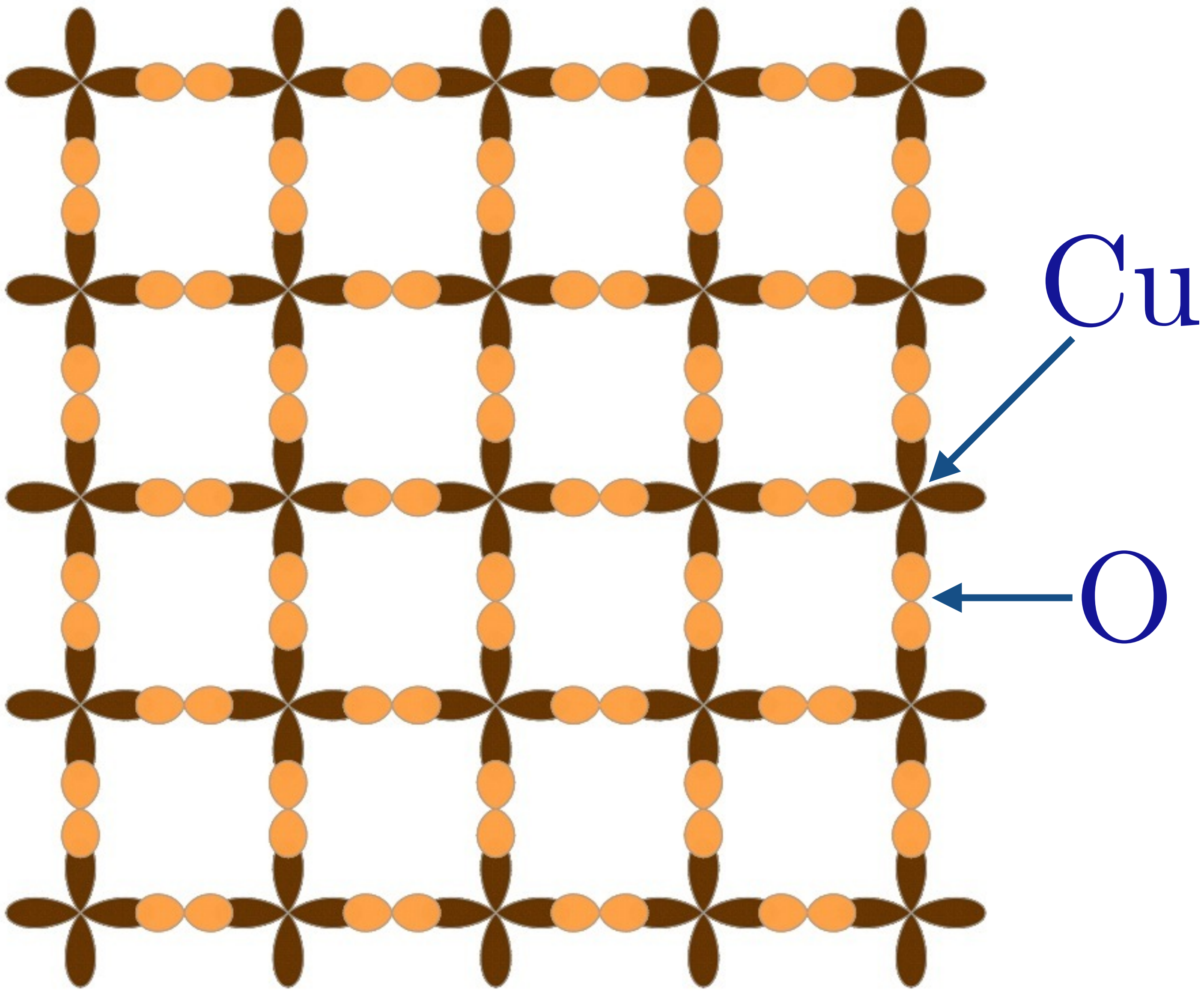
Quantum entanglement

A simple many-particle (SYK) model

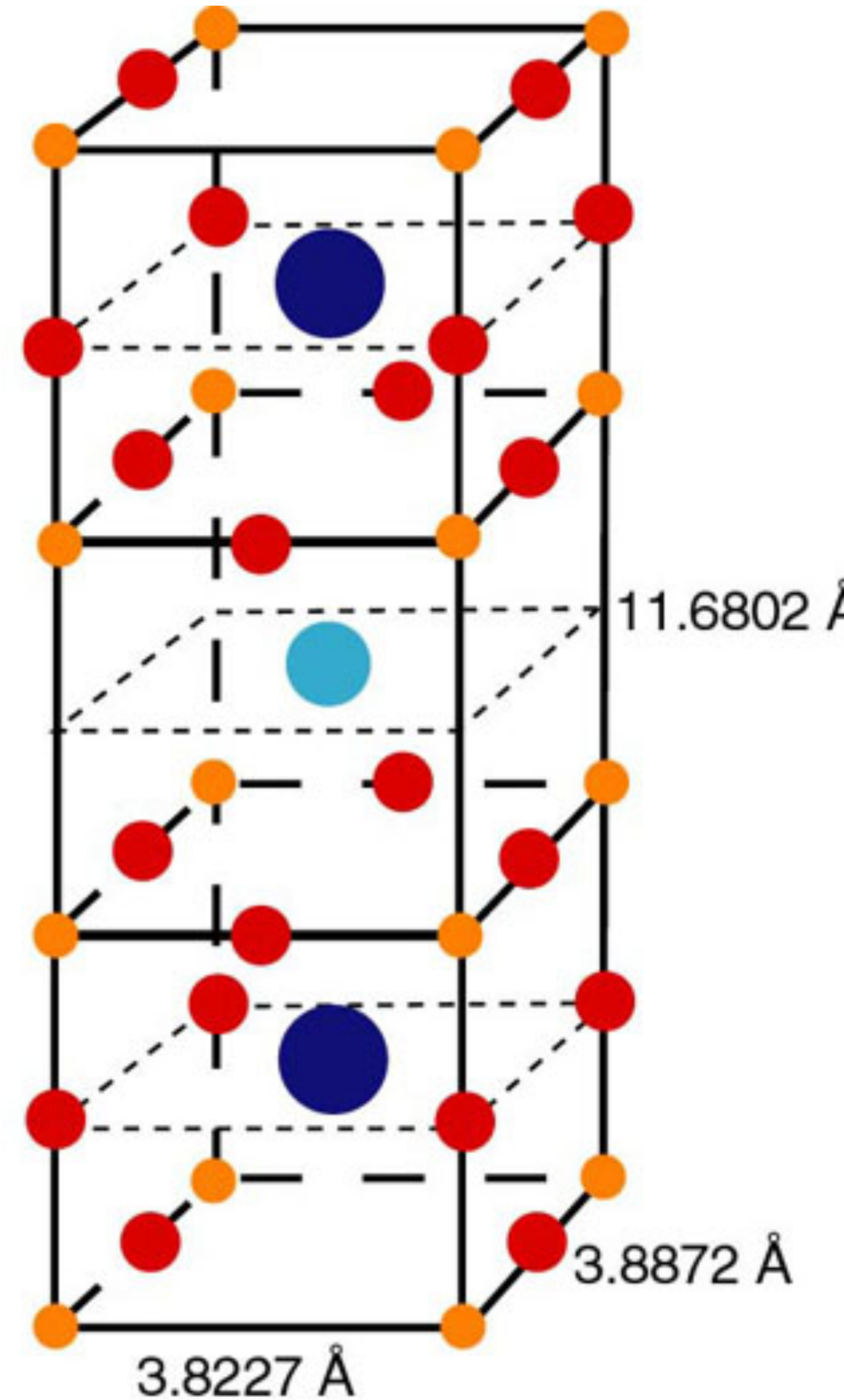
Charged black holes

Copper-based superconductors

High temperature superconductors

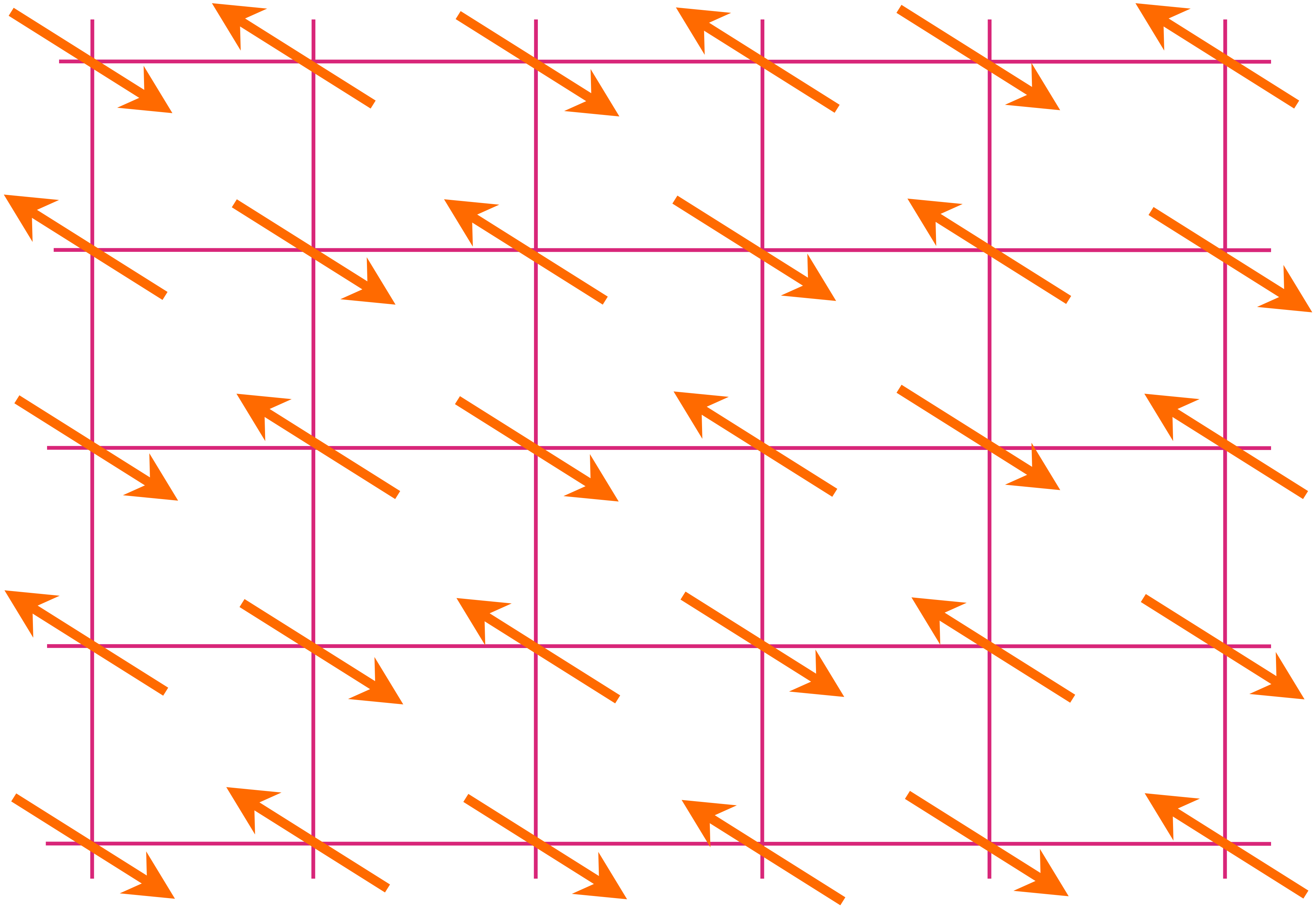


CuO_2 plane

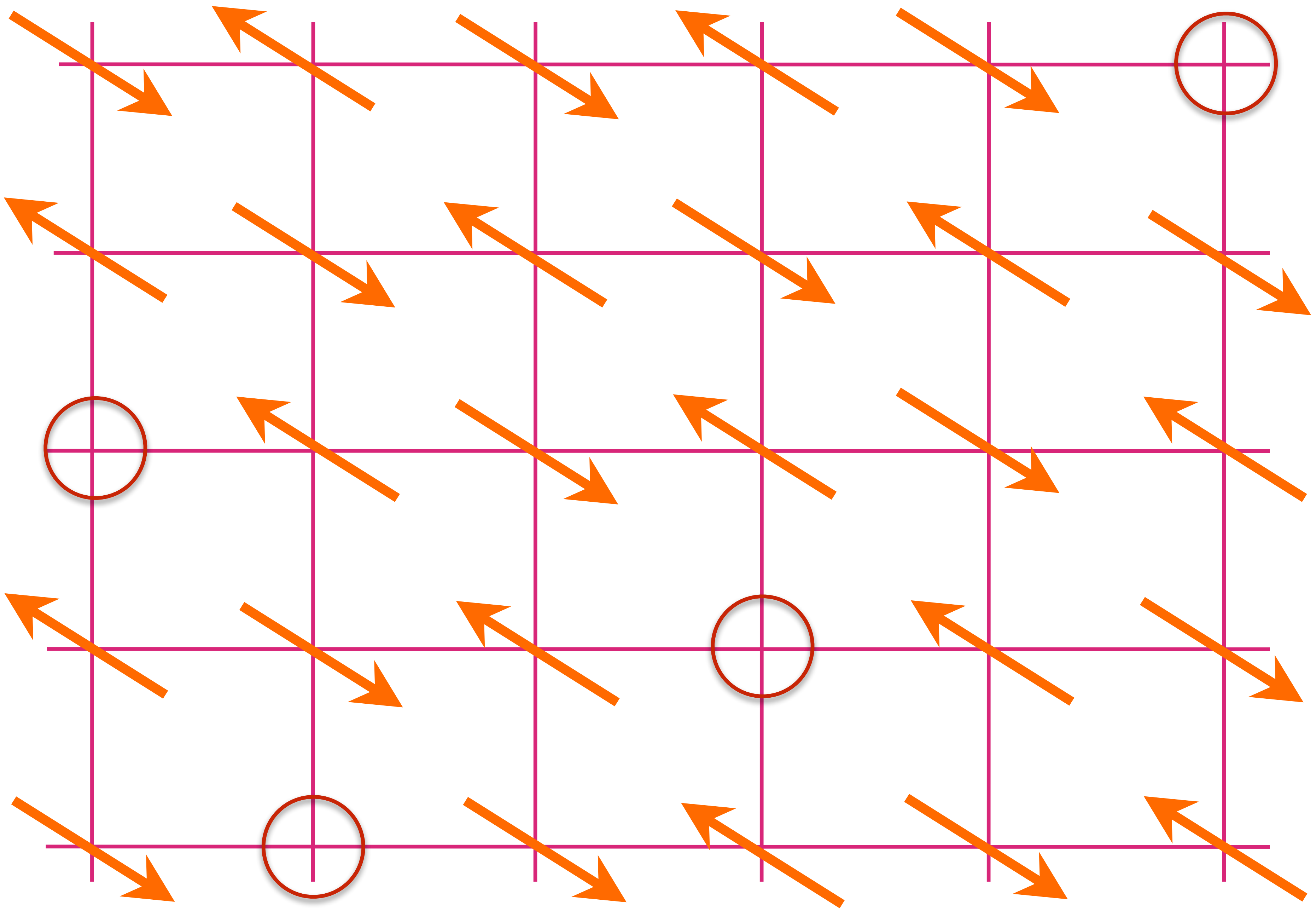


$YBa_2Cu_3O_{6+x}$

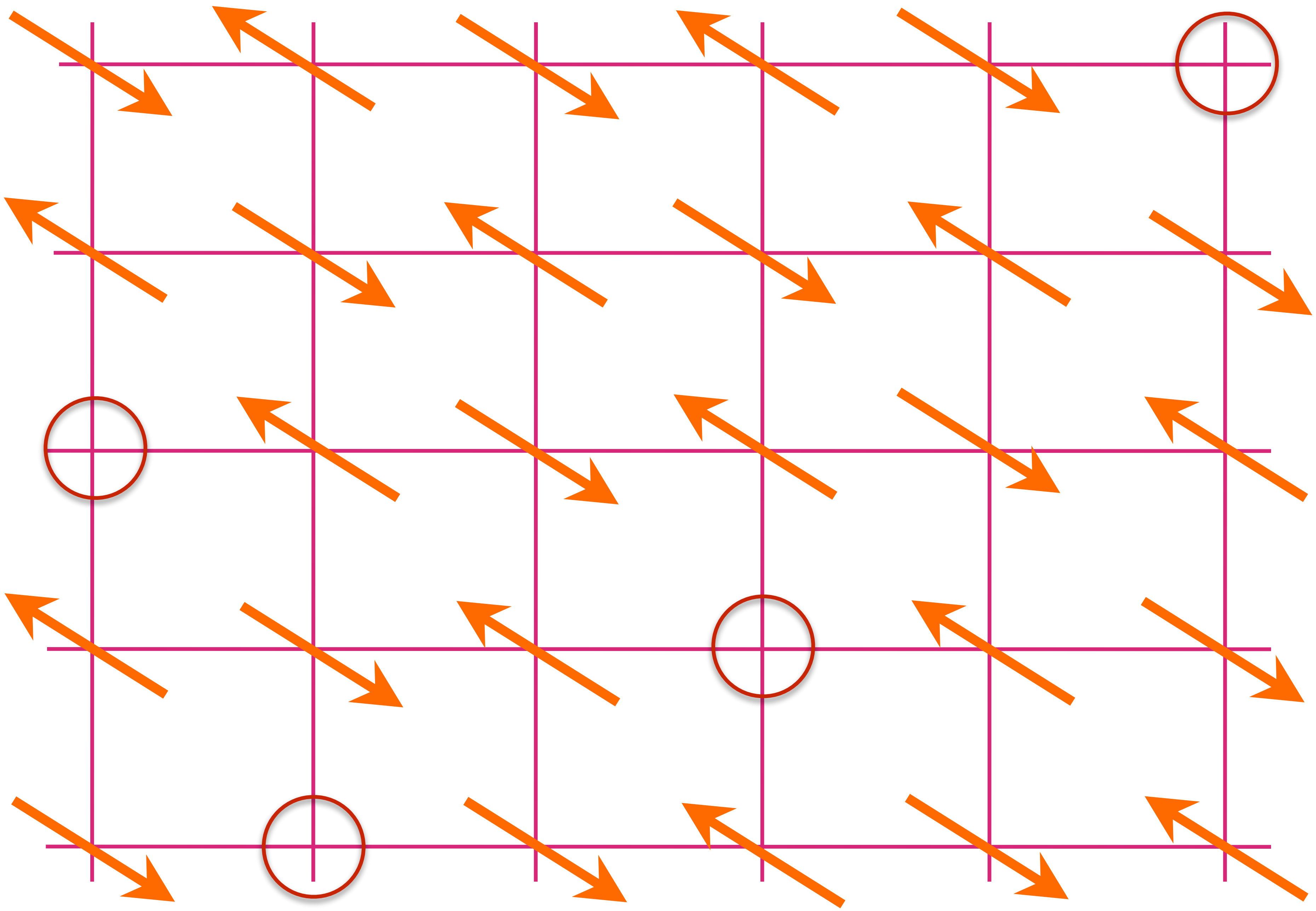
Insulating antiferromagnet



Antiferromagnet doped with hole density p

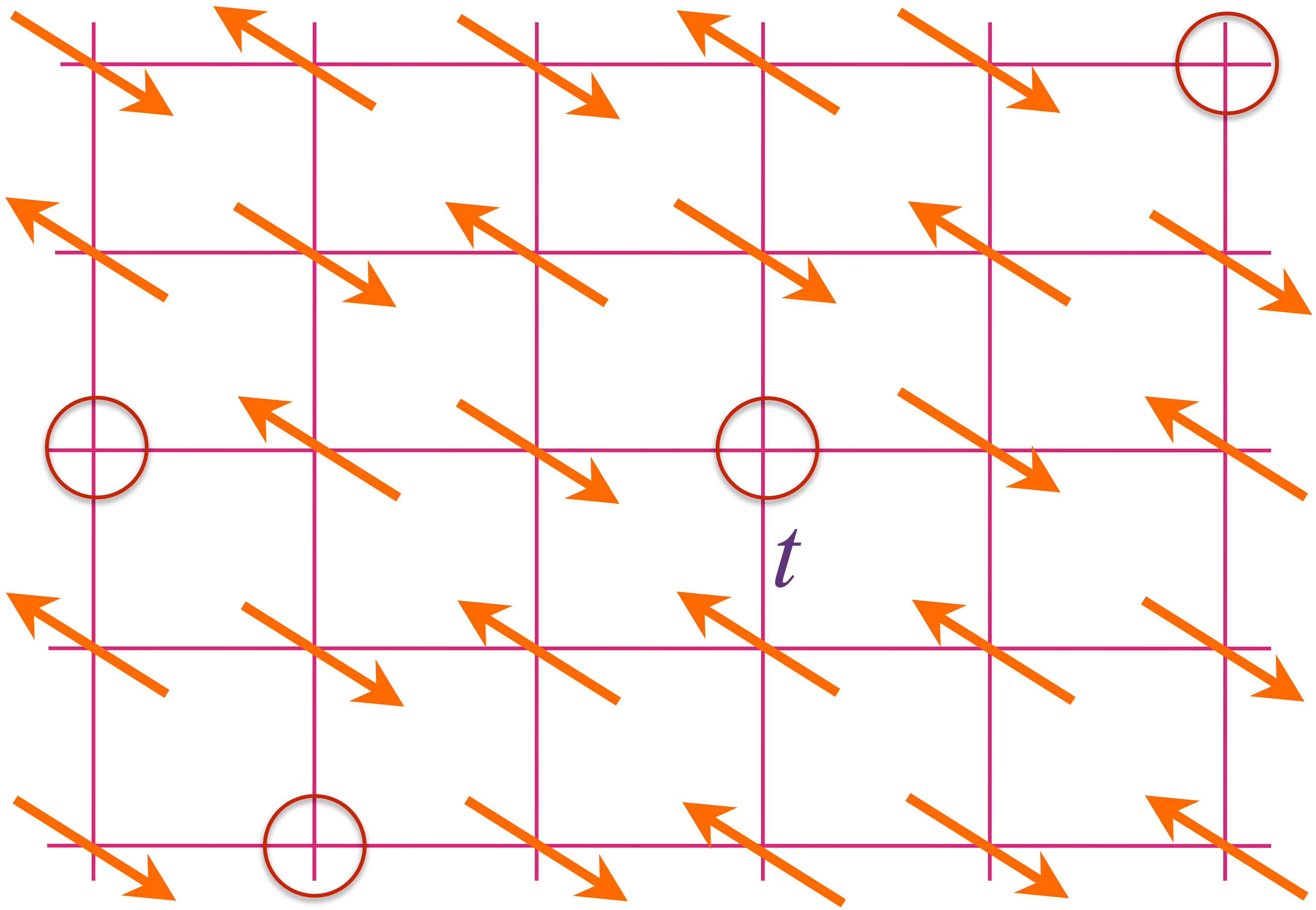


Real-space view



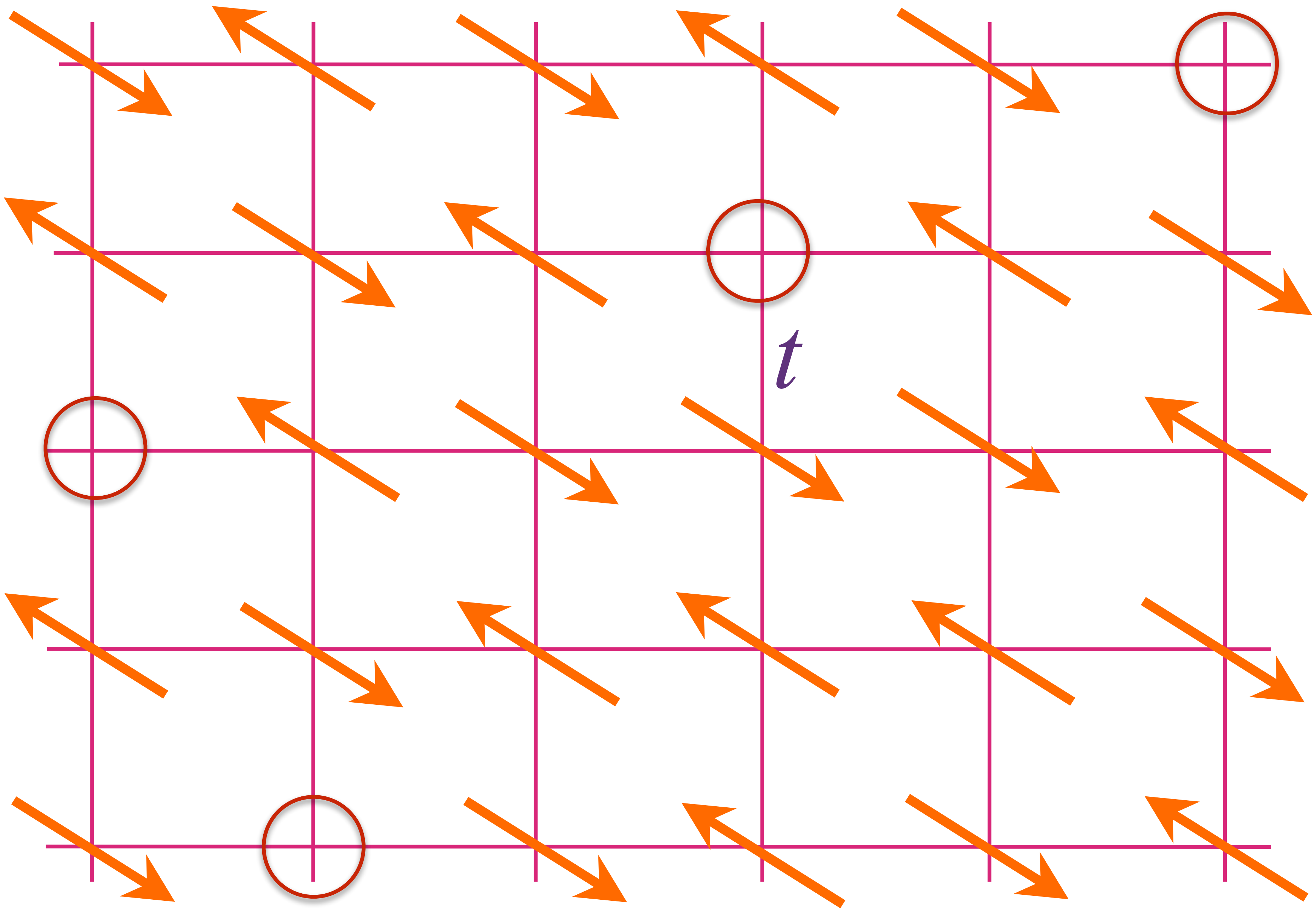
p mobile holes in a background of fluctuating spins

Real-space view



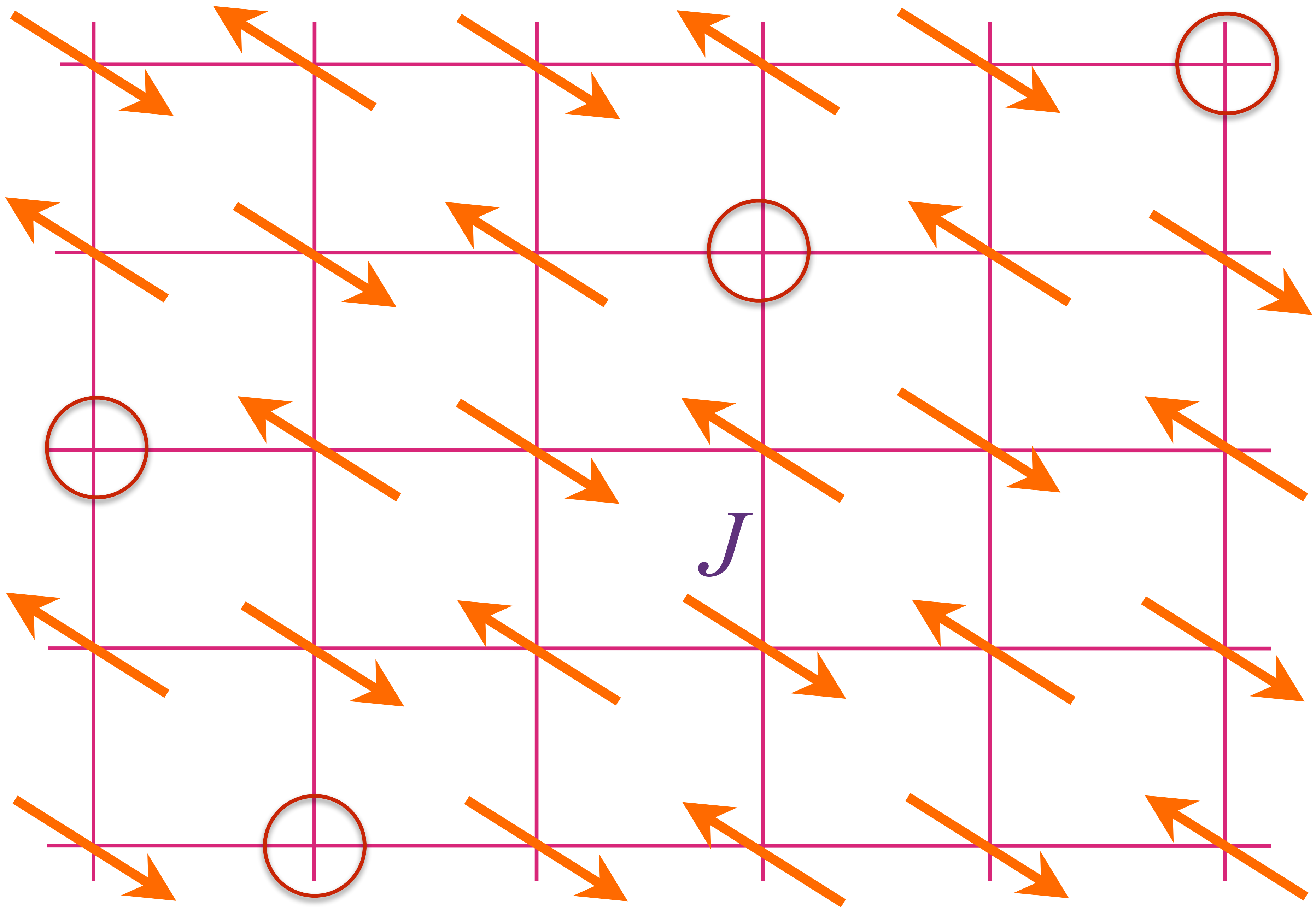
p mobile holes in a background of fluctuating spins

Real-space view



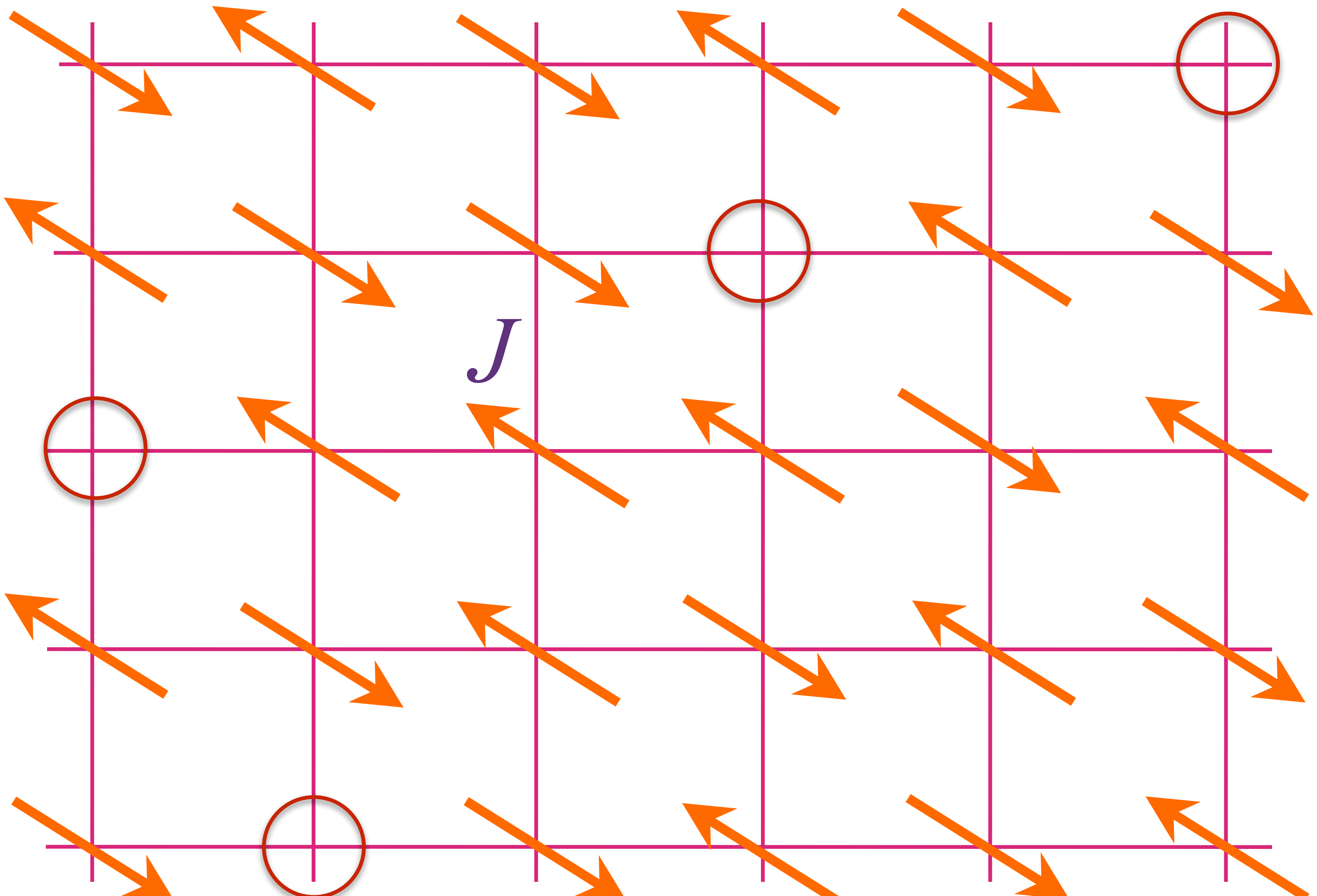
p mobile holes in a background of fluctuating spins

Real-space view



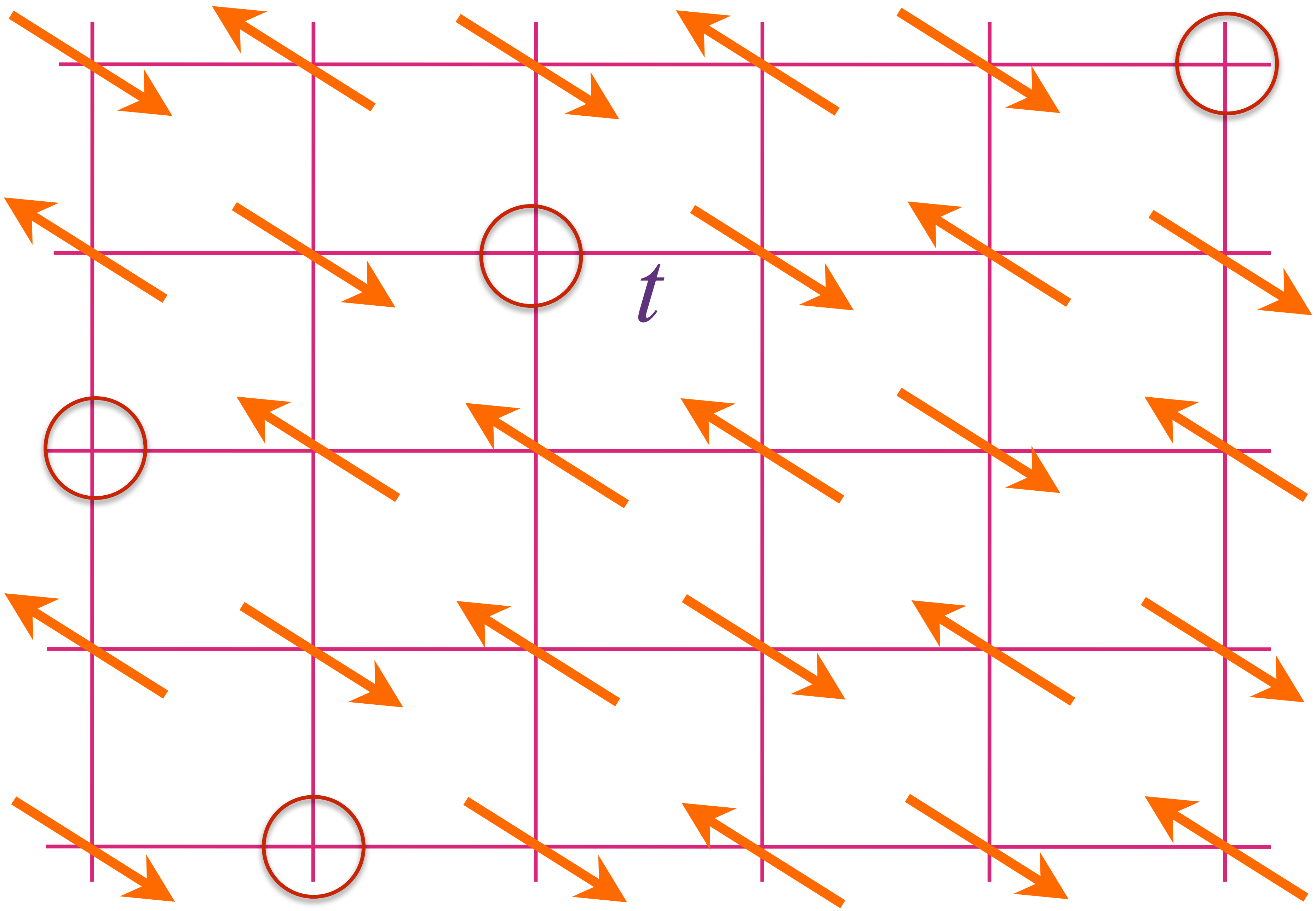
p mobile holes in a background of fluctuating spins

Real-space view



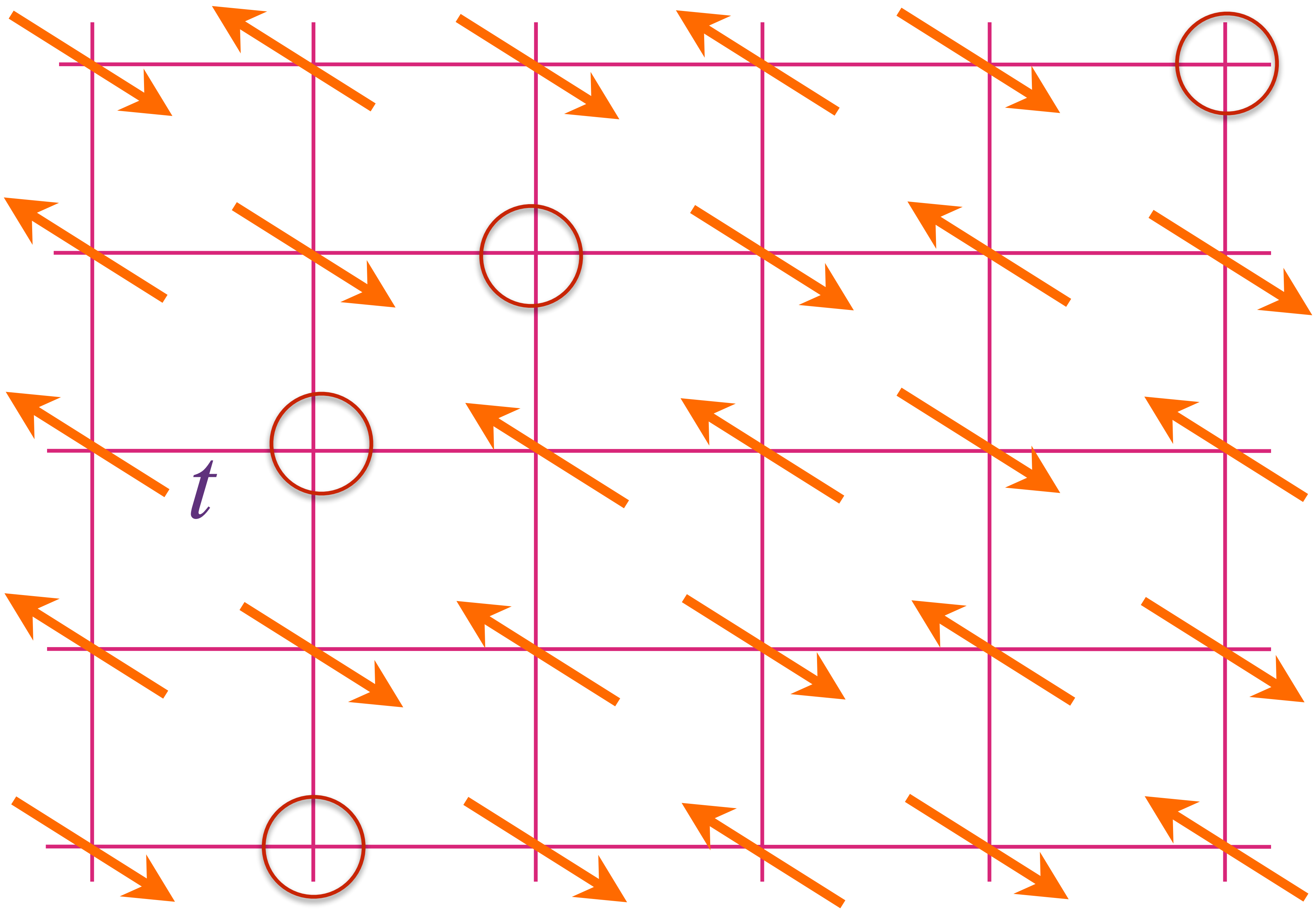
p mobile holes in a background of fluctuating spins

Real-space view



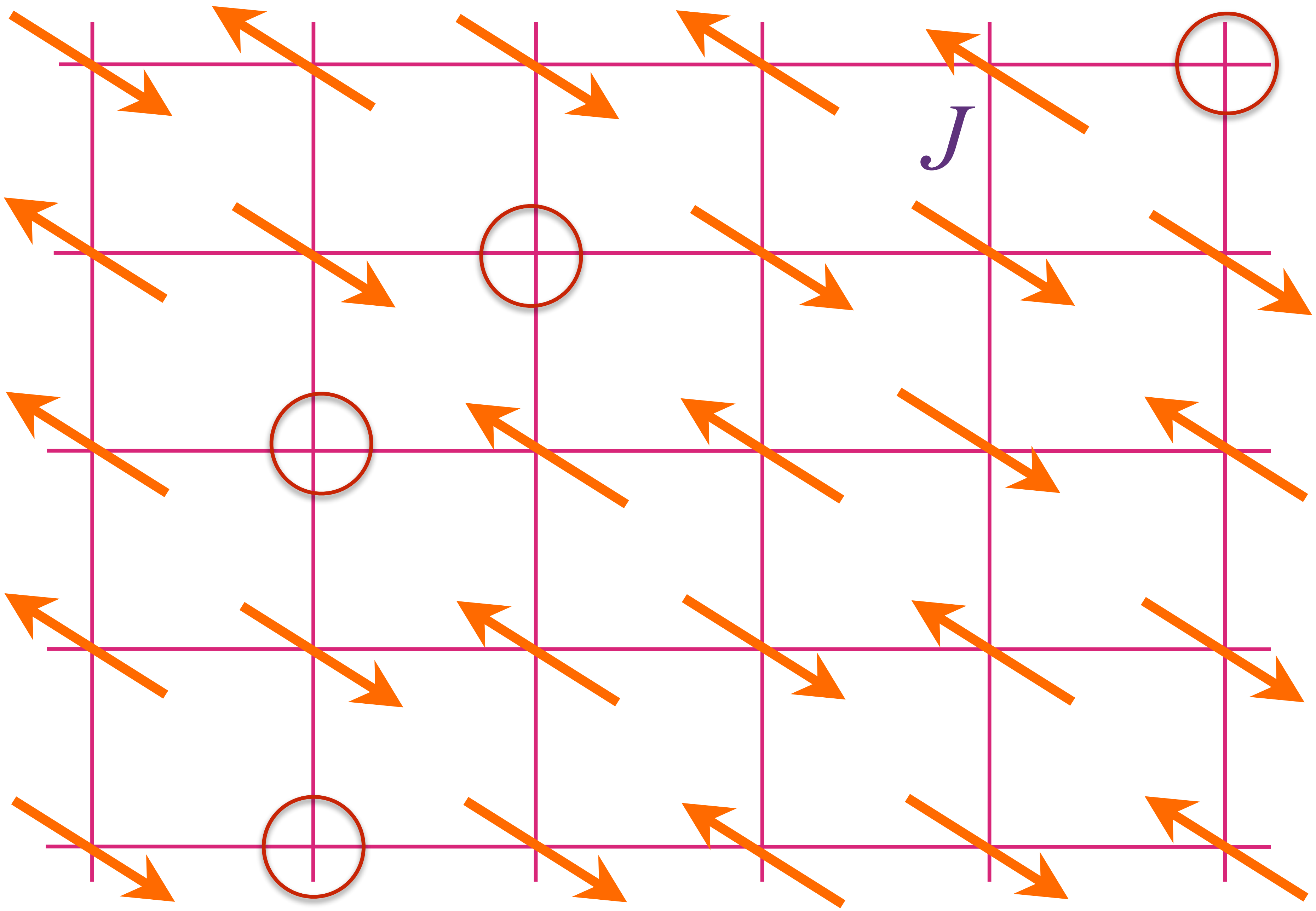
p mobile holes in a background of fluctuating spins

Real-space view



p mobile holes in a background of fluctuating spins

Real-space view



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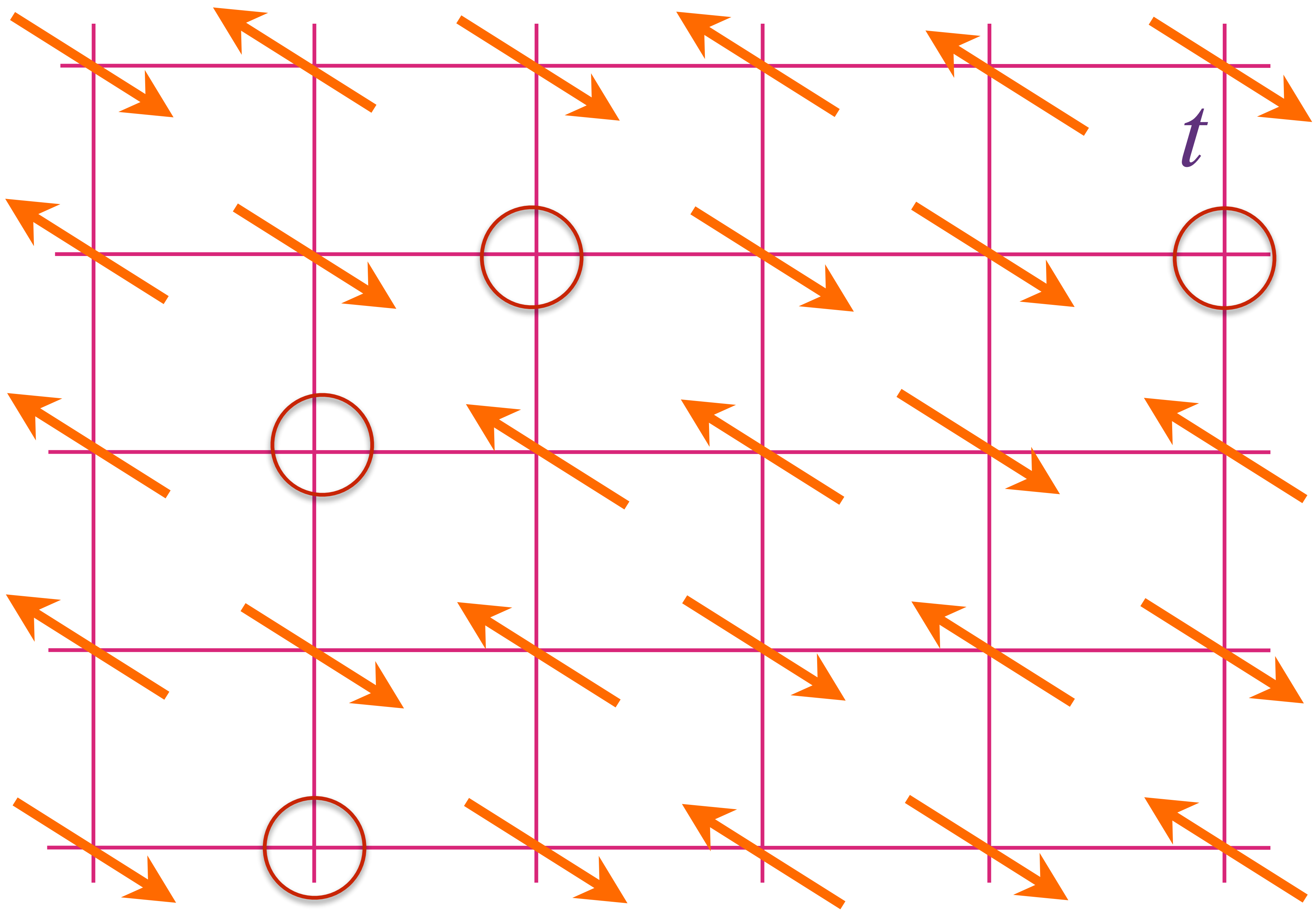
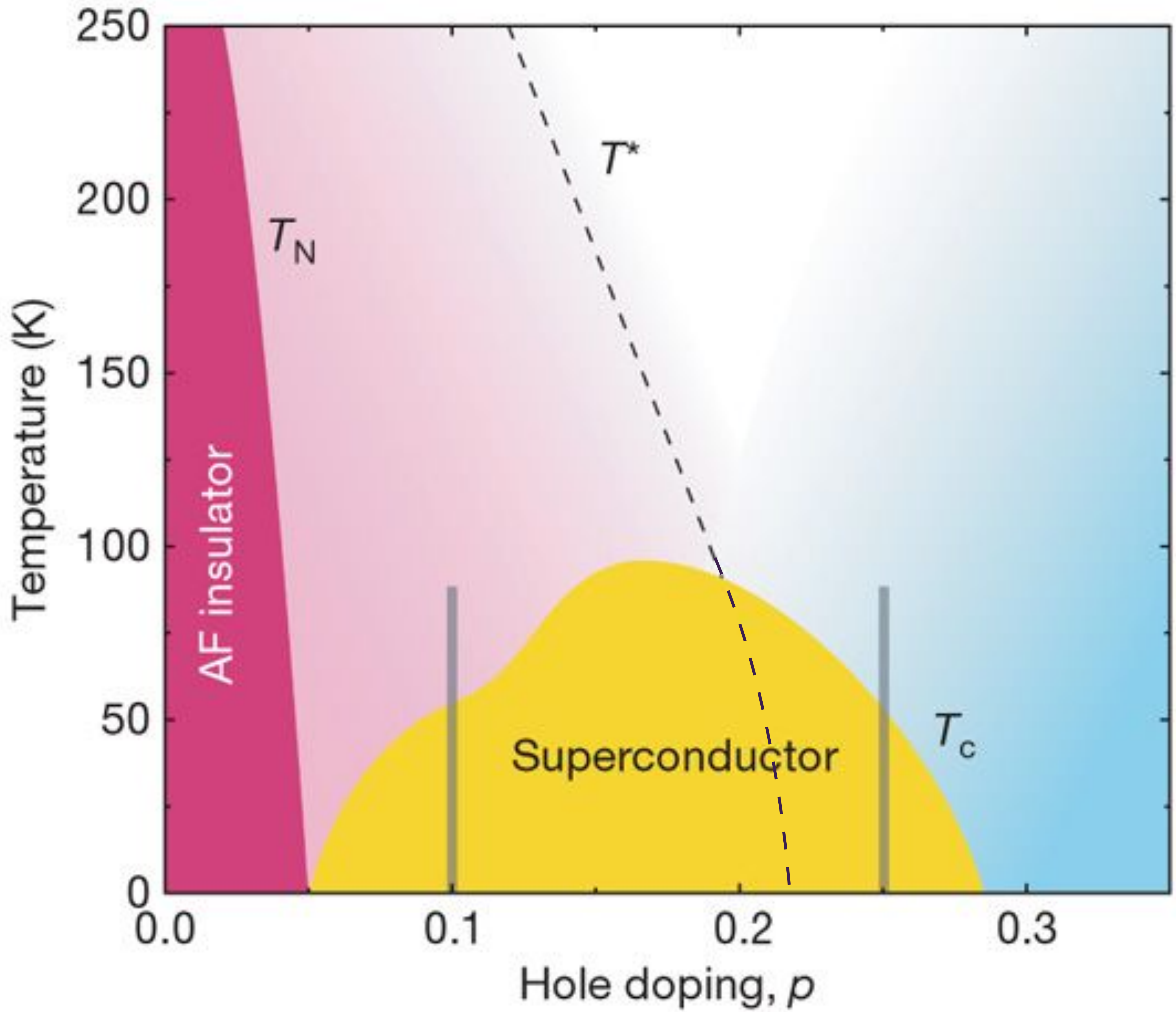
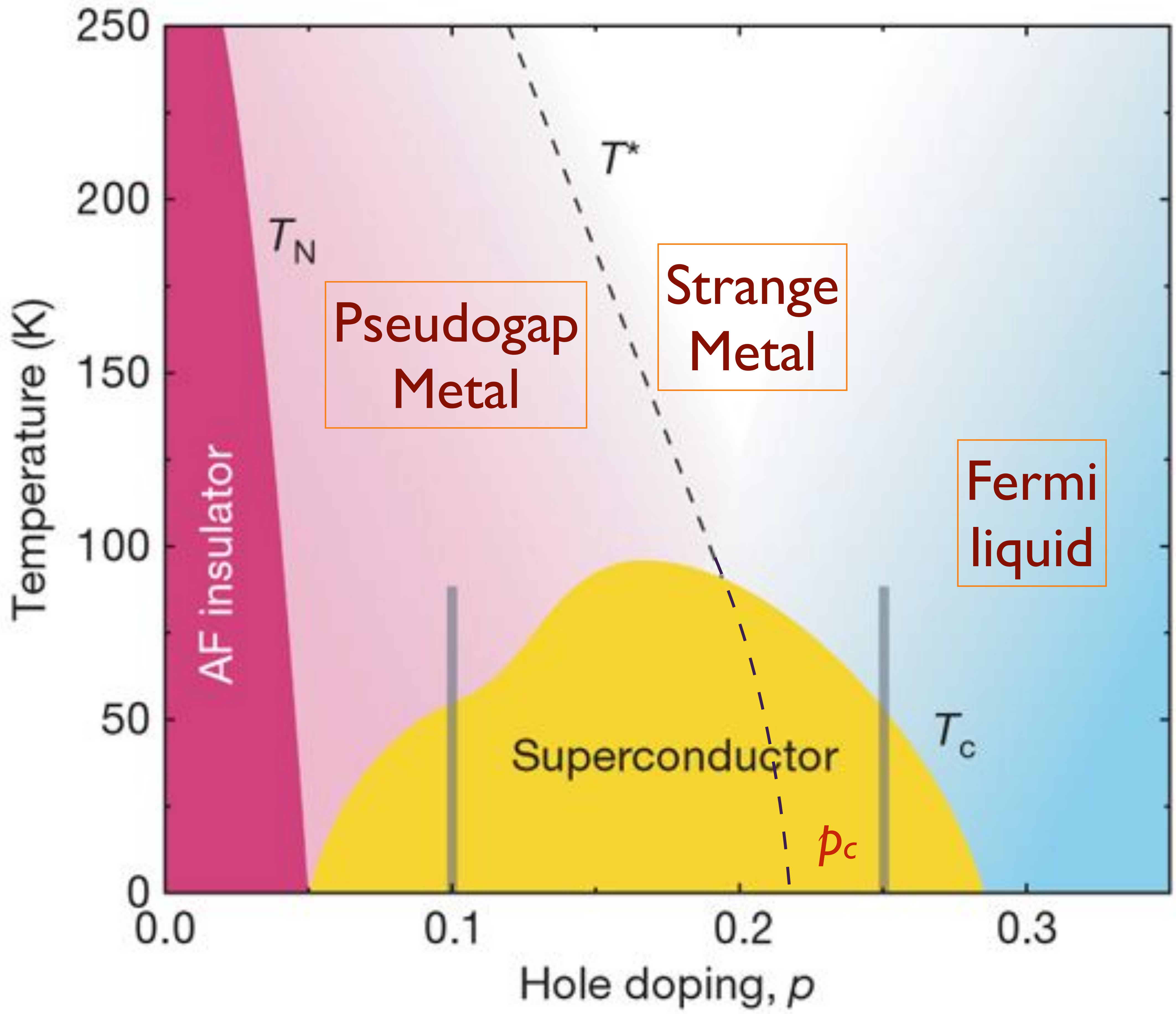
Real-space view



p mobile holes in a background of fluctuating spins

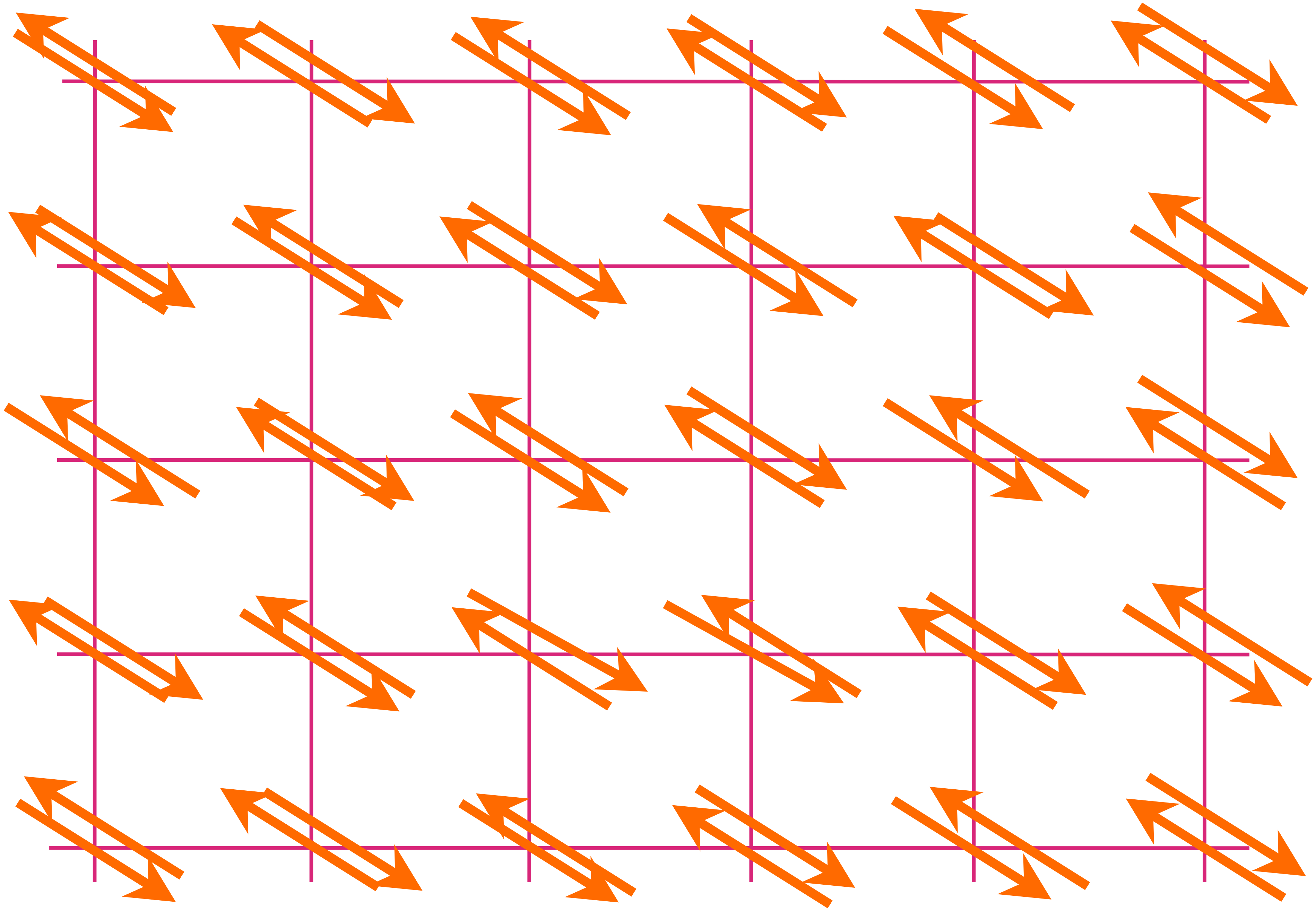


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c



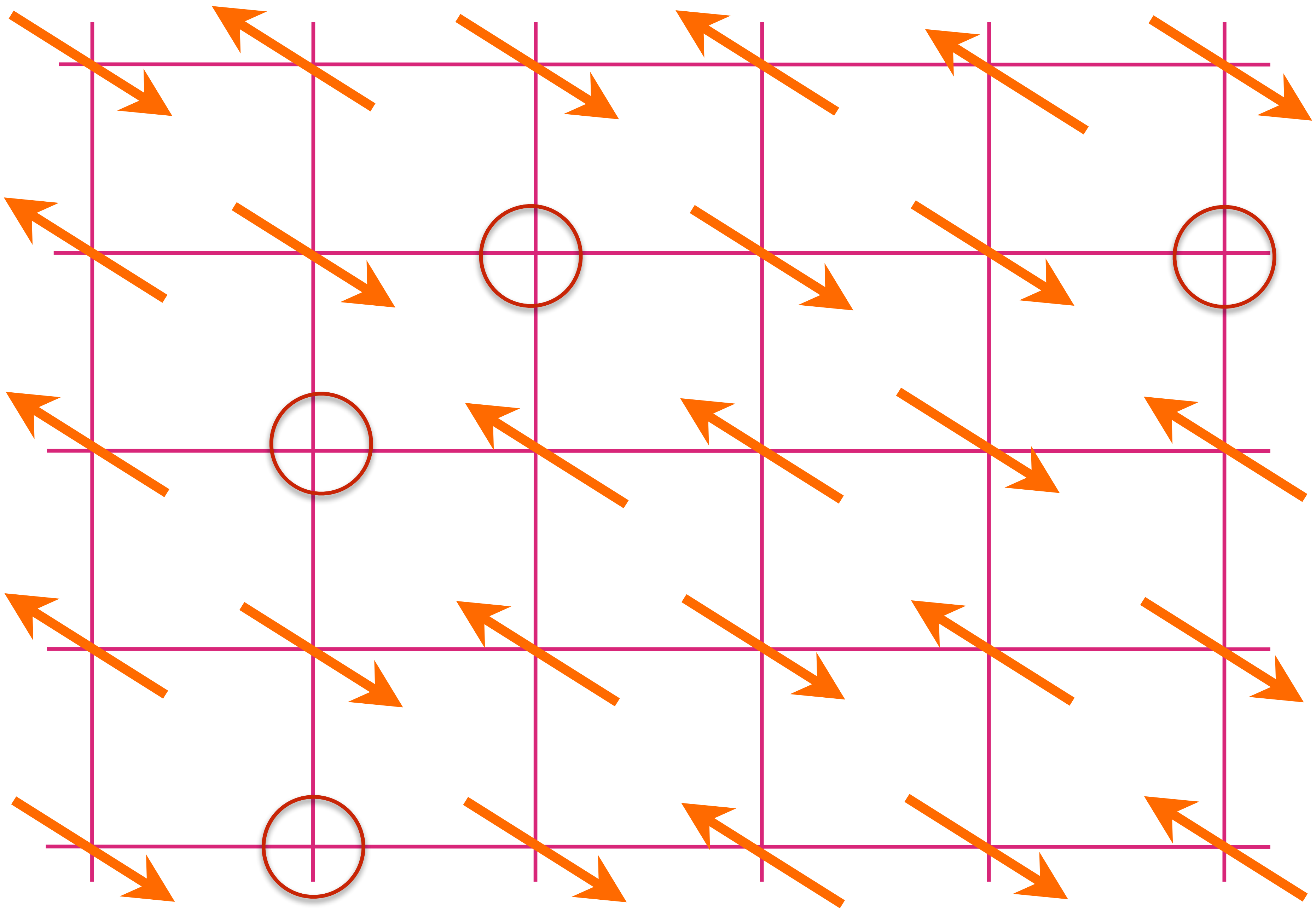
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c

Momentum-space view



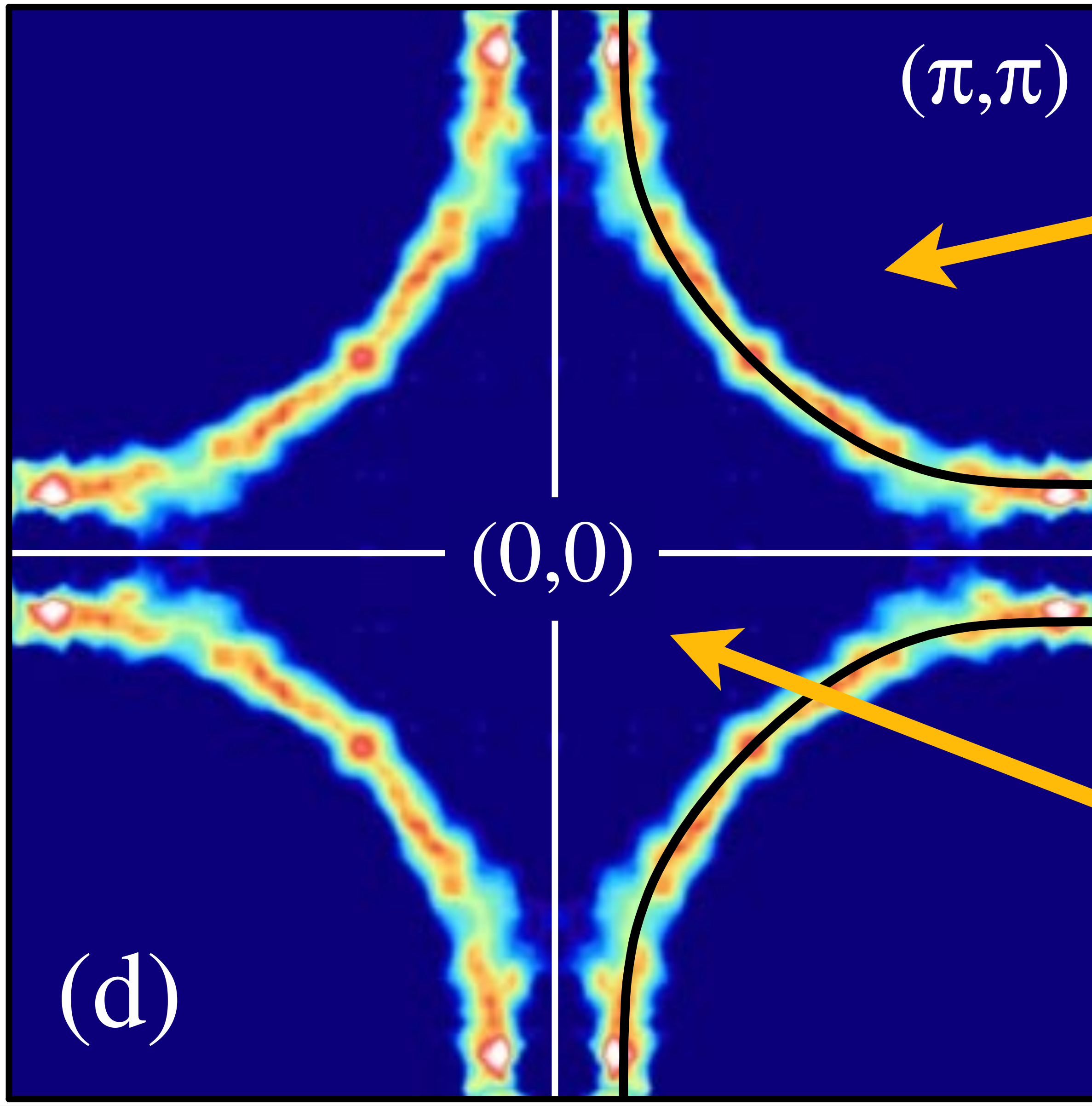
Filled Band

Momentum-space view



1-p mobile electrons =
1+p mobile holes in a filled band

Momentum-space view at large p



$1+p$ holes

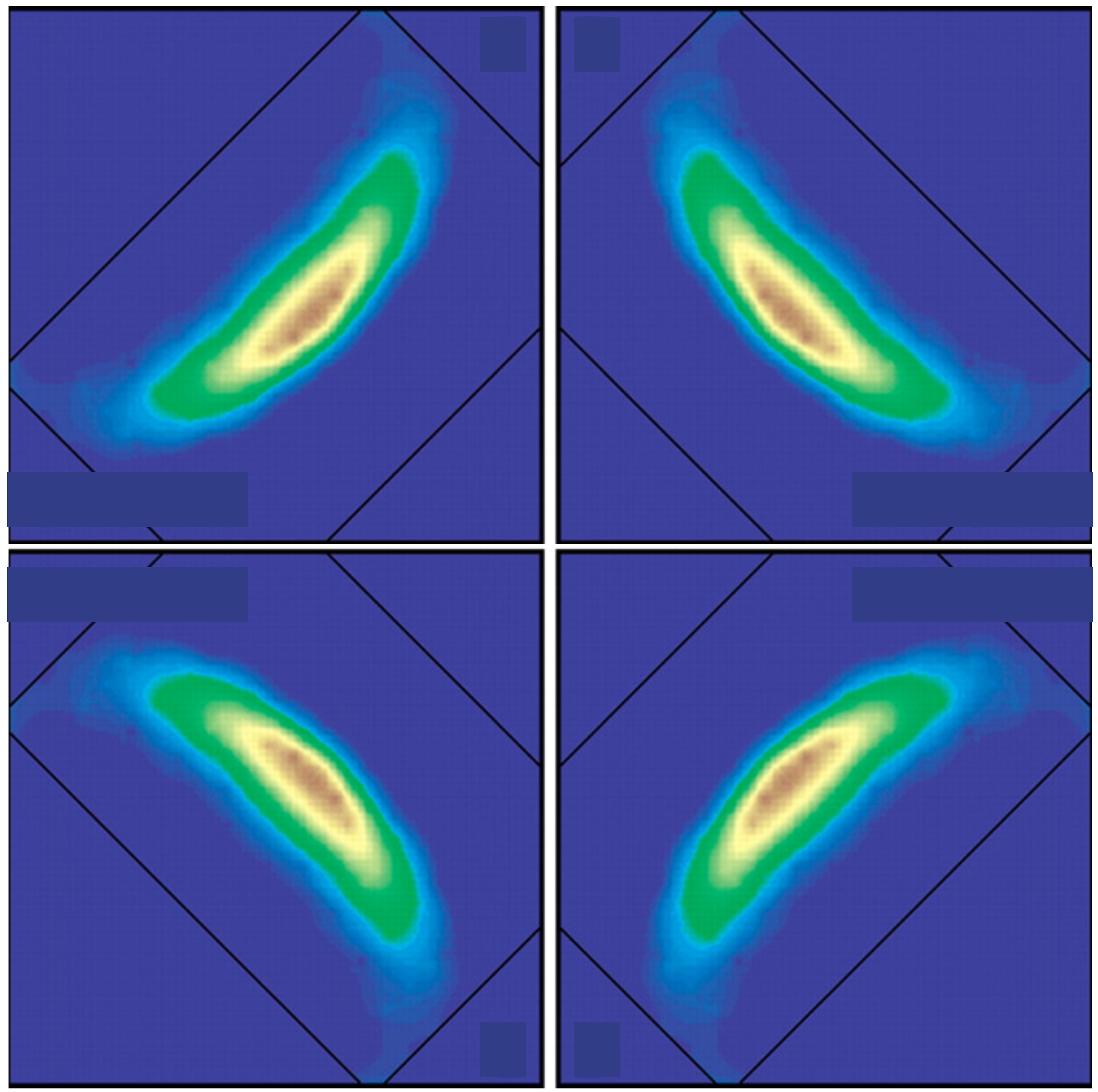
Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

1-p electrons

"Large Fermi surface":
$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Momentum-space view at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

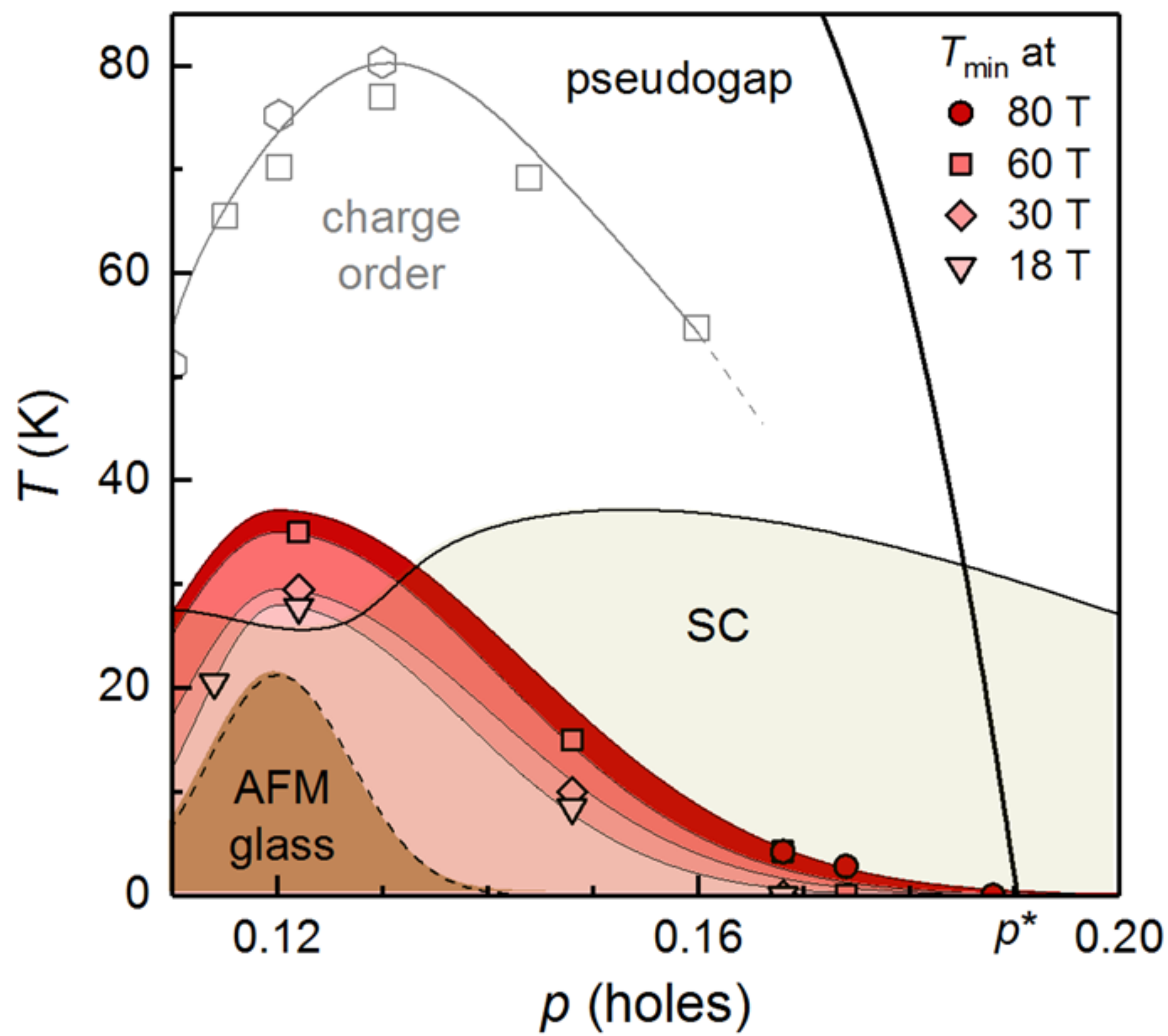
"Fermi arcs" ?

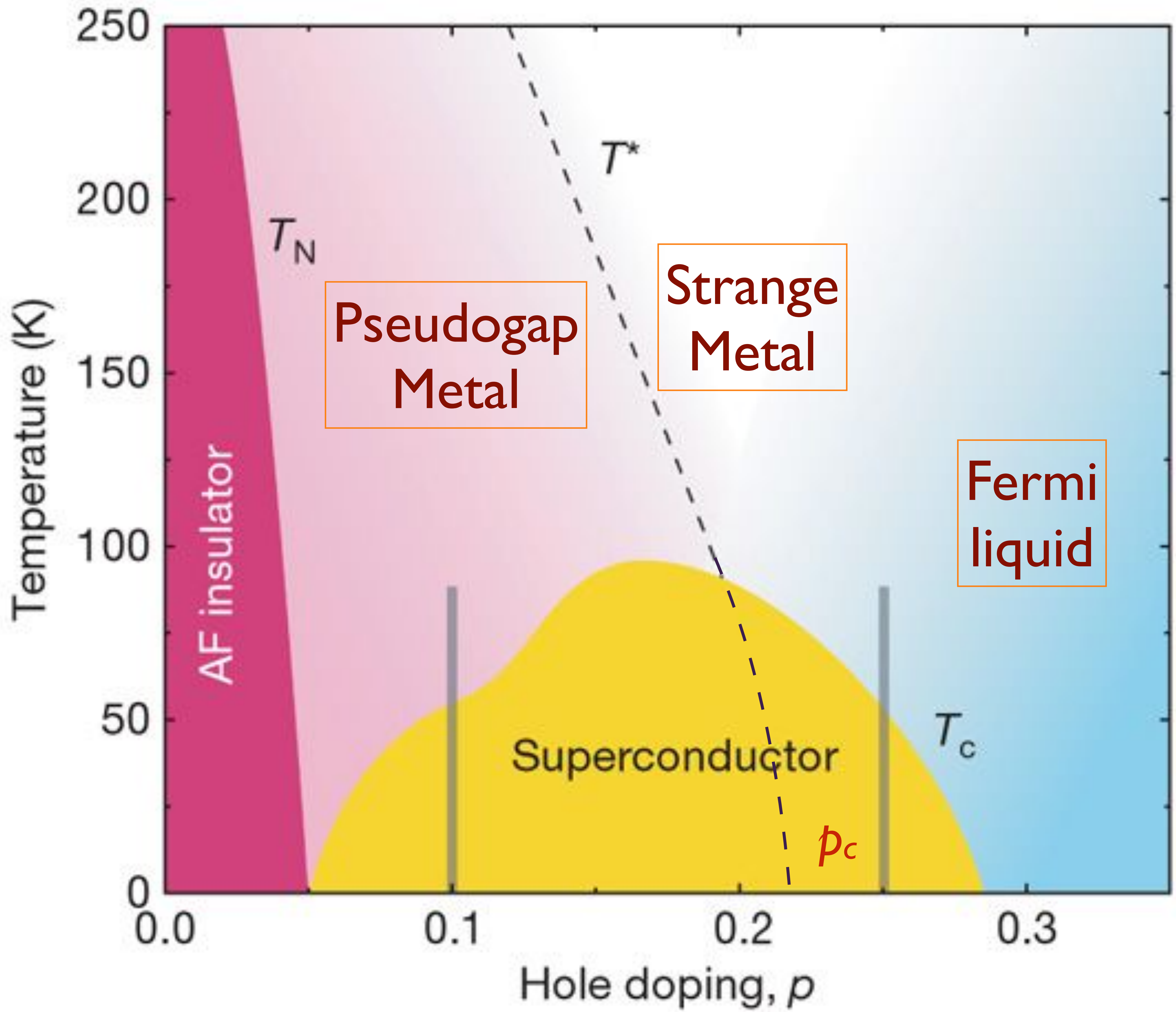
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics 16, 1064 (2020)

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]





250
200
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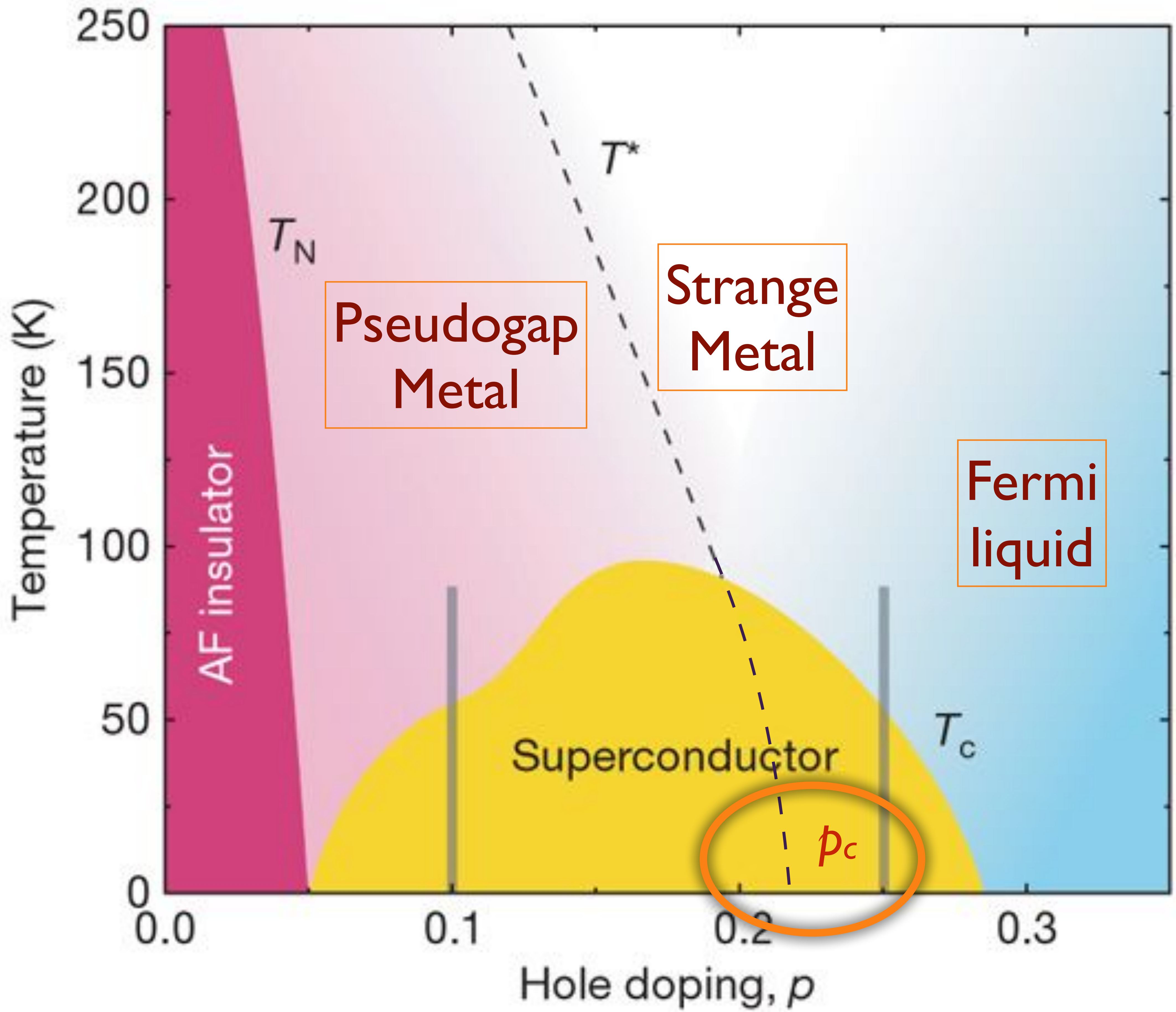
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c



250
200
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Temperature (K)
0.0
0.1
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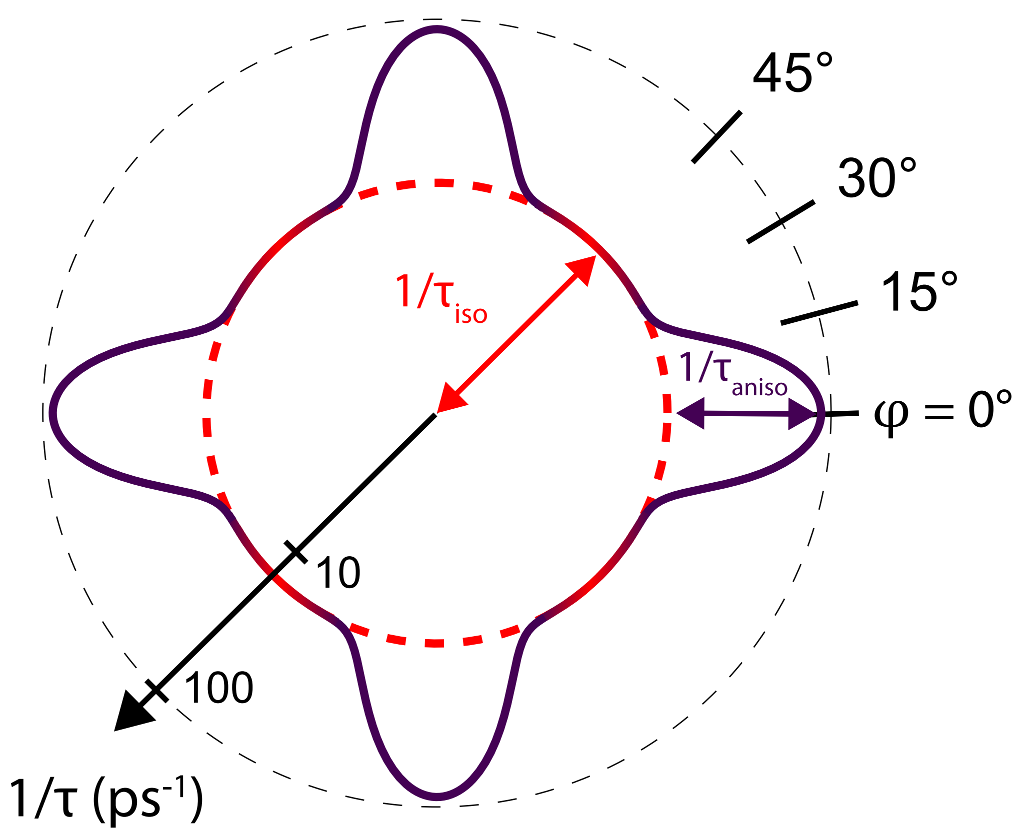
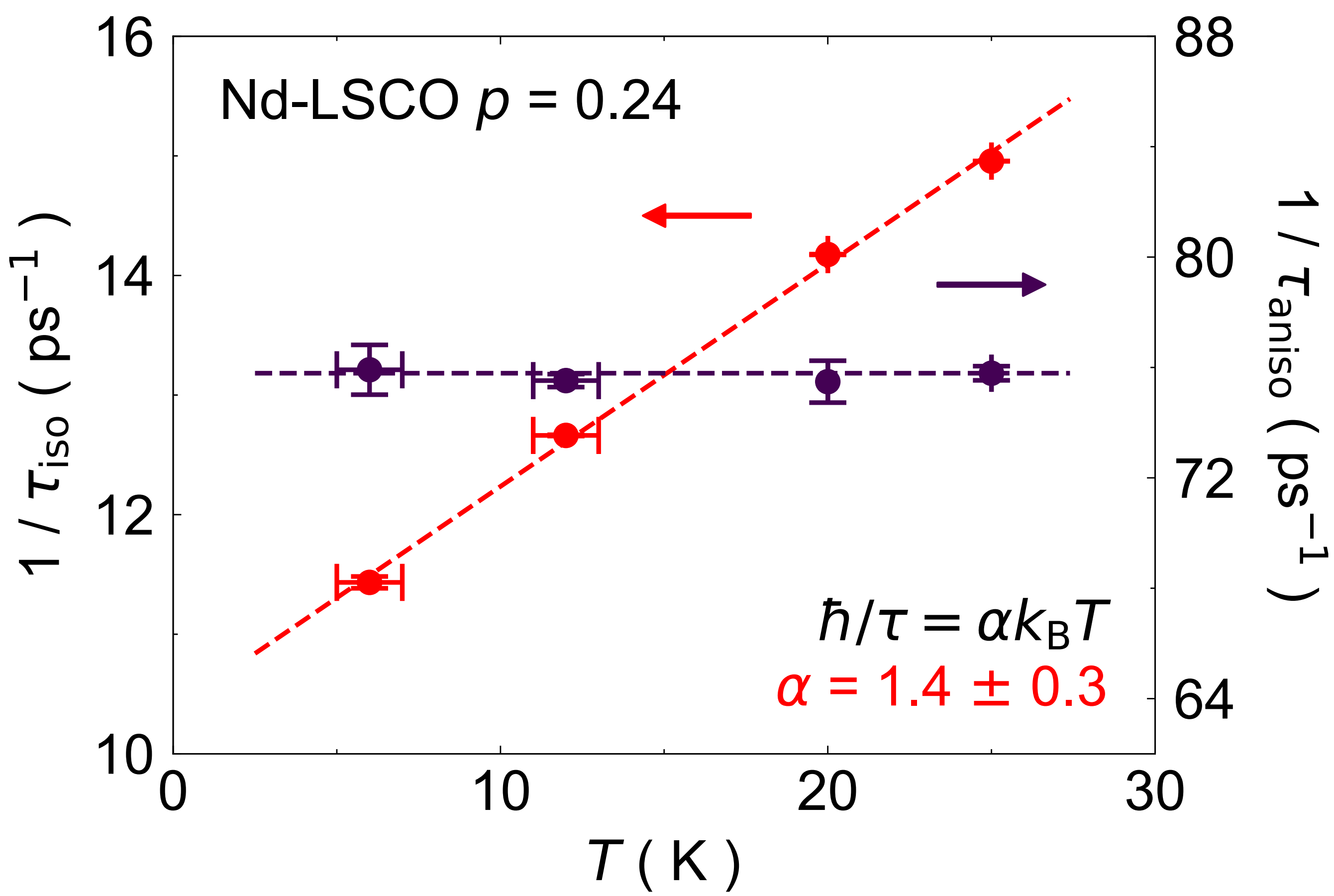
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c

Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.





$$\frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

Alexander Wietek

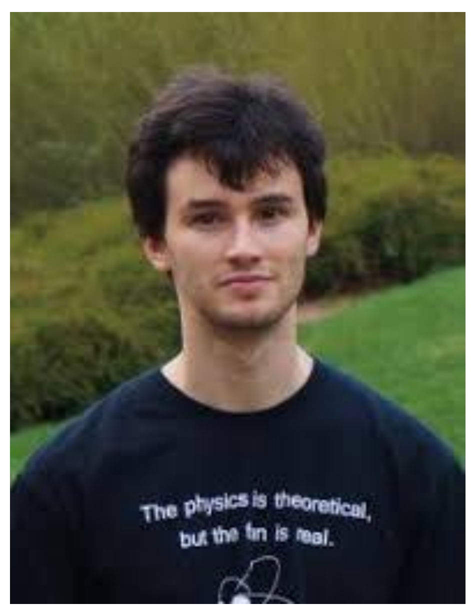
Henry Shackleton

arXiv:2012.06589

Antoine Georges

Maria Tikhanovskaya

Haoyu Guo



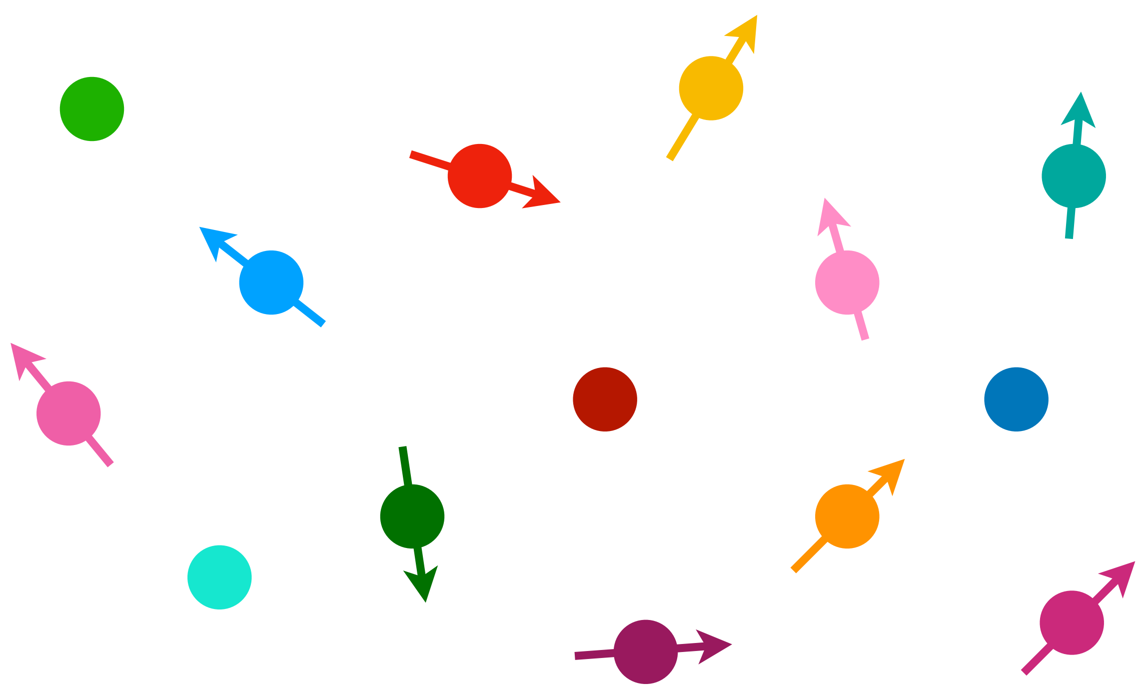
Grigory Tarnopolsky

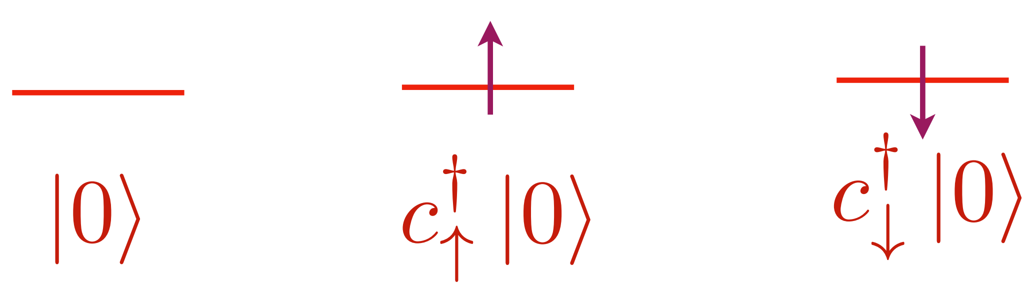
arXiv:2010.09742
arXiv:2012.14449

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





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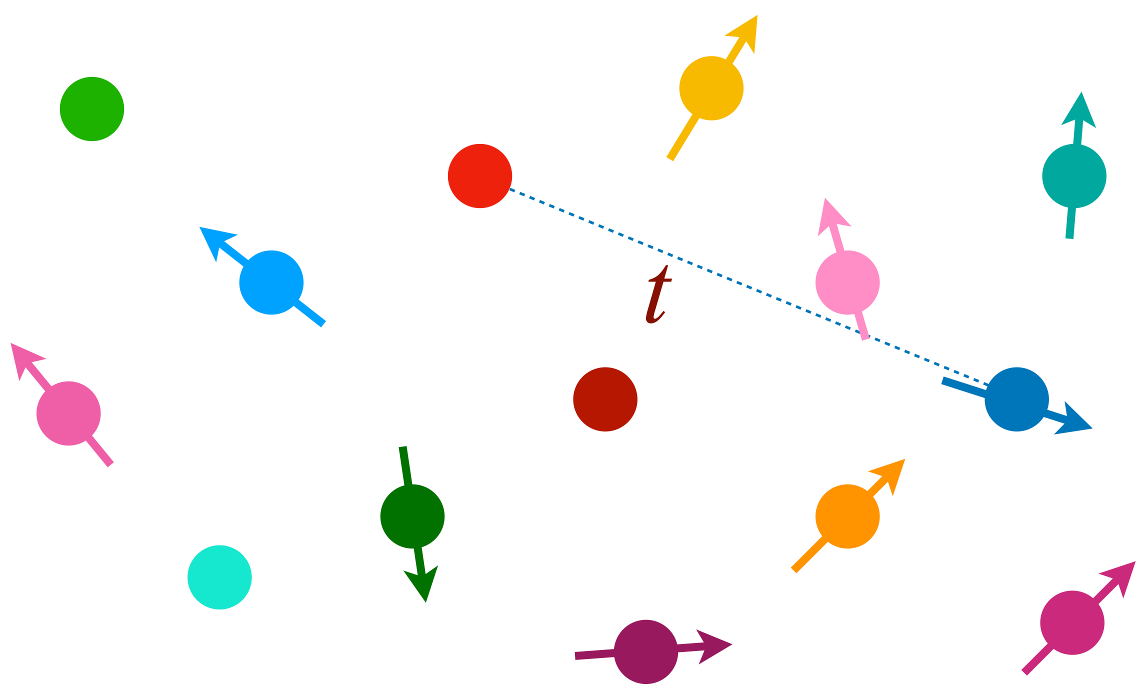

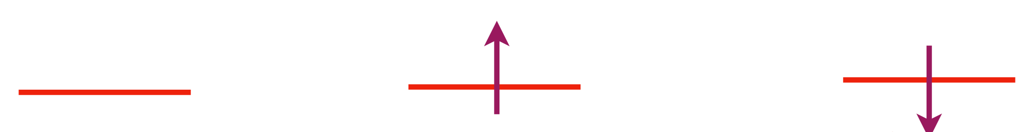

$|0\rangle$ $\quad c_{\uparrow}^{\dagger}|0\rangle$ $\quad c_{\downarrow}^{\dagger}|0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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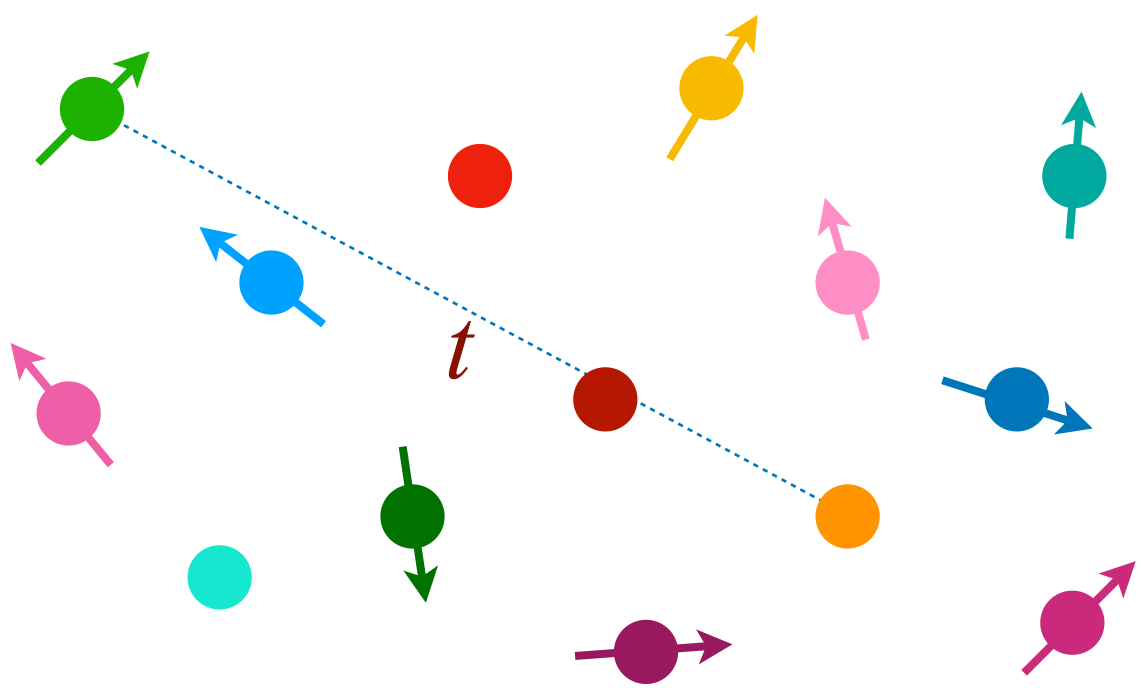

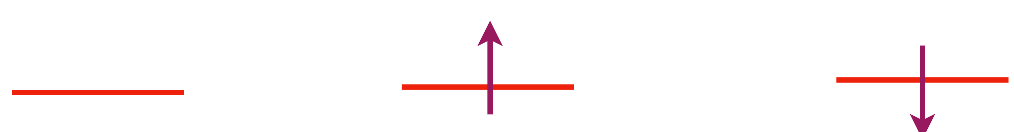

$|0\rangle \qquad c_{\uparrow}^{\dagger}\,|0\rangle \qquad c_{\downarrow}^{\dagger}\,|0\rangle$

Random *t*-*J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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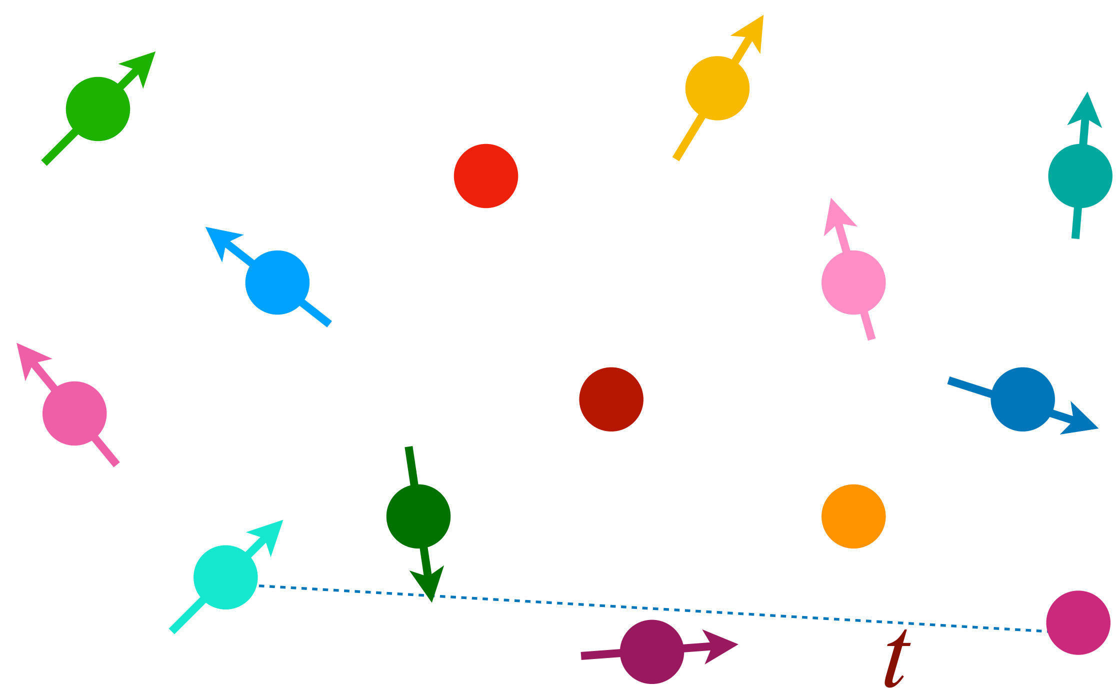

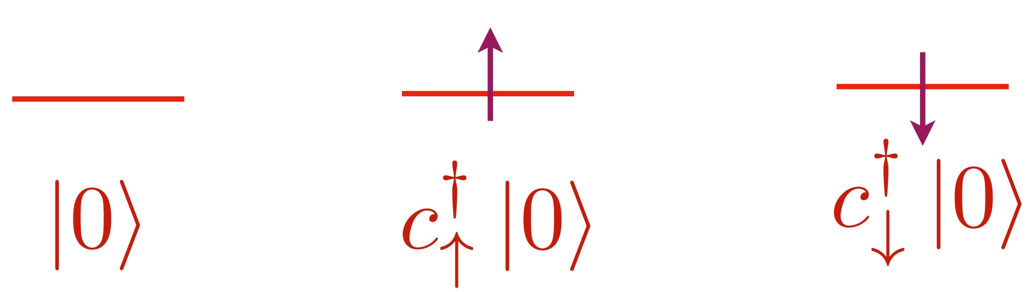

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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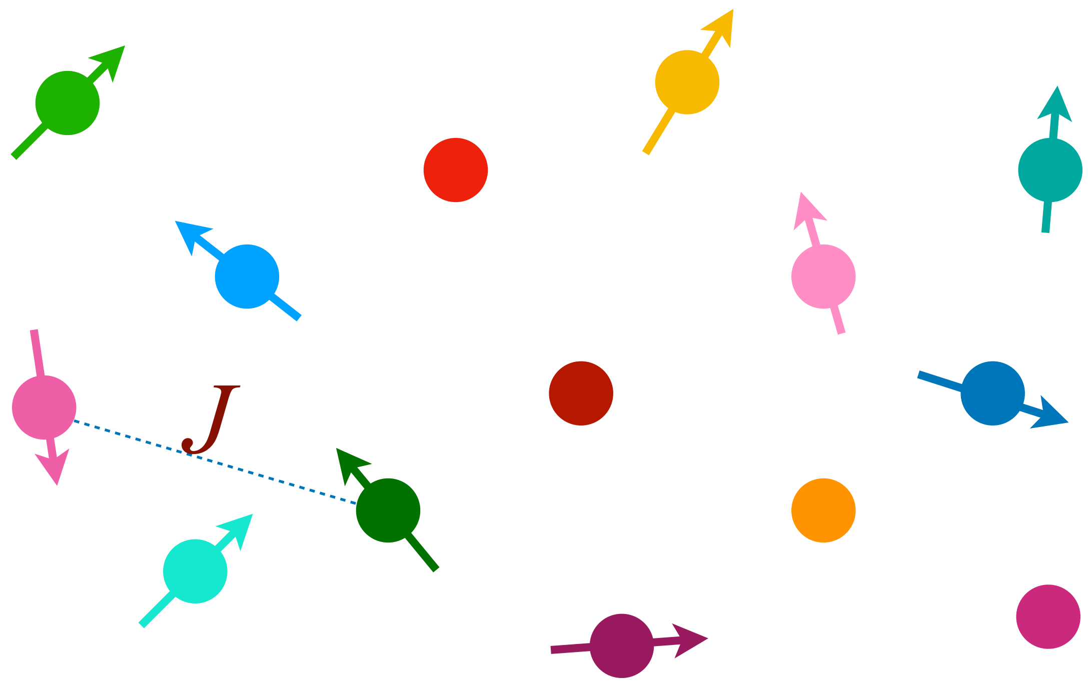
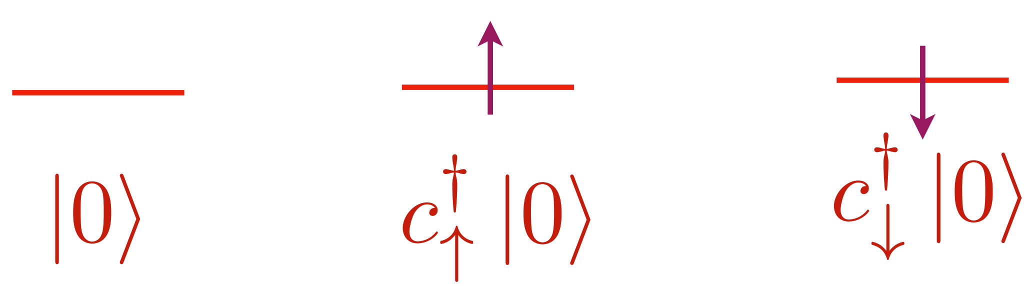
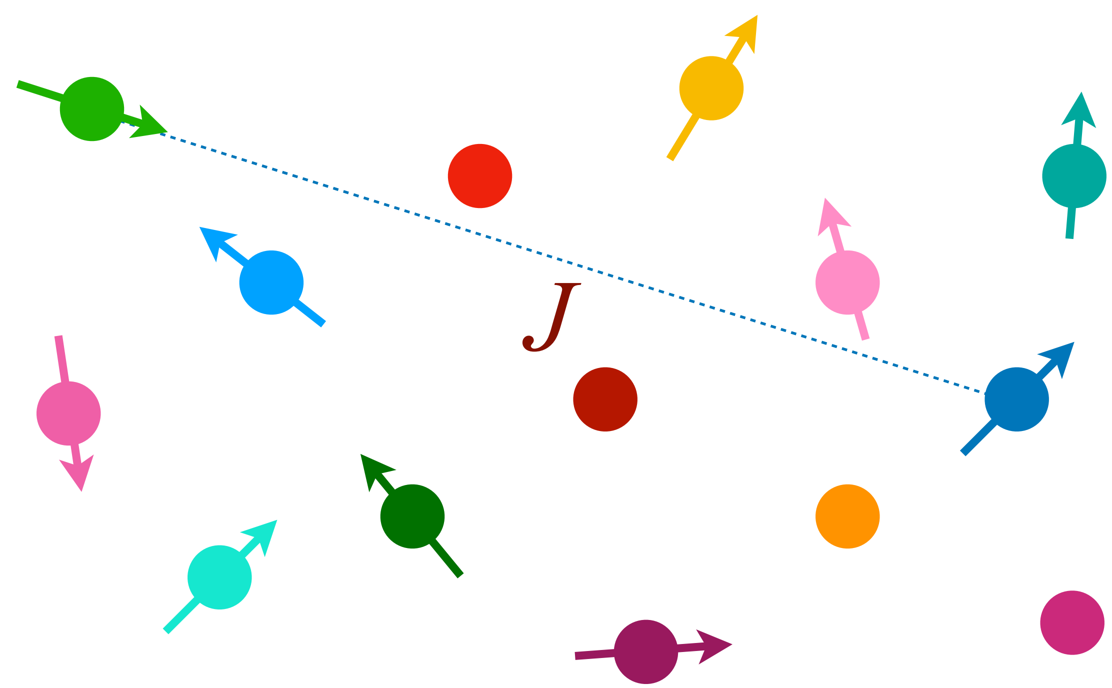


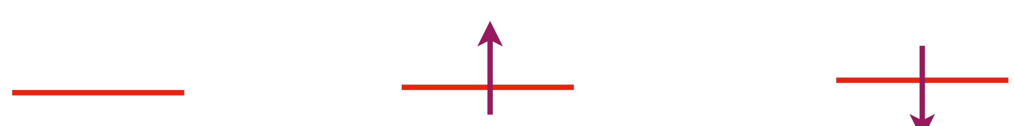


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





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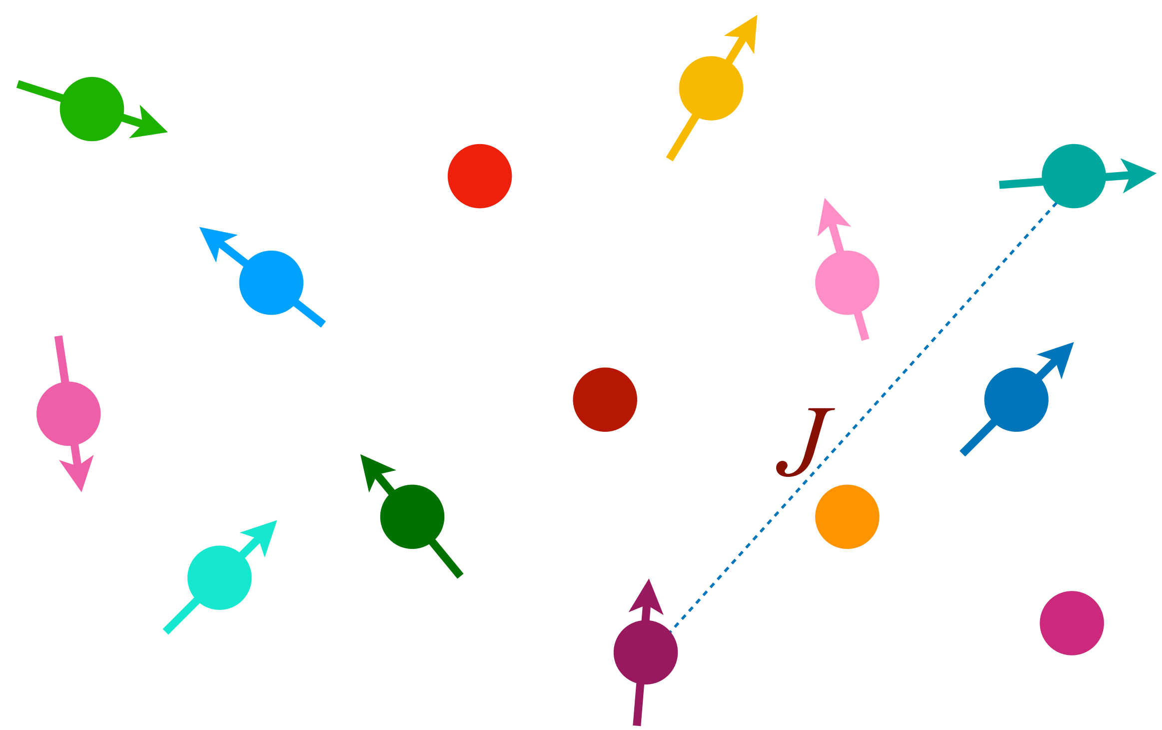

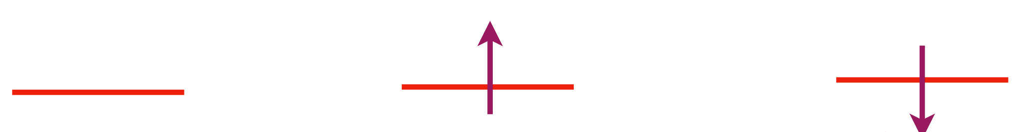

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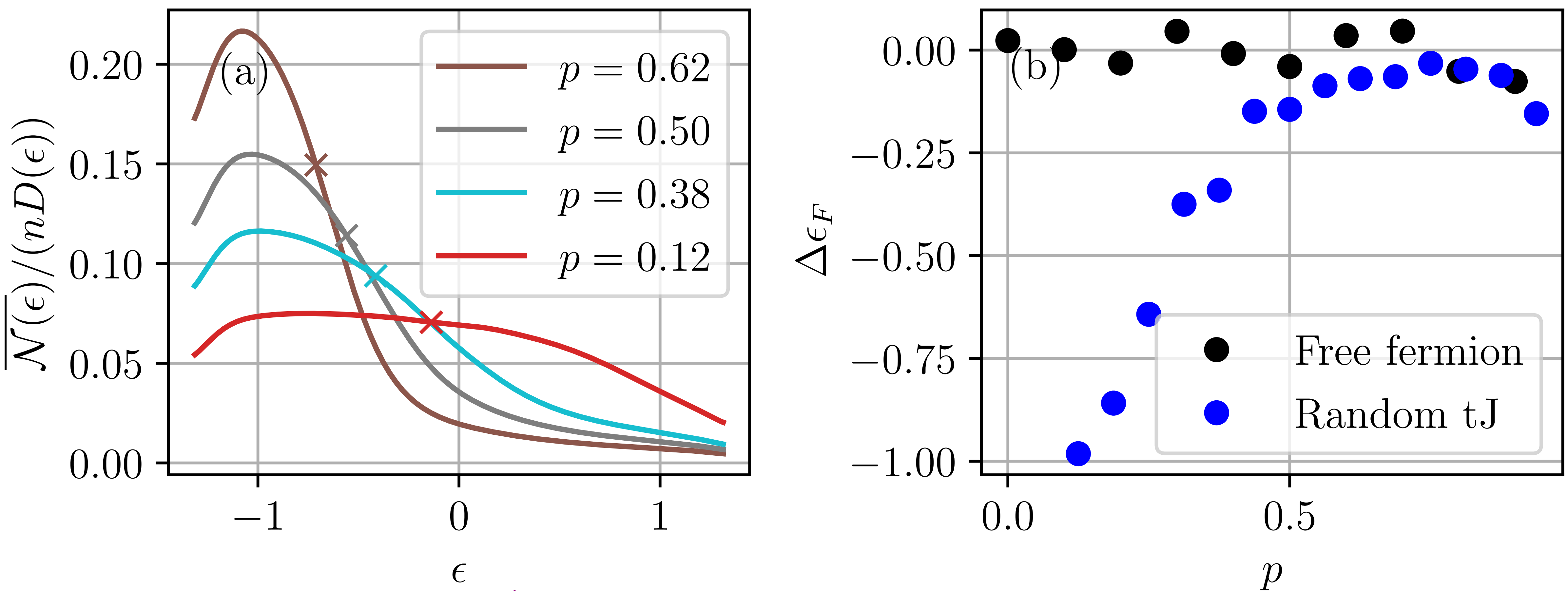
One particle energy distribution function



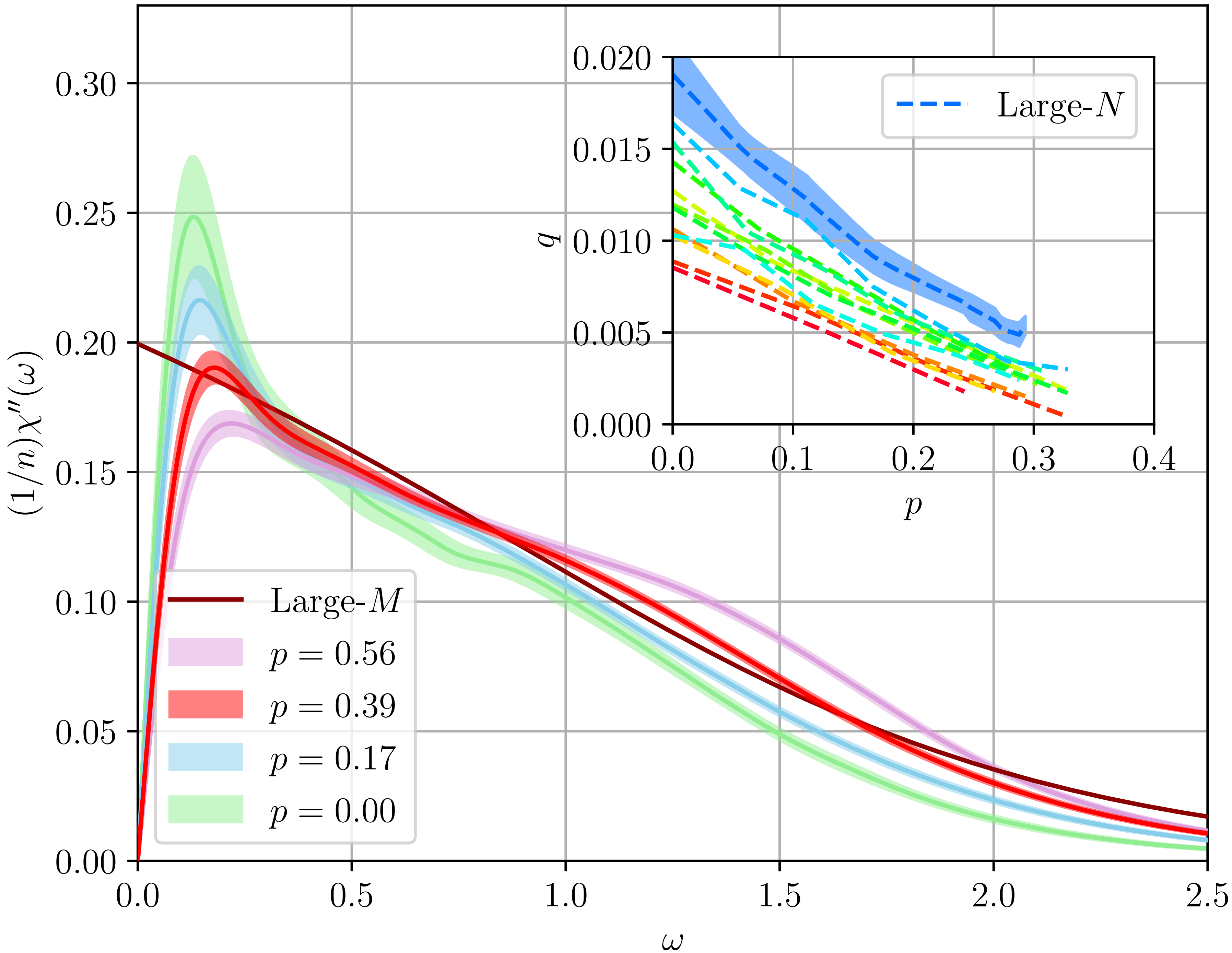
$$\mathcal{N}(\epsilon) = \frac{1}{N}\sum_{\lambda}\delta(\epsilon-\epsilon_\lambda)\sum_{ij\sigma}\langle\lambda|i\rangle\langle c^\dagger_{i\sigma}c_{j\sigma}\rangle\langle j|\lambda\rangle$$

where $|\lambda\rangle$ are one-particle eigenstates of the t_{ij}. In a Fermi liquid, the Luttinger identity implies that $\mathcal{N}(\epsilon)$ has a discontinuity at the free particle Fermi energy ϵ_F. ($D(\epsilon)$ is the Wigner semi-circle density of states.)

Evidence for a *"Large Fermi surface"* for $p > p_c \approx 0.4$

Dynamic spin susceptibility

$$\chi''(\omega) = \sum_n |\langle 0| S_{+i} |n\rangle|^2 \, \delta(\hbar\omega - E_n + E_0), \ (\text{at } T = 0)$$



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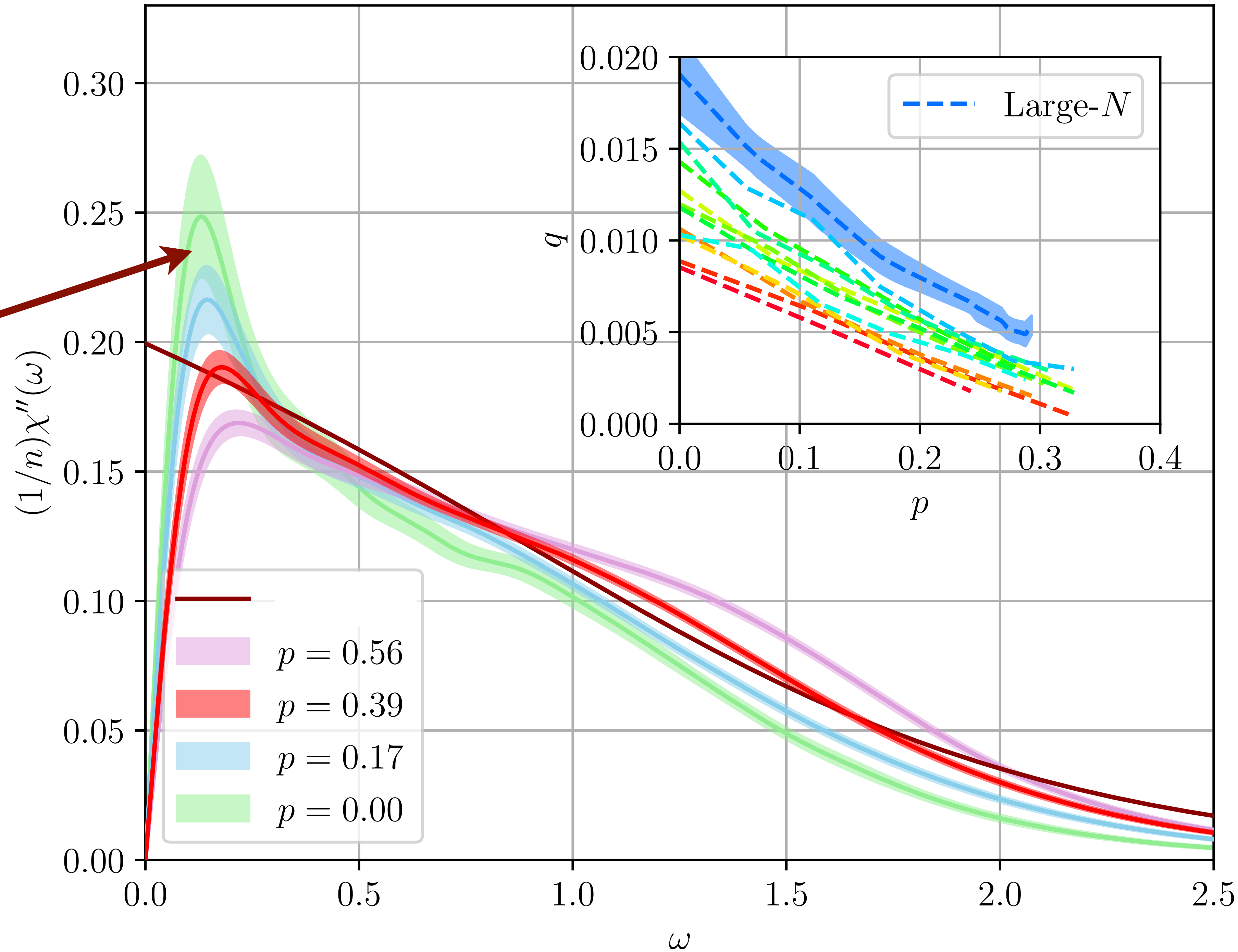
Spin glass order



Spin glass order q non-zero for $p < p_c \approx 0.4$

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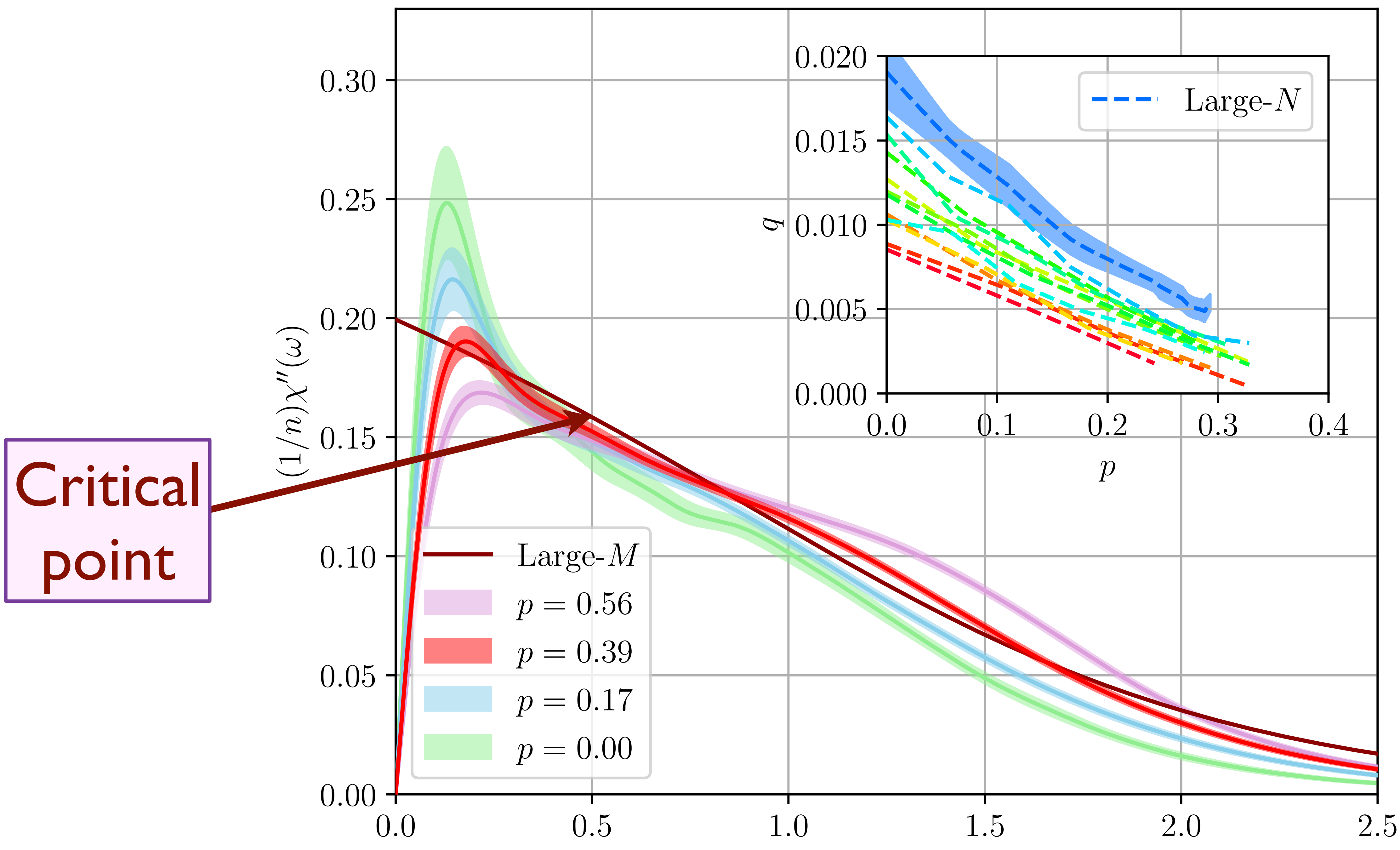


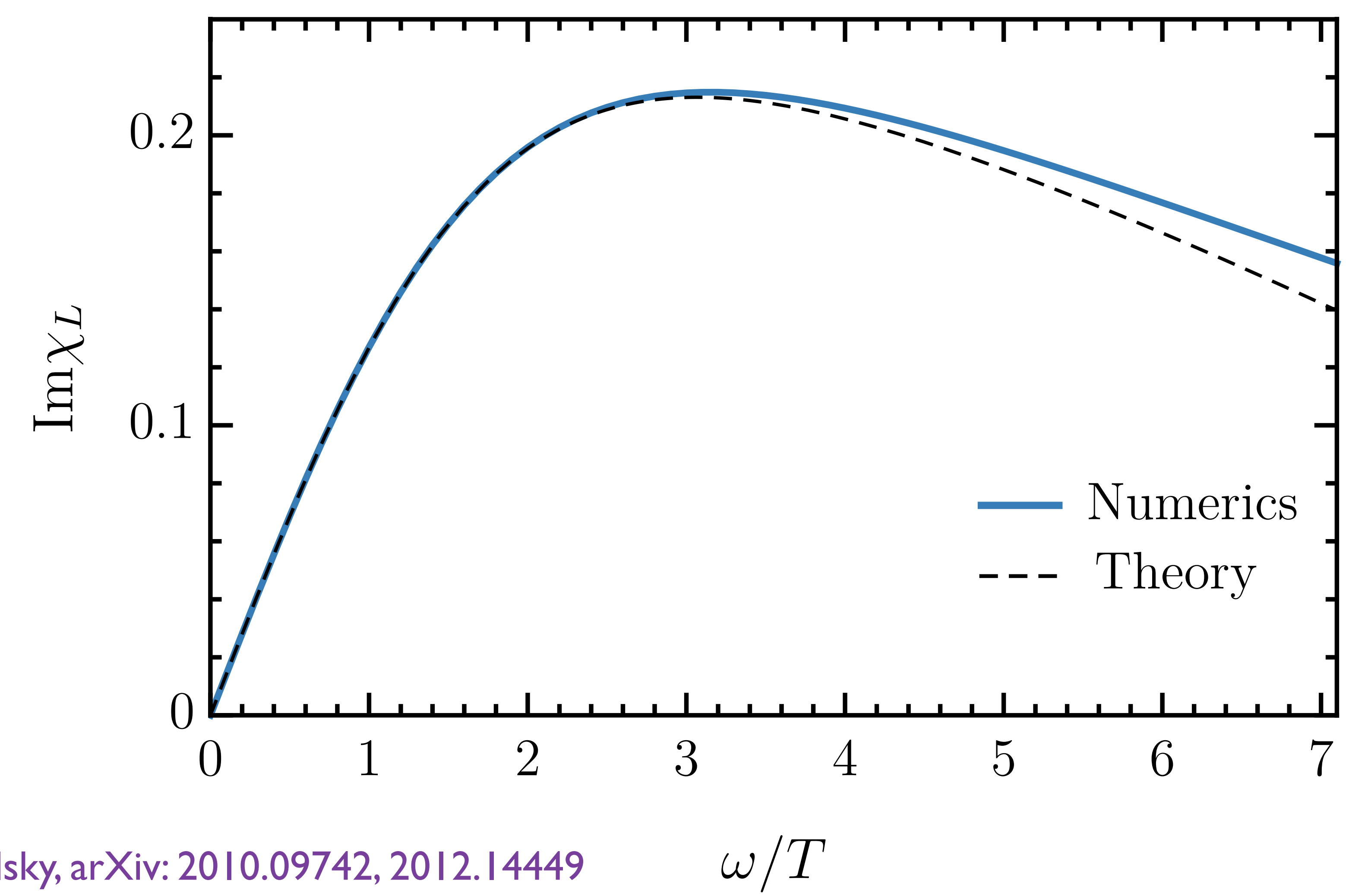
Critical spin susceptibility matches the SYK model!

$$\chi''(\omega) \sim \mathrm{sgn}(\omega) \left[1 - \mathcal{C}\gamma|\omega| + \ldots\right]$$

Consequences of 2D-gravity for the SYK model

$$\chi''(\omega) = \sum_n |\langle 0| S_{+i} |n\rangle|^2 \, \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi''(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right)\left[1 - \mathcal{C}\gamma\,\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \ldots\right]$$



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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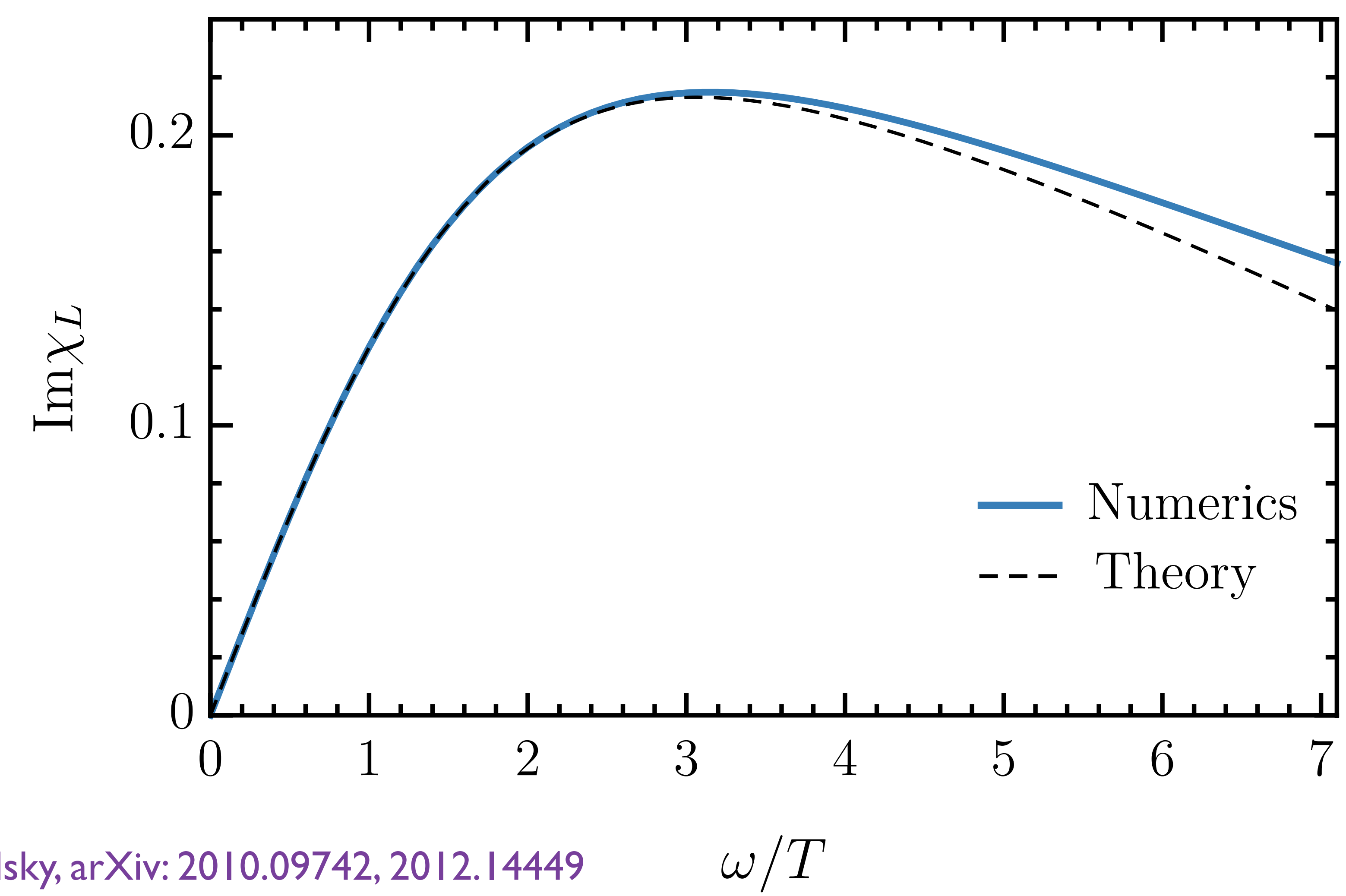
Conformally (SL(2,R)) invariant result with characteristic dissipative time $\sim \hbar/(k_BT)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)



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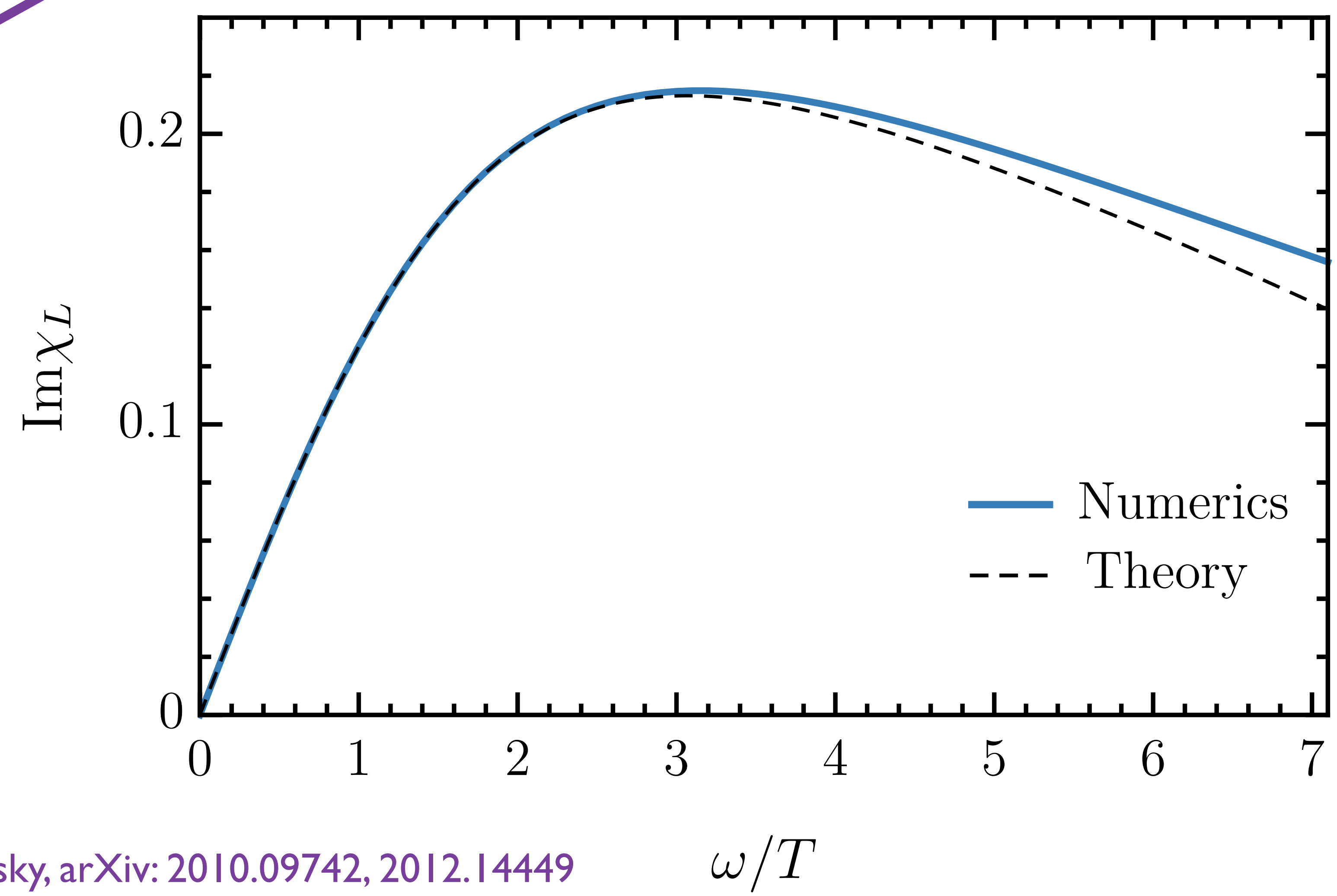
Correction from the boundary graviton



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

The random t-J model has

- Spin glass order for $p < p_c$.
- Fermi liquid with *"large Fermi surface"* for $p > p_c$
- Maxima in entropy, specific heat, and entanglement entropy near $p = p_c$
- SYK-Planckian criticality near p_c.
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- SYK criticality can be understood in a model in which the electron fractionalizes into spinons and holons: then both the t and J terms map onto 4-particle SYK terms.

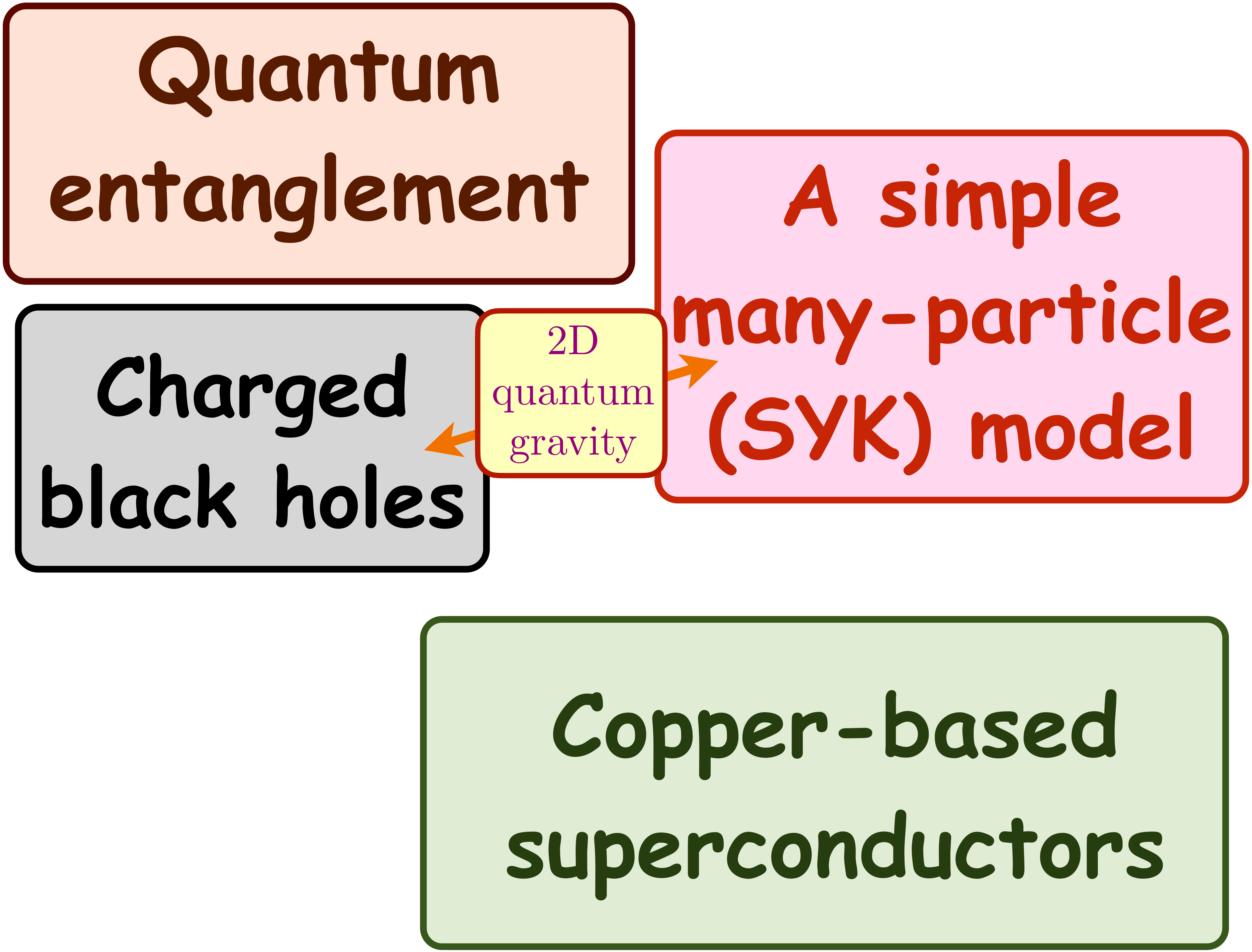
D. G. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, and S. Sachdev, PRX **10**, 021033 (2020)

Quantum entanglement

A simple many-particle (SYK) model

Charged black holes

Copper-based superconductors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Quantum entanglement

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SYK criticality near $p = p_c$

Copper-based superconductors

Complex multi-particle entanglement
leads to quantum systems
without quasiparticle excitations.

Many-body chaos and
thermal equilibration
in the shortest possible
Planckian time $\sim \dfrac{\hbar}{k_B T}$.