1. Review of Fermi liquid theory

Topological argument for the Luttinger theorem

2. Fractionalized Fermi liquid

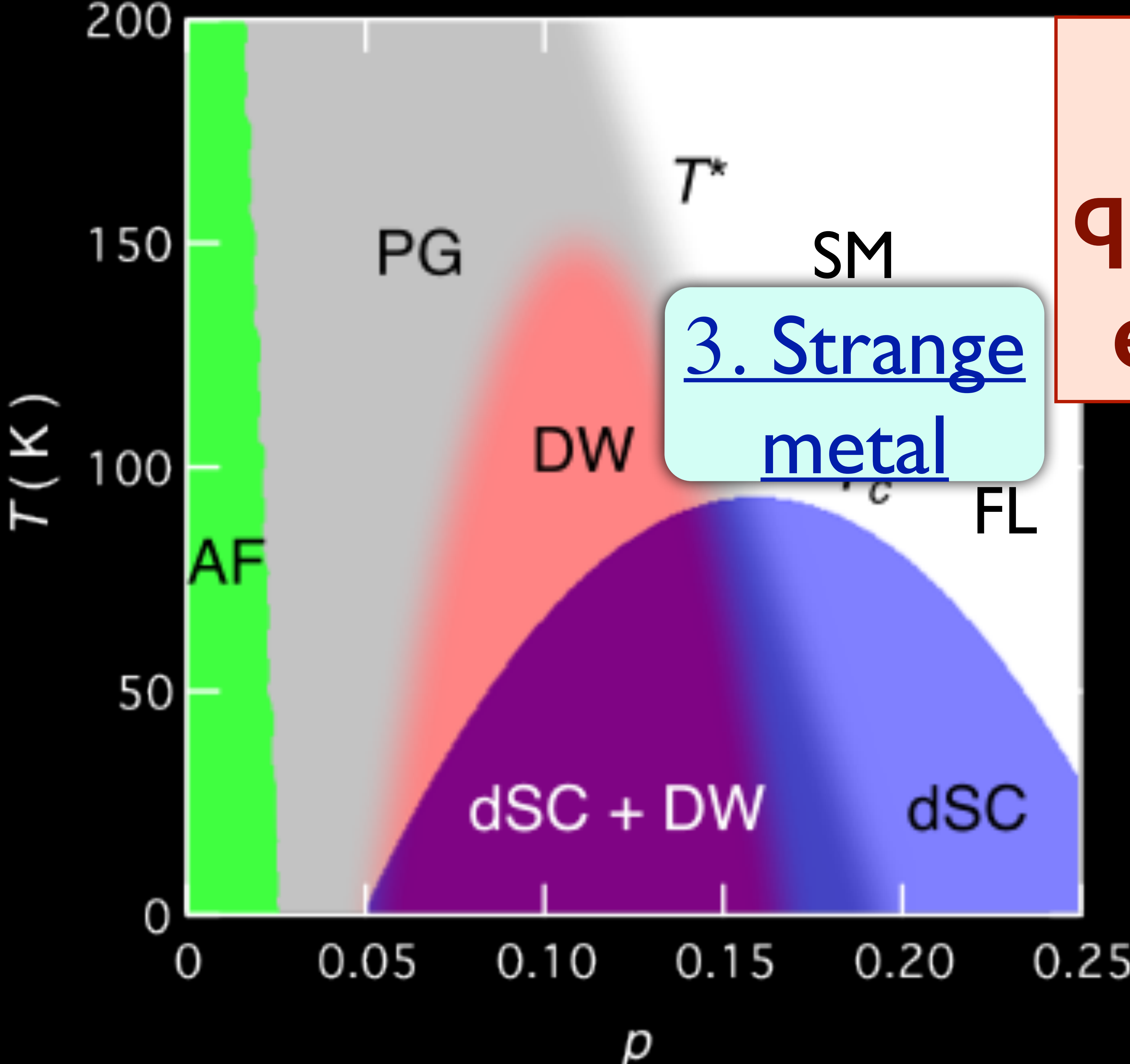
A Fermi liquid co-existing with topological order for the pseudogap metal

3. Quantum matter without quasiparticles

(A) A mean-field model of a non-Fermi liquid, and charged black holes

(B) Field theory of a non-Fermi liquid (Ising-nematic quantum critical point)

(C) Theory of transport in strange metals: application to the (less) strange metal in graphene



3. Strange metal

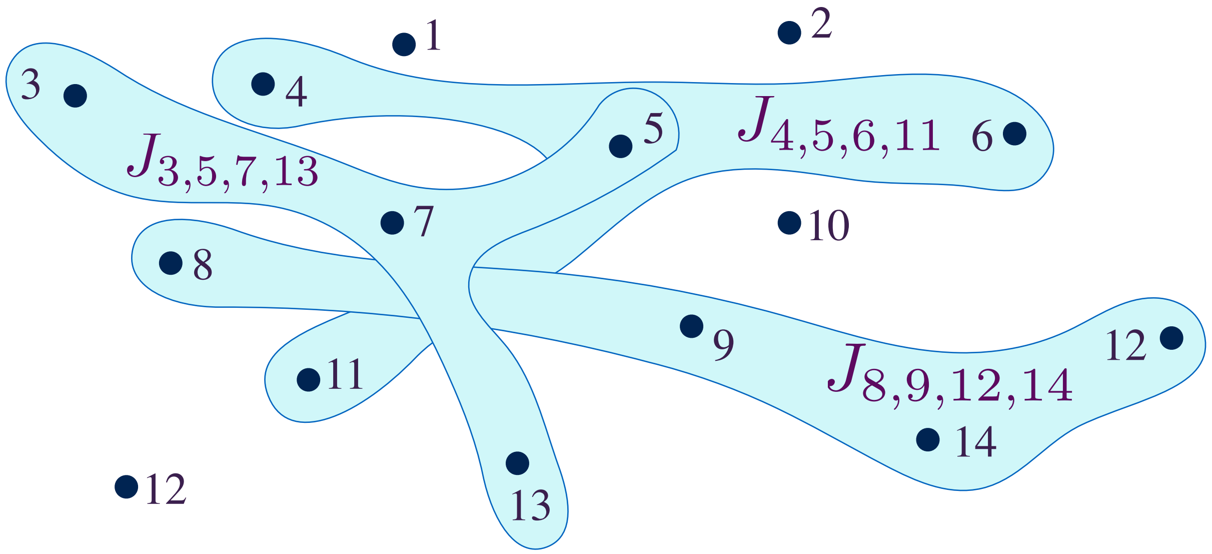
No quasiparticle excitations

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$$c_i c_j + c_j c_i = 0$$

$$c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$J_{ij;k\ell}$ independent random numbers



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

An infinite-range model of a strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

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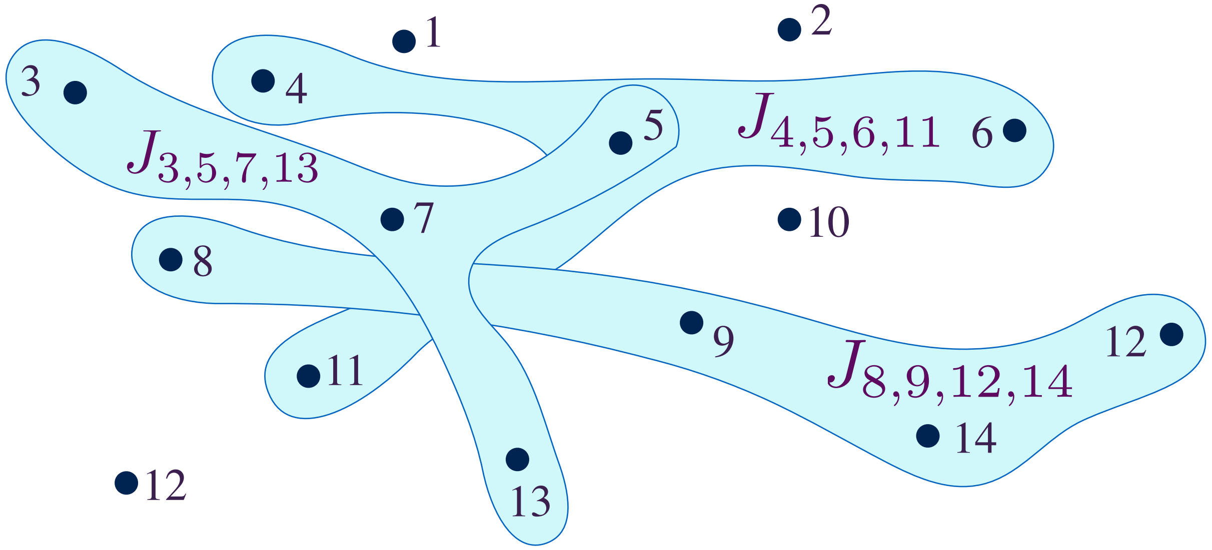

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Local fermion density of states

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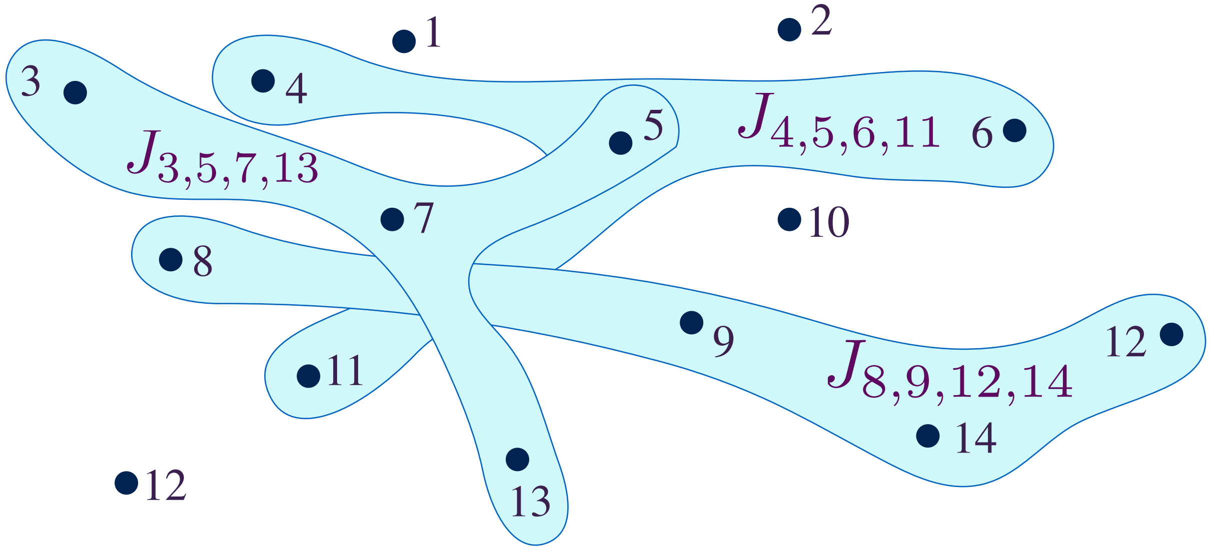

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Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right)$$

A. Georges, O. Parcollet, and S. Sachdev
Phys. Rev. B **63**, 134406 (2001)

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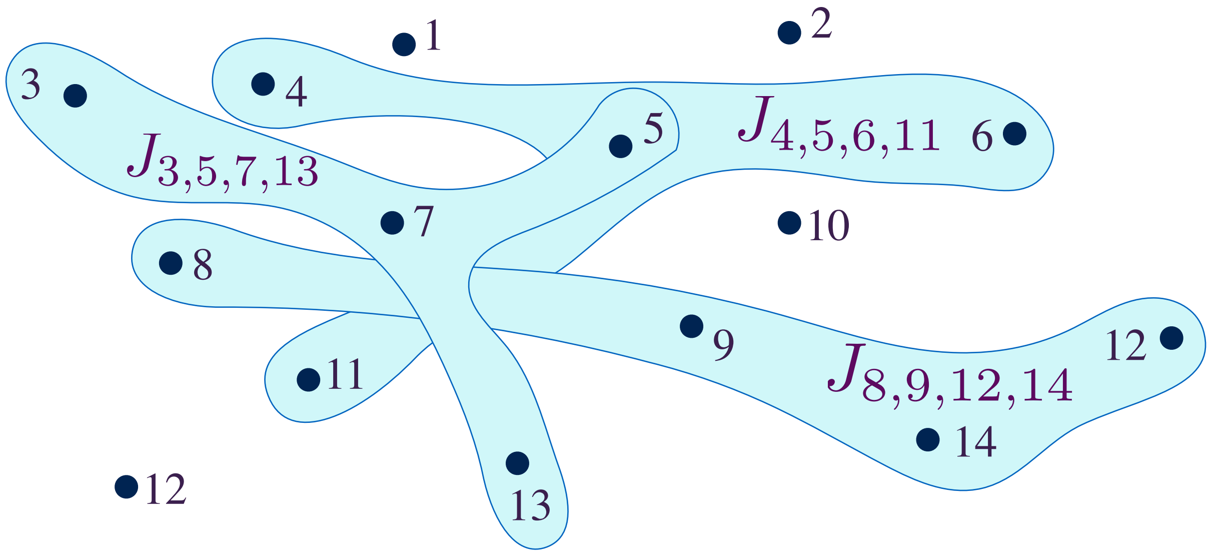

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Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
Phys. Rev. B **58**, 3794 (1998)

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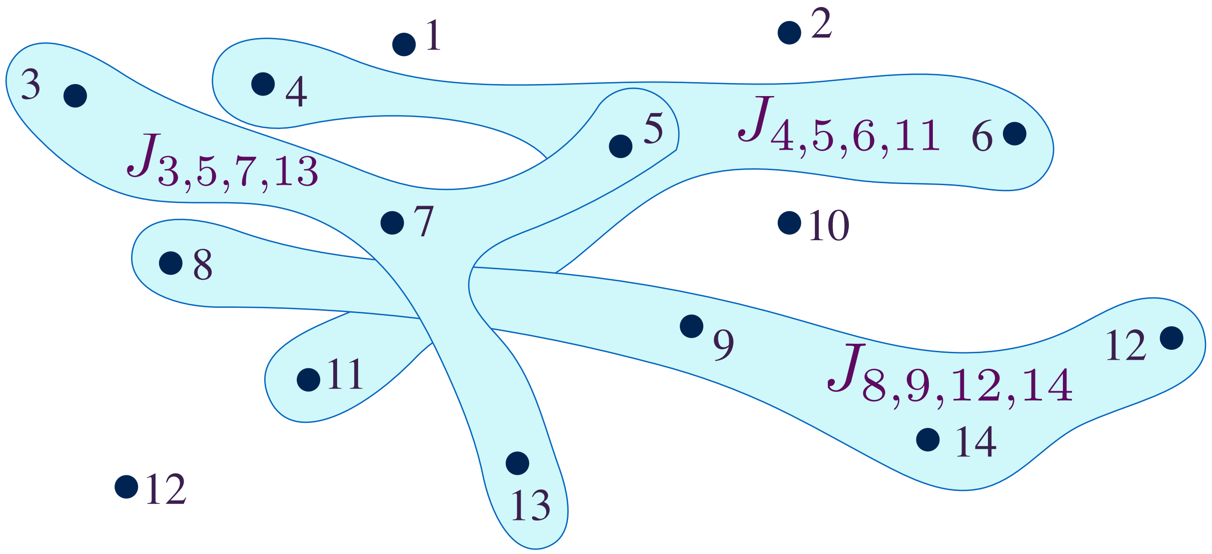

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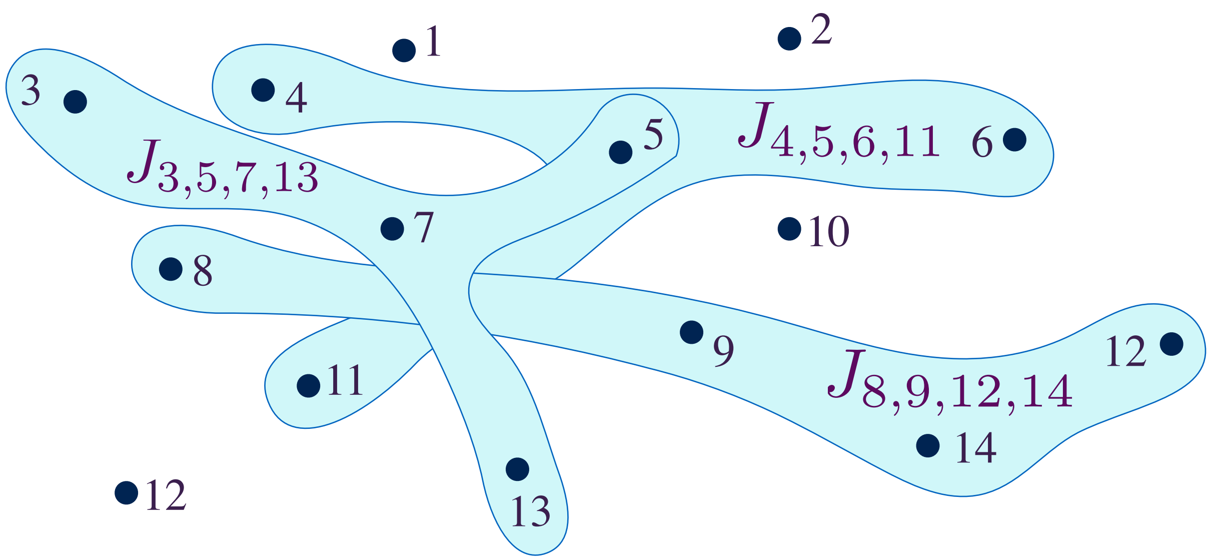

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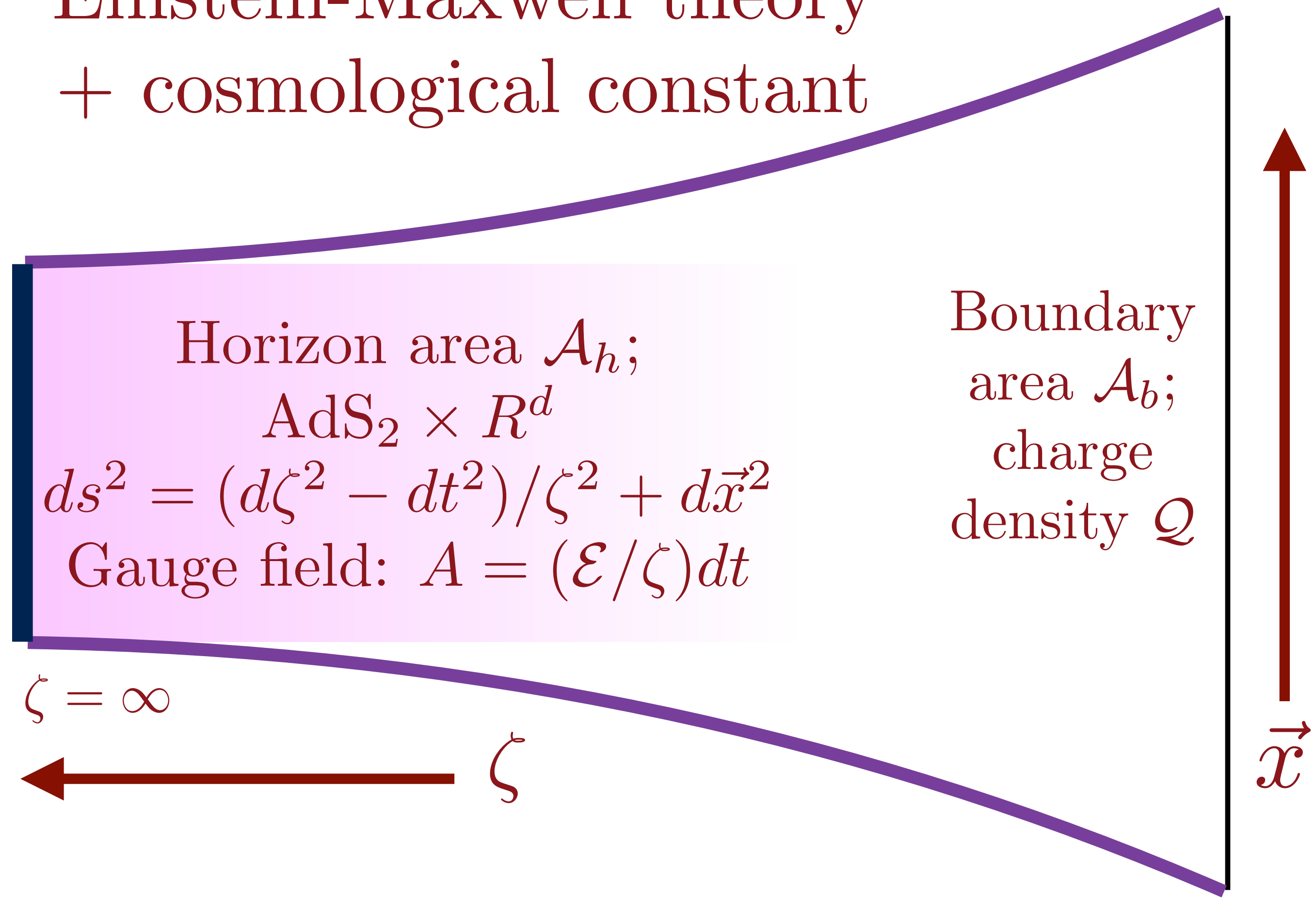

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers
Phys. Rev. D **60**, 064018 (1999)

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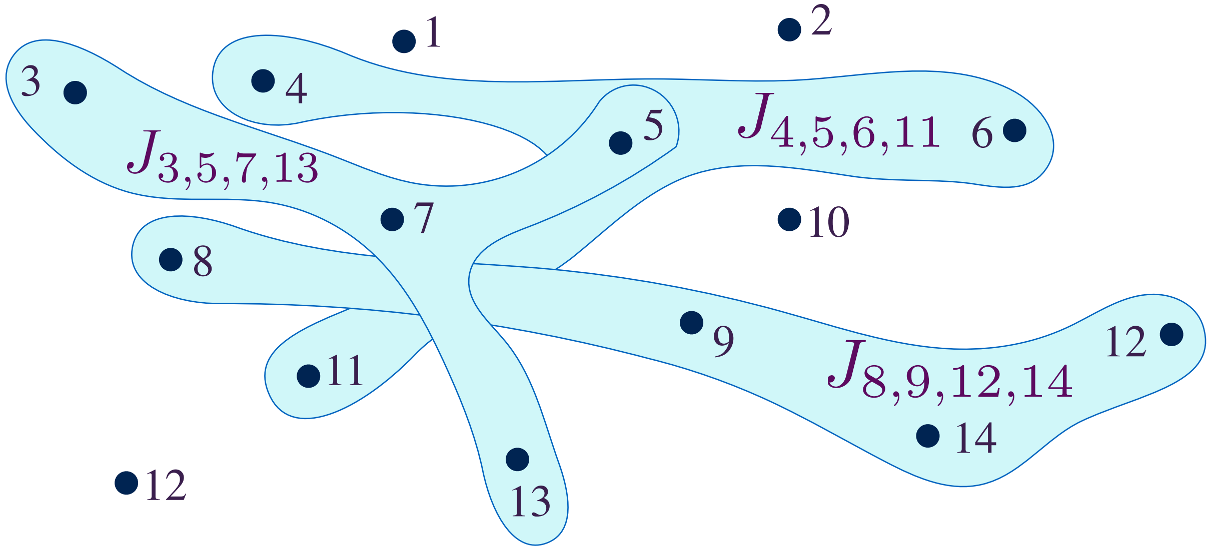

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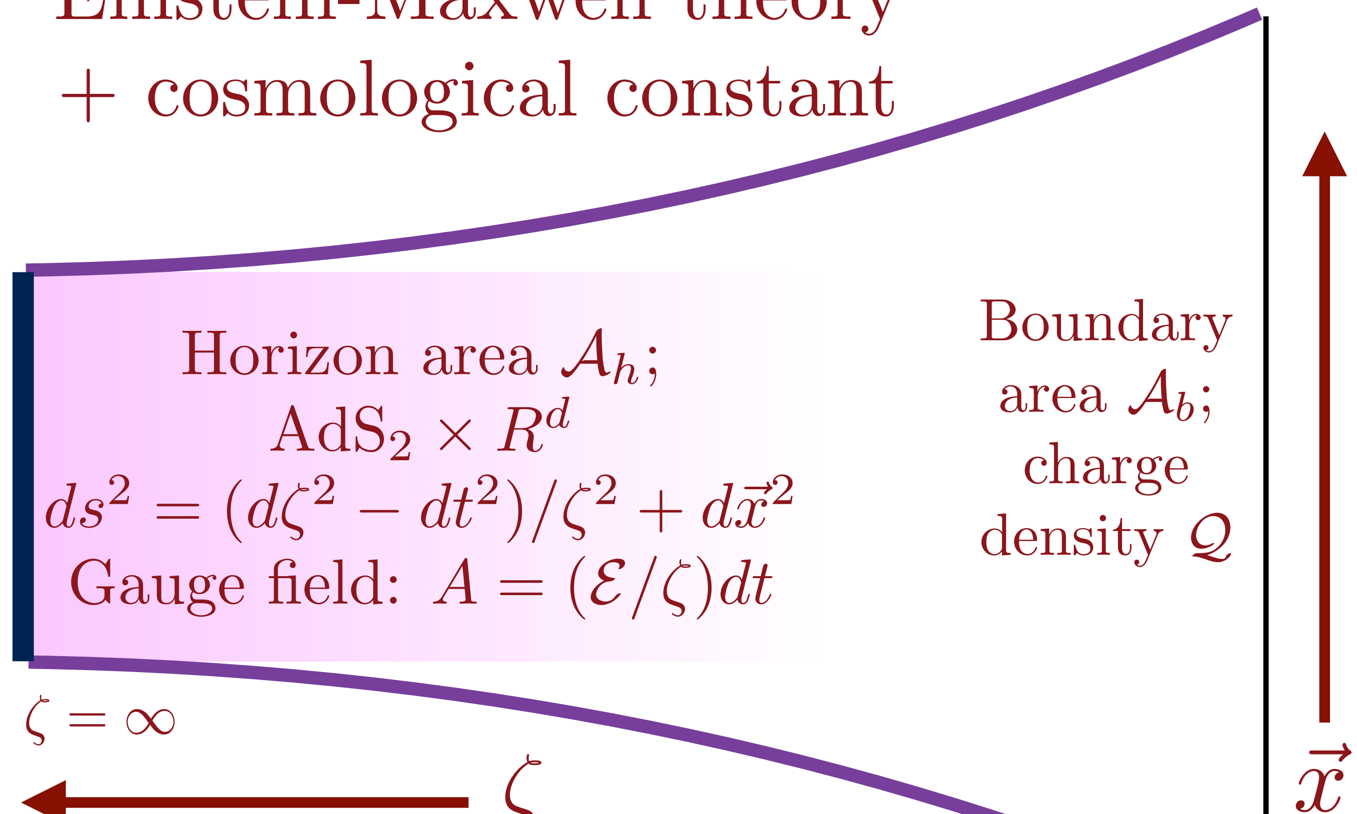

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T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh
Phys. Rev. D **83**, 125002 (2011)

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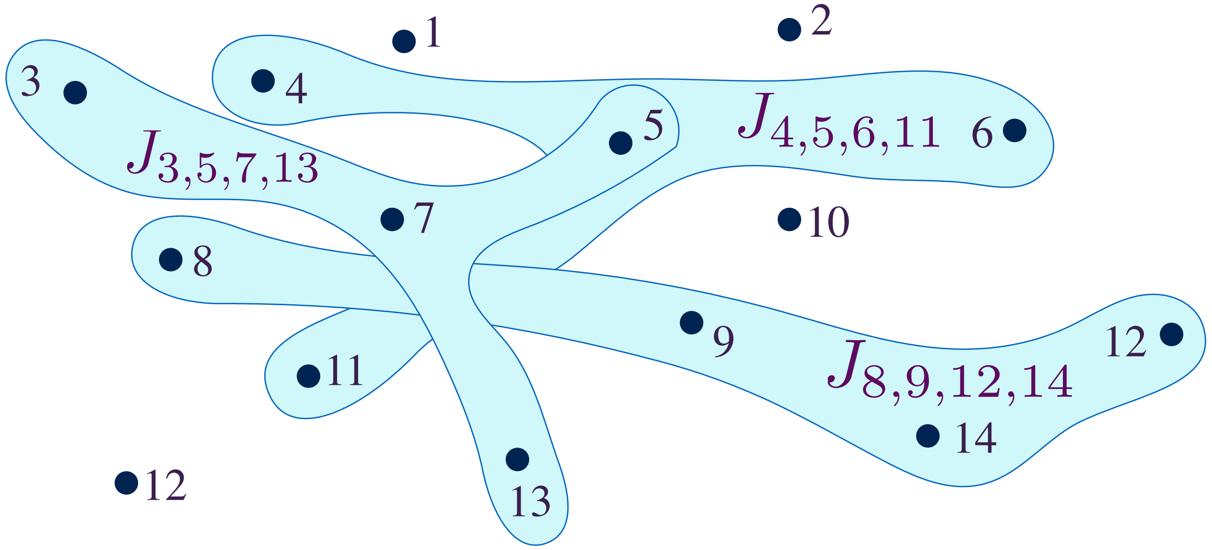

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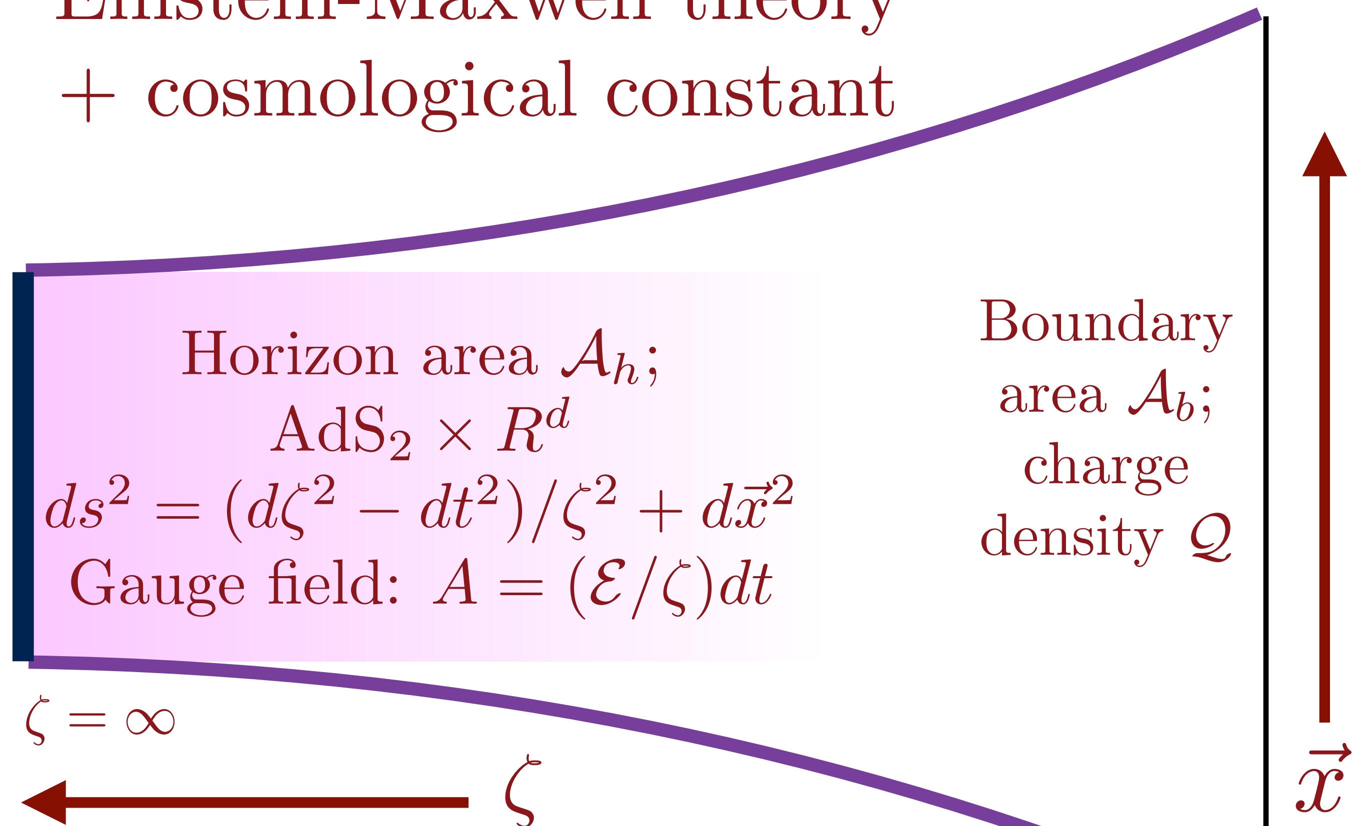

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'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Eliminate r_0 between

$$\mathcal{Q} = \frac{r_0^{d-1}\sqrt{2d\left[(d-1)R^2 + (d+1)r_0^2\right]}}{\kappa^2 g_F}$$

$$\mathcal{E} = \frac{g_F r_0 \sqrt{2d\left[(d-1)R^2 + (d+1)r_0^2\right]}}{2\left[(d-1)^2 R^2 + d(d+1) r_0^2\right]}$$

S. Sachdev, arXiv:1506.05111

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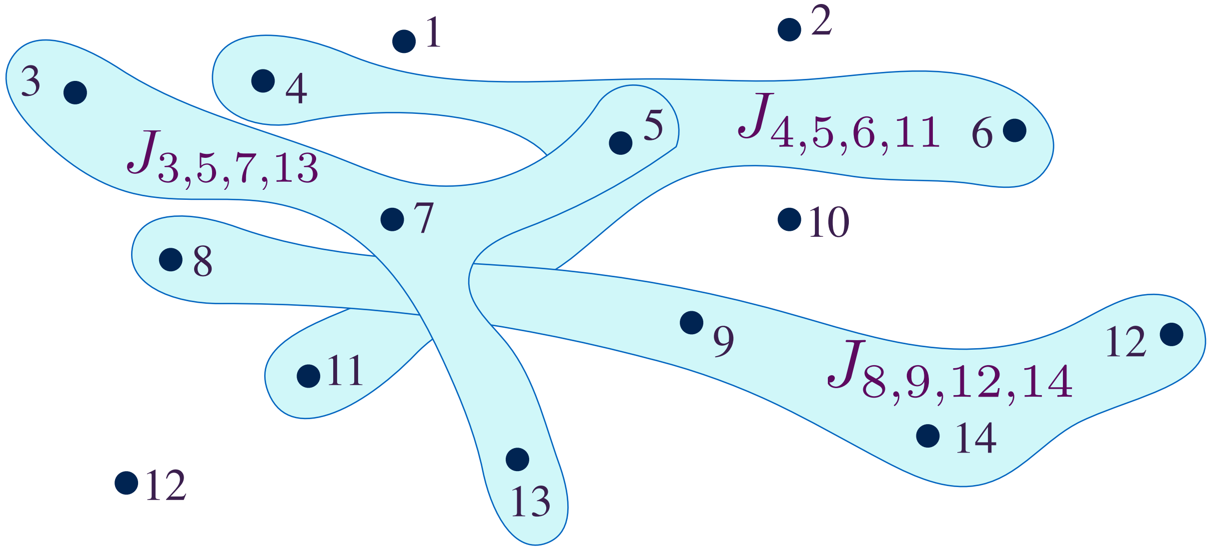

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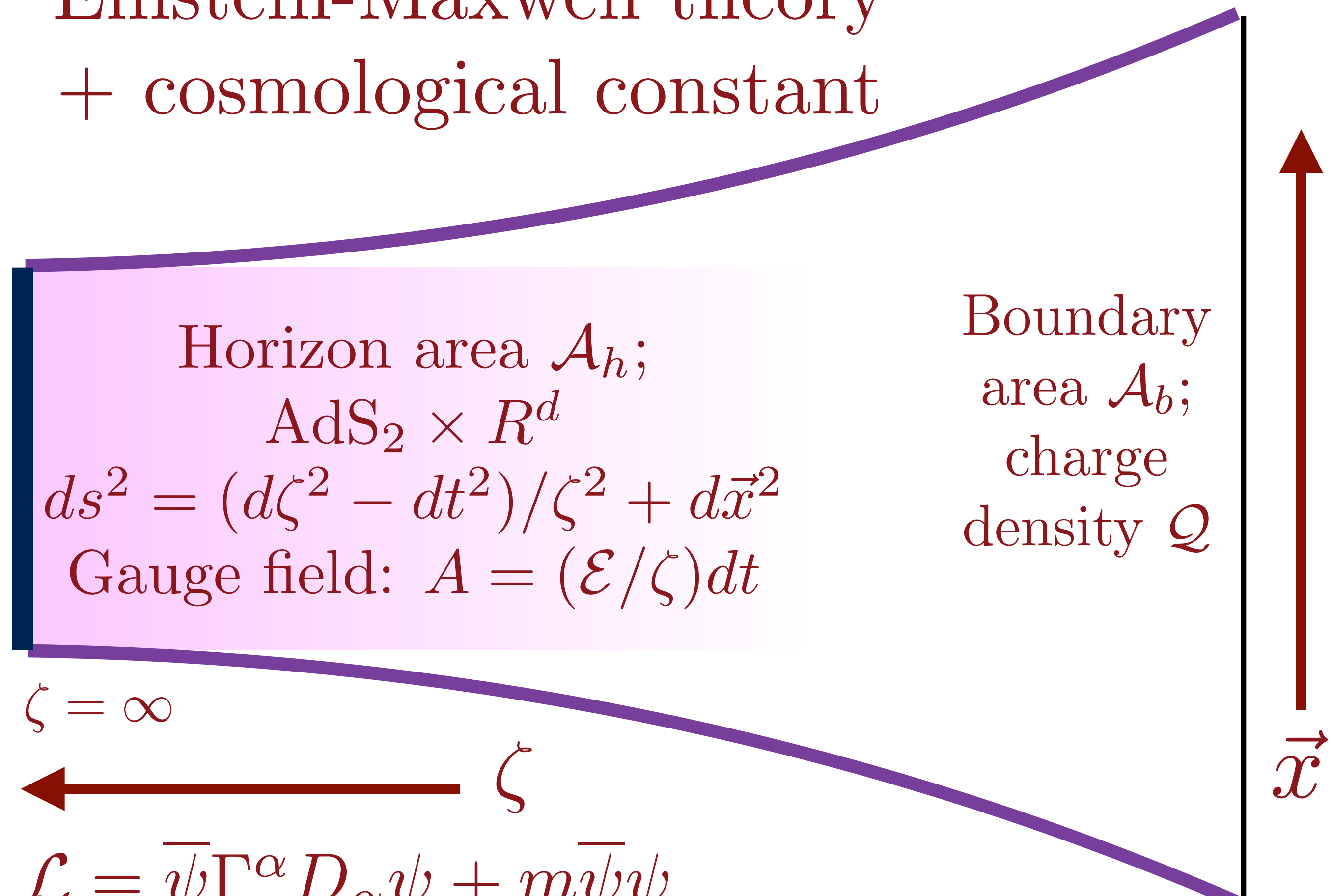

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Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, arXiv:hep-th/0506177; S. Sachdev, arXiv:1506.0511

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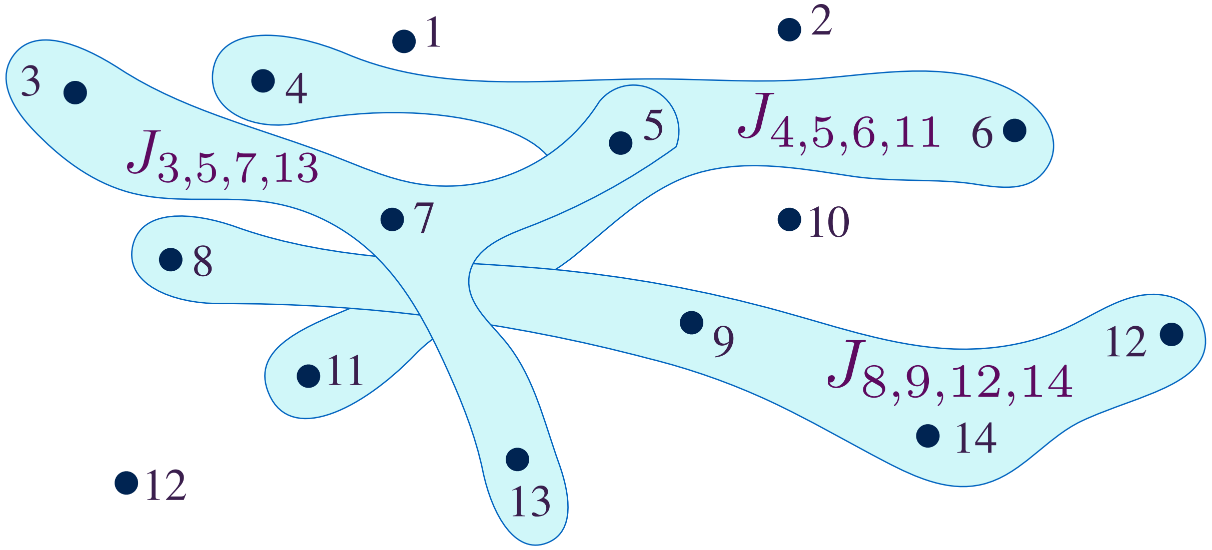

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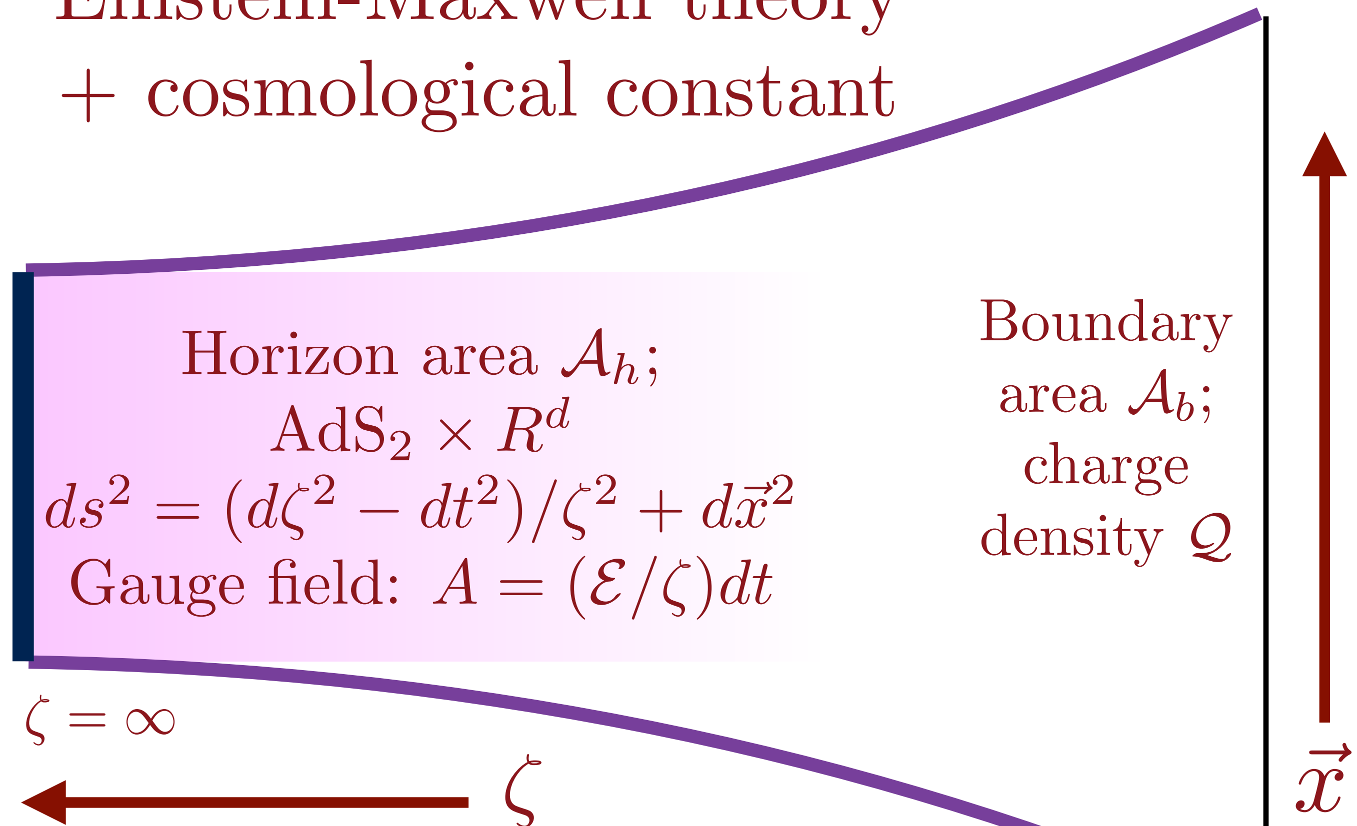

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Evidence for AdS_2 gravity dual of H

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S. Sachdev, arXiv:1506.05111